“Liberté aux professeurs associés!” Interview with Alexandre Aleksandrovich Kirillov

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We are very glad to see you here in Reims at this very special conference dedicated to your 3⁴th birthday. We would like to ask you a few questions. To start with, how did you choose to do mathematics?

You mean, how did I choose to be a mathematician? Well, I think till the fifth grade at school,[1] my dream was to be a pilot. But then my eyesight was not very good; at the end of school I had –4 and at university –5, so the career of a pilot was closed for me. Also, I like very much to dive but I’ve never thought of being a professional diver. I was a member of the university diving team and even won the “All-Union Student’s Game” [“Универсиада” – Universiade], which involved university teams from all over the Soviet Union competing in different forms of sports. That was a very interesting story but not for the official record.

Starting from the 6th grade, I participated in the mathematical Olympiads: my teacher at school said that there was such a thing in the 6th grade, I think. I did not go to the mathematical circles[2] because I was rather shy and in mathematical circles they make you answer questions. In the Olympiads, nobody asked me anything; I just had a problem and I needed to write a solution, that’s it. In the 6th grade, I received a small diploma “Грамота”. The official Olympiad started in the 7th year and it was joint with the 8th year; there, I received an honorary diploma [“Похвальная Грамота”]. In the 8th year, I got the 2nd prize and for the 9th and 10th years, I got the 1st prize. It was natural to go to the Mekhmat – Department of Mechanics and Mathematics, so I did.

Doctor Honoris Causa ceremony. (Photo Vladimir Salnikov)

And your interest in mathematics arose due to the Olympiads or earlier?

In school, I never had any trouble solving mathematical problems. And I realised, starting from the 6th grade or a bit earlier, that I knew mathematics better than my teacher… But I had a rather good teacher who also understood this [laughs], so we were friendly.

Your teachers did not influence you?

Not in school.

Nor your parents?

My parents were not in science. I was the first one. Well, my mother was a doctor but not from a dynasty. She was the first intellectual in her family. At the time, medicine was at Moscow State University but situated in a different place.

But do you think Olympiads are important just to drive curiousity about mathematics or are they important to develop researchers?

No, there were no ideas about research at all. It was more about getting interested in mathematics. You see, I do not believe that any scientist does something because

[1] Fifth year out of 10 (later 11) in the Russian educational system (roughly 12 years old).

[2] A longstanding tradition of mathematical clubs at various stages of school education in the Soviet Union (see, for example, the Malyi Mekhmat article “University Goes to School”, EMS Newsletter 101, September 2016).

Address by M. Pevzner during the ceremony. (Photo Vladimir Salnikov)

he thinks that it is useful for humanity. The only force that brings us to science is curiosity – natural human curiosity.

Could you tell us something about Mekhmat during the time of your studies?

Oh, I can say a lot of things about Mekhmat at the time. It was a great time. I arrived at the university in 1954 at the beginning of the Great Decade ["Великое десятилетие"] (the Khruschev era), which started in 1954 and finished in 1964. And, you see, it is a psychological law that a human being estimates the universe not by an absolute value but by its derivative in time. If you live poorly but things are getting better every day, you are happy. If you live very well but your situation is slightly declining, you are unhappy. And that was the case for a long time, during which the situation in Russia was getting better and better (after the War, at which point it was so low that there was no way for it to get worse). Then, up to the 1960s, the situation got better and better.

It was the second year after Mekhmat had moved to a new building of the university. Before that, it was in the centre, in a very famous building (those who graduated before me liked that one better but for us it was the new one). We have never called it the "University"; we called it the "Temple" ["Храм"]. "Are you going tomorrow to the Temple of Science?" ["Храм Науки"] – it was official – the journalists invented this term. And, somewhat ironically… it really was a temple; it was a paradise. Inside, there was a very good dining room – it was pretty cheap, maybe sponsored by the State, and it was rather good. Later, it got worse and worse but at that time it was excellent. For example, there was a period of friendship with China and there was an "island" in the students' cafeteria especially for Chinese students; the Chinese told me that they had never seen such a thing in China. They knew that it existed but it was affordable only for "big shots" and not for ordinary people. And the Chinese cuisine was of a very high standard and pretty cheap.

And can you speak about a few professors, seminars or activities, maybe someone whom you appreciated?

I think the greatest thing about Moscow University at the time was that there were a lot of good seminars. First of all, there were many seminars not only for professionals but for young mathematicians – for the youngest. Among them, there were two most popular seminars.

