From equations to coastal hazard assessment: mathematical modelling of tsunamis

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Mathematical models play a central role in the understanding and prediction of tsunami hazards. Over the last decades, advances in numerical methods, high-performance computing and geospatial data have enabled the development of operational simulation tools capable of supporting early warning and risk assessment activities. In this article we discuss the mathematical foundations of tsunami modelling and describe how these models have evolved into operational systems used in real-world applications, including the Copernicus Emergency Management Service (Copernicus EMS).

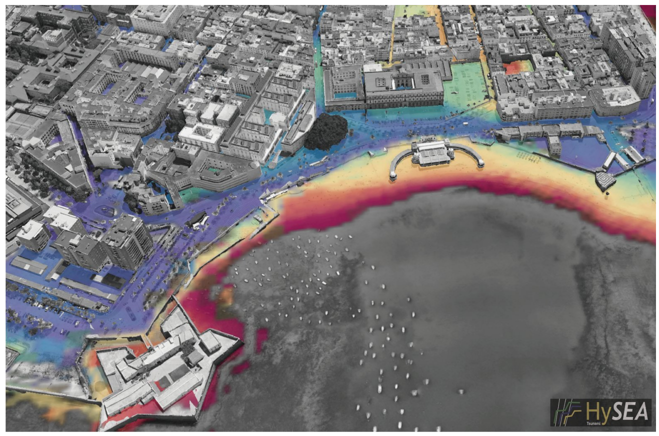

Figure 1. High-resolution tsunami inundation simulation in Cádiz (Spain), showing the interaction between the incoming wave and the urban environment. (Base imagery © Google Earth, Landsat/Copernicus)

1 Introduction

When a tsunami crosses the open ocean, it travels at the speed of a commercial aircraft. Yet far from the coast it may pass almost unnoticed: the wavelength of the wave can reach hundreds of kilometres while its height is often only a few tens of centimetres. Only when the wave approaches shallow coastal waters does this enormous amount of energy become visible, transforming into the destructive surges that have shaped the history of many coastal regions [11].

Behind this dramatic physical phenomenon lies a surprisingly compact mathematical description. Although tsunamis involve processes that extend across entire ocean basins, much of their large-scale behaviour can be captured by depth-averaged models derived from the shallow water equations [6, 10]. These equations describe how gravity, fluid motion and the geometry of the seabed interact to produce waves capable of travelling thousands of kilometres from their source.

Describing the phenomenon mathematically, however, is only the first step. Turning these equations into practical tools requires numerical methods capable of solving them efficiently over domains that range from deep ocean basins to complex coastal environments. This multi-scale nature of the problem poses a significant challenge: simulations must cover very large regions while still resolving coastal processes that may occur on the scale of tens of meters [7].

Over the past decades, advances in numerical methods, high-performance computing and geospatial data have transformed tsunami modelling into a powerful framework for hazard assessment. Numerical simulations are now routinely used to reconstruct historical events, explore hypothetical earthquake scenarios and estimate potential inundation in coastal regions [12, 13]. In many cases these models must produce reliable results within minutes after an earthquake is detected, forming part of the computational infrastructure of tsunami early warning systems [3].

The work described in this article lies at the intersection of these developments. Over the years, a family of numerical models known as HySEA has been developed with the aim of combining robust mathematical formulations with highly efficient computational implementations [8]. These models have been applied to problems ranging from tsunami propagation and coastal inundation to other types of geophysical flows.

Rather than focusing on technical details, our aim here is to illustrate how mathematical modelling and numerical simulation can evolve from theoretical ideas into practical tools that support

