European Mathematical Society

September 2004 Issue 53

Feature
Topological Combinatorics: Kneser Conjecture

p. 16

Anniversary
Henri Cartan 100 years

p. 20

Interview
Michael Atiyah and Isadore Singer

p. 24

ISSN – 1027 - 488X

ERCOM
CRM Barcelona

CENTRE DE RECERCA MATEMÀTICA CRM

p. 37

NEWSLETTER

EUROPEAN MATHEMATICAL SOCIETY

NEWSLETTER No. 53

September 2004

Designed and printed by Armstrong Press Limited
Crosshouse Road, Southampton, Hampshire SO14 5GZ, UK
telephone: (+44) 23 8033 3132 fax: (+44) 23 8033 3134
e-mail: ems@armstrongpress.com

Published by European Mathematical Society
ISSN 1027 - 488X

NOTICE FOR MATHEMATICAL SOCIETIES

Labels for the next issue will be prepared during the second half of November 2004. Please send your updated lists before then to Ms Tuulikki Mäkeläinen, Department of Mathematics and Statistics, P.O. Box 68 (Gustav Hällströmintie 2B), FI-00014 University of Helsinki, Finland; e-mail: tuulikki.makelainen@helsinki.fi

INSTITUTIONAL SUBSCRIPTIONS FOR THE EMS NEWSLETTER

Institutes and libraries can order the EMS Newsletter by mail from the EMS Secretariat, Department of Mathematics and Statistics, P.O. Box 68 (Gustav Hällströmintie 2B), FI-00014 University of Helsinki, Finland, or by e-mail: (tuulikki.makelainen@helsinki.fi). Please include the name and full address (with postal code), telephone and fax number (with country code) and e-mail address. The annual subscription fee (including mailing) is 80 euros; an invoice will be sent with a sample copy of the Newsletter.

EMS Committee

EXECUTIVE COMMITTEE
PRESIDENT
Prof. Sir JOHN KINGMAN (2003-06)
Isaac Newton Institute
20 Clarkson Road
Cambridge CB3 0EH, UK
e-mail: emspresident@newton.cam.ac.uk

VICE-PRESIDENTS
Prof. LUC LEMAIRE (2003-06)
Department of Mathematics
Université Libre de Bruxelles
C.P. 218 – Campus Plaine
Bld du Triomphe
B-1050 Bruxelles, Belgium
e-mail: llemaire@ulb.ac.be
Prof. BODIL BRANNER (2001–04)
Department of Mathematics
Technical University of Denmark
Building 303
DK-2800 Kgs. Lyngby, Denmark
e-mail: b.branner@mat.dtu.dk

SECRETARY
Prof. HELGE HOLDEN (2003-06)
Department of Mathematical Sciences
Norwegian University of Science and Technology
Alfred Getz vei 1
NO-7491 Trondheim, Norway
e-mail: h.holden@math.ntnu.no

TREASURER
Prof. OLLI MARTIO (2003-06)
Department of Mathematics
P.O. Box 4, FIN-00014
University of Helsinki, Finland
e-mail: olli.martio@helsinki.fi

ORDINARY MEMBERS
Prof. VICTOR BUCHSTABER (2001–04)
Steklov Mathematical Institute
Russian Academy of Sciences
Gubkina St. 8, Moscow 117966, Russia
e-mail: buchstab@mendeleevo.ru
Prof. DOINA CIORANESCU (2003–06)
Laboratoire d'Analyse Numérique
Université Paris VI
4 Place Jussieu
75252 Paris Cedex 05, France
e-mail: cioran@ann.jussieu.fr
Prof. PAVEL EXNER (2003-06)
Department of Theoretical Physics, NPI
Academy of Sciences
25068 Rez – Prague, Czech Republic
e-mail: exner@ujf.cas.cz
Prof. MARTA SANZ-SOLÉ (2001-04)
Facultat de Matematiques
Universitat de Barcelona
Gran Via 585
E-08007 Barcelona, Spain
e-mail: sanz@cerber.mat.ub.es
Prof. MINA TEICHER (2001–04)
Department of Mathematics and
Computer Science
Bar-Ilan University
Ramat-Gan 52900, Israel
e-mail: teicher@macs.biu.ac.il

EMS SECRETARIAT
Ms. T. MÄKELÄINEN
Department of Mathematics and Statistics
P.O. Box 68 (Gustav Hällströmintie 2B)
FI-00014 University of Helsinki
Finland
tel: (+358)-9-1915-1426
fax: (+358)-9-1915-1400
telex: 124690
e-mail: tuulikki.makelainen@helsinki.fi
Web site: http://www.emis.de

EMS Agenda

2004

15 November
Deadline for submission of material for the December issue of the EMS Newsletter
Contact: Martin Raussen, e-mail: raussen@math.aau.dk

12-19 December
EMS Summer School and Séminaire Européen de Statistique at Munich (Germany)
The Statistics of Spatio-Temporal Systems
Web site: http://www.stat.uni-muenchen.de/semstat2004/

2005

5-13 February
EMS Summer School at Eilat (Israel)
Applications of braid groups and braid monodromy

16-17 April
EMS Executive Committee meeting at Capri (Italy)

25 June – 2 July
EMS Summer School at Pontignano (Italy)
Subdivision schemes in geometric modelling, theory and applications

17-23 July
EMS Summer School and European young statisticians' training camp at Oslo (Norway)

13-23 September
EMS Summer School at Barcelona (Catalunya, Spain)
Recent trends of Combinatorics in the mathematical context

16-18 September
EMS-SCM Joint Mathematical Weekend at Barcelona (Catalunya, Spain)

25 September – 2 October
EMS Summer School and Séminaire Européen de Statistique at Warwick (UK)
Statistics in Genetics and Molecular Biology

12-16 December
EMS-SIAM-UMALCA joint meeting in applied mathematics
Venue: the CMM (Centre for Mathematical Modelling), Santiago de Chile

2006

22-30 August
International Congress of Mathematicians in Madrid (Spain)
Web site: www.icm2006.org

2007

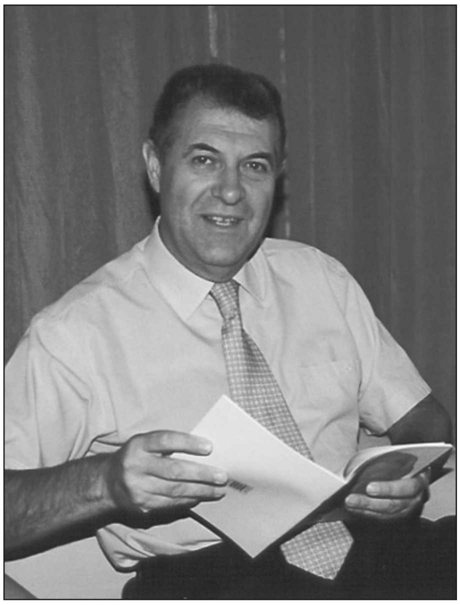
16-20 July
ICIAM 2007, Zurich (Switzerland)

2008

July
5th European Mathematical Congress, Amsterdam (The Netherlands)

Editorial

Manuel Castellet (Barcelona)
Chairman of ERCOM



ERCOM is a committee of the European Mathematical Society created in 1996 consisting of Scientific Directors of European Research Centres in the Mathematical Sciences, or their chosen representatives. Only centres in which the number of visiting staff substantially exceeds the number of permanent and long-term staff and that broadly cover Mathematical Sciences are eligible for representation in ERCOM. The eligibility of centres is decided by the EMS Executive Committee.

The aims of ERCOM are to contribute to the unity of Mathematics, from fundamental to applications, with the purpose of constituting a forum for communication and exchange of information and fostering collaboration and co-ordination between the centres themselves and the EMS, to foster advanced research training on a European level, to advise the Executive Committee of the EMS on matters relating to activities of the centres, to contribute to the visibility of the EMS and to cultivate contacts with similar research centres within and outside Europe.

The 24 centres at present represented in ERCOM have different working and organisational structures, they are also different in size and scope, but they can be grouped in three different classes, as centres mainly for meetings, centres without any permanent researchers, only visitors and post-docs, and centres with a small permanent staff and a large number of visitors. They broadly cover the European Area, from Russia and Sweden to Portugal and Israel. A Chairman and a Vice-chairman, at present myself, as Director of the Centre de Recerca Matemàtica, and Kjell-Ove Widman, Director of the Institut Mittag-Leffler, co-ordinate the annual ERCOM meeting and the other initiatives proposed by the ERCOM members. Sir John Kingman, Director of the Isaac Newton Institute, acts as link between ERCOM and the EMS Executive Committee.

The Editor-in-Chief of the EMS Newsletter, Martin Raussen, recently offered to the ERCOM institutions the opportunity to present themselves in order to give the readers an idea of how they might use the possibilities offered by the centres. EURANDOM, Eindhoven, presented itself in the June issue of the Newsletter and the Centre de Recerca Matemàtica (CRM), Barcelona, is doing so in the present issue.

As Peter Hilton already wrote twenty years ago "A research institute is at least two things at the same time: it is a building and it is an organisation of people working together and dedicated to the pursuit and support of research. But at its best it is more. It starts life as an idea in the mind of one or more persons, of insight and imagination, and then lives and grows by spreading the spirit, imbued by their founders, through the hearts and minds of all those benefiting from its presence and contributing to its future".

In mathematical research, exchange of ideas plays a central role and needs a deep human contact. A contact which is the true mathematical laboratory, and, as Friedrich von Siemens said at the end of the nineteenth century, "Laboratories are the fundamental basis of knowledge and power". The high degree of abstraction of mathematics and the compact way it is presented need direct personal communication, since mathematics is an attempt to develop tools that can be used to achieve a better understanding of the world's measurable aspects and to identify processes that appear in very different natural situations but that are essentially analogous.

But mathematics, apart from this, is a truly international science, perhaps the most international of sciences, since, compared with other disciplines, it is based less on the use of instruments and more on a strong human contact. This is where the research institutes play a crucial role, allowing not only the exchange of ideas between specialists in the same field, but also profound and sometimes surprising links between different lines of research.

In order to foster collaboration and co-ordination between the European research institutes and to provide communication and exchange of information, ERCOM meets yearly, usually by March, in one of the member institutes. Since the 1999 meeting in Cambridge the Administrators have been invited to discuss separately and jointly with the Directors matters of their competence.

Several topics are considered for discussion in the meetings: The European Research Area and the presence of Mathematics in the Framework Programmes of the European Union, the annual call for post-doctoral grants by the European Post-doctoral Institute for the Mathematical Sciences (EPDI), the situation of Mathematics and the mathematicians in Eastern Europe, the INTAS programme, the Digital Math Library, issues arisen from the EMS Executive Committee, etc.

In particular, ERCOM has established an exchange programme for short-term visits of post-doctoral fellows from one ERCOM centre to another, flexible and capable of adapting to different centres and situations, the main goal being to stimulate the exchange and the increase of knowledge and to create synergies enriching research.

The Administrators have prepared a report on the percentage of women doing research in the ERCOM centres. There are about 14% of female researchers conducting long term visits and an average of 16% female researchers conducting short term visits in the ERCOM centres, while, on the other hand, the administrative staff is mostly female and the directorate is mostly male. Last March in the Aarhus meeting the following resolution was adopted: "ERCOM encourages its members to take actions to facilitate the presence of women in their scientific activities, and to collect data regarding the number of applications from female researchers received and approved".

ERCOM, as a committee of the European Mathematical Society, wishes and has the capacity to act as one of the EMS means of scientific outreach, fostering the presence of mathematicians in the new emerging multidisciplinary research domains. With this aim we are preparing a proposal to be submitted to the European Commission as a NEST Support Action through a Co-ordination Action instrument, with the main objective of designing curricula and finding ways to train researchers into emerging specialities of Mathematics, linked especially with System Biology, Neuroscience, Risk Assessment and Data Security, and identifying future research opportunities on the interface between Mathematics and suitable areas of Medicine, Industry and Social Sciences.

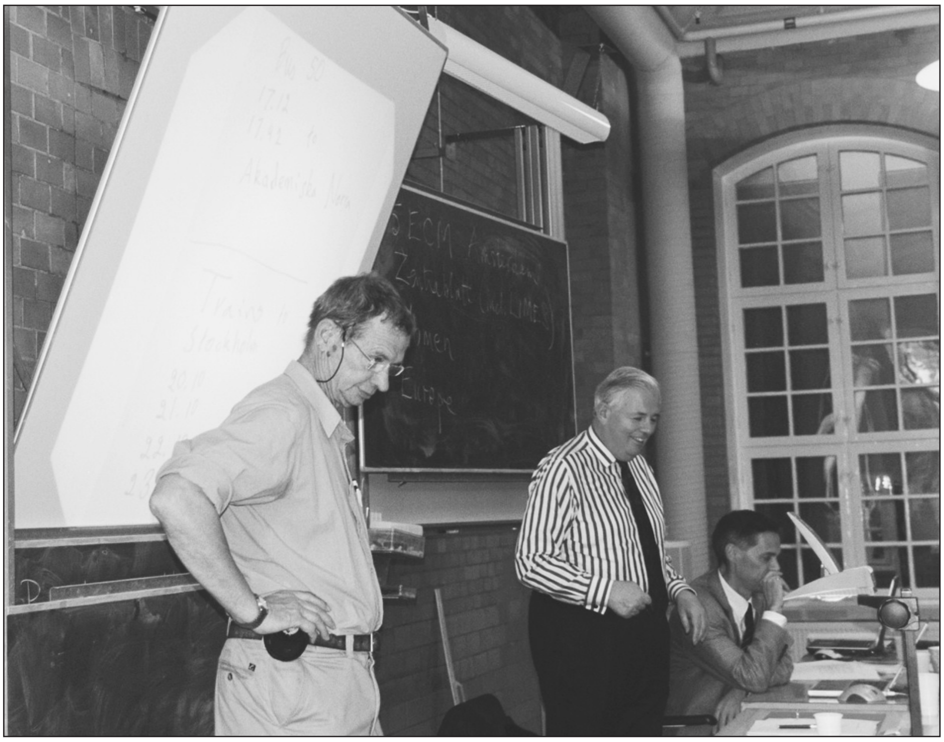
A home page (*http://www.ercom.org*), from which you can reach that of each ERCOM centre, provides information on the ERCOM initiatives and, in particular on open positions offered by the centres and scheduled conferences and training courses.

EMS Council Meeting

Uppsala, Sweden, 26th-27th June 2004

David Salinger, EMS Publicity Officer

The Council of the European Mathematical Society (its "parliament") met in the weekend before the European Congress in Stockholm on the premises of Uppsala University, about 60 kilometres north of the Swedish capital. There were 65 delegates representing 43 Mathematical Societies from all over Europe, 14 delegates representing individual members, and institutions were represented by 9 delegates. The EMS executive committee and several invitees made up the rest of the assembly.



Host Sten Kaijser, Uppsala Univ., and EMS-President Sir John Kingman

Membership issues, budget and elections

Council had its share of routine business, but finished with several lively discussions. The routine business was, however, important, starting with the approval of the membership applications from the Mathematical Societies of Malta, Cyprus and Romania and the Statistical Societies of Belgium, Sweden, Norway and Spain.

The President underlined two important facets of the Society's many activities: its lobbying for mathematics at a European level, in which Luc Lemaire played an invaluable role; and the successful establishment of its own publishing house, for which Rolf Jeltsch had been the driving force. All activities depended on a small group of dedicated people, in particular he warmly thanked Secretary Tuulikki Mäkeläinen, but the Society needed more volunteers as its work expanded. The President commended the willingness of mathematicians to serve on the recently created Scientific Advisory Panel. Council agreed to send congratulations to Henri Cartan on the occasion of his 100th birthday.

The financial statements, presented by Olli Martio, were accepted. The finances were in good health, but a slight rise in the corporate membership fee was needed, which Council approved.

Pavel Exner was elected as Vice-President; Victor Buchstaber, Olga Gil-Medrano, Carlo Sbordone, and Klaus Schmidt as ordinary members of the Committee. The President thanked the outgoing members of the Committee: Bodil Branner, Marta Sanz-Solé, and Mina Teicher.

Reports and Debates

Martin Raussen, editor of the Newsletter reported on a smooth handover from Robin Wilson. Much of the Newsletter would be made available on EMIS, and it was hoped that the Book Reviews and Conferences could be made available in a searchable form. Council thanked Robin Wilson and Martin Raussen for their work for the Newsletter.

There had been some excellent work on behalf of the applied mathematics community, including the AMAM conference in Nice and the focus on applications of mathematics in the forthcoming 4ecm, but the applied mathematics committee had been unable to decide whether to recommend a separate series of JEMS for applied mathematics and there had been a loss of momentum in its activities.

Mina Teicher was the new chair of the Education Committee. She would do some work on the Pisa report and work closely with the education committees of national societies.

Vagn Lundsgaard Hansen talked about the activities of the Committee for Raising Public Awareness and the necessity for mathematicians to engage with politicians, journalists and the general public. This stimulated several suggestions and examples from delegates, such as the forthcoming public activities in Copenhagen during the ICME meeting, links on EMIS and a website for teachers (like that of the DMV).

There was a report from the Special Events Committee and from the Eastern Europe committee, whose principal function remains the allocating of travel grants.

Luc Lemaire reported on the success of the General Meetings Committee in attracting a grant of half a million euros for EMS Summer Schools. A new round of applications was due. The Mathematical Weekends had started with a successful meeting in Lisbon in 2003, with another due in Prague this September and a further one planned for

Delegates and members of the EC during a break

Barcelona in 2005. On the other hand, there had been no suggestions received for EMS lecturers.

A short report on the state of the Bologna process was followed by a stimulating, and occasionally agonised, discussion. The President summed up by saying that the Society should be willing to back the mathematical community in particular countries rather than formulating a general policy.

Thomas Hintermann, managing director of the EMS Publishing House, spoke of its progress. From this year, EMSph was publishing *JEMS, Interfaces and Free Boundaries, Oberwolfach Reports* and, from 2005, *Commentarii Helveticae* and *Elemente der Mathematik* would join the list. Book series had also started. A delegate suggested reduced prices for developing countries.

The Committee on Developing Countries continued to be very active. Herbert Fleischner said the books project was continuing, and the Committee was trying to increase the geographical spread of its work. It had a working fund with contributions from EMS and Zentralblatt-MATH. John Ball spoke of the work the IMU was doing.

Helge Holden talked about the Digital Mathematical Library project. This was being pursued at the national level; the EMS might have a co-ordinating role, but it could not fill it without additional volunteers. John Ball spoke of the importance with which the IMU regards this project, which was moving well.

Council unanimously agreed to hold the fifth European Congress of Mathematics in Amsterdam in July 2008.

The President spoke of the importance of having a viable Zentralblatt-MATH, as a counterweight to Mathematical Reviews-MathSciNet. There were currently some difficulties, in that the financial base (mainly German and French) needed to be broadened and that the project rested heavily on the shoulders of too few people. There was a need for a scientific advisory board, not to manage, but to provide scientific input. An EU funding application (two had failed) should be a secondary activity to support a strategy, but should not be allowed to distort or dominate. Laurent Guillopé spoke of the French government's review of Zentralblatt-MATH.

Reporting on the work of the Committee on Women and Mathematics, Emilia Mezzetti spoke of the limited success of the questionnaire, so that it was not possible to base action on it. But the Committee was collecting data and this did show some improvement in the position of women, particularly at starting levels. The Committee was also involved with mentoring schemes.

Council concluded with a discussion on the position of mathematics within European research programmes. The themed networks were not very good for mathematics. In the Marie Curie and other schemes, the success rate, though low, was fixed to be same in each subject. A delegate spoke passionately about the time-consuming and demeaning bureaucratic hurdles encountered if an application was successful. Though a report had come out in favour of a European Research Council with significant funding, the political will did not appear to be there; nor did this appear to be a high priority for the accession countries.

In his closing remarks, the President appealed to delegates to bore people about the importance of mathematics, particularly the development of abstract mathematics as 'tomorrow's tools'.

Council agreed that the search for a new EMS-president should begin now. The terms of office is four years starting with January 2007. Members of the Society are invited to send suggestions to President Sir John Kingman at emspresident@newton.cam.ac.uk

The Newsletter's Editorial Board held a meeting at Uppsala, too. From left to right: Martin Raussen, Steen Markvorsen, Krzysztof Ciesielski, Robin Wilson.

CALL FOR PROPOSALS

EMS SUMMER SCHOOLS IN FUNDAMENTAL AND INTERDISCIPLINARY MATHEMATICS

Following the success of its first E.U. application (which allows the funding of eight Summer Schools or Conferences in 2004-2005, see the EMS agenda page 2 or *http://www.emis.de/etc/ems-summerschools.html*), the European Mathematical Society is now launching a new call for proposals for such Schools and Conferences for 2006, 2007 and 2008. The deadline for this call is **January 12, 2005**, by e-mail to the address: *llemaire@ ulb.ac.be*.

The deadline will allow the EMS to present a coherent proposal of activities for EU funding, thereby allowing organisers of single meetings to be part of a series of events. EMS direct support being limited, the result of this application will make a major difference to the funding for the meetings selected by EMS. There will be similar calls every two or three years in the future.

This call for proposals concerns all Summer schools or Conferences that any group of mathematicians – pure or applied - would like to run in 2006, 07 or 08 in the EU or associated states.

The EU guidelines for these events must be followed strictly in order to have a reasonable hope for funding (the success rate for EU-applications being extremely low).

Thus, there must be a very strong component of training of young researchers (in the first 10 years of their career) by means of integrated courses and lectures at an advanced level. These can be supplemented by conference type research lectures, but the training component is needed.

Each course must aim at an international audience (no more than 30% of participants should come from a single state; otherwise the event in not eligible for funding).

The EMS will make a selection amongst the proposals received, taking into consideration the criteria applied by the EU, i.e.:

- Scientific quality of the project (high quality, cutting edge subject, speakers of high calibre…)
- Quality of the research training activity (training provided to the young researchers, level of interaction between the main speakers and the young ones…)
- Quality of the host (quality of infrastructure, experience in running conferences, contact making opportunities…)
- Management and feasibility (organisation and management, plans for publicity, dissemination of the results by publications, web sites, any other means…)
- Relevance to the objectives of the EU action and added value to the EU (relevance of the themes in relation with European interests and achievements, measures taken in favour of gender balance, use for integration of less developed regions, plans to favour public understanding of science…).

Concerning the last two criteria, part of the value comes from the EMS organisation, and this will be added in the final application. However, any little point will count, and each organiser can help by stressing qualities.

Proposers are asked to present a project as detailed as possible, with (provisional!) lists of speakers, subjects, and addressing all questions above. They should also include an estimate of the number of eligible participants for each of the three categories (EU or associated states and less than 4 years research experience; less than 10 years; no time limit but a researcher from EU or associated states living outside that zone). For these, no financial estimates should be given. An estimate of "organisational" expenses is also needed, including all expenses of key speakers which are non eligible.

The proposers should be aware of the rules of funding by the EU: the travel and living expenses of all eligible researchers can be covered, up to a rather generous maximum determined by the EU. If a fee is charged to all participants, the EU will reimburse the fees of eligible ones but then subtract this amount from the expenses below. All other expenses, including expenses of non-eligible key speakers, will be covered at a proportion equal to the ratio eligible participants / all participants.

The EU will pay a maximum of 80 % in advance, the last 20 % only when all reports are in, and possibly a year later.

Thus a statement from an organisation (university, conference centre…) must accompany proposals, agreeing that it will cover the complement of organisational expenses, and also be ready to advance 20 % of the funding until payment from the EU has arrived.

To give an idea, for the eight schools in the present project, we had a total of 140 participants of category 1, 155 of category 2, 20 of category 3. Their funding amounts to 366 000 euros, the organisational expenses of each event varies between 12000 and 17 000 euros, of which the EU covers the prescribed percentage.

Note that by a new rule, EU will not fund series of events in a single specific subject. In case that several proposals submit a particular theme, the EMS will have to make a choice or have to determine if a merger between these proposals makes sense.

Luc LEMAIRE
Phone: 00-32-2-6505837
e-mail: *llemaire@ulb.ac.be*

EURYI Call for Proposals

European Young Investigators Awards

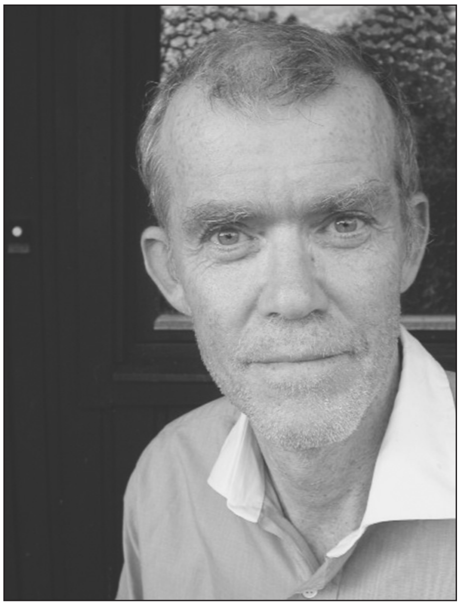
The European Heads of Research Councils (EUROHORCS) in collaboration with the European Science Foundation (ESF) have created the European Young Investigator (EURYI) Awards, launched for the first time in 2003. Awards will last for a 5 year period with a total value normally no less than 750 kEuro. In July 2004, 25 awards were granted after the first call.

Young researchers from anywhere in the world with between 2 and 10 years postdoctoral experience – taking into account career breaks – are eligible. The selection criteria for EURYI Awards are the research quality and potential of the applicant and the originality as well as the groundbreaking character of the research proposal and its feasibility.

Applications must be submitted through a national EUROHORCS organisation no later than November 30, 2004. They will then be assessed first at the national level and later at the European level. Awards are expected to be announced at the end of July 2005. Detailed information can be obtained from *www.esf.org/euryi* .

Stockholm 2004
The Fourth European Congress of Mathematics

Ulf Persson (Göteborg, Sweden)



How it came about

In spite of a difficult start, the fourth European Congress in Mathematics (ECM) took place after all. As its President Ari Laptev revealed in an earlier issue of this newsletter, its eventual location in Stockholm was not initially planned but was the result of a crisis perceived in the summer of 2000. Despite what many outsiders may have assumed, finding sufficient funding from Swedish sources was not a straightforward matter (as the first bid made painfully clear). It was only due to the daring and 'savoire faire' of Ari Laptev and his colleague at Kungliga Tekniska Högskolan (KTH), Anders Lindquist, that sufficiently promising leads were extracted from some of the major Swedish funding institutions, thus enabling the Swedish Mathematical Society to send a small delegation to the executive meeting in London later that fall for negotiations. However, a final commitment to hold the congress was not made until just a week before Christmas - after a rather tense encounter with the then President of the European Mathematical Society (EMS), Rolf Jeltsch. The meeting only came to a desired conclusion after a final short interview with the President of the KTH, Anders Flodström, in which the latter agreed to the necessary financial underwriting. The upshot was that the planning and organization of the congress was shortened by at least a year from what has been customary for this kind of event.

Why big congresses? Was it worth it?

This is obviously not the place to delve deeper into that philosophical question, yet a few reflections may not be amiss. In the good old days, which meant up into the fifties, conferences were few and small, and in particular the International Congresses of Mathematicians (ICM) were rather select and as a consequence not bigger than allowing group portraits to be taken. But when the ICM was held in Stockholm back in 1962, it was the largest scientific congress that had ever been held in Sweden. Since then the ICM have turned from rather exclusive meetings to large affairs attended by the 'mathematical masses'. This fact, together with the recent acceleration in the number of specialized conferences, often referred to as workshops to emphasize their focused and businesslike character, has sown doubts in the minds of many as to the scientific relevance of such gargantuan meetings, which, cynics protest, are more the occasion for tourism than serious scientific interchange. In such a perspective, the establishment of a European Congress may seem redundant. One may point out though that the situation of mathematics is not unique, but is shared by most academic disciplines, and that it is very important, among other things, to counter the fragmentation of a discipline with opportunities for the contemplation of it as a whole. For this to be fully successful there is both a need for the lectures to be directed to a general mathematical audience and for that audience to seek out lectures not primarily within their own speciality. Popular surveys do not rank highly among the priorities of research mathematicians. They are notoriously difficult to do and the rewards are marginal. A scientific committee selects their choices primarily on the basis of scientific excellence and topicality of the subject matter, not on expository skills; and notwithstanding the desirability of the latter this is basically a sound principle, tampering with which would court disaster and seriously undercut the legitimacy of the whole enterprise. Thus selection as a speaker is seen primarily as recognition of scientific merit and not as an opportunity, as well as an obligation, to communicate effectively.

What is needed is a change of culture, something that cannot be decreed but has to evolve slowly. On what constitutes a good lecture, one can of course argue, there obviously being no specific rules on which everyone can agree. And besides, in all kinds of human interaction rules are simply there to be broken. It suffices to point out that a scientific lecture, and especially a mathematical one, is not necessarily made more accessible by the simple process of 'watering down' (i.e. removing technicalities and making unwarranted simplifications). What is required of a lecture is the conveying of an idea (usually one is enough) and some motivation (which should not be confused with justification often of the type: 'this has applications to physics') and placing the subject matter in a wider mathematical context. Apart from that, a good lecture can contain highly technical material, and there is no reason why the audience should understand most of it as long as they have gotten something to take home with them. It would be very dangerous if mathematicians were to abandon their tradition of honesty and aim for merely the 'flashy'. To reflect on why your work is interesting and to try to truly motivate it, should not be seen as a concession to ignorance, but rather as an additional source of inspiration for your own research.

New features

Obviously this is not the place to comment upon how successfully the various speakers performed their task of communicating to a general mathematical audience. Fortunately, although few lectures can be expected to please everyone, the lecture is rare indeed that does not bring rewards to at least a token number in the audience, and one may argue that this is indeed all that is needed to make it worthwhile. However, there was a feeling among the organizers that something extra was needed to justify yet another general conference in mathematics, as well as serving as a rejuvenation of the concept. Two new features were added. One was to invite scientists in natural science, not only in chemistry and physics, but also mathematical biologists, to give lectures. The other was to latch on to the pre-existing structure of the European Networks and thus give to the ECM a natural European anchoring. The first may be seen as somewhat controversial, wedding mathematics and its ultimate justification too tightly to applications. Yet, whether we like it or not, practical mathematical applications are what brings in material resources, and for public relations the importance of the willingness of mathematicians to interact externally, and in the process maybe also acquiring greater public visibility, should not be underestimated. From a less opportunistic standpoint, applications should not merely be seen as necessary justifications, but also as sources of inspiration. The second new feature may hopefully ensure that the new tradition of ECM's continues. The funding and organization of a big international congress is indeed a major undertaking and the difficulties involved may be expected to increase with time rather than decrease. The European Union has already invested a formidable apparatus of networks replete with their own special conferences. What would be more natural than a unifying one, which should bring with it a large body of active participants, and hopefully also channels for necessary funding? At Stockholm, admittedly the network lectures played a rather marginal role, but if the idea is accepted and developed in future ECM's they may provide the core activity onto which various extras may be attached as embellishments.

