

# NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



## Feature

Tell me a Pseudo-Anosov

## Interviews

Mireille Bousquet-Mélou

A. A. Kirillov

## Obituary

Vladimir A. Voevodsky



European  
Mathematical  
Society

December 2017

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**Aims and Scope**

Since its foundation in 1937, *Portugaliae Mathematica* has aimed at publishing high-level research articles in all branches of mathematics. With great efforts by its founders, the journal was able to publish articles by some of the best mathematicians of the time. In 2001 a *New Series of Portugaliae Mathematica* was started, reaffirming the purpose of maintaining a high-level research journal in mathematics with a wide range scope.



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**A journal of the Research Institute for Mathematical Sciences of Kyoto University**

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**Aims and Scope**

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**Aims and Scope**

The Accademia dei Lincei (Lynx), founded in 1603, is the oldest academy dedicated to the study of humanities as well as physics, mathematics and the natural sciences in the world. In 1873 the academy began publishing with the *Atti della Reale Accademia dei Lincei, Transunti*. Since 1990 under the present name, the *Rendiconti Lincei* have been one of the best Italian journals. The journal is dedicated to the publication of high-quality peer-reviewed surveys, research papers and preliminary announcements of important results from all fields of mathematics and its applications.



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**Aims and Scope**

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## Editorial Team

Editor-in-Chief

### Valentin Zagrebnov

Institut de Mathématiques de  
Marseille (UMR 7373) – CMI  
Technopôle Château-Gombert  
39, rue F. Joliot Curie  
13453 Marseille Cedex 13,  
France  
e-mail: Valentin.Zagrebnov@univ-amu.fr

Copy Editor

### Chris Nunn

119 St Michaels Road,  
Aldershot, GU12 4JW, UK  
e-mail: nunn2quick@gmail.com

Editors

### Ramla Abdellatif

(Contacts with SMF)  
LAMFA – UPJV  
80039 Amiens Cedex 1, France  
e-mail: Ramla.Abdellatif@u-picardie.fr

### Jean-Paul Allouche

(Book Reviews)  
IMJ-PRG, UPMC  
4, Place Jussieu, Case 247  
75252 Paris Cedex 05, France  
e-mail: jean-paul.allouche@imj-prg.fr

### Fernando Pestana da Costa

(Societies)  
Depto de Ciências e Tecnolo-  
gia, Secção de Matemática,  
Universidade Aberta,  
Rua da Escola Politécnica, n°  
141-147, 1269-001 Lisboa,  
Portugal  
e-mail: fcosta@uab.pt

### Jean-Luc Dorier

(Math. Education)  
FPSE – Université de Genève  
Bd du pont d'Arve, 40  
1211 Genève 4, Switzerland  
e-mail: Jean-Luc.Dorier@unige.ch

### Javier Fresán

(Young Mathematicians' Column)  
Centre de Mathématiques  
Laurent Schwartz  
École Polytechnique  
91128 Palaiseau, France  
e-mail: javier.fresan@polytechnique.edu



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### Vladimir R. Kostic

(Social Media)  
Department of Mathematics  
and Informatics  
University of Novi Sad  
21000 Novi Sad, Serbia  
e-mail: vladimir.slk@gmail.com

### Eva Miranda

(Research Centres)  
Department of Mathematics  
EPSEB, Edifici P  
Universitat Politècnica  
de Catalunya  
Av. del Dr Marañón 44–50  
08028 Barcelona, Spain  
e-mail: eva.miranda@upc.edu

### Vladimir L. Popov

(Features and Discussions)  
Steklov Mathematical Institute  
Russian Academy of Sciences  
Gubkina 8  
119991 Moscow, Russia  
e-mail: popovvl@mi.ras.ru

### Michael Th. Rassias

(Problem Corner)  
Institute of Mathematics  
University of Zurich  
Winterthurerstrasse 190  
8057 Zürich  
e-mail: michail.rassias@math.uzh.ch

### Volker R. Remmert

(History of Mathematics)  
IZWT, Wuppertal University  
D-42119 Wuppertal, Germany  
e-mail: remmert@uni-wuppertal.de

### Vladimir Salnikov

(Young Mathematicians' Column)  
La Rochelle University  
LaSIE, Avenue Michel Crépeau  
17042 La Rochelle Cedex 1,  
France  
e-mail: vladimir.salnikov@univ-lr.fr

### Dierk Schleicher

(Features and Discussions)  
Research I  
Jacobs University Bremen  
Postfach 750 561  
28725 Bremen, Germany  
e-mail: dierk@jacobs-university.de

### Olaf Teschke

(zbMATH Column)  
FIZ Karlsruhe  
Franklinstraße 11  
10587 Berlin, Germany  
e-mail: olaf.teschke@fiz-karlsruhe.de

# European Mathematical Society

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# EMS Executive Committee

---

## President

### Prof. Pavel Exner

(2015–2018)  
Doppler Institute  
Czech Technical University  
Břehová 7  
11519 Prague 1  
Czech Republic  
e-mail: [ems@ujf.cas.cz](mailto:ems@ujf.cas.cz)

---

## Vice-Presidents

### Prof. Volker Mehrmann

(2017–2018)  
Institut für Mathematik  
TU Berlin MA 4–5  
Strasse des 17. Juni 136  
10623 Berlin  
Germany  
e-mail: [mehrmann@math.TU-Berlin.DE](mailto:mehrmann@math.TU-Berlin.DE)

### Prof. Armen Sergeev

(2017–2020)  
Steklov Mathematical Institute  
Russian Academy of Sciences  
Gubkina str. 8  
119991 Moscow  
Russia  
e-mail: [sergeev@mi.ras.ru](mailto:sergeev@mi.ras.ru)

---

## Secretary

### Prof. Sjoerd Verduyn Lunel

(2015–2018)  
Department of Mathematics  
Utrecht University  
Budapestlaan 6  
3584 CD Utrecht  
The Netherlands  
e-mail: [s.m.verduynlunel@uu.nl](mailto:s.m.verduynlunel@uu.nl)

---

## Treasurer

### Prof. Mats Gyllenberg

(2015–2018)  
Department of Mathematics  
and Statistics  
University of Helsinki  
P.O. Box 68  
00014 University of Helsinki  
Finland  
e-mail: [mats.gyllenberg@helsinki.fi](mailto:mats.gyllenberg@helsinki.fi)

---

## Ordinary Members

### Prof. Nicola Fusco

(2017–2020)  
Dip. di Matematica e Applicazioni  
Complesso Universitario di  
Monte Sant' Angelo  
Via Cintia  
80126 Napoli  
Italy  
e-mail: [n.fusco@unina.it](mailto:n.fusco@unina.it)

### Prof. Stefan Jackowski

(2017–2020)  
Institute of Mathematics  
University of Warsaw  
Banacha 2  
02-097 Warszawa  
Poland  
e-mail: [sjack@mimuw.edu.pl](mailto:sjack@mimuw.edu.pl)

### Prof. Vicente Muñoz

(2017–2020)  
Departamento de Geometría y  
Topología  
Facultad de Matemáticas  
Universidad Complutense de  
Madrid  
Plaza de Ciencias 3  
28040 Madrid  
Spain  
e-mail: [vicente.munoz@mat.ucm.es](mailto:vicente.munoz@mat.ucm.es)

### Prof. Beatrice Pelloni

(2017–2020)  
School of Mathematical &  
Computer Sciences  
Heriot-Watt University  
Edinburgh EH14 4AS  
UK  
e-mail: [b.pelloni@hw.ac.uk](mailto:b.pelloni@hw.ac.uk)

### Prof. Betül Tanbay

(2017–2020)  
Department of Mathematics  
Bogazici University  
Bebek 34342 Istanbul  
Turkey  
e-mail: [tanbay@boun.edu.tr](mailto:tanbay@boun.edu.tr)

---

## EMS Secretariat

### Ms Elvira Hyvönen

Department of Mathematics  
and Statistics  
P.O. Box 68  
(Gustaf Hällströmin katu 2b)  
00014 University of Helsinki  
Finland  
Tel: (+358) 2941 51141  
e-mail: [ems-office@helsinki.fi](mailto:ems-office@helsinki.fi)  
Web site: <http://www.euro-math-soc.eu>

---

## EMS Publicity Officer

### Dr. Richard H. Elwes

School of Mathematics  
University of Leeds  
Leeds, LS2 9JT  
UK  
e-mail: [R.H.Elwes@leeds.ac.uk](mailto:R.H.Elwes@leeds.ac.uk)

# EMS Agenda

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## 2018

### 13–15 April

Meeting of Presidents, Maynooth University, Dublin, Ireland

### 23–24 June

EMS Council, Prague, Czech Republic

### 29–30 July

IMU General Assembly, São Paulo, Brazil

# EMS Scientific Events

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## 2018

### 29 January–9 February

Winter School and Workshop “Riemann–Hilbert  
Correspondences”, Università di Padova, Italy

### 5–9 March

GDM-DMV Annual Meeting 2018, Paderborn, Germany

### 3–6 April

British Congress of Mathematics Education, Warwick, UK

### 4–6 April

Probability, Analysis and Dynamics '18, University of Bristol, UK

### 21–22 May

Nonlinear Analysis and the Physical and Biological Sciences, Edinburgh, UK

### 11–14 June

British Mathematical Colloquium 2018, University of St Andrews, UK

### 11–15 June

International Conference on Complex Analysis, Potential Theory and Applications, University College Dublin, Ireland

### 9–14 July

Young African Scientists in Europe (YASE), Toulouse, France

### 23–27 July

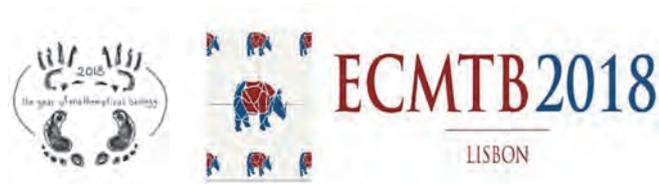
11th European Conference on Mathematical and Theoretical Biology (ECMTB 2018), Lisbon, Portugal

### 1–9 August

ICM 2018  
Rio Centro Convention Center, Rio de Janeiro, Brazil

# Editorial: Year of Mathematical Biology 2018

José A. Carrillo (Imperial College London, UK), Chair of the Applied Mathematics Committee of the EMS and Mats Gyllenberg (University of Helsinki, Finland), Treasurer of the EMS



The year of Mathematical Biology 2018 is a joint venture of the European Mathematical Society (EMS) and the European Society for Mathematical and Theoretical Biology (ESMTB).

The main objectives are to celebrate the huge increase and importance of applications of mathematics to biology and life sciences in the last few years and to foster the feedback loop between life sciences and mathematics for years to come. Applications of mathematics in biology are opening completely new pathways of interactions and they are a huge source of new mathematical problems.

The activities already scheduled during this event are summarised in three large programmes in different aspects of mathematical biology at three ERCOM institutes:

- Simons Semester on Mathematical Biology, December 2017–March 2018, Banach Center, Warsaw, Poland.
- Intensive Research Program in Mathematical Biology, April–June 2018, Centre de Recerca Matemàtica, Spain.
- Thematic Program in Mathematical Biology, September–December 2018, Institut Mittag-Leffler, Sweden.

There are also many other activities across Europe spanning a wide range of current aspects of interest in mathematical biology.

The Year of Mathematical Biology will kick off with an event sponsored by the EMS and the ESMTB: the EMS-Finnish Mathematical Society-ESMTB Joint Mathematical Weekend, 4–5 January 2018, Joensuu, Finland.

Later in the year, the largest European mathematical biology conference series will be organised jointly by the EMS and the ESMTB: the 11th European Conference on Mathematical and Theoretical Biology (ECMTB 2018), 23–27 July 2018, Lisbon, Portugal.

We encourage all our fellow society members with an interest in mathematical biology to get involved in this transversal event and actively participate.

Other activities include the following events (and more are being planned):

- Dynamical systems applied to biology and natural sciences (DSABNS2018), 7–9 February 2018, Torino, Italy.
- Collective dynamics and self-organisation in biological sciences, 30 Apr–4 May 2018, ICMS, Edinburgh, UK.
- Models in population dynamics, ecology and evolution (MPDEE'18), 9–13 April 2018, University of Leicester, UK.
- Mathematical biology modelling days of Besançon, 19–22 June 2018, Besançon, France.
- International conference on mathematical methods and models (BIOMATH 2018), 24–29 June, Sofia, Bulgaria.
- Mathematical perspectives in the biology and therapeutics of cancer, 9–13 July 2018, CIRM, France.
- CEMRACS 2018, Numerical and mathematical modelling for biological and medical applications: deterministic, probabilistic and statistical descriptions, 16 July–24 August 2018, CIRM, Marseille, France.
- The Helsinki summer school on mathematical ecology and evolution, August 2018, Helsinki, Finland.
- Differential equations arising from organising principles in biology, 23–29 September 2018, Mathematisches Forschungsinstitut Oberwolfach, Germany.
- Workshop on mathematical biology, 8–12 October, Institut Mittag-Leffler, Sweden.

An organisation committee for the Year of Mathematical Biology has been set up through the Applied Mathematics Committee of the EMS:

- José A. Carrillo, Imperial College London, UK. (Chair)
- Mathisca de Gunst, University of Amsterdam, The Netherlands.
- Mats Gyllenberg, University of Helsinki, Finland.
- Torbjorn Lundh, Chalmers University, Sweden.
- Anna Marciniak-Czochra, Heidelberg Universität, Germany.
- Roeland Merks, CWI, The Netherlands.
- Marek Niezgodka, ICM, Poland.
- Gael Raoul, École Polytechnique, France.

If you have any suggestions or ideas that you want to share with us, activities to be included or any queries, please contact any member of the committee. We want

to thank everybody involved in the organisation committee and the scientific committee and speakers of each of the events above for the effort put into this endeavour. It is a pleasure to see how this idea has developed over the years from a very small-scale project, with origins in 2014 of celebrating collaborations between mathematics and biology, to a full year of mathematical biology events. This could not be done without the generous effort of a large community who believe in the fantastic outcome that this cross-pollination between disciplines can bring to mathematics as a whole.



*José A. Carrillo holds a Chair of Applied and Numerical Analysis at Imperial College London. He is an expert in Partial Differential Equations, their numerical approximations, and their use in modelling across science and engineering.*



*Mats Gyllenberg is a Professor of Applied Mathematics at the University of Helsinki. He is an expert in population models in mathematical biology and a long serving editor of the Journal of Mathematical Biology.*

## Announcement of the Next Meeting of the EMS Council Prague, June 23 and 24, 2018

The EMS Council meets every second year. The next meeting will be held in Prague, June 23 and 24, 2018 at Balling Hall at the National Library of Technology (Technická 2710/6, 160 80 Praha 6 - Dejvice). The Council meeting starts at 14.00 on June 23 and ends at lunch time on June 24.

### Delegates

Delegates to the Council shall be elected for a period of four years. A delegate may be re-elected provided that consecutive service in the same capacity does not exceed eight years. Delegates will be elected by the following categories of members.

#### (a) Full Members

Full Members are national mathematical societies, which elect 1, 2, 3, or 4 delegates according to their membership class. The membership class is decided by Council, and societies are invited to apply for the new class 4, which was introduced in the 2008 Council. However, the number of delegates for the 2018 Council is determined by the current membership class of the society.

Each society is responsible for the election of its delegates.

There is an online nomination form for delegates of full members. The deadline for nominations for delegates of full members is 15 April 2018.

#### (b) Associate Members

Delegates representing associate members shall be elected by a ballot organized by the Executive Committee from a list of candidates who have been nominated

and seconded by associate members, and have agreed to serve. In October 2017, there were 2 associate members and, according to our statutes, these members may be represented by (up to) one delegate.

There is an online nomination form for delegates of associate members. The deadline for nominations for delegates of associate members is 15 March 2018.

#### (c) Institutional Members

Delegates representing institutional members shall be elected by a ballot organized by the Executive Committee from a list of candidates who have been nominated and seconded by institutional members, and have agreed to serve. In October 2017, there were 44 institutional members and, according to our statutes, these members may be represented by (up to) 4 delegates.

The delegate whose term includes 2018 is Klavdija Kutnar. The delegate who can be re-elected is Alberto Pinto.

There is an online nomination form for delegates of institutional members. The deadline for nominations for delegates of institutional members is 15 March 2018.

#### (d) Individual Members

Delegates representing individual members shall be elected by a ballot organized by the Executive Committee from a list of candidates who have been nominated and seconded, and have agreed to serve. These delegates must themselves be individual members of the European Mathematical Society.

In October 2017 there were 2816 individual members and, according to our statutes, these members may be

represented by (up to) 28 delegates. However, this number may have increased by the time we call the election (if any) for individual members.

Here is a list of the current delegates of individual members whose terms include 2018:

Thierry Bouche  
 Jose Antonio Carrillo  
 Antonio Campillo  
 Piermarco Cannarsa  
 Carles Casacuberta i Vergés  
 Mireille Chaleyat-Maurel  
 Krzysztof Ciesielski  
 Pavel Exner  
 Paul C. Kettler  
 Bostjan Kuzman  
 Ari Laptev  
 Marta Mazzocco  
 Vicente Muñoz  
 José Francisco Rodrigues  
 Marie-Francoise Roy  
 Marta Sanz-Solé  
 Robin Wilson

Here is a list of the delegates of individual members who could be re-elected for the 2018 Council:

Maria Esteban  
 Vincenzo Ferone  
 Luis Narvaález Macarro  
 Jiří Rákosník

There is an online nomination form for delegates of individual members. The deadline for nominations for delegates of individual members is 15 March 2018.

### Agenda

The Executive Committee is responsible for preparing the matters to be discussed at Council meetings. Items for the agenda of this meeting of the Council should be sent as soon as possible, and no later than 15 April 2018, to the EMS Secretariat in Helsinki.

### Executive Committee

The Council is responsible for electing the President, Vice-Presidents, Secretary, Treasurer and other members of the Executive Committee. The present membership of the Executive Committee, together with their individual terms of office, is as follows.

President: Pavel Exner (2015–2018)

Vice-Presidents: Volker Mehrmann (2017–2018)  
 Armen Sergeev (2017–2020)

Secretary: Sjoerd Verduyn Lunel (2015–2018)

Treasurer: Mats Gyllenberg (2015–2018)

Members: Nicola Fusco (2017–2020)  
 Stefan Jackowski (2017–2020)  
 Vicente Muñoz (2017–2020)  
 Beatrice Pelloni (2017–2020)  
 Betül Tanbay (2017–2020)

Members of the Executive Committee are elected for a period of four years. The President can only serve one term. Committee members may be re-elected, provided that consecutive service shall not exceed eight years.

The Council may, at its meeting, add to the nominations received and set up a Nominations Committee, disjoint from the Executive Committee, to consider all candidates. After hearing the report by the Chair of the Nominations Committee (if one has been set up), the Council will proceed to the elections to the Executive Committee posts.

All these arrangements are as required in the Statutes and By-Laws, which can be found here, together with the web page for the Council:

<http://www.euro-math-soc.eu>

The nomination form for full member delegates can be found here

<https://elomake.helsinki.fi/lomakkeet/81087/lomake.html>

The nomination form for institutional, associate and individual member delegates can be found here

<https://elomake.helsinki.fi/lomakkeet/81089/lomake.html>

Secretary: Sjoerd Verduyn Lunel  
 (s.m.verduynlunel@uu.nl)

Secretariat: [ems-office@helsinki.fi](mailto:ems-office@helsinki.fi)

# The EMS Website

Vicente Muñoz (Universidad Complutense de Madrid, Spain) and Armen Sergeev (Steklov Mathematical Institute, Moscow, Russia)

The European Mathematical Society (EMS) has three faces open to the general public, namely: the Newsletter, Social Media (Twitter, Facebook and Google) and the official webpage of the EMS: [www.euro-math-soc.eu](http://www.euro-math-soc.eu).

The website, in its current form, was launched in 2014 by former EMS Vice-President Martin Raussen and developed by Robert Carr, and it was announced in the Newsletter of the EMS, No. 94, December 2014, pp. 6–7. It is based on the content management system Drupal 7, which is currently good enough in terms of convenience and security but should be upgraded in the future to a more advanced and better designed version.

The website contains plenty of material serving as a store of information, presented in an accessible and reliable way. You start with the main page, where the most recent news appears in a nice layout, including a link to the current issue of the Newsletter. At the top of the page, you will find several clickable labels: News, Inside EMS, Membership, Jobs, Services, Scientific Activities, Publishing House and Travel Grants. Each of them has a collection of sub-labels referring to different pages of the site. For the convenience of the readers of the Newsletter, we will give a short overview of the most useful features.

Entering “News”, you will find a list of various current events, such as announcements of mathematical prizes, important mathematical conferences, meetings and so

on. The main source of information consists of feedback from the members of the society and users of the website. Anyone can open an account (members have automatic access to the account through which they can also enter their user’s profile and pay dues online). After logging in, a registered user can post a news item to the website or add a comment to an item. These posts are moderated by the web team (to prevent possible spam) and are shown on the webpage (usually within 24–48 hours). News appears in reverse chronological order, except for a few prominent items, which are shown at the top.

“Inside EMS” contains the various information compiled for the members of the society, such as the structure of the EMS committees, the agenda of the EMS and so on, providing links allowing access to more detailed information. The webpages of the committees are also accessible via this route so that the reader can easily browse to them. Some are hosted on the main site while others are stored on external sites (like the site of the Committee for Raising Public Awareness). These webpages are maintained by the chair or a designated member of the corresponding committee. The statutes and by-laws of the EMS can also be found here.

Under “Membership”, an EMS member (whether individual or corporate) will find methods to pay dues, update personal information and learn about various benefits and discounts.

The European Mathematical Society is a learned society representing mathematicians throughout Europe. It promotes the development of all aspects of mathematics in Europe, in particular mathematical research, relations of mathematics to society, relations to European institutions, and mathematical education. The EMS has as its members around 60 national mathematical societies in Europe, 40 mathematical research centres and departments, and 3000 individuals.

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The label “Jobs” allows a user to browse various job advertisements and positions in mathematics. More than half of the current internet traffic of the EMS goes through this page. Anyone can post a job advertisement (regardless of whether it is a university position or a grant proposal) free of charge unless it is of commercial character. A company advertising a job for mathematicians in the private sector can do the same for a small fee by connecting with the EMS secretary via [ems-office@helsinki.fi](mailto:ems-office@helsinki.fi). Another relevant link is MathHire, a private company based in Europe and offering web based handling of an entire hiring process. MathHire has a cooperation agreement on job advertising with the EMS (with discounts for EMS members).

The label “Services” leads to a collection of many useful items. In particular, “Events” contains announcements of various conferences (including those that are not directly related to the EMS), lecture courses and other mathematical events. The events are shown in calendar form and can be conveniently browsed. There is also an item “Book Reviews” containing reviews on recently published books. These are organised by the team of book reviewers at the Universidad Complutense de Madrid. We want to thank the team, particularly one of the most active reviewers Adhemar Bultheel from Leuven, for their excellent work. We would also like to encourage users who are interested in submitting their own book reviews or in joining the team to contact Vicente Muñoz at [vicente.munoz@mat.ucm.es](mailto:vicente.munoz@mat.ucm.es). The “Discussion Forum” item at the bottom is not currently available but we hope to launch it very soon.

The label “Scientific Activities” focuses on scientific events sponsored or organised by the EMS. Members of the EMS will find application forms and information on the rules for submitting them. The next label “Publishing

House” links to the EMS Publishing House, with information on books and journals published by this company. The meaning of the last item “Travel Grants” is clear from its name.

For internal needs, the EMS organised a web team during a recent meeting in Helsinki in February 2017. It has two coordinators Vicente Muñoz (Executive Committee member) and Armen Sergeev (EMS Vice-President) and includes four additional members: Mats Gyllenberg (EMS Treasurer), Elvira Hyvönen (EMS Secretary), Matti Pauna (administrator) and Albert Ruiz (collaborator). This team is responsible for the everyday functioning and maintaining the up-to-date content of the website. We are thinking of inviting a professional external company to make this part of our job more advanced and sophisticated.

We encourage the users of the EMS website both to look for and send to us relevant information on events, news, jobs, book reviews and so on. You may consult EMS Secretary Elvira Hyvönen at [ems-office@helsinki.fi](mailto:ems-office@helsinki.fi).



*Vicente Muñoz ([vicente.munoz@mat.ucm.es](mailto:vicente.munoz@mat.ucm.es)) is a member of the EMS Executive Committee and a professor at the Universidad Complutense de Madrid. His research areas are differential geometry and algebraic geometry.*



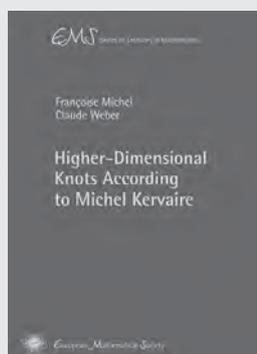
*Armen Sergeev ([sergeev@mi.ras.ru](mailto:sergeev@mi.ras.ru)) is a Vice-President of the EMS and a professor at the Steklov Mathematical Institute in Moscow. His research interests focus on mathematical physics and several complex variables.*



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Françoise Michel (Université Paul Sabatier, Toulouse, France) and Claude Weber (Université de Genève, Switzerland)

### Higher-Dimensional Knots According to Michel Kervaire

ISBN 978-3-03719-180-4. 2017. 144 pages. Softcover. 17 x 24 cm. 32.00 Euro

Michel Kervaire wrote six papers which can be considered fundamental to the development of higher-dimensional knot theory. They are not only of historical interest but naturally introduce to some of the essential techniques in this fascinating theory.

This book is written to provide graduate students with the basic concepts necessary to read texts in higher-dimensional knot theory and its relations with singularities. The first chapters are devoted to a presentation of Pontrjagin's construction, surgery and the work of Kervaire and Milnor on homotopy spheres. We pursue with Kervaire's fundamental work on the group of a knot, knot modules and knot cobordism. We add developments

due to Levine. Tools (like open books, handlebodies, plumbings, ...) often used but hard to find in original articles are presented in appendices. We conclude with a description of the Kervaire invariant and the consequences of the Hill–Hopkins–Ravenel results in knot theory.