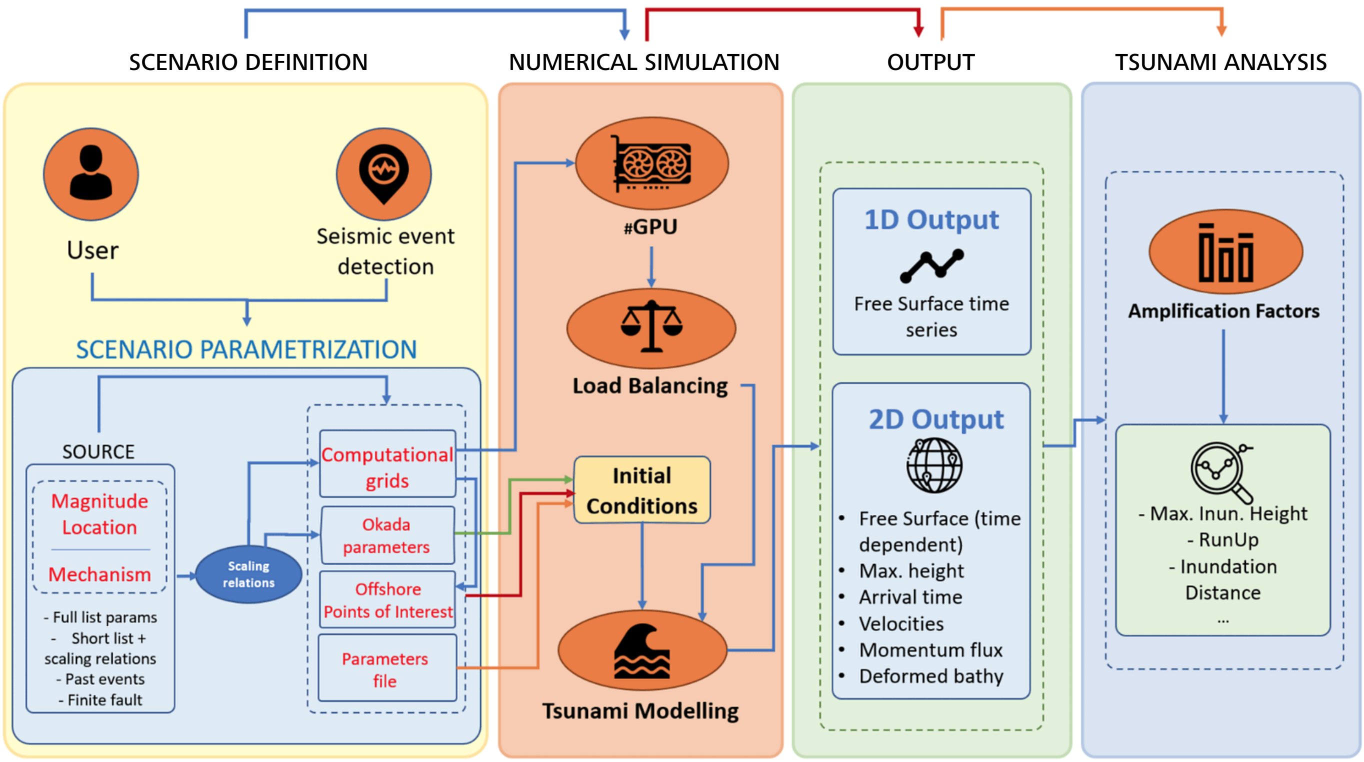

Figure 2. Schematic workflow of tsunami simulation, from scenario definition and source parametrisation to numerical simulation and hazard analysis.

hazard assessment, emergency preparedness and long-term planning in coastal regions.

Before describing the mathematical models and numerical methods in detail, it is useful to outline the typical workflow followed in tsunami simulation and hazard assessment studies.

Starting from the detection or definition of a seismic event, the process involves several stages, including the parametrisation of the source, the construction of suitable computational grids, the numerical simulation of wave propagation and inundation, and the post-processing of the results to extract relevant hazard indicators.

This workflow is summarised in Figure 2, which illustrates how mathematical models, numerical methods and computational tools are combined in practice to produce actionable information for risk assessment.

In the following sections we focus on the mathematical and numerical components of this workflow, and on how they can be translated into efficient computational tools for large-scale tsunami simulation.

2 Mathematical and numerical models for tsunami waves

Tsunami propagation across ocean basins can be described with remarkable accuracy using depth-averaged models. The fundamental observation is that tsunami wavelengths are typically much larger than the local water depth. Under this assumption the vertical structure of the flow can be averaged and the dynamics of the water column can be represented by the shallow water equations, a system of hyperbolic balance laws widely used in geophysical fluid dynamics [6, 10].

These equations describe the evolution of the total water depth and the depth-averaged horizontal momentum under the influence of gravity and bathymetry. Despite their relatively compact mathematical form, they capture the essential mechanisms governing tsunami propagation, including long-distance wave travel, interaction with bottom topography and amplification near coastal regions.

2.1 *The shallow water equations on the sphere*

For simulations at the scale of entire ocean basins it is natural to formulate the shallow water equations directly on the sphere using longitude-latitude coordinates. In this framework the unknowns are the total water depth h and the depth-averaged momentum components $q_\theta = hu_\theta$ and $q_\varphi = hu_\varphi$, where u_θ and u_φ denote the horizontal velocity components.

When modelling tsunami propagation across ocean basins, it is convenient to express the governing equations in spherical coordinates (θ, φ) representing longitude and latitude. The resulting depth-averaged system governing the evolution of the water column is shown below.

$$
\begin{cases}
\partial_t h + \dfrac{1}{R\cos(\varphi)}\left(\partial_\theta q_\theta + \partial_\varphi (q_\varphi \cos(\varphi))\right) = 0, \\
\partial_t q_\theta + \dfrac{1}{R\cos(\varphi)}\partial_\theta\left(\dfrac{q_\theta^2}{h}\right) + \dfrac{1}{R}\partial_\varphi\left(\dfrac{q_\theta q_\varphi}{h}\right) \\
\qquad - 2\dfrac{q_\theta q_\varphi}{Rh}\tan(\varphi) + \dfrac{gh}{R\cos(\varphi)}\partial_\theta h = \dfrac{gh}{R\cos(\varphi)}\partial_\theta H, \\
\partial_t q_\varphi + \dfrac{1}{R\cos(\varphi)}\partial_\theta\left(\dfrac{q_\varphi q_\theta}{h}\right) \\
\qquad + \dfrac{1}{R}\partial_\varphi\left(\dfrac{q_\varphi^2}{h}\right) + \dfrac{(q_\theta^2 - q_\varphi^2)}{hR}\tan(\varphi) + \dfrac{gh}{R}\partial_\varphi h = \dfrac{gh}{R}\partial_\varphi H.
\end{cases}
$$

Two–dimensional shallow water equations in spherical coordinates used for large-scale tsunami simulations.

This system expresses the conservation of mass and momentum in a rotating spherical geometry. The terms involving the bathymetry H account for the influence of the seabed on the flow, while the geometric terms arise from the curvature of the Earth and the use of spherical coordinates. Together, these contributions determine how tsunami waves propagate over large distances and interact with the underlying topography.

In tsunami applications, the system is typically complemented with initial conditions representing the deformation of the ocean surface generated by an earthquake.

