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# Mini-Workshop: Algebraic Foliations: Analytic and Birational Viewpoint

Organized by
Eleonora Di Nezza, Rome
Enrica Floris, Toulouse
Susanna Zimmermann, Basel

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Abstract. The main goal of the mini-workshop was starting strong collaborations between outstanding women in geometry with a broad spectrum of expertise. The focus was on the interplay of three topics in Geometry: analytic methods in Kähler geometry, Foliations and Cremona groups. More precisely, Foliation theory was a unifying theme.

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## Introduction by the Organizers

The mini-workshop *Algebraic foliations: analytic and birational viewpoint*, organized by Eleonora Di Nezza (Rome Tor Vergata), Enrica Floris (Toulouse) and Susanna Zimmermann (Basel) was well attended with 14 participants with broad geographic representation (France, Italy, Germany, USA). This workshop was a nice blend of female researchers with various backgrounds, some expert in complex geometry, others in algebraic or birational geometry.

The main goal of the mini-workshop was starting strong collaborations between outstanding women in geometry with a broad spectrum of expertise. The focus was on the interplay of three topics in Geometry: analytic methods in Kähler geometry, Foliations and Cremona groups. More precisely, Foliation theory was a unifying theme.

Several open problems in foliation theory were presented, of analytic and birational flavour, and connected to the Cremona group. The participants then split into four research groups. For a more precise description of the problems we refer to the talks of Enrica Floris and Susanna Zimmermann.

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## Mini-Workshop: Algebraic Foliations: Analytic and Birational Viewpoint

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# Abstracts

## On algebraic foliations

Enrica Floris

In the last decades the study of foliations from an algebraic point of view has been the center of an intense scientific activity. In particular, the birational geometry of foliations on surfaces has been thoroughly studied and the so-called Minimal Model Program for foliations on threefolds has been completed. A foliation on a projective variety $X$ is the data of a saturated subsheaf of the tangent bundle $\mathcal{F} \subseteq T_X$ which is closed under Lie bracket. The *smooth locus of* $\mathcal{F}$ is the locus where the subsheaf is a subvector bundle. While the first condition can be achieved for any subsheaf after passing to the saturation, the second one is more profound and insures that locally analytically at a point in the smooth locus $\mathcal{F}$ is the pullback of the tangent bundle via a coordinate projection. The local fibres of these projection glue into submanifolds of $X$. The maximal submanifolds with the property are called *leaves* of the foliation. If $f\colon X' \to X$ is a birational morphism, and $\mathcal{F}$ is a foliation on $X$, one can define in a unique way a foliation $\mathcal{F}'$ on $X'$ which coincides with $\mathcal{F}$ where $f$ is an isomorphism.

The determinant of the dual of $\mathcal{F}$ is called the *canonical divisor of the foliation* $\mathcal{F}$ and denoted by $K_{\mathcal{F}}$. Its study sits at the center of the birational approach to the study of foliations, in an analogous way to the case of varieties. For example, we have adjunction formulae for the restriction of $K_{\mathcal{F}}$. A crucial role in the classification of foliations is played by singularities. There is a classical notion of reduced singularities for foliations on surfaces, and a notion of canonical singularities inspired by the minimal model program. While canonical singularities are defined by imposing a certain generality condition on the eigenvalues of the Jacobian of the vector field defining $\mathcal{F}$ at a certain point, canonical singularities are defined as follows: for every birational morphism $f\colon X' \to X$ the difference $K_{\mathcal{F}'} - f^*K_{\mathcal{F}}$ is an effective divisor. Canonical singularities (and klt, lc, terminal) can be defined for higher-dimensional foliations.

If $X$ has dimension 2 and 3 and $\mathcal{F}$ is a foliation on $X$ with mild singularities (defined as above), the curves having negative intersection with $K_{\mathcal{F}}$ can be contracted or flipped and $\mathcal{F}$ admits a minimal model or is birational to a Mori fiber space.

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## Problem session I

Enrica Floris

In the first problem session, after an overview of the MMP results for foliations on surfaces and threefolds, we presented three open problems.

