NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

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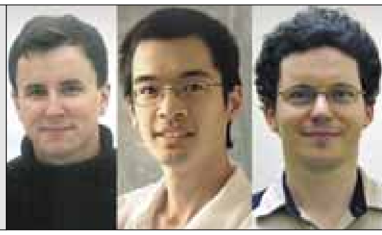

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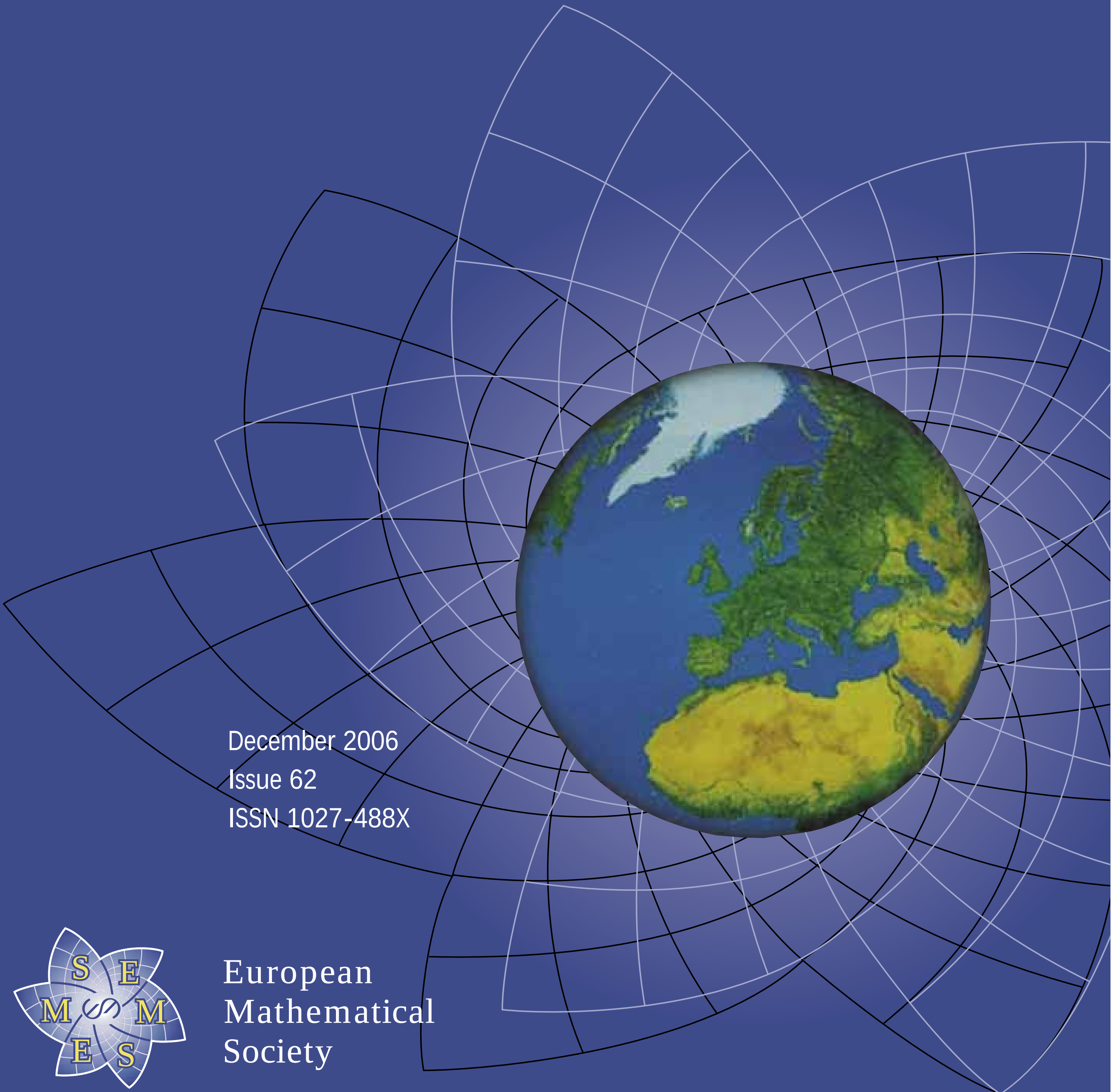

December 2006
Issue 62
ISSN 1027-488X

European
Mathematical
Society

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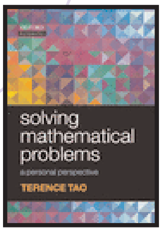
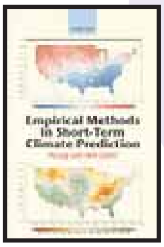
FIELDS MEDAL WINNER 2006
Solving Mathematical Problems
A Personal Perspective
Terence Tao



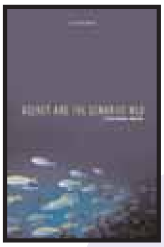
Authored by a leading name in mathematics, this engaging and clearly presented text leads the reader through the tactics involved in solving mathematical problems at the Mathematical Olympiad level.
128 pages | August 2006
0-19-920560-4 | 978-0-19-920560-8 | Pbk | £12.99
0-19-920561-2 | 978-0-19-920561-5 | Hbk | £37.50

Empirical Methods in Short-Term Climate Prediction
Huug van den Dool



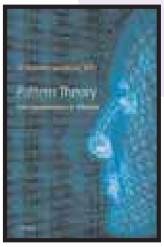
This clear, accessible text describes the methods and advances in short-term climate prediction at time scales of 2 weeks to a year. With an emphasis on the prediction methods themselves and the use of observations, the text is ideal for students and researchers in Meteorology, Atmospheric Science, Geoscience, Mathematics, Statistics and Physics.
240 pages | November 2006
0-19-920278-8 | 978-0-19-920278-2 |Hbk | £49.95

Agency and the Semantic Web
Christopher Walton



This text looks at the construction of the Semantic Web, which will enable computers to automatically and independently consume Web-based information. With numerous programming examples, it is ideal for undergraduates and graduates in mathematics, computer science and logic and researchers interested in Multi-Agent Systems and the Semantic Web.
October 2006 | 272 pages
0-19-929248-5 | 978-0-19-929248-6 | Hbk | £29.95

Pattern Theory
From representation to inference
Ulf Grenander, and Michael Miller



A comprehensive overview of the challenges in signal, data and pattern analysis in speech recognition, computational linguistics, image analysis and computer vision. Includes numerous exercises, an extensive bibliography, and additional resources -- extended proofs, selected solutions and examples -- on a companion website.
608 pages | November 2006
0-19-929706-1 | 978-0-19-929706-1 | Pbk | £50.00
0-19-850570-1 | 978-0-19-850570-9 | Hbk | £100.00

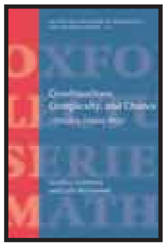
Multi-dimensional hyperbolic partial differential equations
First-order systems and applications
Sylvie Benzoni-Gavage and Denis Serre

Authored by leading scholars, this text presents the state of the art in multi-dimensional hyperbolic PDEs, with an emphasis on problems in which modern tools of analysis are used. Ordered in sections of gradually increasing difficulty and with an extensive bibliography, the text is ideal for graduates and researchers in applied mathematics.
536 pages | November 2006
0-19-921123-X |978-0-19-921123-4 | Hardback | £60.00

The Porous Medium Equation
Mathematical Theory
Juan Luis Vázquez

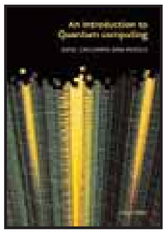
This monograph provides a systematic and comprehensive presentation of the mathematical theory of the nonlinear heat equation usually called the Porous Medium Equation. Each chapter is supplied with a section of notes providing comments, historical notes or recommended reading, and exercises for the reader.
Oxford Mathematical Monographs
October 2006 | 648 pages
0-19-856903-3 |978-0-19-856903-9| Hbk | £65.00

Combinatorics, Complexity, and Chance
A Tribute to Dominic Welsh
Geoffrey Grimmett and Colin McDiarmid



This text provides a review of the consistent themes from Dominic Welsh's influential work in combinatorics and discrete probability. Original articles by key academics are set in a broader context by the inclusion of review material. The text will appeal to all those seeking an introduction to the relevant contemporary aspects of these fields.
January 2007 | 320 pages
0-19-857127-5 |978-0-19-857127-8| Hbk | £39.50

An Introduction to Quantum Computing
Phillip Kaye, Raymond Laflamme, and Michele Mosca



This concise, accessible introduction to quantum computing is aimed at advanced undergraduate and beginning graduate students from a variety of scientific backgrounds. The text is technically detailed and clearly illustrated throughout with diagrams and exercises.
October 2006 | 288 pages
0-19-857000-7 |978-0-19-857000-4 | Hbk |£75.00
0-19-857049-X | 978-0-19-857049-3 | Pbk |£26.50

Proceedings of the International Congress of Mathematicians, Madrid 2006
Marta Sanz-Sole, Javier Soria, Juan Luis Varona and Joan Verdera
American Mathematical Society
December 2006 | 4,500 pages
3-03719-022-1 | 978-3-03719-022-7 | Hbk |£249.50

Lehrbuch der Algebra (Set) 3/e
Heinrich Weber
American Mathematical Society
November 2006
0-8218-3277-8 | 978-0-8218-3277-6 | Hbk |£92.50

Painleve Transcendents: The Riemann-Hilbert Approach
Athanassios S. Fokas, Alexander R. Its, Andrei A. Kapaev and Victor Yu. Novokshenov
American Mathematical Society
November 2006 | 560 pages0
0-8218-3651-X | 978-0-8218-3651-4 | Hbk |£63.50

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European Mathematical Society

Newsletter No. 62, December 2006

The views expressed in this Newsletter are those of the authors and do not necessarily represent those of the EMS or the Editorial Team.

ISSN 1027-488X

Published by the
EMS Publishing House
ETH-Zentrum FLI C4
CH-8092 Zürich, Switzerland.
homepage: www.ems-ph.org

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EMS Calendar

2007

1 February
Deadline for submission of material for the March issue of the EMS Newsletter
Martin Raussen: raussen@math.aau.dk

17–18 March
EMS Executive Committee Meeting at Amsterdam (The Netherlands)
Stephen Huggett: s.huggett@plymouth.ac.uk

1 April
Deadline for proposals for minisymposia at 5ECM
lex@cwi.nl

6–12 May
EMS Summer School – Séminaire Européen de Statistique, La Manga (Cartagena, Spain)
SEM STAT: Statistics for stochastic differential equations models
mathieu.kessler@upct.es or lindner@ma.tum.de

3–10 June
EMS Conference at Będlewo (Poland)
Geometric analysis and nonlinear partial differential equations
B.Bojarski@impan.gov.pl or pawelst@mimuw.edu.pl

9–10 June
EMS Executive Committee Meeting at the invitation of the Euler Society, St. Petersburg (Russia)
Stephen Huggett: s.huggett@plymouth.ac.uk

16–20 July
ICIAM 2007, Zurich (Switzerland)
www.iciam07.ch/

1 November
Deadline for submission of nominations of candidates for EMS prizes
tijdeman@math.leidenuniv.nl

2008

14–18 July
5th European Mathematical Congress, Amsterdam (The Netherlands)
www.5ecm.nl

16–31 August
EMS-SMI Summer School at Cortona (Italy)
Mathematical and numerical methods for the cardiovascular system
dipartimento@matapp.unimib.it

Editorial

Message from the retiring President

It has been a very great privilege to have been, for the last four years, the President of the Society, and the voice of the European mathematical community. European mathematics is bubbling with ideas, and is supplying many of the remarkable advances in the subject and its many applications. It is the task of the EMS to facilitate that ferment of research, and to ensure that mathematics is taken seriously by those who have the power to support it.

There is always a tendency to take mathematics for granted. It underpins so much of modern life, from computer software to the understanding of the human genome, and yet it demands no enormous equipment or massive expenditure. Moreover, many scientists believe that the mathematics they learned in their youth is all that is needed, whereas new technologies often demand radically new mathematical ideas and techniques.

A large part of the agenda that I hand over to my successor has to do with raising the profile of mathematics on the European stage, complementing the activities of national societies in individual countries. We have continually to remind the European Commission, the European Science Foundation and similar organisations that a necessary condition for prosperity and quality of life is the work of mathematicians grappling with old problems and new challenges.

To take just one example, mathematicians are perhaps unusual among scientists in needing access to the literature of past decades and centuries. Nowadays, most current journals can be studied at one's own computer, and it would be a great boon to have such access to the older literature. It is not an impossible ambition to have the whole of mathematics from earliest times so available. The cost would be much less than that of a single project at CERN or the European Space Agency. There are obstacles of copyright and ownership, as well as finance, to be overcome, but the EMS is working with other bodies worldwide to try to achieve a sensible result in finite time.

Meanwhile most mathematicians are worried about the cost of current journals, especially those owned by commercial publishers. It is a bitter irony that publishers can make a profit from charging mathematicians and their universities for the work of other mathematicians who write papers and referee and edit the work of yet others. Learned societies can act as a counterweight, and all mathematicians should think carefully about the choice of journals to which they submit their work.

It was because of these arguments that the EMS, under the leadership of my predecessor Rolf Jeltsch, set up its own publishing house, ably led by Thomas Hintermann. It already has a significant portfolio of good (and reasonably priced) journals, and this should now expand significantly, complemented by high quality textbooks and research monographs.

Space prevents me from going into detail about all the activities of the Society, its conferences and summer schools, its work in raising public awareness of mathematics, its support of national initiatives, and particularly its encouragement of the manifold applications of mathematics, old and new and not least in industry. I want to leave myself room to thank all those whose hard work, energy, imagination and loyalty drive the Society forward and make the life of the President so rewarding.

I wrote in the last issue of the sterling work of Tuulikki Mäkeläinen as head of our Helsinki office from the earliest days, but there are many others to whom I and the Society owe a great debt. It is invidious to pick out individuals, but much of the burden has fallen on the Secretary Helge Holden, the Treasurer Olli Martio, and the Vice-Presidents Bodil Branner, Luc Lemaire and Pavel Exner. I am delighted that Luc is to continue to be our man in Brussels, to put at our service his unrivalled contacts with the European Commission, and that Helge will as Vice-President provide an element of continuity at the top of the Society.

We have benefited from the enthusiasm of David Salinger as Publicity Officer, of Carles Casacuberto as Publications Officer, and as readers will be aware, of Robin Wilson and Martin Raussen as successive Editors of this Newsletter. It has been a great bonus that Past Presidents Jean Pierre Bourguignon and Rolf Jeltsch have remained active, despite their many other commitments, in the life of the EMS. But there are so many others who have, as members of the Executive Committee, as chairs of other committees, and in other ways, contributed to the Society and thus to European mathematical success.

In Torino in July, the Council elected Ari Laptev as President for the four calendar years 2007-2010, and provided him with a strong Executive Committee. I wish them both joy and success as they take up the hard but rewarding work of leading the Society in its next phase of activity. I hope they will have the support of all of us, because it is our interests that the EMS should make its strong and distinctive contribution.

Yours very sincerely,

John Kingman

President, 2003–2006.

5th European Congress of Mathematics

Amsterdam, 14–18 July, 2008

Call for Proposals

At 5 ECM minisymposia on various topics ranging over all of mathematics will be held. Minisymposia typically consist of one 45 minute lecture and 3 half hour lectures. We call for organizers to submit propositions. A proposal should be approximately one page and contain:

– The names of the organizers (or organization, such as research networks)
– A short description of the topic and its importance for contemporary mathematics
– Names of proposed speakers

Proposals should reach the scientific committee before APRIL 1, 2007, e-mail: lex@cwi.nl

Call for Nominations of Candidates for Ten EMS Prizes

Principal Guidelines

Any European mathematician who has not reached his/her 35th birthday on 30 June 2008, and who has not previously received the prize, is eligible for an EMS Prize at 5ecm. A total of 10 prizes will be awarded. The maximum age may be increased by up to three years in the case of an individual with a 'broken career pattern'.

Mathematicians are defined to be 'European' if they are of European nationality or their normal place of work is within Europe. 'Europe' is defined to be the union of any country or part of a country which is geographically within Europe or that has a corporate member of the EMS based in that country.

Prizes are to be awarded for work published before 31 December 2007.

Nominations of the Award

The Prize Committee is responsible for solicitation and evaluation of nominations. Nominations can be made by anyone, including members of the Prize Committee and candidates themselves. It is the responsibility of the nominator to provide all relevant information to the Prize Committee, including a résumé and documentation.

The nomination for each award must be accompanied by a written justification and a citation of about 100 words that can be read at the award ceremony.

The prizes cannot be shared.

Description of the Award

The award comprises a certificate including the citation and a cash prize of 5000 Euro.

Award Presentation

The prizes will be presented at the Fifth European Congress of Mathematics by the President of the European Mathematical Society. The recipients will be invited to present their work at the congress (see www.5ecm.nl).

Prize Fund

The money for the Prize Fund is offered by the Foundation Compositio Mathematica.

Deadline for Submission

Nominations for the prize must reach the chairman of the Prize Committee at the following address, not later than 1 November 2007:

5ECM Prize Committee, Prof. R. Tijdeman, Mathematical Institute,
Leiden University, Postbus 9512, 2300 RA Leiden, The Netherlands.
e-mail: tijdeman@math.leidenuniv.nl
fax: +31715277101, phone: +31715277138

6th European Congress of Mathematics 2012

Call for bids for the 6ECM

Outline bids from possible organizers of the 1012 Congress are now invited and should reach the EMS Secretariat by March 15, 2007. Preferably, they should be submitted by email to:
riitta.ulmanen@helsinki.fi

The postal address of the Secretariat is:
EMS Secretariat, Department of Mathematics & Statistics, P.O.BOX 68
(Gustaf Hällströmink. 2b) 00014 University Helsinki, Finland.

The information below may be helpful to possible organizers. Informal discussions are welcome and should be addressed to any member of the Executive Committee.

General Information on ECMs

European Congresses of Mathematics are organized every four years. In 2008 the Congress will be held in Amsterdam. The next free slot for a Congress, which must be held in Europe, is the year 2012. Experience of previous Congresses suggests that the attendance might be around 1000 mathematicians. The duration has so far been 5 days. There are 10 EMS Prizes to outstanding young European mathematicians awarded at the opening ceremony. The Congress programme should aim to present various new aspects of pure and applied mathematics to a wide audience, to offer a forum for discussion of the relation between mathematics and society in Europe, and to enhance co-operation among mathematicians from all European countries.

The standard format of previous ECMs has been:

– About 10 plenary lectures;
– Section lectures for a more specialized audience, normally several held simultaneously;
– Mini-symposia;
– Film and mathematical software sessions;
– Poster sessions;
– EU network lectures;
– Round tables.

An exhibition space for mathematical societies, booksellers, and so on is required. No official language is specified and no interpretation is needed.

Starting with 4ECM in Stockholm, the Proceedings of the ECMs have been published by the EMS Publishing House.

At this stage bids need only be given in outline, but they must give a clear idea of the proposal and possible sources of financial support.

In addition, they should contain information concerning:

– Size and number of auditoriums; location and equipment.
– Room for exhibitions.
– Hotel rooms and dormitories; location, prices, their number in different categories, and transport to the lecture halls.
– Restaurants close to the Congress site, their number and prices.
– Accessibility and cost of travel from various parts of Europe.
– Financing of the Congress.
– Support to participants from poor countries.
– Financing of the EMS prizes.
– Experience in organizing large conferences.
– Timing of the Congress.
– Social events.
– Plans to use the occasion of the Congress to publicize mathematics.

Prizes

ICIAM 2007

Professor Ian Sloan, President of the International Council for Industrial and Applied Mathematics (ICIAM) announced the winners of the five ICIAM prizes on September, 18. The prizes will be awarded at the Opening Ceremony of the International Congress for Industrial and Applied Mathematics, to be held in Zurich from July 16 to July 20, 2007 (http://www.iciam07.ch).[1] The four-yearly ICIAM Congress is a major international celebration of mathematics in action, and the main event in the applied mathematical calendar.

Deadline for early-bird registration: January 15, 2007.

Prize winners

ICIAM Pioneer Prize: *Ingrid Daubechies* (Princeton Univ., USA) and *Heinz Engl* (Johannes Kepler Univ. Linz, Austria) – joint winners
The Pioneer Prize, established for pioneering work introducing applied mathematical methods and scientific computing techniques to an industrial problem area or a new scientific field of applications. The prize commemorates the spirit and impact of the American pioneers. It was created and is currently funded by SIAM.

ICIAM Collatz Prize: *Felix Otto* (Univ. Bonn, Germany)
The Collatz Prize, established to provide international recognition to individual scientists under 42 years of age for outstanding work on industrial and applied mathematics. It was created and is presently funded by GAMM.

ICIAM Lagrange Prize: *Joseph Keller* (Stanford Univ., USA)
The Lagrange Prize, established to provide international recognition to individual mathematicians who have made an exceptional contribution to applied mathematics throughout their careers. It was created by SMAI and is presently funded by the three societies SMAI, SEMA and SIMAI.

ICIAM Maxwell Prize: *Peter Deuflhard* (ZIB Berlin, Germany)
The Maxwell Prize, established to provide international recognition to a mathematician who has demonstrated originality in applied mathematics. It was created and is presently funded by IMA.

ICIAM Su Buchin Prize: *Gilbert Strang* (MIT, USA)
The Su Buchin Prize, established to provide international recognition of an outstanding contribution by an individual in the application of mathematics to emerging economies and human development, in particular at the economic and cultural level in developing countries. It was created and is presently funded by CSIAM.

This prize is being awarded for the first time; those mentioned before have been awarded since 1999.

Prize citations can be found at www.iciam.org/Prizes/PrizeNominations2007.pdf.

1 The Newsletter plans to publish an interview with ICIAM president Prof. Rolf Jeltsch in 2007.

EURYI

EURYI – the European Young Investigator Awards scheme – is designed to attract outstanding young scientists from any country in the world to create their own research teams at European research centres. Each award is up to € 1,250,000 and comparable in size to the Nobel Prize.

The EURYI Awards are offered by 20 European national research organisations in an open competition, with candidates selected on the basis of their academic and research excellence and their future potential.

Candidates are selected by a two-stage process, firstly at the national level by the relevant Participating Organisation and secondly at the international level by high-level scientific panels managed by ESF.

The EURYI Awards scheme was developed by the European Research Organisations Heads of Research Councils (EuroHORCS) and the European Science Foundation (ESF). ESF's role in the coordination and selection processes of EURYI is supported by funds from the European Commission's Framework Programme 6. More information including a call for applications for 2007: http://www.esf.org/euryi.

In each of the past two years, one of the awardees was a mathematician:

2005: Snorre Christiansen, Oslo Univ., Norway: *Numerical analysis and simulation of geometric wave equations*

2006: Jesper Grodal, Copenhagen Univ., Denmark: *Homotopical group theory – the mathematics of symmetry and deformation*

Letter to the Editor viz. Topology

Bob Ross, Journals Publisher, Elsevier (Cambridge, MA, USA)

This August 2006, Elsevier received the resignation of the Editors of *Topology*. We regret this decision by the Editors as we have appreciated the opportunity to work with them to publish one of the math community's most historically significant journals. We are committed to the long term future of the journal and its archive and to build upon its impressive heritage.

Though we have attempted to address their concerns, it has become clear to us that the editors are no longer interested in working with a commercial publisher. We have made a series of proposals to the Editors of *Topology* and we will build on these going forward.

At a time when publishers have been seeking to offer better value and to meet the needs of universities in consortia collectively purchasing digital access to diverse holdings, many scientists have continued to be focused on price per page as an indicator of value.

Whereas some in the mathematics community might feel *Topology* has become unaffordable, it has never been more available. Over 4000 institutions throughout the world have either print or on-line access to this journal. During 2006, 27,000 downloads have been recorded on this journal alone. Because the majority of our subscribers purchase this journal in a larger set of journals, most are paying a fraction of the institutional subscription price. At the same time, the personal subscription price has been held at $99.

Elsevier has taken steps to moderate price increases. Our average price increase is amongst the lowest in the industry. We have worked with libraries to develop flexible purchasing options that offer significant discounts. We have also made major investments in electronic distribution, archiving and administration. The cost per article downloaded has declined to an average of approximately $2 per article.

Elsevier has invested $160 million in digitizing and maintaining the digital archive of our entire journal program. This investment facilitates and assures electronic access and distribution of the research record, allowing instant access throughout the world or wherever and whenever the internet is available.

A report by CSFB, quoting a case study at the University of California, confirmed that Elsevier provided much better value than a simple comparison of list prices would suggest.[1] More broadly, library statistics from organizations such as the UK's LISU[2] have begun to show increased access to journal literature and falling unit prices.

We want to assure authors – including those with papers currently under review with *Topology* – that the journal will continue. Indeed, subscribers to the 2007 volume will receive the 2008 volume at no further cost. This offer will apply whether they subscribe through a paper subscription or one of the electronic options or packages our customers more commonly choose.

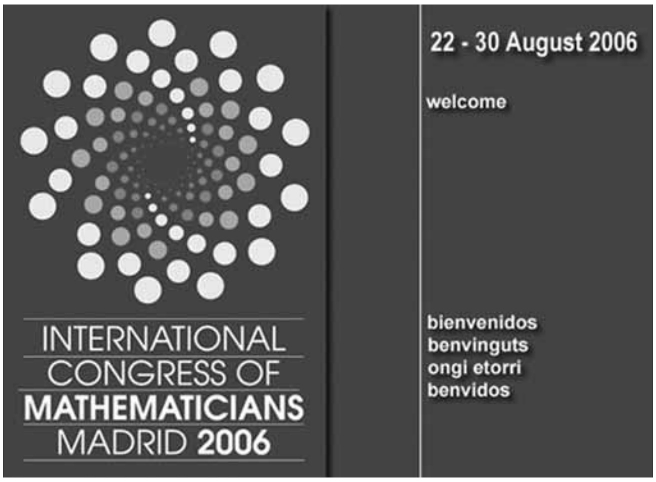
We look forward to engaging the mathematics community to identify how we can work most effectively to serve and meet their needs. Pricing is an issue under continuous discussion here, as it is at all publishers. We again regret the decision of the *Topology* editors but do appreciate their concerns. Elsevier is working hard to inform and work more closely with all of our journal editors and we want to publicly thank those who continue to provide a very necessary service to authors, the community, and to our publishing program.

[1] Credit Suisse First Boston (CSFB), Equity Research, STM publishing sector review, 29 September 2004

[2] LISU annual library statistics 2004: http://www.lboro.ac.uk/departments/ls/lisu/pages/publications/als04.html

ICM2006 in Madrid

Ulf Persson (Göteborg, Sweden)



IMU general assembly

It can scarcely have escaped our readers' notice that the most recent International Congress of Mathematicians took place in Madrid at the end of August. As usual this ICM was preceded by a General Assembly of Representatives of the International Mathematical Union. This assembly took place in Santiago, a medieval city in the lush north-western corner of the Iberian peninsula, a city known chiefly as a popular destination for pilgrims from all over the Catholic world. More than sixty countries took part, including about 130 delegates (not counting the members of the Executive Committee (EC) and the chairmen of the subcommittees). There were no earth-shaking decisions taken, at least not anything comparable to the one taken around the same time by the Astronomers General Assembly when they deprived Pluto of its planetary status, which was duly reported world-wide.

In addition to electing a new EC of the IMU, the assembly confirmed the proposal for the next ICM site to be Hyderabad (India) in 2010. This city is situated at a somewhat elevated altitude in southern India and replaced the original choice of New Delhi for climatic reasons. It is an historical city with a marked Muslim presence and with many architectural remnants from the Mogul past. It will be paired with Bangalore, the Silicone Valley of India, where the next general assembly will meet.

Furthermore, it was decided at the meeting that a new kind of membership will be reinstituted, a temporary, so-called associate membership, with the aim of broadening the international base of the IMU. It should also be noted that a further split between the IMU and its sub-committee, the ICMI of educators, was effected when the latter was granted the right to appoint their own EC at their own general assemblies, this having previously been the privilege of the delegates of the IMU. It should be noted however that the presidency of that organization is still to be chosen by the mathematicians.

Finally, to return to the EC of the IMU, the new president will be László Lóvász, a Hungarian combinatorialist, replacing Sir John Ball, who will nevertheless remain in the EC in his capacity as former president. The secretary of the IMU, Philip Griffiths from the IAS, will also step down and will be replaced by Martin Grötschel, an applied mathematician from Berlin.

