European Mathematical Society

NEWSLETTER No. 21

September 1996

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NOTICE FOR MATHEMATICAL SOCIETIES

Please note labels are prepared during the second half of the month before the next issue. Would you please send your updated lists before this time.

Many thanks. Ms T Mäkeläinen

EDITOR'S NOTE

The slight delay in the appearance of this issue was caused by unexpected IT-related difficulties. We apologise for this. In retrospect, why anyone should be surprised that IT brings with it a series of exasperating technical hitches to overcome is a mystery. Despite the problems, we have enjoyed putting this issue together. We have noted, with regret, that no general article has been supplied for inclusion and would welcome such an article for the December issue.

Prof. R. Bradley

EUROPEAN WOMEN IN MATHEMATICS WEB PAGE

EWM is pleased to announce the launch of its new Web page. The address is

http://www.math.helsinki.fi/EWM

Comments or material for inclusion should be sent to the EWM Web editors at

emw@risc.uni-linz.ac.at

or in case of difficulty to the EWM secretary Riitta Ulmanen at

ulmanen@sophie.helsinki.fi

The next General EWM meeting will be at ICTP, Trieste, Italy, December 12–17, 1997. The general topic will be 'Women in Mathematics – North, South, East and West'. The mathematical topics will be announced later. For more information contact Riitta Ulmanen as above.

EUROPEAN NEWS: Country by Country

GERMANY

Oberwolfach Tagungsprogramm 1997

Übersicht über die bisher geplanten Tagungen in 1997 (Stand: 27. Februar 1996)

05.-11.01. Mathematische Optimierung; John E. Dennis, Houston; Bernhard Korte, Bonn; Klaus Ritter, München

19.-25.01. Finite Fields: Theory and Computation; Joachim von zur Gathen, Paderborn; Igor E. Shparlinski, North Ryde

26.01.-01.02. Mathematical Aspects Of Computational Fluid Dynamics; Claes Johnson, Göteborg; Keith W. Morton, Oxford; Rolf C. Rannacher, Heidelberg

02.-08.02. Punktprozesse: Modelle und Anwendungen; Klaus Krickeberg, Paris; Yoshihiko Ogata, Tokyo; Rolf-Dieter Reiss, Siegen

09.–15.02. Automorphic Forms and Geometry; Stephen S. Kudla, College Park; Joachim Schwermer, Eichstätt

16.-22.02. Adaptive Methoden für partielle Differentialgleichungen; Randolph E. Bank, La Jolla; Gabriel Wittum, Stuttgart; Harry Yserentant, Tübingen

23.02.-01.03. Medical Statistics: Mathematical Models for Diagnosis and Prognosis; Mitchell H. Gail, Rockville; Helmut Schäfer, Marburg

02.-08.03. Mathematische Physik; Klaus Fredenhagen, Hamburg; N. N.

09.-15.03. Mathematische Stochastik; Jürgen Gärtner, Berlin; Richard D. Gill, Utrecht; Enno Mammen, Heidelberg

16.-22.03. Gewöhnliche Differentialgleichungen; Jean Mawhin, Louvain-la-Neuve; Klaus Schmitt, Salt Lake City; Hans-0tto Walther, Giessen

23.-29.03. Reelle algebraische Geometrie; Ludwig Bröcker, Münster; Michel Coste, Rennes; Manfred Knebusch, Regensburg

31.03.-05.04. Schubert Varieties; Geometry, Algebra and Combinatorics; William Fulton, Chicago; Alain Lascoux, Paris; Piotr Pragazc, Bonn/Warszawa

06.-12.04. Arbeitsgemeinschaft mit aktuellem Thema (wird in Heft 1/1997 der DMV-Mitteilungen bekanntgegeben); N.N.

13.-19.04. Numerical Linear Algebra and Scientific Computing; Gene H. Golub, Stanford; Wilhelm Niethammer, Karlsruhe; Richard S. Varga, Kent

20.-26.04. Pseudodifferential Operators and Microlocal Analysis; Michael Beals, New Brunswick; Bernhard Gramsch, Mainz; Bert-Wolfgang Schultze, Potsdam; Harold Widom, Santa Cruz

27.04.-03.05. Gruppentheorie: Strukturtheorie der endlichen einfachen Gruppen und ihre Anwendungen; Martin W. Liebeck, London; Bernd Stellmacher, Kiel; Gernot Stroth, Halle

04.-10.05. Harmonische Analyse und Darstellungstheorie topologischer Gruppen; Roger E. Howe, New Haven; Eberhard Kaniuth, Paderborn; G. Schiffmann, Strasbourg

11.-17.05. Numerische Methoden der Approximationstheorie; Dietrich Braess, Bochum; Larry L. Schumaker, Nashville

18.-24.05. DMV-Seminar; N. N. ; N. N.

18.-24.05. DMV-Seminar; N. N. ; N. N.

25.-31.05. Diskrete Geometrie; Ludwig W. Danzer, Dortmund; Egon Schulte, Boston; Geoffrey C. Shephard, Norwich

01.-07.06. Computational Group Theory; Michael F. Newman, Canberra; Herbert Pahlings, Aachen

08.-14.06. Differentialgeometrie im Grossen; Werner Ballmann, Bonn; Jean-Pierre Bourguignon, Bures; Wolfgang Ziller, Philadelphia

15.-21.06. Galois Groups and Fundamental Groups; David Harbater, Philadelphia; Yasutaka Ihara, Kyoto; B. Heinrich Matzat, Heidelberg

22.-28.06. Mathematical Continuum Mechanics; John M. Ball, Edinburgh; Richard D. James, Minneapolis; Alexander Mielke, Hannover

29.06.-05.07. Nonlinear Evolution Equations; Sergiu Klainerman, Princeton; Michael Struwe, Zürich

06.-12.07. Partielle Differentialgleichungen; L. Craig Evans, Berkeley; Gerhard Huisken, Tübingen; Leon M. Simon, Stanford

13.-19.07. Dynamische Systeme; Helmut W. Hofer, Zürich; Jean-Christophe Yoccoz, Palaiseau; Eduard Zehnder, Zürich

20.-26.07. Algebraische Zahlentheorie; Christopher Deninger, Münster; Gerhard Frey, Essen; Peter Schneider, Münster; Anthony J. Scholl, Durham

03.-09.08. Effiziente Algorithmen; Zvi Galil, New York; Kurt Mehlhorn, Saarbrücken

10.-16.08. Stability for Classical and Non-Newtonian Fluids; Giovanni P. Galdi, Ferrara: Kumbakanam R. Rajagopal, Pittsburgh; Wolf von Wahl, Bayreuth
17.-23.08. Noncommutative Algebra and Representation Theory; Gerhard Michler, Essen; Lance W. Small, La Jolla
24.-30.08. Complex Geometry: Relations with Mathematical Physics; Arnaud Beauville, Paris; Fabrizio Catanese, Pisa; Eduard J.N. Looijenga, Utrecht: Christian Okonek, Zürich
31.08.-06.09. Topologie; Robion C. Kirby, Berkeley; Wolfgang Lück, Mainz; Elmer G. Rees, Edinburgh
07.-13.09. Niedrigdimensionale Topologie; Michel Boileau, Toulouse; Klaus Johannson, Knoxville; Heiner Zieschang, Bochum
14.-20.09. Stochastic Analysis in Finance and Insurance; Paul Embrechts, Zürich; Hans Föllmer, Berlin
21.-27.09. Modelltheorie; Elisabeth Bouscaren, Paris; Alexander Prestel, Konstanz; Martin Ziegler, Freiburg
28.09.-04.10. Random Graphs and Combinatorial Structures; Andrew D. Barbour, Zürich; Bela Bollobas, Cambridge; Ingo Wegener, Dortmund
05.-11.10. Arbeitsgemeinschaft mit aktuellen Thema (wird in Heft 3/1997 der DMV-Mitteilungen bekanntgegeben); N.N.
12.-18.10. DMV-Seminar; N.N.; N.N.
12.-18.10. DMV-Seminar; N.N.; N.N.
19.-25.10. Nonlinear Systems, Solitons and Geometry; Mark J. Ablowitz, Boulder; Benno Fuchssteiner, Paderborn; Martin D. Kruskal, Princeton; Vladimir B. Matveev, St. Petersburg
26.10.-01.11. Combinatorial Convexity and Algebraic Geometry; Günter Ewald, Bochum; Peter McMullen, London; Tadao Oda, Sendai; Richard Stanley, Cambridge
02.-08.11. Einhüllende Algebren und Darstellungstheorie; Walter Borho, Wuppertal; Michel Duflo, Paris; Anthony Joseph, Paris; Rudolf Rentschler, Paris
16.-22.11. DMV-Seminar; N.N; N.N.
23.-29.11. Technisch wissenschaftliches Hochleistungsrechnen; Franz Durst, Erlangen; Christoph Zenger, München; N.N.
07.-13.12. Konvexgeometrie; Paul R. Goodey, Norman; Peter M. Gruber, Wien
14.-20.12. Probability and Analysis in the Context of Mathematical Physics and Biology; Andreas Greven, Erlangen; Frank den Hollander, Nijmegen

Die Teilnehmer an den Tagungen in Oberwolfach werden vom Institut persönlich eingeladen. Ohne eine solche Einladung ist eine Teilnahme nicht möglich.
Interessenten, insbesondere auch jüngere Mathematiker, können sich an das Institut wenden. Da die Anzahl der Teilnehmer an einer Tagung beschränkt ist, können nicht immer alle Interessenten eingeladen werden.

ITALY

UNIVERSITA'DEGLI STUDI DI ROMA

Intensive period of study and research on
New Developments in Enumerative Geometry

An intensive period of study and research on recent developments in enumerative geometry will take place, in the months April-June 1997, at the Department of Mathematics of the University of Rome 'Tor vergata' (Second University of Rome). This activity, sponsored by the Istituto Nazionale di Alta Matematica 'F. Severi', will be organised by Prof Ciro Ciliberto and Prof Corrado de Concini.
There will be a few courses at post-doctorate level and several talks and seminars. So far the following mathematicians have agreed to participate:
Arbarello, Beauville, Ballico, Beherend, Dubrovin, Ellingsrud, Faber, Fantechi, Gintzburg, Givental, Goettsche, Gromov, Kanev, S. Katz, Lascoux, Laksov, Looijenga, Manaresi, Manin, Migliorini, R. Miranda, Piene, Pragacz, Ran, Tian, Tyurin, Vistoli.
For more information, please write to the following e-mail address:

geonum@cibs.sns.it

or to:
Ciro Ciliberto, Università di Roma Tor vergata, Dipartimento di Matematica, Via della Ricerca Scientifica, 00173 Roma
Corrado de Concini, Scuola Normale Superiore, Piazza dei Cavalieri 7, 56100 Pisa

PORTUGAL

Introducing the CIM

The CIM - Centro Internacional de Matemática (in English: International Centre for Mathematics) – has been established in Coimbra, Portugal, since December 1993. It is a non-profit scientific association whose members are the Portuguese Mathematical Society and 14 Portuguese universities or schools of higher learning with departments of Mathematics. It includes 'international' in the name because membership is open to foreign institutions and a Board of Scientific Advisors will be established to which internationally renowned mathematicians will be invited.

According to its charter, 'the aims of the CIM are to promote activities which encourage the development of Mathematical Sciences in general and which foster international co-operation'.

The CIM is supported by its 15 member institutions. It has also received some seed money from JNICT, the Portuguese National Council for Science and Technology. Financial support from other (Portuguese or foreign) sources is being sought.