# Feedback Following the Creation of *Épijournal de Géométrie Algébrique*

Pierre-Emmanuel Chaput (Université de Lorraine, Vandœuvre-lès-Nancy, France), Benoît Claudon (Université de Rennes, France), Damien Mégy (Université de Lorraine, Vandœuvre-lès-Nancy, France), Lucas Fresse (Université de Lorraine, Vandœuvre-lès-Nancy, France), Alain Genestier (Université de Lorraine, Vandœuvre-lès-Nancy, France), Arvid Perego (Université de Lorraine, Vandœuvre-lès-Nancy, France), Matei Toma (Université de Lorraine, Vandœuvre-lès-Nancy, France)\*



A new overlay journal, the *Épijournal de Géométrie Algébrique*, was recently launched. In this note, the launch serves as the starting point of a discussion of overlay journals in general and the Épis-ciences platform<sup>1</sup> in particular. We

then report in more detail on the *Épijournal de Géométrie Algébrique*.

## Overlay journals: a new model of scientific publication

Let us start with a reminder of what an *overlay journal* (or *épijournal* in French) is. It is an open-access scientific journal that is constructed by adding extra structure on top of one (or more) pre-existing open-access archives.

Overlay journals follow exactly the same peer-review process as a traditional mathematical journal. The articles are generally electronically published and also remain on the preprint server where they were initially published.

This description justifies the term “open access”. By its very nature, an overlay journal provides open access to all published articles. In the next section, we discuss the economic model of one overlay platform – Épis-ciences.

## The Épis-ciences platform

An overlay journal is an extra layer on top of one or more open archives, with which it usually connects via a web platform. Épis-ciences is a web platform that has been created to host overlay journals. It currently hosts journals in mathematics and computer science.

This platform is backed by the following open archives for mathematics – arXiv and Hyper Articles en Ligne (HAL) – and is a project of the CCSD (Centre for Direct Scientific Communication). Overlay journals hosted on Épis-ciences are economically ethical since they do not demand publication fees and their costs are paid by public institutions.

\* The authors gratefully acknowledge Catriona Maclean for her help in writing this text.

<sup>1</sup> We also suggest reading <https://gowers.wordpress.com/2013/01/16/why-ive-also-joined-the-good-guys/>.

<sup>2</sup> Overlay journals are not only for mathematicians and computer scientists (see, for example, <http://www.nature.com/news/open-journals-that-piggyback-on-arxiv-gather-momentum-1.19102>).

The platform offers a web infrastructure for managing editorial flow. Note that this platform does not only host new journals: existing journals may decide to switch to overlay journals and transition into the Épis-ciences fold.

At the time of writing (September 2017), the Épis-ciences platform hosts two overlay journals in mathematics (including the *Épijournal de Géométrie Algébrique*), five in computer science and three in the humanities.

## *Épijournal de Géométrie Algébrique*

Observation of difficulties of scientific publishing led the authors to the idea of using the Épis-ciences platform to launch a new mathematical journal. We found it easier to create a specialized journal rather than a general one, particularly for setting up an editorial board, and we soon decided that the scope of our journal would be algebraic geometry.

## Starting the project

Our first tasks were to find a name, form the editorial board and fix our operating rules. We then gathered this information and passed it onto the *épiMaths* committee.<sup>3</sup> The role of this committee is to encourage the creation of new overlay journals and study the applications made for that purpose. Our application was accepted on December 2015.

From January to June 2016, we learned how to use the Épis-ciences platform and started testing it. During that time, we made some requests for new developments to make the platform compatible with our operating rules.

Prior to the launch of our journal, we also had to create website content, keep the editors informed of the platform operating procedures and start prospecting for possible submissions.

## About the journal

As the name suggests, *Épijournal de Géométrie Algébrique* exists to publish original articles in algebraic geometry in a broad sense, from arithmetic geometry to the study of compact Kähler varieties through to the theory of algebraic groups and their representations.

The composition of the editorial board (and more details of the journal) can be found at:

<https://epiga.epis-ciences.org/>.

<sup>3</sup> The composition of this committee is available at <http://epis-ciences.org/page/epimath>.

One of the editors<sup>4</sup> has the special role of coordinator. His role is to assign an editor to each article and to facilitate discussions on the status of a submitted article. Acceptance or rejection decisions are made conjointly by the whole board.

The authors of this note are members of the monitoring committee of the journal. The role of this committee is to ensure that the logistics of the journal work properly (e.g. setting the journal's website, testing the platform interface and developing a LaTeX style journal sheet). Together with the editors, we promote this fledgling journal through relevant mailing lists and by direct article solicitation.

### **The first volume**

The first volume has recently been published and its content can be seen at:

<https://epiga.episciences.org/volume/view/id/267>.

For the time being, it consists of nine articles but other articles accepted in the current year will be added to this first volume. Subsequently, the journal will publish one volume per year.

### **Conclusion**

We have just talked about the conclusion, i.e. soliciting valuable submissions is a task of importance. Any new submission is welcome and we hope that this article will have encouraged this!

Perennial and institutional alternatives to the economic models offered by commercial publishers come in different forms: the Épisiences project is one of

<sup>4</sup> Currently Michel Brion.

those. Let us grasp and expand the opportunities so that researchers willing to publish their work in economically virtuous journals have a wider and wider choice.

*Pierre-Emmanuel Chaput works at the Elie Cartan Institute (IECL), Nancy, France. His research focuses on the interplay between algebraic groups, representation theory, and algebraic geometry.*

*Benoît Claudon works at the Mathematic Institute of Rennes (IRMAR), France. His field of interest is complex algebraic geometry (classification of smooth projective varieties and compact Kähler manifolds).*

*Lucas Fresse works at the Elie Cartan Institute (IECL), Nancy, France. His research topics concern geometric representation theory and Lie theory.*

*Alain Genestier works at the Elie Cartan Institute (IECL), Nancy, France. His research focuses on arithmetic algebraic geometry, with a focus on the consequences in the local Langlands program of the bad reduction behavior of Shimura varieties and shtuka classifying spaces.*

*Damien Mégy works at the Elie Cartan Institute (IECL), Nancy, France. His field of interest is complex algebraic geometry with a focus on Hodge theory.*

*Arvid Perego works at the Elie Cartan Institute (IECL), Nancy, France. His research focuses on hyperkahler manifolds and moduli spaces of semi-stable sheaves (or complexes of sheaves) on surfaces.*

*Matei Toma works at the Elie Cartan Institute (IECL), Nancy, France. His field of research is Complex Geometry.*

## European Prize in Combinatorics 2017

The European Prize in Combinatorics 2017 was awarded to:

**Christian Reiher** (University of Hamburg) for his profound result in extremal and probabilistic combinatorics and particularly for his solution of the Kemnitz conjecture on lattice points and the Lovasz–Simonovits clique density problem; and

**Maryna Viazovska** (EPFL), for her deep contributions to spherical designs and particularly for the solution of the sphere packing problem in dimensions 8 and 24.

The prize ceremony took place at the TU Wien at the Opening of the Eurocomb 2017 Conference on 28 August 2017. Both prize winners gave a prize lecture on 30 August as part of the Eurocomb Conference. The award of 2500 EURO for 2017 is founded with contributions by DIMATIA, the local organisers and Elsevier.

See <http://www.dmg.tuwien.ac.at/eurocomb2017/index.php/2017/09/01/european-prize-in-combinatorics-2/> for more details.

# Maryna Viazovska to Receive the 2017 Sastra Ramanujan Prize

Krishnaswami Alladi (University of Florida, Gainesville, USA), Chair SASTRA Ramanujan Prize Committee



The 2017 SASTRA Ramanujan Prize will be awarded to Dr. Maryna Viazovska of the Swiss Federal Institute of Technology, Lausanne, Switzerland, especially for her stunning solution in dimension 8 of the celebrated sphere packing problem, and for her equally impressive joint work with Henry Cohn, Abhinav Kumar, Stephen D. Miller and Danylo Radchenko resolving the sphere

packing problem in dimension 24, by building upon her fundamental ideas in dimension 8.

The SASTRA Ramanujan Prize was established in 2005 and is awarded annually for outstanding contributions by young mathematicians to areas influenced by the genius Srinivasa Ramanujan. The age limit for the prize has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded during December 21–22, 2017 at the International Conference on Number Theory at SASTRA University in Kumbakonam (Ramanujan's hometown) where the prize has been given annually.

Maryna Viazovska is an extraordinarily gifted mathematician who has made deep contributions to several fundamental problems in number theory. In her outstanding PhD thesis of 2013 written under the direction of Professor Don Zagier at the Max Planck Institute for Mathematics at the University of Bonn, Germany, she resolved the famous Gross–Zagier Conjecture in a substantial number of cases, including the important case pertaining to higher Green's functions that had been open for 30 years. Her thesis work was a tour-de-force making brilliant use of a variety of tools such as Borcherds lifts, Kudla's program on the arithmeticity of theta correspondences, and unusually clever technical calculations. This work of hers was published in the book *Arithmetic and Geometry* (Cambridge University Press, 2016).

Prior to joining the PhD program in Bonn, she had a number of impressive publications in collaboration with several active researchers. Particularly significant is her joint work with Andrii Bondarenko and Danylo Radchenko (a PhD student under Don Zagier in Bonn) which resolved a longstanding conjecture of Korevaar and Meyers on spherical designs, by giving an optimal upper bound for the minimal number of points in a spherical design. This work was published in the *Annals*

*of Mathematics* (Princeton) in 2013. It is with this background that she started thinking about the sphere packing problem while in Bonn, the recent solution to which in dimensions 8 and 24 that propelled her into world prominence.

The sphere packing problem has a long and illustrious history. Johannes Kepler asked for the optimal way to assemble cannon balls (of uniform radius) and conjectured a configuration, but he could not prove it. This is the sphere packing problem in three dimensions, and can be generalized to arbitrary dimensions.

The sphere packing problem in three dimensions is known as Kepler's problem or The Kepler Conjecture. This conjecture in three dimensions was finally resolved by Thomas Hales in 1998 who gave a proof which was tour-de-force that combined ingenious geometric optimization arguments with machine calculations. The sphere packing problem in higher dimensions remained open. The sphere packing problem arises naturally not just in geometry and physics, but also in information theory, where sphere packings are error correcting codes for a continuous communication channel.

In dimension 8 there is  $E_8$ , an exceptional simple Lie group with a root lattice of rank 8, and in dimension 24, we have the Leech Lattice, and both have remarkable structures. This gave some hope that the sphere packing problem could be resolved in dimensions 8 and 24. Indeed Noam Elkies of Harvard University and Henry Cohn of Microsoft Research in Cambridge, Massachusetts, made significant progress by using the Poisson summation formula and linear programming bounds for the sphere packing density. They conjectured the existence of certain magic auxiliary functions in dimensions 8 and 24, which if determined, would resolve the conjecture in those dimensions. But these magic functions remained elusive. Viazovska produced these functions by an ingenious use of modular forms. Her attack was viewed as audacious, but when she succeeded, the mathematical world applauded in disbelief because her proof is remarkably simple. Paul Erdős, a legend of 20th century mathematics, has often joked that God has a Book containing the most beautiful proofs of the most important theorems; Viazovska's proof in dimension 8 is considered to be a proof from *The Book!* Her proof has been published in the *Annals of Mathematics* this year.

Once Viazovska had succeeded in dimension 8, the immediate question was whether her methods could be extended to dimension 24. Indeed, in the span of a week, by working at a furious pace, Viazovska in collaboration with Cohn, Kumar, Miller and Radchenko, successfully resolved the 24 dimensional case by building upon her

ideas in dimension 8. This joint paper has also appeared this year in the *Annals of Mathematics*.

Viazovska's method makes crucial use of the theory of modular forms, which were a favorite of Ramanujan; indeed 24 is a number associated with Ramanujan's work. Viazovska's modular forms techniques are by no means limited to the sphere packing problem or ground states. She has discovered something profound that will play a broader role in discrete geometry, analytic number theory, and harmonic analysis.

The citation for the 2017 SASTRA Ramanujan Prize reads as follows:

*“Maryna Viazovska is awarded the 2017 SASTRA Ramanujan Prize for her stunning and elegant resolution of the celebrated sphere packing problem in dimension 8, the proof of which appeared in her paper in the Annals of Mathematics (2017), and for her joint 2017 paper in the Annals of Mathematics with Henry Cohn, Abhinav Kumar, Stephen D. Miller and Danylo Radchenko, which resolves the sphere packing problem in dimension 24 by building on her ideas in dimension 8. The prize also recognizes her outstanding PhD thesis of 2013 at the University of Bonn in which she resolved significant cases of the Gross–Zagier Conjecture and her work prior to her PhD with A. Bodarenko and D. Radchenko resolving a long standing conjecture of Korevaar and Meyers on spherical designs, that appeared in the Annals of Mathematics in 2013. The prize notes that the modular forms techniques developed by Viazovska will have a significant future impact in discrete geometry, analytic number theory, and harmonic analysis.”*

Maryna Viazovska, now 32 years old, was born in Kiev in the Ukraine on November 2, 1984. She completed her high school education in Kiev in 2001, and her BSc in Mathematics in 2005 at the Kiev National Taras Shevchenko University. She then went to Germany where she obtained a Masters degree in 2007 from the University of Kaiserslautern, after which she joined the University of Bonn that year. She graduated with a PhD from Bonn in 2013 writing a thesis under the direction of Professor Don Zagier. Since her PhD, she has received several awards and recognitions such as the Salem Prize in 2016 and the Clay Research Award in 2017. The SASTRA Ramanujan Prize is now a fitting recognition for her path-breaking work.

The 2017 SASTRA Ramanujan Prize Committee consisted of Professors

Krishnaswami Alladi – Chair (University of Florida),  
 Andrew Granville (University of Montreal)  
 Winfried Kohnen (University of Heidelberg)  
 Philippe Michel (EPF Lausanne)  
 Peter Sarnak (Princeton University and the Institute for Advanced Study)  
 Michael Schlosser (University of Vienna), and  
 Gisbert Wustholz (ETH, Zurich).

Previous winners of the Prize are

Manjul Bhargava and Kannan Soundararajan in 2005 (two full prizes)  
 Terence Tao in 2006  
 Ben Green in 2007  
 Akshay Venkatesh in 2008  
 Kathrin Bringmann in 2009  
 Wei Zhang in 2010  
 Roman Holowinsky in 2011  
 Zhiwei Yun in 2012  
 Peter Scholze in 2013  
 James Maynard in 2014  
 Jacob Tsimerman in 2015, and  
 Kaisa Matomaki and Maksym Radziwill (shared) in 2016.

The award of the 2017 SASTRA Ramanujan Prize to Maryna Viazovska is in keeping with the tradition of recognizing the spectacular contributions by the most brilliant young mathematicians.



*Krishnaswami Alladi is a professor of mathematics at the University of Florida where he was Department Chairman, 1998–2008. He received his PhD from UCLA in 1978. His area of research is number theory. He is the founder and Editor-in-Chief of the Ramanujan Journal published by Springer. He helped create the SASTRA Ramanujan Prize and has chaired the prize committee since its inception.*

# Tell Me a Pseudo-Anosov

Erwan Lanneau (Université Grenoble Alpes, Saint-Martin-d'Hères, France)

Anosov linear homeomorphisms, and more generally Anosov flows, as well as their hyperbolic analogues, have played an important role in the theory of dynamical systems [1, 2, 7].<sup>1</sup>

Their cousins, the *pseudo-Anosov* homeomorphisms, which are also interesting and important, seem to be less well known. In contrast to the theory of Anosov flows, for which we know their contours rather well, there are several fundamental questions about pseudo-Anosov homeomorphisms that so far remain widely open.

## 1 An instructive example

Let us start with a simple example that is, in some sense, more than an example. Any matrix  $A \in \text{SL}(2, \mathbb{Z})$  acts linearly on the plane  $\mathbb{R}^2$ . The induced dynamics are not very interesting (the orbits are either circles or escape to infinity). A method of making things richer is to “pass to the quotient”: since  $A$  bijectively preserves the  $\mathbb{Z}^2$  lattice, that is,  $A(\mathbb{Z}^2) = \mathbb{Z}^2$ , it induces a diffeomorphism  $\psi$  of the torus  $T^2 = \mathbb{R}^2/\mathbb{Z}^2$  given by  $\psi((x, y) + \mathbb{Z}^2) = A(x, y) + \mathbb{Z}^2$ .

The dynamics of  $\psi$  are governed by the eigenvalues  $\lambda, \lambda^{-1}$  of  $A$ . There are three possibilities:

- (1)  $\lambda$  and  $\lambda^{-1}$  are complex conjugates ( $\lambda \neq \pm 1$ ):  $\psi$  is of finite order.
- (2)  $\lambda = \lambda^{-1} = \pm 1$ :  $\psi$  is reducible, that is, it preserves a closed curve on the torus.
- (3)  $\lambda$  and  $\lambda^{-1}$  are distinct irrational numbers:  $\psi$  is of Anosov type.

The second case (parabolic) implies that  $\psi$  arises from a map on a simpler surface (in this case an annulus).

The last case (hyperbolic) is by far the one with the richer dynamics ( $\psi$  has many periodic points, many points of dense orbits, etc.). The cat map  $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ , for which  $\lambda = (3 + \sqrt{5})/2$ , is a nice illustration of this situation (see [5]). These maps, although very simple, capture many properties of elements in an open subset of the set of diffeomorphisms of the torus  $T^2$ : this is the famous Anosov [1] result on structural stability. It states that any diffeomorphism  $\phi$  sufficiently close to an hyperbolic diffeomorphism  $\psi$  in the  $C^1$  topology is topologically conjugated to  $\psi$ : there exists a homeomorphism  $h \in \text{Homeo}(T^2)$  such that  $\phi = h \circ \psi \circ h^{-1}$ . Hence,  $\phi$  and  $\psi$  are the same up to a change of coordinates.

Thus, these Anosov diffeomorphisms provide important information on large open subsets of the group  $\text{Diff}^+(T^2)$ . Their hyperbolic counterparts have occupied mathematicians since then: they are the main actors of  $\text{Diff}^+(S_g)$ , the group of diffeomorphisms of a genus  $g$  surface  $S_g$ .

These Anosov diffeomorphisms are so important that they are also the actors of another family of groups: the modular

groups. In the 1970s, Thurston [8] generalised the analysis carried out on a torus to the case of compact surfaces, thus extending the notion of Anosov maps to that of pseudo-Anosov maps.

## 2 Foliations and pseudo-Anosov homeomorphisms

Measured foliation

An important feature of a linear Anosov of the torus is that it leaves invariant the two foliations  $\mathcal{F}^u$  and  $\mathcal{F}^s$  of “straight lines” of constant slopes (parallel to the directions of the eigenvectors associated to  $\lambda$  and  $\lambda^{-1}$ ). These foliations also come with an additional structure: they are integrable in the sense that we can define them globally as the kernel of a closed 1-form  $dv$ .

Hence, we have a measure  $\mu_s$  defined on arcs  $\alpha$  transverse to the leaves of  $\mathcal{F}^s$ , measuring the total variation of  $\alpha$  in the orthogonal direction:  $\mu_s(\alpha) = \int_{\alpha} dv_s$ .

The measure is invariant in the sense that if we change the extremities of  $\alpha$  in the same leaf, the measure remains unchanged. The data  $(\mathcal{F}^s, \mu_s)$  is a *measured foliation*. Of course, our Anosov preserves these leaves and expands/contracts the measures: we can think that  $\psi$  expands by a factor  $\lambda$  in the direction of  $\mathcal{F}^u$  and contracts by the same factor in the direction of  $\mathcal{F}^s$ .

On a surface of higher genus, the notion of measured foliations also exists but the Gauß–Bonnet formula forces us to extend them to singular foliations. For pairs of transverse measured foliations there is a very elegant way of doing this with the help of half-translation structures.

If  $\Sigma \subset S_g$  is a finite set, a half-translation structure on  $(S_g, \Sigma)$  is an atlas of charts  $\omega = (U_{\alpha}, z_{\alpha})$  of  $S \setminus \Sigma$  for which the changes of charts are of the form  $z \mapsto \pm z + \text{const}$  and such that each point of  $\Sigma$  has a neighbourhood isometric to a finite cover of  $\mathbb{R}^2 \setminus \{0\}$ . The pullback of the horizontal and vertical leaves of  $\mathbb{R}^2$  thus defines a pair of transverse measured foliations on  $S_g$  (the measures are  $dy$  and  $dx$  respectively).

**Example 2.1.** Figure 1 represents, on the left, a half-translation structure on the surface  $S_2$ : we glue together the sides with the same labels. We can verify that the vertices of the  $L$  shaped polygon represent a single point in  $S_2$ , which is singular. It has two obvious measured foliations (horizontal and vertical) with transverse measures  $dy$  and  $dx$  respectively.

Warning! There are measured foliations that do *not* arise from this construction (and so do not admit a transverse measured foliation). In the following example (following Hubbard–Masur), we glue two cylinders, foliated by circles, according to Figure 2: the boundaries of the first cylinder are the arcs  $\gamma_1, \gamma_2$  and  $\gamma_1, \gamma_3, \gamma_4, \gamma_6$  and those of the second cylinder are  $\gamma_5, \gamma_6$  and  $\gamma_2, \gamma_3, \gamma_4, \gamma_5$ . The transverse measure

<sup>1</sup> See the article by A. Bufetov and A. Klimenko in the *Gazette des Mathématiciens* (No. 143, January 2015).

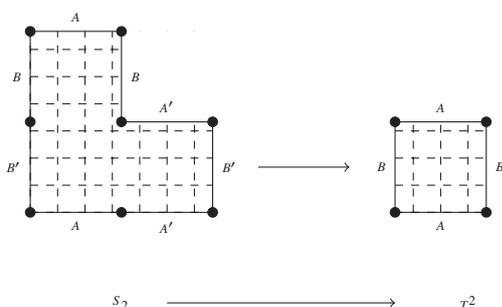


Figure 1. Triple cover of the standard torus: surface with three tiles

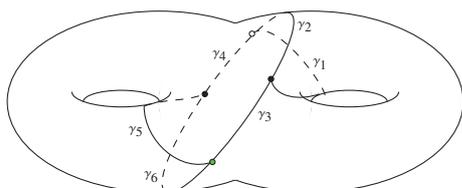


Figure 2. Measured foliation on a surface of genus two with four singularities (according to Hubbard–Masur)

is given by the “height function”. We can observe that a transverse foliation does not exist, otherwise the cylinders would have boundaries with equal lengths. This does not occur since the linear system

$$\begin{cases} |\gamma_1| + |\gamma_2| = |\gamma_1| + |\gamma_3| + |\gamma_4| + |\gamma_6| \\ |\gamma_5| + |\gamma_6| = |\gamma_2| + |\gamma_3| + |\gamma_4| + |\gamma_5| \end{cases}$$

does not admit any strictly positive solutions.

#### Pseudo-Anosov homeomorphisms

A homeomorphism  $\psi : S \rightarrow S$  is a pseudo-Anosov homeomorphism if there exist a pair of measured transverse foliations  $(\mathcal{F}^u, \mu_u)$  and  $(\mathcal{F}^s, \mu_s)$  on  $S_g$ , called unstable and stable respectively, and a number  $\lambda > 1$  (the expansion factor of  $\psi$ ) such that

$$\begin{aligned} \psi \cdot (\mathcal{F}^u, \mu_u) &= (\mathcal{F}^u, \lambda \cdot \mu_u), & \text{and} \\ \psi \cdot (\mathcal{F}^s, \mu_s) &= (\mathcal{F}^s, \lambda^{-1} \cdot \mu_s). \end{aligned}$$

An equivalent way to formulate this is to say that  $\psi$  is an affine diffeomorphism on  $S \setminus \Sigma$  for the Euclidian metric defined above and that its differential  $D\psi = \begin{pmatrix} \pm\lambda & 0 \\ 0 & \pm\lambda^{-1} \end{pmatrix}$  is hyperbolic, that is,  $|\text{tr}(D\psi)| > 2$  (in general,  $\psi$  is not differentiable at the points of  $\Sigma$ ). The group formed by all differentials  $D\psi$  with  $\psi$  affine for the atlas  $\omega$  is called the *Veech group*  $\text{SL}(S, \omega) \subset \text{PSL}(2, \mathbb{R})$ .

Although rather natural, it is not an easy task to construct examples satisfying this definition (at least in genus different from 1). A way of achieving it is to lift linear Anosov maps on the torus to coverings.

**Example 2.2.** The linear Anosov on the torus  $\psi : T^2 \rightarrow T^2$ , with differential  $A = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ , lifts (see Example 2.1) to a pseudo-Anosov  $\tilde{\psi} : S_2 \rightarrow S_2$  such that  $D\tilde{\psi} = A$ , as will be explained in Section 4.

### 3 Modular group

The pseudo-Anosov homeomorphisms are the elementary building blocks for the study of modular groups of surfaces. The group in question is always  $\text{Diff}^+(S_g)$  but this time up to continuous deformation (we shall say up to isotopy). More precisely, the modular group is the quotient group  $\text{Diff}^+(S_g)$  by the group  $\text{Diff}(S_g)_0$  of diffeomorphisms isotopic to the identity:  $\text{Mod}(S_g) = \text{Diff}^+(S_g)/\text{Diff}(S_g)_0$ .

Sometimes, definitions differ from one source to another, e.g., group of diffeomorphisms or group of homeomorphisms. It does not matter: the quotient groups are all isomorphic (even if the groups  $\text{Diff}^+(S_g)$  and  $\text{Homeo}^+(S_g)$  are very different!).

#### Nielsen–Thurston classification

We are now able to state the classification theorem of surface homeomorphisms, which is very close to the one on the torus. Any  $f \in \text{Homeo}^+(S_g)$  is, up to isotopy, either:

- (1) Periodic: there exists  $m$  such that  $f^m = \text{Id}$ .
- (2) Reducible:  $f$  preserves a family of simple closed curves.
- (3) A pseudo-Anosov map.

In the second case, some iterate of  $f$  preserves a subsurface (with boundaries). As we can again apply the theorem to this subsurface, the third case is by far the most interesting one!

#### Classical modular groups

The modular group of the closed disc is rather simple to describe (here our surface has a boundary: we require the homeomorphism to be the identity map on the boundary).