Opening ceremony and prizes

The climactic high-point of any ICM is the opening ceremony. No other event draws such a crowd and as usual the total attendance at this ICM was too large to fit into a single auditorium so a second auditorium was set up where the ceremony could be watched on wide-screen. The most exciting part, as always, was the disclosure of the new Fields medallists, which has been augmented in later years by the Nevanlinna Prize in computer science and for the first time this year in Madrid the newly established Gauss prize, which is intended as a life-time achievement award for mathematical applications; this year the latter was awarded to the Japanese mathematician Kiyoshi Îto. The Spanish king presided at the ceremony, which meant that fairly strict security measures had to be implemented.

We will present interviews with three of the four Fields medallists elsewhere in this issue, while the case of Perelman will be treated in a forthcoming one. Perelman abstained from accepting the medal in spite of entreaties from the mathematical community. In particular, John Ball travelled to St Petersburg earlier this summer to plead with him but Perelman gently but firmly stuck to his commitment not to have anything more to do with the international community of mathematicians. Perelman may refuse to accept the money but cannot of course escape the honour connected with the award. His behaviour has been considered eccentric and as such has attracted the attention of the media, probably making the Madrid congress the best covered congress ever. One may speculate whether this is good or bad for mathematics but I would hazard an opinion that in the long run it is good.

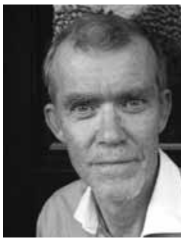
Lectures and discussions

The most visible activity of a congress is the array of lectures. There are too many to be considered systematically but suffice it to say that the topic of the Poincaré Conjecture was covered not only by Richard Hamilton at the first plenary lecture that followed the opening ceremony but also by a more popularly attuned talk delivered by John Morgan. A talk on knots by Étienne Ghys was particularly appreciated as a masterpiece of presentation with a high level of graphics employed for illustration, maybe setting a new trend.

In addition to lectures there were panel-discussions on various topics like education, interaction with the general public, and the supposed widening rift between pure and applied mathematics. In the latter, Yuri Manin stood out brilliantly in my opinion by speaking philosophically of mathematics as model, theory and metaphor, while most other members regretted the rift and emphasized how important it is for the two aspects of mathematics to cooperate. To some panellists, notably Neunzert, the bottom-line of mathematics is simply its industrial character, i.e. its overall economical impact.

Mathematical congresses are too big, at least in comparison to the good old days when group portraits could be taken. This induces many mathematicians to consider them useless; some people even fear that future congresses will split up into a small invited elite and a large mass of mathematical tourists. I do however suspect that most members actually find them more rewarding than they expect (and thus keep coming) and also that one should think of the ICM not as isolated events but as a tradition going back over a hundred years, a tradition that has no such continuous parallel in any of the other sciences.

Guillermo Curbera (Sevilla, Spain) had constructed a very interesting exhibit on all of the ICM's, beginning with Zürich in 1897 (with the actual list of members on display), emphasizing the continuity and illustrating that the whole is greater than the sum of its parts. Curbera will follow this up with an article in a future newsletter issue.



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Should mathematicians communicate to broad audiences? – A panel discussion at the ICM

Lars Døvling Andersen (Aalborg, Denmark)

A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street

(Hilbert, ICM 1900,
quoting "an old French mathematician")

There was a direct contribution by the EMS to the world conference on Wednesday evening, as the society had organized a panel discussion about how, whether, where, when and why mathematicians should communicate to people in general. The meeting was entitled '*Should mathematicians care about communicating to broad audiences? Theory and practice*' and was moderated by former EMS president **Jean-Pierre Bourguignon**. The panellists were

- **Björn Engquist**, KTH, Stockholm, Sweden and the University of Texas, Austin, USA
- **Marcus du Sautoy**, University of Oxford, England
- **Alexei Sossinsky**, Institute for Problems in Mechanics, Russian Academy of Sciences and Independent University of Moscow, Russia
- **Francois Tisseyre**, Atelier ÉcoutezVoir, Paris, France
- **Philippe Tondeur**, University of Illinois, Urbana-Champaign, USA

The meeting began with a welcome by **Luc Lemaire** who, as its vice-president, seized the opportunity to give a brief introduction to the EMS. He could not resist giving his own answer to the question of the title: "Yes, of course!" Then the panel discussion was set off by Jean-Pierre Bourguignon, who introduced the panellists and also stressed that the "why" of the question should not be forgotten; the other answers would depend on the reply to this. Each panellist began by giving a very personal account of his (alas, no women on the panel) thoughts on the topic. Far too many reflections to include here but some of the items that were picked up in the succeeding discussion are mentioned briefly in the following.

Philippe Tondeur said that as mathematics is indispensable for the process of science and the betterment of the human condition, as mathematics is a powerful force for change, and as mathematicians are key partners in these processes, the privilege of developing mathematics gives mathematicians the responsibility of sharing it with their fellow human beings and the next generation. The presentation by Philippe Tondeur can be found on his homepage.

Marcus du Sautoy gave the Hilbert quote stated above and went on to speak about his own very interesting and inspiring work with the media and mathematics. Along the way, he stressed the importance of good relations with people of wealth and power: business people, journalists and politicians. He also mentioned teachers as a very significant target group.

Francois Tisseyre is a film director and not a mathematician. He has made films about the mathematical community (such as a film about the Millennium Meeting) but also films about particular subjects of mathematics and engineering science. He mainly addressed the issue of films and urged people trying to communicate mathematics to know their media well. In particular, a good mathematical film is made somewhere between the fields of a film producer and a mathematician.

Alexei Sossinsky regards mathematics as the most useful and universal of all sciences – and the least known. Least known to the general public, teenagers, the business world, administrators, politicians etc. This condition is something that all mathematicians are responsible for and they should all take part in making mathematics visible and appealing. He advocated a relaxed attitude towards media exposure and suggested that catchy titles and smart icons be used in the communication.

Björn Engquist said that perhaps different target groups such as administrators (both at universities and in government), potential students and the ordinary general audience should have different messages from the mathematical community, although the total communication must be coherent. He encouraged the inclusion of history, perspective, educational issues, recreational mathematics and not least applications of mathematics in the communication. He observed that the topics behind the Nobel prizes are becoming more and more mathematical and he argued that too many mathematicians refrain from contributing to applications and leave the expected use of their basic research for others to realize.

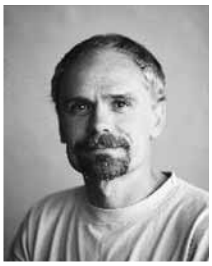
In between and after the introductory statements from the panel there was a lively debate with the audience, mainly in the form of questions to the panellists.

One part of the discussion focussed on how you get the media interested in mathematics, a question also asked directly to Marcus du Sautoy, who has been very successful in this respect. His answer was very thorough and emphasized differences between the various media. Printed media tend to need an actual event, a hook for the article; they are also fond of pictures. For radio, hooks are less important (as are pictures!). Television needs a story but this should not be a problem as mathematics is a narrative topic that lends itself well to story-telling. That the forward moving nature of film requires something like a story was a point also made by Francois Tisseyre. Prompted by a question about his thoughts on stories about mathemat*icians* rather than mathemati*cs* in media, Marcus du Sautoy said that media coverage is always a trade-off: they get personal details and you get to make your own points. In general, a good piece of advice was to maintain regular contact with selected journalists and sub-editors.

The importance of having success stories to sell was stressed by Alexei Sossinsky and this became another topic for discussion, although it was a general consensus of opinion (near-successes were also mentioned, e.g. the recent hurricane Katrina was predicted with great accuracy but necessary action was not taken). Successes can even be internal to the mathematical world. For example, the fact that Terence Tao received a Fields medal at the congress was given much press coverage in Australia and had spurred many mathematical features in the media there. And, as mediator Jean-Pierre Bourguignon pointed out, in this area it is true that nothing succeeds like success; the media world is a very interconnected one, borrowing stories from and citing each other, and a good story will reappear again and again.

An issue dividing the panel was whether there is a danger of overselling mathematics. Some said yes, care must be taken in not overstating the power, and Björn Engquist suggested that when speaking about applications, you should always say that mathematicians are partners in the solution of problems. Philippe Tondeur, on the other hand, said that mathematics cannot be oversold for the time being as it is so miserably funded!

During the discussion, several panellists and audience members suggested the necessity of mathematicians using professionals from the media when communicating to a broad audience. Still, there was big applause near the conclusion of the event when a participant stated that Marcus du Sautoy and Robin J. Wilson (former EMS Newsletter editor and well known provider of popular lectures) are counterexamples to this necessity. Mathematicians should not underestimate themselves! But go out and do it, with or without professionals…



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The Ugly Duckling

Ernesto Aranda (Universidad de Castilla-La Mancha, Spain)

Over the last few days, representatives of the different media groups have given an account of the celebration of the 25th International Congress of Mathematicians. It could be because it is August and we are in the middle of the "silly season" but the presentation of the Fields Medals, whose significance in mathematics is comparable to the Nobel Prize, and the discussion triggered by the rejection of one of these awards, have perhaps surprisingly put mathematicians into the pages of many newspapers.

What I find curious about some of these reports is the somewhat sarcastic attention given to the behaviour and way of dressing of some of the congress participants, who are sometimes referred to as "scruffy" or "crazy-looking", not to mention comments about them attending official sessions in shorts. It is equally curious to observe how the extravagant appearance of some media personalities (like actors, celebrities and sports stars), who are accorded enormous importance in contemporary society, is not only regarded as a secondary factor but somehow makes them more interesting, while the slightly peculiar behaviour of a few mathematicians is supposed to provide irrefutable proof that the equation "mathematics = nuttiness" is indeed true. It appears to be no use giving a counter-example to a society that understands little or nothing of mathematics.

Although the perception of the role played by mathematics seems to be acquiring more importance, the image of mathematicians in society remains that of "strange creatures" who are not only devoted to a subject that sticks in most people's throats but actually seem to enjoy it! All of which reinforces a stereotype that we mathematicians find it very difficult to shake off. This is a pity because if there is one thing that ought to distinguish mathematicians it is the possibility of speaking a single, universal language that rises above cultural differences and external appearances. We must indeed be a very strange species if what really interests us are ideas.

In any case, we mathematicians are used to being the "ugly duckling" of the scientific world and some of us even feel comfortable in the role. However, the distance separating us from other mortals is not without negative consequences. If there are any nine year-olds who should happen to say that they like mathematics, or worse still, that their ambition is to be a mathematician, they will immediately be labelled as "oddballs". Quite the opposite of what would happen if they said they wanted to be engineers, but then again engineers usually attend conferences in jackets and ties. It therefore seems evident that it is up to us to open up our discipline to the rest of society and popularization could well be the right way to do it. Otherwise we should not be surprised to find the faculties of mathematics getting emptier and emptier.

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Keizo Ushio (born in Fukusaki, Japan, 1951) is a significant Japanese artist who has enjoyed international success with his spectacular granite sculptures inspired by mathematics. His work is a perfect example of how topological forms can generate impressive sculptures. Keizo completed his study at the Kyoto City University of Arts in 1976. Since then he has received numerous prizes and has participated in stone sculpture symposia throughout the world. Keizo has exhibited to international acclaim with his work being represented at exhibitions, public locations and private locations in countries such as Japan, Spain, Iceland, Norway, Germany, Israel, India, Australia, New Zealand and the USA.

Sculpting with stone, Keizo Ushio has achieved something of celebrity status from mathematicians and sculptors alike for his magnificent manipulation of geometrical forms such as the Moebius and the torus. One of the main processes in the work of Keizo is the division of a torus to obtain impressive sculptures.

Working with a pneumatic hammer in the middle of the street, Keizo Ushio has been, since the first day of the ICM2006, sculpting a geometrical piece, hewn from black granite and based on a torus, which he has subsequently split into two halves by drilling out a 360º diameter around the circumference in the form of a double twist crossed strip. Ushio has then rearranged the two interlacing halves of the torus to create a new shape which, depending on the angle it is viewed from, is meant to evoke the mathematical symbol for infinity. The work was completed by 30th August and after the congress this mathematical sculpture will stay in the CSIC (Consejo Superior de Investigaciones Científicas) building.

Major trends in Mathematics

Themistocles M. Rassias (Athens, Greece)

In this article we present some ideas regarding the present state and the near future of mathematics, inspired by the ICM 2006 in Madrid. Since assessments and any predictions in this field of science are necessarily subjective I have included the opinions of renowned contemporary mathematicians with whom I have recently come into contact in Madrid (in alphabetical order Thomas Banchoff, Timothy Gowers, Sergei Konyagin, Frank Morgan and Alexei Sossinsky). In addition, since any such assessment by its nature has limited predictability, we have included opinions of famous mathematicians of the past who have spoken about trends in mathematics.

We single out four main directions of current activity:

1. The further unification of mathematics to solve in particular longstanding unsolved problems.
2. The realization that difficult problems require ad hoc tools (e.g. nonlinearity, nonconvexity, discrete mathematics). We are witnessing an increased interest in the interplay between probabilistic ideas and deterministic ones, in particular in such classical branches of mathematics as partial differential equations (Kiyoshi Ito, Adrei Okounkov) and number theory (Timothy Gowers, Terence Tao).
3. Discrete mathematics fuelled by the computer revolution and rekindled interest in number theory (e.g. the Riemann hypothesis and the Goldbach conjecture).
4. The recognition that "good" solutions of many applied problems from other sciences and technology, including contemporary cosmology and biology, often call upon the panorama of mathematics. This fact has contributed to the recent popularization of the strength of mathematics (see also the newly established Gauss prize that is awarded jointly by the German Mathematical Union (GMU) and the International Mathematical Union, and administered by the GMU).

In the spirit of apparent trends in mathematics, the following evaluation of "important problems" perhaps gives an idea of some of the emphasis that will be given in the near future to the problems-related fields of mathematics mentioned above. Specifically the Clay Mathematics Institute (USA) led by its founding scientific board – Alain Connes, Arthur Jaffe, Edward Witten and Andrew Wiles – decided to establish a small set of prize problems. The aim was "to record some of the most difficult issues with which mathematicians were struggling at the turn of the second millennium; to recognize achievement in mathematics of historical dimension; to elevate in the consciousness of the general public the fact that, in mathematics, the frontier is still open and abounds in important unsolved problems; and to emphasize the importance of working toward solutions of the deepest, most difficult problems" (see the introduction to the book: The Millennium Prize Problems, J.Carlson, A.Jaffe, and A. Wiles, Eds., published jointly by the Clay Mathematics Institute and the American Mathematical Society, 2006).

We now present the opinions of mathematicians regarding the trends mentioned above.

Thomas Banchoff (ex-President of the Mathematical Association of America)

"One marvelous modern development for unification in mathematics is scientific visualization. Computer graphics makes it possible to communicate ideas from all parts of mathematics to all people: researchers, artists, students, and the general public".

Timothy Gowers (University of Cambridge, Fields Medal 1998)

"Several current areas of mathematics are less tightly defined than the traditional areas such as algebraic topology or functional analysis. For example, arithmetic combinatorics is a fusion of techniques and insights from analysis, probability, combinatorics and number theory. This tendency, to mix ideas from several branches of mathematics, was clearly visible at ICM2006."

Sergei Konyagin (University of Moscow)

"The process of unification of mathematics in Russia is slow, and there are no evident dramatic changes in the last years. But in my opinion (I am writing about subjects I am familiar with) the most interesting results are based on a combination of ideas from different mathematics areas".

Frank Morgan (Massachusetts, USA)

"Mathematics is finally learning to deal with the singularities which encode the mysteries of the universe".

Alexei Sossinsky (Independent University of Moscow, Institute for Problems in Mechanics – Russian Academy of Sciences)

"One of the crucial developments in recent years has been the disappearance of any boundary between mathematics and theoretical physics. This congress has strikingly demonstrated a new interaction between probability theory and analysis, between structure and randomness".

In earlier times the following mathematicians had specific opinions about the subject of this article (cf. D.J.Albers, G.L.Alexanderson, and C.Reid, *International Mathematical Congresses*, Springer-Verlag, 1986).

Felix Klein. In 1893, as part of the Chicago World's Columbian Exposition, celebrating the 400th anniversary of Columbus's discovery of America, Klein from Göttingen brought a number of European mathematicians to Chicago and opened the congress on "The Present State of Mathematics", in which he underlined the value of world cooperation in mathematics. He said, "The famous investigators of the preceding period … were each great enough to embrace all branches of mathematics and its applications. In particular, astronomy and mathematics were, in their time, regarded as inseparable. With the succeeding generations, however, the tendency to specialization manifests itself. But the developing science departs at the same time more and more from its original scope and purpose and threatens to sacrifice its earlier unity and to split into diverse branches. It became almost the custom to regard modern mathematical speculation as something having no general interest or importance …. Speaking, as I do, under the influence of our Göttingen tradition, and dominated somewhat, perhaps, by the great name of Gauss, I may be pardoned if I characterize the tendency that has been outlined in these remarks as a return to the general Gaussian program …. But our mathematicians must go further still. They must form international unions, and I trust that this … world congress will be a step in that direction".

Adolf Hurwitz. At the opening of the International Congress of Mathematicians (Zurich, 1897) Hurwitz said, "No science, unless perhaps philosophy, has such a brooding and solitary character as mathematics. But nevertheless there lives in the breast of the mathematician the necessity for communication, for conversation with colleagues", i.e. the value at the heart of the creation of the ICM.

Rolf Nevanlinna. During the ICM 1962 in Stockholm, Nevanlinna said, "The development of mathematics (i.e. non-unified) would soon lead to an impossible situation, if it were not for another tendency working against it: the tendency towards synthesis". We mention here that at the ICM congresses since 1982 jointly with the Fields Medals the Rolf Nevanlinna Prize has been awarded.

Stephen Smale. In 1986 at the ICM in Berkeley the 1966 Fields Medallist Stephen Smale predicted the value of future important branches of mathematics, "This subject (i.e. algorithms, complexity, etc) is now likely to change mathematics itself. Algorithms become a subject of study, not just a means of solving problems".

Benoît Mandelbrot. At the ICM 2006, the following quotes were written regarding the new field of mathematics-art that was created by Mandelbrot (see the *Daily News* of the Congress). "His discovery of the so-called Mandelbrot set consisted in a number of visual observations phrased into conjectures demanding rigorous mathematical treatment" … "Mathematics … have always remained isolated and have concentrated on simple figures. I've been very lucky to work with the mathematics of roughness. In a certain sense, the circle has been completed; in many cultures the "rough" has always been considered art. Then this concept arrived to mathematics and afterwards to other sciences, then back to mathematics again, and finally to art once more through fractal art".

In conclusion, we have attempted to give a balanced overview of trends in mathematics based on our own observations at the ICM 2006, made more credible by the independent assessments of world-renowned mathematicians. But who can really guess the future? For example, conceivably the mathematics of biology in the future might be unrecognizable to contemporary colleagues!

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King Juan Carlos with awardees J. Kleinberg, T. Tao, A. Okounkov and W. Werner, ICM 2006

From a press conference following the EMS panel debate (p. 9/10): Marcus du Sautoy, Olga Gil-Medrano, Jean-Pierre Bourguignon

The art of a right decision

Why decision makers may want to know the odds-algorithm

F. Thomas Bruss (Université Libre de Bruxelles, Belgium)

In the case of uncertainty the danger of making a wrong decision is unavoidable. Under certain conditions, however, a new mathematical procedure helps to reduce these dangers to a minimum. This is useful for problems that range from everyday choices to difficult decisions in medicine.

Introduction

If there is one thing that really hurts responsible decision makers it is the following kind of reproaching questions, which, after a wrong decision, are useless and unfair. Managers, doctors, investment specialists, consultants, politicians, and you and I: we all know the questions. Why did you get on this deal when…? Why did you continue the treatment when it was clear that…? Why did you not sell these stocks when they were still at…? Why did you not wait until the election before…?

The physician who finds himself accused of having inflicted meaningless suffering on patients knows why he did not stop the treatment. Things at the time were not so clear to him and the patients were asking for what they saw as their last fragment of hope. Similarly, the manager, the investor, the politician… they all had their reasons to hesitate or to decide on the course of action they did. All that remains afterward is a bitter laugh for those who know better. Yes, it is easier to predict the future once it has happened! In such cases it is not clever to become defensive. A bit of humour is usually better (despite the risk of sounding provocative), such as the response, "Whenever I have the choice between mistakes I choose one I have not tried before".

In such situations, mathematics can be a valuable assistant, both before making a decision and afterwards. We present a new tool (an algorithm) that is surprisingly simple, requiring no more than addition, multiplication and division of the numbers involved. Its first and main objective is to find a strategy for certain situations that maximizes the probability of making a good decision. Secondly, as we will argue, it offers legal support. Therefore, if things go wrong, we can rely on something more substantial than humour.

A mathematical strategy cannot achieve miracles; in particular, it cannot replace knowledge and experience. However, it can make better use of them. Moreover, if a strategy can be shown to be optimal then it should attract our attention, since in non-trivial situations optimal strategies are rare. The strategy we will present here is optimal. But first, what is the problem?

The importance of last opportunities

We begin with a few examples to explain the essence of the problems we are considering. The first one is a simple game.

We throw a die twelve times. As soon as a '6' is thrown we may either declare "This is the last 6" or else say nothing and go on. Only one declaration is allowed. We win if there is at least one 6 among the twelve tosses and if we make the declaration, "This is the last 6", at the correct place. Thus if the results turn out to be 3, 4, 6, 1, 2, 6, 3, 6, 2, 5, 1, 3 then we win if and only if we declared the eighth toss as the last 6. If there is no 6 at all we also lose. What strategy should we apply?

The second example is not a game but a serious and difficult problem. It concerns clinical trials. Patients who are seriously ill sometimes ask for treatments about which little is known. These may bear a high risk or inflict suffering, such as high-dose chemotherapy. Such requests put physicians and researchers of clinical trials into difficult positions. It is not easy for responsible doctors and researchers to establish ethical guidelines for such "compassionate use trials" since success probabilities and adherent risks are typically unknown. This is why a small number of patients are treated one after another so that each patient can profit from the information gained from the preceding experiences. In particular, the sequence of treatments can be stopped when it becomes clear that the expected success rate is too small to justify the suffering of further patients. But when exactly is this the case? What criterium can be defended as an ethical guide-line before patients and doctors?

The third example is a more frequent problem and is again of a different nature. You would like to sell your gorgeous sports car within the month. Potential buyers call and come to see your car, each making an offer. You may either accept or refuse each offer but once you refuse an offer it is lost forever (see also *Spektrum der Wissenschaft*, 5/2004, p 102). You would like to sell your car for the best price so how should you proceed?

Now, what do these problems, which all look so different, have in common?

Firstly, in the dice game the problem is to stop with maximum probability on a last specific event, that is, on the last 6. Here stopping means to declare, "This is the last 6". In the clinical trial example, the doctor has, somewhat surprisingly, the same type of problem, as we will show. Indeed, suppose for a moment that the doctor has prophetic abilities to foresee the total number of successes in the sequence of treatments. Suppose that he/she knows that there are two successes for ten patients. Let "+" stand for a success and "–" for a failure or no visible success. Then, if the sequence of actual treatments starts – + – – +, the doctor would stop on the 5th treatment; he/she would already know that the whole sequence is – + – – + – – – – – –. Stopping earlier would miss at least one precious success whereas stopping later would lead to pointless sufferings of up to five more patients.

– + ? ? ? …?

However, the doctor is no prophet. That is why the best he/she can do is to stop with maximum probability on the last "+" in the sequence of results. Hence the essence of the problem is the same as before. The difference to the dice problem with a fixed success probability of 1/6 is that here the success probabilities are typically unknown and have to be estimated from preceding treatments.

Finally, selling your sports car is just another problem of optimal stopping on a last specific event. To see this it suffices to formulate the problem as follows. Mark the first offer by H. Then mark each following offer by H if it is higher than all preceding ones and L otherwise. The sequence of offers (which you do not know at the beginning) will be a sequence of H's and L's. You would like to stop (sell) on an H, of course, and preferably on the very last H. Indeed, that one is the best offer of all.

As we have seen, whether in a game or in real life situations, a last specific event often plays an important role and there are simple reasons why this is so. Time runs in one direction, many decisions are irrevocable and life is finite. Consequently, the objective of stopping on a last opportunity often has intrinsic interest. Since we have much freedom in the definition of an opportunity, we can address problems of quite different natures: the last 6 is a game objective, the last H stands out as the very best offer while the last + is not distinguished by itself but stands for obtaining all successes without further suffering.

Independence and uncertainty

A second common factor in our examples is independence. Each toss of the die is independent of the other tosses, each patient reacts as an individual independently of others, and there are many more situations where opportunities are independent of each other. This is also true for the sales problem, although admittedly this is far less evident (and must be proved).

Finally, a third common factor is uncertainty. We do not know what the future will bring, that is, when the last 6, the last + or the best offer will appear. Hence deterministic planning must be replaced by a probability model. And it is here that our modelling begins.

Model and algorithm

Let $E_1, E_2, \ldots, E_n$ be a sequence of n independent events. We can observe these events, one after the other, and classify them as either "interesting" or "not interesting". In what follows, each interesting event will be called an "opportunity". We can "stop" on exactly one opportunity but we cannot return to an opportunity over which we have passed.

Let p_k be the probability that E_k is an opportunity. How do we find p_k?

In the dice game each 6 is an opportunity. Hence $p_k = 1/6$ for all $1 \leq k \leq n$. In the sports car example, each H stands for an opportunity. Without further information we assume that all arrival orders of offers are equally likely. Therefore the best among the first k offers is the kth offer with a probability of 1/k. Notice that this is also true for k = 1 (even if it is too low to be taken seriously into consideration it is still the best so far). Hence p_k, which is the probability of getting an H on place number k, equals 1/k. In the clinical trial example, however, p_k is not usually known beforehand (see below).

We are now almost ready for our method. Define $q_k = 1 - p_k$ and $r_k = p_k/q_k$. The ratio of the probability of an event and its complementary probability has a special name, namely the "odds" of the event. r_k is thus the odds for an opportunity at the kth event. These odds r_k for $k = 1,2, \ldots,n$ play a central role in the following algorithm. Hence its name:

Odds-algorithm

Write p_k, q_k and r_k in three lines and write each line in reverse order, that is, beginning with k = n:

(i) $p_n, p_{n-1}, p_{n-2}, \ldots$
(ii) $q_n, q_{n-1}, q_{n-2}, \ldots$
(iii) $r_n, r_{n-1}, r_{n-2}, \ldots$

Each r_k is the quotient of the numbers above it. Now we sum up the odds in line (iii) until the value 1 is reached or just exceeded. This yields the sum $R_s = r_n + r_{n-1} + \ldots + r_s \geq 1$ with a stopping index s (if the sum of odds never reaches 1 then we set s = 1). Then we compute from (ii) the product $Q_s = q_n q_{n-1} \ldots q_s$. This is all we need for the main result.

Optimal strategy and win probability. The optimal strategy is to stop from s onwards on the first opportunity (if any).
The optimal win probability W is the product of R_s and Q_s, that is

$$W = R_s\, Q_s.$$

Note that the odds-algorithm gives us the optimal strategy and optimal value at the same time. Moreover, in the general case no other method could possibly do this more quickly, that is, the algorithm is optimal itself.

Let us apply this to our examples.

Dice problem. Here each 6 is an opportunity. Hence p_k = 1/6 for all k. Therefore q_k = 5/6 and $r_k = p_k/q_k$ = 1/5 for all k. Summing them up backwards yields 1/5 + 1/5 + … and the value 1 is reached (exactly) after the fifth step. Therefore it is optimal to stop on a 6 from the 5th-last toss onwards. The corresponding optimal success probability is then R_s $Q_s = (5/6)^5 = 0.4019$, that is, just above 40%.

Sports car problem. Suppose among the phone calls you received there were eight who showed real interest and made a sufficiently serious impression on you for you to show them your car. Remember that once you refuse an offer it will be lost for good. Without additional information, you may assume that all arrival orders are equally likely. Hence the kth offer will be the best so far with probability $p_k = 1/k$. The independence of opportunities may somewhat surprise us but follows immediately from a well-known theorem on relative ranks of random permutations (from the work of A. Rényî). Hence the odds-algorithm gives the optimal strategy: the series $r_8 + r_7$ + … yields the sum 0.1428 + 0.1666 + 0.2 + 0.25 + 0.3333 = 1.093, where we stop, that is, at the 5th term. Therefore it is optimal to sell for the first H from the 4th client onwards. The corresponding optimal success probability equals W = $R_s\, Q_s$ = 1.093 * 0.375 = 0.4099, thus close to 41%.

Clinical trial problem. If the doctor (or researcher) has good estimates of p_k, the algorithm can be applied as before. However, this is generally not the case. Therefore we need an alternative, which will be discussed later on.