The CIM is located on premises borrowed from the University of Coimbra Astronomical Observatory. Coimbra is home to the oldest university in Portugal, a city with a mild climate and circa 150,000 permanent residents. The Observatory, now, in a sense, a victim of city growth, enjoys a hilltop location on a 25 acre verdant site. Eventually, the CIM will build its own headquarters inside this park. The CIM can easily be reached by public transportation from Downtown Coimbra or the University of Coimbra Department of Mathematics. Ample parking facilities are also available.

To foster mathematical activities in Portugal, the CIM will organise, among other events, workshops, seminars, summer schools, visits by foreign colleagues; a project "Mathematicians in Residency" is in the offing: the goal is to make available to researchers who work in different cities temporary office space and lodging in the vacant houses and apartments formerly occupied by astronomers.

As regards international co-operation, the CIM is willing to work together with similar centres around the world. The CIM believes itself to be in a particularly favourable position to develop joint research projects with mathematicians from Brazil and to help Portuguese speaking countries in Africa, namely Angola, Mozambique, Cape Verde, Guinea-Bissau, and Sdo Tomd e Principe in their efforts to improve their teaching and research facilities in Mathematics.

After a period of 'invisibility', the CIM is now moving ahead; until a President is found, the CIM is run by an organising committee formed by Professors Natália Bebiano (President of the Portuguese Mathematical Society), J.M.S. Simões-Pereira (Univ. of Coimbra), Jorge Almeida and Alberto Pinto (both from Univ. of Porto), Maria do Rosário Grossinho (Univ. of Lisbon) and Estelita Vaz (Univ. of Minho).

For more information you may visit our site on the Internet at

http://www.cim.pt/

or write to:

CIM
Complexo do Observatório Astronómico
Almas de Freire
PT-3040 Coimbra (Portugal)

Problem Corner

Paul Jainta, Werkvolkstr. 10, D-91126 Schwabach, Germany

For reasons beyond our control, the problems in this article have not been edited in the usual way.

No Piece of Stinginess

A Mathematical Challenge from Scotland

Why wander around a distant world when so many good things lie right under your feet? In this case good things are synonymous with European mathematics contests, which have a distinctive flavour. However, the problem columns of respected mathematics magazines often teem with puzzles that have been posed in competitions far from Europe. Why is that - are they more quaint and therefore more attractive? Perhaps experienced problemists seek some esoteric mathematical thrill just as tourists search after some thrill of the unknown.

Surely this has a grain of truth. Perhaps the following observation is more obvious: it seems a reasonable supposition that lethargy or indifference concerning popularising competitions hinders mathematics contests in Europe in gaining publicity. Many years of painful experience have taught me that sometimes it is easier to obtain materials from the most remote parts of, say, Oceania than to get hold of papers from European competitions. It seems that the Old World is frequently poor at logistics even though competitions in the heart of Europe do have an aura of quite exceptional allure. For many years I have been working on problems that I have found in mathematical journals from around the world and so I quickly sense where a problem comes from just by its style. But I'm still dumbfounded as to how different countries produce such different types of problems.

The British have many fussy problems whose numbers don't work out nicely. French problems are formal without cute trappings. Russian problems resemble games: one doesn't need much background knowledge to resolve the riddles, but they still can be difficult. I'm still puzzling over what is typical of German problems. I have seen many of them but I can remember too many exceptions to any generalisations I might make. It is frequently hard, however, to see the characteristics of one's own culture.

Problems are manifestations of the mathematical culture from which they arise. The fact that there are different styles of problems, different traditions in their presentation and solution, reflects a varying appreciation of mathematics as practised in countries throughout the world. It is a historical commonplace to point out the heavily applied nature of British mathematics and to trace this tradition to the importance of Newton in the development of science in Britain; and with perfect consistency, should we be surprised that there are echoes in British contests' problems? One can likewise detect a certain conformity in the traits of formalism found in French mathematics, which can be explained by the influence of Bourbaki, Cauchy and Lagrange, and of other French mathematicians famous for their emphasis on rigorous proof. The Russian whim for problems with little background knowledge as the most popular form is more problematical to understand.

So, there's no denying that there are different styles of mathematics and that some of these styles reflect the culture in which they arose. One could term it the ***host culture*** of mathematical customs and we are free to put forward a series of "laws" concerning the development of mathematics as a process of articulation with this host culture. That's no surprise, for we differ as individuals and differ as national groups as well. This distinctness leads to disparate contributions to mathematics which enrich us all. Therefore it is important to celebrate our differences. Just as a robust biosphere includes a diversity of species, a sturdy mathematical life must encompass divergent national traditions. It is likewise important to exploit our differences by making them public, so that each tradition can derive benefit from and also enrich the others. Hence this Corner.

We could even go a stage further. There are countries in the world — both inside and outside Europe — where local traditions are not well developed or widely publicised. It is of value to discover and offer to their readership such hidden treasures. The rich mathematical traditions of India, say, are not well enough known, and we can all learn something from their way of mathematical thinking.

There are also places in the world where mathematical activity has not yet reached a critical mass. Not enough students in Africa or Latin America are competing in mathematics. Anything we can do to include these countries in our circles of activity will ultimately benefit us all. There are, however, mathematical activities in Europe which, even though effectively excluding many, do an excellent job in challenging pupils in Scottish secondary schools. The Scottish Mathematical Challenge is such a representative of this genre of competitions which doesn't, however, remain hidden from general view.

Mathematical Challenge was introduced by the Scottish Mathematical Council in 1976 as a means of enhancing the interest of young people in Mathematics, and at the same time giving them valuable experience in coping with unfamiliar problems, in understanding what needs to be done to resolve them and in reasoning their way through to

successful conclusions. There are no examinations and contestants are allowed plenty of time to try the problems. Of course there are deadlines for the submission of solutions, but it is the calendar rather than the clock that has to be watched.

Since it began, Mathematical Challenge has run without a break. There have been changes, however, in the form of the competition. In 1976–77, the first year, there were five sets of five problems each. This proved to be too much both for contestants and organisers and so in the second year there were four sets with four problems in each. The competition was open to all students in all Scottish secondary schools though directed mainly at the more senior pupils. It was stipulated that no problem should require technical mathematical knowledge beyond that needed for Ordinary Grade Mathematics.

In 1985, in response to a suggestion that the competition should be more accessible to junior pupils, a major change took place when the number of problems in each of the four sets was changed back to five, but with the difference that the first three problems in each set were chosen so that they could reasonably be attempted by students in first and second years, whose entries were judged on their solutions to these problems alone. This encouraged interest at junior level but left an awkward gap for students in the third and fourth years, because their experience was usually insufficient to enable them to do justice to the more advanced problems.

In 1991 a second major change took place. The problems were to be assigned to three divisions, Junior for years 1 and 2, Middle for years 3 and 4 and Senior for years 5 and 6, with some overlap where the problems were deemed to be suitable for more than one division. The number of problems was reduced to four per set and the number of sets diminished to three. The effect has been to spread the appeal of the competition more evenly although, as might be expected, second year students perform rather better than first year in the Junior Division and fourth year rather better than third year in the Middle Division. Nevertheless, some very good entries have come from first and third year students, and every effort is made to ensure that the problems set in any division are accessible to everyone in that division. Occasionally, a very enterprising contestant submits solutions for a higher section than appropriate for his or her year, but most stick to their own divisions.

Over three complete years since 1991, between 65% and 75% of the entries have been in the Junior Division, between 20% and 26% in the Middle Division and between 5% and 10% in the Senior Division. Records were broken in 1993–94 when 2841 entrants from 262 schools submitted 5215 attempts. New ground was broken at the beginning of 1994–95, when entries from primary schools for the Junior competition were invited and so far the response has been encouraging.

"Mathematics is about basic skills and it is about problem-solving but above all it is about *reasoning*; that is what you need, along with your basic skills, for you to solve problems that come your way". With these words the organisers try to get the participants into the right frame of mind for competing. The contest offers ample scope for those who like to solve problems and work out explanations, who have a determination to succeed and a dislike of having to admit defeat. The questions posed vary considerably in difficulty. Some are easy; others are quite demanding. Needless to say, difficulty is to some extent subjective: one person may find a problem to be very hard, whilst another regards the same problem as trivially easy; yet the same two competitors may find their roles reversed in their attempts on another problem. Often a problem seems at first sight to be impenetrable or even impossible, but when a flash of inspiration brings a solution it can change to being straightforward and obvious.

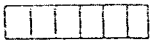
The organisers set great store on problems that can be solved in more than one way, so some solutions are easier than others. The suggested solutions are not necessarily the best possible. The aims are for solutions to be clear and complete and, if possible, simple. But the possibility cannot be ruled out that some of the problems contain traps for the unwary, who have to avoid falling into them.

Indeed, I could hardly stop myself setting six traps, but I suppose no one will become ensnared in these mathematical mantraps. With these problems, chosen from the years 1991 to 1994, I invite a wider audience to participate in the delights of problem-solving in a Scottish style.

Q 51. An automatic light switches on at 6 p.m., remains on for a whole number of minutes, then switches off and remains off for three times as long as it was on. Then it switches on again, repeats the cycle with the same periods as before and continues. It is seen to be off five seconds before 6.11 p.m., to be on five seconds after 9.03 p.m. and on again at 10.15 p.m.

Was it on or off at 11.18 p.m. ? Give your reasons.

Q 52. Two players A and B play the following game. A positive integer N is agreed beforehand. A and B in turn insert a digit 0,1,2,3,4,5,6,7,8 or 9 in a space in the following diagram:



A starts and only one digit is placed in each space. If the resulting 6-digit number is divisible by N, then B wins; otherwise A wins.

For which choices of $N \leq 15$ can B make certain of winning by adopting a suitable strategy ?[A digit may be used more than once and initial zeros are permitted.]

Q 53. A positive integer n is called „nice" if the equation $x^2 + 11y^2 = n$ has a solution in integers x and y.

(i) Prove that if n is nice, then so is 15n.

(Ii) Prove that if 3m is nice (where m is a positive integer), then so is 5m.

Q 54. ABC is an equilateral triangle inscribed in a circle. The distances from a point X on the circle to A, B and C respectively are a , b and c , where $a \geq b$ and $a \geq c$. Prove that $a = b + c$.

Q 55. Suppose that a,b,c,d,e and f are six different integers. Find the least possible value of

$$(a - b)^2 + (b - c)^2 + (c - d)^2 + (d - e)^2 + (e - f)^2 + (f - a)^2$$

Q 56. Three schools, Auchterturra Academy, Bridge of Greetin Grammar School and Craskie Castle High School had a recent athletics meeting, at which each school had just one competitor in each event. The competitors scored points depending on their places, with

first > second > third > 0.

At the end of the meeting Craskie won with 22 pts, whilst Auchterturra and Bridge of Greetin each scored 9 pts. If Auchterturra won the high jump, who won the 1 500 metres ?

SOLUTIONS

Before we turn to some solutions to past problems I would like to present another answer to question 33 submitted by *Maurice Brémond*, Avignon, France.

I refer to Newsletter No. 19 page 28.

We denote x the length of segments AX = BY = CZ and y the length of segments XB = YC = ZA. Then we have successively

$$\frac{1}{2}xy\sin\frac{\pi}{3} = \frac{1}{6}\cdot\frac{1}{2}(x+y)^2\sin\frac{\pi}{3} \Leftrightarrow 6xy = (x+y)^2 \Leftrightarrow \frac{6xy}{y^2} = \left(\frac{x+y}{y}\right)^2 \Leftrightarrow$$

$$6\cdot\frac{x}{y} = \left(\frac{x}{y}+1\right)^2 \Leftrightarrow \left(\frac{x}{y}\right)^2 - 4\cdot\frac{x}{y} + 1 = 0 \Leftrightarrow \frac{x}{y} = 2 \pm \sqrt{2^2-1} = 2 \pm \sqrt{3} \text{ with } (2+\sqrt{3})(2-\sqrt{3}) = 1.$$

Q 40. Consider the real functions f_n defined by $f_1(x) = x$ and $f_n(x) = \sqrt{f_{n-1}(x)} - \frac{1}{4}$,

($n \in \mathbb{N}$, $n \geq 2$).