Such a map  $\phi$  defined on  $\overline{D(0, 1)}$  can easily be deformed by an isotopy acting like  $\phi$  on the small disc of radius  $t < 1$  and being the identity outside. In coordinates, this is

$$F(z, t) = \begin{cases} t\phi(z/t), & \text{if } z \in D(0, t) \text{ and } t \neq 0 \\ z, & \text{otherwise.} \end{cases}$$

We have  $F(\cdot, 0) = \text{Id}$  and  $F(\cdot, 1) = \psi$ . With this idea we easily prove that the modular groups of the disc and of the sphere are trivial.

Although somewhat simplistic, this approach is fundamental: Magnus remarked in 1934 that the action of isotopies on the punctures allows the connection of two *a priori* distinct groups: the modular group on the disc with  $n$  punctures and the braid group on  $n$  strands.

The first nontrivial example of a modular group is the one of the flat cylinder  $C$ . If  $\gamma$  is an oriented, simple closed curve linking the two components of the boundary of  $C$  then the homeomorphism  $T_\gamma$  that twists the cylinder along  $\gamma$  is nontrivial in  $\text{Mod}(C) = \langle T_\gamma \rangle \simeq \mathbb{Z}$ . The homeomorphism  $T_\gamma$  has a very simple expression in the parametrisation  $C = \mathbb{R}/w\mathbb{Z} \times [0; h]$ :

$$T_\gamma(x, y) = (x + w/h \cdot y, y) = (x + \mu^{-1}y, y),$$

where  $\mu = h/w$  is the modulus of the cylinder  $C$ . It is actually a diffeomorphism and  $DT_\gamma = \begin{pmatrix} 1 & \mu^{-1} \\ 0 & 1 \end{pmatrix}$ .

Furthermore, since any surface  $S_g$  contains an annulus  $C$ , we can define by analogy  $T_\gamma \in \text{Mod}(S_g)$  along a simple closed curve  $\gamma$  (since  $T_\gamma$  is the identity on the boundary of the annulus). These elements take an important place in the study of the modular group: we call them *Dehn twists*.

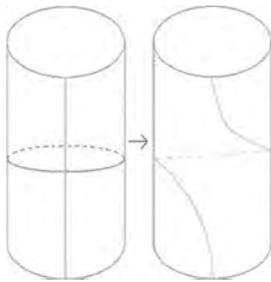


Figure 3. Dehn twist along a curve

Modular group of the torus

Writing  $T^2 = \mathbb{R}/\mathbb{Z} \times \mathbb{R}/\mathbb{Z}$ , we can define two Dehn twists along the two curves  $\alpha = (1, 0)$  and  $\beta = (0, 1)$ : this provides a “large” subgroup of  $\text{Mod}(T^2)$ :  $\langle T_\alpha, T_\beta \rangle = \langle \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \rangle = \text{SL}(2, \mathbb{Z})$  (we identify here a Dehn twist with its differential).

In fact, by letting a homeomorphism of  $T^2$  act on the homology  $H_1(T^2, \mathbb{Z}) = \langle \alpha, \beta \rangle \simeq \mathbb{Z}^2$ , we obtain an isomorphism

$$\text{Mod}(T^2) \simeq \text{SL}(2, \mathbb{Z}) = \text{Aut}(\mathbb{Z}^2)$$

that provides us with a rather precise description of the modular group of genus one surfaces.

Modular group of a surface

Just as we understand  $\text{Mod}(T^2)$  with the help of action on curves, we can study  $\text{Mod}(S_g)$  through the action of  $\text{Diff}^+(S_g)$  on simple closed curves of  $S_g$ . This time it is more complicated than it seems because such a curve can be extremely complicated.

By letting the homeomorphisms act on the homology  $H_1(S_g, \mathbb{Z})$ , we obtain a first “linear” approach of the modular group (choosing a symplectic basis for the intersection form):

$$\text{Mod}(S_g) \rightarrow \text{Sp}(2g, \mathbb{Z}).$$

This homeomorphism is onto (in fact, every element of  $\text{Sp}(2g, \mathbb{Z})$  can be realised by a pseudo-Anosov map, even if we do not always know how to characterise those that fix an orientable measured foliation). On the other hand, if  $g \geq 2$ , its kernel (the Torelli group) is rather large.

We end this section with a result analogous to the well known fact that  $\text{SL}(n, \mathbb{Z})$  is generated by transvection matrices.

The group  $\text{Mod}(S_g)$  is generated by a finite number of Dehn twists (Dehn, 1922).

The (optimal) number of generators is  $2g + 1$  (Humphries, 1977).

#### 4 Several constructions

It is not an easy task to construct pseudo-Anosov homeomorphisms.

Let us give a simple and fruitful idea. An affine Dehn twist  $T_\gamma$  possesses a parabolic differential,  $|\text{tr}(DT_\gamma)| = 2$ . By applying the motto

“a product of parabolic elements is ‘generally’ an hyperbolic element”,

it is possible to show, for well chosen curves  $\gamma$  and  $\eta$ , that  $|\text{tr}(DT_\gamma T_\eta)| > 2$ , that is,  $T_\gamma \circ T_\eta$  is pseudo-Anosov. This is

the Thurston–Veech construction, popularised on the occasion of a talk by John Hubbard at C.I.R.M. in Marseille in 2003. Since then, this construction has sometimes been called the *bouillabaisse* construction.

**Example 4.1.** In Example 2.1, the left surface  $S_2$  is horizontally cut along two cylinders of height 1 with cores  $\alpha_1, \alpha_2$  of length 1 and 2. Thus,  $DT_{\alpha_1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  and  $DT_{\alpha_2} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ . Since each Dehn twist  $T_{\alpha_i}$  is equal to the identity on the boundaries of the cylinders, the “multi-twist”  $T_h = T_{\alpha_1}^2 \circ T_{\alpha_2}$  is a diffeomorphism on  $S_2 \setminus \Sigma$  whose differential is constant and equal to  $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ . By symmetry reasons, the vertical multi-twist  $T_v = T_{\beta_1} \circ T_{\beta_2}^2$  is also affine and has a differential equal to  $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ .

We then check that  $D(T_h \circ T_v) = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$ . This is our Example 2.2, which is pseudo-Anosov!

**Example 4.2** (A more subtle example). Let us consider on a genus 2 surface the multi-curves  $\alpha = \{2a_1, a_2, c_1\}$  and  $\beta = \{b_1, b_2\}$  (represented in Figure 4). The product of the two multi-twists  $T_\alpha \circ T_\beta$ , where

$$T_\alpha = T_{a_1}^2 \circ T_{a_2} \circ T_{c_1} \quad \text{and} \quad T_\beta = T_{b_1} \circ T_{b_2},$$

is an element  $\psi$  of pseudo-Anosov type. Its expansion factor  $\lambda(\psi)$  is the largest real root ( $\approx 1.72$ ) of the polynomial  $X^4 - X^3 - X^2 - X + 1$ .

This idea produces a lot of pseudo-Anosov diffeomorphisms. A beautiful theorem of A. Fathi gives a quantitative version of this motto. Let us consider a family of distinct curves (up to isotopy)  $\{\gamma_1, \dots, \gamma_n\}$  filling  $S$  ( $S \setminus \cup_i \gamma_i$  is a union of discs). Then,

$\exists N \in \mathbb{N}, \forall (n_1, \dots, n_k) \in \mathbb{Z}^k$  : if  $|n_i| \geq N, \forall i$ , then

$T_{\gamma_1} \circ \dots \circ T_{\gamma_k}$  is isotopic to a pseudo-Anosov map.

A surprising corollary is that if  $\psi$  is a pseudo-Anosov and  $\gamma$  is a simple closed curve then  $T_\gamma^n \circ \psi$  is isotopic to a pseudo-Anosov for any non-negative integer  $n$ , with the possible exception of at most seven consecutive values of  $n$ !

There are other constructions that we do not have time to explain, which are algorithmic and, in certain cases, allow us to describe all the pseudo-Anosov maps. Here are a few of them:

- (1) Train track induction.
- (2) The Rauzy–Veech induction.
- (3) Sections of flows on hyperbolic 3-manifolds.

The first induction has been extensively studied by Papadopoulos and Penner.

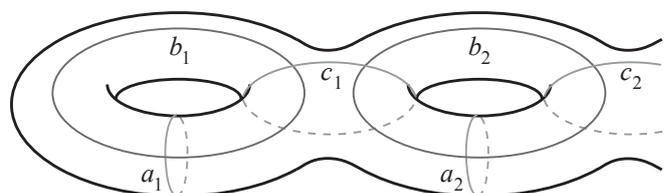


Figure 4. Bouillabaisse construction

## 5 Abundance

We are tempted to say that most of the elements of  $\text{Mod}(S_g)$  are of pseudo-Anosov type. This intuition arises from what happens in genus 1: if we choose a “random” matrix in  $\text{Mod}(S_1) = \text{SL}(2, \mathbb{Z})$ , it has a strong probability of being hyperbolic (the absolute value of its trace is larger than 2). However, we need to precisely formulate the word “random” since all these groups are discrete groups.

A reasonable way to define this is to fix a set of generators of  $\text{Mod}(S_g)$  (for instance, the Dehn twists) and to look at bounded length words (or a ball of radius  $N$  centred at the identity in the Cayley graph).

For some modular groups, and some generating sets, we can show that the proportion of pseudo-Anosov elements in the ball of radius  $N$  tends exponentially fast to 1 as  $N$  tends to infinity (see the work by Caruso-Wiest). There are also versions of this result using the tool of random walks.

## 6 Counting

Another way to show the abundance of pseudo-Anosov diffeomorphisms is to count them. Let us introduce

$$\mathcal{G}_g(T) = \{\text{conjugation classes of } \psi \mid \psi \text{ is pseudo-Anosov and } \log(\lambda(\psi)) < T\}.$$

Veech was the first to study the asymptotic behaviour of  $|\mathcal{G}_g(T)|$  as  $T$  tends to infinity. His work, starting in 1986, eventually culminated in the Eskin–Mirzakhani formula:

$$|\mathcal{G}_g(T)| \sim_{T \rightarrow \infty} \frac{e^{(6g-6)T}}{(6g-6)T}.$$

This formula was generalised later by Eskin–Mirzakhani–Rafi and Hamenstädt. The dynamical techniques that were employed used properties of the geodesic flow on the moduli space  $\mathcal{M}_g$ , inspired by the work of Margulis.

The key point is to make a parallel between the conjugacy class of  $\psi$  and a closed curve on  $\mathcal{M}_g$ , the number  $\log(\lambda(\psi))$  then being the length of this curve for some metric (the Teichmüller metric).

## 7 Expansion factors

Surprisingly, we do not know much about the expansion factors of pseudo-Anosov homeomorphisms.

Realisations of algebraic numbers as expansion factors  
Looking at the action on homology (for a suitable cover), we easily deduce that  $\lambda$  is an eigenvalue of a matrix with integer entries. It is thus an algebraic number (that is, the root of an irreducible polynomial  $P \in \mathbb{Q}[X]$ ) of degree bounded by  $3g - 3$ . In fact, Thurston has shown that it is a bi-Perron number:

$$\forall \alpha \neq \lambda, \lambda^{-1}, P(\alpha) = 0 \implies \lambda^{-1} < \alpha < \lambda.$$

The converse (that is, if a bi-Perron number is an expansion factor) is an open problem. This is the subject of one of the last manuscripts of Thurston [9].

### Minimisation

There are plenty of conjectures on this topic. The easiest ones to state are often about  $\lambda$ . For a fixed  $g$ , an easy argument that

relates roots and coefficients shows that the set

$\text{Spec}_g = \{\lambda(\psi), \psi : S_g \rightarrow S_g \text{ is pseudo-Anosov}\} \subset \mathbb{R}$   
is a discrete subset. What is its smallest element

$$\delta_g = \min(\text{Spec}_g)?$$

This is also an open problem! We know that  $\delta_1 = \frac{3+\sqrt{5}}{2}$  and  $\delta_2 =$  the largest root of  $X^4 - X^3 - X^2 - X + 1 \simeq 1.72$  (compare with Example 4.2) but computing  $\delta_3$  is already an open problem. It is not difficult to get an upper bound for  $\delta_g$  (finding an example is sufficient). It is a little more subtle to get a lower bound. For all  $g \geq 2$ :

$$\frac{\log(2)}{6} \leq |\chi(S_g)| \cdot \log(\delta_g) \leq 2 \cdot \log\left(\frac{3 + \sqrt{5}}{2}\right), \quad (1)$$

where  $\chi(S_g) = 2 - 2g$ . We easily deduce that

$$\limsup_{g \rightarrow \infty} g \log(\delta_g) \leq \log\left(\frac{3 + \sqrt{5}}{2}\right).$$

McMullen conjectured that  $(g \log(\delta_g))_g$  converges but so far there is no proof of this. For a positive answer, one needs a better lower bound (on  $g \log(\delta_g)$ ) than (1).

We present a recent result on matrices in this direction that was, surprisingly, not known before. McMullen [6] has shown that, for all  $g \geq 1$ , the smallest possible value of the spectral radius  $\rho(A)$  of a primitive matrix  $A \in \text{Sp}_{2g}(\mathbb{Z})$  (that is, one for which there exists  $n$  such that all entries of  $A^n$  are strictly positive) is given by the largest root of the polynomial

$$X^{2g} - X^g(1 + X + X^{-1}) + 1.$$

In particular,  $\rho(A)^g \geq \frac{3+\sqrt{5}}{2}$ . Even if this problem is closely related to the previous one, it does not (yet) provide a positive solution to the problem. . .

The discussions in the previous sections evoke a connection between these problems (of geometric nature) and the problem of minimising the eigenvalues of a matrix (of algebraic nature).

### Eigendirections of pseudo-Anosov homeomorphisms

All the questions above are about eigenvalues of matrices (the expansion factor  $\lambda$ ). What about the eigendirections associated to the eigenvectors? This is a very short section since we know almost nothing about it! It seems very difficult to characterise these directions at the moment, even if there are some partial results for genus 2 surfaces and Prym surfaces.

## 8 Lonely guy conjecture

If we choose a “random” flat metric  $\omega$  on a surface  $S_g$  (with respect to some probability measure on the moduli spaces), what kind of group of symmetries  $\text{SL}(S_g, \omega)$  could we expect? The answer that we guess is the trivial group. This is indeed the case (except perhaps if the surface has obvious nontrivial symmetry such as the hyperelliptic involution).

And now, what if we again choose a “random” flat metric  $\omega$  among surfaces already having a symmetry? Again, the answer we expect is that generically the Veech group is cyclic. Surprisingly, this is not the case if the genus of  $S_g$  is two! McMullen has given a quantitative version of this: the group  $\text{SL}(S_g, \omega)$  is very large. Its limit set is the full circle at infinity.

What about when the genus  $g$  is larger than three? This question is widely open. We conjecture that in general the group is (virtually) cyclic ...

## 9 Suspensions and volumes

There is a remarkable connection between the dynamics of pseudo-Anosov homeomorphisms in dimension two and the geometry in dimension three. The relation is given by the (very general) construction of suspension. To each  $f : S_g \rightarrow S_g$  we associate the 3 dimensional object

$$M_f = S_g \times [0, 1]/(1, x) \sim (0, f(x)).$$

Another famous theorem of Thurston states that  $f = \psi$  is pseudo-Anosov if and only if  $M_\psi$  is an hyperbolic 3-manifold. Thus it has a volume, although it is very hard to express it in terms of  $\psi$ . Kojima and McShane have recently established this beautiful inequality relating dynamic and geometric complexities:

$$\log(\lambda(\psi)) \geq \frac{1}{3\pi|\chi(S_g)|} \text{vol}(M_\psi),$$

where  $\chi(S_g) = 2 - 2g$ .

## 10 To learn more about pseudo-Anosov maps

The book by Fathi-Laudenbach-Poenaru [3] is a very good introduction to the topic, containing numerous details. It is based on the work of Thurston [8] on surface homeomorphisms. This book is also available in English.

The book by Farb–Margalit [4] is a more modern introduction to the modular group. It contains all the prerequisites and details for its study.

If one wants to learn more about pseudo-Anosov maps, the literature is rather vast. The recent works by Agol, Hironaka, Leininger and Margalit provide a nice “state of the art” and propose new approaches to the different problems alluded to above.

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Erwan Laneeau is Professor of Mathematics at Grenoble University, France. He has been a researcher working at Institut Fourier, France, since 2012. His main research interests are in geometry, topology, dynamical systems and moduli spaces.

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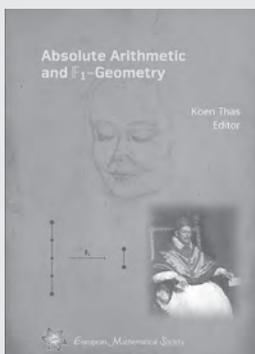
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### Absolute Arithmetic and $\mathbb{F}_1$ -Geometry

Koen Thas (University of Gent, Belgium), Editor

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It has been known for some time that geometries over finite fields, their automorphism groups and certain counting formulae involving these geometries have interesting guises when one lets the size of the field go to 1. On the other hand, the nonexistent field with one element,  $\mathbb{F}_1$ , presents itself as a ghost candidate for an absolute basis in Algebraic Geometry to perform the Deninger–Manin program, which aims at solving the classical Riemann Hypothesis. This book, which is the first of its kind in the  $\mathbb{F}_1$ -world, covers several areas in  $\mathbb{F}_1$ -theory, and is divided into four main parts – Combinatorial Theory, Homological Algebra, Algebraic Geometry and Absolute Arithmetic. Topics treated include the combinatorial theory and geometry behind  $\mathbb{F}_1$ , categorical foundations, the blend of different scheme theories over  $\mathbb{F}_1$  which are presently available, motives and zeta functions, the Habiro

topology, Witt vectors and total positivity, moduli operads, and at the end, even some arithmetic. Each chapter is carefully written by experts, and besides elaborating on known results, brand new results, open problems and conjectures are also met along the way. The diversity of the contents, together with the mystery surrounding the field with one element, should attract any mathematician, regardless of speciality.

# The Art of Counting – Interview with Mireille Bousquet-Mélou

Juanjo Rué (Universitat Politècnica de Catalunya, Spain, and Barcelona Graduate School of Mathematics, Spain)

*Mireille Bousquet-Mélou received the Distinguished Speaker Award in 2017 from the European Mathematical Society. She delivered her talk entitled ‘Functional equations in enumerative combinatorics’ at the Foundations of Computational Mathematics conference in Barcelona, Spain. This interview took place in one of the cloisters at Universitat de Barcelona on 15 July 2017.*

**Dear Mireille, welcome to Barcelona. We are grateful that you could come to the ‘Foundations of Computational Mathematics’ conference in order to talk about combinatorics. We would like to ask you some questions, starting with your mathematical career path. Where do you come from? And as a child, did you have any mathematical reference in your family?**

I was born in 1967 in Albi, a town in the south-west of France, not too far from Toulouse, but I grew up in Pau, where we moved with my family when I was three years old. My parents were both high school teachers, teaching history and geography, and there was no mathematician in the family. I have an elder sister too, who is also a teacher, in design and fine arts. Our tastes were different and that was good because each of us had our own space in the family. Who knows, maybe if she had liked mathematics, I would be good at drawing.

**So, do you have memories about your first contact with mathematics?**

There are a couple of them: in France we have *écoles maternelles* rather than kindergartens, where children are assumed to learn things as in school and not only through play. I entered an *école maternelle* in 1970 when the Bourbaki group was having a huge influence on French society, in particular in school (an influence that the group had probably not sought). So, in *école maternelle* we learnt... set theory. I do not remember this in detail but my parents kept my notebook from my last year at *école maternelle*. There you can see pictures of a rabbit, a cat, a banana... and we had to build the set of animals and give its cardinality. So, my first encounter with mathematics was maybe with this baby version of set theory!

After this, in the first year at primary school, I remember that we learnt to count simultaneously in all bases, not just in base 10 (this was probably again due to Bourbaki’s model in education). We used little cubes to count, arranged first by rows, then piled to form squares, which could then be piled into cubes...

**What was the first moment you got interested in mathematics?**



Mireille Bousquet-Mélou, Barcelona 2017. (Photo: Juanjo Rué)

It took some time: I was the type of good pupil at high school who was decently interested in many topics but not immensely fond of any. I think that my taste for mathematics grew in the last year of high school, where I had a rather peculiar course of mathematics. Our mathematics teacher did not lecture (I think it bored him) but he made us learn the material of the book at home. So the mathematics course at school was based on solving problems. However, the good pupils not only studied the book at home but also worked on the problems. This created a competitive environment where we learnt to work independently. I think this was very good for the good students and probably rather bad for the others. Anyway, I liked it: first working by myself and then discussing with the other pupils.

During high school, I did not have a clear plan of what I wanted to do next but, at that time in France, attending an engineering school sounded like a prestigious thing to do. The good students in my class wanted to become engineers so I used to say the same (even if I did not have any idea of what the job was like). Hence, after high school, I moved to a *classe préparatoire*<sup>1</sup> in Toulouse where I studied mathematics and physics. There I learnt about the *École Normale Supérieure* (ENS). But I heard that students there would become *professeurs*. In France we use the same word (*professeurs*) for university professors and high school teachers. My parents were high school teachers, hence *professeurs*, and (of course) I wanted to do something different, so at the beginning I was not considering taking the entrance examination for the ENS.

<sup>1</sup> French system parallel to university that provides training to enter the so-called *grandes écoles*, most of them being engineer schools.

***But you studied at the École Normale Supérieure anyway. How did you change your mind?***

In the second year of my *classes préparatoires*, some former students who were studying at the ENS came to Toulouse to advertise what they were doing. They talked about research and the atmosphere they described seemed very exciting... and they even had a climbing club! This was clearly irresistible. So they convinced me to take the entrance examination and I was finally selected to go there.

***You moved to Paris to start your studies at the ENS. How was the ENS at the time?***

There were 45 new students entering the school in mathematics and I was the only woman. In fact, this was the first year that the ENS was a mixed school. The head of the school was of course not happy with this proportion (previously there had been 15 positions for women and 30 positions for men).

We could study mathematics of course but we could also move to computer science or physics (and we were somewhat encouraged to do so). But the professors there were keen to keep one woman in mathematics and I went on with mathematics. This was probably the best choice for me, since I realised later that physics is a different world, in which I think I would have felt less comfortable.

I enjoyed my time at the ENS a lot. There were a huge variety of people in (let us say) an educated atmosphere with diverse interests. Indeed, students specialised not only in science but also history, languages and sociology, among other topics. Even if you did not interact much with them, one could feel a different atmosphere compared to the one in the *classes préparatoires* or in engineer schools. Many students had (sometimes unusual) hobbies, ranging from contemporary music to spending nights in the Paris catacombs. It looked a bit as if everyone had to be special in some way (and had to show it). I did not feel such a need. Being a woman in mathematics was probably special enough.

***Who got you into research at the ENS? What were your favourite topics there?***

Research is the “normal” direction to take at the ENS. In the first year, we had to follow a number of courses and one of them was taught by Xavier Viennot on enumerative combinatorics. The type of mathematics he was presenting was very different from the material covered by the other professors. His style was very different too. He was teaching with plastic slides (not on the blackboard), putting six slides on top of each other and proving identities on functions by showing bijections between the objects that these functions counted. It was very attractive. I also took a couple of courses in probability theory by Marc Yor. They were not easy but I liked them a lot. I also remember a great course on complex analysis by Joseph Oesterle. I still use it!

In my second year, Xavier Viennot went abroad and I specialised with a master in probability theory and statistics. However, in the third year (when we were to choose a PhD topic), Xavier Viennot came back to France. He

was living near Bordeaux but coming to Paris once a week to teach. I hesitated between probability and combinatorics. I had seen more of the first topic and only a small piece of combinatorics but this topic was still very tempting. So, in the end, it was combinatorics and I must say that I never had any regrets about this choice (even though I am delighted to be in touch and interacting with probabilists in my work).

***After defending your PhD, you moved with a CNRS position to Bordeaux, at LaBRI (Laboratoire Bordelais de Recherche en Informatique). Can you describe the department when you moved to Bordeaux?***

In France (and other places), combinatorics is at the border between mathematics and computer science. *Laboratoire* in French is similar to department, or institute, so LaBRI is, in fact, the Department of Computer Science at the University of Bordeaux. I was not there from the very beginning but I know that combinatorics played an important role in the early days of LaBRI, together with other aspects of theoretical computer science. The first combinatorialist there was Robert Cori; Xavier Viennot joined LaBRI a bit later. Both of them were former students of Marcel-Paul Schützenberger, who is the “father” (and grandfather and great-grandfather) of many combinatorialists in France. Robert Cori and Xavier Viennot were influential and attracted many people to combinatorics.

Of course, computer science has evolved very rapidly over the years: nowadays, our department combines a larger variety of topics, including parallel computation, image processing, bioinformatics and robotics. But we still have two strong groups in theoretical computer science.

***Can you explain, in general terms, what your research area is and what its connection is with other areas of science?***

I work in a field of discrete mathematics (or is it theoretical computer science...) called *enumerative combinatorics*. Most of the questions that we study start like this: given a set of discrete objects, equipped with a notion of size (say permutations on  $n$  elements), how many objects of size  $n$  are there? Of course you do not want a number for particular values of  $n$  but a formula or, more realistically, a characterisation (e.g. a recurrence relation) valid for general  $n$ . The objects that we (try to) count come from various branches of mathematics, including probability (of course the interaction with this area is particularly strong via discrete probability), algebra (e.g. in connection with representations of classical groups and algebras) and mathematical physics (via the study of discrete models, like the famous Ising model).

Most French combinatorialists work in computer science departments. There are several reasons for that, partly historical but mostly scientific: there is no real boundary between some parts of theoretical computer science (e.g. the study of formal languages) and discrete mathematics. There is also a strong interaction between enumerative combinatorics and the study of the com-

plexity of algorithms, as launched a long time ago by Don Knuth and pursued in France by Philippe Flajolet and his school. The rough idea is that in order to understand the complexity of an algorithm, one has to determine how many entries of a given length get processed in a given time – a well-posed bivariate counting problem.

Let me get back to the nature of enumerative combinatorics: sometimes, more important than getting a counting formula for a certain problem is the fact that to arrive at such a formula requires information about the combinatorial structure under study. Hence, counting is sometimes just a pretext and the important thing is to understand, or discover, a structure in some discrete objects. From this point of view, enumerative combinatorics does not differ much from other branches of mathematics.

