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Alexander Esterov · Ann Lemahieu · Kiyoshi Takeuchi

Corrigendum to "On the monodromy conjecture for non-degenerate hypersurfaces"

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Abstract. We correct several issues in our paper [J. Eur. Math. Soc. 24, 3873–3949 (2022)], detected by Larson, Payne, and Stapledon. The results of the paper are not affected by the corrections, but one update may be of interest for the future research: it is a previously missed case in the classification of 3-dimensional B-facets (Lemma 5.18). It corrects expectations on how this classification might look in higher dimensions.

Keywords: monodromy conjecture, Newton polytope.

We are very grateful to Matt Larson, Sam Payne, and Alan Stapledon for their attention to our text [1] and detecting several subtle issues that we address in this corrigendum.

The results of the paper are not affected by the corrections, but one update may be of interest for the future research: it is a previously missed case in the classification of 3-dimensional B-facets (Lemma 5.18). The corrected full classification is given below; it may adjust the expectations on how B-faces of higher dimension may look.

The corrigendum consists of two parts: part (i) corrects two issues noticed in [2] and is self-contained; part (ii) describes the missing case in Lemma 5.18, and explains why this case does not affect the rest of our results. This part relies on the corrected classification, recently published by Fedor Selyanin [3].

In what follows, we underline the corrections to the original text. When mentioning faces, we always imply faces of the Newton polyhedron of a function for which we wish to prove the monodromy conjecture.

Alexander Esterov: London Institute for Mathematical Sciences, London W1S 4HZ, UK; alexander.esterov@gmail.com

Ann Lemahieu: Université Côte d'Azur, 06108 Nice; UMR CNRS 7351, 06108 Nice, France; ann.lemahieu@unice.fr

Kiyoshi Takeuchi: Institute of Mathematics, Tohoku University, Sendai 980-8578, Japan; takemicro@nifty.com

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(i) Miscellanea

• Proposition 3.7 and consequently 3.8 are not valid for non-compact B_1-facets, because the first phrase in the proof of Proposition 3.7 is clearly erroneous. However, they are valid for compact B_1-facets: see the rest of the proof.

This does not affect the rest of the paper, because we never use these propositions in what follows: as explicitly pointed out in the second paragraph of Section 3.3, the propositions are given only to illustrate the ideas behind subsequent more complicated constructions.

• The underlined part of Definition 4.1.1 below was not present in the original text (making the definition obviously irrelevant to how it was used).

Corrected Definition 4.1 (1). A one-element set $\{i\} \subset \{1, \dots, n\}$ is called a *base direction for a B_1-facet τ* if the i-th coordinate equals 1 for one vertex of τ, equals 0 for the other vertices, and τ is not non-compact for $\{i\}$.

Besides that, it should be stressed that, starting from Definition 4.1, all B-facets are assumed to have at least one base direction.

(ii) Flat borders

We first recall the role and statement of Lemma 5.18, in which one case was found to be missing, and Definition 5.1 of a border, which can be adjusted to repair Lemma 5.18. Then we give corrected versions of Definition 5.1 and Lemma 5.18, and explain how the rest of the paper adjusts to these changes.

An important part of our proof of the main result is to single out certain collections of facets, which cannot alone contribute a pole to the zeta-function. They are recorded in Theorem 5.2; those are collections of B-facets, no two of which are adjacent through a so-called B-border:

Definition 5.1. A bounded triangular face ABD of the Newton polyhedron is called a *border* if up to the order of coordinates, its vertices have the form $A = (1, 1, *, *)$, $B = (0, 0, *, *)$ and $D = (0, 0, *, *)$, and the two adjacent facets are B-facets with the vertex A and the bases in the coordinate hyperplanes $\{v_1 = 0\}$ and $\{v_2 = 0\}$ respectively. ABD is called a *B^2-border* if (up to reordering B and D and the last two coordinates) we have $A = (1, 1, 0, a)$, $B = (0, 0, 0, b)$ and $D = (0, 0, 1, d)$, and a *B-border* otherwise.

So it remains to prove (Theorem 8.25) that the presence of

(a) a non-B-facet, or

(b) a pair of B-facets adjacent through a B-border

ensures that the monodromy eigenvalue they contribute has non-zero multiplicity in the characteristic polynomial of the monodromy operator. This task is quite straightforward for (b), but for (a) it is based on the following statement.

