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Corrigendum to "Nonsymplectic automorphisms of prime order on O'Grady's sixfolds"

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Abstract. In this short note, we correct the lattice theoretic classification in [Rev. Mat. Iberoam. 38 (2022), no. 4, 1199–1218] of the first named author, including some missing cases and explaining how to recover them.

1. Introduction

In this short note, we describe all the cases which are missing in the computation of the invariant and coinvariant sublattices associated to nonsymplectic automorphisms of prime order of hyper-Kähler manifolds of OG6 type [3]. Recall that given a hyper-Kähler manifold X of OG6 type, the second integral cohomology group of X, equipped with its Beauville–Bogomolov–Fujiki quadratic form, is isometric as a lattice to

$$\mathbf{L} := \mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}.$$

Any finite subgroup $G \subset \mathrm{O}(\mathbf{L})$ is called *effective nonsymplectic* if it arises from representing on $\mathbf{L}$ a group of nonsymplectic automorphisms of an OG6 type manifold (see Section 1.2 of [3] for more details). Using the computer program OSCAR [2], we prove the following result.

Theorem 1.1. *There are exactly* 45 *conjugacy classes of effective nonsymplectic groups* $G \subset \mathrm{O}(\mathbf{L})$ *of prime order.*

Proof. According to Proposition 3.4 in [3], it is enough to classify finite subgroups $G \subset \mathrm{O}(\mathbf{L})$ of prime order whose invariant sublattice $\mathbf{L}^G$ has signature $(1, *)$. We proceed as follows. Fix a prime number $p \in \{2, 3, 5, 7\}$ and a primitive pth root of unity ζ. In order to classify, up to conjugacy, the groups G as before, it is enough to determine representatives of conjugacy classes of isometries $g \in \mathrm{O}(\mathbf{L})$ satisfying both of the following:

(1) the invariant lattice $\mathbf{L}^g$ has signature $(1, *)$,

(2) the real quadratic space $\ker((g + g^{-1}) - (\zeta + \zeta^{-1})\,\mathrm{id})$ has signature $(2, *)$.

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In fact, condition (2) indicates that we make g (or g^{-1}) a distinguished generator of the associated cyclic group. We can thus apply[1] Algorithms 1, 2, 3 and 8 in [1], implemented in OSCAR [4] by the second named author, to obtain the results as claimed. We refer to Table 5 for the isometry classes of $\mathbf{L}_G$ and $\mathbf{L}^G$, as well as the number of conjugacy classes of groups with the same associated pairs $(\mathbf{L}^G, \mathbf{L}_G)$, up to isometry. ∎

The theorem can also be proved by first determining a complete list of possible pairs of invariant and coinvariant sublattices $(\mathbf{L}^G, \mathbf{L}_G)$ associated to effective nonsymplectic groups $G \subset \mathrm{O}(\mathbf{L})$, and applying Algorithms 2, 3 and 8 in [1] to this list to obtain the number of conjugacy classes with given invariant and coinvariant sublattices. This list of pairs $(\mathbf{L}^G, \mathbf{L}_G)$ was partially determined in [3]. In what follows, we describe the missing and erroneous cases in *loc. cit.*, and we give corrected versions of Tables 1, 2, 3, 4 and 5 in [3].

2. Corrections

We give a complete list of corrections for Tables 1, 2, 3, 4, and 5 in [3]. We refer to Sections 3 and 4 of [3] for more details on how to perform the computations needed to make such corrections, and to the notebook at

https://github.com/StevellM/Notebooks/tree/main/Corrigendum_Grossi_RMI_2022

to see how one can proceed with a computer program. For the sake of completeness, and for the reader's convenience, we reproduce in the next section Tables 1, 2, 3, 4 and 5 after corrections. In each table, the cases that are missing in [3] appear in a blue row. In Table 3 of [3], an entry (S, T) has not been detected as arising from an involution of $\mathbf{L}$: the blue trefoil "♣" in Table 3 reports this missing case. In Table 5 of [3], there is an erroneous entry: for clarity, we keep it in Table 5 but make it appear in a red row.