Aula Magna

After those general preambles it may be appropriate to describe the Fourth European Congress of Mathematicians as an actual event tied to a physical location at a given slot in time. The basic problem of organizing a conference for one thousand odd expected participants is to find a lecture hall big enough. Unlike the case of the Olympics, the erection of new buildings is not an option. Of course big conferences are legion these days, but commercially available space often comes with price-tags not within the capabilities of mathematical meetings. The Royal Swedish Institute of Technology (Kungliga Tekniska Högskolan - KTH) was not able to provide such a hall on its premises, making a direct collaboration with the University of Stockholm a necessity on this basis alone. The fact that there are two separate departments of mathematics in Stockholm has incidentally been a bone of contention for at least fifty years, and may continue to be so for another fifty years, but this is of course only a matter of local interest. The Aula Magna is the official grand lecture hall of the University of Stockholm, and has of course no direct connection, physical or not, to its department of mathematics, nor to that of the KTH, being about equally (physically) distant from both. It is of fairly recent vintage, located on the main campus of the University of Stockholm, easily accessible by the Stockholm Underground ('T-banan', 'T' as in tunnel). Shaped like an amphitheatre, with options of subdivision, the Aula actually boasts a capacity well in excess of the number of actual participants (around 900), which resulted in the somewhat unfortunate impression of not only the lectures but also the opening ceremony being sparsely attended. As expected, the Aula along with its adjacent hallways became the locus of the meeting. Walking along its perimeter you had immediate access to the young assistants donning yellow T-shirts, as well as to the various bookstalls providing not only opportunities for browsing but also seducing discounts. The Springer stall also supplied piles of copies of the Stockholm Intelligencer (still smelling of fresh print), featuring short articles on Sweden and mathematics, including a short but morbid list of distinguished mathematicians who died here. Climbing a few stairs you could also inspect the various poster-sessions. Additional smaller lecture halls were available at the proverbial stone-throws distance, as well as what in recent years has become an absolute necessity for wayward mathematicians - access to e-mail. A fairly spacious room, accessible only by pressing a code at the entrance, was filled with a sufficient number of computers to keep waiting lines rather than tempers short. Furthermore, a small staff of knowledgeable yellow-shirts was always at hand during opening hours. For those able to resist the allure of the screen at lunchtime, there was the temptation of the official cafeteria situated halfway between the two locations (actually accessible from the Aula Magna remaining indoors the entire way, a godsend in the case of inclement weather). It provided the standard Swedish lunch fare to be expected from that kind of self-serve establishment, confidently assured of a captive audience, not only by its isolated existence but also due to pre-paid lunch coupons.

4ecm staff studying new books at the booth of the EMS publishing house

The opening ceremony

The conference got a head start on Sunday afternoon, on June 27, by providing registration outside the Aula. This involved getting a handy black briefcase, doubling as a rucksack, with the logo of the 4ECM sufficiently discreet to encourage post-conference use. It would be tedious to list its contents of 'goodies', but I am sure that most people appreciated the free public transport passes intended to cover the entire period. One of its more trivial items was a coupon to be exchanged for a single glass of wine (of optional colour) to be served at the hour-long reception starting at six o'clock. The next day, Monday June 28, involved the official opening of the meeting at nine thirty in the morning, preceded by an opportunity for last-minute registrations. Back at the ICM of 1962, the old Swedish King Gustavus VI attended, giving out the Fields Medals. Such a spectacle of royalty at a mathematical meeting was, alas, not repeated this time. The presence of the Swedish Majesty back then has been attributed to the above mentioned fact that at the time, it was the largest scientific meeting ever to have taken place in Sweden. No such distinction could be given to the 4ECM, so neither the old King's grandson, the present King Charles XVI, nor the great-granddaughter, the crown princess Victoria, were able to squeeze the events into their busy schedules. (One should not overestimate the love the general public, including that of royalty, feels for mathematics) Instead two academic notables, Bremer and Franke, provided the required official lustre. The latter, Sigbrit Franke, the chancellor of the university system in Sweden, also served the function of awarding the EMS prizes to the young mathematicians, to which we will return shortly. Both Bremer and Franke, not surprisingly, made a special point in referring to Sofia Kovalevskaya in their short speeches. Kovalevskaya, as can have escaped the notice of few mathematicians, was the first ever woman professor in mathematics, holding her position at the precursor of Stockholm University at the end of the 19th century, and ever since then being a mathematical role model for half the population of the world. Kingman, in his capacity as the President of the EMS, gave the mathematical speech with commendable aplomb, stressing the importance of mathematics, and urging young mathematicians to go for the hard problems, irrespective of whether pure or applied, because you can never tell. He also encouraged the new generation by reminding them that they are as great and imaginative as the great ones of the past, because what puzzled our ancestors does not necessarily puzzle us. The chair of the Prize Committee, Nina Uraltseva from St. Petersburg, who complained that she was a bit too short for the microphone, explained that the work of the prize committee had been very hard. There were fifty nominees, out of which ten had to be selected. To those who did not make it, there is only one thing to say - work harder and try to outdo those who were selected. This is not the place to give a list of the winners, along with short descriptions of their accomplishments. It suffices to add that in addition to the traditional EMS prizes, a so called BIT prize was awarded for work in numerical analysis. No official ceremony is complete without some kind of entertainment. The musical interlude in this case was provided by a small Swedish ensemble specializing in Elizabethan music but also performing, fittingly, some classical Swedish songs. Dressed in period costumes, they did their thing, singing with their own instrumental support, concluding their act by strutting around the stage pilfering the belongings of those unfortunate enough to have the privilege of sitting on the first row.

The week in review

The conference was kicked off by the first plenary lecturer, Oded Shramm, associated with Microsoft Research. An appropriate beginning in view of the fact that President Laptev, earlier in the ceremony, got stuck on his power-point presentation, cursing modern computer technology. Schramm, however, did not address such wordly issues of practicality, but expounded on conformally invariant random processes instead, albeit with many a computer visualization. And then there was time for lunch, and in spite of the customary denial of the existence of such entities, free to all participants. The afternoon was devoted to parallel sessions, four in fact, involving twelve lectures in total. The day was capped off by an EMS reception at the old location for the department of mathematics at KTH, a building commonly referred to as Sing-sing, due to its intimidating lay-out. The reception wisely took place on the ground floor, the limited space of which quickly got extremely crowded. One surmises that afterwards there were only empty wine-bottles among the left-overs. The next day started out with presentations of the prize winners and their work (it should be noted that some of them had also, independently one assumes, been invited as regular speakers as well.) Those were followed by invited lectures and then in the afternoon there were three Science lectures. The last one was that of R.Ernst, a Nobel Prize winner in Chemistry, giving a survey from Fourier to Medical imaging, constituting no doubt a very instructive lecture to the mathematicians, giving them, among other things, a sense of the somewhat alien culture of big science. Tuesday was capped off by a visit to the Town Hall of Stockholm replete with a buffet courtesy of the city of Stockholm. The Town Hall, designed by the Swedish architect Ragnar Östberg and built in the twenties, is one of the most commonly pictorially reproduced landmarks of the city, located at the edge of an island, and commanding a presence on the main waterway. Its design was inspired by the palace of the dodges in Venice and its main feature is the great banquet hall inside, somewhat puzzlingly known as the Blue Hall (guides of the building are just delighted to explain to you the historical reason why), which every December is host to the Nobel banquet following the prize awards at the Concert Hall. This time however, the setting was somewhat less sumptuous. Two grand buffets (presumably identical) were displayed on two long tables on one side. Food was plentiful, but to savour it you needed to do the customary juggle of balancing your wineglass, your plate, and your knife and fork with just two hands, while standing firmly on your feet and trying to make coherent conversation. Afterwards Kingman rose to the occasion thanking the hosts referring to the great success of the conference (so far). He concluded by delivering a splendid celebration of the importance of mathematics, reminding everyone that while back in 1900, some of the scientific accomplishments honoured at this very place involved no mathematics, nowadays this would be almost impossible, and that all scientists should acknowledge this fact. In fact, he reminded us all that the entire human race will benefit from the development of mathematics. In order to gently usher out the mathematical crowd from the premises, a guided tour was offered at the end providing a natural conclusion. For those of us who afterwards lingered on by the waterfront, we may late forget the glorious view made sublime (as one used to say) by the slowly setting sun. The traditional association of Stockholm with Venice, made particularly palpable by the Town Hall, is not plucked out of thin air, but rests on water. Wednesday was taken over by plenary talks in the morning and three additional science lectures in the afternoon. The Austrian Nowak expounded on mathematical biology with special emphasis on evolutionary processes, and Berry presented a cascade of computer generated pictures relating to optics and concomitant singularities. Thursday was only half a day, with invited lectures in the morning and scheduled excursions in the afternoon. In the evening, the French ambassador gave a reception for the notables of the EMS, the organizers and last but not least the young prize-winners. France as a country and culture should be commended for the official respect it accords mathematics and for always recognising mathematical achievement. I fear that the 4ECM may very well have received more publicity in France than in Sweden. Friday was the closing up, with parallel network lectures in the morning and a series of plenary talks in the afternoon, the last fittingly delivered by a local speaker, Johan Håstad at KTH, talking on the difficulty of proving the generally believed NP≠P.

Wrapping up

The concluding ceremony was, as such things tend to be, rather short. Kingman expressed the pride of the EMS to be associated with the ECM and thanked the organizers, and especially its President. There was a lot of applause. Then there was a reference to the upcoming centenary birthday of Henri Cartan, who had an honorary role in the first ECM which took place in Paris 1992, and to whom the entire congress relayed its congratulations. Laptev then referred to the statistics of the event, with the final tally of 930 members, over three hundred posters, and sixty three scheduled talks, of which only one had to be cancelled. As expected he could not refrain from congratulating himself on having arranged such nice weather for the entire duration. No mean feet indeed, and a definite contribution to the general pleasure. Finally the congress was treated to an invitation to the fifth ECM to take place in Amsterdam in 2008. After the usual problems with an unresponsive machine, the audience was eventually exposed to a power-point show by the representative of the next ECM, promising future delights. By the time the audience emerged out of the Aula Magna for the last time, bookstalls were being dismantled, posters taken down, and the last remaining conference briefcases sold off at very reasonable prices. Clearly within hours, no physical traces would be left of the congress. On the other hand, one would hope that mental traces, preferably of the positive and edifying kind, will remain for a very long time.

Ulf Persson [ulfp@math.chalmers.se] has been professor at Chalmers University of Technology in Gothenburg (Göteborg) since 1989. He earned his Ph.D at Harvard 1975 as a student of David Mumford. Works in Algebraic Geometry, especially compact complex surfaces. Hobbies include mathematical pictures programmed directly in PostScript. Has served as the President of the Swedish Mathematical Society and has been actively engaged in public debate on mathematical education. Publishes regularly reviews on scientific-philosophical matters in the Swedish press and is the editor of the newsletter of the aforementioned Swedish Mathematical Society.

Starting the race towards 5ecm at Amsterdam in 2008. From left to right: H.J.J. te Riele, J.O.O. Wiegerinck, C.M. Ran (http://sta.science.uva.nl/ brandts/5ECM/)

EMS Prizes

4ecm STOCKHOLM 2004

The EMS prizes are awarded by the European Mathematical Society in recognition of distinguished contributions in Mathematics by young researchers not older than 35 years. The prizes are presented every four years at the European Congresses of Mathematics.

The Prize Committee is appointed by the EMS and consists of a number of recognized mathematicians from a wide variety of fields. The prizes were first awarded in Paris in 1992 and then in Budapest in 1996 and in Barcelona in 2000.

Each prize winner 2004 received 5,000 Euro.

Prize committee

Enrico Arbarello, - Rome
Victor Buchstaber, - Moscow
John Coates,- Cambridge, UK
Jacek Graczyk, - Orsay
Bertil Gustafsson, - Uppsala
Stefan Hildebrandt, - Bonn
Jean-François Le Gall, - Paris
Vladimir Lin, - Haifa
Leonid Polterovich, - Tel Aviv
Domokos Szasz, - Budapest
Dimitri Yafaev, - Rennes
Eduard Zehnder, - Zürich

EMS Prize winners 2004 with the president of the Prize committee, Nina Uraltseva. Sitting from the left: Xavier Tolsa, Paul Biràn, Sylvia Serfaty, Stefano Bianchini, Otmar Venjakob. Standing from the left: Franck Barthe, Warwick Tucker, Nina Uraltseva (President of the Prize committee), Elon Lindenstrauss, Andrei Oukonkov, Stanislav Smirnov.

EMS Prizes 2004 with citations

Franck Barthe, *Institut de Mathématiques: Laboratoire de Statistique et Probabilités, Toulouse, France*
Barthe pioneered the use of measure-transportation techniques (due to Kantorovich, Brenier, Caffarelli, Mc Cann and others) in geometric inequalities of harmonic and functional analysis with striking applications to geometry of convex bodies. His major achievement is an inverse form of classical Brascamp-Lieb inequalities. Further contributions include discovery of a functional form of isoperimetric inequalities and a recent solution (with Artstein, Ball and Naor) of a long-standing Shannon's problem on entropy production in random systems.

Stefano Bianchini, *Instituto per le Applicazioni del Calcolo "M. Picone", Rome, Italy*
Stefano Bianchini has introduced an entirely new perspective to the theory of discontinuous solutions of one-dimensional hyperbolic conservation laws, representing solutions as local superposition of travelling waves and introducing innovative Glimm functionals. His ideas have led to the solution of the long standing problem of stability and convergence of vanishing viscosity approximations. In his best individual achievement, published in 2003 in Arch.Ration. Mech. Anal., he shows convergence of semidiscrete upwind schemes for general hyperbolic systems. In the technically demanding proof the travelling waves are constructed as solutions of a functional equation, appling center manifold theory in an infinite dimensional space.

Paul Biràn, *School of Mathematical Sciences, Tel-Aviv University, Israel*
Paul Biràn has made fundamental and influential contributions to symplectic topology as well as to algebraic geometry and Hamiltonian systems. His work is characterised by new depths in the interactions between complex algebraic geometry and symplectic topology. One of the earlier contributions is his surprising solution of the symplectic packing problem, completing work of Gromov, McDuff and Polterovich, showing that

compact symplectic manifolds can be packed by symplectic images of equally sized Euclidean balls without wasting volume if the number of balls is not too small. Among the corollaries of his proof, Biràn obtains new estimates in the Nagata problem. A powerful tool in symplectic topology is Biràn's decomposition of symplectic manifolds into a disc bundle over a symplectic submanifold and a Lagrangian skeleton. Applications of this discovery range from the phenomenon of Lagrangian barriers to surprising novel results on topology of Lagrangian submanifolds. Paul Biràn not only proves deep results, he also discovers new phenomena and invents powerful techniques important for the future development of the field of symplectic geometry.

Elon Lindenstrauss, *Clay Mathematics Institute, Massachusetts and Courant Institute of Mathematical Sciences, New York, USA*
Elon Lindestrauss has done deep and highly original work at the interface of ergodic theory and number theory. Although he has worked widely in ergodic theory, his recent proof of the quantum unique ergodicity conjecture for arithmetic hyperbolic surfaces breaks fertile new ground, with great promise for future applications to number theory.

Already, in joint work with Katok and Einsiedler, he has used some of the ideas in this work to prove the celebrated conjecture of Littlewood on simultaneous diophantine approximation for all pairs of real numbers lying outside a set of Hausdorff dimension zero. This goes far beyond what was known earlier about Littlewood's conjecture, and spectacularly confirms the high promise of themethods of ergodic theory in studying previously intractable problems of diophantine approximation.

Andrei Okounkov, *Princeton University, USA*
Andrei Okounkov contributed greatly to the field of asymptotic combinatorics. An extremely versatile mathematician, he found a wide array of applications of his methods. His early results include a proof of a conjecture of Olshanski on the representations theory of groups with infinite-dimensional duals. Okounkov gave the first proof of the celebrated Baik-Deift-Johansson conjecture, which states that the asymptotics of random partitions distributed according to the Plancherel measure coincides with that of the eigenvalues of large Hermitian matrices. An important and influential result of Okounkov is a formula he found in joint work with Borodin, which expresses a general Toeplitz determinant as the Fredholm determinant of the product of two associated Hankel operators. The new techniques of working with random partitions invented and successfully developed by Okounkov lead to a striking array of applications in a wide variety of fields: topology of module spaces, ergodic theory, the theory of random surfaces and algebraic geometry.

Sylvia Serfaty, *Courant Institute of Mathematical Sciences, New York, USA*
Sylvia Serfaty was the first to make a systematic and impressive asymptotic analysis for the case of large parameters in Theory of Ginzburg-Landau equation. She established precisely the values of the first, second and third (with E.Sandier) critical fields for nucleation of one stable vortex, vortex fluids and surface superconductivity. In micromagnetics, her work with F. Alouges and T. Rivière breaks new ground on singularly perturbed variational problems and provides the first explanation for the internal structure of cross-tie walls.

Stanislav Smirnov, *KTH, Sweden and Geneva University, Switzerland*
Stanislav Smirnov's most striking result is the proof of existence and conformal invariance of the scaling limit of crossing probabilities for critical percolation on the triangular lattice. This gives a formula for the limiting values of crossing probabilities, breakthrough in the field, which has allowed for the verification of many conjectures of physicists, concerning power laws and critical values of exponents. Stanislav Smirnov also made several essential contributions to complex dynamics, around the geometry of Julia sets and the thermodynamic formalism.

Xavier Tolsa, *ICREA and Universitat Autònoma de Barcelona, Spain*
Xavier Tolsa has made fundamental contributions to Harmonic and Complex Analysis. His most outstanding work solves Vitushkin's problem about semi-additivity of analytic capacity. The problem was raised in 1967 by Vitushkin in his famous paper on rational approximation in the plane. Tolsa's result has important consequences for a classical (100 years old) problem of Painlevé about a geometric characterization of planar compact sets are removable in the class of bounded analytic functions. Answering affirmatively Melnikov's conjecture, Tolsa provides a solution of the Painlevé problem in terms of the Menger curvature. Xavier Tolsa has also published many important and influential resultsrelated to Calderón-Zygmund theory and rational approximation in the plane.

Warwick Tucker, *Uppsala University, Sweden*
Warwick Tucker has given a rigorous proof that the Lorenz attractor exists for the parameter values provided by Lorenz. This was a long standing challenge to the dynamical system community, and was included by Smale in his list of problems for the new millennium. The proof uses computer estimates with rigorous bounds based on higher dimensional interval arithmetics. In later work, Warwick Tucker has made further significant contributions to the development and application of this area.

Otmar Venjakob, *Mathematisches Institut: Universität Heidelberg, Germany*
Otmar Venjakob has made a number of important discoveries in both the algebraic and arithmetic aspects of non-commutative Iwasawa theory, especially on problems which appeared intractable from the point of view of the classical commutative theory. In arithmetic geometry, Iwasawa theory is the only general technique known for studying the mysterious relations between exact arithmetic formulae and special values of L-functions, as typified by the conjecture of Birch and Swinnerton-Dyer. Venjakob's work applies quite generally to towers of number fields whose Galois group is an arbitrary compact p-adic Lie group (which is not, in general, commutative), and has done much to show that a rich theory is waiting to be developed. His most important results include the proof of a good dimension theory for modules over Iwasawa algebras, and the proof of the first case of a structure theory for modules over these algebras. With Hachimori he discovered the first examples of arithmetic Iwasawa modules which are completely faithful, as well as proving a remarkable asymptotic upper bound for the rank of the Mordell Weil group of elliptic curves in certain towers of number fields over Q whose Galois group is a p-adic Lie group of dimension 2. Very recently, he found the key to the problem of defining, in non-commutative Iwasawa theory, the analogue of the characteristic series of modules over Iwasawa algebras.

Visions of Young Mathematics in Europe

Jon Larsson and Mikael Johansson (Stockholm, Sweden)


Jon Larsson


Mikael Johansson

During the week immediately prior to the European Congress of Mathematics, one of the congress's many satellites took place at the Royal Institute of Technology (KTH) in Stockholm. This particular one differed slightly from most satellite events in that the participants of this conference were aged 14-20 and not yet active research mathematicians.

The Junior Mathematical Congress

The first Junior Mathematical Congress was held in Paris, France, in conjunction with the first ECM. Several hundred young secondary school students, mainly from France but also from several other European countries, gathered and listened to seminars from participants and from invited lecturers. As the time came for the second ECM in Budapest, the JMC was this time held in Miskolc (Northeast Hungary), again with a large international contingent present.

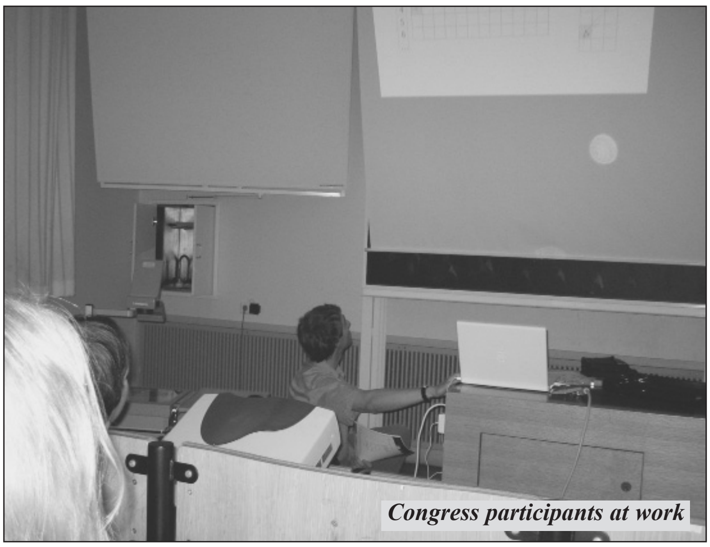
After the Miskolc event, some of the organisers realised that a periodicity of four years is much too sparse for the intended age group, as a student would be able to both start and finish secondary studies - or for that matter higher studies – between congresses and without any contact with the congress activities. Thus, the third JMC was held in Potsdam, in conjunction with the ICM in Berlin. The fourth was supposed to be held in Spain, together with the Barcelona ECM, but the intensity of arranging different activities was too high and thus in 2000, the JMC was held in Miskolc. 2002 again saw a small JMC in Miskolc, this time with mostly Hungarian participants.

In Sweden, home of the authors of this article, the JMC was unheard of until 1998, when Mikael got involved in the preparations for the Potsdam congress during an exchange year in Germany. Two years later, the two of us both attended the fourth JMC in Miskolc. Thrilled and excited after the experience, we were determined to seek out similar events back in Sweden, or - if need be - bring them there.

Stockholm 2004

When news that the 4ECM was to be held in Stockholm reached us three years ago, it was clear that the JMC of 2004 must also take place here - on our home turf. With the help and guidance mainly of Dr Péter Körtesi of the University of Miskolc, and of the Swedish National Centre for Mathematics Education, the Stockholm district of the Swedish Federation of Young Scientists through us started the preparations for the congress.

By the end of June 2004, after having raised EUR 30,000 in funding and gathered 40 participants from seven European countries, it was finally time. A staff of twelve, all just a little older than the participants, got everyone through the week-


Congress participants at work


... and at dinner

long event which, according to the questionnaire at the end, was a great success.

The sixth JMC was indeed a success, but it was also a lot of hard work, uncertainty and confusion. During the course of preparations and the congress itself, we experienced in a very profound way the lack of continuity, established cooperation and well-functioning communication and information networks in the young mathematics movements in Europe. It had been a difficult task at best to reach out to like-minded youngsters throughout the continent and inform them about the JMC's existence and the fact that we wanted them to attend it. Something needed to be done.

Founding a Mathematical Society

In 2000, Péter Körtesi had already formulated the idea of cooperation across national borders on a more general level, and as the work progressed, this idea matured in our discussions. A European Junior Mathematical Society would be well fitted to supply the much needed continuity between congresses by acting as formal principal of the JMC's, keeping records from earlier events and facilitating personal contact between organisers in different countries. On a more general level, the EJMS can act as a common forum and a coordinating body for several national or regional organisations within the field of young mathematics.

The EJMS would not only be an asset for congress organisers and assorted NGO's, but also, perhaps even more importantly, for the individual young mathematicians on the continent. For the teenager interested in mathematics, two of the main problems are often finding others who share their interest, and finding further mathematical stimulation and guidance. These are both problems that could hopefully be alleviated by the existence of a Pan-European network of young mathematicians. That network would also have a natural place in the greater quest to further and increase interest in mathematics among young Europeans in general.

These discussions were brought to a larger audience at the JMC this summer. Many good ideas were brought forth, by youngsters and adults alike. The culmination was reached during the closing ceremony of the congress, when the participants - young mathematicians from seven countries - unanimously declared that it was the wish of the sixth JMC that a European Junior Mathematical Society be founded as soon as possible.

Here in Stockholm, the work has now begun. The formation of a Swedish society for young mathematicians is underway in close cooperation with the Swedish Federation of Young Scientists. With a national organisation in place, we will have a starting point from which we can gather the contacts, support and funding necessary to launch the EJMS.

In closing, we would like to make a call for support to you, the mathematicians of Europe. If you feel that you can help us in any way, be it by founding a Junior Mathematical Society in your country, providing us with funds or simply sharing valuable experience with us, we urge you to contact us by e-mail at info@jmc04.org. We believe the EJMS has great potential and hope that you will share that belief.

Jon Larsson [jon@jmc04.org] is a student of theoretical computer science at the Royal Institute of Technology (KTH) in Stockholm, Sweden, and Chairman of the Stockholm District of the Swedish Federation of Young Scientists.

Mikael Johansson [mikael@jmc04.org] has recently finished his Master thesis at the Department of Mathematics of Stockholm University, Sweden.

Both authors were members of the 4ECM organising committee.

SOME PERSONAL REFLECTIONS ON ICME-10

Vagn Lundsgaard Hansen (Lyngby, Denmark)

In the second week of July 2004, the big quadrennial event in mathematical education, or didactics of mathematics as we say in Europe, took place on the campus of the Technical University of Denmark situated at the outskirts of Copenhagen. We are talking about the "10th International Congress on Mathematical Education", abbreviated as ICME-10.

About 2300 researchers in mathematics, researchers in mathematics education and teachers from all levels in the educational system from primary school to university, discussed new developments in relation to quality in mathematical education. There were participants from 119 different countries. Registration took place on Sunday July 4 in the magnificent celebration hall of the University of Copenhagen, close to the hotels of the participants. The registration day was a beautiful sunny day. During the conference, participants enjoyed rich variations in the Danish weather! At registration, the participants received cards for the public transportation system in the Copenhagen area, which in the following days quickly and easily brought them to the site of the congress.

Researchers and Practitioners

The international congress on mathematical education is *the* event that brings together active research mathematicians, deeply devoted to develop and disseminate their subject, and practitioners, strongly devoted to create a well-functioning and rich teaching environment in mathematics in their countries of origin, under the guidance of researchers in mathematical education, who have contributed significantly to structuring thinking about problems related to the teaching and learning of mathematics. During the congress, I came to realise how important it is that researchers in mathematics and researchers in mathematics education do not loose contact with each other to the confusion of the practitioners in the classroom, who face concrete problems with specific topics from mathematics. More than ever, I now believe that the voices of both groups of researchers are necessary in the debate about the contents, the methods, the presentation and the assessment of mathematics in schools.

From the left; Morten Blomhøj (chair of the LOC), Elin Emborg (administrative secretary for LOC and IPC), Mogens Niss (chair of the IPC), all three IMFUFA, Roskilde University, Denmark and Gerd Brandell (chair of the Nordic Contact Committee for ICME-10), Lund University, Sweden.

Ceremonies and Awards

Many participants will remember the opening ceremony since two politicians made very good speeches and stayed for the whole ceremony. The Danish minister of education, Ulla Tørnæs, faced the problems for mathematics in schools without getting into general political polemic, and the Mayor of Lyngby Municipality, Rolf Aagaard-Svendsen, who holds a Ph.D. in mathematical statistics, made a witty speech where he even showed a page from his thesis. Other speeches were made by professor Hyman Bass, the president of ICMI (International Commission on Mathematical Instruction), professor Mogens Niss, the chair of the international programme committee (IPC), professor Morten Blomhøj, the chair of the local organizing committee (LOC), professor Christian Stubkjær, the dean of research at the Technical University of Denmark, and professor Bernard Hodgson, the general secretary of ICMI. The ceremony was further enlightened by a musical performance by Royal Danish Brass.

The opening ceremony also included the awarding of the first Felix Klein Medal to professor Guy Brousseau, France, for his lifetime contributions to didactics of mathematics, and the first Hans Freudenthal Medal to professor Celia Hoyles, England, for outstanding contributions to research in the domain of technology and mathematics education[1]. With the choices of the first recipients, a high

[1] The citations can be found in issue 52 of the Newsletter.

The president of the International Commission on Mathematical Instruction, Hyman Bass, presents the first Felix Klein Medal to Professor Guy Brousseau (on the right-hand side).

standard has been set for these awards, making them especially prestigious for research in mathematics education.

The Programme

And then we were ready for the conference. The programme was huge: 8 plenary activities, some 80 regular lectures, 29 topic study groups, 24 discussion groups and - as a new thing - a thematic afternoon with 5 choices. There were 5 national presentations during one full afternoon: Korea, Mexico, Romania, Russia and the Nordic countries (Denmark, Finland, Iceland, Norway, and Sweden). Each national presentation had a detailed and comprehensive programme. Obviously, most activities were in parallel sessions with many options. All in all, there were more than half a million possibilities for composing your own programme. To assist the individual planning, participants got an extra copy of the one page schedule, where they could fill in which activities they wanted to attend and where to find them.

For a programme of this size, no single participant can capture it all. But everyone felt the spirit and the energy and compassion with which the many contributors had prepared their contributions. In all fairness it is not possible to single out specific contributions in a general overview. But again you realise that the listeners and the eagerly engaged contributors to discussions are really what make up a good congress. I do think that ICME-10 was a success in that respect.

I attended a thematic afternoon on mathematics and mathematics education that caused a heated debate. Although the tone was not exactly polite, it was very direct and explicit, and it may actually lead to reflections on the various roles of researchers in mathematics and researchers in mathematics education and their possibilities for the shaping of good mathematics teaching in the classroom.

Circus Mathematicus

During the congress, a mathematical circus "Circus Mathematicus" was arranged at the congress site for the general public. The creativity shown was enormous, encompassing among other things: bowling with fractions, a labyrinth, life size puzzles, boomerangs, juggling, origami, wood carving, beautiful Christmas decorations, various rope tricks and construction of kites. The mathematical circus was a great success and helped attract the attention of the news media to the congress in general, including leading Danish newspapers, TV and radio. Also a very interesting exhibition developed in co-operation between ICMI and UNESCO, "Why mathematics", was shown during ICME-10.

Being one of the local organisers of the congress, I must be careful with appraisal. But I do feel that the congress was successful in fulfilling its aim, namely stimulating the continual process towards good teaching of mathematics. As professor Jeremy Kilpatrick assured us under a very interesting plenary interview session with four distinguished mathematical educators: "The strive for good teaching will never stop. There will always be new challenges."

The closing ceremony took place on Sunday, July 11, and the organisers were relieved after more than four interesting years of planning in collaboration with our extremely effective and helpful congress bureau: Congress Consultants.

ICME-11 takes place in Mexico in 2008. You should join it. An ICME congress is worth the effort.

Vagn Lundsgaard Hansen [V.L.Hansen@mat.dtu.dk] is a Professor of Mathematics at the Technical University of Denmark, Lyngby, Denmark. He serves as chairman of the EMS-committee on Raising Public Awareness of Mathematics.

Children and congress participants engaged in mathematical activities in Circus Mathematicus.

