

COMMISSION INTERNATIONALE  
DE L'ENSEIGNEMENT MATHÉMATIQUE  
(THE INTERNATIONAL COMMISSION  
ON MATHEMATICAL INSTRUCTION)

*DISCUSSION DOCUMENT FOR THE TWENTY-FIRST ICMI STUDY*

MATHEMATICS EDUCATION AND LANGUAGE DIVERSITY

EXECUTIVE SUMMARY

Around the world, mathematics is learned and taught in situations of language diversity. Whether through historical multilingualism, migration, colonization, globalization or other factors, mathematics classrooms frequently involve multiple language use, learning through second or additional languages, learning through minority or oppressed languages, or through majority or dominant languages. Increasing recognition and awareness of this long-standing reality have led to a growing body of research that seeks to understand the relationship between different facets of language diversity and mathematics learning and teaching. It is time to review this work critically, consider implications for mathematics classroom practices, and set an agenda for future research.

The principal aims of ICMI Study 21 “Mathematics education and language diversity” are:

- to gather together a community of researchers who are currently addressing issues of language diversity as they relate to mathematics education;
- to reflect on the current state of research on these issues and propose a research agenda for the future;
- to disseminate findings from research to date and issues for future work to the wider mathematics education research community and to practitioners.

In order to fulfil these aims and following the tradition and example of past ICMI Studies, the International Programme Committee is planning and conducting the following activities:

- the publication of this Discussion Document lays out the background, theoretical foundations, focus, rationale, and orientation for the Study, along with sample research questions;
- the organization of an invitational “working conference” to gather researchers who work on language diversity issues in mathematics education;
- the preparation and publication of a Study Volume to present the ‘state of the art’ and set out an agenda for future research;
- the preparation and publication of materials for practitioners to share key findings, concepts and issues and thereby inform mathematics classroom practice.

Note that, although this work is referred to as a ‘study’, neither ICMI nor the IPC will be conducting an empirical research project.

#### A. FOCUS OF THE STUDY

Across the world, the teaching and learning of mathematics occurs in contexts of linguistic and cultural diversity. How do we work with, and work within, this diversity to enhance the learning and teaching of mathematics? In particular, how can the range and complexity of learners’ language backgrounds be most effectively used to promote their mathematical learning? These questions are central to this Study.

Research within the field of mathematics education uses a variety of words to characterise language diversity, including, but not limited to, ‘multilingual’, ‘bilingual’, ‘second language’, ‘additional language’ or ‘minority language’. In this document, for convenience the term ‘multilingualism’ is used.

All of the words mentioned above involve, however, assumptions about the nature of language diversity. These assumptions are often problematic. The Study will, therefore, also seek to examine how constructs about language, discourse and multilingualism that have their origins in language studies are pertinent to mathematics education. Equally, the Study will consider how mathematics educators can conceptualize language diversity in ways that are consistent with the major objectives of mathematics education research and practice.

Given the growth of research in this area and the increasing importance and significance attached to language diversity and multilingualism, this ICMI Study is timely: now is an appropriate time to build on the collective wisdom that exists internationally. ICMI is well placed to support and offer expert guidance on the teaching and learning of mathematics in the context of language diversity in our classrooms.

## B. RATIONALE FOR THE STUDY

One of the most important goals of mathematics education is to facilitate successful mathematical learning. This learning occurs in complex linguistic environments; complexity derives from the multiple language backgrounds, mathematical languages and semiotic systems that are always present. In some cases, learners and teachers feel at home in this environment, perhaps because all participants share similar language backgrounds. In other cases, learners and teachers may feel that they face ‘language barriers’. These barriers may vary from a lack of basic vocabulary to other, less obvious social, cultural or political issues. Whether learners and teachers feel at home in a linguistically diverse environment or not, they routinely find ways to discuss and learn mathematics.

Multilingualism is steadily rising up the political and educational agenda internationally, driven by a combination of economic and political integration. For example, both the European Union and the African Union have policies for the development and management of multilingualism. Economic change leads to migration of people and hence of languages, as well as new demands for foreign language skills, all of which have an impact on education in general, and mathematics education in particular.

In many countries, a global language, such as English, is linked to dominant discourses through high-status jobs or access to the dominant class. As a result, education systems come under pressure to use the global language for learning and teaching. Language in education policies can lead to a class divide in schooling generally, and mathematics education in particular, because students who have learnt mathematics in their local languages are expected to “switch” to the global language at some point in their education. Many students may be ill-prepared to engage in such change.

