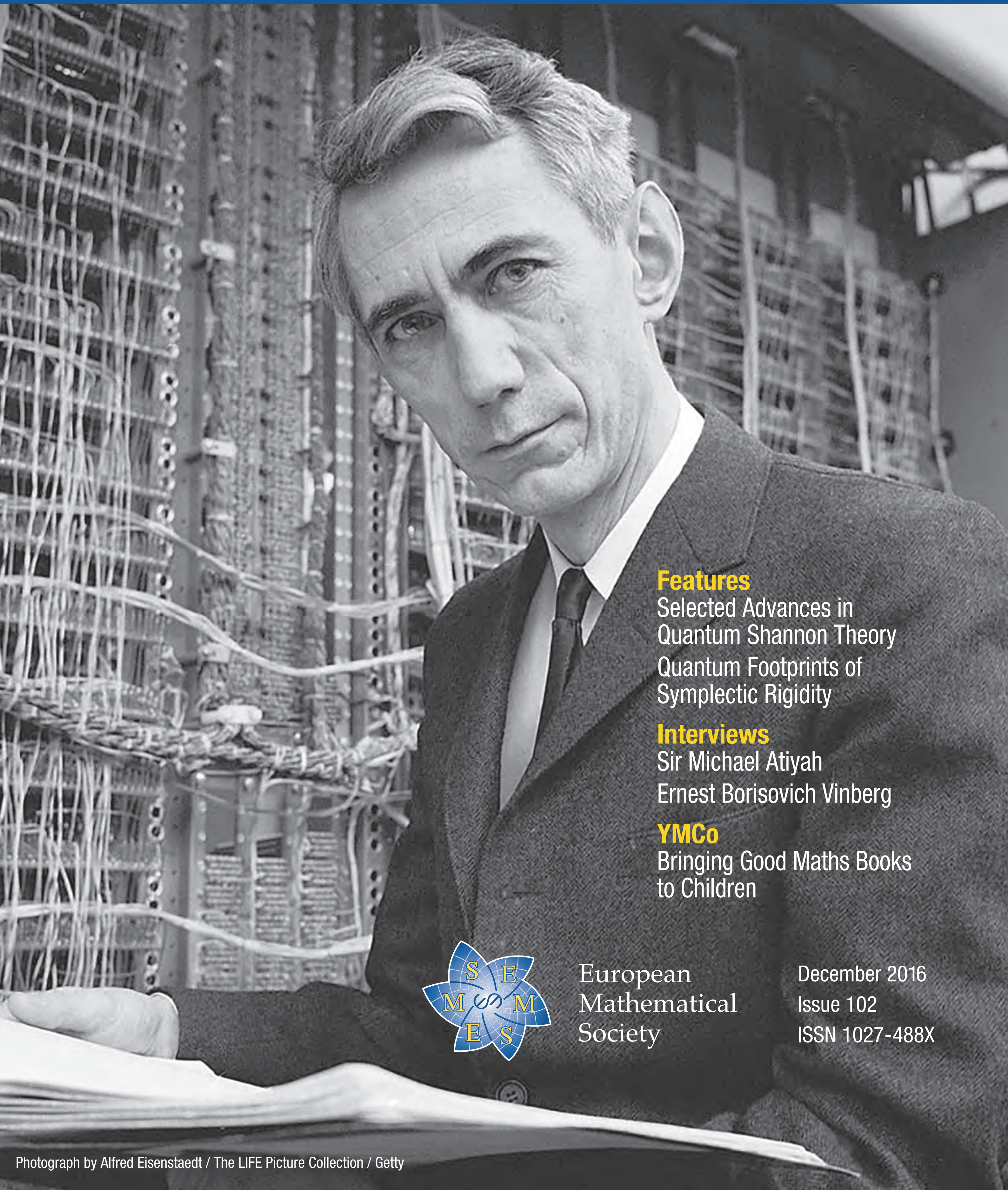
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



Features

Selected Advances in Quantum Shannon Theory

Quantum Footprints of Symplectic Rigidity

Interviews

Sir Michael Atiyah

Ernest Borisovich Vinberg

YMCo

Bringing Good Maths Books to Children

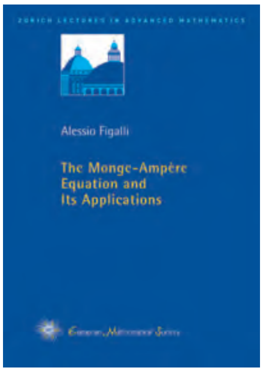
European Mathematical Society

December 2016
Issue 102
ISSN 1027-488X

Photograph by Alfred Eisenstaedt / The LIFE Picture Collection / Getty

New books published by the European Mathematical Society

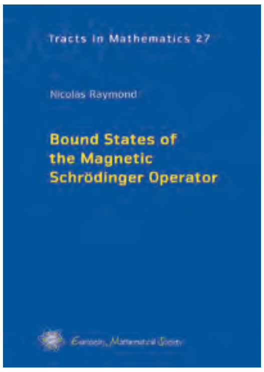
Individual members of the EMS, member societies or societies with a reciprocity agreement (such as the American, Australian and Canadian Mathematical Societies) are entitled to a discount of 20% on any book purchases, if ordered directly at the EMS Publishing House.



Alessio Figalli (ETH Zürich, Switzerland)
The Monge-Ampère Equation and Its Applications (Zürich Lectures in Advanced Mathematics)

ISBN 978-3-03719-170-5. 2017. 208 pages. Hardcover. 17 x 24 cm. 34.00 Euro

The Monge–Ampère equation is one of the most important partial differential equations, appearing in many problems in analysis and geometry.
This monograph is a comprehensive introduction to the existence and regularity theory of the Monge–Ampère equation and some selected applications; the main goal is to provide the reader with a wealth of results and techniques he or she can draw from to understand current research related to this beautiful equation.
The presentation is essentially self-contained, with an appendix wherein one can find precise statements of all the results used from different areas (linear algebra, convex geometry, measure theory, nonlinear analysis, and PDEs).
This book is intended for graduate students and researchers interested in nonlinear PDEs: explanatory figures, detailed proofs, and heuristic arguments make this book suitable for self-study and also as a reference.



Nicolas Raymond (Université de Rennes, France)
Bound States of the Magnetic Schrödinger Operator (EMS Tracts in Mathematics, Vol. 27)

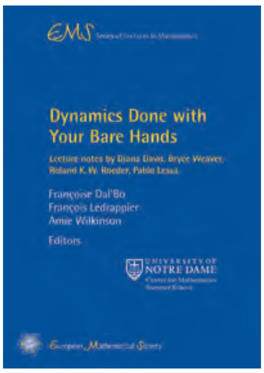
ISBN 978-3-03719-169-9. 2017. 398 pages. Hardcover. 17 x 24 cm. 64.00 Euro

This book is a synthesis of recent advances in the spectral theory of the magnetic Schrödinger operator. It can be considered a catalog of concrete examples of magnetic spectral asymptotics.
Since the presentation involves many notions of spectral theory and semiclassical analysis, it begins with a concise account of concepts and methods used in the book and is illustrated by many elementary examples.
Assuming various points of view (power series expansions, Feshbach–Grushin reductions, WKB constructions, coherent states decompositions, normal forms) a theory of Magnetic Harmonic Approximation is then established which allows, in particular, accurate descriptions of the magnetic eigenvalues and eigenfunctions. Some parts of this theory, such as those related to spectral reductions or waveguides, are still accessible to advanced students while others (e.g., the discussion of the Birkhoff normal form and its spectral consequences, or the results related to boundary magnetic wells in dimension three) are intended for seasoned researchers.

Representation Theory – Current Trends and Perspectives (EMS Series of Congress Reports)
Henning Krause (Universität Bielefeld, Germany), Peter Littelmann (Universität Köln, Germany), Gunter Malle (Universität Kaiserslautern), Karl-Hermann Neeb (Universität Erlangen-Nürnberg) and Christoph Schweigert (Universität Hamburg), Editors

ISBN 978-3-03719-171-2. 2017. 773 pages. Hardcover. 17 x 24 cm. 98.00 Euro

From April 2009 until March 2016, the German Science Foundation supported generously the Priority Program SPP 1388 in Representation Theory. The core principles of the projects realized in the framework of the priority program have been categorification and geometrization, this is also reflected by the contributions to this volume.
Apart from the articles by former postdocs supported by the priority program, the volume contains a number of invited research and survey articles. This volume is covering current research topics from the representation theory of finite groups, of algebraic groups, of Lie superalgebras, of finite dimensional algebras and of infinite dimensional Lie groups.
Graduate students and researchers in mathematics interested in representation theory will find this volume inspiring. It contains many stimulating contributions to the development of this broad and extremely diverse subject.



Dynamics Done with Your Bare Hands. Lecture notes by Diana Davis, Bryce Weaver, Roland K. W. Roeder, Pablo Lessa
Françoise Dal'Bo (Université de Rennes I, France), François Ledrappier (University of Notre Dame, USA) and Amie Wilkinson (University of Chicago), Editors

ISBN 978-3-03719-168-2. 2016. 214 pages. Softcover. 17 x 24 cm. 36.00 Euro

This book arose from 4 lectures given at the Undergraduate Summer School of the Thematic Program Dynamics and Boundaries held at the University of Notre Dame. It is intended to introduce (under)graduate students to the field of dynamical systems by emphasizing elementary examples, exercises and bare hands constructions.
The lecture of Diana Davis is devoted to billiard flows on polygons, a simple-sounding class of continuous time dynamical system for which many problems remain open. Bryce Weaver focuses on the dynamics of a 2×2 matrix acting on the flat torus. This example introduced by Vladimir Arnold illustrates the wide class of uniformly hyperbolic dynamical systems, including the geodesic flow for negatively curved, compact manifolds. Roland Roeder considers a dynamical system on the complex plane governed by a quadratic map with a complex parameter. Pablo Lessa deals with a type of non-deterministic dynamical system: a simple walk on an infinite graph, obtained by starting at a vertex and choosing a random neighbor at each step.

European Mathematical Society

Newsletter No. 102, December 2016

ISSN 1027-488X

Published by the
EMS Publishing House
ETH-Zentrum SEW A27
CH-8092 Zürich, Switzerland.
homepage: www.ems-ph.org

For advertisements and reprint permission requests contact: newsletter@ems-ph.org

EMS Executive Committee

President

Prof. Pavel Exner
(2015–2018)
Doppler Institute
Czech Technical University
Břehová 7
CZ-11519 Prague 1
Czech Republic
e-mail: ems@ujf.cas.cz

Vice-Presidents

Prof. Franco Brezzi
(2013–2016)
Istituto di Matematica Applicata e Tecnologie Informatiche del C.N.R.
via Ferrata 3
I-27100 Pavia
Italy
e-mail: brezzi@imati.cnr.it

Prof. Martin Raussen
(2013–2016)
Department of Mathematical Sciences
Aalborg University
Fredrik Bajers Vej 7G
DK-9220 Aalborg Øst
Denmark
e-mail: raussen@math.aau.dk

Secretary

Prof. Sjoerd Verduyn Lunel
(2015–2018)
Department of Mathematics
Utrecht University
Budapestlaan 6
NL-3584 CD Utrecht
The Netherlands
e-mail: s.m.verduynlunel@uu.nl

Treasurer

Prof. Mats Gyllenberg
(2015–2018)
Department of Mathematics and Statistics
University of Helsinki
P.O. Box 68
FIN-00014 University of Helsinki
Finland
e-mail: mats.gyllenberg@helsinki.fi

Ordinary Members

Prof. Alice Fialowski
(2013–2016)
Institute of Mathematics
Eötvös Loránd University
Pázmány Péter sétány 1/C
H-1117 Budapest
Hungary
e-mail: fialowsk@cs.elte.hu

Prof. Gert-Martin Greuel
(2013–2016)
Department of Mathematics
University of Kaiserslautern
Erwin-Schroedinger Str.
D-67663 Kaiserslautern
Germany
e-mail: greuel@mathematik.uni-kl.de

Prof. Laurence Halpern
(2013–2016)
Laboratoire Analyse, Géométrie & Applications
UMR 7539 CNRS
Université Paris 13
F-93430 Villetaneuse
France
e-mail: halpern@math.univ-paris13.fr

Prof. Volker Mehrmann
(2011–2018)
Institut für Mathematik
TU Berlin MA 4–5
Strasse des 17. Juni 136
D-10623 Berlin
Germany
e-mail: mehrmann@math.TU-Berlin.DE

Prof. Armen Sergeev
(2013–2016)
Steklov Mathematical Institute
Russian Academy of Sciences
Gubkina str. 8
119991 Moscow
Russia
e-mail: sergeev@mi.ras.ru

EMS Secretariat

Ms Elvira Hyvönen
Department of Mathematics and Statistics
P.O. Box 68
(Gustaf Hällströmin katu 2b)
00014 University of Helsinki
Finland
Tel: (+358) 2941 51141
e-mail: ems-office@helsinki.fi
Web site: http://www.euro-math-soc.eu

EMS Publicity Officer

Dr. Richard H. Elwes
School of Mathematics
University of Leeds
Leeds, LS2 9JT
UK
e-mail: R.H.Elwes@leeds.ac.uk

Cover photograph: Claude Elwood Shannon in 1963.

EMS Agenda

2017

17–19 March
EMS Executive Committee Meeting, Bratislava, Slovakia

1–2 April
Meeting of Presidents, Lisbon, Portugal

EMS Scientific Events

2017

29 March–2 April
EUROMATH, Bucharest, Romania
http://euromath.org/index.php?id=262

12–15 June
Meeting of the Catalan, Spanish and Swedish Mathematical Societies (CAT-SP-SW-MATH), Umeå, Sweden
EMS Distinguished Speaker: Kathryn Hess
http://liu.se/mai/catspsw.math?l=en

26–30 June
EMS-ESMTB Summer School 2017: Mathematical Modeling in Neuroscience, Copenhagen, Denmark
http://dsin.ku.dk/calendar/ems-esmtb-summer-school/

26–30 June
EMS summer school: Interactions between Dynamical Systems and Partial Differential Equations, Barcelona, Spain

10–19 July
Foundations of Computational Mathematics (FoCM'17), Barcelona, Spain
EMS Distinguished Speaker: Mireille Bousquet-Mélou
http://www.ub.edu/focm2017/

17–21 July
Summer School: Between Geometry and Relativity, ESI Vienna, Austria
http://www.univie.ac.at/AGESI_2017/school/

24–28 July
31st European Meeting of Statisticians, Helsinki, Finland
EMS-Bernoulli Society Joint Lecture: Alexander Holevo
http://ems2017.helsinki.fi

15–19 November
ASTUCON – The 2nd Academic University Student Conference (Science, Technology Engineering, Mathematics), Larnaca, Cyprus

2018

1–9 August
ICM 2018
Rio Centro Convention Center, Rio de Janeiro, Brazil
http://www.icm2018.org/portal/en/

Editorial
zbMATH – Looking to the Future*

Klaus Hulek (Gottfried Wilhelm Leibniz Universität Hannover, Germany), Editor-in-Chief of zbMath

After 9 years in office as Vice-President for Research at Leibniz Universität Hannover, it had not been my intention to take on another administrative position. Consequently, I was somewhat reluctant (but still honoured) when representatives of FIZ Karlsruhe and Springer Verlag contacted me with the offer to become the new Editor-in-Chief of zbMATH. However, discussions with colleagues soon convinced me that this was not only an interesting and challenging task but also one that could provide a useful service to the mathematical community. I took over my new responsibilities in April 2016, succeeding (after an interlude with Olaf Teschke as acting Editor-in-Chief) Gert-Martin Greuel, who was Editor-in-Chief 2012–2015, and Bernd Wegner, who had held the office for 37 years before that.


ZENTRALBLATT FÜR
MATHEMATIK
UND IHRE GRENZGEBIETE
(REINE UND ANGEWANDTE MATHEMATIK
THEORETISCHE PHYSIK . ASTROPHYSIK . GEOPHYSIK)
HERAUSGEGEBEN VON
P. ALEXANDROFF MOSKAU · J. BARTELS EBERSWALDE · W. BLASCHKE HAMBURG · R. COURANT GÖTTINGEN · H. HAHN WIEN
G. H. HARDY OXFORD · F. HUND LEIPZIG · G. JULIA VERSAILLES · O. KELLOGG CAMBRIDGE/MASS. · H. KIENLE GÖTTINGEN
T. LEVI-CIVITA ROM · R. NEVANLINNA HELSINKI · H. THIRRING WIEN · B. L. VAN DER WAERDEN LEIPZIG
SCHRIFTLEITUNG:
O. NEUGEBAUER-GÖTTINGEN

The Editorial Institutions of zbMATH

zbMATH was founded as "Zentralblatt für Mathematik und ihre Grenzgebiete" by Otto Neugebauer and other prominent mathematicians in 1931 on the initiative of Harald Bohr and Richard Courant, among others. In 1939, it amalgamated with the even older "Jahrbuch über die Fortschritte der Mathematik", which was founded in 1868 by Carl Ohrtmann and Felix Müller. After the Second World War, Zentralblatt was re-established jointly by the Academy of Sciences and Springer Verlag. Even after the partitioning of Germany, Zentralblatt at first remained as one of the few German-German collaborations of that time, edited by both the Academy of Sciences in Berlin (East) and the Heidelberg Academy of Sciences, with Springer remaining the commercial partner. This collaboration was terminated by the GDR in 1977. Currently, zbMATH has three editorial institutions:

- European Mathematical Society (EMS).
- Heidelberg Academy of Sciences.
- FIZ Karlsruhe.

These organisations are jointly responsible for the content and scientific quality of the database. The financial and organisational responsibilities lie with Fachinformationszentrum (FIZ) Karlsruhe, whereas Springer Verlag remains responsible for distribution and marketing.

FIZ Karlsruhe (Leibniz Institute for Information Infrastructure) is a member of the Leibniz Association, a group of independent German research institutes. As such, it receives funding from the federal and state governments but also generates substantial revenues of its own. The German Mathematical Society (DMV) was a founding member of FIZ Karlsruhe (established in 1977). Mathematicians are typically familiar with the Leibniz Association through the Oberwolfach Research Institute (MFO) or the Weierstrass Institute for Applied Analysis and Stochastics (WIAS) in Berlin, both of whom are member institutes.

This structure ensures that zbMATH is not run primarily as a commercial enterprise. The income generated is used to finance the Berlin office of zbMATH with its approximately 30 full-time positions, as well as to maintain and develop the necessary infrastructure for the database. After all, it is crucial that the (currently approximately) 120,000 mathematical publications that appear each year in different sources (such as research monographs, conference proceedings, collected volumes, more than 2,200 journals and roughly 1,000 book series) are documented and reviewed in a timely fashion. For this, the contributions of our 7,000 reviewers are essential. Without these reviewers, zbMATH would not be possible.

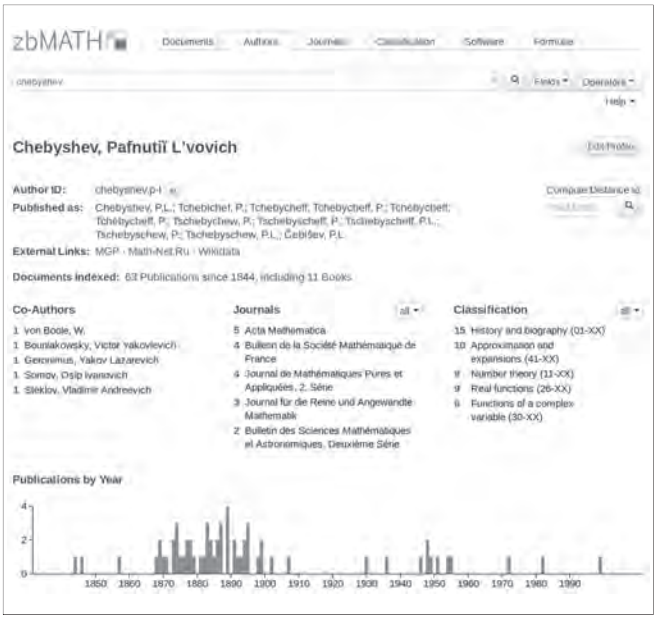
zbMATH caters for the needs of the mathematical community. As such it provides a number of free services:

- Free access for developing countries.
- Free access for individual members of the EMS.
- Free access to author and journal profiles.
- Free information on mathematical software through the database swMATH.
- Free access to three search results, in particular to individual reviews.

The transition from print version to modern database

When I was a young researcher, I went to the library once a week, typically on Friday afternoon, to check Zentral-

* This is the adapted English version of an article that appeared in Mitteilungen der DMV 24, No. 3, 140–143 (2016).



blatt and Mathematical Reviews for recently published articles. At that time, this meant consulting the printed versions, which filled awe-inspiring yellow (Zentralblatt) and red (Mathematical Reviews) shelves in the library. Now that the print versions have been terminated, this is history. Also, it is now the mathematics arXiv that provides, at least in my field of research, much of the information about the latest articles – long before they are published. However, it should be noted that the maths arXiv so far covers 3.5% of all publications listed in zbMATH (even for current research, this figure only stands at 20% [1]).

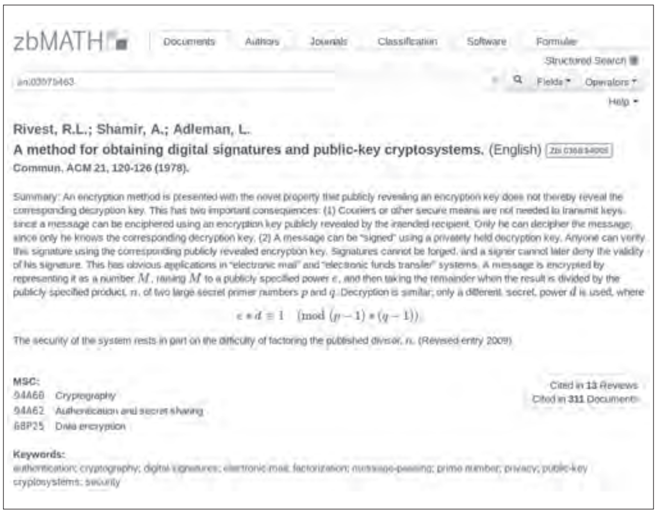
The transition from printed journal to modern database started under the aegis of Bernd Wegner and further important steps were taken in recent years when Gert-Martin Greuel was Editor-in-Chief. The challenges involved are considerable: the digitisation of the content from the 1980s and 1990s required a substantial investment and the same is true for making additional levels of information available.

The first of these is *author disambiguation*. Currently, the zbMATH author database comprises 950,000 entries with more than 6 million authorships, distributed among 3.6 million publications. In more than 20% of these cases, at least two authors share the same surname and abbreviated first name. Even with full names, it often happens that there are multiple possibilities. To take an example, the name "Alexander Schmidt" could be any of seven existing authors or, as an eighth possibility, it could refer to a new author. Similarly, there are many variants of names that could refer to the same person. This could, for example, be because a person has changed their name through marriage or it could be due to different transcriptions: the name Chebyshev, for example, can be found as 15 different variations.

Clearly, this work cannot be done solely intellectually. zbMATH combines advanced algorithms, information provided by users and editorial efforts in order to ensure optimally precise information. While the first version of the author database from 2008 still required substantial corrections, zbMATH has since advanced its algorithms and adapted them specifically to the needs of the zbMATH data. In addition, many manual checks have been carried out. The result is that the percentage of uncertain attributions has dropped to below 4% – and this refers not only to current publications but also to historic data, many of which come with a high share of uncertainty [2].

Another area of increasing importance concerns citation data. Combining references available in the database with full texts provides an additional tool, permitting us to trace mathematical developments. This often allows us to recognise unexpected connections or applications of our own research. Admittedly, it was MathSciNet which initiated this development in 2001, with zbMATH to follow somewhat later. However, in the last few years, the situation has improved significantly: while there were 3 million references available in 2010 for 150,000 articles, of which 30% were linked within the database, the figure now stands at 16 million references for more than 750,000 publications in the period 1873–2016, with 10.5 million linkages within the database. Due to the introduction of a new search interface, the information available through zbMATH can be accessed much more easily and comfortably, e.g. by using the new filter and profile functions.

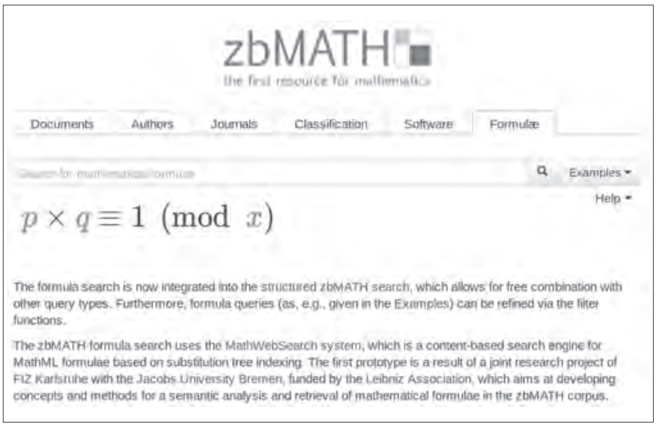
Meanwhile, zbMATH has introduced new functions that allow access to information beyond the realm of classical literature search. One example is the widely used platform swMATH, which originated from a joint project of the Leibniz Association in 2011–2013, involving MFO, FIZ Karlsruhe and other partners. There have been previous collections of mathematical software, such as the "Guide to Mathematical Software" of NIST or the "Oberwolfach References on Mathematical Software". However, it is only the timely connection with the accessible research literature that makes it possible to provide a service allowing not only the researching and highlight-



ing of software use but also access to information about what software has been used for comparable research problems.

When Gert-Martin Greuel took over as Editor-in-Chief, this database was developed further. Since 2013, swMATH has been run by FIZ Karlsruhe, which provides it as a free service to the community. Since then, the service has grown tremendously – it now covers over 15,000 software packages and provides approximately 125,000 records of use in research publications [3]. Under the aegis of the research campus MODAL, the Zuse Institute Berlin (ZIB) has been a cooperation partner of swMATH since 2015.

Another new feature of zbMATH is the search for mathematical formulae. Based on the MathML standardisation of zbMATH contents, this allows a search for contents that are often difficult to describe or locate based solely on a textual description. Variables allow the user to search for potentially substitutable terms. Recently, this feature has been extended to a formula search in more than 120,000 full arXiv texts (see the zbMATH column in this issue [4]). Even though this development is in its infancy, it allows us to glimpse the enormous potential that modern information technology can provide for mathematics. This has found expression in the vision of the Global Digital Mathematics Library: to provide an as complete as possible collection of all mathematical research in a highly standardised form, which, for example, relieves the mathematician of tedious routine tasks by providing access to suitable mathematical software via adapted interfaces [5].



Another important feature of a database such as zbMATH is completeness. This is a challenge that must not be underestimated. Each year, around 300 new journals apply to zbMATH asking to be listed, all of them claiming to publish serious research whose quality is assured by a peer review process. Clearly, some very good new journals have been founded in recent years. At the same time, it is also obvious that not all of the new journals conform to these strict requirements. Indeed, there are fewer than 100 journals a year that are added to zbMATH, although the overall number of journals is growing at an even lower rate, since some journals are discontinued and other journals no longer fulfil the required quality standards. It is a complex challenge to observe the ever-changing landscape of publications and to adjust the necessary decisions concerning their indexing. In addition to this, roughly 3,000 research monographs and conference proceedings are published every year and it has always been a top priority of zbMATH to provide coverage of these as complete as possible.

The Future

With the rapid development of information technology, the conditions under which a database like zbMATH operates are constantly changing. Thus, it is vital that zbMATH constantly reassesses where it stands and where it wants to go. An immediate task in hand is updating the MSC classification. While this system may have lost some of its historic importance as a tool for bibliographic classification, its importance has grown in connection with derived semantics, data analysis and profiling information. For this reason, it is of great importance that new branches of mathematics are adequately represented.

This revision will be undertaken jointly with MathSciNet, as announced at 7ECM and in the Newsletter of the EMS and the Notices of the AMS. The restructuring will be done on the basis of comments received from the mathematical community. You are invited to submit your suggestions either at msc2020.org or by email to feedback@msc2020.org.

There are, however, a number of much more *fundamental challenges*. Information and relevant data on mathematical publications can nowadays be obtained in many ways. Apart from the specialised service provided by the mathematics arXiv, this includes such different sources as Google, Google Scholar, Researchgate, Scopus or the Web of Science. Why then do we still need zbMATH or MathSciNet?

There are several special features concerning information about mathematical research. One is the longevity of mathematical results: the average halftime regarding the citation of mathematical publications is currently 15 years and indicators predict that this will grow even further [6]. Another is the immense importance of a consistent corpus and the necessity of error detection and correction. Finally, mathematical publications are characterised by a high degree of formalisation and an enormous density of information encoded in the mathematical language. It is high quality reviews, written by mathematical experts, summarising the results and putting them into the context of current research, that have hitherto proven an excellent tool in making this information accessible. In future, this will be further supported by the database for software (swMATH) and formula search. But there are many other aspects, and I would like to conclude by briefly mentioning some, which, I believe, will play an important role in the future.

Bibliometric data

We are all acutely aware that bibliometric data are playing an increasingly important role. Many of us are sceptical, and with good reason. At the same time, if we are honest, most of us also use them in some form or another, not least in hiring processes or peer reviews of research proposals. What is important is to know which data are reliable, and for which purposes and in which form they can be used without giving wildly distorted results. On one hand, this requires a reliable high quality database, such as zbMATH, and, on the other hand, it needs the expertise of how these data can be used and how they are to be interpreted. These are questions that can only be answered in collaboration with the mathematical community.

Research data

Research data and big data are buzz words that many mathematicians do not immediately connect with their own research. But this is no longer a tenable attitude. Clearly, mathematics can and will play a major role in the handling and analysis of big data. At the same time, mathematics has started producing its own research data on a large scale. Mathematical statistics is obviously concerned with data from diverse areas, ranging from medicinal data to data from highly sophisticated physical experiments; modelling and simulation requires and generates terabytes of data. Mathematical software, and the resulting research and benchmarks, has become its own ecosystem of data. In the meantime, we have a huge set of mathematical objects that have been collected and described in various forms: these range from integer sequences in the *On-line Encyclopedia of Integer Sequences*, mathematical functions in the *NIST Digital Library of Mathematical Functions*, manifolds in the Manifold Atlas, and lists of Calabi-Yau varieties and modular forms in the *L-functions and modular forms database*, to collections of formalised mathematical definitions, theorems and proofs in systems such as Coq, HOL or Mizar. To produce sustainable access to and linking of this information produces many questions to which we currently have, at best, rudimentary answers.

Non-textual material

Printed or electronic articles and books are now far from being the only ways in which mathematical information is documented and distributed. Numerous lectures (which have always been an essential form of mathematical communication) are now available on the internet, be it as videos or presentations. Some of these, but by no means all, are directly associated to traditional publications and can thus be linked with these. Should zbMATH take this up and incorporate this material in its database? The answer to this question will also depend on how the behaviour of users and working styles develops. The questions raised here concern most of the mathematical community and will, in the future, be discussed at various levels, in private discussions as well as in official committees of professional societies. zbMATH is prepared to face these challenges and I am looking forward to any comments or suggestions you may have.