***How do you decide which combinatorial objects you want to study and which problems you choose to investigate?***

This is a difficult question and, to my shame, I must say that for many years I did not really think about it. Some topics would come along that I liked and that was it. Now, I try to be (sometimes) more selective: there are the problems that you think that you'll be able to solve and, as you get older, you see more of them. But if you do not go further you will not surprise yourself. Surprising oneself by solving something more challenging is more fun... but also more risky. A certain balance is required.

***Can you explain something about your particular favourite combinatorial objects?***

I like them all... Maybe one of my early favourites was the so-called *lecture hall partitions*.<sup>2</sup> In the 1990s, a young Swedish colleague, Kimmo Eriksson, introduced them to me and we discovered surprising identities that we published in three papers. Then I did not work on this topic anymore but it has had ramifications and is still active. Recently, I saw a paper on them that did not even cite the papers with Kimmo: isn't this the sure sign that 'your' objects have grown up?

I like very much *planar maps*, which have recently become a popular topic in probability theory too. Another of my favourites (and not only mine!) is *self-avoiding walks* and our joint results with Nick Beaton, Jan de Gier, Hugo Duminil-Copin and Tony Guttmann. As you know, in 2010, there was this groundbreaking work of Duminil-Copin and Smirnov about the growth constant of self-avoiding walks in the honeycomb lattice, which solved a conjecture from Nienhuis stating that this value was  $\sqrt{2+\sqrt{2}}$ . This was a simple, short and inventive proof. Many people from our community spent time trying to figure out what results could be proved with these new ideas. Tony Guttmann suggested looking at a conjecture dealing with walks in a half plane interacting with the boundary. Finally, after some difficulties, we proved it. I was very glad to be involved in that work.

<sup>2</sup> A sequence of  $n$  integers  $(\lambda_1, \lambda_2, \dots, \lambda_n)$  is a *lecture hall partition* if  $0 \leq \lambda_1 \leq \lambda_2/2 \leq \dots \leq \lambda_n/n$ .



Historical building of Universitat de Barcelona, where FOCM took place. (Image courtesy of Universitat de Barcelona)

***Among your results, which one do you think is your favourite discovery?***

You can like your results for different reasons. For instance, I enjoyed this result about self-avoiding walks because I was doing something a bit different from my usual business, and of course because it is a difficult topic (though the questions are simple to state). I also like my early results about lecture hall partitions. Another topic that I am happy with is the enumeration of lattice walks confined in a cone. With Marni Mishna, we were maybe the first combinatorialists to consider the problem in a unified way, aiming at a complete classification. In the last 10 years, this topic has attracted people from different areas, ranging from computer algebra to differential Galois theory, via, of course, probability theory. In fact, several of these colleagues are here at FOCM. It is always fun when people away from your area get interested in 'your' questions.

***What do you find most rewarding in mathematical research?***

This is, of course, a matter of taste. There are some good things that you can rely on – for instance, I like giving talks. In general, I like writing papers and especially the time spent thinking about how to expose things. In the research itself, what I really like are those very few moments when you start feeling that you are about to understand something (sometimes without even knowing what this something is). You can feel that your ideas are evolving, that your brain is the right place for some alchemy to happen. This has happened to me a few times (at my modest scale) and it is definitely a remarkable feeling.

Of course, once you arrive at a certain result, you very often think: 'How is it that I did not realise this before; this is so simple!' But you still feel happy about having understood something, at last.

***You were an invited speaker at ICM06 (Madrid) in the combinatorics section. Can you explain to us a little about this experience?***

My first surprise was to be invited there. Then, being there was really good for me. I had never seen such a big crowd of mathematicians... This was very stimulating. The strange thing was that, being mostly by myself in

this big crowd, I did not feel isolated: I was moving from plenary lectures to sessions, in probability and combinatorics. I realised that many plenary lectures were very good and understandable. This was when Wendelin Werner and Andreï Okounkov got their Fields Medals. There were presentations of their work, with a strong discrete flavour. There was also a plenary lecture by Richard Stanley, a specialist of enumerative and algebraic combinatorics.

I remember that at some point I was so enthusiastic that I started a long list of topics that I would like to look at. This list is still on my desk, though a bit faded. I have ticked as “done” some of its items but many are left! More than 10 years later, I still have a look at it every now and then. To be frank, a second newer list is now stapled to it.

***Going back to the fact that you were the only woman in your year at the ENS: can you describe how your career path evolved related to the fact that you are a female mathematician?***

Frankly, I did not feel real obstacles due to being a woman. But this does not mean that there are none. What I mean is that if I had had serious difficulties, I would maybe have received less recognition and you would not be interviewing me at the moment. By definition, the women who get exposure are among the happy few...

What strikes me the most in the women/science question is what our societies teach to teenagers: from many sources they hear that science (and especially mathematics) is not for women. I may be pessimistic, and exaggerating a bit, but my impression is that advertisements, movies, etc., still spread the idea that women should take care about their clothes and make-up and that being smart is not a priority. In a sense, it is unfortunate that young people start choosing what they want to study at an age when it is soooooo important to obey gender rules.

Even in our community, there is still this idea that a mathematician is a male mathematician. I would recommend trying the Implicit Association Tests developed by Harvard University:<sup>3</sup> they are cleverly designed and you cannot really cheat them. In my case, I must say that they showed that I strongly correlate science and male gender... I do not know what it means nor whether it implies that my judgment is biased but the correlation is definitely there.

Fortunately, times are slowly changing: 15 years ago this problem was not as explicit as it is today. We are definitely more aware about these biases and we are trying to fight them.

***What would you recommend to a young woman in Europe wanting to start and develop a career in mathematics?***

Hold tight and do what you like! Times seem to be more favourable than before. For young women who are already inside academia, there is at least an effort in the mathematical community to fight bias. And to attract

<sup>3</sup> <https://implicit.harvard.edu/implicit/education.html>.



**Mireille Bousquet-Mélou during her talk at FOCM. July in Barcelona is quite hot and fans provided by the organisation were greatly appreciated by all conference attendees. (Photo: Juanjo Rué)**

more women, female scientists must probably get to talk more to young students at high school, as well as appearing on television and on the internet...

I am not an expert on these questions but I know that many universities explore how to help at various moments of a woman’s career, like after maternity leave (e.g. less teaching when returning and grants to invite collaborators). And of course we should all have the representation of women in mind when organising a conference or gathering together a committee.

***Just to finish, could you tell us the directions that should be followed by enumerative combinatorics in the near future?***

This is a difficult question. I would say that problems interacting with other areas (of mathematics, or even further) will play an important role in combinatorics. But maybe it would be more honest to say that this is the type of combinatorics that I like.

Here at FOCM, I attend talks at the combinatorics workshop of course but also the computer algebra workshop (ah, no; they say ‘symbolic analysis’ so it must be a bit different). I am by no means an expert but I use some computer algebraic tools, for instance to handle classes of formal power series (generating functions, in our combinatorial language). There are also some colleagues here from other areas of mathematics, like number theory, who ask different questions about the same classes of series. I like this mixture. I like to hear about these other perspectives, even if it is likely that I will never work with these colleagues. Well, after all, who knows?

***Thank you very much Mireille.***

Thank you



*Juanjo Rué is an associate professor at the Department of Mathematics of Universitat Politècnica de Catalunya and a faculty member at the Barcelona Graduate School of Mathematics. His research interests are enumerative combinatorics, random graphs, additive combinatorics and extremal combinatorics.*

# “Liberté aux professeurs associés!”

## Interview with Alexandre Aleksandrovich Kirillov

Alice Fialowski (Eötvös Loránd University, Budapest, Hungary), Yury Neretin (University of Vienna, Austria), Michael Pevzner (University of Reims Champagne-Ardenne, Reims, France) and Vladimir Salnikov (La Rochelle University, France)

*This interview took place at the international conference “Representation Theory at the Crossroads of Modern Mathematics”, held in Reims, 29 May–2 June 2017, in honour of Professor Alexandre Alexandrovitch Kirillov. During the conference, he was awarded the degree of Doctor Honoris Causa of the University of Reims Champagne-Ardenne. The conference was organised by A. Borodin (MIT Boston), A. Kirillov (University of New York in Stony Brook), S. Morier-Genoud (Paris 6), A. Okounkov (University of Columbia), V. Ovsienko (CNRS, Reims), M. Pevzner (University of Reims), N. Rozhkovskaya (Kansas State University), M. Schlichenmaier (Luxembourg) and R. Yu (University Reims) and was supported by the University of Reims Champagne-Ardenne, the National Science Foundation, the CNRS, two ANR projects (SC3A and ACORT) and the University of Luxembourg.*

***We are very glad to see you here in Reims at this very special conference dedicated to your 34th birthday. We would like to ask you a few questions. To start with, how did you choose to do mathematics?***

You mean, how did I choose to be a mathematician? Well, I think till the fifth grade at school,<sup>1</sup> my dream was to be a pilot. But then my eyesight was not very good; at the end of school I had  $-4$  and at university  $-5$ , so the career of a pilot was closed for me. Also, I like very much to dive but I’ve never thought of being a professional diver. I was a member of the university diving team and even won the “All-Union Student’s Game” [“Универсиада” – Universiade], which involved university teams from all over the Soviet Union competing in different forms of sports. That was a very interesting story but not for the official record.

Starting from the 6th grade, I participated in the mathematical Olympiads: my teacher at school said that there was such a thing in the 6th grade, I think. I did not go to the mathematical circles<sup>2</sup> because I was rather shy and in mathematical circles they make you answer ques-



Doctor Honoris Causa ceremony. (Photo Vladimir Salnikov)

tions. In the Olympiads, nobody asked me anything; I just had a problem and I needed to write a solution, that’s it. In the 6th grade, I received a small diploma “Грамота”. The official Olympiad started in the 7th year and it was joint with the 8th year; there, I received an honorary diploma [“Похвальная Грамота”]. In the 8th year, I got the 2nd prize and for the 9th and 10th years, I got the 1st prize. It was natural to go to the Mekhmat – Department of Mechanics and Mathematics, so I did.

***And your interest in mathematics arose due to the Olympiads or earlier?***

In school, I never had any trouble solving mathematical problems. And I realised, starting from the 6th grade or a bit earlier, that I knew mathematics better than my teacher... But I had a rather good teacher who also understood this [laughs], so we were friendly.

***Your teachers did not influence you?***

Not in school.

***Nor your parents?***

My parents were not in science. I was the first one. Well, my mother was a doctor but not from a dynasty. She was the first intellectual in her family. At the time, medicine was at Moscow State University but situated in a different place.

***But do you think Olympiads are important just to drive curiosity about mathematics or are they important to develop researchers?***

No, there were no ideas about research at all. It was more about getting interested in mathematics. You see, I do not believe that any scientist does something because

<sup>1</sup> Fifth year out of 10 (later 11) in the Russian educational system (roughly 12 years old).

<sup>2</sup> A longstanding tradition of mathematical clubs at various stages of school education in the Soviet Union (see, for example, the Malyi Mekhmat article “University Goes to School”, EMS Newsletter 101, September 2016).



Address by M. Pevzner during the ceremony. (Photo Vladimir Salmikov)

he thinks that it is useful for humanity. The only force that brings us to science is curiosity – natural human curiosity.

***Could you tell us something about Mekhmat during the time of your studies?***

Oh, I can say a lot of things about Mekhmat at the time. It was a great time. I arrived at the university in 1954 at the beginning of the Great Decade [“Великое десятилетие”] (the Khrushchev era), which started in 1954 and finished in 1964. And, you see, it is a psychological law that a human being estimates the universe not by an absolute value but by its derivative in time. If you live poorly but things are getting better every day, you are happy. If you live very well but your situation is slightly declining, you are unhappy. And that was the case for a long time, during which the situation in Russia was getting better and better (after the War, at which point it was so low that there was no way for it to get worse). Then, up to the 1960s, the situation got better and better.

It was the second year after Mekhmat had moved to a new building of the university. Before that, it was in the centre, in a very famous building (those who graduated before me liked that one better but for us it was the new one). We have never called it the “University”; we called it the “Temple” [“Храм”]. “Are you going tomorrow to the Temple of Science?” [“Храм Науки”] – it was official – the journalists invented this term. And, somewhat ironically... it really was a temple; it was a paradise. Inside, there was a very good dining room – it was pretty cheap, maybe sponsored by the State, and it was rather good. Later, it got worse and worse but at that time it was excellent. For example, there was a period of friendship with China and there was an “island” in the students’ cafeteria especially for Chinese students; the Chinese told me that they had never seen such a thing in China. They knew that it existed but it was affordable only for “big shots” and not for ordinary people. And the Chinese cuisine was of a very high standard and pretty cheap.

***And can you speak about a few professors, seminars or activities, maybe someone whom you appreciated?***

I think the greatest thing about Moscow University at the time was that there were a lot of good seminars. First of all, there were many seminars not only for professionals but for young mathematicians – for the youngest. Among them, there were two most popular seminars.

One was by Anatol Vitushkin, who was a student of A.S. Kronrod, a very well known educator. He more or less imitated the style and strategy of Kronrod and ran his seminar according to those rules. It was an arguable tactic because the idea was that you never had to read anything. If you wanted to know a subject, you had to be given several problems – key problems – and start to solve these problems by yourself. And that’s the only way to study mathematics – no books.

***As far as I know, he was also associated with Konstantinov?***

At that time, Konstantinov was still in the Department of Physics and he did not collaborate with Vitushkin until later.

And the second lecturer was Evgeniy Borisovitch Dynkin, also a known educator and a man with rather original ideas (but a very bright man and someone who was excited about teaching). He was our lecturer in analysis and completely overturned the ideas of how to explain analytic things to students. I even used some of his tricks when I taught mathematics in America but it was not very successful – they are quite different sorts of students.

So, those were two very good seminars for the youngest people – yesterday’s schoolchildren.

And, in my third year, I attended Gelfand’s seminar for the first time. Alik Berezin,<sup>3</sup> my late friend, took me to Gelfand’s seminar and that was also a very nice story (but it would also take time – I will speak about it later maybe).

At that time, the great seminars were Gelfand’s, Kolmogorov’s and Petrovskii’s. I don’t know about algebra seminars at the time; Shafarevich started a bit later... As for topology, certainly there was Pavel Sergeevitch Alexandrov but this was very specialised in set-theoretical topology. Modern topology came with Mikhail Mikhailovitch Postnikov, who was not officially a member of the faculty; he ran a seminar on modern topology and S.P. Novikov was a student of this seminar.

Five years passed very quickly and, after that, my generation started to teach. Every good mathematician of my age, a year before or a year later became a “chef” of a seminar. I had a seminar on representation theory, Arnold had one on Hamiltonian mechanics and differential equations, Manin on algebra, Vinberg on Lie groups... Who else... Sinai, Anosov – probability and dynamical systems. And it is very interesting that there were a cluster of good mathematicians within three years at university who were very bright. And the next such cluster, the next “wave”, appeared after ten years: the age of Kazhdan, Margulis, Katok...

After that, it is already difficult to say; the situation changed globally.

***You’ve mentioned representation theory – so how did you choose your main field? Who influenced your choice?***

<sup>3</sup> The nickname of Felix Alexandrovitch Berezin.

Well, I never chose a subject in mathematics... I followed my teacher Gelfand, who always said that you cannot do analysis, algebra, geometry or mechanics, or something – you must know mathematics and there is no difference between the domains. If you want to be a good expert in representation theory, you have to know everything. So I just tried to solve any interesting problem that I heard of from any direction.

***But how did you do your work on nilpotent Lie groups, i.e. the results that immediately made you famous?***

Ah, that is a concrete question and it is easy to answer. When I was, I think, a third year student, Gelfand said, at a seminar, that there were some interesting papers by Dixmier about irreducible representations of nilpotent Lie groups. And nothing in this direction had been done before. At that time, three or four papers by Dixmier had already appeared.<sup>4</sup>

He asked me to read them and to present what they were about to the seminar. Reading the first paper, I was stuck because, in the paper, it was assumed that the reader knew the notion of induced representations. I understand the word “induced” in a general philological meaning (that something induces something in a similar situation) but the mathematical notion of inducing is very special. You cannot invent it. You cannot reconstruct it without knowing [laughs]. So, it took some time. I asked people. I asked my friends and nobody knew what an induced representation was. Then, I found the definition in an earlier paper by Dixmier and understood that it is a very nice thing. The theory of induced representations of finite groups was invented by Frobenius. It was explained in the collection of his papers and translated into Russian by the Kharkov Mathematical Society in 1938. Gelfand had read this book but I learned about it much later.

***And how did you find your orbit method?***

I did not know at the time that it was called the “orbit method” [laughs].

I had to present what I understood from Dixmier’s papers to Gelfand. And I understood that Gelfand would not like Dixmier’s variant of the exposition. I tried to adapt it to Gelfand’s understanding and, step-by-step, I worked out how I could explain what Dixmier did in more simple and more natural terms. And so I came to coadjoint orbits.

<sup>4</sup> J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, I, *Amer. Journ. Math.* 81, No. 1 (1959), 160–170.  
 J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, II, *Bull. Soc. Math. France* 85 (1957), p. 325–388.  
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 J. Dixmier, Sur les représentations unitaires des groupes de Lie nilpotents, VI, *Canad. Journ. Math.* 12, No. 2 (1960), 324–352.



Guillaume Gellé, president of the University of Reims, awarding the degree to A.A.Kirillov. (Photo L. Amour)

The main problem was to construct irreducible representations and I understood that most representations are induced by 1-dimensional ones. What is a 1-dimensional representation of a Lie group? It is the exponential of a 1-dimensional representation of a Lie algebra, i.e. an exponential of a linear functional. So, the idea was to take a subalgebra, a linear functional, take the corresponding representation and induce. If you do it “by chance”, you get, as a rule, a reducible representation. So, the subalgebra must be big enough to get an irreducible representation but not too big. If it is too big then very few linear functionals produce a representation, since a 1-dimensional representation must vanish on commutators. So, I started to experiment with subalgebras and look for appropriate functionals. Rather soon, I understood that it is better to start with functionals, not with subalgebras. For any functional, you can choose a corresponding subalgebra. And also, even though different functionals produce different representations, they are sometimes equivalent. It is rather evident that the conjugate functionals from the same orbit produce equivalent representations. So, the notion of coadjoint orbits jumps out by itself.

Of course, after that, it was really the discovery that everything, every question, in representation theory in terms of coadjoint orbits can be naturally formulated and sometimes answered (not always, but at least it is the right language for representation theory).

***Was that your first paper or was it later? You had a big paper...***

I never write big papers.

***No, a big paper in “Uspekhi”.***<sup>5</sup>

It was not big: less than 50 pages (and people often write 300 pages). Also, it was an “Uspekhi” paper – where you have to explain your results for beginners and non-experts. But “Doklady” notes that my main results (two or three notes of 3 pages each<sup>6</sup>) were very short.

<sup>5</sup> Unitary representations of nilpotent Lie groups. *Russian Math. Surveys* 17:4 (1962), 53–104.

<sup>6</sup> Doklady Mathematics, Vol. 128, No 5 (1959), Vol. 130, No 6 (1960), Vol. 138, No 2 (1961).



The talk by A.A. Kirillov. (Photo Vladimir Salnikov)

***I would like to go back to the seminars. You said that you started your seminar quite early. How did it start? Who were the first participants?***

I would not say early. I started as soon as I had the chance to do it. [laughs] Sure, I could teach schoolchildren and we were already doing that when I was a first-year student. All my generation of first-year students considered it a duty to teach mathematics to schoolchildren because many of our students (Moscow students) went to the mathematical circles. I did not attend these circles but most of my colleagues did so it was natural to carry out the same activities.

***And you taught the children at the university.***

Yes, usually at the university. Well, it depends. Some bold people like Arnold started their circles at the university from the very beginning and some others first started them at high schools. There are schools that are more or less associated with the university: e.g. the 57th school and some others.<sup>7</sup> I think, in the first year, that I ran a mathematical circle in one of the schools near the university.

***Which was just created at the time, right?***

Yes. And then I switched to the main building.

***And you also wrote some booklets (I know at least one)?***

That was much later,<sup>8</sup> well not much, but later. It was written for the correspondence school of mathematics [“Заочная Школа”] – a quite different “enterprise”. In 1960 something (I don’t remember when exactly), there

<sup>7</sup> See “University Goes to School” – “Moscow University Maths Department for Schoolchildren”, EMS Newsletter 101, September 2016.

<sup>8</sup> И.М. Гельфанд, Е.Г. Глаголева, А.А. Кириллов “Метод координат” М. 1966, 1973.  
С.И. Гельфанд, М.Л. Гервер, А.А. Кириллов, Н.Н. Константинов, А.Г. Кушниренко, Задачи по элементарной математике: последовательности, комбинаторика, пределы М.: Наука, 1965. [Gordon and Breach, 1969, Learn limits through problems.]  
А.А. Кириллов Пределы, М. Наука, 1968, 1973.

was a great idea. Kolmogorov had created a boarding school.<sup>9</sup> Petrovskii asked Gelfand to join but Gelfand said that it did not suit him [laughs]. He, as a polite man, explained why: because Kolmogorov’s boarding school was something like an “elite establishment”. But Gelfand himself was more “wild” and he said that he preferred to organise something for people who do not know anything but who are able and just don’t know that they are able. He proposed a correspondence school, which was accepted. And he did a great administrative job; he convinced many very talented students to participate in it. Some of them, like me, wrote textbooks and some participated by correcting the solutions that were sent by students. And this school, it became a big industry and it was very popular in the mathematical student community, so popular that party officials began to worry.

The point is that, at the time, all students, especially Komsomol [Communist Union of Youth] members, had to do society work [“Общественная Работа” – work of public interest]. And what were the possibilities? You could be active in a Komsomol area or in a profsoyuz [syndicates – labour unions] area or something else of the sort. The correspondence school opened up a new possibility. But why were the party members suspicious? Because work of public interest, by definition, cannot give pleasure [laughs] and these students did it with pleasure. And the correspondence school still exists.

***Speaking about books, how did you get involved in this project?***

When I was attracted by Gelfand to this correspondence school, he said that we must write a good book for the school: “I will write it myself and you will join me.” Our first book had three authors: Gelfand, Glagoleva and Kirillov. Glagoleva was a schoolteacher and she was very good. She passed away this year...

When I joined this committee of three people, they first gave me two or three pages of text. I looked at it, completely rewrote it and gave it back to Gelfand. He gave it to Glagoleva, saying that “our version was bad and this is still worse” [laughs] – he said to please merge it together and get something readable. Glagoleva provided the third variant, then I rewrote it and so on...

***But it was not work of public interest – you enjoyed it?***

Oh, no. [laughs] Well, it was interesting but I would not call it pleasure.

***Back to the science. Could you name some of the first participants of your seminar?***

Well, those who were at school, I do not remember, though maybe some of them became students afterwards, so I remember them as students. But, every year, new students came.

I remember my graduate students – those who had to write a thesis – and just visitors to my seminar (there

<sup>9</sup> See “University Goes to School” – “Mathematics in Kolmogorov’s School”, EMS Newsletter 101, September 2016.

were many more such people). Those who did not write a thesis and left no sort of trace – they are so difficult to remember. I certainly remember those who did PhDs (more than 60) but I can't remember them all. Maybe you do? [to Alice]

***But I came later.***

It was forbidden for me to be your advisor.

***Of course, it was a big problem.***

So you made a tremendous effort to become my student because, again, party officials would say it was impossible for a foreign student to have an advisor who was not a party member.

***I was asked why I chose Kirillov. I said because of his book.***

But you were a citizen of the Socialist Republic of Hungary, you must understand [all laugh].

***And then the Hungarian Embassy had to help.***

***When you mentioned party officials, they were local to the faculty?***

That was a completely local issue. Well, what means local? People like Sadovnichiy, at the time. He started as the Komsomol leader for our department, then he became a party official and then he became a member of a very important thing called the “personal committee”, who decided which of the party members were decent enough and which did not deserve to be members. That was a very important position. He occupied it for many years and then he went up and up. In my time, the highest position he occupied was “twice the first”, namely, the “first deputy of the first Vice-Rector”.

***For your method of leading the seminar, were you inspired by something or somebody?***

Certainly. I think every human being, consciously or unconsciously, imitates what they have seen before. I imitated Dynkin's manner (not completely, but partly) and Vitushkin's manner, Arnold's manner, Manin's... Not Novikov's because I did not like his manner. Who else... Gelfand, of course. Gelfand's seminar was a very special phenomenon in Russian mathematical life. Step-by-step, I created my own method (but not immediately).

***And Olshanski was, for some time, an important person in your seminar. Do you remember how he appeared?***

Olshanski was one of my most talented students but he was so shy, so quiet. For a long time, I did not consider him seriously enough. I knew only that any question I asked to Grisha, he would answer. But he was never the first to say: “Ah, I know.” And he was also the first of my students who started to help other students. For example, here, at this conference, there are Molev, Nazarov, Okounkov, Borodin and maybe ten more... They are practically students of Olshanski and not of me.

They worked together at my seminar but, for example, they got a problem and discussed it for long hours with Olshanski. Then, they said that I gave them a prob-



From left to right: A. A. Kirillov, M. Pevzner, Yu. Neretin, A. Fialowski, V. Salmikov. (Photo L. Amour)

lem and they solved it; actually, it was under the supervision of Grisha Olshanski but he never got any credit for it. The only thing that he received was a letter from Sadovnichiy after Okounkov became a Fields Medallist. Grisha Olshanski got a letter that said (I say it in Russian because it is important):

“Окуньков вырос на Московском Университете, поскольку имел руководителями таких ярчайших математиков, как Ольшанский”.

[Okounkov grew up at Moscow University, since he had, as advisors, some of the brightest mathematicians such as Olshanski.]

I do not remember if he mentioned me or not but Olshanski was certainly mentioned. And I saluted Grisha the next day saying: “Hi, the brightest!” [“Привет, ярчайший!”] [laughs]

I think I must finish with an afterword: Okounkov, at the time, was not a student of the university; he did not have any document saying that he belonged to the university. Being in Moscow, he tried to enter the building at MSU but was stopped by the guard. He showed the letter from Sadovnichiy (exactly this) and the guard said “this is not a document”. [laughs] It was when Andrei was already a Fields Medallist but did not have any position at MSU. Now, he has a position both in Skolkovo and in the Higher School of Economics.

