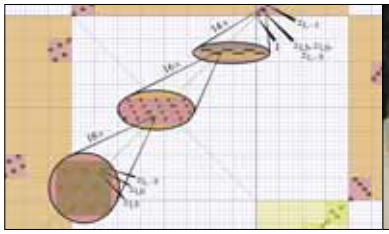
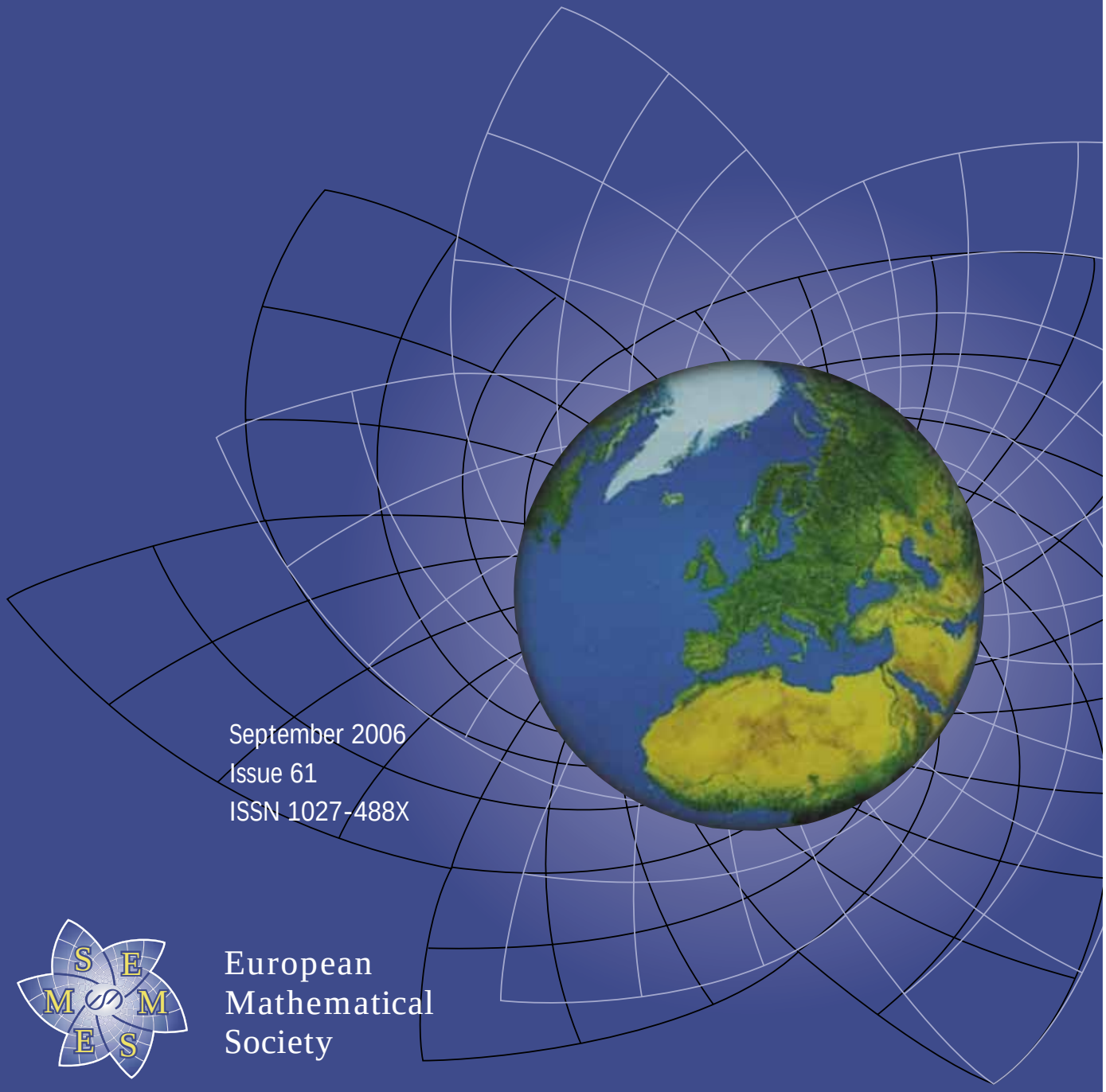
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



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EMS Executive Committee

EMS Calendar

2006

9–23 September
EMS Summer School at Linz (Austria)
Mathematics in molecular cell biology
http://www.ricam.oeaw.ac.at/emsschool; Vincenzo. Capasso@mat.unimi.it or christian.schmeiser@oeaw.ac.at

21–22 October
EMS Executive Committee Meeting at the invitation of the Polish Mathematical Society, Kraków (Poland)
Helge Holden: holden@math.ntnu.no

1 November
Deadline for submission of material for the December issue of the EMS Newsletter
Martin Raussen: raussen@math.aau.dk

2007

6–12 May
EMS Summer School – Séminaire Européen de Statistique, La Manga (Cartagena, Spain)
SEM STAT: Statistics for stochastic differential equations models
mathieu.kessler@upct.es or lindner@ma.tum.de

3–10 June
EMS Conference at Będlewo (Poland)
Geometric analysis and nonlinear partial differential equations
B.Bojarski@impan.gov.pl or pawelst@mimuw.edu.pl

16–20 July
ICIAM 2007, Zurich (Switzerland)
www.iciam07.ch/

2008

14–18 July
5th European Mathematical Congress, Amsterdam (The Netherlands)
www.5ecm.nl

16–31 August
EMS-SMI Summer School at Cortona (Italy)
Mathematical and numerical methods for the cardiovascular system
dipartimento@matapp.unimib.it

Editorial

Tuulikki Mäkeläinen at work

Tuulikki Mäkeläinen

When the European Mathematical Society was set up in 1990, the decision was taken to establish it legally under Finnish law, because the environment in Finland was relatively benign for scientific societies. This made it sensible to have the Society's office in Finland, and an agreement was made for the University of Helsinki to house the secretariat in its mathematics department.

The history of the EMS in the last 16 years has shown how wise these decisions were. The arrangements have had many advantages, but none greater than bringing Tuulikki Mäkeläinen into the affairs of the Society as head of the Helsinki secretariat. She now goes into well earned retirement, carrying with her the warm gratitude of all those who worked with her during this eventful period for European mathematics.

Jean-Pierre Bourguignon, President from 1995 to 1998 writes:

"In 1995 the Society was still in its infancy, and the varieties of opinion on its format that accompanied its creation and led to a compromise structure, between a society of direct members and a federation of national/regional societies, were still in all minds. As a result, the task of the person responsible for getting the Society functioning practically was critical. Tuulikki's dedication, thoroughness, punctuality and reactivity were undoubtedly important ingredients in the subtle chemistry that made the Society take off.

"When Fritz Hirzebruch called me up, to my great surprise, to ask me whether I would run for EMS President and try and be his successor, one of his arguments was that I would not have to worry about the practical side of things because of the excellent work that Tuulikki was doing.

"Tuulikki's discreet but firm personality, together with her efficiency and her commitment, dissolved another potential difficulty, the common tension between volunteers and employees, the one making extravagant requests of the staff, and the other refusing to be flexible."

Similar feelings are expressed by Rolf Jeltsch, the next President (1999–2002):

"I participated for the first time in an Executive Committee meeting in Vienna in 1997. Already Tuulikki impressed me. Everything was organised well ahead of time and perfectly done. She would inform us who would arrive when on which flight, so that we could share transport from the airport. If we had to move between hotel and meeting place, she would get the tickets and ensure that no one got lost.

"I remember arriving at Helsinki airport after midnight. At the meeting point I was confronted by a picture of my face, held up by a taxi driver. She had organised this, she said later, because sometimes after midnight there are no taxis and she wanted to spare me any problems.

"Once I had to give a speech at the Finnish Academy. Sure enough, she very quickly produced my slides.

"During my presidency, she seemed constantly on line. Even on Sundays or late in the evening answers came fast, often within minutes. She knew all the history, the statutes, the presidents of member societies, council delegates, members of committees, … – just amazing! Her minutes were done with perfect precision, with additions showing who should do what. She managed the EMS finances, knowing how to transfer money between all the different countries. It was said that the Finnish word for 'perfect organisation' was 'Tuulikki'.

"And in all this she was always friendly. She never became impatient with my thousands of e-mails. Always polite, always willing to chat about the weather, politics, health, family, her dog …. In all this she became a very good friend."

Tuulikki continued her loyal and efficient service right up to the last Council meeting, in Torino in July. The debt the Society owes to her is incalculable. But the note on which I end is that of friendship. Tuulikki has the rare knack of being firm when necessary, without causing any rancour or any feeling that she has been unfair or arbitrary. I am sure that all those who have collaborated with her over the years, over the whole existence of the Society, will share my feeling of privilege having worked with such a splendid person. In Rolf's words, a very good friend to us all, and to European mathematics.

John Kingman
President, EMS

EMS Council meeting in Torino, 1–2 July 2006

Vasile Berinde, EMS Publicity Officer

Delegates at Council.

The 2006 meeting of the European Mathematical Society's Council was held in the Conference Hall of the Centro Congressi dell'Unione Industriali in Torino. It consisted of two sessions, an afternoon session from 1pm till 6pm on 1st July and a morning session on 2nd July from 9am till 12am.

Around sixty delegates of members were present; the quorum of the Council, which requires two-fifth of its total number of delegates (105), was easily satisfied. Also in attendance were three members of the EC who are not delegates and some invited guests, amongst whom we mention Sir John Ball (President of IMU), Michael Cowling (President of the Australian Mathematical Society), Tuulikki Mäkeläinen and Riita Ulmanen from EMS Helsinki Secretariat, and Thomas Hintermann (Managing Director of the EMS Publishing House).

The meeting was opened by the President who welcomed the delegates and expressed his thanks and very warm gratitude to the local hosts.

Report of the Executive Committee

A brief report on the meetings of the Executive Committee since the summer of 2004 was among the Council papers that were electronically sent to all delegates prior to the meeting. On that issue, the Austrian delegate requested that more detailed accounts of meetings should be given. It was replied that the minutes contain confidential information better kept within the EC, but that EC meetings were regularly reported in broader terms in the EMS Newsletter. The President stressed that the EC wished to keep its members fully informed and invited suggestions for improvement.

Report of the President, Sir John Kingman

The President reported on the continuing efforts of the EMS to convince the public, and decision makers including funding bodies, of the pervasive role of mathematics

in modern society. Mathematics is often taken for granted but new challenges often call for new mathematics. In particular, this case has to be pressed in Brussels, where Luc Lemaire plays an active part. However, there are also other important organizations to be considered, like the European Research Council (EMS Vice-President Pavel Exner is a member of this body), the European Science Foundation (where the EMS sends an observer to the Physical and Engineering Sciences Committee) and UNESCO. The position paper used to brief the incoming EC Commissioner is being revised to take account of criticisms, especially in relation to applied mathematics. The Committee on Applied Mathematics, now reconstituted to embrace a wider range of applications, would be consulted. The President stressed the unity of mathematics.

EMS membership is stable and the only major concern about corporate membership is the small number of national statistical societies that are members of the EMS. Individual membership should be increased to emphasize the commitment of individual mathematicians to European activities. Grateful thanks were extended to the previous EMS Publicity Officer, David Salinger, who was an imaginative and energetic publicity officer. The President reported with regret the forthcoming retirement of Tuulikki Mäkeläinen as Helsinki Secretary and thanked her for her loyal and efficient work for the Society since its earliest years.

Individual Membership

Martin Raussen had prepared a chart of individual membership by country, giving the EMS individual members per capita in each country. The current individual membership is 2151; 2013 of them come from Europe and the rest from USA & Canada (66), Asia (47), Latin America (10), Australia (9) and Africa (6). The chart also showed great variability within countries. The President urged Council delegates to try to urge their colleagues to join the EMS. To this end, the President encouraged member societies to translate the paper „A dozen reasons to join the European Mathematical Society“, written by David Salinger, into their national languages and distribute it among the national membership. The EMS urges the member societies to make it easy to become individual EMS members and pay membership fees. The number of individual members of the EMS is considered very important.

Corporate Membership

The Polish Mathematical Society had asked to upgrade class. At the recommendation of the Executive Committee, the Council agreed to move the Polish Mathematical Society to Class 3 from 1st January 2007. The Kharkov Mathematical Society had informed Council that it withdraws from the Society due to economic difficulties. The Council accepted the withdrawal with regret. The Council had been informed that the Voronez Mathematical Society had ceased to exist and its activities had been taken over by the Research Institute of Mathematics at Voronez University. The Council decided to remove the Voronez Mathematical Society from the membership list.

Report of the Treasurer

The Treasurer commented on the written financial reports and clarified the financial results of the years 2004 and 2005. The financial statements for the years 2004 and 2005 were approved. Despite the financial resources of the Society being healthy and stable, Olli Martio presented a proposal of the Executive Committee and gave reasons for increasing unit fees. The main aim was to increase the reserve fund. It was agreed to raise the unit fee for individual members, denoted y, from EUR 20 to EUR 22 and to increase the basic fee for corporate members, denoted x, from EUR 380 to EUR 400. The EC proposal was accepted by a large majority on a show of hands. The budget proposal was also accepted.

Elections to Executive Committee

There were five vacant officer positions, left open by the ending of the terms of the President, the Vice-President, the Secretary, the Treasurer and one of the ordinary members. The Executive Committee had one nomination for each position and no nominations were made on the floor. The President proposed to vote on each position separately. Ari Laptev was unanimously elected as President of the EMS. In his address of thanks, he briefly described the challenges of being the president and promised to be open to all ideas. Then Helge Holden was elected as Vice-President of the EMS, Stephen Huggett was elected as Secretary and Jouko Vaananen was elected as Treasurer. There were three nominations for the one vacant Member-at-Large seat: Mina Teicher (Israel Mathematical Union), Mireille Martin-Deschamps (French Mathematical Society) and Dorin Popescu (Romanian Mathematical Society). As the Israel Mathematical Union withdrew its proposal, a ballot was taken for the two remaining candidates. Mireille Martin-Deschamps received 46 of 57 valid votes and was elected as Member-at-Large of the EC.

Election of Auditors

Pekka Kaasalainen, CPA, and Johan Weckman, CPA, as deputy, were elected as professional auditors for the accounts of the years 2007 and 2008 (they previously served in this capacity for the years 2003 and 2004). Kazimierz Goebel (re-elected for a second term) and Gregory Makrides (just elected) will be the EMS auditors for the years 2007 and 2008.

Reports and Debates

The Detailed Review of Activities started with the report

of the Newsletter Editor, Martin Raussen. He complemented his written report with important information and, as subscriptions are very few, he urged delegates to try to find subscriptions at their institutes. Martin Raussen was thanked for his excellent work. Thomas Hintermann, Director of the EMS Publishing House also complemented his written report and concluded that the EMS Publishing House has increased its activities substantially and the outlook for the future is good. The other business was related to: JEMS (it has been established as a high-standard journal); Zentralblatt MATH (Sir John Kingman asked delegates to encourage colleagues to use Zentralblatt. It is necessary to keep more than one large database available to keep up competition); and then the written report from the Publicity Officer, David Salinger.

Reports from Society Committees followed, most of the written reports being vivaciously complemented by their Chairs or members.

(a) *Applied Mathematics*. The chair, Mario Primicerio had sent an extensive report. He invited all delegates to join a meeting of the committee after the Council meeting. He offered to add any interested delegate to the committee's mailing list. Norbert Mauser raised a number of concerns about the procedure in relation to this and other committees but received little support;

(b) *Developing Countries*. Tsou Sheung Tsun complemented the report she and Herbert Fleischner had submitted to the Council and stressed the informal internal lecture notes in English from Oxford that were put on CD-ROM and sent to Africa. The plan is to follow the scheme later in French and Spanish;

(c) *Education*. Commenting on her extensive written report, Mina Teicher gave warm thanks to the Bonn Max Planck Institute for their hospitality during the committee's meeting in Bonn. She also announced that the committee wants to be present at ICIAM in Zurich in 2007;

(d) *Electronic Publishing*. Pavel Exner commented that few of the committee members are active. The basis of committee should be broadened. The book section on EMIS needs enlargement. Digitalization of mathematical literature is to be pushed forward. A satellite conference to ICM2006 will take place in Portugal on digitalization;

(e) *ERCOM*. Sir John Kingman reported on the committee and introduced the new chair, Jan Karel Lenstra;

(f) *Group on Relations with European Institutions*. Luc Lemaire made a report on the activities of the group. EMS has been able to gain some influence in the EU. The main task had been improving contact with Brussels but other European institutions were also contacted;

(g) *Meetings*. Luc Lemaire complemented his written report. The major activities included

- Summer Schools, with sizable funding from the EU; the programme is running smoothly.
- EMS Mathematical Weekends had taken place four times and he encouraged the delegates to find new possibilities for future ones.
- No Diderot Mathematical Forums had taken place recently. He made a plea to delegates to be active in finding proposals.
- Joint Meetings with others (SIAM, UMALCA, national societies) had taken place and delegates should discuss with colleagues proposals for future meetings.
- EMS Lectures had not taken place due to lack of proposals. He made a plea to delegates to be active in finding proposals;

(h) *Raising Public Awareness*. Olga Gil-Medrano complemented a report that had been sent by Chair Vagn Lundgaard Hansen with a PowerPoint presentation. Her report was followed by a lively discussion and suggestions for future activities;

(i) *Support of East European Mathematicians*. The written report of this committee was noted. It was also noted that it is a very active committee;

(j) *Women and Mathematics*. Klaus Schmidt highlighted some items of the written report. Among others, he encouraged member societies to think of women when electing delegates. The Newsletter will publish information on the work of this committee.

Closing this matter, the President asked for suggestions from delegates for members of any committees. The EC would adjust terms of office to achieve a better rotation of members.

Congresses of Mathematics

Jan Karel Lenstra reported briefly on the 5th ECM (www.5ecm.nl), 14th–18th July 2008, in Amsterdam. A bid for the ECM in 2012 was announced by Andrzej Pelczar who invited the mathematicians to Krakow in 2012 for the 6th ECM. An official bid would be sent before the end of the year. Other bids are welcome and the President urged the delegates to encourage their national societies to submit bids for the 2012 Congress.

Closing matters

To conclude, the President, who courteously chaired the whole meeting and released the audience from time to time with subtle English humour remarks, thanked the local organizers, Alberto Conte and his colleagues, for their work in collaboration with Tuulikki Mäkeläinen and Helge Holden, resulting in a very successful Council meeting. He also thanked the delegates for attending it.

EMS Executive Committee meeting in Torino

Vasile Berinde, EMS Publicity Officer

EC-members and guests at the Torino meeting.
In the front row on the right: leaving secretary Tuulikki Mäkeläinen and her successor Riitta Ulmanen.

The EMS Executive Committee met on the afternoon of the 30th June 2006 at the Centro Congressi dell'Unione Industriali in Torino, a city where the traces of the recently hosted 20th Winter Olympic Games, 10th–26th February, were still clearly visible.
Committee members present were: Sir John Kingman (President, in the Chair), Pavel Exner and Luc Lemaire (Vice-Presidents), Helge Holden (Secretary), Olli Martio (Treasurer), Viktor Buchstaber, Doina Cioranescu, Olga Gil-Medrano and Klaus Schmidt. Apologies were received from Carlo Sbordone.
Also in attendance (by invitation) were: Vasile Berinde, Stephen Huggett, Ari Laptev, Tuulikki Mäkeläinen, Martin Raussen, Riitta Ulmanen (Tuulikki Mäkeläinen's successor) and Jouko Vaananen.

EMS Council Agenda

Being an EC meeting that preceded the 2006 ordinary EMS Council meeting (held on 1st-2nd July at the same venue) its agenda was dominated by the appropriate preparatory Council matters. As most of the EC meeting issues will therefore be covered to some extent in the material on the Council meeting, they are not reported here. Instead, in the following we shall mention some of the specific matters.
Before the committee scrutinised the EMS Council Agenda, the President welcomed the participants, in particular the Society's new Publicity Officer, Vasile Berinde, and Tuulikki Mäkeläinen's successor, Riitta Ulmanen.

Electronic votes

The EC ratified the results of the following electronic votes that had occurred since the previous meeting:
- Vasile Berinde was voted in as the new EMS Publicity Officer;
- John Toland, Scientific Director of the ICMS in Edinburgh, was voted new Vice Chair of ERCOM for 2006–2009.

EMS Meetings

It was reported that the Joint EMS, SMAI, SMF Mathematical Weekend in Pays de Loire, Nantes (France) was a good meeting but had attracted rather few participants:

about 100, amongst which fifty were local participants.
As the first joint meeting of EMS, SMF, SMAI, and UMI, '*Mathematics and its Applications*', Torino, 3rd–7th July 2006, would start just after the Council meeting closed, the EC authorized the treasurer to provide additional financing for the extra invited speakers at this conference. No new proposals for joint meetings had yet been received.

It was also decided that the next EMS Council meeting will be organized in conjunction with the 5th ECM (http://www.5ecm.nl/) taking place in Amsterdam in 2008.

EMS Bank Accounts

As the authorization of Tuulikki Mäkeläinen to handle the bank accounts for the Society in Finland will cease as of 15th July 2006, the Executive Committee authorized Riitta Ulmanen to handle the EMS accounts as of 15th July 2006.

Subcommittees

It was agreed that the EC should stagger terms of service of committee members. New active persons were needed. EC members should check the terms of the members of the committees that they are responsible for. It would be desirable that a wider variety of countries should be represented in all committees. Klaus Schmidt pointed out that in 2007 the term for several members of the Committee for Women and Mathematics will expire.

EMS Publishing House

The Secretary reported that the European Mathematical Foundation (whose board consists of the EMS President, Secretary, Treasurer, immediate Past President if he/she wishes, and a representative from ETH) had had a good meeting in the morning. A hand-over meeting will be in December 2006 in Zurich. It was reported with satisfaction that the financial status of the Publishing House is positive. EMSPh publishes all material from ICM2006, giving considerable visibility and financially positive results. The EMSPh will also publish the ICIAM2007 proceedings. The posting of the EMS Newsletter is cheaper since the Publishing House takes care of this business. Concerning JEMS, it was noted that the Publishing house, in agreement with the JEMS Editor-in-Chief, has decided to raise the subscription price from EUR 210 to EUR 240, mainly due to the increase in the number of pages per volume from 400 three years ago to 600 pages nowadays.

ICM2006

Sir John Kingman will attend the IMU General Assembly in Santiago de Compostela, 19th–20th August, before the ICM starts, as an observer (the EMS is not a member but an affiliate member of the IMU) and will give a report in Krakow on it.

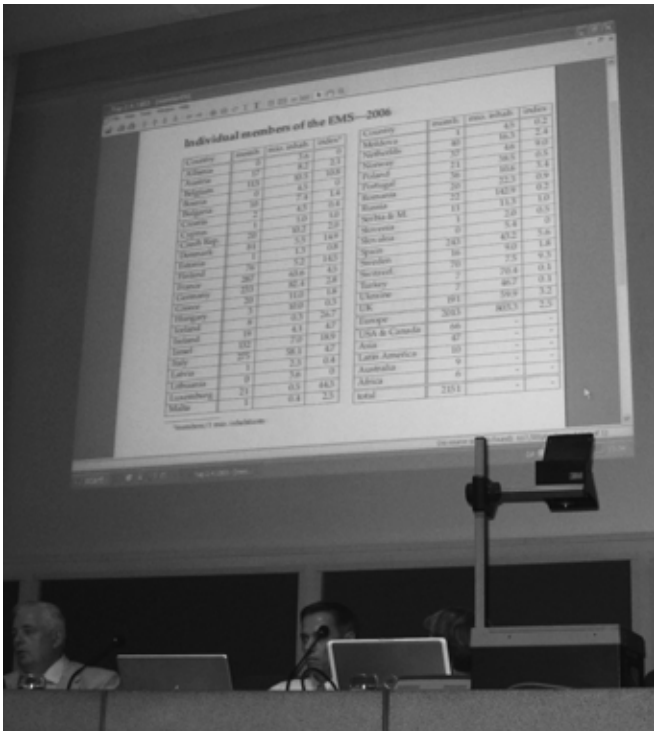
It was noted that during ICM2006 in Madrid, the EMS will have a booth (stand no. 5) organized by Thomas Hintermann and Manfred Karbe from the EMS Publishing House, which will also be the meeting point for EC members. Pavel Exner, Olga Gil Medrano, Ari Laptev, Luc Lemaire and Olli Martio promised to attend the booth at times. Olga Gil Medrano reported on the EMS session organized in Madrid: EMS will have a panel discussion at ICM2006 as a special activity on the topic '*Should mathematicians care about communicating to broad audiences? Theory and Practice*'. This will be moderated by Jean-Pierre Bourguignon.

Miscellanea

The President attended the successful presentation by the Norwegian Academy of the Abel Prize to Lennart Carleson. It was also noted that the EMS succeeded in getting the following eu-domain names: euromath.eu, euromaths.eu and euro-math-soc.eu. The plan for the use of the domain names will be discussed by a committee consisting of Helge Holden, Pavel Exner and Stephen Huggett.

Next EC meeting

After the local organizers, Alberto Conte and Marina Marchisio with their team, had been thanked by the President, the meeting closed.
The next EC meeting will be held in Krakow (Poland), on 21st–22nd October 2006.



Report on the 4th EMS Joint Mathematical Weekend Nantes, 16th–18th June 2006

Laurent Guillopé (Chair of the organising committee)

Angers and Nantes

After Lisbon 2003, Prague 2004 and Barcelona 2005, the fourth EMS Joint Mathematical Weekend [JMW] took place in Nantes, organised by two nearby Mathematical Institutes: the *Laboratoire angevin de recherche mathématique* in Angers and the *Laboratoire de mathématiques Jean Leray* in Nantes. Despite the eighty kilometres between the two cities, there is a strong mathematical collaboration between them including a common graduate programme, joint seminars (algebraic topology, algebraic geometry and probability), co-organised international conferences, etc. These are the two largest cities in Pays de la Loire, which is one piece of the French *21-Régions* puzzle. Both French mathematical societies (*Société mathématique de France* and *Société de mathématiques appliquées et industrielles*) gave their warm support to the organisation of the Weekend.

Angers has played its role in central European history; Princess Hedwige of Anjou was also Queen of Poland (1382–1399) (and gave part of her goods to Krakow University) and Henri de Valois, Duke of Anjou, was elected King of Poland in 1573. This close relationship between Poland and Angers is still alive today. Three professors working in the Angers mathematics department today were born and graduated in Poland! Furthermore, a Jean Leray Centennial Conference opened in Będlewo near Warsaw ten days after the Weekend. Jean Leray was born in Nantes on 7th November 1906 and, as partial differential equations and algebraic topology correspond to ancient research directions in Nantes, it was very natural that the Nantes Mathematical Institute should be named after Jean Leray.

