



European Mathematical Society

NEWSLETTER No. 14

December 1994

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NOTICE FOR MATHEMATICAL SOCIETIES

Please note labels are prepared during the second half of the month before the next issue. Would you please send your updated lists before this time.

Many thanks.

Ms T Mäkeläinen

EUROPEAN MATHEMATICAL SOCIETY

Executive Committee Meeting

Cortona, Italy, 8–9 October 1994

The Committee has become aware of some interest on the part of the European Union in the academic and professional recognition of degrees. The Paris Round Table drew attention to the value in the diversity of mathematical education in Europe. An application may be made to Brussels for funding to compare and to evaluate curricula in mathematics.

Requests continue to arrive for support for meetings, conferences etc.. These are all considered sympathetically but only one or two, at most, are accepted.

Success has been achieved in applications to the European Science Foundation to run Euroconferences. Watch the Newsletter for their announcements!

The next Council meeting will probably be 27–28 July 1996 after, rather than before, the next Congress in Budapest. An important decision will have to be made at this council meeting, namely the site for the Congress in 2000. The Executive Committee will try to draw up a short list next year – interested parties take note!

The Committee, echoing the Council, would like to increase the number of individual members. An increase would help the Society financially and would promote the influence of the Society, but how can the membership be increased?

The vexed question of publication occupied much of the Committee's time. The Committee will proceed very cautiously but intends to pursue negotiations with a publisher. Electronic publishing being very much in the air, a committee has been set up to oversee developments in this field.

Next year a Turn of the Century committee will be set up to formulate plans for the European contribution to World Mathematical Year 2000. The new millennium will soon be upon us, what ideas for appropriate celebration are germinating in people's minds?

Three members retire on 31 December 1994 from the Committee, Professors F. Hirzebruch (President), A. Figà-Talamanca (Vice-President) and E.C. Lance (Secretary). The Committee and Society owes them a profound debt of gratitude for their work which has done much to set the Society on firm foundations which augur well for the future.

ems1/aw/7.11.94

For Council

EUROPEAN MATHEMATICAL SOCIETY

Council Meeting

ETH, Zürich, Switzerland, 12–13 August 1994

The 53 delegates who attended the meeting did so in a happy and eager mood. Congratulations were extended to P.-L. Lions, a member of the Executive Committee, on his award of a Fields medal at the International Congress.

The Council seemed to be content with the report from the Executive Committee which, in the main, was a summary of reports which had already appeared in the Newsletter. It was announced that the first European Mathematical Society Lecturer would be H.W. Lenstra (UC Berkeley) and that the lecture, possibly accompanied by video recording, would take place in Besançon. The delegates were however perturbed to learn that the Stefan Banach Center was in danger of losing its premises although it was hoped that a move to new location could be averted.

The Council approved two alterations to the By-laws of the Society. While the President will continue to be elected by ballot from the members of the Council, it was resolved that the remaining positions on the Executive Committee would be filled by the straightforward balloting procedure which was used, albeit with some constitutional impropriety, at the meeting of the Council in Paris. The other alteration which was agreed empowers the Executive Committee to waive or to reduce the fees of any member. This was considered to be a useful provision in the By-laws which would be used sparingly but which would allow for the conferment of honorary membership status on appropriate individuals.

The corporate membership was still increasing and the Council was pleased to enlarge its membership by electing the Bosnian Mathematical Society, the Byelorussian Mathematical Society, the Society of Mathematicians and Computer Scientists of Macedonia and the Voronezh Mathematical Society.

The elections to the Executive Committee were then held under the new By-laws. Some difficulty was expressed in that members of the Council would have appreciated fuller background information on the candidates than had been provided. There was a strong view that for subsequent elections more information would be required of the candidates. Notwithstanding the difficulty the elections were conducted in a friendly atmosphere. The outcome was that the Executive Committee from 1 January 1995 will be as follows:

President	J.-P. Bourguignon (France)
Vice-Presidents	L. Márki (Hungary) D.A.R. Wallace (United Kingdom)
Secretary	P.W. Michor (Austria)
Treasurer	A. Lahtinen (Finland)
Other Members	E. Bayer-Fluckiger (France) A. Conte (Italy) I.S. Labouriau (Portugal) A. Pelczar (Poland) W.A. Solonnikov (Russia)

The Council was pleased to have a short address from the incoming President in which he identified three major aims, namely, that the EMS should play a role in any restructuring of the Zentralblatt, that the EMS should work to advance mathematics in Europe and that a proper balance should be achieved in the EMS between pure and applied mathematics. In a hackneyed phrase – watch this space!

The finances of the Society are, thanks to the Treasurer, in good, if slender, shape. Nevertheless the level of individual membership is disappointingly low at around 1600. Do encourage your colleagues to join to strengthen the Society!

Very satisfactory relations have been established with the Science Directorate of the European Union. The EMS has become recognised in Brussels and has been able to exert some influence. It is to be hoped that further influence can be exerted by our new President, bearing in mind that the new Commissioner for Education, Training and Research is Edith Cresson, former French Prime Minister, who will take over from Antonio Ruberti on 1 January 1995.

The Council discussed at length the possibility that the EMS might establish a journal in the format suggested in a questionnaire in the Newsletter No. 12 of June 1994. It was reported that this questionnaire had yielded an insufficient response from which to draw conclusions. In the Council many differing views were vigorously expressed although there did seem to be agreement on the future impact of electronic publishing. In the light of the debate the Executive Committee was instructed to pursue, with due caution, the question of publication.

Various sub-committees of the Executive Committee presented reports. 'Women and Mathematics' had been analysing the opportunities for female mathematicians. 'Support of East European Mathematics' had been assigned a budget of 10,000 ECUs but the take-up had been below expectations – do East European mathematicians not wish to have assistance? 'Developing Countries' had been formulating an appropriate agenda – the work in France of Bordeauthèque was relevant and could perhaps be copied in other countries. 'Mathematics Education' had generated several interesting articles in the Newsletter.

Preparations for the Congress in Budapest 1996 were reported to be moving ahead. It was hoped to obtain funding from the European Union to assist with the Congress.

In regard to subsequent Congresses in 2000 and beyond, consideration was given to the procedure by which sites could be selected. The year 2000 has added significance as World Mathematical Year, which delegates felt should be celebrated in an appropriate manner. The Council resolved that all countries in Europe, through the member societies, should be asked to issue mathematical stamps in 2000. The Executive Committee has been enjoined to establish a subcommittee to plan and to coordinate activities for 2000. Any suggestions?

It was stated in the meeting that the initials EMS were not sufficiently well known. What actions are needed to publicise the Society and to encourage more people to join?

The Council concluded in a happy frame of mind with the President-elect thanking the President for all of his work and dedication during his term of office.

ems1b/aw/28.11.94

New Perspectives of a Distributed Electronic Information System for Mathematics – Part II –

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This paper is the continuation of a paper whose first part was published in EMS Newsletter 13, September 1994, 6 – 17.

7. Special Tasks and Pilot Projects

This chapter contains a summary of special tasks and pilot projects. It does not claim completeness. Which activities and which projects will be realized in the Forum will mostly depend on the partners' initiative and, of course, on the financial and technical means the Forum will be able to acquire.

7.1 Special Task: Information Clients and Servers

Partners: Mathematical Departments of Universities; Mathematical Research Institutes; Mathematical Laboratories in Industry; Publishing Houses; Libraries; Museums with Scientific/Technical Departments; DMV; Special Interest Groups of the DMV, and other mathematical societies and institutions; FIZ Karlsruhe, Zentralblatt.

Task: Installation and operation of information stations (one per partner) and clients (on all Internet workstations) at the partners' institutions. Implementation of the corresponding Gopher, WAIS, WWW, Hyper-G responder and client software.

Besides scientific-technical information of the kind described in the following special tasks and pilot projects, the partners also offer information with an organizational character, such as descriptions of institutes and departments, research programs and projects, service facilities and departmental libraries, lectures, seminars and courses, current publications, contact addresses and functions.

Technical Contents: Connection to the Internet and its distributed information services.
Options: WWW-interface to STN-databases;
Electronic information services (discussion lists as in Opt-Net; [Lügger 93]) for special interest groups (of the DMV and other societies).

Effect: Technical infrastructure for information services of the partners; Efficient transfer of information for special interest groups; Informative presentation of the partner institutions; Transparent description of services offered.

7.2 Special Task: Distributed Preprints

Partners: Mathematical Departments of Universities; Mathematical Research Institutes; FIZ Karlsruhe, Zentralblatt; Publishing Houses; Libraries.

Task: Availability of preprints, prepublications of books, lecture notes etc. in the Distributed Information System (information for students to improve studying conditions).

The Zentralblatt integrates the abstracts into the MATH database – with reference to where the full text can be obtained electronically.

The Zentralblatt prepares reviews on selected preprints.

Technical Contents: Data exchange formats;
Standardization;
Organization of accessibility.

Effect: Speeding-up of the reviewing process; Cost Reduction; Improvement of studying conditions; Increased transparency for students.

7.3 Special Task: Distributed Software and Data Collections

Partners: Mathematical Departments of Universities; Mathematical Research Institutes; Mathematical Laboratories in Industry; International Software Suppliers.

Task: The offer of mathematical software and data collections available at research institutes (e.g., concrete data of important applications) will be opened up to the community. The offer will be made available in the Distributed Information System – together with test data and relevant documentation.

Software libraries at the partner institutions and software that is, e.g., available in the electronic library eLib of the Konrad-Zuse-Zentrum (e.g., CodeLib) [eLib], in the NetLib of AT&T [AT&T] and the REDUCE Network Library [RedLib] will be indexed.

These software collections also include algorithm databases (including comparative test results) as well as example and model databases (including open problems). The data collections also contain fact databases (including logical relations between these facts), collections of mathematical formulas and tables, such as computer-readable collections of results from number theory (whose compilation may well have taken months of intensive, interactive work with a symbol manipulation system).

Options: Cooperation with further international partners offering relevant information in the Internet.

Technical Contents: Central entry of distributed software;
WAIS-database.

Access: Internet Gopher, WAIS, WWW, Hyper-G.

Effect: Offer of mathematical software and data collections for mathematics and for other fields of science (natural sciences, engineering etc.); Avoidance of double development work.

7.4 Special Task: Global Information Systems in Mathematics

Partners: Forum for Mathematical Information; Special Centers of Competence, e.g., Euromath Center in Copenhagen or Computer Algebra Netherlands (CAN).

Task: Compilation of mathematical information systems – worldwide – in a web of hypertext documents.

Technical Contents On-line selectibility (by mouse-click) of major “external” mathematical information systems.

Options: Searchable index for external information systems.

Effect: Opening of the German Distributed Information System towards Europe, North America and other active regions.

7.5 Pilot Project: Electronic Reviewing

Partners: FIZ Karlsruhe, Zentralblatt; Reviewers; Akademie der Wissenschaften in Heidelberg; Publishing Houses; Special Interest Groups of the DMV.

Project: Installation of an electronic conference system and an electronic reviewing organization for reviewers of the Zentralblatt. Reviewers with e-mail access write their reviews in a TeX-form that can be directly integrated into databases of the Zentralblatt.

Option: The Zentralblatt editors send articles to its reviewers in electronic form wherever possible.

Precondition: Cooperation with publishing houses, which may supplement the editors of the Zentralblatt by electronic announcements of their products.

Technical Contents: Standards for reviews (TeX, format of the MATH database); SGML exchange format.

Cross Reference: Special Task: Distributed Preprints.

Effect: Speeding-up of the reviewing process; Cost reduction.

7.6 Pilot Project: Electronic Mathematical Journals

Partners: Publishing Houses; Mathematical Research Institutions; Forum for Mathematical Information; DMV; EMS.

Project: A technical and organizational model for the electronic offer of mathematical journals is to be developed. The model for such a journal shall not only be usable at the developing institution itself, but also externally (exportability).

Distribution: via Internet.

Questions: Data exchange formats;
Copyright;
Costs;
Access control, if necessary.

Effect: Testing of new electronic distribution concepts; Cost reduction.

7.7 Pilot Project: Historical Books and Documents

Partners: University and Technical Information Libraries; Museums; Publishing Houses; FIZ Karlsruhe, Zentralblatt; Forum for Mathematical Information.

Project: Valuable mathematical books and historical documents will be scanned and, in this way, republished electronically (in the Internet).

The Zentralblatt prepares special reviews on these reprints.

Options: CD-ROM "Printing on Demand".

Technical Contents: Data exchange formats;
Access control, if necessary.

Questions: Copyright;
Costs.

Effect: Inexpensive access (by electronic reprints) to valuable documents from the libraries' treasuries; Testing of new distribution channels.

7.8 Pilot Project: Living Museum of Mathematics

Partners: University Libraries with Mathematical Archives such as Staats- und Universitätsbibliothek Göttingen with its Manuscript Department (Mathematical Archive); Technical Museums with Scientific/Technical Departments; University Departments and Research Institutes; Mathematical Laboratories in Industry; Forum for Mathematical Information; Individual Mathematicians Interested in History.

Project: Creation of a core organization for an Electronic ("Virtual") Museum of Mathematics and expansion of the concept towards other countries.

The Living Museum of Mathematics compiles in its departments

- * Ancient Mathematics
- * Modern Mathematics
- * Didactics of Mathematics

mathematical ideas in the form of historical documents (scanned), algorithms, visualizations of mathematical models and contemporary computer art as well as mathematical experiments (“living books”) and makes them available in the context of their place in scientific history. In this context, there will also be biographies of great mathematicians, photographs of old mathematical instruments and modern computers, descriptions of modern mathematical fields of research etc.

Options: Mathematical experimental laboratory;
Electronic exhibitions on up-to-date topics (e.g. chaos theory);
Fast-Algorithm Competitions in selected areas, such as the
DIMACS Algorithm Implementation Challenges at Rutgers
University.

Technical Contents: Data exchange formats for graphics, photographs, video
and audio;
Multi-media hypertext protocols and applications.

Influence: The Living Museum of Mathematics will include interested
institutions and research laboratories in Europe,
if not the whole world, into its activities right from the
start (as a broadly-based infrastructural project).

Effect: Inexpensive creation of the first Museum of Mathematics.

The foundations are already available (networks, workstations and visual storage media) or will soon (in approx. 5 years) be available (high bandwidths).

With this Living Museum of Mathematics, for the first time in history, mathematics will receive a worldwide exhibition space for old and modern ideas from the mathematical cosmos.

“Visitors” in this museum will not only be mathematicians, but also appliers of mathematics (natural scientists, engineers, pupils, students, teachers etc.) as well as persons interested in the history of mathematics and mathematical education.

7.9 Pilot Project: Electronic Library Catalogs

Partners: Specialized Libraries of University Departments and Research Institutes; University and Central Information Libraries; Deutsche Bibliothek in Frankfurt.

Project: Access to catalogs of selected specialized and central libraries via Internet. Installation of appropriate clients at all partners for access to these catalogs.

Options: Offer of (scanned) titlepages of articles from journals and of books,
(the latter with table of contents) for retrieval at the partners.

Technical Contents: Universal dialog communication software;
Client-server oriented retrieval;
Z 39.50 protocol.

Questions: Retrieval at distributed databases (offered by selected partners);
Simple formats for bibliographic information;
Universal access via WWW gateway.

Effect: Electronic reference to articles and books (locations) also for traditional specialized information; Improvement of the information offer by enlarging the retrievable contents; Increased transparency of traditional publications for students and teachers.

7.10 Pilot Project: Electronic Document Delivery

Partners: TIB Hannover; University Library Bielefeld; University Departments and Research Institutes; Technical and University Libraries; Publishing Houses.

Project: Electronic delivery of (also of scanned) articles, which (until now) have only been offered in the context of traditional publishing. Selected partners offering or handling a particular (traditional) journal or series supply single articles of these journals electronically (upon electronic request/order).

Technical Contents: Electronic entry, order and supply system;
Data exchange formats (e-mail oriented).

Questions: Contractual realization of corresponding organizational regulations;
Copyright;
Costs.

Cross References: Pilot Project: Electronic Mathematical Journals;
Pilot Project: Historical Books and Documents;
Pilot Project: Electronic Library Catalogs.

