# NEWSLETTER

### OF THE EUROPEAN MATHEMATICAL SOCIETY



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John Pell (1611-1685) and

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## European Mathematical Society

#### Newsletter No. 56, June 2005

EMS Calendar	2
Editorial	3
Editorial Team	4
Open access to journals? – S. Hezlet	6
Abel Prize – Th. Bruss	9
Spin glasses – F. den Hollander & F. Toninelli	13
Why no decent 3D multiplication? – Z. Marciniak	18
Ruy L. Gomez – <i>N. Bebiano</i> &	
J. da Providencia jr.	21
Interview: Arild Stubhaug	23
ERCOM: Institut Mittag-Leffler	29
Book Review: Interpolation and Approximation	31
Personal Column	33
Problem Corner	34
Forthcoming Conferences	38
Recent Books	47

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### **EMS** Calendar

#### 2005

**10–11 June & 14 June** EMS Lectures by Nina Uraltseva *Regularity of free boundaries in parabolic obstacle type problems.* At the conference on Free Boundary Problems 2005 in Coimbra (Portugal) and at the University of Lisbon (Portugal). *Web site: www.fbp-2005.org* 

**25 June–2 July** EMS Summer School at Pontignano (Italy) Subdivision schemes in geometric modelling, theory and applications. Web site: www.subdivision-summerschool.uni-kl.de/

**18–23 July** EMS Summer School and European young statisticians' training camp at Oslo (Norway). Web site: www.ems2005.no

**1 August** Deadline for submission of material for the September issue of the EMS Newsletter. Contact: Martin Raussen: raussen@math.aau.dk

**10–16 September** EMS Summer School and Séminaire Européen de Statistique at Warwick (UK), *Statistics in Genetics and Molecular Biology.* 

Web site: www2.warwick.ac.uk/fac/sci/statistics/news/semstat

**13–23 September** EMS Summer School at Barcelona (Catalunya, Spain) *Recent trends of Combinatorics in the mathematical context.* Web site: www.crm.es/RecentTrends

**16–18 September** EMS-SCM Joint Mathematical Weekend at Barcelona (Catalunya, Spain). Web site: www.iecat.net/institucio/societats/SCMatematiques/emsweekend/

**18–19 September** EMS Executive Committee meeting at Barcelona (Catalunya, Spain) Contact: Helge Holden: holden@math.ntnu.no

#### 2006

**13–17 March** EMS-SIAM-UMALCA *Joint Meeting in applied Mathematics Centre for Mathematical Modelling*, Santiago de Chile

**16–18 June** Joint EMS-SMAI-SMF Mathematical Weekend, Nantes (France)

Contact: Laurent Guillopé: Laurent.Guillope@math.univ-nantes.fr

**30 June** EMS Executive Committee Meeting, Torino (Italy) Contact: Helge Holden: holden@math.ntnu.no

1-2 July EMS Council Meeting, Torino (Italy) Web site: www.math.ntnu.no/ems/council06/

**3–7 July** Mathematics and its Applications: First joint meeting of SIMAI-SMAI-SMF-UMI, under the auspices of the EMS; Torino (Italy)

**22–30 August** International Congress of Mathematicians in Madrid (Spain). Web site: www.icm2006.org/

#### 2007

**16–20 July** ICIAM 2007, Zurich (Switzerland) Web site: www.iciam07.ch/

#### 2008

#### 14–18 July

5th European Mathematical Congress, Amsterdam (The Netherlands). Web site: www.5ecm.nl

### Editorial



#### Pavel Exner (Prague, Czech Republic)

Last year, an executive colleague in the EMS gave a sigh: "Oh my, '05 is approaching. Again we will suffer a lot of physics!" Such a state of mind is emblematic and no doubt there are good reasons for that.

Physicists are noisy. They are not shy about showing the material products of their activity. Some of these are huge, extending for kilometres, and journalists who usually have other priorities than finding close fitting expressions, discover "modern cathedrals" if nothing worse. Other times they are on the contrary small, but the noise is the same. Has a day passed recently without you hearing that something is "nano"? The truth is that we have public relation stars in our team too, but they are not in the same league. The physicists have to compete for attention, and thus for political influence, with biologists, space engineers, etc.

Moreover, most mathematicians find no pleasure in reading physical papers. While we chisel our eternally valid theorems – hoping, of course, that the posterity will one day correct the lack of attention paid by the audience today – physicists use our precious tools carelessly and as a result, physical literature remains a desk that has to be cleaned from time to time.

But there is also another side of the coin. The centenary of the annus mirabilis is a good opportunity to reflect on what happened then and what the consequences were. We find that the worldliness of many mathematicians is firmly interwoven into the history of the Einstein discoveries. It is notoriously known that Poincaré had in his hands the transformation formulae of special relativity, but missed the chance to make the decisive step in their interpretation. On the other hand, when this was done it was Minkowski who first grasped the group structure of the new theory, thus giving it a significant boost. In the following years when Einstein struggled with the Riemannian geometry he benefited from his discussions with Hilbert – so much so that later priority questions arose.

Another 1905 discovery, the equations of Brownian motion, led Wiener to a deep theory of infinite dimensional integration fifteen years later. In the forties, Feynman – by a flagrant misuse of good old integration – proposed an intriguing approach to quantum mechanics. Soon Mark Kac derived from it an immensely useful formula for certain Wiener integrals. To this day, Feynman's misdeed has inspired many strong mathematicians – and it also led a couple of famous ones into a blunder untypical for their other work.

It may at least seem, that no mathematicians were involved in the history of photoeffect. However, a closer look shows that even here a deep connection exists. This discovery of Einstein opened a path that in half a century led to quantum field theories, which were tremendously successful in predicting numerous phenomena, some of which influence our everyday life today. Twenty or thirty years ago, one would add in the same breath that they were a complete mathematical disaster, full of divergent quantities and juggling of various infinities. Today the situation is slightly better, but putting these theories on a firm mathematical ground still remains one of the great challenges.

Looking back on this hundred years perspective at the relationship between the two disciplines one can say that they are like a good family. From the outside you may see noisy quarrels, but if you go below the surface you will find strong brotherly ties. Let us wish it remains so, for the benefit of both parties.

Pavel Exner is Professor in theoretical physics at Charles University at Prague, Czech Republic. He is a Vice-President of the EMS and Chair of its electronic-publishing committee.



#### EMS Council 2006: New Date and Venue

Unfortunately, it proved impossible to hold Council in Madrid at ICM 2006, as was originally planned. Instead, EMS Council 2006 will be held on July 1–2, at Torino (Turin), Italy, preceding the first joint meeting of SIMAI-SMAI-SMF-UMI on Mathematics and its Applications (held under the auspices of the EMS).

### Introducing the editorial team, part II



**Krzysztof Ciesielski** works at the Mathematical Institute of the Jagiellonian University in Krakow (Poland), where he obtained a Ph.D. in 1986 under the supervision of Andrzej Pelczar. Since 1999, he has been the Vice-Head of the Institute,

responsible for teaching. His mathematical interests include dynamical systems, topology, the history of mathematics and popularization. He is the co-author of several books presenting mathematics for a general audience; two of those books were awarded the most prestigious Polish prizes for popularizing science. His activities include mathematics for young people and teachers. He presents lectures, writes articles, and for 25 years he has been involved in the Mathematical Olympic Games for Secondary School Students. He has been a Correspondent for the Mathematical Intelligencer since 1987. He is the Vice-Chairman of the Editorial Committee of the Polish mathematical popular monthly Delta.

His wife Danuta is also a mathematician, working in complex analytic geometry and classical geometry. They have two children. His non-mathematical interests include tourism (especially walking on the mountains), books and music.



Everybody has his own problem area(s). **Paul Jainta's** problem area is the Problem Corner.

Since his school-days, he has been tackling with mathematics problems and enjoyed them greatly. Consequently, he decided to study mathematics and physics at the Friedrich-Alexander University in Erlangen, and is teaching these subjects at a grammar school in Schwabach near Nürnberg (Germany).

Almost all of his life he has encouraged students at schools doing maths outside their regular lessons. From these efforts have grown some mathematics courses or working-groups and two mathematics contests for students; the "Fürther Mathematik Olympiade", established in 1981, with over 2000 participants, and the "Bayerische Landeswettbewerb Mathematik", on whose genesis he has collaborated crucially as secretary. Since then has been known far and wide as the "Father of Competitions"; for in addition he is acting as a marker for the German national event, the "Bundeswettbewerb Mathematik". Recently, he has joined the problems committee of the "Mathematical Olympiad", which is on the starting line throughout Germany, and once again he is a bustling member.

Paul is married and has three children.



**Ana Bela Cruzeiro** is a full professor at the Mathematics Department of the Instituto Superior Técnico (IST), Lisbon (Portugal). She was a student of Paul Malliavin in Paris in the eighties and did her Ph.D. working in analysis on the Wiener space. After

that she worked on statistical hydrodynamics, Euclidean quantum mechanics and, more recently, in several geometric and probabilistic aspects of the path space of a Riemannian manifold. She collaborates with many researchers around the world. Her main research interests are stochastic analysis, Malliavin calculus, infinite dimensional analysis and geometry, and probabilistic methods in mathematical-physics (notably in hydrodynamics and quantum physics).

From 2000 to 2004, she was the President of the Portuguese Mathematical Society (SPM). In September 2003, the first in the series of conferences "EMS Mathematical Weekends" was organized in Lisbon by the EMS and the SPM, thanks to the initiative of a group of Portuguese researchers. She is currently a member of the Meetings Committee of the EMS.

Ana Bela is married (to a mathematical-physicist) and has a son.



Vicente Muñoz is a Researcher at the Department of Mathematics at the Consejo Superior de Investigaciones Científicas in Madrid (Spain). His general background is in Differential and Algebraic Geometry. He completed his Ph.D. at Oxford Univer-

sity in 1996, under the supervision of Professor Simon Donaldson, working in gauge theory and 4-manifold topology, specifically computing the behaviour of Seiberg-Witten and Donaldson invariants of 4-manifolds under the operation of a connected sum along surfaces. One consequence of this work is a proof of (a version of) the Atiyah-Floer conjecture. Moreover, he obtained a proof of the finite type conjecture for all 4-manifolds.

His current research is in the fields of symplectic geometry (asymptotically holomorphic theory and rational homotopy theory for symplectic manifolds) and moduli spaces of vector bundles on Riemann surfaces (representations of fundamental groups and Brill-Noether theory).

Vicente keeps in contact and collaborates with a number of researchers around the world, and he is one of the promoters of the Spanish Research Group on Symplectic and Algebraic Geometry GESTA, that will be organizing a Satellite Conference to the ICM in Madrid 2006.

### Open letter to Professor Summers, President, Harvard University

Mina Teicher (Bar-Ilan University, Israel)

Harvard University President Lawrence Summers caused a firestorm with remarks at a January conference (NBER Conference on Diversifying the Science & Engineering Workforce, Cambridge, Mass., January 14, 2005) suggesting biological differences may partly explain why fewer women reach the very highest-level science jobs. In particular he said: "It does appear that on many, many different human attributes – height, weight, propensity for criminality, overall IQ, mathematical ability, scientific ability – there is relatively clear evidence that whatever the difference in means – which can be debated – there is a difference in the standard deviation, and variability of a male and a female population".

I recall the intellectual stimulation and interest that you raised while visiting Israel back in late 2004. Your position as head of a leading university, with the tradition of absolute excellence, and your personal positions drew attention and respect from all.

Unfortunately, just shortly after that you failed us, by stating wrongly that women are not successful in mathematics for reasons beyond their control. Your statements (as quoted in the press) are harmful to the young generation of talented women who might not choose a career in Science and Technology: your comments discourage them.

Of course, there are low numbers of women in Science and Technology, but not for the biological reasons you mentioned (without biological evidence). I can not elaborate here on the sources for this phenomenon, but can mention briefly the following: legitimate choices of women to choose a career in a field close to their societal values and commitments; need and pressure to raise a family in the very same years as one is judged for tenure; hiring and promotion criteria which are sensitive to gender; prejudice and discrimination in work places.

Your expression might give the impression that we have not progressed much in the last 70 years since the time of Emmy Noether. One of the pillars of science in the previous century, she was a German-Jewish female mathematician who fled from Germany to the US in 1933. Despite her eminence and the originality of her contributions, she accepted a teaching position at Bryn Mawr, a small girls' college in Pennsylvania, whose students rightfully worship her legacy.

I admire Emmy Noether for her contributions to Mathematics and Physics, and for her courage to pursue a career in science at a time when girls were not admitted to these faculties. She was the first female student to be admitted to the Science Faculty in Erlangen. Despite



Emmy Noether (1882-1935)

the strong support of David Hilbert, she never received a professorship in Göttingen, where she worked. In those days she had already made one of the most striking developments in physics, "Noether's theorem", which establishes the connection between symmetries and conservation laws. She is noted for the creation of an entire school of mathematics – commutative algebra – without which modern algebraic geometry would not exist. The research Institute of which I am the director of (a joint venture of the Max-Planck Society in Germany and Bar-Ilan University in Israel) is named after Emmy Noether.

I hope that young girls making a career choice will listen to her silent voice and not to your remarks. Moreover, I call upon you to address young girls, and encourage them to study mathematics. They can do it, and for them it opens the door to all sciences, to high-tech careers, and to a life of economic security. Emmy Noether reached the top of her profession despite many obstacles placed in her way. There are many talented girls today who can go as far if given encouragement.

Sincerely yours,



Prof. Mina Teicher Director, Emmy Noether Research Institute for Mathematics, Israel Chairman of the Education Committee of the European Mathematical Society, Chairman of the Council for the Advancement of Women in S&T of the Israeli Government

### Payment models for journals. Vices and virtues of open access

Suzan Hezlet (London Mathematical Society)

In the last two years there has been an increase of activity in the debate on journal prices and much of this has come from the UK where a Select Committee of the House of Commons has enquired into different payment models for journals. Taken together with the adoption by the US National Institutes of Health of a broadly supportive open access policy, the discussions have sometimes been heated and led to division between the new open access publishers and some librarians on one side, and established publishers and some learned societies on the other. These are strange bedfellows which indicates that the arguments are more complex than we have grown used to in the simple "journals = high prices, publishers = bad, libraries = good" model.

Publishing at the London Mathematical Society began in 1865; it is reported that the publishing programme never made a penny for the first hundred years but recently we have done much better. We are in an awkward position because we are dependent on the income received from publishing to carry out our programmes to support mathematics. At the same time our members might not have access to all the mathematical literature they need and suffer from high journal prices and cuts to the library budgets. Instinctively, one might feel that if open access publishing is a mechanism by which mathematicians could freely access all available literature on the web, surely this is a general benefit that outweighs all others. Is this instinct well-directed or dangerously naïve? The position that the LMS takes on pricing and availability of the journals is the responsibility of the LMS Council, but the views expressed here on this question and the background to it are solely my own.

The aim of this article is to open up the debate on where we go from here. The LMS publishing programme is fully international: papers are sent in from over 80 countries and libraries from a similar number of countries subscribe to the journals. It is important that whatever we do in the future gains the approval of the international community. I hope that the publication of this brief account will encourage mathematicians and librarians to contact us with their views so we may be better informed when it comes to making the big decisions on pricing policy and digitizing our older volumes of the journals.

#### The Background

Many mathematicians equate open access publishing with "free-to-all" publications but much of the political debate has come from the biomedical sciences, to whom open access implies that the author pays for the cost of publication and an author is expected to find about \$1500 per article.

There are at least two reasons why the "free-to-all" model has not been more widely taken up. Firstly, some of the added-value work done in journal production is no longer provided, for example, the texts may not be professionally edited. Secondly, journals of high standing tend to grow beyond that which can be managed by just one or two active voluntary editors. If a substantial number of the 500 or so mathematics journals cited by the ISI became "free-to-all" publications, the workload to the editors who manage the review process and find referees would become impossible and the probability of having a paper accepted within a reasonable time would be subject to even more random factors than is now the case.

The two largest open-access experiments: Public Library of Science (PLoS) and BioMedCentral, have both chosen to charge authors although they will consider waiving fees in exceptional circumstances. Many have criticised the "author-pays" model on the grounds that the charges are like page charges, against which mathematicians argued for many years; and that the quality of the refereeing process will be subverted by the need to receive payments from authors. Neither of these are strong arguments; the quality of an author-pays journal will in the long run depend on what is published and the citation rates will measure the success or failure. Furthermore an open access journal is quite distinct from one that levies page charges where the author receives no extra visibility in return for the payment.

I believe we should look for the answers to two questions before supporting a move to open access publishing:

- a) Will the benefits of increased visibility outweigh the work done by learned societies to promote research?
- b) Even if we assume that learned societies can survive in an open access world; can they survive the revolutionary period during which they would need to compete with open access journals already subsidised by charities and the traditional large commercial publishers? Even if none of us reaches the publishing nirvana of a world full of open access journals, we may have to put up with the revolution.

The second question is a serious concern for us. A few professionally-based societies do not rely on their publishing programmes, but those of us that represent research have few alternative sources of income apart from the raising of membership fees. The very large commercial publishers, governments and large charitable foundations are in strong financial positions to fight their corners even if they sustain losses for a few years, but learned societies cannot do this. Although the publishing programmes of the mathematical societies operate internationally, we only publish mathematics and cannot diversify to weather the storm.

To answer the first question correctly, one would need a comprehensive study of the macroeconomics of research but we can make some simple judgements about the flow of money through the academic world. If a library decides to cut a journal, does the money saved stay in the research pot? I doubt it: most library cuts are made on a permanent basis and the money that is saved by the government or funder is used elsewhere. The answer is even worse if a library cuts a journal owned by a learned society where the profit is kept within the world of research. Not all of the income to learned societies is spent on funding research meetings, but even the money spent on administration is saving mathematicians' time in managing their society business and the choice of how the money is spent by the learned society is made by its members.

#### Arguments specific to mathematics

In the UK, most money for mathematics research originates from the government and salaries and journals are generally paid for out of university budgets rather than the research councils. Therefore, a switch from library subscriptions to author-pays journals should be fairly straightforward, provided the universities are willing to switch funding straight out of library budgets and in to the research authors' pockets. But evidence presented to the UK Select Committee showed that a research-centred university will find it more expensive to fund the publication fees of its authors than the journals to which it subscribes. The argument applies to all research-rich countries in Europe and North America.

One also has to consider the benefits to mathematicians working in isolated circumstances. It is certainly the case that a mathematician in a small remote university would benefit greatly by being able to read research papers freely, and this has been a strong argument in favour of open access publishing. However, a mathematician who wishes to publish in a world of "author-pays" journals will be seriously disadvantaged if he or she does not have funds to pay. This argument applies to mathematicians in developing countries and to those who work in small institutions or who are retired. An author might find some journal owners who are willing to subsidise his or her work, but the author is at the mercy of each journal's charity and may not receive a decision on the waiving of a fee until the lengthy review process is complete.

The traditional journal subscription system adopted by learned societies is not perfect but it benefits two groups: those who benefit directly from our activities and those who submit papers to the journals. The income from publications is recycled into the various programmes and conferences that a society organises or funds indirectly and many mathematicians benefit from these without being aware of the source of the funds; they do not necessarily have to be members of a society or subscribers to the journals. It benefits the second group of authors more directly because under the current setup they do not have to pay to have their papers published in the journals.

Would they benefit more if the learned societies actively adopted an open access policy? Starting a brand new open access journal would simply add one more journal to the excessive number of existing journals, so I looked at the finances of adopting an open access policy on an existing journal. We would have to charge over EUR 2500 per article if we wanted to retain the existing income to the society. If we also wanted to waive fees to authors who could not afford to pay, the price would increase further.

Most publishers and mathematicians agree that the profile of reading and citation of pure mathematics papers is unique among the science subjects. Papers have a long citation half-life of over ten years but are slow to be read. Data on the frequency of paper downloads from



mathematics journals suggests that each individual paper is read by far fewer people than is the case with physics or chemistry papers of similar standing. This distinct profile partly arises from a good international standard of peer review combined with increasingly narrow and specialised interests of mathematics researchers. Learned societies need to support the current standards of peer review in the face of erosion as referees come under pressure from the increased number of papers being written and new journals exacerbate this problem when rejected papers are moved from one journal to another. At the LMS we also hope to maintain and encourage a widening of reading habits through the continued publication of general journals.

#### What the LMS is doing

In the light of all this debate, the LMS Council decided that we could improve access to encourage early reading of the most recently published articles. Currently the two most recently published electronic issues for each of the LMS periodicals: the Proceedings, Journal and Bulletin, are freely available to all readers, regardless of whether they have a subscription. We will operate a "moving wall" so that the papers then move on to the closed access part of the Journal's web site after they have been freely accessible for four months. To begin with, this is a one year trial but the first signs are encouraging and the number of articles downloaded has increased.

The Committee on Electronic Information Communication of the IMU has recently recommended a policy to make older papers (usually over 10 years old) available and this has been adopted by some highly respected mathematics publishers. While I respect their view, I also hope they will understand that there is more than one possible solution to widening access, and that they will support the recent changes we have made to access the LMS journals.

An alternative policy is to offer an open-access option to authors, so that those who wish to pay would have their paper freely accessible while other papers remain closed and are only accessible to those who pay the journal subscription. Springer has adopted such a policy across their journals, although it should be recognised that a substantial number of its journals publish in the bio-medical field where such an option might be popular. For the present, the LMS is not offering this mixed-economy option. Allowing only those authors who can afford to pay an extra service can be considered divisive: distinguishing and advantaging the few who could afford to pay against the rest. However it is possible the LMS Council may reconsider this position if a substantial number of mathematics research funders choose to offer payment.

What of our back archive of volumes that only exist in print? We are looking into the most effective way of "retrodigitizing" these so that it will benefit mathematics without harming the society's income. We have yet to decide on the final mechanism and one possibility is that we will fund the work ourselves over the next few years, and current subscribers to our journals will be able to access the full archive. The idea is to encourage libraries to continue to take an annual subscription to our journals and to benefit them by adding value to the annual subscription. We might also sell copies of the archive for a one-off fee.

Those of us who attend international meetings are frequently asked "what is LMS publishing up to"? We may be up to something else tomorrow, but this is what we are doing at the moment; it is not set in stone and in these revolutionary times it is sensible for us to be adaptable. The best way to adapt in line with what is wanted is to hear your opinion.



Susan Hezlet received her DPhil in theoretical particle physics from Oxford in 1993 and promised not to pursue a career in physics. Instead, she went to work in publishing, moving to the London Mathematical Society in 1998. As Publisher for the LMS, she is responsible for ex-

panding and diversifying the range of publications and retaining the income to the Society from the current list of eleven journals and four book series. She thanks Jim Howie and members of the LMS Publications Committee for help in preparing this article although the views expressed here are her own.

Yakov Pesin (Pennsylvania State University, USA) Lectures on partial hyperbolicity and stable ergodicity

ISBN 3-03719-003-5; July 2004, 128 pages, softcover, 17.0 cm x 24.0 cm; 28.00 Euro

This book is an introduction to the modern theory of partial hyperbolicity with applications to stable ergodicity theory of smooth dynamical systems. It provides a systematic treatment of the theory and describes all the basic concepts and major results that have been obtained in the area since its creation around the early 1970s. It can be used as a textbook for a graduate student course and is also of interest to professional mathematicians working in the field of dynamical systems and their applications.

#### Sun-Yung Alice Chang (Princeton University, USA) Non-linear Elliptic Equations in Conformal Geometry

ISBN 3-03719-006-X; November 2004, 100 pages, softcover, 17.0 cm x 24.0 cm; 24.00 Euro

Non-linear elliptic partial differential equations are an important tool in the study of Riemannian metrics in differential geometry, in particular for problems concerning the conformal change of metrics in Riemannian geometry. In recent years the role played by the second order semi-linear elliptic equations in the study of Gaussian curvature and scalar curvature has been extended to a family of fully non-linear elliptic equations associated with other symmetric functions of the Ricci tensor. A case of particular interest is the second symmetric function of the Ricci tensor in dimension four closely related to the Pfaffian.

In these lectures, starting from the background material, the author reviews the problem of prescribing Gaussian curvature on compact surfaces. She then develops the analytic tools (e.g. higher order conformal invariant operators, Sobolev inequalities, blow-up analysis) in order to solve a fully nonlinear equation in prescribing the Chern-Gauss-Bonnet integrand on compact manifolds of dimension four.





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### Homage to the Abel Prize Homage to Norway

F. Thomas Bruss (Université Libre de Bruxelles, Belgium)

The Abel prize will alter the global mathematical landscape and will raise the visibility of Mathematics in society, something perhaps more important now than ever.

Jacob Palis Past President of the International Mathematical Union

On February 18, 2005, the Royal Norwegian Embassy and the Mission of Norway organized an Abel Prize presentation and reception in Brussels. This took place in the premises and under the auspices of the Royal Academy of Belgium. It was an event which attracted a great deal of interest. The Ambassador of Norway welcomed guests, including the 2004 Abel Prize laureate Professor Atiyah, and the members of the current Abel Prize selection committee. The presence of leading members of the Norwegian Academy of Science, representatives of the Belgian Royal Academy, politicians, and representatives of the European Community, added to the importance of the event. It was interesting to see so many highly distinguished mathematicians among the guests; a few more pedestrian mathematicians like chairmen of Mathematics Departments were also invited. It is safe to say that the event was a great success for all present.

#### **The Abel Prize and Mathematics**

The creation of the Abel Prize in 2002 by the government of Norway has been warmly welcomed by the world's mathematical community. The objective of this letter, stimulated by this event, is to argue that the mathematical community should express more than a warm welcome.

With the creation of the Abel-Prize, mathematicians now feel that they have a "Nobel Prize in Mathematics". This new prize is not likely to impair the glamour of the Fields Medal, the highest distinction in Mathematics so far. The Fields Medal is of a different nature, it is awarded only to those under the age limit of 40. Nobel prizes are not age-limited, and neither is the Abel Prize. They are prizes for life-long scientific achievements. This makes an essential difference, and hence the Fields Medal will keep, independently, its high distinction. Moreover, the existence of the two prizes may give a special edge to Mathematics.

The prize amount for the Abel Prize is the same as for the Nobel prizes. This is another instance of Norway's strategic skill in this matter. Consequently, it is no surprise that the Norwegian Academy of Science shows no wrong pride. It is rather pleased to hear people call the Abel-Prize the "Nobel Prize of Mathematics", as the President of the Norwegian Academy made clear during his address.