The Weekend

Because numerous satellites conferences of the Madrid ICM-2006 were held in August and September, it was chosen to hold the Weekend in June. It took place in Nantes on the large campus of the science faculty on the border of the Erdre River, with good downtown tramway connections and convenient facilities for the Weekend (internet access, wifi connections and the mathematical library in its beautiful modern building). An exhibition of EMS Publishing House books and journals ran over the whole Weekend, as well as an SMF booth with Society publications and another one prepared by a local scientific bookstore displaying an important selection of books related to the different topics of the Weekend.

The Weekend had financial support from the EMS, the CNRS, the *Ministère de l'enseignement supérieur et de la recherche*, the *Institut Universitaire de France*, the Universities in Angers and Nantes, the *Région Pays de Loire* and the *Communauté urbaine de Nantes*. Many sponsors have been very sensitive to the European dimension of this scientific meeting.

The scientific programme

The Weekend focused on five topics corresponding to some significant local research directions: Inverse Problems, Large Scale Stochastics, Complex Algebraic Geometry, Global Analysis, and Real Algebraic Varieties. The scientific committee tried to get a balance between applied and fundamental mathematics with an attempt to find overlaps between different topics (one speaker had been invited to two of the sessions!). Twenty-seven speakers were invited from Russia, the USA, Italy, Germany, Eire, Finland, Japan, Poland, Switzerland and France. The schedule followed the usual plan with plenary lectures and parallel sessions. During the short opening session a welcome was given in the name of the President of Nantes University by Pascal Gressier (Vice-Dean of the Science Faculty for Research) and then Luc Lemaire

(EMS Vice-President) officially opened the Weekend recording the JMW series and the main achievements of the EMS.

The titles of the plenary lectures give some idea of the varied content: Guenadi Henkin lectured on *Electrical tomography of two-dimensional bordered manifolds and complex analysis*, Nobuo Yoshida on *Directed polymers in random environment*, Claire Voisin on *Hodge classes in Kähler and algebraic geometry*, Franck Pacard on *Extremal Kähler metrics on blow ups* and Ilya Itenberg on *Tropical geometry and enumeration of real rational curves*. All titles and abstracts, with a number of pictures, can be found on the website of the Weekend: www.math.sciences.univ-nantes.fr/wems/.

Other activities

The Société mathématique de France asked Pierre Arnoux to set up a round table with the title *Europe, mathematics and young mathematicians*. The panel of colleagues, with the notable European involvement of Jean-Pierre Bourguignon, Luc Lemaire, Jean-Paul Brasselet and Bertrand Monthubert, discussed various critical topics for the strengthening of European research in mathematics: Curie fellowships, Zentralblatt Math, the European Research Council, etc.

The social programme was reduced to a minimum because of the tight schedule of the scientific lectures. On Friday, the participants enjoyed a warm reception in the city hall, a short walk through the downtown district and a (unique) dinner in the poly-arts centre *Lieu unique*. After the end of the round table at 8 pm, Saturday evening was free to visit downtown again.

The participants

About 106 mathematicians registered (there were no registration fees, except for the dinner). Less than half came from Pays de la Loire, others originating from elsewhere in France and fifteen other countries, with every continent represented. The picture on the previous page is a combination of two shots taken on Saturday afternoon.

Participants enjoyed the high scientific level of the Weekend and the fine weather that permitted walking around Nantes, which has been known as the Venice of the West. Mathematicians in Angers and Nantes were very happy with the success of the EMS-JMW. Many of the participants found these three mathematical days very positive, as shown by discussions about the next Weekends.

Laurent Guillopé
[laurent.guillope@univ-nantes.fr] is Professor of Mathematics at Université de Nantes and Director of the Laboratoire de mathématiques Jean Leray (Université de Nantes, CNRS and École centrale de Nantes). His fields of interest are resonances and scattering theory in geometric analysis. As the Vice-director (1995–2004) of the Cellule MathDoc, he participated in the Europeanization of Zentralblatt Math and in launching the digitization programme NUMDAM.

Call for bids: 6th European Mathematical Congress (2012)

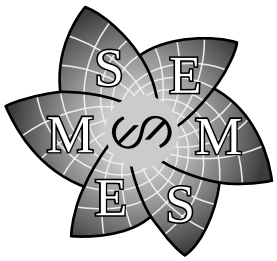
Applications are invited to hold the 6th European Mathematical Congress in the year 2012. Applications should reach the Executive Committee in electronic form before March 15, 2007, to the address riitta.ulmanen@helsinki.fi of the EMS Secretariat.

The decision process is subject to the following guidelines:
The EC appoints a site committee in case there are more than one bid. The site committee makes its inspections during the year 2007; the costs are borne by the bidders. The site committee inspects the auditoriums and the accommodation, plans for the scientific programme, the financial plans and the strength of the mathematical community making the bid. It also takes into consideration the costs for the participants to reach the site and the costs for the stay during the congress. Special attention will be paid to the availability of inexpensive student dormitories.
Decision: In the year 2007 or 2008 the Executive Committee makes a recommendation for the site to the Council and the Council decides in the year 2008.

For guidelines concerning organisation and finances, please consult www.emis.de/ .

Zentralblatt MATH and the EMS

Pavel Exner (Prague, Czech Republic)



The name *Zentralblatt* is one that any mathematician meets regularly throughout his or her career. Some of us use it regularly, for others it is just a long row of yellow volumes somewhere down in the library or an occasional review request that reminds us of another professional duty in our already full schedule. We live in a fast changing world, however, and it is useful to reflect from time to time about the role tools like *Zentralblatt* play in our life and work.

Any branch of science needs a system accumulating the acquired knowledge in a structured way, a database as one says nowadays, both as a reference tool and a source of inspiration. For mathematics the need is probably more vital than for other disciplines because it is the "most eternal" among the sciences. Let the philosophers forgive me saying that we are the only field of science haunted by fundamental questions posed in previous centuries, by which I do not only mean the twentieth. It is only a mild exaggeration to say that a mathematician needs information about everything from Euclid to the present day and needs it readily available.

This need was on the mind of *Zentralblatt* founder Otto Neugebauer when he decided in 1931 that *Jahrbuch über die Fortschritte der Mathematik (JFM)*, which had existed since 1868, was not sufficient for the development of mathematics. As is so often the case, the history of *Zentralblatt* reflects the troubled history of our continent; in 1934 when Hitler came to power, Neugebauer was among the Göttingen mathematicians who had to leave Germany. He went to Copenhagen and continued to edit the journal there. When the Nazis later threatened Denmark in 1939, he moved to Providence in the U.S.A. There *Zentralblatt*'s younger brother, *Mathematical Reviews*, was begotten and edited by Neugebauer from 1940 until 1948.

This historical coincidence meant that the world now had two principal mathematical databases of comparable size. *Zentralblatt*, after including the content of its German predecessor JFM mentioned above, covers all mathematical literature from 1868 up to the present day. The total number of documents available in the database is more than 2.5 million and the annual growth is about 80 thousand items. A journal has to give evidence that the articles published there are subject to a peer-review process in order to be processed by *Zentralblatt*. This applies to about 1800 journals and periodical publications. In addition about 1500 monographs are handled annually. All database entries are recorded with several types of metadata making them searchable with a high level of precision.

Thinking about *Zentralblatt* and *Mathematical Reviews*, the question naturally arises whether Pallas should not spring back into the forehead of Zeus, or maybe vice versa. It was intensely discussed between both services in the eighties and with the EMS at the beginning of the nineties. The participating institutions, Springer as the publisher together with the EMS, FIZ Karlsruhe and the Heidelberg Academy of Sciences, came to the conclusion that the community only benefits from having two wells of knowledge.

The existence of the two sources means, of course, a competition. Since the EMS shares the responsibility for Zentralblatt, we are thinking about ways to make it more efficient, useful and attractive. At the end of last year the EMS formed the *Scientific User Committee* (SCUC), headed by the former President of the Society: Jean-Pierre Bourguignon, as a board at which the database performance would be evaluated and ideas for improvement discussed. The committee, whose other members are Alice Guyonnet, Stephen Huggett, Gil Kalai, Stephan Klaus, Matilde Marcolli, Sergiu Moroianu, Ricardo Perez-Marco and Olof Runborg, started a discussion with the *Zentralblatt* editors who welcomed this help. The experience of the first months shows that SCUC comments and proposals are being taken seriously, laying the foundation for a fruitful collaboration.

Let me briefly mention some of the ideas which are on the table. To begin with, a need is felt for a reliable system enabling the identification of authors, especially in view of the outgrowth of articles coming from Asian mathematicians. The work is progressing and the first offering of checked data should be available by this summer. For a second step, this should be complemented by a database of institutes and departments appearing as the authors' affiliations.

The database entries will improve; links to the full text are regarded as more stable nowadays and will be obligatory. Other interactions with the authors such as links to their own pages are more controversial and require further considerations. The SCUC suggested that it could be useful to have room for more comments on an article, beyond a single review. One possible idea is to have a "second review" of selected articles after a number of years (say a 5–10 year period), which would reflect their impact on current research. This way of providing additional information on articles of interest seems to have advan-

tages over soliciting featured reviews at a time when the article has just been published.

Another way of making the database faster and more complete is to link it to related services such as NumDam, the Mathematical Genealogy Project and various other electronic archives, digitally born or retrodigitized. Such connections exist already and will be gradually strengthened; a priority is given to change to open URL's, which is on the way to being implemented.

More attention has to be paid to the enlargement of the panels for enrolment of new, good reviewers because the quality of the referee corpus is vital for the database and has to be looked after. A collaboration of the national societies could be very useful in this respect and will be actively sought. *Zentralblatt* looks for ways to make the referees proud of their service, e.g. a list of papers reviewed by a particular person can be extracted from the database.

Attention is also paid to the form of the presentation. A task underway is the creation of a facility allowing conversion of the TeX code into MathML on the fly. The result could be immediately processed by all web browsers commonly used. Implementation is scheduled for the summer of this year.

To finish this brief review, let me state the obvious. The discussion about the future of the *Zentralblatt* does not affect only its editors and a small group of senior mathematicians. It should and must be a matter of concern for all mathematicians, particularly those in Europe.

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Mathematics Subject Classification Revision

Kevin F. Clancey (Louisville, USA) and
Bernd Wegner (Berlin, Germany)

Mathematical Reviews (MR) and *Zentralblatt* MATH (Zbl) cooperate in maintaining the Mathematics Subject Classification (MSC), which is used by these reviewing services to categorize items in the mathematical sciences literature. The current version, MSC2000, consists of 63 two-digit classifications refined into over 5000 three- and five-digit classifications. Details of MSC2000 can be found at http://www.ams.org/msc/ and http://www.emis.de/ZMATH/.

MSC2000 was a substantial revision of the 1991 subject classification scheme, developed through collaborative efforts of the editors of MR and Zbl with considerable input from the community. MR and Zbl have initiated the process of revising MSC2000 with the expectation that the revision will begin being used in 2010. From the perspective of MR and Zbl, the five-digit classification scheme is an extremely important device that allows editors and reviewers to process the literature in an appropriate manner. In the era of fully searchable databases, key-word searching appears to be the search method preferred by many users but browsing the MR or Zbl database using a two- or three-digit classification search is an effective method of keeping up with research in specific areas; users of the publications of MR and Zbl exploit the MSC to search the literature in this way.

Based in part on some thoughtful suggestions from members of the community, the editors of MR and Zbl have given preliminary consideration to the scope of the revision of the MSC. It appears there will not be any changes at the two-digit level; however, it is anticipated that there will be refinement of the three- and five-digit levels.

At this point, MR and Zbl welcome additional community input into the process. Comments can be submitted through the Web form http://www.msc2010.org or by email to feedback@msc2010.org . All information about MSC revision is jointly shared by MR and Zbl. This input will be of great value as the process moves forward.

Kevin F. Clancey [kfc@ams.org] is the Executive Editor of Mathematical Reviews. Bernd Wegner [wegner@math.TU-Berlin.DE] is the Editor-in-Chief of Zentralblatt MATH.

Editorial board of journal *Topology* resigns

The entire editorial board of the prestigious mathematical journal Topology has announced its resignation in a letter dated 10 August 2006 addressed to Mr. Robert Ross of Elsevier Science which is reproduced below. The Newsletter will ask the publisher to give comments in a future issue.

We regret to have to tell you that we, the Editorial Board of Topology, are resigning with effect from 31 December 2006.

As you are well aware, the Editors have been concerned about the price of Topology since Elsevier gained control of the journal in 1994. We believe that the price, in combination with Elsevier's politics for pricing mathematical journals more generally, has had a significant and damaging effect on Topology's reputation in the mathematical research community, and that this is likely to become increasingly serious and difficult, indeed impossible, to reverse in the future.

As you know, we have made efforts over the last five to ten years to negate this effect. When the alternative subscription option was introduced a few years ago (electronic access combined with annual print delivery for half the price), we were hopeful that it would help in this regard. However, it made little impact, probably because most university libraries which subscribe to Topology do so through consortia deals.

The journal Topology has an illustrious history with which we, on becoming editors, were extremely proud to be associated. It owed its foundation to the inspiration and vision of the great Oxford topologist JHC Whitehead in the late 1950s, and the Honorary Advisory Editorial Board and also our predecessors on the Editorial Board have included some of the greatest names of 20th century mathematics. We believe that the journal's ethos and structure, based around a group of editors making editorial decisions jointly in Oxford with the expert assistance and advice of highly eminent editors elsewhere around the world, has many strengths and has provided a great service to the mathematical community in the past. However, we feel that Elsevier's policies towards the publication of mathematics research have undermined this legacy.

Therefore, with great reluctance and sadness, we have made the difficult decision to resign.

[signed] Martin Bridson, Ralph Cohen, Nigel Hitchin, Frances Kirwan, Marc Lackenby, Jean Lannes, Wolfgang Lück, John Roe and Ulrike Tillmann.

RICAM – ICIAM grants

The Johann Radon Institute for Computational and Applied Mathematics (RICAM, Linz)
of the Austrian Academy of Sciences announces up to 10 grants which cover the cost of

– a stay of two weeks at RICAM in Linz right before ICIAM 07, and
– the stay at ICIAM 07 in Zürich.

Travel (as cheap as possible), registration to ICIAM and accommodation and subsistence in Linz and Zürich are covered by these grants.

Applications are invited from mathematicians born 1971 or later, who live and work in Central and Eastern Europe including those EU-Countries that joined the Union in 2005. Preference is given to applicants whose scientific expertise is close to the fields represented at RICAM.

Applications should be sent by September 30, 2006 to radon_iciam@ricam.oeaw.ac.at. These applications should include a short statement about scientific interests and achievements, a CV and a list of publications. Also, two letters of support should directly be sent to the e-mail address given.

Successful candidates will be notified around October 15, 2006 and should then submit an abstract to ICIAM. The grant will only be awarded if this abstract is accepted for presentation.

Information:
http://www.ricam.oeaw.ac.at/ , http://www.iciam07.ch/

Profinite Fibonacci Numbers

Hendrik W. Lenstra (Leiden, The Netherlands)

Good recreational mathematics makes you rush for pen and paper. The subject must be simple and contagious, preferably with all kinds of enigmatic properties within reach. Hendrik Lenstra breaks a lance for profinite numbers.

Profinite integers do not enjoy widespread popularity among mathematicians. They form an important technical tool in several parts of algebraic number theory and arithmetic geometry, but their recreational virtues have never been recognized. The purpose of the present paper is to acquaint the casual mathematical reader in an informal way with profinite integers and some of their remarkable properties. The less casual reader is warned that the approach is experimental and heuristic, and that the exact meaning of many assertions may not always be instantly clear. Providing not only precise formulations, but also valid proofs, is a challenge that an expert in p-adic numbers and their analysis can easily face, but that hardly does justice to the entertainment value of the subject.

To define profinite integers, we recall that any positive integer n has a unique representation of the form

$$n = c_k \cdot k! + c_{k-1} \cdot (k-1)! + \ldots + c_2 \cdot 2! + c_1 \cdot 1!,$$

where the 'digits' c_i are integers satisfying $c_k \neq 0$ and $0 \leq c_i \leq i$, for $1 \leq i \leq k$. In the *factorial number system*, the number n is then written as

$$n = (c_k c_{k-1} \ldots c_2 c_1)_!. \tag{1}$$

The exclamation mark distinguishes the factorial representation from the decimal representation. For example, we have $5 = (21)_!$ and $25 = (1001)_!$.

If we allow the sequence of digits to extend indefinitely to the left, then we obtain a *profinite integer*:

$$(\ldots c_5 c_4 c_3 c_2 c_1)_!,$$

where we still require $0 \leq c_i \leq i$ for each i. Usually, only a few of the digits are specified, depending on the accuracy that is required. In this paper, most profinite numbers are given to an accuracy of 24 digits. For example, we shall encounter the following profinite integer:

$$l = (\ldots 6\,0\,4\,6\,3\,8\,0\,4\,7\,6\,8\,0\,0\,4\,9\,0\,0\,0\,1\,2\,0\,1\,0\,0)_!. \tag{2}$$

In this number, the 19th digit has the value 18, but this is written $\,8$ in order to express that it is a single digit. Note that by the 19th digit we mean the 19th digit *from the right*. Likewise, when we speak about the 'first' digits or the 'initial' digits of a profinite number, we always start counting from the right.

One can view each positive integer n as in (1) as a profinite integer, by taking $c_i = 0$ for $i > k$. Also 0 is a profinite integer, with *all* digits equal to 0. The *negative* integers can be viewed as profinite integers as well, for example

$$-1 = (\ldots 24\,23\,22\,21\,20\,19\,18\,17\,16\,15\,14\,13\,12\,11\,10\,9\,8\,7\,6\,5\,4\,3\,2\,1\,0)_!,$$

with $c_i = i$ for all i. In general, negative integers are characterized by the property that $c_i = i$ for all but finitely many i.

The ordinary arithmetic operations can be performed on profinite integers. To add two profinite integers, one adds them digitwise, proceeding from the right; when the sum of the ith digits is found to exceed i, one subtracts $i+1$ from it and adds a carry of 1 to the sum of the $i+1$st digits. The reader may check that in this way one finds that the sum of 1 and -1 equals 0. Subtraction is performed in a similar manner. Multiplication can be done by means of a more elaborate scheme, but it is often more practical to compute products by means of the following rule: for each k, the first k digits of the product of two profinite numbers s and t depend only on the first k digits of s and of t. (This rule is also valid for addition and subtraction.) Using this rule, one reduces the problem of computing products to the case of ordinary positive integers. These operations make the set of all profinite integers into a commutative ring with unit element 1. This ring is denoted $\hat{\mathbf{Z}}$, the *ring of profinite integers*.

Fibonacci numbers

Fibonacci numbers illustrate several features of profinite integers. The nth Fibonacci number F_n is, for $n \geq 0$, inductively defined by $F_0 = 0$, $F_1 = 1$, and

$$F_n = F_{n-1} + F_{n-2} \tag{3}$$

for $n > 1$. It is well known that one can extend the definition to negative n by putting $F_n = (-1)^{n-1} F_{-n}$, and that many familiar identities, such as (3) and

$$F_n F_{m+1} - F_{n+1} F_m = (-1)^m \cdot F_{n-m}, \tag{4}$$

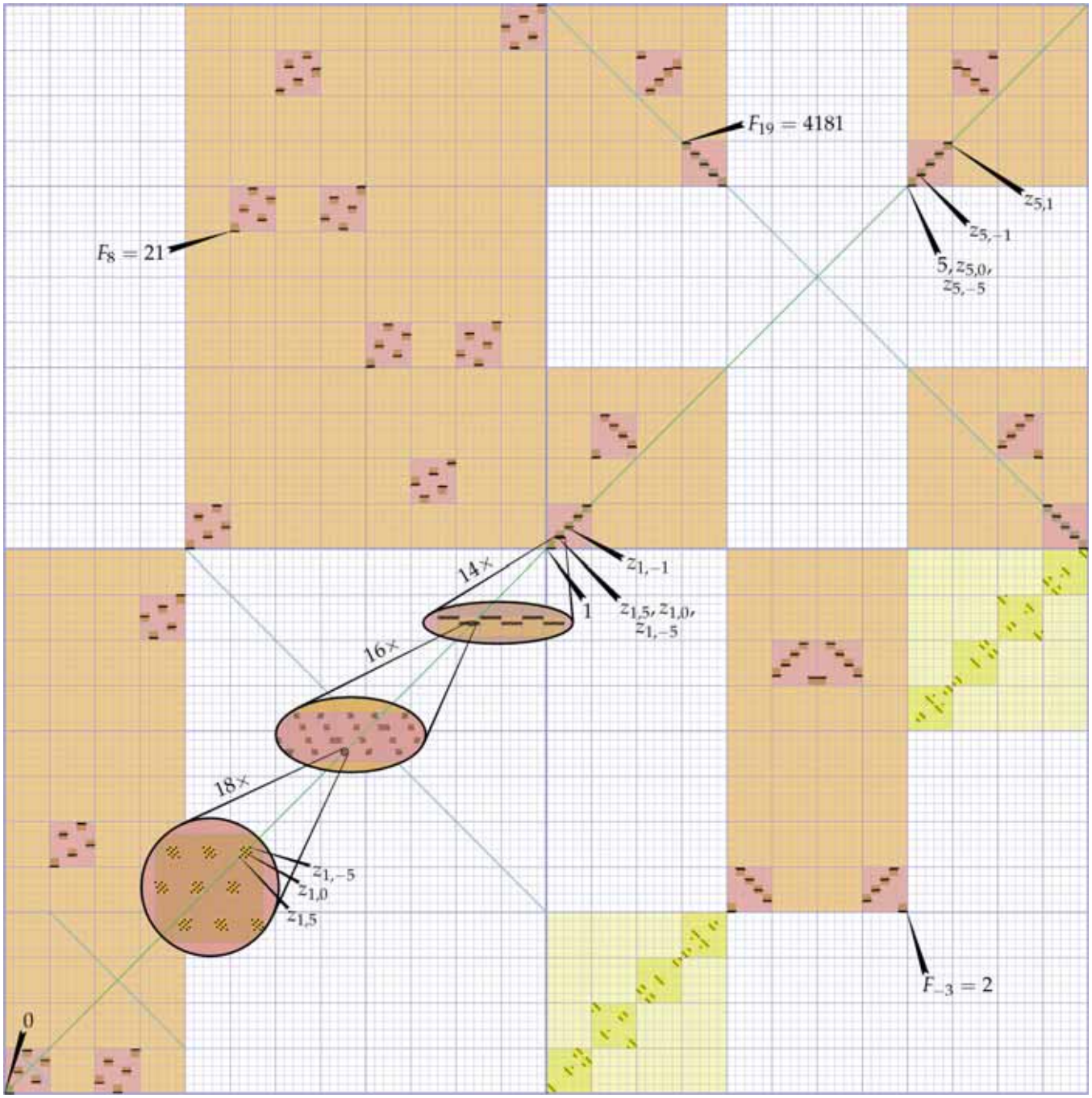
then hold for *all* integers n and m. There is, however, no reason to stop here.

For each profinite integer s, one can in a natural way define the sth Fibonacci number F_s, which is itself a profinite integer. Namely, given s, one can choose a sequence of positive integers $n_1, n_2, n_3, \ldots$ that share more and more initial digits with s, so that it may be said that n_i *converges* to s for $i \to \infty$. Then the numbers $F_{n_1}, F_{n_2}, F_{n_3}, \ldots$ share more and more initial digits as well, and we define F_s to be their 'limit' as $i \to \infty$. This does not depend on the choice of the sequence of numbers n_i.

For example, we can write $s = -1$ as the limit of the numbers $n_1 = (21)_! = 5$, $n_2 = (321)_! = 23$, $n_3 = (4321)_! = 119$, $n_4 = (54321)_! = 719$, ..., so that F_{-1} is the limit of

$$\begin{aligned} F_5 &= 5 = (21)_!, \\ F_{23} &= 28657 = (5444001)_!, \\ F_{119} &= 331164814351698201718008 1 \\ &= (5826\,4\,1\,18\,1\,0\,1\,5\,1\,3\,23418173200001)_!, \\ F_{719} &= (\ldots 3\,1\,6\,6\,98\,1\,6\,1\,2\,51\,1\,1\,4\,3\,1\,149806000001)_!, \\ &\ldots, \end{aligned}$$

which is consistent with the true value $F_{-1} = 1 = (\ldots 000001)_!$.



This picture shows the graph of the Fibonacci function $\hat{\mathbf{Z}} \to \hat{\mathbf{Z}}$. Each element $(\ldots c_3c_2c_1)_! = \sum_{i\geq 1} c_i i!$ of $\hat{\mathbf{Z}}$ is represented by the number $\sum_{i\geq 1} c_i/(i+1)!$ in the unit interval, and the graph $\{(s, F_s) : s \in \hat{\mathbf{Z}}\}$ is correspondingly represented as a subset of the unit square. Successive approximations to the graph are shown in orange, red, and brown. Intersecting the diagonal, shown in green, with the graph, one finds the eleven fixed points 0, 1, 5, $z_{1,-5}$, ..., $z_{5,1}$ of the function. There are two clusters of three fixed points each that are indistinguishable in the precision used. One of these clusters is resolved by a sequence of three blow-ups, with a total magnification factor of $14 \times 16 \times 18 = 4032$. The graph of the function $s \mapsto -s$, shown in blue, enables the viewer to check the formula $F_{-s} = (-1)^{s-1}F_s$. The yellow squares contain the curve $\{(s,t) \in \hat{\mathbf{Z}} \times \hat{\mathbf{Z}} : s \cdot (t+1) = 1\}$, which projects to the group of units of $\hat{\mathbf{Z}}$ on the horizontal axis. (Illustration by Willem Jan Palenstijn.)

For each $k \geq 3$ the first k digits of F_s are determined by the first k digits of s. This rule makes it possible to compute profinite Fibonacci numbers, as we shall see below.

Many identities such as (3) and (4) are also valid for profinite Fibonacci numbers. In order to give a meaning to the sign that appears in (4), we call a profinite integer s *even* or *odd* according as its first digit c_1 is 0 or 1, and we define $(-1)^s = 1$ or -1 accordingly. More generally, one defines a profinite integer s to be *divisible* by a positive integer b if the factorial number formed by the first $b-1$ digits of s is divisible by b. For many b, it suffices to look at far fewer than $b-1$ digits. For example, if k is a non-negative integer, then a profinite integer is divisible by $k!$ if and only if its $k-1$ initial digits are zero. Two profinite numbers s_1 and s_2 are called *congruent* modulo a positive integer b if their difference is divisible by b, notation: $s_1 \equiv s_2 \bmod b$.

