A conversation with Volker Mehrmann

Mathematics brings happiness

Sílvia Barbeiro, Ana Isabel Mendes and Martin Raussen

Volker Mehrmann is a complete mathematician excelling in all areas: research, teaching, knowledge transfer, and management. He is a specialist in numerical linear algebra, algebraic-differential equations, and control theory, and he enjoys conducting research motivated by real-world problems and developing fundamental mathematics that has a significant impact on science and technology. With a striking personality, Mehrmann leaves a remarkable legacy as former president of GAMM (International Association for Applied Mathematics and Mechanics), MATHEON (Mathematical Research Center for Key Technologies), ECMath (Einstein Center for Mathematics in Berlin), and EMS (European Mathematical Society). He is attentive to how mathematics is communicated to fellow mathematicians and to the general public, and he takes care of the working environment of his students.

The three of us (Sílvia Barbeiro, Ana Isabel Mendes and Martin Raussen) held a videoconference meeting with Volker Mehrmann last October. We listened eagerly to his answers to all our questions. At the end, we discovered his secret: mathematics brings happiness.

Figure 1. Math makes you happy and a sign that says do not follow the typical procedures (20 October 2012).

1 High school and University of Bielefeld

Interviewers: *Dear Professor Mehrmann, dear Volker: What and/or who stimulated your interest in mathematics in the first place?*

Volker Mehrmann: Well, it happened in high school. I was at a special high school for math and physics and STEM students. I liked it a lot and I was good at it. This is why I decided I wanted to become a math teacher. And I am.

Int: *Can you tell us a bit more about your school experiences at that time?*

VM: That was in the sixties and seventies, when the German school system was still pretty rigid. Schools were separated into three layers. The Gymnasium or high school took care of the better pupils. Then there were the middle school and the basic school. A high school education took 13 years, the middle school ten, and the other one lasted eight or nine years. I was sent to high school on the advice of my teachers because I was good at math already in elementary school. My parents, who were working class people, agreed, although they were concerned that they would not be able to help me. It was not that easy: I had to go to a different city and had to go by bus every morning. But I liked it, and I was good at school and therefore it was quite clear that I wanted to continue.

Int: *Germany was separated into West and East in those times. What did that mean for you?*

VM: I lived in West Germany, where – at that time – school just began to become a little bit more individualized. Not yet when I arrived at the high school. But later on, there were discussions, and politics became really important. But since I was at a school with emphasis on math and physics, politics didn’t play much of a role. We had more mathematics than other high schools, six hours per week, and I enjoyed it. As far as I know, the East German system was more like the Russian system, and it included a lot of persistence.

Int: *Were you personally interested in politics already at that time, or did that only happen later?*

VM: That came a little bit later…

Int: *Can you tell us about your experiences when you started to study at the university, please?*

VM: Well, it started with a very bad experience. When I entered university, I was still 17, a very young student, which was unusual at that time. After the first semester, I was drafted into the army, so I had to leave my studies for 15 months of service. When I returned, I immediately went back into the third semester as if I hadn't been away. That was not good, since I missed a lot of important stuff that I had to catch up upon. I studied at the University of Bielefeld, which was established as a reform university, one of these newly founded universities in Germany under the social democratic governments. They were based on fantastic ideas. Bielefeld was meant to be highly interdisciplinary, it had centres for interdisciplinary research. But the department of mathematics at the time was, for 90%, very pure. The remaining 10% were asked to work together with economists, chemists, etc. I liked that approach, and therefore I ended up in applied mathematics. The study of mathematics was divided into two different programs, one for teachers' education, and the other, called "Diplom," for students who wanted to work with mathematics outside the schools. The teachers' program included didactics, philosophy, and so on. I was enrolled in that program since I wanted to become a teacher. But before that, I wanted to work as a student tutor at the university, and for that you had to pass an intermediate "Vordiplom" exam, a bit reminiscent of a bachelor's degree nowadays. This is why I took this exam after my third semester.

Int: *Did you have to write a thesis for this exam?*

VM: No, the thesis came only with the final diploma exam; this "Vordiplom" required four oral exams. After I had passed these exams, it happened that a new professor arrived; he was a specialist in combinatorics and graph theory. And as a new tutor, I was assigned to his course in these subjects, a course that I had never attended before. He asked me whether I would be interested in writing a thesis in graph theory with him. But I was already determined to do numerics because I liked that a lot. And I also liked programming.

2 Graduating in the US

VM: I started to work with Professor Ludwig Elsner, within numerics. But when I told him that I wanted to take a teacher's exam and to write a thesis in that program, he had difficulties imagining a teacher's thesis in numerics and suggested that I write a diploma thesis instead. He said that we can certainly have my thesis acknowledged as a teacher's thesis afterwards. By this accident, I ended up writing a diploma thesis. And then I told myself, I might as well take a full diploma exam as well. Shortly after, I also took the teachers' exam, and so I have both.

To work as a teacher at a high school required an additional step at a high school where you had to learn how to teach mathematics to pupils. When I applied, they sent me to a very remote place where I didn't want to go. Being adventurous, I asked Ludwig Elsner whether I could continue studying in a different country. He said: "That's very easy: You take the front page of the journal *Linear Algebra and its Applications*, and you look at all the people that are written on the front page. You write a letter to them asking whether you could go and study abroad." That's what I actually did. I wrote a handwritten letter.

Int: *To each of them?*

VM: Yes, to each of them!

Int: *Who was the first to accept you?*

VM: I got three acceptances: One from Stanford with Gene Golub, one from Calgary with Peter Lancaster, and one with Richard Varga from Kent State University. Richard Varga's answer was in German, and my English was not that good at that time, and his answer also came first. Hence, I agreed to go to Kent State University, a small Midwest college. Careerwise, probably Stanford would have been a bit better, but who knows? I enjoyed it at Kent State.

