NEWSLETTER

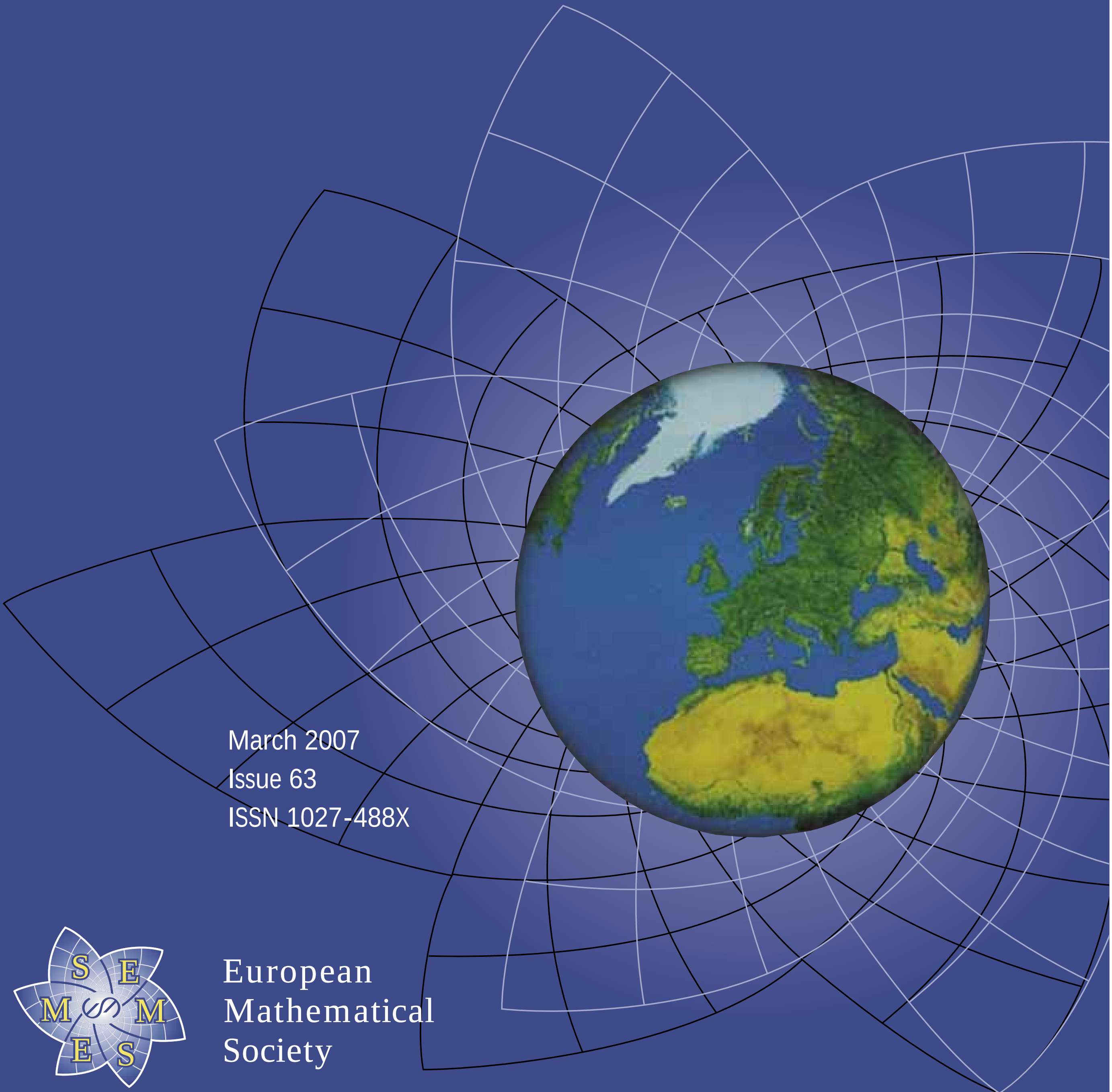
OF THE EUROPEAN MATHEMATICAL SOCIETY

Feature
Google and Linear Algebra
p. 10

History
The ICM through History
p. 16

Interview
J. Morgan, A. Connes, R. Jeltsch
p. 22

ERCOM
Rényi Institute Budapest
p. 36



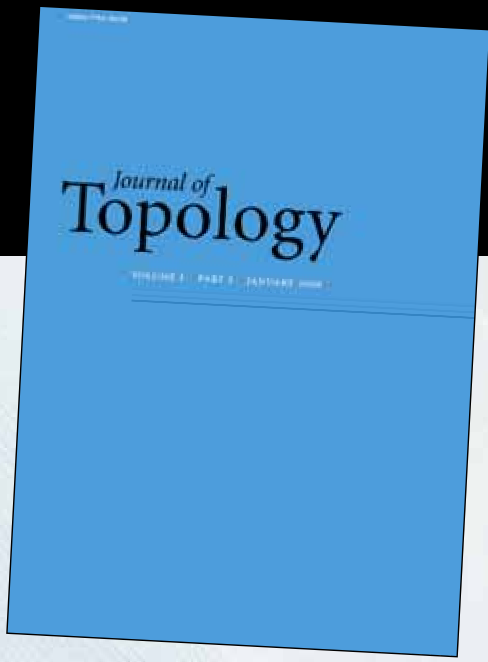
March 2007
Issue 63
ISSN 1027-488X

European
Mathematical
Society

The London Mathematical Society

Journal of Topology

It is with great pleasure that we announce the launch of the ***Journal of Topology***, a new journal that will publish its first issue in January 2008.



The Board of the Journal will be:

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Aims and Scope

The ***Journal of Topology*** will publish papers of high quality and significance in topology, geometry and adjacent areas of mathematics. Interesting, important and often unexpected links connect topology and geometry with many other parts of mathematics, and the editors welcome submissions on exciting new advances concerning such links, as well as those in the core subject areas of the journal. The ***Journal of Topology*** will appear in quarterly issues with articles posted individually online.

Guidelines for Paper Submission

Submissions are currently being welcomed. For information on submitting a paper, please visit **www.lms.ac.uk/publications/jtop.html**
If you have further enquiries about the journal, please contact Susan Hezlet, Publisher LMS, at **hezlet@lms.ac.uk**, or the Managing Editor at **tillmann@maths.ox.ac.uk.**

The price of the Journal for Volume 1 (four issues) including electronic access will be **£300** or **$570** in 2008, although we plan further discounts for libraries who take our other journals. The journal is owned by the London Mathematical Society, a not-for-profit publisher and the pre-eminent British society for research mathematics, and will be published in association with Oxford Journals. All sales enquiries should be made to **jnls.cust.serv@oxfordjournals.org** or to **hezlet@lms.ac.uk**.

European Mathematical Society

Newsletter No. 63, March 2007

ISSN 1027-488X

Published by the
EMS Publishing House
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CH-8092 Zürich, Switzerland.
homepage: www.ems-ph.org

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EMS Calendar

2007

10–11 March
EMS Executive Committee Meeting at Amsterdam
(The Netherlands)

1 April
Deadline for proposals for minisymposia at 5ECM
lex@cwi.nl
www.5ecm.nl/minisymposia.html

1 May
Deadline for submission of material for the June issue of the EMS Newsletter
Martin Raussen: raussen@math.aau.dk

6–12 May
EMS Summer School – Séminaire Européen de Statistique, La Manga (Cartagena, Spain)
SEM STAT: Statistics for stochastic differential equations models
mathieu.kessler@upct.es or lindner@ma.tum.de
http://www.dmae.upct.es/semstat2007/

3–10 June
EMS Conference at Będlewo (Poland)
Geometric analysis and nonlinear partial differential equations
B.Bojarski@impan.gov.pl or pawelst@mimuw.edu.pl

9–10 June
EMS Executive Committee Meeting at the invitation of the Euler Society, St. Petersburg (Russia)
Stephen Huggett: s.huggett@plymouth.ac.uk

16–20 July
ICIAM 2007, Zurich (Switzerland)
www.iciam07.ch/

1 November
Deadline for submission of nominations of candidates for EMS prizes
tijdeman@math.leidenuniv.nl
http://www.5ecm.nl/ecmpc.pdf

2008

1 January
Deadline for the proposal of satellite conferences to 5ECM
top@math.rug.nl; www.5ecm.nl/satellites.html

14–18 July
5th European Mathematical Congress, Amsterdam
(The Netherlands)
www.5ecm.nl

16–31 August
EMS-SMI Summer School at Cortona (Italy)
Mathematical and numerical methods for the cardiovascular system
dipartimento@matapp.unimib.it

Editorial

It has now been a month since I was trusted to take over responsibility for the European Mathematical Society from the former EMS President, Sir John Kingman. It is a great honour as well as an enormous challenge and I shall endeavour to do everything I can to promote mathematics in Europe and make the EMS even more visible and useful to its members.

John has presided over the EMS for the last four years, 2002-2006, with great talent and charm. I am always impressed and delighted by the way he is able to handle meetings with such effortless diplomacy and by his ability to make people feel appreciated and thus motivate them to perform. His brilliant speeches, filled with his inimitable sense of humour, are the highlight of every meeting.

The EMS is a relatively new society that still needs to attract many more individual members by providing a service that will meet the needs of every European mathematician. Although many important aims have already been achieved, there is still much to be done in order to unite European mathematicians. The development of mathematics in Europe will not be possible without the support of all European Sciences. A big step has recently been taken with the creation of the European Research Council (see the article by Pavel Exner in this issue of the Newsletter).

With the enormous development in technology, mathematics has become more and more relevant with regard to its applications. A profound knowledge of mathematics has become crucial in new areas such as nanotechnology, computer science, financial mathematics, etc. Many areas that were previously considered to be pure mathematics have found practical applications. These areas often go their own way and can become quite remote from the mathematical theories from which they originated.

I believe, however, that for future progress it is important to maintain the unity of the various branches of applied and pure mathematics. I am sure that such a unity is beneficial for both sides and many of us feel that it is necessary today to encourage a closer collaboration between pure and applied mathematics. The EMS can play a uniting role in this collaboration.

I also believe that the EMS Applied Mathematics Committee, chaired by Professor Mario Primicerio, will work to unite the applied mathematical societies of the various European countries.

We must not forget to think about the future of the next generation of mathematicians. The teaching of mathematics in schools throughout Europe is not satisfactory. First year students arrive at universities less and less prepared, despite the new theories in pedagogical methods of mathematical education that have been implemented in schools. The gap between what we expect these students to know and what they actually know is growing.

I have noticed that some of my first year students at KTH in Stockholm have never heard of the word „proof" and were most confused when I suggested that they should give a reason why a statement was correct. Mathematics in schools has to be taught in such a way that students are better prepared for University education, i.e. with definitions and proofs. Euclidean geometry should be part of the standard curriculum. School courses should be modernised but not in the way that it has been done over the last few decades, where most of the changes led to a general watering down of the courses.

It would be very valuable if the EMS could help spread the positive experiences of some of our colleagues who have been able to popularise mathematics and attract school children to our beautiful subject. Many such talented enthusiasts can be found in the former USSR and I suggest that we try to learn from their experience.

This general decline in school education is a phenomenon which appears in almost all European countries. Considering that European university education is becoming more and more universal, this problem has to be resolved at the European level. The EMS could really use its influence but only by working together with high school teachers in mathematics throughout Europe.

In order to make European mathematics stronger it would be good to organise a regular forum where all partners could meet. The joint annual meetings of the American Mathematical Society (AMS) and the Mathematical Association of America (MAA) are a good example of collaboration and fruitful interaction between university professors and high school teachers. We too have the possibility of making our forums even more successful by having representatives from European applied mathematics.

In conclusion, I would like to point out that the future success of the EMS depends on the wide support of members of the pure and applied mathematical societies of European countries and on a closer collaboration between the EMS and other mathematical communities and organisations all over the world.

Finally, I should like to wish you all a happy and hard working year for 2007!

February 1, 2007 Ari Laptev

New members of the EMS executive committee

At its meeting in Torino (Italy) in July 2007, EMS council elected five new members to the executive committee of the society; among them a new president (see the Editorial), a new vice-president, a new secretary and a new treasurer. In this issue, three of the new members present themselves; EMS- president Ari Laptev and vice-president Helge Holden will be portrayed in the next issue. The thanks of the society are due to former president Sir John Kingman, vice-president Luc Lemaire, treasurer Olli Martio and to Doina Cioranescu, who all left the executive committee at the end of 2006 after many years of service to the society.

Mireille Martin-Deschamps received a doctorate from the University of Orsay in 1976. She was a researcher at the University of Orsay from 1969 to 1987, then at the University of Paris VII from 1987 to 1990 and at the ENS of Paris from 1991 to 1998. In 1998 she joined the University of Versailles Saint-Quentin as the director of the mathematics laboratory (from 1998 to 2004) and she became a professor there in 2003.

Her research interests lie mainly in complex algebraic geometry (classification of space curves) and commutative algebra, and she has recently been interested in applied algebra and cryptography.

She was president of the French Mathematical Society from 1998 to 2001. Since 2003 she has been a scientific councillor at the Research Directory of the French Ministry of Research and since 2002 she has been a member of the Scientific Council of the City of Paris.

She thinks that today's society is experiencing a growing need for scientists in general and mathematicians in particular. The EMS has a very important role in promoting mathematics and in explaining, at the European level, that they are at the heart of scientific and technological advancement.