One was by Anatol Vitushkin, who was a student of A.S. Kronrod, a very well known educator. He more or less imitated the style and strategy of Kronrod and ran his seminar according to those rules. It was an arguable tactic because the idea was that you never had to read anything. If you wanted to know a subject, you had to be given several problems – key problems – and start to solve these problems by yourself. And that's the only way to study mathematics – no books.

As far as I know, he was also associated with Konstantinov?

At that time, Konstantinov was still in the Department of Physics and he did not collaborate with Vitushkin until later.

And the second lecturer was Evgeniy Borisovitch Dynkin, also a known educator and a man with rather original ideas (but a very bright man and someone who was excited about teaching). He was our lecturer in analysis and completely overturned the ideas of how to explain analytic things to students. I even used some of his tricks when I taught mathematics in America but it was not very successful – they are quite different sorts of students.

So, those were two very good seminars for the youngest people – yesterday's schoolchildren.

And, in my third year, I attended Gelfand's seminar for the first time. Alik Berezin,[3] my late friend, took me to Gelfand's seminar and that was also a very nice story (but it would also take time – I will speak about it later maybe).

At that time, the great seminars were Gelfand's, Kolmogorov's and Petrovskii's. I don't know about algebra seminars at the time; Shafarevich started a bit later… As for topology, certainly there was Pavel Sergeevitch Alexandrov but this was very specialised in set-theoretical topology. Modern topology came with Mikhail Mikhailovitch Postnikov, who was not officially a member of the faculty; he ran a seminar on modern topology and S.P. Novikov was a student of this seminar.

Five years passed very quickly and, after that, my generation started to teach. Every good mathematician of my age, a year before or a year later became a "chef" of a seminar. I had a seminar on representation theory, Arnold had one on Hamiltonian mechanics and differential equations, Manin on algebra, Vinberg on Lie groups… Who else… Sinai, Anosov – probability and dynamical systems. And it is very interesting that there were a cluster of good mathematicians within three years at university who were very bright. And the next such cluster, the next "wave", appeared after ten years: the age of Kazhdan, Margulis, Katok…

After that, it is already difficult to say; the situation changed globally.

You've mentioned representation theory – so how did you choose your main field? Who influenced your choice?

[3] The nickname of Felix Alexandrovitch Berezin.

Well, I never chose a subject in mathematics… I followed my teacher Gelfand, who always said that you cannot do analysis, algebra, geometry or mechanics, or something – you must know mathematics and there is no difference between the domains. If you want to be a good expert in representation theory, you have to know everything. So I just tried to solve any interesting problem that I heard of from any direction.

But how did you do your work on nilpotent Lie groups, i.e. the results that immediately made you famous?
Ah, that is a concrete question and it is easy to answer. When I was, I think, a third year student, Gelfand said, at a seminar, that there were some interesting papers by Dixmier about irreducible representations of nilpotent Lie groups. And nothing in this direction had been done before. At that time, three or four papers by Dixmier had already appeared.[4]

He asked me to read them and to present what they were about to the seminar. Reading the first paper, I was stuck because, in the paper, it was assumed that the reader knew the notion of induced representations. I understand the word "induced" in a general philological meaning (that something induces something in a similar situation) but the mathematical notion of inducing is very special. You cannot invent it. You cannot reconstruct it without knowing [laughs]. So, it took some time. I asked people. I asked my friends and nobody knew what an induced representation was. Then, I found the definition in an earlier paper by Dixmier and understood that it is a very nice thing. The theory of induced representations of finite groups was invented by Frobenius. It was explained in the collection of his papers and translated into Russian by the Kharkov Mathematical Society in 1938. Gelfand had read this book but I learned about it much later.

And how did you find your orbit method?
I did not know at the time that it was called the "orbit method" [laughs].

I had to present what I understood from Dixmier's papers to Gelfand. And I understood that Gelfand would not like Dixmier's variant of the exposition. I tried to adapt it to Gelfand's understanding and, step-by-step, I worked out how I could explain what Dixmier did in more simple and more natural terms. And so I came to coadjoint orbits.

[4] J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, I, *Amer. Journ. Math.* 81, No. 1 (1959), 160–170.
J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, II, *Bull. Soc. Math. France* 85 (1957), p. 325–388.
J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, III, *Canad. Journ. Math*. 10, No. 3 (1958), 321–348.
J. Dixmier, Sur les représentations unitaires groupes de Lie nilpotents, IV, *Canad. Journ. Math*. 11 (1959), 321–344.
J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, V, *Bull. Soc. Math. France* 87, No. 1 (1959).
J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, VI, *Canad. Journ. Math*. 12, No. 2 (1960), 324–352.