Effect: Increased efficiency of the supply of documents for traditional journals; Access to traditional mathematical literature; Rationalization; Cost Reduction.

7.11 Special Task: Electronic Project Organization

Partners: All partners in a community.

Task: Electronic association of all partners (in some project) using the most up-to-date means for electronic communication and cooperation.

Transmission of: Project news;
Project discussions and joint reports;
Questions and solutions etc.

Technical Tools: Electronic Conferencing (Mailing list or News system);
Groupworking.

Options: Realization of electronic WhitePages with contact addresses;
Whois servers;
Project descriptions;
Research funding information.

Effect: Efficient project organization for a distributed project; Informal (efficient) electronic exchange of information.

8. CD-ROM

The Internet is the main distribution medium for the electronic offer of mathematical information. CD-ROM is an alternative and also (relatively) inexpensive medium for the dissemination of specialized electronic information. There are many possibilities to produce CD-ROMs for special mathematics-related purposes. One can think of collections of preprints or collections of important papers on certain topics or of historically interesting documents. CD-ROMs seem to be particularly suited for countries that do not yet have adequate and reliable access to the international electronic networks or are unable to buy books and journals due to very limited budgets, but where a few PCs are available. In particular, CD-ROMs could provide the convenient and inexpensive means to distribute up-to-date mathematical information to developing countries. For customers in such countries, Zentralblatt could offer special prices for its CD-ROM version CompactMath so that at least information about existing mathematical information is available.

9. The Environment and The Chances

Since the economic situation and, in particular, the funding of research and education has declined in many countries (such as Germany) in recent years, innovative ideas are particularly needed today.

One important goal should be that government information and the results of publicly-funded project be disseminated electronically and without charge. It seem a paradox that the access to the catalog FORKAT of projects that may be supported by the German Minister of Research and Technology (BMFT) is as expensive as the access to any other database at FIZ Karlsruhe (presently DM 185.- per hour plus DM 1.10 for each on-line display). We should follow the United States, where all public institution and facilities (e.g., libraries) have come to see it as their natural duty to make the mission, aims, and major results of their institutes available in electronic networks free of charge [Lange 93]. The present situation in our scientific and public libraries (cuts in library budgets) calls for new organizational approaches that are likely to induce cost reductions. It is one of the major aims of the Forum for Mathematical Information to achieve a joint course of action supported by all parties involved, at least for the field of mathematics. Library budgets must be relieved without library services being reduced. In this context, CD-ROM could play an important role as an inexpensive carrier and storage medium in the mathematical publication process. We consider the Internet as the main medium for the distribution of specialized information that is (nearly) free of charge.

The prices for reviewing journals such as Mathematical Reviews (MR) and Zentralblatt (Zbl) have soared in the recent years and reached a level where only libraries in well-to-do countries are able to subscribe. Mathematical production continues to grow and, thus, there is an increasing number of books and articles to be reviewed. It is agreed among all experts that mathematics needs comprehensive and inexpensive reviewing journals and associated searchable electronic databases.

This situation calls for a merger of MR and Zbl to cut costs, improve service, and reduce prices. Unfortunately, a serious attempt of Zbl within the last 15 months to reach an agreement on a joint operation with MR failed due to the unwillingness of MR to share the editorial work and the database. Even support of the merger plans from IMU, EMS, DMV and other European mathematical societies did not help. New organizational concepts and measures are now in the planning phase at Zbl that include various ways of electronic communication and service. Moreover, positive signals from European countries indicate that it may be possible to extend the base of Zentralblatt so that it will become a European reviewing journal. This move should lead to an improvement of Zentralblatt's scope, service and efficiency. It will make heavy use of the "electronic world".

Many examples could be quoted to illustrate a current backwardness in Germany in the field of electronic tools and, in particular, electronic communication. Only very few public authorities in Germany can be reached via e-mail. This state may be improved in the Berlin region in the course of 1994/95 due to a planned "Metropolitan Area Network" (MAN) [ZIB 94]. In Munich and several other important regions corresponding Metropolitan Area Networks are under construction, which will also provide the framework for some libraries to offer information (catalogs and scanned titlepages of papers and books) in a digitalized form [Bayer 94]. The Deutsche Telekom is planning a "data highway" between Berlin and Bonn to connect the two centers of government (BRAVO Project with distributed office organization [Tagesspiegel 93]).

Nevertheless, the day when all important national authorities such as the federal ministries can be addressed by e-mail is not in sight. What is even worse, however, is that the

majority of industrial partners in Germany is not available electronically. If contract proposals and assignments as well as product planning and coordination can only be carried out the slow way (via regular mail), this is likely to develop into a real location disadvantage for the German economy.

We also consider the Distributed Information System for Mathematics as a project to show how to make use of the new technological opportunities on the basis of a model:

Electronic Information and Communication
for Science **AND** Industry.

The point is to create an example of an electronic information infrastructure – also for other fields of science – for database retrieval, electronic mail, electronic conferencing, specialized information networks. In this context, libraries and museums, publishing houses and information centers, industry and commercial enterprises, and public authorities must be included in this process.

The current backwardness compared with the United States must be overcome – not only in mathematics. In the United States, people already talk of the “data superhighway” as a national task. We consider such a “Infobahn” to be of special importance for Germany, too. However, to keep to the image, we also see the necessity to have the corresponding “traffic signs” and “road signals” (organizational regulations) for our field of science.

10. A Vision of “Electronic Prepublishing”

In the past few years, Steven Harnad (Princeton) has developed a special view of electronic publishing [Harnad 90], [Harnad 94], which has been proposed by Andrew Odlyzko (AMS, AT&T) in a similar form for mathematical publishing [Odlyzko 94] and which is of particular importance to us as well. Both authors regard the traditional publication process as extremely slow and expensive and predict that it will be revolutionized in the near future (say, in the next 5 to 10 years).

They point out that today the technical means for a considerably less expensive information dissemination, especially in the field of science, are already available and that with the development of cheap storage media and the distribution of the Internet this process of fundamental change has long since begun. The increasing offer of preprints in the Internet is only a first sign of what might have to be expected: the almost complete substitution of printed material (scientific journals) – in the field of scientific publishing – by the much more efficient and far less expensive electronic offer. The only thing still lacking today is the electronic version of editorial and reviewing structures that have the important function of “filtering”, i.e., extracting high-level publications.

Harnad and Odlyzko propose to publish solely electronically (on principle) and to regard a scientific publication as a “living document”. The life cycle of such a document starts with the preprint, which is made available to everybody in the “public domain”. Every scientist in the same area of research shall now be given the chance to make his/her comments, which will be added to the preprint together with the author’s replies. Revisions shall also be attached to the original preprint as well as “peer reviews”, which are embedded in a particular organizational structure (see below). Whatever is written as a comment on this document and made available electronically shall be preserved. Nothing that has ever been made available electronically shall be withdrawn.

In this way, a global archive of research works will be set up, which will all be classified and indexed in a customary way and can therefore easily be searched for. You may picture this archive as a huge database, however, you are not bound to this notion: the “living document” may just as well be distributed over a network, however, they shall be

available “forever” as to ensure continuity (and storage media are really cheap today). Harnad calls the accessibility of a scientific work in such a public, electronic archive “scholarly skywriting”.

According to Harnad and Odlyzko this horizontal dimension of the archive is to be supplemented by a vertical dimension of “peer expertise”, an electronic hierarchy of editorial and refereeing structure. At the bottom level of this hierarchy there are articles that have not yet been refereed. Every higher level of this hierarchy is linked to special reviews and “peer reviews”, which are to ensure higher quality standards. One could imagine that a publication climbs higher from level to level in this hierarchy and that the best articles in a way reach “top level quality”. Every peer review prepared against this background is attached to the “living document” in the archive. After all, it should be possible to search for documents with a certain level of quality.

One can view it that way that the editors and reviewers on a certain level of quality and in a certain field of science act like editors of an electronic journal that you can subscribe to as reader. If an article is accepted, the subscribers are informed electronically of its acceptance.

Odlyzko substantiates his considerations by an evaluation of the storage requirements for mathematical publications: all mathematical publications of one year (text and pictures), for instance, require approximately 10 GigaByte storage capacity. This means that they could be stored on a single optical disk of the new generation. Disks of this kind only cost a few hundred dollars today and their price is likely to fall even more in the near future. Thus, not the storage of mathematical information is the problem but the compilation of articles (technically: transfer of information) is the real barrier.

In his essay, Odlyzko also asks the question which role printed mathematical journals will play in the future. Surely his reflections are dominated by a technological point of view. Nevertheless, we as well as the traditional actors in the publication process (publishing houses, Fachinformationszentren, libraries) have to face the fact that fundamental social changes in the past have again and again been triggered off by technological breakthroughs.

Acknowledgement

The idea to create a Distributed Information System for Mathematics came up when the Federal Minister for Research and Technology (BMFT) invited the project management of the current DMV Project “Fachinformation” to develop a concept for scientific-technical information with a longterm perspective (beyond the year 2000). An initial concept proposal was presented during a special workshop on “The Future of Electronic Specialized Information in Mathematics” [ZIB 93], [Grötschel 93c] and discussed among the participants.

These early ideas have, in particular, been further developed by the fruitful discussions with the Specialized Information Coordinators of the DMV Project “Fachinformation”, with participants in a similar project in the field of physics [Bischoff 93], and with experts from the field of computer science [GI], e.g., Prof. R. Bayer, regarding his notion of scanning books and documents (see, e.g., [Bayer 93]). New developments from computer sciences in Austria in the field of global hypermedia systems (see, e.g., [Kappe 93] by Dr. F. Kappe, Prof. H. Maurer and their working group let the realization of the first stage of the Distributed Information System seem realistic in the very near future. Furthermore, there is Dr. G. Lange of the Computing Center of the Technische Universität Clausthal-Zellerfeld, who is in charge of the working group “Information Systems” at the DFN,

to be mentioned [Lange 93] as well as A. de Kemp [deKemp 93] of the Springer-Verlag in Heidelberg and Dr. A. Odlyzko of AT&T and AMS, who have illustrated the highly topical subject of electronic publishing in science. We would like to take this opportunity to thank them for their valuable ideas.

The initial concept was then – in a preliminary version – widely distributed electronically (e-mail) among mathematicians, physicists and computer scientists as well as libraries and publishing houses. This has led to a great number of supplements and additions; however, it will not be possible to refer to all contributions now. They also came from other countries, where the discussions about the plans of the Deutsche Mathematiker-Vereinigung had become noticed during a very early phase (e.g., [Lügger 94a], [Lügger 94b]).

With the Gesellschaft für Informatik (GI) and the Deutsche Physikalische Gesellschaft (DPG), close cooperation was agreed, which led to an intensive, mutual exchange of information in the subsequent preparatory phase. In February, 1994 the Council (Präsidium) of the DMV agreed upon the project plannings, which were then (in May, 1994) presented to the Konferenz der Mathematischen Fachbereiche (in Germany) and welcomed unanimously.

Last but not least, we wish to thank Ms. C. Schöning-Walter of the Projektträger Fachinformation (at the GMD in Darmstadt), who is not only handling the current DMV “Fachinformation” Project with great personal commitment, but has also encouraged us again and again to make a contribution to the next BMFT support programme by furnishing new ideas.

Our special thanks are due to Ms. Sybille Mattrisch of the Konrad-Zuse-Zentrum for the translation of the manuscript into English and for the careful typing, setting and corrections.

Call for Cooperation

The set-up of an electronic information and communication system in Germany and the above list of possible projects is currently discussed in detail within the DMV, the mathematical institutions and other interested partners in Germany. It is planned that – after Germany’s policy for furthering the electronic infrastructure has been announced around the end of this year – the DMV seeks (modest) financial government support for some of the plans described before. Surely, only few of the projects and plans have a “national character”. The electronic network clearly is the most important base for international cooperation. But it is not only a base. It is vital to add structure to it so that it will become a versatile and easy to use medium for communication and information retrieval that considerably improves our working conditions. The DMV invites the EMS and other European mathematical societies to discuss the plans sketched above and to cooperate with DMV wherever coordinated efforts seem sensible.

For a detailed list of literature and electronic references please refer to the end of Part I.

Editorial

More than two years have passed since the establishment of the current Committee of Mathematics Education (CME) of EMS. We should therefore appreciate feedback from the readers on the contributions published so far and also hints as to what else might be of interest. Any critique or suggestion should be sent to any one member of the committee as listed below. We look forward to hearing from you soon.

Willi Dörfler, Chair

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NORWAY

PDME III, July 1995

PDME III, Political Dimensions of Mathematics Education Conference, will take place in Bergen, Norway, 24-29 July 1995. The official languages of the conference will be English and Spanish. For information on PDME see the article by Stieg Mellin-Olsen, ICMI Bulletin No.34, 16-17.

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The International Commission on Mathematical Instruction

**PERSPECTIVES ON THE TEACHING OF
GEOMETRY FOR THE 21st CENTURY**

Discussion Document for an ICMI Study

1. Why a study on geometry?

Geometry, considered as a tool for understanding, describing and interacting with the space in which we live, is perhaps the most intuitive, concrete and reality-linked part of mathematics. On the other hand geometry, as a discipline, rests on an extensive formalization process, which has been carried out for over 2000 years at increasing levels of rigour, abstraction and generality.

In recent years, research in geometry has been greatly stimulated by new ideas both from inside mathematics and from other disciplines, including computer science. At present, the enormous possibilities of computer graphics influence many aspects of our lives; in order to use these possibilities, a suitable visual education is needed.

Among mathematicians and mathematics educators there is a wide-spread agreement that, due to the manifold aspects of geometry, teaching of geometry should start at an early age, and continue in appropriate forms throughout the whole mathematics curriculum. However, as soon as one tries to enter into details, opinions diverge on how to accomplish the task. There have been in the past (and there persist even now) strong disagreements about the aims, contents and methods for the teaching of geometry at various levels, from primary school to university.

Perhaps one of the main reasons for this situation is that geometry has so many aspects, and as a consequence there has not yet been found - and perhaps there does not exist at all - a simple, clean, linear, "hierarchical" path from the first beginnings to the more advanced achievements of geometry. Unlike what happens in arithmetic and algebra, even basic concepts in geometry, such as the notions of angle and distance, have to be reconsidered at different stages from different viewpoints.

Another problematic point concerns the role of proofs in geometry: relations between intuition, inductive and deductive proofs, age of students at which proofs can be introduced, and different levels of rigour and abstraction.

Thus the teaching of geometry is not at all an easy task. But instead of trying to face and overcome the obstacles arising in the teaching of geometry, actual school-practice in many countries has simply bypassed these obstacles, cutting out the more demanding parts, often without any replacement. For instance, three-dimensional geometry has almost disappeared or has been confined to a marginal role in the curricula in most countries.

Starting from this analysis, and specifically considering the gap between the increasing importance of geometry for its own sake, as well as in research and in society, and the decline of its role in school curricula, ICMI feels that there is an urgent need for an international study, whose main aims are:

- To discuss the goals of the teaching of geometry at different school levels and according to different cultural traditions and environments.
- To identify important challenges and emerging trends for the future and to analyze their potential didactical impact.
- To exploit and implement new teaching methods.

2. Aspects of geometry

The outstanding historical importance of geometry in the past, in particular as a prototype of an axiomatic theory, is so universally acknowledged that it deserves no further comment. Moreover, in the last century and specifically during the last decades, as Jean Dieudonné asserted at ICME 4 (Berkeley, 1980), Geometry "bursting out of its traditional narrow confines [...] has revealed its hidden powers and its extraordinary versatility and adaptability, thus becoming one of the most

universal and useful tools in all parts of mathematics" (J. Dieudonné: The Universal Domination of Geometry, ZDM 13(1), p 5-7 (1981)).

Actually, geometry, includes so many different aspects, that it is hopeless (and maybe even useless) to write out a complete list of them. Here we mention only those aspects, which in our opinion are particularly relevant in view of their didactical implications:

- Geometry as *the science of space*. From its roots as a tool for describing and measuring figures, geometry has grown into a theory of ideas and methods by which we can construct and study idealized models of the physical world as well as of other real world phenomena. According to different points of view, we get euclidean, affine, descriptive, projective geometry, but also topology or non euclidean and combinatorial geometries.

