



Preface, by ENRICO ARBARELLO, FABRIZIO CATANESE, ALESSIO FIGALLI and UMBERTO ZANNIER

1. ENRICO BOMBIERI AS A MATHEMATICAL RESEARCHER

It would take a long time to describe all the magnificent mathematical achievements of Enrico Bombieri, achievements that earned him the Premio Caccioppoli in 1966, the Fields Medal in 1974, the Premio Feltrinelli in 1976, the Balzan Prize in 1980, the King Faisal International Prize in 2010, the Crafoord Prize in 2020, and many other distinguished acknowledgements.

Since this has already been done elsewhere (see the article by Umberto Zannier in: *I Premi Nobel Italiani*, Vol. I (1906–1959), Vol. II (1963–2007), SEPS, Bologna, 2015), we refrain from doing so again here.

Let us only say that Enrico has always been a remarkable problem solver. His attitude was to choose a difficult problem and to attempt every possible method, rather than selecting the method first or limiting himself to problems solvable with the theories he already mastered. In these times of strict specialization in mathematical research, he has been a guiding light, teaching that the first question one should ask about a new mathematical theory is what it is good for and how it can be applied.

Through this original approach, he has solved difficult problems in many areas, among them number theory, complex analysis, minimal surfaces and geometric measure theory, partial differential equations, algebraic geometry, group theory, arithmetic, Diophantine geometry, special functions, and ordinary differential equations.

As this volume is dedicated to his 85th birthday, we shall therefore indulge more in considerations about him as a master.

2. ENRICO BOMBIERI'S ENGAGEMENT AS A TEACHER AND EDUCATOR

While Enrico's research activity took place all around the globe, his teaching as a professor was mostly concentrated in the ten years 1968–1977 in Pisa, where he was full professor first at the University of Pisa and then, after 1974 (the year when he received the Fields Medal), at the Scuola Normale Superiore (SNS).

He obtained his first chair at the University of Cagliari in 1966, and he began in Pisa with great enthusiasm, teaching the course of *Analisi I* in November 1968, taking

permission from the Italian Air Force (Aeronautica Militare), where he was serving the then compulsory military service at CAMEN in San Piero a Grado (Centro Applicazioni Militari dell'Energia Nucleare).

He would enter the lecture room in his blue military uniform, punctually at 9:25 a.m., ten minutes after the scheduled beginning, a delay that he always tried to make up for, often forcing the late professor Salvatore Ciampa to knock energetically at the door at 10:10.

Particularly illuminating were the examples he gave after explaining the general theory; for instance, how to handle complicated limits using the Taylor development and the order of infinitesimals.

His blue uniform always remained immaculate until the end of the lecture; he was indeed extremely precise, and not only in mathematics¹, but also in games like Bridge. When he was asked whether he had played the less aristocratic game of football (soccer), his smiling reply was: “back in Montepulciano, I had an extremely precise shot.”

He arrived with a Ford Mustang, and soon a brown leather jacket replaced the uniform. His earlier car had been a Lancia Aurelia (the predecessor of the Lancia Flaminia), which he used to drive at high speed between Milano and Montepulciano.

In Milano his father was CEO of the Banca Commerciale Italiana, and from him Enrico learnt the “Bombieri Law of Finance”: *profits are on paper, losses are in cash*.

In Montepulciano, where Enrico attended high school, his family produced an excellent Vino Nobile di Montepulciano according to traditional methods in the vineyards around their villa. When choosing a good bottle for his guests, Enrico carefully inspected how good the “camicia” (the deposit on the side) was.

During oral exams he would read *Topolino* (Mickey Mouse) to release the tension of the students, laughing heartily from time to time, yet listening carefully to all the details.

As most great mathematicians, he was quite generous in judging the students, appreciating their efforts and forgiving their flaws.

Enrico had indeed risen to the rank of “Ispettore del Club di Topolino”; this was well known among students, perhaps less among mathematicians. Dorian Goldfeld (who is contributing a paper to this volume) witnessed a meeting in Bombieri’s office with him and Andrzej Schinzel, both visiting Pisa in 1975. Schinzel (an old-fashioned Polish mathematician wearing gaiters) saw a whole shelf of *Topolino* in Enrico’s library and said: “I didn’t know you were so interested in Topology.” Goldfeld still smiles

(¹) As witnessed by a small comic poem dedicated to him at the end of his military training for becoming an officer: “studia numeri complessi, è campion dei nettaccessi...”

when recalling the shock of Schinzel after opening one of the booklets and seeing a big face of Mickey Mouse.