The following method may be used to compute profinite Fibonacci numbers. Let s be a profinite number, and suppose that one wishes to compute the sth Fibonacci number F_s to an accuracy of k digits, for some $k \geq 3$. Then one first truncates

s to k digits, which gives a non-negative integer n that is usually very large. By the rule mentioned above, F_s and F_n share at least k initial digits, so it suffices to calculate F_n to a precision of k digits. To this end, let ϑ be a symbol that satisfies the rule $\vartheta^2 = \vartheta + 1$. Then for all n one has $\vartheta^n = F_n\vartheta + F_{n-1}$. The left hand side can be quickly calculated by induction, even for very large n, if one uses the identities $\vartheta^{2m} = (\vartheta^m)^2$ and $\vartheta^{2m+1} = \vartheta^{2m} \cdot \vartheta$. All intermediate results are expressed in the form $a\vartheta + b$, where a and b are integers that are only computed to a precision of k digits in the factorial number system. Then in the end one knows F_n to a precision of k digits as well, as required.

The *Lucas numbers* L_n, which are defined by $L_0 = 2$, $L_1 = 1$, $L_n = L_{n-1} + L_{n-2}$ $(n > 1)$, can be generalized to profinite numbers in a completely similar manner. They are expressed in Fibonacci numbers by $L_s = F_{s+1} + F_{s-1}$. One has also $F_s L_s = F_{2s}$ for all $s \in \hat{\mathbf{Z}}$; however, it is not necessarily meaningful to write $L_s = F_{2s}/F_s$, since division is not always well-defined in $\hat{\mathbf{Z}}$.

Power series expansions

A striking property of profinite Fibonacci numbers is that they have *power series* expansions. If $s_0 \in \hat{\mathbf{Z}}$, then the power series expansion for F_s around s_0 takes the shape

$$\begin{aligned} F_s &= F_{s_0} + lL_{s_0}(s-s_0) + 5l^2F_{s_0}\frac{(s-s_0)^2}{2!} \\ &\quad + 5l^3L_{s_0}\frac{(s-s_0)^3}{3!} + 5^2l^4F_{s_0}\frac{(s-s_0)^4}{4!} + \dots \\ &= \sum_{i=0}^{\infty}\Big(5^il^{2i}F_{s_0}\frac{(s-s_0)^{2i}}{(2i)!} + 5^il^{2i+1}L_{s_0}\frac{(s-s_0)^{2i+1}}{(2i+1)!}\Big), \quad (5) \end{aligned}$$

where l is a certain profinite integer that is given by (2). The number l is divisible by all prime numbers except 5. From this it follows that 5^il^{2i} and 5^il^{2i+1} are divisible by $(2i)!$ and $(2i+1)!$, respectively, so that the coefficients in the power series expansions are profinite integers.

No prime number p is known for which l is divisible by p^2. In fact, if p is a prime number, then the number of factors p in l is the same as the number of factors p in $F_{p-1}F_{p+1}$, and no prime number is known for which $F_{p-1}F_{p+1}$ is divisible by p^2. One may, however, reasonably conjecture that there exist infinitely many such primes.

An informal derivation of (5) can be given as follows. Let again ϑ be such that $\vartheta^2 = \vartheta + 1$, and put $\vartheta' = 1 - \vartheta$. Then for all integers n one has $F_n = (\vartheta^n - \vartheta'^n)/(\vartheta - \vartheta')$ and $L_n = \vartheta^n + \vartheta'^n$. This suggests that one has $F_s = (\vartheta^s - \vartheta'^s)/(\vartheta - \vartheta')$ and $L_s = \vartheta^s + \vartheta'^s$ for all profinite integers s as well, and with a suitable interpretation of the powering operation this is indeed correct. Now consider the Taylor series for F_s around s_0:

$$F_s = \sum_{j=0}^{\infty} F_{s_0}^{(j)}\frac{(s-s_0)^j}{j!},$$

where $F_s^{(j)} = \frac{d^jF_s}{ds^j}$ denotes the jth derivative. To calculate these higher derivatives, one first notes that from $\vartheta\vartheta' = -1$ one obtains

$$2(\log\vartheta + \log\vartheta') = 2\log(-1) = \log 1 = 0,$$

and therefore $\log\vartheta = -\log\vartheta'$. This leads to

$$\begin{aligned} \frac{dF_s}{ds} &= \frac{d}{ds}\frac{\vartheta^s - \vartheta'^s}{\vartheta - \vartheta'} = \frac{\log\vartheta}{\vartheta - \vartheta'}\left(\vartheta^s + \vartheta'^s\right) = \frac{\log\vartheta}{\vartheta - \vartheta'}L_s, \\ \frac{dL_s}{ds} &= \log\vartheta \cdot \left(\vartheta^s - \vartheta'^s\right) = \log\vartheta \cdot (\vartheta - \vartheta') \cdot F_s. \end{aligned}$$

Combining this with $(\vartheta - \vartheta')^2 = 5$, one finds

$$F_s^{(2i)} = 5^il^{2i}F_s, \qquad F_s^{(2i+1)} = 5^il^{2i+1}L_s$$

for each $i \geq 0$, where

$$l = \frac{\log\vartheta}{\vartheta - \vartheta'}. \quad (6)$$

This leads immediately to (5).

If one makes this informal argument rigorous, using an appropriate theory of logarithms, then one discovers that the precise meaning of (5) is a little more subtle than one may have expected. Namely, one should interpret (5) to mean that, for each positive integer b, the following is true for every profinite integer s that shares sufficiently many initial digits with s_0: if k is any positive integer, then all but finitely many terms of the infinite sum are divisible by b^k, and the sum of the remaining terms is congruent to F_s modulo b^k. For example, if b divides $5! = 120$, then it suffices for s to share three initial digits with s_0, and if b divides $36!$ then six initial digits are enough.

One application of the power series development is the determination of l to any desired precision. Namely, put $s_0 = 0$, so that $F_{s_0} = 0$ and $L_{s_0} = 2$. Then the power series development reads

$$F_s = 2ls + \frac{2\cdot5\cdot l^3\cdot s^3}{3!} + \frac{2\cdot5^2\cdot l^5\cdot s^5}{5!} + \dots. \quad (7)$$

Suppose that one wishes to determine the first 35 digits of l, or, equivalently, the residue class of l modulo $36!$. Modulo any power of $36!$, the expansion is valid for profinite numbers s of which the first six digits are zero. Choose

$$s = 2^{16}\cdot3^8\cdot5^4\cdot7 = (168133000000000)_!.$$

Using that l is divisible by all prime numbers except 5, one easily sees that in (7) each term on the right beyond the first term is divisible by $2s\cdot36!$. Calculating F_s modulo $2s\cdot36!$ by means of the technique explained earlier, and dividing by $2s$, one finds l modulo $36!$:

$$l = (\dots{}^{2}_{6}{}^{3}_{3}51{}^{1}_{3}{}^{1}_{7}{}^{1}_{1}{}^{2}_{3}471{}^{1}_{6}04{}^{1}_{6}{}^{1}_{3}{}^{1}_{8}{}^{1}_{0}4768{}^{1}_{0}{}^{1}_{0}49000120100)_!.$$

One may also compute l directly from (6), if a good method for computing logarithms is available.

Fixed points

The power series expansion also comes in when one wishes to determine the *fixed points* of the Fibonacci sequence, i.e., the numbers s for which $F_s = s$. It is very easy to see that among the ordinary integers the only examples are $F_0 = 0$, $F_1 = 1$, $F_5 = 5$. In $\hat{\mathbf{Z}}$, there are exactly eight additional fixed points,

namely the following profinite numbers:

$$z_{1,-5} = (\ldots 7^1 1548^1 6^1 786^1 065657871411001)_!,$$
$$z_{1,-1} = (\ldots 1^8 2^1 300813^1 5180733953122001)_!,$$
$$z_{1,0} = (\ldots 1^3 6^1 507^1 6^1 47^1 168^1 133471411001)_!,$$
$$z_{1,5} = (\ldots 1^9 2^1 4^1 8^1 6^1 6^1 16^1 62^1 1290^1 071411001)_!,$$
$$z_{5,-5} = (\ldots 1^2 1^4 08206177^1 02^1 04880000021)_!,$$
$$z_{5,-1} = (\ldots 2^3 3^2 2^1 3^1 4^1 1^1 8^1 4^1 5^1 4^1 1^1 1^1 124871411021)_!,$$
$$z_{5,0} = (\ldots 1^8 1^9 041030^1 23^1 28524400000021)_!,$$
$$z_{5,1} = (\ldots 52^1 8^3 1^4 437^1 10^1 33^1 13^1 10916244021)_!.$$

The notation $z_{a,b}$, for $a \in \{1, 5\}$, $b \in \{-5, -1, 0, 1, 5\}$, is chosen because we have

$$z_{a,b} \equiv a \bmod 6^k, \qquad z_{a,b} \equiv b \bmod 5^k$$

for all positive integers k; this uniquely determines $z_{a,b}$ as a fixed point of the Fibonacci sequence. (For $a = b \in \{1, 5\}$ one may take $z_{a,b} = a$.)

There are several techniques that one may use to calculate the numbers $z_{a,b}$ to any required precision. The first is to start from any number x_0 that satisfies $x_0 \equiv a \bmod 24$, $x_0 \equiv b \bmod 5^k$, where k is at least one quarter of the required number of digits, and $k \geq 2$, and next to apply the iteration $x_{i+1} = F_{x_i}$. This converges to $z_{a,b}$ in the required precision, but the convergence is not very fast. One can accelerate this method by choosing a starting value x_0 for which $x_0 - a$ has a greater number of factors 2 and 3. The second method is to apply a Newton iteration to find a zero of the function $F_s - s$:

$$x_{i+1} = x_i - \frac{F_{x_i} - x_i}{lL_{x_i} - 1}.$$

This requires some care with the division that is involved, and one needs to know l to the same precision. However, it converges much faster, even if the starting value x_0 only satisfies $x_0 \equiv a \bmod 24$, $x_0 \equiv b \bmod 25$.

The eight fixed points $z_{a,b}$ have, imprecisely speaking, the tendency to approximately inherit properties of a, b. For example, each of $a = 1$ and $b = 0$ is equal to its own square, and, correspondingly, $z_{1,0}$ is quite close to its own square, in the sense that the nine initial digits are the same:

$$z_{1,0}^2 = (\ldots 1^3 66^2 0^4 07953^1 02255471411001)_!.$$

Each of $a = 1, b = -1$ has square equal to 1, and this is almost true for $z_{1,-1}$:

$$z_{1,-1}^2 = (\ldots 2^2 1^7 1^0 1^0 00000000000000000001)_!.$$

Studying the expansions of $z_{1,5}$ and $z_{5,1}$, one discovers that for each i with $4 < i \leq 24$ their ith digits add up to i. This is due to the remarkable relation

$$z_{1,5} + z_{5,1} = (\ldots 00000000000000000000100)_!,$$

which reflects the equality $5 + 1 = 6 = (100)_!$. Likewise, $5 \cdot 1 = 5 = (21)_!$ is reflected in

$$z_{1,5} \cdot z_{5,1} = (\ldots 00000000000000000000021)_!.$$

However, greater precision reveals that $z_{1,5} + z_{5,1} \neq 6$ and $z_{1,5} \cdot z_{5,1} \neq 5$:

$$z_{1,5} = (\ldots 2^2 9^2 6^2 4^1 63^1 9^2 14^1 86^1 6^1 16^1 62^1 1290^1 071411001)_!,$$
$$z_{5,1} = (\ldots 26323^1 0^2 252^1 8^3 1^4 437^1 10^1 33^1 13^1 10916244021)_!,$$
$$z_{1,5} + z_{5,1} = (\ldots 55000000000000000000000000000100)_!,$$
$$z_{1,5} \cdot z_{5,1} = (\ldots 2^2 5^2 00000000000000000000000000021)_!.$$

The number $z_{5,-5}$ is the most mysterious of all. By analogy, one suspects its square to be close to $5^2 = (-5)^2 = 25 = (1001)_!$, without being exactly equal to it. Confirming this suspicion requires considerable accuracy:

$$z_{5,-5}^2 = (\ldots 1^4 6^1 7^4 1^1 3^2 2^2 2000000000000000000000000000000000000$$
$$000000000000000000000000000000000000000000$$
$$000000000000000000000000000000000000000000$$
$$000000000000000000000000000000000000000000$$
$$000000000000000000000000000000000001001)_!.$$

Here even the expert may be baffled: given that $z_{5,-5}^2$ is different from 25, is there a good reason for the difference not to show up until after the *two hundredth* digit?

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Lie algebras and automorphic forms

Nils R. Scheithauer (Heidelberg, Germany)

In this article we describe relations between infinite dimensional Lie algebras and automorphic forms.

Let $M_n(\mathbb{C})$ be the algebra of complex $n \times n$ matrices. We can define a new product $[a,b] = ab - ba$ on $M_n(\mathbb{C})$. This product is antisymmetric,

$$[a,b] = -[b,a],$$

and satisfies the Jacobi identity

$$[a,[b,c]] + [b,[c,a]] + [c,[a,b]] = 0.$$

An algebra with such a product is called a Lie algebra. It turns out that there are close relations between certain infinite dimensional Lie algebras and automorphic forms. These are meromorphic functions, which have simple transformation properties under suitable groups.

In this article we describe three examples of the connection between Lie algebras and automorphic forms. We sketch Borcherds' proof of the *moonshine conjecture*. Then we formulate a similar conjecture for Conway's group Co_0. In the last section we describe classification results for infinite dimensional Lie algebras.

Lie algebras

The theory of Lie groups and Lie algebras was introduced in 1873 by S. Lie. A Lie group is a group with the structure of a differentiable manifold such that the multiplication and inverse operations are differentiable. Examples for Lie groups are the spheres $S^1 = \{z \in \mathbb{C} \,|\, |z| = 1\}$ and $S^3 = \{x \in \mathbb{H} \,|\, |x| = 1\}$ and the matrix groups $GL_n(\mathbb{R})$ and $GL_n(\mathbb{C})$. Many geometrical properties of a Lie group can be described in terms of its Lie algebra.

The Lie algebra of a Lie group is the tangent space at the identity. The group structure induces a product on the tangent space that is antisymmetric and satisfies the Jacobi identity. This is the starting point for the abstract definition of a Lie algebra. A Lie algebra is a vector space with an antisymmetric product satisfying the Jacobi identity.

At the end of the 19th century Killing and Cartan classified the finite dimensional simple Lie algebras over the complex numbers. There are 4 infinite families, the classical Lie algebras $A_n = sl_{n+1}(\mathbb{C})$, $B_n = so_{2n+1}(\mathbb{C})$, $C_n = sp_{2n}(\mathbb{C})$ and $D_n = so_{2n}(\mathbb{C})$, and five exceptional Lie algebras G_2, F_4, E_6, E_7 and E_8.

A finite dimensional irreducible representation of a finite dimensional simple Lie algebra decomposes into weight spaces. The representation is characterized by the highest weight in this decomposition. The character of the representation is a series whose coefficients are given by the dimensions of the weight spaces. Weyl's character formula describes the character as the quotient of a sum over the Weyl group and a product over positive roots. Applying Weyl's character formula to the trivial representation we obtain the denominator identity. Outer automorphisms give twisted denominator identities.

We can also associate a matrix, the Cartan matrix, to a finite dimensional simple Lie algebra. The elements on the diagonal and the principal minors of this matrix are positive. By a result of Serre we can reconstruct the Lie algebra from its Cartan matrix by dividing a free Lie algebra by certain relations given by the matrix.

Serre's construction can also be applied to matrices whose principal minors are not necessarily positive. In this way we obtain Kac-Moody algebras [K]. These Lie algebras are in general infinite dimensional but their theory is similar to the finite dimensional theory in many aspects. In particular there is a character formula for irreducible highest weight representations and a denominator identity.

Kac-Moody algebras can only be classified under certain assumptions on the Cartan matrices. For example if we assume that the determinant of the Cartan matrix vanishes and the proper principal minors are positive we obtain the class of affine Kac-Moody algebras. They can be written as tensor products of the finite dimensional simple Lie algebras with Laurent polynomials in one variable. The denominator identities of the affine Kac-Moody algebras give sum expansions of infinite products. For example the denominator identity of the affinization of $sl_2(\mathbb{C})$

$$\prod_{n>0}(1-q^{2n})(1-q^{2n-1}z)(1-q^{2n-1}z^{-1}) = \sum_{n\in\mathbb{Z}}(-1)^n q^{n^2} z^n$$

is Jacobi's triple product identity. The denominator identities of the affine Kac-Moody algebras transform nicely under suitable Jacobi groups, i.e. they are Jacobi forms [K, B2].

Borcherds discovered that the conditions on the Cartan matrix can be weakened further. Applying Serre's construction to matrices whose diagonal elements are not necessarily positive we obtain generalized Kac-Moody algebras. The theory of these Lie algebras is still similar to the finite dimensional theory. In particular a denominator identity holds. We will see that they are sometimes automorphic forms for orthogonal groups. Generalized Kac-Moody algebras have found natural realizations in physics. Some of these Lie algebras describe bosonic strings moving on suitable space times.

Automorphic forms

The modular group $SL_2(\mathbb{Z})$ acts on the upper half plane

$$H = \{\tau \in \mathbb{C} \,|\, \mathrm{Im}(\tau) > 0\}$$

by fractional linear transformations

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tau = \frac{a\tau + b}{c\tau + d}.$$

A meromorphic function on the upper half plane is a modular function of weight k, where k is an integer, if

$$f(M\tau) = (c\tau + d)^k f(\tau)$$

for all $M = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ in $SL_2(\mathbb{Z})$ and f is meromorphic at the cusp $i\infty$. The function f is a modular form if in addition f is holomorphic on H and in $i\infty$.

For example for an even integer $k \geq 4$ the Eisenstein series

$$E_k(\tau) = \frac{1}{2} \sum_{\substack{m,n\in\mathbb{Z} \\ (m,n)=1}} \frac{1}{(m\tau+n)^k}$$

is a modular form of weight k. The transformation behaviour under the modular group follows by reordering the sum. Using the partial fraction expansion of the cotangent we can show that the Fourier expansion of the Eisenstein series is given by

$$E_k(\tau) = 1 - \frac{2k}{B_k} \sum_{n=1}^{\infty} \sigma_{k-1}(n) q^n$$

with $q = e^{2\pi i\tau}$ and $\sigma_{k-1}(n) = \sum_{d|n} d^{k-1}$. The function

$$\begin{aligned} j(\tau) &= 1728 \frac{E_4(\tau)^3}{E_4(\tau)^3 - E_6(\tau)^2} \\ &= q^{-1} + 744 + 196884q + 21493760q^2 + \dots \end{aligned}$$

is a modular function of weight 0.

The definition of a modular form can be generalized in several ways. Jacobi forms are functions on $H \times \mathbb{C}$ that transform in a simple way under the Jacobi group $SL_2(\mathbb{Z}) \ltimes \mathbb{Z}^2$. An example is the Jacobi theta series given above. In general the denominator identities of the affine Kac-Moody algebras are automorphic forms on $H \times \mathbb{C}^n$ for generalizations of the above Jacobi group.

For the theory of generalized Kac-Moody algebras, automorphic forms on orthogonal groups are important. These are meromorphic functions on Grassmannians transforming nicely under discrete subgroups of the orthogonal groups $O_{n,2}(\mathbb{R})$. Borcherds found a map from vector valued modular forms on $SL_2(\mathbb{Z})$ to automorphic forms on orthogonal groups [B3]. These automorphic forms have nice product expansions and are therefore called automorphic products.

The monster

Around 1983 the classification of finite simple groups was achieved. There are 18 infinite families, the cyclic groups of prime order, the alternating groups and the groups of Lie type, and moreover there are 26 sporadic simple groups. The proof of this result comprises the work of more than a hundred mathematicians and consists of several thousand journal pages. The largest sporadic simple group is the monster. This group was predicted by Fischer and Griess in 1973 and constructed by Griess in 1982. The smallest irreducible representations of the monster have dimensions $1, 196883, 21296876, \dots$. McKay noticed the following relations between the coefficients of the function j and the dimensions of the irreducible representations of the monster

$$\begin{aligned} 1 &= 1 \\ 196884 &= 196883 + 1 \\ 21493760 &= 21296876 + 196883 + 1\,. \end{aligned}$$

This observation led Conway and Norton [CN] in 1979, i.e. before the existence of the monster was actually proven, to the conjecture that there should be a module $V = \bigoplus_{n\in\mathbb{Z}} V_n$ for the monster such that the McKay-Thompson series $T_g(\tau) = \sum_{n\in\mathbb{Z}} \mathrm{tr}(g|V_n) q^n$, where $\mathrm{tr}(g|V_n)$ is the trace of the element g in the monster on V_n, are modular functions of weight 0 for genus 0 subgroups of $SL_2(\mathbb{R})$. This conjecture is called the *moonshine conjecture*; in this context moonshine does not refer to the light of the moon but instead means nonsense.

In 1988 Frenkel, Lepowsky and Meurman [FLM] constructed a candidate for V on which the monster acts. The module V has an additional algebraic structure, that of a vertex algebra, which is invariant under the action of the monster. Frenkel, Lepowsky and Meurman showed that the graded dimensions of V are given by the coefficients of $j - 744$, i.e. the McKay-Thompson series of the identity is $j - 744$. But they could not show in general that the McKay-Thompson series are modular functions for genus 0 groups. This was proven by Borcherds applying the theory of generalized Kac-Moody algebras [B1]. He constructed by means of the module V a generalized Kac-Moody algebra, the monster Lie algebra. The denominator identity of this Lie algebra

$$\frac{1}{q_1} \prod_{\substack{n_1>0 \\ n_2\in\mathbb{Z}}} (1 - q_1^{n_1} q_2^{n_2})^{[j](n_1 n_2)} = j(\tau_1) - j(\tau_2)$$

gives a product expansion of the function j. Here the exponent $[j](n)$ denotes the coefficient at q^n in the Fourier expansion of j. Borcherds showed that the monster acts naturally on the monster Lie algebra and calculated the corresponding twisted denominator identities. These identities imply that the McKay-Thompson series are modular functions of weight 0 for genus 0 groups.

Conway's group

A lattice in $\mathbb{R}^n$ is the integral span of n linearly independent vectors. A lattice is called unimodular if the n basis vectors span a parallelotope of unit volume. Furthermore a lattice is even if all vectors in the lattice have even norm. Using the theory of modular forms we can show that even unimodular lattices only exist in dimensions that are divisible by 8. In dimension 8 there is, up to isometry, exactly one such lattice, in dimension 16 there are two, and in dimension 24 there are exactly 24 even unimodular lattices, up to isometry. Thereafter the number of lattices increases very rapidly.

Among the 24 even unimodular lattices in $\mathbb{R}^{24}$ there is exactly one lattice that has no vectors of norm 2. This implies that there is no vector in this lattice such that the reflection in the hyperplane orthogonal to this vector maps the lattice into itself, i.e. the lattice has no roots. This lattice is called the Leech lattice. The automorphism group of the Leech lattice is Conway's group Co_0. Dividing Co_0 by the normal subgroup generated by -1 gives the sporadic simple group Co_1. The characteristic polynomial of an element g of order n in Co_0 can be written as $\prod_{d|n}(x^d - 1)^{b_d}$. The level of g is defined as the level of the modular function $\eta_g(\tau) = \prod_{d|n} \eta(d\tau)^{b_d}$. Conway's group Co_0 acts naturally on the fake monster Lie algebra [B1]. This is a generalized Kac-Moody algebra describing the physical states of a bosonic string moving on a 26-dimensional torus. The twisted denominator identities under the action of Co_0 are probably automorphic forms of singular weight for orthogonal groups. This conjecture is analogous

to Conway and Norton's conjecture. It is proven for elements of squarefree level in [S1, S2, S3]. This theorem implies the following result.

Let N be a squarefree positive integer such that $\sigma_1(N)|24$. Then there is an element g in Co_0 of order N and characteristic polynomial $\prod_{d|N}(x^d-1)^{24/\sigma_1(N)}$. Let Λ^g be the fixpoint lattice of g. Then the twisted denominator identity of g is given by

$$e^{\rho}\prod_{d|N}\prod_{\alpha\in(L\cap dL')^+}(1-e^{\alpha})^{[1/\eta_g](-\alpha^2/2d)} = \sum_{w\in W}\det(w)w\big(\eta_g(e^{\rho})\big),$$

where $L=\Lambda^g\oplus II_{1,1}$ and W is the reflection group of L. This identity defines an automorphic form of singular weight for an orthogonal group and is also the untwisted denominator identity of a generalized Kac-Moody algebra. In this way we obtain 10 generalized Kac-Moody algebras that are very similar to the fake monster Lie algebra.

Classification results

We have already seen that the known classification results of Kac-Moody algebras assume certain properties of the Cartan matrices. In particular the Cartan matrix must be finite. For generalized Kac-Moody algebras this assumption is not reasonable because the most interesting generalized Kac-Moody algebras, the monster Lie algebra and the fake monster Lie algebra, have infinitely many simple roots and therefore infinite Cartan matrices. The fact that the denominator identities of some generalized Kac-Moody algebras are automorphic forms of singular weight for orthogonal groups suggests analysing whether such Lie algebras can be classified. This idea seems to be promising. For example we can show [S4] that the ten Lie algebras constructed above are the only generalized Kac-Moody algebras whose denominator identities are completely reflective automorphic products of singular weight on lattices of squarefree level and positive signature. This classification result relies on properties of the Eisenstein series and the Bernoulli numbers B_k. For example the fake monster Lie algebra owes its existence to the fact that

$$\frac{2k}{B_k}=24$$

for $k=14$. In contrast to the affine Kac-Moody algebras there are only finitely many Lie algebras with automorphic denominator identity in this case.

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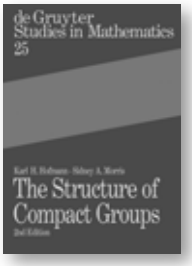

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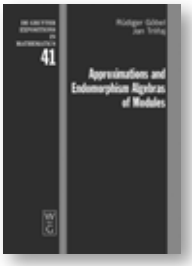

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A walk among the mathematical interests of Miguel de Guzmán (1936–2004)

Eugenio Hernández, Fernando Soria (Universidad Autónoma de Madrid. Spain)

The official retirement day for Miguel de Guzmán would have come at the end of the 2005–06 academic year, almost at the same time as the celebration of the International Congress of Mathematicians in Madrid (22th–30th August 2006). Many have contributed to making such an event possible (for the first time in Spain) but one of the first steps in this direction was taken by Miguel de Guzmán, nurturing a passion for mathematical research that has prevailed in Spain for generations.

The great success of mathematics research in Spain over the last 25 years (as measured by, among other things, the quality of papers published and the international conferences celebrated each year) is due in part to Miguel's determination to, as Isaac Newton said, "climb onto the shoulders of giants" in those areas of mathematics in which he worked, setting an example for many others to come.