(1) In the second part of Table 1 in [3], the case $(\mathbf{\Lambda}_G, \mathbf{\Lambda}^G) = (\mathbf{U}^{\oplus 3} \oplus [-2], \mathbf{U} \oplus [2])$ is missing (see the new entry no. 3 of the second part of Table 1). This correction to Table 1 in [3] gives rise to:
   - the new entry no. 5 $(\mathbf{\Lambda}_G, S) = (\mathbf{U}^{\oplus 3} \oplus [-2], \mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1))$ in Table 2,
   - the new entry no. 4 $(S, T) = (\mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1), [4] \oplus [-2])$ in Table 3, and
   - the new entries no. 2 in Table 4 and in the second part of Table 5.

   Note that the lattice $S = \mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1)$ is already contained in Table 2 of [3], but it is not reported in Table 3 of [3].

(2) In Table 2 of [3], the case $(\mathbf{\Lambda}_G, S) = (\mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}, \mathbf{U}^{\oplus 2} \oplus \mathbf{A}_3(-1))$ is missing (see the new entry no. 2 in Table 2). This correction gives rise to:
   - the new entry no. 2 $(S, T) = (\mathbf{U}^{\oplus 2} \oplus \mathbf{A}_3(-1), [4])$ in Table 3, and
   - the new entries no. 1 in Table 4 and in the second part of Table 5.

[1] The use of Algorithm 8 in [1] is necessary for some technical parts of Brandhorst–Hofmann machinery; namely to compute explicit generators for the image of the map $\mathrm{O}(L, f) \to \mathrm{O}(L^\sharp, f^\sharp)$, given an even indefinite lattice L of rank larger than 2 and a finite order isometry $f \in \mathrm{O}(L)$ (see Section 2.1 of [3] for the notation).

(3) In Table 3 of [3], a possible orthogonal complement to $S = [2]^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [-4]$ in $\mathbf{L}$ is given by the entry no. 3. However, there is a missing possibility, given by $T = \mathbf{U}(2) \oplus [-4]$ (see the new entry no. 6 in Table 3).

(4) In Table 3 of [3], the entries no. 9 and 10 are duplicates of the entry no. 4. This can be seen by remarking the following isometry of lattices: $[2] \oplus [-4] \cong [-2] \oplus [4]$. These two entries are therefore removed in Table 3, and the new entry no. 7 has been decorated with a trefoil “♣”, similarly to the entry no. 10 in Table 3 of [3].

(5) In Table 3 of [3], the entry no. 14 is missing the trefoil “♣” in the last column. Indeed, there exists an involution of $\mathbf{L}$ with coinvariant and invariant sublattices respectively given by $[2] \oplus [-2]^{\oplus 2} \oplus [4]$ and $[2] \oplus [-2]^{\oplus 2} \oplus [-4]$ (see the new entry no. 15 in Table 3). This correction gives rise to the new entries no. 6 in Table 4 and in the second part of Table 5.

(6) In Table 3 of [3], the entries no. 15 and 16 are duplicates of the entries no. 14 and 13 respectively. Again, this can be checked using the identification of lattices $[2] \oplus [-4] \cong [-2] \oplus [4]$. These two entries are therefore removed in Table 3.

(7) In Table 3 of [3], the entries no. 19 and 20 are duplicates of the entry no. 17 and 18 respectively. We use, once again, the identification $[2] \oplus [-4] \cong [-2] \oplus [4]$ to detect those duplicates. These two entries are therefore removed in Table 3, and the new entry no. 17 has been decorated with a trefoil “♣”, similarly to the entry no. 20 in Table 3 of [3]

(8) In Table 3 of [3], possible orthogonal complements to $S = \mathbf{U}(2) \oplus [4]$ in $\mathbf{L}$ are given by the entries no. 21 and 22. However, there is a missing possibility, given by $T = \mathbf{U}(2) \oplus \mathbf{A}_3(-1)$ (see the new entry no. 20 in Table 3). This correction gives rise to the new entries no. 8 in Table 4 and in the second part of Table 5.