(a) Prove that $f_n(x) \leq f_{n-1}(x)$ for all x for which both functions are defined.

(b) For each n, determine all points x in the domain of f_n such that $f_n(x) = x$.

Solution (J.N.Lillington, Winfrith Technology Centre, Dorchester)

Given $f_1(x) = x$, $f_n(x) = \sqrt{f_{n-1}(x)} - \frac{1}{4}$, $n \in \mathbb{N}$, $n \geq 2$ we get

a) $f_{n-1}(x) - f_n(x) = f_{n-1}(x) - \sqrt{f_{n-1}(x)} + \frac{1}{4} = \left(\sqrt{f_{n-1}(x)} - \frac{1}{2}\right)^2 \geq 0$ q.e.d. and

b) $f_1(x) - f_2(x) = \left(\sqrt{f_1(x)} - 0.5\right)^2$;

$\therefore \quad \therefore \quad \therefore$

$f_{n-1}(x) - f_n(x) = \left(\sqrt{f_{n-1}(x)} - 0.5\right)^2$.

Addition of all rows gives $f_1(x) - f_n(x) = \sum_{i=1}^{n-1}\left(\sqrt{f_i(x)} - 0.5\right)^2$. Now $f_n(x) = x \Rightarrow$

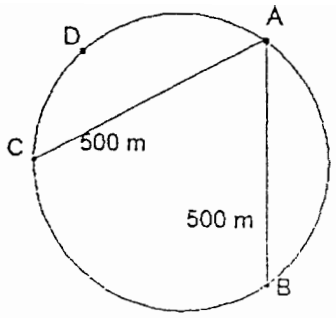
$0 = \sum_{i=1}^{n-1}\left(\sqrt{f_i(x)} - 0.5\right)^2 \Rightarrow \sqrt{f_i(x)} - 0.5 = 0\,,\ 1 \le i \le n-1 \Rightarrow x = \frac{1}{4}$.

Q 42. How many real numbers x , $1 \le x \le 3$, satisfy the following condition:
x^2 and x have the same decimal parts. Give a proof of your answer.

Solution (J.N.Lillington)

From $1 \le x \le 3$ we get $1 \le x^2 \le 9$. The numbers x and x^2 have the same decimal parts if and only if $x = x^2 - n$, $0 \le n \le 6$. Hence $x^2 - x - n = 0$; $x = \frac{1+\sqrt{1+4n}}{2}$ (as $x > 0$) . This formula yields seven solutions $x_1 = 1$; $x_2 = \frac{1+\sqrt{5}}{2}$, $x_3 = 2$; $x_4 = \frac{1+\sqrt{13}}{2}$, $x_5 = \frac{1+\sqrt{17}}{2}$, $x_6 = \frac{1+\sqrt{21}}{2}$ and $x_7 = 3$.

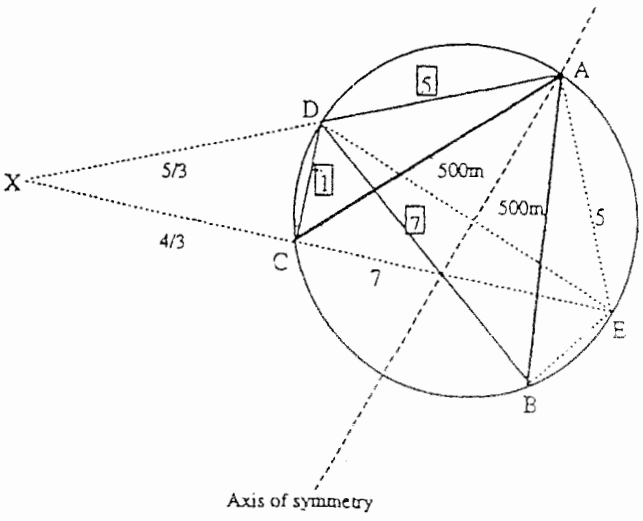
Q 43. Points A ,B,C and D lie on the edge of a circular lake with Radius R, as indicated in the figure. The distances between A and C, between A and B both equal 500 meter. The times a swimmer needs to get from D to C, from D to A and from A to B with the same constant velocity are in proportion to 1, 5 and 7. Determine R.



Solution (Brian R Stonebridge, Department of Computer Science, University of Bristol)
Take normalised units of measurement as shown.
Construct E by reflection in the axis of symmetry through A. Then CE = DB = 7, AE = AD = 5. Angles DCE, DAE are angles in the alternate segment and therefore sum to π.

If one of these angles were less than $\frac{\pi}{2}$ and DE took a value less than $50^{\frac{1}{2}}$ when calculated in that segment, the angle in the alternate segment would be greater than $\frac{\pi}{2}$ giving a value of DE greater than $50^{\frac{1}{2}}$: a contradiction. [Note: $50 = 1^2 + 7^2 = 5^2 + 5^2$]

From this we deduce that both angles must be $\frac{\pi}{2}$.



Construct a 3,4,5 triangle, DCX.
ECX is then a straight line and EAX is a 3,4,5 triangle. By similar triangles EDX, ACX,

$2R = DE = \frac{DX}{CX} \cdot 500 = \frac{5}{4} \cdot 500 = 625.$

Therefore, the radius of the lake, R, is 312,5 m.

Also solved by J.N.Lillington

Q 44. 1. Prove that for all n ∈ N and for all α ∈] 0, π [the equation is satisfied:

$$\sum_{k=0}^{n} \sin k\alpha = \frac{\sin\frac{(n+1)\alpha}{2}\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}.$$

2. Let G(n) be the average length of the diagonals of a regular n - gon (n ≥ 4), inscribed in a circle of radius 1.
Compute $\lim_{n\to+\infty} G(n)$.

Solution (J.N. Lillington)
We proof the first part of the question by induction on n.
Denote the proposition stated P(n).
The result is trivial for n = 1.
Assuming the result for the integer k, we will build on the truth of P(k).

$$P(k) = \sum_{k=0}^{n+1} \sin k\alpha = \frac{\sin\frac{(n+1)\alpha}{2}\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}} + \sin(n+1)\alpha = \text{(by induction hypothesis)}$$

$$= \frac{\left(\sin\frac{n\alpha}{2} + 2\cos\frac{(n+1)\alpha}{2}\sin\frac{\alpha}{2}\right)\sin\left(\frac{n+1}{2}\right)\alpha}{\sin\frac{\alpha}{2}} = \frac{\left(\sin\frac{n\alpha}{2} + \sin\frac{(n+2)\alpha}{2} - \sin\frac{n\alpha}{2}\right)\sin\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}} =$$

$$\frac{\sin\frac{(n+2)\alpha}{2}\sin\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$$

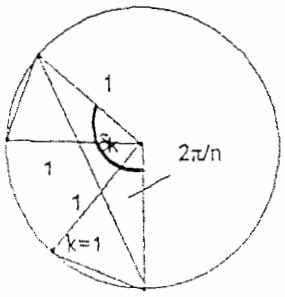
. This is the form for P(k+1), so by induction, the proof of 1. is complete.

Proof of part 2.
We refer to the figure: Denote the length of the diagonal l_k. Then we have

$$G(n) = \frac{\sum_{k=0}^{n} l_k}{n} = \frac{\sum_{k=0}^{n} 2\sin\frac{k\pi}{n}}{n} =$$

$$\frac{2\sin\frac{(n+1)\pi}{2n}\sin\frac{\pi}{2}}{n\cdot\sin\left(\frac{\pi}{2n}\right)} = \frac{2\sin\left(\frac{\pi}{2}+\frac{\pi}{2n}\right)\cdot 1}{n\cdot\sin\frac{\pi}{2n}} =$$

$$\frac{2\cos\frac{\pi}{2n}}{n\cdot\sin\frac{\pi}{2n}} = \frac{2}{n\cdot\tan\frac{\pi}{2n}}.$$



Hence $\lim_{n\to\infty} G(n) = \lim_{n\to\infty} \frac{2}{n\cdot\tan\frac{\pi}{2n}} = \cdot \lim_{n\to\infty} \frac{4}{\pi} \cdot \frac{\frac{\pi}{2n}}{\tan\frac{\pi}{2n}} = \frac{4}{\pi}.$

Q 45. Prove the identity: $\sin^3 18^0 + \sin^2 18^0 = \frac{1}{8}$ (Estonia).

First Solution (Dr Z Reut, London)

Since the angle $18° = \frac{90^0}{5}$, de Moivre`s formula gives $(\cos 18^0 + i\sin 18^0)^5 = \cos 90^0 + i\sin 90^0$ where the right-hand side is equal to i. When the binomial theorem is applied to the left-hand side, the grouping of real and imaginary terms gives two equations.
Since $\cos 18^0 \neq 0$, and the identity $\sin^2 18^0 + \cos^2 18^0 = 1$, the two equations are:
$1 - 12\sin^2 18^0 + 16\sin^4 18^0 = 0$ and $\sin 18^0 (5 - 20\sin^2 18^0 + 16\sin^4 18^0) = 1$.
The elimination of $\sin^4 18^0$ term gives the equation $\sin 18^0 (\frac{1}{2} - \sin^2 18^0) = \frac{1}{8}$; multiplication by $\sin 18^0$ and the elimination of $\sin^4 18^0$ term with the first equation gives $\sin^2 18^0 + \frac{1}{2}\sin 18^0 - \frac{1}{4} = 0$. The solution $\sin 18^0 = \frac{-1+\sqrt{5}}{4}$ reduces the left-hand side of given identity to $\frac{1}{8}$.

Second Solution
A somewhat trickier variant was submitted by Maurice Brémond, Avignon.
Soit $x = \sin 18^0 \in]0, 1[$ (1) . On a: $2+3=5 = \frac{90}{18} \Rightarrow \cos 2\cdot 18^0 = \sin 3\cdot 18^0$, d`ou, en vertu des identités $\cos 2a = 1 - 2\sin^2 a$ et $\sin 3a = 3\sin a - 4\sin^3 a$:
$1 - 2x^2 = 3x - 4x^3 \Leftrightarrow 2 - 2x^2 = 4 - 3 + 3x - 4x^3 \Leftrightarrow 2(x^2 - 1) = 3(1 - x) - 4(1 - x^3)$
$\overset{(1)}{\Leftrightarrow} -2(x+1) = 3 - 4(1+x+x^2) \Leftrightarrow 4x^2 + 2x - 1 = 0 \overset{(1)}{\Leftrightarrow} 4x^3 = x - 2x^2$;
par suite: $8x^3 + 8x^2 = (2x - 4x^2) + 8x^2 = 4x^2 + 2x = 1 \overset{(1)}{\Leftrightarrow} \sin^3 18^0 + \sin^2 18^0 = \frac{1}{8}$.

That completes the Corner for this issue. The Olympiad Season is in full swing. Send me your contest materials, your regional and national Olympiads, comments, suggestions, and your nice solutions to problems posed in the *Corner*.