These deformations are commonly computed using elastic dislocation models, most notably the formulation proposed by Okada [9]. This approach provides a practical way of linking seismic source parameters with the initial state of the hydrodynamic model.

While the governing equations have a relatively compact mathematical structure, their efficient solution over domains that may span entire ocean basins remains a significant challenge. This is due not only to the large spatial scales involved, but also to the need to accurately capture wave propagation across the ocean and its interaction with complex coastal environments.

2.2 *Finite-volume discretisation*

The numerical solution of the shallow water system presents several difficulties. The equations are nonlinear and hyperbolic, and their solutions may develop steep gradients associated with propagating wave fronts. Realistic simulations must also handle highly variable bathymetry and the appearance or disappearance of water near the shoreline during inundation.

A further difficulty comes from the presence of bathymetric source terms. The equations are therefore not purely conservative, and the numerical method must preserve delicate balances

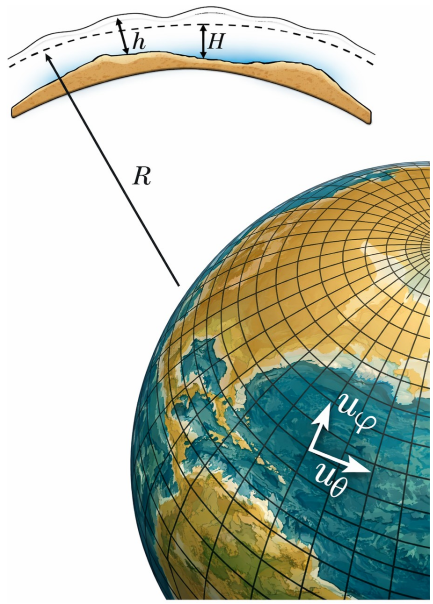

Figure 3. Schematic of the shallow water equations in spherical coordinates.

between fluxes and topography. Finite-volume methods provide a natural framework for this task.

In this setting, the computational domain is divided into control volumes and the governing equations are integrated over each cell. Denoting by U_i^n the average value of the conserved variables in cell i at time t^n, the resulting update takes the form

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}(F_{i+1/2} - F_{i-1/2}) + \Delta t\, S_i.$$

Here the interaction between neighbouring cells is represented through numerical fluxes across cell interfaces, as illustrated in Figure 2.

This formulation provides a robust basis for tsunami simulation over large domains. In particular, well-balanced discretisations are required so that equilibrium states corresponding to water at rest are preserved at the discrete level.

In this formulation the interaction between neighbouring cells occurs through numerical fluxes computed at their interfaces, as illustrated in Figure 4.

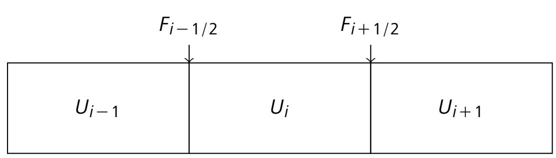

Figure 4. Finite-volume discretisation. The conserved quantities are represented by cell averages and evolve through numerical fluxes exchanged across cell interfaces.

This conservative formulation provides a natural framework for constructing robust numerical schemes capable of accurately propagating long waves across large computational domains. In practice, well-balanced discretisations are incorporated so that stationary solutions corresponding to water at rest are preserved at the discrete level.

2.3 High-resolution schemes for propagation and inundation

Building upon this finite-volume framework, high-resolution numerical schemes are introduced in order to accurately capture steep wave fronts while avoiding spurious oscillations.

Within the HySEA framework, the numerical fluxes are computed with high-resolution finite-volume schemes designed to propagate steep wave fronts accurately while avoiding spurious oscillations. In smooth regions these methods achieve higher-order accuracy, while near sharp gradients they remain stable and robust. In particular, weighted average flux (WAF) formulations combined with total variation diminishing (TVD) limiters are employed to obtain stable and accurate approximations of the shallow water system [1].

In this approach piecewise linear reconstructions of the conserved variables are constructed inside each computational cell and slope limiters are introduced to control the total variation of the numerical solution. The resulting scheme achieves second-order accuracy in smooth regions of the flow while preventing spurious oscillations near steep gradients.

Additional numerical treatments are required in coastal areas in order to deal with inundation processes. In particular, wetting and drying techniques are incorporated to allow the numerical model to track the moving shoreline as the tsunami wave runs up over initially dry land.

By combining high-resolution propagation schemes with robust inundation treatments, the Tsunami-HySEA model provides a unified numerical framework capable of simulating both the propagation of tsunami waves and their complex interaction with coastal environments.

3 From mathematical models to operational tsunami simulation

The mathematical and numerical techniques described in the previous section provide the foundations for tsunami simulation. Transforming these models into tools capable of supporting real-world applications, however, requires addressing additional challenges related to data preparation, computational performance and large-scale simulation workflows.

A particularly illustrative example of this transition arises in tsunami hazard assessment studies carried out along the Spanish coastline within the framework of Copernicus EMS. In these applications, the Tsunami-HySEA model is used to simulate worst-case tsunami scenarios and to quantify their potential impact through high-resolution inundation modelling.

The objective is not only to reproduce wave propagation, but also to provide quantitative information on coastal flooding that can support risk assessment and decision-making processes. This requires the integration of geospatial data, physical models and high-performance numerical simulation into a coherent computational workflow.