- Geometry as *a method for visual representations of concepts and processes* from other areas in mathematics and in other sciences. E.g. graphs and graph theory, diagrams of various kinds, histograms.

- Geometry as *a meeting point* between mathematics as a theory and mathematics as a model resource.

- Geometry as *a way of thinking and understanding* and, at a higher level, as *a formal theory*.

- Geometry as *a paradigmatic example for teaching deductive reasoning*.

- Geometry as *a tool in applications*, both traditional and innovative.

The latter ones include e.g. computer graphics, image processing and image manipulation, pattern recognition, robotic, operations research.

Another distinction should be made with respect to several different approaches according to which one may deal with geometry. Roughly speaking, possible approaches are:

- manipulative
- intuitive
- deductive
- analytic.

Also one may distinguish between a geometry which stresses "static" properties of geometric objects and a geometry where objects are considered in a "dynamic" setting, as they change under the effect of different types of space transformations.

3. Is there a crisis in the teaching of geometry?

During the second half of this century geometry seems to have progressively lost its former central position in mathematics teaching in most countries. The decrease has been both qualitative and quantitative. Symptoms of this decrease may be found for instance in recent national and international surveys on the mathematical knowledge of students. Often geometry is totally ignored or only a very few items concerned with geometry are included. In the latter case questions tend to be confined to some elementary "facts" about simple figures and their properties, and performance is reported to be relatively poor.

What are the main causes of this situation?

- From about 1960 to 1980 a general time pressure on traditional topics has occurred, due to the introduction of new topics in mathematics curricula (e.g. probability, statistics, computer science, discrete mathematics). At the same time the number of school hours devoted to mathematics has gone down. The "modern mathematics movement" has contributed - at least indirectly - to the decline of the role of euclidean geometry, favouring other aspects of mathematics and other points of view for its teaching (e.g. set theory, logic, abstract structures). The decline has involved in particular the role of visual aspects of geometry, both three-dimensional and two-dimensional, and all those parts which did not fit into a theory of linear spaces as, for instance, the study of conic sections and of other noteworthy curves.

- In more recent years there has been a shift back towards more traditional contents in mathematics, with a specific emphasis on problem posing and problem solving activities. However, attempts to restore classical euclidean geometry - which earlier in many parts of the world was the main subject in school geometry - have so far not been very successful. The point is that in traditional courses on euclidean geometry the material is usually presented to students as a ready-made

end product of mathematical activity. Hence, in this form, it does not fit well into curricula where pupils are expected to take an active part in the development of their mathematical knowledge.

- In most countries the percentage of young people attending secondary school has increased very rapidly during the last decades. Thus the traditional way of teaching abstract geometry to a selected minority has become both more difficult and more inappropriate for the expectations of the majority of students of the new generations. At the same time, the need for more teachers has caused, on average, a decline in their university preparation, especially with respect to the more demanding parts of mathematics, in particular geometry. Since younger teachers have learned mathematics under curricula that neglected geometry, they lack a good background in this field, which in turn fosters in them the tendency to neglect the teaching of geometry to their pupils. The situation is even more dramatic in those countries which lack a prior tradition in schooling. In some cases geometry is completely absent from their mathematics curricula.

- The gap between the conception of geometry as a research area and as a subject to be taught in schools seems to be increasing; but so far no consensus has been found how to bridge this gap, nor even whether it could (or should) be bridged through an introduction of more advanced topics in school curricula at lower grades.

4. Geometry as reflected in education

In former sections, we have considered geometry mainly as a mathematical theory and have analyzed some aspects of its *teaching*. Since *learning* is unquestionably the other essential pole of any education project, it is now appropriate to pay due attention to the main variables which may affect a coherent teaching/learning process. Consequently, several different aspects of "dimensions" (considered in their broadest meaning) must be taken into account:

- *The social dimension*, with two poles:

- The cultural pole, i.e. the construction of a common background (knowledge and language) for all the people sharing a common civilization;

- The educational pole, i.e. the development of criteria, internal to each individual, for self consistency and responsibility.

- *The cognitive dimension*, i.e. the process which, starting from reality, leads gradually to a refined perception of space.

- *The epistemologic dimension*, i.e. the ability to exploit the interplay between reality and theory through modelization (make provisions, evaluate their effects, reconsider choices). Thereby axiomatization enables one to get free from reality; this in turn may be seen as a side-step which allows further conceptualization.

- *The didactic dimension*, i.e. the relation between teaching and learning. Within this dimension several aspects deserve consideration. As an example, we list three of them:

- To make various fields interact (both within mathematics and between mathematics and other sciences).

- To make sure that the viewpoints of the teacher and the pupils are consistent in a given study. For instance, to be aware that different distance scales may involve different conceptions and processes adopted by the pupils, even though the mathematical situation is the same: in a "space of small objects", visual perception may help to make conjectures and to identify geometric properties; when dealing with the space where we are used to move around (the classroom, for instance) it is still easy to get local information, but it may be difficult to achieve an overall view; in a "large scale space" (as is the case in geography or in astronomy) symbolic representations are needed in order to analyze its properties.

- To pay due consideration to the influence of tools available in teaching/learning situations (from straightedge and compass, as well as other concrete materials, to graphic calculators, computers and specific software).

It goes without saying that all these dimensions are interrelated with each other and that they should also be related appropriately to different age levels and school types: pre-primary level, primary level, lower secondary level, upper secondary level (where differentiation into academic, technical, vocational tracks usually starts), tertiary (i.e. university) level, including teaching preparation.

5. New technology and teaching aids for geometry

There is a long tradition of mathematicians making use of technological tools, and conversely the use of these tools has given rise to many challenging mathematical problems (e.g. straightedge and compass for geometric constructions, logarithms and mechanical instruments for numerical computations). In recent years new technology, and in particular computers, has affected dramatically all aspects of our society. Many traditional activities have become obsolete, while new professions and new challenges arise. For instance, technical drawing is no longer done by hand. Nowadays, instead, one uses commercial software, plotters and other technological devices. CAD/CAM and symbolic algebra software are becoming widely available.

Computers have also made it possible to construct "virtual realities" and to generate interactively animations or marvellous pictures (e.g. fractal images). Moreover, electronic devices can be used to achieve experiences that in everyday life are either inaccessible, or accessible only as a result of time-consuming and often tedious work.

Of course, in all these activities geometry is deeply involved, both in order to enhance the ability to use technological tools appropriately, and in order to interpret and understand the meaning of the images produced.

Computers can be used also to gain a deeper understanding of geometric structures thanks to software specifically designed for didactical purposes. Examples include the possibility of simulating traditional straightedge and compass constructions, or the possibility of moving basic elements of a configuration on the screen while keeping existing geometric relationships fixed, which may lead to a dynamic presentation of geometric objects and may favour the identification of their invariants.

Until now, school practice has been only marginally influenced by these innovations. But in the near future it is likely that at least some of these new topics will find their way into curricula. This will imply great challenges:

- How will the use of computers affect the teaching of geometry, its aims, its contents and its methods?
- Will the cultural values of classical geometry thereby be preserved, or will they evolve, and how?
- In countries where financial constraints will not allow a massive introduction of computers into schools in the near future, will it nevertheless be possible to restructure geometry curricula in order to cope with the main challenges originated by these technological devices?

6. Key issues and challenges for the future

In this section we list explicitly some of the most relevant questions which arise from the considerations outlined in the preceding sections. We believe that a clarification of these issues would contribute to a significant improvement in the teaching of geometry. Of course we do not claim that all the problems quoted below are solvable, nor that the solutions are unique and have universal validity. On the contrary, the solutions may vary according to different school levels, different school types and different cultural environments.

1. Aims

Why is it advisable and/or necessary to teach geometry?

Which of the following may be considered to be the most relevant aims of the teaching of geometry?

- To describe, understand and interpret the real world and its phenomena
- To supply an example of an axiomatic theory
- To provide a rich and varied collection of problems and exercises for individual student activity
- To training learners to make guesses, state conjectures, provide proofs, and find out examples and counterexamples
- To serve as a tool for other areas of mathematics
- To enrich the public perception of mathematics.

2. Contents

What should be taught?

Is it preferable to emphasize "breadth" or "depth" in the teaching of geometry? And is it possible/advisable to identify a core curriculum?

In the case of an affirmative answer to the second question above, which topics should be included in syllabi at various school levels?

In the case of a negative answer, why is it believed that teachers or local authorities should be left free to choose the geometric contents according to their personal tastes (is this point of view common to other mathematical subjects, or is it peculiar to geometry)?

Should geometry be taught as a specific, separate subject, or should it be merged into general mathematical courses?

There seems to be widespread agreement that the teaching of geometry must reflect the actual and potential needs of society. In particular, geometry of three-dimensional space should be stressed at all school levels, as well as the relationships between three-dimensional and two-dimensional geometry. How could and should the present situation (where only two-dimensional geometry is favoured) therefore be modified and improved?

In which ways can the study of linear algebra enhance the understanding of geometry? At what stage should "abstract" vector space structures be introduced? And what are the goals?

Would it be possible and advisable also to include some elements of non-euclidean geometries into curricula?

3. Methods

How should we teach geometry?

Any topic taught in geometry can be located somewhere between the two extremes of an "intuitive" approach and a "formalized" or "axiomatic" approach. Should only one of these two approaches be stressed at each school level, or should there be a dialectic interplay between them, or else should there be a gradual shift from the former to the latter one, as the age of students and the school level progresses?

What is the role of axiomatics within the teaching of geometry? Should a complete set of axioms be stated from the beginning (and, if so, at what age and school level) or is it advisable to introduce axiomatics gradually, e.g. via a "local deductions" method?

Traditionally, geometry is the subject where "one proves theorems". Should "theorem proving" be confined to geometry?

Would we like to expose students to different levels of rigour in proofs (as age and school level progress)? Should proofs be tools for personal understanding, for convincing others, or for explaining, enlightening, verifying?

Starting from a certain school level, should every statement be proved, or should only a few theorems be selected for proof? In the latter case, should one choose these theorems because of the importance within a specific theoretical framework, or because of the degree of difficulty of their proof? And should intuitive or counterintuitive statements be privileged?

It seems that there is an international trend towards the teaching of analytic methods in increasingly earlier grades, at the expense of other (synthetic) aspects of geometry. Analytic geometry is supposed to present algebraic models for geometric situations. But, as soon as students are introduced to these new methods, they are suddenly projected into a new world of symbols and calculations in which the link between geometric situations and their algebraic models breaks down and geometric interpretations of numerical calculations are often neglected. Hence, at what age and school level should teaching of analytic geometry start? Which activities, methods and theoretical frameworks can be used in order to restore the link between the algebraic representation of space and the geometric situation it symbolizes?

How can we best improve the ability of pupils to choose the appropriate tools for solving specific geometric problems (conceptual, manipulative, technological)?

4. Books, computers, and other teaching aids

Are traditional textbooks as appropriate for teaching and learning geometry, as we would like them to be?

How do teachers and pupils actually use geometry textbooks and other teaching aids? How would

we like pupils to use them?

What changes could and should be made in teaching and learning geometry in the perspective of increased access to software, videos, concrete materials and other technological devices/

What are the advantages, from the educational and geometrical point of view, that can follow from the use of such tools?

Which problems and limitations may arise from the use of such tools, and how can they be overcome?

To what extent is knowledge acquired in a computer environment transferable to other environments?

5. Assessment

The ways of assessment and evaluation of pupils strongly influence teaching and learning strategies. How should we set out objectives and aims, and how should we construct assessment techniques that are consistent with these objectives and aims? Are there issues of assessment which are peculiar to the teaching and learning of geometry?

How does the use of calculators, computers and specific geometric software influence examinations as regards contents, organization and criteria for the evaluation of the answers to the students?

Should assessment procedures be based mainly upon written examination papers (as it seems to be customary in many countries) or else what should be the role of oral communication, of technical drawing and of work with the computer?

What is it exactly that should be evaluated and considered for assessment: The solution outcome? The solution process? The method of thinking? Geometric constructions?

6. Teacher preparation

One essential component of an efficient teaching/learning process, is good teacher preparation, as regards both disciplinary competence and education, epistemological, technological and social aspects. Hence, what specific preparation in geometry is needed (and realistically achievable) for prospective and practising teachers?

It is well known that teachers tend to reproduce in their profession the same models they have experienced when they were students, regardless of subsequent exposure to different points of view. How is it then possible to motivate the need for changes in the perspective of teaching geometry (both from the content and from the methodological point of view)?

Which teaching supplies (books, videos, software,...) should be made available for in-service training of teachers, in order to favour a flexible and open-minded approach to the teaching of geometry?

7. Evaluation of long-term effects

All too often the success (or failure) of a curricular and/or methodological reform or innovation for a certain school system is evaluated on the basis only of a short period of observation of its outcomes. Moreover usually there are no comparative studies on the possible side effects of a change of content or methods. Conversely, it would be necessary to look also at what happens in the long term. For instance:

- Does a visual education from a very young age have an impact on geometric thinking at a later stage?
- How does an early introduction of analytic methods in the teaching of geometry influence the visual intuition of pupils? When these pupils become professionals, do they rely more on the intuitive or on the rational parts of the geometry teaching to which they have been exposed?
- What is the impact of an extensive use of technological tools on geometry learning?

8. Implementation

At ICME 5 (Adelaide, 1984) J. Kilpatrick asked a provocative question: What do we know about mathematics education in 1984 that we did not know in 1980? Recently the same question has been picked up again in the ongoing ICMI study: "what is research in mathematics education and what are its results". (See Newsletter No.8) As for geometry, the possibility of relying on research results would be extremely useful in order to avoid reproposing in the future paths already proved

unsuccessful, and conversely in order to benefit from successful solutions. And, as for still unsettled and relevant questions, we would like research to give us useful information in order to clarify the advantages and drawbacks of possible alternatives.

Hence, a key question might be:

What do we already know from research about the teaching and learning of geometry and what would we want future research to tell us?

7. Call for papers

The ICMI study "**Perspectives on Teaching of Geometry for the 21st Century**" will consist of an invited *Study Conference* and a *Publication to appear* in the ICMI study series, based on the contributions to, and the outcomes of, the Conference.

The conference is scheduled for September 1995 in Catania (Italy). The International program Committee (IPC) for the study hereby invites individuals and groups to submit ideas, suggestions and contributions on major problems or issues related to this discussion document, not later than February 15, 1995.

Although participation in the conference requires an invitation from the IPC, "experts" and "newcomers" interested in contributing to and participating in the conference are encouraged to contact the chair of the IPC. Unfortunately, an invitation to attend does not imply that financial support will be provided by the organizers.

Papers, as well as suggestions concerning the content of the study conference program should be sent to:

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Adults Learning Mathematics - a new field of research

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Whenever you ask adults what they remember from school and especially from their mathematical education they will answer: "I liked math very much!", "The best hours in my life were math lessons!", "All I learned in math I am now using in my everyday life!", "I understand the technological structure of our society very well because I understood mathematics in school!", "I have no problems with computers because I learned logical thinking in the mathematics lessons!", "Now I'm visiting a further education course and I have problems with all subjects except math!"

Oh, you heard or expected other answers? Why? Did you read a research report on the results of teaching mathematics in school? There are only a few such reports. They tell us that many people after school forget nearly all they have known about mathematics. In industrialized countries some

10% (or more) are math illiterates. Our own research and analysis of math knowledge tests show for example the following results (percentages of right solutions):

- Calculating with integers lower than one hundred: addition (88%), subtraction (86%), multiplication (76%), division (90%), combined calculations with a negative number as result (35%)
- Calculating with simple decimal numbers: subtraction (83%), multiplication (60%), division (47%)
- Calculating with simple fractions: addition and subtraction (57%), multiplication (56%), division (39%).

The test was similar to school tests [1].

A second important aspect is the image of mathematics. Only very few people enjoyed their math lessons at school. Many people remember that they did not understand mathematics and that they were stressed by teachers and tests. They think that mathematics is not useful for them and has nothing to do with real life. This way of thinking is dangerous to mathematics teaching.

