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# Corrigendum to "Finiteness of maximal geodesic submanifolds in hyperbolic hybrids"

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**Abstract.** This corrigendum points out an issue with our Angle Rigidity theorem, Theorem 4.1 in [J. Eur. Math. Soc. (JEMS) **23**, 3591–3623 (2021)], in certain codimensions. This effects the proof of our main result on finiteness of maximal totally geodesic submanifolds of $n$-dimensional hyperbolic hybrids, Theorem 1.4 loc. cit., in codimension at least $\frac{n}{2}$. Theorem 1.4 was later proved in greater generality by different methods, and thus holds in the full generality stated in our paper.

*Keywords:* hyperbolic geometry, totally geodesic submanifold, arithmeticity.

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This corrigendum freely uses definitions and notation from [3]. Our primary purpose is to point out a gap in the proof of the following, which is stated as [3, Theorem 4.1].

*Angle Rigidity:* Fix a non-arithmetic hyperbolic manifold $M$ that is built from building blocks, and suppose that two adjacent building blocks $N_1$ and $N_2$ are arithmetic and dissimilar. Let $A \subset M$ be a connected finite-volume immersed totally geodesic submanifold of dimension at least 2 such that $A$ intersects the interior of $N_1$ and $N_2$, i.e., crosses a cutting hypersurface $\Sigma$. Then $A$ meets $\Sigma$ orthogonally.

The proof of this result is not complete if the codimension of $A$ in $M$ is at least $\frac{n}{2}$, where $n = \dim(M)$, and this effects the proof of [3, Theorem 1.4] in this case. It should be emphasized that [3, Theorem 1.4] is correct as stated, since it follows from the (far more general) [1, Theorem 1.1].

The issue with the proof of Angle Rigidity stems from the reliance on the enumeration of totally geodesic subspaces given in [4], which is not complete if $\dim(A) = 3$ or if $A$ has

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codimension at least $\frac{n}{2}$ in $M$. For a correct classification, see [2, §4]. Angle Rigidity holds, with the given proof, when *all* components of $A$ in each $N_j \cap A$ come from those geodesic subspaces enumerated in [4], but Angle Rigidity may in fact fail for the subspaces that [4] omits. Our proof of [3, Theorem 1.2] when $n = 4$ and $\dim(A) = 3$ remains correct; see the paragraph following [2, Theorem 1.2].

Specifically, recall that the arithmetic lattices under consideration arise from the group $\mathrm{SO}(q, F)$, where $F$ is a totally real number field and $q$ is a quadratic form on an $F$-vector space $V$ of dimension $n + 1$. Following [4], in [3] we only considered *subform subspaces*, which arise from decompositions $q_1 \oplus q_2$ associated with $q$-orthogonal subspaces $V_1, V_2 \subset V$. The proof of Angle Rigidity for subspaces of this kind is correct in all codimensions. By [2, Corollary 5.12], all totally geodesic submanifolds of codimension at most $\frac{n-1}{2}$ come from this construction.

In higher codimension, there are other totally geodesic subspaces, namely the *Weil restriction* subspaces. The local arithmetic obstruction that leads to Angle Rigidity does not hold for these subspaces, hence the argument for Angle Rigidity in [3] cannot handle them.

**Remark 1.** The argument given in [3] for Closure Rigidity (Theorem 3.3) also uses [4] in a way that renders the argument incomplete, but that result is still true. We briefly give the argument, replacing $N_1$ there by $N'$ to avoid confusion with the different use of $N_1$ in this corrigendum.

Using the notation from [3], first consider the case where $m \geq 3$, so that $N \cap \Sigma$ is not a union of closed geodesics and cusp-to-cusp geodesics. Before any mention of [4], what is (correctly) proved is that $\Delta = \pi_1(N)$ contains an element $g$ that is not conjugate into the fundamental group $\pi_1(N')$ of the component $N'$ of $N \cap \Sigma$. By hypothesis, $\pi_1(N')$ is a lattice in the stabilizer of the appropriate embedding of $\mathbb{H}^{m-1}$ into $\mathbb{H}^n$. The action of $\pi_1(N)$ on $\mathbb{H}^n$ also stabilizes an embedding of $\mathbb{H}^m$ in $\mathbb{H}^n$ containing this embedded $\mathbb{H}^{m-1}$, since the lift to a map on universal coverings embeds $\widetilde{N}$ as the complement in $\mathbb{H}^m$ of a union of disjoint half-spaces.

The $\mathbb{R}$-Zariski closure $L$ of $\pi_1(N)$ in $\mathrm{SO}(n, 1)$ is then contained in an embedding of $\mathrm{S}(\mathrm{O}(m, 1) \times \mathrm{O}(n - m))$ in $\mathrm{SO}(n, 1)$. Moreover, $L$ contains the lattice $\pi_1(N')$ in a conjugate $W$ of the standard $\mathrm{S}(\mathrm{O}(m - 1, 1) \times \mathrm{O}(n - m + 1))$ and an element $g$ that is not in $W$. In particular, $\pi_1(N')$ is actually contained in the appropriate conjugate of $\mathrm{S}(\mathrm{O}(m - 1, 1) \times \mathrm{O}(n - m))$ that additionally fixes $\mathbb{H}^m$. Therefore,

$$\mathrm{SO}(m, 1) \leq L \leq \mathrm{S}(\mathrm{O}(m, 1) \times \mathrm{O}(n - m)). \tag{1}$$

Taking the $\mathbb{Q}$-Zariski closure $\mathbf{H}$ of $\pi_1(N)$ in the associated $\mathbb{Q}$-algebraic group $\mathbf{G}$ for $\pi_1(M)$ (i.e., the restriction of scalars of the $K$-algebraic group defined in [3]) produces $\mathbf{H}(\mathbb{Z})$ which, after a suitable projection, is the desired lattice in $\mathrm{S}(\mathrm{O}(m, 1) \times \mathrm{O}(n - m))$ containing $\pi_1(N)$.

Now suppose that $m = 2$ and that $N \cap \Sigma$ is a union of closed geodesics and cusp-to-cusp geodesics. The proof in the case where there are only cusp-to-cusp geodesics is correct and does not cite [4]. Indeed, in the language of [2] one explicitly builds a subform subspace of dimension three using a triple of linearly independent isotropic lines associated with endpoints of some pair of cusp-to-cusp geodesics.

Lemma 3.2 in [3] reduces the proof of Closure Rigidity to the case where there is exactly one closed geodesic in the union and the remaining components are cusp-to-cusp geodesics. Then the fundamental group of $\pi_1(N)$ is not cyclic, since otherwise $N$ would deformation retract to that immersed circle and this is ruled out by hypothesis on $N$. It follows that the restriction of $\pi_1(N)$ to the appropriate embedding of $\mathbb{H}^2$ in $\mathbb{H}^n$ is a nonelementary Fuchsian group. Thus, standard results on Fuchsian groups again imply that the $\mathbb{R}$-Zariski closure $L$ of $\pi_1(N)$ satisfies equation (1) with $m = 2$, and the proof of Theorem 3.3 from [3] concludes exactly as before.

**Remark 2.** We also note that [1, §3.1] corrects a base-point shift error for some arguments in [3, §5].

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