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Corrigendum to "Asymptotic stability of planar rarefaction wave to a 2D hyperbolic-elliptic coupling system of the radiating gas on half-space"

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Abstract. This corrigendum corrects the statement of Theorem 2.6 in [J. Eur. Math. Soc. (JEMS) **27**, 3313–3367 (2025)], and also corrects some proofs which are not affect the conclusion.

Keywords: hyperbolic-elliptic coupling system, planar rarefaction wave, L^2-energy method, initial-boundary value problem, asymptotic behavior.

In [1], Theorems 3.1 and 3.2 are correct, but the statement of Theorem 2.6 directly derived from these two theorems is incorrect. We modify the statement of Theorem 2.6 in [1] as follows.

Theorem 1 (Replacement for Theorem 2.6 in [1]). *Assume that* $0 \le f'(u_-) < f'(u_+)$ *holds. Suppose that* $u_0(x,y) - r_0 \in L^2(\mathbb{R}^2_+) \cap L^1(\mathbb{R}^2_+)$ *and* $\nabla u_0 \in H^2(\mathbb{R}^2_+)$*, then there exists a positive constant* δ_0 *such that if*

$$\|u_0(x,y) - r_0\|_{L^2(\mathbb{R}^2_+)} + \|\nabla u_0\|_{H^2(\mathbb{R}^2_+)} + |u_+ - u_-| \le \delta_0,$$

then the initial-boundary value problem (1.1)–(1.8) *in* [1] *has a unique global solution* $(u(x,y,t), q(x,y,t))$ *which satisfies*

$$\begin{cases} u - r \in C^0([0,\infty); L^2(\mathbb{R}^2_+)), \\ \nabla u \in C^0([0,\infty); H^2(\mathbb{R}^2_+)) \cap L^2(0,\infty; H^2(\mathbb{R}^2_+)), \\ q, \operatorname{div} q \in C^0([0,\infty); H^3(\mathbb{R}^2_+)) \cap L^2(0,\infty; H^3(\mathbb{R}^2_+)), \end{cases}$$

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and

$$\sup_{(x,y)\in\mathbb{R}^2_+} |u(x,y,t)-r(x,t)| \to 0 \quad as\ t\to\infty,$$

$$\sup_{(x,y)\in\mathbb{R}^2_+} |q(x,y,t)+r_x(x,t)| \to 0 \quad as\ t\to\infty.$$

There are still some errors in the proof of [1], but these errors do not affect the conclusion. We make corrections as follows:

(1) Replace equation (2.3) in [1] with

$$\begin{cases} \widetilde{w}_t+\widetilde{w}\widetilde{w}_x=\widetilde{w}_{xx}, & x\in\mathbb{R},\ t\geq -t_0,\\ \widetilde{w}(x,-t_0)=w_0^R(x), & x\in\mathbb{R}, \end{cases}$$

where $t_0>0$ is a sufficiently small constant.

(2) In Lemma 2.2 of [1], replace "(iii) $\|w_i(t)-r(t)\|_{L^p(\mathbb{R})}\leq C(1+t)^{-\frac{1}{2}+\frac{1}{2p}}$" by

$$\text{"(iii) } \|w_i(t)-r(t+t_0)\|_{L^p(\mathbb{R})}\leq C(1+t)^{-\frac{1}{2}+\frac{1}{2p}} \quad \text{for } 1<p\leq+\infty\text{"}.$$

And in Lemma 2.3 of [1], replace "(ii) $\|\widetilde{u}(t)-r(t)\|_{L^p(\mathbb{R})}\leq C(1+t)^{-\frac{1}{2}+\frac{1}{2p}}$" by

$$\text{"(ii) } \|\widetilde{u}(t)-r(t+t_0)\|_{L^p(\mathbb{R})}\leq C(1+t)^{-\frac{1}{2}+\frac{1}{2p}} \quad \text{for } 1<p\leq+\infty\text{"}.$$

(3) The inequality for estimating of p_t below (3.117) in [1] does not always hold true because $p_{1xt}p_{2yt}\geq 0$ may not always hold true. We use the following method instead of the original method to estimate it. Integrating (3.117) over $\mathbb{R}^2_+$, we get

$$\begin{aligned} &\iint_{\mathbb{R}^2_+} (\operatorname{div} p_{xt})^2\,\mathrm{d}x\mathrm{d}y+\iint_{\mathbb{R}^2_+} p_{1t}^2\,\mathrm{d}x\mathrm{d}y+2\iint_{\mathbb{R}^2_+}(p_{1xt}^2+p_{1yt}^2)\,\mathrm{d}x\mathrm{d}y\\ &\quad=\iint_{\mathbb{R}^2_+} v_{xt}^2\,\mathrm{d}x\mathrm{d}y, \end{aligned}$$

where we have used the fact that

$$p_{1y}=p_{2x},\quad \operatorname{div} p_t(0,y,t)=0,\quad p_2(0,y,t)=0$$

and

$$\begin{aligned} -2p_{1xt}p_{2yt}&=-\{2p_{1xt}p_{2t}\}_y+\{2p_{1yt}p_{2t}\}_x-2p_{1yt}p_{2xt}\\ &=-\{2p_{1xt}p_{2t}\}_y+\{2p_{1yt}p_{2t}\}_x-2p_{1yt}^2. \end{aligned}$$

Similarly, in the proof of Lemma 3.37 of [1], the inequalities used to get the L^2-estimates of p_{2xyy} and p_{2yyy} do not always hold true. We use the following method instead of the original one to estimate them. We can get from $\partial_y^2(3.89)_2$ that

$$(\operatorname{div} p_{yyy})^2+p_{2yy}^2+2p_{2yyy}^2-2\{\operatorname{div} p_{yy}p_{2yy}\}_y=v_{yyy}^2-2p_{1xyy}p_{2yyy}. \quad (1)$$

Integrating (1) over $\mathbb{R}^2_+$, using $p_{1y} = p_{2x}$, $p_2(0, y, t) = 0$ and

$$-2p_{1xyy}p_{2yyy} = -\{2p_{1xyy}p_{2yy}\}_y + \{2p_{1yyy}p_{2yy}\}_x - 2p^2_{2xyy},$$

we deduce that

$$\iint_{\mathbb{R}^2_+} (\operatorname{div} p_{yyy})^2 \,\mathrm{d}x\mathrm{d}y + \iint_{\mathbb{R}^2_+} p^2_{2yy} \,\mathrm{d}x\mathrm{d}y + 2\iint_{\mathbb{R}^2_+} (p^2_{2xyy} + p^2_{2yyy}) \,\mathrm{d}x\mathrm{d}y$$
$$= \iint_{\mathbb{R}^2_+} v^2_{yyy} \,\mathrm{d}x\mathrm{d}y \le CM^2_0,$$

which completes the L^2-estimates of p_{2xyy} and p_{2yyy}.

References

[1] Zhang, M., Zhu, C.: Asymptotic stability of planar rarefaction wave to a 2D hyperbolic-elliptic coupling system of the radiating gas on half-space. J. Eur. Math. Soc. (JEMS) **27**, 3313–3367 (2025) Zbl 08060121 MR 4911714