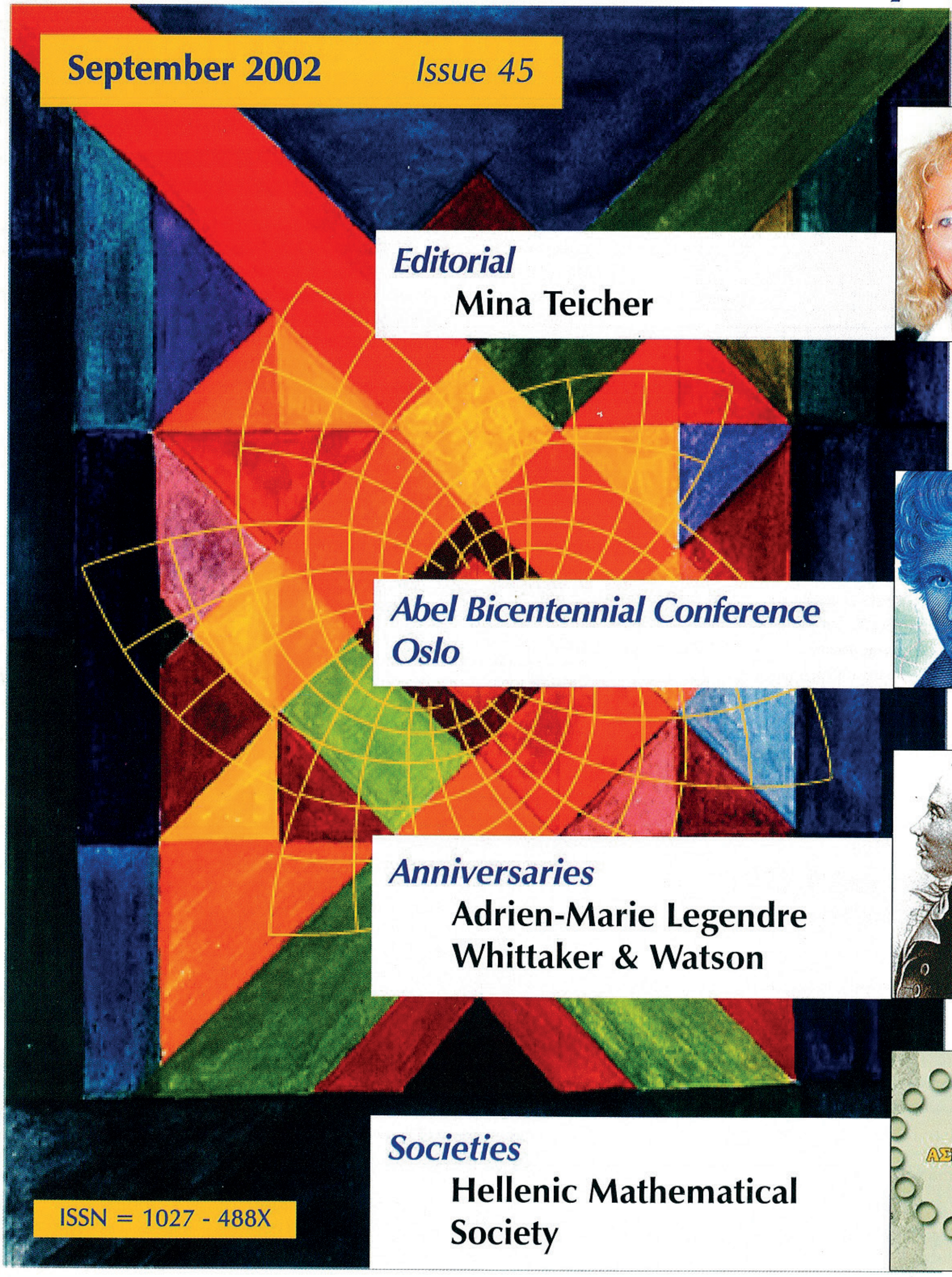
European Mathematical Society

September 2002 Issue 45



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NEWSLETTER

EUROPEAN MATHEMATICAL SOCIETY

NEWSLETTER No. 45

September 2002

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NOTICE FOR MATHEMATICAL SOCIETIES

Labels for the next issue will be prepared during the second half of November 2002. Please send your updated lists before then to Ms Tuulikki Mäkeläinen, Department of Mathematics, P.O. Box 4, FIN-00014 University of Helsinki, Finland; e-mail: makelain@cc.helsinki.fi

INSTITUTIONAL SUBSCRIPTIONS FOR THE EMS NEWSLETTER

Institutes and libraries can order the EMS Newsletter by mail from the EMS Secretariat, Department of Mathematics, P. O. Box 4, FI-00014 University of Helsinki, Finland, or by e-mail: (makelain@cc.helsinki.fi). Please include the name and full address (with postal code), telephone and fax number (with country code) and e-mail address. The annual subscription fee (including mailing) is 65 euros; an invoice will be sent with a sample copy of the Newsletter.

EMS Committee

EMS Agenda

2002

28-30 September
Executive Committee meeting in Stockholm, at the invitation of the Swedish Mathematical Society

30 September
Deadline for proposals for the 2003 EMS Lectures
Contact: David Brannan, e-mail: d.a.brannan@open.ac.uk

15 November
Deadline for submission of material for the December issue of the *EMS Newsletter*
Contact: Robin Wilson, e-mail: r.j.wilson@open.ac.uk

31 December
Deadline for bids for the Fifth European Mathematical Congress, *5ecm*, in 2008
Contact: EMS Secretariat, e-mail: makelain@cc.helsinki.fi
Deadline for the Raising Public Awareness article competition
Contact: Vagn Lundgaard Hansen, e-mail: V.L.Hansen@mat.dtu.dk

2003

8-9 February
Executive Committee meeting in Nice (France), by invitation of the local organisers of AMAM 2003

10-13 February
AMAM 2003: EMS-SMAI-SMF Meeting in Nice (France)
Mathématiques Appliquées - Applications des Mathématiques (Applied Mathematics - Applications of Mathematics)
Contacts: Doina Cioranescu, e-mail: cioran@ann.jussieu.fr and Mireille Martin-Deschamps, e-mail: mmd@math.uvsq.fr, webpage: http://acm.emath.fr/amam/

15 February
Deadline for submission of material for the March issue of the *EMS Newsletter*
Contact: Robin Wilson, e-mail: r.j.wilson@open.ac.uk

1 March
Deadline for proposals for the 2004 EMS Lectures
Contact: Helge Holden, e-mail: holden@math.ntnu.no

18-23 May
IPAM-SIAM-EMS Conference in Los Angeles (UCLA Lake Arrowhead Conference Center), USA
Applied Inverse Problems: Theoretical and Computational Aspects
webpage: http://www.ipam.ucla.edu/programs/aip2003

1-5 July
EMS Summer School, at the University of Porto (Portugal)
Dynamical Systems
Organiser: Maria Pires de Carvalho, e-mail: mpcarval@fc.up.pt

7-12 July
CIME-EMS Summer School at Bressanone/Brixen (Italy)
Stochastic Methods in Finance
Organisers: Marco Frittelli and Wolfgang J. Runggaldier,
e-mail: runggal@math.eunipd.it

12-14 September
SPM-EMS Meeting in Lisbon (Portugal)

2004

25-27 June
EMS Council Meeting, Stockholm (Sweden)

27 June - 2 July
4th European Congress of Mathematics, Stockholm

2-6 September
EMS Summer School, at the Universidad de Cantabria, Santander (Spain)
Empirical processes theory and statistical applications

Editorial: Mina Teicher

(Bar-Ilan University, Israel)

Investing in the future general public

Does the general public attitude towards mathematics depend only on raising the awareness among adults?

How do we ensure a sufficient stream of undergraduate and graduate students to mathematics, as well as creating a positive public and administrative attitude towards mathematics?

Some personal thoughts

In the past few years (since 1997) I have made presentations to different policy makers (journalists, the Minister of Commerce, Army chiefs of staff, etc.). Its goal was to point out how mathematics underlined all existing technologies in daily life: its role in the high-tech revolution, the role of mathematics in the financial world; its future role in advancing science; the interplay between applied mathematics and abstract mathematics and the employment opportunities that will be opened to young individuals by studying mathematics. Raising public awareness of the importance of mathematics to society and to individuals is a long process that needs to be repeated again and again. Nevertheless, it is not clear whether it is too late to address the above high officials in that stage of their life. This is of course a question that is not possible to answer in an affirmative fashion – in other words, was their attitude to mathematics created and fixed while they were high-school students, or even in elementary school?

For many years my department (the Department of Mathematics and Computer Science in Bar-Ilan University in Israel) has been running a special programme for gifted high-school students. The programme involved students from the age group of 14-15 in neighbouring high schools and more remote ones. The students accepted in the programme are the most gifted (and ambitious) students that pass the targeted tests of the programme, as well as personal interviews to determine ambition and dedication. They do not complete their high-school curricula in mathematics with their normal affiliation, but take the programme intensely after school classes and writing their matriculation in mathematics at the age of 16. In the remaining two years of high school they take courses in mathematics

chosen from the regular B.Sc. programme in mathematics of the first and (even) second year (Calculus I, II, Linear Algebra I, II, Set Theory, Algebra, Complex Analysis, and others). This is a very demanding programme – the students participate in the first year B.Sc. mathematics programme while completing their high-school curriculum in other subjects (including writing the relevant matriculation exams); this demands both intellectual and emotional strength. Needless to say, they are very gifted and it is a pleasure to teach them. Some of my Ph.D. students started their mathematical education in this way, and some are already pursuing careers in mathematics, up to full professor.

Lately we have started an outreach project to a different community of high-school students: those who are not ambitious enough and are aiming at the second level of matriculation in mathematics, or even the third level, rather than trying to aim at a higher level (there are three levels of matriculation in mathematics in Israel). Using the results of their normal studies, their teachers' recommendations and personal interviews, we recommend the student to take a special after-school programme that will prepare them in two semesters to move to a higher level. The programme is partially funded by a private foundation, partially by high-tech companies, and partially by public organisations.

The third group we are addressing (not directly, but via promoting the programme in any relevant capacity that I hold in national level), is the very young age group. There is a voluntary group dedicated to raising the level of accomplishment and acceptance of mathematics by teaching it in a conceptual fashion (as well as developing modelling skills) and by providing professional mathematicians to teach mathematics even to first-grade elementary school pupils! It is done as an experiment and started in the early 1990s by a group of mathematicians who immigrated to Israel from the FSU. (Even today, the number of students of Russian origin is relatively high.)

Will these students become more aware of mathematics when they grow up and become informed citizens? Will they be more sympathetic towards providing a budget for mathematics when they become officials of the EU? Will they still eliminate mathematics from the EU's top priorities (as in some programmes of the 6th framework)? As I said in my address in the Luxembourg meeting in Spring 2001 [Celebrating the submitting of the European project 'Reference level - 16-years' (study of the different European programmes for this age-group)]: "A better and targeted education in mathematics from an early age will result in government officials and policy makers who are more favourable towards mathematics."

The floods in Prague

Help us save the mathematical library at Charles University

The Czech Republic, and Prague in particular, was heavily hit by recent floods. The largest Czech mathematical library was located in the building of the Faculty of Mathematics and Physics, Charles University, at Karlin, the most damaged part of Prague. It contained books and journals from all fields of pure and applied mathematics, statistics, numerical analysis and computer science. About two-thirds of the books and journals in the library were heavily damaged and, despite all efforts, are almost surely lost. Among them were about 5000 recent books and monographs and almost 400 journal titles. There were also thousands of historical books from the past three centuries which are impossible to buy – these included, for example, the first editions of the complete works of Cauchy, Weierstrass and Riemann.

Since the whole country was extensively damaged by the flood, we must expect that the reconstruction of the library will be a very difficult task. While we can recover texts needed for undergraduate study, we are not able to repurchase books needed for graduate and postgraduate students or books and journals needed for research.

In our difficult situation, we should like to ask the community of European mathematicians for help. The list of perished books from recent decades and the lost journal series are available on the web page http://www.mff.cuni.cz/povoden/

If anyone is in a position to provide us assistance, a donation of spare copies of books or journals (in particular, from the list on the web page) would be an immense help to us. The books and journal issues can be sent directly to the faculty's library (see address below).

It is also possible to make a financial contribution to the renewal of the library – this is a particularly efficient way to help us to recover exactly what we need. Money can be sent either by cheque, payable to
Charles University in Prague,
Faculty of Mathematics and Physics,
Ke Karlovu 3,
121 16 Praha,
Czech Republic,

or by a money transfer to the following account:
CSOB, Prague 1, Na Porici 24,
account number 01256280/0300.
Our reference: 999,
SWIFT: CEKO CZ PP PRA

Gifts of books or money contributions are free of tax in our country, if we have a letter declaring that the gift was sent as a donation to the library of the Faculty. Any help would be very welcome and highly appreciated.

Ivan Netuka and Vladimir Souček (Editors of the *Recent books* column)

EMS Executive Committee Meeting

Oslo 31 May

Those present were: Rolf Jeltsch (President, in the Chair), David Brannan, Bodil Branner, Victor Buchstaber, Doina Cioranescu, Luc Lemaire, Olli Martio, Marta Sanz-Solé, Mina Teicher. Apologies had been received from Renzo Piccinini. In attendance by invitation were: Carles Casacuberta (Publications Officer), Robin Wilson (Editor-in-Chief, *EMS Newsletter*), David Salinger (Publicity Officer), Saul Abarbanel (Chair of Applied Mathematics Committee), Thomas Hintermann (Managing Director of EMS Publishing House), Sir John Kingman (Nominee for President for 2003-06), Helge Holden (Nominee for Secretary for 2003-06), Dag Normann, President of the Norwegian Mathematical Society, Tuulikki Makelainen, Helsinki Secretariat.

The President thanked the Norwegian Mathematical Society for their invitation and local arrangements, and welcomed all participants.

The minutes of the previous meeting (Brussels, 9-10 February 2002) were accepted and the President signed them.

Electronic votes

The EC ratified the results of the electronic votes since the last meeting:

* Sir John Kingman was elected EMS delegate to PESC (Standing Committee for Physical and Engineering Sciences) Committee.
* EMS would sponsor the conference on *Applied Inverse Problems* at IPAM. Erkki Somersalo would be the member delegated by the EMS to be on the Scientific Committee of the conference. The EMS would give some financial support for the participation of young researchers from Europe.
* Bodil Branner would be the delegate of the EMS to the ESF EURESCO Committee.
* Nina Ural'tseva would be the Chair of the Prize Committee for the EMS Prizes in 2004.
* The following would be members of the EMS Education Committee: Abraham Arcavi (Weizmann Institute, Israel); Gerd Brandell (University of Lulea, Sweden); Willi Doerfler (University of Klagenfurt, Austria); Sava Grozdev (Bulgarian Academy of Sciences, Sofia); Colette Laborde (IREM Grenoble, France); Olli Martio (University of Helsinki, Finland); Rudolf Straesser (University of Bielefeld, Germany); Eva Vasarhelyi (ELTE, Budapest, Hungary); Vinicio Villani (University of Pisa, Italy).
* Michel Jambu, Director of CIMPA, would become a member of CDC for the period 2002-05.
* The following would be members of the EMS Applied Mathematics Committee: Georgios Dassios of the Hellenic Mathematical Society (2002-05); Rüdiger Schultz (2002-05); Vincenzo Capasso (2002-03); Jose-Francesco Rodriguez (2002-05); Erkki Somersalo (2002-05); Jacques Periaux (2002-05); Andras Frank (2002-03); Anders Szepessy (2002-05); Walter Schachermayer (2002-03); Rolf Jeltsch (2002-03).

Officers' Reports

Bodil Branner, Carles Casacuberta, Rolf Jeltsch, Olli Martio, Marta Sanz-Solé, David Salinger and Bernd Wegner would attend the ICM-2002 in Beijing.

It was agreed to recommend to Council a change of class of the Belgium Mathematical Society and the Norwegian Mathematical Society from class 1 to class 2.

EC noted that notice had been given by 30 June 2002 to Springer-Verlag to end the ongoing contract for *JEMS*, and that from the start of 2003 the EMS Publishing House would be responsible for *JEMS*.

4ecm in Stockholm in 2004

The meeting will be held on 27 June-2 July 2004, in Stockholm (Sweden).

It was agreed that only new names should be used for membership of the Prizes Committee members, disjoint from previous Prizes Committee members, that the Prizes Committee should be disjoint from the Scientific Committee, and that one EMS Executive Committee member should be included.

Meetings

A joint meeting of the EMS and the Portuguese Mathematical Society was approved, to be held in Lisbon on 12-14 September 2003. The EMS will give some financial support to the joint meeting, plus a small deficit guarantee.

Bids to EU

A Socrates Project had been submitted by the Education Committee to the EU/EC.

Expressions of Interest had been mailed to the EU on the following:

* Digitisation DML (Digital Mathematics Library)
* Integrated Initiative: web presentation (jobs, conference calendars, e-mailing, new presentation of EMIS)
* *Zentralblatt MATH* improvements: keyboarding back volumes, linking reviews with articles, increase of linking possibilities, and other improvements could be considered as a follow-up of LIMES – making *Zentralblatt MATH* an even more European endeavour.
* Network on Excellence by ERCOM called *INTERMATH*

A proposal for a UNESCO project on Raising Awareness of Mathematics had been received from Mireille Chaleyat-Maurel. It was agreed that EMS would support the project morally and would help to find funding.

Council meeting 2002

There was much useful discussion in preparation for the Council meeting on the following two days!

Future meetings

The next EC meeting will be held in Stockholm 29-30 September 2002, and the following one in Nice (at the invitation from the local organisers of *AMAM2003*) on 9-10 February 2003.

The next Council Meeting will be held in Stockholm on 26-27 June 2004, the 26 June session starting at 10 a.m.

The next GPC meeting will be held in Zurich on 14-15 December 2002, with an EMF Board of Trustees meeting at 14.00-16.00 on the 15 December.

David A. Brannan

A report on the EMS Council Meeting in Oslo, 1-2 June, will appear in the next issue

Announcing AIP2003

Applied Inverse Problems: Theoretical and Applied Aspects
(IPAM-SIAM-EMS Conference, 18-23 May 2003)

The National Science Foundation (NSF) sponsored Institute for Pure and Applied Mathematics (IPAM), will host the second conference by this name during 18-23 May 2003, together with the Society for Industrial and Applied Mathematics (SIAM) and the EMS. This conference is a sequel and it is hoped only the second in a series of conferences following the highly successful conference held at Montecatini (Italy) in June 2001. It is also a lead toward the special quarter on Applied Inverse Problems being held during Fall 2003 at IPAM.

In the last twenty years the field of inverse problems has undergone rapid development. A large international community is engaged in solving these problems where typically the solutions are indirectly related to the available data and where causes are determined for desired or observed effects. The problems are often ill-posed, in that changes in the data can produce large effects in the solution. Furthermore, even questions of whether a solution that corresponds to likely noisy data can exist, and how many and how different solutions there may be that correspond to partial data sets, need to be considered.

The enormous increase in computing power and the development of powerful algorithms has made it possible to consider real-world problems of growing complexity, and has led to a growing appetite to apply the techniques of inverse problems to ever more complicated physical and biological problems. Applications include several medical as well as other imaging techniques, location of oil and mineral deposits in the earth's substructure, creating astrophysical images from telescope data, finding cracks and interfaces within materials, shape optimisation, and model identification in growth processes and more recently in the life sciences. Historically, the model of the physical phenomena was frequently linear with the inverse problem being non-linear; recent work also includes non-linear physical phenomena models.

The goal of this conference is to include a broad spectrum of advancing new problems with presentations on both computational and theoretical issues and for a wide range of applications.

The conference will be held at the UCLA Lake Arrowhead Conference Center which is located in the mountains north of San Bernardino, California, approximately 100 miles north of Los Angeles. The centre, which holds 169 participants, is in a very beautiful location. Response to the conference has been very positive and the organisers anticipate that the conference will fill the centre to capacity with most participants giving talks, either as invited speakers or as mini-symposia participants. There is also a strong emphasis on including participants who are in their early career with a goal of having 40-50 postdoctoral and/or graduate student participants who have completed enough of their work to be able to present a research talk. Conference costs will be favourable for those who have this status, and additional funds for travel reimbursement may be available for early career participants. Travel funds may also be available for participants for whom the travel costs would be a hardship.

A broad spectrum of problems will be presented by the invited and plenary speakers who include: Simon Arridge(London), Mario Bertero (Genova), Brett Borden (U.S. Navy), Raymond Chan (Hong Kong), Tony Chan (UCLA), Margaret Cheney (Rensselaer), Maarten de Hoop (Colorado), David Dobson (Utah), Douglas Gough (Cambridge), Alberto Grunbaum (Berkeley), Martin Hanke (Mainz), Andreas Kirsch (Karlsruhe), Rainer Kress (Göttingen), Bill Kuperman (UCSD), Frank Natterer (Münster), Dianne O'Leary (Maryland), Gary Odell* (Washington), Stanley Osher (UCLA), George Papanicolaou (Stanford), Roy Pike (London), Otmar Scherzer (Innsbruck), Erkki Somersalo (Helsinki), Gunther Uhlmann (Washington).

The organising committee consists of Joyce McLaughlin (Rensselaer) as Chair, Heinz Engl (Linz) and Daniela Calvetti (Case Western). They have sought advice from the Scientific Committee: Erkki Somersalo (Helsinki), Mario Bertero (Genova), Tony Chan (UCLA), David Dobson (Utah), Martijn de Hoop (Colorado), Rainer Kress (Göttingen) and Alberto Grunbaum (Berkeley).

Visit the website at *http://www.ipam.ucla.edu/programs/aip2003* or e-mail questions to *aip2003@ipam.ucla.edu* to learn more about the conference. To learn more about the special fall 2003 semester at IPAM, see *http:///www.ipam.ucla.edu/programs/inv2003*

Journal of the European Mathematical Society

The next issue of *JEMS* (Vol. 4, No. 3) will contain the following articles:

O. Venjakob, *On the structure theory of the Iwasawa algebra of a p-adic Lie group*;

H. M. Soner and N. Touzi, *Dynamic programming for stochastic target problems and geometric flows;*

R. F. Bass, B. M. Hambly and T. J. Lyons, *Extending the Wong-Zakai theorem to reversible Markov processes.*

Abel Bicentennial Conference

Oslo (Norway), June 2002

Don Zagier and Andrew Wiles at Oslo

The Abel Bicentennial Conference was held at the University of Oslo from 3-7 June 2002. The conference was opened by HM King Harald V, and the conference celebrated the launch of the Abel Prize in Mathematics.

The plenary speakers at the conference were F. Catanese, Ciro Ciliberto, Alain Connes, Gerd Faltings, William Fulton, Mark Green, Phillip Griffiths, Genadi Henkin, Christian Houzel, Steve Kleiman, Maxim Kontsevich, Hendrik W. Lenstra, Ken Ribet, Norbert Schappacher, Dennis Sullivan, Andrew Wiles and Don Zagier.

The Proceedings of the Abel Bicentennial Conference will be published by Springer, as a book and in a CD-ROM.

A special feature of the conference was an exhibition of Abel's life and work. The main attraction of the exhibition was the manuscript of the Paris Memoir, considered his most important work. This manuscript belongs to the Moreniana Library in Florence and the Labronica Library of Livorno, and was brought to Oslo especially for the Conference.

HM King Harald V's speech at the Opening Ceremony

Rector, distinguished guests, ladies and gentlemen.

Two hundred years ago our nation was in its childhood. A young nation needs national heroes. Niels Henrik Abel was such a hero. He was young and bright-minded and he became famous in academic circles at home and in Europe. His untimely death was a loss for the young nation, and also for the development of mathematics throughout the world.

This week we shall honour this bright young man, his ideas and achievements. But we shall also honour the mathematics itself, and the development of a great universe of ideas, theories and applications. I sincerely hope that this conference can be another step in this development, that you through discussions and lectures can broaden your experience in your esteemed work. I know that you all have a passionate relationship with mathematics, which can be difficult enough for a non-mathematician as myself to understand.

It is a pleasure for me to see so many professional mathematicians from so many parts of the world gathered here in Oslo. I hope you will all have some spare time to explore our capital and its surroundings during your stay. This time of year is the most beautiful here in the north of Europe, with the long bright summer evenings and you can appreciate the nature waking up after the cold winter.

I wish you success and take pride in declaring the Abel Bicentennial Conference 2002 open.

The short story of Abel

A talk at the opening of the Abel Bicentennial Conference
Arild Stubhaug

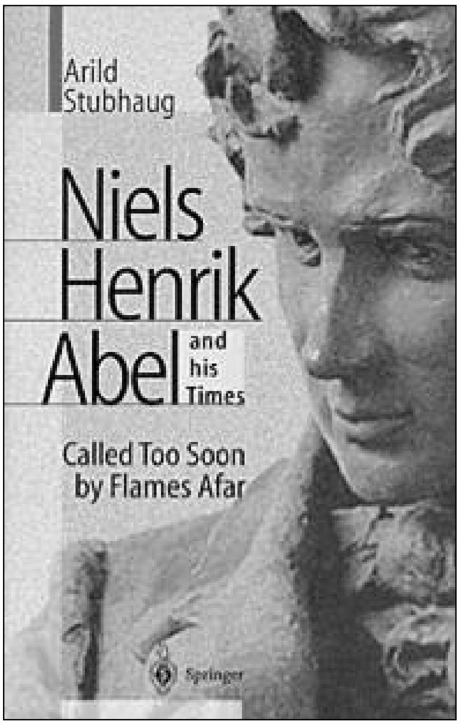
Your Majesty, ladies and gentlemen:

When your grandfather, Your Majesty, our king Haakon VII, for the first time back in 1907 attended a meeting in The Norwegian Academy of Science and Letters, he made the following statement:

Science is in such a fortunate position for not being dependent upon the number of inhabitants, such that one may expect results as great among a people of two million, as among one of twenty. I have only to recall the name of Niels Henrik Abel.

It's true today as well. In the world-wide mathematical landscape Abel is a sparkling figure. But in private: his personal life is a sad story. I am going to tell a little about the life of this outstanding man who only lived for 26 years.