25 years proof of the Kneser conjecture
The advent of topological combinatorics

Mark de Longueville (Berlin)

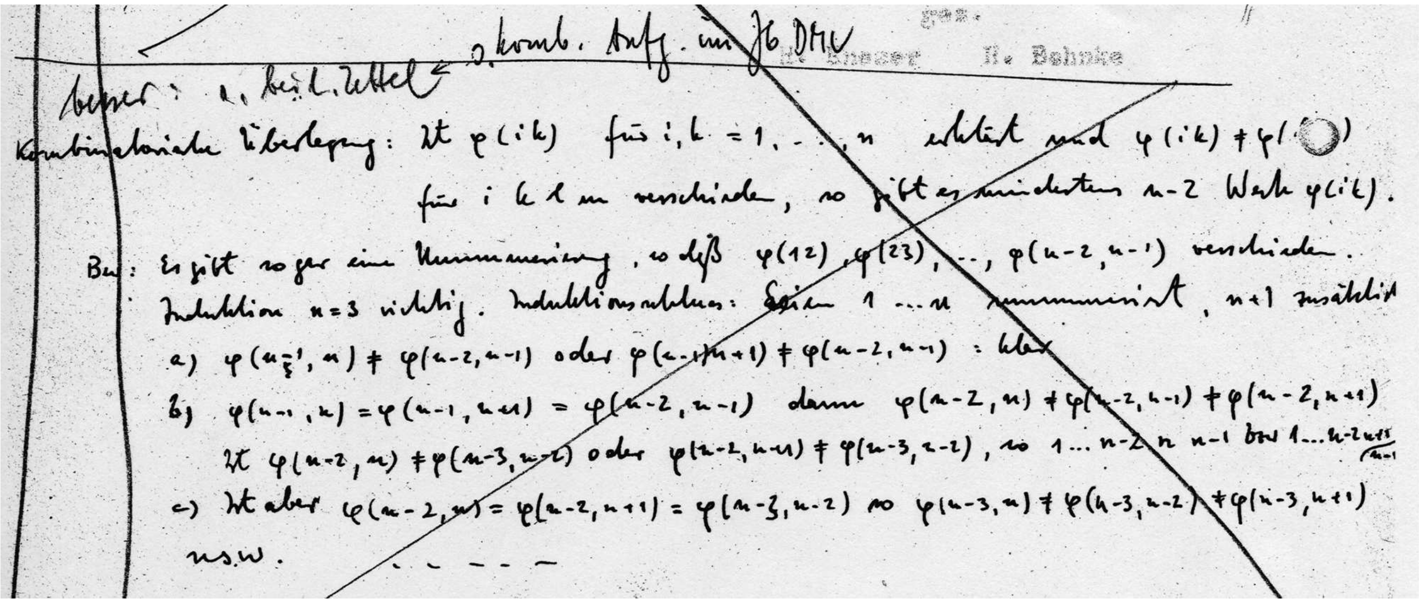


Figure 1: The origin of the Kneser conjecture in the hand writing of Martin Kneser.

László Lovász's proof of the Kneser conjecture about 25 years ago marked the beginning of the history of topological combinatorics: applications of topological methods and theorems to problems of discrete mathematics which did not (seem to) have any connection to topology.

With this article I want to take the opportunity to sketch the development of topological proofs in discrete mathematics beginning with the proof of the Kneser conjecture about 25 years ago, which eventually led to a new discipline: *topological combinatorics.*

In the beginning of the twentieth century the discipline of *combinatorial topology* already made use of combinatorial concepts in topology, finally leading to the emergence of algebraic topology. Meanwhile, discrete mathematics did not make much use of techniques from (algebraic) topology until the seminal proof of the Kneser conjecture. This situation was going to change in an unexpected and fascinating way.

The essence of topological combinatorics can be characterized by a scheme that many proofs in this field pursue. If we want to solve a combinatorial problem by topological means we carry out the following steps.

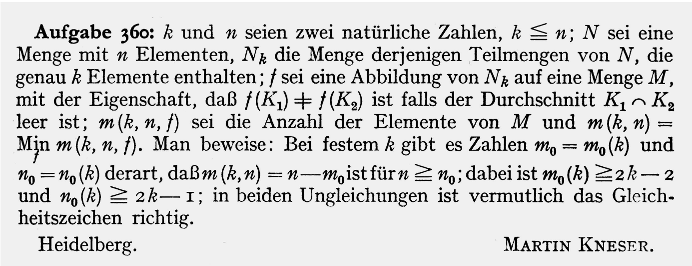
(1) Associate a topological space/continuous map to the given discrete structure such as a graph/graph homomorphism.

(2) Establish a relationship between suitable topological invariants of the space, e.g. dimension, k-connectedness, homology groups, etc. and the desired combinatorial features of the original structure.

(3) Show that the associated space resp. the map has the desired topological properties.

Thus we are concerned with a (functorial) procedure, which is commonly used in mathematics. However, the difference to other fields is that the constructions have to be invented and tailored anew for almost each individual problem. This calls for a certain amount of ingenuity and deeper insight, in particular in steps (1) and (2) of the above procedure.

The first proof of this kind is the proof of the Kneser conjecture by László Lovász. Because of its relevance for the emergence of topological combinatorics, its elegance, and its 25th anniversary, I will sketch this proof and report on the development of topological combinatorics.



Aufgabe 360: k und n seien zwei natürliche Zahlen, $k \leqq n$; N sei eine Menge mit n Elementen, N_k die Menge derjenigen Teilmengen von N, die genau k Elemente enthalten; f sei eine Abbildung von N_k auf eine Menge M, mit der Eigenschaft, daß $f(K_1) \neq f(K_2)$ ist falls der Durchschnitt $K_1 \cap K_2$ leer ist; $m(k, n, f)$ sei die Anzahl der Elemente von M und $m(k, n) = \underset{f}{\mathrm{Min}}\, m(k, n, f)$. Man beweise: Bei festem k gibt es Zahlen $m_0 = m_0(k)$ und $n_0 = n_0(k)$ derart, daß $m(k,n) = n - m_0$ ist für $n \geqq n_0$; dabei ist $m_0(k) \geqq 2k - 2$ und $n_0(k) \geqq 2k - 1$; in beiden Ungleichungen ist vermutlich das Gleichheitszeichen richtig.

Heidelberg. MARTIN KNESER.

Figure 2: From "Jahresbericht der DMV" 1955.

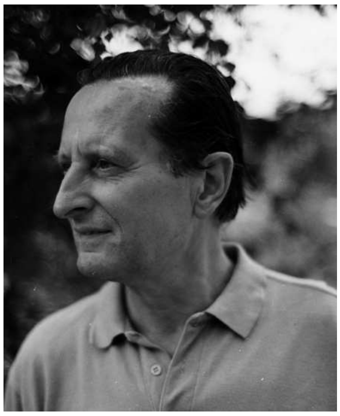


Figure 3: Martin Kneser.

Figure 4: László Lovász.

The Kneser conjecture and its proof

The occupation with an article by Irving Kaplansky on quadratic forms, from 1953, led Martin Kneser to question the behaviour of partitions of the family of k-subsets of an n-set:

> Consider the family of all k-subsets of an n-set. It is easy to partition this family into $n - 2k + 2$ classes $C_1 \cup \cdots \cup C_{n-2k+2}$, such that no pair of k-sets within one class is disjoint. Is it possible to partition the family into $n - 2k + 1$ classes with the same property?

Kneser conjectured that this is not possible! He presented his conjecture in the "Jahresbericht der Deutschen Mathematiker–Vereinigung" in 1955 in the form of an exercise [6] (cf. Figure 2).

We will translate Kneser's conjecture into graph theory language. For that purpose we define the Kneser graph $KG_{n,k}$ in a suggestive manner: the vertices are the k-subsets of an n-set and the edges are given by pairs of disjoint k-sets. Figure 5 shows this graph for the parameters $n = 5$ and $k = 2$. For example, there is an edge between the sets $\{1, 2\}$ and $\{3, 5\}$ because they have empty intersection.

The afore mentioned partitions of the k-subsets of an n-set now correspond to partitions of the vertex set of the graph in so called *colour classes*. The property that no partition set contains a pair of disjoint k-sets now translates into the property that no colour class contains two vertices that are adjacent via an edge in the graph. Such a partition is called a *graph colouring*. The *chromatic number of a graph* is the smallest number of colour classes in a graph colouring. As mentioned above it is easy to define a graph colouring of $KG_{n,k}$ with $n-2k+2$ colour classes. The following example shows $5-2\cdot2+2 = 3$ possible colour classes C_1, C_2, C_3, which define a colouring of $KG_{5,2}$.

$$C_1 = \{\{1,2\},\{1,3\},\{1,4\},\{1,5\}\}$$
$$C_2 = \{\{2,3\},\{2,4\},\{2,5\}\}$$
$$C_3 = \{\{3,4\},\{3,5\},\{4,5\}\}$$

(The example already suggests the general principle for a partition into $n - 2k + 2$ colour classes!) The Kneser conjecture now states that it is impossible to colour the graph with fewer colour classes, i.e., a *lower bound of $n-2k+2$ for the chromatic number of the graph $KG_{n,k}$*.

Twenty three years after Kneser posed his problem László Lovász initiated the development of topological combinatorics with the publication of his proof of the Kneser conjecture [7]. In the following sketch of his proof we will make use of some topological notions that will not be further explained, but which can be found in almost any textbook on topology, such as [11].

Lovász' proof of the Kneser conjecture pursues the scheme that we mentioned in the introduction. Step (1) is based on the invention of a simplicial complex associated with any graph G, the so called neighbourhood complex $\mathcal{N}(G)$. Simplices in the neighbourhood complex are defined by sets of vertices that have a common neighbour in the graph.

As an example we consider the graph G in Figure 6. It defines the neighbourhood complex $\mathcal{N}(G)$:

$$\mathcal{N}(G) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,5\}, \{3,4\}, \{1,2,5\}, \{1,3,4\}\}$$

The inclusion maximal simplices in $\mathcal{N}(G)$ are given by $\{1,2,5\}$, $\{1,3,4\}$ and $\{2,3\}$. In the graph G they correspond to the neighbourhoods of the vertices 3, 2, 1 respectively. Figure 7 shows the topological space that realizes the simplicial complex geometrically. The simplices $\{1,2,5\}$ and $\{1,3,4\}$ correspond to the shaded triangles, the simplex $\{2,3\}$ corresponds to the line segment from 2 to 3.

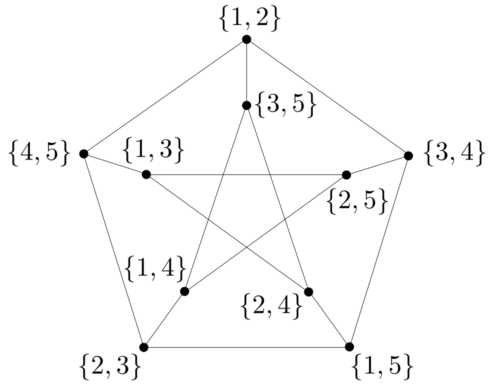


Figure 5: The familiar Petersen graph in the guise of the Kneser graph $K_{5,2}$.

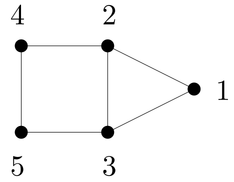


Figure 6: A graph G.

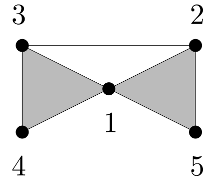


Figure 7: The neighbourhood complex $\mathcal{N}(G)$ associated to G.

In step (2) Lovász applies the *Borsuk–Ulam theorem* from topology, about which Gian–Carlo Rota, in an essay about Stan Ulam, once wrote:

> "While chatting at the Scottish Café with Borsuk, an outstanding Warsaw topologist, he [Ulam] saw in a flash the truth of what is now called the Borsuk–Ulam theorem. Borsuk had to commandeer all his technical resources to prove it."

One of the commonly used versions of this theorem reads:

> If there exists an antipodal continuous map $f : \mathbb{S}^n \to \mathbb{S}^m$ from the n-sphere to the m-sphere, i.e. a continuous map that satisfies $f(-x) = -f(x)$ for all $x \in \mathbb{S}^n$, then $m \geq n$.

Lovász now shows: if $\mathcal{N}(G)$ is topologically m-connected, then G is not $(m+2)$-colourable, i.e., there is no colouring of G with $m+2$ colour classes. In the example of Figure 7, the neighbourhood complex is 0-connected, i.e. connected, but not 1-connected since for example the loop $1\to2\to3\to1$ cannot be contracted to a point. This reflects the fact that G is not 2-colourable but 3-colourable.

We want to sketch this essential part of the proof from a slightly more modern point of view (see e.g. [2] and also quite current is [10]). An $(m+2)$-colouring of a graph G induces a graph homomorphism $G \to K_{m+2}$ from G to the complete graph K_{m+2} on $m+2$ vertices in which all pairs of vertices are connected by an edge. Such a graph homomorphism induces a continuous map $\mathcal{N}(G) \to \mathcal{N}(K_{m+2})$ of topological spaces. It is easy to verify that $\mathcal{N}(K_{m+2})$ is an m-dimensional sphere. Now, if $\mathcal{N}(G)$ is n-connected, then with the help of the map $\mathcal{N}(G) \to \mathcal{N}(K_{m+2})$, one can construct an antipodal continuous map $f : \mathbb{S}^{n+1} \to \mathbb{S}^m$. Hence by the Borsuk–Ulam theorem $m \geq n+1$, resp. $n \leq m-1$ as desired.

In step (3) Lovász completes his proof by verifying that $\mathcal{N}(KG_{n,k})$ is indeed $(n-2k-1)$-connected.

With his proof Lovász had identified the topological core of the problem: the Borsuk–Ulam theorem. In the same year, 1978, a much shorter proof by Imre Bárány followed, which employs the Borsuk–Ulam theorem in a more direct fashion. As recently as 2002, another substantial simplification has been found by the American mathematics student Joshua Greene [4]. For his proof, which can be considered as "Proof from the Book", Greene has been awarded the 2003 AMS-MAA-SIAM Morgan prize.

Topological combinatorics

The role of the Borsuk–Ulam theorem is not restricted to the proof of the Kneser conjecture. General bounds for the chromatic number of a graph, partition results, complexity bounds for algorithmic problems, and much more, have all been established with the help of Borsuk–Ulam's theorem and its generalizations. Maybe one of the most important generalizations is a theorem presented by Albrecht Dold [3] in 1983:

> Let G be a non-trivial finite group which acts freely on (well behaved) spaces X and Y. Suppose X is $(n-1)$-connected and Y has dimension m. If there exists a G-equivariant map from X to Y, then $m \geq n$.

For $X = \mathbb{S}^n$, $Y = \mathbb{S}^m$, and G the two–element group acting via the antipodal map, we re-obtain the Borsuk–Ulam theorem. As a matter of fact, there is such a variety of applications of this theorem and its generalizations that Jiří Matoušek dedicated an entire (and entirely wonderful) book to them with the title "Using the Borsuk–Ulam Theorem" [8].

While Borsuk–Ulam's theorem so far plays the most prominent role in topological combinatorics, most standard tools from algebraic topology have now found their applications in combinatorics, from homology- and cohomology computations, characteristic classes up to spectral sequences. A recent example is the article "Complexes of graph homomorphisms" by Eric Babson and Dmitry Kozlov [1]. Even some methods from differential topology have found a combinatorial analog, e.g. in the invention of *discrete Morse theory* by Robin Forman.

I want to mention a few applications of these methods. Most notably there are *graph colouring problems*. By now a multitude of bounds for the chromatic number of graphs and hypergraphs have been established with topological methods. *Partition results* of different kinds were solved, such as the *necklace problem*, which was solved in its full generality in 1987 by Noga Alon. Moreover, *complexity problems,* such as the complexity of linear decision tree algorithms and the complexity of monotone graph properties in connection with the *evasiveness conjecture* have been addressed with techniques from topological combinatorics. Another huge topic is the *topology of partially ordered sets.* In the early 80s the Swedish mathematician Anders Björner introduced the concept of shellability of a partially ordered set. With a partially ordered set one can associate a simplicial complex and thus a topological space. Shellability of a partial order as defined by Björner implies that the associated topological space is a bouquet of spheres. This combinatorial concept along with its topological consequences found numerous applications, such as in the theory of Bruhat orders and questions in the area of algebraic combinatorics. It should be remarked that using the concept of shellability, it is easy to see that the neighbourhood complex $\mathcal{N}(KG_{n,k})$, as it appears in Lovász' proof of the Kneser conjecture, is indeed $(n-2k-1)$-connected.

Back to combinatorics

Ever since combinatorial theorems were proved with topological methods, the natural question followed as to whether the topological argument could be replaced by a combinatorial one. A first breakthrough in this direction was made by Jiří Matoušek in the year 2000 with a combinatorial proof of the Kneser conjecture [9]. His proof relies on a special case of a combinatorial lemma by A.W. Tucker which is essentially "equivalent" to the Borsuk–Ulam theorem. Combinatorial relatives of the Borsuk–Ulam theorem have also been applied in the construction of approximation algorithms in connection with *fair division problems* . Moreover, the concrete questions in discrete mathematics pointed out the need for explicit methods for the computation of homology groups and other invariants of simplicial complexes which led to efficient programs for the computation of these invariants.

As mentioned in the introduction, combinatorial topology led to the formation of algebraic topology. The term "algebraic topology" was apparently first used in a public lecture by Solomon Lefschetz at Duke University in 1936 (cf. [5]):

> "The assertion is often made of late that all mathematics is composed of algebra and topology. It is not so widely realized that the two subjects interpenetrate so that we have an algebraic topology as well as a topological algebra."

The latter has now also become true for combinatorics and topology.

Addendum: Prof. Dr. Martin Kneser died on February 16, 2004 in Göttingen.

The EMS-Newsletter is grateful to the editors of the Newsletter of Deutsche Mathematiker–Vereinigung for their kind permission to publish the author's English translation of this article which originally appeared in Mitteilungen der DMV, 4/2003 *under the title "25 Jahre Beweis der Kneservermutung".*

Mark de Longueville [delong@math.fu-berlin.de] studied mathematics in Berlin and New Orleans. Supported by a grant from German Research Foundation (DFG), he received his Ph.D. from Technische Universität Berlin. He has been assistant professor at University of Minnesota in Minneapolis and is now assistant professor at Freie Universität Berlin. He is mainly interested in combinatorial problems that potentially have connections to other fields in mathematics.

Literature

[1] Eric Babson and Dmitry N. Kozlov. Complexes of graph homomorphisms. http://arXiv.org/abs/math/0310056, Preprint 2003.

[2] Anders Björner. Topological methods. In R. Graham, M. Grötschel, and L. Lovász, Editors, *Handbook of Combinatorics*, 1819–1872. North Holland, Amsterdam, 1994.

[3] Albrecht Dold. Simple proofs of some Borsuk–Ulam results. *Contemp. Math.*, 19:65–69, 1983.

[4] Joshua Greene. A new short proof of Kneser's conjecture. *Amer.Math.Monthly*, 109:918–920, 2002.

[5] Ioan M. James. From Combinatorial Topology to Algebraic Topology. In I. M. James, Editor, *History of Topology*, 561–573. North Holland, Amsterdam, 1999.

[6] Martin Kneser. Aufgabe 360. *Jahresbericht der DMV*, 58(2):27, 1955.

[7] László Lovász. Kneser's conjecture, chromatic number and homotopy. *J. Combinatorial Theory, Ser. A*, 25:319–324, 1978.

[8] Jiří Matoušek. *Using the Borsuk–Ulam Theorem. Lectures on topological methods in combinatorics and geometry. Written in cooperation with Anders Björner and Günter M. Ziegler.* Universitext, Springer, Heidelberg, 2003.

[9] Jiří Matoušek. A combinatorial proof of Kneser's conjecture. Preprint 2000, *Combinatorica*, to appear.

[10] Jiří Matoušek and Günter M. Ziegler. Topological lower bounds for the chromatic number: A hierarchy. *Jahresbericht der DMV*, Preprint 2003, to appear.

[11] James R. Munkres, *Elements of algebraic topology*, Addison Wesley, 1984.

[12] Rade T. Živaljević. Topological methods. In Jacob E. Goodman et al., Editoren, *Handbook of discrete and computational geometry*, 305–329. CRC Press, Second edition, 2004.

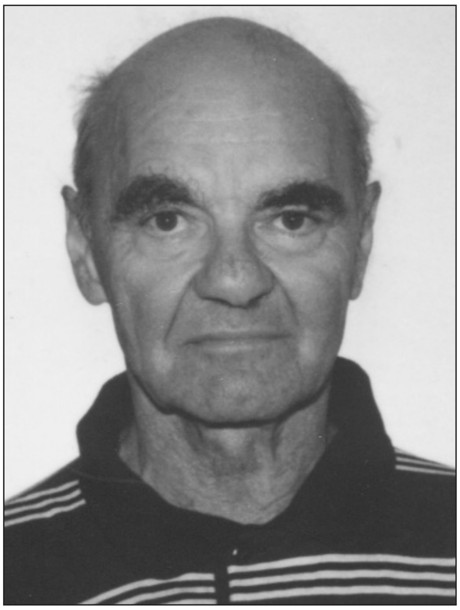
100th Birthday of Henri Cartan

On July 8, 2004, the outstanding French mathematician Henri Cartan turned one hundred years old. The French Mathematical Society held a seminar in honour of Henri Cartan on June 28. Its programme, some of the contributions, a biography, several photos and documents can be found on the web site http://smf.emath.fr/VieSociete/Rencontres/JourneeCartan. The 100th issue of Gazette des Mathématiciens, the news publication of Société Mathématique de France, carried some tributes to Cartan, two of which are reproduced in this issue of the Newsletter along with a resolution from the International Mathematical Union (IMU). At the closing ceremony of the 4th European congress of mathematicians, EMS-president Sir John Kingman sent his cordial congratulations to Prof. Cartan in the name of all the participants.

Three-quarters of a century with Henri Cartan

Jean Cerf (CNRS, Université Paris-Sud, Orsay, France)

This article appeared (in French) in Gazette des Mathématiciens **100***, April 2004, p. 7-8. The Newsletter's thanks go to the author and to the editor of Gazette for the permission to reproduce it and to Larry Siebenmann (Univ. Paris-Sud) for the translation into English.*



In a special section in *Le Monde* of 3 March 2004 entitled "Matière grise: la bataille mondiale" (Grey Matter: the worldwide contest), one of the first observations by the biologist E.E. Baulieu reads: "French mathematicians are among the very best in the world". Above and beyond reservations concerning certain excesses perpetrated under the Bourbaki banner, it is my conviction (hopefully, one widely shared) that this high reputation is in large measure the heritage of that group of young men who, in the thirties of the last century, conceived the Bourbaki project to write a treatise that would develop mathematics from the ground up. In this group, I see, in the front rank, Henri Cartan flanked by his friends André Weil and Jean Dieudonné.

I have had the privilege to frequently cross the path of Henri Cartan. The first occasion (by his own account) was in Strasbourg in 1930; admittedly, I have forgotten this encounter – I was just two years old – but I have retained clear memories from succeeding years. Cartan was a colleague of my father's and a friend of the family, a friend who was admired, but somewhat feared for his caustic turn of mind and tongue; at the same time, his fragile health was a cause for concern! I saw him again in 1939 in Clermont-Ferrand, where the University of Strasbourg had been moved following the outbreak of war, and later in La Bourboule, after the defeat of June 1940, of which he had been one of the few to voice premonitions. Then again in Strasbourg in 1945 after the abyss of war from which my own family was rising safe and sound, but not his. Next in Paris at the École Normale (rue d'Ulm), when he was the professor and I a student: the course for second year students, in which he taught us what a differentiable manifold was, and where, from the back of the lecture room, an unknown person (it was Alexander Grothendieck) ventured to dialog with him as an equal; the first Cartan Seminar on Topology (1948), for which, under his guidance, I presented "exposé no. 3" (thereafter entirely written up by him alone); and frequently in his family gathering, Boulevard Jourdan, on Sunday, where I was a little like one more child. Then as research advisor, who did not *propose* a thesis topic, but who, one day, pointed out to me the article of Feldbau concerning homeomorphisms of spheres, which became the starting point of my thesis.

Even before this last period, I was sufficiently close to him to occasionally glean confidential remarks such as the following that I remember concerning René Thom, whom he 'discovered' before anyone else in spite of their quite different ways of thinking: "Thom is a boy brimming with ideas, but how hard it is to make him write them down!" Then, over a long period: his battles, mostly victorious, to, as he formulated it, "raise the level of the Sorbonne in Mathematics"; his vain efforts to have a chair created at the Collège de France for André Weil. His precocious commitment in favour of Franco-German reconciliation – in a period when that demanded lofty vision, a utopian vision whose realization is today one of our grounds for hope in the face of seemingly insoluble conflicts. His militance in the defence of Human Rights everywhere in the world, and for the construction of Europe as a federal state, an aim to which he attaches such high value.

Jean-Pierre Serre has written: "I believe that the Cartan style is the finest to be found in mathematics".

I would add that the Cartan style is the finest to be found in life in general. Thank you, Monsieur Cartan, for showing, by your example, that it is possible to become with age ever more humane.

Jean Cerf has been Professeur and Directeur de Recherche (CNRS) at Université Paris-Sud (Orsay), France.

Personal souvenirs of Henri Cartan

Pierre Samuel (Université de Paris-Sud, Orsay, France)

This article appeared (in French) in Gazette des Mathématiciens **100***, April 2004, p. 13-14. The Newsletter thanks the editor of gazette for the permission to reproduce it and the author for providing the translation into English.*

My first meeting with Henri Cartan took place in August 1940. I was a candidate at the Ecole Normale Supérieure and he was one of the examiners. Classmates, who had already taken the examination, told me that he was a "nice" examiner as opposed to some examiners at the Ecole Polytechnique who tried to destabilize the candidates. In fact, Henri Cartan wanted to make the candidates reveal as much as possible in order to find out which ones were the most promising. I noticed the same quality when, much later, we sat together in doctorates juries. The good questions he put to the candidates enabled him to predict from their answers which candidates would become first-rate mathematicians. He was never wrong.

I met Henri Cartan again in 1944-1945 at the Ecole Normale Supérieure, when I was preparing for the Agrégation de Mathématiques. At that time, the oral part of this competitive examination consisted in preparing (without documents) and giving lectures on classical subjects (like Euclidian Geometry, Analytic Geometry, Calculus) in front of the jury – just like in front of a high school class. We practiced lecturing in front of Henri Cartan. His criticisms were incisive and to the point and his suggestions were very valuable. Even at this elementary level, we learned a lot of deep Mathematics from him.

I had written to Bourbaki, pointing out mistakes in the exercises of the published volumes, so he took an interest in me and invited René Thom and myself as "guinea pigs" to a Bourbaki congress, which took place in July 1945 at the Ecole Normale Supérieure. This was one of the great experiences in my life.

From 1945 to 1947, I got a research fellowship at Princeton University and I returned with a research project on intersection multiplicities in Algebraic Geometry, mostly inspired by Claude Chevalley. When this work was completed in 1949, I presented it as a thesis to the Université de Paris. I asked Henri Cartan to be in the jury by giving me the subject of the so-called "seconde thèse" (in which the candidate studies and gives an outline

of a topic which has been the subject of recent research, but in a field distinct from the field of the thesis itself). Cartan chose the relations between homology and homotopy and spent a lot of time pointing out the articles to be read and making sure that I was mastering this topic that was new to me.

In the meantime, I became full-fledged member of Bourbaki. During the Congresses, I admired the acuteness of Cartan's comments and his mastery of most branches of Mathematics. If one of his proposals was not accepted immediately, he tried hard to convince other members during the recesses; either they were or the proposals got improved. When travelling by train to the Congresses, he also made us share his enthusiasm for recent discoveries or trends, e.g. the notion of a functor and various topics from S. Eilenberg and his book "Homological algebra".

In spite of the fact that our domains of research were quite distinct, our friendship and our common concerns intensified. Gradually, and especially when I became an environmental activist, he made me share his enthusiasm for the unification of Europe.

I also admired his ceaseless actions for persecuted mathematicians. During the May 1968 movement, we were both open to some demands of the students, e.g. the inclusion of personal studies in the curriculum. In 1970, we both successfully requested to be transferred to the new Université de Paris-Sud (Orsay). There he worked very hard to embody statutes providing a good balance between teaching and research.

Other mathematicians, more competent than me, will surely describe his results on functions of several complex variables, sheaves, algebraic topology, homological algebra, etc. I just know that they are fundamental and that the "Séminaires Cartan", the topics of which were chosen by him each year with a remarkable intuition of what would be important, awakened the vocations of many first-rate mathematicians.

Pierre Samuel is Professeur émérité at Université Paris-Sud (Orsay), France.

On the Occasion of the 100th Birthday of Henri Cartan

It is a great honour for the International Mathematical Union to associate itself with the hundredth birthday of Henri Cartan, on July 8, 2004.

The son of the great mathematician Elie Cartan, his contributions to mathematics have been fundamental, from several complex variables to algebraic topology and homological algebra. A member of the Bourbaki group, his participation in the rejuvenation of the French mathematical school was essential, in particular through his seminar held at the École Normale Supérieure. His roles as teacher and mentor were also exceptional, and were felt well beyond national boundaries.

During the critical years after the second world war, Cartan's enduring friendship with the German mathematician Heinrich Benhke, and his own personal generosity, contributed greatly to the rebirth of German mathematics. He was made an honorary member of the German Mathematical Society (DMV) in 1994.

His natural preoccupation with international cooperation led to his active involvement with the International Mathematical Union, of which he was President from 1967 to 1970. As such he chaired the Fields Medal Committee for the Nice International Congress of Mathematicians in 1970.

He has been actively involved in the defense of mathematicians who were jailed or discriminated against in their countries, and is an ardent defender of European unity.

Mathematicians worldwide unite with respect and admiration in warmly congratulating Henri Cartan, great mathematician and great man, on this happy occasion.

International Mathematical Union

Bringing Math to the Public Frontline

Interview with Nuno Crato, winner of the 2003 'Raising Public Awareness of Mathematics' article competition of the EMS, conducted by José Francisco Rodrigues (Lisbon, Portugal)

N. Crato (left) and J.F. Rodrigues (right)

Nuno Crato *is a mathematician and a well-known science writer in his native Portugal. He is frequently present in public discussions related to research, education, and science popularization. He regularly writes for the press and often appears in radio and television programs. He was recently elected President of the Portuguese Mathematical Society (SPM), and took charge this September.*

Last year, he won the First Prize of the EMS competition Raising Public Awareness (RPA) on Mathematics. The last EMS Newsletter, issue 52, reproduced part of the three-article series on cryptography he submitted to the competition. English and Portuguese versions of these articles are now available online at http://pascal.iseg.utl.pt/~ncrato/EMS. Hungarian and Italian versions will soon be available as well.

Chaired by Vagn Lundsgaard Hansen *and formed by eight mathematicians from an equal number of countries, the RPA committee selected Crato's articles among 26 proposals from 14 different countries.*

José Francisco Rodrigues*, who has been collaborating with the EMS since its foundation (at which he represented the Portuguese Mathematical Society), conducted the conversation that follows.*

Let me first congratulate you, Nuno, on the well-deserved prize. I have been reading your articles in the Portuguese press and one thing that surprises me is the way you chose the topics. They are very diverse and very often are not related to your research. Is this true?

Thanks, José Francisco. My research is in Stochastic Processes and applications, mainly now on long-memory time series, a kind of generalization of fractal Brownian motion. This is necessarily a restricted area and only once I wrote about it to the general public. But I am usually interested in many other subjects, as you are and as everybody who likes mathematics and science usually is. What I find is that I can read an article in, say, *Science,* or *Annals of Statistics,* or even *Scientific American,* faster than most interested readers without a scientific background. As I read journals and magazines very often, just out of my simple curiosity, I very often indeed come across simple ideas on topics. Then I think "I'd like to explain this to someone". It's a natural thing for a teacher. Unfortunately, I can't go to class and say "today, instead of power series, let me explain you how these guys from Scotland are assessing the number of stars in our galaxy using a novel simulation algorithm"…

Then, you decide to write an article on it…

More or less... But I have to do some research on the topic, think about the way to explain it, restrain myself to a limited space, and wait for the right moment…

Couldn't you write immediately?