His first university degree was in philosophy (Munich, Germany, 1961). Years before, while still at high school age, Miguel's brothers, engineering students, introduced him to the book F. G.-M., *Exercises de Géométrie* [F], which so impressed him that he decided to study engineering (1952–54). The influence of this book on the mathematics of Miguel de Guzmán prevailed throughout his life. In the meetings to organize the Saturday morning sessions for talented 13- to 15-year-old students, when the need to prepare new material to nourish their talent arose, Miguel was fond of reminding us that there was a source of never-ending material in the exercises of F. G.-M.'s book. As he wrote and posted on his web page *http://ochoa.mat.ucm.es/~guzman/* "*Elementary Geometry was the great hobby that attracted me to the study of mathematics and has remained with me throughout my life.*"

We cite some examples to highlight his interest in geometry:

1. In 1976 the book entitled "Mirar y ver: nueve ensayos de geometría intuitiva" or "*Look and see: nine essays in intuitive geometry"* [G1] appeared. More adapted to modern times, the book is completely geometric in flavour, even in the proofs of the well known analysis inequalities of Hölder, Minkowski and Jensen.

2. In 1999 he published "*An extension of the Wallace-Simson theorem: projecting in arbitrary directions*" [G2], a problem inspired by the description in F. G.-M.'s book of the original Wallace-Simson result.

3. As part of his effort to popularize mathematics and make his passion for geometry accessible to students and professors, his last publication, "La experiencia de descubrir en matemáticas" or "*The experience of discovering mathematics*" [G3], contains a CD with interactive proofs of many properties relating to the geometry of figures in the plane and in space.

Miguel de Guzmán

This walk among the geometric interests of Miguel de Guzmán has diverted us from his mathematical research on harmonic analysis. After finishing his philosophy degree in 1961, Miguel completed studies in mathematics at the Universidad de Madrid in 1965. The geometric content of the undergraduate courses at Madrid in those years was low and the possibility of doing research in Spain was slim. The opportunity to do graduate work came when Professor Alberto Calderón visited Madrid in 1965 and offered Miguel the chance to apply to the graduate program at the University of Chicago. Working in the celebrated school of Calderón and Zygmund on harmonic analysis and singular integrals would detach Miguel from his beloved subject of geometry. But as we walk through the paths he explored in his research, we will see how he found ways to use geometry in analysis.

His doctoral dissertation, presented in 1968 at the University of Chicago under the direction of Alberto Calderón [G4], treats singular integrals with mixed homogeneity, with inmediate applications to equations of parabolic type. To prove the existence of the principal value of operators of the form

$$Kf(x) = \lim_{\varepsilon \to 0^+} \int_{\rho_0(y)>\varepsilon} k(y)\, f(x-y)\, dy, \qquad (8)$$

where ρ_0 is a translation invariant metric induced by the mixed homogeneity condition on k, Miguel used a lemma of geometric nature similar to those developed by Whitney (see [G5] or Theorem 2.3 in [G6]).

A year of postgraduate work at Washington University in St. Louis, at the invitation of Professor G. Weiss, gave Miguel

the opportunity to work with R. Coifman and to start to develop the Calderón-Zygmund theory in more general settings than the Euclidean case. Their work [CG] was the beginning of the theory of singular integrals in spaces of homogeneous type, a systematic treatment of which would appear years later in [CW].

That same year, this time in collaboration with G. Welland, Miguel started his research on differentiation of integrals in $\mathbb{R}^n$. The starting point is the theorem of H. Lebesgue [L] according to which a locally integrable function satisfies

$$\lim_{r\to 0}\frac{1}{|B_r(x)|}\int_{B_r(x)} f(t)dt = f(x), \quad a.e.\, x\in\mathbb{R}^n, \qquad (9)$$

where $B_r(x)$ denotes the ball of radius r and centre x, and $|B_r(x)|$ its volume. Other families of sets $\mathcal{B}=\bigcup_x \mathcal{B}(x)$ can be considered instead of balls centered at each point. Whether the Lebesgue differentiation property (9) holds for this family of sets depends on the geometric structure of its components. This is perhaps why this set of problems aroused the interest of Miguel.

It was known that weak type inequalities for the maximal operator

$$M_{\mathcal{B}}f(x) = \sup\left\{\frac{1}{|B|}\int_B |f(y)|\,dy:\ B\in\mathcal{B}(x)\right\} \qquad (10)$$

would prove (9) for the family $\mathcal{B}$. It is surprising, as Miguel and G. Welland proved in [GW], that these two facts are equivalent for functions in $L^1(\mathbb{R}^n)$ assuming that $\mathcal{B}$ is invariant under translations and dilations. Proving weak type inequalities for the maximal operator given in (10) requires covering lemmas of a geometric flavor.

Upon returning to the Universidad Complutense de Madrid in 1969, Miguel concentrated his work on the subject of *differentiation of integrals* and for many years conducted a weekly seminar where he invited mathematicians such as A. Calderón, L. Carleson and Y. Meyer and discussed with visitors and students the old and new results in this subject. The fruit of these years of research was an extraordinary monograph published in 1975 and entitled "*Differentiation of integrals on* $\mathbb{R}^n$" [G6], which was a source of inspiration for specialists in this area all over the world. More on the contents of this book can be found in [HS1].

After finishing the book on differentiation of integrals, Miguel undertook the task of collecting the methods most commonly used in Fourier analysis and some of its most recent applications. By discussing old and new results with visitors and students in the weekly seminar at the Universidad Complutense de Madrid, Miguel was able to write a second monograph in 1981 entitled "*Real Variable Methods in Fourier Analysis*" [G7]. His lively presentation of old and new results is still a reference for students and researchers working in harmonic analysis.

One of his more noteworthy achievements from this period was the technique of discretizing a problem given for continuous functions, which is presented in chapter IV of [G7]. To give the reader an example of this type of technique we take the following result from his book [G7]: let $\{K_j\}_{j\geq 1}$ be a family of kernels in L^1 and consider the maximal convolution operator

$$K^*f(x) = \sup_{j\geq 1}|K_j * f(x)|; \qquad (11)$$

then K^* is of weak-L^1 type if and only if K^* satisfies

$$\left|\left\{x : \sup_{j\geq 1}\left|\sum_{k=1}^{N} K_j(x-x_k)\right| > \lambda\right\}\right| \leq C\frac{N}{\lambda} \qquad (12)$$

for certain constant C independent of λ, $N\in\mathbb{N}$, and the sequence of N points $x_1,\ldots,x_N$ chosen in $\mathbb{R}^n$. This result is a tool used by researchers to search for the L^1-weak norm of operators, mainly the Hardy-Littlewood maximal operator. It is also the starting point for the use of combinatorial methods in harmonic analysis. A more detailed account of these results as well as applications can be found in [S].

The year 1982 marks a turning point in the mathematical interests of Miguel. He accepted a position at the Department of Mathematics of the Universidad Autónoma de Madrid, where some of the people he had recommended to do graduate work abroad had returned. But personal reasons conducted him back to the Universidad Complutense de Madrid after two years. At the same time he started a line of research on sets with apparently paradoxical properties, like the Besicovich set [B], which appears in the solution of the Kakeya needle problem, and the Nikodym set [N].

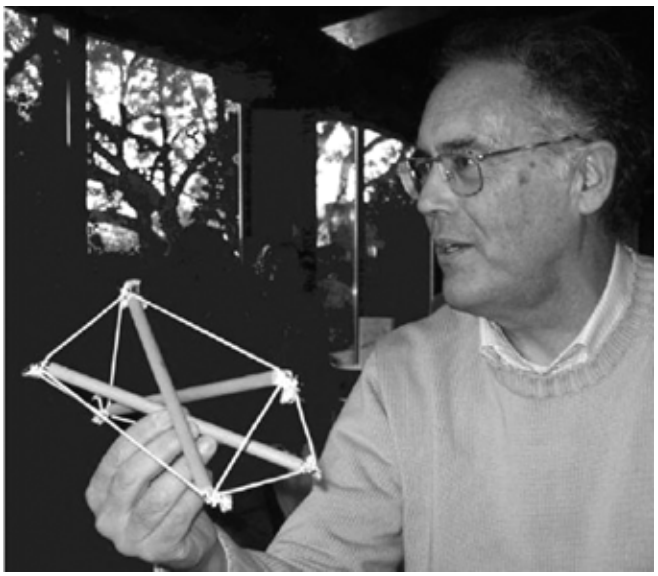
The Japanese mathematician Kakeya proposed, in 1917, the problem of finding the minimum area needed for a segment of length one to move in the plane to a position congruent (and not equal) to the original one. Besicovich [B] proved in 1927 that this can be done in a set of an area as small as one desires. That same year, Nikodym built a set of whole measure in the square $[0,1]^2$ with the property that from each one of its points there is a line that meets the set only at such a point. These sets, and others of a similar nature, play a role in the theory of differentiation of integrals and are the starting point of what is known today as geometric measure theory.

Miguel's approach was to go to the original papers, such as those of Besicovich, Haussdorff and Caratheodory, and work with his students on understanding them to be able to produce their own results once they had "climbed onto the shoulders of giants." The main contribution of Miguel to this area is reflected in the work of his doctoral students. A description of the results obtained in the three theses he directed or co-directed in these years can be found in [MMR].

There are many forests to visit in mathematics and many paths within each forest to explore. By the end of the 1980's Miguel had achieved in little more than twenty years one of his dreams: to have a stable production of high research mathematics going on in Spain. His contributions refer to Fourier analysis and geometric measure theory. But, by this time, the new social and political order in Spain had allowed other branches of mathematics to grow in the country. He then turned to another problem of more social importance.

The teaching of mathematics at all levels was the driving force behind many of the projects Miguel undertook in the last fifteen years of his life. It would take another article as long as this one to describe coherently the amount of activity he undertook in this period to enhance the teaching of mathematics.

Suffice it to say that he published many books to make mathematics accessible to everyone, that he was President of the International Commission on Mathematical Instruction (ICMI) from 1991 to 1998, that he organized courses at all levels for underdeveloped countries and that he founded a



Miguel de Guzmán with a tensegrity model

program for stimulating the talent of young students (known as ESTALMAT). We refer the reader to [HS2] for a more detailed (but still incomplete) description of his teaching-related activities.

His beloved subject of geometry was present all his life. In his last years Miguel was interested in *tensegrity*, systems in which structures stabilize themselves by balancing the forces of compression and tension. His practical mind led him to build several models of tensegrities that he hung from the ceiling and lamps of his study (see accompanying picture of one of his structures). The mathematical relations that create these designs that apparently float in the air were the subject of a still unpublished manuscript that some of his colleagues at the Universidad Complutense de Madrid have promised to publish.

Let us finish this presentation by quoting from [HS2], "The Spanish scientific community has lost an excellent mathematician. For both of us, who had the unique opportunity to meet him as undergraduate students and who belong to that group of "students abroad" that he sent to do graduate work, his death has robbed us of a friend and a teacher." We believe that the celebration of the ICM-2006 to be held in Madrid, 22th–30th August, can be considered a tribute to a man who has made a remarkable contribution to mathematics in Spain both at the research and at the educational level.

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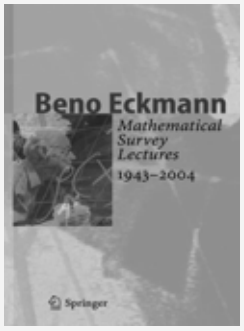

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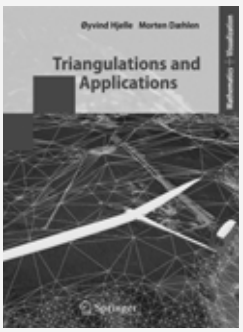

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Boltzmann's Legacy – The Man and his Science

A short biography of Ludwig Boltzmann (1844–1906)

Wolfgang L. Reiter (Vienna, Austria)

Boltzmann was born just outside the city walls of Vienna at what was then 286 Landstrasse on February 20, 1844, the very night marking the passage from Shrove Tuesday to Ash Wednesday.

Boltzmann, ever the wryly ironic rationalist, commented on his birthday that he was 'born between happiness and depression'. Boltzmann was undeniably a product of some of 19th century

Europe's most turbulent years: his life was marked by euphoria and depression, music and an interest in public affairs. His description of his birth date was equally fitting for the decade, something of a cultural and political watershed: in 1844 Johann Strauss the younger (1825–1899) gave his first public performance at the Dommayer Café near the Schönbrunn palace, and 1848 saw failed revolutions in Austria and the rest of Europe which ushered in a period of Habsburg absolutism after the succession of emperor Ferdinand by Franz Joseph I.

The young Ludwig was educated mainly by monks at the Akademisches Gymnasium in Linz, a provincial city in Upper Austria where his father at that time had a post as a civil servant in the state financial administration. Here he took piano lessons from Anton Bruckner (1824–1896) and developed his lifelong interest in music. The early death of his father in 1859 must have left a deep wound in the child's soul. After having passed his final school examinations, the Matura, with distinction in 1863, he enrolled at the University of Vienna to study mathematics and physics. At that time the Vienna Physics Institute was located in the same quarter where he had been born at 104 Landstrasse (now Erdbergerstrasse 15).[1]

Three years later he earned his PhD which, in this era, did not require writing a doctoral thesis. At the Institute of Physics he met Josef Loschmidt (1821–1895), professor of physical chemistry and a dedicated atomist,[2] who exerted a lasting influence during his early years at the Institute. Boltzmann became his assistant in 1867. In 1868 he took the next step in his academic carrier as Privatdozent for mathematical physics, a term used for what later (around 1900) came to be called theoretical physics. Only one year later the twenty-five year old Boltzmann was appointed professor for mathematical physics at the University of Graz, a post he held until 1873.

His first Graz period is marked by two fundamental scientific breakthroughs published in his paper Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen (Further studies on the thermal equilibrium of gas molecules) of 1872, which presents his H-theorem, the very first statistical interpretation of entropy and the famous transport equation, now named after him. The H-theorem raised fierce objections, first in 1875 from his teacher and friend Loschmidt who pointed to a 'reversibility paradox', and later in 1896 by the famous mathematician Ernst Friedrich Ferdinand Zermelo (1871-1953), who pointed to the so-called 'recurrence paradox'.[3] These objections forced Boltzmann to further elaborate on his basic assumptions, finally leading him to his ground-breaking statistical interpretation of the Sec-

[1] Ludwig Boltzmann later wrote the following about this location of the department: 'Erdberg remained throughout my life a symbol of honest and inspired experimental work. When I succeeded in injecting a bit of life into the institute in Graz I used to call it, jokingly, 'Little Erdberg'. By this I did not mean that the available space was scarce, because it was quite ample, probably twice as much as in Stefan's institute; but I had not succeeded in equalling [sic] the spirit of Erdberg as yet. Even in Munich, when young PhDs came to tell me that they did not know what to work on, I used to think: 'How different we were in Erdberg! Today there is beautiful experimental equipment and people are looking for ideas on how to use it. We always had plenty of ideas and were only preoccupied with the lack of equipment.' Quoted in Carlo Cercignani, *Ludwig Boltzmann: The Man Who Trusted Atoms*. (Oxford, New York, Melbourne: Oxford University Press, 1998), p.6.

[2] In Loschmidt's paper of 1866, *Zur Grösse der Luftmolecüle (On the Size of Air Molecules)* he calculated for the first time the number of molecules per unit volume in a gas at standard temperature and pressure, which is generally known as Avogadro's number. Loschmidt's value was off by a factor of 1/30; nevertheless, it provided for the first correct estimate of the size, masses and concentrations of molecules. Ludwig Boltzmann proposed in 1899, on the occasion of the unveiling of Loschmidt's bust at the University of Vienna, that this number be called Loschmidt's number, a term that is sometimes still used in German-speaking countries.

[3] Cf. Stephen G. Brush, *Kinetic theory, Vol. 2, Irreversible processes*. (Oxford: Pergamon Press, 1966). H. Poincaré, pp. 194–202, H. Poincaré, pp. 203–207, E. Zermelo, pp. 208–217, L. Boltzmann, pp. 218–228.

ond Law of Thermodynamics.

In 1873 he met his future wife Henriette von Aigentler (1854–1938), the first female student at the University of Graz. (The letters of the couple during their engagement were published a few years ago by Boltzmann's grandson, Dieter Flamm.[4] That year also brought further professional success: he accepted a post in Vienna as professor of mathematics, a major career step within the Austro-Hungarian academic hierarchy. Boltzmann stayed in Vienna only until 1876, when he went back to Graz for his second and much longer tenure as professor of physics, which lasted until 1890. In 1877 he published his probabilistic interpretation of thermodynamics, termed the 'Boltzmann Principle' by Albert Einstein (1879–1955): the entropy S of a macrostate (determined by pressure, temperature, and other variables) is related to the number W of microstates (determined by the positions and velocities of all atoms) by the famous relation S = k logW (with k the Boltzmann constant).[5] This formula is engraved on Boltzmann's tombstone at the Zentralfriedhof in Vienna.

In 1884 he proved the conjecture of his teacher Josef Stefan (1835–1893), that the total energy emitted by a black body is proportional to the fourth power of its absolute temperature.

Boltzmann's proof provided strong support for Maxwell's theory three years prior to Heinrich Hertz' experimental demonstration of the existence of electromagnetic waves. Stefan had recognized the importance of Maxwell's theory and had introduced Boltzmann, his first and most gifted student, to it. 'In the same year', to quote Carlo Cercignani, 'he also wrote a fundamental paper, generally unknown to the majority of physicists, who by reading only second-hand reports are led to the erroneous belief that Boltzmann dealt only with ideal gases; this paper clearly indicates that he considered mutually interacting molecules as well, with non-negligible potential energy, and thus […] it is he and not Josiah Willard Gibbs (1839–1903) who should be considered as the founder of equilibrium statistical mechanics and of the method of ensembles.'[6] The year Erwin Schrödinger (1887–1961) was born Boltzmann formulated his ergodic hypothesis. By this time he was already a scientific celebrity, attracting students from abroad like the future Nobel laureates Svante August Arrhenius (1859–1927) and Walter Nernst (1864–1941), who came to Graz to study with the leading figure in thermodynamics and kinetic theory on the continent.

By 1876 Boltzmann had married Henriette, and that year he bought and refurbished an old farm house at the outskirts of Graz in Oberkroisbach, where he kept a cow to provide fresh milk for his children. Here was a man flushed with professional success, enjoying a happy family life with his wife, children and a dog. Yet clouds began to gather over this rosy Graz idyll. Several events conspired to undermine Boltzmann's position, and it was at this time that he began his long slide into depression. His beloved mother had died in 1885. In 1888 he was elected to the position of Rektor of the University of Graz and was soon confronted with a month of aggressive protests by German nationalist students. Without informing the Austrian authorities officially, Boltzmann had accepted the prestigious Berlin chair of Gustav Kirchhoff (1824–1887) and then reneged with somewhat unconvincing arguments, citing his myopia and a neglect of significant chapters of mathematical physics in Berlin. However, the most damaging episode was probably the death of his eleven year old son Ludwig in 1889 from misdiagnosed appendicitis.

What was then termed Boltzmann's neurasthenia could no longer be ignored. Restlessness and a sort of escapism probably triggered by the feeling of an increasing isolation at Graz progressively marked his decisions. He renewed his interest in moving to Berlin, announcing his desire to leave Graz: he hoped that a change in life circumstances would calm his mental instability and insecurity. Then, after eighteen years in Graz, he accepted a chair in theoretical physics at the University of Munich in 1890. His restlessness subsided, but he was troubled by homesickness for Austria. When Stefan died in 1892 his Viennese colleagues immediately agreed to persuade Boltzmann to return to his alma mater. It took two years of hesitation, but he finally accepted the appointment in Vienna as Stefan's successor. With his worsening myopia the consideration of the generous retirement scheme offered by the University of Vienna compared to what was offered in Munich may well have been a crucial factor in his decision to return.

As it turned out, his decision to accept the chair in Vienna was a mistake. Compared with the hot bed of physical research that was Munich, his home town was distinctly provincial in this respect: he felt like a school master, training candidates in secondary school education for whom scientific work held little interest; his colleagues were less stimulating and, finally, Vienna harboured a philosophical climate strongly dominated by Ernst Mach's (1838–1916) phenomenological empiricism violently hostile to Boltzmann's atomism, the core of his life's work. When Mach moved to Vienna in 1895 to become professor of philosophy with special emphasis

[4] Dieter Flamm (Ed.), *Hochgeehrter Herr Professor! Innig geliebter Louis! Ludwig Boltzmann, Henriette von Aigentler, Briefwechsel.* (Wien, Köln, Weimar: Böhlau 1995).

[5] Ludwig Boltzmann, Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung, respective den Sätzen über das Wärmegleichgewicht. *Sitzungsberichte der kaiserlichen Akademie der Wissenschaften, Wien, mathematisch-naturwissenschaftliche Klasse (Teil II)*, 76 (1877), pp. 373–435. Albert Einstein, Über einen die Erzeugung und Verwandlung des Lichts betreffenden heuristischen Gesichtspunkt. *Annalen der Physik* 17 (1905), pp. 132–148.

[6] Carlo Cercignani, *Ludwig Boltzmann. The Man Who Trusted Atoms.* (Oxford, New York, Melbourne: Oxford University Press, 1998),

on the history and theory of the inductive sciences[7] both men now performed and competed for attention on the same stage as colleagues at the university and the Imperial Academy of Sciences.

After five short years in Vienna Boltzmann moved to Leipzig as Professor of Theoretical Physics, a decision obviously not taken easily because it caused a nervous breakdown, and he had to be hospitalized in a psychiatric clinic to recover. In Leipzig, Boltzmann was confronted with the most energetic 'energeticist', the chemist and close adherent of Mach's philosophy, Wilhelm Ostwald (1853–1932). Leipzig was another disaster, and it was there that he made the first attempt on his life. His chair in Vienna remained vacant during his Leipzig years, probably because he already started to negotiate his return when leaving for Leipzig.

Since his unsuccessful move to Berlin, Boltzmann had become increasingly trapped in self-conflict regarding his wishes for a change of places. In 1902 he moved back to Vienna and had to promise in writing never to leave Austria again. The Emperor, not a particularly dedicated supporter of the sciences, was not amused by the peregrinations of his famous but unreliable subject. Further disruption followed: because of the urgent need for more space for the Physics Institute, the apartment that the Boltzmann family lived in at Türkenstrasse 3 was appropriated by the university.

Boltzmann's apartment was his official university residence, and to compensate him for its loss, the ministry provided him with a sum of money with which he bought a villa at Haizingergasse 26 in the suburb of Währing in the 18th district, which is still owned by his descendants. Boltzmann lectured on theoretical physics and he finished the second volume of his *Vorlesungen über die Prinzipe der Mechanik*. In 1904 on the occasion of his 60th birthday, he was presented with a famous Festschrift that was edited by his assistant Stefan Meyer (1872–1949) and consisted of 117 contributions from the worldwide community of physicists.[8] 'Daddy gets worse every day. He has lost his faith in our future. I had imagined a better life here in Vienna.'[9] This alarming message was written by Boltzmann's wife to their daughter Ida (1884–1910), who had remained in Leipzig to finish school. Boltzmann suffered from heavy attacks of asthma, headaches and his myopia was near to blindness. An additional heavy burden was his acceptance of the position Mach has left open after suffering a stroke in 1901, a popular lecture course on natural philosophy (*Philosophie der Natur und Methodologie der Naturwissenschaften*) which he gave in 1903 with great success in the beginning but, after a few lectures, was unable to continue.[10] This undermined his self-confidence further.

In May 1906 Boltzmann was retired from his teaching duties. Though Boltzmann's discussion of the philosophy of science and epistemology belong to his less known and little discussed legacy, he made important contributions to the field, presenting a theory of scientific change inspired by Darwin's theory of evolution. Despite his physical and mental health problems during his last years in Vienna he twice crossed the Atlantic in 1904 and 1905, having previously lectured at Clark University, Worcester, Mass., in 1899. St. Louis hosted the world fair of 1904 and Boltzmann was invited to a meeting, and in 1905 he lectured at Berkeley and Stanford. His trip to California resulted in a most typically humorous diary *Reise eines deutschen Professors ins Eldorado*, published as part of his popular lectures in1905 and still a delightful piece of prose.[11] But this was to be his last burst of activity, and was followed by a deep depression and hospitalization. During the winter semester of 1905/06 he gave his last course in theoretical physics. During the following term Boltzmann was unable to fulfil his teaching duties due to his physical and mental suffering. Boltzmann's mental state was officially attributed to a serious form of neurasthenia, a vaguely defined term for some sort of general weakness of the nerve system signifying a broad spectrum of conditions, ranging from anxiety to sleeplessness.[12] In the light of his symptoms it seems likely that the term 'neurasthenia' was a euphemism for a much more serious mental illness: manic depression.

Ludwig Boltzmann ended his life at one of the most scenic spots on the Adriatic coast in Duino, near Trieste, a little village overlooked by the old castle of the noble Torre e Tasso family, where Rainer Maria Rilke wrote his famous *Duino Elegies* seventeen years later. The Viennese newspaper *Die Zeit* of September 7, 1906, reported the tragic news: 'He used a short cord from the crossbar of a window casement. His daughter was the first to discover the suicide.'[13]

A day before Boltzmann's planned return to Vienna on September 6, Ludwig's wife Henriette and their youngest daughter Elsa (1891–1965) had gone for a swim after which Ludwig never rejoined them. Boltzmann had relapsed into the black state of mind that distinguished his last years. He was restless and deeply depressed despite the relaxing atmosphere of the late summer holiday by the sea side he had long promised to spend with his

[7] The German title of Mach's lecture course was *Philosophie, insbesondere Geschichte und Theorie der induktiven Wissenschaften.*

[8] Stefan Meyer (Ed.), *Festschrift Ludwig Boltzmann gewidmet zum sechzigsten Geburtstage 20. Februar 1904* (Leipzig: J. A. Barth, 1904).

[9] Carlo Cercignani, *op. cit.*, p. 30.

[10] Ilse M. Fasol-Boltzmann (Ed.), *Ludwig Boltzmann. Principien der Naturfilosofi. Lectures on Natural Philosophy 1903–1906.* (Berlin/Heidelberg: Springer-Verlag 1990)

[11] Ludwig Boltzmann, *Populäre Schriften. Eingeleitet und ausgewählt von Engelbert Broda.* (Braunschweig/Wiesbaden: Friedr. Vieweg & Sohn 1979), pp. 258–290. Dieter Flamm (Ed.), *op.cit.*, pp. 235–256 (translation into English in Carlo Cercignani, *op. cit.*, pp. 231–250).

[12] Eugen Bleuler, *Lehrbuch der Psychiatrie*, 14. Auflage, neubearbeitet von Manfred Bleuler. (Berlin/Heidelberg/New York: Springer-Verlag 1979), pp. 519–522.

beloved wife. Increasingly short-sighted, anxious about his ability to perform his teaching duties as professor at the University of Vienna for the upcoming term, and beset by the manic depression which had previously led to more than one hospitalization, he ended his life at the Hotel Ples, today part of the international school Collegio del Mondo Unito.[14] His daughter Elsa was sent back to the hotel by her mother to check up on her father and was thus the first to discover his tragic suicide, a shocking and gruesome experience she never talked about for the rest of her life.