The first, stemming from the paper by Marco Brunella [2], concerns surfaces with nef anticanonica divisor. Brunella studies the case of the blow-up $S$ of $\mathbb{P}^2$ along 9 points on a cubic curve and, letting $C$ be the strict transform of the cubic and assuming that $S \setminus C$ does not contain any compact curve, determines necessary and sufficient conditions for the existence on $S$ of a smooth Kähler metric with semipositive Ricci curvature. The existence of examples of surfaces $S$ which do not satisfy the necessary and sufficient condition is not known and the higher-dimensional case remains to be explored.

The second builds on the paper by Marco Brunella [1], in which it is proved that if $\mathcal{F}$ is a foliation on $\mathbb{P}^2$ induced by a non-nilpotent vector field, then every entire map tangent to $\mathcal{F}$ has algebraic image. The sharpness of the bound on the degree of the foliation and on the singularities is not known.

The third problem is an exploration of some consequences of the recent results on the minimal model program for foliations on threefolds. In particular, ut aims at studying the geometry of the augmented base locus of the canonical divisor of a foliation of general type.

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## Overview on (positive) currents

Eleonora Di Nezza

We recall basic notions of currents on a differntiable manifolds. We then talk about positivity notions for smooth forms and currents on complex manifolds. We direct the interested reader to the textbook of J.P. Demailly [1] for a complete treatment.

### 1. Currents

Consider a real oriented manifold $M$ of dimension $m$. A *current* $T$ of degree $q$ (or dimension $m-q$) on $M$ is a continuous linear form on the vector space $\mathcal{D}^{m-q}(X)$ of smooth differential forms of degree $m-q$ with compact support. We denote by $\mathcal{D}'_{m-q}(M)$ (or ${\mathcal{D}'}^{q}(M)$) the space of currents of degree $q$ on $M$, and by $\langle T, \eta \rangle$ the pairing between a test $(m-q)$-form $\eta$ and a current $T$ of degree $q$.

A first example of a current of degree $q$ is the *current of integration* over a smooth closed oriented submanifold $Z$ of dimension $m-q$, which is denoted by $[Z]$ and defined as

$$\langle [Z], \eta \rangle := \int_Z \eta.$$

Observe that, given $\tau$ a $q$-form with coefficients in $L^1_{loc}(M)$, we can associate the current $T_\tau$ of degree $q$ defined as follows:

$$\langle T_\tau, \eta \rangle := \int_M \tau \wedge \eta.$$

As a fact, let us mention that, if we let $(x_1, \cdots, x_m)$ be local coordinates on an open subset $\Omega \subset M$, then every current of degree $q$ can be written in an unique way as

$$T = \sum_{|I|=q} T_I dx_I, \tag{1}$$

where $T_I$ are distributions of order $s$ on $\Omega$.

Given a current $T$ of degree $q$, the wedge product of $T$ with a smooth $p$-form $\beta$ is defined as

$$\langle T \wedge \beta, \eta \rangle := \langle T, \beta \wedge \eta \rangle.$$

One can also define the exterior derivative $dT$ as the $(q+1)$-current satisfying

$$\langle dT, \eta \rangle := (-1)^{q+1} \langle T, d\eta \rangle.$$

A current $T$ is then said to be *closed* if $dT = 0$. We denote by $\{T\}$ the cohomology class defined by the current $T$. By deRham's Theorem, the corresponding cohomology vector space

$$H^{m-q}(M) := \{\text{closed currents of degree}\, q\}/\{dS \mid S \,\text{current of degree}\, q-1\}$$

is isomorphic to the one defined using closed smooth differential forms.

## 2. Positive forms and positive currents

Let now $X$ be a complex manifold of complex dimension $n$. The decomposition of complex valued differential forms according to their bidegrees induces a decomposition at the level of currents.

We say that a current $T$ is of bidegree $(p,q)$ if it is of degree $p+q$ and $\langle T, \eta \rangle = 0$ for any test form $\eta$ of bidegree $(k,l) \neq (n-p, n-q)$. We denote by $\mathcal{D}'^{p,q}(X)$ the space of such currents, and by $H^{p,q}(X)$ the corresponding vector space of cohomology classes.

In the complex case one can define a notion of positivity at the level of forms, hence at the level of currents.

Let $V$ be a complex vector space of dimension $n$ and $(z_1, ..., z_n)$ coordinates on $V$.

