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# Right Sided Ideals and Multilinear Polynomials with Derivations on Prime Rings

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ABSTRACT - Let $R$ be an associative prime ring of char $R \neq 2$ with center $Z(R)$ and extended centroid $C, f(x_1, \dots, x_n)$ a nonzero multilinear polynomial over $C$ in $n$ noncommuting variables, $d$ a nonzero derivation of $R$ and $\rho$ a nonzero right ideal of $R$. We prove that: (i) if $[d^2(f(x_1, \dots, x_n)), f(x_1, \dots, x_n)] = 0$ for all $x_1, \dots, x_n \in \rho$ then $\rho C = eRC$ for some idempotent element $e$ in the socle of $RC$ and $f(x_1, \dots, x_n)$ is central-valued in $eRCe$ unless $d$ is an inner derivation induced by $b \in Q$ such that $b^2 = 0$ and $b\rho = 0$; (ii) if $[d^2(f(x_1, \dots, x_n)), f(x_1, \dots, x_n)] \in Z(R)$ for all $x_1, \dots, x_n \in \rho$ then $\rho C = eRC$ for some idempotent element $e$ in the socle of $RC$ and either $f(x_1, \dots, x_n)$ is central in $eRCe$ or $eRCe$ satisfies the standard identity $S_4(x_1, x_2, x_3, x_4)$ unless $d$ is an inner derivation induced by $b \in Q$ such that $b^2 = 0$ and $b\rho = 0$.

Throughout this paper, $R$ always denotes a prime ring with extended centroid $C$ and $Q$ its two-sided Martindale ring of quotient. By $d$ we mean a nonzero derivation of $R$. For $x, y \in R$, the commutator of $x, y$ is denoted by $[x, y]$ and defined by $[x, y] = xy - yx$. We denote $[x, y]_2 = [[x, y], y] = [x, y]y - y[x, y]$.

A well known result proved by Posner [17] states that $R$ must be commutative if $[d(x), x] \in Z(R)$ for all $x \in R$. In [10] Lanski generalized the Posner's result to a Lie ideal. More precisely Lanski proved that if $L$ is a noncommutative Lie ideal of $R$ such that $[d(x), x] \in Z(R)$ for all $x \in L$,

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then char $R = 2$ and $R$ satisfies $S_4(x_1, x_2, x_3, x_4)$, the standard identity. Note that a noncommutative Lie ideal of $R$ contains all the commutators $[x_1, x_2]$ for $x_1, x_2$ in some nonzero ideal of $R$ ( see [10, Lemma 2 (i), (ii)]). So, it is natural to consider the situation when $[d(x), x] \in Z(R)$ for all commutators $x = [x_1, x_2]$ or more general case $x = f(x_1, \ldots, x_n)$ where $f(x_1, \ldots, x_n)$ is a multilinear polynomial. In [11] Lee and Lee proved that if $[d(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)] \in Z(R)$ for all $x_1, \ldots, x_n$ in some nonzero ideal of $R$, then $f(x_1, \ldots, x_n)$ is central-valued on $R$, except when char $R = 2$ and $R$ satisfies $S_4(x_1, x_2, x_3, x_4)$. Recently, De Filippis and Di Vincenzo (see [7]) consider the situation $\delta([d(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)]) = 0$ for all $x_1, \ldots, x_n \in R$, where $d$ and $\delta$ are two derivations of $R$. The statement of De Filippis and Di Vincenzo's theorem is the following:

THEOREM A ([7, Theorem 1]). *Let $K$ be a noncommutative ring with unity, $R$ a prime $K$-algebra of characteristic different from 2, $d$ and $\delta$ nonzero derivations of $R$ and $f(x_1, \ldots, x_n)$ a multilinear polynomial over $K$. If $\delta([d(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)]) = 0$ for all $x_1, \ldots, x_n \in R$, then $f(x_1, \ldots, x_n)$ is central-valued on $R$.*

*In case $\delta$ and $d$ are two same derivations, the differential identity becomes $[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)] = 0$ for all $x_1, \ldots, x_n \in R$. So, it is natural to ask, what happen in cases $[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)] \in Z(R)$ for all $x_1, \ldots, x_n \in R$ and $[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)] \in Z(R)$ for all $x_1, \ldots, x_n \in \rho$, where $\rho$ is a non-zero right ideal of $R$. In the present paper our object is to study these cases.*

For the sake of completeness we recall some basic notations, definitions and some easy consequences of the result of Kharchenko [8] about the differential identities on a prime ring $R$. First, we denote by $Der(Q)$ the set of all derivations on $Q$. By a derivation word $\Delta$ of $R$ we mean $\Delta = d_1 d_2 d_3 \ldots d_m$ for some derivations $d_i$ of $R$. For $x \in R$, we denote by $x^{\Delta}$ the image of $x$ under $\Delta$, that is $x^{\Delta} = (\cdots(x^{d_1})^{d_2} \cdots)^{d_m}$. By a differential polynomial, we mean a generalized polynomial, with coefficients in $Q$, of the form $\Phi(x_i^{\Delta_j})$ involving noncommutative indeterminates $x_i$ on which the derivations words $\Delta_j$ act as unary operations. $\Phi(x_i^{\Delta_j}) = 0$ is said to be a differential identity on a subset $T$ of $Q$ if it vanishes for any assignment of values from $T$ to its indeterminates $x_i$.

Now let $D_{int}$ be the $C$-subspace of $Der(Q)$ consisting of all inner derivations on $Q$. By Kharchenko's theorem [8, Theorem 2], we have the following result:

Let $R$ be a prime ring of characteristic different from 2. If two nonzero derivations $d$ and $\delta$ are $C$-linearly independent modulo $D_{int}$ and $\Phi(x_i^{\Delta_j})$ is a differential identity on $R$, where $\Delta_j$ are derivations words of the following form $\delta, d, \delta^2, \delta d, d^2$, then $\Phi(y_{ji})$ is a generalized polynomial identity on $R$, where $y_{ji}$ are distinct indeterminates.