Guillaume Gellé, president of the University of Reims, awarding the degree to A.A.Kirillov. (Photo L. Amour)

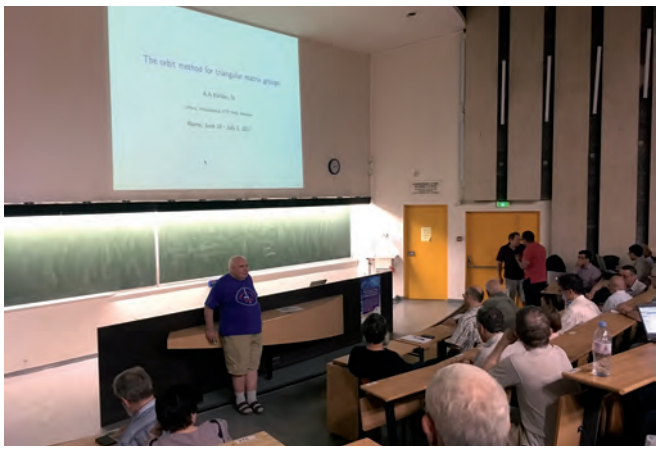
The main problem was to construct irreducible representations and I understood that most representations are induced by 1-dimensional ones. What is a 1-dimensional representation of a Lie group? It is the exponential of a 1-dimensional representation of a Lie algebra, i.e. an exponential of a linear functional. So, the idea was to take a subalgebra, a linear functional, take the corresponding representation and induce. If you do it "by chance", you get, as a rule, a reducible representation. So, the subalgebra must be big enough to get an irreducible representation but not too big. If it is too big then very few linear functionals produce a representation, since a 1-dimensional representation must vanish on commutators. So, I started to experiment with subalgebras and look for appropriate functionals. Rather soon, I understood that it is better to start with functionals, not with subalgebras. For any functional, you can choose a corresponding subalgebra. And also, even though different functionals produce different representations, they are sometimes equivalent. It is rather evident that the conjugate functionals from the same orbit produce equivalent representations. So, the notion of coadjoint orbits jumps out by itself.

Of course, after that, it was really the discovery that everything, every question, in representation theory in terms of coadjoint orbits can be naturally formulated and sometimes answered (not always, but at least it is the right language for representation theory).

Was that your first paper or was it later? You had a big paper…
I never write big papers.

No, a big paper in "Uspekhi".[5]
It was not big: less than 50 pages (and people often write 300 pages). Also, it was an "Uspekhi" paper – where you have to explain your results for beginners and non-experts. But "Doklady" notes that my main results (two or three notes of 3 pages each[6]) were very short.

[5] Unitary representations of nilpotent Lie groups. *Russian Math. Surveys* 17:4 (1962), 53–104.
[6] Doklady Mathematics, Vol. 128, No 5 (1959), Vol. 130, No 6 (1960), Vol. 138, No 2 (1961).



The talk by A.A.Kirillov. (Photo Vladimir Salnikov)

I would like to go back to the seminars. You said that you started your seminar quite early. How did it start? Who were the first participants?
I would not say early. I started as soon as I had the chance to do it. [laughs] Sure, I could teach schoolchildren and we were already doing that when I was a first-year student. All my generation of first-year students considered it a duty to teach mathematics to schoolchildren because many of our students (Moscow students) went to the mathematical circles. I did not attend these circles but most of my colleagues did so it was natural to carry out the same activities.

And you taught the children at the university.
Yes, usually at the university. Well, it depends. Some bold people like Arnold started their circles at the university from the very beginning and some others first started them at high schools. There are schools that are more or less associated with the university: e.g. the 57th school and some others.[7] I think, in the first year, that I ran a mathematical circle in one of the schools near the university.

Which was just created at the time, right?
Yes. And then I switched to the main building.

And you also wrote some booklets (I know at least one)?
That was much later,[8] well not much, but later. It was written for the correspondence school of mathematics ["Заочная Школа"] – a quite different "enterprise". In 1960 something (I don't remember when exactly), there was a great idea. Kolmogorov had created a boarding school.[9] Petrovskii asked Gelfand to join but Gelfand said that it did not suit him [laughs]. He, as a polite man, explained why: because Kolmogorov's boarding school was something like an "elite establishment". But Gelfand himself was more "wild" and he said that he preferred to organise something for people who do not know anything but who are able and just don't know that they are able. He proposed a correspondence school, which was accepted. And he did a great administrative job; he convinced many very talented students to participate in it. Some of them, like me, wrote textbooks and some participated by correcting the solutions that were sent by students. And this school, it became a big industry and it was very popular in the mathematical student community, so popular that party officials began to worry.