His father was a vicar and a prominent person in every way – he gave his six children the best education possible, according to his strong belief in the rationalist spirit of the European Enlightenment.



Niels Henrik's mother was the daughter of a shipping merchant in the lively port town of Risør – and had grown up in luxury and abundance. She very soon became too fond of drinking – even after all her riches had disappeared during bankruptcy that followed the Napoleonic wars.

Father Abel's public and private life ended in misery. Everyone in the village of Gjerstad, where he lived, knew that the vicar and his wife drank, and that they drank separately. Father Abel rapidly went downhill, and he died in the spring of 1820.

When this happened Niels Henrik was a pupil at the Latin school here in the capital – he had just grasped mathematics and was acquiring knowledge with great speed. He became a university student in 1821, but even before that he had begun what was to become his first mathematical feat, his work on the quintic equation.

To the University Senate it was clear that Abel had to go to the world's foremost mathematical milieu at that time – namely to Paris – to gain further education. However he was considered too immature, but got a scholarship and studied on his own, and read all of the considerable mathematical literature available in the University library.

Finally, in the autumn of 1825, he got his travel money and went abroad on the conditions of travelling to Paris and to visit the great Carl Friedrich Gauss in Göttingen, who was considered a mathematical institution in himself. But when Abel arrived in Copenhagen, he changed his plans and went to Berlin instead. The fact that he went to Berlin was however the major fortunate event of his life. In Berlin he met the engineer August Leopold Crelle, who in meeting Abel got the courage to publish a mathematical journal, which would compete with the well established ones in France. In early 1826 the first issue came out of *Journal für die reine und angewandte Mathematik*, and most of what Abel had time to write was published in this journal.

Although Abel now had begun to make his work public in Berlin, he had saved what he considered his best work for the honourable Académie in Paris – being published here would impress the rulers in Norway. As soon as he arrived in Paris, ten months after his departure from Norway, he started work on what has later been called his *Paris Memoir*. Very few, if any, mathematical dissertations have received such acclaim as this Paris dissertation of Abel's. He submitted it at the end of October 1826 and spent the rest of the year waiting for an answer. An answer never came. Abel's work was set aside. For as long as he lived, Abel was convinced that his Paris dissertation had been lost forever. His stay in Paris was a disappointment – he did not feel well: he had a fever, and was told he must be suffering from tuberculosis.

In 'official Norway' Abel's journey abroad was regarded as somewhat of a failure. Nothing had been published in Paris, and he had not visited the great Gauss in Göttingen. Abel had rightly enough had his work printed in Berlin, but what kind of prestige did this new journal in Berlin carry?

Abel had only a year and a half left to live. This time was taken up with an impressive series of dissertations, which he continuously sent to Crelle's *Journal* in Berlin. And Abel had finally realised that accepting a position in Berlin was his only solution – both in order to get his work done and to be able to marry the girl he had been engaged to for four years.

Throughout that last autumn Abel worked here in the capital as intensely and feverishly as before – and when Christmas approached he set out in the cold winter to visit his fiancé in the far-away village of Froland. After a Christmas ball, when he stepped outside to cool off, he began to cough up blood. He stayed in bed – he managed to write one mathematical work: he wrote two or three pages wherein he again attempted to formulate the introductory thoughts to his Paris *Memoir*.

His sick-bed became his death-bed. Abel was only 26 years old. He thought it terrible that it would all soon be over.

On 6 April 1829, life ended for Niels Henrik. Two days later, without any knowledge of what had taken place in Norway, there was correspondence about, and to, Abel in both Paris and Berlin. From Paris the message was that Abel's Paris dissertation had finally been found. From Berlin Crelle wrote in glowing terms that there was now a guaranteed professorship for him in that city: 'You may now regard your future calmly. You belong among us and you will be safe', Crelle wrote, and ended with, 'You are coming to a good country, to a better climate, closer to science and to real friends who appreciate you and are fond of you'.

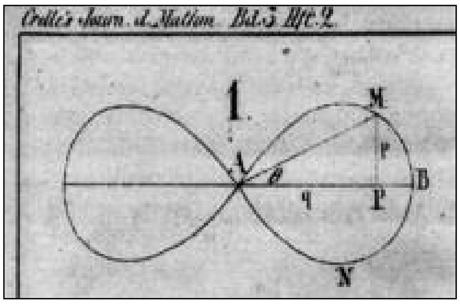
Again your Majesty, we recall the name of Niels Henrik Abel – we are touched by his misfortune and destiny – we pay tribute to his great scientific achievements.

Two new Abel stamps

Two Norwegian stamps were issued during the week of the conference. The 5.50 NOK stamp features the portrait of Abel, based on the painting by John Gørbitz in Paris in 1826, and the lemniscate, the symbol of the conference. The 22 NOK stamp features the cover of the second edition of his collected works and a computer-generated guillochert rosette

.

Abel's lemniscate



The lemniscate, the symbol of the Abel bicentennial conference, is the only mathematical picture in Abel's collected works. Abel was very excited about the lemniscate. In a letter to his former teacher Holmboe, written from Paris in December 1826, he wrote:

You shall see how pretty this is. I have found that one can divide, with the help of ruler and compass, the lemniscate into $2n + 1$ *parts when this number is a prime number. The division depends upon an equation whose degree is* $(2n + 1)^2 - 1$*. But I have found its full solution by means of square roots. In the same vein I have approached the mystery that has surrounded Gauss' Theory of the division of the circle. I see as clearly as day how he has obtained it.*

Abel statue in Oslo

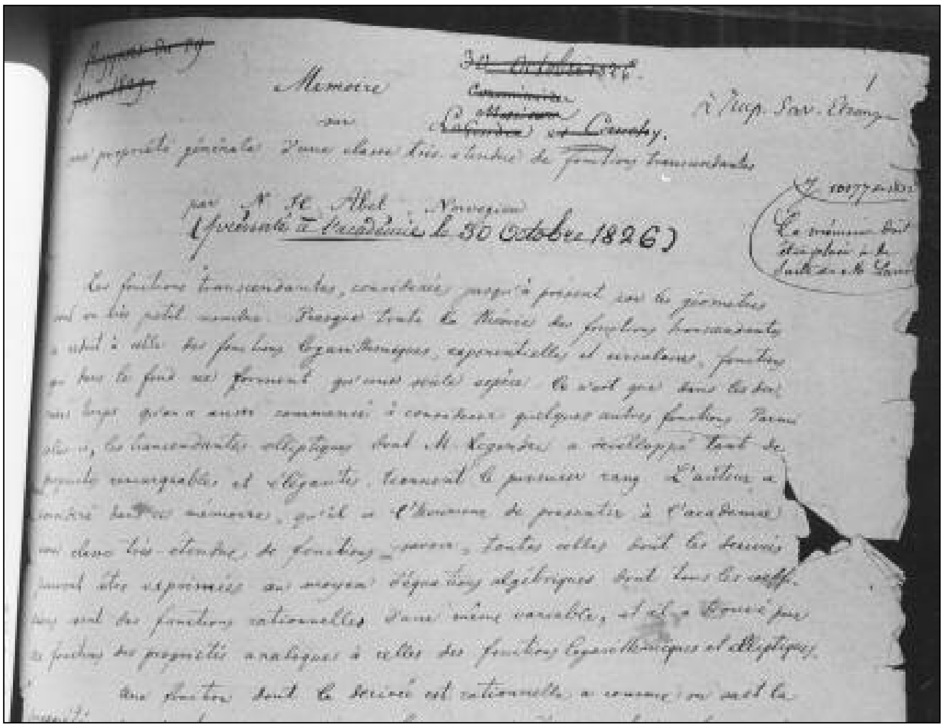
Bicentennial Anniversary Prize awarded to Atle Selberg

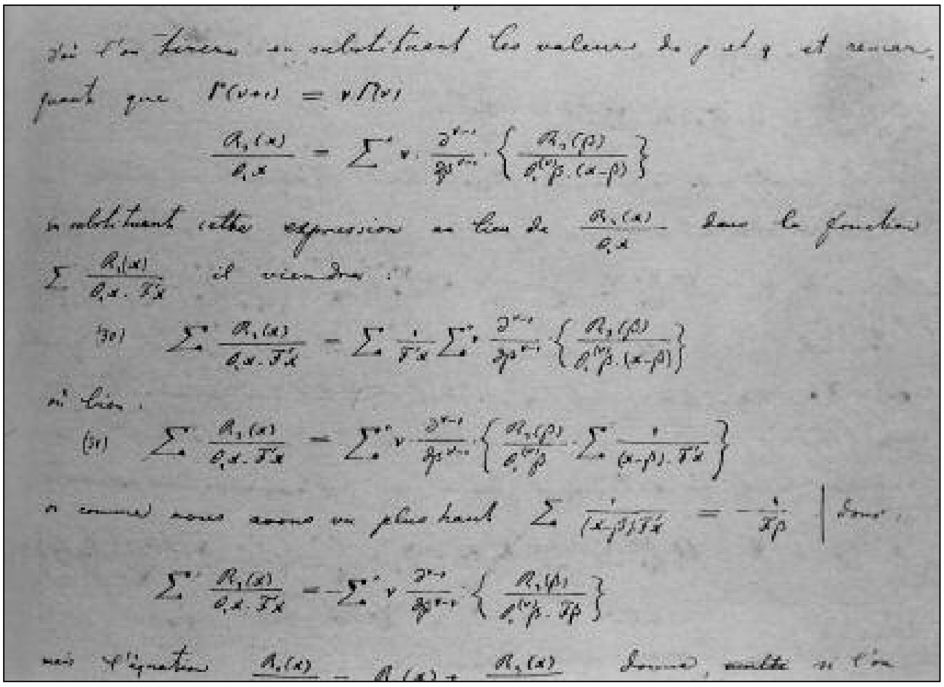
In conjunction with the Abel Bicentennial Conference, the organising committee awarded a special Bicentennial Annviersary Prize to **Atle Selberg**, to honour Selberg's great mathematical contribution and to commemorate Niels Henrik Abel.

Atle Selberg's admiration for Abel was highlighted by Jens Erik Fenstad at the opening ceremony, when he quoted froma letter written by Selberg on 5 January 2002:

I started to study [the collected works of Abel] in middle school when I was 13-14 years old. Nothing made a stronger impression on me than number 27 in the list of his papers [Démonstration d'une propriété générale d'une certaine classe de fonctions transcendantes]. It has always seemd like pure magic to me. Neither Gauss nor Riemann (nor anybody else) has anything that can measure up to this.

Along with Henrik Ibsen and Edvard Munch, Abel is one of the national icons of Norway. To underline this, the organising committee of the Conference selected as the Bicentennial Anniversary prize a first edition of Ibsen's play *Hedda Gabler*, an original etching by Edvard Munch, and a reproduction of the manuscript of Abel's famous *Paris Memoir* from *Abel's Parisian Manuscript*, published in Florence in 2002.





Abel's Paris Memoir

Launch of the Abel Prize

6 June 2002

At the Abel Bicentennial Conference a major event took place for the international mathematical community – the launch of the Abel Prize – to be awarded annually by the Norwegian Academy of Science and Letters, starting in 2003. The launch of the prize took place on Thursday 6 June, and the speeches included the following contributions by Per-Kristian Foss (Minister of Finance), Rolf Jeltsch (EMS president), and Lars Walløe (President of the Norwegian Academy of Science and Letters).

Speech by Per-Kristian Foss, Minister of Finance

Representatives of the mathematical community, Distinguished guests!

It is a great pleasure for me to address the international mathematical community on this occasion.

The year 2002 is an important anniversary for the field of mathematics: the bicentennial of the birth of Niels Henrik Abel. The anniversary has already been duly celebrated through a number of events in all parts of Norway. Mathematics has even become part of our everyday lives through a daily Abel column in one of our major newspapers.

This week's Conference at the University of Oslo is of course one of highlights of the bicentennial. The Conference presents an overview of the mathematical heritage of Niels Henrik Abel and identifies new mathematical trends for the next century.

I am pleased and honoured that so many representatives of the mathematical community have come to Oslo to take part in this event. I can think of no better occasion to present the Niels Henrik Abel Memorial Prize in mathematics.

My colleague Mrs Clemet, Minister of Education and Research, would have wanted to address you today, but could not since she was called to Parliament. I know she would have wanted to emphasise her dedication to the advancement of science, and to the promotion of scientific excellence. She sends her warm regards to all Abel enthusiasts – and hopes to see you next year.

The history of the Abel Prize

As you may know, the question of having an award in mathematics similar to the Nobel Prize is a century-old one. At the centennial Jubilee in 1902, Abel had already reached the position of national symbol and scinetific hero. The celebration committee had two major suggestions: One was to raise a staue of Abel. The other idea was to establish a scientific prize in his name. Only the statue was realized. The chairman of the celebration committee, Mr. Fridtjof Nansen, deeply regretted that the prize had to be abandoned. One of his letters ends as follows:

… unfortunately we have only one Abel; the opportunity will not come again for 100 years.

Nansen was right. The second part of the story started in the year 2000. The new Abel biography (*Called too soon by flames afar*) brought new life to the idea of the Abel prize. This time the idea resulted in a letter to the Ministry of Education and Research, proposing the establishment of a state fund to finance a scientific prize in the name of Abel.

The proposal was backed by numerous declarations of support by Norwegian scientists, from the international mathematical community and from several leading politicians.

With the establishment of the Niels Henrik Abel Memorial Fund in January 2002 a long process was finally brought to a successful end!

The main objectives of the Prize

The main object of establishing the Prize is to honour outstanding scientific work in the field of mathematics. Mathematics has been vital to the development of civilisation. From ancient to modern times it has been fundamental to advances in science, engineering and philosophy. And the application of mathematics in other fields of science keeps expanding.

I therefore believe that there is every reason to introduce an annual prize in mathematics, corresponding to the Nobel Prizes in other areas.

Another important objective is to raise the status of mathematics in society. Above all we need to stimulate the interest of children and young people. The lack of interest in mathematics and the natural sciences is a problem in many countries. Considering the importance of mathematics to virtually all sectors of society, this is a trend we should all try to reverse. I truly believe the Abel Prize may contribute to this end.

Finally, the prize aims to commemorate Norway's most eminent scientist ever. Abel died in 1829, at only 26 years of age. His short life and tragic death have given birth to a number of myths around his person. Some have characterised him as the Mozart of science. Others have even called him 'the James Dean of mathematics'.

But there was nothing glamorous about the life and career of Niels Henrik Abel. He had to rely on short-term loans and scholarships. And his well-deserved professorship in Berlin was not announced until right after his death.

Norway at the time of Abel was a young nation with a small scientific community. The heritage of Abel has been of tremendous importance for future generations. Abel gave our nation scientific traditions and self-confidence. And he formulated new questions that mathematicians are still addressing today.

The establishment of the Abel Prize is a way of saying that we are depply indebted to the work of Niels Henrik Abel. It is somewhat ironic that Abel died with an unsettled personal loan in the central bank of Norway. Now, two centuries later, we have established a fund in his name in the very same bank. (And if Abel's personal loan still has not been settled, I shall personally see to it that it is!)

The Niels Henrik Abel Memorial Fund has an initial capital of 200 million Norwegain kroner (approximately 27 million euro). The annual return on this capital shall be used for three main purposes:

- the award to the Abel laureate, similar to the Nobel Prize in monetary value
- a ceremony in conjunction with the presentation of the prize
- and events targeting children and young people.

Procedures for the Abel Prize

The Abel Prize will be awarded by the Norwegian Academy of Science and Letters on an annual basis, starting from the year 2003.

The Academy will appoint an Abel Board which will be responsible for organising events in conjunction with the presentation of the Prize and for achieving the others objects of the Prize.

In addition the Academy will appoint an Abel Committee. The Abel Committee will consist of five outstanding research scientists in the field of mathematics. The Committee will be responsible for nominating prize candidates and submitting a recommendation to the Norwegian Academy. Both Norwegian and non-Norwegian citizens may be members of the committee.

Concluding remarks

At the beginning of the last century (in 1900), the mathematician David Hilbert concluded his famous speech to the International Congress of Mathematicians by expressing the following wish:

'May the 20th century bring mathematics gifted masters and many enthusiastic disciples.'

Today, at the beginning of a new century, Hilbert's wish deserves to be repeated. The need for mathematical competence is in fact greater than ever.

Our hope is that the Abel Prize will be an inspiration to scientists and students in all parts of the world, and thereby stimulate new gifted masters and enthusiastic disciples.

I thank you for your attention, and we look forward to welcoming you back for the first Abel award ceremony in 2003.

International Mathematical Union endorses 'Best Practices' in Electronic Scholarly Publishing

The Executive of the International Mathematical Union has endorsed a broad ranging set of recommendations on **Electronic Information Communication**. These recommendations, written by its Committee on Electronic Information and Communication (CEIC, www.ceic.math.ca), suggest ways in which mathematicians, librarians and publishers can help shape the future of scholarly communication. The common principle used to formulate recommendations is that those who write, disseminate and store mathematical literature should act in ways that serve the interests of mathematics, first and foremost.

The 15 'best practices' touch on almost every area of scholarly electronic publication, and include such things as:

- suggestions to authors to version their electronic preprints, to post them on publicly available servers, and to become knowledgeable about copyright;
- advice to librarians to make decisions based on journal price and policy, to be alert to the distinction between posted and refereed papers, and to use web statistics with care;
- encouragement to publishers to provide key journal information (abstracts and reference lists) without subscription, to make entire articles similarly available after a suitable period of time, and to archive material in formats that have open standards.

While the recommendations are aimed at the mathematical community, almost all apply to other scholarly disciplines as well.

The advice is meant to ease the transition in scholarly communication both for present scholars and for future generations.

The 15 specific recommendations will be updated in the future, and more detailed information will be added for each. The full text is available at www.ceic.math.ca/Best-Practices.pdf or www.cms.math.ca/bulletins/Best-Practices.pdf

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Gauss Prize

NEW PRIZE IN SCIENCE PROMOTES MATHEMATICS AS A KEY TECHNOLOGY

Mathematics is an important and ancient discipline – no-one doubts that. However, it seems that only the experts know that mathematics is a driving force behind many modern technologies. The *Gauss Prize* has been created to help the rest of the world realise this fundamental fact. The prize is to honour scientists whose mathematical research has had an impact outside mathematics – either in technology, in business, or simply in people's everyday lives.

The Gauss Prize is awarded jointly by the *Deutsche Mathematiker-Vereinigung (DMV = German Mathematical Union)* and the *International Mathematical Union (IMU)*, and administered by the DMV. The prize consists of a medal and a monetary award (currently valued at 10000 euro). The source of the prize is the surplus from the *International Congress of Mathematicians (ICM'98)* held in Berlin.

The official announcement of the establishment of the prize took place on 30 April, the 225th anniversary of the birth of Carl Friedrich Gauss, after whom the award is named. The prize is to be awarded every four years, at the *International Congress of Mathematicians*, with the first award to be presented at the Congress in 2006. The laureates will be chosen by a jury selected by the IMU.

Carl Friedrich Gauss (1777-1855) was one of the greatest mathematicians of all time. He combined scientific theory and practice like no other before him, or since, and even as a young man Gauss made extraordinary contributions to mathematics. His *Disquisitiones arithmeticae*, published in 1801, stands to this day as a true masterpiece of scientific investigation. In the same year, Gauss gained fame in wider circles for his prediction, using very few observations, of when and where the asteroid *Ceres* would next appear. The method of least squares, developed by Gauss as an aid in his mapping of the state of Hannover, is still an indispensable tool for analysing data. His sextant is pictured on the last series of German 10-Mark notes, honouring his considerable contributions to surveying; there, one also finds a bell curve, which is the graphical representation of the Gaussian normal distribution in probability. Together with Wilhelm Weber, Gauss invented the first electric telegraph. In recognition of his contributions to the theory of electromagnetism, the international unit of magnetic induction is the *gauss*.

The IMU has been awarding the Fields Medals – generally considered as the 'Nobel Prize for mathematics' – for fundamental contributions to mathematics since 1936, and the Nevanlinna Prize for outstanding work in the fields of theoretical computer science since 1982. The Nevanlinna Prize and up to four Fields Medals are awarded every four years at the opening of ceremony of the International Congress of Mathematicians. The Gauss Prize will be awarded in the same manner.

With the Gauss Prize, the IMU is broadening the range of its awards, now including the influence of mathematics to other disciplines. The award ceremony will include an overview of the achievements of the prize-winner. The presentation of the mathematical work will be addressed to the general public as well as journalists, so that all may appreciate the importance of mathematics for everyday life.

The statutes of the Gauss Prize can be found at: http://www.mathematik.uni-bielefeld.de/DMV/Gauss/

2002 MSJ Spring Prize

The 2002 Spring Prize of the Mathematical Society of Japan (MSJ) was awarded at the Annual Meeting of the MSJ in Tokyo in March 2002.

The Spring Prize is awarded each year to a mathematician who is not older than forty years old and who has made an outstanding contribution to mathematics.

The 2002 Spring Prize was awarded to Yasuyuki Kawahigashi of Tokyo University for his distinguished contributions to the study on operator algebras.

Yasuyuki Kawahigashi was born in Tokyo in 1962. He received his B.Sc. degree in 1985 from Tokyo University, and his Ph.D. in 1989 from the University of California, Los Angeles, under the direction of M. Takesaki. He also received a Doctor of Science degree in 1990 from Tokyo University. Kawahigashi's main research interests are in operator algebras – in particular, in subfactor theory. His main contribution is the introduction of a new method, called the orbifold construction for subfactors. This method constructs the subfactors corresponding to Coxeter graphs of type D, and gives a part of the complete classification of subfactors with index less than 4, for example. Kawahigashi worked with D.E. Evans to establish the orbifold method for more general subfactors.

As with the introduction of the Jones polynomials for knots, the study of factors deepens the relationship between topology and mathematical physics. By combining analytical and combinatorial methods, Kawahigashi obtained relations between subfactors, topological field theory, and topological invariants for 3-dimensional manifolds. Recently, he has been interested in the relationship between conformal field theory and subfactor theory, and has introduced a new setting for algebraic quantum field theory and clarified the meaning of the modular invariant partition functions from the subfactor viewpoint.

In recognition and support of his contribution to the study of operator algebras, the MSJ presents the 2002 Spring Prize to Yasuyuki Kawahigashi.

Adrien-Marie Legendre

(1752-1833)

JEREMY GRAY

Of all the great mathematicians whose careers straddled the French Revolution, Adrien-Marie Legendre, whose 250th anniversary falls this year, has become the least well known. Unlike Laplace or Lagrange, he has no collected edition of his works (although rumours circulate that something could be done). His achievements are sometimes forgotten, and sometimes held under a cloud. He invented the method of least squares but after Gauss. He attempted to prove the parallel postulate – unsuccessfully, of course, even foolishly. He offered a proof of the theorem of quadratic reciprocity vitiated by a vicious circle. He knew a lot about elliptic integrals, but missed the prize, and the whole topic of elliptic functions was discovered by Abel and Jacobi instead. Yet other mathematicians make mistakes, and even the best see their best results pass into general theories as 'mere' special cases. What was once difficult to discover becomes, in the end, something not too difficult to learn. Legendre's fate is no different from many a major mathematician's, but has been spun rather differently, and perhaps unfairly.

Legendre was born in Paris on 18 September 1752. He was well educated at the Collège Mazarin, and a modest amount of family money allowed him to devote himself mostly to research. He came to the notice of the scientific community at the age of 30, when he won the prize of the Berlin Academy for an essay on ballistics. His career then prospered until the French Revolution wiped out his private income, and he was forced into the risky business of forging a scientific livelihood.

It was to be the educational changes initiated by the Revolution that saved him. These brought into being what eventually became the École Polytechnique and the École Normale Supérieure, and in their wake an immense demand for books to be studied by would-be entrants to French higher education. Moreover, the new system set much more store by mathematics than science, seeing in it the universal education of the citizen. Legendre published the first edition of his *Éléments de Géométrie* for this audience in 1794, and in its numerous subsequent editions it became the dominant French textbook for this market throughout the nineteenth century (the 21st edition came out in 1876). Legendre took the opportunity to move the presentation of geometry back towards its Euclidean roots, and away from the more intuitive style of his predecessor, Clairaut. This led him directly into his unfortunate encounter with the parallel postulate. Legendre naturally believed that the parallel postulate was true, but finding it unpalatable to do what Euclid had done, which was to assume it, he sought instead to prove it. Inevitably, his proofs all failed, and various editions carry various unsuccessful attempts.

In 1798 Legendre published the first edition of his *Essai sur la Théorie des Nombres*. This was expanded in the second edition (1808) and again in the third edition of 1830, which came out in two volumes. Legendre began his account with the method of continued fractions, and then dealt with the theory of quadratic forms. He obtained significant new results in the theory of ternary quadratic forms. He showed, for example, that every odd number of the form $8n + 7$ is a sum of three squares. He had offered a fallacious proof of the theorem of quadratic reciprocity in a paper published in 1785, but this had been criticised by Gauss in the *Disquisitiones Arithmeticae*, and Legendre adopted Gauss's proof in the book of 1808. The 1785 attempt had contained the claim that every arithmetic progression $an + b$, where a and b are relatively prime, has infinitely many primes. This was, of course, not to be proved rigorously until Dirichlet found a proof in 1837.

Legendre had a more painful run-in with Gauss the next year, 1809, when Gauss published his *Theoria motus corporum coelestium*, for there Gauss claimed that the method of least squares, which Legendre had published as his own in his *Nouvelles méthodes pour la détermination des orbites des comètes* in 1806, was his own discovery. The evidence does seem to be that Gauss was right, but typically at the time one only had Gauss's word for it, and the evidence was in unpublished material dating from 1799. Legendre was never convinced of Gauss's priority in the matter, and this contributed to his stout defence of Jacobi's originality in 1827, when he learned that Gauss was claiming to know much of what Jacobi had recently discovered about elliptic integrals (Gauss was again correct, and again referring to unpublished work).

