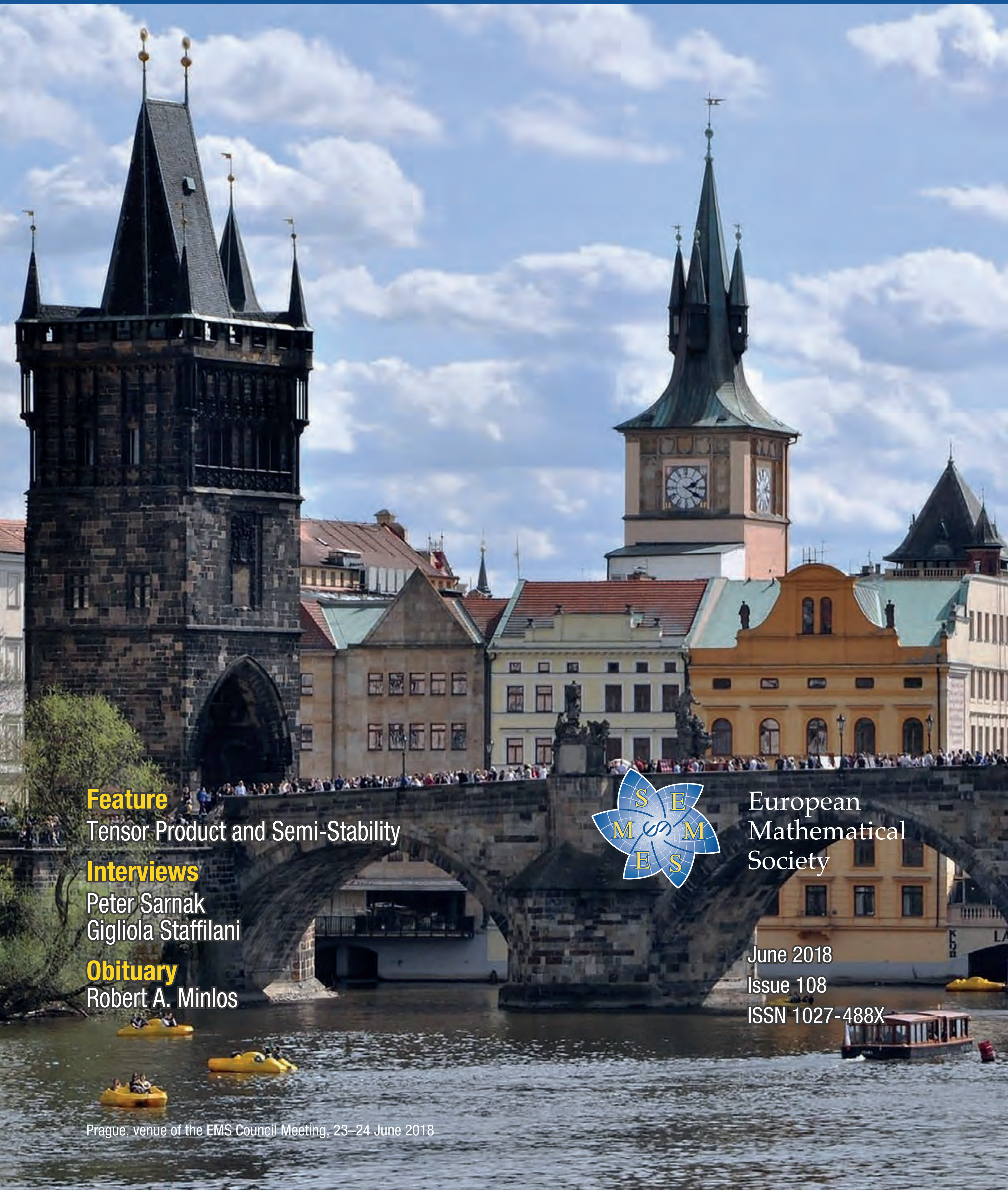
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



Feature
Tensor Product and Semi-Stability

Interviews
Peter Sarnak
Gigliola Staffilani

Obituary
Robert A. Minlos

European Mathematical Society

June 2018
Issue 108
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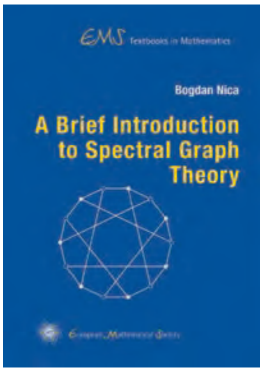
Prague, venue of the EMS Council Meeting, 23–24 June 2018

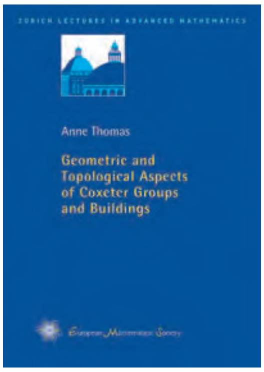
New books published by the European Mathematical Society

Individual members of the EMS, member societies or societies with a reciprocity agreement (such as the American, Australian and Canadian Mathematical Societies) are entitled to a discount of 20% on any book purchases, if ordered directly at the EMS Publishing House.



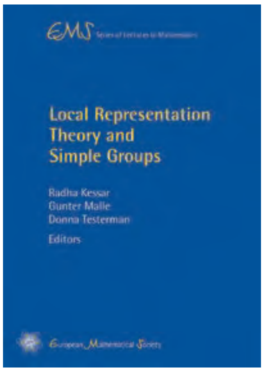
Bogdan Nica (McGill University, Montreal, Canada)
A Brief Introduction to Spectral Graph Theory (EMS Textbooks in Mathematics)
ISBN 978-3-03719-188-0. 2018. 168 pages. Hardcover. 16.5 x 23.5 cm. 38.00 Euro

Spectral graph theory starts by associating matrices to graphs – notably, the adjacency matrix and the Laplacian matrix. The general theme is then, firstly, to compute or estimate the eigenvalues of such matrices, and secondly, to relate the eigenvalues to structural properties of graphs. As it turns out, the spectral perspective is a powerful tool. Some of its loveliest applications concern facts that are, in principle, purely graph theoretic or combinatorial.
This text is an introduction to spectral graph theory, but it could also be seen as an invitation to algebraic graph theory. The first half is devoted to graphs, finite fields, and how they come together. This part provides an appealing motivation and context of the second, spectral, half. The text is enriched by many exercises and their solutions.
The target audience are students from the upper undergraduate level onwards. We assume only a familiarity with linear algebra and basic group theory. Graph theory, finite fields, and character theory for abelian groups receive a concise overview and render the text essentially self-contained.



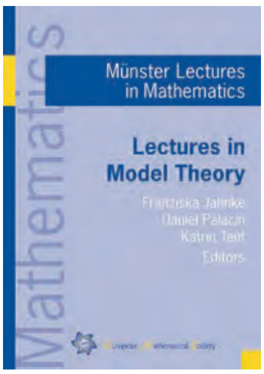
Anne Thomas (The University of Sydney, Australia)
Geometric and Topological Aspects of Coxeter Groups and Buildings (Zürich Lectures in Advanced Mathematics)
ISBN 978-3-03719-189-7. 2018. 160 pages. Softcover. 17 x 24 cm. 34.00 Euro

Coxeter groups are groups generated by reflections, and they appear throughout mathematics. Tits developed the general theory of Coxeter groups in order to develop the theory of buildings. Buildings have interrelated algebraic, combinatorial and geometric structures, and are powerful tools for understanding the groups which act on them.
These notes focus on the geometry and topology of Coxeter groups and buildings, especially nonspherical cases. The emphasis is on geometric intuition, and there are many examples and illustrations. Part I describes Coxeter groups and their geometric realisations, particularly the Davis complex, and Part II gives a concise introduction to buildings.
This book will be suitable for mathematics graduate students and researchers in geometric group theory, as well as algebra and combinatorics. The assumed background is basic group theory, including group actions, and basic algebraic topology, together with some knowledge of Riemannian geometry.



Local Representation Theory and Simple Groups (EMS Series of Lectures in Mathematics)
Radha Kessar (City University of London, UK), Gunter Malle (Universität Kaiserslautern, Germany) and Donna Testerman (EPF Lausanne, Switzerland), Editors
ISBN 978-3-03719-185-9. 2018. 369 pages. Softcover. 17 x 24 cm. 44.00 Euro

The book contains extended versions of seven short lecture courses given during a semester programme on "Local Representation Theory and Simple Groups" held at the Centre Interfacultaire Bernoulli of the EPF Lausanne. These focussed on modular representation theory of finite groups, modern Clifford theoretic methods, the representation theory of finite reductive groups, as well as on various applications of character theory and representation theory, for example to base sizes and to random walks.
These lectures are intended to form a good starting point for graduate students and researchers who wish to familiarize themselves with the foundations of the topics covered here. Furthermore they give an introduction to current research directions, including the state of some open problems in the field.



Lectures in Model Theory (Münster Lectures in Mathematics)
Franziska Jahnke (Universität Münster, Germany), Daniel Palacín (The Hebrew University of Jerusalem, Israel) and Katrin Tent (Universität Münster, Germany), Editors
ISBN 978-3-03719-184-2. 2018. 222 pages. Softcover. 17 x 24 cm. 38.00 Euro

Model theory is a thriving branch of mathematical logic with strong connections to other fields of mathematics. Its versatility has recently led to spectacular applications in areas ranging from diophantine geometry, algebraic number theory and group theory to combinatorics.
This volume presents lecture notes from a spring school in model theory which took place in Münster, Germany. The notes are aimed at PhD students but should also be accessible to undergraduates with some basic knowledge in model theory. They contain the core of stability theory (Bays, Palacín), two chapters connecting generalized stability theory with group theory (Clausen and Tent, Simon), as well as introductions to the model theory of valued fields (Hils, Jahnke) and motivic integration (Halupczok).

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EMS Agenda

2018

29–30 July
IMU General Assembly, São Paulo, Brazil

9–11 November
EMS Executive Committee Meeting, Barcelona, Spain

EMS Scientific Events

2018

2–6 July
The 12th International Vilnius Conference on Probability Theory and Mathematical Statistics and
2018 IMS Annual Meeting on Probability and Statistics
Vilnius, Lithuania

9–13 July
EMS Summer School "4rd Barcelona Summer School on Stochastic Analysis"
Barcelona, Spain

9–14 July
Activity Endorsed by the EMS:
Young African Scientists in Europe (YASE), Toulouse, France

9–14 July
4th EU/US Summer School on Automorphic Forms and Related Topics
Budapest, Hungary

23–27 July
11th European Conference on Mathematical and Theoretical Biology (ECMTB 2018), Lisbon, Portugal

1–9 August
ICM 2018
Rio Centro Convention Center, Rio de Janeiro, Brazil

19–26 August
Helsinki Summer School on Mathematical Ecology and Evolution
Turku, Finland

Editorial – On the Road to MSC 2020

Adam Bannister (FIZ Karlsruhe, Berlin, Germany), Fabian G. Müller (FIZ Karlsruhe, Berlin, Germany), Mark-Christoph Müller (Heidelberg Institute for Theoretical Studies, Heidelberg, Germany) and Olaf Teschke (FIZ Karlsruhe, Berlin, Germany)

Eighteen months ago, the beginning of the revision of the Mathematical Subject Classification was announced.[1] Since then, the mathematical community has already contributed a number of suggestions on the public wiki available at https://msc2020.org/. In this article, we will give a brief overview of the current usage of the MSC, analyse some data related to its effectiveness and precision, relate it to topic clusters generated by data mining techniques and indicate some trends that have become visible in the course of the revision.

Current usage of the Mathematical Subject Classification (MSC)

While the raw scheme had already been introduced in the early volumes of *Mathematical Reviews*, the current shape, as a joint effort of both mathematical reviewing services, evolved in 1980. This came after an initiative of Bernd Wegner to incorporate the system into *Zentralblatt MATH* and to maintain regular collaborative revisions. Since then, MSC has been primarily used by MathSciNet and zbMATH reviewers and editors to classify the mathematical research literature, as well as being adapted by classical and digital libraries and journals. Several recent developments in the zbMATH database, such as author, journal and citation profiles or filter functions, have utilised the subject information beyond its original raison d'être.

How reliable is the MSC?

MSCs are assigned to books and papers by authors, reviewers and editors, with the final classification approved by MathSciNet and zbMATH section editors. Naturally, as a human enterprise, such assignments may be subjective. Hence, it is natural to ask about the degree of subjectivity that comes along with a classification performed by hand and whether it is possible to derive conclusions for the revision from the degree of vagueness. To determine this, a comparison of MSC2010 assignments has been made for 78,063 articles published between 2010 and 2016 in journals indexed cover-to-cover by both MathSciNet and zbMATH. For this corpus, both services coincided at the top level MSC for 62,951 documents, and even for 40,244 at the level of the overall MSC. More precisely, the average F1[2] score for the coincidence of MathSciNet and zbMATH classifications turned out to be 0.83 at the top level, 0.72 at the second level and 0.58 at the third level of the first assigned MSC.[3] The concordance turned out to be significantly larger when permutations were taken into account; indeed, the largest differences by far occurred in the cross-subject MSC sections like 00 (General), 97 (Education), 58 (Global analysis), 19 (K-Theory) and 37 (Dynamical systems). Interestingly, three of them were introduced in the 1991 and 2000 MSC revisions. Hence, while the MSC has overall become less tree-like, with more cross-references introduced in the last revisions, it seems that a large proportion of the articles still fit conveniently into the more classical hierarchical structure of the main subjects. Consequently, it seems justified that there has been no introduction of a new top-level MSC in 2010 and there also seems no need to do so in 2020.

The unreasonable effectiveness of the MSC

The relative reliability of the top-level MSCs can also be derived from the cross-citation Figure 5 in Bannister and Teschke,[4] which shows a strong concentration of references to articles with the same MSC. In this sense, the main subjects can also be seen as most natural clusters of the citation graph. Naturally, due to the interconnected nature of mathematics, this effect is less significant for more granular MSC levels. However, the question remains of whether there are automated ways to organise mathematical literature into subjects. Apart from graph-theory approaches, the last decade has seen tremendous progress in topic modelling by data mining and machine learning techniques. An experiment performed by the Heidelberg Institute of Theoretical Studies (HITS) created several clusters using the ToPMine tool.[5] Human evaluation showed that it performed reasonably well for applied areas (producing, for example, a cluster containing Bayesian inference, posterior distribution and the Gibbs sampler, roughly corresponding to 62F15) but was quite limited for pure mathematics (e.g. it joined the notions of pull back and container loading from category theory and operations research and created the cluster "hyperplane arrangement, traffic jam, speed of light" of hitherto unknown mathematical semantics). Some of the effects may derive from the fact that publication numbers are extremely unevenly distributed in mathematical areas and automated methods tend to underperform for areas with relatively small publication numbers, which are often, however, very important within the mathematical corpus.

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1 E.G. Dunne and K. Hulek, MSC2020 – announcement of the plan to revise the Mathematics Subject Classification. *Eur. Math. Soc. Newsl.* 101, 55 (2016).

2 Weighted harmonic mean of the fractions of MSC codes in one set that also occur in the respective other.

3 MSC codes have three levels of increasing granularity, denoted by two digits, a letter and two more digits.

4 A. Bannister and O. Teschke, An Update on Time Lag in Mathematical References, Preprint Relevance, and Subject Specifics. pp. 41–43. *Eur. Math. Soc. Newsl.* 106, 41–43 (2017).

5 http://illimine.cs.uiuc.edu/software/topmine/.

Developments toward MSC2020

Taking the mentioned limitations into account, quantitative methods such as those mentioned above can be used to create suggestions for the MSC2020 revision. Phrases that have occurred much more frequently since 2010 have often included developments in the applications of mathematics (which tend to be both more numerous in publications and more fluid in topic denomination), for instance "loop quantum gravity" and "PT symmetry" in quantum theory, "scaling limits" arising both in stochastics and physics, "exponential stability" in control theory, "quantum circuits" and "quantum games", as well as "sparse graphs", "spatial graphs", "circulant graphs" and "phylogenetic trees" connected to the rise of network research, "copulae models" in statistics, "character varieties" in algebraic geometry and topology and the cluster "Khovanov/Heegaard-Floer/HOMFLY homology" from topology, along with transcending techniques like "matrix factorisation". Copulae and character varieties have already been independently proposed in the MSC2020 wiki, as well as many new developments not detected by automated methods, such as "numerical algebraic geometry", "higher categories", "topological data analysis" and "computer-assisted proofs". On the other hand, several recent concepts (like homotopy type theory) are still missing, so please engage in the joint effort and contribute to the MSC2020 wiki at https://msc2020.org/, which will remain open until August 2018!

Adam Bannister [adam.bannister@fiz-karlsruhe.de] has a postgraduate diploma in Geographic Information Systems and currently works on the Scalable Author Disambiguation for Bibliographic Databases at zbMath in cooperation with Schloss Dagstuhl and Heidelberg Institute for Theoretical Studies.

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Olaf Teschke [olaf.teschke@fiz-karlsruhe.de] was a member of the Editorial Board of the EMS Newsletter from 2010–2017, responsible for the zbMATH Column.

EMS Secretariat

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If you have never logged in to the EMS membership database before just go to http://www.euro-math-soc.eu/members/ and click the link "Create a new account". Insert your EMS ID number and a password will be sent to your email address that you submitted while registering for the EMS. After that you can login to the database (same address as above) with your EMS ID number and the password given in the email. The EMS ID number can be found in the welcome letter sent to you by the EMS Office or on the address sticker of the EMS Newsletter sent to you by the EMS Publishing House.
If you would like to use the extended services of our webpage, you need to register first and create a user account at our webpage:

http://www.euro-math-soc.eu/user/register

The EMS website is not related to the members database.

Elvira Hyvönen, EMS Secretariat
E-Mail: ems-office@helsinki.fi, Phone: (+358) 2 941 51141

New Editor Appointed

Ulf Persson received his PhD at Harvard in 1975 under David Mumford. His dissertation was entitled "Degenerations of Algebraic Surfaces". Persson's professional publications have been almost exclusively in algebraic geometry and especially on surfaces. He is inordinately proud of having introduced the notion of the 'geography of surfaces', where the notion of 'geography' has caught on in other contexts. Persson has been based in Sweden since 1979 but did many stints as a visitor to a variety of American universities during the 1980s. In recent years his activities have widened. He founded the Newsletter of the Swedish Mathematical Society during his presidentship and has been its main editor for most of the time since then. Ulf Persson has also been an editor of the EMS Newsletter (2010-2014). He is fond of conducting somewhat idiosyncratic interviews with mathematicians, some of them appearing in this Newsletter but the more extreme appearing in the Newsletter of the Swedish Mathematical Society. As is not unusual for people who are aging, he has picked up his youthful interest in philosophy and has published a book and an article on Popper.

Report from the Executive Committee Meeting in Portoroz, 24–26 November 2017

Richard Elwes, EMS Publicity Officer

Last Autumn, the EMS's Executive Committee (EC) plus guests gathered in the Grand Hotel Bernardin in Portoroz on the stunning Slovenian Coast, on the kind invitation of the Society of Mathematicians, Physicists, & Astronomers of Slovenia (DMFA) and the University of Primorska. The choice of venue was significant: in July 2020 the same site will host the 8th European Congress of Mathematics (8ECM). The congress was therefore at the forefront the committee's mind throughout the meeting; in particular, it offered those who had not already done so the opportunity to explore the area and facilities, under the guidance of the congress's local organisers.

On Friday afternoon, the committee was welcomed by Boštjan Kuzman, President of the DMFA's National Committee for Mathematics, who told us about his society's history and activities, as well as some general history of Slovene mathematics. The DMFA was founded in 1949 (when Slovenia was part of Socialist Federal Republic of Yugoslavia). Its first honorary member was Josip Plemelj (1873–1967), the First Rector of University of Ljubljana, famous for his solution to the Riemann–Hilbert problem concerning the existence of a differential equation with a given monodromy group. In 1992, after the break-up of Yugoslavia, the DMFA became a member society of the EMS. Today, the DMFA represents Slovenia in international settings, promotes research, organizes national and international mathematics competitions (including the 2006 International Mathematical Olympiad), publishes the journal Ars Mathematica Contemporanea alongside other journals, textbooks, and problem compendia, organises conferences for researchers, seminars for teachers, and assorted mathematical outreach events. Tomaž Pisanski then introduced some other facets of Slovene mathematical life, including the new Slovenian Discrete and Applied Mathematics Society (SDAMS), which was established in 2017 along with its journal The Art of Discrete and Applied Mathematics whose first edition was published in 2017.

Officers' Reports and Membership

The meeting opened with some words of welcome from the chair, EMS President Pavel Exner, who related his activities since the Spring meeting (many featuring in later items on the agenda). He closed with some reflections on the political situation around the world, noting that the EMS has a duty to raise its voice when mathematics is under threat, and the unique organizational structure of the EMS makes our voice strong. At the same time, the EMS is not principally a political organization, and to maximise our impact, we must be selective in deciding when to speak out.

The EMS Treasurer Mats Gyllenberg presented his report on the society's 2017 income and expenditure. The financial situation is healthy; however less money was spent on scientific projects than intended, something to bear in mind when allocating future funds. The EMS Secretary Sjoerd Verduyn Lunel then reported on preparations for the next EMS Council meeting (23–24 June 2018 in Prague).

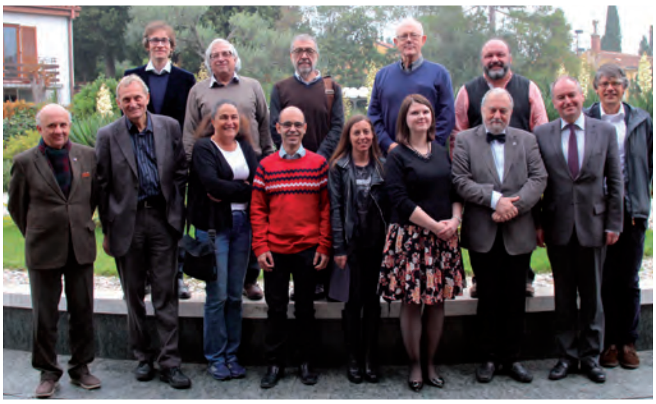
The committee was delighted to approve an application for Institutional Membership from the Mathematical Institute, University of Oxford. Contrastingly, it regretted a notification of withdrawal from the French Statistical Society. Several other member societies remain severely in arrears on their fees, and at the next Council meeting it may sadly be necessary to propose that certain societies have their EMS membership terminated. The committee was pleased to approve a list of 155 new individual members.

Scientific Meetings

The committee heard an encouraging presentation from Klavdija Kutnar, deputy chair of the local organising committee of 8ECM, on plans for the 2020 congress. The President thanked her and chair Tomaž Pisanski for all the work their committee has already done, and wished them continuing success as the congress draws nearer. The EC then began the important discussion of the leadership and personnel of the congress's Scientific and Prize Committees, based on candidates proposed by member societies. The society is of course seeking top mathematicians for these roles, but also individuals with broad mathematical interests, who as part of their duties will attend the ECM in person. Good progress was made, and the discussion will continue at the EC's next meeting in Spring 2018.

The committee then discussed applications for support for scientific events in 2018, based on evaluations from the Meetings committee. The EMS distinguished speakers for 2018 will be Benoit Perthame at the Joint EMS-FMS-ESMTB Mathematical Weekend in Joensuu, Finland (4–5 January 2018) and Andre Neves at the at the 7th Iberoamerican Congress on Geometry in Valladolid, Spain (22–26 January 2018), which will also be supported as an EMS Special Activity. The EC agreed to support the following eight Summer Schools:

- Advanced Techniques in Mathematical Modelling of Tumour Growth, Centre de Recerca Matemàtica (CRM), Barcelona, 3–6 April 2018



- New Results in Combinatorial & Discrete Geometry, CRM, Barcelona, Spain, 7–11 May 2018
- EMS-EWM Summer School "Nonlocal interactions in Partial Differential Equations and Geometry", Institut Mittag Leffler, Stockholm, 21–25 May 2018
- EMS-IAMP summer school in mathematical physics: Universality in probability theory and statistical mechanics, Ischia, Italy, 11–15 June 2018
- Géométrie Algébrique en Liberté XXVI, Strasbourg, France, 18–22 June 2018
- 16th School on Interactions between Dynamical Systems and Partial Differential Equations, CRM, Barcelona, 25–29 June 2018
- 4th Barcelona Summer School on Stochastic Analysis, CRM, Barcelona, Spain, 9–13 July 2018
- "Building Bridges", the 4th EU/US Summer School on Automorphic Forms and Related Topics, Alfréd Rényi Institute, Budapest, 9–14 July 2018
- Helsinki Summer School on Mathematical Ecology and Evolution, Turku, Finland, 19–26 August 2018.

The committee additionally agreed to support the 12th International Vilnius Conference on Probability Theory and Mathematical Statistics, Lithuania 2–6 July 2018, as well as the Emil Artin International Conference, Yerevan, Armenia, 27 May–2 June 2018. The President recalled that the EMS-Bernoulli Society Joint Lecture 2018 will be given at the European Conference on Mathematical and Theoretical Biology in Lisbon, 23–27 July 2018, by Samuel Kou.

Officers, Standing Committees, and Projects

The President reminded the committee that his term and that of Vice-President Volker Mehrmann, along with those of the Treasurer and Secretary, will end at the end of 2018. The next Council meeting in June 2018 will elect a new President and Vice-President; while the current Treasurer and Secretary declared that they would be available for re-election at the Council's pleasure.

Several of the EMS's standing committees also required replenishment of their membership and leadership, and the Executive Committee was pleased to make a number of appointments, including Stéphane Cordier and Koby Rubenstein as Chair and Vice-Chair respectively of the Applied Mathematics Committee, Leif Abrahamsson and Sophie Dabo as Chair and Vice-Chair of the Committee for Developing Countries, Jiří Rákosník and Dirk Werner as Chair and Vice-Chair of the Ethics Committee, Zdenek Strakos as Vice-Chair of the Meetings Committee, and Silvia Benvenutti as Vice-Chair of the Committee for Raising Public Awareness of Mathematics.

The outgoing President of the Applied Mathematics Committee, José Antonio Carrillo, in attendance as a guest, delivered a short report on his committee's activities including the exciting forthcoming Year of Mathematical Biology 2018. On behalf of the society, the President thanked him for his hard work over several years.

Mats Gyllenberg, liaison officer for the Committee for Developing Countries (CDC) delivered a short report on the first round of applications for the EMS-Simons for Africa programme. The EC also discussed the problem of donations to the CDC decreasing over recent years. This is due largely to EMS members increasingly paying their dues through their local societies, rather than through the EMS webpage, where donations to the CDC are solicited.

The Executive Committee also discussed reports from the Committees on Education, Ethics, Meetings, European Solidarity, Publishing and Electronic Dissemination, Raising Public Awareness, and Women in Mathematics.

Discussions followed on other projects the EMS is involved with, including the online Encyclopedia of Mathematics (www.encyclopediaofmath.org), EU-MATHS-IN (European Service Network of Mathematics for Industry and Innovation), the Global Digital Mathematics Library, Zentralblatt MATH (www.zbmath.org). The society's own newsletter was also discussed, with two new appointments made to the editorial team. The EMS's e-news, social media platforms, and other communications and publicity channels were also reviewed.

Funding, Political, and Scientific Organisations

The President gave an update of recent European Research Council (ERC) developments, with 2017 being its tenth anniversary year. The committee welcomed the news that former EMS President Jean-Pierre Bourguignon's term as President of the ERC has been extended.

The committee discussed recent developments regarding Horizon 2020 (and its successor framework), noting the importance of mathematicians registering as possible evaluators of Marie Curie proposals (a separate database from that of ERC evaluators).

Following the recent large increase in the membership fee for the Initiative for Science in Europe, after extensive discussions the committee agreed to terminate the EMS's membership of that body. The committee will investigate other pathways for political lobbying in Europe, given the importance of that task.

The President reported on the latest developments at the International Mathematical Union (IMU), in particular that Carlos Kenig has been proposed as the next IMU President for the period 2019–2022, and that Helge Holden intends to stay for a second term as Secretary. At the 2018 ICM in Rio de Janeiro, the general assembly will decide on the site for ICM 2022. The remaining two bids from Paris and Saint Petersburg, both being European, have each been endorsed by the EMS. Volker Mehrmann (member of the board of the International Council for Industrial and Applied Mathematics (ICIAM)) reported on the latest developments there. The President Elect is Ya-xiang Yuan, the next ICIAM conference will be in 2019 in Valencia (Spain), with ICIAM 2023 to be hosted in Tokyo. The EMS's relationship with various prize committees and research centres was also discussed, including proposing several nominations.

Future Society Meetings and Closing Remarks

The Executive Committee's next official meeting was set for 23–25 Match 2018 in Rome, before which it was agreed to meet for an informal retreat in Joensuu (Finland), 6–7 January 2018 to discuss the society's long-term strategic goals. The annual meeting of Presidents of EMS member societies will take place in Maynooth, Ireland, 14–15 April 2018 on the generous invitation of the Irish Mathematical Society. (In 2019 the corresponding meeting will take place at CIRM in Marseille, France.)

The meeting closed with the committee's warm thanks to the Society of Mathematicians, Physicists and Astronomers of Slovenia and to the University of Primorska, in particular to Klavdija Kutnar, Tomaž Pisanski, and Boštjan Kuzman, both for their impeccable organisation of this meeting and also for their ongoing efforts for the 2020 European Congress, to which the whole society is greatly looking forward. ed at the next Council. The committee was pleased to approve a list of 154 new individual members.

Tensor Product and Semi-stability: Four Variations on a Theme

Marco Maculan (Université Pierre et Marie Curie, Paris, France)

Chevalley proved that in characteristic 0 *the tensor product of semi-simple representations is semi-simple. This result has analogues in rather diverse contexts: three of them are presented here in independent sections, focusing on the differences of the frameworks and the similarities of the proofs. Algebraic groups will play a crucial role, sometimes in unexpected ways.*

1 Representations

Let G be a group and k a field. In this note, a representation of G is a finite-dimensional k-vector space V together with a group homomorphism $\rho\colon G \to \mathrm{GL}(V)$.

Semi-simple representations

A representation is said to be:

- *irreducible* if there are exactly two sub-vector spaces of V stable under the action of G: the zero subspace 0 and the whole vector space V. In particular the zero representation is not considered to be irreducible.
- *semi-simple* if it can be decomposed into irreducible ones: there are irreducible sub-representations $V_1, \dots, V_n$ of V such that $V = V_1 \oplus \cdots \oplus V_n$. This is equivalent to saying that for every G-stable subspace W of V there is a G-stable supplement W'.

Theorem 1 (Chevalley [8], p. 88)**.** *Suppose* $\mathrm{char}(k) = 0$*. The tensor product of semi-simple representations* V_1*,* V_2 *of* G *is semi-simple.*

The proof of Chevalley's theorem is a beautiful application of the theory of linear algebraic groups (that is, groups of matrices defined by polynomial equations), even though the group G may not at all be of this form.

Indeed, in order to prove theorem 1, one may suppose that the field k is algebraically closed and look at $\mathrm{GL}(V_1)$, $\mathrm{GL}(V_2)$ as algebraic groups.

Then one can suppose G to be itself a linear algebraic group. For, it suffices to take the *Zariski-closure* $\bar{G}$ of the image of G in $\mathrm{GL}(V_1) \times \mathrm{GL}(V_2)$, namely the set of points

$$x \in \mathrm{GL}(V_1) \times \mathrm{GL}(V_2)$$

such that $f(x) = 0$ for all polynomial functions f vanishing identically on the image of G. The semi-simplicity of the representation $V_1 \otimes_k V_2$ is equivalent for G and $\bar{G}$, because it is a condition that can be expressed as the vanishing of some polynomials.

Now the theory of linear algebraic groups applies: there is an algebraic subgroup $\mathrm{rad}^u(G)$ of G called the *unipotent radical* which is connected, unipotent (meaning that all the eigenvalues of its elements are 1), normal and contains any other subgroup of G with these three properties. The unipotent radical controls the semi-simplicity of the representations of G:

Theorem 2 (Weyl [33])**.** *Suppose* $\mathrm{char}(k) = 0$*. An algebraic*[1] *representation of a linear algebraic group* G *is semi-simple if and only if every element of the unipotent radical acts as the identity on* V*.*

Applying the preceding fact, the proof of Chevalley's theorem is easily achieved: since the representations V_1 and V_2 are supposed to be semi-simple, an element g of the unipotent radical of G acts as the identity on V_1 and V_2. Therefore g operates trivially on $V_1 \otimes_k V_2$ too.