Abel reception in Brussels (Photo: Didier Vandenbosch)

#### Norway and the Abel Prize

The world's mathematical community is, perhaps somewhat contrary to its reputation, a population endowed with considerable (though subtle) self-esteem. It forms a highly communicative network of people, very internationally-minded, and also much larger in numbers than many outsiders would expect. So, why then should such a community accept that Nobel seems to have, for whatever reason, overlooked its discipline? I think it never has. This is why Norway has made a point in the history of Mathematics, for the future of Mathematics.

As soon as we think slightly deeper about Norway's initiative, and the people who must have been behind it, the conclusion is more than just gratitude. Indeed, it requires not only money to take such an initiative, it requires a lot of courage too. It is inadequate to reason that Norway is a rich country and thus able to do this more easily than others. We should ask, rich with respect to what, or compared to whom? Rich with respect to its population size? Perhaps. But then, why should relative terms here be the relevant measure, and what does this mean in absolute terms? Let us phrase the question differently: Where are the analogues of a Gauss Prize, or a Newton Prize, or a Laplace Prize, or the corresponding prize of any other larger country, including powerful overseas countries?

#### Courage and strategic skill

Putting aside the equivalent sum of some 22 Million Euros to establish a fund (the Abel Memorial Fund) is a sacrifice for any country. Norway was determined to find the money and to show the courage. There must have been strong people in Norway to fight for this cause, with a determination to succeed. We should ask who they were and try to remember their names.

Also, political support on a large scale was clearly needed for this. We can congratulate the political leaders for recognizing that the Abel Memorial Fund is not only an honourable investment but also a very clever investment. Indeed, 22 Million Euros may seem a considerable amount of money, but many countries or companies spend much more for dubious publicity campaigns whose message is forgotten in a year or two. The Abel Prize, in contrast, will, year after year, attract growing attention from both the media and the mathematical community all over the world. Not only that, it is very probable that the Abel Prize, concurrently with the Nobel prizes, can rely on a mutual multiplier effect on the media impact: the Abel Prize on its own and the Abel Prize as a Nobel Prize. It is almost like the best of two worlds.

#### Conclusion

*Alea jacta est.* Other countries could have done it, but it is Norway who has. The Abel Prize exists, and Norway will almost surely have no further competitors in the fu-



Prof. Jan Fridthjof Bernt, president of the Norwegian Academy of Science and Letters (Photo: Didier Vandenbosch)



John Bjørnebye, the Norwegian ambassador to Belgium (Photo: Didier Vandenbosch)

ture. Prize and country make, and will continue to make, their way into the heart of mathematicians. Norway will be from now on, for all of us, a very special country.

We conclude that Norway's future benefit is truly justified, and that our gratefulness towards Norway for the creation of the Abel Prize merits being paired with our profound respect.



F. Thomas Bruss studied Mathematics in Saarbrücken (FRG), Cambridge, and Sheffield (UK). He started his academic career as Assistant and First-assistant in Namur (B). From here, he moved on to the United States as Visiting Associate Professor at UC Santa Barbara, University of Arizona, and UCLA Los Angeles, successively. In 1990, he was

H.J. Jeanrond

appointed Professor at Vesalius College of the Vrije Universiteit Brussels. Since 1993 he has been Professor of Mathematics at the Université Libre de Bruxelles, and is now Chairman of the Mathematics Department. His research is in probability: probabilistic models, optimal stopping, limit theorems and branching processes. He is a member of Tönissteiner Kreis Germany, a fellow of the Institute of Mathematical Statistics, and a fellow of the von-Humboldt Foundation.

### Abel Prize 2005 to Peter D. Lax

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2005 to

Peter D. Lax, Courant Institute of Mathematical Sciences, New York University,

for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions.

Ever since Newton, differential equations have been the basis for the scientific understanding of nature. Linear differential equations, in which cause and effect are directly proportional, are reasonably well understood. The equations that arise in such fields as aerodynamics, meteorology and elasticity are nonlinear and much more complex: their solutions can develop singularities. Think



of the shock waves that appear when an airplane breaks the sound barrier.

In the 1950s and 1960s, Lax laid the foundations for the modern theory of nonlinear equations of this type (hyperbolic systems). He constructed explicit solutions, identified classes of especially well-behaved systems, introduced an important notion of entropy, and, with Glimm, made a penetrating study of how solutions behave over a long period of time. In addition, he introduced the widely used Lax-Friedrichs and Lax-Wendroff numerical schemes for computing solutions. His work in this area was important for the further theoretical developments. It has also been extraordinarily fruitful for practical applications, from weather prediction to airplane design.

Another important cornerstone of modern numerical analysis is the "Lax Equivalence Theorem". Inspired by Richtmyer, Lax established with this theorem the conditions under which a numerical implementation gives a valid approximation to the solution of a differential equation. This result brought enormous clarity to the subject.

A system of differential equations is called "integrable" if its solutions are completely characterized by some

EMS Newsletter June 2005

crucial quantities that do not change in time. A classical example is the spinning top or gyroscope, where these conserved quantities are energy and angular momentum.

Integrable systems have been studied since the 19th century and are important in pure as well as applied mathematics. In the late 1960s a revolution occurred when Kruskal and co-workers discovered a new family of examples, which have "soliton" solutions: single-crested waves that maintain their shape as they travel. Lax became fascinated by these mysterious solutions and found a unifying concept for understanding them, rewriting the equations in terms of what are now called "Lax pairs". This developed into an essential tool for the whole field, leading to new constructions of integrable systems and facilitating their study.

Scattering theory is concerned with the change in a wave as it goes around an obstacle. This phenomenon occurs not only for fluids, but also, for instance, in atomic physics (Schrödinger equation). Together with Phillips, Lax developed a broad theory of scattering and described the long-term behaviour of solutions (specifically, the decay of energy). Their work also turned out to be important in fields of mathematics apparently very distant from differential equations, such as number theory. This is an unusual and very beautiful example of a framework built for applied mathematics leading to new insights within pure mathematics.

Peter D. Lax has been described as the most versatile mathematician of his generation. The impressive list above by no means states all of his achievements. His use of geometric optics to study the propagation of singularities inaugurated the theory of Fourier Integral Operators. With Nirenberg, he derived the definitive Gårding-type estimates for systems of equations. Other celebrated results include the Lax-Milgram lemma and Lax's version of the Phragmén-Lindelöf principle for elliptic equations.

Peter D. Lax stands out in joining together pure and applied mathematics, combining a deep understanding of analysis with an extraordinary capacity to find unifying concepts. He has had a profound influence, not only by his research, but also by his writing, his lifelong commitment to education and his generosity to younger mathematicians.

#### Peter D. Lax

Peter D. Lax was born May 1, 1926 in Budapest, Hungary. He was on his way to New York with his parents on December 7, 1941 when the US joined the war.

Peter D. Lax received his PhD in 1949 from New York University with Richard Courant as his thesis advisor. Courant had founded the Courant Institute of Mathematical Sciences at NYU where Lax served as Director from 1972–1980. In 1950 Peter D. Lax went to Los Alamos for a year and later worked there several summers as a consultant, but already in 1951 he returned to New York University to begin his life work at the Courant Institute. Lax became professor in 1958. At NYU he has also served as Director of the AEC (Atomic Energy Commission) Computing and Applied Math Center.

In nominating Lax as a member of the US National Academy of Sciences in 1962, Courant described him as "embodying as few others do, the unity of abstract mathematical analysis with the most concrete power in solving individual problems".

Peter D. Lax is one of the greatest pure and applied mathematicians of our times and has made significant contributions, ranging from partial differential equations to applications in engineering. His name is connected with many major mathematical results and numerical methods, such as the Lax-Milgram Lemma, the Lax Equivalence Theorem, the Lax-Friedrichs Scheme, the Lax-Wendroff Scheme, the Lax Entropy Condition and the Lax-Levermore Theory.

Peter D. Lax is also one of the founders of modern computational mathematics. Among his most important contributions to High Performance Computing and Communications community was his work on the National Science Board from 1980 to 1986. He also chaired the committee convened by the National Science Board to study large scale computing in science and mathematics – a pioneering effort that resulted in the Lax Report.

Professor Lax's work has been recognized by many honours and awards. He was awarded the National Medal of Science in 1986, presented by President Ronald Reagan at a White House ceremony. Lax received the Wolf Prize in 1987 and the Chauvenet Prize in 1974 and shared the American Mathematical Society's Steele Prize in 1992. He was also awarded the Norbert Wiener Prize in 1975 from the American Mathematical Society and the Society for Industrial and Applied Mathematics. In 1996 he was elected a member of the American Philosophical Society.

Peter D. Lax has been both president (1977–80) and vice president (1969–71) of the American Mathematical Society.

Professor Peter D. Lax is a distinguished educator who has mentored a large number of students. He has also been a tireless reformer of mathematics education and his work with differential equations has for decades been a standard part of the mathematics curriculum worldwide.

Peter D. Lax has received many Honorary Doctorates from universities all over the world. When he was honoured by the University of Technology in Aachen, Germany in 1988, both his deep contribution to mathematics and the importance his work has had in the field of engineering were emphasized. He was also honoured for his positive attitude toward the use of computers in mathematics, research and teaching.

*This presentation is taken from the Abel prize homepage www.abelprisen.no/en/.* 

The Newsletter plans to publish an interview with the laureate in its next issue.



Prof. Peter D. Lax, Abel Laureate 2005

### Spin glasses: A mystery about to be solved

Frank den Hollander (Eindhoven, The Netherlands) Fabio Toninelli (Lyon, France)

The study of spin glasses started some thirty years ago, as a branch of the physics of disordered magnetic systems. In the late 1970's and early 1980's it went through a period of intense activity, when experimental and theoretical physicists discovered that spin glasses exhibit new and remarkable phenomena. However, a real understanding of the behaviour of these systems was not achieved and little progress was made in the next twenty years, especially in mathematical terms. In the 1990's various related systems were studied with mounting success, most notably, neural networks and random energy models. Since a couple of years the field has again entered a phase of exciting development. Some of the main mathematical questions surrounding spin glasses are currently being solved and a full understanding is at hand. In this paper we sketch the main steps in this development, which is interesting not only for the physical and the mathematical relevance of this research field, but also because it is an example where scientific progress follows a tortuous path.

#### Ferromagnets

Let us begin with a brief history of magnetic materials. All matter is composed of a large number of atoms. Atoms carry a spin, i.e., a microscopic "magnetic moment" generated by the motion of the electrons around the nucleus. This spin, which in turn generates a microscopic magnetic field around the atom, can be viewed as a vector in three-dimensional space. To simplify matters, assume that for this vector only two opposite directions are allowed, up and down. In ferromagnets, materials capable of attracting pieces of iron placed in their vicinity, each spin has a tendency to align with the spins in its neighbourhood. At high temperature, the motion of the spins is so erratic that at any time about half of them are pointing up and half are pointing down. Consequently, the net macroscopic magnetisation is zero, i.e., the individual microscopic magnetic fields generated by the spins cancel each other out. As the temperature is lowered, the erratic motion of the spins reduces and the spins become more and more sensitive to their mutual interaction. The characteristic feature of ferromagnets is that there is a *critical temperature*,  $T_c$ , below which the spins exhibit collective behaviour in that a majority of them point in the same direction (either a majority up or a majority down). This phenomenon is called spontaneous *magnetisation* (see Figure 1).

Below  $T_c$  the individual microscopic magnetic fields sum up coherently to create a macroscopic magnetic field, which is what is ultimately responsible for the ferromagnet's capability to attract iron. It is important to emphasize that this seemingly natural picture took a long time to emerge – from 1895 (Curie) until 1944 (Onsager) – and that the genius of many illustrious theoretical physicists and mathematicians was necessary in order to fully establish that this is what actually happens.

The microscopic theory that explains the collective behaviour of atoms is called *statistical physics*. According to this theory, a system in equilibrium is described with the help of an energy functional, called *Hamiltonian*, which associates with each microscopic configuration of the system a macroscopic energy. In our case a configuration means a complete list of the orientations of all the spins. If the spins are located at the sites *x* in a macroscopic box  $\Lambda$ , and if  $s_x \in \{+1, -1\}$ denotes the value of the spin at site *x* (+1 for up and -1 for down), then the configuration is

$$s = \{s_x \colon x \in \Lambda\}$$

and the Hamiltonian of the ferromagnet is

$$H(s) = -\sum_{\substack{x,y\in\Lambda\\x\sim y}} s_x s_y$$

where  $x \sim y$  means that x and y are neighbouring sites. Thus, each pair of neighbouring aligned spins gets energy -1, each pair of neighbouring anti-aligned spins gets energy +1. At a given temperature T, the state of the system is described by the *Gibbs distribution* associated with H,

$$\mu_T(s) = \frac{1}{Z_T} e^{-H(s)/kT}, \qquad s \in \{+1, -1\}^{\Lambda},$$

where k is Boltzmann's constant and  $Z_T$  normalizes  $\mu_T$  to a probability distribution:  $\mu_T(s)$  is the probability that the system assumes configuration s. When T is lowered,  $\mu_T$  tends to concentrate more and more around the configurations having minimal energy, the so-called ground states of the system. For the ferromagnet these ground states are those configurations where all the spins have the same value. Indeed, it is only when  $s_x = +1$  for all x or  $s_x = -1$  for all x that all terms in H(s) give a negative contribution, leading to the maximal value for  $\mu_T(s)$ . This maximum is a pronounced peak when



Figure 1. Spontaneous magnetization: the magnetisation m(T) as a function of the temperature T for a typical configuration of the spins; m(T) is the difference between the number of up-spins and the number of downspins divided by the total number of spins. The characteristic feature of ferromagnets is that there is a *critical temperature*,  $T_c$ , below which the spins exhibit *collective behaviour* in that a majority of them point in the same direction (either a majority up or a majority down). By symmetry, configurations with the opposite magnetisation -m(T) are equally likely.

T is small, explaining why for low temperature in a typical configuration the majority of the spins is aligned.

#### Spin glasses

Now that we have briefly introduced some important concepts from the theory of magnetism, we are in a position to explain what spin glasses are. Consider a system of spins, as before, but assume that some pairs of neighbouring spins prefer to be aligned, while the others prefer to be anti-aligned. The former are said to have a *ferromagnetic* interaction, the latter an *antiferromagnetic* interaction. Say that for any given pair of spins the type of interaction is chosen *randomly* with equal probability. It is because of this randomness in the interactions that such systems are called *disordered*.

In terms of the Hamiltonian, the above model can be defined as

$$H(s) = -\sum_{\substack{x,y\in\Lambda\\x\sim y}} J_{xy}s_xs_y,$$

where, for each  $x \sim y$ ,  $J_{xy}$  can be either +1 (indicating a ferromagnetic interaction) or -1 (indicating an anti-ferromagnetic interaction), with probability  $\frac{1}{2}$  each. This Hamiltonian was introduced in 1975 by Edwards and Anderson [8], in an attempt to describe a class of disordered magnetic systems found a few years earlier by experimental physicists and termed "spin glasses". Examples in this class are disordered magnetic alloys, i.e., metals containing random magnetic impurities, such as AuFe or CuMn.

What is the analogue in this case of the behaviour depicted in Figure 1? Even at low temperature there is no reason why the majority of the spins should be aligned. Indeed, due to the equal competition between ferromagnetic and antiferromagnetic interactions the corresponding magnetisation m(T) will be zero for all T. One might thus conclude that the model simply has no critical temperature and therefore exhibits no interesting phenomena.

However, in the early 1970's it was found *experimentally*, by Cannella and Mydosh [6] and by Tholence and Tournier [19], that there still is a critical temperature below which the system undergoes an *ordering transition*, in the sense that the spins act coherently in some sort of way (see Figure 2). This fact came as a surprise to the physicists.



Figure 2. The magnetic susceptibility  $\chi(T)$  as a function of the temperature *T*.  $\chi(T)$  measures the sensitivity of the system to the application of a magnetic field and shows a cusp at the critical temperature *T*<sub>c</sub>. This cusp signals a freezing of the spins in random directions.

In simplified terms, what happens is the following. Above  $T_c$ , the spins behave essentially independently from one another, i.e., their orientation is hardly influenced by the spins in their neighbourhood. As a result, the typical configurations of the system are those that are completely disordered. This is true both for the ferromagnet and for the spin glass. Below  $T_c$ , however, the spins show cooperative behaviour and can be found in more than one class of typical configurations. In the case of the ferromagnet described above, there are two classes of typical configurations, namely, those having magnetisation +m(T) and -m(T), respectively. These classes of configurations are called *pure states*. In the case of the spin glass, instead, there are *many* pure states, which are not characterised by a non-zero magnetisation, but rather by the occurrence of many "mesoscopic domains" (microscopically large but macroscopically small) in which the spins show some form of "local magnetic order". In fact, a whole "hierarchy" of such domains occurs. At present it is not yet clear what the features of these domains precisely are. The important point, however, is that the existence of a transition at  $T_c$  is experimentally observable.

The Edwards-Anderson model is far too difficult to be analysed theoretically in detail, even today. In fact, condensed matter physicists have been disputing heatedly in the past three decades about what precisely happens at low temperature. In 1975 Sherrington and Kirkpatrick [15] introduced a simplified version of this model. The difference with the Edwards-Anderson model is that each spin is influenced not only by its neighbouring spins, but by *all* the spins in the system. The corresponding Hamiltonian reads

$$H(s) = -\frac{1}{|\Lambda|^{1/2}} \sum_{\substack{x,y \in \Lambda \\ x \neq y}} J_{xy} s_x s_y,$$

where  $J_{xy}$  is +1 or -1, with probability  $\frac{1}{2}$  each, for all  $x \neq y$  (rather than for  $x \sim y$  only), and a factor  $1/|\Lambda|^{1/2}$  is added to normalise the interaction. In statistical physical jargon, the Sherrington-Kirkpatrick model is a *mean-field approximation* of the Edwards-Anderson model. Strange as it may seem, this type of approximation actually makes the model easier.

For a history of spin glasses up to 1986, we refer to Binder and Young [2].

#### Replica symmetry breaking

The article by Sherrington and Kirkpatrick carried the rather innocent title "A solvable model of a spin glass". The authors never imagined that they were giving birth to one of the most exciting enigmas of modern statistical physics. The solution they proposed, assuming so-called "replica symmetry", turned out to be incorrect, and even self-contradictory as they themselves realised very well. It was only a few years later, in 1980, that the Italian theoretical physicist Giorgio Parisi [14] proposed a different solution, known as the *continuous replica symmetry breaking scheme*, which could account for many of the experimental observations (both laboratory experiments and computer simulations).

Replica symmetry breaking theory predicts the existence of a collective behaviour with many *exotic features*, never before observed in any physical system. In sim-

ple words, Parisi's theory predicts that the Hamiltonian of the Sherrington-Kirkpatrick model has many ground states (growing in number as the volume of the system increases), which are highly disordered and which do not seem to be related to one another via simple transformations. In contrast, recall that the ferromagnetic Hamiltonian has only two ground states, one with all spins up and one with all spins down, which are *fully ordered* and which are related to one another via a global inversion of all the spins. Moreover, it turns out that for the Sherrington-Kirkpatrick model, by choosing a different realisation of the disorder (i.e., a different choice for the random interactions  $J_{xy} = \pm 1$ , again with probability  $\frac{1}{2}$ each), the new ground states in general have nothing to do with the old ones. Even more surprisingly, if the disorder realisation is kept fixed but the volume of the system is increased, then the new ground states are not related to the old ones either ("chaotic size dependence"). In spite of this extremely irregular situation, according to Parisi's theory the collection of all the ground states has some regular, highly non-trivial, geometrical structure, called ultrametricity, which is not modified when the disorder realisation is changed. So, what distinguishes the region above the critical temperature  $T_c$  from the one below, for the Sherrington-Kirkpatrick model? Suppose that we take two copies - two replicas - of the system, with the same realisation of the disorder, and compute the overlap between them, i.e.,

$$q(s^{(1)}, s^{(2)}) = \frac{1}{|\Lambda|} \sum_{x \in \Lambda} s_x^{(1)} s_x^{(2)},$$

where  $s^{(1)}$  and  $s^{(2)}$  are the configurations of the first and the second replica, respectively. Then, above  $T_c$  the overlap is zero for typical configurations (typical with respect to the Gibbs distribution and the disorder realisation), while below  $T_c$  it can assume a range of non-zero *random* values. This can be explained as follows. Recall that, at low temperature, the Gibbs distribution is peaked around the ground states of the system. Consequently, the configurations in the two replicas will each be very close to one of the ground states (not necessarily the same one), which causes a non-zero overlap. Due to the erratic nature of the ground states, the overlap does not have a fixed value: it varies randomly with the ground states.

Replica symmetry breaking theory came as a shock to the physics community, not only for the novelty of the phenomena predicted, but also for the way in which it was presented. It happens frequently that theories formulated by physicists are not mathematically rigorous, and contain a number of assumptions and simplifications that need to be justified. Often full mathematical proofs come only much later. Here the situation was more delicate: the works of Parisi and co-workers were not only non-rigorous, they were based on such strange and daring techniques that it was hard to see how the relevant statements could be formulated in a proper mathematical language. This is why part of the mathematics community has regarded Parisi's theory as somewhat magic. Still, the phenomena predicted by the theory were so appealing, and its range of applications so wide, that it soon became a standard tool for theoretical physicists, who were much more excited by its power than worried by its lack of mathematical sense and precision. One could say that Parisi had discovered a new world.

A review of the results of replica symmetry breaking theory up to 1987 can be found in Mézard, Parisi and Virasoro [12].

#### **Towards a solution**

The reader might wonder at this point whether all the excitement about the Sherrington-Kirkpatrick model is really justified. After all, it is only an approximate version of the more difficult - but more realistic - Edwards-Anderson model, which remains unsolved. In fact, it is not yet clear how much we really learn about the Edwards-Anderson model from a detailed analysis of the Sherrington-Kirkpatrick model. According to a scenario put forward by Newman and Stein (see Newman [13]), the behaviour of the two models may well turn out to be qualitatively different: the main phenomena related to replica symmetry breaking may not occur in "short range" models like the Edwards-Anderson model. Still, the excitement is understandable. First, the study of the Sherrington-Kirkpatrick model has taught us a lot and continues to do so. In the attempts to understand this model, new ideas and techniques have been invented and further developed that are extremely interesting and that have turned out to be fruitful for other statistical physical models as well. Second - and more importantly – it has gradually become clear that the knowledge gained through the analysis of the Sherrington-Kirkpatrick model can be applied to a variety of - apparently unrelated - problems in mathematics, physics and engineering. These problems have therefore come to be considered as belonging to the realm of spin glasses. Examples are neural networks (models for memory and learning), error correcting codes (used in communications engineering to recover the information transmitted through a noisy channel) and random combinatorial optimisation (problems of decision in the presence of many mutually competing requirements).

From the moment the replica symmetry breaking theory came into being, trying to prove - or to disprove - the predictions of Parisi and co-workers became an exciting challenge for many among the best mathematical physicists. The task proved to be quite hard and frustrating, and for almost years progress was painfully slow. Much effort was devoted to the search for and the study of mathematical models that would be easier than the Sherrington-Kirkpatrick model, but that would still exhibit replica symmetry breaking effects. In particular, the Generalized Random Energy Model, introduced by Derrida [7] in 1985, shows striking similarities with the Sherrington-Kirkpatrick model, yet is exactly solvable. The structure of the Gibbs distribution in this model has been analysed in full mathematical detail by Bovier and Kurkova [4]. Similarly, extensive rigorous results have been obtained by Bovier, Gayrard and Picco for the Hopfield model of neural networks (see [3] and references therein). The latter is a paradigm for auto-associative memory, i.e., systems that try to recognize words - or patterns - that were previously memorized. In this case, the spins should be interpreted as the states of the neurons located at the various sites:  $s_x = +1$ if the neuron at site x is sending electric pulses,  $s_x = -1$  if it is not. When varying the number of memorized patterns, the behaviour can range from a ferromagnetic type to a spin glass type. For an overview of the expanding panorama of spin glasses up to 1998, see Bovier and Picco [5].

It gradually became clear - more through failures than through positive results - that completely new ideas were needed to make significant progress in the comprehension of replica symmetry breaking. It is only in the last few years that we are witnessing a rapid and unexpected boost in the mathematical understanding of the key questions. Surprisingly, the missing new ideas turned out to be relatively simple, although they were very hard to find. The first steps in this breakthrough were taken in 2000-2002 by the Italian mathematical physicist Francesco Guerra [10], together with Fabio Toninelli [11], building on earlier work by Ghirlanda and Guerra [9]. As a result, some of the mathematical questions that had been tackled in vain in the preceding twenty years could finally be solved. One important result is the existence of the "thermodynamic limit" for the Sherrington-Kirkpatrick model. This means that physical quantities, like the energy of the ground states divided by the volume of the system, converge to a well defined limit when the volume of the system tends to infinity. The proof of this fact is guite standard in statistical physics for models with "short range" interactions, but it is not for mean-field models, especially not for disordered ones. Another important result is that with the help of certain rigorous comparison identities - so-called sum rules the thermodynamic properties of the Sherrington-Kirkpatrick model can be compared with the corresponding expressions given by Parisi's theory. These sum rules concern the free energy  $f(T, |\Lambda|)$  as a function of the temperature T and the volume  $|\Lambda|$ , a quantity of central importance in statistical physics, from which all thermodynamic properties of the system can be deduced. This free energy is related to the Gibbs distribution  $\mu_T$  via the relation  $f(T, |\Lambda|) = -kT \log Z_T$ . The result is that  $f(T, |\Lambda|)$  can be related to the free energy predicted by Parisi's theory via an identity of the type

$$f(T, |\Lambda|) = f^{Parisi}(T, |\Lambda|) + R(T, |\Lambda|),$$

where  $R(T, |\Lambda|)$  is an "error term". Proving the validity of Parisi's theory is equivalent to showing that  $R(T, |\Lambda|)/|\Lambda|$ tends to zero in the thermodynamic limit  $|\Lambda| \to \infty$ . A particularly important fact is that  $R(T, |\Lambda|)$  turns out to be nonnegative, so that Parisi's free energy at least is a lower bound for  $f(T, |\Lambda|)$ , a fact that itself is rich in physical implications (see Toninelli [20]). Subsequently, Aizenman, Sims and Starr [1] obtained Guerra's sum rules through a general variational principle and showed that Parisi's free energy arises from a restriction of this variational principle to "ultrametric structures". This restriction is optimal precisely when replica symmetry breaking theory correctly describes the Sherrington-Kirkpatrick model.