But before that it is important to comment on the success probabilities we obtained above.

Success and pseudo-success. A success probability of around 40%, as we found in the solutions of the dice game and the sales problem may not seem impressive. Many decision-makers may have the feeling that they usually do better. However, we must be careful; feelings and facts are often quite different. If a real estate dealer sells a villa after four weeks for the best offer so far, he looks successful both to himself and his clients. Who cares now that the owners had originally considered a four month horizon for selling the villa. Potential offers from the next three months are now wiped out because there won't be further visits after the sale. Perhaps the next offer would have been higher. Therefore what is currently perceived as a success is in reality only a pseudo-success. In our model, however, a success is a true success, that is, the best offer that would arrive if observations could have been continued; thus it means something much stronger. We can also show that, if there are offers, then the probability of a success or a pseudo-success is always above 63.4% in our model, whatever the number of offers!

Whether you are a politician who would like to find the best time to render an argument efficient, or a manager who would like to pick a good time for a deal, what we said remains true. Our definition of a success is global for the decision period and therefore it is demanding. If we realise such a success in practice then it usually means something substantial.

The odds-algorithm for unknown odds

In the clinical trial example, unlike the two other introductory examples, we can find p_k (and thus the odds) neither by simple logic nor often from other sources. The odds must therefore be estimated sequentially from preceding observations. In many real life problems the situation is quite similar. It is nice to see that even here the odds-algorithm offers solutions that are close to optimal. So let us see how we tackle this example.

One difficulty in the case of unknown odds is that we cannot compute the stopping index from the beginning. Therefore the optimal success probability is, a priori, also unknown. At time k, if there is an opportunity, we must base our decision whether to stop, or go on, on what we know up to time k. Suppose we had at time k a good estimation of the future odds, denoted by $\hat{r}_k, \hat{r}_{k+1}, \hat{r}_{k+2}, \ldots \hat{r}_n$. Then, if the estimates were the true odds we would stop at time k if their sum from k+1 onwards were smaller than 1. Otherwise we would not stop before k+1 because if $\hat{r}_{k+1} + \hat{r}_{k+2} + \ldots + \hat{r}_n \geq 1$ we must at least wait until k+1. This is our guideline. We need not know anything unless we have an opportunity but then we act as if our estimates are the true values of the odds.

Now, how do we estimate the odds? The estimates $\hat{r}_k$ should use all information available, of course. They should also have two important statistical properties: the so-called unbiasedness and consistence qualities. Roughly speaking, these properties mean that on average we estimate precisely and, as k increases, the estimates converge to the true values. Moreover, these estimates should be easy to compute.

Let us first suppose that all p_k are the same, that is $p_k = p$ where p is unknown. Then all r_k are constant with $r_k = r$ (r unknown). The condition $\hat{r}_{k+1} + \ldots + \hat{r}_n < 1$ now becomes $(n - k)\hat{r}_k < 1$, or $n - k < 1/\hat{r}_k$. Let G_k be the number of observed opportunities until time k inclusive. Then we can show that the estimates $\hat{r}_k = G_k/(k + 1 - G_k)$ comprise all the desired properties. This leads to the following approach:

Let p_k be constant, $p_k = p$, where p is unknown. We propose the following algorithm, which we call the odds-algorithm "with updating". Let G_k be the number of observed opportunities until time k inclusive (G_k is known at time k).

Strategy. If time k offers an opportunity (implying that $G_k \geq 1$), then take it if $n - k < (k + 1 - G_k)/G_k$, otherwise go on. This strategy is asymptotically optimal and in general a good approximation to the optimal strategy.

The case of different unknown odds. The problem makes perfect sense as long as r_k can be estimated. An important such case is when p_k is of the form $p_k = p\ f_k$ with p again unknown but where f_k is known. In the clinical trial example we may consider p as the "inner" success probability of a treatment whereas each factor f_k is seen as a factor reducing this success probability (according to the health condition of the kth patient). Here we would choose $0 < f_k < 1$ decreasing in accordance with decreasing general state of health of the patient. In general, the choice of f_k depends on the specific problem. In other problems it may be increasing or even oscillating (as in the case with seasonal factors). We will not discuss the corresponding odds-estimators but point out that the essence remains the same, that is, the corresponding strategy is in analogy to what we have seen before.

Questions of applicability

Following our decision strategy may lead to the following conflict. We may suddenly face an opportunity that seems so extraordinarily favourable to us that it seems foolish not to take it even though we have not yet reached the optimal stopping index obtained from the theory. How should we react to this in practice? Note that, in this case, we are in a situation where an event provides additional information because we recognize the event as exceptional. But this means that we now have more information than we had when we formulated our model. The odds-strategy is optimal with respect to the information that is incorporated in the model! New information implies a new conclusion; we should accept the opportunity.

The clinical trial example merits a few comments on its own. Physicians and researchers in the health sector may view the odds-algorithm with a certain scepticism. The author would agree that decision problems in clinical trials are, in many ways, among the most difficult ones. Two specific points may make things harder. Firstly, the evaluation of a treatment (as a + or a –) may take more time than we have to prepare the next decision. Thus the sequential information comes in with a (random) delay. This slows down the estimation of p_k. However, note that this lies in the nature of the problem. It does not question the model or the method itself. Secondly, some patients may insist on a treatment. The conscientious doctor will repeat and underline his reasoning but if the request of the patient cannot be influenced by such probabilistic arguments then it becomes a confirmed priority of the patient. The doctor is allowed to respect it and will in general do so. But then, this is no longer a medicinal-strategic decision problem.

In general, we must remember that we should see the difficulties in relation to the objective. A physician who tries to obtain with maximal probability all the possible successes without causing avoidable suffering should earn profound respect. Our approach may be no more than a first step but from an ethical point of view the defined objective is not just distinguished; it stands out!

Adapting problems

What can the odds-algorithm offer to you, dear reader, when your own decision problems are quite different from those we have treated? The answer is that it offers some flexibility and this may allow you to adapt your problem or to treat parts of your problem. It also allows the solving of certain problems where n, the total number of observations, is not known in advance. We give a few examples.

Politics. You would like to win the election. During the election compaign you need arguments to win votes. It is important that these arguments are convincing but placing them at the right time is equally important. It so often happens that an argument is forgotten when just a few months ago it attracted so much interest. Repeating it now no longer seems of much use. The counter-arguments of the other party have ripened in the time that has passed. Here again, with a suitable model defining events and opportunities, it is the last opportunity which plays a central role. Who knows in advance how many events may possibly lead to opportunities? It is the simplicity of the model that allows us to overcome this problem. We define opportunities, as before, as interesting events, whereas "non-events" are by definition a trivial form of non-interesting events. It suffices to consider in real life each day k as an experiment that gives rise to an event with probability e_k and which is then (independently) interesting with probability g_k. Independence allows us to set $p_k = e_k\ g_k$ and to apply the odds-algorithm as before. Here is an example. The political argument concerns the problem of unemployment, for example. The other party is cautious and mentions unemployment only if they have to, on an average of once every two weeks. Therefore $e_k = 1/14$. Suppose you estimate your chance as one in three that your argument would fit well as a response to what is said if something is said. Then set $g_k = 1/3$ and hence

$p_k = e_k\, g_k = 1/42$. This would be the simplest model. Often you know more such as the date when the next unemployment figures will be released or when your opponent has to speak before trade unions and so on. Try to fix your ideas on the direction that e_k and/or g_k will change.

Examine your hypotheses but don't be afraid of the details! The impact of some errors in the assumptions is typically softened by a good strategy rather than amplified. But we advise the use of the approach in breadth. Make use of smaller models for each of your good arguments and soon you may have, with a little effort, an arsenal of strategies.

Speculation. This is another field of application. We do not encourage you to play on the stock market but if you want to or have to the following may be of interest. It is the dream objective of each trader or investor to buy cheapest or sell highest in the decision period. This is not an intelligent objective, however, because the success probability is typically much too small. Also, stock prices today are clearly dependent on what they were over the previous days. Hence the independence assumption is strongly violated and thus the odds-algorithm does not make much sense for the dream objective. Consider now a more modest goal: buying in your fixed period in the last "cup" (the price went down but will go up the next day). This is a success because it enables you to sell right away although perhaps with a smaller percentage gain. Firstly, the estimation of cup probabilities (respectively "cap" probabilities if you want to sell) is certainly less difficult than the longer term prediction of the price trajectory. Secondly, the opportunities are now defined in terms of change of prices and not through prices themselves. The independence assumption now makes sense. In other words, we have adapted the problem by choosing a reasonable sub-objective for which the odds-algorithm can be applied.

A look at the Mathematics behind

Why does the odds-algorithm give us the optimal strategy? We now outline the proof. First we confine our interest to the class of strategies that wait until some fixed time (s – 1) and then stop, if possible, on the first opportunity from s onwards. Each such strategy has a success probability that depends on s and of course on $p_1, p_2, \ldots, p_n$. There are several ways to compute it but the following trick (which at first may seem strange) makes things elegant and transparent.

Define for each observation a new variable that takes the value 1 if it is an opportunity and 0 otherwise. Another way of saying that there is exactly one opportunity from s onwards is saying that the sum of the new variables from s onwards exactly equals 1. Using independence and so-called generating functions we obtain a transparent expression for the desired probability. The odds-algorithm does the optimization, that is, it determines the optimal deterministic stopping index s, denoted s*, and the corresponding success probability of this strategy.

But why is the overall optimal strategy in this restricted class of strategies? The essence of the arguments is as follows. Whenever we want to stop on a finite sequence there must exist some index, possibly random, from which we are willing to stop on the first occasion! Hence all relevant strategies are in this enlarged class with possibly random s. The mentioned transparence of probabilities allows us to see that our problem constitutes a so-called monotone case (see Chow et al). This implies that once it is optimal to stop on an opportunity then it will be optimal to do so ever after. Hence it suffices to determine the optimal "beginning", that is, the smallest possible optimal s. Using independence and monotonicity arguments we can see that this smallest index can neither be stricly smaller than s* nor can it be strictly larger. Hence it must coincide with our s* detected by the odds-algorithm.

Legal aspects

The reason for deciding one way or the other often looks like a thin argument as soon as it becomes clear that the decision was wrong. Hence those who make an "error" are typically in a bad position to defend themselves. However unjust a reproach might be, it typically persists.

In the worst case, things can go before an investigation committee. Our model and the odds-algorithm may look modest but they will make a strong point in this situation. You can be sued for fraud, dishonesty, irresponsible negligence and even for incompetence but never for bad luck. If we show that we have modelled our decision and thought about a strategy to deal with it, we give substance to the proof of having truly thought about the problem. An accusation of negligence becomes untenable. Moreover, knowing that the strategy we applied is optimal with respect to our model, we need not fear that somebody may come up with a "better" method we allegedly should have known. Any attempt to accuse us of incompetence would be inconsistent.

Thus, all that remains to our adversary (or plaintiff) is to try to show that our hypotheses were wrong. But such an attempt is, in everyday life, almost surely bound to fail. It is not logical to discuss statements that are, a priori, undecidable. First of all, if one never accepts working with a model where hypotheses are not quite satisfied, one would never use a model at all. Secondly, in a probability model for an unknown future, by definition things go wrong with a positive probability even if our hypotheses are perfectly justified. So then, what would be the plaintiff's point? Refusing models altogether? Making no decisions when it is our task to make them?

These points, taken together, are likely to render our adversary powerless. Even more, the arguments may even backfire. Sometimes, as here, mathematics has strong words to say in everyday life. Therefore, a strategy that is easy to apply and is proven optimal for making a decision should attract our attention, also from the point of view of legal implications.

Post scriptum:

Since its publication, the author has received many remarks and questions (there are now several entries and little programs from independent contributors on the Web; see e.g. "odds algorithm" and "Bruss algorithm".

Questions and comments have proved to be an encouragement to all of us to try to write articles in science communication. I particularly remember an exchange with a German professor in space navigation control and another with two French physicians discussing a medical decision problem. However, I also remember the many young people among whom a group of computer game fanatics went through great efforts to implement the odds-algorithm to become the "guru of the Satyrnokillers", as one of them claims. Although this objective would not figure on my list of examples, I must admire the work of these youngsters to understand and to model their problem. Probably the nicest surprise for myself is a pre-print written by four French scientists working in the Automation Research Center in Nancy who implement the odds-algorithm to propose an innovative automatic online repair concept – a promising idea that most probably would not have come to my mind.

Questions I could not really answer concerned the applicability of the odds-algorithm for search engines. Indeed, we all feel that the odds–algorithm, given its simplicity and speed even for unknown odds, should indeed be useful for never-go-back routing strategies in search engines and expert systems. However, I have no experience in looking at concrete problems of this type.

Editorial comment

This article was first published in German in the magazine *Spektrum der Wissenschaft* (German edition of *Scientific American*), June 2005. Meanwhile, translations have appeared in *Pour la Science* and *Al Oloom* (French and Arab editions of *Scientific American*). It was awarded the 1st prize in the article competition 2005 of the EMS Raising Public Awareness Committee (cf. EMS-Newsletter 60, June 2006).

The present English version is without the original illustrations which are property of *Scientific American*.

References

Sum the Odds to one and Stop, F. Thomas Bruss, Annals of Probability, Vol. 28, 1384–1391, (2000).

A Note on Bounds for the Odds-Theorem of Optimal Stopping, F. Thomas Bruss Annals of Probability, Vol. 31, 1859-1862, (2003).

The Theory of Optimal Stopping, Y.S. Chow, Herbert Robbins and David Siegmund, Dover New York, (1991).

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Picking a random integer

Michał Adamaszek (Warsaw, Poland)

We shall start with the following, spontaneously arising question. For a given positive integer k, what is the probability that a random positive integer is divisible by k? It seems reasonable that this chance equals $\frac{1}{k}$ but at a second glance it is not that simple ...

Probability measures

Our *probabilistic space* is the set $\mathbb{N}$ (assume $\mathbb{N} = \{1,2,\ldots\}$). The task we first encounter is to define a feasible probability measure on $\mathbb{N}$. There are (uncountably) many ways to do this, such as,

$$P(A) = \sum_{n\in A} \frac{1}{2^n}$$

for any $A \subset \mathbb{N}$. It is routine to check that all the axioms of probability are satisfied. Observe, however, that $P(\{2,4,6,8,10,\ldots\}) = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots = \frac{1}{3}$, which is different from what we might intuitively feel. We now see that what we first need is to choose a measure of probability that best suits our problem.

The requirements are as follows. Every set of the form

$$k\mathbb{N} = \{kn : n \in \mathbb{N}\}$$

(numbers divisible by k) should belong to the σ-algebra of sets measured by P. This σ-algebra will in fact be the entire power set $2^{\mathbb{N}}$ due to the identity,

$$\{k\} = k\mathbb{N} \setminus \bigcup_{m=2}^{\infty} (mk)\mathbb{N}$$

Moreover, we want $P(k\mathbb{N}) = \frac{1}{k}$ for all $k \in \mathbb{N}$ since this is what we consider natural. More formally, we are hereby stating the following.

Question. Does there exist a function $P\colon \mathcal{F} \mapsto [0,1]$, with $\mathcal{F} = 2^{\mathbb{N}}$, such that

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n) \qquad (1)$$

for $A_1, A_2, \ldots \in \mathcal{F}$ pairwise disjoint, and

$$P(k\mathbb{N}) = \frac{1}{k} \text{ for every } k \in \mathbb{N} \qquad (2)$$

Answer. This is a classical problem with a *negative answer*. The key hint is the Borel lemma, cf., e.g., [1]. We shall not dwell on the proof of this fact but move on to further questions that arise at this point.

The most natural one is 'what if we give up on one of the above conditions?'. For instance, weakening condition (2) leads to:

Task. Construct a probability on $\mathbb{N}$ that satisfies (2) for prime numbers k.

Densities and finitely additive measures

An alternative path, followed in the rest of this paper, is to abandon countable additivity in condition (1) in favour of finite additivity, preserving our key requirement (2) unchanged. The support of such a function P will be an *algebra* $\mathcal{F}'$ of subsets of $\mathbb{N}$ (a family closed under *finite* sums and complements) generated by the sets $k\mathbb{N}$. Condition (1) is then replaced with,

$$P(A \cup B) = P(A) + P(B) \text{ for disjoint } A, B \in \mathcal{F}' \qquad (3)$$

Notice that the set algebra $\mathcal{F}'$ generated by the sets $k\mathbb{N}$ is not the entire power set $2^{\mathbb{N}}$, in contrast to the previously utilized σ-algebra $\mathcal{F}$. More precisely, consider all finite Boolean formulas ϕ consisting of the operands *'and'*, *'or'* and *not*, with a finite number of clauses of the form $\alpha_i \geq k_i$, where $k_i \geq 0$ is an integer and α_i is a variable. Now $\mathcal{F}'$ is the family of all sets of the form,

$$\left\{n \in \mathbb{N} : n = 2^{\alpha_2} \cdot 3^{\alpha_3} \cdot 5^{\alpha_5} \cdot 7^{\alpha_7} \cdot \ldots, \phi(\alpha_2, \alpha_3, \alpha_5, \alpha_7, \ldots)\right\}$$

for all the formulas ϕ described above. For example

$$\left(\mathbb{N} \setminus 49\mathbb{N}\right) \cup 18\mathbb{N} = \left\{n \in \mathbb{N} : \alpha_7 < 2 \vee \left(\alpha_2 \geq 1 \wedge \alpha_3 \geq 2\right)\right\}.$$

Let us now ask a more philosophical question. Why does our intuition suggest that $\frac{1}{k}$ should be the probability of divisibility by a given k? This is probably due to our attachment to the *ordering* of the set $\mathbb{N}$. We tend to think of what happens in this (smallest) infinite set by restricting it to some (presumably very long) finite interval consisting of the numbers smallest with respect to the natural ordering. This approach is present in the following definition of *density* of a set $A \subset \mathbb{N}$.

$$\rho(A) = \lim_{n\to\infty} \frac{\#(A \cap \{1,2,\ldots,n\})}{n}$$

(as long as the limit exists, we shall say that such an A *has density* and denote the family of all such sets by $\mathcal{D}$). Notice now that $\rho(k\mathbb{N}) = \frac{1}{k}$. It can be seen that density behaves like finite additive probability: $\rho(\emptyset) = 0$, $\rho(\mathbb{N}) = 1$, $0 \leq \rho(A) \leq 1$, and most importantly,

$$\rho(A \cup B) = \rho(A) + \rho(B)$$

for any two disjoint sets $A, B \subset \mathbb{N}$ that have density.

Density, however, has serious drawbacks. First of all, it cannot be defined for all subsets of $\mathbb{N}$. For example, consider the set,

$$M = \bigcup_{n=1}^{\infty} \{2^{2n}, 2^{2n}+1, \ldots, 2^{2n+1}-1\}$$

The lower and upper limits of the sequence in the definition of density for this set are $\frac{1}{3}$ and $\frac{2}{3}$, respectively. To make matters worse consider the sets,

$$M_1 = \bigcup_{n=1}^{\infty} \{2^{2n}+1, 2^{2n}+3, \ldots, 2^{2n+1}-1\} \cup \bigcup_{n=1}^{\infty} \{2^{2n+1}, 2^{2n+1}+2, \ldots, 2^{2n+2}-2\}$$

$$M_2 = 2\mathbb{N}$$

Clearly both of them have density equal to $\frac{1}{2}$. Unfortunately,

$$M_1 \cup M_2 = M \cup (2\mathbb{N})$$

is a set without density, for similar reasons as for M. Hence we may conclude that the family $\mathcal{D}$ of sets that have a density is *not* an algebra. On the other hand after a tedious check one obtains $\mathcal{F}' \subset \mathcal{D}$, so the restriction of ρ to the algebra $\mathcal{F}'$ satisfies the conditions for P.

Instead of proving the last property directly we shall follow a more general approach that will yield the ultimate solution in our quest for a measure P. What we need is a way to deal with divergent sequences corresponding to sets without density (note that such sets are not our main interest; we just need to keep them under control to get a consistent theory). We shall utilize the following beautiful theorem.

Theorem – The Banach limit (cf., e.g., [2, 3]). Every bounded sequence $(a_n)_{n\in\mathbb{N}}$ can be assigned a real number denoted $Lim(a_n)$ in such a way that

$$Lim(\alpha a_n + \beta b_n) = \alpha Lim(a_n) + \beta Lim(b_n)$$

$$\liminf_{n\to\infty} a_n \le Lim(a_n) \le \limsup_{n\to\infty} a_n$$

$$Lim(a_n) = Lim(a_{n+1}).$$

The second condition implies that $Lim(a_n) = \lim_{n\to\infty} a_n$ whenever (a_n) is convergent (this justifies the notation, too). It means that *Lim* is an extension of lim to all bounded sequences that preserves the most crucial properties of lim as outlined in the three conditions above.

There are at least two ways to define *Lim*. One is to use the Hahn-Banach extension theorem from functional analysis ($Lim\colon l^\infty \mapsto \mathbb{R}$ is clearly an extension of the linear functional $\lim\colon c \mapsto \mathbb{R}$ with some extra properties). Another approach uses ultrafilters on $\mathbb{N}$. As one may expect, both methods make substantial use of techniques related to Zorn's lemma.

Now we find it easy to address our main problem. First, define a new, generalized density with the formula,

$$\rho(A) = Lim \frac{\#(A \cap \{1,2,\dots,n\})}{n}$$

for any $A \subset \mathbb{N}$. It follows from the properties of *Lim* that $\rho(A) = \rho(A)$ for all $A \subset \mathbb{N}$ that have density and $0 \le \rho(A) \le 1$ for all $A \subset \mathbb{N}$. Finite additivity of ρ is analogous to finite additivity of ρ. We are now in a position to formulate a theorem that yields even more than first expected: a function P defined not just on $\mathcal{F}'$ but in fact on all subsets of $\mathbb{N}$.

Theorem. There is a finitely additive probability measure P on all subsets of $\mathbb{N}$ (i.e., a function $P\colon 2^{\mathbb{N}} \mapsto [0,1]$ satisfying $P(A\cup B) = P(A) + P(B)$ for any disjoint $A, B \subset \mathbb{N}$, $P(\emptyset) = 0$, $P(\mathbb{N}) = 1$) that extends density, that is, $P(A) = \rho(A)$ for all sets $A \subset \mathbb{N}$ that have density. It is defined by $P(A) = \rho(A)$.

In conclusion let us notice that the whole problem could be solved more easily thanks to proper generalization. With the introduction of condition (3) we entered the world of linear functions and could use the rich theory of functional analysis.

Densities with respect to a permutation

In the context of the previously introduced definition of density the following question also seems interesting. What if we abandon the natural ordering of $\mathbb{N}$? Suppose that for any permutation σ of $\mathbb{N}$ we define the *density with respect to* σ as,

$$\rho_\sigma(A) = \lim_{n\to\infty} \frac{\#(A \cap \{\sigma(1), \sigma(2), \dots, \sigma(n)\})}{n}$$

Let us go back to countable additive probability theory and ask whether there exists a reordering (that is, a permutation σ) of $\mathbb{N}$ and a probability P on $\mathbb{N}$ such that $P(k\mathbb{N}) = \rho_\sigma(k\mathbb{N})$ for every $k \ge 1$? The well-known fact mentioned earlier states that this will not work for any P if $\sigma = id_{\mathbb{N}}$.

We need not restrict ourselves to the sets $k\mathbb{N}$ here. Instead, consider any family $\mathcal{A} \subset 2^{\mathbb{N}}$ ($\mathcal{A} = \{k\mathbb{N}\colon k \in \mathbb{N}\}$ is an example). We are looking for a probability P and a permutation (reordering) σ such that $P(A) = \rho_\sigma(A)$ for all $A \in \mathcal{A}$. In a large number of cases the trivial probability $P = \delta_1$ turns out to be useful. To this end we introduce the following definition.

Definition. We say a family $\mathcal{A} \subset 2^{\mathbb{N}}$ *can be made sparse* if there exists a permutation σ of $\mathbb{N}$ such that $\rho_\sigma(A) = 0$ for every set $A \in \mathcal{A}$.

This notion is useful in the following way. Suppose $\mathcal{A}$ is a family that can be made sparse and additionally $1 \notin A$ for every $A \in \mathcal{A}$. Choose a suitable permutation σ as in the definition above and let $P = \delta_1$. Then $P(A) = \rho_\sigma(A) = 0$ for all $A \in \mathcal{A}$.

The next theorem tells us which countable families can be made sparse. First a terminology note. We say a set $A \subset \mathbb{N}$ has a co-property if its complement $\mathbb{N} \setminus A$ has this property.

Theorem. A countable (possibly finite) family $\mathcal{A} \subset 2^{\mathbb{N}}$ can be made sparse if and only if the union of any finite subfamily of $\mathcal{A}$ is co-infinite.

Proof. The necessary condition is easy. Suppose that for some $A_1, \dots, A_n \in \mathcal{A}$ the set $B = \cup_{i=1}^n A_i$ has finite complement. Then $\rho_\sigma(B) = 1$, regardless of σ. However, for the permutation σ that makes $\mathcal{A}$ sparse we have $\rho_\sigma(A_i) = 0$ for all $i = 1, \dots, n$. On the other hand, density has the following property. If $\rho(X) = \rho(Y) = 0$ then $\rho(X \cup Y) = 0$. This yields a contradiction.

To prove sufficiency we have to construct σ. First of all we may assume that $\bigcup \mathcal{A} = \mathbb{N}$ (otherwise arrange the remaining numbers into singleton sets and add them to $\mathcal{A}$). Let $A_1, A_2, \dots$ be the sequence of all sets in $\mathcal{A}$. Let $X = \{n \in \mathbb{N}\colon A_n \setminus \bigcup_{i=1}^{n-1} A_i \ne \emptyset\}$. Observe that $\mathbb{N} = \bigcup_{n\in X} A_n$, which means that X is infinite (otherwise our assumption that no finite subfamily of $\mathcal{A}$ sums up to a co-finite set would be violated). For each $n \in X$ choose an element $a_n \in A_n \setminus \bigcup_{i=1}^{n-1} A_i$ and arrange these elements into the set R (it is also infinite). Let $S = \{1, 4, 9, \dots\}$ be the set of squares. Choose σ to be any permutation of $\mathbb{N}$ such that $\sigma(\mathbb{N} \setminus S) \subset R$.

To see that σ makes $\mathcal{A}$ sparse fix any $A_k \in \mathcal{A}$. Since $a_n \notin A_k$ for $n > k$, at most a finite number of elements of A_k is contained in the set $\{\sigma(n)\colon n \in \mathbb{N} \setminus S\}$. This implies that $\#(A_k \cap \{\sigma(1), \dots, \sigma(n)\})$ is $O(\sqrt{n})$ as $n \to \infty$. This completes the proof.

Finally let us note that, as a special case, when $\mathcal{A} = \{k\mathbb{N} : k \geq 2\}$ we obtain a solution to our original problem.

Bibliography

[1] P. Billingsley, *Probability and measure*, Wiley, 1979, chap. 1, sect. 4.
[2] John B. Conway, *A course in functional analysis*, Springer-Verlag, 1994.
[3] W. Rudin, *Functional analysis*, McGraw-Hill, 1991.

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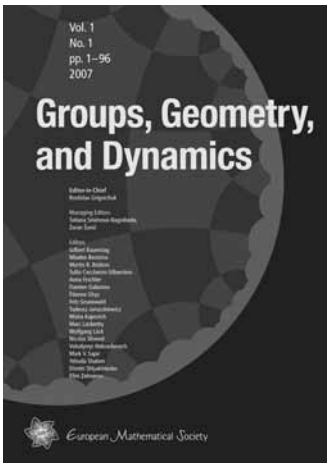
The original Polish version of this article appeared under the title "Losowanie liczby naturalnej" in the journal Delta*, issue 7 (2005). The Newsletter thanks the editors for the permission to reproduce it in English.*



Bruno de Finetti – a great probabilist and a great man

Giuseppe Anichini (Firenze, Italy)

Bruno de Finetti was born in 1906. One hundred years later, in the year 2006, the birth of this mathematician is celebrated everywhere in the (mathematical) world through a series of meetings, events and conferences. In Italy the Accademia Nazionale dei Lincei, the Mathematics Department of the University of Roma "La Sapienza" and the Unione Matematica Italiana (UMI) are going to put in action a series of initiatives. The UMI, for instance, aims to promote a very big event in September and also to publish two volumes in the series "Opere dei Grandi Matematici" in which a Selecta of the most important papers of de Finetti will be collected together. In the first volume there will be about forty papers concerning the scientific work that de Finetti did in probability and statistics; in the second there will be work that de Finetti did in economics, financial science, actuarial science, mathematical analysis, and (mathematical) education and popularization.