The three previous paragraphs highlight different dimensions of language diversity in different contexts: that of classroom practices, that of international policies, and that of national language policies. There are inconsistencies and contradictions between these dimensions. These inconsistencies emerge in relation to the somewhat simplistic, often incompatible, views of language evident across these dimensions. A better theoretical understanding of language as it relates to mathematics education is needed, a perspective that is congruent with these and other dimensions.

## C. FRAMING MULTILINGUALISM

Multilingualism is a contested concept. Furthermore, different theoretical lenses offer particular ways of seeing how multiple language resources interact in any context of practice. Quite frequently multilingualism is defined in terms of *plural monolingualism* — the idea that multiple languages are used discretely and distinctly, even where individuals may be speakers of several languages. Hence boundaries between languages are sharper. Languages may be associated with particular activities, institutions or contexts on a language-by-language basis, so that one language may be seen as ‘for school’, while another language is seen as ‘for home’. An alternative approach is to think in terms of *hybridized multilingualism* — the idea that languages are *interwoven* within the fabric of social life, to the extent that the boundaries between them can be hard to clearly identify (and, indeed, conceptually problematic). Of course, the nature of the hybridity, the emphasis given to different component languages, the

values associated with ‘purity’ or qualifications in one or another language vary from region to region, as well as social context. The hybrid language mix used and valued in mathematics classrooms may be different from the mix used and valued in bars or tea-houses. Of course, the above terms are somewhat idealised, in that language use is more fluid, complex and subtle than a simple two-sided distinction. Besides these two views, there are still other understandings that exist and that could be productive for mathematics education.

In the same way that multilingualism has different meanings, many other related terms such as bilingual, indigenous, native speaker or language learner are also problematic and there is no consensus about their use, meaning or value. We cannot, however, avoid using such terms and they have helped researchers identify the diversity of settings in which multilingualism is apparent. Some examples of these settings are:

- multilingual societies where people are used to using several languages (e.g. South Africa);
- multilingual societies with more than one official language, one of them with a higher status than others (e.g. Catalonia; Wales);
- bilingual societies in which two languages are sometimes used in support of minority language(s) (e.g. Peru) or in which the use of minority languages in the classroom is restricted by law (e.g. some states in the USA);
- societies previously seen as monolingual where immigration has made language diversity more salient (e.g. Europe, Australia);
- societies in which a foreign language is taught through subjects like mathematics (e.g. Czech Republic);
- societies in which mathematics education for indigenous language speakers is conducted in a majority colonising language (e.g. Africa, South America);
- societies where languages of instruction change across primary, secondary, and tertiary levels and/or have changed during specific historical moments (e.g. Pakistan, Algeria).

#### D. MATHEMATICS EDUCATION AND MULTILINGUALISM

Until the 1980s multilingualism has received little attention from many researchers in mathematics education. However, students and teachers regularly make use of different languages available to them in their mathematical work. In recent years, researchers have examined several aspects of this situation, including the role of multilingualism in students’ learning and understanding of mathematics, the relationship between multilingualism and mathematical attainment and the ways in which mathematics teachers deal with multilingualism in their classrooms.

The teaching and learning of mathematics in multilingual contexts is thus a substantive issue in mathematics education, and one around which there has been recent but considerable growth in research and practical knowledge. Participants in this field span a wide range of countries and contexts on every continent, as indicated above.

Research to date on multilingualism and the teaching and learning of mathematics has served to highlight some of the challenges faced by teachers and learners, as well as to identify some successful teaching strategies and approaches to learning. Researchers have consistently argued that multilingualism is potentially an advantage

for learning mathematics and have sought to refute the long-standing assumption that multilingualism is a *problem* in mathematics classrooms. Most studies have been located in single classrooms or in a small number of classrooms and have generally focused on single age groups, although there are some exceptions.

Not enough is known about the specific benefits, challenges and interventions appropriate for different levels of education (pre-primary, primary, secondary and tertiary). And little is known about whether and how the demands of multilingualism and mathematics change with different domains of mathematics (e.g. number, algebra, probability). Tackling these and other issues would lead to valuable additional knowledge with implications for practitioners and policy makers.

Given the widespread occurrence of multilingualism, it is surprising that it is rarely mentioned in published research in mathematics education, mathematics education policy or mathematics curricula. Where multilingualism is mentioned, it is often as an after-thought, a challenge, or a problem to be overcome, but mostly just silence. Multilingualism may therefore be an unmentioned feature, an unmarked factor in much of the work of researching and organising mathematics education. Moreover, this work may also have contributions to make to research and practice in what are considered “monolingual” settings. Thus, we hope this Study will be of interest and value to all those working in mathematics education in general, not just to those with an interest in multilingualism or working in multilingual classrooms.