Not usually. It seems to be better to wait for a moment when the topic gains actuality and becomes interesting for the large public. For example, going back to the articles on the competition, I have been seduced by the marvels of modern cryptography for a while. I have read David Kahn's and Simon Singh's books and a number of expository papers and articles. But I waited. Suddenly, there was a public debate in Portugal on the use of credit cards and electronic purchasing. Some banks even created a special debit card that could be used as a credit card, but only for electronic transactions. They were trying to dispel the fears on the public. It was the ideal moment to talk about cryptography. If you read my first article, you will notice it starts precisely discussing these fears. I wrote it this way to entice the reader. If he or she is hooked with the first sentences, there is a chance the article will be read. If not…

So you try to bring people into science using ordinary subjects as a pretext. Do you consider this work as science popularization or science vulgarization?

Maybe it's both. But I can't give general rules. Sometimes it works the way I described. Sometimes it is different. Some people explain mathematics and science in a straight forward manner; other people use pretexts, as I usually try to do; other people write only chronicles, that is, commentaries on the public scene that are made from a special point of view (in our case, science or math). Everything is useful, when it is done seriously and serves to expand people awareness of science, I think.

Why do you think popularization of mathematics is useful? Some people say it is useless, since the public cannot be educated with light articles that only scratch the surface of things.

I'm sorry: I completely disagree with this idea. We can't confuse formal education with journalism, and science writing is akin to journalism. Popularization of mathematics brings our discipline to public attention. That's it! If some people are enticed by an article and read further, that's fantastic. But if most people only read it and get a general idea, that's also fine. I have nothing against it.

Now, a completely different problem is the rigor we put in the writing. We can't say mistakes and we are always treading dangerous grounds. We have to write things in such a way people are not bored and, at the same time, avoid making errors. How do we do it? I can't give general rules. We just have to be careful and imaginative.

Let me just give an example. If you are talking about something that grows fast, you can't say: "it's an exponential growth", unless it really is. A politician, though, could say it, since "exponential growth" is now a common phrase. But if you are a mathematician or a scientist—people read you in a different way. If you don't want to go into details and explain the difference between polynomial and exponential growth, you can just say: "it is a fast growth".

Let me insist: do you think it is possible to popularize mathematics and science without vulgarizing it?
Many mathematicians and scientists are afraid of vulgarization, I know, because they identify it with oversimplification. I don't like to discuss words, but I have nothing against simplification, that's what we are always doing in life. I condemn errors and try to avoid mistakes, which is another question.

Illustration from the prize-winning articles

But don't you think that, simplifying matters, the popularization of science favors a wrong idea about scientists work?
It may and may not. It all depends on the way things are done. You can say "on a dark night, he suddenly had the idea…" or you can say "after many discussions and many days thinking about the problem, he came to the idea…" My impression, though, is that the public has already a terribly wrong idea about the way science works. Everything you can do to clarify things is positive.

You didn't talk yet about one often cited benefit of mathematics popularization, which is the appreciation of mathematics by the public. How would you rate this benefit?
It's always difficult to do ratings, but this benefit is certainly important. I would add a couple of others, putting in first place the appreciation of the essential ethic of math and science: the intellectual honesty, the critical rationalism, the respect for the reality, the international cooperation, the effort to avoid prejudices.

In general, I would say mathematics and science are an essential part of our culture and they deserve to be on the forefront of public life. This is important for the support of our efforts. And, more importantly, for the creation of a general culture that respects mathematics and science and tries to educate the citizens accordingly.

You have recently been elected President of SPM—Sociedade Portuguesa de Matemática. Do you think our societies should give more attention to scientific popularization?
I can't speak in general. I believe scientific societies should promote research, education, and popularization. All these goals are important, but not all societies can promote them equally.

… as we all should?
I believe scientific organizations should not forget their role in society, which includes popularization of science. But this is a goal for organizations, not for individuals. I think it would be a terrible mistake to try to involve everybody in popularization. Some people may like to do it and may have some aptitude for it. Other people not. The basic thing is research. Teaching comes next. Popularization is at the end of the list.

Isn't it possible to do all three?
It's very difficult, and one activity always harms the other. Very few people are like Ian Stewart, who apparently can write a great book a year, a research paper a month, and still be a dedicated teacher. For most of us, to do science writing necessarily harms research and teaching. It's true that these activities are also complementary. Sometimes, research helps us with ideas for writing to the public, and teaching can give us ideas on ways to explain things. But I never noticed a good research idea coming out a good effort on popularization.

So you think popularization should be left to journalists?
Popularization should be done by people who know what they are talking about and who know how to talk about it. Some journalists have a basic scientific background and the professionalism necessary to do science popularization well. Ideally, we would have plenty of good science journalists. But we do not have so. And it is also refreshing that different worlds and different people communicate and, occasionally and for a while, even trade places.

Illustration from the prize-winning articles

What do you mean?
Think about research and teaching: people who do research can bring to teaching an insider's view and an insight on math problems that other people usually can't.

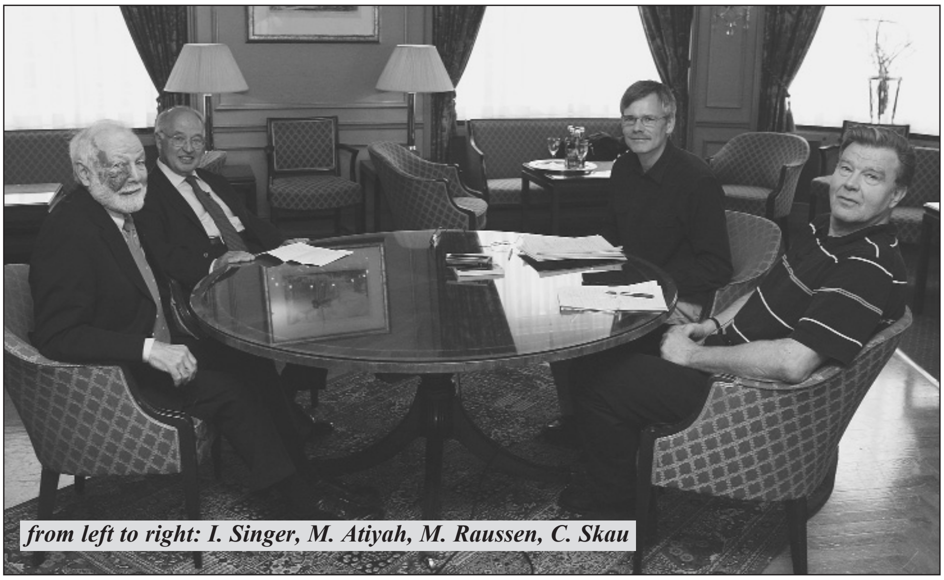
The same way, professional mathematicians can bring to math popularization a rigor and a point of view no good journalist can. So, I believe it's useful for science journalism that some of us, once a while, practice science popularization. The more people talk about math and sciences on the newspapers and the better they do it, the better math and science are appreciated by society. And the better society appreciates science, the better education is. Then, more resources and more people come to mathematics and sciences. And this is good for everybody.

José Francisco Rodrigues [rodrigue@ptmat.fc.ul.pt] is professor of Mathematics at the Faculty of Science of the University of Lisbon, where he directed the Centro de Matemática e Aplicações Fundamentais for several years until 2002. His research interests are in the field of nonlinear PDEs and their applications, in particular, in free boundary problems. He is currently collaborating with the Centro de Matemática of the University of Coimbra, also in Portugal. He co-organized the 1999 Diderot Mathematical Forum on "Mathematics and Music" *and is a member of the EMS Raising Public Awareness of Mathematics Committee. He is a main editor of* "Interface and Free Boundaries"*, an EMS journal.*

Interview with Michael Atiyah and Isadore Singer

Interviewers: Martin Raussen and Christian Skau

The interview took place in Oslo, on the 24th of May 2004, prior to the Abel prize celebrations.



from left to right: I. Singer, M. Atiyah, M. Raussen, C. Skau

The Index Theorem

First, we congratulate both of you for having been awarded the Abel Prize 2004. This prize has been given to you for "the discovery and the proof of the Index Theorem connecting geometry and analysis in a surprising way and your outstanding role in building new bridges between mathematics and theoretical physics". Both of you have an impressive list of fine achievements in mathematics. Is the Index Theorem your most important result and the result you are most pleased with in your entire careers?

ATIYAH First, I would like to say that I prefer to call it a theory, not a theorem. Actually, we have worked on it for 25 years and if I include all the related topics, I have probably spent 30 years of my life working on the area. So it is rather obvious that it is the best thing I have done.

SINGER I too, feel that the index theorem was but the beginning of a high point that has lasted to this very day. It's as if we climbed a mountain and found a plateau we've been on ever since.

We would like you to give us some comments on the history of the discovery of the Index Theorem.[1] Were there precursors, conjectures in this direction already before you started? Were there only mathematical motivations or also physical ones?

ATIYAH Mathematics is always a continuum, linked to its history, the past - nothing comes out of zero. And certainly the Index Theorem is simply a continuation of work that, I would like to say, began with Abel. So of course there are precursors. A theorem is never arrived at in the way that logical thought would lead you to believe or that posterity thinks. It is usually much more accidental, some chance discovery in answer to some kind of question. Eventually you can rationalize it and say that this is how it fits. Discoveries never happen as neatly as that. You can rewrite history and make it look much more logical, but actually it happens quite differently.

SINGER At the time we proved the Index Theorem we saw how important it was in mathematics, but we had no inkling that it would have such an effect on physics some years down the road. That came as a complete surprise to us. Perhaps it should not have been a surprise because it used a lot of geometry and also quantum mechanics in a way, à la Dirac.

You worked out at least three different proofs with different strategies for the Index Theorem. Why did you keep on after the first proof? What different insights did the proofs give?

ATIYAH I think it is said that Gauss had ten different proofs for the law of quadratic reciprocity. Any good theorem should have several proofs, the more the better. For two reasons: usually, different proofs have different strengths and weaknesses, and they generalize in different directions - they are not just repetitions of each other. And that is certainly the case with the proofs that we came up with. There are different reasons for the proofs, they have different histories and backgrounds. Some of them are good for this application, some are good for that application. They all shed light on the area. If you cannot look at a problem from different directions, it is probably not very interesting; the more perspectives, the better!

SINGER There isn't just one theorem; there are generalizations of the theorem. One is the families index theorem using K-theory; another is the heat equation proof which makes the formulas that are topological, more geometric and explicit. Each theorem and proof has merit and has different applications.

Collaboration

Both of you contributed to the index theorem with different expertise and visions – and other people had a share as well, I suppose. Could you describe this collaboration and the establishment of the result a little closer?

SINGER Well, I came with a background in analysis and differential geometry, and Sir Michael's expertise was in algebraic geometry and topology. For the purposes of the Index Theorem, our areas of expertise fit together hand in glove. Moreover, in a way, our personalities fit together, in that "anything goes": Make a suggestion - and whatever it was, we would just put it on the blackboard and work with it; we would both enthusiastically explore it; if it didn't work, it didn't work. But often enough, some idea that seemed far-fetched *did* work. We both had the freedom to continue without worrying about where it came from or where it would lead. It was exciting to work with Sir Michael all these years. And it is as true today as it was when we first met in '55 - that sense of excitement and "anything goes" and "let's see what happens".

ATIYAH No doubt: Singer had a strong expertise and background in analysis and differential geometry. And he knew certainly more physics than I did; it turned out to be very useful later on. My background was in algebraic geometry and topology, so it all came together. But of course there are a lot of people who contributed in the background to the build-up of the Index Theorem – going back to Abel, Riemann, much more recently Serre, who got the Abel prize last year, Hirzebruch, Grothendieck and Bott. There was lots of work from the algebraic geometry side and from topology that prepared the ground. And of course there are also a lot of people who did fundamental work in analysis and the study of differential equations: Hörmander, Nirenberg… In my lecture I will give a long list of names[2]; even that one will be partial. It is an example of international collaboration; you do not work in isolation, neither in terms of time nor in terms of space – especially in these days. Mathematicians are linked so much, people travel around much more. We two met at the Institute at Princeton. It was nice to go to the Arbeitstagung in Bonn every year, which Hirzebruch organised and where many of these other people came. I did not realize that at the time, but looking back, I am very surprised how quickly these ideas moved…

Collaboration seems to play a bigger role in mathematics than earlier. There are a lot of conferences, we see more papers that are written by two, three or even more authors – is that a necessary and commendable development or has it drawbacks as well?

ATIYAH It is not like in physics or chemistry where you have 15 authors because they need an enormous big machine. It is not absolutely necessary or fundamental. But particularly if you are dealing with areas which have rather mixed and interdisciplinary backgrounds, with people who have different expertise, it is much easier and faster. It is also much more interesting for the participants. To be a mathematician on your own in your office can be a little bit dull, so interaction is stimulating, both psychologically and mathematically. It has to be admitted that there are times when you go solitary in your office, but not all the time! It can also be a social activity with lots of interaction. You need a good mix of both, you can't be talking all the time. But talking some of the time is very stimulating. Summing up, I think that it is a good development – I do not see any drawbacks.

SINGER Certainly computers have made collaboration much easier. Many mathematicians collaborate by computer instantly; it's as if they were talking to each other. I am unable to do that. A sobering counterexample to this whole trend is Perelman's results on the Poincaré conjecture: He worked alone for ten to twelve years, I think, before putting his preprints on the net.

ATIYAH Fortunately, there are many different kinds of mathematicians, they work on different subjects, they have different approaches and different personalities – and that is a good thing. We do not want all mathematicians to be isomorphic, we want variety: different mountains need different kinds of techniques to climb.

SINGER I support that. Flexibility is absolutely essential in our society of mathematicians.

Perelman's work on the Poincaré conjecture seems to be another instance where analysis and geometry apparently get linked very much together. It seems that geometry is profiting a lot from analytic perspectives. Is this linkage between different disciplines a general trend – is it true, that important results rely on this interrelation between different disciplines? And a much more specific question: What do you know about the status of the proof of the Poincaré conjecture?

SINGER To date, everything is working out as Perelman says. So I learn from Lott's seminar at the University of Michigan and Tian's seminar at Princeton. Although no one vouches for the final details, it appears that Perelman's proof will be validated.

As to your first question: When any two subjects use each other's techniques in a new way, frequently, something special happens. In geometry, analysis is very important; for existence theorems, the more the better. It is not surprising that some new [at least to me] analysis implies something interesting about the Poincaré conjecture.

ATIYAH I prefer to go even further – I really do not believe in the division of mathematics into specialities; already if you go back into the past, to Newton and Gauss… Although there have been times, particularly post-Hilbert, with the axiomatic approach to mathematics in the first half of the twentieth century, when people began to specialize, to divide up. The Bourbaki trend had its use for a particular time. But this is not part of the general attitude to mathematics: Abel would not have distinguished between algebra and analysis. And I think the same goes for geometry and analysis for people like Newton.

It is artificial to divide mathematics into separate chunks, and then to say that you bring them together as though this is a surprise. On the contrary, they are all part of the puzzle of mathematics. Sometimes you would develop some things for their own sake for a while e.g. if you develop group theory by itself. But that is just a sort of temporary convenient division of labour. Fundamentally, mathematics should be used as a unity. I think the more examples we have of people showing that you can usefully apply analysis to geometry, the better. And not just analysis, I think that some physics came into it as well: Many of the ideas in geometry use physical insight as well – take the example of Riemann! This is all part of the broad mathematical tradition, which sometimes is in danger of being overlooked by modern, younger people who say "we have separate divisions". We do not want to have any of that kind, really.

SINGER The Index Theorem was in fact instrumental in breaking barriers between fields. When it first appeared, many oldtimers in special fields were upset that new techniques were entering their fields and achieving things they could not do in the field by old methods. A younger generation immediately felt freed from the barriers that we both view as artificial.

ATIYAH Let me tell you a little story about Henry Whitehead, the topologist. I remember that he told me that he enjoyed very much being a topologist: He had so many friends within topology, and it was such a great community. "It would be a tragedy if one day I would have a brilliant idea within functional analysis and would have to leave all my topology friends and to go out and work with a different group of people." He regarded it to be his duty to do so, but he would be very reluctant.

Somehow, we have been very fortunate. Things have moved in such a way that we got involved with functional analysts without losing our old friends; we could bring them all with us. Alain Connes was in functional analysis, and now we interact closely. So we have been fortunate to maintain our old links and move into new ones – it has been great fun.

Mathematics and physics

We would like to have your comments on the interplay between physics and mathematics. There is Galilei's famous dictum from the beginning of the scientific revo-

lution, which says that the Laws of Nature are written in the language of mathematics. Why is it that the objects of mathematical creation, satisfying the criteria of beauty and simplicity, are precisely the ones that time and time again are found to be essential for a correct description of the external world? Examples abound, let me just mention group theory and, yes, your Index Theorem!

SINGER There are several approaches in answer to your questions; I will discuss two. First, some parts of mathematics were created in order to describe the world around us. Calculus began by explaining the motion of planets and other moving objects. Calculus, differential equations, and integral equations are a natural part of physics because they were developed for physics. Other parts of mathematics are also natural for physics. I remember lecturing in Feynman's seminar, trying to explain anomalies. His postdocs kept wanting to pick coordinates in order to compute; he stopped them saying: "The Laws of Physics are independent of a coordinate system. Listen to what Singer has to say, because he is describing the situation without coordinates." Coordinate-free means geometry. It is natural that geometry appears in physics, whose laws are independent of a coordinate system.

Symmetries are useful in physics for much the same reason they're useful in mathematics. Beauty aside, symmetries simplify equations, in physics and in mathematics. So physics and math have in common geometry and group theory, creating a close connection between parts of both subjects.

Secondly, there is a deeper reason if your question is interpreted as in the title of Eugene Wigner's essay "*The Unreasonable Effectiveness of Mathematics in the Natural Sciences*"[3]. Mathematics studies coherent systems which I will not try to define. But it studies coherent systems, the connections between such systems and the structure of such systems. We should not be too surprised that mathematics has coherent systems applicable to physics. It remains to be seen whether there is an already developed coherent system in mathematics that will describe the structure of string theory. [At present, we do not even know what the symmetry group of string field theory is.] Witten has said that 21st century mathematics has to develop new mathematics, perhaps in conjunction with physics intuition, to describe the structure of string theory.

ATIYAH I agree with Singer's description of mathematics having evolved out of the physical world; it therefore is not a big surprise that it has a feedback into it.

More fundamentally: to understand the outside world as a human being is an attempt to reduce complexity to simplicity. What is a theory? A lot of things are happening in the outside world, and the aim of scientific inquiry is to reduce this to as simple a number of principles as possible. That is the way the human mind works, the way the human mind wants to see the answer.

If we were computers, which could tabulate vast amounts of all sorts of information, we would never develop theory – we would say, just press the button to get the answer. We want to reduce this complexity to a form that the human mind can understand, to a few simple principles. That's the nature of scientific inquiry, and mathematics is a part of that. Mathematics is an evolution from the human brain, which is responding to outside influences, creating the machinery with which it then attacks the outside world. It is our way of trying to reduce complexity into simplicity, beauty and elegance. It is really very fundamental, simplicity is in the nature of scientific inquiry – we do not look for complicated things.

I tend to think that science and mathematics are ways the human mind looks and experiences – you cannot divorce the human mind from it. Mathematics is part of the human mind. The question whether there is a reality independent of the human mind, has no meaning – at least, we cannot answer it.

Is it too strong to say that the mathematical problems solved and the techniques that arose from physics have been the lifeblood of mathematics in the past; or at least for the last 25 years?

ATIYAH I think you could turn that into an even stronger statement. Almost all mathematics originally arose from external reality, even numbers and counting. At some point, mathematics then turned to ask internal questions, e.g. the theory of prime numbers, which is not directly related to experience but evolved out of it.

There are parts of mathematics where the human mind asks internal questions just out of curiosity. Originally it may be physical, but eventually it becomes something independent. There are other parts that relate much closer to the outside world with much more interaction backwards and forward. In that part of it, physics has for a long time been the lifeblood of mathematics and inspiration for mathematical work. There are times when this goes out of fashion or when parts of mathematics evolve purely internally. Lots of abstract mathematics does not directly relate to the outside world.

It is one of the strengths of mathematics that it has these two and not a single lifeblood: one external and one internal, one arising as response to external events, the other to internal reflection on what we are doing.

SINGER Your statement *is* too strong. I agree with Michael that mathematics is blessed with both an external and internal source of inspiration. In the past several decades, high energy theoretical physics has had a marked influence on mathematics. Many mathematicians have been shocked at this unexpected development: new ideas from outside mathematics so effective in mathematics. We are delighted with these new inputs, but the "shock" exaggerates their overall effect on mathematics.

Newer developments

Can we move to newer developments with impact from the Atiyah-Singer Index Theorem? I.e., String Theory and Edward Witten on the one hand and on the other hand Non-commutative Geometry represented by Alain Connes. Could you describe the approaches to mathematical physics epitomized by these two protagonists?

ATIYAH I tried once in a talk to describe the different approaches to progress in physics like different religions. You have prophets, you have followers – each prophet and his followers think that they have the sole possession of the truth. If you take the strict point of view that there are several different religions, and that the intersection of all these theories is empty, then they are all talking nonsense. Or you can take the view of the mystic, who thinks that they are all talking of different aspects of reality, and so all of them are correct. I tend to take the second point of view. The main "orthodox" view among physicists is certainly represented by a very large group of people working with string theory like Edward Witten. There are a small number of people who have different philosophies, one of them is Alain Connes, and the other is Roger Penrose. Each of them has a very specific point of view; each of them has very interesting ideas. Within the last few years, there has been non-trivial interaction between all of these.

They may all represent different aspects of reality and eventually, when we under

Isadore Singer and Sir Michael Atiyah receive the Abelprize from King Harald. (Photo: Scanpix)

stand it all, we may say "Ah, yes, they are all part of the truth". I think that that will happen. It is difficult to say which will be dominant, when we finally understand the picture – we don't know. But I tend to be open-minded. The problem with a lot of physicists is that they have a tendency to "follow the leader": as soon as a new idea comes up, ten people write ten or more papers on it and the effect is that everything can move very fast in a technical direction. But big progress may come from a different direction; you do need people who are exploring different avenues. And it is very good that we have people like Connes and Penrose with their own independent line from different origins. I am in favour of diversity. I prefer not to close the door or to say "they are just talking nonsense".

SINGER String Theory is in a very special situation at the present time. Physicists have found new solutions on their landscape - so many that you cannot expect to make predictions from String Theory. Its original promise has not been fulfilled. Nevertheless, I am an enthusiastic supporter of Super String Theory, not just because of what it has done in mathematics, but also because as a coherent whole, it is a marvellous subject. Every few years new developments in the theory give additional insight. When that happens, you realize how little one understood about String Theory previously. The theory of *D*-branes is a recent example. Often there is mathematics closely associated with these new insights. Through *D*-branes, *K*-theory entered String Theory naturally and reshaped it. We just have to wait and see what will happen. I am quite confident that physics will come up with some new ideas in String Theory that will give us greater insight into the structure of the subject, and along with that will come new uses of mathematics.

Alain Connes' program is very natural – if you want to combine geometry with quantum mechanics, then you really want to quantize geometry, and that is what non-commutative geometry means. Non-commutative Geometry has been used effectively in various parts of String Theory explaining what happens at certain singularities, for example. I think it may be an interesting way of trying to describe black holes and to explain the Big Bang. I would encourage young physicists to understand non-commutative geometry more deeply than they presently do. Physicists use only parts of non-commutative geometry; the theory has much more to offer. I do not know whether it is going to lead anywhere or not. But one of my projects is to try and redo some known results using non-commutative geometry more fully.

If you should venture a guess, which mathematical areas do you think are going to witness the most important developments in the coming years?

ATIYAH One quick answer is that the most exciting developments are the ones which you cannot predict. If you can predict them, they are not so exciting. So, by definition, your question has no answer.

Ideas from physics, e.g. Quantum Theory, have had an enormous impact so far, in geometry, some parts of algebra, and in topology. The impact on number theory has still been quite small, but there are some examples. I would like to make a rash prediction that it will have a big impact on number theory as the ideas flow across mathematics – on one extreme number theory, on the other physics, and in the middle geometry: the wind is blowing, and it will eventually reach to the farthest extremities of number theory and give us a new point of view. Many problems that are worked upon today with old-fashioned ideas will be done with new ideas. I would like to see this happen: it could be the Riemann hypothesis, it could be the Langlands program or a lot of other related things. I had an argument with Andrew Wiles where I claimed that physics will have an impact on his kind of number theory; he thinks this is nonsense but we had a good argument.

I would also like to make another prediction, namely that fundamental progress on the physics/mathematics front, String Theory questions etc., will emerge from a much more thorough understanding of classical four-dimensional geometry, of Einstein's Equations etc. The hard part of physics in some sense is the non-linearity of Einstein's Equations. Everything that has been done at the moment is circumventing this problem in lots of ways. They haven't really got to grips with the hardest part. Big progress will come when people by some new techniques or new ideas really settle that. Whether you call that geometry, differential equations or physics depends on what is going to happen, but it could be one of the big breakthroughs.

These are of course just my speculations.

SINGER I will be speculative in a slightly different way, though I do agree with the number theory comments that Sir Michael mentioned, particularly theta functions entering from physics in new ways. I think other fields of physics will affect mathematics - like statistical mechanics and condensed matter physics. For example, I predict a new subject of statistical topology. Rather than count the number of holes, Betti-numbers, etc., one will be more interested in the distribution of such objects on noncompact manifolds as one goes out to infinity. We already have precursors in the number of zeros and poles for holomorphic functions. The theory that we have for holomorphic functions will be generalized, and insights will come from condensed matter physics as to what, statistically, the topology might look like as one approaches infinity.

Continuity of mathematics

Mathematics has become so specialized, it seems, that one may fear that the subject will break up into separate areas. Is there

a core holding things together?

ATIYAH I like to think there is a core holding things together, and that the core is rather what I look at myself; but we tend to be rather egocentric. The traditional parts of mathematics, which evolved - geometry, calculus and algebra - all centre on certain notions. As mathematics develops, there are new ideas, which appear to be far from the centre going off in different directions, which I perhaps do not know much about. Sometimes they become rather important for the whole nature of the mathematical enterprise. It is a bit dangerous to restrict the definition to just whatever you happen to understand yourself or think about. For example, there are parts of mathematics that are very combinatorial. Sometimes they are very closely related to the continuous setting, and that is very good: we have interesting links between combinatorics and algebraic geometry and so on. They may also be related to e.g. statistics. I think that mathematics is very difficult to constrain; there are also all sorts of new applications in different directions.

It is nice to think of mathematics having a unity; however, you do not want it to be a straitjacket. The centre of gravity may change with time. It is not necessarily a fixed rigid object in that sense, I think it should develop and grow. I like to think of mathematics having a core, but I do not want it to be rigidly defined so that it excludes things which might be interesting. You do not want to exclude somebody who has made a discovery saying: "You are outside, you are not doing mathematics, you are playing around". You never know! That particular discovery might be the mathematics of the next century; you have got to be careful. Very often, when new ideas come in, they are regarded as being a bit odd, not really central, because they look too abstract.

SINGER Countries differ in their attitudes about the degree of specialization in mathematics and how to treat the problem of too much specialization. In the United States I observe a trend towards early specialization driven by economic considerations. You must show early promise to get good letters of recommendations to get good first jobs. You can't afford to branch out until you have established yourself and have a secure position. The realities of life force a narrowness in perspective that is not inherent to mathematics. We can counter too much specialization with new resources that would give young people more freedom than they presently have, freedom to explore mathematics more broadly, or to explore connections with other subjects, like biology these days where there is lots to be discovered.

When I was young the job market was good. It was important to be at a major university but you could still prosper at a smaller one. I am distressed by the coercive effect of today's job market. Young mathematicians should have the freedom of choice we had when we were young.

The next question concerns the continuity of mathematics. Rephrasing slightly a question that you, Prof. Atiyah are the origin of, let us make the following gedanken experiment: If, say, Newton or Gauss or Abel were to reappear in our midst, do you think they would understand the problems being tackled by the present generation of mathematicians – after they had been given a short refresher course? Or is present day mathematics too far removed from traditional mathematics?

ATIYAH The point that I was trying to make there was that really important progress in mathematics is somewhat independent of technical jargon. Important ideas can be explained to a really good mathematician, like Newton or Gauss or Abel, in conceptual terms. They are in fact coordinate-free, more than that, technology-free and in a sense jargon-free. You don't have to talk of ideals, modules or whatever – you can talk in the common language of scientists and mathematicians. The really important progress mathematics has made within 200 years could easily be understood by people like Gauss and Newton and Abel. Only a small refresher course where they were told a few terms – and then they would immediately understand.

Actually, my pet aversion is that many mathematicians use too many technical terms when they write and talk. They were trained in a way that if you do not say it 100 percent correctly, like lawyers, you will be taken to court. Every statement has to be fully precise and correct. When talking to other people or scientists, I like to use words that are common to the scientific community, not necessarily just to mathematicians. And that is very often possible. If you explain ideas without a vast amount of technical jargon and formalism, I am sure it would not take Newton, Gauss and Abel long – they were bright guys, actually!

SINGER One of my teachers at Chicago was André Weil, and I remember his saying: "If Riemann were here, I would put him in the library for a week, and when he came out he would tell us what to do next."

Communication of mathematics

Next topic: Communication of mathematics: Hilbert, in his famous speech at the International Congress in 1900, in order to make a point about mathematical communication, cited a French mathematician who said: "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street". In order to pass on to new generations of mathematicians the collective knowledge of the previous generation, how important is it that the results have simple and elegant proofs?

ATIYAH The passing of mathematics on to subsequent generations is essential for the future, and this is only possible if every generation of mathematicians understands what they are doing and distils it out in such a form that it is easily understood by the next generation. Many complicated things get simple when you have the right point of view. The first proof of something may be very complicated, but when you understand it well, you readdress it, and eventually you can present it in a way that makes it look much more understandable – and that's the way you pass it on to the next generation! Without that, we could never make progress - we would have all this messy stuff. Mathematics does depend on a sufficiently good grasp, on understanding of the fundamentals so that we can pass it on in as simple a way as possible to our successors. That has been done remarkably successfully for centuries. Otherwise, how could we possibly be where we are? In the 19th century, people said: "There is so much mathematics, how could anyone make any progress?" Well, we have - we do it by various devices, we generalize, we put all things together, we unify by new ideas, we simplify lots of the constructions – we are very successful in mathematics and have been so for several hundred years. There is no evidence that this has stopped: in every new generation, there are mathematicians who make enormous progress. How do they learn it all? It must be because we have been successful communicating it.

SINGER I find it disconcerting speaking to some of my young colleagues, because they have absorbed, reorganized, and simplified a great deal of known material into a new language, much of which I don't understand. Often I'll finally say, "Oh; is that all you meant?" Their new conceptual framework allows them to encompass succinctly considerably more than I can express with mine. Though impressed with the progress, I must confess impatience because it takes me so long to understand what is really being said.

Has the time passed when deep and important theorems in mathematics can be given short proofs? In the past, there are many such examples, e.g., Abel's one-page proof of the addition theorem of algebraic differentials or Goursat's proof of Cauchy's integral theorem.