***I would like to ask a question about students because when I arrived in Moscow and people heard I would be your student, they said I had chosen a very tough leader. Some of your students could not finish.***

I know only two such cases and I think it is not my fault. For example, Sergei Belkin was very exceptional but I think nobody could convince Belkin to write any papers. [laughs] Then, he lived in the student dormitory, not attending any lectures, with many students coming to ask his help on problems of different levels, from an undergraduate exam to a PhD thesis.

***I had a feeling that you gave absolute freedom to your students, with an obligation that they should find their own way – which is perfectly good because these people went around the world, and around Russia or the Soviet Union, and they had to stand on their own two feet. I am grateful for this style.***

Well, it is not universal advice for everybody. I think even good students sometimes need some pushing. For example, I still think that I did not sufficiently push Yura Neretin. He invented, in one moment of his mathematical biography, what I propose to call “Neretin Numbers”. These were parameters of discrete series of representations of Virasoro algebra (which was a very hard subject at the time). And then it was taken on by physicists. Mathematicians, I think, never discovered these parameters. Of course, Victor Kac later explained it but it was post factum, after the physicists had done all the computations. But you were also very close to it.

***It was in my PhD thesis, which mainly was not published.***

***When we were walking here, you mentioned that you were interested in French mathematics, and somehow French mathematics got interested in you, like all these great people who went to your lectures. How did it happen?***

When I went to France, it was 1968. Before that, there was the Moscow Mathematical Congress and, even before that, there was a Congress in Stockholm in 1962. They planned a big delegation of 400 people from the Soviet Union; the idea was to rent a steamboat to get from Leningrad to Stockholm, and the Soviet team would live on this boat and not spend foreign currency [laughs]. Well, the idea was proposed and discussed (it took many years – the preparation of a congress takes four years) and we finished with a delegation of not 400 but 40 people. But, for the first time in history, I think, seven young mathematicians were included. These were Ludwig Faddeev, Yura Manin, me, Arnold, maybe Anosov and who else... You must know that not all the participants were admitted to the congress. The announced list of speakers and the actual list of speakers were not identical.

Many people came to my talk. There were three sorts of talks (it has changed since then): 45 minute talks, 25 minute talks and posters. I had 25 minutes and I was surprised at how many people came to it. I don't know who made the advertisement but the result was there. And, for the first time, I met a lot of mathematicians whose papers I had read: Kadison, Mackey, Mautner, Fell, Atiyah, maybe Singer also, Hirzebruch, a lot of people. And then, in four years, they all came to Moscow and we continued our... not collaboration but discussion. So, when I went to France, I already knew a dozen good mathematicians – not only good but great. At that time, France was a great mathematical country. Arnold considered Bourbaki as something that spoiled mathematics but I think their influence on French mathematics was very strong and positive on the whole.

***Apparently, he was angry about the school reform that was inspired by Bourbaki.***

Maybe, maybe.

***But that is yet another story.***

Nobody is happy about school reforms in any country [laughs].

***So your first long stay abroad was in France 50 years ago. What was your impression?***

You mean my first visit to France. Well, it was 1968 – a very interesting year because it was the year of “La Grande Révolution Française”. And I participated in it. I invented my own slogan: “Liberté aux professeurs associés.” It was a standard slogan of the time. Everybody shouted “Liberté” – to workers, to students. I thought that my duty was to fight for liberté aux professeurs associés. What did it mean? Nobody knew and nobody was interested in it. The main thing was to go onto the street and to shout about it: “Liberté for...” [laughs]

***Do you see any striking differences with nowadays?***

I think yes. You know, one of the fairytales starts like this: “In China, all the inhabitants are Chinese, and the emperor himself is Chinese.” My first impression about France was that “all the citizens are French, and they speak French”. And it was a nice experience because I already knew some French. I like this language a lot. I knew some French mathematicians so for me it was a great pleasure. J'habitais Paris, Cité Universitaire, dans la maison Arménienne. I don't know why.

In Cité Universitaire, each building has a name: maison Arménienne, maison Pays Bas... It was rather close to the IHP, where I was an official member. At the time, Sorbonne had not yet divided into 14 universities; it was one university but the main mathematical organisation was the Institut Henri Poincaré on the rue Pierre et Marie Curie. I liked this place very much until now.

It was a very pleasant situation. French mathematicians are very friendly. I was surprised when Serre told me that all French mathematicians “tutoient” each other. You can say “vous” in France but mathematicians must say “tu”. Maybe a mathematician starting their PhD thesis or an undergraduate student must still respect their teacher but starting after their PhD they must “tutoyer”. Maybe it has changed now – I don't know.<sup>10</sup>

I also consider myself to be extremely lucky because I was in France without any “surveillance”. This was in great contrast to my previous visit to Stockholm. Why? I don't know. The very fact that I entered France was inexplicable.

Actually, I have my own explanation. I have repeated it several times. The year 1968 was not a good year but I learned this only after my departure to Paris. My version is as follows. At the time, the Soviet Union had very good relations with France and personally with President Charles de Gaulle. And French bureaucracy has one remarkable feature: any official paper issued by an organisation must be accompanied by references to all “décrets” of organisations of higher standing that are related to this decision. Therefore, my invitation to France was written like that: I was invited by the IHP, according to the décret of the main person at the IHP, based on the décret of the President of Sorbonne, based on the décret

<sup>10</sup> Indeed, there is some difference for student-professor communications; otherwise, the tradition still exists.

of the Ministry of Education, etc., and, in the end, there was the facsimilé of the signature of de Gaulle.

I imagine very vividly a clerk in the Russian ministry, sitting at a desk and looking at my invitation. His first move would certainly be to decline it immediately because this is the standard reaction. If he approves it, he takes on the responsibility for it; if he declines, he does not risk anything. And then he sees de Gaulle's signature... [laughs] and, thinking not about de Gaulle but about his superior, who would say: "Ah, you declined an invitation signed by de Gaulle..." That's my version. I don't insist on it but I have no other explanation.

***Whom would you like to mention amongst your colleagues of the 1970s and the 1980s at Mekhmat?***

Well, I became a member of the faculty in the 1960s. At the time, Mekhmat was growing because we switched from the old building in the city centre to the new one in Leninskie Gory. In 1961, Petrovskii called Arnold and me and said to us: "I have two positions for young mathematicians and I want to take you." At the time, we were second-year graduate students, after starting graduate school in 1959. He said that graduate school could wait and he had the option of taking us on as members of the faculty. Of course, we agreed with great pleasure and our first position was "Assistent" – a minor position.

So, starting in the Fall semester of 1961, we became members of the faculty. I was the youngest member of my chair (Mekhmat is divided into chairs) of function theory and functional analysis. Arnold went into differential equations because it was Petrovskii who personally took him and I guess that Gelfand was the one who asked Petrovskii to take me as the second.

I was the youngest member of this chair for a long time. But, at the time, there were already Alik Berezin, Bob Minlos, Shilov of course (on our chair) and Schabat, and of course Men'shov and Ul'yanov – two older representatives of the chair. It was non-officially divided into three parts: real analysis, complex analysis and functional analysis. The chief of our chair was Men'shov (a picturesque man) – anyone who saw him once would never forget it. You had the chance to see him?

***Yes, yes – the last time was in 1977.***

[all laugh] A man who looks like don Quixote, as thin, as great and as grey, with a beard and a loud voice. But he invented his own way to cope with party officials. When they told him there was something not so good in his chair (not enough Komsomol activity or something else), he said: "I'm an old man; it is difficult for me – explain to me. I will try." [laughs]

Who else was there from the faculty? At the time, there was, of course, Petrovskii – a very, very busy man but still active in mathematics. He was writing, at the time, his famous paper with Landis, which finally turned out to be erroneous. It proposed the solution of one of Hilbert's problems, about the number of ovals in the algebraic curves, and the solution they proposed was wrong – it turns out that there can be infinitely many cir-



On the campus of the University of Reims. (Photo Vladimir Salnikov)

cles whereas they thought there were finitely many and tried to find the upper bound.

Who else? I do not remember the old algebraists of the university but there was Shafarevitch, who was very active, and a young star Manin. In differential equations, of course, there was Arnold – the main figure. And in topology, Novikov soon took the ruling position. And we were always friendly. Not everybody went to all the seminars but we knew what other people were doing. And also, there was Gelfand's seminar as a club, where everybody came and [laughs] socialised.

***Now, turning toward the present, when you moved to America in the 1990s, you faced a completely different reality. How did you continue your research and teaching there?***

Well, first of all, I never dreamed of moving anywhere. Of course, I was invited many times to many countries. But, in the Soviet period, it was practically impossible. I was invited twice to Israel but both times my application was declined with the reason that it was a very, how to say... There are many "bad words" for other countries and Israel was, of course, one of the worst.<sup>11</sup> So, it was impossible for those who worked at Moscow University, so big and nice, to go to such a 'bad' country as Israel... By the way, at exactly that time, our dean went to Israel but it is, well... [laughs]

In 1990, I got a third invitation. It was at some jubilee of Pyatetski-Shapiro, I think, and I was invited and quite unexpectedly got permission to go "as a private person" (not as a university professor). University professors could still not go to Israel but as a private person it was possible. In the Spring of 1990, I was in Israel and then I decided maybe I could try to go to the United States. It was also a "bad country" but maybe not so bad as Israel. Mark Freidlin, my friend and colleague at Moscow University (we were students of the same year), had a position at Maryland University; he had already invited some of his friends and tried to invite me. And, again, unex-

<sup>11</sup> Speaking of the officials' point of view.

pectedly, I got permission and went for a whole semester to Maryland University. In Israel, it was a short period – one conference, ten days maybe – but this was a one-semester period. It was very interesting. I made the acquaintance of another dozen mathematicians, who were close to me. I liked it very much but never dreamed of staying forever – I finished my semester and went back. But, during my stay in America, I visited four other universities. From Maryland, I went to Philadelphia, Boston and Yale, I think. This was not like Vershik, who, during his first visit to America, visited 20 universities [laughs]. He spent, I think, one month there, or maybe two months. I think it is better to see four universities for one week each than go everywhere for one day. And, after that, I liked Philadelphia and they invited me to come back. I said: “OK, see you soon.”

Then I came back to Moscow. I did not want to go anywhere immediately after that. For me, I decided, to visit the United States once in three years would be a very good practice. So, next time, I went to Philadelphia, in the Fall of 1993. But, at the time, what we called the Second October Revolution was occurring, when the White House<sup>12</sup> was shot at. I was in Philadelphia at the time and my wife was panicking because CNN was showing pictures of the White House in smoke and tanks on the streets of Moscow. She did not like these pictures at all. My wife was very scared and asked me if I could extend my stay in the US to the Spring semester. I went to the chairman and asked if it was possible. He said, sure, but why don't you want to accept a permanent position? I said I was not ready and I would think about it. And I thought about it. My stay in America was extended and I spent one more semester there. After that, my wife convinced me that it was better to accept the position.

Well, as soon as it was known that I was ready to accept a permanent position, I immediately got another invitation from Penn State. Penn State is a bigger university than UPenn but of slightly lower status. It is situated in the middle of Pennsylvania, at the cross of two diagonals of the rectangle (as the English say, “in the middle of nowhere”), where farmers live. And it was founded exactly to bring education to farmers. But this university is very rich and very big, with many more students than UPenn. And (maybe more essentially), it has a good football team.<sup>13</sup> In America, the football competition is very important and, when a crucial match takes place in State College (the city where Penn State is situated), the hotel rooms are booked a year in advance. So, for example, it is impossible to run a mathematical conference at the same time as a football competition. And also, there were already eight Russian mathematicians at Penn State. I thought about it and preferred to go to Philadelphia, which is not far from Washington and New York and where there were no Russians at all. [laughs]

***But there were a number of great mathematicians in your area, like Kadison, Pukanszky, Fell, Wilf and others?***

<sup>12</sup> Russian White House – the seat of the Government.

<sup>13</sup> American Football.

Yes.

Though our football team is not so good [laughs], you know this notion of Ivy League? I was surprised when I found out the origin of the name. I thought the ivy imitated old English universities but, no, the reason is quite different. The point is that Harvard, Penn, Cornell, etc., (the eight old universities) have a very high reputation but rather bad football teams. You see, for example, with Nebraska and Penn State, the match between them is very important because whoever wins guarantees getting a lot of students the next year and whoever loses gets much fewer. The good universities like Yale and Harvard are not so dependent on the results of the football team so they have the option not to hire expensive football coaches. You know that the salary of a football coach is bigger than the salary of a president of a university? But to have no football competition at all, this is also impossible. However good the university is, if there is no baseball, football or basketball then the students will not come to such a university. So they found a new genius idea to organise a special league, a football league, and call it the Ivy League. And so, these eight universities compete between themselves and they have the chance to be the champion of the Ivy League. They have no chance to get a decent place in the whole United States University League but inside the Ivy League, it is quite possible.

***I found a few “traces” of you in Denmark in 1990 because there were three Russians in Denmark before me: one was Peter the Great...***

He was not alone; he brought, I think, several hundreds of people with him...

***But Danish people remember three Russians. So, Peter the Great climbed a horse and rode it to the highest tower in the city. The second one was a young man from Tambov, a mathematician, but the story was about his wife (hence the fourth Russian). And there was also Kirillov and this story was not so bright. Each mathematician in Denmark showed me a café where they drank with Kirillov and the cafés were different.***

[all laugh] I really was in Denmark; it was a conference about the orbit method. It was, I think, the first time that I had been to a capitalist country since my visit to France in 1968. It was in 1988, I think. It was a funny and sad story. The main role in it was played by a device called telex. I think this was something like fax but it was 30 years ago and I think that the fax of today did not yet exist.

You see, I got an invitation for the conference, which lasted one week, and, after that, I gave some lectures so the total invitation was for three weeks: one week for the conference and two weeks of lectures. When I went to the authorities in Moscow, to the foreign division, the chief of this foreign division said that he knew nothing about the invitation for delivering lectures; he only had an invitation for the conference. The invitation for the lectures was in my pocket but I understood that it would be a drastic mistake to show it to him. And I said, OK, I will go to the conference. When I went to the conference,

I asked the equivalent of the foreign officer in Denmark and said that I had two invitations, so could I stay? He said: "Certainly," and wrote something. I took it and went to the Russian Embassy in Copenhagen. They looked very surprised and said: "Who wrote it?" "An official representative of the Danish foreign office." "Who allowed you to talk to this representative? You should immediately leave Denmark!" I said: "I don't have a ticket. So how can I go immediately?" After that, I called my wife Louiza, who was in Moscow. Alesha Gvishiani, a member of our department (and grandson of Kosygin), also wanted to come to this conference. Later, he changed his mind and did not come. But all the documents went together with mine. And Louiza called Alesha and he said that he would try to do something. Then, he said that the telex would arrive at the embassy. So, over one week, I went to the Russian Embassy every morning, as if to work, and asked if they had a telex from Moscow, which would have allowed me to stay in Denmark for another day. [all laugh]

I didn't know whether telex was an electronic device or a pedestrian courier [laughs] but from Moscow to Copenhagen it took ten days. And, every second day, I was told I should leave Denmark immediately. But I came again and again. And then, on the 8th or the 9th day, I came and they said very dryly: "You can stay."

***Moving forward, how could you continue your work and research, and seminars (I mean in Philadelphia during these years)?***

Of course, the life of a mathematician in America is quite different from the life of a mathematician in Moscow. There are different students and different relations with colleagues. Everything is different.

***What can you say about your current research?***

You see, I am not obliged now to publish many papers per year (like young people who must show that they are great). So, I prefer to think for a long time about interesting questions. Right now, I am a member of a team of four people trying to solve one very difficult problem. But I will speak in detail about it tomorrow.<sup>14</sup> We have already spent three years on it, meeting in California where there is the American Institute of Mathematics, which organises work by teams, called "squares". And by the definition of the AIM, a square is a geometric figure that can have from two to eight vertices [laughs]. We form four vertices and have spent ten days every year for three years (but it is finished now). We will continue in Oberwolfach, as a team "in pairs". I do not know the official definition of a "pair"...

***Maybe the very last question is kind of inspiring. What would you like to say as a message to the younger generation?***

Oh... I could say that humanity now has a very big problem: what is the reason of our lives? And most people,

I think, do not have an answer to this question. In poor countries, people suffer from a deficit of everything; in rich countries, like in Scandinavia, the proportion of suicides is growing and growing and people do not know what they are living for. I think the main "raison d'être" – the reason for life – for a human being is to learn about the Universe. And mathematics is one of the ways to understand nature. The liberal arts is another form and there are other forms: certainly not bureaucracy but maybe medicine (although medicine is now half industry and half bureaucracy so I do not advise people to go into medicine). Of course, not everybody gets pleasure from doing mathematics but I think that a non-zero percentage of people are able enough to do mathematics. So, if somebody feels that they can do mathematics, I advise them to do it. Of course, this will put an end to your "American dream", which at the beginning of the [previous] century was to have a million dollars and now is equivalent to having a hundred million dollars. If you are a mathematician, you can be sure you will never get this. But still, it gives a sense of purpose to your life, which may be more important.

***Thank you very much!***



*Alice Fialowski is a professor at the Institute of Mathematics, University of Pécs, and Eötvös Loránd University in Budapest, Hungary. Her research interests are Lie theory, cohomology, representation and deformation theory, with applications in mathematical physics. She is a former student of A.A. Kirillov at Moscow State University.*



*Yury Neretin is a professor at the University of Vienna, the Institute for Theoretical and Experimental Physics (Moscow), Moscow State University and the Institute for Information Transmission Problems (Moscow). His research interests are representation theory and noncommutative harmonic analysis, infinite dimensional groups, classical groups and symmetric spaces, analysis of a matrix variable, special functions, operator theory and mathematical physics. A.A. Kirillov was his scientific adviser.*



*Michael Pevzner is a professor of mathematics at the University of Reims, France. His research interests concern representation theory of Lie groups and its applications in analysis and physics, as well as quantisation theory of homogeneous spaces. A.A. Kirillov was his first scientific advisor.*

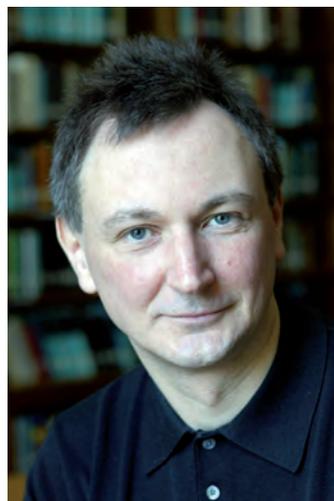


*Vladimir Salnikov is a researcher at CNRS, La Rochelle University, France. His scientific interests are graded and generalised geometry, dynamical systems and integrability, and applications to theoretical physics and mechanics.*

<sup>14</sup> The talk delivered the next day was "Representations of the triangular group over a finite field".

# Vladimir Voevodsky – Work and Destiny

Mark Bickford (Cornell University, Ithaca, USA), Fedor Bogomolov (New York University, USA) and Yuri Tschinkel (New York University and Simons Foundation, New York, USA)



Vladimir Voevodsky, who died in Princeton on 30 September 2017 at the age of 51, was one of the most remarkable and highly original mathematicians of our time. His achievements have been recognised with the highest honour of the profession, the Fields Medal, which he received in 2002. His work transformed several fields of mathematics and theoretical computer science.

Vladimir started his mathematical education as a high school student attending the Shafarevich seminar at the Steklov Institute of the Russian Academy of Sciences in Moscow. He bypassed the “usual” mathematical Olympiad training and moved directly into research. His exceptional talent and focus were already apparent then to all who interacted with him. As an undergraduate student at Moscow State University, he fully immersed himself in the study of Grothendieck’s anabelian geometry, formulated in 1984 in *Esquisse d’un programme*. His early work, jointly with G. Shabat, concerned *Dessins d’enfants*, the study of Galois groups of curves over number fields via their representation by special graphs on Riemann surfaces. The inspiration came partially from a result of Belyi, who proved that all such curves admit special meromorphic functions, with only three ramification points; moreover, the existence of such functions characterises these curves among all complex projective curves. At that time, it seemed that this result might open the door to the solution of major open problems in arithmetic geometry, such as Mordell’s conjecture and Fermat’s last theorem, as well as another important conjecture that is still open: the Section Conjecture of Grothendieck. Vladimir’s interest in this area showed his determination, early on, to tackle the most difficult and challenging conjectures in mathematics.

His next project was a proof of the reconstruction of hyperbolic curves over a natural class of ground fields from their étale fundamental groups. This is the first step toward the Section Conjecture. His joint papers with M. Kapranov, on what seemed to be not very popular issues in category theory ( $n$ -categories,  $\infty$ -groupoids and higher braid groups), turned out to be crucial for his work in algebraic geometry over the next two decades, as

well as for univalent foundations. Already at that time, he was stating casually: “If the categorical framework works out, the Bloch–Kato conjecture will follow trivially.”

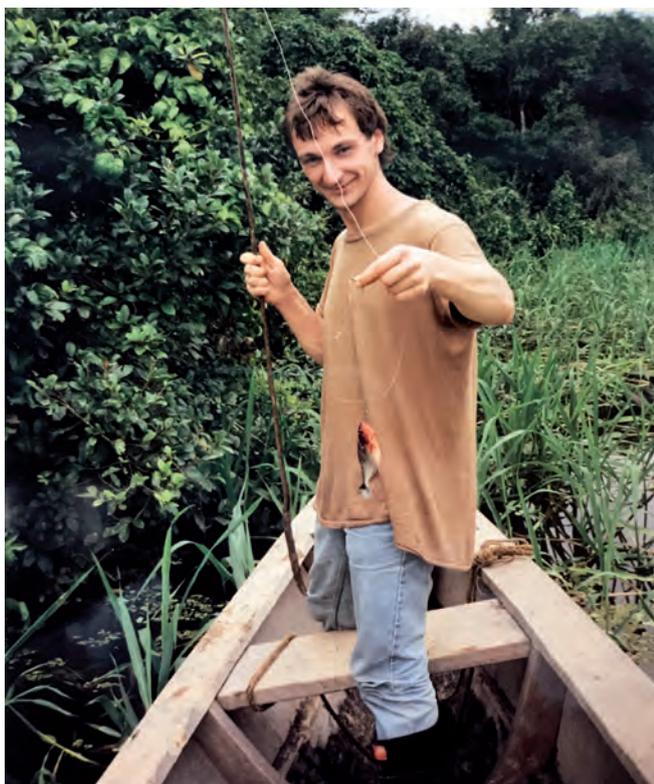
His research interfered with his undergraduate work and he did not show up for classes or exams. After eventually quitting Moscow State University, he moved, in 1990, to Harvard University, where he became a PhD student. He graduated in 1992 and, after one year at the Institute for Advanced Study, he returned to Harvard as a Junior Fellow of the Harvard Society of Fellows.

All these years, he was relentlessly working on foundational problems; his meagre publication record between 1991 and 1995 is in stark contrast with the intensity of his investigations. Then came an avalanche of papers that radically changed algebraic geometry, settling major open conjectures (e.g. the construction of the derived category of motives, the Milnor conjecture and the more general Bloch–Kato conjecture) and introducing powerful new techniques. These conjectures postulated a deep and highly nontrivial connection between the geometry of algebraic varieties and their Galois symmetries. As Voevodsky’s proof showed, this bridge required radically new concepts; no simplifications of his original proof have emerged despite intense efforts by geometers and algebraists.

Voevodsky’s main achievement was the creation of an amalgam of homotopy theory and algebraic geometry. Both theories deal with objects of geometric origin but on the basis of completely different conceptions: while homotopy theory emphasises flexibility, algebraic geometry is rather rigid – algebraic varieties resist small, local perturbations. Mixing these essentially incompatible worlds in a meaningful context required a leap of faith and an enormous, prolonged effort.



In the Harvard Mathematical Department Library (1993).



In the Amazon, with a piranha on the hook (1993).

After a short period of teaching at Northwestern University, Voevodsky moved to the Institute of Advanced Study. In his words, he began to “lose motivation” for work in algebraic geometry around 2003, having completed a vast research programme. He started taking computer science courses at Princeton University; private conversations with him frequently revolved around the nature of correctness, truth and proof in mathematics. This was triggered by the attempts and failures of several mathematicians working on big categorical structures that were too formidable for paper-and-pencil analysis. Voevodsky was led to a more general question of whether mathematicians had the right tools to explore difficult new areas like the highly complex theories he was interested in.

Voevodsky learned that (starting with N.G. de Bruijn) computer scientists and logicians had created automated proof assistants such as Mizar, Coq and Nuprl. However, to be applicable in practice, the mathematical proofs first had to be fully formalised in logical systems that the proof assistants could implement, e.g. set theory or other type theories. This step already presents a daunting obstacle in any minimally nontrivial situation. The analysis of existing proof assistants convinced Voevodsky that computers could, in principle, check mathematical proofs but that none of the available systems were up to this task on a fundamental rather than just a technical level.

With his usual vigour and tenacity, Voevodsky decided to create the foundations for this area at the interface of mathematics and computer science. His main insight was based on his previous experience in mathematics: the introduction of ideas of homotopy theory into the

theory of types. His *univalence axiom* postulates that homotopy-equivalent objects share the same formal properties. Voevodsky was convinced that a systematic use of his *univalent foundations* would lead to the construction of practical proof assistants.

Again, there was a substantial gap in his publication record, followed by a burst of activity, starting in 2014, with 15(!) papers

posted on arXiv, three of them in June of this year alone. The introduction of univalence has already created great excitement in the community working with type theories in mathematics, philosophy and computer science. Initial prototype “univalent” proof assistants were created and Voevodsky and his collaborators embarked on a project to build a comprehensive library of mathematics, based on univalent foundations rather than set theory and rigorously checked by computer.

Voevodsky always said that this work was only a prototype and that the ultimate foundations for computer-checked mathematics had yet to be perfected. His vision sparked many ongoing research efforts, e.g. to find a constructive interpretation of univalence and to explore the use of univalence in various areas of mathematics.

His sudden and untimely death came as a shock to his colleagues and friends. He stands out as one of the giants of modern mathematics. The full impact of his ideas is still to be understood and appreciated.



At the Institut Henri Poincaré, Paris (2014).