Int: *But you succeeded. Did you have to pay for tuition?*

VM: No, they offered me a teaching assistantship right away because I already had a diploma. And I also didn't have to do the qualifying exams, and I could right away start on a thesis. But then I had a very bad experience: I had a topic for a PhD thesis already, within perturbation theory for matrix pencils, and I started to work in this area.

Int: *Was this topic suggested by your supervisor at Kent State?*

VM: No, it came actually from my German master's thesis advisor, who had suggested it as an important open problem, and I was well prepared to work on it. When I came to Kent State, Richard

Varga told me that he would not give a course that semester. I was his only student, and I took a reading course with him. I was on a one to one seminar for two hours every week with Richard Varga. Every week, he gave me a paper to read, and the next week I had to present it to him. For the third week, he gave me a paper by Peter Stuart from Maryland, in which he had solved the problem that was supposed to be my topic!

Richard Varga gave me then a different topic concerning non-negative matrices, specifically, an important problem posed by Olga Taussky, the famous algebraist. This open problem consisted in unifying the theory of the three different types of non-negativity in matrices: matrices can be elementwise non-negative, non-negative with respect to quadratic forms, also known as *Loewner non-negativity.* The third option is total non-negativity, which means that every sub-determinant is non-negative. Olga Taussky had claimed that there must be a uniform way to treat all three types. She had a conjecture using determinental inequalities. I worked on this conjecture, and I proved it for $n = 5$. Then I started trying to find a counterexample with the computer, but I couldn't manage. As we always do in math, instead I proved some other results. I think it was only in 1999 that the conjecture was proved to be false, by a counterexample using 22×22 matrices. Between 5 and 22, the question is still open.

Int: *Who found this counterexample?*

VM: It was found by Olga Holtz at Berkeley. She had the ingenious idea to investigate a special class of matrices, banded Toeplitz matrices, so that she didn't have to worry about that many parameters. I had worked in the general class of $n \times n$-matrices with n^2 parameters and n! determinantal inequalities!

3 Directions in mathematical research

Int: *Can you tell us a bit more about your research, be it as a beginner or quite recently? Are there results that you are particularly fond of?*

VM: Well, there is the saying that the murderer always returns to the place where he committed his crime. In my master's thesis I developed an algorithm towards eigenvalue calculations over the symplectic group, the unitary group, and the orthogonal group with respect to indefinite inner products and the like. I had proved that the only place where the numerics works well was for the unitary group. The rest was unbounded and ill-conditioned. About four years later, after my PhD, I was a second time in the US, and I heard a talk by Ralph Byers who was using the symplectic group in optimal control. And I told myself: "Hey, here's an application for my useless algorithm!" And so, I started to work in control theory, and I have been working in control theory on top of the classical numerical stuff that I was familiar with. That is a lot of fun because it means collaboration with engineers. One learns a lot of different thinking when cooperating with control engineers, since they work with everyday life problems. In every car, in every airplane and in every bicycle, there are issues for which control theory is essential. Over the years, I have always been working in control theory and matrix theory, and also in high-performance computing. These were the key directions.

In 2011, I received an ERC advanced grant for modeling coupled systems from different physical domains. In the beginning, it was a complete failure. I couldn't manage. But then, I heard a talk by the control theorist Arjan van der Schaft from the Netherlands. He suggested that one should go energy based and hence closer to physics. Since 2015, I'm very much into this topic, port-Hamiltonian systems modeling, numerics, linear algebra, and it's really magic. You tell me a question and I put it into the port-Hamiltonan framework. There is nice mathematics in the background, geometry, and physics, of course. Whatever you try, it turns out to work much better than if you don't have this approach. A physicist would certainly say: "Oh well, we knew this all along." But the engineers didn't!

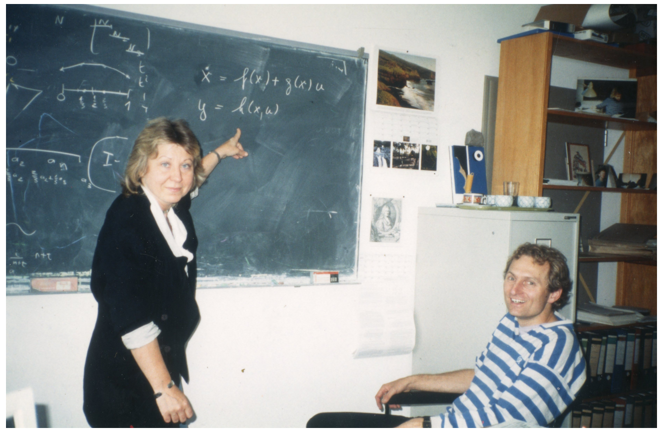

Figure 2. Together with Angelika Bunse-Gerstner in Bielefeld 1991.

Int: *May we come back to the ERC grant? I imagine that you were already very confident about your research since you applied for such a grant. Did you already have a group of people working with you?*

VM: Yes, I had five people on the grant. Each of them had been working on different physical domains, modeling with differential-algebraic equations, that is, differential equations with constraints. In principle, the mathematics for each of them was the same theory. But when you couple mechanical and electric systems, you quickly learn that the variables that are used are so differently scaled that when you start the coupling, the electric system is typically gone. It's just so small or so negligible compared to the mechanical system. This has to do with the scales of variables, but also with the way the coupling is done. Coupling is usually done via position variables or velocity variables, which are not the variables in the other domain, and one had to apply various tricks. But in this port-Hamiltonian framework, you take a new approach: The only uniform quantity in physics is energy, or entropy. Your coupling works via the transfer of energy, which means that you must write your equations in a different way, so that the energy is present in your equations. Usually when you describe the mechanical system, there is kinetic plus potential energy, but you never write it down this way. You use velocity and positions, et cetera, but you forget about the energy. But as soon as you start coupling via the energy, everything fits together in a much nicer way.