Stephen Huggett received a doctorate in mathematics from the University of Oxford in 1980, under the supervision of Roger Penrose. His first research interests were in twistor theory, and he continues to find the interplay between the structure of space-time and ideas from algebraic topology and complex differential geometry completely fascinating. A more recent research interest is in polynomial invariants of knots, graphs, and matroids. He finds the Tutte polynomial intriguing, but even the relatively humble chromatic polynomial contains many wonderful mysteries!

He has taught at King's College London and at Lincoln College Oxford, and is now a Reader in Mathematical Physics at the University of Plymouth. He has also been an Associate Lecturer at the Open University. Together with David Jordan he has written an elementary textbook of geometric topology called *A Topological Aperitif*.

He is the Programme Secretary of the London Mathematical Society, responsible for the Society's meetings and for the Society's research grants. He also chairs the LMS International Affairs Committee. In 2003 he was the Scientific Secretary to the International Review of Mathematics, to assess the standing and potential of mathematics research in UK universities, in comparison with international work in the field.

He enjoys occasional consultancy work, especially with a local company called *Dartcom* on the geometrical principles underlying new software designed to generate map overlays for weather satellite images. One of his favourite activities is walking along the beautiful Cornish coast.

Jouko Väänänen was born in 1950 in Lapland, Finland. He studied in Helsinki, Oslo, Warsaw and Manchester and got his PhD from Manchester University in 1977. He was nominated as an associate professor of mathematics at the University of Helsinki in 1983, was made full professor in 1998 and became a professor of mathematical logic at the University of Amsterdam in 2006.

His research interests lie in logic and the foundations of mathematics, especially set theory, model theory, finite model theory, generalized quantifiers and infinitary logic. He has organized several logic conferences in Finland and he is a member of the editorial boards of the "Notre Dame Journal of Formal Logic" and the "Logica Universalis".

He served as Chairman of the Department of Mathematics and Statistics and as a member of the Senate of the University of Helsinki from 2004 to 2006. Jouko Väänänen has been a member of the Finnish Academy of Science and Letters since 2002. He is a member of the Executive Committee of the Association for Symbolic Logic (ASL) and the chairman of the Committee for Logic in Europe of the ASL.

In his opinion the EMS has a very important role in supporting the mathematical community and emphasizing the role of mathematics in modern society at the European level. More and more research funding is channelled through EC instruments, making it hard for basic research such as mathematics to make its voice heard. The role of mathematics in society is growing, calling for engaging and accessible methods of mathematics education. At the same time the competition for the time and intellectual attention of the mobile phone generation has become harder and harder. In the multi-lingual and multi-cultural European scene the EMS has a special role in both taking its own initiatives and in helping national organizations meet this challenge.

EMS Executive Committee meeting Krakow, 21st–22nd October 2006

Vasile Berinde, EMS Publicity Officer

The EC meeting was held at the Collegium Maius, Jagellonian University (first day) and in the Bursa im. St. Pigonia (second day). Present were: Sir John Kingman, Pavel Exner, Luc Lemaire, Helge Holden, Olli Martio, Viktor Buchstaber, Doina Cioranescu, Olga Gil-Medrano, Carlo Sbordone and (by invitation): Ari Laptev, Stephen Huggett, Jouko Vaananen, Mireille Martin-Deschamps, Riitta Ulmanen, Martin Raussen, Jan Wiegerinck and Vasile Berinde.

The president thanked the local organizers of the EC meeting, in particular Andrzej Pelczar. After the president's report about the IMU General Assembly, which he attended as an invited observer, the EC decided on various EMS financial matters.

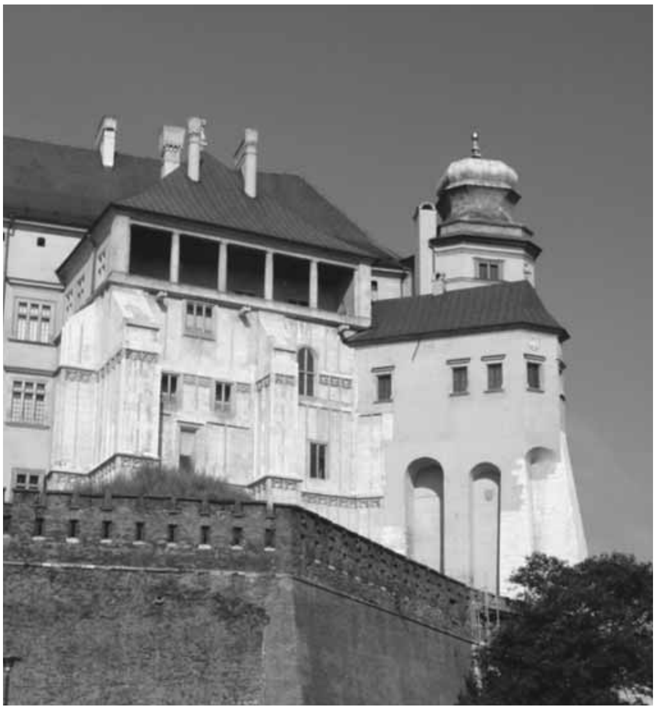
Jan Wiegerinck presented the work for organizing the 5ECM. The conference venue had been reserved but more funds were needed. An intense discussion on this matter focused on the conference fee, exemptions for fees and grants. Bids for the 6ECM to be held in 2012 are requested by March 15, 2007. Krakow (Poland) and Vienna (Austria) have already expressed their interests in organizing the 6ECM. The EC discussed the Prize and Programme Committees: the Chairs of both committees are appointed by the EC. A preliminary list of members of both committees should be proposed by the Chair of each committee and the EMS President for final approval by the EC.

Then various general and specific matters concerning standing EMS Committees were discussed and several reports were given or acknowledged. It was decided to find ways to ensure that terms of office be staggered in order to ensure continuity. Other business considered by the EC: projects, Newsletter (Martin Raussen informed the EC that he plans to step down in 2008 and that the EC has to look for a successor), Zentralblatt, and relations with funding organisations and political bodies and with mathematical organizations. As the present web site of the EMS is shared with the EMIS, it was agreed that the EMS will develop its own web site. Ari Laptev, the new president starting from 1st January 2007, will contact the EMIS to discuss the separation from the EMIS web site. Helge Holden, Stephen Huggett and Pavel Exner will continue to discuss how to develop an EMS web site.

The president thanked Doina Cioranescu, Luc Lemaire, and Olli Martio for their work for the EMS and the EC as their terms of office are coming to an end at the end of 2007. John Kingman also thanked everyone for his four years as President of the EMS and for being the Chair of the Executive Committee.

The next EC meeting will be held in Amsterdam on 10th–11th March 2007.

Courtyard of Collegium Maius, Krakow



Wawel Royal Castle, Krakow

Why ERC?

Pavel Exner (Prague, Czech Republic)

The New Year brought various changes to Europe. Not least among them was the Seventh Framework Programme and, in particular, the section called "Ideas" that will be implemented through the European Research Council. Its creation is a result of a joint several-year effort in which the scientific community has collaborated with European politicians. It is encouraging that among the politicians, there are people capable of thinking on a scale longer than one electoral term.

If there is a political will to support science the next question is how to do so. We all know that top-level science is a competitive matter, often fiercely so. The crucial point is that support for research is efficient only if it is strongly correlated with the quality of the results. It is not my task here to discuss previous EU actions in this field – and there are no doubt many useful things that were done – but their weakest point was a lack of such feedback. In one form or other most of them always contained the principle of *juste retour* by which any country was entitled to roughly the sum it invested.

The European Research Council represents a break with this tradition, being conceived as an institution that will use scientific excellence as a sole criterion. We are convinced that this is the only way in which one can build a system that would rival the NSF and NIH in the U.S., the JSPS in Japan, etc. The need for it is vital; European research has been visibly losing ground in the last decades and resolute actions are required if we do not want to become peripheral to the heightening global competition.

Let me recall what has been done so far without burdening the reader with too many details. Notice that ERC documents can be found at http://erc.europa.eu. The task to formulate the strategy was trusted to the 22-member Scientific Council, nominated by Commisioner Potočnik, which started functioning in October 2005. Note that its members act in their personal capacity and represent neither countries nor societies; nevertheless each of us belongs professionally to some community, to the mathematical one in my case.

The first principle we set was that project selection will be done bottom up. We do not intend to tell the community what is the best science but rather open the door for a *free competition of ideas* in which fresh and strong ones will emerge; pretentiously we hope for breakthroughs of course. The second crucial factor is the peer review scheme. Again we want to avoid the weaknesses of previous European programmes, in this case an uneven quality of the referee corpus. We put the decision into the hands of *panels consisting of respected scientists*; the "mathematical" panel for our first call will be headed by Jean-Pierre Bourguignon and composed to cover the whole breadth of the discipline.

The third deviation from tradition concerns the ways in which the support will be processed. We follow the principle of "*keep it simple, trust the people*". We intend to have a non-complicated grant agreement with a flat 20% overhead and the reporting reduced to a brief summary of essential facts. Naturally, the necessary administrative makeup is being built with our partners from the EC structures who are qualified and dedicated but also experienced in the ways in which these things functioned - everybody knows what a typical Brussels questionnaire looks like. Finding a common language is not always trivial but we feel that the will to do so is not lacking.

As usual one can achieve only what the budget allows. The sum available for the first year is about 300 M Euro which is certainly not much (note that it is less than a third of what the DFG alone spends and about 7% of the NSF budget). On the other hand, the framework programme expects an almost linear growth so that at the end of the programme the annual expenditure will reach 1.7 G Euro. With the money we have at our disposal we decided to concentrate in the first year on a single programme called the *ERC Starting Grant*. In 2008 we will open the next programme to support strong ideas irrespective of the author's age. The first call with the deadline of April 25 is open already (see http://cordis.europa.eu/fp7/dc/).

The starting grants, given for five years in the range of 100–400 k Euro annually, are aimed at young people, up to nine years after PhD, that is at the age when they pass through one or two postdoc positions and look for a more stable one where they could establish themselves as independent research leaders. We feel that this is the biggest bottleneck where Europe loses the brightest young people due to the rigidity of its labour market. Let me stress that we impose restriction neither on the applicant's nationality nor the size or composition of the team; it is important for mathematicians that an individual may apply. The only requirement is that the winners will do their research in Europe.

Let me finally say that from the very beginning the ERC Scientific Council felt strong support from the community, manifested by a prompt and positive reaction to all our calls.

We hope that this attitude will persist (with a realistic appreciation of what can be achieved with the means at our disposal) and that we will live to see Europe again as a world leader in science.

Pavel Exner
EMS Vice President and an ERC Scientific Council member

The 7th RTD Framework Programme of the European Community
First calls for proposals

The seventh Framework Programme of the European Community has now been launched for the period 2007–2013, with a total budget of 50.5 billion Euros.
There are various actions where mathematical research can be supported; and this provides opportunities that should not be missed.

For a description of the programme, see:
http://cordis.europa.eu/fp7/home_en.html
or http://ec.europa.eu/research/fp7/home_en.html
From there, you can access the list of open calls for proposals, on
http://cordis.europa.eu/fp7/dc/index.cfm

The programme is divided in four main parts, namely:

- Cooperation: With a budget of 32.4 billion, this concerns research in ten thematic priorities. To participate, mathematicians should imbed in a network on one of these priorities, together with researchers of that theme.
- Ideas: With a budget of 7.5 billion, this is the most innovative aspect of FP7, since it concerns investigator driven research in all fields, evaluated according to scientific criteria only. This part is handled by the ERC (European Research Council), described in the article. This action should provide the best opportunities for high level European mathematicians.
- People: This part (4.7 billion) is the prolongation of the Marie-Curie actions, with a smaller number of activities. They are Initial training networks, Individual fellowships (either EU funded or co-funded with member states), Industry-academia pathways, International outgoing or incoming fellowships, Marie-Curie Awards. . .
- Capacities: With a budget of 4.1 billion, this component includes notably the support of research infrastructures and special measures to develop the research capacities in the "convergence regions" of the EU.

I believe European mathematicians deserve a good place in this programme, particularly in the Ideas and People actions. So I can only encourage you to apply, following the indications of the calls.

Luc Lemaire, Bruxelles

EWM07: European Women in Mathematics

First Announcement

The 13th general meeting of European Women in Mathematics (EWM), open to members and non-members of EWM, will take place at the Centre for Mathematical Sciences (CMS), University of Cambridge from lunchtime on Monday 3 September to lunchtime on Thursday 6 September 2007. Accommodation has been arranged at Fitzwilliam College, Cambridge. Please consult the website www.maths.cam.ac.uk/ewm for details.

Many have been amazed and encouraged by the experience of attending an EWM conference, never having previously been part of a group of over 100 women listening intently to a talk on state-of-the-art mathematical research, or had the opportunity to meet and talk to women mathematicians in a variety of fields. The conferences have sparked collaborations, follow-on meetings on related themes and, most importantly, have inspired many women from graduate students to professors as they develop their careers as working mathematicians.

Talks at EWM07 will cover a range of mathematical areas. The invited speakers are among the very best in their areas of research, and we hope that there will be something to interest all mathematicians.