Finally, propose problems for which readers will send in solutions. Proposals should, whenever possible, be accompanied by a solution, references and other insights which are likely to be of help to the editor. They can be anything from elementary to advanced, from easy to difficult. Original problems are particularly sought. So please submit any interesting problems you come across, especially those from (problem) books and contests that are not easily accessible. But other interesting problems may also be acceptable provided they are not too well known and references are given as to their provenance. I hereby invite my readers to share them with their colleagues and students.

I welcome your input, and especially problem sets and solutions for use!

Paul Jainta, Werkvolkstr. 10, D-91126 Schwabach

BRIEF REVIEWS

Edited by Ivan Netuka and Vladimír Souček. Books submitted for review should be sent to the following address: Ivan Netuka, MÚUK, Sokolovská 83, 186 00 Praha 8, Czech Republic.

I.R.Shafarevich (Ed.): Algebraic Geometry I. Algebraic Curves. Algebraic Manifolds and Schemes, Encyclopaedia of Mathematical Sciences, vol.23, Springer-Verlag, Berlin, 1994, 307 pp., 49 fig., DM 144, ISBN 3-540-51995-5, ISBN 0-387-51995-5

This volume of the Encyclopaedia consists of two papers. The first, written by V.V.Shokurov, is devoted to Riemann surfaces and algebraic curves. It is an excellent overview of the theory of relations between Riemann surfaces and their models – complex algebraic curves in complex projective spaces. The main topics discussed in the paper include the classification of Riemann surfaces, differentials, divisors and Riemann-Roch formula, Jacobians and abelian varieties. The second paper, written by V.I.Danilov, discusses algebraic varieties and schemes. This survey of the foundations of algebraic geometry introduces the basic objects and main properties. The first part is concerned with the description of the main properties of projective algebraic varieties. In the second part, geometry of an algebraic variety (i.e. the study of properties of geometric objects in it and their positions as well as the intersection theory of cycles) is treated. The last part contains the theory of schemes and their relations to varieties. I can recommend the book as a very good introduction to the basic algebraic geometry. (jbu)

A.Tikhomirov, A.Tyurin (Eds.): Algebraic Geometry and its Applications. Proceedings of the 8th Algebraic Geometry Conference, Yaroslavl' 1992, Aspects of Mathematics, vol. E25, Friedrich Vieweg & Sohn, Braunschweig, 1994, xi+251 pp., ISBN 3-528-06599-0

Conferences in Yaroslavl' on algebraic geometry have a tradition going back to 1979. The one held there in 1992 was the first with foreign participants present. The proceedings of the conference contain 18 contributions on various topics in algebraic geometry, e.g. a geometric quantization scheme for the bosonic string theory in twistor terms, computation of the almost canonical spin-polynomials of a surface S using geometric approximation, a description of exceptional vector bundles, computation of Segre classes of standard vector bundles on Hilbert scheme of points on a surface, birational geometry of manifolds and topology of complex algebraic varieties. (vs)

D.V.Anosov, A.A.Bolibruch: The Riemann-Hilbert Problem. A Publication from the Steklov Institute of Mathematics, Aspects of Mathematics, vol. E 22, Friedrich Vieweg & Sohn, Braunschweig, 1994, ix+190 pp., DM 64, ISBN 3-528-06496-X

The investigation of properties of systems of linear ODEs in a complex domain goes back to Riemann and Hilbert. The problem of whether there always exists a Fuchsian system on the Riemann sphere with prescribed singularities and a given monodromy was included in the famous Hilbert's list of problems (the Hilbert 21st problem). Important partial information was published by Plemelj at the beginning of the century showing (using the theory of singular integral equations) that the solution of the Hilbert problem can be found in a slightly larger class of the so-called regular systems of linear ODEs. Recently, a counterexample was found by Bolibruch showing that the problem (as stated above) has negative answer. The book under review describes both positive and negative results in this area. The Plemelj theorem is proved using the Röhrl approach and the Leiterer proof of the Birkhoff-Grothendieck theorem. Known negative results are described; in the case of systems of 3 by 3 matrices, a complete discussion of the problem is given. The book is nicely written, systematic and pleasant to read. It contains a lot of explanatory remarks and examples (including several historical comments) making it more understandable. It is essentially self-contained (the knowledge of complex function theory being taken for granted), well suited for graduate students or newcomers to the field. (vs)

R.J.Gardner: Geometric Tomography, Encyclopedia of Mathematics and its Applications, vol.58, Cambridge University Press, Cambridge, 1995, xv+424 pp., GBP 45, ISBN 0-521-45126-4

Geometric tomography offers an elegant framework to formulate easily understandable geometrical problems whose solutions are far from trivial. For example, the first section of the book deals with the question, whether a planar convex body is uniquely determined by all its X-rays (chord lengths) of given (finitely many) directions. Similar problems are treated later in higher dimensions and with (parallel) X-rays replaced by projection or section functions, X-rays through given points etc. Though some results are formulated for star-shaped or even general compact sets, the basic framework of the book remains within convex geometry. Each section devoted to a particular sort of problems contains the known results with proofs, a list of related open problems and discussion. Brief biographies of 'classics' of (convex) geometry are appended at the ends of individual sections (J. Steiner, J. Radon, A. Alexandrov, H. Minkowski, W. Süss, W. Blaschke, P. Funk, H. Busemann and H. Hadwiger). The book is self contained, readable and can be warmly recommended not only for specialists. (jra)

W.Schwartz, J.Spilker: Arithmetical Functions. An Introduction to Elementary and Analytic

Properties of Arithmetic Functions and to some of their Almost-Periodic Properties, London Mathematical Society Lecture Note Series 184, Cambridge University Press, Cambridge, 1994, xix+367 pp., GBP 25.95, ISBN 0-521-42725-8

This book covers some parts of the results concerning arithmetical functions (mainly the concepts of convolution, mean-value theorems, the study of spaces of arithmetical functions and the characterisation of some arithmetically interesting functions). The authors are trying to replace some number-theoretical techniques by techniques that are more convenient for most non-specialists. There are many interesting exercises, figures and photographs. Nine chapters cover practically all interesting and important topics in the direction mentioned above (mean-value theorems and multiplicative functions, Ramanujan expansions, uniformly almost periodic arithmetical functions, almost periodic and almost-even arithmetical functions). Appendix (15 pages) contains some general theorems from analysis and functional analysis. The whole book makes very interesting reading and can be warmly recommended to anybody interested in number theory. (bn)

I.R.Porteous: Clifford Algebras and the Classical Groups, Cambridge Studies in Advanced Mathematics 50, Cambridge University Press, Cambridge, 1995, x+295 pp., GBP 30, ISBN 0-521-55177-3

This book covers material which you would hardly find in such a compact form elsewhere. It is very pleasant to have all these things together and so nicely arranged. The starting point are the fields $\mathbf{K} = \mathbf{R}$, $\mathbf{C}$, $\mathbf{H}$, the corresponding double fields ${}^2\mathbf{K} = \mathbf{K} \oplus \mathbf{K}$ (which are not fields) and linear spaces over them. When the linear space is endowed with a binary relation (e.g. bilinear form, hermitian form) we can define the groups preserving this binary relation (e.g. orthogonal group, symplectic group, unitary group), namely the classical groups. (Using a double field we obtain in this framework even the general linear group!) Passing to the spinor groups we must of course take into account the Clifford algebras. This book is by no means a textbook on basic linear algebra. Quite the contrary! You will find here many interesting things and especially many interesting relations. To complete the picture a little, let us mention that you will find here also Grassmannians consisting of isotropic subspaces, the Cayley algebra and the principle of triality for $Spin(8)$. All classifications are complete, the groups are considered not only from the point of view of algebra, but also as topological and Lie groups. You will hardly find a better source for relations among low dimensional classical groups. Even the historical aspect is reflected. The book is designed for the last year of an undergraduate course or the first year of a postgraduate course but I believe that also many mathematicians will enjoy reading in this book. Let us add that each chapter is accompanied with interesting exercises. (jiva)

H.Brezis, J.Lions (Eds.): Nonlinear Partial Differential Equations and Their Applications. College de France. Seminar Vol. XII, Pitman Research Notes in Mathematics Series, vol.302, Longman Scientific & Technical, Harlow, 1994, 235 pp., GBP 28, ISBN 0-582-23801-3

The book contains texts of selected lectures delivered by leading world experts in nonlinear PDEs at the well-known weekly seminar held at Collège de France. The presented selection includes 16 contributions written in English (11 of them) and in French (5 of them). These contributions deal with various aspects of the theory of nonlinear partial differential equations and we shall briefly touch on each of them. In his contribution, G.Allaire studies some new aspects of periodic homogenisation. A method of two-scale convergence is discussed in connection with classical energy methods. C.Bardos discusses the theory of exact controllability and stabilisation for the wave equations. G.Bouchitte and P. Suquet study the role of recession functions in variational problems. A.Capietto, J.Mawhin and F.Zanolin study boundary value problems for forced superlinear second order ODE via a Banach space approach. J.-M.Delort studies the Euler equations in two dimensional space in connection with Majda's conjecture of global existence under some restrictions on vorticity of the data. P.Gerard presents microlocal analysis of compactness using the recent techniques as (microlocal) defect measures, pseudodifferential calculus and application of these to PDEs, especially to problems of homogenisation. D.Grisvard studies maximum norms of complex solutions to differential equations with leading part expressed by the Laplace operator, on polygonal domains, coupled with Dirichlet data. K.Hamdache presents the problem of nonlocal homogenisation for hyperbolic equations. A.Haraux presents an alternative proof to Ball's and Slemrod's theorem on localisation of non harmonic asymptotically lacunary Fourier series. This result is used to study vibrating systems. S.M.Kozlov deals with variational problems in homogenisation theory, discussing the methods of obtaining explicit solutions. M.L.De Cristoforis and S.S.Antman study the equations governing perfect flow past a nonlinearly elastic panel. P.A.Raviart studies paraxial approximations of the Vlasov-Maxwell equations. E.Sanchez-Palencia studies singularities and junctions in elasticity, junctions being understood as couplings between elastic bodies of different dimensions. D.Serre presents a contribution to the theory of nonlinear oscillations for hyperbolic systems in one space dimension. S.Solimini investigates functionals taking into account energy terms expressed by volume integrals and by surface integrals on a singular set. D.W.Stroock deals with estimates for the heat kernel of second order elliptic operators. All contributions are written in a research paper style, presenting interesting and recent topics. This selection

of lectures will therefore be of particular interest to experts in the field as well as to postgraduate students in PDEs. (mr)

J.P.Brasselet: Singularities, London Mathematical Society Lecture Note Series 201, Cambridge University Press, Cambridge, 1994, xvi+419 pp., GBP 25, ISBN 0-521-46631-8

A huge congress on singularities with 230 participants was organised in 1991 in Lille. There were 12 plenary lectures and 36 specialised talks. Sixteen of them are presented in a written form in the book. Four of them correspond to plenary lectures: J.W.Bruce discusses applications of the theory of singularities in 3-dimensional differential geometry; B.Malgrange describes results on meromorphic connections, especially those related to the Riemann problem; R.Thom presents a survey of various types of stratifications of maps and spaces; A.A. du Plessis and C.T.C.Wall have written a survey article on stability questions. Twelve other papers discuss many different aspects of geometry and topology. The book is certainly valuable for anybody specialised in the subject. (vs)

E.Zeidler: Applied Functional Analysis. Applications to Mathematical Physics, Applied Mathematical Sciences 108, Springer-Verlag, New York, 1995, xxix+479 pp., 56 fig., DM 118, ISBN 0-387-94442-7

