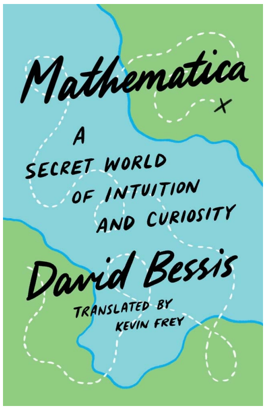
Book review

Mathematica: A Secret World of Intuition and Curiosity
by David Bessis (translated by Kevin Frey)

Reviewed by Mahdi Nouraie



You pilgrims who are on the road,
Where are you? Where?
Come back, come, the beloved is right here!
Your beloved is your neighbor, wall to wall,
You are lost in the desert! What illusion are you under?
– Rumi[1]

This book explores the nature of mathematics by juxtaposing intuitive reasoning with formal logic. It interweaves biographical accounts of figures such as Einstein, Descartes, Gauss, Ramanujan, Thurston, and Grothendieck with examples of exceptional self-taught achievements, including Dick Fosbury's reinvention of high jumping and Ben Underwood's echolocation. The work challenges the conventional view that mathematical ability derives from innate talent or genetic predisposition, presenting mathematics instead as a human activity accessible through deliberate practice and sustained engagement.

From the outset, David Bessis contrasts "official math" with "secret math," arguing that mathematical texts function more as reference manuals than conventional narratives. He asserts that genuine understanding arises from internal cognitive processes not explicitly represented in such texts, which renders mathematical instruction inherently challenging. Bessis links mathematical reasoning to a form of synesthesia, the capacity to associate imagined sensory experiences with abstract concepts, and contends that effective mathematical thinking depends on the ability to "see" and "feel" abstractions rather than merely follow formal procedures. He illustrates this with examples of mathematicians, including himself, who report visual or sensory experiences of mathematical "objects."

Drawing on Daniel Kahneman's distinction between fast intuitive thinking and slow rational thinking as described in [6], Bessis proposes a "System 3," in which intuition is continuously refined through conscious reflection guided by rational thought. Within this framework, logical formalism serves as a precise apparatus for translating intuitive insight into shared mathematical language. Bessis presents this as an organic mode of human learning, characterized by mind elasticity, that distinguishes humans from computational systems. Building on this, he emphasizes writing mathematics during research, in contrast to experimental fields where reports are produced after the fact, arguing that the act of writing itself is integral to mathematical understanding, as articulating ideas precisely compels the revision and refinement of intuition.

The author observes that certain mathematical operations, such as computing one billion minus one, may appear trivial to us because we have learned to "see" 999,999,999; however, such calculations were not straightforward until the modern era. He uses this example to argue that proper education renders mathematics accessible and that operations we now perform mentally should not be taken for granted. The reviewer supplements this point with a historical perspective: concepts that currently seem intuitive, such as averaging data points to estimate the population mean, were not always self-evident. The eminent mathematician Euler, while addressing the "great inequality of Jupiter and Saturn," initially failed to recognize this seemingly obvious principle, worrying that combining data points would multiply errors rather than cancel them [5]. Later, Gauss resolved this issue in detail, demonstrating the assumptions under which errors indeed cancel when data points are combined, thereby laying the foundation for the Gaussian error law.

Bessis introduces the notion of a "child's pose," suggesting that eminent mathematicians approach problems with deliberate innocence and a willingness to ask fundamental questions, echoing the spirit of Socratic inquiry. The reviewer offers a similar perspective based on an observation at the recent Australian Statistical Education Symposium, where Linda McIver provided feedback from

[1] https://rumiroaming.subversivepress.org/pdf/rumi-roaming-ghazal-648-EN-FA.pdf

a former student: "I wouldn't be in this career if you hadn't given me the confidence to play with data like silly putty." This anecdote underscores the relevance of the child's pose, emphasizing that curiosity and intellectual freedom are vital not only in pure mathematics but also in other fields, including statistics and data science.

The primary motivation for writing this review arises from several thought-provoking statements near the conclusion of the book, which can be interpreted as a response to Galileo's famous claim that the universe is written in the language of mathematics:

> *Mathematics isn't the language of the universe. It's the language that allows us to speak with clarity and precision of all the things that we can't point to with our fingers. It's the language that makes us capable of reasoning and doing science. It's the language that's made us what we are, for better and for worse.*

This is followed by a relevant quotation from Thurston and culminates in the provocative assertion:

> *In other words, from a purely practical standpoint, math is indistinguishable from fiction.*

The earliest reference cited by the author in support of this perspective is [4]. The reviewer notes a parallel in probability theory. Although probability has intrigued humans since antiquity, it was only formally studied as a mathematical discipline with the work of Pascal and Fermat [3]. Debates concerning the nature and interpretation of probability remain active in statistics, particularly between the "frequentist" and "Bayesian" schools. The frequentist approach treats probability as an objective property of the world, estimable through experiments and observed frequencies, whereas the Bayesian perspective interprets probability as a measure of plausibility given both data and relevant prior information. Bayesian statistics has historically faced criticism for what Bessis terms the "ingrained suspicion toward anything subjective," a concern he addresses eloquently:

> *But do we really have a choice? When mathematics was just about proving theorems, it was entirely natural to treat the subjective experience as a second-rate citizen, to be covered only informally and anecdotally, if time allowed. But as soon as we realize that mathematics is ultimately about human understanding, this ceases to be an option. Understanding is, in essence, a subjective experience.*

Edwin T. Jaynes, in his posthumously published book on probability theory [5], building on the work of Pierre-Simon Laplace, Harold Jeffreys, George Pólya, and Richard T. Cox, demonstrates that all statements treated as "axioms" in both frequentist and Bayesian schools are, in fact, quantitative rules derivable from general desiderata we impose on the probability function. This perspective reframes the foundations of probability, introducing it as a consistent inference tool for logical reasoning under conditions of incomplete information. Jaynes further cautions against the "mind projection fallacy," whereby one erroneously assumes that probabilistic calculations reflect the objective properties of the world. In [5, Chapter 10], drawing on his background in physics, Jaynes argues that such physical probabilities do not exist in non-quantum settings. Historically, the Pythagoreans, among the earliest groups to study numbers, regarded them as an epistemological tool through which all things are understood, whereas Aristotle's later interpretation that things themselves are numbers became the more widely influential view in scientific discourse.[2]

These perspectives reveal two contrasting views of mathematics: one in which it serves to "discover" universal truths, and another in which it functions to "create" conceptual tools. Bessis emphasizes that the latter reflects the true nature of mathematics: even results that feel like discoveries are, in fact, discoveries of properties of tools created by human beings. From the reviewer's perspective, this distinction is especially apparent in probability and statistics, where the connection to real-world phenomena and the paradoxes that arise expose the fallacy. In contrast, more abstract branches of mathematics may obscure this point due to their greater conceptual distance from empirical experience.

In the final chapters, Bessis presents deep learning as an "effective metaphor" for human learning while maintaining a clear distinction between artificial neural networks and their biological counterparts, a distinction the reviewer considers especially important given current discussions surrounding these technologies. In this context, [7] documents the limitations of one of the contemporary large language models when confronted with elementary mathematical problems from Ancient Greece, while [1] demonstrates that human mathematical understanding engages non-linguistic brain circuits. The latter was recently highlighted by Bessis on his X account and corresponds to what he terms the "language trap" in the book. Fields Medalist Martin Hairer recently highlighted "auto-formalization" as a potential future use of large language models, illustrating this point[3]. In this light, formalism is often mistaken for an end in itself rather than a means, echoing the opening poem of this review. The reviewer also notes [2] as an excellent recent contribution demonstrating how complex mathematical concepts can be effectively conveyed without formal notation.

To conclude, the text is sharply written, with a personal tone. The book consists of short sentences, brief chapters, and concentrated reflections that are often incisive and occasionally fragmentary. Its proselytizing style and frequent bold claims may unsettle

[2] https://plato.sydney.edu.au/entries/pythagoreanism

[3] https://www.youtube.com/watch?v=fbVqc1tPLos

some mathematical readers. The reviewer considers Bessis an exceptional author and finds the book highly engaging, praised for both its wide-ranging topics and its clear, distinctive style. It is an essential read for those aiming to understand mathematics as both an intellectual practice and a lived experience, a perspective endorsed by at least two Fields medalists.[4]

David Bessis (translated by Kevin Frey), *Mathematica: A Secret World of Intuition and Curiosity.* Yale University Press, 2024, 344 pages, Hardcover ISBN 978-0-30027-088-4, Softcover ISBN 978-0-30028-328-0, eBook ISBN 978-0-30027-714-2.

References

[1] M. Amalric and S. Dehaene, Origins of the brain networks for advanced mathematics in expert mathematicians. *Proc. Natl. Acad. Sci. USA* **113**, 4909–4917 (2016)

[2] M. Beckman, *Math without numbers*. Penguin Random House UK, London, UK (2022)

[3] A. Clayton, *Bernoulli's fallacy: Statistical illogic and the crisis of modern science*. Columbia University Press, New York, NY (2021)

[4] R. Hersh, Some proposals for reviving the philosophy of mathematics. *Adv. in Math.* **31**, 31–50 (1979)

[5] E. T. Jaynes, *Probability theory: The logic of science*. Cambridge University Press, Cambridge, UK (2003)

[6] D. Kahneman, *Thinking, fast and slow*. Farrar, Straus and Giroux, New York, NY (2011)

[7] N. Marco and A. J. Stylianides, An exploration into the nature of ChatGPT's mathematical knowledge. *Internat. J. Math. Ed. Sci. Tech.* **56**, 2279–2297 (2025)

Mahdi Nouraie is a PhD candidate in statistics at Macquarie University, Australia. His research focuses on statistical learning and stable variable selection in high-dimensional settings, with applications including gene expression analysis. He is particularly interested in the historical and philosophical foundations of statistics and in communicating statistical ideas with clarity and accessibility.

mahdi.nouraie@hdr.mq.edu.au

DOI 10.4171/MAG/328

[4] https://yalebooks.yale.edu/book/9780300270884/#tab-4