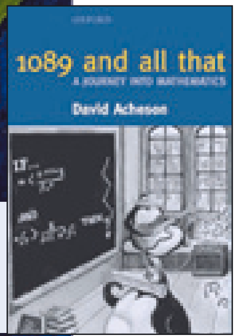
European Mathematical Society

September 2003 *Issue 49*

Feature
1089 and all that



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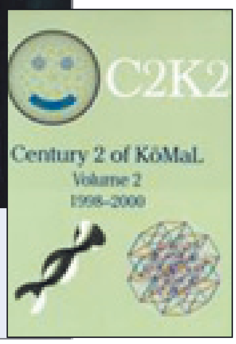
Problem Corner
KöMaL



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NEWSLETTER

EUROPEAN MATHEMATICAL SOCIETY

NEWSLETTER No. 49

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NOTICE FOR MATHEMATICAL SOCIETIES

Labels for the next issue will be prepared during the second half of November 2003. Please send your updated lists before then to Ms Tuulikki Mäkeläinen, Department of Mathematics, P.O. Box 4, FIN-00014 University of Helsinki, Finland; e-mail: makelain@cc.helsinki.fi

INSTITUTIONAL SUBSCRIPTIONS FOR THE EMS NEWSLETTER

Institutes and libraries can order the EMS Newsletter by mail from the EMS Secretariat, Department of Mathematics, P. O. Box 4, FI-00014 University of Helsinki, Finland, or by e-mail: (makelain@cc.helsinki.fi). Please include the name and full address (with postal code), telephone and fax number (with country code) and e-mail address. The annual subscription fee (including mailing) is 80 euros; an invoice will be sent with a sample copy of the Newsletter.

EMS Committee

EMS Agenda

2003

12–14 September
SPM-EMS Weekend Meeting at the Calouste Gulbenkian Foundation, Lisbon (Portugal)
Organiser: Rui Loja Fernandes, e-mail: rfern@math.ist.utl.pt

14–15 September
EMS Executive Committee meeting at Lisbon (Portugal)
Contact: Helge Holden, e-mail: holden@math.ntnu.no

15 November
Deadline for submission of material for the December issue of the EMS *Newsletter*
Contact: Martin Raussen, e-mail: raussen@math.auc.dk

2004

1 January
Closing date for nominations for delegates to EMS Council to represent individual members
Contact: Tuulikki Mäkeläinen, e-mail: tuulikki.makelainen@helsinki.fi

1 February
Deadline for nominations for the EMS Prizes and the Felix Klein Prize, to be awarded at 4ecm
Nominations for the EMS Prizes to 4ecm Organising Committee, Prof. Ari Laptev, Department of Mathematics, Royal Institute of Technology, SE-100 44 Stockholm, Sweden
Nominations for the Felix Klein Prize to EMS Secretariat, Tuulikki Mäkeläinen, Department of Mathematics, P.O.Box 4 (Yliopistonkatu 5), FIN-00014 University of Helsinki, Finland

15 February
Deadline for submission of material for the March issue of the EMS *Newsletter*
Contact: Martin Raussen, e-mail: raussen@math.auc.dk

20 February
Deadline for proposals for EMS Lectures, EMS Joint Mathematical Weekends and EMS Summer Schools (see announcement on page 4)
Contact: Luc Lemaire, e-mail: llemaire@ulb.ac.be

25 June
EMS Executive Committee meeting at Stockholm (Sweden)
Contact: Helge Holden, e-mail: holden@math.ntnu.no

26–27 June
EMS Council meeting at Uppsala (Sweden)
Contact: Helge Holden, e-mail holden@math.ntnu.no
or Tuulikki Mäkeläinen, e-mail: tuulikki.makelainen@helsinki.fi

27 June–2 July
4th European Congress of Mathematics, Stockholm
webpage: http://www.math.kth.se/4ecm

30 August–3 September
EMS Summer School at Universidad de Cantabria, Santander (Spain)
Empirical processes: theory and statistical applications

Editorial

Robin Wilson

In October 1998 I received a letter from Jean-Pierre Bourguignon, then President of the EMS, inviting me to become Editor-in-Chief of the *EMS Newsletter*. I was about to go on a lecture tour of Denmark, and coincidentally there was an EMS Executive Committee meeting in Copenhagen during my visit, so I was able to meet the Committee and find out what the job entailed.

Now, almost five years on, and with nineteen issues totalling over 800 pages behind me, I am hanging up my pen and handing over to a worthy successor, Martin Raussen of the University of Aalborg in Denmark, who has been editing *Matilde*, the newsletter of the Danish Mathematical Society. He and I are jointly producing this issue and the next, and he will take over sole responsibility from the beginning of 2004.

The last five years have been a time of change for the EMS. With the new 1999 team of Rolf Jeltsch as EMS President, David Brannan as Secretary, Olli Martio as Treasurer, and Andrzej Pelczar (and later, Bodil Branner) joining Luc Lemaire as Vice-President, the EMS became increasingly busy, including the following among its many activities:

- hosting *3ecm* (3rd European Congress of Mathematics) in Barcelona in 2000, at which ten EMS prizes for young mathematicians were awarded;
- setting up the European Mathematical Foundation and the EMS Publishing House;
- publishing five volumes of its new journal, *JEMS*;
- being an active participant in World Mathematical Year 2000;
- holding joint meetings on applied mathematics with SIAM (Berlin), SMF/SMAI (Nice) and SIAM/IPAM (Los Angeles);
- holding Diderot Mathematical Forums on *Mathematics and Music* (Paris, Vienna and Lisbon) and *Mathematics and Telecommunications* (Eindhoven, Helsinki and Lausanne);
- organising EMS Summer Schools in the Czech Republic, France, Germany, Israel, Italy, Portugal, Romania, Russia and Scotland;
- organising EMS lecture series by M. Lyubich (St Petersburg, Lyngby and Barcelona), G. Papanicolau (Zurich and Heraklion), M. Vergne (Malta and Rome) and G. Dal Maso (Leipzig and Paris);
- holding two workshops in Berlingen (Switzerland);
- creating reciprocity agreements with several other mathematical societies world-wide;
- providing strong support for the setting up of the Abel prize, awarded for the first time this year;
- welcoming Sir John Kingman (Cambridge) and Helge Holden (Trondheim) as new President and Secretary from 2003.

The *Newsletter* has also undergone change, building on the legacy of the previous editor, Martin Speller, and his team at Glasgow Caledonian University. Following on from their introduction of the now-familiar multicoloured covers in early 1998, we adopted a three-column format from 1999 and included pictures for the first time. The credit for this 'new look' is due to our printer, Jan Kosniowski of Armstrong Press, Southampton, UK, who took on the additional job of designer. He has been a tower of strength throughout and I wish to pay tribute to him for all that he has been doing for the Society and the *Newsletter* over the past five years.

Looking back over the past nineteen issues, it is incredible how much has appeared in its pages. Starting with contributions by Ian Stewart and the late John Fauvel in 1999, we have had a regular stream of feature articles on topics ranging from the early history of the Society, mathematics in biology, and the Hilbert problems, to women and men in mathematics, Pierre de Fermat, and the future role of mathematics in Europe. In addition, there have been interviews with about two dozen distinguished mathematicians, including Tim Gowers, Sergei Novikov, Sir Roger Penrose, Bernhard Neumann, Dmitry Anosov, and Helmut Neunzert.

Newly featured were the *Anniversary articles*, mainly written by our two specialist editors, Jeremy Gray and June Barrow-Green, while Paul Jainta's popular *Problems corners* have continued to include challenging problems from high-school mathematical competitions, usually in Eastern Europe. Another popular series, organised by Krzysztof Ciesielski, includes information about the various national mathematical societies: over twenty of these have appeared since 1999, ranging from Luxembourg to Latvia, Poland to Portugal, and Switzerland to Slovenia.

Other regular features have been the *Forthcoming conferences* section, ably produced by Kathleen Quinn, and more recently by Vasile Berinde, and the extensive *Recent books* section, organised by Ivan Netuka and Vladimír Souček, which incredibly has included reviews of more than eight hundred books since the beginning of 1999. To all these specialist editors and other writers may I say: 'Thank you for your valuable contributions over the past five years: the EMS owes you all a great debt of gratitude'.

Since 1999 I have much enjoyed my involvement with the members of the Executive Committee, attending working weekends in a variety of pleasant venues – the most memorable being a weekend on a ship from Stockholm to Helsinki and back. I have met many fine people at these meetings, and thank them for the continued help and support they have given me: the EMS is in very good hands. I hope to continue my association with the EMS in the future, as an Associate Editor of the *Newsletter* and an EMS Council member.

Finally, it remains for me to welcome Martin Raussen to the Editorship of the EMS *Newsletter*. With his experience of editing mathematical newsletters, and with some exciting new ideas (which he will communicate in due course) the *EMS Newsletter* will continue to thrive in the coming years. I wish him well.

Robin Wilson [r.j.wilson@open.ac.uk] is Head of the Pure Mathematics Department at the Open University, UK.

EMS CALL FOR PROPOSALS

• EMS LECTURES
• EMS JOINT MATHEMATICAL WEEKENDS
• EMS SUMMER SCHOOLS IN FUNDAMENTAL AND INTERDISCIPLINARY MATHEMATICS

The European Mathematical Society is launching a new call for proposals for three activities: EMS lectures, mathematical joint weekends, and summer schools.

The deadline for this call is **20 February 2004**, by e-mail at the address: *llemaire@ulb.ac.be*. However, proposals will be welcome at any time. They need not be in final form, but should include ideas of subject, location, date, main speakers.

The proposals will be examined by the 'General meetings committee' and the scientific panel of the EMS

The deadline will allow the EMS to present a coherent proposal of activities for EU funding, thereby allowing organisers of single meetings to be part of a series of events. EMS direct support being limited, the result of this application will bear on the funding for the meetings selected by EMS. There will be similar calls each year in the future.

Here is a description of these three activities. For any question or tentative project, please contact *llemaire@ulb.ac.be* at any time.

• EMS LECTURES

The EMS is calling for proposals of EMS lectures, in the following new format.

The idea of the EMS lectureship is to allow an institution inside the EMS area to invite a distinguished mathematician (in pure or applied mathematics) to give a series of lectures, and build a small conference around his/her presence.

Typically, he/she would give between 4 and 8 lectures, complemented by talks of the participants to the meeting.

The lectures of the main speaker should lead to a publication in an EMS Lecture Notes Series.

An application should be introduced by a European institution, with agreement of the main lecturer, and include some plan of the meeting built around the course.

The EMS will cover the travel expenses of the main speaker, and a lecture fee upon submission of a manuscript. It will help to obtain support for the meeting, provided that it has a European dimension in participation. The preceding EMS lecturers (in a somewhat different format) have been Professors H. W. Lenstra (Berkeley), M. J. Cutland (Hull), M. Lyubich (Stony Brook), G. Papanicolau (Stanford), M. Vergne (Palaiseau) and G. Dal Maso (SISSA, Trieste).

The aim is to maintain the rhythm of one such course per year, and to help dissemination and development of cutting edge subjects.

• EMS MATHEMATICAL WEEKENDS
(Joint meetings of the EMS with regional or national societies)

The EMS has launched a new format of joint meetings with its corporate member societies, following the model set out by the Portuguese Mathematical Society in the meeting in Lisbon from 12-14 September 2003 (see *http://www.math.ist.utl.pt/ems/*).

These 'EMS joint mathematical weekends' will start on Friday, and finish on Sunday, both at lunchtime, so that they can be easily attended during term-time.

Each would cover around four subjects, chosen by the local organisers to fit the research strengths of the local mathematicians, or new subjects they would want to develop.

For each subject, a plenary lecture and two half-days of parallel sessions will be organised.

Past experience shows that such an internationalisation of the meetings of national societies helps substantially to increase participation.

The EMS will help with scientific organisation, publicity and funding applications.

With more than fifty corporate members, the EMS hopes to see regular meetings of this format. Note that mathematics departments or individual members can also plan such meetings.

• EMS SUMMER SCHOOLS IN FUNDAMENTAL AND INTERDISCIPLINARY MATHEMATICS

The EMS will pursue its programme of Summer schools, aiming at running such schools in pure and applied mathematics.

This call for proposals concerns all schools that any group of mathematicians would like to run in 2005 or later years.

The guidelines for such events are that there must be a very strong component of training of young researchers (in the first 10 years of their career) by means of integrated courses and lectures at advanced level.

This can be supplemented by conference-type research lectures, but the training component is needed.

The courses should aim at an international audience: no more than 30% of the participants should come from a single state.

The EMS will help with advertisement and organisation, as well as the applications for financial support.

Luc LEMAIRE: phone, 00-32-2-6505837; e-mail, llemaire@ulb.ac.be

Call for nominations for the Felix Klein Prize 2004

Principal guidelines

The prize, established in 1999 by the EMS and the endowing organisation, the Institute for Industrial Mathematics in Kaiserslautern, is awarded to a young scientist or a small group of young scientists (normally under the age of 38) for using sophisticated methods to give an outstanding solution, which meets with the complete satisfaction of industry, to a concrete and difficult industrial problem.

Nomination of the award

There are no restrictions on eligibility other than those specified in the Principal Guidelines.

The prize committee is responsible for solicitation and evaluation of nominations. Nominations may be made by anyone, including members of the prize committee or by candidates themselves. It is the responsibility of the nominator to provide all relevant information to the prize committee, including a resumé and documentation of the benefit to industry and the mathematical method used.

The nomination for the award must be accompanied by a written justification and a citation of about 100 words that can be read at the award date. The prize is awarded to a single person or to a small group and cannot be split.

Description of the award

The award comprises a certificate containing the citation and a cash prize of 5000 euro.

Award presentation

The prize is presented every four years at the European Congress of Mathematics. A representative of the endowing Institute for Industrial Mathematics in Kaiserslautern or the president of EMS presents the award. The recipient is invited to present his or her work at the conference.

Prize history

The first Felix Klein Prize was awarded to David C. Dobson (USA) in 2000 during the 3rd European Congress of Mathematics in Barcelona.

Deadline for submission

Nominations for the Prize to be presented at the 4th European Congress of Mathematics, Stockholm (Sweden), 27 June – 2 July 2004, must reach the following address no later than 1 February 2004: EMS Secretariat, Ms. Tuulikki Mäkeläinen, Department of Mathematics, P.O. Box 4 (Yliopistonkatu 5), FI-00014 University of Helsinki, Finland.
Fax: +358-9-1912-3213; e-mail: *tuulikki.makelainen@helsinki.fi*

European Mathematical Society

Comments on the Communication from the Commission:

The role of the universities in the Europe of knowledge

(http://europa.eu.int/eur-lex/en/com/cnc/2003/com2003_0058en01.pdf)

The EMS welcomes the debate launched by the Commission on the role of universities in the future of Europe.

It sees as a paradox the fact that so many political statements press for a future development related to research and higher education, while at the same time financial cuts are seen everywhere in the public sector.

The Communication of the Commission hardly calls for the member states to invest more in their universities (e.g. in the framework of the raise to 3% of the budget of research), or calls for a shift in the European Union's budget towards research and education needs for the future.

Rather, it seems to take as granted that university support will become insufficient, and therefore asks how the universities will handle the situation. In fact, universities in Europe are now asked:

- to provide quality teaching while preserving the model of democratic access to higher education and lifelong learning
- to pursue excellence in research (like the leading American research universities)
- to support innovation and regional economic development
- to find the necessary financial resources outside of public funding and without resort to high enrolment fees for the student.

Briefly put, they are asked to be excellent, of democratic access and more and more self supporting.

In the present debate, the EMS wishes to focus on some basic points.

Fundamental research and teaching

This is the core business of universities, and the communication stresses that 80% of fundamental research in Europe is issued from its universities. Thus the universities are quite efficient (in difficult financial situation) not only in their role of linking research and training, but in research itself.

The vast potential that this research represents is currently under threat, because it does not satisfy short-term profitability criteria. However, it is of course the key to future innovation.

Some programmes of the Commission push for the establishment of very large research centres, leading maybe to a concentration on some locations. This must not be done at the cost of funding for universities, as this would cut the source of those 80% of fundamental research.

In particular, in mathematics, the recognised level of European research is based both on research done in all universities and on a network of research institutes (grouped in the Committee ERCOM). The centres of ERCOM are in fact characterised by the fact that they have more visitors than permanent staff, visitors coming from the universities.

Thus research centres and universities are part of the research efficiency in mathematics and both are necessary.

The booming market in services

The question: 'How can universities be given the necessary flexibility to allow them to take greater advantage of the booming market in services?' seems to take for granted that universities have to shift from a public service enterprise of democratic education and fundamental research to a commercial enterprise of services, in research and education.

It is crucial that this shift be limited, in order to preserve the present level of universities.

Concerning education seen as a private service, the EMS stresses that Europe must defend its tradition of accessible public universities, and not allow higher education to enter international agreements on free trade and services.

This battle must probably be fought repeatedly, to avoid the day when public funding to education will be seen as distortion of concurrence.

Concerning research services, it should be noted that in most cases when European industries pass a research contract with universities, they only pay for the marginal cost, i.e., the supplementary staff and cost, but not the existence of expertise built over the years by the university.

Limited overheads are the only contributions to the general budget of the universities.

This system can only work if the basic funding of the universities remains public funding, unless industry is ready to pay for the basic research they actually use.

Human resources

This is a key issue, and with various return mechanisms the E.U. has taken positive measures.

There is a frustrating paradox in the fact that Europe trains well its researchers up to doctoral level, then presents a lack of career opportunities inciting them to find jobs in the U.S.A. – a country which has not funded their previous education.

The situation will get even more delicate when a whole generation of researchers and teachers will retire in a few years around 2010.

Steps should be taken to keep in activity the young researchers who will be needed to fill those positions.

Excellence and evaluation

The Communication rightly stresses that excellence in research and training is the result of long-term efforts, and that a strong stability in funding mechanism is needed.

This has to go together with evaluation – to ensure the right track is followed. For this, independent peer reviews of the level of research and teaching are needed. These evaluations should be done in sufficient numbers to be useful (and effective) but not so many as to create 'audit fatigue'. They should take care not to penalise emergent research.

Peers know the signs of excellence. No quantified indicator taken in isolation can provide a full judgement.

Interdisciplinarity is to be encouraged, but cannot be an end in itself, but a meeting point for solid disciplinary research, the ability to engineer interaction between separate elements of knowledge, or it faces becoming the weak point of research. So, valid interdisciplinary projects must be encouraged, but not at the cost of funding for fundamental research which is not necessarily interdisciplinary.

Note that mathematics itself can be seen as interdisciplinary, in the sense that the same mathematical structure can find applications in different sciences and technologies. Unfortunately, in most research programmes, no mechanism has been set to support that development.

Contact person: Luc Lemaire (EMS Vice-president), C.P. 218 Campus Plaine ULB, B-1050 Bruxelles, Belgium
phone: 32-2-6505837; fax: 32-2-6505867; e-mail: *llemaire@ulb.ac.be*

EMS Mathematical Weekend

Lisbon, September 12-14, 2003

PLENARY TALKS

Michèle Audin (Strasbourg)
Jean-Michel Bismut (Orsay)
Bernard Dacorogna (Lausanne)
Hans Föllmer (Berlin)
Gilles Lebeau (Nice)

SPECIAL SESSIONS

Symplectic and Related Geometries
Analysis and Geometry
Calculus of Variations
Stochastic Analysis and Mathematical Finance
Non-linear Evolution Equations

LOCAL ORGANIZING COMMITEE

Ana Bela Cruzeiro
Ana Cannas da Silva
Pedro Freitas
Rui Loja Fernandes
José Matias

Organized by the European Mathematical Society and the Portuguese Mathematical Society.

Also sponsored by:

Portuguese Science Foundation

Calouste Gulbenkian Foundation

Center for Mathematical Analysis, Geometry and Dynamical Systems of Instituto Superior Técnico

Mathematical Physics Group of the University of Lisbon

http://www.math.ist.utl.pt/ems

The Council of the European Mathematical Society

David Salinger, EMS Publicity Officer

The EMS Council, which meets every two years, is the governing body of our Society.

The EMS has two distinct categories of members. First, there are the national mathematical societies and academic institutes. Then there are individual members, who generally join through their national societies. As more individual members join, the number of their representatives on Council increases, up to a limit of two-fifths. Currently, Council is composed of 19 delegates elected by the individual membership and 64 delegates of societies.

So much for the theoretical balance of power. In fact, Council delegates don't separate into groups according to the type of membership they represent. They mainly act as individuals, accepting or amending proposals from the Executive Committee, perhaps questioning some of the Society's activities or suggesting new ones. An example of the latter is the joint mathematical weekend the EMS is holding together with the Portuguese Mathematical Society this September.

The atmosphere in Council is generally friendly and constructive. Of course there are differences of approach arising from diverse national traditions, and these do enliven the proceedings from time to time. For instance, in Barcelona there was a debate about the early history of the Society: strong and contradictory views are held about exactly how the Society was brought into being. In Oslo, there were passionate interventions in favour of reduced membership rates for young researchers, but Council was not persuaded. Also in Oslo, Council debated the Bologna process, but was unable to agree a statement acceptable to all delegates.

he next Council meeting will be held in Stockholm in June 2004, during the weekend before the fourth European Congress of Mathematics. There will be vacancies for delegates from the individual membership.

The mathematical library of Charles University, Prague

Vladimír Souček

In August last year, Prague was hit by a flood and many parts of Prague were heavily damaged – including two buildings of the Faculty of Mathematics and Physics at Charles University. During the flood, the library lost more than 13000 books, 468 journal titles, about 6500 textbooks, 2000 diploma theses and hundreds of historical books. About 6000 books and journals were frozen and later dried by a special process: the results were partly successful, depending on the extent of the damage, and a proportion of these books will be restored.

Over the last ten months we have received extensive international solidarity. Many institutions, mathematical societies, university libraries and publishing houses, as well as many individuals, have sent us books, journals and financial contributions to the library. This help has been extremely important for us: the damage was so large that it would have been impossible to recover from the flood's consequences by our own forces alone.

Up to the end of June, the Faculty received the equivalent of 120000 euros as financial gifts from various institutions, as well as from many individuals. The support for Charles University represents the equivalent of 300000 euros for new books and missing journals: of course, the Faculty budget also strongly respects the needs of the library. Starting in September 2002, we have gradually been able to collect in prepared rooms an increasing number of books, monographs and journals from many parts of the world, as well as from our country.

An important problem to be settled was to find means for transporting these publications to Prague from abroad. In Germany and France, this problem was solved by internal transport to the chosen nodes (Berlin, Freiburg, Paris). Then, during the spring, three camions brought them to Prague. Diplomatic services of various countries also gave important assistance in this respect. This process is still far from complete – for example, transporting books from the USA and Austria is planned for the summer period. The number of publications sent to Prague by the end of June includes about 5000 books and 140 titles of journals, and this number is increasing continuously. The registration of new publications and their incorporation into the database is a difficult task for the library personnel and will need many months to complete.

To prevent a possible repetition of the catastrophe, the Faculty has prepared plans for a complete reconstruction of the building, which includes shifting the library rooms to a higher floor. An application for state support for this reconstruction was successful and the work started at the beginning of July.

To summarise, due to extensive international help and state support, the reconstruction of the library is continuing and is expected to be finished soon – probably by the end of the year. Of course, the series of volumes of newly obtained journals still suffer from many gaps and the task to fill them in will be a longer-term challenge. On the other hand, generous gifts from many parts of the world will make it possible to create a distinguished collection of books and monographs in the library, including a great number of recently published books. New rooms to be built for the library are well designed for the purpose and will have new modern equipment.

The Prague mathematical community greatly appreciates the international solidarity offered during the difficult period after the catastrophe. We would like to use this opportunity to express our sincere thanks to all who were willing to spend time and much effort to organise and provide this help.

Editorial Board of JEMS

The following is a list of members of the editorial board of the journal of the European Mathematical Society.

2003 Ferran Sunyer i Balaguer Prize

Two mathematicians from the Universitat de València and one from the Universitat Pompeu Fabra (Barcelona) have won the international 2003 Ferran Sunyer i Balaguer Prize.

The Ferran Sunyer i Balaguer Foundation has awarded the prize for the ninth time, and this year's winning monograph solves a mathematical problem that has been studied for ten years by the best specialists in image processing.

At its meeting on 3 April, the Council of the Ferran Sunyer i Balaguer Foundation approved the proposal made by its Scientific Committee to award the 2003 Prize to **Fuensanta Andreu-Vaillo** and **José M. Mazón**, from the Universitat de València, and to **Vicent Casellas**, from the Universitat Pompeu Fabra, for the work *Parabolic quasilinear equations minimizing linear growth functionals*. The most notable part of the work is a remarkable solution to a problem that has been studied for ten years by the best specialists in image processing, and that amounts to important progress, not only for this subject but also for other areas that can be studied using the same mathematical methods.

The prize was awarded during the ceremony that took place on 24 April at the Institut d'Estudis Catalans (IEC) in Barcelona. The prize consists of 10000 euros and the monograph will be published by Birkhäuser in the series *Progress in Mathematics*.

The Ferran Sunyer i Balaguer Foundation of the IEC (www.crm.es/FerranSunyerBalaguer/ffsb.htm) awards this international prize every year to honour the memory of Ferran Sunyer i Balaguer (1912-67), a Catalan self-taught mathematician who reached international recognition for his research work in mathematical analysis. Sunyer's achievements are the more impressive given his serious physical disabilities since birth.

The present Scientific Committee consists of professors Hyman Bass (University of Michigan), Antonio Córdoba (Universidad Autónoma de Madrid), Paul Malliavin (Université de Paris VI), Joseph Oesterlé (Institut de Mathématiques de Jussieu), and Warren Dicks (Universitat Autònoma de Barcelona).

In 1999, Juan J. Morales-Ruiz won the Prize with the monography *Differential Galois Theory and Non-integratibility of Hamiltonian Systems*, which together with this year's are the only two occasions in which nationals have won the prize.

Henri Poincaré prize

The International Association of Mathematical Physics (IAMP) announces the recipients of the Henri Poincaré Prize sponsored by the Daniel Iagolnitzer Foundation.

The Henri Poincaré Prize recognises outstanding contributions in mathematical physics and contributions that lay the ground for novel developments in this broad field. The prize is awarded by the IAMP at the International Congress on Mathematical Physics held every three years. This year's Congress was held in Lisbon, Portugal, 28 July to 2 August 2003.

The recipients of the Prize are:
Huzihiro Araki, *Research Institute for Mathematical Sciences, Kyoto University*
Citation: For his lifetime contributions to the foundations of quantum field theory, quantum statistical mechanics and the theory of operator algebras. His outstanding achievements at this interface of physics and mathematics are exemplified by his work on the structure of the algebra of local observables and its representations, collision theory, the variational principle in statistical mechanics and the notion of relative entropy for infinite quantum systems.