Reductive groups

Weyl's theorem is usually formulated as follows: an algebraic representation of a *reductive* group, *i.e.* a linear algebraic group whose unipotent radical is trivial, is semi-simple.

Examples of reductive groups are GL_n, SL_n, SO_n, Sp_{2n}. In general, a reductive group is the extension of a semi-simple group by a product of copies of the multiplicative group k^*, the algebraic equivalent of $\mathbb{C}^*$. An example of a non-reductive group is the additive group k [2], which corresponds to the complex Lie group $\mathbb{C}$.

Over the complex numbers, a reductive group G can be seen also as a complex Lie group. Therefore two topologies cohabit in it: the one coming from the topology of $\mathbb{C}$ (the "usual topology"), and the Zariski one (which is coarser). This is true for every algebraic group, but there is a topological property which is specific to reductive groups: they contain subgroups which are dense for the Zariski topology and compact for the usual one.

For instance, a Zariski-dense compact subgroup of $\mathrm{GL}_n(\mathbb{C})$ is the group $\mathbb{U}(n)$ of unitary matrices; on the other hand, the unique compact subgroup of $\mathbb{C}$ is the trivial one (which is not Zariski-dense).

For a semi-simple group G one uses É. Cartan's classification of semi-simple Lie algebras to construct a Zariski-dense compact subgroup K: indeed the Killing form on $\mathrm{Lie}\, G$ is non-degenerate, and K corresponds to a maximal real Lie sub-algebra on which the form is negative definite.

[1] This means that the map $G \to \mathrm{GL}(V)$ is polynomial in the coefficients of the matrices of G.

[2] The additive group k can be seen as a subgroup of $\mathrm{GL}_2(k)$ through the embedding

$$a \longmapsto \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}.$$

Weyl's Unitarian Trick

Let us go back to the proof of theorem 2, in the second reformulation I gave above. Quite often in algebraic geometry the characteristic 0 hypothesis is the shade of transcendental techniques, that is, analysis on real or complex numbers. Theorem 2 is an example of this phenomenon.

Indeed, in order to prove it one reduces to the case where k is $\mathbb{C}$: this is possible because the polynomial equations defining G and its representation $G \to \mathrm{GL}(V)$ involve only finitely many terms that are transcendental over $\mathbb{Q}$.

Given a G-stable subspace W of V, finding a G-stable supplement amounts to construct a G-equivariant linear projection $V \to W$.

Here comes a remarkable argument, refined gradually by Hilbert, Hurwitz and Weyl, who at last named it *Unitarian Trick*. Pick one linear projection $\pi \colon V \to W$ and average it on a Zariski-dense compact subgroup K of G:

$$\tilde{\pi}(v) := \int_K g\pi(g^{-1}v)\, d\mu(g), \quad (v \in V),$$

where μ is the Haar measure of total mass 1 on K. The function $\tilde{\pi}$ is equivariant for the action of K, but by Zariski-density of K, the projection $\tilde{\pi}$ is also G-equivariant.

A *motto* summarizing the previous proof might be: the subgroup K is small for the usual topology (which permits to integrate), but big enough to control the representations of G (which depends only on the Zariski topology).

Characteristic $p > 0$

Over a field k of characteristic $p > 0$ (assumed algebraically closed, for simplicity), both theorems 1 and 2 are false.

On the positive side, every representation V of an algebraic group G is semi-simple in the following cases:

- G is a finite group whose order is prime to p: given a G-stable subspace $W \subset V$ and a projection $\pi \colon V \to W$, the function
$$\tilde{\pi}(v) := \frac{1}{\#G} \sum_{g \in G} g\varphi(g^{-1}v),$$
is G-equivariant;
- G is the multiplicative group k^*: in this case V decomposes as
$$V = \bigoplus_{a \in \mathbb{Z}} V_a,$$
where V_a is the subspace where $t \in k^*$ acts by t^a.

Nagata [11, Théorème IV.3.3.6] shows that there are no other possibilities: given G an algebraic group such that every representation is semi-simple, the identity component G^0 is isomorphic to $\mathbb{G}_m^r$ and G/G^0 has order prime to p.

Concerning theorem 1 one has the following:

Example 3. Let k be a field characteristic $p > 0$ and $V = k^2$. Consider the k-vector space $V(d)$ of homogeneous polynomials of degree d on V: it is of dimension $d + 1$, a basis being given by the monomials

$$x^d, x^{d-1}y, \ldots, xy^{d-1}, y^d.$$

The group $G = \mathrm{SL}_2(k)$ acts on $V(d)$ through the contragradient representation: explicitly, for a polynomial $f(x, y)$ and an invertible matrix

$$g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},$$

with $\alpha, \beta, \gamma, \delta \in k$ such that $\alpha\delta - \beta\gamma = 1$,

$$(gf)(x, y) = f(g^{-1}(x, y)) = f(\delta x - \beta y, -\gamma x + \alpha y).$$

The representation $V(d)$ is irreducible for $d = 1, \ldots, p-1$. For $d = p$ the map $\varphi \colon V(1) \to V(p)$, associating to a linear form f its p-th power f^p, is linear, injective and G-equivariant. The image of φ is the kernel of the map $\psi \colon V(p) \to V(p-2)$ given by

$$f \longmapsto \frac{1}{y}\frac{\partial f}{\partial x}.$$

The exact sequence of representations

$$0 \longrightarrow V(1) \xrightarrow{\varphi} V(p) \xrightarrow{\psi} V(p-2) \longrightarrow 0,$$

obtained in this way does not split if the field k has at least 3 elements. In particular $V(p)$ is not semi-simple. If $d_1, \ldots, d_n$ are integers ranging between 1 and $p - 1$ such that their sum is p, the multiplication map $f_1 \otimes \cdots \otimes f_n \mapsto f_1 \cdots f_n$ induces a surjection $W := V(d_1) \otimes \cdots \otimes V(d_n) \to V(p)$. It follows that W is not semi-simple.

However Serre shows that these problems do not occur as soon as the dimension is small enough:

Theorem 4 (Serre [29])**.** *Let $W_1, \ldots, W_m$ be semi-simple representations of a group G on a field of characteristic $p > 0$. If*

$$\sum_{i=1}^{m} \dim W_i < p + m,$$

then $W_1 \otimes \cdots \otimes W_m$ is semi-simple.

According to Example 3 the condition in the theorem above is sharp. Various generalizations of the result of Serre can be found in [30, 10, 2].

2 Vector bundles on Riemann surfaces

Representations of the fundamental group

Let X be a compact Riemann surface of genus g and $\tilde{X}$ its universal covering. The fundamental group $\pi_1(X, x)$ (with respect to a base point x) is generated by $2g$ loops $a_1, \ldots, a_g, b_1, \ldots, b_g$ satisfying the relation

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} = 1.$$

To a representation $\rho \colon \pi_1(X, x) \to \mathrm{GL}_r(\mathbb{C})$ of the fundamental group, one associates the vector bundle $E(\rho)$ of rank r on X obtained as the quotient of $\tilde{X} \times \mathbb{C}^r$ through the action

$$g(\tilde{x}, v) = (g\tilde{x}, \rho(g)v).$$

Theorem 5 (Narasimhan–Seshadri [25])**.** *Suppose ρ preserves a hermitian norm on $\mathbb{C}^r$. Then:*

1. *$E(\rho)$ is a direct sum of simple bundles, and $E(\rho)$ is simple if and only ρ is irreducible;*
2. *if $E(\rho)$ is simple, then it does not admit non-zero global holomorphic sections;*
3. $\deg E(\rho) = 0$ *and $E(\rho)$ is semi-stable:* $\deg F \leq 0$ *for every sub-vector bundle F of $E(\rho)$.*

Moreover, every stable vector bundle of degree 0 *arises from a unitary representation of the fundamental group of X.*

In the previous statement, a vector bundle E is said to be:

- *simple* if the only endomorphisms of E are homotheties;

- *semi-stable* (resp. *stable*) if for every sub-vector bundle F of E different from 0 and E,

$$\frac{\deg F}{\operatorname{rk} F} \leq \frac{\deg E}{\operatorname{rk} E}, \qquad \text{(resp. <)}.$$

The number $\mu(E) := \frac{\deg E}{\operatorname{rk} E}$ is called the *slope* of E.

Remark that a simple bundle E cannot be written as the direct sum of two proper sub-bundles.

Statements (1) and (2) are consequences of the following isomorphism, for unitary representations $\rho_i \colon \pi_1(X, x) \to \mathrm{GL}(V_i)$ and $i = 1, 2$,

$$\operatorname{Hom}_{\pi_1(X,x)}(V_1, V_2) \xrightarrow{\sim} \operatorname{Hom}(E(\rho_1), E(\rho_2)).$$

Statement (1) is obtained thanks to Weyl's Unitarian Trick, by decomposing the representation ρ into its irreducible components. In order to get a taste of how properties of the representation ρ transfer to those of the vector bundle $E(\rho)$, let me sketch the proof of (3).

The line bundle $L := \bigwedge^r E(\rho)$ is associated to the determinant representation $\det\rho \colon \pi_1(X, x) \to \mathbb{C}^\times$ of ρ, which is unitary. If the degree of L were positive, by the Riemann-Roch theorem, some positive enough multiple $L^{\otimes d}$ of L would admit non-zero global sections: this would contradict (2), as $L^{\otimes d}$ is indecomposable. One concludes by applying the same reasoning to the determinant of the representation ρ^* contragradient to ρ.

Take F to be a sub-vector bundle of $E(\rho)$ and, arguing by contradiction, assume that the degree of F is > 0. One can further suppose that F is a line bundle: if s denotes the rank of F, the s-th exterior power $\bigwedge^s F$ is a sub-line bundle of the vector bundle $\bigwedge^s E(\rho)$, which is associated to the representation $\bigwedge^s \rho$.

As before, by the Riemann–Roch theorem, there is a positive integer $d \geq 1$ such that $L^{\otimes d}$ has non-zero sections. Decompose the unitary representation $\rho^{\otimes d}$ into irreducible ones $\rho_1, \ldots, \rho_n$. This corresponds to writing $E(\rho)^{\otimes d}$ as the sum of the simple vector bundles $E(\rho_i)$. The projection of L onto $E(\rho_i)$ has to be zero for all $i = 1, \ldots, n$: otherwise $E(\rho_i)$ would have non-zero sections, which is impossible according to (2). This implies that F is the trivial bundle, contradicting the hypothesis of having positive degree.

The results of Narasimhan–Seshadri have been for quite a long time the only tool to prove the following result:

Theorem 6. *Let E, F be semi-stable vector bundles on X. Then $E \otimes F$ is semi-stable.*

The key situation is when E and F are stable of degree 0: if this is the case, thanks to the theorem of Narasimhan-Seshadri, E and F are associated to unitary representations ρ_E, ρ_F of the fundamental group. The tensor product representation $\rho_E \otimes \rho_F$ is unitary, thus $E \otimes F$ is semi-stable.

Characteristic $p > 0$

On a field of positive characteristic, compact Riemann surfaces are replaced by smooth projective curves.

To fix ideas, let $f \in \mathbb{F}_p[x_0, x_1, x_2]$ be an irreducible homogeneous polynomial and consider the locus in $\mathbb{P}^2(\bar{\mathbb{F}}_p)$ where it vanishes:

$$X = \Big\{[x_0 : x_1 : x_2] \in \mathbb{P}^2(\bar{\mathbb{F}}_p) : f(x_0, x_1, x_2) = 0\Big\}.$$

Suppose that X does not have singular points: this means that for every point $x \in X$, some partial derivative $\frac{\partial f}{\partial x_i}$ does not vanish at x.

The genus g of X can be computed as for Riemann surfaces: if f is of degree d, then

$$g = \frac{(d-1)(d-2)}{2}.$$

However X carries something that a Riemann surface does not, the *Frobenius endomorphism*. It is the map sending a point $[x_0 : x_1 : x_2] \in X$ to $[x_0^p : x_1^p : x_2^p]$. Note that the latter point still belongs to X because

$$f(x_0^p, x_1^p, x_2^p) = f(x_0, x_1, x_2)^p = 0,$$

(one uses here that f has coefficients in $\mathbb{F}_p$).

There is no obvious way to port unitary representations of the fundamental group into this context. Rather, one takes the point of view of semi-stable vector bundles, whose definition can be translated word by word. Yet the statement analogous to theorem 6 is false and, as one can guess, examples come from symmetric powers:

Example 7. Let E be a vector bundle on X and consider its i-th symmetric power $\operatorname{Sym}^i E$.

The naive idea of embedding E into $\operatorname{Sym}^p E$ by raising sections to the p-th power does not work this time: the map $E \to \operatorname{Sym}^p E$, $(x, s) \mapsto (x, s^p)$, where x is a point of X and s is a section of E over x, is not a bundle map. Nonetheless it induces an inclusion of the pull-back $\mathrm{Fr}_X^* E$ of E (the so-called *Frobenius twist*) as a sub-vector bundle of $\operatorname{Sym}^p E$.

If the genus of X is at least 2, there are plenty of semi-stable vector bundles on X such that their Frobenius twist is not semi-stable[3]: for such a vector bundle E, the above discussion shows that $\operatorname{Sym}^p E$ is not semi-stable.

There is also an analogue of Serre's theorem for semi-stable vector bundles (Balaji–Parameswaran [3], Ilangovan–Mehta–Parameswaran [18]):

Theorem 8. *Let $E_1, \ldots, E_n$ be semi-stable vector bundles on X. If*

$$\sum_{i=1}^n \operatorname{rk} E_i < p + n,$$

then $E_1 \otimes \cdots \otimes E_n$ is semi-stable.

Back to characteristic 0

In view of the preceding example, an algebraic proof of theorem 6 has to break down in positive characteristic. Let me detail the one discovered by Ramanan–Ramanathan [26]: it is particularly interesting from our point of view because it makes intervene algebraic groups (where *a priori* there are none).

Let V_1 and V_2 be semi-stable vector bundles on a compact Riemann surface X, and let W be a sub-vector bundle of the

[3] For instance, consider the push-forward $V := (\mathrm{Fr}_X)_* O_X$ of the trivial bundle O_X. A simple computation shows

$$\deg V = (p-1)(g-1) > 0.$$

By the Harder–Narasimhan filtration, V contains a semi-stable vector bundle E of positive degree. The kernel K of the induced map $\mathrm{Fr}_X^* E \to O_X$ has degree $\geq \deg \mathrm{Fr}_X^* E$. Having smaller rank, the slope of K is bigger than the slope of $\mathrm{Fr}_X^* E$.

is easy to obtain when W is "special" (for instance if W is of the form $W_1 \otimes W_2$ for sub-vector bundles $W_i \subset V_i$) or if it is "generic" (meaning that its intersection with some filtration has small dimension).

This heuristics is made precise by Ramanan–Ramanathan: replacing the locutions "special" and "generic" respectively by unstable and semi-stable in the sense of Geometric Invariant Theory (GIT) permits to derive (1) in both cases. The algebraic group that enters into play is $\mathrm{GL}_{r_1,K} \times \mathrm{GL}_{r_2,K}$ where K is the field of meromorphic functions on X and r_i the rank of V_i.

What goes wrong in characteristic p is the characterization of semi-stability they use (the *Hilbert–Mumford criterion* [17, 24], in the version of Kempf [20] and Rousseau [27]) that needs the field K to be perfect.

If X is a compact Riemann surface, then K is perfect because of characteristic 0. If X is a smooth projective curve over a field k of characteristic p, K is a finite extension of the field $k(t)$ of rational functions in one variable, thus imperfect.

3 Weakly admissible isocrystals

Counting points with eigenvalues

Let $f(x_1, \dots, x_n)$ be a polynomial with integral coefficients. The set of its solutions modulo p,

$$V(\mathbb{F}_p) := \{x = (x_1, \dots, x_n) \in \mathbb{F}_p^n : f(x) = 0\},$$

is finite. What can we say about its cardinality?

There is a fruitful way to look at solutions modulo p. Consider the affine variety defined by f,

$$V := \{x \in \bar{\mathbb{F}}_p^n : f(x) = 0\}.$$

As an element $a \in \bar{\mathbb{F}}_p$ belongs to $\mathbb{F}_p$ if and only if $a^p = a$, a point $(x_1, \dots, x_n) \in V$ lies in $V(\mathbb{F}_p)$ if and only if $x_i^p = x_i$ for all i. In other words, $V(\mathbb{F}_p)$ is the set of fixed points of the Frobenius endomorphism of V (*i.e.* the map raising to the p-th power the coordinates of the points of X).

A brilliant idea of Weil was to relate this point of view with the following form of the "Lefschetz fixed point theorem", known as the Lefschetz–Hopf theorem:

Theorem 9. *Let X be a compact manifold and $F\colon X \to X$ a continuous map. If the set $\mathrm{Fix}(F)$ of fixed points of F is finite, then*

$$\sum_{x \in \mathrm{Fix}(F)} i(x, F) = \sum_{i \geq 0} (-1)^i \operatorname{Tr}(F_* \mid \mathrm{H}_i(X, \mathbb{Q})),$$

where $i(x, F)$ is the index of F at a fixed point x and $\mathrm{H}_i(X, \mathbb{Q})$ the i-th rational homology group.

Weil conjectured that if there were a cohomology theory that behaved well enough (called in this day and age "Weil cohomology theory") one could compute the number of rational points by means of an analogue of the Lefschetz fixed point theorem.

Constructing such a cohomology theory has been one of the driving forces behind the work of Grothendieck and his school. The results in SGA 4 and 5 say that there is actually one for each prime $\ell \neq p$: the *étale ℓ-adic cohomology*, with coefficients in the field $\mathbb{Q}_\ell$ of ℓ-adic numbers. Grothendieck in particular was able to prove an analogue of Lefschetz's fixed point theorem:

Theorem 10 (Grothendieck's trace formula [16])**.** *Let X be a projective[4] variety over $\mathbb{F}_p$. Then,*

$$\# X(\mathbb{F}_p) = \sum_{i \geq 0} (-1)^i \operatorname{Tr}(\mathrm{Fr}_X \mid \mathrm{H}^i_{ét}(X, \mathbb{Q}_\ell)),$$

where $X(\mathbb{F}_p)$ is the set of points of X having coordinates in $\mathbb{F}_p$.

Let us go back to counting the points of $V(\mathbb{F}_p)$. The preceding theorem suggests to pass to the projective closure of V: consider the homogeneous polynomial associated to f,

$$\tilde{f}(x_0, \dots, x_n) := x_0^d f(x_1/x_0, \dots, x_n/x_0),$$

where d is the degree of f, and

$$X := \{x = [x_0 : \dots : x_n] \in \mathbb{P}^n(\bar{\mathbb{F}}_p) : \tilde{f}(x) = 0\}.$$

With this at hand, the philosophy can be restated as follows: in order to compute the number of rational points of X (*i.e.* those with coordinates in $\mathbb{F}_p$), one has to estimate the size of the eigenvalues of the Frobenius acting on the ℓ-adic étale cohomology.

If X is non-singular, the following facts are consequences of Deligne's proof of the last of Weil's conjectures (called *Riemann hypothesis*):

1. the set of eigenvalues of Fr_X on $\mathrm{H}^i_{ét}(X, \mathbb{Q}_\ell)$ does not depend on ℓ;
2. an eigenvalue of Fr_X on $\mathrm{H}^i_{ét}(X, \mathbb{Q}_\ell)$ is an algebraic integer, which is not divisible by ℓ and whose complex absolute value is $\sqrt{p^i}$ (with respect to any complex embedding).

Divisibility properties

A question left aside is the order of divisibility by p of the eigenvalues of the Frobenius, or in other words, their p-adic valuation. A first result of this kind is the following:

Theorem 11 (Chevalley-Warning [28])**.** *Suppose* $\deg f < n$. *Then $\# V(\mathbb{F}_p)$ is divisible by p.*

In order to study such a question it seems appropriate to ask the cohomology theory to produce $\mathbb{Q}_p$-vector spaces. Unfortunately p-adic étale cohomology does not work as one would like and one has to consider *crystalline cohomology*.

From now on suppose X non-singular and that f is not divisible by p. The polynomial f we started with, as well as its homogeneization $\tilde{f}$, have integral coefficients. Consider the projective variety, defined over $\mathbb{Q}$,

$$Y := \{x = [x_0 : \dots : x_n] \in \mathbb{P}^n(\bar{\mathbb{Q}}) : \tilde{f}(x) = 0\}.$$

As a consequence of Berthelot's results on crystalline cohomology, the algebraic de Rham cohomology groups of Y, or better said their extension to $\mathbb{Q}_p$,

$$\mathrm{H}^q_{\mathrm{dR}}(X/\mathbb{Q}_p) := \mathrm{H}^q_{\mathrm{dR}}(Y/\mathbb{Q}) \otimes \mathbb{Q}_p,$$

come equipped with a Frobenius operator Fr_X.

A classical conjecture of Katz proved by Mazur relates the p-adic absolute values of the eigenvalues of Fr_X to the Hodge numbers of Y,

$$h^{i,q-i} := \dim_{\mathbb{Q}} \mathrm{H}^{q-i}(Y, \Omega^i_Y),$$

where Ω^i_Y denotes the bundle of differential i-forms on Y. Order the eigenvalues α_i of Fr_X so that $\mathrm{ord}_p(\alpha_i) \leq \mathrm{ord}_p(\alpha_{i+1})$, and set $\beta_i = h^{0,q} + h^{1,q-1} + \dots + h^{i,q-i}$.

[4] The results holds more generally for any algebraic variety assuming that the cohomology is taken with compact supports.

Theorem 12 (Katz's conjecture [19], Mazur [22, 23]). *With the notations introduced above,*

$$\begin{aligned}&\operatorname{ord}_p(\alpha_1) + \cdots + \operatorname{ord}_p(\alpha_t)\\ &\quad \geq 0 \cdot h^{0,q} + 1 \cdot h^{1,q-1} + \cdots + ih^{i,q-i} + (i+1)(t-\beta_i), \quad (2)\end{aligned}$$

where $\beta_i < t < \beta_{i+1}$.

There is a more geometric way to state theorem 12. Consider the Newton polygon[5] of the characteristic polynomial $\det(\mathrm{id} - \mathrm{Fr}_X \cdot t)$ and the polygon associated to the Hodge filtration of $\mathrm{H}^q_{\mathrm{dR}}(X/\mathbb{Q}_p)$:

$$F^i\,\mathrm{H}^q_{\mathrm{dR}}(X/\mathbb{Q}_p) := \bigoplus_{j=i}^{q} \mathrm{H}^{q-j}(Y, \Omega^j_Y) \otimes \mathbb{Q}_p.$$

Concretely, these polygons are piecewise linear functions starting at $(0,0)$ and with slopes

	Newton polygon	Hodge polygon
slope	$\operatorname{ord}(\alpha_i)$	i
on the interval	$\left[\sum_{j=1}^{i-1} m_j, \sum_{j=1}^{i} m_j\right]$	$\left[\sum_{j=0}^{i-1} h^{j,q-j}, \sum_{j=0}^{i} h^{j,q-j}\right]$

where m_i is the multiplicity of the eigenvalue α_i. Theorem 12 becomes "the Newton polygon lies above the Hodge polygon".

Example 13. Suppose X is a smooth projective curve of genus g. Let me collect here some information on the Newton polygon of X:

- By Serre duality, $h^{0,1} = h^{1,0} = g$.
- Poincaré duality implies that the Newton polygon ends at $(2g, g)$, as the Hodge polygon;
- Let $A = \mathrm{Jac}(X)$ be the jacobian variety of X. For a positive integer n denote by $A[n]$ the subgroup of n-torsion points of $A(\bar{\mathbb{F}}_p)$. There is an integer $0 \leq \mathrm{rk}_p(A) \leq g$, the *p-rank of A*, such that

$$\#A[n] = \begin{cases} n^{\mathrm{rk}_p(A)} & \text{if } n \text{ is a power of } p,\\ n^{2g} & \text{if } n \text{ is prime to } p.\end{cases}$$

The previous information describe completely the Newton polygon of X for $g = 1, 2$. However, starting from $g = 3$ the situation is more involved (see Figure 1).

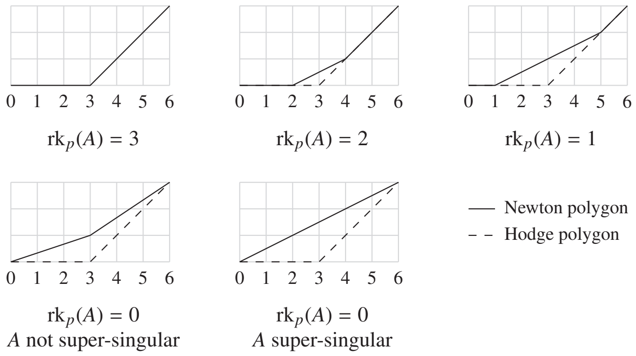


Figure 1: Newton polygons for a curve of genus 3

Filtered isocrystals

A strengthening of Mazur's theorem, conjectured by Fontaine [15] and proved by Faltings [12], states that the inequality (2) holds for every sub-vector space $W \subset \mathrm{H}^q_{\mathrm{dR}}(X/\mathbb{Q}_p)$ stable under the action of the Frobenius endomorphism:

$$\sum_i i \dim_{\mathbb{Q}_p}(W^i/W^{i+1}) \leq \operatorname{ord}_p \det(\mathrm{Fr}_X \mid W),$$

where $W^i := F^i\,\mathrm{H}^q_{\mathrm{dR}}(X/\mathbb{Q}_p) \cap W$. Moreover, the fact that both polygons have the same end-points says that the previous inequality is an identity for $W = \mathrm{H}^q_{\mathrm{dR}}(X/\mathbb{Q}_p)$.

A *filtered isocrystal* over $\mathbb{Q}_p$ is a linear algebraic datum miming the above situation: it is a triple $(V, \varphi, F^\bullet V)$ made of a finite-dimensional $\mathbb{Q}_p$-vector space, a linear map $\varphi\colon V \to V$ and a decreasing filtration $F^\bullet V = (F^i V)_{i\in\mathbb{Z}}$. It is said to be *weakly admissible* if, for every linear subspace $W \subset V$ stable under φ,

$$\sum_i i \dim_{\mathbb{Q}_p}(W^i/W^{i+1}) \leq \operatorname{ord}_p \det(\varphi \mid W),$$

where $W^i := F^i V \cap W$, with equality for $W = V$.[6]

Weakly admissible filtered isocrystals are meant to be one of the possible p-adic analogues of Hodge structures. Because of the Künneth formula for cohomology, it is natural to ask whether the tensor product of weakly admissible isocrystals is weakly admissible. A partial affirmative answer was given by Lafaille [21], while the general result was proven by Faltings [13, 14] and Totaro [32].

The role of semi-stable vector bundles

In order to explain the approaches of Faltings and Totaro, let me consider a slightly different situation.

Let V be a finite dimensional $\mathbb{Q}_p$-vector space, K a finite extension of $\mathbb{Q}_p$ and $F^\bullet V = (F^i V)$ a filtration of the K-vector space $V \otimes_{\mathbb{Q}_p} K$ by K-vector spaces. For a $\mathbb{Q}_p$-linear subspace $W \subset V$ define

$$\deg W = \sum_i i \dim_{\mathbb{Q}_p}(W^i/W^{i+1}), \qquad (3)$$

where $W^i := F^i V \cap (W \otimes_{\mathbb{Q}_p} K)$.

The filtered vector space $(V, F^\bullet V)$ is said to be *semi-stable* if, for all non-zero $\mathbb{Q}_p$-linear subspace $W \subset V$,

$$\mu(W) := \frac{\deg W}{\dim W} \leq \mu(V) := \frac{\deg V}{\dim V}.$$

Example 14. Suppose $V = \mathbb{Q}_p^2$. A point $x \in \mathbb{P}^1(\bar{\mathbb{Q}}_p)$ corresponds to a vector line $L \subset V \otimes_{\mathbb{Q}_p} K$ for a suitable finite extension K of $\mathbb{Q}_p$. The filtration

$$F^2 V := 0 \quad \subset \quad F^1 V := L \quad \subset \quad F^0 V := V \otimes_{\mathbb{Q}_p} K$$

satisfies $\mu(V) = \frac{3}{2}$ and $\mu(W) = 2 - \dim_K L \cap (W \otimes_{\mathbb{Q}_p} K)$ for every $\mathbb{Q}_p$-vector line $W \subset V$. In particular $(V, F^\bullet V)$ is semi-stable if and only if the line L is not defined over $\mathbb{Q}_p$.

The set of semi-stable filtrations on V is *Drinfeld's upper half-plane* $\Omega^1_{\mathbb{Q}_p} := \mathbb{P}^1(\bar{\mathbb{Q}}_p) \smallsetminus \mathbb{P}^1(\mathbb{Q}_p)$. It owes its name to *Poincaré's upper-half plane* $\mathfrak{h} = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ which can be seen as the "upper-half" of $\mathbb{P}^1(\mathbb{C}) \smallsetminus \mathbb{P}^1(\mathbb{R})$.

[5] Given a polynomial $f(t) = a_0 + a_1 t + \cdots + a_d t^d$ with coefficients in $\mathbb{Q}_p$, its Newton polygon is the convex hull of the points $(i, \operatorname{ord}_p(a_i))$, *i.e.* the smallest convex function on the interval $[0, d]$ such that the points $(i, \operatorname{ord}_p(a_i))$ lie above (or on) its graph.

[6] Actually this definition is too restrictive, as one has to let the filtration $F^\bullet V$ to be defined only on a finite extension of $\mathbb{Q}_p$.

To have a picture in mind, Drinfeld's upper half-plane (or better the Berkovich analytic space attached to it) retracts onto the Bruhat-Tits building of $SL_2(\mathbb{Q}_p)$; see Figure 2.

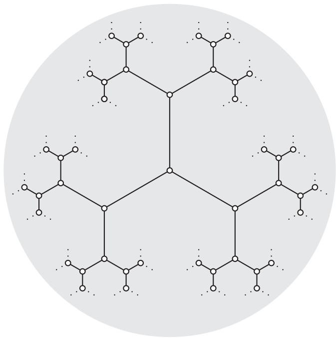


Figure 2: The Bruhat-Tits building of $SL_2(\mathbb{Q}_p)$ **for** $p = 2$

Suppose that K is a Galois extension of $\mathbb{Q}_p$ and consider the filtrations $F_1^\bullet V, \dots, F_n^\bullet V$ conjugated to $F^\bullet V$ under the action of the Galois group $\mathrm{Gal}(K/\mathbb{Q}_p)$. Faltings' idea is to associate to these filtrations a vector bundle $E(V)$ on a compact Riemann surface Y such that $E(V)$ is semi-stable (as a vector bundle) if and only if the filtered vector space $(V, F^\bullet V)$ is. Since this construction is compatible with tensor product, one draws the result thanks to the theorem of Narasimhan–Seshadri.

The proof of Totaro is again based on the tensor product of semi-stable bundles, but instead of relying on the "analytic" result of Narasimhan-Seshadri, it follows the "algebraic" approach of Ramanan–Ramanathan.

Totaro characterizes semi-stability of filtrations in terms of Geometric Invariant Theory. It is just a matter of unwinding the definitions: the reader familiar with GIT recognizes the quantity appearing in (3) as Mumford's μ-coefficient in the case of filtrations [24, §4.4]. Then he uses Ramanan–Ramanathan's dichotomy to derive the wanted inequality.

4 Hermitian vector bundles on arithmetic curves

Euclidean lattices

An *Euclidean lattice* of rank n is a finitely generated subgroup Γ of $\mathbb{R}^n$ such that $\mathbb{R}^n/\Gamma$ is compact. On $\mathbb{R}^n/\Gamma$ there is a unique measure μ_Γ such that, for all continuous functions $f\colon \mathbb{R}^n \to \mathbb{R}$ with compact support,

$$\int_{\mathbb{R}^n} f(x)\, d\lambda(x) = \int_{\mathbb{R}^n/\Gamma} \tilde{f}(\xi)\, d\mu_\Gamma(\xi),$$

where λ is Lebesgue measure on $\mathbb{R}^n$ and $\tilde{f}(\xi) = \sum_{\Gamma + x = \xi} f(x)$. The volume of $\mathbb{R}^n/\Gamma$ with respect to this measure is called the *covolume* of Γ. If $\gamma_1, \dots, \gamma_n$ is a basis of Γ, the covolume $\mathrm{covol}(\Gamma)$ can be computed as $|\det(\gamma_1, \dots, \gamma_n)|$.