Int: *Was that already a key point convincing the jury when you applied to the ERC, or what was the key point actually?*

VM: Well, I don't know, because I was not on the committee myself. Around the year 2000, there arose an initiative to couple different physical domains. That happened in physics, but also in engineering. At that time, we had computers that were good enough for this purpose. The main idea was to establish monolithic systems. Before that time, there were research groups in mechanics and separately in electrical systems. Each of them had their own software. And when you had systems encompassing fluid and structure interaction, you had two codes, and you had to couple the codes. That didn't go too well! A consequence was an initiative towards finding a monolithic modeling the fluid and structure interaction in one type of system. That was my project, and it was open at that time. By the way, I didn't get this grant in mathematics, but at the interdisciplinary panel.

4 Differential-algebraic equations

Int: *Could you please explain to us what differential-algebraic equations (DAE) are all about? They are the ingredients in your most cited work, co-authored with Peter Kunkel…*

VM: A differential-algebraic equation is in essence a differential equation on a manifold. You have a flow associated with a differential equation, but it is constrained. A typical example would be a satellite in orbit, or a robot which has constraints because of the motors in the joints. This came up in the seventies as a new way of investigation. But if you go back in history, it was initiated by Kirchhoff back in 1870 at the Berlin Academy of Science, when he presented the equations for the description of electricity, followed up by Kronecker and Weierstrass, who settled the theory for the linear case already in the late 19th century. But this approach came completely out of fashion. We mathematicians have the great tendency that when we have something that we know how to do, then we take up other things and twist it. ODE theory and dynamical systems were well developed.

Hence, this DAE approach was forgotten until it came back in the 70s when circuit simulation made it necessary to talk about millions of equations that you had to handle automatically. Nobody could resolve the algebraic equations and put them into the ODE framework. William Gear and Linda Petzold from the U.S., actually computer scientists, started research in that direction. It took off pretty fast, but then a lot of difficulties showed up when you need to take constraints into account. For example, take a mechanical system whose position is constrained to a surface. That could be a train on a track. Its position is constrained, but the velocity cannot leave the track either. That means the velocity is also constrained, but that is not written down. These so-called hidden constraints drive the numerical methods crazy. In most realistic systems, there are many hidden constraints.

The collaboration with Peter Kunkel started while I worked at the IBM Research Center in Heidelberg for a year. My boss came with an application from a customer. It was actually an optimal control problem. I implemented it, and I got a wonderful solution. I went for lunch with Peter Kunkel, who worked in the same office. He quickly said, I have a code too. He implemented it and ran it and got a wonderful solution as well. But they were very different!

Int: *What happened then?*

VM: One of them was exponentially growing and the other one was decaying; not a good result! But as a mathematician, you immediately know that the solution is probably not unique. And so, we started to analyze, and that started our cooperation. Actually, it is a wonderful cooperation: we have just finished the second edition of our book that just appeared at EMS Press. This is the most natural way to model. If you want to couple, you write down the coupling as an algebraic or differential or whatever constraint, and then you have more constraints. This was the topic: to get hold on this coupling with differential algebraic equations. But the difficulties are immense and there are not too many good codes available, still today.

Int: *It's the codes rather than the speed of computers that are limiting…*

VM: The problem is that you must differentiate. Numerically, differentiation is bad. Integration is nice, but differentiation is not. There are more changes to consider. If you change the direction a little bit, the derivative comes out completely different, given you implicitly have to differentiate – and the codes have problems. Here is a very nice story: In the seventies, Germany designed ICE trains or high-speed trains, and France did that as well, at the same time. The modeling was performed with ODEs in Germany and with DAEs in France. The German trains would be flying because the constraints weren't there, and at least numerically, the trains would lift off. This is why the German ICE trains were much heavier than the TGVs, at least in the early days.

5 TU Chemnitz

Int: *Interesting… You have been working in many different places. In the U.S., you have, apart from Kent State, been at Madison. You have worked in Aachen and in Chemnitz. And then finally in Berlin for many years. It would be quite interesting to learn a little bit about these different workplaces and how they are characterized. I am personally interested in hearing about Chemnitz, because Chemnitz is situated in the former GDR, which had a very different university system.*

VM: When I was on the job market after my habilitation, it was extremely difficult to get a position. In Germany, hardly any professorship opened up. That's why I took a two-year temporary professorship in Aachen, and I worked a year at IBM in Heidelberg. But then the wall came down. I was still in Aachen, and I got a call from the group in Chemnitz, which I had met before. In Chemnitz, they had had big conflicts in the department between numerics and analysis.

Int: *Going back to GDR times?*

VM: Many of the numerical people were party liners and had important positions, like rector, head of the Disciplinary Committee, and so on. These were the party guys, and the analysts were not of that kind. After unification, the state of Saxony, where Chemnitz is situated, had essentially fired everybody. But they opened new positions, and people could reapply. The people in charge wanted to make sure that the conflict between analysis and numerics wouldn't immediately pop up again. The most important question in my interview became: "Can you imagine working together with the operator theorists and the analysts and building a bridge between the two fields?" But that was what I had been doing before! I was lucky because that's exactly what they wanted.

I moved there in early 1993, just after the whole rehiring had been done. I would say it's one of the best places that I've ever been to for work. The students were fantastic because Chemnitz had been one of these hubs for gifted students. In the GDR, they had schools for gifted students in sports, in math, in music, and in other fields. If you managed to get into such a school, you could go to the university at grade 11. A wonderful example is the Heinrich Hertz Gymnasium in Berlin, with pupils like Peter Scholze and Yuri Tschinkel, now at the Simons Foundation, and similar people. Hence, the students from that particular high school in Chemnitz came to the university at a very young age. And they were fantastic. There were only a few students, around 30 during the first semester. But one half of them were professor candidates!

Int: *Did that system still exist in 1993?*

VM: Yes, it still existed in '93. But then Saxony got a conservative government, and they deleted everything that had a taste of socialism. The elite schools did not have any taste of socialism, but they were related to the old system. The schools are still there. But at least in Saxony, it is no longer possible for gifted students to enter university at a very young age. In Berlin, this is still possible. Gifted students can follow courses during their first year at university while still attending high school.