The overall methodology followed in these studies can be interpreted as a structured pipeline. Starting from the definition of seismic source scenarios, the process includes the construction of consistent topobathymetric models, the generation of initial conditions through seafloor deformation models, and the simulation of tsunami propagation and coastal inundation using multi-resolution computational grids.

The resulting numerical solution is then post-processed to extract quantitative indicators of tsunami impact along the coastline. The different components of this workflow are described in the following subsections, emphasising their role within this hazard assessment framework.

3.1 Geospatial data for coastal hazard assessment

In this workflow, particular attention is paid to the integration of large geospatial datasets describing both the ocean floor and the coastal terrain. In practice, this involves combining bathymetric information from marine surveys with high-resolution topographic data obtained from digital elevation models.

One of the main technical challenges is the construction of a consistent topobathymetric model that seamlessly connects underwater bathymetry with onshore topography. Small inconsistencies in the data, particularly near the shoreline, may introduce numerical artefacts that significantly affect inundation simulations. Considerable effort is therefore devoted to defining a coherent coastline and merging the different datasets into a continuous elevation model.

For the applications considered here, the resulting terrain model covers the entire Andalusian coastline, extending from the Gulf of Cádiz to the Alboran Sea. The combination of high-resolution datasets makes it possible to represent a wide range of coastal environments, from open beaches and estuaries to densely populated urban areas.

A particularly delicate aspect of this process is the definition of a consistent coastline. Since tsunami inundation depends critically on the interaction between the incoming wave and the land-sea interface, small inconsistencies in the representation of the shoreline may lead to significant errors in the estimated inundation extent. This effect is illustrated in Figure 5.

In practice, the coastline is not a sharply defined geometric object, but rather the result of combining bathymetric and

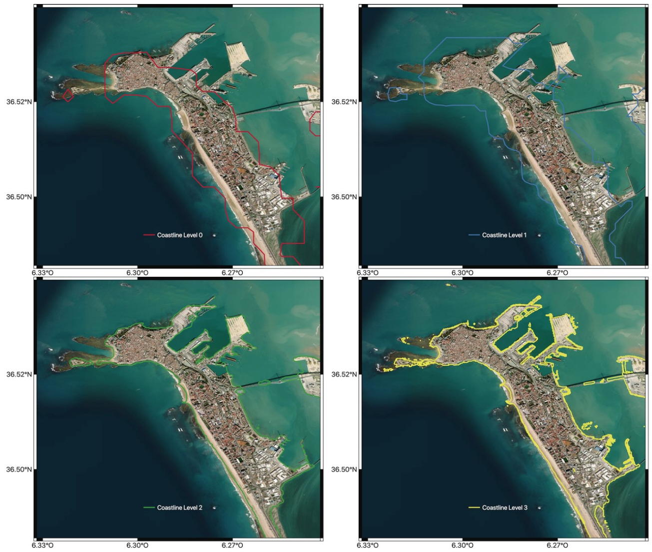

Figure 5. Evolution of the coastline definition in Cádiz as the resolution and consistency of the topobathymetric data improve. From coarse representations (top left) to a refined coastline (bottom right), the definition of the land–sea interface has a direct impact on the accuracy of inundation simulations. (Base imagery © Google Earth)

topographic data with different resolutions and measurement uncertainties. Ensuring a smooth and physically consistent transition between sea and land is therefore an essential step in the modelling workflow.

This example illustrates how the quality and consistency of the underlying geospatial data directly influence the reliability of tsunami inundation modelling, highlighting the importance of careful pre-processing in operational simulation workflows. From a numerical point of view, inconsistencies in the coastline definition may introduce spurious wetting and drying patterns, affecting both stability and accuracy of the solution.

3.2 *Definition of worst-case tsunami scenarios*

Once the terrain model has been constructed, the next step in the workflow is the definition of tsunami generation scenarios. In the context of hazard assessment studies, these scenarios are designed to represent plausible earthquake sources capable of producing significant coastal impact.

Earthquake-generated tsunamis are typically modelled using subfault representations of the seismic source. In this approach, the fault plane is discretised into a set of rectangular patches, each of which is characterised by a small number of parameters describing the relative displacement of the two sides of the fault.

These parameters include the geometrical dimensions of the fault patches, their geographical location and depth, as well as their orientation in space through the strike and dip angles. The direction of slip is defined by the rake angle, while the slip magnitude determines the amount of displacement along the fault. Together, these parameters provide a compact description of the earthquake source that can be directly used in numerical simulations.

The displacement of the seafloor induced by the fault motion is then computed using elastic dislocation models, most commonly the formulation proposed by Okada [9]. This model provides an analytical solution for the deformation of an elastic half-space subject to slip on a finite rectangular fault. The resulting seafloor deformation is assumed to be instantaneously transferred to the ocean surface, providing the initial condition for the shallow water system.

Although this approach relies on simplifying assumptions, such as a flat seafloor and homogeneous elastic properties, it has proven to be a robust and widely used approximation in tsunami modelling. In practice, uncertainties in the fault parameters often dominate over the modelling errors introduced by these assumptions.

Within this hazard assessment workflow, these source models are used to define worst-case tsunami scenarios. For the Atlantic region, the worst-case scenario is defined through a composite fault model involving the Marquês de Pombal and Horseshoe segments [2]. The parameters defining this source are summarised in Table 1. In the Mediterranean region, multiple candidate seismic sources are considered. Their geographical distribution is illustrated in Figure 6.