As a particular case, we have:

If $d$ is a nonzero derivation on $R$ and $\Phi(x_1, \ldots, x_n, x_1^d, \ldots, x_n^d, x_1^{d^2}, \ldots, x_n^{d^2})$ is a differential identity on $R$, then one of the following holds:

(i) either $d \in D_{int}$

or

(ii) $R$ satisfies the generalized polynomial identity $\Phi(x_1, \ldots, x_n, y_1, \ldots, y_n, z_1, \ldots, z_n)$

Denote by $Q *_C C\{X_1, \ldots, X_n\}$ the free product of the $C$-algebra $Q$ and $C\{X_1, \ldots, X_n\}$, the free $C$-algebra in noncommuting indeterminates $X_1, \ldots, X_n$.

Since $f(x_1, \ldots, x_n)$ is a multilinear polynomial, we can write

$$f(x_1, \ldots, x_n) = x_1 x_2 \ldots x_n + \sum_{I \neq \sigma \in S_n} \alpha_\sigma x_{\sigma(1)} \ldots x_{\sigma(n)}$$

where $S_n$ is the permutation group over $n$ elements and any $\alpha_\sigma \in C$. We denote by $f^d(x_1, \ldots, x_n)$ the polynomial obtained from $f(x_1, \ldots, x_n)$ by replacing each coefficient $\alpha_\sigma$ with $d(\alpha_\sigma.1)$. In this way we have

$$d(f(x_1, \ldots, x_n)) = f^d(x_1, \ldots, x_n) + \sum_i f(x_1, \ldots, d(x_i), \ldots, x_n)$$

and

$$\begin{aligned} d^2(f(x_1, \ldots, x_n)) &= d(f^d(x_1, \ldots, x_n)) + d\left(\sum_i f(x_1, \ldots, d(x_i), \ldots, x_n)\right) \\ &= f^{d^2}(x_1, \ldots, x_n) + \sum_i f^d(x_1, \ldots, d(x_i), \ldots, x_n) \\ &+ \sum_i f^d(x_1, \ldots, d(x_i), \ldots, x_n) + \sum_{i \neq j} f(x_1, \ldots, d(x_i), \ldots, d(x_j), \ldots, x_n) \\ &+ \sum_i f(x_1, \ldots, d^2(x_i), \ldots, x_n) \\ &= f^{d^2}(x_1, \ldots, x_n) + 2\sum_i f^d(x_1, \ldots, d(x_i), \ldots, x_n) \\ &+ 2\sum_{i<j} f(x_1, \ldots, d(x_i), \ldots, d(x_j), \ldots, x_n) + \sum_i f(x_1, \ldots, d^2(x_i), \ldots, x_n). \end{aligned}$$

## 1. The case for $\rho = R$.

LEMMA 1.1. *Let $R = M_k(F)$ be the ring of all $k \times k$ matrices over a field $F$ of characteristic $\neq 2$, $b \in R$ and $f(x_1, \ldots, x_n)$ is a multilinear polynomial over $F$. If $k \geq 2$ and $[[b,[b,f(x_1,\ldots,x_n)]], f(x_1,\ldots,x_n)] = 0$ for all $x_1,\ldots,x_n \in R$ or if $k \geq 3$ and $[[b,[b,f(x_1,\ldots,x_n)]], f(x_1,\ldots,x_n)] \in Z(R)$ for all $x_1,\ldots,x_n \in R$, then either $b \in F \cdot I_k$ or $f(x_1,\ldots,x_n)$ is central-valued on $R$.*

PROOF. Let $b = (b_{ij})_{k \times k}$. Let $e_{ij}$ be the usual matrix unit with 1 in $(i,j)$ entry and zero else where. Now we proceed to show that $b \in Z(R)$ if $\in f(x_1,\ldots,x_n)$ is non central valued on $R$.

For simplicity, we write $f(x_1,\ldots,x_n) = f(x)$, where $x = (x_1,\ldots,x_n)$ $R^n = R \times \cdots \times R$ ($n$ times). Then by assumption,

$$[[b,[b,f(x)]], f(x)] = [b^2 f(x) - 2bf(x)b + f(x)b^2, f(x)] \in Z(R)$$

for all $x \in R^n$. Since $f(x_1,\ldots,x_n)$ is assumed to be noncentral on $R$, by [15, Lemma 2, Proof of Lemma 3] there exists a sequence of matrices $r = (r_1,\ldots,r_n)$ in $R$ such that $f(r) = f(r_1,\ldots,r_n) = \alpha e_{ij} \neq 0$ where $0 \neq \alpha \in F$ and $i \neq j$. Thus

$$[b^2 \alpha e_{ij} - 2b\alpha e_{ij} b + \alpha e_{ij} b^2, \alpha e_{ij}] \in Z(R).$$

Since the rank of $[b^2\alpha e_{ij} - 2b\alpha e_{ij}b + \alpha e_{ij}b^2, \alpha e_{ij}]$ is $\leq 2$, $[b^2\alpha e_{ij} - 2b\alpha e_{ij}b + {} + \alpha e_{ij}b^2, \alpha e_{ij}] = 0$. Left multiplying by $e_{ij}$, we get $0 = e_{ij}(-2b\alpha e_{ij} b \alpha e_{ij}) = {} = -2\alpha^2 b_{ji}^2 e_{ij}$. Since char $F \neq 2$, $b_{ji} = 0$. For $s \neq t$, let $\sigma$ be a permutation in the symmetric group $S_m$ such that $\sigma(i) = s$ and $\sigma(j) = t$. Let $\psi$ be the automorphism of $R$ defined by $x^\psi = \left(\sum_{p,q} \xi_{pq} e_{pq}\right)^\psi = \sum_{p,q} \xi_{pq} e_{\sigma(p),\sigma(q)}$. Then $f(r^\psi) = f(r_1^\psi,\ldots,r_n^\psi) = f(r)^\psi = \alpha e_{st} \neq 0$ and we have as above $b_{ts} = 0$ for $s \neq t$. Thus $b$ is a diagonal matrix. For any $F$-automorphism $\theta$ of $R$, $b^\theta$ enjoys the same property as $b$ does, namely, $[[b^\theta,[b^\theta,f(x)]], f(x)] \in Z(R)$ for all $x \in R^n$. Hence, $b^\theta$ must be diagonal. Write $b = \sum_{i=1}^k a_{ii}e_{ii}$; then for each $j \neq 1$, we have

