Book review

Handbook of the History and Philosophy of Mathematical *Practice* by Bharath Sriraman (editor)

Reviewed by Nicolas Michel



From the late 1970s to the late 1980s, a motley crew of mathematicians, logicians, and computer scientists expressed frustration towards what had become the dominant trend in the philosophy of mathematics. "Traditional philosophical formulations," Thomas Tymoczko wrote in the preface to an influential anthology published in 1986 [6, p. ix], fail "to articulate the actual experience of mathematicians." An exclusive emphasis on

metaphysical, logical, and foundational problems, disconnected from the everyday labor and efforts of mathematicians, had allegedly made much of the recent philosophical production at best an uninteresting and self-centered discourse, at worst an artificial construct with no substantial ties to its purported subject-matter. Although widely diverging in the solutions they brought to this shared diagnosis, philosophical essays such as Imre Lakatos' *Proofs and Refutations* (1976) [4] and Philip Kitcher's *The Nature of Mathematical Knowledge* (1984) [3] sought to root their epistemologies in historical studies, and thereby to explore questions a formal presentation of mathematical theories could not even begin to open, for instance the fallibility and empirical character of proofs or the role of authority and community in the collective shaping of mathematical knowledge.

By the early 2000s, these counter-currents – often referred to as the "maverick tradition"¹ – were progressively assimilated into a shared project by a range of mainstream historians and philosophers of mathematics, who wished to take up Tymoczko's challenge all the while softening his wholesale rejection of logical investigations and general analytical philosophy as useful tools for reflecting on mathematical practice. Consensus-building efforts by Paolo Mancosu, José Ferreirós, Jeremy Gray, and many others slowly enabled the formation of a self-identified collective of scholars interested in what is now commonly called "the philosophy of mathematical practice," for which an association was eventually created in 2008.²

Undeniably, the international and interdisciplinary community rallying under this banner nowadays has grown to a consequent size, and it gathers a wide breadth of research interests and methodologies. The research it carries out includes dense micro-historical studies based on the dissection of ancient manuscripts as well as empirical surveys of pedagogical practices; conceptual analyses of themes such as objectivity and pluralism in mathematics as well as sweeping proposals for bringing cognitive sciences into the philosophical arena. What emerged as a counter-reaction to overly systematic and formal views of mathematics is now a multipronged enterprise to study the latter science from all possible angles, so long as it remains rooted in actual practices-be they historically documented, experimentally tested, gathered with the help of sociological surveys, etc. Yet, in many ways, the challenge is now to take stock of the common results achieved by these wildly different methods, and to formulate anew the identity and value of this research programme - if one may call it thus.

Perhaps such was the intention underlying the *Handbook of the History and Philosophy of Mathematical Practice* presently under review. Gathering an astonishing 114 chapters distributed across some 3200 pages and 14 sections (each corresponding to a general philosophical theme, e.g., ontology, proof, signs, etc.), this volume perfectly displays the aforementioned breadth of approaches that has come to characterize the philosophy of mathematical

¹This label originates from *History and Philosophy of Modern Mathematics* (1988) [1, p. 17]; since then, it has taken on a broader and looser meaning.

² See https://www.philmathpractice.org. Two collective volumes which played a structuring role in the emergence of this community are *The Architecture of Modern Mathematics* [2], and *The Philosophy of Mathematical Practice* [5]. The introduction to both of these volumes provides a much more precise genealogy for the tradition collectively labeled "philosophy of mathematical practice" than the space devoted to this review could allow for.

practice. Some of those chapters are written by renowned experts of their respective fields, yet the balance between junior and senior scholars amongst the contributors is to be applauded. At its best, the Handbook thus provides illuminating and stimulating entry points into key themes of the philosophy of mathematical practice, with such leading scholars making the effort of leveraging their expertise towards general yet precise expositions.³ Many other chapters, however, are much closer in form and content to specialized research articles, so that their role in a "handbook" is rather guestionable: what they bring to an audience looking to familiarize itself with the general philosophy of mathematical practice remains wholly unclear.⁴ More troubling still, several chapters have seemingly little to do with mathematical practice: three consecutive chapters, spanning some 170 pages, thus yield erudite analyses of Plato's philosophy of mathematics that could form the basis of a stand-alone monograph, but make no effort whatsoever to connect with the common goals and projects of a philosophy of mathematical practice, let alone to be in the spirit of a handbook.

It remains somewhat unclear what the overarching purpose and target audience of this book is, something which its very short general introduction does not fully address. It must be noted here that some sections have been more tightly edited than others, and therefore near much closer to what could be expected from a handbook than others – notably, the sections on proof, ontology, semiology, and experimental mathematics are to be commended in this regard.

In the first of these three sections, for instance, mathematical proofs are successively approached from a variety of angles: as epistemic devices that can include visual and diagrammatic elements on top or instead of formal ones (Giardino's chapter), as social goods whose validity ought to be collectively negotiated and assessed (Andersen's chapter), and as historiographical categories that played a pivotal role in the shaping of Eurocentric and exclusionary narratives (Chemla's chapter). Read collectively, these three chapters convincingly place mathematical proofs – a classical locus of the epistemology of mathematics – at the nexus of cognitive, social, and cultural processes, thereby building towards a richer picture of proving practices throughout history all the while maintaining profound connections with key philosophical questions.

For all this stimulating material, however, there are as many poorly-edited chapters or contributions which fail to uphold historiographical and philosophical standards of rigor and precision, making this volume an extremely unequal sum which cannot reasonably be recommended to newcomers to the field – especially those looking for a concise introduction to this scholarship. It is a volume from which select chapters and sections can be plucked out and used as valuable surveys of state-of-the-art research questions, the sort of things expected from a handbook, but that selection itself requires familiarity with the field and a great deal of effort to wade through thousands of pages. And so, the search for a synoptic and accessible presentation of the exciting perspectives opened by the philosophy of mathematical practice must continue.

References

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Bharath Sriraman (editor), *Handbook of the History and Philosophy of Mathematical Practice*. Springer, 2024, xxx+3269 pages, Hardcover ISBN 978-3-031-40845-8, eBook ISBN 978-3-031-40846-5.

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DOI 10.4171/MAG/222

³ To cite but a few, see the chapters by Carter, Ferreirós, and Wagner, each of which could be used in graduate teaching or as a primer in (respectively) experimental mathematics, philosophical outlooks on practices, and mathematical semiology.

⁴ Such is the case, for instance, of A. Papadopoulos' obituary for Yuri Ivanovich Manin (pp. 13–34); a moving and interesting tribute which nonetheless seems rather out of place here.

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