The topic of elliptic integrals was the one nearest to Legendre's heart, the one he wrote most about, and the one where he saw himself as a direct descendant of Euler, whose picture he placed at the front of the first volume of his book *Exercises de calcul intégral* (3 vols., 1811-17). His interest in the topic came after the work of Euler, Fagnano and others, and did much more than keep it alive until it was transformed by Abel and Jacobi. It can be seen as a spirited attempt to enlarge the domain of elementary functions by creating a new theory of the functions defined by elliptic integrals, the theory of complete elliptic integrals as functions of the modulus, as well as the theory of Eulerian beta functions. It contains the nowadays familiar reduction of elliptic integrals to three canonical forms, and not only does it describe the differential equations that the complete elliptic integrals satisfy, and give power series expansions for them, it shows how tables of values are to be computed for them and goes on to compute the tables. It also shows at considerable length how these functions solve problems in mechanics.

Among the applied problems that lead naturally to a formulation in terms of elliptic integrals are, Legendre showed in his *Exercises* [1817], the rotation of a solid about a fixed point; the motion either in plane or space of a body attracted to two fixed bodies; and the attraction due to an homogeneous ellipsoid. In his *Traité* [1827] he added four more examples: motion under central forces, the surface area of oblique cones, the surface area of ellipsoids, and the problem of determining geodesics on an ellipsoid. Rather than see all this work as a monumental failure to discover elliptic function theory, it might be more accurate to see it as the creation of a new theory of real functions going all the way from an abstract delineation of the topic to its applications. Indeed, it might well be that the very richness and applicability of the subject obscured the topic we see as central, that of elliptic functions. The functions defined (by the upper end-points of) elliptic integrals are analogous to the arc-sine function, and share its problems. But the inversion that leads to elliptic functions requires that the functions be treated as functions of a complex variable, a considerable extra step to take, and one that leads away from applications. Ironically, of all the major mathematicians of the period, it was Legendre who leant most support to the more minor figures, such as the Français brothers, when they filled up the pages of Gergonne's *Annales* with papers on the nature of complex numbers.

Still, Legendre was very generous in his response to the work of Jacobi, who cultivated him assiduously in letters, and Abel, with whom he corresponded more briefly. He pushed for them to receive the prize of the Académie des Sciences for their work in 1830, by which time Abel had died, and in this way gave his own cherished subject a new lease of life in a form which he surely saw wholly subsumed his own achievements. Abel had met Legendre in Paris in 1826, and described him then (with all the confidence of youth) as hoary with age and as 'old as the stones', but Legendre outlived him and died on 9 January 1833 after a painful illness.

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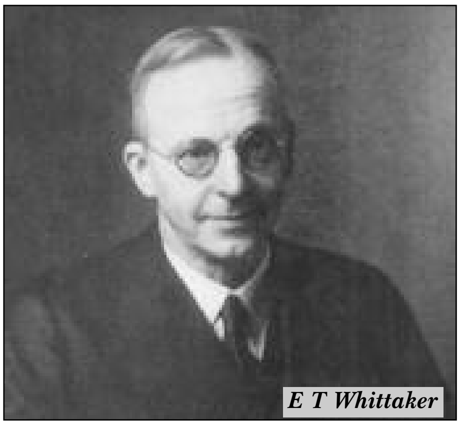
Whittaker and Watson's

'Modern Analysis'

JUNE BARROW-GREEN



E T Whittaker

G N Watson

One of the most enduring of English mathematics textbooks of the twentieth century is Whittaker and Watson's textbook on the theory of functions of a complex variable, *Modern Analysis* – or, to give it its full title, *A Course of Modern Analysis: An Introduction to the General Theory of Infinite Series and of Analytic Functions; With an Account of the Principal Transcendental Functions*.

But *Modern Analysis* did not start out as a joint enterprise: it began life in 1902 under the sole authorship of Edmund Taylor Whittaker (1873-1956), then a lecturer at the University of Cambridge. It was not until some years later that Whittaker joined forces with his former student (George) Neville Watson (1886-1965), and in 1915 a new revised and enlarged edition was published. The book has since been through two further editions, 1920 and 1927, and numerous reprintings, the latest as recently as 1996. For many years it was virtually the only book in English to give an introductory account of the methods and processes of analysis and of the special functions used in mathematical physics. Indeed, such was the paucity of books on the latter that at the time of Whittaker's death, more than 50 years after the first edition was published, the book's discussion of transcendental functions was considered to be 'the only collective account in any language of the ground covered'[1]. That is not to say that specialised accounts of some of the individual functions did not exist by that time – Watson's own treatise on *Bessel Functions* (1922) provides a good example – but there was nothing else by way of a general introduction to the subject.

The book originated, as so many textbooks do, out of a lecture course. In 1896 Whittaker, who had graduated Second wrangler in the mathematical tripos the previous year, was elected to a fellowship of Trinity College, Cambridge, and upon his election he immediately began to lecture. Amongst the courses he gave was one on analysis, or complex function theory, and it was completely new: no-one before him had introduced anything like it into tripos mathematics. Although A. R. Forsyth's *Theory of Functions* had been published in 1895, Forsyth lectured on it only to postgraduates, and the reality was that the general theory of functions as developed by Cauchy, Weierstrass, etc. was barely known in Britain at the time. Indeed, when Whittaker himself was an undergraduate, such was the novelty of the subject that he used to hear Cambridge mathematicians speaking of 'Cocky's' theorem!

Many distinguished mathematicians cut their teeth on Whittaker's course, and his students included such luminaries as G. H. Hardy and J. E. Littlewood, as well as A.S. Eddington, J. H. Jeans, H. W. Turnbull and G. N. Watson. According to Turnbull, Whittaker 'began [his course], as always as far as I know, by introducing broad general ideas, here integration of rational functions, then connection with algebraic curves, with unicursal curves in particular, and leading steadily by pleasant advances to complex integration and the theory of residues. Then came differential equations…"[2]. Since the students had no texts to consult, it was natural that Whittaker himself should fill the gap, and when *Modern Analysis* was published it became the textbook for the course and rapidly made a name for itself. Much of the credit for getting the book to the publishers was due to Whittaker's wife. Not mathematically trained (and only very recently married!), she familiarised herself with the intricacies of mathematical symbolism in order to transcribe a fair copy for the press. And Whittaker's wife was not the only member of his family to get involved with the book. In the preface to the fourth edition, E.T. Copson (1901-80), Whittaker's son-in-law, was thanked for the trouble he had taken to supply the authors with a 'somewhat lengthy' list of errors and misprints.

In 1906 Whittaker left Cambridge to take up the chair of astronomy in the University of Dublin, a post which carried with it the title of Royal Astronomer of Ireland. The move to astronomy was not as surprising as it might seem. Whittaker had a consuming interest in the development of mathematical physics, and while at Cambridge he had lectured on geometrical optics, electricity and magnetism, analytical dynamics, the three-body problem, and astronomy. Nevertheless, as he himself recognised, his own talents lay not in developing these subjects but in producing the sort of mathematics that was needed for their development. In 1912 he succeeded George Chrystal (1851-1911) as professor of mathematics at the University of Edinburgh and he retained the position until he retired in 1946.

Watson's career followed a not dissimilar trajectory to that of Whittaker. A scholar to Trinity, in 1907 he graduated as Senior wrangler and in 1910 became a Fellow of Trinity College. In 1914 he moved to University College London (UCL), where he remained until 1918. During the War he worked in Karl Pearson's computing laboratory at UCL, although it seems that Pearson did not find Watson's manner altogether agreeable[3]. In 1918 Watson became professor of mathematics at Birmingham University, and stayed at Birmingham until 1951 when he retired.

Thus the paths of Whittaker and Watson crossed at Cambridge for only a couple of years, while Whittaker was a junior Fellow and Watson was an undergraduate. And although Whittaker's course must have made an impression upon him, it seems that, initially at least, Watson was more influenced by another lecturer with an interest in complex variable theory, E.W. Barnes (1874-1953). The topics of Watson's early research, finite difference equations, hypergeometric functions, asymptotic expansions, all formed part of Barnes' canon. Of Watson's early work, only his very first paper, 'On the general solution of Laplace's equation in n dimensions' (1906), had any direct connection with the research of Whittaker. Nevertheless, during the early years of the twentieth century, Whittaker was responsible for much of the advance in complex function theory research, so it is not surprising that Watson was drawn to him. The two became good friends and it was not long before Watson offered to share in the preparation of a second edition of Whittaker's book.

As can be seen from the *Table*, the second edition held the same shape as the original, but it contained substantial additions. Watson was responsible for the inclusion of new chapters on Riemann integration, integral equations and the Riemann zeta function. For the third edition a further chapter on 'Ellipsoidal Harmonics and Lamé's Equations' was added. Other additions and improvements to the second and subsequent editions include references to primary sources at the end of each chapter, an appendix on the elementary transcendental functions, a list of authors quoted, and a much improved index – added to which the existing chapters were also largely rewritten, with the extent of the rewriting being apparent from the very first page. The new edition not only provided a more comprehensive survey of the subject – it was almost 50% longer than the original – but the treatment was altogether more rigorous and the style more formal. However, the fresh look was initially not to everyone's taste. It seems that some mathematicians, although by no means all, continued to prefer the original edition with its rather freer style and motivation towards more general results. Nevertheless, the new edition was popular and, in particular, the second part of the book, in which the authors defined the special functions by contour integrals rather than by differential equations, was found to be very useful. And it really came into its own in 1925 and 1926 when wave mechanics was being developed and solutions to the Schrödinger wave equation were being sought for various special problems.

One of the most prominent features of the book is its attention to the historical record. To find a route through the genesis and development of the subject, you have only to look at the footnotes and the list of references given at the end of each chapter. The references also provide a guide to the extent to which, in the second and later editions, Whittaker's and Watson's own original research formed the basis for topics in the second half of the book. For example, the chapter on the *confluent hypergeometric function*, which first appeared in the second edition, grew out of a paper published by Whittaker in 1903 in which he had shown that this function was the general case of several other special functions previously introduced by various authors[4]. (That the London Mathematical Society rejected the paper, later considered to be 'the most fruitful generalisation of the theory of special functions of the past century'[5], so that it was published in America, gives an indication of the general state of knowledge on the subject in Britain at the time.) The same chapter also contains a discussion of the *parabolic cylinder functions*, a topic that Whittaker also first addressed in a paper of 1903[6], and one that Watson considered in papers of 1910 and 1919[7]. The subject of another chapter, *Mathieu Functions*, was originally dealt with by Whittaker in a paper presented at the International Congress of Mathematicians in Cambridge in 1912, on which occasion he took the opportunity to name the functions in honour of their discoverer[8]. And, of course, Watson's work on Bessel functions, which culminated in his eponymous treatise on the subject, informed the chapter of the same name in the book.

During the 1950s Watson felt that much of the book had become outdated, and in his retirement he set to work on an extensive revision of the entire text. But the project was never finished. By the time of his death he had mapped out the contents of fifteen chapters, numbering the first five: I Introduction, II Natural Numbers, III Fractions and Integers, IV Real numbers (including irrational numbers), and V Complex numbers and higher complex numbers. The remaining chapters cover a diversity of topics such as interpolation, Jordan's and Cauchy's theorems, inequalities, cyclotomy, etc. The manuscript, which is in the archives at Birmingham University, is too sketchy to publish, but it is notable for the completeness of the historical footnotes, which reveal the extraordinary depth of Watson's knowledge of his mathematical heritage. It appears that Watson had planned to rewrite the entire book himself, with the exception of a chapter on automorphic functions which he had marked out for Whittaker to write.

1902	1915	1920 & 1927
Part I:	***The Processes of Analysis***	
I. Complex Numbers	I. Complex Numbers	
II. The Theory of Absolute Convergence	II. The Theory of Convergence	
	III. Continuous Functions and Uniform Convergence	III. Continuous Functions and Absolute Convergence
	IV. The Theory of Riemann Integration	
III. The fundamental properties of Analytic Functions; Taylor's, Laurent's, and Liouville's Theorems	V. The fundamental properties of Analytic Functions; Taylor's, Laurent's, and Liouville's Theorems	
IV. The Uniform Convergence of Infinite Series		
V. The Theory of Residues; application to the evaluation of Real Definite Integrals	VI. The Theory of Residues; application to the evaluation of Definite Integrals	
VI. The expansion of functions in Infinite Series	VII. The expansion of functions in Infinite Series	
VIII. Asymptotic Expansions	VIII. Asymptotic Expansions and Summable Series	
VII. Fourier Series	IX. Fourier Series	IX. Fourier Series and Trigonometric Series
	X. Linear Differential Equations	
	XI. Integral Equations	
Part II:	***The Transcendental Functions***	
IX. The Gamma Function	XII. The Gamma Function	
	XIII. The Zeta Function of Riemann	
XI. Hypergeometric Functions	XIV. The Hypergeometric Function	
X. Legendre Functions	XV. Legendre Functions	
	XVI. The Confluent Hypergeometric Function	
XII. Bessel Functions	XVII. Bessel Functions	
XIII. Applications to the Equations of Mathematical Physics	XVIII. The Equations of Mathematical Physics	
XIV. The Elliptic Function P(z)	XIX. Mathieu Functions	
XV. The Elliptic Functions sn z, cn z, dn z.		
XVI. Elliptic Functions; General Theorems	XX. Elliptic Functions. General Theorems and the Weierstrassian Functions	
	XXI. The Theta Functions	
	XXII. The Jacobian Elliptic Functions	
		XXIII. Ellipsoidal Harmonics and Lamé's Equations

The first two columns contain the Chapter headings for the first and second editions; the third column indicates the additions and/or changes to the Chapter headings between the second and fourth editions.

Given their shared interests and the nature of their joint enterprise, one might have expected Watson to have made several trips to Scotland to visit Whittaker. But in fact he only made two trips north of the border, once in July 1914 to attend the Napier Tercentenary Congress, and once in June 1939 to receive his honorary LL.D from Edinburgh University. Apparently Watson used to say that he feared making a third visit as 'each of his two previous visits had precipitated a major European catastrophe'[9].

Today, *Modern Analysis* is more useful as a book of reference than as a course text. However, as the author of this article will testify, anyone who requires an introduction to the topics it contains, or who has an interest in the history of the development of analysis, will find it an invaluable repository of information.

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1 W.H. McCrea 'Edmund Taylor Whittaker' *Journal of the London Mathematical Society* **32** (1957), 234-256, p.244.

2 W.H. McCrea 'Edmund Taylor Whittaker' *Journal of the London Mathematical Society* **32** (1957), 234-256, p.235.

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Fields Medals and the Nevanlinna Prize

International Congress of Mathematicians

Beijing, China, 2002

The 2002 Fields Medals and the 2002 Nevanlinna Prize were awarded in Beijing, China, on Tuesday 20 August at the International Congress of Mathematicians (ICM). The Fields Medal is the world's highest award for achievement in mathematics. The Nevanlinna Prize is among the most prestigious international awards for achievement in theoretical computer science. The medals are presented at the International Congress of Mathematicians (ICM), held every four years at different locations around the world. Although there is no formal age limit for recipients, the Fields Medals have traditionally been presented to mathematicians not older than 40 years of age, as an encouragement for future achievement. The medals are awarded by the International Mathematical Union, on the advice of a selection committee of top mathematicians from around the world.

The 2002 Fields Medallists are:

LAURENT LAFFORGUE, Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, France: he is recognised for making a major advance in the Langlands Programme, thereby providing new connections between number theory and analysis.

VLADIMIR VOEVODSKY, Institute for Advanced Study, Princeton, New Jersey, USA: he is recognised for developing new cohomology theories for algebraic varieties, thereby providing new insights into number theory and algebraic geometry.

The 2002 Nevanlinna Prizewinner is:

MADHU SUDAN, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA: he is recognised for contributions to probabilistically checkable proofs, to non-approximability of optimisation problems, and to error-correcting codes.

LAURENT LAFFORGUE: FIELDS MEDAL

Laurent Lafforgue has made an enormous advance in the so-called Langlands Programme by proving the global Langlands correspondence for function fields. His work is characterised by formidable technical power, deep insight, and a tenacious, systematic approach.

The Langlands Programme, formulated by Robert P. Langlands for the first time in a famous letter to André Weil in 1967, is a set of far-reaching conjectures that make precise predictions about how certain disparate areas of mathematics might be connected. The influence of the Langlands Programme has grown over the years, with each new advance hailed as an important achievement.

One of the most spectacular confirmations of the Langlands Programme came in the 1990s, when Andrew Wiles's proof of Fermat's Last Theorem, together with work by others, led to the solution of the Taniyama-Shimura-Weil Conjecture. This conjecture states that elliptic curves, which are geometric objects with deep arithmetic properties, have a close relationship to modular forms, which are highly periodic functions that originally emerged in a completely different context in mathematical analysis. The Langlands Programme proposes a web of such relationships connecting Galois representations, which arise in number theory, and automorphic forms, which arise in analysis.

The roots of the Langlands programme are found in one of the deepest results in number theory, the law of quadratic reciprocity, which goes back to the time of Fermat in the 17th century and was first proved by Carl Friedrich Gauss in 1801. An important question that often arises in number theory is whether, upon dividing two prime numbers, the remainder is a perfect square. The law of quadratic reciprocity reveals a remarkable connection between two seemingly unrelated questions involving prime numbers p and q: 'is the remainder of p divided by q a perfect square?' and 'is the remainder of q divided by p a perfect square?' Despite many proofs of this law (Gauss himself produced six different proofs), it remains one of the most mysterious facts in number theory. Other reciprocity laws that apply in more general situations were discovered by Teiji Takagi and by Emil Artin in the 1920s. One of the original motivations behind the Langlands Programme was to provide a complete understanding of reciprocity laws that apply in even more general situations.

The global Langlands correspondence proved by Lafforgue provides this complete understanding in the setting, not of the ordinary numbers, but of more abstract objects called function fields. One can think of a function field as consisting of quotients of polynomials; these quotients can be added, subtracted, multiplied, and divided just like the rational numbers. Lafforgue established, for any given function field, a precise link between the representations of its Galois groups and the automorphic forms associated with the field. He built on work of 1990 Fields Medallist Vladimir Drinfeld, who proved a special case of the Langlands correspondence in the 1970s. Lafforgue was the first to see how Drinfeld's work could be expanded to provide a complete picture of the Langlands correspondence in the function field case.

In the course of this work, Lafforgue invented a new geometric construction that may prove to be important in the future. The influence of these developments is being felt across all of mathematics.

Laurent Lafforgue was born on 6 November 1966 in Antony, France. He graduated from the Ecole Normale Supérieure in Paris (1986). He became an attache de recherche of the Centre National de la Recherche Scientifique (1990) and worked in the Arithmetic and Algebraic Geometry team at the Université de Paris-Sud, where he received his doctorate (1994). In 2000 he was made a permanent professor of mathematics at the Institut des Hautes Etudes Scientifiques in Bures-sur-Yvette, France.

About the work of Lafforgue:

Dana Mackenzie, Fermat's Last Theorem's First Cousin, *Science* **287** (No. 5454), 4 February 2000, pp. 792-3.

VLADIMIR VOEVODSKY: FIELDS MEDAL

Vladimir Voevodsky made one of the most outstanding advances in algebraic geometry in the past few decades by developing new cohomology theories for algebraic varieties. His work is characterised by an ability to handle highly abstract ideas with ease and flexibility and to deploy those ideas in solving quite concrete mathematical problems.

Voevodsky's achievement has its roots in the work of 1966 Fields Medallist Alexandre Grothendieck, a profound and original mathematician who could perceive the deep abstract structures that unite mathematics. Grothendieck realised that there should be objects, which he called 'motives', that are at the root of the unity between two branches of mathematics, number theory and geometry. Grothendieck's ideas have had widespread influence in mathematics and provided inspiration for Voevodsky's work.

The notion of cohomology first arose in topology, which can be loosely described as 'the science of shapes'. Examples of shapes studied are the sphere, the surface of a doughnut, and their higher-dimensional analogues. Topology investigates fundamental properties that do not change when such objects are deformed (but not torn). On a very basic level, cohomology theory provides a way to cut a topological object into easier-to-understand pieces. Cohomology groups encode how the pieces fit together to form the object. There are various ways of making this precise, one of which is called singular cohomology. Generalised cohomology theories extract data about properties of topological objects and encode that information in the language of groups. One of the most important of the generalised cohomology theories, topological K-theory, was developed chiefly by another 1966 Fields Medallist, Michael Atiyah. One remarkable result revealed a strong connection between singular cohomology and topological K-theory.

In algebraic geometry, the main objects of study are algebraic varieties, which are the common solution sets of polynomial equations. Algebraic varieties can be represented by geometric objects like curves or surfaces, but they are far more 'rigid' than the malleable objects of topology, so the cohomology theories developed in the topological setting do not apply here. For about forty years, mathematicians worked hard to develop good cohomology theories for algebraic varieties; the best understood of these was the algebraic version of K-theory. A major advance came when Voevodsky, building on a little-understood idea proposed by Andrei Suslin, created a theory of 'motivic cohomology'. In analogy with the topological setting, there is a strong connection between motivic cohomology and algebraic K-theory. In addition, Voevodsky provided a framework for describing many new cohomology theories for algebraic varieties. His work constitutes a major step toward fulfilling Grothendieck's vision of the unity of mathematics.

One consequence of Voevodsky's work, and one of his most celebrated achievements, is the solution of the Milnor Conjecture, which for three decades was the main outstanding problem in algebraic K-theory. This result has striking consequences in several areas, including Galois cohomology, quadratic forms, and the cohomology of complex algebraic varieties. Voevodsky's work may have a large impact on mathematics in the future by allowing powerful machinery developed in topology to be used for investigating algebraic varieties.

Vladimir Voevodsky was born on 4 June 1966 in Russia. He received his B.S. in mathematics from Moscow State University (1989) and his Ph.D. in mathematics from Harvard University (1992). He held visiting positions at the Institute for Advanced Study, Harvard University, and the Max-Planck-Institut für Mathematik before joining the faculty of Northwestern University in 1996. In 2002 he was named a permanent professor in the School of Mathematics at the Institute for Advanced Study in Princeton, New Jersey.

About the work of Voevodsky:
Allyn Jackson, The Motivation Behind Motivic Cohomology, *What's New in Math*, American Mathematical Society web site *http://www.ams.org/new-in-math/mathnews/motivic.html*

MADHU SUDAN: NEVANLINNA PRIZE

Madhu Sudan has made important contributions to several areas of theoretical computer science, including probabilistically checkable proofs, non-approximability of optimisation problems, and error-correcting codes. His work is characterised by brilliant insights and wide-ranging interests.

Sudan has been a main contributor to the development of the theory of probabilistically checkable proofs. Given a proof of a mathematical statement, the theory provides a way to recast the proof in a form where its fundamental logic is encoded as a sequence of bits that can be stored in a computer. A 'verifier' can, by checking only some of the bits, determine with high probability whether the proof is correct. What is extremely surprising, and quite counter-intuitive, is that the number of bits the verifier needs to examine can be made extremely small. The theory was developed in papers by Sudan, S. Arora, U. Feige, S. Goldwasser, C. Lund, L. Lovász, R. Motwani, S. Safra, and M. Szegedy: for this work, these authors jointly received the 2001 Gödel Prize of the Association for Computing Machinery.

Also together with other researchers, Sudan has made fundamental contributions to understanding the non-approximability of solutions to certain problems. This work connects to the fundamental outstanding question in theoretical computer science: does P equal NP? Roughly, P consists of problems that are 'easy' to solve with current computing methods, while NP is thought to contain problems that are fundamentally harder; the term 'easy' has a technical meaning related to the efficiency of computer algorithms for solving problems. A fundamentally hard problem in NP has the property that a proposed solution is easily checked but that no algorithm is known that will easily produce a solution from scratch. Some NP hard problems require finding an optimal solution to a combinatorial problem such as the following: given a finite collection of finite sets, what is the largest size of a subcollection such that every two sets in the subcollection are disjoint? What Sudan and others showed is that, for many such problems, approximating an optimal solution is just as hard as finding an optimal solution. This result is closely related to the work on probabilistically checkable proofs. Because the problems in question are closely related to many every-day problems in science and technology, this result is of immense practical as well as theoretical significance.

The third area in which Sudan made important contributions is error-correcting codes. These codes play an enormous role in securing the reliability and quality of all kinds of information transmission, from music recorded on CDs to communications over the Internet to satellite transmissions. In any communication channel, there is a certain amount of noise that can introduce errors into the messages being sent. Redundancy is used to eliminate errors due to noise by encoding the message into a larger message. Provided the coded message does not suffer too many errors in transmission, the recipient can recover the original message. Redundancy adds to the cost of transmitting messages, and the art and science of error-correcting codes is to balance redundancy with efficiency. A class of widely used codes is the Reed-Solomon codes (and their variants), which were invented in the 1960s. For 40 years it was assumed that the codes could correct only a certain number of errors. By creating a new decoding algorithm, Sudan demonstrated that the Reed-Solomon codes could correct many more errors than previously thought possible.

Madhu Sudan was born on 12 September 1966, in Madras (now Chennai), India. He received his B.Tech. degree in computer science from the Indian Institute of Technology in New Delhi (1987) and his Ph.D. in computer science at the University of California at Berkeley (1992). He was a research staff member at the IBM Thomas J. Watson Research Center in Yorktown Heights, New York (1992-7). He is currently an associate professor in the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology.

About the work of Sudan:
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FURTHER INFORMATION
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http://www.icm2002.org.cn
International Mathematical Union:
http://www.mathunion.org
Fields Medals and Nevanlinna Prize:
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Women and Men in Mathematics:

Then and Now – Part 2

Andrea E. Abele, Helmut Neunzert, Renate Tobies and Jan Krüsken

[This article is an English translation, by Angela Rast-Margerison, of 'Frauen und Männer in der Mathematik' which appeared in DMV-Mitteilungen 2/2001. It is in two parts: Part 1 appeared in the previous issue.]

Diploma versus Staatsexamen

The comparison between diploma graduates and Staatsexamen graduates is only possible in our modern sample, because the diploma was not introduced until 1942. Today students choose at the beginning of their courses of study whether they will study as diploma students or Staatsexamen students. The aims and contents of the two courses of studies are different.

