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On 2-dominated operators on ℓ_p spaces

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Abstract. We use the Maurey factorization theorem to obtain a new characterization of 2-dominated operators on a product of ℓ_p spaces. This new characterization is used to find the necessary and sufficient conditions for some diagonal operators on a product of ℓ_p spaces to be 2-dominated.

1. Introduction, notation, and background

The main aim of this paper is to use the Maurey factorization theorem in the study of 2-dominated operators on ℓ_p spaces. Let us recall the famous Maurey factorization theorem, see [10, Proposition 43, page 68 and Proposition 44, page 70], [8, page 252], [6, page 233], [5] or [17].

Maurey factorization theorem. Let $1 \leq p < 2 \leq r < \infty$ be such that $\frac{1}{p} = \frac{1}{2} + \frac{1}{r}$ and (Ω, Σ, μ) a measure space. If X has type 2, then each bounded linear operator $U : X \to L_p(\mu)$ has a factorization of the form $X \xrightarrow{V} L_2(\mu) \xrightarrow{M_g} L_p(Y)$, V bounded linear, $g \in L_r(\mu)$ and $M_g(f) = gf$. Moreover, $\|U\| \leq \|V\| \|g\|_r \leq \frac{1}{A_p} T_2(X) \|U\|$, A_p is the Khinchin constant and $T_2(X)$ is type 2 constant of X.

For the measure space $(\mathbb{N}, \mathcal{P}(\mathbb{N}), card)$, the Maurey factorization theorem gives us the following corollary.

Corollary 1. *Let $1 \leq p < 2 \leq r < \infty$ be such that $\frac{1}{p} = \frac{1}{2} + \frac{1}{r}$. If X has type 2, then each bounded linear operator $U : X \to \ell_p$ has a factorization of the form $X \xrightarrow{V} \ell_2 \xrightarrow{M_a} \ell_p$, V bounded linear and $a \in \ell_r$. Moreover, $\|U\| \leq \|V\| \|a\|_r \leq \frac{1}{A_p} T_2(X) \|U\|$.*

Above $M_a : \ell_2 \to \ell_p$ is the multiplication operator $M_a(x) = ax$ where, if $a = (a_n)_{n\in\mathbb{N}}$, $b = (b_n)_{n\in\mathbb{N}}$ are two scalar sequences $ab := (a_n b_n)_{n\in\mathbb{N}}$.

Let us fix some notation and notions. In this paper, all spaces considered are Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. For a finite system $(x_i)_{1\leq i\leq m} \subset X$, we write $w_2((x_i)_{1\leq i\leq m})$ to denote by $\sup_{\|x^*\|\leq 1} (\sum_{i=1}^m |x^*(x_i)|^2)^{\frac{1}{2}}$. A bounded linear operator $U : X \to Y$ is 2-summing if there exists a constant $C \geq 0$ such that, for every $x_1, \ldots, x_m \in X$ the following

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relation holds:

$$\left(\sum_{i=1}^{m}\|U(x_i)\|^2\right)^{\frac{1}{2}} \le C w_2((x_i)_{1\le i\le m}),$$

and the 2-summing norm of U is $\pi_2(U) = \inf\{C \mid C \text{ as above}\}$, see [8, page 234]. A bounded linear operator $U : X \to Y$ is 2-dominated if there exists a constant $C \ge 0$ such that for every $x_1, \ldots, x_m \in X$, $y_1^*, \ldots, y_m^* \in Y^*$ the following relation holds:

$$\sum_{i=1}^{m}|y_i^*(U(x_i))| \le C w_2((x_i)_{1\le i\le m}) w_2((y_i^*)_{1\le i\le m}),$$

and the 2-dominated norm of U is $\Delta_2(U) = \inf\{C \mid C \text{ as above}\}$, see [12, page 236]. Let n be a natural number and $X_1, \ldots, X_n$, Y Banach spaces. A bounded n-linear operator $U : X_1 \times \cdots \times X_n \to Y$ is called 2-dominated if there exists $C \ge 0$ such that for each $(x_i^1, \ldots, x_i^n)_{1\le i\le m} \subset X_1 \times \cdots \times X_n$ the following relation holds:

$$\left(\sum_{i=1}^{m}\|U(x_i^1, \ldots, x_i^n)\|^{\frac{2}{n}}\right)^{\frac{n}{2}} \le C w_2((x_i^1)_{1\le i\le m}) \cdots w_2((x_i^n)_{1\le i\le m})$$

and $\Delta_2(U) = \inf\{C \mid C \text{ as above}\}$, see [11]. The class of the all 2-dominated operators has the so-called ideal property, that is, if in the diagram

$$E_1 \times \cdots \times E_n \xrightarrow{(A_1,\ldots,A_n)} X_1 \times \cdots \times X_n \xrightarrow{U} Y \xrightarrow{V} Z,$$

U is 2-dominated and $A_1, \ldots, A_n$ are bounded linear and V is bounded linear, then the composition $V \circ U \circ (A_1, \ldots, A_n)$ is 2-dominated and

$$\Delta_2(V \circ U \circ (A_1, \ldots, A_n)) \le \|V\| \Delta_2(U) \|A_1\| \cdots \|A_n\|.$$

Let us observe that a bounded linear operator $U : X \to Y$ is 2-dominated if and only if the bilinear functional associated to it $\psi_U : X \times Y^* \to \mathbb{K}$ defined by $\psi_U(x, y^*) = y^*(T(x))$ is 2-dominated. Moreover, $\Delta_2(U) = \Delta_2(\psi_U)$.

All notation and notions used and not defined in this paper are standard in Banach space theory, e.g., see, [6, 8, 12, 20].

2. The main result

In a series of the papers, the Maurey factorization theorem is used in the study of various classes of the operators. Thus, in [7, Lemma 4.5] it is used in the study of unconditionality of some tensor norms, in [16, 18] is used in the study of multiple 2-summing operators and in [14] is used in the study of almost summing operators, in [15] is used in the study of 2-summing operators. In the next result, we use again the Maurey factorization theorem to give a characterization of 2-dominated operators on a Cartesian product of ℓ_p spaces.