Table 1. Parameters defining the Atlantic worst-case tsunami scenario, based on a composite fault model combining the Marquês de Pombal and Horseshoe segments.

Fault	Lon (°)	Lat (°)	Depth (km)	L (km)	W (km)	Str. (°)	Dip (°)	Rake (°)	Slip (m)	M_W
MdP	10.25	36.64	5	120	80	20	35	90	12	8.8
HS	10.32	35.92	5	170	100	42	35	90	14	

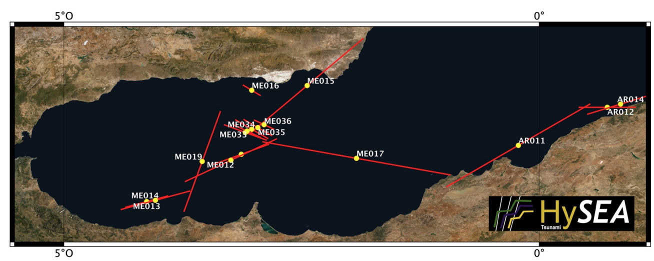

Figure 6. Seismic source scenarios considered in the Mediterranean region, derived from the QAFI database [5] and used to define a worst-case envelope of tsunami impact. (Basemap © Esri World Imagery)

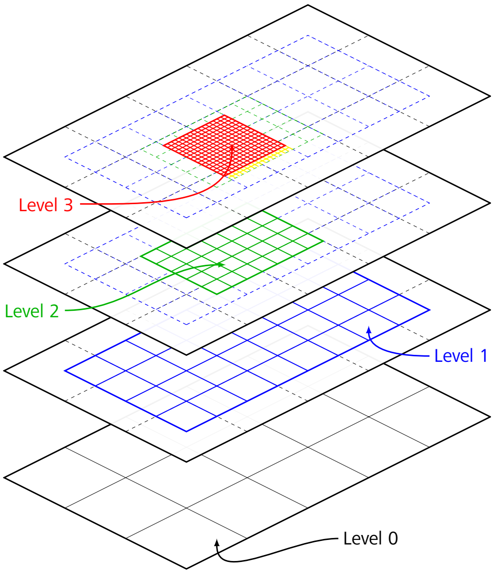

Figure 7. Schematic representation of nested grid strategies used in tsunami simulations. Coarse grids capture large-scale propagation, while finer grids resolve coastal inundation processes.

3.3 Nested grids for coastal inundation modelling

Tsunami propagation involves a wide range of spatial scales, from ocean-basin dynamics to highly localised inundation processes near the coastline. From a numerical perspective, resolving this range of scales within a single computational grid would be prohibitively expensive, as it would require using a very fine resolution over the entire domain.

To overcome this limitation, tsunami simulations rely on nested grid strategies in which multiple computational meshes with different resolutions are coupled together. In this approach, coarse grids are used to simulate wave propagation across the open ocean, while progressively finer grids are introduced in coastal regions where higher accuracy is required.

Figure 7 illustrates this multi-resolution strategy. The nesting procedure allows the model to focus computational resources on regions of interest while maintaining an accurate representation of large-scale wave propagation.

In the Tsunami-HySEA model, a two-way nesting approach is employed. In contrast to one-way nesting, where information flows only from the coarse grid to the finer grids, two-way nesting allows feedback from the fine grids to the coarser levels. This improves the consistency of the numerical solution across different spatial scales, particularly in regions where coastal processes may influence the larger-scale flow.

Within the hazard assessment workflow, nested grids are constructed in a hierarchical manner, starting from a basin-scale propagation grid and refining progressively towards the coastline.

For the simulations considered here, four nested grid levels are employed. The outermost grid, with a resolution of approximately 320 m, captures the large-scale propagation of tsunami waves. This resolution is progressively refined to 80 m and 20 m in intermediate coastal grids, and further down to about 5 m in selected high-resolution target areas. At this finest scale, corresponding to 5 m per pixel, the computational domain comprises a total of 55,439,678 control volumes for the Atlantic region and 155,904,079 for the Mediterranean region.

The configuration used in these simulations is illustrated in Figure 8, which shows the nested grid structure applied to the Spanish coastline within the Copernicus framework.

Such high spatial resolution is essential for accurately representing tsunami inundation in complex coastal environments.

