Elem. Math. 68 (2013) 88 – 88
0013-6018/13/020088-1
DOI 10.4171/EM/225

Elemente der Mathematik

Rezensionen

M. Jarnicki, P. Pflug: First Steps in Several Complex Variables: Reinhardt Domains. viii+359 pages, €58. EMS Textbooks in Mathematics, European Mathematical Society, Zürich, 2008; ISBN 978-3-03719-049-4.

As the authors notice in their preface, in contrast to the case of holomorphic functions of one variable, there are very few textbooks on functions of several complex variables, especially introductory texts. The idea of introducing holomorphic functions of several variables from the viewpoint of Reinhardt domains is interesting for at least two reasons. The first one is the relative simplicity of the geometry of Reinhardt domains: they are those domains in $\mathbb{C}^n$ which are invariant under all transformations of the form $(z_1, \dots, z_n) \mapsto (e^{i\theta_1} z_1, \dots, e^{i\theta_n} z_n)$, and thus such a domain D is completely characterized by its image in $\mathbb{R}^n_+$ given by $\{(|z_1|, \dots, |z_n|) : (z_1, \dots, z_n) \in D\}$. The second reason is related to power series $S(z) = \sum_{\alpha \in \mathbb{Z}^n_+} a_\alpha z_1^{\alpha_1} \cdots z_n^{\alpha_n}$ (centered at the origin): the *domain of convergence* of such a series is the set $\mathcal{D}_S$ of elements $z \in \mathbb{C}^n$ which have a neighbourhood V_z where the series converges absolutely. Then such a domain, whenever non empty, contains 0 and is a Reinhardt domain. The fact that every holomorphic function defined in a neighbourhood of 0 admits a power series expansion as above motivates the study of Reinhardt domains.

Before describing the content of the textbook, it should be noted that here holomorphic functions are mostly considered as a tool for the study of Reinhardt domains and that they are not much studied for themselves. The first chapter is the longest one, it extends up to a half into the book. It is devoted to the definition and the main properties of holomorphic functions of several variables and to the basic properties of Reinhardt domains. It contains theorems on holomorphic functions that show fundamental differences with the case of one complex variable: Hartogs extention theorem, Riemann removable singularities theorem, and a characterization of those Reinhardt domains that are domains of holomorphy, to quote just some results. I just regret the absence of the Weierstrass preparation theorem, though. Its right place would have been in Chapter 1. The second chapter studies biholomorphic domains and their groups of automorphisms, with emphasis on the biholomorphism equivalence of Reinhardt domains. In the third chapter, the authors consider Fréchet spaces $\mathcal{S}(D)$ of holomorphic functions (for instance the space of all bounded holomorphic functions on D) for domains D that are maximal domains of existence of functions from $\mathcal{S}$, and they get characterizations of Reinhardt domains of holomorphy for suitable Fréchet spaces $\mathcal{S}$. In order to describe the content of the last chapter, let us observe that if we wish to prove that two given domains D_1 and D_2 are not biholomorphically equivalent, we can try to prove that their automorphism groups $\mathrm{Aut}(D_j)$ are not isomorphic, or that they do not have the same amount of holomorphic functions of some type (for instance bounded holomorphic functions). Biholomorphically invariant pseudodistances provide such a tool, and it is the main subject of Chapter 4.

Finally, it is worth saying that the book is well written, that it contains a lot of illuminating examples and stimulating exercises which force the reader to have an active and enriching attitude in studying this fascinating area of mathematics.

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