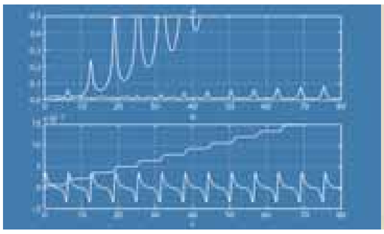
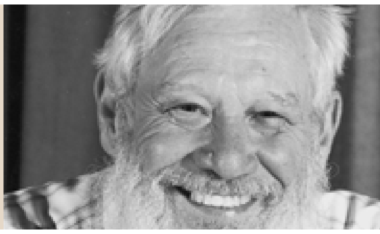
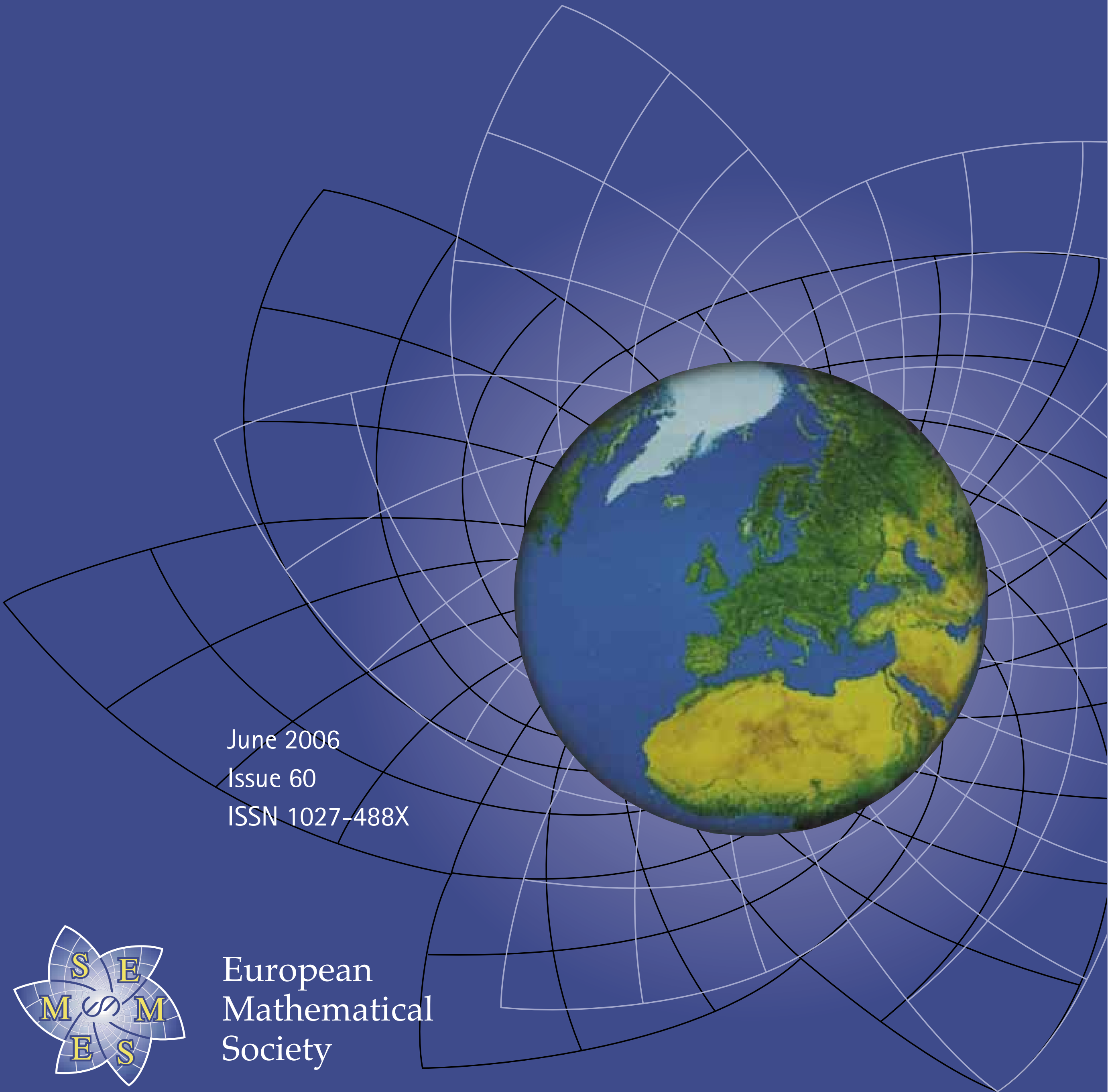
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY





June 2006
Issue 60
ISSN 1027-488X

European
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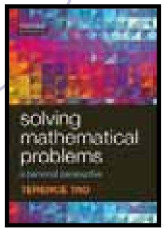
Solving Mathematical Problems



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Terence Tao

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128 pages | August 2006
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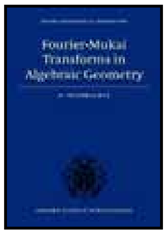


Daniel Huybrechts

This seminal text by a leading researcher is based on a course given at the Institut de Mathematiques de Jussieu. Aimed at students with a basic knowledge of algebraic geometry, the key aspect of this book is the derived category of coherent sheaves on a smooth projective variety. Full proofs are given and exercises aid the reader throughout.

Oxford Mathematical Monographs. | May 2006 | 280 pages
0-19-929686-3 |978-0-19-929686-6 | Hardback | £50.00

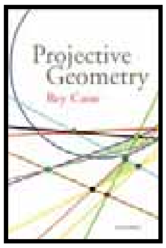
Hilbert Modular Forms and Iwasawa Theory

Haruzo Hida

The 1995 work of Wiles and Taylor-Wiles opened up a whole new technique in algebraic number theory and, a decade on, the waves caused by this incredibly important work are still being felt. This book, by a leading researcher, covers both this general area and that of Iwasawa Theory, which is currently enjoying a resurgence in popularity

Oxford Mathematical Monographs. | June 2006 | 400 pages
0-19-857102-X |978-0-19-857102-5| Hbk | £65.00

Projective Geometry



An introduction

Rey Casse

This lucid, accessible text provides an introductory guide to projective geometry, an area of mathematics concerned with the properties and invariants of geometric figures under projection. Including numerous examples and exercises, this text is ideal for year 3 and 4 mathematics undergraduates.

216 pages | August 2006
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New in Paperback...

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European Mathematical Society

Newsletter No. 60, Juni 2006

ISSN 1027-488X

Published by the
EMS Publishing House
ETH-Zentrum FLI C4
CH-8092 Zürich, Switzerland.
homepage: www.ems-ph.org

Editor: Dr. Manfred Karbe

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EMS Calendar

2006

16–18 June Joint EMS–SMAI–SMF Mathematical Weekend in Pays de Loire, Nantes (France)
www.math.sciences.univ-nantes.fr/WEM2006

30 June EMS Executive Committee Meeting, Torino (Italy)
Helge Holden: holden@math.ntnu.no

1–2 July EMS Council Meeting, Torino (Italy)
www.math.ntnu.no/ems/council06/

3–7 July *Mathematics and its Applications*: First joint meeting of EMS, SIMAI, SMF, SMAI, and UMI, Torino (Italy)
www.dm.unito.it/convegniseminari/mathsandapps

10–15 July EMS Conference at CRM Barcelona (Catalunya, Spain)
Recent developments in the arithmetic of Shimura varieties and Arakelov geometry
http://www.crm.es/svag; svag@crm.es

10–22 July EMS Summer School at the Renyi Institute in Budapest and at Lake Balaton (Hungary)
Horizon of combinatorics
http://www.renyi.hu/conferences/horizon/; ervin@renyi.hu or veve@renyi.hu

2–23 July EMS-SMI Cortona Summer School (Italy)
A geometric approach to free boundary problems
http://www.matapp.unimib.it/smi/;
dipartimento@matapp.unimib.it

1 August Deadline for submission of material for the September issue of the EMS Newsletter
Martin Raussen: raussen@math.aau.dk

22–30 August International Congress of Mathematicians, Madrid (Spain)
www.icm2006.org/

23 August Panel discussion at ICM2006 organised by EMS
Should mathematicians care about communicating to broad audiences? Theory and Practice.
http://www.icm2006.org/scientificprogram/specialactivities/#ems

9–23 September EMS Summer School at Linz (Austria)
Mathematics in molecular cell biology
http://www.ricam.oeaw.ac.at/emsschool; Vincenzo.Capasso@mat.unimi.it or christian.schmeiser@oeaw.ac.at

2007

6–12 May EMS Summer School – Séminaire Européen de Statistique, La Manga (Cartagena, Spain)
SEM STAT: Statistics for stochastic differential equations models
mathieu.kessler@upct.es or lindner@ma.tum.de

3–10 June EMS Conference at Będlewo (Poland)
Geometric analysis and nonlinear partial differential equations
B.Bojarski@impan.gov.pl or pawelst@mimuw.edu.pl

16–20 July ICIAM 2007, Zürich (Switzerland)
www.iciam07.ch/

Editorial

Mario Primicerio (Florence, Italy)

Somebody likes it… Techno

Being mathematicians, we like to clearly define the objects we are dealing with. Therefore, we feel uncomfortable when we talk about 'applied mathematics', which is much more characterized by a mental attitude (rooted in training and in personal taste) than by its specific contents.

Nevertheless, stressing the applied side of mathematical research and promoting it among scientists, opinion leaders and decision makers are among the aims and scope of the 'Applied Mathematics Committee' that has been established by the EMS. Being in charge of coordinating its activities, I will here try to present my views on a subset of applied mathematics that seems more easily definable: industrial mathematics or technomathematics. The temptation is to say that this could be defined as the mathematics that companies ask for in order to better understand, to optimize or, at least, to improve their performance. Consequently they are ready to pay for it and to finance research and education.

This definition could however be misleading, for at least two reasons:

- not all the mathematics requested by industry lies in the realm of technomathematics, which is characterized by the novelty and relevance of models (as simple as possible but as complex as necessary) and of methods (both theoretical and computational);
- alongside research aimed at pursuing a specific and concrete objective (i.e. giving an answer to a precise request), technomathematics also includes research aimed at improving the performance of a method (one might, for example, think to accelerate a numerical method). Of course we have to note that only companies with larger scientific and managerial dimensions, wider views and larger budgets are sometimes disposed to pay for this kind of industrial mathematics.

Despite the ambiguities indicated above, we can state with surety that in Europe there have been many positive experiences of cooperation between mathematicians and industry, with distinctive characteristics and traditions in each country. It is also worthwhile recalling the twenty years of experience of the ECMI (European Consortium for Mathematics in Industry), which has done remarkable work from an educational point of view by establishing ECMI diplomas at Masters level and has maintained a network of very active research groups.

Why can mathematicians be successful when they cooperate with industrial researchers? Because they master a universal language that is a prerequisite for the interdisciplinary scientific activity inherent in the complexity of problems modern industry has to deal with. They are also able to 'sense' (at the mathematical level) affinities between problems from areas that may appear to have little or nothing in common.

What EMS can do is to bridge the gap that exists between technomathematics and more theoretically oriented research. However, this can only be successful through an increase in trust, respect and genuine interest for the activities on the other side of the existing divide. Indeed, questions encountered in dealing with industrial problems may need competencies from any branch of mathematics. And it is a common experience that industry can be the source of novel theoretical developments. Applied mathematics (and the subset of it that we have called technomathematics) is not a mathematics that is "son (or daughter) of a lesser god" either from the point of view of difficulty or from that of innovative capacity! Any supercilious attitude in this sense is unjustified and culturally superficial; the real discrimination should be between good mathematics and poor mathematics.

On the other hand, it is necessary to put a caveat on mathematicians who try to 'smuggle' a paper through as applied research even when the application does not go beyond the title and the aim is just to 'sell' an earlier obtained result. This goes against the interests of the author but also of mathematics as a whole!

Any knowledge-based economy (as the European Union claims is its mission) necessarily involves mathematics as a basic factor. In the last century it was common to measure the degree of maturity of a branch of science by means of the amount of mathematics it used. Nowadays we can say that in most of the productive sectors mathematical content is a measure of how technologically advanced a European company is and thus how strong its position in facing global competition. Last, but not least, a common effort in technomathematics will accelerate the integration of the new member states of the E.U. at both the scientific and the industrial level.

In conclusion, I believe that a common effort of the whole mathematical community in Europe has to be made in order to expand mathematics in European research programmes. Too often we are forced to ask for hospitality (or rather for asylum!) in different applied areas to find a program where a research project of ours can fit. We all know that the European Framework Programs are shaped as if scientists from other disciplines know all the mathematics that can be useful (whilst the rest is simply "cultural curiosity"). And we also know that sometimes our attitudes paradoxically play in favour of the strategies that eschew mathematics.

To regain the role (and the support) for mathematics that it deserves is not an easy or a short-term task; it has to be pursued in each country and at the international level. And what the Applied Mathematics Committee of EMS can do is to try to stimulate and give an impetus to this progression.

Mario Primicerio [primicer@math.unifi.it] is Professor in Rational Mechanics at the Faculty of Sciences of the University of Florence. He was the Mayor of Florence in the period 1995–1999. Since the beginning of this year, he has been the Chair of the EMS Applied Mathematics Committee.

EC-Meeting, London, March 2006

The Executive Committee held a meeting in London on the 11th and 12th of March 2006, in the newly extended Hardy Room in De Morgan House, home of the London Mathematical Society.

The Treasurer presented a healthy financial report to the Committee. The Society‘s contracts with the EU for summer schools are very significant (financially). So far they have been extremely successful, and the Society has every reason to be proud of them, but we would be rather vulnerable if something went wrong. So the Committee decided to prepare a risk analysis for its October meeting. It was also agreed that improvements in the Newsletter, and inflation, make necessary a proposal to the Torino Council that we should raise the unit fee to 22 euros for individual members and 400 euros for corporate members. Finally, under the Treasurer‘s business, various means of increasing individual membership were briefly discussed.

The Executive Committee then went on to discuss the various events being held under the auspices of the EMS. Planning for the Mathematical Weekend in Nantes in June 2006 seems to be going well, and it was agreed that we would welcome suggestions for future Mathematical Weekends. Victor Buchstaber announced a conference in honour of Euler (born 1707) to be held in St Petersburg in August 2007.

The Standing Committees of the EMS are all very active (which has not always been the case), and the Executive Committee was pleased to receive and endorse their various reports. In particular, we discussed the article competition organized by the Committee for Raising Public Awareness. That Committee had made very careful judgments of the articles submitted, and the Executive Committee was happy to agree with its recommendation that Thomas Bruss, Tom Apostol, and Hansjörg Geiges should be the winners.

Under publishing, the Committee was pleased to note that the EMS publishing house is now becoming well established, both in terms of high quality and increased volume. It was also agreed that the Newsletter is now settled and very successful. Suggestions for future directions for Zentralblatt were coming from the coordinating committee and from the Scientific Users’ Committee (chaired by Jean-Pierre Bourguignon).

The European Research Council is potentially of very great significance, but its formation is still in the early stages and its budget is not settled. It was noted that the 7th Framework should be ready in September, and that in general, mathematicians do not apply enough for EU grants. The Committee agreed that the EMS web site should encourage more such applications.

Vasile Berinde (North University of Baia Mare, Romania) was proposed as the new Publicity Officer, to succeed David Salinger. [Since the meeting Vasile has been formally appointed, and he will attend the Council in Torino in his new role.]

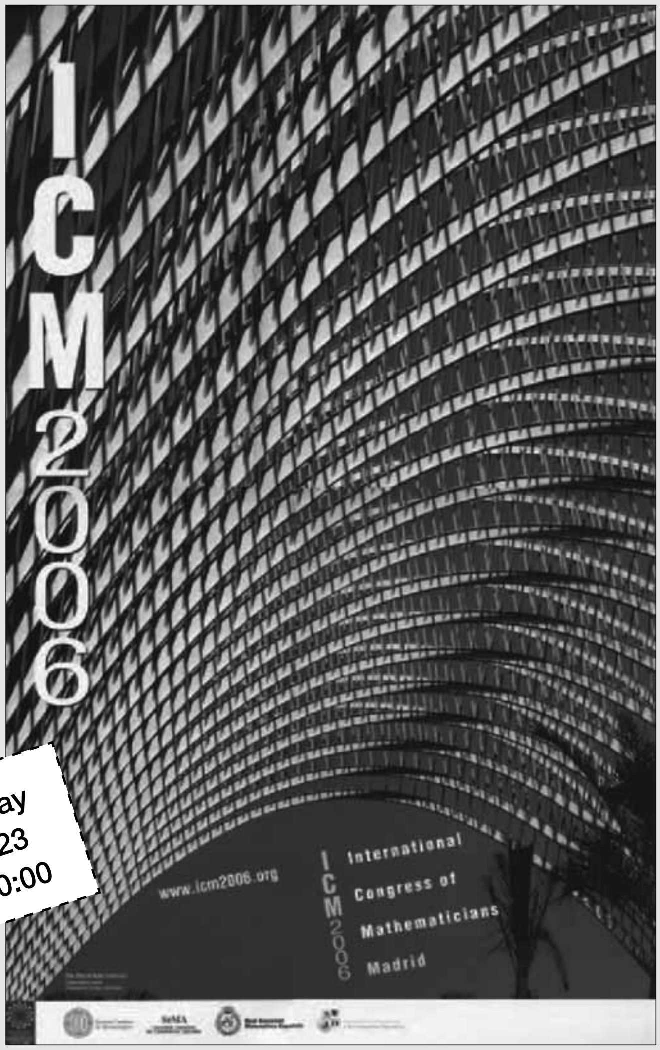
After reviewing the various preparations for the Torino Council, the President thanked the London Mathematical Society for its hospitality and closed the meeting.

Executive Committee and guests at the de Morgan House

Panel discussion organised by the EMS

Special Activity of the ICM2006

Should mathematicians care about communicating to broad audiences? Theory and Practice.



Wednesday August 23 18:00–20:00

Moderator

Jean-Pierre Bourguignon
CNRS and Institut d'Hautes Études Scientifiques, Bures sur Yvette, France

Panelists

Björn Engquist
Royal Institute of Technology, Stockholm, Sweden and University of Texas at Austin, USA

Marcus du Sautoy
Oxford University, UK

Alexei Sossinsky
Institute for Problems in Mecha- nics, Russian Academy of Sciences, and Independent University of Moscow, Russia

François Tisseyre
Film Director, France

Philippe Tondeur
University of Illinois at Urbana-Champaign, USA

In most countries, mathematics is not present at par in the media with other basic sciences. This is especially true as far as the communication of outstanding new results, their significance and perspectives of development is concerned. The main purpose is to nurture the debate on why an efficient communication about mathematics, as a thriving part of science, is needed and how it can be achieved. More information at http://www.icm2006.org/scientificprogram/specialactivities/

Results from the EMS Article Competition 2005

Vagn Lundsgaard Hansen (Chairman of the EMS committee for Raising Public Awareness of Mathematics)

As one of the ways to raise public understanding and awareness of mathematics, the European Mathematical Society occasionally organizes competitions to write articles of a mathematical theme that enhance knowledge about the nature of mathematics and what mathematicians do. The articles should be written for as wide an audience as possible but the exact target audience may vary with each competition. The competitions are organized by the EMS committee for Raising Public Awareness of Mathematics, which also functions as the jury for the competitions.

Members of the RPA committee

Ehrhard Behrends, Germany
Chris J Budd, UK
Mireille Chaleyat-Maurel (Vice chair), France
Michele Emmer, Italy
Andreas Frommer, Germany
Olga Gil-Medrano, Spain
Vagn Lundsgaard Hansen (Chair), Denmark
Osmo Pekonen, Finland.
José Francisco Rodrigues, Portugal
Marta Sanz-Solé, Spain
Robin Wilson, The Open University, UK.

Further details about the composition of the EMS committee for Raising Public Awareness of Mathematics can be found at http://www.mat.dtu.dk/people/V.L.Hansen/rpa_com.html .

Running the EMS Article Competition 2005

During the first EMS Article Competition (2003) it became apparent that it would be necessary to distinguish between articles written for a completely general audience in a daily newspaper and articles written for an educated audience in a more specialized magazine. For the EMS Article Competition 2005, the target audience for the articles was chosen to be the educated layman.

On the deadline for submissions, 1st August 2005, the RPA committee had received ten articles for consideration. Through a careful selection procedure, four articles were chosen for further evaluation to choose the prize winners. In this phase, the RPA committee put particular emphasis on the general interest and readability of an article in the light of its mathematical level. The recommendations of the RPA committee were approved by the Executive Committee of the European Mathematical Society at its meeting in London, 11th-12th March 2006.

Prizewinners of the EMS Article Competition 2005

First prize: Professor F. Thomas Bruss, Dépt. de Mathématiques, Université Libre de Bruxelles, Campus Plaine, CP 210, B-1050 Bruxelles, Belgium, for his article "Die Kunst der richtigen Entscheidung" published in the magazine *Spektrum der Wissenschaft* (Scientific American, German Ed.), Juni 2005. It has also been published in French in *Pour La Science* (Scientific American, French Ed.), Septembre 2005 and in Arabic in *Al Oloom* (Scientific American, Arabic Ed.), Novembre 2005. An English version of the article will appear in the EMS Newsletter and will also be made available at http://homepages.ulb.ac.be/~tbruss/ .

Citation: Thomas Bruss receives the first prize for an appealing and highly original article on how to base decision-making on reasoning. The article presents among others recent research of the author and is very attractively written for an audience of educated laymen. The article has already been translated into several languages.

Second prize: Professor Tom M. Apostol, Project "Mathematics", California Institute of Technology, Pasadena, CA 91125 USA, for his article "A Visual Approach to Calculus Problems" published in the magazine *Engineering & Science* (published quarterly by the California Institute of Technology), Volume LXIII, Number 3, pp. 22–31, 2000 (see http://pr.Caltech.edu/periodicals/EandS/archives/LXIII_3.html).

Citation: Tom M. Apostol receives the second prize for a wonderful piece of exposition showing how old problems from calculus can be developed with fresh eyes and may be given new dimensions using modern technology. The article especially appeals to scientists and engineers who enjoyed calculus courses and maintain a certain curiosity for mathematics.

Third prize: Professor Hansjörg Geiges, Mathematisches Institut, Universität zu Köln, Weyertal 86–90, D-50931 Köln, Germany, for his article "Christiaan Huygens and Contact Geometry" published in the Dutch magazine

Nieuw Archief voor Wiskunde (5th series), Vol. 6, pp.117–123, 2005 (see http://www.mi.uni-koeln.de/~geiges/naw05.pdf).

Citation: Hansjörg Geiges receives the third prize for an interesting article presenting a nice combination of history, culture and mathematics related to contact geometry. The article originates in the author's inaugural lecture at the University of Köln (chair in geometry) and it is especially appealing to mathematicians and physicists.

Runner up: Professor Aviezri S. Fraenkel, Faculty of Mathematics and Computer Science, Weizmann Institute of Science, 76100 Rehovot, Israel, for his article "Why are Games Exciting and Stimulating?" accepted for publication in the magazine *Math Horizons*, which is published by the Mathematical Association of America (to appear February 2007 in a special games issue). It has also been published in German (translation by Niek Neuwahl) in connection with a games exhibition that took place in Göttingen, 17 July–21 August, 2005 (see http://www.wisdom.weizmann.ac.il/~fraenkel/Papers/niek_2.pdf).

Citation: Aviezri S. Fraenkel was a very close runner up with an article on games. The article is appealing to the many laypersons interested in games, in particular to those who like to analyse the games.

Final remarks on the EMS Article Competition 2005

The RPA committee appreciated the many valuable articles we received for consideration.

In addition to the articles already mentioned, the RPA committee enjoyed an excellent article on chaotic behaviour in dynamical systems and another one on wavelets. These articles were not, however, directed towards the educated layman and required considerable mathematical maturity in the reader. There was also a very interesting article on the page rank algorithm used by Google and a nice short article on the application of prime numbers in cryptography. Some of the articles from the competition will appear in later issues of the EMS Newsletter.

It is encouraging to see the energy and inspiration with which members of the mathematical community communicate their enjoyment of mathematics. It is not an easy task to write a good article on a mathematical topic for a general audience presenting mathematical ideas in a proper way. The EMS article competitions have demonstrated that it is possible to write such articles and to write them on a wide range of mathematical subjects.

As chairman of the RPA committee it is my pleasure to congratulate the prize winners and to express my sincere thanks to all those who contributed to the success of the EMS Article Competition 2005.

Decisions in Economics and Finance

Call for Papers

On the occasion of the birth centenary of Bruno de Finetti, Decisions in Economics and Finance (DEF) is seeking for contributions (both theoretical and applied) in Decision Theory, Economics and Finance that bring some relationship to the work of de Finetti.

The DEADLINE for submission is September 30. 2006. The papers must contain original unpublished work that is not being submitted for publication elsewhere. Manuscripts submitted will be refereed according to standard procedures for DEF. The editorial process is expected to proceed very rapidly, and lead to publication within May 2007.

Manuscripts can be emailed as PDF files to:

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E-mail: holzer@units.it
WWW: http://link.springer.de/link/service/journals/10203/index.htm

Abel Prize 2006

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2006 to Lennart Carleson, Royal Institute of Technology, Sweden,

for his profound and seminal contributions to harmonic analysis and the theory of smooth dynamical systems.

In 1807, the versatile mathematician, engineer and Egyptologist Jean Baptiste Joseph Fourier made the revolutionary discovery that many phenomena, ranging from the typical profiles describing the propagation of heat through a metal bar to the vibrations of violin strings, can be viewed as sums of simple wave patterns called sines and cosines. Such summations are now called Fourier series. Harmonic analysis is the branch of mathematics that studies these series and similar objects.

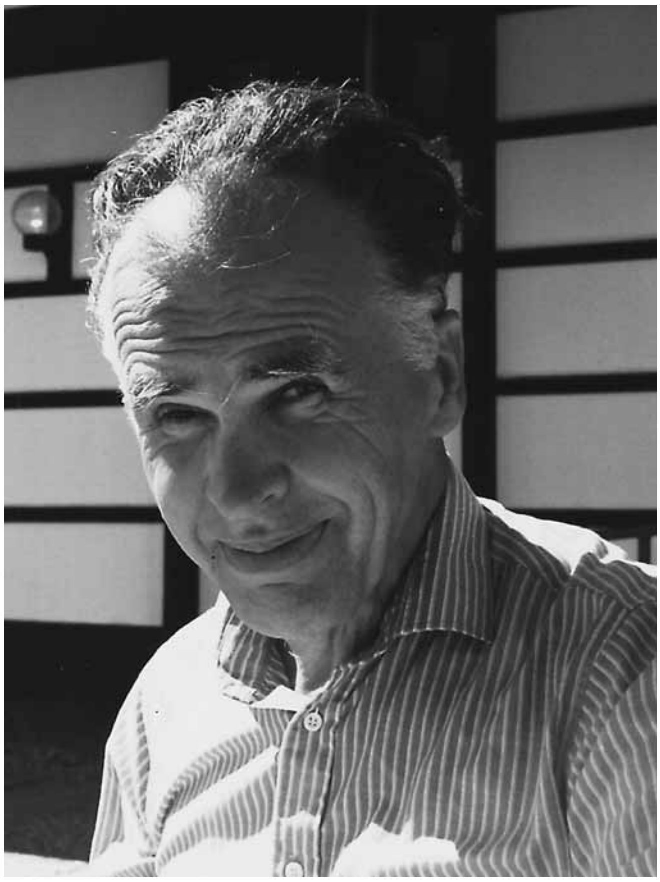
For more than 150 years after Fourier's discovery, no adequate formulation and justification was found of his claim that every function equals the sum of its Fourier series. In hindsight this loose statement should be interpreted as regarding every function for which "it is possible to draw the graph", or more precisely, every continuous function. Despite contributions by several mathematicians, the problem remained open.

In 1913 it was formalized by the Russian mathematician Lusin in the form of what became known as Lusin's conjecture. A famous negative result of Kolmogorov in 1926, together with the lack of any progress, made experts believe that it would only be a matter of time before someone constructed a continuous function for which the sum of its Fourier series failed to give the function value anywhere. In 1966, to the surprise of the mathematical community, Carleson broke the decades-long impasse by proving Lusin's conjecture that every square-integrable function, and thus in particular every continuous function, equals the sum of its Fourier series "almost everywhere".

The proof of this result is so difficult that for over thirty years it stood mostly isolated from the rest of harmonic analysis. It is only within the past decade that mathematicians have understood the general theory of operators into which this theorem fits and have started to use his powerful ideas in their own work.

Carleson has made many other fundamental contributions to harmonic analysis, complex analysis, quasiconformal mappings, and dynamical systems. Standing out among them is his solution of the famous corona problem, so called because it examines structures that become apparent "around" a disk when the disk itself is "obscured", poetically analogous to the corona of the sun seen during an eclipse. In this work he introduced what has become known as Carleson measures, now a fundamental tool of both complex and harmonic analysis.

The influence of Carleson's original work in complex and harmonic analysis does not limit itself to this. For example, the Carleson-Sjölin theorem on Fourier multipliers has become a standard tool in the study of the "Kakeya problem", the prototype of which is the "turning needle problem": how can we turn a needle 180 degrees in a plane, while sweeping as little area as possible? Although the Kakeya problem originated as a toy, the description of the volume swept in the general case turns out to contain important and deep clues about the structure of Euclidean space.



Lennart Carleson, Abel Laureate 2006

Dynamical systems are mathematical models that seek to describe the behaviour in time of large classes of phenomena, such as those observed in meteorology, financial markets, and many biological systems, from fluctuations in fish populations to epidemiology. Even the simplest dynamical systems can be mathematically surprisingly complex. With Benedicks, Carleson studied the Hénon map, a dynamical system first proposed in 1976 by the astronomer Michel Hénon, a simple system exhibiting the intricacies of weather dynamics and turbulence. This system was generally believed to have a so-called strange attractor, drawn in beautiful detail by computer graphics tools, but poorly understood mathematically. In a great tour de force, Benedicks and Carleson provided the first proof of the existence of this strange attractor in 1991; this development opened the way to a systematic study of this class of dynamical systems.

Carleson's work has forever altered our view of analysis. Not only did he prove extremely hard theorems, but the methods he introduced to prove them have turned out to be as important as the theorems themselves. His unique style is characterized by geometric insight combined with amazing control of the branching complexities of the proofs.

Carleson is always far ahead of the crowd. He concentrates on only the most difficult and deep problems. Once these are solved, he lets others invade the kingdom he has discovered, and he moves on to even wilder and more remote domains of Science.

The impact of the ideas and actions of Lennart Carleson is not restricted to his mathematical work.

He has played an important role in the popularization of mathematics in Sweden. He wrote the popular book *Matematik för vår tid* ("Mathematics for our time"), and he has always been interested in mathematical education.

Carleson has had 26 Ph. D. students, many of whom became professors at universities in Sweden and elsewhere. As Director of the Mittag-Leffler Institute near Stockholm from 1968 to 1984, he realized the original vision of Mittag-Leffler, building the Institute as we now know it, a foremost international research centre in mathematics. He also placed special emphasis on the role of the Institute in the mentoring of young mathematicians, a tradition that continues to this day.

As president from 1978 to 1982 of the International Mathematical Union (IMU), Carleson worked hard to have the People's Republic of China represented. He also convinced the IMU to take the contributions of computer science to mathematics into account and was instrumental in the creation of the Nevanlinna Prize, rewarding young theoretical computer scientists. As president of the Scientific Committee of the fourth European Congress in Mathematics, in 2004, he started the initiative of the Science Lectures, where distinguished scientists discuss the most relevant aspects of mathematics to science and technology.

Lennart Carleson is an outstanding scientist with a broad vision of mathematics and its role in the world.

The 2006 Ferran Sunyer I Balaguer Prize

On 24th April 2006, the Fundació Ferran Sunyer i Balaguer awarded the fourteenth Ferran Sunyer i Balaguer Prize to Xiaonan Ma of the Centre de Mathématiques Laurent Schwartz, École Polytechnique (France), and George Marinescu of the Institut für Analysis und matematische Physik, J.W.Goethe Universität (Frankfurt, Germany), for the monograph "Holomorphic Morse inequalities and Bergman kernels".

The monograph gives a self-contained and unified approach to holomorphic Morse inequalities and the asymptotic expansion of the Bergman kernel on manifolds using the heat kernel, from the viewpoint of local index theory. The book presents various applications and illustrates several active areas of research in complex, Kähler and symplectic geometry.

A large number of applications are included, e.g. an analytic proof of the Kodaira embedding theorem, the solutions of the Grauert-Riemenschneider and Shiffman conjectures, the compactification of complete Kähler manifolds of pinched negative curvature, Berezin-Toeplitz quantization, the weak Lefschetz theorems, and asymptotics of the Ray-Singer analytic torsion.