In our modern sample, the motivation for study was quite different between diploma students and Staatsexamen students, independent of gender. For instance, Staatsexamen graduates rated the compatibility between work and private life as a more important reason, and rated the scientific interest in mathematics as less important, for having chosen mathematics as study major than diploma graduates.

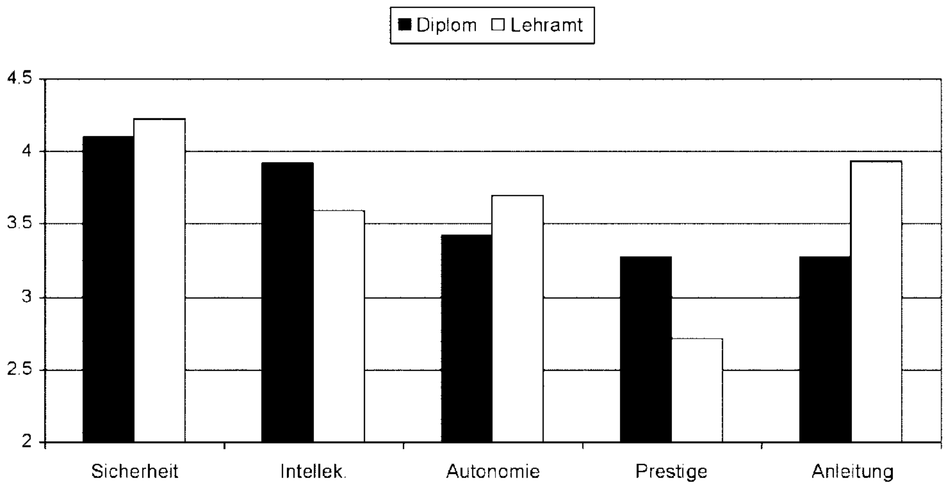
In retrospect, the Staatsexamen graduates assessed their time at university more negatively than the diploma graduates. They felt less well prepared for their future work; they thought that they had been less able to develop their scientific interests; they rated the learning experiences during their studies more negatively; they were generally less interested in mathematical science, and they intended much less frequently to obtain a doctoral degree (9% compared to 30%) than the diploma graduates. All these differences are independent of gender.

Work values also differ between Staatsexamen and diploma graduates. The figure shows how diploma and Staatsexamen graduates rate the importance of five different work value areas: job-security and work environment- oriented values (such as job security and trust among colleagues); intellectual/creative values, (developing new ideas, being creative, working on difficult and challenging issues); autonomy-oriented values (decision competence); prestige-oriented values (prestigious position, high salary) and guidance-oriented values (leading and guiding others). These values were rated on 5-point scales ranging from 'unimportant' (1) to 'very important' (5). As the figure shows, Staatsexamen graduates rate security-oriented values, autonomy-oriented work values and especially guidance-oriented work values as more important than diploma graduates, whereas diploma graduates have higher ratings on intellectual-creative work values and prestige-oriented values.

Women versus Men

Women more often study mathematics in order to become schoolteachers (Staatsexamen), whereas men more often choose diploma studies that lead to career opportunities in other fields.

In the historical sample, this hypothesis was irrelevant, because at the beginning of the 20th century, both women and men studied mathematics mainly to become schoolteachers: there were almost no other occupational fields for them at that time.



The modern sample clearly confirms this hypothesis. For diploma graduates the percentage of women today is about one-quarter (1998: 26.4%), whereas for Staatsexamen graduates it is nearly two-thirds (59.3%). Contrary to popular stereotypes, women *are* interested in mathematics. However, their career plans are directed towards an occupation that allows the combination of work and family, sometimes referred to as 'soft careers' [3]. Such a soft career is more easily possibly as a schoolteacher, because they can work part-time and can also more easily continue their careers after a baby break than in other mathematical fields. These findings replicate previous research we have conducted with university graduates [4]. In that study we also found that women are as interested in science as men. However, if they choose mathematics, physics or chemistry as their study major, they do it with the perspective of becoming a schoolteacher, whereas men do it with the intention of pursuing other careers. The probability is high that, in the near future, there will be more women than men teaching mathematics at secondary schools. How this change might influence girls' and boys' interest development in mathematics is an important question, but not one with which we can deal here.

At the time of exams and at the start of a career, there are (despite popular stereotypes) hardly any differences between women and men.

Both the historical sample and the modern one show that there are no performance differences between men and women in grades achieved, in PhDs passed, or in the intention to pass a PhD. In the modern sample there were only small differences in the topics the graduates had worked on in their diploma theses or wanted to work on in their intended PhD dissertations. (This is similar to the results of a recent study we carried out, analysing all mathematical dissertations defended in Germany from 1907 to 1945.) Apart from a high similarity in performance, there was also a similarity in the degree of self-confidence expressed by women and men in our modern sample. Self-confidence is, in its turn, an important psychological determinant of career development [5]. Finally, the modern data shows that the motivation to pursue a career is equal for women and men: for example, women and men with diplomas in mathematics had the same number and types of professional goals (see [6]).

This high degree of congruence between women and men can be traced back to three influences: *selection*, *socialisation* and *age*. *Selection* means that, as far as interest in mathematics is concerned, the women and men in our sample form a highly selected and homogeneous group: thus, there were no gender differences in the answers given to questions concerning mathematics. *Socialisation* means that the participants have had similar experiences by successfully completing a course of studies in mathematics. This common experience may also have led to similar answers to questions on career decisions. Finally, *age* is important, because at the age of graduation a possible conflict that might arise in women between career wishes and the wish to start a family is not yet acute. Women academics are currently over thirty years old when they have their first child, three years older than the women in our modern sample.

We expect that in later studies with our modern sample the homogenising selection effect will remain the same, while the homogenising socialisation effect will weaken because our participants will have followed different career paths. The age effect will disappear because, up to a certain age, combining career and family will become more important the older a woman becomes.

The present marginal differences between men and women at the time of their final exams will have bigger effects in the future.

At the end of their time at university, the differences between women and men who have studied mathematics are very slight. It is possible, however, that these slight differences will accumulate and eventually lead to more differences in career development between female and male mathematicians.

Regarding our historical sample, the archival records do not include statements on motivation, interests or intentions. However, the above-mentioned reasons for quitting jobs show clear gender differences. Men left for reasons of bad health, or moved up the career ladder (college or university positions or jobs in industry). Women left because they had to choose between career and marriage/family. If women changed their jobs, then they did not move up, but rather left to teach at public schools. (There are a few isolated examples of women continuing to work when they had children, where the husband was not employed in the civil service: one such example was Frieda Nugel-Hahn (1884-1966), the first women to obtain a degree in mathematics from the University of Halle, and mother of four [7].)

In the modern sample we find a few hints on slight gender differences, that all point into the same direction. For instance, in retrospect, women judge their time at university in a somewhat more negative light than men. They rated opportunities for interest development during studies somewhat more negatively and felt somewhat less well prepared for their occupational life than men. Women were less interested in 'career goals' in a narrow sense, but were more interested in learning goals and intrinsic aspects of their work than men (see [6]). This means that, on average, the women in our modern sample were less career-oriented in a strict sense than the men. This could be one reason for different career paths in the future (see [1, 2]).

A further aspect is the higher willingness of the women to do 'family work'. In our modern sample, 63% women say they are prepared to reduce their career engagements if they have a child, while only 27% of the men say they would be prepared to do so. This effect – fewer career-oriented women and a greater willingness to fulfil family obligations – is, not specific to mathematics, however, but applies to students of all other subjects, too (see [1, 2]). With regard to an academic career, women were slightly less interested than the men in 'doing science'. When asked about future plans, they mentioned a dissertation slightly less often than men (women 23%, men 27%). However, once the decision was taken, there were no differences between men and women in motivation, reasons and the plans for a dissertation.

Will there be more than 8% women in top mathematics positions in the year 2015?

We are optimistic that the number of women professors of mathematics will rise in the near future. Women who study mathematics have the potential – and had it a hundred years ago – to pursue successful careers; they are certainly capable of doing so.

Compared with the strong discrimination against working women in earlier times, such as the celibacy law for civil servants that forced women to give up their work if they wanted to marry, the generally lower salaries of women civil servants compared with men civil servants, and the regulations on the proportions of female and male teachers in girls' secondary schools – women today have a far better chance to succeed in a career *and* have a private life: of course, they are allowed to do so. Whether barriers exist today will be shown in a later phase of our project. Even though the combination of career and family has become much easier today, there are still barriers and prejudices against working mothers.

The question of whether women want to pursue top careers in mathematics may be answered with a 'yes, but ...'. The 'yes' is supported by the high professional motivation with which young women finish university and their rapid integration into the workforce; there is no difference from men. The 'but' arises from the following data. On the one hand, women's stronger orientation towards the 'Staatsexamen' and teaching, and thus their weaker orientation towards science, means that there will not be a rapid rise in the number of female university professors in mathematics. On the other hand, with regard to women concluding their studies with a diploma, there are a number of small differences which, if accumulated, could prevent the number of women in mathematical top positions from increasing by much. These are the problems of combining career and family which are still more pronounced for women than for men; the somewhat lower emphasis that women place on a PhD and on career prospects in a narrow sense, and also their somewhat lower interest in doing mathematical science.

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Heinz Hopf Lecture at the ETH Zürich

Urs Stammbach

On 4 December 2001 the first Heinz Hopf Lecture at the ETH Zürich took place. The speaker was Don B. Zagier, Max Planck Institute for Mathematics, Bonn, and Collège de France, Paris; he spoke on *Diophantische L-Werte*.

Heinz Hopf was professor at ETH Zürich from 1931 until 1965, when he retired. During that time he established a very successful school in algebraic topology; among the many students he led to a Ph.D. at ETH are J. J. Stoker, E. Stiefel, B. Eckmann, H. Samelson, W. Gysin, E. Specker, M. Kervaire, K. Voss. In the years after the war, F. Hirzebruch, J. Tits, L. Nirenberg, J. Milnor came to Zürich to work with Heinz Hopf. His international recognition and esteem reached the highest level in 1955 when he was elected president of the International Mathematical Union.

In 2000 the ETH Zürich received a donation from Alfred Aeppli and his wife Dorothee to institute a regularly organised Heinz Hopf Lecture. Alfred Aeppli, Professor Emeritus at the University of Minnesota, was a student of Beno Eckmann at the ETH, and hence is a 'mathematical grandson' of Heinz Hopf. In his letter Alfred Aeppli wrote:

To me, it is a great idea to gather from time to time in memory of Heinz Hopf, at the Swiss Federal Institute of Technology in Zürich, for some special lecture on current mathematical research. We remember Heinz Hopf as one of our finest professors at ETH. He will always be an inspiration to many of us in mathematics and in mathematics education at the university level. He was a mind at work who thrived in the world of disciplined curiosity full of intellectual challenges. He was a mathematical thinker who taught us to think.

Heinz Hopf demonstrated doing mathematics in talks, lecture courses, discussions and in his publications. I remember in the introductory linear algebra course in 1947/8, the impression Heinz Hopf made on us students was outstanding and unforgettable. A student, by attending a course by Hopf, received a good answer to the question 'What is mathematics?' The notions had to be clear, the intellectual tools of the highest calibre, the arguments compelling and impeccable.

The first of the Heinz Hopf Lectures took place on 4 December 2001. In a splendid talk Don Zagier presented new spectacular results from number theory, about *L*-functions and elliptic curves. In the evening a formal dinner in the Restaurant Sonnenberg was held. On that occasion Beno Eckmann, Hopf's colleague over many years at the ETH, and his close friend, gave a speech. It is a moving tribute to this eminent mathematician. We reproduce it here in an English translation.

After-dinner speech by Beno Eckmann on the occasion of the Heinz Hopf Lecture 2001 at the ETH Zürich

Our guest of honour Don Zagier was twenty years old, when he spent the first year after his PhD at the Forschungsinstitut of the ETH in Zürich. That was 29 years ago. Heinz Hopf died 30 years ago and so Heinz Hopf and Don Zagier never met.

With the passing away of each human being a mystery disappears from the world, a mystery that nobody else will ever be able to rediscover (Friedrich Hebbel).

Unfortunately only very few people remain who ever met Heinz Hopf personally. Both mathematicians and theoretical physicists speak frequently about Hopf algebras, about Hopf fibrations, and about other concepts that carry his name. Moreover, a great part of the mathematics which was developed in the second half of the last century reflects ideas of Heinz Hopf, far beyond the borders of his own field, and in circumstances which he could not have foreseen: I refer particularly to his very characteristic way of looking at the relationship between the concrete and the abstract.

From 1931 until 1965 Heinz Hopf's centre of activity was the ETH in Zürich. One has to remember how difficult communication was in those years: it became more and more difficult before and during the war, and only after the war was it again possible to write and receive letters. Telephone calls were so expensive, nobody ever really thought of using them for ordinary things! Under these conditions it is astonishing how Heinz Hopf's name and work were already then receiving such strong international attention and recognition. After the war he received and accepted invitations to make long visits to Princeton and Stanford; and in 1955 he became President of the International Mathematical Union. In principle reluctant to exercise power, his personality made it possible, only a few years after a deadly and terrible war, to form a worldwide community of mathematicians free from any political restrictions.

I believe that these achievements were possible not solely thanks to his mathematics, great as it was, but that they were also in no small part due to his whole personality.

What was his secret, the secret of success of this small, modest, almost inconspicuous man with his impeccable manners? It is easy to list a few attributes: kind, cordial, open, and easy going, but they are somehow inadequate. Perhaps it gives a clearer picture if one says that everybody liked him. He carried with him an aura of human warmth, his judgements on other people and other people's work were completely fair and objective, and – this is very important – he had a great sense of humour.

I have used the word 'modest', and in fact this word occurs whenever one speaks about Heinz Hopf. Yes, his style of living was modest, and one should remember that the salary of a professor, up to the time of his retirement, was indeed modest, too. He was free from conceit, arrogance and condescension. But nevertheless he was well aware of the value and power of his own mathematical ideas. His judgement was fair and absolutely objective; it was thus easy for him to recognise and to admire the work of others. In his presence one felt relaxed and surrounded by an atmosphere of calm and friendship. With one small word of appreciation, or with a slight expression of doubt he could give discussions and ideas a new direction; and this applies both to mathematics and to private matters.

He was a citizen of the world, in the best meaning of the phrase. However, in Zürich, in Zollikon where he lived, and at the ETH he felt at home. He was incredibly helpful: he helped refugees before, during and after the war, he helped children who had suffered during the war, he helped friends who were in need, and he did all this almost beyond his own economic capacities.

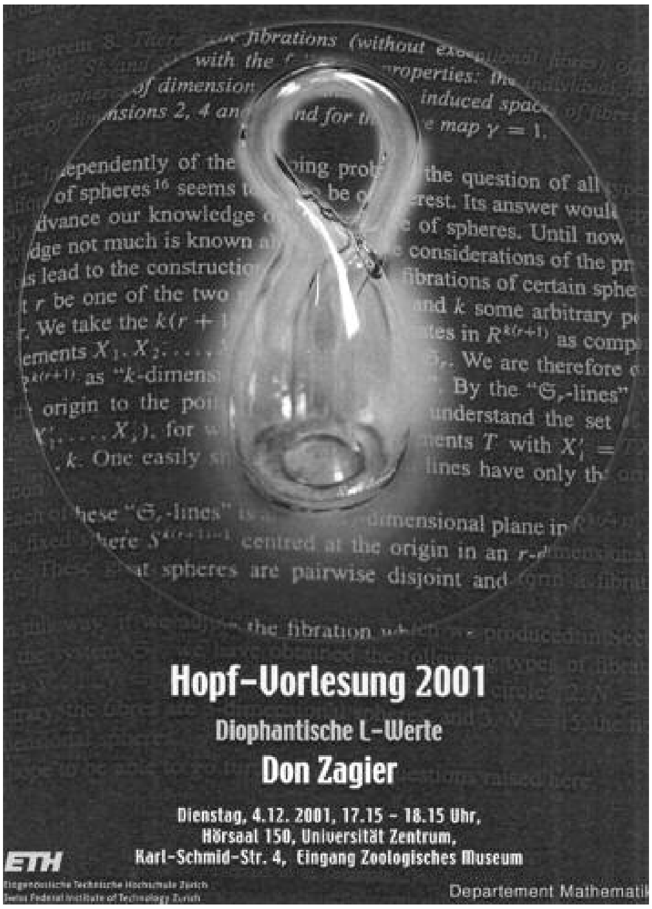
These are just words, and words are clearly inadequate to describe the personality of Heinz Hopf: the mystery of Heinz Hopf remains.

One could easily relate numerous stories about Heinz Hopf; as you know, I was his student, his assistant, his colleague and his friend. We talked not just about mathematics – and about mathematicians, we also talked about 'God and the world', about Thomas Mann, Christian Morgenstern, Rainer Maria Rilke, etc. But this goes beyond the limits of a dinner speech.

Let me instead tell you a story that came to my mind during my work editing Heinz Hopf's *Collected Papers*. In 1947, at a relatively early age, he received an honorary degree from Princeton University. My wife and I joined him there only a short time after the ceremony. In his humorous way Hopf told us that he had always believed that an honorary degree from Princeton would be something full of dignity. But this could not possibly be true, he said, with the cap and gown he had to wear: the gown was much too long for him, so that he was always in danger of tripping on it. After the ceremony a whole crowd of journalists had arrived, and he confessed that he had thought: 'Am I really that famous?' Only a moment later he realised that all these journalists did not come because of him, but because of James Stewart who had received an honorary degree at the same time. In later years he expressed the opinion that an honorary degree is simply a reward for not publishing more papers.

He and Wolfgang Pauli often took walks in the woods together. Once he commented: 'Today we had a heated discussion as to why man was created, to do Pure Mathematics or to do Applied Mathematics. However we did not resolve the problem.'

After the war he was invited to Cambridge, and after his return we asked him whether he had had success. 'Not really', he said. At the meal at Trinity College he was treated as guest of honour and accordingly he was seated right next to the Master of Trinity, who at that time was a famous Nobel prize winner. Trying to make conversation the Master had remarked to Hopf: 'Nice weather today, isn't it?' And Hopf had answered: 'No, not nice.' In retrospect, and judging from the reaction of the Master, he thought that this must have been a tactless reply, even though it really had been raining the whole day. After this first incident the Master had asked him where he was from. 'From Zürich'. 'From which University?' 'Not a University, from the Swiss Technical High School.' And that – Hopf said – had been the end of the conversation for the whole evening. Evidently 'High School' and even 'technical', that was too much.



Hopf often asked his students and his colleagues the question: 'Suppose that you are offered the solution of all mathematical problems as a gift, but under the condition that you don't tell anybody. Would you accept that gift?' For him the answer was absolutely clear: 'No, never!' For Heinz Hopf, mathematics involved interaction with other people and joint efforts in thinking and working. Knowing the solution was only the end but not the main part of doing mathematics.

We are very happy that the memory of Heinz Hopf is honoured in such a nice and appropriate way, with the Heinz Hopf Lectures. Perhaps this can help to preserve in our mathematical community the harmonious atmosphere that Heinz Hopf was able to establish during his lifetime.

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Thanks are due to Peter J. Hilton for generous help in translating the German text of Beno Eckmann's speech.

Marian Rejewski, Polish cryptologist

6 December 2001

(sent by Krzysztof Ciesielski)

Marian Rejewski 1905-1980

A few months ago, a special memorial plaque was unveiled at the tomb of the Polish mathematician Marian Rejewski. The event was organised by the Polish Mathematical Society, and the plaque was unveiled by Bolesław Szafirski, President of the Society. Rejewski made great advances with the breaking of codes (especially the German 'Enigma' code), which helped a lot with the fight against Hitler during the Second World War. Several important Army officers were present, and part of the ceremony was shown on Polish television; the event gave valuable publicity to mathematics in Poland.

The beginning of the ceremony

The speech by the President was as follows.

Ladies and Gentlemen,

I come from the Jagiellonian University, which for more than 600 years has followed the maxim *Plus ratio quam vis* (Reason over force): Marian Rejewski's place of eternal rest carries the same message.

We have gathered here to commemorate an outstanding Polish cryptologist, mathematician and soldier who fought in the war of the mind against the forces of evil, waged in the 1930s and 1940s. This war was fought to check the spread of evil and to stamp out a malady which had engulfed a large part of the world. Hardly a shooting war, it was fought by means of broadcast intercepts and mathematics.

The makers of the German cipher machine were sure its codes could not be broken. They were wrong. Marian Rejewski created a mathematical method for breaking the German 'Enigma' code; this allowed the Allies to discover and forestall German military plans. The importance of military intelligence cannot be underestimated.

Marian Rejewski obtained his mathematical education at the University of Poznań. Subsequently, following a wise suggestion from the Cipher Bureau of the General Staff of the Polish Army, he turned his energies to the mathematical methods used in cryptology. His studies quickly brought about extraordinary and surprising results. It does great credit to the Cipher Bureau officers to have realised so soon the potential of mathematics in cryptological research.

In 1983, a poster was published on the occasion of the International Congress of Mathematicians held in Warsaw. It showed the photographs of the greatest Polish mathematicians – among them, Marian Rejewski.

The Polish mathematical community bestowed upon Marian Rejewski its

The moment of unveiling the plaque by the President, Bolesław Szafirski

supreme accolade – the title of Honorary Member of the Polish Mathematical Society. We are proud that he was one of us.

May this plaque we unveil today be a symbol of our gratitude and admiration for a man who significantly altered – through mathematics – the course of the Second World War.

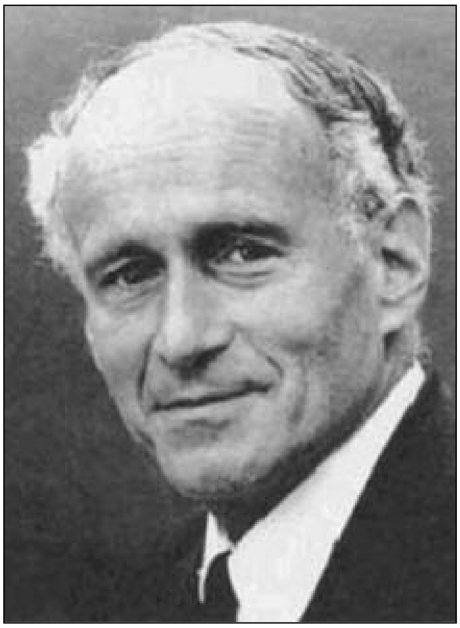
May it remind us all that reason is more powerful than force, more powerful than evil, more powerful than any form of violence or captivity. *Plus ratio quam vis*.

Bolesław Szafirski

The plaque after the unveiling

Laurent Schwartz

1915-2002



'I am a mathematician. Mathematics have filled my life': thus begins the autobiography [1] of Laurent Schwartz, who died in July. But the lucidity he brought to mathematics he turned also to account in his active political life.

Laurent Schwartz was born to a well-off family (his father was a surgeon), was a brilliant student at school (in Latin as well as maths) and studied mathematics at the Ecole Normale Superieure in Paris. His mathematical career was slow to take off, because of military service and the onset of war, but he found his feet in Clermont-Ferrand after the shattering defeat of France in 1940.

The University of Clermont-Ferrand was host to the nucleus of the Bourbaki group (including Henri Cartan, Jean Dieudonné and, in due course, Schwartz himself) and the rigorous approach of that group was, in Schwartz's words 'a revelation'. According to him, this approach was just what he needed to develop the theory of distributions. In November 1944, in liberated Paris, he experienced, in 'the most beautiful night of my life', the fundamental insight which was to lead to his definition of distributions as the elements of the dual space of the non-metrisable space of smooth functions of compact support. He gives a fascinating account of the stages of his discovery in [1]. Now, when distributions play a routine role in partial differential equations, harmonic analysis, symbolic calculus, currents on a manifold, and when topological vector spaces are taught to beginning graduate students, it takes an effort to appreciate the mathematical barriers that Laurent Schwartz had to overcome and the power of the advance that distributions represent. Before his time, physicists and engineers used the Heavyside calculus and differentiated the Dirac delta function because 'it worked'. Schwartz's discovery gave these procedures rigour and made precise their scope.

This is not the place for a full evalution of his mathematical work, but I cannot resist mentioning his discovery that a sphere in Euclidean 3-space is a set of 'non-synthesis' (for the algebra of Fourier transforms of Lebesgue integrable functions on 3-space) – in fact, the first example of this phenomenon. The main idea is to differentiate (in the sense of distributions) the equidistributed measure on the sphere [3].

In 1950, Schwartz was awarded the Fields medal, but it required the combined efforts of the US mathematical community and a threatened boycott of the ICM by French mathematicians to get him a visa to allow him to enter the USA. To understand why, we need to consider Laurent Schwatz's political activism.

Though he inherited rather conservative views, the friends he made and the rigour of their arguments led to his joining a Trotskyist party in the 1930s. During the war, he was in double jeopardy, because of his Resistance activity and his Jewish descent. He and his wife, Marie-Hélène, survived under the cover of false names hiding in a small village. He left his party in 1947, but was proud to be called an 'ex-Trotskyist. I shall necessarily be one all my life and don't regret it'. Though he came to regard his activism during the war years as ineffective, he later made very significant interventions in French political life. Immediately after the war, he had a post in Nancy, then moved to Paris. Nominated to a post at the Ecole Polytechnique, he was a signatory of a petition at the time of the Algerian war calling on soldiers to disobey the order to fight. The Ecole Polytechnique being (formally) a military academy, this cost him his job there. The army minister considered that it would be 'against common sense and honour' for Schwartz to continue in post. His reply was scathing [2]: 'If I signed the declaration, it was in part because for years I had seen torture unpunished and the torturers rewarded. My pupil Maurice Audin was tortured and murdered in June 1957, and it was you, Minister, who signed the order ... [promoting some of those responsible to high rank in the Legion of Honour]. Coming from a Minister who has taken upon himself such responsibilities, your considerations of honour can only leave me cold.' He was reinstated two years later. On retiring from the Ecole Polytechnique, he returned to a chair at Paris VII.

An active organiser of support for all wars of colonial liberation, Laurent Schwartz served on the Russell Tribunal at the time of the Vietnam war, and on the committee which campaigned for the liberation of mathematical and scientific political prisoners in the USSR and elsewhere.