This is an extremely valuable introduction to functional analysis. The presentation is self-contained, application-oriented, emphasising the unity and usefulness of mathematics. It combines linear functional analysis, nonlinear functional analysis, numerical functional analysis and important applications related to the main stream of mathematics and physics. Fixed-point theorems are applied to integral and differential equations, Hilbert space techniques are shown to be well adapted to treatment of boundary value problems (including numerical methods), the theory of compact symmetric operators is used in connection with boundary-eigenvalue problems. The last chapter on self-adjoint operators, the Friedrichs extension and PDEs of mathematical physics provides applications to boundary-eigenvalue problems for the Laplace equation, heat equation, wave equation, Schrödinger equation, to quantum mechanics and to quantum statistics. It also includes an overview of scattering theory, Feynman's path integral, Dirac calculus, solitons and inverse scattering theory. There is no doubt that the book will be very popular with teachers as well as students of mathematics and physics. (in)

V.P.Maslov: The Complex WKB Method for Nonlinear Equations I. Linear Theory, Progress in Physics, vol.16, Birkhäuser Verlag, Basel, 1994, vii+300 pp., sFr 168.80, ISBN 3-764-35088-1, ISBN 0-817-65088-1

The book is a translation of (the first part of) the Russian original published in 1977. The subject of the book is the study of certain linear and nonlinear equations of mathematical physics which depend on a small parameter and construction of their asymptotic solutions. Asymptotic solutions are supposed to have a form $\varphi(x)\exp\{iS(x)/h\}$, $x \in \mathbf{R}^3$, where $S(x)$ is a complex valued function and $\operatorname{Im} S(x) \geq 0$. The case $\operatorname{Im} S(x) = 0$ is known in quantum mechanics under the name WKB-method. The construction of such solutions can be reduced to a search for solutions of the corresponding canonical system of first order equations – the Hamilton-Jacobi equation and the transport (Liouville) equation. The main topic of the book is the study of properties of approximate solutions of canonical systems in the case (important for laser technology) of infinitely narrow beams concentrated around k-dimensional surfaces in $\mathbf{R}^n$. The main geometric objects in the phase space used for the constructions are incomplete Lagrangian manifolds with complex germ. This complex germ method is also applied in the last chapter to spectral problems of quantum mechanics. There are three new appendices. The first one describes a new topological approach to semiclassical quantization by the complex germ method; in the second, the case of purely imaginary phase is considered; the third one contains certain heuristic considerations in the case of asymptotics of oscillatory decreasing type. The topic discussed is quite interesting and important; it is certainly useful to have the book available in English. (vs)

R.W.Bruggeman: Families of Automorphic Forms, Monographs in Mathematics, vol.88, Birkhäuser Verlag, Basel, 1994, x+317 pp., sFr 138, ISBN 3-764-35046-6, ISBN 0-817-65046-6

The theory of holomorphic automorphic forms on the upper half plane is an old and well known subject. The study of basic properties of real analytic automorphic forms is more recent. In this book, the author studies the behaviour of families of automorphic forms on the upper half plane with singularities and with exponential growth at the cusps for general cofinite discrete subgroups. The main results describe meromorphic continuation of Eisenstein and Poincaré series in all parameters (eigenvalues as well as the multiplier system). The first chapter offers a nice summary of properties of automorphic forms which can be useful also for nonspecialists. In the main part of the book, a general theory is developed for cofinite discrete groups and for the group of multiplier systems of any dimension. The last three chapters illustrate the general theory by special examples of the modular group, the theta group and the commutator subgroup of the modular group. The book is certainly very useful for mathematicians working in the field. (vs)

W.Kaup, K.McCrimmon, H.Petersson (Eds.): Jordan Algebras. Proceedings of the Conference held in Oberwolfach, Germany, August 9-15, 1992, Walter de Gruyter, Berlin, 1994, viii+339 pp., DM 198, ISBN 3-110-14251-1

These proceedings consist of fourteen contributions presented by authors expert in the theory of Jordan algebras and related topics. There are two long survey papers on nonassociative normed algebras (Jordan structures in analysis by A.Rodrigues Palacios) and on structure theorems for JB^*-triples (by B.Russo); several shorter review papers on Jordan algebras in differential geometry (Algebraic systems in differential geometry by J.Dorfmeister), in incidence geometry (Geometry and algebraic structures by J.R.Faulkner), in harmonic analysis and quantization (Jordan algebras, complex analysis and quantization by H.Upmeier) and on Albert algebras (by H.P.Peterson and M.L.Racine). The book also contains several shorter research contributions concerning various topics connected with Jordan algebras: Structurable algebras and the construction of simple Lie algebras (by B.M.Allison), Fonctions speciales sur une algebre de Jordan (by J.Faraut), Bounded symmetric domains and the JB^* - triple structures in physics (by Y.Friedman), Recent results on finiteness conditions in Jordan pairs (by O.Loos), Max Koecher's work on Jordan algebras (by H.P.Peterson), The classification of the simple Lie algebras over algebraically closed field with positive characteristic (by H.Strade), Algebraic structures and differential equations (by S.Walcher), On linear groups and Lie algebras over arbitrary rings (by E.Zelmanov). A valuable book offering a good overview of the subject. (jbu)

V.V.Jikov, S.M.Kozlov, O.A.Oleinik: Homogenization of Differential Operators and Integral Funcionals, Springer-Verlag, Berlin, 1994, xi+568 pp., 13 fig., DM 178, ISBN 3-764-54809-2, ISBN 0-387-54809-2

It was mainly during the last two decades that the theory of G- and Γ- convergences as well as the theory of homogenisation or averaging of partial differential equations emerged as a distinct mathematical discipline. It has become increasingly important with a number of applications in many branches of physics and modern technology. This well organised book covers a wide scale of problems connected with the theory of homogenisation. It starts with homogenisation of second order elliptic operators with periodic coefficients and with analogous procedures for parabolic operators and Maxwell equations. Abstract notions of G-convergence of differential operators and Γ- convergence of variational functionals associated with nonlinear second order elliptic operators are thoroughly studied. Special care is devoted to operators with random coefficients, homogenisation in perforated random domains and homogenisation and percolation. Particular attention is paid to homogenisation in elasticity theory. A very interesting feature is the exposition of authors' results. The choice of material and clear and precise way of exposition make the book accessible to a wide audience. (jsta)

M.Fuchs: Topics in the Calculus of Variations, Advanced Lectures in Mathematics, Friedrich Vieweg & Sohn, Braunschweig, 1994, vii+145 pp., $ 28, ISBN 3-528-06623-7

These lecture notes, split into two parts, reflect in a consistent form the work of the author and his co-workers in the last decade. Their main aim is to present several techniques (as isoperimetric inequality, the method of linearisation, monotonicity-like and blow-up arguments, etc.) that are useful in the investigation of the regularity question of minimisers of non-quadratic potentials, and the existence and smoothness of manifolds of prescribed mean curvature. The first part, devoted to degenerate variational integrals with nonlinear side condition (as the restriction on the range of mappings or the requirement on Jacobian to be strictly positive), provides partial regularity methods for p-minimisers. The studied problems are motivated by model examples in nonlinear elasticity. In the second part, the classical Plateau problem is generalised to the case of arbitrary dimension and codimension within the framework of geometric measure theory. (jmal)

H.Leptin, J.Ludwig: Unitary Representation Theory of Exponential Lie Groups, de Gruyter Exposition in Mathematics, vol.18, Walter de Gruyter, Berlin, 1994, x+200 pp., DM 198, ISBN 3-110-13938-3

The description of the set $\hat{G}$ of all unitary irreducible representations of a given group G is one of the basic problems in representation theory. The Kirillov orbital method describes $\hat{G}$ using the set $\underline{g}^*/G$ of all coadjoint orbits. The Kirillov conjecture says that the corresponding bijective map is not only continuous but even a homeomorphism (with respect to the natural topologies on both sets). The conjecture was proved for nilpotent groups by I.D.Brown in 1973. The main purpose of the book is to prove it for a larger subclass of connected, simply connected solvable Lie groups (the so called exponential groups, ones for which the exponential map is a diffeomorphism). The first chapter reviews necessary fundamental notions. The main method used in the proof is to consider the whole problem in a broader framework of variable Lie groups and algebras (i.e. families of Lie groups with the same underlying space G and varying product structure); these notions are described in the second chapter. The last chapter presents a proof that the Kirillov map is a homeomorphism. The book is basically self-contained, well organised and can be recommended to people interested in the subject. (vs)

H.E.Rose: A Course in Number Theory. Second Edition, Clarendon Press, Oxford, 1994, xv+398 pp., ISBN 0-198-53479-5

The first edition of this nice book appeared in 1988 (see MR 89f.11002). This edition contains some new proofs and corrections and, more importantly, two new additions: the author extends Chapters 14 and 15, and adds a new Chapter 16 dealing with diophantine equations and the role of the theory of elliptic curves

(a connection with Wiles' proof of the FLT). Moreover, he briefly shows the contemporary role of computational number theory (coding theory, RSA algorithm, Lenstra's factorisation method). The remaining chapters (1-13) cover basic elementary number theory, properties of algebraic numbers, continued fractions theory and Diophantine approximations, transcendence (Hermite, Lindemann and Gelfond-Schneider), the theory of quadratic forms, partitions, an elementary proof of the prime number theorem, Dirichlet theorem. About 350 interesting problems (with hints provided) as well as an index and some tables complete the book. It is of interest both for students and teachers working in this field. (bn)

H.Skoda, J.M.Trépreau (Eds.): Contributions to Complex Analysis and Analytic Geometry. Dedicated to Pierre Dolbeault, Aspects of Math., vol.E26, Friedrich Vieweg & Sohn, Braunschweig, 1994, xi+250 pp., ISBN 3-528-06633-4

The colloquium on Complex analysis and analytic geometry organised at the occasion of retirement of Pierre Dolbeault was held in Paris in 1992. The book contains written versions of contributions presenting recent research results in various branches of complex analysis. Two longer papers discuss boundary behaviour of holomorphic functions of several complex variables and removable singularities inside the boundary (E.M.Chirka, E.L.Stout) and subvarieties of a complex manifold equipped with an holomorphic action of a Lie group (A.Huckleberry). The other topics included are a construction of an analogue of the de Rham complex on a reduced analytic space (V.Ancona, B.Gaveau); estimates for the $\overline{\partial}$-operator (B.Berndtsson); regularisations of closed positive currents (J.-P.Demailly); bounded pseudoconvex domains (K.Diederich, G.Herbort); Riemann surfaces with a prescribed boundary in $\mathbf{CP}^n$ (P.Dolbeault, G.Henkin); multidimensional residues (M.Passare, A.Tsikh, O.Zhdanov) and extension theorems of Hartogs' type for meromorphic maps (B.Shiffman). A very interesting book for mathematicians working in complex analysis. (vs)

A.S.Schwarz: Topology for Physicists, Grundlehren der mathematischen Wissenschaften. A Series of Comprehensive Studies in Math. vol.308, Springer-Verlag, Berlin, 1994, xi+296 pp., 54 fig., DM 178, ISBN 3-540-54754-1, ISBN 0-387-54754-1

The Russian edition of the author's book on quantum field theory and topology appeared in 1989. The first two thirds of it (applications in QFT) was previously published by Springer under the title "Quantum field theory and topology" in 1993. The present book contains an expanded version of the last part of the Russian original. It covers the main topics in topology and geometry needed for physicists (homotopy, homology and cohomology, fundamental group and covering spaces, manifolds, differential forms, fibred spaces, the degree of maps, Lie groups and algebras and gauge fields). The main parts of the book are written in a very intuitive and understandable way and explain basic notions and the most important properties. To add detail and mathematical rigour, additional paragraphs are included in most of the chapters (they are carefully marked and can be skipped at first). Additional material is contained in 77 problems of various level of difficulty. The book is very useful as an introduction to topology and related geometry, not only for physicists but also for readers from other branches of mathematics interested in these fields. (vs)

