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First-order model theory and Kaplansky's stable finiteness conjecture for surjunctive groups

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Abstract. Using algebraic geometry methods, the third author proved that the group ring of a surjunctive group with coefficients in a field is always stably finite. In other words, every group satisfying Gottschalk's conjecture also satisfies Kaplansky's stable finiteness conjecture. Here, we present a proof of this result based on the first-order model theory.

To Slava Grigorchuk on his 70th birthday

1. Introduction

A group G is called *surjunctive* if, for any finite set A, every injective G-equivariant map $\tau: A^G \to A^G$ which is continuous with respect to the prodiscrete topology on A^G is surjective (see Section 2 for more precise definitions). Since every injective self-mapping of a finite set is surjective, it is clear that all finite groups are surjunctive. More generally, Gromov [13] and Weiss [22] proved that every sofic group is surjunctive. A conjecture going back to Gottschalk [12] is that every group is surjunctive. Although Gottschalk's conjecture is believed to be false by several experts, no example of a non-surjunctive group, not even of a non-sofic group, has been found up to now.

A ring R is said to be *stably finite* if, for any integer $d \geq 1$, every one-sided invertible square matrix of order d with entries in R is two-sided invertible. Kaplansky [15, p. 122], [14, Problem 23] proved, using techniques from the theory of operator algebras, that the group ring $K[G]$ is stably finite for any group G and any field K of characteristic 0. Kaplansky also asked whether this property remains true for fields of positive characteristic. The claim that $K[G]$ is stably finite for any group G and any field K is known as "Kaplansky's stable finiteness conjecture". Elek and Szabó [10, Corollary 4.7] proved that every sofic group satisfies Kaplansky's stable finiteness conjecture (see also [4, Corollary 1.4], [1, Corollary 7.7], and [2, Corollary 1.9]).

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In [21, Theorem B], the third author of the present paper obtained the following result as a consequence of a generalization of Kaplansky's direct finiteness conjecture for algebraic cellular automata.

Theorem 1.1. *Let G be a surjunctive group and let K be a field. Then the group ring $K[G]$ is stably finite.*

Observe that Theorem 1.1, combined with the theorem of Gromov and Weiss on the surjunctivity of sofic groups, yields the theorem of Elek and Szabó on the stable finiteness of group rings of sofic groups with coefficients in arbitrary fields.

Several alternative proofs of Theorem 1.1 are known (see Remark 4.3). The goal of the present note is to provide a proof relying on model theory. Our proof does not make use of Kaplansky's result on the stable finiteness of group rings in characteristic 0.

2. Background material

2.1. Cellular automata and Gottschalk's conjecture (see [6, Chapter 1])

Let G be a group and let A be a set. Consider the set A^G consisting of all maps $x\colon G \to A$. A *cellular automaton* over the group G and the *alphabet* A is a map $\tau\colon A^G \to A^G$ satisfying the following property: there exist a finite subset $S \subset G$ and a map $\mu\colon A^S \to A$ such that

$$(\tau(x))(g) = \mu((x \circ L_{g^{-1}})|_S) \tag{2.1}$$

for all $x \in A^G$ and $g \in G$, where $L_{g^{-1}}\colon G \to G$ denotes the left multiplication by g^{-1} and $(x \circ L_{g^{-1}})|_S$ denotes the restriction of $x \circ L_{g^{-1}}$ to S.

The *prodiscrete uniform structure* on A^G is the product uniform structure on A^G obtained by taking the uniform discrete structure on every factor A of $A^G = \prod_{g \in G} A$. The *prodiscrete topology* on A^G is the topology associated with the prodiscrete uniform structure on A^G. The prodiscrete topology is also the product topology obtained by taking the discrete topology on every factor of A^G. Equip A^G with the left action of G defined by $(g, x) \mapsto x \circ L_{g^{-1}}$ for all $g \in G$ and $x \in A^G$. A map $\tau\colon A^G \to A^G$ is a cellular automaton if and only if it is G-equivariant and uniformly continuous with respect to the prodiscrete uniform structure on A^G [6, Theorem 1.9.1]. In the case when A is a finite set, a map $\tau\colon A^G \to A^G$ is a cellular automaton if and only if it is G-equivariant and continuous with respect to the prodiscrete topology on A^G (this is the celebrated Curtis–Hedlund–Lyndon theorem, cf. [6, Theorem 1.8.1]). Thus, Gottschalk's conjecture amounts to saying that, for any group G and any finite set A, every injective cellular automaton $\tau\colon A^G \to A^G$ is surjective.