He was no less active or controversial in scientific politics, setting up and chairing the Committee for Evaluating Universities. His report on the state of French universities pulled no punches.

'To make discoveries, a mathematician must upset all taboos. To find something in mathematics is to overcome an inhibition and a tradition. One cannot advance without being subversive ... It is a revolution a little like the fall of the Berlin wall. Suddenly you tell yourself: that won't work. I must change ... To start with, it's very difficult, but, once you've begun, you see that the change is liberating. I think that people don't understand this: they view mathematics too much as fixed or achieved.'

Throughout his life, Laurent Schwartz found time to increase his collection of butterflies. His friends and pupils attest to his personal charm and humanity. He was an outstanding teacher and placed great importance on communicating mathematics to mathematicians and users of mathematics alike.

In a fine tribute, his friend and colleague Michel Broué writes: 'Witness and moving spirit in a period and of a generation which have given us some of the greatest figures of humankind, Laurent Schwartz was a great man in all senses of the term. He was exceptional, with what a personal style! Rigour and precision lived in symbiosis with self-mockery and humour; the apparent detachment and objectivity in regard to his own life, even when it became dramatic, hid neither an immense tenderness nor occasions of deep indignation ... His out-of-the-ordinary personality, his qualities of uprightness and rigour, of generosity and humanity, were a point of reference for more than one generation of scientists and militants.'

David Salinger

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The quotations here were translated from

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The full tribute by Michel Broué is to be found at *http://www.diderotp7.jussieu.fr/2002/pres020712.htm*

The Hellenic Mathematical Society

The Hellenic Mathematical Society was founded in 1918. Its main goal was to encourage the study of, and research in, the science of mathematics and its many applications, as well as the evolution of mathematical education. Among its main objectives are:

- the advancement of the science of mathematics
- the evolution of free interchange of information between mathematicians, scientists and the public
- the development and preservation of scientific integrity and abilities of its members
- the substantial and continuous improvement of mathematical education and the progress of general education
- to approach Greek mathematicians, inform them of recent progress in science and technology, and offer them practical assistance in matters that may occur during their educational and didactic work.

The achievement and accomplishment of the above objectives premise the continuous improvement of the various projects of the Society. The Administrative Board and the relevant committees of the Society work methodically, with coherence and responsibility, offering their spare time, ideas and knowledge for the causes of the Society.

There is always an 'open' invitation towards all colleagues who wish to help the Society, by taking part in the various editing, conference, competition, educational, computer, and other committees. Today, the Hellenic Mathematical Society continues to be a healthy scientific organisation. The conservation of this state is a very difficult task, especially since daily actions affect scientific societies. It is easy to ascertain, from the number of members and the scientific activities of the Society, that the H.M.S. holds a distinguished scientific position, even through tough economic times.

The H.M.S. today has more than 14,000 members and 30 regional branches, with an outstanding presence in the scientific and cultural area. The number of new members of the H.M.S. is expected to rise, since there is currently a systematic promotion to all prospective members of the H.M.S., outlining its activities and the benefits of being a member.

Mathematicians have a natural desire and responsibility to form a lively and active part of the scientific community, which is associated with the important educational problems of mathematical science, the occupation of the mathematician, and new technologies on a national and international base.

The specific and scientific activities of a scientific society are not among the main objectives. The greater activities of the Society cover the needs and interests of mathematicians and offer many interesting activities, such as mathematics education seminars and training in new technological subjects, where mathematics and computers represent critical factors.

What does the H.M.S. have to offer?

The main areas of activity are publications, conferences, special meetings and seminars, and competitions.

The H.M.S. issues a research magazine (*Bulletin*), which is of international prestige and contains original research papers in mathematics and applied mathematics. The *Mathematical Survey* is another publication, which covers general subjects from contemporary mathematics. There are also three *Euclides* magazines issued: *Euclides A* for junior high-school students, *Euclides B* for senior high-school students, and *Euclides C* for teachers, containing material on education and the teaching of mathematics. Lastly, there is a publication whose purpose is to notify its members (*Briefing*) and a magazine (*Astrolabe*) with material on computer science. The Society also publishes selected mathematical books and translations of articles related to mathematics, applied mathematics and the history of mathematics.

The H.M.S. organises annual conferences on mathematical education, and national and international mathematics and computer conferences. The Society has also organised the Panhellenic Mathematical Competition (since 1931) and the National Mathematical Olympiad (since 1984), and takes part in the International Mathematical Olympiad and the Balkan Mathematical Olympiad.

The H.M.S. organises training sessions in mathematics and computing for mathematics graduates and, recently, computer seminars for high-school students. These seminars enable small groups to participate actively on up-to-date computers in basic programming languages, and to learn the fundamentals of arithmetic calculation and methods, algorithms and data structures, data bases and applications, artificial intelligence, geometry on computers, etc.

The H.M.S. constantly keeps a watch on national and international scientific events as they happen, noting the impact that they may have on mathematics. Special committees study the reports and guidelines in order to inform the members on subjects such as mathematical education and culture. The Society has formed a Centre of Computers and Mathematical Software, whose main goals and interests are mathematics and computers, applied mathematics, and the planning and development of educational and mathematical software programs. The Centre of Education and Training has as its main goals the general subjects of mathematical education, training and culture.

The Society maintains close ties with other Greek scientific societies, such as the Greek Physical, Chemical, Geological and Biological Societies, and all other Greek scientific organisations and media. It is, by law, a part of the Congress of Higher Education and the Congress of Technological Education.

Mathematical education

Mathematical education, from primary school to the postgraduate level, is considered very important for those who study mathematics and its applications. Academic researchers and those working on various mathematical applications

understand the need for an adequate number of well-trained mathematicians.

Education is one of the H.M.S.'s main goals. The Society preserves and furthers continuous communication with the Minister and the relevant organs of the Ministry of Education, with pedagogical, educational and research institutes, universities, technical universities and public and private educational organisations.

The H.M.S. provides representatives for special committees that reform the mathematical educational programme at all levels of education. It also takes part in national and international meetings and discussions regarding mathematical educational programmes, training and postgraduate studies for teachers and research programmes of international collaboration.

The H.M.S. is a founding member of the European Mathematical Society, the International Mathematical Union and the Balkan Mathematical Society, and maintains close scientific ties with many national mathematical societies in Europe, Asia and America. Through its publications the H.M.S. continually promotes and publishes its opinions and viewpoints on critical issues occupying the members of the Greek and International Mathematical Community,

Who can become a member of the H.M.S.?
If you are a university graduate of a Greek or foreign mathematical department and you teach or use mathematics in your work, you may become a member of the H.M.S. Every scientific and occupational association has different categories of members. The basic category for graduates is that of tactical member, and there are also associate members and corresponding members.

Administration of the H.M.S.
The activities of the H.M.S. are controlled by the Administrative Board (AB), which is elected by the members of the Society. Amongst the former presidents and members of the AB are distinguished university professors of mathematics, and the members of the AB are renowned members of

the mathematical community with experience and contribution to higher and middle education (public and private) and in sections of commerce and industry. A majority of the members of the H.M.S. are middle-school mathematics teachers. The H.M.S. has over 30 regional branches all over Greece that are controlled by the regional Administrative Board, which is elected by the members of the H.M.S. in that particular area. The branches organise scientific conferences, symposia and social activities.

The Society has created tutorial centres, with pertinent administrative organs. These centres cover subjects such as computers, education (didactics of mathematics and applied mathematics) and their applications.

The H.M.S.'s collaborations
The H.M.S. has very close relations with many scientific societies and occupational scientific and technological unions, both within Greece and beyond. The Society cooperates with many mathematical societies in Europe, Asia and America, and also has relations with the International Mathematical Union, the European Mathematical Society and the Balkan Mathematical Society. It takes part in the organisation of international conferences, in collaboration with other national mathematical societies, and plays an active role in the organisation of international conferences.

What are the benefits of joining?
A member of an active scientific society has two main commitments – to pay dues and participate in the society's activities: the actual gain that members of the Society have is in proportion to their contributions. With a basic subscription, each member receives issues of *Euclides A* and *Euclides B*. The authors of articles published in the H.M.S.'s publications receive many letters, showing the interest of the readers.

The members of the H.M.S. can participate in conferences organised by the Society at a lower cost than non-members, and may purchase research publications and scientific magazines and books at special prices.

The members can communicate with colleagues, including those who work in different cognitive fields. They are also welcome to take an active part in scheduled or exclusive scientific activities of the H.M.S. Scientific and occupational information is available to members who communicate with the Society, for various problems, questions and needs.

H.M.S. members have special advantages and benefits regarding personal and occupational progress: information about occupational settlement, organisation of special seminars, purchasing of goods at reduced prices, etc.

For more information and/or subscription applications, please contact the main offices of the Hellenic Mathematical Society. (Website: *http://www.hms.gr*)

Towards a common framework for Mathematics degrees in Europe

THE MATHEMATICS TUNING GROUP

In the wake of the *Bologna Declaration* [1], signed in 1999 by Ministers responsible for Higher Education from 29 European countries, and its follow up, the *Prague Communiqué* [2], a group of universities established the project 'Tuning educational structures in Europe' [4, 5]. It was co-ordinated by the Universities of Deusto and Groningen and benefited from the financial support of the European Union. As its name suggests, the main objective of the project was to study how to 'tune' (*not* to make uniform) educational structures in Europe, and thereby aid the development of the European Higher Education Area. This in turn should help mobility and improve the employability of European graduates.

Mathematics was one of the areas included in Tuning, and this paper reflects the unanimous consensus of the mathematics group of the project. But since the group does not pretend to have any representative role, we think it is necessary to make this document available for comment to the wider community of European mathematicians. We believe that any kind of action along the lines we sketch will only be possible and fruitful when a broad agreement has been reached. Indeed any mathematician member of the group welcomes comments on the document. E-mail addresses appear at the end.

The Mathematics Tuning Group is happy to express its thanks to the co-ordinators of the Tuning Project, Julia González (Universidad de Deusto) and Robert Wagenaar (Rijksuniversiteit Groningen), as well as to the European Commission, for creating the conditions for fruitful and pleasant interactions between its members.

Summary

§ This paper refers only to universities (including technical universities), and none of our proposals apply to other types of institutions.

§ The aim of a 'common framework for mathematics degrees in Europe' is to facilitate an automatic recognition of degrees in order to help mobility.

§ The idea of a common framework must be combined with an accreditation system.

§ The two components of a common framework are similar (although not necessarily identical) structures and a basic common core curriculum (allowing for some degree of local flexibility) for the first two or three years.

§ Beyond the basic common core curriculum, and certainly in the second cycle, programmes could diverge significantly. Since there are many areas in mathematics, and many of them are linked to other fields of knowledge, flexibility is of the utmost importance.

§ Common ground for all programmes will include calculus in one and several real variables and linear algebra.

§ We propose a broad list of further areas that graduates should be acquainted with in order to be easily recognised as mathematicians. It is not proposed that all programmes include individual modules covering each of these areas.

§ We do not present a prescriptive list of topics to be covered, but we do mention the three skills we consider may be expected of any mathematics graduate:
(a) the ability to conceive a proof,
(b) the ability to model a situation mathematically,
(c) the ability to solve problems using mathematical tools.

§ The first cycle should normally allow time to learn some computing and to meet at least one major area of application of mathematics.

§ We should aim for a wide variety of flavours in second cycle programmes in mathematics. Their unifying characteristic feature should be the requirement that all students carry out a significant amount of individual work. To do this, a minimum of 90 ECTS credits[1] seems necessary for the award of a Master's qualification.

§ It might be acceptable that various non-identical systems coexist, but large deviations from the standard (in terms of core curriculum or cycle structure) need to be grounded in appropriate entry level requirements, or other program specific factors, which can be judged by external accreditation. Otherwise, such degrees risk not benefiting from the automatic European recognition provided by a common framework, even though they may constitute worthy higher education programmes.

1. A common framework: what it should and shouldn't be or do

1.1 The only possible aim in agreeing a 'common European framework' should be to facilitate the automatic recognition of mathematics degrees in Europe in order to help mobility. By this we mean that when somebody with a degree in mathematics from country A goes to country B:
(a) he/she will be legally recognised as holding such a degree, and the Government of country B will not require further proof of competence.
(b) a potential employer in country B will be able to assume that he/she has the general knowledge expected from somebody with a mathematics degree.

Of course, neither of these guarantees employment: the mathematics graduate will still have to go through whatever procedures (competitive exams, interviews, analysis of his/her curriculum, value of the degree awarding institution in the eyes of the employer, ...) are used in country B to obtain either private or public employment.

1.2 One important component of a common framework for mathematics degrees in Europe is that all programmes have similar, although not necessarily identical, structures. Another component is agreeing on a basic common core curriculum while allowing for some degree of local flexibility.

1.3 We should emphasise that by no means do we think that agreeing on any kind of common framework can be used as a tool for automatic transfer between Universities. These will always require consideration by case, since different programmes can bring students to adequate levels in different but coherent ways, but

1 ECTS stands for 'European Credit Transfer System'. ECTS credits measure the learning outcomes attained by students. The basic general assumption is that the learning outcomes that an average full time student is expected to attain in one academic year are worth 60 ECTS credits. Therefore, the workload required to get 60 ECTS credits should correspond to what an average full time student is expected to do in one academic year.

an inappropriate mixing of programmes may not.

1.4 In many European countries there exist higher education institutions that differ from universities, both in the level they demand from students and in their general approach to teaching and learning. In fact, in order not to exclude a substantial number of students from higher education, it is essential that these differences be maintained. We want to make explicit that *this paper refers only to universities (including technical universities)*, and that any proposal of a common framework designed for universities would not necessarily apply to other types of institutions.

2. Towards a common core mathematics curriculum

2.1 General remarks

At first sight, mathematics seems to be well suited for the definition of a core curriculum, especially so in the first two or three years. Because of the very nature of mathematics, and its logical structure, there will be a common part in all mathematics programmes, consisting of the fundamental notions. On the other hand, there are many areas in mathematics, and many of them are linked to other fields of knowledge (computer science, physics, engineering, economics, etc.). Flexibility is of the utmost importance to keep this variety and the interrelations that enrich our science.

There could possibly be an agreement on a list of subjects that must absolutely be included (linear algebra, calculus/analysis) or that should be included (probability/statistics, some familiarity with the mathematical use of a computer) in any mathematics degree. In the case of some specialised courses, such as mathematical physics, there will certainly be variations between countries and even between universities within one country, without implying any difference of quality of the programmes.

Moreover, a large variety of mathematics programmes exist currently in Europe. Their entry requirements vary, as do their length and the demands on the student. It is extremely important that this variety be maintained, both for the efficiency of the education system and socially, to accommodate the possibilities of more potential students. To fix a single definition of contents, skills and level for the whole of European higher education would exclude many students from the system, and would, in general, be counter-productive.

In fact, the group is in complete agreement that programmes could diverge significantly beyond the basic common core curriculum (e.g., in the direction of 'pure' mathematics, or probability/statistics applied to economy or finance, or mathematical physics, or the teaching of mathematics in secondary schools). The presentation and level of rigour, as well as accepting there is and must continue to be variation in emphasis and, to some extent, content, even within the first two or three years, will make all those programmes recognisable as valid mathematics programmes.

As for the second cycle, not only do we think that programmes could differ, but we are convinced that, to reflect the diversity of mathematics and its relations with other fields, all kinds of different second cycles in mathematics should be developed, using in particular the specific strengths of each institution.

2.2 The need for accreditation

The idea of a basic core curriculum must be combined with an accreditation system. If the aim is to recognise that a given programme fulfils the requirement of the core curriculum, then one has to check on three aspects:

§ a list of contents
§ a list of skills
§ the level of mastery of concepts

These cannot be reduced to a simple scale.

To give accreditation to a mathematics programme, an examination by a group of peer reviewers, mostly mathematicians, is considered essential. The key aspects to be evaluated should be:

(a) the programme as a whole
(b) the units in the programme (both the contents and the level)
(c) the entry requirements
(d) the learning outcomes (skills and level attained)
(e) a qualitative assessment by both graduates and employers

The group does not believe that a (heavy) system of European accreditation is needed, but that universities in their quest for recognition will act at the national level. For this recognition to acquire international standing, the presence on the review panel of mathematicians from other countries seems necessary.

3. Some principles for a common core curriculum for the first degree (Bachelor) in mathematics

We do not feel that fixing a detailed list of topics to be covered is necessary, or even convenient. However, we do think that it is possible to give some guidelines for the common content of a 'European first degree in mathematics', and more important, for the skills that all graduates should develop.

3.1 Contents

3.1.1 All mathematics graduates will have knowledge and understanding of, and the ability to use, mathematical methods and techniques appropriate to their programme. Common ground for all programmes will include

§ calculus in one and several real variables
§ linear algebra.

3.1.2 Mathematics graduates must have knowledge of the basic areas of mathematics, not only those that have historically driven mathematical activity, but also others of more modern origin. Therefore graduates should normally be acquainted with most, and preferably all, of the following:

§ basic differential equations
§ basic complex functions
§ some probability
§ some statistics
§ some numerical methods
§ basic geometry of curves and surfaces
§ some algebraic structures
§ some discrete mathematics

These need not be learned in individual modules covering each subject in depth from an abstract point of view. For example, one could learn about groups in a course on (abstract) group theory or in the framework of a course on cryptography. Geometric ideas, given their central role, could appear in a variety of courses.

3.1.3 Other methods and techniques will be developed according to the requirements and character of the programme, which will also largely determine the levels to which the developments are taken. In any case, all programmes should include a substantial number of courses with mathematical content.

3.1.4 In fact, broadly two kinds of mathematics curricula currently coexist in Europe, and both are useful. Let us call them, following [3][2], 'theory based' and 'practice based' programmes. The weight of each of the two kinds of programmes varies widely depending on the country, and it might be interesting to find out whether most European university programmes of mathematics are 'theory based' or not.

Graduates from theory-based programmes will have knowledge and understanding of results from a range of major areas of mathematics. Examples of possible areas are algebra, analysis, geometry, number theory, differential equations, mechanics, probability theory and statistics, but there are many others. This knowledge and understanding will support the knowledge and understanding of mathematical methods and techniques, by providing a firmly developed mathematical context.

Graduates from practise-based programmes will also have knowledge of results from a range of areas of mathematics, but the knowledge will commonly be designed to support the understanding of models and how and when they can be applied. Besides those mentioned above, these areas include numerical analysis, control theory, operations research, discrete mathematics, game theory and many more. (These areas may of course also be studied in theory-based programmes.)

3.1.5 It is necessary that all graduates will have met at least one major area of application of their subject in which it is used in a serious manner, and this is considered essential for a proper appreciation of the subject. The nature of the application area

2 This document was considered extremely useful and met with unanimous agreement from the group. In fact, we have quoted it almost verbatim at some points.

and the manner in which it is studied might vary depending on whether the programme is theory-based or practice-based. Possible areas of application include physics, astronomy, chemistry, biology, engineering, computer science, information and communication technology, economics, accountancy, actuarial science, finance and many others.

3.2 Skills

3.2.1 For a standard notion like integration in one variable, the same 'content' could imply:

§ computing simple integrals
§ understanding the definition of the Riemann integral
§ proving the existence and properties of the Riemann integral for classes of functions
§ using integrals to model and solve problems of various sciences.

So, on one hand the contents must be clearly spelled out, and on the other various skills are developed by the study of the subject.

3.2.2 Students who graduate from programmes in mathematics have an extremely wide choice of career available to them. Employers greatly value the intellectual ability and rigour and the skills in reasoning that these students will have acquired, their firmly established numeracy, and the analytic approach to problem-solving that is their hallmark.

Therefore, the three key skills that we consider may be expected of any mathematics graduate are:

(a) the ability to conceive a proof,
(b) the ability to model a situation mathematically,
(c) the ability to solve problems using mathematical tools.

It is clear that, nowadays, solving problems should include their numerical and computational resolution. This requires a sound knowledge of algorithms and programming and the use of available software.

3.2.3 Note also that skills and level are developed progressively through the practice of many subjects. We do not start a mathematics programme with one course called 'how to make a proof' and one called 'how to model a situation', with the idea that those skills will be acquired immediately. Instead, it is through practice in all courses that these develop.

3.3 Level

All graduates will have knowledge and understanding developed to higher levels in particular areas. The higher-level content of programmes will reflect the title of the programme. For example, graduates from programmes with titles involving statistics will have substantial knowledge and understanding of the essential theory of statistical inference and of many applications of statistics. Programmes with titles such as mathematics might range quite widely over several branches of the subject, but nevertheless graduates from such programmes will have treated some topics in depth.

4. The second degree (Master) in mathematics

We have already made explicit our belief that establishing any kind of common curriculum for second cycle studies would be a mistake. Because of the diversity of mathematics, the different programmes should be directed to a broad range of students, including in many cases those whose first degree is not in mathematics, but in more or less related fields (computer science, physics, engineering, economics, etc.). We should therefore aim for a wide variety of flavours in second cycle programmes.

Rather than the contents, we think that the common denominator of all second cycles should be the level of achievement expected from students. A unifying characteristic feature could be the requirement that all second cycle students carry out a significant amount of individual work. This could be reflected in the presentation of a substantial individual project.

We believe that, to be able to do real individual work in mathematics, the time required to obtain a Master's qualification should be the equivalent of at least 90 ECTS credits. Therefore, depending on the national structure of first and second cycles, a Master would typically vary between 90 and 120 ECTS credits.

5. A common framework and the Bologna agreement

5.1 How various countries implement the Bologna agreement will make a difference on core curricula. In particular, 3+2 may not be equivalent to 5, because, in a 3+2 years structure, the 3 years could lead to a professional diploma, meaning that less time is spent on fundamental notions, or to a supplementary 2 years, and in that case the whole spirit of the 3 years programme should be different.

5.2 Whether it will be better for mathematics studies to consist of a 180 ECTS Bachelor, followed by a 120 ECTS Master (a 3+2 structure in terms of academic years), or whether a 240+90 (4+1+project) structure is preferable, may depend on a number of circumstances. For example, a 3+2 break up will surely facilitate crossing between fields, where students pursue Masters in an area different from that in which they obtained their Bachelor degree.

One aspect that cannot be ignored, at least in mathematics, is the training of secondary school teachers. If the pedagogical qualification must be obtained during the first cycle studies, these should probably last for 4 years. On the other hand, if secondary school teaching requires a Master (or some other kind of postgraduate qualification), a 3-years Bachelor may be adequate, with teacher training being one of the possible postgraduate options (at the Master's level or otherwise).

5.3 The group did not attempt to solve contradictions that could appear in the case of different implementations of the Bologna agreement (i.e., if three years and five years university programmes coexist; or different cycle structures are established: 3+1, 3+2, 4+1, 4+1+project, 4+2 have all been proposed). As we said before, it might be acceptable that various systems coexist, but we believe that large deviations from the standard (such as a 3+1 structure, or not following the principles stated in Section 3) need to be grounded in appropriate entry level requirements, or other programme-specific factors, which can be judged by external accreditation. Otherwise, such degrees risk not benefiting from the automatic European recognition provided by a common framework, even though they may constitute worthy higher education programmes.

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3. The Benchmark document on Mathematics, Statistics and Operational Research, from the UK Quality Assurance Agency for Higher Education, http://www.qaa.ac.uk/crntwork/benchmark/phase2/mathematics.pdf.
4. The official sites of the project *Tuning educational structures in Europe*: http://www.relint.deusto.es/TuningProject/index.htm, http://www.let.rug.nl/TuningProject/index.htm
5. Information on the project *Tuning educational structures in Europe* in the European Commission site: http://europa.eu.int/comm/education/tuning.html

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Problem Corner:

Contests from Bulgaria, I

Paul Jainta

A scribbler is always pleased to encourage others in his specific focus of interest: here it is the allure of the national promotion of mathematically gifted adolescents. An opportunity like this seldom occurs, but as a sequel to my articles on the educational situation in Romania, where youngsters grow up into ace mathematicians or scientists, an inquiry reached me from Bulgaria. The questioner wanted to know whether I could include some companion pieces on the Bulgarian scene. I agreed forthwith, for here I have hooked a big fish. It was Prof. *Sava Ivanov Grozdev*, from the Institute of Mechanics at the Bulgarian Academy of Sciences in Sofia, who holds the office of Secretary of the Union of Bulgarian Mathematicians, is his country's representative in the EMS, coaches the Bulgarian teams participating in the IMO, leads the Scientific Group for Preparing Talented Bulgarian Students, and presides at the Bulgarian National Olympiad Committee for Mathematics. Besides this, he has a journalistic bent as Editor-in-Chief of 'Mathematics Plus', the leading Bulgarian mathematics journal for students and is on the boards of other international maths magazines ('Tangent', Serbia, 'Sigma', Macedonia, or 'Integral', Armenia). He is clearly the right person to describe Bulgaria's way of identifying natural mathematical talent. The first part of his series starts with a problem.

The mathematical prehistory (Sava Grozdev)

> *Given a set $A = \{a_1, a_2, \ldots, a_{10}\}$, find 30 subsets A_i, each with 6 elements, such that each element of A belongs to exactly 10 of the subsets A_i.*

This combinatorial problem was solved by the Bulgarian mathematician Ivan Salabashev in 1879, and arose from a curious historical event. In 1878 the Russian-Turkish war ended and Bulgaria was liberated from 500 years domination by the Ottoman Empire. Following the peace treaty, the Bulgarian territory was divided into two parts: the Bulgarian Principality and Eastern Rummelia; unification and independence were subsequently proclaimed in 1885. In 1879, Ivan Salabashev, who had earlier graduated in mathematical sciences from the University of Prague, was elected a deputy in the Parliament of Eastern Rummelia. Under the Constitution, the Parliament had to elect a Standing Committee of 10 members. Each deputy could vote for 6 persons, and 47 deputies out of 56 took part in the vote. It turned out that 30 of them were of Bulgarian nationality and the goal was to bring as many of them as possible into the Standing Committee. Using his mathematical skills, Salabashev solved the above combinatorial problem, and as a result of his optimal solution all places in the Standing Committee were taken by Bulgarian deputies; this historical event brought enormous popularity to the young Salabashev, who later served several governments as a minister. In 1898, he became the first President of the newly founded Union of Bulgarian Mathematicians, which now includes more than 8000 Bulgarian teachers in mathematics and computer science, university professors and professionals. A considerable part of its activities is connected with the holding of mathematics olympiads and competitions, as well as with the scientific preparation of Bulgarian participants in all international events of this kind.