(9) In Table 3 of [3], possible orthogonal complements to $S = [2]^{\oplus 2} \oplus [-4]$ in $\mathbf{L}$ are given by the entries no. 23 and 24. However, there are two missing possibilities, given by $T = \mathbf{U} \oplus \mathbf{A}_3(-1)$ and $T = \mathbf{U}(2) \oplus \mathbf{A}_3(-1)$ (see the new respective entries no. 23 and 24 in Table 3).

(10) In Table 3 of [3], a possible orthogonal complement to $S = [2] \oplus [4]$ in $\mathbf{L}$ is given by the entry no. 27. However, there is a missing possibility, given by $T = \mathbf{U} \oplus \mathbf{A}_3(-1) \oplus [-2]$ (see the new entry no. 28 in Table 3). This correction gives rise to the new entries no. 11 in Table 4 and in the second part of Table 5.

(11) In the first part of Table 5 in [3], the case $(\mathbf{L}_G, \mathbf{L}^G) = (\mathbf{U} \oplus [2] \oplus [-2]^{\oplus 3}, \mathbf{U}(2))$ is missing (see the new entry no. 3 in the first part of Table 5). Note that the lattice $\mathbf{L}_G = \mathbf{U} \oplus [2] \oplus [-2]^{\oplus 3}$ is already contained in the first part of Table 5 in [3], but the possible orthogonal complement given by $\mathbf{U}(2)$ is missing.

(12) In the first part of Table 5 in [3], the entry no. 24 is not correct (see the colored entry no. 25 in the first part of Table 5). Indeed, there is no primitive extension $([2]^{\oplus 2}) \oplus (\mathbf{U}(2) \oplus \mathbf{D}_4(-1)) \subseteq \mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}$.

3. Corrected tables

Table 1. Pairs $(\mathbf{\Lambda}^G, \mathbf{\Lambda}_G)$ for $G \subset \mathrm{O}(\mathbf{\Lambda})$ of prime order $p = 2$ and $\mathrm{sgn}(\mathbf{\Lambda}_G) = (2, \mathrm{rk}(\mathbf{\Lambda}_G) - 2)$, or $p = 2$ and $\mathrm{sgn}(\mathbf{\Lambda}_G) = (3, \mathrm{rk}(\mathbf{\Lambda}_G) - 3)$, or $p \in \{3, 5, 7\}$ and $\mathrm{sgn}(\mathbf{\Lambda}_G) = (2, \mathrm{rk}(\mathbf{\Lambda}_G) - 2)$.