2.2. Stably finite rings (see [16, Section 1.B])

All rings are assumed to be associative and unital. The zero element of a ring is denoted by 0 and its unital element is denoted by 1.

A ring R is called *directly finite* if $ab = 1$ implies $ba = 1$ for all $a, b \in R$. If K is a field and V is a vector space over K, then the endomorphism ring of V is directly finite if and only if V is finite-dimensional. A ring R is called *stably finite* if the ring $\mathrm{Mat}_d(R)$ of $d \times d$ matrices with entries in R is directly finite for every integer $d \geq 1$. Every stably finite ring R is directly finite since the ring $\mathrm{Mat}_1(R)$ is isomorphic to R. All finite rings, all commutative rings, all fields, all division rings, all one-sided Noetherian rings, and all unit-regular rings are stably finite (and therefore directly finite). There exist directly finite rings that are not stably finite (see, for instance, [17, Exercise 1.18]).

2.3. Group rings (see [19])

Let G be a group and let K be a field. The set K^G, which consists of all maps $\alpha\colon G \to K$, has a natural structure of a vector space over K. The *support* of $\alpha \in K^G$ is the subset of G consisting of all $g \in G$ such that $\alpha(g) \neq 0$. Let $K[G]$ denote the vector subspace of K^G consisting of all $\alpha \in K[G]$ having finite support. The *convolution product* $\alpha\beta$ of two elements $\alpha, \beta \in K[G]$ is defined by

$$(\alpha\beta)(g) := \sum_{\substack{h_1, h_2 \in G \\ h_1 h_2 = g}} \alpha(h_1)\beta(h_2)$$

for all $g \in G$. Equipped with the convolution product, the vector space $K[G]$ is a K-algebra. For $g \in G$, consider the element $\delta_g \in K[G]$ defined by $\delta_g(g) = 1$ and $\delta_g(h) = 0$ for all $h \in G \setminus \{g\}$. Then $\delta_{1_G} = 1 \in K[G]$. As $\delta_{gg'} = \delta_g \delta_{g'}$ for all $g, g' \in G$, we deduce that the map $g \mapsto \delta_g$ defines a group embedding of G into the group of units of $K[G]$. We have $\alpha = \sum_{g \in G} \alpha(g)\delta_g$ for all $\alpha \in K[G]$, so that the family $(\delta_g)_{g \in G}$ is a vector basis for $K[G]$.

The *group ring* of G with coefficients in K is the ring underlying the K-algebra $K[G]$.

2.4. Linear cellular automata and stable finiteness of group rings (see [4], [6, Chapter 8])

Let K be a field and let V be a vector space over K. The set V^G has a natural product structure of a vector space over K. A cellular automaton $\tau\colon V^G \to V^G$ is called a *linear cellular automaton* if τ is a K-linear map. The stable finiteness of group rings admits the following interpretation in terms of linear cellular automata.

Theorem 2.1. *Let G be a group and let K be a field. Then the ring $K[G]$ is stably finite if and only if, for any finite-dimensional vector space V over K, every injective linear cellular automaton $\tau\colon V^G \to V^G$ is surjective.*

Proof. See [6, Corollary 8.15.6]. ∎

2.5. Model theory of algebraically closed fields (see [18], [13, Section 5])

Two fields are called *elementary equivalent* if they satisfy the same first-order sentences (i.e., first-order formulae without free variables in the language of rings). Isomorphic fields are always elementary equivalent, but the converse does not hold in general. For example, the algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$ and the field $\mathbb{C}$ of complex numbers are not isomorphic since $\overline{\mathbb{Q}}$ is countable while $\mathbb{C}$ is uncountable. However, the fields $\overline{\mathbb{Q}}$ and $\mathbb{C}$ are elementary equivalent by the *Lefschetz principle* whose general formulation is as follows.

