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Algebraic Geometry. – *Singularities of base loci on abelian varieties*, by Giuseppe Pareschi, accepted on 30 January 2026.

Abstract. – We prove that the log canonical threshold of the base ideal of a complete linear system on a complex abelian variety is ≥ 1, and that equality holds if and only if the base locus has divisorial components. Consequently, the same assertions hold for the ideal of the intersection of translates of theta divisors by the points of a finite subgroup.

Keywords. – abelian varieties, linear systems.

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1. Introduction

A well-known theorem of Kollár [17, Theorem 17.13] asserts that for a complex principally polarized abelian variety, the pair (A, Θ) is log-canonical. This simple and fundamental result has been subsequently sharpened by Ein and Lazarsfeld in [9], and there are still important open conjectures in this field, see e.g. [6], [23, Section H.29], [29, Section 9] and references therein. In turn, the theorems of Kollár and Ein–Lazarsfeld have been extended to other integral and/or $\mathbb{Q}$-divisors on abelian varieties in [3, 8, 13, 14, 16, 24].

In this note, Kollár's result is extended in a different direction: we are looking at base loci of complete linear systems, and intersections of translates of theta divisors by the points belonging to a finite subgroup. We work over $\mathbb{C}$. For an effective line bundle L on an abelian variety A we denote $\mathfrak{b}_L$ as the base ideal sheaf of the complete linear system $|L|$, namely, the image of the twisted evaluation map $H^0(A, L) \otimes L^{-1} \to \mathcal{O}_A$. This is our result.

Theorem 1. $\operatorname{lct}(\mathfrak{b}_L) \geq 1$ *and equality holds if and only if the zero locus of* $\mathfrak{b}_L$ *has divisorial components.*

(When there are divisorial components, the situation is completely understood, see the last part of the proof below). By a standard argument, it follows that the conclusion of the theorem holds also for the (scheme-theoretic) intersection of translates of theta divisors of a principally polarized abelian variety A by all points of a finite subgroup $H \subset A$.

Here and in the sequel, given a morphism $f : X \to Y$ and an ideal sheaf $\mathfrak{a}$ on Y, we will denote by $f^{-1}\mathfrak{a}$ the inverse image ideal sheaf on X, i.e., the ideal sheaf generated by the pullbacks of local sections of $\mathfrak{a}$. The translation by a point $x \in A$ is denoted as t_x.

COROLLARY 1. *Let A be a principally polarized abelian variety, Θ a theta divisor, and H a finite subgroup of A. Let*

$$\mathfrak{a}_{\Theta,H} = \sum_{h \in H} t_h^{-1} \mathcal{O}_A(-\Theta) \subset \mathcal{O}_A.$$

Then,

$$\mathrm{lct}(\mathfrak{a}_{\Theta,H}) \geq 1$$

and equality holds if and only if the zero locus of $\mathfrak{a}_{\Theta,H}$ has divisorial components.

When there are no divisorial components, we do not expect our results to be sharp or even close to it, and a better understanding of the log canonical threshold (and more generally of the jumping numbers) of base ideals on abelian varieties seems to be an interesting and difficult problem.

The motivation for this question comes from the attempt to show that $\mathfrak{b}_L$ is a radical ideal sheaf; i.e., the base scheme of L is reduced, or at least, as conjectured by Debarre in [7, Conjecture 2], that the 2-codimensional components of the base scheme are generically reduced (see the erratum in [1]). Although our result does not imply this, we hope that the method we use will be helpful in future developments.

2. Background material on the Fourier–Mukai transform and generic vanishing

2.1. Notation

Given an abelian variety A, we set $g = \dim A$. Moreover, we set $\widehat{A} = \mathrm{Pic}^0 A$ and $\mathcal{P}$ the Poincaré line bundle on $A \times \widehat{A}$. Given a point $\alpha \in \widehat{A}$, we denote by P_α the corresponding line bundle on A. All sheaves appearing in the sequel are *coherent* sheaves.