There is a more intrinsic definition of a Euclidean lattice: it is a couple $\bar{E} = (E, \|\cdot\|_E)$ made of a free abelian group E of finite rank together with a hermitian norm $\|\cdot\|_E$ on $E \otimes_{\mathbb{Z}} \mathbb{R}$. Following conventions that have become usual in Arakelov geometry, define:

the *rank* of $\bar{E}$	$\mathrm{rk}(\bar{E}) := \dim_{\mathbb{R}} E \otimes_{\mathbb{Z}} \mathbb{R}$,
the *degree* of $\bar{E}$	$\widehat{\deg}(\bar{E}) = -\log \mathrm{covol}(E)$,
the *slope* of $\bar{E}$	$\hat{\mu}(\bar{E}) = (\mathrm{rk}\, \bar{E})^{-1}\, \widehat{\deg}\, \bar{E}$.

Diophantine approximation

Before diving into the analogy with vector bundles, let me explain how these concepts arise in Diophantine approximation.

Theorem 15 (Thue [31])**.** *Let $\alpha \in \mathbb{R}$ be an algebraic number of degree $d \geq 2$. Given $\varepsilon > 0$ there are only finitely many rational numbers $p/q \in \mathbb{Q}$ such that*

$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{q^{\frac{d}{2}+1+\varepsilon}},$$

where $(p, q) = 1$ and $q > 0$.

The exponent $\frac{d}{2} + 1$ has been successively sharpened by Siegel, Dyson and Roth respectively to $2\sqrt{d}$, $\sqrt{2d}$ and 2, the latter being optimal because of Dirichlet's theorem on approximation of real numbers. However all these proofs follow the scheme of Thue's argument, which can be roughly split in four steps:

1. Construct a polynomial f in n variables with small integral coefficients vanishing "as much as possible" on the point $(\alpha, \dots, \alpha)$.
2. Prove that the polynomial f does not vanish too much on n-tuples $x = (x_1, \dots, x_n)$ made of rational approximations. Or better said some derivative g of f does not vanish at x.
3. Bound from above $|g(x)|$ in terms of the order of vanishing of g at $(\alpha, \dots, \alpha)$ by looking at its Taylor expansion.
4. Bound from below $|g(x)|$ by using that a positive integer is ≥ 1.

When there are too many good approximations of α to exist, bounds in 3 and 4 are in contradiction.

In his Bourbaki report on the work of Masser and Wüstholz on periods and isogenies of abelian varieties (having among its consequences Mordell's Conjecture, by the time already a theorem of Faltings), Bost reinterprets in terms of Euclidean lattices the preceding steps.

The point is to use a simple fact on the behaviour of slopes with respect to linear maps:

Lemma 16 (Slopes inequality)**.** *Let $\bar{E}$, $\bar{F}$ be Euclidean lattices and $\varphi\colon E \to F$ an injective map of abelian groups. Then,*

$$\hat{\mu}(\bar{E}) \leq \hat{\mu}_{\max}(\bar{F}) + \log \|\varphi\|_{\sup},$$

where $\hat{\mu}_{\max}(\bar{F})$ is the maximum of the slopes of sub-lattices of $\bar{F}$ and $\|\varphi\|_{\sup}$ is the operator norm of φ.

Thue's four steps argument is translated as follows. Step 2, which is the one with geometric content, corresponds to the injectivity[7] of the map φ. Step 3 corresponds to the upper bound of $\|\varphi\|_{\sup}$ and step 4 to bounding the slopes of $\bar{E}$ and $\bar{F}$.

A step that seems gone missing is the first one, which is indeed a little different. Instead of picking a particular polynomial, the slopes inequality allows to consider the whole space of polynomials with the wanted vanishing property. In the concrete case of Masser and Wüstholz this permitted Bost to replace fine considerations on theta functions by geometric arguments of Moret–Bailly.

Semi-stable lattices

A Euclidean lattice $\bar{E}$ is said to be *semi-stable* if $\hat{\mu}(\bar{F}) \leq \hat{\mu}(\bar{E})$ for all non-zero sub-lattices $F \subset E$ with the induced Euclidean norm.

[7] In this situation φ is the evaluation map of a polynomial at α (or better, some truncated Taylor expansion of f around α).

Question (Bost). *Is the tensor product[8] of semi-stable lattices semi-stable?*

Results towards a positive answer to this question have been proved by André [1], Bost, de Shalit–Parzanovski, Chen [7] and Bost–Künneman [6]. The best available result is the following:

Theorem 17 (Bost-Chen [5]). *Let $\bar{E}, \bar{F}$ be semi-stable Euclidean lattices. Then,*

$$\hat{\mu}_{\max}(\bar{E} \otimes \bar{F}) \leq \hat{\mu}(\bar{E}) + \hat{\mu}(\bar{F}) + \frac{1}{2} \min\{\ell(\operatorname{rk} E), \ell(\operatorname{rk} F)\},$$

where, for an integer $n \geq 2$, $\ell(n) = \sum_{i=2}^{n} \frac{1}{i} \leq \log n$.

Furthermore, if $\operatorname{rk} E \cdot \operatorname{rk} F \leq 9$, then $\bar{E} \otimes \bar{F}$ is semi-stable.

Analogy with the projective line?

The field of rational numbers ressembles somehow to the field of rationals functions in one variable. This analogy works by making correspond points of $\mathbb{P}^1$ to equivalence classes of absolute values on $\mathbb{Q}$. Stressing this point of view, the avatar over $\mathbb{Q}$ of vector bundles are Euclidean lattices.

Nonetheless the theory of vector bundles on $\mathbb{P}^1$ is rather poor: a vector bundle on $\mathbb{P}^1$ is a direct sum of line bundles[9], and semi-stable vector bundles are all of the form $O(d)^{\oplus n}$.

This is not at all the situation for Euclidean lattices: the set of isomorphism classes of Euclidean lattices of rank n is the double quotient $\mathrm{GL}_n(\mathbb{Z})\backslash \mathrm{GL}_n(\mathbb{R})/\mathrm{O}_n(\mathbb{R})$, where $\mathrm{O}_n(\mathbb{R})$ is the group of orthogonal matrices of size n. Moreover, if one associates to a point τ in the upper half plane the lattice generated by 1 and τ, then the region of semi-stable lattices of rank two is the closed exterior of the Ford circles in the strip $0 < \operatorname{Im}\tau \leq 1$ [1].

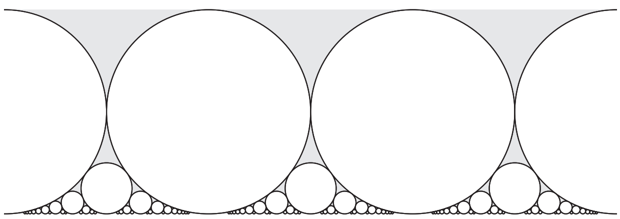


Figure 3: The exterior of Ford circles

Irreducible Euclidean lattices

Given a Euclidean lattice $\bar{E}$, denote by $\mathrm{Aut}(\bar{E})$ the group of elements $g \in \mathrm{GL}(E)$ respecting the Euclidean norm: it is a finite group. A Euclidean lattice $\bar{E}$ is said to be *irreducible* if the representation $\mathrm{Aut}(\bar{E}) \to \mathrm{GL}(E \otimes_{\mathbb{Z}} \mathbb{Q})$ is.

Example 18. Consider the real plane

$$H = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\},$$

inheriting the standard scalar product of $\mathbb{R}^3$. Consider the lattice $E = H \cap \mathbb{Z}^3$. Identifying H with the standard Euclidean plane $\mathbb{R}^2$, E corresponds to the vertices of a planar tessellation by equilateral triangles.

The group of isometries of E is the group of permutations on 3 elements, which acts irreducibly on H.

Other examples of irreducible Euclidean lattices come from reductive groups! Indeed, over an algebraically closed field, reductive groups are classified by a combinatorial datum called the *root system*. Simple Lie algebras correspond to irreducible root systems, which give rise to irreducible Euclidean lattices: the example above is the root system of SL_3.

Theorem 19 (Bost). *1. An irreducible Euclidean lattice is semi-stable.*

2. The tensor product of irreducible Euclidean lattices is irreducible.

The first statement follows from the existence of the Harder–Narasimhan filtration: there is a sub-lattice $\bar{E}_{\max}$ realizing the biggest slope and which is maximal for this property. By maximality, it is unique hence stable under the action of $\mathrm{Aut}(\bar{E})$. Therefore, if $\bar{E}$ is irreducible, $\bar{E} = \bar{E}_{\max}$.

The second is a straightforward consequence of the following classical result:

Theorem 20. *Let k be a field of characteristic 0 and V_i an irreducible representation of a group G_i over k ($i = 1, 2$). Then $V_1 \otimes_k V_2$ is an irreducible representation of $G_1 \times G_2$.*

Theorem 20 follows from the formula

$$\mathrm{End}_{G_1 \times G_2}(V_1 \otimes_k V_2) = \mathrm{End}_{G_1}(V_1) \otimes \mathrm{End}_{G_2}(V_2) \qquad (4)$$

and Schur's Lemma: if k is algebraically closed and V is an irreducible representation of a group G, then $\mathrm{End}_G(V) = k$.

Formula (4) is a special case of Jacobson's density theorem. Another proof, similar to that of Chevalley's theorem, goes as follows. One may assume that G_1 and G_2 are reductive groups and the ground field is $\mathbb{C}$. For $i = 1, 2$ let K_i be a Zariski-dense compact subgroup of G_i. Write a G-equivariant endomorphism φ of $V_1 \otimes_k V_2$ as

$$\varphi = \sum_{\lambda=1}^{N} \alpha_{1\lambda} \otimes \alpha_{2\lambda},$$

with $\alpha_{i\lambda} \in \mathrm{End}(V_i)$. By G-equivariance of φ and Fubini's theorem:

$$\begin{aligned} \varphi(x_1 \otimes x_2) &= \int_{K_1 \times K_2} g\varphi(g^{-1}(x_1 \otimes x_2))\, d\mu_1 \boxtimes d\mu_2(g) \\ &= \sum_{\lambda=1}^{N} \tilde{\alpha}_{1\lambda} \otimes \tilde{\alpha}_{2\lambda}(x_1 \otimes x_2), \end{aligned}$$

where μ_i is the Haar measure of total mass 1 on K_i and

$$\tilde{\alpha}_{i\lambda}(x) := \int_{K_i} g_i \alpha_{i\lambda}(g_i^{-1} x_i) d\mu_i(g_i),$$

is G_i-equivariant.

[8] The Euclidean norm on the tensor product is defined as follows: given Euclidean spaces V, W with scalar products $\langle \cdot, \cdot \rangle_V$ and $\langle \cdot, \cdot \rangle_W$, the tensor product $V \otimes_{\mathbb{R}} W$ is given the structure of Euclidean space by defining

$$\langle v \otimes w, v' \otimes w' \rangle_{V \otimes W} = \langle v, v' \rangle_V \cdot \langle w, w' \rangle_W.$$

[9] In the modern literature, this theorem is sometimes referred to as the Grothendieck–Birkhoff decomposition. However, it is easily deduced from the Elementary Divisors theorem for the ring $k[t, t^{-1}]$, due to Dedekind–Weber [9], or its complex analytic analogue discovered by Birkhoff [4].

where μ_i is the Haar measure of total mass 1 on K_i and

$$\tilde{\alpha}_{i\lambda}(x) := \int_{K_i} g_i \alpha_{i\lambda}(g_i^{-1} x_i) d\mu_i(g_i),$$

is G_i-equivariant.

Ramanan–Ramanathan method, again

Chen translated the argument of Ramanan–Ramanathan, as elaborated by Totaro, in the context of Arakelov geometry in order to give the following bound:

$$\hat{\mu}_{\max}(\bar{E}_1 \otimes \cdots \otimes \bar{E}_n) \leq \sum_{i=1}^{n} (\hat{\mu}(\bar{E}_i) + \log \operatorname{rk} E_i).$$

The error term is Chen's inequality comes for a very simple reason: the operator norm of the projection $V^{\otimes \dim V} \to \det V$ is $\dim V!$.

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Interview with Peter Sarnak

Michael Th. Rassias (University of Zürich, Switzerland)

Peter Sarnak.

Celebration at the conference in honor of Peter Sarnak's 61st birthday at Princeton.

Peter Sarnak is the Eugene Higgins Professor of Mathematics at Princeton University, as well as a Professor at the School of Mathematics of the Institute for Advanced Study, Princeton. Born in 1953 in Johannesburg, South Africa, he studied at the University of Witwatersrand, obtaining a BSc in 1974 and a postgraduate BSc (Hons) in 1975.

He obtained his PhD from Stanford University in 1980 under the direction of Paul Cohen.

Sarnak is known for his groundbreaking contributions to number theory and problems of mathematical analysis motivated by number theory. The techniques he employs in his research, as well as his interests, are surprisingly wide-ranging, from zeta functions and automorphic forms to mathematical physics and quantum computation.

Throughout his career, he has received several prestigious awards, including the Pólya Prize (1998), the Ostrowski Prize (2001), the Levi L. Conant Prize (2003), the Cole Prize (2005) and the Wolf Prize (2014), and has been awarded with honorary doctorates from several universities. He has also been elected as a member of several important academies and societies, including as a Member of the National Academy of Sciences (USA) and a Fellow of the Royal Society (UK). Peter Sarnak has also supervised more than 50 PhD students!

M. Th. Rassias: If I remember correctly, as a high school student you were very much involved with chess competitions and you did not get really interested in mathematics until your undergraduate years. Is this related to how mathematics was taught to you in your early life in South Africa or was it just a matter of taste at the time? Provided with other inputs, might you have been participating in mathematics problem-solving competitions during your high school years instead?

P. Sarnak: The mathematics that I was exposed to in high school in South Africa in the late 1960s was mostly routine and, while it came easily to me, I wasn't aware of any of the challenges that mathematics had to offer. On the other hand, chess was a challenge and one that was decided with an immediate outcome. I was drawn to it from an early age and probably hit my peak aged 16. My father was very supportive of my involvement in chess competitions until I declared my intention of going abroad after school and trying to make it as a professional chess player. He insisted that I go to university first to study and, indeed, once I got there (at the University of Witwatersrand in Johannesburg) and was introduced to real mathematics and especially abstraction, I was quickly drawn to its beauty and challenge. I carried on playing chess competitively until my early 20s but, after that and until today, mathematics has been my passion.

Were you exposed to any other scientific disciplines before choosing mathematics? (If yes, was that beneficial?)

As I mentioned, the subjects that came easily to me in high school were mathematics and science (the latter meaning basic physics and chemistry). Being a professional mathematician was not on my radar (or of those around me). So, I was planning to major in physics. However, in my first year, I had a miserable time in the laboratory and a good friend of mine Eddie Price, who was one of the top chess players in South Africa at the time and also a lecturer in the physics department of the university, told me that if it is the theoretical side of physics that I enjoy then I would be better off doing applied mathematics. So, I majored in mathematics and applied mathematics and, while it is "pure" mathematics that has driven my main interests and research, my early introduction to applied mathematics played a key role in me having broad mathematical interests and an appreciation of its applications.

Do you believe that the strategic way of thinking you cultivated as a professional chess player helped you when approaching mathematical problems and, if so, in what way?

The deductive analytic and positional reasoning, and the rich chess theory that human chess players employ (I say

Peter Sarnak with Henryk Iwaniec.

human since, if I understand correctly, the recent computer programs such as the machine learning "alpha go", which are stronger than any program, apparently use very little "rich human chess theory"), have a lot of similarities to parts of mathematics. It is certainly very good training for the mind and for mathematics, in a similar way to mathematics competitions, but like the latter it is neither necessary nor sufficient for being a good mathematician.

Was there a specific paper, book, lecture or even theorem you came across that won you over to mathematics? What was the spark?

What won me over as a first year undergraduate studying mathematics was abstraction and specifically that conceptual thinking can make the solution of a problem and understanding of a theory completely transparent. I remember the first course in abstract linear algebra as a spark, and also a topology course that drew me to want to learn and understand much more.

One may state that mathematics has witnessed a great expansion over the last, say, 100 years, with many different areas emerging and various methods discovered, bridging seemingly different fields. You once said that: "Mathematics is really very small, not big. There aren't that many great ideas and people use the same idea over and over again in different contexts." As a mathematician who has worked in various areas of mathematics and who has used a variety of techniques in order to tackle difficult problems, do you believe that the more one matures in mathematics, the greater the unity one sees in it?

I continue to hold the opinion in the quote that you mention, though now, some years later, with the explosion of papers on the arXiv, it seems harder to hold that view. Mathematical research is looking more like other sciences, with many papers having multiple authors and even some research being done in what looks like mathematics laboratories. Some of this is natural, thanks to various tools becoming very specialised and complicated and so it is not surprising that technical projects draw contributions from people with different expertise. However, when one gets down to understanding as to how an argument works, you find that the fundamental ideas and tools are much more limited than what might be apparent at first. In Hilbert's time, say, one person, like himself or his student Weyl, could have a good understanding of large portions of central mathematics. While today this might seem impossible, our mathematical universe is still small and the cornerstones from which major developments and changes are taking place at any given period are quite limited.

Is there a mathematician who influenced you the most, either through collaborations or interactions or even by studying their work?

Firstly, Paul Cohen, who was my thesis advisor, had a major influence on my mathematical taste, knowledge, insight and intuition. His view of the unity of mathematics (and that one really need not stick to a small sub-field) made a big impression on me. Together, Paul and I studied a good portion of Selberg's works, and the core of my own work is very much shaped and influenced by Selberg's ideas.

I remember you saying to us in a work group seminar at Fine Hall, Princeton, around three years ago, that you believed that the problem of factoring integers into primes must not be as hard as we think and that at some point someone will find an easy way to factor in polynomial time. Why is that?

There is no theoretical evidence that factoring is difficult and if an efficient algorithm to factor were found, no problem that is expected to be difficult (e.g. an NP complete problem) would follow. At this point in time and as far as is known in the public domain, there is no known efficient factoring algorithm, and the evidence that is offered to it being "hard" is that smart people have tried and failed. If our attitude in mathematics is that smart people have failed to solve a problem therefore it cannot be solved, we would be out of business as far as attacking the central unsolved problems.

Us mathematicians working in the trenches must have some beliefs as to what is true and what can be proved in order to proceed in our efforts, and my belief as far as factoring goes is that it can be done efficiently. In fact, one can take Shor's quantum factoring algorithm as evidence that factoring can be done efficiently with a classical computer.

Speaking about primes, you have formulated a very intriguing conjecture related to the Möbius function, which has captured the interest of many mathematicians. Would you like to describe it for the readers who do not belong to the world of number theory?

The parity of the number of prime factors of a number n is an elusive quantity that carries a lot of information (even the complexity of computing this parity appears to be as difficult as factoring n).

If n is square-free, the Möbius function $\mu(n)$ is $(-1)^{\wedge}$(parity), while $\mu(n)$ is zero for numbers that are not square-free. As a function of n, $\mu(n)$ is apparently very random. For example, the partial sums of $\mu(n)$ for n up to N are of order roughly sqrt(N) if and only if the Riemann hypothesis is true. More generally, the sum of $\mu(n).f(n)$ for n up to N is expected to have some cancellation for a bounded function $f(n)$ if the latter is of "low complexity"; this heuristic is known as the Möbius randomness principle and goes back at least to I.M. Vinogradov. My "Möbius disjointness conjecture" that you mention makes this heuristic precise by realising $f(n)$ as an observable sequence in a (topological) dynamical system.

If the system is deterministic (i.e. has zero entropy) then there should be cancellation. The primary tool for estimating such sums when f(n) is not a multiplicative function of n ($f(n)$ is multiplicative if $f(mn) = f(m).f(m)$ when m and n are relatively prime) is Vinogradov's bi-linear method. For $f(n)$ an observable in a dynamical system, Vinogradov's bi-linear sum is a Birkhoff sum for the joining of the system with itself.

This allows for dynamical ideas to be brought into the study of these sums and, as a consequence, there has been a lot of progress proving this Möbius disjointness conjecture for many deterministic systems. For logarithmically averaged versions of the conjecture, there is even more progress, thanks to works of Matomaki/Radziwill, Tao and Frantzikinakis and Host. In the latter, notions from Furstenberg's non-conventional ergodic averages and his dynamical disjointness play a central role.

It is really impressive that in your work you are able to use techniques from seemingly independent areas in order to solve a problem you are working on. In that manner, you have studied and discovered some fascinating interconnections. One example is the association of the problem of sums of squares in number theory with objects from quantum computation, such as the so-called Golden Gates. Would you like to discuss this topic?

Yes, I would be happy to elaborate on this topic. It turns out that for the construction of universal quantum gates, one is faced with the problem of providing an optimal set of topological generators for the groups G = SU(2) and SU(4) (the first for single qubits and the second for two qubits). By optimal, we mean that the words of length m in the generators cover G (with its bi-invariant metric) optimally for all large m and that there is an efficient algorithm to find the best approximation of length m for any g in G. Standard quantum computation textbooks give specific generators for SU(2), such as "Clifford matrices plus T". It turns out that these generate an S-arithmetic unitary group defined over a real quadratic number field and it is precisely this that makes them good generators. In recent work, Parzanchevski and I show how, for each of the (finite) symmetry groups of the Platonic solids, one can add an involution yielding an essentially optimal generating set for SU(2). Interestingly, the proof of the optimal (almost) covering property makes use of the Ramanujan conjectures established by Deligne. An heuristic algorithm developed by Ross and Selinger to navigate with the Clifford and T matrices can be adapted for all these "arithmetic Golden Gates", as I like to call them, and leads to efficient navigation.

Peter Sarnak with young mathematicians.

While the mathematics of S-arithmetic unitary groups is available and provides such Golden Gates for SU(4) (Parzanchevski), efficient navigation remains open in this case.

How do you see the future of applications of number theory to other fields of science?

In Hardy's "Mathematicians Apology", he points to the theory of numbers as the epitome of "pure mathematics" and being very far from applications.

The modern digital and computer world has proven him to be quite mistaken. The applied area that is often pointed to as far as applications of number theory go is cryptography (for example, RSA and factoring). However, it runs much further than that. In fact, any fundamental problem that is discrete in nature (e.g. quantum mechanics) will, when studied down to its finest features, become one of number theory. There are many examples, such as the quantum gates above, where number theory problems that emerge are also ones that have been identified by number theorists as fundamental. I like to think that the reason for this is that in number theory (and in mathematics more generally), mathematicians are looking for the deeper truths about whole numbers and, while we are not motivated by applications, these come naturally in cases where our insights are fundamental features of the objects that we study. In any case, when things work out this way, it is particularly pleasing.

My colleague Michail Aizenman (a mathematical physicist) once commented in a lecture on random matrix theory and zeros of the zeta function that "number theory is the final frontier of science". He doesn't have to convince me of that!

Mentoring young researchers can be an important aspect of the life of a mathematician. Hilbert, for example, supervised 69 PhD students throughout his life. It is said that he enjoyed interacting with students and used to go for long walks with them to discuss mathematics. You have already supervised more than 50 PhD students and I have personally witnessed the very close relations you have with them. Would you like to talk about this aspect of your mathematical life?

Yes, I have guided quite a number of PhD students over the years and I am very fortunate to have had this opportunity. For me, teaching, communicating and mentoring are an integral part of doing mathematics. Very often, I learn as much from these exchanges as do those being mentored. Over time, this reciprocal activity of guiding many PhD students has allowed me to learn and appreciate a much wider landscape of mathematics and it has opened doors to finding unexpected connections between disparate areas.

Directly and indirectly, the students I have mentored have played a big role in what I have managed to do. My role as a senior mentor is mostly that of being a coach: one provides encouragement and makes sure that the person being mentored is working on interesting problems and that they are aware of the basic tools that are available and what is known.

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Michael Th. Rassias is on the Editorial Board of the EMS Newsletter.

Mathematics as a Positive Mental Place – An Interview with Gigliola Staffilani

Roberto Natalini (Consiglio Nazionale delle Ricerche, Rome, Italy, Chair of the Raising Public Awareness Committee of the EMS)

Gigliola Staffilani is an Italian mathematician working in the USA as the Abby Rockefeller Mauze Professor of Mathematics at the Massachusetts Institute of Technology. Her research concerns harmonic analysis and partial differential equations. In 2014, she was elected to the American Academy of Arts and Sciences.

Roberto: Gigliola, could you describe your background, your early education and your life as a child in Italy?

Gigliola: I was born in a small town in Abruzzo. My parents were farmers and I lived with my family and that of my dad's brother. I really enjoyed playing outdoors with my dear friend Lina, who lived next door.

I was very good in school, in particular math. I was also very competitive and I was very unhappy if I did not get the highest marks. My brother is 10 years older than me and he was the first in the family to go to high school and then university. There were no books at home but he subscribed to "Le Scienze" and so, from very early on, I started reading about amazing discoveries in science. I couldn't understand much of what I was reading but I loved the short biographies of the scientists. It was during this time that I learned about Princeton, Stanford, Harvard, MIT… I loved my childhood but unfortunately, when I was 9, my father, who was 43, got sick with advanced colon cancer and died in less than a year. I was devastated and I lost my carefree spirit. In order not to think about my loss during my spare time I decided to start solving math problems from my school book and I continued doing it well into high school (and somehow this worked).

What did you think about mathematics when you were a child?

I loved the fact that mathematics was completely logic – no surprises there. I liked the fact that I could control it and that a proof was not subjective or emotional. I had enough negative emotions around me and I just needed a mental place where no emotions were taking over everything.

Gigliola Staffilani – image from the video "Truth Values: One Girl's Romp Through M.I.T.'s Male Math Maze", as part of the institute's 150th anniversary celebration, December 2012.

Could you describe the beginning of your career in mathematics in the United States? Was it hard to start as an emigrant in a new and unknown country?

I think that the first and probably biggest "culture shock" came when I moved from my little town to Bologna. The move from Bologna to the University of Chicago was in a way simpler, in spite of the fact that I encountered an endless lists of obstacles. I didn't have to prove anything to anybody in Chicago and I was completely naive about American culture. The first obstacle I encountered once I arrived on campus was that I couldn't register as a student because I had not taken the TOEFL exam. In fact, I didn't know any English. As a consequence, I only had a "prospective student visa" – not enough to register. Since I couldn't register, I couldn't live as a student in the International House, so I found myself basically homeless in one of the most dangerous campuses in the US. I decided to look for a room to rent as I had saved some money working during the previous two Summers. I found one unfurnished and I took it; at least I had a roof over my head. Fortunately, though, the chair of the math department allowed me to register as a student in spite of the visa; he was just hoping that I would give up after a couple of weeks and solve the problem. But I stayed for a whole month. Then, a second obstacle appeared. I didn't receive the first cheque of my fellowship. I learned later that this was again a consequence of the fact that officially I was a "prospective student". At this point, I was ready to give up and, with enormous disappointment, I decided to use the public phone in the math department to make a reservation to fly back to Italy. While at the phone, my registration advisor, Professor Sally, walked by. He noticed that I was upset and he signalled to me to end the phone call and talked to me. With my broken English, I explained the situation and with total ease he walked me to his office and gave me a personal cheque of about $1,500, which was the equivalent of my first month of fellowship. Very often, I think about this event: a completely random and lucky coincidence that may not have happened at all. If so, my life would have been completely different!

You are now a worldwide recognised expert in harmonic analysis and dispersive partial differential equations. How did it happen that you started to work in this area? Why do you like it?

I started working on harmonic analysis as a student in Bologna when I was writing my thesis on certain Green's functions. I like analysis; for me, it is way more flexible than algebra. Harmonic analysis, in particular, allows you to reduce many problems to understanding a variety of interactions between simple functions and then reassemble them in a clever way to deduce properties for general functions. I think it is a very powerful and flexible tool.

Gigliola Staffilani during a workshop at the Institut des Hautes Études Scientifiques, Paris, July 2016.

Could you explain, for an educated but not specialist audience, the core of these works about dispersive equations you performed in the first part of your academic career?

When I started talking to my advisor at the University of Chicago, Carlos Kenig, he explained to me that I could work in one of two areas that he was an expert in: elliptic equations, on which incredible progress had been made in the preceding years and where the problems left open were really hard, or dispersive equations, on which he had started working more recently and where many problems were completely open. He added that he didn't really know if this direction would become central in analysis. I decided to take the second option and I am glad I did because, indeed, thanks also to the work in this area by Jean Bourgain and Terence Tao, dispersive equations became very important. The main questions that I addressed with collaborators were on existence, uniqueness and stability (well-posedness) of rough solutions to dispersive equations, such as the Schrödinger of the KdV equations. We were interested in rough solutions because one would like to assume that only the mass (L^2 norm) or the energy (related to the H^1 norm) are bounded for these solutions. As a first step, one would prove well-posedness in a small interval of time but the next and harder step is to understand what happens when time evolves arbitrarily far. To answer this question, with my collaborators Colliander, Keel, Takaoka and T. Tao, we invented the concept of "almost conservation laws", which was then developed in many different contexts by us and other researchers.

What are the main contributions you have made in your field – the main original ideas?

I would say that the idea of the "almost conservation laws" is what I like the most.

Could you mention the most important of your mathematical results and why it is important to you?

For me, the proof of global well-posedness for the energy critical nonlinear Schrödinger equation in 3D is my most important result. I think it is important because we had to find a missing ingredient, now known as the interaction Morawetz inequality, which is actually a fundamental identity that had not been discovered till then.

You collaborate with some other well-known mathematicians, such as James Colliander, Markus Keel, Hideo Takaoka and Terence Tao, and I read that you are known as the "I-team". Could you explain the meaning and the origin of this name?

We are called the I-team because, in one of the original papers on "almost conservation laws", we used a multiplier operator that for no special reason we called "I". I guess we had run out of other good letters by that point.

You were appointed as a professor of pure mathematics at MIT. I believe you are one of the few women to get this kind of position. What is your feeling about that? Is the situation changing?

When I arrived at MIT, there was only another woman in applied mathematics; I was the only one in pure mathematics. Today we are a total of five. So, a little improvement but not much if one considers that there are a total of 53 professors in our department.

How is the environment in your department and how is it important to you for your work?
I love my department; it is very "democratic" and people listen. Of course, there are discussions but they are constructive. There are no groups fighting against each other and everybody is invested in having the best set-up for students, postdocs and professors that we can possibly have. This, for me, is absolutely fundamental. I need to feel happy when I go to my office; otherwise, I would be a terrible researcher, teacher and mentor.

What about more recent problems you have considered? What is the core of your activity nowadays?
Recently, I have been introducing a little more probability into my work. Often, when working with rough data, one can prove that there are special counterexamples for well-posedness. But, if one is a little less greedy, one may be happy to claim that for "almost all initial data", well-posedness is available. Of course, one has to make sense of the "almost all" but this is what probability is for. I have also been working on the integrability structure for a certain hierarchy of so-called dispersive equations that model Bose-Einstein condensation in the framework of Gross-Pitaevskii theory.

You have been awarded many honours and prizes. Which one is the most important to you?
I would say being elected to the American Academy of Arts and Sciences has been really great. It is such an historical organisation that I feel like I am part of history itself. Also, as a member, I get to discuss possible directions in education that may one day affect many people, so it is a bit like "giving back" to society.

How much in your work is intuition and how much is just hard work?
I think, in my case, it is 50%-50%. I believe that intuition comes when you have cleared up your brain to receive it. To clear up your brain, you need to work hard to eliminate all those attempts that do not lead anywhere.

How do you organise your work? Do you follow a routine or does it vary a lot according to external conditions?
Recently, I have been working a lot with senior collaborators and postdocs. It is difficult to juggle everything so I try to set aside certain times with certain groups that are essentially fixed every week. So, I would say that I follow a routine.

According to you, what is the situation of women in mathematics around the world? Is there any difference between Europe and the United States?
I can compare maybe Italy and the US. I think in both countries there are too few women at the level of full professor. But, in Italy, I do not think that people believe that the reason is that women do not have the same talent as men. Unfortunately, in the US, people still think that women are not good at math in general and that not being good at math for a girl is totally acceptable. This social belief is really difficult to change I am afraid.

Gigliola Staffilani and her family, from the interview on the Italian site MaddMaths!, December 2010.

You are committed to reducing the gap between women and men in mathematics. What are your actions in this direction?
I strongly believe in diversity, in all its shades: gender, race and family background. I believe that when there are no role models, it is very difficult to imagine yourself in a certain position, so I am a strong supporter of having role models as mentors. At MIT, I organise a lunch seminar, where I invite senior women mathematicians, working in academia or industry, to come and recount to the women in the department (from undergraduates onward) how they arrived to the place they are now. In doing so, they also explain, in general terms, the mathematics they use in their research or their job.

What do you do outside math? Do you have hobbies?
I really do not have much time for myself but when I do I like to go hiking, take care of my small city garden and, most of all, spend time chatting with my kids and my husband.