*Dr. Mark Bickford (PhD 1983 in mathematics and logic from UW Madison) is currently a visiting scientist at Cornell University, computer science. His interests are in constructive mathematics and type theory. He is a user and developer of Nuprl and is formalizing the constructive model for Univalence in Nuprl.*



*Fedor Bogomolov is Professor of Mathematics at the Courant Institute, NYU. He works in algebraic geometry.*



*Yuri Tschinkel is Professor of Mathematics at the Courant Institute, NYU, and Director of Mathematics and the Physical Sciences at the Simons Foundation. His research interests are in algebraic geometry and number theory.*

# The Icelandic Mathematical Society

Sigurður F. Hafstein (University of Iceland, Reykjavík, Iceland)

The Icelandic Mathematical Society (IMS), or “Íslenska stærðfræðafélagið” as its name is in Icelandic, was founded 70 years ago on 31 October 1947. It was founded on the 70th birthday of Dr Ólafur Daniélsson at his home and in his honour. There were 15 founding members and eight of them were mathematicians; the others included three physicists, two engineers and two astronomers. The declared purpose of the society was to be a platform for Icelanders, who had a university education in mathematics or a related subject, to meet and discuss subjects related to mathematics. It should be noted that in 1947, Iceland was a very backward country with just 133,000 inhabitants. It had only been independent for three years and the dominant economies were agriculture and fishing. The University of Iceland, the oldest and largest university in Iceland, was founded in 1911 but teaching and research in mathematics first started in 1943 when Dr Leifur Ásgeirsson was hired to the engineering department. Ásgeirsson had studied mathematics at the Georg-August University in Göttingen and received his Dr. rer. nat. degree from Dr Richard Courant in 1933. He is mainly known for his mean value theorem for ultrahyperbolic partial differential equations and was one of the founders of the IMS.

The IMS has declared three honorary members: Dr Ólafur Daniélsson (1877–1957), who was a pioneer in mathematics education in Iceland, Dr Leifur Ásgeirsson (1903–1990), who was the first mathematics professor in Iceland, and Dr Sigurður Helgason (b. 1927), who is an emeritus professor of mathematics at Massachusetts Institute of Technology, USA. In celebration of his 90th birthday last September, Helgason founded an annual monetary prize to be awarded to exceptional undergraduate students of mathematics at the University of Iceland. Over the years, the IMS has been involved in some publishing work, both together with the other Nordic Mathematical Societies (*Mathematica Scandinavia* and *Nordisk Matematisk Tidskrift*) and on its own. In an old tradition, foreign words are given special Icelandic names and one of the publications of the IMS is an English-Icelandic dictionary for mathematical words. In the dictionary, the editors have shown much originality and imagination in inventing numerous Icelandic words for mathematical concepts. As an example, the Icelandic word for “dual space” is “nykurrúm” (“rúm” is “space” and “nykur” is an old mythical creature similar to a horse but with the hooves reversed).

Today, the IMS has close to 300 members and serves as the umbrella organisation for mathematicians in Iceland. This is in contrast to the mathematical societies of the other Nordic countries, which are much more connected to the universities than to the lower levels of education. One of the main activities of the IMS is the

monthly meetings, where a member or a guest gives a talk and the members traditionally meet before the talk to have coffee and a Danish and discuss mathematics. Some well known mathematicians who have given guest talks at these meetings are Dr André Weil and Dr Paul Erdős, the latter of whom gave three talks on his first visit to Iceland and one on his second visit. To give some idea about the diversity of these talks, the last three were: “Mathematics: professional development for teachers and enrichment for students”, given by Dr Uwe Leck and Dr Ian Roberts (who are affiliated to the Europa-University in Flensburg, Germany, and the Charles-Darwin University in Darwin, Australia, respectively); “The Social Cost of Carbon Dioxide – Mitigating Global Warming Whilst Avoiding Economic Collapse”, given by Dr Christopher Kellett from the University of Newcastle, Australia; and “Convey’s napkin problem”, given by Dr Anders Claesson from the University of Iceland. Additionally, the IMS is involved in organising mathematical contests for gymnasium pupils and giving book prizes to exceptional pupils graduating gymnasium. Since 2001, the IMS has organised the biannual conference “Stærðfræði á Íslandi” or “Mathematics in Iceland”. The conference is usually attended by 40-50 people and the programme is in Icelandic. The talks include topics from mathematical research, teaching and the use of mathematics in industry.

As a final note, the word used for mathematics in most languages is some version of the Greek original “μάθημα”. In Icelandic, however, the word for mathematics is “stærðfræði”, which translates to “teachings of sizes”. It was introduced into the Icelandic language by Dr Guðmundur Finnbogason, a psychologist and philosopher, in his 1931 translation of Dr Alfred Whitehead’s *An Introduction to Mathematics*.



*Sigurður Freyr Hafstein is a professor for applied mathematics at the University of Iceland. He studied mathematics and physics at the University of Iceland, the Georg-August University Göttingen Germany, and the Gerhard-Mercator University Duisburg Germany, from which he holds a Dr. rer. nat. title in mathematics. His research interests include dynamical systems and computational methods. He has been in the managing board of the Icelandic Mathematical Society since 2013 and chaired it since 2015.*

# An Exchange of Messages Between Two Authors and a Journal

Adolfo Quirós (Universidad Autónoma de Madrid, Spain), Chair of the EMS Ethics Committee

## Background

This note was prepared by the members of the EMS Ethics Committee. We shall describe the experience of two young European mathematicians when they submitted a manuscript as a pdf file to a mathematical journal. We shall not mention any identifying details.<sup>1</sup> We shall refer to this mathematical journal as “the Journal”. It is a journal published by a large commercial online-only publisher.

## The events

A manuscript is submitted to the Journal. The next day, the submission is acknowledged by the Managing Editor. Fourteen days after submission, the authors are informed that their manuscript has been accepted for publication and they are asked to provide the Word or LaTeX file. They are also told how to pay the publication charge. For the purpose of this exposition, we shall assume that the charge is EUR 1000. The actual charge is slightly less.

The Managing Editor sends a reminder 18 days later.

The authors do not respond immediately. Five weeks after the submission date, and thus three weeks after acceptance, they write to the Managing Editor withdrawing their submission. They have not sent the Word or LaTeX file to the Journal.

Three days later, the Managing Editor responds that the Journal is an open-access journal and relies on publication charges to run its organisation. The authors are then offered a 20% reduction on the publication charge.

The authors respond the same day, maintaining that they have withdrawn their submission.

Two days later, they are informed that the Journal has finished the publication process and that the charge for withdrawing the submission is 50% of the publication charge. They are given the payment information.

The authors object immediately, saying that they do not understand how the journal can finish the publication process without the Word or LaTeX file. They insist that they have withdrawn their manuscript. They state that they intend to submit it to another journal.

The Managing Editor replies that the authors did not react immediately to the information that the manuscript had been accepted and then writes:

*“...as DOI link have been generated you cannot submit your article in any of another Journals.”*

<sup>1</sup> The Ethics Committee has had access to the full correspondence. The quotes given are verbatim.

This time they are offered a 30% reduction in the publication charge.

A new person now enters the correspondence. It is someone from the management of the organisation, reminding them that they have not paid the invoice for their publication charge. This invoice is for the original amount of EUR 1000. The message is sent two weeks after the previous message from the Managing Editor.

The authors respond, stating that they have withdrawn their manuscript. The person from the management responds by offering a 20% reduction to the publication charge. He continues:

*“If you still wish to publish your article in our esteemed journal kindly quote the amount that you can afford, so that we can respond you positively include your valuable submission for the next issue.”*

At this point, the authors decide to check the Journal. The manuscript has already been published. According to the information in the Journal, the manuscript was accepted eight days after submission and published 11 days after submission. The published version is the submitted pdf file.

They write again to the Journal pointing out that they have not transferred copyright to the Journal, but they do not receive a reply immediately.

The response that they finally receive is from a different Managing Editor. It is sent 69 days after submission. They are informed (again) that their manuscript has been accepted for publication. Two referee reports are included. The full text is (with the topic replaced by X):

*Comments from the Editors and Reviewers:*

*Reviewer #1: I will accept this paper.*

*Reviewer #2: Dear Author, Discussion of X is very good.*

In this mail, the publication charge is not mentioned.

## Lessons to be learned

When you submit a manuscript to a journal, you should check what kind of journal it is and what conditions it imposes.

In this example, the authors did not realise that there was a publication charge since they did not check the Journal carefully. The Journal also clearly states its policy on withdrawal:

*“...an author is free to withdraw an article at no charge – as long as it is withdrawn within 10 days of its initial submission.”*

We note that the publication date in the Journal is 11 days after submission.

The Journal has an Editorial Board, and some prominent mathematicians are listed as members. One could wonder whether these people know this and are fully aware of the practices of this journal.

Finally, let us point out two relevant items from the Comments on the Code of Practice on the EMS homepage. We cite them here:

[PE1] Those colleagues who allow their names to be used, for example as “Editorial advisers”, to assure the mathematical public of the quality of a journal, have an obligation to be well aware of, and content with, the journal’s goals, policies, standards, and pricing.

We caution that the policies of journals may change with time, for example, when new owners take over, but that journals may not trouble to inform editorial advisers that their policies are changing.

[PE5] It is gross misconduct for an editor or publisher to accept for publication a submitted article without seeking to verify that the article does not plagiarize an existing article or work and is mathematically correct.



*Adolfo Quirós has been a member of the EMS Ethics Committee since it was established in 2010, and serves as its Chair in 2016-2017. He obtained his Ph. D. from the University of Minnesota and is Professor Titular at the Department of Mathematics of Universidad Autónoma de Madrid (Spain). His area of research is Arithmetic Algebraic Geometry and he is also involved in popularization activities, like the Mathematical Challenges run (on video) by the Spanish daily El País. He has been Vice-president of Real Sociedad Matemática Española and is currently editor of its members’ journal, La Gaceta.*

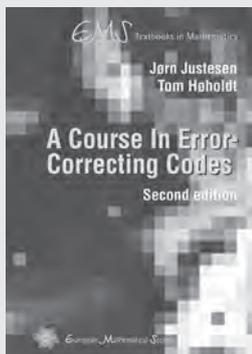


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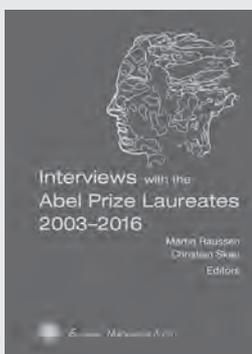
Jørn Justesen and Tom Høholdt (both Technical University of Denmark, Lyngby, Denmark)

**A Course In Error-Correcting Codes. Second edition**

ISBN 978-3-03719-179-8. 2017. 226 pages. Hardcover. 16.5 x 23.5 cm. 39.50 Euro

This book, updated and enlarged for the second edition, is written as a text for a course aimed at 3rd or 4th year students. Only some familiarity with elementary linear algebra and probability is directly assumed, but some maturity is required. The students may specialize in discrete mathematics, computer science, or communication engineering. The book is also a suitable introduction to coding theory for researchers from related fields or for professionals who want to supplement their theoretical basis. The book gives the coding basics for working on projects in any of the above areas, but material specific to one of these fields has not been included. The chapters cover the codes and decoding methods that are currently of most interest in research, development, and application. They give a relatively brief presentation of the essential results, emphasizing the interrelations

between different methods and proofs of all important results. A sequence of problems at the end of each chapter serves to review the results and give the student an appreciation of the concepts. In addition, some problems and suggestions for projects indicate direction for further work. The presentation encourages the use of programming tools for studying codes, implementing decoding methods, and simulating performance. Specific examples of programming exercises are provided on the book’s home page.



**Interviews with the Abel Prize Laureates 2003–2016**

Martin Raussen (Aalborg University, Denmark) and Christian Skau (Norwegian University of Science and Technology (NTNU), Trondheim, Norway), Editors

ISBN 978-3-03719-177-4. 2017. 301 pages. Softcover. 17 x 24 cm. 24.00 Euro

The Abel Prize was established in 2002 by the Norwegian Ministry of Education and Research. It has been awarded annually to mathematicians in recognition of pioneering scientific achievements.

Since the first occasion in 2003, Martin Raussen and Christian Skau have had the opportunity to conduct extensive interviews with the laureates. The interviews were broadcast by Norwegian television; moreover, they have appeared in the membership journals of several mathematical societies.

The interviews from the period 2003–2016 have now been collected in this edition. They highlight the mathematical achievements of the laureates in a historical perspective and they try to unravel the way in which the

world’s most famous mathematicians conceive and judge their results, how they collaborate with peers and students, and how they perceive the importance of mathematics for society.

# ICMI Column

Michèle Artigue (Université Paris Diderot-Paris 7, France)

## Jean-Pierre Kahane (1926– 2017): His legacy to mathematics education



Plenary speaker at the ICMI Study 16 conference: “Challenging mathematics in and beyond the classroom”, Trondheim, Norway, 28 June–02 July 2006.

On 21 June 2017, the mathematical community suffered a great loss with the death of Jean-Pierre Kahane at the age of 90. He was a student at the *Ecole Normale Supérieure* (ENS), defended his doctorate under the supervision of Mandelbrojt in 1954 and was appointed as *Maitre de Conférences* (1954–57) the same year and then professor (until 1961) at the University of Montpellier, before getting a position in Orsay (University Paris Sud), where he pursued his career until he retired in

1994, remaining as a professor emeritus until his death.

He was a world-renowned specialist in harmonic analysis, chaos theory and Brownian movement, and won several prizes: *Peccot* (1957), *Maurice Audin* (1960), *Carrière de mathématiques* (1964), *Servant* (1972), *Grand Prix d'Etat des Sciences Mathématiques et Physiques* (1980) and *Médaille Emile Picard* (1995). He was nominated at the French *Académie des Sciences* in 1988 and was made *Grand-officier de la Légion d'honneur*, *Chevalier de l'ordre national du mérite* and *Commandeur des palmes académiques*.

He was not only a brilliant mathematician but was also very active in many connected areas, in which his engagement was recognised for his cleverness and accuracy, as well as for his humanity, enthusiasm and eagerness. Among several responsibilities, he served as Secretary General of the French Union of Higher Education (1962–65), President of the French Mathematical Society (SMF) (1972–73), President du French National Committee of Mathematicians (CNFM) (1974–78), the second President of *Université Paris-Sud* (1975–78), President of the *Mission interministérielle de l'information scientifique et technique* (MIDIST) (1982–86), President of the International Commission on Mathematical Instruction (ICMI) (1983–90), President of the scientific committee of the *Instituts de recherche sur l'enseignement des mathématiques* (IREM) (1997–1999), President of the *Commission de réflexion sur l'enseignement des mathématiques* (CREM) (1999–2002) and President of the *Union Rationaliste* (2001–2004). Politically, he was engaged in 1946 with the French Communist Party (PCF), to which he remained faithful all his life. In particular, he was head of the scientific journal “*Progressistes*” since 2014.



“Coup d’œil sur l’analyse de Fourier”, conference at Ecole Polytechnique, May 2011.

A man of conviction, he has always been a source of inspiration for generations of young mathematicians to whom he stayed close until the very last days of his extraordinary life, a life of service to mathematics in all its forms, delivered with a fantastic openness of spirit.

### Jean-Pierre Kahane and mathematics education

Jean-Pierre Kahane was not only an eminent researcher but also a mathematician who, throughout his career and up to the last days of his life, sought to pass on his passion for mathematics and to share his profound conviction that it is a powerful tool for understanding the world, both in its complexity and beauty and in the fight against all forms of obscurantism.

He was an outstanding teacher, perhaps because very early on, as a student at the ENS, he understood how much education is an opportunity to learn. He recounted this in an interview for the centenary of the International Commission on Mathematical Instruction (ICMI), which he chaired from 1983 to 1990 (<http://www.icmihistory.unito.it/clips.php>).

It was at the request of Lennart Carleson, who was then President of the International Mathematical Union (IMU) and whom he knew well, that he decided to engage with the ICMI, accepting the presidency and the challenge to revitalise the commission. As he explained in the interview mentioned above, the main lines of action were established during a meeting in December 1982 at Orsay University with Geoffrey Howson (who was going to accompany him as Secretary of the ICMI during his two terms of office), Bent Christiansen (a Danish educator and already Vice-President of the ICMI) and Ed Jacobsen (a mathematics specialist at UNESCO), where they decided to launch a series of studies (which would become an essential activity of the ICMI), as well as defining the objective and the

structure. It was also here that the first five themes were decided. Three of them, Studies 1, 3 and 5, were particularly relevant to Kahane: “The influence of computers and informatics on mathematics and its teaching”, “Mathematics as a service subject” and “The popularization of mathematics”. The model developed at the time generally remains the one followed today, 30 years and 25 studies later.

During the presidency of Jean-Pierre Kahane, with thanks also to Ed Jacobsen, relations with UNESCO were strengthened. UNESCO supported the development of the ICMI studies and, in 1992, republished an updated version of the first study, as well as the ICMI’s various activities regarding developing countries.

Jean-Pierre Kahane’s presidency was noteworthy, as Geoffrey Howson testified in the text sent for the colloquium held at Orsay University in honour of his 90th birthday:<sup>1</sup>

*“Then and throughout the eight years in which he and I co-operated his knowledge, his leadership, his network of friends, his status within the mathematics community, his ability to raise funds, and his personal charm and efficiency never failed to impress me and others. Personally, it was not only a pleasure to work with and, indeed, just to be with, Jean-Pierre, but also to learn so much from him – his great erudition, his ability to supply simple explanations and guidance, and his wide interests. When he stepped down as President, ICMI was in a much stronger position than when he had taken over.”*

Jean-Pierre Kahane’s commitment to teaching and disseminating mathematics was, of course, not limited to his work at the ICMI. It has been a constant of his professional life, as evidenced by the various responsibilities he has exercised (as recalled at the beginning of this notice). We would like to mention here more particularly his role as President of the CREM (the commission on reflection on the teaching of mathematics) from 1999 to 2002. This commission was created at the request of associations of teachers and learned societies alarmed by the statements of the French Minister of Education of the time, Claude Allègre, on mathematics and its teaching. The commission was given charge of an in-depth reflection on the teaching of mathematics and, in particular, on the relations between mathematics and computer science. To lead this commission, which was composed of strong personalities, in a climate of tension generated by curricular reforms in progress, was not an easy task. Jean-Pierre Kahane, through his intelligence, his listening and his sense of dialogue and synthesis, combined with a determination without flaw, managed this leadership perfectly. Daniel Perrin expresses this well in the text he sent for the Orsay colloquium:

<sup>1</sup> The various testimonies quoted here are all accessible online on the website of the *Commission Française pour l’Enseignement des Mathématiques* (CFEM): <http://www.cfem.asso.fr/actualites/deces-de-jean-pierre-kahane>.



**During the prizes ceremony for the junior contest organised by the French mathematical society SMF, 10 June 2017 (11 days before he died).**

*“In retrospect, I think he carried out this difficult task, where we had to manage the strong and diverse personalities, with both subtlety and firmness. Subtlety, because he quickly sensed people in their complexity, which enabled him to anticipate conflicts and solve some of them before they even broke out. Firmness, because he knew how to put a stop when the slips began.*

*In any case, and I have precisely this memory about the elaboration of the report on geometry, he knew both how to trust people by giving them a real job, to help them from his great culture by being at their side and to realize and sustain them morally by constantly encouraging them. It was a great experience for me.”*

From the different works he supervised as the President of the CREM, the reports on the teaching of geometry, computation, statistics and probabilities and computer science (Kahane, 2001)<sup>2</sup> are still considered as texts of reference and are regularly cited.

Jean-Pierre Kahane was also a loyal supporter of the network of research institutes on the teaching of mathematics (IREM), created at the turn of the 1970s and a unique institution, where university mathematicians, researchers in mathematics education, teachers and teacher educators can collaborate together (<http://www.univ-irem.fr>). He became the president of the scientific committee of the IREM network in 1997, at a critical moment for this institution, and managed to mobilise the national and international mathematical community for their defence and to plead the cause with government delegates. He also encouraged the scientific committee to carry out a thorough reflection on the missions of the IREMs and how to effectively fulfil them in a context of deeply renewed teacher training. He succeeded in making the scientific committee a place of debate and reflection open on the “outside”. The structure that he then imparted to the functioning of the scientific committee continues to this day.

<sup>2</sup> The reports are accessible on the website of the Paris IREM: [http://www.irem.univ-paris-diderot.fr/articles/document\\_rapport\\_et\\_annexes\\_de\\_la\\_commission\\_kahane](http://www.irem.univ-paris-diderot.fr/articles/document_rapport_et_annexes_de_la_commission_kahane).

Jean-Pierre Kahane's activities in the service of mathematics education also included innumerable lectures for all kinds of public groups, up until just days before his death, at the presentation of prizes for the junior competition organised by the French Mathematical Society (SMF) on 10 June 2017, support for the activities of associations such as MATH.en.JEANS (<https://www.mathen-jeans.fr>), of which he was a member of the scientific committee, his participation on the French commission for the teaching of mathematics (<http://www.cfem.asso.fr>), the French sub-commission of the ICMI, of which he was honorary president, and his interventions with the Academy of Sciences.

He always surprised us with his unwavering combativeness, the strength and clarity of his ideas and his insatiable intellectual curiosity, without forgetting, as Cath-

erine Combelle wrote in a message sent for the Orsay colloquium, the "amused and always benevolent malice of his eyes". He was a great man!



*Michèle Artigue is emeritus professor at the mathematics department of the University Paris Diderot-Paris 7. After a PhD in logic, she progressively moved to mathematics education which has been her main field of research since the mid-eighties. She has been vice-president, then president of ICMI from*

*1998 to 2009, and was awarded the Felix Klein Medal for her life-long research achievements in 2013, and the Luis Santalo Medal for her contribution to the development of mathematics education in Latin America in 2015.*

## Open Problems in Mathematics with John Nash\*

Michael Th. Rassias (Institute of Mathematics, University of Zurich, Switzerland & Institute for Advanced Study, Program in Interdisciplinary Studies, Princeton, USA)

Looking back on life, there are experiences that are considered important and that stand out, whereas others just become memories that fade away. Sometimes, experiences are so precious or even surreal that it takes time to digest that they were even part of your life in the first place, perhaps even experiences that may then influence aspects of your future life. Spending time and collaborating with John F. Nash, Jr., was one such experience.

It all started in September 2014, in one of the afternoon coffee/tea meetings that take place on a daily basis in the common room of Fine Hall, the building housing the Mathematics Department of Princeton University. John Nash silently entered the room, poured himself a cup of decaf coffee and then sat alone in a chair close by. That was when I first approached him and had a really pleasant chat about problems in the interplay of game theory and number theory. From that day onwards, our discussions became ever more frequent. From the common room to his office, to the library and to the beautiful parks of Princeton, our discussions about various topics of mathematics often led us – in one way or another – to some intriguing open problems in mathematics. On one of those occasions, we happened to chat about David Hilbert's famous list of 23 problems. This very math-

ematical/philosophical dialogue influenced our later decision to collaborate on the preparation of a book entitled *Open Problems in Mathematics*. Of course, as we also explain in the preface of the book, we intend neither to compare (in any degree!) nor to associate our list of open problems with that of the great Hilbert. After all, this would almost be blasphemy. Among the readers of this article, the mathematicians surely know about Hilbert's list and the non-mathematicians probably don't. Therefore, a few words follow for the latter group to clarify this mysterious list of 23 problems and how it came about.

Every four years, in one city of the world, the International Mathematical Union hosts the International Congress of Mathematicians (ICM), which is the largest and most prestigious conference devoted to the field of mathematics. The first (official) ICM was held in Zürich in 1897. There were just a few exceptions when the ICM was not organised after a period of four years. The first such exception<sup>1</sup> was 1900, when the 2nd ICM took place in Paris. This happened so that it would coincide with the "Exposition universelle" in Paris and most importantly so that this ICM would mark the opening of the new century of mathematics.

\* This article was previously an invited contribution to "The Institute Letter" of the Institute for Advanced Study (IAS), Princeton, Summer Issue, 2016. It is republished here with the kind permission of the IAS.

<sup>1</sup> The other exceptions when the ICM was not held in its standard four-year cycle were those during World Wars I and II, as well as the one scheduled for 1982 in Warsaw, which was postponed until 1983 due to political turmoil in Poland.



A picture of Nash and M. Th. Rassias, which was captured in the office of John Nash at Fine Hall – the building housing Princeton’s Mathematics Department – around the beginning of their collaboration for the book *Open Problems in Mathematics* (ca. October 2014).

A couple of years prior to this event,<sup>2</sup> the great French mathematician Henri Poincaré proposed to Hilbert the preparation of a list as well as an elaborate presentation of open problems that Hilbert considered to be the most – or among the most – important open problems in the entire field of mathematics at the time: a list of open problems that would help guide generations of bright mathematicians for the coming century. Hilbert accepted Poincaré’s proposition and, for the ICM of 1900, prepared his celebrated list of 23 Problems.<sup>3</sup> This collection of open problems has been extremely influential and has channelled a great deal of important research ever since. Several prominent figures in the history of mathematics from 1900 onwards have invested years of research in their efforts to solve one or other of Hilbert’s problems. One of those luminaries was Nash, who, independently of Ennio de Giorgi, solved Hilbert’s 19th problem.

At the time of my discussion with Nash in September 2014 about Hilbert’s problems, we decided to prepare together the book *Open Problems in Mathematics*. The content of that dialogue can be mainly summarised by the following part of the preface we jointly composed later for our book:

*“It has become clear to the modern working mathematician that no single researcher, regardless of his knowledge, experience and talent, is capable anymore of overviewing the major open problems and trends of Mathematics in its entirety. The breadth and diversity of Mathematics during the last century has witnessed an unprecedented expansion. [...] Perhaps*

<sup>2</sup> The following historical remark was communicated by Dirk Struik (1894–2000) to Themistocles M. Rassias at MIT in 1980. Struik had the privilege of obtaining this information from David Hilbert (1862–1943) himself!

<sup>3</sup> At the actual conference, he presented 10 of the problems, whereas the entire list was published a little later (see: David Hilbert, *Mathematische Probleme*, Vortrag, gehalten auf dem internationalen Mathematiker-Kongress zu Paris 1900, Göttingen, 1900, and an English version: David Hilbert, *Mathematical Problems*, Bulletin of the American Mathematical Society, 8(10)(1902), 437–479).