About the prize

Ferran Sunyer i Balaguer was a self-taught Catalan mathematician who was very active in classical mathematical analysis. Since April 1993, the Ferran Sunyer i Balaguer Foundation has awarded this prize to authors of mathematical monographs of an expository nature. The prize amounts to 12000 euros and the publication of the monograph by Birkhäuser Verlag.

For more on the Ferran Sunyer i Balaguer Foundation and a list of previous winners, see http://ffsb.iec.cat/ .

ICMI Awards – The 2005 Felix Klein and Hans Freudenthal Medals

The International Commission on Mathematical Instruction (ICMI) of the International Mathematical Union has announced recipients of its 2005 awards.

The Felix Klein Medal, named for the first president of ICMI (1908-1920), honors a lifetime achievement. The Felix Klein Medal for 2005 is awarded to **Ubiratan D'Ambrosio**, Emeritus Professor at UNICAMP, in Brasil. This distinction acknowledges the role Ubiratan D'Ambrosio has played in the development of mathematics education as a field of research and development throughout the world, above all in Latin America. It also recognises Ubiratan D'Ambrosio's pioneering role in the development of research perspectives which are sensitive to the characteristics of social, cultural, and historical contexts in which the teaching and learning of mathematics take place, as well as his insistence on providing quality mathematics education to all, not just to a privileged segment of society.

The Hans Freudenthal Medal, named for the eight president of ICMI (1967–1970), recognizes a major cumulative program of research. The Hans Freudenthal Medal for 2005 is awarded to **Paul Cobb**, Professor at Vanderbilt University, in the US. This distinction acknowledges his outstanding contribution to mathematics education: a rare combination of theoretical developments, empirical research and practical applications, which has had a major influence on the mathematics education community and beyond.

Ubiratan d'Ambrosio

Paul Cobb

The ICMI Awards represent the judgement of an (anonymous) jury of distinguished scholars of international stature. The jury for the 2005 awards was chaired by Professor Michèle Artigue, of the Université de Paris 7. Presentation of the medals, and invited addresses of the medallists, will occur at ICME-11 in Monterrey, México, July 2008.

Citations of the work of the medallists can be found at http://www.mathunion.org/ICMI/Awards/2005/

Statistics on Women in Mathematics

Catherine Hobbs and Esmyr Koomen (Oxford Brookes University, UK)

Introduction

Data on women in mathematical research in Europe was last collected in 1993 by the Women in Mathematics Committee of the European Mathematical Society. This illustrated a perhaps surprising distribution of the proportions of women in mathematical research across the EU, with considerable differences between different regions. In particular, 'southern' countries, such as Italy, Portugal and Spain, had a much higher proportion of women in mathematics than 'northern' countries such as Germany and Sweden.

In 2005, funding from the UK Royal Society Athena Awards enabled us to repeat this data collection exercise. We used a variety of sources for data collection, including European Women in Mathematics regional co-ordinators, national statistics agencies and the internet. Our aim was to collect data to compare with the 1993 study, but we also collected further data from countries not included in the original study and tried to obtain more detailed information about career grading than had been possible in 1993. In fact it is hard to obtain truly comparative data since countries in Europe often have very different academic career grades and also the distinction between research mathematics, educational mathematics and research in related areas such as physics and statistics makes it hard to distinguish just the women in mathematics research. However, we have made as good an attempt as possible and present our data and analysis here.

Table 1: Comparative data, 1993 and 2005

	1993 data						2005 data					
	Mathematicians			Full Professors			Mathematicians			Full Professors		
Country	Total	F	% F	Total	F	% F	Total	F	% F	Total	F	% F
North												
Denmark	121	4	3.3%	19	1	5.3%	237	23	9.7%	53	2	3.8%
Finland	127	3	2.4%	34	1	2.9%	440	64	14.5%	71	2	2.8%
Iceland	10	0	0.0%	4	0	0.0%	13	0	0.0%	7	0	0.0%
Norway	103	9	8.7%	45	3	6.7%	134	16	11.9%	53	2	3.8%
Sweden	150	7	4.7%	21	0	0.0%	637	79	12.4%	166	7	4.2%
West												
Austria	762	54	7.1%	79	0	0.0%	228	21	9.2%	32	1	3.1%
Belgium	219	30	13.7%	134	8	6.0%	337	84	24.9%	38	3	7.9%
France			20–25%			8.0%	3740	860	23.0%	1168	120	10.3%
Germany[1]	1500	40	2.7%	490	4	0.8%	4116	600	14.6%	1388	95	6.8%
Ireland	135	7	5.2%	9	0	0.0%	173	18	10.4%	45	0	0.0%
Netherlands	437	19	4.3%	88	1	1.1%	458	45	9.8%	120	3	2.5%
Switzerland	141	3	2.1%	91	0	0.0%	119	8	6.7%	97	3	3.1%
UK[2]	1379	97	7.0%	267	3	1.1%	2909	519.7	17.9%	561.2	15.8	2.8%
East												
Czech Rep[3]	500	60	12.0%	65	2	3.1%	1013	267	26.4%	138	3	2.2%
Estonia	109	32	29.4%	8	0	0.0%	250	88	35.2%	19	2	10.5%
South												
Italy	1727	609	35.3%	646	84	13.0%	2476	867	35.0%	823	124	15.1%
Portugal			40–45%			5%	906	431	47.6%	84	27	32.1%
Spain	1075	168	15.6%	279	12	4.3%	1331	358	26.3%	139	18	12.9%

Data

Table 1 shows the comparison between 1993 data and 2005 data, divided into four regions of Europe. The 1993 data included only the distinction between mathematicians (which we took to include researchers, lecturers and senior lecturers, but not PhD students) and full professors (which we took as the most senior career grade in any academic system –- in some countries most academics are called professors and we counted only the most senior in this category). Note that UK data, which is collected by a government agency (the Higher Education Statistics Authority), counts part-time staff as fractional appointments, hence the UK data is sometimes not a whole number.

Table 2 shows the fuller data we were able to collect in 2005, which included more countries and a more complete breakdown of different categories of staff. Note that the total number of mathematicians does not include Ph.D. students. We included professors, senior research staff and Heads of Department in the category `Professor‘, senior lecturers, principal lectures, senior researchers and associate professors in the category `Senior Lecturer‘ and lecturers and research staff in the category `Lecturer‘. We used this particular breakdown as it closely reflects the categories used in the UK, where we are based. The data is sorted from largest percentage of women mathematicians to least.

Table 3 shows some additional data we were able to collect showing the numbers of women in mathematical research in some non-European countries. The career categories are as described above.

Analysis

It is clear from the comparative data that in almost all countries the proportion of women in mathematical research has increased in the 12 years between the surveys, in many cases dramatically. Some of this increase can be explained geographically: for example, the figures for Germany in 1993 only included former West Germany. With

Table 2: 2005 data, ordered by decreasing percentage of women mathematicians

	Mathematicians			Professors			Senior Lecturers			Lecturers		
Country	T	F	%F	T	F	%F	T	F	%F	T	F	%F
Portugal	906	431	47.6%	84	27	32.1%	231	106	45.9%	591	298	50.4%
Estonia	250	88	35.2%	19	2	10.5%	76	27	35.3%	155	59	38.1%
Malta	28	9	32.1%	3	0	0.0%	5	1	20.0%	20	8	40.0%
Italy	2476	867	35.0%	823	124	15.1%	895	361	40.3%	758	382	50.4%
Spain	1331	358	26.9%	139	18	12.9%	586	152	25.9%	606	188	31.0%
Czech Rep	1013	267	26.4%	138	3	2.2%	244	29	11.9%	631	235	37.2%
Belgium	337	84	24.9%	38	3	7.9%	108	21	19.4%	191	60	31.4%
France	3740	860	23.0%	1168	120	10.3%	2230	685	30.7%	342	55	16.1%
UK	2909	519.7	17.9%	561.2	15.8	2.8%	644.1	81.2	12.6%	1703.7	422.7	24.8%
Lithuania	321	56	17.4%	56	1	1.8%	135	26	19.3%	130	29	22.3%
Germany	4116	600	14.6%	1388	95	6.8%	434	57	13.1%	2294	448	19.5%
Finland	440	64	14.5%	71	2	2.8%	69	7	10.1%	300	55	18.3%
Azerbaijan	1449	194	13.4%	224	11	4.9%	0	0	0.0%	1225	183	14.9%
Sweden	637	79	12.4%	166	7	4.2%	311	40	12.9%	160	32	20.0%
Norway	134	16	11.9%	53	2	3.8%	49	6	12.2%	32	8	25.0%
Ireland	173	18	10.4%	45	0	0.0%	34	7	20.6%	94	11	11.7%
Netherlands	458	45	9.8%	120	3	2.5%	136	10	7.4%	202	32	15.8%
Denmark	237	23	9.7%	53	2	3.8%	128	10	7.7%	56	11	19.6%
Austria	228	21	9.2%	32	1	3.1%	104	3	2.9%	92	17	18.5%
Switzerland	119	8	6.7%	97	3	3.1%	7	1	14.3%	15	4	26.7%
Iceland	13	0	0.0%	7	0	0.0%	3	0	0.0%	3	0	0.0%

[1] Data was just for former West Germany in 1993 but for unified Germany in 2005.
[2] Figures obtained from the UK Higher Education Statistics Agency.
[3] Data was for Czechoslovakia in 1993.

	Mathematicians			Professors			Senior Lecturers			Lecturers		
Country	T	F	%F	T	F	%F	T	F	%F	T	F	%F
Canada	2077	290	14.0%	670	59	8.8%	447	53	11.9%	960	178	18.5%
New Zealand	179	27	15.1%	57	3	5.3%	61	9	14.8%	61	15	24.6%
Australia	1006	170	16.9%	418	15	3.6%	227	52	22.9%	361	103	28.5%
South Africa	264	56	21.2%	59	4	6.8%	87	12	13.8%	118	40	33.9%
Japan	494	13	2.6%	193	3	1.6%						

the unification of Germany, many more women mathematicians from former East Germany are now included in the data. Other increases can be explained by changes in the counting system. For example, in the UK the 1993 data did not include mathematicians working at the former polytechnics, which all became universities in 1992/93. There also appear to be some anomalous figures, such as the very large number of mathematicians recorded in Austria in 1993. Since we did not collect the 1993 data we cannot vouch for its accuracy or be sure that the same concept of a 'mathematician' was applied across all countries.

Even so, we observe that in many European countries the numbers of women in mathematics has doubled or even trebled, particularly where the percentage in 1993 was very low. In the countries where women were already well-represented the increase has been much less significant. This may suggest a drift towards a mean of around 40-50% representation of women in mathematics.

The data broken down by region shows that there are distinct profiles of the proportion of women in mathematics in different parts of Europe. There seems to be a clear difference between 'western/northern European' systems and 'southern/eastern' regions. The data for non-European countries is in some sense consistent with this as one could regard the academic systems and cultures of countries such as Canada, Australia and New Zealand to be more closely related to western European culture than to southern/eastern Europe.

Conclusions

The data presents a positive trend – the proportion of women in mathematical research is increasing. However, the regional differences show that in many countries there is a long way to go, particularly those in northern and western Europe.

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Esmyr Koomen [esmyr@koomen.demon.co.uk] has a BSc first class honours in Computing and Psychology. She combines part-time teaching in computing with co-running an agency for the placement of volunteers in socio-economic development projects. She was the administrator for the European Women in Mathematics web-based mentoring scheme 2002–2004.

Geometric integration and its applications

Chris Budd (Bath, UK)

Geometric integration is the general term for a set of numerical algorithms for solving differential equations that aim to reproduce qualitative features in the solution. These can be conservation laws, symmetries, symplectic structure, singularities or long time asymptotic behaviour. Often these properties are fundamental to the physical system being modelled, and to reproduce them requires more than a careful local error control. In this article I will describe some of these methods, looking in particular to an application of them to solving the harmonic oscillator and to the Kepler problem in celestial mechanics, in which the advantages of them over more traditional numerical methods becomes clear.

What is geometric integration?

It is natural when computing the behaviour of a physical system governed by a system of differential equations, such as a large collection of molecules or the weather, to expect the numerical approximation to have the same qualitative features as the underlying solution. However, the historical development of numerical methods has not always been in this direction. A traditional approach to the design of a numerical algorithm to solve a system of differential equations is to insist that the differential operators in the equation are carefully approximated. Typically this is by using a finite difference, finite element, spectral or finite volume method. This is essentially a *local* process operating on a small region of phase space. Whilst it can give a close approximation to the solution over moderate time scales (especially when coupled to an error control scheme) it does not pay any attention to the qualitative features of the solution, which often have a much more global nature. In contrast geometric integration methods are designed to exactly reproduce certain qualitative features, often by directly implementing some of the physical processes underlying the differential equation. The result is methods which have many nice features, for example the ability to reproduce the behaviour of the system over very long time intervals, or being able to cope with singular or highly oscillatory solutions.

So, what sort of qualitative features might we be interested in? The most obvious example are the *conservation laws* which pervade many physical systems (and are often discovered long before the underlying differential equations). These may include properties of the *whole system* such as energy, linear and angular momentum, or may be features conserved *along a trajectory* such as vorticity in the two dimensional Euler equations of fluid motion or *isospectral flows* in which eigenvalues of an operator are conserved. Another qualitative feature of many physical systems are *symmetries* which may be *discrete* such as reflexions (eg. time reversibility) or *continuous* such as rotations, translations, scalings or deeper Lie group symmetries. These qualitative features often arise from deeper structures of the system, such as a Lagrangian or Hamiltonian formulation (for example a symmetry of the Lagrangian can lead directly to a conservation law). The moral of this is that if we can design a numerical method which respects this deeper (geometric structure) then we may well get nice qualitative properties for free, and without having to do complicated error analysis. It is this philosophy which lies of the heart of the various numerical methods in the geometric integration toolbox.

Integrating the Harmonic Oscillator using both traditional and geometric methods

To motivate the geometric integration approach we will look at the performance of a number of different numerical schemes on the test problem

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -x. \tag{1}$$

This is of course the Harmonic oscillator, which has bounded periodic solutions $(x,y) = (\sin(t), \cos(t))$ for which the Hamiltonian energy $H = (x^2+y^2)/2$ is conserved. It has other geometric features, for example it is reversible, so that flows forward and backward in time look identical.

The simplest numerical scheme to approximate the solutions is the *Forward Euler Method*. In this method we take a constant time step h and approximate $(x(t), y(t))$ at the time $t_n = nh$ by (X_n, Y_n). A single step of this method then takes the form

$$X_{n+1} = X_n + hY_n, \quad Y_{n+1} = Y_n - hX_n.$$

A conventional analysis shows that if h is small then every time this method is used it makes an error proportional to h^2. However, this local analysis gives no insight into how these errors accumulate or what the geometric properties of the scheme are. A simple way of seeing this, however is to see how the Hamiltonian $H_n = (X_n^2 + Y_n^2)/2$ changes from one step to the next. It is not difficult to show that $H_{n+1} = \left(1+h^2\right) H_n$. From the perspective of a local analysis this is fine, as H_n only changes by $O(h^2)$ at each stage. However from a *global* perspective it is a disaster. The energy increases at each time step and rapidly gets very large. There is no way that the solutions can be periodic, and the numerical method is completely non-symmetric, with the magnitude of the solution increasing in forward time and decreasing in backwards time.

A common fix when dealing with an unstable numerical calculation which is often used in applications, is to use the

Backward Euler Method. In this case we take

$$X_{n+1} = X_n + hY_{n+1}, \quad Y_{n+1} = Y_n - hX_{n+1}.$$

In this case we find that $H_{n+1} = H_n/\left(1+h^2\right)$. In contrast to the Forward Euler Method, the Backward Euler Method takes energy *away* from the system at each step. Again this is a qualitative crime, and the numerical scheme is again non-periodic and non-symmetric.

Now let's look at a combination of these two method called the *Implicit Mid-Point Rule* (and is equivalent to the Trapezium rule for this problem) for which

$$X_{n+1} = X_n + \frac{h}{2}\left(Y_n + Y_{n+1}\right), \quad Y_{n+1} = Y_n - \frac{h}{2}\left(X_n + X_{n+1}\right).$$

A direct calculation shows that in this case the Hamiltonian is conserved with $H_{n+1} = H_n$ and also the method is symmetric with the numerical integrations both forwards and backwards in time looking identical. In fact things are even better than this. An exact solution of the numerical scheme is given by

$$X_n = \sin(\omega t_n), \quad Y_n = \cos(\omega t_n),$$

where $\omega = 1 - h^2/12 + O(h^4)$.

This has qualitatively very similar features to the underlying solution. Although it has a local error proportional to h^2 the errors do not accumulate in the same way as they did for the other two methods, and the solutions remain bounded and periodic for all time with a *phase error* proportional to $h^2 t$.

Modified equation analysis

Very significantly, the values taken by the *numerical solution* are precisely the same as the values of the functions $\left(X(t), Y(t)\right)$ at the times t_n where X and Y are the *exact solution* of the differential equation

$$\frac{dX}{dt} = \omega^2 Y, \quad \frac{dY}{dt} = -\omega^2 X.$$

This differential equation is the *original equation* perturbed by an error proportional to h^2. This shows us why the Implicit Mid-Point Rule is so effective. Its solutions exactly satisfy a differential equation which is a modified version of the original. We say that the solutions of the original system are *shadowed* by those of the modified equation. The behaviour of the numerical scheme can thus be analysed by using all of the theory of perturbed differential equations rather than local error analysis. For example it is obvious that in common with the original equation, the modified equation has also got bounded periodic solutions. A comparison of the three methods described above (with $h = 1$) when acting on a circle of initial data (indicated by a dashed line) is given in the Figure. 1 (note that different scales have been used for the thre figures). In this figure we see the circle growing in size and spiralling out when we use the Forward Euler method. In contrast it contracts and spirals in when using the Backward Euler method. However it stays constant in area and shape and gives a periodic solution when using the Implicit Mid-point rule.

Symplectic methods

There is a further, deeper, reason, why the Implicit Mid-Point Rule has performed so well. The Harmonic oscillator is the simplest example of a *Hamiltonian differential equation* with Hamiltonian H. When acting on the (x,y) phase-space it generates a map which is *symplectic*. This map preserves area in phase space. Poincaré recognised that it was this single property which above all others governed the qualitative properties of the differential equation. Any numerical scheme also defines a map on the same phase space. However the symplectic property is only given by a few numerical schemes. The Implicit Mid-Point rule is an example of a *symplectic method* which has this crucial feature. (There are many other such schemes, but this rule is one of the simplest examples). All such methods share the feature that (apart from a very small error proportional to $\exp(-h^*/h)$) the points (X_n, Y_n) are the sample values of the *exact* solution of a Hamiltonian differential equation which is a small perturbation of the original (as we saw in the Harmonic oscillator problem). We can deduce many properties of the solutions of the numerical scheme (in particular its long time behaviour) from this fundamental observation.

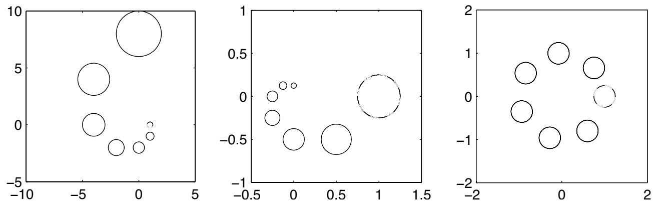

Figure 1. A comparison of the Forward Euler Method, the Backward Euler Method and the Implicit Mid-Point rule on the same set of initial data indicated by the dashed circle (note the use of different scales).

The Kepler Problem

A more challenging example which shows some more of the power of the geometric approach to computation, is given by the *Kepler problem* in celestial mechanics describing the motion of a (single) planet around the sun. If the planet is at the position (x,y) with respective momenta $(u,w) = (\dot{x}, \dot{y})$ then these satisfy the differential equations

$$\ddot{x} = -\frac{\mu x}{(x^2+y^2)^{3/2}}, \quad \ddot{y} = -\frac{\mu y}{(x^2+y^2)^{3/2}} \tag{2}$$

This system has a number of distinctive qualitative features. Like the Harmonic oscillator it is *Hamiltonian* with Hamiltonian

$$H = \frac{1}{2}\left(u^2 + w^2\right) - \frac{\mu}{(x^2+y^2)^{1/2}},$$
$$\dot{x} = \partial H/\partial u, \dot{u} = -\partial H/\partial x \quad \text{etc.}$$

It is invariant under continuous translations in time and rotations in space leading to conservation of both the Hamiltonian and of the *angular momentum* $L = xw - yu$. The latter conservation law being *Kepler's second law*. The angular momentum is a *quadratic invariant* which is relatively easy to conserve exactly using a numerical method. In contrast the Hamiltonian is much harder to conserve exactly, although we can remain close to it. A further symmetry of the system is its invariance under the *scaling symmetry* group

$$t \to \lambda t, \quad (x,y) \to \lambda^{2/3}(x,y), \quad (u,w) \to \lambda^{-1/3}(u,w), \quad \lambda > 0. \tag{3}$$

This property of the Kepler system was discovered before it was formulated as a differential equation. It is none other than *Kepler's Third Law* and is equivalent to the statement that any periodic orbit with period T and maximum radius R can be mapped to another of period $\lambda^3 T$ and maximum radius $\lambda^2 R$.

Splitting methods

We are now faced with an apparent choice. Of the three qualitative properties comprising the two conservation laws and the scaling symmetry, which should we preserve in a numerical method. The best answer is, of course, all three. However, this is not, in general, possible, and certainly a method with a *fixed step size* has no chance of preserving the scaling symmetry. As the problem is Hamiltonian it is natural to consider using a symplectic method such as the Implicit Mid-Point rule. However a disadvantage of this method is the fact that it is implicit and we have to solve nonlinear equations at each stage. This is awkward for the Kepler problem, and essentially impossible if we look at much larger gravitating systems with many particles. However, there is a very nice way around this problem because for a set of important physical problems it is possible to use *explicit* symplectic methods, which have all of the nice features of a symplectic method, whilst also being easy to use. These exploit the fact that the Hamiltonian for the Kepler problem, and many other problems, splits into two parts, T and V corresponding to the Kinetic and Potential energies of the system, so that $H = T + V$, with $T = (u^2 + w^2)/2$ a function of u and w only, and $V = -\mu/(x^2 + y^2)^{1/2}$ a function of x and y only. Problems in molecular dynamics also have this very nice feature, although complex mechanical systems tend not to. Symplectic methods can exploit this by splitting the evolution of the whole system, alternating between evolving the system using the kinetic and the potential components. An example of such a scheme is a symplectic version of the Forward Euler method called the *Symplectic Euler Method* in which

$$(X_{n+1}, Y_{n+1}) = (X_n, Y_n) + h(U_n, W_n),$$

and

$$(U_{n+1}, W_{n+1}) = (U_n, W_n) - h\mu(X_{n+1}, Y_{n+1})/(X_{n+1}^2 + Y_{n+1}^2)^{3/2}.$$

Note that we don't need to solve any nonlinear equations to advance the solution using this method.

Now, let's compare the performance of the Symplectic Euler Method with that of the Forward Euler Method. The first thing that we can look at is how well each deals with the conservation of angular momentum and of the Hamiltonian. It is not difficult to show that the Symplectic Euler method *exactly conserves the angular momentum*. This is a very desirable property. In contrast, when using the Forward Euler method, the error in the angular momentum increases with time at a rate proportional to h. We now look at the evolution of the Hamiltonian using each method. This is is not exactly conserved by the Symplectic Euler Method, however, if H_n is the Hamiltonian of the discrete approximation, then H_n differs from H by a *bounded quantity* proportional to h. In fact the method exactly conserves a discrete energy which is very close to H and it also has *periodic solutions* which have a phase error proportional to ht. In contrast, the Hamiltonian

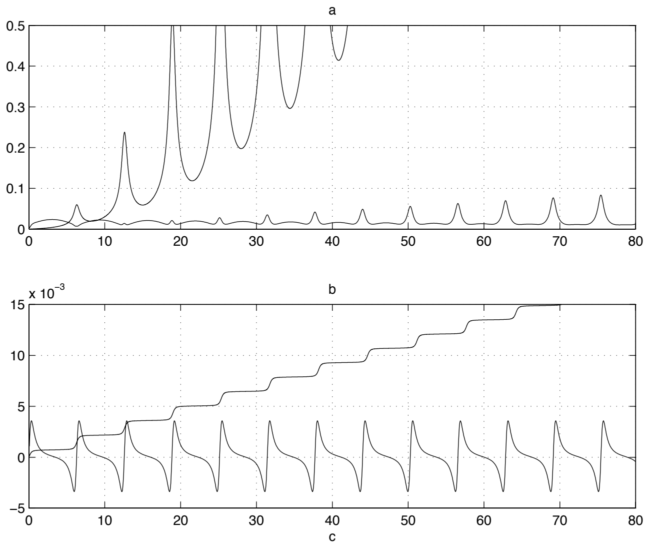

Figure 2. A comparison of (a) the solution error and (b) the error of the Hamiltonian between the Forward Euler method (larger error) and the Symplectic Euler method

of the Forward Euler approximation increases with time and the solutions of this method are not periodic. A comparison of the two methods when computing an elliptical orbit with eccentricity $e = 0.5$ is given in Figure 2. In this figure we plot first the solution error and then the error in the Hamiltonian. In both cases the error of the Symplectic Euler Method lies below that of the Forward Euler Method with the symplectic method having a solution error growing at a much slower rate than that of the non symplectic method, with the Hamiltonian error bounded for all time.

Adaptivity and other methods

Interestingly, the error in the Symplectic Euler Method has peaks, corresponding to close approaches of the elliptical orbit. This error increases rapidly as the eccentricity of the orbit approaches unity and the close approaches become more acute. It is possible to significantly reduce this error by using an *adaptive method* in which the value of the time-step h is chosen adaptively so that it is small during the close approaches. This process has to be done very carefully indeed when using symplectic methods, to ensure that the nice conservation properties of the method, which follow from the shadowing of the solution given by the modified equation analysis, still apply. It is possible, but not easy to do this. In fact, the use of adaptivity allows the resulting method to be invariant under the scaling symmetries described earlier. This means that it is possible to have adaptive, symplectic methods which also obey Kepler's third law. These methods perform very well!

The Symplectic Euler Method is not especially accurate, and there are much more accurate symplectic methods which are based on splitting and are just as easy to use. A famous example is the *Störmer-Verlet method*. This was invented by Verlet early in the 20th Century to tackle problems in celestial mechanics, and then rediscovered by Störmer to solve problems in molecular dynamics. It is a beautiful method which is explicit, accurate, symplectic, reversible and has excellent conservation properties. It forms the basis of much of modern molecular dynamics simulation software. Symplectic methods have also been used to compute the evolution of the solar system for times in excess of a billion years.

Other applications of geometric integration

The above examples have shown how useful it is to use a geometric approach when the underlying problem has structure we can exploit. Geometric methods are used in many other applications, for example when computing the solution of the equations governing the weather, when looking at the behaviour of robotic systems where we can exploit Lie group structures, when looking at systems with very rapid oscillations, systems which evolve on manifolds and when solving integrable partial differential equations such as the nonlinear Schrödinger equation. The range of applications of these methods is growing rapidly, but much work still needs to be done to apply them to many of the challenging problems arising in physics and engineering, particularly those which are governed by partial differential equations. The future development of geometric methods for such problems will involve a team effort between scientists, numerical analysts, pure and applied mathematicians all contributing equally. I hope that I have whetted your appetite. If you want to learn more then some surveys of the field are given in the following references. The reference [1] surveys the theory of these methods in the context of ODEs and reference [2] extends this to PDEs, whilst reference [3] shows how geometric methods can be used in many different applications.

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[2] C.J. Budd and M.D. Piggott, *Geometric integration and its applications*, in Handbook of Numerical Analysis, **XI**, eds. P.G. Ciarlet & F. Cucker, (2003), North-Holland

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There is also a web-site for the *Geometric Integration Interest Group* which has many examples of uses of geometric integration in many applications and can be found at http://www.focm.net/gi/.

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Poincaré conjecture and Ricci flow An outline of the work of R. Hamilton and G. Perelman: Part II

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*In the first section of this text (EMS-Newsletter **59**, pp. 11–15), we explained how to study singularities of the Ricci flow with sequences of parabolic rescaling. We showed that these sequences have limits with very strong geometric constraints and that these limits are called κ-solutions. We begin the final part of the article with their classification.*

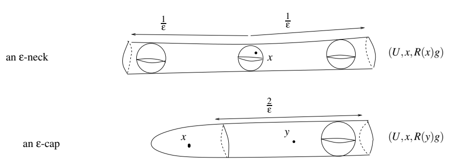
κ-solutions

Recall that a 3-dimensional κ-solution is a solution to the Ricci flow $(M^3, g(t))$, defined for at least $t \in (-\infty, 0]$, such that for any $t \leq 0$, $g(t)$ is a complete Riemannian metric with positive non-flat bounded sectional curvature. Moreover, it is κ-non-collapsed at any scale. For example, $\mathbb{S}^3$ and $\mathbb{S}^2 \times \mathbb{R}$ with their standard flows are κ-solutions while $\mathbb{S}^2 \times \mathbb{S}^1$ is not (when $t \to -\infty$, the $\mathbb{S}^1$ factor is very small compared to the sphere $\mathbb{S}^2$). Perelman makes the full classification of 3-dimensional κ-solutions in chapter 11 of [PeI] and 1.5 in [PeII].

Theorem 1. *A 3-dimensional κ-solution is isometric to:*
(A) $\mathbb{S}^2 \times \mathbb{R}$ *with cylindrical flow,*
(B) $\mathbb{B}^3$ *or* $\mathbb{RP}^3 - \overline{\mathbb{B}^3}$ *with Ricci flow of strictly positive sectional curvature,*
(C) $\mathbb{S}^3/\Gamma$, *where* Γ *is a finite subgroup of isometries of the round sphere and the Ricci flow has strictly positive curvature.*

Local description

We can describe the local geometry of these κ-solutions with two fundamental building blocks: ε-necks and ε-caps. Fixing some $\varepsilon > 0$ (where ε is small), an ε-neck is a pointed Riemannian manifold (U, x, g) that is ε-close, after dilation by $R(x)$, to the pointed Riemannian manifold $\big(\mathbb{S}^2 \times (-\frac{1}{\varepsilon}, \frac{1}{\varepsilon}), \{*\} \times \{0\}, \mathrm{can}\big)$, where the metric has scalar curvature 1. An ε-cap is a pointed Riemannian manifold (U, x, g) where $U = \mathbb{B}^3$ or $\mathbb{RP}^3 - \overline{\mathbb{B}^3}$ and any point outside a compact set is in an ε-neck.



Remark 2. *If ε is sufficiently small, we have the following property. Let (M, g) be a compact Riemannian manifold.*
(1) If (M, g) is covered by ε-necks, then M is diffeomorphic to $\mathbb{S}^2 \times \mathbb{S}^1$.
(2) If (M, g) is covered by ε-necks and at least one ε-cap, then M is diffeomorphic to S^3, $\mathbb{RP}^3$ or a connected sum of two $\mathbb{RP}^3$ (in fact there are exactly two caps, which can be $\mathbb{B}^3$ or $\mathbb{RP}^3 - \overline{\mathbb{B}^3}$).

The local geometry of κ-solutions obeys the following result:

Theorem 3. *There exist $\kappa_0 > 0$ and $\eta > 0$ such that any 3-dimensional κ-solution is a κ_0-solution or a quotient of the round sphere. Moreover, oscillations of the scalar curvature are controlled by the equations*

$$|\nabla R| \leq \eta R^{\frac{3}{2}}, \quad \left|\frac{\partial R}{\partial t}\right| \leq \eta R^2. \tag{4}$$

Additionally, for any $\varepsilon > 0$ sufficiently small, there exists $C(\varepsilon) > 0$ such that, for any $t \leq 0$, any point x of any 3-dimensional κ-solution has a neighborhood U such that, for some

$$r \in \left[\frac{C^{-1}}{\sqrt{R(x,t)}}, \frac{C}{\sqrt{R(x,t)}}\right],$$

one has $B(x,t,r) \subset U \subset B(x,t,2r)$ and U is one of the following:
(A) an ε-neck,
(B) an ε-cap,
(C) a compact manifold of strictly positive sectional curvature.