Boltzmann passed from this world without leaving a suicide note, but the preface of his lectures on mechanics (*Vorlesungen über die Prinzipe der Mechanik*), signed Abbazia, August 3, 1897, opens with a motto carefully chosen by Boltzmann in summing up his position on science and life:

Bring' vor was wahr ist;
Schreib' so, dass es klar ist
Und verficht's, bis es mit Dir gar ist.

The funeral ceremony took place on Saturday, September 8, 1906, at the Döblinger cemetery at an unusually late hour. It was seven o'clock in the evening. The train from Trieste with Boltzmann's coffin had been delayed for hours caused by heavy weekend traffic. The last of the mourners to speak at the open grave was his assistant Stefan Meyer. He ended on behalf of Boltzmann's students with words their teacher had used: 'Reinheit und Klarheit im Kampf um die Wahrheit!'[15]

This article appeared originally in the first Newsletter ESI-NEWS of the Erwin Schrödinger International Institute for Mathematical Physics, Vienna, Austria. We thank the editor for the permission to reprint it.

[13] 'Aus Triest kommt uns die Meldung: Hofrat Prof. Dr. Ludwig Boltzmann, der zum Sommeraufenthalt mit seiner Tochter in Duino weilte, wurde gestern als Leiche in seinem Zimmer aufgefunden. Er hatte sich mit einem kurzen Strick am Fensterkreuz erhängt. Seine Tochter war die erste, die den Selbstmord entdeckte.' Wiener Neuigkeiten. Selbstmord des Prof. Boltzmann. In: *Die Zeit* (Vienna), Nr. 1420, 7. September 1906, p.1.

[14] Several versions have been presented for the location where Boltzmann committed suicide. The version of the mathematician and colleague of Boltzmann, Franz Mertens (1840–1927), that Boltzmann took his life in the church of Duino, can be excluded, because no re-consecration of the church is documented. Support for the version that the location has to be identified with his hotel room is given in a short notice Die Ausführung des Selbstmords in the Neue Freie Presse (Vienna), Nr. 15102, 7. September 1906, p. 3, but without naming the hotel. The name of the hotel presented here is a private communication by Franz Gammer, Vienna.

[15] Hofrat Prof. Boltzmann. Das Leichenbegängnis in Wien. In: Die Zeit (Vienna), Nr. 1422, 9. September 1906, p. 4. Hofrat Prof. Boltzmann. Das Leichenbegängnis. Neues Wiener Tagblatt (Vienna), Nr. 49, 9. September 1906, pp. 6–7.

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cofounder and vice president of the Erwin Schrödinger Institute for Mathematical Physics – studied physics, mathematics, and philosophy and earned his Ph.D. at the University of Vienna. Until recently, he was director of the Natural Sciences Unit of the Federal Ministry for Education, Science and Culture. He is interested in the history of physics and the forced emigration of scientists from Austria in the thirties and forties.

Interview with Abel Prize recipient Lennart Carleson

Interviewers: Martin Raussen (Aalborg, Denmark) and Christian Skau (Trondheim, Norway)

The interview was conducted in Oslo on May 22nd prior to the Abel prize celebration and was later shown on Norwegian TV. The first two questions and their answers were originally phrased in the three Scandinavian languages: Norwegian, Danish and Swedish. They are reproduced here translated into English.

From left to right: Lennart Carleson, Martin Raussen, Christian Skau. (Photo: Terje Bendiksby/Scanpix)

Introduction

On behalf of the Norwegian and Danish mathematical societies, we want to congratulate you on winning the Abel prize for 2006.

This year we commemorate the 100th centenary of the death of the Norwegian dramatist and poet Henrik Ibsen. He passed away on the 23rd of May just a stone's throw away from this place. The longest poem he ever wrote is called "Balloon letter to a Swedish lady" and it contains a verse which reads as follows:

> ***"aldri svulmer der en løftning***
> ***av et regnestykkes drøftning***
> ***ti mot skjønnhet hungrer tiden"***

Translated into English this becomes:

> ***"never arises elation***
> ***from the analysis of an equation***
> ***for our age craves beauty"***

Without drawing too far-reaching conclusions, Ibsen seems to express a feeling shared by many people, i.e. that mathematics and beauty or art are opposed to each other, that they belong to different spheres. What are your comments to this view?

I do not think that Ibsen was very well-oriented about beauty in mathematics, which you certainly can find and enjoy. And I would even maintain that the beauty of many mathematical arguments can be easier to comprehend than many modern paintings. But a lot of mathematics is void of beauty. Maybe particularly in modern mathematics, where problem areas have often gotten extremely complex and complicated, with the result that the solution can only be formulated on several hundreds of pages. And that can scarcely be called beautiful. But in classical mathematics you find many striking theorems and arguments that hit you as something really original. It is reasonable to use the term beauty for those.

Mathematicians all over Scandinavia are proud of counting one of their own among the very first recipients of the Abel Prize. How would you characterize and evaluate Scandinavian, and particularly Swedish, mathematics in an international perspective?

I think that Scandinavia does quite well in this respect. In Sweden, we have a fine new generation of young mathematicians. And I think it looks very much alike in the other Scandinavian countries. It is difficult to perceive a new Abel on the horizon, but that is probably too much to hope for.

Could you please characterize the unique contribution that the Finnish/Swedish school of Lindelöf, M.Riesz, Carleman, R.Nevanlinna, Phragmen, Beurling and Ahlfors brought to analysis in the first half of the 20th century, which was formative and decisive for your own contribution to hard analysis?

In your list, you miss another Scandinavian mathematician: J. L. Jensen. The importance of "Jensen's inequality" can hardly be exaggerated. He and Lindelöf started the Scandinavian school, building of course on Riemann's approach to complex analysis rather than that of Cauchy-Weierstrass; Nevanlinna and Carleman continued, followed by Ahlfors and Beurling, a remarkable concentration of talent in Scandinavia. My lecture tomorrow will give more details.

Mathematical achievements in context

Abel first thought that he had solved the general quintic by radicals. Then he found a mistake and subsequently he proved that it was impossible to solve the quintic algebraically. The famous and notoriously difficult

problem about the pointwise convergence almost everywhere of L^2-functions, that Lusin formulated in 1913 and actually goes back to Fourier in 1807, was solved by you in the mid-1960s. We understand that the prehistory of that result was converse to that of Abel's, in the sense that you first tried to disprove it. Could you comment on that story?

Yes, of course. I met the problem already as a student when I bought Zygmund's book on trigonometric series. Then I had the opportunity to meet Zygmund. He was at Harvard in '50 or '51. I was at that time working on Blaschke products and I said maybe one could use those to produce a counterexample. Zygmund was very positive and said "of course, you should do that". I tried for some years and then I forgot about it before it again came back to me. Then, in the beginning of the '60s, I suddenly realized that I knew exactly why there had to be a counterexample and how one should construct one. Somehow, the trigonometric system is the type of system where it is easiest to provide counterexamples. Then I could prove that my approach was impossible. I found out that this idea would never work; I mean that it couldn't work. If there were a counterexample for the trigonometric system, it would be an exception to the rule.

Then I decided that maybe no one had really tried to prove the converse. From then on it only took two years or so. But it is an interesting example of 'to prove something hard, it is extremely important to be convinced of what is right and what is wrong'. You could never do it by alternating between the one and the other because the conviction somehow has to be there.

Could we move to another problem, the so-called Corona problem that you solved in 1962? In this connection, you introduced the so-called Carleson measure, which was used extensively by other mathematicians afterwards. Could you try to explain why the notion of the Carleson measure is such a fruitful and useful notion?

Well, I guess because it occurs in problems related to the general theory of BMO and H^1-spaces. I wish this class of measures had been given a more neutral name. In my original proof of the Corona problem, the measures were arc lengths on the special curves needed there. Beurling suggested that I should formulate the inequality for general measures. The proof was the same and quite awkward. Stein soon gave a natural and simple proof and only then the class deserved a special name.

I'll move to another one of your achievements. Hardy once said that mathematics is a young mans game. But you seem to be a counterexample; after you passed sixty years of age, you and Michael Benedicks managed to prove that the so called Hénon map has strange attractors exhibiting chaotic behaviour. The proof is extremely complicated. It's a tour de force that took many years to do. With this as a background, what is your comment on mathematical creativity and age?

I guess and hope that you don't get more stupid when you get older. But I think your stamina is less, your perseverance weakens (keeping lots of facts in your mind at the same time). Probably this has to do with the circulation of the blood or something like that. So I find it now much harder to concentrate for a long period. And if you really want to solve complicated problems, you have to keep many facts available at the same time.

Lennart Carleson receives the Abel Prize for 2006 from Queen Sonja. (Photo: Knut Falch/Scanpix, © Det Norske Videnskaps-Akademi/ Abelprisen)

Mathematical Problems

You seem to have focused exclusively on the most difficult and profound problems of mathematical analysis. As soon as you have solved any one of these, you leave the further exploration and elaboration to others, while you move on to other difficult and seemingly intractable problems. Is this a fair assessment of your mathematical career and of your mathematical driving urge?

Yes, I think so. Problem solving is my game, rather than to develop theories. Certainly the development of mathematical theories and systems is very important but it is of a very different character. I enjoy starting on something new, where the background is not so complicated. If you take the Hénon case, any schoolboy can understand the problem. The tools also are not really sophisticated in any way; we do not use a lot of theory.

The Fourier series problem of course used more machinery that you had to know. But that was somehow my background. In the circles of dynamical systems people, I always consider myself an amateur. I am not educated as an expert on dynamical systems.

Have there been mathematical problems in analysis that you have worked on seriously, but at which you

have not been able to succeed? Or are there any particular problems in analysis that you especially would have liked to solve?
Yes, definitely. There is one in dynamical systems, which is called the standard map. This is like the Hénon map but in the area preserving case. I spent several years working on it, collaborating with Spencer for example, but we never got anywhere.

If you want to survive as a mathematician, you have to know when to give up also. And I am sure that there have been many other cases also. But I haven't spent any time on the Riemann hypothesis… and it wouldn't have worked either.

Characterization of great mathematicians

What are the most important features, besides having a good intellectual capacity of course, that characterize a great mathematician?
I don't think they are the same for everybody. They are not well defined really. If you want to solve problems, as in my case, the most important property is to be very, very stubborn. And also to select problems which are within reach. That needs some kind of intuition, I believe, which is a little closer to what we talked about initially, about beauty. You must somehow have a feeling for mathematics: What is right, what is wrong and what is feasible. But, of course, there are many other mathematicians who create theories and they combine results into new buildings and keep other people working. It is a different kind of a mathematician. I don't think you should try to find a simple formula for people.

For several decades, you have worked hard on problems that were known to be exceptionally difficult. What drove you and what kept you going for years, with no success guaranteed? What drives a person to devote so much energy to an arcane subject that may only be appreciated by a handful of other mathematicians?
Yes, that's a big issue. Stubbornness is important; you don't want to give up. But as I said before, you have to know when to give up also. If you want to succeed you have to be very persistent. And I think it's a drive not to be beaten by stupid problems.

Your main research contribution has been within mathematical analysis. What about your interest in algebra and topology/geometry?
Geometry is of course very much part of the analysis. But I have no feeling for algebra or topology, I would say. I have never tried to... I should have learned more!

Mathematics of the future

What do you consider to be the most challenging and exciting area of mathematics that will be explored in the 21st century? Do you have any thoughts on the future development of mathematics?
Yes, of course I have had thoughts. Most of the influence comes from the outside. I think we are still lacking a good understanding of which kind of methods we should use in relation to computers and computer science. And also in relation to problems depending on a medium sized number of variables. We have the machinery for a small number of variables and we have probability for a large number of variables. But we don't even know which questions to ask, much less which methods to use, when we have ten variables or twenty variables.

This leads to the next question. What is the significance of computers in mathematics? Is it mainly checking experimentally certain conjectures? Or is it completing proofs by checking an enormous amount of special cases? What are your thoughts on computers in mathematics?
There are a few instances that I have been involved with. I had a student, Warwick Tucker, who proved that the Lorenz attractor exists. The proof was based on explicit computations of orbits. And in that case you could get away with a finite number of orbits. This is very different from the Hénon map, where you could never succeed in that way. You could never decide whether a parameter was good or bad. But for the Lorenz attractor he actually proved it for the specific values that Lorenz had prescribed. Because it is uniformly expanding, there is room for small changes in the parameter. So this is an example of an actual proof by computer.

Of course then you could insist on interval arithmetics. That's the fine part of the game so to say, in order to make it rigorous for the people who have very formal requirements.

But what about computers used, for instance, for the four colour problem, checking all these cases?
Probably unavoidable, but that's okay. I wouldn't like to do it myself. But it's the same with group structures, the classification of simple groups, I guess. We have to accept that.

The solution of the 350 year old Fermat conjecture, by Andrew Wiles in 1994, uses deep results from algebraic number theory. Do you think that this will be a trend in the future, that proofs of results which are simple to state will require a strong dose of theory and machinery?
I don't know. The striking part in the proof of the Fermat theorem is the connection between the number theory problem and the modular functions. And once you have been able to prove that, you have moved the problem away from what looked like an impossible question about integers, into an area where there exists machinery.

Career. Teachers.

Your CV shows that you started your university education already at the age of 17 and that you took your PhD at Uppsala University when you were 22 years old. Were you sort of a wunderkind?
No, I didn't feel like a wunderkind.

Can you elaborate about what aroused your mathematical interests? And when did you become aware

that you had an exceptional mathematical talent?
During high school I inherited some books on calculus from my sister. I read those but otherwise I didn't really study mathematics in any systematic way. When I went to university it was natural for me to start with mathematics. Then it just kept going somehow. But I was not born a mathematician.

You already told us about your PhD advisor, Arne Beurling, an exceptional Swedish mathematician, who is probably not as well known as he deserves. Could you characterize him as a person and as a researcher in a few sentences? Did he have a lasting influence on your own work?
Yes, definitely. He was the one who set me on track. We worked on the same type of problems but we had a different attitude towards mathematics. He was one of the few people about whom I would use the word genius. Mathematics was part of his personality somehow. He looked at mathematics as a piece of art. Ibsen would have profited from meeting him. He also considered his papers as pieces of art. They were not used for education and they were not used to guide future researches. But they were used as you would use a painting. He liked to hide how he found his ideas. If you would ask him how he found his result, he would say a wizard doesn't explain his tricks. So that was a rather unusual education. But of course I learned a lot from him. As you said, he has never been really recognized in a way which he deserves.

Apart from Arne Beurling, which other mathematicians have played an important part in your development as a mathematician?
I have learnt from many others, in particular from the people I collaborated with and in particular from Peter Jones. I feel a special debt to Michel Herman. His thesis, where he proved the global Arnold conjecture on diffeomorphisms of the circle, gave me a new aspect on analysis and was my introduction to dynamical systems.

You have concentrated your research efforts mainly on topics in hard analysis, with some spices from geometry and combinatorics. Is there a specific background for this choice of area?
I don't think so. There is a combinatorial part in all of the three problems we have discussed here. And all of them are based on stopping time arguments. You make some construction and then you stop the construction, and you start all over again.

This is what is called renormalization?
Yes, renormalization. That was something I didn't learn. Probability was not a part of the Uppsala school. And similarly for coverings, which is also part of the combinatorics.
Which mathematical area and what kind of mathematical problems are you currently the most interested in?
Well, I like to think about complexity. I would like to prove that it's harder to multiply than to add.

That seems to be notoriously difficult, I understand.

Lennart Carleson thanks the Norwegian people for the Abel prize. (Photo: Knut Falch/Scanpix, © Det Norske Videnskaps-Akademi/ Abelprisen)

Well, I am not so sure. It's too hard for me so far.

You have a reputation as a particularly skilful advisor and mentor for young mathematicians; 26 mathematicians were granted a PhD under your supervision. Do you have particular secrets on how to encourage, to advise and to educate young promising mathematicians?
The crucial point, I think, is to suggest an interesting topic for the thesis. This is quite hard since you have to be reasonably sure that the topic fits the student and that it leads to results. And you should do this without actually solving the problem! A good strategy is to have several layers of the problem. But then many students have their own ideas. I remember one student who wanted to work on orthogonal polynomials. I suggested that he could start by reading Szegö's book. "Oh, no!" he said, "I don't want to have any preconceived ideas".

Publishing mathematics

I would like to move to the organization of research. Let's start with the journal Acta Mathematica. It is a world famous journal founded by Gösta Mittag-Leffler back in 1882 in Stockholm as a one-man enterprise at that time. It rose very quickly to be one of the most important mathematical journals. You were its editor in chief for a long period of time. Is there a particular recipe for maintaining Acta as a top mathematical journal? Is very arduous refereeing most important?
It is the initial period that is crucial, when you build up a reputation so that people find it attractive to have

a paper published there. Then you have to be very serious in your refereeing and in your decisions. You have to reject a lot of papers. You have to accept being unpopular.

Scientific publication at large is about to undergo big changes. The number of scientific journals is exploding and many papers and research results are sometimes available on the internet many years before they are published in print. How will the organization of scientific publication develop in the future? Will printed journals survive? Will peer review survive as today for the next decades?

I've been predicting the death of the system of mathematical journals within ten years for at least 25 years. And it dies slowly, but it will only die in the form we know it today. If I can have a wish for the future, I would wish that we had, say, 100 journals or so in mathematics, which would be very selective in what they publish and which wouldn't accept anything that isn't really finalized, somehow. In the current situation, people tend to publish half-baked results in order to get better promotions or to get a raise in their salary.

The printing press was invented by Gutenberg 500 years ago in order to let information spread from one person to many others. But we have completely different systems today which are much more efficient than going through the printing process and we haven't really used that enough.

I think that refereeing is exaggerated. Let people publish wrong results and let other people criticize. As long as it's available on the net it won't be any great problem. Moreover, referees aren't very reliable; it doesn't really work anyway. I am predicting a great change, but it's extremely slow in coming. And in the meantime the printers make lots of money.

Research Institutions

I've just returned from a nice stay at the Institute Mittag-Leffler, which is situated in Djursholm, north of Stockholm; one of the leading research institutes of our times. This institute was, when you stepped in as its director in 1968, something that I would characterize as a sleeping beauty. But you turned it into something very much different, very active within a few years. By now around thirty mathematicians work together there at any given time but there is almost no permanent staff. What was the inspiration for the concept of the Institute Mittag-Leffler as it looks like today? And how was it possible to get the necessary funds for this institute? Finally, how would you judge the present activities of the institute?

To answer the last question first, I have to be satisfied with the way it worked out and the way it continues also. I just hope that it can stay on the same course.

In the '60s, there was a period when the Swedish government (and maybe also other governments) was willing to invest in science. There was a discussion about people moving to the United States. Hörmander had already moved and the question was whether I was going to move as well. In this situation, you could make a bargain with them. So we got some money, which was of course the important part. But there was a rather amusing connection with the Acta, which is not so well known. From Mittag-Leffler's days, there was almost no money in the funds of the academy for the Mittag-Leffler institute. But we were able to accumulate rather large sums of money by selling old volumes of the Acta. Mittag-Leffler had printed large stocks of the old Acta journals which he never sold at the time. They were stored in the basement of the institute. During the '50s and early '60s one could sell the complete set of volumes. I don't remember what a set could be sold for, maybe 1000 dollars or so. He had printed several hundred extra copies, and there were several hundred new universities. If you multiply these figures together you get a large amount of money. And that is still the foundation of the economy of the institute.

A bit later, you became the president of the International Mathematical Union, an organization that promotes international cooperation within mathematics. This happened during the cold war and I know that you were specifically concerned with integrating Chinese mathematics at the time. Could you share some of your memories from your presidency?

Well, I considered my main concern to be the relation to the Soviet Union. The Chinese question had only started. I went to China and talked to people in Taiwan, and to people in mainland China. But it didn't work out until the next presidential period and it simply ripened. The main issue was always whether there was to be a comma in a certain place, or not, in the statutes.

It was somehow much more serious with the Russians. You know, they threatened to withdraw from international cooperation altogether. The IMU committee and I considered that the relation between the West and the East was the most important issue of the International Mathematical Union. So that was exciting. Negotiations with Pontryagin and Vinogradov were kind of special.

Did these two express some anti-semitic views also?

No, not officially. Well they did, of course, in private conversation. I remember Vinogradov being very upset about a certain Fields Medal being given to somebody, probably Jewish, and he didn't like that. He said this is going to ruin the Fields Prize forever. Then I asked him if he knew who received the first Nobel Prize in literature. Do you? It was a French poet called Sully Prudhomme; and that was during a period when Tolstoy, Ibsen and Strindberg were available to get the prize. Well, the Nobel Prize survived.

Mathematics for our times

You wrote a book, "Matematik för vår tid" or "Mathematics for our times", which was published in Sweden in 1968. In that book, you took part in the debate on so-called New Mathematics, but you also described concrete mathematical problems and their solutions.

Abel laureates Peter Lax (2005), Lennart Carleson (2006), and Jean-Pierre Serre (2003) (Photo: Scanpix)

Among other things you talked about the separation between pure and applied mathematics. You described it as being harmful for mathematics and harmful for contact with other scientists. How do you see recent developments in this direction? What are the chances of cross-fertilization between mathematics on the one side and, say, physics, biology or computer science on the other side? Isn't computer science somehow presently drifting away from mathematics?

Yes, but I think we should blame ourselves; mathematics hasn't really produced what we should, i.e. enough new tools. I think this is, as we talked about before, really one of the challenges. We still have lots of input from physics, statistical physics, string theory, and I don't know what. I stand by my statement from the sixties.

But that book was written mostly as a way to encourage the teachers to stay with established values. That was during the Bourbaki and New Math period and mathematics was really going to pieces, I think. The teachers were very worried and they had very little backing. And that was somehow the main reason for the book.

If you compare the sixties with today, mathematics at a relatively elevated level is taught to many more people and other parts of the subject are emphasized. For example the use of computers is now at a much higher state then at that time, where it almost didn't exist. What are your main points of view concerning the curriculum of mathematics at, say, high school level and the early years of university? Are we at the right terms? Are we teaching in the right way?

No, I don't think so. Again, something predictable happens very slowly. How do you incorporate the fact that you can do many computations with these hand-computers into mathematics teaching?

But in the meantime, one has also expelled many things from the classroom which are related to the very basis of mathematics, for example proofs and definitions and logical thinking in general. I think it is dangerous to throw out all computational aspects; one needs to be able to do calculations in order to have any feeling for mathematics.

You have to find a new balance somehow. I don't think anybody has seriously gotten there. They talk a lot about didactics, but I've never understood that there is any progress here.

There is a very strong feeling in school, certainly, that mathematics is a God-given subject. That it is once and for all fixed. And of course that gets boring.

Public awareness

Let us move to public awareness of mathematics: It seems very hard to explain your own mathematics to the man on the street; we experience that right now. In general pure mathematicians have a hard time when they try to justify their business. Today there is an emphasis on immediate relevance and it's quite hard to explain what mathematicians do to the public, to people in politics, and even to our colleagues from other sciences. Do you have any particular hints on how mathematicians should convey what they are doing in a better way?

Well, we should at least work on it; it's important. But it is also very difficult. A comment which may sound kind of stupid is that physicists have been able to sell their terms much more effectively. I mean who knows what an electron is? And who knows what a quark is? But they have been able to sell these words. The first thing we should try to do is to sell the words so that people get used to the idea of a derivative, or an integral, or whatever.

As something mysterious and interesting, right?

Yes, it should be something mysterious and interesting. And that could be one step in that direction, because once you start to talk about something you have a feeling about what it is. But we haven't been able to really sell these terms. Which I think is too bad.

Thank you very much for this interview on behalf of the Norwegian, the Danish and the European Mathematical Societies!

ERCOM: Centro di Ricerca Matematica Ennio De Giorgi

Matematica nelle Scienze Naturali e Sociali

Centro di Ricerca Matematica Ennio De Giorgi

Institutional framework

The "Centro di Ricerca Matematica Ennio De Giorgi" was established at the end of 2001 as a centre of the Scuola Normale Superiore, with the character of an inter-university centre, by the three University institutions of Pisa: the University of Pisa, the Scuola di Studi Superiori Sant'Anna and the Scuola Normale Superiore.

The centre has a council, composed of five professors (two from the University, two from the Scuola Normale and one from the Scuola Sant'Anna) and a scientific committee, named jointly by the director of the Scuola Normale, the rector of the University and the director of the Scuola Sant'Anna. The scientific committee presently consists of Jean-Pierre Bourguignon (IHES, Bures-sur-Yvette), Luis Caffarelli (Univesity of Texas, Austin), Stefan Hildebrandt (University of Bonn), Alan Kirman (Groupment de Recherche en Economie Quantitative, Marseille), Richard Schoen (Stanford University) and Jean-Christophe Yoccoz (Collège de France, Paris). The centre has a director named by the director of the Scuola Normale in consultation with the rector of the University and the director of the Scuola Sant'Anna. From the outset the director has been Mariano Giaquinta.

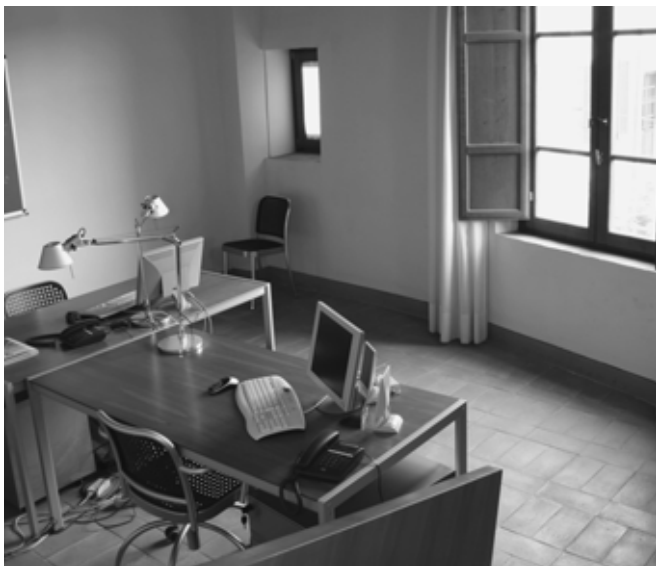
The centre is named after Ennio De Giorgi (1928–1996), who was one of the most prominent figures in the renaissance of mathematics in Pisa in the second half of the last century. Ennio De Giorgi was called to the Scuola Normale in 1959 and he remained in Pisa until his death on 25th October 1996. He is noted for having solved one of the problems formulated by David Hilbert in the early 1900s, for his contributions to the theory of minimal surfaces and for having developed in the 1970's the very innovative theory of 'Gamma convergence', which provides a setting to unify many phenomena in mathematics and physics.

Since 2003 the Centro De Giorgi has been part of ERCOM (European Research Centres on Mathematics), a committee consisting of directors of European mathematical research centres. It has also established agreements for academic collaboration with several prestigious institutions.