Theorem 1. *Let $n \in \mathbb{N}$, $1 \le p_j < 2 \le r_j < \infty$ be such that $\frac{1}{p_j} = \frac{1}{2} + \frac{1}{r_j}$ for all $1 \le j \le n$, $k \in \mathbb{N}$ and $U : \ell_{p_1} \times \cdots \times \ell_{p_n} \times X_1 \times \cdots \times X_k \to Y$ a bounded $n+k$-linear operator. Then, the following assertions are equivalent:*

(i) *U is 2-dominated,*

(ii) *$U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k}) : \ell_2 \times \cdots \times \ell_2 \times X_1 \times \cdots \times X_k \to Y$ is 2-dominated for each $a_1 \in \ell_{r_1}, \ldots, a_n \in \ell_{r_n}$. Moreover,*

$$\sup_{\|a_1\|_{r_1} \le 1, \ldots, \|a_n\|_{r_n} \le 1} \Delta_2(U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k})) \le \Delta_2(U)$$
$$\le \frac{1}{A_{p_1} \cdots A_{p_n}} \sup_{\|a_1\|_{r_1} \le 1, \ldots, \|a_n\|_{r_n} \le 1} \Delta_2(U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k})).$$

Proof. (i)⇒(ii) It follows from the ideal property of the class of 2-dominated operators together with the inequality

$$\sup_{\|a_1\|_{r_1} \le 1, \ldots, \|a_n\|_{r_n} \le 1} \Delta_2(U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k})) \le \Delta_2(U).$$

(ii)⇒(i). From (ii) and the uniform boundedness principle, it follows that

$$L_{\Delta_2} = \sup_{\|a_1\|_{r_1} \le 1, \ldots, \|a_n\|_{r_n} \le 1} \Delta_2(U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k})) < \infty.$$

For $1 \le j \le n$ and $(x_i^j)_{1 \le i \le m} \subset \ell_{p_j}$, let us define $S_j : \ell_2 \to \ell_{p_j}$ in the standard way, i.e., $S_j(\xi) = \sum_{i=1}^m \langle \xi, e_i \rangle x_i^j$ and note that, as is well-known $\|S_j\| = w_2((x_i^j)_{1 \le i \le m})$, see [12, Proof of Theorem 17.5.3]. Since ℓ_2 has type 2 (with $T_2(\ell_2) = 1$) and $1 \le p_j < 2$, from the Maurey factorization theorem, it follows that there exist bounded linear operators $V_j : \ell_2 \to \ell_2$ and $a_j \in \ell_{r_j}$ such that $S_j = M_{a_j} \circ V_j$ and $\|V_j\| \|a\|_{r_j} \le \frac{\|S_j\|}{A_{p_j}}$. We note that

$$U \circ (S_1, \ldots, S_n, I_{X_1}, \ldots, I_{X_k})$$
$$= U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k}) \circ (V_1, \ldots, V_n, I_{X_1}, \ldots, I_{X_k}).$$

Let us take also $(v_i^1, \ldots, v_i^k)_{1 \le i \le m} \subset X_1 \times \cdots \times X_k$. By (ii), the ideal property of the class of 2-dominated operators and the definition of L_Δ, we deduce that

$$\Delta_2(U \circ (S_1, \ldots, S_n, I_{X_1}, \ldots, I_{X_k}))$$
$$\le \Delta_2(U \circ (M_{a_1}, \ldots, M_{a_n}, I_{X_1}, \ldots, I_{X_k})) \|V_1\| \cdots \|V_n\|$$
$$\le L_{\Delta_2} \|a_1\|_{r_1} \cdots \|a_n\|_{r_n} \|V_1\| \cdots \|V_n\| \le \frac{L_{\Delta_2}}{A_{p_1} \cdots A_{p_n}} \|S_1\| \cdots \|S_n\|$$
$$= \frac{L_{\Delta_2}}{A_{p_1} \cdots A_{p_n}} w_2((x_i^1)_{1 \le i \le m}) \cdots w_2((x_i^n)_{1 \le i \le m}).$$

Then, by the definition of 2-dominated operators and $w_2((e_i)_{1\le i\le m};\ell_2)=1$, we get

$$\begin{aligned}&\left(\sum_{i=1}^{m}\|U\circ(S_1,\ldots,S_n,I_{X_1},\ldots,I_{X_k})(e_i,\ldots,e_i,v_i^1,\ldots,v_i^k)\|^{\frac{2}{n+k}}\right)^{\frac{n+k}{2}}\\&\quad\le\Delta_2(U\circ(S_1,\ldots,S_n,I_{X_1},\ldots,I_{X_k}))[w_2((e_i)_{1\le i\le m};\ell_2)]^n\\&\qquad\times w_2((v_i^1)_{1\le i\le m})\cdots w_2((v_i^k)_{1\le i\le m})\\&\quad=\Delta_2(U\circ(S_1,\ldots,S_n,I_{X_1},\ldots,I_{X_k}))w_2((v_i^1)_{1\le i\le m})\cdots w_2((v_i^k)_{1\le i\le m}),\end{aligned}$$

i.e.,

$$\begin{aligned}&\left(\sum_{i=1}^{m}\|U(x_i^1,\ldots,x_i^n,v_i^1,\ldots,v_i^k)\|^{\frac{2}{n+k}}\right)^{\frac{n+k}{2}}\\&\le\frac{L_{\Delta_2}}{A_{p_1}\cdots A_{p_n}}w_2((x_i^1)_{1\le i\le m})\cdots w_2((x_i^n)_{1\le i\le m})w_2((v_i^1)_{1\le i\le m})\cdots w_2((v_i^k)_{1\le i\le m}).\end{aligned}$$

This means that U is 2-dominated and $\Delta_2(U)\le\frac{L_{\Delta_2}}{A_{p_1}\cdots A_{p_n}}$. ∎

3. The case of diagonal operators on a Cartesian product of ℓ_p spaces

Let $n\ge 2$ be a natural number and $1\le p_1,\ldots,p_n<\infty$, and let Y be a Banach space and $y=(y_i)_{i\in\mathbb{N}}$ a sequence in Y such that for all $(\xi_1,\ldots,\xi_n)\in\ell_{p_1}\times\cdots\times\ell_{p_n}$ the series $\sum_{i=1}^{\infty}\langle\xi_1,e_i\rangle\cdots\langle\xi_n,e_i\rangle y_i$ is convergent. In this case, define the operator $D_y:\ell_{p_1}\times\cdots\times\ell_{p_n}\to Y$ by

$$D_y(\xi_1,\ldots,\xi_n)=\sum_{i=1}^{\infty}\langle\xi_1,e_i\rangle\cdots\langle\xi_n,e_i\rangle y_i$$