These two points and our experiences with different types of math courses for adults motivated us to undertake various research projects in this area. We investigated mathematical knowledge and belief systems about mathematics, evaluated math answers, analyzed curricula and organizational structures. We used tests, questionnaires, made interviews and had informal discussions with learners, teachers and organizers of courses. With respect to the beliefs about mathematics [2] we found that many adult learners consider mathematics as a toolbox they can use to solve job- or everyday-life problems. Simultaneously they agree that mathematics is a big theory with exact definitions and proofs, but only some of them characterize mathematics as a game with numbers, formulae and special rules for manipulation. More than 75% believe that working with mathematics fosters logical thinking.

Why do more and more adults want to learn mathematics?

The most important motive for adult learners to start taking a course involving math is their job: they need better qualifications to remain employed or to improve their chances of obtaining a higher income. In order to get a new and better job some people need a formal graduation (for example a "technical master" ("*Meister*") degree). But there is another strong motive as well, beside the job-related ones. Participants of math courses are eager to learn, to generally improve their knowledge and abilities.

These personal motivations match structural tendencies of our technological society. All the new technologies are based on mathematics. All computers, any hard- and software, are products of mathematical technology [3]. More and more parts of our society are structured mathematically. In this situation further education in mathematics is not "only" useful for individuals and their employment, but also forms an important basis for a better orientation in and understanding of the technological world and its rules.

Cooperation

Mathematics education for adults is a new field of research within the didactics of mathematics. We have a lot of questions and only partial results. Therefore we are interested in cooperation: Have you worked in this field? Do you know anything about adults' learning of math? If you would like to exchange questions and answers with us, please contact us.

Literature

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Problem Corner

Paul Jainta

When I was given occasion to set up a *Problem Corner* in the NEWSLETTER at the start of the year I considered the pros and cons of what - apart from problems - to present within this new department. Suddenly the penny dropped. I remembered that I had been conducting a section in the German mathematical journal for students at schools, called α , comparable to that launched by the EMS. There in my *Olympiad Corner* I've devoted myself to missionary work in reporting on projects to promote mathematically interested and gifted pupils. This corner lives on the support of other experienced competition organisers and is such that organisers of new competitions do not have to *re-invent the wheel*. As long as there were two German states existing together, both were worlds apart in encouraging their young mathematical élite. In the Western part, a deep chasm yawned between those who teach in schools and those who do mathematical research. University staff who worked with high school students or younger children did so at the risk of their reputation and even their tenure. The result has been that gifted young mathematicians did not get enough encouragement in their studies. Their teachers got little exposure to new developments in mathematics, and both suffered from a lack of status in society at large. In Eastern Germany things had been different. The Government of the day invested heavily in education. Here as I see it there was much more interplay between those who teach mathematics and those who create it. Teaching itself was regarded as a more creative activity than it was in the Western part of Germany, and some of the best mathematicians of the German Democratic Republic had spent much time and effort working with high school students. A rich culture of mathematical cooperation among all levels of teaching had developed, including curriculum development and teacher training.

So, it was no accident that East Germany was present to found the International Mathematical Olympiad which started as a contest among the Eastern bloc countries - and this contest was only the most prominent of a dense and wide network of mathematical competitions. And of course the former GDR had its own magazine, *alpha*, written for pre-college students by mathematicians and university professors. Among other journals like *kvant*, *Kömal*, or the Polish *Delta*, this periodical made important and lasting contributions to the literature of elementary mathematics.

Well, after German reunification one could have thought that mathematical circles and summer camps in which teachers, students and mathematicians form close relationships, would spread to all of Germany rapidly. But this process is making only slow progress. Western mathematics just puts a premium of publishing, not on teaching or working with students. Nevertheless, some of the old German federal states have brought new forms of promotion into being and now five new states have joined with multifarious and long experience in offering encouragement to mathematically interested pupils. Gradually, it has got around among the leading minds in the field of education that it would be desirable to preserve an organic variety of methods, structures and demands of effective measures of promotion or attractive competitions. At last, the general consensus is that quite a lot of work must be done to foster pupils in the junior forms of schools, keen to develop their mathematical knowledge and to acquire new skills, from place to place and nation-wide.

On reflection I think all this should be valid on a European scale, too. Thus this Problem Corner! But unfortunately I am bound to say that evidently it is easier to reach firsthand impressions of maths events from the back of beyond than from the central part of Europe. I am therefore grateful to Dr. Tony Gardiner (University of Birmingham), the Secretary of the British Mathematical Olympiad Committee (BMOC), for forwarding two booklets to me. The first booklet contains full solutions and notes of both rounds of the 1994 BMO held in the UK on Wednesday 19th January and Thursday, 24th February. The other publication is *UK Junior Mathematical Olympiads 1989-1993 (Problems-Results-Comments, Hints and Solutions)*.

The olympiad-type set of problems for this issue is the 1994 BMO, round 1. The British Mathematical Olympiad is the final stage to make a selection from the best contestants to work as a team at the IMO. The BMOC is an independent body set up as a result of cooperation between the four participating societies - the *Edinburgh Mathematical Society*, the *Institute of Mathematics and its Applications*, the *London Mathematical Society* and the *Mathematical Association*, with the support from the *Royal Society* and the *Department for Education*. Its constitution charges it among other things to the specific task to *initiate, promote and manage competitions for such students in the senior*

years of schooling, select, prepare and make arrangements for participants to represent the United Kingdom in the International Mathematical Olympiad.

The 1994 British Mathematical Olympiad Round 1 was taken by around 650 students, many of whom showed commendable knowledge and skill in their attempts at the questions. The first two problems on the paper are chosen so that all candidates can appreciate what is asked of them, and can be solved without any great degree of specialised knowledge. This does not mean that they are easy. In question 1, for instance, a large number of students found all possible answers and may wonder why they failed to score many marks; some fluked the answers by the use of faulty algebra, many others simply failed to demonstrate that these constituted all possible answers; others, however, merely found one and guessed the others. The later questions are designed to allow candidates to show clear mathematical reasoning, and the final problem is specifically chosen to discriminate between those who can reason clearly and carefully and those who can spot a pattern but prove nothing. This does not mean that problem 5 is the hardest question on the paper - many preferred it to the two earlier geometric problems.

The following are the five questions of Round 1.

- Q11. Starting with any three digit number n we obtain a new number $f(n)$ which is equal to the sum of the three digits of n , their three products in pairs, and the product of all three digits.
- Find the value of $nf(n)$ when $n = 625$.
 - Find all three digit numbers such that the ratio $nf(n) = 1$.
- Q12 In triangle ABC the point X lies on BC.
- Suppose that $\angle BAC = 90^\circ$, that X is the midpoint of BC, and that $\angle BAX$ is one third of $\angle BAC$. What can you say (and prove!) about triangle ACX?
 - Suppose that $\angle BAC = 60^\circ$, that X lies one third of the way from B to C, and that AX bisects $\angle BAC$. What can you say (and prove!) about triangle ACX?
- Q13 The sequence of integers $u_0, u_1, u_2, u_3, \dots$ satisfies $u_0 = 1$ and $u_{n+1} \cdot u_{n-1} = k \cdot u_n$ for each $n \geq 1$, where k is some fixed positive integer. If $u_{2000} = 2000$, determine all possible values of k .
- Q14 The points Q,R lie on the circle γ , and P is a point such that PQ, PR are tangents to γ . A is a point on the extension of PQ, and γ' is the circumcircle of triangle PAR. The circle γ' cuts γ again at B, and AR cuts γ at the point C. Prove that $\angle PAR = \angle ABC$.
- Q15 An increasing sequence of integers is said to be alternating if it starts with an odd term, the second term is even, the third term is odd, the fourth is even, and so on. The empty sequence (with no term at all!) is considered to be alternating. Let $A(n)$ denote the number of alternating sequences which only involve integers from the set $\{1,2,\dots,n\}$. Show that $A(1) = 2$ and $A(2) = 3$. Find the value of $A(20)$, and prove that your value is correct.
- Q16 *Proposed by François Sigrist, Neuchâtel, Switzerland.*
An equilateral triangle is covered by five smaller equilateral triangles congruent to each other. Prove that it is possible to move the smaller triangles so as to cover the big triangle with four of them.

Propose problems for which readers will send in solutions. Proposals should, whenever possible, be accompanied by a solution, references, and other insights which are likely to be of help to the editor. They can be anything from elementary to advanced, from easy to difficult. Original problems are particularly sought. So, please submit any interesting problem you come across, especially those from (problem) books and contests that are not easily accessible. I hereby invite my readers to share them with their colleagues and students.

SOLUTIONS

François Sigrist, Institut de mathématiques, Université de Neuchâtel, Switzerland, has sent in tersely expressed solutions to all parts of Q9 and Q10, so I will give a more detailed summary of his results.

Q9 *Facts:* A proper divisor of a positive integer n is a positive integer smaller than n which exactly divides n . One is a proper divisor of every positive integer. A perfect number is a positive integer whose proper divisors sum to the original number. For example, 6 is the smallest perfect number since its proper divisors are 1,2,3, and $1+2+3=6$. The next smallest perfect number is 28.

Definitions: A magical set is a multiset of three or more positive integers, not necessarily distinct, such that each number in the set exactly divides the sum of the remaining numbers. We also require that these numbers have no common divisor except 1. Hence the set $\{1,2,6,9\}$ is magical since $1+2+6=9$ is a multiple of 9, $1+2+9=12$ is a multiple of 6, $1+6+9=16$ is a multiple of 2, and $2+6+9=17$ is a multiple of 1. The set $\{2,4,12,18\}$ is not magical since all the numbers are divisible by 2.

Problems:

- i. Show that the set $\{1,1,2,4,\dots,2^n\}$ is magical for all $n \geq 1$.
- ii. Show that all the proper divisors of a perfect number form a magical set.
- iii. Find all magical sets with exactly three numbers.
- iv. Find all magical sets with four numbers whose smallest elements are 1 and 3, i.e. of the form $\{1,3,m,n\}$ with $m,n \geq 3$.
- v. Prove that given any magical set, one can include an additional number in the set so that this new set is also magical.

Part (i). The definition for magical sets is a little cumbersome as it is stated. It would be much easier to work with the sum of all the numbers in a magical set. Indeed, we note that in the set $\{1,2,6,9\}$, the example given in the definition above, the sum of all elements is 18, and each element of the set divides 18. This observation motivates the following

Lemma: *A set is magical if and only if each element in the set divides the sum of all the elements and the GCD of all elements is 1.*

I leave it to the reader to prove this lemma. Thus the rest of part (i) follows readily. If $M=\{1,1,2,\dots,2^n\}$, then the sum of all elements is $S=1+1+2+\dots+2^n = 2^{n+1}$. Clearly each element of M divides the sum S and since one of the elements is 1 the GCD of all the elements is also 1, so M must be a magical set by the lemma.

Part (ii). Let $M = \{d_1, d_2, \dots, d_n\}$ be the set of all proper divisors of some perfect number P . Then $P = d_1 + d_2 + \dots + d_n$ by the definition of a perfect number, and each d_i is a divisor of P . Thus M is a set in which every element divides the sum of all the elements and again 1 must be in the set since 1 is a divisor of every perfect number, so by the lemma, M is a magical set.

Part (iii). Let the magical set be $\{a,b,c\}$ and let the largest element be c . The $1 \leq a \leq c$ and $1 \leq b \leq c$ so $2 \leq a+b \leq 2c$, but $a+b$ must be a multiple of c (it's a magical set), so either $a+b=c$ or $a+b=2c$.

The latter case is easily disposed of, since the only way to achieve $a+b=2c$ given $a \leq c$ and $b \leq c$ is if $a=c$ and $b=c$, yielding the set $\{c,c,c\}$. We require the GCD to be 1, so the only solution of this form is $\{1,1,1\}$.

In the former case, where $a+b=c$, we now check the parity of a and b . There are three subcases. If a and b are both even, then $c=a+b$ is also even, so 2 is a common divisor of $\{a,b,c\}$, contradicting the definition of a magical set.

The next subcase is if one of a,b is odd and the other is even; say a is odd and b is even. Write $b = 2k$. Since the set is magical we know that $a \mid b+(a+b)$ and $b \mid a+(a+b)$, which is equivalent to $a \mid 2b$ and $b \mid 2a$, or $a \mid 4k$ and $2k \mid 2a$ once we substitute $b=2k$. However,

a is odd so $a \mid 4k$ means $a \mid k$. Also, if $2k \mid 2a$ then $k \mid a$. Hence $a=k$, so $b=2a$, $c=a+b=3a$, and our set is $\{a,2a,3a\}$. Requiring that the GCD be 1 yields $\{1,2,3\}$ as the only solution of this type.

Finally, if both a and b are odd then in the same manner as before we can conclude that $a \mid 2b$ and $b \mid 2a$ since the set is magical. But this time a and b are odd so $a \mid 2b$ implies $a \mid b$ and similarly $b \mid a$. Therefore $a=b$, and $c=a+b=2a$, whereupon our set becomes $\{a,a,2a\}$. The usual GCD requirement yields the single solution $\{1,1,2\}$.

Part (iv). A quick solution to this problem involves the following observation: if the set is $\{1,3,m,n\}$ with $m \leq n$ then we must have $n=m+4$ or $n=2(m+4)$, which can be shown as follows. Our set is magical so $n \mid 1+3+m$, which can be written $4+m=kn$. But $n \geq m$ so $kn \geq km$, which substituted into the last equation yields $4+m \geq km$, or $k \leq 1+4/m$. Since $m \geq 3$ this only allows $k=1,2$, so $n=4+m$ or $2(4+m)$.

If $k=1$ then our set looks like $\{1,3,m,m+4\}$. We must also have $m \mid 1+3+(m+4)$, which means $m \mid 8+m$ or $m \mid 8$. As $m \geq 3$ the only two possibilities are $m=4$ or 8 , leading to the two sets $\{1,3,4,8\}$ and $\{1,3,8,12\}$. Only the second of these is magical, upon checking.

Recall that $k \leq 1+4/m$, so the only way to have $k=2$ is if $m=3$ or 4 . This leads to the two sets $\{1,3,3,7/2\}$ and $\{1,3,4,4\}$. The first doesn't really have a chance of being magical, but the second is. Therefore the only two such sets are $\{1,3,4,4\}$ and $\{1,3,8,12\}$.

Part (v). One candidate for an additional element is the sum of the elements in the original set. If this sum is S , then the sum of the elements in the new set is clearly $2S$. By the lemma, we need only show that each element of the new set divides $2S$. The proof is trivial. So the new set will be magical as well.

* * * * *

Q10 *(Algebraic proof by the proposer)*

Within my garden I've a pond that's round
 Whose surface equal to 5028 $\frac{4}{7}$ square feet is found:
 In th' midst of which, a pole stands just upright
 Above the plain, one hundred feet in height:
 This pole being broke into two parts,
 Come, tell this query now, ye men of arts.
 The broken piece fell just at the pond's brink:
 How long is then the piece left, do you think?

If $\pi=22/7$ then the square of the radius of the circle is $5028 \frac{4}{7} \cdot 7/22 = 1600$. Put $c=100$, the whole height of the pole, and x = the part standing. Then $c-x$ will be the part broken off, and is the hypotenuse of a right-angled triangle whose two legs are $a=40$ and x ; consequently due to *Pythagoras's Theorem* $40^2+x^2=(c-x)^2$ and hence $x = \frac{10000-1600}{2 \cdot 100} = 42$, the length required.

* * * * *

Nothing remains but to insert two remarks. One reader found fault with the solution to Question 4. Of course, note that the measure of $\angle A$ is irrelevant. Please, show lenience with my carelessness! Finally, another cluster of correct solutions to the first problems, sent in by Dr. Tony Gardiner, the leader of the team representing the U.K. in the IMO. Unfortunately, they were too late to be taken into consideration.

This completes the solutions submitted for problems from the summer 1994 number, and this is all the space we have in this issue. Send me your contests, Olympiads, pre-Olympiads, and especially your nice solutions from across Europe!