In the wake of 7ECM, the EMS has conducted a zbMATH user survey. While a detailed analysis will be given in the zbMATH column of a subsequent issue, I would like to mention two interesting conclusions here. The first, and for us a gratifying result, is that the community values the great effort zbMATH has made over the last few years to update its web interface and to provide new services such as author and citation profiles and swMATH. The second, and this was not at all clear to us, is that formula search seems to be considered a valuable asset, in particular in connection with search in full texts. We look forward to giving a detailed analysis of the feedback in a future column of this newsletter.

References

[1] F. Müller and O. Teschke, Will all mathematics be on the arXiv (soon)? Eur. Math. Soc. Newsl. 99, 55–57 (2016).

[2] M. Jost, N. Roy and O. Teschke, Another update on the collaboration graph, Eur. Math. Soc. Newsl. 100, 58–61 (2016).

[3] G.-M. Greuel and W. Sperber: swMATH – ein neuer Service für die Suche nach mathematischer Software, Mitteilungen der DMV, 21, 12–13 (2013).

[4] F. Müller and O. Teschke, Discovering formulae in arXiv full-texts via zbMATH, Eur. Math. Soc. Newsl. 102, (2016).

[5] https://en.wikipedia.org/wiki/Global_digital_Mathematics_Library.
[6] Th. Bouche, O. Teschke and K. Wojciechowski, Time lag in mathematical references. Eur. Math. Soc. Newsl. 86, 54-55 (2012).

Klaus Hulek studied mathematics at the University of Munich and the University of Oxford. He completed his PhD in Erlangen in 1979 and spent the academic year 1982/83 at Brown University, Providence, RI, USA. He received his habilitation in Erlangen in 1984. Klaus Hulek was a professor of mathematics in Bayreuth from 1985 to 1990, when he moved to Hanover. He held the position of Vice-President of Research of Leibniz Universität Hannover from 2005 to 2014, and from 2007 to 2014 he represented the German Rectors' Conference (HRK) at the Research Policy Working Group of the European University Association (EUA). Klaus Hulek was a member of the Institute for Advanced Study (IAS) in Princeton in 2015. His field of research is algebraic geometry.

Report from the EMS Council Meeting, Berlin, 16–17 July 2016

Richard Elwes, EMS Publicity Officer

The EMS Council is our society's governing body and meets every two years, most recently in 2014 in Donostia/San Sebastián, Spain. When the year coincides with a European Congress of Mathematics (ECM), as happened this Summer, the council meeting traditionally immediately precedes the ECM. Thus, on 16–17 July 2016, 82 delegates gathered with 17 guests at Humboldt University in sunny Berlin, to hear reports, debate motions and hold elections, as well as to set the direction of our society for the coming years.

Reports & Finance

After welcoming the delegates and guests, EMS President Pavel Exner briefly reported on his activities. The Secretary Sjoerd Verduyn Lunel then summarised the activities of the Executive Committee over the last two years (reports from all its meetings have appeared in in this newsletter). The Treasurer Mats Gyllenberg followed with an account of the society's finances, including auditor reports for 2014–15. Broadly speaking, the society's financial position is very healthy. The EMS Publishing House supports the EMS by providing the newsletter and the Mathematical Surveys at no cost to the society. With interest rates currently low, around half the society's financial resources are currently invested in a scheme with a 'medium' risk profile. Given this positive situation, the treasurer proposed to increase the budget for scientific activities and not to increase membership fees. The council approved the budget for 2017–18 and also appointed auditors.

Membership

Discussion then turned to the topic of membership. The EMS currently has 2721 individual members (an increase from 2445 in 2014). After a presentation by the President of the Armenian Mathematical Union (AMU) Tigran Harutunyan, the council approved by acclamation the application of the AMU to become a full member of the EMS. Four new institutional members have also joined: the Department of Mathematical Sciences at Aalborg University (Denmark), the Basque Center for Applied Mathematics (Bilbao, Spain), the University of Primorska's Faculty of Mathematics, Natural Sciences and Information Technologies (Koper, Slovenia) and the Department of Mathematics, Stockholm University (Stockholm, Sweden).

By-laws

The council approved modifications to Rules 8 and 23 of the society's by-laws, along with a more significant change to Rule 30 on student membership, which now reads: "Anyone who is a student at the time of becoming an individual EMS member, whether PhD or in a more junior category, shall enjoy a three-year introductory period with membership fees waived. All the standard benefits will be granted during this time, except printed copies of the Newsletter."

Elections to the Executive Council

The day-to-day running of the society is by its executive committee of ten members, all elected by the council. On this occasion, there were vacancies for two vice-presidents and five members-at-large. Two candidates for vice-president were proposed by the executive committee: Volker Mehrmann and Armen Sergeev (both currently members of the executive committee). Each gave a short presentation before leaving the room. Both were then elected unopposed by a show of hands. Volker Mehrmann will therefore serve as vice-president for the term 2017–2018 and Armen Sergeev will serve for the term 2017–2020.

The elections for members-at-large were more competitive, with 13 candidates contesting 5 positions. Each candidate delivered a short presentation and delegates then voted in two rounds. The final result saw Nicola Fusco, Stefan Jackowski, Vincente Muñoz, Beatrice Pelloni and Betül Tanbay elected to the executive committee for the term 2017–2020. The president extended the society's sincere gratitude to all candidates who had put themselves forward.

ECMs 7 & 8

One major decision to be taken by the council was the venue of the 8th ECM, to be held in 2020. Two bids representing Seville (Spain) and Portorož (Slovenia) were presented and discussed. Whilst each was of an impressively high quality, the bids were strikingly different in style, presenting delegates with an intriguing dilemma. The president conveyed the society's thanks to everyone involved in preparing both bids and expressed regret that one must fail. He encouraged the unsuccessful bidders to try again in the future. In the subsequent vote, Portorož won by 45 votes to 33. Thus, the European mathematical community will gather for ECM8 on the beautiful Slovenian coast, 5–11 July 2020.

The council also heard from organiser Volker Mehrmann on the preparations for ECM7, starting imminently in Berlin. The delegates expressed their enthusiastic thanks to all the local organisers for their tireless efforts and for an exciting meeting in prospect.

Committees

The council next heard reports from the 11 standing committees of the EMS (excluding the executive committee): Applied Mathematics, Developing Countries, Education, Electronic Publishing, ERCOM (Scientific Directors of European Research Centres in the Mathematical Sciences), Ethics, European Solidarity, Meetings, Publications, Raising Public Awareness and Women. The members of these committees carry out many, varied and valuable tasks for the European mathematics community. The council expressed warm gratitude for all their hard work.

European Projects

The president reported on the latest developments around Horizon 2020, with the recent EU open consultation on mathematics due to be discussed in September. With Jean-Pierre Bourguignon (President of the European Research Council (ERC)) absent, the president also led a discussion on the ERC report, recalling that its inauguration was a successful outcome of lobbying by the Initiative Science Europe (ISE), of which the EMS is a member. With the midterm evaluation of Horizon 2020 due shortly, it is important that the ERC portfolio continues to grow and stabilise over the coming years.

Maria Esteban delivered a presentation on EU-MATHS-IN (the European Service Network of Mathematics for Industry and Innovation), reporting that the national networks that make up EU-MATHS-IN are now active and cooperative. On 15 July, shortly before the council meeting, there was some welcome news that the proposal "Mathematical Modelling, Simulation and Optimization for Societal Challenges with Scientific Computing" had been granted under the Horizon 2020 programme on user-driven e-infrastructure innovation.

The EMS is supporting a bid from Paris to host the International Congress of Mathematicians in 2022. If another serious European bid materialises, the society will support this as well.

Jiří Rákosník gave a presentation on the European Digital Mathematics Library (EuDML), reporting that a Scientific Advisory Board of EuDML with Frédéric Hélein as chair was installed earlier this year. The Encyclopedia of Mathematics (www.encyclopediaofmath.org) was also discussed and Vice-President Martin Raussen delivered a short presentation on a new collaboration between the EMS and MathHire (www.mathhire.org) to provide a new job portal for recruitment in mathematics.

Close

On behalf of all the delegates and guests, the president expressed the society's appreciation, to the local organisers, to Humboldt University and to those who had assisted by acting as scrutineers for the elections, for a very successful council meeting. He also thanked the delegates and guests for their participation and expressed hope that the discussions and decisions taken over the two days will set the society on a positive trajectory for the next two years.

50th Sophus Lie Seminar, Będlewo, Poland, September 2016

Katarzyna Grabowska (University of Warsaw, Poland)

The 50th jubilee Sophus Lie Seminar took place at the Mathematical Research and Conference Centre of the Institute of Mathematics, Polish Academy of Sciences. The centre is located in a beautiful 19th century palace in the village of Będlewo near Poznań, Poland. In the wonderful surroundings of the palace, gardens and nearby forest and lakes of the Greater Poland National Park, 75 participants from 21 countries worked on various aspects of Lie theory and celebrated the 25th anniversary and 50th meeting of the renowned seminar.

Mathematical Research and Conference Center in Będlewo, Poland. (All photos in this article by Janusz Grabowski.)

The Sophus Lie Seminar was founded around 1989–1990 after political changes in Eastern Europe made it possible to establish contact between mathematicians from the German Democratic Republic and the Federal Republic of Germany. The collaboration was initiated by mathematicians from four universities in Darmstadt, Erlangen, Greifswald and Leipzig. Initially, meetings of the seminar took place in Germany in one of the founding universities. Quite soon, however, the seminar started to grow, including researchers from more countries from both Eastern and Western Europe. Now it is clearly an international event, this year even intercontinental since participants came from Europe, North America and Asia.

Due to its special character, the 50th Seminar Sophus Lie was longer than usual. Instead of a two or three day weekend meeting, the conference lasted for full five days form Monday to Friday. There was the chance to listen to 15 invited lectures, with 12 contributed lectures and a poster session of 12 posters. There was a vote on the best poster and, on the last day, a talk from the winner. Invited speakers presented both their new achievements on various aspects of Lie theory as well as survey lectures to serve simultaneously as a summary and an introduction to research on specific topics. There were lectures on classical subjects of Lie theory such as representations of Lie groups and Lie algebras, addressing structural and classification questions and infinite dimensional Lie groups. Fairly new branches of mathematics associated to Lie theory were represented as well, e.g. Lie groupoids and algebroids, and the even more contemporary supergeometry and supergroups. For this jubilee meeting, participants could listen to talks about aspects of Lie theory that have not been covered at previous seminars, like lectures on Lie-Sheffers systems of differential equations. This branch of Lie theory is very close to works of Sophus Lie himself, since Lie groups were initially invented as a tool for understanding and solving systems of differential equations. One of the most interesting lectures, "Short SL(3)-structures on Lie algebras", was given by this year's European Mathematical Society distinguished speaker, Professor Ernest B. Vinberg from Moscow State University. The opportunity to listen to his lecture and discuss with him during the conference was a great honour and pleasure for all the members of this anniversary meeting. One of the invited lectures was given by Professor Karl Strambach, who was among the founders of the Sophus Lie Seminars 25 years ago. His historical lecture gave the participants an insight into this series of seminars.

Professor Ernest B. Vinberg.

It was a common opinion of many of the participants of the conference that the scientific level of the meeting was very high. It was therefore decided that it would be a good idea to share the lectures with the broader mathematical community. We will be working toward preparing a special volume of Banach Center Publications with the proceedings of the conference.

I would like to point out one more value of the 50th Sophus Lie Seminar that cannot be seen by looking at the programme of the conference or even reading the upcoming proceedings. As a member of the local organising committee, I was mostly occupied by practical matters, running from lecture room to reception and dining hall. And what I saw was that throughout the rooms of the conference centre, including the poster room, there were groups of people gathering by every piece of blackboard and discussing their work with each other. I am convinced that these informal meetings are perhaps even more important than the carefully prepared lectures and posters. I sincerely hope that besides the proceedings, there will be one more outcome of this jubilee seminar – new collaborations between people who had the chance to meet and share their interests with each other. Looking forward to future meetings of the Seminar Sophus Lie, we should keep in mind its history. It is probably unbelievable to younger participants of the seminar that not that long ago there were times when the possibility of meeting researchers with similar interests from countries from the other side of the Iron Curtain was a luxury, accessible to very few, if any.

Participants of the 50th Sophus Lie Seminar.

The organisation of the jubilee seminar would not have been possible without the financial support of several scientific institutions. The Scientific and Organising Committee is very grateful to the Banach Center of the Polish Academy of Sciences, the Warsaw Center of Mathematics and Computer Sciences, the University of Luxembourg, the European Mathematical Society and the Alexander von Humboldt Foundation for supporting this event. More details, including the programme of the conference and abstracts of all the talks and posters, can be found at the webpage http://50sls.impan.pl/.

Finally, let me say a few words about the International Banach Center. The centre was founded in 1972. Initially, it was established by seven academies of science of Eastern European countries. From the very beginning, it was a part of the Institute of Mathematics of the Polish Academy of Sciences. The aim of the centre has always been the promotion and stimulation of international cooperation in mathematics. After political changes, the agreement between academies was terminated but the centre continues to run successfully. Its scientific council consists of renowned mathematicians from the founding countries as well as representatives from the European Mathematical Society and the Polish Academy of Sciences. The palace in Będlewo became a part of the Banach Center at the beginning of the 21st century. The palace itself was renovated and new hotel buildings were added to create a lovely and quiet place to host international mathematical conferences. The centre offers accommodation and full board for about 80 participants. The staff of the conference centre are experienced, professional and very helpful. For the typically weekly conferences, groups of scientists can apply with a detailed project, including proposed participants, scientific and local organising committees, shape of conference and budget, and the desired time period (with a possible second choice). It is advised to look for financial support. If approved, the Banach Center can cover up to one third of the planned budget. The registration fee is then decided by the organisers, depending on what kind of additional support becomes available. The fee covers all local expenses (lodging and dining). The deadline for submitting applications is 15 April for the following year. In order to avoid time conflicts, it is advisable to send proposals as early as possible. If you plan to apply for a smaller conference, two of these can run simultaneously. Shorter conference applications are also welcome. Application forms and all details about procedures are available on the webpage of the Banach Center https://www.impan.pl/en/activities/banach-center. More information about Będlewo, including photos of the palace and the surroundings, can be found on the webpage https://www.impan.pl/en/activities/bedlewo-conference-center/about-center.

Katarzyna Grabowska [konieczn@fuw.edu.pl] is an assistant professor at the Department of Mathematical Methods in Physics, Department of Physics, University of Warsaw. Her current research interests lie in applications of differential geometry to classical and quantum physics. She worked as the Head of the Local Organising Committee of the 50th Sophus Lie Seminar. She wants to thank Alice Fialowski and Vladimir Salnikov for their help in the preparation of this document.

Selected Advances in Quantum Shannon Theory

Dedicated to the 100th Birthday of Claude E. Shannon

Nilanjana Datta (Statistical Laboratory, University of Cambridge, UK)

The last few years have witnessed various significant advances in quantum Shannon theory. In this article, we briefly review the salient features of three of them: a counterexample to the additivity conjecture, superactivation of the quantum capacity of a channel and one-shot quantum information theory. The first two pertain to information-transmitting properties of quantum channels whilst the third applies to a plethora of information-processing tasks, over and above information transmission.

The biggest hurdle in the path of information transmission is the presence of noise in communication channels, which can distort messages sent through them and necessitates the use of error-correcting codes. There is, however, a fundamental limit on the rate at which information can be transmitted reliably through a channel. The maximum rate is called the capacity of the channel and was originally evaluated in the so-called *asymptotic, memoryless (or i.i.d.) setting*. In this setting, it is assumed that the channel is: (*i*) available for an unlimited number of uses (say, n) and (*ii*) memoryless, i.e. there is no correlation in the noise acting on successive inputs to the channel. Classically, such a channel is modelled by a sequence of independent and identically distributed (i.i.d.) random variables. The capacity of the channel is the optimal rate at which information can be reliably transmitted through it in the asymptotic limit ($n \to \infty$).

The capacity of a memoryless classical channel was derived by Claude Shannon in his seminal paper of 1948 [1], which heralded the birth of the field of classical information theory. His *Noisy Channel Coding Theorem* gives an explicit expression for the capacity of a discrete memoryless channel $\mathcal{N}$. Such a channel can be completely described by its conditional probabilities $p_{Y|X}(y|x)$ of producing output y given input x, with X and Y denoting discrete random variables characterising the inputs and outputs of the channel. Shannon proved that the capacity $C(\mathcal{N})$ of such a channel is given by the formula

$$C(\mathcal{N}) = \max_{\{p_X(x)\}} I(X:Y), \qquad (1)$$

where $I(X:Y)$ denotes the mutual information of the random variables X and Y, and the maximisation is over all possible input probability distributions $\{p_X(x)\}$.

In contrast to a classical channel, a quantum channel has many different capacities. These depend on various factors, e.g. on the type of information (classical or quantum) being transmitted, the nature of the input states (entangled or not), the nature of the measurements made on the outputs of the channel (collective or individual) and whether any auxiliary resources are available to assist the transmission. Auxiliary resources, like prior shared entanglement between the sender and the receiver, can enhance the capacities of a quantum channel. This is in contrast to the case of a classical channel, where auxiliary resources, such as shared randomness between the sender and the receiver, fail to enhance the capacity.

Let us briefly recall some basic facts about quantum channels. For simplicity of exposition, we refer to the sender as Alice and the receiver as Bob. A quantum channel $\mathcal{N}$ is mathematically given by a linear, completely positive trace-preserving (CPTP) map, which maps states (i.e. density matrices) ρ of the input quantum system A to states of the output system B. More generally, $\mathcal{N} \equiv \mathcal{N}^{A\to B} : \mathcal{B}(\mathcal{H}_A) \to \mathcal{B}(\mathcal{H}_B)$, where $\mathcal{H}_A$ ($\mathcal{H}_B$) denote the Hilbert spaces associated with the system A (B) and, in this article, they are considered to be finite-dimensional. By Stinespring's dilation theorem, any such quantum channel can be seen as an isometry followed by a partial trace, i.e. there is an auxiliary system E, usually referred to as the *environment*, and an isometry $U_\mathcal{N} : \mathcal{H}_A \to \mathcal{H}_B \otimes \mathcal{H}_E$, such that $\mathcal{N}(\rho) = \mathrm{Tr}_E\, U_\mathcal{N} \rho U_\mathcal{N}^\dagger$. This, in turn, induces the complementary channel $\mathcal{N}_c \equiv \mathcal{N}_c^{A\to E} : \mathcal{B}(\mathcal{H}_A) \to \mathcal{B}(\mathcal{H}_E)$ from the system A to the environment E, given by $\mathcal{N}_c(\rho) = \mathrm{Tr}_B\, U_\mathcal{N} \rho U_\mathcal{N}^\dagger$. Physically, the complementary channel captures the environment's view of the channel. A quantum channel is said to be *anti-degradable* if there exists a CPTP map $\mathcal{E} : \mathcal{B}(\mathcal{H}_E) \to \mathcal{B}(\mathcal{H}_B)$ so that the composition of the maps $\mathcal{N}_c$ and $\mathcal{E}$ satisfies the identity $\mathcal{N} = \mathcal{E} \circ \mathcal{N}_c$. So, an eavesdropper (Eve), who has access to the environment of the channel, can simulate the channel from A to B by locally applying the map $\mathcal{E}$. An anti-degradable channel has zero quantum capacity since it would otherwise violate the so-called *no-cloning theorem*, which forbids the creation of identical copies of an arbitrary unknown quantum state. This can be seen as follows. Suppose there is an encoding and decoding scheme for Alice to communicate quantum information reliably at a non-zero rate over such a channel. Then, by acting on the output that she receives by the CPTP map $\mathcal{D} \circ \mathcal{E}$, where $\mathcal{D}$ is the decoding map that Bob uses, Eve could obtain the quantum information sent by Alice. However, the ability for both Bob and Eve to obtain Alice's information violates the no-cloning theorem. Hence the quantum capacity of an anti-degradable channel must be zero. In contrast, a quantum channel is said to be degradable if there exists a CPTP map $\mathcal{E}' : \mathcal{B}(\mathcal{H}_B) \to \mathcal{B}(\mathcal{H}_E)$ such that $\mathcal{N}_c = \mathcal{E}' \circ \mathcal{N}$. In this case, Bob can simulate the complementary channel from A to E by locally applying the map $\mathcal{E}'$.

The problem of determining the different capacities of a quantum channel have only been partially resolved, in the sense that the expressions obtained for most of them thus far are regularised ones. They are therefore intractable and cannot be used to determine the capacities of a given channel in any effective way. If entanglement between inputs to successive uses of a quantum channel is not allowed, its capacity for transmitting classical information is given by an entropic quantity, $\chi^*(\mathcal{N})$, called its Holevo capacity [2]. The general classical capacity of a quantum channel, in the absence of auxiliary resources and without the above restriction, is given by the following regularised expression:

$$C(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n}\chi^*(\mathcal{N}^{\otimes n}). \tag{2}$$

Similarly, the capacity $Q(\mathcal{N})$ of a quantum channel for transmitting quantum information (in the absence of auxiliary resources) is also known [3] to be given by a regularised expression:

$$Q(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n} I_c(\mathcal{N}^{\otimes n}), \tag{3}$$

where, for any quantum channel $\widetilde{\mathcal{N}}$, $I_c(\widetilde{\mathcal{N}})$ is an entropic quantity referred to as its coherent information.

Another important capacity of a quantum channel is its private capacity $P(\mathcal{N})$, which is the maximum rate at which classical information can be sent through it in a way such that an eavesdropper, Eve, who has access to the environment of the channel, cannot infer the transmitted information. The private classical capacity $P(\mathcal{N})$ of a quantum channel is also given by the regularisation of an entropic quantity, which we denote $P^{(1)}(\mathcal{N})$. Unfortunately, these intractable, regularised expressions are in general useless for computing the actual capacities of a channel. Regarding the quantum capacity, an exception to this is provided by so-called degradable channels, for which the coherent information is additive and so the quantum capacity reduces to a single-letter formula. Other than the Holevo capacity, there are only a few other capacities which have a single-letter (and hence not-regularised) expression for any arbitrary quantum channel. The most important of these is the entanglement-assisted classical capacity [4], which is the maximum rate of reliable classical communication when Alice and Bob are allowed to make use of entangled states that they initially share.

An important property of the capacity of a classical channel is its additivity on the set of channels. Given two classical channels $\mathcal{N}_1$ and $\mathcal{N}_2$, the capacity of the product channel $\mathcal{N}_1 \otimes \mathcal{N}_2$ satisfies $C(\mathcal{N}_1 \otimes \mathcal{N}_2) = C(\mathcal{N}_1) + C(\mathcal{N}_2)$. In fact, many important questions in information theory can be reduced to the purely mathematical question of additivity of certain entropic functions on the set of channels. In particular, the regularised expressions for the classical, quantum and private capacities of a quantum channel $\mathcal{N}$ would reduce to tractable single-letter expressions if its Holevo capacity, coherent information and $P^{(1)}(\mathcal{N})$ were respectively additive. However, it has been proved that the coherent information and $P^{(1)}(\mathcal{N})$ are not necessarily additive for all channels. It was conjectured that the Holevo capacity of a quantum channel is indeed additive, i.e. for any two quantum channels $\mathcal{N}_1$ and $\mathcal{N}_2$,

$$\chi^*(\mathcal{N}_1 \otimes \mathcal{N}_2) = \chi^*(\mathcal{N}_1) + \chi^*(\mathcal{N}_2).$$

This conjecture is directly related to the important question: *'Can entanglement between successive input states boost classical communication through a memoryless quantum channel?'* The answer to this question is "no" if the Holevo capacity of the channel is additive, since in this case $C(\mathcal{N}) = \chi^*(\mathcal{N})$, i.e. the general classical capacity reduces to the classical capacity evaluated under the restriction of unentangled input states. The additivity conjecture had been proved for several channels (see, for example, [5] and references therein). However, proving that it is true for *all* quantum channels had remained an important open problem for more than a decade. Shor [6] provided useful insights into the problem by proving that the additivity conjecture for the Holevo capacity was equivalent to additivity-type conjectures for three other quantities arising in quantum information theory, in the sense that if any one of these conjectures is always true then so are the others. One of these conjectures concerns the additivity of the minimum output entropy (MOE) of a quantum channel, which is defined as

$$H_{\min}(\mathcal{N}) = \min_{\rho} H(\mathcal{N}(\rho)),$$

where, for any state σ, $H(\sigma) := -\operatorname{Tr}(\sigma \log \sigma)$ is its von Neumann entropy. The additivity conjecture for the MOE is that, for any pair of quantum channels $\mathcal{N}_1$, $\mathcal{N}_2$, the minimum entropy of the product channel $\mathcal{N}_1 \otimes \mathcal{N}_2$ satisfies

$$H_{\min}(\mathcal{N}_1 \otimes \mathcal{N}_2) = H_{\min}(\mathcal{N}_1) + H_{\min}(\mathcal{N}_2). \tag{4}$$

Note that we always have $\leq$ in (4). This can be seen by considering the product state $\rho_1 \otimes \rho_2$ as input to $\mathcal{N}_1 \otimes \mathcal{N}_2$, with ρ_1 and ρ_2 being the minimisers for MOEs of $\mathcal{N}_1$ and $\mathcal{N}_2$ respectively. The conjecture amounts to the claim that we cannot get a smaller MOE by entangling the inputs to $\mathcal{N}_1 \otimes \mathcal{N}_2$.

These longstanding additivity conjectures were finally resolved in 2008 by Hastings [7], who built on prior work by Hayden and Winter [8]. He proved the existence of a pair of channels for which the above conjecture is false. By Shor's equivalence, this in turn implied that all the additivity conjectures (including that for the Holevo capacity) are false. Hence, we can conclude that there exist quantum channels for which using entangled input states can indeed enhance the classical capacity.

The product channel considered by Hastings has the form $\mathcal{N} \otimes \overline{\mathcal{N}}$, where $\mathcal{N}$ is a special channel called a *random unitary channel*, and $\overline{\mathcal{N}}$ is its complex conjugate. This means that there are positive numbers $\nu_1, \nu_2, \ldots, \nu_d$, with $\sum_{i=1}^{d} \nu_i = 1$, and unitary $n \times n$ matrices $U_1, U_2, \ldots, U_d$, chosen at random with respect to the Haar measure, such that for any input state ρ,

$$\mathcal{N}(\rho) = \sum_{i=1}^{d} \nu_i U_i \rho U_i^\dagger \quad ; \quad \overline{\mathcal{N}}(\rho) = \sum_{i=1}^{d} \nu_i \overline{U}_i \rho \overline{U}_i^\dagger .$$

The probabilities ν_i are chosen randomly and depend on the integers n and d, where n is the dimension of the input space of the channel and d is the dimension of its environment. Hastings' main result is that for n and d large enough, there are random unitary channels for which $H_{\min}(\mathcal{N} \otimes \overline{\mathcal{N}}) < H_{\min}(\mathcal{N}) + H_{\min}(\overline{\mathcal{N}})$, thus disproving (4).

A key ingredient of Hastings' proof is the relative values of the dimensions, namely $n >> d >> 1$. The details of Hastings' original argument were elucidated later by Fukuda, King and Moser [9]. These authors also derived explicit lower bounds to the input, output and environment dimensions of a quantum channel for which the additivity conjecture is violated. A simplified proof of Hastings' result was given by Brandao and Horodecki [10] in the framework of concentration of measure. They also proved non-additivity for the overwhelming majority of channels consisting of a Haar random isometry followed by partial trace over the environment, for an environment dimension much bigger than the output dimension, thus extending the class of channels for which additivity can be shown to be violated. Remarkably, in 2010, Aubrun, Szarek and Werner [11] proved that Hastings' counterexample can be readily deduced from a version of *Dvoretzky's theorem*, which is a fundamental result of Asymptotic Geometric Analysis – a field of mathematics concerning the behaviour of geometric parameters associated with norms in $\mathbb{R}^n$ (or equivalently, with convex bodies) when n becomes large. However, the violation to additivity in Hastings' example is numerically small and the question of how strong a violation of additivity is possible is the subject of active research.

The year 2008 also saw the discovery of a startling phenomenon in quantum information theory, again related to the question of additivity of capacities. Smith and Yard [12] proved that there are pairs of quantum channels each having zero quantum capacity but which have a non-zero quantum capacity when used together. Hence, even though each channel in such a pair is by itself useless for sending quantum information, they can be used together to send quantum information reliably. This phenomenon was termed "superactivation", since the two channels somehow "activate" each other's hidden ability to transmit quantum information. Superactivation is a purely quantum phenomenon because classically if two channels have zero capacity, the capacity of the joint channel must also be zero. This follows directly from the additivity of the capacity of a classical channel, which in turn ensures that the capacity of a classical channel is an intrinsic measure of its information-transmitting properties. In the quantum case, in contrast, the possibility of superactivation implies that the quantum capacity of a channel is strongly non-additive and does not adequately characterise its ability to transmit quantum information, since the usefulness of a channel depends on what other channels are also available. A particular consequence of this phenomenon is that the set of quantum channels with zero quantum capacity is not convex.

Superactivation of quantum capacity continues to be the subject of much research and is still not completely understood. However, it seems to be related to the existence of channels, called "private Horodecki channels", which have zero quantum capacity but positive private capacity. The key ingredient of Smith and Yard's proof of superactivation is a novel relationship between two different capacities of a quantum channel $\mathcal{N}$, namely, its private capacity $P(\mathcal{N})$ and its assisted capacity $Q_A(\mathcal{N})$. The latter is the quantum capacity of the product channel $\mathcal{N} \otimes \mathcal{A}$, where $\mathcal{A}$ is a *symmetric channel*. Such a channel maps symmetrically between its output and its environment, i.e. for any input state ρ, the joint state $\sigma_{BE} := U_{\mathcal{A}} \rho U_{\mathcal{A}}^\dagger$ of the output and the environment after the action of the channel $\mathcal{A}$ is invariant under the interchange of B and E. A symmetric side channel is anti-degradable and hence has zero quantum capacity. Smith and Yard proved that

$$Q_{\mathcal{A}}(\mathcal{N}) \geq \frac{1}{2} P(\mathcal{N}).$$

This in turn implies that any private Horodecki channel, $\mathcal{N}_H$, has a positive assisted capacity and hence the two zero-quantum-capacity channels $\mathcal{N}_H$ and $\mathcal{A}$ exhibit superactivation:

$$Q_A(\mathcal{N}_H) = Q(\mathcal{N}_H \otimes \mathcal{A}) > 0.$$

The particular symmetric side channel that Smith and Yard considered was a 50% *erasure channel*, which, with equal probability, faithfully transmits the input state or outputs an erasure flag.