One of the mathematics competitions that takes place in Bulgaria is the 'Ivan Salabashev Tournament', which takes place annually in November and involves students from second to twelfth grade; the venue is Stara Zagora, the birthplace of Ivan Salabashev. Each year, Stara Zagora brings together about 1000 students from all over the country. The regulations provide for 30 short-answer problems to be solved within 2 hours. The type of problem resembles those of the annual International Mathematics Competition 'Kangourou sans Frontières', organised by the European Association and based in Paris. The problems are often realistic and model concrete real-life situations – no wonder that these problems are very popular with students! As a sign of this approval, one notes the considerable number of participants in the 'European Kangaroo', about 10000 per year, and in total more than 2.5 million students from 30 countries. (In comparison, Bulgaria has only 8 million inhabitants.) It is interesting to note that, if they prefer, Bulgarian students can use the original French version of the problem texts from the European Kangaroo. Stimulating such participants is of importance because Bulgaria is a small country and one of the prime goals of the Bulgarian educational system is to learn foreign languages, so that Bulgarian youngsters can communicate easily with Europe and the rest of the world.

In 1993 the European Kangaroo competition was organised solely in French, with participating countries mostly from the Francophone community. At that time the Bulgarian student Mladen Dimitrov was in tenth grade studying English, while French was completely unknown to him. His determination to take part in the European Kangaroo stimulated him to learn enough French words for him to understand the French formulations of the problems. In spite of his lack of language skills, he scored the highest marks, not only in his age group but among all participants. Later he graduated from the Louis Le Grand Collège in Paris and the Ecole Normale. In this case, mathematics provided the stimulus for learning French, while the competition was the cause for the self-taught youngster to develop his mathematical skills and eventually to become a top-quality professional mathematician.

These mathematics competitions are not the only ones carried out in Bulgaria. Other events are the 'Chernorizets Hrabar Tournament', performed annually, and the 'Hitter Peter' competition, based in Gabrovo and given to students from grades 4 to 7. Both are multiple-choice tests and the problems often have a practical orientation. Junior students are extremely enthusiastic about them: this interest is of importance because mathematics is a science for young people. One of the goals of education is to identify talented students with a flair for mathematics as early as possible. However, mathematics competitions are not the only instruments for this purpose: journals, periodicals and mathematics publications are also essential to this process.

PROBLEM CORNER

Here are five problems produced by Bulgarian students.

132 Find the smallest positive integer n such that the number 1 can be represented as a sum of n periodic decimal fractions consisting only of the digits 0 and 7. *Created by G. Bunova while a student in 11th grade; she is now at the Massachusetts Institute of Technology (MIT).*

133 The number $A = 11\ldots1$ has n digits (each 1), and is divisible by 7. Find the sum of all the digits of the number $A/7$. *Created by G. Grigorov while in 10th grade; he is now a Ph.D. student at Princeton.*

134 Given an acute triangle ABC, let M be an interior point of $\angle ABC$ but an exterior point of ΔABC. If H is the orthocenter of ΔABC and M_1, M_2 are symmetric to M with respect to AB, BC, prove that M lies on the circumcircle of ΔABC if $H \in M_1M_2$. *Created by A. Marinova while in 10th grade.*

135 Given a triangle ABC with $\angle ABC = 60^0$, let T be the common point of the incircle and the side AB, and let C_1 be symmetric to C with respect to T. If the common point of the perpendicular bisector BC_1 and the angular bisector of $\angle BAC$ is A_1, prove that ΔA_1BC_1 is equilateral. *Created by T. Tselkov while in 11th grade; he is now a student at Harvard.*

136 Given n points in the plane, no 3 being collinear, prove the existence of at least $[n(n-1)/6]$ triangles with vertices formed by these points, such that no triangle is contained in any other triangle. ($[x]$ denotes the integer part of x.) *Created by D. Jechev while in 10th grade; he is now at Harvard.*

137 Prove that, for each positive integer n, there exists a polynomial $P_n(x)$ with integer coefficients, such that the numbers $P_n(1), P_n(2), \ldots, P_n(n)$ are different powers of 2. *Created by V. Barzov while in 11th grade; he is now at MIT.*

Solutions to some earlier problems

128 Given 21 distinct natural numbers selected from the set $\{1, 2, \ldots, 2046\}$, show that there exist three chosen numbers a, b, c such that $bc < 2a^2 < 4bc$.

***Solution** by **Niels Bejlegaard**, Copenhagen (Denmark); also solved by Erich N. Gulliver, Schwäbisch Hall (Germany) and Dr Z Reut, London (UK).*

First we note that, if b, c are two positive real numbers with $b < c$, then

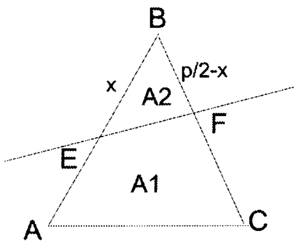
$(b, c) \subset (1/\sqrt{2}\cdot\sqrt{bc}, \sqrt{2}\cdot\sqrt{bc})$ if $b > c/2$;

for, $1/\sqrt{2}\cdot\sqrt{bc} < b$ implies that $b > c/2$ and $\sqrt{2}\cdot\sqrt{bc} > c$ gives $b > c/2$.
Now, let the 21 numbers from the set $\{1,2, \ldots, 2046\}$ be $a_1 < a_2 < \ldots < a_{21}$.
If any of the intervals $(a_{19}, a_{21}), (a_{17}, a_{19}), \ldots, (a_1, a_3)$ satisfies the above condition, we are done, since we can take one of the numbers $a_2, a_4, \ldots, a_{20}$ as the third number.
Otherwise, for any interval the conditions $a_{19} < ½\, a_{21}, \ldots, a_1 < ½\, a_3$ must be valid.
This means that $a_1 < ½\, a_3 < \ldots < (½)^{10} a_{21} < {}^{2046}/_{1024} = 1 + {}^{1022}/_{1024}$.
This forces $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, since $a_3 = 4$ would imply that the maximum number of the given set had to be at least $4\cdot 2^9 = 2048$, a contradiction.
However, $a_2 = 2$ will work as $1/\sqrt{2}\cdot\sqrt{1\cdot 3} < 2 < \sqrt{2}\cdot\sqrt{1\cdot 3}$, so we are finished.

129 Let ABC be an equilateral triangle. A straight line divides this triangle into two parts with the same perimeter but different areas A_1 and A_2. Prove that

$7/9 \le A_1/A_2 \le 9/7.$

***Solution** by **J.F. Lillington**, Dorchester (UK); also solved by Niels Bejlegaard, Erich N. Gulliver, Michael Mudge, (Wales) and Dr Z Reut.*



Let ABC have perimeter p, with the line meeting AB and BC in E and F, respectively (see the figure).
Let $BE = x$, $p/6 \le x \le p/3$; by hypothesis, $BF = p/2 - x$. Then
$A_1/A_2 = [½\,(p/3)^2 \sin 60^0 - ½\,x(p/2 - x) \sin 60^0]$
$/[½\,x(p/2 - x) \sin 60^0]$
$= p^2/[9x\,(p/2 - x)] - 1$
$= p^2/[9\,(p^2/16 - (x - p/4)^2] - 1.$
For $p/6 \le x \le p/3$, A_1/A_2 is a minimum at $x = p/4$ and $A_1/A_2 = 7/9$.
At $x = p/6$ or $x = p/3$, $A_1/A_2 = 1$.
Since A_1 and A_2 can be interchanged, we obtain the double inequality

$7/9 \le A_1/A_2 \le 9/7.$

130 Find all non-negative integers n such that

$\sqrt{(x-1)} + \sqrt{(x-2)} + \ldots + \sqrt{(x-n)} < x$

is valid for all $x \ge n$.

***Solution** by **Gerald A. Heuer** (USA); also solved by Niels Bejlegaard, Pierre Bornsztein (France), Erich N. Gulliver, J.F. Lillington and Dr Z Reut.*

The only possibilities are $n = 1$ or 2.
The case $n = 1$ and any $x > 1$ follows immediately from

$x^2 - x + 1 = (x-1)^2 + x > 0$, so $x^2 > x\ \ 1$ and $x > \sqrt{(x-1)}$.

The case $n = 2$ can be deduced from the fact that

$\sqrt{(x-1)} + \sqrt{(x-2)} < 2\sqrt{(x-1)}$, for $x \ge 2$.

If $n = 0$, the sum on the left is empty (conventionally taken to be 0), and the statement '$0 < x$ for all $x \ge 0$' is false.

If $n \geq 3$, and $x = n + 1$, we have

$\sqrt{(x-1)} + \sqrt{(x-2)} + \ldots + \sqrt{(x-n)}$
$= \sqrt{n} + \sqrt{(n\ 1)} + \ldots + \sqrt{3} + \sqrt{2} + \sqrt{1}$
$> (n-3) + (\sqrt{3} + \sqrt{2}) + 1$
$> (n-3) + 3 + 1 = x.$

131	Consider the polynomial $p(x) = ax^3 + bx^2 + cx + d$ with rational coefficients a, b, c, d and roots x_1, x_2, x_3. Show that if x_1/x_2 is a rational number different from 0 and 1, then all roots of $p(x)$ must be rational.

***Solution** by **Dr Z Reut**; also solved by Niels Bejlegaard, Erich N. Gulliver and J.F. Lillington.*

If x_1/x_2 is a rational number, then x_1 and x_2 are either both rational numbers or both irrational numbers.
In the former case, the third root x_3 is also rational, because $x_1 + x_2 + x_3 = b/a$, where the right-hand side is a rational number, for a □ 0.
In the latter case, both roots x_1 and x_2 have equal absolute values but with opposite signs (which is excluded by the condition $x_1/x_2 \neq 1$), or are rational multiples of the same irrational number (which is excluded, for example, by the Cardano form of solution, and in particular cases by the condition $x_1/x_2 \neq 0$).
(Ed. Pierre Bornsztein and Erich N. Gulliver have indicated that a misprint occurred in the wording of this problem, where 1 *appeared instead of* –1. *We apologise for this mistake.)*

132	Let f: $\mathbf{R} \to \mathbf{R}\backslash\{3\}$ be a function with the property that there exists $k > 0$ such that $f(x+k) = (f(x) - 5) / (f(x) - 3)$, for all x in $\mathbf{R}$. Prove that f is periodic.

***Solution** by **Pierre Bornsztein**; also solved by Niels Bejlegaard, Erich N. Gulliver, Gerald A. Heuer, J.F. Lillington and Dr Z. Reut.*

Let f be such a function.
For all x in $\mathbf{R}$, since $f(x + k)$ □ 3, we have $f(x) \neq 2$.
Then, $f(x + 2k) = (f(x + k)\ 5) / (f(x + k)\ 3)$
$= [(f(x) - 5) / (f(x) - 3) - 5]\ [(f(x) - 5) / (f(x) - 3) - 3]$
$= (5\ 2f(x)) / (2\ f(x)) = 2 + 1/(2\ f(x))$.
Thus, $f(x + 2k) - 2 =\ 1/(2 - f(x))$.
It follows that

$f(x + 4k) - 2 = 1/(2 - f(x + 2k)) = f(x) - 2$;

Thus, $f(x + 4k) = f(x)$, and f is periodic with period $4k$.

133	Find all numbers of the form $m(m + 1)/3$ that are perfect squares.

***Solution** by **Erich N. Gulliver**; also solved by Niels Bejlegaard, Pierre Bornsztein, J. F. Lillington.*

There are infinitely many numbers m produced by the algorithm

$x_1 = 2, y_1 = 1, x_{n+1} = 2x_n + 3y_n, y_{n+1} = x_n + 2y_n$ and $m_n = 3y_n^2$ $(n \geq 1)$.

This gives $m = 3, 48, 675, 9408, \ldots$.
To prove this result we first show that the numbers m, where $m(m + 1)/3$ is a square, are precisely the numbers $3y^2$ with x, y integers satisfying the conditions

$x > 0, y > 0, x^2\ 3y^2 = 1$ (*).

Secondly, we will show that the solutions of (*) are those numbers arising from the enumeration above.
If x, y are solutions of the equation in question and $m = 3y^2$, then $m(m + 1)/3 = (xy)^2$.
Conversely, let m be a positive integer such that $m(m + 1)/3$ is a square.
Since m and $m + 1$ are coprime, one of them must be a square, and the other must be three times a square. But (–1) is not a square modulo 3, so the only possibility is $m + 1 = x^2$ and $m = 3y^2$ for some positive integers x, y. Then x, y satisfy (*).
Secondly, the numbers x_n, y_n defined by the above recursion also fulfil (*), because x_1, y_1 do and $x_{n+1}^2 - 3y_{n+1}^2 = x_n^2 - 3y_n^2$.
If any solutions were not contained in this sequence, let x, y denote the solution with the smallest possible y.
Let $u = 2x - 3y$ and $v = 2y - x$. Then again $u^2 - 3v^2 = x^2 - 3y^2 = 1$.
Also, $y \neq y_1$, so $y > 1$ and $(2y)^2 = 4y^2 > 3y^2 + 1 = x^2$,
so $v > 0$, and $(2x)^2 - (3y)^2 = 4x^2 - 9y^2 = 4 + 3y^2 > 0$, so $u > 0$.
Hence u, v constitute another solution of (*), and $x^2 = 1 + 3y^2 > y^2$ implies that $x > y$,
so $v < y$, which contradicts the minimality of y.
Hence, there do not exist solutions other than the enumerated ones.

Ed. Neils Bejlegaard, Pierre Bornsztein and J. F. Lillington have indicated that the required numbers m are all solutions of Pell's equation $x^2 - 3y^2 = 1$.
A well-known theorem states that the general equation $x^2 - dy^2 = 1$ *(with positive integer d) has infinitely many positive solutions. If* (x_1, y_1) *is the least positive solution, then for n = 1, 2, 3, ... define* $x_n + y_n\sqrt{d} = (x_1 + y_1\sqrt{d})^n$; *the pairs* (x_n, y_n) *are then all the positive solutions of Pell's equation.*
Here, $(x_1, y_1) = (2, 1)$ *is the least positive solution of the equation* $x^2 - 3y^2 = 1$.
So all positive solutions are (x_n, y_n), *where* $x_n + y_n\sqrt{3} = (2 + \sqrt{3})^n$.
Now, $x_n - y_n\sqrt{3} = (2 - \sqrt{3})^n$.
It follows that $y_n = 1/(2\sqrt{3})[(2 + \sqrt{3})^n - (2 - \sqrt{3})^n]$, *and this produces an infinite sequence of numbers* $\{m_n\}$, *satisfying the condition in the statement, described by*

$m_n = \frac{1}{4}[(2 + \sqrt{3})^n - (2 - \sqrt{3})^n]$.

The formula yields m = 3, 48, 675, 9408, ...

Forthcoming conferences

Compiled by Kathleen Quinn

This is the last *Forthcoming conferences* to be compiled by Kathleen Quinn. The Editor would like to thank her for carrying out the task so efficiently over the past four years. Her place will be taken by Vasile Berinde of the University of Baia Mare, Romania [*vberinde@univer.ubm.ro*] to whom all future notices of conferences should be sent.

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to vberinde@univer.ubm.ro. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

Ocober 2002

9-11: 7th Conference on Shell Structures, Theory and Applications (SSTA2002), Gdańsk-Jurata, Poland
Information: web site: http://www.pg.gda.pl/ssta2002
[For details, see EMS Newsletter 42]

10-13: Third International Conference on Applied Mathematics, Baia Mare and Borsa, Romania
Topics: algebra, geometry, mathematical analysis, differential equations, numerical methods, computer science, applied mathematics
Main speakers: D. Cioranescu (France), R. Jeltsch (Switzerland), G. Still (Netherlands) and others
Call for papers: refereed proceedings of the conference will be published in the journal Bul. Stiint. Univ. Baia Mare Ser. B, Mat-Inf. (reviewed by both MR and Zbl). Authors are invited to send their papers in TeX (preferably LaTeX) on a diskette or by e-mail, accompanied by a paper copy. For a regular lecture 8 pages are expected, while for a key lecture up to 12 pages are reserved. All papers to be published will be reviewed by at least two referees
Scientific committee: I. Banicescu, C.Corduneanu, M. Paprzycki (USA), S. Bilchev (Bulgaria), D. Cioranescu, M. Geck (France), R. Jeltsch (Switzerland), T. Laffey, G. Pfeifer (Ireland), Y. Nosenko (Ukraine), V. Soltes (Slovakia), J. Steiner (Australia), G. Still (Netherlands), T. Zamfirescu (Germany), D. Acu, V. Berinde, I. Pavaloiu, I. Coroian, F. Boian, G. Micula, P.T. Mocanu, R. Precup, I. Purdea, I.A. Rus, D.D. Stancu, G. Groza, I. Tomescu (Romania)
Organising committee: V. Berinde (Chair), G. Ardelean, L. Balogh, M. Bancos, D. Barbosu, I. Coroian, O. Cozma, L. Iancu, G. Kovacs, L. Kozma, C. Mustata, I.Pavaloiu, A. Pop, M. S. Pop, N. Pop, H. Sass, I. Tascu, I. Zelina
Secretariat: I. Buhan, M. Gâta (all from North University of Baia Mare, Romania)
Proceedings: to be published
Location: Baia Mare and Borsa, Romania
Deadlines: 1 September for registration, 15 September for submitting papers
Information: e-mail: icam@univer.ubm.ro or vberinde@univer.ubm.ro,
web site: http://super.ubm.ro/ubm/site-ro/facultati/departament/manifestari/icam3/index.html

December 2002

19-21: 3rd WSEAS International Conferences on Acoustics, Music, Speech and Language Processing (ICAMSL 2002); Mathematics and Computers in Biology and Chemistry (MCBC 2002); Mathematics and Computers in Business and Economics (MCBE 2002); Automation and Information (ICAI 2002) , Tenerife, Spain
Information: web site: http://www.wseas.org

29-31: WSEAS International Conferences on Mathematical Methods and Computational Techniques in Electrical Engineering (MMACTEE 2002); Non-Linear Analysis, Non-Linear Systems and Chaos (NOLASC 2002); Wavelet Analysis and Multirate Systems (WAMUS 2002), Athens, Greece
Information: web site: http://www.wseas.org

February 2003

5-7: 4th IMACS Symposium on Mathematical Modelling, Vienna, Austria
Information: web site: http://simtech.tuwien.ac.at/MATHMOD
[For details, see EMS Newsletter 41]

5-7: 4th MATHMOD Vienna 4th IMACS Symposium on Mathematical Modelling, Vienna, Austria
Information: e-mail: inge.troch@tuwien.ac.at, web site: http://simtech.tuwien.ac.at/MATHMOD
[For details, see EMS Newsletter 44]

13-15: 4th WSEAS International Conferences on Neural Networks and Applications (NNA '03), Fuzzy Sets and Fuzzy Systems (FSFS '03), Evolutionary Computation (EC '03), Lanzarote Island, Spain
Information: web site: http://www.wseas.org

May 2003

11-16: International Conference on General Control Problems and Applications (GCP-2003), Tambov, Russia
Information: e-mail: aib@tsu.tmb.ru, uaa@hmb.nnn.tstu.ru,
web site: http://www.opu2003.narod.ru
[For details, see EMS Newsletter 44]

11-18: Conference on Topological Algebras, their Applications, and Related Topics, Bedlewo, Poland
Information: e-mail: ta2003@amu.edu.pl, web site: http://main.amu.edu.pl/~ta2003
[For details, see EMS Newsletter 44]

26-30: Fifth International Conference on Sampling Theory and Applications (SampTA03), Strobl, Austria
Topics: non-uniform sampling, numerical methods and fast numerical algorithms, sampling and interpolation in spline-type spaces, effective bandwidth and noise reduction, frames, non-orthogonal expansions, and applications, greedy algorithms and thresholding methods, radial basis functions, wavelet and Gabor methods in sampling theory, sampling topics related to wavelet and Gabor theory
Call for papers: please visit the conference web site (see below)
Programme committee: A. Aldroubi, J. J. Benedetto, C. Berenstein, P. Butzer, P. Casazza, O. Christensen, H. G. Feichtinger, P. Ferreira, S. Godsill, K. Gröchenig, C. Heil, J. R. Higgins, A. Iske, A. J. E. M. Janssen, A. Jerri, Y. Lyubarskii, W. Madych, F. Marvasti, F. Natterer, T. Randen, M. Unser, K. Seip, R. L. Stens, B. Torresani, T. Strohmer, M. Vetterli, D. Walnut, A. Zayed, Y. Zeevi
Organising committee: H. G. Feichtinger and NuHAG (Numerical Harmonic Analysis Group), Department of Mathematics, University of Vienna, Austria
Location: Bundesinstitut für Erwachsenenbildung St. Wolfgang, Austria (www.bifeb.at)
Deadline: 1 November for abstracts
Information: web site: www.univie.ac.at/NuHAG/SampTA03/

June 2003

23-27: Workshop on Extremal Graph Theory (Miklos Simonovits is 60), Lake Balaton, Hungary
Organising committee: Zoltán Füredi, Ervin Györi, Cecilia Kulcsar, Dezsö Miklós, János Pach, Attila Sali, Krisztian Tichler (secretary)
Sponsors: János Bolyai Mathematical Society, Alfred Rényi Institute of Mathematics, Paul Erdös Center of Mathematics
Information: e-mail: extgr03@renyi.hu, web site: http://www.renyi.hu/~extgr03

24-27: Days on Diffraction '03, St Petersburg, Russia
[preceding 'Waves 2003', 30 June - 4 July 2003, Askyla, Finland, http://www.mit.jyu.fi/emsh/waves2003]
Topics: mathematical and applied aspects of various nature wave phenomena
Format: four-day conference, including plenary session, parallel sessions, poster sessions
Sponsors: Russian Foundation for Basic Research, IEEE ED/MTT/AP St. Petersburg Chapter
Proceedings: to be published
Information: e-mail: grikurov@mph.phys.spbu.ru,
web site: http://mph.phys.spbu.ru/DD

25-28: 7th WSEAS CSCC International Multiconference on Circuits; Systems; Communications and Computers, Corfu Island, Greece
Information: web site: http://www.wseas.org/conferences/2003/corfu

Recent books

edited by Ivan Netuka and Vladimír Souček

Books submitted for review should be sent to the following address:
Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic.

T. M. Atanackovic and A. Guran, *Theory of Elasticity for Scientists and Engineers,* *Birkhäuser, Boston, 2000, 374 pp., DM 148, ISBN 0-8176-4072-X and 3-7643-4072-X.*

It is a challenging task to write a new textbook in the field, where so many excellent treaties already exist, but despite that, the authors have done a remarkably good job. This comprehensive text of 370 pages provides an up-to-date presentation of most of the fundamental topics in the classical theory of elasticity and its basic applications.

After a clear exposition in three sections of the concepts of stress, strain (including the finite strain aspects), and the elastic and thermo-elastic constitutive relations, Section 4 presents the boundary-value problems of the elasticity, including a brief review of relevant methods of solutions. The theoretical part of the book is accompanied by a description of energy methods in the theory of elasticity, presented in an application-oriented form (Section 7), and the basic aspects of elastic stability, with a vivid description of various possible definitions of stability, in Section 10.

The general approach is oriented toward the formulation and solution of problems, thus providing the reader with the necessary background for tackling application topics. Section 5 contains analytical solutions of numerous problems in two and three dimensions, including a solution of the classical problem of plane harmonic waves. Section 6 is devoted to plane problems and the corresponding complex variable methods. Section 8 deals with the plate theory, including von Karman's and the Reisner-Mindlin plate models. Section 9 presents basic contact problems. Each chapter is accompanied by a number of nicely selected exercises. (jakr)

K. Binmore and J. Davies, *Calculus: Concepts and Methods,* *Cambridge University Press, Cambridge, 2002, 554 pp., £25.95, ISBN 0-521-77541-8*

This is a textbook of mathematics for students of economics, and is based on lectures given by the authors for several years at prestigious economics schools in the UK and the USA. It covers first-year university mathematical topics: linear algebra, differential and integral calculus, and an introduction to differential equations.

The material is divided into fourteen chapters; the applications accompanying Chapters 10, 11 and 13 present a brief introduction to the basic notions of probability and statistics. The topics are presented here in a very clear and neat way. It is shown step by step that mathematics can model various real-life situations, to forecast their course and outcome. The book not only broadens one's mathematical scope and improves one's mathematical skill but, even more usefully, it shows the reader how powerful are the tools that mathematics offers, and that mathematics allows us participate in processes where our presence is impossible: through mathematics we can enter forbidden worlds, through mathematics we can see the future. Applications in economics are stressed, but the book leaves no doubt that mathematics is equally powerful in other sciences.

The text is accompanied by a considerable number of exercises, and students are strongly encouraged to try them. Most chapters are accompanied by numerous applications. The reader is supposed to have a basic knowledge of linear algebra and functions of one variable. Topics that are supposed to be known, as well as the more challenging ones, are marked for the reader's convenience. Few pages have no diagrams, graphs or figures. At the end are hints and solutions to the starred exercises. Although the book contains no formal definitions, theorems or proofs, it is a genuine mathematical textbook showing how and why the explained methods and concepts work. One hardly can imagine a better textbook of this type. (jdr)

R. Blei, *Analysis in Integer and Fractional Dimensions*, *Cambridge Studies in Advanced Mathematics* ***71****, Cambridge University Press, Cambridge, 2001, 556 pp., £65, ISBN 0-521-65084-4*

This book is a comprehensive exposition of a subject that mixes harmonic analysis, functional analysis and probability theory. In contrast with classical, linear functional analysis, the issues studied in this book are multi-linear. Starting from inequalities and other results on Cartesian and tensor products, the focus is extended to objects which can have fractional dimension. Here the notion of dimension is much different from (e.g.) Hausdorff dimension. The combinatorial dimension, used through the book, describes the degree of freedom of the set. Roughly speaking, a typical object of combinatorial dimension k in X^n is the graph of a map f from X^k to X^{n-k}.