and call it a diagonal operator. Let us mention that in [2] in the particular case $p_1=\cdots=p_n$ and $Y=\mathbb{K}$ are also introduced in the diagonal mappings. A particular case of the diagonal operators is the multiplication operators; namely, if $n\in\mathbb{N}$, $1\le p_1,\ldots,p_n,p_{n+1}<\infty$ and a is a sequence of scalars, we define $M_a:\ell_{p_1}\times\cdots\times\ell_{p_n}\to\ell_{p_{n+1}}$ by the formula $M_a(\xi_1,\ldots,\xi_n)=a\xi_1\cdots\xi_n$. As is well known, M_a is well defined if and only if $a\in\ell_s$ in the case $\frac{1}{p_{n+1}}>\frac{1}{p_1}+\cdots+\frac{1}{p_n}$, where $\frac{1}{p_{n+1}}=\frac{1}{p_1}+\cdots+\frac{1}{p_n}+\frac{1}{s}$, or $a\in\ell_\infty$ in the case $\frac{1}{p_{n+1}}\le\frac{1}{p_1}+\cdots+\frac{1}{p_n}$. Since $\xi_1\cdots\xi_n=\sum_{i=1}^{\infty}\langle\xi_1,e_i\rangle\cdots\langle\xi_n,e_i\rangle e_i$, we get that $M_a(\xi_1,\ldots,\xi_n)=\sum_{i=1}^{\infty}\langle\xi_1,e_i\rangle\cdots\langle\xi_n,e_i\rangle a_ie_i$, that is, $M_a=D_y$, where $y=(a_ie_i)_{i\in\mathbb{N}}$. The main aim of this section is to apply Theorem 1 to find the necessary and sufficient conditions for the diagonal operators to be 2-dominated. For other different results, the reader can consult the papers [2, 4]. Let us mention that in the papers of D. Carando, V. Dimant, P. Sevilla–Peris [2, 3] and V. Dimant, R. Villafañe [9] various properties for diagonal operators are studied. The main novelty of our results comparatively with those is that our results use mainly the Maurey factorization theorem.

Proposition 1. *Let $n \geq 2$ be a natural number, where $2 \leq p_1, \ldots, p_n < \infty$. Then, the diagonal operator D_y is 2-dominated if and only if $\sum_{i=1}^{\infty} \|y_i\|^{\frac{2}{n}} < \infty$. Moreover, $\Delta_2(D_y) = (\sum_{i=1}^{\infty} \|y_i\|^{\frac{2}{n}})^{\frac{n}{2}}$.*

Proof. Let us suppose that D_y is 2-dominated. Since for $p \geq 2$, $w_2(e_i; \ell_p) = \|\ell_{p^*} \hookrightarrow \ell_2\| = 1$, we deduce that $(\sum_{i=1}^{\infty} \|D_y(e_i, \ldots, e_i)\|^{\frac{2}{n}})^{\frac{n}{2}} \leq \Delta_2(D_y)$, that is, $(\sum_{i=1}^{\infty} \|y_i\|^{\frac{2}{n}})^{\frac{n}{r}} \leq \Delta_2(D_y)$. For the converse, we note first that, for all $(\xi_1, \ldots, \xi_n) \in \ell_{p_1} \times \cdots \times \ell_{p_n}$, we have

$$
\begin{aligned}
\|D_y(\xi_1, \ldots, \xi_n)\| &\leq \sum_{i=1}^{\infty} |\langle \xi_1, e_i\rangle \cdots \langle \xi_n, e_i\rangle| \|y_i\| \\
&\leq \left(\sum_{i=1}^{\infty} |\langle \xi_1, e_i\rangle \cdots \langle \xi_n, e_i\rangle|^{\frac{2}{n}} \|y_i\|^{\frac{2}{n}} \right)^{\frac{n}{2}},
\end{aligned}
$$

since $\frac{2}{n} \leq 1$. Hence, $\|D_y(\xi_1, \ldots, \xi_n)\|^{\frac{2}{n}} \leq \sum_{i=1}^{\infty} |\langle \xi_1, e_i\rangle|^{\frac{2}{n}} \cdots |\langle \xi_n, e_i\rangle|^{\frac{2}{n}} \|y_i\|^{\frac{2}{n}}$. For all $(\xi_j^1, \ldots, \xi_j^n)_{1 \leq j \leq m} \subset \ell_{p_1} \times \cdots \times \ell_{p_n}$, by Hölder's inequality,

$$
\begin{aligned}
\sum_{j=1}^{m} |\langle \xi_j^1, e_i\rangle|^{\frac{2}{n}} \cdots |\langle \xi_j^n, e_i\rangle|^{\frac{2}{n}} &\leq \left(\sum_{j=1}^{m} |\langle \xi_j^1, e_i\rangle|^2 \right)^{\frac{1}{n}} \cdots \left(\sum_{j=1}^{m} |\langle \xi_j^n, e_i\rangle|^2 \right)^{\frac{1}{n}} \\
&\leq \left[w_2((\xi_j^1)_{1 \leq j \leq m}) \cdots w_2((\xi_j^n)_{1 \leq j \leq m}) \right]^{\frac{2}{n}}.
\end{aligned}
$$

We deduce that

$$
\begin{aligned}
\sum_{j=1}^{m} \|D_y(\xi_j^1, \ldots, \xi_j^n)\|^{\frac{2}{n}} &\leq \sum_{i=1}^{\infty} \|y_i\|^{\frac{2}{n}} \sum_{j=1}^{m} |\langle \xi_j^1, e_i\rangle|^{\frac{2}{n}} \cdots |\langle \xi_j^n, e_i\rangle|^{\frac{2}{n}} \\
&\leq \left(\sum_{i=1}^{\infty} \|y_i\|^{\frac{2}{n}} \right) \left[w_2((\xi_j^1)_{1 \leq j \leq m}) \cdots w_2((\xi_j^n)_{1 \leq j \leq m}) \right]^{\frac{2}{n}};
\end{aligned}
$$

that is, D_y is 2-dominated and $\Delta_2(D_y) \leq (\sum_{i=1}^{\infty} \|y_i\|^{\frac{2}{n}})^{\frac{n}{2}}$. ∎

In our next applications, we need the following well-known result whose proof is omitted.

Proposition 2. *Let $n \in \mathbb{N}$, $0 < r_1, \ldots, r_n, r < \infty$ be such that $\frac{1}{r} > \frac{1}{r_1} + \cdots + \frac{1}{r_n}$ and define $0 < s < \infty$ by $\frac{1}{r} = \frac{1}{r_1} + \cdots + \frac{1}{r_n} + \frac{1}{s}$. Let a be a sequence of scalars. Then, the following assertions are equivalent:*

(i) $ax_1 \cdots x_n \in \ell_r$ *for all* $x_1 \in \ell_{r_1}, \ldots, x_n \in \ell_{r_n}$,

(ii) $a \in \ell_s$.