These new ideas provoked great excitement in the scientific community, and new feverish work began. The last part of this story is still in progress and is keeping the excitement high. In July 2003 the French mathematician Michel Talagrand, who has been working on the problem intensively and who has introduced many new ideas in this field since the mid 1990's (see [16]), announced (see [17]) that he was able to complete the mathematical proof of Parisi's solution, extending the method of sum rules invented by Guerra. The details of the proof were made public only in April 2004 [18] and will be published in 2005. It is not hard to imagine the impression this announcement has produced on the experts. It seems that the full mathematical justification of Parisi's theory, explaining the mysterious features of the Sherrington-Kirkpatrick model, is finally at hand. Currently, research in this rapidly evolving field is being carried out by a number of groups, including the Random Spatial Structures group at EU-RANDOM, the European institute for research on stochastic phenomena located at the Technical University of Eindhoven, The Netherlands.

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### Why is there no decent multiplication in 3D?

#### Zbigniew Marciniak (Warszawa, Poland

The famous Irish mathematician William Rowan Hamilton was born in Dublin almost 200 years ago, on 4 August 1805. Among many other things, he is famous for the invention of quaternions – which occured to him while he was walking along the Royal Canal in Dublin with his wife, to preside at a Council meeting of the Royal Irish Academy on October 16, 1843. Previously, he had tried for some time to define an algebra of "triplets", and this had become an obsession that plagued him for many years. It is said that Hamilton was so preoccupied with the triplets that even his children were aware of it. Every morning they would inquire: "Well, Papa can you multiply triplets?", but he had to admit that he could still only add and subtract them.<sup>1</sup>

This article explains in an elementary way why Hamilton could not succeed.

We all learned how to add and multiply real numbers at school, so we can easily enumerate the properties of these two operations:

- (a) Addition + is associative, commutative and has a neutral element 0. Moreover, for any element *a* there is an opposite element -a such that a + (-a) = 0.
- (b) Multiplication  $\cdot$  is associative, commutative and has an identity element 1. Moreover, any non-zero element *a* has an inverse  $a^{-1}$  such that  $a \cdot a^{-1} = 1$ .
- (c) Multiplication distributes over addition.

Besides the set **R** of real numbers, we also have the set **N** of natural numbers, the set **Z** of integers and the set **Q** of rational numbers. Even though addition and multiplication can be performed in any one of them, only **Q** and **R** have the properties (a), (b) and (c). To keep things short, mathematicians say that **Q** and **R** are *fields*, whereas **N** and **Z** are not.

The set of complex numbers **C** is an example of a field that is bigger than **R**. Recall that complex numbers are expressions of the form a + bi with  $a, b \in \mathbf{R}$ . We can add and multiply such numbers just as we do with polynomials, using the relation  $i^2 = -1$ . For example,

$$(2+3i) + (4+2i) = 6+5i$$
  
 $(2+3i)(4+2i) = 8+16i+6i^2 = 2+16i$ 

It is not difficult to verify that operations so defined satisfy the properties (a),(b) and (c), that is, **C** is a field. Every complex number a+bi is basically made up of an ordered pair (a,b) of



Sir William Rowan Hamilton (1805-1865)

real numbers, hence we can identify the set C with the plane  $\mathbf{R}^2$  equipped with addition and multiplication of points. Under this interpretation, the operations can be described by the following formulae:

$$(a,b) + (c,d) = (a+c,b+d)$$
$$a,b) \cdot (c,d) = (ac-bd,ad+bc).$$

Notice that such an addition operation can be easily and naturally generalized to the 3-dimensional space  $\mathbf{R}^3$ : just add triples of real numbers coordinatewise as if they were vectors:

$$(a,b,c) + (d,e,f) = (a+d,b+e,c+f).$$

Obviously such an operation again has property (a).

A natural question arises: can the 3-dimensional space  $\mathbb{R}^3$  be equipped with a multiplication which together with this addition would possess all the properties (a), (b) and (c)? We could also demand that this operation be consistent with multiplication by real scalars, well defined in  $\mathbb{R}^3$ . In other words, we might expect that for any  $v, w \in \mathbb{R}^3$  and  $t \in \mathbb{R}$ ,

$$(tv) \cdot w = v \cdot (tw) = t(v \cdot w).$$

A multiplication operation with this property is said to be *bi-linear*.

Clearly coordinatewise multiplication does not work, since the product of two non-zero elements may become zero. For example,

$$(1,0,0) \cdot (0,1,0) = (0,0,0).$$

This cannot happen in any field.

We will now prove the following result:

**Theorem.** There is no bilinear multiplication in  $\mathbf{R}^3$  that satisfies the conditions (a), (b) and (c) together with coordinate-wise addition.

Indeed, suppose the contrary, that is, that such a multiplication in  $\mathbb{R}^3$  exists. It is convenient to think of the elements of  $\mathbb{R}^3$  as vectors with a tail at 0 = (0,0,0). Property (b) implies that one of these vectors is the identity for multiplication; call it *e*. Let *L* be the straight line extending this vector. Then

$$L = \{te \in \mathbf{R}^3 : t \in \mathbf{R}\}$$

For each vector  $v \in \mathbf{R}^3 \setminus L$ , let P(v) be the plane spanned by the vectors e and v. First of all, observe that the following property holds.

**Lemma 1.** Let v be a vector in  $\mathbf{R}^3$ . If its square  $v^2$  belongs to the plane P(v) then this plane is a subset of  $\mathbf{R}^3$  closed with respect to addition, subtraction, multiplication and inversion.

In this case we say that the subset P(v) is a *subfield* of  $\mathbb{R}^3$ .

*Proof.* It is clear that a plane containing the point 0 is closed with respect to addition and subtraction of vectors. Every element of P(v) is of the form se + tv for some real numbers s and

*t*. When two such elements are multiplied, the result is a linear combination of the vectors e, v and  $v^2$ , so if each of these vectors belongs to P(v), then so does the product. It remains to prove that if  $w \in P(v)$  and  $w \neq 0$ , then the inverse  $w^{-1}$  is again an element of P(v). Now, if w = te, then  $w^{-1} = t^{-1}e$ is also an element of P(v). Otherwise,  $w \in P(v) \setminus L$  and consequently P(w) = P(v). Due to the plane P(v) being closed with respect to multiplication, as we already know, we can infer that  $w^2 \in P(v) = P(w)$ , that is,  $w^2 = se + tw$  for some real numbers s and t. If s = 0, then  $w^2 = tw$  and therefore  $w \cdot (w - te) = 0$ . This, however, is impossible, both factors

$$w \cdot (s^{-1}w - ts^{-1}e) = e,$$

being non-zero. Thus  $s \neq 0$ , which further implies

and this leads to the conclusion that  $w^{-1} = s^{-1}w - ts^{-1}e$  belongs to P(v). This completes the proof of lemma 1.

Now we will see that the situation described in Lemma 1 can never occur.

**Lemma 2.** If  $v \in \mathbf{R}^3 \setminus L$  then  $v^2$  does not belong to P(v).

*Proof.* Suppose  $v^2 \in P(v)$  and take an arbitrary vector  $w \in \mathbf{R}^3 \setminus P(v)$ . Then the three vectors e, v and w generate the entire space  $\mathbf{R}^3$ . In particular, the vector  $w \cdot v$  can be represented as their linear combination:

 $w \cdot v = re + sv + tw.$ 

But then w(v - te) = re + sv and further

$$w = (re + sv) \cdot (v - te)^{-1}.$$

Lemma 1 implies that the element on the right-hand side is in P(v). But then w must also belong to P(v), contrary to the initial assumption.

Now we can complete the proof of the theorem. Take an arbitrary vector  $v \in \mathbf{R}^3 \setminus L$ . By lemma 2, the vectors e, v and  $v^2$  generate the entire space  $\mathbf{R}^3$ , so the vector  $v^3$  is their linear combination:  $v^3 = re + sv + tv^2$ . Consider the mapping

$$f: \mathbf{R} \longrightarrow \mathbf{R}, \quad x \mapsto x^3 - r - sx - tx^2$$

Since f(x) < 0 for sufficiently small negative arguments and f(x) > 0 for sufficiently large positive arguments, there is a

real number c such that f(c) = 0, so f(x) can be factored as follows:  $f(x) = (x - c)(a + bx + x^2)$  for some real numbers a and b. Now substituting the vector v for the variable x we get

$$0 = f(v) = (v - ce)(ae + bv + v^{2}).$$

By the assumptions and lemma 2, neither factor of this product is null. This, however, is a contradiction. Thus the proof is completed.

A remark for the experts: We did not use multiplication commutativity in the proof. Therefore  $\mathbf{R}^3$  does not even have the structure of what is known as *non-commutative algebra* with division.

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#### Note

1. Taken from the Hamilton biography by J.J. O'Connor and E.F. Robertson, available at the MacTutor History of Mathematics archive, University of St Andrews, http://www-groups.dcs.st-and.ac.uk/~history/



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# Tribute to Ruy Luís Gomes on his 100th Birthday

Natália Bebiano and João da Providência, jr (Coimbra, Portugal)



Ruy Luís Gomes (1905-1984)

The Portuguese mathematician Ruy Luís Gomes was born in O'Porto on 5 December 1905. His father, Doctor António Luís Gomes, was an academic who held a government position as minister of the first Portuguese Republic. He was also Rector of the University of Coimbra, one of the oldest universities in Europe.

Gomes entered the Faculty of Sciences of the University of Coimbra in 1922, and graduated from there in Mathematical Sciences in 1927. He was a brilliant student, and in 1928 he obtained a doctoral degree from the University of Coimbra with a thesis entitled "Desvio das trajectórias holónomas" ("Deviation of holonomic trajectories"), a topic that became his main area of research for the first years of his scientific life.

Gomes started his academic career as an assistant professor at Coimbra. In 1929 he moved to the University of O'Porto as assistant professor, where for one year he taught Algebra and Projective Geometry, and then Mathematical Physics. He was director of the Gabinete de Astronomia (Astronomical Department), and promoted the foundation of an Astronomical Observatory at Monte da Virgem, O'Porto. During his career, Gomes taught courses on Infinitesimal Calculus, Relativity, Hilbert Spaces and Quantum Mechanics, Measure Theory and Integration.

In 1933, Gomes applied for the position of full professor at O'Porto, having presented a dissertation entitled "Sobre a estabilidade dos movimentos de um sistema holónomo" ("On the problem of the stability of the movements of a holonomic system"). His dissertation was inspired by the paper of Tullio Levi-Civita (1873–1941), "Sur L'écart géodésique", published in *Mathematische Annalen*. In Gomes' dissertation, the theme was treated for the first time by following Levi-Civita's methods of absolute differential calculus. The Italian mathematician became friends with Gomes and ensured the publication of twelve of his articles, from the fertile period between 1930 and 1937, in the *Rendiconti della Academia dei Lincei*. Two of these papers, namely, "Quelques considérations sur l'équation fondamentale de la nouvelle conception de la lumière de Louis de Broglie", 1935, pp. 358–364 (vol. 21) and "Sur la propriété de l'opérateur H de Louis de Broglie", 1935, pp. 499–501, dealt with concepts introduced by the Nobel Prize winner Louis de Broglie. De Broglie referred to these papers in his classes at the Institut Henri Poincaré, which impressed the Portuguese PhD students attending these classes. Some of Gomes' articles were published in the review *Journal de Physique et Radium*, for example, his "Sur la Cynématique rélativiste des systèmes" in 1935.

In the 1930s the totalitarian regime of Salazar became firmly established in Portugal. Several teachers and students were persecuted and arrested. Gomes was an ardent defender of democracy and his revolutionary opposition to the dictatorship of Salazar caused him several difficulties. He was persecuted and arrested several times, for instance when his colleague Abel Salazar (1889–1946) was expelled from the University of Porto for political reasons, or on the occasion of the invasion of the Portuguese colonies of Goa, Damão and Dio by India. In 1957, he was arrested for his position against the repressive regime and sentenced to twenty-four months imprisonment.

Ruy L. Gomes was a distinguished member of a group of young Portuguese scientists who, in the late nineteen thirties and forties, conceived the project of the rebirth of Portuguese mathematics. His participation in the rejuvenation of the scientific life of the country was crucial. Gomes organized seminars, was a co-founder of the research journal *Portugaliae Mathematica* (1937) and of the journal for the popularization of mathematics, *Gazeta de Matemática* (1939), and was one of the founders of the Portuguese Mathematical Society (1940).

As in other countries, there were anti-relativistic manifestations in Portugal. Gomes maintained controversy on this topic in cultural newspapers and reviews (such as Seara Nova) with Admiral Gago Coutinho, the man who had made the first crossing of the South Atlantic Ocean in an aircraft. In 1937, Ruy L. Gomes gave a series of five lectures on special relativity at the Instituto Superior Técnico (Technical University of Lisbon). These conferences were published in 1938 under the title "Teoria da Relatividade Restrita" (Theory of Special Relativity) in the second issue of a promising collection of science popularization edited by the so called "Núcleo de Matemática, Física e Química", which unfortunately ended with the publication of the third issue. This book explains the fundamental principles of special relativity and includes exercises and notes. It has four chapters, with the follow-

#### History

ing titles: Chapter I - The fundamental equations of electrodynamics and its invariance group; Chapter II - Time in relativity; Chapter III - Relation between two Einstein referentials; Chapter IV - Composition of movements. Fundamental relation. The velocity of light as an upper bound of the velocities of material points.

In the Introduction, the author explains that the theory is developed according to the canons of the logical empiricism of the School of Vienna, and frequently quotes its main thinkers. He makes it clear that a physical theory is a logical construction, the adjective physical being used "when its statements as a whole, or at least in part, have their roots in physical reality and succeed in fact to explain it" (p. 12). In the positivist sense, "any scientific proposition will, therefore, be verifiable in principle; or else it has no sense – it will be a metaphysical proposition or a pseudo-proposition" (p. 15). Breaking with the traditional distinction of scientific and philosophical domains in human thought authorized by the "increasing number of scientistsphilosophers, among them the distinguished group of the School of Vienna", Ruy L. Gomes states that he "dares to talk of space and time without being afraid of violating the old canons or of invading the territory of others" (p. 12).

Besides writing on special relativity interpreted in the light of logical positivism (from which the above-mentioned book is an example), Gomes wrote several popularizations of relativity, and did original research work in this area (such as the characterization of a solid body in relativity). Among his technical articles on special relativity that were published in international magazines, we point out those that appeared in the *Rendiconti della Accademia del Lincei* and in the *Journal de Physique et Radium*. He also investigated general relativity (for example, he studied the concepts of absolute space and absolute time for certain cosmological metrics). In 1954, he published his *Teoria da Relatividade – Espaço, Tempo, Gravitação* (Theory of Relativity – Space, Time, Gravitation).

In 1941, Gomes founded the Centro de Estudos Matemáticos do Porto (Research Centre of Mathematical Studies of O'Porto), of which he was the first director. In 1943 he was a co-founder of Junta de Investigação Matemática (Council of Mathematics Research), whose main objectives were to coordinate Portuguese mathematical research, to develop mathematics among young people and to promote the collaboration of the Portuguese mathematical community with the international community. The civil war in Spain, and then the Second World War, brought serious obstacles to communication. John von Neumann and Maurice Fréchet, among other famous mathematicians, published some of their papers in Portugaliae Mathematica, and the exchange of this journal with international scientific journals was essential for the progress of mathematics in Portugal.

Gomes strongly promoted cooperation with foreign mathematicians, and obtained scholarships from the Portuguese government for young graduates so that they could prepare their doctoral dissertations in the best mathematical centres in Europe.

In 1947, Gomes was dismissed from the Portuguese University for political reasons. In 1951 he was proposed as a candidate for the presidency of the Portuguese Republic, against the candidate of the regime, but the State Council rejected his candidature. In 1953, he won a Prize from the Academy of Sciences of Lisbon for his original contributions to mathematics. In 1958 he emigrated to Argentina, where he got a position at the Instituto de Matemática of the Universidad Nacional del Sur in Bahia Blanca. In 1962, he moved to the UniversidadeFederal de Pernambuco (UFPE), Brazil, where he taught courses at graduate and post-graduate level. In 1967, Gomes and his exiled Portuguese colleague J. Morgado (1921–2003) founded a Masters course in Mathematics in UFPE which obtained remarkable reputation and which contributed to the formation of many Masters and Ph.D. degrees.

Ruy L. Gomes returned to Portugal after the Revolution of 25 April 1974. He was distinguished by being nominated for the Conselho de Estado (State Council). He was also Rector and, after his retirement, Rector honorary of the University of O'Porto until his death in 1984.

Gomes had many students, many of whom have become distinguished mathematicians. His human warmth, utopian vision of a fraternal world of peace and justice, his broad cultural interests, and his militancy in the defence of Human Rights enriched the lives of those who had the privilege to know him. His roles as teacher, mentor and citizen were exceptional, and were felt beyond Portugal, in Brazil and Argentina.

Gomes developed a persistent political intervention that deviated him from his first vocation of researcher and professor. The process of change of the social and cultural structures of Portugal seemed primordial for the development of a profound and stable scientific activity. In the period from 1948–1958, he restricted his mathematical works and dedicated his efforts and creativity to politics against the Salazar regime. In his struggle for cultural and scientific promotion he was a benchmark.



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### Interview with Arild Stubhaug

Conducted by Ulf Persson (Göteborg, Sweden)



Arild Stubhaug, who is known among mathematicians for his biography of Abel, has also produced a noted biography of Sophus Lie and is now involved in the project of writing a biography of the Swedish mathematician Mittag-Leffler and the mathematical period in which he was influential. Following up the interview, we will also have the privilege of giving a sample of the up-coming work in a forthcoming issue of the Newsletter.

Arild Stubhaug

You are an established literary writer in Norway, and if I recall correctly you already had your first work published by the age of twenty-two. You have written poetry as well as novels, what made you start writing biographies of Norwegian mathematicians in recent years?

In recent years..., that is not exactly correct. I started the biography on Abel back in 1988 and during the eight years I worked on it I also concomitantly published three collections of poetry. I started to write about Abel and his times for many different reasons. To write poetry - however exciting it may be by itself - has turned into a narrow groove of work, not to say a marginal one. It is difficult to get new poetry properly recognised and appreciated, maybe because of the steadily diminished importance and influence it exerts in the development of language, in particular as regards to innovative constructions. Thus I wanted to try out alternative means of expression. Mathematics has always been a source of fascination. Abel was somewhat of a hero early on, and the historical interest was kindled when I lived in a town called Arendal in the south of Norway, where the past is still alive and moreover kept alive in a special way.

#### What are your qualifications for writing about mathematicians? As a young man you studied a variety of subjects at university, including mathematics and the history of religion. Am I correct, and if so would you be able to explain what to most of us may appear to be a strange combination.

It is correct. Mathematics was initially my primary interest. But this was back in 1968, and many other things caught my interest and engagement – especially the great priority that was accorded verbal expression at the time. Whatever could be caught and formulated in language became more important than anything else. By and by,



Arild Stubhaug with a statue of Gösta Mittag-Leffler

language turned out to be the most interesting subject of work for me. After mathematics I studied Latin, history of literature and eastern religion purely out of personal curiosity and desire. At that time such a combination could never be part of a regular university degree, hence I have never been regularly employed.

But why Mittag-Leffler? Abel is one of the greatest mathematicians ever, this is an uncontroversial fact, and his short life has all the ingredients of romantic tragedy. Lie may not be of the same exalted stature, but of course the notion of "Lie", in group-theory and algebra, is a mathematical household word. But Mittag-Leffler? I think that any mathematician would be hard pressed to come up with a single significant result that has been attributed to Mittag-Leffler. The theorem of Mittag-Leffler is of course duly mentioned and can be seen as an elementary precursor of sheaf-theory but clearly it is rather light-weight and hardly anything to get very excited about.

Many people have indeed asked me: is Mittag-Leffler really worth such an ambitious biography? The reason for expressing such doubts may be that one easily confuses biography with a celebration of genius. Many have a romantic tendency to make its subject into an object of wonder rather than simply trying to understand the individual concerned as a human being. It is as if one would conceive biography solely in terms of an adventure story, neglecting its more mundane aspects. I do not believe that the differences between us humans are necessarily where we usually look for them: As if somebody by virtue of genius would live on a different planet. The crucial differences are of a far less grandiose nature, consisting in the way key decisions are made or points of views are formed by the individual, rather than by a fundamental otherness. But it is true that the main motivation to write about Mittag-Leffler was of course not because of his direct scientific contribution, but you can exert a crucial influence on mathematics without proving any theorems. Gårding, in his book on Swedish mathematicians until 1950, hails Mittag-Leffler as the father of Swedish mathematics. What makes it different to write about mathematicians rather than artists, politicians, explorers and other classical subjects of the

genre, is that the subject matter of mathematicians is incomprehensible to most readers. This puts even tougher demands on the approaches one chooses to present the subject. I believe that by the choice of Mittag-Leffler you will be able to write a biography of a mathematician in a way that can hold the interest of non-mathematicians, as I do not only want to give a portrait of Mittag-Leffler the man, but also to use him as a springboard to describe an era and a concomitant mentality. Behind this lies a basic conception of what constitutes the human. An individual is to a great extent a mirror of his or her times. If we take ourselves or people to whom we are very close as our point of view, we realise that the way we express ourselves is largely determined by whom we express ourselves to, i.e. our contemporaries. Not writing with the corresponding perspective means depriving the reader of a necessary sounding-board for understanding, automatically present whenever we read about our own contemporaries.

To write a biography of Abel must have been a challenge, because there is so little documentation, but with Mittag-Leffler the challenge is of quite a different sort, as we find here an embarrassment of riches instead. With Abel one got the impression that you included everything you had manage to ferret out, but this will of course be impossible when Mittag-Leffler is concerned. In fact the habit of writing letters and keeping diaries has, much to the consternation of historians, waned during the 20th century. Thus ironically the subjects of which we can really present full-rounded biographical pictures are those of the 19th century, and of course Mittag-Leffler is exceptional even among those as he really kept a systematic account of his epistolarian output, keeping copies of essentially all the letters he ever penned. Are you first going to produce a preliminary text, a gross version so to speak, say a thousand pages long, out of which you will distil a net version of suitable length?

Finally, you have earlier told me that you are able to follow Mittag-Leffler day by day, something I doubt that you can do with your own life. But with all that documentation, are you really able to see the forest for all the trees? In particular do you feel that you really get to know Mittag-Leffler, or in spite of all the writing does nothing remain but exalted verbiage hiding the man and his innermost thoughts?

Let me confront those questions one by one. It is true that Mittag-Leffler's Nachlass is impressive, some 75 meters of correspondences, diary-notes, articles and drafts thereof, of which about 60 meters are archived at Kungliga Biblioteket.<sup>1</sup> There are about 3000 correspondents and I estimate the number of letters to be around 20000. It certainly takes time to go though so much material and to try and digest it. This is why I feel that this biography cannot be rushed, there is so much potential that I feel must be realised before I let go of the work. Besides the idea of having at some later date to start all over again and redo in greater detail is just too daunting to be contemplated. As to the final version it may happen that in the end I will be forced to make a distillation out of a gross version, but for the time being I have not committed myself to any a priori length, and I am still working under the assumption that the format will have to comply with the contents. The ultimate aim is to weave together the different strands of history relating to that of the individual, especially his intellectual development and that of mathematics. The enormous material available simply forces such a biography; namely that of writing Mittag-Leffler, the father of modern Swedish mathematics as noted above, into a context, in the same way in which great artists, politicians, barons of commerce etc have traditionally always been understood.

As to Mittag-Leffler remaining opaque in spite of all the things he wrote I do not agree. In fact I feel that I know him inside and out. Especially in his early letters, he is not committed to any preconceived image of himself that he wants to live up to and sustain. On the contrary, they testify to a desire to express any kind of anxious excitement arising out of his encounters with new people and new thoughts. The young Mittag-Leffler looked upon his own self as exciting a subject of investigation as anything else. Perceptions, feelings, ideas, ways of thinking... nothing was too insignificant or too grand for that matter to be touched upon in letters or in diary notes.

This is the fourth person I am writing a biography on, and I must admit that on the whole I feel that I know those people better than those in my actual vicinity. And Mittag-Leffler, by virtue of the rich and extensive documentation available, maybe in even higher degree than those previously portrayed.

#### The British biographer Peter Ackroyd<sup>2</sup> claims somewhat paradoxically that the writing of a biography makes more demands on your imagination than the writing of fiction. Would you care to comment on that?

It reminds me of a remark that at its time was attributed to the French writer and philosopher Voltaire, to the effect that Archimedes displayed more imagination than Homer. A statement that needless to say epitomizes the opposition between a classical concept of erudition and a more modern one based on scientific methods and paradigms of thought, in recent years actualized by the discussion of the two cultures of Snow.

Ackroyd's statement is interesting, provided one defines imagination not only as unfettered fabulation but as the power to survey and deal with a large, amorphous and many-facetted load of material; because if so the writer of biographies is in more need of it than a weaver of fiction. The more constraints are imposed on the ways imagination can be articulated, the greater the necessity for surveying and balancing. The narrower the latitude, the more the demand for an imagination of precision. It becomes like comparing a tightrope walker constrained

<sup>&</sup>lt;sup>1</sup>Literally the Royal Library in Swedish

<sup>&</sup>lt;sup>2</sup>Known among other things for his biographies of T. S. Eliot and Charles Dickens and in later years of the city of London

to his suspended line of rope with one who is free to walk on the surface of the ground, the latter may sway and dither, it does not matter much, while the former must engage his complete concentration to avoid falling off...

As a writer of biographies one may work as a historian most of the time, seeking out the relevant sources, reading and summarizing. Do you find that this aspect of your work takes almost all of your time, or will there at least in the final write-up be enough time for fashioning a literary narrative? If you had the option of choice what would you prefer, the historically correct narration you have been assigned to produce, or a freer dramatization of his life?

I would claim that what might appear a straightforward account in practice will put the same kind of demands on writing skills as that of a dramatization. To arrange facts in such a way that they form a wave in which the reader is carried away by the feeling of making his own discoveries, conclusions and drawing of parallels with his own life, is a form of dramatization that demands its due share of work. To structure the extensive material in such a way that all components fit seamlessly together as strands of the great warp which will constitute the final book, I consider as a truly literary challenge.

#### For whom is this biography really written? Is it for the mathematician, and thus we are inevitably talking about an international audience? Or is it for the educated Swedish public, because much of the subject matter lends itself to the painting of a panorama of the Oscarian period<sup>3</sup> of much concern and interest to the Swede, but maybe of less interest to a wider public.