Biography

As a member of the committee appointed (by the UMI) to choose the papers of the Selecta, I would like to recall some facts relating to the life and scientific thinking of this famous mathematician.

Bruno de Finetti was born in Innsbruck to Italian parents; they were also Austrian citizens as he himself wrote in an autobiographic note accompanying the book [1] edited by his former students and friends on the occasion of his 75th birthday. In 1906 his father was working in Innsbruck as a railway constructor; he was an engineer as was his father before him. Thus it was no surprise when in 1923 Bruno de Finetti enrolled at Milano Polytechnic. There he discovered his true passion for mathematics and during his third year at Milano Polytechnic, perhaps inspired by a paper by the biologist Carlo Foà, he started research in the field of population genetics, which soon led him (aged twenty years) to the first of almost three hundred writings. It was the first example of a model with overlapping generations in population genetics and it was many years ahead of its time. Even today, bioscience researchers quote the results of the young de Finetti.

He then moved to the recently founded University of Milan and there, in 1927, he graduated in mathematics with a dissertation on affine geometry. Among his teachers at the University of Milan, it is worth mentioning Oscar Chisini, who is well known in statistics for his general definition of "Chisini's mean".

At the time de Finetti received his degree, a position was waiting for him in Roma at the Italian Central Statistical Institute, which was founded and directed by Corrado Gini. De Finetti remained there until 1931, after which he moved to Trieste and started working for a big insurance company. There he worked as an actuary and also on the mechanization of some actuarial services. In the following years, he supplemented his work with several academic appointments both in Trieste and Padova. In 1947 he became a full professor firstly at the University of Trieste and subsequently at "La Sapienza" University of Roma, where he remained until the end of his career.

Numerous letters, memoranda, newspaper clippings, articles and court documents give evidence of de Finetti's political and social activism. It is worth considering his vital interest in civil aspects and social justice [6]. His longing for social justice caused him in the 1970's to be a candidate in several elections and he was also arrested for his antimilitarist position. At the time of his death in 1985, Bruno de Finetti was an honorary fellow of the Royal Statistical Society as well as a member of the UMI, a member of the International Statistical Institute and a fellow of the Institute of Mathematical Statistics. Additionally, in 1974, he had been elected a corresponding member, and then a full member, of the Accademia dei Lincei. Many details on his life are given in the papers of M.D. Cifarelli and E. Regazzini [5] (in which a broad picture of the scientific milieu in which Bruno de Finetti took the first steps of his scientific career is given) as well as by L. Daboni [6], who was appointed by the UMI to the official commemoration for the Bollettino of the UMI (the society's major journal). Some significant flashes of the history of probability in Italy, in which de Finetti played the main role, are given in [7] and [8].

A summary of de Finetti's scientific contributions

Bruno de Finetti is known worldwide as one of the most important probabilists and statisticians of the 20th century. In fact, even in his former position in Roma, he was laying the foundations for his principal contributions to probability theory and statistics: the subjective approach to probability (i.e., the *operational subjective* conception of probability), the definition and analysis of sequences of exchangeable events, the definition and analysis of processes with stationary independent increments and infinitely decomposable laws, and the theory of mean values (it is worth remarking that in this period he qualified as a university lecturer of mathematical analysis; the examiners were Giuseppe Peano, Mauro Picone and Salvatore Pincherle). De Finetti started working on probability and statistics in a period of tremendous development for these subjects. For instance A. N. Kolmogorov and P. Lévy were giving their decisive contributions to the modern theory of probability and R.A. Fisher was setting out the basic technical concepts for his new approach to statistics. In Italy

At the 2nd Berkeley Symposium, from left to right: M. Loeve, P. Lévy, W. Feller, B. de Finetti

Guido Castelnuovo, Francesco Paolo Cantelli, Corrado Gini and young de Finetti became impressed by this cultural revival. Moreover a very big event took place over these years, i.e., the International Mathematical Union (IMU) Congress in Bologna. The former president of the UMI, Salvatore Pincherle, successfully worked, at the end of the world war, to get together all the people who had a keen interest in mathematics irrespective of nationality. So around 840 mathematicians assembled in Bologna, among them de Finetti and the most famous probabilists and statisticians of the time: Maurice Fréchet, Aleksandr Y. Khinchin, Paul Lévy, Jerzy Neyman, Ronald A. Fisher and George Pólya.

Let us summarize the main scientific contributions of Bruno de Finetti: Major research topics studied by de Finetti were probability (subjective theory, calculus, Bayes's theorem) and statistics but also mechanization, genetics, mathematical analysis, mathematics applied to economics (game theory, financial and actuarial mathematics), and popularization and educational mathematics.

Mechanization: as stated above, working in an insurance company probably contributed to making him one of the first mathematicians to be aware of the possibilities offered by computing machinery. Later on, after 1950, in the position of adviser to the Italian Research Council (CNR), he was instrumental in getting the first electronic computer to the INAC (National Institute for Applied Computation) in Roma, whose director at the time was Mauro Picone.

Genetics: even today a de Finetti diagram is used to graph the genotype frequencies of populations where there are two alleles and the population is diploid. It is based on an equilateral triangle and on the theorem that from any point within the triangle the sum of the lengths of the three lines from that point to the sides of the triangle, where these lines are perpendicular to the sides, is equal.

Mathematical analysis: at the beginning of his scientific career de Finetti studied the characteristic properties of vectorial analysis with regard to the case of projective homographies (bijective maps between linear spaces). Subsequently he considered some very important topics in mathematical analysis like measures in abstract spaces, the Riemann-Stieltjes integral and convex stratifications. The latter are now known as quasiconvex functions (or quasi concave as W. Fenchel named them later on). Since then convex and quasiconvex analysis has been widely applied in many fields such as optimization theory, game theory, and linear and nonlinear programming.

Economics: Bruno de Finetti's interest in economics was innate and led him during his first year at the Milano Polytechnic to attend the lectures given there by Ulisse Gobbi, who was later the dean of the important economics and financial studies at "Bocconi" University. The lectures, in turn, confirmed his radical position, which he himself summarised as follows in his autobiographical note [1]:

> …the only directive of the whole of economics, freed from the damned game and tangle of individual and group egoisms, should always be the realisation of a collective Pareto optimum inspired by some criterion of equity.

Educational: it is worth emphasizing the devotion of such a great scientist to mathematical education topics. As Carla Rossi (one of de Finetti's pupils) said in [8] the substance of de Finetti's approach and ideas of teaching can be found in his every scientific paper even more than in the many works specifically devoted to that issue. And about problems to take into consideration he used to say

> … before approaching a problem to solve you need to see it, in order that a subject of study, specifically Mathematics, does not appear sterile, obscure and useless, it should always be presented so that studying it is fully and genuinely justified.

Involvement in school reform and teaching methodology was one of his major interests throughout his life. A wide variety of materials illustrate de Finetti's efforts to improve science and mathematics teaching, teacher education, and school curricula, e.g., his writings *Il Saper vedere in matematica* (Know-how in Mathematics) and *Perchè la matematica?* (Why Mathematics?), his plans for educational movies, and a project for an educational centre for teachers.

Now let us shortly outline the major contributions given by de Finetti in the fields of probability and statistics theory.

Probability and statistics

The classic exposition of his distinctive theory are the papers [2] and [3], in which he discussed probability founded on the coherence of betting odds and the consequences of exchangeability. A "summa" of Bruno de Finetti's revolutionary ideas

Bruno de Finetti (1979)

on probability can be found in the two volumes [4] of his best known book *Teoria della Probabilità* (1970), which was translated into English in 1975. However, his contributions to probability and statistics do not reduce to his subjective approach; they include important results on finitely additive measures, processes with independent increments, sequences of exchangeable variables and associative means (see the review [5] for details on these).

(a) The concepts of probability in de Finetti's time: until the 1920's it was tacitly assumed that the frequentist interpretative ideas of probability played the main role in the various applications of the discipline, or at least the most popular methods of assessment were based on a combinatorial approach or on an observed frequency. The idea of subjective probability was almost surely undermined by the developments in physics after a few considerations on it given by a couple of French scientists (E. Borel (1924) and P. Lévy (1925); see [10] for a suitable reference).

Since his early days as a mathematician, Bruno de Finetti revitalized the theory of subjective probability in a very different spirit with respect to the past. De Finetti probabilism (as he called it in [2]) is the "*true heir of the empiricist philosophical tradition in the spirit of David Hume … de Finetti was a prodigy who could make his philosophical and conceptual ideas match his mathematical developments*" [10]. In an interesting correspondence with M. Fréchet in the 1930's de Finetti, discussing some papers concerning the almost sure convergence of a sequence of independent and identically distributed bernoullian random variables, states that the problems concerning stochastic convergence are mere signs of a deeper problem concerning the correct mathematical definition of probability. As a matter of fact, in his opinion, the definition has to adhere to the intuitive notion of probability as it is conceived by every one of us in usual everyday life. He maintains that one has no right to make arbitrary use of the properties introduced to give a mathematical definition of probability. Indeed, these very properties have to be not only formally consistent but also intrinsically necessary with respect to a meaningful interpretation of probability. De Finetti shares Fréchet's opinion implying that each concept, even of a mere mathematical nature, is more or less directly triggered by intuition. Nevertheless, this definition can effectively be arbitrary, provided that one confines oneself to deduce purely formal conclusions from it. This turns out to be the case in the definition of measure. A different case is connected with the definition of weight, since *we cannot force a pair of scales to work according to our definition*.

De Finetti proposed a thought experiment along the following lines (a philosophical gambling strategy): you must set the price of a promise to pay 1 (lira in de Finetti's time) if, for instance, there was life on Mars one billion years ago and 0 if there was not, and tomorrow the answer will be revealed. You know that your opponent will be able to choose either to buy such a promise from you at the price you have set, or require you to buy such a promise from them, still at the same price. In other words, you set the odds but your opponent decides which side of the bet will be yours. The price you set is the operational subjective probability that you assign to the proposition on which you are betting. This price has to obey the probability axioms if you are not to face certain loss, as you would if you set a price above 1 (or a negative price). It is seen that in any application of probability theory we can interpret the probabilities as personal degrees of belief of a rational agent; this is the term reserved for a person who will not accept a Dutch book. By considering bets on more than one event de Finetti could justify additivity. Prices, or equivalently odds, that do not expose you to certain loss through a Dutch book are called coherent. Probability will be the *degree of belief* assigned by you to the *occurrence* of an event.

The mathematical formulation of probability $\mathbb{P}$ was given in [2]. Given a class E of events and an element A of the class, any $p \in [0,1]$ represents a coherent assessment on A. After defining a probability de Finetti proves that the usual rules of the calculus of probability are necessary for the coherence of $\mathbb{P}$ on E, i.e., he states the well known properties (except σ-additivity):

If $\mathbb{P}$ is a probability on a class E, we have:

1. $A \in E \Longrightarrow \mathbb{P}(A) \in [0,1]$;
2. $\Omega \in E \Longrightarrow \mathbb{P}(\Omega) = 1$ (here Ω is the certain event);
3. if $A_1, \cdots, A_n \in E, \cup_{k=1}^n A_k \in E$ and $A_i \cap A_j = \emptyset$ for $i \neq j$ then $\mathbb{P}(\cup_{k=1}^n A_k) = \sum_{k=1}^n \mathbb{P}(A_k)$.

These classical properties, i.e., the fact that $\mathbb{P}$ is a function whose range lies between 0 and 1 (these two extreme values being assumed by, but not kept only for, the *impossible* and the *certain* events respectively) and which is additive for mutually exclusive events, constitute the starting point in the axiomatic approach; so de Finetti can rightly claim that the subjective view can only enlarge and never restrict the practical purport of probability theory.

Subsequently, in 1949, with regard to the problem of existence of at least a probability on a given class of events, he provided the following extension theorem:

If A and B are classes of events such that $A \subset B$ and $\mathbb{P}_1$ is a probability on A, then there is a probability $\mathbb{P}_2$ on B such that $\mathbb{P}_1 = \mathbb{P}_2$ on A.

He also showed that the previous methods (i.e., the combinatorial and the frequentist methods) can be recovered if some useful (even if very particular) methods of coherent evaluation are considered; they are subjective as well and they are unnecessarily restricted to the domain of applicability. De Finetti also makes absolutely clear the distinction between the subjective character of the notion of probability and the objective character of the elements (i.e., events) to which it refers.

Although there is no reason why different interpretations (senses) of a word cannot be used in different contexts, there is a history of antagonism between the followers of de Finetti (sometimes called Bayesians) and frequentists, with the latter often rejecting the subjective interpretation as ill-grounded. The groups have also disagreed about which of the two senses reflects what is commonly meant by the term probable. In the preface of many books concerning probability theory there is a wide trace of this controversial dispute. Today the long wave of the subjective approach of de Finetti is growing more and more in the field of assessments of probability. All the work of de Finetti exhibits an intuitionist and constructivist view, with a natural bent for submitting the mathematical formulation of probability theory only to the needs required by any practical application.

(b) Stochastic processes with independent increments: the crisis of determinism and of the casuality principle introduces a novelty into the scientific method. Rigid laws stating that a certain fact is bound to occur in a certain way are being replaced by probabilistic or statistical laws stating that a certain fact can occur depending on a variety of ways governed by probability laws. Thus, given a scalar quantity whose temporal evolution is described by $X = X(t)$, $t \geq 0$, one assumes that the values taken by $X(t)$ are known for $t \leq t_0$ and considers the conditional increment $\{(X(t) - X(t_0)/X(u), u \leq t_0, t > t_0\}$. As far as the probability distribution function $F(\cdot)$ of such an increment is concerned, de Finetti considers the three cases:

1. $F(\cdot)$ is independent of $X(u)$ for every $u \in [0,t_0]$ - ($F(\cdot)$ is called known);
2. $F(\cdot)$ is independent of $X(u)$ for every $u \in [0,t_0)$ - ($F(\cdot)$ is called differential);
3. $F(\cdot)$ is dependent on $X(u)$ on $[0,t_0]$ - ($F(\cdot)$ is called integral).

De Finetti deals with the problem of characterizing the probability distribution of $X(t)$: if $X(0) = 0$ and ϕ_t, ψ_t denote the probability distribution function and the characteristic function of $X(t)$ respectively, then $\{\psi_{\frac{1}{n}}(\cdot)\}^n$ is the characteristic function of the sum of n independent increments, identically distributed according to the law $X(t) - X(0)$. In modern literature these processes are known as processes with homogeneous independent increments and $\psi_1[= (\psi_{\frac{1}{n}})^n]$ is called the infinitely decomposable characteristic function; de Finetti shows that ψ_t is continuous whenever X is continuous on $[0,+\infty)$ and $X(t)$ is different from ct. Moreover the examples chosen to emphasize the relevance of the continuity for X are very noteworthy: the Poisson process and the compound Poisson process. The method de Finetti uses here is quite innovative with respect to the past. Finally he achieves the well known result:

The class of infinitely decomposable laws coincides with the class of distributions limits of finite convolutions of distributions of Poisson type.

This result was a starting point for a subsequent series of papers by A. N. Kolmogorov and P. Lévy.

(c) Exchangeability: the works of P. Levy and G. Castelnuovo (from 1925 to 1928) taught him the analytical tools for arriving at one of the most important results in the theory of probability, i.e., the concept of exchangeability of events (1928), followed (in 1929) by the probability laws of continuous time random processes.

With regard to the connections between the subjective viewpoint and the objective one, which in a different way characterizes the classical approach and the frequentist approaches, these procedures are, according to de Finetti, not necessarily conducive to the existence of an objective probability. But, if the classical probability assignment can be justified immediately by judging the events equally probable, the analysis of the frequentistic point of view is more complex. To do that de Finetti broke the analysis down into two steps (explaining their subjective foundations): the first deals with the relations between the assignments of probabilities and the prevision of future frequencies; the second concerns the relationship between the observation of past frequencies and the prevision of future frequencies.

Let us consider a sequence of events $E_1, E_2, \cdots$ relative to a sequence of trials and suppose that, under the hypothesis H_N stating a certain result of the first N events, a person considers equally probable the events $E_{N+1}, E_{N+2}, \cdots$. Then, denoting by f_{H_N} the prevision of the random relative frequency of the occurence of n events $E_{N+1}, E_{N+2}, \cdots, E_{N+n}$ conditional to H_N, the well known properties of a prevision yield $p_{H_N} = f_{H_N}$, where p_{H_N} indicates the probability of each $E_{N+1}, E_{N+2}, \cdots$ conditional to H_N. But when is it possible to estimate f_{H_N} in such a manner? De Finetti's answer is: when the events considered are supposed to be elements of a stochastic process whose probability law, conditional on a large sample, admits, as prevision of the future frequencies, a value approximately equal to the frequency observed in these samples. Since the choice of the probability law governing the stochastic process is subjective, the prevision of a future frequency based on the observation of those past is naturally subjective. This procedure is perfectly admissible when the process is *exchangeable*, that is when only information about the number of successes and failures is relevant, irrespective of exactly which trials are successes or failures.

De Finetti defines a sequence of events to be *equivalent* (the word "exchangeable" was proposed later by Pólya) in a communication at the above mentioned IMU Congress of Bologna. Subsequently de Finetti was able to justify the evaluation of f_{H_N} via past frequencies thanks to some important representation theorems (see [5] for a suitable reference).

(d) The de Finetti–Kolmogorov–Nagumo theorem: de Finetti worked in the field of statistics firstly by approaching descrip-

tive statistics and afterwards inductive reasoning. We confine ourselves to the first argument leaving to the considerations illustrated in the previous section the main ideas of statistical inference. "Reasoning by induction" means, according to de Finetti's interpretation, learning from experience, and this thought provoking remark is clear enough and wholly pervasive.

In a paper of 1931 de Finetti obtained a significant extension of a theorem independently proved by Kolmogorov and Nagumo. To this aim he extends Chisini's definition of a mean to distribution functions in the following way: *given a class F of frequency distribution functions on $\mathbb{R}$ and a real valued function f on F, a mean of ϕ in F, with respect to the evaluation of f, is any number ρ such that $f(\phi) = f(D_\rho)$ where D_ρ denotes the probability distribution function which degenerates at x.*

Subsequently let $A, B, A < B$ be real numbers and let $F = F(A,B)$ denote the class of all distribution functions whose support is included in $[A,B]$; moreover he defines $m : F \to \mathbb{R}$ through $f(D_{m(\phi)}) = f(\phi)$ for any distribution $\phi \in F$. Finally the result is given:

Suppose that $m : F[A,B] \to \mathbb{R}$ is a consistent, strictly increasing and associative mean. Then there is a function ψ, continuous and strictly increasing in $[A,B]$, for which $m(\phi) = \psi^{-1}(\int_{\mathbb{R}} \psi(x) d\phi(x))$, $(\phi \in F(A,B))$. Moreover ψ is uniquely determined up to linear transformations. Conversely, if m is defined as before for a function ψ with the properties stated, then it satisfies consistency, strict monotonicity and associativity.

The latter properties of consistency, strict monotonicity and associativity identify with a well know definition, in terms of random gains and of stochastic dominance.

Final remark

It is strange that the summary of a lifetime of work on the theory of something should begin by the declaration that something does not exist but so begins de Finetti's Theory of Probability [4]: *My thesis, paradoxically, and a little provocatively, but nonetheless genuinely, is simply this: Probability does not exist.* This conveys his idea that probability is an expression of the observer's view of the world and as such it has no existence of its own. As a consequence of the subjective approach, statistical inference is no longer an empirical process producing opinions from data but it becomes a logical-psychological process selecting opinions compatible with data among the available ones. In de Finetti's theory, bets are for money, so your probability of an event is effectively the price that you are willing to pay for a lottery ticket that yields 1 unit of money if the event occurs and nothing otherwise; de Finetti used the (Italian) notation 'Pr' to refer interchangeably to Probability, Price and Prevision (foresight) and he treated them as alternative labels for a single concept. The appeal of his money based definition is that it has the same beauty and simplicity as theories of (modern) physics; the measurements are direct and operational, they involve exchanges of a naturally conserved quantity and their empirical laws are deducible from a single governing principle, namely the principle of coherence or non-arbitration. The coherence condition can also be shown to be very useful for welfare evaluations, where it provides a natural foundation for utilitarianism. Starting from [3], where the famous argument of de Finetti on decision under uncertainty is presented, economists tried to develop an argument stating a natural condition that turns out to imply the existence of coherent subjective probabilities and can justify a model of choice based on them; de Finetti's idea served later as a point of departure for Savage's theory of subjective expected utility (see [9]).

We conclude by quoting de Finetti himself.

> The only relevant thing is uncertainty – the extent of our knowledge and ignorance. The actual fact of whether or not the events considered are in some sense determined, or known by other people, and so on, is of no consequence.

Bibliography

[1] B. de Finetti *Scritti (1926–1930),* Cedam, Padova, 1981.

[2] B. de Finetti, *Sul Significato Soggettivo della Probabilità,* Fundamenta Mathematicae, vol. 17, 1931, 298–329.

[3] B. de Finetti, *La prévision: ses lois logiques, ses sources subjectives*, Annales de l'Institute Henri Poincaré, vol. 7, 1937, 1–68.

[4] B. de Finetti, *Theory of Probability*, (translation of 1970 book) 2 volumes, New York, Wiley, 1974–75.

[5] D. M. Cifarelli and E. Regazzini, *de Finetti's contribution to probability and statistics*, Statistical Science, vol. 11, 1996, 253–282. Or: D. M. Cifarelli and E. Regazzini, *Il contributo di de Finetti alla probabilità e alla statistica*, Pristem/Storia, Note di Matematica, Storia e Cultura, vol. 2, pag 55–113, Springer Italia, 1999.

[6] L. Daboni, *Bruno de Finetti*, Bollettino dell'Unione Matematica Italiana, Serie VII, vol. 1-A, 1987, 283–308.

[7] E. Regazzini, *Teoria e Calcolo delle Probabilità*, La Matematica Italiana dopo l'Unità. Gli anni tra le due guerre mondiali. Ed. S. di Sieno, A. Guerraggio, P. Nastasi; Marco y Marcos, Milano, 1998.

[8] C. Rossi, *Bruno de Finetti: the mathematician, the statistician, the economist, the forerunner*, Statistics in Medicine, vol. 20, 2001, 3651–3666.

[9] L. J. Savage, *The Foundation of Statistics*, Dover Publications, 1972.

[10] J. von Plato, *Creating Modern Probability*, Cambridge Studies in Probability, Induction, and Decision Theory, Cambridge University Press, 1994.

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Kurt Gödel (1906–1978)

Ralf Schindler (Münster, Germany)

Kurt Gödel is certainly the most important logician of the first half of the 20th century. Mathematical logic and set theory were still in their infancy in the 1920s and 30s and it was during this time that Gödel achieved some seminal results that had influence far into the future: the Completeness Theorem, the Incompleteness Theorems and his theorems on the consistency of the Axiom of Choice and of Cantor's Continuum Hypothesis. He developed methods that are still used today in contemporary logic and set theory, albeit in a refined and significantly extended manner.

Kurt Gödel was born one hundred years ago on 28th April 1906, 75 miles north of Vienna in the Czech town of Brno, which was then the capital of the Austrian-Hungarian margraviate Moravia. The "Moravian Compromise" that was made at the time was intended to relieve tensions between the Czechs and Germans in Moravian Parliament but the irascibility of nationalism completely incapacitated the Cisleithanian Parliament around 1907 so that from 1909 there was an authoritarian rule by Law of Decree. Gödel's family belonged to the German minority. The Gödels were wealthy; Kurt's father was the director of a textile factory and he was the first in Brno to own a Chrysler. In 1918, after the corrupt Habsburg regime had collapsed, the Czechoslovakian Republic was founded and the Gödels became Czechoslovakian citizens. "Sudenten German" parties continued to act aggressively and they would later find maniac support from the National Socialists.

After leaving his German high school in Brno, Kurt Gödel goes to Vienna to study physics, mathematics and philosophy. From 1926 he starts attending the Thursday evening meetings of the Vienna Circle, which are organized by Moritz Schlick. Other participants are Rudolf Carnap, Herbert Feigl, Hans Hahn and Otto Neurath. It is likely that the atmosphere of the Vienna Circle stimulated Gödel's interest for the foundational questions of mathematics. The key questions that were discussed in those days were exactly the kind of ones that Gödel would focus on in his research and which would receive complete answers in his PhD thesis and his *Habilitationsschrift*.

In 1928 he first meets Adele Nimbursky, nee Porkert, who works as a dancer in a Viennese night club, the "Nachtfalter". Apparently their relationship caused some uproar within the Gödel family. It is not until ten years later that Adele und Kurt get married. In 1929, Gödel is granted Austrian citizenship. On 6th July 1929 Gödel is awarded his PhD with his thesis, "Über die Vollständigkeit des Logikkalküls" or "On the completeness of logic calculus", supervised by Hans Hahn. However, it was probably Carnap (a student of Frege's) who drew Gödel's attention to the issue of completeness. A proof of a sentence j from a given system of axioms G (i.e. a set of sentences) is a finite sequence of sentences with final member j such that each member of the sequence is either an element of G or else results from previous sentences in the sequence via rules from some fixed calculus of logic. All the standard logic calculi for first order logic prove the very same sentences. Gödel's *Completeness Theorem* says that there is a reason for this and that there is no need for extending any such logic calculus: if φ is true in all models of Γ then there is actually a proof of φ from Γ. As proofs are finite, an immediate corollary is the *Compactness Theorem*: if Γ is a system of axioms such that every finite subset of Γ has a model, then Γ itself has a model. Using the Compactness Theorem it is very easy to construct non-standard models of Peano Arithmetic or Analysis or to construct countable models of set theory.

Gödel with his wife Adele on their wedding day in Vienna in 1938

Hilbert and Ackermann formulated the problem of completeness of first order calculus very clearly. The arguments of Skolem (who wrote in the 20s) almost yield a proof of the Completeness Theorem but they display a certain lack of Platonic courage and do not construct the necessary model. Gödel himself mentions an unpublished manuscript by Carnap that presents a Completeness Theorem for some other system but it is Gödel himself who deserves the credit for having shown what we now call *the* Completeness Theorem. Gödel talks about his result at a meeting in Königsberg (now Kaliningrad) on 6th September 1929 but its significance is not immediately clear. Gödel's original proof was simplified later by Leon Henkin and G. Hasenjäger, among others, who wrote their dissertations on this area in 1947 and 1950 respectively (twenty years after Gödel's discovery!).

Present-day model theory has spectacularly refined the methods of model construction as first seen in the proof of the Completeness Theorem. Such tools led E. Hrushovski to a proof of the geometric Mordell Lang conjecture.

Gödel's father dies in 1929. Kurt Gödel now lives off the inheritance. In 1932, he submits his *Habilitations-*

schift and he becomes a *Privatdozent* on 11th March 1933. Gödel never holds a more serious position than this at the University of Vienna; he teaches and receives the fees that the students have to pay for attending his courses. Gödel's first (and only) permanent position will be the one he will get at the Institute for Advanced Studies.

David Hilbert had asked for a finitistic proof of the consistency of the axioms of number theory (where 'finitistic' was never formally defined). This task, i.e. Hilbert's second problem, is also known as "Hilbert's Program". Gödel showed in his *Habilitationsschrift* that such a consistency proof is impossible in principle: let Γ be a consistent and recursively enumerable axiom system that is sufficiently strong in that it contains a certain fragment of number theory (the required fragment of number theory is weaker than the usual system of Peano Arithmetic). Gödel's First Incompleteness Theorem says that there is a sentence φ such that neither j nor its negation ¬φ is provable in Γ. His Second Incompleteness Theorem gives an example of such a sentence φ. The proof exploits the diagonal argument that Cantor had used for showing that there are more reals than integers. With the help of what we now call 'Gödelization', i.e. a simply computable identification of expressions of the formal language with integers, Gödel produces a sentence φ in the language of number theory that expresses "I am not provable in Γ", which cannot then be provable in G and which is therefore also true. In fact, φ = "I am not provable in Γ" is not provable in Γ if and only if j is true if and only if Γ is consistent. Therefore the consistency of Γ is not provable in Γ.

As a corollary, Gödel's proof yields the fact that the truth predicate cannot be definable in formal language, as otherwise we could consider the sentence expressing "I am not true". Since these issues become more exciting the less they are understood, Gödel is now ubiquitous in popular mathematics literature where his achievements are not always reported accurately.

Gödel speaks about his incompleteness results at a meeting of the German Mathematical Society in Bad Elster in September 1931. Ernst Zermelo (the father of what is now the standard axiomatization of set theory, ZFC), whom Gödel meets there for the first time, is not able to fully grasp the content of Gödel's new theorem. Others have problems as well. Hilbert had learned about Gödel's Incompleteness Theorem in 1931 and he immediately realized its impact on his program. In fact, the task of showing the consistency of the axioms of number theory by finitistic means completely collapses in the light of Gödel's result! The first textbook presentation of the Incompleteness Theorem was produced by Hilbert and P. Bernays in 1939.