Moreover, $\sup_{\|x_1\|_{r_1} \leq 1, \ldots, \|x_n\|_{r_n} \leq 1} \|ax_1 \cdots x_n\|_r = \|a\|_s$.

In the rest of the paper for $1 \leq p \leq \infty$, we denote by p^* its conjugate that is $\frac{1}{p} + \frac{1}{p^*} = 1$.

Proposition 3. *Let $n \geq 2$ be a natural number and $1 \leq p_1, \ldots, p_n < 2$. Then, the diagonal operator D_y is 2-dominated if and only if $y \in \ell_s(Y)$, where $\frac{1}{s} = \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*}$. Moreover, $\|y\|_s \leq \Delta_2(D_y) \leq \frac{1}{A_{p_1} \cdots A_{p_n}} \|y\|_s$.*

Proof. By Theorem 1, D_y is 2-dominated if and only if $D_y \circ (M_{a_1}, \ldots, M_{a_n}) : \ell_2 \times \cdots \times \ell_2 \to Y$ is 2-dominated for each $a_1 \in \ell_{r_1}, \ldots, a_n \in \ell_{r_n}$, where $\frac{1}{p_j} = \frac{1}{2} + \frac{1}{r_j}$ for all $1 \leq j \leq n$. Since

$$D_y \circ (M_{a_1}, \ldots, M_{a_n})(\xi_1, \ldots, \xi_n) = \sum_{i=1}^{\infty} \langle \xi_1, e_i \rangle \cdots \langle \xi_n, e_i \rangle z_i,$$

where $z_i = \langle a_1, e_i \rangle \cdots \langle a_n, e_i \rangle y_i$, by Proposition 1 this is equivalent to $z \in \ell_{\frac{2}{n}}(Y)$, and moreover, $\Delta_2(D_y \circ (M_{a_1}, \ldots, M_{a_n})) = \|z\|_{\frac{2}{n}}$. Hence, D_y is 2-dominated if and only if $a_1 \cdots a_n \|y\| \in \ell_{\frac{2}{n}}$ for all $a_1 \in \ell_{r_1}, \ldots, a_n \in \ell_{r_n}$. Now, note that the condition $\frac{1}{\frac{2}{n}} > \frac{1}{r_1} + \cdots + \frac{1}{r_n}$ is equivalent to $\frac{n}{2} > (\frac{1}{p_1} - \frac{1}{2}) + \cdots + (\frac{1}{p_n} - \frac{1}{2})$ or $\frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*} > 0$. By Proposition 2, D_y is 2-dominated if and only if $\|y\| \in \ell_s$, where $\frac{1}{\frac{2}{n}} = \frac{1}{r_1} + \cdots + \frac{1}{r_n} + \frac{1}{s}$, $\frac{1}{s} = \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*}$. ■

Since the multiplication operators are diagonal operators from Proposition 3, we get the following corollary.

Corollary 2. *Let $n \geq 2$ be a natural number, $1 \leq p_1, \ldots, p_n < 2$, $M_a : \ell_{p_1} \times \cdots \times \ell_{p_n} \to \ell_p$ (or c_0) the multiplication operator defined by $M_a(\xi_1, \ldots, \xi_n) = a\xi_1 \cdots \xi_n$. Then, M_a is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*}$.*

Proposition 4. *Let $n \geq 2$ be a natural number, $1 \leq p < \infty$, $a = (a_k)_{k \in \mathbb{N}}$ such that $a \in \ell_\infty$ for $p \leq n$ or $a \in \ell_{\frac{p}{p-n}}$ for $p > n$ and let $D_a : \underbrace{\ell_p \times \cdots \times \ell_p}_{n\text{-times}} \to \mathbb{K}$ be the diagonal operator defined by $D_a(\xi_1, \ldots, \xi_n) = \sum_{i=1}^{\infty} \langle \xi_1, e_i \rangle \cdots \langle \xi_n, e_i \rangle a_i$. Then, D_a is 2-dominated if and only if $\sum_{i=1}^{\infty} |a_i|^{\frac{\max(p^*,2)}{n}} < \infty$.*

Proof. First proof. If $1 \leq p < 2$, from Proposition 3, D_a is 2-dominated if and only if $\sum_{i=1}^{\infty} |a_i|^{\frac{p^*}{n}} < \infty$. If $p \geq 2$, from Proposition 1, D_a is 2-dominated if and only if $\sum_{i=1}^{\infty} |a_i|^{\frac{2}{n}} < \infty$.

Second proof. The following proof was shown to us by one of the reviewers of our paper and is included here with his permission. In Section 2 of [3], the limit orders of scalar-valued r-dominated multilinear forms from ℓ_p are obtained. These limit orders give, essentially, the inverse of the value of s for which D_a is r-dominated on ℓ_p whenever a belongs to ℓ_s. For $r = 2$ (i.e., for 2-dominated multilinear forms), equation (2.1) in [3] states that the limit order is $\frac{n}{p^*}$ if $2 \leq p^* \leq \infty$ and $\frac{n}{2}$ if $1 \leq p^* \leq 2$. This means that corresponding s is precisely $\frac{\max(p^*,2)}{n}$. ■

From Proposition 4, we deduce that for every $n \geq 2$ and every $1 \leq p < \infty$ there exist polynomials of degree n on ℓ_p which are not 2-dominated. For the case of infinite dimensional Banach spaces, see [1, Theorem 2.2].

The next results are natural companions of Proposition 3.

Proposition 5. *Let $n, k \in \mathbb{N}$, $1 \leq p_1, \ldots, p_n < 2 \leq p_{n+1}, \ldots, p_{n+k}$ and $D_y : \ell_{p_1} \times \cdots \times \ell_{p_n} \times \ell_{p_{n+1}} \times \cdots \times \ell_{p_{n+k}} \to Y$ be the diagonal operator defined by*

$$D_y(\xi_1, \ldots, \xi_n, \xi_{n+1}, \ldots, \xi_{n+k}) = \sum_{i=1}^{\infty} \langle \xi_1, e_i \rangle \cdots \langle \xi_{n+k}, e_i \rangle y_i.$$

Then, D_y is 2-dominated if and only if $y \in \ell_s(Y)$, where $\frac{1}{s} = \frac{k}{2} + \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*}$.