The point is that, at the time, all students, especially Komsomol [Communist Union of Youth] members, had to do society work ["Общественная Работа" – work of public interest]. And what were the possibilities? You could be active in a Komsomol area or in a profsoyuz [syndicates – labour unions] area or something else of the sort. The correspondence school opened up a new possibility. But why were the party members suspicious? Because work of public interest, by definition, cannot give pleasure [laughs] and these students did it with pleasure. And the correspondence school still exists.

Speaking about books, how did you get involved in this project?
When I was attracted by Gelfand to this correspondence school, he said that we must write a good book for the school: "I will write it myself and you will join me." Our first book had three authors: Gelfand, Glagoleva and Kirillov. Glagoleva was a schoolteacher and she was very good. She passed away this year…

When I joined this committee of three people, they first gave me two or three pages of text. I looked at it, completely rewrote it and gave it back to Gelfand. He gave it to Glagoleva, saying that "our version was bad and this is still worse" [laughs] – he said to please merge it together and get something readable. Glagoleva provided the third variant, then I rewrote it and so on…

But it was not work of public interest – you enjoyed it?
Oh, no. [laughs] Well, it was interesting but I would not call it pleasure.

Back to the science. Could you name some of the first participants of your seminar?
Well, those who were at school, I do not remember, though maybe some of them became students afterwards, so I remember them as students. But, every year, new students came.

I remember my graduate students – those who had to write a thesis – and just visitors to my seminar (there

[7] See "University Goes to School" – "Moscow University Maths Department for Schoolchildren", EMS Newsletter 101, September 2016.

[8] И.М. Гельфанд, Е.Г. Глаголева, А.А. Кириллов "Метод координат" М. 1966, 1973.
С.И. Гельфанд, М.Л. Гервер, А.А. Кириллов, Н.Н. Константинов, А.Г. Кушниренко, Задачи по элементарной математике: последовательности, комбинаторика, пределы М.: Наука, 1965. [Gordon and Breach, 1969, Learn limits through problems.]
А.А. Кириллов Пределы, М. Наука, 1968, 1973.

[9] See "University Goes to School" – "Mathematics in Kolmogorov's School", EMS Newsletter 101, September 2016.

were many more such people). Those who did not write a thesis and left no sort of trace – they are so difficult to remember. I certainly remember those who did PhDs (more than 60) but I can't remember them all. Maybe you do? [to Alice]

But I came later.
It was forbidden for me to be your advisor.

Of course, it was a big problem.
So you made a tremendous effort to become my student because, again, party officials would say it was impossible for a foreign student to have an advisor who was not a party member.

I was asked why I chose Kirillov. I said because of his book.
But you were a citizen of the Socialist Republic of Hungary, you must understand [all laugh].

And then the Hungarian Embassy had to help.
When you mentioned party officials, they were local to the faculty?
That was a completely local issue. Well, what means local? People like Sadovnichiy, at the time. He started as the Komsomol leader for our department, then he became a party official and then he became a member of a very important thing called the "personal committee", who decided which of the party members were decent enough and which did not deserve to be members. That was a very important position. He occupied it for many years and then he went up and up. In my time, the highest position he occupied was "twice the first", namely, the "first deputy of the first Vice-Rector".

For your method of leading the seminar, were you inspired by something or somebody?
Certainly. I think every human being, consciously or unconsciously, imitates what they have seen before. I imitated Dynkin's manner (not completely, but partly) and Vitushkin's manner, Arnold's manner, Manin's… Not Novikov's because I did not like his manner. Who else… Gelfand, of course. Gelfand's seminar was a very special phenomenon in Russian mathematical life. Step-by-step, I created my own method (but not immediately).

And Olshanski was, for some time, an important person in your seminar. Do you remember how he appeared?
Olshanski was one of my most talented students but he was so shy, so quiet. For a long time, I did not consider him seriously enough. I knew only that any question I asked to Grisha, he would answer. But he was never the first to say: "Ah, I know." And he was also the first of my students who started to help other students. For example, here, at this conference, there are Molev, Nazarov, Okounkov, Borodin and maybe ten more… They are practically students of Olshanski and not of me.