$$(1 + e_{1j})b(1 - e_{1j}) = \sum_{i=1}^k a_{ii}e_{ii} + (b_{jj} - b_{11})e_{1j}$$

diagonal. Therefore, $b_{jj} = b_{11}$ and so $b$ is a scalar matrix.

LEMMA 1.2. *Let $R$ be a prime ring of characteristic different from 2 and $f(x_1,\dots,x_n)$ a multilinear polynomial over $C$. If for any $i=1,\dots,n$,*

$$[f(x_1,\dots,z_i,\dots,x_n), f(x_1,\dots,x_n)] = 0$$

*for all $x_1,\dots,x_n,z_i \in R$, then the polynomial $f(x_1,\dots,x_n)$ is central-valued on $R$.*

PROOF. Let $a$ be a noncentral element of $R$. Then replacing $z_i$ with $[a,x_i]$ we have that for any $i=1,\dots,n$

$$[f(x_1,\dots,[a,x_i],\dots,x_n), f(x_1,\dots,x_n)] = 0$$

and so

$$\left[\sum_{i=0}^{n} f(x_1,\dots,[a,x_i],\dots,x_n), f(x_1,\dots,x_n)\right] = 0$$

which implies, $[a, f(x_1,\dots,x_n)]_2 = 0$ for all $x_1,\dots,x_n \in R$. By [11, Theorem], $f(x_1,\dots,x_n)$ is central-valued on $R$.

THEOREM 1.3. *Let $R$ be a prime ring of characteristic different from 2, $d$ a nonzero derivation of $R$, $f(x_1,\dots,x_n)$ a multilinear polynomial over $C$. If*

$$[d^2(f(x_1,\dots,x_n)), f(x_1,\dots,x_n)] \in Z(R) \quad \text{for all } x_1,\dots,x_n \in R,$$

*then either $f(x_1,\dots,x_n)$ is central-valued on $R$ or $R$ satisfies the standard identity $S_4(x_1,x_2,x_3,x_4)$.*

PROOF. Let $I$ be any nonzero two-sided ideal of $R$. If for every $r_1,\dots,r_n \in I$, $[d^2(f(r_1,\dots,r_n)), f(r_1,\dots,r_n)] = 0$, then by [14], this generalized differential identity is also satisfied by $Q$ and hence by $R$ as well. By Theorem $A$, $f(r_1,\dots,r_n)$ is then central-valued on $R$ and we are done. Now we assume that for some $r_1,\dots,r_n \in I$, $0 \neq [d^2(f(r_1,\dots,r_n)), f(r_1,\dots,r_n)] \in I \cap Z(R)$. Thus $I \cap Z(R) \neq 0$. Let $K$ be a nonzero two-sided ideal of $R_Z$, the ring of central quotients of $R$. Since $K \cap R$ is a nonzero two-sided ideal of $R$, $(K \cap R) \cap Z(R) \neq 0$. Therefore, $K$ contains an invertible element in $R_Z$ and so $R_Z$ is a simple ring with identity 1.

By assumption, $R$ satisfies the differential identity

$$
\begin{aligned}
& g(x_1,\dots,x_n,d(x_1),\dots,d(x_n),d^2(x_1),\dots,d^2(x_n)) \\
& = [[f^{d^2}(x_1,\dots,x_n) + 2\sum_i f^d(x_1,\dots,d(x_i),\dots,x_n) \\
& \quad + 2\sum_{i<j} f(x_1,\dots,d(x_i),\dots,d(x_j),\dots,x_n) \\
& + \sum_i f(x_1,\dots,d^2(x_i),\dots,x_n), f(x_1,\dots,x_n)], x_{n+1}].
\end{aligned}
$$

If $d$ is not $Q$-inner, then by Kharchenko's theorem [8],

$$
\begin{aligned}
& \Bigg[\Bigg[f^{d^2}(x_1,\dots,x_n) + 2\sum_i f^d(x_1,\dots,y_i,\dots,x_n) \\
(1) \qquad & \quad + 2\sum_{i<j} f(x_1,\dots,y_i,\dots,y_j,\dots,x_n) \\
& \quad + \sum_i f(x_1,\dots,z_i,\dots,x_n), f(x_1,\dots,x_n)\Bigg], x_{n+1}\Bigg] = 0
\end{aligned}
$$

for all $x_i, y_i, z_i, x_{n+1} \in R$ for $i = 1,2,\dots,n$. In particular, for any $i$, assuming $y_1 = \dots = y_{i-1} = y_{i+1} = \dots = y_n = 0, z_1 = \dots = z_n = 0$, we have

$$[[f^{d^2}(x_1,\dots,x_n) + 2f^d(x_1,\dots,y_i,\dots,x_n), f(x_1,\dots,x_n)], x_{n+1}] = 0$$

and so

$$\Bigg[\Bigg[f^{d^2}(x_1,\dots,x_n) + 2\sum_i f^d(x_1,\dots,y_i,\dots,x_n), f(x_1,\dots,x_n)\Bigg], x_{n+1}\Bigg] = 0$$

for all $x_i, y_i, x_{n+1} \in R$, $i = 1,2,\dots,n$. Thus from (1), we obtain

$$
\begin{aligned}
& \Bigg[\Bigg[2\sum_{i<j} f(x_1,\dots,y_i,\dots,y_j,\dots,x_n) \\
(2) \qquad & + \sum_i f(x_1,\dots,z_i,\dots,x_n), f(x_1,\dots,x_n)\Bigg], x_{n+1}\Bigg] = 0
\end{aligned}
$$

for all $x_i, y_i, z_i, x_{n+1} \in R$ for $i = 1,2,\dots,n$.