Confirmed speakers so far include:
Ana Achucarro
Lorenz Institute, Leiden: Theoretical Physics
Natalia Berloff
Cambridge, UK: Quantum fluids
Lenore Blum
Carnegie Mellon University, USA: Theoretical Computer Science
Simone Gutt
Univ. Libre de Bruxelles: Symplectic Geometry
Eleny Ionel
Stanford, USA: Symplectic Geometry
Dusa McDuff
Stonybrook, USA: Symplectic Geometry
Cheryl Praeger
University of Western Australia: Group Theory
Vera Sos
Budapest: Combinatorics
Ramdorai Sujatha
Tata Institute, Mumbai: Number Theory.

There will also be a session on mathematical education, with a talk given by Toni Beardon (Cambridge, UK), speaking on the impact of computers and the internet on globalising mathematics education.

In addition there will be an afternoon devoted to contributed short talks and posters, which will have parallel sessions.

A discussion on the role and future of EWM is also planned.

A conference banquet has been arranged at Trinity College, Cambridge on Wednesday 5th September. There will also be receptions at Newnham College and at the Master's Lodge, Trinity College on the other evenings. Participants will have the chance to go on a punting trip on the river in Cambridge.

We aim to provide some childcare during the conference – let us know if you might require this.

Costs and Funding

There is a registration fee of £60 (£30 for students) for the conference. The conference dinner costs around an extra £35 per person. We may be able to reduce these costs if sufficient sponsorship is found.

Accommodation at Fitzwilliam College costs between £29–£41.60 per night (42.50–61 Euros) for bed and breakfast, depending on facilities.

The Organising Committee hopes to obtain some funding for participants, particularly students and participants from developing countries.

In addition, opportunities are available for women from developing countries to visit the ICTP in Trieste for a few weeks around the time of this conference. Please let us know if you would be interested in applying for such a visit.

Organizing Committee:

Eva Bayer (Lausanne, Switzerland)
Anne Davis (Cambridge, UK)
Catherine Hobbs (Oxford Brookes, UK)
Marjo Lipponen (Turku, Finland)
Ursula Martin (Queen Mary, London, UK)
Sylvie Paycha (Blaise Pascal, Clermont Ferrand, France)
Caroline Series (Warwick, UK)

If you are interested in attending the meeting, do visit our website and send an email to the conference administrator, Amanda Stagg, at *ewm07@maths.cam.ac.uk* with your name and contact details.

Google's secret and Linear Algebra

Pablo Fernández Gallardo (Madrid, Spain)

1. Introduction

Some months ago newspapers all around the world considered Google's plan to go public. The significance of this piece of news was not only related to the volume of the transaction, the biggest since the dot-com "irrational exuberance" in the 90's, but also to the particular characteristics of the firm. A few decades ago there was a complete revolution in technology and communications (and also a cultural and sociological revolution), namely the generalization of use and access to the Internet. Google's appearance has represented a revolution comparable to the former as it became a tool that brought some order into this universe of information, a universe that was not manageable before.

The design of a web search engine is a problem of *mathematical engineering*. Notice the adjective. First, a deep knowledge of the context is needed in order to translate it into models, into mathematics. But after this process of abstraction and after the relevant conclusions have been drawn, it is essential to carry out a thorough, detailed and efficient design of the computational aspects inherent in this problem.

2. The Google engine

The Google search engine was designed in 1998 by Sergei Brin and Lawrence Page, two computer science doctorate students at Stanford – two young men, now in their thirties, who have become multimillionaires. The odd name of the firm is a variation of the term *googol*, the name that somebody[1] invented to refer to the overwhelming number 10^{100}. This is one of those numbers that mathematicians are comfortable with but are perhaps bigger than the number of particles in the whole universe.

The scale of the question we are concerned with is also immense. In 1997, when Brin and Page were to start working on Google's design, there were about 100 million web pages. Altavista, the most popular search engine in those days, attended to 20 million daily queries. Today, these figures have been multiplied; Google receives some hundred million daily queries and indexes several billion web pages.

Therefore, the design of a search engine must efficiently solve some computational aspects, namely the way to store that enormous amount of information, how it is updated, how to manage the queries, the way to search the databases, etc. But, although interesting, we are not going to treat these questions here. The point of interest can be formulated in a simple manner. Let us suppose that, after a certain query, we have determined that, say, one hundred web pages enclose information that might, in some sense, be relevant to the user. Now, *in which order should they be displayed?* The objective, as explicitly posed[2] by Brin and Page (see [6]), is that in the majority of attempts, at least one of, say, the first ten displayed pages contains useful information for the user.

We now ask the reader (quite possibly a *google-maniac* himself) to decide, from his own experience, whether Google fulfils this objective or not. We are sure the common response will be affirmative ... and even amazingly affirmative! It seems to be magic[3] but it is just mathematics, mathematics requiring no more than the tools of a first year graduate course, as we will soon see.

To tackle our task, we need an **ordering criterion**. Notice that if we label each web page with symbols $P_1,\dots,P_n$, all we want is to assign each P_j a number x_j, its **significance**. These numbers might range, for example, between 0 and 1. Once the complete list of web pages, along with their significances, is at our disposal, we can use this ordering each time we answer a query; the selected pages will be displayed in the order as prescribed by the list.[4]

3. The model

Let us suppose that we have collected all the information about the web: sites, contents, links between pages, etc. The set of web pages, labelled $P_1 \dots P_n$, and the links between them can be modelled with a (directed) **graph** G. Each web page P_j is a vertex of the graph and there will be an edge between vertices P_i and P_j whenever there is a link from page P_i to page P_j. It is a gigantic, overwhelming graph, whose real structure deserves some consideration (see Section 8).

When dealing with graphs, we like to use drawings in the paper, in which vertices are points of the plane, while edges are merely arrows joining these points. But, for our purposes, it is helpful to consider an alternative description, with matrices. Let us build an $n \times n$ matrix $\mathbf{M}$ with zero-one entries, whose rows and columns are labelled with symbols $P_1,\dots,P_n$. The matrix entry m_{ij} will be 1 whenever there is a link from page P_j to page P_i and 0 otherwise.

The matrix $\mathbf{M}$ is, except for a transposition, the *adjacency matrix* of the graph. Notice that it is not necessarily symmetric because we are dealing with a directed graph. Observe also that the sum of the entries for P_j's column is the number of P_j's outgoing links, while we get the number of ingoing links by summing rows.

We will assume that the significance of a certain page P_j "is related to" the pages linking to it. This sounds reasonable; if there are a lot of pages pointing to P_j, its information must have been considered as "advisable" by a lot of web-makers.

The above term "related to" is still rather vague. A **first attempt** to define it, in perhaps a naïve manner, amounts to supposing that the significance x_j of each P_j *is proportional to the number of links to* P_j. Let us note that, whenever we have the matrix $\mathbf{M}$ at our disposal, the computation of each x_j is quite simple; it is just the sum of the entries of each row P_j.

This model does not adequately grasp a situation deserving attention, i.e., when a certain page is cited from a few *very relevant* pages, e.g., from www.microsoft.com and www.

amazon.com. The previous algorithm would assign it a low significance and this is not what we want. So we need to enhance our model in such a way that a strong significance is assigned both to highly cited pages and to those that, although not cited so many times, have links from very "significant pages".

Following this line of argument, the **second attempt** assumes that the significance x_j of each page P_j *is proportional to the sum of the significances of the pages linking to* P_j. This slight variation completely alters the features of the problem.

Suppose, for instance, that page P_1 is cited on pages P_2, P_{25} and P_{256}, that P_2 is only cited on pages P_1 and P_{256}, etc., and that there are links to page P_n from P_1, P_2, P_3, P_{25} and P_{n-1}. Following the previous assignment, x_1 should be proportional to 3, x_2 to 2, etc., while x_n should be proportional to 5. But now, our assignment $x_1,\dots,x_n$ must verify that

$$\begin{aligned} x_1 &= K\,(x_2+x_{25}+x_{256})\,,\\ x_2 &= K\,(x_1+x_{256})\,,\\ &\;\vdots\\ x_n &= K\,(x_1+x_2+x_3+x_{25}+x_{n-1})\,, \end{aligned}$$

where K is a certain proportionality constant. In this way, we face an enormous system of linear equations, whose solutions are all the admissible assignments $x_1\dots x_n$. Below these lines we write the system of equations in a better way, using matrices.

$$\begin{pmatrix}x_1\\x_2\\\vdots\\x_n\end{pmatrix} = K \begin{array}{c} \begin{array}{ccccccccccccccc} P_1 & P_2 & & & & P_{25} & & & & P_{256} & & & & P_{n-1} & \\ \downarrow & \downarrow & & & & \downarrow & & & & \downarrow & & & & \downarrow & \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0\\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & 1 & 1 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \end{array} \begin{pmatrix}x_1\\x_2\\\vdots\\x_n\end{pmatrix}$$

Let us call the significance vector $\mathbf{x}$. The $n\times n$ matrix of the system is exactly the matrix $\mathbf{M}$ associated with the graph. So we can state that the significance assignment is a solution of

$$\mathbf{Mx} = \lambda\mathbf{x}\,.$$

We have already used the symbol λ for the constant of proportionality. This is so because, as anyone who has been exposed to a linear algebra first course will recognize, the question has become a problem of **eigenvalues** and **eigenvectors**; the significance vector $\mathbf{x}$ is no more than an eigenvector of the matrix $\mathbf{M}$. You might recall that this matrix contains all the information about the web structure, i.e., the vertices and adjacency relations.

Perhaps this is not enough to arouse the reader's enthusiasm yet. Alright, an eigenvector. But which one? There are so many. And also, how could we compute it? The matrix is inconceivably huge. Remember, it is built up of a thousand million rows (or columns). Patience, please. For the time being, it sounds reasonable to demand the entries of our vector (the significance of the web pages) to be non-negative (or, at least, with the same sign). This will be written as $\mathbf{x}\geq 0$. We ask the reader to excuse this abuse of notation. But also, since we intend the method to be useful, we need this hypothetical non-negative vector to be **unique**. If there were more than one, which of them should be chosen?

4. The random surfer

Google's approach to the question follows a slightly different point of view. At the present stage, a page P_j distributes a "1" to every page where there is an outgoing link. This means that pages with many outgoing links have a great influence, which surely is not reasonable. It is more fair to assign each page P_j a "total weight" 1, which is equally distributed among the outgoing links. So we should consider a new matrix instead of $\mathbf{M}$ (the matrix of the graph, with entries 0 and 1). Let N_j be the number of P_j's outgoing links (that is, the sum of the entries in the column labelled P_j in $\mathbf{M}$). The new matrix $\mathbf{M}'$ is built from the original $\mathbf{M}$ by replacing each entry m_{ij} by $m'_{ij} = m_{ij}/N_j$. The entries of $\mathbf{M}'$ will be non-negative numbers (between 0 and 1) and the sum of the entries for *each column* will be 1. And now we are interested in the non-negative vector of the corresponding[5] problem $\mathbf{M}'\mathbf{x} = \lambda\mathbf{x}$. The matrix $\mathbf{M}'$ is called a **stochastic (or Markovian) matrix**.

This new point of view leads us to a nice interpretation. Let us imagine a user surfing the web. At some moment he will reach some page, say P_1. But, probably bored with the contents of P_1, he will jump to another page, following P_1's outgoing links (suppose there are N_1 possibilities). But, to which one? Our brave navigator is a *random* surfer – and needless to say, also blond and suntanned. So, in order to decide his destination, he is going to use chance, and in the most simple possible way: with a regular (and virtual, we presume) die, which has the same number of faces as the number of outgoing links from P_1. In technical terms, the choice of destination follows a (discrete) *uniform* probability distribution in $[1,N_1]$. Say, for instance, that there are three edges leaving P_1 to vertices P_2, P_6 and P_8. Our navigator draws his destination, assigning a probability of 1/3 to each vertex.

Our model is no longer *deterministic* but *probabilistic*. We do not know where he will be a moment of time later but we do know *what his chances are* of being in each admissible destination. And it is a *dynamic* model as well because the same argument may be applied to the second movement, to the third one, etc. In our example, if the first movement is from P_1 to P_2 and there are four edges leaving P_2, then he is to draw again, now with probability 1/4 for each possible destination. Our surfer is following what is known as a **random walk** in the graph.

And what about the matrix $\mathbf{M}'$? Let us say that the surfer is on page (vertex) P_k at the beginning, that is in probabilistic terms, he is on page P_k with a probability of 100 %. We represent this initial condition with the vector $(0,\dots,1,\dots,0)$, the 1 being in position k. Recall that the surfer draws among the N_k destinations, assigning probability $1/N_k$ to each of them. But when we multiply the matrix $\mathbf{M}'$ by this initial vector, we get $(m'_{1k}, m'_{2k},\dots,m'_{nk})$, a vector with entries summing to 1: the m'_{jk} are either 0 or $1/N_k$ and there are exactly N_k non-zero entries. Notice that the vector we get exactly describes the probability of being, one moment later, on each page of the web, assuming he began at P_k. More than that, in order to know the probabilities of being on each page of the web after *two* moments of time, it is enough to repeat the process. That is, to multiply $(\mathbf{M}')^2$ by the initial vector. And the same for the third movement, the fourth, etc.

Following the usual terminology, we consider a certain number of **states**, in our case just being the vertices of the graph G. The matrix $\mathbf{M}'$ is (appropriately) called the **transition matrix** of the system; each entry m'_{ij} describes the probability of going from state (vertex) P_j to state (vertex) P_i. And the entries of the successive powers of the matrix give us transition probabilities between vertices as time goes by. The well versed reader may have already deduced the relation with the previous ideas: the stationary state of this Markov chain turns out to be precisely the non-negative vector of the problem $\mathbf{M}'\mathbf{x} = \lambda\mathbf{x}$.