Paul Jainta, Werkvolkstr. 10, D-91126 Schwabach, Germany

EUROPEAN NEWS: Country by Country

CATALONIA

Collected Works of Pere Menal

Edited by the Societat Catalana de Matemàtiques, Institut d'Estudis Catalans, Carme 47,
E-08001 Barcelona, Spain. Tel: +34 3 3185516 Fax: +34 3 4122994

Distributed by Birkhäuser Verlag AG

At the time of his death in a traffic accident at the age of thirty-nine, **Pere Menal** held the Chair of Algebra at the Universitat Autònoma de Barcelona, was an editor of *Communications in Algebra*, had directed six Ph.D. theses, and had written some forty papers in ring theory, several of them in collaboration with ring theorists from many parts of the world.

This book collects together all of his papers on ring theory, both the published and the unpublished. The earlier articles deal with group rings; later articles deal with von Neumann regular rings and C^* -algebras. A theme common to many of the articles is the behaviour of general linear groups, from such viewpoints as residual finiteness or the description of normalizers. This book should be of interest to mathematics libraries, and to mathematicians interested in ring theory, von Neumann regular rings, C^* -algebras, or general linear groups.

CZECH REPUBLIC

Spring Schools in Analysis

23 - 29 April 1995

Topic: **Recent trends in Banach Spaces**
Lectures delivered by Gideon Schechtman (*Rehovot, Israel*), Yoav Benyamini (*Haifa, Israel*), Piotr Mankiewicz (*Warsaw, Poland*), Per Enflo.

4 - 10 June 1995

Topic: **Mathematical models of microstructure**
A series of lectures delivered by Stefan Müller (*Freiburg*)

Address: Katedra matematické analýzy
Matematicko-fyzikální fakulta UK
Sokolovská 83, 186 00 Praha 8, Czech Republic
Tel/Fax 42-2-231 76 62
Email: obdrzal@karlin.mff.cuni.cz (for April conference)
or maly@karlin.mff.cuni.cz (for June conference)

There will also be a conference on Function Spaces, Differential Operators and Non Linear Analysis at Paseky. This is a continuation of two conferences on the same topic held in Sodenkyla (*Finland*) 1988 and Friedrichrode (*Germany*) 1992. For details contact

Dr. Jiří Rákosník, Mathematical Institute, Academy of Sciences of the Czech Republic,
Žitna 25, 11567 Praha 1, Czech Republic
Fax: +42-2-24227633 E-mail: rakosnik@earn.cvut.cz

ESTONIA

Estonian Mathematical Society

The new President of Estonian Mathematical Society is

Professor Mati Abel.

ITALY

ICTP, Trieste

Some mathematical conferences in 1995

- 13 - 24 March** **CONFERENCE ON TOPOLOGICAL AND GEOMETRICAL PROBLEMS RELATED TO QUANTUM FIELD THEORY**
Co-sponsor: CEC
Directors: P. Braam, C. De Concini, R. Dijkgraaf
Local Organizer: M.S. Narasimhan
Deadline for requesting participation: 31 August 1994
E-mail address: smr847@ictp.trieste.it
- 27 March - 7 April** **SPRING SCHOOL AND WORKSHOP ON STRING THEORY, GAUGE THEORY AND QUANTUM GRAVITY**
In cooperation with INFN and SISSA
Directors: R. Dijkgraaf, R. Iengo, I. Klebanov, K.S. Narain, S. Randjbar-Daemi
Local Organizers: R. Iengo, K.S. Narain, S. Randjbar-Daemi
Deadline for requesting participation: 31 January 1995
E-mail address: smr848@ictp.trieste.it
- 10 - 12 April** **CONFERENCE ON RECENT DEVELOPMENTS IN STATISTICAL MECHANICS AND QUANTUM FIELD THEORY**
Co-sponsor: SISSA
Directors: L. Bonora, J.L. Cardy, G. Mussardo, S. Randjbar-Daemi, H. Saleur
Local Organizers: L. Bonora, G. Mussardo, S. Randjbar-Daemi
Deadline for requesting participation: 31 January 1995
E-mail address: smr849@ictp.trieste.it
- 22 May - 2 June** **WORKSHOP ON DYNAMICAL SYSTEMS**
Co-sponsor: CEC
Directors: J. Palis, Y. Sinai, J.-C. Yoccoz
Local Organizer: G. Vidossich
Deadline for requesting participation: 15 November 1994
E-mail address: smr855@ictp.trieste.it
- 5 - 9 June** **TRIESTE CONFERENCE ON PHYSICAL AND MATHEMATICAL IMPLICATIONS OF MIRROR SYMMETRY**
Directors: E. Gava, K.S. Narain, C. Vafa
Local Organizers: E. Gava, K.S. Narain
E-mail address: smr856@ictp.trieste.it
- 21 August - 1 September** **CONFERENCE ON PARTIAL DIFFERENTIAL EQUATIONS & APPLICATIONS TO GEOMETRY**
Co-sponsor: CEC
Directors: K.-C. Chang, M. Giaquinta
Local Organizer: G. Vidossich
Deadline for requesting participation: 28 February 1995
E-mail address: smr869@ictp.trieste.it
- 22 - 25 August** **RANDOMNESS STOCHASTICITY AND NOISE (Adriatico Research Conference)**
Co-sponsor: CEC
Directors: H. Cerdeira, Hu Gang, F. Marchesoni
Deadline for requesting participation: 15 April 1995
E-mail address: smr868@ictp.trieste.it
- 29 August - 1 September** **INFORMATION THEORY IN CLASSICAL AND QUANTUM SYSTEMS (Adriatico Research Conference)**
Co-sponsor: CEC
Directors: H. Cerdeira, H. Haken, A.N. Proto
Deadline for requesting participation: 15 May 1995
E-mail address: smr870@ictp.trieste.it
- 4 - 15 September** **WORKSHOP ON GENERAL THEORY OF PARTIAL DIFFERENTIAL EQUATIONS & MICROLOCAL ANALYSIS**
Co-sponsor: CEC
Directors: Qi Min-you, L. Rodino
Local Organizer: G. Vidossich
Deadline for requesting participation: 28 February 1995
E-mail address: smr871@ictp.trieste.it

UNITED KINGDOM

47th British Mathematical Colloquium

Heriot-Watt University, Edinburgh.

3-6 April 1995

The 47th BMC, supported by a grant from the London Mathematical Society and by sponsorship from Shell Research, takes place at Heriot-Watt on the above dates.

Principal Speakers: J-P Serre, College de France (Monday); G. Pisier, Paris (Tuesday); and E. Bombieri, Princeton (Wednesday).

Morning Speakers: R.J. Archbold, R.W. Carter, J.E. Cremona, A.M. Etheridge, N.D. Gilbert, W.T. Gowers, M.R. Jerrum, J.T. Lewis, L.J. Mason, J.B. Paris, A.W. Reid, M. Reid, C. Ringel, R.L. Taylor, J.F. Toland.

Special Sessions: Functional Analysis (organised by A. M. Sinclair), and Ring Theory (K. A. Brown).

Applications The LaTeX source file of the application form is available by anonymous ftp from ftp.ma.hw.ac.uk (log on as ftp, give your full user-id as password, and see the file READ.ME in the directory pub/bmc for further instructions), or by email on request from bmc@ma.hw.ac.uk.

Erasmus Instructional Conference

An Erasmus Instructional Conference on Algebra and Related Topics (including Knot Theory and Operator Spaces) will be held at Edinburgh University immediately before the BMC. Further details are available from Dr T. H. Lenagan (erasmus@maths.ed.ac.uk; 0131-650 5078).

Further Information: Can be found in a World-Wide Web page on <http://www.ma.hw.ac.uk> (under Department of Mathematics : Conferences and Seminars), which will be kept up-to-date.

Enquiries: Professor J. Howie, Department of Mathematics, Heriot-Watt University, Edinburgh EH14 4AS (+44 131-451 3240; bmc@ma.hw.ac.uk).

THE NETHERLANDS

Arithmetic and geometry of abelian varieties: a conference in honour of Frans Oort

Date: 5 - 9 June 1995

Location University of Utrecht, Utrecht, The Netherlands

The Conference: The meeting will be held to celebrate the 60th birthday of Frans Oort.

Organizers: The Mathematical Research Institute MRI. MRI is a joint initiative of the mathematics departments of the universities of Groningen, Nijmegen, Twente and Utrecht in The Netherlands. Its aim is to promote research, organize graduate courses and seminars, and stimulate international contacts and exchange.

Program: Talks will start in the morning of Monday the 5th, and will end in the afternoon of Friday the 9th. On each day of the conference we expect to have two lectures by invited speakers. There will also be time for contributed lectures.

Confirmed speakers: C.-L. Chai (*University of Pennsylvania*), C. Deninger (*Universitaet Muenster*), B. Gross (*Harvard University*), H.W. Lenstra, jr. (*University of California*), Yu. Manin (*Max Planck Institut Bonn*) K. Ueno (*Kyoto University*), D. Zagier (*Max Planck Institut Bonn and Univ. Utrecht*).

Organizing committee: Bas Edixhoven (*Rennes*) (edix@univ-rennes1.fr)
 Johan de Jong (*Utrecht*) (dejong@math.ruu.nl)
 Ben Moonen (*Utrecht*) (moonen@math.ruu.nl)
 Jozef Steenbrink (*Nijmegen*) (steenbri@sci.kun.nl)
 Jaap Top (*Groningen*) (top@math.rug.nl)

Contact: B.J.J. Moonen, Vakgroep Wiskunde, P.O.Box 80.010, 3508 TA Utrecht, The Netherlands. Electronic mail: moonen@math.ruu.nl

GERMANY

Oberwolfach Tagungsprogramm 1995

Die Tagungen dauern jeweils eine Woche. Zur Abkürzung ist nur der Beginn angegeben.

- 1.1. Symmetrien; Jose M. Montesinos, Madrid; Reinhold Remmert, Münster; Peter Slodowy, Hamburg
- 8.1. Mathematische Optimierung; Bernhard Korte, Bonn; Klaus Ritter, München
- 15.1. Enumerative Combinatorics and the Symmetric Groups; George E. Andrews, University Park; Christine Bessenrodt, Magdeburg; Jörn Börling Olsson, København
- 22.01. Numerical Methods for Singular Perturbations; Pieter Hemker, Amsterdam; Hans-G. Roos, Dresden; Martin Stynes, Cork
- 29.01. Applied and Computational Convexity; Peter Gritzmann, Trier; Victor L. Klee, Seattle; Peter Kleinschmidt, Passau
- 05.02. Algebraic and Geometric Combinatorics; Anders Björner, Stockholm; Gil Kalai, Jerusalem; Günter M. Ziegler, Berlin
- 12.02. Qualitative Aspects of Partial Differential Equations; Henri Berestycki, Paris; Bernhard Kawohl, Köln; Giorgio Talenti, Firenze
- 19.02. Medical Statistics: Multivariate Models for Longitudinal Data; Niels Keiding, København; Martin Schumacher, Freiburg
- 26.02. Große diskrete Systeme; Thomas Beth, Karlsruhe; Martin Grötschel, Berlin
- 05.03. Mathematische Stochastik; Hans-Georg Müller, Davis; Dietrich Stoyan, Freiberg
- 12.03. DMV-Seminar: Statistische Wavelets; David L. Donoho, Berkeley; Iain M. Johnstone, Stanford
- 12.03. DMV-Seminar: Composite Materials and Homogenization; Luc Tartar, Pittsburgh; Francois Murat, Paris
- 19.03. Gewöhnliche Differentialgleichungen: Topological and Geometrical Methods in O.D.E's; Jean Mawhin, Louvain-la-Neuve; Klaus Schmitt, Salt Lake

- City; Hans-Otto Walther, München
- 26.03. Constructive Methods in Complex Analysis; Dieter Gaier, Gießen; Richard S. Varga, Kent
- 02.04. Mathematische Logik; Walter Felscher, Tübingen; Helmut Schwichtenberg, München; Anne S. Troelstra, Amsterdam
- 02.04. DMV-Seminar: Mori-Theorie; Thomas Peternell, Bayreuth; Yoichi Miyaoka, Tokyo
- 09.04. Arbeitsgemeinschaft mit aktuellem Thema: (wird in Heft 1/1995 der DMV-Mitteilungen bekanntgegeben);
- 16.04. Orders in Arithmetic and Geometry; Jürgen Ritter, Augsburg; Martin J. Taylor, Manchester
- 23.04. Algebraische Gruppen; Peter Slodowy, Hamburg; Tonny A. Springer, Utrecht; Jacques Tits, Paris
- 30.04. Multilevel Methods and Applications; Wolfgang Hackbusch, Kiel; Pieter Hemker, Amsterdam; Gabriel Wittum, Stuttgart
- 07.05. Linear Operators and Applications; Israel C. Gohberg, Tel Aviv; Bernhard Gramsch, Mainz; Helmut H. Schaefer, Tübingen
- 14.05. Mathematical Models in Phase Transitions; Hans Wilhelm Alt, Bonn; Paul C. Fife, Salt Lake City
- 21.05. Computational Aspects of Commutative Algebra and Algebraic Geometry; David Eisenbud, Waltham; Günter Scheja, Tübingen; Frank-Olaf Schreyer, Bayreuth
- 28.05. Computational Number Theory; Hendrik W. Lenstra Jr., Berkeley; Michael E. Pohst, Berlin; Horst G. Zimmer, Saarbrücken
- 04.06. Freiformkurven und Freiformflächen; Robert E. Barnhill, Tempe; Wolfgang Boehm, Braunschweig; Josef Hoschek, Darmstadt
- 11.06. Differentialgeometrie im Großen; Werner Ballmann, Bonn; Jean Pierre Bourguignon, Palaiseau; Wolfgang Ziller, Philadelphia
- 18.06. Quadratische Formen; Manfred Knebusch, Regensburg; Walter Pfister, Mainz; Winfried Scharlau, Münster
- 25.06. Bifurkation und Symmetrie; Eugene Allgower, Fort Collins; Klaus Böhmer, Marburg; Martin Golubitsky, Houston
- 02.07. Algebraic Methods in Multivariate Statistical Analysis; Michael D. Perlman, Seattle; Friedrich Pukelsheim, Augsburg
- 09.07. Schrödingeroperato-

- ren; Michael Demuth, Clausthal-Zellerfeld; Volker Enß, Aachen
- 16.07. Dynamische Systeme; Helmut W. Hofer, Zürich; Jürgen Moser, Zürich; Eduard Zehnder, Zürich
- 23.07. Darstellungstheorie endlich-dimensionaler Algebren; Maurice Auslander, Waltham; Claus Michael Ringel, Bielefeld
- 30.07. Einhüllende Algebren und Darstellungstheorie; Walter Borho, Wuppertal; Michel Duflo, Paris; Anthony Joseph, Paris; Rudolf Rentschler, Paris
- 06.08. Wavelets (Signalverarbeitung); Ingrid Daubechies, Murray Hill; Alfred K. Louis, Saarbrücken; Yves Meyer, Paris
- 13.08. Partielle Differentialgleichungen; Gerhard Huisken, Tübingen; Leon M. Simon, Stanford; Michael Struwe, Zürich
- 20.08. DMV-Seminar: L^2 -Invariants of Manifolds and Applications; Wolfgang Lück, Mainz; N.N.
- 20.08. Ganzzahlige quadratische Formen und Gitter; Eva Bayer-Fluckiger, Besancon; Helmut Koch, Berlin; Boris Borisovic Venkov, St. Petersburg
- 27.08. Complex Geometry: Vector Bundles in Geometry and Physics; Nigel J. Hitchin, Coventry; Klaus Hulek, Hannover; Robert Lazarsfeld, Los Angeles
- 03.09. Topologie; John Jones, Coventry; Ib Madsen, Aarhus; Elmar Vogt, Berlin
- 10.09. Knotentheorie; Joan S. Birman, New York; Maxim Kontsevich, Bonn
- 17.09. Informationstheorie; Rudolf Ahlswede, Bielefeld; Jacobus H. van Lint, Eindhoven; James Massey, Zürich
- 24.09. Nonlinear and Stochastic Systems; Ludwig Arnold, Bremen; Werner Schiehler, Stuttgart; Walter Wedig, Karlsruhe
- 01.10. Mathematische Methoden der Geodäsie; Willi Freeden, Kaiserlautern; Erik W. Grafarend, Stuttgart
- 08.10. Arbeitsgemeinschaft mit aktuellem Thema: (wird in Heft 3/1995 der DMV-Mitteilungen bekanntgegeben);
- 15.10. Geometrie; Victor Bangert, Freiburg; Ulrich Pinkall, Berlin
- 22.10. Empirical Processes: Theory and Applications; P. Laurie Davies, Essen; Peter Gaenssler,

- München; Winfried Stute, Gießen
- 29.10. Differential-Algebraic Equations, Related Fields of Theory and Applications; Roswitha März, Berlin; Linda R. Petzold, Minneapolis
- 05.11. Algebraic K-Theory and Homotopy Theory; Thomas Goodwillie, Providence; Friedhelm Waldhausen, Bielefeld
- 12.11. Allgemeine Ungleichungen; William Norrie Everitt, Birmingham; Laszlo Losonczy, Debrecen; Wolfgang Walter, Karlsruhe
- 19.11. DMV-Seminar: Infinite Dimensional Kaehler Manifolds; Aleksander A. Kirillov, Moscow; Alan T. Huckleberry, Bochum
- 19.11. Fortbildungslehrgang für Bibliothekare;
- 03.12. Topologische Methoden in der Gruppentheorie; Herbert Abels, Bielefeld; Peter Kropholler, London
- 10.12. Kinematik und Robotik; Hans Georg Bock, Heidelberg; Manfred Hiller, Duisburg; Josef Hoschek, Darmstadt; Friedrich Pfeiffer, München
- 17.12. Branching Processes; Peter E. Ney, Madison; Anton Wakolbinger, Frankfurt

Die Teilnehmer an den Tagungen in Oberwolfach werden vom Institut persönlich eingeladen. Ohne eine solche Einladung ist eine Teilnahme nicht möglich.