Theorem 2.2 (Lefschetz principle). *Any two algebraically closed fields of the same characteristic are elementary equivalent.*

Proof. This is a classical result in model theory which can be rephrased by saying that the theory of algebraically closed fields of a fixed characteristic is complete (see [18, Proposition 2.2.5 and Theorem 2.2.6]). ∎

The Lefschetz principle is one of the key ingredients in our proof. We shall also make use of the following.

Theorem 2.3. *Let ψ be a first-order sentence in the language of rings which is satisfied by some (and therefore any) algebraically closed field of characteristic* 0*. Then there exists an integer N such that ψ is satisfied by any algebraically closed field of characteristic $p \geq N$.*

Proof. This is (iii) $\Rightarrow$ (v) in [18, Corollary 2.2.10]. ∎

3. Proof of Theorem 1.1

Let us first establish some auxiliary results.

Lemma 3.1. *Let G be a group, $d \geq 1$ an integer, and $S \subset G$ a finite subset. Then there exists a first-order sentence $\psi_{d,S}$ in the language of rings such that a field K satisfies $\psi_{d,S}$ if and only if there exist two matrices $A, B \in \mathrm{Mat}_d(K[G])$ such that*

(1) *the support of each entry of A and B is contained in S;*

(2) *$AB = 1$ and $BA \neq 1$.*

Proof. Since d and S are fixed, we can quantify over $d \times d$ matrices in $\mathrm{Mat}_d(K[G])$ whose support of each entry is contained in S by quantifying over the coefficients of every entry of the matrix. Consequently, the existence of two matrices $A, B \in \mathrm{Mat}_d(K[G])$ satisfying (1) and (2) can be expressed by a $2d^2|S|$-variables first-order sentence $\psi_{d,S}$ in the language of rings, depending only on the group multiplication table of the elements in S.

For the sake of completeness, we give below an explicit formula for $\psi_{d,S}$. We represent the entries at the position (i, j) of the matrices A and B by $\sum_{s\in S} x_{i,j,s}s$ and $\sum_{s\in S} y_{i,j,s}s$, respectively. For $1 \le i, j \le d$, and $g \in S^2 = \{st : s, t \in S\} \subset G$, let

$$P(i,j,g) := \sum_{k=1}^{d} \sum_{\substack{s,t\in S \\ st=g}} x_{i,k,s} y_{k,j,t} \quad \text{and} \quad Q(i,j,g) := \sum_{k=1}^{d} \sum_{\substack{s,t\in S \\ st=g}} y_{i,k,s} x_{k,j,t}.$$

Let $D := \{(i,i,1_G) : 1 \le i \le d\}$. Then the properties $AB = 1$ and $BA = 1$ can be, respectively, expressed by the first-order formulae P and Q, where

$$P = \left(\bigwedge_{(i,i,1_G)\in D} P(i,i,1_G) = 1 \right) \wedge \left(\bigwedge_{g\in S^2,(i,j,g)\notin D} P(i,j,g) = 0 \right)$$

$$Q = \left(\bigwedge_{(i,i,1_G)\in D} Q(i,i,1_G) = 1 \right) \wedge \left(\bigwedge_{g\in S^2,(i,j,g)\notin D} Q(i,j,g) = 0 \right).$$

Hence, we can take $\psi_{d,S} := \exists\, x_{i,j,s}, y_{i,j,s}\ (1 \le i,\ j \le d,\ s \in S), P \wedge \neg Q$. ∎

Lemma 3.2. *Let G be a group and suppose that K and L are elementary equivalent fields. Then $K[G]$ is stably finite if and only if $L[G]$ is stably finite.*