Int: *Did you move to Chemnitz with your family?*

VM: Yes, I moved there with my partner at that time, and we had a baby just after I arrived there. But she couldn't stand it. She was also in math, in graph theory. It was not a good place; Chemnitz was a dreadful city. When I left Chemnitz in 2000, I was asked: "Why? You have a great career here!" And I said: "Look, there are other parts of life beyond mathematics and if you cannot make friends…" Chemnitz was a very special place because it was the role model town in GDR. They had changed the name to Karl Marx Stadt. The party and the Secret Service were dominant everywhere. Due to this, professors didn't have much social contact with each other.

Int: *Fearing that there might be an informant.*

VM: Yes. And there was.

Int: *Even after '93?*

VM: No. Because they were fired and as soon as they found something in the records that a person had been an informant, they were not letting them in. Very different in other states.

6 TU Berlin and MATHEON

Int: *Berlin was a completely different story, I believe.*

VM: It's just the opposite. Open, international… Chemnitz wasn't international. Speaking English was the exception. Bringing in foreign students, or post-docs was not easy. Berlin was just the opposite. I must say, I was afraid. Because Berlin is a completely different scale of town, and I was already 45 then in 2000. But the people in Berlin are so open. I was immediately welcomed.

Int: *And then MATHEON came into place.*

VM: Yeah, that was funny. In 1999, I was elected as chair of the Math Committee of the German Science Foundation, the panel that decides about projects. I was elected as an Easterner because I was from Chemnitz. Just after I arrived in Berlin, the administration called me and said, there is this proposal upcoming for new research centers. And it would be great if there would be one in mathematics. It was clear that they didn't want to have pure math. They wanted to introduce something more directly relevant to society.

After only four weeks in this place, this was a good challenge to start. I caught up with a few of the people: Peter Deuflhard, Jürgen Sprekels, and Martin Grötschel. We had six weeks of time to write a shorter exposé. We met in my office, and after about 20 minutes, I became a typist because of these guys, the ideas just jumped out of them. In the first round, we didn't make it. It went from medicine to physics and to ocean sciences. But they immediately opened a second round, and in the second round in 2002, we got it. I must have been a good typist, since Martin Grötschel asked me to be the vice-chair.

Int: *And MATHEON carried on for many years.*

VM: Yeah. We had 12 years of MATHEON. During six years, Martin was the chair, and then I became the chair. I liked being Martin's vice-chair because Martin is a fantastic politician. And he knew how to present mathematics to the public in a much better way than I could have done it. But when he said that we should switch now, I became the chair. Next step was the Einstein Center for five years. That continued to the excellent initiative. But then I stepped down.

Int: *Now it's called "Math+".*

VM: It's a new program, and they always want a new name. Math+ is a little bit more than MATHEON because it includes a mathematical graduate school, an American style graduate school. You can come from a bachelor's and go directly for a PhD. Both MATHEON and the Einstein Center for Mathematics, and also Math+, are a joint effort of the three universities in Berlin, and of two research centers: the Zuse Institut Berlin, which was originally a high-performance computing center and the Weierstrass Institut, which is a former Academy of Science of the GDR. Our biggest success is that we managed to get the entire Berlin math community together. Which looks great to the outside! But I can tell you, it caused a lot of friction by the different university administrators. They didn't like this always. They liked that we got it, but they wanted to make decisions, but they couldn't.

Figure 3. MATHEON received the prize "Germany land of ideas" (2007).

7 Teaching mathematics

Int: *Let us talk about your teaching career. You told us already that you were trained as a high school teacher, and you then became a university teacher. I know that you have taught Linear Algebra to first year students many times. You have even written a book about linear algebra according to your philosophy. But you have also trained a lot of PhD students.*

VM: Yes, indeed.

Int: *And there's a lot in between. May we have some comments, please?*

VM: Right away from the beginning, I always had the ambition to make mathematics digestible, not only for the three or four geniuses, but for the average, good mathematicians, as well. And that means that you must think in a slightly different way. How does mathematics enter our brains? As a high school teacher, it's clear you have to teach everybody. But when I came to the university, most of the teachers had only the three or four geniuses in mind. They taught linear algebra in a very abstract way, immediately in terms of modules and finite fields and things like that. That took a long time to enter my brain. And it's getting worse because high school education has been degenerating since then. And, in particular, abstract linear algebra has a problem. With analysis, it is a little bit easier because functions and derivatives and things like that, you can grasp that. But the abstract concept of a group or an algebra or things like that? I had difficulties producing a picture in my brain.

I find it very important in this learning process that you get a picture of what the abstract thing you're talking about, what these pictures mean, before you can learn about them. You can certainly learn to operate the whole apparatus. But does it sit there somewhere?

For the average high school student who arrives at the university now, linear algebra is a pain. That's why I went back and reflected about what would you do with such a high school student, and that's the approach that this linear algebra book takes.

We start with matrices rather than with vector spaces and maps, homomorphisms between vector spaces. That all comes, but it comes after the image of a linear mapping being a matrix times a vector is already encoded. And the feedback is really encouraging. Just recently, we made the fourth edition of the book, and the English version has 4.5 million downloads. And the next linear algebra book is in the hundred thousand. I think this is a very important step. I know that many of my colleagues don't like that. It's far too concrete to have a matrix and then be forced to be coordinate based. But I think it enters the thinking like that a little bit better than the fully abstract theory right from the beginning. For the very good students, it doesn't really matter. I use the same philosophy in my courses on numerics and differential algebraic equations. I always tried to have the concrete application on the side so that one can imagine what it would mean to have a method working. On the other hand, I like the abstract stuff very much. So, I also teach it. But I think from the point of view of communication about mathematics, it's helpful to start far more concrete.