Later, Brandao, Oppenheim and Strelchuk [13] proved that superactivation even occurs for pairs of channels $(\mathcal{N}_H, \mathcal{N})$ where $\mathcal{N}$ is anti-degradable but not necessarily symmetric. Specifically, they proved the occurrence of superactivation for two different choices of $\mathcal{N}$: (i) an erasure channel that outputs an erasure flag with probability $p \in [1/2, 1)$ and faithfully transmits the input state otherwise; and (ii) a depolarising channel that completely randomises the input state with probability $p \in [0, 1/2]$ and faithfully transmits the input state otherwise. It is known that the output of any arbitrary quantum channel can be mapped to that of a depolarising channel by an operation known as "twirling". The latter consists of Alice applying some randomly chosen unitary on the input state before sending it through the channel and informing Bob as to which unitary operator U she used, with Bob subsequently acting on the output state of the channel by the inverse operator $U^\dagger$. This special feature of the depolarising channel and the fact that it can be used for superactivation, suggests that superactivation is a rather generic effect. Superactivation has also been proven for other capacities of a quantum channel (see, for example, [14] and references therein), namely its zero-error classical and quantum capacities, which are, respectively, the classical and quantum capacities evaluated under the requirement that the probability of an error being incurred in transmitting the information is strictly zero (and doesn't just vanish asymptotically).

All the capacities mentioned above were originally evaluated in the limit of asymptotically many uses of a memoryless channel. In fact, optimal rates of most information-processing tasks, including transmission and compression of information, and manipulation of entanglement, were originally evaluated in the asymptotic, memoryless setting. As mentioned above, in this setting, one assumes that there is no correlation in successive uses of resources (e.g. information sources, channels and entanglement resources) employed in the tasks, and one requires the tasks to be achieved perfectly in the limit of asymptotically many uses of the resources. These asymptotic rates, e.g. the various capacities discussed above, are seen to be given in terms of entropic functions that can all be derived from a single parent quantity, namely, the quantum relative entropy.

In reality, however, the assumption of resources being uncorrelated and available for an unlimited number of uses is not necessarily justified. This is particularly problematic in cryptography, where one of the main challenges is dealing with an adversary who might pursue an *arbitrary* (and unknown) strategy. In particular, the adversary might manipulate resources (e.g. a communications channel) and introduce undesired correlations. A more general theory of quantum information-processing tasks is instead obtained in the so-called *one-shot scenario* in which resources are considered to be finite and possibly correlated. Moreover, the information-processing tasks are required to be achieved only up to a finite accuracy, i.e. one allows for a fixed, non-zero but small error tolerance. This also corresponds to the scenario in which experiments are performed since channels, sources and entanglement resources available for practical use are typically finite and correlated, and transformations can only be achieved approximately.

The last few years have witnessed a surge of research leading to the development of *one-shot quantum information theory*. The birth of this field can be attributed to Renner (see [15] and references therein) who introduced a mathematical framework, called the *smooth entropy framework,* which facilitated the analysis of information-processing tasks in the one-shot scenario. He and his collaborators introduced new entropy measures of states, called *smooth min- and max-entropies*, which depend on a parameter (say, ε), called the smoothing parameter. The smooth entropies $H^{\varepsilon}_{\min}(\rho)$ and $H^{\varepsilon}_{\max}(\rho)$ of a state ρ can be defined as optimisations of the relevant non-smooth quantities, the (non-smooth) min- and max-entropies, over a ball $B^{\varepsilon}(\rho)$ of neighbouring states, which are at a distance of at most ε from ρ, measured in an appropriate metric. For a bipartite state ρ_{AB}, they also define conditional min- and max-entropies.

Subsequently, it was proved (see, for example, [16]) that these conditional and unconditional smooth min- and max-entropies characterise the optimal rates of various information-processing tasks in the one-shot scenario, with the smoothing parameter corresponding to the allowed error tolerance. For example, the one-shot ε-error quantum capacity of a channel, which is the maximum amount of quantum information that can be transmitted over a single use of a quantum channel with an error tolerance of ε, has been proven to be given in terms of a smooth conditional max-entropy [22, 17]. Note that a single use of a channel can itself correspond to a finite number of uses of a channel with arbitrarily correlated noise. Hence the one-shot analysis indeed includes the consideration of finite, correlated resources. Furthermore, one-shot rates of all the different information-processing tasks studied thus far readily yield the corresponding known rates in the asymptotic limit, in the case of uncorrelated (i.e. memoryless) resources. Moreover, they also yield asymptotic rates of tasks involving correlated resources via the so-called Quantum Information Spectrum method (see, for example, [18] and references therein). Hence, one-shot quantum information theory can be viewed as the fundamental building block of quantum information theory and its development has opened up various new avenues of research.

In [20], we defined a generalised relative entropy called the *max-relative entropy*, from which the min- and max-entropies can be readily obtained, just as the ordinary quantum (i.e. von Neumann) entropies are obtained from the quantum relative entropy. Hence, the max-relative entropy plays the role of a parent quantity for optimal rates of various information-processing tasks in the one-shot scenario, analogous to that of the quantum relative entropy in the asymptotic, memoryless scenario. Moreover, it has an interesting operational interpretation, being related to the optimal Bayesian error probability in determining which one, of a finite number of known states, a given quantum system is prepared in. The max-relative entropy also leads naturally to the definition of an entanglement monotone, which is seen to have an interesting operational interpretation in the context of entanglement manipulation [19]. The different information-processing tasks in the one-shot scenario were initially studied separately. However, we subsequently proved [22] that a host of these tasks can be related to each other and conveniently arranged in a family tree, thus yielding a unifying mathematical framework for analysing them. Recently, we introduced a two-parameter family of generalised relative entropies, called the $\alpha - z$ relative Rényi entropies, from which the various different relative entropies (including the quantum relative entropy and the max-relative entropy) that arise in quantum information theory can be derived. This family provides a unifying framework for the analysis of properties of these different relative entropies, which are both of mathematical interest and of operational significance.

References

[1] C. E. Shannon, *Bell System Technical Journal*, **27**, 379 (1948).
[2] A. S. Holevo, *IEEE Trans. Inf. Theory*, **44**, 269 (1998); B. Schumacher & M. D. Westmoreland, *Phys. Rev. A*, **56**, 131 (1997).
[3] I. Devetak, *IEEE Trans. Inf. Theory*, **51**, **44** (2005); S. Lloyd, *Phys. Rev. A*, **55**, 1613 (1997); P. W. Shor, MSRI Workshop on Quantum Computation (2002).
[4] C. H. Bennett, P. W. Shor, J. A. Smolin & A. V. Thapliyal, *IEEE Trans. Inf. Theory*, **48**, 2637 (2002); A. S. Holevo, *J. Math. Phys.*, **43**, 4326 (2002).
[5] C. King, *J. Math. Phys.* **43**, 4641 (2002); *IEEE Trans. Inf. Theory*, **49**, 221, 2003; C. King, K. Matsumoto, M. Nathanson and M. B. Ruskai, *Markov Process and Related Fields*, **13**, 391 (2007); N. Datta, M. Fukuda and A. S. Holevo, *Quan. Inf. Proc.*, **5**, 179 (2006).
[6] P. W. Shor, *Comm. Math. Phys.* **246**, 453 (2004).
[7] M. B. Hastings, *Nature Physics* **5**, 255 (2009).
[8] P. Hayden & A. Winter, *Comm. Math. Phys.* **284**, 263 (2008).
[9] M. Fukuda, C. King & D. Moser, *Comm. Math. Phys.* **296**, 111 (2010). 2007.
[10] F. G. S. L. Brandao & M. Horodecki, *Open Syst. Inf. Dyn.* **17**, 31 (2010).
[11] G. Aubrun, S. Szarek & E. Werner, *Comm. Math. Phys.* **305**, 85 (2011).
[12] G. Smith & J. Yard, *Science* **321**, 1812 (2008); G. Smith, J. A. Smolin, & J. Yard, *Nature Photonics*, **5**, 624 (2011).
[13] F. G. S. L. Brandao, J. Oppenheim & S. Strelchuk, *Phys. Rev. Lett.* **108**, 040501 (2012).

[14] J. Chen, T. S. Cubitt, A. W. Harrow & G. Smith, *Phys. Rev. Lett.* **107**, 250504 (2011); J. Chen, T. S. Cubitt & A. W. Harrow, *IEEE Trans. Inf. Th.*, **57**, 8114 (2011).

[15] R. Renner, *PhD thesis*, ETH Zurich, 2005; R. Renner & R. Koenig, *Proc. of TCC* 2005, LNCS, Springer, **3378** (2005).

[16] F. Buscemi & N. Datta, *J. Math. Phys.*, **51**, 102201, 2010; F. Buscemi & N. Datta, *Phys. Rev. Lett.* **106**, 130503 (2011), F. Buscemi & N. Datta, *IEEE Trans. Inf. Th.*, **59**, 1940 (2013); M. Berta, M. Christandl & R. Renner, *Commun. Math. Phys.*, **306**, 579 (2011); L. Wang & R. Renner, *Phys. Rev. Lett.*, **108**, 200501 (2012); J. M. Renes & R. Renner, *IEEE Trans. Inf. Th.*, **58**, 1985 (2012); N. Datta & M. H. Hsieh, *IEEE Trans. Inf. Th.*, **59**, 1929 (2013).

[17] F. Buscemi & N. Datta, *IEEE Trans. Inf. Theory*, **56**, 1447 (2010).

[18] N. Datta & R. Renner *IEEE Trans. Inf. Theory* **55**, 2807 (2009).

[19] F. Brandao & N. Datta, *IEEE Trans. Inf. Theory*, **57**, 1754 (2011).

[20] N. Datta, *IEEE Trans. Inf. Theory*, **55**, 2816 (2009).

[21] N. Datta & M.-H. Hsieh, *New J. Phys.*, **13** 093042 (2011).

[22] K. M. R. Audenaert & N. Datta *Jour. Math. Phys.*, **56**, 022202 (2015).

Nilanjana Datta [n.datta@statslab.cam.ac.uk] is an affiliated lecturer of the Statistical Laboratory, Centre for Mathematical Sciences, University of Cambridge, and a fellow of Pembroke College. She completed her PhD in mathematical physics at ETH Zurich in 1996. She has been working on quantum information theory since 2001 and has written papers on various topics in this field. These include quantum channels and the additivity problem, quantum data compression, perfect transfer of quantum states and entanglement theory. In the last few years, her research has focused on developing one-shot quantum information theory, which concerns the analysis of information processing tasks involving finite, correlated resources.

Quantum Footprints of Symplectic Rigidity

Leonid Polterovich (Tel Aviv University, Tel Aviv, Israel)

Suddenly the result turned out completely different from what he had expected: again it was 1 + 1 = 2. *"Wait a minute!" he cried out, "Something's wrong here." And at that very moment, the entire class began whispering the solution to him in unison: "Planck's constant! Planck's constant!"*

After M. Pavic, Landscape Painted with Tea, 1988

In this note, we discuss an interaction between symplectic topology and quantum mechanics. The interaction goes in both directions. On one hand, ideas from quantum mechanics give rise to new notions and structures on the symplectic side and, furthermore, quantum mechanical insights lead to useful symplectic predictions when topological intuition fails. On the other hand, some phenomena discovered within symplectic topology admit a meaningful translation into the language of quantum mechanics, thus revealing quantum footprints of symplectic rigidity.

1 What is . . . symplectic?

A symplectic manifold is an even-dimensional manifold M^{2n} equipped with a closed differential 2-form ω that can be written as $\sum_{i=1}^{n} dp_i \wedge dq_i$ in appropriate local coordinates (p, q). For an oriented surface $\Sigma \subset M$, the integral $\int_\Sigma \omega$ plays the role of a generalised area, which, in contrast to the Riemannian area, can be negative or vanish.

To have some interesting examples in mind, think of surfaces with an area form and their products, as well as complex projective spaces equipped with the Fubini-Study form, and their complex submanifolds.

Symplectic manifolds model the phase spaces of systems of classical mechanics. Observables (i.e. physical quantities such as energy, momentum, etc.) are represented by functions on M. The states of the system are encoded by Borel probability measures μ on M. The simplest states are given by the Dirac measure δ_z concentrated at a point $z \in M$.

The laws of motion are governed by the *Poisson bracket*, a canonical operation on smooth functions on M, given by $\{f, g\} = \sum_j \left(\frac{\partial f}{\partial q_j}\frac{\partial g}{\partial p_j} - \frac{\partial f}{\partial p_j}\frac{\partial g}{\partial q_j}\right)$. The evolution of the system is determined by its energy, a time-dependent function $f_t : M \to \mathbb{R}$ called its *Hamiltonian*. Hamilton's famous equation describing the motion of a system is given, in the Heisenberg picture, by $\dot{g}_t = \{f_t, g_t\}$, where $g_t = g \circ \phi_t^{-1}$ stands for the time evolution of an observable function g on M under the Hamiltonian flow ϕ_t. The maps ϕ_t are called *Hamiltonian diffeomorphisms*. They preserve the symplectic form ω and constitute a group with respect to composition.

In the 1980s, new methods, such as Gromov's theory of pseudo-holomorphic curves and the Floer-Morse theory on loop spaces, gave birth to "hard" symplectic topology. It detected surprising symplectic rigidity phenomena involving symplectic manifolds, their subsets and diffeomorphisms. A number of recent advances show that there is yet another manifestation of symplectic rigidity taking place in function spaces associated to a symplectic manifold. Its study forms the subject of function theory on symplectic manifolds, a rapidly evolving area whose development has led to the interactions with quantum mechanics described below.

2 The non-displaceable fiber theorem

In 1990, Hofer [21] introduced an intrinsic "small scale" on a symplectic manifold: a subset $X \subset M$ is called *displaceable* if there exists a Hamiltonian diffeomorphism ϕ such that $\phi(X) \cap X = \emptyset$.

Let us illustrate this notion in the case when $M = S^2$ is the two-dimensional sphere equipped with the standard area form. Any disc lying in the upper hemisphere is displaceable: one can send it to the lower hemisphere by a rotation. However, the equator (a simple closed curve splitting the sphere into two discs of equal area) is non-displaceable by any area-preserving transformation (see Figure 1). This example demonstrates the contrast between symplectic "smallness" and measure theoretic "smallness": the equator has measure 0, yet it is large from the viewpoint of symplectic topology.

The central result discussed in this note brings together (non)-displaceability and Poisson commutativity.

Theorem 2.1 (Non-displaceable fiber theorem, [12])**.** Let $\vec{f} = (f_1, \ldots, f_N) : M \to \mathbb{R}^N$ be a smooth map of a closed symplectic manifold M whose coordinate functions f_i pairwise Poisson commute: $\{f_i, f_j\} = 0$. Then, $\vec{f}$ possesses a non-displaceable fiber: for some $w \in \mathbb{R}^N$, the set $\vec{f}^{-1}(w)$ is non-empty and non-displaceable.

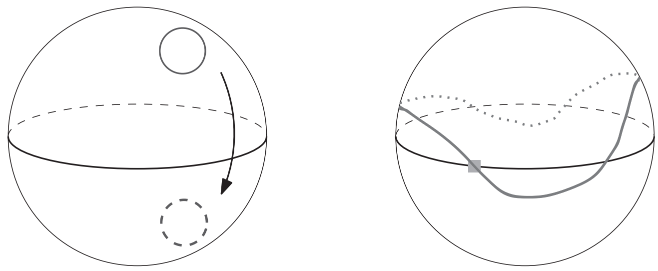


Figure 1. (Non)-displaceability on the sphere

Maps with pairwise commuting components naturally arise in the theory of integrable systems and in Hamiltonian tori actions (the moment maps). Roughly speaking, the theorem above states that each such map necessarily possesses a symplectically large fiber. Let us note that interesting classes of maps that necessarily possess "large" fibers appear in several seemingly remote areas of mathematics, from combinatorics to Riemannian geometry [20]. It would be interesting to explore this analogy.

Detecting non-displaceability of subsets of symplectic manifolds is a classical problem going back to Arnold's seminal Lagrangian intersections conjecture. In fact, the very existence of subsets that can be displaced by a volume preserving diffeomorphism but not by a Hamiltonian one is a manifestation of symplectic rigidity. Theorem 2.1 provides a useful tool in the following situation. Assume that we know *a priori* that all but possibly one fiber of a map $\vec{f}: M \to \mathbb{R}^N$ with Poisson-commuting components are displaceable. Then that particular fiber is necessarily non-displaceable.

As an illustration, assume that $M = S^2 \times S^2$ is the product of two spheres, and $\vec{f} = (z_1, z_2)$, where z_i is the height function on the i-th copy of S^2. One can see that all the fibers, except possibly the *Clifford torus* L given by the product of the equators, are displaceable (see Figure 2). Therefore, L is non-displaceable (see [29]).

The non-displaceable fiber theorem has the following equivalent formulation.

Theorem 2.2 (Rigidity of partitions of unity, [13])**.** A finite open cover of a closed symplectic manifold by open displaceable sets does not admit a Poisson-commuting partition of unity.

Interestingly enough, both the proof and the interpretation of this result involve quantum mechanics.

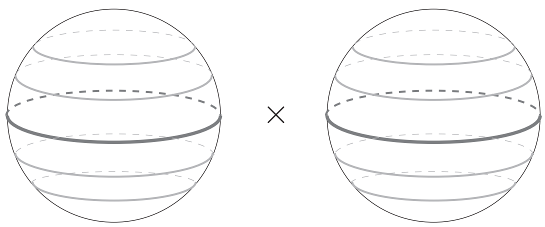


Figure 2. The Clifford torus

3 Encountering quantum mechanics

The mathematical model of quantum mechanics starts with a complex Hilbert space H. In what follows, we shall focus on finite-dimensional Hilbert spaces only, as they are quantum counterparts of closed symplectic manifolds. Observables are represented by Hermitian operators whose space is denoted by $\mathcal{L}(H)$. The states are provided by *density operators*, i.e. positive trace-one operators $\rho \in \mathcal{L}(H)$. *Pure states* are given by rank-one projectors. They are usually identified with the unit vectors in H generating their images and defined up to a phase factor.

Quantization is a formalism behind quantum-classical correspondence, a fundamental principle stating that quantum mechanics contains the classical one as a limiting case when the Planck constant $\hbar \to 0$. Mathematically, the correspondence in question is a linear map $f \mapsto T_\hbar(f)$ between smooth functions on a symplectic manifold and Hermitian linear operators on a complex Hilbert space H depending on $\hbar$. The dimension of H grows to ∞ as $\hbar \to 0$. The map $T_\hbar$ is assumed to satisfy a number of axioms, some of which are summarised in Table 1. Let us emphasise that the quantum-classical correspondence is not precise. It holds true up to an error that is small with $\hbar$.

Table 1. Quantum-classical correspondence

	Classical	Quantum
	Symplectic manifold (M, ω)	$\mathbb{C}$-Hilbert space H
Observables	$f \in C^\infty(M)$	$T_\hbar(f) \in \mathcal{L}(H)$
States	Probability measures on M	$\rho \in \mathcal{L}(H), \rho \geq 0, \mathrm{trace}(\rho) = 1$
Bracket	$\{f, g\}$	$-(i/\hbar)[T_\hbar(f), T_\hbar(g)]$

In finite-dimensional quantum mechanics, observables take a finite number of values only. Given an observable A, let $A = \sum_{i=1}^k \lambda_i P_i$ be its spectral decomposition, where the P_i's are pairwise orthogonal projectors. According to the main statistical postulate, in a state ρ the observable A takes the values λ_i with probabilities $\mathrm{trace}(P_i\rho)$.

The finiteness is well illustrated by the famous Stern-Gerlach experiment dealing with the deflection of a beam of atoms passing through a specially chosen magnetic field. This experiment highlighted the following phenomenon: the vertical component of the angular momentum of the atoms takes only two values ± 1, as opposed to the classical prediction that its values fill the interval $[-1, 1]$. Recall that the angular momentum $L = (L_1, L_2, L_3)$ is an attribute of a rotating body depending on its angular velocity and shape. Its phase space is the unit sphere $S^2 \subset \mathbb{R}^3$. Its components satisfy the commutation relation $\{L_1, L_2\} = L_3$ and its cyclic permutations. In quantum mechanics, the components of the angular momentum correspond to 2×2 Pauli matrices whose commutator relations are (up to a factor) the same as of L_i's and whose eigenvalues are ± 1. This perfectly explains the Stern-Gerlach phenomenon. However, we immediately arrive at the following paradox [33]. Look (on the quantum side) at the projections of L on three unit vectors u, v and w in $\mathbb{R}^3$, with $u + v + w = 0$. By symmetry considerations, each of these projections also takes a value ± 1, while their sum equals 0, and we get a contradiction! One of the resolutions of this paradox is as follows: the quantum-classical correspondence takes these projections to pairwise non-commuting matrices, hence we cannot measure them simultaneously. In quantum mechanics, we face a new role of the bracket: it controls uncertainty. According to the Heisenberg uncertainty principle, for every pair of observables $A, B \in \mathcal{L}(H)$ and a state ρ,

$$\mathrm{Variance}(A, \rho) \cdot \mathrm{Variance}(B, \rho) \geq \frac{1}{4} \cdot |\mathrm{trace}([A, B] \cdot \rho)|^2 .$$

4 From quantum indeterminism to quasi-states

In his foundational 1932 book [42], von Neumann defined quantum states as real valued functionals $\rho : \mathcal{L}(H) \to \mathbb{R}$ satisfying three simple axioms: $\rho(\mathbb{1}) = 1$ (*normalization*),

Figure 3. Grete Hermann (1901–1984)

$\rho(A) \geq 0$ if $A \geq 0$ (*positivity*) and *linearity*. Next, he showed that each such functional can be written as $\rho(A) := \text{trace}(\rho A)$, where ρ is a density operator. Interpreting $\rho(A)$ as the expectation of the observable A in the state ρ, von Neumann concluded that for any quantum state ρ, there exists an observable A such that the variance $\rho(A^2) - \rho(A)^2$ is strictly positive. In other words, in sharp contrast to Dirac δ-measures in classical mechanics, there are no quantum states in which the values of all observables are deterministic.

This conclusion, known as the impossibility to introduce hidden variables in quantum mechanics, caused a passionate discussion among physicists. According to Grete Hermann (1935), a German physicist and philosopher, the linearity axiom only makes sense for observables A, B that can be measured simultaneously, i.e. that commute: $[A, B] = 0$. It should be mentioned that Hermann's criticism went unnoticed for almost three decades until the work of Bell, perhaps because the times were tough and Hermann had no opportunity to properly advertise her ideas as she became active in the underground movement against the Nazis. An account of this story is given in a lovely book by L. Gilder [16]; one could also see the Wikipedia article about Grete Hermann (from where the photo in Figure 3 has been sourced) and references therein. An attempt to incorporate Hermann's criticism leads to the following definition. A *quantum quasi-state* is a functional $\rho\colon \mathcal{L}(H) \to \mathbb{R}$ that satisfies the normalization and positivity axioms, while linearity is relaxed as follows: ρ is linear on every commutative subspace of $\mathcal{L}(H)$ (*quasi-linearity*).

However, in 1957, Gleason proved the following remarkable theorem. *If H has complex dimension 3 or greater, any quantum quasi-state is linear, that is, it is a quantum state.* This confirms Neumann's conclusion. Citing Peres [33, p. 196], "Gleason's theorem is a powerful argument against the hypothesis that the stochastic behavior of quantum tests can be explained by the existence of a subquantum world, endowed with hidden variables whose values unambiguously determine the outcome of each test."

Let us now mimic the definition of a quantum quasi-state in classical mechanics, using the quantum-classical correspondence and keeping in mind that commuting Hermitian operators correspond to Poisson-commuting functions. Let (M, ω) be a closed symplectic manifold. A *symplectic quasi-state* on M is a functional $\zeta\colon C(M) \to \mathbb{R}$ such that $\zeta(1) = 1$ (*normalization*), $\zeta(f) \geq 0$ for $f \geq 0$ (*positivity*) and ζ is linear on every Poisson-commutative subspace (*quasi-linearity*).

In contrast to quantum mechanics, *certain symplectic manifolds admit nonlinear symplectic quasi-states.* Starting from [12], this "anti-Gleason phenomenon" in classical mechanics has been established for various manifolds, including complex projective spaces and their products, toric manifolds, blow ups and coadjoint orbits [32, 40, 15, 6].

In terms of the existence mechanism for symplectic quasi-states, there is a mysterious dichotomy (vaguely resembling the rank-one versus higher rank dichotomy in Lie theory). In dimension 2 (i.e. for surfaces), symplectic quasi-states exist in abundance. Their construction is provided by the theory of topological quasi-states by Aarnes [1], whose motivation was to explore the validity of the Gleason theorem for algebras of functions on topological spaces, where the quasi-linearity is understood as linearity on all singly-generated subalgebras. In fact, in dimension 2, topological and symplectic quasi-states coincide. However, all known nonlinear symplectic quasi-states in higher dimensions come from Floer theory.

Interestingly enough, Floer-theory quasi-states come with a trove of additional features, which make them useful for various applications in symplectic topology. In particular, $\zeta(f) = 0$ for every function f with displaceable support (*vanishing property*). This immediately yields the rigidity of partitions of unity, which in turn is equivalent to the non-displaceable fiber theorem (see Theorems 2.2 and 2.1 above). Indeed, assume that $f_1, \ldots, f_N$ are pairwise commuting functions with displaceable supports that sum to 1. By the vanishing property, $\zeta(f_i) = 0$. By normalization and quasi-linearity, $1 = \zeta(\sum f_i) = \sum \zeta(f_i) = 0$, and we get a contradiction.

5 Quasi-states from Floer theory

Here, the reader is invited to catch a glimpse of Floer theory, with a focus on the construction of symplectic quasi-states. To this end, it is time to reveal the main secret of "hard" symplectic topology: the actual object of study is not the symplectic manifold itself but the space LM of all contractible loops $z : S^1 \to M$. The symplectic structure ω induces a functional $\mathcal{A} : LM \to \mathbb{R}$ as follows. Given a loop z, take any disc $D \subset M$ spanning z (see Figure 4) and put $\mathcal{A}(z) = -\int_D \omega$. Since ω is a closed form, the integral does not change under homotopies of the disc with fixed boundary and, therefore, $\mathcal{A}$ is well defined up to the homotopy class of D, an ambiguity we shall ignore. Its critical points are degenerate: they form the submanifold of all constant loops. In order to resolve this degeneracy, fix a time-dependent Hamiltonian $f_t : M \to \mathbb{R}$, $t \in S^1$, and define a perturbation $\mathcal{A}_f : LM \to \mathbb{R}$ of $\mathcal{A}$ by

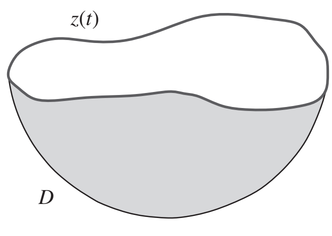


Figure 4. A disc spanning a loop

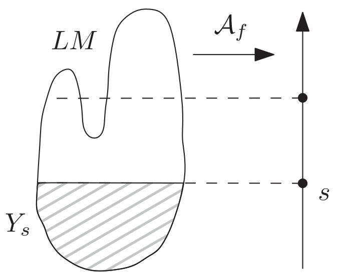


Figure 5. A topological "phase transition"

$\mathcal{A}_f(z) = \mathcal{A}(z) + \int_0^1 f_t(z(t))dt$. This is the *classical action functional*. Ironically, the perturbations become the main object of interest.

Roughly speaking, Floer theory is the Morse theory for $\mathcal{A}_f$. The space LM carries a special class of Riemannian metrics associated to almost complex structures on M. Pick such a metric and look at the space of the gradient trajectories of $\mathcal{A}_f$ connecting two critical points (i.e. two periodic orbits). Note that in M such a trajectory is a path of loops, i.e. a cylinder. It was a great insight of Floer [14] that these cylinders satisfy a version of the Cauchy-Riemann equation with asymptotic boundary conditions and, in particular, they can be studied within Gromov's theory of pseudo-holomorphic curves [18]. Even though the gradient flow of $\mathcal{A}_f$ is ill-defined, this asymptotic boundary problem is well posed and Fredholm. With this key ingredient at hand, one can build a meaningful version of the Morse theory of the action functional on the loop space. It is powerful enough to detect topological "phase transitions" of the sublevel sets $Y_s := \{\mathcal{A}_f < s\}$ as s runs from $+\infty$ to $-\infty$ (see Figure 5). They happen at special critical values $s = c(f)$ of $\mathcal{A}_f$, called spectral invariants, which were discovered and studied by C. Viterbo [41], M. Schwartz [38] and Y.-G. Oh [30] (also see [28, 36, 31]). The symplectic quasi-state ζ introduced in [12] captures such transitions for high energies: $\zeta(f) := \lim_{E\to+\infty} c(Ef)/E$.

6 An example: The median quasi-state

In general, Floer-homological quasi-states do not admit a simple description. However, there is one exception. First, we define a quasi-state $\zeta : C(S^2) \to \mathbb{R}$ on smooth Morse functions $f \in C^\infty(S^2)$, where the sphere S^2 is equipped with the area form ω of total area 1. Recall that the *Reeb graph* Γ of f is obtained from S^2 by collapsing connected components of the level sets of f to points (see Figure 6). In the case of S^2, the Reeb graph is necessarily a tree. Denote the natural projection by $\pi : S^2 \to \Gamma$. The push-forward of the symplectic area on the sphere is a probability measure on Γ. It is not hard to show

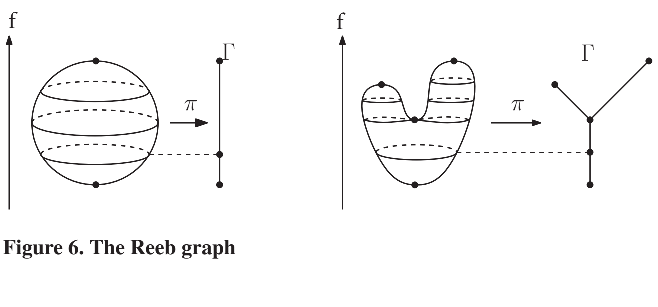


Figure 6. The Reeb graph

(and, in fact, it is well known in combinatorial optimisation) that there exists a unique point $m \in \Gamma$, called the *median* of Γ, such that each connected component of $\Gamma \setminus \{m\}$ has measure $\leq \frac{1}{2}$ (see [11, Section 5.3]). Define $\zeta(f)$ as the value of f on the level $\pi^{-1}(m)$. It turns out that ζ is Lipshitz in the uniform norm and its extension to $C(M)$ is a nonlinear quasi-state – the one which comes from Floer theory on S^2.

7 Noise-localization uncertainty

Recall that a finite open cover of a closed symplectic manifold by open displaceable sets does not admit a Poisson-commuting partition of unity. Can the functions entering the partition of unity "almost commute"? It turns out that the rigidity of partitions of unity phenomenon admits the following quantitative version. Throughout this section, we fix a finite cover $\mathcal{U} = \{U_1, \ldots, U_N\}$ of M by open displaceable sets.