Proof. By Theorem 1, D_y is 2-dominated if and only if $D_y \circ (M_{a_1}, \ldots, M_{a_n}, I, \ldots, I) : \ell_2 \times \cdots \times \ell_2 \to Y$ is 2-dominated for each $a_1 \in \ell_{r_1}, \ldots, a_n \in \ell_{r_n}$, where $\frac{1}{p_j} = \frac{1}{2} + \frac{1}{r_j}$ for all $1 \leq j \leq n$. Since $D_y \circ (M_{a_1}, \ldots, M_{a_n}, I, \ldots, I)(\xi_1, \ldots, \xi_{n+k}) = \sum_{i=1}^{\infty} \langle \xi_1, e_i \rangle \cdots \langle \xi_{n+k}, e_i \rangle z_i$, where $z_i = \langle a_1, e_i \rangle \cdots \langle a_n, e_i \rangle y_i$, by Proposition 1 this is equivalent to $z \in \ell_{\frac{2}{n+k}}(Y)$, and moreover, $\Delta_2(D_y \circ (M_{a_1}, \ldots, M_{a_n}, I, \ldots, I)) = \|z\|_{\frac{2}{n+k}}$. Hence, D_y is 2-dominated if and only if $a_1 \cdots a_n \|y\| \in \ell_{\frac{2}{n+k}}$ for all $a_1 \in \ell_{r_1}, \ldots, a_n \in \ell_{r_n}$. The condition $\frac{1}{\frac{2}{n+k}} > \frac{1}{r_1} + \cdots + \frac{1}{r_n}$ is equivalent to $\frac{n+k}{2} > (\frac{1}{p_1} - \frac{1}{2}) + \cdots + (\frac{1}{p_n} - \frac{1}{2})$ or $\frac{k}{2} + \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*} > 0$. By Proposition 2, D_y is 2-dominated if and only if $\|y\| \in \ell_s$, where $\frac{1}{\frac{2}{n+k}} = \frac{1}{r_1} + \cdots + \frac{1}{r_n} + \frac{1}{s}$, $\frac{1}{s} = \frac{k}{2} + \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*}$. ∎

In the case of the multiplication operators, Proposition 5 gives us the following corollary.

Corollary 3. *Let n, k be natural numbers, where $1 \leq p_1, \ldots, p_n < 2 \leq p_{n+1}, \ldots, p_{n+k}$. Then, the multiplication operator $M_a : \ell_{p_1} \times \cdots \times \ell_{p_n} \times \ell_{p_{n+1}} \times \cdots \times \ell_{p_{n+k}} \to \ell_p$ (or c_0) defined by $M_a(\xi_1, \ldots, \xi_{n+k}) = a\xi_1 \cdots \xi_{n+k}$ is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{k}{2} + \frac{1}{p_1^*} + \cdots + \frac{1}{p_n^*}$.*

Remark 1. Let n, k be a natural numbers, where $2 \leq p_1, \ldots, p_n$, $1 \leq p_{n+1}, \ldots, p_{n+k} < 2$ and let $D_y : l_{p_1} \times \cdots \times l_{p_n} \times l_{p_{n+1}} \times \cdots \times l_{p_{n+k}} \to Y$ be the diagonal operator defined by

$$D_y(\xi_1, \ldots, \xi_n, \xi_{n+1}, \ldots, \xi_{n+k}) = \sum_{i=1}^{\infty} \langle \xi_1, e_i \rangle \cdots \langle \xi_{n+k}, e_i \rangle y_i.$$

Then, D_y is 2-dominated if and only if $y \in l_s(Y)$, where $\frac{1}{s} = \frac{n}{2} + \frac{1}{p_{n+1}^*} + \cdots + \frac{1}{p_{n+k}^*}$.

In the next example, we give the necessary and sufficient conditions for the linear multiplication operator between two ℓ_p spaces to be 2-dominated, see also [9]. Let us mention that L. Schwartz in [19] gives the necessary and sufficient conditions for the multiplication operators between two ℓ_p spaces to be 2-summing and that in [13] the results of L. Schwartz were extended to the case of vector-valued ℓ_p spaces.

Proposition 6. *Let* $1 \le p_1, p_2 < \infty$ *and* $M_a : \ell_{p_1} \to \ell_{p_2}$ *be the multiplication operator.*

(i) *If* $1 \le p_1 < 2$, $2 < p_2$, *then* M_a *is* 2*-dominated if and only if* $a \in \ell_s$, *where* $\frac{1}{s} = \frac{1}{p_1^*} + \frac{1}{p_2}$.

(ii) *If* $1 \le p_1 < 2$, $1 \le p_2 \le 2$, *then* M_a *is* 2*-dominated if and only if* $a \in \ell_s$, *where* $\frac{1}{s} = \frac{1}{p_1^*} + \frac{1}{2}$.

(iii) *If* $2 \le p_1$, $1 < p_2 < 2$, *then* M_a *is* 2*-dominated if and only if* $a \in \ell_1$.

(iv) *If* $2 \le p_1$, $2 \le p_2$, *then* M_a *is* 2*-dominated if and only if* $a \in \ell_s$, *where* $\frac{1}{s} = \frac{1}{2} + \frac{1}{p_2}$.

Proof. The case $1 < p_2 < \infty$. By definition, M_a is 2-dominated if and only if $\psi_{M_a} : \ell_{p_1} \times \ell_{p_2^*} \to \mathbb{K}$ defined by $\psi_{M_a}(\xi, \eta) = \langle M_a(\xi), \eta\rangle = \sum_{k=1}^{\infty} \langle \xi, e_k\rangle\langle \eta, e_k\rangle a_k$ is 2-dominated.

(i) In this case, $1 \le p_1 < 2$, $1 < p_2^* < 2$ and by Proposition 3, ψ_{M_a} is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{p_1^*} + \frac{1}{p_2}$.

(ii) In this case, $1 \le p_1 < 2$, $2 \le p_2^*$ and by Proposition 5, ψ_{M_a} is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{p_1^*} + \frac{1}{2}$.

(iii) In this case, $2 \le p_1$, $2 < p_2^*$ and by Proposition 1, ψ_{M_a} is 2-dominated if and only if $a \in \ell_1$.

(iv) In this case, $2 \le p_1$, $1 < p_2^* < 2$ and by Remark 1, ψ_{M_a} is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{2} + \frac{1}{p_2}$.