The exposition, after a historical prologue, begins with inequalities in multi-dimensional analysis, including Khintchin's, Littlewood's and Orlicz's inequality, and particularly Grothendieck's inequality. The multi-dimensional extensions of this inequality are studied later. Grothendieck's factorisation theorem gives a representation of a bilinear functional on the product of two spaces of continuous functions. Multi-dimensional measure theory, studying Fréchet measures (separately countably additive set functions on product of algebras), is developed. The treatment of harmonic analysis on $\{-1, 1\}^N$ includes the Walsh system, p-Sidon sets and various inequalities. A multi-dimensional version of Grothendieck's inequality is derived, and products of Fréchet measures are investigated.

Then the exposition turns to probability theory. The Wiener process and subsequent chaos processes are studied from the point of view of the theme of the book. A general class of processes called *integrators* is investigated, in connection with stochastic integration and Fréchet measures. Then fractional-dimensional Cartesian products are constructed and combinatorial dimension is studied. Finally, further applications and connections are presented and perspectives of the field are discussed.

The exposition is clear and self contained. Each chapter ends with many exercises, and numerous open problems are mentioned throughout. Starting from significant results on Grothendieck-type inequalities and the solution of the p-Sidon set problem, the author develops a nice part of analysis and introduces natural new notions such as combinatorial dimension and fractional Cartesian product. This book is directed at the wide mathematical community and other scientists interested in mathematical methods. In particular, the book can be used as a textbook for graduate students. (jama)

A. A. Borichev and N. K. Nikolski (eds.), *Systems, Approximation, Singular Integral Operators, and Related Topics*, *IWOTA 2000, Operator Theory Advances and Applications* ***129****, Birkhäuser, Basel, 2001, 527 pp., DM 304, ISBN 3-7643-6645-1*

All twenty papers in these Proceedings are expanded versions of lectures delivered at the 11th workshop named above, held at Bordeaux in June 2000. Since at least seven contributions are surveys, the book gives non-experts a good review of the current state of operator theory and its relation to complex function theory and harmonic analysis, parts of system theory (including stochastic control), spectral perturbations and applications to differential equations and mathematical physics. The other papers have the character of research reports, but also contain long introductory sections. All contributions are presented at postgraduate level and can be recommended to those whose interests are connected with modern operator theory. Experts will find here many new results and new insights, together with non-obvious internal relations. The book is well printed. (jmil)

J. J. Callahan, *The Geometry of Spacetime,* *Undergraduate Texts in Mathematics, Springer, New York, 2000, 451 pp., DM 98, ISBN 0-387-98641-3*

This book gives a unified picture of special and general relativity in geometric terms. It is addressed mainly to a mathematical audience and assumes only a basic preliminary knowledge of mathematics (some calculus and linear algebra of matrices, mainly of size 2×2). The physical sources of special relativity are discussed with great care.

The book starts with a detailed discussion of basic notions, beginning with the concept of spacetime. (It illustrates this with amusing reprints of old train timetables.) Then it proceeds with a thorough, clearly written (but still elementary) discussion of the principles of special relativity. Most of the book is then devoted to an introduction to general relativity, starting with a discussion of arbitrary coordinate frames and the equivalence principle. The author explains carefully, but at an elementary level, more advanced and technical notions from differential geometry, starting with Euler's *theorema egregium* and other results of intrinsic geometry. Finally, after developing the necessary geometric notions, the author presents the basic facts of general relativity: equations of motion, vacuum-field equations, matter-field equations, and explains some famous consequences of general relativity.

In this book, great care has been taken to make the subject accessible to an undergraduate with only a very basic knowledge of mathematics. The more advanced topics needed for relativity theory are developed in an understandable way. Interesting historical reminiscences, good organisation, clear graphical presentation, many illustrations and a careful presentation make the reading of the book really enjoyable. The only imperfection I found was an extra 'n' in a reference to the treatise by Dubrov(n)in, Fomenko and Novikov. (mzahr)

A. Carbone and S. Semmes, *A Graphic Apology for Symmetry and Implicitness*, *Oxford Mathematical Monographs, 2000, 501 pp., £69.50, ISBN 0-19-850729-1*

This book aims broadly to confirm that 'complicated objects, which admit comparatively short descriptions, have some kind of compensating internal symmetry'. This philosophy is approached via mathematical logic in a way different from Kolmogorov complexity (called, by the authors, 'middle ground'), which is related to a more syntactical approach such as 'cut-elimination' and recursive aspects of computational complexity. Most of the book is devoted to graphical models and the study of 'visibility graphs' (trees generated by the reachability relation).

The authors are motivated by formal proof theory and by geometry: these connections are the most interesting part of the book. The language and terminology are sometimes unusual – for example, Chapter 2 discusses 'morphisms in logic' and Chapter 10 'mappings and graphs', while only for algebras do they reserve the usual term 'homomorphisms'. In many places one finds *ad hoc* terminology and notation (for example, complete graphs are denoted by *Ck*). The combinatorial part is mostly easy, but suffers from complicated concepts and notation: one expects more elegance at this level of generality. Yet the topic is interesting and this is certainly one of the first treatments of these problems, which deserve more attention. (jneš)

C. Casacuberta *et al.* (eds.), *European Congress of Mathematics, Barcelona, July 10-14, 2000, I, II*, *Progress in Mathematics* ***201, 202***, *Birkhäuser, Basel, 2001, 582 and 641 pp., DM 224 and 224, ISBN 3-7643-6417-3, 3-7643-6419-X and ISBN 3-7643-6418-1, 3-7643-6419-X*

This two-volume set of Proceedings gives an overview of the state of the art in many fields of mathematics at the end of the twentieth century. Volume I includes speeches delivered at the opening and closing ceremonies, the list of lectures, and brief summaries of the work of the prizewinners. Articles by eight plenary speakers and 27 parallel speakers also appear here. Volume II includes eight articles by prizewinners (S. Alesker, R. Cerf, D. Joyce, V. Lafforgue, M. McQuillan, S. Nemirovski, P. Seidel and W. Werner) and articles presented at various mini-symposia: computer algebra (5 articles), curves over finite fields and codes (6), free boundary problems (9), mathematical finance: theory and practice (5), quantum chaology (4), quantum computing (1), string theory and *M*-theory (5), symplectic and contact geometry and Hamiltonian dynamics (7) and wavelet applications in signal processing (6).

These Proceedings will be of interest to all professional mathematicians and is warmly recommended as a guide to a large part of contemporary mathematicians. (in)

A. Deitmar, *A First Course in Harmonic Analysis*, *Universitext, Springer, New York, 2002, 151 pp., EUR 44,95, ISBN 0-387-95375-2*

The author's aim is to provide beginners with a short introduction to the basic facts of harmonic analysis. It differs from other similar texts: it does not use Lebesgue integral and requires only understanding of metric spaces – no abstract topology is needed. The material is chosen and organised in such a way that almost everything is proved.

In the first fifty pages the author presents the necessary elementary facts on Fourier series and Fourier transforms, together with the elements of Hilbert spaces, using ideas of Riemann integration only. Later the author is less strict: *special cases* of the dominated convergence theorem and the monotone convergence theorem are proved for an integral over the real line. This means that some less elementary tools are used: the 'first Rudin' is an appropriate background. Generalised sums are used, Haar integration and the gamma function are introduced in a few lines. For a deeper understanding of the contents of the book, one needs additional sources; on the other hand, for a first acquaintance with harmonic analysis, this short text is useful. (jive)

C. Faber, G. van der Geer and F. Oort (eds.), *Moduli of Abelian Varieties*, *Progress in Mathematics* ***195***, *Birkhäuser, Basel, 2001, 518 pp., DM 196, ISBN 3-7643-6517-X*

This book is based on material presented during the 3rd Texel Conference on Moduli of Algebraic Varieties, held in April 1999, but some articles here are unrelated to lectures at the conference. It consists of seventeen contributions: the main topics are moduli spaces of higher-dimensional abelian varieties, their compactifications, and links with other fields such as algebraic groups, representation theory and number theory. There are two longer review papers – one on Mumford's uniformisation and Neron models of Jacobians of semistable curves over complete rings (by F. Andreatta), and the other on stratification of moduli space of abelian varieties (by F. Oort). Other contributions include a paper by Yu. Manin on mirror symmetry and quantisation of Abelian varieties, containing a discussion of various aspects of commutative and non-commutative geometry of tori and abelian varieties and quantisation of abelian varieties. A paper by A. Beilinson and A. Polishchuk shows that Fourier transform on the Jacobian of a curve interchanges '-function' on the curve and the theta-divisor, the Torelli theorem being an easy consequence. Most of the contributions present results for moduli spaces in positive characteristics.

This book can be recommended particularly to algebraic geometers and number theorists interested in the theory of moduli spaces of abelian varieties and related topics. (jbu)

H. M. Farkas and I. Kra, *Theta Constants, Riemann Surfaces and the Modular Group*, *Graduate Studies in Mathematics* ***37***, *American Mathematical Society, Providence, 2001, 531 pp., US$69, ISBN 0-8218-1392-7*

The subtitle 'An introduction with application to uniformisation theorems, partition identities and combinatorial number theory' shows the main direction developed in this book. It contains applications and connections between the theory of Riemann surfaces to some directions of combinatorial number theory, mainly centred around representations of integers as sums of elements of certain arithmetical sequences.

The early chapters deal mainly with the theory of functions on Riemann surfaces (such as theta functions and theta identities, meromorphic functions and differentials, and particular cases of modular curves). The reader then meets such topics as properties of the partition function, the j-function, Ramanujan congruences, the relation of Lambert and Euler series and continued fractions to theta functions. The book is written in a very clear and readable style and will interest anyone concerned with such classical topics as theta functions, modular groups and curves, the application of uniformisation theorems, etc. The authors supply many examples. The book contains new material not published elsewhere, and can be warmly recommended to readers interested in these beautiful applications of classical function theory. (špor)

Y. Félix, S. Halperin and J.-C. Thomas, *Rational Homotopy Theory*, *Graduate Texts in Mathematics* ***205***, *Springer, New York, 2001, 535 pp., DM 119, ISBN 0-387-95068-0*

For over twenty years rational homotopy theory has been a well-established part of topology, and the above authors have substantially contributed to its development, a fact that guarantees the top quality of this monograph.

We remark that rational homotopy provides us with less information than the ordinary homotopy, but this drawback is highly compensated by the possibility of nice concrete computations that can be performed within the framework of rational homotopy, using various types of models. This phenomenon makes the book attractive even for specialists in other parts of mathematics and for graduate students. (The techniques from rational homotopy theory have been successfully applied in local commutative algebra, for example).

The book is very carefully written and the authors have tried to make it as self-contained as possible. As a result, the necessary prerequisites are quite modest: basic facts from general topology, homology and cohomology, homotopy groups, and some linear algebra. But generally, this book is a basic monograph on rational homotopy theory. Compared with others, it seems to be the most compact, most complete and most modern, and is good for references. Many results have new, or shorter, proofs, and every chapter is followed by exercises.

The book covers the central part of rational homotopy theory, without extensions and generalisations. The authors limit themselves to simply connected spaces, even when most of the results hold also for spaces with nilpotent fundamental group: since it is a topology book, the above-mentioned results in local commutative algebra are not included. But readers interested in other parts of rational homotopy theory can find relevant references in the introduction. This book can be strongly recommended. (jiva)

G. Fischer, *Plane Algebraic Curves*, *Student Mathematical Library* ***15***, *American Mathematical Society, Providence, 2001, 231 pp., US$35,* *ISBN 0-8218-2122-9*

The aim of this book is to describe (real or complex) algebraic curves from the point of view of basic algebraic geometry and invariant theory. In Chapters 1-5, the geometry of algebraic curves (tangents, singularities, inflection points, etc.) is reviewed from an elementary viewpoint: the theory of resultants is used as the main technical tool for dealing with intersection theory. This part culminates in Bezout's theorem on the number of intersection points of two algebraic curves and the Plücker formulas describing the relation between geometric invariants of a given curve and its dual curve. Chapters 6-8 introduce local complex analysis – the theory of convergent power series and holomorphic functions of several complex variables: the emphasis is on the study of properties of the ring of convergent power series. In the last chapter, the local parametrisations considered so far are patched together to form the Riemann surface of an algebraic curve. This part contains classical nineteenth-century results, such as the Riemann-Hurwitz and genus formulas.

This book contains the classical material on the algebraic geometry of algebraic curves and should serve as a good source for readers wishing to become acquainted with this topic. (pso)

M. Foreman, A. S. Kechris, A. Louveau and B. Weiss (eds.), ***Descriptive Set Theory and Dynamical Systems****, London Mathematical Society Lecture Note Series* ***277****, Cambridge University Press, Cambridge, 2000, 291 pp., £27.95, ISBN 0-521-78644*

The aim of this book is to present interconnections between descriptive set theory and the theory of dynamical systems. It consists of survey papers on ergodic theory of non-singular transformations (J. Aaronson), recurrence theorems in ergodic theory (V. Bergelson), cocycles for non-singular group actions (S. Bezuglyi), and the structure theory of minimal dynamical systems (E. Glasner). The role of cocycles in generic topological dynamics is investigated by V. Ya. Golodets, V. M. Kulagin and S. D. Sinel'shchikov, while M. Foreman's paper includes an introduction to descriptive set theory of Polish spaces and ergodic theory. Connections between these two fields are also presented: A. S. Kechris deals with actions of Polish groups from the point of view of descriptive set theory, and shows some of the main methods used in this area (Baire category arguments and changing topology technique). A. B. Ramsay's paper is devoted to Polish groupoids. The book concludes with B. Weiss on generic dynamics. (mzel)

M. Geck and G. Pfeiffer, ***Characters of Finite Coxeter Groups and Iwahori-Hecke Algebras****, London Mathematical Society Monographs New Series* ***21****, Clarendon Press, Oxford, 2000, 446 pp., £65, ISBN 0-19-850250-8*

As its title indicates, the authors develop the theory of conjugacy classes and irreducible characters for finite Coxeter groups and the associated Iwahori-Hecke algebras. Finite Coxeter groups are generated by reflections, and the Iwahori-Hecke algebras are introduced as certain quotients of the group algebra of the braid group (universal monoids related to Matsumoto's theorem). Based on earlier results of Michel and the authors, they develop a new way to classify conjugacy classes of a finite Coxeter group in terms of parabolic subgroups and cuspidal classes. The book contains a clear introduction to the character theory of finite Coxeter groups and their description using induction of characters from parabolic subgroups. After developing the structural theory of the Iwahori-Hecke algebras, the reader will find formulas for the determination of values of the irreducible characters for each series of irreducible Coxeter groups, and methods for computing generic degrees and character tables of generic Iwahori-Hecke algebras.

The book is written so that each chapter can be read independently, with a comprehensive introduction and ending with bibliographical remarks and exercises. The appendix contains tables of exceptional types. It is written in an engaging and intelligible style, and is well structured and clearly printed. (špor)

Hai-Tao Cai and Jian-Ke Lu, ***Mathematical Theory in Periodic Plane Elasticity****, Asian Mathematical Series* ***4****, Gordon and Breach, Amsterdam, 2000, 153 pp., US$58, ISBN 90-5699-242-2*

This monograph is a detailed exposition of periodic boundary-value problems in plane elasticity, solved by methods of complex variables, and most of the contents arise from research of the authors. The theory is illustrated by a number of analytically solved examples. In the most general case, the stresses are assumed to be periodic and the displacement to be quasi-periodic. Periodic welding problems are studied by reducing them to periodic Riemann boundary-value problems. Various periodic problems of elastic half-space are treated and solved by reduction to the Riemann-Hilbert boundary-value problems with discontinuous coefficients. Periodic crack problems are solved by transforming them to singular integral equations, and in addition, doubly-periodic problems are briefly examined. A good knowledge of theory of linear elasticity and complex analysis is necessary. (jakr)

D. Haskell, A. Pillay and C. Steinhorn (eds.), ***Model Theory, Algebra, and Geometry****, Mathematical Sciences Research Institute Publications* ***39****, Cambridge University Press, Cambridge, 2000, 227 pp., £30, ISBN 0-521-78068-3*

This book contains nine contributions on three main topics: model theory of fields, dimension theory and geometry. The first paper contains an introduction to elementary model theory, and the next three articles are devoted to model theory of classical, differential and difference fields. It is shown how differential fields enter into Hrushovski's model-theoretic proof of the Mordell-Lang conjecture for function fields of characteristic 0. The fifth and sixth chapters are devoted to dimension theory, an analysis of complexity of definable sets based on Morlay rank illuminated by a pregeometry given by model-theoretic dependence. The discussion also includes the important notions of minimality, o-minimality, etc. The last three papers are 'geometric', indicating applications of model theory to various types of geometry (algebraic, analytic, diophantine, *p*-adic). They include, for example, results describing behaviour semialgebraic and subanalytic sets. The book refers to a wide variety of types of model theory (such as quantifier elimination, stability, forking or simplicity) and contains some illuminating discussions of open problems. It is a useful source for graduate students and researchers. (jmlè)

T. Hawkins, ***Emergence of the Theory of Lie Groups, Sources and Studies in the History of Mathematics and Physical Sciences****, Springer, New York, 2000, 564 pp., DM 159, ISBN 0-387-98963-3*

This book presents the history of Lie group theory from 1869 to 1926. It is divided into four parts, each being connected with name of a leading mathematician of the period.

The first part is devoted to the geometric and analytic origins of theory of continuous transformations groups created by S. Lie. In the second part, the central figure is W. Killing, who discovered the main concepts and principal results of the structure and classification of semisimple Lie algebras. The third part is devoted to representation theory of semisimple Lie algebras, with É. Cartan as the main personality. The last part describes the fundamental role played by H. Weyl, who completed earlier work and opened new ways for further research. Much comment and explanation on the development of the theory can be found throughout the book, such as a description of the key role played by F. Klein in the development of relations between group theory and geometry (the Erlangen programme), the letters exchanged by W. Killing and F. Engel showing the practical background to the evolution of the theory. Other mathematicians influencing the origins of the subject were members of the Lie school (E. Study, G. Fano, A. Cayley and G. Kowalewski) and mathematicians from the Göttingen school of D. Hilbert (H. Minkowski and E. Schmidt, among others). The important theory of characters was developed by G. Frobenius and A. Hurwitz, while the theory of invariants was extended by I. Schur and A. Ostrowski. The end of the book discusses the role of Lie groups and representation theory in the discovery of the theory of relativity, this period being connected with the names of H. Weyl and A. Einstein. The concept of symmetry contributed to fundamental ideas in the evolution of mathematics and physics, and representation theory was the language developed for its efficient application. The beginning of the theory described in the book forms a basis for further enormous evolution during the last century.

The book will be useful for a wide variety of mathematicians and physicists, for students, researchers and anybody interested in the history and philosophy of science. (jbu)

T. Hida, R. L. Karandikar, H. Kunita, B. S. Rajput, S. Watanabe and J. Xiong (eds.), ***Stochastics in Finite and Infinite Dimensions****, Trends in Mathematics, Birkhäuser, Boston, 2001, 410 pp., DM 238, ISBN 0-8176-4137-8 and 3-7643-4137-8*

In this book, a number of leading experts have written research articles highlighting progress and new directions in diverse areas related to Kallianpur's former pioneering and influential work. The list of these areas is impressive: non-linear prediction and filtering problems, consistent estimation, control theory, solutions to SDE and SPDE and their numerical approximations, random iteration of quadratic maps, Feynman integrals, diffusions in infinite dimensions, reproducing kernel Hilbert space theory and 0-1 laws for Gaussian processes, fractional Brownian motion, Chern-Simons integral, and, recently, stochastic finance, stochastic fluid mechanics and environmental pollution problems.

Contributors to the volume presented their work at a special conference, held in Kallianpur's honor in Calcutta in December 2000. The book includes a nice biographical article by B. V. Rao and a list of Kallianpur's publications. This book should find its place in the library of any probabilist and mathematician with an interest in the rapidly developing areas of probability reflected in this volume. (mzah)

G. James and M. Liebeck, *Representations and Characters of Groups, 2nd edn.*, *Cambridge University Press, Cambridge, 2001, 458 pp., £24.95, ISBN 0-521-00392-X*

This is a beautiful well-balanced introduction to representations and characters of finite groups over the complex field, suitable for advanced undergraduate and beginning graduate students. Written in a style similar to Ledermann's book, the reader becomes acquainted with all basic results, with the possible exception of the representations of symmetric groups. The pace is leisurely (Maschke's theorem is on page 70, for instance), but steady. Frequent explanations, going beyond the proofs, offer an early insight into the theory, and the solved exercises and summary at the end of every chapter are also of great help. Most of the material is developed in terms of modules, rather than matrices, but once characters are introduced they become the focus of the book. The authors methodically construct character tables and show how they shed light on the subgroup structure (simplicity, normal subgroups, commutators, involutions, etc.) of the group in question. No mention is made of fields other than **R** and **C**.

The reviewer enjoyed the exposition tremendously and strongly recommends this already popular book both as a textbook and for self-study. The second edition contains two new chapters, on character tables of $GL(2,q)$ and on characters of permutation groups. More space is devoted to real representations and to the use of character theory within the classification of finite simple groups. (ad)

M. Kapovich, *Hyperbolic Manifolds and Discrete Groups*, *Progress in Mathematics* ***183***, *Birkhäuser, Boston, 2001, 467 pp., DM 168, ISBN 0-8176-3904-7 and 3-7643-3904-7*

This book presents the first complete proof of the Thurston hyperbolisation theorem (also called 'the Big Monster'), which can be stated as follows. If M is a compact atoroidal Haken 3-manifold with zero Euler characteristic, then the interior of M admits a complete hyperbolic metric of finite volume. This theorem establishes a strong link between the geometry and topology of 3-manifolds and the algebra of discrete subgroups of the isometry group of hyperbolic three-dimensional space, and several further interesting consequences can easily be deduced. A short history of Thurston's theorem is presented and various ingredients of its proof are discussed. For the proof, it was necessary to collect material from different parts of mathematics: geometry and topology of 3-manifolds, geometry of hyperbolic space, Kleinian groups, orbifold theory, etc.: the book can thus be used as a nice textbook on these topics.

The book is self-contained and includes many examples, exercises and open problems for further research. It can be used either as a textbook for advanced seminars or as a good and comprehensive reference. (jbu)

A. Khrennikov, *Superanalysis*, *Mathematics and its Applications* ***470***, *Kluwer Academic Publishers, Dordrecht, 1999, 347 pp., ISBN 0-7923-5607-1*

This book is devoted to one of the possible mathematical formulations of superspace and supersymmetry, introduced and developed by physicists during recent decades. The approach used here closely follows the original ideas of Schwinger from the 1950s. The aim is to develop calculus on a superspace, similar to Newton's original differential calculus. The basic setting closely follows the ideas of V. S. Vladimirov and I. V. Volovich in the 1980s, and their approach is summarised in the first chapter, which introduces the basic notions of a (super)-commutative superalgebra (CSA) and a superspace over it: the main examples are various subalgebras of an infinite-dimensional Grassmann algebra over a unital associative commutative algebra. The topology on such a CSA is supposed to be given by a norm, and a superspace is then defined as a finite product of even and odds parts of the appropriate Banach CSA. In this setting, an analogue of standard differential and integral calculus in Banach spaces is developed. The next chapters contain a systematic exposition of the theory of generalised functions and the theory of pseudodifferential operators and probability theory on a superspace, as developed by the author in the early 1990s. There is also a chapter on non-Archimedean superanalysis. The last two chapters describe the author's ideas on possible generalisations to non-commutative basic algebra and possible applications to theoretical physics. At the end of each chapter (as well as in the text itself), the author formulates may open questions and problems, indicating that the theory is far from being finished. (vs)

J. Koslowski and A. Melton (eds.), *Categorical Perspectives*, *Trends in Mathematics, Birkhäuser, Boston, 2001, 281 pp., DM 196, ISBN 0-8176-4186-6 and 3-7643-4186-6*

The book may be considered as the proceedings of a conference on category theory held at Kent State University in August 1998 in honour of George E. Strecker's 60th birthday, and also contains introductory chapters to category theory and expository papers. The research papers are devoted to interplay of category theory and topology, but there are also papers of an algebraic or combinatorial nature. An overview of Strecker's research in categorical topology is included, with a number of references. In any case, one should consult Strecker's seven paragraphs entitled '10 Rules for surviving as a mathematician and teacher'. (rb)

K.-Y. Lam, I. Sharplinski, H. Wang and C. Xing (eds.), *Cryptography and Computational Theory*, *Progress in Computer Science and Applied Logic* ***20***, *Birkhäuser, Basel, 2001, 378 pp., DM 196, ISBN 3-7643-6510-2*

This volume contains the refereed proceedings of a Workshop on Cryptography and Computational Number Theory held in Singapore in 1999, designed to stimulate collaboration and active interaction between mathematicians, computer scientists, practical cryptographers and engineers in academia, industry and government. The papers cover various topics: new cryptographic systems and protocols, new attacks on the existing cryptosystems, new cryptographic paradigms such as visual and audio cryptography, pseudorandom number generator and stream ciphers, primality proving and integer factorisation, fast algorithms, cryptographic aspects of the theory of elliptic and higher genus curves, polynomials over finite fields, and analytical number theory. Several papers survey current number theoretic research areas significant to cryptography, while others describe original results and new ideas. (jtu)

Ch. Laurent-Thiébaut, *Théorie des fonctions holomorphes de plusieurs variables*, *Mathématiques, EDP Sciences, Les Ulis, 1997, 244 pp., FRF 280, ISBN 2-7296-0660-2 and 2-271-05501-6*

This book is based on lectures to advanced undergraduate and postgraduate students at University of Grenoble, and offers an introduction to the modern theory of several complex variables. It starts with standard basic facts on holomorphic functions of several variables, complex manifolds, differential forms in a complex setting and the Dolbeault complex. The author also introduces a modern language of currents and defines their Kronecker index; these notions are needed in the rest of the book.