It might happen that some pages have no outgoing links at all (with only zeros in the corresponding columns). This would not give a stochastic matrix. We will discuss Google's solution to this problem in Section 8.

5. Qualifying for the playoffs

We will illustrate the ordering algorithm[6] with the following question. Let us imagine a sports competition in which teams are divided in groups or conferences[7]. Each team plays the same number of games but not *the same number of games against each other*; it is customary they play more games against the teams from their own conference. So we may ask the following question. Once the regular season is finished, which teams should classify for the playoffs? The standard system computes the number of wins to determine the final positions but it is reasonable (see [10]) to wonder whether this is a "fair" system or not. After all, it might happen that a certain team could have achieved many wins just because it was included in a very "weak" conference. What should be worthier: the number of wins or their "quality"? And we again face Google's dichotomy!

Say, for example, that there are six teams, $E_1, \dots, E_6$, divided into two conferences. Each team plays 21 games in all: 6 against each team from its own conference, 3 against the others. These are the results of the competition:

	E_1	E_2	E_3	E_4	E_5	E_6	
E_1	–	3/21	0/21	0/21	1/21	2/21	$\to$ 6/21
E_2	3/21	–	2/21	2/21	2/21	1/21	$\to$ 10/21
E_3	6/21	4/21	–	2/21	1/21	1/21	$\to$ 14/21
E_4	3/21	1/21	1/21	–	2/21	2/21	$\to$ 9/21
E_5	2/21	1/21	2/21	4/21	–	2/21	$\to$ 11/21
E_6	1/21	2/21	2/21	4/21	4/21	–	$\to$ 13/21

To the right of the table, we have written the number of wins of each team. This count suggests the following ordering: $E_3 \to E_6 \to E_5 \to E_2 \to E_4 \to E_1$. But notice, for instance, that the leader team E_3 has collected a lot of victories against E_1, the worst one.

Let us now assign *significances* $\mathbf{x} = (x_1, \dots, x_6)$ to the teams with the mentioned criterion: x_j is proportional to the number of wins of E_j, *weighted* with the significance of the other teams. If $\mathbf{A}$ is the above table, this leads, once more, to $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$. And again, we want to find a non-negative eigenvector of $\mathbf{A}$ (a unique one, if possible).

Even in such a simple example as this one, we need to use a computer. So we ask some mathematical software to perform the calculations. We find that the moduli of the six eigenvalues are 0.012, 0.475, 0.161, 0.126, 0.139 and 0.161. So $\lambda = 0.475$ is the biggest (in modulus) eigenvalue, its associated eigenvector being

$$\mathbf{x} = (0.509, 0.746, 0.928, 0.690, 0.840, 1).$$

And this is the **only** eigenvector that has real non-negative entries! The components of the vector suggest the following ordering: $E_6 \to E_3 \to E_5 \to E_2 \to E_4 \to E_1$. And now E_6 is the best team!

Let us summarize. In this particular matrix with non-negative entries (that might be regarded as a small-scale version of the Internet matrix) we are in the best possible situation; there is a unique non-negative eigenvector, the one we need to solve the ordering question we posed. Did this happen by chance? Or was it just a trick, an artfully chosen matrix to persuade the unwary reader that things work as they should? The reader, far from being unwary, is now urgently demanding a categorical response. And he knows that it is time to welcome a new actor to this performance.

6. Mathematics enters the stage

Let us distil the common essence of all the questions we have been dealing with. Doing so, we discover that the only feature shared by all our matrices (being stochastic or not) is that all their entries are **non-negative**. Not a lot of information, it seems. They are neither symmetric matrices nor positive definite nor... Nevertheless, as shown by Perron[8] at the beginning of the 20th century, it is enough to obtain interesting results:

Theorem (Perron, 1907). *Let* $\mathbf{A}$ *be a square matrix with positive entries,* $\mathbf{A} > 0$. *Then*
(a) *there exists a (simple) eigenvalue* $\lambda > 0$ *such that* $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$*, where the corresponding eigenvector is* $\mathbf{v} > 0$*;*
(b) λ *is bigger (in modulus) than the other eigenvalues;*
(c) *any other positive eigenvector of* $\mathbf{A}$ *is a multiple of* $\mathbf{v}$.

Perron's result points to the direction we are interested in but it is not enough because the matrices we deal with might contain zeros. So we need something else. The following act of this performance was written several years later by Frobenius[9] where he deals with the general case of non-negative matrices. Frobenius observed that if we only have that $\mathbf{A} \geq 0$ then, although there is still a *dominant* (of maximum modulus) eigenvalue $\lambda > 0$ associated to an eigenvector $\mathbf{v} \geq 0$, there might be other eigenvalues of the same "size". Here is his theorem:

Theorem (Frobenius, 1908–1912). *Let* $\mathbf{A}$ *be a square matrix with non-negative entries,* $\mathbf{A} \geq 0$. *If* $\mathbf{A}$ *is* ***irreducible***,[10] *then*
(a) *there exists a (simple) eigenvalue* $\lambda > 0$ *such that* $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$*, where the corresponding eigenvector is* $\mathbf{v} > 0$. *In addition,* $\lambda \geq |\mu|$ *for any other eigenvalue* μ *of* $\mathbf{A}$.
(b) *Any eigenvector* ≥ 0 *is a multiple of* $\mathbf{v}$.
(c) *If there are k eigenvalues of maximum modulus, then they are the solutions of the equation* $x^k - \lambda^k = 0$.

Notice firstly that Frobenius' theorem is indeed a generalization of Perron's result, because if $\mathbf{A} > 0$, then $\mathbf{A}$ is ≥ 0 and irreducible. Secondly, if $\mathbf{A}$ is irreducible then the question is completely solved: there exists a unique non-negative eigenvector associated to the positive eigenvalue of maximum modulus (a very useful feature, as we will see in a moment).

These results, to which we will refer from now on as the Perron–Frobenius Theorem, are widely used in other contexts (see Section 9). Some people even talk about "Perron–Frobenius Theory", this theorem being one of its central results.

The proof is quite complicated and here we will just sketch an argument (in the 3×3 case) with some of the fundamental ideas. Let us start with a non-negative vector $\mathbf{x} \geq 0$. As $\mathbf{A} \geq 0$, the vector $\mathbf{Ax}$ is also non-negative. In geometric terms, the matrix $\mathbf{A}$ maps the positive octant into itself. Let us consider now the mapping $\boldsymbol{\alpha}$ given by $\boldsymbol{\alpha}(\mathbf{x}) = \mathbf{Ax}/\|\mathbf{Ax}\|$. Notice that $\boldsymbol{\alpha}(\mathbf{x})$ is always a unit length vector. The function $\boldsymbol{\alpha}$ maps the set $\{\mathbf{x} \in \mathbb{R}^3 : \mathbf{x} \geq 0, \|\mathbf{x}\| = 1\}$ into itself. Now, applying the Brouwer Fixed Point Theorem[11], there exists a certain $\tilde{\mathbf{x}}$ such that $\boldsymbol{\alpha}(\tilde{\mathbf{x}}) = \tilde{\mathbf{x}}$. Therefore,

$$\boldsymbol{\alpha}(\tilde{\mathbf{x}}) = \frac{\mathbf{A}\tilde{\mathbf{x}}}{\|\mathbf{A}\tilde{\mathbf{x}}\|} = \tilde{\mathbf{x}} \quad \Longrightarrow \quad \mathbf{A}\tilde{\mathbf{x}} = \|\mathbf{A}\tilde{\mathbf{x}}\|\,\tilde{\mathbf{x}}.$$

Summing up, $\tilde{\mathbf{x}}$ is an eigenvector of $\mathbf{A}$ with non-negative entries associated to an eigenvalue > 0. For all other details, such as proving that this eigenvector is (essentially) unique and the other parts of the theorem, we refer the reader to [1], [4], [13] and [14].

7. And what about the computational aspects?

The captious reader will be raising a serious objection: Perron–Frobenius' theorem guarantees the existence of the needed eigenvector for our ordering problem but *says nothing* about how to compute it. Notice that the proof we sketched is not a constructive one. Thus, we still should not rule out the possibility that these results are not so satisfactory. Recall that Google's matrix is overwhelming. The calculation of our eigenvector could be a cumbersome task!

Let us suppose we are in an ideal situation, i.e., in those conditions[12] that guarantee the existence of a positive eigenvalue λ_1 *strictly bigger* (in modulus) than the other eigenvalues. Let $\mathbf{v}_1$ be its (positive) eigenvector. We could, of course, compute *all* the eigenvalues and keep the one of interest but even using efficient methods, the task would be excessive. However, the structure of the problem helps us again and make the computation easy. It all comes from the fact that the eigenvector is associated to the dominant eigenvalue.

Suppose, to simplify the argument, that $\mathbf{A}$ is diagonalizable. We have a basis of $\mathbb{R}^n$ with the eigenvectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, the corresponding eigenvalues being decreasing size ordered: $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_n|$. We start, say, with a certain $\mathbf{v}_0 \geq 0$ that may be written as $\mathbf{v}_0 = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$, where the numbers $c_1, \ldots, c_n$ are the $\mathbf{v}_0$ coordinates in our basis. Now we multiply vector $\mathbf{v}_0$ by matrix $\mathbf{A}$ to obtain $\mathbf{Av}_0 = c_1\lambda_1\mathbf{v}_1 + c_2\lambda_2\mathbf{v}_2 + \cdots + c_n\lambda_n\mathbf{v}_n$ because the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are eigenvectors of $\mathbf{A}$. Let us repeat the operation, say k times: $\mathbf{A}^k\mathbf{v}_0 = c_1\lambda_1^k\mathbf{v}_1 + c_2\lambda_2^k\mathbf{v}_2 + \cdots + c_n\lambda_n^k\mathbf{v}_n$. Let us suppose that $c_1 \neq 0$. Then,

$$\frac{1}{\lambda_1^k}\mathbf{A}^k\mathbf{v}_0 = c_1\mathbf{v}_1 + c_2\left(\frac{\lambda_2}{\lambda_1}\right)^k\mathbf{v}_2 + \cdots + c_n\left(\frac{\lambda_n}{\lambda_1}\right)^k\mathbf{v}_n \xrightarrow{k\to\infty} c_1\mathbf{v}_1$$

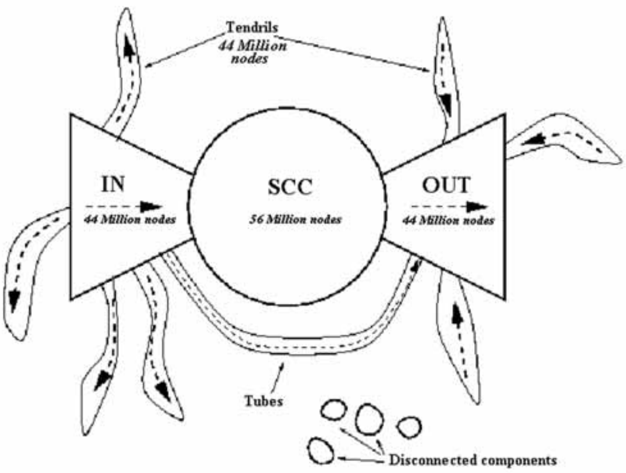
since $|\lambda_j/\lambda_1| < 1$ for each $j = 2, \ldots, n$ (recall that λ_1 was the dominant eigenvalue).

Therefore, when repeatedly multiplying the initial vector by the matrix $\mathbf{A}$, we determine, more precisely each time, the *direction* of interest, namely the one given by $\mathbf{v}_1$. This numerical method is known as the **power method** and its rate of convergence depends on the *ratio* between the first and the second eigenvalue (see in [8] an estimate for Google's matrix).

Our problem is finally solved, at least if we are in the best possible conditions (a non-negative irreducible matrix). The answer does exist, it is unique and we have an efficient method to compute it at our disposal (according to Google's web page, a few hours are needed). But …

8. Are we in an ideal situation?

To make things work properly, we need the matrix $\mathbf{M}$ associated to the web-graph G to be irreducible. In other words, we need G to be a strongly connected graph[13]. As the reader might suspect, this is not the case. Research developed in 1999 (see [7]) came to the conclusion that, among the 203 million pages under study, 90 % of them laid in a gigantic (weakly connected) component, this in turn having a quite complex internal structure, as can be seen in the following picture, taken from [7].



This is a quite peculiar structure, which resembles a biological organism, a kind of colossal amoeba. Along with the central part (SCC, Strongly Connected Component), we find two more pieces[14]: the IN part is made up of web pages having links to those of SCC and the OUT part is formed by pages pointed from the pages of SCC. Furthermore, there are sort of tendrils (sometimes turning into tubes) comprising the pages not pointing to SCC's pages nor accessible from them. Notice that the configuration of the web is something dynamic and that it is evolving with time. And it is not clear whether this structure has been essentially preserved or not[15]. We refer here to [3].

What Google does in this situation is a standard trick: try to get the best possible situation in a reasonable way[16]. For instance, *adding* a whole series of transition probabilities to all the vertices. That is, considering the following matrix,

$$\mathbf{M}'' = c\mathbf{M}' + (1-c)\begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix}(1, \ldots, 1),$$

where $p_1, \dots, p_n$ is a certain probability distribution ($p_j \geq 0$, $\sum_j p_j = 1$) and c is a parameter between 0 and 1 (for Google, about 0.85).