The case $p_2 = 1$. By definition, M_a is 2-dominated if and only if $\psi_{M_a} : \ell_{p_1} \times \ell_\infty \to \mathbb{K}$ defined by $\psi_{M_a}(\xi, \eta) = \langle M_a(\xi), \eta\rangle = \langle a\xi, \eta\rangle = \sum_{k=1}^{\infty} \langle \xi, e_k\rangle\langle \eta, e_k\rangle a_k$ is 2-dominated. If $1 \le p_1 < 2$, by Remark 5 ψ_{M_a} is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{p_1^*} + \frac{1}{2}$. If $p_1 \ge 2$, by Proposition 1 ψ_{M_a} is 2-dominated if and only if $a \in \ell_1$. ∎

Proposition 7. (i) *Let* $1 \le p < \infty$ *and* $M_a : \ell_p \to \ell_p$ *be the multiplication operator. Then, for* $1 \le p < 2$, M_a *is* 2*-dominated if and only if* $a \in \ell_s$, *where* $\frac{1}{s} = \frac{1}{p^*} + \frac{1}{2}$ *and for* $p \ge 2$, M_a *is* 2*-dominated if and only if* $a \in \ell_s$, *where* $\frac{1}{s} = \frac{1}{2} + \frac{1}{p}$.

(ii) *Let* $P_a : \ell_p \to \ell_p$ *be the polynomial multiplication operator of degree* $n \ge 2$ *that is* $P_a(x) = ax^n$. *Then,* P_a *is* 2*-dominated if and only if* $a \in \ell_{\frac{\max(p^*,2)}{n}}$.

Proof. (i) is a particular case of Proposition 6.

(ii) By Theorem 6 in [11], $P_a : \ell_p \to \ell_p$ is 2-dominated if and only if its symmetric bounded multilinear associated $M_a = \widehat{P_a} : \ell_p \times \cdots \times \ell_p \to \ell_p$ is 2-dominated. The equality $M_a = \widehat{P_a}$ is clear. If $1 \le p < 2$, by Proposition 3, this is equivalent to $a \in \ell_s$, where $s = \frac{p^*}{n}$. If $2 \le p < \infty$, by Proposition 1, this is equivalent to $a \in \ell_{\frac{2}{n}}$. ∎

Proposition 8. (i) *Let* $M_a : c_0 \to c_0$ *be the multiplication operator. Then,* M_a *is* 2*-dominated if and only if* $a \in \ell_2$.

(ii) *Let* $P_a : c_0 \to c_0$ *be the polynomial multiplication operator of degree* $n \ge 2$ *that is* $P_a(x) = ax^n$. *Then,* P_a *is* 2*-dominated if and only if* $a \in \ell_{\frac{2}{n}}$.

Proof. (i) *First proof.* By definition, M_a is 2-dominated if and only if $\psi_{M_a} : c_0 \times \ell_1 \to \mathbb{K}$ defined by $\psi_{M_a}(\xi, \eta) = \langle M_a(\xi), \eta\rangle = \langle a\xi, \eta\rangle = \sum_{k=1}^{\infty} \langle \xi, e_k\rangle\langle \eta, e_k\rangle a_k$ is 2-dominated. By Proposition 1, ψ_{M_a} is 2-dominated if and only if $a \in \ell_2$.

Second proof. If M_a is 2-dominated, then is 2-summing, and hence, $a \in \ell_2$. If $a \in \ell_2$, then $\psi : c_0 \times \ell_1 \xrightarrow{(M_a,J)} \ell_2 \times \ell_2 \xrightarrow{\langle,\rangle} \mathbb{K}$ is a factorization of ψ_{M_a}. Since $a \in \ell_2$, $M_a : c_0 \to \ell_2$ is 2-summing and the canonical inclusion $J : \ell_1 \hookrightarrow \ell_2$ is 2-summing, we get that ψ_{M_a} is 2-dominated.

(ii) By Theorem 6 in [11], $P_a : c_0 \to c_0$ is 2-dominated if and only if $M_a = \widehat{P_a}$: $c_0 \times \cdots \times c_0 \to c_0$ is 2-dominated. The equality $M_a = \widehat{P_a}$ is clear. By Proposition 1, this is equivalent to $a \in \ell_{\frac{2}{n}}$. ■

Proposition 9. *Let* $1 \le p < \infty$ *and* $M_a : \ell_p \to c_0$ *be the multiplication operator. Then,* M_a *is 2-dominated if and only if* $a \in \ell_{\max(p^*,2)}$.

Proof. By definition, M_a is 2-dominated if and only if $\psi_{M_a} : \ell_p \times \ell_1 \to \mathbb{K}$ defined by $\psi_{M_a}(\xi, \eta) = \langle M_a(\xi), \eta\rangle = \langle a\xi, \eta\rangle = \sum_{k=1}^{\infty} \langle \xi, e_k\rangle\langle \eta, e_k\rangle a_k$ is 2-dominated. If $1 \le p < 2$, by Proposition 3 ψ_{M_a} is 2-dominated if and only if $a \in \ell_{p^*}$. If $p \ge 2$, by Proposition 1 ψ_{M_a} is 2-dominated if and only if $a \in \ell_2$. ■

Proposition 10. *Let* $1 \le p < \infty$ *and* $M_a : c_0 \to \ell_p$ *be the multiplication operator. Then, the following statements hold.*

(i) *For* $1 \le p \le 2$, M_a *is 2-dominated if and only if* $a \in \ell_1$.

(ii) *For* $p > 2$, M_a *is 2-dominated if and only if* $a \in \ell_s$, *where* $\frac{1}{s} = \frac{1}{2} + \frac{1}{p}$.

Proof. By definition, M_a is 2-dominated if and only if $\psi_{M_a} : c_0 \times \ell_{p^*} \to \mathbb{K}$ defined by $\psi_{M_a}(\xi, \eta) = \langle M_a(\xi), \eta\rangle = \sum_{k=1}^{\infty} \langle \xi, e_k\rangle\langle \eta, e_k\rangle a_k$ is 2-dominated.

(i) In this case, by Proposition 1, ψ_{M_a} is 2-dominated if and only if $a \in \ell_1$.

(ii) In this case, by Proposition 1, ψ_{M_a} is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{2} + \frac{1}{p}$. ■

We give in the sequel the necessary and sufficient conditions for some general operators to be 2-dominated.

Proposition 11. *Let* $n \ge 2$ *be a natural number,* $1 \le p < \infty$, $B : c_0 \to Y$ *a bounded linear operator and* $T : \underbrace{\ell_p \times \cdots \times \ell_p}_{n\text{-times}} \to Y$ *defined by* $T(\xi_1, \ldots, \xi_n) = B(\xi_1 \cdots \xi_n)$. *Then,* T *is 2-dominated if and only if* $\sum_{i=1}^{\infty} \|B(e_i)\|^{\frac{\max(p^*,2)}{n}} < \infty$.