#### E. ORIENTATION

As the preceding sections show, thinking about language diversity in the learning and teaching of mathematics involves a good deal of complexity. In this section, we highlight some of the different perspectives from which this learning and teaching can be examined. Each perspective reveals slightly different issues, features and questions; each highlights a different part of the complexity. We see these different perspectives as relevant and important and we expect to see them informing the Study as it progresses.

*The mathematical aspects of learning and teaching mathematics in multilingual classrooms.* As noted above, the existing literature focuses on multilingualism and the learning and teaching of mathematics in general. But there has been little systematic focus on whether and how the demands of multilingualism and mathematics change with different domains of mathematics. In the mathematics classroom, we are not teaching and learning undefined and vague objects and processes, but *mathematical* objects and processes, with their own differing and specific natures and structures. Is research in this area sufficiently focused on the specificity of the mathematics in our on-going research discourse? Is it the same to have multilingualism in a class where the teacher is introducing objects such as the idea of a “vertical asymptote,” in comparison to a class where the teacher is introducing non-mathematical objects, mathematical objects from other domains, or mathematical objects being presented with non-symbolic language? How do we include consideration of the intersection of the language of mathematics with the mathematics itself in the multilingual classroom? And finally, of course, we must be clear about the mathematical practices that are intervening in our empirical studies. Various analytic tools may contribute to this aspect, including, for example, work on semiotics or functional linguistics.

*The psychological aspects of learning and teaching mathematics in multilingual classrooms.* Any research concerning issues like learning, understanding, thinking, feeling or knowing necessarily involves some kind of psychological perspective. Many mathematics educators have been informed to a greater or lesser extent by socio-cultural psychology, derived from the work of Vygotsky, although this is by no means the only perspective. Equally, research on the psychology of language learning, processing and use may offer useful insights, such as research in the psycholinguistics of bilingualism or multilingualism.

*The sociolinguistic aspects of learning and teaching mathematics in multilingual classrooms.* Multilingual mathematics classrooms do not exist in a vacuum; they are influenced by the society in which they are situated. Sociolinguistics offers ideas on and insights into the nature of language use in multilingual settings. Such work includes, for example, research on code-switching, as well as research into the distribution of different languages and language varieties across different groups, professions, locations, etc. Such work is relevant to multilingual mathematics classrooms, where the use of multiple/mixed languages is potentially a relevant feature of interaction, and which, therefore, needs to be carefully conceptualised.

*The political aspects of learning and teaching mathematics in multilingual classrooms.* Languages are implicated in issues of power and access, whether to education, employment or positions of authority. Language use is often a marker of social class, with English often (though not always) having a particular cachet. The politics of language influence multilingual mathematics classrooms in many ways, including, for example, the choice of 'official' language, the desire of students or their parents to be taught in a particular language, or the challenges of accommodating competing languages. In some cases, language policy or practice in multilingual mathematics classrooms (including the mathematics curriculum) may be implicated in the maintenance or suppression of threatened languages or languages of marginalised groups. These languages may include 'non-standard' languages, such as creoles, or various kinds of 'street' language.

*The cultural aspects of learning and teaching mathematics in multilingual classrooms.* Language and culture are intertwined. In any culture, different quantitative and spatial (i.e. mathematical) relationships and practices can be generated, organized and transmitted informally to solve immediate needs. In this way mathematics and language are incorporated into the core of the learning-by-doing processes of a community and are therefore part of what we call culture. From this standpoint, language and mathematics are generated within multiple cultural communities, which include, but are not limited to, academic mathematics. The transition from mathematical ideas embedded in learning-by-doing processes of any culture to more abstract mathematical objects is of particular interest.

*The discursive aspects of learning and teaching mathematics in multilingual classrooms.* The business of learning and teaching mathematics is conducted largely through interaction, including talk, as well as the use of written symbols, diagrams, charts and other texts. In multilingual classrooms, these various texts may involve the use of several languages. Discursive perspectives on this interaction can provide fine-grained analysis of how the joint construction of mathematical thinking is mediated by

these various texts and languages. These aspects also relate mathematical language and discourse. Ways of communicating in a mathematics classroom to construct knowledge meanings and ideas are different from those in other domains or classrooms. While mathematical language and discourse may be expressed in the language of instruction, communication processes such as reading, writing and talking specifically about mathematics have their own features.