As an example, we could choose a uniform distribution, $p_j = 1/n$, for each $j = 1, \dots, n$ (and the matrix would have positive entries). But there are other reasonable choices and this degree of freedom gives us the possibility of making "personalized" searches. In terms of the random surfer, we are giving him the option (with probability $1-c$) to get "bored" of following the links and to jump to any web page (obeying a certain probability distribution)[17].

9. Non-negative matrices in other contexts

The results on non-negative matrices that we have seen above have a wide range of applications. The following two observations (see [13]) may explain their ubiquity:

- In most "real" systems (from physics, economy, biology, technology, etc.) the measured interactions are positive, or at least non-negative. And matrices with non-negative entries are the appropriate way to encode these measurements.
- Many models involve linear iterative processes: starting from an initial state $\mathbf{x}_0$, the generic one is of the form $\mathbf{x}_k = \mathbf{A}^k \mathbf{x}_0$. The convergence of the method depends upon the size of $\mathbf{A}$'s eigenvalues and upon the ratios between their sizes, particularly between the biggest and all the others. And here is where Perron–Frobenius' theorem has something to say, as long as the matrix $\mathbf{A}$ is non-negative.

The probabilistic model of Markov chains is widely used in quite diverse contexts. Google's method is a nice example, but it is also used as a model for population migrations, transmission of diseases, rating migrations in finance, etc. But, as mentioned before, Perron–Frobenius' Theory also plays a central role in many other contexts (we refer the reader again to [13]). Let us mention just a pair:

Biological models: a well known population model, in some sense a generalization of the one developed by Fibonacci, is encoded with the so called *Leslie matrices*. Their entries are non-negative numbers, related to the transition fractions between age classes and survival rates. If λ_1 is the dominant eigenvalue then the system behaviour (extinction, endless growth or oscillating behaviour) depends upon the precise value of λ_1 ($\lambda_1 > 1$, $\lambda_1 = 1$ or $\lambda_1 < 1$ being the three cases of interest).

Economic models: in 1973, Leontief was awarded the Nobel Prize for the development of his *input-output model*. A certain country's economy is divided into sectors and the basic hypothesis is that the j^{th} sector's input of the i^{th} sector's output is proportional to the j^{th} sector's output. In these conditions, the existence of the solution for the system depends upon the value of the dominant eigenvalue of the matrix that encodes the features of the problem.

Finally, there are several extensions of Perron–Frobenius' Theory that the reader might find interesting:

Cones in $\mathbb{R}^n$: the key point of Perron–Frobenius' theorem is that any $n \times n$ matrix with non-negative entries preserve the "positive octant". There is a general version dealing with (proper convex) cones[18] (see [1, 4]).

Banach spaces: those readers versed in functional analysis and spectral theory will be aware of the generalization to Banach spaces known as the Krein–Rutman theorem (see [12] and [5]). And those engaged in partial differential equations will enjoy proving, using Krein–Rutman Theorem, that the first eigenfunction of the Laplacian in the Dirichlet problem (in an open, connected and bounded set $\Omega \subset \mathbb{R}^n$) is *positive* (see the details in the appendix to Chapter 8 of [9]).

10. Coda

The design of a web search engine is a formidable technological challenge. But in the end, we discover that the key point is mathematics: a wise application of theorems and a detailed analysis of the algorithm convergence. A new confirmation of the *unreasonable effectiveness* of mathematics in the natural sciences, as Eugene Wigner used to say – as in so many other fields, we might add. We hope that these pages will encourage the readers to explore for themselves the many problems we have briefly sketched here – and hopefully, they have been a source of good entertainment. And a very fond farewell to Perron–Frobenius' theorem, which plays such a distinguished role in so many questions. Let us bid farewell with a humorous (but regretfully untranslatable[19]) *coplilla manriqueña*:

Un hermoso resultado
que además se nos revela
indiscreto;
y un tanto desvergonzado,
porque de Google desvela
su secreto.

11. To know more

The following book is an excellent and very recent reference: *Google's page rank and beyond: the science of search engine rankings*. (Amy N. Langville and Carl D. Meyer. Princeton University Press, 2006).

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Notes

1. The inventor of the name is said to be a nephew of the mathematician Edward Kasner. Kasner also defined the *googolplex*, its value being 10^{googol}. Wow!
2. They also intended the search engine to be "resistant" to any kind of manipulation, like commercially-oriented attempts to place certain pages at the top positions on the list. Curiously enough, nowadays a new "sport", *Google bombing*, has become very popular: to try to place a web page in the top positions, usually as only a recreational exercise. Some queries such as "miserable failure" have become classics.
3. Not to mention the incredible capacity of the search engine to "correct" the query terms and suggest the word one indeed had in mind. This leads us to envisage supernatural phenomena... well, let us give it up.
4. Although we will not go into the details, we should mention that there are a pair of elements used by Google, in combination with the general criterion we will explain here, when answering specific queries. On one hand, as is reasonable, Google does not give the same "score" to a term that is in the title of the page, in boldface, in a small font, etc. For combined searches, it will be quite different if, within the document, the terms appear "close" or "distant" to each other.
5. This is indeed a new model. Notice that, in general, matrices $\mathbf{M}$ and $\mathbf{M}'$ will not have the same spectral properties.
6. The ideas behind Google's procedure can be traced back to the algorithms developed by Kendall and Wei [11, 15] in the 1950's. At the same time that Brin and Page were developing their engine, Jon Kleinberg presented his HITS (Hypertext Induced Topic Search) algorithm, which followed a similar scheme. Kleinberg was awarded the Nevanlinna Prize at the recent ICM 2006.
7. The NBA competition is a good example although the dichotomy of "number of wins" *versus* their "quality" could also be applied to any competition.
8. The German mathematician Oskar Perron (1880–1975) was a conspicuous example of mathematical longevity and was interested in several fields such as analysis, differential equations, algebra, geometry and number theory, in which he published several text-books that eventually became classics.
9. Ferdinand Georg Frobenius (1849–1917) was one of the outstanding members of the Berlin School, along with distinguished mathematicians such as Kronecker, Kummer and Weierstrass. He is well known for his contributions to group theory. His works on non-negative matrices were done in the last stages of his live.
10. An $n \times n$ matrix $\mathbf{M}$ is *irreducible* if all the entries of the matrix $(\mathbf{I}+\mathbf{A})^{n-1}$, where $\mathbf{I}$ stands for the $n \times n$ identity matrix, are positive. If $\mathbf{A}$ is the adjacency matrix of a graph then the graph is *strongly* connected (see Section 8).
11. Notice that the part of the 2-sphere that is situated in the positive orthant is homeomorphic to a 2-disc.
12. A matrix $\mathbf{A}$ is said to be **primitive** if it has a dominant eigenvalue (bigger, in modulus, than the other eigenvalues). This happens, for instance, when, for a certain positive integer k, all the entries of the matrix $\mathbf{A}^k$ are positive.
13. Let us consider a directed graph G (a set of vertices and a set of directed edges). G is said to be strongly connected if, given any two vertices u and v, we are able to find a sequence of edges joining one to the other. The same conclusion, but "erasing" the directions of the edges, lead us to the concept of a weakly connected graph. Needless to say, a strongly connected graph is also a weakly connected graph but not necessarily the reverse.
14. Researchers put forward some explanations: The IN set might be made up of newly created pages with no time to get linked by the central kernel pages. OUT pages might be corporate web pages, including only internal links.
15. A lot of interesting questions come up about the structure of the web graph. For instance, the average number of links per page, the mean distance between two pages and the probability $P(k)$ of a randomly selected page to have exactly k (say ingoing) links. Should the graph be *random* (in the precise sense of Erdös and Rényi) then we would expect to have a binomial distribution (or a Poisson distribution in the limit). And we would predict that most pages would have a similar number of links. However, empirical studies suggest that the decay of the probability distribution is not exponential but follows a *power law*, $k^{-\beta}$, where β is a little bigger than 2 (see, for instance, [2]). This would imply, for example, that most pages have very few links, while a minority (even though very significant) have a lot. More than that, if we consider the web as an evolving system, to which new pages are added in succession, the outcome is that the trend gets reinforced: "the rich get richer". This is a usual conclusion in competitive systems (as in real life).
16. "Reasonable" means here that it works, as the corresponding ranking vector turns out to be remarkably good at assigning significances.
17. In fact, Google's procedure involves two steps: firstly, in order to make matrix $\mathbf{M}'$ stochastic, the entries of the zero columns are replaced by $1/n$. This "new" matrix $\mathbf{M}'$ is then transformed into $\mathbf{M}''$ as explained in the text. Notice that the original $\mathbf{M}'$ is a very sparse matrix, a very convenient feature for multiplication. In contrast, $\mathbf{M}''$ is a dense matrix. But, as the reader may check, all the vector-matrix multiplications in the power method are executed on the sparse matrix $\mathbf{M}'$.
18. A set $C \subset \mathbb{R}^n$ is said to be a *cone* if $a\mathbf{x} \in C$ for any $\mathbf{x} \in C$ and for any number $a \geq 0$. It will be a *convex cone* if $\lambda\mathbf{x} + \mu\mathbf{y} \in C$ for all $\mathbf{x}, \mathbf{y} \in C$ and $\lambda, \mu \geq 0$. A cone is *proper* if (a) $C \cap (-C) = \{0\}$, (b) $\mathrm{int}(C) \neq \emptyset$; y (c) $\mathrm{span}(C) = \mathbb{R}^n$.
19. More or less: "a beautiful result, which shows itself as indiscreet and shameless, because it reveals... Google's secret".

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The ICM through History

Guillermo Curbera (Sevilla, Spain)

It is Wednesday evening, 15th July 1936, and the City of Oslo is offering a dinner for the members of the *International Congress of Mathematicians* at the Bristol Hotel. Several speeches are delivered, starting with a representative from the municipality who greets the guests. The organizing committee has prepared speeches in different languages. In the name of the German speaking members of the congress, Erhard Schmidt from Berlin recalls the relation of the great Norwegian mathematicians Niels Henrik Abel and Sophus Lie with German universities. For the English speaking members of the congress, Luther P. Eisenhart from Princeton stresses that "*mathematics is international … it does not recognize national boundaries*", an idea, although clear to mathematicians through time, was subjected to questioning in that era. Lastly, the French mathematician, Gaston Julia, a professor at Sorbonne University in Paris, takes the stand as the French voice. After praising the country of Amundsen, Ibsen, and Grieg, he evokes a personal story:

"*Twenty years ago, a young, wounded officer was taken after having surgery at night to a room. He was falling asleep when he was awakened by his own blood overflowing in his mouth: an artery had just reopened. He barely had time to cry for help before losing consciousness.*

When he recovered consciousness he recognized by him the nurse in charge of the service. In absence of the surgeon, who had left the hospital, and of the night doctor, occupied elsewhere, at the moment without hesitating she stopped and cured the bleeding with security and determination, reanimating that fainting body. When the doctor was back he realized everything was well done and praised the decision and ability of the nurse.

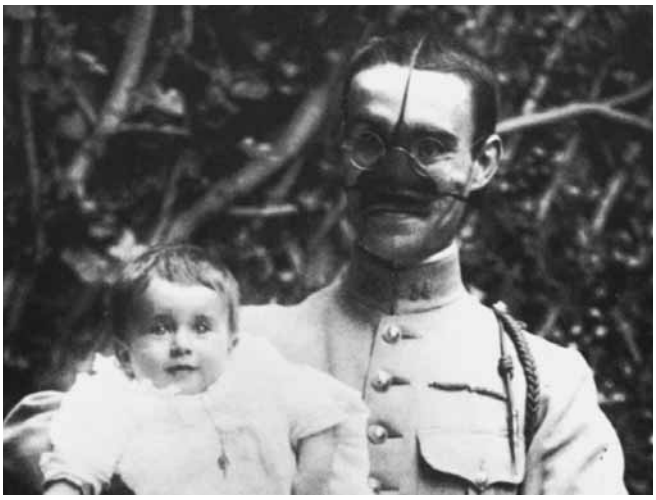
On the fear of the accident occurring again, in a spontaneous and charitable gesture, that generous lady decided to stay all that difficult night by the wounded soldier. I will never forget that long night in which, almost unable to speak, broken by the bleeding, and unable to get sleep, I felt relieved by the presence of that woman who, sitting by my side, was sewing in silence under the discreet circle of light from the lamp, listening at regular intervals to my breathing, taking my pulse, and scrutinizing my eyes, which only by glancing could express my ardent gratitude.

Ladies and gentlemen. This generous woman, this strong woman, was a daughter of Norway."

Beyond the impressive intensity of the personal tribute contained in these words, the scene has a deep significance when interpreted within the history of the international cooperation in mathematics. Let us collect all the facts needed for proper understanding.