They worked together at my seminar but, for example, they got a problem and discussed it for long hours with Olshanski. Then, they said that I gave them a problem and they solved it; actually, it was under the supervision of Grisha Olshanski but he never got any credit for it. The only thing that he received was a letter from Sadovnichiy after Okounkov became a Fields Medallist. Grisha Olshanski got a letter that said (I say it in Russian because it is important):

From left to right: A.A.Kirillov, M. Pevzner, Yu. Neretin, A. Fialowski, V. Salnikov. (Photo L. Amour)

> "Окуньков вырос на Московском Университете, поскольку имел руководителями таких ярчайших математиков, как Ольшанский".
> *[Okounkov grew up at Moscow University, since he had, as advisors, some of the brightest mathematicians such as Olshanski.]*

I do not remember if he mentioned me or not but Olshanski was certainly mentioned. And I saluted Grisha the next day saying: ''Hi, the brightest!" ["Привет, ярчайший!"] [laughs]

I think I must finish with an afterword: Okounkov, at the time, was not a student of the university; he did not have any document saying that he belonged to the university. Being in Moscow, he tried to enter the building at MSU but was stopped by the guard. He showed the letter from Sadovnichiy (exactly this) and the guard said "this is not a document". [laughs] It was when Andrei was already a Fields Medallist but did not have any position at MSU. Now, he has a position both in Skolkovo and in the Higher School of Economics.

I would like to ask a question about students because when I arrived in Moscow and people heard I would be your student, they said I had chosen a very tough leader. Some of your students could not finish.
I know only two such cases and I think it is not my fault. For example, Sergei Belkin was very exceptional but I think nobody could convince Belkin to write any papers. [laughs] Then, he lived in the student dormitory, not attending any lectures, with many students coming to ask his help on problems of different levels, from an undergraduate exam to a PhD thesis.

I had a feeling that you gave absolute freedom to your students, with an obligation that they should find their own way – which is perfectly good because these people went around the world, and around Russia or the Soviet Union, and they had to stand on their own two feet. I am grateful for this style.

Well, it is not universal advice for everybody. I think even good students sometimes need some pushing. For example, I still think that I did not sufficiently push Yura Neretin. He invented, in one moment of his mathematical biography, what I propose to call "Neretin Numbers". These were parameters of discrete series of representations of Virasoro algebra (which was a very hard subject at the time). And then it was taken on by physicists. Mathematicians, I think, never discovered these parameters. Of course, Victor Kac later explained it but it was post factum, after the physicists had done all the computations. But you were also very close to it.

It was in my PhD thesis, which mainly was not published.

When we were walking here, you mentioned that you were interested in French mathematics, and somehow French mathematics got interested in you, like all these great people who went to your lectures. How did it happen?

When I went to France, it was 1968. Before that, there was the Moscow Mathematical Congress and, even before that, there was a Congress in Stockholm in 1962. They planned a big delegation of 400 people from the Soviet Union; the idea was to rent a steamboat to get from Leningrad to Stockholm, and the Soviet team would live on this boat and not spend foreign currency [laughs]. Well, the idea was proposed and discussed (it took many years – the preparation of a congress takes four years) and we finished with a delegation of not 400 but 40 people. But, for the first time in history, I think, seven young mathematicians were included. These were Ludwig Faddeev, Yura Manin, me, Arnold, maybe Anosov and who else… You must know that not all the participants were admitted to the congress. The announced list of speakers and the actual list of speakers were not identical.

Many people came to my talk. There were three sorts of talks (it has changed since then): 45 minute talks, 25 minute talks and posters. I had 25 minutes and I was surprised at how many people came to it. I don't know who made the advertisement but the result was there. And, for the first time, I met a lot of mathematicians whose papers I had read: Kadison, Mackey, Mautner, Fell, Atiyah, maybe Singer also, Hirzebruch, a lot of people. And then, in four years, they all came to Moscow and we continued our… not collaboration but discussion. So, when I went to France, I already knew a dozen good mathematicians – not only good but great. At that time, France was a great mathematical country. Arnold considered Bourbaki as something that spoiled mathematics but I think their influence on French mathematics was very strong and positive on the whole.

Apparently, he was angry about the school reform that was inspired by Bourbaki.

Maybe, maybe.

But that is yet another story.

Nobody is happy about school reforms in any country [laughs].

So your first long stay abroad was in France 50 years ago. What was your impression?

You mean my first visit to France. Well, it was 1968 – a very interesting year because it was the year of "La Grande Revolution Française". And I participated in it. I invented my own slogan: "Liberté aux professeurs associés." It was a standard slogan of the time. Everybody shouted "Liberté" – to workers, to students. I thought that my duty was to fight for liberté aux professeurs associés. What did it mean? Nobody knew and nobody was interested in it. The main thing was to go onto the street and to shout about it: "Liberté for…" [laughs]

Do you see any striking differences with nowadays?