ATIYAH I do not think that at all! Of course, that depends on what foundations you are allowed to start from. If we have to start from the axioms of mathematics, then every proof will be very long. The common framework at any given time is constantly advancing; we are already at a high platform. If we are allowed to start within that framework, then at every stage there are short proofs.

One example from my own life is this famous problem about vector fields on spheres solved by Frank Adams where the proof took many hundreds of pages. One day I discovered how to write a proof on a postcard. I sent it over to Frank Adams and we wrote a little paper which then would fit on a bigger postcard. But of course that used some K-theory; not that complicated in itself. You are always building on a higher platform; you have always got more tools at your disposal that are part of the lingua franca which you can use. In the old days you had a smaller base: If you make a simple proof nowadays, then you are allowed to assume that people know what group theory is, you are allowed to talk about Hilbert space. Hilbert space took a long time to develop, so we have got a much bigger vocabulary, and with that we can write more poetry.

SINGER Often enough one can distil the ideas in a complicated proof and make that part of a new language. The new proof becomes simpler and more illuminating. For clarity and logic, parts of the original proof have been set aside and discussed separately.

ATIYAH Take your example of Abel's Paris memoir: His contemporaries did not find it at all easy. It laid the foundation of the theory. Only later on, in the light of that theory, we can all say: "Ah, what a beautifully simple proof!" At the time, all the ideas had to be developed, and they were hidden, and most people could not read that paper. It was very, very far from appearing easy for his contemporaries.

Individual work style

I heard you, Prof. Atiyah, mention that one reason for your choice of mathematics for your career was that it is not necessary to remember a lot of facts by heart. Nevertheless, a lot of threads have to be woven together when new ideas are developed. Could you tell us how you work best, how do new ideas arrive?

ATIYAH My fundamental approach to doing research is always to ask questions. You ask "Why is this true?" when there is something mysterious or if a proof seems very complicated. I used to say – as a kind of joke – that the best ideas come to you during a bad lecture. If somebody gives a terrible lecture, it may be a beautiful result but with terrible proofs, you spend your time trying to find better ones, you do not listen to the lecture. It is all about asking questions – you simply have to have an inquisitive mind! Out of ten questions, nine will lead nowhere, and one leads to something productive. You constantly have to be inquisitive and be prepared to go in any direction. If you go in new directions, then you have to learn new material.

Usually, if you ask a question or decide to solve a problem, it has a background. If you understand where a problem comes from then it makes it easy for you to understand the tools that have to be used on it. You immediately interpret them in terms of your own context. When I was a student, I learned things by going to lectures and reading books – after that I read very few books. I would talk with people; I would learn the essence of analysis by talking to Hörmander or other people. I would be asking questions because I was interested in a particular problem. So you learn new things because you connect them and relate them to old ones, and in that way you can start to spread around.

If you come with a problem, and you need to move to a new area for its solution, then you have an introduction – you have already a point of view. Interacting with other people is of course essential: if you move into a new field, you have to learn the language, you talk with experts; they will distil the essentials out of their experience. I did not learn all the things from the bottom upwards; I went to the top and got the insight into how you think about analysis or whatever.

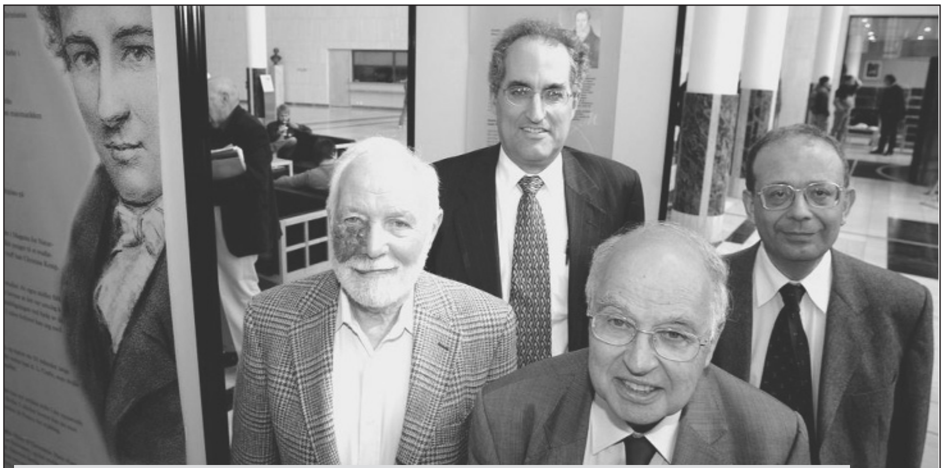
SINGER I seem to have some built-in sense of how things should be in mathematics. At a lecture, or reading a paper, or during a discussion, I frequently think, "that's not the way it is supposed to be." But when I try out my ideas, I'm wrong 99% of the time. I learn from that and from studying the ideas, techniques, and procedures of successful methods. My stubbornness wastes lots of time and energy. But on the rare occasion when my internal sense of mathematics is right, I've done something different.

Both of you have passed ordinary retirement age several years ago. But you are still very active mathematicians, and you have even chosen retirement or visiting positions remote from your original work places. What are the driving forces for keeping up your work? Is it wrong that mathematics is a "young man's game" as Hardy put it?

ATIYAH It is no doubt true that mathematics is a young man's game in the sense that you peak in your twenties or thirties in terms of intellectual concentration and in originality. But later you compensate that by experience and other factors. It is also true that if you haven't done anything significant by the time you are forty, you will not do so suddenly. But it is wrong that you have to decline, you can carry on, and if you manage to diversify in different fields this gives you a broad coverage. The kind of mathematician who has difficulty maintaining the momentum all his life is a person who decides to work in a very narrow field with great depths, who e.g. spends all his life trying to solve the Poincaré conjecture – whether you succeed or not, after 10-15 years in this field you exhaust your mind; and then, it may be too late to diversify. If you are the sort of person that chooses to make restrictions to yourself, to specialize in a field, you will find it harder and harder – because the only things that are left are harder and harder technical problems in your own area, and then the younger people are better than you.

You need a broad base, from which you can evolve. When this area dries out, then you go to that area – or when the field as a whole, internationally, changes gear, you can change too. The length of the time you can go on being active within mathematics very much depends on the width of your coverage. You might have contributions to make in terms of perspective, breadth, interactions. A broad coverage is the secret of a happy and successful long life in mathematical terms. I cannot think of any counter example.

SINGER I became a graduate student at the University of Chicago after three years in the US army during World War II. I was older and far behind in mathematics. So I was shocked when my fellow graduate students said, "If you haven't proved the


The Abel lectures were given by I. Singer, E. Witten, M. Atiyah and J.- M. Bismut

Riemann Hypothesis by age thirty, you might as well commit suicide." How infantile! Age means little to me. What keeps me going is the excitement of what I'm doing and its possibilities. I constantly check [and collaborate!] with younger colleagues to be sure that I'm not deluding myself – that what we are doing is interesting. So I'm happily active in mathematics. Another reason is, in a way, a joke. String Theory needs us! String Theory needs new ideas. Where will they come from, if not from Sir Michael and me?

ATIYAH Well, we have some students…

SINGER Anyway, I am very excited about the interface of geometry and physics, and delighted to be able to work at that frontier.

History of the EMS

You, Prof. Atiyah, have been very much involved in the establishment of the European Mathematical Society around 1990. Are you satisfied with its development since then?

ATIYAH Let me just comment a little on my involvement. It started an awful long time ago, probably about 30 years ago. When I started trying to get people interested in forming a European Mathematical Society in the same spirit as the European Physical Society, I thought it would be easy. I got mathematicians from different countries together and it was like a mini-UN: the French and the Germans wouldn't agree; we spent years arguing about differences, and – unlike in the real UN – where eventually at the end of the day you are dealing with real problems of the world and you have to come to an agreement sometime; in mathematics, it was not absolutely essential. We went on for probably 15 years, before we founded the EMS.

On the one hand, mathematicians have much more in common than politicians, we are international in our mathematical life, it is easy to talk to colleagues from other countries; on the other hand, mathematicians are much more argumentative. When it comes to the fine details of a constitution, then they are terrible; they are worse than lawyers. But eventually – in principle – the good will was there for collaboration.

Fortunately, the timing was right. In the meantime, Europe had solved some of its other problems: the Berlin Wall had come down – so suddenly there was a new Europe to be involved in the EMS. This very fact made it possible to get a lot more people interested in it. It gave an opportunity for a broader base of the EMS with more opportunities and also relations to the European Commission and so on.

Having been involved with the set-up, I withdrew and left it to others to carry on. I have not followed in detail what has been happening except that it seems to be active. I get my Newsletter, and I see what is going on.

Roughly at the same time as the collapse of the Berlin Wall, mathematicians in general – both in Europe and in the USA – began to be more aware of their need to be socially involved and that mathematics had an important role to play in society. Instead of being shut up in their universities doing just their mathematics, they felt that there was some pressure to get out and get involved in education, etc. The EMS took on this role at a European level, and the EMS congresses – I was involved in the one in Barcelona – definitely made an attempt to interact with the public. I think that these are additional opportunities over and above the old-fashioned role of learned societies. There are a lot of opportunities both in terms of the geography of Europe and in terms of the broader reach.

Europe is getting ever larger: when we started we had discussions about where were the borders of Europe. We met people from Georgia, who told us very clearly, that the boundary of Europe is this river on the other side of Georgia; they were very keen to make sure that Georgia is part of Europe. Now, the politicians have to decide where the borders of Europe are.

It is good that the EMS exists; but you should think rather broadly about how it is evolving as Europe evolves, as the world evolves, as mathematics evolves. What should its function be? How should it relate to national societies? How should it relate to the AMS? How should it relate to the governmental bodies? It is an opportunity! It has a role to play!

Apart from mathematics…

Could you tell us in a few words about your main interests besides mathematics?

SINGER I love to play tennis, and I try to do so 2-3 times a week. That refreshes me and I think that it has helped me work hard in mathematics all these years.

ATIYAH Well, I do not have his energy! I like to walk in the hills, the Scottish hills – I have retired partly to Scotland. In Cambridge, where I was before, the highest hill was about this (gesture) big. Of course you have got even bigger ones in Norway. I spent a lot of my time outdoors and I like to plant trees, I like nature. I believe that if you do mathematics, you need a good relaxation which is not intellectual – being outside in the open air, climbing a mountain, working in your garden. But you actually do mathematics meanwhile. While you go for a long walk in the hills or you work in your garden – the ideas can still carry on. My wife complains, because when I walk she knows I am thinking of mathematics.

SINGER I can assure you, tennis does not allow that!

Thank you very much on behalf of the Norwegian, the Danish, and the European Mathematical Societies!

The interviewers were Martin Raussen, Aalborg University, Denmark, and Christian Skau, Norwegian University of Science and Technology, Trondheim, Norway.

1 More details were given in the laureates' lectures.

2 Among those: Newton, Gauss, Cauchy, Laplace, Abel, Jacobi, Riemann, Weierstrass, Lie, Picard, Poincaré, Castelnuovo, Enriques, Severi, Hilbert, Lefschetz, Hodge, Todd, Leray, Cartan, Serre, Kodaira, Spencer, Dirac, Pontrjagin, Chern, Weil, Borel, Hirzebruch, Bott, Eilenberg, Grothendieck, Hörmander, Nirenberg.

3 Comm. Pure App. Math. **13**(1), 1960.

Oslo 2004: The Abel Prize celebrations

Nils Voje Johansen and Yngvar Reichelt (Oslo, Norway)

On 25 March, the Norwegian Academy of Science and Letters announced that the Abel Prize for 2004 was to be awarded to *Sir Michael F. Atiyah* of the University of Edinburgh and *Isadore M. Singer* of MIT.

This is the second Abel Prize awarded following the Norwegian Government's decision in 2001 to allocate NOK 200 million to the creation of the Abel Foundation, with the intention of awarding an international prize for outstanding research in mathematics. The prize, amounting to NOK 6 million, was instituted to make up for the fact that there is no Nobel Prize for mathematics. In addition to awarding the international prize, the Foundation shall contribute part of its earnings to measures for increasing interest in, and stimulating recruitment to, mathematical and scientific fields.


Nils Voje Johansen


Yngvar Reichelt

The first Abel Prize was awarded in 2003 to the French mathematician Jean-Pierre Serre for playing a key role in shaping the modern form of many parts of mathematics. In 2004, the Abel Committee decided that Michael F. Atiyah and Isadore M. Singer should share the prize for:

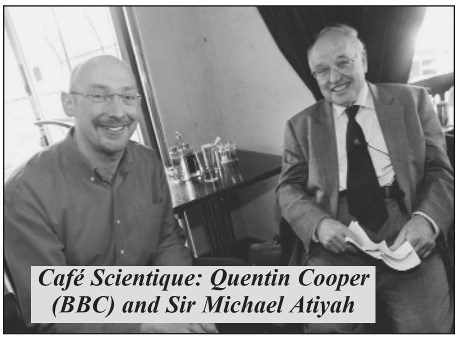
> *their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics.*

This year's committee consisted of Erling Størmer (Oslo, Leader), David Mumford (Brown University), Jacob Palis (IMPA, Brazil), Gilbert Strang (MIT) and Don Zagier (Max-Planck-Institut für Mathematik, Germany).

The Abel Prize for 2004 was presented on 25 May, the occasion being marked by a number of associated events in Oslo.

Café Scientifique

On Sunday 23 May, Sir Michael Atiyah participated in the first event in connection with the award of this year's Abel Prize. In collaboration with the Norwegian Association of Young Scientists, the British Council arranged a Café Scientifique at the Kafé Rust in Oslo, in which Atiyah gave an informal lecture on his chosen subject: *Man versus machine – the brain and the computer*, with the subtitle "Will a computer ever be awarded the Abel Prize?" Quentin Cooper, one of the BBC's most popular radio presenters, chaired the meeting, in which Sir Michael spoke for an hour to an audience of about 50 people. He pointed out that while computers are extremely adept at following pre-determined rules and that he himself is not surprised that, for example, very good chess programs have been developed, what is surprising is that people are still able to play chess on an equal footing with machines. In other words, computers are good at following rules, but what they are not able to do is to break the rules in a creative manner. As an example, he cited Niels Henrik Abel's proof of the impossibility of solving the general quintic equation. A computer would have continued to search for the solution, and would never have been able to break the rules, as Abel did, and look at the inverse of the problem. As a mathematician you have to know the rules, but to create something new you have to break those rules creatively, just like an artist or a musical composer.


Café Scientique: Quentin Cooper (BBC) and Sir Michael Atiyah

After a brief interval, Quentin Cooper invited questions from the audience and a number of points were brought up that Atiyah addressed thoroughly and professionally.

After a highly successful meeting lasting almost two hours, Atiyah answered his own question, "Will a computer ever be awarded the Abel Prize?": *– Only if the Abel Prize Committee is replaced by computers*.

Youth and maths in the celebration of Niels Henrik Abel

Monday 24 April was Youth Day, on which the Abel Committee invited the winners of various mathematical competitions for young people to Oslo. In addition to the winners of the Abel Competition for Norwegian upper secondary schools and of KappAbel, the mathematics competition for schools, the winners of mathematics competitions in Berlin and France were also invited. The main event took place at Oslo Cathedral School – the school at which Abel himself was a pupil. An audience of 200 was assembled, mostly consisting of the school's own mathematics pupils. After a brief introduction by Paul Jasper, headmaster of the school, one of the pupils, Jon Strand, took over and led the proceedings with capable hands. First the French students presented their winning

Awards to the winners of the Abel competition

entry to the competition, "*Niels Henrik Abel in the French tradition*". The audience were then given an introduction to the winning project in this year's KappAbel competition, and Sir Michael Atiyah and Professor Isadore Singer awarded a book prize to the young participants. After this, Per Manne, the leader of the Norwegian Mathematics Council, took the floor and presented information about a recently established prize for good mathematics teaching, in memory of *Bernt Michael Holmboe*. Holmboe was the teacher who discovered Niels Henrik Abel's exceptional talent and who was his early tutor. The Norwegian Mathematics Council arranges the Holmboe Prize, with financial support from the Abel Foundation. The prize is to be awarded annually to one or more teachers who have distinguished themselves through high quality and inspiring mathematics teaching. In conjunction with the award of the prize, a symposium on mathematics and mathematics teaching will also be held.

Wreath laying at the Abel monument

At 5 p.m. there was a wreath laying ceremony at Gustav Vigeland's monument to Abel, which stands in front of the Royal Palace in Oslo. The ceremony began with a display by a troop from His Majesty's Corps of Signals. Then the leader of the Abel Committee, Jens Erik Fenstad, held a short speech in which he explained about the origin of the Abel monument. The ceremony culminated in the wreath laying by Atiyah and Singer. Afterwards, the younger contingent retired to a restaurant, while the two prize winners and the members of the Norwegian Academy of Science and Letters were invited to dinner at the Academy.

The prize ceremony

After an audience with Their Majesties King Harald and Queen Sonja earlier in the day, it was time for the prize ceremony in the Great Hall of Oslo University. The main street of Oslo, Karl Johans gate, was decorated for the occasion with colourful Abel Prize banners. This year's prize winners arrived at the packed hall to the sound of Klaus Sanvik's recently-composed *Abel Fanfare*, performed by Sidsel Walstad on electric harp, followed by the arrival of the King and Queen.

Lars Walløe, the President of the Norwegian Academy of Science and Letters, welcomed those present to the ceremony. Before the official presentation, the audience were given a surprise in the form of a new arrangement of Michael Jackson's *Billy Jean*, performed by Sidsel Walstad (electric harp), Mocci Ryen (vocals) and Børre Flyen (percussion). The lively and youthful performance was appreciated, at least by Isadore Singer, who tapped his foot enthusiastically in time with the music.

The leader of the Abel Committee, Erling Størmer, briefly explained the reasons for the selection of Atiyah and Singer as this year's prize winners: "*The Atiyah-Singer index theorem is one of the most important mathematical results of the twentieth century. It has had an enormous impact on the further development of topology, differential geometry and theoretical physics. The theorem also provides us with a glimpse of the beauty of mathematical theory in that it explicitly demonstrates a deep connection between mathematical disciplines that appear to be completely separate."*

After the address, His Majesty King Harald presented the Abel Prize to the two winners.

Sir Michael Atiyah commenced his acceptance speech by thanking colleagues who had made important contributions to the work, mentioning in particular Fritz Hirzebruch, Raoul Bott, Graeme Segal and Nigel Hitchin. He went on to explain that right from the time of Newton to that of Einstein there has been a close relationship between mathematics and the exploration of the natural world. *"One of the unexpected joys of my partnership with Is Singer has been that these links with physics have been reinforced during our time"* In conclusion, Atiyah said that in his opinion, "*Abel was really the first modern mathematician. His whole approach, with its generality, its insight and its elegance, set the tone for the next two centuries. If Abel had lived longer, he would have been the natural successor to the great Gauss: a statement with which I fully concur except for the qualification that Abel was a much nicer man, modest, friendly and likeable. I am proud to have a prize that bears his name.*"

Abel Fanfare (Photo: Anne Lise Flavik)

After that it was the turn of Isadore Singer, who started with a confession. *"Outside of the university environment it is difficult to be a pure mathematician. No one in my family understands what I do. At parties, when someone learns I am a mathematician, they frown and say; "Oh, I never could understand calculus", and they turn away."* After this description, all too familiar to many, Singer described how mathematicians are fascinated by the beauty, logic and power of mathematics. The index theorem itself had "*provided new insight in such fields as Gauge theory and String theory. Breakthroughs in physics needed new mathematics, and the index theory frequently supplied what was needed. Mathematicians and physicists began talking to each other again. Now we take for granted this new discipline of mathematical physics."* Finally, Singer stated that the establishment of the Abel Prize attracts the attention of the world and emphasises the fundamental role which mathematics plays in modern living.

The ceremony was concluded with Edvard Grieg's *Halling* before the King and Queen and the Abel Prize winners left the hall.

Press conference

Following the prize ceremony, a press conference was held in the "Annen Etage" restaurant at the Hotel Continental. Jens Erik Fenstad and Jacob Palis commenced by providing information about the involvement of the Abel Foundation and the IMU with regard to the developing countries, after which the floor was open to the Press to put their questions to the prize winners. There was also an opportunity to taste something new. Morten Hallan, the hotel's Chef, had created a completely new Abel cake, which was on sale in the legendary *Theatercafeen* during "Abel Week". The recipe is of course secret, but the different layers of the cake are as follows: First a base of chocolate cake soaked in blackcurrant liqueur, followed by a layer of chocolate truffle and then a layer of blackcurrant preserve, covered by nut meringue, and topped off with

Italian meringue. The cake is decorated with white and dark chocolate and caramel cornets filled with blackcurrants. Bon appétit! The cake will also be served in connection with future prize ceremonies.

The Abel cake with the logo of the Abel Prize (Photo: Knut Falch/Scanpix)

Banquet

At 7 p.m. the same day, the Norwegian Government held an Abel banquet at Oslo's historic Akershus Fortress, hosted by Kristin Clemet, the Minister of Education and Research. The banquet was attended by the King and Queen, Norwegian and foreign mathematicians, eminent politicians and members of Norwegian society. Many people recognised the mathematician John Donaldson, father of Danish Crown Princess Mary Donaldson, who was specially invited to the Abel Prize ceremony by King Harald at the wedding of the Crown Prince of Denmark some weeks before. After a welcoming cocktail in Christian IV's Hall, the party proceeded to Skriverstuen (the Scribes' Hall), where all were welcomed by the Minister of Education and Research, Sir Michael and Professor Singer. Dinner was then served in the Romerike Hall. The menu consisted of Norwegian trout and turbot, veal fillet and caramel mousse.

In her address, the Minister, Kristin Clemet, mentioned that there was one winner (Jean-Pierre Serre) at the first Abel Prize ceremony, while at the second there were two winners. Based on this, one could perhaps draw the conclusion that the number of prize winners would be determined by the equation $x = n$, where n is the number of years the prize has existed. Having reminded the audience of the story of Descartes, who died of pneumonia when he visited Scandinavia, she touched on the effect of the Abel Prize on the recruitment of future mathematicians. The Holmboe Prize has now been created to honour the best mathematics teachers – those with the ability to impart understanding of the beauty of mathematics to their students. In connection with this she quoted Sir Michael: *By exploring the whole country of mathematics you get to the top of Mount Everest and look around. It's a long route, and when you get to the top, it's a big scene you can see*. The Minister went on to express her delight with the international response to the Abel Prize: it had been extremely positive, and she noted that the selection of winners had gained widespread support.

She then gave the floor to the president of the European Mathematical Society, Sir John Kingman.

In his speech, Sir John touched on the criteria for the successful establishment of the Abel Prize: "*The great name of Niels Henrik Abel is important. Of course the actual value of the prize is important. It is also important that those who select the prize winners select people that future prize winners will be proud to follow*."

He went on to point out that such a prize could have a positive effect on recruitment, which is of considerable importance since we need "*new mathematicians doing new mathematics. Whether it is what we call pure mathematics, mathematics for its own sake, or whether it is pursuing interesting new applications, applying the techniques which the so-called pure mathematicians have invented*."

Exhibition

In connection with the award of this year's Abel Prize, an exhibition was staged with the objective of informing laypersons about the work of the prize winners. The exhibition was based on simple concepts in the fields of topology, geometry and analysis, and demonstrated how the links between the disciplines had been discovered. An example of such a link is illustrated by the Gauss-Bonnet theorem. By means of the Atiyah-Singer index theorem, an overall unification of the disciplines was acquired. In turn, this new insight formed the origin of countless applications in the field of theoretical physics. The exhibition paid special attention to applications in the fields of gauge theory and string theory. The exhibition was located in the foyer outside the auditorium in which the Abel Lectures were held and was the result of collaboration between the Departments of Mathematics and Physics at the University of Oslo.

The mathematicians' own party

On the evening of 26 May, it was time to drop the formalities. The Abel days in Oslo were wound up with a party at the Norwegian Academy of Science and Letters. Mathematicians from near and far were invited, in addition to other people who had worked with this year's events. The atmosphere was pleasant and everybody had an opportunity to meet old friends and make new acquaintances. As a contribution to the convivial atmosphere, refreshments were served and throughout the venerable building musical treats of various types could be enjoyed.

Nils Voje Johansen [nilsvo@math.uio.no] and Yngvar Reichelt [reichelt@math.uio.no] are associated to the Department of Mathematics at the University of Oslo. Both were members of a group of mathematicians that formulated the suggestion to the Norwegian government to install the Abel Prize in commemoration of Niels Henrik Abel's bicentenary, which was finally adopted by the Norwegian parliament.

The Abel Prize 2005

King Harald of Norway will present the Abel Prize for 2005 to the winner on May 24th in the Aula of the University of Oslo. The deadline for nominating candidates is the 15th of November 2004.

Nominations letters should contain a CV and a description of the candidate's work, together with names of distinguished specialists in the field of the nominee who can be contacted for independent opinion. The letter should be marked "Abel Prize Nomination" and addressed to:

The Norwegian Academy of Science and Letters
Drammensveien 78
NO-0271 OSLO
Norway

Detailed information is obtainable from the web site *www.abelprize.no* .

THE GREEK MATHEMATICAL SOCIETY

Its predecessors, its founders, and some highlights from its life

(Themistocles M. Rassias, Athens, Greece)

The beginnings

The history of "modern" Greek mathematics begins with Nicholas Nicolaidis (1826-1889). Born during the Greek Revolution in the very center of Arcadia, officer-engineer in the Greek Army, he went with a scholarship to Paris and became a Docteur d'Etat es Sc. Math., with O. Bonnet in his Committee. He published several original works in the Editions of Gauthier-Villars. When he returned to Greece he became professor at the University of Athens. His research work continues to enjoy some interest even in the present time (he is known, in particular, for the term "Nicolaidis' theorem" in Kinematics). With him begins our story.

Cyparissos Stephanos

John Hadzidakis

The next to appear in Greek mathematics were the professors Cyparissos Stéphanos (1857-1917) from the island of Kea, and John Hadzidakis (1844-1921) from Crete. Stéphanos was a Docteur d'Etat es Sc. Math. from Paris, with Charles Hermite (1822-1901) as a member of his Committee. He was a geometer and algebraist, with references made to his work by a number of mathematicians including the young (at that time) David Hilbert (1862-1943). He also served as a member of a number of Honorary Committees of International Congresses. J. Hadzidakis is known for his work in Geometry with the notion "Hadzidakis transformation", that is cited in M. Spivak's "Differential Geometry" and in J. McCleary's "Geometry from a Differential Viewpoint". He was a pupil of the Paris school and of Karl Weierstrass (1815-1897) in Berlin. He is also well known in Greece for writing good books for all levels of mathematical education.

J. Hadzidakis and C. Stéphanos established the mathematical tradition of the University of Athens and were the teachers of the founders of the Greek Mathematical Society (G.M.S.). These founders, all three professors at the University of Athens, were Nicholas Hadzidakis (1872-1942), a son of J. Hadzidakis, George Remoundos (1878-1928) from Athens, and Panayotis Zervos (1878-1952) from Cephalonia. All of them had been pupils of the Paris School, in particular N. Hadzidakis of David Hilbert. Following the geometric approach of Jean Gaston Darboux (1842-1917), N. Hadzidakis introduced Kinetic Geometry, as well as the study of complexes of lines and surfaces, to Greece. Remoundos contributed a lot in function theory and in particular with his generalization of Picard's theorem to algebraic functions. In the year 1905, Jacques Hadamard (1865-1963) proposed to P. Zervos the study of Monge's conjecture (sometimes called Monge's problem) in the field of Partial Differential Equations, which was posed in 1784 and has not been solved since. In 1905, P. Zervos was the first mathematician to notice the falsity of Monge's conjecture. In the year 1912, after some intensive work by P. Zervos, E. Goursat, and O. Botasso among others, D. Hilbert solved a major part of Monge's problem. Elie Cartan's seminal paper of the year 1914 on Pfaff's problem, begins as follows: "In a recent paper (1913), P. Zervos generalizes a theorem of D. Hilbert…" (Bull. de la Soc. Math. de France, 1914).

Foundation of the G.M.S.

In the year 1918, N. Hadzidakis, G. Remoundos and P. Zervos founded the G.M.S. as well as the Research Seminars in Mathematics at the University of Athens**.** In addition, the year after, in 1919, they started the international journal: *Bulletin of the G.M.S***.** The first issue of the Bulletin of the G.M.S. published P. Zervos' inaugural lesson entitled "Relation of Mathematics to the other Sciences and to Philosophy" as well as some "technical" articles in mathematics by the above three mathematicians and others. Zervos had been in the audience of Poincaré's lectures on Celestial Mechanics in Paris during the period 1903-1905, and he was also the translator into Greek of Poincaré's celebrated book "Science and Hypothesis". In 1935, the Bulletin of the G.M.S. published a paper by Nicholas Criticos (1894-1985) that was devoted to a new proof of the Jordan

Curve Theorem. In the context of two lectures given by him in the G.M.S., a kind of first advanced text in "*Point Set Topology*" was provided, written in Greek.

Papakyriakopoulos and Carathéodory

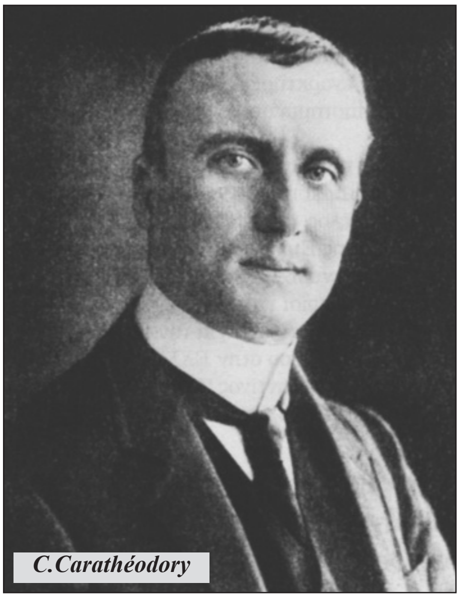
A rigorous presentation of Mathematical Analysis on the basis of Dedekind Cuts and Cantor's Set Theory had been made by P. Zervos in his masterpiece: "Infinitesimal Calculus". Both of the above mentioned texts, by N. Criticos and P. Zervos, contributed greatly to the orientation in mathematical studies of the well-known topologist Christos D. Papakyriakopoulos (Athens, 1913 - Princeton, 1976). The doctoral thesis of Papakyriakopoulos, entitled "A new proof of the invariance of the homology groups of a complex", was published in the year 1943 in the Bulletin of the G.M.S. The 154-page thesis of Papakyriakopoulos that was published in the Bulletin also constitutes the first text written in Greek on Algebraic Topology. The (unofficial) referee for the Thesis was C. P. Papaioannou (1899-1979), who was Professor of Mechanics at the University of Athens. Papaioannou had the ability to immediately foresee Papakyriakopoulos' brilliant research career. Another prophetic vision of Papaioannou appears in his inaugural speech at the Academy of Athens in the year 1965, in which he sees the universe as Platon's dodecahedron. This is something that was later very beautifully presented in a modern mathematical language by William P. Thurston (Fields Medal in Topology, 1982) in the Scientific American. This seems to be a major subject of research by satellites (cf. Notices of the A.M.S., June-July 2004).