The day before, Tuesday 14th July 1936, the International Congress of Mathematicians (ICM) had been inaugurated. Four hundred and eighty seven mathematicians from thirty-six countries attended the congress. The inauguration ceremony took place in the Aula of the University of Oslo, in the presence of King Haakon VII of Norway. A well known photograph shows this moment, with the king seated in the corridor and several known mathematicians in the front rows. After the opening speeches and the election of Carl Størmer as President of the Congress, a new feature in ICM protocol was included: the first two Fields medals were awarded. Elie Cartan acting as President of the *Fields Commission* explained to the congress:

"*At the closing session on the 12th September 1932, the International Congress of Mathematicians of Zurich decided to accept the legacy of the regretted Professor Fields which allowed the awarding, at each international con-*



Gaston Julia (1893–1978)

Opening ceremony at the Aula of Oslo University, 1936

Presentation of the medals in Oslo, as reported by the press

gress, of two gold medals to two young mathematicians to be distinguished by particularly remarkable works. At the same time, a commission was named in charge of designating the two laureates by the Oslo Congress, and composed by Mr Birkhoff, Mr Carathéodory, Mr Cartan, Mr Severi, and Mr Tagaki. This commission was presided over by Mr Severi who, not having being able to attend the Oslo Congress, has asked me to replace him in the presidency. The commission has come to the agreement of designating Mr Lars Alhfors from the University of Helsinki and Mr Jesse Douglas from the Massachusetts Institute of Technology as the two first awardees of the Fields medals. Mr Carathéodory has agreed to report on the work of the two laureates; he will give reading to his report"

After the report by Carathéodory, Cartan presented the medals to the awardees. Douglas' medal was collected by Norbert Wiener in Douglas' name since, as explained in the proceedings of the congress, "*Mr Douglas was strained and was not able to receive himself the medal to him assigned*". (The precise cause of this lack of attendance is still one of the mysteries of the history of the ICMs and the Fields medals.)

The Oslo congress was the tenth in the series of ICMs. The series had begun in Zurich, where *der erste Internationale Mathematiker-Kongress* took place from the 9th to the 11th of September 1897. The first invitation letter had an impressive list of signatories, including Adolf Hurwitz, Felix Klein, Andrei Markoff, Hermann Minkowski, Gösta Mittag-Leffler and Henry Poincaré. The congress was a success both with its attendance and its scientific level. Two hundred and eight mathematicians attended, among them Émile Borel, Georg Cantor, Felix Hausdorff, Charles de la Vallée Poussin (who lectured *Sur la théorie des nombres premiers*), Ernst Lindelöf, Émile Picard and Vito Volterra. Four of them were the plenary speakers: Hurwitz from Switzerland, Klein from Germany, Giuseppe Peano from Italy and Poincaré from France (who eventually did not attend but whose lecture, *Sur les rapports de l'analyse pure et de la physique mathématique*, was read at the congress). A total of thirty-four lectures were delivered. It is worth noting the fine equilibrium of nationalities of the chosen plenary speakers. But there was also another success: the atmosphere of the congress. This is very well illustrated by the regulations that were approved. The first article established the objectives of the congress. The first two were:

a) *To promote personal relations between mathematicians of different countries.*
b) *To review, in reports and lectures, the current state of the different branches of mathematics and to provide the occasion to discuss issues of recognized importance.*

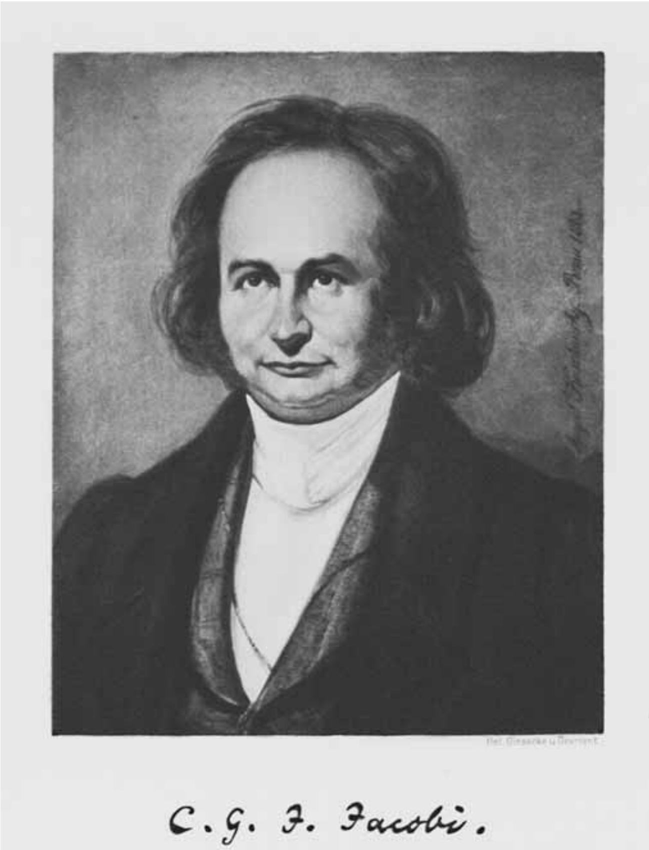
It is quite interesting to note the importance (for the development of mathematics) assigned to the role of personal relations between mathematicians.

From this congress comes the well known and beautifully coloured lithograph that was included in the proceedings, featuring five great Swiss mathematicians: Leonhard Euler, Daniel Bernoulli, Jakob Bernoulli, Johann Bernoulli and Jakob Steiner, and showing the Eidgenössiches Polytechnikum, a building that has hosted three ICMs.

Starting a tradition that was to occur for many congresses to come, at the closing session the offer presented by the Societé Mathématique de France for organizing the next congress was accepted. Thus, the *deuxième Congrès International des Mathematiciens* took place three years later in Paris. Charles Hermite was named honorary president, while the acting president was Poincaré. The four plenary speakers were Moritz Cantor (an historian of mathematics from Heildelberg), Poincaré, Mittag-Leffler from Sweden and Volterra from Italy. The celebrated lecture where David Hilbert presented his renowned list of twenty-three problems should have been a plenary lecture but, due to late submission, it was delivered in the Bibliography and History of Mathematics session and contained only ten of the problems. (When the proceedings were published, it was included among the plenary lectures with the title *Sur les problèmes futurs des Mathématiques* and contained all twenty-three problems.) At the closing session of the congress, Poincaré lectured *Sur le rôle de l'intuition et de la logique en la Mathématique* and Mittag-Leffler gave a tribute to his master, presenting *Une page de la vie de Weierstrass*.

At the invitation of the Deutscher Mathematiker-Verenigung, the city of Heidelberg hosted in 1904 *der dritte Internationale Mathematiker-Kongress*. Once again there were four plenary speakers: Alfred Greenhill from Great Britain, Paul Painlevé from France, Corrado Segre from Italy and Wilhem Wirtinger from the Austro-Hungarian Empire.

There is a legendary episode that occurred during this congress, caused by the lecture *Zum Kontinuum Problem*, by Jules König from Budapest, where it was claimed that Cantor's beloved conjecture was false. The section of the proceedings of the congress known as *Bericht über die Tätigkeit der Sektionen* (report on the activity of the ses-



Engraving of C. G. J. Jacobi, from the 1904 biography

sions) allows us to imagine the disturbance caused by the lecture. Indeed, it is explained that after the lecture there was a discussion in which Cantor, Hilbert and Schönflies participated. It is not written in the proceedings but that evening Felix Klein had to explain to the Grossherzog of Baden (who, together with Kaiser Wilhelm II, had covered the expenses of the congress) what could cause such unrest at a mathematical congress. Cantor's suffering was short; it is said that soon after Zermelo found the error in König's argument.

In this congress we find one of the classic features of older ICMs: a dedication for looking to the past of mathematics beyond the ever present section on the history of mathematics. The congress coincided with the centenary of the birth of Carl Gustav Jacob Jacobi. As a commemoration, the congress was opened with a biographical sketch of Jacobi given by Leo Königsberger from Heidelberg. The editor B. G. Teubner, who had also helped finance the congress, published a detailed biography of Jacobi by Königsberger and offered the book to the congress members at a third of its selling price. The dedication to Jacobi's commemoration even went so far as to inspire a discussion of the abandonment of Jacobi's grave in Berlin.

Carl Runge, from Munich, also looked back in history (and also forward!) when he lectured on the calculating machine designed by Leibniz in 1674. There was no way he could imagine the future development of calculating machines but with his lecture he initiated the presence of these devices at ICMs.

The congress also witnessed the presentation of the *Encyclopédie de sciences mathématiques*, a far reaching scientific project that was the expanded French version of the *Encyklopädie der mathematischen Wissenschaften*, which was first published in 1894.

Four years later, in 1908, the *IV Congresso Internazionale dei Matematici* took place in Rome. The organizing institutions were the great Italian scientific institution, the *Reale Accademia dei Lincei*, one of whose first members was Galileo Galilei, and the *Circolo Matematico di Palermo*. The congress was magnificent in its display: the inauguration was presided over by the king of Italy and took place in the Campidoglio beside the gorgeous frescos of the *Sala degli Orazi e Curiazo*; the Academia dei Lincei offered the lecture rooms of the Renaissance style Villa Farnesina by the Tiber river; and a reception was offered at the Philosopher's Room of Villa Adriana in Tivoli.

The increase in the size of the congress shows that the ICMs were gaining popularity. This time there were nine plenary lectures and five hundred and thirty five mathematicians attending. The scientific profile of the congress reflected the traditional tendency of Italian mathematics towards the study of physical phenomena: two of the plenary speakers were Antoon Hendrik Lorentz (who had received the Nobel Prize in Physics in 1902) and Simon Newcomb (the American astronomer); the congress was presided over by the Italian physicist Pietro Blaserna; and apart from the traditional sessions on arithmetic, algebra, analysis, geometry, history, philosophy and didactics, new sessions on mathematical physics and geodesy were added. Of all the lectures delivered, a particular highlight was *Grundlagen der Aritmetik und Analysis* by Ernst Zermelo.

A singular character of the Rome congress was the Italian mathematician Giovanni Guccia. In 1884 he founded the *Circolo Matematico di Palermo* and he was the person responsible for running and supporting the journal *Rendicontti del Circolo Matematico di Palermo*. Guccia supported the congress with two offerings: an award and the publication of the proceedings. The award, *the Medaglia Guccia*, was for a memoir on algebraic curves. The jury for assigning it comprised Corrado Segre, Max Noether and Poincaré, and they awarded the medal to Francesco Severi. Unfortunately, when Guccia passed away so did his fortune and the medal was never awarded again. Regarding the publications of the proceedings, a strike of Sicilian typographers prevented their publication in the *Rendicontti*; they were later published by the *Accademia dei Lincei*.

Still one of main countries in the history of mathematics had not organized an ICM. Thus, in 1912, the *V International Congress of Mathematicians* was held at the University of Cambridge. As had happened in Rome, the program of the congress also reflected the British tendency towards applications: the honorary president was Lord Rayleigh (who received the Nobel Prize in Physics in 1902), the president of the congress was George Darwin (astronomer and son of Charles Darwin) and new sessions were added on astronomy, economy, actu-

Giovanni Guccia, founder of the Circolo Matematico di Palermo

The Cambridge Scientific Instrument Company

arial sciences and statistics. Even the congress' scheduled visit had an applied flavour: the Cambridge Scientific Instrument Company, a leading company devoted to manufacturing high precision machinery. (In later times, this peculiarity of the ICM scientific program of reflecting the national trend of the mathematics of the host country was to disappear when the International Mathematical Union took over control of the program.)

Other features that had been seen at previous ICMs remained in the Cambridge congress. Looking to the past, members of the congress visited Mill Road Cemetery where a wreath was laid on Arthur Cayley's grave. Calculating machines also appeared; indeed, there was an exhibition at the Cavendish Laboratory of *books, models and machines (chiefly calculating machines)*.

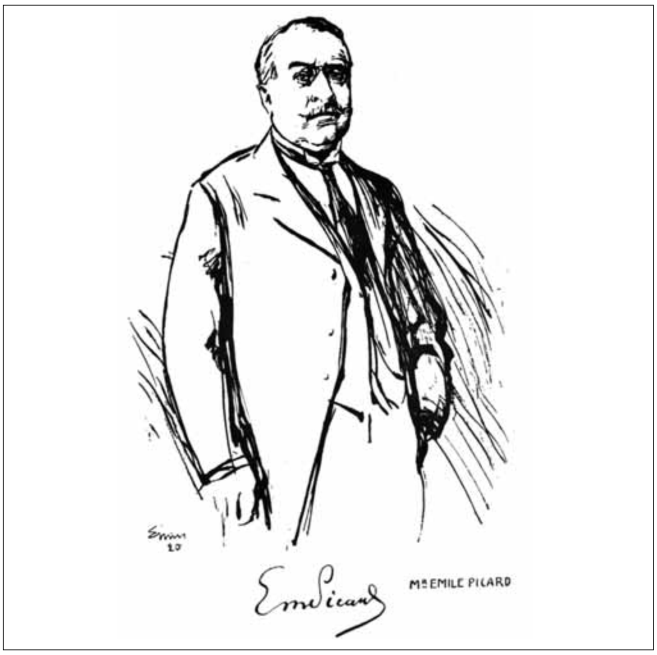
The Cambridge congress marked a high point in the History of the ICMs. Almost six hundred participants, coming from twenty-eight countries, seemed to assure a splendid future of international cooperation in mathematics. Four years before, at the Rome congress, cooperation in mathematical education had begun with the creation of the *Commission Internationale de l'Enseignement Mathématique* (originally presided over by Felix Klein). Even the idea of creating an international association of mathematicians was considered. It is remarkable that mathematicians could be so detached from the general atmosphere of increasing international tension.