I think yes. You know, one of the fairytales starts like this: "In China, all the inhabitants are Chinese, and the emperor himself is Chinese." My first impression about France was that "all the citizens are French, and they speak French". And it was a nice experience because I already knew some French. I like this language a lot. I knew some French mathematicians so for me it was a great pleasure. J'habitais Paris, Cité Universitaire, dans la maison Armenienne. I don't know why.

In Cité Universitaire, each building has a name: maison Armenienne, maison Pays Bas… It was rather close to the IHP, where I was an official member. At the time, Sorbonne had not yet divided into 14 universities; it was one university but the main mathematical organisation was the Institut Henri Poincaré on the rue Pierre et Marie Curie. I liked this place very much until now.

It was a very pleasant situation. French mathematicians are very friendly. I was surprised when Serre told me that all French mathematicians "tutoient" each other. You can say "vous" in France but mathematicians must say "tu". Maybe a mathematician starting their PhD thesis or an undergraduate student must still respect their teacher but starting after their PhD they must "tutoyer". Maybe it has changed now – I don't know.[10]

I also consider myself to be extremely lucky because I was in France without any "surveillance". This was in great contrast to my previous visit to Stockholm. Why? I don't know. The very fact that I entered France was inexplicable.

Actually, I have my own explanation. I have repeated it several times. The year 1968 was not a good year but I learned this only after my departure to Paris. My version is as follows. At the time, the Soviet Union had very good relations with France and personally with President Charles de Gaulle. And French bureaucracy has one remarkable feature: any official paper issued by an organisation must be accompanied by references to all "decréts" of organisations of higher standing that are related to this decision. Therefore, my invitation to France was written like that: I was invited by the IHP, according to the decrét of the main person at the IHP, based on the decrét of the President of Sorbonne, based on the decrét

[10] Indeed, there is some difference for student-professor communications; otherwise, the tradition still exists.

of the Ministry of Education, etc., and, in the end, there was the facsimilé of the signature of de Gaulle.

I imagine very vividly a clerk in the Russian ministry, sitting at a desk and looking at my invitation. His first move would certainly be to decline it immediately because this is the standard reaction. If he approves it, he takes on the responsibility for it; if he declines, he does not risk anything. And then he sees de Gaulle's signature… [laughs] and, thinking not about de Gaulle but about his superior, who would say: "Ah, you declined an invitation signed by de Gaulle…" That's my version. I don't insist on it but I have no other explanation.

Whom would you like to mention amongst your colleagues of the 1970s and the 1980s at Mekhmat?

Well, I became a member of the faculty in the 1960s. At the time, Mekhmat was growing because we switched from the old building in the city centre to the new one in Leninskie Gory. In 1961, Petrovskii called Arnold and me and said to us: "I have two positions for young mathematicians and I want to take you." At the time, we were second-year graduate students, after starting graduate school in 1959. He said that graduate school could wait and he had the option of taking us on as members of the faculty. Of course, we agreed with great pleasure and our first position was "Assistent" – a minor position.

So, starting in the Fall semester of 1961, we became members of the faculty. I was the youngest member of my chair (Mekhmat is divided into chairs) of function theory and functional analysis. Arnold went into differential equations because it was Petrovskii who personally took him and I guess that Gelfand was the one who asked Petrovskii to take me as the second.

I was the youngest member of this chair for a long time. But, at the time, there were already Alik Berezin, Bob Minlos, Shilov of course (on our chair) and Schabat, and of course Men'shov and Ul'yanov – two older representatives of the chair. It was non-officially divided into three parts: real analysis, complex analysis and functional analysis. The chief of our chair was Men'shov (a picturesque man) – anyone who saw him once would never forget it. You had the chance to see him?

Yes, yes – the last time was in 1977.

[all laugh] A man who looks like don Quixote, as thin, as great and as grey, with a beard and a loud voice. But he invented his own way to cope with party officials. When they told him there was something not so good in his chair (not enough Komsomol activity or something else), he said: "I'm an old man; it is difficult for me – explain to me. I will try." [laughs]

Who else was there from the faculty? At the time, there was, of course, Petrovski – a very, very busy man but still active in mathematics. He was writing, at the time, his famous paper with Landis, which finally turned out to be erroneous. It proposed the solution of one of Hilbert's problems, about the number of ovals in the algebraic curves, and the solution they proposed was wrong – it turns out that there can be infinitely many cir-

On the campus of the University of Reims. (Photo Vladimir Salnikov)

cles whereas they thought there were finitely many and tried to find the upper bound.

Who else? I do not remember the old algebraists of the university but there was Shafarevitch, who was very active, and a young star Manin. In differential equations, of course, there was Arnold – the main figure. And in topology, Novikov soon took the ruling position. And we were always friendly. Not everybody went to all the seminars but we knew what other people were doing. And also, there was Gelfand's seminar as a club, where everybody came and [laughs] socialised.