The 1943 issue of the Bulletin of the G.M.S. was, naturally, dedicated to Constantin Carathéodory at the occasion of his 70-th birthday. Carathéodory was born in Berlin, of Greek parents, on 13 September 1873, and he died in Munich on 2 February 1950. His work covered several subjects of Mathematics, including the calculus of variations, function theory, measure and integration, as well as applied mathematics. Carathéodory has also done very fundamental work in mechanics, thermodynamics, geometrical optics and relativity theory. An example of Carathéodory's wide ranging influence in the international mathematical community was seen during the first Fields Medals awards at the International Congress of Mathematicians, Oslo, 1936. The selection committee consisted of George D. Birkhoff, Elie Cartan, C. Carathéodory, F. Severi, and T. Takagi. Two medals were awarded, one to Lars V. Ahlfors (Harvard University) and one to Jesse Douglas (M.I.T). It was C. Carathéodory who presented the work of both medalists during the opening of the International Congress (see *Constantin Carathéodory: An International Tribute, Vols I & II, World Scientific Publ. Co., 1991, edited by Th. M. Rassias*).

Now let us come back to our story. Remoundos' pupil Theodore Varopoulos (1894-1957), professor of the University of Thessaloniki (father of the well-known Paris Analyst Nicholas Th. Varopoulos), the closest pupil of Paul Montel, had a special interest in the Geometry of Polynomials. From time to time one could find articles in the Bulletin of the G.M.S. devoted to polynomials (from both an algebraic and a geometric approach) influenced directly or indirectly by Th. Varopoulos. For instance, in the thirties one can find articles by Constantine Yannopoulos and by John Papademetriou. Around 1950, one has to mention the thesis on polynomials written in Athens by Th. Varopoulos' research student Dionysios Vythoulcas. Moving further and with more essential results in the context of polynomials and especially their localization of roots (zeros), Spyros P. Zervos (a son of P. Zervos) wrote his Thèse d'Etat in Paris (Ann. Sc. de l'Ecole Normale Sup., 1960) in the spirit of Modern Mathematics. Furthermore, some interesting work in the classical version of polynomials and related subjects was obtained by the late I. Ferentinou-Nicolacopoulou and was published in the Bulletin of the G.M.S.

Other activities

Until now, we have been devoted to the scientific side of the subject and have not covered the general activity of the G.M.S. In this connection, we recall the Panhellenic Competition in Mathematics for High School Students, inaugurated by the G.M.S. in the year 1931. The first winner was George Tsamis from Patras, at that time a schoolboy, who was to become a very influential High School Teacher in mathematics. In the sixties, the Mathematical Competition got a new lease of life by Aristide Pallas, a man who was entirely devoted to the G.M.S and at that time its president.



C.Carathéodory

In 1975, the new Council of the G.M.S., under the presidency of Professor S.P.Zervos, enlarged the domain of the activities and publications of the G.M.S. In that year, S.P. Zervos conceived the idea for the foundation of a "University of Aegean", with a Pythagorean Department of Mathematics, on the island of Samos, and other Departments (Schools) in various subjects on other islands. This idea was materialized later on. Some distinguished mathematicians delivered lectures during the "Pythagorean Days", in Athens and in Samos. Among the mathematicians who delivered such lectures are the following: K. Borsuk (Warszawa), L. Iliev (Sofia), A. Kawaguchi (Japan), M. Krasner (Paris), K. Kuratowski (Warszawa), Dj. Kurepa (Beograd), M. Loi (Paris), O. Onicescu (Bucharest).

A subsequent article will deal with the 'discovery' of the 'Olympiads' for High Schools in Greece, as well as to activities of the G.M.S. with emphasis to the last three decades.

Themistocles M. Rassias [trassias@math.ntua.gr] studied mathematics at the University of California at Berkeley, where he received his Ph.D with Steven Smale. He has published more than 170 papers and six research books and he has edited 24 volumes on different subjects in Mathematical Analysis, Geometry/Topology and their applications. His research work has found more than 2000 citations. His work is known in the field of Mathematical Analysis with the term "Hyers-Ulam-Rassias Stability" and in Geometry with the term "Alexandrov - Rassias Problem". He has lectured extensively and conducted research in various academic institutions in Europe and North America. More than ten mathematical journals count him as a member of their Editorial Board. He is Professor at the National Technical University of Athens, Greece. He is married and has two children.

FRENCH-POLISH COOPERATION IN APPROXIMATION THEORY AND REAL AND COMPLEX ANALYSIS

Wiesław Pleśniak (Jagiellonian University, Kraków, Poland)

There has always been a great tradition of academic exchange and contact between France and Kraków in the field of mathematics. The origin of the formal French-Polish cooperation between the Institute of Mathematics at the Jagiellonian University and French universities in the domain of approximation theory and real and complex analysis is almost anecdotic. During his stay at the Paul Sabatier University (Toulouse) in 1985, Prof. Wiesław Pleśniak learned that Prof. Jacek Gilewicz (Université de Toulon et du Var and Centre de Physique Théorique, Marseille-Luminy) and the physicists from the University of Warsaw had been intending to organize an international conference on rational approximation at the Castle of Łańcut, the home town of Pleśniak. This inspired Prof. Pleśniak to go to Marseille and meet Prof. Gilewicz. As a consequence of that meeting, Prof. Jacek Gilewicz was invited to the Institute of Mathematics at the

The participants of the 1992 conference in Niedzica Castle.

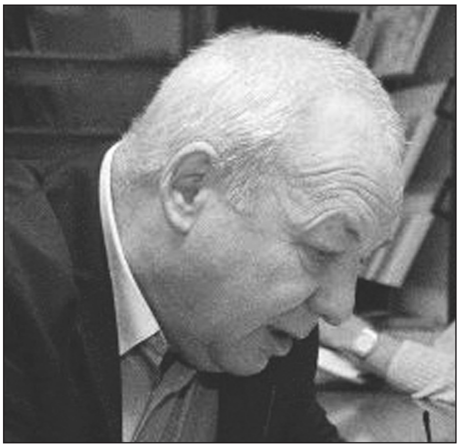
Jagiellonian University and spent two months there in 1986. During this visit, Prof. Gilewicz and Prof. Pleśniak had succeeded in attracting interest from the French Embassy in Warsaw. It ended in the cooperation that yielded a formal cooperation project under the ATP Programme (Action Thématique Programmée) supported by the French Ministry of Foreign Affairs and the Polish State Committee for Scientific Research, The French universities were represented by the Universite de Toulon et du Var, Universite Paul Sabatier (Toulouse), Université de Paris XI (Orsay), Université des Sciences et Technologies de Lille, and then the Centre International des Rencontres Mathématiques (Marseille-Luminy). The project was coordinated by Professor Jacek Gilewicz and then by Professor Jean-Paul Brasselet (Centre International des Rencontres Mathématiques) and the undersigned Wiesław Pleśniak (Institute of Mathematics of the Jagiellonian University).

Thanks to the financial support from the French Embassy in Poland and the Institute of Mathematics of the Jagiellonian University, the coordinators had succeeded in organising a series of seven French-Polish meetings in Poland (Karniowice near Kraków, the Niedzica Castle, and then Krynica) under the common theme: *Le français des mathématiciens.* These meetings served as the permanent forum of the presentation of the progress of both Polish and French participants of the ATP Project in research concerning the subject of cooperation. The meetings were also opportunities of first contacts with French scientific language and French culture for young Polish mathematicians that were planning for invited stays in French universities.

First, we should make special note of an important French-Polish meeting at the Centre of the Polish Academy of Sciences in Paris in November of 1995 that was organized by Prof. Jacek Gilewicz and Prof. Wiesław Pleśniak, At this meeting, the programme of future co-operation in the above mentioned subject was scheduled. The next important step was the common "Réseau-Formation-Recherche" project that was implemented 1995-1998 under the coordination of Prof. Anne-Marie Chollet (Université des Sciences et Technologies de Lille) and Prof. Wiesław Pleśniak. Thanks to this project, Polish partners from the Jagiellonian University and the Adam Mickiewicz University of Poznań obtained 56 months of French Government fellowships for Polish doctorate students and globally, more than five hundred thousand French Francs for the exchange programme.

Other common projects included the work-

Ceremony at Toulon, 23 October 2003. Wiesław Pleśniak, Jacek Gilewicz, Józef Siciak, Mirosław Baran.



shop on Differential Analysis at the International Banach Center of Warsaw (September 2000), (organized by Prof. Anne-Marie Chollet, Prof. Krzysztof Kurdyka (Université de Savoie a Chambery), and Prof. Wiesław Pleśniak. Moreover, the workshop on Approximation Theory (January 2002) was organised at the same International Banach Center (organised by Wiesław Pleśniak with the cooperation of Prof. Anne-Marie Chollet, Vincent Thilliez (Université des Sciences et Technologies de Lille), and Jacques Chaumat (Université de Paris-Sud).

Since 1998, the cooperation continues under the "Polonium" Project "Real and Complex Analysis" coordinated by Anne-Marie Chollet and Wiesław Pleśniak which has been accepted by the French-Polish Government Commission for implementation in 2004.

Wiesław Pleśniak [plesniak@im.uj.edu. pl], born in 1944, is a professor of the Jagiellonian University at Kraków, Poland, where he holds the Chair in Approximation Theory at the Department of Mathematics. His main mathematical interest is approximation theory and complex analysis. He is a member of Editorial Boards of three journals: Annales Polonici Mathematici, Commentationes Mathematicae and East Journal on Approximation. For some years he served as the President of the Kraków branch of the Polish Mathematical Society; moreover, he was the Vice-Dean of the Faculty. Currently he is the Vice-President of the Mathematical Committee of the Polish Academy of Sciences.

Wiesław Pleśniak has received a doctorate honoris causa from the University of Toulon in France. He is a member of Société Royale des Sciences de Liège and of the Polish Academy of Arts and Sciences.

Among other distinctions, he was awarded the Mazurkiewicz Great Prize and the Zaremba Great Prize of the Polish Mathematical Society (1977, 1989), and he has been awarded scientific prizes from the Polish Academy of Science three times.

CRM

Centre de Recerca Matemàtica, Barcelona

Institutional framework

The CRM was created in 1984 by the *Institut d'Estudis Catalans* (Institute for Catalan Studies), an academic, scientific and cultural body whose aim it is to promote research in science, technology and humanities, while supporting all aspects of Catalan culture. Since 2001, the CRM has been a consortium, with its own legal status, integrated by the *Institut d'Estudis Catalans* and the Catalan Government. It is a research institute associated with the *Universitat Autònoma de Barcelona* and located on the Bellaterra campus, about 15 km from Barcelona.

The Governing Board of the CRM has eight members and is chaired by the Minister of Universities, Research and the Information Society of the Catalan Government. In 2002, the Governing Board re-elected Manuel Castellet as Director and appointed a new Scientific Advisory Board, with the following members: Joan Bagaria, Àngel Calsina, Carles Casacuberta, Vicent Caselles, Alberto Facchini, Evarist Giné, Joan Girbau, Antoni Huerta, Jaume Llibre, Xavier Massaneda, M. Pilar Muñoz, Joan C. Naranjo, David Nualart, Pere Pascual, Joan Porti, Jordi Quer, Oriol Serra, and Juan L. Vázquez. Since 2004, Carles Casacuberta and Jordi Quer have acted as Vice-Directors.

Aims and scope

The CRM gives support to local research groups in all areas of mathematics and fosters emerging research directions by inviting outstanding mathematicians and organising specialised research programmes, conferences, workshops, and advanced courses.

It hosts post-doctoral fellows (thirteen in 2003), including several Marie Curie grant holders, and it has also hosted doctoral students as a Marie Curie Training Site of the European Union from 2000 to 2004. Between 2000 and 2003, the CRM was a node of an EC Training Network entitled *Modern Homotopy Theory*, jointly with teams in Aarhus, Aberdeen, Lille, Louvain-la-Neuve, Paris, and Sheffield.

Research programmes

In 2002, the Governing Board of the CRM approved a quadrennial plan that includes two research programmes every year, together with complementary activities. Each research programme offers the following positions during one academic year: one full-time local researcher, one full-time visiting researcher, two post-doctoral fellows, and 24 months of visiting researchers for periods of one to three months. Activities include a weekly seminar, a conference or a workshop and an advanced course at a doctoral or post-doctoral level. Partial funding for visitors and activities is provided by the Department of Universities, Research and the Information Society (DURSI) of the Catalan Government, under a contract programme that is revised every year, and by means of competitive calls. Other sources of funding, through competitive applications, are the Spanish Ministry of Education and Science and the European Commission.

An open call for research programmes is made every year. Each programme has to be approved by the CRM Governing Board. Proposals are presented by the Director on the grounds of evaluation reports prepared by the Scientific Advisory Board.

During the academic year 2003-2004, a research programme on Set Theory was carried out. The organisers were Joan Bagaria (ICREA and Universitat de Barcelona) and Stevo Todorcevic (CNRS and Université Paris 7). For the academic year 2004-2005, a research programme on the Geometry of the Word Problem is planned, under the direction of Josep Burillo and Enric Ventura (Universitat Politècnica de Catalunya), and Noel Brady (University of Oklahoma). Shorter specialised programmes will be held on Control, Geometry and Engineering, co-ordinated by Miguel C. Muñoz (UPC), and on Contemporary Cryptology, co-ordinated by Jorge Villar and Carles Padró (UPC). The Scientific Advisory Board selected two programmes for the academic year 2005-2006: one on Arakelov Geometry and Shimura Varieties, co-ordinated by José I. Burgos (Universitat de Barcelona) and Jörg Wildeshaus (Université Paris 13), and another one on Hilbert's 16th Problem, co-ordinated by Armengol Gasull and Jaume Llibre (Universitat Autònoma de Barcelona), together with Chengzhi Li and Jiazhong Yang (Beijing University).

Excursion to Cabrera 1991

CRM Research Programmes

2003-2004
Set Theory
2004-2005
Geometry of the Word Problem
2005-2006
Arakelov Geometry and Shimura Varieties
On Hilbert's 16th Problem

Scientific meetings

The following conferences and courses were scheduled for 2004 and 2005: *Advanced Course on Ramsey Methods in Analysis* (January 19 to 28, 2004); *Workshop on the Foundations of Set Theory* (June 9 to 12, 2004); *Advanced Course on Contemporary Cryptology* (February 2 to 13, 2004); *Conference on Mathematical Foundations of Learning Theory* (June 18 to 23, 2004); *Workshop on Non-Linear Differential Galois Theory* (June 28 to July 2, 2004); *Advanced Course on Automata Groups* (July 5 to 16, 2004); *HYKE Conference on Complex Flows* (October 6 to 9, 2004); *4th Congress of the European Society for Research in Mathematics Education* (February 17 to 21, 2005); *Barcelona Conference on Geometric Group Theory* (June 8 to July 2, 2005); *Advanced Course on the Geometry of the Word Problem for Finitely Generated Groups* (July 5 to 15, 2005); *Advanced Course on Recent Trends on Combinatorics in the Mathematical Context* (September 12 to 23, 2005).

Around the International Congress of Mathematicians (ICM2006) in Madrid, the CRM will organise a three-month research programme on Analysis and an advanced course on Computational Geometry, both co-ordinated jointly by researchers from Barcelona and Madrid. An ICM satellite conference is also planned.

Links with European entities

Since 2003, the CRM has been an institutional member of the EMS. It is also a member of ERCOM (European Research Centres on Mathematics), a committee of the EMS consisting of scientific directors of European research centres in mathematics. In fact, the current CRM Director, Manuel Castellet, has been the chair of ERCOM since 2002.

Lecture room

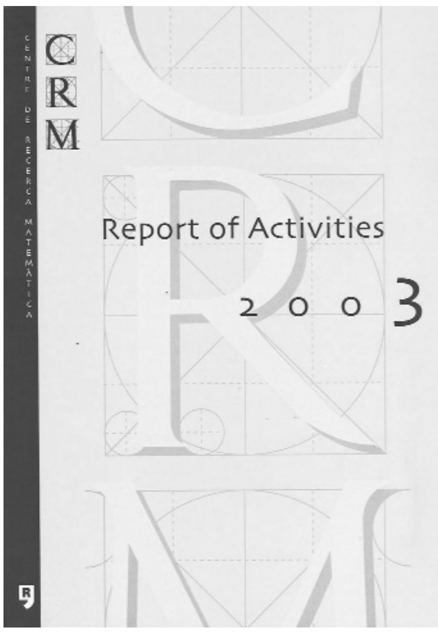
In a similar vein to other ERCOM centres, the CRM undertakes actions to reinforce the role of mathematics in the thematic priorities of the 6th Framework Programme of the European Commission. Funding is offered to young mathematicians in order to foster the development of the following topics, which were selected on the basis of reports prepared by local teams: Life sciences, genomics, and biotechnology for health; nanotechnologies and nanosciences; information society technologies; sustainable development, global change and ecosystems. Several activities, including doctoral training and workshops, are planned on neuroscience and cryptology for the years 2004, 2005 and 2006.

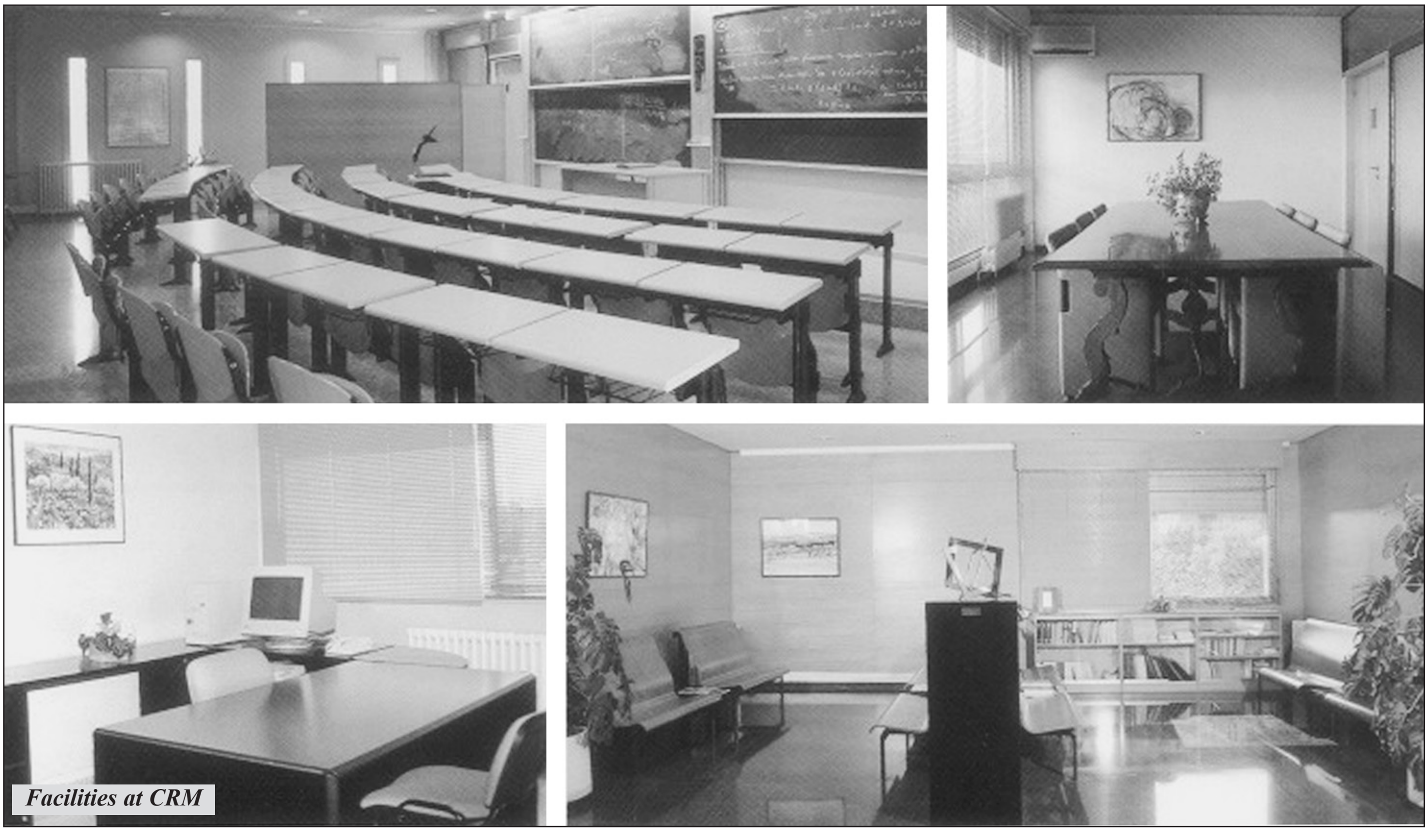
The CRM is a member of the EPDI (European Post-Doctoral Institute for Mathematics), a network of nine European research institutes that jointly offer post-doctoral fellowships in mathematics and mathematical physics every year.

Publications

Besides its twenty-year old preprint collection the CRM has published, since 2001, a monograph series entitled *Advanced Courses in Mathematics CRM Barcelona*, which is produced and distributed by Birkhäuser Verlag (Basel). The series is especially addressed to doctoral and post-doctoral students. Volumes contain carefully edited notes written by lecturers at CRM advanced courses. The following volumes were published in 2003 and 2004: *Symplectic Geometry and Integrable Hamiltonian Systems*, by M. Audin,

Congress at Institut d'Estudis Catalans. Andrew Barron, Steve Smale, and José M. Matías





Facilities at CRM

A. Cannas da Silva and E. Lerman; *Global Riemannian Geometry: Curvature and Topology*, by S. Markvorsen and M. Min-Oo; *Proper Group Actions and the Baum-Connes Conjecture*, by G. Mislin and A. Valette; *Polynomial Identity Rings*, by V. Drensky and E. Formanek.

Facilities and infrastructure

The CRM occupies 940 square metres in the Science Building of the *Universitat Autònoma de Barcelona*. Office space allows the allocation of up to fifteen guests. Two lecture rooms are used for seminars and meetings. All offices and rooms are fully equipped and air conditioned. The CRM has a LAN Ethernet with twenty-five working stations and four printers. Access to major bibliographic data bases is provided.

In addition, visitors have free access to the scientific infrastructure of the Catalan universities, including the use of the UAB Science and Engineering Library. The CRM has several furnished apartments at its disposal in the *Vila Universitària* of the Bellaterra campus and in the nearby town of Sant Cugat.

20th anniversary

In the Autumn of 2004, the CRM will celebrate its 20th anniversary. On this occasion, Jean-Pierre Serre will deliver a lecture during an academic event on November 9, which will be presided by the foremost academic and political authorities of Catalonia. The history of the CRM during its first twenty years of existence will be published in a commemorative volume and a CD. In this period, the CRM hosted 969 visitors from 58 different countries, including 44 post-doctoral fellows. Twenty-four congresses were organised, together with 19 workshops and 23 advanced courses. These events were attended by a total number of 3,480 participants, coming from 72 countries. Many of them, hopefully all, enjoyed Barcelona, the Catalan countryside, or the hospitality of their Catalan colleagues, and keep pleasant memories from their stay.

20th Anniversary of the CRM
Jean-Pierre Serre, Collège de France
Groupes finis: Choix de théorèmes
November 9, 2004
Institut d'Estudis Catalans, Barcelona

Web site

Additional information about the CRM, including full lists of visitors and activities, can be found at the internet address *www.crm.es*.

Centres at present represented in ERCOM, ordered by decreasing geographical latitude

Euler International Mathematical Institute, St. Petersburg (59º55')
Institut Mittag-Leffler, Stockholm (59º20')
Network in Mathematical Physics and Stochastics, Aarhus (56º10')
International Centre for Mathematical Sciences, Edinburg (55º57')
Isaac Newton Institute for Mathematical Sciences, Cambridge (52º21')
Centrum voor Wiskunde en Informatica, Amsterdam (52º21')
Stefan Banach International Mathematical Center, Warszawa (52º15')
Thomas Stieltjes Institute for Mathematics, Leiden (52º10')
Lorentz Center, Leiden (52º10')
Mathematical Research Institute, Nijmegen (51º50')
EURANDOM, Eindhoven (51º26')
Max Planck Institute for Mathematics in the Sciences, Leipzig (51º20')
Max-Planck Institut für Mathematik, Bonn (50º44')
Institut Henri Poincaré, Centre Emile Borel, Paris (48º52')
Institut des Hautes Études Scientifiques, Bures-sur-Ivette (48º52')
Mathematisches Forschungsinstitut Oberwolfach (48º18')
Erwin Schrödinger International Institute for Mathematical Physics, Wien (48º13')
The Abdus Salam International Center for Theoretical Physics, Trieste (45º39')
Centro di Recerca Matematica Ennio Di Giorgi, Pisa (43º43')
Centre International de Rencontres Mathématiques, Luminy (43º18')
Instituto Nazionale di Alta Matemática Francesco Severi, Roma (41º53')
Centre de Recerca Matemática, Barcelona (41º25')
Centro Internacional de Matemática, Lisboa (38º44')
Emmy Noether Research Institute for Mathematics, Ramat-Gan (32º35')

Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses ***vberinde@ubm.ro*** *or* ***vasile_berinde@yahoo.com****.*
Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

September 2004

8–11: Dixièmes journées montoises d'informatique théorique, à Liège (Tenth Mons theoretical computer science days, in Liège)
Information:
e-mail: M.Rigo@ulg.ac.be
Web site : www.jm2004.ulg.ac.be
[For details, see EMS Newsletter 50]

13–18: 3rd CIME Course - Stochastic Geometry, Martina Franca, Taranto, Italy
Information:
e-mail: cime@math.unifi.it
Web site:
http://www.math.unifi.it/CIME
Address: Fondazione C.I.M.E. c/o Dipart.di Matematica "U. Dini"
Viale Morgagni, 67/A - 50134 FIRENZE (ITALY)
Phone: +39-55-434975 / +39-55-4237111
Fax: +39-55-434975 / +39-55-4222695
[For details, see EMS Newsletter 51]

20–24: 12th French-German-Spanish Conference on Optimization, Avignon, France
Information:
e-mail: alberto.seeger@univ-avignon.fr
Web site:
http://www.fgs2004.univ-avignon.fr
[For details, see EMS Newsletter 50]

23–26: 4th International Conference on Applied Mathematics (ICAM-4), Baia Mare, Romania (previous editions in 1998, 2000 and 2002)
Information: e-mail:
marietag@ubm.ro; icam4@ubm.ro
Web site: http://www.ubm.ro/site-ro/facultati/departament/manifestari/icam4/index.html
[For details, see EMS Newsletter 49]

26–October 1: Potential theory and related topics, Hejnice, Czech Republic
Information:
e-mail: jvesely@karlin.mff.cuni.cz
Web site:
http://www.karlin.mff.cuni.cz/PTRT04
[For details, see EMS Newsletter 52]

29–October 2: XII Annual Congress of the Portuguese Statistical Society, Evora, Portugal
Information:
e-mail: spe2004@uevora.pt
Web site:
www.eventos.uevora.pt/spe2004
Address: Comissao Organizadora XII Congresso SPE, Departamento de Matematica, Universidade de Evora, Rua Romao Ramalho 59, 7000-671 EVORA, PORTUGAL
Phone: +351-266745370
Fax: +351-266745393
[For details, see EMS Newsletter 52]

October 2004

6–9: HYKE Conference on Complex Flows, Centre de Recerca Matematica, Bellaterra, Spain
Information:
e-mail: ComplexFlows@crm.es
Web site:
http://www.crm.es/ComplexFlows
[For details, see EMS Newsletter 52]

24–30: Partial Differential Equations in Mathematical Physics (*in memory of Olga A. Ladyzhenskaya*), Trento, Italy
Organizer: CIRM, Centro Internazionale per la Ricerca Matematica, Trento, Italy
Scientific Committee: H. Beirao da Veiga (Pisa), G. Seregin (St. Petersburg), V. Solonnikov (Ferrara), N. Uraltseva (St. Petersburg), A. Valli (Trento)
Location: Grand Hotel Bellavista, Levico Terme, Trento, Italy
Information:
e-mail: michelet@science.unitn.it
Web sites: www.science.unitn.it/cirm/ ,
http://www.science.unitn.it/cirm/Ladylecture.html

January 2005

6–9: 24th Nordic and 1st Franco-Nordic Congress of Mathematicians, Reykjavik, Iceland
Aim: The main goal of the congress is to bring together mathematicians to present recent results in several important areas of research in the Nordic countries and France. Around half of the programme will be devoted to 3 main themes: Algebraic Geometry, Geometric Analysis, and Probability. Satellite meetings will be organized in the days before the congress.
Scientific committee: H. Thorisson (chair) (Iceland), J. Kr. Arason (Iceland), G. Grubb (Denmark), Ch. Kiselman (Sweden), J.-F. Le Gall (France), P. Koskela (Finland), Mireille Martin-Deschamps (France), R. Piene (Norway), and M. Waldschmidt (France)
Plenary speakers: S. Helgason (Iceland/USA), M. Yor (France), M. Audin (France), S. Neshveyev (Norway), H. Schlichtkrull (Denmark), C. Villani (France), F. Loeser (France), A. Buch (Denmark), M. Passare (Sweden), Xiao Zhong (Finland), Olav Kallenberg (Sweden/USA)
Sessions and their organizers:
1. Arithmetic theory of differential equations and q-difference equations, Y. André (France); 2. Combinatorics, E. Steingrimsson (Iceland/Sweden); 3. Geometric topology, J. Dupont (Denmark); 4. Group theory, R.G. Moller (Iceland); 5. Homological methods in algebra and topology, B. Dundas and I. Reiten (Norway); 6. Lie groups and harmonic analysis, F. Rouviere (France); 7. Mathematical physics, A. Kupiainen (Finland); 8. Nonlinear partial differential equations, K. H. Karlsen (Norway); 9. Real algebraic geometry and applications (theme: Algebraic Geometry), Marie-Francoise Coste-Roy (France); 10. Singularities (theme: Algebraic Geometry), B. Teissier (France); 11. Foliations in algebraic and arithmetical geometry (theme: Algebraic Geometry), T. Ekedahl (Sweden); 12. Moduli spaces (theme: Algebraic Geometry), C. Faber (Sweden); 13. Complex geometry (theme:

Geometric Analysis), A. Tsikh (Russia/Sweden); 14. Quasiconformal techniques in analysis (theme: Geometric Analysis), M. Zinsmeister (France); 15. Spectral geometry (theme: Geometric Analysis), R. Nest (Denmark); 16. Analysis on metric spaces (theme: Geometric Analysis), Juha Kinnunen (Finland). 17. Percolation and spatial interaction (theme: Probability), O. Haggstrom (Sweden); 18. Applied probability and stochastic networks (theme: Probability), F. Baccelli (France); 19. Branching processes (theme: Probability), P. Jagers (Sweden); 20. Large random matrices (theme: Probability), Alice Guionnet (France).
Information: e-mail: FrancoNordic Congress@raunvis.hi.is
Web site: http://www.raunvis.hi.is/ 1FrancoNordic Congress/

28–30: 4th Mediterranean Conference on Mathematics Education, Palermo-Italy
Description: The Fourth Mediterranean Conference on Mathematics Education with International participation follows a successful series of three such conferences since 1997. At this stage educators and researchers who may be interested in attending the conference with or without a paper should send a message with full address, fax and e-mail to the organizing committee.
Themes: Mathematics in the Modern World; Mathematics and Didactics; Mathematics and Society; Mathematics and Talent; Mathematics and Sciences; Mathematics and Technology; Mathematics and Motivation; Mathematics and Statistics; History of Mathematics
Deadline: for full papers is 15 October 2004
Organizers: Cyprus Mathematical Society and University of Palermo
Format: The conference will consist of several sessions covering multiple themes but all under the general aim of Mathematics Education.
Grants: The conference will offer a two day free accommodation to one presenter of each paper provided that their papers are accepted by the scientific committee. All papers will be reviewed by at least two referees.
Local Organising Committee: Chair: F. Spagnolo, University of Palermo; Gianna Manno; Claudia Sortino; A. Scimone; Teresa Marino (all GRIM, Department of Mathematics, University of Palermo)
International Organizing Committee: Chair: G. Makrides, Vice-Chair: F. Spagnolo; Secretary A: Gianna Manno; Secretary B: Katerina Nicolaou; Treasurer A: Claudia Sortino; Treasurer B: A. Philippou; Members: MATHEU partners; S. Antoniou
Scientific/Program Committee: A. Gagatsis; F. Spagnolo; B. D'Amore; N. Alexandris; S. Grozdev; G. Makrides; C. Christou; P. Damianou; Demetra Pitta; A. Brigaglia; U. Bottazzini
Information:
e-mail: spagnolo@math.unipa.it;
Web sites: http://math.unipa.it/~grim/mediterranean_05.htm;
www.cms.org.cy

February 2005

5–13: Applications of braid groups and braid monodromy, EMS Summer School, Eilat, Israel
Information: Web site: www.emis.de /etc/ems-summer-schools.html

March 2005

9–April 1, 2005: 14th INTERNATIONAL WORKSHOP ON MATRICES AND STATISTICS (IWMS-2005), Massey University, Albany Campus, Auckland, New Zealand
Aim: to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between statistics and matrix theory. The Workshop will provide a forum through which statisticians may be better informed of the latest developments and newest techniques in linear algebra and matrix theory and may exchange ideas with researchers from a wide variety of countries. The International Workshops in Matrices and Statistics are held annually in different locations.

