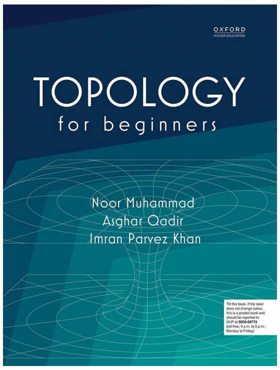
Book review

Topology for Beginners by Noor Muhammad, Asghar Qadir and Imran Parvez Khan

Reviewed by M. Ali Khan



There are wonderful books in topology and considerable material on hand on the internet, and so this particular reviewer may be forgiven for approaching this volume with some skepticism even if, or rather especially if, note is taken of the word "beginners" in its title. The question as to what the proverbial beginner would/could get from this book initially loomed large when texts such as those of Dugundji, Engelking, Kelley, Munkres, Schubert, Simmons, Ward, Willard are all easily available. There is also Wilder's classic piece on "Topology: its nature and significance," in *Mathematical Teaching*, **55(6)**, 462–475 (1962). The authors cite Steen–Seebach on examples – why does it not suffice as a text for a term? Why the need for another book? Can none of the texts listed above adequately serve the beginner? Do they require more commitment than a beginner can give? What is then unique and singular to what this book does offer?

The authors furnish a five-fold answer to the one question that underlies this medley already in their preface. They point to their use of examples (and counterexamples) to illustrate definitions, and to their explications of theorems by illustrating their use on well-chosen applications, excursions into applying the theory. Their third reason nods to history in the assertion that the "*raison d'être* for any definition really comes out by exploring changes in it." Yes, every theorem has its past. Surely, none of these reasons are controversial, let alone original, but the final two reasons concerning style and signature stand out for this reviewer: Wittgenstein's pictorial theory of meaning, and to his emphasis on the language of the everyday. However, there is a note of mild defensiveness when the authors note these.

> Early intuition is based in geometrical concepts which are best explained by "pictures," or diagrams, which may often be quite misleading. Students need to be gradually "weaned" away from using them rather than plunged into a bewildering world of great abstraction.

And as they move on to style and language, the authors rely on anthropomorphizing the book:

> [The] style of presentation is extremely *informal*, not to say *downright chatty*. We, as serious mathematicians, would not have it so, but that is the way the book insisted on being written, and we had to go along with it.

One imagines that they have the style of more demanding classical texts like that of Hardy–Littlewood–Pólya's on inequalities, or Fremlin's treatise on measure theory, in their minds as a counterpoint. However, speaking only for myself, I am glad that their book asserted itself. It knew its own strength.

The introductory chapter is most interesting. As the authors write, "it is as brief as the first chapter is long, and as simple and heuristic as that chapter." With the aid of 19 pictures/diagrams, the chapter is sectioned into twelve parts, and the individual titles succinctly convey what all is in the chapter. If permitted a little gloss on the titles, they may be listed as follows: (i) topology in the broad context of mathematics, (ii) the development of mathematics, (iii) the advent of topology, (iv) the mathematical background for it, (v) a psychology aside, (vi) topological constructions, (vii) the need for a language of sets, (viii) a diversion into logic, (ix) more on the language of sets, (x) cardinal numbers and counting, (xi) transfinite numbers and uncountability, (xii) more general topological transformations. This reviewer found some sections especially fascinating and provocative, and crying out for serious engagement for all interested in mathematics as language and its unreasonable effectiveness, not only in the natural sciences but also in the social sciences and the humanities. I shall resist this temptation in this short review, other than to send the readers of this newsletter to the standard references. I do so in response to the authors saying that "mathematical argument is there for all to see and judge without any ambiguity and without inducing any prejudices." If only the sociology of knowledge was as simple as that.

The material in Chapters 2 to 7 is standard. The concluding Chapter 8 is to whet the beginner's appetite by talking in the language the beginner has so far been talked to. In the concluding paragraph of the first section on further topological directions, while asserting that topology is "a *must* for theoretical physics and can be expected to rapidly extend its domain of influence," the authors write:

> Topology is required in Economics and is becoming important in Game Theory and Decision Making. Part of the reason lies in its use of the Bolzano–Weierstrass theorem, which is used for finding optimal solutions, or proving that there is no optimal solution available.

Relevant to optimization and economics is also the earlier statement:

> Since all Dynamics derives especially from the minimization of a function called the Lagrangian, the study of Dynamics and Dynamical Systems is based on the study of the connectedness of the space of permissible functions.

They might also want to confront the algebraic and the topological approaches to additive representations, as is done by Peter Wakker in the *Journal of Mathematical Psychology* **32**, 421–435 (1988).

I have only one minor quibble: it is that *compactness* and *upper semicontinuity* of the associated topology on the choice set, and the objective function respectively for the existence of an optimum, and the need for a *convex* structure for its uniqueness, may be as important as that for *connectedness*. Pushing a bit further, the applications of topology in the social sciences have by now gone considerably beyond the Bolzano-Weierstrass theorem which, strictly speaking, belongs more to analysis than to topology. This then leads to a plea to the authors, with their admirable expository style, to follow up this volume, if not with another written on topology for beginners in social sciences, at least to include another chapter in a second edition. The authors might enjoy Paul Samuelson's 1944 *Foundations of Economic Analysis* and his 1960 tribute to Harold Hotelling on the "structure of a minimum equilibrium system" in Ralph Pfouts (ed.) *Essays in Economics and Econometrics*, North Carolina Press. Samuelson was an avid reader of classical theoretical physics. They might also want to look at the reviewer's work with Metin Uyanik on a "deconstruction and integration of the continuity postulate" and on "On an extension of a theorem of Eilenberg and a characterization of topological connectedness": the first in the *Journal of Mathematical Economics* and the second in *Topology and its Applications*.

Let me conclude my strong recommendation of this book to interested beginners by drawing attention to its thirty-three references: fourteen are on physics, ten on topology, nine to articles of general interest, five of which concern mathematical issues. This reviewer, coming as he does from a background in the human sciences, found invaluable the references to Cabrera on superconductivity, to Glashow–Weinberg on unified theory, to Guth–Linde on an inflationary universe, to Misner–Thorne–Wheeler on gravitation, to Penrose on laws, and to Qadir himself on Einstein's relativity. What more could a beginner want in 160 pages of symbols, pictures and prose?

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