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Corrigendum to “Asymptotic stability of planar rarefaction wave to a 2D hyperbolic-elliptic coupling system of the radiating gas on half-space”

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Abstract. This corrigendum corrects the statement of Theorem 2.6 in [J. Eur. Math. Soc. (JEMS) 27, 3313–3367 (2025)], and also corrects some proofs which are not affect the conclusion.

Keywords: hyperbolic-elliptic coupling system, planar rarefaction wave, L^2 -energy method, initial-boundary value problem, asymptotic behavior.

In [1], Theorems 3.1 and 3.2 are correct, but the statement of Theorem 2.6 directly derived from these two theorems is incorrect. We modify the statement of Theorem 2.6 in [1] as follows.

Theorem 1 (Replacement for Theorem 2.6 in [1]). *Assume that $0 \leq f'(u_-) < f'(u_+)$ holds. Suppose that $u_0(x, y) - r_0 \in L^2(\mathbb{R}_+^2) \cap L^1(\mathbb{R}_+^2)$ and $\nabla u_0 \in H^2(\mathbb{R}_+^2)$, then there exists a positive constant δ_0 such that if*

$$\|u_0(x, y) - r_0\|_{L^2(\mathbb{R}_+^2)} + \|\nabla u_0\|_{H^2(\mathbb{R}_+^2)} + |u_+ - u_-| \leq \delta_0,$$

then the initial-boundary value problem (1.1)–(1.8) in [1] has a unique global solution $(u(x, y, t), q(x, y, t))$ which satisfies

$$\begin{cases} u - r \in C^0([0, \infty); L^2(\mathbb{R}_+^2)), \\ \nabla u \in C^0([0, \infty); H^2(\mathbb{R}_+^2)) \cap L^2(0, \infty; H^2(\mathbb{R}_+^2)), \\ q, \operatorname{div} q \in C^0([0, \infty); H^3(\mathbb{R}_+^2)) \cap L^2(0, \infty; H^3(\mathbb{R}_+^2)), \end{cases}$$

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and

$$\begin{aligned} \sup_{(x,y) \in \mathbb{R}_+^2} |u(x, y, t) - r(x, t)| &\rightarrow 0 \quad \text{as } t \rightarrow \infty, \\ \sup_{(x,y) \in \mathbb{R}_+^2} |q(x, y, t) + r_x(x, t)| &\rightarrow 0 \quad \text{as } t \rightarrow \infty. \end{aligned}$$

There are still some errors in the proof of [1], but these errors do not affect the conclusion. We make corrections as follows:

(1) Replace equation (2.3) in [1] with

$$\begin{cases} \tilde{w}_t + \tilde{w}\tilde{w}_x = \tilde{w}_{xx}, & x \in \mathbb{R}, t \geq -t_0, \\ \tilde{w}(x, -t_0) = w_0^R(x), & x \in \mathbb{R}, \end{cases}$$

where $t_0 > 0$ is a sufficiently small constant.

(2) In Lemma 2.2 of [1], replace “(iii) $\|w_i(t) - r(t)\|_{L^p(\mathbb{R})} \leq C(1 + t)^{-\frac{1}{2} + \frac{1}{2p}}$ ” by

$$\text{“(iii) } \|w_i(t) - r(t + t_0)\|_{L^p(\mathbb{R})} \leq C(1 + t)^{-\frac{1}{2} + \frac{1}{2p}} \quad \text{for } 1 < p \leq +\infty\text{”}.$$

And in Lemma 2.3 of [1], replace “(ii) $\|\tilde{u}(t) - r(t)\|_{L^p(\mathbb{R})} \leq C(1 + t)^{-\frac{1}{2} + \frac{1}{2p}}$ ” by

$$\text{“(ii) } \|\tilde{u}(t) - r(t + t_0)\|_{L^p(\mathbb{R})} \leq C(1 + t)^{-\frac{1}{2} + \frac{1}{2p}} \quad \text{for } 1 < p \leq +\infty\text{”}.$$

(3) The inequality for estimating of p_t below (3.117) in [1] does not always hold true because $p_{1xt}p_{2yt} \geq 0$ may not always hold true. We use the following method instead of the original method to estimate it. Integrating (3.117) over \mathbb{R}_+^2 , we get

$$\begin{aligned} &\iint_{\mathbb{R}_+^2} (\operatorname{div} p_{xt})^2 \, dx dy + \iint_{\mathbb{R}_+^2} p_{1t}^2 \, dx dy + 2 \iint_{\mathbb{R}_+^2} (p_{1xt}^2 + p_{1yt}^2) \, dx dy \\ &= \iint_{\mathbb{R}_+^2} v_{xt}^2 \, dx dy, \end{aligned}$$

where we have used the fact that

$$p_{1y} = p_{2x}, \quad \operatorname{div} p_t(0, y, t) = 0, \quad p_2(0, y, t) = 0$$

and

$$\begin{aligned} -2p_{1xt}p_{2yt} &= -\{2p_{1xt}p_{2t}\}_y + \{2p_{1yt}p_{2t}\}_x - 2p_{1yt}p_{2xt} \\ &= -\{2p_{1xt}p_{2t}\}_y + \{2p_{1yt}p_{2t}\}_x - 2p_{1yt}^2. \end{aligned}$$

Similarly, in the proof of Lemma 3.37 of [1], the inequalities used to get the L^2 -estimates of p_{2xyy} and p_{2yyy} do not always hold true. We use the following method instead of the original one to estimate them. We can get from $\partial_y^2(3.89)_2$ that

$$(\operatorname{div} p_{yyy})^2 + p_{2yy}^2 + 2p_{2yyy}^2 - 2\{\operatorname{div} p_{yy}p_{2yy}\}_y = v_{yyy}^2 - 2p_{1xyy}p_{2yyy}. \quad (1)$$

Integrating (1) over \mathbb{R}_+^2 , using $p_{1y} = p_{2x}$, $p_2(0, y, t) = 0$ and

$$-2p_{1xyy}p_{2yyy} = -\{2p_{1xyy}p_{2yy}\}_y + \{2p_{1yyy}p_{2yy}\}_x - 2p_{2xyy}^2,$$

we deduce that

$$\begin{aligned} & \iint_{\mathbb{R}_+^2} (\operatorname{div} p_{yyy})^2 \, dx dy + \iint_{\mathbb{R}_+^2} p_{2yy}^2 \, dx dy + 2 \iint_{\mathbb{R}_+^2} (p_{2xyy}^2 + p_{2yyy}^2) \, dx dy \\ &= \iint_{\mathbb{R}_+^2} v_{yyy}^2 \, dx dy \leq CM_0^2, \end{aligned}$$

which completes the L^2 -estimates of p_{2xyy} and p_{2yyy} .

References

- [1] Zhang, M., Zhu, C.: [Asymptotic stability of planar rarefaction wave to a 2D hyperbolic-elliptic coupling system of the radiating gas on half-space](#). *J. Eur. Math. Soc. (JEMS)* **27**, 3313–3367 (2025) [Zbl 08060121](#) [MR 4911714](#)