For a finite collection $\vec{f} = (f_1, \ldots, f_N)$ of smooth functions on M, define the quantity

$$\nu(\vec{f}) = \max_{x,y\in[-1,1]^N} \Big\| \Big\{ \sum_i x_i f_i, \sum_i y_i f_i \Big\} \Big\| ,$$

which measures the magnitude of non-commutativity of these functions. Here $\|\cdot\|$ stands for the uniform norm.

Now introduce the Poisson bracket invariant [34] of the cover $\mathcal{U}$:

$$\mathrm{pb}(\mathcal{U}) = \inf_{\vec{f}} \nu(\vec{f}) ,$$

where the infimum is taken over all partitions of unity subordinated to $\mathcal{U}$. It measures the minimal possible magnitude of non-commutativity of a partition of unity subordinated to $\mathcal{U}$.

Next, define *the symplectic size* $\mathrm{Size}(U)$ of a displaceable subset $U \subset M$ as the minimal time T needed in order to displace U with unit energy, i.e. by using a Hamiltonian f_t, $t \in [0, T]$, with $\|f_t\| = 1$ for all t. The size is a fundamental symplectic invariant (which is usually called the *displacement energy*) introduced by Hofer [21]. It is an important fact, proved in full generality by Lalonde and McDuff [24], that the size of a ball of sufficiently small radius r in M is $\sim r^2$. Define the size of the cover $\mathcal{U}$ by $\mathrm{Size}(\mathcal{U}) := \max_i \mathrm{Size}(U_i)$.

It turns out that

$$\mathrm{pb}(\mathcal{U}) \cdot \mathrm{Size}(\mathcal{U}) \geq C > 0 , \quad (1)$$

where the constant C depends, roughly speaking, on the local geometry and combinatorics of the cover. We refer to works by the author [34], Seyfaddini [39] and Ishikawa [22] for more information about this constant.

An interpretation of this result comes from the phase localization problem in quantum mechanics. Here we think of the cover $\mathcal{U} = \{U_1, ..., U_N\}$ as a small scale coarse-graining of M. Given a particle z on M, we wish to localize it in the phase space, i.e. to provide an answer to the following question: to which of the sets U_i does z belong? Of course, the question is ambiguous due to overlaps between the sets U_i. Following an idea of I. Polterovich, we illustrate this by a toy model of a cellular communication network consisting of a collection of access points $u_1, \ldots, u_N$. Each access point u_j can be reached from a domain U_j, the so-called *location area*. The location areas U_j cover some territory M (e.g. Europe). Your phone at a given location $z \in M$ must register in exactly one access point u_j, whose location area U_j contains z (see Figure 7).

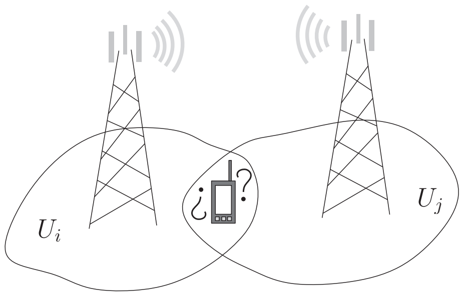


Figure 7. The registration procedure

In order to resolve the ambiguity, let us make the required assignment $z \mapsto U_j$ at random: fix a partition of unity $\vec{f} = (f_1, \dots, f_N)$ subordinated to $\mathcal{U}$ and register z in U_j with probability $f_j(z)$. Since f_j is supported in U_j, this procedure provides "the truth, but not the whole truth".

Now, consider the quantum version of the registration procedure. Let us assume that the quantum-classical correspondence takes a function f_j to a *positive* Hermitian operator A_j on a Hilbert space H. This holds, for instance, in the framework of Berezin-Toeplitz quantization [2, 3, 19, 4, 25, 7], (see [37] for a beautiful survey). With this assumption at hand, in a state ρ the probability of registration of the particle in the set U_j equals trace$(T_\hbar(f_j)\rho)$.

The quantum registration procedure exhibits *noise* ("an increment of variances"), which is governed by the operator norms of the commutators $\|[A_i, A_j]\|_{op} \approx \hbar\|\{f_i, f_j\}\|$. The noise and the symplectic size turn out to be related by the following *noise-localization uncertainty relation* [34]:

$$\text{Noise} \times \text{Size}(\mathcal{U}) \geq C\hbar\ ,$$

where the constant C is related to the one in (1). Its physical meaning fits the uncertainty principle: a sufficiently fine localization yields a large noise. In fact, inequality (1) was discovered after a translation into quantum language: a purely symplectic intuition yields a much worse lower bound.

We refer to a survey by Bush, Lahti and Werner [5] for a discussion of noise in quantum measurements, and to Kalai's article [23] for an intriguing link between quantum noise production and non-commutativity in the context of quantum computing.

8 Classical vs. quantum speed limit

How long does it take to displace with unit energy a state concentrated in a subset U of the phase space? In the classical framework, this *speed limit* is governed by the symplectic size introduced above.

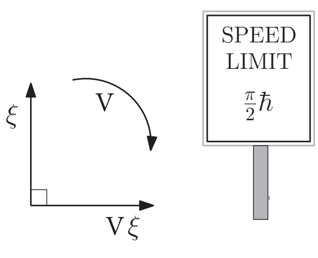


Figure 8. "Displacing" a pure quantum state

In the quantum world, the dynamics of a system with (time-dependent) energy $F_t \in \mathcal{L}(H)$ are described by the unitary evolution $V_t : H \to H$ satisfying the Schrödinger equation $\dot{V}_t = -(i/\hbar)F_t V_t$. The displacement (at least, for pure states $\xi \in H$, $|\xi| = 1$) corresponds to orthogonalisation: a unitary transformation V displaces ξ if $\langle V\xi, \xi\rangle = 0$ (see Figure 8). Interestingly enough, the universal quantum speed limit was discovered by the physicists Mandelstam and Tamm [26] as early as 1945 and refined by Margolus and Levitin [27] in 1998. It turns out that the minimal possible orthogonalisation time with unit energy, i.e. with $\|F_t\|_{op} = 1$ for all t, equals $(\pi/2)\hbar$. Thus (carrying out reverse engineering of the past in the spirit of the Ministry of Truth), one can argue that the definition of symplectic size could have been found over four decades before Hofer if somebody would have taken the trouble to dequantize it!

On a more serious note, in a recent work with Charles [9], we addressed a question about semiclassical "displacement" of semiclassical states. It turns out that if such a state is concentrated in a ball of radius $\sim \hbar^\epsilon$, $\epsilon \in [0, 1/2)$, the minimal displacement time is $\sim \hbar^{2\epsilon}$. Thus, on the scale exceeding the quantum length scale $\sim \sqrt{\hbar}$, the semiclassical speed limit is more restrictive. The proofs involve both symplectic topology and semiclassical analysis and, in particular, the sharp remainder estimates for Berezin-Toeplitz quantization found in [8].

9 Epilogue

Nowadays, symplectic geometry is a well developed subject. Its many facets include, in particular, "hard" symplectic topology (notably, Floer theory) and quantization. In this note, we have described some first steps toward understanding the interactions between these areas, which highlight quantum mechanics as a playground for testing and applications of "hard" symplectic methods. In general, a meaningful translation of symplectic rigidity phenomena involving subsets and diffeomorphisms into the language of quantum mechanics faces serious analytical and conceptual difficulties. However, such a translation becomes possible if one shifts the focus from subsets and morphisms of manifolds to function spaces. The latter exhibit interesting structures and features such as symplectic quasi-states and rigidity of partitions of unity, which are systematically studied within function theory on symplectic manifolds.

Acknowledgments.
In parts, the results surveyed above are in partnership with Laurent Charles, Michael Entov and Frol Zapolsky; I cordially thank them all. I am very grateful to Karina Samvelyan for the skilful preparation of the figures and useful comments, as well as to Andrei Iacob for superb copyediting. This note is an abridged version of my paper [35]. This research was partially supported by the Israel Science Foundation grant 178/13 and the European Research Council Advanced grant 338809.

References

[1] J. Aarnes, Quasi-states and quasi-measures. *Adv. Math.* **86** (1991), 41–67.

[2] F. Berezin, General concept of quantization. *Comm. Math. Phys.* **40** (1975), 153–174.

[3] M. Bordemann, E. Meinrenken and M. Schlichenmaier, Toeplitz quantization of Kähler manifolds and $gl(N), N \to \infty$ limits. *Comm. Math. Phys.* **165** (1994), 281–296.

[4] D. Borthwick and A. Uribe, Almost complex structures and geometric quantization. *Math. Res. Lett.* **3** (1996), 845–861.

[5] P. Busch, P. Lahti and R. Werner, Colloquium: Quantum root-mean-square error and measurement uncertainty relations. *Reviews of Modern Physics* **86** (2014): 1261.

[6] A. Caviedes Castro, Calabi quasimorphisms for monotone coadjoint orbits. arXiv:1507.06511, to appear in the Journal of Topology and Analysis.

[7] L. Charles, Quantization of compact symplectic manifolds, *Journal of Geometric Analysis* **26** (2016), 2664–2710.

[8] L. Charles and L. Polterovich, Sharp correspondence principle and quantum measurements, arXiv:1510.02450, Preprint, 2015.

[9] L. Charles and L. Polterovich, Quantum speed limit vs. classical displacement energy, arXiv:1609.05395, Preprint, 2016.

[10] M. Entov, Quasi-morphisms and quasi-states in symplectic topology. In *International Congress of Mathematicians*, Seoul, 2014.

[11] M. Entov and L. Polterovich, Calabi quasimorphism and quantum homology. *Int. Math. Res. Not.* (2003), 1635–1676.

[12] M. Entov and L. Polterovich, Quasi-states and symplectic intersections. *Comment. Math. Helv.* **81** (2006), 75–99.

[13] M. Entov, L. Polterovich and F. Zapolsky, Quasi-morphisms and the Poisson bracket. *Pure Appl. Math. Q.* **3** (2007), 1037–1055.

[14] A. Floer, Symplectic fixed points and holomorphic spheres. *Comm. Math. Phys.* **120** (1989), 575–611.

[15] K. Fukaya, Y.-G. Oh, H. Ohta and K. Ono, Spectral invariants with bulk, quasimorphisms and Lagrangian Floer theory. arXiv:1105.5124, Preprint, 2011.

[16] L. Gilder, *The Age of Entanglement: When Quantum Physics Was Reborn*. Vintage Books USA, 2009.

[17] A. M. Gleason, Measures on the closed subspaces of a Hilbert space. *J. Math. Mech.* **6** (1957), 885–893.

[18] M. Gromov, Pseudoholomorphic curves in symplectic manifolds, *Invent. Math.* **82** (1985), 307–347.

[19] V. Guillemin, Star products on compact pre-quantizable symplectic manifolds. *Lett. Math. Phys.* **35** (1995), 85–89.

[20] L. Guth, The waist inequality in Gromov's work. In *The Abel Prize 2008–2012*, pp. 181–195.

[21] H. Hofer, On the topological properties of symplectic maps. *Proc. Roy. Soc. Edinburgh Sect. A* **115** (1990), 25–38.

[22] S. Ishikawa, Spectral invariants of distance functions. *J. Topol. Anal.* **8** (2016), 655–676.

[23] G. Kalai, The quantum computer puzzle. *Notices Amer. Math. Soc.* **63** (2016), 508–516.

[24] F. Lalonde and D. McDuff, The geometry of symplectic energy. *Ann. of Math.* **141** (1995), 349–371.

[25] X. Ma and G. Marinescu, Toeplitz operators on symplectic manifolds. *J. Geom. Anal.* **18** (2008), 565–611.

[26] L. Mandelstam and I. Tamm, The uncertainty relation between energy and time in nonrelativistic quantum mechanics. *J. Phys.(USSR)* **9**, no. 249 (1945): 1.

[27] N. Margolus and L. B. Levitin, The maximum speed of dynamical evolution. *Physica D: Nonlinear Phenomena* **120** (1998), 188–195.

[28] D. McDuff and D. Salamon, *J-holomorphic Curves and Symplectic Topology.* Second edition, American Mathematical Society Colloquium Publications, 52, American Mathematical Society, Providence, RI, 2012.

[29] Y.-G. Oh, Floer cohomology of Lagrangian intersections and pseudo-holomorphic disks. II: $(\mathbb{C}P^n, \mathbb{R}P^n)$, *Comm. Pure Appl. Math.* **46** (1993), 995–1012.

[30] Y.-G. Oh, Construction of spectral invariants of Hamiltonian diffeomorphisms on general symplectic manifolds, in: *The breadth of symplectic and Poisson geometry*, Birkhäuser Boston, Inc., Boston, MA, 2005, pp. 525–570.

[31] Y.-G. Oh, *Symplectic Topology and Floer Homology.* Cambridge University Press, 2015.

[32] Y. Ostrover, Calabi quasi-morphisms for some non-monotone symplectic manifolds. *Algebr. Geom. Topol.* **6** (2006), 405–434.

[33] A. Peres, *Quantum Theory: Concepts and Methods.* Fundamental Theories of Physics, 57, Kluwer Academic Publishers Group, Dordrecht, 1993.

[34] L. Polterovich, Symplectic geometry of quantum noise. *Comm. Math. Phys.,* **327** (2014), 481–519.

[35] L. Polterovich, Symplectic rigidity and quantum mechanics, submitted to Proceedings of 7ECM, Berlin, 2016, available at https://sites.google.com/site/polterov/miscellaneoustexts/symplectic-rigidity-and-quantum-mechanics.

[36] L. Polterovich and D. Rosen, *Function Theory on Symplectic Manifolds.* American Mathematical Society, 2014.

[37] M. Schlichenmaier, Berezin–Toeplitz quantization for compact Kähler manifolds. A review of results. *Adv. Math. Phys.* 2010, 927280.

[38] M. Schwarz, On the action spectrum for closed symplectically aspherical manifolds, *Pacific J. Math.* **193** (2000), 419–461.

[39] S. Seyfaddini, Spectral killers and Poisson bracket invariants. *J. Mod. Dyn.* **9** (2015), 51–66.

[40] M. Usher, Deformed Hamiltonian Floer theory, capacity estimates and Calabi quasimorphisms. *Geom. Topol.* **15** (2011), 1313–1417.

[41] C. Viterbo, Symplectic topology as the geometry of generating functions, *Math. Ann.* **292** (1992), 685–710.

[42] J. von Neumann, *Mathematical Foundations of Quantum Mechanics.* Princeton University Press, Princeton, 1955. (Translation of Mathematische Grundlagen der Quantenmechanik, Springer, Berlin, 1932.)

Leonid Polterovich [polterov@runbox.com] participated in Sinai's and Arnold's seminars at Moscow State University prior to his move to Israel in 1990 and received his PhD degree from Tel Aviv University in 1990. Currently, he holds the Gordon Chair in Dynamical Systems and Symplectic Topology at Tel Aviv University and a visiting professorship at the University of Chicago. This article follows his plenary talk at the 7th European Congress of Mathematics in Berlin, 2016.

Interview with Sir Michael Atiyah Fields Medal 1966 and Abel Prize 2004

Oscar García-Prada (Consejo Superior de Investigaciones Científicas, Madrid, Spain)

Michael Atiyah and his collaborators have changed the face of mathematics in recent decades. In his work, one could single out, among other fundamental works, the index theorem (in collaboration with Isadore Singer) and the study of the geometry of the Yang–Mills equations, with important applications in theoretical physics. His contributions wonderfully illustrate the unity of mathematics and show, in particular, the importance of the interaction between geometry and physics. He is a key actor with tremendous influence on the work of the scientific community devoted to these subjects. Among other prizes, he has been awarded the Fields Medal in 1966, the Copley Medal in 1988 and the Abel Prize in 2004. He was also one of the promoters for the foundation of the European Mathematical Society.

We were with Sir Michael Atiyah in the French city of Brest on 10 July 2014, immersed in a conference on real vector bundles organised by the Centre Henri Lebesgue. This theme has its origins in a seminal paper of Sir Michael from more than 50 years ago.

Sir Michael Atiyah the day of the interview.

Michael, since your work has produced fundamental chapters in the mathematics of the 20th and 21st centuries that are very well known, I think it would be nice if we could speak about the people that you have met in your mathematical career.
Yes, sure. I like talking about people.

Yes, your memories and recollection of some of these people. I'd like to start with your supervisors when you were at university, or even school – I mean your mentors, but especially your supervisors Todd and Hodge. What can you tell us about them?
Yes, well, I went to school in Cairo – it was an English school – and also in Alexandria. I had my teachers there; I had quite a good mathematics teacher (but a bit old-fashioned and not very sophisticated). I mean, I got a good education but nothing special in mathematics. I was always the youngest in the class, by two years actually. I was the small boy in the class. When you're at school and you're two years younger than everybody else, what happened was that I would help the older boys with their homework and in return they would defend me. So, I had powerful friends; they were big but they were not so clever so I would help with their homework and in return I got bodyguards [laughs], which is important if you are small, you know. At school you can get bullied if you're small and everybody's older. So that was very good.

In my last year at school in Egypt, in Alexandria, we had a mathematics teacher who was old-fashioned. He was quite good. In fact, he hadn't trained as a mathematician; he had trained as a chemist but he was a good teacher: very severe and disciplined. I have vague memories of a teacher who had a French education in a very different style. I think he was Greek (his name was Mouzouris). I remember he actually gave me some books on modern analysis that he had studied in France. That was the first time I met such things but it didn't make very much impression on me.

Then, afterwards, I went to school in Manchester in England. There, I went to a very good school. Well, my father asked how to prepare for university; he asked what was the best school for mathematics and everybody said Manchester Grammar School, which was a sort of intellectually elite school – we had a very dedicated maths teacher. He had been in Oxford in 1910 or something – old-fashioned but very inspiring in a way. So, I worked very hard there because we were working on fairly hard examinations to get into Cambridge, which was very competitive. So I worked harder there than at any time in my life, probably.

How old were you then?
Seventeen. I went there at the age of 16/17 and we were all very well trained so we all got scholarships into Cambridge. I arrived in Cambridge with a very good background. Of course, you don't know when you arrive at university how good you are compared to everybody else because everybody is the best person of their school. At the end of the first year, I came top of the university so I realised I was good from that point of view and I had a lot of friends who were very good mathematicians. Many of them became quite famous afterwards, not only in mathematics but in other fields. So it was a good environment. I went to Trinity College, which is famous for Isaac Newton and many other people: Ramanujan, Hardy,

Littlewood – so there was a strong mathematical tradition in the college. Eventually, I came back as the Master of the College 50 years later [laughs].

So, I had very good training. The lectures were rather average, with one or two very good lecturers but most rather nondescript, and one or two very bad lecturers. But there were one or two very good lecturers; then I did the courses and I did accelerate. I went to many lectures to progress very fast and published my first paper as a second year undergraduate. I went to some courses by Todd. There was a nice problem in classical geometry; I made a little contribution and he encouraged me to publish it. Just a two-page note, you know, but I was a second year student and it gave me tremendous pride to publish a paper! I am probably more proud of this paper than anything else. So, that was a good start and then, after that, I did my graduate studies. I had to select a supervisor for my PhD. I had been taught as an undergraduate by Todd, who was a good mathematician but very shy. He didn't speak when I would go to see him; he would discuss the problems but then nothing else. So I had to go along with a long list of extra questions to ask him to keep the conversation going.

I decided not to work with him but to work with Hodge, who was much more famous for his work and had an international reputation. I was impressed by him. I thought he would have a bigger vision and he did but he was also a very different person to Todd. He was a very gregarious, extrovert, friendly person. If you met him, you'd think he wasn't a mathematician; he looked like a grocer running a shop. In fact, I discovered afterwards he came from a family that had grocer shops! Grand shops [laughs]! He was the only one who went into mathematics; all the others were doing business in their shops and so on. But he was very affable and very friendly so he had a big influence on me and gave me good direction. So it was a good start to my career and I was lucky to arrive at a good time. I had good fellow students and the mathematical world was just changing after the war. New things were happening in Paris and in Princeton. I used to go to the library every week to see the latest issue of the Comptes Rendus: new papers by Serre, Cartan. And Hodge had contacts in Princeton, I would hear. So I was quite quickly in touch with these movements. This helped me get started and I went to Princeton.

What was the mathematical problem that you tackled in your thesis? Was it Hodge who suggested this problem to you?

Well, I did two quite separate things in my thesis. One, I picked by myself. It was to do with what geometers call ruled surfaces. These are surfaces which are families of lines arising in classical geometry. I got interested in them from one point of view, relating them to vector bundles and sheaf cohomology methods. I used modern methods to start a classification but these were the early days. It became a big industry afterwards. I wrote the first paper on the subject in 1953–54 and wrote it more or less by myself. In my second year of research, Hodge, with whom I had been working, saw how to use modern methods to attack the whole problem he had been interested in in algebraic geometry integrals. So, he gave me the idea to start with, which I developed, and then we wrote a joint paper together on this, which became quite well known. So, I did two quite separate things in my thesis. One was entirely my own work and the other was really in conjunction with my supervisor. By the end of the second year, I had more or less finished.

Where did Hodge come from mathematically?

Well, Hodge was a Scot and Scotland has a very good tradition. He graduated from Edinburgh University, which is actually where I am now. He went from there to Cambridge to finish a degree and so he had a good background in mathematics and physics, which was actually relevant to his work (Hodge theory). Then, in Cambridge, he was in a very strong school of geometry (old-fashioned geometry) and he forged his own way, away from this feeling of ideas. He was very much influenced by Lefschetz, who was revolutionising algebraic geometry by using topology methods. He wasn't present – it was action at a distance; he followed Lefschetz's books and works and eventually he met him. So, by entirely his own choice, he made his name without wanting to and, of course, he was young and went to Princeton. Interestingly enough, when he first met him, Lefschetz refused to believe that he had proved what he had proved. He kept arguing he was wrong and it took Hodge a long time to persuade him he was correct; eventually, he used Lefschetz's ideas in a more complicated way. Lefschetz had a very strong personality and when he was finally persuaded that Hodge was right, he reversed himself and became a strong supporter. From being a strong opponent, he became a strong supporter and he got Hodge a chair; he was a great support. At first, you know, it was all rubbish. Then, after a while: 'Ah! Magnificent!' He was a very colourful personality. I met Lefschetz when I first went to Princeton because I was Hodge's student, and he was very aggressive. By that time, he was doing other things but he looked at my paper with Hodge and he said: "But where's the theory? Come on, tell me." He was sort of aggressive, trying to say there was nothing in the paper of importance. I think it was a matter of style, anyway. We became good friends later on but he was a very strong personality.

Of the people you met after your thesis in Cambridge when you went to Princeton, is there anybody you would like to mention?

Yes, I went to the Institute for Advanced Study. There were a lot of distinguished permanent professors but I arrived just too late to meet people like Hermann Weyl, von Neumann and Einstein. They all died more or less just as I arrived. Besides the permanent professors, they had a large number of brilliant young people that came as post-docs and, because it was just shortly after the war, there was a large backlog of people whose education had been changed by the war – many generations were sort of compressed together. So, there I met Hirzebruch, Serre, Singer, Kodaira, Spencer, Bott – all of these – and I spent a year and a half at Princeton. That was the time when I really met most mathematical talents. I learnt things I'd never heard before, like Lie groups and topology.

They were all in Princeton?
They were all in Princeton Institute, yes, exactly. Kodaira and Spencer were respected professors and the others were all post-docs. We spent a year or two together and some of them had been at Princeton before, so it was a very good meeting place for young people. We learnt a lot from each other. We didn't go to lectures together at university. I'd learnt by myself from the French school of mathematics in France, and while I was at Cambridge, but at Princeton there was personal contact and the influence of people. I'd say I got very friendly with them all. I learnt a tremendous amount in just over a year. It was like reaching adulthood; suddenly I became a sort of professional mathematician. We learnt new ideas; it was one of the top places in the world and there were all sorts of things happening and new advances every week: new theories, characteristic classes, cohomology. It was an ideal time to come in and I made my own contributions.

I got to know Hirzebruch and then, when he came back to Europe, I carried on meeting him and meeting people in Bonn, that sort of thing, so it was very good. It was the ideal time to arrive at Princeton, in that period, and then come back to Europe. Things were happening there too. You know, the war finished in 1945 and I went to Princeton in 1955 (enough time for things to settle down) but many of my colleagues had been not exactly fighting in the war but had been called up. Singer served in the US Navy. Bott was trained and about to enter the war. Hirzebruch was in the German army as a young man and was captured as a prisoner of war by the Americans but only for a few months; he was 17 and he escaped. So I was just on the tail-end. The people who were caught up in the war were older and there for a long time as well. By the time I went to Princeton, it was ten years over. People had recovered and so it was a very good time.

And you came back to Europe after two years, right?
Yes, I had a year and a half and then I came back. I had a job in Cambridge. I came back to a job and I spent a few more years in Cambridge and then I moved to Oxford.

So why don't you say something about your students, both in Cambridge and in Oxford?
Well, in Cambridge I didn't have many students because I left Cambridge young but I had a couple of students I had inherited from Hodge, my supervisor. He had taken on students and by this time he was a very busy man. He didn't have time; his own career had been sort of spoilt by the war. He had become famous when still young before the war and then during the war he'd had to stay in the college and do a lot of administration. By the time the war was over, he was a bit out of touch so he took students but he passed them onto me. So, my first two students were handovers and they were OK. They both did their theses with me. It was good preparation for me; I had to learn how to handle students. It's not so obvious and, of course, you realise after a while that some students teach themselves, some are independent, but many need a lot of help because they come with many different levels of ability. Some are very strong, some are rather weak. So, I had these two students who were with me before I went to Oxford. In Oxford, I was there for very much longer and I gradually got more students over time. When you are young, you wonder why they would want to come and work with you, you see. You have to become a bit older and a bit more famous and then students come. I had a large number of students, altogether around 50 students. Well, it's difficult to count students because the face of a student is not so well defined – or somebody else's student is really, de facto, your student – but between 40 and 50 students over a period, over a lifetime. At a given time, I would have five or six students doing their PhDs with me, two in each year, and so that was good. Then, I went to Princeton as a researcher and had four students there.

You mean that while you were in Oxford, you went to Princeton again?
Yes. I went to Oxford first in 1961 and in 1969 I went to Princeton. So I was in Oxford for eight years and then I went to Princeton for three and a half years and then I came back to Oxford. One advantage at Princeton was that you could invite people to come and work with you, so you had some choice. One person who came with me, originally from Oxford, was George Luzstig, a very young man from Romania; he was a brilliant student. He was my student in Princeton. And I could also invite people as my assistants so I had Nigel Hitchin as my assistant.

He had already been your student in Oxford, right?
He had been my student (or de facto student). He had been officially working for somebody else but he worked as I suggested and I kept in touch with him. So, he was really my student as well. Before that I had Graeme Segal. He had been another student of Hodge for a year.

Hodge sent him to Oxford?
Well, I think he sent himself to Oxford [laughs]. He came to Oxford to work with me. By that time, I was collecting students. In Princeton, I had a few and when I came back to Oxford I got a large number of students because, by this time, I suppose, I was better known. I got many students from Cambridge, many students from abroad, several from India. Ah! Patodi was a very young Indian; he came and worked with me as a de facto student. Then, later on, I had some very brilliant students: Simon Donaldson and so on. It frightens me; I went through a period where I was thinking I'm not getting very good students. I'm not doing very well. Maybe I should stop taking students. I'm no longer sufficiently active. And then something changes and suddenly you find half-a-dozen brilliant students and it's very much, sort of, a chance event. Of course, you learn from your students, the very good students. Donaldson was there. He gave some lectures after a while. I went to his lectures, even when he was just barely doing his PhD. So, yes, you learn quite a lot and with so many students you give them some thesis to work

on, you encourage them, you tell them which direction to go, you give them various degrees of help and sometimes they do everything themselves, sometimes you do the work for them and sometimes it's a collaboration. So, it's a very positive experience and I enjoyed that. When I went to Princeton Institute, I didn't really have students; there was no formal university, you know. For Oxford university students, some were local and some would come from outside to do PhDs (specifically with me or some by themselves) and then there were some from countries like Australia (like Graeme Segal), America, India, yes, quite international.

So, you collaborated with some of your students, like Nigel Hitchin.

Yes, I collaborated, usually after they had finished their PhDs, as colleagues, junior colleagues. But, because they had worked with me, they worked in the same area. So it was natural that I continued together on joint papers with Nigel Hitchin and Graeme Segal. Usually, I liked to have my students working in slightly different areas: some in differential geometry, some in algebraic geometry and some in topology – so they weren't all in the same field. So, I would collaborate with them and they would also have their own individual personality and mathematical tastes – they would be different. They would be going in slightly different directions, which is very good. You get to broaden – some more with analysis, some geometry, some more with topology – and that way you learn with these 20-year-old students because they become more expert. Segal became more expert in homotopy theory, Hitchin became more expert in differential geometry… So, it's a way of learning. When you start off, you learn something, but when you're teaching, you don't have much time to go back and study so you have to learn in a different way and one way to learn is through your students, in collaborating with your students.

Perhaps you can say something about the main collaborators you have had throughout your career.

Yes, among my main collaborators (senior collaborators, my age or older) there was Hirzebruch, who was just two years older than me. He seemed older than me; I went into the army and did two years there – he didn't do that. He got promoted very young. He was a professor when I was just finishing my PhD but we were quite close in age really so we collaborated for quite a long time because I used to go to Bonn. Work developed there; it was natural that we should write papers together. Then, the other two people I worked with were Bott and Singer. They were in America, in Harvard and MIT, and I used to meet them in Princeton or I would go to MIT, or they would come to Oxford. We spent a lot of time together. We all wrote papers together. We all had common interests and had different strengths. Hirzebruch was very much close to me in many ways but I learnt from him. He was an expert in characteristic classes and algebraic topology. Bott was more into differential geometry and Lie groups and things like this, and Singer was more from an analytical background and functional analysis and Hilbert space theory. So, they all had slightly different areas of expertise but they all overlapped and so we had a lot of common interests, which was very good. I was able to write many papers. They were experts – well, not only were they experts but they knew the real experts. Singer had a lot of good friends who were leading figures in differential equations and so on, and Bott knew a lot of people in topology and he knew a lot of people through Bonn, so they all had very wide intellectual networks of contacts and students. Smale and Quillen were students of Bott, so this gives you a good network.

From left to right: Henrik Pedersen, Nigel Hitchin, Nedda Hitchin, Sir Michael Atiyah, the author of the interview, Graeme Segal, Jacques Hurtubise and Jean-Pierre Bourguignon (10 September 2016 at the celebration of Nigel Hitchin's 70th birthday conference in Oxford).