Now, turning toward the present, when you moved to America in the 1990s, you faced a completely different reality. How did you continue your research and teaching there?

Well, first of all, I never dreamed of moving anywhere. Of course, I was invited many times to many countries. But, in the Soviet period, it was practically impossible. I was invited twice to Israel but both times my application was declined with the reason that it was a very, how to say… There are many "bad words" for other countries and Israel was, of course, one of the worst.[11] So, it was impossible for those who worked at Moscow University, so big and nice, to go to such a 'bad' country as Israel… By the way, at exactly that time, our dean went to Israel but it is, well… [laughs]

In 1990, I got a third invitation. It was at some jubilee of Pyatetski-Shapiro, I think, and I was invited and quite unexpectedly got permission to go "as a private person" (not as a university professor). University professors could still not go to Israel but as a private person it was possible. In the Spring of 1990, I was in Israel and then I decided maybe I could try to go to the United States. It was also a "bad country" but maybe not so bad as Israel. Mark Freidlin, my friend and colleague at Moscow University (we were students of the same year), had a position at Maryland University; he had already invited some of his friends and tried to invite me. And, again, unex-

[11] Speaking of the officials' point of view.

pectedly, I got permission and went for a whole semester to Maryland University. In Israel, it was a short period – one conference, ten days maybe – but this was a one-semester period. It was very interesting. I made the acquaintance of another dozen mathematicians, who were close to me. I liked it very much but never dreamed of staying forever – I finished my semester and went back. But, during my stay in America, I visited four other universities. From Maryland, I went to Philadelphia, Boston and Yale, I think. This was not like Vershik, who, during his first visit to America, visited 20 universities [laughs]. He spent, I think, one month there, or maybe two months. I think it is better to see four universities for one week each than go everywhere for one day. And, after that, I liked Philadelphia and they invited me to come back. I said: "OK, see you soon."

Then I came back to Moscow. I did not want to go anywhere immediately after that. For me, I decided, to visit the United States once in three years would be a very good practice. So, next time, I went to Philadelphia, in the Fall of 1993. But, at the time, what we called the Second October Revolution was occurring, when the White House[12] was shot at. I was in Philadelphia at the time and my wife was panicking because CNN was showing pictures of the White House in smoke and tanks on the streets of Moscow. She did not like these pictures at all. My wife was very scared and asked me if I could extend my stay in the US to the Spring semester. I went to the chairman and asked if it was possible. He said, sure, but why don't you want to accept a permanent position? I said I was not ready and I would think about it. And I thought about it. My stay in America was extended and I spent one more semester there. After that, my wife convinced me that it was better to accept the position.

Well, as soon as it was known that I was ready to accept a permanent position, I immediately got another invitation from Penn State. Penn State is a bigger university than UPenn but of slightly lower status. It is situated in the middle of Pennsylvania, at the cross of two diagonals of the rectangle (as the English say, "in the middle of nowhere"), where farmers live. And it was founded exactly to bring education to farmers. But this university is very rich and very big, with many more students than UPenn. And (maybe more essentially), it has a good football team.[13] In America, the football competition is very important and, when a crucial match takes place in State College (the city where Penn State is situated), the hotel rooms are booked a year in advance. So, for example, it is impossible to run a mathematical conference at the same time as a football competition. And also, there were already eight Russian mathematicians at Penn State. I thought about it and preferred to go to Philadelphia, which is not far from Washington and New York and where there were no Russians at all. [laughs]

But there were a number of great mathematicians in your area, like Kadison, Pukanszky, Fell, Wilf and others?

Yes.

Though our football team is not so good [laughs], you know this notion of Ivy League? I was surprised when I found out the origin of the name. I thought the ivy imitated old English universities but, no, the reason is quite different. The point is that Harvard, Penn, Cornell, etc., (the eight old universities) have a very high reputation but rather bad football teams. You see, for example, with Nebraska and Penn State, the match between them is very important because whoever wins guarantees getting a lot of students the next year and whoever loses gets much fewer. The good universities like Yale and Harvard are not so dependent on the results of the football team so they have the option not to hire expensive football coaches. You know that the salary of a football coach is bigger than the salary of a president of a university? But to have no football competition at all, this is also impossible. However good the university is, if there is no baseball, football or basketball then the students will not come to such a university. So they found a new genius idea to organise a special league, a football league, and call it the Ivy League. And so, these eight universities compete between themselves and they have the chance to be the champion of the Ivy League. They have no chance to get a decent place in the whole United States University League but inside the Ivy League, it is quite possible.