How is your relation with Italy now? Are you still in touch with your country?
I love to visit Italy, either for work or personal reasons. I am in touch with a few mathematicians there and I have lectured in a few Summer Schools as well. In July, in fact, I will be in Rome for a week!

Roberto Natalini has been the Director of the Istituto per le Applicazioni del Calcolo "Mauro Picone" of the National Research Council of Italy since 2014. His research interests include fluid dynamics, road traffic, semiconductors, chemical damage of monuments and biomathematics. He is Chair of the Raising Awareness Committee of the European Mathematical Society.

Robert Adol'fovich Minlos (1931–2018) – His Work and Legacy

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Robert Adol'fovich Minlos. (Courtesy of A. Kassian)

On 9 January 2018, the renowned mathematician Professor Robert Adol'fovich Minlos passed away at the age of 86. An eminent researcher and outstanding teacher, he was a world-renowned specialist in the area of functional analysis, probability theory and contemporary mathematical physics.

R.A. Minlos was born on 28 February 1931 into a family associated to the humanities. His father Adol'f Davidovich Miller was known as a lecturer and author of English dictionaries and manuals. His mother Nora Romanovna (Robertovna) Minlos was an historian-ethnographer. Her brother Bruno Robertovich Minlos, with a PhD in historical sciences, was a specialist in the history of Spain. This is perhaps why Robert Adol'fovich loved poetry, wrote verses himself, was a fervent theatre-goer from his school years and was seriously occupied with painting from the age of 40.

Nothing foretold a mathematical future but when he was 15, the young Robert accidentally saw a poster about the Moscow Mathematical Olympiad for schoolchildren. He participated in it, obtained the second prize and, inspired by that, began to attend the school club led by E.B. Dynkin. In 1949, Robert joined the Faculty of Mechanics and Mathematics of the Moscow State University. He continued to participate in Dynkin's seminar, which, together with A.S. Kronrod's seminar, had a great influence on him as an undergraduate student.

R.A. Minlos prepared his first scientific paper (equivalent to a Master's degree thesis) in 1950 while participating in the Moscow State University seminar on the theory of functions of a real variable (under the leadership of A.S. Kronrod). However, the real scientific interests of the young mathematics student began to form after he became acquainted to I.M. Gelfand. Their joint publication "Solution of the equations of quantum fields" (*Doklady Akad. Nauk SSSR*, n.s., 97, 209–212, 1954) became Minlos' diploma thesis in mathematics. It was devoted to the *functional* or, in mathematical physics language, the *path* integral, which has a direct relation to quantum physics.

As Minlos himself admitted: "My further life in mathematics was predetermined by that work because I was subsequently mainly occupied with mathematical physics." There were, nevertheless, more works on random processes, on measure theory and on functional analysis. Very soon, one of his papers, "Extension of a generalised random process to a completely additive measure" (*Doklady Akad. Nauk SSSR*, 119, 439–442, 1958), brought Minlos to worldwide fame. It became the basis of his Candidate (equivalent to PhD) Dissertation "Generalised random processes and their extension to a measure", which was published in *Trudy MMO*, 8, 497–518, 1959. This result, which is important for the theory of random processes, as well as for functional analysis, is now known as the *Minlos theorem* on the extension of cylindrical measures to Radon measures on the continuous dual of a nuclear space, i.e. the continuation of a process to a measure on spaces adjoint to nuclear spaces.

The connection of Minlos to mathematical physics at that time manifested in the publication (jointly with I.M. Gelfand and Z.Ya. Shapiro) of the monograph *Representations of the rotation and Lorentz groups and their applications* (1958), which was later translated from the Russian by Pergamon, London, in 1964. Note that the monograph appeared in 1958, just at the time when the need for physicists to understand representation theory was strongly motivated by the discovery of elementary particle symmetries, as well as the role of their spins and symmetries related to relativistic Lorentz transformations.

Seminar of F. A. Berezin and R. A. Minlos at the Faculty of Mechanics and Mathematics – MSU, 1959. (Courtesy of N.D. Vvedenskaya)

From 1956 to 1992, R.A. Minlos was employed by the Department of the Theory of Functions and Functional Analysis of the Faculty of Mechanics and Mathematics at Moscow State University (MSU). In that period, there was a need to organise a joint seminar with F.A. Berezin, primarily to discuss the mathematical problems of quantum mechanics and quantum field theory.

A real advance of activity in the field of mathematical physics at the Faculty of Mechanics and Mathematics of MSU was achieved with the organisation in 1962 by R.A. Minlos and R.L. Dobrushin of a seminar on statistical physics. It soon became widely known in the Soviet Union and abroad as the *Dobrushin–Malyshev–Minlos–Sinai* seminar. The quantum aspects of statistical mechanics at the seminar were primarily associated to the name of R.A. Minlos. The seminar lasted until 1994 and had a huge impact on the world of modern mathematical physics. Almost all the celebrated specialists in the field visited Moscow during the lifespan of the seminar. Besides traditional scientific contacts with Socialist European countries, a fruitful collaboration was also established with colleagues from other countries. However, the most intensive contacts were within the country, involving almost all the republics. For example, many of Minlos' PhD students came from Uzbekistan.

The beginning of the 1960s was extremely fruitful for Robert Adol'fovich. Initially, there were new results obtained jointly with L.D. Faddeev on the quantum mechanical description of three particles (1961). This was followed by two articles devoted to the study of the thermodynamic limit in classical statistical physics (1967). There, R.A. Minlos suggested the first rigorous mathematical definition of the limiting Gibbs distributions for an infinite system of interacting classical particles and also analysed the properties of such distributions (*Funct. Anal. Appl.*, 1, 140–150 and 206–217, 1967). His approach was very close to the classical Kolmogorov construction of random processes (fields). This result anticipated the origin of the Markovian understanding of Gibbs random fields in the sense of Dobrushin–Lanford–Ruelle (1968).

The result (together with Ya.G. Sinai) of the appearance of phase separation in lattice systems at low temperatures (*Math. USSR-Sb.*, 2, 335–395, 1967; *Trudy MMO*, 17, 213–242, 1967, and 19, 113–178, 1968) was of fundamental importance for the mathematical theory of phase transitions. It formed the basis of Minlos' doctoral dissertation, which he submitted for habilitation in 1968. In another joint work with Ya.G. Sinai (*Theor. Math. Phys.*, 2, 167–176, 1970), the foundation was laid for a new approach to the study of spectral properties of many-particle systems. In combination with cluster expansions, this approach drove significant progress in the description of properties of such infinite systems, including the spectrum of elementary particles of quantum fields and the mathematical description of the *quasi-particle* picture in statistical physics.

The new powerful method of cluster expansions was, from the very beginning, a central one in the list of interests of Robert Adol'fovich. The results of a large series of papers in this topic by R.A. Minlos, V.A. Malyshev and their students have been summarised in two monographs: *Gibbs Random Fields: Cluster Expansions* (Springer 1991, translation of the 1985 Russian edition) and *Linear Infinite-particle Operators* (Amer. Math. Soc. 1995, translation of the 1994 Russian edition). As was outlined in the book *Gibbs Random Fields*, the method of cluster expansions provides, besides a construction of the limiting Gibbs measure, a cluster representation of the projections of the limiting Gibbs measure onto bounded regions.

A famous peculiarity of the Dobrushin–Malyshev–Minlos–Sinai seminar was not only its duration of about four hours, which was amazing for foreign guests, or the assertive directness in communicating with lecturers but also the opportunity to obtain from the discussions some interesting problems to be solved. In essence, the seminar was functioning as a *machine*, generating questions and a possible way to convert them into answers. Robert Adol'fovich was always one of the sources of interesting questions and open problems. The list of projects thus originated includes, for example, cluster expansions and their applications to the problem of uniqueness/non-uniqueness of Gibbs states, the quantum three-particle problem, the Trotter product formula for Gibbs semigroups, the study of infinite-particle operators spectra, the analysis of the quasi-particle picture in statistical physics and many others.

The Dobrushin–Malyshev–Minlos–Sinai seminar had a tradition of studying and discussing new important publications on mathematical results and problems in statistical physics. The Trotter product formula problem for Gibbs semigroups was originated after discussing the Pavel Bleher report about new techniques, *reflection positivity* and *infrared bound estimates* (launched by Fröhlich–Simon–Spencer (1976–1978) to prove the existence of phase transitions). In the case of quantum systems, this technique involves the Trotter product formula approximation of the Gibbs density matrix. In this context, Robert Adol'fovich posed a question about the topology of convergence of the Trotter product formula because, to obtain the infrared bound, one has to interchange the trace and the limit of the Trotter approximants. In fact, this operation is not harmful for quantum spin systems since the underlying Hilbert spaces are finite-dimensional but it does produce a problem, for example, in the case of *unbounded* spins. A typical example is the problem of the proof of infrared bounds for the case of structural phase transitions in one-site double-well anharmonic quantum crystals with harmonic interaction between sites (Fröhlich, 1976). Then, the interchange is possible only when the Trotter product formula converges in trace-norm topology. For a particular case of anharmonic quantum crystals, the convergence of the Trotter product formula in the trace-norm topology was proved via the Feynman-Kac representation for Schrödinger (Gibbs) semigroups. The abstract result, which also includes a generalisation to trace-norm Trotter–Kato product formula convergence, is due to H. Neidhardt and V.A. Zagrebnov (1990). So, the answer to the question posed by Robert Adol'fovich was solved affirmatively in favour of trace-norm topology for the case of Gibbs semigroups.

At the end of the 1990s, Robert Adol'fovich returned to the question of quantum phase transitions, in the area of anharmonic quantum crystals (which was already well-known to him) but from the opposite direction. It is known that in contrast to classical systems, phase transitions in their quantum analogues may disappear due to intrinsic quantum fluctuations, which may lead to important tunnelling in the double-well potential. A particular manifestation of that is the elimination by these fluctuations of the order parameter even at zero temperature, whereas it is non-zero in the classical limit when the Planck constant $\hbar = 0$. A typical example is the above structural phase transition in one-site double-well potential anharmonic quantum crystals with harmonic interaction between sites for particles of mass m in each site. Moreover, experimental data for crystals close to this model manifest a so-called "isotopic effect": the order parameter of the structural phase transition for samples with light masses $m<m_c$ disappears, a fact which warms up interest in the mathematical aspect of this phenomenon.

R.A. Minlos with co-authors N. Angelescu and V.A. Zagrebnov on a visit to Dublin Institute for Advanced Studies, 2000.

During his visits to Dublin, Leuven and Marseilles, Robert Adol'fovich, in collaboration with E.A. Pechersky, A. Verbeure and V.A. Zagrebnov, addressed the proof of the existence of a critical mass m_c such that below this threshold the quantum state of the system is in a certain sense *trivial*, or at least the order parameter is trivial. In two papers, with A. Verbeure and V.A. Zagrebnov (2000) and then with E.A. Pechersky and V.A. Zagrebnov (2002), R.A. Minlos proposed using cluster expansion techniques for the small parameter $\xi = \sqrt{m}/\hbar$. Then, the classical limit corresponds to $\xi \to \infty$ and the quantum regime, with zero order parameter for any temperature, corresponds to $\xi < \sqrt{m_c}/\hbar$. Since the structural phase transition in the model manifests as the displacement order parameter, these papers consider projection of the full quantum state on the commutative coordinate *-subalgebra $\mathfrak{A}_q$ of bounded functions of displacements on the lattice. Then, the Feynman–Kac–Nelson formula for the Gibbs semigroup kernel allows one to show that this projection reduces to a classical ensemble of weakly interacting (for $m<m_c$) Ornstein–Uhlenbeck trajectories. Using the cluster expansion technique, the exponential mixing of the limit state with respect to the lattice group translations was proven for all temperatures, including zero, if $m<m_c$ (2000). To check that in this domain of light masses (high *quantumness*) the order parameter is zero for all temperatures, including the ground state, R.A. Minlos and his coauthors, in the 2002 paper, used the external sources h conjugated to the local displacements instead of localising the trajectories boundary conditions, as in the 2000 paper. This allows the analysis, for any temperature θ, of the free-energy density function $f(\theta,h)$ for free or periodic boundary conditions. It is proved in the 2002 paper that there exists a radius $h_0(m)$ such that $h \mapsto f(\theta,h)$ is holomorphic in the disc $\{h\in\mathbb{C}$: $|h| < h_0(m)\}$ for any $\theta \geq 0$ if $m<m_c$. Moreover, the Gibbs expectations: $h \mapsto \langle A \rangle(\theta,h)$, are holomorphic in the same disc for any bounded operator A of a quasi-local *-algebra $\mathfrak{A}$ of observables. The analyticity, in particular, yields that the displacement order parameter is equal to zero for $h=0$ and for any temperature $\theta \geq 0$ if $m<m_c$.

This was also a period when, during his visits to KU Leuven and CPT Marseilles, Robert Adol'fovich got into an argument with A. Verbeure about the mathematical sense of the notion of *quasi-particles* in many-body problems and of the *corpuscular* structure of infinite system excitations. In the framework of quantum statistical mechanics, an attractive way to promote this notion was based on the non-commutative central limit theorem for *collective* excitations. This concept yields a plausible (for physics) picture of *boson* quasi-particles excitations (phonons, magnons, plasmons, etc.) in the corresponding Fock spaces (A. Verbeure et al. (1995)).

On the other hand, in their book *Linear infinite-particle operators*, V.A. Malyshev and R.A. Minlos proposed the description of a quasi-particle picture based on the construction by cluster expansions of the lower branches of the spectrum of infinite many-body systems with good clustering. This idea goes back to the paper by R.A. Minlos and Ya.G. Sinai "Investigation of the spectra of some stochastic operators arising in the lattice gas models" (1970). There, a new approach to studying the spectral properties of the transfer matrix in general lattice models at high temperatures was developed. For translation-invariant systems, the lowest branch of the spectrum enumerated by momentum corresponds to one-quasi-particle excitations above the ground state. Then, in the simplest case, the energy of these excited states is completely defined by the momentum. This is called a dispersion law for quasi-particles, which is also well-known for boson quasi-particles. If the system possesses a good clustering, one can construct separated translation-invariant two-, three- and more (interacting) quasi-particles excited states, which are combinations of branches with bands of continuum spectra. Robert Adol'fovich called this property of excitations "The 'corpuscular' structure of the spectra of operators describing large systems" (the title of his paper in *Mathematical Physics* 2002, Imperial Coll. Press, 2000).

The technique developed by V.A. Malyshev and R.A. Minlos allowed the study of the *corpuscular* structure of generators of stochastic dynamics. Their approach was also applied to generators of stochastic systems: Glauber dynamics, the stochastic models of planar rotators, the stochastic Ising model with random interaction and other lattice stochastic models with compact and non-compact spin spaces, as well as stochastic dynamics of a continuous gas and other stochastic particle systems in the continuum. Using this technique, one can find spectral gaps and construct lowest one-particle invariant subspaces of the generator that determine the rate of convergence to the equilibrium Gibbs state. Moreover, it also allowed the study, in detail, of the spectrum branches of infinite-particle operators on the leading invariant subspaces and, in particular, the construction of two-particle bound states of the cluster operators. These results led to the understanding that a wide class of linear infinite-particle operators of systems in a regular regime have a corpuscular structure.

Visiting Leuven and Marseilles, Robert Adol'fovich proposed elucidating the concept of corpuscular structure of spectral branches for several particular models on the lowest level of one-particle elementary excitations. This programme was performed in papers with N. Angelescu and V.A. Zagrebnov (2000, 2005) and then all together with J. Ruiz (2008), for lattice models and for polaron-type problems. More activity and results in this direction were due to intensive collaboration of Robert Adol'fovich with the group of H. Spohn, where he studied spectral properties of Hamiltonians for quantum physical systems, in particular for Nelson's model of a quantum particle coupled to a massless scalar field.

Another long-term and fruitful collaboration of R.A. Minlos was with the Bielefeld group, essentially with Yu.G. Kondratiev and his co-authors and pupils. Firstly, they generalised the original method (adapted for lattices) to functional spaces to control general particle configurations. This allowed the extension of their analysis from lattice to *continuous* systems. In the paper "One-particle subspace of the Glauber dynamics generator for continuous particle systems" (2004), they studied, in detail, the spectrum of the generator of Glauber dynamics for continuous gas with repulsive pair potential. To this end, the invariant subspaces corresponding to the corpuscular structure were constructed.

The ideas and technical tools elaborated in this paper were used in a number of other projects (Yu. Kondratiev, E. Zhizhina, S. Pirogov and O. Kutoviy) on equilibrium and non-equilibrium continuous stochastic particle systems. This, in particular, concerns a delicate continuous models problem of thermodynamic limit. One unexpected application of this technique concerns image restoration processing. Robert Adolfovich, in collaboration with X. Descombes and E. Zhizhina, actively participated in the INRIA project on the mathematical justification of a new stochastic algorithm for object detection problems. The result was summarised in the article "Object extraction using stochastic birth-and-death dynamics in continuum" (2009).

In addition to the Dobrushin–Malyshev–Minlos–Sinai seminar in the 1970s, there was also a regular tutorial seminar, which was led by Robert Adol'fovich once a week. This was a very good opportunity to learn elements of topological vector spaces, in particular the Minlos theorem about the extension of a generalised random process to a measure on spaces adjoint to nuclear spaces. The seminar also covered elements of mathematical statistical physics in the spirit of the famous "Lectures on statistical physics" in *Uspekhi Math. Nauk* (1968). These lectures of Robert Adol'fovich very quickly became a textbook for many students and scientists interested in mathematical statistical physics. In these lectures, Robert Adol'fovich systematically used the notion of configuration space, which appeared in his earlier work, where he gave the mathematical definition of the limiting Gibbs measure as a measure on the space of locally finite configurations in $\mathbb{R}^d$. This concept is technically very useful and is close to modern random point process theory.

R.A. Minlos, Ya.G. Sinai and R.L. Dobrushin were often invited by the Yerevan State University and the Institute of Mathematics of the Armenian Academy of Sciences to give lecture courses on statistical mechanics. They all had PhD students working at the Institute of Mathematics in Yerevan. This was the main motivation for the Institute of Mathematics to organise regular conferences (every 2–3 years) in Armenia under the name "Probabilistic methods in modern statistical physics". The first one was held in 1982 and the last one in 1988, three years before the collapse of the Soviet Union.

R.A. Minlos with participants of the conference "Probabilistic methods in modern statistical physics", Yerevan, Lake Sevan, 2006.

The conferences restarted in 1995 at the international level. Robert Adol'fovich participated (as a rule, with his students) in all of them, including the conference in Lake Sevan in 2006. He always supported the conferences in Armenia by being a permanent member of the programme committee and one of the main speakers, formulating new problems and generating interesting ideas, questions and discussions. Unfortunately, he was not able to participate at the conferences after 2006.

In the early 1990s, Robert Adol'fovich began his collaboration with Italian institutions and mathematicians. He was a guest of the Department of Mathematics at the University of Rome "La Sapienza" many times

R.A. Minlos with participants of the conference: S. Lakaev, V. Zagrebnov, H. Suqiasian and B. Nahapetian, Yerevan, Lake Sevan, 2006.

and he also visited other institutions in Trieste, Naples, L'Aquila and Camerino. During his stay at "La Sapienza", he read a course on the mathematical foundations of statistical mechanics, which was published as a book by the American Mathematical Society in 2000 under the title *Introduction to mathematical statistical physics*'. In Rome, he began a collaboration with C. Boldrighini and A. Pellegrinotti on models of random walks (RW) in interaction with a random environment fluctuating in time ("dynamic environment").

At that time, several important results on random walks in a fixed environment had already been obtained, by Solomon, Kesten, Sinai and others, but very little was known for dynamic environments. Following the usual terminology, the behaviour of RW for a fixed choice of the history of the environment is called "quenched" and its distribution induced by the probability measure of the environment is called "annealed". A first result had been obtained by C. Boldrighini, I.A. Ignatyuk, V.A. Malyshev and A. Pellegrinotti on the annealed model of a discrete-time random walk on a d-dimensional lattice in mutual interaction with a dynamic random environment. Robert Adol'fovich proposed applying the results that he had obtained, together with V.A. Malyshev and their students, on the spectral analysis of the transfer matrix for perturbed homogeneous random fields. The approach proved to be very fruitful and in the following years 1993–1996, several results (C. Boldrighini et al. (1994)) were obtained on the annealed RW, on the convergence to a limiting measure for the field "as seen from the particle", on the decay of the space-time correlation for the random field in interaction with the RW and on the RW of two particles in mutual interaction with a random environment.

It was then possible, with the help of some tools of complex analysis of which Robert Adol'fovich had a deep knowledge, to deal with the quenched model of the RW. After the first results of a perturbative approach (C. Boldrighini et al. (1997)), a complete non-perturbative answer could be obtained for the case when the components of the dynamic environment $\xi = \{\xi(x,t):(x,t) \in \mathbb{Z}^d \in\times \mathbb{Z}\}$ are independent, identically distributed random variables (C. Boldrighini et al. (2004)). Unlike the case with fixed environment, the quenched RW in dynamic environment behaves almost surely as the annealed RW in all dimensions $d \geq 1$. In low dimension $d=1,2$, the random correction to the leading term of the RW asymptotics is of an "anomalous" large size. A quenched local limit theorem was also obtained, with an explicit dependence on the field as seen from the particle. The results were then extended to models of directed polymers in dimension $d>2$ below the stochastic threshold. Results for a quenched model of RW in a dynamic environment with Markov evolution were also obtained in dimension $d>2$ by cluster expansion methods (C. Boldrighini et al. (2000)). Further results on models of RW in a dynamic environment were obtained by Zeitouni, Rassoul-Agha, Liverani, Dolgopyat and others.

Later on (in collaboration with F.R. Nardi), it was possible to derive Ornstein–Zernike asymptotics for the correlations of a Markov field in interaction with a RW (C. Boldrighini et al. (2008)) and also for a general "two-particle" lattice operator (C. Boldrighini et al. (2011)).

R.A. Minlos with A. Pellegrinotti and C. Boldrighini, Yerevan, 2006.

In the last few years, the interest of Robert Adol'fovich in the study of random walks in a dynamic random environment did not fade and several difficult problems were solved. They concern extensions to continuous space (C. Boldrighini et al. (2009)), to continuous time and to the case of long-range space correlations for the environment (in collaboration with E.A. Zhizhina).

Robert Adol'fovich was a wonderful teacher and a patient and wise mentor. Directness, accessibility and enthusiasm attracted numerous students and followers to him. Many of his later PhD students made their first acquaintance with special branches of mathematics and mathematical physics due to the tutorial seminar at the Faculty of Mechanics and Mathematics at MSU. There, they benefited from direct generous contact with the *Master*. This student seminar was combined with lectures and scientific seminars guided by Robert Adol'fovich, together with F.A. Berezin and then with V.A. Malyshev. The lecture notes gave rise to many nice and popular tutorial books, for example *Introduction to mathematical statistical physics*, published by R.A. Minlos in Univ. Lect. Series, Vol. 19, AMS 2000. Many of Minlos' former students successfully continue research in different branches of mathematics and mathematical physics, for example: S.K. Poghosyan and E.A. Zhizhina – spectral theory of infinite systems and mathematical problems of statistical mechanics; S. Lakaev – operator spectral theory and mathematical quantum mechanics; A. Mogilner

– mathematical biology; E. Lakshtanov – infinite particle systems; and D.A. Yarotsky – random processes and spectral theory of infinite systems.

Besides the students and the tutorial seminar, Robert Adol'fovich was in contact with followers and co-authors assisting the crowded Dobrushin–Malyshev–Minlos–Sinai research seminar. There, Minlos initiated a number of projects, often related to discussions during the seminar. Always attentive and gentle, Robert Adol'fovich shared his enthusiasm to encourage followers in solving the problems.

In this way, Minlos launched the project "Cluster expansions" with V.A. Malyshev. In fact, this happened by accident when they were both in a lift while attending a seminar in the tall main MSU building. Less unusual were the origins of the projects "On the spectral analysis of stochastic dynamics" with E.A. Zhizhina and "Gibbs semigroups" with V.A. Zagrebnov, which in fact started from questions formulated during and after the seminar. The origins of many of them were due to active contacts made by Robert Adol'fovich travelling to other research centres. This is, for example, the case for the project "Application of the spectral analysis of the stochastic operator to random walks in dynamic random environments" with C. Boldrighini and A. Pellegrinotti and "Spectral properties of multi-particle models" with H. Spohn, as well as "Infinite dimensional analysis" and "Stochastic evolutions in continuum" with Yu.G. Kondratiev.

To his students and collaborators, Robert Adol'fovich was a *Master*, who, like a brilliant sculptor, could create a mathematical masterpiece from a shapeless block by cutting off the excess. Sometimes, it brought not just a feeling of amazement but a sense of miracle when, as a result of some incredible expansions, evaluations, virtuoso combinations with various spaces and other technical refinements, complex infinite-dimensional and infinite-particle systems took an elegant, precise and easily understandable form.

In this connection, problems related to the theory of operators and to quantum physics should be especially noted. This theme began in his joint paper with I.M. Gelfand and, since then, it has permanently been the focus of Minlos' attention. In 2010, together with his old co-author and friend V.A. Malyshev, he turned to a fundamental question in quantum chemistry: what is the interaction between atoms? (*Theoretical and Mathematical Physics*, 162, 317–331, 2010.) However, his favourite subject since the 1960s and until recently has been the quantum three-body problem and point interaction. A long paper ("A system of three quantum particles with point-like interactions", Russian Math. Surveys, 69, 539–564, 2014) was published by R.A. Minlos on this topic.

A recent paper by Robert Adol'fovich was dedicated to another of his favourite subjects: the random walk in a random environment ("Random walk in dynamic random environment with long-range space correlations", *Mosc. Math. J.*, 16:4 (2016), 621–640, with C. Boldrighini and A. Pellegrinotti). His very last manuscript (with C. Boldrighini, A. Pellegrinotti and E.A. Zhizhina) was also on this subject: "Regular and singular continuous time random walk in dynamic random environment".

Robert Adol'fovich selflessly served science and, in everyday life, was a generous and friendly person. He gladly shared his enthusiasm and energy with his students and colleagues. In addition to the accuracy of reasoning and complicated techniques involved, there is always a beautiful idea and harmony in his works. It is interesting to mention his response to the question of Natasha Kondratyeva: "What three mathematical formulas are the most beautiful to you?" He gave the answer: "The Gibbs formula, the Feynman–Kac formula and the Stirling formula." And those are the formulae that were widely used by Robert Adol'fovich in his works.

R.A. Minlos, Moscow 2016 (Courtesy of E. Gourko).

Robert Adol'fovich was notable for his figurative Russian language and good wit, often with subtle mathematical humour. In the 1980s, in a conversation with Roland Dobrushin at the Fourth Vilnius Conferences on Probability Theory and Mathematical Statistics (1985), he expressed his doubt that "the life of a Soviet citizen is *complete* with respect to the *norm* of the anti-alcohol campaign". A campaign was ongoing at that time in the country under the slogan "Sobriety is the norm of our life!" and was visible everywhere on white-red streamers. Since then, this allusion to the completeness of life and normed spaces has entered into the folklore of the mathematical community.

Always surrounded by relatives and intimates, and also by loving pupils, colleagues and friends, Robert Adol'fovich Minlos lived a complete life. In each of those who knew Robert Adol'fovich, he left a bright drop of memory of himself.

Supporting Young Researcher Families in Switzerland

Maria Podkopaeva and Olga Chekeres (both University of Geneva, Switzerland)

The life of a PhD student or postdoc in mathematics (or any other subject) can be challenging in many ways, and more so if you have a family, especially if you have young children. In Switzerland, there exist several institutions that offer help in such situations. Some measures, specifically tuned to young researchers in mathematics and theoretical physics, are proposed by NCCR SwissMAP.

The Swiss National Centre of Competence in Research (NCCR) SwissMAP[1] is a programme supporting mathematics and theoretical physics in Switzerland, created in the context of the NCCR projects of the Swiss National Science Foundation. It brings together around 40 professors from Swiss universities and CERN and their groups (altogether around 200 people). In addition to its research curriculum, the goal is to attract more talented young people to mathematics and theoretical physics in Switzerland. This is done via a full cycle of educational activities: from a wide variety of outreach projects for schools, some of which have been described in a previous issue of the EMS Newsletter,[2] through master classes (year-long in-depth Master's level programmes for internationally selected students), to opportunities and training for PhD students and postdocs. NCCR SwissMAP also has several programmes for supporting young researchers with families.

Dual careers

Everybody knows that the lifestyle of a postdoc (or a student involved in an exchange programme) is not very well suited for someone with a family. The positions are short-term and spouses have their own careers to follow. Obviously, dual career measures are more effective at a higher level but it is not very often that both partners work within the relatively narrow field on the border of mathematics and theoretical physics. When it does happen (so far, in four years of SwissMAP's existence, we've had one case, with a Master's student in mathematics married to an exchange PhD student in physics, and one more case is being discussed right now), we are happy to help by providing dual career scholarships or supporting other arrangements.

Young children

Being a PhD student or a postdoc with a baby can be tough. If you plan to pursue a career in maths, you have to work on your CV, publish papers and participate in conferences, schools and workshops, often abroad. You are probably also expected to teach, since part of your salary usually comes from being hired in your department as a teaching assistant. All this can be a major challenge if you are a young mother: first, you are out of the game for the several months of your maternity leave and then, even if you have a place at kindergarten, you still need (and want!) to spend lots of time with your child. During their first year, there are all-night feedings, teething, upset stomachs, etc., and later on, you need to keep up with an energetic toddler. This does not leave much space for research.

To help, we have introduced several schemes of support. In the mobility aspect, if you want to attend a conference or a school, SwissMAP proposes covering the extra costs for travelling with your baby or temporary childcare while you are away. The same is offered to external researchers who visit events organised by SwissMAP.

Last year, we launched the "buy out of teaching" grants.[3] Once again, when you are an assistant at university, in addition to your research you have teaching

[1] http://nccr-swissmap.ch/.
[2] EMS Newsletter No. 100, June 2016.
[3] http://nccr-swissmap.ch/articles/family-support-grants.

duties, which often take a lot of time: the TA sessions themselves, plus the preparation, plus correcting homework… With SwissMAP family support grants, young mothers can be liberated from their teaching (or other non-research) duties so that they do not have to sacrifice their research time, which is already decreased because of their new function as a mother. After all, research is the priority of a scientist and, unlike teaching, it is hard to catch up if you take a long break.

The first person to receive such a grant was Olga Chekeres, a PhD student in Geneva, whose son was born in February 2017. Here is what she thinks about the mother/PhD student duality.

You are a mother of two, one of whom is still a baby. How does this affect your work as a PhD student? What are the main challenges?
As a mother of two children, with one still a baby, I experience great difficulties managing time, like any working mom I guess. Every day, I have to bring the baby to the daycare in the morning and then not to be late to pick him up in the evening; my effective working hours are reduced. Also, when I am at home, normally there is no way I can do anything for my research – do some reading or write down something – and this is a lot of time gone from my PhD.

Another problem is psychological – to manage making plans. It seems simple, but it took me some time to realise that I don't belong to myself. I have to fix it in my mind that when I leave the office I allow myself not to work, because this time belongs to my kids. And if it happens that the baby goes to bed early and easily and gives me time to enjoy doing something for my project, I would rather take this as a happy bonus, instead of making plans to get him to sleep at 21:00 and then being upset if this doesn't work.

And, of course, finances. The nursery consumes a significant part of the income.

Were SwissMAP programmes for supporting families useful for you? In what way?
SwissMAP programmes were really useful. One of them liberated me from giving classes, compensating the teaching part of my salary. It really helps to focus more on my research, since my working time is reduced.

Another programme reimburses the expenses for a daycare centre or a baby-sitter if I attend a conference, which is really a great thing, allowing me to stay updated and to participate in academic life.

Do you see other aspects of reconciling family life and your career, where SwissMAP or your university could help?
Actually, any financial support is always useful.

Organising a parking space for working moms might be helpful. For me, this problem was solved with the help of a friend, otherwise renting a place or paying per hour is extremely expensive. Without a car, there is no mobility to come quickly to the nursery if anything happens,

to bring the baby to a doctor and then return back to work, etc., etc. And even on a daily basis, just the trajectory home – nursery – office would consume much more working time, which is already reduced.

Our aim is to be as flexible as possible when it comes to supporting young researchers. The schemes described above are already formalised and functioning but we always keep our eyes open for new ideas and try to find a way to help in any special situations (thankfully, the Swiss Science Foundation is usually quite forthcoming with respect to such a personalised approach).

Maria Podkopaeva is a research associate at the University of Geneva working as a science officer for NCCR SwissMAP. She obtained her PhD at the University of Geneva in 2012.