By localizing $R$ at $Z(R)$, we obtain that (2) is also an identity of $R_Z$. Since $R$ and $R_Z$ satisfy the same polynomial identities, in order to prove that $R$ satisfies $S_4$, we may assume that $R$ is a simple ring with 1. Thus $R$ satisfies the identity (2). Now putting $y_i = [b, x_i] = \delta(x_i)$ and $z_i = [b, [b, x_i]] =$

$= \delta^2(x_i)$, $i = 1, 2, \ldots, n$ for some $b \notin Z(R)$, where $\delta$ is an inner derivation induced by some $b \in R$, we obtain that $R$ satisfies

$$[[\delta^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)], x_{n+1}] = 0.$$

Thus by Martindale's theorem [16], $R$ is a primitive ring with a minimal right ideal, whose commuting ring $D$ is a division ring which is finite dimensional over $Z(R)$. However, since $R$ is simple with 1, $R$ must be Artinian. Hence $R = D_{k'}$, the ring of $k' \times k'$ matrices over $D$, for some $k' \geq 1$. Again, by [9, Lemma 2], it follows that there exists a field $F$ such that $R \subseteq M_k(F)$, the ring of all $k \times k$ matrices over the field $F$, and $M_k(F)$ satisfies

$$[[\delta^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)], x_{n+1}] = 0.$$

If $k \geq 3$, then by Lemma 1.1, we have $b \in Z(R)$, a contradiction. Thus $k = 2$, that is, $R$ satisfies $S_4(x_1, x_2, x_3, x_4)$.

Similarly, the same conclusion can be drawn in case $d$ is an $Q$-inner derivation induced by some $b \in Q$.

## 2. The case for one-sided ideal.

We begin with the following lemmas

LEMMA 2.1. *Let $\rho$ be a nonzero right ideal of $R$ and $d$ a derivation of $R$. Then the following conditions are equivalent:*

(i) *$d$ is an inner derivation induced by some $b \in Q$ such that $b\rho = 0$;*
(ii) *$d(\rho)\rho = 0$.*

For its proof, we refer to [2, Lemma].

LEMMA 2.2. *Let $R$ be a prime ring, $\rho$ a nonzero right ideal of $R$, $f(x_1, \ldots, x_t)$ a multilinear polynomial over $C$, $a \in R$ and $n$ a fixed positive integer. If $f(x_1, \ldots, x_t)^n a = 0$ for all $x_1, \ldots, x_t \in \rho$, then either $a = 0$ or $f(\rho)\rho = 0$.*

For its proof, we refer to [3, Lemma 2 (II)].

LEMMA 2.3. *Let $R$ be a prime ring. If $[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)] \in Z(R)$ for all $x_1, \ldots, x_n \in \rho$, then $R$ satisfies nontrivial generalized polynomial identity unless $d$ is an inner derivation induced by $b \in Q$ such that $b^2 = 0$ and $b\rho = 0$.*

PROOF. Suppose on the contrary that $R$ does not satisfy any nontrivial generalized polynomial identity (GPI). Thus we may assume that $R$ is noncommutative, otherwise $R$ satisfies trivially a nontrivial GPI. Now we consider the following two cases:

CASE I. Suppose that $d$ is a $Q$-inner derivation induced by an element $b \in Q$ such that $b^2 \neq 0$. Then for any $x_0 \in \rho$

$$[[b,[b,f(x_0X_1,\ldots,x_0X_n)]],f(x_0X_1,\ldots,x_0X_n)] \in Z(R)$$

that is

$$\begin{aligned}(3)\qquad &[[b^2f(x_0X_1,\ldots,x_0X_n)-2bf(x_0X_1,\ldots,x_0X_n)b\\ &+f(x_0X_1,\ldots,x_0X_n)b^2,f(x_0X_1,\ldots,x_0X_n)],x_0X_{n+1}]\end{aligned}$$

is a GPI for $R$, so it is the zero element in $Q *_C C\{X_1,\ldots,X_{n+1}\}$. Denote $l_R(\rho)$ the left annihilator of $\rho$ in $R$. Suppose first that $\{1,b,b^2\}$ are linearly $C$-independent modulo $l_R(\rho)$, that is $(\alpha b^2+\beta b+\gamma)\rho=0$ if and only if $\alpha=\beta=\gamma=0$. Since $R$ is not a GPI-ring, a fortiori it can not be a PI-ring. Thus, by [13, Lemma 3] there exists $x_0 \in \rho$ such that $\{b^2x_0, bx_0, x_0\}$ are linearly $C$-independent. Then we have that

$$\begin{aligned}&[[b^2f(x_0X_1,\ldots,x_0X_n)-2bf(x_0X_1,\ldots,x_0X_n)b\\ &+f(x_0X_1,\ldots,x_0X_n)b^2,f(x_0X_1,\ldots,x_0X_n)],x_0X_{n+1}]=0\end{aligned}$$

is a nontrivial GPI for $R$, a contradiction.