Given a sheaf $\mathcal{F}$ on A, its cohomological support loci are defined by

$$V^i(A, \mathcal{F}) = \{\alpha \in \mathrm{Pic}^0 A \mid h^i(A, \mathcal{F} \otimes P_\alpha) > 0\}.$$

2.2. (Symmetric) Fourier–Mukai transform associated with the Poincaré line bundle

Denoting $D(X)$ as the bounded derived category of the category of coherent sheaves on a projective variety X, the Fourier–Mukai functor

$$\Phi_{\mathcal{P}}^{A \to \widehat{A}} : D(A) \to D(\widehat{A})$$

(introduced in the seminal paper [21]) is an exact equivalence. Its quasi-inverse is $\Phi_{\mathcal{P}^\vee[g]}^{\widehat{A} \to A}$.

For our purposes, a version of this equivalence, introduced by Schnell in [28], will be more practical. The *symmetric* Fourier–Mukai transform is defined as the contravariant functor

$$FM_A : D(A) \to D(\widehat{A})^{\mathrm{op}}, \quad FM_A = \Phi_{\mathcal{P}}^{A \to \widehat{A}} \circ \Delta_A$$

where $\Delta_A : D(A) \to D(A)^{\mathrm{op}}$ is the dualizing functor $\Delta_A = \mathbf{R}\mathcal{H}om(\cdot, \mathcal{O}_A[g])$. Also FM_A is an exact equivalence, with quasi-inverse $FM_{\widehat{A}}$ [28, Theorem 1.1].

Let $\pi : A \to B$ be a surjective homomorphism with connected fibers of abelian varieties and let $i : \widehat{B} \to \widehat{A}$ be the inclusion dual to π. By [28, Proposition 1.1], the following properties hold:

$$FM_A \circ \pi^* = i_* \circ FM_B, \tag{2.1}$$

$$FM_B \circ Li^* = R\pi_* \circ FM_A. \tag{2.2}$$

2.3. *Vanishing conditions*

Following [28], we have the following vanishing conditions on sheaves on abelian varieties.

(a) A sheaf $\mathcal{F}$ on A is said to be a *GV sheaf* (i.e., satisfies generic vanishing) if $FM_A(\mathcal{F})$ is a sheaf (in degree zero). If this is the case, we denote

$$FM_A(\mathcal{F}) = \widehat{\mathcal{F}}. \tag{2.3}$$

It follows that if $\mathcal{F}$ is a GV sheaf on A, then also $\widehat{\mathcal{F}}$ is a GV-sheaf on $\widehat{A}$ and $\widehat{\widehat{\mathcal{F}}} = \mathcal{F}$. If $\mathcal{F}$ is a GV sheaf, then the fiber of $\widehat{F}$ at a point $\alpha \in \widehat{A}$ is $H^0(A, \mathcal{F} \otimes P_{-\alpha})^\vee$ [28, (10)]. It follows that the (reduced structure of the) support of $\widehat{\mathcal{F}}$ is the subvariety $-V^0(A, \mathcal{F})$.

(b) $\mathcal{F}$ is said to be *M-regular* if $\mathcal{F}$ is GV and $\widehat{\mathcal{F}}$ is a torsion-free sheaf. Since the support of $\widehat{\mathcal{F}}$ is $-V^0(A, \mathcal{F})$, if $\mathcal{F}$ is M-regular, then $V^0(A, \mathcal{F}) = \widehat{A}$, i.e.,

$$h^0(A, F \otimes P_\alpha) > 0, \quad \text{for all } \alpha \in \widehat{A}.$$

(c) $\mathcal{F}$ is said to be *an IT0 sheaf* (i.e., satisfies the index theorem with index 0) if $\mathcal{F}$ is GV and $\widehat{\mathcal{F}}$ is a locally free sheaf.