Proof. Since $\xi_1 \cdots \xi_n = \sum_{i=1}^{\infty} \langle \xi_1, e_i\rangle \cdots \langle \xi_n, e_i\rangle e_i$, we get that

$$T(\xi_1, \ldots, \xi_n) = \sum_{i=1}^{\infty} \langle \xi_1, e_i\rangle \cdots \langle \xi_n, e_i\rangle B(e_i).$$

If $1 \le p < 2$, by Proposition 3, T is 2-dominated if and only if $\sum_{i=1}^{\infty} \|B(e_i)\|^{\frac{p^*}{n}} < \infty$. If $p \ge 2$, by Proposition 1, T is 2-dominated if and only if $\sum_{i=1}^{\infty} \|B(e_i)\|^{\frac{2}{n}} < \infty$. ■

We recall that an infinite matrix of scalar numbers $(\alpha_{kj})_{(k,j)\in\mathbb{N}\times\mathbb{N}}$ is called a method of summability if for every sequence $(x_j)_{j\in\mathbb{N}} \in c_0$ it follows that all the series $y_k = \sum_{j=1}^{\infty} \alpha_{kj} x_j$ are convergent and the sequence $(y_k)_{k\in\mathbb{N}} \in c_0$. It induces a bounded linear operator $B : c_0 \to c_0$ defined by $B(\xi) = (\sum_{j=1}^{\infty} \alpha_{kj} \langle \xi, e_j \rangle)_{k\in\mathbb{N}}$.

Proposition 12. *Let $n \ge 2$ be a natural number, $1 \le p < \infty$, $(\alpha_{kj})_{(k,j)\in\mathbb{N}\times\mathbb{N}}$ a method of summability and $T : \ell_p \times \cdots \times \ell_p \to c_0$ defined by*

$$T(\xi_1, \ldots, \xi_n) = \left(\sum_{j=1}^{\infty} \alpha_{kj} \langle \xi_1, e_j \rangle \cdots \langle \xi_n, e_j \rangle \right)_{k\in\mathbb{N}}.$$

Then, T is 2-dominated if and only if $\sum_{i=1}^{\infty} (\sup_{k\in\mathbb{N}} |\alpha_{ki}|)^{\frac{\max(p^*,2)}{n}} < \infty$.

Proof. Let $B : c_0 \to c_0$ be defined by $B(\xi) = (\sum_{j=1}^{\infty} \alpha_{kj} \langle \xi, e_j \rangle)_{k\in\mathbb{N}}$. Now, note that $T(\xi_1, \ldots, \xi_n) = B(\xi_1 \cdots \xi_n)$. We have $B(e_i) = (\alpha_{ki})_{k\in\mathbb{N}}$ and $\|B(e_i)\|_{c_0} = \sup_{k\in\mathbb{N}} |\alpha_{ki}|$. We apply Theorem 11. ■

Let us prove two particular cases of Proposition 12.

Corollary 4. *Let $n \ge 2$ be a natural number, $1 \le p < \infty$ and let $C : \ell_p \times \cdots \times \ell_p \to c_0$ be the Cesarò operator defined by*

$$C(\xi_1, \ldots, \xi_n) = \left(\frac{\langle \xi_1, e_1 \rangle \cdots \langle \xi_n, e_1 \rangle + \cdots + \langle \xi_1, e_k \rangle \cdots \langle \xi_n, e_k \rangle}{k} \right)_{k\in\mathbb{N}}.$$

Then, C is 2-dominated if and only if $p < \frac{n}{n-1}$.

Proof. Let us take

$$\alpha_{kj} = \begin{cases} \frac{1}{k} & \text{if } j \le k, \\ 0 & \text{if } j \ge k+1, \end{cases}$$

and note as is well known that this is the so-called Cesaro method. Since $\sup_{k\in\mathbb{N}} |\alpha_{ki}| = \frac{1}{i}$, $i \in \mathbb{N}$, by Proposition 12, C is 2-dominated if and only if $\sum_{i=1}^{\infty} \frac{1}{i^{\frac{\max(p^*,2)}{n}}} < \infty$, that is, $\frac{\max(p^*,2)}{n} > 1$. Since, $n \ge 2$ this is equivalent to $p^* > n$, $p < n^* = \frac{n}{n-1}$. ■

Proposition 13. *Let $n \ge 2$ be a natural number, $1 \le p < \infty$ and let $T : \ell_p \times \cdots \times \ell_p \to c_0$ be the operator defined by*

$$T(\xi_1, \ldots, \xi_n) = \left(\frac{\frac{\langle \xi_1, e_1 \rangle \cdots \langle \xi_n, e_1 \rangle}{1} + \frac{\langle \xi_1, e_2 \rangle \cdots \langle \xi_n, e_2 \rangle}{2} + \cdots + \frac{\langle \xi_1, e_k \rangle \cdots \langle \xi_n, e_k \rangle}{k}}{\ln(k+1)} \right)_{k\in\mathbb{N}}.$$

Then, C is 2-dominated if and only if $p < \frac{n}{n-1}$.

Proof. The matrix

$$\alpha_{kj} = \begin{cases} \frac{1}{j\ln(k+1)} & \text{if } j \leq k, \\ 0 & \text{if } j \geq k+1, \end{cases}$$

generates the so-called logarithmic method. Since $\sup_{k\in\mathbb{N}} |\alpha_{ki}| = \frac{1}{i\ln(i+1)}$, $i \in \mathbb{N}$, by Proposition 12 T is 2-dominated if and only if

$$\sum_{i=1}^{\infty} \frac{1}{(i\ln(i+1))^{\frac{\max(p^*,2)}{n}}} < \infty,$$

that is, $\frac{\max(p^*,2)}{n} > 1$ or equivalent to $p < \frac{n}{n-1}$. ∎

Proposition 14. *Let $1 \leq p < \infty$, Y a Banach space, $(y_i)_{i\in\mathbb{N}} \subset Y$ a bounded sequence and let $D_y : \ell_{p^*} \times \ell_p \to Y$ be the diagonal operator defined by $D_y(\xi,\eta) = \sum_{i=1}^{\infty} \langle \xi, e_i\rangle\langle \eta, e_i\rangle y_i$.*

(i) *If $1 \leq p < 2$, then D_y is 2-dominated if and only if $\sum_{i=1}^{\infty} \|y_i\|^s < \infty$, where $\frac{1}{s} = \frac{1}{2} + \frac{1}{p^*}$.*

(ii) *If $2 \leq p < \infty$, then D_y is 2-dominated if and only if $\sum_{i=1}^{\infty} \|y_i\|^s < \infty$, where $\frac{1}{s} = \frac{1}{p} + \frac{1}{2}$.*

Proof. The case $p = 2$ was proved in Proposition 1.

(i) If $1 \leq p < 2$, by Proposition 1, D_y is 2-dominated if and only if $\sum_{i=1}^{\infty} \|y_i\|^s < \infty$, where $\frac{1}{s} = \frac{1}{2} + \frac{1}{p^*}$.