I found a few "traces" of you in Denmark in 1990 because there were three Russians in Denmark before me: one was Peter the Great…

He was not alone; he brought, I think, several hundreds of people with him…

But Danish people remember three Russians. So, Peter the Great climbed a horse and rode it to the highest tower in the city. The second one was a young man from Tambov, a mathematician, but the story was about his wife (hence the fourth Russian). And there was also Kirillov and this story was not so bright. Each mathematician in Denmark showed me a café where they drank with Kirillov and the cafés were different.

[all laugh] I really was in Denmark; it was a conference about the orbit method. It was, I think, the first time that I had been to a capitalist country since my visit to France in 1968. It was in 1988, I think. It was a funny and sad story. The main role in it was played by a device called telex. I think this was something like fax but it was 30 years ago and I think that the fax of today did not yet exist.

You see, I got an invitation for the conference, which lasted one week, and, after that, I gave some lectures so the total invitation was for three weeks: one week for the conference and two weeks of lectures. When I went to the authorities in Moscow, to the foreign division, the chief of this foreign division said that he knew nothing about the invitation for delivering lectures; he only had an invitation for the conference. The invitation for the lectures was in my pocket but I understood that it would be a drastic mistake to show it to him. And I said, OK, I will go to the conference. When I went to the conference,

[12] Russian White House – the seat of the Government.
[13] American Football.

I asked the equivalent of the foreign officer in Denmark and said that I had two invitations, so could I stay? He said: "Certainly," and wrote something. I took it and went to the Russian Embassy in Copenhagen. They looked very surprised and said: "Who wrote it?" "An official representative of the Danish foreign office." "Who allowed you to talk to this representative? You should immediately leave Denmark!" I said: "I don't have a ticket. So how can I go immediately?" After that, I called my wife Louiza, who was in Moscow. Alesha Gvishiani, a member of our department (and grandson of Kosygin), also wanted to come to this conference. Later, he changed his mind and did not come. But all the documents went together with mine. And Louiza called Alesha and he said that he would try to do something. Then, he said that the telex would arrive at the embassy. So, over one week, I went to the Russian Embassy every morning, as if to work, and asked if they had a telex from Moscow, which would have allowed me to stay in Denmark for another day. [all laugh]

I didn't know whether telex was an electronic device or a pedestrian courier [laughs] but from Moscow to Copenhagen it took ten days. And, every second day, I was told I should leave Denmark immediately. But I came again and again. And then, on the 8th or the 9th day, I came and they said very dryly: "You can stay."

Moving forward, how could you continue your work and research, and seminars (I mean in Philadelphia during these years)?

Of course, the life of a mathematician in America is quite different from the life of a mathematician in Moscow. There are different students and different relations with colleagues. Everything is different.

What can you say about your current research?

You see, I am not obliged now to publish many papers per year (like young people who must show that they are great). So, I prefer to think for a long time about interesting questions. Right now, I am a member of a team of four people trying to solve one very difficult problem. But I will speak in detail about it tomorrow.[14] We have already spent three years on it, meeting in California where there is the American Institute of Mathematics, which organises work by teams, called "squares". And by the definition of the AIM, a square is a geometric figure that can have from two to eight vertices [laughs]. We form four vertices and have spent ten days every year for three years (but it is finished now). We will continue in Oberwolfach, as a team "in pairs". I do not know the official definition of a "pair"…

Maybe the very last question is kind of inspiring. What would you like to say as a message to the younger generation?

Oh… I could say that humanity now has a very big problem: what is the reason of our lives? And most people, I think, do not have an answer to this question. In poor countries, people suffer from a deficit of everything; in rich countries, like in Scandinavia, the proportion of suicides is growing and growing and people do not know what they are living for. I think the main "raison d'être" – the reason for life – for a human being is to learn about the Universe. And mathematics is one of the ways to understand nature. The liberal arts is another form and there are other forms: certainly not bureaucracy but maybe medicine (although medicine is now half industry and half bureaucracy so I do not advise people to go into medicine). Of course, not everybody gets pleasure from doing mathematics but I think that a non-zero percentage of people are able enough to do mathematics. So, if somebody feels that they can do mathematics, I advise them to do it. Of course, this will put an end to your "American dream", which at the beginning of the [previous] century was to have a million dollars and now is equivalent to having a hundred million dollars. If you are a mathematician, you can be sure you will never get this. But still, it gives a sense of purpose to your life, which may be more important.

Thank you very much!

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[14] The talk delivered the next day was "Representations of the triangular group over a finite field".