First I would like to repeat what I mentioned before. The material cries out to be articulated according to its intrinsic nature, and this of course has been my leading star so to speak. It is true that Mittag-Leffler knew everybody who was somebody, he was active not only scientifically, but also knew all the main artists, writers and intellectuals in general in Sweden. As the 20th century broke he was down in Egypt consorting with the great writer Selma Lagerlöf. Thus the biography ought to be of interest to any educated Swede, and in fact it will appear by a Swedish publisher that has brought out many works on Swedish history. But I believe that the micro-cosmos I present will intrigue readers with no previous acquaintance. After all, there have been successful popular histories of the late Habsburg Empire, which present similar intimate hotbeds of intellectual ferment, albeit on a grander scale. Then of course Mittag-Leffler had a unique perspective on mathematics, he did indeed know all the important players and maintained personal friendships with a few of them and

extensive correspondences with a wider circle. Such matters will inevitably pander to the curiosities of mathematicians in general.

One thing that surely is going to interest an international audience is the relationship between Mittag-Leffler and Nobel and how it might have influenced the (unfortunate?) fact that there is no Nobel Prize in mathematics. The story that Mittag-Leffler had an affair with Nobel's wife can of course be discounted (for obvious reasons) right here and now, but that does not invalidate the general question. Personally I believe that Nobel was a practical man and that his ambitions for the prize were very down-to-earth, and that he in fact never had an inkling of the scientific prestige the prize would eventually be accorded. The thought to award the esoteric subject of mathematics must never have entered the mind of the businessman Nobel.

It is of course true, as you indicate that Nobel was never married. But it is not true that Nobel and Mittag-Leffler never had anything to do with each other. I have unearthed previously unknown correspondence between the two, and although their exchanges were polite they were not particularly cordial. When Nobel announced that he was going to make a major donation,<sup>4</sup> Mittag-Leffler wrote him a long letter pleading for support for a professorship for Sonja Kowalevski. Nobel wrote back that the donation was made to the memory of his mother and thus his intentions were more of supporting charities than scientific advancement. And he also added, which I find remarkable in its gratuity, that Mlle Kowalevski would be much better served staying in St-Petersburg, a "milieu" far better suited to a lady of her gifts and abilities, rather than to remain a winged bird in a cage in provincial Stockholm. I do not think it is utterly unreasonable to suspect that there might have been some kind of rivalry between Mittag-Leffler and Nobel as regards to Kowalevski, who as a beautiful lady was accustomed to expect attention of a gallant kind.

Also, it is not true that the prize was even initially thought of as a practical one, and the fact that Nobel had neglected both what would later turn into the University of Stockholm<sup>5</sup> and mathematics in his will was commented upon early on, leading to speculations unfavourable to Mittag-Leffler. I doubt that the issue will ever be fully resolved, just like most other historical bones of contention, but personally I do not hold it unreasonable that the relation between Mittag-Leffler and Nobel did in fact influence the latter to the detriment of mathematics. In fact once, at the very end of his life whilst dining at a restaurant, Nobel caught sight of Mittag-Leffler walking in the street outside. He is then reported to have remarked that there goes the worst scoundrel in the country, meaning in matters financial.

<sup>&</sup>lt;sup>3</sup>King Oscar II, grandson of Bernadotte erstwhile Napoleonic Marshal and later an almost unwitting founder of the present Swedish Royal dynasty, as well as great great grandfather of the present King, reigned from 1872 to his death in 1907, a period thus coinciding with the late Victorian period, and as the case of his British relative, the personality of the King very much epitomized it for better and for worse.

<sup>&</sup>lt;sup>4</sup>One in 1890 and thus not to be confused with his ultimate donation in his will

<sup>&</sup>lt;sup>5</sup>Stockholm's Högskola, only in the 1950s formally designated as a University, was an independent institution of higher learning guided by very progressive ambitions

The name of Sonja Kowalevski is also one to which Mittag-Leffler is ineffably linked. Can we mathematicians not take pride in the fact that we pioneered the introduction of women into science? And the fact that there are so few women in mathematics is not due to any oppression of the same, a conclusion otherwise so easy to jump at. Mathematical talent is supposedly very easy to recognise, so although Mittag-Leffler, as a publicist friend of mine has remarked, may have been something of an arch-conservative, he recognised talent where it was due, and thus ironically in this regard at least can be seen as ahead of his times.

It is true that Mittag-Leffler did everything he was capable of to promote Sofia Kowalevski. He also tried to get her elected to the Swedish Academy of Sciences, but its President infamously remarked that if we are going to include women, where on the scale of creation would we then stop? That president was not, needless to say, a mathematician. Mittag-Leffler also took a very keen interest in the literary career of his sister, and he once remarked to her in a letter, how important it is that works of art should not be viewed from a perspective of gender, warning her about being identified with the parochial concerns of the blue-stockings. What is much less known than his championship of Kowalevski, is that he also intervened for the benefit of Marie Curie. Initially it looked like only her husband and Bequerel would have received the prize, but Mittag-Leffler had actually written to Pierre Curie, explicitly asking about his wife's contribution and the lengthy reply which he got, he forwarded to the Nobel Committee. Awarding Marie Curie the prize was a pivotal decision, and I think that Mittag-Leffler deserves a lot of credit for it. In fact to return to the issue of the Nobel Prize, Mittag-Leffler played a very active role. He unsuccessfully tried to lobby for Henri Poincare (in connection to which I have discovered a wonderful picture taken of Poincare on his visit to Djursholm in 1905) and later for Einstein successfully. For many years he arranged a dinner for the laureates at his sumptuous Villa in Djursholm on the day after the awards (although when Marconi got his prize in 1909 he scheduled it the day before, as not to have to invite what he thought was a humbug).

Mittag-Leffler is often presented as an imposing but vain figure, appearing not a little ridiculous. He is also censored for his faibless for high-society. His relations to women, although correct, seem formal and artificial. In his letters as a boy and a young man he comes across as exemplary and very chaste telling his mother everything. It is hard to reconcile this dependant and introverted young man with the extroverted figure that won the confidence and respect of so many of the leading mathematicians of the day, and successfully, not to say brutally, brokered many a business deal.

It is true that his letters to his mother are very intimate and honest. Definitely more so than to his wife, although the latter had their fair share of glow of conventional passion, at least during the initial courting stage. In fact there is nothing that he is able to confide to friends or to his diary that he is not also able to confide to his mother. To

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One of Mittag-Leffler's notebooks - written on almost transparent paper

some extent this might illustrate the tenor of his times, in which women, especially as mothers, were objects of adulation. But Mittag-Leffler clearly goes beyond this. One may partly explain this by his being stricken as a child by a serious disease, through which his mother nursed him back to health. This must have created a strong bond. Relations with his father were more distant, as was not unusual at the time, and they certainly were not helped by his father's mental collapse when Mittag-Leffler was in his early twenties. His father was restrained to mental institutions for the rest of his life and was a source of worry and maybe above all of embarrassment. Mittag-Leffler was quite clear about his strong relation to his mother. He writes to her that any marriage he would conceivably enter was to be one of convention, giving explicitly as an explanation his strong attachment to her. He eventually married a young and beautiful girl, who was also very rich. But the marriage was not particularly happy and resulted in no issue, which one surmises must have been a source of common disappointment not to say sorrow. His relation to his sister, who like his well-known protégé died rather young, was also very close, and he took a great interest in her writing career. Her death, as well as that of Kowalevski which had preceded it, left him shattered.

Admittedly early on in his career he cultivated useful relations with nobility. It is revealing to learn of his initial scepticism not to say distaste and the ease with which he discarded such reservations. As to why he managed to establish such fruitful ties with the leading mathematical lights, one simply should ascribe this to his personal charm. When he travelled on the continent in his late twenties he was a striking figure, tall and handsome, able to carry on cultivated conversations, and also, although not in a historical sense, a more than competent mathematician. He became a personal friend with several of his teachers (Hermite, Kronecker, and of course Weierstrass, just to mention a few), participated in several scientific conferences, and established a net of contacts with many of his contemporaries. In short he was brought ajour with international mathematical research on the cutting edge, or more precisely, he established solid personal contacts with the greatest mathematicians and their schools, giving him standards of excellence he was to maintain for the rest of his life.

It is also true that he did amass a fortune, although the First World War seriously eroded it, but I think one should not conceive this ambition in purely personal terms. His worldly and financial success had a definite purpose, namely that of promoting mathematics. I believe that those standards of excellence he acquired in his first encounter with continental mathematics, this awareness and conviction of what a first-rate mathematician or scientist really represented, had a deep impact on Mittag-Leffler, and provided him with a basis from which to both judge his contemporaries and to determine his own positions on various issues. Of course it could appear arrogant and disparaging when he would apply those standards of excellence to his colleagues in the north and the scientific scene in which he found himself. Naturally many people around him thought that he was living in his own world with his head in the clouds. As founding editor of Acta Mathematica (from 1882), Mittag-Leffler confirmed his claim as an arbitrator of mathematical taste and importance, and the journal quickly became one of the leading ones in the world, providing the fundaments of his international standing.

Mittag-Leffler was a scientist at heart, he strongly believed in the Victorian concept of progress, especially the scientific one. He had inscribed over the fireplace words to the effect "by the emergence of number thought was born and beyond the number thought does not reach".<sup>6</sup> An inscription that has inspired much later scorn, but to me it illustrates his deeply set idealism. Man was not just a tabula rasa on which experience and external stimuli was scratched, but was endowed with a higher spirit actively engaged in the world and its understanding.

#### Mittag-Leffler left a tangible legacy. His Villa, in which we now find ourselves, thanks to the efforts of Lennart Carleson, has served for almost forty years as the kind of institution he had envisioned. Do you think he looks down from his heaven, or wherever his ultimate destination happens to be, with satisfaction?

The question is of course impossible to answer, at least literally. However, I think that an institution of mathematics with no examinations was his dream. In fact he tried to turn Stockholm's Högskola into such a one. Mittag-Leffler was not a classical scholar, and the requirements of learning Latin had been an ordeal to him, and to his mind an utterly meaningless hurdle in the pursuit of mathematics. Such personal experiences strongly coloured his view of education, which must be thought of as progressive, once again disproving his archconservative image.

### If you were asked to make a comparison between the writing on Abel, Lie and Mittag-Leffler respectively, what would you then emphasize?

The main differences are not primarily to be found in the



The mathematical correspondence of Mittag-Leffler is kept in locked closets at the Mittag-Leffler institute

actual work of writing, although the difficulty may be even greater this time around, but that we are talking about three profoundly different personalities. What strikes one first is Mittag-Leffler's gradually acquired consciousness of and faith in his position as a prominent scientist, and the uses to which he put it. Abel never really understood his position and influence; he was standing outside, banging at the door, but was never let in. Lie kicked in the door by brute force and appropriated the position that clearly was his due. Mittag-Leffler simply had the key.



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Swedish Mathematical Society and has been actively engaged in public debate on mathematical education. Publishes occasionally reviews on scientific-philosophical matters in the Swedish press and is the editor of the newsletter of the aforementioned Swedish Mathematical Society.

<sup>&</sup>lt;sup>6</sup>It comes out much better in Swedish with the ambiguity of the word number that can also mean speech. "Talet är tänkandets början och slut. Med tanken föddes talet. Utöver talet når tanken icke."

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### Institut Mittag-Leffler, Stockholm

#### History of the institute

As mathematics research institutes go, Institut Mittag-Leffler (IML) is one of the oldest. On his 70th birthday in 1916, Gösta Mittag-Leffler and his wife announced that they would bequeath their villa, their library and their considerable fortune to the Royal Swedish Academy of Sciences, in order that a research institute for mathematics be established on the premises. The will also included instructions for the running of the institute. The model he used was the Pasteur Institute, which he thought was a good example of an institution that devoted itself to pure research, without having to spend time on undergraduate teaching. At the time, the couple was still very rich, so the prospects for the new institute must have seemed quite bright.

Unfortunately, during the latter part of WWI, and in the years after, the fortune dwindled. In fact in 1922, Mittag-Leffler was on the verge of bankruptcy (his wife Signe had died in 1920) but the situation was saved, in part due to a loan from the Swedish government. When he died in 1927, however, there was not enough money in the estate to run a research institute on the lines Mittag-Leffler had suggested, so the Academy decided to appoint a caretaker director, Torsten Carleman, to see to the buildings and library. Other activities were to be put on hold until more money could be found. Carleman was not a very practical man, so apart from one or two seminars, neither fund raising nor much in the way of mathematics was undertaken in the years up to Carleman's death in 1949.

A new effort to breathe some life into the Institute was then made by the Academy. Lars Ahlfors and Arne Beurling were offered the directorship, but when both declined, the effort was given up, and a new caretaker director was appointed, Otto Frostman.

It was not until the mid 60s, when Lennart Carleson was appointed to a so-called personal professorship by the Swedish government – a very rare occurrence – that the wishes of Mittag-Leffler could start to be realized. The personal professorship meant that Carleson could move to Djursholm and take up the directorship. He also managed to secure money from the Walleberg foundation and from an insurance company to have parts of the main building renovated and rebuilt to be able to function as a research institute, and to build a number of apartments for visiting researchers.

#### Format of the institute

The format of the Institute, conceived by Carleson, was more precise and certainly more practicable than Mittag-Leffler's original ideas, and has been kept even after Carleson left as director. In fact, it has also been the model for other institutes. Carleson's ideas, stature, and contacts in the world of mathematics were instrumental



The Mittag-Leffler villa

in the formative years of the Institute, and he is to be credited for shaping it and making it into an institution which enjoys considerable respect in and out of mathematics and Academia.

The IML runs year-long or semester-long research programs; topics are chosen by the board of the Institute. For each program a committee is appointed to draw up the specifications of the program and to decide on whom to invite. One or two specialists are also nominated as research directors for the program, and are expected to be present during the semester/year. Mathematicians are invited to spend time at the Institute, without any duties other than to be present and devote themselves to their science. Short visits are frowned upon, everybody is expected to stay for at least a month, and we always try to invite one or two to remain for the whole program. The Institute offers free housing, and contributes to the increased expenses of having to live abroad. At any given time about 15 senior researchers are in attendance, and with few exceptions, they all live on the premises.

Concurrently with the senior visiting program, there is a junior visiting program for post docs and advanced graduate students. About 10 are usually present. A number of scholarships are offered for this category. In particular, Scandinavian participants can often subsist on scholarships, grants or salaries from home, plus a small contribution from the institute to cover the extra expenses.

The junior visitor program is a strong part of the raison d'être of IML. The requirement that everybody stay for a substantial amount of time makes it less daunting for the young to get to know the established names, and to start working with them. It is easier to approach a famous professor in the congenial atmosphere of the Institute than having to run him down in the corridor of a university department on his way to a faculty meeting.

During the long reign of Lennart Carleson, who was director for 16 years, the topics of the scientific programs were close to his interests. It is a testimony to the enormous breadth of his mathematical appetite that, despite this, the



**Guest buildings** 

programs of the Institute covered such a wide range. Over the last 20 years, even more diversification has taken place, with topics from algebra, combinatorics, logic and various branches of applied mathematics having been added to the long list of analysis programs. The next few years will see PDE (wave theory), algebraic geometry, moduli spaces, and stochastic differential equations join the list.

#### Facilities

With few exceptions, all visitors can find accommodation in the Institute's own apartments. In addition to two older houses, and the apartments built at the end of the 60s, a new set of flats was built five years ago. Although of varying type, they are all comfortable, and some also have room for accompanying spouses and families. In all, from 100 to 150 visitors spend time at the Institute each year.

The pride of IML is its library. Gösta Mittag-Leffler was an avid book collector, and the mathematics part of it was left to the Institute. The collection of older books is admirable, and about 250 of the most important journals are subscribed to. The so-called upper library room is a delight to the eye as well as to the mind. It was designed by the architect Ferdinand Boberg, and must have been one of the most beautiful private library rooms of its day. Directly adjoining this room is Gösta's former study, with a spectacular view over the Baltic inlet of Askrikefjärden.

For a small organisation, keeping up the computer system is always a problem. We have been lucky enough to have our own systems manager for several years now. All visitors get their own desk with a work station (or laptop connection), although most will have to share an office.

The IML also publishes two well-known scientific journals, the *Acta Mathematica* and the *Arkiv för matematik*. The *Acta* was started by Gösta Mittag-Leffler in 1882, while the *Arkiv* was taken over from the Royal Swedish Academy in 1971. Although subscription handling and distribution have now been out-sourced, the editorial work and part of the typesetting is done within the Institute's walls.

#### **Budget and Administration**

Compared with some of our sister institutes, IML is run on a shoestring budget. The overall costs amount to about EUR 1,000,000, for which we think we do quite a lot. This figure includes staff salaries, upkeep of buildings, library acquisitions, and the scientific program. The Mittag-Leffler Foundation, which under the auspices of the Academy has grown considerably in the 78 years since Mittag-Leffler's death, provides money for the infrastructure, while the scientific programs are financed through contributions from the research councils of the Nordic countries and various foundations and programs. The staff of seven are all dedicated to taking care of the visitors and the house and grounds.

Although formally a part of the Royal Swedish Academy of Sciences, the IML has always led a rather independent life. The Academy of course keeps an eye on us and our finances, but rarely interferes. Oversight and leadership is provided by the Board of the Institute, comprised of the members of the mathematics class of the Academy, and representatives from the other Nordic countries: Denmark, Finland, Iceland, and Norway. In fact, Gösta Mittag-Leffler insisted on the Institute being a Nordic institution, with equal opportunities for all. On average, about one third of the visitors are from the Nordic countries. To get an input of a more international kind, the Board has retained the kind help of advisors, currently William Fulton and Peter Jones.

#### The location

The original Mittag-Leffler villa, built in 1891 and rebuilt and added to in 1897, 1903 and 1907, is a magnificent house with an almost Italian renaissance facade and a total area of some 900 square meters. A few rooms are kept essentially in their original state, and provide an Art nouveau atmosphere of a rather unique kind. Of course, the offices, both of the staff and the visitors, are of a more prosaic nature. The surrounding park, some 25,000 square meters, is home to owls and deer, lots of wild flowers and oak trees. Most visitors seem to enjoy the rather secluded location in a suburb of Stockholm. There are shops and a couple of restaurants nearby, but those needing specialty stores or nightlife activities have to go to the City, which can be reached in about half an hour via bus and the underground.

#### **Future programs**

Fall 2005: Wave motion Spring 2005: Algebraic Topology 2006/2007: Moduli Spaces Fall 2007: Stochastic Partial Differential Equations

#### **Detailed information**

can be found on the institute's web site http://www.ml.kva.se

### Book review

Andrei Martínez Finkelshtein (Almeria, Spain)



George M. Phillips Interpolation and Approximation by Polynomials CMS Books in Mathematics Vol. 14 Springer Verlag 2003 Pages: 312 ISBN: 0-387-00215-4

Approximation theory is a field of mathematics nourished by analysis (functional, real and complex), and plays a major role in numerical computation. In consequence, when writing a book on approximation one needs to allot attention between "purely" analytic aspects and algorithmic questions of implementation. For the book under review it is evident, however, that in the author's scientific taste, classical analysis prevails (bias which is occasionally corrected by the insertion of some algorithm written in pseudo code).

The title brings to mind reminiscences of the famous monograph of P. J. Davis *Interpolation and Approximation* (and the author admits that this is indeed a tribute to it), but is without any doubt on an introductory level, and the intended audience is supposed to be less prepared. The text is easy reading, although sometimes by omitting some tedious proofs. The inclusion of problems will be welcomed both by lecturers and by self-learning students. There are also several interesting new aspects that are usually not included in texts of this level, such as the multivariate interpolation, the analysis of the Peano kernels, q-integers (mentioned in 5 of the 8 chapters of the book) and others.

Let us briefly see the content by chapters. The (univariate) polynomial interpolation is one of the cornerstones of polynomial approximation in one variable, and its low numerical importance does not prevent it from appearing as the first chapter of most of the books on approximation. It is less frequent to find a detailed discussion of the construction of the interpolation polynomials by means of the solution of a system of linear equations with Vandermonde matrix using the LU factorization. This is one of the examples of theoretic questions which are interesting but of little use for computation (indeed, this approach is often "forbidden" to our students of numerical analysis, by showing them the disastrous condition of the problem). Less understandable is the decision of the author to restrict the study of Hermite interpolation to the case when only the value of a function and of its derivative at each node is given, since from the Newton formula the general case is in sight.

Chapter 2 is devoted to the best approximation (both in least squares and uniform). It starts with an exposition of the properties of the orthogonal polynomials, first of Legendre and later of Chebyshev. The methodological value of this decision is arguable, taking into account that the Chebyshev polynomials, as the Aspirin, are ubiquitous. On the positive side, there is a simple proof that the equioscillation property is sufficient for the extremality in the uniform norm (although inexplicably this theorem is not attributed explicitly to Chebyshev), and a short discussion of Remez algorithms (scarce in books of this level). At the end of the chapter we find an elegant exposition of the Lebesgue function describing the approximative properties of the interpolation polynomials, and a two page description of the modulus of continuity and Jacson theorems (rather to "comply" and without going into many complications).

Chapters 3 and 4 respectively deal with numerical integration and the Peano kernel. The proof of the Euler-Maclaurin summation formula (chapter 3) and the deduction of the Peano kernel and its application to the error formulas are highly recommended. This part of the book reveals with more strength the "analytical" (vs. "numerical") bias. For example, it is stated that the nodes of the Gaussian quadrature are the zeros of the corresponding orthogonal polynomials, which is totally correct. But this information is incomplete in applications to real computations: it would be essential to show the existing relation between the parameters of the quadrature formula and the eigenvalues and eigenvectors of the corresponding Jacobi matrices.

I was pleasantly surprised to find also a chapter devoted to the multivariate interpolation, both in rectangular grids (which in practice reduce to the univariate case) and in triangular regions. The latter require more sophisticated techniques, like methods of projective geometry and homogeneous coordinates. In chapter 5, we find the celebrated result of Chung and Yao on the set of unisolvence for the polynomial interpolation, the generalization of the scheme of Newton interpolation, the interpolation on the q-integers and elements of cubature formulas on triangles. All of this is new and infrequent material.

Splines (piecewise polynomial functions with some degree of smoothness) could not be missed in this book (chapter 6). In contrast to the first part of the book, here the treatment is more constructive. In addition to the standard material we find the definition and basic prop-

erties of the B-splines; however we miss some other frequent topics, like the Holladay identity and the minimization of energy that yields the physical interpretation of splines (and gives sense to the name). Splines with equidistributed nodes and nodes at the q-integers are analysed in some detail, together with a method for their computation.

Chapter 7, entitled "Bernstein polynomials", contains some of the jewels related to the previous material. For instance, for the uniform approximation we have the proof of the convergence of these polynomials to a continuous function, followed by a proof of Korovkin theorem (I confess that I would have interchanged the order), which is applied to Riesz-Fejer interpolation. The theory of splines benefits from the discussion of the total positivity and of its applications to the conservative approximation. As it could be expected, Bernstein polynomials are generalised to q-integers.

And finally we reach Chapter 8, entitled..., yes, "Properties of the q-integers". It deals with more specific issues related to non-uniform nets and their applications to multivariate interpolation. This is a clear tribute (explicitly mentioned in the introduction) to the author's scientific interests.

The bibliography, comprised of 55 entries only, is not absolutely exhaustive, but it is sufficient for a textbook. The publisher has taken great care in producing a highquality edition, as usual in Springer Verlag. Summarizing, this is a medium level book, recommended for everyone interested in the fundamental constructive aspects of approximation theory in one or several real variables, especially, as announced in the title, by polynomials (or splines), and it is written by a wellknown specialist in the field.

This book review appeared originally (in Spanish) in "La Gaceta de la Real Sociedad Matemática española" 7, no. 3, 2004. We thank Vicente Muñoz for the translation into English and the editors for the permission to republish it here.



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### Personal column

Please send information on mathematical awards and deaths to the editor.

#### Awards

**Christof Teuscher** from Switzerland was presented the 5,000 Euro Cor Baayen Award for his research on unconventional biologically-inspired machines during a ceremony at the ERCIM fall meetings in Malaga, Spain.

**Luigi Ambrosio** (Pisa) received the prix Fermat Recherche for his contributions to the calculus of variations and to geometric measure theory and their connections to partial differential equations.

The Karl Georg Christian von Staudt-Prize was awarded to **Günter Harder** (Bonn) and Friedrich **Waldhausen** (Bielefeld).

**Friedrich Hirzebruch** (Bonn) has been awarded the 2004 Georg Cantor Medal from the German Mathematical Society in recognition of his remarkable achievements.

The German Mathematical Society gave their Media Prize 2004 to **Christoph Pöppe** and their Journalist Prize 2004 to **Hubertus Breuer**.

**Martin Vingron** (Berlin) and **Eugene W. Myers** (Berkeley) are the two winners of the 2004 Max Planck Research Prize from the Alexander von Humboldt Foundation and the Max Planck Society, for their work in computational molecular biology in the analysis of the genome in the field of bioinformatics.

The 2005 Wolf Prize has been awarded to **Gregory A. Margulis** (Yale) and to **Sergei P. Novikov** (Maryland & Moscow). A short presentation was given in issue 55 of this *Newsletter*.

The Autumn Prize of the Mathematical Society of Japan (MSJ) 2004 was awarded to **Arai Toshiyasu** (Kobe) for his distinguished contributions to Hilbert's second problem.

The 2004 MSJ Geometry Prize was awarded to **Seiichi Kamada** (Hiroshima) and to **Shin Nayatani** (Nagoya).

The 2004 MSJ Analysis Prize was awarded to Masafumi Akahira (Tsukuba), Katsunori Iwasaki (Kyushu) and to Takaaki Nishida (Kyoto). **Peter D. Lax** (New York) received the Abel Prize 2005 of the Norwegian Academy of Sciences. See the announcement in this issue for details.

The 2004 SEMA (Spanish Applied Mathematics Society) Prize, "Popularization of Applied Mathematics", has been awarded to **Pablo Fernández** (Madrid) for his article "Google's secret and Linear Algebra".

The Young research SEMA Prize 2004 was awarded to **Marco Antonio** Fontelos (Madrid) for his work in fluid mechanics.

**Vladimir Maz'ya** (Ohio State & Liverpool) received the Celsius Gold Medal of the Royal Society of Sciences of Uppsala for his outstanding research in partial differential equations and hydrodynamics.

**Ben Green** (Cambridge) received a 2004 Clay Research Award in recognition of his work on arithmetic progressions of prime numbers.

**Gérard Laumon and Bao-Chau Ngo** (Paris-Sud) received a 2004 Clay Research Award in recognition of their proof of the fundamental lemma for unitary groups.