Remark 4. *A neighborhood of type C contains all the manifold, which is thus diffeomorphic to a spherical manifold. Moreover, one has estimates on the minimum of the sectional curvature.*

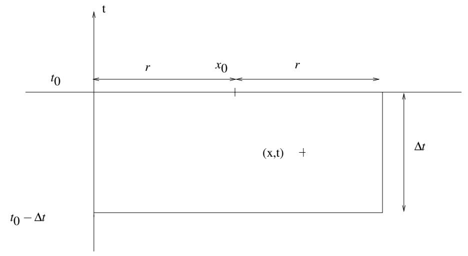
The main tools for this classification are a compactness theorem and a splitting theorem for κ-solutions. We now return to the study of the flow $(M, g(t))$ itself, with the Canonical Neighborhood Theorem (CNT), which was the main achievement of Perelman's first paper.

G. Perelman lecturing

Canonical Neighborhood Theorem

Essentially, this result states that any point of sufficiently large scalar curvature has a neighborhood in space-time of controlled size that is close to a piece of a κ-solution. Thus the geometry near the point is almost canonical. This result is crucial in determining the topology of regions of large curvature and in defining the surgery. A *parabolic neighborhood at* (x_0, t_0) is the set

$$P(x_0, t_0, r, \Delta t) = B(x_0, t_0, r) \times [t_0 - \Delta t, t_0].$$



Theorem 5. *Let* $\varepsilon, \kappa > 0$. *There exists a constant* $r_0(\varepsilon, \kappa) > 0$ *with the following property. Let* $(M^3, g(t))$ *be a Ricci flow defined on* $[0, T)$, κ*-non-collapsed at scale* r_0 *and* (x_0, t_0) *a point such that* $t_0 \geq 1$ *and* $Q \equiv R(x_0, t_0) \geq r_0^{-2}$. *Then the parabolic neighborhood* $P(x_0, t_0, \frac{1}{\sqrt{\varepsilon Q}}, -\frac{1}{\varepsilon Q})$ *is* ε*-close, after parabolic rescaling by Q, to the corresponding subset of a* κ*-solution.*

The proof of the CNT is an ingenious proof by contradiction. One considers a sequence $r_k \to 0$, Ricci flows $(M_k, g_k(t))$ satisfying the assumptions, and in each flow a "bad point" (x_k, t_k) such that $Q_k \equiv R(x_k, t_k) \geq r_k^{-2}$ but whose parabolic neighborhood is not ε-close, after parabolic rescaling by Q_k, to the corresponding subset of a κ-solution. The goal is to prove that a subsequence of the parabolic rescaled pointed flows $\big(M_k, \bar{g}_k(t) = Q_k g_k(t_k + \frac{t}{Q_k}), (x_k, 0)\big)$ converge to a κ-solution, which is a contradiction for large k. The difficulty is that we have no a priori bound on the sectional curvatures. The trick is to choose bad points with almost maximal curvature so that points with larger curvature will have canonical geometry. In some sense, this argument is an induction on the scale of curvature. The proof is highly involved and will not be given in this text.

An important conclusion of the theorem is that any point satisfying the assumptions has a neighborhood that is an ε-neck, an ε-cap or is diffeomorphic to a spherical manifold. In particular, estimates (4) on the spatial and temporal derivatives of scalar curvature hold near the point; that is, oscillations of curvature are controlled when the curvature is large. We say that a Ricci flow satisfies the canonical neighborhood assumption at scale r_0 if any point of scalar curvature $\geq r_0^{-2}$ has such a neighborhood (the constant $\varepsilon > 0$ is supposed to be small and fixed).

Ricci flow with surgery

In this section we define the Ricci flow with surgery on a 3-dimensional compact manifold. When the Ricci flow hits a singularity, the idea is to cut the manifold and throw away pieces with large curvature. Perelman uses a technique developed by Hamilton on 4-dimensional manifolds. We begin with the description of the Ricci flow at a singular time.

We suppose that $(M^3, g(t))$ is a compact 3-dimensional Ricci flow on a maximal time interval $[0, T)$, where $T < \infty$ and $T > 1$ (one can assume this by dilation). Let $\kappa > 0$, $\varepsilon > 0$ and $r_0 > 0$, where κ is given by the NLCT (see the first section of this text) and r_0 is the scale of the canonical neighborhoods given by the CNT.

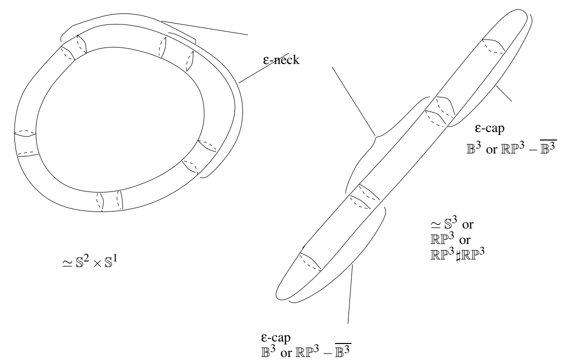
The Ricci flow at a singular time

Since $T < \infty$, there exists $x \in M$ such that $R(x, t) \to \infty$ as $t \to T$. Let

$$\Omega = \{x \in M,\ R(x, \cdot) \leq c(x)\}, \tag{5}$$

which is the set of points with bounded curvature. Suppose initially that Ω is empty, that is the scalar curvature explodes to $+\infty$ at each point. One says that the flow extincts. This situation looks like the situation in theorem [Ha82] but without hypothesis on Ricci curvature.

1st case: Ω is empty; the flow extincts. *M* is therefore diffeomorphic to $\mathbb{S}^3/\Gamma$, $\mathbb{S}^2 \times \mathbb{S}^1$ or a connected sum of two $\mathbb{RP}^3$. Indeed, there exists $t_0 < T$ close to T such that $R(x, t_0) \geq r_0^{-2}$ for any $x \in M$. Then $(M, g(t_0))$ is covered by canonical neighborhoods that are ε-necks, ε-caps or are diffeomorphic to a spherical manifold. Using remark 2, we see that *M* is diffeomorphic to a spherical manifold, $\mathbb{S}^2 \times \mathbb{S}^1$ or $\mathbb{RP}^3 \# \mathbb{RP}^3$.



Remark 6. *If M is simply connected, one concludes that M is diffeomorphic to* S^3.

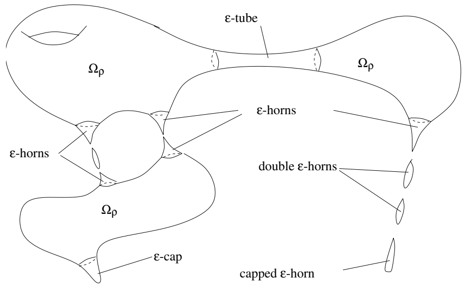
2nd case: Ω is non-empty. With estimates (4) one can show that Ω is an open set with $g(t)$ converging to a smooth metric $g(T)$. Moreover, $(\Omega, g(T))$ satisfies the canonical neighborhood assumption (perhaps after a slight increase of ε). To understand the boundary of Ω, one can consider a scale $\rho < r_0$ and the set

$$\Omega_\rho = \{x \in M\,;\, R(x, T) \leq \rho^{-2}\}.$$

The set $\Omega \setminus \Omega_\rho$ is covered by ε-necks and ε-caps. To describe the connected components, Perelman introduced the following terminology. An ε*-tube* is a metric on $\mathbb{S}^2 \times I$ such that any point is in an ε-neck and the scalar curvature is bounded (similarly ε*-horns* are bounded at one end only and *double* ε*-horns* are unbounded at both ends). A *capped* ε*-horn* is an ε-cap with unbounded curvature, that is a metric on $\mathbb{B}^3$ or $\mathbb{RP}^3 - \overline{\mathbb{B}^3}$ such that outside a compact set any point is in an ε-neck and such that the scalar curvature is unbounded at the end. Considering possible combinations of ε-necks and ε-caps, one can see that any point of $\Omega \setminus \Omega_\rho$ is in one of the following sets:

(a) an ε-tube with boundary in Ω_ρ,
(b) an ε-horn with one end in Ω_ρ,
(c) an ε-cap with boundary in Ω_ρ,
(d) a capped ε-horn,
(e) a double ε-horn.

The last two components are disconnected from Ω_ρ. The figure below illustrates these components.



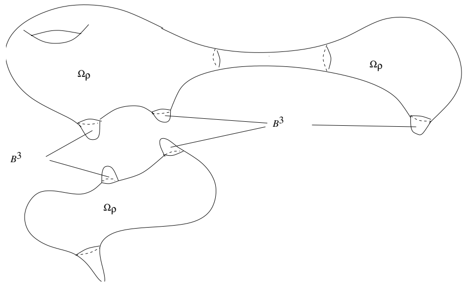
The set Ω can be very complicated. In particular, there can be an infinite number of double ε-horns. However, there can only be a finite number of connected components of Ω intersecting Ω_ρ.

Remark 7. *If Ω_ρ is empty, $(M, g(t))$ is covered by canonical neighborhoods thus, as in the first case, M is diffeomorphic to $\mathbb{S}^3/\Gamma$, $\mathbb{S}^2 \times \mathbb{S}^1$ or $\mathbb{RP}^3 \# \mathbb{RP}^3$. In this case, one says that the flow extincts; this will be justified by the definition of surgery.*

The surgery

If Ω is non-empty, one performs a topological surgery as follows:

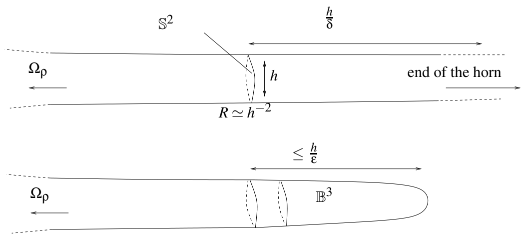
(1) throw away connected components of Ω disconnected from Ω_ρ,
(2) cut the ε-horns (that are connected to Ω_ρ), throw away the unbounded part and paste a ball $\mathbb{B}^3$ on the other part.



The new manifold, called M_1, has a finite number of connected components. For any time $t < T$ close to T, $(M \backslash \Omega(\rho), g(t))$ is covered by canonical neighborhoods and one can therefore compare the topology of M and M_1. One can see that M is the connected sum of the components of M_1 and a finite number of $\mathbb{S}^2 \times \mathbb{S}^1$ and projective spaces $\mathbb{RP}^3$. In the figure below, the surgery eliminates one $\mathbb{S}^2 \times \mathbb{S}^1$ and one $\mathbb{RP}^3$ if the capped ε-horn is $\mathbb{RP}^3 - \overline{\mathbb{B}^3}$.

Remark 8. *If the set Ω_ρ is empty, the surgery above makes sense. In this case, all the manifold is thrown away.*

This surgery can be defined metrically. One cuts the ε-horns in the middle of a δ-neck, for some parameter $0 < \delta << \varepsilon$. Indeed, Perelman showed that for any $0 < \delta < \varepsilon$, there exists $h(\delta, \rho, \varepsilon) > 0$ such that in any ε-horn any point of scalar curvature larger than h^{-2} is in a δ-neck. One performs surgery along a sphere of curvature approximatively h^{-2} and then one pastes a ball $\mathbb{B}^3$ with a standard metric ε-cap like. The two metrics are interpolated on a cylinder $\mathbb{S}^2 \times [0, \lambda h]$, where λ is a constant, in a metric close to the standard one. The length of the added ε-cap remains less than $\frac{h}{\varepsilon}$. An important point is that this surgery and pasting can be done so that it preserves Hamilton-Ivey pinching, the κ-non-collapsing and the scale r_0 of the canonical neighborhoods.



Note that the volume lost in the surgery is larger than $\frac{h^3}{\delta} - \frac{h^3}{\varepsilon}$. This surgery is called surgery with (r_0, δ)-cutoff (the parameter ρ is fixed by $\rho = \delta \cdot r_0$).

The new compact manifold M_1 has a metric $g_1(T)$ and one can therefore start the Ricci flow again on each component. A natural question is whether we can repeat this procedure enough to decompose M completely.

Ricci flow with surgery

In [PeII], Perelman showed that the procedure defined above can be repeated indefinitely with parameters r_0 and δ dependent upon time. Firstly, we give a definition of Ricci flow with surgery.

Definition 9. *Let $r(t)$ and $\delta(t)$ be strictly positive functions on $[0, +\infty)$. Ricci flow with surgery with parameters (r, δ) is:*

- *a sequence $(t_k)_{0 \le k \le N \le \infty}$ in $[0, +\infty)$ that is strictly increasing and discrete, and for each integer k,*
- *a compact manifold M_k that is possibly non-connected or empty,*
- *a Ricci flow $g_k(t)$ on $M_k \times [t_k, t_{k+1})$ that is singular at time t_{k+1}, satisfying the canonical neighborhood assumptions at scale $r(t)$,*

such that $(M_{k+1}, g_{k+1}(t_{k+1}))$ is the result of the surgery with (r, δ)-cutoff at time t_{k+1} on $(M_k, g_k(t))$.

In particular, there is only a finite number of surgeries on finite time intervals. If M_k^j, $j \in \{1, \cdots, N(k)\}$, are the connected components of M_k then, taking into account the possible extinction of components, one has that $M = M_0$ is diffeomorphic to the connected sum of M_k^j and a finite number of $\mathbb{S}^2 \times \mathbb{S}^1$ and finite quotients of $\mathbb{S}^3$.

Remark 10. *If M_k is empty (the flow extincts on all components), one sets $t_{k+1} = +\infty$. The manifold M is thus diffeomorphic to a connected sum of a finite number of $\mathbb{S}^2 \times \mathbb{S}^1$ and finite quotients of $\mathbb{S}^3$. If it is simply connected, one can conclude that M is diffeomorphic to S^3.*

To find universal functions r and δ, one takes a metric g_0 at the origin that is normalized. That is:

(a) The sectional curvatures are bounded in absolute value by one.
(b) The volume of each unit ball is at least half that of a Euclidean one.
(c) The metric g_0 satisfies Hamilton-Ivey pinching.

Then, with some neatening, we have

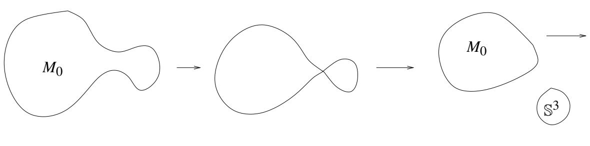
Theorem 11 ([PeII]). *Let $\varepsilon > 0$. There exist decreasing functions $r(t) > 0$ and $\delta(t) > 0$ defined on $[0, \infty)$ with the following property. For any initial normalized data (M, g_0), the Ricci flow with surgery with parameters $(r(t), \delta(t))$ is defined on $[0, \infty)$.*

The proof of this result is highly technical and will not be explained in this text.

The proof of the Poincaré conjecture

According to remark 10, it is sufficient to prove that if M is a 3-dimensional, compact and simply connected manifold, the Ricci flow with surgery with initial data M extincts in finite time. In fact, it is sufficient to argue the result on irreducible manifolds, where the proof is simpler. *One says that a 3-dimensional manifold N is irreducible if any embedded sphere $S^2 \subset N$ is the boundary of a ball $B^3 \subset N$.* It implies that any connected sum decomposition of N is trivial: if $N = N_1 \# N_2$, then one of N_i is diffeomorphic to N and the other is diffeomorphic to $\mathbb{S}^3$. By the Kneser theorem, any 3-dimensional, compact manifold has a unique connected sum decomposition into irreducible manifolds and $\mathbb{S}^2 \times \mathbb{S}^1$. Thus if M is simply connected, it has a unique decomposition into irreducible, simply connected manifolds. It is sufficient to prove that these manifolds are diffeomorphic to $\mathbb{S}^3$.

Hence we consider a compact, irreducible, simply connected manifold M_0. We specify a normalized metric and start the Ricci flow with surgery. Now M_0 survives at each surgery, up to diffeomorphism, so we can consider that M_0 has a piecewise smooth Ricci flow.



It is sufficient to prove that the flow extincts on M_0.

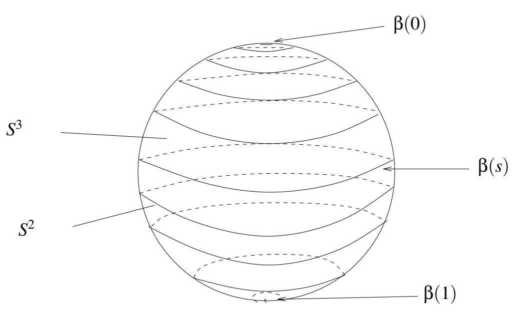
Extinction in finite time

We give the argument of T. Colding and W. Minocozzi [CM], which is technically simpler than Perelman's [PeIII]. They show that the width of $(M_0, g(t))$, a strictly positive geometric quantity calculated by sweeping M_0 with spheres S^2, decreases sufficiently fast along the Ricci flow to reach zero in finite time. This relies on the fact that the space of maps from S^2 to M_0 is not simply connected.

We consider the space of all maps $S^2 \to M_0$, continuous with bounded energy $H = C^0 \cap L^2_1(S^2, M_0)$, where the energy $E(f)$ of a map $f : S^2 \to M_0$ is defined by

$$E(f) = \int_{S^2} |df|^2_g dV_{S^2}.$$

Let $i(M_0) \subset H$ be the set of constant maps that send S^2 to a point in M_0. By classical topological arguments, the fundamental group $\Pi_1(H, i(M_0))$ is non-trivial. That is, there exists a path of maps $S^2 \to M_0$, $\beta : [0,1] \to H$, with constant maps at extremities, that is homotopically non-trivial. Consider the following picture of $\mathbb{S}^3$:



Given the class $[\beta]$, one defines the width of (M_0, g) by a minimax principle: one takes the minimum among the representatives γ of $[\beta]$ of the maximal energy of spheres S^2 along $\gamma(s)$:

$$W([\beta], g) = \inf_{\gamma \in [\beta]} \sup_{s \in [0,1]} E\big(\gamma(s)\big).$$

We then have,

Theorem 12 (J. Jost [J]). $W([\beta], g) > 0$.

Also, T. Colding and W. Minicozzi [CM] have shown the following:

Proposition 13. *If $g(t)$ is a smooth Ricci flow on M_0, then*

$$\frac{dW\big([\beta], g(t)\big)}{dt} \leq -4\pi + \frac{3}{4(t+C)} W([\beta], g(t)),$$

where C depends on $R_{min}(0)$. This inequality implies that the width of $(M_0, g(t))$ reaches zero in finite time.

Recall that we have a piecewise smooth Ricci flow $\big(M_0, g(t)\big)$. The inequality above holds on each smooth interval of the flow. The constant C may change at each surgery but in fact it improves as $R_{min}(t)$ increases. To conclude, it suffices to show that the width of $(M_0, g(t))$ decreases at each singularity. This is a consequence of the following lemma.

Lemma 14. *Let T be a singular time of the piecewise smooth Ricci flow $g(t)$ on M_0. Then, there exists for $t < T$ close to T a diffeomorphism that is $(1+\chi(t))$-Lipschitz between $(M_0, g(t))$ and $(M_0, g(T))$, where $\chi(t) \to 0$ as $t \to T$. Thus*

$$\lim_{t \to T_-} W([\beta], g(t)) \geq W([\beta], g(T)).$$

Indeed, consider the figure on page 7 and go back slightly into the past of $t < T$. On the left of the central sphere and on a part of the δ-neck, the metric $g(t)$ is very close to $g(T)$, whereas the end of the horn is replaced by a ball $\mathbb{B}^3$. We define the diffeomorphism as the identity on the left of the central sphere and a part of the δ-neck, and the right part diffeomorphically goes to the end of the ε-cap. One can see that the map is $(1+\chi(t))$-Lipschitz on the left side and a part of the δ-neck, and very contracting on the complementary. Now this map decreases energy, up to a multiplicative factor $\big(1+\chi(t)\big)^2$:

$$W\big([\beta], g(T)\big) \leq \big(1+\chi(t)\big)^2 W\big([\beta], g(t)\big).$$

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A Scientific Duo: Reflections on the Interplay between Mathematics and Physics 1809–1950

Jesper Lützen (Copenhagen, Denmark)

The interaction between mathematics and physics is often described as an application of the former to the latter. Its indisputable effectiveness has been characterized as unreasonable by Eugene Wigner. In particular, mathematics seems to have almost magical powers to predict new phenomena in modern elementary particle physics. This poses a philosophical problem that cannot easily be dismissed let alone solved. However, in order to emphasize the unreasonableness, there is a widespread tendency to present the nature of the interaction between the two sciences in a way that puts the philosophical problem in a false light. This tendentious account goes as follows: mathematics is an a priori product of the human mind, uncontaminated by external factors. Physics is a description of natural phenomena, whose objects are things in nature. Mathematics can be applied to physics in the sense that it provides a precise quantitative language to describe (understand) the natural phenomena and even predict new phenomena. Why this can be done successfully is the philosophical problem that Wigner pointed to.

The above account of the interplay between mathematics and physics suffers from at least three problems having to do with the nature of mathematics, physics and applications. Firstly, there is no unmathematical physics to which mathematics can be applied. Our Western attempt to understand nature is permeated with mathematics from the start. Even the basic concepts of physics are deeply mathematical. One may perhaps argue that the concepts of space (distance), time and mass are non-mathematical in nature, but their quantifications are mathematical, and all other concepts of physics have developed in close interaction with mathematics. For example, a concept like velocity needed two types of mathematical input in order to be made into a precise workable physical quantity. In order to quantify even uniform motion it was necessary to develop a theory of ratios of quantities of different kinds, such as spaces and times, a theory that was not developed by the ancient Greeks. And the idea of velocity of a non-uniform motion was developed in close connection with the invention of calculus. Most other physical concepts needed even more mathematical input. For example the conceptualization of energy around 1850 was based on the observation (fifty years earlier) that Newtonian forces could be derived as the gradient of a scalar quantity, the potential. In this way a rather deep mathematical theorem was a precondition for the formation of a physical concept that is today considered a simple everyday term.



Joseph Liouville **Heinrich Hertz** **Laurent Schwartz**

Secondly, the idea of mathematics as a creation of pure intellect is also problematic. Philosophically, such an idea may be defensible and it is in agreement with the popular formalistic conception of mathematics, according to which this science deals with structures, such as groups, Hilbert spaces etc. defined by hopefully consistent but otherwise arbitrary axiom systems and progressing by cumulative deductions by way of arbitrary rules of deduction. However, even philosophers of the late 20th century have called attention to the unsatisfactory nature of the formalistic philosophy. In particular, it does not explain the dynamics that drive the development of mathematics. In order to uncover these dynamics, philosophers have turned to historical cases. For example, in his book *Proofs and Refutations*, Lakatos has proposed the following scheme for mathematical development: it starts with a conjecture and its proof follows by a counterexample. This results in a new and finer conjecture, after which the process of proofs and refutations is repeated. This scheme rejects the simple cumulative nature of mathematical development but it still depicts the dynamic as a very internal process. Indeed, apart from the initial conjecture, which may be suggested by things outside of mathematics, the whole process of proofs and refutations makes no appeal to anything outside of mathematics proper. Thus, according to philosophers like Lakatos, mathematics has a historic root in reality but it goes far back to our forefathers from the stone or bronze age who began to count apples and sheep and to measure the fields in Egypt (as Herodotus has it), but then proceeds by dynamics that are internal to mathematics. I will admit that such schemes can account for the development of some mathematical theories such as number theory. However, for many other parts of mathematics, and in particular for those parts that are most useful in physics, such explanations do not suffice. Indeed, mathematics continuously receives impressions from the outside world and although each impression may not be great, their total effect is decisive for the course of the science.

One may liken the development of mathematics to the rolling of a ball on a slightly bumpy surface. Here inertial motion, which one can liken to the internal dynamics of mathematics, will account for the local motion of the ball, but the small bumps (the external influences) are important if one wants to find out where the ball ends up. This mechanical image and in particular its likening of internal progress to inertia may be considered offensive to "pure" mathematicians. Of course I do not want to suggest that internal developments are automatic or simple, so in order to acknowledge that such developments are important and require much ingenuity and work I could replace the ball in the previous image with a complicated mechanical system perturbed by small but secular external perturbations. That will not change my main point, namely that many parts of mathematics continually interact with the non-mathematical world in a way which is decisive for its long term development.

Thirdly, as is already implied by the previous remarks, the interaction between mathematics and physics is not appropriately described by the term "application". It is certainly not an application in the same sense as a Stone Age man applies a stone to crack an oyster. Where the stone and the oyster were developed independently of each other, I have argued that mathematics in fact owes much of its content and form to physics. One may rather liken the application of mathematics to nature to the application of an axe to a tree, the axe having been formed with this purpose in mind. However, even that image does not capture the application of mathematics to physics. Indeed, where it seems reasonable to claim that nature was developed independently of mathematics just as trees were developed independently of axes, I have argued that physics was by no means developed independently of mathematics. So if the term "application" should be upheld, it should rather be likened to the application of a screwdriver to a screw. Here the screwdriver would not exist without the screw nor would the screw exist without the screwdriver. In the same way, mathematics and physics depend upon each other. Of course, one could imagine a mathematics that had not been influenced by physics but it would probably have looked different from the mathematics that is used in modern physics. Similarly, one could imagine a description of nature in non-mathematical terms, and indeed many cultures have developed such descriptions, but they look very different from Western physics.

Rather than talking about applications of mathematics to physics we should perhaps talk about encounters between the two sciences. When such an encounter takes place, both sciences are usually changed leading to the development of a new branch of mathematical physics. I shall not refute that there may exist instances where entirely pure mathematics met with physics, such as when number theory was recently applied in coding theory, but I am sure it has happened very rarely. And it is probably even rarer that mathematics has not benefited from the encounter with physics.

It would be desirable if philosophers would try to develop theories about what happens when mathematics and physics meet. As a historian of mathematics, I shall not venture to characterize this process generally but I shall give some historical considerations of the changing conditions for the interaction between mathematics and physics in the period 1809 to 1950 and give a series of less well known cases where physics has influenced mathematics. These cases are drawn from my own research on the mathematics of Joseph Liouville (1809–1882) (this is the reason for the odd first year in the title) on Heinrich Hertz's mechanics (1894) and on the prehistory of the theory of distributions (?–1950).

Liouville's mathematics

During the first decades of the 19th century, Paris was the unrivalled centre of mathematics and here a highly applied view of mathematics prevailed. Thus it is symptomatic that it was an engineering school, the École polytechnique, that offered the highest mathematics education in the world. As a student at this school, Liouville was brought up with the applied view of mathematics and all through his life he continued to emphasize that the most interesting results of mathematics are indebted to physics and in particular to mechanics. He contributed to many areas of applied mathematics, such as mechanics and potential theory, and his contributions to apparently purer areas of mathematics were often inspired by physics. For example, he proved his theorem about conformal mappings of space in connection to William Thomson's (the later Lord Kelvin's) ideas about electrical images. He also developed a very interesting but mostly unpublished spectral theory for a special type of integral operators that was directly inspired by his work concerning figures of equilibrium of rotating masses of fluid.

Even his theory of differentiation of arbitrary (i.e. non-integer) order had its roots in physics. His notebooks reveal that he developed the theory in order to be able to solve certain integral equations that were of central importance in a research program that Laplace had launched. Laplace had suggested that just as the motions of the celestial bodies could be explained from Newton's gravitational forces between the smallest parts of matter so all other physical phenomena should be explained by actions at a distance between these parts and various imponderable fluids. Along similar lines, Ampère had suggested that all electromagnetic phenomena should be deduced from an elementary force between pairs of infinitesimal conducting elements. Liouville, who was a student of Ampère, pointed out that if this elementary force should be deduced from macroscopic experiments, one was led to integral equations that he showed were solvable using the theory of differentiation of fractional order. The physical origin of the theory shows up in the final theory, not least in the definition of fractional derivatives that only works as intended if the considered functions tend to zero at infinity, as potentials and forces do.

This example shows that something as intangible as a physical research program (here Laplacian physics) can influence which mathematical theories are developed and even influence the form of the mathematical theory.

A less surprising but more important example of a piece of physics-inspired mathematics is the theory of differential equations that Liouville developed together with his friend Charles Sturm and that was later named after them. They were both interested in Fourier's and Poisson's theory of heat conduction in homogeneous materials (and the theory of trigonometric series developed for this purpose) and attempted to generalize it to heterogeneous materials. However, in the general case they could not find usable explicit expressions for the solutions (eigenfunctions) so they had to 'make do with' deducing their properties directly from the differential equation. The properties they investigated were qualitative ones such as the number and behaviour of the zeroes (Sturm's comparison and oscillation theorems). Based on Sturm's results, Liouville succeeded in proving the generalized Fourier series of an 'arbitrary' function[1].

Thus it was the generality of the physical problem that forced the two friends to take a great step in the direction of a qualitative theory of differential equations. Such a development from a formula based quantitative mathematics toward a more qualitative and concept based mathematics has been emphasized as an important characteristic of the development of mathematics in the 19th century. So here we have an example of a physical problem that has contributed to a fundamental change of course in mathematics. Later the problem of the stability of the solar system made Poincaré go further in this direction.

Hertz's mechanics

From around 1840, the centre of mathematics shifted towards Germany where a neo-humanistic conception of mathematics, the product of and a tribute to the human spirit, overshadowed its applicability. This led to a larger separation between pure and applied mathematics, a separation that has sometimes been exaggerated by historians of mathematics. Even though mathematics achieved greater autonomy vis-à-vis the natural world, its applications, in particular in physics, remained a constant source of inspiration. For example, when earlier histories of mathematics describe the invention of non-Euclidean and Riemannian geometry as the result of a purely mathematical analysis of the axiom system of geometry, and in particular the role of the parallel postulate, they give a totally distorted picture of the actual historical driving forces behind the development. The central actors in the history: Gauss, Lobachevski, Riemann and Helmholtz, were not interested in axiom systems but wanted to investigate the nature of space (the physical one) by first examining the mathematically possible geometries and subsequently empirically investigate which of those geometries describes the space we live in.

When the new non-Euclidean geometry in its Riemannian form became generally known around 1870, mathematicians like Lipschitz and Darboux immediately used it to give a new differential geometric formulation of mechanics. Expressed in a slightly modernized language, they showed how the motion of a system of point masses acting on each other with conservative forces can be described as a geodesic motion of one point in a high dimensional Riemannian manifold whose metric is defined in terms of the potential defining the forces. Also in this case the new physical application gave rise to new mathematical concepts and results.

The physicist Heinrich Hertz rather independently took a similar step. During the last three years of his life, he wrote a book on the *Principles of Mechanics* in which he formulated a foundation of physics that did not operate with force or energy as fundamental concepts but was instead formulated in differential geometric terms. In the introduction to the book he stated that the physical content and the mathematical form of the book were in principle independent of each other. However, his surviving notes and drafts of the book reveal that while he worked on the book the mathematics and the physics infected each other in more decisive ways than the finished book may suggest. For example, in the third draft of the book Hertz defined the "reduced component of a vector quantity in a given direction", corresponding to the modern concept of the covariant component of the vector, because it allowed him to formulate Lagrange's and Hamilton's equations in the usual way. So here is an instance where an important differential geometric concept was introduced for physical reasons.

There were also instances where the mathematical form influenced the physical theory Hertz put forward. For example, he suggested an image (a model) of matter as consisting of infinitely many infinitely small equal mass-parts. Philosophers have been interested in this image because it became the source of inspiration for certain elements of Wittgenstein's philosophy. Now, Hertz's drafts show that he introduced the mass-parts because they allowed him to deduce his fundamental non-Euclidean line element in configuration space from the Euclidean distance between the mass-parts. Thus, the mass-parts were introduced because they served a purpose in the geometric formalism.

These examples indicate that one can only understand a work like Hertz's mechanics if one takes both the physical theory and its mathematical form serious.

[1] Sturm's theorem concerning the number of real roots of a polynomial in an interval was also a result of these investigations. It can be viewed as a discrete version of Sturm's oscillation theorem.

The theory of distributions

Until around 1900, mathematics was philosophically closely tied to the real world. Plato considered mathematics as a description of the ideal world, Aristotle considered it an abstract description of the world and Kant maintained that mathematics is an a priori construction in our intuition of space and time forming a basis for structuring our perceptions of the world. However, partly as a result of the invention of non-Euclidean geometry, mathematics began to be viewed as a creation of the human intellect, independent of the real world. During the 20th century the preferred formalistic philosophy of mathematics declared that mathematics is a game with symbols according to fixed but arbitrary rules. However, one must not forget that even Hilbert, "the father" of formalistic philosophy, repeatedly stated that axiomatization is an act of logically tidying up already developed theories. Axioms are not invented freely by the mathematician. They are a part of the dynamics that drive mathematics in which problems and interaction with other fields, not least physics, continues to play an important role. The creation of the theory of distributions is a good example of this interplay.