Although not directly used in the sequel, it is perhaps useful to keep in mind the following well-known characterizations of the above conditions in terms of the cohomological support loci (which are more commonly adopted as definitions in the literature):

A coherent sheaf $\mathcal{F}$ is GV (resp., M-regular, IT0) if and only if $\operatorname{codim}_{\widehat{A}} V^i(A, \mathcal{F}) \geq i$ *(resp., $> i$, empty) for all $i > 0$.*

For the GV condition, this is [25, Corollary 3.12] (but one direction was proved earlier in [15]). For the M-regularity condition, this is [26, Proposition 2.8], and for the IT(0) condition in one direction, it follows easily from base change, and in the other one, it follows e.g. from [25, Remark 3.13].

2.4. *Chen–Jiang decompositions and theorem*

Chen–Jiang decompositions were introduced by J. Chen and Z. Jiang in the paper [5, Theorem 1.1]. A sheaf $\mathcal{F}$ on an abelian variety A is said *to admit a Chen–Jiang decomposition* if

$$\mathcal{F} \cong \bigoplus_i (\pi_{B_i}^* \mathcal{F}_i) \otimes P_{\alpha_i},$$

where $\pi_{b_i} : A \to B_i$ are surjective homomorphisms with connected fibers, $\mathcal{F}_i$ are M-regular sheaves on B_i, and $\alpha_i \in \widehat{A}$ are points of finite order.

Some remarks about this notion are as follows:

(a) Chen–Jiang decompositions are essentially unique ([20, Lemma 3.5] and comments after Proposition 3.3, or [4, Section 9]).

(b) Unless $\dim B_i = \dim A$ (i.e., the homomorphism with connected fibers π_i is an isomorphism), the pullbacks $\pi_{B_i}^* \mathcal{F}_i$ of the M-regular sheaves $\mathcal{F}_i$ are GV but not M-regular (in fact, by (2.1), $FM_A(\pi_{B_i}^* \mathcal{F}_i) = i_{\widehat{B_i}*} \widehat{F_i}$ is a torsion sheaf).

(c) Therefore, a sheaf $\mathcal{F}$ admitting a Chen–Jiang decomposition is always GV, but it is M-regular if and only if the decomposition has $\mathcal{F}$ itself as the only summand.

(d) A direct summand of a sheaf admitting a Chen–Jiang decomposition admits a Chen–Jiang decomposition [28, Proposition 3.6].

The theorem of Chen–Jiang [5, Theorem 1.1] (together with the subsequent generalization in [27]) summarizes and strengthens a good deal of the generic vanishing theorems of Green–Lazarsfeld and Hacon [11, 12, 15]. It states that the following:

If $f : X \to A$ is a generically finite morphism from a smooth variety to an abelian variety, then the sheaf $f_*(\omega_X)$ admits a Chen–Jiang decomposition.

3. Proofs

3.1. *A preliminary lemma*

Let L be an effective line bundle on an abelian variety A. Let $K(L)$ be the subgroup $\{x \in A \mid t_x^* L \cong L\}$, acting on A by translations. The line bundle L is ample if and only if $K(L)$ is a finite group.

LEMMA 3.1.1. *Let L be an ample line bundle on an abelian variety A. Let $\mathcal{I} \subset \mathcal{O}_A$ be a $K(L)$-invariant ideal sheaf, i.e., $t_x^{-1}\mathcal{I} = \mathcal{I}$ for all $x \in K(L)$. Then, for $y \in A$, either $H^0(A, t_y^*L \otimes \mathcal{I}) = H^0(A, t_y^*L)$ or $H^0(A, t_y^*L \otimes \mathcal{I}) = 0$. Moreover, if $\mathcal{I}$ is non-trivial, then the Zariski open set of $y \in A$ such that $H^0(A, t_y^*L \otimes \mathcal{I}) = 0$ is non-empty.*