Therefore, $\{1,b,b^2\}$ are linearly $C$-dependent modulo $l_R(\rho)$, that is there exist $\alpha,\beta,\gamma \in C$, not all zero, such that $(\alpha b^2+\beta b+\gamma)\rho=0$. Suppose that $\alpha=0$. Then $\beta\neq 0$, otherwise $\gamma=0$. Thus by $(\beta b+\gamma)\rho=0$, we have that $(b+\beta^{-1}\gamma)\rho=0$. Since $b$ and $b+\beta^{-1}\gamma$ induce the same inner derivation, we may replace $b$ by $b+\beta^{-1}\gamma$ in the basic hypothesis. Therefore, in any case we may suppose $b\rho=0$ and then from (3), $R$ satisfies $x_0X_{n+1}f^2(x_0X_1,\ldots,x_0X_n)b^2=0$. Since $R$ does not satisfy any nontrivial GPI, $b^2=0$, a contradiction.

Next suppose that $\alpha\neq 0$. In this case there exist $\lambda,\mu \in C$ such that $b^2x_0=\lambda bx_0+\mu x_0$ for all $x_0 \in \rho$. If $bx_0$ and $x_0$ are linearly $C$-dependent for all $x_0\in\rho$, then again we obtain $b\rho=0$ and so $b^2=0$. Therefore choose $x_0\in\rho$ such that $bx_0$ and $x_0$ are linearly $C$-independent. Then replacing $b^2x_0$ with $\lambda bx_0+\mu x_0$, we obtain from (3) that $R$ satisfies

$$\Big[\big\{(\lambda b+\mu)f^2(x_0X_1,\dots,x_0X_n)-2bf(x_0X_1,\dots,x_0X_n)bf(x_0X_1,\dots,x_0X_n)$$
$$+f(x_0X_1,\dots,x_0X_n)(\lambda b+\mu)f(x_0X_1,\dots,x_0X_n)\big\}$$
$$-\big\{f(x_0X_1,\dots,x_0X_n)(\lambda b+\mu)f(x_0X_1,\dots,x_0X_n)$$
$$-2f(x_0X_1,\dots,x_0X_n)bf(x_0X_1,\dots,x_0X_n)b+f^2(x_0X_1,\dots,x_0X_n)b^2\big\},x_0X_{n+1}\Big].$$

This is a nontrivial GPI for $R$, because the term

$$\big(\lambda bf^2(x_0X_1,\dots,x_0X_n)-2bf(x_0X_1,\dots,x_0X_n)bf(x_0X_1,\dots,x_0X_n)\big)x_0X_{n+1}$$

appears nontrivially, a contradiction.

CASE II. Suppose that $d$ is an inner derivation induced by an element $b\in Q$ such that $b^2=0$. Thus we have that $[-2bf(X_1,\dots,X_n)b, f(X_1,\dots,X_n)]\in Z(R)$ is satisfied by $\rho$. In case there exists $x_0\in\rho$ such that $\{bx_0,x_0\}$ are linearly $C$-independent, we have that $[[-2bf(x_0X_1,\dots,x_0X_n)b, f(x_0X_1,\dots,x_0X_n)],x_0X_{n+1}]$ is a non trivial GPI for $R$, a contradiction. Hence $\{bx_0,x_0\}$ are linearly $C$-dependent for all $x_0\in\rho$, that is there exists $\alpha\in C$ such that $(b-\alpha)\rho=0$. Thus we have that $[\alpha f^2(X_1,\dots,X_n)(\alpha-b),X_{n+1}]$ is satisfied by $\rho$, in particular $R$ satisfies:

$$[\alpha f^2(x_0X_1,\dots,x_0X_n)(\alpha-b),f(x_0X_1,\dots,x_0X_n)]=\alpha f^3(X_1,\dots,X_n)(\alpha-b)$$

for any $x_0\in\rho$. Since $R$ is not GPI, it follows that either $b=\alpha\in C$, which is a contradiction, or $\alpha=0$ which means $b\rho=0$, as required.

CASE III. Suppose that $d$ is an inner derivation induced by an element $b\in Q$ such that $b\rho=0$. Thus we have that $[-f^2(X_1,\dots,X_n)b^2,X_{n+1}]$ is satisfied by $\rho$, in particular $R$ satisfies:

$$[-f^2(x_0X_1,\dots,x_0X_n)b^2,f(x_0X_1,\dots,x_0X_n)]=f^3(x_0X_1,\dots,x_0X_n)b^2$$

for any $x_0\in\rho$. Again since $R$ is not GPI we conclude that $b^2=0$.

CASE IV. Next suppose that $d$ is not $Q$-inner derivation. By our assumption we have that $R$ satisfies

$$0=\Big[\big[f^{d^2}(xX_1,\dots,xX_n)+2\sum_i f^d(xX_1,\dots,d(x)X_i+xd(X_i),\dots,xX_n)$$
$$+2\sum_{i<j} f(xX_1,\dots,d(x)X_i+xd(X_i),\dots,d(x)X_j+xd(X_j),\dots,xX_n)$$
$$+\sum_i f(xX_1,\dots,d^2(x)X_i+2d(x)d(X_i)+xd^2(X_i),\dots,xX_n),f(xX_1,\dots,xX_n)\big],X_{n+1}\Big].$$

By Kharchenko's theorem [8],

$$\Big[\big[f^{d^2}(xX_1,\dots,xX_n)+2\sum_i f^d(xX_1,\dots,d(x)X_i+xr_i,\dots,xX_n)$$
$$+2\sum_{i<j} f(xX_1,\dots,d(x)X_i+xr_i,\dots,d(x)X_j+xr_j,\dots,xX_n)$$
$$+\sum_i f(xX_1,\dots,d^2(x)X_i+2d(x)r_i+xs_i,\dots,xX_n), f(xX_1,\dots,xX_n)\big],X_{n+1}\Big]=0$$

for all $X_1,\dots,X_n,r_1,\dots,r_n,s_1,\dots,s_n\in R$. In particular, for $r_1 = = r_2 = \cdots = r_n = 0$, we have

$$\Big[\big[f^{d^2}(xX_1,\dots,xX_n)+2\sum_i f^d(xX_1,\dots,d(x)X_i,\dots,xX_n)$$
$$2\sum_{i<j} f(xX_1,\dots,d(x)X_i,\dots,d(x)X_j,\dots,xX_n)+\sum_i f(xX_1,\dots,d^2(x)X_i,\dots,xX_n)+$$
$$+\sum_i f(xX_1,\dots,xs_i,\dots,xX_n), f(xX_1,\dots,xX_n)\big],X_{n+1}\Big]=0.$$

Hence $R$ satisfies the blended component

$$[[f(xs_1,\dots,xX_n), f(xX_1,\dots,xX_n)],X_{n+1}]=0$$

which is a nontrivial GPI for $R$, a contradiction.