This Workshop will be the fourteenth in the series, with the previous Workshops held at the following locations: 1) Tampere, Finland, August 1990; 2) Auckland, New Zealand, December 1992; 3) Tartu, Estonia, May 1994; 4) Montreal, Quebec, Canada, July 1995; 5) Shrewsbury, England, July 1996; 6) Istanbul, Turkey, August 1997; 7) Fort Lauderdale, Florida, USA, December 1998; 8) Tampere, Finland, August 1999; 9) Hyderabad, India, December 2000; 10) Voorburg, The Netherlands, August 2001; 11) Lyngby, Denmark, August 2002; 12) Dortmund, Germany, August 2003 (http://www.statistik.uni-dortmund.de/IWMS/main.html); 13) Bedlęwo, Poland, August 2004 (http:// matrix04.amu.edu.pl/).
IWMS-2005 is a Satellite Conference to the 55th Biennial Session of the International Statistical Institute to be held in Sydney, April 5 - 12, 2005.
Format: The Workshop will include invited and contributed talks.
Proceedings: It is intended that refereed Conference Proceedings will be published.
Local Organising Committee: Jeff Hunter (Chair) <j.hunter@massey.ac.nz>
International Organizing Committee: George Styan (Chair) <styan@math.mcgill.ca>; Hans Joachim Werner (Vice-Chair) <werner@united.econ.uni-bonn.de> and Simo Puntanen <Simo.Puntanen@uta.fi>
Information: Web site: http://iwms2005.massey.ac.nz/

June 2005

12–24: Foliations 2005, Łodz, Poland
Description: The conference is the fourth in a series devoted to the theory of foliations. The previous three took place in 1990 (Łodz), 1995 and 2000 (both Warszawa). The main purpose of the conference is to interchange new ideas in all aspects of foliation theory and related topics: contact and symplectic structures, confoliations, Engel and Goursat structures, groups and pseudogroups of action on manifolds, holonomy groups and pseudogroups of foliations etc.
Format: 2 mini-courses, invited lectures and contributed talks
Invited speakers: V. Kaimanovich (mini-course), S. Matsumoto (mini-course), J. Alvarez Lopez, M. Asaoka, S. Fenley, V. Grines, X. Gomes Mont, J. Heitsch, Y. Kordyukov, H. Minakawa, K. Richardson, E. Zhuzhoma. Confirmation of speakers is in progress, see the conference web site: http://fol2005.math.uni.lodz.pl
Organizers: Katedra Geometrii Uniwersytetu Łodzkiego (Łodz), Banach Centre (Warszawa)
Organizing Committee: S. Hurder (Chicago), R. Langevin (Dijon), T. Tsuboi (Tokyo), P. Walczak (Łodz) and M. Czarnecki, secretary, (Łodz)
Location: Uniwersytet Łodzki, Łodz, Poland
Grants: Several EU grants for young mathematicians will probably be available
Information:
e-mail: fol2005@math.uni.lodz.pl
Web site: http://fol2005.math.uni.lodz.pl

25–July 2 : Subdivision schemes in geometric modelling, theory and applications, EMS Summer School, Pontignano, Italy
Information:
Web site: www.emis.de/etc/ems-summer-schools.html

Recent books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to the following address:
Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic

B. Beckman: *Codebreakers: Arne Beurling and the Swedish Crypto Program during World War II,* *American Mathematical Society, Providence, 2002, 259 pp., $39, ISBN 0-8218-2889-4*
All stories about cryptanalytical work prior and during the World War II read like a thriller. The book under review is no exception. The author, the former head of the cryptanalytical section of FRA (The Radio Agency of the Swedish Defence Forces), presents a well of information. Its highlight is the detailed description of Arne Beurling's solution of the German Siemens machine, one of the most magnificent achievements of cryptanalysis of that time. During the previous century, in the thirties and the first half of the forties, the cryptanalytical work of cryptographic "superpowers" (Poland, England, and USA) became dominated by mathematicians. The transition of secret services to the search of cryptanalytical talents among graduates of mathematics departments probably started in Poland and people like Marian Rejewski became the founders of modern mathematically based cryptology. Well known are the contributions of Alan Turing to the war efforts in Bletchley Park. Arne Beurling is another, though less known, example of a top class mathematician serving his country as a cryptanalytician during the war times.

The book starts with an explanation of what a code and a cipher is. Then the author overviews the Swedish cryptanalytical history to the end of the World War II. Most of the book is based on declassified Swedish documents and it traces the work of Arne Beurling in solving various ciphers used by different countries during the war, including the top secret German ciphers. For the reviewer it is particularly interesting to read about the Beurling solution of the cipher used by a Czech resident in Sweden named Vaněk to communicate with the Czechoslovak Government exiled in London. The second part of the book is dedicated to the life and mathematical work of Arne Beurling, a close friend and collaborator of Lars Ahlfors. The book can be highly recommended to anyone interested in the role of mathematics in classical cryptology and in the history of the 20th century. (jtu)

J. Borwein, D. Bailey: *Mathematics by Experiment. Plausible Reasoning in the 21st Century,* *A. K. Peters, Natick, 2003, 350 p., $45, ISBN 1-56881-211-6*
J. Borwein, D. Bailey, R. Girgensohn: *Experimentation in Mathematics. Computational Path to Discovery,* *A. K. Peters, Natick, 2004, 350 p., $49, ISBN 1-56881-136-5*
Despite the slightly different titles, the books form two volumes of the same publication. Mathematical experiments performed on computers are more and more important factors of further development in the mathematics. The text is carefully written and the plentiful short remarks on related topics are pleasant to read. The material is mostly accessible without knowledge of advanced parts of modern mathematics. A certain familiarity with computer algebra programs like *Maple* or *Mathematica* is helpful for a reader willing to try these things in practice. The first volume of the work contains a gentle introduction to experimental mathematics in its historical context and to its methodology, using a series of well-chosen examples. The first chapter shows it as a rapidly developing field of mathematics, the second contains further numerous illustrative examples. These quite often provoke a reader to work or play with the examples on his own PC. The third chapter describes the progress in computing π, where both authors made significant contributions (BBP formula for computing π). A chapter on normality of numbers deals with another fascinating problem in which experimental mathematics promises at least some path to future discoveries. In contrast to the fact that the book introduces to the reader a highly up-to-date part of mathematics, the next chapter offers another look at classical themes like the fundamental theorem of algebra, the Gamma function or Stirling's formula. After an exposition on basic tools, the book is closed by a chapter with the title 'Making sense of experimental mathematics'.

For those wanting to know more on the progression from numerical experiments to hypotheses and finally to deep mathematical theorems proven within the frame of "classical mathematics", the authors prepared the second volume of the work together with R. Girgensohn. In fact, both volumes can be read independently.

The second volume starts with a chapter on sequences, series, products and integrals, which is followed by the chapter on Fourier series and integrals. These chapters form one third of the book and will be of interest to any open-minded person with a deeper interest in mathematics, equipped with a basic knowledge of undergraduate mathematics. The following chapters on zeta functions and multi-zeta functions, partition, powers, primes and polynomials are more specialised. The final two chapters invite a reader to explore deeper methods and tools of experimental mathematics. Each chapter in both volumes is closed by commentaries and additional examples. The amount of collected material is tremendous. It is impossible to describe the content of the whole work in detail in just a few lines. These are very nice and provoking books showing that experiments both were and are an important part of the development of mathematics. New computer-based tools are broadly used mainly "outside" mathematics and the book shows "... how today, the use of advanced computing technology provides mathematicians with an amazing, previously unimaginable 'laboratory', in which examples can be analysed, new ideas tested, and patterns discovered." I strongly recommend visiting e.g. web-site on URL *http://www.expmath.info*, where some parts of the first volume are also available. Also the very good typesetting and nice graphical appearance of the whole work are worth mentioning. It can be recommended not only to libraries but to all members of the mathematical community. (jive)

T. Brzezinski, R. Wisbauer: *Corings and Comodules,* *London Mathematical Society Lecture Note Series 309, Cambridge University Press, Cambridge, 2003, 476 pp., £37,95, ISBN 0 521-53931-5*
This is a first comprehensive monograph on corings and comodules. Corings are generalizations of coalgebras: while a coalgebra is an R-module over a commutative associative unital ring R, equipped with an R-linear coproduct and a comultiplication, a coring is an (A,A)-bimodule over an associative, but not necessarily commutative, unital R-algebra A, equipped with a (A,A)-bilinear coproduct and comultiplication. Corings can also be viewed as coalgebras in a particular monoidal category (the 'tensor category'). As observed by Takeuchi, important examples of corings are the so-called entwining structures connecting R-algebras with R-coalgebras. Moreover, in the particular case of bialgebra entwinings, entwined modules are exactly the classical Hopf modules. So - and this is the point of the book - the theory of corings and comodules is a natural common generalization of several theories connecting algebra with category theory, noncommutative geometry, and quantum physics.

The book consists of six chapters and an Appendix. After developing coalgebra and comodule theory from the module-theoretic point of view in Chapter 1, the authors deal with bialgebras, classical Hopf algebras and their modules in Chapter 2. The module theoretic approach of Chapter 1 pays back in Chapter 3, where basics of the coring and comodule theory are developed. Chapter 4 deals with an important class of corings coming from ring extensions (together with the BOCS's of Rojter and Kleiner, these were the first examples of corings truly generalizing coalgebras). Chapters 5 and 6 deal with the relation to entwinings and weak entwinings. The Appendix recalls basics on the module category $\sigma[M]$, which is one of the main tools for the theory. The reason is that any right comodule over a coring C is a left module over the left dual ring ${}^*C$, and, for example, the so called 'left α-condition' just says that the category of all right C-comodules coincides with $\sigma[{}^*{}_C C]$. The book provides a unified and general treatment of a theory whose pieces were originally developed by people working in rather distinct areas of algebra, category theory, noncommutative geometry, and mathematical physics. The book is a welcome addition to the literature on this young and rapidly developing subject. (jtrl)

V. M. Buchstaber, T. E. Panov: *Torus Actions and Their Applications in Topology and Combinatorics,* *University Lecture Series, vol. 24, American Mathematical Society, Providence, 2002, 144 pp., $29, ISBN 0-8218-3186-0*
This book is an outgrowth of an extensive paper "Torus actions, Combinatorial topology and homological algebra", written by the authors and published in Uspekhi Mat. Nauk (Russian Math. Surveys) in 2000. It is yet another contribution to the recent rich interplay of combinatorics and convex geometry with algebraic geometry and topology. In Chapters 1-5, the authors review basic notions of both the combinatorial and topological side with a slight preference for combinatorial notions (which they consider less familiar to a prospective reader). The last three chapters are devoted to their own contribution, which is centred around moment angle complexes, their cohomology and to subspace arrangements. This is a useful book that will be of interest to the (algebraic) combinatorics community as well as to researchers in algebraic geometry and topology. It is a carefully written state of the art book. (jneš)

B. Buffoni, J. Toland: *Analytic Theory of Global Bifurcation,* *Princeton Series in Applied Mathematics, Princeton University Press, Princeton, Oxford, 2003, 169 pp., £29,95, ISBN 0 691-11298-3*
In the book, bifurcation problems for non-linear operator equations in infinite dimensional spaces are studied. To read the book requires certain knowledge, hence the first three chapters are devoted to a review (without proofs) of basic notions and facts from linear functional analysis, nonlinear functional analysis (e.g., the implicit function theory) and from the theory of analytic operators in Banach spaces. Main facts on holomorphic functions of several complex variables, real analytic functions of several real variables and on (finite-dimensional) analytic varieties, can be found in the next three chapters respectively. Analytic sets (in the complex setting, or their real version) have a nice, distinguished structure, which can be used in a study of analytic operator equations in infinite dimension. A tool needed for such a relation is a suitable version of the implicit function theorem, reducing infinite dimensional questions to finite dimension. The last two chapters contain applications of previous methods to steady periodic water waves. (vs)

H. Cabral, F. Diacu, Eds.: *Classical and Celestial Mechanics: The Recife Lectures,* *Princeton University Press, Princeton, 2002, 385 pp., £35, ISBN 0-691-05022-8*
This book has its origin in a lecture series hosted by the Federal University of Pernambuco in Recife, Brazil, between 1993 and 1999. Immediately after opening the book, one gets a feeling of the great enthusiasm of its editors, organizers of the corresponding series of lectures. It is a really attractive book, presenting many important topics from Hamiltonian dynamics and celestial mechanics in an accessible way. The editors and the mathematical centre in the equator, colonial city of Recife, deserve great admiration for producing such an impressive output - a record of their long term seminar, devoted to classical mechanics. (Several contributions to the seminar were published independently, before the publication of this book.)

The book contains the following contributions: Central configurations and relative equilibria for the N-body problem (by D. Schmidt), Singularities of the N-body problem (by F. Diacu), Lectures on the two-body problem (by A. Albouy), Normal forms of Hamilton systems and stability of equilibria (by H. E. Cabral), The Poincaré compactification and applications to celestial mechanics (by E. Pérez-Chavela), The motion of the moon (by D. Schmidt), Lectures on geometrical methods in mechanics (by M. Levi), Momentum maps and geometric phases: Overview, classical adiabatic angles, holonomy for gyrostats, microswimming (by J. Koiller et al.), and Bifurcation from families of periodic solutions (J. K. Hale and P. Táboas). (mzah)

A. Candel, L. Conlon: *Foliations II,* *Graduate Studies in Mathematics, vol. 60, American Mathematical Society, Providence, 2003, 545 pp., $79, ISBN 0-8218-0809-5*
This is the second volume of the two-volume series with the title Foliations. It has three independent parts, describing three special topics in the theory of foliations: Analysis on foliated spaces, Characteristic classes of foliations and Foliated 3-manifolds. Each part contains a description of a topic in foliation theory and its relation to another field of contemporary mathematics. In the first part, the C^*- algebras of foliated spaces are studied and some of the classical notions from Riemannian geometry (heat flow and Brownian motion) are generalized to foliated spaces. Necessary analytic background can be found in three appendices. The second part is devoted to characteristic classes and foliations. Here the reader can find constructions of exotic classes based on the Chern-Weil theorem, vanishing theorem for Godbillon-Vey classes and a discussion on obstructions to existence of a foliation transverse to the fibres of circle bundles over surfaces. In the third part, compact 3-manifolds foliated by surfaces are studied. Special methods of 3-manifolds topology yield existence theorems and further results unique for dimension three. There is an appendix with a proof and further discussion of Palmeiras theorem, which says that the only simply connected n-manifold foliated by leaves diffeomorphic to R^{n-1} is R^n. The book contains a lot of interesting results and can be recommended to anybody interested in the topic. (jbu)

X. Chen, K. Guo: *Analytic Hilbert Modules,* *CRC Research Notes in Mathematics 433, Chapman & Hall/CRC, Boca Raton, 2003, 201 pp., $99,95, ISBN 1-58488-399-5*
The main theme of the book is the structure of modules over function algebras of holomorphic functions in several complex variables. The book concentrates mainly on topics developed by both authors. The book starts with a nice introduction summarizing main results described in the book. Chapter 2 describes the technique of characteristic space theory for analytic Hilbert modules, which was developed by K. Guo. Chapter 3 shows that analytic Hilbert modules in several variables have a much more rigid structure than in one variable. The equivalence problem for Hardy submodules in the cases of the polydisk or the unit ball is treated in Chapter 4. The next chapter describes the structure of the Fock space, or more generally, reproducing function spaces on C^n. Chapter 6 contains a discussion of modules over the Arveson space of square integrable functions on the unit ball in C^n. In the last chapter, the authors describe the extension theory of Hilbert modules over function algebras. The book is written in a clear and systematic way and it describes an interesting part of the recent development in the field. Each chapter ends with useful bibliographic comments. (vs)

I. Dolgachev: *Lectures on Invariant Theory,* *London Mathematical Society Lecture Note Series 296, Cambridge University Press, Cambridge, 2003, 220 pp., £29,95, ISBN 0-521-52548-9*
In many problems in mathematics, the structure of the set of all orbits of a group acting on a suitable space is described by means of invariants of the group action. In particular, the case of the action of a linear algebraic group G on an algebraic variety X is the case studied in classical invariant theory. The present book is intended for beginners as a first introduction to the theory. Knowledge of fundamental notions and basic facts from algebraic geometry is expected. The purpose of the book is to offer a short description of main ideas of the classical invariant theory illustrated by many specific examples. Every chapter ends with a set of exercises and with hints for further reading.

The first two chapters treat the classical example of the action of the group $GL(V)$ on homogeneous polynomials of degree m on the space E, where E itself is the space of homogeneous polynomials of degree d on V. In the next chapter, the Nagata theorem on the algebra of invariant polynomials on the space of a linear rational representation of a reductive algebraic group is proved. The next chapter is devoted to linear rational representations of a non-reductive algebraic group, including the Nagata counterexample to Hilbert's 14th problem. Covariants of the action are treated in Chapter 5. Categorical and geometric quotients and linearization of the action, a notion of stability of an algebraic action together with numerical criterion of stability are described next. The last three chapters treat some special cases (hypersurfaces in projective space, ordered sets of linear subspaces in projective space, and toric varieties). (vs)

R. M. Dudley: *Real Analysis and Probability, Cambridge* *Studies in Advanced Mathematics 74, Cambridge University Press, Cambridge, 2002, 555 pp., £32,95, ISBN 0-521-80972-X*
This book serves as a clear, rigorous, and complete introduction to modern probability theory using methods of mathematical analysis, and a description of relations between the two fields. The first half of the book is devoted to an exposition of real analysis. Starting with basic facts of set theory, the book treats e.g. the real number system, transfinite induction, and problems of cardinality, touching both the continuum hypothesis and axiom of choice together with its equivalences. General topology is discussed, including compactness and compactification, completion and completeness, and metric. Measure theory and integration is treated carefully, because it serves as the most important tool for probability theory. Among more advanced topics of real and functional analysis, we can find here an introduction to functional analysis on Banach and Hilbert spaces, convex functions, convex sets and dualities, and measures on topological spaces. The second half of the book contains a description of modern probability theory, including convergence laws, central limit theorems and laws of large numbers. Ergodic theory, as well as martingales, is studied. More advanced topics include convergence laws on separable metric spaces, stochastic processes and Brownian motion. The book can be considered as a textbook. It is a self-contained text and all relevant facts are proved. The appendices show that the author carefully filled all gaps in mathematical background needed later. The book contains a number of exercises helping to understand the contents. It could be very useful for students interested in learning both topics, it can also serve as complementary reading to standard lectures. Teachers preparing their graduate level courses can use the book as an excellent, rigorously written and complete source. (mrok)

J. Faraut, F. Rouviére, M. Vergne: *Analyse sur les groupes de Lie et théorie des représentations,* *Séminaires & Congrés 7, Société Mathématique de France, Paris, 2003, 177 pp., €40, ISBN 2-85629-142-2*
The book is based on lectures presented at the summer school on analysis on Lie groups and representation theory, held in 1999 in Kénitra, Morocco. It contains a written form of three series of lectures. The first one (given by M. Vergne, recorded by S. Paycha) contains a description of the equivariant cohomology of a manifold, which was introduced independently by N. Berline and M. Vergne, E. Witten and M. F. Atiyah and R. Bott in the 80's. In the special case of S^1-action with isolated fixed points, the Paradan formula is used for a description of the equivariant cohomology in terms of fixed points of the action. As an important application, it is shown how to obtain the Duistermaat-Heckman stationary phase for-

mula for a U(1)-action on a symplectic manifold.

The second series of lectures (given by F. Rouviére) was devoted to the Damek-Ricci spaces. A manifold is harmonic if the mean value theorem holds for harmonic functions. The Damek-Ricci spaces are harmonic manifolds but they are not symmetric spaces. Basic facts from geometry and harmonic analysis on the Damek-Ricci spaces are described in the course.

The third series of lectures (given by J. Faraut) is devoted to Hilbert spaces of holomorphic functions invariant under the action of a group of automorphisms of a complex manifold. The set of such Hilbert spaces forms a convex cone and it is possible to use methods of the Choquet theory for a description of its structure and for an integral representation of their reproducing kernels. The last parts are devoted to the case of invariant domains in the complexification of a compact symmetric space. A part of the school's program was reserved for lectures on the research work of participants, the list of lectures and their abstracts can be found in the book. The book brings nice reviews of very interesting areas of the field. (vs)

M. Feistauer, J. Felcman, I. Straškraba: *Mathematical and Computational Methods for Compressible Flow,* *Numerical Mathematics and Scientific Computation, Clarendon Press, Oxford, 2003, 535 pp., £59,95, ISBN 0-19-850588-4*

The book deals with the numerical solution of equations describing motion of compressible fluids. Classical as well as modern numerical schemes are summarized here. At the beginning, the governing equations are determined and the mathematical problems are defined. Some theoretical results, such as existence and uniqueness of solutions, are mentioned. The main part of the book is concerned with the numerical solution of inviscid as well as viscous compressible flows. Applications of various numerical methods (finite volume method, finite element method, discontinuous Galerkin method and their variants) for the Euler and Navier-Stokes equations are discussed. Convergence properties of numerical schemes are mostly derived for a scalar nonlinear convection-diffusion equation. Several types of mesh adaptation techniques are presented as well. In the book the reader can find a lot of numerical examples demonstrating the efficiency of described methods. The book is suitable for researchers as well as for students dealing with the numerical solution of compressible flows. (jdol)

W. Fenchel, J. Nielsen: *Discontinuous Groups of Isometries in the Hyperbolic Plane,* *de Gruyter Studies in Mathematics 29, Walter de Gruyter, Berlin, 2003, 364 pp., €78,50, ISBN 3-1-017526-6*

In 1920's and 1930's, J. Nielsen published three long papers in Acta Mathematica on discontinuous groups of isometries of the non-euclidean plane. His further research in the field led him to the idea to describe the theory of discontinuous groups of isometries systematically and in full generality. After the Second World War, he started a project along these lines together with W. Fenchel and they prepared the first version of the manuscript. W. Fenchel was able to continue (with his collaborators) the work on the project after J. Nielsen's death and the typewritten version of the manuscript was ready at the end of the 80's.

The book under review is the final version of the manuscript, which was prepared for publication by A. L. Schmidt after W. Fenchel's death in 1988. It offers a systematic geometric treatment of discontinuous groups of isometries. It starts with a careful description of Möbius transformations of the Riemann sphere and their use in non-euclidean geometry. The second chapter contains a detailed description of discontinuous groups of motions of the unit discs with its hyperbolic metric. In the third chapter, associated surfaces and invariants needed for a classification are discussed. Elementary groups and elementary surfaces, the decompositions of the discontinuous groups and their normal forms, are all treated in the fourth chapter. The final chapter is devoted to isomorphisms of discontinuous groups, homeomorphisms of the corresponding discs and their extensions to the boundary. A specific feature of the book is that it is based entirely on geometric arguments. The Fenchel-Nielsen manuscript has been famous for a long time already and its final publication is a valuable edition to mathematical literature. (vs)

M. Georgiadou: *Constantin Carathéodory, Mathematics and Politics in Turbulent times,* *Springer, Heidelberg, 2004, 651 pp., €89,95, ISBN 3-540-44258-8*

This is an excellent biography of a man who belongs to the most renowned mathematicians of the first half of the 20th century. More than 50 years after his death – much later than his contribution to the science deserves – this book brings his personality and work to life again. It depicts them in full context with that period, so rich on far reaching revolutionary events, in a very lively and suggestive way. Being a German mathematician born in an outstanding Greek family of diplomats, Constantin Carathéodory was attached to German intellectual tradition by his German education, as well as to Greece by his emotions. Moreover, a man of his intellectual influence must have also been a man of political significance. Because of these facts and in connection with the dramatic period in which he lived, he is very suitable subject for a biography. The author has gathered with exceptional care almost all accessible material that has a bearing to the life of this scientist, and has built a narrative that will fascinate everyone who opens the book. (jdr)

M. Gidea, C. P. Niculescu: *Chaotic Dynamical Systems: An Introduction,* *Centre for Nonlinear Analysis and its Applications 3, Universitaria Press, Craiova, 2003, 239 pp., €10, ISBN 973-8043159-9*

The book aims to be an introduction to the theory of dynamical systems on the graduate level. It covers all classical topics of the subject, including structural stability, Lyapunov exponents, the horse-shoe dynamics, hyperbolic and symbolic dynamics, chaotic attractors, entropy, ergodicity and invariant measures. The particular emphasis is given to the concept of chaos and the various mathematical tools for its understanding. The exposition is clear and concise, yet perfectly rigorous. Many simple examples and a number of exercises serve excellently to the purpose of the book to be an elementary introduction. The only minor setback seems to be a somewhat lesser quality of the print. (dpr)

M. Gross, D. Huybrechts, D. Joyce: *Calabi-Yau Manifolds and Related Geometries,* *Universitext, Springer, Berlin, 2003, 239 pp., €49,95, ISBN 3-540-44059-3*

Summer schools in Nordfjordeid, Norway, have been organized regularly since 1996. Topics vary each year but there are always three series of lectures by invited experts with evening exercises. The school held in June 2001 was devoted to recent interaction between differential and algebraic geometry. The book consists of notes written by lecturers of the corresponding three series of lectures. The first contribution (by D. Joyce) is devoted to the introduction and study of Riemannian properties. The last contribution (by D. Huybrechts) describes compact hyperkähler manifolds. The class of compact hyperkähler manifolds form an interesting class of Ricci-flat manifolds, playing an important role in different branches of mathematics and mathematical physics. In this part, many interesting topics are presented (including theory of holomorphic symplectic manifolds, deformation theory of complex structures, cohomology and other properties of compact hyperkähler manifolds, twistor space and moduli space of a hyperkähler manifold and a discussion of the projectivity of hyperkähler metrics). The themes of all three contributions are interrelated and together they give a nice introduction into a very interesting field of research on the border between mathematics and physics. I would like to strongly recommend the book to anybody interested in the topic. (jbu)

B. C. Hall: *Lie groups, Lie algebras, and representations: An Elementary Introduction,* *Springer, Heidelberg, 2003, 351 pp., €59,95, ISBN 0-387-40122-9*

To present a circle of ideas around Lie groups, Lie algebras and their representations, it is necessary to make a few principal choices. The first question is how to describe a relation between Lie groups and Lie algebras. To make the book accessible to a broader audience, the author does not suppose knowledge of the theory of manifolds. He restricts the attention to matrix groups. Lie algebra of G is then defined using simple properties of the exponential map. As for the correspondence between Lie group homomorphisms and Lie algebra homomorphisms, the author is using the Baker-Campbell-Hausdorff theorem for its description.