PROOF. Let us briefly recall, somewhat informally, the following fundamental construction introduced in [22] (we refer to Mumford's paper for the appropriate invariant treatment). Let $\mathcal{G}(L)$ be the *theta-group* of L, namely, the group of pairs (x, φ), where $x \in K(L)$ and $\varphi : L \to t_x^*L$ is an isomorphism. This theta-group sits in a central extension

$$0 \to \mathbb{C}^* \to \mathcal{G}(L) \to K(L) \to 0. \tag{3.1}$$

The group $K(L)$ does not act naturally on the space of global sections $H^0(A, L)$ but $\mathcal{G}(L)$ does, in such a way that $\mathbb{C}^*$ acts with weight 1, i.e., by homoteties. Mumford's theorem [22, Section 1, Proposition 3] states that, up to isomorphism, this is the only irreducible representation of $\mathcal{G}(L)$ where $\mathbb{C}^*$ acts with weight 1. Of course, $K(t_y^*L) = K(L)$ for all points $y \in A$, and the extensions (3.1) for L and t_y^*L are naturally isomorphic, as well as the representations $H^0(A, L)$ and $H^0(A, t_y^*L)$. If $\mathcal{I}$ is a $K(L)$-invariant ideal sheaf, then, for all $y \in A$, $H^0(A, t_y^*L \otimes \mathcal{I})$ is a representation of $\mathcal{G}(L)$ where $\mathbb{C}^*$ acts with weight one, and $h^0(A, t_y^*L \otimes \mathcal{I}) \leq h^0(A, L)$. Therefore, by Mumford's theorem, either $h^0(A, t_y^*L \otimes \mathcal{I})$ equals $h^0(A, L)$ or it is zero. If $\mathcal{I}$ is non-trivial, then $h^0(A, t_y^*L \otimes \mathcal{I})$ must be zero for some y; otherwise, the zero scheme of $\mathcal{I}$ would be contained in the base locus of all translates of L. ■

3.2. *Proof of the theorem*

Given a rational number $c > 0$, we denote by $\mathcal{J}(\mathfrak{b}_L^c)$ the multiplier ideal sheaf associated with c and $\mathfrak{b}_L$. We refer to [19] for generalities about multiplier ideal sheaves.

Theorem 1 states that $\mathcal{J}(\mathfrak{b}_L^c)$ is trivial as soon as $c < 1$, and that $\mathcal{J}(\mathfrak{b}_L)$ is trivial if and only if the zero locus of $\mathfrak{b}$ has no divisorial components.

In order to prove this, in the first place, we can assume that the effective line bundle L is ample. Indeed otherwise, as it is well known, there is a surjective homomorphism with connected fibers $\pi : A \to B$ and an ample line bundle M on B such that $L = \pi^*M$ (one takes $B = A/K_0(L)$, where $K_0(L)$ is the neutral component of $K(L)$). Therefore,

$$H^0(A, L) = \pi^* H^0(B, M);$$

hence, $\mathfrak{b}_L = \pi^{-1}\mathfrak{b}_M$. Thus, $\mathcal{J}(\mathfrak{b}_L^c) = \pi^{-1}\mathcal{J}(\mathfrak{b}_M^c)$ [19, Example 9.5.45].

The proof of the first assertion of the theorem is standard. Let L be an ample line bundle and $c < 1$. By Nadel vanishing $V^i(L \otimes \mathcal{J}(\mathfrak{b}_L^c)) = \emptyset$ for $i > 0$. Therefore, the sheaf $L \otimes \mathcal{J}(\mathfrak{b}_L^c)$ is ITO and the transform $\widehat{L \otimes \mathcal{J}(\mathfrak{b}_L^c)}$ is locally free of rank

$$h^0(A, L \otimes \mathcal{J}(\mathfrak{b}_L^c) \otimes P_\alpha) := h^0.$$

Let us recall that, for all $x \in A$, $t_x^* L \cong L \otimes P_\alpha$ for some $\alpha \in \widehat{A}$. If $\mathcal{J}(\mathfrak{b}_L^c)$ were not trivial, then $h^0 = 0$, because the zero locus of $\mathcal{J}(\mathfrak{b}_L^c)$ cannot not be contained in the base locus of all translates of L. Therefore,

$$\widehat{L \otimes \mathcal{J}(\mathfrak{b}_L^c)} = 0;$$

hence, $L \otimes \mathcal{J}(\mathfrak{b}_L^c) = 0$. Since this is impossible, $\mathcal{J}(\mathfrak{b}_L^c)$ is trivial.