THEOREM 2.4. *Let $R$ be an associative prime ring of char $R\neq 2$ with center $Z(R)$ and extended centroid $C$, $f(x_1,\dots,x_n)$ a nonzero multilinear polynomial over $C$ in $n$ noncommuting variables, $d$ a nonzero derivation of $R$ and $\rho$ a nonzero right ideal of $R$. If $[d^2(f(x_1,\dots,x_n)), f(x_1,\dots,x_n)]=0$ for all $x_1,\dots,x_n\in\rho$ then $\rho C=eRC$ for some idempotent $e$ in the socle of $RC$ and $f(x_1,\dots,x_n)$ is central-valued on $eRCe$ unless $d$ is an inner derivation induced by $b\in Q$ such that $b^2=0$ and $b\rho=0$.*

PROOF. Suppose $d$ is not a $Q$-inner derivation induced by an element $b\in Q$ such that $b^2=0$ and $b\rho=0$.

Now assume first that $f(\rho)\rho=0$, that is $f(x_1,\dots,x_n)x_{n+1}=0$ for all $x_1,x_2,\dots,x_{n+1}\in\rho$. Then by [12, Proposition], $\rho C=eRC$ for some idempotent $e\in soc(RC)$. Since $f(\rho)\rho=0$, we have $f(\rho R)\rho R=0$ and hence $f(\rho Q)\rho Q=0$ by [4, Theorem 2]. In particular, $f(\rho C)\rho C=0$, or equivalently, $f(eRC)e=0$. Then $f(eRCe)=0$, that is, $f(x_1,\dots,x_n)$ is a PI for $eRCe$ and, a fortiori, central valued on $eRCe$.

Next assume that $f(\rho)\rho\neq 0$, that is $f(x_1,\dots,x_n)x_{n+1}$ is not an identity for $\rho$ and then we derive a contradiction. By Lemma 2.3, $R$ is a GPI-ring and so is also $Q$ (see [1] and [4]). By [16], $Q$ is a primitive ring with $H = soc(Q) \neq 0$. Moreover, we may assume $f(\rho H)\rho H \neq 0$, otherwise by [1] and [4], $f(\rho Q)\rho Q = 0$, which is a contradiction. Choose $a_0, a_1, \ldots, a_n \in \rho H$ such that $f(a_1, \ldots, a_n)a_0 \neq 0$. Let $a \in \rho H$. Since $H$ is a regular ring, there exists $e^2 = e \in H$ such that

$$eH = aH + a_0H + a_1H + \cdots + a_nH.$$

Then $e \in \rho H$ and $a = ea, a_i = ea_i$ for $i = 0, 1, \ldots, n$. Thus we have $f(eHe) = f(eH)e \neq 0$. By our assumption and by [14, Theorem 2], we also assume that

$$[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)]$$

is an identity for $\rho Q$. In particular $[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)]$ is an identity for $\rho H$ and so for $eH$. It follows that, for all $r_1, \ldots, r_n \in H$,

$$0 = [d^2(f(er_1, \ldots, er_n)), f(er_1, \ldots, er_n)].$$

We may write $f(x_1, \ldots, x_n) = t(x_1, \ldots, x_{n-1})x_n + h(x_1, \ldots, x_n)$, where $x_n$ never appears as last variable in any monomials of $h$. Let $r \in H$. Then replacing $r_n$ with $r(1-e)$, we have

$$0 = [d^2(t(er_1, \ldots, er_{n-1})er(1-e)), t(er_1, \ldots, er_{n-1})er(1-e)]. \tag{4}$$

Now, we know the fact that $d(x(1-e))e = -x(1-e)d(e)$ and $(1-e)d(ex) = = (1-e)d(e)ex$ and so

$$\begin{aligned}(1-e)d^2(ex(1-e))e &= (1-e)d\{d(e)ex(1-e) + ed(ex(1-e))\}e\\ &= (1-e)d(e)d(ex(1-e))e + (1-e)d(e)d(ex(1-e))e\\ &= -2(1-e)d(e)ex(1-e)d(e).\end{aligned}$$

Thus using this facts, we have from (4),

$$\begin{aligned}0 &= (1-e)[d^2(t(er_1, \ldots, er_{n-1})er(1-e)), t(er_1, \ldots, er_{n-1})er(1-e)]\\ &= (1-e)d^2(t(er_1, \ldots, er_{n-1})er(1-e))t(er_1, \ldots, er_{n-1})er(1-e)\\ &= -2(1-e)d(e)t(er_1, \ldots, er_{n-1})er(1-e)d(e)t(er_1, \ldots, er_{n-1})er(1-e)\\ &= -2((1-e)d(e)t(er_1, \ldots, er_{n-1})er)^2(1-e).\end{aligned}$$