In the main part of the book, finite dimensional representations of classical semisimple Lie groups are classified by their highest weights. Their construction is given in three different ways (as quotients of Verma modules, or by the Peter-Weyl theorem, or by the Borel-Weil realization). The proof of the complete reducibility is based on properties of representations of compact groups. The Weyl character formula and the classification of complex semisimple Lie algebras end the main part of the book. To keep prerequisites minimal, the author also offers a few appendices. The book is written in a systematic and clear way, each chapter ends with a set of exercises. The book could be valuable for students of mathematics and physics as well as for teachers, for the preparation of courses. It is a nice addition to the existing literature. (vs)

U. Hertrich-Jeromin: *Introduction to Möbius Differential Geometry,* *London Mathematical Society Lecture Note Series 300, Cambridge University Press, Cambridge, 2003, 413 pp., £29,95, ISBN 0-521-53569-7*

The book is an introduction to the geometry of submanifolds of the conformal n-sphere. A Möbius transformation is a conformal (i.e., angle preserving) transformation of the sphere. The sphere can be considered as a homogeneous space of the group of Möbius transforms, hence it is a model of Möbius geometry from F. Klein's point of view. There are several other models of Möbius geometry (projective model, quaternionic model and Clifford algebra model). Classically, a conformal structure is represented by a Riemannian metric (modulo a multiplication by a positive function). The change of Riemannian properties under conformal change is described and the Weyl and Schouten tensors are introduced. Conformal flatness is discussed in details. The projective model and related objects (congruencies, etc.) are introduced in the first chapter. The Cartan method of moving frames is often used for computations. As an application of projective model constructions, conformally flat hypersurfaces, isothermic and Willmore surfaces are discussed. A quaternionic model introduced in the next chapter is used for a study of isothermic surfaces in the four-dimensional case, while in higher dimensions, the Clifford algebra model is used. At the end of the book, the reader can find a discussion of triply orthogonal systems, their Ribaucour transformations and isothermic surfaces of arbitrary codimension. The book is a well-written survey of classical results from a new point of view and a nice textbook for a study of the subject. (jbu)

J. Ize, A. Vignoli: *Equivariant Degree Theory,* *de Gruyter Series in Nonlinear Analysis and Applications 8, Walter de Gruyter, Berlin, 2003, 361 pp., €98, ISBN 3-11-017550-9*
The aim of the book is the development and applications of the degree theory in the context of equivariant maps. (Equivariant simply means that the mapping has certain symmetries, e.g., being even/odd, periodic, rotational invariant, etc.). The theory is developed both in finite and infinite dimension. The first chapter gives necessary preliminaries. The second chapter brings the definition of the degree and studies its basic properties. As the definition is somewhat abstract (the degree is defined as an element of the group of equivariant homotopy classes of maps between two spheres), it is useful to compute the degree in various particular cases. This is accomplished in Chapter 3. The last and also the longest chapter, deals with applications to particular ODE's and to bifurcation theory. The aim of the authors was to write a book that would be easily accessible even to non-specialists, thus the exposition is accompanied by a number of examples and the use of abstract special tools is limited. It is also worth noting that each chapter is accompanied by detailed bibliographical remarks. (dpr)

C. U. Jensen, A. Ledet, N. Yui: *Generic Polynomials: Constructive Aspects of the Inverse Galois Problem,* *Mathematical Sciences Research Institute Publications 45, Cambridge University Press, Cambridge, 2003, 258 pp., £45, ISBN 0-521-81998-9*
The inverse Galois problem is to determine, for a given field K and a given finite group G, whether there exists a Galois extension of K, whose Galois group is isomorphic to G. And if there is such an extension, to find an explicit polynomial over K, whose Galois group is the prescribed group G. The authors present a family of "generic" polynomials for certain finite groups, which give all Galois extensions having the required group as their Galois group. The existence of such generic polynomials is discussed and a detailed treatment of their construction is given in those cases, when they exist. (jtu)

D. S. Jones, B. D. Sleeman: *Differential Equations and Mathematical Biology*, *Chapman & Hall/CRC Mathematical Biology and Medicine Series, Chapman & Hall/CRC, Boca Raton, 2003, 390 pp., \$71,96, ISBN 1-58488-296-4*
The book is written with two aims: firstly, to be an introduction both to ordinary and partial differential equations; secondly, to present main ideas on how to model deterministic (and mostly continuous) processes in biology, physiology and ecology. The style of writing is subordinated to these purposes. It is remarkable that without the classical scheme (definition, theorem and proof) it is possible to explain rather deep results like properties of the Fitz-Hugh-Nagumo model of nerve impulse transmission or the Turing model of pattern formation. This feature makes the reading of this text pleasant business for mathematicians also. There exists a similar book written by J. D. Murray (Mathematical Biology), which contains more biological models. In comparison with it, the book under review is also a textbook on differential equations. It can be recommended for students of mathematics who like to see applications, because it introduces them to problems on how to model processes in biology, and also for theoretically oriented students of biology, because it presents constructions of mathematical models and the steps needed for their investigations in a clear way and without references to other books. (jmil)

J. W. Kammeyer, D. J. Rudolph: *Restricted Orbit Equivalence for Actions of Discrete Amenable Groups,* *Cambridge Tracts in Mathematics 146, Cambridge University Press, Cambridge, 2002, 201 pp., £35, ISBN 0-521-80795-6*
The main topic of the book belongs to ergodic theory and measurable dynamics. It studies suitable notions of similarity of two dynamical systems using structure of orbits of corresponding systems. The authors discuss free and ergodic actions of countable discrete amenable groups. The key notions in the book are an orbit relation $O=\{x,T_g(x)\}_{g\in G}\subset X \times X$ generated by the action, an orbit equivalence (a measure preserving map carrying one orbit relation to the other), and arrangements and rearrangements of orbits. Basic definitions and examples can be found in Chapter 2 (the vocabulary of arrangements and rearrangements, and m-equivalence classes of an arrangement). Fundamental results by Ornstein and Weiss on ergodic theory of actions of amenable groups are reviewed in Chapter 3. The next two chapters include key technical lemmas and entropy theory for restricted orbit equivalences. Chapter 6 contains a construction of a topological model for arrangements and rearrangements using a notion of Polish spaces and Polish actions. The last chapter contains a formulation and a proof of the equivalence theorem. Similar questions were studied carefully in the last decades for Z-actions as well as for Z_d -actions (d≥ 1). Relations of the results described in the book to results obtained in these special cases can be found in the Appendix. (vs)

V. V. Kravchenko: *Applied Quaternionic Analysis,* *Research and Expositions in Mathematics, vol. 28, Heldermann, Lemgo, 2003, 127 pp., €24, ISBN 3-88538-228-8*
Function theory for the Dirac equation has grown substantially during the last several decades. The case of dimension four is special in two ways – it is a physical dimension and spinor fields can be identified with quaternionic functions of a quaternionic variable. Obtained results were applied to a wide spectrum of problems in mathematical physics. The first part contains a summary of basic facts from quaternionic analysis, including integral formulae for unbounded domains. The selection of topics is guided by their possible applications. Boundary problems for Maxwell equations in homogeneous media and the Dirac equation for a free particle are treated in the second part. The last part contains a discussion of Maxwell equations for inhomogeneous media, the Dirac equation with potentials, and a generalization of the Riccati equation to a nonlinear quaternionic equation. The book can be useful not only for mathematicians interested in the field but also for engineers (the book is based on a course given by the author to future engineers). (vs)

T. Lawson: *Topology: A Geometric Approach,* *Oxford Graduate Texts in Mathematics 9, Oxford University Press, Oxford, 2003, 388 pp., £45, ISBN 0-19-851597-9*
The book is a nice introduction to topology with an emphasis on its geometric aspects. It is written for two purposes. In the first part, consisting of three chapters, there is material suitable for a one semester basic course of topology. It starts with basic point set topology with special attention to the topology of R^n, followed by a description of the classification of surfaces. The last topic introduced and studied in this part is the fundamental group of a space, including its application to surfaces and the vector field problem in the plane and on surfaces. To compute fundamental groups, the Seifert-van Kampen theorem is introduced and proved.

The second part contains an extension of the material of the first part to a full-year course. It starts with the description of covering spaces, covering transformations and universal covering space, followed by a study of CW-complexes and their properties from a homotopy point of view. Simplicial complexes and Δ-complexes for CW-complexes are described as well. The last chapter is devoted to the homology theory with special attention to its relations to homotopy. There are about 750 exercises of different levels, which can attract students to a more active study of the subject. Solutions of selected exercises are included as an appendix (solutions to all exercises are available to the instructor in electronic form on application to Oxford University Press). The book is an excellent introduction to the subject and be can recommended to anybody interested in geometry and topology as his/her first reading. (jbu)

J. Lewin: *An Interactive Introduction to Mathematical Analysis,* *Cambridge University Press, Cambridge, 2003, 492 pp., £27,95, ISBN 0-521-01718-1*
The book under review provides an introduction to mathematical analysis using possibilities of computers. The book starts with some background material concerning quantifiers, sets, logic formulas, and methods of proofs. The second part deals with the set of real numbers, limits of sequences and functions, differentiation of functions, the Riemann integral, and infinite series. The next chapters are devoted to improper integrals, sequences and series of functions and integration of functions of two variables. The enclosed CD forms an important on-screen part of the book. The CD contains a lot of exercises with solutions. Some parts of the book are explained here in a more detailed way and some chapters are added (e.g., calculus of several variables or complex variable calculus) or are treated in an alternative (advanced) way. This book together with the CD will be useful to many students. (mzel)

E. L. Lima: *Fundamental Groups and Covering Spaces,* *A. K. Peters, Natick, 2003, 210 pp., \$49, ISBN 1-56881-131-4*
The book is an introduction to a part of algebraic topology. It concentrates on the circle of ideas around homotopy theory, fundamental groups and

covering spaces. The first part of the book introduces a general concept of a homotopy, together with a particular case of path homotopy. The fundamental group is defined and computed for many examples, including real and complex projective spaces and classical matrix groups. The fundamental group of the circle is related to winding numbers of plane curves. The second part of the book is devoted to basic properties of (differentiable) covering spaces and their relations to fundamental groups. Each chapter ends with exercises. The book starts from the beginning and its reading requires only a basic knowledge. The book is a pleasure to read, it contains a lot of illustrations and presentation is clear and systematic. (vs)

M. Métivier: *Semimartingales. A Course on Stochastic Processes,* *de Gruyter Studies in Mathematics 2, Walter de Gruyter, Berlin, 1982, 287 pp., €68, ISBN 3-11-008674-3*

The presented monograph is an advanced book on general martingale theory. A reader with a good knowledge of probability and discrete time processes will appreciate the deep results contained in the book. Theorems are formulated for quasimartingales, some parts of the book are devoted to Hilbert space valued processes, and the theory is not restricted to continuous square integrable martingales as is usually the case. Due to these facts, the book is a necessity for all researchers working with general stochastic processes.

The book consists of two parts. In the first part the general theory of martingales is given. It starts with basic definitions and facts from stochastic processes; filtration, measurability, adapted and predictable processes, stopping time, and decomposition. The next chapters continue with the martingale property. We find many classical results in a quite general context like Doob's inequalities or convergence theorems. The first part is concluded by a chapter devoted to square integrable semimartingales, quadratic variation and Meyer's process. Here we also find the theory of Hilbert space valued martingales and stochastic integrals with respect to them. In the second part, stochastic calculus is the focus. First, the stochastic integral is built and a semimartingale version of Itô transformation theorem is proved. Then, as an application, the Brownian and Poisson processes are considered and changes of probability, as well as Girsanov formula, are presented. The book culminates with a chapter on SDE. This is not a book which I would recommend as an introduction to stochastic processes and martingales. But for any rigorous work in the theory of martingales, namely non-continuous processes, semimartingales and multidimensional martingales, it is the book that should be consulted first. It is not easy to write a concise book on general martingale theory but in my opinion Michel Métivier has done great job. (dh)

I. Moerdijk, J. Mrčun: *Introduction to Foliations and Lie Groupoids,* *Cambridge Studies in Advanced Mathematics 91, Cambridge University Press, Cambridge, 2003, 173 pp., £30, ISBN 0-521-83197-0*

This is just a small book nicely covering principal notions of the theory of foliations and its relations to recently introduced notions of Lie groupoids and Lie algebroids. After the first chapter, containing a definition of a foliation and main examples and constructions, the authors introduce the key notion of holonomy of a leaf, a definition of an orbifold and they prove the Reeb and the Thurston stability theorems. Chapter 3 contains the Haefliger theorem (there are no analytic foliations of codimension 1 on S^3) and the Novikov theorem (concerning existence of compact leaves in a codimension 1 transversely oriented foliation of a compact three-dimensional manifold). The Molino structure theorem for foliations defined by nonsingular Maurer-Cartan forms is treated in Chapter 4. A holonomy groupoid of a foliation is a basic example of so called Lie groupoids. The last two chapters describe properties of Lie groupoids, a notion of weak equivalence between Lie groupoids, a special class of étale groupoids, and a Lie algebroid as an infinitesimal version of a Lie groupoid. The book is based on course lecture notes and it still keeps its qualities and nice presentation. (vs)

T. Mora: *Solving Polynomial Equation Systems I: The Kronecker-Duval Philosophy,* *Encyclopedia of Mathematics and its Applications 88, Cambridge University Press, Cambridge, 2003, 423 pp., £60, ISBN 0-521-81154-6*

This is an excellent book for readers interested in algebraic methods. A part of the content will not be new to most of them; its usefulness lies in the fact that so much is brought together in one book. In the first part of the book, the author describes the Kronecker-Duval approach to the solution of systems of polynomial equations. The second part contains a discussion of factorisation of polynomials. The author says: "It is my firm belief that the best way of understanding a theory and an algorithm is to verify it through computation …" Accordingly, the book contains 52 numerical examples provided with solutions and 27 programs. I enjoyed the author's language enormously. The author's words from the preface "the number of hidden mistakes in a draft is always larger than the number of the found ones" are true. I spotted a few, e.g., in Theorem 5.2.3 as well as in Theorem 5.5.6, one has to add the assumption that the degree of considered polynomials is at least one. This criticism of the text is a little more than a quibble and, in any case, is greatly outweighed by its virtues.
(lber)

J. Nestruev: *Smooth Manifolds and Observables,* *Graduate Texts in Mathematics, vol. 220, Springer, New York, 2003, 222 pp., €64,95, ISBN 0-387-95543-7*

Main themes of the book are manifolds, fibre bundles and differential operators acting on sections of vector bundles. A classical treatment of these topics starts with a coordinate description of a manifold M; the algebra of smooth functions on M is a derived object in this approach. The present book is based on an alternative point of view, where calculus on manifolds is treated as a part of commutative algebra. In particular, the initial object is a commutative associative unital algebra F with certain additional properties. The corresponding smooth manifold is reconstructed as the spectrum of F. (A generalization to the non-commutative case, which is usually called non-commutative geometry, is based on this point of view. This generalization, however, is not treated in the book.)

The first few chapters describe properties of algebras that correspond to smooth manifolds, introduce a notion of charts and atlases and define smooth maps between such manifolds. The authors (J. Nestruev is an invented name hiding a group of authors) then show equivalence of the algebraic definition with the usual one. In the second part of the book, the authors define tangent and cotangent fibre bundles of a manifold, jet bundles and they introduce linear differential operators in this algebraic setting. As explained throughout the book, and in particular in the appendix (written by A. M. Vinogradov), it is possible to give a motivation coming from classical mechanics for basic notions treated in the book. The commutative algebra F is related to the laboratory itself, elements of F to measuring devices and points in the spectrum of F to states of an observed physical system. The book contains quite a few exercises and many useful illustrations. (vs)

W. K. Nicholson, M. F. Yousif: *Quasi-Frobenius Rings,* *Cambridge Tracts in Mathematics 158, Cambridge University Press, Cambridge, 2003, 307 pp., £55, ISBN 0-521-81593-2*

Quasi-Frobenius algebras provide the basic setting for modular representation theory of finite groups. Indeed, the group algebra of any finite group is quasi-Frobenius. The presented book is a very accessible introduction to basic properties of quasi-Frobenius rings and the modules over them. Rather than dealing with classical representation theory, the authors consider a more general setting of mininjective rings and show that basics of the classical theory can be developed using only elementary module theory in a more general setting. (A ring is right mininjective, if any isomorphism between minimal right ideals is induced by a left multiplication. By a theorem of Ikeda, quasi-Frobenius rings are exactly the right and left mininjective, right and left artinian rings).

While basic notions and results on (weak) self-injectivity, *CS*- and *C2*-conditions, AB5*, and dualities are developed through Chapters 2-7, the reader is gradually introduced into three challenging open problems: the Faith Conjecture (whether every left or right perfect right self-injective ring is quasi-Frobenius), the FGF-Conjecture (whether the condition that every finitely generated module embeds in a free module implies that the ring is quasi-Frobenius), and the Faith-Menal Conjecture (asking whether any right strongly Johns ring is quasi-Frobenius. A ring R is right Johns, if R is right noetherian and each right ideal of R is an annihilator; R is right strongly Johns, if all full matrix rings $M_n(R)$, $n \geq 1$ are Johns). The latter conjecture is investigated in Chapter 8, where an example is given that the conjecture fails if the term 'strongly' is omitted. Chapter 9 deals with the Faith Conjecture, providing a generic construction of examples using particular $3x3$-upper triangular matrix rings with coefficients in bimodules over division rings.

The book concludes with three Appendices: on Morita theory of equivalence, on Bass' theory of perfect rings, and on the Camps-Dicks Theorem (proving that the endomorphism ring of any artinian module is semilocal). The authors have achieved two seemingly incompatible goals: to provide an elementary introduction to the classical theory of quasi-Frobenius rings, and to bring the reader up to the current research in the field. This makes the book interesting both for graduate students and researchers in contemporary module theory. (jtrl)

L. I. Nicolaescu: *The Reidemeister Torsion of 3-Manifolds,* *de Gruyter Studies in Mathematics, vol. 30, Walter de Gruyter, Berlin, 2003, 249 pp., €84, ISBN 3-11-017383-2*

This is a book on the Reidemeister torsion and its (mostly Turaev's) generalizations. When comparing it with Turaev's book (Introduction to

Combinatorial Torsions, Lectures in Mathematics, ETH Zürich, Birkhäuser, 2001), which appeared recently, we can immediately see that it has a different character. While Turaev's book can also serve as a kind of introduction to the subject (as well as an introduction to the contemporary research in the field), the book under review is devoted to a wide range of applications of the torsion, and its reading requires certain prerequisites.

In the first chapter, which makes the reader familiar with the necessary algebraic notions, the reader is supposed to have some topological (and also algebraic) background. The author modestly states in the introduction that this is a computationally oriented little book. But let us note that the "computations" we find here are very clever computations, and the wide variety of applications presented here do not support the description of a little book. The book will be indispensable for specialists in the field, and I think that it is very good that it exists together with Turaev's book. It is very well written, with many examples and also many exercises. The wide range of applications will be interesting not only for topologists but also for differential geometers. (jiva)

S. P. Novikov, V. A. Rohlin (Eds.): ***Topology II, Homotopy and Homology, Classical Manifolds,*** *Encyclopedia of Mathematical Sciences, Springer, Berlin, 2004, 257 pp., ISBN 3-540-51996-3*

The volume under review consists of three chapters: Introduction to Homotopy Theory (by O. Ya. Viro and D. B. Fuchs), Homology and Cohomology (again by O. Ya. Viro and D. B. Fuchs), and Classical Manifolds (by D. B. Fuchs). The original Russian text was translated by C. J. Shaddock. The presentation of notions and results in all three chapters is really very nice. The first two chapters contain quite detailed exposition, while the third chapter has a more encyclopedia like character. This means that the first two chapters can also serve very well as a textbook. I would even recommend them for this purpose, because the presentation is on one hand a detailed one (as already mentioned), and on the other hand it is not too long. The choice of material is very good, the text is saturated with many examples, and we find here information about further developments and recommendations for further study. Naturally, because this is an encyclopedia, we find no exercises. The last chapter contains information about the topology of classical manifolds, and I do not think that information of this type, in such a compact form and to such an extent, can be found elsewhere. Generally, the whole volume makes a very good impression, and I would say that it is very clearly written. (jiva)

H. Pajot: ***Analytic Capacity, Rectifiability, Menger Curvature and the Cauchy Integral,*** *Lecture Notes in Mathematics 1799, Springer, Berlin, 2002, 118 pp., €22,95, ISBN 3-540-00001-1*

The question of characterization of removable sets for bounded analytic functions is known as the Painlevé problem and has attracted the interest of mathematicians for more than 115 years. In a reformulation due to L. Ahlfors, the point is to describe for which sets the analytic capacity vanishes. In 1977, A. P. Calderón proved the Denjoy Conjecture that one-dimensional rectifiable curves are removable if and only if their Hausdorff length is zero. The proof was based on estimates of singular integrals, namely of the Cauchy operator. The version of the Vitushkin conjecture that a purely unrectifiable set of finite Hausdorff length is removable has been proved in 1998 by G.David, after significant steps forward by M. Melnikov, V. Verdera, P. Mattila, M. Christ and others. In this research the Menger curvature appeared to be relevant. In 2003, X. Tolsa characterized removable sets in terms of the Menger curvature. In this volume, the author presents the above mentioned results and related developments.

The first two chapters are devoted to the Hausdorff measure and rectifiability, including beta numbers and uniform rectifiability. The Menger curvature is explained in Chapter 3. In Chapter 4, the theory of singular integrals is applied to the Cauchy operator. The Painlevé problem and the analytic capacity are discussed in Chapter 5. In Chapter 6, the proofs of the Denjoy conjecture and of the Vitushkin conjecture as well as related results are presented. Some very recent results including the Tolsa theorem are given in Chapter 7. The book excellently explains this beautiful theory, which is a subtle mix of complex analysis, harmonic analysis and geometric measure theory. The text is almost self-contained. The history of this development is nicely reviewed. At the end, main open problems of the theory are listed. (jama)

J. Roe: ***Lectures on Coarse Geometry,*** *University Lecture Series, vol. 31, American Mathematical Society, Providence, 2003, 175 pp., $39, ISBN 0-8218-3332-4*

The book is based on lectures given by the author at Penn State University. The notion of 'coarse geometry' was invented to keep track of large scale properties of metric spaces. The first part of the book introduces an abstract notion of a coarse structure, a notion of a bounded geometry coarse space, its growth and a notion of an amenable metric space, and discusses coarse algebraic topology. The main topic in the middle part is the Mostow rigidity theorem saying that if two compact hyperbolic manifolds of dimensions at least 3 are homotopy equivalent, they are isometric. The last part of the book contains a discussion of a notion of asymptotic dimensions and uniform embeddings into Hilbert spaces, together with relations to the Kazhdan property of discrete groups. The book offers a very readable description of a circle of ideas around the notion of coarse geometry. (vs)

C. Sabot: ***Spectral Properties of Self-Similar Lattices and Iteration of Rational Maps,*** *Mémoires de la SMF 92, Société Mathématique de France, Paris, 2003, 104 pp., €25, ISBN 2-85629-133-3*

The method of a renormalization group, widely used in physics, produces some considerable mathematical difficulties, when applied to systems on usual, regular lattices. Hierarchical, self-similar lattices are much better suited for a detailed mathematical development of this method. The present book gives a detailed treatment of the problems, which were first studied by Rammal and Toulouse on the Sierpiński gasket, and puts them in a more general perspective. The goal of the book is to study systematically, in a mathematically rigorous way, the spectral properties of Laplace operators on self-similar sets. A new renormalization map, which is a rational map defined on a smooth projective variety, is introduced. Then, the characteristics of the spectrum of these operators are related with the characteristics of the dynamics of iterates of such a renormalization map. An explicit formula for the density of states is given, and it is shown that the spectral properties of the operator depend substantially on the asymptotic degree of the renormalization map. The contents (a slightly shortened list of the chapters of the book) are: Definitions and basic results (self-similar Laplacian, density of states); Preliminaries (Grassmann algebra, trace of a Dirichlet form); The renormalization map (construction, the main theorem in the lattice case); Analysis of the psh function G (the dichotomy theorem, asymptotic degree, regularity of the density of states, some related rational maps); Examples; Remarks, questions and conjecture; and Appendix (plurisubharmonic functions and positive currents, dynamics of rational maps on projective space, iteration of meromorphic maps on compact manifolds, G - Lagrangian Grassmanian). (mz)

I. Stewart: ***Galois Theory, third edition,*** *Chapman & Hall/CRC Mathematics, Chapman & Hall/CRC, Boca Raton, 2003, 288 pp., $44,96, ISBN 1-58488-393-6*

This is the third edition of the classic textbook of Galois theory, first published in 1972. But it is not just a reprint of earlier editions. Those who know the first two editions will be surprised by a radical change of presentation. The author reversed the original Bourbakiste approach expressed by a slogan "from general to concrete" and now presents the theory in the direction "from concrete to general".

Thus after a historical chapter, he starts with solutions in radicals of polynomial equations of degree 2, 3, 4, and presents a quintic equation solvable in radicals. Factorization of complex polynomials is developed from theory of polynomials with complex coefficients and the fundamental theorem of algebra. Field extensions of rational numbers follow including the definition of rational expressions and the degree of an extension. As a digression, the author proves non-existence of ruler-and-compass solutions of classical geometric problems of squaring the cube, trisecting the angle and squaring the circle. Then the Galois theory starts. After a short explanation of Galois groups according to Galois, he presents modern definitions of the Galois correspondence, splitting fields, normal and separable extensions, and field automorphisms. The fundamental Galois correspondence between the subfields of a field extension and subgroups of the automorphism group of the extension is proved. An example of the correspondence resulting from a quartic equation is also given. Solvable and simple groups are introduced and the Galois theorem about solvability of equations in radicals is proved.

After all this, abstract rings and fields are introduced and the abstract theory of field extensions is developed. The last part of the book contains some applications, e.g., the construction of finite fields, constructions of regular polygons, circle divisions (including cyclotomic polynomials) and an algorithm on how to calculate the Galois group of a polynomial equation. The penultimate chapter is about algebraically closed fields and the last chapter, on transcendental numbers, contains "what-every-mathematician-should-see-at-least-once", the proof of transcendence of π. The book is designed for the second and third year undergraduate courses. I will certainly use it. (jtu)

D. Stirzaker: ***Elementary probability,*** *Second edition, Cambridge University Press, Cambridge, 2003, 536 pp., £65, ISBN 0-521-83344-2*

This book provides an introduction to elementary probability and some of its applications. The word elementary means that the book does not need the abstract Lebesgue measure and integration theory, only elementary calculus is used. The strong feature of the textbook is a choice of good examples. Each theoretical explanation is accompanied by a large number of examples and followed by worked examples incorporating a cluster of exercises. The examples and exercises illustrate the treated topics and help the student solve the kind of problems typical of examinations. Each chapter concludes with problems. Solutions to many of these appear in an appendix, together with solutions to most of exercises.

The second edition of the book contains some new sections. A new section provides a first introduction to elementary properties of martingales, which is now occupying a central position in modern probability. Another section provides an elementary introduction to Brownian motion, diffusion, and the Wiener process, which has underpinned much of classical financial mathematics, such as the Black-Scholes formula for pricing options. Optional stopping and its applications are introduced in the context of these important stochastic models, together with several associated new examples and exercises. The list of chapters: Introduction, Probability, Conditional Probability and Independence, Counting, Random Variables: Distribution and Expectation, Random Vectors: Independence and Dependence, Generating Functions and Their Applications, Continuous Random Variables, Jointly Continuous Random Variables, and Markov Chains. Some sections can be omitted at a first reading (e.g. Generating Functions, Markov Chains). The book is suitable for a first university course in probability and very useful for self-study. (br)

E. B. Vinberg: *A Course in Algebra,* *Graduate Studies in Mathematics, vol. 56, American Mathematical Society, Providence, 2003, 511 pp., $89, ISBN 0-8218-3318-9*

The book covers all topics, which are usually included in basic courses on linear algebra and algebra in the first two years of study. In addition, it also includes a lot of non-standard and interesting material going into several different directions. The book is based on the long time teaching experience of the author. At the beginning, main algebraic structures are introduced (groups, rings, fields, algebras, vector spaces). Polynomial algebra is studied in detail and basic facts of group theory are covered. Linear algebra is covered in Chapter 2, Chapter 5 and Chapter 6, together with bilinear and quadratic functionals. Chapter 8 contains a description of general multilinear algebra. The last four chapters are devoted to commutative algebra (principal ideal domains, Noetherian rings, algebraic extensions, and affine algebraic varieties), groups (Sylow theorems, simple groups Galois extensions and Galois theory), linear representations of associative algebras (complete reducibility, invariants, division algebras) and linear Lie groups (the exponential map, the adjoint representation, basic facts on linear representations). The book is beautifully written, the choice of topics and their order is excellent and the book is very carefully produced. It contains a huge number of exercises and it appeals to geometric intuition whenever possible. It can be highly recommended for independent reading or as material for preparation of courses. (vs)

C. Voisin: *Hodge Theory and Complex Algebraic Geometry I,* *Cambridge Studies in Advanced Mathematics 76, Cambridge University Press, Cambridge, 2002, 322 pp., £55, ISBN 0-521-80260-1*

As the title clearly implies, the book deals with the basics of Hodge theory and its relation with complex algebraic geometry. The first four chapters are devoted to preliminaries. The main result of the first chapter is the Riemann and Hartogs Theorems on extension of holomorphic functions and existence of local solutions to the Cauchy-Riemann equations. The second chapter contains an introduction to (holomorphic) vector bundles and the Dolbeault complex on differentiable manifolds with complex structure (determined from an almost complex structure via Newlander-Nirenberg theorem). The third chapter contains a self-contained introduction to Kähler geometry, Kähler metrics and Chern connections on a holomorphic vector bundle equipped with a Kähler metric. The introductory part ends with the fourth chapter describing theory of sheaves, cohomology of a topological space with values in a sheaf and theory of acyclic resolutions leading to a proof of the de Rham theorem.

The second part of the exposition is devoted to a proof of the Hodge and Lefschetz decomposition theorems of cohomology of a complex manifold. The first two chapters of this part summarize a necessary background from analysis on Hilbert spaces used to define (formal) adjoints of elliptic operators, their Laplace operators and finally, as an application, harmonic forms and their relation to cohomology. The following two chapters give conceptual applications of these results. The notion of a polarized Hodge structure is explained and applied to the Kodaira embedding theorem, which states that a complex manifold is projective if it admits an integral polarization. The next chapter is devoted to the holomorphic de Rham complex and the interpretation of the Hodge Theorem in terms of degeneracy of the Frölicher spectral sequence. The chapter ends with an introduction to holomorphic de Rham complex with logarithmic singularities for quasi-projective smooth varieties with mixed Hodge structure on their cohomology. The third part is devoted to a study of variations of Hodge structures. The notion of a family of complex differentiable manifolds leads to the construction of the Kodaira-Spencer map and Gauss-Manin connection associated to the local system of cohomology of fibers of this family. Using the Kodaira-Spencer map and the cup product in Dolbeault cohomology, the period map and its differential are described. In the last chapter, various "cycle classes" are studied. In particular, Hodge classes arising in the study of morphisms of Hodge structures are described in more detail. The Deligne cohomology and the Abel-Jacobi map are introduced, although they are studied in more detail in the next volume of the series. (pso)

C. Voisin: *Hodge Theory and Complex Algebraic Geometry II,* *Cambridge Studies in Advanced Mathematics 77, Cambridge University Press, Cambridge, 2003, 351 pp., £60, ISBN 0-521-80283-0*

The book is the second volume of a two-volume treatise on Hodge theory and its applications. The volume consists of three parts. The first part illustrates various aspects of the qualitative influence of Hodge theory on the topology of algebraic varieties. In particular, the Deligne's theorem on the degeneration of Leray spectral sequence of rational cohomology of a projective fibration at E_2 and its consequences, e.g. surjectivity of the map from rational cohomology of the total space to (the base generated) monodromy invariant subspace of rational cohomology of the fiber, are discussed.

The second part is devoted to the study of infinitesimal variations of Hodge structure for a family of smooth projective varieties and its applications, in particular those concerning the case of complete families of hypersurfaces of complete intersections of a given variety. The main explicit result in this part is Nori's connectivity theorem.

The third (and final) part of the volume is devoted to relations between Hodge theory and algebraic cycles. Using infinitesimal techniques from the second part, certain cycle class maps and equivalence relations (rational, homological, algebraic and Abel-Jacobi equivalence) are established. In particular, variations on the theme of the relation of Chow groups and Hodge theory of smooth complex varieties are reviewed. (pso)

V. A. Zorich: *Mathematical analysis I,* *Universitext, Springer, Berlin, 2004, 574 p., €53,45, ISBN 3-540-40386-8*

V. A. Zorich: *Mathematical analysis II,* *Universitext, Springer, Berlin, 2004, 681 p., €53,45 , ISBN 3-540-40633-6*

This is a very nice textbook on mathematical analysis, which will be useful to both the students and the lecturers. The list of chapters is as follows: Vol. I. 1) Some General Mathematical Concepts and Notation, 2) The Real Numbers, 3) Limits, 4) Continuous Functions, 5) Differential Calculus, 6) Integration, 7) Functions of Several Variables, 8) Differential Calculus in Several Variables, Some problems from the Midterm Examinations and Examination Topics. Vol. II. 9) Continuous Mappings (General Theory), 10) Differential Calculus from a General Viewpoint, 11) Multiple integrals, 12) Surfaces and Differential forms in R_n, 13) Line and Surface Integrals, 14) Elements of Vector Analysis and Field Theory, 15) Integration of Differential Forms on Manifolds, 16) Uniform Convergence and Basic Operations of Analysis, 17) Integrals Depending on the Parameter, 18) Fourier Series and Fourier Transform, 19) Asymptotic Expansions, Topics and Questions for Midterm Examinations and Examination Topics.

About style of explanation one can say that the definitions are motivated and precisely formulated. The proofs of theorems are in appropriate generality, presented in detail and without logical gaps. This is illustrated in many examples (many of them arise in applications) and each section ends with a list of problems and exercises, which extend and supplement the basic text. Finally, one can make several remarks on the approaches used. Real numbers are introduced axiomatically, the general concept of limits with respect to (filter) base is explained and used, e.g. in the definition of Riemann integral, and real powers of a positive number are introduced as limits of rational powers. Trigonometric functions are firstly introduced intuitively using the unit circle, then as sums of some power series and finally as inverse functions to functions arcsine and arccosine, which are defined after definition of the length of a curve. The multiple integral is firstly introduced as a Riemann integral over an n-dimensional interval (analogous to the 1-dimensional case). Then for the bounded set, D is defined as the integral over an interval containing D of the product of the given function and the characteristic function of the set D. The Lebesgue necessary and sufficient criterion for integrability is proved and frequently used. Line and surface integrals are introduced as integrals of differential forms over surfaces. Smooth and piecewise smooth surfaces are considered. Fundamental integral formulas (including the general Stokes formula) are proved. This material is explained in more detail in Chapter 15. In Chapter 14, vector versions of fundamental integral formulas are stated and vector fields having a potential are also studied. (jkop)