Olga Chekeres is a PhD student at the University of Geneva under the supervision of Anton Alekseev. Her research field is mathematical physics.

The Floer Center of Geometry

Alberto Abbondandolo (Ruhr University, Bochum, Germany)

The Floer Center of Geometry has been active at the Ruhr University of Bochum since 2011. It is named after Andreas Floer, who was first a student and later a professor at the Faculty of Mathematics in Bochum and whose ideas profoundly influenced the fields of symplectic geometry and low dimensional topology.

Andreas Floer

Andreas Floer was born in 1956 in the German town of Duisburg, where the river Ruhr meets the Rhine, in the middle of a densely populated mining and industrial area. At the time of his birth, there was not a single university in the whole Ruhr area but that situation was soon going to change. Local and national governments had understood that coal and heavy industry were not forever and had started to diversify. One of the first effects of this diversification was the foundation of the Ruhr University of Bochum in 1962, the "oldest of the new universities", that is, the first of the new public universities that were built in Germany after World War II. Classes in Bochum started in 1965 and, a decade later, Floer enrolled as a student in mathematics. He specialised in algebraic topology and dynamical systems, two fields that were investigated in Bochum under the chairs of Heiner Zieschang and Eduard Zehnder. He received his diploma in 1982 under the supervision of Ralph Stöcker.

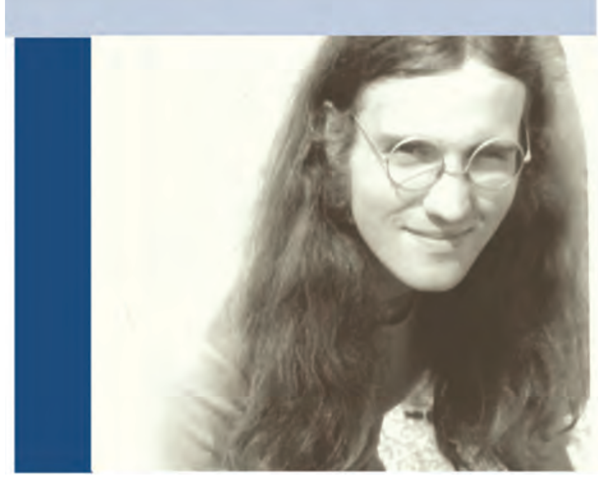
He then spent a year and a half in Berkeley, where he started his PhD studies working with Clifford Taubes and Alan Weinstein. He then returned to Bochum, where he received his doctoral degree in 1984 under the supervision of Eduard Zehnder. He later had research positions at SUNY in Stony Brook and at the Courant Institute of New York University, before becoming an assistant professor at Berkeley in 1988. In the Autumn of 1990, he returned to Bochum as a full professor. He tragically took his life on 15 May 1991.

Floer's work had a deep impact in symplectic geometry, gauge theory and low dimensional topology. Most of the recent progress in these fields would be simply unthinkable without the seminal ideas that shape what is now known as Floer homology. Floer homology is not a single theory but rather a new approach to the study of critical points of certain geometric functionals and a way of producing algebraic invariants out of them.

The first version of Floer's theory was motivated by the Arnold conjecture on the number of fixed points of Hamiltonian diffeomorphisms. In order to attack this conjecture, one has to prove lower bounds on the number of critical points of a functional (the action functional from Hamiltonian mechanics), which presents severe analytical difficulties. It is a functional defined on the space of closed loops on a symplectic manifold and its second differential at critical points is strongly indefinite, meaning that maximal subspaces on which this bilinear form is positive or negative definite are both infinite dimensional. Floer observed that a suitable gradient equation for this functional is given by an elliptic partial differential equation, which is a lower order perturbation of the nonlinear Cauchy-Riemann equation that Gromov had introduced and studied in 1985. Starting from this observation, Floer developed a Morse theory for the action functional based on an algebraic counting of the spaces of finite energy solutions of this elliptic equation, and eventually proved the Arnold conjecture for a class of closed symplectic manifolds. Together with Helmut Hofer, he also realised that his Floer homology could also be used to produce new powerful invariants for domains inside symplectic manifolds.

Floer soon realised that his ideas could also be applied to the study of the Chern–Simons functional, a functional on the space of SU(2) and SO(3)-connections over a three-manifold whose critical points are flat connections. This led him to the definition of the instanton Floer homology of a three-manifold, a theory which plays a major role in the study of four-manifolds with boundary.

A quarter of a century after Floer's work, his ideas permeate symplectic geometry and low dimensional topology. Lagrangian intersection Floer homology, contact homology, embedded contact homology, Heegaard Floer homology and Seiberg-Witten Floer homology are just a few of the theories that grew out of the work of this extremely talented mathematician.



The Floer Center

The Floer Center of Geometry was opened in Bochum in 2011, 20 years after Andreas Floer's tragic death. It is based at the Faculty of Mathematics of the University of Bochum and brings together the research activity of professors, postdocs and PhD students working in algebra, algebraic geometry, complex geometry, differential geometry, dynamical systems, symplectic geometry and topology.

The opening ceremony took place in December 2011, with colloquium talks by Helmut Hofer, Stefan Nemirovski and Andrew Ranicki, who were introduced by Gerd Laures (initiator of the foundation of the Floer Center of Geometry and its first director until 2013).

One of the main events of the Floer Center of Geometry is the annual Floer Lectures, which, over the last few years, have been given by Kai Cieliebak, Alex Ritter, Claude Viterbo, Frol Zapolski, Yakov Eliashberg, Hansjörg Geiges, Stefan Nemirovski, Ralph Cohen, Thomas Kragh, Michael Weiss and Thomas Willwacher.

In February 2017, the Floer Center of Geometry hosted a very special edition of "Geometric Dynamic Days", an annual event involving several German universities. The speakers of this edition, Helmut Hofer and Eduard Zehnder, have been close collaborators and friends of Andreas Floer since his undergraduate years. Their lectures on "Pseudoholomorphic Curves in Hamiltonian Dynamics and Symplectic Geometry" and "The beginnings of symplectic topology at the RUB" can be watched online at

http://www.ruhr-uni-bochum.de/ffm/Lehrstuehle/Lehrstuhl-VII/gdd17.html.

The next edition of the Floer Lectures will take place on 21–22 June 2018, with four talks by Victor Guillemin and Paul Seidel.

Since 2017, the Floer Center of Geometry offers Floer Postdoctoral Research Fellowships: three-year postdoc positions for young mathematicians working in the core fields of the centre. The first postdoctoral fellow Lara Simone Suarez, an expert in Lagrangian Floer homology and Lagrangian cobordism, was hired in October 2017.

The Floer Center of Geometry is funded by the University of Bochum through the Faculty of Mathematics and by diverse grants of the Deutsche Forschungsgemeinschaft (DFG). It serves also as a platform to coordinate the activities of the Collaborative Research Centre on "Symplectic Structures in Geometry, Algebra and Dynamics", a collaboration scheme funded by the DFG and based at the Universities of Bochum and Cologne.

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The Mathematical Society of Serbia

Vladimir Mićić and Zoran Kadelburg (University of Belgrade, Serbia)

This year, the Mathematical Society of Serbia (MSS) (or Društvo Matematičara Srbije in Serbian) celebrated its 70th anniversary. But is this a long or short period?

Preliminaries

Liberation and restoration of the State of Serbia (in the first half of the 19th century) was followed by a serious movement toward educational and scientific development in the Principality as well as the Kingdom of Serbia. A number of Serbian mathematicians had PhDs from European scientific centres (e.g. Paris, Wien and Budapest) and provided an essential contribution to this development, which included the advancement of mathematical sciences at university. In the final decades of the 19th century and the first four decades of the 20th century, one can see the presence of Serbian mathematicians (e.g. Mihailo Petrović, Nikola Saltikov, Anton Bilimović, Radivoje Kašanin, Tadija Pejović and Jovan Karamata) as authors of papers in scientific mathematical journals, as well as participants of the ICM and the ICME, with Mihailo Petrović present at the founding meeting of the IMU in 1920. In 1926, a "Mathematical Club" at the University of Belgrade was founded, directed by Anton Bilimović; it dealt with the scientific achievements and discussions of members, including their applications for publication in journals. In 1937, this club had turned into the "Yugoslav Mathematical Society", gathering together approximately 100 members with Tadija Pejović as its president. An additional list of mathematical publications, intended for secondary school students ("Matematički list") and researchers in mathematics ("Matematički vesnik"), as well as activities of the Union of Mathematics Students, complete this sketch of mathematical life in Serbia during the period before the Second World War.

Foundation of the MSS – the first years

The foundational meeting of the society took place on 4 January 1948 in Beograd; Tadija Pejović was elected as its first president. There was a basic decision to realise three parallel activities: scientific work in mathematics, mathematics education, and popularisation of mathematics. As physicists were included, the same scheme was also projected for physics (till 1981). At the very beginning, the scientific journal *Vesnik Društva matematičara i fizičara Srbije* was established and Jovan Karamata was named the editor-in-chief; its publication started in 1949. As *Matematički Vesnik*, it has been a permanent presence in the scientific life of Serbia since 1964. The MSS initiated the foundation of the Union of Yugoslav Societies of Mathematicians and Physicists (UYSMP) and organised the first Congress of Mathematicians and Physicists of Yugoslavia. These initiatives were realised in 1949 and, later on, UYSMP represented all the societies on international boards and organisations, including the IMU, the ICMI and the BMU (Balkan Mathematical Union). Publishing the necessary journals and other publications (intended for the whole country) was initiated and promptly realised. *Nastava matematike i fizike*, covering problems of education at all levels, has been published in Beograd since 1952 (it is now modified and entitled *Nastava matematike*).

Tadija Pejović

Jovan Karamata

The years that followed

The original scheme of MSS activities, enriched and modernised, has been preserved in subsequent years.

Scientific section

In the scientific domain, the MSS has been permanently present as a responsible standard-bearer in a wide range of activities (including international connections and collaboration), such as: publication of the scientific journal; regularly organised congresses of mathematicians; symposia and seminars (national and international) considering actual problems in several mathematical disciplines; organisation of a number of scientific sessions; and coordination of collaborations between various participants in the scientific life of the country (mathematical departments, institutes and other partners).

The scientific journal *Matematički Vesnik* mentioned in the introductory section has been in existence for 69 years (with 210 issues so far) and has been an important stage for the publication of original research papers, the presentation of new results and achievements in mathematical sciences and the provision (till 1980) of information about the activities of the MSS. The editorial board has been (and is) of international character, as is the majority of invited referees. The list of authors has varied from predominantly domestic to predominantly foreign authors; at present, it is somewhat balanced. The complete list of editors-in-chief is Jovan Karamata, Dragoljub Marković, Zlatko Mamuzić, Dušan Adnadjević, Zoran Kadelburg, Mila Mršević, Ljubiša Kočinac and Neda Bokan. The journal is available in electronic form and can be found at www.vesnik.math.rs.

Scientific meetings, organised by the MSS itself or by some other institution with an essential MSS contribution, can be classified into (a) national congresses (13 so far with the 14th in preparation), Yugoslav as well as Serbian, with significant international participation, and (b) international seminars and symposia. The latter have covered: Differential and Partial Differential Equations; Numerical Solution of Differential Equations; Coordination of Mathematics and Physics Instruction; the 5th Balkan Congress of Mathematicians; Topology and its Applications (five symposia); Complex Analysis and Applications (three symposia); and Mathematical Analysis and its Applications (two symposia with the third in preparation). One should take into account that the majority of these meetings were organised during the time of the Cold War (before 1990), a period when contact, even between scientists, was controlled and, in a way, extremely reduced. As Yugoslavia (and therefore Serbia) was a non-alignment country, we were able to invite colleagues from both sides of the "Iron Curtain" and to initiate creative official and private meetings and further collaboration. In the shortened list of foreign participants, one can recognise the names of numerous significant mathematicians of this period: L. Ahlfors, L. Aizenberg, P. S. Aleksandrov, T. Ando, M. Antonovskij, V.G. Boltjanskij, K. Borsuk, G. Choquet, R. Courant, A. Csaszar, W. Hengartner, G. Henkin, E. Hille, Y. Komatzu, I. Korevaar, W. Rudin, G. Sansone, W. Sierpinski, M.H. Stone, M. Vuorinen and V.A. Zorič. Since the 10th congress (Beograd 2001), the scientific committees of the congresses have awarded special prizes for young researchers (to Danko Jocić, Vladimir Dragović, Božidar Jovanović and Dragana Cvetković-Ilić).

Another in the range of scientific activities led by the MSS was a serious contribution to originating a number of "schools", characterising topics of mathematics achievement in Serbia, such as topology and set theory, complex analysis and distribution theory.

Particular thanks go to certain members of the MSS: Vojin Dajović, Djuro Kurepa, Konstantin Orlov, Zlatko Mamuzić, Bogoljub Stanković and Milica Ilić-Dajović, due to whose enthusiasm and essential contribution the meetings and activities mentioned have been successfully realised.

Vojin Dajović

Djuro Kurepa

Teaching section

Systematic efforts toward improvement of teaching and learning in mathematics have been present in MSS activities from the very beginning. In the early 1960s, thanks to the initiative and personal devotion of Vojin Dajović, an official document of the state authorities was accepted containing explicit recommendations for "paying special care to the mathematics education and development of the mathematical culture". This was a strong impulse for general affirmation of mathematics. It has been followed by further confirmation of programmes with a science-mathematical specialisation in high schools, including the foundation in Beograd of a specialised Mathematical High School and an increase of interest in studying mathematics. The first steps introducing numerical methods and computer programming into the curricula of secondary schools and universities were made during these years, too.

Nastava matematike, the MSS Serbian language journal (and its bulletin) continued to be published, with the aim of including theoretical problems of learning and teaching practices in mathematics, the exchange of good practices, information about tendencies in mathematics educational practices around the world, etc. The list of editors of the journal over 65 years of mathematical life in Serbia is: Ivan Bandić, Djuro Kurepa, Milica Ilić-Dajović, Miroslav Živković, Milosav Marjanović, Vladimir Mićić and Zoran Kadelburg. The MSS scientific journal *The Teaching of Mathematics* was founded in 1998 for research papers in mathematics teaching and, over the past 20 years, has acquired wide support in Serbia and abroad. The international character of both the editorial board and the majority of invited referees can be taken as an additional guarantee of permanency. The complete list of editors is relatively short: Milosav Marjanović, Vladimir Mićić and Zoran Kadelburg. The active site of the journal is www.teaching.math.rs.

In the early 1960s, traditional "state seminars for professional improvement of primary and secondary school mathematics teaching" were established (from the late 1980s also including informatics). Their programmes consist of a few plenary lectures, several special working sections and a number of master classes. In recent years, since the process of systematic licensing of teachers within the educational system has started, the character of these seminars has been, in some way, formalised and their role in the professional progress of mathematics and informatics teachers has been significantly improved. They are supplemented by regional seminars of similar character. The permanent increase in the number of master classes in informatics is evident.

Popularisation section

Ever since the founding of the MSS, systematic activities have been present for the identification, fostering and development of students, at all levels, who are gifted and interested in mathematics. A range of "out of the classroom" activities have been realised. Following the practice of mathematical societies and other institutions in many countries, the necessary literature was published and a system of competitions was organised. For secondary school students, the journal *Tangenta* has been published since 1995. The same function for primary school pupils was assigned to *Matematički list*, published in Beograd since 1967. These two journals have been accompanied by 55 issues of the series *Materijali za mlade matematičare*, which is intended for young mathematicians. At least twice a year, MSS organises a "Summer (Winter) school for young mathematicians", a kind of master class for interested students. They have been organised by MSS branches from mathematical centres of Serbia, the majority of them by the Valjevo branch and managed by Vojislav Andrić, the current president of the MSS.

From the first unofficial competition for secondary school students (Beograd, 1958) to participation at the 7th IMO (Poland, 1963), the route was direct and prompt. This was similar for higher primary school pupils, aged between 9 and 15, who started with a Serbian competition in 1967. Their first participation in an international event took place in 1997 at the Junior Balkan Mathematical Olympiad, held in Beograd and organised by the MSS. Young mathematicians of Serbia, through the MSS, are regular participants of the famous international competition "Kangaroo without frontiers" (the number of participants has increased to 36,000 in 2018).

Thanks to its specific geopolitical position, the MSS, as the host of the 9th IMO (1967), was able to invite several countries from Western Europe to participate; with the presence of young mathematicians from the United Kingdom, France, Italy and Sweden, the IMO seized the opportunity for a mathematical competition with representation from more than just Eastern European countries. The next contribution of the MSS to the broadening of the IMO movement was realised in 1977, when the MSS, as host of the 19th IMO, invited mathematicians from some non-European countries; with the presence of young mathematicians from Algeria, Brazil and Cuba, the IMO turned into a worldwide event. These organisational steps were accompanied by some very good results for Serbian students at the IMOs, including 15 gold medals, with Teodor von Burg achieving four of them, which resulted in him holding first position in the Hall of Fame of the IMO for several years.

The Mathematical Society of Serbia will continue all of its activities over the forthcoming years. In particular, in 2018, the 14th Serbian Mathematical Congress will be organised, the 35th Balkan Mathematical Olympiad will be held in Beograd and there will be an event marking the 150th anniversary of our great mathematician Mihailo Petrović.

Vladimir Mićić (1936) [vladimic@mts.rs] is a professor of advanced engineering mathematics and didactics of mathematics at the University of Belgrade. He has been active with the MSS for 55 years. He is an ex-president of the society and is its first Honoured Member.

Zoran Kadelburg (1950) [kadelbur@matf.bg.ac.rs] is a professor of mathematical analysis and functional analysis and is an Ex-Dean of the Faculty of Mathematics at the University of Belgrade. He has been involved in the complete range of MSS activities for 50 years and is an ex-president of the society.

Is the System of Scientific Publications on the Eve of a Revolution? And if so, Toward What?

Marie Farge (École Normale Supérieure, Paris) and Frédéric Hélein (Université Paris Diderot - Paris 7, France)

Today, researchers benefit from an extremely powerful electronic distribution system for scientific articles, which allows any researcher (whose institution has paid the subscription fees) to access, via the internet, most of the articles they need, instantaneously and regardless of one's location. Electronic publishing has indeed enabled libraries to subscribe electronically to a very large number of journals, through so-called big deals, at rates that were initially affordable, especially in cases where these libraries partnered in consortia to negotiate with publishers.

However, behind this simplicity for the researcher lies a digital infrastructure much more complex than that of traditional libraries acquiring and preserving printed articles on paper and, behind the apparent impression of gratuity that the researcher may have, there are exorbitant and constantly increasing costs and contracts that have become opaque. The creation of journal bundles has resulted in pricing that no longer has any relation to the real cost of production for the publisher, and the dematerialisation of articles has allowed commercial publishers to multiply tolls according to usage: pay to read the papers to which your library has subscribed this year, pay to read the archives of journals to which it subscribed in previous years, pay to publish in Open Access, etc. At the same time, electronics has allowed large publishers to make enormous reductions in the costs of producing an electronic paper (and any researcher knows that the raw material and its evaluation are provided free of charge).

In economic terms, this development has mainly benefited big publishers, whose profit margins far exceed those of pharmaceutical companies or banks, but it is increasingly weighing on public finances. Indirectly, it is also the small publishing houses, private or academic, that bear the costs because it is by cancelling subscriptions with these small publishers that libraries manage to balance their budgets. This maintains a vicious circle, which results in a growing concentration of scientific journals in the hands of the largest publishers. These economic mechanisms have had an impact on the quality of scientific publications, as major publishers have a commercial interest in encouraging the proliferation of journals and articles. A visible effect of this is that many scientific communities are saturated by the number of papers and the quality of peer review has fallen. Another effect is the multiplication of predatory journals (there are nearly 10,000 today, all disciplines combined).

The Open Access project

Since the early 2000s, and particularly since an initiative [1] launched in Budapest in 2002, researchers have dreamed of building a model for distributing articles free of charge for all (thanks to the internet) and thus freeing themselves from the big publishers. Although at first glance this dream seemed within reach, it is far from being realised as, even today, more than 80% of articles are not published in Open Access. The first reason is that any system allowing rapid dissemination (*e.g.*, the arXiv) has a cost, even if it is small in comparison to the one charged by publishers.

Publishers, who initially feared the Open Access movement, have managed to turn the Open Access project to their advantage by proposing a model in which the author pays publication fees, often called "APC" (for *Article Processing Charges*), so that their article is immediately Open Access. However, it would be more correct to call APC a *licence fee* (which we will do in the following) because the amount usually has no relation to the real cost of the service.

There is therefore still a long way to go to build an Open Access publishing model, in which researchers and their institutions regain full possession and free use of the data they themselves have produced, and in which the costs associated with their publication and dissemination are charged at the right price. Indeed, the transition is tremendously complex to set up between, on the one hand, extremely well organised multinationals and, on the other hand, scientists and institutions, divided according to their disciplines, their institutions and their countries and where dialogue is not necessarily well organised nor does information circulate very well.

Moreover, a fundamental obstacle is that scientific journals combine several essential functions, inherited from the age of printing, namely:

- Evaluation process. This task is mainly carried out by researchers, as members of editorial boards or as referees. It should, however, be noted that this task involves secretarial work, which is usually carried out by a secretary, but also sometimes by researchers, who may be assisted by software. Secretariat funding is provided by the publisher or a public institution, in varying proportions depending on the journals.
- Label provided by the reputation of the journal.
- Referencing and notification of an article. Several actors contribute to this function: the journal itself, data-

bases, such as *Web of Science* and *Scopus* for most disciplines and *MathSciNet* and *Zentralblatt* for mathematics, and libraries, as well as institutional reference systems that are developing.
- Diffusion and promotion.
- Archiving.

The latter two functions can be performed by journals or by archive platforms, such as *arXiv*, but also, as far as archiving is concerned, by public institutions and non-profit organisations, such as *CLOCKSS* [2] and *PKP-LOCKSS* [3], not forgetting libraries for printed versions.

It can be seen that almost all of these functions are services that can be the subject of a call for competition on a market, or be taken over by public institutions or non-profit organisations. All of them are services except … the label of a journal, which is attached to recognition by the community and which is thus unique. This is the reason why the market for scientific journals is essentially without competition, and this is the main explanation for prices unrelated to the cost of services. As long as journals combine these functions, it will always be difficult to bring subscription prices down to reasonable levels.

Where do we stand?

After two rather calm decades, it seems that we have reached an unstable situation, and are probably on the eve of great upheaval. Indeed, tough negotiations with publishers are multiplying (as we will see later), institutions are unsubscribing from big deals (see a list of institutions that have cancelled their subscription to big deals in [4]) and the Sci-Hub pirate platform is enjoying worldwide success, with hundreds of thousands of illegal downloads of articles per day. In Europe, the pressure is increasing because the European Union has set, under Horizon 2020, the objective of free dissemination of European scientific production. Finally, the development of alternative solutions is accelerating, as we shall see next.

An overview of countless innovations

Open Archive platforms like arXiv [6], which was created in 1991 by Paul Ginsparg, were long confined to certain science disciplines, such as high-energy physics, computer science, mathematics and economy.[1] Today, the situation is changing rapidly as new platforms are being created, such as bioRxiv [12] in biology in 2013, ChemRxiv [13] in chemistry in 2016, and EarthArXiv [14] and ESSOAr [15] in geosciences in 2018. In addition, recent agreements between institutions to accelerate the development of these archives and to federate them, together with initiatives such as ASAPbio [16], are aimed at encouraging researchers in biology and medicine to deposit their preprints on public archives. These initiatives have an international dimension.

This rise in the power of Open Archives can be largely, but not only, explained as a reaction to abusive tariffs by major publishers or to *licence fees* for publishing in Open Access. It also appears that this flexible and commercially unfettered method of dissemination is incomparably faster than the journal system and better protects researchers from plagiarism.

Open Access journals without licence fee, often referred to as Diamond Open Access journals [17], continue to grow. The existence of such journals is most often based on projects, such as Episciences [18] or Cedram [19] (whose activities are expanding with the creation of the Centre Mersenne [20]) for mathematics. Similar projects exist in all disciplines (especially in the humanities, which is at the forefront of this movement), such as: the Public Knowledge Project [21], which develops *Open Journal Systems*, an open source software to publish journals; organisations such as Knowledge Unlatched [22] and Open Library of Humanities [23], which propose crowdfunding for the publication of Open Access books and journals; OpenEdition [24], which provides platforms for books, journals and blogs; and the OA Cooperative Project [25]. In biology, the mega journal PLOS One [26], supported by a non-profit organisation, is an Open Access journal that was free of publication fees in its early days (but unfortunately now charges publication fees of the order of $1,500 per article). In Latin America, the SciELO [27] platform, founded by Brazil, includes 1285 journals, most of which are Open Access. These projects are supported by various foundations and organisations, such as LingOA [28], MathOA [29], the Foundation Compositio Mathematica [30], SPARC [31] and SCOSS [32].

These Open Access journals can either be a new journal, or an already existing title whose editorial board resigns or leaves a commercial publisher so that the journal can continue in an open framework. This is called *emancipation* of a journal [33]. The most notable example of emancipation of a mathematical journal was *Topology*, which became in 2006 the *Journal of Topology*, but there have been other journals before and after, including *Acta Mathematica* (since 2017). A list of emancipated Open Access journals can be found on the Journal Declaration of Independance [34]. A list of Open Access mathematical journals without publication fees can be found on the Cimpa website [35]. (See also, for example, the Directory of Open Access Journals [36] for all fields.)

[1] Open archive platforms were developed in the last century by researchers to share their preprints and reprints (note that those years publishers where providing reprints to authors for free and asked them to send them to colleagues for advertising). The SPIRES High Energy Physics database, developed at SLAC-Stanford University in the ’70s, was the first made accessible via the Web in 1991 and replaced by Inspire [10] in 2012. It was followed by the database ADS [5] developed for astrophysics in 1988 and transferred to the Web in 1994. In 1991 arXiv [6] was created by Paul Ginsparg at Los Alamos National Laboratory for physics. In 1994 Michael Jensen founded SSRN [7] for social sciences and humanities (in 2013 it was the largest open repository in the world and in 2016 it was sold to Elsevier). In 1997 PubMed [8] was designed for medicine and RePEc [9] for economy.

Lastly, there are some very interesting innovations in terms of evaluation, including Open Peer Review [17, 37]. The principle is to organise the evaluation of an article in an open way, i.e. by making public the reports of the referees, the authors' responses to the referee and even the contributions of other researchers. Such a system may give rise to fears, but many variants exist, and the result will depend on the quality of the editorial board that oversees this process and on the adjustment of its details (in particular, one can choose to keep references anonymous and to make public only the positive reports, in which case one should really speak of recommendations).

Among the first experiments on Open Peer Review are Copernicus in 2001 [38] and F1000Research [39] (a non-profit organisation currently with low publication fees) and SciPost [40] (without publication fees); they seem to lead to a significant improvement in the quality of reviews. A similar experience, perhaps even more innovative, is "Peer Community in…" [41], a recommendation platform that is not a journal, offering positive evaluations of preprints or articles. These projects take up an idea already proposed in the Self Journal of Science project [42], which has unfortunately had difficulties getting off the ground. The interest in these latter projects is to decouple evaluation (and its associated label) from the rest of the services.

We could also mention the European Digital Mathematical Library [43], and the platform Dissemin [44] which detects papers behind pay-walls and invites their authors to upload them in one click to an open repository, in order to boost open access while respecting legal constraints [44]. Another interesting innovation is the DOAI (Digital Open Access Identifier) which is an alternate DOI (Digital Object Identifier) resolver that points to a free version of the requested article, when available, instead of its version under pay-wall [45].

The previous list of innovations, institutions and tools is by no means exhaustive since this would be impossible, as new innovations are emerging every month. The term *bibliodiversity* has been proposed to refer to this proliferation and led to the "The Jussieu Call" [46], an initiative aimed at supporting these alternative solutions, which has been signed by more than 100 institutions from 13 countries.

The temptation to contract with major publishers for Open Access publishing

(This section reproduces a large portion of the article by the authors, "Transition vers l'Accès Libre: le piège des accords globaux avec les éditeurs", which was published in the French newspaper Mediapart on 14 April 2018 [47].)

A worrying policy in some countries is to conclude national agreements with large multinational publishing companies to pre-pay licence fees for Open Access publishing. As it happens, the countries at the forefront of this movement are essentially those in which these global publishing giants are mostly established, namely the United Kingdom, the Netherlands and Germany. We are therefore entitled to ask ourselves whether this is a coincidence and we propose here hypotheses to interpret this "coincidence".

Let us put ourselves in the shoes of a multinational scientific publishing company

To understand this situation, it is useful to adopt the point of view of the major commercial publishing groups and ask what would be the most effective strategy for these groups to adapt to these changes whilst maintaining or increasing their profits:

- Firstly, *preserve current income*, without rushing but simply by taking advantage of the inertia of the system that exists today, with more than 80% of articles published in journals with subscription, whilst multiplying the tolls to access articles on the electronic platforms of the publishers.
- Then, invest for the future by developing, just like Google, new "services" based on algorithms to mine the large amount of data accumulated. This is the direction taken by Elsevier, the largest multinational publishing company, which buys back several start-ups that have developed such services each year. The amount of data processed can come both from the vast scientific corpus contained in papers and from the data concerning the researchers themselves (as authors or experts). In this "market", Elsevier competes with Clarivate (formerly Thomson–Reuters).
- Finally, build an Open Access model that is stable and that guarantees a firm's profits to be as large as those they currently make with subscriptions. We will see how delicate this operation is.

Indeed, for the major publishing groups, Open Access has remained, since 2000, both a *source of additional income* and a *source of concern*.

- It is a *source of additional income* thanks to the payment of *licence fees* by researchers (or their institutions) for each article published in Open Access (noting, in passing, that these fees may correspond to articles that are either published in journals that are totally Open Access or *hybrid journals*, i.e. for which libraries already pay for subscriptions!).
- It is a *source of concern* because of the risk that researchers and institutions might emancipate themselves from the current system of journals by building their own Open Access system (for example, this concern can be seen directly through Elsevier's share price, which fell at the time of the *Budapest Initiative* in 2001 and then on other similar occasions). To avoid this risk, these multinationals must build a stable Open Access publishing model that allows them to collect fees as cost-effectively as the current subscription model but, at the same time, does not provoke a hostile and destabilising reaction from researchers and their institutions.

With the current subscription system, a publisher receives, on average, more than 5000 euros per article published, this revenue being even higher in the case of *Elsevier*. Therefore, to maintain comparable profit margins with the Open Access publishing system, such a publisher would have to charge royalties at an average price of the same order of magnitude. On a laboratory scale, such rates would result in an extremely high bill (and in mathematics, even charging 2000 or 2500 euros, the fees for all items in some laboratories, would absorb the entire budget!). Such a solution is therefore unrealistic. Moreover, even if the budgetary limits of the laboratories were disregarded, this would have a devastating effect on the publisher's image because it would reveal to researchers, in a concrete way, the scandalous level of fees charged, of which, in the majority of cases, they are unaware. Moreover, the fees give those who control the credits excessive power, which is likely to degrade relations between colleagues. Finally, the development of such a model risks, even before it has come to an end, provoking an acute awareness and reaction from the scientific community, which would compromise the commercial publisher's projects. Indeed, there are now more and more initiatives from the scientific community that could well be the premise for such a reaction.

The best solution for these publishers therefore remains to obtain payment of fees from their traditional interlocutors, libraries, which already have large budgets capable of supporting expensive subscriptions, and national agencies or institutions. Thus, the fees will not weigh directly on laboratories' budgets, will remain invisible to researchers and, even better, will "smoothly"replace the astronomical sums that libraries are used to paying. Publishers will thus be able to continue their "business as usual" in complete security.

The first country where such a model was tested was the United Kingdom. The British have been engaged since 2013 in a transition programme toward Open Access, combining the use of Open Archives and the payment of fees to publishers. This required the establishing of a complex protocol for the institutions and the creation of a special fund to finance the additional costs involved. Despite a political will to control overall costs (subscriptions and fees), it is clear that no expenditure could be contained. Worse still, the system of reimbursement of fees to universities by the special fund has created a bureaucracy whose cost has added to the bill. There is therefore quite a bit of discontent with this system in the United Kingdom. This experience has encouraged the major publishing groups and countries tempted by this direction to move towards global agreements on a country scale, the bill of which would be paid by libraries.

This is the path taken by the Netherlands, by concluding a first agreement at the end of 2014 with Springer that integrates a subscription to a bundle of journals with the right for Dutch researchers to publish Open Access at no additional cost (these are therefore included in the subscription invoice). Similar agreements have since been concluded with other publishers and in other countries: in Austria and Germany at the end of 2015, in Sweden in 2016, in Finland in 2018, etc. The type of contract varies but there is a shift toward contracts in which the proportion of fees for Open Access publications is becoming increasingly important. Thus, from the end of 2016, Germany embarked on an even more radical path: instead of wanting to conclude mixed agreements, concerning both subscriptions to read and fees to publish, Germany believes that it is no longer a question of paying to read but just paying to publish.