This implies

$$0 = -2\{(1-e)d(e)t(er_1, \ldots, er_{n-1})er\}^3$$

that is

$$0 = -2\{(1-e)d(e)t(er_1, \ldots, er_{n-1})eH\}^3.$$

By [6], $(1-e)d(e)t(er_1,\dots,er_{n-1})eH=0$ which implies

$$(1-e)d(e)t(er_1e,\dots,er_{n-1}e)=0.$$

Since $eHe$ is a simple Artinian ring and $t(eHe)\neq 0$ is invariant under the action of all inner automorphisms of $eHe$, by [5, Lemma 2],$(1-e)d(e)=0$ and so $d(e)=ed(e)\in eH$. Thus $d(eH)\subseteq d(e)H+ed(H)\subseteq eH\subseteq \rho H$ and $d(a)=d(ea)\in d(eH)\subseteq \rho H$. Therefore, $d(\rho H)\subseteq \rho H$. Denote the left annihilator of $\rho H$ in $H$ by $l_H(\rho H)$. Then $\overline{\rho H}=\dfrac{\rho H}{\rho H\cap l_H(\rho H)}$, a prime $C$-algebra with the derivation $\overline{d}$ such that $\overline{d}(\overline{x})=\overline{d(x)}$, for all $x\in\rho H$. By assumption, we have that

$$[\overline{d}^2(f(\overline{x_1},\dots,\overline{x_n})),f(\overline{x_1},\dots,\overline{x_n})]=0$$

for all $\overline{x_1},\dots,\overline{x_n}\in\overline{\rho H}$. By Theorem $A$, either $\overline{d}=0$ or $f(\overline{x_1},\dots,\overline{x_n})$ is central-valued on $\overline{\rho H}$.

If $\overline{d}=0$, then $d(\rho H)\rho H=0$ and so $d(\rho)\rho=0$. By Lemma 2.1, $d$ is an inner derivation induced by an element $b\in Q$ such that $b\rho=0$. Then for all $x_1,\dots,x_n\in\rho$, we have by assumption that

$$0=[[b,[b,f(x_1,\dots,x_n)]],f(x_1,\dots,x_n)]=-f^2(x_1,\dots,x_n)b^2.$$

By [3, Lemma 4], either $b^2=0$ or $f(\rho)\rho=0$. In both cases we have contradiction.

If $f(\overline{x_1},\dots,\overline{x_n})$ is central-valued on $\overline{\rho H}$, then $\rho H$, as well as $\rho$, satisfies $[f(x_1,\dots,x_n),x_{n+1}]x_{n+2}=0$. Then $\rho C=eRC$ for some idempotent element $e\in soc(RC)$ by [12, Proposition] and $f(x_1,\dots,x_n)$ is central-valued on $eRCe$ and we are done.

THEOREM 2.5. *Let $R$ be an associative prime ring of char $R\neq 2$ with center $Z(R)$ and extended centroid $C$, $f(x_1,\dots,x_n)$ a nonzero multilinear polynomial over $C$ in $n$ noncommuting variables, $d$ a nonzero derivation of $R$ and $\rho$ a nonzero right ideal of $R$. If $[d^2(f(x_1,\dots,x_n)),f(x_1,\dots,x_n)]\in Z(R)$ for all $x_1,\dots,x_n\in\rho$ then $\rho C=eRC$ for some idempotent $e$ in the socle of $RC$ and either $f(x_1,\dots,x_n)$ is central-valued on $eRCe$ or $eRCe$ satisfies $S_4(x_1,x_2,x_3,x_4)$ unless $d$ is an inner derivation induced by $b\in Q$ such that $b^2=0$ and $b\rho=0$.*

PROOF. Suppose $d$ is not a $Q$-inner derivation induced by an element $b\in Q$ such that $b^2=0$ and $b\rho=0$.

If $[f(\rho),\rho]\rho=0$, that is $[f(x_1,\dots,x_n),x_{n+1}]x_{n+2}=0$ for all

$x_1, x_2, \ldots, x_{n+2} \in \rho$, then by [12, Proposition], $\rho C = eRC$ for some idempotent $e \in soc(RC)$ and $f(x_1, \ldots, x_n)$ is central-valued on $eRCe$.

So, assume that $[f(\rho), \rho]\rho \neq 0$, that is $[f(x_1, \ldots, x_n), x_{n+1}]x_{n+2}$ is not an identity for $\rho$ and then we derive that $eRCe$ satisfies $S_4$. By Lemma 2.3, $R$ is a GPI-ring and so is also $Q$ (see [1] and [4]). By [16], $Q$ is a primitive ring with $H = soc(Q) \neq 0$. Moreover, we may assume $[f(\rho H), \rho H]\rho H \neq 0$, otherwise by [1] and [4], $[f(\rho Q), \rho Q]\rho Q = 0$, which is a contradiction. Choose $a_1, \ldots, a_{n+2}, b_1, \ldots, b_5 \in \rho H$ such that $[f(a_1, \ldots, a_n), a_{n+1}]a_{n+2} \neq 0$ and $S_4(b_1, b_2, b_3, b_4)b_5 \neq 0$. Let $a \in \rho H$. Since $H$ is a regular ring, there exists $e^2 = e \in H$ such that

$$eH = aH + a_1H + \cdots + a_{n+2}H + b_1H + \cdots + b_5H.$$

Then $e \in \rho H$ and $a = ea, a_i = ea_i$ for $i = 1, \ldots, n+2$, $b_i = eb_i$ for $i = 1, \ldots, 5$. Thus we have $f(eHe) = f(eH)e \neq 0$. Moreover, by [14, Theorem 2], we may also assume that

$$[[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)], x_{n+1}]$$

is an identity for $\rho Q$. In particular, $[[d^2(f(x_1, \ldots, x_n)), f(x_1, \ldots, x_n)], x_{n+1}]$ is an identity for $\rho H$ and so for $eH$. It follows that, for all $r_1, \ldots, r_{n+1} \in H$,

$$0 = [[d^2(f(er_1, \ldots, er_n)), f(er_1, \ldots, er_n)], er_{n+1}].$$

We may write $f(x_1, \ldots, x_n) = t(x_1, \ldots, x_{n-1})x_n + h(x_1, \ldots, x_n)$, where $x_n$ never appears as last variable in any monomials of $h$. Let $r \in H$. Then replacing $r_n$ with $r(1-e)$ and $r_{n+1}$ with $r_{n+1}(1-e)$, we have

$$(5) \quad 0 = [[d^2(t(er_1, \ldots, er_{n-1})er(1-e)), t(er_1, \ldots, er_{n-1})er(1-e)], er_{n+1}(1-e)].$$