Interessenten, insbesondere auch jüngere Mathematiker, können sich an das Institut wenden. Da die Anzahl der Teilnehmer an einer Tagung beschränkt ist, können nicht immer alle Interessenten eingeladen werden.

OTHER CONFERENCES

BAHRAIN

An International Conference on Pure and Applied Mathematics

Date: 19 - 22 November 1995

Location: University of Bahrain

Contact: Professor A.Q.M.Khaliq, Conference Secretary - ICPAM95. Department of Mathematics, University of Bahrain, PO Box 32038, Isa Town, BAHRAIN
Tel: (+973) 688348 Fax: (+973) 682582 email: ICPAM95@isa.cc.uob.bh

BRIEF REVIEWS

Edited by Ivan Netuka and Vladimír Souček. Books submitted for review should be sent to the following address: Ivan Netuka, MÚUK, Sokolovská 83, 186 00 Praha 8, Czech Republic.

W.Scharlau (Ed.): *Mathematische Institute in Deutschland 1800-1945*, Dokumente zur Geschichte der Mathematik, Band 5, Friedrich Vieweg & Sohn, Braunschweig, 1990, vi+291 pp., DM 72.00, ISBN 3-528-08992-X

The book contains basic information about 38 mathematical institutes at German universities and technical colleges in 1800-1945. For every university or college, there is a short description of its history and the development of its mathematical institute and a list of its mathematicians (professors, associate professors, senior lecturers etc.) with essential data. The volume ends with the article of G.Schubring: *Zur strukturellen Entwicklung der Mathematik an den deutschen Hochschulen 1800-1945* (13 pp.) and with an index of names. The book can be recommended not only to mathematicians but also to historians as a source of information about German mathematics, mathematicians, universities and institutes. (jbe)

I.M.Gelfand, A.Shen: *Algebra*, Birkhäuser, Boston, 1993, 153 pp., DM 42.00, ISBN 3-764-33677-3, ISBN 0-817-63677-3

This book is the third book in a series which is intended for high school students. It is a nicely written introduction to the basic notions of elementary algebra. The authors' idea of the book is briefly expressed by Gelfand's words: "It was not our intention that all of the students who study from these books or even completed the School by Correspondence should choose mathematics as their future career. Nevertheless, no matter what they would later choose, the results of this mathematical training remain with them. For many, this is a first experience in being able to do something completely independently of a teacher." A further characterization of the book can be given by a sample of the topics covered: The Use of Parentheses. Dealing with Fractions. How to Explain the Square of the Sum Formula to Your Younger Brother or Sister. Division of Polynomials in One Variable, the Remainder. The Sum of a Geometric Progression. Quadratic equations. How to Confuse the Student on the Exam. The Arithmetic and Geometric Mean of Several Numbers. Many interesting elementary problems are included; the intent of the authors concerning their role is explained in the introduction: "The main part of the book is formed by problems. The best way to deal with them is: solve the problem by yourself - compare your solution with the solution in the book (if it exists) - go to the next problem." The book can be warmly recommended both to beginners as well as to those who prepare texts in elementary maths for beginners. (lbe)

Zhe-xian Wan: *Introduction to Abstract and Linear Algebra*, Chartwell-Bratt, Bromley, 1992, v+369 pp., GBP 29.95, ISBN 0-862-38316-1

This is an introductory text in algebra and linear algebra for undergraduate engineering students. The presentation is self-contained, assuming only high school knowledge of mathematics. The text contains all the standard material about groups, fields, matrices, and polynomials. The parts on linear algebra which are of a more geometric nature (e.g. Euclidean and unitary spaces) are not included. The focus of the presentation are finite fields, polynomials and commutative rings. Section 4.3 contains a beautiful presentation of Chinese Remainder Theorem, going back to Sun-Zi, 3-5 century AD. The highlights of the text are structure theorems for finite fields and a detailed analysis of polynomials over finite fields, their factorization (Berlekamp's algorithm) and irreducibility. Moreover, tables of irreducible polynomials over \mathbf{Z}_2 are included. The presentation deserves one more chapter dedicated to an introduction to linear codes and the role the above mentioned notions play there. Such chapter would surely increase the attractiveness of this nice book to students of engineering. (It seems likely that there was a chapter like this in the original text "Algebra and Coding Theory" (in Chinese), the translated, selected and revised version of which constitutes the present book.) (jt)

I.Chavel: *Riemannian Geometry: A Modern Introduction*, Cambridge Tracts in Mathematics, vol.108, Cambridge University Press, Cambridge, 1993, xii+386 pp., GBP 35.00, ISBN 0-521-43201-4

The book consists of 7 chapters: 1) Riemannian manifolds, 2) Riemannian curvature, 3) Riemannian volume, 4) Riemannian coverings, 5) The kinematic density, 6) Isoperimetric inequalities, 7) Comparison and finiteness theorems. This work is certainly not an elementary introduction for the beginners. The standard elementary topics are treated rather briefly and many of them are just left to the reader as exercises. The main emphasis is on the global properties of Riemannian manifolds, mainly on various integral formulas and global inequalities. In this respect, the material presented in this work is enormous and the number of topics, either discussed in detail or only briefly touched, is impressive. (Let us mention the spectrum of the Laplacian, measure theory, symplectic geometry, discretization of a Riemannian manifold, exponential volume growth, the Lichnerowicz conjecture on harmonic space, among many others). What is also impressive is the way the author is able to bridge, in a natural way, the whole historical period from the classics (Blaschke, Bonnet, Gauss, Jacobi, Liouville) to

the modern authors (Berger, Cheeger, Gromov, Milnor, Santalo, Yau and many others). The important part of the book is the large number of nontrivial exercises (with hints), notes and bibliographical remarks. The title of the book might well have been "What is hot in global Riemannian geometry". (ok)

R.Hermann: Constrained Mechanics and Lie Theory, Interdisciplinary Mathematics, vol.27, Math Sci Press, Brookline, 1992, 288 pp., ISBN 0-915692-43-0

This is another volume in the long series of books written by R. Hermann (mostly devoted to geometrical methods used in mathematical physics) which contains a mixture of three very different topics. The first one is non-holonomic geometry and the theory of non-holonomic dynamical systems (a historical survey of this interesting subject from a modern point of view together with an extensive bibliography can be found in EMS, vol. 16, Springer-Verlag, see the review below). The second part contains a detailed discussion of one of Lie's papers on symmetries of differential equations developed later by Vessiot and Pommaret. The last part treats a quite different subject - generalized function algebras and their connection to renormalization in Quantum Field Theory. The book is selfcontained - the author reviews briefly many known subjects at various parts of the book (e.g. distributions on manifolds, moving frames, affine connections, jets, Lie algebra valued forms, Maurer-Cartan equations). Nevertheless, these parts are not very useful for specialists and references to a full description of these topics in standard monographs would be better for a beginner. (vs)

V.I.Arnol'd, S.P.Novikov (Eds.): Dynamical Systems VII. Integrable Systems. Nonholonomic Dynamical Systems, Encyclopaedia of Mathematical Sciences, vol.37, Springer-Verlag, Berlin, 1994, 341 pp., 9 fig., DM 144.00, ISBN 3-540-18176-8, ISBN 0-387-18176-8

The book consists of three parts. The first one contains a discussion of nonholonomic dynamical systems. The subject is closely connected to classical problems in mechanics. Moreover, an interesting connection with the general theory of PDEs (hypoelliptic equations) and to optimal control problems have been recently established. The book contains the first part of a survey of the subject, namely the part devoted to non-integrable distributions on manifolds and to nonholonomic variational problems on Riemannian manifolds. The use of the language of modern differential geometry for the description of classical results is especially valuable. The second topic discussed is the description of certain integrable Hamiltonian systems based on group theory. Many examples of integrable Hamiltonian systems were found in classical mechanics. During last decades, a group theoretical scheme for their construction was found. The construction is based on a choice of a simple Lie algebra, a Riemannian symmetric pair or a Kac-Moody algebra and as a result, we get

a lot of classical integrable systems together with some new examples. The connection with algebraic geometry and a description of solutions using theta functions is given. Poisson Lie groups are used for discussion of difference Lax equations. The survey ends with a study of quantum Toda lattices. The third part of the book is devoted to integrability of Hamiltonian systems on homogeneous spaces and contains a topological theory of integrable systems. The book is a very valuable survey of the above topics and can be warmly recommended to anybody interested in the field. (vs)

L.M.Batten, A.Beutelspacher: The Theory of Finite Linear Spaces, Cambridge University Press, Cambridge, 1993, x+214 pp., 30.00, ISBN 0-521-33317-2

This is the first comprehensive treatment of finite linear (not vector!) spaces. The study includes important particular cases of affine, semi-affine and projective spaces. The approach is mostly purely combinatorial, except for the last chapter, where some deep results of the theory of finite groups are used. Besides many exercises and an appendix containing a list of all linear spaces with at most 9 points, one finds also lists of open problems and references after each chapter. This makes the book valuable both for researchers and graduate students. (jatr)

S.H.Lehnigk: The Generalized Feller Equation and Related Topics, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol.68, Longman Scientific & Technical, Harlow, 1993, xvi+292 pp., GBP 46.00, ISBN 0-582-08736-8

The generalized Feller equation is a linear parabolic partial differential equation of Fokker-Planck type for a function of one space variable. The equation is used for a description of diffusion-type models in various branches of physics and chemistry. The book contains a systematic discussion of properties of its solutions. A fundamental solution is constructed and initial and boundary value problems are solved. The expansion of solutions into series of special functions and related probability density functions are discussed. The book can be useful for mathematicians interested in applications, probability and statistics and for engineers. (vs)

D.Eisenbud, L.Robbiano (Eds.): Computational Algebraic Geometry and Commutative Algebra, Symposia Mathematica, vol.XXXIV, Cambridge University Press, Cambridge, 1993, x+298 pp., GBP 29.95, ISBN 0-521-44218-4

The book describes a part of the present state of computational algebraic geometry and shows many of the interesting trends and developments in the field. It contains two articles on open problems in commutative algebra, algebraic geometry and classification of algebras; four survey articles describing the most interesting current work and four original research papers. There is also a well-written introduction

to the theory of Groebner bases, their descriptions and their use in computation. The book could be useful for computer scientists interested in symbolic computation, robotics or Groebner basis as well as for mathematicians interested in algebraic geometry and commutative algebra. (jbu)

S.Iyanaga: Collected Papers, Iwanami Shoten, Tokyo, 1994, xx+361 pp., ISBN 4-00-005376-0.

This volume commemorates Shokichi Iyanaga's 88th birthday (a more significant milestone in Japan than the multiples of 10 celebrated in the West). It contains his mathematical papers written in English, French and German (including some major contributions to algebraic number theory), personal reminiscences of eminent mathematicians such as Chevalley, Delsarte, Hua and Takagi, and some fascinating insights into the development of 20th-century Japanese mathematics. The volume closes with a historical commentary by Iyanaga, full of interesting views on his life as a student in Tokyo, Hamburg and Paris, and on his later career as one of the most prominent figures in algebraic number theory and in the Japanese and international mathematical communities. (gaj)

P.Karow: Font Technology. Methods and Tools, Springer-Verlag, Berlin, 1994, xiv+457 pp., 306 fig., DM 128.00, ISBN 3-540-57223-6, ISBN 0-387-57223-6

This is a translation of the German book "Schrifttechnologie" by the same author. In 1987 Peter Karow wrote a book "Digital Formats and Typefaces" devoted to the subject. The book under review has a non-empty overlap with it but it also contains a lot of new material. The transition of typography to the use of digitalized typefaces brought a lot of interesting problems and some of them involve a portion of mathematics and/or informatics. For those who would like to learn the basic principles of creating nice fonts, the book represents an invaluable source of information. Parts of the book were written by other authors - to give an idea what is covered, here are the titles of some chapters: Typeface Market, Font Production in past and present (by G. Flake), Intelligent Font Scaling, Letter Spacing (by B. Kämmle), Type Quality, Legibility (by D. Wendt), Classifying Typefaces according to DIN (by K. H. Warkentin), Copyright (by P. Rosenfeld). Appendices A - E contain additional information on standards, a list of literature, index, etc. (jive)

W.A.Day: Entropy and Partial Differential Equations, Pitman Research Notes in Mathematics Series, vol.295, Longman Scientific & Technical, Harlow, 1993, xiv+108 pp., GBP 22.00, ISBN 0-582-22928-6

The aim of this book is to use the basic ideas of thermodynamics like the notion of entropy and the Clausius(-Duham) inequality to study some qualitative properties of positive solutions (representing a temperature, for example) of partial differential equations. Starting from a simple elliptic equation, the

positiveness and smoothness of a solution is employed to arrive at the entropy balance equality saying that entropy supply F in a bounded domain is equal to rate of generation of entropy G in this domain plus entropy flux H outward across the boundary of A . Studying the behavior of F , G , H and their appropriate generalizations for other types of equations leads to some interesting results. For example, it is shown for the heat equation that in an isolated system the entropy cannot decrease, a physically relevant fact. The book is suitable for mathematicians interested in understanding some of the physical principles underlying PDEs. The book should be understandable at PhD level. However, it is not meant as a first course in PDEs or in thermodynamics. Some basic knowledge of PDEs is recommended and some in thermodynamics is useful. Formally, there is just one point the reviewer found rather inconvenient, namely that no theorem is formulated throughout the book. Therefore, the search for information sometimes results in having to read a section of the book from top to bottom to understand the context of a result. (mr)

V.I.Arnold, V.V.Kozlov, A.I.Neishtadt: Dynamical Systems III. Mathematical Aspects of Classical and Celestial Mechanics. Second Edition, Encyclopaedia of Mathematical Sciences, vol.3, Springer-Verlag, Berlin, 1993, xiv+291 pp., 81 fig., DM 144.00, ISBN 3-540-57241-4, ISBN 0-387-57241-4

The book presents neither an introduction to dynamical systems nor a textbook for learning the topic. Rather, it provides a very well-written overview of the mathematical aspects of classical mechanics while also giving the reader the corresponding physical background. Basic principles of classical mechanics are discussed, like relativity, oscillations, Lagrangian and Hamiltonian mechanics, Hamilton's principle, dynamics of ideal gases, ending up with more advanced mathematical/physical topics like the theory of invariant manifolds, perturbation theory for integrable systems and the theory of small and quasi-random oscillations. Fans of the classical mathematical approach to dynamical systems should be satisfied too, finding sections devoted to the study of phase portraits and stability of equilibria states for corresponding systems. An important part of the book is devoted to the questions of integrability and nonintegrability. The reviewer appreciates the topological and geometrical aspects of (non)integrability discussed. Historical remarks and useful examples throughout the book make it even more interesting to read. The book is well-written both from a scientific and a formal point of view. It's a book for any season (... to be compared with "This is a book to curl up with in front of a fire on a cold winter's evening ..." - SIAM Reviews, Sept. 1989) (mr)