**Elias M. Stein** (Princeton) has been awarded the 2005 Stefan Bergman Prize.

#### Deaths

We regret to announce the deaths of:

Jerzy Baksalary (8.3.2005) Shiing-Shen Chern (3.12.2004) Michael Grüter (13.11.2004) Andrew Charles King (13.1.2005) Frédéric Poupaud (13.10.2004) Martin Schröder (17.11.2004) René Taton (9.8.2004) Jozef Joachim Telega (28.1.2005) Hansjoachim Walther (17.1.2005) Tom Willmore (20.2.2005) Michajlo Yadrenko (28.9.2004)

### Problem Corner mathematics Olympiads held in Kiev

#### Paul Jainta (Schwabach, Germany)

Last autumn, the Ukraine became the centre of the world's interest for a short time. The capital Kiev, and its citizens who have demonstrated for free elections, have been in the limelight facing world opinion. It watched in shocked amazement the fascinating struggle for power, which in the Western media was portrayed as a conflict between the forces of dictatorship and democracy and between an autocratic and a democratic opposition.

The 2004 presidential election was the most important event in Ukraine since independence was achieved in 1991. The November 21 runoff determined whether Ukraine fulfilled its quest for democracy and integration into the Euro-Atlantic community or maintained its corrupt status-quo drifting increasingly toward an authoritarian system along the Eurasian model. The result was what some have dubbed the "Chestnut Revolution" – named after the chestnut trees that line the boulevards of Kiev. Others called it the "Orange Revolution" due to the opposition's campaign colour.

As everybody knows, on 28 December 2004, the Central Election Commission announced that opposition candidate Viktor Yuschenko was the official winner in Ukraine's repeat presidential vote on 26 December. A reformist widely regarded as pro-Western by pundits at home and abroad, Yuschenko defeated Prime Minister Viktor Yanukovych 51.99 percent to 44.19 percent. Ballots were reportedly cast against both candidates by 2.34 percents of voters. Yuschenko vowed to mount a legal challenge to the results of the 26 December election as returns showed his reformist opponent with an insurmountable lead.

The Ukraine is back to Europe! Ukrainian mathematics was established within the heart of European sciences a long time ago and was praised by experts from the start. Today's Corner will throw some light on mathematics contests that are resident at the Universities of Kiev. Professor **Alexander Kukush**, Faculty of Mechanics and Mathematics of the National Taras Shevchenko University, and his student **Volodymyr Brayman**, have drawn up a short report giving some details.

#### Mathematics Olympiads in Kiev

Mathematics competitions have been held at the Faculty of Mechanics & Mathematics of Kiev National Taras Shevchenko University since 1974. During the last seven years, it has become an Open Olympiad, free for students registered at a university of Kiev. Most of the winners are from the Faculty of Mechanics & Mathematics for obvious reasons, but students from the National Technical University and the Faculty of Cybernetics are successful as well, and they are extremely efficient at the All-Ukrainian round of the competitions, usually held in the city of Lviv, and at the International Olympiads for University Students.

The competition is organized for students of grades 1-2 and 3-4, separately. Usually, there are 80-100 participants, most of them

junior students. The competitors have to sit a 3.5 hour paper, working on 6 to 10 problems, so they can choose their favourite topics to deal with. The problems on calculus compose the main part of the task and are the most popular with the participants. Traditionally, there are some interesting probability problems, which is not really surprising because Kiev has been known as a famous centre of Probability Theory. The problems are not too technical by nature and they admit short and elegant solutions. However, one or two problems are more difficult and need really tough mathematical research from time to time. Almost all problems are original and composed by teachers, PhD students, and senior students of the Faculty of Mechanics & Mathematics. The results of the Olympiads, together with instructions for solutions, are published in the journal *In the World of Mathematics* which is edited at the faculty.

For more than twenty years, till 1997, the well-known statistician, Professor Anatoly Dorogovtsev (1935–2004), had been the head of the jury. He has proposed hundreds of problems on calculus, measure theory, and functional analysis, and extended the competition to a real festival for the involved students. One of his successors, the outstanding probabilist and Corresponding Member of National Academy of Sciences of Ukraine, Mykhailo Yadrenko (1931– 2004), contributed a lot of ingenious original problems on Probability Theory and Discrete Mathematics until death. We also remember some other organizers of the competition, Vladimir Anisimov, Corresponding Member of National Academy of Sciences of Ukraine, and not to forget the Professors Alexei Konstantinov, Vladimir Mazorchuk, and Volodymyr Nekrashevich. Since 1999, the President of the Board of Mathematical Analysis, Professor Igor Shevchuk, has been head of the jury.

Several famous mathematicians have been among the winners of past competitions. For example, Professor Sasha Reznikov (1960– 2003) was a member of the London Mathematical Society and applied hard analysis in modern geometry, see http://www.lms.ac. uk/newsletter/323/323\_09.html. Professor Andrey Dorogovtsev received the State Award in 2003 for his monograph on stochastic calculus. We also mention the Professors Alexey Daletskiy, Paul Etingof, Nikolay Kartashov, Yury Kondratyev, Alexander Kukush, Vladimir Lyubashenko, Yuliya Mishura, and Boris Tsygan, and apologize if we missed some notable persons.

Now you are invited to solve some selected problems from the Olympiads. We hope you'll take a fancy to their ingenuity. The authors are given in brackets.

**170** Find all functions  $f \in C(\mathbb{R})$  such that the following identity holds for every  $x, y, z \in \mathbb{R}$ :

$$f(x) + f(y) + f(z) = f\left(\frac{3}{7}x + \frac{6}{7}y - \frac{2}{7}z\right) + f\left(\frac{6}{7}x - \frac{2}{7}y + \frac{3}{7}z\right) + f\left(-\frac{2}{7}x + \frac{3}{7}y + \frac{6}{7}z\right).$$

(V. Brayman)

171 For every positive integer *n* consider the function

 $f_n(x) = n^{\sin x} + n^{\cos x}, x \in \mathbb{R}.$ 

Prove that there exists a sequence  $\{x_n\}$  such that for every  $n \ge 1$ ,  $f_n$  has a global maximum at  $x_n$  and  $x_n \to 0$  as  $n \to \infty$ . (A. Kukush)

**172** Find all continuous odd functions  $f : \mathbb{R} \to \mathbb{R}$  that satisfy the functional equation f(f(x)) = x for every  $x \in \mathbb{R}$  (A. Kukush)

**173** Let A, B, C, D be (not necessarily square) real matrices such that  $A^T = BCD$ ,  $B^T = CDA$ ,  $C^T = DAB$ ,  $D^T = ABC$ . For S = ABCD prove that  $S^2 = S$ . (V. Brayman)

**174** Let  $R(z) = \frac{z^2}{2} - z + \ln(1+z), z \in \mathbb{C}, z \neq -1$ . Prove that for every  $x \in \mathbb{R}$  the following inequality holds:  $|R(ix)| \leq \frac{|x|^3}{3}$ . ("ln" denotes the value of logarithm corresponding to the branch where  $\ln 1 = 0$ ) (D. Mitin)

**175** A croupier and two players are playing the following game. The croupier chooses an integer in the interval [0, 2004] with uniform probability. The players guess the integer in turn. After each guess the croupier indicates whether the chosen integer is bigger or smaller or has just been guessed. The player who guesses the integer first wins. Prove that both players can develop a winning strategy such that their chances of guessing are at least  $\frac{1}{2}$ . (S. Shklyar)

The first item in our 'solutions' section is an answer to problem 158, proposed in the March 2004 issue of the Corner. All questions were hatched in Bulgaria. Judging by the dripping incoming submissions, I have a suspicion that this breed was rather temperamental.

**158** Given an *n*-gon (not necessarily convex) and a point *S* in its interior, a light source is located at S such that no side of the polygon can be illuminated by the source entirely. What polygon with the smallest number of edges ( $n \ge 3$ ) satisfies this property?



Solution by J. N. Lillington, Wareham, UK. The polygon ABCDEF has 6 sides with the desired property that no side can be entirely illuminated from S, from the interior. Thus  $n \le 6$ . Now suppose n < 6 and that a polygon exists with the desired property. Choose 3 points A, C, E as vertices, and consider the sectors SAC, SEC, SEA. Since there are at most two additional vertices

if n < 6, there exists at least one sector, say SEA, without a vertex. But then S can floodlight the whole of EA which is against the hypothesis. Thus n = 6.

**159** Find all prime numbers p and q, such that the number  $\frac{(5^p-2^p)(5^q-2^q)}{n^q}$  is an integer.

*Solution by Prof. Dr. W. Fensch, Karlsruhe, Germany.* We use the following two well-known results from number theory:

1. 'If p|ab then p|a or p|b, p a prime' and

2. 'If *p* is a prime number that does not divide the number *a*, then  $a^{p-1} \equiv 1 \pmod{p}$ ' (Fermat's Little Theorem).

So, if p is prime and  $p|(5^p - 2^p)$ , Fermat's Little Theorem implies  $5^p \equiv 2^p \pmod{3}$ , and thus  $5^3 - 2^3 = 3 \cdot 3 \cdot 13$ . There are three possible values for q: q = 3, q = 13 or  $q|(5^q - 2^q)$ . The last possibility implies q = 3, and we have the following solutions: (3,3), (3,13) and (13,3).

Now, let p = 3. Because the congruence  $5^q \equiv 2^q \pmod{3}$  is satisfied for every q, we may have  $5^3 \equiv 2^3 \pmod{q}$  or  $5^q \equiv 2^q \pmod{q}$ . q).  $5^3 \equiv 2^3 \pmod{q}$  implies q = 3 or q = 13, thus we focus at  $5^q \equiv 2^q \pmod{q}$  (mod q) when  $q \neq 2, 3, 5$ . In this case there exists an integer 1 < x < q satisfying  $5 \equiv 2x \pmod{q}$ . Then it follows that  $5^q \equiv 2^q \cdot x^q \pmod{q}$ , and  $5^q \equiv 2^q \pmod{q}$  would cause  $1 \equiv x^q \pmod{q}$ . Therefore we have  $x^{q-1} \equiv x^q \pmod{q}$  from Fermat's Little Theorem, thus  $1 \equiv x \pmod{q}$ , and this is a contradiction to x > 1. The only values of *q* matching p = 3 are q = 3 or q = 13.

Let  $p \neq 3$ ,  $q \neq 3$  and  $p|(5^q - 2^q)$ ,  $q|(5^p - 2^p)$ . As shown, p = q is impossible. The previous argumentation gives  $1 \equiv x^q \pmod{p}$  and  $1 \equiv y^p \pmod{q}$  with integers 1 < x < q, 1 < y < p. Fermat's Little Theorem now yields q|(p-1) and p|(q-1), which is also impossible.

So, there are no other pairs (p,q) satisfying the congruence  $(5^q - 2^q)(5^p - 2^p) \equiv 0 \pmod{pq}$  than those given above.

**160** If *a*, *b*, *c* are positive real numbers with abc = 1, prove that  $\frac{1}{1+a+b} + \frac{1}{1+b+c} + \frac{1}{1+c+a} \leq \frac{1}{2+a} + \frac{1}{2+b} + \frac{1}{2+c}$ , with equality only if a = b = c = 1.

Solution by J. N. Lillington. (Ed. Laboured algebraic transformations will rearrange the expressions on the left and right side of the inequality into a kind of mammoth term.)

$$\frac{1}{2+a} + \frac{1}{2+b} + \frac{1}{2+c} = \frac{(2+b)(2+c) + (2+a)(2+c) + (2+a)(2+b)}{(2+a)(2+b)(2+c)} = \frac{12+4(a+b+c) + (ab+ac+bc)}{9+4(a+b+c)+2(ab+ac+bc)}$$
(1)

$$\frac{\frac{1}{1+a+b} + \frac{1}{1+b+c} + \frac{1}{1+c+a}}{(1+a+c)(1+b+c) + (1+a+b)(1+b+c) + (1+a+b)(1+a+c)}}{(1+a+b)(1+a+c)(1+b+c)}$$

$$= \frac{3+4(a+b+c) + (ab+ac+bc) + (a+b+c)^2}{(a+b+c)^2}$$

 $\frac{1}{2(a+b+c) + (ab+ac+bc) + (a+b+c)^2 + (a+b+c)(ab+ac+bc)}$ (2)

Now let x = a + b + c, y = ab + ac + bc. Then  $x \ge 3$ ,  $y \ge 3$ ,  $x^2 \ge 3y$  by the arithmetic mean – geometric mean inequality with equality only if x = y = 3.

To prove the given inequality, it is sufficient to show:

$$\frac{12+4x+y}{9+4x+2y} - \frac{3+4x+y+x^2}{2x+y+x^2+xy} \ge 0, x \ge 3, y \ge 3$$

It is further sufficient to prove that

$$f(x,y) = (12+4x+y)(2x+y+x^2+xy) -(9+4x+2y)(3+4x+y+x^2) \ge 0.$$

Simplifying, we get

$$f(x,y) = -27 - 24x - 3y - 5x^2 - y^2 + 6xy + 3x^2y + xy^2.$$

Now,

$$\frac{\delta f(x,y)}{\delta x} = -24 - 10x + 6y + 6xy + y^2$$
  
$$\geq -24 - 10x + 6y + 18x + y^2 > 0 \text{ if } x, y \geq 3$$

and

$$\frac{\delta f(x,y)}{\delta y} = -3 - 2y + 6x + 3x^2 + 2xy > 0 \text{ if } x, y \ge 3.$$

Thus,  $f(x,y) \ge f(3,3) = -27 - 72 - 9 - 45 - 9 + 54 + 81 + 27 = 0$ with equality if and only if x = y = 3.

Also solved by Dr. W. Fensch.

**161**  $A_1, A_2, A_3$  are n equidistant collinear points in the plane. They are coloured in two colours. What is the smallest number of points  $(n \ge 3)$ , such that there exists three points  $A_i, A_j, A_{2j-i}$  ( $1 \le i < 2j - i \le n$ ), which are of the same colour? Solution by J. N. Lillington. Let the points be coloured r(ed) and b(lack). We prove that for n = 8, there are possible combinations without three points of the same colour, but for n = 9, there exist three points with the same colour for all combinations.

For n = 8, the allowable indices i, j, 2j - i are (1, 2, 3), (1, 3, 5), (1, 4, 1), (2, 3, 4), (2, 4, 6), (2, 5, 8), (3, 4, 5), (3, 5, 7), (4, 5, 6), (5, 6, 7) and (6, 7, 8).

For n = 9, the additional indices are  $(i, j, 2j - i) \in \{(1, 5, 9), (3, 6, 9), (5, 7, 9), (7, 8, 9)\}.$ 

Consider all paths of the tree of possibilities, a cross *x* means a particular combination will give three allowable points of the same colour. The following paths show all combinations exhausted:

rrb—rrbr—rrbrr—rrbrrb—rrbrrbrx rrb-rrbr-rrbrrb-rrbrrbb-rrbrrbbrx rrb—rrbrbmrbb—rrbrbrx rrb—rrbrb—rrbrbb—rrbrbbrx rrb—rrbbr—rrbbrr—rrbbrrb—rrbbrrbrx rrb—rrbb—rrbbrr—rrbbrrb—rrbbrrbb—rrbbrrbbrx rrb—rrbb—rrbbr—rrbbrb—rrbbrbr—rrbbrbr rrb—rrbb—rrbbrb—rrbbrbr—rrbbrbrb—rrbbrbrbr rrb—rrbbr—rrbbrb—rrbbrbrb—rrbbrbrbbrbbx rrb—rrbb—rrbbr—rrbbrbb—rrbbrbbrx rbr—rbrr—rbrrb—rbrrbr—rbrrbrrx rbr—rbrrr—rbrrbr—rbrrbrb—rbrrbrbbx rbr—rbrrb—rbrrbb—rbrrbbrx rbr-rbrb-rbrbx rbr—rbrbb—rbrbbr—rbrbbrr—rbrbbrrbx rbr—rbrbb—rbrbbr—rbrbbrb—rbrbbrbbx

Thus n = 9 is the smallest number of points such that there exist three points  $A_i$ ,  $A_j$ ,  $A_{2j-i}$  of the same colour.

**162** Prove that there exists no function  $f : \mathbb{R}^+ \to \mathbb{R}^+$ , which satisfies for any  $x, y \in \mathbb{R}^+$  the inequality  $(f(x))^2 \ge f(x+y) \cdot (f(x)+y)$ .

Solution by Prof. Steve Maybank, School of Computer Science and Information Systems, London. It is shown that the inequality  $(f(x))^2 \ge f(x+y)(f(x)+y)$  (1) leads to a contradiction.

It follows from (1) that f(x) > f(x+y) thus f is strictly monotonic decreasing.

Choose any number a > 0 and define the sequence  $u_1, u_2, ..., u_n$ by  $u_1 = a, u_2 = f(a), u_3 = f(a + f(a)), ..., u_n = f(u_1 + u_2 + ... + u_{n-1}).$ 

Define  $S_n$  by  $S_n = u_1 + u_2 + \ldots + u_n$ . On setting y = f(x) in (1) it follows that

$$(f(x))^2 \ge 2 \cdot f(x+f(x)) \cdot f(x)$$

thus

$$\frac{f\left(x+f\left(x\right)\right)}{f\left(x\right)} \leqslant \frac{1}{2}.$$
(2)

It follows from (2) that if  $n \ge 3$ , then

$$\frac{u_n}{u_{n-1}} = \frac{f(S_{n-2} + f(S_{n-2}))}{f(S_{n-2})} \leqslant \frac{1}{2}.$$

So, the sequence  $S_n$  converges to a limit S. The terms  $u_n$  are all strictly positive, thus  $S > S_n$ . The function f is strictly monotonic decreasing, and  $f(S) < f(S_n) = u_{n+1}$ .

The term  $u_{n+1}$  tends to zero as n tends to infinity. It follows that  $f(S) \leq 0$ , which is the required contradiction.

Also solved by J. N. Lillington, and Dr Z. Reut, London.

**163** Given a triangle with side lengths a,b and the lengths of the corresponding angular bisectors  $l_a, l_b$ , find the smallest rational number k, such that  $\frac{l_a+l_b}{a+b} < k$ .

*Solution by Prof. Dr. W. Fensch, Karlsruhe.* We start with the following well-known trigonometric identities and relations between sides and angles of a triangle:

$$\sin \alpha = 2\sin \frac{\alpha}{2}\cos \frac{\alpha}{2}; \cos \alpha = 2\cos^2 \frac{\alpha}{2} - 1;$$
  

$$\sin \left(\pi - \frac{\alpha}{2} - \beta\right) = \sin \left(\frac{\alpha}{2} + \beta\right) = \sin \frac{\alpha}{2}\cos \beta + \cos \frac{\alpha}{2}\sin \beta;$$
  

$$\frac{\sin (\pi - \alpha - \beta)}{c} = \frac{\sin \beta}{c} \Rightarrow \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{c}{b}\sin \beta;$$

$$\frac{\sin\left(\pi - \frac{\alpha}{2} - \beta\right)}{c} = \frac{\sin\beta}{l_a} \Rightarrow l_a$$

$$= \frac{\sin\beta}{\sin\frac{\alpha}{2}\cos\beta + \cos\frac{\alpha}{2}\sin\beta} \cdot c$$

$$= \frac{2\cos\frac{\alpha}{2}\sin\beta}{2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\cos\beta + 2\cos^2\frac{\alpha}{2}\sin\beta} \cdot c \Rightarrow l_a$$

$$= \frac{2\sin\beta\cos\frac{\alpha}{2}}{(\sin\alpha\cos\beta + \cos\alpha\sin\beta) + \sin\beta} \cdot c$$

$$= \frac{2\cos\frac{\alpha}{2}}{\frac{c}{b} + 1} \cdot c = \frac{2bc}{b + c} \cdot \cos\frac{\alpha}{2}$$

Without loss of generality we let c = 1, assuming a = b.

Because we are looking at triangles, we must have 1 < 2a. Thus we get:

$$\lim_{a \to \frac{1}{2} + 0} \frac{4 \cdot \frac{a}{a+1} \cos \frac{\alpha}{2}}{2a} = \frac{2 \cos 0}{\frac{1}{2} + 1} = \frac{4}{3}$$

Obviously the following inequalities hold:

a

$$\frac{1}{2} < a \Rightarrow 2 < 4a \Rightarrow 6 < 4a + 4 \Rightarrow \frac{2}{a+1} < \frac{4}{3} \Rightarrow \frac{2\cos\frac{\alpha}{2}}{a+1} < \frac{4}{3}$$

This shows  $k = \frac{4}{3}$  if a = b.

Now let  $a \neq b$ , and 1 < a + b, a < 1 + b, b < 1 + a. Define *a*, *b* as  $a = \frac{1}{2} + x$ ,  $b = \frac{1}{2} + y$ .

Then we have x + y > 0, and we can stack up step by step:

$$0 < x + y + 2x^{2} + 2y^{2}; x + y < 2x + 2x^{2} + 2y + 2y^{2};$$
  
$$1 + x + y < 2 \cdot \left(\frac{1}{4} + x + x^{2}\right) + 2 \cdot \left(\frac{1}{4} + y + y^{2}\right);$$

$$\begin{split} a+b &< 2a^2+2b^2; \quad 2a+2b+4ab < 4ab(a+b)+4a^2+4b^2; \\ 6ab+6b+6ab+6a &< 4a^2b+4ab^2+8ab+4a^2+4b^2+4a+4b; \\ 6b(a+1)+6a(b+1) &< 4(a^2b+ab^2+2ab+a^2+b^2+a+b); \end{split}$$

$$\begin{aligned} 6b\,(a+1)\cos\frac{\alpha}{2} + 6a\,(b+1)\cos\frac{\beta}{2} &< 4\,(a+b)\,(a+1)\,(b+1)\,;\\ \frac{2b\cos\frac{\alpha}{2}}{b+1} + \frac{2a\cos\frac{\beta}{2}}{a+1} &< \frac{4}{3}\,(a+b)\,,\\ \text{and finally } \frac{l_a+l_b}{a+b} &< \frac{4}{3}\,. \end{aligned}$$

Thus we have proven, that the term in question is smaller than  $\frac{4}{3}$  for  $a \neq b$ , too. So we have  $k = \frac{4}{3}$ .

Also solved by J. N. Lillington.

That completes the *Corner* for this issue. Send me your nice solutions to recent problems for use in upcoming issues as well as Olympiad Contests.



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# Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses **vberinde@ubm.ro** or **vasile\_ber**inde**@yahoo.com**. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

June 2005

1–7: International Conference Constructive Theory of Functions, Varna, Bulgaria Information: e-mail: ctf2005@math.bas.bg Web site http://www.math.bas.bg/CTF-2005 [For details, see EMS Newsletter 55]

2–10: Seminario de Algorítmica y Criptografía cuánticas, Madrid, Spain

Information: http://www.dma.eui.upm.es/seminario/

**2–10:** Seventh International Conference on Geometry, Integrability and Quantization with a special session on Multisymplectic Geometry and Classical Field Theory, Sts. Constantine and Elena resort (near Varna), Bulgaria

*Information*: For more information please contact I. M. Mladenov (e-mail: mladenov@obzor.bio21.bas.bg), A. C. Hirshfeld (e-mail: hirsh@physik.uni-dortmund.de), Manuel de Leon (email: mdeleon@imaff.cfmac.csic.es) or visit the Conference *Web page: http://www.bio21.bas.bg/conference/*. [For details, see EMS Newsletter 55]

#### **3–5: International Conference on Mathematics Education,** Svishtov, Bulgaria.

Dedicated to the 50th anniversary of the Department of Mathematics Education at the Faculty of Mathematics and Informatics – Sofia Univ.

*Information*: e-mail: smb@math.bas.bg [For details, see EMS Newsletter 55]

6–10: Congreso Internacional Mediterráneo de Matemáticas Almería 2005 (CIMMA 2005), Almería, Spain Information: Web site: http://www.ual.es/Congresos/CIMMA2005/

8–11: Second advanced course in Operator Theory and Complex Analysis, Sevilla, Spain *Information*: Web site: http://www.us.es/ceacyto2/

**8–12: Computability in Europe – New Computational Paradigms,** University of Amsterdam, The Netherlands *Description*: CiE 2005 is an interdisciplinary venue for researchers from computer science and mathematics focusing on New Computational Paradigms. These include prominently connections between computation and physical systems but also higher mathematical models of computation. Researchers from different communities will exchange ideas, approaches and techniques in their respective work, thereby generating a wider community for work on computational issues that allows uniform approaches to diverse areas, the transformation of theoretical ideas into applicable projects, and general crossfertilization transcending disciplinary borders

*Format*: There will be two three-hour tutorials, eight plenary talks, six special sessions with altogether 25 talks and around 65 contributed talks covering the entire range of research in computability theory

Plenary speakers: S. Abramsky (Oxford), H. Buhrman (Amsterdam), J.D. Hamkins (New York), A. Hodges (Oxford), U. Kohlenbach (Darmstadt), J. van Leeuwen (Utrecht), Y. Matiyasevich (St. Petersburg), Y. Moschovakis (Athens/Los Angeles CA), G. Paun (Bucharest), U. Schoening (Ulm), K. Weihrauch (Hagen) Special sessions: with organisers, Biological Computation (T. Baeck, Leiden); Complexity (E. Mayordomo, Zaragoza); Epistemology and Methodology of Computing (H. Fitz, Amsterdam; G. Tamburrini, Pisa); Proofs and Computation (A. Beckmann, Swansea; L. Crosilla, Firenze); Real Computation (A. Edalat, Imperial College, London), Relative Computation (Barry Cooper, Leeds; A. Sorbi, Siena) Registration: http://www.illc.uva.nl/CiE/index.php?page=13 Sponsorship: The conference is sponsored/supported by the Association for Symbolic Logic (ASL), European Association for Theoretical Computer Science (EATCS), Netherlands Organisation for Scientific Research (NWO), Royal Netherlands Academy of Arts and Sciences (KNAW)

Programme committee: K. Ambos-Spies (Heidelberg), A. Atserias (Barcelona), B. Cooper (Leeds, co-chair), S. Goncharov (Novosibirsk), B. Loewe (Amsterdam, co-chair), D. Normann (Oslo), H. Schwichtenberg (Muenchen), A. Sorbi (Siena), I. Soskov (Sofia), L. Torenvliet (Amsterdam), J. Tucker (Swansea), J. van Benthem (Amsterdam / Stanford), P. van Emde Boas (Amsterdam), J. Wiedermann (Praha) Information: e-mail : bloewe@science.uva.nl, Wab site: http://www.ile.uva.nl/CiE/

Web site: http://www.illc.uva.nl/CiE/

12-24: Foliations 2005, Łodz, Poland

*Information*: e-mail: fol2005@math.uni.lodz.pl *Web site*: http://fol2005.math.uni.lodz.pl [For details, see EMS Newsletter 53]

### 13–17: Computational Methods and Function Theory CMFT 2005, Joensuu, Finland

*Information*: e-mail: cmft@joensuu.fi *Web site*: http://www.joensuu.fi/cmft [For details, see EMS Newsletter 54]

**15–18: Algebraic and Topological Methods in non-classical Logics II,** Barcelona, Spain *Information*: Web site: http://mat.ub.es/~logica/meeting2005/

**20-22: Workshop on Mathematical Problems and Techniques in Cryptology,** Centre de Recerca Matemàtica, Campus of the Universitat Autònoma de Barcelona, Spain *Information*: e-mail: work-cryptology@crm.es *Web site*: http://www.crm.es/jornadas\_criptologia [For details, see EMS Newsletter 54]

#### 25–July 2: European Mathematical Society Summer School on Subdivision Schemes in Geometric Modelling-Theory and Applications, Pontignano, Tuscany, Italy

*Theme*: Subdivision Schemes in Geometric Modelling-Theory and Applications

*Aim:* This summer school is one of eight summer schools promoted by the European Mathematical Society and supported by the European Union for the year 2004-2005. It is a training program mainly for research students and post-doc students. The school will consist of five training days and one day of social activities.