A $(q,q)$-form $\beta$ is *strongly positive* if it writes as a convex combination of forms of type

$$(i\alpha_1 \wedge \bar{\alpha_1}) \wedge \cdots \wedge (i\alpha_q \wedge \bar{\alpha}_q)$$

with $\alpha_j \in V^*$, for $j = 1, \cdots, q$. For example, any positive multiple of the volume form

$$(idz_1 \wedge d\bar{z}_1) \wedge ... \wedge (idz_n \wedge d\bar{z}_n)$$

is a strongly positive $(n, n)$-form.

A $(p, p)$-form $\eta$ is said to be *positive* if for all strongly positive (n-p, n-p)-form $\beta \in V^*$, we have that $\eta \wedge \beta$ is a positive multiple of the volume form.

Equivalently, a form of bidegree $(p, p)$ is positive if and only if its restriction to every $p$-dimensional subspace $S \subset V$ is a positive volume form on $S$.

The set of positive $(p, p)$-forms is a closed convex cone in $\bigwedge^{p,p} V^*$ and its dual cone in $\bigwedge^{n-p,n-p} V^*$ is the *strongly positive cone.* Observe that functions (forms of degree zero) or $(n, n)$-forms the two notion of positivity coincide.

Of course, we are interested in the case $V = T_xX$, $x \in X$. In this way, we are able to define, at each $x \in X$, a notion of positivity for smooth forms on $X$, and so we can then give a notion of positivity for currents. A current $T$ of bidimension $(p, p)$ is *positive* if $\langle T, \beta\rangle \geq 0$ for all strongly positive test forms $\beta \in \mathcal{D}^{p,p}(X)$.

Two extreme examples of positive currents are currents of integration along analytic subsets of dimension $p$ and positive smooth differential forms of bidegree $(p, p)$.

Again as a fact, we mention that, any positive $(p, p)$-current $T$ is a real current of order 0, i.e. in (1) the coefficients are distributions of order 0. More precisely, $T$ can be written (in local holomorphic coordinates in $\Omega \subset X$) as

$$T = i^{(n-p)^2} \sum_{|I|=|J|=p} T_{I,J} dz_I \wedge d\bar{z}_J, \tag{2}$$

where the coefficients $T_{I,J}$ are complex measures in $\Omega$ satisfying $\bar{T}_{I,J} = T_{J,I}$ and $T_{I,I}$ are positive Borel measures in $\Omega$.

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## Introduction to Cremona groups

Susanna Zimmermann

**Introduction.** A broad objective of Algebraic Geometry is to classify algebraic varieties up to isomorphisms. This is, however, much to difficult. More reasonable is to classify up to isomorphisms between open dense sets; these are called birational maps. In the late 19th century, a program was started to classify plane curves, and the goal consisted to find some type of normal form. A variable replacement is a birational map of the curve, and soon it was noticed that, up to symmetry and scaling, mostly replacements of the form $(u, v) \mapsto (u, uv)$ were necessary [1]. Its inverse is the map $(u, v) \mapsto (u, \frac{v}{u})$, which is not well-defined at the origin and is an isomorphism away from the origin. This very observation opend doors and windows to the study of the plane Cremona group.

The Cremona group $\mathrm{Bir}(\mathbb{P}^{\mathrm{n}})$ is the group is the group of birational maps of $\mathbb{P}^n$. A rational map of $\mathbb{P}^n$ is a "map" of the form

$$\mathbb{P}^n \dashrightarrow \mathbb{P}^n, \quad [x_0 : \cdots : x_n] \mapsto [f_0(x_0, \dots, x_n) : \cdots : f_n(x_0, \dots, x_n)]$$

where $f_0, \dots, f_n \in \mathbb{C}[x_0, \dots, x_n]$ are homogeneous polynomials of the same degree. It is not a map, because it is not defined where all $f_i$ vanish simultaneously. However, instead of making the domain more precise, we put a dashed arrow. The rational map $f$ is birational, if there is a rational map $g$ of $\mathbb{P}^n$ such that $g \circ f = \mathrm{id}_{\mathbb{P}^{\mathrm{n}}}$ and $f \circ g = \mathrm{id}_{\mathbb{P}^{\mathrm{n}}}$. Dehomogenizing yields a birational map of $\mathbb{A}^n$. Here are some examples:

**Example 1** (affine and projective version)

$$\sigma\colon (x, y) \mapsto \left(\frac{1}{x}, \frac{1}{y}\right), \text{ and } \sigma\colon [x_0 : x_1 : x_2] \mapsto [x_1x_2 : x_0x_2 : x_0x_1]$$

**Example 2** (affine and projective version)

$$\tau_p\colon (x, y) \mapsto \left(\frac{p(y)}{x}, y\right) \text{ and } [x_0 : x_1 : x_2] \mapsto [P(x_1, x_2) : x_0x_1x_2^{\deg p-2} : x_1x_2^{\deg p-1}]$$

where $p \in \mathbb{C}[y]$ and $P(y, 1) = p(y)$.

