J. Eur. Math. Soc. 28, 4515–4518 (2026) DOI 10.4171/JEMS/1802

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Corrigendum to "Free Banach lattices"

Received October 8, 2025; revised January 12, 2026

Abstract. Theorem 2.8 from [J. Eur. Math. Soc. 28, 4387–4514 (2026)] regarding the order density of $\mathrm{FVL}[E]$ in $\mathrm{FBL}^{(p)}[E]$ only holds when the underlying Banach space E is finite-dimensional. We correct this issue and re-evaluate the minor consequences on the paper.

Keywords: free Banach lattice, p-convex Banach lattice, AM-space, p-summing map, lattice homomorphism.

This corrigendum freely uses the terminology from [3]. Our main purpose is to provide a correction to [3, Theorem 2.8] stating that $\mathrm{FVL}[E]$ is always order dense in $\mathrm{FBL}^{(p)}[E]$. The argument given there is only valid when E is finite-dimensional. In fact, we will show below that the result only holds in that case.

Here is the corrected version of the theorem.

Theorem 1. *For a Banach space E and any $1 \leq p \leq \infty$, $\mathrm{FVL}[E]$ is order dense in $\mathrm{FBL}^{(p)}[E]$ if and only if E is finite-dimensional.*

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Mathematics Subject Classification 2020: 46B42 (primary); 46A22, 46A40, 46B80, 47B60 (secondary).

Proof. Suppose first that E is finite-dimensional. In this case, $\mathrm{FBL}^{(p)}[E]$ is lattice isomorphic to $C(S_{E^*})$. Take a non-zero $f \in \mathrm{FBL}^{(p)}[E]_+$; our goal is to show the existence of $h \in \mathrm{FVL}[E] \setminus \{0\}$ with $0 \leq h \leq f$.

Since $f \neq 0$, there exists $x_0^* \in S_{E^*}$ with $f(x_0^*) > 0$. Hence there exists $\varepsilon > 0$ and an open neighborhood U of x_0^* in S_{E^*} such that $f \geq \varepsilon$ on U. By [3, Lemma 2.9], for any $y_0^* \in U$, there exists $g \in \mathrm{FVL}[E]$ such that $g(y_0^*) > 0$, g vanishes outside U and $g \leq \varepsilon$ on U. Hence, $0 < g \leq f$ and $\mathrm{FVL}[E]$ is order dense (alternatively, one may think of $\mathrm{FVL}[E]$ as the order dense sublattice of $C(S_{E^*})$ generated by the piecewise linear functions).

Now suppose that E is infinite-dimensional. Let E_0 be a separable infinite-dimensional subspace of E, and let $(e_k)_{k=0}^\infty$ be a countable dense subset of the unit sphere of E_0. Since $|\delta_{e_k}|$ lies in the unit ball of $\mathrm{FBL}^{(p)}[E]$, the series $\sum_{k=1}^\infty 2^{-k-1}|\delta_{e_k}|$ converges in $\mathrm{FBL}^{(p)}[E]$. Hence, we can take

$$f = \Big(|\delta_{e_0}| - \sum_{k=1}^{\infty} 2^{-k-1}|\delta_{e_k}|\Big)_+,$$

which clearly belongs to $\mathrm{FBL}^{(p)}[E]$. Also, there exists $e^* \in E^*$ such that $\|e^*\| = 1$ and $\langle e^*, e_0\rangle > 3/4$, hence

$$f(e^*) > \frac{3}{4} - \sum_{k=1}^{\infty} 2^{-k-1} = \frac{1}{4},$$

which shows that $f > 0$.

Now, let $g \in \mathrm{FVL}[E]$ with $g > 0$. Write g as a lattice expression in $(\delta_{y_j})_{j=1}^N$ for certain $(y_j)_{j=1}^N \subset E$. Find $x^* \in E^*$ with $g(x^*) = \alpha > 0$. Note that $e_0^\perp \cap (\bigcap_{j=1}^N y_j^\perp)$ has finite codimension in E^*. Also, since $E^*/E_0^\perp$ is isomorphic to E_0^*, we see that $E^*/E_0^\perp$ is infinite-dimensional. Therefore, we can pick a non-zero $y^* \in [e_0^\perp \cap (\bigcap_{j=1}^N y_j^\perp)] \setminus E_0^\perp \subset E^*$.