Let us turn to the second assertion of the theorem. Assume that $\mathcal{J}(\mathfrak{b}_L)$ is non-trivial. We will prove that the zero locus of $\mathfrak{b}_L$ has divisorial components. To begin with, we recall the following: *There exists a smooth projective variety X, and a generically finite morphism $f : X \to A$ such that $L \otimes \mathcal{J}(\mathfrak{b}_L)$ is a direct summand of $f_* \omega_X$*. Let us review this. In fact,

$$\mathcal{J}(\mathfrak{b}_L) = \mathcal{J}\left(\frac{1}{k} E\right),$$

where $k > 1$ and $E = E_1 + \cdots + E_k$, where the E_i's are general divisors in $|L|$, see [19, Propositions 9.2.22 and 9.2.26]. Thus, E is a divisor in $|kL|$ and it follows from the work of Esnault–Viehweg [10, (3.13)] that the sheaf $\mathcal{J}(\frac{1}{k} E) \otimes L$ is a direct summand of the pushforward of a canonical bundle. A very readable account of this is [8, Case $t = 1$, p. 226].

Thus, by the theorem of Chen–Jiang plus remark (d) of Section 2.4, the (indecomposable!) sheaf $\mathcal{J}(\frac{1}{k} E) \otimes L$ has a Chen–Jiang decomposition. Therefore, there exist a surjective homomorphism of abelian varieties $\pi_B : A \to B$ with connected fibers, an M-regular sheaf $\mathcal{G}$ on B, and a finite-order point $\beta \in \widehat{A}$ such that

$$\mathcal{J}(\mathfrak{b}_L) \otimes L \cong P_\beta \otimes \pi_B^* \mathcal{G}. \tag{3.2}$$

The base ideal $\mathfrak{b}_L$ is $K(L)$-invariant. Hence, also the multiplier ideal $\mathcal{J}(\mathfrak{b}_L)$ is $K(L)$-invariant, because, according to [18, Section 3.3], one can take a $K(L)$-invariant log resolution of $\mathfrak{b}_L$. Therefore, by Lemma 3.1.1, $H^0(A, \mathcal{J}(\mathfrak{b}_L) \otimes L \otimes P_\alpha) = 0$ for α general in $\widehat{A}$. Hence, the GV sheaf $J(\mathfrak{b}_L) \otimes L$ is not M-regular (item (b) of Section 2.3). Thus, $\dim B < \dim A$ in (3.2).

Since $\mathcal{G}$ is an M-regular sheaf on B, we have that $V^0(B, \mathcal{F}) = \widehat{B}$, and therefore it follows from (3.2) and projection formula that

$$V^0\big(A, L \otimes \mathcal{J}(\mathfrak{b})\big) = i_{\widehat{B}}(\widehat{B}) - \beta,$$

where $i_{\widehat{B}} : \widehat{B} \to \widehat{A}$ is the inclusion dual to the surjective homomorphism π_B. In (3.2), one can assume $\beta = \widehat{0}$. Indeed, since $\mathfrak{b}_L \subset \mathcal{J}(\mathfrak{b}_L)$, $H^0(A, L \otimes \mathcal{J}(\mathfrak{b}_L)) = H^0(A, L)$, and therefore $\widehat{0} \in V^0(A, L \otimes \mathcal{J}(\mathfrak{b})) = i_{\widehat{B}}(\widehat{B}) - \beta$, i.e., $\beta \in i_{\widehat{B}}(\widehat{B})$. Hence, we can assume that (3.2) takes the form