The 1912 congress was closed with the invitation by Mittag-Leffler to host the 1916 ICM in Stockholm. The passion of Mittag-Leffler for his journal motivated this invitation to be issued in the name of the Swedish Academy and the journal *Acta Mathematica*. Two other invitations were presented: Budapest for the 1920 congress and Athens for the 1924 congress. The decisions on them were postponed until the Stockholm congress.

The Great War crushed this enthusiasm. The war and its aftermath had a tremendous impact on all aspects of social life, and science was not immune to it. In Brussels in 1919, the Allied Powers created the *International Research Council* (IRC) with the declared aim of promoting the creation of international scientific unions but whose scarcely hidden objective was to eliminate the preeminence that German science had in many fields. Many of the current scientific unions were then created, such as the International Union of Pure and Applied Physics (IUPAP), the International Union of Pure and Applied Chemistry (IUPAC) and the International Union for Astronomy (IUA). Well known mathematicians had a significant role in the IRC: Picard was the president until its dissolution in 1931 and Volterra was the vice-president.

Following IRC instructions, in 1920 the *Union Mathématique Internationale* (UMI) was created. The decision to celebrate the next ICM in Stockholm was overturned and an option more in tune with the Versailles Treaty was taken. The congress would to be held in Strasbourg, capital of the Alsace region, which had been regained by France after its loss to Germany in the Franco-Prussian war of 1870-71. This congress was the one with the least number of participants in the history of the ICMs. The reason was twofold. Firstly, an exclusion of mathematician from the former Central Powers (Germans, Austrians, Hungarians and Bulgarians) was imposed by the IRC and secondly, there was opposition from certain mathematicians, still a minority at that point, to this exclusion policy. The post-war tone of the congress as described in the proceedings is startling. There was a visit to the mausoleum of the Maréchal de Saxe, Genéral Taufflied lectured on *La Science en Alsace* and the ode *Salut à Straboug* was recited. There were also special regulations for the lunch of those congressmen who were reserve officers of the Allied Armies. The peak of this atmosphere came in the closing speech by Picard, who said, "*we assume the fine words of Cardinal Mercier during the war: to pardon certain crimes is to become accomplice with them*".

At Strasbourg it was decided to hold the next congress in New York in 1924. As the time got closer, it was clear that under the IRC exclusion policy it would not be possible to obtain the necessary support from the American mathematical community, which had long standing ties



A drawing of Émile Picard from the proceedings of the Strasbourg congress

Biberbach and Sierpinski on the steps of the ETH

with German mathematicians. At that moment, the continuity of the series of the ICM was seriously in danger. It was finally saved by the Canadian mathematician John Charles Fields who offered to organize the congress in Toronto. The congress, still with the absence of German, Austrian, Hungarian and Bulgarian mathematicians, was not able to avoid the influence of the war. The Belgian mathematician Charles de la Vallèe Poussin, President of the UMI, laid a wreath at the Soldier's Memorial Tower (the proceedings show a picture of the event) and in the opening session explained the meaning of the Strasbourg congress:

"What then mattered was not only a scientific congress but a symbol and a feast, the celebration of the liberation of Alsace and also, as I then said, the liberation of science from the sacrilegious hands that for so long had used it for their criminal aims."

When Salvatore Pincherle started to organize the 1928 ICM in Bologna, he was confronted with a serious dilemma; on the one hand there was insistence from the UMI and the IRC for the continuation of the exclusion policy and on the other hand there was a strong stand by several mathematical societies, such as the American Mathematical Society and the London Mathematical Society, and many individual mathematicians, who threatened not to attend if the congress was not truly international. The final outcome was that the congress was held not under the auspices of the UMI but those of the University of Bologna and was open to all mathematicians, regardless of their nationality. After two congresses in absence, German, Austrian, Hungarian and Bulgarian mathematicians could now attend. The entrance in the Aula Magna of the Archiginnasio of Bologna of Hilbert preceding the German delegation is legendary; the congress as a whole rose and applauded. Very few mathematicians represented so well as Hilbert the essence of the ICMs, that of being rooted in open international cooperation. Indeed, his plenary lecture *Probleme der Grundlegung der Mathematik* opened the congress.

The congress was an absolute success. Eight hundred and thirty-five mathematicians attended (of which Germans were the second largest national group) and the high number of communications can be easily judged by the size of the proceedings, which amounts to six large volumes. The congress retained some old traditions: two commemorative plaques were unveiled, one in the family house of Scipione dal Ferro and another in the church where Bonaventura Cavalieri was prior, and the book *Prefazione ai libri inediti dell' Algebra di Rafael Bombelli* by the historian of mathematics Ettore Bortolotti was presented at the congress and offered to the congress members.

The 1932 congress was held in Zurich again. After the turmoil of the previous ones, this was a sweet congress. The international mathematical community was reunited again and the deep dangers that were soon going to threaten the world had not yet risen. Two pictures display this pleasant atmosphere. One is the well know picture of Waclaw Sierpinski from Warsaw, chatting at the steps of the Eidgenössiche Technischen Hochshule with Ludwig Bieberbach from Berlin, who was later to become one of the promoters of the so called Deutsche Mathematik.

The other is related to the controversy on whether or not to consider the Strasbourg and Toronto congresses as true ICMs due to the restrictions they had on their attendance. The proceedings of the Bologna congress had labeled it as the sixth ICM, the one after the fifth of 1912 in Cambridge. At the opening ceremony in Zurich, Hermann Weyl said:

"We attend here to an extraordinary improbable event. For the number of n, corresponding to the just opened International Congress of Mathematicians, we have the inequality 7 ≤ n ≤ 9; unfortunately our axiomatic foundations are not sufficient to give a more precise statement"

The consequence of this controversy was influential; the ICMs have not been numbered since.

Regarding the UMI, its statutes had expired in 1931 and at the Zurich congress the decision to dissolve the Union was taken. The strong feeling of mathematicians forming a united community had won against the intervention of the IRC. This situation is unique among the scientific unions. The UMI disappeared due to the abandonment of its members. (The re-foundation of the union after World War II is a different and happier story).

When Gaston Julia stood up at the Bristol Hotel in Oslo and referred to his war experience, he was speaking to mathematicians of different nationalities, many of whom had participated in the war and lived in its aftermath. He himself was a living example of the horrors of the war; he carried a mask partially covering his face for the rest of his life. But international cooperation in mathematics had survived the war and its aftermath.

This story illustrates the richness contained in the history of the ICMs, beyond its extraordinary mathematical content.

These and many other stories are told in the exhibition, *The ICM through History* (organized for the ICM-2006 in Madrid) which is a chronicle, based on graphical materials, of the ICM as a human endeavor. The exhibition is now the property of the Spanish mathematical societies and it is stored in the University of Sevilla. It has been conceived to be exhibited elsewhere. The institutions interested in exhibiting it should contact the author of this article.

The exhibition *The ICM through History* at the ICM-2006 in Madrid

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Questions about the proof – an interview with John Morgan

conducted by Ulf Persson (Göteborg, Sweden)
in connection with the International Congress of Mathematicians (ICM) in Madrid, 2006

Poincaré Conjecture – topology, geometry and analysis

John Morgan at ICM2006

In the early eighties Freedman proved what was often referred to as the 4-dimensional Poincaré conjecture (PC). At the time there were comments from the topological community that this was much easier than the 3-dimensional case. Would you care to comment on this?

To me it is impossible to directly compare Freedman's 4-dimensional proof with Perelman's 3-dimensional one. Freedman's proof involved pushing down well-understood techniques from dimension five and higher to four dimensions, isolating the main difficulty, namely that of the existence of Whitney discs. Once you had that everything else would follow by standard techniques. Freedman in fact showed that these discs did exist topologically but in order to fit them in so to speak he had to crinkle them uncontrollably. This was an incredible technical tour de force of topological arguments.

Perelman's proof on the other hand is of a totally different nature. He employs powerful analytic and geometric results in order to prove the existence of 'good metrics'. From there it is easy to get the PC.

This is what strikes the outsider in the proof of the PC, namely the intrusion of extra-topological methods. During the years there have been very many attempts at the PC and I guess most of them have tried to stay within the topological fold. I suspect that you mainly identify yourself as a topologist; when did you hear about Hamilton's approach and did it interest you at all before it was brought to conclusion by Perelman?

For one hundred years people tried direct topological approaches. As far as I am concerned I considered the fact that Freedman's argument was so difficult and so much at the border of the possible as a positive proof that one would not be able to solve the PC by direct topological arguments. It is true that I have known of Hamilton's theorem (1982), to the effect that positive constant Ricci curvature implies spherical, for a long time but I never seriously studied it nor did I think it would lead to a proof of the PC.

One would suspect that you as a topologist would find the techniques employed such as partial differential equations and Riemannian differential geometry not part of your expertise. Do you feel that you eventually understood those aspects or would it have been impossible for you to have written this exposition without the assistance of somebody like Gang Tian? Nevertheless I suspect that the meat of the argument is really topological – surgery, although the initial approach is 'differential' (equational or geometrical), and this is why you got involved in it. Would that be correct?

No, in fact almost all the argument is of an analytic and differential geometric kind. Indeed this is why I started working with Tian. The actual topology involved is really quite elementary; the heart of the argument is geometric/analytic. I first got involved, as I noted earlier, because I wanted to get a feeling for the arguments used to resolve the most famous topological conjecture.

You (and Tian) have really made a very ambitious attempt to understand and explain (the two activities cannot be easily separated) the Perelman proof. From a personal career point of view such an attempt must be considered very unselfish as you tend only to get credit for your own creative contributions. So why did you do it? To vindicate Perelman?

I was motivated by the desire to understand the argument and thereafter by the beauty and power of what I saw. Writing the book was meant as a service to the community. It never ever occurred to me that Perelman needed neither vindication nor protection.

Correctness of proofs. Credits.

At the ICM you formally announced that the proof is correct and that Perelman has proved the PC. Would you consider this, by virtue of your invitation to the ICM, as an authoritative statement?

I was speaking only for myself at the ICM.

This leads to the next question. When is a proof correct? If you take the point of view that mathematics is simply a human activity, then the matter of correctness is simply a question of consensus. If you take a more Platonist point of view, a statement is true or false and the point of a proof is to convince ourselves of forming a belief about its truth or falsity, and our belief has no

bearing upon the matter of truth. In practice there is of course very little difference between the two standpoints but from a philosophical point of view, there is a crucial difference. Mathematicians are often derided for taking the second point of view. What is your opinion on the matter?

I think the confidence of the community in proof evolves with time. Perelman posts paper one; clearly he has broken new ground and has made major advances. He claims that the same arguments will prove not only the PC but also Thurston's Geometrization conjecture. I am dubious. Now Perelman posts paper two, giving the extensions of his results to Ricci flow with surgery. Arguments are more intricate and even more condensed. But clearly it is serious business. Paper two ends with the statement of a Theorem on collapsing space. Proof is promised in an upcoming 3rd paper yet to be written. Will this be an issue? Now the community begins to come to grips with paper one – it looks OK. The most delicate analytic points are verified by others (often many others). Paper two is harder but no obvious errors are being reported. Perelman posts paper three giving a short-cut to the PC. People are finding paper two really hard going. Colding and Minicozzi post a result analogous to Perelman's third paper, so that is probably OK.

In September 2004, Kleiner, Lott, Tian and myself organize a small workshop with about a dozen people in attendance in order to work through paper two. We then all believe that it is correct and that we understand the argument. We are then convinced that paper one and two are correct and that either paper three or with the other promised result Perelman has it all. Tian and I then decide to write up what we understand. For the next two years we worked through every detail as well as re-arranging and presenting the material coherently. When we finished in May 2006, we, at least, are convinced. The final stage of general acceptance will happen when people closely examine what we have done, what Kleinert-Lott have done and what Cao-Zhu have done, and come to the conclusion that it is OK. I think the Clay rule of waiting two years after publication is about right.

As to the philosophical point, I think there is an abstract notion of proof (assuming mathematics is consistent), but we will never completely get there. We always take short-cuts. We never reduce arguments to logical atoms and complete explicitness. But there is a consensus on where you are close enough to be sure that the argument is correct.

Ostensibly the point of a proof is to decide whether something is true or not but as mathematicians we are probably more interested in explanations, of why something is true. This is why we find computer-assisted proofs so unsatisfactory. Obviously your conviction of the soundness of the proof does not rest on having checked every detail but rather on having been brought to understand a few key ideas. Would you be able to pin-point a few of those that really illuminated why it had to work?

Richard Hamilton

Grigori Perelman

The notion of Ricci-flows is often presented as a key insight and its analogies to the heat-equation stressed. Is the latter just a formal analogy or does it carry explanatory power and in fact structure the entire argument, at least that pertaining to differential equations?

Indeed the analytic theory of Ricci flow certainly rests on the analogy with the heat equation, as the smoothing property of the latter served as inspiration for the hope that the Ricci flow would make the metric better. But there is nevertheless an essential difference. The Ricci flow equation is non-linear and the non-linearity leads to singularity formations. I would identify Perelman's chief insight as the introduction of the length function and its use to show non-collapsing. Once he had that, everything else was a matter of technical power. How he ever came across the notion of the length function I simply can not imagine.

The question of justly assigning credit to the proof of a conjecture is a tricky one. Say that the main conjecture on modular elliptic curves had already been established (by Wiles) and only later the connection with Fermat's theorem had been pointed out by Frey. Would he (or maybe rather Ribet) have been credited with solving Fermat?