Int: *You mentioned to us that you were recently "recycled", you are teaching again.*

VM: Yes. I retired a year ago. But at the moment, we have a shortage of numerical analysis teachers. So, they asked me to come back, I volunteered, and I enjoy it.

Int: *We just want to add that even for a genius, it's an advantage to have good examples in mind. You have not only taught freshmen, but also many Ph.D. students. The Mathematics Genealogy Project counts 34. What is your philosophy for them?*

VM: It's actually fourty PhD students already. In the German system you can be an advisor only if you are a professor. I advised several students, but the professor had to sign. A PhD is the first step in becoming an independent researcher who is standing on his or her own feet. On the other hand, the field of mathematics, like any other scientific field, is growing exponentially in all kinds of directions. I always found it very important that PhD students have somebody to talk to all the time so that they don't end up in this black hole of not making any progress. Moreover, I also tried to encourage them to talk to other PhD students or postdocs. As a PhD student, already as a master or teacher student, you should be curious to learn new things, curious to make connections, curious to learn more. You shouldn't just be focused on a single direction. This is what I tried to teach to my students, that they should be curious and that they should also discuss with other PhD students

Figure 4. MATHEON received the prize "Germany land of ideas" (2007).

and other groups. To foster that, I tried to provide a good working climate, organising parties and hiking events and the like.

Int: *Spend time together.*

VM: Yeah. And that helps a lot! In particular, when you're in this crisis, which we all from time to time drop into, when we don't make progress on the stuff that we're working on. And this has worked very well. They all talked to each other, and they wrote papers together, and I support that very much. So again, pretty much the same philosophy, working together. Try to show your curiosity about science and math and their developments and, if necessary, learn new directions that you haven't learnt. And my commitment is that I'm always there for them. I still have four PhD students at the moment, very different ones. One of them is at the Potsdam Institute for Climate Impact Research, working on stabilization of power networks. One is together with the mechanics department, working on dynamical systems and instabilities in cars and brakes and things like that. A third, who is working on saving energy in automated train driving for the subway system of the city of Nuremberg. It is an advantage if you are in a more applied field, it's usually much easier to get funding for a PhD student.

Int: *But in these applied topics, do you also develop fundamental mathematics?*

VM: Yes, the thesis always includes fundamental mathematics. My experience is, if you get a really nice problem from applications, the mathematics that you need to tackle it, is not well developed. I can give you a very nice example. So, in 2004 a guy from a company working on the layout of the high-speed train tracks from Frankfurt to Cologne approached me. They wanted to travel at 350 kilometers per hour, and they were not at all sure that this was possible. The vibrations of the tracks might lead to derailing and things like that. They performed a lot of simulations and calculations, but they couldn't get any correct digits with the experiment; then you get really nervous as an engineer! They came to me with the problem. And I said, hey, the eigenvalues are lambda and lambda bar. There must be some symplecticity there. We investigated the situation, and it became very clearly essential. And then I said, there must be a generalization of symplecticity. These are polynomial eigenvalue problems. There must be a generalization of symplecticity to matrix polynomials. We wrote a few papers, and then it exploded immediately. Now, there are lots of papers, in particular in Spain and Italy. This came out of an entirely applied problem and ended up with entirely pure mathematics. It's now done over finite fields and the like…

8 EMS presidency and communication in mathematics

Int: *Can we spend a few minutes on your presidency of the European Mathematical Society? You told us already that it was difficult times for two reasons. There was the COVID period during which in-person meetings were impossible. And then, at the end of your presidency, we witnessed the aggression of Russia into Ukraine. That must have taken emphasis from the subject itself.*

VM: Yeah.

Int: *What could you achieve despite all of these complex difficulties?*

VM: I certainly had an agenda. The first point on the agenda involved unity of mathematics. I very much believe in the unity of mathematics. But I do believe that some of our colleagues don't, and they don't like it. Because I think it's much more comfortable to stay in your own community of 20 or 30 researchers worldwide who understand each other. Communicating about mathematics is not their strength typically. And they also feel threatened by it. All of a sudden, you have to talk to engineers or physicists. This was one of my key objectives. Let's try to keep the unity intact as much as possible! But let's also try to make an effort to be able to communicate better with each other. I think I partially succeeded, but only partially.

Int: *You gave us already many examples of the importance of the pipelines between application-driven mathematics and curiosity-driven mathematics. You have problems that come from applications and to tackle them you need to apply tools from the pool that already exists. On the other hand, you have also given examples showing that pure mathematics pops up as soon as you are curious enough to take problems for applied areas seriously. Back in history, a lot of mathematics has arisen from origins in physical considerations.*

VM: I think we need better communication, and I don't think that we have too many mathematicians in the world who can be translators. Take an abstract result in algebraic geometry or take Peter Scholze perfectoids, or the ABC conjecture claimed proven by Mochizuki. I do not think there are many people who can take what is in there. I am sure there are a lot of things in there translated or transferred to something that can be used by engineers or physicists. The strength of mathematics is abstraction. And it becomes more and more abstract. The problem arises if you stay up there and do not do the translation that brought you to the abstract level. This is what is strongly missing. And it is a pity. This is what I would like to have the community realize and to stop being arrogant if somebody else does not understand what you're talking about because you haven't studied algebraic geometry for 15 years. Make an effort and try to explain!