I'm very gregarious. I like to talk, you see [laughs] and I love mathematical discussion. We would get to the blackboard and we would exchange ideas and I like this. It is very stimulating. After we talked, we would think and we would go back and discuss again. So, it's a very social process and so you make good friends too. A working relationship is very intimate in that sense. So, they were my main collaborators. Then I had younger collaborators like Graeme Segal, Nigel Hitchin and, later, younger ones like Frances Kirwan. I wrote quite a few papers with Hitchin and Kirwan. This was a similar relationship just inverted because I was the teacher and they were the students. We had common interests and, again, their interests were paralleled by the interests of someone older. They were quite a new generation with new ideas, so it was a very good network.

You also had very good friends in the physics community, in particular with Witten, right?

That was later, yes. I remember meeting Witten when I went to America in the early 1970s. We had just realised then that there was some overlap between what the physicists were doing and what Singer and I were doing. So, I went and had a meeting with a group of four physicists from MIT – these older people and one young chap sat in the chair and, at the end, after the discussion, I realised he was a really bright guy. He understood much more of the mathematics I was trying to explain – and that was

Ed Witten. He was a junior fellow. After that, I invited him to Oxford for a few weeks; I got to know him well. So, I've known him since he was a young fellow at Harvard and he was always tremendously impressive. I learnt an enormous amount from him and I tried to read almost every paper he wrote. He writes an incredible amount and I think that one of my main contributions was to introduce the mathematical world to the ideas coming out of physics through people like Witten and his collaborators. In the early days, a lot of mathematicians were suspicious of physicists. They said physics was nothing to do with mathematics: 'They don't prove theorems', 'It was a doubtful business.' So, I got a bad reputation for mixing with bad company, you know [laughs]! I think that even with Witten mathematicians were sceptical but they understood he could do things they couldn't do – he opened up many doors and got the Fields medal. So, following his development was really part of my education and, in the end, I became like his graduate student (but this was many years later). I spent a term with him in Caltech and it was a bit like being a graduate student again. I would go and see him in the morning, we would have an hour discussing each problem and then go away and think about it for 23 hours, before coming back. Meanwhile, he would do everything else. I would come back the next day and we would carry on the discussion. I had to work to keep up with him…

You wrote a paper.
Yes, a 100-page paper. I wrote parts of it. He decided we should work on this, probably because it had some relationship with what I had done before. But he had ideas about it. He pushed and he was so good that we would occasionally have arguments about the mathematical side of the results and he would usually be right [laughs] and I would be wrong, yes! It was quite an experience, usually; by this time, I was already getting old, well, advanced in years anyway, but it was like being a student – really exciting. Even now in Edinburgh, among the people I collaborate with there are many physicists, mathematical physicists – physicists of the new generation. I do more and more mathematics in connection with physics.

Going back in time, you also interacted a lot with Roger Penrose.
Yes, well, Roger Penrose was my fellow student. He came as a student from London and started his PhD the same time as me, as a student of Hodge, but he didn't get on very well with Hodge; his interests were different and so, after one year, he switched to Todd.

The reverse thing that you did.
Yes, well, I had been taught by Todd. It was ironic because Todd was doing more algebra and geometry. We lost touch when he finished Cambridge and went elsewhere. So, then he became seriously interested in physics. We met again when he came to Oxford as a professor of mathematical physics, after I came back from Princeton. Then we managed to rebuild our connections. We had this common root in algebraic geometry and he was able to explain to me what he was doing and, after a while, I realised the modern ideas of sheaf theory were really what he needed. I introduced his group to new ideas in physics and that went off very well. I wrote a paper with one of his students, Richard Ward, so that went very well. Interestingly enough, when I was at Princeton at that time, before going back to Oxford, I talked with Freeman Dyson and we discussed Roger Penrose and he said: "Oh! Roger Penrose did some very good things about black holes, which I always admired, but he did some very funny things about twistors. I didn't understand, so maybe, when you go to Oxford, you'll understand what twistors are." And he was right, exactly right [laughs]. That was the connecting link.

It was connected to your common background in algebraic geometry, right?
Of course, we learnt about the Klein representation of lines and Grassmanians. We knew classical geometry well, so it was a good relationship and we got on well. He had a large number of students; he worked with a team of students and he met Hawking when he was a younger man, so I had good links with that group of physicists and I learnt a lot – also through Singer. Both Singer and Bott had degrees outside mathematics originally. Bott trained as an electrical engineer and Singer trained in physics. They got into mathematics after. Singer went into physics and then decided physics wasn't rigorous enough, you know. But Bott was trained as an electrical engineer and got into mathematics through Hermann Weyl, who pushed him in the right direction, in a way. Yes, they came from different backgrounds because, in those days, mathematics wasn't really a profession. Your father didn't think you should do that; you should train in a job, like engineering, that would give you some money [laughs]. To be a mathematician wasn't regarded as an occupation where you could get a job. Of course, it has changed a bit now but in those days it was very much so.

Singer and Bott knew Chern very well. Chern was a very good friend of Yang because he had taught them in Chicago. They were both Chinese so there was a link – Yang, Lee, Jim Simons and Chern, and Singer – and that gave us entry into modern physics at the same time, when things were happening. But it was coincidental. It was very funny. At Princeton, they had this big School of Mathematics and Natural Science, which had originally been one and then had been broken up. The first appointments in Princeton were all big figures: Hermann Weyl, Von Neumann, Gödel – people like Pauli were also there. Ah, but later on, mathematics became a different kind of mathematics; they were rather Bourbaki type, rather pure mathematics and physics. They just drifted away from physics so when I arrived, they were totally divorced; they didn't talk to each other. Dyson could have been a link because he started life as a mathematician and became a physicist, but physicists and mathematicians had, by that time, gone down different paths. They were pursuing different things, it may be said, and mathematicians were not very sympathetic to physics. They thought physics was a messy subject, not really rigorous, and the physicists themselves had similar views about mathematics. Modern mathematics was very abstract, so they really had no link. By the time things had changed and Witten came on the scene, it was totally different. It was more interactive; they had some seminars together but they still kept some distance.

But, if we go back to the 1950s, was it really an accident that physicists were developing Yang–Mills theories and mathematicians were simultaneously developing the theories of bundles, Chern classes, connections and all that? What was the connection?

Well, it's a very interesting story. I mean, the lynchpin would really have been Hermann Weyl. He was the person who introduced gauge theory to physics. He wrote the first paper on how to use gauge theory methods. He was the grand man of mathematics and he was at the institute very early on. But he died in 1955, the year I arrived. Yang–Mills theory was developed, more or less, by that time. I met Mills, who was there as a visitor. One would think that Yang and Hermann Weyl would have spoken while Weyl was still interested in physics.

They overlapped in Princeton but I believe they never had a chance to discuss.

Well, by this time Weyl was a bit older and his interest in physics had been 20 years before. Modern physics had moved in very different directions; he was doing quite different things. New particles had been discovered and he wasn't much into that. But he was the grand old man and if they had just talked to Herman Weyl, he would have told them all about connections and about Lie groups. So, it was just an accident of age and time that he didn't and I really find it mysterious that he and Yang didn't make some contact. So that opportunity was missed. Simultaneously, by the way, one of my contemporaries in Cambridge, Ronald Shaw, wrote his thesis on this. He independently discovered the theory but his superior said it was "not worth publishing" – poor chap, he never published it. But, at that time there were physical objections to the theory, which made it not so popular, so it was dropped. It was some years later when people re-looked at it. They still had to make some use of it, a proper physical use, and then it became popular. But it was probably 15 years later, in the 1970s, that it was taken up again and, in those intervening years, they were chasing different things. They were chasing symmetries, particle representations, classifications… They were doing quite different sorts of things and Yang–Mills theory was left behind. When it resurfaced, that was the time when Singer and I got involved and interested because we were doing mathematics that was related. But Hermann Weyl knew it all, the physics and the mathematics, and he was there before the physicists. But the physicists never emphasised the geometrical side.

But one gets the feeling that there is a missing link that makes it more mysterious, that they were developing similar objects and they took time to realise this.

Well, you see, the story is that Hermann Weyl used gauge theory in order to unify magnetism with Einstein's theory of relativity. When he writes his paper, it was pointed out by Einstein that it was physically nonsense because what Weyl was doing was working with real line bundles where the change of scale took place. Gauge theory was to do with scale and his idea was that if you went round a path in a magnetic field, you would alter the length and scale of things. Einstein said this was nonsense. If that were the case then all hydrogen atoms would not have the same mass because they would have different histories. Despite this, the paper was published; this is what I find interesting. The paper was published because Weyl insisted he was still right, and Einstein's objections appeared as an appendix. So, Weyl knew about it but it was only a few years later, when quantum mechanics appeared, that they reinterpreted the length of a phase. Then, the physical objection disappeared and the theory became standard, a modern standard. By that time, Weyl had left the subject, he had gone off, so he wasn't actually doing that any more. But he knew, of course, that it was all his theory, although the non-abelian version didn't take off until after his death. If he had lived longer, he could have been the main missing link.

Sir Michael Atiyah (right) and the author of the interview the day of the interview.

But it's also interesting that in the mathematics community the non-abelian theory was being developed.

Yes, but that's almost inevitable. The point is that the theory of bundles is an offshoot of Riemannian geometry. That was all developed by Riemann and the Italian geometers – differential geometry, parallel transport. That was for the tangent bundle, for the metric, not for a super-structure of bundles, which is actually easier. The case of a metric is more difficult.

When Einstein presented the theory of relativity, there was a great deal of interest from differential geometers. That gave a big spurt to differential geometry. Parallel transport was all part of general relativity, so this was very natural. What was new was taking vector bundles on top of the space. This was excellent. But the whole notion of parallel transport was familiar to geometers and, shortly after that, Chern and Weil brought it to bundle theory and characteristic classes. In maths, they had been doing this for a long time. They had been doing it ever since Riemann and Betti in differential geometry. Einstein's relativity theory tagged onto differential geometry and Yang-Mills came into it for bundle theory.

This was all part of mathematics. What happened is that Singer and I made links to the Dirac equation, differential equations of the kind familiar to physicists: spin, spinors and so on. That was a new bit of mathematics that hadn't been done before, not seriously. Who knew about it? So, I think mathematics was always there. The physicists had just touched on it here and there and then became seriously interested later. Then Hermann Weyl died. It is an interesting story but, like most things in life, the development of the facts is not what you expect, not what you reap if you do it retrospectively. You'd have done it differently. It's a bit accidental. It depends on the fashions of the time, the people of the time and their personalities. So, it's very interesting, you know. It's not predictable. It's not automatic. It's a bit by chance.

The panorama of theoretical physics has changed enormously after those exciting years, your contributions and those of your collaborators and your school. For example, moduli spaces are now ubiquitous in physics.
Yes, we started off with that and, of course, they came under algebraic geometry, and I knew about those. So, physicists then got seriously interested in string theory and became much more mathematical, and they took over large amounts of mathematics that had been done by everybody else. My students got drawn into Donaldson theory so the interaction increased enormously after that episode in the 1970s and has been enormously influential (and still is). Physics and mathematics are still feeding off each other.

I wanted to ask you about that. How do you feel about things currently? Are there exciting things that you feel are happening?
Yes. As you get older, of course, you get a bit out of touch with what is happening. I get to hear a bit about it indirectly. I read some of the new papers written. There are some developments in Chern–Simons theory. As part of the story that I was interested in, there is knot theory and so on, and I try to follow it to some extent, although less now. The mathematics often gets more sophisticated. There are more abstract things, like derived categories – things that older people don't like. But the interaction is still very close and there's a whole generation of people who are now into both mathematics and physics. It's very hard to distinguish if they are physicists or mathematicians; they are a mixture, hybrids, which means that they have some problems because physicists don't regard them as physicists and mathematicians don't regard them as mathematicians. So, it's difficult for them to get jobs sometimes. I mean, who is going to give you a job if you're neither a fish nor a fowl. But I think this is something that is very healthy and there are some centres where they encourage these hybrid ideas, like string theory. So, there's no question it's still a very active area. Exactly what does it mean to physics? Physics and mathematics have a close relationship but there are differences: physics is looking for a unique solution to the universe while mathematics is exploring all possible universes or possible theories. So we get a lot of ideas. Some of them die in physics because they prefer new ideas, but for mathematicians: they can work on everything so it's a different sort of relationship. You never know with physics.

I have my own ideas. I follow what's going on but I try to be a bit independent. I think there's no point in trying to follow exactly what the young guys are doing. I like to have some thoughts that are a bit more out of the box, so to say, or a bit more original. I play with new ideas that are a bit unorthodox. I am working on some things that are different from what other physicists are currently doing. I mean, nobody knows in physics whether there is a final theory or if we're close to the final theory or whether, in fact, they'll be totally different views in five years time, or whether the series will evolve and there'll be quite radical changes. Some of the ideas at present will be absorbed, some of them will be kicked out, some of them will change but mathematics will benefit from it all, whether it's good physics or bad physics. It has mathematical content and mathematicians have learnt a lot. Mirror symmetries and string dualities are ideas that came from physics. So there's a lot. I think it was Witten's propaganda which said that string theory is a branch of mathematics from the 21st century accidentally discovered in the 20th century. So, it's now coming into its own and it's not quite clear what this is a theory of, but it is bringing new ideas which are transforming mathematics. We're in the middle of a sort of maelstrom of ideas, like swirling winds all around. You don't know what's going to happen. It's hard to predict and you don't want to predict because I always say that if you can predict it's uninteresting. Interesting things are the new developments and if you could predict them they wouldn't be so exciting. So, you have to be prepared for surprises. You have to look for surprises and every now and again there is a surprise.

I'm astonished at how dynamic you are in the conference we're having here. You're still thinking and producing work. Tell me, what is it that occupies your days, nowadays?
Well, unfortunately, at the moment, I'm getting old and my wife is also getting older. She has a lot of problems. I have to spend a lot of time looking after my wife. It happens to us all, in one form or another. So, she occupies 75% of my time. When I come to conferences like this, it's rather a rare event. I get a holiday talking about science. When I'm at home, I just barely survive and I have physics friends I meet once or twice a week to discuss my ideas. For the last year or two, I've been busy writing biographical articles about Hirzebruch. I'm also involved in one for the London Mathematical Society and one for the Royal Society (it's not finished but it has taken up a lot of my time). It was obviously a priority: I had to do it while I am still here.

Outside, I have these crazy ideas that I'm trying to pursue. I talk with younger people because you need young guys to follow it through and so on. And some of them…

well, this year's conference is a bit of an accident because I was into some of these ideas a long time ago and I didn't realise so many people were working on real vector bundles. So, I came in and I found I could follow some but not all of it. And much of it is derived from a paper I wrote 50 years ago. It's a funny experience, you know. I have this experience now. I go to a conference, a big lecture theatre like this. I sit at the top because it's very easy to get in and out. The young guys are at the bottom and they're talking busily about me and my work from 50 years ago. I feel as though I'm living above in the sky, looking down on my past. I'm floating up, closer and closer to heaven. It's a very bizarre experience sometimes. The guys there don't ever know I'm there [laughs]. Also, when you look back on your own work 50 years later, you know, it's a funny experience because you have difficulty following your own papers. When you're a young man, you're very quick and fast. I try to read my own papers and they're quite hard, you know [laughs]. Even though in principal I know them, I've forgotten some of the technicalities and I wouldn't be able to do it now. There are some terrible problems with signs you have to watch out for. So, it's a funny experience looking and it's quite gratifying to find things I did finish years ago that are still alive, you know. Many times things move on and what people do is forgotten, but some of the things I did 50 years ago are still being used and rediscovered or redeveloped by young guys and being pushed in new directions. So, that's very encouraging. I can't say I follow all the stuff but I can see that it's going in a good direction and trying to progress things.

It's been quite nice coming here to this particular event – a small scale event, I mean. I go to other meetings too but I don't have so many chances. I go, of course, to lectures and seminars. I recently went to a festival in Italy. The Italians like festivals, where they have music, poetry and mathematics; it's very nice, sort of a mixed culture. The Italians like this sort of thing. They do a lot of it. Renaissance ideas! I've been to Rome, Milan… The last one was in the south of Naples and I met interesting people. I think it was the one in Rome where I met Boris Spassky – you know, the chess player. We talked about chess and things like that. And then I also met Nash, the mathematician, who got the Noble Prize for Economics. He was there and he was interviewed. I knew him a little bit in Princeton when he was a bit crazy but now he's recovered remarkably well. But, of course, he's an old man – older than me now. [When this interview took place Nash was still alive.]

Did you have a chance to talk?
Yes, He was interviewed, for example, about his life and about the film they made of his life, and I was there. It was interesting but it's a sad case, of course. At least he's recovered from his years of illness. So, you meet interesting people at these events. I met a chap when I stayed at another hotel: Paolo Coelho, the Brazilian writer. He was very famous. He happened to appear on the same stage, in the same performance as me. He didn't care about mathematics. He was a big figure. Yes, so you meet an interesting mixture of people: musicians, poets…

You have recently written a paper on relations between mathematics and beauty, right?
I have a friend of mine who I collaborate with. He's a neuro-physiologist. But he's Lebanese, like me. He's of Lebanese origin so we do Lebanese food together and we meet and, for some time now, we have had discussions. He's interested a lot in art. He's written a book about art and vision, comparing what painters try to achieve with art and what processes happen in the brain. He does scanning and so we got into this question of mathematics. I asked him, you know, when people think about mathematics what happens in their brain. And we wrote about it. So, we had some previous work. The most recent one was about beauty. When mathematicians talk about beauty they know what they mean but is it the same kind of beauty as you see in art and music? Is it the same physiological phenomenon? And, basically, the experiments he did with his team show that, yes, there is a common part of the brain that lights up, whether you're talking about beauty in mathematics or in art or anything else. Of course, other parts of the brain light up depending on the context. There's a common part. So, the abstract notion of beauty is built into the brain and, whether talking about mathematics or painting or music, it is a common experience. So, it's correct to use the word beauty.

So have you experienced this link between mathematics and other art?
Well, we all know what we mean by beauty. We appreciate it through music and art. We also know how to appreciate it in mathematics and I think they are the same but you don't know if this is very objective. Now there is a proof, a scientific proof that it isn't subjective. The notion of beauty is physiologically based on the same kind of experience. So, when we wrote this paper it became immediately famous worldwide. There were articles in the *New York Times*, the *London Times* and one in Madrid. Everyone could understand what it said. So it became instantly quite famous. We originally had difficulty getting it published because for the orthodox guys these sorts of things tend not to be so acceptable. For the general public, of course, it's fascinating.

So, do you think people can get as moved seeing or proving a beautiful theorem as listening or playing a wonderful piece of music?
Yes, absolutely. I mean, obviously they're different but if you compare music and painting, for example, they're not the same; there's a big difference between them but there's a common aspect to the appreciation of art, I think.

But it's more difficult with mathematics, isn't it?
It's more difficult, yes, but that's the whole point. We were unsure if the word was correctly used but as mathematicians we know what we mean by beauty and I think the beauty in mathematics is comparable to the beauty in music. They're not the same but they are comparable, there's no question about it. We know what a really beautiful theorem is [laughs]. It's a subjective feeling but it's true. Now, Hermann Weyl made the following quote: "Most of

my life my two objectives were searching for truth and beauty but when in doubt I always chose beauty." Now people think this is ridiculous but why should you be worried about the truth? In fact, I argue this, you see: truth is something you never reach; you find other things while searching for the truth. What you have at any given moment is an approximation of the truth – partial truth. It may even be an illusion. But beauty is subjective, an immediate experience. You see beauty, you know. I'd like to say that beauty is the torch that guides you towards the truth. You can see it. It throws light. It shows you the direction. You follow that and experience has shown that beautiful things lead to true results. So, I think it is a very interesting connection between truth and beauty. I think Hermann Weyl would've agreed with that. People say it was a joke but I'm sure that he meant it.

Talking about beauty, we have dinner very soon.
Yes [laughs].

A light dinner. So I don't want to take up more of your time. Thank you.
Okay. Good. Thank you very much.

I really thank you for this. I enjoyed enormously listening to you.
Yes, I also enjoyed talking about all this.

Thank you very much, Michael.

Acknowledgements
The author wishes to thank Eva Miranda for her help with the preparation of this interview. This interview first appeared in Spanish in "La Gaceta de la RSME, Vol. 19 (2016), Núm. 2, Págs. 275–291". Thanks are due to Leo Alonso and Ana Jeremías for their help and to the Xunta de Galicia and FEDER grant GRC2013-045, which covered the transcription of the original interview.

Oscar García-Prada is a CSIC research professor at the Instituto de Ciencias Matemáticas— ICMAT. He obtained a DPhil in mathematics at the University of Oxford in 1991 and had postdoctoral appointments at the Institut des Hautes Études Scientific (Paris), the University of California at Berkeley and the University of Paris-Sud, before holding positions at the University Autónoma of Madrid and the École Polytéchnique (Paris). In 2002, he joined the Spanish National Research Council (CSIC). His research interests lie in the interplay of differential and algebraic geometry with differential equations of theoretical physics, specifically in the study of moduli spaces and geometric structures. He participates regularly in public outreach activities on mathematics and their interactions with physics and music, collaborating with the newspaper EL PAIS, Spanish Radio Exterior and Spanish television RTVE.

Discoveries, not Inventions – Interview with Ernest Borisovich Vinberg

Alice Fialowski (University of Pécs and Eötvös Loránd University, Hungary), Joachim Hilgert (Paderborn University, Germany), Bent Ørsted (Aarhus University, Denmark) and Vladimir Salnikov (Luxembourg University, Luxembourg)

Ernest Borisovich Vinberg received the Distinguished Speaker Award in 2016 from the European Mathematical Society. He delivered his talk at the 50th Sophus Lie Seminar in Będlewo, Poland. On this occasion, we asked him to give an interview for the EMS Newsletter. The interview took place in Będlewo, Poland, on 27 September 2016.

Professor Vinberg, we are very happy that you came here to this 50th Seminar "Sophus Lie" and we would like to ask a few questions.
To begin with, who got you into your research?
In fact, I had two advisors: Evgeny Borisovich Dynkin and Ilya Iosifovich Piatetski-Shapiro. They were both distinguished mathematicians. Dynkin was a brilliant lecturer, attracting a lot of young people, but Piatetski-Shapiro posed the problem which was kind of a challenge. This problem concerned homogeneous bounded domains. The question posed by Élie Cartan was whether any such domain is symmetric, and Piatetski-Shapiro gave a counterexample in dimension 4. Then a problem arose to give a classification of homogeneous bounded domains in complex spaces. And this turned out to be related to classification of homogeneous convex cones in real vector spaces, which was the subject of my PhD thesis.[1]

And when was it?
I entered the graduate school in 1959 but I began my scientific research some time earlier. My first published paper was my diploma work on invariant linear connections in homogeneous spaces, which was fulfilled under the guidance of Dynkin in the academic year 1958/59.

Did you continue working with Dynkin and Piatetski-Shapiro? Tell us something about your two advisors.
I never really worked with Dynkin because he had completely switched to probability theory by that time. I continued to work on homogeneous Kähler manifolds with Piatetski-Shapiro and another of his students, Simon Gindikin, up to 1965. After that, I turned to other problems. Some of them were influenced by my previous work, some of them not, but I became more or less independent at the age of about 27. But the topic of my doctoral thesis[2] was also related to Piatetski-Shapiro's question. It concerned the Selberg conjecture about arithmeticity of discrete subgroups, so-called lattices, in semisimple Lie groups. Piatetski-Shapiro was interested in this problem but he did not manage to solve it, although he had some results in this direction. I developed a theory of hyperbolic reflection groups, which permitted me to construct many counterexamples at rank one. After that, Margulis proved his famous theorem for higher rank, answering affirmatively the Selberg conjecture. My work on hyperbolic reflection groups had two sources: one was Dynkin's theory of simple roots, which is tightly related to finite reflection groups; and the other was the theory of automorphic forms for lattices in semisimple Lie groups, which was a favourite subject of Piatetski-Shapiro. So in a sense I continued studying some questions they posed to me (both Dynkin and Piatetski-Shapiro). In 1966, there was the International Congress

Ernest Borisovich Vinberg, Będlewo 2016. (All photos in this article by Janusz Grabowski.)

1 Кандидатская диссертация in Russian.

2 Докторская диссертация in Russian – roughly equivalent to habilitation in some countries.

Interviewing Ernest Borisovich.

of Mathematicians in Moscow and I gave two talks there. One of them was related to our common activity with Piatetski-Shapiro and Gindikin, and another was about hyperbolic reflection groups.

The Moscow school at that time was really famous. Who were your colleagues from the same generation?

Yes, it was a really remarkable time; some people call it the Golden Age of Moscow Mathematics. And especially our course was very strong. Many people from our course were hired to teach at our mathematics faculty in 1961. It was an initiative of academician Kolmogorov. Among them, apart from me, there were Kirillov, Arnold, probability theorists Shur and Tutubalin, topologists Arkhangelski, Pasynkov and Ponomarev, and some others. I was very happy that this happened.

How do you choose a problem to work on? I mean, how do you decide if the problem you think of is worth your time? You are known as having solved many different problems; you are not a researcher who goes in one direction only. What do you need to like in a problem to get started?

Yes, I worked on different problems but they were somehow related. All the areas of mathematics are related. It is difficult for me to say how I choose a problem. If something is interesting for me and I feel that I can do something in this direction, I try to do it. If it is not interesting for me, I don't try to do it.

Can you give something like a criterion of what makes it interesting for you?

I think there are two types of mathematical results: inventions and discoveries. I realise the importance of inventions but I prefer discoveries. Sometimes, when I obtain a result, I have a strong feeling that I am discovering something existing in nature. And I don't have such a feeling with respect to some other works and mathematical results; they seem to be more inventions – creations of the human brain – rather than something really existing in nature.

What is your favourite discovery?

I hope I have not made it yet.

If you have to choose among those you have discovered already?

I think these are, first, the theory of hyperbolic reflection groups, second, my development of invariant theory, which I call the "effective invariant theory" (related to graded Lie algebras) and maybe also my work about invariant orderings on semisimple Lie groups. Quite recently I began to study the so-called Chevalley-type theorems. There is a famous theorem of Shephard-Todd-Chevalley about the criterion for the algebra of invariants of a finite linear group to be free. It says that it is exactly the case when the group is generated by complex reflections. The same question can be posed for infinite reflection groups and the natural setting is to study complex reflection groups in symmetric domains, namely in complex balls and so-called future tubes, which are the only symmetric domains admitting reflections. Recently I obtained some results in this direction, together with Osip Schwarzman, my former student. Maybe it will be my favourite result for the moment.

You've conducted a very famous seminar for several decades, together with Onishchik, and produced a book out of it. Can you tell us more about this seminar?

Yes. It was, in fact, a continuation of the famous seminar ran by Dynkin – our common teacher – after he had switched to probability theory. He was a student of Kolmogorov – a famous probabilist and the creator of axiomatic probability theory. And he started with some work in probability theory. But after that, he attended a seminar of Gelfand, and Gelfand suggested he prepare a talk on the classification of simple Lie algebras. Preparing this talk, Dynkin discovered his famous simple roots. At that time, he was a student in the fourth year (it was during the war in 1944). After that, Dynkin got interested in the theory of simple Lie algebras and produced his famous works, which made him a classic in Lie theory. About 1955, he switched to probability theory and eventually became a classic in this theory, too. By the way, it is interesting that several years before his death in 2014, he did some joint work with my student Andrei Minchenko on simple Lie algebras.

Coming back to your question, our seminar started in 1961, when we were both young teachers in our department. First, we tried to study all mathematics, starting from Cantor's set theory. But soon we realised that we had been quite naive and concentrated on Lie group theory. At that time, the famous Chevalley seminar was available and the Chevalley monograph "Theory of Lie groups" appeared. We understood that we should study algebraic groups in connection with Lie groups.

There were many enthusiastic young people attending our seminar. Everything was going on in the following way: we (me and Onishchik) presented some theory, mostly in the form of a series of problems. All the participants – there were about 25 of them – split into several groups, discussing the problems and the solutions between two sessions of the seminar, and the most interesting solutions were presented at the next session. Then we continued presenting some new theory.

As a result of this study, our book *Seminar on Lie Groups and Algebraic Groups* appeared (in the English translation, "Seminar" was omitted). First, it was prepared with the help of some participants of our seminar, who wrote down what we told them and the problems, and so on. Then me and Onishchik rewrote all this, edited it, and in this way our book appeared. It retains the style of the seminar. The theory is presented in a series of problems, which the reader is supposed to try to solve on his/her own, but there are some hints at the end of each chapter. There are also some exercises. This book was used by several generations of students and graduate students of our department.

After that, we switched to invariant theory. We studied it together, the leaders and the participants. As a result, some of us became experts in this theory and made some contributions. During this period, V. Popov joined me and Onishchik as a leader of the seminar.

Apart from these, many other topics were also presented at our seminar. One year we studied supermathematics. Perhaps you know that one of the founders of supermathematics was Berezin, who was our elder scientific relative, a student of Gelfand. And also some other topics appeared, e.g. discrete groups and applications of Lie theory in mathematical physics. Dmitry Alekseevsky, one of the participants of our seminar, understood mathemetical physics very well and he presented a series of talks.

The seminar lasted in this way for about 50 years. But these days, Onishchik is not able to help with this anymore and there are new leaders: my younger students (nowadays colleagues) Timashev and Arzhantsev. Unfortunately, this is going on in a much worse way because fewer students come to study mathematics. The younger generation is less motivated in this and this is quite sad…

You were the founder of the Journal "Transformation Groups". Could you tell us about the first steps?

We founded this journal together with my former student Vladimir Popov, with active participation and help of Ann Kostant, the mathematics editor of Birkhäuser at that time, and have been running it (I hope, successfully) for 20 years. The first managing editors were (apart from me and Popov) C. De Concini, G. Margulis, A. Onishchik, G. Schwarz and M. Vergne. The whole editorial board consists of more than 30 mathematicians and is gradually being renovated. At different times, the managing editors have been M. Brion, P. Etingof, E. Frenkel, V. Ginzburg, W. Goldman, M. Kapovich, A. Kleshchev, I. Mirkovic, H. Nakajima, A. Premet and A. Zelevinsky. We reject more than half the submitted papers.

As was written in the preface to the first issue, the concept of a transformation group reflects the symmetry of the world, which is perceivable so far as it is symmetric (but we do not know why it is so remarkably symmetric). All my personal work is related to different kinds of transformation groups.

During the interview.

I first knew you as a teacher – during my first years at the Mathematics Department in Moscow. So my question is: do you have any teaching philosophy, any principles that you would like to share?

My first principle is that it is not so important what to teach but the most important thing is how to teach. Because it is clear that most of the theorems that we teach to our students will never be needed for them when they graduate from university. But we have to teach them the right thinking. The second principle is that one should try to avoid tedious calculations, replacing them with the ideas that permit getting the same result without calculations.

And Mechmat[3] is not the only place where you can learn mathematics in Moscow.

It used to be essentially the only place. But then the Independent University appeared and, in recent years, the Faculty of Mathematics of the Higher School of Economics has appeared.

Would you like to say something about the Independent University?