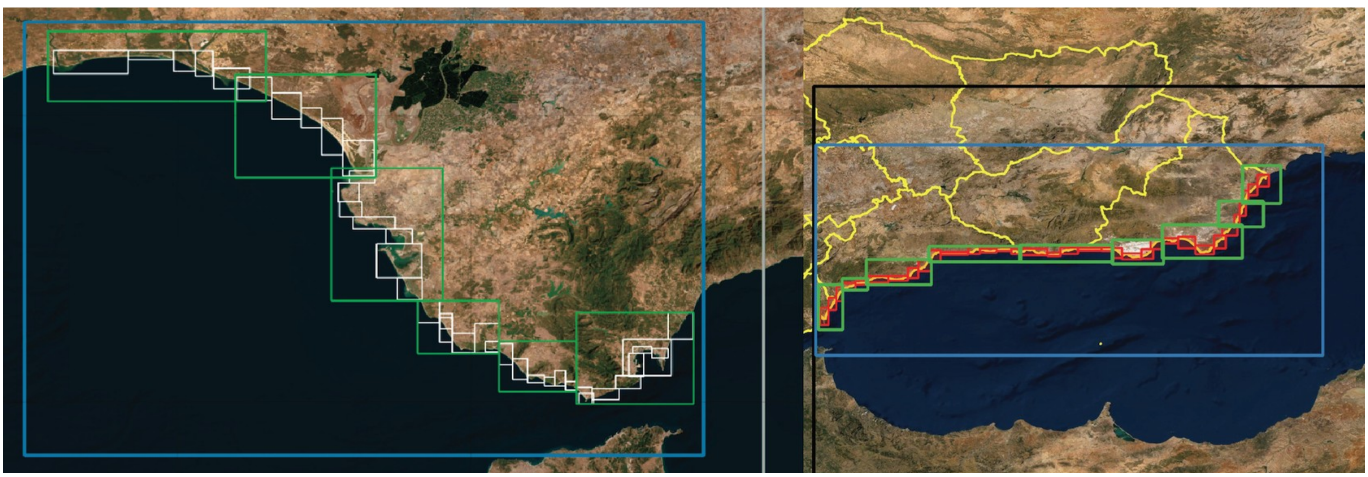

Figure 8. Spatial extent and distribution of the nested computational meshes defining the simulation domain, with resolutions ranging from 320 m (black) and 160 m (blue) to progressively finer grids of 20 m (green) and 5 m (white/red). Atlantic coast (left) and Mediterranean coast (right). (Basemap © Esri World Imagery)

At resolutions of a few metres, the numerical model begins to resolve the interaction of the flow with coastal features such as harbours, breakwaters and urban infrastructure, which play a key role in determining the extent and intensity of flooding.

From a computational perspective, nested grids provide a balance between accuracy and efficiency, allowing large-scale simulations to be performed while retaining the level of detail required for operational hazard assessment.

3.4 *Large-scale simulations for risk assessment*

The combination of high-resolution terrain models, nested grids and multiple tsunami scenarios leads to numerical simulations of considerable size. Individual simulations may involve tens of millions of computational cells and require the solution of the governing equations over thousands of time steps.

To make these simulations feasible, the model have been implemented on modern high-performance computing architectures. In particular, GPU-based implementations allow the large number of finite-volume operations to be executed efficiently in parallel, significantly reducing computation times [4, 8].

These developments have been supported by several European HPC initiatives. The HySEA models are part of the ChEESE (Centre of Excellence for Exascale in Solid Earth) project, which focuses on developing advanced simulation tools for geophysical hazards. More recently, they have also been integrated into the eFlows4HPC project, aimed at combining high-performance simulation with data-driven workflows.

Within this framework, large-scale tsunami hazard assessments have been carried out along the Spanish coastline, reaching spatial resolutions of up to five metres in coastal areas. These simulations provide detailed estimates of inundation extent and water depth, which are essential for risk assessment and emergency planning.

The final outcome of this workflow is a set of high-resolution hazard products, including maps of maximum inundation depth, run-up and flood extent. These products are delivered within the Copernicus Emergency Management Service and are used by national and regional authorities to support coastal risk assessment, emergency planning and mitigation strategies.

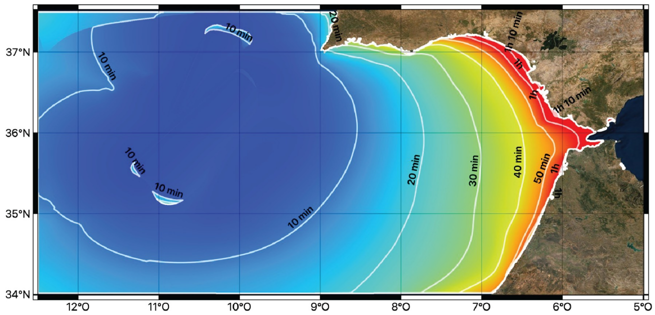

Figure 9. Arrival times of the tsunami wave for the selected Atlantic scenario. (Basemap © Esri World Imagery)

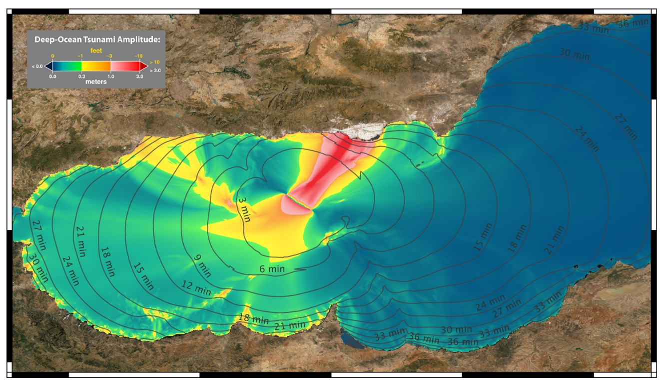

Figure 10. Arrival times along the coastline for the Averroes fault scenario in the Mediterranean region. (Basemap © Esri World Imagery)

3.5 *Hazard indicators and simulation outputs*

The numerical simulations provide a detailed description of tsunami propagation and coastal impact through a set of physically meaningful indicators derived from the solution of the shallow water system. These quantities characterise both the large-scale dynamics of the wave and its local impact on coastal areas.

At the basin scale, the simulations allow the analysis of tsunami propagation through arrival times and maximum wave height. Arrival time maps provide a global view of the evolution of the tsunami front, highlighting the time available for potential warning and response.

For instance, Figure 9 shows the spatial distribution of arrival times for the Atlantic worst-case scenario, revealing the progressive propagation of the tsunami wave towards the coastline.