Int: *We talked already about the European Congress of Mathematics. You were the main organizer of the Congress that took place in Berlin eight years ago. And the experience that you had was probably more or less the same. At least some of the talks were not understandable to a large audience. Do you have ideas for what can be done to change this attitude?*

VM: I really do not know how to convince some of the people who are top stars to leave their ivory tower and try to explain at least the idea behind what they're doing. We tried it at that meeting, we sent everybody a memo with what we would like them to do. But…

Int: *You did not have too much success…*

VM: Let me tell you a story. My son is an algebraic topologist. Very abstract: categories and homology. He applied for stipend, a PhD stipend of the State of Baden-Württemberg. He asked me: "Can you have a look at my proposal?" I answered: "Sure. But you have to take my criticism." And then I had to tell him: "Look, I've read this three-page proposal. Several times I felt offended because you wrote: 'It's obvious that if you do this and that, then it's clear that blah, blah, blah, …' Think about it. You want to sell your proposal to a committee that wants to give you money. A committee for all fields: math, physics, sociology, philosophy, with at most one mathematician in that committee. If this guy is more applied or a statistician and he doesn't understand a single word, then you lost right from the beginning. Why don't you try on at least one of the three pages to explain what your overall way of thinking is? Nobody will blame you then for going into the details on the remaining pages."

There used to be an unwritten rule for talks: the first part for everybody, the next part for experienced people, and the last part is for yourself. It is not like that anymore. Many lecturers seem to expect that you know everything beforehand.

Int: *But we have to say that your legacy as EMS president was great! One of us participated in the EMS congress this year and met many people from applied mathematics. Engaging lots of people, also applied mathematicians, in the EMS, is a fine achievement.*

VM: True, but I went to all the invited and plenary talks at the Seville conference that I could go to. And I would still say that about 30 or 40% gave talks that made no effort whatsoever for the audience to understand, and that is talking to mathematicians. I don't understand it. If you do that, you drive away anybody outside of your inner circle…

Étienne Ghys gave a wonderful talk, a master example. He talked about something completely abstract, but he still could convey the message. This is what I would like people to do. But there is still a certain arrogance.

Int: *It should be perhaps part of training as a mathematician.*

VM: I would think so. You always have students who tend to speak over the heads of the audience. When that happens, I criticize it.

9 EMS YA and TAGs

Int: *What else was important for you as EMS president?*

VM: The second point on the agenda was to make EMS a more balanced society, balanced in the sense of gender, but also balanced in the sense of participation of smaller and less wealthy countries and societies. Martin, you were a vice-president, you know, when the committees were formed, and names were thrown on the table these were almost always French, German and British, and sometimes somebody from Denmark or Poland or so. Some countries never showed up in committees. My goal was to get a little bit better together, over all Europe. The third important point on my agenda was to bring the young people into the Society and make sure that they felt at home in the mathematical community and that they contribute to bringing the mathematical community forward. These were the points at the head of my agenda.

Let us start with the balance. I think I partly succeeded. During the last EMS congress, EMS prizes were given to four women out of ten, which was pretty good. In the editorial boards, improvements are still needed when it comes to participation of people from Southeastern Europe or the former East. I think the war completely killed that initiative because their representatives wouldn't talk to each other anymore. And you can only be elected if you have supporters. That didn't work out well! The Russian community was very strong in the EMS, it is terrible that they are really set aside now.

The EMS Young Academy (YA) seems to work well. They have now a seat on the executive committee. They have to compete against others, but they can nominate one position. I have talked to a lot of these young people and they're eager to get active. I have actually stolen this idea from GAMM.[1]

Int: *You were the president of GAMM.*

VM: Yes, I was the president of GAMM from 2011 to 2014. We established the GAMM Juniors. Becoming a member is in fact a title, providing some prestige. But these young members get active. They organize many events, and they push us old guys in front of them. For example, the annual GAMM meeting was always scheduled in Easter week. But Easter week is the worst week for a meeting if you have small children. What can you do with your small children? They have organized career events; not only for academic careers, but also for careers outside of the university. They took action.

I hope that this will happen in a similar way in the EMS. The members already talk to each other. I think that is the most important thing, that the community already starts to talk to each other when they're young and not established superstars already.

Another goal I wanted to implement was to establish groups working in a certain subfield of mathematics within Europe so that they meet each other regularly. We formed these topical activity groups (TAG)s. I have to admit that I am pretty disappointed that there was no proposal for an activity group in anything from pure math. I think, we inside Europe should get together and know each other better and better. We have so much to offer.

10 EMS press and S2O

VM: The biggest success was in fact handed over to me by my predecessor, Pavel Exner. In the last years when he was president, he was badly trying to find a succession for the EMS publishing house. There were at least five or six different options that were investigated, and in the end none of them worked out. I had this young rebel André Gaul, who was a student from our numerically linear algebra group. He said: "Let's forget about all this publishing. Let's do it differently, let's do it grassroots, let's do it open!" He started a company where you could download a paper in an Internet forum, you could make comments on papers and books, and so on. I had lots of discussions with him. My point was that this might work in fields that are read by a lot of people, but I do

[1] Gesellschaft für Angewandte Mathematik und Mechanik.

not think it would work in a highly specialized field. People would not write bad or critical comments. They might do it in a referee report, but not in the forum.

I asked this guy to apply to become the new director of the publishing house. And the committee chose him! We planned how to set up EMS Press in a completely different way. Now, it's a company that is owned by the EMS as the only shareholder. The company is oriented towards the community, but it should also make some money, that then helps the EMS.

And since André Gaul is so much in favour of open access and everything is open, we immediately started to look into the current ways of publishing by commercial publishers, which is really terrible. They essentially try to make as much money as possible and drop quality, if necessary, when it comes to getting another paper accepted with which they make money. André Gaul came up with the idea of going to Subscribe to Open (S2O). And I think it is a real success!!! He imagined it would work this way. On top of that, he's a fantastic organizer, he does a lot of great stuff.

Int: *Could you please explain this S2O concept further?*

VM: Subscribe to Open is a new publishing model where the community, in this case, the math community, the mathematicians, the societies, and the libraries work together. The idea is: the libraries subscribe to the journal like they did in the past. When the journal has enough subscribers, it goes open, completely.

That means that the subscribers pay for the rest of the world, and then science is open!!! This is completely different from the model of article processing charges where you pay per article, where some government organization or you must pay. To give you a figure, if you want to subscribe to all the 22 journals of EMS Press, this costs you as a package something like €6,000 per year. If you want to publish an article in Springer or Elsevier, you must pay €2,500 per article.