S.D.Chatterji, B.Fuchssteiner, U.Kulisch, R.Liedl (Eds.): Jahrbuch Überblicke Mathematik 1993, Jahrbuch Überblicke Mathematik, Friedrich Vieweg & Sohn, Braunschweig, 1993, viii+225 pp., DM 58.00, ISBN 3-528-06492-7

S.D.Chatterji, B.Fuchssteiner, U.Kulisch, R.Liedl (Eds.): Jahrbuch Überblicke Mathematik 1994, Jahrbuch Überblicke Mathematik, Friedrich Vieweg & Sohn, Braunschweig, 1994, viii+264 pp., DM 62.00 ISBN 3-528-06578-8

This is a series of year - books devoted to survey articles and introductions to topical fields of mathematics and the history of mathematics. The articles are presented in English or German. In the 1993 volume articles on the following topics are included: geometric duality and combinatorial optimization (M. Jünger and W. Pulleybank), creation of equi- distributed random numbers (U. Dieter), algorithms for propagation in artificial intelligence (B. Faltings), numerical computations for bounds of π (W. Krämer), asymptotic inversion of convolution operators (B. Silbermann), recent developments of quantum groups (P.P. Kulish), mathematics in the design of a control circuit (N. Dourdumas). There is a report on the European Mathematical Congress in 1992 in Paris and news of mathematics in the period 1991 - 1992. A biography of Norbert Wiener is presented by S.D. Chatterji. The volume for 1994 contains articles on functional equations (J. Schwaiger), algorithms in discrete mathematics and probabilistic arguments (H. J. Prömel), semigroups in population dynamics (J. Prüss and W. Schappacher), numerics for some equations in mechanics (P. Kaps), algebraic eigenvalue problems and methods of verification of the results (G. Mayer), applied algebra (M. Hazewinkel). There is a report on the situation in scientific publications and electronic information by M. Grötschel, J. Lügger and W. Sperber. Historical aspects are touched on in articles on the Fredholm alternative, the Bieberbach conjecture, the history of ETH in Zürich and mathematics in Berlin in the period 1700 - 1993. (ss)

I.Stewart: The Problems of Mathematics. Second Edition, Oxford University Press, Oxford, 1992, x+347 pp., 42 fig., GBP 8.99, ISBN 0-192-19262-0, ISBN 0-192-86148-4

This is a revised edition of the book published in 1987. Both specialists and nonspecialists alike will gain new insights into problems of mathematics. The author explains in everyday language many of the great problems and techniques. The book is beautifully written and can be used in various ways for making effective propaganda for mathematics. (ss)

W.F.Pfeffer: The Riemann Approach to Integration: Local Geometric Theory, Cambridge Tracts in Mathematics, vol.109, Cambridge University Press, Cambridge, 1993, xv+302 pp., GBP 30.00, ISBN 0-521-44035-1

A detailed exposition of the generalized Riemann (Riemann-Stieltjes) integrals discovered by J. Kurzweil, R. Henstock (and by E.J.McShane for the absolutely convergent form) is presented in the book. Generalized integrals are based on Riemann (Riemann-Stieltjes) integral sums with respect to partitions of the figure over which the integral is to be defined. While the one-dimensional generalized integration is relatively simple, the theory of generalized integration over higher-dimensional sets is more sophisticated. For obtaining an integral with nice properties (e.g., an integral with a nice Gauss-Green Theorem) the definition given in the one-dimensional case has to be modified. These modifications concern mainly the geometric properties of the partitions. Pfeffer's book is a well organized monograph reflecting one branch of the new viewpoint on integration based on Riemann type integral sums and leading to very general integration theories. (ss)

A.J.P.Watkins: Derive-based Investigations for Post-16 Core Mathematics. 2nd Edition, Chartwell-Bratt, Bromley, 1993, 102 pp., GBP 9.95, ISBN 0-862-38312-9

Derive, A Mathematical Assistant, is one of several packages intended for both symbolical and numerical calculations providing the user with reasonable graphical possibilities. This booklet is designed to introduce the reader to this package and to some of the concepts of advanced mathematics. The topics covered are as follows: Elementary graphics and elementary calculus; The graphical features of reciprocal, trigonometric, exponential and logarithmic functions; Limits, differentiations and differential equations; Combining oscillations; Numerical methods; Selected functions and their series expansions. The level is very elementary, say that of the better scientific calculator. For each chapter the exercises with solutions are available. This book is completely unsuitable for those wishing to master the DERIVE package for solving difficult problems. On the other hand, it can be used for students who are starting mathematical analysis courses at technical universities. I regret that I did not find either the contents or index. (jant)

M.Giaquinta: Introduction to Regularity Theory for Nonlinear Elliptic Systems, Lectures in Mathematics, Birkhäuser, Basel, 1993, viii+130 pp., DM 38.00, ISBN 3-764-32879-7, ISBN 0-817-62879-7

This book is intended for graduate students and researchers whose interests may include partial differential equations, calculus of variations and related areas. The main aim of this book is to investigate the behavior of minimizers of variational integrals and of solutions of elliptic systems. Chapters 1 and 2 are devoted to the so-called Hilbert space approach to existence and regularity of solutions of nonlinear systems in divergence form. The classical "Dirichlet-principle" and "Direct methods

in the calculus of variations" are described in simple situations. Both interior and boundary regularity in Sobolev spaces are studied by means of the difference-quotient-method. The Morrey and the Campanato spaces are introduced in Chapter 3 as useful tools for investigation of Hölder-continuity of the derivatives of solutions. In Chapter 4, L^p estimates for linear systems are presented. The most beautiful results and techniques are described in Chapter 5. The classical regularity result on second order elliptic equations with bounded measurable coefficients is presented. The author provides two different proofs of this famous theorem: using of De Giorgi class of functions and Moser's iteration technique. Chapter 6 may be regarded as a brief introduction to the "vector-valued" case. To avoid technical difficulties, the author presents only simple model situations and therefore the book can be recommended to beginners in the field. (otta)

D.W.Stroock: Probability Theory, an Analytic View, Cambridge University Press, Cambridge, 1993, xvi+512 pp., GBP 30.00, ISBN 0-521-43123-9

This is a new contribution to the collection of basic probability texts. What are the main features of this textbook compared to classical treatises of authors like Feller or Loève? Firstly, it is the author's emphasis on relations of probability theory to analysis (e.g., singular integral operators and harmonic analysis). Also, many new results which have emerged in recent years are discussed. The presentation of all the basic themes (like the Law of Large Numbers and Large Deviations, Central Limit Theorem, Infinitely Divisible Laws, various aspects of the Wiener Measure, Martingales, Diffusion, basics of Potential Theory) is very instructive, precise technical descriptions being complemented also by more condensed text somewhere and by many intuitively appealing comments. Compared to the wealth of information and insights on these subjects and their relation to analysis, the relations to linear algebra and statistical physics are stressed to a lesser extent and notions like entropy, percolation or Poisson point field are not included in the book. Acknowledging these limitations in the aim of the book, it is a pleasure to read and one can certainly recommend this very well written and very well organized book both to beginners and specialists in the subject. (mz)

J.-P.Kahane: Some Random Series of Functions. Second Edition, Cambridge Studies in Advanced Mathematics 5, Cambridge University Press, Cambridge, 1993, xiii+305 pp., GBP 19.95, ISBN 0-521-45602-9, ISBN 0-521-24966-X

This is a book on applications of random methods in harmonic analysis. It deals with objects like random series in Banach or Hilbert space, Taylor or Fourier series with random coefficients (either of the "Rademacher" or gaussian type), random point masses on the circle, random coverings of the circle and Brownian motion. It is the second, expanded and updated edition (1985,

paperback 1993) of the previous book (1968) of the author describing the present status of the field which started in the papers by Paley, Wiener, Zygmund, ..., in 1930's. The book is a concise, selfcontained and very useful source of information on the subject, even for a reader looking "randomly" at some of the many interesting topics discussed there. The introductory chapter makes the book accessible even to the readers lacking preparation in probability theory. (mz)

S.Yakubov: Completeness of Root Functions of Regular Differential Operators, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol.71, Longman Scientific & Technical, Harlow, 1994, 245 pp., GBP 43.00, ISBN 0-582-23692-4

The monograph is a systematic explanation of non-selfadjoint operators and operator pencils. The first chapter contains auxiliary material with the emphasize on interpolation theory. In Chapter 2, the discreteness of the spectra of perturbed unbounded operators as well as the completeness of root vectors (i.e., the eigenvectors and the associated vectors) are studied. Further, the same aspects are considered in the case of operator polynomial pencils. (The operator pencils play approximately the same role in the theory of differential equations with constant operator coefficients as characteristic polynomials play in theory of linear differential equations with constant coefficients.) In what follows (Chapters 3 - 6), the questions of the completeness of elementary solutions of problems of mathematical physics are investigated. The book is intended for scientists and graduate students in Functional Analysis, Differential Equations and related topics. It contains a lot of the author's original results. Reference Notes as well as 97 references are included. (oj)

Ch. Swartz: An Introduction to Functional Analysis, Monographs and Textbooks in Pure and Applied Mathematics, vol.157, Marcel Dekker, Inc., New York, 1992, xi+600 pp., \$ 55.00, ISBN 0-824-78643-2

This is a textbook on functional analysis for graduate students showing the interplay between the abstract theory and its applications. The choice of the main topics reflects the taste of the author who assumes only a basic knowledge of measure theory and topology. The book starts with the standard theory of topological vector spaces. Then three basic principles of functional analysis are introduced (the Hahn-Banach theorem with applications to Banach limits and the moment problem, the uniform boundedness principle based on the basic matrix theorem of Antosik and Mikusinski which requires no completeness assumptions on the domain space, and the open mapping and closed graph theorems). The next chapter is devoted to locally convex spaces (mainly duality, weak topologies, Krein-Milman theorem, quasi-barrelled and bornological spaces). It follows the theory of linear operators (barreled spaces, the Krein-Smulian theorem, vector integration, Schwartz

distributions, compact, weakly compact and absolutely summing operators, the Dvoretzky-Rogers theorem, etc.) and a final chapter on spectral theory (also Lomonosov's theorem, the spectral theory for hermitian operators, Banach algebras). (jl)

T.W.Palmer: Banach Algebras and the General Theory of *-Algebras. Volume I: Algebras and Banach Algebras, Encyclopedia of Mathematics and its Applications, vol.49, Cambridge University Press, Cambridge, 1994, xi+794 pp., GBP 60.00, ISBN 0-521-36637-2

This volume of the Encyclopedia of Mathematics and its Applications presents a detailed modern account of basic Banach algebra theory. The main chapters of the bulky monograph of 794 pages concern normed algebras, the notion of spectrum, commutative algebras and functional calculus, ideals, representations and radicals, approximate identities and factorization, automatic continuity, structure spaces and algebras with minimal ideals. The focus is on the algebraic, and sometimes the geometric, background of the analytic theory. The book is accompanied by plenty of historical remarks, background material and examples. The volume also contains previously unpublished results provided with complete proofs at a level suitable for graduate students. An extensive bibliography is attached. The book provides the background for Volume II which will deal mainly with *-algebras. (jl)

I.Grattan-Guinness: Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences, vol. 1 & 2, Routledge, London, 1993, 2648 pp., GBP 150.00, ISBN 0-415-09238-8, ISBN 0-415-09239-6

The *Companion Encyclopedia* is a collection of 176 articles (133 authors of 18 nationalities) describing the development of all areas of pure and applied mathematics (each article is self-contained and has its own bibliography). It aims to recover our mathematical heritage, examining the history and philosophy of the mathematical sciences in a cultural context, and tracing their evolution from ancient times up to the twentieth century. The book is divided into thirteen parts. Part 1 treats mathematics in ancient and non-Western cultures from ancient up to medieval and Renaissance times (Babylon, Egypt, Greece, Byzantium, Islamic civilization, Africa, China, Japan, Korea, Tibet, ...). Part 2 deals with development of mathematics during the medieval and Renaissance periods (Euclidean and Archimedean traditions, practical geometry, abacus arithmetics, logarithms, mechanics, astronomy, optics, technology and machines, ...). Parts 3-10 are divided into the main branches along which mathematics developed from the early seventeenth century onwards. Parts 11-12 presents the history of mathematics from other points of view (institutions of higher education, and organization in various countries, women and mathematics, mathematical journals, mathematical

games, recreational mathematics, art and architecture, mathematics and poetry, monuments to mathematics and mathematicians, ...). Part 13 consists of a general bibliography of important books and periodicals in the history of mathematics, a chronological table including some of the principal events in the development of mathematics (up to 1931), a concise record of basic facts about the great mathematicians, a list of authors and board members and a general index of persons and subjects. The *Companion Encyclopedia* contains much rich introductory material concerning the development of mathematics. It will be useful for mathematics students as well as teachers. (jibe)

K.Bierstedt, A.Pietsch, W.M.Ruess, D. Vogt (Eds.): Functional Analysis. Proceedings of the Essen Conference, Lecture Notes in Pure and Applied Mathematics, vol.150, Marcel Dekker, Inc., New York, 1994, xviii+525 pp., \$ 150.00, ISBN 0-824-79066-9

The present volume is based on lectures presented at the international Symposium on Functional Analysis held in Essen, Germany, 1991 covering fields such as the geometry of Banach spaces and operator ideals, Fréchet spaces with applications to analysis and partial differential equations, semigroups of operators and evolution equations. The main lecture series were given by Stefan Heinrich (Random approximation in numerical analysis), Reinhold Meise (Continuous linear right inverses for linear partial differential operators) and Philippe Bényan (Main results and open problems in the theory of nonlinear evolution equations). This volume contains 26 further articles, the Conference programme and the list of Conference participants. (jl)

L.W.Baggett: Functional Analysis. A Primer, Monographs and Textbooks in Pure and Applied Mathematics, vol.153, Marcel Dekker, Inc., New York, 1992, xi+267 pp., \$ 55.00, ISBN 0-824-78598-3

This book represents an excellent primary text for graduate students taking a one-year course assuming only a basic knowledge of measure theory and topology. The author states relatively few theorems with proofs and invites the reader to complete the further material in the form of exercises. For the sake of simplicity the most general cases of theorems and definitions are not presented. For example, the author considers only second countable locally compact spaces, σ -finite measures and separable Hilbert spaces. After preliminaries and the presentation of the Riesz representation theorem and the Hahn-Banach extensions theorems, the author introduces topological vector spaces (including the Schwartz space, Krein-Milman theorem and Choquet's theory), Banach spaces and duality. The next chapter contains applications to classical analysis (integral operators, convolution, Green's function, Fourier and Hilbert transform, Poisson and Gauss kernels, etc.). The second half of the book concerns the spectral theory on Hilbert spaces (the Gelfand approach for bounded normal operators). Of

course, standard classes of operators (compact, Hilbert-Schmidt, trace class, unbounded selfadjoint) are also studied. As a motivation the author briefly presents a set of axioms for a mathematical model of experimental sciences and introduces the foundations of quantum mechanics. The final chapter is devoted to a basic introduction to nonlinear functional analysis and the infinite-dimensional calculus culminating in the implicit function theorem. (jl)

J.L.Doob: *Measure Theory*, Graduate Texts in Mathematics, vol.143, Springer-Verlag, New York, 1994, xii+210 pp., DM 88.00, ISBN 0-387-94055-3, ISBN 3-540-94055-3