Modern-day proof theory studies in detail by what means we may prove the consistency of systems of number theory or analysis. Using the seminal work of Gentzen in the 30s, we know that having the resources of induction along a well-ordering of length ε_0 suffices (and this is the best possible).

Gödel spends the academic year 1933–34 at the Institute for Advanced Studies in Princeton, New Jersey. He has to get psychiatric treatment in the fall of 1934; he spends at least one week in a hospital in Purkersdorf near Vienna. It was not to be his last such stay.

A relative of the Incompleteness Theorem is Church's theorem on the undecidability of the set of all sentences that are provable in an axiom system G that is sufficiently strong. Gödel was always interested in questions of computability. In the 30's, Alan Turing started investigating what was to become the foundation of theoretical computer science. Gödel's paper, "On the lengths of proofs", from 1936 deals with questions of complexity theory, again anticipating later developments.

In 1934 the Austrian government declares all political parties to be illegal except for the "Vaterländische Front" or "Patriotic front". Dollfuß aims for an alliance with fascist Italy against the German Reich. In the course of the attempted National Socialist coup on 25th July 1934 Dollfuß is killed. Germany stops its support as Italian troops deploy at the Brenner.

During this turbulent time Gödel becomes interested in set theory and Hilbert's first problem. In 1934 he succeeds in showing the relative consistency of the Axiom of Choice, AC, with the remaining axioms of the standard axiom system of set theory, ZF. This proves that despite the fact that AC has apparently paradoxical consequences, like Hausdorff's decomposition of the unit ball, the addition of AC to ZF does not produce inconsistencies. During the night of 14th June 1937 Gödel finishes his proof, according to which the Generalized Continuum Hypothesis, GCH, is also consistent relative to ZFC = ZF + AC. This result is published in 1938 in the Proceedings of the National Academy of Sciences, USA.

Both of his relative consistency results are shown with the help of the inner model, L, of all constructible sets, which he isolated for the purpose of these proofs. L is the closure of the class of all ordinals under a finite class of simple set theoretical functions (namely, the generators for the rudimentary functions). This implies that L is in fact the least (transitive) model of ZF that contains all the ordinals. L may be stratified in a uniformly shaped hierarchy that allowed Gödel to prove not only ZF but also AC in L. The Condensation Lemma for L finally leads to an elegant proof for (a strong version of) the Generalized Continuum Hypothesis to be true in L.

In much the same way as Gödel's Completeness Theorem and Incompleteness Theorems resulted in the end of an era of naivety in logic, Gödel's discovery of L turned set theory into a highly non-trivial subject. In the 70's Ronald Jensen started the task of refining Gödel's investigations into constructability. Jensen's resulting "Fine Structure Theory" proves some spectacular theorems. This theory and the Theory of Inner Models, both of which are very active branches of contemporary set theory, yield important and surprising insights into the local and global structure of the mathematical universe.

During the years 1935–38 Gödel frequently visits the US. His phobia problems seem to be worsening. He develops a strong fear of being poisoned, especially through his food. He never recovers from this mental illness and it eventually proves fatal.

Gödel with Einstein in Princeton in 1950

In 1938, Schuschnigg visits Hitler in Berchtesgaden, where the two agree on installing Seyß-Inquart as the new Austrian Minister for the Interior. 12th March 1939 sees Austria's annexation ("Anschluß") to Germany and on 2nd April 1939 Hitler arranges a celebration on the *Heldenplatz* in Vienna. Gödel automatically becomes German.

Kurt Gödel is politically naive. In 1939 he is attacked by a party of young National Socialists near the *Strudlhofstiege* in Vienna. Rumour has it that Adele successfully defends him with the help of her umbrella. During March 1939 the title of "Privatdozent" is abandoned and "Dozent neuer Ordnung" ("docent of new order") is established. A seemingly hopeless war of paperwork starts that is supposed to arrange Gödel's emigration to the US. Adele and Kurt finally leave Vienna on 18th January 1940. They travel by train to Vladivostok via Berlin and Moscow. They depart from Yokohama by steamer to San Francisco where they arrive on 4th March 1940. From here they continue to Princeton, New Jersey. Kurt Gödel will never again travel back to Europe.

Gödel becomes a US citizen in 1948. He now works at the Institute for Advanced Studies but he only becomes a full professor in 1953. His attempts to show the independence of the Continuum Hypothesis from ZFC fail. He becomes interested in the theory of general relativity (Einstein is his colleague at the IAS) as well as philosophy. Being a Platonist, Gödel never holds the view that the Continuum Problem would be settled if the Continuum Hypothesis were shown to be independent from ZFC.

Exactly this is achieved by Paul Cohen, who harks from the field of harmonic analysis. He invented the method of "forcing" by which real numbers may be adjoined to countable models of ZFC in such a way that the extended models still satisfy ZFC but also the negation of the Continuum Hypothesis. There is thus finally a mathematically significant statement that observes Gödel's First Incompleteness Theorem with respect to ZFC. Present-day set theory offers an amazing variety of such statements. Cohen picked up the 'stamp of approval' from Gödel himself. In 1966, Cohen received the only Fields Medal which has been given so far to honour work in mathematical logic. Later, Cohen lost interest in set theory. However, his achievement and its further development by R. Solovay and others turned set theory into a difficult and active area of research. Set theory, as we know it now, is only forty years young!

Gödel expected that some day convincing axioms would be isolated that would refute the Continuum Hypothesis. In the 70's, he speculated about candidates for such axioms in his works on "scales". His "square axioms" state, among other things, that for each integer n the cofinality of the set of all functions from ω_n to ω_n modulo the Fréchet filter is equal to ω_{n+1}. Unfortunately, Gödel's experiments were unsuccessful. It seems that Gödel held the opinion (at least at certain times) that there should be $\aleph_2$ real numbers.

The question about the size of the continuum is still one of the strong (but sometimes hidden) driving forces of contemporary set theory research. The "Proper Forcing Axiom", which generalizes Martin's Axiom and which is very useful in set theoretical topology, implies that there are $\aleph_2$ real numbers. The arguments that show this implication are somewhat similar to the arguments Gödel tried to exploit in his work on "scales". W. Hugh Woodin recently presented an elaborate line of reasoning that tries to refute the Continuum Hypothesis, relying on facts about forcing absoluteness and deep insights in descriptive set theory, which in part are obtained by methods of constructability. Woodin's arguments were attacked by M. Foreman and J. Steel but they are certainly a stimulus for present-day research.

In the US Gödel was no longer able to produce mathematical results at the same level as he did in Vienna. He receives a lot of honours, for instance the "National medal of science" but in June 1977 Adele Gödel needs surgery and without his wife to feed him Gödel refuses to eat at all. He dies on 14th January 1978 from "malnutrition and inanition caused by personality disturbance". His wife dies three years later.

Oskar Morgenstern writes in his diary: "He [Gödel] was very funny in his mixture of depth and unworldliness." Gödel's outstanding legacy is omnipresent in logic and set theory. His collected works appear as

Gödel, Kurt, *Collected works*, 5 volumes, S. Feferman et al, eds., Oxford University Press, 1986ff.

*This article first appeared in German in DMV-Mitteilungen **14**–1(2006), pp. 42–45. It was translated into English by the author. He thanks Gunter Fuchs for his comments.*

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Interviews with three Fields medallists

Interviewers: Vicente Muñoz (Madrid, Spain) and Ulf Persson (Göteborg, Sweden)

Andrei Okounkov

How did you get interested in mathematics?

The most important part of becoming a mathematician is learning from one's teachers. Here I was very fortunate. Growing up in Kirillov seminar, I had in its participants, especially in Grisha Olshanski, wonderful teachers who generously invested their time and talent into explaining mathematics and who patiently followed my first professional steps. I can't imagine becoming a mathematician without them. So it must be that in this respect my professional formation resembles everybody else's.

What was perhaps less usual is the path that led me to mathematics. I didn't go through special schools and olympiads. I came via studying economics and army service. I had family before papers. As a result, my mind is probably not as quick as it could have been with an early drilling in math. But perhaps I also had some advantages over my younger classmates. I had a broader view of the universe and a better idea about the place of mathematics in it. This helped me form my own opinion about what is important, beautiful, promising etc.

It also made mathematics less competitive for me. Competition is one of those motors of human society that will always be running. For example, we are having this interview because of the outcome of a certain competition. But I believe it distracts us from achieving the true goal of science, which is to understand our world.

So, you wouldn't say that competition is the best way to do mathematics?

I think it is a mistake that competition is actively promoted on every level of math. While kids take solving puzzles perhaps a bit too seriously, grown-ups place the ultimate value on being the first to prove something. A first complete proof, while obviously very important, is only a certain stage in the development of our knowledge. Often, pioneering insights precede it and a lot of creative work follows it before a particular phenomenon may be considered understood. It is thrilling to be the first, but a clear proof is for all and forever.

How do you prefer to work on mathematical problems? Alone or in collaboration?

Perhaps you can guess from what I said before that I like to work alone, I equally like to freely share my thoughts, and I also like to perfect my papers and talks.

There may well be alternate routes, but I personally don't know how one can understand something without both thinking about it quietly over and over and discussing it with friends. When I feel puzzled, I like long walks or bike rides. I like to be alone with my computer playing with formulas or experimenting with code. But when I finally have an idea, I can't wait to share it with others. I am so fortunate to be able to share my work and my excitement about it with many brilliant people who are at the same time wonderful friends.

And when it comes to writing or presenting, shouldn't everyone make an effort to explain? Wouldn't it be a shame if something you understood were to exist only as a feeble neuron connection in your brain?

Do you prefer to solve problems or to develop theories?

I like both theory and problems, but best of all I like examples. For me, examples populate the world of mathematics. Glorious empty buildings are not my taste. I recall my teacher Kirillov saying that it is easier to generalize an example than to specialize a theory. Perhaps he did not mean this 100% seriously, but there is a certain important truth in those words. Understanding examples links with ability to compute. Great mathematicians of the past could perform spectacular computations. I worry that, in spite of enormous advances in computational methods and power, this is a skill that is not adequately emphasized and developed. Any new computation, exact or numeric, can be very valuable. The ability to do a challenging computation and to get it right is an important measure of understanding, just like the ability to prove is.

Much of your work has deep connections to physics. Does that mean that you find it essential that mathematics is related to the natural word, or that you would even think of it as subservient to the other natural sciences?

When I said "our world" earlier I didn't mean just the tangible objects of our everyday experience. Primes are as real as planets. Or, in the present context, should I say that celestial bodies are as real as primes? Throughout their history, natural sciences were a constant source of deep and challenging mathematical problems. Let's not dwell now on the obvious practical importance of these problems and talk about something else, namely the rich intuition that comes with them. This complex knowledge was derived from multitude of sources by generations of deep thinkers. It is often very mathematical. Anyone looking to make a mathematical discovery needs a problem and a clue. Why not look for both in natural sciences?

This doesn't make mathematics a subordinate of other sciences. We bring, among other things, the power of abstraction and the freedom to apply any tools we can think of, no matter how apparently unrelated to the problem at hand. Plus what we know we really do know.

So we can build on firmer foundations, hence higher. And look – mathematics is the tallest building on campus both in Princeton and in Moscow.

There is a common view of the public that computers will make mathematicians superfluous. Do you see a danger in that? And in particular what is your stand on computer-assisted proofs? Something to be welcomed or condemned?

Computers are no more a threat to mathematicians than food processors are a threat to cooks. As mathematics gets more and more complex while the pace of our lives accelerates, we must delegate as much as we can to machines. And I mean both numeric and symbolic work. Some people can manage without dishwashers, but I think proofs come out a lot cleaner when routine work is automated.

This brings up many issues. I am not an expert, but I think we need a symbolic standard to make computer manipulations easier to document and verify. And with all due respect to free market, perhaps we should not be dependent on commercial software here. An open source project could, perhaps, find better answers to the obvious problems such as availability, bugs, backward compatibility, platform independence, standard libraries, etc.

One can learn from the success of TEX and more specialized software like Macaulay2. I do hope that funding agencies are looking into this.

The age of the universalists is gone. Nowadays mathematics is very diverse and people tend to get mired into sub-specialities. Do you see any remedy to this?

Mathematics is complex. Specialization, while inevitable, doesn't resolve the problem. Mathematics is a living organism; one cannot simply chop it up. So how we both embrace and resist specialization?

We can be better neighbours. We shouldn't build high fences out of sophisticated words and "you wouldn't understand" attitude. We should explain what we know in simplest possible terms and minimal generality. Then it will be possible to see what grows in the next field and use the fruits of your neighbour's labour.

A good social contact makes good neighbours. Effective networks are hard to synthesize but they may be our best hope in the fight against fragmentation of mathematics. I, personally, wouldn't get anywhere without my friends/collaborators. I think there is a definite tendency in mathematics to work in larger groups, and I am certain this trend will continue.

Terence Tao

When did you become interested in mathematics?

As far as I can remember I have always enjoyed mathematics, though for different reasons at different times. My parents tell me that at age two I was already trying to teach other kids to count using number and letter blocks.

Who influenced you to take the path of mathematics?

I of course read about great names in mathematics and science while growing up, and perhaps had an overly romanticised view of how progress is made; for instance, E.T. Bell's "Men of mathematics" had an impact on me, even though nowadays I realise that many of the stories in that book were overly dramatised. But it was my own advisors and mentors, in particular my undergraduate advisor Garth Gaudry and my graduate advisor Eli Stein, who were the greatest influence on my career choices.

What was your feeling when you were told about being a medallist?

I had heard rumours of my getting the medal a few months before I was officially notified –- which meant that I could truthfully deny these rumours before they got out of hand. It was still of course a great surprise, and then the ceremony in Madrid was an overwhelming experience in many ways.

Do you think that the Fields Medal will put too high expectations on you, thus coming to have an inhibiting influence?

Yes and no. On the one hand, the medal frees one up to work on longer term or more speculative projects, since one now has a proven track record of being able to actually produce results. On the other hand, as the work and opinions of a medallist carry some weight among other mathematicians, one has to choose what to work on more carefully, as there is a risk of sending many younger mathematicians to work in a direction which ends up being less fruitful than first anticipated. I have always taken the philosophy to work on the problems at hand and let the recognition and other consequences take care of themselves. Mathematics is a process of discovery and is hence unpredictable; one cannot reasonably try to plan out one's career, say by naming some big open problems to spend the next few years working on. (Though there are notable exceptions to this, such as the years-long successful attacks by Wiles and Perelman on Fermat's last theorem and Poincaré's conjecture respectively.) So I have instead pursued my research organically, seeking out problems just at the edge of known technology whose answer is likely to be interesting, lead to new tools, or lead to new questions.

Do you feel the pressure of having to obtain results quickly?

I have been fortunate to work in fields where there are many more problems than there are people, so there is little need to competitively rush to grab any particular problem (though this has happened occasionally, and has usually been sorted out amicably, for instance via a joint paper). On the other hand, most of my work is joint with other collaborators, and so there is an obligation to finish the research projects that one starts with them. (Some projects are over six years old and still unfinished!) Actually, I find the "pressure" of having to finish up joint work to be a great motivator, more so than the more abstract motivation of improving one's publication list, as it places a human face on the work one is doing.

What are your preferences when attacking a problem?
It depends on the problem. Sometimes I just want to demonstrate a proof of concept, that a certain idea can be made to work in at least one simplified setting; in that case, I would write a paper as short and simple as possible, and leave extensions and generalisations to others. In other cases I would want to thoroughly solve a major problem, and then I would want the paper to become very systematic, thorough, and polished, and spend a lot of time focusing on getting the paper just right. I usually write joint papers, but the collaboration style varies from co-author to co-author; sometimes we rotate the paper several times between us until it is polished, or else we designate one author to write the majority of the article and the rest just contribute corrections and suggestions.

Do you spend a lot of time on a particular problem?
If there is a problem which I ought to be able to solve, but somehow am blocked from doing so, that really bugs me and I will keep returning to the problem to try to resolve it. (Then there are countless problems which I have no clue how to solve; these don't bother me at all).

Do you prefer to solve problems or to develop theories?
I would say that I primarily solve problems, but in the service of developing theory; firstly, one needs to develop some theory in order to find the right framework to attack the problem, and secondly, once the problem is solved it often hints at the start of a larger theory (which in turn suggests some other model problems to look at in order to flesh out that theory). So problem-solving and theory-building go hand in hand, though I tend to work on the problems first and then figure out the theory later.

Both theory and problems are trying to encapsulate mathematical phenomena. For instance, in analysis, one key question is the extent to which control on inputs to an operation determines control on outputs; for instance, given a linear operator T, whether a norm bound on an input function f implies a norm bound on the output function Tf. One can attack this question either by posing specific problems (specifying the operator and the norms) or by trying to set up a theory, say of bounded linear operators on normed vector spaces. Both approaches have their strengths and weaknesses, but one needs to combine them in order to make the most progress.

How important is physical intuition in your work?
I find physical intuition very useful, particularly with regard to PDE – I need to see a wave and have some idea of its frequency, momentum, energy, and so forth, before I can guess what it is going to do – and then, of course, I would try to use rigorous mathematical analysis to prove it. One has to keep alternating between intuition and rigour to make progress on a problem, otherwise it is like tying one hand behind your back.

What point of view is helpful for attacking a problem?
I also find it helpful to anthromorphise various mathematical components of a problem or argument, such as calling certain terms "good" or "bad", or saying that a certain object "wants" " to exhibit some behaviour, and so forth. This allows one to bring more of one's mental resources (beyond the usual abstract intellectual component of one's brain) to address the problem.

Many mathematicians are Platonists, although many may not be aware of it, and others would be reluctant to admit it. A more "sophisticated" approach is to claim that it is just a formal game. Where do you stand on this issue?
I suppose I am both a formalist and a Platonist. On the one hand, mathematics is one of the best ways we know to try to formalise thinking and understanding of concepts and phenomena. Ideally we want to deal with these concepts and phenomena directly, but this takes a lot of insight and mental training. The purpose of formalism in mathematics, I think, is to discipline one's mind (and filter out bad or unreliable intuition) to the point where one can approach this ideal. On the other hand, I feel the formalist approach is a good way to reach the Platonic ideal. Of course, other ways of discovering mathematics, such as heuristic or intuitive reasoning, are also important, though without the rigour of formalism they are too unreliable to be useful by themselves.

Is there nowadays too much a separation between pure and applied mathematics?
Pure mathematics and applied mathematics are both about applications, but with a very different time frame. A piece of applied mathematics will employ mature ideas from pure mathematics in order to solve an applied problem today; a piece of pure mathematics will create a new idea or insight which, if the insight is a good one, is quite likely to lead to an application perhaps ten or twenty years in the future. For instance, a theoretical result on the stability of a PDE may lend insight as to what components of the dynamics are the most important, and how to control them, which may eventually inspire an efficient numerical scheme for solving that PDE computationally.

Mathematics is often described as a game of combinatorial reasoning. If so, how would it differ from a game say like chess?
I view mathematics as a very natural type of game, or conversely games are a very artificial type of mathematics. Certainly one can profitably attack certain mathematical problems by viewing them as a game between against some adversary who is trying to disprove the result you are trying to prove, by selecting parameters in the most obstructive way possible, and so forth. But other than the fact that games are artificially constructed, whereas the challenge of proving a mathematical problem tends to arise naturally, I don't see any fundamental distinctions between the two activities. For instance, there are both frivolous and serious games, and there is also frivolous and serious mathematics.

Do you use computers for establishing results?
Most of the areas of mathematics I work in have not yet been amenable to systematic computer assistance, because the algebra they use is still too complicated to be easily formalised, and the numeric works they would need (e.g. for simulating PDE) is still too computationally expensive. But this may change in the future; there are already some isolated occurrences of rigorous computer verification of things such as spectral gaps, which are needed for some arguments in analysis.

Is a computer-assisted proof acceptable from your point of view?
It is of course important that a proof can be verified in a transparent way by anyone else equipped with similar computational power. Assuming that is the case, I think such proofs can be satisfactory if the computational component of the proof is merely confirming some expected or unsurprising phenomenon (e.g. the absence of sporadic solutions to some equation, or the existence of some parameters which obey a set of mild conditions), as opposed to demonstrating some truly unusual and inexplicable event which cries out for a more human-comprehensible analysis. In particular, if the computer-assisted claims in the proof already have a firm heuristic grounding then I think there is no problem with using computers to establish the claims rigorously. Of course, it is still worthwhile to look for human-readable proofs as well.

Is mathematics becoming a very dispersive area of knowledge?
Certainly mathematics has expanded at such a rate that it is no longer possible to be a universalist such as Poincaré and Hilbert. On the other hand, there has also been a significant advance in simplification and insight, so that mathematics which was mysterious in, say, the early 20th century now appears routine (or more commonly, several difficult pieces of mathematics have been unified into a single difficult piece of mathematics, reducing the total complexity of mathematics significantly). Also, some universal heuristics and themes have emerged which can describe large parts of mathematics quite succinctly; for instance, the theme of passing from local control to global control, or the idea of viewing a space in terms of its functions and sections rather than by its points, lend a clarity to the subject which was not available in the days of Poincare or Hilbert. So I remain confident that mathematics can remain a unified subject in the future, though our way of understanding it may change dramatically.

What fields of mathematics you foresee will grow in importance, and maybe less positively, fade away?
I don't think that any good piece of mathematics is truly wasted; it may get absorbed into a more general or efficient framework, but it is still there. I think the next few decades of mathematics will be characterised by interdisciplinary synthesis of disparate fields of mathematics; the emphasis will be less on developing each field as deeply as possible (though this is of course still very important), but rather on uniting their tools and insights to attack problems previously considered beyond reach. I also see a restoration of balance between formalism and intuition, as the former has been developed far more heavily than the latter in the last century or so, though intuition has seen a resurgence in more recent decades.

There are lots of definitions of randomness. Do you think there is a satisfying way of thinking of randomness?
I do see the dichotomy between structure and randomness exhibiting itself in many fields of mathematics, but the precise way to define and distinguish these concepts varies dramatically across fields. In some cases, it is computational structure and randomness which is decisive; in other cases, it is a statistical (correlation-based) or ergodic concept of structure and randomness, and in other cases still it is a Fourier-based division. We don't yet have a proper axiomatic framework for what a notion of structure or randomness looks like (in contrast to, say, the axioms for measurability or convergence or multiplication, which are well understood). My feeling is that such a framework will eventually exist, but it is premature to go look for it now.

If you would not have been a mathematician, what career would you have considered?
I think if I had not become a mathematician, I would like to be involved in some other creative, problem-solving, autonomous occupation, though I find it hard to think of one which matches the job satisfaction I get from mathematics.

Wendelin Werner

Were you always interested in mathematics?
Well, as far as I can remember, maths was always my preferred topic at school, and I was a rather keen board-games player in my childhood (maybe this is why I now work on 2D problems?). As a child, when I was asked if I knew what I wanted to later, I responded "astronomer". In high school, just because of coincidences, I ended up playing in a movie[1] and having the possibility to try to continue in this domain, but I remember vividly that I never seriously considered it, because I preferred the idea to become a scientist, even if at the time, I did not know what it really meant. When it was time to really choose a subject, I guess that I realized that mathematics was probably closer to what I liked about astronomy (infiniteness, etc.)

Have you known about the Fields medal since an early age, and did it in anyway motivate you? In particular what was your feeling when you were told about being a medallist?
I learnt about the existence of the Fields medal quite late (when I graduated roughly). In fact, I remember some

1 'La passante du sans-souci' released in 1982 with Romy Schneider starring in her last performance.

friends telling me half-joking, half-seriously that "you will never get the Fields medal if you do this" when I told them that I was planning to specialize in probability theory (it is true that this field had never been recognized before this year).

It is of course a nice feeling to get this medal today, but it is also very strange: I really do not feel any different or "better" than other mathematicians, and to be singled out like this, while there exist so many great mathematicians who do not get enough recognition is almost embarrassing It gives a rather big responsibility, and I will now have to be careful each time I say something (even now). But again, it is nice to get recognition for one's work, and I am very happy. Also, I take it as a recognition for my collaborators (Greg Lawler and Oded Schramm) and for the fact that probability theory is a nice and important field in contemporary mathematics.

I guess that all these feelings and thoughts were present in my head when I hang up the phone after learning from John Ball in late May that I was awarded the medal. I knew that it was a possibility, but nevertheless took me by surprise.

Are there some mathematicians that you admire particularly?

I am not a specialist of history of mathematics, but I find it amazing what the great 19th century mathematicians (Gauss, Riemann,...) managed to workout – I certainly feel like a dwarf compared to giants. I have also greatest respect for those who shaped probability theory into what it is now (Kyoshi Îto, Paul Lévy, Ed Harris, Harry Kesten to mention just a few). Also, I owe a lot to the generation of probabilists that are just a little older than me (just look at the list of Loève prize winners for instance. I really felt very honoured to be on that list!) and opened so many doors.

Do you fear that the Fields Medal will inhibit you by putting up too high expectations for future work?

It is true that in a way, the medal puts some pressure to deliver nice work in the future, and that it will be probably be more scrutinized than before. On the other hand, it gives a great liberty to think about hard problems, to be generous with ideas and time with PhD students for instance. We shall see how it goes.

As you pointed out, your chosen subject has never been awarded before. Is it because it has been considered as "applied mathematics"? Would you call yourself an "applied mathematician"?

Probability theory has long been considered as part of applied mathematics. Maybe also because of some administrative reasons (in the US, probabilists often work in Statistics Departments that are disjoint from the Mathematics Departments). This has maybe let to a separate development of this field, slightly isolated. Now, people realize how fruitful interactions between probabilistic ideas and other fields in mathematics can be, and this automatic "applied" notion is fading away (even if probability can be indeed fruitfully applied in many ways). In a way, the field that I am working on has been really boosted by the combination of complex analytic ideas with probability (for instance Schramm's idea to use Loewner's equation in a probabilistic context to understand conformally invariant systems). I personally never felt that I was doing "applied" mathematics. It is true that we are using ideas, intuitions and analogies from physics to help us to get some intuition about the concepts that we working on. Brownian motion is a mathematical concept with something that resonates in us, gives us some intuition about it and stimulates us.

Is there any risk that computers will make mathematicians obsolete, say by providing computerized proofs? Or do you believe this will stimulate mathematics instead?

Well, one of my brothers is precisely working on computer-generated or computer-checked proofs. I have to be careful about what I say, especially since my own knowledge on this is very thin. I do personally not really use the computer in my work, besides TEX and the (too long) time spent with emailing. It can very well be that some day soon, computers may be even more efficient than now in helping understanding and proving things. The past years have shown how things that looked quite out of reach ended up being possible with computers.

Do you have any other interests besides mathematics?

I often go to concerts (classical music) and play (at non-professional level though) the violin. Often, I hear people saying "yes, maths and music are so similar, that is why so many mathematicians are also musicians". I think that this is only partially true. I can not forget that many of those I was playing music with as a child simply had to stop playing as adults because their profession did not leave any time or energy to continue to practise their instruments: doctors have usually much more working hours than us. Also, music is nicely compatible with mathematics because –– at least for me – it is hard to concentrate on a math problem more than 4-5 hours a day, and music is a good complementary activity: it does not fill the brain with other concerns and problems that distract from maths. It is hard to do maths after having had an argument with somebody about non-mathematical things, but after one hour of violin scales, one is in a good state of mind.

Also, but this is a more personal feeling, with the years, I guess that what I am looking for in music becomes less and less abstract and analytical and more and more about emotions –– which makes it less mathematical...

But I should also mention that, as far as I can see it, mathematics is simultaneously an abstract theory and also very human: When we work on mathematical ideas, we do it because in a way, we like them, because we find something in them that resonates in us (for different reasons, we are all different). It is not a dry subject that is separate from the emotional world. This is not so easy to explain to non-mathematicians, for which this field is just about computing numbers and solving equations.