Already in 1822, in connection with his investigation of heat conduction in solids, Fourier interchanged summation and integration in the expression of a Fourier series of an arbitrary function and concluded that the function

$$\frac{1}{\pi}\left(\frac{1}{2}+\sum_{n=1}^{\infty}\cos nx\right)$$

has the property that if it is multiplied by f (x) and integrated between $-\pi$ and π the result will be f (0). However, when infinite quantities and divergent series were banished from mathematics later in the 19th century and the modern function concept became generally accepted such a function had to be rejected. Thus when Darboux published Fourier's collected works he added a footnote explaining that the above sum had no meaning.

Yet a 'function' with such properties continued to turn up in different areas of physics. In electrostatics, it was difficult to avoid it in connection with the description of point charges and more generally in connection with the so-called Green's function. Kirchhoff introduced the function explicitly in 1882 and it became a central element in Heaviside's analysis of telegraphy and electric networks. In this connection, it was defined as the derivative of the Heaviside step function. It got its name, the delta function, when Dirac used it in a decisive way in his account of quantum mechanics. However, all these applications took place outside of mainstream mathematics and mathematicians continued to point out that no function with these properties exists. Indeed, when von Neumann developed a new formalism for quantum mechanics, this was primarily because he was dissatisfied with Dirac's use of the illegitimate delta function.

It is well known that Laurent Schwartz was the first mathematician to develop a rigorous foundation for the delta function[2]. However, his extension of the function concept was more directly a result of considerations concerning generalized solutions of differential equations. In classical analysis $f\,(x{-}t)$ is only a solution of the wave equation if it is twice differentiable. This is inconvenient since one can easily imagine waves with corners. From the end of the 19th century, several methods were suggested to generalize the concept of a solution of a differential equation such that non-differentiable functions could also be solutions of e.g. the wave equation.

In his autobiography, Schwartz explains that he was not entirely satisfied with these methods because it remained a mystery what the derivatives of the non-differentiable functions were. This was one of the problems that he clarified with his theory of distributions developed in the period 1945–1950. Another problem concerned the Fourier transformation that had been used from the 18th century onwards to solve differential equations and more specifically by Wiener in an attempt to rescue Heaviside's operational calculus. The problem here was that the Fourier transformation only applies to functions that tend to zero sufficiently fast at $\pm\infty$. Hahn, Wiener and in particular Bochner had devised various ways around this problem but Schwartz's solution by way of tempered distributions was much more general and elegant.

Thus, the problems that led to the theory of distributions came from physics and applied mathematics but the solutions came from the newly created functional analysis. This branch of mathematics was developed as a pure mathematical discipline but its importance had undoubtedly been boosted by its almost immediate application in quantum mechanics. And once functional analysis was applied to distribution theory, it was itself enriched. Indeed, Dieudonné suggested to Schwartz that he ought to anchor his concept of convergence of distributions in a topology on the space of distributions. This gave rise to the LF-topologies and other more general topological concepts.

Conclusion

During the century between Liouville and Schwartz, the interplay between mathematics and physics changed both sociologically and philosophically. As far as sociology is concerned, it is worth remarking that while the early 19th century scientist was often mathematician and physicist in one person the mathematical and physical communities have been almost disjoint during the 20th century. Fourier for example spent much time in the laboratory constructing experiments related to the phenomena of heat conduction, whose mathematical description led

[2] Sobolev had already invented distributions a decade before Schwartz but he did not notice that they could be used to legitimize the delta function and he did not develop the theory as far as Schwartz did.

him to Fourier series and the delta function. Schwartz, on the other hand, only knew of the physical problems at second hand. Philosophically, mathematics in Liouville's time had its root in reality whereas 20th century mathematics was, in principle, liberated from such ties. Still, the complicated interplay between mathematics and physics continued during the entire century in a form that benefited them both. This mutual enrichment is undoubtedly a decisive condition for the effectiveness of mathematics in the description of nature.

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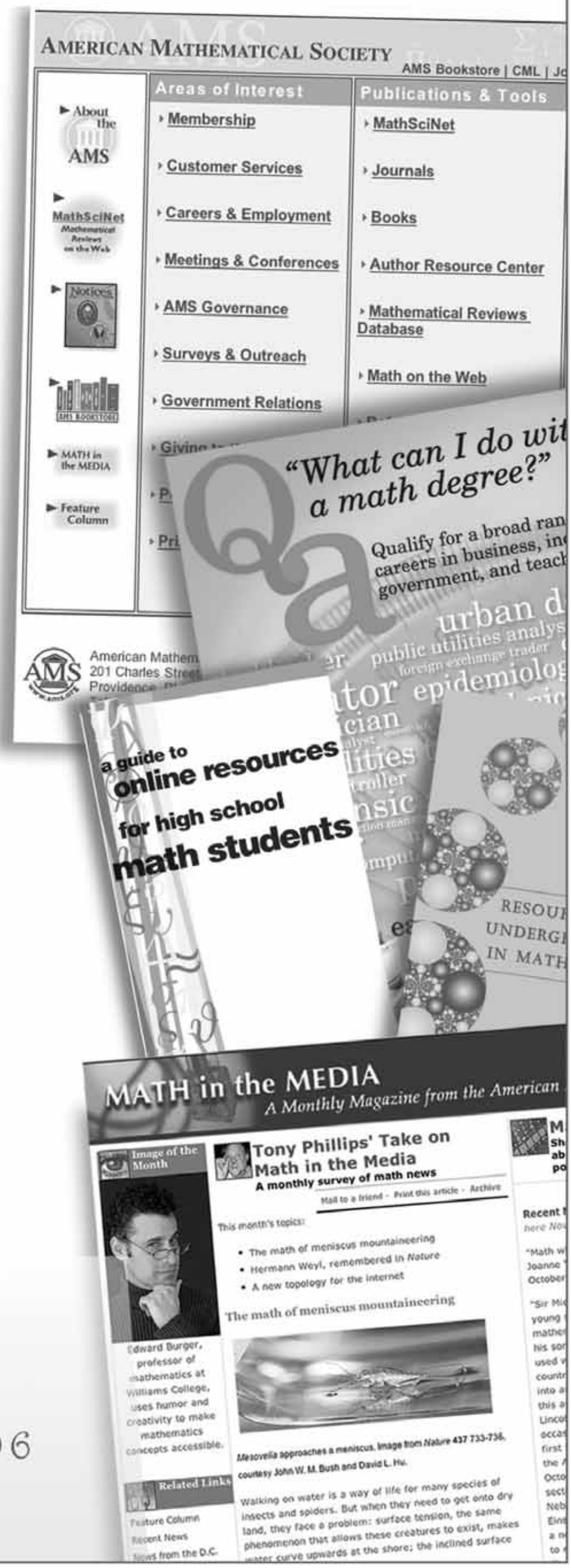

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An interview with Robert Aumann

Sergiu Hart (Jerusalem)

The Royal Swedish Academy has awarded the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel, 2005, jointly to ***Robert J. Aumann****, Center for the Study of Rationality, Hebrew University of Jerusalem, Israel, and to* ***Thomas C. Schelling****, Department of Economics and School of Public Policy, University of Maryland, College Park, MD, USA, "for having enhanced our understanding of conflict and cooperation through game-theory analysis". The Newsletter is glad to be able to publish excerpts of an interview that Sergiu Hart (Aumann's colleague at the Center for the Study of Rationality) conducted with Aumann in 2004. The complete interview was published in the journal* Macroeconomic Dynamics ***9*** *(5), pp. 683–740 (2005), ©* *Cambridge University Press. We thank Prof. Aumann, Prof. Hart and Cambridge University Press for the reproduction permission and Prof. Schulze-Pillot from DMV-Mitteilungen for the compilation of these excerpts.*

Who is Robert Aumann? Is he an economist or a mathematician? A rational scientist or a deeply religious man? A deep thinker or an easygoing person?

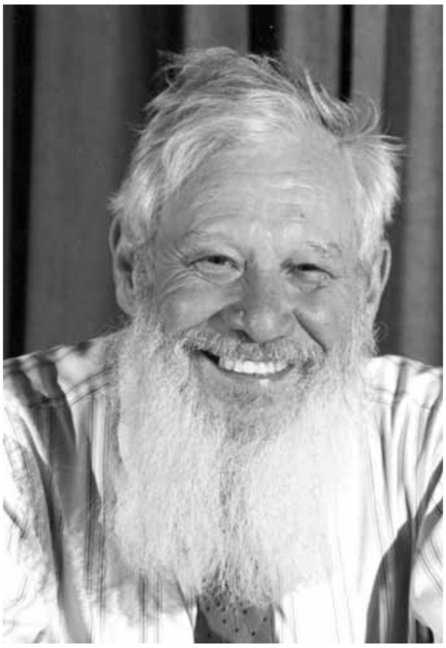
These seemingly disparate qualities can all be found in Aumann; all are essential facets of his personality. A pure mathematician who is a renowned economist, he has been a central figure in developing game theory and establishing its key role in modern economics. He has shaped the field through his fundamental and pioneering work, work that is conceptually profound, and much of it mathematically deep. He has greatly influenced and inspired many people: his students, collaborators, colleagues, and anyone who has been excited by reading his papers or listening to his talks.

Aumann promotes a unified view of rational behavior, in many different disciplines: chiefly economics, but also political science, biology, computer science, and more. He has broken new ground in many areas, the most notable being perfect competition, repeated games, correlated equilibrium, interactive knowledge and rationality, and coalitions and cooperation.

But Aumann is not just a theoretical scholar, closed in his ivory tower. He is interested in real-life phenomena and issues, to which he applies insights from his research. He is a devoutly religious man; and he is one of the founding fathers – and a central and most active member – of the multidisciplinary Center for the Study of Rationality at the Hebrew University in Jerusalem.

Aumann enjoys skiing, mountain climbing, and cooking – no less than working out a complex economic question or proving a deep theorem. He is a family man, a very warm and gracious person – of an extremely subtle and sharp mind.

This interview catches a few glimpses of Robert Aumann's fascinating world. It was held in Jerusalem on three consecutive days in September 2004. I hope the reader will learn from it and enjoy it as much as we two did.



Bob Aumann, circa 2000

Sergiu HART: Good morning, Professor Aumann. Let's start with your scientific biography, namely, what were the milestones on your scientific route?

Robert AUMANN: I did an undergraduate degree at City College in New York in mathematics, then on to MIT, where I did a doctorate with George Whitehead in algebraic topology, then on to a post-doc at Princeton with an operations research group affiliated with the math department. There I got interested in game theory. From there I went to the Hebrew University in Jerusalem, where I've been ever since. That's the broad outline.

Now to fill that in a little bit. My interest in mathematics actually started in high school – the Rabbi Jacob Joseph Yeshiva (Hebrew Day School) on the lower east side of New York City. There was a marvelous teacher of mathematics there, by the name of Joseph Gansler. The classes were very small; the high school had just started operating. He used to gather the students around his desk. What really turned me on was geometry, theorems and proofs. So all the credit belongs to Joey Gansler.

Then I went on to City College. Actually I did a bit of soul-searching when finishing high school, on whether to become a Talmudic scholar, or study secular subjects at a university. For a while I did both. But after one semester it became too much for me and I made the hard decision to quit the yeshiva and study mathematics.

At City College, there was a very active group of mathematics students. A lot of socializing went on. There was a table in the cafeteria called the mathematics table. Between classes we would sit there and have ice cream and –

Sergiu Hart, Mike Maschler, Bob Aumann, Bob Wilson, and Oskar Morgenstern, at the 1994 Morgenstern Lecture, Jerusalem

H: Discuss the topology of bagels?

A: Right, that kind of thing. A lot of chess playing, a lot of math talk. We ran our own seminars, had a math club. Some very prominent mathematicians came out of there – Jack Schwartz of Dunford–Schwartz fame, Leon Ehrenpreis, Alan Shields, Leo Flatto, Martin Davis, D. J. Newman. That was a very intense experience. From there I went on to graduate work at MIT, where I did a doctorate in algebraic topology with George Whitehead.

Let me tell you something very moving relating to my thesis. As an undergraduate, I read a lot of analytic and algebraic number theory. What is fascinating about number theory is that it uses very deep methods to attack problems that are in some sense very "natural" and also simple to formulate. A schoolchild can understand Fermat's last theorem, but it took extremely deep methods to prove it. Another interesting aspect of number theory was that it was absolutely useless – pure mathematics at its purest.

In graduate school, I heard George Whitehead's excellent lectures on algebraic topology. Whitehead did not talk much about knots, but I had heard about them, and they fascinated me. Knots are like number theory: the problems are very simple to formulate, a schoolchild can understand them; and they are very natural, they have a simplicity and immediacy that is even greater than that of Fermat's last theorem. But it is very difficult to prove anything at all about them; it requires really deep methods of algebraic topology. And, like number theory, knot theory was totally, totally useless.

So, I was attracted to knots. I went to Whitehead and said, I want to do a PhD with you, please give me a problem. But not just any problem; please, give me an open problem in knot theory. And he did; he gave me a famous, very difficult problem – the "asphericity" of knots – that had been open for twenty-five years and had defied the most concerted attempts to solve.

Though I did not solve that problem, I did solve a special case. The complete statement of my result is not easy to formulate for a layman, but it does have an interesting implication that even a schoolchild can understand and that had not been known before my work: alternating knots do not "come apart," cannot be separated.

So, I had accomplished my objective – done something that i) is the answer to a "natural" question, ii) is easy to formulate, iii) has a deep, difficult proof, and iv) is absolutely useless, the purest of pure mathematics.

It was in the fall of 1954 that I got the crucial idea that was the key to proving my result. The thesis was published in the *Annals of Mathematics* in 1956; but the proof was essentially in place in the fall of 1954.

That's Act I of the story. And now, the curtain rises on Act II – fifty years later, almost to the day. It's 10 p.m., and the phone rings in my home. My grandson Yakov Rosen is on the line. Yakov is in his second year of medical school. "Grandpa," he says, "can I pick your brain? We are studying knots. I don't understand the material, and think that our lecturer doesn't understand it either. For example, could you explain to me what, exactly, are 'linking numbers'?" "Why are you studying knots?" I ask. "What do knots have to do with medicine?" "Well," says Yakov, "sometimes the DNA in a cell gets knotted up. Depending on the characteristics of the knot, this may lead to cancer. So, we have to understand knots."

I was completely bowled over. Fifty years later, the "absolutely useless" – the "purest of the pure" – is taught in the second year of medical school, and my grandson is studying it. I invited Yakov to come over, and told him about knots, and linking numbers, and my thesis.

Moving into Game Theory

H: Okay, now that we are all tied up in knots, let's untangle them and go on. You did your PhD at MIT in algebraic topology, and then what?

A: Then for my post-doc, I joined an operations research group at Princeton. This was a rather sharp turn because algebraic topology is just about the purest of pure mathematics and operations research is very applied. It was a small group of about ten people at the Forrestal Research Center, which is attached to Princeton University.

H: In those days operations research and game theory were quite connected. I guess that's how you –

A: – became interested in game theory, exactly. There was a problem about defending a city from a squadron of aircraft most of which are decoys – do not carry any weapons – but a small percentage do carry nuclear weapons. The project was sponsored by Bell Labs, who were developing a defense missile.

At MIT I had met John Nash, who came there in '53 after doing his doctorate at Princeton. I was a senior graduate student and he was a Moore instructor, which was a prestigious instructorship for young mathematicians. So he was a little older than me, scientifically and also chronologically. We got to know each other fairly well and I heard from him about game theory. One of the problems that we kicked around was that of dueling – silent duels, noisy duels, and so on. So when I came to Princeton, although I didn't know much about game theory at all, I had heard about it; and when we were given this problem by Bell Labs, I was able to say, this sounds a little bit like what Nash was telling us; let's examine it from that point of view. So I started studying game theory; the rest is history, as they say.

Repeated Games

H: Since you started talking about these topics, let's perhaps move to Mathematica, the United States Arms Control and Disarmament Agency (ACDA), and repeated games. Tell us about your famous work on repeated games. But first, what are repeated games?

A: It's when a single game is repeated many times. How exactly you model "many" may be important, but qualitatively speaking, it usually doesn't matter too much.

H: Why are these models important?

A: They model ongoing interactions. In the real world we often respond to a given game situation, not so much because of the outcome of that particular game, as because our behavior in a particular situation may affect the outcome of future situations in which a similar game is played. For example, let's say somebody promises something and we respond to that promise and then he doesn't keep it – he double-crosses us. He may turn out a winner in the short term, but a loser in the long term: if I meet up with him again, I won't trust him. Whether he is rational, whether we are both rational, is reflected not only in the outcome of the particular situation in which we are involved today, but also in how it affects future situations.

Another example is revenge, which in the short term may seem irrational; but in the long term, it may be rational, because if you take revenge, then the next time you meet that person, he will not kick you in the stomach. Altruistic behavior, revengeful behavior, any of those things, make sense when viewed from the perspective of a repeated game, but not from the perspective of a one-shot game. So, a repeated game is often more realistic than a one-shot game: it models ongoing relationships.

In 1959 I published a paper on repeated games (*Contrib GameTh* IV). The brunt of that paper is that cooperative behavior in the one-shot game corresponds to equilibrium or egoistic behavior in the repeated game. This is to put it very simplistically.

H: There is the famous "Folk Theorem." In the seventies you named it, in your survey of repeated games. The name has stuck.

A: The Folk Theorem is quite similar to my '59 paper, but a good deal simpler, less deep. I called it the Folk Theorem because its authorship is not clear, like folk music, folk songs. It was in the air in the late fifties and early sixties.

H: Yours was the first full formal statement and proof of something like this. Even Luce and Raiffa, in their very influential '57 book, *Games and Decisions*, don't have the Folk Theorem.

A: The first people explicitly to consider repeated non-zero-sum games of the kind treated in my '59 paper were Luce and Raiffa. But as you say, they didn't have the Folk Theorem. Shubik's book *Strategy and Market Structure*, published in '59, has a special case of the Folk Theorem, with a proof that has the germ of the general proof.

At that time people did not necessarily publish everything they knew – in fact, they published only a small proportion of what they knew, only really deep results or something really interesting and nontrivial in the mathematical sense of the word – which is not a good sense. Some very important things would be considered trivial by a mathematician.

For example, take the Cantor diagonal method. Perhaps it really is "trivial." But it is extremely important; inter alia, Gödel's famous incompleteness theorem is based on it.

So, even within pure mathematics the trivial may be important. But certainly outside of it, there are interesting observations that are mathematically trivial – like the Folk Theorem. I knew about the Folk Theorem in the late fifties, but was too young to recognize its importance. I wanted something deeper, and that is what I did in fact publish. That's my '59 paper. It's a nice paper – my first published paper in game theory proper. But the Folk Theorem, although much easier, is more important. So it's important for a person to realize what's important. At that time I didn't have the maturity for this.

Quite possibly, other people knew about it. People were thinking about long-term interaction. There are Shapley's stochastic games, Everett's recursive games, the work of Gillette, and so on. I wasn't the only person thinking about repeated games. Anybody who thinks a little about repeated games, especially if he is a mathematician, will very soon hit on the Folk Theorem. It is not deep.

H: That's '59; let's move forward.

A: In the early sixties Morgenstern and Kuhn founded a consulting firm called Mathematica, based in Princeton, not to be confused with the software that goes by that name today. In '64 they started working with the United States Arms Control and Disarmament Agency on a project that had to do with the Geneva disarmament negotiations: a series of negotiations with the Soviet Union, on arms control and disarmament. The people on this project included Kuhn, Gerard Debreu, Herb Scarf, Reinhard Selten, John Harsanyi, Jim Mayberry, Mike Maschler, Dick Stearns (who came in a little later), and me. What struck Maschler and me was that these negotiations were taking place again and again; a good way of modeling this is a repeated game. The only thing that distinguished it from the theory of the late fifties that we discussed before is that these were repeated games of incomplete information. We did not know how many weapons the Russians held, and the Russians did not know how many weapons we held. What we – the United States – proposed to put into the agreements might influence what the Russians thought or knew that we had, and this would affect what they would do in later rounds.

H: What you do reveals something about your private information. For example, taking an action that is optimal in the short run may reveal to the other side exactly what your situation is, and then in the long run you may be worse off.

A: Right. This informational aspect is absent from the previous work, where everything was open and above board, and the issues are how one's behavior affects future interaction. Here the question is how one's *behavior* affects the other player's *knowledge*.

So Maschler and I, and later Stearns, developed a theory of repeated games of incomplete information. This theory was set forth in a series of research reports between '66 and '68, which for many years were unavailable.

H: Except to the aficionados, who were passing bootlegged copies from mimeograph machines. They were extremely hard to find.

At the 1994 Morgenstern Lecture, Jerusalem. Bob Aumann (front row), Don Patinkin, Mike Maschler, Ken Arrow (second row, left to right), Tom Schelling (third row, second from left); also Marshall Sarnat, Jonathan Shalev, Michael Beenstock, Dieter Balkenborg, Eytan Sheshinski, Edna Ullmann-Margalit, Maya Bar-Hillel, Gershon Ben-Shakhar, Benjamin Weiss, Reuben Gronau, Motty Perry, Menahem Yaari, Zur Shapira, David Budescu, Gary Bornstein

A: Eventually they were published by MIT Press in '95, together with extensive postscripts describing what has happened since the late sixties – a tremendous amount of work. The mathematically deepest started in the early seventies in Belgium, at CORE, and in Israel, mostly by my students and then by their students. Later it spread to France, Russia, and elsewhere. The area is still active.

H: What is the big insight?

A: It is always misleading to sum it up in a few words, but here goes: in the long run, you cannot use information without revealing it; you can use information only to the extent that you are willing to reveal it. A player with private information must choose between not making use of that information – and then he doesn't have to reveal it – or making use of it, and then taking the consequences of the other side finding it out. That's the big picture.

H: In addition, in a non-zero-sum situation, you may *want* to pass information to the other side; it may be mutually advantageous to reveal your information. The question is how to do it so that you can be trusted, or in technical terms, in a way that is incentive-compatible.

A: The bottom line remains similar. In that case you can use the information, not only if you are willing to reveal it, but also if you actually *want* to reveal it. It may actually have positive value to reveal the information. Then you use it *and* reveal it.

The Continuum in Economic Theory

H: Let's move to another major work of yours, "Markets with a Continuum of Traders" (*Econometrica* 1964): modeling perfect competition by a continuum.

A: At Princeton in '60–'61, the Milnor–Shapley paper "Oceanic Games" caught my fancy. It treats games with an ocean – nowadays we call it a continuum – of small players, and a small number of large players, whom they called atoms. Then in the fall of '61, at a conference at which Henry Kissinger and Lloyd Shapley were present, Herb Scarf gave a talk about large markets. He had a countable infinity of players. Before that, in '59, Martin Shubik had published a paper called "Edgeworth Market Games," in which he made a connection between the core of a large market game and the competitive equilibrium. Scarf's model somehow wasn't very satisfactory, and Herb realized that himself; afterwards, he and Debreu proved a much more satisfactory version, in their *IER* 1963 paper. The bottom line was that, under certain assumptions, the core of a large economy is close to the competitive solution, the solution to which one is led from the law of supply and demand. I heard Scarf's talk, and, as I said, the formulation was not very satisfactory. I put it together with the result of Milnor and Shapley about oceanic games, and realized that *that* has to be the right way of treating this situation: a continuum, not the countable infinity that Scarf was using. It took a while longer to put all this together, but eventually I did get a very general theorem with a continuum of traders. It has very few assumptions, and it is not a limit result. It simply says that the core of a large market is the *same* as the set of competitive outcomes.

H: Indeed, the introduction of the continuum idea to economic theory has proved indispensable to the advancement of the discipline. In the same way as in most of the natural sciences, it enables a precise and rigorous analysis, which otherwise would have been very hard or even impossible.

A: The continuum is an approximation to the "true" situation, in which the number of traders is large but finite. The purpose of the continuous approximation is to make available the powerful and elegant methods of the branch of mathematics called "analysis," in a situation where treatment by finite methods would be much more difficult or even hopeless – think of trying to do fluid mechanics by solving n-body problems for large n.

H: The continuum is the best way to start understanding what's going on. Once you have that, you can do approximations and get limit results.

A: Yes, these approximations by finite markets became a hot topic in the late sixties and early seventies. The '64 paper was followed by the *Econometrica* '66 paper on existence of competitive equilibria in continuum markets; in '75 came the paper on values of such markets, also in *Econometrica*. Then there came later papers using a continuum, by me with or without coauthors, by Werner Hildenbrand and his school, and by many, many others.

Sergiu Hart and Bob Aumann, at the 2005 Nobel Award Ceremony, Stockholm

The Center for Rationality

H: Let's make a big jump. In 1991, the Center for Rationality was established at the Hebrew University.

A: Yoram Ben-Porath, who was rector of the university, asked Menahem Yaari, Itamar Pitowsky, Motty Perry, and me to make a proposal for establishing an interdisciplinary center. What came out was the Center for Rationality, which you, Sergiu, directed for its first eight critical years; it was you who really got it going and gave it its oomph. The Center is really unique in the whole world in that it brings together very many disciplines. Throughout the world there are several research centers in areas connected with game theory. Usually they are associated with departments of economics: the Cowles Foundation at Yale, CORE in Louvain, the late Institute for Mathematical Studies in the Social Sciences at Stanford. The Center for Rationality at the Hebrew University is quite different, in that it is much broader. The basic idea is "rationality": behavior that advances one's own interests. This appears in many different contexts, represented by many academic disciplines. The Center has members from mathematics, economics, computer science, evolutionary biology, general philosophy, philosophy of science, psychology, law, statistics, the business school, and education. There is nothing in the world even approaching the breadth of coverage of the Center for Rationality.

It is broad but nevertheless focused. There would seem to be a contradiction between breadth and focus, but our Center has both – breadth and focus. The breadth is in the number and range of different disciplines that are represented at the Center. The focus is, in all these disciplines, on rational, self-interested behavior – or the lack of it. We take all these different disciplines, and we look at a certain segment of each one, and at how these various segments from this great number of disciplines fit together.

H: Can you give a few examples? Readers may be surprised to hear about some of these connections.

A: I'll try; let's go through some applications. In computer science we have distributed computing, in which there are many different processors. The problem is to coordinate the work of these processors, which may number in the hundreds of thousands, each doing its own work.

H: That is, how processors that work in a decentralized way reach a coordinated goal.

A: Exactly. Another application is protecting computers against hackers who are trying to break down the computer. This is a very grim game, but it is a game. Still another kind comes from computers that solve games, play games, and design games – like auctions – particularly on the Web.

Biology is another example where one might think that games don't seem particularly relevant. But they are! There is a book by Richard Dawkins called *The Selfish Gene*. This book discusses how evolution makes organisms operate as if they were promoting their self-interest, acting rationally. What drives this is the survival of the fittest. If the genes that organisms have developed in the course of evolution are not optimal, are not doing as well as other genes, then they will not survive. There is a tremendous range of applications of game-theoretic and rationalistic reasoning in evolutionary biology.

Economics is of course the main area of application of game theory. The book by von Neumann and Morgenstern that started game theory rolling is called *The Theory of Games and Economic Behavior*. Psychology – that of decision-making – has close ties to game theory; whether behavior is rational or irrational – the *subject* is still rationality.

There is much political application of game theory in international relations. There also are national politics, like various electoral systems. Another aspect is forming a government coalition: if it is too small – a minimal winning coalition – it will be unstable; if too large, the prime minister will have too little influence. What is the right balance?

Law: more and more, we have law and economics, law and game theory. There are studies of how laws affect the behavior of people, the behavior of criminals, the behavior of the police. All these things are about self-interested, rational behavior.

Biography

H: Let's move now to your personal biography.

A: I was born in 1930 in Frankfurt, Germany, to an orthodox Jewish family, the second of two boys. My father was a wholesale textile merchant – a fine, upright man, a loving, warm father. My mother was extraordinary. She got a bachelor's degree in England in 1914, at a time when that was very unusual for women. She was a medal-winning long-distance swimmer, sang Schubert lieder while accompanying herself on the piano, introduced us children to nature, music, reading. We would walk the streets and she would teach us the names of the trees. At night we looked at the sky and she taught us the names of the constellations. When I was about twelve, we started reading Dickens's *A Tale of Two Cities* together – until the book gripped me and I raced ahead alone. From then on, I read voraciously. She even introduced me to interactive epistemology; look at the "folk ditty" in *GEB* '96. She always encouraged, always pushed us along, gently, unobtrusively, always allowed us to make our own decisions.

We got away in 1938. Actually we had planned to leave already when Hitler came to power in 1933, but for one reason or another we didn't. People convinced my parents that it wasn't so bad; it will be okay, this thing will blow over. The German people will not allow such a madman to take over, etc., etc. A well-known story. But it illustrates that when one is in the middle of things it is very, very difficult to see the future. Things seem clear in hindsight, but in the middle of the crisis they are very murky.

H: Especially when it is a slow-moving process, rather than a dramatic change: every time it is just a little more and you say, that's not much, but when you look at the integral of all this, suddenly it is a big change.

A: That is one thing. But even more basically, it is just difficult to see. Let me jump forward from 1933 to 1967. I was in Israel and there was the crisis preceding the Six-Day War. In hindsight it was "clear" that Israel would come out on top of that conflict. But at the time it wasn't at all clear, not at all. I vividly remember the weeks leading up to the Six-Day War, the crisis in which Nasser closed the Tiran Straits and massed troops on Israel's border; it wasn't at all clear that Israel would survive. Not only to me, but to anybody in the general population. Maybe our generals were confident, but I don't think so, because our government certainly was not confident. Prime Minister Eshkol was very worried. He made a broadcast in which he stuttered and his concern was very evident, very real. Nobody knew what was going to happen; people were very worried, and I, too, was very worried. I had a wife and three children and we all had American papers. So I said to myself, Johnny, don't make the mistake your father made by staying in Germany. Pick yourself up, get on a plane and leave, and save your skin and that of your family; because there is a very good chance that Israel will be destroyed and the inhabitants of Israel will be wiped out totally, killed, in the next two or three weeks. Pick yourself up and *GO*.

I made a conscious decision not to do that. I said, I am staying. Herb Scarf was here during the crisis. When he left, about two weeks before the war, we said good-bye, and it was clear to both of us that we might never see each other again.

This illustrates that it is very difficult to judge a situation from the middle of it. When you're swimming in a big lake, it's difficult to see the shore, because you are low, you are inside it. One should not blame the German Jews or the European Jews for not leaving Europe in the thirties, because it was difficult to assess the situation.

We did get away in time, in 1938. We left Germany, and made our way to the United States. In this passage, my parents lost all their money. They had to work extremely hard in the United States to make ends meet, but nevertheless they gave their two children, my brother and myself, a good Jewish and a good secular education.

When the State of Israel was created in 1948, I made a determination eventually to come to Israel, but that didn't actually happen until 1956. In 1954 I met an Israeli girl, Esther Schlesinger, who was visiting the United States. We fell in love, got engaged, and got married. We had five children; the oldest, Shlomo, was killed in action in Lebanon in 1982. My other children are all happily married. Shlomo's widow also remarried and she is like a daughter to us. Shlomo had two children, the second one born after he was killed. Altogether I now have seventeen grandchildren and one great-grandchild. We have a very good family relationship, do a lot of things together. One of the things we like best is skiing. Every year I go with a different part of the family. Once in four or five years, all thirty of us go together.

The end

H: Any closing "words of wisdom"?

A: Just one: Game theory is ethically neutral. That is, game theorists don't necessarily advocate carrying out the normative prescriptions of game theory. Bacteriologists do not advocate disease, they study it; similarly, studying self-interested behavior is different from advocating it. Game theory says nothing about whether the "rational" way is morally or ethically right. It just says what rational – self-interested – entities will do; not what they "should" do, ethically speaking. If we want a better world, we had better pay attention to where rational incentives lead.

H: That's a very good conclusion to this fascinating interview. Thank you.

A: And thank *you*, Sergiu, for your part in this wonderful interview.

The interviewer, Sergiu Hart [hart@huji.ac.il, http://www.ma.huji.ac.il/hart], is a Professor of Mathematics, a Professor of Economics, and the Kusiel-Vorreuter University Professor at the Hebrew University of Jerusalem. He was the Founding Director of its Center for the Study of Rationality (http://www.ratio.huji.ac.il). He is now the President of the Israel Mathematical Union (http://imu.org.il).

The *Atractor Project* in Portugal

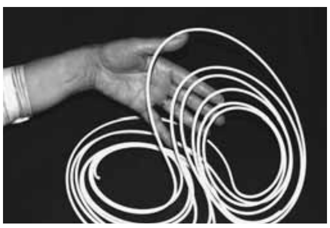
Manuel Arala Chaves (Porto, Portugal)

Atractor is the Portuguese word for attractor and it was the name chosen for a non-profit association created in April 1999 in Portugal for the popularisation of mathematics. Its purpose is to *attract* people to mathematics at different levels, trying to reach the broadest possible cross-section of the public. It tries to do so by giving a sense not only of the beauty and importance of mathematical ideas and some kind of understanding of them but also of the relevance of mathematical applications in all other sciences and in everyday life. Its methodology consists of getting people actively involved by proposing physical or virtual interactive exhibits.