The Independent University was a very important and useful project. But I would like to say that it is not really independent. I know only one person (Valentina Kirichenko) who graduated from the Independent University and did not study at the Maths Faculty of Moscow State University. The Independent University is rather a system of advanced courses: they do not teach elementary algorithms. But I think it was very important for several generations of young people and also for talented mathematicians who could not go to teach at Moscow State University for some reason.

You have supervised a lot of students: more than 40 PhD theses and several habilitations. Do you have any strategy? What is your approach to advising students?

Well, I have no special strategy. I just try to make them get interested in mathematics. I try to find some interesting problems, which they can solve. But I'm afraid I don't pay enough attention to my students.

[3] Faculty of Mechanics and Mathematics of MSU.

You have been an excellent teacher and an outstanding researcher for all your life. And you are one of the people who stayed in Moscow…
Yes, I never considered the possibility of emigrating. I stayed in Moscow. But over the last 20 years, I have visited Germany, namely Bielefeld University, every Summer, for two to three months. They initiated this, nominating me for the Alexander von Humboldt Prize, which I won in 1997. After that, they continued inviting me in the framework of their SFB (Sonderforschungsbereich). By the way, this university has successfully nominated many Russian mathematicians for the Humboldt prize: Alexander Merkurjev, Sergei Adjan, Vladimir Platonov and others. In Bielefeld, I collaborated with local mathematicians J. Mennicke, H. Helling and H. Abels, and also with other guests from all over the world. Visiting Bielefeld University is sort of my second life. Being in Moscow, I am always quite busy with many things; many people disturb me and want something from me. And when I go to Bielefeld, I relax and reflect on problems that I am interested in, I talk to my colleagues and so on. So my life is divided into two different parts, each of them being very important for me.

You told us before that right now you are working on complex reflection groups. Could you share some other things that you currently find interesting, that you are working on at the moment? Some projects perhaps? Your current activities?
I am always working simultaneously on two to three topics. For the moment, I am working on complex reflection groups as you said and on some "non-abelian gradings" of simple Lie algebras, on which I am going to talk tomorrow at this conference. I am also reflecting about some problems of equivariant symplectic geometry, continuing my previous results.

And we've heard that you are also involved right now in teaching as always. What classes do you teach?
The course of algebra for the second year undergraduate students and the advanced course in invariant theory for students starting from their third year. I am also running two seminars: one research seminar we have already discussed and another one for students, which is called "Algebra and Geometry". At the latter one, we try to show relations between algebra and geometry. For example, for several years we studied relations between algebraic invariant theory and the theory of automorphic forms. Namely, due to Torelli type theorems, automorphic forms can be studied by means of geometric invariant theory, realising arithmetic quotients of symmetric domains as the moduli spaces of some classes of algebraic varieties.

And nowadays, how many students attend those seminars, since you said there are fewer interested students?
Yes. Unfortunately, only a few students attend our research seminar in Lie groups and only 10–12 students attend our seminar "Algebra and Geometry". But, from time to time, I still have talented students coming to me,

E. B. Vinberg delivers his EMS lecture.

who are interested in doing mathematics and not business, and some of them are not even going to emigrate.

Thank you very much!
Thank you for your interest!

P.S. Off the record, Ernest Borisovich told us about two of his interviews with Dynkin in the USA in 1992 and 1999. A huge collection of interviews of Dynkin with many Russian and Western mathematicians who came to visit him in the United States, is now available at the online library of the Cornell University: http://dynkincollection.library.cornell.edu/.

© Noel Tovia Matoff

Alice Fialowski is professor of mathematics at the Institute of Mathematics, University of Pécs, and at the Eötvös Loránd University in Budapest, Hungary. Her research interests are Lie theory, cohomology, representation and deformation theory, with applications in mathematical physics.

Joachim Hilgert is professor of mathematics at the Institut für Mathematik at Paderborn University, Germany. His research interests are harmonic analysis, representations of Lie groups, symplectic geometry, and supermanifolds.

Bent Ørsted is professor of mathematics at the Department of Mathematics, Aarhus University, Denmark. His research interests are harmonic analysis, representations of Lie groups, conformal geometry, spectral geometry.

Vladimir Salnikov is a senior researcher in mathematics at the RMATH, University of Luxembourg, Luxembourg. His research interests are graded and generalized geometry, dynamical systems and integrability, applications to theoretical physics and mechanics.

Bringing Good Maths Books to Children

Nguyen Tien Zung (University of Toulouse, France)

This article is about the *Sputnik Bookcase*, a project that I founded with some colleagues to bring inspirational, high-quality, educational books in mathematics and other subjects to children in Vietnam (with an idea to expand internationally). From February 2015 to October 2016, we printed 25 books, totalling about 100,000 copies, with many more books in the pipeline.

Why do children and adults hate maths?

Vietnam is a poor country with a GDP per capita of just 2,000 USD (PPP) but it often ranks in the top 10 in International Mathematical Olympiads (IMOs), higher than France and Germany, for example. This result does not mean that Vietnam has a better mathematical education system than Europe but, somehow, reflects the fact that the education system in Vietnam is too exam-oriented: children waste a lot of time on learning by rote, trying to memorise formulas and solutions to typical problems in order to get high scores in exams and competitions. They often go to additional private classes many times a week and repeat lessons until very late in the night.

While exam-oriented learning may be good for getting high scores in exams, it is very expensive in terms of time and money and has detrimental long-term consequences: students become passive, lack creativity and critical thinking, do not really understand what they learn and even risk depression due to lack of sleep and physical activity.

Since this kind of maths education is mostly disconnected from the real world and does not show children how joyful and useful mathematics really is, a majority of them naturally come to hate mathematics. When asked, most adults would say that high school mathematics is useless for them, especially the more advanced topics like integrals and complex numbers, and many people think that such topics should be deleted from the programme. This opinion about the uselessness of maths is widespread not only in Vietnam but probably in many other countries as well, including France.

Maths books to make learning joyful and useful

In their exam-oriented learning, most children and students in Vietnam only use textbooks and exercise books. Maths notions in these books are often introduced in a formal, unintuitive and even dogmatic way. For example, instead of saying that a rational number is the quotient of two integers, they give the following definition: a rational number is a decimal number whose expression is either finite or infinite periodic.

One notorious professor who had a lot of influence in Vietnam bragged that he could teach higher mathematics to young schoolchildren. How did he do it? As an example, he taught group theory by making children learn by heart all the axioms of a group and then check that these axioms are satisfied on some finite sets with given tables of multiplication, claiming that the children "knew group theory" after these formal lessons. Needless to say, education reforms proposed by such professors were a disaster.

In our opinion, maths notions should not be introduced *formally* but *naturally* and *intuitively*, with a lot of motivation and explanation about how and why they were invented and what they were invented for. And it's not enough to have textbooks and exercise books; children also need other kinds of interesting maths books, e.g. maths novels, history of maths, applications of maths, recreational maths, etc., books that can inspire and show them how natural, joyful and useful maths really is.

The birth of Sputnik Education

I live in France but am very worried about the situation in my native country. I have written numerous articles advocating political and educational reforms in Vietnam but, as usual, they have fallen on deaf ears. I also wanted to do more concrete things and so I founded a small education company in Vietnam in 2014 together with five friends, two of whom were business-oriented (Phan Thanh Do and Hoang Thi Thai Thanh) and three of whom were reputed mathematicians: Professor Ha Huy Khoai (formerly Director of the Hanoi Institute of Mathematics), Professor Do Duc Thai (Head of the Department of Mathematics at Hanoi National University of Education) and Dr Tran Nam Dung (a famous trainer of mathematical Olympiad teams).

Later on, some other key members joined, who now form the new management of the company (I'm not an official manager, just a founder and the informal 'Editor-in-Chief').

Nguyen Tien Zung, Tran Nam Dung, Ha Huy Khoai & Do Duc Thai.

We needed a name for our company and, after much thought, chose "Sputnik Education". Why Sputnik? Firstly because this is the name of the celebrated artificial satellite that marked the beginning of a new era of humanity. Secondly, the Russian word "sputnik" means "companion" and our company is a "companion for joyful learning". Thirdly, the Russian maths education system is one of the best in the world and five of the six founders of Sputnik Education happened to have studied in Russia.

The first purpose of our company is to produce the *Sputnik Bookcase*, a series of high-quality educational books. When the company was formed, we had five books ready. They are (Vietnamese translations of) *The man who counted* by Malba Tahan, *Three days in Karlikania* by Vladimir Levshin, *Combinatorics and induction* by N. Ia. Vilenkin, *169 interesting maths problems* by Tran Nam Dung and my book *Maths lessons for Mirella*.

The current laws of Vietnam prohibit private publishing houses and we need to make contracts with state-controlled publishing houses (who charge us a fee) in order to print our books. After many months of looking around, our first five books finally appeared in early 2015.

Living on a shoestring

Since none of us were rich, our "garage-based business" started with less than 40,000 euros of capital and we used a room in one of our houses as an office and warehouse. In theory, we could raise more capital but then we would risk losing control of the company to get-rich-quick people who do more harm than good to the education system.

That small amount of money was enough to print about 10 books (3,000 copies per book) and we had to employ a few people (even though most of the work was originally unpaid and carried out by ourselves) and operate in a very bleak market. Looking at official figures, the whole book market of Vietnam was only 90 million euros in 2015, i.e. about 1 euro per person. Add to that the very low book prices (equivalent to about a fifth of the international prices), widespread pirating, closures of bookstores, etc., and many book companies end up losing money. We needed profit in order to survive whilst maintaining high quality so we had to follow a set of criteria for choosing books:

- *Correctness*. The book should be scientifically sound, without serious errors or inaccuracies.
- *Attractiveness*. The book should be clearly written, easy to understand, attractive and inspirational for the reader.
- *Diversity*. Besides textbooks and exercise books, we also want maths novels, maths in real life, maths modelling, recreational maths, maths and logic puzzles, history of maths, etc.
- *Profitability*. Our books should be easy to sell and not too expensive to make. We have to avoid, for example, very good university-level books because most students in Vietnam have other priorities and don't buy books –they just make photocopies of the books that they need.

Other barriers to be overcome

The language barrier. "Traduire, c'est trahir." With ridiculously low book prices, publishers in Vietnam cannot afford to pay translators well enough. As a consequence, it is very difficult to find good translators and too many translated books contain serious errors on every page. It often happens that after someone translates a book, we have to re-translate it to correct the errors. For some "difficult" books, it is not possible to find a translator at all. For example, over two years we gave Abbott's *Flatland* to four different translators and they all gave up after a few months.

The copyright barrier. Copyright fees themselves are not *the* problem; instead, it is more about making contracts with foreign authors. This is because no one is experienced in this matter in our company and we can't afford to hire someone just for that. Therefore, we are losing a lot of time and energy on it. For example, after more than a year, we could not finalise a contract for the Vietnamese version of Wendy Lichtman's book *Secrets, Lies and Algebra*. We wish we could buy the publishing rights as easily as buying food from a grocery store. We are also in contact with some other authors, e.g. Ian Stewart for his popular maths books, and hope that things will go more smoothly. In some lucky cases, when the authors don't ask for royalties, things are easier for us and, in those cases, we give books to charities instead of royalties.

Accounting mess. Even with a small company like ours, accounting can be a serious problem. We had a part-time accountant from the beginning but things were not done properly and so we recently had to hire an external expert and pay him well to help us with accounting and tax filing.

Bestsellers to the rescue

Fortunately, we had some (international) bestsellers that kept us afloat, despite all the troubles we had faced. Here are a few of these bestsellers:

The Man Who Counted (in Portuguese: O Homen Que Calculava) by Talba Mahan (real name: Júlio César de Mello e Souza (1895–1974), "the only Brazilian mathematician who was as famous as a soccer star") is a maths novel for children that sold more than 2 million copies in Brasil alone. We printed it twice and sold about 5,000 copies.

Kiselev's *Geometry* (*I: Planimetry*; *II: Stereometry*). Leonid Polterovich (Tel Aviv) and Alexander Goncharov (Chicago) recommended this book to us and Alexander Givental (UC Berkeley) gave us permission to use his English version (2006). People are probably right when they say that Kiselev's *Geometry* is still the best geometry textbook: the presentation in the book is extremely clear, precise and still very modern for a book written a century ago. Our first print of Kiselev's *Planimetry* sold out in four months.

Around The Rotations by Waldemar Pompe is another extremely interesting elementary geometry book. It shows how to use symmetries to arrive at very elegant solutions to many difficult problems in planar geometry. It has been recommended to us and translated from Polish to Vietnamese by Nguyen Hung Son and Nguyen Sinh Hoa, who are professors in Warsaw.

Vladimir Levshin's *Trilogy: Three days in Karlikania*, *Black Mask from Al-Jabr* and *Fregate of Captain Unit*. This trilogy is a wonderful and gentle introduction to elementary arithmetic, algebra, geometry and scientific thinking in general. As a child, I somehow got hold of the book *Black Mask from Al-Jabr* and it was because of it that I enjoyed learning equations. These books had been translated from Russian to Vietnamese before and sold very well in Vietnam. However, the old translation contained many inaccuracies so we decided to make a new one.

A Day's Adventure in Math Wonderland by Jin Akiyama and Mari-Jo Ruiz. Jin Akiyama is one of the most famous maths popularisers in the world and he has founded a mathematical museum in Hokkaido (Japan), called "Math Wonderland". This book is an exciting virtual excursion to his wonderland. It has been recommended to us and translated by Vuong Hoa, a young lady who happened to know Akiyama while studying at his university.

The first 20 books of Sputnik Bookcase; cover of "Maths and Arts"; journal "Buddhist Culture", which contains a review article of the book "Maths and Arts".

Maths and Arts (in Vietnamese: Toán học và nghệ thuật) by Nguyen Tien Zung. In May 2016, I gave a public lecture and then wanted to write a book about the relations between mathematics and the arts (including the visual arts, music, prose and poetry), in memory of my late father. The book came out in August 2016 and quickly sold about a thousand copies over the first month. Two newspapers/journals printed review articles about it (written by readers who liked the book), which was a first for Sputnik Bookcase.

Reputation and advertisements help

One of the challenges for us is to build up a good distribution network, which is still weak at the moment. Nevertheless, all of our books are doing well, thanks in part to our good reputation. Many readers like Sputnik Bookcase so much that they buy the whole series – every book in it.

Of course, good advertisements also help. Recently, we started buying advertisements on social networks and saw the number of customers (at least those buying books directly on http://shop.sputnikedu.com) increase significantly. We have also participated in many science popularisation activities (e.g. maths open days, STEM fests, public lectures and exhibitions) and charity programmes (e.g. the UNESCO-prizewinning programme "Books for the rural area" by Nguyen Quang Thach, where we donated hundreds of books), which of course helped the children and, at the same time, enhanced our reputation.

What next?

We have many ideas on how to grow Sputnik Education into a major education company. One of them is, of course, to continue to publish many books, not only in mathematics but in other subjects as well, keeping our high quality standards. Other ideas include: educational toys, online interactive education, international expansion, etc. For example, there are now many video lectures available online but they are often of low pedagogical quality. Whatever product or service we make, we want it to be of high educational value.

In the book market, textbooks form the most lucrative niche: they often sell by hundreds of thousands of copies. This niche has been the monopoly of the Ministry of Education but they have decided to open up the market next year. So, we will have the opportunity to publish modern maths textbooks, which fit our philosophy of joyful, meaningful and useful education.

Nguyen Tien Zung was born in Vietnam in 1970 and graduated from Moscow State University in 1991. He defended his PhD thesis in Strasbourg in 1994 on the topology of integrable Hamiltonian systems and worked as a "chargé de recherches CNRS" in Montpellier from 1995 to 2002 before becoming a professor at the University of Toulouse in 2002 and being promoted to "professeur de classe exceptionnelle" (distinguished professor) in 2015.

Mathematical Etudes: Evolution from Multimedia to a Book

N. N. Andreev, N. P. Dolbilin, S. P. Konovalov and N. M. Panyunin (Steklov Mathematical Institute, Moscow, Russia

An exceptional feature of mathematical popularisation in Russia from its very beginning has been the main role always played by leading scientists. The popularisation movement started from extracurricular clubs called "mathematical circles", which appeared in Leningrad (now Saint Petersburg) in the early 1930s. This maths club activity was accompanied by the first Russian citywide Mathematics Olympiad for high school pupils, organised in 1934 by the prominent mathematician and populariser Boris Nikolaevich Delaunay. In the post-war period, the maths circles and mathematics Olympiad movement became a mass phenomenon. Many outstanding mathematicians (including A. N. Kolmogorov, P.S. Alexandrov, I.M. Gel'fand, D.K. Faddeev and V.I. Arnold) were engaged in this activity: giving lectures, writing popular science books for children and organising Olympiads. It was understood (and this fact also became Russian tradition) that to awaken a child's interest in some occupation, it had to be taught by leading scientists.

Following these Russian traditions in popularising mathematics, the Steklov Mathematical Institute, through the initiative of its director academician V.V. Kozlov, launched in 2010 a separate Laboratory of Popularisation and Promotion of Mathematics (though preparatory work preceding its creation began in 2002). This Laboratory became a pioneer in the promotion of mathematics in Russia, setting new standards in the popularisation of mathematics and stimulating the development of this field.

During the first years of the Lab, several multimedia projects were established that are now well known in Russia. A list of these projects is presented in more detail below.

- Mathematical Etudes (ME). This is a series of more than 60 movies, made using modern 3D computer graphics, devoted to some solved and unsolved mathematical problems.
- Mechanisms by Tchebyshev. This is a set of movies and other materials on mechanisms suggested and constructed by this great Russian mathematician of the 19th century.
- Apps for mobile devices with mathematical content.

Several electronic online libraries were also organised. The first of them is the archive of the Publishing House "Mathesis", which was the first publishing house in Russia publishing the best popular science books on physics and mathematics printed in Europe and Russia from 1904 to 1925.

The second online library is the archive of the *Experimental Physics and Elementary Mathematics Bulletin* journal, which was published 24 times a year from 1886 to 1917. This journal was a pre-revolutionary analogue of the famous magazine *Kvant*, which was organised under the guidance of the eminent scientists I.M. Kikoin and A.N. Kolmogorov in 1970 and still exists to this day. The magazine *Quantum*, published in the US from 1989 to 2002, consisted of selected materials from *Kvant*.

It is surprising (taking into account that a century has passed since then) how many interesting materials on mathematics and the other sciences can still be found in these books and journals. These materials remain useful for the popularisation of both science and education. (Unfortunately, the last two projects are only available in Russian.)

"Mathematical Essence"

For the first 13 years, the team of the Laboratory showed up mostly on the internet, even though they also delivered hundreds of lectures for schools and university students, and teachers and educators in various regions of Russia. It should be emphasised that these lectures were all based on the multimedia content created in the Laboratory.

However, in 2015, the Laboratory found it necessary to prepare a book called *Mathematical Essence*. Its publication was supported by the Scientific Council of Steklov Mathematical Institute. The Russian title of the book is not easy to translate adequately into English, giving several possible alternative English versions: "Mathematical Component", "Mathematical Feature" or "Mathematical Constituent".

The fact that mathematics is the language and an important tool of science and technology is well known to any reader. The experienced reader will know that mathematics plays a great role in physics, in the implementation of space flight, in the taming of atomic energy and in the computer world. The importance of mathematics in other fields such as medicine or linguistics is less known to the general public.

But even the reader who has heard about the significant mathematical "component" in various spheres of human activity is often not aware of the degree of dependence of these sciences on mathematics. The main reason for such ignorance is the complexity of mathematical tools designed particularly for a given application. Verbally recognising the role of mathematics, people usually do not ponder the mathematical "filling" of the objects and phenomena surrounding us. Sometimes they simply do not notice it.

The main purpose of the book *Mathematical Essence* is precisely to bring to the surface the mathematical "essence" of some of the greatest achievements of our civilization, as well as to manifest the mathematical "content" inside usual, everyday things.

There is one more problem in modern day life that should be taken into account by popularisers of mathematics: information overload, resulting in an almost universal unwillingness to learn anything that is not related to a daily necessity. From this point of view, presentation of mathematics as a necessary and essential part of world knowledge may produce a "personal" interest in the potential reader for studying mathematics.

The authors of the book are well known as actively working mathematicians. It is very important that the reader obtains all the scientific information from "the table" of leading scientists. A popular descriptive style of presentation (with minimal use of formulas) is specially designed for a wide range of readers.

It was clear to the editors that the publication of such a book was absolutely necessary and timely, noting that the range of readers prepared to read popular books on mathematics has been rapidly decreasing over recent decades and we should respond in some way to this sad tendency. One of the purposes of the book is to show that mathematics is not an isolated science but an essential (although sometimes hidden) part of many important phenomena and objects of the world. The illustrations presented and the Russian website http://book.etudes.ru give an idea of the topics collected in the book.

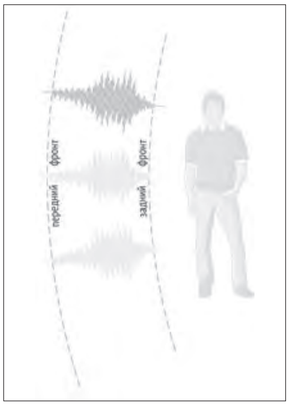
Why did we decide to return to the classical format of a book after working for over 10 years on the creation of multimedia presentations of mathematical topics? The main reason is that all the Laboratory materials were designed for a thinking person and it is much more convenient to think while reading a book rather than clinging to the monitor screen.

There is another psychological advantage of a book compared to multimedia and interactive sites: a reader, looking through the contents of the book, gets an immediate idea of the book as a whole.

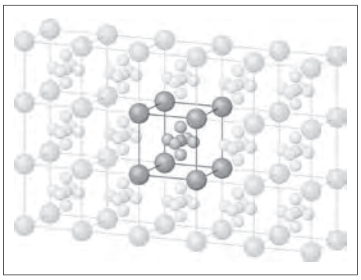
And finally, to illustrate one more reason for the evolution from multimedia to a book, one could say that the difference between a website and a book is the same as the difference between a young and a vintage wine. The contents of a book (and the authors themselves) need to mature before a decent publication.

In Russia, "Mathematical Essence" has become very popular and has won a prestigious prize for popular scientific literature. We hope that the potential translation of the book into other languages will expand the circle of its readers.

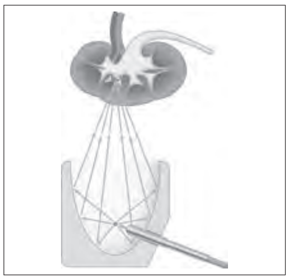
Computer tomography (by A. G. Sergeev).



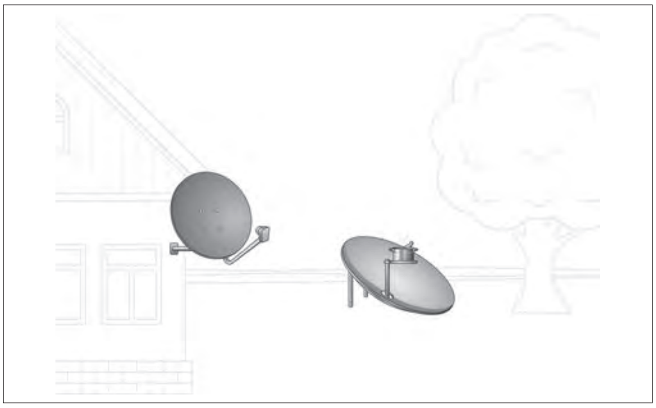
Sound wave propagation (by M. V. Feigin).



Geometric crystallography (by N. P. Dolbilin).



Kidney stone treatment (optical properties of an ellipse).

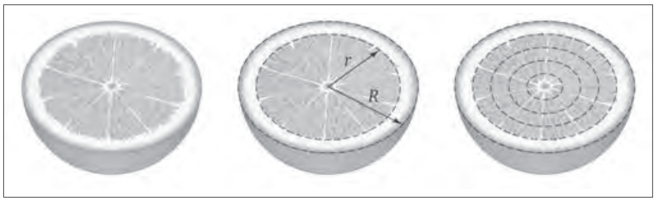


Parabolic antenna (optical properties of a parabola).

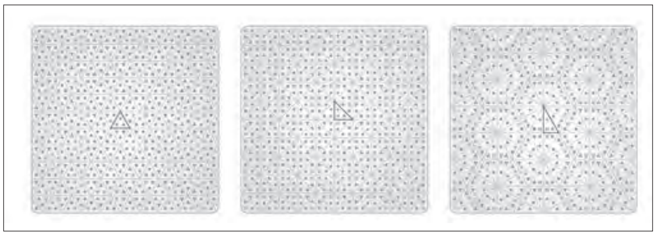
The musical scale (geometric progression).

Railway wheel sets (properties of a cone).

Distance to the horizon (the Pythagorean theorem).



Volume of oranges (properties of the volume of a ball).



Kaleidoscope (reflection groups).

“Mathematical Etudes”

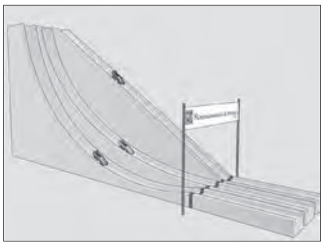
More than 60 movies on different topics in pure and applied mathematics have been created throughout 13 years of the main Laboratory project “Mathematical Etudes” (http://etudes.ru). Some of the movies are based on purely mathematical ideas; the others deal with its applications to technology and engineering. There are also movies that describe the historical context of a discovery and present the scientists and engineers who made crucial contributions. Each movie is accompanied by a short popular description of the problem and appropriate references and links. The main goal is to attract viewers to mathematics, to show its intrinsic beauty and present ideas for a deeper understanding of the world.

This project differs from the other, more traditional forms of popularisation by the intensive use of modern tools in the presentation. The main tool is the use of 3D computer graphics, which has been chosen not only because it makes the mathematical ideas easier to understand but also because it is much more attractive for today’s young people.

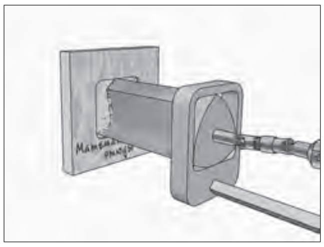
Some highly experienced computer professionals have gathered together for this project: the 3D graphics were produced by Mikhail Kalinichenko, and Roman Koksharov created the 2D graphics, as well as designing and programming the websites (not only for this project but also for our other projects).

Cubist parquet (nets of the cube).

And this is a net?! (the theory of nets of polyhedra).

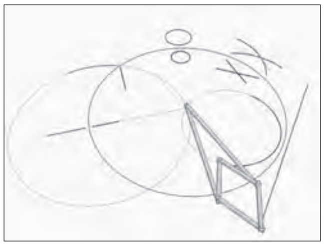


Cycloid (the brahistohrone problem).

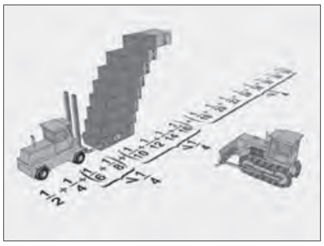


Drilling of square holes (figures of constant width).

The folded rouble (V.I. Arnold’s problem).

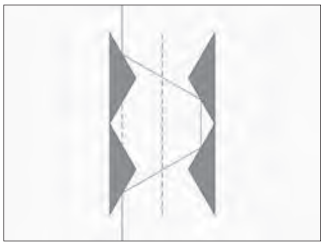


Lipkin inversor (how to draw a straight line with linkage).



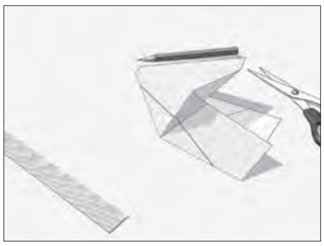
A ladder to infinity (the divergence of the harmonic series).

Contact numbers (the theory of coding).

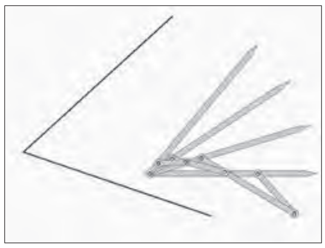


The invisible (construction of invisible bodies).

The sandwich problem (the Bolzano–Cauchy theorem).



With a single cut (cutting out an arbitrary polygon with one incision).



Angle trisection (the theory of linkages).

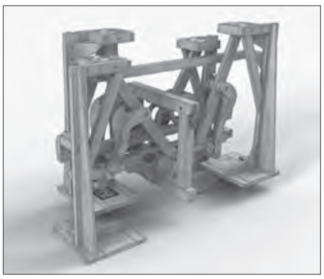
"Mechanisms by Tchebyshev"

An extensive and important topic, designed as a separate subproject, is "Mechanisms by Tchebyshev" (http://tcheb.ru). All 33 planar linkage mechanisms devised by the great Russian mathematician P.L. Tchebyshev (or Chebyshev in modern writing), as well as various other mechanisms based on these, are demonstrated, explained and modelled using computer graphics. These devices include the world's first walking machine (called a "plantigrade" machine by Tchebyshev), a "sorting machine", a "wheelchair" and a "paddling" mechanism.

Some of these devices and mechanisms are stored in the Polytechnical Museum of Moscow, the Museum of History of Saint Petersburg University, the Museum of Arts and Metiers of Paris and the Science Museum of London. By agreement with these museums, the Laboratory has created computer realisations of these mechanisms based on precise measurements of the original models. Preserving the size of the models, as well as other details of the mechanisms, makes it possible to manufacture (in principle) exact copies of Tchebyshev devices. Mechanisms that appeared to be lost have been reconstructed according to archive documents.

The movies of the project demonstrate both how the mechanisms operate and where they originate from (i.e. their mathematical background). In particular, the idea of an approximation of a desired curve (a line segment, an arc of a circle, a full circle and so on) by a linkage curve played an important role in Tchebyshev's discoveries. He started from a problem, posed by James Watt, of how to construct a linkage between the circular motion and the straight one. Tchebyshev, however, failed to find a precise solution of this problem and then started to develop approximation theory. It is quite remarkable that this technical part of Tchebyshev's activity resulted in the discovery of the celebrated Tchebyshev polynomials, thus initiating a completely new chapter in mathematics! We also hope that the Tchebyshev mechanisms project of our Lab will set a new standard of scientific internet-museums.

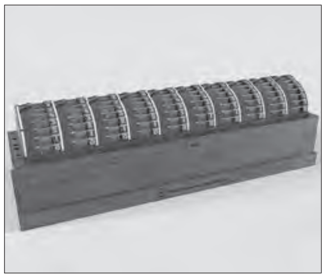
We are happy to have a cupboard in our Lab that used to belong to Tchebyshev, decorated with Tchebyshev's initials and his family's coat of arms. We consider it a symbolic bridge between the era of Tchebyshev and the present time. However, it has not only symbolic meaning but also a quite realistic allocation. In his time, Tchebyshev used it to keep his mechanisms inside. Now, next to the cupboard, we exhibit some of the models described below.



Plantigrade machine.

Sorting mechanism.

Wheelchair.