P.Malliavin: Integration and Probability, Graduate Texts in Mathematics, vol.157, Springer-Verlag, New York, 1995, xxi+322 pp., DM 74, ISBN 0-387-94409-5

This is a well written book including in balanced proportions abstract theories and concrete problems. Probability appears here as an integral part of real analysis and deep links among various theories are emphasised in such a way that one understands analysis as a whole and not as a union of individual independent parts. The exposition is condensed, but clear. Abstract measure theory and integration is contained in Chapter I and Borel and Radon measures in Chapter II (including Riesz representation theorem, Lebesgue measure, signed measures, vague and weak convergence of measures). Chapter III deals with Fourier analysis (locally compact abelian groups, spectral synthesis, Sobolev spaces, distributions, pseudodifferential operators). Chapter IV presents a Hilbertian treatment of basic laws of probability including Doob's martingale convergence theorem. Chapter V is devoted to Gaussian Sobolev spaces and Malliavin's stochastic calculus of variations. On the last 50 pages (from a total of 318), 116 exercises (without solutions) to all of five chapters are presented. The book is an invaluable contribution to the existing literature and is recommended to teachers as well as to students of analysis and probability and to anybody who wants to be acquainted with modern mathematics. (in)

J.G.Ratcliffe: Foundations of Hyperbolic Manifolds, Graduate Texts in Mathematics, vol.149, Springer-Verlag, New York, 1994, xi+747 pp., 164 fig., DM 79, ISBN 3-540-94348-X, ISBN 0-387-94348-X

The book under the review covers several facets of hyperbolic geometry. It starts with the classical story of Euclid's fifth postulate and with the spherical and hyperbolic geometry showing its independence on other axioms. The next topic is a study of classical discrete groups of isometries of S^n, R^n, H^n and tesselations of these spaces. The next part of the book contains the theory of hyperbolic manifolds. It culminates with the Mostow rigidity theorem and a description of geometrically finite hyperbolic manifolds. The last chapter brings together both previous parts in the theory of hyperbolic orbifolds. There is a large number of exercises (at the end of individual sections). The book contains more than 400 items in the bibliography and each chapter ends with historical comments. The book is heartily recommended as a textbook for both short or

long courses on hyperbolic geometry. (vs)

H.O.Cordes: The Technique of Pseudodifferential Operators, London Mathematical Society Lecture Note Series 202, Cambridge University Press, Cambridge, 1995, xii+382 pp., GBP 22.95, ISBN 0-521-37864-8

The approach to the theory of pseudodifferential operators, as presented in this book, can be viewed from at least two directions. Firstly, the calculus of pseudodifferential operators provides a useful method to solve (initial-)boundary-value problems for elliptic, parabolic and hyperbolic partial differential equations facilitating the extension of the Fourier-Laplace method to the equations with nonconstant coefficients, as well as applications to more general domains (as manifolds with boundary or domains with noncompact boundary). Secondly, the study of pseudodifferential operators leads naturally to the investigation of certain type of Banach algebras of linear operators. In particular, the connections to the theory of C^*-algebras, invariance properties of algebras of pseudodifferential operators under conjugation with evolution operators and the relation of hyperbolic theory to the propagation of maximal ideals are studied in detail. The last chapter deals with the applications of this theory to the Dirac equation. The book can be appreciated by the researchers in PDEs and mathematical physics.

E.C.Lance: Hilbert C* - modules. A toolkit for operator algebraists, London Mathematical Society Lecture Note Series 210, Cambridge University Press, Cambridge, 1995, ix+130 pp., ISBN 0-521-47910-X

Hilbert C^*-modules are, roughly speaking, objects similar to Hilbert spaces; the only difference being that they are equipped with inner products with values not only in the scalars but in C^*-algebras. This point of view was adapted at the beginning of the theory by Kaplanski in 1953. However, the result is very rich and interesting algebraic (and also topological) structure, where the methods of the theory of Hilbert spaces, Banach spaces, bounded and unbounded operators and operator algebras come in use. Natural examples are: spaces of sections of vector bundles with fibres formed by Hilbert spaces, C^*-algebras itself, tensor products of Hilbert spaces and C^*-algebras and the examples aiming to the C^*-algebraic theory of quantum groups. The latter are discussed in the last three chapters. The author collects together for the first time a lot of material in the field. The book is pleasant to read and is suitable for the first information as well as for specialists. (kj)

L.C.G.Rogers, D.Williams: Diffusions, Markov Processes, and Martingales. Volume 1: Foundations. Second Edition, J.Wiley & Sons, Inc., Chichester, 1994, xx+386 pp., GBP 49.95, ISBN 0-471-95061-0

It is hardly possible to characterise the intent of the book better than is done in the author's preface to the first edition (written by D.Williams only): "My aim ... is to sharpen your intuition to a point where the advanced abstract literature becomes accessible, enjoyable and 'relevant'. ... this is a missionary tract not a theological treatise." The first edition of the book appeared in 1979, the new version is a result of a rather substantial revision. Retaining the original vivid style, the authors provide many topics with a more systematic treatment including full proofs, and added new material. The book comprises three chapters. In the first one, properties of the Brownian motion (and, to a lesser extent, of general Gaussian and Lévy processes) are discussed from the very beginning to very deep results in an informal manner. Chapter 2 is devoted to the basic probability theory (with an emphasis on martingales) as needed in the rest of the book, whilst the third chapter can be viewed as a self-contained introductory text on the modern theory of Markov processes. The second volume of the book ("Itô Calculus") was issued in 1987, also by Wiley. (jis)

S.E.Schmidt: Grundlegungen zu einer allgemeinen affinen Geometrie, Birkhäuser, Basel, 1995, xiii+118 pp., DM 44, ISBN 3-764-35171-3

The content of the book is divided into two parts. In the first, general affine geometry is built up, based on a generalised notion of a linear space – linear systems. Assuming that the so called triangle axiom holds, a relation of parallelism is introduced, satisfying the Euclidean postulate and the axiom of monotonicity. The morphisms (affine collineations) are defined in a natural way. After introduction of affine lattices, many different categories of canonical equivalences are shown; for example the category of affine lattices and category of affine linear systems. In the second part, the relation of independence of points is added and the affine spaces are introduced via modules. The so called great and small Desargues' theorems are generalised into the n-argues theorems. As expressed in the introduction, the author's intention is to demonstrate the relations among various different approaches to the construction of an abstract axiomatic affine geometry which have appeared independently of each other during the last fifty years. The book under the review is an example of a relationship between the geometry of affine spaces and algebra. (lbo)

S.David (Ed.): Number Theory. Séminaire de Théorie des Nombres de Paris 1992-3, London Mathematical Society Lecture Note Series 215, Cambridge University Press, Cambridge, 1995, 291 pp., GBP 24.95, ISBN 0-521-55911-1

The book contains sixteen papers covering a part of thirty six lectures held in the above mentioned outstanding "Séminaire..." between October 5th, 1992 and June 28th, 1993: among them papers by N.Boston, P.Cohen; E.Faury and M.Ram Murty (Supersingular primes common to two elliptic curves); T.N.Shorey; R.Tijdeman (Decomposition of the integers as direct sum of two subsets), K.Alladi (the same title). (bn)

P.Rowlinson (Ed.): Surveys in Combinatorics, 1995, London Mathematical Society Lecture Note Series 218, Cambridge University Press, Cambridge, 1995, 231 pp., ISBN 0-521-49797-3

This volume contains 9 principal lectures presented at 15th British Combinatorial Conference which took place in June 1995 at University of Stirling. The topics covered are related to most areas of contemporary combinatorics. One can divide the 1995 contributions into two main groups: 4 papers (C.D. Godsil: Euclidean geometry of distance regular graphs; T.S.Griggs, A.Rosa: Large sets of Steiner Triples; R.Hill: Searching with lies; E.Spence: Construction and classification of combinatorial designs) are related to various aspects of "regular configurations" thus marking the 100th anniversary of death of Rev. T.P.Kirkman - by now the recognised father of the area. Also the paper F. Jaeger: Spin models for link invariants is related. 4 remaining papers (M.Jerrum: Computational Polya theory; L.Lov sz, P.Winkler: Mixing of random walks and other diffusions on a graph; A.Lubotzky: Cayley graphs: eigenvalues, expanders and random walks; J.Spencer: Modern probabilistic methods in combinatorics) cover various aspects of the remarkable development of probabilistic methods in combinatorics in contexts including group theory and theoretical computer science. In the good tradition of British Combinatorial Conferences, these are carefully prepared survey articles by some of the leading experts in their fields. A remarkable volume. (jne)

M.Reid: Undergraduate Commutative Algebra, London Mathematical Society Student Texts 29, Cambridge University Press, Cambridge, 1995, xiii+153 pp., GBP 11, ISBN 0-521-45255-4, ISBN 0-521-45889-7

Reid's book covers standard material on commutative algebra (noetherian rings, Noether normalisation, Nullstellensatz, localisation, primary decomposition and DVRs) but from a particular perspective. The author takes care to explain the geometric and number theoretic meaning of the algebraic methods and results presented. This makes the book perhaps more demanding, but surely much more interesting than the standard ones. The final chapter contains a thought-provoking "anti-Bourbaki" essay on the current role and presentation of abstract algebra. (jtrl)

S.P.Novikov (Ed.): Topology I. General Survey, Encyclopaedia of Mathematical Sciences vol.12, Springer-Verlag, Berlin, 1996, v+319 pp., 78 fig., DM 148, ISBN 3-540-17007-3, ISBN 0-387-17007-3

A.V.Archangel'skii (Ed.): General Topology II. Compactness, Homologies of General Spaces, Encyclopaedia of Mathematical Sciences vol.50, Springer-Verlag, Berlin, 1996, vii+256 pp., DM 148, ISBN 3-540-54695-2, ISBN 0-387-54695-2

A.V.Archangel'skii (Ed.): General Topology III. Paracompactness, Function Spaces, Desriptive Theory, Encyclopaedia of Mathematical Sciences, vol.51, Springer-Verlag, Berlin, 1995, 229 pp., DM 148, ISBN 3-540-54698-7, ISBN 0-387-54698-7

The series of three volumes consists of translations from Russian originals published in 1986 (vol. I) and 1989 (vol. II and III). It covers important parts of general and algebraic topology and theory of manifolds and knots. All the three volumes are very convenient as reference books both for specialists and nonspecialists in topology. The exposition is not encyclopaedic, so the books are useful also for those who want to learn main topics from topology (basic ones as well as advanced and deep theories). The first volume (319 pages including literature and index) is updated by using footnotes in the text and by adding an appendix on the recent development in the topology of 3-manifolds and knots. The next volumes are not updated. The volume I was written by S.P.Novikov and contains a discussion of fibrations, homotopies, complexes, homology and cohomology, coverings and sheaves, obstructions, fibre bundles, K-theory, bordism and cobordism theory, smooth manifolds and knots. The first part (117 pages) of volume II was written by A.V.Archangel'skii and deals with compactness and its generalisations (together with classical notions, it contains also function spaces and Stone-Weierstrass, Krein-Milman, Alaoglu theorems, Milyutin and Dugundji compact spaces, Stone and Pontryagin dualities, almost periodic functions, fragmentable compacta). The second part (127 pages) is about homology and cohomology theories of general spaces and was written by E.G.Sklyarenko; it contains cohomology theories of Čech and Alexander-Spanier, sheaf theory, axiomatic approach to homology and cohomology, and special results on tautness, local theories, homological dimension and dualities. The last volume consists of three parts: The first one (paracompactness and metrization, written by A.V.Archangel'skii) describes the method of covers in the classification of spaces as well as Čech-complete spaces and p-spaces (70 pages). The second one (spaces of mappings and rings of continuous functions, written by A.V.Archangel'skii) deals with Eberlein compacta and linear homeomorphisms of function spaces (86 pages). The third one (descriptive set theory and topology, written by M.M.Choban) has 63 pages. (mh)