*Topics*: Subdivision schemes for univariate functions and for curves, Subdivision schemes for surfaces, Applications of surface schemes *Main speakers*: N. Dyn (Tel Aviv Univ.), L. Kobbelt (RWTH Aachen), D. Levin (Tel Aviv Univ.), U. Reif (TU Darmstadt), M. Sabin (Cambridge Univ.), P. Schroeder (California Institute of Technology)

*Format*: The program for the training days consists of four 50-minute lectures in the morning and two and a half hours of computer-lab and exercises in the afternoon. The teachers will be available to the students during the free time of the training days *Sessions*: - Subdivision schemes for univariate functions and for curves, 26th–27th June, Dyn/Levin/Sabin/Reif/Schroeder;

- Subdivision schemes for surfaces, 28th June + 1st July, Levin/Sabin/Reif/Kobbelt;
- Applications of surface schemes, 2nd July, Kobbelt/Schroeder Organising committee: C.Conti (Univ. of Florence), N.Dyn (Tel-Aviv Univ.), L.Kobbelt (RWTH Aachen), R.Morandi
- (Univ. of Florence), M.Sabin (Cambridge Univ.)
- Sponsors: European Union, EuroGraphics
- Location: Pontignano, Tuscany, Italy

*Grants*: Students form EU or associated countries can apply for support through the application form at http://www.subdivision-summerschool.uni-kl.de/application/ appform.html *Deadlines*: for student applications: May 1st 2005; for registration: June 1st 2005

*Information*: Further information can be found at http://www.subdivision-summerschool.uni-kl.de/

### 27–29: XI Encuentros de Geometría Computacional, Santander, Spain

Information: Web site: http://www.matesco.unican.es/egc05/

#### **28–July 2: Primeras Jornadas de Teoría de Números,** Vilanova i La Geltrú (Barcelona, Spain)

Information: Web site: http://anduril.epsevg.upc.es/~jtn05

#### 28-July 2: Barcelona Conference on Geometric Group

**Theory,** Centre de Recerca Matemàtica, Campus of the Universitat Autònoma de Barcelona, Spain

*Information: e-mail*: GeometricGroupTheory@crm.es *Web site*: http://www.crm.es/GeometricGroupTheory [For details, see EMS Newsletter 54]

### **30–July 9: Foundations of Computational Mathematics 2005,** Santander, Spain

Information: Web site: http://www.damtp.cam.ac.uk/user/na/ FoCM/FoCM05/

#### July 2005

4–6: Special Functions: Asymptotic Analysis and Computation, Santander, Spain

Information: Web site:http://www.sf05.unican.es/

#### 4–7: XII Jornadas para el Aprendizaje y Enseñanza de las Matemáticas (J.A.E.M.), Albacete, Spain *Information*: http://www.albacete.org/xiijaem

**5–15: Advanced Course on the Geometry of the Word Problem for finitely generated groups,** Centre de Recerca Matemàtica, Campus of the Universitat Autonoma de Barcelona, Spain

*Information*: e-mail: WordProblem@crm.es *Web site: http://*www.crm.es/WordProblem [For details, see EMS Newsletter 54]

#### 7–9: OTFUSA2005: Conference on Operator Theory, Function Spaces and Applications, Dedicated to the 60th birthday of Professor F.-O. Speck, Aveiro, Portugal *Information*: http://www.mat.ua.pt/otfusa2005/ [For details, see EMS Newsletter 55]

#### **17–August 14: Summer School of Atlantic Association for Research in the Mathematical Sciences,** Campus of Dalhousie University in Halifax, Nova Scotia, Canada

*Information*: e-mail: tony@mathstat.dal.ca; renzo@matapp.unimib.it; renzo@mathstat.dal.ca [For details, see EMS Newsletter 54]

### **17–23: European young statisticians training camp, EMS summer school**, Oslo, Norway

associated with the European Meeting of Statistics (see below). *Information*: Web site: www.emis.de/etc/ems-summer-schools. html#2005

#### 24-28: 25th European Meeting of Statisticians (EMS 2005),

Oslo, Norway Information: e-mail: ems2005@nr.no Web site: ems2005.no [For details, see EMS Newsletter 55]

#### 30-August 6: Groups St. Andrews 2005, University of St.

Andrews, St. Andrews, Scotland Information: e-mail: gps2005@mcs.st-and.ac.uk Web site: http://groupsstandrews.org [For details, see EMS Newsletter 54]

#### August 2005

#### 21–26: SEMT 05 (International Symposium on Elementary Mathematics Teaching, Charles University in Prague, Faculty

of Education, Czech Republic

*Information*: e-mail: jarmila.novotna@pedf.cuni.cz *Web site*: http://www.pedf.cuni.cz/kmdm/index.htm [For details, see EMS Newsletter 54]

**28–2 Sept: Workshop "Arithmetic Geometry and High Energy Physics",** Lorentz Center (Leiden, The Netherlands)

Information: http://www.lc.leidenuniv.nl/ [For details, see EMS Newsletter 55]

#### **28–9 Sept: Summer school and workshop on partial differential equations, optimal design and numerics 2005**, Benasque, Spain

*Description*: The activity is mainly intended for young scientists as PhD students and post-docs, and will count with the participation of world leader mathematicians. The topics include partial differential equations and their applications to shape optimization, optimal control problems, singularities in fracture mechanics and fluid dynamics, and numerical analysis

Information: http://benasque.ecm.ub.es/2005pde/2005pde.htm

### 29–September 2: Conference on Differential Geometry and Physics, Budapest, Hungary

Information: e-mail: csikos@cs.elte.hu; Web site: http://www. cs.elte.hu/geometry/DGP05/

[For details, see EMS Newsletter 55]

#### **31–September 2: International Workshop on Topological Groups**, Pamplona, Spain

*Description*: This workshop is directed to graduate students and all researchers interested on topological groups. There will be three minicourses and other plenary lectures. Each minicourse consists on three sessions introducing the state of the art in a concrete area of topological groups. Participants are encouraged to contribute with a short talk or a poster.

*Organizers*: Maria Jesus Chasco (Univ. de Navarra), S. Ardanza-Trevijano (Univ. de Navarra), S. Hernandez (Univ. Jaume I)

*Location*: Universidad de Navarra *Deadline*: for abstracts, July 31, 2005

Information: www.unav.es/topology

September 2005

### 1–December 31: The Scientific Revolutions of the XVI and XVII Century, Centro di Ricerca Matematica Ennio De Giorgi

– Scuola Normale Superiore, Pisa, Italia.

Information: crm@crm.sns.it

[For details, see EMS Newsletter 55]

6-10: Spanish relativistic meetings (ERE 2005), University of Oviedo, Spain

Information: http://fisi24.ciencias.uniovi.es/ere05.html

#### 7–9: 4th IMA International Conference on Mathematics in Transport [A Conference in Honour of Richard Allsop], London, UK

Information: e-mail: conferences@ima.org.uk

*Web site*: www.ima.org.uk/Conferences/mathsintransport.htm [For details, see EMS Newsletter 55]

### **10–16: EMS Summer School and Séminaire Européen de Statistique, Statistics in Genetics and Molecular Biology**, Warwick, UK

Information: Web site: http://www2.warwick.ac.uk/fac/sci/statistics/news/semstat/

[For details, see EMS Newsletter 54]

**13–23: Advanced Course on Combinatorics,** Centre de Recerca Matemática, Campus of the Universitat Autonoma de Barcelona, Spain *Information*: e-mail: RecentTrends@crm.es *Web site*: http://www.crm.es/RecentTrends

[For details, see EMS Newsletter 54]

### 14–16: XIV Encuentro de Otoño de Matemática y Física, Bilbao, Spain

Information: Web site: http://www.ehu.es/wgp2005

### 16–18: EMS-Catalan Mathematical Society Joint Mathematical Weekend, Barcelona, Spain

*Information*: http://www.iecat.net/scm/emsweekend [For details, see EMS Newsletter 55]

#### **16–21: Geometric Representation and Invariant Theory: Algebraic Quantization and Deformations,** Sol Cress Conference Centre, Spa, Belgium

*Organizers*: F. van Oystaeyen (University of Antwerp, BE, Chair), B. Torrecillas (University of Almeria, ES, Vice-Chair)

*Location*: The conference will take place at the Sol Cress Conference Centre in Spa, which is about 50 km from Liège. Spa is located in the Ardennes region and is a small city surrounded by some magnificent forests. Detailed information on location, access to the site, services provided and photos will be provided in due course.

*Grants*: A certain number of grants – covering the conference fee and possibly part of the travel expenses – will be available upon request. Grant requests should be indicated by ticking appropriate fields in the paragraph "Grant application" of the application form.

Deadline: for application and for abstract submission: 17 June 2005

Information: Web site: http://www.esf.org/generic/2210/2005197pp.pdf

### 20–22: 7th Hellenic-European Research on Computer Mathematics and its Applications (HERCMA 2005) Conference, Athens, Greece

*Information*: http://www.aueb.gr/conferences/hercma2005 [For details, see EMS Newsletter 55]

**23–October 2: Braid Groups – Applications to Geometry, Cryptography and Computation – Second Phase**, Eilat, Israel *Program*: The School is a continuation of the EMS School in Israel from February 19–27, 2005. It will be composed of three parts: Part I: During the first two days there will be a short presentation of the results and methods presented in the first School. Part II: This part will be an algorithmic and problems session. Part III: This part will include updates given by speakers from the first School as well as two additional lectures given by guest speakers invited to address this School. You are strongly advised to bring a laptop with you for the tutoring session. Two poster sessions will allow the participants to present their work.

*Organizers*: Dr. Boris Kunyavski and Dr. Tzachi Ben-Itzhak (Emmy Noether Research Institute for Mathematics)

*Admittance*: The school has a limited capacity and we therefore ask all interested parties to fill in an application form (see Info below) and send it (by email) to the organizing committee

WWW.DEGRUYTER.DE

#### Just published



#### ■ Finite Groups 2003

Proceedings of the Gainesville Conference on Finite Groups, March 6–12, 2003 Ed. by Chat Yin Ho / Peter Sin / Pham Huu Tiep / Alexandre Turull 2004. XV, 417 pages. 1 Frontispiece. Cloth.

€ [D] 168,- / sFr 269,- / for USA, Canada, Mexico US\$ 168,95. ISBN 3-11-017447-2

This is a volume of research articles related to finite groups. Topics covered include the classification of finite simple groups, the theory of p-groups, cohomology of groups, representation theory and the theory of buildings and geometries.

As well as more than twenty original papers on the latest developments, which will be of great interest to specialists, the volume contains several expository articles, from which students and non-experts can learn about the present state of knowledge and promising directions for further research.

The Finite Groups 2003 conference was held in honor of John Thompson. The profound influence of his fundamental contributions is clearly visible in this collection of papers dedicated to him.



Hans Föllmer / Alexander Schied

#### Stochastic Finance

An Introduction in Discrete Time

2<sup>nd</sup> rev. and extend. ed. 2004. XI, 459 pages. Cloth. € [D] 58.00 / sFr 93.00 / for USA, Canada, Mexico US\$ 59.95. ISBN 3-11-018346-3

This book is an introduction to financial mathematics.

The first part of the book studies a simple one-period model which serves as a building block for later developments. Topics include the characterization of arbitrage-free markets, preferences on asset profiles, an introduction to equilibrium analysis, and monetary measures of risk.

In the second part, the idea of dynamic hedging of contingent claims is developed in a multiperiod framework. Such models are typically incomplete: They involve intrinsic risks which cannot be hedged away completely. Topics include martingale measures, pricing formulas for derivatives, American options, superhedging, and hedging strategies with minimal shortfall risk.

In addition to many corrections and improvements, this second edition contains several new sections, including a systematic discussion of law-invariant risk measures and of the connections between American options, superhedging, and dynamic risk measures.



Prices are subject to change.

along with a CV and a letter of recommendation. These must be sent no later than July 25th.

*EMS Financial support*: Candidates from EU countries (and associate countries such as Israel, Norway and Switzerland) who have received their MSc. or Diploma degree (or equivalent) within the past ten years are eligible for reimbursement of the following costs: travel, living expenses and registration fees. Europeans currently living outside the EU are eligible for financial support without time constraints.

Information: e-mail: debbyspero@hotmail.com

#### 25–29: SYNASC 2005, 7th International Symposium on Symbolic and Numeric Algorithms for Scientific Comput-

ing, Timisoara, Romania

Theme: symbolic and numeric computations

*Aim*: to stimulate the interaction between the two scientific communities of symbolic and numeric computing, with the purpose of exhibiting interesting applications of these areas both in theory and in practice

*Scope*: to present the state of the art and design the current trend in the field

*Topics*: symbolic techniques applied to numerics; numerics and symbolics for geometry; automated reasoning; computer algebra; solving systems of nonlinear equations; parallel, distributed and web computing for symbolics and numerics; formal system verification; software quality assessment; programming with constraints, narrowing; artificial intelligence in numeric solving; multi-agent systems for complex problem solving; scientific visualization; grid middleware and applications; soft computing; data mining

*Format*: invited talks, contributed presentations, software demo Sessions: Proposals for workshops and technical sessions are invited to be submitted until May 6. The workshops and special sessions should have a focus and topic content that relates to the SYNASC05 topics.

*Organizers*: West Univ. Timisoara (Romania); Johannes Kepler Univ. Linz (Austria), Institute e-Austria Timisoara. General Chairs: B. Buchberger (Johannes Kepler Univ.), S. Maruster (West Univ. Timisoara)

*Programme committee*: Program chairs: T. Jebelean (Johannes Kepler Univ., Linz), V. Negru (West Univ. Timisoara)

*Call for papers*: Papers of up to 12 pages must be submitted electronically to synasc05@info.uvt.ro. Extended abstracts (up to 4 pages) may be submitted for short papers and system presentations

*Organising committee*: Local Chairs: D. Petcu, D. Zaharie (West University of Timisoara)

*Proceedings*: The papers accepted for presentation at SYN-ASC'05 will be published in electronic pre-proceedings (CD with ISBN) available during the symposium. Revised versions of papers selected from those presented at SYNASC'05 will further be published by an international publisher (e.g. IEEE Press)

Location: West University of Timisoara, Romania

*Deadlines*: Workshops proposals (extended deadline): May 6, 2005; Submission deadline: May 16, 2005; Notification of acceptance: July 1, 2005; Final paper: September 1, 2005; Registration: September 1, 2005; Symposium starts: September 25, 2005; Revised papers for postproceedings: October 30, 2005

*Information*: e-mail: synasc05@info.uvt.ro;

Web site: http://synasc05.info.uvt.ro;

tel/fax: +40-256-2448234

*Contact*: Dana Petcu: petcu@info.uvt.ro, Computer Science Department, West University of Timisoara B-dul Vasile Parvan 4, 300223 Timisoara, Romania

#### October 2005

### 14–16 Conference on Applied and Industrial Mathematics (CAIM 2005), Pitesti, Romania

*Theme*: Mathematical and computational modelling in mechanics, physics, biology, medicine, economics and engineering

*Aim:* To provide an updated overview of new techniques and results in pure and applied mathematics

*Scope*: To present the state of the art in some mathematical domains and their applications to problems in science and engineering

*Topics*: Algebra, logic, topology; Ordinary differential equations and finite dimensional dynamical systems; Functional analysis and partial differential equations; Analytical and numerical methods in mechanics; Industrial mathematics; Theoretical computer sciences; Education

*Main speakers*: M. Ciobanu (Tiraspol), A. Georgescu (Pitesti), B. Loginov (Ulyanovsk), L. Palese (Bari), M. Stefanescu (Constanta), M. Rata (Chisinau), L. Restuccia (Messina), V. Trenogin (Moscow)

Format: Plenary lectures, communications

Sessions: 4 plenary lectures, 4 parallel sessions

Languages: English and Romanian

*Call for papers*: If you wish to present a contributed presentation, please submit an abstract (LATEX)

Organizers: Romanian Society of Applied and Industrial Mathematics, University of Pitesti

*Organising committee*: M. Abrudeanu (Bucharest), R. Georgescu, A. Ion, C. Ion, M. Macarie, Gh. Nistor, L. Sandulescu, Gh. Secara, D. Sarbu, M. Trifan (all from Pitesti)

Scientific committee: S. Basarab (Bucharest), V. Branzanescu (Bucharest), I. Burdujan (Jassy), A. Carabineanu (Bucharest), M. Ciobanu (Tiraspol), S. Cleja-Tigoiu (Bucharest), C. Fetecau (Jassy), A. Georgescu (Pitesti), B. Loginov (Ulyanovsk), L. Palese (Bari), T. Petrila (Cluj-Napoca), E. Petrisor (Timisoara), M. Popa (Chisinau), M. Rata (Chisinau), L. Restuccia (Messina), V. Sava (Jassy), M. Stefanescu (Constanta), N. Suciu (Cluj-Napoca), V. Trenogin (Moscow), H. Vereecken (Jülich) *Proceedings*: To be published

*Location*: The building of the Faculty of Mathematics and Computer Science, University of Pitesti, Targu din Vale Street, 1, Pitesti, Romania

*Deadlines*: For registration: 31 August 2005; For abstracts: 31 August 2005; For papers: 30 September 2005

*Information*: e-mail: cgeorge@univ-ovidius.ro, geocv@yahoo. com, averionro@yahoo.com, adelinageorgescu@yahoo.com

### **17–21: Nonlinear Parabolic Problems**, Helsinki, Finland *Information*:

*Web site*: http://www.math.helsinki.fi/research/FMSvisitor0506 [For details, see EMS Newsletter 54]

November 2005

25-December 1: 8th International Conference of The Mathematics Education into the 21st Century Project,

Hotel Eden Garden, Johor Bharu, Malaysia

The Mathematics Education into the 21st Century Project has just completed its seventh successful international conference in Ciechocinek, Poland, following conferences in Egypt, Jordan, Poland, Australia, Sicily and the Czech Republic. The next conference will be in southern Malaysia

*Theme*: "Reform, Revolution and Paradigm Shifts in Mathematics Education"

*Aim:* to provide an international overview of innovative ideas and materials for the teaching of mathematics in schools

*Topics*: mathematical modelling, technology, equity, teacher training

Main speakers: to be announced.

*Format*: keynote lectures, round table plenary, paper presentations, workshops and an open forum of ideas

Organizers: The Mathematics Education into the 21st Century Project

*Programme committee chairpersons*: A. Rogerson (Poland), F. Mina (Egypt)

Organising committee chairperson: A. Zanzali (Malaysia)

*Proceedings*: to be published as hard copy and on our conference website

*Grants*: available for students, teachers and participants from countries in a difficult economic situation

*Deadlines*: to be advised in the First Announcement available from the email address below

*Information*: e-mail: arogerson@vsg.edu.au for all information Web sites of previous conferences only: http://math.unipa. it/~grim/21project.htm

May 2006

#### **30–June 6: 8th international Spring School on Nonlinear Analysis, Function Spaces and Applications (NAFSA 8),** Prague, Czech Republic

Invited speakers: A. Carbery (Univ. of Edinburgh, UK), V. Kolyada (Karlstad Univ., Sweden), M. Ruzicka (Univ. of Freiburg, Germany), Y. Sagher (Florida Atlantic Univ., Boca Raton, USA), H.-J. Schmeisser (Friedrich Schiller Univ., Jena, Germany), G. Sinnamon (Univ. of Western Ontario, London, Canada) *Description*: The spring school will consist of series of lectures delivered by the invited speakers.

Organizers: A. Kufner, B. Opic, L. Pick (Czech Republic) Proceedings: (containing main talks) to be published Information: e-mail: nafsa8@math.cas.cz Web site: http://www.math.cas.cz/~nafsa8

http://adela.karlin.mff.cuni.cz/nafsa/2006

#### June 2006

### 13–16: Mathematics of Finite Elements and Applications (MAFELAP 2006), Brunel University, UK

*Dedication*: In view of his immense contributions to finite elements, his contributions to the MAFELAP Conferences, and on account of his association with Brunel University, MAFE-LAP 2006 will honor Professor Ivo Babuska, who will attain the age of 80 in 2006.

*Format*: We are encouraging colleagues to propose Mini-Symposia which they would like to organize at MAFELAP. Briefly, the parameters for a mini-symposium are that we shall allocate

a time-slot to the organizers, during which they can have four speakers, each giving a presentation (probably 30 minutes, 25 minutes talk plus 5 minutes for questions). If organizers want more than four talks, then we could give them two two-hour slots. If you are interested in running a mini-symposium, please submit a title to us as soon as possible, with a tentative list of speakers.

Information: Web site: www.brunel.ac.uk/bicom/mafelap2006

#### **21–23: 6th International Conference on Mathematical Problems in Engineering and Aerospace Sciences**, Budapest, Hungary

*Theme*: Theory, methods (includes experimental, computational) and applications concerned with mathematical problems in various disciplines of Engineering and Aerospace Sciences

*Aim*: To promote understanding, and collaboration among engineers, mathematicians and scientists who work on various technical and theoretical aspects of problems related to mathematics, engineering and aerospace sciences

*Scope*: to present the state of the art and design the current trend in the field

*Topics*: mathematical problems in engineering and aerospace sciences

Main speakers: For updated list please visit the conference website

*Format*: keynote lectures, invited talks and contributed presentations

Sessions: there will be plenary lectures and many parallel sessions

*Call for papers*: Please submit abstract through conference website abstract submit section

*Organizers*: IFNA, IFIP, IEEE, AIAA, Budapest University of Technology and Economics, Hungary

Programme committee: Please visit the conference website program section

*Organizing committee*: K.T. Alfriend (USA), A.V. Balakrishnan (USA), S. Balint (Romania), P. Borne (France), E.A. Fedosov (Russia), P. Friedman (USA), N. Goto (Japan), M.J. Grimble (UK), L. Gruyitch (France), S. Joshi (USA), T. Katayama (Japan), V. Lakshmikantham (USA), D. Lainiotis (USA), W.G. Luber (Germany), V. Matrosov (Russia), M. Maurice (USA), D. McLean (UK), A. Miele (USA), V. Modi (Canada), Y.Y. Nie (China), K. Ninomiya (Japan), F.Quagliotti (Italy), J. Rohacs (Hungary), S. Sivasundaram (USA), S. Sliwa (USA), M. Vidyasagar (India), A. Zellweger (USA)

*Proceedings*: to be published

*Location*: Budapest University of Technology and Economics, Budapest, Hungary

Grants: Pending on funding

*Deadlines*: for registration, abstracts, deadline please visit the conference website

*Information*: e-mail: info@icnpaa.com ; seenithi@aol.com Web site: www.icnpaa.com The Faculty of Science of the University of Fribourg (Switzerland) invites applications for a position of a

### FULL PROFESSOR IN STATISTICS OR PROBABILITY THEORY

at the Department of Mathematics starting from March 1, 2006, or at the earliest convenience.

We are seeking candidates with an internationally recognized research record in any area of statistics or probability theory and proven ability to direct research of high quality. Duties of the new professor include teaching of mathematics at undergraduate and graduate level, in particular in statistics and probability theory, and the management of the statistical consulting service. He or she will also have to teach in French or German (if necessary after a convenient time of adaptation), to become acquainted with both languages and to assume administrative duties.

Applications with curriculum vitae, a list of publications and a short outline of the current and planned research should be sent by July 31, 2005, to the Dean of the Faculty of Science, University of Fribourg, Perolles, CH-1700 Fribourg, Switzerland. Further information may be obtained from Prof. Jean-Paul Berrut (Jean-Paul.Berrut@unifr.ch) or at http://www.unifr.ch/math/OpenPosition.

The Faculty of Science encourages female candidates to apply.

The Faculty of Science of the University of Fribourg (Switzerland) invites applications for a position of a

### FULL PROFESSOR IN MATHEMATICS

at the Department of Mathematics starting from October 1, 2006.

We are seeking candidates with an internationally recognized research record in any area of algebra, geometric topology or mathematical physics and proven ability to direct research of high quality. The candidate should reinforce the existing research groups. Duties of the new professor include teaching of mathematics at undergraduate and graduate level, in particular in algebra and geometry. He or she will also have to teach in French or German (if necessary after a convenient time of adaptation), to become acquainted with both languages and to assume administrative duties.

Applications with curriculum vitae, a list of publications and a short outline of the current and planned research should be sent by July 31, 2005, to the Dean of the Faculty of Science, University of Fribourg, Perolles, CH-1700 Fribourg, Switzerland. Further information may be obtained from Prof. Ruth Kellerhals (Ruth.Kellerhals@unifr.ch) or at http://www.unifr.ch/math/OpenPosition.

The Faculty of Science encourages female candidates to apply.