Elliott H. Lieb, *Princeton University*
Citation: For his lifetime achievements in quantum mechanics, statistical mechanics and analysis. His work has encompassed the exact solution of the Ice model, an unceasing quest for a complete understanding of the stability of matter, Thomas-Fermi theory and quantum spin systems, and the discovery of a remarkable range of fundamental inequalities.

Oded Schramm, *Microsoft Research*
Citation: For his contributions to discrete conformal geometry, where he discovered new classes of circle patterns described by integrable systems and proved the ultimate results on convergence to the corresponding conformal mappings, and for the discovery of the Stochastic Loewner Process as a candidate for scaling limits in two dimensional statistical mechanics.

ESF vision on a European Research Council

Jens Degett

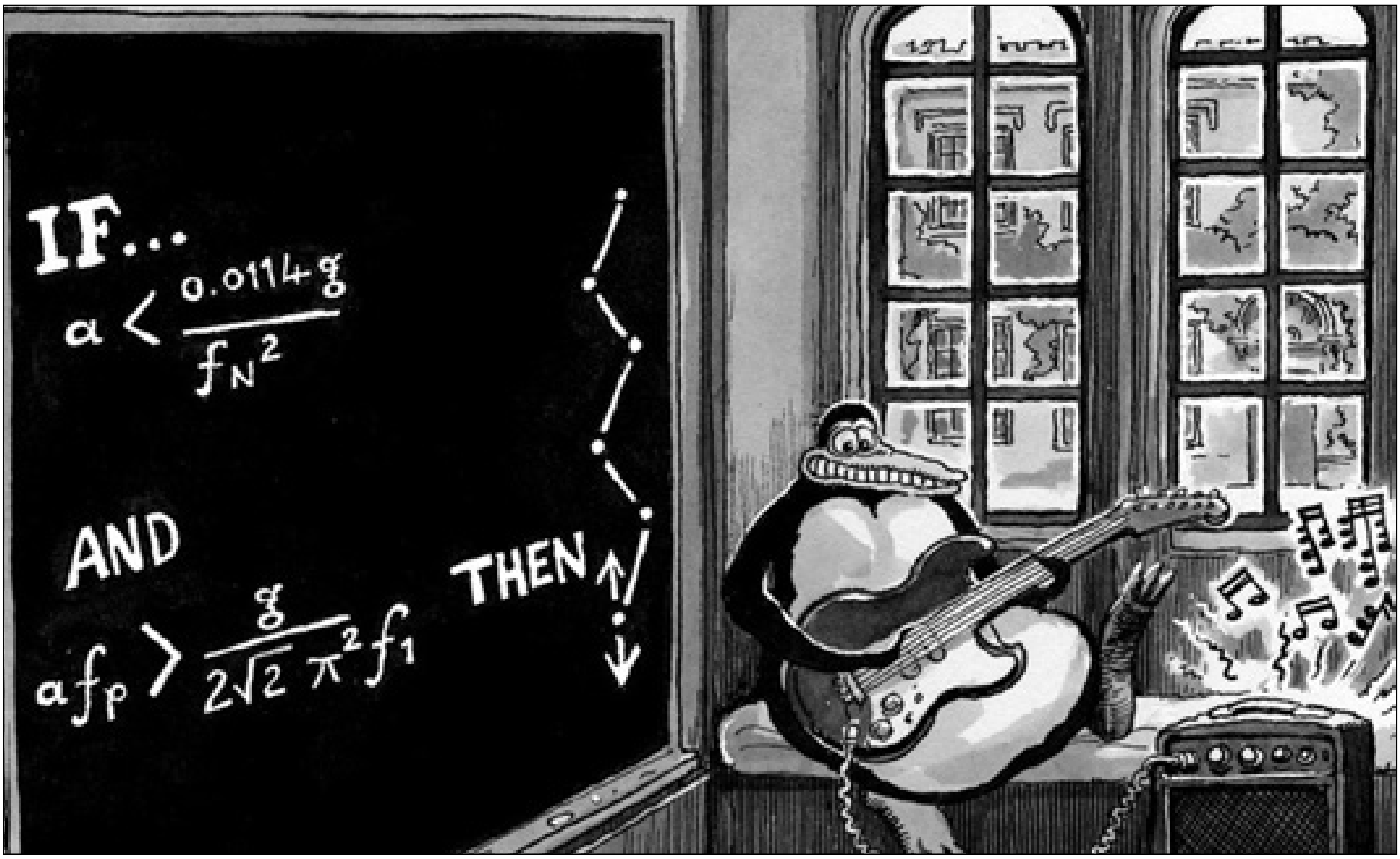
Last year, the European Science Foundation (ESF) started a debate on the construction of a new European research funding structure, with substantial funding capacity for research in the forefront of knowledge in Europe. This idea of creating a European Research Council would complement the European Commission's vision of a true European Research Area with more open competition and collaboration between European researchers and a strengthening of the funding structures of research in Europe.

An ESF High Level Working Group, chaired by Sir Richard Sykes, Rector of Imperial College, London, has now produced a report, which advocates the need for an ERC and shapes the scope of its remit and basic principles, its mode of operation, institutional development, and its funding sources. A PDF version of the report available at *http://www.esf.org/publication/159/ercpositionpaper.pdf*

ESF policy paper on science communication
The ESF paper on science communication states that the strategic goal from Lisbon and Barcelona is not only a question of increasing research and education budgets, streamlining the patent laws and supporting knowledge-based enterprises, but is also necessary to create a culture of public interest in science and technology in Europe if the ambitious plan is to have any chance of success. A public culture of science in Europe has to start with the scientists themselves; they should be more aware of the importance of communicating science to the broader public.

A number of recommendations on how to strengthen science communication at the national and European levels are given in the report. One of the more important recommendations is a target of at least 1% of all free research money to be spent on communication and educational activities. A PDF version of the policy paper is at *http://www.esf.org/medias/section_5/61/ESPB20.pdf* All ESF policy papers and reports can be downloaded from the ESF website, *www.esf.org*

Jens Degett [jdegett@esf.org] is ESF Head of Communications: tel: +33-3-88-76-71-32.



1089 and all that

The element of surprise in mathematics

David Acheson (Oxford)

Why do so many people say they *hate* mathematics?

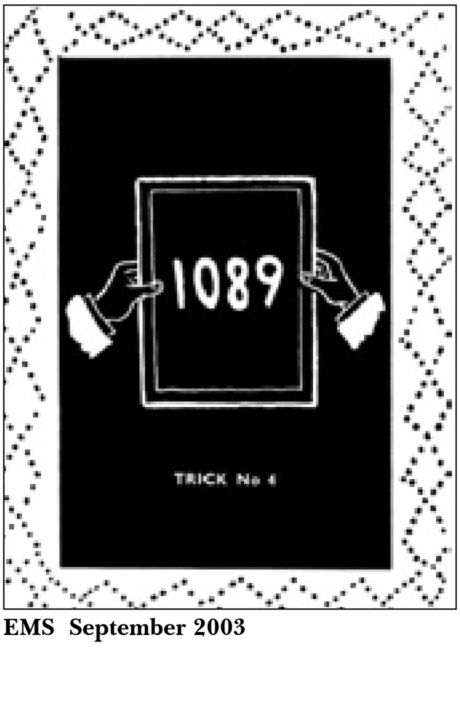
All too often, the real truth is that they have never been allowed anywhere near it, and I believe that mathematicians could do more, if they wanted, to bring some of the ideas and pleasures of their subject to a wide public.

One way of doing this might be to emphasise the element of *surprise* that often accompanies mathematics at its best.

Everybody likes a nice surprise.

A trick with numbers

I had my first mathematical surprise in 1956, at the age of 10. I was keen on magic tricks at the time, and one day I came across a 'trick with numbers' in an article called *Uncle Jack turns you into a Conjuror!*



Take any 3-digit number in which the first and last digits differ by 2 or more.

Reverse the number, and subtract the smaller of the two numbers from the larger (e.g. 782 – 287 = 495).

Then reverse the result and add (thus 495 + 594 = 1089).

The remarkable thing with this is: the result is *always* 1089, no matter which 3-figure number you start with.

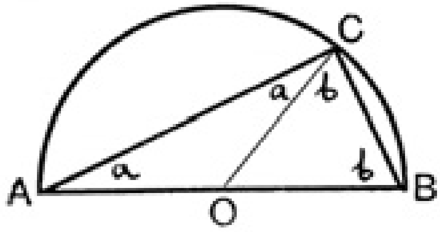
I realise now, of course, that this '1089 trick' is mathematically lightweight and of relatively little consequence. But I must tell you this: if you first see it as a 10-year old boy in 1956, it blows your socks off.

Surprising geometry

A little later on I had my first surprise in geometry.

One day at school we were told that if AB is a diameter of a circle, and C is any point on the circumference, then the angle ACB is a right angle. I remember finding this rather difficult to believe: it seemed to me that moving the point C along the circumference would almost certainly change the angle ACB, especially as C moved closer and closer to B.

And then came the proof that it doesn't.



First, join C to the centre of the circle O by a straight line. Then OA = OB = OC. So triangle AOC is isosceles, so the angles marked a in the diagram are equal.

By the same argument applied to the triangle BOC, the angles marked b are equal.

But the angles of the triangle ACB add up to 180°, so $a + (a + b) + b = 180°$.

So $a + b = 90°$, and angle ACB is therefore a right angle.

Even now, I still find this one of the most devastatingly effective and revealing proofs in the whole of mathematics.

Great mistakes

The whole idea of *proof* is, perhaps, one of the major difficulties in communicating mathematics to the wider public. Mathematicians can easily appear unduly obsessed with it, and I believe we have to be willing to explain just why it is so important.

One way of doing this is to point out how, in the absence of proof, mathematicians can sometimes get things badly wrong. And even the greatest of mathematicians are not immune.

Leonhard Euler (1707-83)

In 1753, for instance, Euler proved the $N = 3$ case of Fermat's Last Theorem. He proved, in other words, that there are no whole number solutions a, b, c to the equation

$$a^3 + b^3 = c^3;$$

in short, 2 cubes cannot add up to a cube.

But a little later he went on to conjecture that, in a similar way, it would be impossible for 3 fourth powers to add up to a fourth power, and more generally, that it would be impossible for m–1 mth powers to add up to an mth power.

For nearly *two hundred years* no one could find anything wrong with this proposition. Nobody could actually prove it, either, but it had been around for a very long time, and it had been proposed, after all, by a very respected authority … .

And then, in 1966, L. J. Lander and T. R. Parkin found a counter-example, 4 fifth powers that add up to a fifth power:

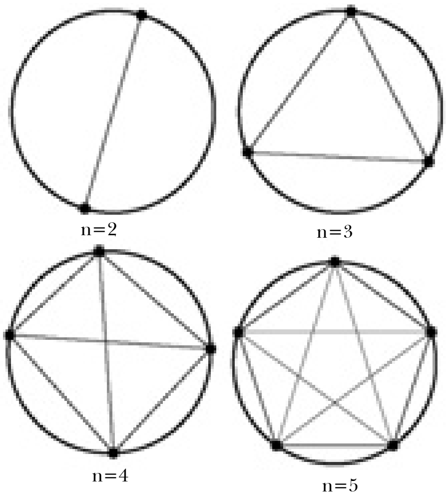
$$27^5 + 84^5 + 110^5 + 133^5 = 144^5$$

And 20 years after that, the $m = 4$ case was demolished by N. Elkies:

$$2682440^4 + 15365639^4 + 18796760^4 = 20615673^4$$

Attempting to *generalize* in mathematics on the basis of one or two special cases is always a risky business, of course, and one of the most telling examples I know of this involves an apparently innocent little problem in geometry.

Take a circle, mark n points on the circumference, and join each point to all the others by straight lines. This divides the circle into a number of different regions, and the question is: *how many*? (It is assumed that no more than two lines intersect at any point inside the circle)

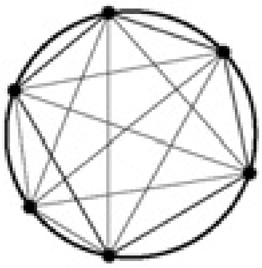


Now, for the first few values of n – namely, 2, 3, 4 and 5 – the number of regions follows a very simple pattern: 2, 4, 8, 16.

And, in my experience, it is possible at this point to lure virtually anybody into 'deducing' that when $n = 6$ the number of regions will be 32.

But it isn't.

It's 31:



And the general formula for the number of regions isn't 2^{n-1} …

… it's $\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24)$.

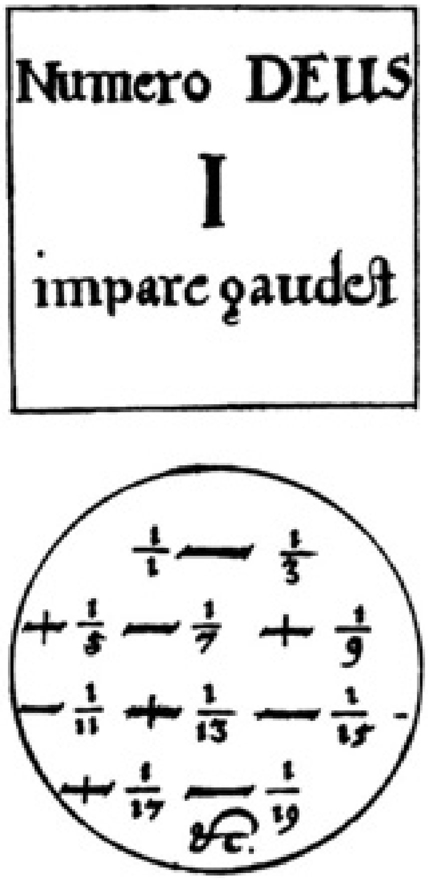
Surprising connections

In higher mathematics, some of the deepest surprises come about from unexpected connections between different parts of the subject.

Soon after learning calculus, for instance, we come across the famous Gregory-Leibniz series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

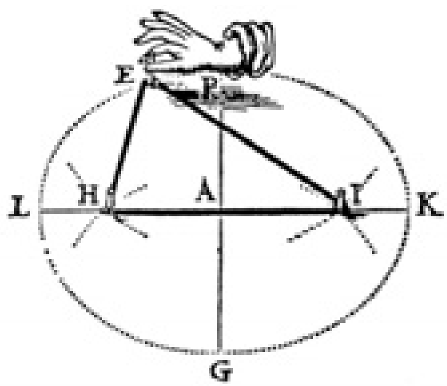
And even a rigorous proof does little, in my view, to diminish the sense of wonder and surprise at this extraordinary result. After all, when we first meet the number π it is all about circles, and I have yet to meet anyone who can explain the connection between circles and the reciprocals of the odd numbers in truly simple terms.



Illustrations from Leibniz's 1674 paper

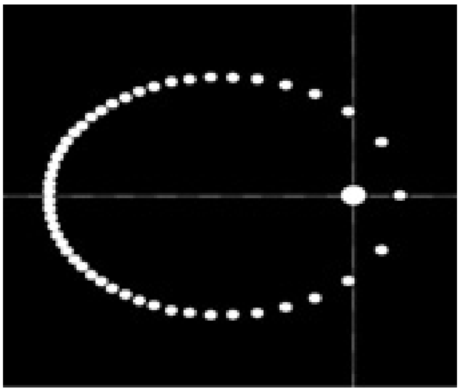
Sometimes, however, the surprising connection lies not within mathematics itself, but between mathematics and the real world.

The ellipse, for instance, is a curve that was well known to Greek mathematicians, and it can be constructed by pulling a loop of string round two fixed points. These points, H and I, are called *focal points*.



Ellipse, from van Schooten's Exercitationum mathematicorum (1657)

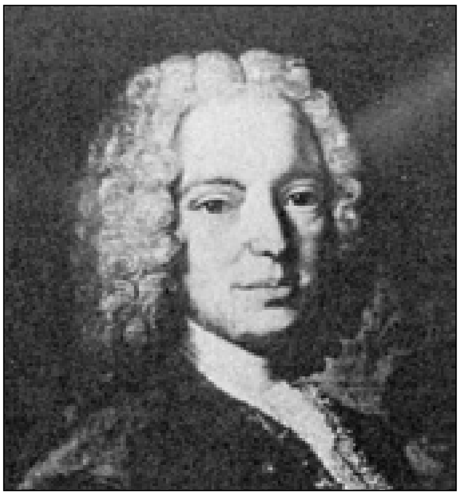
At first sight, perhaps, this is 'just' geometry. Yet, if we leave the world of pure geometry and model a planet (or a comet) as a point mass, and assume that it is subject to an inverse-square-law gravitational force towards a fixed 'Sun', and solve the resulting differential equations, we find not only that any closed orbit must be an ellipse, but that the Sun *is always at one of the focal points*!

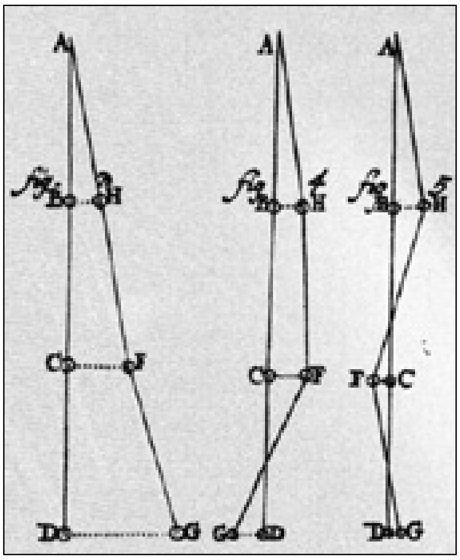


Not Quite the Indian Rope Trick

I suppose the greatest mathematical surprise I've ever had came one rainy November afternoon in 1992, when I found myself proving a strange new theorem.

I was trying to give a new twist to an old problem in dynamics, first studied by Daniel Bernoulli in 1738. He had considered a hanging chain of N linked pendulums, all suspended from one another, and discovered N different modes of oscillation. In the lowest mode, at frequency f_1, the pendulums swing to and fro





The 3 modes of oscillation of a triple pendulum, from Daniel Bernoulli's original paper of 1738

together, much as if they form one long single pendulum, while in the highest mode, with frequency f_N, adjacent pendulums swing in opposite directions at any given moment.

My theorem showed how it is possible to take the N linked pendulums, turn them upside-down, so that they are all precariously balanced on top of one another, and then *stabilise them in that position by vibrating the pivot up and down*. Provided that f_N^2 is much greater than f_1^2 (as is usually the case), the criteria for stability turned out to be

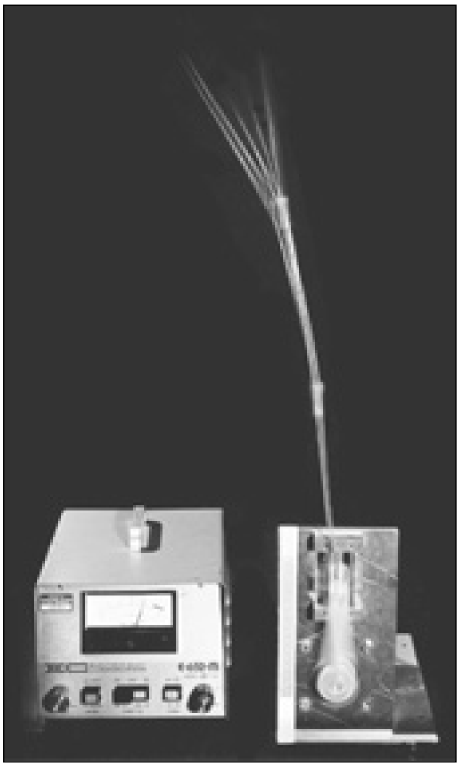
$$a < 0.0114 g / f_N^2$$

and

$$a f_p > g / (2\sqrt{2}\, \pi^2 f_1),$$

where a denotes half the total distance through which the pivot moves up and down, f_p denotes the frequency of the pivot vibration, and g is the acceleration due to gravity.

In this way, then, the theorem depends very simply on only the two numbers f_1 and f_N which characterise how the pendulums oscillate about the downward state when the pivot is fixed. And the upshot is that the 'trick' can always be done if the pivot is vibrated up and down by a small enough amount and at a high enough frequency.



Computer simulations suggested that the upside-down state could in fact be very stable indeed, and this was borne out when a colleague of mine, Tom Mullin, confirmed the theorem experimentally. The photograph above shows a 50 cm inverted triple pendulum, with the pivot vibrating through 2 cm or so, at about 40 cycles per second, and the chain of pendulums is seen wobbling back towards the upward vertical after a fairly severe initial disturbance.

As soon as we started calling our lectures on this subject 'Not Quite the Indian Rope Trick' we found that newspapers, radio and TV all began getting interested, and Tom and I have had a great deal of fun with this topic over the years. My scientific papers on the subject have even been acquired by the archives of the Magic Circle in London, which would surely have astonished a certain 10-year old boy in 1956. (The papers are kept, I understand, in a box file called *Sundry Ephemera*).

In the end, though, it is not all that important whether mathematics might or might not explain a particular magic trick.

What matters, surely, is the extent to which surprising results like this may help persuade the wider public that mathematics, at its best, has a certain magic of its own.

Reference

David Acheson, *1089 and all that: A journey into mathematics*, Oxford University Press, 2002. (Chapter 15, in particular, gives further information about the 'Indian rope trick'.)

David Acheson is a Fellow in Mathematics at Jesus College, Oxford. This article is a shortened version of his 2003 London Mathematical Society Popular Lecture: 'Mathematics, magic and the electric guitar'. *His book* '1089 and all that' *is an original attempt at bringing some of the ideas and pleasures of mathematics to a wide public, and further details may be found at* www.jesus.ox.ac.uk/~dacheson

A. N. Kolmogorov (1903-1987)

N. H. BINGHAM

The great Russian (and Soviet) mathematician Andrei Nikolaevich Kolmogorov was born on 25 April 1903 in Tambov to Maria Yakovlevna Kolmogorova. His mother died in childbirth, and the baby boy was adopted by his mother's sister Vera, who brought him up in the village of Tunoshna, near Yaroslavl on the Volga.

The three sisters Kolmogorova – Maria, Vera and Nadejda – were independent and educated women, which in the climate of those Tsarist times inclined them towards dissent. Both Maria and Vera were imprisoned, and family history has it that forbidden revolutionary literature was once hidden under Andrei's cot during a search. Kolmogorov's father, Nikolai Matveevich Kataev, was a scientific man – an agronomist and statistician, who held a government post in agricultural education after the Revolution. He died (or was killed) during the Civil War, in the Denikin Offensive of 1919. (Denikin was a White Russian general who – with British and French naval and military support, and while Yudenich advanced on St Petersburg – advanced from the south, taking Kiev and Kursk and marching on Moscow before the Whites' lack of coordination enabled the Red Army to defeat them piecemeal.)

It was common in Tsarist times for progressively-minded couples to signal their dissent from the established social order by not formally marrying. This, and perhaps Maria's untimely death, led to the name of Kolmogorov rather than Kataev being immortalised in the history of mathematics and science.

Kolmogorov stayed in Yaroslavl till 1910, already discovering mathematics. He then moved with Vera to Moscow, where he obtained a good secondary education, despite the upheaval and privations of the time. He was able to enter the Faculty of Physics and Mathematics of Moscow University in 1920. His first field of interest was metallurgy, but he soon turned to mathematics. He impressed Luzin, his lecturer on analytic functions, by finding an error in one of Luzin's lectures while still in his first year. Later he similarly impressed Alexandrov (who became a close and lifelong friend), and Urysohn.

During his student years of 1922-25, Kolmogorov supplemented his income by teaching in a secondary school. His interest in education at school level lasted throughout his life, and came to be his central focus in his later years.

In his second year Kolmogorov turned to Fourier series, in Stepanov's seminar. He made his mathematical debut in 1922 with his famous construction of a Fourier series that diverges almost everywhere (strengthened to divergence everywhere in 1926). He also worked on analytic sets under Suslin and Luzin, although some of this work was only published much later. He graduated in 1925 and finished his postgraduate studies in 1929 with 18 papers to his credit. This precocious beginning was continued by his appointment to a chair in Moscow in 1931; he received his doctorate in 1935 for his published work without submitting a thesis.

Kolmogorov's first work on probability was his paper of 1925 with Khinchin, also a pupil of Luzin's. This concerns the convergence of random series (whose terms are independent random variables). In this and subsequent papers, one finds the Khinchin-Kolmogorov theorem and Kolmogorov's three-series theorem on random series, the Kolmogorov (maximal) inequalities, and conditions for the validity of the law of large numbers, in weak (convergence in probability) and strong (convergence almost surely) forms. Kolmogorov himself dated the beginning of probability theory proper to James Bernoulli's classic book *Ars Conjectandi* of 1713, where the first form of the law of large numbers occurs. This – the person in the street's 'Law of Averages' – found its definitive form in Kolmogorov's strong law of large numbers 220 years later.

The question of fluctuations in the law of large numbers – originating in number theory via decimal expansions – led Khinchin to his famous Law of the Iterated Logarithm in 1924. Kolmogorov generalised this, giving his own form of the Law of the Iterated Logarithm in 1926.

One of Kolmogorov's most famous papers, 'Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung' ('analytic methods') appeared in 1931 in *Math. Annalen*. This focuses on the analytic aspects of stochastic processes – specifically, on the transition probabilities of what are now called Markov processes and diffusions, obtaining the Chapman-Kolmogorov equation and the Kolmogorov forward and backward partial differential equations for them.

Perhaps Kolmogorov's most famous single achievement is his classic book of 1933, *Grundbegriffe der Wahrscheinlichkeitsrechnung* (Foundations of Probability Theory). Here Kolmogorov succeeds in turning probability theory at last from a jumble of isolated results and imprecise reasoning into a mature and rigorous branch of mathematics. The necessary machinery to do this was measure theory, created around the turn of the century by Lebesgue, Borel, Baire and others. Borel, Fréchet and Lévy had made progress in harnessing measure theory to the needs of probability theory. Another impetus was in Nikod'ym's completion in 1930 of Radon's work. The Radon-Nikod'ym theorem gave Kolmogorov the tool he needed to give a rigorous treatment of *conditioning*. In Kolmogorov's hands, a conditional expectation is a random variable which 'integrates the right way over the relevant sets'. This provides a general and flexible tool to handle probability statements given (or conditional on) partial information – and in particular, on information flow unfolding in time. This is the key to modern probability and stochastics, and has been rightly called by David Williams 'the central definition of modern probability theory'. Other highlights of the *Grundbegriffe* include the Daniell-Kolmogorov theorem (roughly, the use of infinitely many consistent finite-dimensional distributions to define one infinite-dimensional distribution – essentially, a stochastic process), and the final 'if and only if' form of Kolmogorov's strong law of large numbers.

Kolmogorov's later probabilistic work in the 1930s included:

- the Kolmogorov-Petrovskii-Piskounov semi-linear PDE, governing the spread of an advantageous gene (Kolmogorov was much influenced by R. A. Fisher's work on genetics);
- his work on non-parametric statistics, leading to the Kolmogorov-Smirnov test of goodness of fit;
- his contributions to the theory of infinite divisibility, leading to the Lévy-Khintchine formula;
- his studies of 1936-37 on differentiability properties of transition probability functions of Markov chains;
- his work on branching processes, on mathematical geology, on ergodic theorems, on Brownian motion and on the reversibility of the arrow of time, and on crystallisation.