Translation of some extracts from the initial document [1] proposing the creation of Atractor:
«for an important part of the population, even considered cultivated by usual standards (i.e. with an humanistic culture), it is very hard to imagine what else can be done (or discovered) in mathematics and for many people the word mathematics hardly means more than something related with calculations; (...) still worse, the word mathematics often has a strong negative impact related with generalised failure in school (...) and the idea that mathematics is a «finished» if not «dead» science, where all that existed to be discovered has already been discovered, is not uncommon». Atractor «may help to (...) change the attitude (...) towards mathematics (...)»

Attractors are important mathematical objects, very often with unusual and appealing shapes. They could not therefore be absent from Atractor's activities! Here are two stages in the production by Atractor (2000) of a real physical model of an orbit of a Lorenz attractor using stereolitography (the orbit begins to emerge from the powder)

and here is the final model.



The Sierpinski attractor is also present in one exhibition: each visitor throws a three colored die and a point is added to the gradually forming Sierpinski triangle.

Since Atractor's conception, an important aim has been the creation of an Interactive Mathematics Centre with a permanent exhibition and regular activities targeting schools and the general population. This centre would also be used as a privileged place for observation of attitudes concerning the proposed exhibits and activities and in a way would allow some sort of didactic experimentation. The intention is that this centre should not concentrate exclusively on itself; it should organise activities and travelling mathematical exhibitions for the whole country.

A guarantee for the national scope of its activities was given by the composition of Atractor itself; present institutional members include as founders the Association of Mathematical Teachers (APM) and the Portuguese Mathematical Society (SPM), Faculties of Science from Coimbra, Lisbon and Porto and the Universities of Aveiro and Porto. These groups were joined later by Ciência Viva (a pioneering program for developing experimental scientific activities in schools) and the Higher School for Technology of Bragança Polytechnic Institute. The Town Council of Ovar was also a founder member since the project for the intended centre was in the small town of Ovar.

Mathematics Alive

Some temporary exhibitions were organised in various towns during 1999 and 2000. The first one took place even before the legal existence of Atractor but by far the most important one was conceived, developed and built after an invitation in March 2000 from Ciência Viva and the Minister of Science and Technology to organise a temporary exhibition to commemorate the World Year for Mathematics. This Exhibition, called *Matemática Viva* [2] (Mathematics Alive) was inaugurated in November 2000 at Pavilhão do Conhecimento (Pavilion of Knowledge) in Lisbon and was due to last for 4–6 months; it was a great success and is still there six years later. Pavilhão do Conhecimento has always had exhibitions (temporary and permanent) from well-known science centres in Europe and USA (Exploratorium, La Villette, Eureka, Deutsches Museum, etc) but Atractor's Matemática Viva was the first one to have been entirely conceived and built in Portugal.

Hyperbolic billiards

The organisation of Matemática Viva was a very hard task considering the short period, the absence of structures and the lack of previous experience of Atractor for a task of such dimension. But it had the advantage that this was group effort by colleagues from many different institutions and this also gave increased visibility to Atractor.

The three billiard tables of *Matemática Viva* (for the three conics)

The principle of running interactive mathematical exhibitions was not consensual. And even for those in favour there were different possible approaches. I quote two written statements from that time.

It is not evident, even for some mathematicians, that mathematics is adequate for «interactive» presentations and on the other hand some feel that such presentations may not have a scientific quality and may pervert the ideas (or the problems) they want to present.
The plan should not follow the mathematics curricula for any academic degree, although school population will certainly form a large part of the visitors. But (...) the exhibition should not be reduced to a mere tool for guided school visits.
The exhibits, although they may have an informative component, should be planned mainly to awake the visitor's active curiosity (...). Leading the visitor to voluntarily make some effort, small though it may be, to grasp an idea is certainly better than to give him/her a higher volume of encyclopaedic knowledge or mathematical results. (...) The question of scientific rigour (...) is controversial. (...) Although it is obviously necessary to avoid use of technical, specialized vocabulary (...) it is crucial to avoid actually distorting the ideas to transmit under the temptation of making popularisation accessible at any cost.

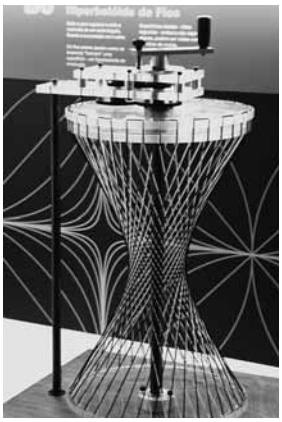
The general philosophy when designing the Matemática Viva Exhibition was to follow these general principles.

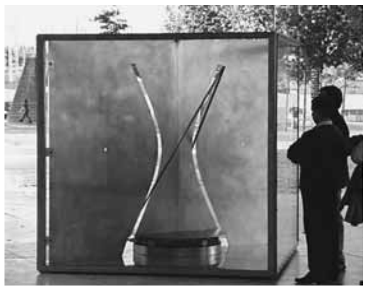
Part of Atractor's team in the real size Ames Room built for Matemática Viva; notice the relationship between size and order...
On the right the spinors exhibit.

It is not possible in this text to give an idea of the whole exhibition (50–60 different exhibits) but I shall give just a few examples. There are small non-technical exhibits like this binary mechanical counting machine and another one about pi (which is longer: the straight line or the circular one?) There are more sophisticated exhibits like the one in which you have two musical keyboards and obtain the Lissajous curves corresponding to the frequencies of the two sounds. On the left side a table with these different curves allows you to get the ratio for the frequencies of both sounds. Close to this there is a

"composer" for the (many) different tunes you can get with the well-known Mozart dice game rules.

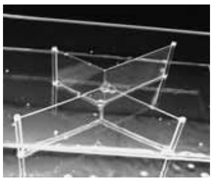
Geometry is present in many different contexts. Two closely related exhibits are illustrated in the pictures below:



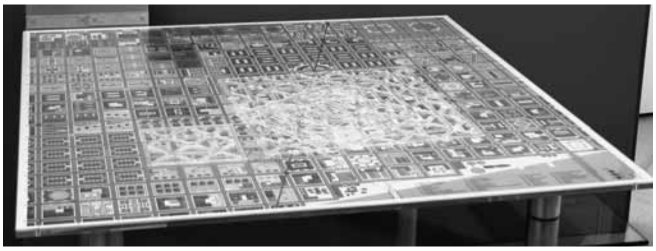


On the right a big exhibit outside the Pavilhão do Conhecimento has two rotating rods generating an invisible hyperboloid whose traces on a plane are visible hyperbolic slits through which pass the rotating rods. On the left a mechanism allows the visitor to move a handle that simultaneously deforms two families of wires to produce a variable hyperboloid with the two families of generators.

Of course many examples of experiments with soap films are available, including the Möbius band (shown in one of the pictures) and the classical *application* to solving minimal graph problems (also shown). It is striking how visitors are delighted to discover that one can get



solutions to *practical* questions (like minimising material for connecting different places with electrical cables) with analogical methods. This is the case even if they are aware that these are not examples of real applications in the sense that nobody in practice uses this method for solving these particular problems.



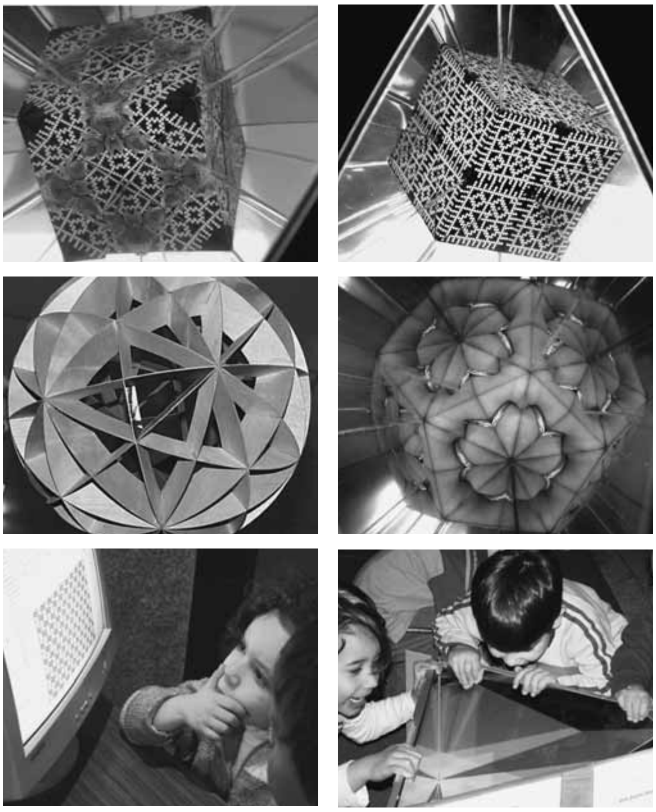
An imaginary squared town is the pretext for some problems and proposals concerning the taxi metrics...

There are several anamorphoses.

A big deformed picture on the floor gives the Sierpinski cube on a cylindrical mirror around a pillar in the building.

Symmetry – playing with mirrors

Immediately after its creation (1999) Atractor participated in a joint international program financed by the European Commission, which lasted up until 2003. One important consequence of this kind of program is very often the creation or strengthening of links among participants from different countries. This has happened with Atractor and other participants and in one of the cases, i.e. the University of Milan, the collaboration lasted well after the end of the project and is still alive, now extended to the recently created inter-university structure Matematita, which includes the Department of Mathematics of the University of Milan. The collaboration with Italian colleagues led to the construction of Atractor's second important exhibition *Simetria – jogos de espelhos* [3] (Symmetry – playing



Three beautiful images from kaleidoscopes with triangular holes, a model of the fundamental regions for dodecahedron symmetry and children attracted by symmetry.

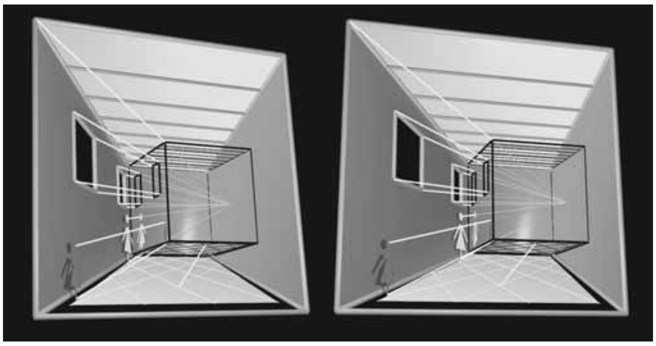
with mirrors) a slightly enlarged version of an exhibition of a similar name at the Department of Mathematics of the University of Milan. The exhibition was built in two versions: one fixed, which is intended to be included in the future Atractor's Centre and is for the moment at the University of Porto, and the other, which was organised as a travelling exhibition that has meanwhile visited more than three dozen institutions from the extreme north of Portugal (Bragança and Viana do Castelo) to the southern Algarve. The construction of this exhibition was supported by the University of Porto, its Faculty of Science and by Ciência Viva. The fixed exhibition was inaugurated in March 2002. The support for the travelling exhibition is organised by Atractor and the training of monitors and teachers who will care locally for the exhibition in the locations visited is given at the fixed exhibition in Porto by Atractor's staff. Sometimes the visits are complemented by lectures and workshops connected to different aspects of symmetry.

Virtual production

The construction of interactive virtual material has been used by Atractor since the beginning and not only on its site. For instance, the Matemática Viva Exhibition has an internal network of computers with some interactive material complementing the physical exhibits of the exhibition. The idea was also to have, where possible, information at different levels of difficulty, in order to allow different approaches to the same activity, according to the background and age of the visitors. Given the short period of time for the preparation of the Exhibition, this program was carried out at different levels for the different exhibits and a Linux version of a program produced by the Geometry Centre was implemented at the time and made freely available [4].

More recently the ability to use programs directly written in Java has given more freedom in the choice of applications to be produced. A good example is the well-known «15»-puzzle, which can be played not only on a plane as usual but also on a cylinder, a torus, a Möbius band or even on a Klein bottle. On the plane it is well-known that one can only get even permutations but what happens on the other surfaces? Does the answer depend only on the surface? You can even play with an image of your choice that you yourself send through the web [5].

One of the areas we developed in the last few years was the production of stereographic material: images, flash animations and applets. Two systems were used: one requiring two video projectors, a double graphics board, polarised filters and polarising glasses, gives very high quality for projection; the other one, present on Atractor's site [6], uses two images filling a computer display but requires a special prismatic kit developed specially by Atractor. This allows practically everybody to see the stereo effect, which does not happen with stereograms, including dot-stereograms.



Stereogram making clear why the Ames room works

Stereogram for the Alexander horned sphere

Another very big and ambitious project is the production of an interactive DVD giving an idea of the application of Thurston's orbifolds to the study of patterns and friezes. This is in the final stages of production and will be available with text in Portuguese, English, French, Italian and Spanish. A Java interactive version (still at the revision stage) of part of the DVD can be found at [7].

The Möbius band stamping a pmg pattern (from Alhambra)

Supports

Of course all the activity mentioned above would not have been possible without continuity of the people working on the project and this required regular institutional support. This kind of support came from a variety of sources. Firstly there was the support from the Ministry of Education, which assigned two secondary teachers to work full-time on Atractor from the start. Then there was the Foundation for Science and Technology (FCT), which accorded two continued grants. This was very important for the local development of competency in using specific tools for creating interactive virtual material, which allowed Atractor to start ambitious work in this area. More recently Atractor has had some grants from the Gulbenkian Foundation that were crucial in complementing and giving continuity to the work already started in that area. There was also sporadic support from projects in which Atractor participated, for instance the previously mentioned project financed by the European Commission and one that is now active, called Pencil. At present there are eight young people working on Atractor (six of them full-time), all graduates in mathematics, even those who specialised in the use of informatic tools such as POV-Ray, LiveGraphics3D, Flash or Java. All these people with complementary competencies form a determined and enthusiastic group. Last but not least, Atractor has counted on the support of the Faculty of Science of the University of Porto, which allocated a place for this group to work in its main building over all these years.

Making a balance

Looking back, we cannot avoid some feeling of frustration concerning one of our main goals: the creation of an interactive centre entirely devoted to mathematics and its applications. At the time of writing this text (9th April) Atractor is still waiting for a decision concerning the centre in Ovar. Excluding this (important) aspect, I would say that in all others Atractor's activity has exceeded the most optimistic initial expectations.

Concerning the future, a proposal for developing interactive material for blind children was recently approved. This idea arose from a very successful visit by blind children for activities we had specially prepared for them. We also plan to build a big interactive exhibition about topology and dynamical systems. And we have a lot of work in a final stage of production for our site and many plans for new material.

(A color PDF version of this article can be found in [8]).

References

[1] /http://www.atractor.pt/geral/atrac5wp.html/
[2] /http://www.atractor.pt/matviva/
[3] /http://www.atractor.pt/simetria/
[4] /http://www.atractor.pt/soft/kaleido/kaleido.htm/
[5] /http://www.atractor.pt/mat/puzzle-15/
[6] /http://www.atractor.pt/geral/fr-stereoP.htm/
[7] /http://www.atractor.pt/mat/orbifolds/
[8] /http://www.atractor.pt/div/ems.pdf/

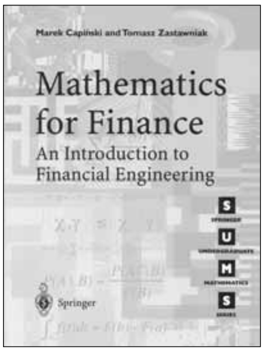
Manuel Arala Chaves [machaves@ fc.up.pt] was Full Professor of Mathematics at the Faculty of Science of Porto from 1973 until 2003, when he decided to retire in order to work fulltime on the Atractor project.

Book review

Pablo Fernández Gallardo (Madrid, Spain)

Marek Capiński and Tomasz Zastawniak
**Mathematics for Finance:
An Introduction to Financial Engineering**

Springer Undergraduate Mathematical Series (2003)
310 pages
Springer Verlag
ISBN: 1-85233-330-8



1973 was a key year in the development of financial mathematics. The Chicago Board Options Exchange was founded and at the same time Fischer Black and Myron Scholes published the paper "The Pricing of Options and Corporate Liabilities" [1], while Robert Merton published the article "Theory of Rational Option Pricing" [2].

These works contained a new methodology for the valuation of derivatives (financial instruments whose payoffs depend upon the value of other instruments) and developed the famous Black-Scholes pricing formulas for calls and puts.

In 1997 the Royal Swedish Academy of Sciences awarded the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel (the Nobel Prize for Economics) to Merton and Scholes "for a new method to determine the value of derivatives" (Black had passed away some years before).

Black, Merton and Scholes' framework rests upon two main pillars. The first is the modelling of stock prices with geometric Brownian motion (as Mandelbrojt and Nobel Prize winner Samuelson had done some years before). The second, and here lies their contribution, is a principle of equilibrium: the absence of arbitrage opportunities (and the corresponding replicating portfolio arguments). Nowadays, their arguments might sound natural, yet it is worth noting that Black and Scholes' paper, which has become one of the most cited papers in the scientific literature of this field, was rejected by some prestigious economics journals for some years.

The history of derivatives is not as recent a topic as one might think. There are references to the use of these financial contracts (mainly futures) throughout history. It is said that Thales of Miletus made a fortune trading with the rights of use of oil mills. In the 17th century there was a speculative boom in the tulip futures market in Amsterdam. However, the first mathematical attempt to model the financial markets was made in 1900, in the doctoral thesis "Théorie de la Spéculation" by Louis Bachelier, one of Poincaré's students.

Black, Merton and Scholes' ideas and valuation formulae were immediately understood and accepted by the markets. This is an amazing example of vertiginous transference of knowledge between the academic world and the 'real' world; as they allowed risks to be translated into prices and different derivatives to be compared, the formulae actually worked!

Since then, the range of applications of stochastic processes and partial differential equations in finance (modelling and designing of financial instruments, pricing, hedging, risk management, etc.) has been increasing day by day. Thus, it has become a standard topic in undergraduate courses in mathematics and economics. The book, "Mathematics for Finance: An Introduction to Financial Engineering", is a textbook for an introductory course on three basic questions: the non-arbitrage option pricing theory, the Markowitz portfolio theory and the modelling of interest rates.

The book

The book is divided into eleven chapters. The first four serve as an initial approach to financial markets. The first chapter describes some of the basic hypotheses, illustrated with the one-step binomial model. A detailed account of riskless assets (time value of the money, bonds, etc.) is included in the second chapter. The third is devoted to the dynamics of risky assets, with particular emphasis on the binomial tree model, although the trinomial tree model is also analysed and a brief discussion on the continuous time limit is included as well. The fourth chapter ties together the previous concepts in the general framework of discrete models. Questions relating to portfolio management (Markowitz theory, efficient frontiers, CAPM) are treated in detail in the fifth chapter. Chapters six and seven are dedicated to explain some basic characteristics of derivatives such as futures, forward contracts and options.

Chapter eight deals with option pricing. The Cox-Ross-Rubinstein binomial model is used to price European and American options. A brief outline of the arguments that lead to the Black-Scholes formula is also included. Issues dealing with the use of derivatives in risk management are described in the ninth chapter: hedging, risk measures as value-at-risk (VaR), speculating strategies with options, etc. These questions are treated by making use of a case study approach. Finally, chapters ten and eleven are devoted to interest rates including term structure, the binomial model and a brief account of interest rates derivatives (swaps, caps, floors).

A textbook on financial mathematics like the one we are reviewing unavoidably faces a dilemma, namely the balance between mathematical rigor and financial concepts and their practical implementation.

In most chapters, Capiński and Zastawniak's work succeeds in moving away from some advanced mathematical language, such as stochastic calculus and partial differential equations, and focuses on discrete time models, so that only basic notions of calculus, linear algebra and probability are needed to read it. At the same time, despite its mathematically formal prose, the text is very readable. The relevant concepts are introduced gradually; often these concepts are preceded by numerical examples and in the end they are all given their corresponding formal definitions. Almost all the main results are accompanied by corresponding proofs. A good deal of worked examples and remarks elucidate the associated financial concepts. Each chapter contains a collection of useful exercises (interspersed in the text). Some of the quite detailed solutions are included in an appendix. The reader will find other solutions (in Excel files) on the web page www-users.york.ac.uk/~tz506/m4f/index.html (see also www.springeronline.com/1-85233-330-8). Some typos are listed on the aforementioned web page.

Thus, we can say that the book strikes a balance between mathematical rigor and practical application and we recommend it as a textbook for an introductory course on financial mathematics.

References

[1] Black, F. and Scholes, M.: "The Pricing of Options and Corporate Liabilities". *Journal of Political Economy* 81 (1973), 637–654.

[2] Merton, R.: "Theory of Rational Option Pricing". *The Bell Journal of Economics and Management Science* 4 (1973), 141–183.

Pablo Fernández Gallardo [pablo.fernandez@uam.es] got his PhD in 1997 from the Universidad Autónoma de Madrid and he is currently working as an Assistant Professor in the Mathematics Department there. His interests range from harmonic analysis to discrete mathematics and mathematical finance.

ERCOM: Max Planck Institute for Mathematics, Bonn (Germany)

History

The Max Planck Institute for Mathematics (MPIM) in Bonn was established in 1981 by its first director Friedrich Hirzebruch, it arose from the Sonderforschungsbereich (SFB, special research area) "Theoretische Mathematik" at the University of Bonn. Until the present day there remains a close collaboration with that university. At first the institute was even located within the university buildings, but in January of 1982 it moved to its own quarters in Bonn-Beuel on the right bank of the Rhine. In 1999 the institute moved into more spacious facilities located in Bonn centre, not far from the University (both on the left bank of the Rhine).

Don Zagier and Günter Harder were appointed as scientific members in 1984. Gerd Faltings (1994) and Yuri Manin (1993) also accepted positions as scientific members. When Hirzebruch retired from his directorship in 1995, these four became the directors with always one of them being "managing director", a position which rotates every two years. Manin retired from directorship in March 2005 and Harder in April 2006. Presently, Faltings is managing director.

The MPIM in Bonn is part of the Max Planck Society (MPG). The MPG is an independent, non-profit research organisation, financed mainly by the government, that primarily promotes and supports research at its own institutes. There are Max Planck Institutes located all over Germany focusing on scientific topics in the research fields of Biology and Medicine, Chemistry, Physics and Technology, and Humanities. The Max Planck Institute most closely related to the MPIM is the "MPI for Mathematics in the Sciences" in Leipzig (founded in 1996) which focuses more on applied mathematics.

On March 31, 2006, the MPIM celebrated its 25th anniversary. At this occasion several people from politics and science gave short speeches, Friedrich Hirzebruch described the founding history of MPIM and Sir Michael Atiyah gave a more mathematical lecture. On March 31 also a book (in German) describing for a general audience highlights of 25 years of research at MPIM was presented. This day of celebration was followed by a five day mathematical conference

Aims

The MPIM in Bonn is a guest research institute, which means that nearly all of the researchers work only for a short time at the institute, from several days to one year (longer in exceptional cases). The primary goal of the MPIM is to bring together mathematicians from all over the world and to give them the opportunity to work in a lively and stimulating atmosphere apart from the duties at their home universities. Often, new collaborations arise at the institute. In addition, the MPIM wants to spread new ideas. Therefore, seminars about important recent research results are organised.

Organisation of scientific work

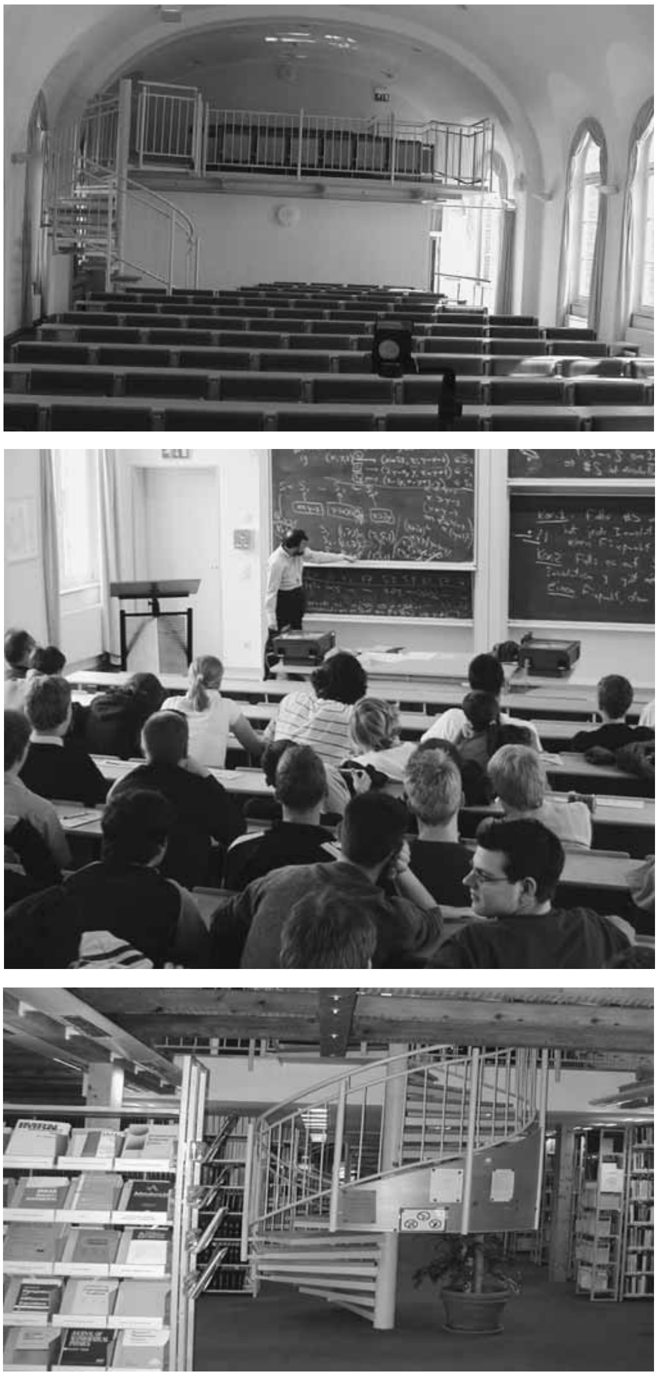
In contrast to some other research institutes, the MPIM does not have programmes. In fact any mathematician can apply for a visit and it depends only on the mathematical quality of the applicant as to whether or not (s)he is invited for a stay. Every year, about 350 to 400 mathematicians are invited to come to the MPIM. There are always approximately 80 researchers present at the institute half of which are young scientists ranging in experience from PhD students to postdocs up to 8 years past their PhD.

The scientific work at the MPIM is not scheduled, the guests may work individually or in small groups, they can participate in seminars or organise their own seminars or workshops. Also they have the option of taking an active part in the teaching at the University of Bonn. Upon consent of the managing director guests may invite collaborators for a period of stay up to two weeks. Every year, workshops and activities on special topics are offered. For example, the MPIM and Bonn University host the Arbeitstagung, a famous meeting which has been held in the odd numbered years since 1957. Every week, an "Oberseminar" takes place where the visitors present their work in a way that should be understandable to all MPIM visitors, even to those who are working in other areas. In addition to seminars, workshops and activities, the MPIM also disseminates mathematical results through its preprint series.

The MPIM has a special interest in supporting young mathematicians. Thus, there are several efforts for supporting them. One of these is the "International Max Planck Research School (IMPRS) for Moduli Spaces", another one the "European Postdoctoral Institute" (EPDI), a common project of the ERCOM institutes. At IMPRS, which is similar to an American graduate school, young mathematicians can work at their PhD under the guidance of a MPIM director or a Bonn University professor. Many different nationalities are represented at IMPRS. Courses given for them are well attended by other MPIM guests.

Research areas

Mathematicians at MPIM work in all areas of pure mathematics. However, the following fields of mathematics are particularly well represented:



Main lecture hall (with Prof. D. Zagier lecturing) and library.

Algebraic geometry and number theory, Arithmetic algebraic geometry, Automorphic forms, Algebraic groups and arithmetic subgroups, Representation theory, Singularities, Complex analysis, Algebraic topology, Differential topology, Differential geometry and Mathematical physics.

Location and environment

Since 1999, the MPIM has been located in a nice, old building directly in the centre of Bonn. Part of the building is a former palace, the Fürstenberg Palais. It is near to the University of Bonn and very close to the main train station.

The surroundings of the MPIM are populated by many pubs, restaurants and shops. A ten minute walk leads to the Rhine and a popular beer garden (Biergarten). A five minute walk leads to the Hofgarten, a small park in front of the main university building. With good weather, there will be people playing soccer or frisbee, picknicking or sunbathing. On the other side of the Rhine, a little bit to the south, there is the Siebengebirge - a hilly area with medieval ruins which is popular for hiking. Moreover, the area of the Rhine immediately south of Bonn is especially nice, steeped in castles, keeps, vineyards, and picturesque villages.

The opera building of Bonn one reaches in a ten minute walk. The Beethovenhalle (music hall) is a bit farther away. The house where Beethoven was born (a seven minute walk) also has an auditorium. His birth-house is now a small museum. Bonn used to be the capital of Germany and as a result there are many museums, especially along the "Museumsmeile".

The Bonn-Cologne airport can be reached in half an hour. In less than half an hour one reaches Köln (Cologne) by train. This city has about a million inhabitants and a good deal of culture to offer. There are several museums and a music auditorium in the immediate vicinity of the main train station of Köln.

Facilities

The MPIM has 108 working places on 3500 m2 usable surface, an auditorium with 120 seats and whiteboards everywhere.

The scientific work is supported by a few employees who work in the administration, the library and the computer group. The library provides about 13 000 monographs, more than 160 journals and preprint collections, a website for investigation in electronic journals, use of mathematical databases and a library catalogue that is worldwide available. In addition the library of the University of Bonn may be used by the guests. The computer department owns about 30 servers, it makes about 140 workstation computers available with broadband-connection to the German science-network and the computer team helps with solutions for special scientific requirements.

The administration helps with visa, insurance, housing and many other practical problems the guests might have.

Website

A bit more detailed information about the MPIM is provided in the article by Allyn Jackson 'Bonn's Max Planck Institute: a new building and a new era', Notices Amer. Math. Soc. 45 (1998), no. 5, 582–588. For further information and contact please visit our website www.mpim-bonn.mpg.de

The website also provides information on how to apply for visiting scholarships or the IMPRS Research Graduate School.

Personal column

Please send information on mathematical awards and deaths to the editor.

Awards

Wendelin Werner (Univ. Paris-Sud) has been awarded the 2005 Line and Michel Loève International Prize in Probability.
http://www.stat.berkeley.edu/~aldous/Research/werner-citation.html

Felix Otto (Univ. Bonn, Germany) was awarded one of the Gottfried Wilhelm Leibniz Prizes in 2006 by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) for his work on the analysis of partial differential equations.
http://www.dfg.de/aktuelles_presse/preise/leibniz_preis/2006/otto.html

Günter Ziegler and **Florian Pfender** (TU Berlin, Germany) have been honoured with the 2006 Chauvenet Prize of the Mathematical Association of America (MAA) for their article "Kissing numbers, sphere packings, and some unexpected proofs" (AMS-Notices, Sept. 2004).
http://www.maa.org/awards/JMM06.pdf

The Autumn Prize of the Japan Mathematical Society Prize for 2005 was awarded to **Kaoru Ono** (Hokkaido Univ.) for his distinguished contributions to the study of symplectic geometry, especially for his proof of the flux conjecture based on Floer-Novikov cohomology.

The Geometry Prizes in the year 2005 were awarded to **Koji Fujiwara** (Tohoku Univ.) in recognition of his fundamental research work on geometric group theory, and to **Ryushi Goto** (Osaka Univ.) for his outstanding research work on a new unified approach to the deformation theory of Ricci-flat geometric structures.

The Analysis Prizes of MSJ in the year 2005 were awarded to **Kenji Nakanishi** (Nagoya Univ.), **Hidenori Fujiwara** (Kinki Univ.), and to **Nobuo Yoshida** (Kyoto Univ.) in recognition of their outstanding contributions in analysis.

The Takebe Senior Prize was awarded to **Nobuhiro Honda** (Tokyo Inst. of Technology), **Hiroki Yagisita** (Tokyo Univ. of Science), and to **Katsutoshi Yamanoi** (Kyoto Univ.).