A complementary perspective is obtained by considering one-dimensional profiles of arrival times along the coastline. An example associated with the Averroes fault in the Mediterranean region is shown in Figure 10, where the spatial variability of arrival times can be clearly observed.

A key difference between the Atlantic and Mediterranean scenarios lies in the characteristic time scales and wave amplitudes. In the Atlantic case, represented by a Lisbon-type worst-case event, the tsunami wave reaches the coast after approximately 50 minutes to one hour, with maximum wave heights of the order of several metres, reaching values close to 8 m in some locations.

In contrast, the Mediterranean scenarios are characterised by much shorter travel distances between the source and the coastline. As a result, the arrival time of the first wave can be as short as

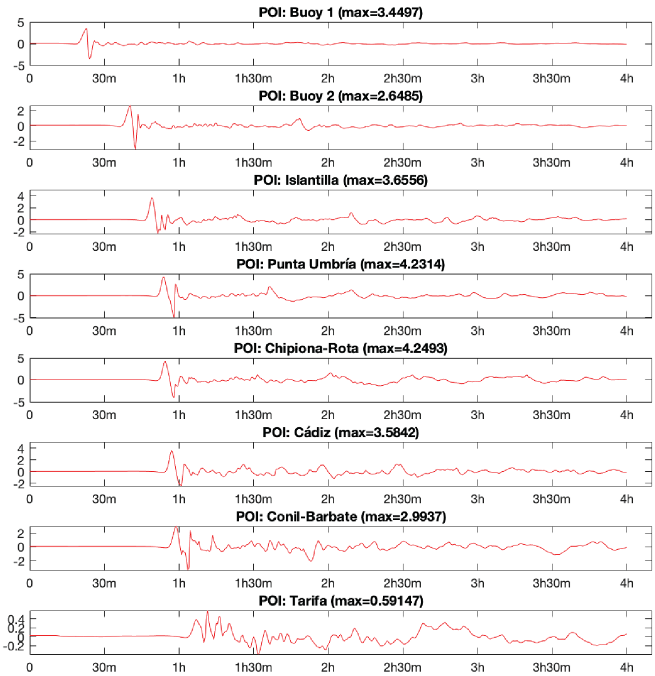

Figure 11. Time series of the free surface elevation at selected points of interest along the coastline.

approximately 15 minutes in the most exposed areas. However, these events typically generate lower wave amplitudes compared to the Atlantic case.

Moreover, due to the more complex seismic setting in the Mediterranean, hazard assessment is based on the analysis of multiple candidate sources. In this work, several scenarios are simulated and combined through an envelope approach, retaining the maximum impact obtained across the most critical events.

These differences have important implications for risk assessment and emergency planning. While the Atlantic scenario is dominated by higher wave amplitudes and longer warning times, the Mediterranean case is characterised by shorter response times, which place stronger constraints on early warning and evacuation strategies.

Complementary to this global description, time series of the free surface elevation are computed at virtual gauges distributed along the coastline. These signals provide detailed information on the temporal evolution of the tsunami, including wave amplitude, arrival sequence and duration of the event. In particular, they make it possible to identify multiple wave arrivals and resonance effects in specific locations, as illustrated in Figure 11.

As the tsunami approaches the coast, the analysis shifts from propagation to inundation processes. High-resolution simulations allow the computation of the maximum water depth over the flooded area, which determines both the spatial extent and the severity of the inundation. This variable is one of the most relevant indicators for hazard assessment, as it is directly related to potential damage to infrastructure and exposed assets. An example of maximum inundation depth in a coastal area is shown in Figure 12.

At finer spatial scales, the numerical model resolves the interaction between the flow and the built environment. This allows a detailed analysis of flooding patterns in urban areas, where local topography and infrastructure strongly influence the flow paths and water accumulation. Such simulations reveal complex features such as flow channelling along streets, local amplification in confined areas, and shadowing effects behind obstacles.

These effects are illustrated in Figures 1 and 13, which show high-resolution inundation patterns in representative coastal environments.

As the analysis moves from wave propagation to coastal inundation, high-resolution simulations provide detailed information on the extent and intensity of flooding. The combination of digital elevation models and simulated inundation patterns allows the identification of flood-prone areas with a high level of spatial accuracy.

These inundation maps form the basis for defining evacuation strategies. On the one hand, the horizontal extent of flooding supports the design of evacuation routes and the identification of safe meeting points outside the affected areas. On the other hand, the spatial distribution of water depth provides key information

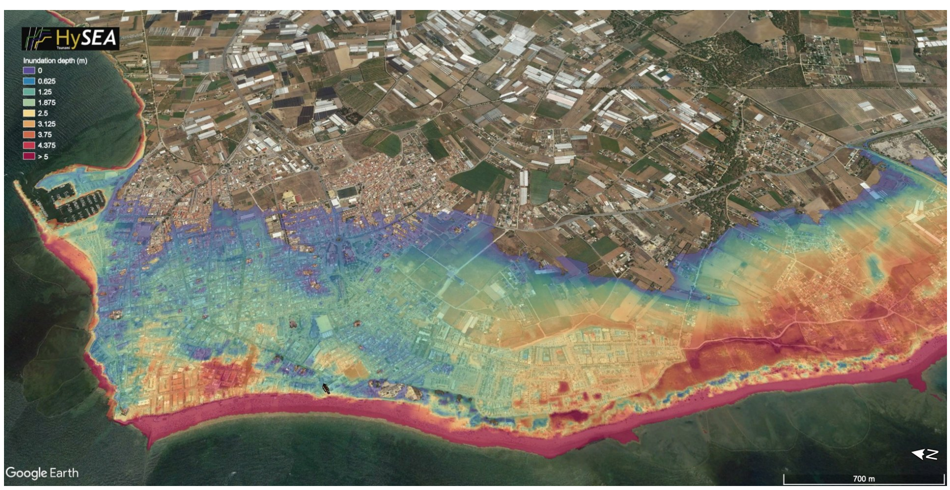

Figure 12. Maximum inundation depth in Chipiona (Cádiz). (Base imagery © Google Earth)