**Example 3** (affine verison)

$$\tau_{\alpha\beta\gamma\delta}\colon (x, y) \mapsto \left(x, \frac{\alpha(x)y + \beta(x)}{\gamma(x)y + \delta(x)}\right)$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}(x)$ with $\alpha\delta - \beta\gamma \neq 0$. These are so-called Jonquières maps. They fix the pencil of lines $(x = cst)$.

Notice that Example 1 and Example 2 are involutions.

**Properties of birational maps.** Let us notice some properties of birational maps. A birational map $f\colon \mathbb{P}^n \dashrightarrow \mathbb{P}^n$ is not a map, because it is not defined on $(f_0 = \cdots = f_n = 0)$. This is a codimension two subset of $\mathbb{P}^n$. More over, if $f^{-1}$ is given by coordinates $f^{-1}\colon [x_0 : \cdots : x_n] \mapsto [g_0(x_0, \dots, x_n) : \cdots : g_n(x_0, \dots, x_n)]$, then $f$ induces a diffeomorphism

$$f\colon \left(\det\left(\frac{\partial f_i}{\partial x_j}\right) \neq 0\right) \xrightarrow{\sim} \left(\det\left(\frac{\partial g_i}{\partial x_j}\right) \neq 0\right)$$

The union of hypersurfaces $\left(\det\left(\frac{\partial f_i}{\partial x_j}\right) = 0\right)$ is contracted onto the codimension two set $(g_0 = \cdots = g_n = 0)$. In the above examples, this looks as follows:

- (Ex 1) $\sigma$ contracts the three lines $(x_0x_1x_2 = 0)$ onto the points $[1 : 0 : 0], [0 : 1 : 0], [0 : 0 : 1]$.
- (Ex 2) $\tau_p$ contracts the curve $(P = 0)$ onto the points $\{[0 : y : 1] \mid P(y, 1) = 0\}$ and the line $(x_1 = 0)$ onto $[1 : 0 : 0]$.

Notice that in all examples, the contracted curves are rational. This is not a coincidence and follows from the fact that every birational map between smooth projective surfaces is a composition of blow-ups and blow-downs of points. In fact, the following theorem holds particularly for birational maps of the plane.

**Theorem** [3] The group $\mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}})$ is generated by $\sigma\colon (x,y) \mapsto (1/x, 1/y)$ and $\mathrm{Aut}(\mathbb{P}^2)$.

The $\mathbb{C}$ is added on purpose: the above theorem holds over any algebraically closed field (of any characteristic), and it is false over non-closed fields. In fact, if in Example 2, the polynomial $P$ is chosen to be irreducible of degree at least two, then $\tau_p$ contracts an irrational curve and cannot be contained in the group generated by $(x,y) \mapsto (1/x, 1/y)$ and $\mathrm{Aut}(\mathbb{P}^2)$.

**Corollary** $\mathrm{Bir}(\mathbb{P}^2_{\mathbb{C}})$ is generated by involutions.

This corollary holds over any perfect field [6].

**Big disappointement / opportunity:** We do not know any *reasonable* generating set of $\mathrm{Bir}(\mathbb{P}^{\mathrm{n}})$ for $n \geq 3$.

**Theorem** [7] For $n \geq 4$, there is a surjective group homomorphism $\mathrm{Bir}(\mathbb{P}^{\mathrm{n}}) \to \mathbb{Z}$. In particular, $\mathrm{Bir}(\mathbb{P}^{\mathrm{n}})$ is not generated by elements of finite order.

We do not know what happens for $n = 3$.

**Problems:**

(1) Let $\omega$ be a meromorphic contact structure on $\mathbb{P}^n$. Study the group $\mathrm{Bir(X)}_{\mathrm{c}(\omega)}$ of birational maps of $X$ preserving the contact structure $\omega$. This is done in [4] for the case $n = 3$.