The success of this approach is clear. More and more societies and publishing houses are moving to S2O. We are getting requests every week that a journal wants to come under the umbrella of EMS Press and to join Subscribe to Open.

Why is S2O so successful? It is in the key interests of the libraries that their old-style subscription model is kept. With processing charges, libraries will become obsolete. It will all be on the internet, but it's better if the library provides it.

Moreover, one should never sacrifice quality for money, particularly not in science. This is what this model can achieve. The publishing house is really doing well, making money for EMS and at the same time changing the world of publishing.

Int: *We wish you well, that is really very important for our community!! Also, in relation to money: When you started as president of the EMS, one of your goals was to go to Brussels to get better funding for mathematics.*

VM: That is one of the goals where I did not succeed, for several reasons. When I started, or actually a few years before, several new hypes were created: artificial intelligence, quantum computing, et cetera. Math is just not as sexy as artificial intelligence or quantum computing.

Int: *Although it underpins both.*

VM: Yeah, exactly, it completely underpins both. But the money is going to these sexy topics. We had tried very hard to play digital twins as a topic in the European Commission. No success. If you know a little bit about how Brussels works, it is a permanent struggle to find somebody to talk to you. Unfortunately, if you want as much money as we mathematicians need, we are not considered seriously. We want a few millions. The quantum guys request billions, and this is a completely different level.

11 AI and computing

Int: *Talking about AI, do you think that it will influence the future of professional mathematicians? Will we still work in Math as before AI?*

VM: Certainly, some things will change. We had this discussion in the Congress in Seville. AI can certainly help with automatic proving. Image recognition in medical fields, will not be possible without AI in the near future, I think.

One big hype has already died. It's PINN: Physics informed neural networks. It was completely hyped by a guy from Princeton collecting all the prizes. PINN was everywhere.

Essentially, the idea was you take the PDE's from physics and then you use neural networks to solve the problem, but it didn't work out. For linear ODEs it worked out, but we don't need AI for that purpose. We can do that ourselves. [laughs]

What would change? I think we have to insert topics about the mathematics of artificial intelligence into our curricula, and we must train our students. I always tell my numerical analysis students, that, apart from doing nice algorithms and solving problems, they are the good conscience of the engineer. Because they can analyze. I tell them: You can analyze, you can trust the result, you can do an error analysis, you can do a convergence analysis. You can do estimates about how good the result would be. You can do uncertainty quantification.

In the near future, I think it's the mathematicians who have to be the good conscience of AI, because somebody has to tell when and where AI is really giving you something successful or whether AI is making a lot of money and giving you shitty results.

Int: *That would probably require deeper mathematical understanding around how and when AI actually works well, as it seemingly often does.*

VM: That's why we must train our students in this topic. Several universities have started already. But, you know, the students vote with their feet when they must choose between just applying AI to solve a problem or analyzing AI. Just applying it to solve a problem is so much easier; it is also sexier!

I'm just teaching this course numerical analysis. I am struggling, because a lot of people believe that what is taught in this course can be done by AI right away. Moreover, we are creating new computers for AI. These computers don't follow the standards that we're used to in numerical analysis, e.g., IEEE arithmetic, so that we can analyze the problem. So, what does that mean?

That means that a typical computer for AI, which the big companies produce, uses four digits. You can't do reasonable numerics with four digits. I would bet that in ten years from now a regular computer on which you can run MATLAB with 16 digits will be expensive.

Mainly, because all the money is going into the development of hardware which is written for AI and not for standard engineering, mathematics, or physics. This worries me.

Int: *Four digits can be a real danger…*

VM: Yes, it is.

Int: *So how can we convince people that for some problems, it is really bad…*

VM: As long as we have alternatives available, this is not a problem. I'll give you another example: Around 2000, people started to work on graphic cards (GPU computing). The graphics card doesn't follow the IEEE standard, only to a certain part. They do not have overflow and underflow, and they have bitflips. So, they have an array where small units do the calculation, and you will never find out whether one of the small units has flipped a bit into the wrong way. That means you cannot verify the results, and if you run the same program twice, you can get different results. Now, we as mathematicians know how to deal with this! If we have uncertainty in our data and in our algorithms, we do statistics, right?

You run the same program 100 times and plot a Gauss curve, you may say, it's pretty nice, has a small tail, and then you know that this is probably the result that you should go for. But that would mean that you must do all the calculations 100 times, and they are already wasting energy like crazy for AI. Ecologically and sustainably, this is really a bad idea, and I am worried about this.

I tell my students: "Look, it is your job later on. You will work in a company and the boss will tell you to use this AI computer that was just bought. It is your job to tell them: 'Did you solve this linear system or this optimization problem?'" Only mathematicians can do that, because in the background there is deep approximation theory, deep stochastics and optimization. I have no clue about automatic proving or verification, what is behind. But for the numerics, I'm worried. I cannot teach numerics in the usual way, and we will have to integrate AI somehow.

At the University of Erlangen, they started a program in mathematics of data science. And now they have 10% of the new students in mathematics and 90% in mathematics of data science.

Int: *How will we see this in the future? How much mathematics and statistics will there be in data science courses?*

It really depends. The simplistic idea of AI is as follows: you have data, and you want to find a function in several variables that interpolates the data so that you can evaluate it in between. In numerics, we call that interpolation. We have points and we want to find a curve which supports them. If your function that you started with is arctangent of an exponential function, for example, then we have polynomials or piecewise polynomials and Weierstrass tells us that if you take a degree that is high enough, you get convergence.

And what is AI? It uses a neural network. And what is a neural network? It is a function $Ax + b$, then $G(Ax + b)$, and then you iterate eight times or more. That is a neural network!! There is a theorem using very nice mathematics from functional analysis that tells you that every continuous function can be arbitrarily well approximated by a neural network. You got it. It is no longer based on polynomials but another class of functions. It all relies on an approximation result!