In any event, it is striking to observe that this movement toward global contracts including Open Access fees (which, as we have seen, is certainly the most satisfactory solution for multinational publishing companies) is developing mainly from the United Kingdom, Germany and the Netherlands and in the geographical area around these countries. But it is also striking to note that, with the exception of the American Chemical Society, the main publishing companies (Elsevier, Springer Nature, Wiley and Taylor & Francis) are precisely located in these three countries. It should also be noted that small scientific, commercial or academic publishers are not involved in these agreements and risk, once again, paying the price for these developments.

There may be several explanations for this coincidence: the result of lobbying by these publishing firms cannot of course be excluded, but the most plausible explanation is the conflict of interest situation in which these three countries naturally find themselves. Indeed, public institutions in these countries must certainly try to reduce, if not contain, the bill they pay to publishers but, at the same time, we can assume that it is difficult for them to make choices that would harm the multinationals based in these countries, not so much for fear of weakening them as for fear that these companies would threaten to relocate their activities to other countries.

In Germany, the Max Planck Society (*Max Planck Gesellschaft*), the main German research institution, which has played an active role in promoting Open Access since 2003, is also in an ambiguous situation. Stefan von Holtzbrinck, who owns more than half of Springer Nature's capital, is a member of its Board of Directors. Stefan von Holtzbrinck is also the president and co-founder of the *Max Planck Förderstiftung*, a foundation that financially supports the Max Planck Society.

With regard to Germany, it is important to distinguish two trends that are developing in independent directions:

- On the one hand, the German institutions have joined forces within the DEAL consortium [48] to negotiate hard with Elsevier, Springer Nature and Wiley, in order to obtain significant price reductions and to obtain transparent contracts (since, until now, the details of these contracts have remained confidential, an opacity that benefits publishers). Having failed to reach an agreement with Elsevier at the end of 2016, the consortium decided that, as of January 2017, all contracts with Elsevier that were due to expire would be terminated, resulting in a standoff with this publisher. Of course, we can only applaud this exemplary determination.
- On the other hand, as we have seen, the Germans demand contracts that guarantee them free access to journals but they agree to pay to publish in Open Ac-

cess. This radical position is inspired by a strategy developed by the Max Planck Society, which is the subject of a promotional campaign inviting institutions around the world to rapidly switch to an Open Access publication system, even if this means paying licence fees in advance. The Max Planck Society thus proposes to institutions around the world to commit to this path by signing the Initiative *OA2020* for *Open Access 2020* [49] – not to be confused with the *H2020* programme of the European Union for financing research! Note that this initiative also calls for transparency of costs – currently refused by publishers – and for a certain diversity of models.

For the moment, no final agreement with the publishers has been concluded. It seems that the reason for this is, not so much the reverse model (paying to publish instead of paying to read), but the price to pay and the complexity of the deals. As long as publishers are not asked to significantly reduce their turnover, they have every reason to be satisfied to the extent that they are offered a stable solution. Indeed, Springer and Wiley affirm their will to bring these negotiations to a successful conclusion and, even if relations with them are much more tense, Elsevier clearly indicated in a note [50], made public in September 2017, that they do not disagree with the principle but on the price and details of the implementation of the changeover.

On the other hand, the selling point [51] underlying the *OA2020* initiative, promising a significant reduction in tariffs, unfortunately seems simplistic and it is hard to see what serious element could support it once one market without competition is replaced by another market without competition. On the contrary, the risk seems great that the result will be the creation of a new Open Access publishing model in which large companies will be able to continue to dictate their financial terms (see [52]).

As a conclusion

As we have seen, Germany has been engaged since January 2017 in a tough negotiation with Elsevier. Likewise, the French institutions, grouped within the Couperin consortium, have recently followed a similar approach with Springer (but for a big deal subscription contract that does not include Open Access). Failing to obtain a significant reduction in Springer's rates, these institutions terminated their contracts in January 2018 … until this publisher become reasonable. Finally, many institutions [53] around the world have simply cancelled their subscriptions to big deals. All these steps may signal a change in the power balance. It is interesting to note that in Germany, after cutting access to institutions that had terminated their contracts for a few weeks, Elsevier finally reinstated them in February 2017. Similarly, in France, Springer has not cut off access to French institutions since January 2018.

On the other hand, there is a risk that in many countries, decision-makers and library managers, eager to make a transition to Open Access, will give in to temptation to follow the examples of northern European countries by signing agreements with major publishing groups that would include the advance payment of licence fees for Open Access publishing. We must avoid this scenario, which would further strengthen the hold of these large groups.

In addition, the transition to Open Access must not replace commercial barriers to read with barriers to publish. Research results and, more generally, data produced by public institutions must stop being privatised, which does not exclude using private providers to disseminate and make them visible.

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Can Statistics Predict the Fields Medal Winners?

Adam Bannister and Olaf Teschke (FIZ Karlsruhe, Berlin)

With the upcoming ICM in Rio de Janeiro, the seasonal speculation about who will receive the 2018 Fields Medals at the opening ceremony is once more in full swing. With the big data industry measuring us in all possible ways, a natural question might be whether statistical approaches could possibly predict the committee's choice of the Fields Medallists. We undertake some experiments here to see which predictions are provided by standard approaches based on data from zbMATH and linked data sources.

Fields Medal is small data

One obvious obstacle, however, is that the set of Fields Medallists is small by its very nature and may easily defy statistics with all kinds of outliers. As a nice example, one could recommend reading Borjas-Doran's study on a statistical decline of Fields Medallists' productivity [BD14] and Kollár's amusing review [K15]. Without further discussing the fundamental problem of measuring a mathematician's productivity by publication and citation numbers – the fallacies of this approach have been frequently discussed in the newsletter, for example in [BT17] – we just note here the very last observation in [K15]:

> *"The limits of statistics are illustrated by the numbers contained in the penultimate line of [BD14, Table 1]. (It is not commented on in the paper.) While most of the Fields Medallists and contenders are happily alive, Figure 3 shows a disturbing pattern about those who have passed away […Namely, an average age of death of 74.0 for Fields Medallists compared to only 66.3 for contenders…] Thus, if you got a Fields Medal, you can expect to enjoy your extra US$120,000 per year for almost eight more years."*

Firstly, we may take this as an illustration of how seemingly exact science is often perturbed by possibly unreliable data. It was, for us, impossible to reproduce the average age of death of 74 from [BK14] ([K15] does not comment on this figure). Submitted in 2014 before the death of Grothendieck, the nine Fields Medallists deceased at that time reached an average age of 78.5. (A closer look at the appendix of [BK14] reveals that the 1936 medallists Ahlfors and Douglas seem to have been excluded from the study but that has almost no effect on this average). Secondly, as we are all sadly aware, this figure has been significantly affected since then by the passing of several medallists, reducing the average age at death by more than four years and wiping out a large part of this statistical effect.

Which data can reasonably be taken into account?

There is basically no formal limitation to being a Fields Medal candidate except for the famous age rule. However, even this simple condition requires some work – for most of the (approximately) one million authors in zbMATH, the age is simply unknown. Reasonable estimates are often possible based on publication history but in many cases may lead to gross errors: while quite a few mathematicians have already published relevant research in their teens, others may be over 30 at their first zbMATH entry. This happens frequently in border areas when most publications are outside the scope of the database or for mathematicians suffering from political suppression, e.g. from the Nazi regime, Stalinist terror or during the Cultural Revolution in China.

Needless to say, birth dates derived from publication history are therefore not suitable to check the rather clear-cut Fields Medal age limit. Fortunately, the zbMATH author database is linked to many other collections, some of which – like Wikipedia, MacTutor, GND and MathNet.Ru – provide birth date information. Additionally, the communities pursuing these services usually do a reliable job in relevance decisions. Overall, zbMATH contains links to data resources providing sufficient age information for almost 13,000 authors – only a fraction of the overall million but covering all Fields Medallists and likely candidates. Only 252 of them are at most 40 years old.

The harder task is to identify relevant features for the statistical model. Modern databases like zbMATH offer various facets that can be taken into account – not just quantitative publication and citation information but also granular subject information, or co-author and reference networks. Unfortunately, many of these quantities can be dual-edged: a high publication number may be obtained by people like Yau, Bourgain or Tao, or by more notorious representatives of the class of prolific writers.[1] High citation numbers may be related to a lasting impact of results but they are also much affected by subject and community custom. Even more importantly, they come with a massive time delay [BT17]. Subject information is certainly valuable (statistically, the Fields Medals are far from evenly distributed within MSC subjects) but will usually not reflect breakthroughs that may define new areas in the future. Publication sources are certainly meaningful – prize winners will almost inevitably have a distinctive record in the Annals, Inventiones, etc. – but are significantly limited by publication delay, with many relevant results appearing only after several years (the committee of course being aware of them). Close collaboration or citation distance to former prize winners may indicate that you are actively involved in pursuing cutting-edge research but could also be an indication of a supporting role rather than a unique individual effort qualifying for the medal.

Less ambiguous features would be existing prizes like the EMS Prizes (which are also connected to an age limit and have a distinct overlap with later Fields Medallists from Europe) but many prizes have a shorter history than the Fields Medal, as well as regional restrictions, thereby further complicating the involved statistics. Fortunately, a substantial list of prize winners is available for analysis via the zbMATH connection with Wikidata; others (like the EMS prizes) have been added manually. The same holds for the information on being an invited speaker to an ICM, which may reasonably be treated like winning a global prize.

Finally, we emphasise that no data generated by user searches were taken into account due to our strict data protection policy [HT14]. As outlined there, one could expect rather distinctive results, especially if IP information were analysed (which is ruled out). A rough approximation might be obtained by taking Google search data into account (although this would most likely not reflect the committee's procedure well). Currently, this would see Simon Brendle, Hugo Duminil-Copin, Alessio Figalli, Ciprian Manolescu, Fernando Codá Marques, Sophie Morel, Peter Scholze, Maryna Viazovska and Geordie Williamson as the most likely candidates (in alphabetic order, with Peter Scholze leading).[2] A closer look at the trends indicates that most of the queries are correlated to prize announcements, hence one might expect that this is covered by the above features.

Methodology

Educated humans will usually overcome most of these obstacles, e.g. a closer look will easily distinguish deep results from superficial mass publications with bulk references. Automatic recognition is, however, still limited in addressing such questions. Approaches like neural networks have made tremendous progress over the past years but still encounter problems, for example in distinguishing art from pornography (a somehow related question), despite the fact that technology in image processing has become more advanced and much more data are available. Some tools to recognise "maths pornography" might help editors, reviewers and readers but there has not been much activity toward this yet. Moreover, big data approaches would ideally require billions of samples as training data, far more than the currently available mathematical publications (although several groups of authors, in an often undervalued effort, are very active in enlarging the available datasets). The problem of scarce data applies not just to bibliometrics but even more to the other features mentioned, so there currently seems no hope of applying neural network technology to the Fields Medal prediction.

[1] Currently, Yau, the Fields Medallist with the most publications in the zbMATH database (authoring on average a paper every two weeks over the past few years), ranks only around 50th place in this list.

[2] This kind of crowd-sourced projection also agrees well with certain internet polls, e.g. https://poll.pollcode.com/44839318_result?v.

Instead, we just put the available data into a support vector machine model. We defined data slices for the information available at the time of the congress. First, we trained a model based on previous years' winners. We then used this model to analyse our candidates for this year. We repeated this procedure 20 times and averaged the results to remove any outliers created by an imbalance in the splitting of testing and training data. This averaging of the results is due purely to the small sample of data available to train a model; in some runs, we could be unlucky enough to get no positive examples in our training set.

As we have used prizes first awarded in the 1990s, we also had to limit our Fields years to 1994 or later. We took into consideration the EMS Prizes, the Bôcher Memorial Prize, the Coxeter-James Prize, the Fermat Prize, the SASTRA Ramanujan Prize, the Oswald Veblen Prize, the Clay Research Award, the Wolf Prize and the Salem Prize.

As a by-product, we obtained measures for the significance of the different features.

Results

Not unexpectedly, sole bibliometric features turn out to be almost non-predictive. In a model that takes just citation figures, journals or MSC subjects into account (the latter two features should be at least included to adjust citations numbers [BT17]), one can generate a high-dimensional (due to the variety of journals medallists have published in) linear model that is adjusted to the past but generates only individual winning probabilities of about 1% and less in the projection (with Jeremy Blanc, Anton Koroshkin, Luis Pedro Montejano and Evgeny Sevostyanov leading by slim digits a basically even field). Hence, citation-based hiring will most likely lead to missing a future Fields Medallist (actually, it will perform at most marginally better than randomly picking a mathematician younger than 40 years with a Wikipedia entry).

In contrast, other prizes and ICM invitations are the most predictive sole features, which produce distinctive projections and were more than 97% successful for past test sets. By adding further features like collaboration and citation distance to former Fields Medallists, prize winners and invited speakers, the success rate for test sets can be improved further (as is natural when dimensions are added) up to greater than 99.3% but with decreasing sharpness of prediction. The differences also indicate a possible bias toward more collaborative communities, after adding the distance features, and a bias against recent, yet unpublished achievements. Table 1 shows the figures for the leading contenders in the respective models.[3]

From this, one might reasonably predict that Peter Scholze is a strong favourite to win a Fields Medal but the others remain highly competitive, with different models producing very different outcomes. Geordie Williamson and Bo'az Klartag seem to have the most consistent statistical chances from the field.

Does the committee's composition matter?

Of course, the decision is solely made by the committee members, whom we can expect to weight mathematical achievement over superficial facets. Since assuming responsibility for the committee, the IMU has put much effort into creating a balanced composition of prize committee with respect to aspects like geography or research area, and the difficulty of obtaining significant projections may serve as a good illustration. Of course, the composition of the 2018 committee cannot be used for projections since it is revealed only at the ICM (except for IMU president Shigefumi Mori, who is an ex-officio

[3] Important caveat: Since we didn't use 4-years age slices of in the model to avoid more sparsity effects, the resulting probability reflects the chance of winning a Fields Medal in the future, not necessarily at the next Congress.

Table 1. Projected winners in different models

	Prize	Prize + Invitation	Prize + Invitation + Coauthor	Prize + Invitation + Coauthor + Citation	All features
Peter Scholze	64%	81%	91%	34%	86%
Geordie Williamson	56%	82%	25%	10%	2%
Bo'az Klartag	50%	38%	15%	14%	16%
Simon Brendle	49%	30%	2%	<1%	1%
Hugo Duminil-Copin	5%	6%	14%	11%	20%
Peter Pal Varju	5%	6%	11%	2%	16%
Sophie Morel	6%	6%	9%	<1%	6%
Alessio Figalli	5%	6%	9%	4%	2%
Ciprian Manolescu	5%	6%	9%	3%	<1%
Maryna Viazovska	1%	1%	9%	1%	<1%
Fernando Coda Marques	<1%	2%	1%	<1%	3%

member). However, one may ask whether the knowledge of the composition of past committees[4] would have significantly influenced the projections. The figures show only modest changes when adding the committee information, hence a significant "committee bias" cannot be confirmed via this statistical approach.

Conclusions

There are several facets of public information available that may serve as features for statistical predictions about Fields Medal winners but many come with certain disadvantages. Taking different reasonable models into account, the formal statistical approach may provide some educated guesses with reasonable probabilities but a rather high uncertainty remains, certainly sufficient to keep the tension about the disclosure of the winners at the ICM.

Perhaps the most important caveat is, however, that the statistical method will only succeed in carrying forward past trends to the future. As is well known, this is one of its major drawbacks, which may preserve or even worsen existing discriminations [O16]. Due to these effects, we didn't include available data features like gender or country of origin into the model because this would almost certainly generate further intrinsic bias. Since the composition of the Fields Medallists has grown significantly more diverse over the past few years (reflecting the development of the mathematical community), statistical predictions will most likely have a conservative bias compared to the actual decisions and the committee will likely succeed in proving statistical guesses at least partially wrong.

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See page 4 for a photo and biography of the authors.

ICMI Column – The Mathematical Legacy of Jean-Pierre Kahane

Julien Barral (Université Paris 13, Villetaneuse, France), Jacques Peyrière (Université Paris-Sud, Orsay, France) and Hervé Queffélec (Université des Sciences et Technologies de Lille, Villeneuve d'Ascq, France)

Jean-Pierre Kahane (1926–2017), an alumnus of École Normale Supérieure, was appointed as a professor at Montpellier University just after his PhD thesis (prepared at CNRS) and at the Faculté des Sciences d'Orsay in 1962, which was then a part of the Sorbonne and later linked to Université Paris-Sud. He has been the recipient of numerous awards (the enumeration of which would be tedious) and he was a member of the Académie des Sciences de Paris.

His particularly intense and fruitful scientific activity did not prevent him from assuming various positions to the benefit of the community: for instance, he served as the President of Université Paris-Sud, the President of the MIDIST (a Government agency) and the President of the CIEM. Again, it would be tedious to list all the responsibilities he has assumed.

He was fully concerned with the teaching of mathematics. Indeed, he thought of it as intimately linked with research. We refer to the preceding article by Michèle Artigue for an account on this subject. He also had a special interest in the history of mathematics. In particular, he revisited some of Plato's writings.

Jean-Pierre Kahane acted as the advisor for numerous PhD theses. He was a patient and attentive advisor; he fully respected the personality of his students. He was a talented organiser and coordinator. He is one of the founders of the mathematics department at Orsay. He created and energised a seminar and a team. Due to his international fame, he was able to attract the most world-wise renowned visitors to Orsay.

His first studies deal with the connections between the mean periodic functions of Laurent Schwartz, entire functions and quasi-analyticity. Of course, this echoes the preoccupations of Szolem Mandelbrojt, his PhD advisor. From him, he inherited a taste for Dirichlet series, on which he kept an eye throughout his career.

Soon, he won international renown, as shown by the list of his early collaborators. Nevertheless, in the 1960s, Jean-Pierre Kahane was (in France) a somewhat singular mathematician, or at least he felt so. The preface of his book with Raphael Salem, "Ensembles parfaits et séries trigonométriques", is the best description of his view of mathematics. Here is an excerpt:

> Il y a quelques dizaines d'années, ce livre aurait pu se passer de cette préface, qui est écrite en guise d'apologie. Aujourd'hui, venant à un moment où la plupart des mathématiciens – et les meilleurs – s'intéressent surtout aux questions de structure, il peut paraître suranné et ressembler en quelque sorte à un herbier. Les auteurs se doivent donc d'expliquer que leur propos n'est en aucune façon réactionnaire. Ils savent la beauté des grandes théories modernes, et que leur puissance est irremplaçable, car sans elles on serait souvent condamné (comme l'a dit Lebesgue) à renoncer à la solution de bien des problèmes à énoncés simples posés depuis fort longtemps. Mais ils pensent que, sans ignorer l'architecture qui domine les êtres mathématiques, il est permis de s'intéresser à ces êtres eux-mêmes qui, pour isolés qu'ils puissent paraître, cachent souvent en eux des propriétés, qui, considérées avec attention, posent des problèmes passionnants …

The objects that this text refers to are generically called thin sets (Kronecker, Sidon, Helson sets, uniqueness sets, etc.), the study of which has been very intensive in his team for more than a decade.

This book was published in 1962, with a second issue in 1994. It is worth noting that such a preface would nowadays be unnecessary; indeed, this view of mathematics, in particular due to Jean-Pierre Kahane, is widespread.

He left a huge volume of work, of which the main topics are:

- Harmonic analysis and Fourier series, in connection with functional analysis and number theory.
- Probabilistic methods in analysis: fine structure of Brownian motion and construction of fractal sets.
- Harmonic analysis methods in analytic number theory.
- Mandelbrot martingales and multiplicative chaos.

Rather than try and describe all of his contributions in this limited space, it is preferable to put the stress on his works that have had the greatest influence. Most of his works are easily accessible via the book "Selected works"(Kendrick press, 2009).

The Kahane–Katznelson–de Leeuw theorem

If the sequence $(c_n)_{n\in\mathbb{Z}}$ is square summable, there exists a continuous 2π-periodic function such that, for all $n \in \mathbb{Z}$, $|\widehat{f}(n)| \geq |c_n|$. This brings a definitive conclusion to lots of previous works. The proof mixes probability and combinatorics, a cocktail that he was fond of. Probabilities (the Khintchine inequalities) give $|\widehat{f}(n)| \geq |c_n|$ and $f \in \bigcap_{p<\infty} L^p$. The difficulty is to get $f \in L^\infty$. This is achieved via the use of combinatorics.

This result teased several followers: Kisliakov showed that this result still holds for the disc and bidisc algebras. The problem seems to still be open in higher dimension; later on, in 1997, Françoise Piquard proved a similar result for infinite matrices.

The Khintchine–Kahane inequalities

This is a highly non-straightforward extension of Khintchine inequalities to vector random variables. The idea is expressed "à la Kahane" in his book "Some random series of functions":

> If the probability that a sum of the type Rademacher be large is small, then the probability it be very large is very small.

More precisely, if M is the supremum of the norms of partial sums of a Rademacher series $\sum r_n x_n$ (where the x_n lie in an arbitrary Banach space) then, for $t > 0$ large enough, one has $\mathbb{P}(M > 2t) \leq \mathbb{P}(M > t)^2$. It results that the sum S of this series belongs to the Orlicz space L^{ψ_1}, where $\psi_1(x) = e^x - 1$. This seems not to be as good as in the scalar case, where the correct Orlicz function is $\psi_2(x) = e^{x^2} - 1$. But, as Kwapien showed, this result is self-improving so as to give the optimal Orlicz function ψ_2.

Slow points of Brownian motion $\mathbf{B(t)}$

This is a long story. One dimensional Brownian motion was known to have, with probability 1, continuous trajectories, almost uniformly Hölder $1/2$ (not $1/2$, of course, because of the iterated logarithm law). It was also known (Paley–Wiener–Zygmund) that it is a.s. everywhere non-differentiable and even everywhere non-Hölder α if $\alpha > 1/2$. Dvoretzky, asked by Kahane about the case $\alpha = 1/2$, proved in 1963 the following result. Set

$$L(t) := \limsup_{h\to 0} \frac{|B(t+h) - B(t)|}{|h|^{1/2}}.$$

Then, a.s. for all t, $L(t) > 0$. In 1974, Orey and Taylor proved that $L(t) = \infty$, and a little more, can happen (such points are so-called "fast points"). Shortly afterwards, by using an astonishingly inventive method, Kahane proved that slow points exist, i.e. points at which the Hölder exponent is $1/2$. The proof is detailed in the second issue of his book "Some random series of functions".

Solution of the Bateman–Diamond conjecture

J.-P. Kahane has always been fascinated by the issue, in the job of a mathematician, of the direction in which one should look for an answer? *No*, and you look for a counterexample. *Yes*, and you look for a general proof … In this respect, he often quoted Carleson's theorem (the solution of the Lusin conjecture) on almost everywhere convergence of Fourier series of square-summable functions on the circle. According to him, Carleson rather believed the answer to be negative. But each attempt in this sense had run into a stone wall. Here was Kahane's comment in 2011:

> Finally, the walls he ran into nearly built a POSITIVE answer to the Lusin conjecture!

Quite in this spirit, he obtained, in 1997, when over 70, a *positive answer* to a conjecture of Bateman and Diamond about generalised prime numbers of Beurling. Here are some details. You start from a discrete and multiplicatively independent subset P of the half-line $(1, +\infty)$ (the generalised prime numbers) and then consider the multiplicative semi-group N generated by P (the generalised integers), as well as their respective counting functions

$$P(x) = \operatorname{card} \mathsf{P} \cap [1, x] \quad \text{and} \quad N(x) = \operatorname{card} \mathsf{N} \cap [1, x].$$

Beurling showed that by setting $N(x) = Dx + x\varepsilon(x)$, with D a positive constant, the assumption $\varepsilon(x) = O((\log x)^{-\alpha})$ for some $\alpha > 3/2$ is enough to imply the Prime Numbers Theorem (PNT):

$$P(x) \sim \frac{x}{\log x}.$$

In this context, the less demanding condition

$$\int_1^\infty (\varepsilon(x) \log x)^2 \, \frac{dx}{x} < \infty \qquad (1)$$

naturally emerges. Bateman and Diamond asked whether Condition (1) implies the PNT. After failing to build a counterexample and having analysed the reasons of that failure, Kahane was led, in Carleson's style, to a positive answer to the conjecture. The (quite elegant) method uses Fourier analysis and, notably, the non-vanishing of ζ, the zeta function naturally associated with N, on the line $\Re s = 1$; this is by interpreting the hypothesis as the membership of $t \mapsto \zeta(1+it)$ in the Sobolev space H^1 and making use of the local properties of functions in that space, notably the property

$$f \in H^1 \Rightarrow |f(t_0+h) - f(t_0)| = o(|h|^{1/2}).$$

(Surprisingly enough, this is somewhat reminiscent of slow points of Brownian motion.) Jean-Pierre Kahane was particularly proud of this achievement.

The Mandelbrot martingales

Let $c \geq 2$ be an integer and W a non-negative random variable of expectation 1. We consider a family $W_{j_1,j_2,\cdots,j_n}$, indexed by the finite sequences of numbers between 1 and c, of independent random variables equidistributed with W. Each sequence $j_1, j_2, \cdots, j_n$ determines a c-adic subinterval $I_{j_1,j_2,\cdots,j_n}$ of $[0,1]$. Let μ_n be the measure whose density with respect to the Lebesgue measure on each interval $I_{j_1,j_2,\cdots,j_n}$ is the constant $W_{j_1}W_{j_1,j_2}\cdots W_{j_1,j_2,\cdots,j_n}$. Its total mass is the random variable

$$Y_n = c^{-n} \sum_{1\leq j_1 j_2 \cdots j_n \leq c} W_{j_1}W_{j_1,j_2}\cdots W_{j_1,j_2,\cdots,j_n}. \tag{2}$$

This is a non-negative martingale so it converges a.s. to a random variable Y. Of course, the expectation of Y does not exceed 1. It may happen that $Y = 0$ with probability 1. When the expectation of Y equals 1, the sequence of measures μ_n has, with probability 1, a non-trivial weak limit μ. This construction was devised by B. Mandelbrot to statistically describe turbulent fluids – in this case, μ is interpreted as energy dissipation – or, more generally, to account for intermittency. So, it was crucial to determine whether the expectation of Y differs from 1 and Mandelbrot constructed a few conjectures. These have been answered positively by Kahane. Here are the results.

- For $p > 1$, the martingale (2) converges in L^p if and only if $\mathbb{E}\,W^p < c^{p-1}$ (1974).
- The martingale converges in L^1 if and only if $\mathbb{E}\,W \log W < \log c$ (1976).
- When this martingale does not converge in L^1, it completely degenerates, i.e. $\mathbb{E}\,Y = 0$.

The 1974 result was already difficult but the one in 1976 is a true feat of strength.

These works had longlasting influence. Other proofs have been devised and lots of generalisations have been studied.

Multiplicative chaos

In 1974, B. Mandelbrot introduced the previous martingales as a toy model for energy dissipation in fully developed turbulence. The more realistic model he had considered consisted of a basis-free lognormal multiplicative process, which was rigorously founded by Kahane in "Sur le chaos multiplicatif" in 1985, where Gaussian multiplicative chaos theory is elaborated. Then, Kahane unified both models in the more abstract T-martingale setting, of which there is a short presentation here.

One considers $(\Omega, \mathcal{A}, \mathbb{P})$ a probability space, (T,d) a locally compact metric space and

$$Q = (Q_n : T \times \Omega \to \mathbb{R}_+)_{n\geq 1}$$

a sequence of measurable maps such that for all $t \in T$, $(Q_n(t,\cdot))_{n\geq 1}$ is a martingale of expectation 1.

Given σ any Radon measure on T, the sequence $(Q_n\sigma)_{n\geq 1}$ vaguely converges almost surely to a Radon measure $Q\sigma$ and, as distributions $\mathbb{E}(Q_n\sigma)$ and σ satisfy

$$\mathbb{E}(Q\sigma) \leq \sigma \quad \text{(by Fatou's Lemma)},$$

if $Q\sigma \neq 0$ with positive probability, one says that $Q\sigma$ is non-degenerate. Basic questions are then:

1. When is $Q\sigma$ non-degenerate and when does the equality $\mathbb{E}(Q\sigma) = \sigma$ hold? In the case of degeneracy, how can $Q\sigma_n$ be renormalised to get a non-trivial limit, at least in law?
2. Suppose $Q\sigma$ is non-degenerate.
   - Which moments of the total mass of $Q\sigma$ restricted to compact sets are finite?
   - What is the Hausdorff dimension of $Q\sigma$?
   - What is the multifractal nature of $Q\sigma$, i.e. what are the Hausdorff dimensions of the sets

$$E(\alpha) = \left\{ t \in T : \liminf_{r\to 0^+} \frac{\log\left(Q\sigma(B(t,r))\right)}{\log(r)} = \alpha \right\}?$$

Gaussian mutiplicative chaos as defined by Kahane corresponds to

$$Q_n(t) = \prod_{k=1}^{n} P_k(t), \quad \text{with } P_n(t) = \exp\Big(X_n(t) - \frac{1}{2}\mathbb{E}X_n^2(t)\Big),$$

where $X_n(t)$ are independent Gaussian centered random functions on T. The distribution of P_n depends only on the correlation function

$$p_n(t,s) = \mathbb{E}(X_n(t)X_n(s))$$

and the distribution of $Q\sigma$ only depends on

$$q(t,s) = \sum_{n=1}^{\infty} p_n(t,s)$$

and not on the order of summation if the p_n are non-negative. This non-trivial achievement is based on fundamental inequalities that Kahane obtained in this context. Let $(X_i)_{1\leq i\leq n}$ and $(Y_i)_{1\leq i\leq n}$ be two centered Gaussian vectors such that

$$\forall\, i, j, \quad \mathbb{E}(X_iX_j) \leq \mathbb{E}(Y_iY_j).$$

Then, for all non-negative weights $(p_i)_{1\leq i\leq n}$ and all convex (resp. concave) functions $F : \mathbb{R}_+ \to \mathbb{R}$, with almost polynomial growth at infinity,

$$\mathbb{E}\left(F\Big(\sum_{i=1}^{n} p_i e^{X_i - \frac{1}{2}\mathbb{E}(X_i^2)}\Big)\right) \leq \text{(resp. } \geq)\mathbb{E}\left(F\Big(\sum_{i=1}^{n} p_i e^{Y_i - \frac{1}{2}\mathbb{E}(Y_i^2)}\Big)\right).$$

A case of particular interest is when the Gaussian field $\sum_{n=1}^{\infty} X_n(t)$ (seen as a distribution) is log-correlated, i.e.

$$q(t,s) = u \log^+ \frac{1}{d(t,s)} + O(1) \quad (u > 0). \tag{3}$$

Then, if

$$\sup_B N\Big(\frac{1}{2}\mathrm{diam}(B), B\Big) < \infty,$$

where $N(\delta, B)$ stands for the covering number of the closed ball B by closed balls of radii δ, Kahane shows that:

(1) If $\dim T < u/2$ then for all σ one has $Q\sigma = 0$.
(2) If $\dim T > u/2$ then the infimum of the dimensions of random Borel sets on which the non-vanishing measures $Q\sigma$ are concentrated is $\dim T - u/2$.

Moreover, when (T, d) is the Euclidean space $\mathbb{R}^d$ and σ = Lebesgue then $\dim T < u/2$ can be replaced by $\dim T \leq u/2$ in (1). These results are partly based on Peyrière's approach to the dimensions of limits of Mandelbrot martingales and Kahane's observation of a deep principle of composition for multiplicative chaos.

Questions of non-degeneracy, moments, dimension and multifractal analysis were then solved for various classes of models of T-martingales (works by Kahane, Holley and Waymire, Molchan, Falconer, Fan, Barral and Mandelbrot, Bacry and Muzy, Rhodes, Sohier and Vargas, and Barral and Jin), including log-correlated Gaussian multiplicative chaos.