The Takebe Junior Prize was awarded to Tetsushi Ito and **Shin-ichi Ohta** (both Kyoto Univ.), to **Shunsuke Takagi** (Kyushu Univ.), **Masatomo Takahashi** (Hokkkaido Univ.) and to **Toru Nakajima** (Shizuoka Univ.).

The International Commission for the History of Mathematics (ICHM) has awarded the 2005 Kenneth O. May Prize and Medal to **Henk Bos** (Utrecht Univ., the Netherlands).
http://www.unizar.es/ichm/reports/henkvale.html

Adam Skalski (Łódž, Poland) was awarded the Prize of the Polish Mathematical Society for young mathematicians.

Jakub Gismatullin and **Paweł Kawa** (Wrocław, Poland) were awarded the first Marcinkiewicz Prizes of the Polish Mathematical Society for students' research papers.

The Clay Mathematics Institute (CMI) has presented its Clay Research Awards for 2005 to **Manjul Bhargava** (Princeton Univ.) and to **Nils Dencker** (Lund Univ., Sweden).
http://www.claymath.org/annual_meeting/2005_Annual_Meeting/

The second European Prize in Combinatorics was awarded to **Dmitry Feichtner-Kozlov** (Zürich, Switzerland) for deep combinatorial results obtained by algebraic topology and particularly for a solution of a conjecture of Lovasz.
http://www.math.tu-berlin.de/EuroComb05/prize.html

Simon Donaldson (Imperial College, London, UK) and **M.S. Narasimhan** (Tata Instit. for Fund. Research, Mumbai, India) have been jointly awarded the 2006 King Faisal International Prize for Science.
http://www.imperial.ac.uk/P7202.htm

The University of Cambridge has awarded the Adams Prize 2005 to **Jonathan Sherratt** (Heriot-Watt Univ., Edinburgh, UK) for major contributions to several areas of mathematical biology.
http://www.maths.cam.ac.uk/news/adams-results-2005.html

Deaths

We regret to announce the deaths of:

Thomas Beth (Germany, 17.8.2005)
Sir Hermann Bondi (UK, 10.9.2005)
Aurel Cornea (Germany, 3.9.2005)
Dick Dalitz (UK, 13.1.2006)
Tadeusz Dłotko (Poland, 10.10.2005)
Kazimierz Głazek (Poland, 25.9.2005)
Alfred W. Goldie (UK, 8.10.2005)
Ulrich Hirsch (Germany, 30.4.2005)
Hans Georg Kellerer (Germany, 14.7.2005)
Konrad Königsberger (Germany, 4.10.2005)
Gerd Laßner (Germany, 24.8.2005)
Jaak Lõhmus (Estonia, 23.2.2006)
Pere Rodriguez Mumbru (Spain, 28.7.2005)
Sir Harry Raymond Pitt (UK, 8.10.2005)
Walter Roelcke (Germany, 24.12.2005)
C. Ambrose Rogers (UK, 5.12.2005)
Helmut Schaefer (Germany, 16.12.2005)
Charles Thomas (UK, 16.12.2005)

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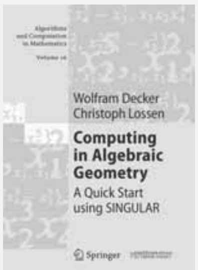
New Textbooks from Springer

An Introduction to Infinite-Dimensional Analysis

G. Da Prato, Scuola Normale Superiore, Pisa, Italy

In this revised and extended version of his course notes from a 1-year course at Scuola Normale Superiore, Pisa, the author provides an introduction – for an audience knowing basic functional analysis and measure theory but not necessarily probability theory – to analysis in a separable Hilbert space of infinite dimension. Moreover, some details have been added as well as some new material on dynamical systems with dissipative nonlinearities and asymptotic behavior for gradient systems.

2006. Approx. 200 p. (Universitext) Softcover
ISBN 3-540-29020-6 ► **€ 44,95 | £34.50**



Computing in Algebraic Geometry

A Quick Start using SINGULAR

W. Decker, Universität des Saarlandes, Saarbrücken, Germany; **C. Lossen**, Technische Universität Kaiserslautern, Germany

Originating from a number of intense one week schools taught by the authors, the text is designed so as to provide a step by step introduction which enables the reader to get started with his own computational experiments right away. The authors present the basic concepts and ideas in a compact way, omitting proofs and detours, and they give references for further reading on some of the more advanced topics.

2006. XV, 327 p. (Algorithms and Computation in Mathematics, Vol. 16) Hardcover
ISBN 3-540-28992-5 ► **€ 39,95 | £30.50**

The IMO Compendium

A Collection of Problems Suggested for The International Mathematical Olympiads: 1959-2004

D. Djukic, University of Toronto, ON, Canada; **V. Z. Jankovic**, Belgrade University, Serbia and Montenegro; **I. Matic**, University of California, Berkeley, CA, USA; **N. Petrovic**, Institute of Physics, Belgrade, Serbia and Montenegro

2006. XIV, 740 p. 200 illus. (Problem Books in Mathematics) Hardcover
ISBN 0-387-24299-6 ► **€ 69,95 | £54.00**

Qualitative Theory of Planar Differential Systems

F. Dumortier, Limburgs Universitair Centrum, Diepenbeek, Belgium; **J. Llibre**, Universitat Autònoma de Barcelona, Bellaterra, Spain; **J. C. Artés**, Universitat Autònoma de Barcelona, Bellaterra, Spain

The book deals essentially with systems of polynomial autonomous ordinary differential equations in two real variables. The emphasis is mainly qualitative, although attention is also given to more algebraic aspects as a thorough study of the center/ focus problem and recent results on integrability.

2006. Approx. 380 p. (Universitext) Softcover
ISBN 3-540-32893-9 ► **€ 39,95 | £30.50**

Compact Riemann Surfaces

3RD EDITION

An Introduction to Contemporary Mathematics

J. Jost, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Although Riemann surfaces are a time-honoured field, this book is novel in its broad perspective that systematically explores the connection with other fields of mathematics. It can serve as an introduction to contemporary mathematics as a whole as it develops background material from algebraic topology, differential geometry, the calculus of variations, elliptic PDE, and algebraic geometry.

3rd ed. 2006. XIV, 277 p. (Universitext) Softcover
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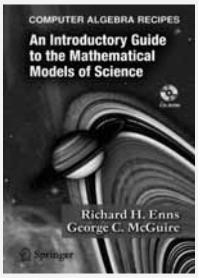


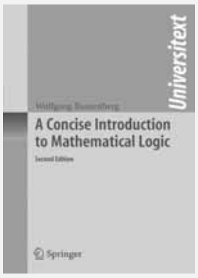
Computer Algebra Recipes

An Introductory Guide to the Mathematical Models of Science

R. H. Enns, Simon Fraser University, Burnaby, BC, Canada; **G. C. McGuire**, University College of the Fraser Valley, Abbotsford, BC, Canada

Computer algebra systems have revolutionized the study of mathematically intensive subjects in science and engineering. The heart of this text is a large number of computer algebra worksheets or "recipes" that have been designed using MAPLE (Version 10) not only to provide tools for problem solving, but also to stimulate the reader's imagination.

2006. X, 430 p. With CD-ROM. Softcover
ISBN 0-387-25767-5 ► **€ 49,95 | £38.50**



2ND EDITION

A Concise Introduction to Mathematical Logic

W. Rautenberg, Freie Universität Berlin, Germany

Traditional logic as a part of philosophy is one of the oldest scientific disciplines. Mathematical logic, however, is a relatively young discipline. This book is much more concise than most others, treating the material in a streamlined fashion which allows the instructor to cover many important topics in a one semester course. The author has provided exercises for each chapter, as well as hints to selected exercises.

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Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses vberinde@ubm.ro or vasile_berinde@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

June 2006

1–3: International Conference on Computers and Communications, Baile Felix Spa, Oradea, Romania
Information: idzitac@univagora.ro; http://www.iccc.univagora.ro/index.htm

1–3: Fields Institute Applied Probability Workshop, Carleton University, Ottawa, Canada
Information: programs@fields.utoronto.ca;
http://www.fields.utoronto.ca/programs/scientific/05-06/applied_probability/

2–9: Formal theory of partial differential equations and their applications, Mekrijärvi Research Station, University of Joensuu, Finland
Information : pdes2006@joensuu.fi;
http://www.joensuu.fi/mathematics/PDEworkshop2006/index.html

4–10: Dynamics, Topology and Computations DyToComp2006, Będlewo, Poland
Information: http://www.ii.uj.edu.pl/DyToComp2006/

4–10: Workshop on Commutative Rings, Cortona, Italy
Information: cortona2006@mat.uniroma3.it;
http://www.mat.uniroma3.it/users/cortona/cortona_2006.html

5–9: Workshop on Fourier Analysis, Geometric Measure Theory and Applications, Barcelona, Spain
Information: http://www.crm.es/Research/0506/AnalysisEng.htm

6–9: Conference on Lattice Theory. In honour of the 70th birthday of George Grätzer and E. Tamás Schmidt, Budapest, Hungary
Information: http://www.renyi.hu/conferences/grasch.html

7–9: Boltzmann's Legacy 2006, Erwin Schroedinger Institute for Mathematical Physics
Information: secr@esi.ac.at; http://www.esi.ac.at/activities/Boltzmann2006.html

7–10: Recent advances in nonlinear partial differential equations and applications. A conference in honor of Peter D. Lax and Louis Nirenberg on the occasion of their 80th birthday, Toledo, Spain
Information: http://www.mat.ucm.es/~ln06

8–10: Xenakis Legacies Symposium, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca

8–11: Carthapos2006 a Workshop-conference on Positivity at Carthage, Carthage, Tunisia
Information: http://www.cck.rnu.tn/carthapos2006/

9–14: Eight International Conference on Geometry, Integrability and Quantization, Sts. Constantine and Elena resort (near Varna), Bulgaria
Information: mladenov@obzor.bio21.bas.bg;
http://www.bio21.bas.bg/conference/

10–16: Discontinuous change in behavior issues in partial differential equations, Anogia Academic Village, Crete, Greece
Information: athan@tem.uoc.gr; http://www.iacm.forth.gr/news/anogia06.html

10–20: Mathematical Modeling of Infectious Diseases Summer School, Fields Institute, York University, Toronto, Canada
Information: summer2006@yorku.ca; http://www.liam.yorku.ca/sc06/

11–14: ICMSE 2006 – International Conference in Mathematics, Sciences and Science Education, Aveiro, Portugal
Information: icmse@mat.ua.pt; http://gag.mat.ua.pt/ICMSE/

12–16: Function Theories in Higher Dimensions, Tampere University of Technology, Tampere, Finland,
Information: sirkka-liisa.eriksson@tut.fi; http://www.tut.fi/fthd/

13–16: SDS2006 – Structural dynamical systems. Computational Aspects, Hotel Porto Giardino, Capitolo, Monopoli, Italy
Information: delbuono@dm.uniba.it;
http://www.dm.uniba.it/~delbuono/sds2006.htm

13–16: Mathematics of Finite Elements and Applications (MAFELAP 2006), Brunel University, UK
Information: www.brunel.ac.uk/bicom/mafelap2006
[For details, see EMS Newsletter 56]

19–23: Conference „Modern stochastics: theory and applications", Kyiv National Taras Shevchenko University, Kyiv, Ukraine
Information: probab.conf.2006@univ.kiev.ua;
http://www.mechmat.univ.kiev.ua/probability/ Events/2006/informletterengl.html
[For details, see EMS Newsletter 57]

19–23: Harmonic Analysis and Related Problems (HARP 2006), Zaros, Crete, Greece
Information: http://fourier.math.uoc.gr/~harp2006/

19–23: Quantile Regression, LMS Method and Robust Statistics in the 21st Century, Edinburgh, UK
Information: enquiries@icms.org.uk;
http://www.icms.org.uk/meetings/2006/quantile

19–24: Hodge Theory, Venice International University, Island of San Servolo, Italy
Information: hodge@math.unipd.it;
http://www.mat.uniroma3.it/GVA/HTVIU/

19–24: Pseudo-Differential Operators, Quantization and Signals. CIME school, Cetraro (Cosenza), Italy
Information: cime@math.unifi.it; http://web.math.unifi.it/users/cime/Courses/2006/03.php

19–30: Third Banach Center Symposium – CAUSTICS'06, Warsaw, Poland
Information: http://alpha.mini.pw.edu.pl/~janeczko/Caustics'06.html

21–23: 6th International Conference on Mathematical Problems in Engineering and Aerospace Sciences, Budapest, Hungary
Information: info@icnpaa.com; seenithi@aol.com; www.icnpaa.com
[For details, see EMS Newsletter 56]

23–26: 2006 International Conference on Topology and its Applications, Aegion, Greece
Information: http://www.math.upatras.gr/~aegion/

25–30: 9th International Vilnius Conference on Probability Theory and Mathematical Statistics, Vilnius, Lithuania
Information: http://www.mii.lt/vilconf9/; conf2006@ktl.mii.lt

26–29: ACA 2006 12th International Conference on Applications of Computer Algebra, Varna, Bulgaria
Information: http://www.math.bas.bg/artint/mspirid/ACA2006/

26–30: Applied Asymptotics and Modelling, Edinburgh, UK
Information: enquiries@icms.org.uk; http://www.icms.org.uk/meetings

26–July 1: Mixed Finite Elements, Compatibility Conditions, and Applications. CIME school, Cetraro (Cosenza), Italy
Information: cime@math.unifi.it; http://web.math.unifi.it/users/cime/Courses/2006/03.php

26–July 8: Course on On Limit Cycles of Differential Equations, Barcelona, Spain
Information: http://www.crm.es/Conferences/0506/Limitcycles/limitcycles.htm

27–30: Perspectives of System Informatics. Sixth International Andrei Ershov Memorial Conference, Novosibirsk, Russia
Information: http://www.iis.nsk.su/PSI06

29–July 4: 21th International Conference on Operator Theory, Timisoara, Romania
Information: http://www.imar.ro/~ot/

July 2006

1–December 31: Thematic Program on Cryptography, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/

3–7: Function Spaces VIII, Będlewo, Poland
Information: M.Nowak@wmie.uz.zgora.pl; http://www.fs8.wmie.uz.zgora.pl/

3–7: LMS Regional Meeting and Workshop in Functional Analysis Bounded and unbounded operators on Banach and Hilbert spaces, Leeds, Yorkshire, United Kingdom
Information: j.r.partington@leeds.ac.uk; http://www.maths.leeds.ac.uk/pure/analysis/lms/

3–22: Valuation Theory and Integral Closures in Commutative Algebra, Fields Institute, University of Ottawa, Ottawa, Canada
Information: programs@fields.utoronto.ca; http://www.mathstat.dal.ca/%7Efaridi/integral-closure.html

3–7: XII Meeting on Real Analysis and Measure Theory (CARTEMI), Ischia, Italy
Information: http://www.dma.unina.it/~cartemi/

5–8: Numerical Analysis and Approximation Theory (NAAT 2006), Cluj-Napoca, Romania
Information: naat2006@cs.ubbcluj.ro; http://www.cs.ubbcluj.ro/~naat2006/

6–8: 4th Portuguese Finance Network (PFN) Finance Conference, Porto, Portugal
Information: pfn2006@fep.up.pt; http://www.pfn2006.org

6–9: ISPDC 2006 – 5th International Symposium on Parallel and Distributed Computing, Timisoara, Romania
Information: ispdc06@info.uvt.ro; http://ispdc06.info.uvt.ro

7–10: 2nd International Conference „From Scientific Computing to Computational Engineering“ (2nd IC-SCCE 2006), Athens, Greece
Information: ic-scce2006@upatras.gr; http://ic-scce2006.upatras.gr

8–10: Pioneers of Bulgarian Mathematics, Sofia, Bulgaria
Information: http://profot.fmi.uni-sofia.bg/

9–12: International Symposium on Symbolic and Algebraic Computation (ISSAC 2006), Genova, Italy
Information: http://issac2006.dima.unige.it/

9–15: Which Mathematics for Biology? Anogia Academic Village, Crete, Greece
Information: manoussaki@gmail.com; http://www.iacm.forth.gr/news/anogia06.html

9–22: Horizon of Combinatorics, Budapest, Hungary
Information: comb06@renyi.hu; http://www.renyi.hu/conferences/horizon/

10–12: 6th Meeting on Game Theory and Practice dedicated to development, natural resources and the environment, Zaragoza, Spain
Information: http://www.iamz.ciheam.org/GTP2006/

10–14: New Directions in Applied Probability: Stochastic Networks and Beyond, Edinburgh, UK
Information: enquiries@icms.org.uk; http://www.icms.org.uk/meetings

10–14: WavE 2006, EPFL Lausanne, Switzerland
Information: http://wavelet.epfl.ch/

10–14: CMPI-2006 Campus Multidisciplinar en Percepción e Inteligencia, Albacete, Spain
Information: caballer@info-ab.uclm.es; http://www.info-ab.uclm.es/ cmpi/

10–15: Conference on Arithmetic of Shimura Varieties and Arakelov Geometry, Barcelona, Spain
Information: http://www.crm.es/Conferences/0506/ ConferenceShimura/conferenceshimura.htm

12–13: Eighth International Workshop on Deontic Logic in Computer Science (DEON2006), Utrecht, The Netherlands
Information: http://www.cs.uu.nl/deon2006/

17–19: Geometric Aspects of Integrable Systems, Coimbra, Portugal
Information: geomis@mat.uc.pt; http://www.mat.uc.pt/~geomis

17–21: Stochastic Processes and Applications (SPA XXXI), Paris, France
Information: spa2006@math-info.univ-paris5.fr; http://www.proba.jussieu.fr/pageperso/spa06/index.html

17–21: Extremal Kähler Metrics and Stability, Edinburgh, UK
Information: enquiries@icms.org.uk; http://www.icms.org.uk/meetings

17–21: Eleventh International Conference on Hyperbolic Problems Theory, Numerics, Applications, Lyon, France
Information: hyp2006@math.univ-lyon1.fr; http://math.univ-lyon1.fr/~hyp2006/

18–21: 13th ILAS Conference, Amsterdam, The Netherlands
Information: http://staff.science.uva.nl/~brandts/ILAS06/

23–28: 7th Algorithmic Number Theory Symposium (ANTS VII), Berlin, Germany
Information: http://www.math.tu-berlin.de/~kant/ants/

24–27: Joint GAMM-SIAM Conference on Applied Linear Algebra (ALA 2006), Düsseldorf, Germany
Information: http://www.ala2006.de/

24–27: Modeling and Optimization: Theory and Applications (MOPTA 06), Fields Institute, University of Waterloo, Waterloo, Canada
Information: hwolkowicz@uwaterloo.ca; http://www.stats.uwaterloo.ca/stats_navigation/Mopta/index.shtml

24–28: 2nd SIPTA School on Imprecise Probabilities, Madrid, Spain
Information: enrique.miranda@urjc.es; http://bayes.escet.urjc.es/~emiranda/sipta

24–28: New trends in viscosity solutions and nonlinear PDE, Lisboa, Portugal
Information: dgomes@math.ist.utl.pt; http://www.math.ist.utl.pt/~dgomes/newtrends/

24–August 4: Computational and Combinatorial Commutative Algebra, Fields Institute, Toronto, Canada
Information: ragnar@math.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/comalgebra/

25–30: 9th International Vilnius Conference on Probability Theory and Mathematical Statistics, Vilnius, Lithuania
Information: conf2006@ktl.mii.lt; http://www.mii.lt/vilconf9/

31–August 11: ESSLLI 2006 18th European Summer School in Logic, Language and Information, Màlaga, Spain
Information: http://esslli2006.lcc.uma.es/

August 2006

1–5: Information-MFCSIT'06, Cork, Ireland
Information: info-mfcsit@ucc.ie; http://www.ucc.ie/info-mfcsit/

7–10: The Oxford Conference on Topology and Computer Science in Honour of Peter Collins and Mike Reed, Oxford, United Kingdom
Information: http://www.maths.ox.ac.uk/~knight/Conference/

7–10: SECRYPT 2006 – International Conference on Security and Cryptography, Setubal, Portugal
Information: secretariat@secrypt.org; http://www.secrypt.org

7–11: Partial Differential Equations on Noncompact and Singular Manifolds, University of Potsdam, Germany
Information: pdensm@math.uni-potsdam.de; http://pdensm.math.uni-potsdam.de/

7–12: Algebraic Theory of Differential Equations, Edinburgh, UK
Information: enquiries@icms.org.uk; http://www.icms.org.uk/meetings

7–13: Third European Summer School in Mathematics Education (YESS-3), University of Jyväskylä, Finland
Information: http://ermeweb.free.fr/news.php

11–21: Methods of Integrable Systems in Geometry. London Mathematical Society Durham Symposium, Durham, United Kingdom
Information: http://maths.dur.ac.uk/events/Meetings/LMS/2006/IS/

13–19: Workshop on Triangulated Categories, University of Leeds, UK
Information: tholm@maths.leeds.ac.uk; http://www.maths.leeds.ac.uk/pure/algebra/TriCat06.html

13–19: 10th Prague Topological Symposium, Prague, Czech Republic
Information: http://toposym.mff.cuni.cz/

14–16: Canadian Computational Geometry Conference (CCCG), Fields Institute, Queen's University, Kingston, Canada
Information: programs@fields.utoronto.ca; http://www.cs.queensu.ca/cccg/

14–18: International Conference on Spectral Theory and Global Analysis, Carl von Ossietzky University, Oldenburg, Germany
Information: stga@mathematik.uni-oldenburg.de; http://www.mathematik.uni-oldenburg.de/personen/grieser/stga/

14–18: MCQMC 2006 Seventh International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing, Ulm, Germany
Information: http://mcqmc.uni-ulm.de/

16–19: Geometric Methods in Group Theory, Fields Institute, Carleton University, Ottawa, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/group_theory/

16–19: VII Workshop on Symplectic and Contact Topology, Satellite Conference of ICM2006, Madrid, Spain
Information: gesta@vilma.upc.edu; http://www.ma1.upc.edu/gesta/

17–19: The XIV Conference On Applied And Industrial Mathematics. Satellite Conference of ICM2006, Chisinau, Moldova

Information: mathconf@mail.md; http://www.usm.md/math/CAIM/

18–19: The 1st International Workshop on Multi-Objective Cybernetics (MOC 2006), Helsinki, Finland
Information: moshaiov@eng.tau.ac.il; http://plectics.org

19–26: Continuum Theory, Prague 2006 Open Problem Workshop, Prague, Czech Republic
Information: pyrih@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/~pyrih/open/workshop2006.htm

20–24: Sixth International Conference on Numerical Methods and Applications – NM&A'06, Borovets, Bulgaria
Information: http://www.fmi.uni-sofia.bg/nma06/

20–25: International Conference on Set-theoretic Topology, Kielce, Poland
Information: topoconf@pu.kielce.pl; http://www.pu.kielce.pl/~topoconf/

22–30: International Congress of Mathematicians (ICM2006), Madrid, Spain
Information: icm2006@unicongress.com; http://www.icm2006.org

26–29: Fourth Conference on Finite Difference Methods: Theory and Applications, Lozenetz, Bulgaria
Information: fdm06@ru.acad.bg; http://www.ru.acad.bg/fdm06

28–September 1: Evolution Equations 2006: in memory of G. Lumer, Mons and Valenciennes, Belgium and France
Information: snicaise@univ-valenciennes.fr; http://www.univ-valenciennes.fr/lamav/eveq06

28–September 1: 8th French-Romanian Colloquium in Applied Mathematics, Chambery, France
Information: 8cfr-ioan@univ-savoie.fr; http://www.lama.univ-savoie.fr/8cfr

28–September 2: Young Topologists and New Topology, Będlewo, Poland
Information: http://www.uni-math.gwdg.de/schick/young_topologists06 Będlewo.html

31–September 2: Geometry and Topology of Low Dimensional Manifolds, El Burgo de Osma, Spain
Information: http://mai.liu.se/LowDim/

31–September 2: Advanced Course on Combinatorial and Computational Geometry: trends and topics for the future, Alcalá de Henares, Spain
Information: http://www.crm.es/Conferences/0607/CCGeometry/combinatorial _index.htm

31–September 2: Lorenz-Gini Type Asymptotic Methods in Statistics, Fields Institute, Carleton University, Ottawa, Canada
Information: programs@fields.utoronto.ca;
http://www.fields.utoronto.ca/programs/scientific/

31–September 4: Geometric and Asymptotic Group Theory with Applications, Manresa (Barcelona), Spain
Information: gagta@epsem.upc.edu; http://www.epsem.upc.edu/~gagta/

31–September 5: Workshop on Geometric and Topological Combinatorics, Alcalá de Henares, Spain
Information: http://www2.uah.es/gtc06/

31–September 5: Advanced Course on Combinatorial and Computational Geometry: trends and topics for the future, Alcalá de Henares, Spain
Information: http://www.crm.es/Conferences/0607/CCGeometry/combinatorial_index.htm

September 2006

1–4: Topics in Mathematical Analysis and Graph Theory, Belgrade, Serbia and Montenegro
Information: pefmath@etf.bg.ac.yu; http://magt.etf.bg.ac.yu

1–4: Conference on Mathematical Neuroscience, Universitat d'Andorra, Principat d'Andorra
Information: CMathNeuroscience@crm.es;
http://www.crm.es/CMathNeuroscience

1–4: SCRA 2006-FIM XIII. Thirteenth International Conference of the Forum for Interdisciplinary Mathematics on Interdisciplinary Mathematical and Statistical Techniques, Lisbon-Tomar, Portugal
Information: cmac@fct.unl.pt ; mishra@jaguar1.usouthal.edu; http://scra2006.southalabama.edu/

1–5: International Summer School and Workshop of Operator Algebras, Operator Theory and Applications 2006, Lisbon, Portugal
Information: woat@math.ist.utl.pt; http://woat2006.ist.utl.pt/

2–5: 37th Annual Iranian Mathematics Conference, Azarbaijan University of Tarbiat Moallem, Tabriz, Iran
Information: aimc37@azaruniv.edu; http://www.azaruniv.edu

3–9: Summer School on General Algebra and Ordered Sets, Radejov, Czech Republic
Information: ssaos@math.muni.cz; http://www.math.muni.cz/~ssaos/

4–6: Optimal Discrete Structures and Algorithms (ODSA 2006), Rostock, Germany
Information: odsa@uni-rostock.de; http://www.math.uni-rostock.de/odsa/

4–8: Groups in Geometry and Topology Málaga 2006. The First Group Action Forum Conference, Màlaga, Spain
Information: http://agt.cie.uma.es/~ggt06/

4–8: Barcelona Analysis Conference (BAC06), Barcelona, Spain
Information: bac06@imub.ub.es; http://www.imub.ub.es/bac06/

4–8: International Conference on Arithmetic Algebraic Geometry, El Escorial (Madrid), Spain
Information: aag2006@uam.es; http://www.uam.es/otros/aag2006/

4–8: International Seminar on Applied Geometry in Andalusia, Granada, Spain
Information: isaga06@ugr.es; http://gigda.ugr.es/isaga06/

4–8: Derived Algebraic Geometry Workshop, Oslo, Norway
Information: http://www.math.uio.no/~rognes/suprema/dag-workshop.html

4–8: Stochastic Analysis in Mathematical Physics, Grupo de Física-Matemática, Complexo Interdisciplinar da Universidade de Lisboa, Lisboa, Portugal
Information: abcruz@math.ist.utl.pt;
http://gfm.cii.fc.ul.pt/events-en/samp2006/

4–9: International Conference on Applied Analysis and Differential Equations, Iasi, Romania
Information: icaade@uaic.ro; http://www.math.uaic.ro/~icaade

4–9: From a Microscopic to a Macroscopic Description of Complex Systems. CIME school and workshop, Będlewo, Poland
Information: http://web.math.unifi.it/users/cime/Courses/2006/03.php

4–29: The Painleve Equations and Monodromy Problems, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK. Topic: ODE and Dynamical Systems
Information: s.wilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/PEM/

5–8: CDDE 2006, Colloquium on Differential and Difference Equations, Brno, Czech Republic
Information: cdde@math.muni.cz; http://www.math.muni.cz/~cdde/2006

5–8: Uncertainty: Reasoning about probability and vagueness, Prague, Czech Republic
Information: http://www.flu.cas.cz/Logica/Aconf/col2006.html

5–8: CDDE 2006, Colloquium on Differential and Difference Equations, Brno, Czech Republic
Information: cdde@math.muni.cz; http://www.math.muni.cz/~cdde/2006

6–8: The Second International Workshop on Analysis and Numerical Approximation of Singular Problems, Karlovassi, Samos, Greece
Information: iwanasp06@aegean.gr; http://www.tech.port.ac.uk/staffweb/makrogla/conf/IWANASP06/samos06.html

8–10: International Conference on Modules and Comodules. Dedicated to Robert Wisbauer, Porto, Portugal
Information: http://www.fc.up.pt/mp/clomp/ModulesAndComodules/

8–12: 1st Dolomites Workshop on Constructive Approximation and Applications. Dedicated to Walter Gautschi for his 50 years of professional activity, Alba di Canazei (Trento), Italy
Information: dwcaa06@sci.univr.it; http://www.sci.univr.it/~dwcaa06

10–17: Parabolic and Navier-Stokes Equations, Będlewo, Poland
Information: http://www.impan.gov.pl/~parabolic/

11–13: 21st British Topology Meeting, Powys, Wales, United Kingdom
Information: http://www-maths.swan.ac.uk/btm21/

11–16: XV Fall Workshop on Geometry and Physics, Puerto de la Cruz (Tenerife, Canary Islands), Spain
Information: jcmarrer@ull.es;
http://www.gt.matfun.ull.es/15iwgp2006/index.htm

11–17: Quantum transport: modelling, analysis and asymptotics. CIME school and workshop, Cetraro (Cosenza), Italy
Information: cime@math.unifi.it; http://web.math.unifi.it/users/cime/Courses/2006/03.php

12–17: International Conference on Differential Equations, Lviv, Ukraine
Information: http://www.franko.lviv.ua/faculty/mechmat/Departments/Conf/index.htm

14–16: Recent Advances in Free Boundary Problems and Related Topics, Levico, Italy
Information: pierluigi.colli@unipv.it; http://fbp2006.math.unifi.it/

15–19: International Conference of Numerical Analysis and Applied Mathematics 2006 (ICNAAM 2006), Hersonnisos, Crete, Greece
Information: icnaam@uop.gr; http://www.icnaam.org/

18–20: Algebraic curves in cryptography (The 10th Workshop on Elliptic Curve Cryptography (ECC 2006)), Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/#ECC2006

19–21: Credit Risk under Lévy Models, Edinburgh, UK
Information: enquiries@icms.org.uk; http:///www.icms.org.uk/meetings

21–24: Fifth International Conference on Applied Mathematics (ICAM5). In honour of Professor Ioan A. Rus at the occasion of his 70th birthday, Baia Mare, Romania
Information: marieta.gata@rdslink.ro; vberinde@ubm.ro;
http://www.ubm.ro/ro/icam5

22–January 29: Conference on Geometry and Dynamics of Groups and Spaces. In Memory of Alexander Reznikov, Bonn, Germany
Information: gdgs06@mpim-bonn.mpg.de; web-site:
http://www.mpim-bonn.mpg.de/Events/This+Year+and+Prospect/Geometry+and+Dynamics+of+Groups+and+Spaces/

26–29: SYNASC 2006 – 8th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, Timisoara, Romania
Information: synasc06@info.uvt.ro; http://synasc06.info.uvt.ro

October 2006

2–6: Quantum cryptography and computing, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca;
http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/quantum/

4–6: International Conference on Multifield Problems, University of Stuttgart, Germany
Information: mehrfeld@mathematik.uni-stuttgart.de;
http://www.icmp.uni-stuttgart.de

7–10: PDE Approaches to Image Processing, a Workshop sponsored by the ESF Programme "Global and Geometric Aspects of Nonlinear Partial Differential Equations", Cologne, Germany
Information: http://www.mi.uni-koeln.de/~jhorak/workshop/

23–December 15: Stochastic Computation in the Biological Sciences, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK
Information: s.wilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/SCB/

30–November 3: Computational challenges arising in algorithmic number theory and cryptography, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/number_theory/

November 2006

23–25: ESF Workshop Entropy Methods in PDE Theory and in Stochastics, Mainz, Germany
Information: http://www.numerik.mathematik.uni-mainz.de/ESF/workshop.html

27–December 1: Cryptography: Underlying Mathematics, Provability and Foundations, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/crypto_foundations/

December 2006

17–20: Seventh International Conference on Mathematics in Signal Processing, Cirencester, Gloucestershire, United Kingdom
Information: conferences@ima.org.uk; http://www.ima.org.uk/Conferences/7InternationalConfonMathsinSignalProcessing.htm

January 2007

1–June 30: Geometric Applications of Homotopy Theory, Fields Institute Thematic Program, Toronto, Canada
Subprograms: Higher categories and their applications (January-February), Homotopy theory of schemes (March-April), Stacks in geometry and topology (May-June)
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/ index.html

8–June 29: Analysis on Graphs and its Applications, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/AGA/index.html

15–July 6: Highly Oscillatory Problems: Computation, Theory and Application, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/HOP/index.html

22–26: Winter School "Geometric Measure Theory, Random Sets and Digital Stereology", Sandbjerg Estate, Sønderborg, Denmark
Information: oddbjorg@imf.au.dk; http://www.thiele.au.dk/winterschool07/

March 2007

26–30: Workshop: Homotopy theory of schemes, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

May 2007

6–12: Semstat 2007, Statistics for stochastic differential equations models, La Manga del Mar Menor, Cartagena, Spain
Information: mathieu.kessler@upct.es; http://www.dmae.upct.es/semstat2007

14–18: Workshop: Stacks in geometry and topology, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

June 2007

9–13: Workshop: Higher categories and their applications, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

25–26: Mathematical Modelling in Sport, Manchester, United Kingdom
Information: http://www.ima.org.uk/Conferences/conferences.htm

July 2007

1: Summer Conference on Topology and its Applications 2007, Castellón, Spain
Information: http://www.sumtop07.uji.es

9–11: MCP 2007 Vienna – 5th international conference on multiple comparison procedures, Vienna, Austria
Information: http://www.mcp-conference.org

16–20: 6th International Congress on Industrial and Applied Mathematics (ICIAM 07), Zurich, Switzerland
Information: http://www.iciam07.ch

23–December 21: Strong Fields, Integrability and Strings, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/SIS/index.html

September 2007

3–December 21: Phylogenetics, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/PLG/index.html

Recent books

edited by Ivan Netuka and Vladimir Souček (Prague)

Books submitted for review should be sent to the following address: Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic

M. J. Ablowitz, B. Prinari, A. D. Trubatch: Discrete and Continuous Nonlinear Schrödinger Systems*, London Mathematical Society Lecture Note Series 302, Cambridge University Press, Cambridge, 2004, 269 pp., GBP 37,50, ISBN 0-521-53437-2*
Solitons (localized and stable waves interacting almost like elastic objects) in nonlinear systems are a fascinating theme. They were initially studied (over a century ago) in relation to water waves, where they appear as solutions of the corresponding nonlinear differential equations (the Korteweg-de Vries equation). These phenomena have been intensively studied over the last thirty years. This book is devoted to the study of solitons for nonlinear Schrödinger systems in nonlinear optics. Such systems describe, among others, wave transmission in optical fibers, which have technological applications of critical importance.