(2) Let $\mathcal{F}$ be a foliation on $\mathbb{P}^n$. Study the group $\mathrm{Bir}(\mathcal{F})$ of birational maps of $\mathbb{P}^n$ preserving $\mathcal{F}$. This is achieved in [2] for the case $n = 2$.

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## Presentation of Marco Brunella's paper: On Kähler surfaces with semipositive Ricci curvature

Yifan Chen

Let $\mathbf{p} := p_1, p_2, \ldots, p_9$ be nine points passing through a nonsingular cubic curve $C_0$ inside the complex projective plane $\mathbb{P}^2$. Blowing up those 9 points we get a surface $S$. The anticanonical bundle of $S$ is given by $-K_S = -\pi^*(K_{\mathbb{P}^2}) - \sum_i E_i = \pi^*(C_0) - \sum_i E_i$, which is the strict transform of $C_0$, denoted by $C$. Since $C$ has zero self-intersection and by Hodge index theorem we know that $-K_S$ is nef.

In algebraic geometry, the well known Abundance conjecture says that if the canonical bundle $K_X$ of a compact Kähler manifold $X$ is nef, then it is semiample. A weaker notion of semiample in analytic point of view is semipositive, i.e., it admits a hermitian metric with semipositive curvature form. One can also ask same question for anticanonical bundle $-K_X$, i.e. if the canonical bundle of a compact Kähler manifold $X$ is nef, then it is semiample or semipositive in a weaker sense. However, this is not true as the example shown in [2] and they adjust this question by adding the condition of rationally connectedness of $X$. As a special case, [2] asked the following question:

**Question:** Does $S$ always has a semipositive anticanonical bundle?

There are three known cases with affirmative answer depending on the position of those 9 points $\mathbf{p}$ in $\mathbb{P}^2$. Let $N$ be the normal bundle of $C$ inside $S$, which is $\pi^*(O_{C_0}(3H - \sum p_i))$ and a flat line bundle in $\mathrm{Pic}^0(C) \simeq \mathbb{T}^2$.

(1) *Elliptic fibration*: this is the case written in [2]. When $N$ is a torsion point in $\mathrm{Pic}^0(C) \simeq \mathbb{T}^2$ with order $m$, there exists a degree $3m$ polynomial $R_m$ vanishing at order $m$ on $\mathbf{p}$. Thus, $S$ admits an elliptic fibration $\phi_m$ over $\mathbb{P}^1$ and $C$ is a multiple fiber of multiplicity $m$, so $-mK_S = mC = \phi^*(O(\mathbb{P}^1))$ is semipositive. Noticing that torsion points are dense in $\mathrm{Pic}^0(C)$, they propose that the approximation by a sequence of torsion configurations $\mathbf{p}_m$ will lead to a holomorphic foliation as the limit of elliptic fibrations $\phi_m$ with a closed leaf $C$.

(2) *Nonelliptic surface*: In the example constructed by [5], there are special choices of $\mathbf{p}$ such that $S$ is not a elliptic fibration, while locally $C$ has nonrational étale neighborhoods.

(3) *Pseudoflat neighborhood*: this case is studied in [3]. When $N$ has infinite order in $\mathbb{T}^2$ but satisfies a Diophantine approximation condition defined by [3] like *finite irrationality measure* in number theory, [3] proved that there exists a neighborhood $V$ of $C$ and a multi-valued holomorphic function $u$ on $V$ as a defining function of $C$ which descends down to a single-valued function on $V$. This $V$ is also called as a *pseudoflat neighborhood* of $C$. By a cut–off argument one can construct a smooth semipositive representative of $-K_S$.

The last pseudoflat neighborhood condition on $N$ is generic in the sense that the compliment of this set in $\mathrm{Pic}^0(C)$ has Lebesgue measure 0. Brunella proves that this is almost the only case by adding a stronger condition: the absence of compact curve in $S \setminus C$.

**Theorem 1.**[1] If $S \setminus C$ has no compact curve, then $-K_S$ is semipositive if and only if $C$ has a pseudoflat neighborhood.

From pseudoflat neighborhood of $C$ to semipositivity of $-K_S$ is again by a cut–off argument and could also work in higher dimension. By Aubin-Calabi-Yau theorem we then have a Kähler metric with semipositive Ricci curvature. In this case, the support of Ricci curvature lies in the interpolating part of the cut–off function and thus is an "annulus" around $C$. Brunella also asks about how large or small this support could be if we choose $\mathbf{p}$ properly, while, as a comparison, this is quite different from the fibration case, where Ricci curvature supports on whole $S$.