Of course, the above perspectives all overlap — they are ways of understanding the same situation. Indeed, at a meta-level, we also need to account for the epistemological orientation of this Study. As the range of perspectives summarised above shows, our work potentially draws on many areas of expertise, notably various branches of applied linguistics. Applied linguists do not often visit mathematics classrooms, and even when they do, they rarely have any mathematical training. We cannot, therefore, expect applied linguistics to provide ‘off-the-shelf’ concepts that we can straightforwardly apply. On the other hand, applied linguists can provide important expertise concerning the nature of language diversity in society. We need to make use of this expertise to enhance our own work. Ultimately, we hope that this work will lead to the development of tools and ideas that offer mathematics researchers, teachers and educators a way forward in developing teaching and promoting learning in multilingual mathematics classrooms. Of course, it could mean that ideas, tools or categories from linguistics are integrated with mathematics education and developed into new or modified ideas that help us see the way forward for mathematics teaching and learning in multilingual classrooms.

#### F. THEMES FOR THE STUDY

Provided below is a series of themes proposed by the International Programme Committee. Around each theme is a selection of questions that can serve as an initial focus for potential papers for the Study. These themes and questions are a starting point and could be adapted, modified or developed as the Study progresses and is finalised.

##### 1. *Teaching mathematics in diverse language contexts*

This theme focuses on language issues in the teaching of mathematics in different language contexts. The assumption here is that language issues that emerge in different contexts are not only shaped by the complexities of the language of mathematics, but are also shaped by the linguistic contexts in which mathematics is taught and learned:

- a) What strategies and resources can educators in multilingual classrooms use to teach mathematics effectively to learners who are in the process of learning the language of instruction?
- b) How do assessment and curriculum systems relate to mathematics teaching and language policies in diverse language contexts?
- c) Which current teaching practices are sensitive to the relationship between multilingualism and mathematics learning?
- d) What are the relationships between teaching language and teaching mathematics?

## 2. *Teacher education for diverse language contexts*

The focus here is on issues in and for teacher education in diverse language contexts. An assumption is that teacher education principles and practices are rooted in the real world of the classrooms, and therefore must take into account the different language contexts in which mathematics is taught:

- a) Which current practices in teacher education are sensitive to the relationships between multilingualism and mathematics teaching and learning?
- b) What can be done to prepare teachers to teach mathematics effectively in multilingual classrooms?
- c) What kind of data from multilingual classrooms would be useful in designing teacher education programs?
- d) What knowledge and skills do teachers need to teach in multilingual classrooms and what are teachers' perspectives on this question?

## 3. *Researching mathematics teaching and learning in multilingual contexts*

This theme focuses on the theories and methods that relate to doing research in multilingual contexts:

- a) What types of theories and methods enable the development of research in this area?
- b) What ethical issues arise in pursuing this kind of research and how can researchers address them?
- c) On what basis can researchers interpret the mathematical worlds of students who come from linguistic backgrounds with which they are not familiar?
- d) To what extent is mathematics education research sensitive to linguistic diversity?

## 4. *Mathematics, multilingualism and society*

Diverse language settings reflect the broader social, cultural and political issues. When considering classroom dynamics, an assumption is that they are shaped by and go on to shape the broader social and political settings:

- a) To what extent and for what purposes does research in multilingual contexts need to address multicultural issues?
- b) What role does teaching mathematics in diverse language settings play in reproducing or challenging prevailing social patterns?
- c) How can researchers engage productively with policy makers involved in mathematics education to address language diversity?
- d) What is the relationship between the teaching and learning of mathematics in multilingual settings, and wider social discourses?

## 5. *Student mathematics learning and experiences in multilingual classrooms*

This theme brings forth issues regarding students' learning and students' experiences of learning mathematics in different language contexts. An underlying assumption is that it is important to focus on learners in order to support them to learn mathematics effectively:

- a) What are the characteristics of students' mathematical discussions and explanations in different languages, in multiple classroom contexts, and in multiple mathematical domains?

- b) What are the demands on multilingual students learning mathematics in different mathematical domains (i.e. algebra, geometry, etc.) and at different ages ?
- c) How do students themselves see and describe their experiences in multilingual mathematics classrooms ?
- d) What are students' strengths and resources, what can we learn from successful students, and how can instruction build on these resources, strengths, and successes in linguistically diverse settings ?

#### G. PARTICIPATION IN THE STUDY

*Design of the Study.* The ICMI Study 21 on Mathematics education and language diversity is designed to enable researchers and practitioners around the world to share research, theoretical work, projects descriptions, experiences and analyses. It will consist of two components: the *Study Conference* and the *Study Volume*.

1. The *Study Conference* will be held in São Paulo, Brazil, on 16–20 September, 2011, the number of participants to be invited being limited to at most 120. It is hoped that the Conference will attract not only established researchers but also some “newcomers” to the field and mathematics teachers with interesting and refreshing ideas or promising work in progress, as well as participants from countries usually under-represented in mathematics education research meetings.