In genetics, he also championed the Mendelian implications of the work of Ermolaeveva. This attracted criticism from Lysenko, a scientific charlatan in favour with Stalin, because of the supposed political implications of his views ('communism can alter nature'). This incident casts light both on the hysteria of the Stalinist repression of the time (1939) and on Kolmogorov's personal courage and scientific integrity.

In the 1940s, Kolmogorov turned to problems of prediction and time series. The mathematical context was the use of spectral methods to pass between the 'time domain' and the 'frequency domain' for time series, and the systematic use of

Hilbert-space methods for this purpose. The applied context was problems of fire control for artillery, particularly anti-aircraft artillery – this under the pressure of what is called World War II (1939-45) in the West and the Great Patriotic War (1941-45) in the former Soviet Union. Similar work was being done at the same time for the same reason by Wiener in the USA. The 'Kolmogorov-Wiener filter' is expounded in, for example, the last chapter of J. L. Doob's classic 1953 book, *Stochastic Processes.*

Kolmogorov is famous in the aerodynamics and hydrodynamics communities for his work on turbulence. His published work here dates from the early 1940s, although his interests here predate this. Turbulence is a chaotic (and very complicated) phenomenon, and Kolmogorov saw that one should use stochastic methods, viewing fields of hydrodynamic characteristics as random functions in time and space and averaging over ensembles. This leads to his work on turbulent flows of fluids with large Reynolds number, and his 'two-thirds power' law.

The book by B. V. Gnedenko and Kolmogorov, *Limit distributions for sums of independent random variables,* which appeared in Russian in 1949 (English translation, with annotations, by Kai-Lai Chung, in 1954), gives a definitive treatment of the 'central limit problem' of probability theory. We know that if one sums independent random variables with the same mean, variance and law, changes location and scale (to mean 0 by subtraction and to variance 1 by division) and goes to the limit, one obtains the standard normal or Gaussian law as limit distribution. This is the 'Central Limit Theorem' (the precise form of the 'Law of Errors' of the 'physicist in the street'). The book of Gnedenko and Kolmogorov takes this circle of ideas to its ultimate generality, while still preserving independence, and is an enduring classic. Weak dependence can be handled (martingale, Markov or mixing dependence, for example), but this is harder and belongs to a later period.

We pick out three main themes from Kolmogorov's work in the 1950s.

(1) Ergodic and stability properties of dynamical systems. The motivation goes back to Poincaré's work early in the century on the stability of the solar system. The motions of the planets Jupiter and Saturn are observed to influence each other dramatically, and this reflects a 'small denominators' phenomenon, where integer combinations of frequencies being small result in deviations from mean orbit being large. Kolmogorov's work was continued by his pupil V. I. Arnol'd and the late Jurgen Moser – Kolmogorov-Arnol'd-Moser (or KAM) theory involving invariant tori.

(2) Information theory, originated by Shannon, its use of the concept of entropy, and its application to various ergodic questions. The use of 'epsilon-entropy' had a great impact in approximation theory, in particular in the work of G. G. Lorentz. The use of entropy as an isomorphism invariant in ergodic theory was taken up by Kolmogorov's pupil Sinai in the USSR and by Donald Ornstein in the USA.

(3) Kolmogorov's solution of Hilbert's Thirteenth Problem, which he himself regarded as his most technically difficult achievement. This is concerned with the representation of (continuous) functions of many variables by superpositions of (continuous) functions of one variable.

During the 1950s, Kolmogorov was also developing his interest in algorithms, and wrote his book with Fomin.

In the 1960s, Kolmogorov turned to what is now called Algorithmic Information Theory. To give a brief glimpse of the subject-matter, consider a large number (a thousand, say, or a million) of (independent) tosses of a fair coin. There are 21000 outcomes, all with the same probability 2–1000, by symmetry. According to classical probability theory, *of which Kolmogorov was the principal creator,* all these outcomes are on the same footing – so the one that you produced by actually tossing a coin a thousand times is on the same footing as a sequence of a thousand zeros. But it is evident to the person in the street, if not to the classical probabilist, that these two outcomes are *not* on the same footing. Kolmogorov saw that there *is* a difference, and w*here it lies.* Your outcome takes a thousand bits of information to describe it; a sequence of a thousand zeros takes only two – 'all zeros'. Kolmogorov's second approach to probability theory, via Algorithmic Information Theory ('Kolmogorov Mark II', in place of the classical 'Kolmogorov Mark I') *identifies randomness with maximal complexity.* It turns out to be possible to build an extensive and powerful theory encapsulating these ideas. For details, we must refer to his 1987 paper with Uspenskii (*Theory of Probability and Applications*, **32**), which gives a version of the strong law of large numbers in this setting (also the paper by Vovk following it, which gives the corresponding version of the law of the iterated logarithm), and the third volume of his selected works, *Information Theory and the Theory of Algorithms.*

Further themes from Kolmogorov's work in the 1960s include the statistical study of metre in Russian verse, mathematical statistics, and turbulence.

In 1963, four special schools for mathematics and physics were opened, one (School No. 18) being attached to Moscow University. Kolmogorov devoted much of his time to it thereafter, so it is often known as 'Kolmogorov's school'. Here the interest in teaching matters – at school rather than university level – which had been part of Kolmogorov's outlook since his student days found full expression.

Kolmogorov's work was recognised by many honours. From the Soviet Union he received seven Orders of Lenin, the title Hero of Socialist Labour, the Stalin Prize and the Lenin Prize. He received many honorary degrees. He was elected a full member of the USSR Academy of Sciences (1939), an honorary member of the Royal Statistical Society (1956), a Fellow of the Royal Society of London (1964) and a member of the National Academy of Sciences, USA (1967).

Kolmogorov produced over 500 publications, including a number of books. His Selected Works appeared in three volumes in Russian – *Mathematics and Mechanics* (1985), *Theory of Probability and Mathematical Statistics* (1986) and *Theory of Information and Theory of Algorithms* (1987), or MM, PS and IA – published by Nauka in Moscow. These were later translated into English and published by Kluwer. Both general tributes to Kolmogorov, and detailed studies of aspects of his work, may be found in articles in *Theory of Probability and its Applications* **34** (1989), *Annals of Probability* **17** (1989) and the *Bulletin of the London Mathematical Society* **22** (1990): the piece in *BLMS* is by D. G. Kendall. Some insight into the sheer scope of Kolmogorov's work is conveyed by the fact that no fewer than ten other authors contributed on aspects of Kolmogorov's work: G. K. Batchelor (turbulence), N. H. Bingham (probability limit theorems), W. K. Hayman (Fourier series), J. M. E. Hyland (logic), G. G. Lorentz (entropy), H. K. Moffatt (KAM theory), W. Parry (ergodic theory), A. A. Razborov (complexity of algorithms), C. A. Robinson (cohomology) and P. Whittle (prediction).

Kolmogorov married Anna Dmitrievna Egorova, a friend from his schooldays. She survived him; there were no children. His adopted mother Vera, who died in 1950, lived to see her son become one of the great mathematicians of the twentieth – or any other – century.

It is sad to record that – perhaps because Trotsky was Jewish – life in the Soviet Union was deformed by official anti-semitism, and that in mathematics this attitude was shared by two great and influential mathematicians, I. M. Vinogradov and L. S. Pontryagin. It was not shared by Kolmogorov. For many years Kolmogorov organised meetings of the Moscow Mathematical Society, inviting speakers on merit. The Society was accordingly referred to scathingly by Pontryagin as Kolmogorov's Jewish Mathematical Society.

Kolmogorov had a great many pupils, many of whom have become distinguished mathematicians in their own right. Those close to him always speak of his human warmth, broad cultural interests, and love of the outdoors, sports and hiking. He enriched the lives of all who knew him. Even those who merely saw him (I heard him speak at the International Congress of Mathematicians in Nice, 1970) treasure the memory. His life and work have permanently enriched mathematics, and it is a pleasure to salute his memory on the anniversary of his birth.

Nick Bingham [nick.bingham@brunel.ac.uk] is Professor in the Department of Mathematical Sciences, Brunel University, Middlesex, UK.

Address by Sir John Kingman, EMS President,

on the opening of the International Conference

to celebrate the centenary of the birth of A. N. Kolmogorov

Kolmogorov and Contemporary Mathematics

It is a great pleasure to represent the European mathematical community on this occasion of celebration of the birth a hundred years ago of Andrei Nikolaevich Kolmogorov, one of a very small number of really great mathematicians of the twentieth century. Although Kolmogorov spent his long career in Russia, his influence was global, and few mathematicians in Europe or beyond can have failed to be influenced by his profound and seminal work.

This work has many facets, which are well represented in the talks planned for this meeting. He is admired by geometers, topologists and analysts. The logician knows his work on computational complexity. To the applied mathematician he is the man who made the single most important contribution to the impossibly difficult problem of understanding turbulent fluid flow. With Arnold and Moser he revolutionised the theory of dynamical systems. With Sinai he added a new invariant to ergodic theory. With Smirnov he gave statisticians a new tool for non-parametric testing. And so on, and so on.

But there was a unifying thread to all his work, which was his fascination with the mathematics of probability. That was the subject of his early work, the title of his professorial chair, and it remained a major interest throughout his long mathematical life. Moreover, many of his successful forays into other parts of the subject were in effect applications of his deep understanding of stochastic phenomena. I hope therefore that I am not being unduly partisan if, as a probabilist myself, I concentrate here on the impact of Kolmogorov on modern probability theory.

Probability calculations go back for centuries before Kolmogorov. They began to assume modern form with the work of Fermat and Pascal, the normal distribution of de Moivre and Laplace gave Gauss the tool for his theory of errors, and later the Russian school of Chebyšev, Bernstein and Markov showed how complex a calculus could be developed.

But these calculations were distrusted by many mathematicians well into the twentieth century. When I was a student in Cambridge in the 1950s I was discouraged from studying probability because it was neither proper pure mathematics nor the application of mathematics to well-defined physical phenomena. It dealt with 'random variables', which were entities subject to random variation about which all one could say was that probability statements could be made about them. What these random variables were, or what the probability statements meant, was left obscure. The basic tools were an 'addition law of probability' and a 'multiplication law of probability', which had a formal similarity, but one of which was an axiom and the other a circular definition.

Had we but known, the answer to these difficulties was contained in a little book, little in size but gigantic in importance, that had been published in German in 1933 as *Grundbegriffe der Wahrscheinlichkeitsrechnung*, but not until 1950 in an English translation. Seventy years later, this book remains a remarkably modern account of the theory of probability; with a few changes of notation it could be used as the basis of an excellent lecture course today.

Mathematicians tend to be bored by foundations, regarding them as best left to a few eccentric pedants, while the real mathematics is done according to accepted canons which the logicians may find inadequate. Probability is somewhat different, and experience shows that the subject needs to go back regularly to its basis, and its basis is that laid down by Kolmogorov in 1933, neither more nor less. If you think about it, that is a very remarkable statement to make about a 30-year old, and I want to illustrate it in relation to the development of probability theory over the 70 years since the book appeared.

As everyone knows, the fundamental concept is that of a probability space. This is an abstract set whose elements (denoted by small Greek letters) are thought of as the outcomes of some random phenomenon, but of much greater importance is a distinguished class of subsets (denoted by italic capitals) which are the events about which probability statements can be made. Every event has associated with it a number between 0 and 1 which is its probability, and it is assumed that the class of events is closed under countable set-theoretic operations (that is, it is a Borel field), and that probability is a countably additive function from the Borel field of events onto the unit interval (a probability measure).

He saw that this formulation made a probability space an abstraction of the Borel-Lebesgue theory of measure and integration that had already transformed the integral calculus of Newton and Riemann. Random variables could now be defined as measurable functions from the probability space to the real line (or indeed to more complicated spaces): they induced probability measures as distributions and joint distributions and thus validated the calculations of probabilists from Fermat to Markov.

A powerful extension theorem allowed the construction of probability spaces whose elements are functions of a (time or space) parameter, and this is the foundation of the whole theory of random processes. The modern definition of conditional probability, based on the Radon-Nikodym theorem, is there in almost full generality. Independence is properly presented as a special case, and a good version of the strong law of large numbers is proved, as is a general zero-one law.

One of the most perceptive insights in the book is the importance of the Borel field of events. In the Borel-Lebesgue theory of measure on Euclidean space, the notion of a measurable set is a way of excluding pathological subsets. Every set that the analyst encounters is, if not Borel measurable, at least Lebesgue measurable, and indeed it is possible to do most real analysis in an axiom system in which every set is Lebesgue measurable.

Kolmogorov could have followed the same path, but he realised that a much more useful approach was to regard the Borel field as describing the information available in a particular context. For instance, in a random process evolving in time, we have at any time t a Borel field depending on the value of t. The events in this field are those of which we can say with confidence at time t whether or not they have occurred. As t increases, the field gets bigger and bigger, and we have the concept of a filtration, an increasing family of Borel fields, which is basic to the modern theory of temporal random processes. Combined with his definition of conditional probability it facilitated, in the hands of Doob and others, the theory of martingales and stopping times that is essential to the development of Markov processes.

Markov processes are only referred to in passing in the *Grundbegriffe*, but they were central to Kolmogorov's later work in probability. Markov had studied them as a simple generalisation of independent sequences of random events, in which the probability of one event could depend on its predecessor but not further back. He had set up the fundamental equation, a special case of what we now call the Chapman-Kolmogorov equation, and had seen that a matrix formulation of this equation permitted useful calculations and limiting results. (The mysterious name Chapman is that of a distinguished geophysicist Sydney Chapman, who derived a very special case of the equation in a particular physical problem.)

In fact, the Markov property can now be seen as the exact analogue for random processes evolving in time of the property of a well-posed dynamical system. If, for instance, we model the evolution of the orbits of the planets around the Sun, we need enough dynamical variables that we can write down differential equations giving the rates of change of all the variables as functions of some or all of them. The system of differential equations should then predict the way in which the dynamics will evolve in time, although in practice instability and chaos may limit the value of such predictions.

Likewise, in a system evolving randomly in time, we need to specify the state at time t in enough detail that the probability of events after t, conditional on the past up to t, depends only on the present state. In principle this can always be done (take the state at t to be the conditional probability given the whole past), but useful calculations depend, as in the deterministic case, on being able to summarise this complexity into a reasonably simple format. In other words, we must reduce the problem to a Markov process on a reasonably concrete state space.

It is one sign of a great mathematician to choose the right level of abstraction and generality. Too special, and the results are parochial and lack application. Too abstract, and they are shallow and superficial. Kolmogorov saw that the most interesting case was that of the Markov process, in discrete or in continuous time, with a countable infinity of states. He proved the fundamental limit theorem that determines the stationary distribution (when it exists) of the state after a long time, thus anticipating the Erdős-Feller-Pollard theorem on recurrent events. He showed that the process could recur infinitely often to its starting state without having a stationary distribution, or could wander off to infinity (and could have different destinations at infinity, so that the countable state space had an intrinsic compactification).

Markov processes in continuous time have the convenient property that their stochastic properties are often determined by their transition rates, so that one can neglect transitions that in time intervals of small length h have probabilities of smaller order than h. Kolmogorov gave conditions for this to be true, but he also produced remarkable examples of processes where this property fails. His work, rigorous and analytic, complemented the highly intuitive description by Paul Levy of the possible behaviour of these processes. It led in all sorts of fascinating directions, to the Hille-Yosida theory of infinitesimal generators, to the Levy-Trotter theory of local time, to Chung's definitive treatment of the analytic properties of Markov transition functions and the subsequent characterisation of Markov transition functions, to the martingale analysis of Markov processes that culminated in the intrinsic compactification of the state space, and so on.

But Kolmogorov's approach to Markov's theory also made applied probability possible. When the operational researcher builds models of complex queueing systems, when the biologist analyses the evolution of a genetically diverse population, when the statistician uses Markov chain Monte-Carlo techniques of modern Bayesian analysis, they are basing their calculations on a foundation that Kolmogorov made secure.

There is much else that Kolmogorov achieved in probability theory and that I have no time to describe. Just to mention one more example, he worked independently of Wiener on the prediction and smoothing of stationary time series, and arrived at essentially the same conclusions; it was one of the tragedies of the Second World War that these two powerful mathematicians could not collaborate.

The most important lasting legacy of Kolmogorov to the study of probability, whether seen as an elegant mathematical calculus or as an effective tool for analysing practical problems, is however a way of thinking. His picture of the underlying probability space, in which the most important entities are the particular subsets to which probabilities can be assigned, has become the working paradigm of all users of the theory. Conditional probability and expectation, the construction of measures on function spaces from finite-dimensional distributions, and the forward and backward differential equations that bear his name, are tools that we use every day.

Kolmogorov could have devoted his whole time to any of the fields he touched, to Fourier series, to mathematical logic, to hydrodynamics, to ergodic and information theory, to mathematical statistics. I am deeply grateful that, despite his contributions to these many fields, he retained his fascination for his first love, and gave so much for his successors to build on in the mathematics of probability.

Oxford doctorate for Jean-Pierre Serre

In June, Jean-Pierre Serre received an honorary doctorate from Oxford University. For those readers who wish to try out their Latin, the citation by the Public Orator Jasper Griffin was as follows; a translation appears below. We thank Jasper Griffin for permission to reprint his citation.

PROFESSOR JEAN-PIERRE SERRE
Honorary Professor at the Collège de France

Deum ipsum semper in re geometrica versari a Platone philosophorum omnium praeclarissimo discimus, qui super Academiae suae fores inscribendum curavit Μηδειζ αγεωμετοζ εισιτω, *Nemo intrare audeat nisi qui geometriae vacavit: quo interdicto cum nonnulli nostrum ne excludantur iure vereantur, tum optimo iure, Platone autem ipso plaudante, hunc quem produco fas est introduci, qui tam diu ex cathedra celeberrima geometriae studiosos erudit. quinquaginta iam fere anni sunt ex quo hic vixdum e pueris egressus praemio est ornatus amplissimo, cum et formarum indolem qualem tractant geometriae et numerorum ipsorum indolem, quos aliquo modo abstrusa totius mundi natura subesse sentimus, tamquam digitos suos cognorit amplectatur ceteris explicet. arti mathematicae plurimi post Pythagoram se dediderunt; plurimi pertransierunt et scientia multiplicata est; quorum argutissimus quisque cum multa observarit plura tamen posterioribus reliquit excogitanda. fingite, quaeso, animis superficiem annularem quandam nesciocuius materiae ita detorqueri ut nunc calicis, nunc placentae formam cepisse videatur: quid demum erit quod post metamorphoses illis Ovidianis magis admirandas immutatum maneat atque integrum? huius labores istas quaestiones multo tractabiliores reddiderunt; sed vir acutissimus in eo quoque excellit quo homines mathematici figurarum proprietates ea subtilitate definiunt quam flagitant homines algebraistae, et quidem in re algebraica quae vocatur se omnium principem praestitit. huius praecipue operae acceptum referimus quod his temporibus geometriae aurea quaedam aetas illuxit; numerorum autem rationes sic fere describit ut nemo alius, et quidem olim, nomine suo dissimulato, simul cum conlegis, Nicolae Bourbaki, si dis placet, persona adsumpta,de ipsis rei mathematicae fundamentis verba fecerit luculenta quae obscuritatem dissiparunt, intromiserunt Lucretiana ista lucida tela diei, in re algebraica, provincia scilicet mathematicae artis abstrusissima, praecipue insignis est. auctoritatem iure consectus amplissimam, civilitatem egregiam praestat, quo ceteris auxilium opem consilium largiatur.*

Praesento mathematicorum principem ingeniosissimum, figurarum aestimatorem perspicacissimum, numerorum ipsorum examinatorem subtilissimum, Iohannem Petrum Serre, praemiis plurimis insignitum, Abeliano omnium primum ornatum, Societatis Regiae Sodalem honoris causa adscitum, ut admittatur honoris causa ad gradum Doctoris in Scientia.

Admission by the Chancellor
Geometrarum insignissime, mathematicorum eminentissime, qui tuis egregiis laboribus ceteris mortalibus tantum luminis attulisti, ego auctoritate mea et totius Universitatis admitto te ad gradum Doctoris in Scientia honoris causa.

Paraphrase in English
Plato, greatest of philosophers, assures us that God is always doing geometry; he actually had inscribed over the entrance to his Academy the words: No entry for the ungeometrised. Many of us, I fear, might be excluded by such a prohibition, but Professor Serre would be admitted with flying colours and with Plato's enthusiastic support. It is some fifty years since he won the Fields Medal, a most prestigious award, at a very tender age. He is a master both of the science of geometrical forms and also of numbers, which we realise in some obscure way underlie the structure of the universe; he knows them like the back of his hand, and he shares that knowledge with panache. Since the days of Pythagoras there have been many devoted mathematicians; in the words of Daniel, very many have passed through, and knowledge has been multiplied. Even the greatest of them has still left many problems to be tackled by his successors. Imagine the surface of an object in the shape of a ring; then imagine it distorted in such a way that it resembles a cup, or a doughnut. After such changes of form, more startling than the *Metamorphoses* of Ovid, what is there that remains the same? The work of Professor Serre has provided profound answers to such questions. His intellectual energy has been fruitfully turned to translating geometrical figures into algebraic rigour, and Professor Serre is an algebraist of exceptional power. This has also brought in a golden age of geometry. As for the properties of numbers, he has shown unique insight into them, ever since he was one of the group of researchers who concealed their identity under the collective name of Nicolas Bourbaki. He is the author of most illuminating accounts of the fundamental concepts of mathematics, which have done much to dispel the darkness of ignorance with what the poet Lucretius calls 'the brilliant shafts of day'. Above all, perhaps, he excels in algebra, the most abstract area of mathematics. He has rightly attained very great influence in the subject, but he remains eminently approachable, and he is generous in advice to younger colleagues.

I present Professor Jean-Pierre Serre, FRS, a brilliant leader among mathematicians, an eagle-eyed investigator of geometrical figures, a most penetrating critic of numbers; the recipient of many honours and first winner of the Abel prize, for admission to the honorary degree of Doctor of Science.

Admission by the Chancellor
Outstanding among algebraists, most eminent of mathematicians, your distinguished work has brought to the rest of us a flood of illumination. Acting on my own authority and on that of the University as a whole, I admit you to the degree of honorary Doctor of Science.

Award of the first Abel Prize to Jean-Pierre Serre

Martin Raussen
Aalborg (Denmark)

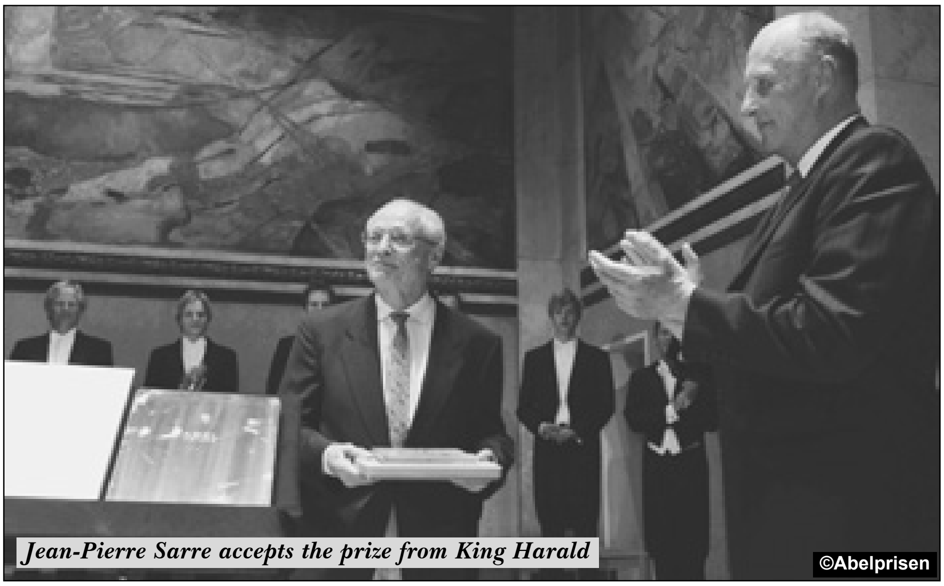
In the June issue we reported that the first Abel prize had been awarded to Jean-Pierre Serre. Here we summarise the events that took place in Oslo in June. A Danish version of this article appeared in the Newsletter of the Danish Mathematical Society, Matilde ***17****. Thanks are due to Jette Matthiesen for help with the English translation.*

Unfortunately, Alfred Nobel never donated a prize for mathematicians. The Fields Medals, awarded every four years during the International Congress of Mathematicians, have given its laureates Nobel prize-like prestige, but only among mathematicians. The medals have not received the same attention by the public, and the prize amount is modest.

In 2001 the Norwegian parliament Stortinget decided, at the request of the government, to grant NOK 200 million to the new Niels Henrik Abel Memorial Fund. They determined that the return of the fund should cover the award of an international Abel prize in mathematics, the events in conjunction with the presentation of the prize, and activities that aimed at increasing interest in the natural sciences among young people. The prize (amounting to NOK 6 million annually) was officially established at a bicentennial conference last year in conjunction with the 200th anniversary of Abel's birth. An Abel Committee was appointed, consisting

Jean-Pierre Serre accepting the Abel prize

of Erling Størmer (Oslo, Chair), John Macleod Ball (Oxford), Friedrich Hirzebruch (Bonn), David Mumford (Brown U, Fields medal 1974) and Jacob Palis (IMPA, Brazil). The committee decided to award the first Abel prize to the 76-year old French mathematician Jean-Pierre Serre, who had already received the Fields medal for his pioneering work in homotopy theory in 1954, when he was not even 28 years old.



Jean-Pierre Sarre accepts the prize from King Harald

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Events in Oslo

It was apparent from the main Karl Johan Avenue that something special was going to happen. Both sides of the boulevard were decorated with Bordeaux-red standards with the Abel prize logo.

The events started in bright sunshine in Oslo on Sunday, 1 June with a simple ceremony at the Abel Monument in Slottsparken. After Jens Erik Fenstad, Chair of the Abel Board (organiser of the prize events), had given a short speech, the Abel laureate Jean-Pierre Serre laid a wreath at the monument.

Abel lecture

On Monday 2 June the scientific programme started in Georg Sverdrup's house, the wonderful new library at the University of Oslo in Blindern. The organisers published a daily 'Abel-Gazette' with all the necessary information. An interesting exhibition, bridging the earliest Nordic evidence of mathematical literature and Serre's research areas and fields of interest, had been arranged on the first floor, in Gallery Sverdrup. Among many other things, a computer graphical explanation of the group structure in an elliptic curve was displayed.

Jean-Pierre Serre was introduced before he started his Abel lecture. To the amusement of the audience, he was twice called the Nobel Prizewinner in mathematics. Serre's lecture was entitled *Prime numbers, equations and modular forms*. He fully lived up to his reputation as a master expositor, lecturing in the old-fashioned way with chalk on the blackboard, and impressed everybody with a very clear presentation without notes.