Then for any $t > 0$, $g(tx^* + y^*) = g(tx^*) = t\alpha > 0$. On the other hand, there exists k such that $\langle y^*, e_k\rangle > 0$. Then

$$\begin{aligned} f(tx^* + y^*) &\leq \big(|\langle tx^* + y^*, e_0\rangle| - 2^{-k-1}|\langle tx^* + y^*, e_k\rangle|\big)_+ \\ &\leq \big(t(|\langle x^*, e_0\rangle| + 2^{-k-1}|\langle x^*, e_k\rangle|) - 2^{-k-1}|\langle y^*, e_k\rangle|\big)_+, \end{aligned}$$

which vanishes for t small enough. Hence, $g \not\leq f$ and since this holds for arbitrary $g \in \mathrm{FVL}[E]$ with $g > 0$ we conclude that $\mathrm{FVL}[E]$ cannot be order dense. ■

Remark 2. The gap in the proof of [3, Theorem 2.8] seems to be in the last paragraph of the proof, where it is shown that $h(x^*) < f(x^*)$ for x^* in the intersection of a certain weak* open set U with the unit ball B_{E^*}. By positive homogeneity the same inequality holds in the cone generated by this intersection. It is claimed that this inequality holds in the cone generated by U; however, this need not coincide with the cone generated by the intersection $U \cap B_{E^*}$.

In what follows we check the validity of those results from [3] in whose proof [3, Theorem 2.8] was playing a role. These correspond precisely to Corollary 2.10, Proposition 3.2 (1) and Theorem 3.4.

Recall that [3, Corollary 2.10] claims that for every $1 \le p \le \infty$, every disjoint collection of elements of $\mathrm{FBL}^{(p)}[E]$ is at most countable. This fact still holds as it was already proved in [1].

Concerning [3, Proposition 3.2 (1)], the original proof can be adapted replacing the role of the weak* topology with the *bounded weak* topology*, but we also provide an alternative proof:

Proposition 3. *Let $T : F \to E$ be a bounded linear operator and let $\overline{T} : \mathrm{FBL}^{(p)}[F] \to \mathrm{FBL}^{(p)}[E]$ be its unique extension to a lattice homomorphism. Then T is injective if and only if $\overline{T}$ is.*

Proof. If $\overline{T}$ is injective, then clearly so is T. For the converse, proceeding as in [3, Proposition 3.2 (1)], we have to show that, if $f \in \mathrm{FBL}^{(p)}[F]_+$ satisfies $f(T^*x^*) = 0$ for any $x^* \in E^*$, then $f = 0$. To this end, let $A_0 = T^*(E^*)$, and define a transfinite sequence of *derived sets* A_α, indexed by ordinals. Specifically, if α has an immediate predecessor β, then let $A_\alpha = \bigcup_{n\in\mathbb{N}} \overline{A_\beta \cap nB_{F^*}}^*$ (the weak* closure). If α is a limit ordinal, let $A_\alpha = \bigcup_{\beta<\alpha} A_\beta$.

By the weak* continuity of addition and scalar multiplication, (A_α) is an increasing sequence of linear subspaces of F^*. This sequence eventually stabilizes: for some α, $A_\alpha = A_{\alpha+1}$. For every n, $A_\alpha \cap nB_{F^*}$ must be weak* closed. Thus, by the Krein–Shmul'yan Theorem, A_α is weak* closed; it must be the weak* closure of A_0, which, by the injectivity of T, coincides with F^*.

If f vanishes on A_β, then, by weak* continuity on bounded sets, it also vanishes on $A_{\beta+1}$. By transfinite induction, f vanishes on A_α for any α, hence on the entirety of F^*. ∎

Finally, [3, Theorem 3.4] claims that if F is a closed subspace of the Banach space E and $\iota : F \to E$ denotes the inclusion map, then the extension $\bar{\iota} : \mathrm{FBL}^{(p)}[F] \to \mathrm{FBL}^{(p)}[E]$ is order continuous. We note that the current proof only works when F is complemented in E. Luckily, this result has not been used at any further point in the paper or the literature.

In the recent preprint [2], the authors also provide alternative proofs for Theorem 1 and Proposition 3, as well as a version of [3, Theorem 3.4] for $p = \infty$.

Acknowledgements. We thank Youssef Azouzi for calling our attention to the gap in the proof of [3, Theorem 2.8] and for sharing the preprint [2].

Funding. T. Oikhberg was supported by the NSF award 1912897. The majority of this work was completed when M. A. Taylor was affiliated with the University of California, Berkeley. P. Tradacete was partially supported by MCIN/AEI/ 10.13039/501100011033 under grants PID2020-116398GB-I00, PID2024-162214NB-I00 and CEX2023-001347-S. V. G. Troitsky was supported by NSERC grant RGPIN-2020-04855.

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