*Hilbert was among the last great mathematicians who could talk about Mathematics as a whole, presenting problems which covered most of its range at the time. One can claim this, not because there will be no other mathematicians of Hilbert’s caliber, but because life is probably too short for one to have the opportunity to expose himself to the allness of the realm of modern Mathematics. Melancholic as this thought may sound, it simultaneously creates the necessity and aspiration for intense collaboration between researchers of different disciplines. Thus, overviewing open problems in Mathematics has nowadays become a task which can only be accomplished by collective efforts.”*

The above excerpt basically manifests the ideology with which this project was initiated. The day we made the decision to prepare this book, Nash turned to me and said with his gentle voice: “I don’t want to be *just a name* on the cover though. I want to be really involved.” After that, we met almost daily and discussed for several hours at a time, examining a vast number of open problems in mathematics ranging over several areas. During these discussions, it became even clearer to me that his way of thinking was very different from that of almost all other mathematicians I have ever met. He was thinking in an unconventional, most creative way. His quick and distinctive mind was still shining bright into his late 80s. He still had this spark, the soul of a young mathematician. The fact that he moved slowly and talked with a quiet voice had nothing to do with the enthusiasm with which he did mathematics. The scope of the book we were preparing was to publish invited survey papers by world experts presenting the status of some essential open problems in pure and applied mathematics, including old and new results as well as methods and techniques used toward their solution. One “expository” paper is devoted to each problem or constellation of related problems.

After being asked to contribute to this article about the experience of working with John Nash, I started recollecting all those moments from my privileged year as his collaborator and frequent companion. Among all those memories, I recalled a freezing winter day at Princeton that still makes me shiver. It was late January 2015, classes/seminars were cancelled and the university had advised all its members to remain at home due to an upcoming snowstorm. Nash and I also postponed our meeting until the storm had passed. While working from home that night, I received from John Nash an email, which was a kind of account or even a testimony of his career as a problem-solver. Interestingly enough, he didn’t mention his work on game theory for which he is more widely known. It also surprised me that he signed the email with his full name rather than just “John” as he would normally do in our correspondence. This email is enclosed below (see following page).

Months went by, winter passed and our almost daily discussions continued and remained deeply interesting, as well as a source of everlasting inspiration for me. The book was almost ready before John and Alicia Nash left in May for Oslo, where he was awarded the 2015 Abel

Date: 26/01/2015 (22:15:05 EST)  
 From: John F. Nash Jr.  
 To: michailrassias@math.princeton.edu  
 Cc: John F Nash

You replied to all recipients of this message on 26/01/2015 22:45:28.

Text (3 KB)

Dear Michail,

Now, of the little family here, we are clustered in our house for the storm (which I hope will really not be so bad!).

I recently thought of how I can be thought of as a "problem solver" in my mathematical career. There WAS, actually, an existing open problem about the representability of geometrically defined entities through means of algebraic varieties.

So I achieved that, at an early time in my career, and this was a known problem solved.

And then later I published "The Imbedding Problem for Riemannian Manifolds". (This had an error in Part D which was for non-compact manifolds.)

That was certainly an existing "problem" but it was not, for example, defined to be a "Hilbert" problem.

Well, I hope that you and we (in our house), in the next days will have good luck experiencing the snow storm!!

John F. Nash, Jr.

Prize from the Norwegian Academy of Science and Letters. We had even prepared the preface of this volume, which he was so looking forward to seeing published.

On this occasion, I would also like to say just a few words about the man behind the mathematician. In the famous movie *A Beautiful Mind*, which portrayed his life, he was presented as a really combative person. It is true that in his early years he might have been, also having to battle with the demons of his illness. Being almost 60 years younger than him, I had the chance to get acquainted with his personality in his senior years. All the people around him, including myself, can avow that he was a truly wonderful person: very kind and disarmingly simple,

as well as modest. This is the reason why, among friends at Princeton, I used to humorously say that the movie should have been called *A Beautiful Mind and a Beautiful Person*. What was certainly true though was the dear love between John and Alicia Nash, who together faced and overcame the tremendous challenges of John Nash's life. It is somehow a romantic tragedy that fate bound them to leave this life together on their return from Oslo where Nash had received the Abel Prize in May 2015.

One can say that among the mathematicians who have reached greatness, there are some – a selected few – who have gone beyond greatness to become legends. John Nash is one such legend. During a celebration organised at the Department of Mathematics of Princeton University in March 2015 for the announcement that Nash and Louis Nirenberg would share the 2015 Abel Prize, I remember Morgan Kelly from the university's office of communications asking me what it was like to collaborate with John Nash. What I felt then about my collaboration with Nash is what I still feel now. If you were a musician and had an opportunity to work with Beethoven and compose music with him, it would be astonishing. This was the same thing. If a mathematician of the stature of John Nash so generously invests his time and energy in a researcher more than half a century younger than him, it makes you wonder what we should do, when the time comes, for the younger generations of scientists eager to learn and explore.

*Michael Th. Rassias is a member of the Editorial Board of the EMS Newsletter. He was introduced in issue 103 (March 2017) with a short biography.*

## ERME Column

Jason Cooper (Weizmann Institute of Science, Rehovot, Israel), Alison Clark-Wilson (University College London Institute of Education, UK), Hans Georg Weigand (University of Würzburg, Germany)

### YESS-9

The 9th ERME Summer School for young researchers will take place on 20-25 August 2018 in Montpellier, France. The school provides a unique opportunity for young researchers (graduate students and researchers up to 3 years after receiving a PhD) to discuss their research in mathematics education with a board of experts and with fellow students. For more details, please visit the website (<https://yess9.sciencesconf.org/>).

### Introducing CERME Thematic Working Groups 15 and 16 – Teaching and Learning Mathematics with Technology and Other Resources

Group leaders: Alison Clark-Wilson and Hans Georg Weigand

The European Society for Research in Mathematics Education (ERME) holds a biennial conference (CERME), in which research is presented and discussed in Thematic Working Groups (TWGs). We continue here the initiative of introducing the working groups (which began in the September 2017 issue of the Newsletter) focusing on ways in which European research in the field of mathematics education may be interesting or relevant for research mathematicians. Our aim is to extend the ERME community with new participants, who may benefit from hearing about research methods and findings and who may contribute to future CERMEs.

TWGs 15 and 16 are concerned with the roles of technology and other resources (e.g. textbooks and manipulatives) in mathematics education and, in particular,

with the transformative impact of technology on the ways in which mathematics is taught and learned at all levels of education. CERME has been concerned with these issues since its inception in 1999. The separation of teaching and learning was a necessity due to the high level of participation, reflecting the growing prominence of research in the field.

The growth and evolution of these groups is founded upon the inextricable link between mathematical knowledge and the capability afforded by mathematical digital tools such as computer algebra systems, and dynamic geometry and dynamic graphing software applications. The nature of the tools, and the underlying principles that guide their design, are leading to new forms of mathematical representation and syntax; in a sense, technology is transforming what it means to “know” mathematics. Hence, in its early days, the technology group’s analysis of “tools” was framed by two main concerns: *interactions between tool and knowledge* and *interactions among knowledge, tool and the learner* [1].

As these tools began to find their way into mathematics classrooms, they came to have considerable impact on the mathematical practices that teachers and students engaged in. Accordingly, research has moved from experimental “lab” environments, where new tools have been tried and analysed, to real classrooms, creating a third concern: *integration of a tool in a mathematics curriculum and in the classroom* [1].

Digital tools for education have further developed to support the organisation of various aspects of learning and teaching, including digital curricula, communication tools, digital learning environments and learning management systems. There are also tools for the assessment of learning, which include digital testing and automatic analysis and reporting of results. The proliferation of such tools has led to wide interest and involvement amongst teachers and researchers.

Existing digital tools for education bridge and connect many mathematical topics: software such as Geogebra<sup>1</sup> combines computer algebra systems, function plotters, spreadsheets and dynamic geometry software, making multi-modal representations of mathematical objects such as functions (linked numeric, symbolic and graphic representations) readily available; 3D programmes such as Cabri3D<sup>2</sup> provide new access to 3D geometry; and new developments such as augmented or virtual reality are not only opening new perspectives on geometry but are also supporting connections with other sciences such as physics, biology, chemistry and engineering, and are providing the basis for the emerging field of *embodied cognition*.

The range of topics that this academic field addresses includes: design of tools and resources (e.g. linked multi-modal representations); impact (epistemic and affective) of digital tools on students’ and teachers’ experiences (e.g. implications of dynamic mathematical applications); ongoing critique of the nature of mathematical and cur-

ricular knowledge in light of digital tools and resources (e.g. a trend to link mathematics and computer science through the inclusion of technology-based algorithmics in mathematics curricula); and implications of the above for teacher preparation and for ongoing professional support.

Collaboration among communities is common, in an attempt to bridge the diverse domains of expertise that the field draws upon. Often, seeds of ideas for new digital tools originate in mathematics classrooms or through curricular design, ideas that are then honed and developed through longer term collaborative projects. However, participants in TWGs 15 and 16 have been almost exclusively from the field of mathematics education. As technology makes its way into universities, the relevance of this field for higher education is growing. Teachers of university-level mathematics have much to contribute to this field of research and also much to gain from its findings. Both TWGs warmly welcome participation and contributions from the wider mathematics community.

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*Jason Cooper is a research fellow at the University of Haifa’s Faculty of Education. He is also a researcher at the Weizmann Institute’s Department of Science Teaching. His research concerns various aspects of teacher knowledge, including roles of advanced mathematical knowledge in teaching and contributions of research mathematicians to the professional development of teachers. He has been a member of the ERME Board since 2015.*



*Alison Clark-Wilson is a principal researcher in mathematics education at the University College London Institute of Education. Her research focuses on the design, enactment and impact of information technology on mathematical learning at upper primary and secondary levels, on teachers’ knowledge and pedagogical use of information technology and on models for teachers’ collaborative learning.*



*Hans Georg Weigand is a professor of mathematics education at the University of Würzburg (Germany). He studied mathematics and physics and taught for six years at a German grammar school (Gymnasium). He is interested in the use of new digital technologies in mathematics education and in teacher education. He has written books on didactics of algebra, geometry and calculus, and on the use of computers in mathematics education.*

<sup>1</sup> [www.geogebra.org](http://www.geogebra.org).

<sup>2</sup> <http://cabri.com/en/>.

# An Update on Time Lag in Mathematical References, Preprint Relevance, and Subject Specifics<sup>1</sup>

Adam Bannister and Olaf Teschke (both FIZ Karlsruhe, Berlin, Germany)

Almost five years ago, we reported in this column on what turned out to be the most extensive study at the time of citation delay of mathematics publications [1]. Back in 2012, we were quite satisfied to have reference data available for about 50,000 EuDML articles and 170,000 zbMATH articles; this already accounted for a larger proportion of the mathematical literature when compared to commercial citation databases, which tend to be less comprehensive for the field of mathematics. Today, however, the situation has changed considerably. Further digitisation efforts and improved availability of reference data now allow the interrogation of about 20 million references for more than 900,000 mathematical articles in zbMATH. Furthermore, linking them to available zbMATH entries and arXiv submissions also facilitates an analysis of subject specifics and preprint effects. While a detailed investigation is beyond the limits of this column, we take the opportunity to outline some aspects that become visible when taking extended data into account.

## Growing longevity confirmed

With much more data available, the trends discovered in [1] have been confirmed. The graph in Figure 1, covering

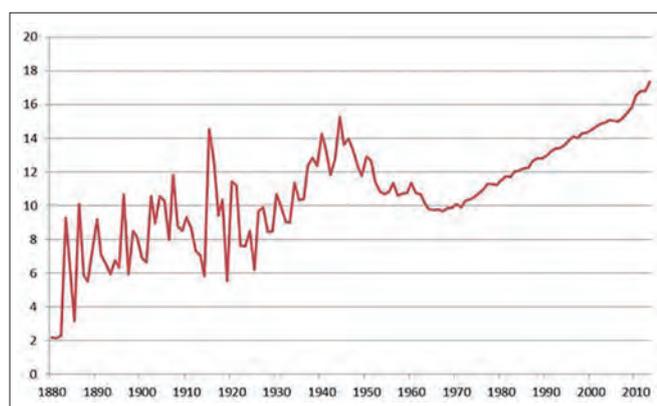


Figure 1. Average reference lag per year based on about 11 million matched zbMATH data.

<sup>1</sup> Several aspects of this report were discussed in the framework of a panel discussion on bibliometrics on Sep 12th, 2017, at the 19th ÖMG Congress and Annual DMV Meeting held in Salzburg. At this panel, representatives of the DMV, EMS, ÖMG, UMI, and zbMATH discussed the various challenges connected with the increased use of bibliometrical measures throughout Europe, and outlined practical approaches. Results of this discussion will also be published in a forthcoming article at the *Mitteilungen der Deutschen Mathematiker-Vereinigung*.

the extended period from 1880 to 2017, for which reference data are now available, looks very similar to the one from [1]. However, some additional effects have become evident: the extremal points are slightly smoothed and, in particular, war effects, which were extensively discussed in [1], are less emphasised. This can be explained by the current database being broader, the restriction to publication years originating from matched references (in 2012, we had to extract plausible publication years from the reference strings since the matched data were still too sparse) and the bias originating from a dominance in 2012 of Springer data (which tended to be more affected by the World Wars). Still, the conclusion from 2012 holds that since World War II there has been a steady increase in the average citation longevity. Indeed, the prediction of further growth we made back then has become a reality.

## Half-life: Infinity?

An interesting effect resulting from both the increased number of publications (and hence references) and growing longevity is that the notion of (absolute) citation half-life doesn't seem to be reasonable for mathematical publications. This is due to the fact that the distribution is heavily skewed toward a thick, long tail, without any indication of a convergence to zero. Hence, formal computation of half-lives actually leads to numbers much larger than half the period since publication, which still grow by about eight months every year. Figure 2, showing the distribution of references to publications for some fixed publication years, is typical. One should note, however, that this is mainly influenced by publication

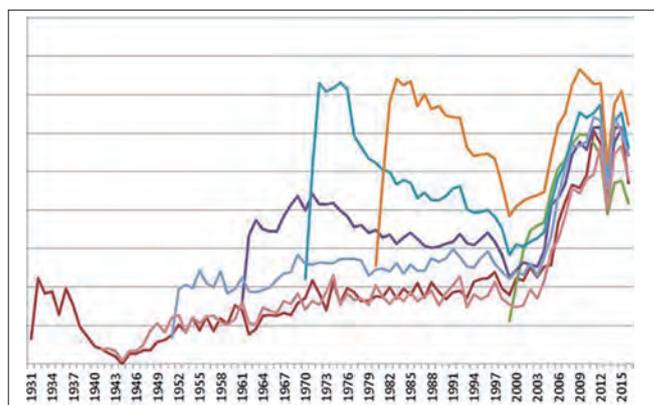


Figure 2. Long-term distribution of absolute citation numbers for fixed publication years.

and corresponding citation growth; the relative citation frequency (adjusted for the available number of references) is indeed declining in the long term. Some aspects of this will be addressed in the last section.

It might be worth noting that this is contrary to the general development in the sciences: a study [2] considering the fields of medicine, molecular biology, chemistry and physics found that there has recently been a quicker decay of citation numbers, which, according to the authors, is due to a general increase in the number of published papers that supersede earlier results.<sup>2</sup> There is no indication of a similar effect in mathematics.

### Preprint citations

While the long-term pattern seems to be very stable, it goes without saying that the publication landscape has changed significantly over the last few decades. The diagrams above, showing an average citation lag larger than 15 years for traditional publications, indicate that it may yet be too early to see the effects of changing publication behaviour at this level. It is natural to ask about these patterns when arXiv preprints are included. As shown earlier in this column [3], the arXiv has established itself as the standard preprint repository for many areas in mathematics, often preceding the actual publication by several years. Taking arXiv submission years into account, one might be able to get rid of backlog effects affecting the publication year. Since the arXiv version is matched to the zbMATH entry and it is easy to identify the arXiv submission year, one might wonder about the results when taking preprints into account.

The comparison, however, shows no significant difference in long-term citation behaviour. Of course, the average citation lag is initially much smaller for references to the arXiv (which, by definition, has no submission years before 1991) but it very closely resembles the behaviour

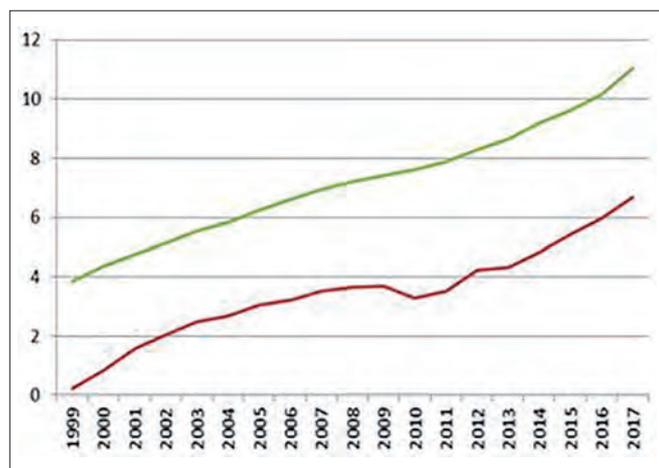


Figure 3. Time lag for references to arXiv submissions compared to publications after 1991.

<sup>2</sup> It is not fully clear whether this study might actually miss a large number of references due to not comprehensively covering commercial citation databases. Computed formally, the “half-lives” of mathematical publications also seem to be declining rapidly for recent work, if only due to the fact that the bulk of citations are likely to happen in the future.

of references to traditional publications when publications before 1991 are omitted (see Figure 3). Any local differences can be linked to the fact that the arXiv corpus has grown more quickly and shows a different subject pattern (as explained in [3]). In particular, the distribution of references to arXiv submissions for a fixed year shows the same right-skewed pattern related to the “immortality” of mathematical research (Figure 4). As a conclusion, this would support handling citations to the arXiv on an equal footing with those of traditional publications, taking advantage of avoiding the publication gap associated to journal backlog.<sup>3</sup>

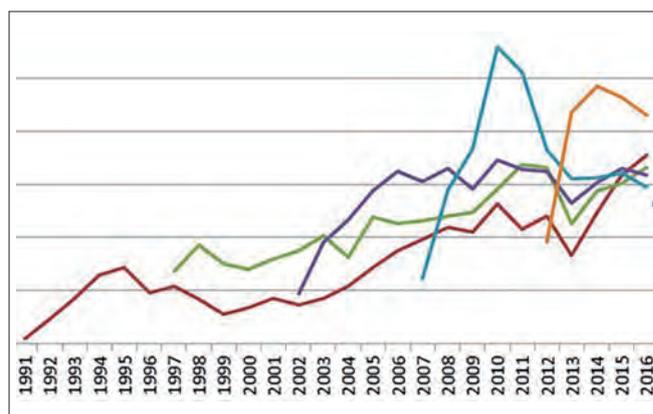


Figure 4. Absolute citation numbers for fixed arXiv submission years.

### One step further: subject specifics

One might wonder whether it is possible to differentiate this general picture further by taking mathematical subjects into account. Matching citations to zbMATH provides MSC information and perhaps the first natural question is whether the topic is reflected by the citation network. Figure 5 shows that there is indeed a strong concentration in the diagonal (which means that the bulk of references go to papers with the same MSC), although there obviously exist further cross-references that should

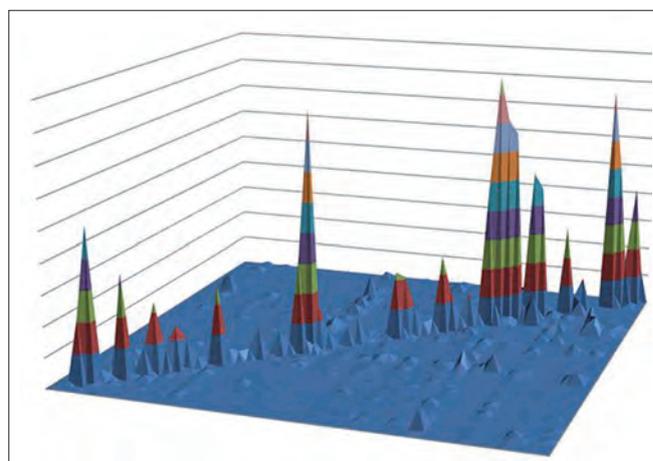


Figure 5. Cross-MSC citation map.

<sup>3</sup> One caveat to be aware of is that arXiv submissions are usually not yet peer-reviewed, whereas only citations of arXiv submissions by peer-reviewed papers were taken into account.

not be neglected in detailed studies. For a first impression, however, it might be justified to restrict to MSC-preserving citations.

### Subject specifics

Since the number of publications and references is very unevenly distributed for different mathematical subject classes, it makes sense to study long-term referencing behaviour within the main MSC classes subject to availability of citation data. This adjustment also aims to eliminate the growth effects mentioned in Section 2. Figure 6 shows the relative distribution of references for several mathematical subjects in relation to the gap years (from 0 to 24).

It should be noticed that, at least for the relative citation frequencies, there is a long-term decay and also a clearly visible long tail. The only clearly different distribution belongs to quantum mechanics, where the initial relative citation rate is much higher before descending much more quickly. For the remaining subjects (with such diverse areas as number theory, algebraic geometry, partial differential equations, functional analysis, mathematical statistics and mathematical programming), the

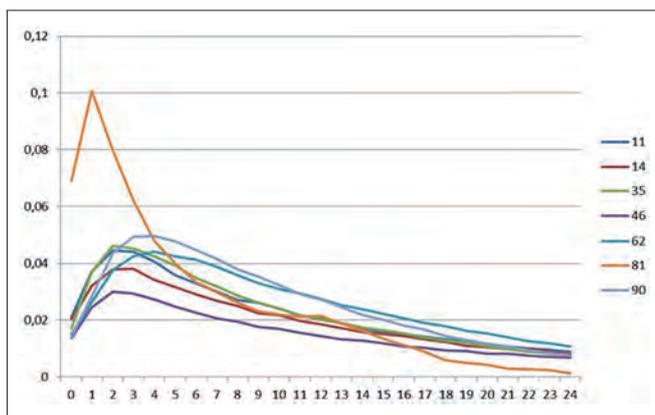


Figure 6. Relative time lags for MSC-preserving citations.

long-term behaviour is surprisingly similar, although there exist significant initial differences for the relative citation frequency. Therefore, a computation of relative citation half-lives on this basis yields somewhat different results for the mathematical subjects (mostly between 7 and 10 years, with the exception of mathematical physics, as shown in Figure 7). Even in this setting, it once more becomes obvious that the most widely used citation metrics (like impact factors, which usually consider a span of at most five years) miss the bulk of relevant information.

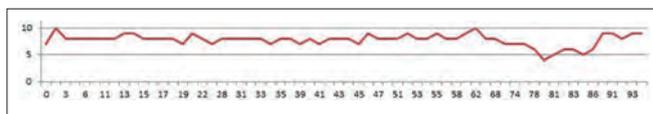


Figure 7. Relative half-lives of MSC-preserving citations.

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Adam Bannister [[adam.bannister@fiz-karlsruhe.de](mailto:adam.bannister@fiz-karlsruhe.de)] has a postgraduate diploma in Geographic Information Systems and currently works on the Scalable Author Disambiguation for Bibliographic Databases at zbMath in cooperation with Schloss Dagstuhl and Heidelberg Institute

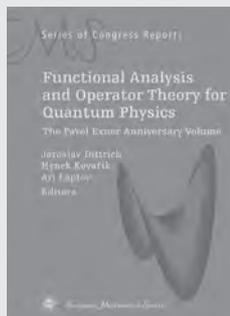
for Theoretical Studies.

Olaf Teschke [[olaf.teschke@fiz-karlsruhe.de](mailto:olaf.teschke@fiz-karlsruhe.de)] is a member of the Editorial Board of the EMS Newsletter, responsible for the zbMATH Column.



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#### Functional Analysis and Operator Theory for Quantum Physics. The Pavel Exner Anniversary Volume

Jaroslav Dittrich (Czech Academy of Sciences, Rez-Prague, Czech Republic), Hynek Kovařík (Università degli Studi di Brescia, Italy) and Ari Laptev (Imperial College London, UK), Editors

ISBN 978-3-03719-175-0. 2017. 595 pages. Hardcover. 17 x 24 cm. 98.00 Euro

This volume, dedicated to Pavel Exner on the occasion of his 70th anniversary, collects contributions by numerous scientists with expertise in mathematical physics, particularly in problems arising from quantum mechanics. The questions addressed cover a large range of topics. A lot of attention was paid to differential operators with zero range interactions, which are often used as models in quantum mechanics. Several authors considered problems related to systems with mixed-dimensions such as quantum waveguides, quantum layers and quantum graphs. Eigenvalues and eigenfunctions of Laplace and Schrödinger operators are discussed too, as well as systems with

adiabatic time evolution. The book provides a wide variety of techniques from functional analysis and operator theory. Altogether the volume presents a collection of research papers which will be of interest to any active scientist working in one of the above mentioned fields.

# Book Reviews



Jean-Pierre Serre

**Finite Groups:  
An Introduction**

International Press, 2016

190 p.

978-1-57146-327-2

Reviewer: Jean-Paul Allouche

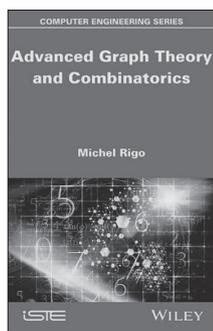
I have always been fascinated by finite groups. Chief among my favourite properties is the influence of the “arithmetic nature” of the cardinality of a group on its structure. The most basic property is that every group of prime order must be cyclic. Possibly less well known are: the result that there exists only one group of order  $n$  (hence the cyclic group of order  $n$ ) if and only if  $n$  is prime to the number of integers less than  $n$  and prime to  $n$ , i.e.,  $\gcd(n, \varphi(n)) = 1$ , where  $\varphi$  is the Euler function; and the result that all groups of order  $n$  are abelian if and only if  $n$  is cube-free and  $\gcd(n, \psi(n)) = 1$ , where, for distinct prime numbers  $p_j$ , one has, by definition,  $\psi(\prod p_j^{a_j}) = \prod (p_j^{a_j} - 1)$ . Possibly even less well known is the fact that arithmetic characterisations exist for integers  $n$  such that all groups of order  $n$  have property  $\mathfrak{P}$ , where  $\mathfrak{P}$  is one of the properties: to be nilpotent, to have an ordered Sylow-tower, to be supersolvable or to be metacyclic [see G. Pazderski, Die Ordnungen, zu denen nur Gruppen mit gegebener Eigenschaft gehören, *Arch. Math. (Basel)* **10** (1959), 331–343, and the survey L. Crew, On the characterization of the numbers  $n$  such that any group of order  $n$  has a given property, <https://arxiv.org/abs/1501.03170> (2015)]. A last result of this kind is the famous (non-)characteristic property of odd integers that every group of odd order is solvable: this result is the Feit-Thomson theorem, W. Feit, J. G. Thompson, Solvability of groups of odd order, *Pac. J. Math.* **13** (1963), 775–1029.