The lecture was based on two equations, $x^3 - x - 1 = 0$ and $y^2 - y = x^3 - x^2$. Using the first, Serre explained how the number of solutions (minus 1) modulo a prime number p can be calculated as a coefficient in a power series (an Euler product), and as a coefficient in a Dirichlet series corresponding to a Galois representation, and thereby with a basis in modular forms (of weight 1). From the second equation, the prototype of an elliptic curve with a simple group structure, it is interesting to examine the number of solutions modulo p (or rather the number $a_p = 1 + p$ – the number of solutions: that this number has good arithmetical properties is explained by Lefschetz's fixed-point formula from topology). After a tour de force through 1-adic cohomology theory, we ended again with modular forms, now of weight 2. Finally, Serre explained the so-called Sato-Tate conjecture: if a_p is written as $2p^{1/2}$ cosh (φ_p), then the angles φ_p are evenly distributed with respect to the Sato-Tate-measure, which describes the distribution of angles around the rotating axis in a three-dimensional rotation – illustrated by means of the movement of a ping-pong ball.

Maths Tivoli

On Monday afternoon, the second purpose of the fund was in focus: to stimulate young people's interest in mathematics. To this end, a Maths Tivoli was established in front of the main building of the university, featuring Math-bingo, Math-bowling, the tower of Hanoi, and many other activities. Instructors wearing Abel prize T-shirts helped 400 children aged 10-12 from invited school classes through a course of playful mathematical activities. All participants received a book prize.

The winners of KappAbel (a contest for grade 9 pupils) and of an Abel Competition (a contest for upper secondary school pupils – the winners will participate in the Mathematics Olympics in Tokyo) were awarded prizes by Jean-Pierre Serre, assisted by Kristin Clemet, Minister of Education and Research, who stressed the importance of mathematics as a basis for knowledge and technological innovation and of the support to talented mathematicians. A young enthusiastic entertainer and a rapper provided the setting for the events.

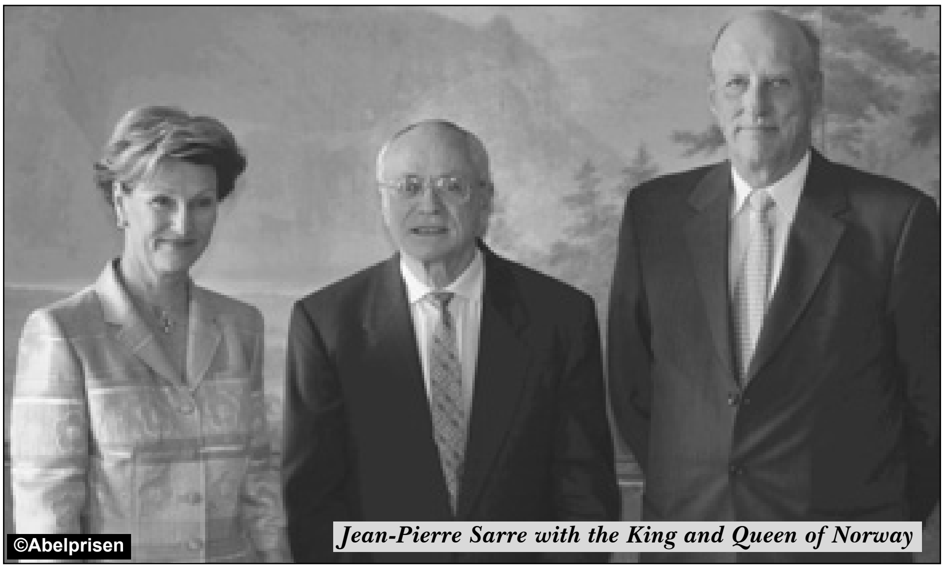
Press Contacts

Later in the afternoon, Jean-Pierre Serre received several parties of journalists for interviews at the Hotel Continental, where he was staying in the Abel Suite: the interview in this issue of the *Newsletter* was recorded on that occasion. On Tuesday morning, Serre and representatives of the Abel Committee and the Abel Board met the world press – 10 journalists from Norway, England, France and Germany – at the hotel. After this, Serre was received by the Queen and King of Norway at the Royal Palace.

Presentation of the prize

On Tuesday afternoon, the prize was presented at a ceremony – with due pomp and circumstance. In my entire life, I have never seen so many mathematicians in suits and ties, and I myself had to accept this unaccustomed role, too. At the entrance to the hall of the University of Oslo – with decorations by Edward Munch – Prince Christian August's Grenadiers played on fifes, horns and drums.

The ceremony opened with a procession through the central gangway, in which the Abel Board, the Abel Committee, the board of the Norwegian Academy of Sciences, and of course Jean-Pierre Serre, all participated. King Harald and Queen Sonja were then led into the hall. In her speech the president of the Norwegian Research Academy, Inger Moen, stated the double purpose of the Abel Prize: to strengthen mathematical research and to increase interest in mathematics and natural sciences among young people. The Abel Committee's chairman, professor



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Jean-Pierre Sarre with the King and Queen of Norway

Erling Størmer, motivated the choice of Serre as the first prize laureate: Serre has been a 'locomotive' for the modern shape of many mathematical fields, and has thus contributed to the solution of many outstanding problems in geometry and number theory.

The King then presented the prize to Jean-Pierre Serre. As the speeches so far had been in Norwegian, Serre reckoned that he was within his rights to give parts of his speech of thanks in his native French. He spoke warmly about the mathematical family and expressed hope that the Abel prize, both the 'big one' and the competitions for pupils, would increase the understanding of science among the public. The excellent Norwegian solo choir and a trumpeter provided a beautiful frame for the ceremony, with modern settings of Shakespeare pieces.

In the evening, the Norwegian government hosted an Abel banquet at Akershus Castle for VIPs. King Harald and Queen Sonja attended; both Prime Minister Kjell Bondevik and the IMU chairman, John Ball, gave speeches.

Abel Symposium

The professional programme ended on Wednesday with an Abel symposium at the University of Oslo. In her introduction, Ragni Piene explained that the symposium would contain lectures related to the prizewinner's work. Jean-Pierre Serre himself opened with a lecture on 'Finite subgroups of Lie groups'; at the end he determined the maximum orders for p-subgroups of the Lie-group E_8 over the rational numbers. Tony Springer (Utrecht) lectured on 'The compactification of a semisimple group' about the consequences of the properties of a group (such as its distribution into conjugacy classes) for its compactification. After that, Peter Sarnak (Princeton) lectured on 'L-functions and equidistributions' and on new connections to ergodic theory. The last lecturer, Barry Mazur (Harvard), connected in his lecture 'Spectra and L-functions' two of Serre's working fields, topology and number theory, by linking the Kervaire-Milnor form for the number of smooth structures of a sphere with p-adic L-functions by p-adic interpolation.

Finally, the mathematicians and their friends gathered for a wonderful garden party at the premises of the Norwegian Academy – and the weather was glorious! Only the future will show whether these events help the campaign for mathematics among the public. The serious Norwegian press described the prize award and its background in some detail – although the events were surely overshadowed by the expectations of a football match against a certain small neighbouring country to the south. The Norwegian mathematical family has certainly made great efforts for the public image of mathematics during the Abel events.

Martin Raussen [raussen@math.auc.dk], Department of Mathematical Sciences, Aalborg University, Denmark

Interview with Jean-Pierre Serre

Interviewers: Martin Raussen and Christian Skau

This interview took place in Oslo on 2 June 2003 during the Abel Prize celebrations.

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Topology

First, we congratulate you on winning the first Abel Prize. You started your career with a thesis that centred on algebraic topology. This was then (at least in France) a very new discipline and not a major area. What made you choose this topic?

I was participating in the Cartan Seminar, on Algebraic Topology. But Cartan did not suggest research topics to his students: they had to find one themselves; after that he would help them. This is what happened to me. I found that Leray's theory (about fibre spaces and their spectral sequence) could be applied to many more situations than was thought possible, and that such an extension could be used to compute homotopy groups.

The methods and results that you created in your thesis revolutionised homotopy theory and shaped it in its modern look ...

They certainly opened up lots of possibilities. Before my thesis, homotopy groups of spheres were almost entirely *terra incognita*; one did not even know that they are finitely generated!

One interesting aspect of the method I introduced was its algebraic character. In particular, one could make "local" computations, where the word "local" here is taken as in number theory: relative to a given prime number.

Is it true that one of the crucial points in this story was to identify something that looks like a fibre space without it being on the nose?

Indeed, to apply Leray's theory I needed to construct fibre spaces which did not exist if one used the standard definition. Namely, for every space X, I needed a fibre space E with base X and with trivial homotopy (for instance contractible). But how to get such a space?

One night in 1950, on the train bringing me back from our summer vacation, I saw it in a flash: just take for E the space of paths on X (with fixed origin a), the projection $E \to X$ being the evaluation map: path $\to$ extremity of the path. The fibre is then the loop space of (X, a). I had no doubt: this was it! So much so that I even waked up my wife to tell her ... (Of course, I still had to show that $E \to X$ deserves to be called a "fibration", and that Leray's theory applies to it. This was purely technical, but not completely easy.) It is strange that such a simple construction had so many consequences.

Work Themes and Work Style

This story about your sudden observation is reminiscent of Poincaré's flash of insight when stepping into a tramway: this is told in Hadamard's booklet The psychology of invention in the mathematical field. ***Do you often rely on sudden inspiration or would you rather characterise your work style as systematic? Or is it a mixture?***

There are topics to which I come back from time to time (l-adic representations, for instance), but I do not do this in a really systematic way. I rather follow my nose. As for flashes, like the one Hadamard described, I have had only two or three in more than 50 years. They are wonderful ... but much too rare!

These flashes come after a long effort, I guess?

I would not use the word "effort" in that case. Maybe a lot of thinking. It is not the conscious part of the mind which does the job. This is very well explained in Littlewood's charming book "A Mathematician's Miscellany".

Most of your work, since the 'topology years', has been devoted to number theory and algebraic geometry.

You see, I work in several apparently different topics, but in fact they are all related to each other. I do not feel that I am really changing. For instance, in number theory, group theory or algebraic geometry, I use ideas from topology, such as cohomology, sheaves and obstructions.

From that point of view, I especially enjoyed working on l-adic representations and modular forms: one needs number theory, algebraic geometry, Lie groups (both real and l-adic), q-expansions (combinatorics style) ... A wonderful mélange.

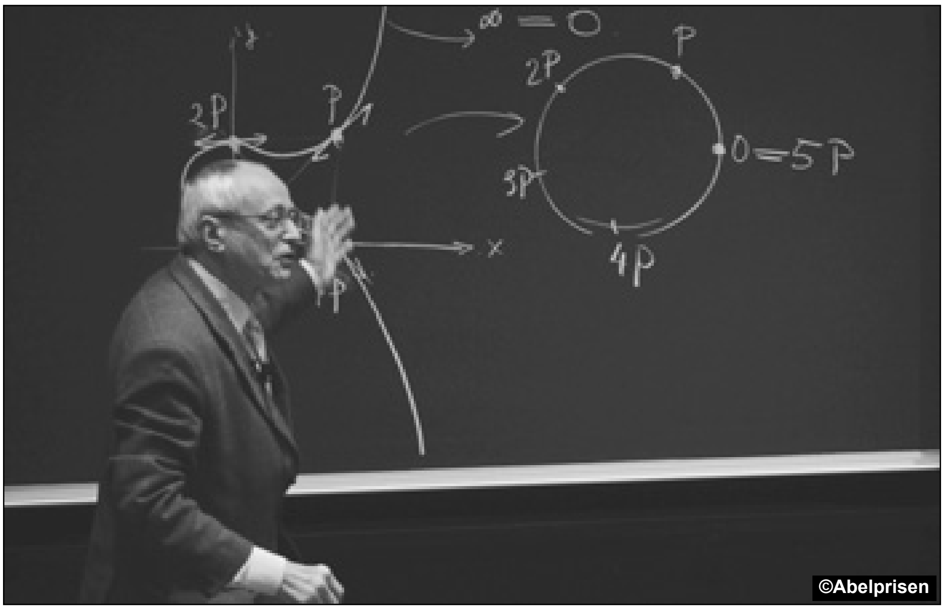
Do you have a geometric or an algebraic intuition and way of thinking – or both?
I would say algebraic, but I understand the geometric language better than the purely algebraic one: if I have to choose between a Lie group and a bi-algebra, I choose the Lie group! Still, I don't feel I am a true geometer, such as Bott, or Gromov.

I also like analysis, but I can't pretend to be a true analyst either. The true analyst knows at first sight what is "large", "small", "probably small" and "provably small" (not the same thing). I lack that intuitive feeling: I need to write down pedestrian estimates.

You have had a long career and have worked on many different subjects. Which of your theories or results do you like most? Which are most important to you?
A delicate question. Would you ask a mother which of her children she prefers? All I can say is that some of my papers were very easy to write, and some others were truly difficult. In the first category, there is FAC ("faisceaux algébriques cohérents"). When I wrote it, I felt that I was merely copying a text which already existed; there was almost no effort on my part. In the "difficult" category, I remember a paper on open subgroups of profinite groups, which gave me so much trouble that, until the very end, I was not sure whether I was proving the theorem or making a counter-example! Another difficult one was the paper dedicated to Manin where I made some very precise (and very daring) conjectures on "modular" Galois representations (mod p); this one was even painful; after I had finished it, I was so exhausted that I stopped publishing for several years.
On the pleasure side, I should mention a paper dedicated to Borel, on tensor products of group representations in characteristic p. I had been a group theory lover since my early twenties, and I had used groups a lot, and even proved a few theorems on them. But the theorem on tensor products, obtained when I was in my late sixties, was the first one I really enjoyed. I had the feeling that Group Theory, after a 40 years courtship, had consented to give me a kiss.

You have been active in the mathematical frontline for more than 50 years. Hardy made the often quoted remark that 'Mathematics is a young man's game'. Isn't that wrong – aren't you a perfect counterexample?
Not a perfect one: have you noticed that most of the quotations of the Abel Prize are relative to things I had done before I was 30?

What is true is that people of my generation (such as Atiyah, Borel, Bott, Shimura, ...) keep working longer than our predecessors did (with a few remarkable exceptions such as Elie Cartan, Siegel, Zariski). I hope we shall continue.



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Relations to mathematical history

Since you've won the Abel Prize, we'd like to ask some questions with a background in Abel's time. The algebraic equations that Abel and Galois studied, coming from the transformation theory of elliptic functions, turned out to be very important much later for the arithmetic theory of elliptic curves. What are your comments on this remarkable fact, especially in connection with your own contribution to this theory?
Yes, elliptic curves are very much in fashion (with good reasons, ranging from Langlands' program to cryptography). In the 60s and 70s I spent a lot of time studying their division points (a.k.a. Tate modules) and their Galois groups. A very entertaining game: one has to combine information coming from several different sources: Hodge-Tate decompositions, tame inertia, Frobenius elements, finiteness theorems à la Siegel, ... I like that.

Hermite once said that Abel had given mathematicians something to work on for the next 150 years. Do you agree?
I dislike such grand statements as Hermite's. They imply that the person who speaks knows what will happen in the next century. This is hubris.

In the introduction of one of his papers Abel writes that one should strive to give a problem a form such that it is always possible to solve it – something which he claims is always possible. And he goes on, saying that by presenting a problem in a well-chosen form the statement itself will contain the seeds of its solution.
An optimistic point of view! Grothendieck would certainly share it. As for myself, I am afraid it applies only to algebraic questions, not to arithmetic ones. For instance, what would Abel have said about the Riemann hypothesis? That the form in which it is stated is not the good one?

The role of proofs

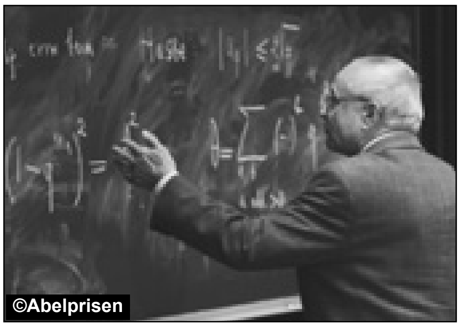
When you are doing mathematics, can you know that something is true even before you have the proof?
Of course, this is very common. But one should distinguish between the genuine goal (say, the modularity of elliptic curves, in the case of Wiles), which one feels is surely true, and the auxiliary statements (lemmas, etc), which may well be untractable (as happened to Wiles in his first attempt) or even downright false (as happened similarly to Lafforgue).

Do proofs always have a value in themselves? What about, for example, of the proof of the four-colour theorem.
We are entering a grey area: computer-aided proofs. They are not proofs in the standard sense that they can be checked by a line by line verification. They are especially unreliable when they claim to make a complete list of something or other.

[I remember receiving in the 90s such a list for the subgroups of given index of some discrete group. The computer had found, let us say, 20 of them. I was familiar with these groups, and I easily found "by hand" about 30 such. I wrote to the authors. They explained their mistake: they had made part of the computation in Japan, and another part in Germany, but they had forgotten to do some intermediate part ... Typical!]

On the other hand, computer-aided proofs are often more convincing than many standard proofs based on diagrams which are claimed to commute, arrows which are supposed to be the same, and arguments which are left to the reader.

What about the proof of the classification of the finite simple groups?
You are pushing the right button. For years, I have been arguing with group theorists who claimed that the "Classification Theorem" was a "theorem", i.e. had been proved. It had indeed been announced as such in 1980 by Gorenstein, but it was found later that there was a gap (the classification of "quasi-thin" groups). Whenever I asked the specialists, they replied something like: "Oh no, it is not a gap, it is just something which has not been written, but there is an incomplete unpublished 800 pages manu



script on it ”.

For me, it was just the same as a “gap”, and I could not understand why it was not acknowledged as such. Fortunately, Aschbacher and Smith have now written a long manuscript (more than 1200 pages) in order to fill in the gap. When this will have been checked by other experts, it will be the right moment to celebrate.

But if a proof is 1200 pages long, what use is it?

As a matter of fact, the total length of the proof of the classification is much more than 1200 pages; about 10 times more. But that is not surprising: the mere statement of the theorem is itself extremely long, since, in order to be useful, it has to include the detailed description, not only of the Chevalley groups, but also of the 26 sporadic groups.

It is a beautiful theorem. It has many very surprising applications. I don’t think that using it raises a real problem for mathematicians in other fields: they just have to make clear what part of their proof depends on it.

Important mathematical problems

Do you feel that there are core or mainstream areas in mathematics – are some topics more important than others?

A delicate question. Clearly, there are branches of mathematics which are less important; those where people just play around with a few axioms and their logical dependences. But it is not possible to be dogmatic about this. Sometimes, a neglected area becomes interesting, and develops new connections with other branches of mathematics.

On the other hand, there are questions which are clearly central for our understanding of the mathematical world: the Riemann hypothesis and the Langlands program are two obvious cases. There is also the Poincaré conjecture – which may well stop being a conjecture, thanks to Perelman!

Do you have more information, or a hunch, about the correctness of the proof?

Hunch? Who cares about hunches? Information? Not really, but I have heard that people at IHES and MIT are very excited about this sketch of proof. An interesting aspect of Perelman’s method is that it uses Analysis, for what is a purely topological problem. Very satisfying.

We have already moved a little into the future with our discussion of the Poincaré conjecture. Which important mathematical problems would you like to see solved in the near future? And do you agree with the primary importance of the Clay Millennium Prize Problems?

Ah, the million dollars Clay problems! A strange idea: giving so much money for one problem ... but how can I criticise it, just after having received the Abel prize? Still, I feel there is some risk involved, namely that people would shy from discussing their partial results, as already happened ten years ago with Fermat’s theorem.

As for the choice of questions made by the Clay Institute, I feel it is very good. The Riemann hypothesis and the Birch & Swinnerton-Dyer conjecture are rightly there. The Hodge conjecture, too; but for a different reason: it is not clear at all whether the answer will be yes or no; what will be very important will be to decide which (I am hoping, of course, that it will not turn out to be undecidable ...). The P = NP question belongs to the same category as Hodge, except that there would be many more applications if the answer turned out to be “yes”.

Can you think of any other problems of the same stature?

I already told you that the Langlands program is one of the major questions in mathematics nowadays. It was probably not included in the Clay list because it is very hard to formulate with the required precision.

Besides your scientific merits, you are also known as a master expositor, as we witnessed during your lecture today.

Thanks. I come from the South of France, where people like to speak; not only with their mouth, but with their hands, and in my case with a piece of chalk.

When I have understood something, I have the feeling that anybody else can understand it too, and it gives me great pleasure to explain it to other mathematicians, be they students or colleagues.

Another side of the coin is that wrong statements make me almost physically sick. I can’t bear them. When I hear one in a lecture I usually interrupt the speaker, and when I find one in a preprint, a paper or in a book I write to the author (or, if the author happens to be myself, I make a note in view of a next edition). I am not sure this habit of mine has made me very popular among lecturers and authors ...

Accessibility and importance of mathematics

Mathematics witnesses an explosion of subjects and disciplines, making it difficult to master even the minor disciplines. On the other hand – as you demonstrated today in your lecture – it is very important that disciplines cross-fertilise each other. How can young mathematicians, in particular, cope with this explosion of knowledge and come up with something new?

Oh yes, I have already been asked that question in my Singapore interview, reproduced by *Intelligencer*[1]. My answer is that, when one is truly interested in a specific question, there is usually very little in the existing literature which is relevant. This means you are on your own.

As for the feeling of “explosion” of mathematics, I am convinced that Abel felt the same way when he started working, after Euler, Lagrange, Legendre and Gauss. But he found new questions and new solutions. It has been the same ever since. There is no need to worry.

Another current problem is that many young and talented people – and also public opinion leaders – don’t think that mathematics is very exciting.

Yes. Sadly enough, there are many such examples.

A few years ago, there was even a French minister of Research who was quoted as saying that mathematicians are not useful any more, since now it is enough to know how to punch a key on a computer. (He probably believed that keys and computer programs grow on trees ...)

Still, I am optimistic about young people discovering, and being attracted by, mathematics. One good aspect of the Abel festivities is the Norwegian Abel competitions, for high school students.

Sports and literature

Could you tell us about your interests besides mathematics?

Sports! More precisely: skiing, ping-pong, and rock climbing. I was never really good at any of them (e.g. when I skied, I did not know how to slalom, so that I would rather go “schuss” than trying to turn); but I enjoyed them a lot.

As luck has it, a consequence of old age is that my knees are not working any more (one of them is even replaced by a metal-plastic contraption), so that I had to stop doing any sport. The only type of rock-climbing I can do now is a vicarious one: taking friends to Fontainebleau and coaxing them into climbing the rocks I would have done ten years ago. It is still fun; but much less so than the real thing.

Other interests:

- movies (“Pulp Fiction” is one of my favourites – I am also a fan of Altman, Truffaut, Rohmer, the Coen brothers ...);
- chess;
- books (of all kinds, from Giono to Böll and to Kawabata, including fairy tales and the “Harry Potter” series).

Prof. Serre, thank you for this interview on behalf of the Danish and the Norwegian Mathematical Societies.

[1] An interview with J-P. Serre, *The Mathematical Intelligencer* **8** (1986), 8-13.

Consult also: -

‘Jean-Pierre Serre’, in *Wolf Prize in Mathematics Vol. II* (eds. S. S. Chern and F. Hirzebruch), World Sci. Publ. Co. (2001) 523-551;

‘Jean-Pierre Serre, medalla Fields’ by Pilar Bayer, *La Gaceta* **4** (1) (2001), 211-247.

The interviewers were Martin Raussen, Aalborg University, Denmark, and Christian Skau, Norwegian University of Science and Technology, Trondheim, Norway.

Interview with Philippe Tondeur

The future of the mathematical sciences

interviewer : Luc Lemaire (Bruxelles)

Philippe Tondeur's lifelong involvement with mathematics started with a Ph.D. in Zurich, followed by various research and teaching positions in Paris, Harvard, Berkeley and many other places. He joined the mathematics department of the University of Illinois at Urbana-Champaign in 1968, where he is now Emeritus professor. From 1999 to 2002 he was Director of the Division of Mathematical Sciences at the U.S.A. National Science Foundation (NSF).

Thanks to his work and power of vision, the annual NSF budget for mathematics has approximately doubled in four years, whereas over the preceding ten-year period it had increased at an annual rate of 1.5%. He is obviously a key witness and actor in the development of mathematical research.

How do you see the future of mathematics, in this period of rapid societal changes?

Mathematics is a key science for the future, both through its fundamental development and through its enabling role for science, engineering and technology. This is illustrated by dramatic advances in communications, bioinformatics, the understanding of uncertainty, and dealing with large data sets. All these developments are fuelled by ongoing advances in fundamental mathematics and the impact of computing.

Mathematics is thriving as a discipline, but its future is fragile – because the worldwide flow of talent into this science is at risk. One impediment to this necessary talent flow is the decreasing interest of young people in this exciting, but admittedly difficult, discipline.

A functional modern society requires a massive improvement in mathematics education at all levels. Mathematical scientists certainly share responsibility in this endeavour, but the endeavour is a huge challenge for mankind. Higher education in the mathematical sciences can contribute by developing in all students a sense of the embeddedness of mathematics in the broader science enterprise, and in doctoral and postdoctoral students a sense of the stewardship responsibility for the mathematical sciences.

In the future the increasing importance of interdisciplinary research will require more flexibility in career pathways, and a greater adaptability of academic research portfolios to evolving research agendas. Society's interest in an adaptive scientific workforce is paramount. No other investment will have more significant long-term impact on education, health, and the economy.

How could governments steer the development of mathematical research?

In my view, governmental support for the mathematical sciences should not be focussed on trying to guess future successful themes, but should rather concentrate on the development of human resources and facilitating infrastructure. This corresponds to Alexander von Humboldt's idea of the University, where research and education are inextricably intertwined. Society's interest is in the development of a diverse globally competitive and globally engaged workforce of scientists and engineers, and in a broader sense, a scientifically literate citizenry. The mathematical sciences are key to this development. Science innovation and the integration of the disciplines into the evolving science machinery is inseparably bound to the education process at university level.

What could be done to improve future developments?

I want to list some practical recommendations for the improvement of the health of the mathematical sciences. They resulted from a 1998 report circulating widely as the *Odom report*, named after the chair of the committee issuing the report; this report is accessible under the title *Report: International Assessment of the US Mathematical Sciences* at the webpage *http://www.nsf.gov/mps/dms/* under Research Highlights). The recommendations have been influential guidelines for action at the U.S. National Science Foundation over the last few years. They are tailored to the specific higher education structure of the U.S., but with proper interpretation seem to me to make sense for a large portion of higher education throughout the world:

- undergraduate students have to believe that a career in the mathematical sciences offers exciting opportunities.
- graduate trainee and postdoctoral positions are needed to attract and support the best talent.
- undergraduate and graduate education in the mathematical sciences should be broadened by increased exposure to other fields.
- interactions among university-based mathematical scientists and users of mathematics and statistics in other disciplines, in industry and government, should be encouraged and supported.