This book results from the convincing attempt to organize and present standard measure theory to the benefit of a would-be analyst not only as an intrinsic part of his education in pure mathematics but also as a reliable bridge to the probability side of mathematics. The author's project "to introduce probability concepts in their appropriate place, not to consign them to a ghetto" is performed excellently; the colorful language of probability is used as frequently as possible to enliven the mathematical reasoning, concepts like independency and the martingale property are promoted to the inner cycle of important concepts of measure theory. Doubtless, students of probability theory will find the book useful and instructive also, nevertheless, the missing crucial parts of probabilistic measure theory (weak convergence is introduced for locally compact spaces only, for example) will perhaps make the book less important from their point of view. Professor Doob presents his mathematics beautifully as always, making his technical reasoning as condensed as possible, stressing the ideas and their "non mathematical presentation". The book is well equipped with examples taken mostly from probability and potential theory. The author prefers to use pseudometrics instead of metrics for spaces of sets and functions to eliminate equivalence classes, a concept which he finds to be artificial for the purposes of the book. Pseudometric space properties are applied also in the context of measure extension problems: Outer measures are used to pseudometrize pavings of sets and the extension of a measure from an algebra to the minimal-algebra becomes a problem to find a closure of a subset of a pseudometric space. Contents: Operation on sets, Classes of subsets of a space, Set functions, Measure spaces, Measurable functions, Integration, Hilbert space, Convergence of measure sequences, Signed measures, Measures and functions of bounded variation on \mathbb{R} , Conditional expectations: Martingale theory. (jos)

D.L.Cohn: *Measure Theory*, Birkhäuser, Boston, 1993, ix+373 pp., Sfr 74.00 ISBN 3-764-33003-1, ISBN 0-817-63003-1

This book brings a straightforward and precise treatment of classical measure theory in the style of P.Halmos which is needed in analysis and probability. It is enriched by three excellent chapters on locally compact

spaces integration, Polish, Souslin and Lusin spaces, the topics that have become increasingly important recently for both probabilists and analysts. The first five chapters of the book on abstract measure and integration theory follow well-tried concepts, Lebesgue measure on \mathbb{R}^d constructed in Chapter 1 being the basic motivation and example thereafter. Chapter 6, a very useful one for a beginner, provides an elegant treatment of changes of variables and basic results on the differentiation of functions and measures on \mathbb{R}^d . Probabilists might miss a presentation of measure theory on infinite dimensional product spaces in this part of the book. Chapter 7 deals with the integration on locally compact Hausdorff spaces, the duality and Riezs representation theorems being the central topic. The construction and properties of Haar measure together with a brief introduction to convolution semigroups of integrable functions on locally compact topological groups constitute the main part of Chapter 9. Of course, what makes this book so attractive for the reviewer is the content of Chapter 8, i.e., Polish, Souslin and Lusin spaces topology and measure theory. The corresponding parts in L.Schwartz's "Radon measures..." were roughly sketched to the satisfaction of a well-trained topologist, D.L.Cohn offers a neat text that is presentable to students and will surely serve as a useful reference. Five appendices are attached to the main body of the text to summarize some basic facts from set theory, algebra, calculus and topology, to introduce briefly the Bochner integral. Each section of the book ends with exercises, those needed later in the text itself are provided with hints. The author wrote the first six chapters for a reader with a knowledge acquired in a standard course on mathematical analysis and algebra, while Chapters 7, 8 and 9 presuppose perhaps a familiarity with the essentials of general topology. The text is clearly arranged so it may be used either for personal studies or as good reference text for courses in analysis and probability. (jos)

R.L.Crole: *Categories for Types*, Cambridge Mathematical Textbooks, Cambridge University Press, Cambridge, 1993, xvii+335 pp., GBP 14.95, ISBN 0-521-45701-7, ISBN 0-521-45092-6

This book is devoted to categorical type theory. The core of the contents of the book is formed by examining four formal type theories, namely algebraic type theory, functional type theory, polymorphic functional type theory (2...x-Theory) and higher order polymorphism (...x-Theories). The book has a very clear structure: the first chapter is devoted to ordered sets, Boolean and Heyting algebras, domains and their properties as an introduction to category theory and as a base for the domain-theoretic model of polymorphism in the last two chapters. The second chapter presents basic notions, ideas and results of category theory needed later (adjoints and adjoint functor theorem, Yoneda lemma, Cartesian closed categories, indexed categories). The remaining Chapters 3,4,5,6 are devoted in turn

to the above four formal type theories. In all these four cases, the author starts with motivating ideas, mostly arising from programming and programming languages. Then he describes the syntax and categorical semantics. The models of algebraic type theories are built in categories with finite products, the models of functional type theories in Cartesian closed categories and the models of 2..x-Theories and of ...x-Theories are built in more complicated categorical structures. The soundness theorem and the completeness are shown for all the four formal systems. A classifying categorical structure with a generic model is constructed for each of these theories and a one-to-one correspondence (up to an equivalence) between the formal type system and the corresponding categorical structure is shown. For polymorphic theories also the important models, PER model and Domain model, are presented. (vt)

R.Bulirsch, D.Kraft (Eds.): Computational Optimal Control, International Series of Numerical Mathematics, vol.115, Birkhäuser, Basel, 1994, vii+382 pp., DM 148.00, ISBN 3-764-35015-6, ISBN 0-817-65015-6

These proceedings contain thirty selected papers from the workshop "Control Applications of Optimization" which was held in Munich in 1992 and was the ninth in a series of biennial meetings. The topic of the workshop was very applied as can be seen from the following five main parts of this collection: surveys on computational optimal control (OC), theoretical aspects of OC and nonlinear programming (this part contains five articles), algorithms and software for OC calculations, and applications of OC. These applications arise from astronautics, aeronautics and mostly from control of robots. There is also a paper on control of an activator-inhibitor enzymatic system. This perfectly designed book is interesting not only for experts but for anybody who likes to know the present state of optimization and nonlinear programming. (jm)

D.Hilbert: Theory of Algebraic Invariants, Cambridge University Press, Cambridge, 1993, xiv+191 pp., GBP 25.00, ISBN 0-521-44457-8, ISBN 0-521-44903-0

This book (published simultaneously in hardcover and paperback edition) belongs to the Cambridge Mathematical Library which provides a relatively inexpensive new edition of classic titles. This volume contains an English translation of the notes of Hilbert's course on invariant theory taken by his student Sophus Marxsen. The course consists of 51 lectures which were read from April 27 to August 6, 1897 at the University of Göttingen. The contents of the course includes, e.g., the characterization of invariants and covariants, Hilbert's Finiteness Theorem, an algorithm for computing a full system of invariants (including Hilbert's Nullstellensatz), Hilbert's Syzygy Theorem and several applications of invariant theory to algebraic geometry. This text makes a bridge between nineteenth-

and twentieth-century mathematics and can serve as a self-contained introduction to invariant theory but it is especially invaluable as a historical source for the foundations of modern mathematics. (pn)

J.Adámek, J.Rosický: Locally Presentable and Accessible Categories, London Mathematical Society Lecture Note Series 189, Cambridge University Press, Cambridge, 1994, xiv+316 pp., GBP 25.00, ISBN 0-521-42261-2

The monograph is devoted to some aspects of a categorical theory of mathematical structures, with the locally presentable categories and the accessible categories playing an important rôle. The locally presentable categories, introduced by Gabriel and Ulmer in 1971 in a purely categorical way, are categories which appear in universal algebra and model theory in many connections; many current categories are locally presentable. In the monograph, many characterizations of accessible and locally presentable categories are presented, e.g., by means of first order logic, by means of orthogonality, specified free cocompletions of small categories, and others. The monograph has six chapters and an Appendix. In Chapter I and Chapter II, basic properties of locally presentable and accessible categories are examined. Chapter III is of algebraic nature; in Chapter IV, the authors investigate injectivity, Chapter V is devoted to the description of various types of logics and the locally presentable and accessible categories are characterized as the categories of models of certain theories in these logics. "Vopěnka's" principle is the title of Chapter VI. This set-theoretical statement has a surprisingly strong influence on the properties of locally presentable categories. In the Appendix "Large Cardinals", the authors explain the position of Vopěnka's principle in set theory: Vopěnka's principle implies that measurable cardinals exist and if huge cardinals exist, then Vopěnka's principle is consistent. The monograph starts at advanced level, the authors expect that the reader is familiar with the basic category-theoretical concepts. The monograph not only compiles the important modern part of category but also enriches it by many new results of the authors. Some of these results are published in the monograph for the first time. (vt)

S.Zeidman: Topics in Abstract Differential Equations, Pitman Research Notes in Mathematics Series, vol.304, Longman Scientific & Technical, Harlow, 1994, xii+188 pp., GBP 26.00, ISBN 0-582-23744-0

These research notes are a continuation of the previous author's book "Abstract Differential Equations" (the same series 36(1979)) and is based on several of the author's recent papers. The main attention is paid to existence and uniqueness of solutions of the Cauchy problem for linear, semilinear and singular (i.e. $Bu=Au$) abstract differential equations and also to the existence and uniqueness of periodic, almost periodic and asymptotically almost periodic solutions

to nonhomogeneous equations. Preliminary knowledge on semigroups of operators and evolution equations is required and therefore the book can be recommended mainly to postgraduate students and research workers. Unfortunately, the book is not clearly arranged and moreover contains no index. (jm)

F.S.Macaulay: The Algebraic Theory of Modular Systems, Cambridge Mathematical Library, Cambridge University Press, Cambridge, 1994, xxxi+112 pp., GBP 12.95, ISBN 0-521-45562-6

This book is a new edition of a classic title by F.S.Macaulay, first published in 1916. The first two chapters deal with the question of finding solutions to systems of polynomial equations and are devoted to the study of the resultant and the resolvent. The remaining two chapters are concerned with the structure of ideals in polynomial rings, including, e.g., the Unmixedness Theorem, primary decomposition, the Hilbert basis theorem and the "inverse system" of an ideal (in modern terms, Hom....). The book is equipped with an extensive introduction by P.Roberts. In this introduction, Macaulay's ideas are placed in their historical and mathematical context showing also their development since those days. (pn)

W.Ebeling: Lattices and Codes. A Course Partially Based on Lectures by F.Hirzebruch, Advanced Lectures in Mathematics, Friedrich Vieweg & Sohn, Braunschweig, 1994, xii+178 pp., DM 48.00, ISBN 3-528-06497-8

This book presents a nice investigation of relations between the theory of error-correcting codes and the theory of integral lattices (discrete subgroups of n -dimensional Euclidean space with compact quotient). In particular, a deep analysis of the Golay code and the Leech lattice is given. The results are also generalized to weight enumerators of self-dual linear codes over the prime field F_p , p being an odd prime number. The book is thoroughly written and requires only minimal prerequisites (basic algebra and complex analysis). It shows in a nice way how seemingly distant branches of mathematics are intrinsically closely interconnected. The book could well serve as a base for a course both in coding theory and the theory of integral lattices. (pn)

R.Lidl, H.Niederreiter: Introduction to Finite Fields and Their Applications. Revised Edition, Cambridge University Press, Cambridge, 1994, xi+416 pp., GBP 29.95, ISBN 0-521-46094-8

This book is a revised edition (first published 1986) and is meant as a textbook for undergraduate students. The first part of the book presents an algebraic background, the structure theory of finite fields, the theory of polynomials over the fields and several factorization algorithms. The second part deals with the most popular applications of finite fields (algebraic coding theory, cryptography, information theory). Various links with finite geometries and combinatorics are also mentioned. The book is comfortably readable and the material exposed is well illustrated by numerous worked out examples and exercises. (tk)

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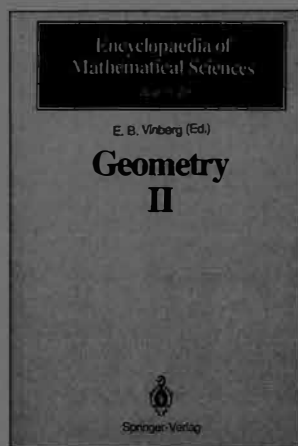
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Geometry II Spaces of Constant Curvature

With contributions by D.V. Alekseevskij,
O.V. Shvartsman, A.S. Solodovnikov,
E.B. Vinberg

1993. IX, 254 pp. 87 figs. (Encyclopaedia of
Mathematical Sciences, Vol. 29)
Hardcover DM 144,- ISBN 3-540-52000-7

This book contains a systematic and
comprehensive exposition of Lobachevskian
geometry and the theory of discrete groups of
motions in Euclidean space and Lobachevsky
space. The authors give a very clear account of
their subject describing it from the viewpoints of
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and methods for computing volumes of non-
Euclidean polyhedra. Part II includes the theory of
crystallographic, Fuchsian, and Kleinian groups
and an exposition of Thurston's theory of
deformations.

R.V. Gamkrelidze (Ed.)

Geometry I Basic Ideas and Concepts of Differential Geometry

With contributions by D.V. Alekseevskij,
V.V. Lychagin, A.M. Vinogradov

1991. V, 264 pp. 62 figs. (Encyclopaedia of
Mathematical Sciences, Vol. 28)
Hardcover DM 144,- ISBN 3-540-51999-8

Since the early work of Gauss and Riemann,
differential geometry has grown into a vast network
of ideas and approaches, encompassing local
considerations such as differential invariants and
jets as well as global ideas, such as Morse theory
and characteristic classes. In this volume, the
authors give a tour of the principal areas and
methods of modern differential geometry.

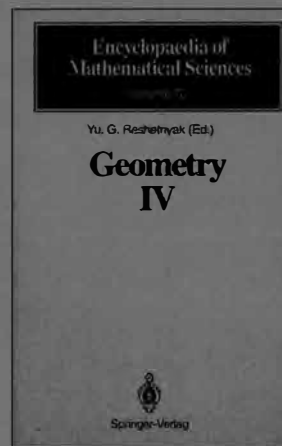
The book is structured so that the reader may
choose parts of the text to read and still take away
a completed picture of some area of differential
geometry. Beginning at the introductory level with
curves in Euclidean space, the sections become
more challenging, arriving finally at the advanced
topics which form the greatest part of the book:
transformation groups, the geometry of differential
equations, geometric structures, the equivalence
problem, and the geometry of elliptic operators.
Several of the topics are approaches which are
now enjoying a resurgence, e.g. G-structures and
contact geometry.

Y.D. Burago, V.A. Zalgaller (Eds.)

Geometry III Theory of Surfaces

1992. VIII, 256 pp. 80 figs. (Encyclopaedia of
Mathematical Sciences, Vol. 48)
Hardcover DM 144,- ISBN 3-540-53377-X

The theory of surfaces in Euclidean spaces is
remarkably rich in deep results and applications,
e.g. in the theory of non-linear partial differential
equations, physics and mechanics. This theory has
great clarity and intrinsic beauty, and differs in
many respects from the theory of multidimensional
submanifolds, therefore a separate volume is
devoted to surfaces. It is concerned mainly with the
connection between the theory of embedded
surfaces and two-dimensional Riemannian
geometry, and, above all, with the influence of
properties of intrinsic metrics on the geometry of
surfaces.



Y.G. Reshetnyak (Ed.)

Geometry IV Non-regular Riemannian Geometry

With contributions by V.N. Berestovskij,
I.G. Nikolaev, Y.G. Reshetnyak

1993. VII, 250 pp. 58 figs. (Encyclopaedia of
Mathematical Sciences, Vol. 70)
Hardcover DM 144,- ISBN 3-540-54701-0

This volume contains two articles, which give a
survey of modern research into non-regular
Riemannian geometry. The first article written by
Reshetnyak is devoted to two-dimensional
Riemannian manifolds of bounded curvature.
Concepts of Riemannian geometry, such as the
area and integral curvature of a set, and the length
and integral curvature of a curve are also defined
for these manifolds. Some fundamental results of
Riemannian geometry like the Gauss-Bonnet
formula are true in the more general case
considered in the book.

The second article by Berestovskij and Nikolaev
is devoted to the theory of metric spaces whose
curvature lies between two given constants. The
main result is that these spaces are in fact
Riemannian. This result has important applications
in global Riemannian geometry.

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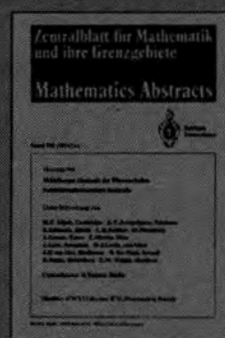
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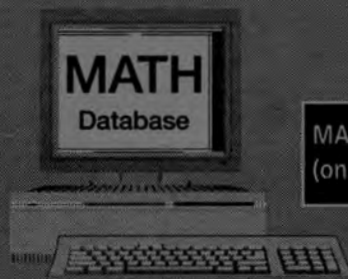
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