Of course, though the proof of the result just cited took over 250 pages, there is much more in the theory of finite groups than the links between the structure of a finite group and the arithmetic properties of its cardinality. A bible on finite groups is the book of B. Huppert and N. Blackburn, which consists of three volumes, together about 1800 pages.

The first volume “Endliche Gruppen” (by B. Huppert alone) only exists in German – there was a discussion in the 1970s between C. Chevalley and two students about the possibility of translating this book into French but this was never accomplished (the two students were B. Randé and the author of this review). Clearly, such a huge bible is not adapted for beginners. Among books on finite groups that are more accessible to students is the book under review, which is based on a course given in 1978–1979 by J.-P. Serre at the École Normale Supérieure de Jeunes Filles. At that time, there were only male students at the École Normale Supérieure (de la rue d’Ulm), while the female students were at the École Normale Supérieure de Jeunes Filles (in Sèvres); similarly, there was the École Normale Supérieure de Saint-Cloud for men and the École Normale Supérieure de Fontenay-aux-Roses for women (these two became the École Normale Supérieure de Lyon), while the École Normale Supérieure de Cachan was for both women and men.

The book under review succeeds in giving, in about 160 pages, a nice, fairly comprehensive view of what a (good) student should know about finite groups. The author skips the definition of a group, which is supposedly well known, and starts from group actions, filtrations, the Jordan-Hölder theorem, the Goursat lemma and the Ribet lemma. After these preliminaries, there is a chapter on Sylow theorems, a chapter on solvable and nilpotent groups, a chapter on group extensions, a chapter on Hall subgroups, a chapter on Frobenius groups, one on transfers, one on characters, one on the finite subgroups of  $GL_n$  and one on “small” groups. We cannot resist citing an application given in Chapter 7 to a “baby case of the Feit-Thomson theorem”, namely that *any group of odd order < 2000 is solvable*. Each chapter is carefully constructed and written, and each is followed by several exercises that either exemplify/illuminate the notions and results just explained or go further by stating some nice results not given in the corresponding chapter. The reader can therefore practise with finite groups (and possibly guess the author’s favourite exercises). This book is highly recommended to students who want to learn the theory of finite groups but also to colleagues who would like to refresh their knowledge of the subject: they might well learn some nice results that they were not expecting to be interested in and they might even become addicted to finite groups.

*Jean-Paul Allouche is a member of the Editorial Board of the EMS Newsletter, responsible for the Book Reviews section.*



Michel Rigo

**Advanced graph theory and combinatorics.**

John Wiley & Sons, 2016

xiv, 268 p.

ISBN 978-1-84821-616-7

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Reviewer: Venkataraman Yegnanarayanan

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*The Newsletter thanks zbMATH and Venkataraman Yegnanarayanan for the permission to republish this review, originally appeared as Zbl 1369.05001.*

It is not an easy task nowadays to write a standard text book and earn a huge readership. It is because of very stiff neck to neck competition and availability of any amount of information on anything through very convenient free e-source. Given such a background it is definitely a laudable attempt by the author and very carefully he has chosen the topics in modern graph theory and gave it a fascinating treatment using his rich experience in both teaching and research. A careful and clever proof reading and the comments he has solicited from both the experts as well his peers has made this book a wonderful master piece.

His journey in this book, by beginning his narration on Chapter 1, his first encounter with graphs covering digraphs, regular graphs, multigraphs, unoriented graphs, algorithmic treatment of Hamiltonian and Eulerian graphs with lot of illustrations from practical real life situations, for instance, Google's PageRank graphs associated with social networks like Facebook or Twitter, collaboration graphs and finding shortest path for GPS device, problem of assigning IP addresses, problems related to Cayley graphs like: given any two words written over an alphabet of generators and their inverses is it possible to decide algorithmically whether two words represent the same element of the group etc., and a crisp proof on associated results have provided a clear message for the readers regarding what is in store in the remaining chapters.

In Chapter 2, the author has just provided a superficial view of complexity theory and explained briefly the class P, class NP, class NP-complete and NP-hard problems and listed a set of nine problems from the original set of sixty five graph theory problems listed in the famous M. R. Garey and D. S. Johnson's book [*Computers and intractability. A guide to the theory of NP-completeness*. San Francisco: W. H. Freeman and Company (1979; Zbl 0411.68039)]. The nine problems are vertex cover, dominating set, planarity, clique, independent set, graph homomorphism, degree-bounded connected subgraph, degree-constrained spanning tree, and monochromatic triangle and the reason for the same is they will be used widely in the subsequent chapters.

In Chapter 3, the author concentrates only on Hamiltonian graphs and hence the graphs are mostly simple. Besides giving the standard results of Dirac, Ore, and Chvatal the author also uses de-Bruijn words to devise a nice magic trick. That is, a magician asks a group of spectators to cut a deck of playing cards. Then each of six spectators picks one card, each time, the first of the heap without revealing anything to the magician. The first five loudly announces only the color of their card. The magician then guesses the exact value of the sixth card. He can even find the exact value of the other five distributed cards. How it is a de-Bruijn word of order 5. Inquisitive readers are invited to read to know further.

In Chapter 4, the author covers BFS and DFS while discussing trees and acyclic graphs. He also proves an interesting result that a finite multigraph has a topological sort if and only if it has no cycle.

In Chapter 5, he gives a new treatment which makes this book different from other such books, in explaining how to extract new graphs from old ones. Another new concept not discussed that often in other similar books is that of unravelling of a finite digraph from a randomly chosen vertex on it. It is actually an infinite tree (assuming  $G$  has at least one cycle) whose vertices are in 1-1 correspondence with the set of walks in  $G$  starting from the chosen vertex.

In Chapter 6, it is a usual discussion on planar graphs with routine results and a noteworthy among them is: a graph  $G$  is a skeleton of a convex polyhedron if and only if it is planar and 3-vertex connected. He also discussed a special case of the so called Robertson–Seymour theorem namely, a multigraph  $G$  is planar if and only if it does not have  $K_5$  and  $K_{3,3}$  as a minor.

Chapter 7 is devoted for graph coloring, chromatic number, chromatic polynomial and Ramsey numbers. An interesting lemma rarely found in other similar books is: There is a 1-1 correspondence between the set of simple graphs with  $n$  vertices and the set of edge-colorings of  $K_n$  with two colors.

From Chapter 8 onwards, the author jumps to more advanced treatment of the subject by making use of the powerful concept of associating a matrix with a graph and the machinery of linear algebra and matrix computations. He provides necessary prerequisites and takes readers to Hoffman's theorem. A multigraph  $G$  with no loop is connected and regular if and only if a square matrix  $J$  with all entries equal to 1 belongs to the algebra of polynomials of adjacency matrix of  $G$  over the set of complex numbers  $C$ . He concludes this chapter with a deep result: For a digraph  $G$  with no loops and indegree of each of its vertices at most 1, the  $(v,u)$  minor of the indegree matrix  $D(G)$  is equal to 1 if  $G$  is an arborescence rooted at  $v$ .

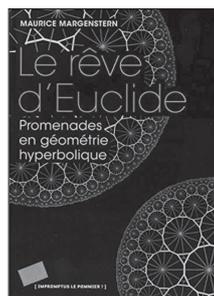
Perron–Frobenius theory is discussed in Chapter 9 and this is a major highlight of this wonderful book. It covers topics such as Perron eigenvalue, irreducible graphs, asymptotic properties of a primitive matrix, graphs with primitive components. etc. He moves towards Perron–Frobenius theorem and its applications through some exciting concluding examples.

In the last Chapter, he explains very crisply Google's PageRank concept. He defines nicely the Goggle matrix and said how to harvest the primitivity of the Google matrix He also provided probabilistic interpretation and remarked that Google does not follow blindly the ranking computed with the PageRank.

Overall it is a very exciting learning experience for the readers especially the researchers and I strongly recommend this book as a compulsory course book for those who pursue research in the area of computer science and engineering leaning on the ideas of advanced graph theory.



*Prof. Dr. V. Yegnanarayanan earned his Ph.D degree in Mathematics from Anna-malai University in 1996. His area of specialization is graph theory. He has 29 years of experience in teaching and research. He has so far authored 156 research papers in referred international/national journals and in proceedings of international and national conferences. He has delivered a number of invited talks and finished research projects funded by Government of India in his area of expertise.*



Maurice Margenstern

**Le rêve d'Euclide. Promenades en géométrie hyperbolique**

Éditions Le Pommier, 2015  
220 p.  
ISBN 978-2-7465-0775-3

Reviewer: Faruk Abi-Khuzam

*The Newsletter thanks zbMATH and Faruk Abi-Khuzam for the permission to republish this review, originally appeared as Zbl 1367.51003.*

Take a triangular piece of paper or cardboard. Nip off the three corners with scissors, and line up the pieces. A straight edge becomes apparent. Repeat the experiment with different triangular shapes, and the same thing occurs. This is the experimental way to see that “the sum of the angles of a triangle is a straight angle”. But it took the deep insight of Euclid to see that, while this statement was experimentally plausible, it, or something logically equivalent to it, had to be assumed in setting down the axiomatic foundations of geometry. Thus, the fifth postulate in his “*Elements*” was born. It is the axiom equivalent to the following:

*“Through a point outside a straight line, there is one and only one straight line parallel to the given straight line.”*

Hyperbolic geometry is the history of this postulate, its birth, the inspiration it gave to those who wanted to reverse it, the attempts of many to subdue it, and the break up of its antagonists into two camps: those who did not believe in the uniqueness of parallels, and those who did not believe even in the existence of parallels. The first group comprised among its adherents a distinguished list of mathematicians, whose re- searches ultimately led to discovery, or invention, of Non-Euclidean geometry. Undoubtedly a monumental invention, not only because of its originality, but also because of its far reaching effects on liberating the mind from established and rigid authority. Naturally, the story of this most mag-

nificent creation cannot be told without going chronologically through the contributions of the first group, and the author most certainly does this. Although it might be said that the history is by now very well known, the old paths extremely well troden, and yet somehow, at least for the enthusiast, there always appears an element of surprise or drama in retelling the story of what is now called hyperbolic geometry.

In his book, the author has as one of his goals, to paint, for the layman as well as for the expert, a portrait of the fascinating aesthetic aspects, as well as prevalent social forces, that went into shaping the history of this most magnificent invention of the human mind. In so doing, the author is led to describe the tension that ensues between hard held, almost religious beliefs, and the gradual dawning of the truth on the players, beckoning them to reconsider. Until finally, helped by previously sown seeds, someone breaks loose, and a revolution occurs.

The author starts, gently, by taking us on a journey in time, from the original inceptions implicit in works of Posidonius (135 B.C., 51 B.C.), Omar Khayyam (1048–1123), Nasir-al-Din Tusi (1201–1274), and John Wallis (1616–1703): Then, he presents the serious attempts by Saccheri (1667–1733), and Lambert (1728–1777), to prove the fifth postulate from the first four axioms, all the way to the formal and serious introduction of a new geometry by Lobatchevsky and Bolyai the son, whose father was a friend of Gauss, the dominating scientific figure at the time. The proposed new geometry of Lobatchevsky and Bolyai, allows for more than one parallel to a given line from an outside point. It is a geometry where the angle sum of a triangle is always less than a straight angle, and it is, supposedly, void of any contradiction.

The question of consistency – is the new geometry void of contradictions – could not be answered. Instead, two of its models, later supplied by Poincaré and Beltrami serve to prove that it is no less and no more consistent than the good old Euclidean geometry. So it is more realistic to ask, as Poincaré suggests, which geometry better describes the physical world. But if one were to use hyperbolic geometry for this description, one had to be ready to face the views of the the Church and Kant. Bolyai and Lobatchevsky were not to be hindered from their ownership of this new invention, by these two formidable

authorities. But the drama, for one of them, Bolyai, was to come from an unexpected source: Gauss! For Bolyai the father, had written to Gauss about the discovery of his son. To his dismay, no acknowledgement was to be forthcoming from Gauss who wrote claiming to have considered, much earlier, ideas on geometry similar to those of Bolyai. But Gauss had added, in order to justify his refrain from having published such work, a curious remark on “younger researchers” who “do not have clear ideas on such questions”, and about his fear of “misunderstanding” if he were to publish his findings. Thus, even the great Gauss was not immune to the dictates of the two main authorities at the time. In this connection, the reader will be gratified reading the author’s investigations on the question: did Gauss really make the discovery of hyperbolic geometry before Lobatchevsky and Bolyai?

The story does not end here. For then comes Riemann who considers the other alternative of no parallels. He replaces the infinite by the notion of absence of boundary, and lays the foundations for elliptic, and more generally, differential geometry.

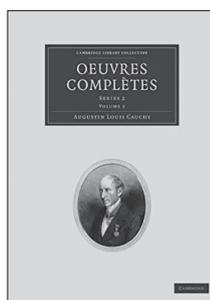
The book has a lot to offer both to the amateur and the expert. The reader interested in technical details of this new geometry will find ample details in Chapter 4, “in the heart of hyperbolic geometry”. This would be

quite educational if combined with the discussion of classical problems of Euclidean geometry in the new setting. If interested only in the history, Chapter 2 will be a pleasure to read and learn of the drama as it unfolded. There is also an exposition of what the author calls “hyperbolic cellular automata”, and a most appealing approach to the tiling of the hyperbolic plane, as in the Poincaré disk model. The reader need not be an expert to admire the beauty in this approach, as well as, in the colored pictures illustrating this idea and punctuating the pages of this book.

In this thought provoking delightful book, the reader will be led to consider deep philosophical and scientific questions, that lie at the heart of human thought, how it evolves, how it struggles, and in rare historical moments, succeeds in surmounting formidable difficulties, and reveals the existence of a world never anticipated or even imagined before.



*Faruk F. Abi-Khuzam is professor of mathematics at the American University of Beirut. His research interests are in complex analysis and geometric Inequalities, with occasional excursions into harmonic analysis and minimal surfaces.*



Augustin-Louis Cauchy

**Œuvres complètes.  
Series 2. Vol. 2**

Cambridge University Press, 2009  
421 p.  
ISBN 978-1-108-00291-2

Reviewer: Ádám Besenyei

*The Newsletter thanks zbMATH and Ádám Besenyei for the permission to republish this review, originally appeared as Zbl 1367.00002.*

The complete works of Augustin-Louis Cauchy, one of the most prolific mathematicians in history, fill 27 volumes in two series, the publication of which took nearly a century from 1882 to 1974. The next to last volume in this process was Volume 2 of Series 2 in 1958 edited by the French historian of science, René Taton. It contains the articles that were published by Cauchy in Liouville’s *Journal de Mathématiques pures et appliquées*, *Bulletin de Férussac*, *Bulletin de la Société philomatique*, Gergonne’s *Annales de mathématiques* and *Correspondance sur l’École Polytechnique*.

The present book under review is an unaltered reprint of the original Volume 2 of Series 2. The majority of the papers are connected to mathematics and cover vari-

ous topics from number theory and through differential equations to complex analysis.

Though, Cauchy published few in these journals – he preferred the periodical of the French Academy and the journal founded by himself –, the reader might find some attractive papers in this volume. In this regard, we underline the article *Sur le développement des fonctions en séries et sur l’intégration des équations différentielles ou aux différences partielles* where Cauchy discussed some anomalies concerning power series expansions and this was the first appearance of the function  $\exp(-1/x^2)$  (extended continuously to  $x = 0$ ) as a remarkable example of a non-zero function possessing the null series as its expansion about the origin.

It happened often that Cauchy published short abstracts or extracts in the aforementioned journals as announcements of results that appeared in full memoirs elsewhere. Even the modern reader might find many of these brief notes interesting:

- *Sur les polyèdres* is one of Cauchy’s earliest published notes as a student reporting on his results concerning Euler’s polyhedral formula and the existence of only four regular star polyhedra.
- *Démonstration générale du théorème de Fermat sur les nombres polygones* is a communication of a general proof of Fermat’s conjecture on polygonal numbers which made Cauchy famous in 1815.
- *Sur les intégrales définies prises entre des limites imaginaires* from 1825 is an extract of Cauchy’s fundamental pamphlet dealing with complex integrals.

- *Sur la mécanique céleste et sur un nouveau calcul qui s'applique à un grand nombre de questions diverses* is a preliminary abstract of Cauchy's famous first Turin memoir where he developed a technique for estimations of power series remainders, known today as the method of majorants [F. Smithies, *Cauchy and the creation of complex function theory*, Cambridge: Cambridge University Press (1997; Zbl 0883.01020)].

Besides the mathematical papers, some works connected to the theory of light and continuum mechanics can also be found in this volume. In fact, Cauchy's first published paper in the subject of continuum mechanics was the brief extract *Recherches sur l'équilibre et le mouvement intérieur des corps solides ou fluides, élastiques ou non élastiques* where he proved the existence of a stress tensor and some of its properties [C.A. Truesdell, *Rev. Hist. Sci.* 45, No. 1, 5–24 (1992; Zbl 0757.01014)]. As Freudenthal remarked [H. Freudenthal, *Cauchy*, Augustin-Louis, in: Charles C. Gillispie (ed), *Dictionary of Sci-*

*entific Biography*, 3, New York: Charles Scribner's Sons, pp. 131–148 (1970–1980)]: “Rarely has a broad mathematical theory been as fully explained in as few words with as striking a lack of mathematical symbols. Never had Cauchy given the world a work as mature from the outset as this.”

Apparently, this volume provides an essence of the diversity of Cauchy's works. Readers interested in Cauchy's original papers and in 19th century mathematics will certainly benefit from the book.



*Ádám Besenyei earned his Ph.D. in applied mathematics in 2009 from Eötvös Loránd University in Budapest, Hungary and he is currently an associate professor there. His mathematical interests include inequalities, differential equations and history of mathematics. He enjoys reading primary sources and often tries to incorporate them in his teaching.*

## Personal Column

Please send information on mathematical awards and deaths to [newsletter@ems-ph.org](mailto:newsletter@ems-ph.org).

### Awards

**Denis Serre**, professor at the University Claude Bernard in Lyon, was awarded the **Jacques-Louis Lions Prize 2017** of the French Academy of Science dedicated to an outstanding mathematician working in the area studied by Jacques-Louis Lions.

**Gabriel Peyré**, professor at the Ecole Normale Supérieure in Paris, was awarded the **Blaise Pascal Prize 2017** of the French Academy of Science for his major contribution to the study of image processing.

**Marc Bordenave**, CNRS researcher at the Institut de Mathématiques de Toulouse, was awarded the **Marc Yor Prize 2017** sponsored by the French Academy of Science. This prize rewards a young mathematician working in probability in France.

**Karine Beauchard**, professor at the Ecole Normale Supérieure de Rennes, was awarded the **Michel Montpetit Prize 2017** of the French Academy of science for her major contribution to the study of the control and analysis of partial differential equations in particular the Schrödinger equations and hypoelliptic equations.

**Hugo Duminil-Copin** was awarded the **Prix Line et Michel Loève 2017** and the **Grand Prix Jacques Herbrand 2017** of the Académie des Sciences for his results in probability theory.

The Austrian Mathematical Society awarded its **2017 Promotion Prize for Young Scientists** to **Michael Eichmair** for out-

standing achievements in differential geometry and general relativity.

The **European Prize of Combinatorics 2017** was awarded to **Christian Reiher** (University of Hamburg) for his profound result in extremal and probabilistic combinatorics, particularly for his solution of the Kemnitz conjecture on lattice points and the Lovasz-Simonovits clique density problem, and to **Maryna Viazovska** (EPFL) for her deep contributions to spherical designs and particularly for the solution of the sphere packing problem in dimensions 8 and 24.

Professor **Gerd Faltings**, director at the Max Planck Institute for Mathematics in Bonn and member of the Hausdorff Center for Mathematics is receiving the **Cantor Medal 2017** of the German Mathematical Society (Deutsche Mathematiker-Vereinigung, DMV) for his outstanding scientific achievements over many years.

### Deaths

We regret to announce the deaths of:

**Ymer Merovci** (9 April 2017, Pristina, Kosovo)

**Jean-Pierre Kahane** (21 June 2017, Paris, France)

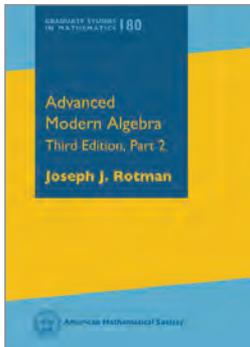
**Seppo Uolevi Rickman** (16 August 2017, Helsinki, Finland)

**Johan Ernest Mebius** (24 August 2017, Rotterdam, The Netherlands)

**Gérard Tronel** (25 August 2017, Ussel (Corrèze), France)

**Heinz Günther Tillmann** (28 August 2017, Münster, Germany)

**Vladimir Alexandrovich Voevodsky** (30 September 2017, Princeton, USA)



## ADVANCED MODERN ALGEBRA

### Third Edition, Part 2

Joseph J. Rotman, *University of Illinois at Urbana-Champaign*

This is the second part of the new edition of *Advanced Modern Algebra*. Compared to the previous edition, the material has been significantly reorganized and many sections have been rewritten. The book presents many topics mentioned in the first part in greater depth and in more detail. The five chapters of the book are devoted to group theory, representation theory, homological algebra, categories, and commutative algebra, respectively. The book can be used as a text for a second abstract algebra graduate course, as a source of additional material to a first abstract algebra graduate course, or for self-study.

*Graduate Studies in Mathematics, Vol. 180*

Oct 2017 549pp 9781470423117 Hardback €100.00



### Part 1

*Graduate Studies in Mathematics, Vol. 165*

2015 709pp 9781470415549 Hardback €95.00

### Parts 1 and 2

*Graduate Studies in Mathematics, Volume 165/180*

Oct 2017 1264pp 9781470441746 Hardback €184.00

## ALICE AND BOB MEET BANACH

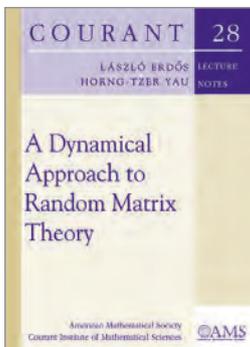
### The Interface of Asymptotic Geometric Analysis and Quantum Information Theory

Guillaume Aubrun, *Université Claude Bernard Lyon 1* & Stanislaw J. Szarek, *Case Western Reserve University*

*Alice and Bob Meet Banach* is aimed at multiple audiences connected through their interest in the interface of QIT and AGA: at quantum information researchers who want to learn AGA or apply its tools; at mathematicians interested in learning QIT, especially the part that is relevant to functional analysis/convex geometry/random matrix theory and related areas; and at beginning researchers in either field. Moreover, this user-friendly book contains numerous tables and explicit estimates, with reasonable constants when possible, which make it a useful reference even for established mathematicians generally familiar with the subject.

*Mathematical Surveys and Monographs, Vol. 223*

Oct 2017 413pp 9781470434687 Hardback €123.00



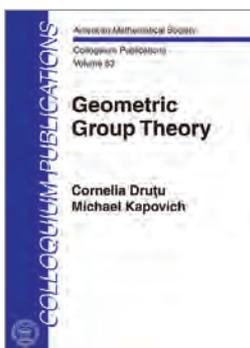
## A DYNAMICAL APPROACH TO RANDOM MATRIX THEORY

László Erdős, *Institute of Science and Technology Austria* & Horng-Tzer Yau, *Harvard University*

Offers a concise and self-contained introduction to recent techniques to prove local spectral universality for large random matrices. Random matrix theory is a fast expanding research area, and this book mainly focuses on the methods that the authors participated in developing over the past few years. The authors present key concepts that they believe are the core of these methods.

*Courant Lecture Notes, Vol. 28*

Sep 2017 226pp 9781470436483 Paperback €46.00



## GEOMETRIC GROUP THEORY

Cornelia Drutu, *Mathematical Institute, Oxford* & Michael Kapovich, *University of California*

Filling a big gap in the literature, this book contains proofs of several fundamental results of geometric group theory, such as Gromov's theorem on groups of polynomial growth, Tits's alternative, Stallings's theorem on ends of groups, Dunwoody's accessibility theorem, the Mostow Rigidity Theorem, and quasiisometric rigidity theorems of Tukia and Schwartz.

*Colloquium Publications, Vol. 63*

Nov 2017 814pp 9781470411046 Hardback €143.00

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## EMS Monograph Award – Call for Submissions

The EMS Monograph Award is assigned every year to the author(s) of a monograph in any area of mathematics that is judged by the selection committee to be an outstanding contribution to its field. The prize is endowed with 10,000 Euro and the winning monograph will be published by the EMS Publishing House in the series “EMS Tracts in Mathematics”.

Previous prize winners were

- Patrick Dehornoy et al. for the monograph *Foundations of Garside Theory*,
- Augusto C. Ponce (*Elliptic PDEs, Measures and Capacities. From the Poisson Equation to Nonlinear Thomas–Fermi Problems*),
- Vincent Guedj and Ahmed Zeriahi (*Degenerate Complex Monge–Ampère Equations*), and
- Yves de Cornulier and Pierre de la Harpe (*Metric Geometry of Locally Compact Groups*).

All books were published in the Tracts series.

The deadline for the next award, to be announced in 2019, is **30 June 2018**.

### Submission of manuscripts

The monograph must be original and unpublished, written in English and should not be submitted elsewhere until an editorial decision is rendered on the submission. Monographs should preferably be typeset in TeX. Authors should send a pdf file of the manuscript by email to:

award@ems-ph.org

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