The implications for financial support of the mathematical sciences are to:

- increase the support for undergraduate, graduate, and postdoctoral students;
- support collaborative research in addition to the traditional support of individual researchers;
- encourage cross-disciplinary research and training;
- improve the support of the infrastructure.

One exciting infrastructure project in need of international support is the Digital Mathematics Library (digitisation of the entire mathematical research literature), adopted by the Committee on Electronic Information Communication of the International Mathematical Union: see the documentation at *http://www.library.cornell.edu/dmlib* and *http://www.ams.org/ewing/Documents/Twenty_centuries.pdf*).

You have clearly convinced the NSF of the importance of mathematics. Do you have a secret recipe?

You have to advocate what the mathematical sciences can do for society. Mathematics is a key science for the future, both through its fundamental development and through its enabling role for science, engineering and technology.

How do you see the classical dichotomy between 'pure' and 'applied' mathematics?

I do consider fundamental and interdisciplinary mathematics as overlapping, mutually reinforcing agendas: they are inseparably intertwined, intellectually as well as budgetarily. I think that this is a more realistic view of the dynamics of the science enterprise than the classical dichotomy, which I strive to avoid. The NSF leadership completely concurs with this view.

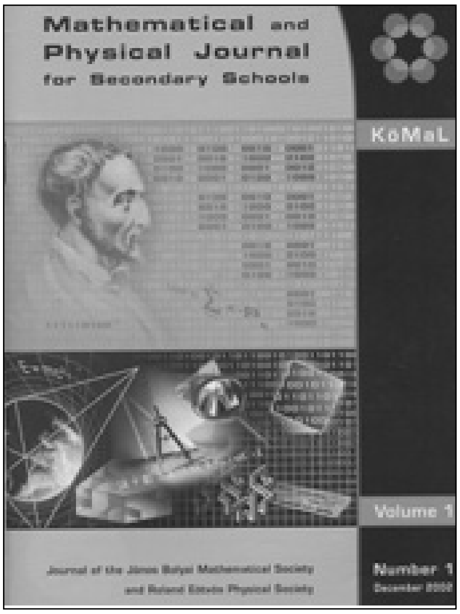
A conclusion?

The opportunities for the mathematical sciences at the beginning of this century are fantastic. This century is going to be one of unprecedented pervasiveness of mathematical thought throughout the sciences and our learning culture. In a data-driven world, mathematical concepts and algorithmic processes will be the primary navigational tools. The challenge for the mathematical sciences community is to seize this opportunity, and thereby help to shape the world of tomorrow.

Problem Corner

KöMaL – jazzing up an aged journal

Paul Jainta



Half a year ago an American friend of Hungarian extraction told me about a scientific magazine of advanced years that has produced offspring. The name of the aged journal is KöMaL, which appeared for the first time in 1894: KöMaL is an abbreviation of *Középiskolai Matematikai és Fizikai Lapok* (High School Mathematics and Physics Journal). Since December 2002 the parent journal has been joined by a periodical in English, called ***Mathematical and Physical Journal for Secondary Schools*** and producing two issues per year.

Except for the interruptions caused by the World Wars, KöMaL has been alive and well for a century. It managed to survive the Peace Treaty of Trianon, which dismembered the former Hungary by reducing its territory by 70% and its population by 60%. It even gained considerably during the communist era, when opportunities for intellectual freedom in other fields were severely limited. Nevertheless, it is enlightening to note that, among the seventeen cities from which solutions were first sent in, only eight are in present-day Hungary (Budapest, Debrecen, Győr, Kaposvár, Kisújszállás, Nyíregyháza, Pécs and Sátoraljaújhely). The remaining nine cities are now located in Slovakia (Losonc, Nyitra and Bratislava) and Romania (Arad, Déva, Gilad, Kolozsvár, Nagyenyed and Székelyudvarhely).

It has been said that Hungary's per capita output of mathematicians is the largest in the world. The same claim probably holds in other sciences and engineering as well, especially if one includes those who received their high-school education in Hungary but later emigrated. Most of them were avid readers of KöMaL and active participants in its year-round problem-solving competition. They learned the skills of problem solving by perusing the solutions published and emulating them to the best of their abilities.

KöMaL now produces nine issues per year, each of 64 pages. There are 22 new problems in each number: four practice exercises (marked 'C'), eight problems for students in Grades 9-10 (marked 'G' for 'gyakorlat', exercise), six problems for students in Grades 11-12 (marked 'F' for 'feladat', problem), and four more challenging teasers (marked 'N'). Students in lower grades may submit solutions to problems of a higher level, but those in upper grades are restricted to the more difficult queries. Since the mid-1960s, the problems have also appeared in English.

Among all participants, there is a year-round competition for each grade level in each problem category. Those contestants who accumulate most points during the school year receive a year's subscription to KöMaL. The solutions to all problems appear in later issues, usually attributed to the students whose submissions often require little or no editing. For each problem the solver gets at most five points, but grading is hard because only complete solutions are awarded full credit; it is done mostly by former contestants while pursuing their university studies. Sometimes contestants can win up to two extra points for an alternative solution if it is markedly different from the original one. Contestants, teachers, university students and professors often propose new questions for consideration by the mathematics Editorial Board.

In addition to problems and solutions, KöMaL contains regular reports on regional, national and international competitions, as well as articles of general interest and book reviews. KöMaL and its young offspring are published jointly by the János Bolyai Mathematical Society and the Roland Eötvös Physical Society, with financial support by the Ministry of Education, and have a circulation of 5000 copies.

To subscribe to *Mathematical and Physical Journal for Secondary Schools*, use the electronic subscription form on the of KöMaL website *http://www.komal.hu* or contact the e-mail address: *mail.elft@mtesz.hu*

The subsidiary journal contains the most interesting problems and articles from KöMaL. The two lead stories of the first issue were 'Cardan and cryptography' and 'Poncelet's theorem'. From a wide range of problems I've selected two examples, 152 and 157. The remaining teasers come from former issues of KöMaL.

152 The first four terms of an arithmetic progression of integers are a_1, a_2, a_3, a_4. Show that $1 \cdot a_1^2 + 2 \cdot a_2^2 + 3 \cdot a_3^2 + 4 \cdot a_4^2$ can be expressed as the sum of two perfect squares.

153 Is it possible to get equal results if $\sqrt{(10^{2n} - 10^n)}$ and $\sqrt{(10^{2n} - 10^n + 1)}$ are rounded to the nearest integer? (n is a positive integer.)

154 An Aztec pyramid is a square-based right truncated pyramid. The length of the base edges is 81m, the top edges are 16m and the lateral edges are 65m long. A tourist access is designed to start at a base vertex and to rise at a uniform rate along all four lateral faces, ending at a corner of the top square. At what points should the path cross the lateral edges?

155 The billposters of the Mathematician's Party observed that people read the posters standing 3 meters far away from the centres of the cylindrical advertising pillars that have 1.5m diameter. The Party wants to achieve that, after sticking the posters around a pillar, a whole poster will be visible from any direction. How wide should the posters be?

156 Find a simpler expression for the sum $S = 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \ldots + 100 \cdot 3^{100}$.

157 Given two non-negative numbers x and y, satisfying the inequality $x^3 + y^4 \leq x^2 + y^3$, prove that $x^3 + y^3 \leq 2$.

Solutions to some earlier problems

140 Find the smallest positive integer n such that the number 1 can be represented as a sum of n periodic decimal fractions consisting only of the digits 0 and 7.

Solution by the Con Amore problem group, The Danish University of Education, Copenhagen.

Also solved by Pietro Fanciulli, Porto S. Stefano, Italy, Niels Bejlegaard, Copenhagen, J. N. Lillington, Wareham, UK, and Dr Z. Reut, London.

The problem turns to be equivalent to representing the fraction $^1/_7$ as a sum of n periodic decimal fractions consisting only of the digits 0 and 1. Because $^1/_7 = 0.142857\ldots$, in order to write it as required, n must be at least 8, since 8 appears among the digits that determine the period.

However, $n = 8$ is enough, since we can choose 1 as the fourth decimal place in each period of the eight summands of decimal fractions, and then choose 1 as the first decimal place in exactly one of them, 1 as the second decimal place in exactly four of them, and so on, and 0 otherwise. If we exchange every digit 1 in each of these eight decimal fractions with digit 7, we get a representation of the type required, and it cannot be done with fewer than eight such decimal fractions.

Finally, we give an example of a representation of $^1/_7$ as a sum of eight periodic decimal fractions with as many digits 1 as possible in a 'greedy' way. We have:

$$0.142857\ldots = 0.111111\ldots + 0.011111\ldots + 0.010111\ldots + 0.010111\ldots + 0.000111\ldots + 0.000101\ldots + 0.000101\ldots + 0.000100\ldots$$
$$= {}^1/_9 + {}^{11111}/_{999999} + {}^{10111}/_{999999} + {}^{10111}/_{999999} + {}^1/_{9009} + {}^{101}/_{999999} + {}^{101}/_{999999} + {}^{100}/_{999999} = {}^1/_7.$$

From this we can derive the sums

$$0.999999\ldots = 0.777777\ldots + 0.077777\ldots + 0.070777\ldots + 0.070777\ldots + 0.000777\ldots + 0.000707\ldots + 0.000707\ldots + 0.000700\ldots$$
$$= {}^7/_9 + {}^{11111}/_{142857} + {}^{10111}/_{142857} + {}^{10111}/_{142857} + {}^1/_{1287} + {}^{101}/_{142857} + {}^{101}/_{142857} + {}^{100}/_{142857} = 1.$$

141 The number $A = 11\ldots1$ has n digits (each 1), and is divisible by 7.
Find the sum of all the digits of the number $A/7$.

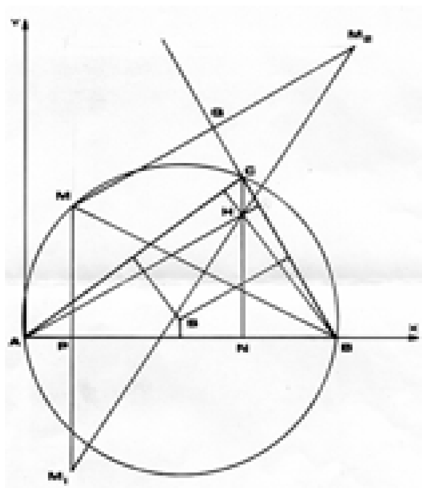
Solution by Pietro Fanciulli, Porto S. Stefano, Italy.

Also solved by Con Amore problem group, Niels Bejlegaard, J. N. Lillington, and Dr Z. Reut.

Let $A_n = 11\ldots1 = 10^{n-1} + 10^{n-2} + \ldots + 1 = (10^n - 1)/9$.
If A_n is divisible by 7, then the condition of the problem implies that $10^n \equiv 1 \pmod 7$.
By Fermat's Theorem we thus have $10^6 \equiv 1 \pmod 7$, and so 6 must divide n – so $n = 6k$ ($k = 1, 2, \ldots$).
Now, for $k = 1$, $A_6/7 = 15873 = N$; $A_{12}/7 = 10^6N + N = (10^6 + 1)N$; $A_{18}/7 = (10^{12} + 10^6 + 1)N$; and so on.
In general, we can write $A/7 = A_{6k} = (10^{6k-6} + 10^{6k-12} + \ldots + 1)N = 15873\ 015873\ldots015873$.
Thus, the block of numbers 015873 is repeated k times.
Let $s(N) = 24 \times$ the digit sum of N; then $s(A_{6k}/7) = 24k$, for $k = 1, 2, \ldots$.

142 Given an acute triangle ABC, let M be an interior point of $\angle ABC$ but an exterior point of ΔABC.
If H is the orthocenter of ΔABC and M_1, M_2 are symmetric to M with respect to AB, BC,
prove that M lies on the circumcircle of ΔABC if $H \in M_1M_2$.

Solution by Dr Z. Reut. Also solved by Niels Bejlegaard and J. N. Lillington.



Let the triangle ABC have internal angle β at the vertex B and the adjacent sides $AB = c$ and $BC = a$, and refer the positions of points with respect to rectangular coordinates (x, y) with the origin at the vertex A (see the figure).
Assume that the point $M(x, y)$ is on the circumcircle with radius R and centre at $S(\frac{1}{2}c, (R^2 - (\frac{1}{2}c)^2)^{1/2})$, and that the points $M_1(x, -y)$ and M are symmetric with respect to AB.
The triangle BPM has a right angle at P; let the angle $PBM = \beta^*$, so

$\sin \beta^* = y / BM$ and $\cos \beta^* = (c - x) / BM$.

The triangle BQM has a right angle at Q, and the angle $QBM = \beta - \beta^*$, so $\sin(\beta - \beta^*) = MQ / BM$.

It follows that

$MQ = [\sin\beta\cos\beta^* - \cos\beta\sin\beta^*]\cdot BM = (c - x)\sin\beta - y\cos\beta$,

and $MM_2 = 2MQ$, because of symmetry with respect to BC.

Since the angle PMQ is the supplement of β, because of MP being perpendicular to AB and MQ to BC, the coordinates of the point M_2 are

$(x + MM_2\sin\beta, y + MM_2\cos\beta) = [x + 2(c - x)\sin^2\beta - 2y\sin\beta\cos\beta, 2(c - x)\sin\beta\cos\beta + y - 2y\cos^2\beta]$.

The coordinates of the orthocentre H are $(c - a\cos\beta, (c - a\cos\beta)\cot\beta)$,

and H is on the line M_1M_2 when M_1H is collinear with the line.

The slopes of the line segments M_1H and M_1M_2 must be equal, leading to the condition

$[(c - a\cos\beta)\cot\beta + y] / (c - a\cos\beta - x] = [(c - x)\cos\beta + y\sin\beta] / [(c - x)\sin\beta - y\cos\beta)$,

which reduces to the equation

$x^2 - cx + y^2 - y(a - c\cos\beta) / \sin\beta = 0$, for $\sin\beta \neq 0$.

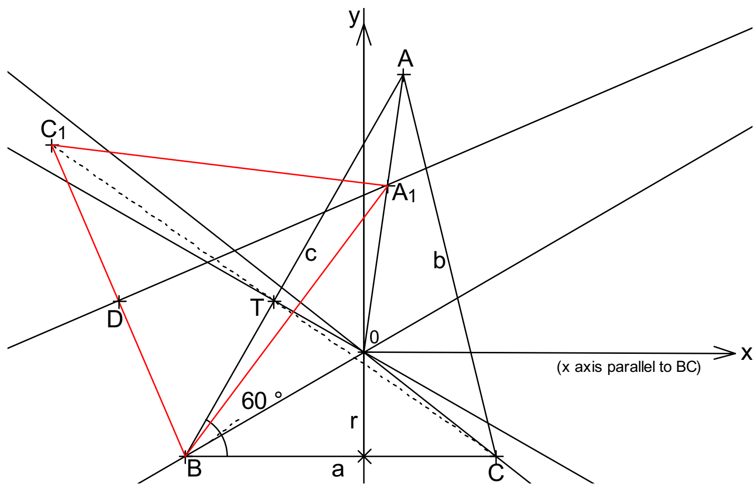
The equation of the circumcircle is $(x - ½c)^2 + (y - (R^2 - (½c)^2)^{1/2})^2 = R^2$,

which reduces to $x^2 - cx + y^2 - 2y(R^2 - (½c)^2)^{1/2} = 0$.

Comparing these two equations gives $2(R^2 - (½c)^2)^{1/2} = (a - c\cos\beta) / \sin\beta$,

and further, $R = [a^2 + c^2 - 2ac\cos\beta]^{1/2} / (2\sin\beta)$, giving the radius of the circumcircle.

143 Given a triangle ABC with $\angle ABC = 60°$, let T be the common point of the incircle and the side AB, and let C_1 be symmetric to C with respect to T. If the common point of the perpendicular bisector BC_1 and the angular bisector of $\angle BAC$ is A_1, prove that ΔA_1BC_1 is equilateral.



Solution by J.N. Lillington. Also solved by Niels Bejlegaard.

Choose a coordinate system with the centre of the incircle at the origin (0, 0).

Let $BC = a$, $AB = c$, and let r be the radius of the incircle.

Then A is $(-r\sqrt{3} + ½c, -r + ½c\sqrt{3})$ (for $x_A = c\cos 60° - r/\tan 30°$ and $y_A = c\sin 60° - r$),

B is $(-r\sqrt{3}, -r)$, and C is $(a - r\sqrt{3}, -r)$.

Further, T is $((-½r\sqrt{3}), ½r)$, C_1 is $(-a, 2r)$, D is $(½(-a - r\sqrt{3}), ½r)$.

Now $½r(a + b + c) = ½ac\sin 60°$ and $b^2 = a^2 + c^2 - 2ac\cos 60°$.

Eliminating b gives $c = (12r^2 - 4\sqrt{3}ar) / (4\sqrt{3}r - 3a)$.

Thus A can also be written as $((\sqrt{3}ar - 6r^2) / (4\sqrt{3}r - 3a), (-3ar + 2\sqrt{3}r^2) / (4\sqrt{3}r - 3a))$.

The equation of DA_1 is $y - [(a - \sqrt{3}r) / 3r]x - a^2/6r = 0$;

the equation of OA is $y + (3ar - 2\sqrt{3}r^2) / [(\sqrt{3}ar - 6r^2)]x = 0$.

Then the coordinates of A_1 are $(-½a + \sqrt{3}r, ½\sqrt{3}a - r)$, and $A_1B^2 = A_1C_1^2 = BC_1^2 = a^2 - 2\sqrt{3}ar + 12r^2$.

This shows that the triangle A_1BC_1 is equilateral.

144 Given n points in the plane, no three being collinear, prove the existence of at least $[n(n-1)/6]$ triangles with vertices formed by these points, such that no triangle is contained in any other triangle. ($[x]$ denotes the integer part of x).

Solution by *J . N. Lillington. Also solved by Niels Bejlegaard.*

Proof by induction for $n \geq 3$.

If $n = 3$, $[n(n - 1)/6] = 1$, and the result is true.

Suppose now that the result is true for n, and we consider $n + 1$ points.

For each of these $n + 1$ points, consider the remaining n points.

In this set there are at least $[n(n - 1) / 6]$ triangles not contained in any other triangle, by the induction hypothesis.

Adding these for each point gives a total of $(n + 1)[n(n - 1) / 6]$ triangles.

Each distinct triangle appears $n + 1 - 3 = n - 2$ times, so the total number N of distinct triangles with the required property is $N = ((n + 1) / (n - 2))\ [n(n - 1) / 6]$.

If n = 6k, $6k + 1$, $6k + 3$, $6k + 4$ (for $k \geq 0$), then

$$N = ((n + 1) / (n - 2))\ (n(n - 1) / 6) > (n + 1)n / 6 \geq [n(n + 1) / 6].$$

If $n = 6k + 2$, $N = ((n + 1) / (n - 2))\ [(6k + 2)(6k + 1) / 6]$

$$> ((n + 1) / (n - 2))\ [(6k + 2)6k / 6] = ((n + 1) / (n - 2))\ n(n - 2) / 6 \geq [(n + 1)n / 6].$$

If $n = 6k + 5$, $N = ((n + 1) / (n - 2))\ [(6k + 5)(6k + 4) / 6] = ((n + 1) / (n - 2))\ [(36k^2 + 54k + 20) / 6]$

$$> ((n + 1) / (n - 2))\ [(36k^2 + 54k + 18) / 6] = ((n + 1) / (n - 2))\ [(6k + 6)(6k + 3) / 6]$$

$$= ((n + 1) / (n - 2))^2\ (n - 2) / 6 > (n + 1)n / 6 \geq [n(n + 1) / 6].$$

145 Prove that, for each positive integer n, there exists a polynomial $P_n(x)$ with integer coefficients, such that the numbers $P_n(1), P_n(2), \dots , P_n(n)$ are different powers of 2.

Solution by the Con Amore problem group. Also solved by Niels Bejlegaard and J. N. Lillington.

Let $M(n) = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 4 & \dots & 2^{n-1} \\ 1 & 3 & 9 & \dots & 3^{n-1} \\ & & \dots & & \\ 1 & n & n^2 & \dots & n^{n-1} \end{pmatrix}$,

and $d = \det(M(n))$; then $d \neq 0$.

Further let $d = 2^a b$, where b is odd.

We have $M(n)\begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$, so $M(n)^{-1}\begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ (1)

The elements $\mu_{ij}(n)$ of $M(n)^{-1}$ can be written as $\mu_{ij}(n) = t_{ij}/d$, and then (1) means that

$t_{11}/d + \dots + t_{1n}/d = 1$ (2)

and $t_{i1}/d + \dots + t_{in}/d = 0 \quad (i = 2, \dots , n)$ (3)

Now let p be the period of 2 in the multiplicative group modulo b.

Then for any k in $\mathbf{Z}$, there is a c_k in $\mathbf{Z}$ such that $2^{kp} = 1 + c_k b$. (4)

Furthermore, let $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{n-1} \end{pmatrix} = M(n)^{-1} \cdot \begin{pmatrix} 2^a \\ 2^{a+p} \\ 2^{a+2p} \\ \vdots \\ 2^{a+(n-1)p} \end{pmatrix}$ (5)

We have (due to (4))

$a_0 = t_{11} / d\, 2^a + t_{12} / d\, 2^a (1 + c_2 b) + \dots + t_{1n} / d\, 2^a\ (1 + c_n b)$

$= (t_{11} / d + \dots + t_{1n} / d)\ 2^a + t_{12}\, c_2\, 2^a\, b / d + \dots + t_{1n}\, c_n\, 2^a\, b/d$

$= 1.2^a + t_{12}\, c_2 + \dots + t_{1n}\, c_n$ (because of condition (2)), so a_1 is in $\mathbf{Z}$.

Similarly, for $i = 2, \dots, n$, using (3) and (4), we get $a_i = 0.2^a + t_{12}\, c_2 + \dots + t_{1n}\, c_n$, and a_i is in $\mathbf{Z}$, too.

Finally, (5) implies that

$$\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 4 & \dots & 2^{n-1} \\ 1 & 3 & 9 & \dots & 3^{n-1} \\ & & \dots & & \\ 1 & n & n^2 & \dots & n^{n-1} \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} 2^a \\ 2^{a+p} \\ 2^{a+2p} \\ \vdots \\ 2^{a+(n-1)p} \end{pmatrix},$$

so for $n = 1, 2, \dots$, the polynomial $P_n(x) = a_0 + a_1 x + \dots + a^{n-1} x^{n-1}$ is as desired.

Finally, it remains for me to mention a set of solutions to problems 134-135 and 137-139, submitted by *Pierre Bornsztein,* Pontoise, France, that didn't reach me on time, so they were omitted.

That completes the *Corner* for this issue.

Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses ***vberinde@ubm.ro*** *or* ***vasile_berinde@yahoo.com****. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.*

December 2003

16-18: Ninth International Conference on Cryptography and Coding, Cirencester, UK
Theme: The mathematical theory and practice of cryptography and coding
Aim: Facilitating the growth of data communications and data networks of various types
Topics: Data compression, channel coding, error control coding, combined coding and modulation, coding for multi-media, quantum coding, crypt-analysis, public key cryptography, authentication, key management, data integrity, secure and reliable protocols, interaction between cryptography, coding, combinatorics and other relevant topics
Main speakers: P. Farrell (Lancaster, UK), T. Johansson (Lund, Sweden), V. Niemi (Nokia), R. Walton (Royal Holloway, UK)
Format: Invited lectures and contributed presentations
Sessions: Plenary
Call for papers: Deadline passed
Organisers: The Institute of Mathematics and its Applications
Organising committee: K. Paterson, M. Darnell, M. Ganley, B. Honary, C. Mitchell, M. Parker, M. Walker (all UK)
Sponsors: Hewlett-Packard Laboratories, Vodafone, the IEEE UKRI Communications Chapter
Proceedings: To be published in Springer's *Lecture Notes in Computer Science* series
Location: The Royal Agricultural College, Cirencester, UK
Information: conferences@ima.org.uk
Website: www.ima.org.uk/mathematics/conferences.htm

January 2004

19-28 : Advanced Course on Ramsey Methods in Analysis, Bellaterra (Barcelona)
Aim: To offer young researchers working in analysis and combinatorics an opportunity to learn about the recent applications of Ramsey methods to outstanding problems in analysis, ranging from fundamental questions in point-set topology to well-known problems in the theory of Banach spaces.
Topics: Some applications of Ramsey theory to Banach spaces and an introductory course on Ramsey theory
Main speakers: Spiros Argyros (Athens) ; Stevo Todorcevic (CNRS, Paris 7)
Coordinator: Joan Bagaria (ICREA-UB)
Organising Committee: Joan Bagaria (ICREA-UB) ; Jimena Llopis (Universitat Pompeu Fabra) ; Jordi López-Abad (Université Paris 7)
Grants: a limited number of grants for registration and accommodation for young researchers
Deadlines: 3 November for applications for financial support
15 December for registration and payment
Information:
http://www.crm.es/RamseyMethods
e-mail: RamseyMethods@crm.es

February 2004

2-13: Advanced Course on Contemporary Cryptology, Campus Nord, Universitat Politècnica de Catalunya
Aim: To provide training for young researchers in different problems in cryptology
Topics: Specific multiparty computations protocols, general multiparty computation, foundations of cryptography, provable security for public key schemes, and secure and efficient algorithms for public key cryptography
Main speakers: Dario Catalano (Paris); Ivan Damgard (Aarhus); Giovanni Di Crescenzo (Morristown, NJ); David Pointcheval (Paris); Tsuyoshi Takagi (Darmstadt)
Coordinator: Paz Morillo (Universitat Politècnica de Catalunya)
Scientific and Organising Committee: Paz Morillo, Ignacio Gracia, Sebastià Martín, Jaume Martí, Germán Sáez, Jorge Luis Villar, Carles Padró (all Universitat Politècnica de Catalunya)
Deadlines: 20 November for applications for financial support
31 December for registration and payment
Information:
Website: http://www.crm.es/ContemporaryCryptology
e-mail:
Contemporary-Cryptology@crm.es

March 2004

31-2 April: Quantitative Modelling in the Management of Healthcare IV, Salford, UK
Theme: The nuts and bolts of modelling: a discussion of how modellers can get started, and of what software and data are available – for example, the Hospital Episodes Statistics (HES) database.
Topics: Experience with established methodologies and issues relating to their implementation.
Main speakers: S. M. Dixon, S. Gallivan, E. Wolstenholme and T. Young (all UK).
Format: Keynote lectures, contributed papers and an exhibition of software
Call for papers: Abstracts to be submitted by 30 September
Organisers: The Institute of Mathematics and its Applications
Organising committee: R. Baker, S. Brailsford, T. Chaussalet, R. Davies, P. Millard, G. Murphy, M-M. Nelson (all UK)
Location: University of Salford, UK
Deadline: 30 September for abstract submission
Information: www.ima.org.uk/mathematics