From semipositivity of $-K_S$ to a pseudoflat neighborhood heavily relies on dimension 2 and the special structure of $S$. We sketch the proof here: the smooth Hermitian metric $h$ on $-K_S$ defines a plurisubharmonic function $F$ on $S \setminus C$ growth at order $-\log|s|$, where $s$ is a defining function of $C$. The level set of $F$, diffeomorphic to $\mathbb{T}^3$, bounds domains approximating $S \setminus C$. A topological theorem by [4] gives the Levi-flatness of those 3-tori and from some cohomology argument one can constructs a pseudoflat neighborhood gluing the Levi-flat $\mathbb{T}^3$.

Moreover, Brunella also showes the following restriction of the Kähler metric with semipositive Ricci:

**Theorem 2.**[1] If $S \setminus C$ has no compact curve, then any smooth Kähler metric with semipositive Ricci curvature cannot be real analytic.

The real analytic condition extend the foliation around $C$ globally $S$, which contradicts to the nonexistence of holomorphic foliation on $S$ tangent to $C$ and is smooth along $C$.

**Open questions:** Let us end this note by summarizing open problems mentioned in Brunella's paper:

(1) Find 9 points p on a smooth elliptic curve $C_0$ such that $3H|_{C_0} - \sum p_i$ representing a point in $\mathrm{Pic}^0(C_0)$ without Diophantine approximation, prove that in this case there is no pseudoflat neighborhood around $C$ and study the semipositivity of $-K_S$ in this case.
(2) Find 9 points p such that the closure of pseudoflat neighborhood $U$ of $C$ has full support.
(3) One byproduct of the proof is that $S \setminus C$ is not stein under the assumption of semipositive $-K_S$. Hartshorne conjectures that $S \setminus C$ is never stein no matter how we choose those nine points.

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*Reporter: Yifan Chen*

# Participants

**Dr. Irina Bobrova**
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig
GERMANY

**Dr. Yifan Chen**
Department of Mathematics
University of Rome, Tor Vergata
00133 Roma
ITALY

**Prof. Dr. Eleonora Di Nezza**
Dipartimento di Matematica
Università di Roma Tor Vergata
00133 Roma
ITALY

**Prof. Dr. Enrica Floris**
Institut de Mathématiques de Toulouse
Université de Toulouse
118 route de Narbonne
31000 Toulouse
FRANCE

**Prof. Dr. Cécile Gachet**
Fakultät für Mathematik
Ruhr-Universität Bochum
44801 Bochum
GERMANY

**Prof. Dr. Ursula Hamenstädt**
Mathematisches Institut
Universität Bonn
Endenicher Allee 60
53115 Bonn
GERMANY

**Prof. Dr. Lena Ji**
Department of Mathematics
University of Illinois Urbana-Champaign
1305 West Green Street
214 Harker Hall
Urbana, IL 61801
UNITED STATES

**Romy Marie Merkel**
Mathematisches Institut
Universität Münster
Einsteinstraße 62
48149 Münster
GERMANY

**Prof. Dr. Rita Pardini**
Dipartimento di Matematica
Università di Pisa
Largo Bruno Pontecorvo, 5
56127 Pisa
ITALY

**Dr. Elvira Pérez Callejo**
Campus Universitario Miguel Delibes.
Facultad de Ciencias
Departamento de Algebra, Análisis
Matemático, Geometría y Topología
Universidad de Valladolid
P.º de Belén, 7.
47011 Valladolid
SPAIN

**Francesca Rizzo**
Institut de Mathématiques de Jussieu
Paris Rive Gauche
Université Paris Cité
Bâtiment Sophie Germain
Case 7012
75205 Paris Cedex 13
FRANCE

**Stefania Vassiliadis**
The London School of Geometry
and Number Theory
Departement of Mathematics
King's College London
Strand
London WC2R2LS
UNITED KINGDOM

**Liqingjing Wang**
Université Côte d'Azur
Parc Valrose
06108 Nice Cedex 2
FRANCE

**Prof. Dr. Susanna Zimmermann**
Department of Mathematics and
Computer Science
Universität Basel
Spiegelgasse 1
4051 Basel
SWITZERLAND