The answer to the renormalisation question came recently for Mandelbrot martingales (in works by Aidekon and Shi, Webb, Madaule, and Barral, Rhodes and Vargas) and for some log-correlated Gaussian multiplicative chaos on $\mathbb{R}^d$ (in works by Duplantier, Rhodes, Sheffield and Vargas, and Madaule, Rhodes and Vargas). Both situations led to similarly beautiful results. In the latter case, if we take into account the dependence of Q_n in the parameter u, if $X_n \sim \mathcal{N}(0, 1)$ and σ is the Lebesgue measure then, in the critical case $u = 2d$, the normalised sequence $\sqrt{n}(Q_{u,n}\sigma)$ weakly converges in probability to a positive continuous random measure μ of dimension 0, while if $u > 2d$ then $n^{\frac{3\sqrt{u}}{2\sqrt{2d}}} e^{(\frac{\sqrt{u}}{\sqrt{2}} - \sqrt{d})^2 n}(Q_{u,n}\sigma)$ weakly converges in law to a purely atomic stable random measure, whose atoms are placed according to a Poisson point process with intensity given by μ. Note that complex T-martingales and their renormalisation have also been investigated.

Gaussian multiplicative chaos theory was further developed over the last decade with two related motivations: constructing a theory associating random measures to nonnegative kernels $q(t, s)$ of positive type but not necessarily of σ-positive type (works by Robert and Vargas, Duplantier and Sheffield, Rhodes and Vargas, Shamov and Berestycki); and developing a rigorous probabilistic theory of quantum field theory (works by Duplantier, Miller and Sheffield, David, Kupianen, Rhodes and Vargas, and Aru, Hung and Sun). It turns out that, in certain cases, not only can one characterise the finiteness of moments but one can determine the law of the total mass! (See recent works by Kupianen, Rhodes and Vargas, and Rémy.)

Random coverings, multiplicative chaos and subordinators

Multiplicative chaos is related to some random covering problems, which were among the favourites of Kahane. Along with his students Billard and El Hélou, he was among the main protagonists (with Erdös, Hawkes, Mandelbrot and Shepp) in the study and solution of a random covering problem on the circle raised by Dvoretzky in 1956, and of its extension to tori. Kahane also worked on the closely related and more tractable Poisson covering problem in Euclidean spaces introduced by Mandelbrot (1972). Let us discuss the latter model in dimension 1.

Denote the Lebesgue measure on $\mathbb{R}$ by λ and fix a positive Borel measure μ on $(0, \infty)$. Consider a Poisson point process of intensity $\lambda \otimes \mu$, which is a random discrete subset of $\mathbb{R} \times (0, \infty)$ such that the number of its point in a given Borel set B is a Poisson r.v. of parameter $\lambda \otimes \mu(B)$ and the number of points associated with disjoint Borel sets are independent. Denote the points of this process by (x_i, y_i) and associate to them the open intervals $(x_i, x_i + y_i)$. Let $G = \bigcup_i (x_i, x_i + y_i)$. The problem is to determine whether $G = \mathbb{R}$ a.s. or not. The answer was given by Shepp (1972), simultaneously with the answer to Dvoretzky's problem:

$$G = \mathbb{R} \text{ a.s. or } G \neq \mathbb{R} \quad \text{a.s. according to } \int_0^\infty e^{-t} k(t)\, dt = \infty \text{ or not,}$$

where

$$k(t) = \exp \int_t^\infty \mu(y, \infty) dy.$$

Kahane revisits this result in the late 1980s. He gives a new proof based on a stopping time idea due to Janson and also uses a potential theory approach to get the following finer result. If K is a compact subset of $\mathbb{R}$ then $K \subset G$ a.s. or $K \not\subset \bigcap_{n\geq 1} \bigcup_{i:\, y_i < 1/n} (x_i, x_i + y_i)$ a.s. according to whether $\mathrm{Cap}_k(K) = 0$ or not. Multiplicative chaos comes into play when $\mathrm{Cap}_k(K) > 0$, i.e. there exists a Borel probability measure σ on K such that

$$\int_{K^2} k(t - s)\, d\sigma(t) d\sigma(s) < \infty. \tag{4}$$

Indeed, for $n \geq 1$, setting $G_n = \bigcup_{i:\, y_i \geq 1/n} (x_i, x_i + y_i)$ and

$$Q_n(t) = \frac{1 - \mathbf{1}_{G_n}(t)}{1 - \mathbb{P}(t \in G_n)},$$

(4) is equivalent to L^2-convergence of the K-martingale $(Q_n\sigma)(K)$.

Now, consider $G_+ = \bigcup_{i:\, x_i > 0} (x_i, x_i + y_i)$. In the same period, Kahane shows that when

$$\int_0^1 \exp\Big(\int_x^1 \mu(y, \infty) dy \Big) dx < \infty,$$

there exists some Radon measure σ supported on $\mathbb{R}_+$ such that $Q\sigma$ is supported on $\mathbb{R}_+ \setminus G_+$, the set of uncovered points, and $Q\sigma$ is the image of the Lebesgue measure by some Lévy subordinator. This implies a result by Fitzsimmons, Firstedt and Shepp (partially proved by Mandelbrot), claiming that, in this case, $\mathbb{R}_+ \setminus G_+$ is the closure of the range of this subordinator but provides a new interpretation in terms of multiplicative chaos. In particular, an interesting consequence of this approach is that local times of Brownian motion are the indefinite integrals of limits of $\mathbb{R}$-martingales. It should be noted that the topic of random coverings is currently particularly active.

J.-P. Kahane's influence turned out to be amazing. Firstly, he was always ready to discuss and talk about mathematics. We remember J. P. Serre asking him: "We were wondering if we could get from you a talk on Brownian motion next week?" He instantly replied: "I agree." And he delivered several talks at the Bourbaki seminar, notably on Beurling's works, to whom he first asked for agreement. Facing a lack of reply from the latter, he sent a message: "In three days, I present your work at the Bourbaki seminar," no longer asking for agreement. He then received agreement with an abrupt

"OK". Much more recently (in 2014), J. P. Kahane delivered a talk on Brownian motion, white noise and Langevin's equation at the "BNF" (Bibliothèque Nationale de France), in front of a large audience comprised of many teenagers. His aptitude for "doing mathematics" and writing equations, while remaining sufficiently general to avoid the young listeners getting lost, was impressive.

J.-P. Kahane wrote four advanced books:

- "Ensembles parfaits et Séries Trigonométriques" with R. Salem (1963); second edition in 1994. This book has not aged. In particular, it provides a clear account on capacities and Hausdorff dimension.
- "Séries de Fourier absolument convergentes" (1970). This book contains magnificent applications of Banach algebras (Shilov idempotents theorem) to the zero sets of analytic functions in the Wiener class.
- "Some random series of functions" (1968, reissued in 1985). This book is still an obligatory reference.
- "Séries de Fourier et Ondelettes" with P. G. Lemarié (1998, reissued in 2016).

This last work shows the taste and ability of J.-P. Kahane for history, focusing on the development of ideas rather than on persons. Before the wavelet part, written by P.-G. Lemarié, he writes about the contributions of the great precursors: Euler, Fourier, Dirichlet, Weierstrass, Cantor, Riemann and Lebesgue. His analysis of their ideas and achievements is very exciting.

Jean-Pierre Kahane will remain an example thanks to his enthusiasm, his quest for invention and his preoccupation to explain and transmit his knowledge and experience. His influence will continue for a long time.

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ERME Column

Renaud Chorlay (Sorbonne Université, Paris, France) and Jason Cooper (Weizmann Institute of Science, Rehovot, Israel)

ERME Thematic Working Groups

The European Society for Research in Mathematics Education (ERME) holds a biennial conference (CERME), where research is presented and discussed in Thematic Working Groups (TWGs). We continue here the initiative (which began in the September 2017 issue) of introducing the working groups, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for research mathematicians. Our aim is to extend the ERME community with new participants, who may benefit from hearing about research methods and findings and who may contribute to future CERMEs.

Introducing CERME's Thematic Working Group 12 – History in Mathematics Education

Group leader: Renaud Chorlay

Even though the inception of this TWG is recent (CERME6, 2009), it has deep institutional roots within the mathematics education research community. Indeed, the *History and Pedagogy of Mathematics* study group (HPM [1]) was founded at the 1972 ICME conference; it has been organising satellite conferences of the ICME meetings since 1984 and has several active regional branches (e.g. HPM-Americas and European Summer Universities). At CERME10 (2017), 16 papers and two posters were presented in TWG12, covering a large range of European countries (from Ireland to Russia) and beyond (Brazil, Mexico and the U.S.). A survey has shown that this TWG has attracted newcomers to the CERME community from the HPM community, since nine participants were CERME first-timers yet only two had never attended an HPM-related event. The strength of the historical and HPM communities varies greatly among countries and these meetings play a crucial role for researchers working in relative isolation.

The work carried out in TWG12 lies at the intersection of two different fields of academic research: mathematics education and history of mathematics, a specific situation which calls for versatility and methodological vigilance [2]. The historical approach enables researchers to consider mathematics not only as a collection of facts and methods but as a multifaceted human endeavour. At the interface between the two fields, TWG12 meetings allow the dissemination of recent results and renewed perspectives from historical research, including: empirical and theoretical investigations into the variety of science-makers, the nature of the work collectives and the epistemological cultures shaping the engagement with mathematics; studies on the forms and the meaning of algorithmic and diagrammatic thinking; studies on the role of examples, numerical tables and problems, either in themselves or as organised collections. The underlying rationale is not that of a parallelism between historical "development" and cognitive development of the learner but that – on a par with mathematical knowledge – historical knowledge provides relevant tools and insights for all facets of didactical research. This was reflected in the papers presented at CERME10, with an emphasis on argumentation in numerical and algebraic contexts. It should be noted that there was little intersection with what was covered in TWG8 (Affects and mathematical thinking) and TWG10 (Diversity and mathematics education), in spite of the fact that it is not uncommon for outsiders to the HPM research community – including policymakers and curriculum-designers – to ascribe such pedagogical goals to the historical perspective in teaching.

For a few years now, research perspectives on the design of tasks using historical documents – either in the classroom or in teacher-training – have evolved significantly. Even though there is still room for new insights into the "why" question and for accounts of small-scale design-processes, the emphasis has shifted to place greater attention on the "how" question [3], on the theoretical analysis of the nature of the expected educational effects and on the importance of empirical studies into actual effects. As far as higher education is concerned, the latest developments of the TRIUMPHS [4] project were discussed at CERME10. This five-year project, funded by the National Science Foundation in the U.S., will create 25 full-length Primary Source Projects (PSPs) and 30 one-day "mini-PSPs", allowing students to study "from the masters" (such as Euler, Cauchy and Cantor). The project includes an extensive "research with evaluation" study, which will enable both formative and summative evaluation of the project activities. By the end of the project, it is expected that some 50 instructors and over 1000 students will have participated.

Two other promising research topics were discussed at the conference. One covers the "how" question, prompting us to carry out empirical investigations into the actual practices of "ordinary" teachers attempting to integrate historical elements in the classroom. The other highlights the potential of studies combining history of mathematics education and history of didactical theories for the networking of theories in mathematics education.

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Our Missing Teachers

John Ewing (Math for America, New York, USA)

It was another amiable conference, scheduled over two days in a hotel that adjoined the campus of a large university. The topic was mathematics education, from elementary through to high school. The roughly 30 speakers were almost all people I knew well, some for many years. They were distinguished mathematics or education faculty from universities across the United States, with a few from Europe and Asia. Their talks and panels covered many topics from curriculum to pedagogy, from elementary school to high school and from policy to practice. I had been looking forward to the event.

As I settled into my seat near the back of the room, I looked at the list of participants. One aspect of the conference was strikingly familiar – the speakers included not a single practising classroom teacher. Only one was on a panel and that was because he was connected to a university teacher training programme.

Something similar happens at most conferences and workshops on K-12 mathematics education held in the United States (and many, although not all, other countries). The speakers are university faculty, education reformers, superintendents of districts, CEOs of corporations, even politicians – almost anyone other than practising teachers, the people who carry out the day-to-day work of K-12 education. When a handful of teachers *are* included, it's because they have some other role.

I have spent most of my life in or associated to universities. For most of that time, I never saw anything unusual about the missing classroom teachers. I worried regularly that research mathematicians were often missing from education events and projects. Reforming K-12 education should be a partnership between mathematics education and mathematics faculty, I insisted. Education policy disconnected from content loses its way, giving rise to "school mathematics" rather than "mathematics." But I never thought that failing to involve classroom teachers might be a problem.

The problem

The missing classroom teachers *are* a problem, of course, and for the same reason that missing mathematicians are. Talking about education, either policy or practice, without actual practitioners is just as nutty as talking about education without experts in content. It's easy to lose perspective, to misunderstand the consequences of actions and to misjudge the difficulty of success. Why leave out people who can provide such perspective?

But the absence of classroom teachers is a symptom of a more serious problem: we omit teachers because we do not think of teachers as professionals – masters of a discipline with special expertise and craft. We don't think of them in the same way we think of, say, medical doctors, engineers, or university mathematicians. Indeed, we don't think of teaching as a profession in the way we think about other professions. The public often sees classroom teachers as workers who follow instructions provided by someone else – the real experts. Teachers are like education contractors, not education architects. And because we don't *think* of them as professionals, we don't *treat* them that way.

At this point, some readers will be shaking their heads. "Not all classroom teachers *are* professionals," they protest. Of course, they are right, some are not. There are a great variety of classroom teachers. In many countries, including the United States, most teachers in lower grades are generalists, with little specialised training in mathematics. While they may be professional teachers, they are hardly professional *mathematics* teachers (or professional X teachers for any value of X). How can they contribute to policy discussions about mathematics education? Teachers in the upper grades vary as well. Some have only modest backgrounds in mathematics and often teach routine courses in routine ways. Some have lost their edge over the years and become dull. Some may be knowledgeable in their subject but awful in their craft, unable to unpack ideas that are familiar to them. Some may be truly dreadful. Yes, yes, yes.

But *some* mathematics teachers *are* professionals. In the lower grades, they know their subject in surprisingly deep ways and are, in every respect, teachers of mathematics, inspiring young students. In upper grades, they are not only teachers of mathematics but mathematicians as well, with both insight and devotion to their subject. They are experts in their craft. They inspire their students. They guide and mentor their colleagues. They continue to learn, both mathematics and pedagogy, throughout their careers. These teachers have all these characteristics and they are professionals in every sense of the word. They deserve to be treated accordingly. When we treat them otherwise, we send a message that their professionalism isn't valued.

If we don't value our most accomplished teachers, they will not stay in teaching. If we don't treat them as professionals, the profession itself becomes unattractive and we will lose future accomplished teachers as well. If we want *more* high-quality teachers, we had better value the ones we have.

A secondary problem

This situation illustrates a second problem in education. When we look at teachers, we almost always focus on the weakest – the ones who are deficient in some way, who need repair, who represent some failure. It is hard to think of teachers as professionals who can contribute to education more broadly because we are focused on that awful algebra teacher who barely knew mathematics and ruined mathematics for our son or daughter. We may remember a few great teachers in our own lives but when we talk about education, we think about the teachers who are deficient and need repair.

This is an unconscious theme in modern K-12 education. Reform has become a simple formula: find what's broken and fix it. Find the weakest teachers, the poorest schools, the most troubled students. Expose them, repair them, get rid of them and education will get better. This is educational reform today.

But it's a remarkably short-sighted view, strangely peculiar to K-12 education. In the business world, one doesn't only focus on what doesn't work well. To build a thriving business, you fix things to be sure, but you also find things that already work well. You build great businesses on those things, using them as the cornerstones. Similarly, a university president would never arrive on campus and immediately focus on faculty who are weakest. Great universities get built using the most prestigious and accomplished. Why do we try to improve education by focusing only on the rubble of failure? In life, excellence is built on excellence. Education is no different.

Math for America

Math for America (M*f*A) sets out to address these problems. The idea is embarrassingly simple – find genuinely accomplished teachers, give them opportunities modelled on professional life in universities, and trust them to take advantage of the opportunities. In short, find teachers who deserve to be treated as professionals and treat them that way.

("Math for America" is a misnomer in two ways. The programme includes science teachers as well as mathematics, in roughly equal numbers. Also, it is only for teachers in New York City rather than all America.)

M*f*A offers a 4-year master teacher fellowship for experienced mathematics and science teachers who are currently in the classroom (and who continue to teach). Teachers apply, are selected, and join a community of maths and science teachers spanning all the grades, although predominantly in grades 6-12. The community tries to model scholarly life in a vibrant university – workshops, seminars and lectures, covering maths or science, policy or pedagogy and abstract or practical topics. It also offers the opportunity to exchange ideas and information with colleagues from a wide range of schools. Fellows participate actively and enthusiastically in ways that suit their individual professional needs. They also receive an annual stipend (currently US $15,000), which is meant to recognise their achievement and to compensate for their substantial commitment. Teachers can also apply to renew their fellowship.

New York City's school system is large: 1.2 million students in 1800 schools taught by about 75,000 teachers. MfA now has over 1000 teachers – about 10% of the maths and science teachers in the city. Is that the right percentage? Perhaps. It seems about right in New York City. The larger the number, the greater the effect, of course. On the other hand, it is essential to have accomplished teachers to make this programme work. In New York, 10% seems to strike the right balance.

While the basic idea is simple, the details are more complicated. They are important! It would be easy to allow a programme like this to drift into something resembling traditional education reform, for example, by selecting teachers who were not accomplished or by converting the workshops into "teacher repair". The lure of traditional reform is powerful and one must safeguard against it. Here are some details:

Selection

The selection process is important because M*f*A's success depends upon finding accomplished teachers. Selection begins with a lengthy application consisting of information about a teacher's education (including transcripts) and career. Applicants submit a short essay and a lesson study. They have three people write detailed letters of recommendation. They take a standard (undemanding) test of content that is specific to their specific discipline. One element is notably and deliberately missing from the selection process – test scores from a teacher's students. The application is structured in a way that helps teachers decide whether the fellowship is right for them and encourages them to stop if it's not. We don't encourage applications simply so we can brag about our low acceptance rates.

Applicants who make it through the first part of the process are invited for an interview. At that interview, they are first assigned to work in small groups, creating and presenting a piece of mathematics or science they

Typical evening at M*f*A.

An M*f*A workshop on puzzles.

can choose themselves (because it's interesting and not because they teach it). Afterwards, they converse about the presentations, asking each other questions, answering them and offering observations. A three-person team (a mathematician, an educator and an M*f*A representative) observe all this. The team then interviews each candidate separately, following up on what's been observed. The process allows us to gauge with remarkable accuracy a teacher's mathematical or scientific knowledge, their ability to communicate that knowledge and their approach to collaborative work. At the end, we know each candidate's strengths and weaknesses.

Final acceptance is carried out by M*f*A staff, using the complete dossier for each applicant and a set of carefully developed rubrics, along with sound judgment.

Old fashioned string art.

Scholarship

During most evenings, the M*f*A teachers swarm throughout our New York City offices, which include a lounge, a small library and a number of seminar rooms and breakout areas. This is meant to approximate the facilities of a high-quality maths or science department.

Some might describe what M*f*A teachers do during these evenings as "professional development". I avoid that term. Traditional professional development is often dull and dreary, disconnected from a teacher's discipline, and aimed at fixing deficiencies or providing tips on how to improve test scores. M*f*A's workshops, seminars and mini-courses are meant to be intellectually engaging. Some are directly connected to instruction, but many are about maths or science – a recent research result or an interesting topic – or about education challenges and policies. A few are single sessions but most are given in sequences of three or more sessions. In the 2017-18 academic year, M*f*A offered over 400 "courses" like this, taking place in nearly 1000 sessions.

Here is the most important part: two-thirds of these courses are created and led by the teachers themselves. M*f*A provides the infrastructure. We put together the catalogue, determine the schedules and provide the facilities, but the teachers themselves drive most of this forward – in the same way that faculty and students drive seminars and colloquia in a healthy university.

Of course, some workshops are run by people from the outside, including mathematicians, scientists and educators. But even these workshops are inspired by teacher interest and aimed at intriguing, rather than fixing, teachers.

For many teachers, especially those in small schools with few colleagues in their field, M*f*A's scholarly community serves as their essential connection to their discipline. It makes them feel like mathematicians and scientists. It makes them feel professional.

Interaction

Teaching is a lonely job. People are sometimes surprised when I say this. They picture teachers surrounded by dozens of students and they wonder how anyone can be lonely with all those students. But teachers interact professionally with teachers and the modern structure of schools makes these professional interactions more and more difficult.

One of the benefits of the M*f*A community is the opportunity to interact with other teachers in new ways: mathematics teachers talk to science teachers; middle school and elementary teachers talk to high school teachers; and teachers at exclusive schools (in NYC they are called "exam schools") talk to those enrolling mainly high-needs students. They all come together. This kind of professional interaction builds connections, which creates networks that teachers can draw on for years to come. It is a kind of interaction that seldom happens in the everyday lives of teachers. At M*f*A, it happens naturally.

We also foster such interaction. Part of our programme (about 15%) consists of teachers who are in the first few years of their teaching careers. We pair them with master teachers who serve as mentors, not merely in name but in fact. Both junior and senior teachers profit from these relationships. Many of our workshops are offered in a special format – Professional Learning Teams (PLTs) – that bring together 15–20 teachers over the course of a semester or year. PLTs are always co-led by a pair of teachers and adhere to a special format designed to involve all participants in the ongoing discussions. We even run workshops on how to run workshops, inside and outside of M*f*A. We encourage M*f*A teachers to start their own PLTs in their schools.

University mathematicians are often unaware that such routine interaction with their colleagues plays an essential role in their professional lives. It is an unremarkable part of life in a good university. For classroom teachers, these interactions are a new experience that change the way that teaching feels.

Trust

None of this works without one final ingredient: trust. M*f*A fellows are required to participate in a modest number of workshops, roughly one session per month. (Most participate in far more!) We don't require them to learn any particular content. We don't ask them to acquire any particular skills. We are not fixing any particular deficiency. We trust them to decide for themselves how they want to grow professionally.

Sample Mathematics Workshops
Spring 2018

Applying Ideas from Modern Algebra to Secondary Teaching (*MINI*)
Combinatorial Game Theory (*SSW*)
Delving Deeper into Fraction Subconstructs and Processes (*PLT*)
Dynamic Lesson Planning Using Geometer's Sketchpad (*SSW*)
Exploring Rational Tangles (*SSW*)
Fostering a Growth Mindset in Mathematics (*PLT*)
How to explain hard "Why" questions in Algebra and Geometry using Calculus (*MINI*)
Introduction to Category Theory (*MINI*)
Investigating Calculus Teaching and Learning (*PLT*)
Islamic Art and Geometry (*MINI*)
The Mathematics of Gerrymandering (*MINI*)
Multi-criteria Decision Analysis for High School (*SSW*)
Made You Look – Statistics through Data Visualizations (*TT*)
Using Mathematical "Magic" to Engage Students in Mathematics (*MINI*)
Vertical Alignment in High School Math (*IG*)

Similarly, we invite teachers to submit proposals for workshops and courses based on their own ideas and not ours. We vet these proposals, of course, but we trust the teachers to come up with good ideas. And they do!

Trust is a crucial ingredient in changing the way we think about teachers. It is often confused with education laissez faire – the proposition that teachers should do whatever strikes their fancy in the classroom. But laissez faire is impractical in most settings (including, it should be added, in universities!). Professional trust is different and more subtle. It means trusting teachers to control their own professional lives, deciding what's most important to them and how they want to develop their own careers. Extending that trust is important.

For many of our teachers, this is their first experience with this kind of trust. Teachers' careers often progress from one mandated professional development experience to the next, many of them useless and some of them dreadful. They hear public figures proclaim how much they "respect" teachers, without extending even a modicum of trust in this sense. They are controlled, regimented and evaluated by a system designed with the least able teachers in mind. M*f*A tries to change that, at least in one part of their professional lives. It is an important change.

One aspect of M*f*A is not as essential as the previous four but still deserves a mention. Master teachers can renew their fellowship after four years.

Renewals are not automatic. They require both an application and an interview (each different in nature from the initial ones). We expect more from master teachers when they renew. They are meant to be leaders, contributing more to M*f*A, their colleagues and (possibly) their own schools. We make this a requirement but not an overbearing one. The renewal itself depends on our assessment of a teacher's ability to take on this role. Not all master teachers choose to renew and not all who apply are accepted.

If we did not permit renewals, we could offer fellowships to more teachers. But we would lose part of what makes M*f*A thrive. The senior master teachers who stay on for two or more fellowships play a special role in our community. They mentor those who are new to M*f*A: They take the lead in proposing workshops and running them. They model what an active professional life looks like; and they often carry their M*f*A experience outside to their schools and to the rest of public education.

While renewals may not be essential, they make M*f*A more effective.

Advocating

M*f*A has evolved over time. The original programme was conceived by Jim Simons and a group of financial mathematicians in 2004. The original implementation created a fellowship that enticed mathematically talented individuals to become teachers – a year of training, four years of commitment, substantial stipends, and a community. The programme was supported by an annual poker benefit together with the Simons Foundation.

A 3D printer in the M*f*A lounge.

A Professional Learning Team (PLT).

MfA teachers in a breakout.

Gradually, MfA discovered that many highly accomplished mathematics teachers were already in classrooms. Many of them were leaving and keeping them seemed more efficient than creating brand new teachers. The master teacher programme began to grow in 2012. A similar programme was created for teachers early in their careers. Science was added to broaden the appeal. The scope was extended to include maths and science elementary teachers. Now, after 14 years of evolution, M*f*A has slightly more than 1000 teachers in its fellowships – about 10% of the maths and science teachers in New York City. About half are in maths and half are in science. Most are master teachers and some are early career. Watching them work together each evening is inspirational.

M*f*A in New York City could serve as a model for other programmes elsewhere. We have tried to persuade others to create similar programmes in other locations with limited success. A few arose in other cities in the U.S., with the largest in Los Angeles (nearly 100 teachers). A large, publicly-funded programme began several years ago in the rest of New York State (which has roughly the same population as New York City). It currently has over 900 teachers at nine sites around the state. We are working with other states to help them create similar publicly-funded programmes, not identical but similar to M*f*A.

Advocating is a tough job, however. People sympathise with the goal but the approach can be jarring. That we focus on excellent teachers seems counter-intuitive to many people, especially education reformers. Why waste resources on teachers who are already accomplished? Why not concentrate on teachers who need help? And many people find it hard to trust teachers in this way. They can only remember that dreadful teacher they (or their child) recently encountered at school. Surely *that* teacher doesn't deserve a stipend or our trust!

If we really want to improve the quality of mathematics and science teachers, however, we need to find a way to overcome these obstacles.

Coda

Does MfA work? It's too early to tell for sure. Unlike traditional education reform, M*f*A is not about fixing teachers. There is no "treatment" that can be withheld from a control group to see whether the dosage is correct. There is no simple statistic that measures what we want to achieve. Some things are hard to measure with numbers. Professionalism is one of them.

In two respects, though, M*f*A is already successful:

- In the U.S. today, experienced teachers leave teaching for other careers at an alarming rate (about 8% each year). M*f*A teachers leave at a far lower rate (3%). We want experienced, accomplished teachers to stay in teaching even if it is only for four additional years.
- Throughout the world, teachers complain frequently about shallow and useless professional development. Teacher-led professional development that treats teachers as professionals is a welcome change. M*f*A teachers thrive in such an environment. Even the most accomplished teacher wants to grow professionally and in M*f*A they do.

But the ultimate goal is to change perceptions – to convince the public and teachers themselves that teaching is not merely standing in front of a classroom and that it is a profession requiring mastery of content and craft, which takes place over many years and is motivated by curiosity, ambition and pride. Teaching is not preparing students for tests. It is not following instructions. It is not reciting facts or procedures. It is a profession and we should welcome its accomplished professionals into the mathematics community … and treat them like the professionals they are.

Perceptions only change over time, however. Achieving this goal will require persistence and patience – qualities that are frequently missing from educational reform today. Fortunately, M*f*A has both.

(For more details and background, see the Math for America website at www.mathforamerica.org.)

All photo credits: Michael Lisnet, MfA photographer.

John Ewing is the President of Math for America, an organisation associated to and supported by the Simons Foundation in New York City. He was formerly the Executive Director of the American Mathematical Society and, before that, he was on the faculty of Indiana University. Since joining MfA in 2009, he has gained both humility about the difficulty of mathematics education and certainty that improving it requires us all to work together – mathematicians, maths educators and classroom teachers – as equal partners.

News on the zbMATH Interface

Octavio Paniagua Taboada (FIZ Karlsruhe, Berlin, Germany)

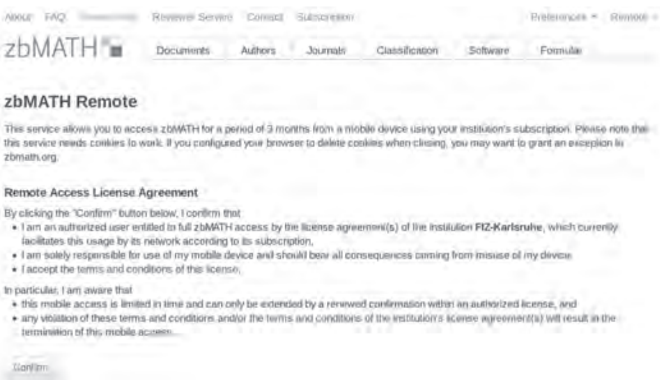
Our staff of developers and editors have updated and improved several important features in zbMATH. All these updates aim to provide our users a rich and modern database interconnected with other worldwide databases and services (e.g. digital libraries, Wikidata, ORCID and links to discussions in MathOverflow).

Reviewer service and compensation

During the last few months, we have updated several components of the reviewer service and submission tool. It is now possible to check the account balance and order Springer books, as well as make donations to the EMS book programme for developing countries via the interface. From January 2018, the financial compensation per review will be increased to 3.00 EUR from the traditional amount of 2.56 EUR. When this amount is used to order Springer books, a discount of 50% is applied.

zbMATH Remote

zbMATH Remote (https://zbmath.org/remote_access/) provides access to zbMATH from your mobile device using your institution’s subscription, even when you are not logged into the institution’s network. This access is valid for a period of three months and can be renewed whenever logged into your institution’s network. When connected to a licensing network, you can activate remote access to zbMATH by clicking on the ‘‘Remote’’ link located at the top right menu of the zbMATH website.

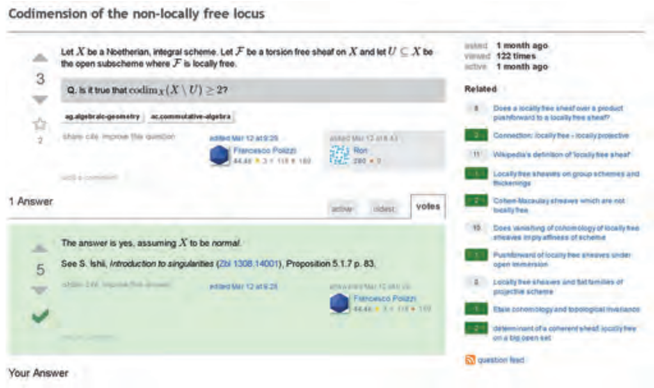


zbMATH interface features

Search results can now be sorted using customised criteria such as publication year, document or review citation, and volume number.

Additional filters have been added. Formula search has been extended by the integration of arXiv full-texts, making more than 160 million mathematical formulas retrievable. A new interface available at MathOverflow allows users to insert a citation into any question, answer or comment, and enables linking back from zbMATH to a discussion there.

Here is an example of such a citation.



MSC2020

zbMATH and Mathematical Reviews have worked together to revise and improve the Mathematics Subject Classification (MSC) schema, which is used by these reviewing services and publishers to categorise items in mathematics literature. Comments and suggestions can be submitted through the website http://msc2020.org/.

zbMATH Atom Feed

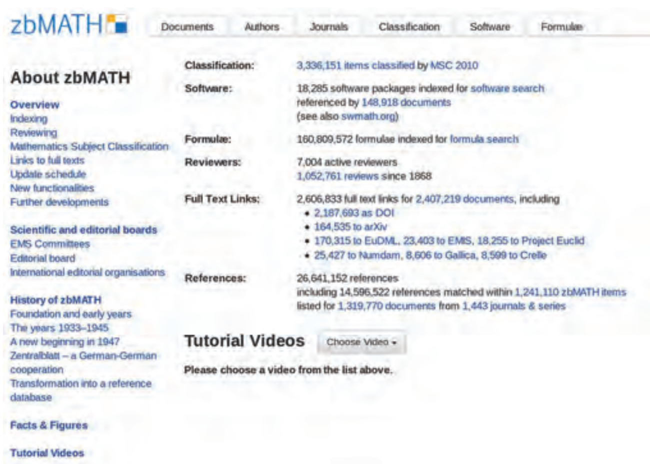
zbMATH now offers an additional way of keeping researchers up-to-date with mathematical developments in their areas of interest via an atom feed. Every two weeks, researchers receive an update of recently included items and reviews in electronic form.