Aims of the Centre

The centre intends to:

- gather Italian and non-Italian researchers, both junior and senior, with the idea of fostering collaboration and the exchange of ideas and producing researchers specializing in interdisciplinary endeavours.
- organize intensive research programmes with a focus on research areas of particular relevance, including pure mathematics and applications in the natural and social sciences (e.g. physics, biology, finance and economics). These periods of specialization bring together Italian and non-Italian scientists and also provide an environment for the training of researchers from developing countries.
- promote new ideas and research at the frontiers of interdisciplinary studies.
- promote the formation of collaborative research nuclei from among those present, to advance particular areas of mathematics, their application to the natural and social sciences and their application to problems in the industrial and technological arenas.



Office at the research centre

Research activities

The centre operates by means of several programmes:
– Intensive research periods
– Workshops
– Schools
– Research in pairs
– Junior and senior visiting programmes
– Small research groups

Since its foundation, fourteen intensive research programmes (duration between two weeks and five months, consult the table below) and eighteen workshops and schools (duration between one and seven days) have been organized.

Classroom

Intensive research programmes 2002–2005

– Research Trimester on Dynamical Systems
– Financial Markets
– Interacting Particles and Computational Biology
– Topics in Complex and Real Geometry
– Probability and Statistical Mechanics in Information Science
– Summer School in Harmonic Analysis
– Intensive Research Period on Geometric Analysis
– Phase Space Analysis of Partial Differential Equations
– Harmonic Analysis
– Differential Geometry and Topology
– Two Weeks on Global Analysis
– Diophantine Geometry
– The Scientific Revolutions of the 16th and 17th Centuries
– Stochastic Analysis, Stochastic Partial Differential Equations and Applications to Fluid Dynamics and Particle Systems

Detailed information on these and other activities (scientific committees, invited speakers, lists of participants, reports etc.) can be found on the centre's web page http://www.crm.sns.it/ .

Not necessarily connected with the topics of the intensive research programmes, each year young visitors are invited to spend a period at the centre. Since 2004, several post-doctoral fellowships, called junior visiting positions, have been awarded by the centre to young researchers. Each year, during October and November, the centre announces two or more visiting positions (with deadlines for applications at the beginning of January), each one providing a salary of 25 000 Euro per year (plus a research grant of 5000 Euro). Since 2004, nine junior visitor positions have been awarded. Senior visitors are usually invited for shorter periods; in the last three years more than 22 outstanding mathematicians visited the centre as senior visitors.

At the end of 2005, the centre started a "Research in Pairs" programme, providing facilities such as office space, computers, library access, lodging and meals. Apparently the programme has been very welcome and we have had to reject many good applications.

The plan of forthcoming activities for this year includes an intensive research programme on "Calculus of Variations and Partial Differential Equations", to be held from September through till December 2006. The scientific committee is composed of Luigi Ambrosio (Scuola Normale Superiore), Sun-Yung (Alice) Chang (Princeton University), Gianni Dal Maso (SISSA), Lawrence Craig Evans (Berkeley University), Felix Otto (Universität Bonn) and Michael Struwe (Swiss Federal Institute of Technology).

There will also be the following school and workshop:
– "School on Neuromathematics of Vision" 4th–9th September 2006.
– "Four mini courses on fine properties of solutions of Partial Differential Equations" 11th–15th September 2006

The programme for 2007 includes an intensive research programme "Dynamical Systems and Number Theory" (16.4.–13.7.) and workshops on "Local holomorphic dynamics" (22.1.–26.1.), "La cultura in Italia: una, dua, nessuna" (5.3.–10.3.), "Quantum Information and Many-Body Quantum Systems" (26.3.–31.3.), "Many-body theory of inhomogeneous superfluids" (9.7.–29.7.), "Non-linear hyperbolic equations and related topics" (3.9.–8.9.), "Stochastic Networks and Internet Technology" (16.9.–22.9.), and "Structures of the mechanics of complex bodies" (1.10.–7.10.).

Research Groups:

Six small research groups operate at the Centre:
– Evolution of interfaces
– Measure theoretic aspects of partial differential equations
– Probability in information science
– Quantum information
– Harmonic analysis on Lie groups
– Phase space analysis of partial differential equations

Blackboards

Palazzo Puetano

At regular intervals members of each group meet at the centre and organize workshops and seminars.

More information can be found on the centre's web pages.

Cultural programs

The centre also contributes to the 'cultural mathematical formation' of teachers and layman, organizing a series of public lectures and seminars for teachers.

Publications

Proceedings of the scientific activities and monographs instigated by activities at the centre are published in the CRM-Series of the *Edizioni della Normale*.

Facilities and infrastructure

The Centro De Giorgi is located in the Palazzo Puteano, the former seat of the "Collegio Puteano", which was founded in 1604 by the Archbishop of Pisa. The building is part of the Piazza dei Cavalieri, which was designed by Giorgio Vasari in 1561 for Cosimo I de' Medici.

There are ten offices available for visitors with a total of sixteen workstations. Moreover, four public workstations and two printers are located in the common areas of the building.

Visitors can access the library of the Scuola Normale Superiore and various books and periodicals are available in the common areas of the centre.

To know more

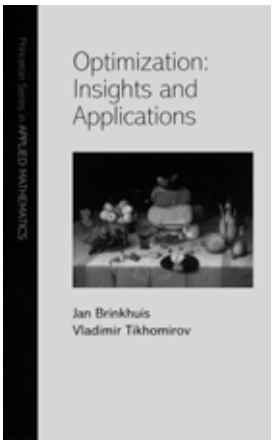
consult http://www.crm.sns.it/ .

Book review

Miguel A. Goberna (Alicante, Spain)

J. Brinkhuis and V. Tikhomirov
Optimization: Insights and Applications

Princeton Series in Applied Mathematics (2005)
658 pages, £ 51.95/hbk
Princeton University Press
ISBN 0-691-10287-0, ISBN 978-0-691-10287-2



This book provides an excellent overview of the theory, methods and applications of continuous optimization problems. The book deals with one-dimensional optimization (chapter 1), unconstrained optimization (chapter 2), optimization under equality constraints (chapter 3), optimization under inequality constraints (chapter 4), linear programming (chapter 6), convex optimization (chapter 7), mixed smooth-convex optimization (chapter 10), dynamic programming in discrete time (chapter 11), and dynamic optimization in continuous time (chapter 12). The following four-step procedure, conveniently adapted to each of the above families of continuous optimization problems, is proposed for computing an optimal solution of those problems that are analytically solvable (i.e. by a formula):

1. Establish the existence of global solutions.
2. Write down the first order necessary conditions.
3. Investigate these conditions.
4. Write down the conclusions.

The main tools in step 1 are the classical Weierstrass theorem (applied to some bounded non-empty sublevel set) and its extension to coercive functions. Step 2 is based upon the so-called Fermat-Lagrange principle (i.e. conditions of multiplier type) for finite-dimensional optimization problems and the Pontryagin principle for dynamic optimization problems. The particular versions of the Fermat-Lagrange principle for the different classes of mathematical programming problems considered in this book (all of them involving differentiable and/or convex functions, i.e. functions admitting linear approximations) are obtained via the tangent space theorem (closely related to the implicit function theorem) for smooth problems and by means of the supporting hyperplane theorem (related to the separation theorem) in the case of convex problems. Typically, step 3 provides a list of candidates and step 1 allows the identification of optimal solutions among them if the given problem is solvable. The second order conditions in chapter 5 give some insight although they do not play a crucial role in this approach because they can seldom be checked in practice. A similar point of view is adopted regarding the constraint qualifications.

Concerning those optimization problems that cannot be solved analytically but which admit a convex reformulation, the authors propose to solve them by means of interior point methods (such as the self-concordant barrier method) or cutting plane methods (such as the ellipsoid method). These and other complementary methods (e.g. line search methods and linear optimization methods) are described in chapters 6 and 7.

Each chapter contains a rich list of selected applications (many of them rare in the standard textbooks) that are presented as either motivating examples or as complementary exercises. Classical optimization problems (most of them relating to geometric objects), physical laws and economic models usually admit analytic solutions whereas real life (or pragmatic) applications may require the use of numerical methods. The solutions are given in appendix H. Chapters 8 and 9 are devoted to economic and mathematical applications, many of them surprising.

This description of the content of the book suggests that it is just one more of the many good textbooks that have been written on continuous optimization. Nevertheless, this book presents the following novel features:

1. The presentation of the material is as friendly, simple and suggestive as possible, including many illustrative examples and historical notes about results, methods and applications. These notes illustrate the important contributions of Russian mathematicians to optimization and related topics that have been ignored up to now in the occidental academic world.
2. The book is totally self-contained, although some familiarity with basic calculus and algebra could help the reader. The first explanation of each topic is intuitive, showing the geometric meaning of concepts (including those of basic calculus), results and numerical methods by means of suitable pictures. For instance, all the main ideas about one-dimensional optimization in the book are sketched in chapter 1 and this section could be read by college students. Subsection 1.5.1, entitled '*All methods of continuous optimization in a nutshell*', is an excellent introduction to this topic. In order to understand the basic proofs and to be able to obtain analytic solutions, the reader must recall some elements of linear algebra, real analysis and continuity (Weierstrass theorem) that can be found in appendices A, B and C respectively.
3. The basic part of the book is written in a personal way, emphasizing the underlying ideas that are usually hidden behind technical details. For instance, in the authors' opinion, the secret power of the Lagrange multiplier method consists in reversing the order of the two tasks of elimination and differentiation and not in the use of multipliers (as many experts think). In the same vein, personal anecdotes are also given, e.g. mention is made of some open problems (most of them extending classical optimization problems)

that they posed in their lectures and were brilliantly solved by their students.

4. The book provides new simple proofs of important results on optimization theory (such as the Pontryagin principle in appendix G) and also results on other mathematical fields that can be derived from optimization theory (such as the fundamental theorem of algebra in chapter 2). Most formal proofs are confined to the last two appendices, which are written in a fully analytic style.

This book can be used for different courses on continuous optimization, from introductory to advanced level, for any field for which optimization is relevant: mathematics, engineering, economics, physics, etc. The introduction of each chapter describes a "royal road" containing the essential tools for problem solving. Examples of possible courses based on materials contained in this textbook are:

Basic optimization course: chapters 1, 2, 3, 4, and appendix D (*Crash course on problem solving*).

Intermediate optimization course: chapters 5, 6, 7, 10 and appendices E and F.

Advanced-course on the applications of optimization: chapters 8 and 9.

Advanced-course on dynamic optimization: chapters 11, 12 and Appendix G.

The website of the first author (people.few.eur.nl/brinkhuis) contains references to implementations of optimization algorithms and a list of corrections for the few shortcomings that are unavoidable in a first edition, despite its careful production. In my opinion, this stimulating textbook will be for the teaching of optimization what Spivak's "Calculus" was for the teaching of that subject (and even real analysis) in the '70s.

Miguel A. Goberna [mgoberna@ua.es] received his PhD in 1979 from the Universidad de Valencia and he is currently a professor at the Universidad de Alicante (Spain). He has published more than sixty research papers on optimization and related topics (inequality systems, convexity and set-valued analysis) and several books, such as Linear Semi-Infinite Optimization, J. Wiley, 1998 (with M.A. López), Semi-Infinite Programming: Recent Advances, Kluwer, 2001 (with M.A. López, Eds.), and Linear Optimization, McGraw-Hill, 2004 (in Spanish, with V. Jornet and R. Puente).

Problem Corner: Mathematics Competitions in Spain

Marco Castrillón López (Madrid, Spain)

The mathematical Olympic torch was first lit in Spain in 1963 when the Royal Spanish Mathematical Society organized the "I Olimpiada Matemática Española". Since then, this contest has been held annually and is generally divided into two rounds: a local one and a national one. Some years later, in 1983, the winners of this contest participated in the IMO (International Mathematical Olympiad) held in Paris, thus starting the regular participation of Spain in this major international competition. Since the beginning, the interest of the country in this kind of contest for young students has steadily grown. In fact, one could speculate about the link between the healthy expansion of mathematical scientific activity in Spain over the last two decades and the improving mathematical problem solving activities for youngsters organized all around the country.

Spanish teenagers now have an interesting variety of contests alongside the Spanish Mathematical Olympiad. For example, a national contest for 14-year-old students has been organized every year since 1985 by the Spanish Federation of Mathematics Teachers. Some cities and regions (Castilla-León, Cataluña, Baleares) participate in the Kangaroo network. A similar contest ("Concurso de Primavera") is organized in Madrid. Moreover, we have the "Puig Adam" contest, named after a famous Spanish mathematician of the 20th century. It is worth pointing out the number of participants of these events. For instance, the "Concurso de Primavera" has almost 21,000 participants in the first round (the in-school round) and 2,000 participants in the second round, taking place in the Faculty of Mathematics of the Universidad Complutense de Madrid. The main goal of the organizers is always to foster the taste for mathematical problems in our society as well as to offer to students the possibility of competing, testing and enjoying mathematical skills.

Some years ago, in 1988, Prof Miguel de Guzmán founded a project to detect and nurture mathematical abilities of gifted young students. Nowadays, the project is well established in many cities of Spain. Even though the goal of this project is not the preparation for mathematical contests, there is an interesting link; the students taking part in the project enjoy participating in math contests. In fact, the basic tools of the fostering programme of Miguel de Guzman are obviously aimed at problem solving. Thus, a contest represents an ideal place where all these techniques can be put into practice. Hence, the students participate and perform remarkably well. This is because teenagers do actually like maths problems if they can find the right motivation. This is, perhaps, one of the weak points to be improved by mathematicians in current society.

Within the international network of maths competitions other than the IMO, Spain is enrolled in the Iberoamerican Mathematical Olympiad. The participants of this Olympiad are students from Latin-American countries as well as from Portugal and Spain. The first of these competitions was organized in Colombia in 1985 and Spain has subsequently been the host of two of them. We have included some of the problems from the second competition held in Spain below. Although the participant countries are culturally closed, it is fascinating how maths problems reveal themselves as a common ground among nations and how the participants build cultural bridges, souvenirs which often last for life. Another international event is the Mediterranean Olympiad. This is a competition in which each country organizes the contest for its participants and sends the results to a central headquarters. A similar system is used in the "Olimpiada de Mayo". Furthermore, there are some international competitions for university students but in this case the participation relies basically on the sole interest of each student and there is not a concrete supporting organization.

The maturity of the Spanish participation in international competitions has now materialized with the organization, in the city of Granada, of IMO 2008. The Royal Spanish Mathematical Society has enthusiastically accepted this challenge that, we hope, will fulfil our expectations and the quality of the previous IMOs.

In summary, the increasing offer of competitions seems to be a positive symptom of today's new mathematical generation. We are not speaking about the quality of the education system, a complicated issue that would fill many more pages than this article. We speak instead of showing teenagers the powerful attraction of mathematics and the excitement of problem-solving. There is, however, still much to be done.

Finally, a problem corner cannot finish without some maths problems. We wish you a pleasant time with them.

XIX Olimpiada Iberomaericana de Matemáticas Castellón, Spain, 2004.

Problem 1

We colour some cells of a 1001×1001 board according to the following rules:
1) Whenever two cells share a common edge, at least one of them must be coloured.
2) Every six consecutive cells (in a column or a row) must have at least two neighbouring cells coloured.
Find the minimum numbers of cells to be coloured.

Problem 2

Let O be the centre of a circle of radius r and let A be an exterior point. Let M be any point on the circle and N be diametrically opposite to M. Find the locus of the centres of the circles passing through A, M and N as M moves along the circle.

Problem 3

For n and k positive integers such that n is odd or n and k are even, prove that there exist integers a and b such that $\gcd(a, n) = \gcd(b, n) = 1$ and $k = a+b$.

Problem 4

Find all couples (a,b) of 2-digit positive integers such that $100a + b$ and $201a + b$ are 4-digit perfect squares.

Problem 5

Given a scalene triangle ABC, let A' be the intersection point of the angle bisector of A with the segment BC. B' and C' are defined similarly.
We consider the following points: A'' is the intersection of BC and the perpendicular bisector of AA'; B'' is the intersection of AC and the perpendicular bisector of BB'; and C'' is the intersection of AB and the perpendicular bisector of CC'. Prove that A'', B'' and C'' lie on a line.

Problem 6

For a set H of points in the plane, we say that a point P is a *cut point* of H if there are four different points A, B, C and D belonging to H such that lines AB and CD are different and meet at P. Given a finite set A_0, we consider the sequence of sets $\{A_1, A_2, A_3, \ldots\}$; A_{j+1} being the union A_j with the set of its cut points. If the union of all the sets of the sequence is finite, prove that $A_j = A_1$ for all j.

To know more

Royal Spanish Mathematical Society:
http://www.rsme.es
Ibero-American Organization:
http://www.campus-oei.org/oim/
Spanish Federation of Mathematics Teachers:
http://www.fespm.org/
Miguel de Guzman's project:
http://www.estalmat.org

Marco Castrillón López [mcastri@mat.ucm.es] received his PhD from the Universidad Complutense de Madrid in 1999. After a period as assistant at École Polythechnique Fédérale de Lausanne (Switzerland) he is now Associate Professor at the Department of Geometry and Topology of Universidad Complutense de Madrid. He is a member of the Olympic Committee of the Royal Spanish Mathematical Society and has been the tutor of the Spanish Olympic teams under several international competitions.

Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses vberinde@ubm.ro or vasile_berinde@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

September 2006

1–4: Topics in Mathematical Analysis and Graph Theory, Belgrade, Serbia
Information: pefmath@etf.bg.ac.yu; http://magt.etf.bg.ac.yu

1–4: Conference on Mathematical Neuroscience, Universitat d'Andorra, Principat d'Andorra
Information: CMathNeuroscience@crm.es; http://www.crm.es/CMathNeuroscience

1–4: SCRA 2006-FIM XIII. Thirteenth International Conference of the Forum for Interdisciplinary Mathematics on Interdisciplinary Mathematical and Statistical Techniques, Lisbon-Tomar, Portugal
Information: cmac@fct.unl.pt ; mishra@jaguar1.usouthal.edu; http://scra2006.southalabama.edu/

1–5: International Summer School and Workshop of Operator Algebras, Operator Theory and Applications 2006, Lisbon, Portugal
Information: woat@math.ist.utl.pt; http://woat2006.ist.utl.pt/

1–December 15: Calculus of Variations and Partial Differential Equations, Centro di Ricerca Matematica Ennio De Giorgi, Pisa, Italy
Information: crm@crm.sns.it; http://www.crm.sns.it/cgi-bin/pagina.pl?Tipo=elencoeventi&&TipoEvento=programs

2–5: 37th Annual Iranian Mathematics Conference, Azarbaijan University of Tarbiat Moallem, Tabriz, Iran
Information: aimc37@azaruniv.edu; http://www.azaruniv.edu

3–9: Summer School on General Algebra and Ordered Sets, Radejov, Czech Republic
Information: ssaos@math.muni.cz; http://www.math.muni.cz/~ssaos/

4–6: Optimal Discrete Structures and Algorithms (ODSA 2006), Rostock, Germany
Information: odsa@uni-rostock.de; http://www.math.uni-rostock.de/odsa/

4–8: Groups in Geometry and Topology Málaga 2006. The First Group Action Forum Conference, Málaga, Spain
Information: http://agt.cie.uma.es/~ggt06/

4–8: Barcelona Analysis Conference (BAC06), Barcelona, Spain
Information: bac06@imub.ub.es; http://www.imub.ub.es/bac06/

4–8: International Conference on Arithmetic Algebraic Geometry, El Escorial (Madrid), Spain
Information: aag2006@uam.es; http://www.uam.es/otros/aag2006/

4–8: International Seminar on Applied Geometry in Andalusia, Granada, Spain
Information: isaga06@ugr.es; http://gigda.ugr.es/isaga06/

4–8: Derived Algebraic Geometry Workshop, Oslo, Norway
Information: http://www.math.uio.no/~rognes/suprema/dag-workshop.html

4–8: Stochastic Analysis in Mathematical Physics, Grupo de Física- Matemática, Complexo Interdisciplinar da Universidade de Lisboa, Lisboa, Portugal
Information: abcruz@math.ist.utl.pt; http://gfm.cii.fc.ul.pt/events-en/samp2006/

4–9: International Conference on Applied Analysis and Differential Equations, Iasi, Romania
Information: icaade@uaic.ro; http://www.math.uaic.ro/~icaade

4–9: From a Microscopic to a Macroscopic Description of Complex Systems. CIME school and workshop, Będlewo, Poland
Information: http://web.math.unifi.it/users/cime/Courses/2006/03.php

4–29: The Painleve Equations and Monodromy Problems, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK
Information: s.wilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/PEM/

5–8: CDDE 2006, Colloquium on Differential and Difference Equations, Brno, Czech Republic
Information: cdde@math.muni.cz; http://www.math.muni.cz/~cdde/2006

5–8: Uncertainty: Reasoning about Probability and Vagueness, Prague, Czech Republic
Information: http://www.flu.cas.cz/Logica/Aconf/col2006.html

6–8: The Second International Workshop on Analysis and Numerical Approximation of Singular Problems, Karlovassi, Samos, Greece
Information: iwanasp06@aegean.gr; http://www.tech.port.ac.uk/staffweb/makrogla/conf/IWANASP06/samos06.html

8–10: International Conference on Modules and Comodules. Dedicated to Robert Wisbauer, Porto, Portugal
Information: http://www.fc.up.pt/mp/clomp/ModulesAndComodules/

8–12: 1st Dolomites Workshop on Constructive Approximation and Applications. Dedicated to Walter Gautschi for his 50 years of professional activity, Alba di Canazei (Trento), Italy
Information: dwcaa06@sci.univr.it; http://www.sci.univr.it/~dwcaa06

10–17: Parabolic and Navier-Stokes Equations, Będlewo, Poland
Information: http://www.impan.gov.pl/~parabolic/

11–13: 21st British Topology Meeting, Powys, Wales, UK
Information: http://www-maths.swan.ac.uk/btm21/

11–16: XV Fall Workshop on Geometry and Physics, Puerto de la Cruz (Tenerife, Canary Islands), Spain
Information: jcmarrer@ull.es;
http://www.gt.matfun.ull.es/15iwgp2006/index.htm

11–17: Quantum Transport: Modelling, Analysis and Asymptotics. CIME school and workshop, Cetraro (Cosenza), Italy
Information: cime@math.unifi.it; http://web.math.unifi.it/users/cime/Courses/2006/03.php

12–17: International Conference on Differential Equations, Lviv, Ukraine
Information: http://www.franko.lviv.ua/faculty/mechmat/Departments/Conf/index.htm

14–16: Recent Advances in Free Boundary Problems and Related Topics, Levico, Italy
Information: pierluigi.colli@unipv.it; http://fbp2006.math.unifi.it/

15–19: International Conference of Numerical Analysis and Applied Mathematics 2006 (ICNAAM 2006), Hersonnisos, Crete, Greece
Information: icnaam@uop.gr; http://www.icnaam.org/

17–23: 11th International Conference on Functional Equations and Inequalities, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

18–20: Algebraic Curves in Cryptography (The 10th Workshop on Elliptic Curve Cryptography (ECC 2006)), Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/#ECC2006

19–21: Credit Risk under Lévy Models, Edinburgh, UK
Information: enquiries@icms.org.uk; http://www.icms.org.uk/meetings

21–24: Fifth International Conference on Applied Mathematics (ICAM5). In honour of Professor Ioan A. Rus on the occasion of his 70th birthday, Baia Mare, Romania
Information: marieta.gata@rdslink.ro; vberinde@ubm.ro;
http://www.ubm.ro/ro/icam5

22–29: Conference on Geometry and Dynamics of Groups and Spaces. **In Memory of Alexander Reznikov**, Bonn, Germany
Information: gdgs06@mpim-bonn.mpg.de; web-site:
http://www.mpim-bonn.mpg.de/Events/This+Year+and+ Prospect/Geometry+and+Dynamics+of+Groups+and+Spaces/

24–29: Harmonic Analysis and Orthogonal Expansions, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–29: SYNASC 2006 – 8th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, Timisoara, Romania
Information: synasc06@info.uvt.ro; http://synasc06.info.u vt.ro

29–October 5: Non-Commutative Harmonic Analysis with Applications to Non-Commutative Probability, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

October 2006

2–6: Quantum Cryptography and Computing, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca;
http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/quantum/

4–6: International Conference on Multifield Problems, University of Stuttgart, Germany
Information: mehrfeld@mathematik.uni-stuttgart.de;
http://www.icmp.uni-stuttgart.de

7–10: PDE Approaches to Image Processing, **a Workshop sponsored by the ESF Programme "Global and Geometric Aspects of Nonlinear Partial Differential Equations"**, Cologne, Germany
Information: http://www.mi.uni-koeln.de/~jhorak/workshop/

23–27: 9th International Conference "Intelligent Systems And Computer Sciences", Moscow, Russia
Information: intsys2006@intsys.msu.ru; http://intsys.msu.ru/en/science/conference/

23–December 15: Stochastic Computation in the Biological Sciences, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK
Information: s.wilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/SCB/

27–29: ICVL 2006 – 1st International Conference on Virtual Learning Virtual Environments for Education and Training-Software and management for education, Bucharest, Romania
Information: icvl@fmi.unibuc.ro; http://fmi.unibuc.ro/icvl/

27–November 1: International Conference of Computational Methods in Sciences and Engineering 2006 (ICCMSE 2006), Chania, Crete
Information: iccmse@uop.gr; http://www.iccmse.org/

30–November 3: Computational Challenges arising in Algorithmic Number Theory and Cryptography, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca;
http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/number_theory/

November 2006

1–3: Workshop on Global Integrability of Field Theories and Applications – GIFT 2006, Daresbury, United Kingdom
Information: http://iaks-www.ira.uka.de/iaks-calmet/gift2006/

27–December 1: Cryptography: Underlying Mathematics, **Probability and Foundations**, Fields Institute, Toronto, Canada
Information: programs@fields.utoronto.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/crypto_foundations/

December 2006

17–20: Seventh International Conference on Mathematics in Signal Processing, Cirencester, Gloucestershire, United Kingdom

Information: conferences@ima.org.uk; http://www.ima.org.uk/Conferences/7InternationalConfonMathsinSignalProcessing.htm

January 2007

8–June 29: Analysis on Graphs and its Applications, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/AGA/index.html

15–July 6: Highly Oscillatory Problems: Computation, Theory and Application, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk; http://www.newton.cam.ac.uk/programmes/HOP/index.html

22–26: Winter School "Geometric Measure Theory, Random Sets and Digital Stereology", Sandbjerg Estate, Sønderborg, Denmark
Information: oddbjorg@imf.au.dk; http://www.thiele.au.dk/winterschool07/

22–26: Local Holomorphic Dynamics, Centro di Ricerca Matematica Ennio de Giorgi, Pisa, Italy
Information: crm@crm.sns.it; http://www.mat.uniroma2.it/~tovena/