$$\mathcal{J}(\mathfrak{b}_L) \otimes L \cong \pi_B^* \mathcal{G}. \tag{3.3}$$

Next, we claim that

$$\mathcal{G} \cong \pi_{B*} L. \tag{3.4}$$

To prove this, we apply the symmetric Fourier–Mukai transform to (3.3). Using (2.1), we have

$$\widehat{\mathcal{J}(\mathfrak{b}_L) \otimes L} = i_{\widehat{B}*} \widehat{\mathcal{G}} \tag{3.5}$$

(where $\widehat{\mathcal{G}}$ is understood to be $FM_B(\mathcal{G})$). On the other hand, we claim that from Lemma 3.1.1 it follows that

$$\widehat{\mathcal{J}(\mathfrak{b}_L) \otimes L} = i_{\widehat{B}*} i_{\widehat{B}}^* \widehat{L}. \tag{3.6}$$

Indeed applying the functor FM_A to the inclusion $\mathcal{J}(\mathfrak{b}) \otimes L \hookrightarrow L$, one gets a surjection

$$\widehat{L} \to \widehat{\mathcal{J}(\mathfrak{b}) \otimes L} \to 0. \tag{3.7}$$

As recalled in Section 2.3, the fiber $\widehat{\mathcal{J}(\mathfrak{b}_L) \otimes L}(\alpha) = \widehat{G}(\alpha)$ over a point $\alpha \in \widehat{B}$ is identified to the vector space $H^0(A, L \otimes \mathcal{J}(\mathfrak{b}_L) \otimes P_\alpha)^\vee$ which, by Lemma 3.1.1, is equal to $H^0(A, L \otimes P_\alpha)^\vee$, i.e., the fiber of $\widehat{L}(\alpha)$ over the point α. By Nakayama's lemma, it follows that the surjection (3.7), restricted to $i_{\widehat{B}}(\widehat{B})$, is the identity. This proves (3.6).

Comparing (3.6) and (3.5), we get

$$\widehat{\mathcal{G}} \cong i_{\widehat{B}}^* \widehat{L}. \tag{3.8}$$

Finally, applying the functor $FM_{\widehat{A}}$ to (3.8) and using (2.2), we get what is claimed, i.e., (3.4).

Finally, plugging (3.4) into (3.3), we get

$$L \otimes \mathcal{J}(\mathfrak{b}_L) \cong \pi_B^* \pi_{B*} L.$$

Since $\pi_B^* \pi_{B*} L$ is locally free, $\mathcal{J}(\mathfrak{b}_L)$ must be a line bundle. Since $\mathcal{J}(\mathfrak{b}_L)$ is assumed to be non-trivial, $\mathfrak{b}_L$ has some divisorial components. This proves one direction of the second assertion of the theorem.

Conversely, if L is ample and $\mathfrak{b}_L$ has some divisorial components, it is known that $A \cong B \times C$, where B is a principally polarized abelian variety, C is an abelian variety, and $L = \mathcal{O}_B(\Theta_B) \boxtimes L_C$, where L_C is an ample line bundle on C without base divisors [2, Theorem 4.3.1]. It follows that $\mathfrak{b}_L = \mathcal{O}_B(-\Theta_B) \boxtimes \mathfrak{b}_{L_C}$. Hence, by the above,

$$\mathcal{J}(\mathfrak{b}_L) = p_B^* \mathcal{O}_B(-\Theta_B).$$

This concludes the proof of the theorem.

3.3. *Proof of the corollary*

We prove more generally the following.

Corollary 3.3.1. *Let L be an effective line bundle on an abelian variety and let H be a finite subgroup of A. Then,*

$$\operatorname{lct}\Big(\sum_{h \in H} t_h^{-1} \mathfrak{b}_L\Big) \geq 1$$

and equality holds if and only if the zero scheme of $\sum_{h\in H} t_h^{-1}\mathfrak{b}_L$ has some divisorial components.