Participation in the Study Conference is by invitation only, based on a submitted contribution. Proposed contributions will be reviewed and selections made according to the quality of the work, the potential to contribute to the advancement of the Study, with explicit links to the themes and related questions outlined in this Discussion Document, and the need to ensure diversity among the perspectives. Accepted papers will appear in the Conference Proceedings that will be published by ICMI as a CDROM, and will form the basis of the Study's scientific work.

An invitation to the Conference does not imply that an oral presentation of the submitted contribution will be made during the Conference, as the International Programme Committee (IPC) may decide to organize it in other ways that facilitate the Study's effectiveness and productivity. This will be a working conference, every participant being expected to make a scientific contribution. We therefore hope that the participants will represent a variety of backgrounds, expertise, experience and nationalities that will lead to a suitable coverage of the Study theme, its different topics and the related questions.

It is the IPC's hope that the Conference will attract mathematics education researchers, mathematics teacher educators, policy makers and linguists who are interested and do work in the area of mathematics education and language diversity as well as mathematics teachers in multilingual classrooms.

Unfortunately an invitation to participate in the Conference does not imply financial support from the organisers, and participants should finance their attendance at the Conference. It is hoped that this invitation will help participants to get appropriate support from their own countries. Funds are being sought to provide partial support for participants from non-affluent countries, but the number of such grants will be limited.

2. The *Study Volume*, a post-conference publication, will appear in the New ICMI Study Series (NISS), published by Springer. Acceptance of a paper for the Conference does not ensure automatic inclusion in this book. The Study Volume will be based on selected contributions as well as on the outcome of the Conference. The exact format of the Study Volume has not yet been decided but it is expected to be an edited coherent book that can hopefully serve as a standard reference in the field for a foreseeable future. A report on the Study will be presented during the 12<sup>th</sup> International Congress on Mathematical Education (ICME-12), to be held in Seoul, Korea, on July 8–15, 2012.

*Call for Contributions.* The International Programme Committee hereby invites individuals or groups to submit original contributions on specific questions, problems or issues related to the topic of the Study for consideration by the Committee. A submission should represent a significant contribution to knowledge about the Study topic and may address questions from one or more of the Study themes (see section F above), or further issues relating to these, but it should clearly identify its primary focus. The IPC welcomes high-quality proposals from researchers and practitioners who can make solid practical and scientific contributions to the Study. New researchers in the field, teachers and participants from countries under-represented in mathematics education research meetings are especially encouraged to submit contributions. To ensure a rich and varied scope of resources for the Study, participation from countries with different economic levels or with different cultural heritage and practices is encouraged.

Those who would like to participate should prepare a paper addressing matters raised in this document or other issues related to the topic of the Study. Papers concerning work that is ongoing or yet to be carried out are also welcome. Research questions should be carefully stated and expected results should be formulated, if possible with reference to earlier and related work. These papers should be submitted *no later than 30 November 2010*, to both co-chairs of the Study by e-mail. All such documents will be regarded as input to the planning of the Study Conference and will assist the IPC in making decisions on the invitations, to be issued no later than 28 February, 2011.

*General guidelines on submission of papers.* The papers must be written in English. The format of papers must be as follows:

- A maximum of 8 pages, including references and figures.
- Times New Roman 14-point font, 16-point line space, and 6 points between paragraphs; occupying a frame of 170 by 247 mm.
- The title (in 16-point bold capitals), author(s) name(s) (in 14-point bold), and affiliation(s) of author(s) (in 14-point italics) should appear in this order centered, all in Times New Roman.
- The paper must begin with an abstract of up to 10 lines, single-spaced, in italics.
- Video clips may be referred to in the paper and a link should be provided.

Further technical details about the format of submissions will be available on the Study website <http://www.icmi-21.com>, which will be progressively updated with all study and travel information.

## H. STUDY TIMELINE

28 February 2010: Launch of study website  
 30 November 2010: Deadline for submission of conference papers  
 28 February 2011: Letters of acceptance of papers and invitation to authors  
 15 April 2011: Closing date for registration and submission of final versions of papers  
 16–20 September 2011: Study Conference (in São Paulo, Brazil)

## I. INQUIRIES

Inquiries on all aspects of the Study and suggestions concerning the content of the Study Conference should be sent to both co-chairs:

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## J. MEMBERS OF THE INTERNATIONAL PROGRAMME COMMITTEE

Mamokgethi SETATI (South Africa), College of Science, Engineering and Technology, University of South Africa, *co-chair*

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