### Recent books

edited by Ivan Netuka and Vladimir Souček (Prague)

#### E. R. Berlekamp, J. H. Conway, R.K. Guy: Winning Ways for Your Mathematical Plays, vol. 3, second edition, A.K. Peters, Natick, 2003, 461–801 pp., USD 49, ISBN 1-56881-143-8

The book under the review is the third of four parts of the second edition of the book *Winning Ways for Your Mathematical Plays*, which was successfully published a quarter of century ago. In the second edition, some new parts are added. The third volume is devoted to games that are played in Clubs, these games usually use coins or paper and pencil. In the book, a lot of games is discussed, usually with their winning strategies. More famous games like chess and go are also considered, the authors describe their rules and some facts concerning their history, however, they do not discuss their strategy. The book is full of pictures and diagrams, which makes the reading of the book quite comfortable. Although the topic belongs to recreational mathematics, all studied problems are treated very precisely. (rč)

#### *F. Bernon: Propriétés de l'intégrale de Cauchy Harish-Chandra pour certaines paires duales d'algebres de Lie, Mémoires de la SMF, no. 93, Société Mathématique de France, Paris, 2003, 136 pp., EUR 25, ISBN 2-85629-137-6*

In the book, the following problem is considered. Let W be a real vector space with a symplectic product and let Sp(W) be the corresponding Lie group. Consider a dual pair (G,G') in Sp(W) in the sense of R. Howe. There is the so called Cauchy Harish-Chandra integral, which maps test functions with compact support on the Lie algebra of G to functions defined on the set of regular elements of the Lie algebra of G'. The aim of the book is to study properties of functions in the image of the Cauchy Harish-Chandra integral. For pairs formed by general linear groups over R, C or H, it is proved that functions in the image are orbital integrals. For pairs formed by unitary groups having the same rank, it is shown that they behave locally as orbital integrals and the author proves also suitable jump relations. (vs)

#### J. Bewersdorf: Luck, Logic and White Lies: The Mathematics of Games, A.K. Peters, Wellesley, 2004, 486 pp., USD 49, ISBN 1-56881-210-8

The book is divided into three parts roughly corresponding to topics indicated in the title of the book. Games of chance are investigated in the first part. The influence of randomness is typical for roulette and various dice and card games. In the middle part, the second type of games is analyzed (combinatorial games, e.g., chess and go). The book ends up with strategic games. Rock-paper-scissors is the most simple and well-known game considered in this part. The author reviews mathematical methods that have been developed for different types of games according to their character. For games of chance an important mathematical tool is provided by probability theory, which can help to answer question what is a probability to win for a particular player as well as more complicated questions. Combinatorial games are investigated by means of combinatorial gametheory. Due to a large number of combinations involved, it is necessary to look for algorithms and computational procedures in order to solve specific problems. Mathematical game theory is applied to analysis of strategic games. Many other examples of more or less known games are used to explain mathematical methods considered (e.g., backgammon, Monopoly, blackjack, nim, domino, memory, mastermind, poker, checkers, Hex, Le Her, baccarat, nine men's morris, go-moku). The book is well-written and can be recommended to all readers with interest in game theory. Although a lot of mathematics is used in the text, it doesn't require a deep mathematical background. There are many references (mostly in German) helping to find more detailed information about considered topics. (zpw)

### J. Bierbrauer: Introduction to Coding Theory, Discrete Mathematics and Its Applications, Chapman & Hall/CRC, Boca Raton, 2004, 390 pp., USD 79,95, ISBN 1-58488-421-5

The book is divided into two parts. The first one, An elementary introduction to coding, can serve as a textbook for a one-semester undergraduate course of error-correcting codes. It starts with the concept of coding and presents the classical spherepacking bound. Then it deals with binary codes, general linear codes, Reed-Solomon codes, binary Golay codes and their relation to combinatorial designs. There are also chapters on Shannon entropy, universal hashing, asymptotic results and 3dimensional codes. The second part starts with subfield codes and trace codes, then it proceeds with cyclic codes. There is also a chapter on the applications of orthogonal arrays in statistics and computer science including linear shift registers and cryptography. Chapters on the geometric description of codes and additive codes contain some advanced topics like quantum codes. The final chapter explores relationships of codes to sphere packings, designs, presents also the linear-programming bound and some algebraic-geometric codes. Because of the second part, the book can be used also as supplementary reading for a graduate course. (jtu)

# *C. Birkenhage, H. Lange: Complex Abelian Varieties,* second edition, Grundlehren der mathematischen Wissenschaften, vol. 302, Springer, Berlin, 2004, 635 pp., EUR 89,95, ISBN 3-540-20488-1

Abelian varieties play an important role in several branches of mathematics, e.g., in number theory, class field theory, dynamical systems, mathematical physics, etc. Their importance for algebraic geometry lies in the fact that there are natural ways how to associate an abelian variety X with a (smooth) projective algebraic variety and how to investigate its properties by means of a study of X. In order to be able to present more advanced results, the authors restricted themselves to abelian varieties over the field C of complex numbers. The main advantage is that a line bundle on a complex torus can be described by factors of automorphy on its universal covering. After a few introductory chapters containing classical and standard results, the book proceeds with its main topics. Projective embeddings of an abelian variety, their equations and geometric properties are discussed in Chapters 7 and 10. Several moduli spaces of abelian varieties with an additional structure are constructed in Chapters 8 and 9 and some applications to the theory of algebraic curves are given in Chapters 11 and 12. (pso)





#### Call for the 14th edition of the

### Ferran Sunyer i Balaguer Prize

The prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in Mathematics, in which the applicant has made important contributions.

#### **Conditions of the Prize**

- The monograph must be original, written in English, and of at least 150 pages. The monograph must not be subject to any previous copyright agreement. In exceptional cases, manuscripts in other languages may be considered.
- > The winning monograph will be published in Birkhäuser Verlag's series "Progress in Mathematics", subject to the usual regulations concerning copyright and author's rights.
- > The prize, amounting to 12,000 euros, is provided by the Ferran Sunyer i Balaguer Foundation.
- > The prize-winner will be announced in Barcelona in April, 2006.

#### **Scientific Committee**

- > A. Córdoba (Universidad Autónoma de Madrid)
- > P. Malliavin (Université de Paris VI)
- > J. Oesterlé (Institut de Mathématiques de Jussieu)
- > O. Serra (Universitat Politècnica de Catalunya)
- > A. Weinstein (University of California at Berkeley)

### Deadline for Submission\*

December 2on, 2005

\*For further information check the Prize's web page at: http://www.crm.es/FSBPrize/ffsb.htm

#### R. H. Bisseling: Parallel Scientific Computation: A Structured Approach using BSP and MPI, Oxford University Press, Oxford, 2004, 324 pp., GBP 45, ISBN 0-19-852939-2

The book deals with implementations of parallel algorithms in scientific computing using Bulk Synchronous Parallel (BSP) approach. It is a realistic abstraction of a real parallel computer. The first chapter introduces the BSP approach to parallel programming and BSPlib as the corresponding tool. An example of inner product illustrates how to use and analyze this approach. The next three chapters are devoted to a study of three essential algorithms. They contain a study of LU decomposition of a matrix, an analysis of FFT and a study of sparse matrix-vector multiplication. The author develops then some versions of parallel algorithms, real implementations using BSP, and experiments on real systems. The book ends with bibliographic notes and exercises. There are three appendices describing auxiliary functions from the package BSPEDUPACK, a quick reference guide to BSP lib and hints how to use MPI tools for programming in BSP style. The book will be useful for students and researchers in the field of computer science, numerical analysis and high performance computing. (pmay)

K. A. Brown, K. R. Goodearl: Lectures on Algebraic Quantum Groups, Advanced Courses in Mathematics CRM Barcelona, Birkhäuser, Basel, 2002, 348 pp., EUR 39,80, ISBN 3-7643-6714-8 The book is devoted to algebraic aspects of theory of quantum groups. The attention is concentrated to two main objects - the quantized enveloping algebras  $U_q(G)$  and quantize coordinate rings  $O_q(G)$ , which are dual to each other. The first introductory part of the book contains a systematic construction of  $U_q(G)$  and  $O_q(G)$  and a description of their basic properties for the case, when G is a classical simple Lie group and q is not a root of unity. It includes also a summary of basic facts on semisimple Lie algebras and appendices describing various algebraic tools needed in the book. Then the book splits into two parts, which are to a certain extent independent. The first one is devoted to the generic case, while the other treats the cases, when q is a root of unity. In the generic case, discussion concentrates mainly on a study of prime and primitive ideals in  $O_q(G)$ , in an analogy to the case of the coordinate ring of an affine algebraic variety. In the case of roots of unity, the structure and representations of  $O_q(G)$  are described. The book offers a very nice introduction to the subject, containing many illuminating comments and remarks, a lot of exercises and it contains also a formulation of open directions for research from this algebraic point of view. (vs)

#### D. Bump: Lie Groups, Graduate Texts in Mathematics, vol. 225, Springer, New York, 2004, 451 pp., 50 fig., EUR 64,95, ISBN 0-387-21154-3

Reductive Lie groups and their representations form a very broad field. The aim of the book is to select essential topics for a year course for (good) graduate students and it offers several possibilities. The simplest one is to concentrate on compact groups and their representations. But the book also includes many other beautiful topics, which can be combined with the main line. A short introductory part explains quickly basic properties of representations of compact groups (Schur orthogonalilty, the Peter-Weyl theorem). The main topics for a course are contained in the second part (Lie groups and the

corresponding Lie algebras, universal enveloping algebra, Weyl integration formula, root systems, the Weyl group, Weyl character formula). The second part also contains a discussion of reductive complex groups and real simple Lie groups and symmetric space (including Satake diagrams, Iwasawa decomposition and a discussion of embeddings of Lie groups). The last part adds an array of very interesting other topics. The major theme is the Frobenius-Schur duality and its various applications. It includes random matrix theory, branching formulae, the Cauchy identity, Gel'fand pairs, Hecke algebras, cusp forms and cohomology of flag varieties. The book is nicely written and efficiently organized. There are many books covering basic facts of finite dimensional representations of simple Lie groups and algebras. The presented book brings a beautiful selection of a number of further important additional topics, which are worth to include into a course. It is a very important addition to existing literature on the subject. (vs)

#### *M. Cabanes, M. Enguehard: Representation Theory of Finite Reductive Groups*, *Cambridge University Press, Cambridge*, 2004, 436 pp., *GBP* 65, *ISBN* 0-521-82517-2

Finite reductive groups form, together with derived subgroups of symmetric groups and sporadic groups, the class of non-commutative finite simple groups. Their representation theory went through an enormous development during the last 30 years. The main goal of the monograph is an exposition of recent Bonnafé-Rouquier affirmative answer to one of the Broué conjectures. The Morita equivalence between blocks of finite reductive groups and blocks defined by unipotent characters motivates the second essential aim of the book - an investigation of modular aspects of representation theory by means of unipotent blocks and unipotent characters.

The book is divided into five parts. Part I includes a description of a general concept of finite BN-pairs and Hecke algebras, an auto-equivalence of the derived category of the category of modules over a group algebra and a determination of simple modules in the natural characteristic. The second part is devoted to algebraic-geometric aspects of representation theory. An exposition of Deligne-Lusztig methods is completed by the Bonnafé-Rouquier proof of the Morita equivalence between the category of blocks of finite reductive groups and the category of unipotent blocks. Part III deals with unipotent characters. The reader can find here a complete proof of the Lusztig theorem on restrictions of irreducible characters to a commutator in case of connected reductive groups. The main theme of Part IV is the author's exposition of the Dipper-James theory describing modular aspects of representation theory. The final part of the book is devoted to further development of local methods, which makes it possible, for example, to prove a version of Fong-Srinivasan theorems on defect groups and ordinary characters of unipotent blocks. The monograph ends with three very compact appendices describing basic categorical and algebraic tools used in the book (e.g., étale cohomology, derived categories, sheaves and varieties). Material of the monograph is well-arranged and developed in a logical order, which makes the book very suitable as a reference for researchers. The text contains a lot of examples and every chapter is followed by a rich collection of exercises. The monograph can hence be also used as an introduction to the representation theory of finite reductive groups for graduate students. (jž)

#### J. Carlson, S. Müller-Stach, Ch. Peters: Period Mappings and Period Domains, Cambridge Studies in Advanced Mathematics 85, Cambridge University Press, Cambridge, 2003, 446 pp., GBP 65, ISBN 0-521-81466-9

The book serves as an introduction to theory of period mappings for variation of a Hodge structure. It contains three chapters. The first part is devoted to a basic overview of the subject, starting from classical results for period integrals of elliptic curves, Hodge structure on cohomology of complex manifolds, holomorphic families of invariants related to cohomology, and it culminates in the central concept of the book illustrated by the role of monodromy for Lefschetz pencils. The second part treats some technical details, which allow to tie up loose ends of the first section. The chapter starts with the machinery of spectral sequences and goes on to Koszul complexes, normal functions and Nori's theorem as a tool to investigate algebraic cycles. The final chapter of the book turns to purely differential aspects of period domains. The main goal is an explanation of the curvature properties relevant for period maps. (pso)

#### **R. R. Colby, K. R. Fuller: Equivalence and Duality for Module Categories (with Tilting and Cotilting for Rings)**, Cambridge Tracts in Mathematics 161, Cambridge University Press, Cambridge, 2004, 152 pp., GBP 35, ISBN 0-521-83821-5

Tilting and cotilting theory appeared first in representation theory of finite dimensional algebras as a generalization of the classical Morita theory of equivalence and duality. The generalization still has the strength and beauty of the classical theory but it makes it possible to understand new (tilted, or iterated tilted) algebras that are far from being Morita equivalent to original algebras. While the focus in the finite dimensional algebra case is on finite dimensional modules, many results hold for general modules over general rings. This has lead to a development of tilting and cotilting theory for rings, in works of the authors of this monograph and of the Padova school, since the late 80's. The monograph provides a unified treatment of the theory, in particular, of its relations to equivalence and duality. The key facts are presented here, often with simplified proofs. Indeed, the authors approach tilting and cotilting modules through more general notions of a comodule and a costar module.

Chapters 2 and 3 of the monograph deal with representable equivalences. Here, modules are shown to be finitely generated, and the notions of a quasi-progenerator and a tilting module to occur naturally as particular instances of \*-modules. The main result is a general tilting theorem providing a pair of representable equivalences between large subcategories of modules. Chapters 4 and 5 deal with representable dualities. The focus is on cotilting modules: Bazzoni's proof of their pure-injectivity is presented as well as the Colpi cotilting theorem. Notions of a weak Morita duality and a generalized Morita duality are studied in detail. Working at this level of generality, the authors need interesting classes of non-artinian examples. These are provided by modules over noetherian serial rings. For the convenience of the reader, there is an appendix with preliminaries on these rings, and another appendix on adjoint functors and equivalence. The monograph fills in a gap in the literature by covering tilting and cotilting theory in general setting of arbitrary modules over arbitrary associative rings. The theory has recently gained importance by its connection to finitistic dimension conjectures. There is no doubt that the monograph will become a basic reference in this active area of abstract algebra. (jtrl)

#### J. Cuntz, G. Skandalis, B. Tsygan: Cyclic Homology in Non-Commutative Geometry, Encyclopedia of Mathematical Sciences, vol. 121, Springer, Berlin, 2004,137 pp., EUR 84,95, ISBN 3-540-40469-4

Cyclic homology was introduced independently by A. Connes (whose motivation comes from *K*-homology and index pairing with *K*-theory) and by B. Tsygan (who was motivated by algebraic *K*-theory and Lie algebra cohomology). The book contains three contributions. The first one (by J. Cuntz) discusses basic aspects of cyclic theory with emphasis on closely related topological *K*-theory, bivariant *K*-theory, and locally convex and *m*-algebras. The second contribution (by B. Tsygan) studies various algebraic structures on Hochschild and cyclic cochains and homologies. The third contribution, written by G. Skandalis, explains some aspects of cyclic cohomology related to operator theoretic index formula (e.g., the transversal signature operator) and diffeomorphism invariant pseudodifferential calculus. (pso)

#### S. D. Cutkosky: Resolution of Singularities, Graduate Studies in Mathematics, vol. 63, American Mathematical Society, Providence, 2004, 186 pp., USD 39, ISBN 0-8218-3555-6

An algebraic variety X over an algebraically closed field Kneed not be a smooth manifold. Its set of singular points is a proper closed subset of X. The famous result of Hironaka from 60's says that there is a resolution of singularities of X, if K is a field of characteristic zero. It means that there exist a non-singular variety Y and a projective morphism  $\pi$  of Y on X, which is an isomorphism away from the singular locus of X. The existence of a resolution of singularities has a number of important applications in other fields of mathematics and mathematical physics. The presented book contains a discussion of resolution of singularities in various cases. The first six chapters of the book contain the proof of the Hironaka desingularization theorem based on canonical resolutions. The last three chapters cover additional topics (including resolutions of singularities for surfaces in positive characteristic and resolutions of surface singularities through local uniformization of valuations). The reader is assumed to know basic facts from algebraic geometry (schemes) and commutative algebra. (vs)

#### A. Del Centina, A. Fiocca: The Archive of Guglielmo Libri From Its Dispersal to the Collections at the Biblioteca Moreniana, Cultura e Memoria 32, Leo S. Olschki, Firenze, 2004, 407 pp., EUR 42, ISBN 88-222-5394-9

This excellent book written in Italian and English is devoted to the description of the Archive of Guglielmo Libri, which is deposited in the Biblioteca Moreniana in Firenze. The first part written by Andrea del Centina and Alessandra Fiocca is divided into four chapters. The book starts with a short biography of Guglielmo Libri (1802–1869), who was a mathematician, historian, patriot, journalist, proponent of science, collector and seller of antique books and manuscripts. There is a description of the complicated history of Libri's archive, which was one of the greatest and best stocked private archive in Europe and which later dispersed through series of auctions in London between 1859 and 1864. His archive and library included not only historical family documents but also hundreds of letters from scientists, historians, writers, politicians, mathematical manuscripts as well as many documents of great scientific interest. The third chapter gives further information about relationships between Libri and contemporary Italian mathematicians (for example Manzoni, Gherardi, Boncompagni) as well as European mathematicians, Libri's part in the buying and later sale of the volume containing the so called "Mathematical challenge papers" (letters exchanged between Ferrari and Tartaglia), the discovery of Fermat's and Leonardo da Vinci's manuscripts and the circumstances surrounding their sale. The fourth chapter contains the list of some letters (e.g., letters from Cauchy, Gauss, Betti, etc.) that were not been studied in detail up to now.

The second part (written by G. Adini and M. L. Tanganelli, in Italian) contains the catalogue of the so called fund "Carte Libri", which is in the Biblioteca Moreniana in Firenze. There is a list of more than four hundred letters, two volumes of very rare manuscripts, many books and papers. Two alphabetical indexes are included at the end. This book will be very interesting and helpful for the historians of mathematics who are interested in the development of mathematics in the first half of the 19th century. (mnem)

### V. Drensky, E. Formanek: Polynomial Identity Rings, Advanced Courses in Mathematics CRM Barcelona, Birkhäuser, Basel, 2004, 200 pp., EUR 29,96, ISBN 3-7643-7126-9

The book consists of lecture notes for the advanced course on polynomial identity (PI-) rings held in CRM Barcelona in July 2003. PI-rings form a large class, which includes all finite dimensional algebras, all commutative rings, and the Grassmann algebra. Methods used in PI-ring theory are mainly combinatorial (a study of the ideal of polynomial identities satisfied by a ring) and structural (a study of ring theoretic properties of rings satisfying a polynomial identity). The combinatorial methods are presented in part A of the book, written by V. Drensky. Here, Razmyslov's construction of central polynomials for matrices is explained as well as the Nagata-Higman theorem on nilpotency of nil algebras of bounded index, and the Shirshov and the Regev theorems. Part B, written by E. Formanek, describes fundamental theorems of Kaplansky and Posner on the structure of primitive and prime PI-rings, respectively, and the Artin theorem on Azumaya algebras. These (and many other) classical results are presented clearly and in detail. The book also contains several open problems and comments on recent results (for example, negative solution of the Specht problem, growth of PI-algebras, and center of the generic division ring). The book will definitely be useful for anyone interested in PIring theory, and more generally, in combinatorial and structural aspects of contemporary associative algebra. (jtrl)

### *P. Duren: Harmonic Mappings in the Plane*, *Cambridge Tracts in Mathematics 156*, *Cambridge University Press, Cambridge*, 2004, 212 pp., GBP 40, ISBN 0-521-64121-7

A complex valued function of a complex variable is called a harmonic function if it is a solution of the Laplace equation componentwise. Harmonic mappings are then univalent harmonic functions. Any harmonic function can be locally decomposed as a sum of an analytic function and a co-analytic function. If it is a harmonic mapping, then one of those parts is strictly majorized by the other one. If, say, the analytic part prevails, then the mapping is sense preserving. In many aspects, the theory reminds theory of conformal mappings, but the class of harmonic mappings is much less stable. For example, an inverse of a harmonic mapping typically fails to be harmonic. The theory of harmonic mappings, besides of its own interest, has many applications (e.g., to theory of minimal surfaces). After several introductory results, the Radó-Kneser-Choquet theorem is presented. This shows that any homeomorphism of the unit circle onto a boundary of a convex domain can be extended to a harmonic mapping of the full disc onto the closure of the domain. Of course, the extension is nothing else than the solution of the Dirichlet problem but the main point of the theorem is to show that the solution is univalent. The shear construction, which leads to interesting examples of explicit harmonic mappings, is described and applied. Another class of explicit harmonic mappings with dilatation of type of a Blaschke product is used to map the disc onto a convex polygon. The harmonic Koebe function is a very interesting mapping, which is extremal for many problems.

A part of the book is devoted to a study of analogues of the Riemann mapping theorem. The situation is much more complicated here than in the conformal case. It is also interesting to observe what is known for multiply connected domains. Estimates in Hardy spaces are represented by a few results. Many nice results relating coefficients of the Taylor expansion of the analytic and coanalytic part with the image of the mapping are formulated. The last part of the book shows the way how the theory can be applied to minimal surface problems. The Weierstrass-Enneper representation of minimal surfaces is explained and minimal graphs are studied. The theory of harmonic mappings is applied to curvature estimates of minimal surfaces. The roots of the theory of harmonic mappings can be considered as classical, its development is fluent and interesting problems still wait for their solution. The book of Peter Duren is the first comprehensive treatment of the topic. Any friend of complex analysis will admire the beauty of this extension of the theory so nicely presented in the volume. (jama)

#### A. Facchini, E. Houston, L. Salce, Eds.: Rings, Modules, Algebras, and Abelian Groups, A Dekker Series of Lecture Notes in Pure and Applied Mathematics, vol. 236, Marcel Dekker, New York, 2004, 497 pp., USD 185, ISBN 0-8247-4807-7

These are proceedings of the "Algebra Conference – Venezia 2002", held at Venice International University in June 2002. The volume contains recent results in six main areas of contemporary algebra presented at the conference: abelian groups, algebras and their representations, commutative rings, module theory, ring theory, and topological algebraic structures. The volume consists of 38 research papers, whose authors and coauthors include some leading experts in the area (including L. Fuchs, R. Gilmer, and S. Shelah). Since the commutative ring section of the conference was dedicated to the 30-year anniversary of publication of Gilmer's "Multiplicative Ideal Theory", the volume also contains a 27-page survey by Gilmer of main developments in commutative ring theory in past decades. The book is indispensable for anyone interested in current trends and methods of ring and module theory. (jtrl)

#### L. H. Harper: Global Methods for Combinatorial Isoperimetric Problems, Cambridge Studies in Advanced Mathematics 90, Cambridge University Press, Cambridge, 2004, 246 pp., GBP 40, ISBN 0-521-83268-3

The main theme of the book is the edge-isoperimetric problem (EIP), which means the problem to find a set of vertices of a given size and with minimum number of edges leaving the set in a given graph. The first chapter presents an elementary (though not easy) solution of EIP for hypercube and an application to the wirelength problem. The next chapter describes a reduction to the problem of finding a minimal path in a certain network. This brings us to global methods mentioned in the title. A morphism between networks (a pathmorphism) is defined and shown to preserve the minimum path. In the next chapter, two basic constructions of a pathmorphism (stabilization and compression) are presented. This uses a symmetry or the product decomposition of the graph and in many instances leads to a tremendous simplification of the problem. In particular, the EIP for hypercube becomes trivial. The fourth chapter repeats the whole process for vertex-isoperimetric problem, elements of the Coxeter theory of groups are presented and used to facilitate stabilization in the fifth chapter.