Now, we know the fact that $d(x(1-e))e = -x(1-e)d(e)$, $(1-e)d(ex) = (1-e)d(e)ex$ and $(1-e)d^2(ex(1-e))e = -2(1-e)d(e)ex(1-e)d(e)$. Thus using these facts, we have from (5),

$$\begin{aligned}
0 &= [[d^2(t(er_1, \ldots, er_{n-1})er(1-e)), t(er_1, \ldots, er_{n-1})er(1-e)], er_{n+1}(1-e)] \\
&= [d^2(t(er_1, \ldots, er_{n-1})er(1-e)), t(er_1, \ldots, er_{n-1})er(1-e)]er_{n+1}(1-e) \\
&\quad - er_{n+1}(1-e)[d^2(t(er_1, \ldots, er_{n-1})er(1-e)), t(er_1, \ldots, er_{n-1})er(1-e)] \\
&= -t(er_1, \ldots, er_{n-1})er(1-e)d^2(t(er_1, \ldots, er_{n-1})er(1-e))er_{n+1}(1-e) \\
&\quad - er_{n+1}(1-e)d^2(t(er_1, \ldots, er_{n-1})er(1-e))t(er_1, \ldots, er_{n-1})er(1-e) \\
&= t(er_1, \ldots, er_{n-1})er(1-e)d(e)t(er_1, \ldots, er_{n-1})er(1-e)d(e)r_{n+1}(1-e) \\
&\quad + er_{n+1}(1-e)d(e)t(er_1, \ldots, er_{n-1})er(1-e)d(e)t(er_1, \ldots, er_{n-1})er(1-e).
\end{aligned}$$

Replacing $r_{n+1}$ with $t(er_1, \ldots, er_{n-1})er$ in the above relation, we get

$$2t(er_1, \ldots, er_{n-1})er((1-e)d(e)t(er_1, \ldots, er_{n-1})er)^2(1-e) = 0.$$

This implies

$$2((1-e)d(e)t(er_1, \ldots, er_{n-1})er)^4 = 0$$

that is

$$2\{(1-e)d(e)t(er_1, \ldots, er_{n-1})eH\}^4 = 0.$$

By [6], $(1-e)d(e)t(er_1, \ldots, er_{n-1})eH = 0$ which implies

$$(1-e)d(e)t(er_1e, \ldots, er_{n-1}e) = 0.$$

Since $eHe$ is a simple Artinian ring and $t(eHe) \neq 0$ is invariant under the action of all inner automorphisms of $eHe$, by [5, Lemma 2],$(1-e)d(e) = 0$ and so $d(e) = ed(e) \in eH$. Thus $d(eH) \subseteq d(e)H + ed(H) \subseteq eH \subseteq \rho H$ and $d(a) = d(ea) \in d(eH) \subseteq \rho H$. Therefore, $d(\rho H) \subseteq \rho H$. Denote the left annihilator of $\rho H$ in $H$ by $l_H(\rho H)$. Then $\overline{\rho H} = \dfrac{\rho H}{\rho H \cap l_H(\rho H)}$, a prime $C$-algebra with the derivation $\overline{d}$ such that $\overline{d}(\overline{x}) = \overline{d(x)}$, for all $x \in \rho H$. By assumption, we have that

$$[[\overline{d}^2 f(\overline{x_1}, \ldots, \overline{x_n}), f(\overline{x_1}, \ldots, \overline{x_n})], \overline{x_{n+1}}]] = 0$$

for all $\overline{x_1}, \ldots, \overline{x_n} \in \overline{\rho H}$. By Theorem 1.3, either $\overline{d} = 0$ or $f(\overline{x_1}, \ldots, \overline{x_n})$ is central-valued on $\overline{\rho H}$ or $\overline{\rho H}$ satisfies the standard identity $S_4(\overline{x_1}, \ldots, \overline{x_4})$.

If $\overline{d} = 0$, then as in the proof of Theorem 2.4, we have $d(\rho)\rho = 0$ and hence by Lemma 2.1, $d$ is an inner derivation induced by an element $b \in Q$ such that $b\rho = 0$. Thus for all $r_1, \ldots, r_n \in \rho H$,

$$[d^2(f(r_1, \ldots, r_n)), f(r_1, \ldots, r_n)] = -f(r_1, \ldots, r_n)^2 b^2 \in C.$$

Commuting both sides with $f(r_1, \ldots, r_n)$, we obtain $f(r_1, \ldots, r_n)^3 b^2 = 0$. In this case by Lemma 2.2, since $b^2 \neq 0$, $f(\rho H)\rho H = 0$. If $f(\rho H)\rho H = 0$, then $[f(\rho H), \rho H]\rho H = 0$, a contradiction.

If $f(\overline{x_1}, \ldots, \overline{x_n})$ is central-valued on $\overline{\rho H}$, then we obtain that

$$[f(x_1, \ldots, x_n), x_{n+1}]x_{n+2}$$

is an identity for $\rho$, a contradiction.

Finally, if $S_4(\overline{x_1}, \ldots, \overline{x_4})$ is an identity for $\overline{\rho H}$, $S_4(x_1, \ldots, x_4)x_5$ is an identity for $\rho H$ and so for $\rho C$ and this contradicts the choices of the elements $b_1, \ldots, b_5 \in \rho H$. Therefore, we conclude that in any case $\rho C$ satisfies a polynomial identity, hence by [12, Proposition], there exists an idempotent $e \in Soc(RC)$ such that $\rho C = eRC$, as desired.

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