No.	$\lvert G\rvert$	$\mathbf{\Lambda}_G$	$\mathbf{\Lambda}^G$	$\mathrm{sgn}(\mathbf{\Lambda}_G)$	a	δ
1	2	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 3}$	$[2]^{\oplus 3}$	$(2,5)$	3	1
2	2	$\mathbf{U} \oplus [-2]^{\oplus 3} \oplus [2]$	$[2]^{\oplus 3} \oplus [-2]$	$(2,4)$	4	1
3	2	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2}$	$\mathbf{U} \oplus [2]^{\oplus 2}$	$(2,4)$	2	1
4	2	$[2]^{\oplus 2} \oplus [-2]^{\oplus 3}$	$[-2]^{\oplus 2} \oplus [2]^{\oplus 3}$	$(2,3)$	5	1
5	2	$\mathbf{U} \oplus [-2]^{\oplus 2} \oplus [2]$	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]$	$(2,3)$	3	1
6	2	$\mathbf{U}^{\oplus 2} \oplus [-2]$	$\mathbf{U}^{\oplus 2} \oplus [2]$	$(2,3)$	1	1
7	2	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$(2,2)$	4	1
8	2	$\mathbf{U}(2)^{\oplus 2}$	$\mathbf{U} \oplus \mathbf{U}(2)^{\oplus 2}$	$(2,2)$	4	0
9	2	$\mathbf{U} \oplus [2] \oplus [-2]$	$\mathbf{U}^{\oplus 2} \oplus [2] \oplus [-2]$	$(2,2)$	2	1
10	2	$\mathbf{U} \oplus \mathbf{U}(2)$	$\mathbf{U}^{\oplus 2} \oplus \mathbf{U}(2)$	$(2,2)$	2	0
11	2	$\mathbf{U}^{\oplus 2}$	$\mathbf{U}^{\oplus 3}$	$(2,2)$	0	0
12	2	$[2]^{\oplus 2} \oplus [-2]$	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [2]$	$(2,1)$	3	1
13	2	$\mathbf{U} \oplus [2]$	$\mathbf{U}^{\oplus 3} \oplus [-2]$	$(2,1)$	1	1
14	2	$[2]^{\oplus 2}$	$\mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}$	$(2,0)$	2	1
1	2	$\mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}$	$[2]^{\oplus 2}$	$(3,5)$	2	1
2	2	$\mathbf{U}^{\oplus 2} \oplus [2] \oplus [-2]^{\oplus 2}$	$[2]^{\oplus 2} \oplus [-2]$	$(3,4)$	3	1
3	2	$\mathbf{U}^{\oplus 3} \oplus [-2]$	$\mathbf{U} \oplus [2]$	$(3,4)$	1	1
4	2	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$(3,3)$	4	1
5	2	$\mathbf{U} \oplus \mathbf{U}(2)^{\oplus 2}$	$\mathbf{U}(2)^{\oplus 2}$	$(3,3)$	4	0
6	2	$\mathbf{U}^{\oplus 2} \oplus [2] \oplus [-2]$	$\mathbf{U} \oplus [2] \oplus [-2]$	$(3,3)$	2	1
7	2	$\mathbf{U}^{\oplus 2} \oplus \mathbf{U}(2)$	$\mathbf{U} \oplus \mathbf{U}(2)$	$(3,3)$	2	0
8	2	$\mathbf{U}^{\oplus 3}$	$\mathbf{U}^{\oplus 2}$	$(3,3)$	0	0
9	2	$[-2]^{\oplus 2} \oplus [2]^{\oplus 3}$	$[2]^{\oplus 2} \oplus [-2]^{\oplus 3}$	$(3,2)$	5	1
10	2	$\mathbf{U} \oplus [-2] \oplus [2]^{\oplus 2}$	$\mathbf{U} \oplus [2] \oplus [-2]^{\oplus 2}$	$(3,2)$	3	1
11	2	$\mathbf{U}^{\oplus 2} \oplus [2]$	$\mathbf{U}^{\oplus 2} \oplus [-2]$	$(3,2)$	1	1
12	2	$\mathbf{U}(2) \oplus [2]^{\oplus 2}$	$\mathbf{U} \oplus \mathbf{U}(2) \oplus [-2]^{\oplus 2}$	$(3,1)$	4	1
13	2	$\mathbf{U} \oplus [2]^{\oplus 2}$	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2}$	$(3,1)$	2	1
14	2	$[2]^{\oplus 3}$	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 3}$	$(3,0)$	3	1
1	3	$\mathbf{U}^{\oplus 2} \oplus \mathbf{A}_2(-1)$	$\mathbf{U} \oplus \mathbf{A}_2$	$(2,4)$	1	–
2	3	$\mathbf{U} \oplus \mathbf{U}(3) \oplus \mathbf{A}_2(-1)$	$\mathbf{U}(3) \oplus \mathbf{A}_2$	$(2,4)$	3	–
3	3	$\mathbf{U}^{\oplus 2}$	$\mathbf{U}^{\oplus 3}$	$(2,2)$	0	–
4	3	$\mathbf{U} \oplus \mathbf{U}(3)$	$\mathbf{U}^{\oplus 2} \oplus \mathbf{U}(3)$	$(2,2)$	2	–
5	3	$\mathbf{A}_2$	$\mathbf{U}^{\oplus 3} \oplus \mathbf{A}_2(-1)$	$(2,0)$	1	–
1	5	$\mathbf{U} \oplus \mathbf{H}_5$	$\mathbf{U}^{\oplus 2} \oplus \mathbf{H}_5$	$(2,2)$	1	–
1	7	$\mathbf{U}^{\oplus 2} \oplus \mathbf{K}_7$	$\mathbf{U} \oplus \mathbf{K}_7(-1)$	$(2,4)$	1	–

Table 2. Orthogonal complements of $[4] \hookrightarrow \mathbf{\Lambda}_G$.