In the second half, the theory is extended in various ways. Two chapters deal with hypergraphs and with infinite graphs. Two chapters are devoted to the maximum weight ideal problem. This leads to the Ahlswede-Cai theorem and to the Bezrukov-Das-Elsässer theorem (EIP for powers of the Petersen graph). In the last chapter, a continuous approximation of EIP is described, in particular, a theorem by Bollobás and Leader is presented. Besides presentation of the theory, the book contains well-chosen exercises to help reader's comprehension and insightful comments including author's biographical remarks on developing this area of combinatorics. No prior knowledge is assumed. The book may be used for a semester graduate course. It is an illustrative and innovative application of the notion of a morphism to concrete combinatorial problems and as such, it may be recommended to a general audience. It is a real pleasure to read the book, because it offers to the reader not only a description of the theory but also an explanation how theory was developed. (ršam)

### **D.** Huybrechts: Complex Geometry: An Introduction, Universitext, Springer, Berlin, 2005, 309 pp., EUR 49,95, ISBN 3-540-21290-6

The book is based on a year course on complex geometry and its interaction with Riemannian geometry. It prepares a basic ground for a study of complex geometry as well as for understanding ideas coming recently from string theory. The book starts with a summary of facts from several complex variables (properties of holomorphic functions, properties of analytic sets, including the Nullstellensatz), algebraic preliminaries on the Grassmann algebra (needed in a study of Kähler manifolds) and properties of the Dolbeault complex. The second chapter treats complex manifolds and holomorphic vector bundles (including a discussion of divisors and global sections of holomorphic line bundles, properties of the complex projective space and its use for a complex surgery and the Newlander-Nierenberg theorem). Kähler manifolds are studied in the third chapter (including the Hodge theory and a discussion of the Hodge conjecture). In the fourth chapter, the author describes basic tools of complex analysis on complex manifolds (connections and their curvature, Chern classes). Central results in complex algebraic geometry (the Hirzebruch-Riemann-Roch theorem, the Kodaira vanishing and embedding theorems) are contained in the fifth chapter. The last chapter introduces local aspects of classification of complex structures on a given smooth manifold and the deformation theory of complex manifolds. The case of Calabi-Yau manifolds is treated using the language of differential algebras. The book is a very good introduction to the subject and can be very useful both for mathematicians and mathematical physicists. (vs)

#### **D. Husemöller: Elliptic Curves,** second edition, Graduate Texts in Mathematics, vol. 111, Springer, New York, 2004, xxi+487 pp., 42 fig., EUR 84,95, ISBN 0-387-95490-2

The book covers standard material of the theory of elliptic curves. In first six chapters, the Mordell theorem on the finite generation of rational points on elliptic curves defined over rational numbers is proved by elementary methods. This part grew out of Tate's 1961 Haverford Philips Lectures. The next part, consisting of Chapters 7, 8, surveys Galois theory and then recasts arguments used in the proof of the Mordell theorem into the context of Galois representations and descent theory. Remaining series of sections contains an introduction to scheme theoretic properties of classifying spaces for families of elliptic curves. The topics include *l*-adic representations, *L*functions, Birch and Schwinnerton-Dyer conjecture, etc. The book ends with three appendices containing applications of Calabi-Yau manifolds in string theory, applications of elliptic curves in cryptography and in a study of spectra of topological modular forms. The book contains a number of misprints (e.g., on page 180, the integral formula for the  $\Gamma$ -function should contain  $exp(-t^2)$  instead of exp(-1), or on page 190, there should be  $\pi$  in exponentials instead of *n*'s). (pso)

### *H. Iwaniec, E. Kowalski: Analytic Number Theory*, AMS Colloquium Publications, vol. 53, American Mathematical Society, Providence, 2004, 615 pp., USD 99, ISBN 0-8218-3633-1

The book under review shows the reader a variety of techniques and tools used in analytic number theory. It covers a broad spectrum of topics starting with results on arithmetic functions and elementary theory of prime numbers, through classical results of analytic number theory on *L*-functions, primes in arithmetical progression or circle method to more modern parts as sieve methods, Kloosterman sums, sums over finite fields or automorphic forms only to mention some other theme. The book is written in a very lively and nicely readable style, and requires only standard prerequisites from real and complex calculus or theory of Fourier series. It is intended to graduate students but it can be very useful for everyone who is interested in many facets of methods of analytic number theory. In particular, the book contains a very well chosen and balanced material. (špor)

### **Y. Katznelson: An Introduction to Harmonic Analysis**, third edition, Cambridge Mathematical Library, Cambridge University Press, Cambridge, 2004, 314 pp., GBP 50, ISBN 0-521-54359-2

This is the third edition of the famous book (firstly published in 1968 by John Wiley & Sons), which earned the author the 2002 Leroy P. Steele prize for mathematical exposition. During the last thirty five years, the textbook has been used by generations of graduate students as a friendly introduction to central ideas of harmonic analysis. These are demonstrated on classical problems in theory of Fourier series on circle (Chapters I–V) and the Fourier transform of functions, measures and classes of tempered distributions on line (Chapter VI). The attention is per-

manently paid to preparation for generalizations to harmonic analysis on locally compact commutative groups and commutative Banach algebras, which are briefly investigated in Chapters VII and VIII. Spectrum of  $L^{\infty}$ -functions is described in Chapter VI and spectral synthesis in regular Banach algebras later on in Chapter VIII. The author uses both real and complex methods simultaneously for studying either Hardy spaces in Chapter III, or interpolation of operators in Chapter IV. Typical features of author's style are indications how one dimensional methods and results can be used in other parts of analysis, e.g., to get a spectral theorem for unitary operators or to characterize function spaces (e.g., Sobolev or Besov spaces) by trigonometric approximations. For the third edition the author added some facts, which are closely related to the subject of the book. Since harmonic analysis belongs to the core of analysis, this carefully written book can be highly recommended to anybody who is interested in analysis. The reading of it has been and will be a pleasure both for students and experts. (jmil)

#### T. W. Körner: A Companion to Analysis. A second First and First Second Course in Analysis, Graduate Studies in Mathematics, vol. 62, American Mathematical Society, Providence, 2004, 590 pp., USD 79, ISBN 0-8218-3447-9

This interesting book is intended "for those students who might find rigorous analysis a treat". They should read it after the first (undergraduate) course or before the second (graduate) course in analysis. It is assumed that the reader has a basic knowledge of linear algebra and has an experience with use and manipulation of limits. The book is not a usual systematic textbook. The author explains mainly the hardest problems, which must be resolved in order to obtain a rigorous development of the calculus and easier facts are not mentioned or are left to the reader as exercises. The author starts with a discussion of real numbers, he emphasizes how important is completeness of real line. Almost all standard topics of differential calculus of one and more variables are discussed then. Theory of one-dimensional Riemann integral is described in detail. Improper integral, integral of two variables and the Riemann-Stieltjes integral are briefly discussed. Basic theory of complete metric spaces with applications to differential equations is also presented. Moreover, some more special advanced topics are discussed in an interesting elementary way. Among them are: Shannon's theorem, geodesics, first steps in calculus of variations, Green's function for ordinary differential equations of second order, and the idea of a constructive analysis. Each chapter contains a number of exercises directly related to the exposition. Further 345 exercises (whose solutions are electronically available) are contained in the appendix, many of them lead the reader through standard pieces of theory. The book is written in a very personal style and contains many witty comments, quotations and historical remarks. The exposition is very precise, but it is interesting and enthusiastic. The book can be warmly recommended to gifted and hardworking students and can be also useful for teachers of analysis. (lzaj)

#### S. G. Krantz: A Handbook of Real Analysis. With Applications to Differential Equations and Fourier Analysis, Springer, Berlin, 2004, 201 pp., EUR 49,95, ISBN 0-8176-4329-X

The book is a carefully written handbook of real analysis, which will be very helpful for engineers, physicists and anybody who

wants to use mathematics in applications. It contains description of many important results with illustrative examples. Starting with basics of real analysis, it explains topology, continuity, integration, up to uniform convergence, the Weierstrass theorem and similar topics. Special attention is paid to applications in differential equations and in Fourier analysis. On the other hand, deeper parts of theory are omitted. The last 25 pages contain a glossary of terms from real analysis, detailed list of notations, a guide to literature, index and bibliography. For a student of mathematics, it could serve as a survey of results, which he should master during undergraduate studies. (jive)

#### A. C. Krinik, R. J. Swift: Stochastic Processes and Functional Analysis. A Volume of Recent Advances in Honor of M.M. Rao, A Dekker Series of Lecture Notes in Pure and Applied Mathematics, vol. 238, Marcel Dekker, New York, 2004, 493 pp., USD 185, ISBN 0-8247-5404-2

An AMS special meeting in honour of a distinguished probabilist M. M. Rao was held in 2002 and the present volume contains most of talks given at the meeting. More than 20 original papers reflect Rao's broad scientific interests in probability, stochastic processes, Banach space theory, measure theory and (stochastic) differential equations. His introductory paper Stochastic analysis and function spaces with a broad bibliography serves and will serve as an excellent survey of recent research. Contributed papers discus following topics: Sinkhorn balancing to counting problem (Beichl, Sullivan), nonlinear filtering of an equation with Ornstein-Uhlenbeck noise (Bhatt, Rajput, Xiong), hyperfunctionals and generalized distributions (Burgin), process measures and their stochastic integrals (Dincunuleanu), invariant sets for nonlinear operators (G. Goldstein, J. A. Goldstein), immigration-emigration catastrophe model (Green), approximating scale mixtures (Hamdan, Nolan), cyclostationary arrays (Hurd, Koski), operators and pseudoergodicity in information channels (Kakihara), birth-deaths processes (Krinik, Mortensen, Rubino), integrated Gaussian processes (Lukič), prediction for multidimensional harmonizable processes (Mehlman), double-level averaging on a stratified space (O'Bryant), optimal asset allocation with stable distributed returns (Rachev, Ortobelli, Schwartz), computations for nosquare constants of Orlicz spaces (Ren), Asymptotically stationary processes (Screiber), Superlinearity and Sobolev spaces (Shapiro), doubly stochastic operators, Birkhoff's problem no. 111 (King, Shiflet), harmonizable isotropic random fields (Swift), geographically-uniform coevolution (Switkes, Moody) and time delay oscillators and delay couplings (Wirkus). The volume is completed with a biography and bibliography of M. M. Rao, a remarkable collection of personal reminiscences (written by his former students) adds a human dimension to this fine book. (jste)

# A. Lascoux: Symmetric Functions and Combinatorial Operators on Polynomials, CBMS Regional Conference Series in Mathematics, no. 99, American Mathematical Society, Providence, 2003, 268 pp., USD 55, ISBN 0-8218-2871-1

In the first chapter, the Schur functions are defined using minors of Toeplitz matrices or using the Cauchy kernel. Multiplicative structure of the ring of symmetric functions is described by Pieri formulae. The technique of  $\lambda$ -rings, which is the main tool used in the book, is introduced in the second chapter. The

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CAMBRIDGE UNIVERSITY PRESS reader can see that it gives a possibility to deduce many different classical formulae in a unified way. Next chapters describe different applications of the theory of symmetric functions to several field of mathematics (the Euclidean division, continued fractions, division, Padé approximants, orthogonal polynomials). To apply such constructions to non-symmetric polynomials, it is necessary to replace Schur functions by Schubert polynomials. The method used here goes back to Newton, a Cauchy-type kernel can be again used to obtain Schubert polynomials. The last chapter shows (without proofs) how it is possible to define non-commutative Schur functions, the Robinson-Schensted construction and the plactic algebra are used to define products of two such Schur functions. The book contains a substantial amount of exercises at the end of individual chapters, their solutions are given in the last 60 pages of the book. The book is a nice illustration of the fact that a key problem for a description of a field of mathematics is to find the most natural and appropriate notions and tools, best suited for the given field. The book offers a huge amount of interesting material for any working mathematician. (vs)

#### O. A. Laudal, R. Piene, Eds.: The Legacy of Niels Henrik Abel. The Abel Bicentennial, Oslo, 2002, Springer, Berlin, 2004, 784 pp., 37 fig., EUR 79,95, ISBN 3-540-43826-2

The core of the book consists of contributions presented at the Abel bicentennial conference held at the University of Oslo, June 3-8, 2002 and commemorating the 200th anniversary of Niels Henrik Abel's birth. The volume does not contain all contributions of invited speakers at the conference and not all of the contributors attended the conference. The book contains the opening address of King Harald V and 25 papers devoted to various aspects of Abel's work. However, the reader can find here also papers treating topics, which can be considered as mathematics of the next generations. This includes Manin's paper on applications of non-commutative geometry in Abelian class field theory for real quadratic fields, Fulton's paper on quantum cohomology of homogeneous varieties, Kassel's contribution on Hopf-Galois extensions to non-commutative algebras from the point of view of topology, van den Bergh's paper on non-commutative crepant resolutions of singularities and Chas and Sullivan's joint paper on closed string operators in topology leading to Lie bialgebras and higher string algebras. The other contributions describe many aspects of Abel's work as well as parts of number theory, analysis, algebra and geometry having roots in Abel's work. The book contains a huge amount of information of historical and mathematical nature. It can be recommended not only to those working in fields having roots in one of Abel's versatile contributions to mathematics but also to anybody who likes to read how ideas can influence the future development. (špor)

# *P. McMullen, E. Schulte: Abstract Regular Polytopes,* Encyclopedia of Mathematics and its Applications 92, Cambridge University Press, Cambridge, 2002, 551 pp., GBP 75, ISBN 0-521-81469-0

Regular polytopes are described with all details in classical monographs by H. S. M. Coxeter. The main theme of the book – abstract regular polytopes – form a generalization of classical regular polytopes. The automorphism groups of abstract regular polytopes can be characterized as the so called string *C*-groups. After a description of basic notions, the authors discuss Coxeter groups, amalgamation of abstract regular polytopes, their realizations and regular polytopes on space-forms. Various techniques for a construction of new regular polytopes (mixing or twisting) are studied then. There is a strong relation between regular polytopes and real quadratic forms. Similar relations hold also for an important class of abstract regular polytopes (real forms are replaced by hermitian forms). The next three chapters contain a description of locally toroidal regular polytopes. The classification problem for locally toroidal regular polytopes. The book ends with a description of certain families of regular polytopes to linear groups. There is a rich bibliography containing a half thousand items. (vs)

C. Menini, F. Van Oystaeyen: Abstract Algebra: A Comprehensive Treatment, Pure and Applied Mathematics, vol. 263, Marcel Dekker, New York, 2004, 754 pp., USD 95, ISBN 0-8247-0985-3 This is a comprehensive treatment of basic parts of abstract algebra. Starting from scratch, the authors gradually present fundamentals of classical theory of groups, rings, fields and polynomials. Then they proceed to Galois theory and its applications to ruler and compass constructions. The book also contains basics on linear codes, and an introduction to representation theory of groups. The presentation is clear and detailed. Moreover, there are numerous examples and exercises of varying level of difficulty accompanying each chapter. The book ends with an interesting chapter on history of algebra. Material presented here has been chosen in such a way that it includes a complete (abstract) algebra course on the 'bachelor' level taught at European universities. It has been field-tested by its use in the Erasmus educational project in past years. The experience of the authors together with a careful presentation of material makes this book a valuable addition to modern textbooks in algebra. (jtrl)

# S. Mukai: An Introduction to Invariants and Moduli, Cambridge Studies in Advanced Mathematics 81, Cambridge University Press, Cambridge, 2003, 503 pp., GBP 65, ISBN 0-521-80906-1

The aim of the book is to provide a concise introduction to algebraic geometry and algebraic moduli theory. On the way to this goal, the author explains some of fundamental contributions of Caley, Hilbert, Nagata, Grothendieck and Mumford, keeping proofs as elementary as possible or avoiding them completely. The author works in the category of algebraic varieties instead of schemes and sheaves (regarded as functors) and uses quotients of affine algebraic varieties by general linear group GL instead of geometric invariant quotients of projective varieties by PGL. In constructing the moduli of vector bundles on an algebraic curve, the Grothendieck Quot scheme is replaced by certain explicit affine variety consisting of matrices with polynomial entries. Important analytic viewpoints represented by Hodge and Kodaira-Spencer theories are not treated in the book. (pso)

D. F. Parker: Fields, Flows and Waves: An Introduction to Continuum Models, Springer Undergraduate Mathematics Series, Springer, Berlin, 2003, 270 pp., 90 fig., EUR 24,95, ISBN 1-85233-708-7 The book is a first introduction to a mathematical description of fields, flows and waves. It is designed for undergraduate students in mathematics, mathematical physics and engineering. It shows students how many mathematical topics are useful in constructing, analyzing and interpreting phenomena in the real world, in physics and in biology. It develops concepts of flux, conservation law and boundary value problems through simple examples of heat flow, electric potentials and gravitational fields. The ideas are developed through worked examples and a range of exercises (with solutions provided). The first seven chapters contain material for an introductory lecture course. It includes the continuum description, unsteady heat flow, fields and potentials, the Laplace and the Poisson equations, motion of an elastic string, fluid flow and elastic deformations. The next chapters are on waves in fluids, solids and electromagnetism, and on biomathematics. They show how an extension of earlier ideas leads to a description and explanation of important topics in modern technology and science. (zvl)

#### W. P. Petersen, P. Arbenz: Introduction to Parallel Computing: A Practical Guide with Examples in C, Oxford Texts in Applied and Engineering Mathematics, Oxford University Press, Oxford, 2004, 278 pp., GBP 80, ISBN 0-19-851576-6, ISBN 0-19-851577-4

In the book, implementations of basic algorithms of linear algebra are studied for most popular architectures of today's computers and tools for their implementations are discussed. The first chapter contains a discussion of memory and data dependencies. The second chapter presents some applications (e.g., LAPACK, sparse iterative methods, FFT and Monte Carlo methods). Their parallel implementations are discussed. The third chapter is devoted to SIMD processors, the fourth chapter studies shared memory computers, OpenMP is described. The fifth chapter explains MIMD architectures, MPI and PETSc tools are presented. The book has seven appendices. Appendix A describes SSE instructions of Intel processors, Appendix B describes the Altivec intrinsic for Motorola/IBM/Apple G-4 chip. Appendix C shows OpenMP commands and Appendix D presents MPI commands. Communication between C and Fortran codes is shown in Appendix E. The book will be of interest to students and researchers in computer science, applied and numerical mathematics who deals with design and implementation of parallel codes. (pmay)

#### L. Rabinowitz: Elementary Probability with Applications, A

*K Peters, Wellesley, 2004, 198 pp., USD 35, ISBN 1-56881-222-1* The author presents a textbook based on his elementary probability courses for students of specializations outside mathematics. It suits well for classroom use as well as for a self-study. The key areas described in the book are basic concepts of probability (including sample spaces and random events), conditional probability and independence, random variables, their distributions and expectation values, sampling with and without replacement and some simple statistical tests. Only discrete random variables and models are introduced. Explanation is based on examples rather than a theory so that only prerequisite is knowledge of basic high school algebra. Examples are mainly oriented to practical situations (such as different problems in economy, sports, elections, drug testing, cryptology, etc.). Each chapter contains a variety of exercises and problems to solve. The chosen approach is practical and entertaining. The book is a useful tool for teachers and for anybody interested in basic ideas and applications of classical probability theory. (jzich)

#### *S. Ramanan: Global Calculus*, Graduate Studies in Mathematics, vol. 65, American Mathematical Society, Providence, 2005, 316 pp., USD 55, ISBN 0-8218-3702-8

The book presents an unusual and fresh way, how to organize a course on global analysis on manifolds for graduate or postgraduate students. The author starts with basic definitions and facts from theory of smooth manifolds and continues with a discussion of differential operators on manifolds. He introduces a notion of a connection on a vector bundle and its curvature and gives more details on complex and symplectic manifolds and Riemannian, resp. spin geometry. The last two chapters contain a discussion of elliptic operators on compact manifolds (including necessary tools, e.g., the Fourier transform, the Sobolev theory, interior regularity and symbol calculus) and corresponding vanishing theorems. An unusual feature of the book is the fact that explanation of many facts is not standard or they are discussed in a different order. The first and the main tool introduced (even before a definition of a differential manifold) is a notion of a sheaf on a topological space. Differential operators are defined as linear operators on sheaves. The cohomology with values in a sheaf is introduced then and systematically used. (vs)

# *M. Reid, A. Skorobogatov, Eds.: Number Theory and Algebraic Geometry,* London Mathematical Society Lecture Note Series 303, Cambridge University Press, Cambridge, 2004, 300 pp., GBP 35, ISBN 0-521-54518-8

The volume presents a collection of papers written in honor of Peter Swinnerton-Dyer on the occasion of his 75 birthday. The collection contains 14 papers discussing contributions of this versatile personality in number theory and algebraic geometry and also his "political commitments". The papers are written by leading personalities in the subject and the reader can find here, for instance, a survey material on delicately intertwined aspects between analytic, geometric and cohomological methods from the point of view of their role in solution of fundamental questions of theory of Diophantine equations (e.g., rational points on algebraic varieties, Shafarevich-Tate groups, Zagier conjecture, Diophantine approximation, properties of Abelian and Fano varieties, etc.). Presented papers contain a description of a variety of important results and techniques and will provide an invaluable help not only for specialists in the field but also for those who would like to find a quick and competent answer to specific questions or an orientation in this important part of contemporary mathematics. (špor)

### S. Russ: The Mathematical Works of Bernard Bolzano, Oxford University Press, Oxford, 2004, 698 pp., GBP 120, ISBN 0-19-853930-4

The book contains an English translation of nine Bolzano's most significant mathematical works from the geometry, analysis and theory of numbers. Some of them are translated to English for the first time. There is information about primary sources and German or English printed editions of Bolzano's work, which was used by the author. The main aim of the book is to present a representative selection of Bolzano's mathematical works for a broad English speaking audience. Interesting prefaces containing interpretation of the historical circumstances, mathematical sources and achievements of Bolzano's works are added to each translation. Important detailed critical commentaries on context of Bolzano's works and his mathematical achievements are included. The book ends with subject and name indices. The book will be very interesting and helpful for historians of mathematics who are interested in the development of mathematics in the first half of the 19th century. The translations may encourage historian and philosophers of mathematics, potential researchers and students to study the works of Bolzano and his contemporaries from many points of view (mathematical, linguistic, historical, etc.) (mnem)

#### *B. Simon: Orthogonal Polynomials on the Unit Circle. Part 1: Classical Theory*, *AMS Colloquium Publications, vol. 54, part 1, American Mathematical Society, Providence, 2005, 466 pp., USD 89, ISBN 0-8218-3446-0*

*B. Simon: Orthogonal Polynomials on the Unit Circle. Part 2: Spectral Theory, AMS Colloquium Publications, vol. 54, part 2, American Mathematical Society, Providence, 2005, 1044 pp., USD 99, ISBN 0-8218-3675-7* 

This two-volume monograph is a comprehensive treatment of theory of probability measures on unit circle, viewed from perspective of orthogonal polynomials defined by these measures. Part I discusses primarily main topics in the subject between 1920 and 1985 with addition of the CMV (Cantero, Moral and Velásquez) matrix representation. Part II deals with a presentation of the theory of orthogonal polynomials on the unit circle as a spectral theory problem analogous to spectral theory for Schrödinger operators or Jacobi matrices.

The book establishes a connection between Verblunsky coefficients (coefficients of the recurrence equation for orthogonal polynomials) and measures, an analogue of the spectral theory of one-dimensional Schrödinger operators. Among topics discussed here, the reader can find a study of asymptotics of Toeplitz determinants, limit theorems for the density of zeros of orthogonal polynomials, matrix representations for multiplications by CMV matrices, periodic Verblunsky coefficients from the point of view of meromorphic functions on hyperelliptic surfaces, and connections between theories of orthogonal polynomials on the unit circle and on the real line. Summarizing, the book is intended as a companion to basic literature on orthogonal polynomials on the unit circle. It consists of 13 chapters with remarks and historical notes and some appendices. The list of references contains more than one thousand references. The reviewer is convinced that the book will offer an inspiration for further research. It can be strongly recommended to mathematicians specializing in theory of Schrödinger operators and theory of orthogonal polynomials. (knaj)

# *H. Strade: Simple Lie Algebras over Fields of Positive Characteristic: I. Structure Theory*, *de Gruyter Expositions in Mathematics*, vol. 38, Walter de Gruyter, Berlin, 2004, 540 pp., EUR 128, ISBN 3-11-014211-2

The first volume of a two-volume monograph contains a complete review of methods and results of classification of simple Lie algebras over an algebraically closed field of characteristic p>3. A substantial classification scheme in the remaining characteristics p=2, 3 is also presented. Indispensable tools for classification scheme in this case, mainly a *p*-envelope and absolute toral rank, special derivations and their Witt algebra, are discussed in first three chapters. In the fourth chapter, simple algebras of classical, Cartan and Melikian type are introduced. It is shown that the latter two algebras carry a distinguished natural filtration. The reader can find here also a list of all presently known simple Lie algebras in characteristic 3. The fifth chapter contains, employing cohomology theory, various recognition theorems based on an observation that a graded Lie algebra is determined by its non-positive part. In the remaining chapters, a complete solution of the isomorphism problem of classical, Cartan and Melikian algebras is given. Also, derivation algebras and automorphism groups of the latter two algebras are determined. (pso)

#### *G. Walschap: Metric Structures in Differential Geometry*, *Graduate Texts in Mathematics, vol. 224, Springer, New York,* 2004, 226 pp., EUR 69,95, ISBN 0-387-20430-X

This text should be an elementary introduction to differential geometry. It consists of six Chapters. In Chapter 1 ("Differentiable manifolds") the author treats, besides the standard concepts and results about manifolds, multilinear algebra and tensor fields as well as the de Rham cohomology and the Stokes theorem. Chapter 2 is devoted to fiber bundles and it is culminating by fundamental theorems about Grassmannians as universal bundles. Chapter 3 deals with homotopy groups and bundles over spheres. Chapter 4 is devoted to theory of connections and curvature on manifolds and principal vector bundles. Chapter 5 has the title "Metric structures". It deals with Riemannian connection, Riemannian curvature and related curvatures, isometric immersions and Riemannian submersions, variational calculus connected with minimizing properties of geodesics, theorems by Hadamard-Cartan and Bonnet-Myers, and finally, with actions of compact Lie groups on Riemannian manifolds. The most advanced chapter is Chapter 6, which discusses characteristic classes. It involves all most fundamental concepts and results, which one should expect from an introductory text. The style is rather concise and many facts are shifted to 165 nontrivial exercises. The book is very well written and can be recommended to those who want to learn the topic quickly and actively. I have only three comments: i) The title of the book is a bit misleading. One would expect under this title something like the reference [16], i.e., the direction developed by Gromov and his collaborators. ii) The (subject) index is not enough complete. iii) The bibliography (which is just a list of text-books about differential geometry) is sometimes not written carefully. For example, the book [17] by Helgason was published in 1978, not in 1962. The reference [21] to Kobayashi and Nomizu is referring only to the first volume, not to the second volume which appeared in 1969. (ok)

#### W. J. Wickless: A First Graduate Course in Abstract Algebra, Pure and Applied Mathematics, vol. 266, Marcel Dekker, New York, 2004, 234 pp., USD 85, ISBN 0-8247-5627-4

The title of the book is self-explanatory. The first five chapters cover the basics, groups, rings, modules, vector spaces, and fields (including the Galois theory). The remaining three chapters are independent and can be treated as optional in a one-year course. They cover non-commutative rings, group extensions, and abelian groups. There is no introductory chapter reviewing what-everyone-reading-this-book-should-know. The corresponding material is placed where it is needed. There are many exercises throughout the text and there are problem sets at the end of each chapter. A novelty is an early introduction of the tensor product and the concept of projective modules. These concepts are used to simplify some standard proofs. The book can serve well as a basis for a one year graduate course. (jtu)

*R. Wienands, W. Joppich: Practical Fourier Analysis for Multigrid Methods, Numerical Insights, vol. 4, Chapman & Hall/ CRC, Boca Raton, 2004, xx+217 pp., USD 79,95, ISBN 1-58488-*492-4

The book uses a detailed and systematic description of local Fourier k-grid (k=1, 2, 3) analysis for general systems of partial differential equations to provide a framework that answers questions related to the quality of convergence, whether a development will pay out, whether multigrid will work for a particular

application, and what the numerical properties are. Accompanying software confirms written statements about convergence and efficiency of algorithms and is easily adapted to new applications. The book enables an understanding of basic principles of multigrid and local Fourier analysis, and also describes theory important to those who need to delve deaper into the details of the subject. The book consists of two parts. The first part (Chapters 1-4) provides facts, which are necessary for understanding basic principles of multigrid and local Fourier analysis and for an efficient use of the accompanying software. The second part (Chapters 5-7) describes theory used in the software and is important for those readers, who want to understand details. Summarizing, the book is intended as a companion to basic multigrid literature with a focus on quantitative convergence estimates, which are crucial for development of new multigrid software. The volume will be of interest not only for those interested in theory but also to a wide range of practically oriented researchers, not limited to multigrid specialists. (knaj)

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The European Congress of Mathematics, held every four years, has established itself as a major international mathematical event. Following those in Paris, 1992, Budapest, 1996 and Barcelona, 2000, the Fourth European Congress of Mathematics took place in Stockholm,

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