Proof. As K and L play symmetric roles, it suffices to show that if $K[G]$ is not stably finite then $L[G]$ is not stably finite. So, let us assume that $K[G]$ is not stably finite. This means that there exist an integer $d \ge 1$ and two square matrices A and B of order d with entries in $K[G]$ such that $AB = 1$ and $BA \ne 1$. If $S \subset G$ is a finite subset containing the support of each entry of A and B, the field K satisfies the sentence $\psi_{d,S}$ given by Lemma 3.1. Since K and L are elementary equivalent by our hypothesis, the sentence $\psi_{d,S}$ is also satisfied by the field L. Consequently, the group ring $L[G]$ is not stably finite either. ∎

The following result follows from Theorem 2.1 and [20, Theorem 6.3].

Lemma 3.3. *Let G be a group and suppose that the group ring $K[G]$ is stably finite for every finite field K. Then the group ring $K[G]$ is stably finite for any field K.*

Proof. We divide the proof into four cases.

Case 1. The field K is the algebraic closure of the field $F_p := \mathbb{Z}/p\mathbb{Z}$ for some prime p. For every integer $n \ge 1$, let K_n denote the subfield of K consisting of all roots of the polynomial $X^{p^{n!}} - X$. In other words, denoting by $\phi\colon K \to K$ the Frobenius automorphism, K_n is the subfield of K consisting of all fixed points of $\phi^{n!}$. We have $K_n \subset K_{n+1}$ for all $n \ge 1$ and $K = \bigcup_{n\ge1} K_n$. Moreover, K_n is a finite field (of cardinality $p^{n!}$) for every $n \ge 1$. Let A and B be square matrices of order d with entries in $K[G]$ such that $AB = 1$.

Then there exists $n_0 \geq 1$ such that all entries of A and B are in $K_{n_0}[G]$. Since $K_{n_0}[G]$ is stably finite by our hypothesis, we deduce that $BA = 1$. This shows that $K[G]$ is stably finite.

Case 2. The field K is algebraically closed of characteristic $p > 0$. This follows from Case 1, Lemma 3.2, and Theorem 2.2.

Case 3. The field K is algebraically closed of characteristic 0. Suppose by contradiction that $K[G]$ is not stably finite. This means that K satisfies the sentence $\psi_{d,S}$ given by Lemma 3.1 for some integer $d \geq 1$ and some finite subset $S \subset G$. By applying Theorem 2.3, we deduce that there exists an integer $N \geq 1$ such that $\psi_{d,S}$ is satisfied by any algebraically closed field of characteristic $p \geq N$. This implies that $L[G]$ is not stably finite whenever L is an algebraically closed field of characteristic $p \geq N$, in contradiction with Case 2.

Case 4. The field K is arbitrary. Let $\overline{K}$ denote the algebraic closure of K. Then $\overline{K}[G]$ is stably finite by Case 2 and Case 3. As $K[G]$ is a subring of $\overline{K}[G]$, we deduce that $K[G]$ is itself stably finite. ∎

Remark 3.4. We could also deduce Case 3 from the theorem of Kaplansky mentioned above asserting that $K[G]$ is stably finite for any group G and any field K of characteristic 0.

Proof of Theorem 1.1. Suppose first that the field K is finite. Let V be a finite-dimensional vector space over K. Then V is finite (of cardinality $|V| = |K|^{\dim(V)}$). Since G is surjunctive, every injective cellular automaton $\tau \colon V^G \to V^G$ is surjective. In particular, every injective linear cellular automaton $\tau \colon V^G \to V^G$ is surjective. Therefore, $K[G]$ is stably finite by Theorem 2.1.