The book follows an earlier monograph by Ablowitz and Clarkson (Solitons, Nonlinear Evolutions Equations and Inverse Scattering, LMS Lecture Notes Series 149, Cambridge University Press, 1991). The central tool used in the study of these systems, which is thoroughly explained in the book, is the inverse scattering transform (IST): a method that can be viewed as a nonlinear version of the Fourier transform and which allows one to linearize certain classes of nonlinear evolution equations. This method is applied to four different types of nonlinear Schrödinger systems in 1+1 and 1+2 dimensions: Nonlinear Schrödinger equations (NLS), integrable discrete NLS, matrix NLS and integrable discrete matrix NLS. To summarize, this valuable book provides a detailed and self-contained presentation of an extremely important tool used in the study of NLS systems. (mzahr)

S. Altmann, E. L. Ortiz, Eds.: Mathematics and Social Utopias in France: Olinde Rodriques and His Times, History of Mathematics*, vol. 28, American Mathematical Society, Providence, 2005, 168 pp., USD 49, ISBN 0-8218-3860-1*
This book is written by a team of mathematicians, historians of mathematics, and historians of culture and society and it deals with the life and work of B. O. Rodrigues (1794-1851) who was a fascinating figure in Paris in the first half of the nineteenth century. He was born to a Jewish family, which was almost certainly of Portuguese and Spanish origin. Rodrigues studied at the École Normale, where he obtained a doctorate in mathematics in 1816. Then he became a prosperous banker and supported the development of the French railway system. But he was also a social reformer within the Saint-Simon utopian socialist movement. His interests were wide; he wrote on mathematics, politics, banking, social reforms and he discussed moral questions such as the position of women in society, the treatment of workers, etc.

The authors have put together, for the first time, archival resources and documents on B. O. Rodrigues, which are scattered throughout a variety of archives, to show different aspects of Rodrigues‘ fascinating life and to describe his interesting mathematical works and his influence on mathematics in France. The authors corrected some inaccurate references and mistakes that have been repeatedly made about Rodrigues' life. After a description of his scientific, cultural and social backgrounds, Rodrigues' life and career, including his contributions to orthogonal polynomials, combinatorics, groups of transformations, rotations, and applications of quaternions, are discussed from both a mathematical and an historical point of view. The book draws attention to the first half of the 19th century, to a period of French history that was very creative and influential on European intellectual and social development. The reader can find here many roots of modern mathematics as well as foundations of utopian ideas, including the beginning of an understanding of housing and banking transport. The book is recommended for people interested in the roots of modern mathematics and modern society. (mbec)

W. O. Amrein, A. M. Hinz, D. B. Pearson, Eds.: Sturm-Liouville Theory: Past and Present*, Birkhäuser, Basel, 2005, 335 pp., EUR 68, ISBN 3-7643-7066-1*
The conference commemorating the 200th anniversary of the birth of C. F. Sturm took place at Geneva in September 2003. This book contains twelve survey articles, which are expanded versions of contributions to the conference. They show how Sturm's ideas have been developing up to the present day.

Three articles were written by N. Everitt. The first one describes improvements of the classical Sturm and Liouville (S-L) results made by H. Weyl, M. Stone and E. Titchmarsh. The second one presents a catalogue of more than fifty examples of differential equations of S-L type and their properties. The third paper (written jointly with C. Benewitz) refers to the eigenfunction expansion problem and methods based on function theory.

D. Hinton and B. Simon describe various aspects and generalizations of the Sturm oscillation theorem. Investigations of spectral properties of S-L operators have stimulated a great deal of research in operator theory and vice versa. This fact can be followed in the contributions written by D. Gilbert (the link between asymptotics of solutions and spectral properties and, in particular, the concept of subordinacy), Y. Last (discrete and continuous Schrödinger operators), R. del Rio (influence of boundary conditions upon spectral properties) and J. Weidmann (approximation of singular problems by regular ones). Results on the inverse spectral theory for systems of S-L equations are described by M. Malamud. There are also two contributions devoted to nonlinear equations; bifurcation phenomena are presented by C-N. Chen and the overview on evolution of zero sets for solutions of nonlinear parabolic equations is written by V. Galaktionov and P. Harwin.

The level of presentation of all the surveys is accessible to graduate students. They will find here comments on the current state of research and information about the most important ordinary differential equations. Since the articles are also accompanied by many references, reading of the book is a good starting point for research in differential equations and/or functional analysis. Any mathematician will find the historical contextual information useful. (jmil)

P. L. Anderson: Business Economics and Finance with MATLAB, GIS, and Simulation Models*, Chapman & Hall/CRC, Boca Raton, 2004, 472 pp., USD 79,95, ISBN 1-58488-348-0*
This book explains how to apply economic principles to practical problems in business, finance, public policy and other fields. In 17 chapters, it covers topics like mathematical and simulation models in business economics, MATLAB and Simulink design guidelines, importing and reporting data, economic and fiscal models, tax revenue and policy, regional economics, business valuation and applications to finance. Most of the chapters are accompanied by MATLAB code in the appendices. "Think first, calculate later" is the leitmotif of the book. It follows that the text is more a philosophical treatise than a mathematical discourse. Only a few formulas can be found. There are many useful hints, methodological suggestions and a lot of web references in the book. We will mention some of them: methodology of organizing models, conversion of data from various sources (like HTML) to a more useful format, clear explanation of the fiscal system, and many other recommendations that may also be applied to other fields. The book is nicely readable and is recommended for readers interested in philosophical problems of modelling phenomena in business and finance. (jh)

S. A. Argyros, S. Todorcevic: Ramsey Methods in Analysis*, Advanced Courses in Mathematics CRM Barcelona, Birkhäuser, Basel, 2005, 263 pp., EUR 38, ISBN 3-7643-7264-8*
This book consists of two parts prepared by the authors for the Advanced Course on Ramsey Methods in Analysis held at the Centre de Recerca Matemàtica in January 2004. It is aimed at graduate students and researchers interested in the topic, which has developed significantly over the last fifteen years. Big progress was achieved in the 1990's, when several long-standing problems in Banach space theory were solved. In 1991, T. Schlumprecht published his solution of the distortion problem and T. Gowers with W. T. Maurey published their solution of the unconditional basic sequence problem. Their constructions gave rise to more general methods, which turned out to be fruitful in further research and which brought about a number of other deep and interesting results.

The first part of the book (written by S. Argyros) is an exposition of methods of construction of peculiar Banach spaces developed by a unified attitude to the particular examples of Banach spaces given by B. S. Tsirelson and T. Schlumprecht. Applications include constructions of strongly singular extensions, quasi-reflexive hereditarily indecomposable spaces, nonseparable hereditarily indecomposable spaces, as well as the study of operators on such spaces. The second part (by S. Todorcevic) is devoted to various forms of Ramsey theory. The application of finite-dimensional theory to finite representability of the unit basis of _p and co spaces in a basic sequence of a Banach space is explained. Infinite-dimensional Ramsey theory of finite and infinite sequences of Nash-Williams is used to get results on summability in Banach spaces and topological Abelian groups. The Ramsey theory of finite and infinite block sequences in Banach spaces by T. Gowers concludes the exposition. (phol)

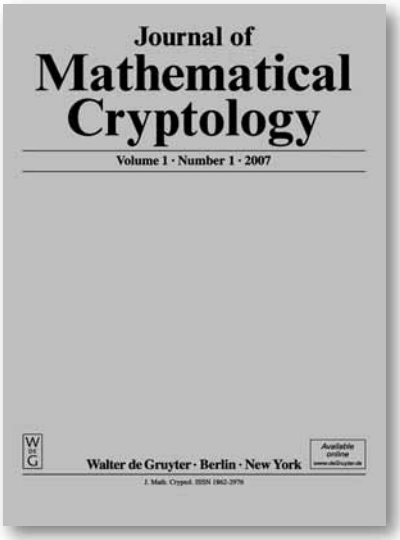
A. Bátkai, S. Piazzera: Semigroups for Delay Equations*, Research Notes in Mathematics, vol. 10, A.K. Peters, Wellesley, 2005, 259 pp., USD 49, ISBN 1-56881-243-4*
This book deals with the semigroup approach to delay equations in Banach spaces, building the theory from scratch. In the first part, semigroup theory and spectral theory for semigroups are presented. These results are required for the rest of the book. The second part deals with the well-posedness of problems containing a leading undelayed term and a delayed term that is in some sense smaller and can therefore be handled as a small perturbation. The third part is concerned with stability. The authors investigate asymptotic behavior via the spectral mapping theorem and critical spectrum, show how things become easier when positivity is involved, and discuss the influence of small perturbations and the size of the delay on stability. Part IV is devoted to second order equations with and without delay in Hilbert spaces and to delay equations where the delay terms contain derivatives of the same order as the leading undelayed terms.

The book describes the semigroup approach to delay equations in a well-arranged and understandable way. Moreover, it contains many references for different approaches to delay equations. Theoretical results are always followed by illustrative examples. Therefore, the book will be useful for graduate students as well as for researchers in evolution equations. (tb)

D. Beltita: Smooth Homogeneous Structures in Operator Theory*, Monographs and Surveys in Pure and Applied Mathematics, vol. 137, Chapman & Hall/CRC, Boca Raton, 2005, 302 pp., USD 89,95, ISBN 1-58488-617-X*
This book summarizes (and acquaints the reader with) some methods of differential geometry that are applied to functional analysis, mainly operator theory. Homogeneous spaces are the focus of interest, i.e. the orbits of group actions endowed with the structure of smooth manifolds. These spaces appear in many settings in the theory of operators and operator algebras. Only a few books have been written on this topic and the present one is very valuable, in part because it presents in terse and clear form the recent results that have previously only been available as journal articles. The author also raises new ideas, e.g. the investigation of operator ideals from the point of view of Lie theory.

Part 1 of the book is an introduction to Lie theory in infinite dimensions. In part 2, geometry of homogeneous spaces is studied. In part 3, the orbits are presented as manifolds, where differential geometric structure carries a lot of operator theoretic information. Many questions concerning further development of this field are set forth. The author also suggests tools that can be used to approach the problems studied. The book is very well arranged. It brings new and fresh ideas and is therefore a challenge and encouragement to those interested in the field. (jdr)

G. Betsch, K.H. Hofmann, Eds.: Hellmuth Kneser: Gesammelte Abhandlungen/Collected Papers*, Walter de Gruyter, Berlin, 2005, 923 pp., EUR 248, ISBN 3-11-016653-4*
As indicated by the title, the book consists of the collected papers that were written by a prominent German mathematician, Hellmuth Kneser, one of few mathematicians who were able to keep pace with the fast development of science and to retain a global view of mathematics at the beginning of the last century. His influence and role were significant and there is a stunning variety of topics to which he has contributed, including theoretical physics, topology, the theory of functions, Lie groups, or-



Starting in 2007

■ Journal of Mathematical Cryptology

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ISSN 1862-2976

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The Journal of Mathematical Cryptology (JMC) is a forum for original research articles in the area of mathematical cryptology. Works in the theory of cryptology and articles linking mathematics with cryptology are welcome. Submissions from all areas of mathematics significant for cryptology are invited, including but not limited to, algebra, algebraic geometry, coding theory, combinatorics, number theory, probability and stochastic processes. The scope includes mathematical results of algorithmic or computational nature that are of interest to cryptology. While JMC does not cover information security as a whole, the submission of manuscripts on information security with a strong mathematical emphasis is explicitly encouraged.

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dinary differential equations and partial differential equations. He also had a great part to play in helping German mathematics catch up with the more mathematically developed countries by filling the gap created after the Second World War in some fields, e.g. in mathematical economy. He did a lot to show to the general public how important mathematics is for modern society and he contributed substantially in improving teaching.

The first chapter (576 pp.), containing Kneser's mathematical articles, is followed by a chapter of review articles (20 pp.). The next chapter (with the title Wissenschaftliche Grundlagen der Schulmathematik, 185 pp.) is devoted to arithmetic, algebra, analysis and geometry. These three chapters are written in German. The rest of the book contains chapters with commentaries on H. Kneser‘s papers on topology and complex function theory, and on his contributions to convexity, foundations and education, and miscellaneous topics by D. Gabai, W. H. Kazez, C. McA. Gordon, K. H. Hoffman, G. Betsch, A. Huckleberry, R. Remmert, M. Range, I. N. Baker, G. Thorbergsson, J. Kindler and G. Pickert. (jdr)

N. Bourbaki: Elements of Mathematics: Integration II. Chapters 7–9*, Springer, Berlin, 2004, 326 pp., EUR 99,95, ISBN 3-540-20585-3*

This volume of the new English edition of the famous series of monographs contains chapters 7, 8, and 9, which are devoted to integration theory. The first six chapters can be found in Integration I of the new English edition, which also appeared recently. Chapter 7 is an exposition of the theory of Haar measures. It contains the basic theory on locally compact groups, an abstract theory, applications and examples. Exercises conclude this as well as the other chapters. The theory of convolutions and linear representations of groups is the subject of chapter 8. Measures on Hausdorff topological spaces are investigated in the last chapter. Besides the general theory, which covers results on products, disintegration, inverse limits and tight convergence of measures, a paragraph is devoted to measures on locally convex spaces. It includes the theory of Fourier transforms, Gaussian measures, measures on Hilbert spaces, etc.
Knowledge of an undergraduate course of mathematics and of some parts of Integration I is necessary for good understanding. Some preliminary experience with concrete examples of the abstract objects under consideration may be useful as well. Due to an abstract, general and exact exposition, the textbook may also be of interest for specialists in the topic. (phol)

J. Bourgain: Green‘s Function Estimates for Lattice Schrödinger Operators and Applications*, Annals of Mathematics Studies, no. 158, Princeton University Press, Princeton, 2004, 172 pp., USD 35, ISBN 0-691-12098-6*

This book deals mainly with the localization problem (i.e. with a description of the properties of the spectrum, the eigenstates and the resolvent) for quasi-periodic lattice Schrödinger operators. The origins of the field can be traced to the seminal works of Sinai and collaborators in the eighties and to the work of Fröhlich, Spencer and Wittwer. The main goal of the book is to give an overview of some of the important lines of current research in the field and to present it in a unified way. Some parts of the text thus follow original research literature, in particular recent articles written by the author and his collaborators.

This determines the organization of the book, which consists of twenty short chapters. Both nonperturbative and perturbative results are treated. Ideas on the latter (from chapter 14 onwards) come from KAM theory and they have a wide range of applications. The book also contains new results, e.g. results concerning regularity properties of the Lyapunov exponent and integrated density of states in chapter 8. Chapters 18–20 deal with the problem of quasi-periodic solutions for infinite Hamiltonian systems given by nonlinear Schrödinger or wave equations. A new general method presented by the author can be used to consider the problem for systems that are perturbations of the linear ones in any dimension. The Figotin-Pastur approach for Schrödinger operators associated to strongly mixing dynamical systems is explained in the appendix. The whole subject is still an ongoing area of research and some methods (the renormalization group by Helfer and Sjöstrand) are omitted in the book. Still, it gives an up to date review of this important field of current research. (mzahr)

C. Browne: Connection Games: Variations on a Theme*, A. K. Peters, Wellesley, 2005, 402 pp., USD 48, ISBN 1-56881-224-8*

Connection games constitute certain kinds of board game that share a common feature: the main goal is to develop or complete a connection of some sort before the other players do. The games have plenty of applications in many areas, in particular in economy. The classic connection game is called Hex, invented in the 40's by Danish mathematician Piet Hein and reinvented shortly afterwards by the famous John Nash. The subject of connection games had been studied with growing interest through the second half of the twentieth century and has seen a real boom since 2000. By the outbreak of 21st century, connection games had finally earned certain recognition as a distinct genre of board game. Connection games can be described mathematically but they can also be enjoyed by players with no mathematical education whatsoever. The introduction of connection games to the public was established through Martin Gardner's Scientific American article 'Concerning the Game of Hex, which May be Played on the Tiles of the Bathroom Floor', which was published in 1957.

It seems that time has come for a comprehensive book dedicated purely to connection games to appear. And here it is! This book by C. Browne is a true masterpiece in an area in which such a thing was desperately needed and there is no doubt that it will become a classic. Moreover, it is likely to attract many people not only to connection games or strategy games, but, in general, to mathematics. What a blessing! The book is a skilful blend of history, graph theory, psychology, philosophy, strategy and tactics, and, most of all, it is an encyclopedia of games, plenty of which haven‘t been published before. The book is unbearably entertaining and dangerous too; one should certainly not open it unless one has a free two hours or so.

First of all, the book describes a precise definition of what is exactly meant by a connection game. Examples illustrate why for example Go or Tic Tac Toe are not connection games. The notions of adjacency, connection, pattern, duality, switching games, graphs, edges, vertices, deadlocks and more are explained thoroughly, correctly and completely, and yet the expository is carefully kept light enough in order not to scare away math-phobic readers. The main part of the text is devoted to a detailed description of the games (Part II). Each game has its own section in which the rules, strategy and tactics, history and variants are described together with interesting notes. Although the games are examined in detail, the emphasis is on exploring their philosophy, fundamental nature and what distinguishes them from other games.

Many games are published here for the first time. Most of them have been invented since 2000, many by the author of the book (such as the intriguing three-dimensional game Druid). One of the things I like most about this book is that it concentrates solely on board games. Computer games are excluded unless they have a reasonable board version. OK. Enough! Don't read this review. Read the book. It is simply a great one. (lp)

L. Caffarelli, S. Salsa: A Geometric Approach to Free Boundary Problems*, Graduate Studies in Mathematics, vol. 68, American Mathematical Society, Providence, 2005, 270 pp., USD 49, ISBN 0-8218-3784-2*

This book offers a comprehensive treatment of the subject, rich in methods and results. It provides a presentation of elliptic and parabolic free boundary problems and techniques used for treating basic results. Part 1 concerns elliptic problems. After a formulation of the problem and its description in an introductory chapter, viscosity solutions are introduced and studied in chapter 2. Chapters 3–5 are dedicated to the regularity of a free boundary. In chapter 6, the existence theory for viscosity solutions of free boundary problems in a smooth (Lipschitz) domain is constructed. Part 2 is devoted to evolution problems, including the chapters: 7) Parabolic free boundary problems; 8) Lipschitz free boundary: weak results; 9) Lipschitz free boundary: strong results; 10) Flat free boundary are smooth. In Part 3 (Complementary chapters: main tools), the boundary behaviour of harmonic functions (chapter 11) and the boundary behaviour of caloric functions (chapter 13) in Lipschitz domains are studied. Monotonicity formulas and their applications are considered in chapter 12.

As the explained material becomes quickly rather complicated, it might be useful to add a list of symbols to the next edition; it would be highly appreciated by any reader who needs to study some separate "internal" parts of this important and valuable monograph. (oj)

S.-Y. A. Chang: Non-linear Elliptic Equations in Conformal Geometry*, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2004, 92 pp., EUR 24, ISBN 3-03719-006-X*

A study of curvature invariants in conformal geometry and related nonlinear partial differential equations has a long tradition going back to a study of behaviour of Gauss curvature under a conformal change of metric in two dimensions. Recently, a considerable effort has concentrated around a higher dimensional generalization of Gauss curvature called Q-curvature and its relations to higher order conformally invariant differential operators.

This book contains a discussion of many related topics. It starts with a description of the case of surfaces, in particular the equation prescribing Gauss curvature on compact surfaces. After a discussion of conformal invariants and conformally invariant partial differential equations in higher dimensions, the author concentrates on the dimension four case. The key role is played here by a special conformally invariant fourth order operator called the Paneitz operator. Its relation to the second elementary symmetric function of the Ricci tensor in dimension four is carefully discussed in the book, which is based on lectures given by the author at ETH in Zürich. The formula for Q-curvature and many results around it are due to Tom Branson. It is a sad fact that he unexpectedly died a few months ago. But the impact of his ideas and work is clearly illustrated in the book and they will stay with us. (vs)

M. Chipot, J. Escher, Eds.: Nonlinear Elliptic and Parabolic Problems*, Progress in Nonlinear Differential Equations and Their Applications, vol. 64, Birkhäuser, Basel, 2005, 536 pp., EUR 168, ISBN 3-7643-7266-4*

This book is the proceedings of a conference held in Zürich, Swirtzerland, in 2004. It contains about thirty articles, mostly dealing with areas where H. Amann made an important contribution. It could be said that the mathematical problem behind all the articles is the nonlinear elliptic/parabolic equation $\partial t\, u - \Delta u + f(x,u, \text{grad } u) = 0$ or a system of such equations. The articles could be roughly divided into three groups. (i) Papers dealing with a single equation of the above type in its simplest form. In such a case, quite a detailed description of the qualitative behaviour can usually be given. (ii) Systems of equations of the above type that have a clear physical, technical or biological application (the incompressible, homogeneous Navier-Stokes equations are a particular example). (iii) Abstract functional analytic problems, bifurcations, and the properties of function spaces arising in connection to an equation of the above type. The papers usually contain new results but many of them also provide a longer survey of results for the particular area. (dpr)

H. Cohen, G. Frey: Handbook of Elliptic and Hyperelliptic Curve Cryptography*, Discrete Mathematics and Its Applications, Chapman & Hall/CRC, Boca Raton, 2005, 808 pp., USD 99,95, ISBN 1-58488-518-1*

While the title refers to elliptic curves, the handbook covers many further aspects. Alternative titles might be "Mathematics of discrete logarithm systems and applications" or "Treatise on public-key cryptography and its mathematical background". To illustrate this fact, let us mention that 120 pages are devoted to algorithms of arithmetic operations (integers, finite fields, p-adic numbers), 25 pages to the algebraic and number-theoretic background and 25 pages to an overview of algorithms for computing the discrete logarithm without using curves (there are 30 pages that exhibit algorithms of this kind that do use curves). The last part of the handbook deals with hardware, smart cards, random number generators and various cryptological attacks (invasive, non-invasive and side channel). In this part only about 20 pages out of 120 are directly concerned with curves. Nevertheless, the bulk of the handbook is true to the title.

The background (varieties, pairings, Weil descent) is covered in about 100 pages, the arithmetic of curves taking a further 120 pages (two thirds of which deal with hyperelliptic curves, Koblitz curves and some other special classes). Furthermore there are chapters on implementation of parings, point counting and complex multiplication (which is needed to compute the class polynomials). The section called "Applications" is concerned with the choice of parameters for efficient systems, their construction, usual heuristics and protocols. Pairing-Based Cryptography is then treated in more detail. There is also a chapter on compositeness and primality testing and factoring that gives methods based on curves as one approach amongst others that are used as a standard. Many algorithms and theorems in central chapters come with proofs. Elsewhere the reader is confronted with lists of lemmas, propositions, theorems, algorithms and examples that often appear without much justification. This is of course dictated by the nature of the publication.

The book seems to be a very useful, mainly because of the breadth and depth of the material it covers. A successful effort has been made to trace every result and algorithm to its mathematical origins. Theoretically one could start using the book equipped with a very limited mathematical background. However, then one would have to be able to digest a lot of mathematics that is stated as a fact without proof. The idea seems to have been to address a very wide audience. I am not totally convinced that the field is well suited for such an approach, a question the authors must have been asking themselves as well. The amount of applications that involve elliptic curves seems to be the main reason why such a widely based enterprise got realized. (ad)

J. Copeland, Ed.: Alan Turing's Automatic Computing Engine*, Oxford University Press, Oxford, 2005, 553 pp., GBP 55, ISBN 0-19-856593-3*

This book is an excellent collection of papers covering a major part of Alan Turing's contribution to computer development. Contrary to the title, only the invention of a "universal computing machine" and its implementation as the ACE machine is covered. The detailed calculation and schematics of individual parts of the machine brings us into the mystery of computing at end of the Second World War and provides the reader with a complete picture of the time of the "computer pioneers". The concentration of the editor on the ACE machine unfortunately leaves other important contributions of Alan Turing (e.g. the Colossus machine and his artificial intelligence papers) uncovered. Nevertheless the amount of information collected by the editor is huge and it offers interesting reading for anybody with a deep interest in computer history. (vjir)

S. Cordier et al., Eds.: Numerical Methods for Hyperbolic and Kinetic Problems*, IRMA Lectures in Mathematics and Theoretical Physics 7, European Mathematical Society, Zürich, 2005, 359 pp., EUR 44,50, ISBN 3-03179-012-4*

This proceedings-like contribution presents the results obtained during and after (as an output of) the CEMRACS summer research center held at CIRM in Luminy in 2003. The brief was to study problems arising in kinetic and hyperbolic theory from a numerical point of view. This includes a wide variety of industrial and engineering problems, including multi-phase flows, plasma physics problems, quantum particle dynamics, radiative transfer, sprays and aero-acoustics. From a mathematical point of view, in altogether 14 contributions, we learn about problems connected with numerical solutions of the Vlasov equation, the nonlinear Schrödinger equation, the conservative bifluid model, two-phase flows and phase transition modeling, etc. As to the numerical methods used, the authors solve the problems listed above generally by Particle-In-Cell methods and the time splitting spectral scheme (kinetic problems), and explicit finite volume methods and the discontinuous Galerkin method (hyperbolic problems). This contribution will be valuable for researchers, engineers and graduate students seeking information on the topic. (mrok)

B. N. Delone: The St. Petersburg School of Number Theory*, History of Mathematics, vol. 26, American Mathematical Society, Providence, 2005, 278 pp., USD 59, ISBN 0-8218-3457-6*

This book (published by the Academy of the USSR in 1947) is an English translation of the Russian original written by B. N. Delone (1890–1980). He was a distinguished member of the St. Petersburg school and the main aim of the book is to present classical works of number theory written by Russian mathematicians and members of the school: P. L. Chebyshev (1821–1894), A. N. Korkin (1837–1908), E. I. Zolotarev (1847–1878), A. A. Markov (1856–1922), G. F. Voronoi (1868–1908) and I. M. Vinogradov (1891–1983). B. N. Delone treats the life and work of these six mathematicians in turn, starting each with a short biography and continuing with a description of their most significant number theory contributions. He first presents their papers in the authors' original terminology and notation and then he adds commentaries of greater or lesser breadth and depth. The author is strictly focused on their number theory works and he does not discuss their important contributions to other branches of mathematics.

The English text is essentially unaltered, with a few footnotes and references added to clarify the author's exposition. Although the mathematical expositions are much longer than the biographies and historical notes, we get a good impression of their professional lives as well as a good description of the beginnings and development of the St. Petersburg mathematical school. At the end, the reader will find a chronological list of works in number theory written by the six mathematicians. The book should be very interesting to a broad mathematical community, in particular mathematicians working in number theory as well as historians of mathematics concentrating on the development of mathematics in the second part of the 19th century. (mbec)

C. Eck, J. Jarůšek, M. Krbec: Unilateral Contact Problems: Variational Methods and Existence Theorems*, Pure and Applied Mathematics 270, Chapman & Hall/CRC, Boca Raton, 2005, 398 pp., USD 99,95, ISBN 1-57444-629-0*

Contact problems (when two bodies touch each other) arise naturally in many areas of mechanical engineering, machine dynamics and manufacturing, and when predicting earthquakes. In this book, they are studied by mathematical methods. Despite the fact that the notion of contact is quite natural, the corresponding frictional contact problems are rather delicate. The problem has a variational formulation in terms of variational inequalities. The frictional functional turns out to be neither monotone nor compact. Moreover, the Euclidean norm makes the friction non-smooth. The basic method to overcome these difficulties is to approximate the given problem by another problem with a simpler structure. In most cases, the Coulomb law of friction is substituted by a given friction. A solution of the original problem is then obtained by the fixed point theorem. The second method is to penalize Signorini contact conditions and to get a solution of the original problem by taking a limit of the penalty parameter. In both these methods, additional a priori estimates of solutions to approximated problems are needed. Typically, some estimates in Besov spaces are derived by the method of tangential translation.

The necessary background from the theory of Besov spaces is explained in chapter 2. In chapter 3, results for static contact problems obtained by time discretization as well as for quasi-static contact problems (if the motion of the body is very slow) are presented. The existence results for problems with one or two bodies are given provided the coefficients of friction are small enough. The size of the coefficients is then calculated. Chapter 4 is devoted to dynamic contact problems. The authors start with results for purely elastic bodies, which are limited to strings, polyharmonic problems and to special problems on half-space. In the rest of the chapter, an additional viscosity is added to properties of the body. Materials with short and long memory are also studied.

In the last chapter, the authors return to problems involving the Coulomb law for materials with a short memory. In order to overcome the lack of regularity, they replace Signorini conditions by their first order approximations with respect to time. Also existence is obtained if the friction coefficient is small and its size is explicitly computed. As there is dissipation of energy due to viscosity, it is natural to add an equation for this dissipated energy to the system. This is done in the last section of this chapter.

The book provides a self contained overview of the state of research in the given area. It will surely be appreciated by scientists working in this field and it will motivate them to further research. As there are explicitly computed bounds for the friction coefficient, the book will also be useful for engineers working on numerical approximation of contact problems. (pkap)

S. Fischler, E. Gaudron, S. Khémira, Eds.: Formes modulaires et transcendance*, Séminaires & Congres, Société Mathématique de France, Paris, 2005, 269 pp., EUR 41, ISBN 2-85629-176-7*
This is a carefully written introduction to the (very few) existing results on transcendence and algebraic independence of values of modular forms, based on a series of mini-courses delivered at a conference organized by a group of young French mathematicians in 2003. The book consists of four chapters. The first one gives a survey of modular forms with special emphasis on objects relevant to transcendence proofs, such as the Rankin-Cohen brackets, quasi-modular and quasi holomorphic modular forms. The second chapter is devoted to Nesterenko's result on algebraic independence of values of certain Eisenstein series. Chapter 3 presents a proof of an earlier result (the "St. Etienne theorem" on transcendence of J(q)) from a more conceptual and geometric perspective. The final chapter offers an introduction to Hilbert modular forms in the classical language (including generalizations of the Rankin Cohen brackets). Various methods of construction of such forms are made explicit for the field $Q(\sqrt{5})$. (jnek)

A. Gueraggio, P. Nastasi: Italian Mathematics Between the Two World Wars*, Birkhäuser, Basel, 2005, 299 pp., USD 119, ISBN 3-7643-6555-2.*
The beginning of the 20th century is often considered to be the golden age of Italian mathematics. Together with Germany and France, Italy was one of the three "mathematical powers" of the world. Things have changed in the period between the two world wars. The authors describe events that influenced Italian mathematics such as the rise of fascism to power in the 1920s, the anti-Semitic laws, and international isolation of Italy in the late 1930s. The main characters of Italian inter-war mathematics were V. Volterra, F. Enriques, T. Levi-Civita, F. Severi, L. Tonelli and M. Picone. We learn about their relationships, ambitions, political opinions and of course their professional interests. Mathematical research was concentrated around three disciplines, which had a long tradition in Italy: algebraic geometry, analysis and mathematical physics. While still producing important results, the Italians found it too difficult to maintain the high level that they had reached at the beginning of the century. This fact was perhaps related to a certain lack of interest in new theories such as functional analysis, modern algebra and topology, which flourished in other countries. (asl)

R. Hardt, Ed.: Six Themes on Variation*, Student Mathematical Library, vol. 26, American Mathematical Society, Providence, 2004, 153 pp., USD 29, ISBN 0-8218-3720-6*
To attract talented young students to research in mathematics is a problem facing all universities. A purpose of the meeting organized at Rice University was to present various aspects of calculus of variation in a way that was understandable to undergraduate students. The workshop was very successful and the reader can find a written version of lectures in this small booklet. The lectures are very well prepared and their topics are quite attractive.