No.	$\mathbf{\Lambda}_G$	$S = [4]^{\perp_{\Lambda_G}}$
1	$\mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}$	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [-4]$
2	$\mathbf{U}^{\oplus 3} \oplus [-2]^{\oplus 2}$	$\mathbf{U}^{\oplus 2} \oplus \mathbf{A}_3(-1)$
3	$\mathbf{U}^{\oplus 2} \oplus [2] \oplus [-2]^{\oplus 2}$	$\mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1)$
4	$\mathbf{U}^{\oplus 2} \oplus [2] \oplus [-2]^{\oplus 2}$	$\mathbf{U} \oplus [2] \oplus [-2]^{\oplus 2} \oplus [-4]$
5	$\mathbf{U}^{\oplus 3} \oplus [-2]$	$\mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1)$
6	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [-4]$
7	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$\mathbf{U} \oplus [-2]^{\oplus 2} \oplus [4]$
8	$\mathbf{U} \oplus \mathbf{U}(2)^{\oplus 2}$	$\mathbf{U}(2)^{\oplus 2} \oplus [-4]$
9	$\mathbf{U} \oplus \mathbf{U}(2)^{\oplus 2}$	$\mathbf{U} \oplus \mathbf{U}(2) \oplus [-4]$
10	$\mathbf{U}^{\oplus 2} \oplus [2] \oplus [-2]$	$\mathbf{U} \oplus [2] \oplus [-2] \oplus [-4]$
11	$\mathbf{U}^{\oplus 2} \oplus \mathbf{U}(2)$	$\mathbf{U} \oplus \mathbf{U}(2) \oplus [-4]$
12	$\mathbf{U}^{\oplus 2} \oplus \mathbf{U}(2)$	$\mathbf{U}^{\oplus 2} \oplus [-4]$
13	$\mathbf{U}^{\oplus 3}$	$\mathbf{U}^{\oplus 2} \oplus [-4]$
14	$[2]^{\oplus 3} \oplus [-2]^{\oplus 2}$	$[2] \oplus [-2]^{\oplus 2} \oplus [4]$
15	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]$	$[2]^{\oplus 2} \oplus [-2] \oplus [-4]$
16	$\mathbf{U} \oplus [2]^{\oplus 2} \oplus [-2]$	$\mathbf{U} \oplus [-2] \oplus [4]$
17	$\mathbf{U}^{\oplus 2} \oplus [2]$	$\mathbf{U} \oplus [2] \oplus [-4]$
18	$\mathbf{U}(2) \oplus [2]^{\oplus 2}$	$\mathbf{U}(2) \oplus [4]$
19	$\mathbf{U}(2) \oplus [2]^{\oplus 2}$	$[2]^{\oplus 2} \oplus [-4]$
20	$\mathbf{U} \oplus [2]^{\oplus 2}$	$[2]^{\oplus 2} \oplus [-4]$
21	$\mathbf{U} \oplus [2]^{\oplus 2}$	$\mathbf{U} \oplus [4]$
22	$[2]^{\oplus 3}$	$[2] \oplus [4]$

Table 3. Orthogonal complements of $S \hookrightarrow \mathbf{L}$.

No.	S	T	♣
1	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [-4]$	$[4]$	–
2	$\mathbf{U}^{\oplus 2} \oplus \mathbf{A}_3(-1)$	$[4]$	♣
3	$\mathbf{U} \oplus [2] \oplus [-2]^{\oplus 2} \oplus [-4]$	$[4] \oplus [-2]$	–
4	$\mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1)$	$[4] \oplus [-2]$	♣
5	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [-4]$	$[-2]^{\oplus 2} \oplus [4]$	–
6	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2} \oplus [-4]$	$\mathbf{U}(2) \oplus [-4]$	–

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Table 3, follows from previous page