ERCOM: CIM – Centro Internacional de Matemática

Origins

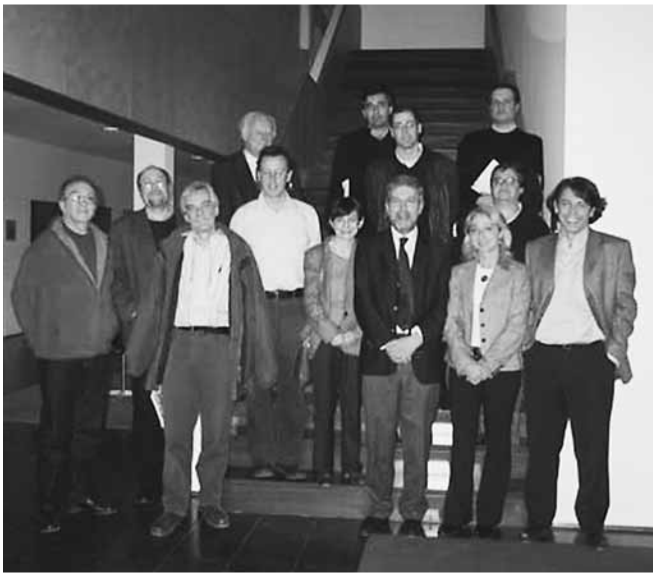
The preliminary steps in the creation of the CIM (Centro Internacional de Matemática) took place in 1991, with the support of several mathematicians such as J. M. Lemaire (Director of CIMPA) from Nice (France), Angelo Marzollo of UNESCO, and F. Hirzebruch from Bonn (Germany), who expressed his support on behalf of the European Mathematical Society. The Portuguese Mathematical Society was quite enthusiastic about the project and an ad hoc committee was formed in Coimbra, chaired by Sampaio Martins and with Craveiro de Carvalho, Paula Oliveira, João Queiró and Simões Pereira as members. This committee steered the whole process until the centre was formally established on 3rd December 1993. Since then and until the election of its first president, the CIM was run by an organizing committee formed of Natália Bebiano (president of the Portuguese Mathematical Society), Simões Pereira (University of Coimbra), Maria do Rosário Grossinho (University of Lisbon), Jorge Almeida and Alberto Adrego Pinto (both from Porto University), and Estelita Vaz (University of Minho). António St Aubyn (former president of the Portuguese Mathematical Society), Fernando C. Silva (University of Lisbon) and Isabel Labouriau (Porto University) were also members of this committee. The first president of the CIM was José Perdigão da Silva (University of Lisbon), who was elected at the general assembly on 16th July 1996 for a term of four years. Luís Trabucho (University of Lisbon) was the second president and his term ran from July 2000 until July 2004.

Mission

The CIM aims to promote all kinds of mathematical activities in Portugal, which help to improve the level of mathematics in this country by fostering cooperation with similar centres and with internationally renowned research mathematicians. In addition, the centre expects to assist mathematicians in Third World countries, with priority given to Portuguese speaking countries in Africa (Angola, Mozambique, Cape Verde, Guine-Bissau, São Tomé and Príncipe) and also East Timor, in their efforts to develop their teaching and research capabilities. The CIM mission is fulfilled by exchanging experiences and information, co-organizing mathematical events (such as workshops, summer schools, short courses and scientific meetings) with similar centres, universities, research institutions, and mathematical societies, encouraging visits of foreign mathematicians to Portugal, and editing available documentation pertaining to mathematical activities.

Organization and financial resources

The CIM is a private, scientific, not-for-profit association, set up according to Portuguese law and based in Coimbra, Portugal. The founder members are Portuguese institutions but membership is not restricted to these. The CIM has three types of members: regular, sponsor and honorary. Regular and sponsor members are institutions whereas honorary members are individuals. Regular members are institutions with a stake in mathematics or mathematical activities of any kind. They are grouped according to the amount of their dues and have a voting weight depending on the group they belong to. Sponsor members are institutions that give substantial donations to the CIM and they have voting rights similar to regular members. Honorary members are distinguished mathematicians or those who have rendered valuable services to the CIM. During its lifetime, the CIM has established several protocols with private and public institutions from which it has received financial support for specific scientific goals. The Calouste Gulbenkian Foundation and the Portuguese Science Foundation (FCT) are notable examples.



Most of the members of the Scientific Council during their meeting in 2006

Guidelines for action stem from the general assembly comprising all the members. The general assembly meets at least once a year and elects its president and a board of directors, formed of five members appointed for a four year term. The board of directors may appoint an executive secretary. Currently the president of the CIM is Joaquim Júdice (University of Coimbra) and the board of directors includes Ivette Gomes (University of

Lisbon), Filomena Dias d'Almeida (University of Porto), Domingos Cardoso (University of Aveiro) and José Miguel Urbano (University of Coimbra). João Patrício is the executive secretary.

The audience of the seminars of the 2006 meeting of the scientific council

There is a scientific council, whose task is to advise the board of directors by annually approving the scientific meetings that are supported, organized or co-organized by the CIM. Portuguese and non-Portuguese mathematicians belong to this committee, whose President is Irene Fonseca (Carnegie-Mellon University).

Main activities

Since its foundation, the CIM has supported many conferences, workshops, thematic schools, seminars and several other research activities. The following activities from 2006 should be noted: Follow-up Workshop "Mathematics and the Environment", 27th–28th January in Coimbra; Seminars of the Annual Scientific Council Meeting, 11th February in Coimbra; Aveiro Workshop on Graph Spectra, 10th–12th April in Aveiro; Workshop "From Lie Algebras to Quantum Groups", 28th–30th June in Coimbra; Workshop on Mathematics in Chemistry, 19th–21st July in Lisbon; 3rd International Workshop on Mathematical Techniques and Problems in Telecommunications, 4th–8th September in Leiria; 2nd Summer School on "Mathematics in Biology and Medicine", 11th–15th September in Oeiras; COMPImage – International Symposium on Computational Modelling of Objects Represented in Images, 20th–21st October in Coimbra; Iberian Conference in Optimization, 16th–18th November in Coimbra.

The participants of the Aveiro workshop on graph spectra in April 2006

The CIM has been organizing working afternoons with the Portuguese Mathematical Society on several mathematical topics such as geometry, partial differential equations, dynamical systems, statistics, optimization, numerical analysis and algebra.

The Research in Pairs program supports visits of non-Portuguese researchers to Coimbra to cooperate with Portuguese mathematicians. For a two week period, the CIM offers accommodation in the observatory residence, as well as all the necessary working conditions.

The CIM bulletin is published twice a year and includes news from associates, reports about scientific meetings supported by the CIM, informative mathematical articles and interviews with prestigious mathematicians. Furthermore, monographs and proceedings of some conferences are often published by the CIM.

Location and facilities

The CIM facilities are located in the main building of the Astronomical Observatory of the University of Coimbra. The facilities include seminar rooms, offices with computer facilities and a residence for visitors.

Astronomical observatory of the University of Coimbra

The place is very quiet, on the top of a hill with a nice view over Coimbra. Public transport is available from the observatory to the city centre, as well as to all the departments and faculties of the University of Coimbra, in particular the Mathematics Department.

All of you are welcome to visit the CIM and enjoy its appealing atmosphere and to find out about our challenges and projects.

More detailed information about the CIM and its current activities may be found at http://www.cim.pt.

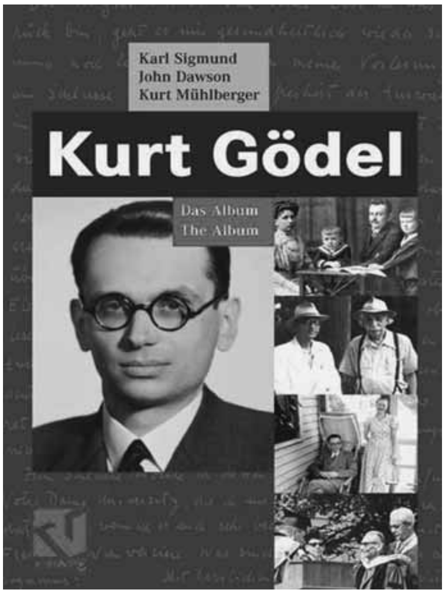
Book Review

Ralf Schindler (Münster, Germany)

Karl Sigmund, John Dawson, Kurt Mühlberger

Kurt Gödel
Das Album – The Album

225 pages, EUR 29.90
Vieweg und Sohn Verlag, Wiesbaden 2006
ISBN 978-3-8348-0173-9



A great book! It is at least as valuable as the exhibition that it catalogues. It will be appreciated by anybody who is interested in or curious about Kurt Gödel.

The life's work of Gödel ranks among the highest from the point of view of pure science. At the same time it must be seen in the context of the intellectually productive Viennese atmosphere that was present in the first decades of the 20th century and of the following political disaster.

The catalogue is divided into three parts: Gödel's life, Gödel's work and Gödel's Vienna. It is beautifully illustrated, with photographs, documents and letters. We have never been given a closer look at the true Gödel; we see a copy of a school report of the eleven-year-old Kurt that exhibits only the best grades, with the only exception being a second best in mathematics! We also see copies of official documents concerning Gödel's PhD and his Habilitation, and we see photographs of Adele, who was seven years older than Kurt and who, according to O. Morgenstern, "saved his life".

Between 1929 and 1937, Gödel produced his breathtaking results on the completeness of the calculus for 1st order logic (his PhD topic), on the incompleteness of formal systems (the topic of his *Habilitationsschrift*) and on the relative consistency of the axiom of choice and the generalized continuum hypothesis. But not only did he never get a permanent position at the university of Vienna, he was also deprived of his 'venia legendi' by the National Socialists. The relevant paperwork reproduced in the catalogue shows an oppressive reality. In June 1940, Gödel was again granted the title of "Dozent"; he never picked up the certificate, though, having emigrated to the US earlier that same year.

Other emigrants from Vienna included Karl Menger, O. Morgenstern, Olga Taussky, Otto Neurath and Rudolf Carnap, to mention just a few who are discussed in the catalogue.

In Princeton, Gödel focused on philosophy and the theory of general relativity. He reconstructed a formal version of the ontological proof of the existence of God, which he did not want to be published for fear that people would conclude that he really believed in God. Gödel also presented a solution to the field equations allowing time travel into the past. His letters to his mother give a detailed account of his intellectual hikes. Photographs show him chatting with Albert Einstein and enjoying himself in the garden of his house.

Some of the material as well as background information may be found at www.goedelexhibition.at/. We must congratulate the organizers on creating such an impressive exhibition and the authors on producing this catalogue. I wish there could be more books like this, which make mathematicians comprehensible in their cultural and political contexts.

Personal column

Please send information on mathematical awards and deaths to the editor

Awards

The Association for Computing Machinery (ACM) has named **Peter Naur** (Copenhagen, Denmark) as the winner of the 2005 A.M. Turing award.
http://www.naur.com/full.html

The d'Alembert Prize of the Société Mathématique de France has been awarded to **Philippe Boulanger**, Editor-in-Chief of Pour la Science.

The 2006 Henri Poincaré Prize, recognizing outstanding contributions in mathematical physics, was awarded to **Ludvig D. Faddeev** (St. Petersburg, Russia), **David Ruelle** (IHES, Bures-sur-Yvette, France) and **Edward Witten** (IAS Princeton, USA).
http://www.iamp.org/poincare/

Ramdorai Sujatha (Tata Institute of Fundamental Research, India) will receive the Ramanujan Prize for 2006 on December 18th at the International Centre for Theoretical Physics in Trieste, Italy, in recognition of her work on the arithmetic of algebraic varieties and her substantial contributions to non-commutative Iwasawa theory.
http://www.abelprisen.no/en/

The 2005 John von Neumann Theory Prize has been awarded to **Robert J. Aumann** (Hebrew University of Jerusalem, Israel) in recognition of his fundamental contributions to game theory and related areas.

The 2006 Spring Prize of the Mathematical Society of Japan has been awarded to **Takuro Mochizuki** (Kyoto University) for his distinguished contributions to the study of the asymptotic behaviour of harmonic bundles.

The 2005 Algebra Prize of the Mathematical Society of Japan was awarded to **Masaki Hanamura** (Tohoku University) for his outstanding contribution to the study of mixed motives and to **Hiroyuki Yoshida** (Kyoto University) for his outstanding contribution to the study of automorphic forms and periods.

Javier Parcet (Consejo Superior de Investagaciones Científicas) is the 2005 "Jose Luis Rubio de Francia" prize winner. This prize is awarded every year by the Real Sociedad Matemática Española with the aim of recognising and stimulating the scientific work of young researchers in mathematics in Spain.

Dennis Hejhal (Uppsala University, Sweden, and University of Minnesota, Minneapolis, USA) has been awarded the Eva and Lars Gårding Prize in Mathematics by the Royal Physiographic Society in Lund, Sweden.

The 2006 Sylvester medal of the Royal Society, London, has been awarded to Sir Peter **Swinnerton-Dyer** (Cambridge, UK) for his fundamental work in arithmetic geometry and his many contributions to the theory of ordinary differential equations. Moreover, Sir Peter has been awarded the Pólya Prize of the London Mathematical Society.

Miles Reid (University of Warwick, UK) has been awarded the Senior Berwick Prize of the LMS.

Michael Weiss (University of Aberdeen, UK) has been awarded the Fröhlich Prize of the LMS for his use of algebraic topological methods to solve a number of different geometric problems.
http://www.lms.ac.uk/activities/prizes_com/citations06.html

(Information about the prizes awarded by the IMU and the ICIAM, and the EURYI-prizes, can be found in separate articles of this issue.)

Deaths

We regret to announce the deaths of:

Adriana Berechet (Romania, 3.2006)
Bernhard Hornfeck (Germany, 16.6.2006)
Zalman Rubinstein (Israel, 7.9.2006)
Vitaliy M. Usenko (Ukraine, 6.3.2006)
William Parry (UK, 20.8.2006)
Karl Zeller (Germany, 7.7.2006)

Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e–mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses ***vberinde@ubm.ro or vasile_berinde@yahoo.com****. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.*

December 2006

17–20: Seventh International Conference on Mathematics in Signal Processing, Cirencester, Gloucestershire, UK
Information: conferences@ima.org.uk; http://www.ima.org.uk/Conferences/7InternationalConfonMathsinSignalProcessing.htm

January 2007

8–June 29: Analysis on Graphs and its Applications, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/AGA/index.html

15–26: Advanced Course on Analytic and Probabilistic Techniques in Combinatorics, Centre de Recerca Matematica, Barcelona, Spain
Information: ACProbabilistic@crm.es; http://ww.crm.cat/ACProbabilistic

15–July 6: Highly Oscillatory Problems: Computation, Theory and Application, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/HOP/index.html

22–26: Winter School "Geometric Measure Theory, Random Sets and Digital Stereology", Sandbjerg Estate, Sønderborg, Denmark
Information: oddbjorg@imf.au.dk; http://www.thiele.au.dk/winterschool07/

22–26: Local Holomorphic Dynamics, Centro di Ricerca Matematica Ennio de Giorgi, Pisa, Italy
Information: crm@crm.sns.it; http://www.mat.uniroma2.it/~tovena/

26–28: MEDCONF. 5th Mediterranean Conference on Mathematics Education, Rhodes Island, Greece
Information: makrides.greg@usa.net, http://www.cms.org.cy

29–February 2: Diophantine and Analytical Problems in Number Theory, Moscow, Russia
Information: gelfond–100@mi.ras.ru; http://gelfond–100.mi.ras.ru

29–February 3: Operator Algebras and Topology, Moscow State University, Moscow, Russia
Information: oat2007@higeom.math.msu.su; http://higeom.math.msu.su/oat2007/

30–February 10: Advanced Course on Quasideterminants and Universal Localization, Centre de Recerca Matematica, Barcelona, Spain
Information: ACQuasideterminants@crm.es; http://www.crm.cat/ACQuasideterminants

February 2007

5–8: IV International Meeting on Lorentzian Geometry, Santiago de Compostela, Spain
Information: xtedugr@usc.es; http://xtsunxet.usc.es/gelosantiago/

26–March 10: School on Quantum Potential Theory: Structure and Applications to Physics, Greifswald, Germany
Information: http://www.math–inf.uni–greifswald.de/algebra/qpt/

March 2007

6–11: SEEMOUS (South–Eastern European Mathematical Olympiad for University Students), Cyprus
Information: makrides.greg@usa.net, cms@cms.org.cy

21–25: MAT–TRIAD 07, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–30: Structured Perturbations and Distance Problems in Matrix Computations, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–30: Workshop: Homotopy theory of schemes, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06–07/homotopy/index.html

27–April 4: MATHEU (Identification, Motivation and Support of Mathematical Talents in European Schools) Training Course (Comenius 2.2), Bulgaria
Information: makrides.greg@usa.net; www.matheu.org; http://ec.europa.eu/education/trainingdatabas/

April 2007

4–18: TAMTAM'07, Tendances dans les Applications Mathématiques en Tunisie, Algérie, Maroc; Algiers, Algeria
Information: http://tamtam07alger.ifrance.com/

10–14: Workshop on Control Theory & Finance, Lisbon, Portugal
Information: wmctf@iseg.utl.pt;
http://srv–ceoc.mat.ua.pt/conf/wmctf2007/

23–27: Dynamics in Perturbations. On the occasion of the 60th birthday of Freddy Dumortier, Hasselt University (Campus Diepenbeek), Belgium
Information: patrick.bonckaert@uhasselt.be; http://www.uhasselt.be/dysy/dynper/

30–May 6: Advances in Mathematics of Finance, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

May 2007

6–12: Semstat 2007, Statistics for Stochastic Differential Equations models, La Manga del Mar Menor, Cartagena, Spain
Information: mathieu.kessler@upct.es;
http://www.dmae.upct.es/semstat2007

14–18: Workshop: Stacks in Geometry and Topology, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06–07/homotopy/index.html

14–18: Conference on Cryptography and Digital Content Security, Centre de Recerca Matematica, Barcelona, Spain
Information: ContentSecurity@crm.es; http://www.crm.cat/ContentSecurity

20–26: Convex and Fractal Geometry, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

27–June 1: Stochastic Networks and Related Topics, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

27–June 2: Spring School on Analysis: Function Spaces, Inequalities and Interpolation, Paseky nad Jizerou, Czech Republic
Information: pasejune@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/katedry/kma/ss/jun07/

28–June 2: Advanced Course on Group–Based Cryptography, Centre de Recerca Matematica, Barcelona, Spain
Information: ACGroupBased@crm.es; http://www.crm.cat/ACGroupBased

2**8–June 2: Workshop on Finsler Geometry and its Applications**, Balatonfuldvar, Hungary
Information: kozma@math.klte.hu; http://www.math.klte.hu/finsler2007

29–June 1: XIIth Applied Stochastic Models and Data Analysis International Conference (ASMDA2007), Chania, Crete, Greece
Information: skiadas@ermes.tuc.gr; http://www.asmda.com/id7.html

30–June 6: MATHEU (Identification, Motivation and Support of Mathematical Talents in European Schools) Training Course (Comenius 2.2), Cyprus
Information: makrides.greg@usa.net; www.matheu.org; http://ec.europa.eu/education/trainingdatabas/

June 2007

1–30: Geometric Applications of Homotopy Theory, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06–07/homotopy/ index.html

3–10: Geometric Analysis and Nonlinear Partial Differential Equations, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

9–13: Workshop Higher Categories and their Applications, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06–07/homotopy/index.html

11–15: Barcelona Conference on C*–Algebras and Their Invariants, Centre de Recerca Matematica, Barcelona, Spain
Information: OAlgebras @crm.es; http://www.crm.cat/OAlgebras

12–16: Complex Function Theory and Geometry, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

16–22: Fifth International Workshop on Optimal Codes and Related Topics (OC 2007). Dedicated to the 60th anniversary of the Insitute of Mathematics and Informatics, Hotel White Lagoon, Balchik, Bulgaria
Information: oc2007@moi.math.bas.bg; http://www.moi.math.bas.bg/oc2007/oc2007.html

18–23: CiE 2007 (Computability in Europe 2007 – Computation and Logic in the Real World), University of Siena, Italy
Information: http://www.amsta.leeds.ac.uk/~pmt6sbc/cie07.html

18–24: Algebraic Topology: Old and New (M. M. Postnikov Memorial Conference), Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

19–20: Fifth Computer Algebra in Mathematics Education Symposium (CAME–5), Pecs, Hungary
Information: http://matserv.pmmf.hu/cadgme/

24–30: Nonlocal and Abstract Parabolic Equations and thei Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

24–30: Lyapunov Memorial Conference. International Conference on the occasion of the 150th birthday of Aleksandr Lyapunov, Kharkiv, Ukraine
Information: lmc07@ilt.kharkov.ua; http://www.ilt.kharkov.ua/lmc07/

24–30: Seventh International Conference "Symmetry in Nonlinear Mathematical Physics", Kiev, Ukraine
Information: appmath@imath.kiev.ua; http://www.imath.kiev.ua/~appmath/conf.html

25–26: Mathematical Modelling in Sport, Manchester, UK
Information: http://www.ima.org.uk/Conferences/conferences.htm

25–29: Conference on Enumeration and Probabilistic Methods in Combinatorics, Centre de Recerca Matematica, Barcelona, Spain
Information: Enumeration@crm.es; http://www.crm.cat/Enumeration

25–30: Topics in Geometric Group Theory, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

25–30: ERLOGOL–2007. Intermediate problems of Model theory and Universal algebra, Novosibirsk–Altai–Novosibirsk, Russia
Information: algebra@nstu.ru

26–29: Biennial Conferences on Numerical Analysis, Dundee, Scotland, UK

26–30: 3rd International Conference Computational Methods in Applied Mathematics (CMAM–3), Minsk, Belarus
Information: http://www.cmam.info/conferences

27–29: Fifth Italian Latinoamerican Conference on Industrial and Applied Mathematics, Trieste, Italy

28–July 4: 6th Congress of Romanian Mathematicians, Bucharest, Romania
Information: congmatro@imar.ro; http://www.imar.ro/~purice/announcements.html

July 2007

1: Summer Conference on Topology and its Applications 2007, Castellón, Spain
Information: http://www.sumtop07.uji.es

1–7: Groups and Their Actions, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

2–4: The 2007 International Conference of Applied and Engineering Mathematics, Imperial College London, London, U.K.
Information: williamyoung@iaeng.org;
http://www.iaeng.org/worldeng2007/ICAEM2007.html

2–4: Algebraic Biology 2007, RISC, Castle of Hagenberg, Austria
Information: http://www.risc.uni–linz.ac.at/about/conferences/ab2007/

2–6: 2nd European Conference for Aerospace Sciences, Brussels, Belgium
Information: http://www.vki.ac.be/eucass2007/

4–8: International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA–2007), Cluj–Napoca, Romania
Information: nodeacj@math.ubbcluj.ro; http://www.math.ubbcluj.ro/~mserban/confan.html

8–11: EURO XXII, 22nd European Conference on Operations Research, Prague, Czech Republic
Information: http://euro2007.vse.cz/

9–11: MCP 2007. 5th international conference on multiple comparison procedures, Vienna, Austria
Information: http://www.mcp–conference.org

9–11: Eccomas Thematic Conference on Meshless Methods, University of Porto, Portugal
Information: http://paginas.fe.up.pt/~meshless/

9–12: International Conference on Preconditioning Technique, Toulouse, France
Information: http://www.precond07.enseeiht.fr/

9–13: The First European Set Theory Meeting, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

9–13: Dynamics Days Europe, Loughborough University, UK
Information: http://www.lboro.ac.uk/dynamicsdays07

9–13: SciCADE'07 International Conference on SCIentific Computation And Differential Equations, Saint–Malo, France
Information: http://scicade07.irisa.fr/

9–13: 9th International Meeting on Fully Three–Dimensional Image Reconstruction in Radiology and Nuclear Medicine, Lindau, Germany
Information: http://www.fully3d.org/2007/

11–13: International Conference on Approximation Methods and Numerical Modelling in Environment and Natural Resources, Granada, Spain
Information: http://www.ugr.es/local/mamern07

16–20: 6th International Congress on Industrial and Applied Mathematics (ICIAM 07), Zürich, Switzerland
Information: http://www.iciam07.ch

22–25: OPTIMIZATION 2007, University of Porto, Portugal.
Information: opti2007@fep.up.pt; http://www.fep.up.pt/opti2007/

22–28: Topological Theory of Fixed and Periodic Points (TTFPP 2007), Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

23–December 21: Strong Fields, Integrability and Strings, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/SIS/index.html

23–27: 23rd IFIP TC 7 Conference on System Modelling and Optimization, Cracow, Poland
Information: http://ifip2007.agh.edu.pl/

23–27: Waves 2007. The 8th International Conference on Mathematical and Numerical Aspects of Waves, Reading, UK
Information: http://www.waves2007.org/

August 2007

14–19: Workshops Loops '07, Prague, Czech Republic
Information: loops07@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/~loops07/workshops.html

19–25: Loops '07, Prague, Czech Republic
Information: loops07@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/~loops07

19–26: XXIInd International Workshop on Differential Geometric Methods in Theoretical Mechanics, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

September 2007

2–8: Linear and Non–Linear Theory of Generalized Functions and its Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

3–6: 13th General Meeting of European Women in Mathematics (EWM07), Cambridge, UK
Information: www.maths.cam.ac.uk/ewm

3–December 21: Phylogenetics, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/PLG/index.html

9–15: Measure Theory–Edward Marczewski Centennial Conference, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

10–14: Fifth Symposium on Nonlinear Analysis, Torun, Poland
Information: sna2007@mat.uni.torun.pl;
http://www–users.mat.uni.torun.pl/~sna2007/index.html

Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to: Ivan Netuka, MÚUK, Sokolovská, 83, 186 75 Praha 8, Czech Republic.

***M. Aguiar, S. Mahajan: Coxeter Groups and Hopf Algebras**, Fields Institute Monographs, vol. 23, American Mathematical Society, Providence, 2006, 181 pp., USD 59, ISBN 0-8218-3907-1*

This monograph is the first to present structural interactions between the theory of Coxeter groups and the theory of Hopf algebras. These interactions occur naturally in various parts of science, notably in algebra, combinatorics, geometry and theoretical physics. The monograph is divided into two parts. The first part (chapters 1–3) introduces basic concepts and properties of Coxeter groups, left regular bands and Hopf algebras. The second part (chapters 4–8) consists mostly of original work by the authors. Firstly, it deals with the descent theory for Coxeter groups. Then it proceeds to constructions of Hopf algebras related to Coxeter groups via certain diagrams of semigroup algebras. The theory is then applied to the Hopf algebra of pairs of permutations, in particular proving its freeness and cofreeness. Finally, cofreeness of the Hopf algebras of (pointed) faces, and of quasi-symmetric functions, is established. It is not just the results but also a unified conceptual treatment of the theory that make this monograph a very valuable one. Moreover, in a number of places further ideas are suggested and new results are announced; more details will appear in a follow-up monograph announced by the authors in the preface. (jtrl)

***J. Akahori et al., Eds.: Stochastic Processes and Applications to Mathematical Finance**, Proceedings of the 5th Ritsumeikan International Symposium, World Scientific, New Jersey, 2006, 217 pp., USD 88, ISBN 981-256-519-1*

The International Colloquium on Stochastic Processes and Applications to Mathematical Finance was held at the Ritsumeikan University during March 2005. This volume contains the original papers presented at the colloquium that were devoted to applications of the theory of stochastic processes and stochastic analysis to financial problems.

The first paper by E. Barucci, P. Malliavin and M. E. Mancino describes how to compute the volatility of a semimartingale based on Fourier series. The method allows the computation of instantaneous and integrated volatility and is therefore useful for high frequency data. T. R. Bielecki, M. Jeanblanc and M. Rutkowski present alternative mathematical techniques used to derive hedging strategies for credit derivatives in models with totally unexpected defaults. In this paper, two alternative approaches are introduced: the stochastic calculus approach in order to establish abstract characterization results for hedgeable contingent claims in a general set-up and the partial differential equation approach in a Markovian setting. A. Kohatsu-Higa and A. Sulem define a forward integral and show that the forward integral is suitable for applications on a portfolio maximization problem. Y. Miyahara introduces the geometric Lévy processes and minimal entropy martingale measure pricing model as a pricing model for an incomplete financial market. The author proves useful properties of the model and presents several examples of applications of this model.