Now, 99% of the computing time is spent fitting the parameters. To do this, you use the trial-and-error method, the stochastic gradient. You go in one direction, you try to find the minimum in that direction, then you choose a different one. You waste tons of computing time by trying to do this. It will finally work if you run 100 million times in all different directions in your city. At some stage, you might end up in the shop that you wanted to go to in the first place.

Int: *Don't you think that it is extremely important that people understand exactly what you said?*

VM: I am hundred percent sure that it is important, and that we have to teach this.

Int: *People have started talking about this now in the newspapers, in particular about the waste of energy…*

VM: But go to a standard math department and suggest that, instead of Algebraic Topology II or Algebraic Geometry II or III, students could follow a course on the mathematics of artificial intelligence. I'm not so sure that you'll be able to get it accepted by the faculty.

I think it is a big problem that the mathematical community drove away the computer scientists 40 years ago. In many universities, also the statisticians were driven out of the math departments. And I am afraid that we will do the same with artificial intelligence.

There is a new player in town, and we will have to give something from us to integrate this new player.

Int: *And the new player has a lot of support. From the outside, it's much easier to get support if you are in this part of science…*

VM: And you may get way more recognition by the president of the university.

12 Mathematics globally

Int: *Leaving European mathematics for a moment, countries that we have considered as developing countries have become important players in mathematics. With China, that's very obvious. But there are also other countries that have become driving forces in mathematics by now. How do you see this development and how can we support it if necessary? How can we tackle Chinese mathematics, for example?*

VM: This is a very difficult question. Twenty years ago, we could have integrated them much better, perhaps. But now they're so strong and they have so much money and they put far more money into science than we do in Europe or in the Western world. Similar things will happen in India, and perhaps also in Northern Africa.

We have to be open, but there is a real problem…. For example, in my journal, Linear Algebra and Applications, we get about 1400 submissions per year. About half of them come from China. But the refereeing is done to 80% by people from the West. Because we do not know these people. We have editors in China, but it is a totally different market, and it is completely money driven.

Some of these universities have incentives, if you write a paper in the top tier journal, if you publish in a journal such as *Annals* or *Inventiones,* you get a year salary on top.

Int: *In China?*

VM: In China, yes! What will scientists do as a consequence? They will try to get in and bombard the system with papers. And if you want to be non-biased, you should review them but, as a consequence, we will waste a lot of time of our colleagues. I don't know how to resolve this dilemma. The number of mathematicians in China is exploding and the number of incremental papers is also exploding.

I am hoping that we will at some stage know people in these communities better and we can then integrate them better into our evaluation system. After all, they all want to go to Europe or to the US, at least for the career jump. Then we know them! On the other hand, we cannot use most of them in teaching because their German, French, Portuguese is typically not adequate. It is a weird transition period; I don't know where it goes. EMS has for a while tried to organise conferences with China, with Japan and with India. Covid stopped all that, and I do not know how it will develop.

I will tell you another story. A very famous colleague, in Princeton, was once visited by the FBI because he had so many contacts with China, with Chinese people. Now he has left Princeton. He went to the Chinese Academy of Science and got money for hundreds of people to do PDEs and AI. If you have 100 people, even if there are only ten very good ones, that should be enough!

13 Private interests

Int: *Let us finally leave mathematics! I would like to ask you about your personal interests outside of mathematics. What are you delving into in your spare time?*

VM: In the recent five years, I have started gardening, growing my own vegetables. As you know, I am a vegetarian. It is easy in Berlin to be a vegetarian, but it's not easy if you're not in Berlin. Portugal was okay, but not spectacular. In Spain I spent a very difficult time. Now, I am growing my own vegetables. I have a nice house in the countryside, where I grow my vegetables and do mathematics.

Also, I am doing a lot of cycling. This is my hobby for my body. Gardening is actually not so good for the body. You always have a pain in your back. [laughs]

I have a lot of friends and like to go to the movies and music and to cultural events, which is so easy in Berlin.

A friend of mine, who started training on the trombone at age 52, asked me: "Why don't you, now that you retired, start music?" Well, I said: "I would be interested in playing rock guitar, but I have no time."

I was not too unhappy when I was asked to teach again. Moreover, I have one extra job that is on hold at the moment. In 2011, I applied for an interdisciplinary research center. Something like the MATHEON with a building where groups of people from different fields can come together. It was actually granted in 2013. And so, we are building a new house. And since we are in Berlin, we are a bit slow as usual. The building is finished now, and we will move in, at the beginning of next year. The university asked me to coordinate that enterprise, and to facilitate so that the people really do interdisciplinary research projects together. In a way, this is one of the babies of my career.

Figure 5. Hiking in the Saxonian Switzerland with the Chemnitz and Berlin numerics groups (June 2005).

Int: *Integrating research, science and administration which you have been into for many, many years.*

VM: Yes. And I managed to set it up in such a way that the administration of the university has no say [laughs]. It is kind of a dream that we can make this work. And it goes back to the time when I studied, when I was at a university at which interdisciplinary research was very highly valued. That's why I'm saying the murderer always comes back to the place where he committed the crime.

Int: *How were you able to combine high level research, high level teaching, a high level of science and administration and a family life in one life? This is very impressive. What is your trick?*

VM: Well, the trick is very simple. All these are things that I like to do. It's fun… When you do things that you like and that you enjoy, and where you get recognition and people say: Great, you managed! Then I think we can harvest a lot of energy out of that. That is my kind of secret. Almost everybody who is successful in this respect has this experience. You cannot do it in a job that you don't like. It must be fun!

Int: *This is very inspiring indeed. I think this is a good moment to finish this interview. But if you have something else that you'd like to share with us…*

VM: Unfortunately, I don't have this T-shirt with me, but I have a wonderful T-shirt from the Math Museum in Giessen that states: "Maths makes you happy."

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