April 2004

19-24: CHT-04 International Symposium on Advances in Computational Heat Transfer; on cruise ship MS Midnatsol between Kirkenes and Bergen, Norway
Theme: all aspects of computational heat transfer and fluid dynamics
Aim: to provide a forum for the exposure and exchange of ideas, methods and results in computational heat transfer
Topics: biological heat transfer, boundary layer flow and heat transfer, combustion and fire modelling, computational methods in CHT, double diffusive convection, internal flow and heat transfer, micro- and nano-scale heat transfer, radiative heat transfer, single and multiphase flow and heat transfer, solidification and melting, turbulent heat transfer and turbulence modelling, validation of computational solutions
Keynote speakers: Lars Davidson (Chalmers University), David Gosman (Imperial College), Janicka (TU

Darmstadt), Patrick Le Quere (LIMSI), Brian Milton (UNSW), Dimos Poulikakos (ETH Zurich), Akshai K. Runchal (ACRiCFD), Wen Quan Tao (Xi'an Jiaotong University), Alexander Zhmakin (Russia)
Format: Keynote lectures; poster sessions; panel discussion on Verification and Validation; mini-symposium on Computational Combustion
Sessions: Keynote lectures and poster session; no parallel sessions
Call for papers: Extended (3-4 page) abstracts are now invited.
Organisers: International Centre for Heat and Mass Transfer; CFD Research Lab., UNSW Australia.
Organising committee: co-chairs: Graham de Vahl Davis and Eddie Leonardin (UNSW)
Proceedings: CD-ROM of all papers; archival quality papers will be submitted to relevant journals
Location: Cruise ship MS Midnatsol between Kirkenes and Bergen, Norway.
Grants: Some support available: priority will be given to students and to scientists early in their career from developing countries
Deadlines: Extended abstracts due 1 October; full papers due 15 January
Information: School of Mechanical & Manufacturing Engineering,
UNSW, Sydney, Australia 2052.
fax: +61-2-9663-1222;
e-mail: cht04@cfd.mech.unsw.edu.au
Website: http://cht04.mech.unsw.edu.au

May 2004

30-6 June: SPT2004: Symmetry and Perturbation Theory, Cala Gonone, Sardinia, Italy
(follows SPT96, SPT98, SPT2001 and SPT2002 conferences)
Theme: Symmetry and perturbation theory and cognate topics, such as classical and quantum dynamical systems, integrable systems, symmetry of differential equations, etc.
Aim: to discuss interrelations between symmetry and perturbation theory, with special emphasis on cognate fields
Scope: to compare recent advances obtained by different approaches and to foster interdisciplinary collaboration
Topics: Symmetry, perturbation theory, dynamical systems, integrable systems, classical and quantum mechanics, differential equations
Main speakers: to be announced
Format: to be announced
Call for papers: open to contribution by participants; call will appear on the conference website
Organisers: A. Degasperis (Roma), G. Gaeta (Milano), B. Prinari (Lecce)
Programme (scientific) committee: S. Abenda (Bologna), D. Bambusi (Milano), G. Cicogna (Pisa), A. Degasperis (Roma), G. Gaeta (Milano), V. Kuznetsov (Leeds), G. Marmo (Napoli), P. Olver (Minneapolis), J.P. Ortega (Nice), S. Rauch (Linkoping), E. Sousa Dias (Lisboa), F. Verhulst (Utrecht), S. Walcher (Aachen), B. Zhilinskii (Dunquerque)
Proceedings: to be announced; previous SPT conference proceedings were published by World Scientific
Location: Hotel Palmasera, Cala Gonone, Italy
Grants: to be announced
Deadlines: none
Information:
website: http://www.sptspt.it

June 2004

2-4: Mathematical problems in Engineering and Aerospace Sciences, The West University of Timisoara, Romania (ICNPAA2004)
Theme: Theory, methods (includes experimental, computational) and applications
Aim: Mathematical problems in engineering and aerospace science have stimulated cooperation among scientists from a variety of disciplines; developments in computer technology have additionally allowed for solutions of mathematical problems; this international forum will extend scholarly cooperation and collaboration, encouraging the dissemination of ideas and information
Scope: Conference sponsors seek a spectrum of theoretical, computational, and experimental inquiries concerned with mathematical problems in engineering and aerospace science
Programme: keynote addresses, invited and contributed lectures, contributed presentations, and communication with remote sites
Topics: Areas of interest include, but are not limited to: aeroacoustics, adaptive and smart structures, aerodynamics, computational fluid dynamics, air traffic control design, non-linear filtering, artificial intelligence, aviation management, atmospheric dynamics, atmospheric sciences, atmospheric flight mechanics, computational structures, propulsion and combustion, human factors modelling, multidisciplinary design, navigation, guidance, stability, and control, control of defence systems, non-linear system modelling and chaos, hybrid systems, neural networks; neural, fuzzy control, modelling and simulation; parallel programming, high performance computing, communication, computer security and cryptography, pattern recognition; image processing; virtual reality, optimisation, reliability, validation and verification
Main speakers: visit the web site for an updated list of speakers
Sessions: There will be plenary lectures and parallel invited and contributed paper sessions
Call for papers: to present a contributed presentation, please submit an abstract to organisers via the website or directly
Programme (scientific) committee: K. T. Alfriend (USA), A. V. Balakrishnan (USA), S. Balint (Romania), P. Borne (France), E. A. Fedosov (Russia), P. Friedman (USA), N. Goto (Japan), M. J. Grimble (UK), L. Gruyitch (France), S. Joshi (USA), D. Lainiotis (USA), V. Lakshmikantham (USA), W. G. Luber (Germany), V. Matrosov (Russia), D. McLean (UK), A. Miele (USA), V. Modi (Canada), Y. Y. Nie (China), K. Ninomiya (Japan), J. Rohacs (Hungary), S. Sivasundaram (USA), S. Sliwa (USA), K. Tsuchiya (Japan), F. E. Udwadia (USA), S. N. Vassilyev (Russia), M. Vidyasagar (India), A. Zellweger (USA)
Organising committee: General chair: Seenith Sivasundaram; Administrative chair: Jose A. Ruiz-Vega, Local organising chair: Stefan Balint; Local organising committee: Horia Ene, Agneta Balint, Victoria Iordan, Constantin Chilarescu, Silviu Birauas, Eva Kaslik; Conference banquet chair: Agneta Balint
Location: The West University of Timisoara, Romania
Sponsors: IFNA: International Federation of Nonlinear Analysts; IFIP: International Federation of Information Processing; IEEE: Institute of Electrical and Electronic Engineers Inc.; AIAA: American Institute of Aeronautics and Astronautics; The West University of Timisoara, Romania
Proceedings: to be published
Grants: to be posted on the website
Deadlines: 30 August for the title of the session, name of the organisers
30 November for the title of the talks and speakers
30 January, final deadline for abstracts of the talks
15 July, final deadline for completed paper for the proceedings
Information: e-mail:
SeenithI@aol.com;
website: www.icnpaa.com

7-11: Conference on Poisson Geometry, Luxembourg City, Grand-Duchy of Luxembourg
Topics: This is the 4th conference on Poisson geometry and related areas (Warsaw 1998, Luminy 2000, Lisbon 2002). It focuses on Poisson structures and generalisations, notions of equivalence, normal forms, hamiltonian systems and generalised moment maps, Poisson Lie groups, Poisson groupoids and dynamical Poisson groupoids, Poisson homogeneous and symmetric spaces, Lie and Courant algebroids, deformation quantisation
Main speakers: these include A. Alekseev (Geneva), A. S. Cattaneo (Zurich), R. L. Fernandes (Lisbon), P. Foth (Arizona), J. Grabowski (Warsaw), J. Huebschmann (Lille), J.-H. Lu (Arizona), K. C. H. Mackenzie (Sheffield), Y. Maeda (Keio), P. Monnier (Bordeaux), S. Parmentier (Lyon), O. Radko (Los Angeles), T. S. Ratiu (Lausanne), P. Severa

(Bratislava), I. Vaisman (Haïfa), A. Weinstein (Berkeley), P. Xu (Penn State), N. T. Zung (Toulouse)
Scientific Committee: D. Arnal (Dijon), J.-P. Dufour (Montpellier), J. Grabowski (Warsaw), S. Gutt (Bruxelles), Y. Kosmann-Schwarzbach (Palaiseau), P. Lecomte (Liège), Y. Maeda (Keio), V. Ovsienko (Lyon), A. Weinstein (Berkeley)
Organising Committee: C. Molitor-Braun (Luxembourg), N. Poncin (Luxembourg)
Contact: N. Poncin, Luxembourg University, e-mail,
Location: University of Luxembourg, avenue de la Faïencerie, 162A, 1511 Luxembourg City
Deadlines: grants (limited): 31 December; posters: 31 March; registration: 30 April
Sponsors: Fonds National de la Recherche (FNR Luxembourg), Centre Universitaire de Luxembourg, Société mathématique du Luxembourg
Information: http://www.cu.lu/Poisson2004

July 2004

26-31: 6th World Congress of the Bernoulli Society and the 67th Annual Meeting of the Institute of Mathematical Statistics, Barcelona, Spain
Theme: Probability and mathematical statistics
Aim: To present the recent developments and the state of the art in probability and statistics and their applications
Topics: Probability theory, stochastic processes, statistical inference, statistics in biology, mathematical finance, random matrices, etc.
Main speakers: David Aldous (Kolmogorov Lecture), Wendelin Werner (Lévy Lecture), Jun Liu (Bernoulli Lecture), Steffen Lauritzen (Laplace Lecture), Iain Johnstone (IMS Wald Lecture, 3 sessions), Peter Bickel (IMS Rietz Lecture), IMS Medallion Lecturers: Vladimir Koltchinskii, Evarist Giné,Cun-Hui Zhang, Alison Etheridge
Format: keynote lectures, invited sessions and contributed talks
Sessions: plenary lectures and parallel sessions
Call for papers: please check the webpage
Organisers: Bernoulli Society, Institute of Mathematical Statistics, Institute of Mathematics of the University of Barcelona (IMUB)
Programme (Scientific) committee: G. Ben Arous (Courant, NY), D. Brillinger (Berkeley), R. Dahlhaus (Heidelberg), M. Delecroix (ENSAI, Bruz), E. Giné (Connecticut), W. González Manteiga (Santiago de Compostela), W. S. Kendall (Warwick) (Chair), R. Lyons (Indiana), E. Mammen (Heidelberg), T. Mikosch (Copenhagen), S. Murphy (Michigan), D. Nualart (Barcelona), D. Nychka (NCAR, Boulder), Y. Ogata (Tokyo), C. Rogers (Cambridge), R. Schonmann (UCLA, Los Angeles), M. Sørensen (Copenhagen), S. Tavaré (USC, Los Angeles), S. van de Geer (Leiden), A. Wakolbinger (Frankfurt), O. Zeitouni (Technion, Haifa)
Organising committee: Joan del Castillo (UAB), José M. Corcuera (UB), Arturo Kohatsu-Higa (UPF), David Márquez-Carreras (UB), David Nualart (UB) (Chair), Carles Rovira (UB), Marta Sanz-Solé (UB), Frederic Utzet (UAB).
Sponsors: Idescat, Forum Barcelona 2004, Borsa de Barcelona
Location: Historic building of the University of Barcelona (UB), Gran Via de les Corts Catalanes 585, 08007 Barcelona, Spain
Grants: probably support for participants from countries of the European Union
Deadlines: for abstracts, 10 February; for registration, 15 April
Information: e-mail: wc2004@imub.ub.es
website: http://www.imub.ub.es/events/wc2004/

August 2004

24-27: International Conference on Nonlinear Operators, Differential Equations and Applications (ICN-ODEA-2004), Cluj-Napoca, Romania
Theme: Non-linear operators, differential equations and applications
Aim: To bring together experts in nonlinear analysis and promote contacts between Romanian mathematicians and mathematicians from all over the world, in order to discuss recent advances in the field
Topics: Fixed point theory; ordinary differential equations; non-linear integral equations; partial differential equations; multivalued analysis; optimal control; biomathematics; mathematical economics; approximation and numerical methods
Main speakers: see the conference website for an updated list
Format: plenary lectures, invited section lectures, short communications, poster session
Sessions: plenary lectures and parallel sessions
Call for papers: please check the webpage
Organisers: I. A. Rus, Gh. Micula, A. Petrusel, R. Precup ('Babes-Bolyai', University of Cluj-Napoca, Romania)
Location: 'Babes-Bolyai', University of Cluj-Napoca, Romania
Deadline: 30 April for registration
Information: e-mail: nodeacj@math.ubbcluj.ro
website: http://www.math.ubbcluj.ro/~mserban/confan.htm

September 2004

23-26: 4th International Conference on Applied Mathematics (ICAM-4), Baia Mare, Romania (previous conferences in 1998, 2000 and 2002)
Theme: In recent decades significant insights have been made in several areas of computational mathematics and their applications to sciences and engineering. There is a permanent need of new methods and methodologies based on interdisciplinary and multidisciplinary interactions, in order to solve an increasing amount of challenging problems facing the contemporary world
Aim: To bring together computational scientists (mathematicians, computer scientists, engineers, etc.) whose areas of interest cover several disciplines, in order to share ideas, methods, methodologies and concrete experience in the field. In the spirit of its predecessors, ICAM-4 aims to promote contacts between Romanian scientists and researchers from all over the world, in order to discuss recent advances in the field circumscribed by the theme
Topics: these include (but are not limited to) mathematical modelling, solving ODE and PDE models, mathematical methods in sciences and engineering, industrial mathematics, computational mathematics, computational physics, chemistry, biology and medicine, computational mechanics, computational economics, computational finance and statistics, scientific computing, high performance computing, parallel numerical algorithms, parallel processing in computational mathematics, computer-aided design and software tools, network programming and neural networks
Main speakers: see the conference website for a continuously updated list
Format: plenary lectures, invited section lectures, short communications, poster session
Call for papers: please check the website
Organisers: V. Berinde and N. Pop (North University of Baia Mare, Romania)
Sponsors: Romanian Mathematical Society - Maramures Branch; SINUS Foundation
Proceedings: to be published
Location: North University of Baia Mare (registration) and the mountain Suior Touristic Resort (3-star hotels, 20 km from Baia Mare), Maramures, Romania
Grants: Expected financial support for young scientists (mainly) from Eastern European countries (not yet confirmed)
Deadlines: 30 June for registration
Information: e-mail: marietag@ubm.ro; icam4@ubm.ro
website: http://www.ubm.ro/site-ro/facultati/departament/manifestari/icam4/index.html

Recent books

edited by Ivan Netuka and Vladimír Souček

Books submitted for review should be sent to the following address:
Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic.

D. Acheson, *1089 and All That: A Journey into Mathematics*, *Oxford University Press, 2002, 178 pp., £12.99, ISBN 0-19-851623-1*
Think of a three-figure number such that its first and last digit differ by two or more. Now, reverse the number, and subtract the smaller of the resulting two numbers from the larger: for example, 782 – 287 = 495. Finally, reverse the new three-digit number, and add: 495 + 594 = 1089. Whatever your initial choice, the result is always 1089. This wonderful and rather surprising trick serves as an opening of the lovely book by David Acheson, a professor at Jesus College, Oxford, and a jazz guitar player, according to whom this '1089-trick' had been the first piece of mathematics that had really impressed him when he learnt it back in 1956. The trick even made the title of the book.

This simple algebraic idea is a well-chosen appetiser, giving the reader an idea of what to expect from this nice little book. The author pinpoints, in a witty and reader-friendly style, certain interesting facts from mathematics and related subjects. The book is obviously aimed at an audience far wider than just professional mathematicians, but each reader, whether mathematician or keen layman, will be delighted. David Acheson writes in his own style, the main feature of which is the fascinating fact of how much he is able to communicate in a rather small amount of words. Naturally, he has something to say about many of the notorious and often publicised pieces from mathematics, history, applications and, of course, famous open problems (or famous ex-open problems that had been open for a long time and have been solved recently). Well-known topics are not missing in the book, but even though typical readers will have at least some knowledge of all these, they will always find something new and interesting in this book.

Several chapters of the book are outstanding and deserve extra mention here. First, from the past, is mathematical analysis (calculus) with its great explanation of the Leibniz d*y*/d*t* notation, and one of the subsequent chapters in which the origin of the leopard's spots is claimed to be governed by a certain differential equation. Another is the chapter on the magical Indian rope trick [*see David Acheson's article in this issue: ed*]: a length of a rope is thrown up in the air and stays there, defying gravity. A small child then climbs the rope. The author obtained a rather sophisticated mathematical description of this remarkable phenomenon and, to demonstrate it, persuaded a friend to construct an upside-down double pendulum. Equipped for both theory and experiment, they then proved that the trick really works. They demonstrated unbelievable stability in the pendulum: after being pushed over by as much as 40 degrees, it would gradually wobble back to the upward vertical.

There are more fascinating things in the book that cannot be described here. So, here is the message to all potential readers of this type of mathematical writing: even though you have doubtless read everything by Keith Devlin, Simon Singh, Martin Gardner, Raymond Smullyan, Lewis Carroll and you-name-it, this wonderful work is yet another 'must' for your bookshelf! (lp)

J. Agler and J. E. McCarthy, *Pick Interpolation and Hilbert Function Spaces*, *Graduate Studies in Mathematics **44**, American Mathematical Society, Providence, 2002, 308 pp., US$49, ISBN 0-8218-2898-3*
This book is devoted to a study of the Pick interpolation problem via operator theory. The original problem was posed by G. Pick (professor of mathematics in Prague) in 1916 and was then studied independently in 1919 by Nevanlinna, apparently unaware of Pick's work. A new look at the problem was taken in the 1960s by D. Sarason, a pioneer of the operator theory approach. He noticed that the Pick interpolation problem can be viewed as a question about the multiplier algebra of the Hardy space H_2.

The exposition follows a one-semester course by the second author to an audience of PhD students and faculty staff. The book is geared so that it is accessible to graduate students interested in operator theory or spaces of holomorphic functions, and therefore contains background results as well as material based on research papers by the authors, which appears in book form for the first time. The approach taken here is not as straightforward as the classical one.

Three chapters (Chapters 2-4) build the necessary background on Hardy spaces and other indispensable spaces of holomorphic functions, and another one (Chapter 10) brings in standard results from operator theory and model theory. The pay-off is that the results obtained can be extensively generalised. The main results of the book include a proof that the Hardy space has the Pick property (with particular emphasis on the role of the positivity of the Pick matrix), a study of qualitative properties of the solutions to the Pick problem, a characterisation of spaces with the matrix-valued Pick property, a proof that Dirichlet and Sobolev spaces have the Pick property, the existence of a universal kernel with the Pick property, a study of the Pick problem in uniform algebras, the development of a hereditary functional calculus and its applications to a characterisation of operators that can be modelled by the adjoint of a given multiplication operator, and, a proof that the complete Pick property is equivalent to a certain localisation property for dilations. (lp)

J. Aguadé, C. Broto and C. Casacuberta (eds.), *Cohomological Methods in Homotopy Theory*, *Progress in Mathematics **196**, Birkhäuser, Basel, 2001, 415 pp., DM 196, ISBN 3-7643-6588-9*
This is a collection of 25 research articles, assembled in 1998 and 1999 during a semester organised by Centre de Recerca Matemàtica, and especially during the 1998 Barcelona Conference on Algebraic Topology. The potential reader should note that these are research articles and not expository ones: nevertheless, we strongly encourage the reader to read them, as they give a very good picture of contemporary research in modern homotopy theory and are written by prominent specialists in the field. Each article contains a short introduction and enough references to provide orientation in the subject. They will be indispensable for researchers in homotopy theory and attractive for postgraduate students starting to work in this direction.

The main topics covered are: abstract stable homotopy, model categories, homotopical localisations and cellular approximations, *p*-compact groups, modules over the Steenrod algebra, classifying spaces for proper actions of discrete groups, *K*-theory and other generalised cohomology theories, cohomology of finite and profinite groups, Hochschild homology, configuration spaces, Lusternik-Schnirelmann category, stable and unstable splittings. (jiva)

S. Alinhac and P. Gérard: *Opérateurs pseudo-différentiels et théorème de Nash-Moser*, *Mathématiques, EDP Sciences, Les Ulis, 2000, 188 pp., FRF 230, ISBN 2-7296-0364-6 and 2-222-04535-7*
The book is based on an advanced university lecture course providing an extensive study of basic properties of pseudo-differential operators and some of their principal applications, with particular emphasis on the Nash-Moser theorem. The exposition is written in a very attractive way, which should appeal to experts in the field as well as to PhD students (or even gifted undergraduates) with some basic knowledge of elementary functional analysis, Fourier analysis and, perhaps, the theory of distributions. The book is self-contained, and the authors have taken a lot of trouble to make their exposition as reader-friendly as possible.

The book is divided into three chapters (plus an introductory Chapter 0). Chapter I is an exposition of the theory of pseudo-differential operators (the authors call this part a 'minimal theory', but it is in fact quite comprehensive). The material includes the concept of a symbol, its use in operator calculus, the action of operators on Sobolev spaces and the invariance under change of variables. Chapter II is divided into three themes, covering (among other topics) the Littlewood-Paley theory of dyadic decomposition of distributions (with such interesting facts as a characterisation of the Hölder and Sobolev spaces), 'micro-local analysis' and some energy estimates. The last chapter, 'The implicit function theorems' treats the role of implicit functions in elliptic problems, examples of applications of fixed-point theorems to semilinear hyperbolic systems, and a thorough and comprehensive exposition of the Nash-Moser theorem on the existence and properties of a solution to the equation $\varphi(u) = \varphi(u_0) + f$. (lp)

A. I. Ban and S. G. Gal, *Defects of Properties in Mathematics: Quantitative Characterizations*, *Series on Concrete and Applicable Mathematics **5**, World Scientific, New Jersey, 2002, 352 pp., US$48, ISBN 981-02-4924-1*
This book is devoted to a research method, called by the authors the quantitative study of the defects of properties. The authors recall some basic definitions from various fields of mathematics. Instead of defining 'object x has property P', the authors define a quantity $E(x)$, called the defect of property P, which measures 'how far' x is from having the property P.

Chapter 1 is a detailed overview of the topics discussed in the rest of the book: this is handy for the readers new to the theory, who can get the whole picture, skipping the more technical parts on a first reading. In the next relatively independent chapters the authors study properties in set theory, topology, measure theory, real function theory, functional analysis and algebra. In the final chapter, the authors study properties in complex analysis, geometry, number theory and fuzzy logic. The book deals with a great variety of problems in a very readable way. New ideas are motivated by many examples and remarks, and most of the chapters conclude with applications. The only small objection I have is that applications should have deserved more space. In the bibliographical remarks in every chapter, the authors give full references and explicitly state which results are new.

This book is a good overview of the theory of quantitative characterisations, the introduced concepts are elegant and the methods of proof are (the authors believe) 'rather elementary'. This makes the material accessible to undergraduate and graduate students, while researchers may find the new concepts very motivating. (mbe)

M. Bitbol *et al.*, *De la méthode. Recherches en histoire et philosophie des mathématiques*, *Presses Universitaires Franc-Comtoises, Besançon, 2002, 356 pp., 22 euros, ISBN 2-84867-000-2*

This book contains ten interesting and original articles from the history and philosophy of mathematics, written by well-known mathematicians and historians of mathematics (eight from France, one from the UK). The articles have their origin in lectures at the *Séminaire d'épistémologie de l'IREM de l'Université Paris VII* and the *Colloque de philosophie des mathématiques*, organised by Michel Serfati, and discuss certain historical topics in classical and modern mathematics from mathematical, historical, philosophical and psychological viewpoints.

The first part (*La force de la méthode*) describes the force of some mathematical methods, their historical backgrounds and their influence on the development of mathematics. It was written by M. Serfati, A. Douady, R. Langevin, A. Revuz, O. Hurdy and I. Grattan-Guinness. The second part (*L'existence en mathématiques*) contains four articles by A. Michel, M. Serfati, M. Bitbol and J. Mosconi, which have deep psychological and philosophical aspects.

This book can be recommended to all scientists and teachers of mathematics, history and philosophy of mathematics, as well as those interested in the history of science. (mnem)

M. Boileau and J. Porti, *Geometrization of 3-Orbifolds of Cyclic Type*, *Astérisque* ***272***, *Société Mathématique de France, Paris, 2001, 208 pp., FRF 250, ISBN 2-85629-100-7*

This research monograph is devoted to the geometry and topology of 3-dimensional manifolds (or, more generally, 3-dimensional orbifolds). This field was enormously influenced and advanced by work of W. Thurston, and the authors give a full proof of the Thurston orbifold theorem in the case where all local isotropy groups are cyclic subgroups of SO(3). As a consequence, they can prove the Thurston geometrisation conjecture for compact orientable irreducible 3-manifolds with a non-free symmetry. The first appendix is written in collaboration with M. Heusner. (vs)

L. Carbone and R. De Arcangelis, *Unbounded Functionals in the Calculus of Variations. Representation, Relaxation, and Homogenization,* *Monographs and Surveys in Pure and Applied Mathematics* ***125***, *Chapman & Hall/CRC, Boca Raton, 2002, 394 pp., US$84.95, ISBN 1-58488-235-2*

Mathematical modelling of stationary problems often leads to minimising energy functionals of the type $F(\Omega, u) = \int_\Omega f(x, \partial u)\,dx$. In standard situations, the functional is locally bounded and lower semicontinuous in some Sobolev space, or in the space of functions of bounded variation. However, a more careful analysis allows one to consider functionals for which the reasonable function spaces remain the same, but the functional can be unbounded on bounded subsets of the space, or attain infinite values. In this book, the authors are primarily motivated by examples in which even the energy density function f may be somewhere infinite: this appears as a natural way of handling the presence of a constraint.

To keep the volume as self-contained as possible, five chapters of preliminaries are included. Here the reader can find the foundations of measure and integration theory and an introduction to some function spaces and variational methods in a very abstract setting. Chapter 6 surveys some classical results for finite-valued functionals, as a counterpart to later results in the 'unbounded' theory, and three physically motivated examples are formulated: elastoplastic torsion problems, modelling of non-linear elastomers and electrostatic screening. Chapter 7 starts a systematic treatment of 'unbounded' functionals. Abstract regularisation and Jensen's inequality yield a very general result on lower semicontinuity of convex functionals. The unique extension problem, like the relaxation problem, asks for an extension of a functional which is well defined for smooth functions but lacks meaning for more general functions: its analysis is performed in an axiomatic setting. The problem of integral representation asks which functionals on a given function space can be represented as integrals in the form shown above (or, in a more general form, with a part singular with respect to the Lebesgue measure). The general integral representation theory helps to answer such questions for functionals arising as relaxed functionals or Γ-limits. The relaxation problem is studied for Neumann and Dirichlet boundary data and the answer is given with full identification of the integrand. In Chapters 11-14, the homogenisation theory for unbounded functionals is developed: this studies the limit behaviour of fine periodical structures as the period tends to 0, and simulates a macroscopic view of microstructured materials. In this book, the main feature of the analysis is 'unboundedness' of the functional. The limit process is described by the Γ-convergence and applied to convergence of minima and minimisers. The homogenisation procedure is investigated in the general case, then in case of special constraints, and finally for the particular models arising from physics.