(ii) If $2 < p$, by Proposition 5, D_y is 2-dominated if and only if $\sum_{i=1}^{\infty} \|y_i\|^s < \infty$, where $\frac{1}{s} = \frac{1}{2} + \frac{1}{p}$. ∎

In the next proposition, $(r_i)_{i\in\mathbb{N}}$ denotes the sequence of the Rademacher functions.

Proposition 15. *Let $1 \leq p, s < \infty$, $a = (a_i)_{i\in\mathbb{N}} \in \ell_\infty$ and $D_a : \ell_{p^*} \times \ell_p \to L_s[0,1]$ be the operator defined by $D_a(\xi,\eta) = \sum_{i=1}^{\infty} \langle \xi, e_i\rangle\langle \eta, e_i\rangle a_i r_i$.*

(i) *If $1 \leq p < 2$, D_a is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{2} + \frac{1}{p^*}$.*

(ii) *If $2 \leq p < \infty$, D_a is 2-dominated if and only if $a \in \ell_s$, where $\frac{1}{s} = \frac{1}{p} + \frac{1}{2}$.*

Proof. Let $(\xi,\eta) \in \ell_{p^*} \times \ell_p$ and note that by Khinchin's inequality, we have

$$A_s\left(\sum_{i=1}^{\infty} |\langle \xi, e_i\rangle\langle \eta, e_i\rangle a_i|^2\right)^{\frac{1}{2}} \leq \|D_a(\xi,\eta)\|_s \leq B_s\left(\sum_{i=1}^{\infty} |\langle \xi, e_i\rangle\langle \eta, e_i\rangle a_i|^2\right)^{\frac{1}{2}}$$

or $A_s\|M_a(\xi,\eta)\|_2 \leq \|D_a(\xi,\eta)\|_s \leq B_s\|M_a(\xi,\eta)\|_2$, where $M_a : \ell_{p^*} \times \ell_p \to \ell_2$ is the multiplication operator. Hence, D_a is 2-dominated if and only if M_a is 2-dominated. We apply Proposition 14. ∎

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References

[1] G. Botelho, D. Pellegrino, and P. Rueda, Dominated polynomials on infinite dimensional spaces. *Proc. Amer. Math. Soc.* **138** (2010), no. 1, 209–216 Zbl 1208.46046 MR 2550185

[2] D. Carando, V. Dimant, and P. Sevilla-Peris, Limit orders and multilinear forms on l_p spaces. *Publ. Res. Inst. Math. Sci.* **42** (2006), no. 2, 507–522 Zbl 1151.46031 MR 2250071

[3] D. Carando, V. Dimant, P. Sevilla-Peris, and R. Villafañe, Diagonal extendible multilinear operators between ℓ_p-spaces. *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM* **108** (2014), no. 2, 541–555 Zbl 1320.47059 MR 3249960

[4] R. Cilia and J. M. Gutiérrez, Dominated, diagonal polynomials on l_p spaces. *Arch. Math. (Basel)* **84** (2005), no. 5, 421–431 Zbl 1076.46033 MR 2139545

[5] A. Defant, Variants of the Maurey–Rosenthal theorem for quasi Köthe function spaces. *Positivity* **5** (2001), no. 2, 153–175 Zbl 0994.47036 MR 1825653

[6] A. Defant and K. Floret, *Tensor norms and operator ideals*. North-Holland Math. Stud. 176, North-Holland, Amsterdam, 1993 Zbl 0774.46018 MR 1209438

[7] A. Defant and D. Pérez-García, A tensor norm preserving unconditionality for $\mathcal{L}_p$-spaces. *Trans. Amer. Math. Soc.* **360** (2008), no. 6, 3287–3306 Zbl 1159.46045 MR 2379797

[8] J. Diestel, H. Jarchow, and A. Tonge, *Absolutely summing operators*. Cambridge Stud. Adv. Math. 43, Cambridge University Press, Cambridge, 1995 Zbl 0855.47016 MR 1342297

[9] V. Dimant and R. Villafañe, Diagonal multilinear operators on Köthe sequence spaces. *Linear Multilinear Algebra* **67** (2019), no. 2, 248–266 Zbl 1415.46030 MR 3890846

[10] B. Maurey, *Théorèmes de factorisation pour les opérateurs linéaires à valeurs dans les espaces* L^p. Astérisque 11, Société Mathématique de France, Paris, 1974 Zbl 0278.46028 MR 0344931

[11] Y. Meléndez and A. Tonge, Polynomials and the Pietsch domination theorem. *Math. Proc. R. Ir. Acad.* **99A** (1999), no. 2, 195–212 Zbl 0973.46037 MR 1881812

[12] A. Pietsch, *Operator ideals*. Mathematische Monographien 16, VEB Deutscher Verlag der Wissenschaften, Berlin, 1978 MR 0519680

[13] D. Popa, 2-summing multiplication operators. *Studia Math.* **216** (2013), no. 1, 77–96 Zbl 1288.47022 MR 3078637

[14] D. Popa, Almost summing and multiple almost summing multilinear operators on l_p spaces. *Arch. Math. (Basel)* **103** (2014), no. 3, 291–300 Zbl 1314.46057 MR 3266372

[15] D. Popa, Multiple summing, dominated and summing operators on a product of l_1 spaces. *Positivity* **18** (2014), no. 4, 751–765 Zbl 1321.46048 MR 3275365

[16] D. Popa, Multiple summing operators on l_p spaces. *Studia Math.* **225** (2014), no. 1, 9–28 Zbl 1320.47060 MR 3299394

[17] D. Popa, An extension of the Maurey factorization theorem. *Linear Multilinear Algebra* **68** (2020), no. 2, 304–322 Zbl 1503.47025 MR 4048058

[18] D. Popa, Multiple 2-summing operators from a Cartezian product of $l_1(H)$-spaces with values in a Hilbert space. *Linear Multilinear Algebra* **72** (2024), no. 18, 3112–3127 Zbl 07960519 MR 4832695

[19] L. Schwartz, Les applications 0-radonifiantes dans les espaces de suites. In *Séminaire Laurent Schwartz 1969–1970: Applications radonifiantes*, p. article no. 26, École Polytech, Paris, 1970 MR 0632040

[20] N. Tomczak-Jaegermann, *Banach–Mazur distances and finite-dimensional operator ideals*. Pitman Monographs and Surveys in Pure and Applied Mathematics 38, Longman Scientific & Technical, Harlow; copublished in the United States with John Wiley, New York, 1989 MR 0993774

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