Arithmometer, first model.

Cupboard of P.L. Tchebyshev.

Models

In various scientific museums around the world, one can see models demonstrating numerous physical, chemical and biological phenomena directly and in a natural way. However, it is much more difficult to present mathematical models in the same way due to the high level of abstraction of mathematics compared to the other natural sciences.

This makes good mathematical models very significant: they demonstrate the essence of mathematical concepts or statements and they show viewers the hidden mathematical meaning via a series of impressions and sensations, both visual and memorable.

With today's low level of general mathematical education in schools, a visual model and its accessible explanation may help viewers open the gates to the fascinating world of genuine mathematics.

In our collection, we have gathered together some remarkable models made by a real master: Alexander Leshchynskyi. We use some of them in our on-site lectures. Certainly, manipulation with models is a favourite occupation of children and their parents visiting the Laboratory.

Not surprisingly, these models also attract professional mathematicians to the Lab. In particular, during the meeting of the Executive Committee of the European Mathematical Society held in Moscow, all the members of Executive Committee enjoyed the visit to the Lab.

However, since the Laboratory can only accept a restricted number of guests, we have organised the Models Section on the website of "Mathematical Etudes" (http://etudes.ru/ru/models). In this section, we are also going to create a complete electronic catalogue of mathematical models, i.e. bring together all available knowledge of such models including our own achievements. Each model should be illustrated by photos, instructions of how to construct these models and animated representations made using 3D computer graphics. Such animations are already themselves a good teaching tool.

Naturally, our virtual collection should go beyond pure entertainment. Its aim is to help the visitor learn something new from mathematics, to explain mathematically what they see or read. The superficial comments are often the weakest point of modern interactive museums. We hope that the development of a public directory of mathematical models will contribute to the appearance of visual aids in mathematical classrooms and interactive museums.

1001 Problems for Oral Calculations.

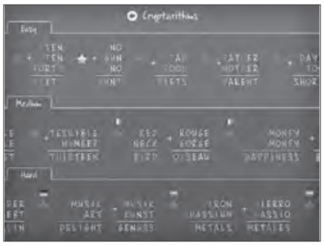
AppStore Apps

Mobile devices have become our everyday companions. This opens up new possibilities in mathematics for those who are still not involved, those for whom the traditional ways of teaching mathematics turned out to be ineffective. By developing applications for mobile devices, each user can become the owner of a permanent "mathematical sputnik" – an interlocutor and mentor. We will give several examples of the Laboratory's outcomes in this field.

In Russia, there is a classical book *1001 Problems for Oral Calculations* that is well known among mathematicians and mathematics educators. This collection of fascinating arithmetical problems was composed by the outstanding Russian teacher Sergei A. Rachinskii in the late 19th century. Rachinskii was a professor at Moscow University but left it to teach children in a rural school. One of Rachnskii's former students, the Russian artist N.P. Bogdanov-Belsky, painted a rural classroom scene (this painting can now be found in the Moscow Tretyakov Gallery). It shows Rachinskii teaching the peasant students oral arithmetic. A problem is written on the blackboard: $(10^2+11^2+12^2+13^2+14^2)$ / 365. The peasant children are meant to compute the resulting quantity without pencil, paper or chalk.

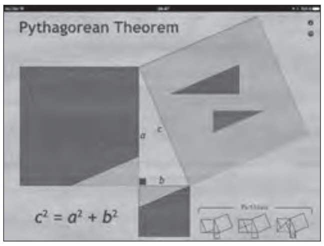
The Laboratory has digitised "1001 Problems" and the interactive app for iPhone and iPad has been downloaded by more than 2,000,000 users of ru-Appstore. This is a significant number for the online store, which is on a much smaller scale than its American analogue. Such a great number of paper copies would obviously be impossible in Russia – or indeed anywhere today.

There are several other ME applications (the programming was done by Anton Fonarev and Mark Pervovskiy) available in all regions of the appstore. The Cryptarithms App is a set of mathematical puzzles containing numbers in which digits are substituted by characters. The problem is to replace the characters with digits in order to obtain the correct answer. This is a rather popular entertainment in mathematical circles. The Four Colours app concerns the famous four colour theorem: here one can try to colour various given maps of different countries using only four colours. The Pythagoras App and Pythagoras HD App are puzzles where one tries to prove the Pythagoras theorem.



Cryptarithms.

Four Colours.



Pythagoras HD.

The Classroom and Beyond

All the movies are available for free on the "Mathematical Etudes" website. Some of them are translated into Italian, French and English. Others are still waiting to be translated (though one can try to use the automatic translator). The site is visited by more than 15,000 users daily. It is also the entry point to the other Lab projects described above. Different parts of our projects have been demonstrated in more than 700 popular science lectures for school pupils, teachers and university students in different regions of Russia. A standard talk lasts two hours. However, the attending schoolchildren very often ask for an additional class lesson to accompany this lecture. (We all know that maths lessons in school are usually considered the most hard and least understandable!) The Laboratory quite regularly conducts maths lectures on Russian TV based on the Lab products.

Mathematics school teachers in Russia widely use materials of our projects in their classes, as well as in their additional class lessons, in maths circles, at popular-science conferences for school pupils and so on.

Though our Lab projects are mainly directed at school pupils, teachers, university students and educators, our experience has shown that even professional mathematicians and other scientists express their interest in the content of projects made in the Laboratory of Popularisation and Promotion of Mathematics of the Steklov Mathematical Institute. We are trying our best to support communication with all visitors to the "Mathematical Etudes" site. We hope that no visitor to this site will be disappointed.

Nikolai Andreev [andreev@etudes.ru] heads the Laboratory of Popularisation and Promotion of Mathematics at the Steklov Mathematical Institute of the Russian Academy of Sciences (RAS) in Moscow. He received his degree in mathematics at the Mechanics and Mathematics Department of the Lomonosov Moscow State University and has worked at the Steklov Institute since 2000. His interests involve approximation theory, coding theory, mathematics popularisation and teaching.

Nikolai Dolbilin [dolbilin@mi.ras.ru] received his PhD in mathematics under B. N. Delaunay from the Steklov Mathematical Institute in Moscow and has worked there since 1969. He is also a professor of mathematics at Moscow State University. His interests involve geometry (tilings and Delaunay sets, crystallographic groups, quasicrystalline structures, the theory of polytopes, including parallelohedra), the Ising model, mathematics popularisation and teaching.

Sergei Konovalov [serk@mi.ras.ru] is a senior scientific researcher at the Laboratory of the Steklov Mathematical Institute and an associate professor in higher mathematics at the Moscow Institute of Physics and Technology. He is a member of the editorial board of "Kvant", the popular-science journal for scholars and students. His scientific interests include function theory, Lie group analysis of differential equations and the history of mathematics. His hobbies include chess and tennis.

Nikita Panyunin [panyunin@mi.ras.ru] received his degree in mathematics at the Mechanics and Mathematics Department of the Lomonosov Moscow State University. From 2006 to 2010, he worked at the Scientific Research Institute for System Studies of the RAS. Since then, he has worked at the Laboratory of the Steklov Mathematical Institute. In his spare time, he enjoys skiing, running, backpacking trips and playing piano.

Find and Advertise Jobs for Mathematicians at MathHire.org

Daniel Lütgehetmann and Sebastian Meinert (both Freie Universität Berlin, Germany)

The World Wide Web now hosts a new website for jobs for mathematicians, launched earlier this year at MathHire.org. In collaboration with professors, graduate students and university administrations, a new tool has been designed, programmed and tested that smoothly connects employers and jobseekers in mathematics.

We knew about 'mathjobs' offered by the AMS and other existing websites but we wanted to give the concept a complete overhaul and significantly improve the user experience. After months of hard work, we are convinced that MathHire leaves little to be desired: job listings are published in quick time, the job search is highly intuitive and the platform is finely crafted with attention to detail.

In the current phase, we are primarily focusing on the European market but if the tool is well received, we will strive to expand our services.

Universities and research institutions may publish advertising-only job listings free of charge but the system delivers its full strength to employers who accept applications through MathHire.org for a fee. Employers then benefit from a sophisticated web interface to review and evaluate received applications (see https://mathhire.org/showcase). Letters of recommendation are confidentially managed through MathHire. And last but not least, employers may establish a digital hiring committee and grant colleagues the right to review and evaluate applications in teams.

After initial test runs and collaboration on central features of the platform, the EMS has signed a cooperation agreement with MathHire to help the new website to grow. As part of the cooperation, the institutional members of the EMS receive one free listing per year and a 20% discount on every paid listing thereafter. In order to claim these benefits, representatives are asked to visit https://mathhire.org/ems and follow the instructions.

Daniel Lütgehetmann [daniel@mathhire.org] is a graduate student in mathematics at Freie Universität Berlin. He is studying Teichmüller spaces under the supervision of Holger Reich.

Sebastian Meinert [sebastian@mathhire.org] studied mathematics at Freie Universität Berlin, where he received his PhD in 2014. He studied deformation spaces of trees, also under the supervision of Holger Reich.

ICMI Column

Jean-Luc Dorier (University of Geneva, Switzerland)

Renewal of the ICMI Executive Committee

The new ICMI Executive Committee was elected during the ICMI General Assembly, which took place on 24 July just before ICME 13 in Hamburg (Germany).

This new committee (below) will be in service from 1 January 2017 for four years.

President:	Jill Adler	(South Africa)
Vice-Presidents:	Merrylin Goos	(Australia)
	Luis Radford	(Canada)
Secretary General:	Abraham Arcavi	(Israel)
Members-at-large:	Xu Binyan	(China)
	Jean-Luc Dorier	(Switzerland)
	Zahra Gooya	(Iran)
	Anita Rampal	(India)
	Yuriko Yamamoto Baldin	(Brazil)

Moreover, the Ex-President of the ICMI Ferdinando Arzarello (Italy), as well as IMU President Shigefumi Mori (Japan) and Secretary Helge Holden (Norway), are members ex-officio.

Alicia Dickenstein (Argentina), Vice-President of the IMU, is the liaison for the ICMI.

13th International Congress on Mathematical Education in Hamburg – the biggest ICME so far

About 3,500 participants from 105 countries participated in the 13th International Congress on Mathematical Education, which took place 24–31 July 2016 at the University of Hamburg and the Hamburg Congress Centre, making it the biggest congress in world congress series so far. ICME-13 was hosted by the Society of Didactics of Mathematics (Gesellschaft für Didaktik der Mathematik – GDM) and took place under the auspices of the International Commission on Mathematical Instruction (ICMI). The German community is the first international mathematics educational community to host an ICME more than once; as early as 1976, the Third International Congress took place in Karlsruhe (Germany). For this special event, a thematic afternoon took place devoted to describing development over the last 40 years with a European and historical perspective. The presentations focused on European didactic traditions, German-speaking traditions in mathematics education research and the legacy of Felix Klein.

At the opening ceremony, awards of the ICMI were presented to Michèle Artigue and Alan Bishop (Felix-Klein award), Jill Adler and Frederick Leung (Hans-Freudenthal award) and Hugh Burkhardt and Malcolm Swan (Emma-Castelnuovo award).

The heart of the congress took the form of 54 topic study groups, devoted to major themes of mathematics education, in which about 745 presentations were given. In attached oral communications, around 931 shorter papers were presented, complemented by 533 posters presented in two sessions. A large variety of other activities also took place, such as two plenary panels, four plenary lectures and 64 invited lectures. Initiated by congress participants, 38 discussion groups and 42 workshops were offered. Reflecting specific ICMI traditions, five ICMI survey teams described the state-of-the-art on their themes and three ICMI studies were also presented, in addition to six national presentations.

About 230 scholars from less affluent countries were supported by solidarity grants, making up a considerable proportion of the congress budget.

In addition, special activities for teachers held in German were attended by 250 teachers from all over Germany, despite the school vacation having already begun. Before the congress, an Early Career Researcher Day was offered, tackling specific themes for this group. A large number of early career researchers (450) participated in this congress, which made it a particular asset for ICME-13.

ICME-13 was clouded by the dramatic political events in Turkey; out of 100 registered participants, only 17 could come. However, about 45 were able to give their presentations via video and nine posters were presented. At the closing ceremony, the congress participants expressed their solidarity with the mathematics educators in Turkey by adopting a solidarity address.

ERME Topic Conference: Mathematics Teaching, Resources and Teacher Professional Development

5–7 October 2016, Humboldt-Universität zu Berlin, Germany

Stefan Zehetmeier (University of Klagenfurt, Austria), Bettina Rösken-Winter (Humboldt-Universität zu Berlin, Germany, Despina Potari (National and Kapodistrian University of Athens, Greece) and Miguel Ribeiro (Campinas State University (UNICAMP), Campinas, Brazil)

Mathematics teaching and mathematics teacher professional development are areas where research has increased substantially in the last years. For the last ERME conferences, a large number of proposals was related to this research area (e.g., three topic groups were formed at CERME9 in 2015: TWG18 on mathematics teacher education; TWG19 on mathematics teaching; and TWG20 on resources for teaching).

In this ongoing field of research, many issues need further investigation. We need to better understand the underlying characteristics of mathematics teacher education and the professional development contexts that have a positive impact on teachers' professional learning, even with respect to sustainability. Also, further discussion and research are needed on how to *link* research findings and how to *bridge* theoretical and methodological approaches to mathematics teacher pre-service and in-service education.

Studying mathematics teaching goes beyond teachers' classroom behavior. It encompasses teachers' actions and meaning-making as these relate to instruction. This includes, amongst others, task selection and design, classroom communication and assessment as well as the interplay between goals and actions as classroom interactions unfold in the context of broader institutional, educational, and social settings. A central question for investigation is what kind of methodological and theoretical tools are necessary to address this complexity.

In terms of resources, the focus of research for the last decades has been on teachers' beliefs and knowledge. More recently, teachers' identity, tasks, and teaching resources have received attention. Moreover, mathematics teacher educators' knowledge and development has been an emerging field. Aiming at achieving a better understanding, characterizing and/or evaluating the content of teachers' knowledge, several theoretical and methodological frameworks have been developed and discussed. Yet, further discussion seems to be needed in order to better describe the content of such knowledge, its relationships with (and influence on) teachers' beliefs, goals and identity as well as with mathematics teaching.

These three strands (mathematics teacher education, teaching and resources) are far from being disconnected. The ERME Topic conference "Mathematics Teaching, Resources and Teacher Professional Development" (5–7 October 2016, Humboldt-Universität zu Berlin, Germany) served as a platform for investigating in what ways these strands are linked – as regards research questions, methodologies and theoretical perspectives. The International Programme Committee was chaired by Stefan Zehetmeier (Austria), Miguel Ribeiro (Brazil), Bettina Rösken-Winter (Germany), and Despina Potari (Greece).

The conference focused on exchanging participants' knowledge and experiences, and on networking between scholars from different countries and cultures. In sum 69 scholars (60 from Europe) from 16 countries (12 from Europe) participated in this conference and submitted 37 papers and 14 posters. All submissions were peer-reviewed and a selection was made according to the quality of the work and the potential to contribute to the conference themes. Finally, 27 papers and 12 posters were accepted to be presented at the conference. Pre-conference proceedings were published online on the conference website (https://www.hu-berlin.de/de/einrichtungen-organisation/wissenschaftliche-einrichtungen/zentralinstitute/pse/erme/erme-topic-conference). A selection of extended papers will be published within the ERME book series.

Stefan Zehetmeier is an associate professor at the University of Klagenfurt (Austria). His research interests include teacher education, school development, evaluation and impact analysis of teacher professional development programmes.

Bettina Rösken-Winter is a professor for Design-Based Research and Mathematics Education at the Humboldt-Universität zu Berlin (Germany). Her main research interests are related to developing and evaluating courses to support and enhance mathematics teachers' continuous professional development.

Despina Potari is a professor in mathematics education at the University of Athens (Greece). Her main research interests have been on the development of mathematics teaching and learning and teacher development at different educational levels. Her current research focus is on the research-practice relation and on the role of different contexts and tools in classroom and PD settings.

Miguel Ribeiro concluded his PhD with a research focusing on teachers' knowledge, beliefs and mathematical goals. His research interests include kindergarten to secondary mathematics teachers specialized knowledge and teacher trainers specialized knowledge.

Faculty of Mathematics National Research University Higher School of Economics

Sergei Lando and Vladlen Timorin (both NRU HSE, Moscow, Russia)

Established in 2008, the Faculty of Mathematics at the National Research University Higher School of Economics (FM HSE) has gained considerable international reputation in mathematics and mathematical education. Not only our faculty members but also our students and graduates are starting to become recognised internationally.

In July 2016, the FM HSE moved to a new building in a quiet street not far from Moscow city centre. The new location has three times as much area as the previous one and we finally have room for off-curriculum activities for students.

The new building of the FM HSE.

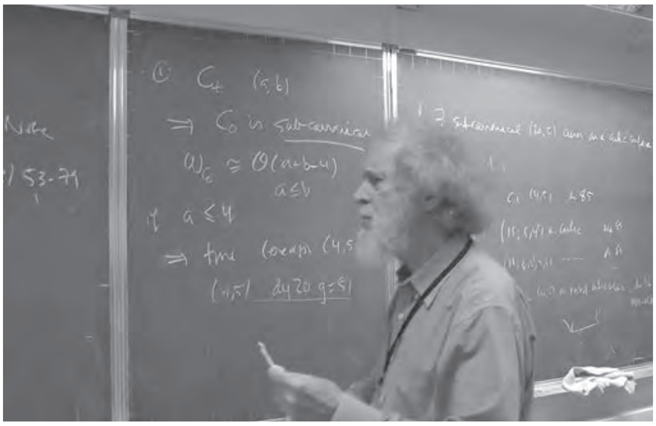
Research and Teaching

The National Research University Higher School of Economics (HSE)[1] is a national leader in economic and social sciences. This was the original intention of the founders back in 1992. However, the university's ambitions extended much further; HSE transformed into a classical (comprehensive) university and has, in fact, outgrown its name. In 2007, the HSE administration suggested that the Independent University of Moscow[2] [2] (a non-government, open educational organisation aimed at training professional mathematicians) should help create a world-level department of mathematics. And so it happened, with students majoring in fundamental mathematics entering the HSE in 2008. As of now, we are called the Faculty of Mathematics (despite our smaller size, we have the same status as the Faculty of Humanities and the Faculty of Social Sciences). The FM HSE now includes two international research units and three joint departments with the Russian Academy of Science. Alongside about 380 students, we have about 150 professors and research fellows and we offer educational programmes at all levels (BSc, MSc and PhD). The concentration of talented and highly motivated students is arguably the highest among all Russian programmes in fundamental mathematics. More than a half of our undergraduates are winners of prestigious contests for high school students, including the International Mathematical Olympiad.

In its teaching practices, the Faculty of Mathematics attributes weight to individual interactions between professors and students. There are two mechanisms to keep this interaction active: the so-called "mathematical practicum" (students individually discuss their solutions of special assignments with faculty members and teaching assistants) and coursework (preparation of term papers) during every year of study. The first two years of the 4-year Bachelor of Science programme consist mostly of core courses, whilst the last two years are spent according to individual learning trajectories. Our students are engaged in actual research projects and some of them produce publishable results.

[1] https://www.hse.ru/en/
[2] http://ium.mccme.ru/english/index.html

Sergei Fomin giving a lecture at HSE.



Robin Hartshorne giving a lecture at HSE.

Except for the initial composition of the faculty, all faculty members have been hired from around the world; active researchers from many different countries compete for positions at the HSE. Fourteen faculty members have been invited speakers at International Congresses of Mathematicians, including three plenary speakers. At ICM 2014 (Seoul, Korea), there were only four invited session speakers from Russia and three of them are affiliated with the FM HSE or associated laboratories: Alexander Kuznetsov (Algebraic and Complex Geometry session), Grigori Olshanski (Combinatorics session) and Misha Verbitsky (Algebraic and Complex Geometry session).

Partners

The FM HSE collaborates with leading research institutes of the Russian Academy of Sciences (RAS) through joint departments. These provide an interface between researchers of the RAS and students of the HSE (for project and thesis supervision, special topic courses and seminars). We have joint departments with the Steklov Mathematical Institute (headed by Victor Vassiliev), the Kharkevich Institute for Information Transmission Problems (headed by Alexander Krasnoselskii) and the Lebedev Physics Institute (headed by Andrei Marshakov).

Two research units, the so-called international laboratories, are associated with the Faculty of Mathematics. International laboratories unite researchers from the HSE with international researchers holding principal affiliations at different universities in different countries. Our best students also work in the laboratories as research assistants. The Laboratory of Algebraic Geometry and its Applications[3] was created in 2010 as a centre of excellence, funded by a mega-grant from the Russian Federation Government. It is headed by Fedor Bogomolov (Courant Institute) and it continues its operation as an international laboratory funded by the university. The Laboratory of Representation Theory and Mathematical Physics[4], headed by Andrei Okounkov (University of Columbia, Fields Medal winner in 2006), was created in 2015. Both laboratories invite visitors and organise seminars, conferences and summer schools.

Among international partners of the FM HSE are the Universities of Kyoto, Tokyo, Leiden, Nantes and Luxemburg, as well as the "Grande Écoles" in Paris. We have regular student and faculty exchanges with these universities and a number of additional cooperative agreements are being negotiated.

International Advisory Board

The International Advisory Board[5] of the FM HSE consists of leading external experts in mathematics and the ex-officio membership of the Dean. The board evaluates the overall performance of the faculty and provides recommendations for the HSE administration. Until 2015, the external members of the board included Stanislav Smirnov (Head), Sergei Fomin, Pierre Deligne, Tetsuji Miwa and Andrei Okounkov. In 2015, Okounkov became a faculty member and Nikita Nekrasov took his place on the board.

In 2012, the FM HSE filed the first report to the International Advisory Board. The members of the board studied the report, visited the HSE and had long talks with the students, the professors and the administration. This resulted in the report of the board to the HSE administration. The main conclusions of the board were:

- The Bachelor's programme is on a par with the world's best Bachelor's programmes.
- Research at the FM HSE is on a level with the top 100 mathematics departments in the world.
- The postgraduate programmes (MSc and PhD) are subject to further improvement that would allow them to reach the level of the Bachelor's programme.

As a by-product of the advisory board visits, HSE students have a remarkable opportunity not only to attend lectures by Fields Medallists but also to directly communicate with them. In April 2016, the second faculty report was sent to the International Advisory Board.

Cooperation with the IUM

Being an offspring of the Independent University of Moscow (IUM), the Faculty of Mathematics retains a

[3] https://ag.hse.ru/en/
[4] https://mf.hse.ru/en/
[5] https://math.hse.ru/en/experts

tight connection with it. Several projects initiated by the IUM are continuing jointly with the HSE, the *Moscow Mathematical Journal* and the *Math in Moscow* programme (MiM) among them.

The *Moscow Mathematical Journal*[6] (MMJ) was founded in 2000; as of 2014, it was the leading Russian journal in terms of Scopus SJR (the highest rank achievable amongst all Russian journals and not just mathematical ones). The MMJ is published in English and has an international editorial board. The journal is distributed by the American Mathematical Society.

The Math in Moscow programme[7] is aimed at international students getting in touch with the best traditions of the Moscow mathematical school. MiM is a fee-paying programme. Its participants mostly come from North American universities. Recently, there have also been a number of students from China. Credit points of the MiM are transferable. The US National Science Foundation, as well as the Natural Sciences and Engineering Research Council of Canada, offer several stipends each year to cover participation by US and Canadian students in the MiM.

[6] http://www.ams.org/distribution/mmj/
[7] https://math.hse.ru/en/MiM-en

Sergei Lando [lando@hse.ru] is a professor of mathematics; he was the first Dean of the FM HSE, 2008–2015.

Vladlen Timorin [vtimorin@hse.ru] is a professor of mathematics; he has been the Dean of the FM HSE since April 2015.

Research in University Mathematics Education: The khdm

Rolf Biehler (Universität Paderborn, Germany), Reinhard Hochmuth (Leibniz Universität Hannover, Germany), Hans-Georg Rück (Universität Kassel, Germany), Robin Göller (Universität Kassel, Germany), Axel Hoppenbrock (Universität Paderborn, Germany), Michael Liebendörfer (Leibniz Universität Hannover, Germany) and Juliane Püschl (Universität Paderborn, Germany)

The khdm[1] (German Centre for Higher Mathematics Education) was founded in 2010. In 2010, there had been a call *Bologna – Zukunft der Lehre* (future of teaching) by the German foundations VolkswagenStiftung and Stiftung Mercator that intended to provide financial support for projects aiming to optimise teaching and learning at traditional universities and universities for applied sciences. The proposal of the khdm was among the few successful ones. It was submitted by the Universities of Paderborn and Kassel with Rolf Biehler and Reinhard Hochmuth responsible. Together with Hans-Georg Rück from the University of Kassel, they form the current board of managing directors. After an intermediate period, where the University of Lüneburg joined the khdm, the khdm is now about to get the University of Hanover as a third partner university. The financial support by the foundations ended in 2015 but the centre will continue to operate with financial support from the three universities and further third party grants.

The main objective of the khdm is the realisation of research and development projects in cooperation with mathematicians and mathematics educators. Altogether, 15 professors from Kassel, Paderborn and Hanover work together with about 15 research assistants, most of whom are PhD students.

[1] Kompetenzzentrum Hochschuldidaktik Mathematik.

The starting point was a network of projects that were strongly focused on specific needs in various courses of study: the education of primary and lower secondary school mathematics teachers, mathematics majors and future high school teachers (Gymnasium), economists and engineers. Further projects focused on e-learning issues, in particular in the context of the secondary-tertiary transition. These five domains correspond to five working groups established within the khdm.

In order to give an insight into the research carried out by the khdm, we will describe two projects in more detail.

A first example is a cooperation project between mathematicians and mathematics educators. At the University of Kassel, this cooperation has led to a modification of the first year curriculum of mathematics majors, including a newly established course that focuses on mathematical thinking and working styles. In addition, the project f-f-u[2] (integration of mathematics and mathematics education at university), led by Andreas Eichler, Maria Specovius-Neugebauer and Hans-Georg Rück, develops teacher oriented exercises for these courses that are appropriate to illustrate connections between mathematics at university

[2] Vernetzung fachwissenschaftlichen, fachdidaktischen und unterrichtspraktischen Wissens im Bereich Mathematik.

level and school mathematics. These exercises should encourage teacher students to learn university mathematics with more engagement and motivation because they will better see its benefits for their future teaching. It is part of a larger project named PRONET[3] (professionalisation through interconnection) at the University of Kassel, funded by the BMBF (the German Federal Ministry of Education and Research).

A second example is a project carried out in cooperation with 14 other universities, the WiGeMath[4] project, which is a BMBF-financed joint project of the Universities of Hanover and Paderborn, led by Rolf Biehler, Reinhard Hochmuth and Niclas Schaper and running from 2015 to 2018 (Colberg et al., 2016). It evaluates different types of projects for supporting students in university mathematics, including bridging courses, mathematics support centres, redesigned lectures and support measures accompanying regular courses (e.g. special tutorials or the provision of online learning material) in programmes for secondary school mathematics teachers as well as regular mathematics and engineering programmes. One part of the evaluation uses a programme evaluation approach (Chen, 1994) that aims at reconstructing and evaluating goals, measures and their boundary conditions from the point of view of the involved protagonists. Moreover, some of the intended effects of the projects are investigated by control group designs that follow a quantitative empirical research paradigm.

Whereas these projects concentrate on innovations in whole courses and effects on the participants of courses, a considerable number of PhD students carry out their research within the khdm. Many of them study student learning processes related to specific topics such as convergence of sequences, derivatives, the vector concept and vector spaces, as well as mathematical activities such as proving and problem solving. Moreover, research on fostering student motivation, interest and learning strategies are the focus of other PhD projects.

Besides doing research and developing teaching, one major task of the khdm is to provide specific benefits and services for its home universities. Firstly, the khdm provides a natural context for interdisciplinary and collaborative research studies within and across our universities. Secondly, for the PhD students, it provides professional development on research methods and design, as well as regular forums for the exchange of practical experience and results of their research. Beyond that, there are also more personal exchanges on individual development processes, such as starting as a mathematician with some interest in teaching and becoming a researcher in higher mathematics education. In addition, the khdm provides some service and advisory competence for teaching mathematics at our home universities. Rather important is that the khdm builds a critical mass for applying and getting external funding through grants.

[3] Professionalisierung durch Vernetzung.
[4] Wirkung und Gelingensbedingungen von Unterstützungsmaßnahmen für mathematikbezogenes Lernen in der Studieneingangsphase.

Last but not least, the khdm is an actor in the European research community that has recently developed. The khdm organised a workshop at the Oberwolfach Research Institute for Mathematics (MFO Mathematisches Forschungsinstitut Oberwolfach) in December 2014 under the title "Mathematics in Undergraduate Study Programs: Challenges for Research and for the Dialogue between Mathematics and Didactics of Mathematics" (Biehler et al., 2014). Another conference organised by the khdm was the international conference on "Didactics of Mathematics in Higher Education as a Scientific Discipline" in Schloss Herrenhausen, Hanover, in November 2015 (Göller et al., in press). Moreover, we are planning common activities with our colleagues from England, France, Norway, Denmark, Spain and other countries in the context of INDRUM, the International Network for Didactic Research in University Mathematics (indrum2016.sciencesconf.org).

Since its beginning, the khdm has been critically and constructively accompanied by an international scientific board, consisting of Tommy Dreyfus, Willibald Dörfler, Daniel Grieser, Lisa Hefendehl-Hebeker (chair), Holger Horz, Jürg Kramer and Johannes Wildt.

More information can also be found at the website http://www.khdm.de/en/.

References

Biehler, R., Hochmuth, R., Hoyles, C., & Thompson, P. W. (2014). Mathematics in Undergraduate Study Programs: Challenges for Research and for the Dialogue between Mathematics and Didactics of Mathematics. *Oberwolfach Reports*, 11(4), 3103–3175. doi:10.4171/OWR/2014/56 [https://www.mfo.de/document/1450/OWR_2014_56.pdf].

Chen, H.-T. (1994). Theory-driven evaluations: Need, difficulties, and options. *Evaluation Practice*, 15(1), 79–82.

Colberg, C., Biehler, R., Hochmuth, R., Schaper, N., Liebendörfer, M., & Schürmann, M. (2016). Wirkung und Gelingensbedingungen von Unterstützungsmaßnahmen für mathematikbezogenes Lernen in der Studieneingangsphase. In: Institut für Mathematik und Informatik Heidelberg (eds.), *Beiträge zum Mathematikunterricht* 2016. Münster: WTM-Verlag.

Göller, R., Biehler, R., Hochmuth, R., & Rück, H.-G. (in press). *Didactics of Mathematics in Higher Education as a Scientific Discipline.* Conference Proceedings of the Herrenhausen 2015 Conference. To appear as khdm report 16-05, Kassel, Lüneburg and Paderborn.