Proof. The argument is standard and similar to [1, Corollary 8]. Let $\varphi_L : A \to \operatorname{Pic}^0 A$ be the homomorphism associated with L and let $K := \varphi_L(H)$. Let us consider the isogeny

$$f : \operatorname{Pic}^0 A \to (\operatorname{Pic}^0 A)/K$$

and let

$$\pi_K : B \to A$$

be the dual isogeny. Then,

$$H^0(B, \pi_K^* L) \cong \bigoplus_{\alpha \in K} \pi_K^* H^0(A, L \otimes P_\alpha).$$

Therefore,

$$\mathfrak{b}_{\pi_K^* L} = \pi_K^{-1}\Big(\sum_{\alpha \in K} \mathfrak{b}_{L \otimes P_\alpha}\Big) = \pi_K^{-1}\Big(\sum_{h \in H} t_h^{-1} \mathfrak{b}_L\Big).$$

Corollary 3.3.1 follows from the theorem because $\mathcal{J}(\mathfrak{b}_{\pi_K^* L}) = \pi_K^{-1} \mathcal{J}(\sum_{h\in H} t_h^{-1}\mathfrak{b}_L)$ [19, Example 9.5.44]. ■

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REFERENCES

[1] E. ARBARELLO – G. CODOGNI – G. PARESCHI, Characterizing Jacobians via the KP equation and via flexes and degenerate trisecants to the Kummer variety: an algebro-geometric approach. *J. Reine Angew. Math.* **777** (2021), 251–271. Zbl 1473.14091 MR 4292866

[2] C. BIRKENHAKE – H. LANGE, *Complex abelian varieties*. 2nd edn., Grundlehren Math. Wiss. 302, Springer, Berlin, 2004. Zbl 1056.14063 MR 2062673

[3] F. BLONDEAU, *Singularities of low-degree divisors on abelian varieties*. Ph.D. thesis, Tor Vergata University, 2022.

[4] F. CAUCCI – L. LOMBARDI – G. PARESCHI, Derived invariance of the Albanese relative canonical ring. *Adv. Math.* **419** (2023), article no. 108965. Zbl 1514.14019 MR 4561583

[5] J. A. CHEN – Z. JIANG, Positivity in varieties of maximal Albanese dimension. *J. Reine Angew. Math.* **736** (2018), 225–253. Zbl 1387.14033 MR 3769990

[6] G. CODOGNI – S. GRUSHEVSKY – E. SERNESI, The degree of the Gauss map of the theta divisor. *Algebra Number Theory* **11** (2017), 983–1001. Zbl 1450.14009 MR 3665643

[7] O. DEBARRE, Update on: Singularities of divisors on abelian varieties, https://perso.imj-prg.fr/olivier-debarre/wp-content/uploads/sites/34/2025/05/DivVarAb2024.pdf visited on 6 February 2026.

[8] O. DEBARRE – C. D. HACON, Singularities of divisors of low degree on abelian varieties. *Manuscripta Math.* **122** (2007), no. 2, 217–228. Zbl 1140.14040 MR 2295113

[9] L. EIN – R. LAZARSFELD, Singularities of theta divisors and the birational geometry of irregular varieties. *J. Amer. Math. Soc.* **10** (1997), no. 1, 243–258. Zbl 0901.14028 MR 1396893

[10] H. ESNAULT – E. VIEHWEG, Dyson's lemma for polynomials in several variables (and the theorem of Roth). *Invent. Math.* **78** (1984), 445–490. Zbl 0545.10021 MR 0768988

[11] M. GREEN – R. LAZARSFELD, Deformation theory, generic vanishing theorems, and some conjectures of Enriques, Catanese and Beauville. *Invent. Math.* **90** (1987), no. 2, 389–407. Zbl 0659.14007 MR 0910207