M. Yamazato explains important properties related to gamma processes, defines subclasses of the class of infinitely divisible distributions, which are generated by mixtures and convolutions of gamma distributions, and studies their properties. H. Hashimoto, T. Tsuchiya and T. Yamada treat Tanaka's equation in the case of symmetric stable processes and discuss a uniqueness result in the one-dimensional case. The second part of the paper delivers some results concerning the comparison problem and in the last part, a sufficient condition is proposed that guarantees pathwise uniqueness in the d-dimensional case. Watanabe presents a martingale representation theorem for the cases of discreet and continuous time and the Clark-Bismut-Ocone formula with a proof based on the Wiener chaos expansion. The volume contains a lot of interesting theoretical results together with their applications. It is a good source of information for research in the field of stochastic processes in financial mathematics. (jste)

***L. Ambrosio et al., Eds.: Ennio De Giorgi – Selected Papers**, Springer, Berlin, 2006, 888 pp., EUR 149,95, ISBN 3-540-26169-9*

This volume contains a well-balanced choice of papers written by Ennio De Giorgi, an outstanding Italian mathematician of the 20th century. Many important papers were written by Ennio De Giorgi in Italian, hence the main motivation behind the project was to make them available to a larger public. In addition to their English translations, some of papers are also included in their Italian versions in order to give the reader a feeling of De Giorgi's original style (e.g. this is the case with the celebrated article, "Sulla differenziabilita e l'analiticita delle estremali degli integrali multipli regolari", one of the most important contributions to regularity theory). After a short biography, there is a description of the scientific work of Ennio De Giorgi, ending with contributions by Louis Nirenberg (remarks on some analytic works of Ennio De Giorgi) and Louis Caffarelli (De Giorgi's contribution to the regularity theory of elliptic equations). Then there is a collection of 43 papers (out of 152). In my opinion the volume, which covers all the main directions of the author's activities (measure theory, the 19th Hilbert problem, minimal hypersurfaces, G- and Γ-convergence, and foundations of mathematics) should be on the shelves of every mathematical library. (oj)

***V. Ancona, B. Gaveau: Differential Forms on Singular Varieties – De Rham and Hodge Theory Simplified**, Pure and Applied Mathematics, vol. 273, Chapman & Hall/CRC, Boca Raton, 2005, 312 pp., USD 99,95, ISBN 0-8493-3739-9*

A mixed Hodge structure on the cohomology of algebraic varieties was constructed by P. Deligne. For this construction, he used the Hironaka desingularization theorem, Leray spectral sequences, residues for forms with logarithmic singularities and his cohomological descent theory. The main theme of this book is a systematic and elegant theory of differential forms on spaces with singularities. The authors develop a general theory and then use it to give an alternative treatment of mixed Hodge structures, which avoids the use of the cohomological descent theory.

The first part of the book presents the classical Hodge theory on compact Kähler manifolds and the residue theory on a smooth divisor. A summary of properties of differential forms on complex manifolds, sheaf cohomology, the de Rham Laplacian, complex spaces and spectral sequences is included in this first part. The second part of the book describes a systematic theory of differential forms on complex spaces with singularities. Suitable filtrations on the space of forms are defined recursively using induction on dimension. Induced filtrations on the cohomology and the associated spectral sequence then lead to a construction of a mixed Hodge structure on compact spaces. The last part of the book treats mixed Hodge structure on noncompact spaces. It contains a description of the Leray residue theory and its applications in the construction of mixed Hodge structures on noncompact spaces. The book is essentially self-contained and offers a clear, understandable and systematic approach to the theory of mixed Hodge structures. It is a book to be recommended to anybody interested in the topic. (vs)

A. Ash, R. Gross: Fearless Symmetry – Exposing the Hidden Patterns of Numbers*, Princeton University Press, Princeton, 2006, 272 pp., USD 24,95, ISBN 0-691-12492-2*

Number theory is an old and difficult subject. It is possible to have a problem that is easy to formulate (e.g. Fermat's last theorem) but very difficult to solve. A variety of different methods have been developed over the centuries. This book is devoted to methods in number theory connected with (hidden) symmetries, their realizations by means of groups and their representations. The book is designed for a wide audience of non-specialists. The authors were willing to make an attempt to explain exciting discoveries in mathematics for a larger public without special training in mathematics. They did a splendid job. They are able to describe an eminent role of symmetries in number theory by carefully explaining what it means to represent something (a group) by something else. The first part of the book reviews basic algebraic notions and introduces the Legendre symbol and the law of quadratic reciprocity. The Galois group and its role in number theory is the main topic of the second part. The last part treats various reciprocity laws. It also indicates the prominent role played by ideas of symmetry in the proof of Fermat's last theorem. The book describes very nicely and in simple terms key ideas of the field so that they can be appreciated by people with no particular mathematical education. It is also inspiring and useful for general mathematicians who are not specialized in the field. (vs)

S. Awodey: Category Theory*, Oxford Logic Guides 49, Clarendon Press, Oxford, 2006, 256 pp., GBP 65, ISBN 0-19-856861-4*

This book is intended as a text book on category theory not only for students of mathematics but also, as the author says in the preface, "for researchers and students in computer science, logic, cognitive sciences, philosophy and students in other fields that now make use of it". Very few mathematical prerequisites are expected of the reader. Illustrative examples are concentrated on various aspects of posets and monoids (e.g. the poset as a category on one side and the category of posets and their monotone maps on the other side, and analogously for monoids). No example is given from topology (even in the paragraph "Stone duality" no topology is mentioned).

The author has been giving courses on category theory at Carnegie Mellon University over the last ten years. The lecture course, based on the material in this book, consists of two 90 minute lectures every week for fifteen weeks. The author himself says that "the selection of material was easy. There is a standard core that must be included: categories, functors, natural transformations, equivalence, limits and colimits, functor categories, representables, Yoneda's lemma, adjoints, and monads. That nearly fills a course. The only 'optional' topic included here is Cartesian closed categories and the lambda-calculus, which is a must for computer scientists, logicians and linguists." The book is written with the pedagogical mastership of a skilled teacher trying to help the reader as much as possible. This excellent textbook can be recommended to everybody who would like to learn the basis of category theory. (vtr)

F. Beichelt: Stochastic Processes in Science, Engineering and Finance*, Chapman & Hall/CRC, Boca Raton, 2006, 417 pp., USD 89,95, ISBN 1-58488-493-2*

This book presents an introduction to stochastic processes. It covers background for point processes, Markov chains in discrete and continuous time, Brownian motion, and a small section on martingales. The presentation is suitable for students and researchers in the applied sciences and engineering because a knowledge of measure theory is not assumed. On the other hand, the text pays a lot of attention to applications in science, finance, engineering, operations research and computer science. Omitting a number of proofs, it is not a full university mathematical text but its strength is in a development of thinking towards a solution of real-life problems of random nature. Applied topics like queuing models and networks, actuarial risk analysis and maintenance, and many exercises formulated in a variety of practical situations, help the reader to understand the usefulness of probabilistic models of temporal processes. Solutions to most of the exercises are provided in the appendix. The book is highly recommended to students of technical and economical schools. (vbe)

W. Benz: Classical Geometries in Modern Contexts – Geometry of Real Inner Product Spaces*, Birkhäuser, Basel, 2005, 244 pp., EUR 78, ISBN 3-7643-7371-7*

The main hero of this book is a real vector space X (possibly infinite dimensional) with a chosen (positive definite) scalar product. The approach of the author follows the spirit of the Klein Erlangen program, where geometry of a space is encoded in the appropriate transformation group G. In the cases studied in the book, the group G is a semidirect product of an orthogonal group O (defined by means of a given scalar product) and a translation group T specific for a given geometry. The whole approach is coordinate free and covers uniformly finite as well as infinite dimensional cases. The first chapter is devoted to a description of the translation group T. The main result here shows that a chosen definition of the translation group implies that the resulting geometry is either Euclidean or hyperbolic. The next chapter is devoted to a study of invariants (under the group G) for these two geometries. The third chapter treats the Lie sphere geometries. The last chapter contains a discussion of the geometry of Minkowski space (including the proof of the Alexandrov theorem, which characterizes the conformal Lorentz transformations and a description of the Einstein and de Sitter space-times). Almost no prerequisites are needed to read the book. (vs)

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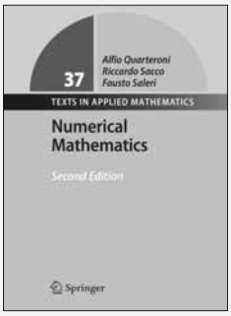

Numerical Mathematics

A. M. Quarteroni, École Polytechnique Fédérale de Lausanne, Switzerland; **R. Sacco**, Politecnico di Milano, Milan, Italy; **F. Saleri**, Università di Milano, Milan, Italy

From the reviews of the first edition ► *This is an excellent and modern textbook in numerical mathematics! It is primarily addressed to undergraduate students in mathematics, physics, computer science and engineering. But you will need a weekly 4 hour lecture for 3 terms lecture to teach all topics treated in this book! Well known methods as well as very new algorithms are given. The methods and their performances are demonstrated by illustrative examples and computer examples. Exercises shall help the reader to understand the theory and to apply it. MATLAB-software satisfies the need of user-friendliness. [....] In the reviewers opinion, the presented book is the best textbook in numerical mathematics edited in the last ten years* ► **Zentralblatt für Mathematik 2001, 991.38387**

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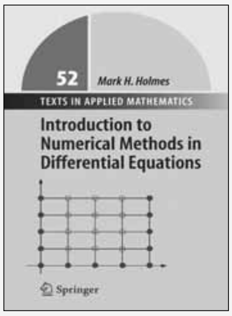

Introduction to Numerical Methods in Differential Equations

M. H. Holmes, Rensselaer Polytechnic Institute, New York, NY, USA

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J. L. Berggren, R. S. D. Thomas: Euclid's Phaenomena: A Translation and Study of a Hellenistic Treatise in Spherical Astronomy*, History of Mathematics, vol. 29, American Mathematical Society, Providence, 2006, 132 pp., USD 29, ISBN 0-8218-4072-X*

This book contains the English translation of Euclid's Phaenomena, one of the masterpieces of Hellenistic exact sciences, which was written in the third century B.C. and which was studied as a major part of the mathematical training of astronomers from the third century B.C. up until the works of N. Kopernik and J. Kepler.

The book is divided into four parts. The first part is an extended introduction describing the history of Euclid's geometry and its applications to astronomy. The authors place Euclid's work in its historical context for readers who are not familiar with Greek history of thinking and science. The second part contains mathematical and astronomical prerequisites, which are necessary for an understanding of Phaenomena. In the third part, the authors describe the history of the oldest Latin printed translations as well as new translations into the modern languages. They also add remarks concerning their translation (sources of their translation, their difficulties with Greek texts, technical problems with using the brackets, italics and other kind of writing, figures and diagrams). The fourth part contains an English translation of Phaenomena. It begins with the introduction and it contains eighteen propositions set out in a geometric style. The authors add many interesting notes and comments to the Euclid text. At the end of the book, there are English and Greek glossaries of selected technical terms and phrases, a selected bibliography, and an index of names and subjects, all included to help with understanding. The presented English translation could be very useful for historians of science as well as astronomers and mathematicians who are interested in Greek astronomy but who are not able to read the original texts in old Greek. (mbec)

B. C. Berndt: Number Theory in the Spirit of Ramanujan*, Student Mathematical Library, vol. 34, American Mathematical Society, Providence, 2006, 187 pp., USD 35, ISBN 0-8218-4178-5*

The purpose of this booklet is to provide an introduction to a small fraction of Ramanujan's work on theta functions and q-series and their connection to number theory. Among the main topics are q-series and theta-functions, congruences for τ and partition functions, sums of squares and triangular numbers, Eisenstein series, hypergeometric functions and their relation to theta functions, and Rogers-Ramanujan continued fractions. Each chapter ends with notes containing various historical and bibliographic comments. There is also a short account of Ramanujan's life in the introduction. The book would be suitable for junior and senior undergraduates and recent graduates. (pso)

E. Bézout: General Theory of Algebraic Equations*, Princeton University Press, Princeton, 2006, 337 pp., GBP 32,50, ISBN 0-691-11432-3*

This book contains the first English translation of the monograph, "The General Theory of Algebraic Equations", which was published in French in 1779 by Etienne Bézout (1730–1783) under the name, "Théorie Générale des Équations Algébriques". The book became very popular after its publication. The monograph was translated by Eric Feron, Professor of Aerospace Engineering at Georgia Institute of Technology. The book presents the Bézout approach to the problem of how to solve systems of polynomial equations in several variables and his new notation for the polynomial multiplier, which simplified the problem of variable elimination by reducing it to a system of linear equations. The book describes the major Bézout's result called "the Bézout theorem", his deep analysis of systems of algebraic equations, his uses of determinants for finding a solution of systems of linear equations, his approach to integration and differentiation of functions, etc. The English translation can be recommended to everyone who is interested in problems of solving systems of polynomial equations and inequalities because it describes this mathematical problem from a historical perspective. (mbec)

J. A. Bucklew: Introduction to Rare Event Simulation*, Springer Series in Statistics, Springer, New York, 2004, 260 pp., EUR 74,95, ISBN 0-387-20078-9*

This is a very interesting monograph that attempts to present a unified theory of rare events simulation. Two basic tools used are importance sampling and the theory of large deviations. This framework allows an assortment of simulation problems to be viewed from a single unified perspective and gives a great deal of insight into the fundamental nature of rare events simulation. After a short summary of random number generation and simulation of selected stochastic models (such as Markov chains, processes and fields) the author presents basic results of large deviation theory and importance sampling methodology. A key part of the book is chapter 5, which deals with the large deviation theory of importance sampling. It shows the way to cover efficiently rare events simulations and includes, in the form of examples, many important models from different fields of statistics. The rest of the book is devoted to special applications of the methodology, including conditional importance sampling, Chernoff's bound for rare events simulation, level crossing and queuing models. One small point I would like to make is that, unlike the author, I am a bit skeptical about the blind simulation as described in chapter 12. I recommend the book to everybody who is interested in rare events and/or Monte Carlo simulation. (jant)

G. Chartrand, L. Lesniak: Graphs & Digraphs*, fourth edition, Chapman & Hall/CRC, Boca Raton, 2004, 386 pp., USD 89,95, ISBN 1-58488-390-1*

This is another edition of a popular introductory book to graph theory. Like the previous versions, it is aimed at mathematics students and it treats the topic with rigorous but easy to read pace. The topics covered include basic notions such as trees, cycles and connectivity, and it gently moves to more involved issues of planarity and graph embeddings on surfaces, graph colourings, the Ramsey theory, extremal graph theory, matchings and factorizations, domination in graphs and a sample of applications of the probabilistic method. The sections that have been most thoroughly revised compared to the previous editions are those on graph colouring (including list colouring and modern related trends in the chromatic theory of graphs), Ramsey theory (with generalized and rainbow Ramsey theorems) and domination theory (introducing independent domination and other variants of this concept). All these revisions follow the most recent development in their fields. (jkrat)

***M. Coornaert: Dimension topologique et systèmes dynamiques**, Cours spécialisés 14, Société Mathématique de France, Paris, 2005, 126 pp., EUR 26, ISBN 2-85629-177-5*

This booklet (comprising 129 pages) is a readable and concise introduction to the classical theory of topological (covering) dimension, completed with a treatment of a new concept of the mean topological dimension of a dynamical system. The first chapter introduces basic concepts and it shows that in normal spaces the dimension of a countable union of closed subspaces is the supremum of their dimensions. The second chapter shows that in Hausdorff spaces (called 'les espaces séparés') there is a sequence of concepts of increasing strength: (covering) dimension zero; existence of a clopen base (les espaces éparpillés); totally separated; totally disconnected (these concepts are all equivalent in compact Hausdorff spaces). Chapter 3 treats the dimension of polyhedra and proves the Lebesgue lemma, which implies dim $R^n = n$. Chapter 4 shows that any compact metric space of dimension n can be embedded into R^{2n+1}. Chapter 5 presents counter-examples of Knaster-Kuratowski and Tichonov.

The rest of the book treats the theory of mean topological dimension introduced recently by Gromov. The concept works for dynamical systems (self-homeomorphisms) of a normal topological space. It is defined analogously to topological entropy but the size of degree is used instead of the power of the cover. If the topological dimension of the underlying space is finite, the mean topological dimension is zero, so the theory is intended for infinite-dimensional dynamical systems. Chapter 6 introduces basic notions and chapter 7 shows that the mean topological dimension can be any nonnegative real number or infinity. Chapter 8 proves the Javorski theorem, which says that each dynamical system without periodic points on a compact metric space of a finite dimension is conjugated to a subshift of R^Z. The counter-example of Lindenstrauss-Weiss shows that the assumption of the finite topological dimension is necessary. (pku)

***S. Dineen: Probability Theory in Finance – A Mathematical Guide to the Black-Scholes Formula**, Graduate Studies in Mathematics, vol. 70, American Mathematical Society, Providence, 2005, 294 pp., USD 55, ISBN 0-8218-3951-9*

The primary aim of this book is to offer a rigorous probabilistic treatment to more delicate topics of financial mathematics, exemplified by the Black-Scholes formula: a well known and extremely efficient tool to analyze a wide range of options prizes. The author builds up the necessary finance theory and mathematics simultaneously. Having started with concepts such as money, interest rates, markets, hedging and arbitrage, he moves on to the basics of measure theory (σ-fields, filtrations, measures, measurability and convergences, expectations, independence, product measures and conditioning). Short excursions both into finance and mathematical analysis are offered and inserted whenever it might be convenient for the reader and natural in the course of the presentation. The call options and hedging on the financial side and the continuous and convex functions and Riemann integration on the side of analysis may serve as examples. The central limit theorem, discrete martingales, the Wiener process and its exponential, the elements of stochastic integration (including the Girsanov change of measure) are treated rigorously to prepare the reader for the final goal of the book, which is the Black-Scholes formula. The considerable collection of exercises with solutions available has an important role in the neatly and lively written text that may be warmly recommended to undergraduates and graduate students in mathematics and finance. (jste)

***Y. Dodge, Ed.: The Oxford Dictionary of Statistical Terms**, Oxford University Press, Oxford, 2006, 498 pp., GBP 12,99, ISBN 0-19-920613-9, ISBN 978-0-19-920613-1*

The first edition of this dictionary appeared in 1957 and contained 1,700 terms. It became a well-respected reference book. The revision for the current sixth edition started in 1998. A website was constructed where all the terms of the fifth edition were listed. The statistical community suggested 320 terms for elimination and 1,067 terms for addition. It was decided to eliminate 265 entries and to add 640 new entries. The sixth edition now contains 3,540 terms. For some entries a reference to the literature is given. The list of references is quite long (pp. 439-498) and it is one of the positive features of the dictionary. The readers and users of the dictionary are invited to send their comments and suggestions to a website of the International Statistical Institute. The address of the website is introduced in the preface.

I would just like to make the comment that I am puzzled by the explanation of the term *Rayleigh distribution* presented on page 339: "A χ^2 distribution with two degrees of freedom, so called because it was considered by Rayleigh in some physical situations". In statistical textbooks and papers we read something different. For example, in Lindgren B. W. (1993): *Statistical Theory*, 4th edition, Chapman and Hall, p. 186, we find the following information: "if X and Y are iid $N(0, \sigma^2)$ random variables, then $R=\sqrt{(X^2+Y^2)}$ has the Rayleigh distribution. Since X^2+Y^2 has the χ^2 distribution with two degrees of freedom, the distribution of R cannot be χ^2 ". Despite this small point, I must say that the book covers a broad area of mathematical statistics and it is a useful reference for fast orientation on statistical terms. (ja)

***P. Ebenfelt, B. Gustafsson, D. Khavinson, M. Putinar, Eds.: Quadrature Domains and their Applications – The Harold S. Shapiro Anniversary Volume**, Operator Theory – Advances and Applications, vol. 156, Birkhäuser, Basel, 2005, 277 pp., EUR 108, ISBN 3-7643-7145-5*

This book is an expanded version of talks presented at a conference held at Santa Barbara in 2003 to celebrate the 75th birthday of Professor H. S. Shapiro. Quadrature domains are related to Gaussian type quadratures for classes of integrable analytic or harmonic functions. The origin goes back to specific problems of classical potential theory. However, the research developed intensively since the seventies has an interdisciplinary character: univalent functions, approximation theory, fluid mechanics, variational problems for partial differential equations, free boundaries, etc. The introductory article, 'What is a quadrature domain', written by B. Gustafsson and H. S. Shapiro, gives an informative introduction to the subject. The remaining twelve papers cannot be described in detail here. One deals with fluid dynamics, one with the Brownian motion aspects of quadrature domains, four papers are associated to potential theory and six papers are linked to complex analysis. A selected bibliography by H. S. Shapiro and an open problems section are included.

The book gives a nice overview of recent achievements in the field of quadrature domains. (in)

***G. Farin, D. Hansford: Practical Linear Algebra – A Geometry Toolbox**, A. K. Peters, Wellesley, 2005, 384 pp., USD 67, ISBN 1-56881-234-5*

This book covers all the fundamental topics of linear algebra and outlines some of its applications. Unlike the majority of mathematical textbooks, it does not use the standard theorem-proof approach. The authors explain linear algebra by means of examples and its practical and geometric applications. The reader is led to an intuitive understanding of the notions of the subject and provided with a nice survey of their applications. The basic topics (vectors, linear maps, linear systems, affine maps, and eigenvalues and eigenvectors) are explained first in two dimensions. Then the same concepts are retained and extended in a three dimensional setting, and finally general linear systems of equations and general vector spaces are defined and studied. In addition, several chapters are devoted to geometric and practical applications including the analysis of conics, polygons, triangulations, numerical methods, and curves. The book also contains a brief postscript tutorial chapter and solutions to selected problems. The book is designed for students of fields using linear algebra, such as engineering or computer science. The text could help specialists in these branches to understand the mathematical background of the methods they use. It is also an inspiring book for pure mathematicians who would like to learn more about applications of linear algebra. It is definitely a useful source of many nice examples and applications for teachers of linear algebra. (pru)

***A. Giambruno, C.P. Milies, S.K. Seghal, Eds.: Groups, Rings and Group Rings**, A Series of Lecture Notes in Pure and Applied Mathematics, vol. 248, Chapman & Hall/CRC, Boca Raton, 2006, 350 pp., USD 179,95, ISBN 1-58488-581-5*

This is the proceedings of the conference on "Groups, Rings, and Group Rings" held at Ubatuba, Brazil in July 2004 in honour of Professor Polcino Milies. The volume consists of 29 original research papers and four surveys, written by some of the leading experts on group-ring theoretic methods of (non-commutative) group theory. Further topics include Lie and Jordan (super) algebras and applications to coding theory. The book presents a variety of tools and results of the contemporary theory of group rings. It will be of considerable interest for researchers as well as graduate students in this active branch of modern algebra. (jtrl)

***R. Godement: Analysis II: Differential and Integral Calculus, Fourier Series, Holomorphic Functions**, Universitext, Springer, Berlin, 2005, 443 pp., 20 fig., EUR 44,95, ISBN 3-540-20921-2*

The choice of topics included in this book and their order is far from the traditional approach. Elementary considerations are mixed with deeper results and historical comments. The book starts with the theory of Riemann integrals but within a few pages the reader will find uniform convergence, Fourier series and power series. Chapter 5 (the first in the second volume) ends with approximation theorems (Weierstrass), Radon measures in R and C and Schwarz distributions. The remaining two chapters are devoted to an introduction to Lebesgue theory and harmonic analysis and holomorphic functions. The last chapter contains elementary facts on Fourier series, analytic functions and holomorphic functions (while facts depending on the Cauchy integrals over arbitrary curves are omitted) and it closes with a study of Fourier integrals. The final part of the book is called '*Science, technology, arms*' and it contains almost fifty pages of the author's description of the role of mathematics and applied mathematics in the development of the technical background to certain arms, like the A- and H-bombs. (jive)

***P. Goeters, O.M.G. Jenda, Eds.: Abelian Groups, Rings, Modules, and Homological Algebra**, A Series of Lecture Notes in Pure and Applied Mathematics, vol. 249, Chapman & Hall/CRC, Boca Raton, 2006, 317 pp., USD 169,95, ISBN 1-58488-552-1*

This is the proceedings of the special "Abelian Groups, Rings and Modules Conference" held at Auburn University in September 2004 in honour of Professor Edgar E. Enochs. The volume consists of 25 original research papers and one survey, related to a variety of topics of modern algebra inspired by, or closely related to, the influential research of Enochs over the past decades. Of course, the focus is naturally on module theory over commutative rings. Given the number of interesting contributions in this volume, it is impossible to list them all in a short review. We only mention four of them: a survey on Gorenstein homological algebra written by Enochs and Jenda presenting some of the recent ideas and challenges of this topical area, a paper by Fuchs, Heinzer and Olberding developing commutative ideal theory without finiteness conditions, a paper by Goebel and Shelah characterizing all torsionless linearly compact abelian groups, and a paper by Hassler and Wiegand presenting a method of construction of large indecomposable modules over hypersurface singularities. The book is yet another proof of the lasting impact of Enochs on a number of areas of contemporary module theory. It will definitely be of interest both for researchers and for graduate students in algebra. (jtrl)

***R. A. Hibschweiler, T. H. MacGregor: Fractional Cauchy Transforms**, Monographs and Surveys in Pure and Applied Mathematics, vol. 136, Chapman & Hall/CRC, Boca Raton, 2005, 248 pp., USD 89,95, ISBN 1-58488-560-2*

Fractional Cauchy transforms form a family of integral transformations depending on a parameter α generalizing the classical Cauchy transform. The concept was introduced by V. P. Havin in the 1960s. The formula reduces to the well known Cauchy formula for $\alpha = 1$. The present book studies the class F_α from several perspectives. The relation to Hardy and Besov spaces is treated in chapter 3. The behaviour near the boundary (radial limits) and the distribution of zeros are studied in chapters 4 and 5. Chapters 6 and 7 deal with multipliers, i.e. with functions g with the property that g maps (by multiplication) F_α to F_α. Compositions of F_α with analytic functions are studied in chapter 8. Chapter 9 is devoted to univalent functions. An analytic characterization of the class F_α is proved in chapter 10. However, in the general case, such a characterization remains an important open problem of the theory. The study of F_α goes back to the 60's but most of the results in the book are of recent origin. Though the exposition is to a large extent self-contained, the book is clearly aimed at specialists; good knowledge of complex and functional analysis is required for reading. (dpr)

P. G. Hjorth, C. L. Petersen, Eds.: Dynamics on the Riemann Sphere: A Bodil Branner Festschrift*, European Mathematical Society, Zürich, 2006, 222 pp., EUR 68, ISBN 3-03719-011-6*

The papers collected in this volume were written to celebrate Bodil Branner's 60th birthday. Most of them were presented at the 'Bodil Fest', a symposium on holomorphic dynamics held in June 2003 in Holbæk, Denmark. The main research theme of Bodil Branner is the iteration of cubic polynomials. Together with John H. Hubbard, she described the global topology of the parameter space C^2 of polynomials of the form $P_{a,b}(z) = z^3 - a^2z + b$. Several decompositions of the parameter space have been considered. The first splitting is to separate the connectedness locus (where both critical points have bounded orbit and the Julia set is therefore connected) from the escape locus (where at least one critical point escapes to infinity). The second splitting is to foliate the escape locus into different hyper-surfaces, each one corresponding to a fixed maximal escape rate of the critical points. A particular way of constructing Teichmüller almost complex structures, which are invariant under Pa,b, was introduced as wringing and stretching of the complex structure; this technique is now referred to as Branner-Hubbard motion. In the volume, Branner-Hubbard motion is described in the papers of C. L. Petersen, Tan Lei, and A. Douady. A survey paper of J. Milnor treats Lattès maps. A. Avila and M. Lyubich give examples of infinitely renormalizable quadratic maps whose Julia sets have Hausdorff dimension arbitrarily close to one. A. Chéritat studies the linearizability of the family $P_\theta(z) = e2\pi i\theta z + z^2$. Two papers (written by P. Blanchard et al. and P. Roesch) treat the parameter space of the family $f_\lambda(z) = z^2 + \lambda/z^2$. T. Kawahira studies small perturbations of geometrically finite maps into other geometrically finite maps that are (semi)-conjugate on the Julia set to the original map. W. Jung constructs by quasiconformal surgery a class of homeomorphisms of subsets of the Mandelbrot set. N. Fagella and Ch. Henriksen study Arnold tongues in the complexification of analytic diffeomorphisms $f_{a,t}(x) = x + t + (a/2\pi)\sin(2\pi x)$ of the circle. (pku)

A. Kanel-Belov, L. H. Rowen: Computational Aspects of Polynomial Identities*, Research Notes in Mathematics, vol. 9, A.K. Peters, Wellesley, 2005, 378 pp., USD 69, ISBN 1-56881-163-2*

In 1950 Wilhelm Specht asked whether every T-ideal of a free F-algebra on the set of letters X is finitely based. An affirmative answer implies that every identity of a PI-algebra is a consequence of a finite set of identities. Various aspects of the Specht problem have been studied by many researchers. The main purpose of this monograph is to present a complete and concise answer to the Specht question.