I.Adamson: A General Topology Workbook, Birkhäuser, Boston, 1995, viii+152 pp., DM 48, ISBN 0-817-63844-X, ISBN 3-764-33844-X

In the words of the author, "This book has grown from my attempts to provide a self-learning introduction to general topology ...". The first part of the book consists of basic definitions followed by exercises (217 of them, many of them followed by hints). All solutions and proofs appear in the second part of the book. The contents of the first part are Topological spaces, Mappings of topological spaces, Induced and coinduced topologies, Convergence, Separation axioms, Compactness, Connectedness. Although the book is elementary, it uses tools like ultrafilters, initial and final

structures, and the results go up to, for instance, Čech-Stone compactifications. The book is convenient not only to students but also to any mathematician who wants to learn by himself basics of general topology. The best use of the Workbook is in a group of students led by a specialist. (mh)

F.Giannasi, R.Low: Math for Computing and Information Technology, Essential Maths for Students, Longman Scientific & Technical, Harlow, 1995, viii+259 pp., GBP 13.99, ISBN 0-582-23654-1

This introductory text was written for students in computer science and information technology. The text provides a first acquaintance with essential mathematical ideas. The basic mathematical tools are explained in the first four chapters. They are devoted to propositional calculus, theory of sets, predicate calculus and mathematical proofs. The next chapters deal with finite state automata, probability, matrices, co-ordinate geometry and calculus. The text is very elementary. The book is well-organised. It contains a lot of illustrating examples and exercises with solutions. (mzel)

W.V.Petryshyn: Generalized Topological Degree and Semilinear Equations, Cambridge Tracts in Mathematics, vol.117, Cambridge University Press, 1995, x+240 pp., GBP 40, ISBN 0-521-44474-8

This monograph develops the generalised topological degree theory for densely defined approximation-proper operators and uses it to study the solvability and the structure of the solution set of a class of abstract equations in Banach spaces. A-proper mappings arise when solving equations in Banach spaces via finite dimensional approximation. The theory enlarges the class of equations which can be solved by classical theory, since no compactness or condensing property of operators are required. The first chapter is an outline of the Brouwer and the Leray-Schauder degree theory and introduces the notion of an A-proper map. The second chapter presents the theory of the generalised degree for densely defined A-proper maps and shows how the theory can be applied to the solvability and global bifurcation problems of abstract equations. The abstract results are applied to periodic ordinary differential equations and semilinear elliptic partial differential equations. The last chapter is devoted to applications of abstract results of the fourth chapter to semilinear elliptic partial differential equations. The book presents new and well-known results in a unified approach. (efa)

R.E.Miles: Symmetric Bends: How to Join Two Lengths of Cord, Series on Knots and Everything, vol.8, World Scientific, Singapore, 1995, xii+163 pp., GBP 23, ISBN 9-810-22194-0

Roger E. Miles, an Australian mathematician well known especially for his pioneering work in the field of stochastic geometry and stereology, has written a book which might be a surprise for the mathematical community. Inspired probably by his life-long interest in mountain climbing and sailing, he has created a mathematically based theory of how to tie knots. Equipped with 60 coloured pictures of nice symmetric bends, the book is interesting also for non-mathematicians, people who like to tie knots for practical or aesthetical reasons. The mathematical background concerns mainly the geometrical (special types of symmetry) and topological properties of knots. (jra)

M.Pollicott, K.Schmidt (Eds.): Ergodic Theory of Z^d Actions. Proceedings of the Warwick Symposium 1993-4, London Mathematical Society Lecture Note Series 228, Cambridge University Press, Cambridge, 1996, viii+484 pp., GBP 29.95, ISBN 0-521-57688-1

This book represents the proceedings of the symposium of the same title held in Warwick, 1993-94. It consists of 8 survey contributions and 12 research papers. The contributions cover many different aspects and connections of the recent development in the dynamics of lattice models in higher dimensions, as for example the use of the Ramsey approach known from number theory for ergodic systems, the concept of fractal dimension from geometric measure theory for the entropy analysis etc. The book will serve as a valuable resource of information and motivation for specialists. (jra)

C.Broto, C.Casacuberta, G.Mislin (Eds.): Algebraic Topology: New Trends in Localization and Periodicity. Barcelona Conference on Algebraic Topology, Sant Feliu de Guíxols, June 1-7,1994, Progress in Mathematics, vol.136, Birkhäuser, Basel, 1996, ix+400 pp., DM 118, ISBN 3-764-35333-3, ISBN 0-817-65333-3

These are proceedings of the Barcelona Conference on Algebraic Topology, Sant Feliu de Guíxols, Spain, June, 1-7, 1994, which was part of the activities during a semester devoted to algebraic topology at the Centre de Recerca Matemàtica. The book contains 29 articles characterised in the foreword as reports on current progress in algebraic topology, focusing on advances in localisation and periodicity in homotopy theory, as a central but not exclusive topic. It is a remarkable collection of first class articles written by leading specialists in this rapidly developing area. I am sure that every topologist will find here a lot of interesting material. Every article contains a good introduction which can really help the beginner in algebraic topology find his own orientation. I agree with the statement at the back cover of the book that the text is accessible not only to the professional mathematician but also to the advanced student. It was tempting for me to give the complete list of articles in the proceedings but it would take too much space. Therefore, I feel it is my duty to recommend to every topologist to obtain these proceedings for themselves. (jiva)

M.Lübke, A.Teleman: The Kobayashi-Hitchin Correspondence, World Scientific, Singapore, 1995, viii+254 pp., GBP 35, ISBN 9-810-22168-1

Take a complex manifold X with a hermitian metric g

and a differentiable complex vector bundle E over X endowed with a hermitian metric h. The Kobayashi-Hitchin correspondence is a very interesting natural bijection map between two moduli spaces. The first moduli space $M_g^{HE}(E,h)$ is the space of gauge equivalence classes of h-unitary g-hermitian-Einstein connections in E, and the second one $M_g^{st}(E)$ is the space of isomorphism classes of g-stable holomorphic structures in E. The main aim of the book is to provide a thorough introduction of both the moduli spaces and the Kobayashi-Hitchin correspondence and present a detail investigation of their properties. The reader is supposed to have certain minimal knowledge of complex manifolds and basic knowledge of algebraic geometry. With these prerequisites he will find the presentation very nice and easy to understand. For the reader's convenience the authors included five appendices where more difficult notions are explained. The book contains many interesting examples and applications of the correspondence to the study of the relevant moduli spaces. The fact that the correspondence is a real analytic isomorphism (which was part of the second author's thesis) is published here for the first time. The reader gets a very good survey of this new part of holomorphic bundle theory. Moreover, the authors pay great attention to the explanation of the history and development of the subject, and show many further connections, analogues and generalisations of the Kobayashi-Hitchin correspondence, showing thus possible ways of further research. The literature is relatively complete and has 155 items. (jiva)

J.L.Troutman: Variational Calculus and Optimal Control. Optimization with Elementary Convexity. Second Edition, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1996, xv+461 pp., 87 fig., DM 84, ISBN 0-387-94511-3

This textbook is a thorough modern exposition of classical fields, being a refinement and an extension of author's two previous texts about variational calculus (from 1983) and optimal control (1986). The first part ("Basic theory" in Chapters 1–6) starts with representative samples of standard optimisation problems, then introduces theory of linear spaces and Gâteaux variations, minimisation of convex functions and lemmas of Lagrange and Du Bois-Reymond. It ends with local extrema in normed linear spaces and Euler-Lagrange equations. The second part ("Advanced topics" in Chapters 7–9) deals with variational problems having piecewise C^1 solutions and with classical necessary conditions such as the Weierstrass, the Legendre, and the Weierstrass-Erdman conditions. Furthermore, various variational principles in mechanics, such as the Hamilton, the Bernoulli and the Jacobi principles, are covered. The third part ("Optimal control" in Chapters 10–11) deals with optimal control problems for ordinary differential equations and, in particular, involves the Pontryagin maximum principle. Ancillary mathematical apparatus is summarised in the appendix. Each chapter is accompanied by a lot of examples, some of them answered at the end of the book. Altogether, being nicely written, the book may very well serve not only for undergraduate students but also for graduate students interested in optimisation or variational methods and as a good introductory book for experts in other branches of applied mathematics. (trou)

S.C.Countinho: A Primer of Algebraic D-modules, London Mathematical Society Student Texts 33, Cambridge University Press, Cambridge, 1995, xii+207 pp., GBP 13.95, ISBN 0-521-55199-6, ISBN 0-521-55908-1

The Weyl algebra (i.e. Dirac's quantum algebra) was a centre of interest since the advent of quantum mechanics. More recently, an algebraic approach to the study of systems of PDEs was developed within the theory of D-modules and has found a number of important applications in different branches of mathematics. The book treats modules over the Weyl algebra from the view-point of D-modules. There are already comprehensive treatments of D-modules theory available (e.g. M.Kashiwara, P.Schapira: Sheaves on manifolds, Springer, 1990) but the advantage of the present book is that it explains what D-modules are on an elementary level avoiding the more advanced tools of homological algebra and algebraic geometry needed in the full treatment of the theory. The book is very nicely written and carefully prepared to be understandable already at the beginning of graduate study. A lot of exercises (including computational ones) at the end of individual chapters are quite helpful. An excellent small book recommended warmly to students or to nonspecialists for first introduction to the subject. (vs)

V.Lakshmikantham(Ed.): World Congress of Nonlinear Analysts '92. Proceedings of the First World Congress of Nonlinear Analysts Tampa, Florida, August 19-26, 1992, 4 volumes, Walter de Gruyter, Berlin, 1995, xlvi+3954 pp., 436 fig., DM 1198, ISBN 3-110-13215-X

These four volumes contain about 350 contributions selected out of some seven hundred presented at the "First World Congress of Nonlinear Analysts" held during August 1992 in Tampa, Florida, under the auspices of the International Federation of Nonlinear Analysts. Over one thousand scientists from eighty countries participated in this congress which involved about 40 principle speakers, over 80 organised sessions, several short communications, and also "Lyapunov's and Poincare's Centenary sessions" and "Round Table meetings". The congress had many purposes: to promote co-operation in the world community of nonlinear scientists from various disciplines, to influence unification of similar ideas used in different branches, to highlight recent advances and to share ideas for future trends. The 1st volume of the proceedings consists of various aspects of partial differential

equations including problems in physics, fluid mechanics, combustion theory, structural mechanics, hysteresis, and atmospheric and marine sciences. The 2nd volume contains problems in (both deterministic and stochastic) ordinary differential equations as well as difference equations and dynamical systems, and also neural networks, problems in superconductivity and microelectronic devices, computer vision and numerical and computational methods. The 3rd volume involves theoretical aspects of nonlinear analysis such as nonlinear operators, nonconvex analysis, control theory and optimisation, fixed point theory and evolution equations, and additionally models in economic theory, manufacturing systems, engineering applications, chaos, bifurcation and artificial intelligence in nonlinear electronics. Finally, the 4th volume deals with various aspects of biomathematics and ecology including biochemical systems, environmental problems, bursting rhythms and biomedicine. Altogether, one can say that the huge 4-volume proceedings genuinely reflect the present state of the pure and applied nonlinear analysis to the maximum possible and will certainly be found to be very useful by everybody in a wide community of experts meeting nonlinear phenomena both in their theoretical research and in unlimited practical applications. (trou)