[12] M. GREEN – R. LAZARSFELD, Higher obstructions to deforming cohomology groups of line bundles. *J. Amer. Math. Soc.* **4** (1991), no. 1, 87–103. Zbl 0735.14004 MR 1076513

[13] C. D. HACON, Divisors on principally polarized abelian varieties. *Compositio Math.* **119** (1999), no. 3, 321–329. Zbl 0980.14031 MR 1727134

[14] C. D. HACON, Fourier transforms, generic vanishing theorems and polarizations of abelian varieties. *Math. Z.* **235** (2000), no. 4, 717–726. Zbl 1041.14019 MR 1801582

[15] C. D. HACON, A derived category approach to generic vanishing. *J. Reine Angew. Math.* **575** (2004), 173–187. Zbl 1137.14012 MR 2097552

[16] Z. JIANG – H. LIU, Singularities of divisors of small degrees on abelian varieties. *Math. Z.* **309** (2025), no. 2, article no. 29. Zbl 1566.14020 MR 4843068

[17] J. KOLLÁR, *Shafarevich maps and automorphic forms*. M. B. Porter Lectures, Princeton University Press, Princeton, NJ, 1995. Zbl 0871.14015 MR 1341589

[18] J. KOLLÁR, *Lectures on resolution of singularities*. Ann. of Math. Stud. 166, Princeton University Press, Princeton, NJ, 2007. Zbl 1113.14013 MR 2289519

[19] R. LAZARSFELD, *Positivity in algebraic geometry. II. Positivity for vector bundles, and multiplier ideals*. Ergeb. Math. Grenzgeb. (3) 49, Springer, Berlin, 2004. Zbl 1093.14500 MR 2095472

[20] L. LOMBARDI – M. POPA – C. SCHNELL, Pushforwards of pluricanonical bundles under morphisms to abelian varieties. *J. Eur. Math. Soc. (JEMS)* **22** (2020), no. 8, 2511–2536. Zbl 1467.14047 MR 4118614

[21] S. MUKAI, Duality between $D(X)$ and $D(\hat{X})$ with its application to Picard sheaves. *Nagoya Math. J.* **81** (1981), 153–175. Zbl 0417.14036 MR 0607081

[22] D. MUMFORD, On the equations defining abelian varieties. I. *Invent. Math.* **1** (1966), 287–354. Zbl 0219.14024 MR 0204427

[23] M. MUSTAŢĂ – M. POPA, Hodge ideals. *Mem. Amer. Math. Soc.* **262** (2019), no. 1268, v+80. Zbl 1442.14004 MR 4044463

[24] G. PARESCHI, Singularities of divisors of low degree on simple abelian varieties. *Int. Math. Res. Not. IMRN* **2021** (2021), no. 21, 16726–16735. Zbl 1481.14014 MR 4338231

[25] G. PARESCHI – M. POPA, GV-sheaves, Fourier-Mukai transform, and generic vanishing. *Amer. J. Math.* **133** (2011), no. 1, 235–271. Zbl 1208.14015 MR 2752940

[26] G. PARESCHI – M. POPA, Regularity on abelian varieties III: Relationship with generic vanishing and applications. In *Grassmannians, moduli spaces and vector bundles*, pp. 141–167, Clay Math. Proc. 14, American Mathematical Society, Providence, RI, 2011. Zbl 1236.14020 MR 2807853

[27] G. PARESCHI – M. POPA – C. SCHNELL, Hodge modules on complex tori and generic vanishing for compact Kähler manifolds. *Geom. Topol.* **21** (2017), no. 4, 2419–2460. Zbl 1374.14008 MR 3654112

[28] C. Schnell, The Fourier-Mukai transform made easy. *Pure Appl. Math. Q.* **18** (2022), no. 4, 1749–1770. Zbl 1509.14040 MR 4504037

[29] C. Schnell – R. Yang, Higher multiplier ideals. [v1] 2023, [v6] 2025, arXiv:2309.16763v6.

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