This feed is an enhanced version of the classic web feed format RSS and is supported by all current news readers.

If you are interested in a specific author, you can access this news feed through the author ID (displayed at the top of each author profile, e.g. https://zbmath.org/atom/ai/leibniz.gottfried-wilhelm). You can also access the atom feed via MSC code, which can be defined as complex and specific as you wish (e.g. https://zbmath.org/atom/cc/81,17B,57R56 for quantum theory, Lie algebras and Lie superalgebras, and topological quantum field theories).

Tutorial videos

A series of videos have been produced in order to help users exploit all of zbMATH's potential. There are examples with complex queries, tricks and hints. New videos will be added to cater for newly developed features in zbMATH. You can watch these videos at the very bottom of the "About" page https://zbmath.org/about#id_5.



Octavio Paniagua Taboada is an editor for zbMATH at the Berlin office of FIZ Karlsruhe. He received a bachelor's degree and a Master's in mathematics from the National Autonomous University of Mexico (UNAM) and a PhD in mathematics from the Université Paris-Sud XI, Orsay, France. He has been an editor of zbMATH since January 2014 and of the EMS Newsletter since 2018.

Book Reviews

Jacques Sauloy

Differential Galois Theory through Riemann-Hilbert Correspondence: An Elementary Introduction

AMS, 2016
275 p.
ISBN 978-1-4704-3095-5

Reviewer: Julien Roques

Linear differential equations are at the crossroads of several areas of mathematics. This book starts with an exploration of analytic theory of (regular singular) linear differential equations and concludes with an invitation to the associated differential Galois theory. It should be stressed that Sauloy's intention is not to expose differential Galois theory in its full generality, for linear differential equations with coefficients in an arbitrary differential field K, but rather to focus on the case $K = \mathbb{C}(z)$, taking advantage of the analytic tools at our disposal in this context and making apparent some beautiful interplay between analysis and algebra. The intention of Sauloy is clearly to explain the meaning of the adage "differential Galois theory is what algebra can say about dynamics" and to convince the reader that modern differential Galois theory has its roots in analytic theory of differential equations (and, more precisely, in Riemann's *monodromy representations*). And it is a success.

Here follows a summary of the contents of the book.

Part I is an introduction to complex analysis and a prelude to the analytic study of linear differential equations undertaken in Parts II and III.

Parts II and III are mainly concerned with analytic theory of regular singular differential equations on $\mathbb{P}^1(\mathbb{C})$, say, of the form

$$a_n(z)\, y^{(n)}(z) + \cdots + a_1(z)\, y'(z) + a_0(z)\, y(z) = 0, \quad (1)$$

with $a_0(z), \ldots, a_n(z) \in \mathbb{C}(z)$ and $a_n(z) \neq 0$, and, more specifically, with their monodromy representation. Before describing in more detail what can be found in Parts II and III, here are a few words about the concept of monodromy, which is the hero of the book. The starting point is the so-called Cauchy theorem. Denoting by $\mathcal{S}$ the set of singularities in $\mathbb{P}^1(\mathbb{C})$ of (1) and by U its complement in $\mathbb{P}^1(\mathbb{C})$, Cauchy's theorem reads: for any $z_0 \in U\$$, the complex vector space V_{z_0} of germs of analytic functions at z_0 that are solutions of (1 has dimension n. These solutions possess a marvellous property: any $f \in V_{z_0}$ can be continued analytically along any loop γ in U based at z_0 and the germ of analytic function f^γ resulting from this process is again a solution of (1), i.e. $f^\gamma \in V_{z_0}$. In this way, we get a $\mathbb{C}$-linear action of the fundamental group $\pi_1(U,z_0)$ on V_{z_0}, given, for all $\gamma \in \pi_1(U,z_0)$ and $f \in V_{z_0}$, by $[\gamma]\, f := f^\gamma$. In other words, we get a finite-dimensional linear representation

$$\rho : \pi_1(U,z_0) \to \mathrm{GL}(V_{z_0}).$$

This is the monodromy representation attached to (1). The following two questions are natural:

- Is sole knowledge of the monodromy representation sufficient to reconstruct the differential equation we started with?
- Is any finite-dimensional linear representation of the fundamental group $\pi_1 (U, z_0)$ the monodromy representation of some differential equation?

In full generality, the answer to the first question is negative but becomes positive if we restrict our attention to a special class of differential equations, namely, to those having only regular singularities. The answer to the second question is positive even if we restrict our attention to differential equations having only regular singularities. These facts are usually formalised as a certain equivalence of categories, known as Riemann–Hilbert correspondance.

We are now in a position to describe the content of Parts II and III. Part II starts with Cauchy's theorem, continues with the proof of the analytic continuation property of the solutions mentioned above and introduces the monodromy representation. Part III starts with the notion of regular singularity and with a detailed study of this type of singularity. A local version of Riemann–Hilbert correspondance is then stated and proved. The statement and a sketch of the proof of (global) Riemann–Hilbert correspondance (over $\mathbb{P}^1(\mathbb{C})$) are then given. It is worth noting that Part III also contains the determination of the monodromy representation of hypergeometric differential equations (following Riemann's original method).

With Part IV, the reader enters the world of differential Galois theory. The first few pages of this chapter contain the definition of local differential Galois groups and a proof of the fact that they are complex linear algebraic groups. The transition from Parts II and III to Part IV is carried out in a very natural and smooth way via the so-called Schlesinger density theorem: the monodromy is Galoisian and gives rise to Zariski-dense subgroups of the above mentioned local differential Galois groups in the regular singular case.

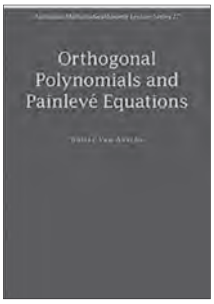
The rest of Part IV is essentially concerned with the regular singular local universal Galois group; roughly speaking, this universal Galois group is the algebraic hull of the local topological fundamental group and it can be used to formulate an "algebraic Riemann–Hilbert correspondance".

The book ends with a selection of further developments and readings.

This is an accessible book, well-suited for students. It is mainly self-contained. Some "advanced concepts" are used, such as sheaves, categories and linear algebraic groups but these concepts are introduced progressively throughout the book, when needed, without any attempt of systematic presentation or maximal generality. This is a nice and efficient choice. Several exercises can also be found throughout the book.

The book is warmly recommended for those looking for an accessible text about the analytic theory of regular singular differential equations and the associated differential Galois theory.

Julien Roques is a maître de conférences at the Institut Fourier of the Université Grenoble Alpes. He works on algebra, number theory and functional equations, and on their interactions and applications.



Walter van Assche

Orthogonal Polynomials and Painlevé Equations

Cambridge University Press, 2018
xii, 179 p.
ISBN 978-1-108-44194-0

Reviewer: Francisco Marcellán

The Newsletter thanks zbMATH and Francisco Marcellán for the permission to republish this review, originally appeared as Zbl 06811550.

At the end of the 19th century some relevant mathematicians like Poincaré, Fuchs, Picard, Painlevé, among others, were interested to find those nonlinear ordinary differential equations (ODE) such that their general solutions are free from movable branch points (Painlevé property). The locations of possible branch points and critical essential singularities of solutions can be independent on the initial values. In the case of first order differential equations the Painlevé property only gives linear differential equations, the Riccati differential equation and the equation associated with the Weierstrass elliptic function. For second order differential equations, Picard raised the description of those nonlinear differential equations $y''(x) = R(x, y, y')$, where R is a rational function, such that the Painlevé property holds. Paul Painlevé found that, up to some simple transformations, there are 50 canonical forms for such ODEs. 44 of them can be reduced to linear ODEs, Riccati equation or Weierstrass case. The remaining six equations of such a list are called Painlevé ODEs and their solutions are known as Painlevé transcendents. For them, the only movable singularities are poles (no essential singularities). A remarkable overview on these topics is presented in [P.A. Clarkson, *Lect. Notes Math*. 1883, 331–411 (2006; Zbl 1100.33006)].

On the other hand, discrete Painlevé equations are nonlinear recurrence relations for which the continuous limit is one of the Painlevé ODE. A classification of discrete Painlevé equations based on rational surfaces associated with affine root systems was suggested by H. Sakai [*Commun. Math. Phys*. 220, No. 1, 165–229 (2001; Zbl 1010.34083)].

The book under review is focused on the relationship between Painlevé equations and orthogonal polynomials. The basic fact is that the coefficients of the three term recurrence relations (TTRR) the sequences of polynomials with respect to measures supported on the real line together with some differential properties of the measure (Pearson equation) yield discrete Painlevé equations. Moreover, if the measure depends on some time parameter, then you can deduce Painlevé ODE for the coefficients of the recurrence relation. This fact is connected with some integrable systems (Toda, Langmuir, among others).

Two blocks constitute the skeleton of this book. In the first one (five chapters) the author deals with the Painlevé equations associated with the coefficients of the TTRR that some families of orthogonal polynomials satisfy. In the second one (two chapters), the attention is focussed on rational solutions of Painlevé ODE which appear for some choices of the parameters involved in ODEs. Furthermore, the asymptotic behavior of orthogonal polynomials near critical points is presented by using the Riemann-Hilbert approach.

Chapter 1 provides a basic background about orthogonal polynomials with a special emphasis on those associated with the so called semiclassical weights. Next, the description of the continuous and discrete Painlevé equations is presented.

In Chapter 2, some examples of Freud weights are studied. First, in the case $w(x) = \exp(-x^4 + tx^2)$ a $d-P_I$ equation appears for the coefficients of the TTRR. The unicity of the positivity solution of such an equation with an initial condition $x_0 = 0$ is analyzed and, consequently, the value of x_1 is determined. On the other hand, a differential-difference equation for the coefficients of the TTRR is deduced and, by using the above equations, a Painlevé IV equation for such coefficients is obtained.

Chapter 3 deals with orthogonal polynomials associated with the measure $w(z) = \exp((z + 1/z)\, t/2)$ supported on the unit circle. Now, the parameters of the recurrence relation that the corresponding orthogonal polynomials satisfy (the so called Verblunsky coefficients) are related to a $d-P_{II}$ equation. They also satisfy a Painlevé V equation, that can be reduced to a Painlevé III. Examples of discrete orthogonal polynomials (generalized Charlier) are also analyzed and the corresponding Painlevé equations for some choices of the parameters are obtained. The unicity of solutions of $d-P_{II}$ and its positivity according to some initial conditions is proved.

By using the formulation of the Riemann–Hilbert problem for orthogonal polynomials a new proof of a classical result about ladder operators for exponential weights (see [Y. Chen and M.E.H. Ismail, *J. Phys. A, Math. Gen*. 30, No. 22, 7817–7829 (1997; Zbl 0927.33011)]) is given. This approach is very useful in order to analyze Painlevé equations for different families of semiclassical weights in continuous and discrete cases as done in Chapters 4 and 5.

In Chapter 6 the author focuses the attention on those Painlevé equations such that, for some choices of the parameters, either rational or special function solutions appear. For the Painlevé II and III they are given in terms of the logarithmic derivative of two consecutive Yablonskii- Vorobiev polynomials and Umemura polynomials, respectively. In other cases they can expressed as Wronskians of classical orthogonal polynomials. The Painlevé equations II–VI also have solutions which can be expressed in terms of classical special functions (Airy, Bessel, parabolic cylinder, Kummer and Gauss functions, respectively).

Finally, in Chapter 7 the connection between the density of zeros of orthogonal polynomials associated to weight functions with singularities, the construction of a local parametrix at certain critical points in the asymptotic behavior of orthogonal polynomials and random matrices is stated for Painlevé I, II, IV and V.

The presentation of this book is very friendly for a general audience interested in the theory of orthogonal polynomials, nonlinear ODE and integrable systems. The contents of the chapters are very pleasant taking into account the main results and their proofs are given with a smart distribution. Many of the results described in the first block are based on the contributions by the author and co-workers.

The book is based on lecture notes of courses and seminars in several higher education institutions. A list of 16 exercises with their solutions allows a dynamical approach to the techniques described therein. Finally, an updated list of 156 references invites the reader to advance in the learning of many questions contained in this nice book.

Francisco Marcellán is professor of Applied Mathematics at Universidad Carlos III de Madrid, Spain, as well as a senior researcher at Instituto de Ciencias Matemáticas (ICMAT) CSIC-UAM-UCM-UC3M. His research interests are in orthogonal polynomials, special functions, approximation theory, matrix analysis and their applications to integrable systems and signal theory. At present he is the President of the Royal Spanish Mathematical Society.

Dan Burghelea

New Topological Invariants for Real- and Angle-valued Maps. An Alternative to Morse–Novikov Theory

World Scientific, 2018
xvi, 242 p.
ISBN 978-981-4618-24-3

Reviewer: Dorin Andrica

The Newsletter thanks zbMATH and Dorin Andrica for the permission to republish this review, originally appeared as Zbl 06793944.

In the early 1920s, M. Morse discovered that the number of critical points of a smooth real-valued function on a manifold is closely related to the topology of the manifold. This became a starting point of the Morse Theory which is now one of the basic and active parts of differential topology. We refer for instance to the book of L.I. Nicolaescu [*An invitation to Morse theory*. 2nd ed. Berlin: Springer (2011; Zbl 1238.57001)]. Circle-valued Morse theory is originated from a problem in hydrodynamics studied by S.P. Novikov in the early 1980s. It is a constantly growing field of contemporary mathematics, also known as Novikov Theory, with applications and connections to many geometrical problems such as Arnold's conjecture in the theory of Lagrangian intersections, fibrations of manifolds over the circle, dynamical zeta functions, and the theory of knots and links in the three-dimensional sphere (see the monograph of A. Pajitnov [*Circle-valued Morse theory*. Berlin: De Gruyter (2006; Zbl 1118.58007)]).

The present book presents a very interesting alternative to the above mentioned Morse–Novikov Theory, symbolically called **AMN** Theory. The theory presented by the author is of great interest in topology and dynamics, it has provided inspiration and has applications outside of mathematics, especially in data analysis and shape recognitions in physics and computer science. In this book a *nice space* is a locally compact ANR (Absolute Neighborhood Retract). Remark that finite dimensional simplicial complexes and finite dimensional topological manifolds are nice spaces but the class is considerably larger. The author considers *tame maps* to be proper continuous maps $f: X \to \mathbb{R}$ or $f: \to \mathbb{S}^1$, defined on a nice space X, which satisfies the following properties: (i) each fiber of f is a neighborhood deformation retract; (ii) away from a discrete set $\Sigma \subset \mathbb{R}$ or $\Sigma \subset \mathbb{S}^1$ the restriction of f to $X \setminus f^{-1}(\Sigma)$ is a fibration. In particular for $t \notin f^{-1}(\Sigma)$ there exists a neighborhood U of t such that for any $t' \in U$ the inclusion $f^{-1}(t') \subset f^{-1}(U)$ is a homotopy equivalence. Note that all proper simplicial maps and proper smooth generic maps defined on a smooth manifold, in particular proper real or angle valued Morse maps, are tame. At least for spaces homeomorphic to simplicial complexes the set of tame maps is residual in the space of all continuous maps and weakly homotopy equivalent to the space of all continuous maps (equipped with the compact-open topology). Some refinements of this notion are also considered in the book. The main idea of **AMN** Theory is to consider instead of critical points, critical values for real-valued or circle-valued functions. Instead of critical points and instantons between critical points, the **AMN** Theory introduces and calculates barcodes. The closed trajectories are replaced by Jordan cells or Jordan blocks whose direct sums calculate the homological monodromies. It is remarkable that most of the fundamental algebraic topology invariants which can be recovered in the Morse–Novikov Theory from critical points, instantons, and closed trajectories, can equally well recovered in the **AMN** Theory from barcodes and Jordan blocks. The derived invariants are finite, computable by implementable algorithms in case the underlying space of the map has a triangulation and the map is simplicial, and are, in some sense, the analogues of the set of trajectories between rest points and of closed trajectories of a generic vector field (which admits a Lyapunov closed one form) on a smooth manifold. The book presents remarkable properties such as stability and Poincaré-duality of these invariants, and it relates them to the global algebraic topology of the space.

To give a general presentation of the results contained in the book, we give an overview on the content. The material is organized into nine chapters including Chapter 1 as the author description.

Preparatory Material: It reviews the linear algebra of matrices and of linear relations, as well as few concepts and results in topology: matrices, Fredholm maps and Fredholm cross-ratio, an algorithm to calculate $R(A,B)_{\mathrm{reg}}$, ARNs, tameness, regular and critical values, compact Hilbert cube manifolds, infinite cyclic covers, simplicial complexes, cell complexes and incidence matrices, configurations, algebraic topology of a pair $(X, \xi \in H^1(X;\mathbb{Z}))$.

Graph Representations: generalities on graph representations, the idecomposable representations (two basic constructions, the $\$\kappa[t^{-1}\},t]$-module associated to a G_{2m}-representation, the matrix $M(\rho)$ and the representation ρ_u), calculation of indecomposables (an algorithm) (elementary transformations, algorithm for deriving barcodes from $M(\rho)$, implementation of $T_1(i), T_2(i), T_3(i), T_4(i)$).

Barcodes and Jordan Blocks via Graph Representations: the graph representation associated to a map, barcodes and Jordan blocks of a tame map (the configurations δ^f_r, the **AM** and **AN** spaces, the relevant exact sequences), barcodes, Jordan cells, and homology, barcodes and Borel-Moore homology, calculations of barcodes and Jordan cells.

Configurations δ^f_r and $\hat{\delta}^f_r$ (Alternative Approach): general considerations, the case of real-valued maps, the case of angle-valued maps.

Configurations γ^f_r: general considerations, the case of real-valued functions, the case of angle-valued functions.

Monodromy and Jordan Cells: general considerations, geometric r-monodromy via linear relations, an algorithm for the calculation of Jordan cells of a simplicial angle-valued map.

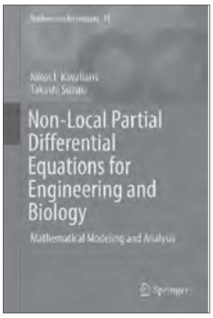
Applications: relations with the classical Morse and Morse–Novikov theories (the Morse complex, the Novikov complex, chain complexes of vector spaces, the **AM** and **AN** complexes for a Morse map), a few computational applications (Novikov–Betti numbers in relation with Jordan cells, Alexander polynomial of a knot and generalizations).

Comments: relation to persistence theory, a measure-theoretic aspect of the configurations of δ_r^f, γ_r^f, an invitation.

The book ends with a rich bibliogrphy containing 56 suggestive references for the subject, nine of them representing the author's contributions. A List of Figures and an useful Index are also included.

The book under review is a very nice and valuable text on the new **AMN** Theory. It is written in a clear manner and it can help anyone who wants to learn this new theory as well as its recent developments and applications.

Dorin Andrica is professor at Babes-Bolyai University in Cluj-Napoca, Romania, where he received his PhD in 1992 with a thesis on critical point theory with applications to the geometry of differentiable submanifolds. His scientific interests are in Differential Topology (critical point theory and applications, Morse theory with applications), Differential Geometry, Number Theory, Problem Solving, Mathematics for competitions and Olympiads. He published more than 170 research papers, 35 books in Romanian, English, Arabic, Portuguese, Japanese, Korean, and proposed more than 1000 original problems of various level of difficulty to different mathematical competitions. Professor Dorin Andrica is well known for his conjecture concerning the gap of the consecutive primes, presently called "Andrica Conjecture".



Nikos I. Kavallaris, Takashi Suzuki

Non-Local Partial Differential Equations for Engineering and Biology. Mathematical Modeling and Analysis.

Springer, 2018
xix, 300 p.
ISBN 978-3-319-67942-6

Reviewer: Teodora-Liliana Rădulescu

The Newsletter thanks zbMATH and Teodora-Liliana Rădulescu for the permission to republish this review, originally appeared as Zbl 06817103.

This book presents some new results concerning non-local models arising in mathematical analysis. Basic concepts in mechanics, thermodynamics, game theory, and theoretical biology are examined in detail. It starts with a review and summary of the basic ideas of mathematical modeling frequently employed in sciences and engineering. The first part of this monograph is devoted to the investigation of some non-local models linked with applications from engineering.

Chapter 1 focuses on the study of non-local models associated with electrostatic micro-electro-mechanical-systems (MEMS) control. The authors describe the two main physical problems which build up the operation of an idealized MEMS device: the elastic and the electric problem. Next, the authors are concerned with the structure of the set of radially symmetric steady-state solutions, which are investigated along with their stability. Then, they study the circumstances under which finite-time quenching occurs.

Chapter 2 discusses some non-local models describing Ohmic heat production in various industrial processes. In the first part of the chapter, the process of food sterilization through Ohmic heating is considered on the basis of a one-dimensional non-local model. Next, and under different circumstances, a hyperbolic approach with a non-local convection velocity is built up. The second part of this chapter deals with another application of Ohmic heating process in a thermistor device.

Chapter 3 deals with an application arising in the process of linear friction welding applied in metallurgy. Next, a similar non-local model is derived for the hard-material case where the exponential nonlinearity is replaced by $f(u) = (-u)^p$, for $p = 1/\alpha$.

Chapter 4 discusses a degenerate non-local model which is associated with the industrial process of resistance spot welding and the unknown \$u\$ represents the temperature in the welding area.

Part II of this monograph deals with some applications of non-local models coming from biology. The authors are concerned with the Gierer-Meinhardt system, an application arising in evolutionary game dynamics, biological phenomena arising in chemotaxis, and a mathematical model in cell biology that describes the evolution of protein dimers within human cells.

The models developed in this book are based on various laws of physics such as mechanics of continuum, electromagnetic theory, and thermodynamics. For these reasons, the arguments come from many areas of mathematics such as calculus of variations, dynamical systems, integrable systems, blow-up analysis, and energy methods. The book under review is mainly addressed to researchers and upper grade students in mathematics, engineering, physics, economics, and biology.

Teodora-Liliana Radulescu is professor of mathematics at the "Fratii Buzesti" National College in Craiova, Romania. She received her Ph.D. in 2005 from Babes-Bolyai University of Cluj-Napoca. Her research interests are problem solving and methods in nonlinear analysis. She co-authored the book "Problems in Real Analysis. Advanced Calculus on the Real Axis", Springer, 2009.

Personal Column

Please send information on mathematical awards and deaths to newsletter@ems-ph.org.

Awards

The Unione Matematica Italiana has awarded the following prizes for 2017:
the **Mario Baldassarri Prize** to **Andrea Seppi**;
the **Giuseppe Bartolozzi Prize** to **Andrea Mondino**;
the **Guido Castelnuovo Prize** to **Chiara Andrà**, **Nicola Parolini** and **Marco Verani** for their project BetonMath;
the **Stefania Cotoneschi Prize** to **Daniele Pasquazi**;
the **Bruno de Finetti Prize** to **Alessandro Foschi**

David Pérez García (Universidad Complutense de Madrid, Spain) received the **Prize Miguel Catalán 2017** in recognition of his contributions in the area of quantum technologies.

The **2017 Stefan Bergman Prize** was awarded to **Bo Berndtsson** (University of Gothenburg and Chalmers University, Sweden) and **Nessim Sibony** (Université Paris-Sud Orsay, France) for their many fundamental contributions to several complex variables, complex potential theory, and complex geometry.

David Alonso Gutiérrez (Universidad de Zaragoza, Spain) received the **2017 Research Award** from Real Academia de Ciencias de Zaragoza.

The **Richard-von-Mises-Prize** was awarded to **Marc Avila** (Bremen, Germany) in acknowledgment of his scientific achievements in the area of applied mathematics and mechanics.

The Israel Mathematical Union awards the **2018 Anna and Lajos Erdős Prize in Mathematics** to **Ronen Eldan** (Weizmann Institute, Rehovot, Israel) and the **2018 Haim Nessyahu Prize** to **Sara Tukachinsky** (Hebrew University, Jerusalem, Israel) and **Eliran Subag** (Weizmann Institute, Rehovot, Israel).

Assaf Naor (Princeton University, USA) has been awarded the **2018 Frederic Esser Nemmers Mathematics Prize** for "his profound work on the geometry of metric spaces, which has led to breakthroughs in the theory of algorithms."

The **Prize Marc Yor 2018** has been awarded to **Christophe Garban** (Université Lyon 1, France), for his work in probability and statistical physics. The prize was instituted by SMAI (Society for Applied and Industrial Mathematics) and SMF (Société Mathématique de France) and it is patronized by the French Académie des sciences.

The Norwegian Academy of Science and Letters has decided to award the **Abel Prize for 2018** to **Robert P. Langlands** (Institute for Advanced Study, Princeton, USA) "for his visionary program connecting representation theory to number theory".

The CNRS (France) has awarded the **médaille d'argent** to **Grégory Miermont** (UMPA), the **médaille de bronze** to Anne-Laure Dalibard (LJLL) and the **médaille de cristal** to **Elisabeth Kneller** (Bibliothèque mathématique Jacques Hadamard).

The **Rolf Schock Prize 2018** of the Royal Swedish Academy of Sciences has been awarded **Ronald Coifman** (Yale University, USA) "for his fundamental contributions to pure and applied harmonic analysis".

The **2018 Wolf Prize for Mathematics** has been awarded to **Alexander Beilinson** and **Vladimir Drinfeld** (both University of Chicago, USA) "for their groundbreaking work in algebraic geometry, representation theory, and mathematical physics".

Sir John M. Ball (University of Oxford, UK) is the winner of the **2018 King Faisal Prize for Science** for his outstanding contributions to mathematics. He also received the **European Academy of Sciences Leonardo da Vinci Award 2018**.

The **2018 Breakthrough Prize in Mathematics** was awarded to **Christopher Hacon** (University of Utah, USA) and **James McKernan** (University of California San Diego, USA) for transformational contributions to birational algebraic geometry, especially to the minimal model program in all dimensions.

Martin Branda, **Jan Šaroch** (both Charles University, Prague, Czech Republic) and **Michal Doucha**, **Václav Mácha** (both Institute of Mathematics of the CAS) received in 2018 the **Prize for Young Mathematicians** awarded by the Czech Mathematical Society.

Mickaël Launay is winner of the 2018 award of the SMF. He received the **Prix d'Alembert** for his contributions to the popularisation of mathematics.

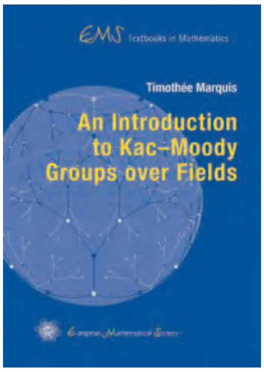
Deaths

We regret to announce the deaths of:

Rudolf Gorenflo (20 October 2017, Berlin, Germany)
Klaus Keimel (18 November 2017, Darmstadt, Germany)
Jan-Erik Roos (15 December 2017, Uppsala, Sweden)
Robert Adol'fovich Minlos (9 January 2018, Moscow, Russian Federation)
Jean-Louis Koszul (12 January 2018, Grenoble, France)
Ulrich Dieter (25 January 2018, Graz, Austria)
Manfred Stern (1 February 2018, Halle, Germany)
Alan Baker (4 February 2018, Cambridge, UK)
Herbert Heyer (8 February 2018, Tübingen, Germany)
Thomas Friedrich (27 February 2018, Berlin, Germany)
John Roe (9 March 2018, State College, USA)
Stephen William Hawking (14 March 2018, Cambridge, UK)
Willi Törnig (2 April 2018, Darmstadt, Germany)
Miguel Ángel Revilla Ramos (2 April 2018, Valladolid, Spain)
Bernardo Cascales Salinas (5 April 2018, Murcia, Spain)

New books published by the European Mathematical Society

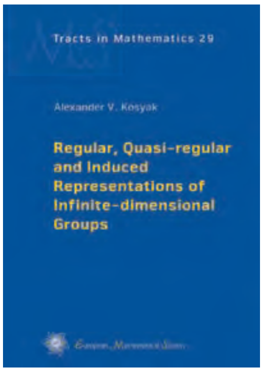
Individual members of the EMS, member societies or societies with a reciprocity agreement (such as the American, Australian and Canadian Mathematical Societies) are entitled to a discount of 20% on any book purchases, if ordered directly at the EMS Publishing House.



Timothée Marquis (Université Catholique de Louvain, Louvain-la-Neuve, Belgium)
An Introduction to Kac–Moody Groups over Fields (EMS Textbooks in Mathematics)

ISBN 978-3-03719-187-3. 2018. 341 pages. Hardcover. 16.5 x 23.5 cm. 48.00 Euro

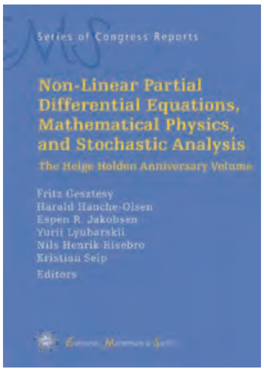
The interest for Kac–Moody algebras and groups has grown exponentially in the past decades, both in the mathematical and physics communities, and with it also the need for an introductory textbook on the topic.
The aims of this book are twofold:
- to offer an accessible, reader-friendly and self-contained introduction to Kac–Moody algebras and groups;
- to clean the foundations and to provide a unified treatment of the theory.
The book starts with an outline of the classical Lie theory, used to set the scene. Part II provides a self-contained introduction to Kac–Moody algebras. The heart of the book is Part III, which develops an intuitive approach to the construction and fundamental properties of Kac–Moody groups. It is complemented by two appendices, respectively offering introductions to affine group schemes and to the theory of buildings. Many exercises are included, accompanying the readers throughout their journey.
The book assumes only a minimal background in linear algebra and basic topology, and is addressed to anyone interested in learning about Kac–Moody algebras and/or groups, from graduate (master) students to specialists.



Alexander V. Kosyak (National Academy of Science of Ukraine, Kiev, Ukraine)
Regular, Quasi-regular and Induced Representations of Infinite-dimensional Groups (EMS Tracts in Mathematics Vol. 29)

ISBN 978-3-03719-181-1. 2018. 587 pages. Hardcover. 17 x 24 cm. 98.00 Euro

Almost all harmonic analysis on locally compact groups is based on the existence (and uniqueness) of a Haar measure. Therefore, it is very natural to attempt a similar construction for non-locally compact groups. The essential idea is to replace the non-existing Haar measure on an infinite-dimensional group by a suitable quasi-invariant measure on an appropriate completion of the initial group or on the completion of a homogeneous space.
The aim of the book is a systematic development, by example, of noncommutative harmonic analysis on infinite-dimensional (non-locally compact) matrix groups. We generalize the notion of regular, quasi-regular and induced representations for arbitrary infinite-dimensional groups. The central idea to verify the irreducibility is the Ismagilov conjecture. We also extend the Kirillov orbit method for the group of upper triangular matrices of infinite order.
In order to make the content accessible to a wide audience of nonspecialists, the exposition is essentially self-contained and very few prerequisites are needed. The book is aimed at graduate and advanced undergraduate students, as well as mathematicians who wish an introduction to representations of infinite-dimensional groups.



Non-Linear Partial Differential Equations, Mathematical Physics, and Stochastic Analysis. The Helge Holden Anniversary Volume (EMS Series of Congress Reports)
Fritz Gesztesy (Baylor University, Waco, USA), Harald Hanche-Olsen (The Norwegian University of Science and Technology, Trondheim, Norway), Espen R. Jakobsen (The Norwegian University of Science and Technology, Trondheim, Norway), Yurii I. Lyubarskii (The Norwegian University of Science and Technology, Trondheim, Norway), Nils Henrik Risebro (University of Oslo, Norway) and Kristian Seip (Norwegian University of Science and Technology, Trondheim, Norway) , Editors

ISBN 978-3-03719-186-6. 2018. 502 pages. Hardcover. 17 x 24 cm. 98.00 Euro

This volume is dedicated to Helge Holden on the occasion of his 60th anniversary. It collects contributions by numerous scientists with expertise in non-linear partial differential equations (PDEs), mathematical physics, and stochastic analysis, reflecting to a large degree Helge Holden's longstanding research interests. Accordingly, the problems addressed in the contributions deal with a large range of topics, including, in particular, infinite-dimensional analysis, linear and nonlinear PDEs, stochastic analysis, spectral theory, completely integrable systems, random matrix theory, and chaotic dynamics and sestina poetry. They represent to some extent the lectures presented at the conference *Non-linear PDEs, Mathematical Physics and Stochastic Analysis*, held at NTNU, Trondheim, July 4–7, 2016 (https://wiki.math.ntnu.no/holden60).
The mathematical tools involved draw from a wide variety of techniques in functional analysis, operator theory, and probability theory.
This collection of research papers will be of interest to any active scientist working in one of the above mentioned areas.