S.Sternberg: Group Theory and Physics, Cambridge University Press, Cambridge, 1995, xiii+429 pp., GBP 19.95, ISBN 0-521-24870-1, ISBN 0-521-55885-9

This is a remarkable book. From the point of view of pure mathematics we can say that it deals mainly with the representation theory of finite groups, compact Lie groups and Lie algebras. But this is not the feature which makes the book so attractive. The beauty of the book consists in the constant presence of physics during the exposition. The author shows how physical considerations lead naturally to mathematical notions and their investigations, and how mathematical achievements influence the development of physics. It is often the case that though mathematicians know perfectly well the mathematical machinery the physicists use, they are not quite so sure in their physical understanding of just what the physicists are describing, even less of how such a description arises. Furthermore, even if a mathematician tries to gain a deeper understanding of the physical theories, it may be quite difficult to find the necessary information in a compact form. In such a case, I can strongly recommend the book under review. To be more explicit, let us add (especially for geometers who are as a rule familiar with relativity) that the physical side of the book covers first of all nuclear physics, particle physics and crystallography. On the other hand, I expect that the book will be attractive also for physicists because the explanation (with many examples) will satisfy their requirements. The prerequisites for reading of the book are by no means very high: some linear algebra, group theory and operators in Hilbert spaces. At the end of the book there are detailed indications of further reading. Finally, we must mention the many very competent historical excursions which the book contains. (jiva)

R.Russo (Ed.): Mathematical Problems in Elasticity, Series on Advances in Mathematics for Applied Sciences, vol.38, World Scientific, Singapore, 1996, vii+185 pp., GBP 26, ISBN 9-810-22576-8

This 38th volume of Series on Advances in Mathematics for Applied Sciences collects five papers giving a sample of the problems and results in the mathematical theory of elasticity which characterise recent trends and advances. The volume starts with "Collected results on finite amplitude plane waves in deformed Mooney-Rivlin materials" by Ph. Boulanger and M. Hayes, and "Decay estimates for boundary-value problems in linear and nonlinear continuum mechanics" by C.O. Horgan, which is of relevance to the Saint-Venant principle. Then R. Russo and G. Starita deal with "Traction problem in incompressible linear elasticity for unbounded domains", deriving existence, uniqueness and continuous-dependence results. Further, T. Valent contributes "An abstract perturbation problem with symmetries suggested by live boundary problems in elasticity", i.e., by external forces depending on the unknown deformation, and finally L.T. Wheeler discusses applications of scalar-valued maximum principles to "Maximum principles in classical elasticity", giving also a number of counterexamples. Altogether, the book will be appreciated by experts in mathematics of solid-phase continuum mechanics as well as graduate students in this field. (trou)

E.Haier, G.Wanner: Analysis by Its History, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1996, x+374 pp., 173 fig., DM 68, ISBN 0-387-94551-2

This is a non-traditional, useful, interesting and well written book on basic analysis. Unlike usual books on analysis, this text respects the historical order: from introduction to analysis of the infinite via differential and integral calculus to foundations of classical analysis and to calculus in several variables. A lot of material, attractive from both mathematical as well as historical point of view, has been included. The exposition is clear and pleasant, there are many figures, pictures, drawings, tables, graphs, quotations from the literature, bibliographical indications and also exercises. The book can be warmly recommended to analysis teachers as a valuable source of elegant mathematics viewed from historical perspective as well as to students who will find not only a collection of basic results of analysis but will learn at the same time about the origin, motivation and further development. (in)

Ch.Kassel: Quantum Groups, Graduate Texts in Mathematics, vol.155, Springer-Verlag, New York, 1995, xii+531 pp., 88 fig., DM 79, ISBN 0-387-94370-6, ISBN

3-540-94370-6

This is a fundamental book on quantum groups designed first of all for mathematicians. It has developed from the author's two graduate courses at the Department of Mathematics of the Université Louis Pasteur in Strasbourg in the years 1990-92. From the very beginning, the author pays great attention to the presentation of the theory. The prerequisites are quite modest - standard linear algebra and some basic topology. (For illustration, let us mention that you can find a complete chapter on tensor products of vector spaces.) Actually, even undergraduate students will be able to read this book provided they have enough perseverance. However, I do not want to create the impression that this book is only a very well written introduction to quantum groups; reading further you will come to very high levels of mathematical abstraction and to interesting and very deep connections. The author himself states in the Preface that "The aim of this book is to provide an introduction to the algebra behind the words "quantum groups" with emphasis on the fascinating and spectacular connections with low-dimensional topology". The book is divided into four parts: 1. Quantum $SL(2)$, 2. Universal R-matrices, 3. Low-dimensional topology and tensor categories, 4. Quantum groups and monodromy. We note that even the relations among the isotopy invariant polynomials associated with links, solutions of the Yang-Baxter equations and various tensor categories are very clearly explained. It is true that the relevant techniques are often rather complicated but the leading idea is always clear. Moreover, it seems that the level of abstraction on which the whole theory is presented is at present the maximal possible. It is also worth mentioning that in the last part of the book the reader can find relations among braid group representations and the monodromy of the Knizhnik-Zamolodchikov equations and construction of universal knot invariants of finite type. Each of the 20 chapters is followed by Exercises and Notes. The exercises are mostly theoretical in character and extend the reader's knowledge of the relevant part of the theory. The notes also include comments on the history of the subject and indications for further reading. The references contain 202 items. (jiva)

H.Grauert: Selected papers, vol. I, II, Springer-Verlag, Berlin, 1994, xii+923 pp., DM 398, ISBN 0-387-57107-8, ISBN 3-540-57107-8

The influence of the mathematical work of H. Grauert on the shape of the theory of several complex variables was enormous. The two volumes of his selected papers bring together all his papers in this field. The collection of papers (in original versions, mostly in German) is split into eight subfields. At the beginning of each one, there are several pages of commentaries describing the contents of papers, showing their role in the evolution of the subject together with many other references and historical remarks. These commentaries form a really important part of the books and to read just them can be very useful for nonspecialists. The first volume contains papers on complex spaces, sheaf theory, vector bundles, the Mordell conjecture, the Levi problem, convexity and Stein spaces. The topics of the second volume are direct images, analytic and meromorphic decompositions, q-convexity and cohomology and hyperbolic complex spaces. The last two commentaries on non-Archimedean function theory and on discrete geometry describe some papers not included in the book. Both volumes can be well used independently of each other, complete contents and the complete bibliography of H.Grauert are contained in both volumes. The books are very carefully arranged and can be really useful to anybody interested in the field. (vs)

H.Amann: Linear and Quasilinear Parabolic Problems. Volume I. Abstract Linear Theory, Monographs in Mathematics, vol. 89, Birkhäuser, Basel, 1995, xxxv+335 pp., DM 172, ISBN 3-764-35114-4, ISBN 0-817-65114-4

During the last few decades, partial differential equations of parabolic type describing irreversible processes in physics, chemistry or biology have become an important subject of study. This first volume of the treatise is devoted to linear problems concentrating on functional analytic and dynamical system-type approaches to general nonautonomous and noncoercive problems and on the qualitative analysis of their solutions. The main idea consists of an interpretation of a parabolic problem as an ordinary differential equation in a suitably chosen Banach space. It requires a deep study of underlying parts of functional analysis: function spaces, semigroups and interpolation theory. This background represents the contents of Chapter I. Chapter II is devoted to the detailed study of evolution operators, their connections with integral equations, existence and stability. The maximal regularity theory both in Hölder and Sobolev spaces is explained in Chapter III. The following Chapter IV solves analogous problems in the case when the domains of generators $A(t)$ may depend on time variable t. In the last Chapter V an abstract general framework is developed for evolution equations in continuous interpolation - extrapolation scales of Banach spaces. The book represents a modern and systematic treatment of an attractive topic applicable in many situation occurring both in theory and applications. It contains a wide range of material starting with careful exposition of constant domain case and including recent results (many of them by the author). The treatise will be indispensable for everybody working in the field and will undoubtedly become a standard reference. (jsta)

P. de Bartolomeis, F. Tricerri, E. Vesentini (Eds.): Manifolds and Geometry, Symposia Mathematica vol.XXXVI, Cambridge University Press, Cambridge, 1996, viii+321 pp., GBP 40, ISBN 0-521-56216-3

This volume is the Proceedings of the conference on dif-

ferential geometry held in Pisa in September 1993 and dedicated to the honour of E.Calabi. The invited lectures published here have been presented by the following mathematicians: D.V.Alekseevsky and M.M.Graev, M.Berger, J.P.Burguignon, B.L.Bryant, D.Burns and R.Mazzeo, F.Catanese, Y.Eliashberg and H.Hofer, N.J.Hitchin, J.Jorgenson and A.Todorov, S.Kobayashi, F.Labourie, P.Lu and G.Tian, S.M.Salamon, Y.-T.Siu. The main article devoted to the scientific activity of Eugenio Calabi is written by M.Berger. (ok)

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Applications are invited for the above vacant post within the newly-formed School which comprises the Departments of Mathematics, Applied Mathematics and Statistics. Applicants should have an internationally recognised record of achievement in research, preferably in an area of main-stream Applied Mathematics such as fluid or solid mechanics, electromagnetism and optics, acoustics and wave phenomena, or dynamical systems but other applied areas involving mathematical model building will also be considered. An interest in nonlinear problems will be especially welcome. Substantial teaching experience will also be sought. The appointee will be expected to provide leadership by example and initiative, to promote excellence in teaching, research and consultancy, and to undertake administration. The post will include a commitment to act as Head of the Department of Applied Mathematics or as Chairman of the School of Mathematics, Applied Mathematics and Statistics, if appointed by the Governing Body.

The main areas of interest in the current Department of Applied Mathematics include solid and fluid mechanics, computational mechanics, applied numerical analysis, nonlinear optics, modelling and computation, mathematically intensive software development There is a commitment to build on current strengths, and two other permanent academic appointments will be made in conjunction with the Chair. The school offers well-established undergraduate and postgraduate degree programmes in the Faculties of Science and Arts, as well as contributing to many other courses across a wide range of disciplines.

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Applicants should complete a standard application form and send it, together with a curriculum vitae and the names and brief biographies of three referees, to

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University College, Cork, Ireland
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from whom application forms and other official details may be obtained.

See also the following Worldwide web address: http://www.ucc.ie/ucc/appts/acad/jobs.html
where application forms can be accessed. A brief statement describing the significance of the candidate's research, and copies of up to three of his/her most important publications, should also be included. For informal or technical discussions, please contact the Chairman of the Search committee

Professor Patrick D Barry
Department of Mathematics
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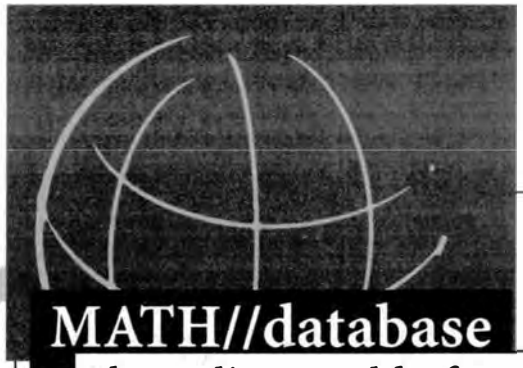

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