Figure 13. High-resolution inundation simulation in Málaga. (Base imagery © Google Earth, Landsat/Copernicus)

for vertical evacuation strategies, where the safety of upper floors in buildings can be assessed by combining inundation depth with building data and cadastral information.

This level of detail is only achievable through high-resolution simulations, where the interaction between the flow and the urban environment is explicitly resolved. In this context, inundation depth becomes a central variable, as it directly determines both the extent of flooding and the potential impact on exposed assets.

Building on this, these results have been translated into operational products for coastal risk management. In particular, the combination of high-resolution inundation maps with evacuation planning information has led to the development of interactive visualisation platforms.

An example of such a product is provided by the PEMAREA platform (Plan de Emergencia ante el Riesgo de Maremotos de Andalucía, Andalusian Tsunami Emergency Plan), which integrates tsunami hazard information with emergency planning layers at the regional scale.

Figure 14 shows an example of these products for the coastal town of Chipiona. The map combines the simulated inundation extent for worst-case scenarios with evacuation routes and designated safe meeting points, providing a comprehensive view of the potential impact and the corresponding response strategy.

In this representation, the inundation footprint derived from the numerical simulations defines the areas at risk, while the evacuation routes are designed to guide the population towards safe zones

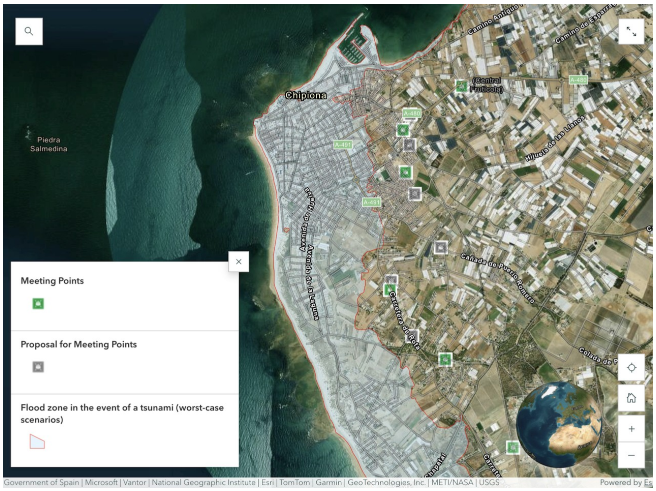

Figure 14. Example of an operational tsunami hazard product for Chipiona (Cádiz), derived from the PEMAREA platform, combining inundation extent, evacuation routes and designated meeting points. (Basemap © National Geographic Institute of Spain, Earthstar Geographics, Esri, TomTom, Garmin, FAO, NOAA, USGS)

outside the flooded region. The location of meeting points is determined by considering both accessibility and safety with respect to the maximum expected water levels.

This integration of simulation results with territorial information allows local authorities to design evacuation plans adapted to the specific characteristics of each coastal area. In particular, the combination of inundation depth with building data enables the assessment of vertical evacuation options, identifying structures where upper floors remain above the predicted flood levels.

From an operational perspective, these products provide a direct link between numerical modelling and emergency planning. Rather than being an abstract representation of tsunami dynamics, the simulation results are translated into actionable information that can be used to guide evacuation procedures and improve preparedness at the local scale.

4 Conclusions

In this work we have described the development of a modelling framework that connects mathematical theory, numerical simulation and operational applications in the context of tsunami hazard assessment.

Starting from depth-averaged descriptions of geophysical flows, we have shown how well-balanced finite-volume schemes, together with multi-resolution strategies and high-performance computing, make it possible to simulate tsunami propagation and inundation across a wide range of spatial scales. These developments allow both ocean-basin dynamics and highly localised coastal impact to be captured within a single computational framework.

Beyond the mathematical and numerical aspects, a key contribution of this work lies in the integration of these models into a complete workflow, including geospatial data processing, scenario definition and the generation of hazard indicators. This workflow has been implemented in operational hazard assessment contexts, where high-resolution simulations are used to create products supporting coastal risk assessment and emergency planning.

The results presented here illustrate how the solution of a system of conservation laws can be translated into actionable information, such as inundation maps, evacuation routes and hazard indicators. In this process, mathematical models provide not only a description of the physical phenomenon, but also a framework for decision-making.

A distinctive feature of tsunami modelling is that these decisions must be taken under significant uncertainty and, in some cases, within very limited time scales. The reliability, efficiency and robustness of the underlying numerical methods therefore become essential, as they directly influence the quality of the information available for risk assessment and emergency response.

More broadly, this work exemplifies how the interaction between mathematical modelling, numerical analysis, geospatial data and high-performance computing can lead to practical tools with direct societal impact. In the context of tsunami hazard assessment, this impact is measured not only in terms of scientific understanding, but also in the capacity to support preparedness and, ultimately, to contribute to the protection of human lives.

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