26–28: MEDCONF. 5th Mediterranean Conference on Mathematics Education, Rhodes Island, Greece
Information: makrides.greg@usa.net, www.cms.org.cy

March 2007

6–11: SEEMOUS (South-Eastern European Mathematical Olympiad for University Students), Cyprus
Information: makrides.greg@usa.net, cms@cms.org.cy

21–25: MAT-TRIAD 07, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–30: Structured Perturbations and Distance Problems in Matrix Computations, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–30: Workshop: Homotopy theory of schemes, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

27–April 4: MATHEU (Identification, Motivation and Support of Mathematical Talents in European Schools) Training Course (Comenius 2.2), Bulgaria
Information: makrides.greg@usa.net; www.matheu.org; http://ec.europa.eu/education/trainingdatabas/

April 2007

4–18: TAMTAM'07, Tendances dans les Applications Mathémati-ques en Tunisie, Algérie, Maroc; Algiers, Algeria
Information: http://tamtam07alger.ifrance.com/

10–14: Workshop on Control Theory & Finance, Lisbon, Portugal
Information: wmctf@iseg.utl.pt; http://srv-ceoc.mat.ua.pt/conf/wmctf2007/

30–May 6: Advances in Mathematics of Finance, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

May 2007

6–12: Semstat 2007, Statistics for Stochastic Differential Equations models, La Manga del Mar Menor, Cartagena, Spain
Information: mathieu.kessler@upct.es; http://www.dmae.upct.es/semstat2007

14–18: Workshop: Stacks in Geometry and Topology, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

20–26: Convex and Fractal Geometry, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

27–June 1: Stochastic Networks and Related Topics, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

30–June 6: MATHEU (Identification, Motivation and Support of Mathematical Talents in European Schools) Training Course (Comenius 2.2), Cyprus
Information: makrides.greg@usa.net; www.matheu.org; http://ec.europa.eu/education/trainingdatabas/

June 2007

1–30: Geometric Applications of Homotopy Theory, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/ index.html

3–10: Geometric Analysis and Nonlinear Partial Differential Equations, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

9–13: Workshop: Higher Categories and their Applications, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

12–16: Complex Function Theory and Geometry, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

18–24: Algebraic Topology: Old and New (M. M. Postnikov Memorial Conference), Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

24–30: Nonlocal and Abstract Parabolic Equations and their Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

24–30: Lyapunov Memorial Conference. International Conference on the occasion of the 150th birthday of Aleksandr Lyapunov, Kharkiv, Ukraine
Information: lmc07@ilt.kharkov.ua; http://www.ilt.kharkov.ua/lmc07/

25–26: Mathematical Modelling in Sport, Manchester, UK
Information: http://www.ima.org.uk/Conferences/conferences.htm

25–30: Topics in Geometric Group Theory, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

28–July 4: 6th Congress of Romanian Mathematicians, Bucharest, Romania
Information: congmatro@imar.ro; http://www.imar.ro/~purice/announcements.html

July 2007

1: Summer Conference on Topology and its Applications 2007, Castellón, Spain
Information: http://www.sumtop07.uji.es

1–7: Groups and Their Actions, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

4–8: International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA-2007), Cluj-Napoca, Romania
Information: nodeacj@math.ubbcluj.ro; http://www.math.ubbcluj.ro/~mserban/confan.html

9–11: MCP 2007 Vienna – 5th international conference on multiple comparison procedures, Vienna, Austria
Information: http://www.mcp-conference.org

9–13: The First European Set Theory Meeting, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

16–20: 6th International Congress on Industrial and Applied Mathematics (ICIAM 07), Zürich, Switzerland
Information: http://www.iciam07.ch

22–28: Topological Theory of Fixed and Periodic Points (TTFPP 2007), Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

23–December 21: Strong Fields, Integrability and Strings, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/SIS/index.html

August 2007

14–19: Workshops Loops, 07, Prague, Czech Republic
Information: loops07 AT karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/~loops07/workshops.html

19–26: XXIInd International Workshop on Differential Geometric Methods in Theoretical Mechanics, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

September 2007

2–8: Linear and Non-Linear Theory of Generalized Functions and its Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/
3–December 21: Phylogenetics, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/PLG/index.html

9–15: Measure Theory – Edward Marczewski Centennial Conference, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

July 2008

14–18 : Fifth European Congress of Mathematics (5ECM), Amsterdam, The Netherlands
Information: www.5ecm.nl

Recent books

edited by Ivan Netuka and Vladimir Souček (Prague)

Books submitted for review should be sent to the following address: Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic

***A. Adolphson et al., Eds.: Geometric Aspects of Dwork Theory, vol. I**, Walter de Gruyter, Berlin, 2004, 554 pp., EUR 328, ISBN 3-11-017478-2*
***A. Adolphson et al., Eds: Geometric Aspects of Dwork Theory, vol. II**, Walter de Gruyter, Berlin, 2004, 555-1129 pp., EUR 328, ISBN 3-11-017478-2*
These two volumes collect together lectures given at the intensive "Dwork Trimester in Italy", which was held from May to July 2001 at the University of Padova. As these facts indicate, the focus of the papers collected in the proceedings is on the many p-adic facets of Dwork's contributions to mathematics. The collection of papers comprises more than a thousand pages, the papers being written by leading experts in the field. The reader will find here not only overview and introductory papers but also research articles on some very attractive applications of p-adic techniques in algebraic geometry, the theory of modular form, differential or difference equations and in homology algebra. The range of topics covered by the papers is wide, including variations of p-adic cohomology of families of exponential sums, p-adic analogues of hypergeometric functions, the Dwork trace formula, the celebrated Grothendieck's conjecture on differential linear equations and N. Katz's reformulation. The proceedings will be useful for a wide range of readers interested in modern applications of p-adic methods. (spor)

***S. Alaca, K.S. Williams: Introductory Algebraic Number Theory**, Cambridge University Press, Cambridge, 2004, 446 pp., GBP 24,99, ISBN 0-521-54011-9, ISBN 0-521-83250-0*
This book provides a nice introduction to classical parts of algebraic number theory. The text follows a standard scheme of introductions and discusses unique factorization, Euclidean rings, Dedekind rings, integral bases, ideal factorization, Dirichlet's unit theorem, the class group, etc. Analytical techniques, such as L-series or the class number formula, are not covered. The attention paid to cubic extensions, which was motivated by applications in Diophantine equations, is a bit unusual. Another unusual but positive feature of the book is that besides stand-

ard supplements to each chapter (such as a list of exercises), the reader will find an annotated bibliography with suggestions for further reading. The text is written in a lively style and can be read without any prerequisites. Therefore the book is very suitable for graduate students starting mathematics courses or mathematicians interested in introductory reading in algebraic number theory. The book presents a welcome addition to the existing literature. (spor)

***J. Bergen, S. Catoiu, W. Chin, Eds.: Hopf Algebras**, A Dekker Series of Lecture Notes in Pure and Applied Mathematics, vol. 237, Marcel Dekker, New York,2004,262 pp., USD 185, ISBN 0-8247-5566-9*

This volume contains eighteen refereed papers that were presented at the International Conference on Hopf Algebras held at DePaul University, Chicago, Illinois in 2002. Among the results presented is a new proof of the Skolem-Noether Theorem, saying that for any simple algebra R that is finite dimensional over its centre F, all F-linear skew derivations and all automorphisms are inner. There is also a summary of results on the classification of Hopf algebras of dimension pq over an algebraically closed field of characteristics zero, where p and q are different odd prime numbers. Another paper deals with categories of infinitesimal Hopf modules and bimodules over an infinitesimal bialgebra. Further, we find an illustration of the realization of bialgebras using the Myhill-Nerode Theorem and Fliess' Theorem, a paper on integrals for bialgebras and almost commutative Hopf algebras, representations of two-parameter quantum groups and an analog of Schur-Weyl duality. (jtu)

***J.-P. Brasselet, T. Suwa, Eds.: Singularités Franco-Japonaises**, Séminaires & Congrès Collection SMF no. 10, Société Mathématique de France, Paris, 2005, 460 pp., EUR 57, ISBN 2-85629-166-X*

This volume represents the proceedings of the conference "Singularités Franco-Japonaises", which took place at CIRM (Marseille-Luminy) in September 2002 (it was already the second meeting of this type; the proceedings of the first meeting are also available, appearing in the series "Advanced Studies in the Pure Mathematics" of the Japanese Mathematical Society as volume 29, 2000). The whole conference was devoted to singularities of algebraic varieties and the volume consists of 26 articles on this topic. The articles are written by leading specialists in the field, who provide substantial contributions to our knowledge of the subject, and can serve as starting points for further investigations. They also present a very good overview of the state of contemporary research.

Naturally, we can hardly try to describe here the contents of 26 articles but the reader can use the abstracts (written in both English and French), which can be found at the beginning of the volume, as a quick orientation. A collection of articles of such a high quality is obviously indispensable for specialists. But I believe that even non-specialists with a good attitude towards algebraic geometry and singularity theory will find these articles interesting. Let me mention here as an illustration that I was attracted by the article "T. Suwa: Residues of Chern classes on singular varieties" and by the article "S. Yokura: Generalized Ginzburg-Chern classes". (jiva)

***K. Burns, M. Gidea: Differential Geometry and Topology – With a View to Dynamical Systems**, Studies in Advanced Mathematics, Chapman & Hall/CRC, Boca Raton, 2005, 389 pp., USD 89,95, ISBN 1-58488-253-0*

This book offers a nice introduction to major topics in differential geometry and differential topology and their applications in the theory of dynamical systems. It starts with a chapter on manifolds (including Sard's theorem), followed by a discussion of vector fields, the Lie derivative and Lie brackets, and discrete and smooth dynamical systems. The following chapters treat Riemannian manifolds, affine and Levi-Civita connections, geodesics, curvatures, Jacobi fields and conjugate points and the geodesic flow. The chapter on tensors and differential forms includes integration of differential forms, Stokes theorem and a discussion of de Rham and singular homology. Chapter 7 contains a description of the Brouwer degree, intersection numbers, Euler characteristics, and the Gauss-Bonnet theorem. Chapter 8 treats Morse theory and the final chapter discusses hyperbolic dynamical systems and geodesic flows. The book is nicely written and understandable, with many illustrations and intuitive comments. It is very suitable as an introduction to the field. (vs)

***G. Cain, G. H. Meyer: Separation of Variables for Partial Differential Equations**, Studies in Advanced Mathematics, Chapman & Hall/CRC, Boca Raton, 2005, 281 pp., USD 89,95, ISBN 1-58488-420-7*

The authors give the foundation of an eigenfunction approach (separation of variables) to the boundary value and initial boundary value problems for linear partial differential equations. The method is applied to three basic equations in the plane with non-trivial right-hand sides. In contrast to basic courses on partial differential equations where the method is usually explained very superficially, in this book it is done in full detail. Possibilities and advantages of the method are also considered in detail in this very carefully written text. Solved examples as well as many exercises of various degrees of difficulty accompany each chapter. In my opinion, the book is very inspiring and it offers a lot of themes for theses at the Bachelors and Masters level. (oj)

***L. Capogna, C. E. Kenig, L. Lanzani: Harmonic Measure – Geometric and Analytic Points of View**, University Lecture Series, vol. 35, American Mathematical Society, Providence, 2005, 155 pp., USD 35, ISBN 0-8218-2728-6*

This book is based on a series of lectures given by Carlos Kenig in March 2000 at the University of Arkansas. The Dirichlet problem for the Laplace equation can be solved (without any regularity restriction on the domain) by integration with respect to the harmonic measure of the domain. On good domains, the harmonic measure may have density with respect to the surface measure; this is the Poisson kernel. It requires hard analysis to relate regularity properties of the harmonic measure to the geometry of the domain.

The main goal of this book is to present recent deep results by Kenig and Toro. They completely settle two questions. The first one is when the harmonic measure is asymptotically locally doubling. This roughly means that the ratio $\mu(B(x,\sigma r))/\mu(B(x,r))$ tends to σ^n as r approaches zero. The answer is in terms of the flatness of the domain (vanishing Reichenberg

flatness). The second question is when the Poisson kernel exists and has VMO logarithm (VMO means the well known space of functions with vanishing mean oscillation). The answer is in terms of the flatness and asymptotic behaviour of the surface (perimeter) measure and the condition is called the vanishing chord arc. All main results are in general dimension. The presentation is mostly developed in two stages, first the scheme of the proof is outlined and then the exposition proceeds to complete details. This helps in understanding the idea of the proof. The results mentioned above mainly occur as natural end points of previously known results of Kellogg, Alt, Caffarelli, Dahlberg, Jerison, Kenig and others. These results, as well as two-dimensional results, are presented in the form of a survey. Some background is included to make the exposition more self-contained and accessible. (jama)

F.-X. Dehon: Cobordisme complexe des espaces profinis et foncteur T de Lannes*, Mémoires de la Société Mathématique de France, no. 98, Société Mathématique de France,, Paris, 2004, 138 pp., EUR 26, ISBN 2-85629-162-7*

This volume represents a continuation of the paper F.-X. Dehon, J. Lannes, Sur les espaces fonctionnels dont la source est le classifiant d'un groupe de Lie compact commutatif, Publ. Math. Inst. Hautes Études Sci. 89 (1999), 127–177, and also of the paper N. Kuhn, M. Winstead, On the torsion in the cohomology of certain mapping spaces, Topology 39 (1996), 875–881. Its aim is the following: let p be a prime, let π be a compact commutative Lie group, and let MU denote the spectrum representing the complex cobordism. We take a space X such that its cohomology with coefficients in the ring of p-adic integers has no torsion.

The authors consider the functional space consisting of continuous mappings whose source is the classifying space $B\pi$ of the group π and the target is the profinite p-completion of the space X. They prove that the continuous MU-cohomology completed at the prime p of this functional space can be identified with the image under a functor $T\mathrm{B}\pi$ of the p-completed MU-cohomology of the target space. The functor $T\mathrm{B}\pi$ is an analogy of the Lannes' functor T, where π is a cyclic group of order p. Let us mention that the authors work in the simplicial setting. This highly interesting text is intended primarily for specialists. In order to make the approach to the results easier, the authors attached an appendix with some explanations. (jiva)

J. J. Duistermaat, J. A. C. Kolk: Multidimensional Real Analysis*, 2 Volume Hardback Set, Cambridge Studies in Advanced Mathematics 86, 87, Cambridge University Press, Cambridge, 2004, 798 pp., GBP 75, ISBN 0-521-82930-5*

This two-volume set is devoted to a quite well known and many times presented topic – differential and integral calculus for functions of several real variables. There are a considerable number of textbooks explaining this subject at various levels of mathematical rigour. Nevertheless, these books bring a new and fresh point of view to an old theme. The first important feature is that these books do not follow the usual strict division into analysis, geometry and algebra; they combine ideas from these different branches (together with those coming from mathematical physics, Lie groups theory, number theory, probability, and special functions) whenever it is suitable. Another unusual feature is that half of the content is constructed of exercises (altogether more than 550) ranging from standard ones to those indicating relations to other fields of mathematics and to challenging ones (useful for accompanying seminars). A logical dependence inside the set of exercises is carefully indicated. It is assumed that the reader has a good understanding of differential and integral calculus with one real variable, otherwise everything is fully proved. The book offers a nice solution to the usual problem of how to present vector calculus and the Stokes type theorem in a basic course of analysis.

The first book is devoted to differential calculus in several variables. The first chapter studies topology of R^n and continuous maps between Euclidean spaces (distance, open, connected and compact sets, and continuous maps). The topics treated in the second chapter are differentiable maps between Euclidean spaces (approximation by linear maps, partial derivatives, critical points, the Taylor formula, and commutativity of a limit process with differentiation and integration). The inverse function theorem and the implicit function theorem are the main tools for a description of local behavior of differentiable maps, which are studied in chapter 3. All that having been covered, smooth submanifolds in R^n are defined in chapter 4 (the Morse lemma and a discussion of critical points of functions is included here) and their tangent spaces are studied in chapter 5 (including a study of curves in R^3, Gaussian curvature of a surface and Lie algebras of linear Lie groups).

The second book is devoted to integral calculus in R^n. It starts with Riemann integration in R^n (Fubini's theorem and change of variables, with several proofs) and continues with applications involving the Fourier transform and the basic limit theorems in integral calculus. Chapter 7 contains the theory of integration with respect to a density over a submanifold in R^n, the main point being the Gauss divergence theorem. The last chapter is devoted to differential forms and a general form of the Stokes theorem. As a whole, the monograph is an excellent addition to the existing literature. Its most valuable feature is that it preserves (and presents) natural relations to many other fields of mathematics, which are cut off in standard treatments. The carefully worked out and comprehensive set of exercises will be very useful for teachers in their lectures and seminars. These books can be very much recommended to any teacher of real analysis as well as to a general mathematical audience. (vs)

V. Ya. Eiderman, M. V. Samokhin, Eds.: Selected Topics in Complex Analysis – The S. Ya. Khavinson Memorial Volume, *Operator Theory: Advances and Applications, vol. 158, Birkhäuser, Basel, 2005, 222 pp., EUR 108, ISBN 3-7643-7145-5*

This book is dedicated to the memory of S. Ya. Khavinson. The introductory note is written by V. P. Havin. It recalls the main results of S. Ya. Khavinson, in particular results in linear programming and complex analysis (Khavinson's approach to extremal problems), extremal problems with supplementary restrictions (a Tchebyshev-like phenomenon), spaces of analytic functions in multiply connected domains (zero logarithmic capacity of removable sets and factorization problems), analytic capacity (Khavinson's measure and Cauchy potentials) and approximation problems. A complete bibliography and several photos are included. Moreover, the book contains twelve research and expository papers covering the following topics: extremal problems, approximation problems, the Cauchy integral, analytic ca-

pacity, Cantor sets, meromorphic functions, the Carathéodory inequality, and the integral representation of functions and the shift operator. The book is suitable for graduate students and researchers in analysis. (pp)

***P. Etingof, F. Latour: The Dynamical Yang-Baxter Equation, Representation Theory, and Quantum Integrable Systems**, Oxford Lecture Series in Mathematics and its Applications 29, Oxford University Press, Oxford, 2005, 137 pp., GBP 35, ISBN 0-19-853068-4*

This nice small book, based on a course given at MIT in 2001, is devoted to connections of quantum dynamical Yang-Baxter equations with certain integrable systems and representations of semisimple Lie algebras and of quantum groups. A condense summary of these interrelations can be found in the introductory chapter of the book. To make it more accessible, the authors review basic facts on finite-dimensional representations of semisimple Lie algebras and the generalization to the case of quantum groups. The main topics treated in the book are intertwining operators, fusion operators and exchange operators. It leads quickly to quantum dynamical Yang-Baxter equation and the ABRR equation. The transfer matrix construction shows how to create a quantum integrable system corresponding to an R-matrix. In particular, it leads to integrable systems of Macdonald-Ruijsenaars type. It is then possible to give an interpretation of certain Macdonald polynomials in terms of representation theory. The book ends with a description of the theory of the dynamical Weyl group for a semisimple Lie algebra. To read the book, it is helpful to understand the theory of finite-dimensional representations of semisimple Lie algebras (a short summary is included). The contents of the book is very nicely presented and the book offers a readable introduction to a very interesting, interdisciplinary field. (vs)

***I. Gohberg, P. Lancaster, L. Rodman: Indefinite Linear Algebra and Applications**, Birkhäuser, Basel, 2005, 357 pp., EUR 38, ISBN 3-7643-7349-0*

This book is dedicated to relatively recent results in linear algebra of spaces with indefinite inner product. Within this framework, it presents the theory of subspaces and orthogonalization and then goes on to the theory of matrices, perturbation and stability theory. The book also includes applications of the theory to a study of matrix polynomials with selfadjoint constant coefficients, to differential and difference equations with constant coefficients, and to algebraic Ricatti equations.

After the notation and conventions, the book starts with basic geometric ideas concerning spaces with an indefinite inner product, the main topics here being orthogonalization, classification of subspaces and orthogonal polynomials. Further sections are devoted to a study of the classification of linear transformations in indefinite inner product spaces. H-selfadjoint, H-unitary and H-normal transformations together with their canonical forms are of particular interest. Functional calculus is discussed in the next chapter, where special attention is paid to the logarithmic and exponential functions. One chapter is used for a detailed analysis of the structure of H-normal matrices in spaces with an indefinite inner product. Following this, perturbation and stability theories for H-selfadjoint and H-unitary matrices are studied. This topic is important in applications involving the stable boundedness of solutions of differential and difference equations. One section is devoted to applications involving differential equations of the first order, the other for equations of higher orders. The last chapter contains the theory of algebraic Ricatti equations. The appendix serves as a refresher of some parts of linear algebra and matrix theory used in the main body of the book.

Each chapter ends with a series of examples that illustrates the discussed topics. The book has the structure of a graduate text in which chapters on advanced linear algebra form the core. This, together with many significant applications and an accessible style, makes it useful for engineers, scientists and mathematicians alike. (jsp)

***R. E. Greene, S. G. Krantz: Function Theory of One Complex Variable**, third edition, Graduate Studies in Mathematics, vol. 40, American Mathematical Society, Providence, 2006, 504 pp., USD 79, ISBN 0-8218-3962-4*

A review of the second edition appeared in Issue 44 of this Newsletter (June 2002). The third edition follows the arrangement of topics of the second edition. However, a considerable number of proofs have been simplified or clarified, and many exercises have been revised and rearranged. In the preface, the authors say, "The mathematical roads that this new edition follows are the same as before, but we hope that the ride is considerably smoother". I share the opinion of the reviewer of the second edition that this book can be recommended as a text for a basic course (the first third of the text), as well as for an advanced course at graduate level. (in)

***L. Guillopé, D. Robert: Actes des journées mathématiques à la mémoire de Jean Leray**, Séminaires & Congrès, Société Mathématique de France, Paris, 2004, 207 pp., EUR 41, ISBN 2-85629-160-0*

This volume contains an introduction by Yves Meyer and ten contributions illustrating various aspects of the scientific work of Jean Leray. They stem from lectures given at the meeting organised by the Laboratory of Mathematics of the University of Nantes in June 2002 in order to celebrate the memory of Jean Leray. A very interesting introduction, "Jean Leray et la recherche de la vérité", collects keystones of Jean Leray's personal and scientific life accompanied by his memories, comments and opinions.

Research contributions include: S. Agmon, "On the asymptotics of Green functions of elliptic operators with constant coefficients"; D. Barlet, "Singularités réelles isolées et développements asymptotiques d'intégrales oscillantes"; P. Bolley, Pham The Lai, "Réduction au bord d'un problème modèle de Kelvin"; R. Camalès, "Problème de Cauchy ramifié pour une classe d'opérateurs dont les racines caractéristique sont en involution"; J. Y. Chemin, "Le système de Navier-Stokes incompressible soixante dix ans après Jean Leray"; Y. Choquet-Bruhat, "Asymptotic solutions of nonlinear wave equations and polarized null conditions"; M. Fontes, E. Saksman, "Optimal results for the two dimensional Navier-Stokes equations with lower regularity on the data"; J. L. Loday, "Scindement d'associativité et algèbres de Hopf"; P. Schapira, "Sheaves: from Leray to Grothendieck and Sato"; J. C. Sikorav, "Dual elliptic planes". The volume contains a short abstract in French and in English for each paper; five of the papers are written in English and five in French. (jsta)

***A. Jeffrey: Complex Analysis and Applications**, second edition, Chapman & Hall/CRC, Boca Raton, 2005, 579 pp., USD 79,95, ISBN 1-58488-553-X*

This book is a substantially rewritten edition of a textbook aimed at students of applied mathematics and engineering. In this version, applications of complex analysis are more concentrated and they cover some important two-dimensional boundary value problems. While the first four chapters are devoted to a detailed exposition of the classical material, culminating in the residue theorem and elementary conformal mapping, the last one deals with applications to fluid mechanics, temperature distribution and electrostatics. The reader is led step by step through the basics of complex function theory. The book contains numerous examples and exercises and selected exercises are provided with solutions. Practical use of the theory is accented; for example the Cauchy theorem forms an optional item. The text is accompanied with many pictures. The book is recommended for technical universities; the exposition is clear and intuitive without going into details of a theoretical nature. (jive)

***V. A. Kaimanovich, Ed.: Random Walks and Geometry**, Walter de Gruyter, Berlin, 2004, 532 pp., EUR 168, ISBN 3-11-017237-2*

This volume collects selected contributions from the workshop on random walks, which was held at the Schrödinger Institute in Vienna in 2001. Topics of this special semester included probability and random walks on groups and graphs, products of random matrices, potential theory, Brownian motion on manifolds, combinatorial and spectral properties of random walks on graphs and fractals. The volume concentrates on geometric applications like hyperbolic geometry, Lie groups, and geometric group theory. The book is divided into two parts, longer surveys and articles (pp. 1–295) followed by shorter research communications (pp. 296–532). Some additional keywords of the main contributions to this book are Lyapunov spectrum, stochastic Loewner equation, expander graphs, quantum chaos, cellular automata and symbolic dynamical systems. The volume is dedicated to the memory of Martine Babillot. (mzahr)

***A. W. Knapp: Basic Real Analysis and Advanced Real Analysis** (set), Cornerstones, Birkhäuser, Boston, 2005, 1176 pp., EUR 72, ISBN 0-8176-4407-5*

Both these books appear in the series called 'Cornerstones'. In the preface of the first book the author says, "This book and its companion Advanced Real Analysis systematically develop concepts and tools in real analysis that are vital to every mathematician, whether pure or applied, aspiring or established. The two books together contain what the young mathematician needs to know about real analysis in order to communicate well with colleagues in all branches of mathematics". The textbook has an encyclopedic character; it is carefully written and contains almost all one can imagine after reading the statement quoted above. There are some exceptions, e.g. complex analysis and surface integration are not included. After a short review of the basics of calculus of one real variable, the first book deals with metric spaces (ca 650 pp.), calculus of several variables and the theory of ordinary differential equations and their systems. This part is followed by an exposition of measure theory, Lebesgue integration and its differentiation on the real line and the Fourier transform in Euclidean space. After a treatment of L^p spaces, topological spaces and a chapter on integration in locally compact spaces, the first book ends with a chapter devoted to Banach and Hilbert spaces.