The first six chapters of the book develop tools needed for the Kemer proof of the Specht conjecture in characteristic zero. The choice and exposition of the topics of this part of the book (such as the Schirschov height theorem, the Razmyslov-Kemer-Braun theorem for the nilpotency of the Jacobson radical of finitely generated PI-algebras, the development of Kemer polynomials, and the theory of Grassmann algebras and their connection to superalgebras) is guided by this aim. Chapter 7 contains counterexamples to the Specht question in positive characteristics and related theory. The final five chapters of the monograph are devoted to further development of themes that were established on the way to the Kemer proof. This part of the book deals with notions of Hilbert series and Gelfand-Kirillov dimension as well as the theory of cocharacters, trace identities, and the general theory of identities. The book ends with exercises, the list of main theorems, examples and counterexamples, and a list of open problems. All topics of the monograph are well-arranged and developed in a clear way. The book is suitable not only as a useful reference for researchers but also as part of a course on PI-algebras for graduate students. (jzem)

W. Kocay, D.L. Kreher: Graphs, Algorithms, and Optimization, *Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2004, 482 pp., USD 89,95, ISBN 1-58488-396-0*

The authors decided to write a textbook that would introduce the realm of graph theory to computer science students. In many ways they have undoubtedly succeeded. Rigorously stated mathematical theorems and proofs continuously interlace with tables of programming codes formally describing the algorithms presented. Spice is added to the menu now and then by slight detours to the theory of NP-completeness and a few samples of polynomial reductions, a few more advanced techniques of proof exploiting linear algebra, and a slightly informal but illustrative exposition of embeddings of graphs on surfaces of higher genus. The topics covered include basic notions of degree sequences, trees, cycles and connectivity, all illustrated from an algorithmic point of view. Further topics include matchings and Hamilton cycles, network flows and Menger theorems, and graph colorings. The relatively slow pace of the introductory chapters becomes a bit faster towards the end of the textbook and in the end the final chapters face an ambitious task of introducing linear programming to the reader. It is hard to imagine that all the material could be covered in a single one-semester course but, if the lecturer invests just a little bit of effort in careful selection, this textbook can be used to teach a very nice course on computer science. (jkrat)

S. G. Krantz: Geometric Function Theory – Explorations in Complex Analysis*, Cornerstones, Birkhäuser, Boston, 2006, 314 pp., EUR 58, ISBN 0-8176-4339-7*

Standard courses in complex analysis do not guide students to choose this seemingly closed subject as their research topic. However, this book presents one part of modern complex analysis: geometric function theory, not as a closed subject but instead as a fruitful synthesis of many different areas: functional analysis, invariant geometry, harmonic analysis, differential geometry, partial differential equations, the automorphism groups of domains and abstract algebra. It will capture the imagination of advanced undergraduate and graduate students and other mathematicians and it shows how to formulate problems. Moreover, the book is nicely-written, readable and a methodologically designed showroom of modern mathematics. It contains many exercises, examples and an extensive bibliography. The statement, "There is hardly any other analysis text that offers such a variety and synthesis of mathematical topics", (cited from the preface) is not exaggerated. The book will be useful for a wide range of students and mathematicians. (zv)

S. B. Kuksin: Randomly Forced Nonlinear PDE's and Statistical Hydrodynamics in 2 Space Dimensions*, Zurich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2006, 92 pp., EUR 28, ISBN 3-03719-021-3*

This booklet is based on the author's lecture-course at ETH Zürich and it addresses the incompressible isothermal Navier-Stokes equation in a two-dimensional domain with periodic boundary conditions forced randomly by the right-hand side. The topic is related to turbulence theory but heuristic considerations are suppressed only to discussion while the focus is on rigorously proved mathematical results. The author shows, for example, that statistical characteristics of a turbulent flow stabilize with time growing to characteristics independent of an initial velocity profile, that the time average of any characteristic of a turbulent flow equals the ensemble average, and that the turbulent flow is a Gaussian process at large time scales. A short introductory section on function spaces and partial differential equations (focused on the Navier-Stokes equation) is included but the reader's probability background is assumed. The book is primarily intended for experts in probability theory and partial differential equations with a focus on theoretical fluid dynamics. (trou)

P. Kunkel, V. Mehrmann: Differential-Algebraic Equations: Analysis and Numerical Solution*, EMS Textbooks in Mathematics, European Mathematical Society, Zürich, 2006, 377 pp., EUR 58, ISBN 3-03719-017-5*

Differential-algebraic equations (also called implicit equations or singular systems) are equations of the form $F(t,x,x')=0$ that cannot be brought into an explicit form $x'=f(t,x)$. Such equations form a classical topic going back to Kronecker and Weierstrass but this field still attracts a lot of research interest and it has many applications. Both the analytic and numerical study of an implicit equation revolves around the concept of the index. There are several ways of counting the index; the authors prefer the strangeness index, which measures 'how far' the given system is from a system of explicit ordinary differential equations decoupled with a purely algebraic system. The aim of the book is to present a self-contained and elementary introduction to the topic at the level of a graduate mathematics course. The first part of the book is devoted to analysis. Basic theory for linear problems with constant coefficients, problems with variable coefficients, and general nonlinear problems are developed. Though most of the exposition remains in the classical setting, generalized solutions and equations on manifolds are discussed as well. Some attention is given to control problems. The second part of the book is concerned with the numerical treatment of differential-algebraic equations. Runge-Kutta and BDF methods are developed first for systems with a low index while numerical index reduction is studied in later chapters. The final section gives an extensive list of available numerical packages. (dpraz)

A. Ledet: Brauer Type Embedding Problems*, Fields Institute Monographs, vol. 21, American Mathematical Society, Providence, 2005, 171 pp., USD 52, ISBN 0-8218-3726-5*

The main theme of this book is a study of the embedding of the Galois group of an extension of the field K to the Galois group of a different extension satisfying appropriate conditions. In the simple cyclic case, it is possible to solve the problem in an elementary way. In general, it needs quite advanced methods of algebraic number theory. The book concentrates on embedding problems of the Brauer type and starts with Galois theory. Chapter 2 introduces obstructions as elements in a suitable cohomology group. The next three chapters review the main tools: Brauer groups, group cohomology and quadratic forms. Decompositions of obstructions are studied in chapter 6. The last two chapters contain a discussion of solvability of certain embedding problems and their possible reduction. An introduction to pro-finite Galois theory is contained in the appendix. The book will be useful as a reference book or as an introduction to the field. (vs)

B. Malgrange: Systèmes différentiels involutifs, *Panoramas et Synthèses, no. 19, Société Mathématique de France, Paris, 2005, 106 pp., EUR 26, ISBN 2-85629-178-3*

The origin of the theory of involutive systems of partial differential equations goes back to Élie Cartan, who studied them in terms of exterior differential systems. During the last fifty years, methods of homological algebra were applied with success to study such systems. In the first part of the book, the author reviews the theory of involutive systems from the point of view of partial differential equations (a relation to the approach by exterior differential systems is explained in appendix B). In the main part of the book (chapters 4 and 5) the author introduces a notion of D-analytic spaces, proves the finiteness theorem and proves the involutiveness of the system for a generic case. In such a way, the author gives a precise sense to the old statement of Cartan that "by a prolongation, a differential system becomes eventually involutive". This is a small booklet with a very interesting content related to many questions studied recently in the realm of partial differential equations with algebraic or analytic coefficients. (vs)

J. Milnor: Dynamics in One Complex Variable*, third edition, Annals of Mathematics Studies, no. 160, Princeton University Press, Princeton, 2006, 304 pp., USD 45, ISBN 0-691-12488-4*

This very nice book is devoted to the dynamics of an iterated holomorphic mapping f from a Riemann surface to itself. The emphasis is put on the study of rational maps. As a key object, Fatou and Julia sets are defined in chapter 2. The author studies these sets on different Riemann surfaces, investigates the dynamics in the neighbourhood of fixed points and cycles and finally describes the structure of Fatou and Julia sets. In the appendices, auxiliary results and some extensions are collected. The exposition of the book is very clear. The studied problems are illustrated by many pictures and if reasonable, they are explained from different points of view. In particular, if there are independent interesting proofs for a statement, all proofs are presented. Since this is already the third edition of a book that appeared originally in 1999, many researchers in the field surely have it in their personal libraries. Those who do not will surely welcome this new edition. But the book is not just for experts in dynamics in one complex variable. Due to the clarity of presentation, it can also serve as an introduction to problems for students with a basic knowledge of complex variable theory. (pkap)

S. Montiel, A. Ros: Curves and Surfaces*, Graduate Studies in Mathematics, vol. 69, American Mathematical Society, Providence, 2005, 376 pp., USD 59, ISBN 0-8218-3815-6*

At present, quite a few textbooks are available describing the classical theory of curves and surfaces in Euclidean three-dimensional space. This book is special in the sense that together with the standard local theory, a lot of attention is paid to global problems. The first three chapters of the books contain a description of the local theory for curves and surfaces. Chapter 4 is devoted to the topology of the space. A higher dimensional version of the well known Jordan curve theorem is discussed. The proof is based on a version of degree theory. Existence of a tubular neighbourhood is used here as well as in chapter 5, where surface integration is treated. This chapter also includes the Gauss theorem and the Brower fixed point theorem. Gauss' Theorema Egregium is explained in chapter 7. Global geometry of surfaces with a positive Gauss curvature, the Alexandrov theorem and the isoperimetric inequality are described in chapter 6. The book culminates with the Gauss-Bonnet theorem (chapter 8). The last chapter is then devoted to the global geometry of curves. Each chapter ends with a set of exercises and hints for their solutions. As a prerequisite for reading the book, a few basic facts from linear algebra, topology and ordinary differential equations are needed, together with Lebesgue integration. This textbook is well organized and nicely written. It is suitable for self-study and also for teachers preparing courses on the theory of curves and surfaces. (vs)

M. Moubachir, J.-P. Zolésio: Moving Shape Analysis and Control, *Applications to Fluid Structure Interactions, Pure and Applied Mathematics, vol. 277, Chapman & Hall/CRC, Boca Raton, 2006, 291 pp., USD 99,95, ISBN 1-58488-611-0*

This book presents and develops mathematical tools needed for a description of the motion of two and three-dimensional domains. Such problems have a number of applications in practice (such as free surface flows, shape optimization and contact problems). The authors concentrate on fluid-structure interaction problems and the presentation is based on the Eulerian approach. The book consists of eight chapters and four appendices. The first, introductory chapter presents several typical problems that arise when designing a fluid-structure interaction system. Chapter 2 deals with the identification of a moving boundary that separates a solid and a liquid phase (inverse Stefan problem). Chapter 3 is focused on the weak evolution of measurable sets described by the convection equation for characteristic functions. The concept of transverse variations makes it possible to differentiate functionals associated to evolution tubes. Chapter 4 recalls the concept of the shape differential equation. Applications to a simple shape control problem are given. In addition, it is shown how to proceed when domains are parameterized via the level set formulation.

Chapter 5 is devoted to the dynamic shape control of the Navier-Stokes equations by using the non-cylindrical Eulerian moving shape analysis. In contrast to the previous chapter, the Lagrangian shape analysis approach is used in chapter 6 for solving fluid-solid interaction problems. Chapter 7 presents a complex analysis of an inverse problem arising in the study of bridge deck aeroelastic stability. Finally in chapter 8, the results of the previous chapter are extended to the case of an elastic solid under large displacements inside an incompressible fluid flow. The book ends with four appendices summarizing basic information on function spaces, regularity of domains and the Fourier transform, which is needed for better understanding of the text. The book is intended for researchers and graduate students who are interested in the control of systems involving moving boundaries. A good preliminary knowledge of the topic is required for some parts of the book. (jhas)

V. E. Nazaikinskii, A. Yu. Savin, B.-W. Schulze, B. Yu. Sternin: Elliptic Theory on Singular Manifolds, Differential and Integral Equations and Their Applications, vol. 7, Chapman & Hall/CRC, Boca Raton, 2005, 356 pp., USD 99,95, ISBN 1-58488-520-3

The classical index theorem discovered by M. Atiyah and I. Singer was gradually generalized and extended ever since its formulation half a century ago. One possible line of generalizations tries to extend the index theorem to manifolds with singularities. The main aim of this book is to describe systematically such an extension in the case of a manifold with a suitable type of singularities. The first part of the book introduces elliptic operators on singular manifolds (i.e. on manifolds with conical singularities or on manifolds with edges). In the second part, the reader can find a review of the classical theory of pseudodifferential operators on manifolds and its extension to singular manifolds. A generalization of index theory to the case of singular manifolds is described in the third part of the book. The last part describes various additional topics and applications (Fourier integral operators on singular manifolds, relative elliptic theory, index theorems on manifolds with cylindrical ends, homotopy classifications of elliptic operators and the Lefschetz type formulae). Spectral flows and eta invariants are treated in two appendices. Manifolds with special types of singularities are appearing more and more often in various parts of mathematics. Hence the book will be very useful for mathematicians from different branches of mathematics and also for theoretical physicists. (vs)

H. T. Nguyen: An Introduction to Random Sets, *Chapman & Hall/CRC, Boca Raton, 2006, 257 pp., USD 49,99, ISBN 1-58488-519-X*

In many fields of probability theory and mathematical statistics, there are several books on similar topics. The theory of random sets is presented much less, partly because it is young, partly because of its complexity. The book by Hung Nguyen demonstrates the variety of tools necessary for development in this field. It begins with some motivating examples of well known random sets in statistics. Then finite random sets are discussed (which are technically easier). When applied in decision making or when studying set valued functions by means of incidence algebras, the beauty of the topic is clearly visible. General random closed set theory starts classically with hit-or-miss topology, the capacity functional and the Choquet theorem. Then a solid background to the Choquet integral (with respect to nonadditive set functions) is built and applied to the investigation of convergence in the distribution of random closed sets in terms of their capacity functionals. The final chapter returns to statistical applications, namely to coarse data analysis. The book contains many exercises and would provide graduate students with an excellent course before studying advanced papers on the topic. (vbe)

D. O'Regan, Y. J. Cho, Y.-Q. Chen: Topological Degree Theory and Applications, *Series in Mathematical Analysis and Applications, vol. 10, Chapman & Hall/CRC, Boca Raton, 2006, 221 pp., USD 79,95, ISBN 1-58488-648-X*

After introducing the Brouwer degree theory in R^n, the authors consider the Leray-Schauder degree for compact mappings in normed spaces. A description of degree theory for condensing mappings (chapter 3) is followed by a chapter dedicated to studies of degree theory for A-proper mappings. The focus then turns to the construction of the Mawhin coincidence degree for L-compact mappings, degree theory for mappings of class $(S+)$ and their perturbation with other monotone-type mappings. The last chapter is dedicated to fixed point index theory in a cone of a Banach space and presents a new fixed point index for countably condensing maps. Each chapter is accompanied by important applications illustrating the reason that it was necessary to change the previous concept of topological degree to a more general one. Many examples and exercises conclude each chapter. The book forms a good text for a self-study course or special topic courses and it is an important reference for anybody working in differential equations, analysis or topology. In summary, we can say that the book is an up-to-date exposition of the theory and applications of an important part of mathematics. (oj)

J. M. Plotkin, Ed.: Hausdorff on Ordered Sets, *History of Mathematics, vol. 25, American Mathematical Society, Providence, 2005, 322 pp., USD 69, ISBN 0-8218-3788-5*
This book is devoted to seven of Hausdorff's articles on Cantorian set theory (mostly about ordered sets) that were published during the period 1900–1910. Each article, translated by the author, is preceded by an essay that offers an illuminating commentary including references on the relationship to the work of others. Furthermore, a translation of Hausdorff's article, "Sums $1_\aleph$ Sets", from 1936 is in the appendix; it belongs to the circle of problems in question. Moreover, we can find in the preface a short outline of the substantial historical period of the birth and initial development of set-theoretically based modern mathematics. Recall that in the years 1900–1910 it was mostly E. Zermelo and F. Hausdorff who traced out two basic trends in the still new field of set theory. While E. Zermelo can be seen as the father of abstract set theory (remember his list of axioms of formal set theory) F. Hausdorff developed set theory as a useful tool and background to a study of concrete mathematical problems. In this spirit, he influenced both measure theory and the foundations of general topology. Thus the book gives material concerning fundamental knowledge of several branches of modern mathematics and it can be recommended as an illuminating description of the field, both historically and mathematically. (jmlc)

M. M. Rao: Conditional Measures and Applications, *second edition, Pure and Applied Mathematics, Chapman & Hall/CRC, Boca Raton, 2005, 483 pp., USD 99,95, ISBN 1-57444-593-6*
This monograph presents a broad overview on conditioning with historical notes. It starts with principles, ideas and concepts for conditional probability measures. It then proceeds to a general concept based on conditional expectations that coincides with the Radon-Nikodým derivative. Some axiomatic definitions of conditioning are introduced. Later the conditional expectation is shown to be a particular case of a projection operator acting on functional spaces. The remaining theory, conditions and properties of conditioning are presented and discussed with several different backgrounds. The author is also interested in computational aspects of conditioning and in applications to mathematical statistics and random processes.

One chapter is devoted to sufficient statistics, which are closely related or, more correctly, defined by means of conditioning. Later chapters deals with martingales, submartingales and Markov processes. For interested readers, the last two chapters describe the impact of conditioning on modern analysis and present conditioning concepts in general spaces as some subset of functions. The latter chapters give advanced reading on the subject from a general point of view. This comprehensive monograph on conditioning can be a valuable help for anyone interested in probability theory and the connection to abstract analysis. I recommend the book to any reader interested in probability and measure theory oriented to conditioning. The book will be convenient as a textbook for introductory, tutorial and advanced courses on conditioning. (pl)

L. Robbiano, C. Zuily: Strichartz Estimates for Schrödinger Equations with Variable Coefficients, *Mémoires de la Société Mathématique de France, no. 101–102, Société Mathématique de France, Paris, 2005, 208 pp., EUR 41, ISBN 2-85629-180-5*
This book is devoted to the proof of regularity results for the solution of the Schrödinger equation $i\, \partial t\; u - A\; u + V(x) = 0$. Coefficients of the second order differential operator A are allowed to depend on the space variable x but A is assumed to be asymptotically (as $|x| \to +\infty$) a perturbation of the Laplacian. For solutions of the equation the authors show the Strichartz estimate. The main step of the proof is to express the solution for an auxiliary problem via a Fourier integral operator with complex phase, which is described in chapter 6. It relies on a careful study of phase and transport equations, presented in chapter 4 and 5. When the authors have the explicit formula for the solution, they then deduce the dispersion estimate and use a general result [M. Kell, T. Tao: End point Strichartz estimate, Amer. J. Math. 120 (1998), p. 955–980] to conclude the proof in chapter 7. The proof of the theorem is quite technically demanding but the book is well ordered and carefully written. It is very interesting and experts in the field will surely appreciate it. (pkap)

C. Sabbah: Polarizable Twistor D Modules, *Astérisque 300, Société Mathématique de France, Paris, 2005, 208 pp., EUR 46, ISBN 2-85629-174-0*
The theory of polarizable Hodge modules was developed by M. Saito at the end of the last century. The theory of twistor structures was then developed by C. Simpson. It is expected that many parts of the Saito theory will have an appropriate analogue in a more general case of mixed twistor structures. In particular, there is the Kashiwara conjecture for push-forwards and vanishing cycles in the case of semisimple holonomic D-modules. This book is devoted to the Kashiwara conjecture for the category of polarized regular twistor D-modules (equivalent to the category of semisimple perverse sheaves). It introduces a notion of regular twistor D-modules and it contains proofs of the corresponding versions of the decomposition and the vanishing cycles theorems. The main strategy used for the proof is similar to that of Saito theory. One of the tools used in the proofs is the Mellin transform for distributions developed by D. Barlet and M. Kashiwara. There is no doubt that the work available now in the revised book presents an important step on the way to the proof of the Kashiwara conjecture. (vs)

F. Schlenk: Embedding Problems in Symplectic Geometry, *de Gruyter Expositions in Mathematics 40, Walter de Gruyter, Ber-*

lin, 2005, 250 pp., EUR 98, ISBN 3-11-017876-1

Let ω_0 denote the canonical symplectic 2-form on R^{2n}. A smooth map Φ defined on an open subset U of R^{2n} is called symplectic if $\Phi^*\omega_0 = \omega_0$. A symplectic map is called a symplectic embedding if it is injective. The study of symplectic embeddings is the main goal of this book. It is a relatively new, interesting and promising topic that has not existed for more than two decades. The book has nine chapters and five appendices but the main information is contained within chapter 1. Roughly speaking, all the other chapters contain proofs of the results announced in chapter 1 and further development of the theory. The main attention is devoted to various symplectic embedding constructions (lifting, wrapping and folding). These relatively elementary constructions are then used for proofs of various symplectic embedding results.

The prerequisites required for reading this book are very modest. One must be familiar with the notion of a differentiable manifold and with differential forms on manifolds. In a way, the whole theory is not at all easy and it requires a lot of techniques. Nevertheless the author's presentation is excellent and I do not think it could be written better. The book deals with a new and young area. Consequently, it contains many open problems and conjectures. The author mentions that it is addressed to mathematicians interested in geometry and dynamics and to physicists working in a field related to symplectic geometry. I think it will be interesting in particular for young mathematicians who will discover a new, wide and attractive area for their research. (jiva)

D. Stirzaker: Stochastic Processes and Models*, Oxford University Press, Oxford, 2005, 331 pp., GBP 65, ISBN 0-19-856813-4*

This book presents an introduction to stochastic processes. It includes material on Markov chains in discrete and continuous time, martingales and diffusion processes. The mathematical tools are reduced so that there is no need for measure theory in the explanations. Therefore the text is suitable also for non-mathematicians, e.g. engineers, economists, and students and researchers from applied sciences. The main aim is to give ideas and applications of probabilistic models, not always with full proofs of statements and theorems. There are easy exercises at the end of each section and more challenging problems at the end of chapters. In this manner, advanced topics are also investigated, such as the reversibility of Markov chains, the Wiener process and functions of it, martingale methods and an introduction to stochastic calculus (including the Itô formula). Applications of Poisson and branching processes, renewals, birth-death processes and queues, ruin probability, the stock price model, etc., are used to demonstrate the use of various models. The textbook may serve as an undergraduate course for students with a mathematical background in various types of schools. (vbe)

C. van Aart: Organizational Principles for Multi-Agent Architectures*, Whitestein Series in Software Agent Technologies, Birkhäuser, Basel, 2004, 204 pp., EUR 34, ISBN 3-7643-7213-3*

The author of this book introduces the main issues of agent-oriented architectures and gives their solutions accompanied by practical examples. The explanations are based on similarities with issues and solutions in human organizations. Each of the main issues is discussed in a separate chapter. Firstly, the agent organization framework: decomposition, structures, organizational design activities, and an example (supply chain management) are presented. Then coordination strategies are discussed as problem solving techniques, their methods are classified, and implications to their design are deduced. The 'five capabilities model' is then described in its dimensions and then clarified on a complex insurance example. The chapter, 'Interoperation within complex multi-agent strategies', is based on experience with the IRBOW project. The chapter also concerns levels of interoperability with emphasis on the semantic one. The last chapter, 'Message content ontologies', recalls some useful ontologies and issues concerning ontology development and applications. The book may be useful for people willing to increase their understanding of agents and their coordination. (mzeml)

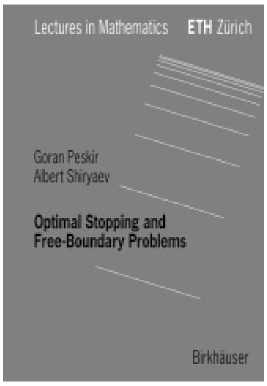
List of reviewers for 2006

The Editor would like to thank the following for their reviews this year.

J. Anděl, T. Bárta, M. Bečvářová-Němcová, V. Beneš, L. Boček, M. Bubeníková, J. Bureš, A. Drápal, J. Drahoš, M. Feistauer, J. Haslinger, S. Hencl, D. Hlubinka, P. Holický, J. Hurt, J. Ježek, V. Jírovský, O. John, O. Kalenda, P. Kaplický, J. Kofroň, J. Kratochvíl, A. Kučera, P. Kůrka, P. Lachout, J. Lukeš, J. Malý, M. Mareš, J. Milota, J. Mlček, I. Mrázová, K. Najzar, J. Nekovář, J. Nešetřil, I. Netuka, A. Pultr, L. Pick, Š. Porubský, D. Pražák, P. Pyrih, J. Rataj, M. Rokyta, T. Roubíček, P. Růžička, I. Saxl, A. Slavík, P. Somberg, V. Souček, J. Spurný, D. Stanovský, J. Stará, J. Štěpán, J. Trlifaj, V. Trnková, J. Tůma, J. Vančura, Z. Vlášek, J. Veselý, M. Zahradník, J. Žemlička, M. Žemlička, K. Žitný.

All of the above are on the staff of the Charles University, Faculty of Mathematics and Physics, Prague, except J. Vanžura (Mathematical Institute, Czech Academy of Sciences), M. M. Bečvářová-Němcová, Š. Porubský (Technical University, Prague) and J. Nekovář (University Paris VI, France).

New Books

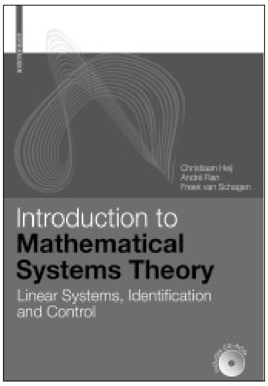


Peskir, G., The University of Manchester, UK / **Shiryaev, A.**, Steklov Mathematical Institute, Moscow, Russia

Optimal Stopping and Free-Boundary Problems

2006. XXII, 500 p. Hardcover
€ 48.– / CHF 78.–
ISBN 978-3-7643-2419-3
LM – Lectures in Mathematics ETH Zürich

The book aims at disclosing a fascinating connection between optimal stopping problems in probability and free-boundary problems in analysis using minimal tools and focusing on key examples. The general theory of optimal stopping is exposed at the level of basic principles in both discrete and continuous time covering martingale and Markovian methods. Methods of solution explained range from classic ones (such as change of time, change of space, change of measure) to more recent ones (such as local time-space calculus and nonlinear integral equations). A detailed chapter on stochastic processes is included making the material more accessible to a wider cross-disciplinary audience. The book may be viewed as an ideal compendium for an interested reader who wishes to master stochastic calculus via fundamental examples. Areas of application where examples are worked out in full detail include financial mathematics, financial engineering, mathematical statistics.



Heij, C., Erasmus University Rotterdam, The Netherlands / **Ran, A.** / **van Schagen, F.**, both Vrije Universiteit Amsterdam, The Netherlands

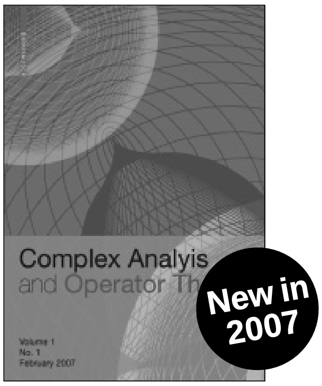
Introduction to Mathematical Systems Theory

Linear Systems, Identification and Control

2007. VII, 166 p. With CD-ROM. Softcover
€ 38.– / CHF 62.–
ISBN 978-3-7643-7548-5

This book provides an introduction to the theory of linear systems and control for students in business mathematics, econometrics, computer science, and engineering. The focus is on discrete time systems, which are the most relevant in business applications, as opposed to continuous time systems, requiring less mathematical preliminaries. The subjects treated are among the central topics of deterministic linear system theory: controllability, observability, realization theory, stability and stabilization by feedback, LQ-optimal control theory. Kalman filtering and LQC-control of stochastic systems are also discussed, as are modeling, time series analysis and model specification, along with model validation. Exercises using MATLAB, presented on an accompanying CD, enhance the main concepts and techniques in the text.

New Journals



Complex Analysis and Operator Theory

Complex Analysis and Operator Theory is devoted to the publication of current research developments in the closely related fields of complex analysis and operator theory as well as in applications to system theory, harmonic analysis, probability, statistics, learning theory, and other related fields. Articles using the theory of reproducing kernel spaces are in particular welcomed.

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ISSN 1661-8254 (print)
ISSN 1661-8262 (electronic)
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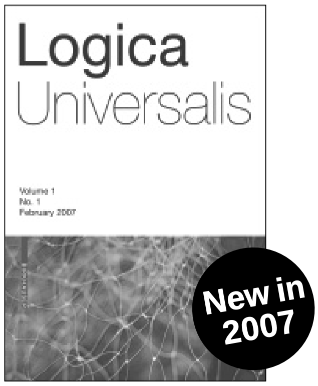

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Logica Universalis publishes peer-reviewed research papers related to universal features of logics. Topics include general tools and techniques for studying already existing logics and building new ones, the study of classes of logics, the scope of validity and the domain of application of fundamental theorems, and also philosophical and historical aspects of general concepts of logic.

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ISSN 1661-8297 (print)
ISSN 1661-8300 (electronic)
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