It was rumoured that Enrico was a world-class stamp collector, and the rumour was confirmed when he sold his collection of Turkish stamps. Around 1974 he had a beautiful house with garden built in Pisa, Porta a Lucca, by Gae Aulenti, who together with Renzo Piano was one of the top Italian architects of the 20th century.

When he came to Pisa he was young, full of energy and enthusiasm, and he was working hard in several quite different fields of mathematical research: among them, beyond his beloved number theory, algebraic geometry (where he began working with Aldo Andreotti), complex variables, and geometric measure theory, i.e., hypersurfaces of minimal area, where he collaborated with Ennio De Giorgi and Enrico Giusti.

In his early years he was a real fighter in problem-solving, sometimes working even 15 hours in a row, also in fields where he was a newcomer and not yet an expert.

As was customary for him, he tried all possible ways to find the desired counterexample to the Bernstein regularity theorem in dimension at least 8, showing that the cone

$$\left\{ \sum_{i=1}^4 x_i^2 = \sum_{i=5}^8 x_i^2 \right\} \subset \mathbb{R}^8$$

has minimal 7-dimensional area.

De Giorgi discussed with him and listened to his proposals with care, pointing out gaps and difficulties, but the legend is that one weekend Ennio called him from a *Rifugio* (mountain hut) on the Alpi Apuane, where he had ascended along a trail (first degree of difficulty), telling him: “yes, I think that this time it is working!”

The paper by Bombieri–De Giorgi–Giusti was a big hit and still stands as a “Muster” (model) on the back cover of the journal *Inventiones Mathematicae*.

His lectures on analytic number theory were excellent and full of insight, yet for non-expert students, accustomed to simple patterns such as “definition followed by theorem,” the intricate interdependence of the Riemann hypothesis, the generalized Riemann hypothesis, the Lindelöf hypothesis, and so on, was hard to digest at first. Afterwards, the assistants explained that Enrico had been “born” with this problem, and that his father had promised him a Ferrari and a bottle of Cognac Lagrange of 1830 if he solved the Riemann hypothesis (after Wolfgang Pauli had advised the young boy that he seemed to be more talented for mathematics than for physics).

After one year abroad, he returned in 1974 with the Fields Medal in his pocket, and at the SNS he ran, each Monday, three seminars consecutively from 3 p.m. to 8 p.m.: first on number theory, then on varieties of minimal volume, and finally on algebraic surfaces (discussing in 1975–76 his joint work with Mumford, extending the Enriques classification to positive characteristic). Sometimes he finished even later; he was a

slow starter, but he was truly playing “live on the stage,” and always made a point of giving a complete proof on the spot, even when he had forgotten some detail at the beginning.

During the breaks he played table tennis, which had become his new passion after meeting Jean-Pierre Serre (probably at Harvard). He became very competitive in this new sport, which he continued to practice also at the Institute for Advanced Study in Princeton, N.J., where he moved in September 1977.

At the IAS he had more free time, which enabled him not only to be more engaged in research, completing several long-term projects, but also to find time for relaxation, developing hobbies such as painting (“The kite-eating tree,” portraits of friends such as Dorian Goldfeld, etc.) or working on precious stones like opals, while keeping his love for stamps. At that time he dedicated himself to ancient stamps from New Zealand, occasionally traveling to auctions at Christie’s in London to acquire them.

He continued mentoring younger researchers and collaborating with them. Some of his students back in Pisa complained that it was difficult to get him to answer their questions immediately: first they had to listen to all the details of what he was trying to do in order to solve the problems he was working on. And if the listener’s attention began to fade, Enrico’s trick was the opposite of what a typical Italian would do (talking louder or shouting): he would begin to whisper.

Even if somewhat shy, he was a merry person, with a very fine sense of humour, and he could spend hours on “Carabinieri” jokes (the Italian analogue of “Polish jokes”), such as: *Why do the cars of the Carabinieri have the word “Carabinieri” written on the side doors? Easy: to prevent them from trying to get in through the trunk!*

Even now that he has grown old, wiser than ever (though not always completely square!), he has lost neither his sense of humour nor his hair, nor his wits and good spirits. And he has certainly left a durable imprint on Italian mathematics.

The present volume is intended to give a small sample of his many diverse interests, even though number theory was always deeply rooted in his heart and the Riemann hypothesis remained his ultimate goal, even at times when possessing and driving a Ferrari had become less important to him.

Enrico, this is our modest way to acknowledge your achievements, your merits, your deeds, and your service to the Italian community, as well as the legacy you have left.

Thanks, Enrico!