Rolf Biehler, Reinhard Hochmuth and Hans-Georg Rück are the current managing directors of the khdm. All three studied mathematics. Rolf Biehler is a professor of didactics of mathematics at the University of Paderborn, previously in Kassel. Reinhard Hochmuth has been a professor of didactics of mathematics at the University of Hanover since 2014 and was a professor of mathematics at the Universities of Lüneburg and Kassel before. Hans-Georg Rück is a professor of mathematics at the University of Kassel. Robin Göller, Axel Hoppenbrock, Michael Liebendörfer and Juliane Püschl are researchers at the khdm doing their PhDs but working at the same time on the management team of the kdhm. Robin, Michael and Juliane joined the khdm after having finished their diplomas or state examinations in mathematics. Axel had been teaching mathematics at school level for several years before he joined the khdm.

Full Text Formula Search in zbMATH

Fabian Müller and Olaf Teschke (FIZ Karlsruhe, Berlin, Germany)

Three years ago, formula search has been introduced in zbMATH[1]. Formula retrieval is based on three ingredients: digitisation, content extraction, and a math-aware search engine. Our aim is to give an update on its status and developments.

The search uses the MathWebSearch engine[2] developed by the KWARC group[3] at Jacobs University Bremen, which leverages a technique called *substitution tree indexing*[4]. This method enables high-performance structural searching in a large corpus of formulae using query expressions that may contain free variables or placeholders. The latter are denoted by a leading question mark and will match arbitrary subexpressions of any complexity. When occurring multiple times in the input query, they will be substituted with the same concrete expression for each occurrence. Thus a query like “-?a \leq ?b \leq ?a” would match the formula “$-\sqrt{a} \leq f(u) \leq \sqrt{a}$”, but not “$0 \leq x \leq 1$”.

In order to be indexed, a LaTeX document must first be converted to MathML, which is handled by the LaTeXML converter developed by Bruce Miller at NIST[5]. After conversion, the formulae contained in each document are extracted by the indexing engine and can then be retrieved using an XML-based query syntax. The formula search interface features an interactive preview converting the user’s LaTeX input on-the-fly to MathML that is then displayed in the browser.

As outlined in[1], the resulting indexes are quite huge, and processing the vast amount of zbMATH formulae requires considerable resources. Hence, it has been a non-trivial (though hidden) achievement to transform the project prototype into a sustained feature that is now updated daily along with the ever-growing inflow of documents. Likewise, the integration of new versions of LaTeXML went certainly unnoticed, though Bruce Miller’s efforts have significantly improved on capturing underlying semantics supporting a more precise retrieval. However, the difficult challenge of extracting semantic content from LaTeX information still remains unsolved: E.g., searching for ?a^?n + ?b^?n=?c^?n may produce results involving Diophantine equations, a two-dimension eikonal equation, or norms fulfilling 1/p+1/q=1/r. Defining the domain of the variables would help, but this is currently not feasible; instead, the more pragmatic approach of specifying the mathematical area often serves the same purpose. Hence, it was a useful improvement to enable combined search of metadata and formulae in zbMATH two years ago, which allows for refining formula search results by terms or subjects.

The main remaining challenge to improve formula search is digitisation. Since LaTeX is required, the search was initially restricted to zbMATH reviews and abstracts that are available in this format. The situation for full texts is worse: Even articles of the last decades are usually only available as pdf (older just as scans). Though some approaches for LaTeX conversion exist[6,7], the results often lack the precision required for seamless formula indexing. Publishers could provide a tremendous support for math retrieval by making LaTeX sources or derived XML data available.

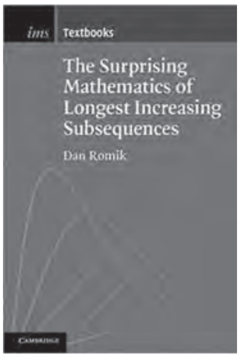
Fortunately, this is already the case for the arXiv. The recent indexing of about 120,000 arXiv full text links[8] within the zbMATH database enabled us to extend the formula search considerably. Even this small fraction of the 3.7 million zbMATH documents pushed the number of indexed formulae to more than 100 million. It is interesting to note that for a sample of frequent formulae, the number of search results increased only by an average of 30%, indicating that relevant formulae are frequently mentioned in the reviews. On the other hand, there is now a long tail of rare expressions available for searching which did not show up in the corpus before.

An interesting aspect of the zbMATH user survey conducting during this year’s ECM[9] was that formula search is among the least frequently used though potentially most promising future features of zbMATH. This discrepancy is not surprising: the first reaction of most mathematicians encouraged to test formula search is that they believe such a system could not work yet. Taking the mentioned obstacles into account, there may be some justification for this; however, the progress made during the last years has surpassed our expectations, so we believe it is worth to stay up-to-date with this feature and experiment from time to time by searching for your favourite formula.

For the authors’ CVs and photos we refer to the zbMATH column of the EMS Newsletter No. 99 by the same authors.

1 M. Kohlhase et al., Eur. Math. Soc. Newsl. 89, 56–58 (2013; Zbl 1310.68217).
2 https://github.com/KWARC/mws
3 https://kwarc.info/
4 P. Graf, “Substitution tree indexing”, Lect. Notes Comp. Sci. 914, 117—131 (1995; doi:10.1007/3-540-59200-8_52)
5 http://dlmf.nist.gov/LaTeXML/
6 Infty, http://www.inftyproject.org/en/index.html
7 Maxtract, http://www.cs.bham.ac.uk/research/groupings/reasoning/sdag/maxtract.php
8 see F. Müller and O. Teschke, Eur. Math. Soc. Newsl. 99, 55–56 (2016; Zbl 1345.68267)
9 A detailed report will be given in the next column.

Book Reviews



Dan Romik

The Surprising Mathematics of Longest Increasing Subsequences

Cambridge University Press, 2015
363 p.
ISBN 978-1-107-42882-9

Reviewer: Manjil Saikia

The Newsletter thanks zbMATH and Manjil Saikia for the permission to republish this review, originally appeared as Zbl 1345.05003.

The author has written a wonderful book, both in terms of the subject matter and the style and presentation. Despite choosing one question on which to base an entire book, the text is never boring and is a very important addition to the literature in analytic combinatorics. The author states that the book is set at about graduate level and graduate level probability theory is absolutely necessary to understand the contents of the book. However, even with the recommended level of knowledge, this book is not light reading. The way the topics are introduced makes the job a bit easier for the reader, with boxed information on some of the topics and references for further or background study providing enough motivation for the reader to continue on. A very nice feature is that the author proves almost all of the results he needs to prove the main theorem presented in each chapter. This makes the book more self-contained than is the norm for this type of book.

The main point of interest in writing the book is the famous Baik-Deift-Johansson theorem, which is the subject matter of Chapter 2. The theme, as is evident from the title, is longest increasing subsequences in a random permutation. If σ is a permutation in S_n then we denote by $L(\sigma)$ the length of the maximal increasing subsequence of σ. Similarly, $D(\sigma)$ denotes the length of the maximal decreasing subsequence of σ. The object of interest is the quantity

$$\ell_n = \frac{1}{n!} \sum_{\sigma \in S_n} L(\sigma),$$

the average of $L(\sigma)$ over all permutations of order n. The main result discussed is the asymptotic behaviour of this quantity as n grows large. *S. Ulam* [“Monte Carlo calculations in problems of mathematical physics”, in: Modern mathematics for the engineer. 2nd series. New York, NY: McGraw Hill. 261–281 (1961)] was the first to suggest studying the statistical distribution of maximal monotone subsequence lengths in random permutations. This task was taken up by *J. M. Hammersley* [in: Proc. 6th Berkeley Sympos. Math. Statist. Probab., Univ. Calif. 1970, 1, 345–394 (1972; Zbl 0236.00018)] and hence the problem is referred to as the Ulam-Hammersley problem in the book.

The final answer (or at least one form of the answer) is presented in Chapter 2, which is the famous result of *J. Baik* et al. [J. Am. Math. Soc. 12, No. 4, 1119–1178 (1999; Zbl 0932.05001)].

For each $n \geq 1$, let σ_n denote a uniformly random permutation of order n. Then, for any $x \in \mathbb{R}$, as $n \to \infty$, we have that

$$\mathbb{P}\left(\frac{L(\sigma_n) - 2\sqrt{n}}{n^{1/6}} \leq x\right) \to F_2(x).$$

Here, $F_2(t) = \det(I - A_{|L^2(t,\infty)})$ and A is the Airy kernel.

But, before that, in Chapter 1, the author proves the first step toward the Ulam-Hammersley problem, namely the following result, which was proved by *A. M. Vershik* and *S. V. Kerov* [Sov. Math., Dokl. 18, 527–531 (1977; Zbl 0406.05008); translation from Dokl. Akad. Nauk SSSR 233, 1024–1027 (1977); Funct. Anal. Appl. 19, 21–31 (1985); translation from Funkts. Anal. Prilozh. 19, No. 1, 25–36 (1985; Zbl 0592.20015)] and independently by *B. F. Logan* and *L. A. Shepp* [Adv. Math. 26, 206–222 (1977; Zbl 0363.62068)].

We have the limit

$$\frac{\ell_n}{\sqrt{n}} \to 2$$

as $n \to \infty$. Also, for each n, if σ_n denotes a uniformly random permutation in S_n then $L(\sigma_n)/\sqrt{n} \to 2$ in probability as $n \to \infty$.

The distinguishing feature of this book and the range and depth of subjects related to the problem are very clear from the first chapter. Even to prove the result of Vershik–Kerov and Logan–Shepp, we meet Poisson point processes, the Robinson-Schensted algorithm (the full Robinson–Schensted–Knuth algorithm also appears later in the book), the hook-length formula, plancheral measures (which is an integral part of the book) and limit shapes (presented in a nice way toward the end of the book). The techniques that are used to prove auxilliary lemmas come from calculus of variations, probability theory and a little bit of fractional calculus. The author not only proves the result mentioned above but even proves a limit shape theorem for Plancheral-random Young diagrams, which is of independent interest.

In the proof of the Baik–Deift–Johansson theorem, we get to meet many different mathematical concepts, like the Tracy–Wildom distribution, determinantal point processes and the classical special function, and Bessel and Airy functions. The book gets difficult at this point and we are given a crash course on many different objects while obtaining the proof of the main result. As in the first chapter, the author actually proves a much stronger result than what is needed. This is a recurrent theme in the book and the reviewer feels that this is a nice motivation for the reader to learn a bit more than what is expected.

At this point, the reader may be a bit worried about what the remaining three chapters cover if the main result has already been proved. But, as with a fine dinner, so also with this book, the latter section (the reviewer here alludes to the dessert) is as interesting if not more so than what was covered in the first part.

Chapter 3 is devoted to a special class of permutations called Erdős–Szekeres permutations. These are permutations where the longest monotone subsequence is the shortest possible, thus demonstrating extremal cases. The name comes from the celebrated theorem of Erdős and Szekeres, which is also discussed in Chapter 1. The methods that were developed in Chapter 1 now come in handy in this chapter as well. The first two sections of this chapter are devoted to characterising the permutations combinatorially. The rest of the chapter is focused on limit shape theorems for random Erdős Szekeres permutations and for random square Young tableaux. It is advisable to read Chapter 1 before reading this chapter, to get a sense of what the author is trying to accomplish. Sometimes if one is not careful, it is easy to get lost and forget the main purpose of all the hard work; this could particularly be the case for non-experts in the field. The chapter ends with a short description of the so-called Arctic Circle phenomenon, which appears when some results are interpreted by the asymptotic behaviour of an interacting particle system. The Arctic Circle phenomenon for square Young tableau jump processes is proved at the end of the chapter.

Chapters 4 and 5 are devoted to corner growth processes and their limit shapes (in Chapter 4) and distribution (in Chapter 5). The corner growth process is a random walk on the Young graph, which is defined by the rule that each new cell is always added in a position chosen uniformly randomly among the available places. This is a different type of random walk than the one already considered in the first chapter of the book, to which we did not allude earlier. The major focus in Chapter 4 is a result of *H. Rost* [Z. Wahrscheinlichkeitstheor. Verw. Geb. 58, 41–53 (1981; Zbl 0451.60097)] on the limit shape for the corner growth process.

For a Young diagram $\lambda = (\lambda_1, \ldots, \lambda_k)$, let set_λ denote the planar set associated with λ, defined as

$$\text{set}_\lambda = \cup_{1\leq j\leq k, 1\leq j\leq \lambda_i}([i-1, i] \times [j-1, j]).$$

Let $(\lambda^{(n)})_{n=1}^{\infty}$ denote the corner growth process and define the set Λ_{CG} by

$$\Lambda_{CG} = \left\{(x, y) \in \mathbb{R}^2 : x, y \geq 0,\ \sqrt{x} + \sqrt{y} \leq 6^{1/4}\right\}.$$

Then, for any $0 < \epsilon < 1$, as $n \to \infty$, we have that

$$\mathbb{P}\Big((1-\epsilon)\Lambda_{CG} \subseteq \frac{1}{\sqrt{n}}\text{set}_{\lambda^{(n)}} \subseteq (1+\epsilon)\Lambda_{CG}\Big) \to 1.$$

Whilst proving this theorem, we are led into interesting mathematics about Legendre transforms, last-passage percolation and exclusion processes. Toward the end of the chapter, some results on multicorner growth processes are also discussed.

In Chapter 5, the work of *K. Johansson* [Commun. Math. Phys. 209, No. 2, 437–476 (2000; Zbl 0969.15008)] on a connection between corner growth processes and longest increasing subsequences in generalised permutations is considered. This connection is studied via the Robinson-Schensted-Knuth (RSK) algorithm and this connection might also assuage any reader who was wondering what the corner growth process has to do with the subject of the book. Whilst proving the main theorem of this chapter, the RSK algorithm is discussed, as well as semistandard Young tableaux and orthogonal polynomials. Random matrix theory rears its head here and this may be a good motivation for some readers (like the reviewer himself) to learn a bit more in this area, which is finding an increasing number of applications in mathematics and physics.

The book as a whole is a wonderful addition for specialists as well as motivated non-specialists. Although a thorough background in probability theory is essential for understanding the book, it should also be noted that the book is not for the faint of heart. There are numerous exercises after each chapter, which adds to some of the things discussed in the book. The exercises are marked in order of difficulty with coffee cups, ranging from one to five (five for research level problems). It should be noted that some of the problems would probably require many more cups of coffee to solve than is recommended by the author.

Manjil Saikia [manjil.saikia@univie.ac.at] is a research fellow at the University of Vienna, Austria. His research interests are in enumerative and algebraic combinatorics. He runs a website on mathematics (http://gonitsora.com) focused on a student audience.

Herbert Bruderer

Meilensteine der Rechentechnik. Zur Geschichte der Mathematik und der Informatik

De Gruyter, 2015
XXXII, 818 p.
ISBN 978-3-11-037547-3

Reviewer: Steven Deckelman

This review originally appeared in MAA Reviews[1] and is being published with permission of the Mathematical Association of America.

This impressive new book by Herbert Bruderer is an extensive in-depth scholarly history of mathematics and computer science with a focus on computing technology in German lands. Computing technology is defined in the most general sense. Under this definition can be included any tool that facilitates computation. This runs the gamut from tallying sticks and bones to fingers, pebble stones, pencil and paper, slide rules and to machines, including both mechanical, electronic and even quantum devices. Also, ideas (algorithms) relating to computation and the books that preserved and transmitted them are included among these tools. For example, the Liber Abaci of Leonardo of Pisa as well as John Napier's logarithms would be included.

As a work by a professional historian, the book poses questions, presents evidence (in the form of historical machines, documents, drawings and pictures) and proposes interpretations as well as raises further research questions. Some of the historical questions include

What kind of device is it?
What was the origin of the device?
How old is the device?
How did the device work?
What technology was the device based on?
For whom and for what purpose was the device used?
How was the device discovered?

among others.

The book consists of 818 pages with 8 chapters along with an extensive 225 page multi-lingual biography exceeding 3000 entries, mostly from the German, French and English literature. It is very rich in detailed historical references. There are many pictures, tables and timelines. The book also includes new primary source material on recently discovered computing devices since 2009 and of new documents on the relationship between German computing pioneer Konrad Zuse and the ETH Zurich concerning Z4 and Ermeth (Elektronische Rechenmaschine der ETH).

[1] http://www.maa.org/press/maa-reviews/meilensteine-der-rechentechnik

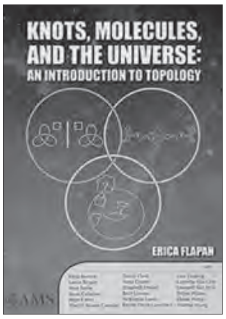
This book will be of particular interest to historians of mathematics and computer science. Those who teach undergraduate history of mathematics and possibly ethnomathematics courses and who would like to supplement their course with some episodes from the history of computer science will also find a wealth of material for student projects. For example, students may find it interesting to learn about the Curta, a high quality mechanical calculator invented by Buchenwald concentration camp inmate Curt Herzstark as a possible gift for the Führer, or about the many forms of the slide rule, or abacus, and how they were used. This book contains detailed instructions about how the Curta actually worked.

As a non-expert reviewing this book, I found it both surprising and fascinating how many open questions there are even about relatively recent (twentieth century) history. For example John Von Neumann's 1945 paper introducing Von Neumann architecture contains no reference to Alan Turing's 1936 paper on the universal Turing machine. Was Von Neumann influenced by Turing or were these discoveries independent? Von Neumann was at the Institute for Advanced Study during the time Turing was at Princeton. With whom do the distinctions between control unit, ALU, memory, as well as input and output devices, originate? Prior to Von Neumann this had already been anticipated by Charles Babbage and Konrad Zuse. Who wrote the first computer program? Ada Lovelace was certainly the first woman programmer but whether she was the first programmer is in dispute among historians, some of whom argue it was Charles Babbage. In chapter 4, at least a dozen such open historical questions are mentioned.

One topic that I would have liked to have seen but which was omitted was a detailed description of Chebyshev's calculating machine. But given the 818 pages as well as its stated focus on German lands, perhaps that was a reasonable omission. Chebyshev's machine is mentioned and references are given.

This book is a must-have for anyone interested in the history of mathematics and computer science as well engineering (especially mechanical and electrical), technology and the history of science.

Steven Deckelman is a professor of mathematics at the University of Wisconsin-Stout, where he has been since 1997. He received his Ph.D from the University of Wisconsin-Madison in 1994 for a thesis in several complex variables written under Patrick Ahern. Some of his interests include complex analysis, mathematical biology and the history of mathematics.



Erica Flapan

Knots, Molecules, and the Universe. An Introduction to Topology

AMS, 2016
xvii, 386 p.
ISBN 978-1-4704-2535-7

Reviewer: Claus Ernst

The Newsletter thanks zbMATH and Claus Ernst for the permission to republish this review, originally appeared as Zbl 1343.57001.

This book can serve as an elementary introduction to topics in topology and geometry for students with a minimal background in formal mathematics. It touches on topics that are recent applications of geometry and topology in the field of molecular biology and chemistry. As such it can be used to teach a variety of different courses: to math majors in their freshman year, to future secondary teachers, or as an interdisciplinary course – for example an honors program seminar – linking topology and geometry to physics, biology and chemistry. One could even imagine to use this book to motivate gifted high school students to enter the fields of mathematics. In this book, Flapan and her co-authors have taken a very unique approach to these subjects, focusing on intuition and on recent applications. Technical details and a more rigorous presentation of the material has been avoided to make the topics available to a wide audience. Most exercises concentrate on specific examples and how to apply the results of a given section. Proofs in the text and in the exercises are avoided – unless a very elementary and short proof exists. Many theorems of deep and recent results are only stated and their implications are explained while the reader is told to take a more advanced course to see proofs of these facts. Readers who want to know more details and are not at the level of mathematical seniors can go to other sources such as C. C. Adams excellent introduction to knot theory [The knot book. An elementary introduction to the mathematical theory of knots. Revised reprint of the 1994 original. Providence, RI: American Mathematical Society (2004; Zbl 1065.57003)] or E. Flapan's book [When topology meets chemistry. A topological look at molecular chirality. Cambridge: Cambridge University Press (2000; Zbl 0977.92034)] for the applications in chemistry and molecular biology. Overall Erica Flapan and her co-authors have created a very unique book that can serve as an entry into the world of modern mathematics while avoiding analytic and algebraic arguments that might turn off some students from entering the field of mathematics.

Formally, the book consists of three parts. The first and the largest part serves as an introduction to the question "What are possible shapes of a universe?". It contains an introduction to the classification of surfaces, with an extension to geometries (Euclidean, hyperbolic and spherical) on surfaces. The readers learn how to glue boundaries of various objects together, to form not only surfaces but also three manifolds. The reader is introduced to the concept of higher dimensions and how "life" would be in these potential shapes of universes, through fictive characters called "A. 3D-girl", "A. Square" or "B. Triangle". Much of the material and topics of this first part is motivated by the book by J. R. Weeks [The shape of space. 2nd, revised and expanded ed. New York, NY: Marcel Dekker (2002; Zbl 1030.57001)]. The second part of the book is an elementary introduction to knot theory with an emphasis on knot symmetries (chirality and invertibility) and some knot invariants such as the Jones polynomial. The topics of knot theory introduced concentrate on topics that are needed in the third part of the book. This second part is the shortest of the three parts and is just enough to wet the readers' appetite to study the field of knot theory. The last part – which aligns with Flapan's own research – concentrates on applications of topology and geometry to molecular biology and chemistry. It discusses symmetries and geometrical, chemical and topological chirality of molecules. Techniques are explained how one can prove that a molecule is topological chiral using techniques from knot theory and graph theory. Next a chapter is devoted to the topology and geometry of DNA molecules, covering topics such as the linking number, twist and writhe of a DNA molecule and how the tangle model can be used to analyze enzyme action of site specific recombination experiments that create DNA knots. The final chapter discusses the topology and geometry of proteins. In particular it explains what we mean by knots or Möbius ladders in proteins.

In summary, this is a wonderful introduction to geometry and topology and their applications to the sciences. The book contains a unique collection of topics that might entice young readers to continue their academic careers by learning more about the world of mathematics.

Claus Ernst (claus.ernst@wku.edu) is a professor at the Department of Mathematics at Western Kentucky University, Ky, USA. He received is PhD in mathematics in 1988 (Florida State University). His research interests are in knot theory, in particular applications of knot theory to the physical sciences. He became a Western Kentucky University Distinguished Professor in 2013.

Personal Column

Please send information on mathematical awards and deaths to newsletter@ems-ph.org.

Awards

The Austrian Mathematical Society has awarded its 2016 **Promotion Prize for Young Scientists** to **Aleksey Kostenko** for outstanding achievements in spectral theory and mathematical physics.

Ernest Borisovich Vinberg received in 2016 the **Distinguished Speaker Award** from the European Mathematical Society.

Lithuanian Mathematical Society has decided to award the **Lithuanian Mathematical Society Young Mathematician Prize** for 2016 to **Vytautas Paškūnas** (Universität Duisburg-Essen) for his research on p-adic Langlands program.

José Bonet Solves (Universidad Politécnica de Valencia, Spain), **María Gaspar Alonso-Vega** (Universidad Complutense de Madrid, Spain) and **María Teresa Lozano Imízcoz** (Universidad de Zaragoza, Spain) have been awarded the 2016 **Medals of the Real Sociedad Matemática Española**.

In September 2016 **Marián Fabian** and **Vladimír Müller** (Institute of Mathematics of the CAS, Prague) received the **Honorary Bolzano Medals for Merits in Mathematical Sciences** awarded by the Czech Academy of Sciences.

The 2016 laureate of the **Ramiro Melendreras Prize** 2016 from the Spanish Society of Statistics and Operations Research is **Beatriz Sinova Fernandez** for the contribution "Comparative Analysis of M-estimators and Trimmed Means for Fuzzy Set-Valued". This award recognizes the work of young researchers in Statistics and Operations Research.

The Royal Society of Sciences at Uppsala has awarded Professor **Svante Janson** (Uppsala University) the 2016 **Celsius Medal** for his outstanding work in combinatorial probability.

The researchers **Roger Casals**, **Francesc Castellà**, **Leonardo Colombo**, **José Manuel Conde Alonso**, **Martín López García** and **Jesús Yepes Nicolás** have been awarded the prize **Premios Vicent Caselles** 2016 by the RSME-FBBVA.

The 2016 laureats of the **Tullio Levi-Civita Prize** of the International Research Center for Mathematics & Mechanics of Complex Systems of the Università dell'Aquila are **Mauro Carfora** (Università degli Studi di Pavia) and **Tudor S. Ratiu** (École Polytechnique Fédérale de Lausanne).
The 2016 **Grand Prix en Sciences mathématiques** of the Institute Grand-ducal de Luxembourg / Prix de la Bourse de Luxembourg has been attributed to **Martin Schlichenmaier** (University of Luxemburg).

Tomaž Pisanski awarded in 2016 the **Donald Michie and Alan Turing Prize** for lifetime achievements in Slovenian information society.

Marius Crainic (Univeristy of Utrecht) was awarded in 2016 the first **De Bruijn Prize**. This biennial prize is attributing for the most influential recent publication (or series of publications) by a mathematician affiliated to an institute in the Netherlands.

The 2015 **SIMAI Prize** has been awarded to **Paola Antonietti** (Department of Mathematics of Politecnico di Milano). The prize is attributed to young researcher, who has given outstanding contributions in the field of applied and industrial mathematics.

Roger Casals (Department of Mathematics MIT, USA) has been awarded the **José Luis Rubio de Francia Prize** 2015 from the Royal Spanish Mathematical Society. This award recognizes and encourages the work of young researchers in mathematics.

The 2015 **Federico Bartolozzi Prize** has been attributed to **Spadaro Emanuele** (Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig).

Deaths

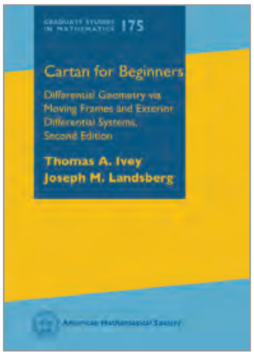
We regret to announce the deaths of:

Javier Cilleruelo (15 May 2016, Madrid, Spain)
Ante Mimica (9 June 2016 , Mimice, Croatia)
Sibe Mardešić (18 June 2016 , Zagreb, Croatia)
Roman Dwilewicz (29 July 2016, St.Louis, USA)
Ilona Kopocińska (3 August 2016, Wrocław, Poland)
Paweł Domański (4 August 2016, Poznań, Poland)
Asunción Sastre (24 August 2016, Elche, Spain)
Antonio Giraldo (24 August 2016, Elche, Spain)
Emil Vitásek (28 August 2016, Prague, Czech Republic)
Jean-Christophe Yoccoz (3 September 2016, Paris, France)
Marcel Berger (15 October 2016, Paris, France)
Bolesław Szafirski (6 November 2016, Cracow, Poland)
Eberhard Zeidler (18 November 2016, Leipzig, Germany)

Erratum

Unfortunately there was a typing error in the paper "Wrinkles: From the Sea to Mathematics" by François Laudenbach in the September 2016 issue of the EMS Newsletter (No. 101). The first subtitle on page 15 should read as follows: Wrinkles and sailing.
We apologize for the mistake.

AMS
AMERICAN MATHEMATICAL SOCIETY



CARTAN FOR BEGINNERS

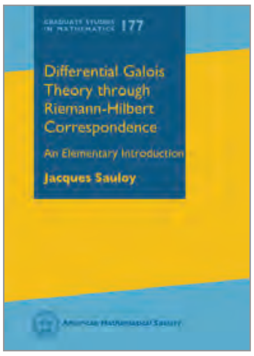
Differential Geometry via Moving Frames and Exterior Differential Systems, Second Edition

Thomas A. Ivey, College of Charleston & Joseph M. Landsberg, Texas A&M University

Two central aspects of Cartan's approach to differential geometry are the theory of exterior differential systems (EDS) and the method of moving frames. This book presents thorough and modern treatments of both subjects, including their applications to both classic and contemporary problems in geometry. Key concepts are developed incrementally, with motivating examples leading to definitions, theorems, and proofs.

Graduate Studies in Mathematics, Vol. 175

Jan 2017 455pp 9781470409869 Hardback €99.00



DIFFERENTIAL GALOIS THEORY THROUGH RIEMANN-HILBERT CORRESPONDENCE

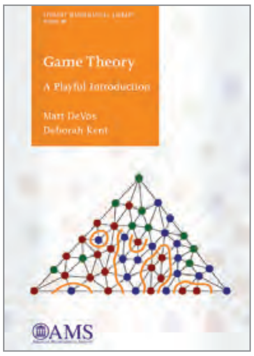
An Elementary Introduction

Jacques Sauloy, Institut de Mathématiques de Toulouse

Offers a hands-on transcendental approach to differential Galois theory, based on the Riemann-Hilbert correspondence. Along the way, it provides a smooth, down-to-earth introduction to algebraic geometry, category theory and tannakian duality.

Graduate Studies in Mathematics, Vol. 177

Jan 2017 279pp 9781470430955 Hardback €99.000



GAME THEORY

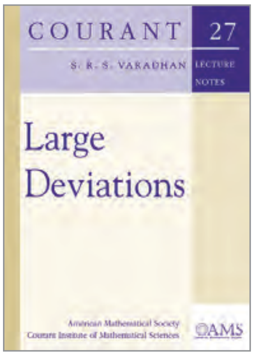
A Playful Introduction

Matt DeVos, Simon Fraser University & Deborah Kent, Drake University

Offers a gentle introduction to the mathematics of both sides of game theory: combinatorial and classical. The combination allows for a dynamic and rich tour of the subject united by a common theme of strategic reasoning. Instructors, students, and independent readers alike will appreciate the flexibility in content choices as well as the generous sets of exercises at various levels.

Student Mathematical Library, Vol. 80

Jan 2017 360pp 9781470422103 Paperback €55.00



LARGE DEVIATIONS

S.R.S. Varadhan, Courant Institute of Mathematical Sciences

The theory of large deviations deals with rates at which probabilities of certain events decay as a natural parameter in the problem varies. This book, which is based on a graduate course on large deviations at the Courant Institute, focuses on three concrete sets of examples: (i) diffusions with small noise and the exit problem, (ii) large time behaviour of Markov processes and their connection to the Feynman-Kac formula and the related large deviation behaviour of the number of distinct sites visited by a random walk, and (iii) interacting particle systems, their scaling limits, and large deviations from their expected limits.

A co-publication of the AMS and Courant Institute of Mathematical Sciences at New York University.

Courant Lecture Notes, Vol. 27

Dec 2016 104pp 9780821840863 Paperback €40.00

Free delivery worldwide at **eurospanbookstore.com/ams**

AMS is distributed by **Eurospan** | group

CUSTOMER SERVICES:
Tel: +44 (0)1767 604972
Fax: +44 (0)1767 601640
Email: eurospan@turpin-distribution.com

FURTHER INFORMATION:
Tel: +44 (0)20 7240 0856
Fax: +44 (0)20 7379 0609
Email: info@eurospangroup.com