This book contains old and new results from a significant part of the calculus of variations. The authors are well-known experts in the field whose approach is unified and elegant. The book is primarily aimed at graduate students and researchers in mathematics, but may be useful and comprehensive for a broader community of applied mathematicians, physicists and engineers. (jama)

V. V. Chepyzhov and M. I. Vishik, *Attractors for Equations of Mathematical Physics*, *Colloquium Publications* ***49***, *American Mathematical Society, Providence, 2002,363 pp., US$69, ISBN 0-8218-2950-5*

This book evolves around the notion of the global attractor, its dimension, and various generalisations of these concepts. The concrete equations studied throughout the book include general reaction-diffusion equations, the Navier-Stokes equations and hyperbolic equations of dissipative type.

The first part reviews classical material on the attractors and its dimension in the autonomous setting; several results concerning lower bounds for the dimension are also given. In the second part, these results are generalised to the non-autonomous setting. The concept of 'process' as a generalisation of semigroup is introduced, and a notion of Kolmogorov entropy (a generalisation of the fractal dimension) is studied. One chapter covers a lot of material concerning translation compact spaces of functions. Finally, in the third part, the new concept of 'trajectory attractor' is introduced in order to study evolutionary problems with possible non-uniqueness: notable examples are the three-dimensional Navier-Stokes system and the wave equation with supercritical non-linearity. (dp)

P.-A. Cherix, M. Cowling, P. Jolissaint, P. Julg and A. Valette, *Groups with the Haagerup Property*, *Progress in Mathematics* ***197***, *Birkhäuser, Basel, 2001, 126 pp., DM 104, ISBN 3-7643-6598-6*

A second countable locally compact group has the Haagerup property if it has a proper continuous isometric action on an affine Hilbert space. The class of groups with this property is quite broad: examples include compact groups, $SO(n,1)$, $SU(n,1)$, Coxeter groups, free groups and amenable groups.

The book contains a series of papers concerning these groups, starting with a short introduction by A. Valette, and various equivalent definitions by P. Jolissaint. A geometric proof of the Haagerup property for $SO(n,1)$ and $SU(n,1)$ is given by P. Julg, and the classification of groups with the Haagerup property is presented by P.-A. Cherix, M. Cowling and A. Valette. The classification asserts that such a group is locally isomorphic to a direct product of an amenable group with a finite number of copies of $SO(n,1)$ and $SU(n,1)$ (dimensions can vary). M. Cowling's studies groups with the radial Haagerup property. Some hereditary results (mainly for discrete groups) are discussed in the paper by P. Jolissaint, P. Julg and A. Valette. The book ends with a list of open problems prepared by A. Valette. (vs)

R. Churchhouse, *Codes and Ciphers. Julius Caesar, the Enigma and the internet*, *Cambridge University Press, 2002, 240 pp., £14.95, ISBN 0-521-81054-X and 0-521-00890-5*

This is a popular introduction to the basic concepts and methods of cryptology, the science and art of secret communication.

The first seven chapters are concerned with classical cryptography and cryptoanalysis, starting with simple substitution and polyalphabetic ciphers such as the Vigenère cipher and various transposition ciphers. Examples of monograph to digraph substitutions, digraph to digraph substitutions, and their combinations are presented. Each historical chapter starts with the description of a cipher and then presents detailed examples of its cryptoanalysis: important events in the history of cryptoanalysis (breaking the Zimmermann telegram in World War I, solution of the Enigma cipher of the Germans in World War II) are also described. Another chapter is concerned with the description and cryptoanalysis of one of the Hagelin machines. Three chapters deal with modern cryptography: one explains random and pseudorandom sequences

of numbers and letters and how to produce them; another explains the basic concepts and ideas of public key cryptography, including the Diffie-Hellman key exchange system; the last chapter is on security of internet communications, explaining DES and its implementation, authentication and signature verification.
The book is written in a lively and amusing style but not at the expense of mathematical rigour. It contains 28 mathematical appendices, where mathematical concepts used in the text are explained in more detail. (jtu)

F. U. Coelho and H. A. Merklen (eds.), *Representations of Algebras*, *Lecture Notes in Pure and Applied Mathematics* **224**, *Marcel Dekker, New York, 2002, 282 pp., US$150, ISBN 0-8247-0733-8*
This is the proceedings of the CRASP conference at the University of Sao Paulo. It covers a wide range of topics in representation theory, including a study of almost split sequences (Angeleri-Hügel and Smalo), a description of strongly simply connected derived tubular algebras (Assem), an exploration of Koszul algebras and the Gorenstein condition (Martínez-Villa), a characterisation of hereditary categories containing simple objects (Happel and Reiten), a study of derived tame coil algebras (de la Pena and Tomé), an analysis of Hopf algebras (Artamonov, and Zhang and Li), and many other results. It is an excellent source on recent progress in the representation theory of Artin algebras and related areas of mathematics and physics. Researchers and graduate students will benefit from it. (jtrl)

J. S. Cohen, *Computer Algebra and Symbolic Computation: Elementary Algorithms*, *A. K. Peters, Natick, 2002, 323 pp., US$50, ISBN 1-56881-158-6*
This is the first volume of a two-volume introduction to symbolic computation. The first volume tries to bridge the gap between software manuals that only explain how to use computer algebra programs such as *Mathematica* or *Maple*, and graduate texts which only describe algorithms. It is aimed at undergraduate students of mathematics, computer science and engineering. The mathematical prerequisites are the usual undergraduate two-year course of mathematical analysis, elementary linear algebra and applied ordinary differential equations. The computer science prerequisites are experience with a computer programming language and the skills in problem-solving and algorithm development from a beginning programming course. The presentation is based on an algorithmic language called *mathematical pseudolanguage (MPL)*.

After introducing the language in Chapter 2, the author investigates the internal tree structure of mathematical expressions in Chapter 3. Chapter 4 contains description of elementary algorithms, while Chapter 5 deals with recursion as a programming technique and gives a number of examples that illustrate its advantages and limitations. Chapter 6 is concerned with algorithms that analyse and manipulate polynomials and rational expressions. The final chapter deals with algorithms manipulating exponential and trigonometric functions. The book comes with a CD with the entire text, active hyperlinks, and complete algorithms. It can be recommended to anyone interested in the *magic* of computer algebra systems. (jtu)

M. J. Collins, B. J. Parshall and L. L. Scott (eds.), *Modular Representation Theory of Finite Groups*, *Walter de Gruyter, Berlin, 2001, 262 pp., DM 216, ISBN 3-11-016367-5*
This is the proceedings of a symposium on modular representation theory, held at the University of Virginia in 1998. Its main topics include:
(1) the modular representation theory of groups of Lie type in non-defining characteristic;
(2) the relationship of q-Schur algebras to quantisation of other algebras;
(3) connections with modular representation theory of symmetric groups;
(4) Broué's conjectures on the equivalence of derived categories of modules in blocks with abelian defect group.

The first part of the book contains three comprehensive surveys on recent developments, stressing functorial and q-Schur algebra methods – by M. Geck on (1) and (2), J. Brundan and S. Kleshchev on (3), and R. Rouquier on (4). The second part comprises research papers dealing with particular aspects of the theory, by R. Boltje (a new reformulation of Alperin's weight conjecture), M. Cabanes and J. Rickard (Broué's conjecture on the Alvis-Curtis duality), S. R. Doty, D. K. Nakano, J. Du, C. Hoffman, N. J. Kuhn, K. Maagard and P. H. Tiep.

The book will serve as an invaluable source of recent progress in modular representation theory, both for established researchers and for graduate students. (jtrl)

A. Connes, A. Lichnerowicz and M. P. Schützenberger, *Triangle of Thoughts*, *American Mathematical Society, Providence, 2001, 179 pp., £18, ISBN 0-8218-2614-X*
This extraordinary and very unusual book is based on discussions between the three authors, all of whom are scientists of the highest level in their fields. The topics of their discussions are of general interest – logic and its role in mathematics, the role of mathematics in the world around us, interactions between physics and mathematics, the grand unification of physics, interpretation of quantum mechanics, general relativity and cosmology, quantum gravity, string theory, etc. Their views and opinions on various questions represent varying approaches to these questions, which can often be found in the scientific community. The discussion avoids all technicalities, and is thus understandable to a general reader interested in these questions.

This small book contains a wealth of interesting ideas for further reflection and can be heartily recommended to all interested readers. (vs)

P. Dalgaard, *Introductory Statistics with R, Statistics and Computing*, *Springer, New York, 2002, 267 pp., 29,95 euros, ISBN 0-387-95475-9*
This is a nice book on statistical methods and statistical computing in R, a language and environment for statistical computing and graphs: this dialect of the S language is available as free software through internet. The fact that R is based on a formal computer language gives it tremendous flexibility, which is very useful for *ad hoc* model building in analysis of a complex data. The book is not a manual of R, but introduces a number of basic concepts and techniques that should allow the reader to get started with practical statistics.

The book covers the curriculum for a course in basic statistics. It presents one- and two-sample tests (t-tests with their distribution-free counterparts), linear models (ANOVA, simple and multiple linear regression), contingency tables, power calculations and computation of the sample size. It also presents some methods that are not typical for elementary statistical courses: logistic regression and survival analysis. The appendices describe how to obtain and install R, a systematic description of original data sets used in the book, and a compendium of R functions and commands. Explanation of statistical methods, together with an interpretation of statistical concepts, is the prevailing style of the text. They are illustrated by plenty of practical examples, all computed using R. This book will be useful for novices in applied statistics or in computing in R. (kzv)

B. A. Davey and H. A. Priestley, *Introduction to Lattices and Order, 2nd edn*, *Cambridge University Press, 2002, 298 pp., £19.95, ISBN 0-521-78451-4*
The primary aim of this second edition is to serve as a textbook devoted to ordered sets and lattices and to their contemporary applications. The level is suitable for advanced undergraduate and first-year graduate students. The only prerequisites are a knowledge of elementary abstract algebra and the notation of set theory.

Starting with elementary concepts of ordered sets, lattices and complete lattices, the book proceeds to a brief description of formal concept analysis, to elements of the structure theory of lattices and of their representations in the finite case. Several topics important for theoretical computer science are also studied, including Galois connection, fixed-point theorems for ordered sets, and applications of domains in information systems. The book contains numerous exercises and reflects the lively lecturing style of both authors. It can be recommended as a valuable source to anyone who needs to use ordered structures in any context. (jt)

M. Fabian *et al.*, *Functional Analysis and Infinite-Dimensional Geometry*, *CMS Books in Mathematics, Springer, New York, 2001, 449 pp., DM 181,79, ISBN 0-387-95219-5*
This monograph is based on an earlier publication of P. Habala, P. Hájek and V. Zizler (*Introduction to Banach Spaces I, II*) published by Matfyzpress, Prague, in 1996. An enlarged team of authors have expanded the contents of these lecture notes, adding updated information in the geometric theory of Banach spaces.

The first three chapters on basic concepts in Banach spaces present the central material of the book: the Hahn-Banach theorem, the open mapping theorems and weak topologies disseminated by such refinements as the Krein-Milman theorem, James boundary, Ekeland's variational principle and the Bishop-Phelps theorem, together with material on the Riesz-Schauder theory of compact operators. Chapter 4 deals with locally convex spaces (Mackey topologies, Choquet's representation theorem and Kaplansky's theorem). The structure of Banach spaces (projections, complementability and Schur's property) and Schauder bases are discussed in Chapters 5 and 6. Chapters 8 and 9 are about the differentiability of norms (Šmulyan's test, Fréchet differentiability of convex functions) and uniform convexity and uniform smoothness (also local reflexivity, superreflexivity, Enflo's renorming). Smoothness and structure (variational principles, smooth approximation, Lipschitz homeomorphisms, etc.) form the subject of Chapter 9. The monograph contains a huge number of exercises, some for students' drill and others is to extend the theory presented in the text.

This book can be warmly recommended to everyone interested in functional analysis, and Banach space theory in particular. It serves also as a textbook in courses for students in probability, physics, or engineering. Graduate students and researchers surely will find a lot of material from the field, as well as a source of inspiration. (jl)

H. Föllmer and A. Schied, *Stochastic Finance: An Introduction in Discrete Time*, *de Gruyter Studies in Mathematics* ***27****, Berlin, 2002, 422 pp., 54 euros, ISBN 3-11-017119-8*

This text on probabilistic methods in finance is intended for graduate students in mathematics with some background in probability: stochastic models in discrete time are considered.

The exposition has two parts. In the first part (Chapters 1-4), the one-period model of a financial market is considered. Basic principles of mathematical finance are explained: portfolio, market efficiency, arbitrage, martingale measure, options, contingent claims, perfect hedge, complete and incomplete market. The mathematical theory of utility is then presented, and used for portfolio optimisation and risk measuring. The second part (Chapters 5-10) deals with multi-period models of financial market that are characterised by stochastic processes in discrete time. Chapter 5 is of fundamental importance and the true core of the book: dynamic arbitrage theory, based on martingales and equivalent martingale measures; European contingent claim and the problem of pricing of this financial derivative; the Cox, Ross and Rubinstein binomial model; limit theory, leading to the well-known Black-Scholes pricing formula in continuous time. The remaining chapters are devoted to various hedging strategies. An appendix summarises some results from probability, convex and functional analysis.

This is an excellent textbook. Since considerations in discrete-time stochastic financial models are simpler, the exposition proceeds quickly to key problems in the theory of pricing and hedging of financial derivatives. However, the authors formulate their models on general probability spaces, so the text also captures some interplay between probability theory and functional analysis. (zp)

J.-P. Gauthier and I. Kupka, *Deterministic Observation Theory and Applications*, *Cambridge University Press, 2001, 226 pp., £47.50, ISBN 0-521-80593-7*

The main topic in this book is so-called observation theory. The aim is to reconstruct complete information about a solution of a non-linear system of ordinary differential equations (depending in general on a parameter u in $\mathbf{R}^d$, possibly coupled to a system of functional equations in the same variables) from the knowledge of suitable partial data. The authors introduce a new concept of observability (resp. strong observability). The character of the problem depends strongly on a relation between the number of functional equations and the dimension of the parameter u, and develop a systematic method for treating such problems. The methods used include transversality theorems, the theory of subanalytic sets, Whitney stratification results, some facts from several complex variables and Lyapunov's theorems. The book ends with two applications in chemical engineering, and will be of interest to researchers in control theory. (vs)

M. Grosser, E. Farkas, M. Kunzinger and R. Steinbauer, *On the Foundations of Nonlinear Generalized Functions I and II*, *Memoirs of the American Mathematical Society* ***729****, Providence, 2001, 93 pp., £31.50, ISBN 0-8218-2729-4*

The main drawback of the Colombeau generalised functions is that the canonical embedding of the space of Schwartz distributions into the algebra (and sheaf) of generalised functions is not intrinsic. This canonical imbedding is not preserved by coordinate diffeomorphisms, so generalised functions cannot be defined on a manifold although the distributions can. How to remove this inconvenience by a slight change of definition of the Colombeau generalised functions was outlined by Colombeau and Meril in 1994 and elaborated by Jelínek in 1999.

In the first part, the Colombeau-type algebra defined by Jelínek (called diffeomorphism invariant) is carefully examined, and its description is corrected and completed. Colombeau-type algebra of generalised functions is defined as a quotient algebra $G = E_M / I$, where E_M is an algebra of so-called moderate representatives and I is an ideal of negligible representatives: some equivalent definitions of these notions are presented. In the second part, other ways of defining moderateness and negligibility are studied and only two diffeomorphism invariant algebras are found among them. The assumption that they are different is neither proved nor disproved. (jjel)

B. Hart and M. Valeriote (eds.), *Lectures on Algebraic Model Theory*, *Fields Institute Monographs* ***15****, American Mathematical Society, Providence, 2002, 111 pp., US$30, ISBN 0-8218-2705-7*

This book contains the notes of three lecture series presented din the first semester of a year-long programme in algebraic model theory, organised by the Fields Institute for research in mathematical sciences during the academic year 1996-97. The lectures on differential fields (A. Pillay), *o*-minimality (P. Speisegger) and tame congruence theory (M. Clasen and M. Valeriote) present recent developments in these areas: in particular, the second lecture discusses the problem of adding the exponential function to certain *o*-minimal expansion of real numbers, and the third lecture also contains applications to the study of residually small varieties. A minimal knowledge of these topics is assumed. (jmlc)

F. Hausdorff, *Felix Hausdorff - Gesammelte Werke, Band IV: Analysis, Algebra und Zahlentheorie*, *Springer, Berlin, 2001, 554 pp., 74,95 euros, ISBN 3-540-41760-5*

This is the first of eight planned volumes devoted to the work of F. Hausdorff (including physics and his literary work). The pattern is the same throughout: Hausdorff's writing (published or unpublished) is followed by a commentary by a current expert in the area. The part devoted to analysis occupies about 80% of the book, and consists of 13 of Hausdorff's published research papers from 1914-31 and 19 items (from 1914-42) devoted to analysis and found in his inheritance. We find here classical papers on measure theory and the moment problem; among unpublished papers we notice the introduction of the long line in 1915, results on trigonometric series, extensions of measures, orthogonal polynomial functions, and functions with special properties. In a letter to L. Bieberbach, Hausdorff suggested a simplification of one of Bieberbach's results concerning holomorphic functions. The second part devoted to algebra contains three published papers and one unpublished paper. The number theory part consists of two published papers. The book concludes with an extensive author index and two subject indexes, German and English. (mih)

E. P. Hsu, *Stochastic Analysis on Manifolds*, *Graduate Studies in Mathematics* ***38****, American Mathematical Society, Providence, 2002, 281 pp., US$44, ISBN 0-8218-0802-8*

There is a deep and well-known relation between probabilistic objects that are studied in stochastic analysis (typically, Brownian motion) and some analytic objects (the Laplace operator). The main purpose of this book is, roughly speaking, to explore the connection between Brownian motion and analysis in the area of differential geometry (in particular, the concept of curvature).

The basic facts about stochastic differential equations on manifolds are explained in Chapter 1, the main result being the existence and uniqueness up to explosion time for the Itô equation on a manifold. Chapter 2 studies horizontal lift and stochastic development, two concepts that are central to the Eells-Elworthy-Malliavin construction of Brownian motion on a Riemannian manifold, which is thoroughly studied in Chapter 3. Chapter 4 explores the connection between the heat kernel and Brownian motion, and considers stochastic completeness, the Feller property and recurrence and transience of the heat semigroup. Other chapters study the short-time behaviour of the heat kernel and Brownian motion, and probabilistic proofs of the Gauss-Bonnet-Chern theorem and the Atiyah-Singer index theorem. Some further applications of Brownian motion to geometric problems also appear.

The book is mainly intended for probabilists interested in geometric applications, and a basic knowledge of Euclidean stochastic analysis is assumed; differential geometry is reviewed, but the reader should have a grounding in the basic definitions. It will be useful especially for advanced graduate students and researchers interested in stochastic analysis and stochastic methods in differential geometry. (bm)

A. Huckleberry and T. Wurzbacher (eds.), *Infinite Dimensional Kähler Manifolds*, *DMV Seminar* ***31****, Birkhäuser, Basel, 2001, 375 pp., DM 76, ISBN 3-7643-6602-8*

This book is based on a DMV seminar held at Oberwolfach in 1995: the main topics come from recent intensive interactions between theoretical physics and mathematics. It contains six survey papers systematically explaining many topics connected with infinite-dimensional Lie groups and Lie algebras, their representations and infinite-dimensional homogeneous spaces.

The first contribution (by A. Huckleberry) is a useful summary of basic facts needed from the finite-dimensional situation (differentiable manifolds, fibre bundles, symplectic and complex geometry and their equivariant versions, the Borel-Weil realisations of irreducible representations of compact Lie groups). The first contribution by K.-H. Neeb describes a generalisation of infinite-dimensional groups and their representations to the setting of groups modelled over sequentially complete locally convex spaces. This is then used in his second paper devoted to the Borel-Weil theory for positive energy representations of loop groups. The coadjoint representations of the Virasoro algebra and its generalisations is treated by V. Yu. Ovsienko: it describes a geometrical realisation of these coadjoint representations in terms of representations of suitable Lie superalgebras on spaces of linear differential operators and their connection to the Adler-Gelfand-Dickey bracket. C. Paycha describes renormalisation techniques needed to define determinant bundles in the infinite-dimensional situation and their use in quantisation of gauge field theories. T. Wurzbacher describes a relation between the fermionic second quantisation and the geometry of restricted Grassmannians, including the **C***-algebra approach to the Grassmannian and its determinant bundle.

This book can be recommended to a wide range of interested readers, students and researchers, from mathematics and from theoretical and mathematical physics. (vs)

I. James, ***Remarkable Mathematicians****, Cambridge University Press, 2003, 433 pp., £19.95, ISBN 0-521-52094-0*

This interesting book contains life stories of many remarkable mathematicians born in the period 1700-1903. It is divided into ten chapters and there are six biographies in each of them, giving sixty profiles altogether. The author has chosen important mathematicians who influenced mathematics by their ideas, teaching, or in some other way. The biographies are arranged chronologically, and the authors' attention is devoted more to individual life stories than to details of their mathematical achievements. Each profile includes a portrait, a quotation from a personal letter, memories and estates. A short epilogue, a list of recommended references and a collection of the most important mathematical works are included.

This book can be recommended to all scientists and teachers of mathematics, the history and philosophy of mathematics, as well as to anyone interested in history of science. There is a lot of interesting information about people who contributed substantially to the development of mathematics. (mnem)

J. Jorgenson and S. Lang, ***Spherical Inversion on SLn (R),*** *Springer Monographs in Mathematics, New York, 2001, 426 pp., DM 181,79, ISBN 0-387-95115-6*

The aim of this book is to give an introduction to Harish-Chandra's inversion for spherical functions, concentrating $SL_n(\mathbf{R})$ and, where possible, using a list of axioms that is readily verified in this particular case. This makes the book accessible to graduate students and it enables one to read its parts independently. All required material is developed from the beginning, while keeping the book to a reasonable length. This approach is to be contrasted with other modern accounts that are completely general (treating all semisimple groups at once), at the expense of being impenetrable to outsiders.

The book starts with a description of various decompositions (Iwasawa, Bruhat, etc.), computing corresponding Jacobian determinants, and with the study of invariant differential operators. The Casimir operator is discussed as a useful tool. The book culminates with Rosenberg's proof of Harish-Chandra's inversion on Paley-Wiener spaces and with Anker's proof of the inversion on Schwartz spaces. The final chapter shows how the theory simplifies on $SL_n(\mathbf{C})$. (pš)

D. D. Joyce, ***Compact Manifolds with Special Holonomy****, Oxford Mathematical Monographs, 2000, 436 pp., £55, ISBN 0-19-850601-5*

The main subject of this book is a study of compact Riemannian manifolds with special holonomy groups: particular attention is devoted to the holonomy groups $SU(m)$, $Sp(m)$, $G2$ and Spin(7) (the corresponding metrics are Ricci-flat in all these cases). Examples of such manifolds are hard to find, but the reader can learn many important constructions and examples of such manifolds here.

The book has two parts. The first reviews the basic tools needed for later constructions. It is a very nice introduction to the field, covering many important topics (connections, their curvatures and holonomy groups, both in the principal bundle and vector bundle cases; connections on tangent bundles and their torsions; G-structures on manifolds; Riemannian holonomy groups and their classification; Kähler manifolds and their curvature, exterior algebra of Kähler manifolds; some important facts about complex algebraic varieties, line bundles and divisors; the Calabi-Yau conjecture and its full proof; Calabi-Yau manifolds, orbifolds and their resolutions; hyperkähler manifolds and other quaternionic geometries). Some parts of this material will be useful when preparing courses for graduate or postgraduate students. The second part contains a wealth of new research material concerning constructions of compact manifolds with special (resp. exceptional) holonomy and computation of their Betti numbers. The $SU(m)$ and $Sp(m)$ holonomy are treated first, and manifolds with $G2$ holonomy and Spin(7) holonomy follow. The proofs here are involved and difficult, and are placed at the end of individual chapters so that one can postpone reading them until later.

The book is written in a very clear and understandable way, with careful explanation of the main ideas and many remarks and comments, and it includes systematic suggestions for further reading. The topic of the book has been inspired by the recent intensive interaction between theoretical physics and mathematics, and the book is really outstanding. It can be warmly recommended to mathematicians (in geometry and global analysis, in particular) as well as to physicists interested in string theory. (vs)

G. Kalai and G. M. Ziegler (eds.), ***Polytopes – Combinatorics and Computation****, DMV Seminar* ***29****, Birkhäuser, Basel, 2000, 225 pp., DM 58, ISBN 3-7643-6351-7*

This volume is a collection of ten papers, most of which formed contributions to the workshop *Polytopes and optimization* held in Oberwolfach in November 1997. It offers a wide panorama of relations of polytope theory to other fields. It starts with a remarkable paper on the combinatorics and geometry of 0/1-polytopes, written by Günter M. Ziegler, who says that it is meant as an introduction and invitation rather than an extensive survey. Different aspects of the complexity of higher-dimensional 0/1-polytopes are presented in an elegant way, accessible also to non-specialists.

The collection contains the following papers: E. Gawrilow and M. Joswig: POLYMAKE: a framework for analyzing convex polytopes; G. Kalai, P. Kleinschmidt and G. Meisinger: Flag numbers and FLAGTOOL; A. Höppner and G. M. Ziegler: A census of flag-vectors of 4-polytopes; O. Aichholzer: Extremal properties of 0/1-polytopes of dimension 5; B. Büeler, A. Enge and K. Fukuda: Exact volume computation for polytopes: a practical study; H. Achatz and P. Kleinschmidt: Reconstructing a simple polytope from its graph; M. Joswig: Reconstructing a non-simple polytope from its graph; D. Avis: A revised implementation of the reverse search vertex enumeration algorithm; S. G. Bartels: The complexity of Yamnitsky and Levin's simplices method. (pv)

R. Kane, ***Reflection Groups and Invariant Theory****, CMS Books in Mathematics, Springer, New York, 2001, 379 pp., DM 60,39, ISBN 0-387-98979-X*

The main theme of this book is a study of the properties of reflection groups (finite groups generated by reflections in a Euclidean space), their generalisations and invariant theory connected with them. Suitable generalisations of reflections in Euclidean spaces are pseudo-reflections in a vector space V: these are linear maps f from V to V of the form $f(u) = u - (a^*, u)a$, with a^* in V^* and a in V. Groups generated by pseudo-refelections are called pseudo-reflection groups.