The second book starts with an introduction to boundary-value problems. Just a list of chapters does not give a complete picture; for example in a chapter on Euclidean Fourier analysis, one can find tempered distributions, Sobolev spaces, harmonic functions and H^p theory, the Calderon-Zygmund theorem with its applications, and multiple Fourier series. From the remaining chapters, we point out some topics: distributions, Fourier analysis and compact groups, and pseudodifferential operators on manifolds. The book (at about 460 pages) closes with a chapter on the foundations of probability, which includes the strong law of large numbers. Each chapter is followed by a number of problems (altogether 306 + 190), which often include further important results. All problems are provided with hints at the end of both books and the first book also contains ten appendices on facts that a reader might like to recall without searching through the literature. Well-balanced indexes, bibliographical notes, abstracts at the beginning of each chapter, schema of dependence among chapters and a guide for the reader make both books easier to handle, but they require not only reading but also active cooperation of the reader. Both books can be recommended for the mathematical libraries of universities. (jive)

***E. Kowalski: Un cours de théorie analytique des nombres**, Cours spécialisés 13, Société Mathématique de France, Paris, 2004, 232 pp., EUR 41, ISBN 2-85629-161-9*

This book consists of lecture notes given by the author at the University of Bordeaux in 2001/2002. It aims to give an introduction into analytic number theory with an emphasis on multiplicative structures. Its main theme is centered on the theorem of prime numbers in arithmetic progression. All standard notions and results about Dirichlet and Poincaré series, L-functions, Riemannian geometry and automorphic forms are reviewed in the text. The only exception is the fifth chapter on sieves in number theory, containing a recent result of Duke-Friedlander-Iwaniec on oscillating sieves. The lecture series is written in French. (pso)

***S. S. Kutateladze, Ed.: A. D. Alexandrov: Selected Works, Part II. Intrinsic Geometry of Convex Surfaces**, Chapman & Hall/CRC, Boca Raton, 2005, 426 pp., USD 129,95, ISBN 0-415-29802-4*

After Carl Friedrich Gauss, Aleksandr Danilovich Alexandrov is considered the second most important contributor to surface theory. He laid the foundation of non-smooth intrinsic geometry. The second volume of his selected works presents a thorough introduction to the intrinsic theory of convex surfaces and is comprehensible to non-specialists. The key notion here is that of an intrinsic metric; given a metric space (in the topological sense), its metric is called intrinsic if the distance of any two points equals the infimum of lengths of curves joining them. One of the main results of the book is the following "realizability" theorem; a metric space homeomorphic to a two-dimensional sphere is isometric to a convex surface (in R^3) if and only if the metric is intrinsic and has positive curvature (i.e. locally, the sum of interior angles of any geodesic triangle is at least π). Further, the Gauss-Bonnet formula is derived for non-smooth

surfaces of positive curvature; the curvature (measure) is defined via the spherical map. All notions used are very natural and have been used and developed in non-smooth geometry over the last few decades. (jrat)

***R. Lazarsfeld: Positivity in Algebraic Geometry I. Classical Setting – Line Bundles and Linear Series**, Springer, Berlin, 2004, 387 pp., EUR 34,95, ISBN 3-540-22528-5*
***R. Lazarsfeld: Positivity in Algebraic Geometry II. Positivity for Vector Bundles, and Multiplier Ideals**, Springer, Berlin, 2004, 385 pp., 20 fig., EUR 34,95, ISBN 3-540-22531-5*
The main theme of this two volume monograph is a comprehensive description of the fields of complex algebraic geometry connected with the notion of positivity. The first volume in the set offers a systematic presentation of ideas connected with classical notions of linear series and ample divisors on a projective variety from a modern point of view. Individual chapters are devoted to the basic theory of positivity for line bundles and Castelnuovo-Mumford regularity, asymptotic geometry of linear systems, geometric properties of projective subvarieties of small codimension, vanishing theorems for divisors and the theory of local positivity.

The second volume contains the second and the third parts of the monograph. The main theme of the second part is positivity for vector bundles of higher ranks. It contains a systematic exposition of the theory (formal properties of ample and nef bundles, higher rank generalizations of the Lefschetz hyperplane theorem and the Kodaira vanishing theorem and numerical properties of ample bundles). The third part describes ideas and methods connected with multiplier ideals (Kawamta-Viehweg vanishing theorems for **Q**-divisors, multiplier ideals and their applications, and asymptotic multiplier ideals). The book is written for mathematicians interested in the modern development of algebraic geometry. A knowledge of basic notions is assumed (a standard introductory courses is sufficient). (vs)

***J. Lefort: L'aventure cartographique**, Éditions Belin, Paris, 2004, 319 pp., EUR 34, ISBN 2-84245-069-8*
This monograph gives an overview of the development of cartography from antiquity through to present day. The title describes the content quite well; it is an exciting piece on the search for methods to represent reality on a map, on an attempt to make measurements more precise and to invent instruments to make this possible, on triangulation of near inaccessible areas of mountains, on discoveries of far-away countries and continents, on determining the shape of the Earth, etc. The book is complemented with a large number of illustrations and also by two appendices, the first one on the movement of the Earth and on measures and units, the second one on various projections of the sphere onto the plane that are used in cartography. The mathematical explanations are quite precise for a popular work. In a minimal number of cases they are not very comprehensive or they contain a mistake (for example the explanation of the principle of sextant on page 106 contains several misprints and mistakes, which make it less understandable). (okal)

***D. Lehmann: Mathematical Methods of Many-Body Quantum Field Theory**, Chapman & Hall/CRC Research Notes in Mathematics, vol. 436, Chapman & Hall/CRC Press, Boca Raton, 2004, 253 pp., USD 89,95, ISBN 1-58488-490-8*
This book originated from a series of lecture notes (prepared for courses given to students of mathematical physics at TU Berlin) and the Habilitationsschrift by the same author.

It develops the basic mathematical tools needed for a description of quantum many-body systems: the method of second quantization (rewriting of the many-body Hamiltonians for electron systems in terms of annihilation and creation operators), perturbation theory (expansion of the exponential of the perturbed Hamiltonian into the corresponding Wick series), and the concept of "Grassmann integration" of anticommuting variables and Gaussian measures (with respect to bosonic or Grassmann, i.e. commuting or anticommuting, variables, therefore giving a unified approach, with the same combinatorics, to determinants, permanents, Pfaffians and unsigned sums over pairings, all these objects appearing as expectations of suitable monomials).

Then some more physical themes are treated in the rest of the book: the bosonic functional integral representation (obtained from the Hubbard-Stratonovich transformation of the Grassmann integral representation), the BCS theory of superconductivity, an introduction to the fractional quantum Hall effect, Feynman diagrams, renormalization group methods, and re-summation of Feynman diagrams. The book concludes with a list of the author's favourite unsolved "millenium" problems. The presentation is mathematically rigorous, where possible. The author's aim was to create a book containing enough motivation and enough mathematical details for those interested in this advanced and important field of contemporary mathematical physics. (mzahr)

***E. Lesigne: Heads or Tails – An Introduction to Limit Theorems in Probability**, Student Mathematical Library, vol. 28, American Mathematical Society, Providence, 2005, 150 pp., USD 29, ISBN 0-8218-3714-1*
This book is an excellent guide to the most important limit theorems in probability for anyone who is able to follow a basic calculus course. The author uses as his "invitation to probability" a simple model of coin tossing to show the beauty and power of limit theorems in probability. There is no need to have any knowledge about measure or probability theory before reading the book. Only elementary calculus is used for proofs, making the book accessible to anyone interested in the essentials of the probability asymptotic.

The text is divided into 14 chapters. Chapters 1 to 4 contain preliminary definitions and results of discrete probability related to random variables, independence and the binomial distribution. Each of the next chapters contains one well known limit theorem stated for the sequence of independent random variables with the relevant distribution. Besides the weak law of large numbers and the central limit theorem (results that are widely known and used), more advanced theory is represented by the large and moderate deviations, the law of the iterated logarithm and the arcsine law. The results are illustrated with examples and applications in statistics are discussed to show both the utility and beauty of probability theory. The book concludes with the recurrence of random walks and the last chapter is an overview of the generalization of the results to more general random variables. The book can be recommended for students of any area of science or engineering who want to learn about the limit laws of probability. It can be also recom-

mended to teachers as an inspiration for undergraduate introductory courses of probability or for researchers who want to have a reference book showing the strength and wide range of limit theorems in probability. (dh)

***M. Loday-Richaud (Ed.): Analyse complexe, systèmes dynamiques, sommabilité des séries divergentes et théories galoisiennes I**, Astérisque 296, Société Mathématique de France, Paris, 2004, 269 pp., EUR 57, ISBN 2-85629-167-8*
***M. Loday-Richaud (Ed.): Analyse complexe, systèmes dynamiques, sommabilité des séries divergentes et théories galoisiennes II**, Astérisque 297, Société Mathématique de France, Paris, 2004, 231 pp., EUR 57, ISBN 2-85629-168-6*
These two volumes cover the conference, 'Complex Analysis, Dynamical Systems, Summability of Divergent Series and Galois Theories', which was held in Toulouse (22th–26th Sept, 2003) on the occasion of J.-P. Ramis' 60th birthday. For this reason many contributions pay tribute to the impact of J.-P. Ramis' work. There are surveys on his contribution to the theories of linear and nonlinear differential and functional equations and also papers devoted to further prospects of his results. The spectrum is broad and ranges from functional and differential equations and partial differential equations, through p-adic analysis to summation processes and various algebraic constructions connected to these theories. The proceedings present a very interesting collection of surveys on the versatility of ideas used in this important part of mathematical analysis and mathematics as a whole. The collection can be recommended to those actively working in the fields related to those mentioned in the title of the proceedings. (spor)

***S. Macias: Topics on Continua**, Chapman & Hall/CRC, Boca Raton, 2005, 361 pp., USD 89,95, ISBN 0-8493-3738-0*
This book is an important complement to the existing literature on continuum theory. It covers four topics: inverse limits, the Jones set function T, homogenous continua, and n-fold hyperspaces. The book starts with some topology background material (assuming a one year course on general topology). It covers continuous decompositions, the fundamental group and hyperspaces. Chapter 2 is devoted to inverse limits on continua proving commutativity with finite products, cones and hyperspaces. In chapter 3, the Jones set function T is introduced with relations to connectedness "im Kleinem", i.e. local connectedness. Some applications are presented as well. Chapters 4 and 5 cover homogenous continua. Decomposition theorems are proved using the Jones set function T and the Effros theorem. Some nontrivial examples of homogeneous continua are presented. Chapter 6 is devoted to general properties of n-fold hyperspaces. In chapter 7, the book provides a list of open questions on each topic studied. The book is very well written and will be a useful tool for the continuum theory community and an excellent source of research topics. (pp)

***M. Marcolli: Arithmetic Noncommutative Geometry**, University Lecture Series, vol. 36, American Mathematical Society, Providence, 2005, 136 pp., USD 29, ISBN 0-8218-3833-4*
The main focus of these lecture notes is an interpretation of the arithmetic of modular curves and archimedean fibers of arithmetic varieties in the realm of non-commutative geometry. The first chapter reviews the basic notions of non-commutative geometry. The second chapter discusses properties of modular curves from the point of view of a description of their boundary strata in the framework of non-commutative geometry. The third chapter includes a discussion of the non-commutative space of commensurability classes of Q-lattices and its relation to class field theory. The fourth and last chapter involves the non-commutative geometry of the fibers at arithmetic infinity of varieties (surfaces) over number fields. The book aims to introduce and link together a variety of general concepts from geometry, number theory and physics. Many proofs have not been included in the text.(pso)

***M. Marino: Chern-Simons Theory, Matrix Models, and Topological Strings,** International Series of Monographs on Physics, 131, Oxford University Press, Oxford, 2005, 197 pp., GBP 49,50, ISBN 0-19-856849-5*
The main topics of this book are correspondences (dualities) between topological string theories and gauge theories. The first prototypes in this direction are the Gopakumar-Vafa duality between Chern-Simons theory and topological string theory on a special Calabi-Yau manifold called resolved conifold and the Dijkgraaf-Vafa duality between topological string theory and matrix model theory. The first part of the book is devoted to an introduction into gauge theories and their 1/N-expansions. The second part concentrates on topological string theories with an emphasis on non-compact Calabi-Yau manifolds. The third part presents relations between gauge theories and topological string theories as discussed in the first and the second part of the text. The book grew out of graduate level courses given by the author in 2003/2004. (pso)

***G.-V. Nguyen-Chu: Intégrales orbitales unipotentes stables et leurs transformées de Satake**, Mémoires de la Société Mathématique de France, Société Mathématique de France, Paris, 2004, 110 pp., EUR 26, ISBN 2-85629-157-0*
Topics treated in this book belong to harmonic analysis on classical groups defined over a p-adic field. Let $\mathbf{G}$ be a classical Lie group and let $G = \mathbf{G}(F)$ be its set of F-rational points. The book covers the space of stable unipotent distributions and their Satake transformations in the case when F is a p-adic field. This space is well understood in the case of $\mathbf{GL}(n)$. The author concentrates on the case of the group $\mathbf{Sp}(2n)$ and he studies relations of these spaces to Satake transformations of certain characters of representations of $\mathbf{GL}(2n+1)$. Using explicit calculations for $n = 2$, he is able to formulate precise conjectures for the case of general dimension n. The book will be of interest for mathematicians working in the field. (vs)

***C. Niculescu, L.-E. Persson: Convex Functions and Their Applications – A Contemporary Approach**, CMS Books in Mathematics, Springer, Berlin, 2006, 255 pp., EUR 59,95, ISBN 0-387-24300-3*
This is a nice little book, providing a new look at the old subject of convexity and treating it from different points of view. As the authors suggest, parts of it can be used for a course on convexity for first year graduate students. The book consists of four chapters, the last of which is devoted to Choquet theory. Appendices present a necessary background on the separation of convex sets in LC spaces, the theory of semi-algebraic sets, applications to elliptic boundary value problems and the Horn

conjecture concerning Weyl's problem on spectra of the sum of Hermitean matrices. While elementary parts of the book can be presented even in a basic course of analysis, the book goes rather deeply into the subject. A unified approach based on an application of means of different types is combined with many classical and recent results to show to the reader convexity as one of central notions of mathematics. The book well documents sources of ideas and the origins of results and will be of interest even for specialists in the field; it may also be used as a reference book on the subject. (jive)

***D. Pei: Authentication Codes and Combinatorial Designs**, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2006, 244 pp., USD 89,95, ISBN 1-58488-473-8*

This book is concerned with two authentication schemes. In both of them there is a sender, a receiver and an opponent. The scheme consists of a set of source states, a set of messages and a set of keys. The goal is to transmit a source state by means of sending a message. Each key is a one-to-one mapping from a subset of the message set into the set of source states. The sender and receiver agree on some key and the opponent tries to mislead the receiver with a fraudulent message. If the sender and the receiver are supposed to trust each other, one speaks about an authentication scheme with three participants. The scheme with four participants involves an arbiter between the sender and receiver in the case when they are not regarded as trustworthy. The main line of the book consists of considering the designs that appear when a key is associated with a set of admissible messages (i.e. with the range of a mapping with which we identify a given key).

In the beginning the author uses probability and entropy computations to show that the perfect 3-party authentication schemes with uniform probability distributions correspond to strong partially balanced t-designs. The definition of a partially balanced design differs from that of a t-design with parameters (v, b, k, λ) by allowing the existence of k-sets that do not extend to any t-set. Such a design is called strong when it is also a partial balanced r-design for every r smaller than t. The conditions for perfect 4-party authentications are of a similar type but much more technically complicated. Much of the content of the book is a survey of various designs that can be used for the studied authentication schemes. A lot of material is taken from other sources and the exposition is very careful, often in an elementary manner. Even some well known topics are covered in detail, like Steiner triples or mutually orthogonal Latin squares. There is an overview of constructions of orthogonal arrays and a description of interesting families of authentication schemes based on curves and on finite geometries. (ad)

***V. Runde: A Taste of Topology**, Universitext, Springer, Berlin, 2005, 176 pp., EUR 32,95, ISBN 0-387-25790-X*

This book is an introductory text for a study of topology. In general, the book is oriented to second-year undergraduates. It presents the basic language used in various fields of modern mathematics. The book covers classical topics of topology. Chapter 1 is about set theory. Chapter 2 covers metric spaces including continuity, compactness and completeness. These notions are studied in chapter 3 from the point of view of topological spaces. Separation properties are included as well. Chapter 4 is devoted to systems of continuous functions, covering the Urysohn lemma, the Stone-Čech compactification and the Stone-Weierstrass theorems. Chapter 5 presents basics of algebraic topology, containing a study of homotopy, the fundamental group and covering spaces. The text also presents several non-typical approaches to various topics. For instance, the Baire theorem is derived from Bourbaki's Mittag-Leffler theorem, the Tychonoff theorem is proved intuitively using nets, and the complex Stone-Weierstrass theorem is obtained using a short and elegant approach. The book is also a source of exercises on basic topological notions. (pp)

***M. Sanz-Solé: Malliavin Calculus with Applications to Stochastic Partial Differential Equations**, Fundamental Sciences: Mathematics, Chapman & Hall/CRC (EPFL Press), Boca Raton, 2005, 162 pp., USD 84,95, ISBN 0-8493-4030-6*

Professor Sanz-Solé (University of Barcelona) has written an inspiring text in an alert style, which can be used for many different purposes from individual reading to graduate level seminars; the interest of potential users is continuously kept alive. She demonstrates how Malliavin calculus can provide penetrating insights into deep aspects of probability theory. The book gives a self-contained introduction to the background material (including the Ornstein-Uhlenbeck operator and the representation of Wiener functionals). However, the choice of topics is highly selective, with emphasis on those frequently used in research and inspired by special problems discussed in the later chapters. These include questions related to stochastic partial differential equations. Several results are published here for the first time in book form. The quick progress from fundamentals to applicable topics ensures that both graduate students and young researchers will benefit from the book. (kzit)

***K.-G. Steffens: The History of Approximation Theory – From Euler to Bernstein**, Birkhäuser, Boston, 2006, 219 pp., EUR 68, ISBN 0-8176-4353-2*

This nice little book is devoted to the early development of approximation theory. It mainly concentrates on the role of Russian mathematicians but it confronts it with the development of approximation theory in the West. It first explains the approximation results of Euler connected with geodesy (he constructed a best possible approximation for a map of the Russian Empire satisfying certain side conditions) and Laplace's contribution to the determination of the best ellipsoidal approximation of the planets. Then more than a quarter of the book is devoted to Chebyshev and his results and another part of about the same size to Saint Petersburg mathematical school (among his direct successors one finds the names Bessel, Sochocki, Zolotarev, A. A. and V. A. Markov, Lyapunov, Voronoy and others). The list of Western mathematicians who contributed to approximation theory includes Klein, Blichfeldt, Kirchberg, Weierstrass, Runge, Borel, Young, Fréchet, Fejér and Jackson. The author also tries to present a closer look at the personalities of those he mentions; we learn many facts about their lives and their philosophical attitudes to different approximation results. He quite often quotes Russian sources (with translations), which will help most readers gain understanding of their personalities. The last chapter describes the work of Pscheborski and Bernstein and their role in the creation of the Kharkov School. The book will be of interest for historians of mathematics and for specialists in approximation theory. (jive)

***S. Tabachnikov: Geometry and Billiards**, Student Mathematical Library, vol. 30, American Mathematical Society, Providence, 2005, 176 pp., USD 35, ISBN 0-8218-3919-5*

This small booklet is based on lecture notes from a course in a special semester for advanced undergraduate students. It treats various mathematical disciplines connected with a description of an ideal (mathematical) billiard. In the book, connections to geometry and optics are stressed. Diverse methods are discussed and used (including symplectic geometry, calculus of variation, evolutes and involutes, a mathematical theory of rainbows, the Morse theory, 4-vertex theorem, and symplectic reduction). Various forms of billiards are studied (for instance completely integrable billiards, periodic billiards and dual billiards). The book is the first in a future series connected with the special semesters mentioned above. A good grounding in analysis, algebra, differential geometry and topology is helpful (but not necessary) for the reading of the book. (vs)

***A. N. Vasil'ev: The Field Theoretic Renormalization Group in Critical Behavior Theory and Stochastic Dynamics**, Chapman & Hall/CRC, Boca Raton, 2004,681 pp., USD 99,95, ISBN 0-415-31002-4*

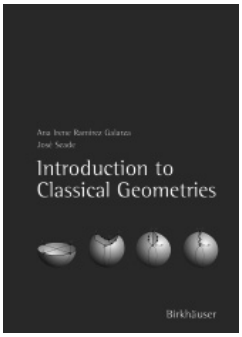
This outstanding, extensive monograph deals with the phenomenon of scale invariance, which is a property observed in the critical behaviour of systems with an infinite number of degrees of freedom studied in quantum field theory, statistical physics, the theory of turbulence and other fields. The method making it possible to understand such behaviour (at least on the "physical level of rigor") is the famous renormalization group (RG) technique, developed successively over the last fifty years. From a purely mathematical point of view, there are still many hard problems in the very foundations of this theory left to be clarified and the corresponding results rigorously proven. The book is based on a series of lectures for graduate students on quantum RG techniques given by the author at St. Petersburg University. It is intended not only as a general and self-contained textbook but also as a reference text containing the final, most accurate results of deep calculations often with a long history of development. The prime goal of the book is to give a thorough and detailed explanation of all essential computational techniques.

The first chapter ("Foundations of the Theory of Critical Phenomena") outlines the general scheme of the RG method in the theory of critical phenomena of statistical physics. It contains an introductory section with a nicely written historical review of the subject. This material is greatly expanded later in chapter 4 ("Critical Statistics"), which is the major part of the book. It deals with many aspects of RG analysis, including the ε and $1/n$ expansions and the problem of critical conformal invariance. Chapters 2 ("Functional and Diagrammatic technique of Quantum Field Theory") and 3 ("Ultraviolet Renormalization") deal with techniques of the RG in quantum field theory. Chapter 5 is devoted to problems of critical dynamics with stochastic Langevin equations, while chapter 6 deals with an important example of non Langevin type equations: the stochastic theory of turbulence. In conclusion, this is a monograph containing an impressive wealth of material, explaining in detail one of the most universal, nontrivial and powerful computational methods of contemporary theoretical physics. (mzahr)

***J. Williamson: Bayesian Nets and Casuality**, Oxford University Press, Oxford, 2004, 239 pp., GBP 40, ISBN 0-19-853079-X*

This book has two central claims, namely the exploration of the basic scientific and philosophical concepts probability and causality and the relationship between Bayesian nets and the maximum entropy principle as applied in computational methods. The text is based on a number of the author's studies and papers published between 1999 and 2004 and must be considered as an important, well thought out contribution to both of the topics above. The range of philosophical and mathematical views connected with causality and probability, interpretations of experience and human mental characteristics, like the obsessive urge to think out cause-effect relationships for any event, is extreme and covers a long period of time.

The reader is confronted with views from personalities such as Francis Bacon, David Hume, Immanuel Kant and Jakob Bernoulli as well as Frank P. Ramsey, John M. Keynes, Bruno de Finneti, Richard von Mises, Karl Popper, Rudolf Carnap and many others. The book opens with a short overview of various interpretations of probability. Bayesian nets are then introduced and their use in causal reasoning is proposed in chapters 2–4. An objective Bayesian interpretation of probability and its application to net construction is examined in the two following chapters. Epistemic and recursive causalities and their close relations to causal graphs and Bayesian nets are discussed in chapters 9 and 10 and logical reasoning is covered in chapter 11. The analogies between causal and logical influence are identified and it is shown that the Bayesian net formalism is applicable to logical implications. In the closing twelfth chapter, the author shows that the changes in belief can be handled as the language changes. The book will certainly be appreciated by researchers and graduate students in computer science, mathematics and philosophy and, in particular, by all interested in the complicated relations between subjective and objective interpretations of probabilistic phenomena. (isaxl)



Ramirez Galarza, A.I. / **Seade, J.**, both Universidad Nacional Autónoma de México

Introduction to Classical Geometries

2006. Approx. 264 p. Softcover
ISBN 3-7643-7517-5
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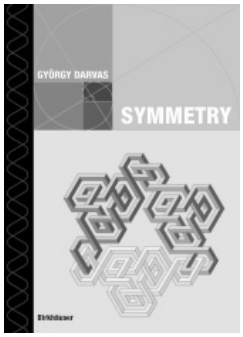
This book follows Klein's proposal for studying geometry by looking at the symmetries (or rigid motions) of the space in question. This approach introduces in turn all the classical geometries: Euclidean, affine, elliptic, projective and hyperbolic. Numerous illustrations are included to reinforce understanding.
Once plane geometry is well understood, it is much easier to go into higher dimensions. Therefore, the main focus of this text is on the mathematically rich two-dimensional case, although some aspects of 3- or *n*-dimensional geometries are included.
The book appeals to, and helps to further develop the geometric intuition of the reader. Some basic notions of algebra and analysis are also used to convey more complete understanding of various concepts and results. Concepts of geometry are presented in a very simple way, so that they become easily accessible; this leads on to advanced concepts such as manifolds and Lie groups, group actions, curvature and foliations. The only pre-requisits are calculus, linear algebra and basic analytic geometry.



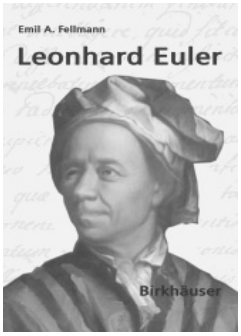
Darvas, G., Hungarian Academy of Sciences, Budapest, Hungary

Symmetry

2007. Approx. 456 p. Softcover
ISBN 3-7643-7554-X
Coming soon

Did you know, that symmetry is a universal concept, that appeared in all cultures since the prehistoric ages, and followed us in the history of mankind? What important role played asymmetries, like the one-handedness of the neutrino, in the formation of the matter since the assumed Big Bang up to the asymmetries of the human brain? This book tries to explain these, and several related questions.

In the recent half century symmetry (and symmetry breaking) became a leading principle in physics, and in all sciences that deal with the structure of matter (like particle physics, high energy physics, structural chemistry, theory of valences); as well as in biochemistry of proteins, in the study of the genetic code, in brain research (where functional asymmetries have been revealed) and their consequences in the education/learning process, in psychology; and also in developing architectural structures, and business decision making, to mention only a few examples. The book is looking for the common regularities beyond these apparently distant phenomena. It covers most achievements reached in the sciences, to which science-art mutual influences along symmetry (or just its lack) gave a new impetus in the recent decades.



Fellmann, E.A.

Leonhard Euler

Translated by Erika Gautschi and Walter Gautschi
2007. Approx. 160 p. Hardcover
ISBN 3-7643-7538-8
Coming soon

Euler was not only by far the most productive mathematician in the history of mankind, but also one of the greatest scholars of all time. Cosmopolitan in the truest sense of the word; he lived during his first twenty years in Basel, was active altogether for more than thirty years in Petersburg and for a quarter of a century in Berlin; he attained, like only a few scholars, a degree of popularity and fame which may well be compared with that of Galilei, Newton, or Einstein.
This biography is published in commemoration of the Tercentenary of his birth. Written in a brilliant style, accessible to a wide range of readers without losses in precision and depth.

Contents.:
- Prologue
- Basel 1707–1727
- The first Petersburg period 1727–1741
- The Berlin period 1741–1766
- The second Petersburg period 1766–1783
- Epilogue

Leonhard Euler
(Basel 1707 – St. Petersburg 1783)

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