No.	S	T	♣
7	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[4]$	$[2]\oplus[-2]\oplus[-4]$	♣
8	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[4]$	$\mathbf{U}\oplus[-4]$	–
9	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[4]$	$\mathbf{U}(2)\oplus[-4]$	–
10	$\mathbf{U}\oplus\mathbf{U}(2)\oplus[-4]$	$\mathbf{U}(2)\oplus[-4]$	♣
11	$\mathbf{U}\oplus\mathbf{U}(2)\oplus[-4]$	$[-2]^{\oplus 2}\oplus[4]$	–
12	$\mathbf{U}^{\oplus 2}\oplus[-4]$	$[-2]^{\oplus 2}\oplus[4]$	–
13	$\mathbf{U}^{\oplus 2}\oplus[-4]$	$\mathbf{U}\oplus[-4]$	♣
14	$[2]\oplus[-2]^{\oplus 2}\oplus[4]$	$\mathbf{U}\oplus[-2]\oplus[-4]$	–
15	$[2]\oplus[-2]^{\oplus 2}\oplus[4]$	$[2]\oplus[-2]^{\oplus 2}\oplus[-4]$	♣
16	$\mathbf{U}\oplus[-2]\oplus[4]$	$[2]\oplus[-2]^{\oplus 2}\oplus[-4]$	–
17	$\mathbf{U}\oplus[-2]\oplus[4]$	$\mathbf{U}\oplus[-2]\oplus[-4]$	♣
18	$\mathbf{U}(2)\oplus[4]$	$\mathbf{U}(2)\oplus[-2]^{\oplus 2}\oplus[-4]$	–
19	$\mathbf{U}(2)\oplus[4]$	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[-4]$	–
20	$\mathbf{U}(2)\oplus[4]$	$\mathbf{U}(2)\oplus\mathbf{A}_3(-1)$	♣
21	$[2]^{\oplus 2}\oplus[-4]$	$[-2]^{\oplus 4}\oplus[4]$	–
22	$[2]^{\oplus 2}\oplus[-4]$	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[-4]$	♣
23	$[2]^{\oplus 2}\oplus[-4]$	$\mathbf{U}\oplus\mathbf{A}_3(-1)$	–
24	$[2]^{\oplus 2}\oplus[-4]$	$\mathbf{U}(2)\oplus\mathbf{A}_3(-1)$	–
25	$\mathbf{U}\oplus[4]$	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[-4]$	–
26	$\mathbf{U}\oplus[4]$	$\mathbf{U}\oplus\mathbf{A}_3(-1)$	♣
27	$[2]\oplus[4]$	$\mathbf{U}\oplus[-2]^{\oplus 3}\oplus[-4]$	–
28	$[2]\oplus[4]$	$\mathbf{U}\oplus\mathbf{A}_3(-1)\oplus[-2]$	♣

Table 4. Invariant and coinvariant sublattices of nonsymplectic groups $G\subset\mathrm{O}(\mathbf{L})$ of order 2 and $|G^\sharp|=2$ on manifolds of OG6 type.

No.	$\mathbf{L}_G$	$\mathbf{L}^G$	example
1	$\mathbf{U}^{\oplus 2}\oplus\mathbf{A}_3(-1)$	$[4]$	$\begin{pmatrix} -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&0&0&1&2&1&1\\ 0&0&0&0&2&1&1&1\\ 0&0&0&0&-2&-2&-2&-1\\ 0&0&0&0&-2&-2&-1&-2 \end{pmatrix}$
2	$\mathbf{U}\oplus[2]\oplus\mathbf{A}_3(-1)$	$[4]\oplus[-2]$	$\begin{pmatrix} -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&0&1&2&1&1\\ 0&0&0&0&2&1&1&1\\ 0&0&0&0&-2&-2&-2&-1\\ 0&0&0&0&-2&-2&-1&-2 \end{pmatrix}$

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Table 4, follows from previous page