Lemma 5.18. *Every bounded non-B-facet contains a non-B-simplex.*

However, the authors of [2] found a counterexample to this statement. Fortunately, it turns out that the notion of B-facet can be extended to keep Lemma 5.18 valid (roughly, by allowing B-borders to be triangles in facets of the Newton polyhedron, rather than faces of this polyhedron, and including their ambient facets in the notion of B-facet). Such an extension was conjectured by the first named author and proved in [3]:

Corrected Lemma 5.18. *Every bounded non-B-facet of the Newton polyhedron of a non-Morse singularity contains a non-B-simplex or a flat border, in the sense of corrected Definition* 5.1.

Corrected Definition 5.1. A lattice triangle ABD in the boundary of the Newton polyhedron is called a *border* if, up to the order of coordinates, its vertices have the form $A = (1, 1, *, *)$, $B = (0, 0, *, *)$ and $D = (0, 0, *, *)$, and the two adjacent facets (or the two pieces into which ABD splits its unique ambient facet) are B-facets with vertex A and with bases in the coordinate hyperplanes $\{v_1 = 0\}$ and $\{v_2 = 0\}$ respectively.

The triangle ABD is called a B^2*-border* if (up to reordering B and D and the last two coordinates) we have $A = (1, 1, 0, a)$, $B = (0, 0, 0, b)$ and $D = (0, 0, 1, d)$, and a B*-border* otherwise.

The border ABD is called *flat* if ABD itself is not a face of the Newton polyhedron. For the order of coordinates as above, the vertex A is called the *apex* of the flat border, and the 3rd and 4th coordinate hyperplanes its *bases* (note that different choices of the order of coordinates for the same flat border may result in different vertices called the apex). From now on, we include the facets containing flat borders into the notion of a B-facet (along with B_1- and B_2-facets).

Note that, upon these corrections, non-flat borders have the same meaning as before.

It is a good luck that the only lost case in the corrected Lemma 5.18 (flat borders) is just a degenerate version of the one that we already treated (non-flat borders). Namely, we only need to include the treatment of flat B-borders in the proof of Theorem 8.25 and flat B^2-borders in the statement of Theorem 5.2. However, the original proofs of both statements for non-flat borders apply to flat ones without changes, except for the following (purely linguistic) adjustments.

Adjustments to Section 5 in the presence of flat B^2-borders

In Section 5.1, Definition 5.3 should be preceded with the following new paragraph.

Note that, if the border τ is flat, then it is not a face of the Newton polyhedron, and the same may happen to some of its edges e. In this case, referring to their dual cones (denoted by τ° or e° respectively), we always imply the dual cone to the minimal face of the Newton polyhedron containing τ or e. (It is still the set of all linear functions, whose restriction to the polyhedron attains its minimal value at the points of τ or e respectively.)

In Section 5.2, the following corrections will not change anything in the absence of flat borders, but ensure that a critical flat border is considered not very critical. Then the remaining part of Section 5 applies in the presence of flat borders without changes.

Addition after the first paragraph: Besides that, we fix the choice of the preferred apex and bases for every flat border (see corrected Definition 5.1 above).

Correction to the last sentence in Definition 5.8: Similarly, the preferred apex $a_F = a_{F^\circ} \subset \{1, 2, 3, 4\}$ of a critical face F is the set of the preferred apices on the side of F for all B-facets containing F. We say that the face F and its dual cone are *very critical* if $|a_F| \geq 2$.

Adjustments to Section 8 in the presence of flat B-borders

Corrections to the proof of Lemma 8.14: In the first case, the V-edge of the border contributes to the sought eigenvalue (see the proof of Lemma 8.31), and, if the border is flat, it triangulates its ambient facet. In the second case, triangulate the contributing non-B-facet by Lemma 5.18, and, in both cases, extend this triangulation arbitrarily to the whole $\Gamma_+(f)$.

With this triangulation, all subsequent arguments in Section 8 involving borders (in particular, part 3 of the proof of Theorem 8.25 and the proof of Lemma 8.31) apply literally to the case of a flat B-border.

References

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