By applying Lemma 3.3, we conclude that $K[G]$ is stably finite for any field K. ∎

4. Final remarks

Remark 4.1. Following [6, Definition 8.14.1], we say that a group G is *L-surjunctive* if, for any field K and any finite-dimensional vector space V over K, every injective linear cellular automaton $\tau \colon V^G \to V^G$ is surjective. The following definition was introduced by the third author in [20]. A group G is called *linearly surjunctive* if for every finite-dimensional vector space A over a finite field K, all injective linear cellular automata $\tau \colon A^G \to A^G$ are surjective. Every L-surjunctive group is obviously linearly surjunctive. The converse (cf. [20, Corollary 7.3]) is also true by Lemma 3.3 and Theorem 2.1. Thus, a group is L-surjunctive if and only if it is linearly surjunctive. Every surjunctive group is clearly linearly surjunctive, since every finite-dimensional vector space over a finite field is finite, but there might exist linearly surjunctive groups that are not surjunctive. Observe that the hypothesis that G is linearly surjunctive is sufficient in the first part of the proof of Theorem 1.1. Thus, the conclusion of Theorem 1.1 remains valid for all linearly surjunctive groups. From Theorem 2.1 and the discussion above, we deduce that the following conditions are equivalent for a group G: (i) G is linearly surjunctive; (ii) G is L-surjunctive; and (iii) G satisfies Kaplansky's stable finiteness conjecture.

Remark 4.2. Kaplansky's *direct finiteness conjecture* asserts that the ring $K[G]$ is directly finite for any group G and any field K. Since stable finiteness implies direct finiteness, Kaplansky's direct finiteness conjecture is a weakening of Kaplansky's stable finiteness conjecture. In [9, Theorem 2.2], Dykema and Juschenko have shown that, for any field K and any group G, the ring $K[G]$ is stably finite if and only if the ring $K[G \times H]$ is directly finite for every finite group H. It follows that Kaplansky's direct finiteness conjecture is, in fact, equivalent to Kaplansky's stable finiteness conjecture.

Remark 4.3. In [3, p. 10], using the fact that every field embeds in an ultraproduct of finite fields (an observation credited to Pestov), the authors prove that, given a group G, the ring $K[G]$ is directly finite for any field K as soon as $K[G]$ is directly finite for any finite field K. They then deduce that $K[G]$ is directly finite for any surjunctive group G and any field K. As every virtually surjunctive group is surjunctive (see [7, Exercise 3.26]), this last result, combined with the result of Dykema and Juschenko mentioned in Remark 4.2, implies Theorem 1.1. Recently, Bradford and Fournier-Facio [2, Corollary 3.25] gave another proof of Theorem 1.1. Note that their proof makes use of Kaplansky's result on the stable finiteness of group rings in characteristic 0. Other alternative proofs for Theorem 1.1 have been privately communicated to us by Benjamin Steinberg and Andreas Thom.

Remark 4.4. There are also three famous conjectures attributed to Kaplansky about the structure of group rings of torsion-free groups: the unit conjecture, the zero-divisor conjecture, and the idempotents conjecture. Kaplansky's unit (resp. zero-divisor, resp. idempotent) conjecture asserts that, for every torsion-free group G and any field K, the ring $K[G]$ has no non-trivial units (resp. no zero-divisors, resp. no non-trivial idempotents). It is well known that if $K[G]$ has no non-trivial units then it has no zero-divisors, and that if $K[G]$ has no zero-divisors then it has no non-trivial idempotents. On the other hand, if a ring R has no non-trivial idempotents, then it is directly finite (observe that if $a, b \in R$ satisfy $ab = 1$, then ba is an idempotent). Thus, any torsion-free group satisfying Kaplansky's idempotent conjecture also satisfies Kaplansky's direct finiteness conjecture. Recently, Gardam [11] disproved Kaplansky's unit conjecture by exhibiting a non-trivial unit in the group ring of the Promislow group with coefficients in F_2 (the Promislow group is the fundamental group of the unique flat 3-dimensional closed manifold which is a real homology sphere). By replacing $\psi_{d,S}$ by a suitably chosen first-order sentence ψ_S in the language of rings and adapting the proofs of the three lemmas in the previous section, one sees that, given a torsion-free group G, the ring $K[G]$ has no non-trivial units (resp. no zero-divisors, resp. no non-trivial idempotents) for any field K as soon as this is true for any finite field.

Added in proof. In [8], we have extended our model-theoretic arguments to the monoid setting and showed that monoid algebras of surjunctive monoids (cf. [5]) are stably finite.

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