No.	$\mathbf{L}_G$	$\mathbf{L}^G$	example
3	$\mathbf{U}\oplus\mathbf{U}(2)\oplus[-4]$	$\mathbf{U}(2)\oplus[-4]$	$\begin{pmatrix}-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$
4	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[4]$	$[2]\oplus[-2]\oplus[-4]$	$\begin{pmatrix}-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$
5	$\mathbf{U}^{\oplus 2}\oplus[-4]$	$\mathbf{U}\oplus[-4]$	$\begin{pmatrix}-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$
6	$[2]\oplus[-2]^{\oplus 2}\oplus[4]$	$[2]\oplus[-2]^{\oplus 2}\oplus[-4]$	$\begin{pmatrix}0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$
7	$\mathbf{U}\oplus[-2]\oplus[4]$	$\mathbf{U}\oplus[-2]\oplus[-4]$	$\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$
8	$\mathbf{U}(2)\oplus[4]$	$\mathbf{U}(2)\oplus\mathbf{A}_3(-1)$	$\begin{pmatrix}0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&0&0&-1&-2&-1&-1\\0&0&0&0&-2&-1&-1&-1\\0&0&0&0&2&2&2&1\\0&0&0&0&2&2&1&2\end{pmatrix}$
9	$[2]^{\oplus 2}\oplus[-4]$	$\mathbf{U}\oplus[-2]^{\oplus 2}\oplus[-4]$	$\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$

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Table 4, follows from previous page

No.	$\mathbf{L}_G$	$\mathbf{L}^G$	example
10	$\mathbf{U} \oplus [4]$	$\mathbf{U} \oplus \mathbf{A}_3(-1)$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -2 & -1 & -1 \\ 0 & 0 & 0 & 0 & -2 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 \end{pmatrix}$
11	$[2] \oplus [4]$	$\mathbf{U} \oplus \mathbf{A}_3(-1) \oplus [-2]$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -2 & -1 & -1 \\ 0 & 0 & 0 & 0 & -2 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 \end{pmatrix}$

Table 5. Invariant and coinvariant sublattices of effective nonsymplectic groups $G \subset \mathrm{O}(\mathbf{L})$ on manifolds of OG6 type.

No.	$\|G\|$	$\|G^\sharp\|$	$\mathbf{L}_G$	$\mathbf{L}^G$	number of classes
1	2	1	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 3}$	$[2]$	1
2	2	1	$\mathbf{U} \oplus [2] \oplus [-2]^{\oplus 3}$	$[2] \oplus [-2]$	1
3	2	1	$\mathbf{U} \oplus [2] \oplus [-2]^{\oplus 3}$	$\mathbf{U}(2)$	1
4	2	1	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2}$	$\mathbf{U}$	1
5	2	1	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2}$	$[2] \oplus [-2]$	1
6	2	1	$\mathbf{U}^{\oplus 2} \oplus [-2]^{\oplus 2}$	$\mathbf{U}(2)$	1
7	2	1	$[2]^{\oplus 2} \oplus [-2]^{\oplus 3}$	$[2] \oplus [-2]^{\oplus 2}$	1
8	2	1	$\mathbf{U} \oplus [-2]^{\oplus 2} \oplus [2]$	$\mathbf{U} \oplus [-2]$	1
9	2	1	$\mathbf{U} \oplus [-2]^{\oplus 2} \oplus [2]$	$[2] \oplus [-2]^{\oplus 2}$	3
10	2	1	$\mathbf{U}^{\oplus 2} \oplus [-2]$	$[-2]^{\oplus 2} \oplus [2]$	1
11	2	1	$\mathbf{U}^{\oplus 2} \oplus [-2]$	$\mathbf{U} \oplus [-2]$	1
12	2	1	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$\mathbf{U} \oplus [-2]^{\oplus 2}$	1
13	2	1	$[2]^{\oplus 2} \oplus [-2]^{\oplus 2}$	$[-2]^{\oplus 3} \oplus [2]$	2
14	2	1	$\mathbf{U}(2)^{\oplus 2}$	$\mathbf{U}(2) \oplus [-2]^{\oplus 2}$	1
15	2	1	$\mathbf{U} \oplus [2] \oplus [-2]$	$[2] \oplus [-2]^{\oplus 3}$	1
16	2	1	$\mathbf{U} \oplus [2] \oplus [-2]$	$\mathbf{U} \oplus [-2]^{\oplus 2}$	1
17	2	1	$\mathbf{U} \oplus \mathbf{U}(2)$	$\mathbf{U}(2) \oplus [-2]^{\oplus 2}$	1
18	2	1	$\mathbf{U} \oplus \mathbf{U}(2)$	$\mathbf{U} \oplus [-2]^{\oplus 2}$	1
19	2	1	$\mathbf{U}^{\oplus 2}$	$\mathbf{U} \oplus [-2]^{\oplus 2}$	1
20	2	1	$[2]^{\oplus 2} \oplus [-2]$	$[-2]^{\oplus 4} \oplus [2]$	1
21	2	1	$[2]^{\oplus 2} \oplus [-2]$	$\mathbf{U} \oplus [-2]^{\oplus 3}$	2
22	2	1	$\mathbf{U} \oplus [2]$	$\mathbf{U} \oplus [-2]^{\oplus 3}$	1
23	2	1	$[2]^{\oplus 2}$	$\mathbf{U} \oplus [-2]^{\oplus 4}$	1