The book is in three parts. The first part contains a study of reflection groups and their associated Coxeter systems, using root systems as a tool: Weyl groups and their crystallographic root systems form an important special case. The second important tool used in the book is invariant theory. A natural framework for a study of invariant theory is the concept of pseudo-reflection group, and this theme is carefully investigated in the second part. The final part contains a discussion of conjugacy classes of elements and subgroups for reflection groups (such as a relation of Coxeter elements to the underlying root system), as well as a use of invariant theory in the study of eigenvalues of elements from pseudo-reflection groups.

The topics treated in the book play a key role in other important branches of mathematics (Lie groups and Lie algebras and their applications in geometry and analysis or theory of algebraic groups). The book is nicely organised and written in a very understandable way. The main ideas are clearly explained at the beginning of each chapter or section, so it is easy to learn the main facts first and to fill in the details later, when needed. For the main part of the book, only a basic knowledge of linear algebra and algebra is needed, and short summaries of more advanced knowledge are enclosed for the convenience of the reader. The book is pleasant to read, and can be heartily recommended to a general mathematical audience, starting with graduate students. (vs)

B. Khoussainov and A. Nerode, ***Automata Theory and its Applications****, Progress in Computer Science and Applied Logic* ***21****, Birkhäuser, Boston, 2001, 430 pp., DM 180, ISBN 0-8176-4207-2 and 3-7643-4207-2*

The aim of this book is to present a theory of several types of automata and applications of these facts in logic, concurrency and algebra. It contains lectures concerning the theory and applications of finite automata, Büchi automata, games on finite graphs, and Rabin automata. The chapter on finite automata is extended by finite automata recognisable relations and finite automata with equational constraints. The final section of this chapter proves the decidability of the monadic second-order logic of finite strings. The chapter on Büchi automata contains basic results and results on the relationship to Müller automata and sequential Rabin automata. Among the applications, the authors prove that the monadic second-order theory of one successor (S1S) is decidable. The purpose of the chapter on games is to give a game-theoretic models for concurrent processes of infinite duration: this chapter introduces the concept of the last visitation record, which plays an important role in proofs of some fundamental results. The chapter on Rabin automata uses game-theoretic techniques from the previous chapter: special (Rabin) automata are also studied in this chapter. The final chapter contains examples of applications of Rabin automata to mathematical theories, such as the monadic second-order theory of *n* successors. The book contains a representative bibliography, divided according to individual applications.

The book contains suitable material for a two-semester course for students of computer science or mathematics. It is completely self-contained and one can really enjoy reading it. It is strongly recommended for researchers and postgraduate students interested in logic, automata and/or concurrency. (mpl)

M. Křížek, F. Luca and L. Somer, ***17 Lectures on Fermat Numbers. From Number Theory to Geometry***, *CMS Books in Mathematics* ***9***, *Springer, New York, 2001, 257 pp., 69,95 euros, ISBN 0-387-95332-9*

Quoting from the foreword by Alena Šolcová: 'At the beginning of the third millennium, Fermat's name is held in great respect throughout the whole mathematical community. This book illustrates how one of Fermat's famous challenges led to developments in mathematics from number theory to geometry.' Let us also quote the authors: 'This book was written on the occasion of the 400th anniversary of Pierre de Fermat's birth (1601) … We followed an old Chinese proverb that a picture is worth a thousand words.'

The first three chapters of this book deal with basic results on Fermat numbers. In the chapter on 'The most beautiful theorems on Fermat numbers', we find theorems of Goldbach, Gauss, Abel, Lucas and Suyama and the crucial result that the fifth Fermat number is divisible by 641. The text covers other aspects of the subject, including open problems and a Euclidean construction of the regular heptadecagon. The reader is required to know only some elementary facts from algebra. This makes the book ideal for a general mathematical audience, students, and researchers from other fields interested in number theory. (lber)

J. Kurzweil, ***Integration between the Lebesgue Integral and the Henstock-Kurzweil Integral: Its Relation to Local Convex Vector Spaces***, *Series in Real Analysis* ***8***, *World Scientific, Singapore, 2002, 140 pp., £19, ISBN 981-238-046-9*

The main topic of this book is integration of Riemann type on a compact interval on the real line with respect to various integration bases. This concept covers many types of integration, including the Lebesgue integration, the Denjoy integration in the restricted sense, the integration introduced by Pfeffer and Bongiorno, and many others. After the general theory of integration with respect to various integration bases in Chapters 1 and 2, two chapters present a natural locally convex topology on the space of sequences of primitive functions. The problem of completeness and other topics related to this topology are discussed in detail in Chapters 5-9: particular attention is devoted to the Lebesgue integral. The last part of the book introduces a general type of differentiation, again with respect to various integration bases, and clarifies the relation between integration and differentiation.

The book is self-contained, and will be of interest to specialists in the field of real functions. It can also be read by students, since only the basics of mathematical analysis and vector spaces are required. (jsp)

O. Lafitte, ***The Wave Difracted by a Wedge with Mixed Boundary Conditions***, *Mémoires de la Société Mathématique de France, Société Mathématique de France, Paris, 2002, 167 pp., 31 euros, ISBN 2-85629-118-X*

This book studies diffraction of a wave by a wedge in $\mathbf{R}^2$. The faces of the wedge are described by analytic functions and are equipped with mixed boundary conditions of impedance type. The solutions are shown to be locally a sum of two reflected waves and one diffracted wave, and explicit formulas in terms of the Fourier transform are given. Moreover, the leading order term is the same as that corresponding to the linearised wedge. Results of the book generalise previously known results concerning straight wedges and curved wedges with Dirichlet boundary conditions.

It is not easy to read this book which has a rather concise style, many technical computations, and partial dependence (both in ideas and notation) on previous papers. (dp)

G. Lyubeznik (ed.), ***Local Cohomology and its Applications***, *Lecture Notes in Pure and Applied Mathematics* ***226***, *Marcel Dekker, New York, 2002, 342 pp., US$150, ISBN 0-8247-0741-9*

This book presents recent developments and applications of Grothendieck's local cohomology theory. The book is an expanded and updated version of two minicourse lectures and surveys, delivered at a workshop at CIMAT (Mexico) in 1999. The first lectures, on local cohomology and equivariant theory, are by J. Greenlees; the second, on local cohomology and duality, are by J. Lipman. M. Brodman then surveys recent results on cohomology of projective schemes, and G. Lyubeznik the development in local cohomology in the 1990s. H. Tsai and A. Leykin deal with algorithmic aspects of local cohomology via D-modules, E. Miller and K. Yanagawa discuss local cohomology over graded rings, A. Singh deals with associated primes of the local cohomology module, and I-Chiau Huang with applications to combinatorial analysis.

This book will be of considerable interest to algebraists and geometers, and also to topologists and specialists working in combinatorics and the theory of D-modules. (jtrl)

B. A. Magurn, ***An Algebraic Introduction to K-Theory***, *Encyclopaedia of Mathematics and Its Applications* ***87***, *Cambridge University Press, 2002, 676 pp., £75, ISBN 0-521-80078-1*

This is a fine introduction to algebraic *K*-theory, requiring only a basic preliminary knowledge of groups, rings and modules. While developing the theory, the author provides numerous examples and presents the basics of closely related areas where *K*-theory is applied: these include Dedekind domains, classical groups, character theory, quadratic forms, tensor products, completion and localisation, symmetric and tensor algebras, central simple algebras, and Brauer groups of fields.

Part I deals with K_0, which creates a kind of substitute for the dimension of arbitrary modules. The Grothendieck group $K_0(R)$ of the category of finitely generated projective modules is first introduced as an abelian group, and the related notion of stability is discussed in detail. The ring structure on $K_0(R)$ is then introduced via tensor product in the case when R is commutative; as examples, the Witt-Grothendieck ring and the Witt ring are considered in detail and applied to a classification of forms.

Part II deals with applications of K_0 to number theory and the representation theory of finite groups. The relation between $K_0(R)$ and the class group of a Dedekind domain R is established. Among other things, the structure of semisimple rings and Maschke's theorem are proved, as well as basic properties of characters of finite groups.

Part III deals with $K_1(R)$, which consists of the row-equivalence classes of invertible matrices over R. $K_1(R)$ is used to study (generalised) determinants. It is shown that the stable rank of R leads to a bound on the dimensions of the matrices needed to represent $K_1(R)$. As an application, the solution of the congruence subgroup problem over Dedekind domains of arithmetic type is presented.

Part IV introduces $K_2(R)$ as a fine measure for row reduction of matrices over R. A method of its computation via Steinberg symbols is developed, and the 'relative exact sequence' is presented, linking K_0, K_1 and K_2 for R and R/J, where J is an ideal of R. Finally, Matsumoto's theorem relating $K2(R)$ to symbol maps is proved.

Part V, 'Sources of K_2' presents various symbol maps occurring in number theory and non-commutative algebra, thereby linking these areas with *K*-theory. The book culminates with a discussion of the relation between the Brower group of a field F and $K_2(F)$, presenting (without proof) the Tate-Merkurjev-Suslin isomorphisms and their consequences for the structure of Azumaya F-algebras. Given the wide range of applications of *K*-theory presented, the book will certainly be of interest to algebraists, and to number theorists, topologists, geometers and functional analysts. (jtrl)

C. Martín-Vide and V. Mitrana (eds.), ***Grammars and Automata for String Processing: From Mathematics and Computer Science to Biology, and back***, *Topics in Computer Mathematics* ***9***, *Taylor & Francis, London, 2003, 422 pp., £55, ISBN 0-414-29855-7*

This is a collection of 40 articles written in honour of Gheorghe Paun on the occasion of his 50th birthday. The first part (Grammars and grammar systems) treats contextual grammars, descriptional complexity, parsability approaches for contextual grammars and context-free grammars, and the power of limitation of regulated rewriting in image generation. The second part (Automata) is concerned with probabilistic cellular automata, directable automata, *X*-machines, finite automata and real-time automata. There are applications in software engineering, linguistics and ecology. The third part (Logics, languages and combinatorics) treats homomorphic characterisations of the language classes in the Chomsky hierarchy, languages for picture descriptions, semilinear power series and DOL power series, unary languages, relationships between different classes of languages, and the languages associated with rewriting systems. The last part (Models of molecular computing) presents models of metabolic reactions in bacteria, entropies of DNA-based computing models, Adelman's model, splicing systems, and tiling shifts. (pku)

J.-C. Nédélec, ***Acoustic and Electromagnetic Equations. Integral Representations for Harmonic Problems***, *Applied Mathematical Sciences* ***144***, *Springer, New York, 2001, 316 pp., DM 160,39, ISBN 0-387-91155-5*

This book investigates special solutions to two classical hyperbolic equations – the wave equations and the Maxwell equations on Minkowski space. In both cases, the author considers only solutions that are harmonic in the time variable t, and so can be written as a product of a function of space variables with function exp(ikt). The wave equation for such functions reduces to the Helmholtz equation, and similarly for solutions of the Maxwell equations. The second chapter is devoted to properties of the solutions of the Helmholtz equation, including preparatory sections describing classical spherical harmonics and interior and exterior problems for the Laplace equation in $\mathbf{R}^3$. Chapter 3 describes integral representations for Helmholtz equations and the corresponding integral equations, while Chapter 4 is devoted to basic facts on singular integral operators. The final chapter contains a treatment of solutions of the Maxwell equation. The choice of material is guided by the needs of applications in mathematical physics and engineering, and the book is suitable for graduate students. (vs)

C. A. Pickover, *The Mathematics of Oz: Mental Gymnastics from Beyond the Edge*, *Cambridge University Press, 2002, 351 pp., £21.95, ISBN 0-521-01678-9*
This is an excellent collection of mathematical problems and puzzles at all levels of mathematical sophistication. The tasks are presented to little Dorothy by Dr. Oz, a mathematically obsessed alien. Dorothy tries to solve problems, each described via a nice story: this motivates the reader to solve the challenge. Full solutions are presented in the 'Further Exploring' part of the book.

The problems cover various topics: geometry and mazes, sequences, series, sets, arrangements, probability and misdirection, number theory, arithmetic and the physical world. As an example, consider the following problem: 'Can you divide a triangle into finitely many triangles to produce all acute internal angles?' The reader is guided to think about many interesting areas, and various methods are exploited and presented in a non-traditional way. The author has written many interesting books, and his website is well known: (pp)

W. Rindler, *Relativity. Special, General and Cosmological*, *Oxford University Press, 2001, 428 pp., £24,95, ISBN 0-19-850836-0 and 0-19-850835-2*
This book covers many important topics from special relativity, general relativity and cosmology. The language of tensor fields is introduced early on and used systematically, and the necessary knowledge is recalled in a short and clearly written summary. The reader will find here a clear and understandable explanation of the physical principles behind the theory of relativity and its use in cosmology. Each chapter of the book ends with exercises, around 300 in total, which form an important part of the book. The special relativity part starts with the Lorentz group, relativistic kinematics and optics. Four-vectors are used to describe relativistic particle mechanics, and the Maxwell equations are formulated in terms of Minkowski tensor fields. After introducing basic ideas of general relativity and formulating the Einstein equations in terms of (pseudo)-Riemannian geometry, the author discusses properties of the Schwarzschild space, black holes, plane waves, (anti-)de Sitter spaces, and linearised gravity. The book ends with three chapters on cosmology, including many properties of Friedman-Robertson-Walker universes.

This book is a pleasure to read. It will be an excellent source, allowing the reader to build a proper intuition and to understand the basic facts of the theory. The level of mathematical knowledge required is very modest, so it should be useful for students at graduate level, both physicists and mathematicians. (vs)

D. J. S. Robinson, *An Introduction to Abstract Algebra*, *de Gruyter Textbook, Walter de Gruyter, Berlin, 2003, 282 pp., 42,95 euros, ISBN 3-11-017544-4*
The principal structures of modern algebra and its applications form the main topic of the book. In twelve chapters, the author explains the main topics in abstract algebra with many useful examples.

The first two chapters introduce fundamental concepts (set, class, properties of integers, GCD, Euclidean algorithm, the fundamental theorem of arithmetics). The concept of a group is explored in Chapters 3, 4, 5 and 9. The exposition follows a historical development, and the author uses the group of permutations as a motivating example. Lagrange's theorem, the isomorphism theorems and other classic facts concerning normal and quotient groups are included as a standard part of a university algebra course. Group actions are used to investigate the group structure. Algebraic structures with two binary operations are introduced in Chapter 6: rings, Euclidean domains, roots of polynomials, and splitting fields. Vector spaces are used in many applications: they are discussed in a separate chapter with many examples. Chapters 10 and 11 contain a description of fields and Galois theory, with applications to orthogonal latin squares, Steiner systems and the solvability of equations by radicals. The final chapter contains some additional material useful for applications. This book will find its readers among specialists in mathematics, physics and informatics, who need to acquire a basic knowledge of algebra and its applications. The book should be useful for university students attending algebra courses. (mer)

W. T. Ross and H. S. Shapiro, *Generalized Analytic Continuation*, *University Lecture Series **25**, American Mathematical Society, Providence, 2002, 149 pp., US$31, ISBN 0-8218-3175-5*
The authors study various kinds of generalised analytic continuations (GAC) of meromorphic functions. They present a broad scope of methods used to solve very interesting problems of GAC. The methods presented are closely related to methods of summability of divergent series. It is worth noting that the methods of GAC included here do not provide, in general, the same results.

The chapters cover the following topics: the Poincaré example, non-tangential boundary values, Borel's ideas, superconvergence, Borel series, Gonèar continuation, pseudocontinuation, a problem of Walsh and Tumarkinn, the Darlington synthesis problem, gap theorems, non-continuability, formal multiplication of series, spectral properties, uniqueness and an axiomatic approach. Another approach to GAC developed in potential theory (Fuglede's finely holomorphic functions) is not mentioned.

The book is very clearly written, and contains a large number of results (including proofs) and historical comments. The presented solutions of GAC are far from the final exhausting answer, and several open problems are presented. The book will be useful for working mathematicians in GAC and complex function theory. The authors will place updates, corrections and additions at . (pp)

L. Saloff-Coste, *Aspects of Sobolev-Type Inequalities*, *London Mathematical Society Lecture Note Series **289**, Cambridge University Press, 2001, 190 pp., £24.95, ISBN 0-521-00607-4*
Sobolev-type inequalities play a fundamental role in many areas of function space theory and in partial differential equations. The purpose of this volume is to extend results on Sobolev-type inequalities from the classical version (domains in Euclidean space) to the framework of Riemannian manifolds.

The first chapter presents fundamental Sobolev inequalities for embedding into Lebesgue spaces, together with two classical proofs using alternatively Gagliardo inequality and Riesz potentials. Related results (embeddings into Hölder spaces, best embedding constants and the Poincaré inequality) are also discussed. Possibilities of introduced techniques are illustrated in Chapter 2 in a discussion of Moser's famous proof of Harnack's inequality for weak positive solutions of elliptic equations in divergence form. Chapter 3 concentrates on Sobolev-type inequalities on Riemannian manifolds. After explaining basic facts about Riemannian manifolds, relations between the Sobolev and Poincaré inequalities and the volume growth of the manifold are established. Strong and weak forms of the Sobolev inequality are proved, together with an interesting form of pseudo-Poincaré inequality. The fourth chapter is devoted to applications: ultracontractivity of the heat equation, Gaussian heat kernel estimates, the Rozenblum-Lieb–Cwikel inequality and the Birman-Schwinger principle. The final chapter studies a parabolic version of the Harnack inequality and characterises manifolds that satisfy a scale invariant parabolic Harnack principle.

This book is a well-written and self-contained account of the topic. It is accessible to advanced graduate students and to researchers. (jsta)

G. Scheja and U. Storch, *Regular Sequences and Resultants*, *Research Notes in Mathematics **8**, A. K. Peters, Natick, 2001, 142 pp., US$30, ISBN 1-56881-151-9*
The main purpose of this monograph is to investigate eliminations in weighted projective spaces over commutative noetherian rings. Among the important algebraic tools introduced and generalised, regular sequences and complete intersections play the key role in the theory developed by the authors.

The book has four chapters. Chapter I contains the general concept of Kronecker extensions of a ring and its modules, and presents an investigation of numerical monoids that turn up in a natural way as monoids of positive weights of indeterminates in polynomial algebras. Chapter II deals with regular sequences and complete intersections. The authors present special (but for elimination theory essential) cases of regular sequences, which are characterised by combinatorial means (sequences of generic polynomials and generic Laurent polynomials). Chapter III is a study of the main case of elimination with respect to projective spaces. In particular, the structure of elimination ideals of ground rings of algebras is investigated. It is proved (for regular sequences) that the elimination ideal over an integrally closed noetherian domain is a divisible ideal, and that the elimination ideal over a factorial noetherian ring is principal. The concept of resultant ideals for a regular sequence of homogeneous polynomials is introduced in the Chapter IV, where the construction and basic properties of resultants (canonical generators of resultant ideals) are presented.

This monograph is written carefully and lucidly in a fresh mathematical style. All topics are arranged very clearly. Each section contains a supplement in which additional details and examples are presented. The book is suitable as a useful reference for researchers with an interest in commutative algebra, and for those interested in applications in applied and computational algebra and in algebraic geometry. (jz)

J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis, 3rd edition*, *Texts in Applied Mathematics* **12**, *Springer, New York, 2002, 744 pp., 69,95 euros, ISBN 0-387-95452-X*

This is the third edition of a famous work on the basics of numerical analysis. It is a well-written textbook for advanced undergraduate/beginning graduate students containing both classical methods and modern approaches to numerical mathematics. The theory is illustrated by many interesting examples, and carefully selected exercises lead the reader to a better understanding of the topics discussed. References at the end of each chapter, and a list of monographs on numerical methods at the end of the book, motivate a deeper study of explained techniques.

The book starts with a discussion of the general effects of input and round-off errors on the result of a calculation. Interpolation is considered – in particular, trigonometric interpolation and interpolation by polynomials, rational functions, and splines. Here, B-splines are also treated, including their use in the context of multiresolution methods. After a description of methods of numerical quadrature, direct methods for solving systems of linear equations are thoroughly discussed, including the Gaussian elimination, the Choleski decomposition, the simplex method, and orthogonalisation techniques with their use in solving linear least-squares problems, and elimination methods for sparse matrices are mentioned. The next chapter is concerned with iterative methods suitable for finding zeros and minimum points of a given function, and contains a detailed discussion of Newton's method. The authors then describe various normal forms of matrices and several methods for reducing matrices to (tri)diagonal form or Hessenberg form and explain the main algorithms for computing eigenvalues and eigenvectors, including the LR and QR algorithms. A long chapter is devoted to numerical solutions of ordinary differential equations, where the authors treat initial-value problems (one-step and multistep methods) and boundary-value problems (simple and multiple shooting methods), and sensitivity analysis and handling of discontinuities are considered. The final chapter describes iterative methods for solutions of large systems of linear equations, including a nice description of various Krylov space methods (including preconditioning techniques), such as CG, GMRES, QMR, Bi-CG, Bi-CGSTAB, and the presentation of multigrid methods.

The thrid edition contains new material and several improved passages and will be useful also for those who already use the previous editions. (pkn)

J.-P. Tignol, *Galois' Theory of Algebraic Equations*, *World Scientific Publishing, Singapore, 2001, 333 pp., £26, ISBN 981-02-4541-6*

The main purpose of Tignol's book is, in his own words, the methodology of mathematics, and the theme used to illustrate the general methodology is the theory of algebraic equations. The main stages of its evolution starting from its origins in Babylonian times to its completion by Galois are reviewed and carefully discussed. Although the statement of the basic problems of the theory of algebraic equations is elementary, requiring only high-school mathematics, its long and eventful development led to profound ideas and to the fundamental concepts of modern algebra. It thus seems to be an ideal topic for the purpose of explaining how mathematics is made: this is why the author relies more on the individual experiences of great mathematicians rather than on the precise development of Galois theory up to modern standards. A pleasant feature is the use of modern notation and terminology to explain the original ideas of Cardano, Viète, Descartes, Newton, Lagrange, Waring, Gauss, Ruffini, Abel, Galois, etc., making the book accessible to any undergraduate student of mathematics, and to any mathematician interested in the historical development of mathematical ideas. (jt)

D. Wolfe and T. Rodgers (eds), *Puzzlers' Tribute. A Feast for the Mind*, *A. K. Peters, Natick, 2002, 420 pp., US$35, ISBN 1-56881-121-7*

In the early 1990s, Tom Rodgers conceived the idea of hosting a weekend gathering in honour of Martin Gardner, to bring together people interested in problems and puzzles. Members of the three communities of mathematicians, magicians and puzzlers met to honour their forefront man and nexus, and so far four 'Gatherings for Gardner' (known as G4G1 through G4G4) have been organised. At these gatherings, held in 1993, 1994, 1998 and 2002, participants share problems and puzzles, knowledge and ideas. In 1999, 'The mathemagician and pied puzzler' was published as a tribute to Martin Gardner, based on contributions to G4G1. This is a second volume.

In the preface, the work of Martin Gardner and his contribution to the popularisation of mathematics is briefly reviewed: in the words of John Conway, 'Martin Gardner has brought more mathematics, to more millions, than anybody else'. The book continues with a section dedicated to the memory of Mel Stover, Harry Eng and David Klarner, three G4G participants who have passed away. Called 'The toast tributes', this section contains several short moving articles commemorating their lives and ideas. The rest of the book contains 56 contributions of G4G participants, divided into six sections. This presents an incredible range of beautiful material, witty ideas, tantalising puzzles, intriguing problems, and surprising ingenious solutions. Anyone interested in mind-boggling problems will find plenty to interest them here. The famous authors in the area, such as Barry Cipra, Roger Penrose, Raymond Smullyan, Keith Devlin, John Conway, and many more, are among the contributors. The material covers all kinds of problems that one can possibly think of: card problems, labyrinth problems, topological games, knots, floor-tiling problems, spider-and-fly problems, pentacube towers, chess, magic squares, pure arithmetic problems, geometry, trigonometry, Japanese puzzles and much, much more. (lp)

Y. Yang, *Solitons in Field Theory and Nonlinear Analysis*, *Springer Monographs in Mathematics, Springer, New York, 2001, 553 pp., DM 181,79, ISBN 0-387-95242-X*

In recent decades, the study of non-linear integrable systems of partial differential equations has advanced greatly, and many interesting methods have been developed. But most of the non-linear systems of PDEs arising in modern theoretical physics are not integrable, and instead of solutions described in a closed form, general existence theorems are proved using modern tools of functional analysis. A particularly interesting feature of many non-linear systems is the existence of solitons, locally concentrated solutions of the corresponding equations. This monograph is devoted mainly to them.

Included here are many important examples of such systems – sigma models, self-dual Yang-Mills fields, generalised abelian Higgs equations, Chern-Simons equations, the Salam-Weinberg theory of electro-weak interactions and their multivortex solutions, static solutions representing dyons in various non-abelian gauge field theory models, radially symmetric solutions of a general scalar equation, strings in cosmology, vortices and anti-vortices in abelian gauge theory, and field equations arising from the classical Born-Infeld electromagnetic theory. The author mainly uses the language of theoretical physics, and these parts will be more understandable by mathematical physicists. However, many results are formulated in the language of theorems and their proofs, as is standard in the mathematical literature. Individual chapters end with notes containing open problems for research. For convenience, classical field theory is summarised in Chapter 1, and another chapter explains the classification of simple Lie algebras, presented in the language of theoretical physics. The material should be of interest to mathematicians and mathematical physicists at postgraduate level. (vs)

M. Zeman, *Inner Models and Large Cardinals*, *de Gruyter Series in Logic and its Applications* **5**, *Walter de Gruyter, Berlin, 2001, 369 pp., DM 276, ISBN 3-11-016368-3*

This book introduces the theory of inner models of set theory constructible relative to coherent extender sequences: such models admit a fine structure analogous to that of Gödel's constructible universe. Chapters 1-3 introduce the general fine structure theory of acceptable structures, developed abstractly without reference to large cardinals or inner models. Chapters 4-7 present full core model theory for measures of order 0, and Chapter 8 indicates a generalisation for models that can contain up to one strong cardinal. A 'linear' iterability (a possibility of forming certain required iterated ultrapowers), associated to the models in question, plays a crucial role here. As Chapter 9 explains, this iterability is highly non-linear in the case of theory of Jensen extender models that are beyond one strong cardinal.

The exposition is relatively self-contained, but some elementary knowledge (for example, concerning acceptable structures) is assumed. The book can serve as an introductory text for those unfamiliar with the subject and as a textbook for graduate students. (jmlc)

Reviewers: *lp = L. Pick, mbe = M. Beneš, mnem = M. Němcová, vs = V. Souček, jama = J. Malý, dp = D. Pražák, jtu = J. Tůma, jtrl = J. Trlifaj, mer = M. Ernestová, kzv = K. Zvára, jl = J. Lukeš, zp = Z. Prášková, jjel = J. Jelínek, jmlc = J. Mlcek, mih = M. Hušek, bm = B. Maslowski, pv = P. Valtr, mpl = M. Plátek, lber = L. Beran, jsp = J. Spurný, pku = P. Kůrka, pp = P. Pyrih, jsta = J. Stará, jž = J. Žemlička, pkn = P. Knobloch.*