They include a discussion of Euler-Lagrange equations, an introduction to Morse theory, a discussion of minimal surfaces, an explanation of isoperimetric problem and the double bubble conjecture, a presentation of new extraordinary minimal surfaces, a study of string vibrations and of decay of their amplitudes, and differential equations modelling traffic flow. Themes described in the book would need difficult and advanced mathematics for a complete treatment. Nevertheless, the authors succeed in presenting the main ideas of the chosen topics at an understandable and elementary level. The book is recommended to an audience of undergraduate students as well as to teachers looking for inspiration for their own lectures. (vs)

I. N. Herstein: Noncommutative rings*, The Carus Mathematical Monographs 15, The Mathematical Association of America, Washington, 2004, 202 pp., GBP 20, ISBN 0-88385-015-X*
This is the fourth printing of a classic Carus Mathematical Monograph first published in 1968. The book is not a treatise on noncommutative rings. Its aim is to present several fundamental ideas of the theory and some classical theorems. Prerequisities are a first course on abstract algebra and a basic knowledge of field theory as covered in the introductory chapter on fields in van der Waerden's Modern Algebra. After the first two introductory chapters on the structure of noncommutative rings, which are focused on the Jacobson radical and semisimple rings, the authors cover basic results on commutativity of division rings, simple algebras, representations of finite groups and polynomial identities. The two final chapters present the Goldie theorem and the Golod-Shafarevitch theorem. The book is an excellent introduction to the theory of noncommutative rings for advanced students as well as for non-specialists. (jtu)

***L. Hodgkin, Ed.: A History of Mathematics. From Mesopotamia to Modernity**, Oxford University Press, Oxford, 2005, 281 pp., GBP 39,50, ISBN 0-19-852937-6*

This excellent book on the history of mathematics describes the evolution of mathematics from Babylon, through time and across most of the important Eastern and Western cultures and civilizations, up to the modern era. The way of thinking and diligence, and the achievements and heritage of Greek mathematics are covered in detail. The development of Chinese and Islamic mathematics and their influence on European mathematics are studied in detail as well. Early Western mathematics (Middle Age, Renaissance) is presented only in a global context of scientific revolution. The leading figures in mathematics and its history (Galileo, Newton, Leibniz, Helmholtz, Hilbert and many others) are introduced with their mathematical achievements, results and theories. The history of modern mathematical disciplines (including computer science, topology and category, chaos and fractals) and the story of the solution of Fermat's Last Theorem are also included. The author briefly and clearly explains mathematical methods, theories and algorithms of respective fields of mathematics. It is a pity that no attention is given to mathematics in Egypt and Japan.

The book contains more than 100 illustrations, pictures and figures, and many exercises (with solutions). An extensive bibliography with cross-references will be very helpful for students and readers. The book will be interesting for undergraduate and postgraduate students of mathematics and other readers interested in the history and philosophy of mathematics. (mbec)

***G. E. Karniadakis, S. Sherwin: Spectral/hp Element Methods for Computational Fluid Dynamics**, second edition, Numerical Mathematics and Scientific Computation, Oxford University Press, Oxford, 2005, 657 pp., GBP 60, ISBN 0-19-852869-8*

This book provides the reader with a suitably detailed, extensive and mathematically precise treatment of spectral/hp element methods applied to numerical solutions of problems in computational fluid dynamics (CFD). It contains a detailed explanation of fundamental concepts, derivation of methods, and their analysis and realization, accompanied by a number of applications. The book comprises ten chapters and six appendices.

The first chapter is concerned with models of compressible and incompressible flow. In chapter 2, fundamental concepts of spectral/hp techniques are formulated in the framework of one-dimensional models. Chapter 3 deals with an extension of fundamental concepts to multidimensional situations. In chapter 4, implementation aspects are treated. In particular, the authors are concerned with the so-called local operations, global operations and pre- and post-processing. In chapter 5, attention is paid to the use of spectral/hp methods for the solution of diffusion equations. It contains discretization of the Helmholtz equation, temporal discretization, investigation of eigenspectra and the iterative solution of the weak Laplacian, and problems in non-smooth domains. Advection and advection-diffusion problems are treated in chapter 6. Here, among other, Galerkin and discontinuous Galerkin techniques are explained.

Chapter 7 is concerned with non-conforming elements. It contains the treatment of interface conditions, constrained approximation, mortar patching and various types of discontinuous Galerkin methods. Chapter 8 is devoted to explanation of algorithms for solution of incompressible flow models based on the use of primitive variables as well as velocity-vorticity formulations. Chapter 9 presents several examples of incompressible flow simulations using simple laminar flow benchmarks, which possess exact solutions and allow one to ensure that spectral/hp codes give the correct results. One also finds here direct numerical simulation (DNS) and large-eddy simulation (LES) of viscous incompressible flow. Finally, in chapter 10, spectral/hp techniques for hyperbolic problems are treated.

The book contains a large amount of material, including a number of exercises, examples and figures. The book will be helpful to specialists coming into contact with CFD, applied and numerical mathematicians, engineers, physicists and specialists in climate and ocean modeling. It can also be recommended for advanced students of these disciplines. (mf)

***M. Kreck, W. Lück: The Novikov Conjecture: Geometry and Algebra**, Birkhäuser, Basel, 2004, 266 pp., EUR 38, ISBN 3-7643-7141-2*

This volume stems from an Oberwolfach seminar that was held in January 2004. It will attract attention from both beginners and specialists. Postgraduate students will find here a very clearly described motivation for the Novikov conjecture and they will be pleased by the fact that many notions, many of which they may not be familiar with, are described in the subsequent chapters with quite a few details. There are chapters treating the signature and the signature theorem, cobordism theory, the Whitehead group and the Whitehead torsion, several chapters concerning the surgery, a chapter on spectra and a chapter on classifying spaces of families. The book includes a proof of the Novikov conjecture for finitely generated free abelian groups.

It is very useful that all this material is concentrated in one volume but what I mostly appreciate is that the reader always knows why he or she is studying this or that theory and where these theories have their applications. There is no doubt that it is very stimulating. For better understanding, the book concludes with a whole chapter of exercises and one more chapter containing hints to these exercises. The book has an index as well as an index of notations and a long list of references consisting of 258 items. On the other hand, a specialist in topology will find here information about the recent developments concerning the Novikov conjecture. A lot of attention is also devoted to other conjectures closely related to the Novikov conjecture, especially to the Farell-Jones and Baum-Connes conjectures. (jiva)

***N. Lauritzen: Concrete Abstract Algebra: From Numbers to Gröbner Bases**, Cambridge University Press, Cambridge, 2003, 240 pp., GBP 19,99, ISBN 0-521-53410-0, ISBN 0-521-82679-9*

This book introduces basic notions and results from abstract algebra together with their concrete applications to cryptography and factorization of numbers and polynomials. Its first chapter is called "Numbers"; it starts from scratch and ends with the RSA method and with algorithms for prime factorization. Chapter 2 (Groups) develops basics of group theory and ends with the Sylow Theorems. Chapter 3 (Rings) presents basic facts on commutative rings, with emphasis on unique factorization. Chapter 4 (Polynomials) is a preparation for the final chapter, which is called "Gröbner bases". There, the classical Hilbert basis theorem is proved using Gröbner bases, and the Buchberger algorithm for computing Gröbner bases is presented in detail, together with simple applications to solving systems of polynomial equations. Unlike many books on abstract algebra, this one is written in a very lively style, with emphasis on illuminating examples and applications. This makes the book a valuable addition to undergraduate literature on this topical subject. (jtrl)

M. Liebeck: A Concise Introduction to Pure Mathematics*, second edition, Chapman & Hall/CRC Mathematics, Chapman & Hall/CRC, Boca Raton, 2005, 204 pp., USD 44,95, ISBN 1-58488-547-5*

Here follows a brief review of the second edition of this book (for a detailed review of the first edition, see Newsletter issue 38). The aim of this well-written book is to fill the gap between high-school mathematics and mathematics at university. The author shows to the reader what it means to prove something rigorously. The book includes sections about number systems, combinatorics and geometry, and gives a basic introduction to analysis and set theory. The second edition contains more exercises and new chapters about RSA codes and permutations. (shen)

S. Mac Lane: A Mathematical Autobiography*, A. K. Peters, Wellesley, 2005, 358 pp., USD 39, ISBN 1-56881-150-0*

This book is an excellent autobiography written by the one of the greatest 20th century mathematician Saunders Mac Lane (1909–2005). The book is an account of his personal and scientific life. In fifteen parts subdivided into 64 chapters, the author describes his life from his youth and studies in the USA, through his studies in Göttingen (where he was close to D. Hilbert) up to his stays at many important universities (including Yale, Harvard and Chicago). He explains the academic, cultural and political atmosphere and traditions in the USA, Europe and China in the 20th century. His memoirs contain many interesting reflections on his scientific work and teaching concepts, relations between the political situations and scientific activities in the USA and Europe, and the relationship between his professional and personal lives.

He presents his inspiring style of teaching, which influenced many leading mathematicians of our time in the USA. He explains his scientific activities and ideas (research in modern algebra, category theory, homological algebra, algebraic functions, geometric mechanics and the philosophy of mathematics). He also describes his work on the monographs 'A Survey of Modern Algebra' (with G. Birkhoff) and 'Homology', which influenced the development of modern mathematical research. Some chapters are devoted to his activities as part of the American Mathematical Society (he was its influential president) and the National Academy of Science. He mixes professional observations with deeply personal commentaries. Many photographs of Mac Lane, his family and friends are included. The book was written not only for mathematicians but also for laymen interested in mathematics, mathematicians and their lives. (mbec)

D. Mumford: Selected Papers. On the Classification of Varieties and Moduli Spaces*, Springer, New York, 2004, 796 pp., EUR 99,95, ISBN 0-387-21092-X*

This book contains a collection of reprints of important classical papers written by David Mumford. The book is divided into three parts, each of them coming with commentaries by leading experts in the field. The first part contains the Mumford study of various aspects of the moduli space of algebraic curves. He was the first to formulate a purely algebraic approach (based on geometric invariant theory and valid in all characteristics) to the description of the moduli space of Riemann surfaces as a quasi-projective variety. It also applies to Chow rings, tautological classes, enumerative geometry and compactification of the moduli spaces of algebraic curves by stable (nodal) algebraic curves.

The second part is devoted to Mumford's work on (finite) theta functions and equations of Abelian varieties, families of Abelian varieties and their degeneracies, theta characteristics, the Horrocks-Mumford bundle, Prym varieties and compactifications of bounded symmetric domains. The third series of Mumford's articles focuses on the classification of surfaces and other special varieties. It involves vanishing theorems and pathologies in positive characteristic for complex projective varieties and a classification theory of surfaces in positive characteristics. The Iitaka and the Mori minimal model program as well as many explicit (non-)existence results are included in a series of articles on pathologies in algebraic geometry. (pso)

A. J. Milani, N. J. Koksch: An Introduction to Semiflows, Monographs and Surveys in Pure and Applied Mathematics*, vol. 134, Chapman & Hall/CRC, Boca Raton, 2004, 386 pp., USD 89,95, ISBN 1-58488-458-4*

This book is devoted to a study of the asymptotic behaviour of semilinear dissipative evolution equations. Attracting sets of interest for such equations include attractors, in particular exponential ones, and inertial manifolds. Existence of these sets is studied in the book. The first two chapters contain a brief introduction to dynamical systems and their dissipativity properties. The core of the book consists of the next three chapters. Existence of global attractors is proved in chapter 3 for a semilinear reaction-diffusion equation and a dissipative wave equation. These results are refined in chapter 4, where geometric conditions for exponential attractivity are described. The regularity (Lipschitz continuity) of attractors is examined in chapter 5 in a general context. The notions of a squeezing property, cone invariance and a spectral gap condition are introduced. With their help, the construction of an inertial manifold is presented.

As concrete examples, semilinear parabolic and hyperbolic equations with one-dimensional space variables are investigated. As a counterpoint, the result (due to Mora and Sola-Morales) on non-existence of inertial manifolds for hyperbolic equations is proved in chapter 7. In chapter 6, various types of attractor are constructed for famous nonlinear equations (Cahn-Hilliard, extensible beam, two dimensional Navier-Stokes and Maxwell).

The book is carefully written and it can be strongly recommended to graduate students as a guide before reading papers oriented to specialists. The book also contains eight short appendices that include basic facts about differential equations and functional analysis. The text will also be of use to researchers in applied fields, since they will find here a rigorous treatment of basic notions and results on dissipative systems together with many references. (jmil)

Moreno, S. S. Wagstaff, Jr.: Sums of Squares of Integers*, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2005, 354 pp., USD 89,95, ISBN 1-58488-456-8*

Sums of squares is an important topic in number theory; it represents a melting pot for various methods, extending from elementary ideas to advanced analytic and algebraic techniques. This book presents many facets of ideas developed in the past to solve problems connected with the representation of integers as sums of squares. Starting with elementary methods that have been used since the field was young and ranging to Liouville's contribution, the book continues through the theory of modular forms and the analytical methods used to count the representations. An interesting extension (in comparison with other books on the subject) is the two chapters devoted to arithmetic progressions and applications to real life problems. The first one deals with the theorem of van der Waerden, Roth and Szemerédi (probably the first book that gives a proof of this deep result) and the second one covers connections to factorization and applications in the areas of microwave radiation, diamond cutting and cryptanalysis.

The book is written in a very fresh style and it is essentially self-contained (e.g. the Riemann-Roch theorem is used but not proved) and it includes about a hundred interesting exercises of various levels to test the reader's understanding of the text. The book can be recommended to those interested in the development of ideas in the subject and their context. (spor)

***S. Oakes, A. Pears, A. Rice: The Book of Presidents 1865–1965**, London Mathematical Society, London, 2005, 157 pp., GBP 19, ISBN 0-9502734-1-4*

This book concentrates on the history of the London Mathematical Society, which was established in 1865 during the reign of Queen Victoria and which became one of the most important scientific societies and well respected in Europe and America. In the introduction, the authors describe its history using the lives of its eminent members as a basis. The second part contains short biographies and scientific evaluations of the presidents of the London Mathematical Society from 1865 up to 2003 (from A. De Morgan up to F. C. Kirwan), together with more than eighty photographs and a list of presidential addresses given at the conclusion of their respective presidencies. The third part contains a list of the De Morgan medalists from 1884 (A. Cayley) up to 2004 (R. Penrose) with their photographs and short biographies. At the end, a list of honorary members from 1867 (M. Chasles) up to 2004 (I. M. Singer) is added. The short glossary explains the history of prizes, sizeships, medals, orders and "chairs" of the society.

This is an excellent book, which can be recommended to readers willing to learn about and appreciate the rich history of the London Mathematical Society and to know which branches of mathematics were popular in the second part of the 19th century through to the 20th century. (mbec)

***E. M. Ouhabaz: Analysis of Heat Equations on Domains**, London Mathematical Society Monographs, vol. 31, Princeton University Press, Princeton, 2004, 284 pp., GBP 38,95, ISBN 0-691-12016-1*

Although the title suggests that heat equations would be the only topic of the book, the reader soon observes that the book is devoted to a more general study of the L_p theory of evolution equations associated with non-self-adjoint operators in divergence form. The author uses the technique of sesquilinear forms and semigroup theory, which avoids using Sobolev embeddings and therefore does not need smoothness properties of the boundary. On the other hand, this means that the question of regularity is not addressed by this approach.

One of the aims of the author was to write a self-contained contribution, which will be useful for a majority of readers. First we learn some necessary background material from functional analysis, semigroup theory, sesquilinear forms and the theory of evolutionary partial differential equations needed to understand the topic. The author proceeds to study semigroups associated with sesquilinear forms, both for uniformly elliptic and degenerate elliptic operators. Finally, an even more general approach is presented. The L_p estimates for the Schrödinger and wave-type equations are given in the setting of abstract operators on domains of metric spaces. This framework includes operators on general Riemannian manifolds, sub-Laplacians on Lie groups or Laplacians on fractals. The book is intended not only for specialists in partial differential equations but also for graduate students who want to learn about the sesquilinear form technique and the semigroup approach to partial differential equations containing second-order elliptic operators in divergence form. (mrok)

***A. Papadopoulos: Metric Spaces, Convexity and Nonpositive Curvature**, IRMA Lectures in Mathematics and Theoretical Physics 6, European Mathematical Society, Zürich, 2004, 285 pp., EUR 48, ISBN 3-03719-010-8*

A lot of standard notions known from Riemannian geometry have their counterparts for metric spaces. Busemann gave a definition of a nonpositively curved metric space using convexity properties of the distance function. It is also possible to study geodesics on a metric space. This book offers a systematic description of these points. In the first part, the author reviews basic notions about metric spaces (lengths of paths, length spaces and geodesic spaces, and distances). The second part of the book studies questions related to convexity in vector spaces. In the last part of the book, the author discusses Busemann spaces, locally convex metric spaces, their convexity properties and some further questions (including properties of convex functions, isometries, asymptotic rays and visual boundaries). An important role is played in the book by suitable examples, in particular by the Teichmüller space. Prerequisites needed to read the book are modest (basic facts on hyperbolic space and Teichmüller space). The book can be of interest for mathematicians working in analysis, geometry and topology. (vs)

***H.-J. Petsche: Grassmann**, Vita Mathematica, vol. 13, Birkhäuser, Basel, 2006, 326 pp., EUR 58, ISBN 3-7643-7257-5*

This book (written in German) describes the life and mathematical achievements of Hermann Günther Grassmann (1809–1877). In the first chapter, the author presents Grassmann's life from his childhood up to his death, a life played out on the background political, social and cultural situation in Germany during the first half of the 19th century. His family, his studies at the gymnasium in Stettin and his studies at the University of Berlin are described in a lot of detail. It may be surprising that Grassmann had no formal university training in mathematics; he took courses on theology, classical languages, philosophy and literature to become a minister in the Lutheran church in Stettin. After completing his studies in 1830, he became a teacher. After one year of mathematical studies, he took an examination to become a teacher of mathematics at gymnasium level. But his works and his knowledge were not sufficient, so he obtained permission to teach at a lower gymnasium level only.

The author shows Grassmann's pedagogical activities and his mathematical production during the two periods (1830–1840, 1840-1848) during which his most important works were written. The second chapter describes Grassmann's mathematical and philosophical academic background. His father was an excellent professor of mathematics and physics in Stettin, writing several textbooks on mathematics, mineralogy and physics. His younger brother Robert also became a teacher of mathematics and he collaborated with Hermann on some projects and publications. The influence that F. D. E. Schleiermacher (a great philosopher, theologist, politician and teacher) had on the development of Grassmann's philosophical thinking is described here.

The third chapter gives a brief survey of the mathematical achievements from the 17th century up to the 19th century that could have influenced Grassmann's mathematical thinking and methods. His important publications are very carefully analysed. Grassmann's contributions to linear algebra, algebraic forms, the theory of algebras, differential geometry, analysis and number theory are presented. The fourth chapter explains the philosophical principles and the background of Grassmann's most important and inspired monograph 'Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik'.

The book contains many interesting pictures, photographs, reproductions and notes. It will be very helpful for historians and philosophers of mathematics, for teachers at universities and secondary schools, and students as well as researchers in mathematics and history. It can be recommended to people who are interested in the roots of modern mathematics. (mbec)

M. Rédei, Ed.: John von Neumann: Selected Letters, History of Mathematics*, vol. 27, American Mathematical Society, Providence, 2005, 301 pp., USD 59, ISBN 0-8218-3776-1*

This book is a collection of selected letters written by John von Neumann (1903–1957) to his colleagues, friends, government officials, etc. The book starts with a short description of von Neumann's life, career and results in mathematics (logic, foundations of mathematics, theory of operator algebras and unbounded operators), physics (quantum mechanics and ergodic theorem), computer science and game theory. His letters are arranged in alphabetic order of recipient (we find included here the names G. Birkhoff, P. Dirac, K. Gödel, P. Jordan, I. Kaplansky, E. Schrödinger, E. Segre and N. Wiener). Neumann's original text, punctuation and writing style are carefully maintained. In some cases when typographical errors and misspellings would lead to misunderstanding, the reader will find helpful footnotes.

The letters provide readers with a glimpse of Neumann's thinking about mathematics, physics, computer science, management, education, politics and war. Some of them contain technical problems or difficult topics from mathematics and physics, and they are not easily understandable. Some others concern daily or general problems and would be understandable to a broad cross-section of the public. At the end of the book, the reader can find biographical notes on the recipients of Neumann's letters (69 scientists, politicians, educational reformers, publishers, etc.), a list of references and a list of Neumann's publications mentioned in the letters and in the introductory comments. John von Neumann was one of the most influential mathematicians of the twentieth century. The reader will find in the book a description of the background and the development of modern science and many interesting notes on leading scientists and their works. (mbec)

V. Scheidemann: Introduction to Complex Analysis in Several Variables*, Birkhäuser, Basel, 2005, 171 pp., EUR 25, ISBN 3-7643-7490-X*

The theory of several complex variables is a broad, beautiful and (nowadays) classical part of mathematics. There are several excellent books available that describe the theory in full detail. However, what is often needed for students and researchers who come from other fields of mathematics is an introduction to the basic properties of functions of several complex variables without going into the full and difficult theory. This is what the reader can find here.

The book starts with the basics (definitions, the Cauchy integral formula, properties of rings of holomorphic functions, power series expansions and Reinhardt domains). Properties of the Dolbeault complex are used in the proof of a version of the Hartogs theorem. A chapter is devoted to the implicit and inverse function theorems, the Riemann mapping theorem and to properties of biholomorphic maps. Analytic continuation, domains of holomorphy, holomorphically convex domains, the Bochner theorem and the Cartan-Thullen theorems are treated in two chapters. The book also includes a description of basic properties of analytic sets and the proof of the Nullstellensatz for principal ideals. Hence basic features of the theory are introduced and illustrated in a relatively small space (of course, difficult parts of the theory are not treated here). The book contains a lot of examples and exercises. (vs)

J. Shao: Mathematical Statistics: Exercises and Solutions*, Springer, Berlin, 2005, 359 pp., EUR 39,95, ISBN 0-387-24970-2*

This book is a companion to the author's textbook 'Mathematical Statistics' (2nd ed., Springer, 2003), which contains over 900 exercises. This collection consists of 400 exercises and their solutions. Most of the exercises (over 95%) are introduced in the cited textbook. The reader should have a good knowledge of advanced calculus, real analysis and measure theory. The book is divided into seven chapters: 1. Probability theory (measure and integral, distribution functions, random variables), 2. Fundamentals of statistics (sufficiency, risk functions, admissibility, consistency, Bayes rule), 3. Unbiased estimation (uniformly minimum variance unbiased estimators, Fisher information, U-statistics, linear models), 4. Estimation in parametric models (conjugate priors, posterior distributions, minimum risk invariant estimators, least squares estimators, maximum likelihood estimators, asymptotic relative efficiency), 5. Estimation in nonparametric models (Mallows' distance, influence function, L-functionals, Hodges-Lehmann estimator), 6. Hypothesis test (uniformly most powerful tests, likelihood ratio tests), 7. Confidence sets (Fieller's confidence sets, pivotal quantity, uniformly most accurate confidence sets).

The collection is a stand-alone book. It is written very rigorously and solutions are presented in detail. It can be recommended as a source of solved problems for teachers and students of advanced mathematical statistics.

I will finish with two remarks. First, I think that it may be of some interest to reproduce a very simple exercise (Ex. 9, p. 7): Let F be a cumulative distribution function on the real line and a a real number. Show that $\int [F(x + a) - F(x)]\, dx = a$. Second, the reviewer would like to point out a misprint to prove that he studied the book: the expression $m<k$ on p. 96[3] should probably read $m > k$. (ja)

***K. Tapp: Matrix Groups for Undergraduates**, Student Mathematical Library, vol. 29, American Mathematical Society, Providence, 2005, 166 pp., USD 29, ISBN 0-8218-3785-0*

This book is written as a textbook for undergraduate students familiar with linear algebra and abstract algebraic structures. It could be used as an excellent textbook for a one semester course at university and it will prepare students for a graduate course on Lie groups, Lie algebras, etc. The author begins with basic facts on matrices (definition, operations and matrices as linear transformations), quaternions, general linear groups and change of basis. In the eight following chapters he explains matrix groups, orthogonal groups, topology of matrix groups, Lie algebras, matrix exponentiation, matrix groups as manifolds, Lie brackets and maximal tori. The book combines an intuitive style of writing (with many examples and a geometric motivation) with rigorous definitions and proofs, giving examples from fields of mathematics, physics and other sciences, where matrices are successfully applied. The book will surely be interesting and helpful for students of algebra and their teachers. (mbec)

***S. A. Tavares: Generation of Multivariate Hermite Interpolating Polynomials**, Pure and Applied Mathematics, vol. 274, Chapman & Hall/CRC, Boca Raton, 2005, 672 pp., USD 99,95, ISBN 1-58488-572-6*

This book describes approximate solutions of differential equations in the case when a solution can be expanded into a basis of polynomials with all derivatives up to a predefined order specified at the boundary. The book is divided into three sections. It begins with a thorough examination of constrained numbers, which form a basis for constructions of interpolating polynomials. The author develops their geometric representation in coordinate systems in several dimensions and he presents generating algorithms for each level number. As an application, it is possible to compute the derivative of a product of functions of several variables and to construct an expression for n-dimensional natural numbers.

Section II focuses on a construction of Hermite interpolating polynomials, from their characterizing properties and generating algorithms to a graphical analysis of their behaviour. The final section is devoted to applications of Hermite interpolating polynomials to linear and nonlinear differential equations of one or several variables. An example based on the author's thermal analysis of the space shuttle during re-entry to the Earth's atmosphere is particularly interesting. He uses here polynomials developed in the book to solve the heat transfer equations for heating of the lower surface of the wing. The author presents a lot of algorithms and pseudo-codes for generating constrained numbers and Hermite interpolating polynomials. The book can offer an inspiration for further research and will be of interest for graduate students, researchers and software developers in mathematics, physics and engineering. (knaj)

***G. van Brummelen, M. Kinyon, Eds.: Mathematics and the Historian's Craft: The Kenneth O. May Lectures**, CMS Books in Mathematics, Springer, Berlin, 2005, 357 pp., EUR 89,95, ISBN 0-387-25284-3*

This collection of papers contains extended versions of selected lectures delivered at the annual meeting of the Canadian Society for the History and Philosophy of Mathematics, which was held from 1996 up to 2003. The book contains the following papers written by twelve outstanding historians of mathematics (in alphabetical order of the authors):

T. Archibald, L. Charbonneau: Mathematics in Canada before 1945: A Preliminary Survey; J. Bennet: Mathematics, Instruments and Navigation, 1600–1800; J. W. Dauben: The Battle for Cantorian Set Theory; J. Grabiner: Was Newton's Calculus a Dead End? The Continental Influence of Maclaurin's Treatise of Fluxions; I. Grattan-Guinnesss: History or Heritage? An Important Distinction in Mathematics and for Mathematics Education; A. Hibner-Koblitz: Mathematics and Gender: Some Cross-Cultural Observations; A. Jones: Ptolemy's Mathematical Models and their Meaning; K. H. Parshall: The Emergence of the American Mathematical Research Community; V. Peckhaus: 19th Century Logic Between Philosophy and Mathematics; R. Thiele: The Mathematics and Science of Leonhard Euler (1707–1783); R. Thiele: Hilbert and his Twenty-Four Problems; S. Shanker: Turing and the Origins of AI.

These papers present a wide variety of original work in the history and philosophy of mathematics. They are full of interesting quotations from original sources, attractive illustrations and photographs, and copies of historical and very rare documents (including title pages and frontispieces of the first editions of the most important mathematical works and views of the cities). Each paper is supplemented with a bibliography. The papers are written not only for historians of mathematics but also, in particular, for others who are interested in the nature and beauty of mathematics. The book ends with the subject and name indices. The book will be very helpful for historians and philosophers of mathematics, for teachers at universities and secondary schools, and students as well as researchers in mathematics and history. (mbec)

***L. M. Wapner: The Pea and the Sun: A Mathematical Paradox**, A. K. Peters, Wellesley, 2005, 218 pp., USD 34, ISBN 1-56881-213-2*

Every year in November we organize at Charles University the so-called Day of Open Doors. The interested public is encouraged to come and listen to lectures and watch physical experiments, shows and exhibitions, etc. Departments send their representatives to advertise their subjects. In our department (mathematical analysis), people take turns in trying to prepare an interesting lecture on analysis. The choice of topic largely depends on the personal taste of the lecturer, but there is one issue that is almost always present: the Banach-Tarski paradox. The message to a student goes somehow as follows: if you decide to study analysis, we will teach you in the second year measure course how to partition a 100 Crown banknote to pieces and then to reassemble them to form a 1000 Crown banknote (special scissors are recommended).

The Banach-Tarski paradox is something really special. Some call it the most surprising result in theoretical mathematics. Many respectable citizens feel irate and call for a law against it. Even among professional mathematicians the claim ignited a lot of controversy immediately after the result was published. The remarkable Banach-Tarski theorem was obtained by two great stars of 20th century Polish mathematics, Stefan Banach and Alfred Tarski, in 1924. The two had very little in common otherwise and this was their only meeting point, their only joint work (imagine what they could have discovered had they worked together longer!). Banach‘s main subject was analysis; he is considered to be the founder of functional analysis and the list of objects bearing his name is rather impressive. Tarski was an equally spectacular logician. When the general public gradually learned about the theorem, the controversy mentioned above spread and people (including mathematicians) formed two antagonistic camps, one accepting the beautiful discovery and tolerating its counterintuitive nature and the other rejecting it right from the very start as outrageous nonsense.

A good part of the controversy and the misunderstandings was caused by the fact that the result is hardly accessible for a layman. It is a deep measure-theoretic theorem whose understanding requires a thorough study of several abstract disciplines and perfect knowledge of the Axiom of Choice and non-measurable sets. Therefore, naturally, despite the avalanche of highly technical papers in academic journals, little has been written for the general public. Most experts would consider it a very difficult task, many impossible. Well, not any more. The book by Leonard M. Wapner shows that the task is possible and it achieves the goal in a most satisfactory way. One does not need to have a degree in mathematics in order to follow the lively and readable, highly intriguing story of the paradox. Yet the exposition is serious, correct and comprehensive, and it presents a detailed proof of the result. The presentation is light-hearted, highly entertaining and illustrated with many examples, puzzles, etc. This book is (already) a classic in an area that needed one. (lp)

R. M. Weiss: Quadrangular Algebras*, Mathematical Notes 46, Princeton University Press, Princeton, 2005, 131 pp., GBP 29,95, ISBN 0-691-12460-4*

This book introduces a new class of non-associative algebras related to certain algebraic groups and their associated buildings. It develops a theory of these algebras, opening the first purely algebraic approach to the exceptional Moufang quadrangles. Based on their relationship to exceptional algebraic groups, quadrangular algebras belong to a series together with alternative and Jordan division algebras. Formally, the notion is derived from that of a pseudo-quadratic space over a quaternion division ring. Chapters 1–9 develop the complete classification of quadrangular algebras; in particular, every quadrangular algebra is shown to be either special, improper, regular or defective. The classification is completely elementary and (except for a few standard facts about Clifford algebras) self-contained. The book closes with a chapter on isotopes and the structure group of a quadrangular algebra. (jjez)