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Table 5, follows from previous page

No.	$\lvert G\rvert$	$\lvert G^\sharp\rvert$	$\mathbf{L}_G$	$\mathbf{L}^G$	number of classes
24	2	1	$[2]^{\oplus 2}$	$\mathbf{U} \oplus \mathbf{D}_4(-1)$	1
25	2	1	$[2]^{\oplus 2}$	$\mathbf{U}(2) \oplus \mathbf{D}_4(-1)$	0
1	2	2	$\mathbf{U}^{\oplus 2} \oplus \mathbf{A}_3(-1)$	$[4]$	1
2	2	2	$\mathbf{U} \oplus [2] \oplus \mathbf{A}_3(-1)$	$[4] \oplus [-2]$	1
3	2	2	$\mathbf{U} \oplus \mathbf{U}(2) \oplus [-4]$	$\mathbf{U}(2) \oplus [-4]$	1
4	2	2	$\mathbf{U} \oplus [2] \oplus [-2] \oplus [-4]$	$[2] \oplus [-2] \oplus [-4]$	1
5	2	2	$\mathbf{U}^{\oplus 2} \oplus [-4]$	$\mathbf{U} \oplus [-4]$	1
6	2	2	$[2] \oplus [-2]^{\oplus 2} \oplus [4]$	$[2] \oplus [-2]^{\oplus 2} \oplus [-4]$	1
7	2	2	$\mathbf{U} \oplus [2] \oplus [-4]$	$\mathbf{U} \oplus [-2] \oplus [-4]$	1
8	2	2	$\mathbf{U}(2) \oplus [4]$	$\mathbf{U}(2) \oplus A_3(-1)$	1
9	2	2	$[2]^{\oplus 2} \oplus [-4]$	$\mathbf{U} \oplus [-2]^{\oplus 2} \oplus [-4]$	1
10	2	2	$\mathbf{U} \oplus [4]$	$\mathbf{U} \oplus \mathbf{A}_3(-1)$	1
11	2	2	$[2] \oplus [4]$	$\mathbf{U} \oplus \mathbf{A}_3(-1) \oplus [-2]$	1
1	3	1	$\mathbf{U}^{\oplus 2} \oplus \mathbf{A}_2(-1)$	$[-2] \oplus [6]$	1
2	3	1	$\mathbf{U}^{\oplus 2}$	$\mathbf{U} \oplus [-2]^{\oplus 2}$	1
3	3	1	$\mathbf{U} \oplus \mathbf{U}(3)$	$\mathbf{U}(3) \oplus [-2]^{\oplus 2}$	1
4	3	1	$\mathbf{A}_2$	$\mathbf{U} \oplus \mathbf{A}_2(-1) \oplus [-2]^{\oplus 2}$	1
1	5	1	$\mathbf{U} \oplus \mathbf{H}_5$	$[-2] \oplus [-10] \oplus \mathbf{U}$	1
1	7	1	$\mathbf{U}^{\oplus 2} \oplus \mathbf{K}_7$	$[-2] \oplus [14]$	1

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