The main tools used are certain integral formulae. First is the Bochner-Martinelli kernel and its generalisation to differential forms, discovered by Koppelman; this is immediately used to show the exactness of the Dolbeault complex for forms with compact support, as well as the regularity of the CR operator. The author then introduces CR functions on the boundary of bounded domains and proves the Bochner theorem and theorems on extensions for CR functions; these theorems are then extended to a complex manifold setting, using the Cauchy-Fantappie formula. The next two chapters are devoted to the classical ideas surrounding domains of holomorphy and pseudoconvexity: the Levi problem is discussed first in C_n, and then on complex manifolds, and the final chapter is devoted to removable singularities for CR functions. Three appendices summarise the facts needed on manifolds, differential forms, their integration and Stokes' theorem, sheaf cohomology and functional analysis.

The book is very well written and nicely organised and presents the basic facts of the theory in a reasonable number of pages. Brief comments at the ends of individual chapters are helpful and the bibliography contains only basic references. The book is a useful addition to the already rich literature on the subject. (vs)

P. Le Calvez, *Décomposition des difféomorphismes du tore en applications déviant la verticale*, *Mémoires de la Société Mathématique de France* ***79***, *SMF, Paris, 1999, 148 pp., FRF 150, ISBN 2-85629-080-9*

This book studies the properties of fixed points and periodic orbits of diffeomorphisms of the torus, homotopic to the identity. A diffeomorphism F of the two-dimensional torus T can be written as a composition of an even number ($2n$) of positive and negative twist maps; such a decomposition, together with a lift f of F to the plane, induces a certain vector field on a suitable $2n$-dimensional manifold.

The main topic of the book is a study of singularities of this vector field and their relations to the set of fixed points of *F*, which are lifted to fixed points of *f*. An appendix (written jointly with J.-M. Gambaudo) gives an alternative proof of certain results of J. Frank on the existence of periodic orbits (with arbitrarily large periods) for area-preserving diffeomorphisms of the closed annulus having at least one fixed point. This book will be of interest to mathematicians working in the field. (vs)

L. Lipshitz and Z. Robinson, ***Rings of Separated Power Series and Quasi-Affinoid Geometry***, *Astérisque **264**, Société Mathématique de France Paris, 2000, 171 pp., FRF 250, ISBN 2-85629-084-1*

The main topic of this book is rigid analytic geometry over an ultrametric field *K*, based on the commutative algebra of power series rings $S_{m,n}$. The first chapter is the longest (more than 100 pages), and contains the first systematic treatment of the ring $S_{m,n}$ of separated power series and the local theory of quasi-affinoid varieties. The results collected in the first part are then used in the next three chapters (about 20 pages each). The main results of Chapter 2 are elimination theorems and their applications to the theory of sub-analytic sets. In Chapter 3, the authors define a category of quasi-affinoid varieties and build the fundamentals of a sheaf theory: the main result here is the quasi-affinoid acyclicity theorem. Chapter 4 (written by Z. Robinson) contains a rigid analytic approximation theorem, linking properties of affinoid and quasi-affinoid algebras. (vs)

M. Lothaire, ***Combinatorics on Words***, *Cambridge Mathematical Library, 1997, 238 pp., £17.95, ISBN 0-521-59924-5 and 0-201-13516-7*

The field of combinatorics on words has developed immensely since the appearance of Lothaire's first book (*Combinatorics on Words*, 1983). The present book will be welcomed by many mathematicians because it provides a unified exposition of various important topics that were previously available mainly in journals. Many results are presented together in a broader algebraic framework.

An introductory chapter includes the basics of automata theory and topics on generating series and symbolic dynamical systems. Each subsequent chapter is self-contained and covers important basic results before turning to more advanced material. This makes the book suitable for introductory reading, as well as a possible teaching text that can be used at the advanced undergraduate level. Each chapter ends with problems, ranging from easy exercises to difficult open questions, and notes provide the necessary bibliographical and historical background. The topics range from a deep treatment of classical subjects to theories that did not appear in the previous book. Among the most basic topics are codes (enriched by ω-codes) and periodicity, while the other end of the spectrum can be represented by a rather specialised combinatorial subject of sesquipowers. An exposition of independent systems of equations concentrates around the defect effect and the compactness property: these notions are then considered with respect to general semigroups. Then there is unavoidability – an area with many beautiful results; in the Preface the authors state that a pattern is unavoidable if there exist infinitely many words that do not encounter this pattern: this is a mistake, since in this way one defines an avoidable pattern. Two chapters are dedicated to the combinatorial properties of permutations that are considered as words; this topic includes statistics on words. The basics of *q*-calculus are also explained and applied. One chapter is devoted to Makanin's algorithm, designed to determine whether a word equation with constants has a solution; the algorithm is very complicated and this treatment is probably the best available. Recently, new approaches to this area have appeared, and this chapter can thus be regarded as an exposition of a classical theory. An algebraic generalisation of words to polynomials and power series is suitable for the study of commutation properties. The most elementary connection between words and number theory is that of the positional number system. An interesting modification of this classical notion appears when an arbitrary real is taken for the base. The resulting β-expansions have many nice properties that are described in the second part of the chapter. Sturmian words form an attractive topic that has aroused much interest in recent decades. One chapter deals with the plactic monoid, which is the algebraic structure found behind Young tableaux; this theory has many significant applications in various areas.

This is an excellent book, essential for anybody working in the field. Although written by several authors, who constitute the collective author M. Lothaire, the book makes a surprisingly compact impression. As already mentioned, it has all prerequisites for an important reference tool. (ad, šh)

J. McCleary, ***A User's Guide to Spectral Sequences***, *Cambridge Studies in Advanced Mathematics **58**, Cambridge University Press, Cambridge, 2001, 561 pp., £21.95, ISBN 0-521-56141-8 and 0-521-56759-9*

The spectral sequence is a rather complicated algebraic structure needed for dealing with topological problems. The book is consequently divided into three parts. The first part, *Algebra*, presents the fundaments of filtered differential graded algebras and modules, exact couples and double complexes, in the framework of homological algebra; it also gives a gentle introduction to the first quadrant spectral sequence, its convergence and various comparison theorems. The second part, *Topology*, deals with applications of the spectral sequence in homotopy theory, with an account of Leray-Serre, Eilenberg-Moore, Cartan-Leray, Lyndon-Hochschild-Serre, Bockstein, Adams, and many other spectral sequences. More advanced topics include nilpotent spaces, the homology of groups and *H*-spaces. The third part, *Sins and Omissions*, provides a concise list of examples and applications of spectral sequences in other branches of mathematics, such as commutative algebra, algebraic geometry, algebraic *K*-theory, analysis, and even mathematical physics. A particularly nice feature of the book is that it contains many explicitly solved examples, as well as many exercises. (pso)

R. R. Phelps, ***Lectures on Choquet's Theorem, 2nd edn.***, *Lecture Notes in Mathematics **1757**, Springer, Berlin, 2001, 124 pp., DM 49,11, ISBN 3-540-41834-2*

This is the second edition of Phelps' monograph, originally published by Van Nostrand in 1966. The present text is a somewhat revised and expanded version of the first edition and provides a very readable introduction to Choquet theory and its applications.

Let us describe the main additions in the second edition. As an application of the Choquet-Bishop-de Leeuv theorem, Haydon's theorem on an extreme point criterion for separability of a dual Banach space is proved. Chapters 9 and 11 are new: Korovkin sets in $C(X)$ are characterised in Chapter 9, which gives an abstract and elegant approach to Korovkin's theorem, while in Chapter 11, properties of the resultant map are studied and a selection theorem of M. Rao and G. Vincent-Smith is proved.

The final chapter, Additional Topics, is expanded. It contains a survey of recent literature, as well as five new sections on topics not covered in the first edition: the Poulsen simplex, a geometrical characterisation of simplices, faces and Edward's separation theorem, unique representations in the complex cases and Choquet theorems for non-compact sets. No proofs are included, but results are stated and explained and references are provided. The list of references in the book has increased from 38 items to 83, but unfortunately, the page references in the index are incorrect.

The book will be appreciated by a broad mathematical community and is warmly recommended, especially to mathematicians interested in modern analysis. (in)

A. O. Pittenger, ***An Introduction to Quantum Computing Algorithms***, *Progress in Computer Science and Applied Logic **19**, Birkhäuser, Boston, 2000, 138 pp., DM 98, ISBN 0-8176-4127-0*

In the field of quantum computing, there are two major parts – the theory of quantum computing and an implementation (realisation) of quantum computers: both are based on quantum mechanics. But quantum mechanics has two faces as well – the physical theory, describing certain part of physical world, and the mathematical theory, which has its own structure and rules and is not directly related to real world.

The present book is a purely mathematical presentation, written for computer scientists. It is a brief and very clearly written introduction to quantum computing algorithms, containing only the strictly necessary parts of the mathematics of quantum theory. In the first chapter, there is a simplest possible description of the states of a quantum computer (qubits) and the operations used to transform these states (unitary transformations): this is the basis of a (mathematical) quantum theory which uses finite-dimensional complex Hilbert spaces and unitary operators acting on them. In the second chapter, elementary instruments used in quantum computing are introduced: tensor products of Hilbert spaces, quantum gates and quantum subroutines, including a teleportation circuit. The basic strategy of quantum computing – quantum parallelism based on the superposition principle – is introduced. Quantum algorithms, such as the Deutsch-Jozsa, Simons, Grower and Shors algorithms, are described in Chapter 3, where there is also a description of the finite Fourier transform; this main part of the book is written in a clear algorithm-oriented style. Each algorithm is written as a sequence of steps and the procedure is then analysed. Each quantum algorithm contains unitary and projection parts, and both are clearly defined. The last chapter contains a topic usually omitted from brief descriptions of quantum computing – quantum error-correcting codes. The importance of automatic error-correction for the construction of computers is well known. In the construction of a quantum computer, the construction of error-correcting codes is the first necessity. The treatment is purely mathematical and includes a discussion of the theory of abstract quantum error-correcting codes. (jis)

Qing Liu, *Algebraic Geometry and Arithmetic Curves*, *Oxford Graduate Texts in Mathematics **6**, Oxford University Press, 2002, 576 pp., £45, ISBN 0-19-850284-2*

This book introduces (arithmetic) algebraic geometry in the language of schemes.

The first part has seven chapters. The first two introduce basic notions (scheme, morphism, tensor product, flatness, formal completion, etc). In Chapter 3, the fibered product and base change for schemes are studied. Chapter 4 treats local properties of schemes and morphisms, such as normality and smoothness. Global aspects of schemes through coherent sheaves and their Čech cohomology groups are discussed in Chapter 6, which deals with particular coherent sheaves – the sheaf of differentials and the relative dualising sheaf, culminating in Grothendieck duality theory. Chapter 7 starts with divisor theory, quickly passing to the Riemann-Roch and Hurwitz theorems, and concluding with a detailed study of the Picard group of (generally singular) projective curves over an algebraically closed field.

The general theory appears in the second part, with a study of arithmetic surfaces and the reduction of algebraic curves. Chapter 8 begins with blow-ups and, after digressing to Cohen-Macaulay, Nagata and excellent rings, proceeds to the desingularisation of (arithmetic) surfaces. Chapter 9 treats intersection theory on arithmetic surfaces. The final chapter studies the reduction theory of algebraic curves and related problems of stable reduction.

This book will be useful to graduate students as an introduction to arithmetic algebraic geometry, and to more advanced readers and experts in the field. (pso)

B. V. Rajarama Bhat, G. A. Elliott and P. A. Fillmore (eds.), *Lectures on Operator Theory*, *Fields Institute Monographs **13**, American Mathematical Society, Providence, 1999, 323 pp., US$69, ISBN 0-8218-0821-4*

This book is a collection of articles on operator algebras and their applications, most of which are records of lectures presented during a two-semester research programme at the Fields Institute in Waterloo, Ontario, in 1994-5.

The first part is devoted to C^*-algebras. It starts from basic definitions and includes discussions of positivity in C^*-algebras, K-theory, tensor, crossed and free products, dilation theory and relations to several complex variables and mathematical physics. Part 2 contains papers on von Neumann algebras, including an introduction with emphasis on II1 factors, an exposition of Connes' fundamental results, introductory topics in the theory of subfactors and an introduction to Voiculescu's work on free products. Five papers in the third part are devoted to classifications of various types of C^*-algebras (AF-algebras, amenable C^*-algebras, and simple purely infinite C^*-algebras). Part 4 contains classification results for a class of hereditary subalgebras of certain simple non-real rank 0 C^*-algebras, based on the Ph.D. thesis of I. Stevens. The last 80 pages provide a record of lectures by A. Ocneanu on paths on Coxeter diagrams and their relations to minimal models and subfactors. There is no overall bibliography: short reference lists are included in most of the individual lectures. (vs)

R. Rebolledo (ed.), *Stochastic Analysis and Mathematical Physics*, *Trends in Mathematics, Birkhäuser, Boston, 2000, 166 pp., DM 128, ISBN 0-8176-4185-8 and 3-7643-4185-8*

These Proceedings reflect recent research in the area of quantum probability, related to a seminar on Stochastic Analysis and Mathematical Physics at the Catholic University of Chile in Santiago, mostly presented at the ANESTOC '98 Workshop. The main topics include quantum flows and semigroups, domains of these semigroups, the conservativity property (preserving of the unit), asymptotic analysis, quantum anharmonic oscillators and oscillator algebras and Fock spaces – in particular, exponentials of quadratic field polynomials of field operators on these spaces. In addition, some problems of classical stochastic analysis (Feller semigroups, commutative wave maps, Korovkin operators) are discussed, partly motivated by the quantum point of view. A contribution on Bernstein processes finishes this volume.

This book is aimed at mathematical physicists and probability and operator algebraists. The contributing authors are R. Carbone, A. M. Chebotarev, J. C. García, R. Quezada, M. Corgini, A. B. Cruzeiro, L. Wu, J. C. Zambrini, F. Fagnola, C. Fernández, K. B. Sinha, A. Guichardet, E. B. Nielsen, O. Rask, R. Rebolledo, J. A. Van Casteren and W. von Waldenfels. (mzahr)

A. M. Robert, *A Course in p-adic Analysis*, *Graduate Texts in Mathematics **198**, Springer, New York, 2000, 437 pp., DM 109, ISBN 0-387-98669-3*

This is a beautifully written introduction to p-adic analysis and to the p-adic world in general. From a p-adic point of view, it starts from scratch, assuming no knowledge of p-adic numbers.

The rather lengthy introductory chapter introduces these numbers and develops their properties. Chapters 2 and 3 treat the algebra of p-adic numbers, dealing with finite extensions of the field of p-adic numbers and with the universal p-adic fields; these chapters assume a familiarity with linear algebra and some ring and field theory. Chapters 4, 5 and 6 constitute the core of the book. Here the author proceeds in analogy with the classical real and complex analysis, emphasising the similarities and differences between classical and p-adic analysis; even readers with only a knowledge of basic calculus will understand much of this. The last chapter is devoted to arithmetic applications (special functions and congruences).

Each chapter concludes with a series of exercises. The book is very attractive, contains recent results, and will be interesting both to specialists in the p-adic world and to mathematicians working in analysis, number theory, algebraic geometry and algebraic topology. (jiva)

P. Rothmaler, *Introduction to Model Theory*, *Algebra, Logic and Applications Series **15**, Gordon and Breach, Singapore, 2000, 305 pp., $38, ISBN 90-5699-313-5*

This book is an introduction to model theory of first-order logic and its applications to algebra. The first four parts cover the necessary topics of pure model theory, such as compactness, axiomatisable classes, ultraproducts, elimination, chains, types, omitting types, saturation, atomic models and categoricity. There is also an exposition of ordinal and cardinal numbers, and some classic important algebraic applications, such as Hilbert's Nullstellensatz and a Chevalley theorem about projections of constructible sets. The last part is oriented algebraically and consists of two chapters. The first of these, Strongly minimal theories, deals with models of such theories, which can be seen as a natural generalisation of the theory of algebraically closed fields; a notion of dimension plays a substantial role here. The other treats a concrete theory – a complete theory of abelian groups of integers. There are illuminating remarks and exercises in the text and suggestions for further reading. Some elementary algebra is assumed, but a knowledge of mathematical logic is not required. (jmlč)

K. Saxe, *Beginning Functional Analysis*, *Undergraduate Texts in Mathematics, Springer, New York, 2002, 197 pp., EUR 44,95, ISBN 0-387-95224-1*

The author offers a course of functional analysis for beginners in less than 200 pages; the prerequisites are a basic knowledge of linear algebra and real analysis. Lebesgue integration is explained, followed by the elements of metric, normed and inner product spaces. The other three chapters are devoted to Fourier analysis in Hilbert spaces, elements of linear operator theory, and the Stone-Weierstrass theorem, together with three principles of linear functional analysis. Two appendices deal with complex numbers and basic set theory. All chapters are accompanied by numerous exercises (more than 160 in total), where the reader is sometimes asked to fill in details omitted from the text. Short biographies of Fréchet, Hilbert, Banach, Lebesgue, F. Riesz, Fourier, Enflo, Stone and J. von Neumann are included, as well as 130 references and a short index.

The author's style is stimulating, and she includes numerous comments on the development of the discipline. The book can also serve (rather early) for reading in a seminar for students with a deeper interest in mathematics. (jive)

M. Ó. Searcóid, *Elements of Abstract Analysis*, *Undergraduate Mathematics Series, Springer, London, 2002, 298 pp., EUR 34,95, ISBN 1-85233-424-X*

This book covers the elements of abstract analysis, treated mostly within the context of topological spaces. The author's aim is to build up the theory from axioms and to provide an overview of known notions and methods, providing a solid background for functional analysis. Thus, the basic concepts of convergence, continuity, completeness, countability, connectedness and compactness are studied at a greater level of abstraction. This is a carefully written book, starting from set-theoretic axioms, and containing about 180 problems and exercises with hints and solutions. It is an ideal source of information for independent student study. There is a comprehensive index. (jive)

M. J. Shaiharan, *The Mysteries of the Real Prime*, *London Mathematical Society Monographs New Series **25**, Clarendon Press, Oxford, 2001, 240 pp., £45, ISBN 0-19-*

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The mysterious real prime in the title is the 'prime' responsible for the completion of rationals leading to the real numbers. Instead of trying to base the theory of p-adic numbers on the reals, the author does the opposite, by 'implementing' the p-adic attitudes to the reals to 'unveil' the mysteries of this prime. The result is a highly original monograph interweaving many topics and requiring from the reader a broad range of knowledge, such as Markov chains, Heisenberg group and quantum algebra, q-series and various special functions. The book ends, as the author writes in the introduction, 'By giving a heuristic proof of the Riemann hypothesis. This proof works in the function fields case and shows that the proof of the Riemann hypothesis is distinctly related to the mysteries of the real prime'.

This book provides interesting and inspiring reading, fusing various original observations, and as such can be recommended to anyone interested in the interplay of a wide range of mathematical ideas. (špor)

I. Stewart and D. Tall, ***Algebraic Number Theory and Fermat's Last Theorem, 3rd edn.,*** *A. K. Peters, Natick, 2002, 313 pp., US$38, ISBN 1-56881-119-5*

This edition is completely revised edition from the second one from 1987. In the first part, the authors develop the basic material of algebraic number theory – factorisation, ideals and algebraic number fields. The second part is devoted to geometric methods (lattices, Minkowski's theorem, the geometric representation of algebraic numbers, the finiteness of class group). In Part III these tools are applied to show the reader the approach to Fermat's last theorem, based on the development of ideas started by Kummer. The last part is devoted to Wiles' breakthrough.

This book cannot, of course, present the full proof, but the reader can find a description and introduction to the milestones on 'the path to the final breakthrough'. There are two appendices – on quadratic residues and on Dirichlet's unit theorem. The chapters include many exercises. As its predecessors, the book is written in a nice readable style, supplemented with lively historical comments. It can be warmly recommended to anybody interested in algebraic number theory. (špor)

J. Stillwell, ***Mathematics and its History*****, *2nd edn.,*** *Undergraduate Texts in Mathematics, Springer, New York, 2002, 542 pp., 181 fig., EUR 59,95, ISBN 0-387-95336-1*

The present edition of this book, first published in 1989, has several improvements: it contains three new chapters and many additional exercises, and bibliographical data and the index have been enlarged.

This book is not an attempt to cover the history of every part of mathematics, but it deals with those that are not treated in detail in other similar books. Although it contains 36 biographical items, the book is focused on the mathematics, which is presented in a lively style without unnecessary detail. It is very stimulating and will be appreciated not only by students. Much attention is paid to problems and to the development of mathematics before the end of the nineteenth century and of an 'algebraic nature', but selected topics from geometry and analysis are fairly covered: the roots of analytic and projective geometry, calculus, elliptic functions, the problem of the vibrating string, non-Euclidean geometry, algebraic number theory, geometric topology, sets, measure, logic and computability.

This book brings to the non-specialist interested in mathematics many interesting results. It can be recommended for seminars and will be enjoyed by the broad mathematical community. (jive)

H. P. F. Swinnerton-Dyer, ***A Brief Guide to Algebraic Number Theory*****,** *London Mathematical Society Student Texts* ***50****, Cambridge University Press, Cambridge, 2001, 146 pp., £15.95, ISBN 0-521-80292-X and 0-521-00423-3*

This book provides a brief introduction to some aspects of algebraic number theory. Starting with basic tools (ideals, factorisation, discriminant and valuations), it includes basic properties and results concerning idèles and adèles. Basic notions and results are then illustrated using basic algebraic number fields of degree 2, 3, 4, and cyclotomic fields. The booklet culminates with analytic tools, such as zeta functions, L-series, class field theory, the general reciprocity law and the Kronecker-Weber theorem. The reader is expected to be familiar with basic results on finite extensions, Galois theory, etc. However, the book contains appendices with basic facts about such useful topics as Haar measure, Fourier transforms and the Galois theory of infinite extensions.

The reader will find in this subtle booklet much relevant material in this important part of number theory, which has influenced many areas of mathematics. It is written in a nice (but necessarily concise) style, containing exercises to broaden the reader's knowledge and skills. The result is a very solid introduction to the subject. (špor)

P. Tondeur, ***Collected Papers of K.-T. Chen*****,** *Contemporary Mathematicians, Birkhäuser, Boston, 2001, 737 pp., DM 460, ISBN 0-8176-4005-3 and 3-7643-4005-3*

The results of Kuo-Tsai Chen are very well known among topologists and differential geometers. His deeply original and bright ideas have brought us the theory of iterated integrals and principal bundles with the structure group being the 'Lie group' of formal power series, together with the theory of connections on these bundles. He has very effectively and explicitly used these techniques to obtain fundamental results on the cohomology of loop spaces. His approach is closely related to the theory of minimal models, developed by D. Sullivan, and it is known that Sullivan was partially inspired by Chen's work.

This book is the complete collection of Chen's papers, presented in their original form. (The papers were reproduced photographically.) An introductory article on his life and work was written by Richard Hain and Philippe Tondeur, famous specialists in the field. Included are lists of Chen's papers and of his students. (jiva)

W. T. Tutte, ***Graph Theory*****,** *Encyclopedia of Mathematics and its Applications* ***21****, Cambridge University Press, Cambridge, 2001, 333 pp., £19.95, ISBN 0-521-79489-7*

This is a classic textbook (first published by Addison-Wesley in 1984), written by one of the pioneers and most distinguished contributors to the field, and dealing with many of the central themes of graph theory: Menger's theorem and network flows, the reconstruction problem, the matrix-tree theorem, the theory of factors (or matchings) in graphs, chromatic polynomials, Brooks' theorem, Menger's theorem, planar graphs and Kuratowski's theorem. The treatment of all these topics is unified by the author's highly individual approach: this is not just another textbook in graph theory!

This book has much to offer to any reader interested in graph theory. It draws together some important themes from the rapidly growing literature on the subject, while also providing an excellent preparation for some slightly more specialised topics, such as the theory of planar enumeration and chromatic polynomials of maps, Tutte's theorem on Hamiltonian circuits in 4-connected planar graphs, the five-flow conjecture, and the interaction between matroids and graphs. (jtu)

A. Vasy, ***Propagation of Singularities in Three-Body Scattering*****,** *Astérisque 262, Société Mathématique de France, Paris, 2000, 151 pp., FRF 150, ISBN 2-85629-082-5*

Excerpts from the summary: The author takes a compact manifold X with a boundary equipped with a scattering metric g and with a collection of disjoint closed embedded submanifolds of ∂X. The Hamiltonian $H = \Delta + V$ is then considered, where Δ is the Laplacian of g and V is a sufficiently smooth potential, vanishing at the lift of ∂X. Three-body scattering of Coulomb type provides the standard example of such a set-up. The author then analyses the propagation of singularities of generalised eigenfunctions of H, showing that this is essentially a hyperbolic problem having some similarities with the Dirichlet and transmission problems for the wave operator.

The book is divided into nineteen chapters, named (slightly shortened) introduction, differential operators, three-body scattering calculus, restriction to the boundary, composition of operators, the normal operator, commutators, mapping properties, wavefront set, functional calculus, Hamiltonian, Mourre estimate, the basic commutator estimate, propagation of singularities in normal (resp. tangential) directions, bound states with negative energy, radial sets, the resolvent, and the scattering matrix. Three appendices (on the construction of plane waves near the initial point, the absence of positive eigenvalues, and positive operators) conclude the book, which is based on the author's doctoral thesis, submitted at MIT. It will be of interest to specialists in scattering theory. (mzahr)

W. D. Wallis, ***A Beginner's Guide to Graph Theory*****,** *Birkhäuser, Boston, 2000, 230 pp., DM 78, ISBN 0-8176-4176-9*

This book is yet another contribution to the growing literature on graphs. The material covered is standard and the presentation stays on the level, which corresponds to the state of the art in the 1960s. Thus, for example, the minor vertex is introduced, but neither minor nor matroid are mentioned. On the other hand, the general Ramsey theorem is stated (but not proved), so the selection of topics seems to be somewhat unbalanced. But it is a nicely written book, which contains enough material for a semester course. It includes computational considerations (Chapter 13) and some short hints, answers and solutions as an appendix. (jneš)