In the case of Hamilton and Perelman there is no such clear-cut case. I do not believe that Hamilton ever formulated a theorem A, claiming that if A was proved so would PC, and that Perelman came along proving A. Yet to an outsider it is easy to get the impression that Hamilton did the crucial work, setting up an approach, but getting mired in some technical details, which Perelman simply clarified.

What would you specifically point at to dispel such a misconception?

Hamilton did indeed lay out the approach and understood that surgery was required. But he was stuck. He was not going to get there, without some coup de genie. Perelman added new and deep insights which were absolutely essential. They are just not 'technical details'. To me the correct analogy is Grothendieck-Deligne and the Weil conjectures. In the ethos of mathematics one says that Deligne proved the Weil Conjectures and Grothendieck revolutionized algebraic geometry. Here we will say that Perelman proved the PC and Hamilton developed the powerful method of Ricci flow.

Conceptual ideas and formal arguments

At your press-conference at the ICM you pointed out some interesting phenomena, no doubt well-known to all mathematicians and also the source of much didactic confusion. You encounter a statement, you do not understand it, then finally you work it out, only to find out in the end that if you would now explain it succinctly and clearly to the mathematician in the street, you would more or less use the same words and the same formulations, which were so opaque to you initially. This illustrates that mathematical statements, like all other statements, do not live in isolation but are only meaningful in a context. Do you see any way around this?

In particular a proof consisting of a chain of correct statements is not that satisfying, what you need is also a context, and I believe that in many proofs that explanatory context is missing. In other words there are in fact often (in spite of hand waving) too many details in a proof and not enough motivation setting the reader on the right track. Would you have any comments on this? Or do you think that there simply are no shortcuts, sooner or later a mathematician has to work things out in privacy and there is little outsiders can do to assist.

I have thought much about this. To me it is a question of internalizing the argument, making it your own. It could be having a picture to go with the formal argument. At the end of the day I think you need both a formal argument with details and the conceptual idea of the proof. I do not believe something until I have both. In writing the correct balance between these two aspects is difficult to find and maintain. Some mathematicians, notably Milnor, succeed; most of us do not. Papers can be as unreadable if they have too much detail as having not enough. In the end you always have to work things out for yourself.

Perelman and the Press

Perelman has received a lot of extra-mathematical attention, much of it I believe actively promoted by some of our colleagues. The question is whether this kind of attention (now further stimulated by Yau threatening to sue the New Yorker for the Naser article) is good or bad for mathematics? On one hand, as people in the media well understand, bad publicity is better than no publicity. On the other hand does not this kind of publicity confirm many people in the standard misconceptions of mathematics (and mathematicians)?

I have concluded that it is after all good for math. I do not think it feeds the stereotype. Yau is certainly not an asocial nerd and certainly does not come across as one in the piece. People relate to human drama and I think it helps to form the impression that what we do is important enough, at least to us, to fight about. I also think the public will come away with the sense of the power and accomplishment of the subject.

At the aforementioned press-conference you got the question whether Perelman was a genius. You disavowed such a label, claiming that it did not explain anything. You were instead talking about incredible talent, of being insightful, and of power. On the face of it those words seem almost as general as the much abused notion of 'genius'. I guess that what you wanted to convey was that Perelman is not extraordinary; he has the same qualities as all other mathematicians, but to a sharper degree.

Would you care to be more specific what you really mean by talent in mathematics, as opposed to having insights? And what do you mean by 'power' as being different from the other two? A sense of perseverance, 'going that extra mile'?

I think of 'genius' as being basically synonymous with extraordinarily high I.Q. And I could easily list hundreds of mathematicians with that quality. Powerful? This aspect of mathematical talent is probably close to I.Q, namely the ability to work through incredibly complex arguments with many interlocking pieces and keep it all straight, sensing all along when you are making progress, not unlike that of a juggler who is able to keep lots of balls in the air and not losing track. Powerful mathematicians are those who can work out arguments requiring many intricately interlaced ideas. Originality on the other hand is the rarest and the most important - the ability to get out of old habits of thought and see new possibilities, or to make new unsuspected connections. Of course in addition to this there is a matter of perseverance, confidence, clarity of thought, self-reliance. They are all important.

Plans for the future

You identify yourself as a topologist. But in mathematics all branches are interconnected, something we as mathematicians learn at our peril. Yet no man today has the capacity to be a universalist and specialization is both a necessity and a curse. How do you look upon yourself?

I have always worked at the interface of topology and other subjects – algebraic geometry, mathematical physics, differential geometry – but I have an interest far away in algebra and number theory.

The natural question is what do you plan to do now? Will you pursue other things in topology or do you feel tempted to apply some of the new things you learned from Perelman? And if so, what would be the natural questions to consider?

I would love to be able to meld the Ricci flow and the Perelman techniques with the elliptic techniques that have produced invariants of 4-manifolds. This is terra incognita and I hope that what I have learned in three dimensions will be helpful in four dimensions.

Mathematics is a competitive subject, especially when you are young. What is your attitude to this? Is it inevitable and should be encouraged, or is it something that

actually is counter-productive to most mathematicians? In this context, how would you look upon the Fields Medals and the Clay Prize (the latter can of course be seen as a promotion thing, while the Fields medals are strictly a matter of internal appreciation)?

Mathematics is hard and in some ways competitive. But I think that the healthiest subjects, and the most fun to work in, are those where the competition is not the primary feature but rather the advancement of the subject and our understanding of it. Then one attains a genuine respect and admiration for the accomplishments of ones 'competitors'.

The Fields medal has, I think, become somewhat artificial because of the age restriction. To my mind the winners over the last twenty years have been of widely varying quality. The Clay prize is for publicizing mathematics and it has worked remarkably well. I worry that introducing real money into mathematics may cause more of the kind of scenes we saw this summer (though I do not think money was the cause of that). If so, it would be a terrible shame.

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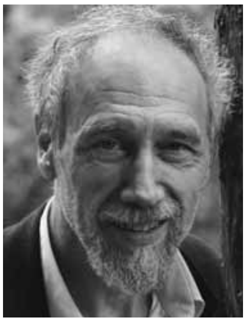
An interview with Alain Connes

Part I: conducted by Catherine Goldstein and Georges Skandalis (Paris)



Alain Connes

Are there mathematicians of the past you feel close to?

Close to, I would not say but there is one I admire in particular: Galois. There's a very striking characteristic in his writings; their formulation is amazingly simple. For instance, '*Take an equation with n different roots. Then, first statement, there is a rational function of these roots which takes $n!$ different values when you permute the roots; and, second statement, the roots are rational functions of this function*'.

In spite of the deceiving simplicity of their formulation, using these statements Galois succeeds in going extremely far. He writes down the equation whose roots are the $n!$ different values of the rational function, he splits it into irreducible factors and chooses one of them, he writes how the roots of the original equation depend on the roots of this factor and he sees a group. And he shows that this group is independent of all the choices made along the way… To achieve this he characterizes the group abstractly by a unique property: '*A function of the roots is rationally determined if and only if it is invariant by this group*'.

It is so simple. What I find fabulous is this kind of leap using the power of abstraction, this enormous step in conceptualizing things. The power of Galois' intuition is not based on the idea of symmetry but on a concept of ambiguity. Naively, you might say he studied the invariance group of certain functions. But Galois' first step is just the opposite: he breaks the symmetry as much as possible by choosing a function which has no invariance at all. The mathematicians before him – Cardano, Lagrange – worked with symmetric functions of roots. Galois, in the footsteps of Abel, does the opposite: he chooses a function with the least symmetry possible. And this is the function he starts with.

What strikes me is the fecundity of these ideas; the various formalisms we have developed to catch them do not yet exhaust their power. Galois' ideas have a clarity, a lightness, a thought provoking potential which remains untamed to this day and finds an echo in the minds of mathematicians till now. They have generated great concepts like Tannakian categories or the Riemann–Hilbert correspondence... These ideas are very pretty but they are often set out with such pedantry that they look like heavy yokes and you don't get the impression they have been freed to the point Galois had freed them. Other avatars of Galois' ideas are the differential Galois theory and the theory of motives, which can be seen as a higher-dimensional analogue of Galois theory.

But have we really understood what Galois had in mind when he wrote:

«Mes principales méditations depuis quelque temps étaient dirigées sur l'application à l'analyse transcendante de la théorie de l'ambigüité. Il s'agissait de voir a priori dans une relation entre des quantités ou fonctions transcendantes quels échanges on pouvait faire, quelles quantités on pouvait substituer aux quantités données sans que la relation pût cesser d'avoir lieu. Cela fait reconnaitre tout de suite l'impossibilité de beaucoup d'expressions que l'on pourrait chercher. Mais je n'ai pas le temps et mes idées ne sont pas encore assez développées sur ce terrain qui est immense.»

There are other examples of mathematicians who really helped me at an early stage as a source of inspiration.

It isn't that I feel close to them at all in what I do but I admire what they do. At first, I was fascinated by Jacobi because I found his way of computing marvellous. And by von Neumann – the depth of what he had discovered and the way he talked about it... And by Tomita of course. I was fascinated by Tomita's mysterious personality; he's someone who has succeeded in avoiding all the traps that society tends to set for someone extremely original. He became deaf at the age of two. When he started his research, his thesis advisor gave him a huge book telling him, "Come back and see me once you have read this book". Tomita met accidentally his thesis advisor two years later and the latter asked him, "How is the book going?" to which Tomita replied, "Oh, I lost it after one week"… But I think the freshest, the most limpid source, is Galois. It's very odd but I have never separated Galois from this powerful mixture of simplicity and fecundity.

Would you like to say something about Choquet?

I remember the first years I was doing research; I worked alone at home, but every Thursday I attended Choquet's seminar. And he shone by his intelligence, his wit. There were questions bursting out, it was extremely open. This shaped me, in depth. And Choquet had something unique: he had been very close to the Polish school of mathematics before the war. And so he knew a lot of things which do not make up the usual curriculum of mathematicians, but which in fact are quite interesting.

It is only with Choquet for instance that I learnt the theory of ordinals. You might think that this theory is useless, but that's absolutely false. For instance, I remember once, the IHES had an open-door day. There was a first grade class, little kids, and among them a girl with shining intelligence. And so, after the subject of undecidability had been brought up, I gave them an example from the theory of ordinals, the story of the hare and the tortoise. You take a number N, not too big (they had taken 5 or something like that). They had learnt to write numbers in various bases: 2, 3, etc. I explained to them that one writes the number in base 2, then the hare comes and replaces all the 2's by 3's. Thus $5 = 2^2 + 1$ gets replaced by $3^3 + 1 = 28$…, and the tortoise just subtracts 1. Then one writes the result in base 3 and the hare comes and replaces all the 3's by 4's and the tortoise subtracts 1 again, etc. Well, the extraordinary phenomenon which comes from the theory of ordinals is that the tortoise wins. After a finite number of steps and even though you have the impression that the hare makes absolutely gigantic jumps each time, you get 0!

And what's hard to believe is that this cannot be proved in the framework of Peano arithmetic. The proof uses the theory of ordinals! You can in fact show that the number of steps required before the tortoise wins is growing faster than any function of N you can write explicitly. You can see on the computer how many steps it takes to reach 0. But the proof that the tortoise wins takes one line with the theory of ordinals. What do you do? You take the first number and instead of replacing the 2's by 3's, then by 4's, etc., you replace them by the ordinal ω. For example 5 is 2^2+1, so you write $\omega^\omega + 1$. This is an ordinal and an ordinal is a well-ordered set and every decreasing sequence of ordinals necessarily stops. Now when you do the hare's move, it doesn't change anything, but the tortoise's move subtracts 1, and you obtain in this way a strictly decreasing sequence of ordinals. This has to stop, and you have the proof. And this proof uses ω^{ω^ω} … so it is not so surprising that it goes beyond Peano arithmetic. This is typical of the kind of things we discussed at Choquet's seminar.

And this is a partly forgotten mathematical culture, but which is in fact an extremely rich one. We live in a mathematical world which is more and more monocultural. We proclaim principles to say which mathematics is important and which isn't. I try to defend diversity. I believe it is crucial to let schools blossom. This is very important for the health of mathematics.

Operator algebras and coincidences: how did it all begin?

In 1970, I went to the Les Houches summer school [in physics], sent by Choquet. At that time, I had been working on non-standard analysis but after a while I had found a catch in the theory… The point is that as soon as you have a non-standard number, you get a non-measurable set. And in Choquet's circle, having well studied the Polish school, we knew that every set you can name is measurable. So it seemed utterly doomed to failure to try to use non-standard analysis to do physics. But it suited me as a passport to Les Houches in 1970.

And from there, I was taken on as a fellow at the Battelle Institute and I got an invitation for Seattle. I accepted it mostly to visit the States – I did not even look at the program. And the coincidence that occurred is that I stopped in Princeton to visit my brother and I bought a book, at random, at the Princeton bookstore. I hesitated among several books until I came across one which fascinated me, by Takesaki on Tomita's theory. And as I knew I was going to have a long train trip, I bought the book. And I contemplated the book – I can't say I read it, it was really too hard – during the trip through the plains of the Middle West. And the most extraordinary coincidence was that, when I arrived in Seattle, the first day I went and saw the program of the conference and there was Takesaki lecturing on Tomita's theory. From that day, I said to myself, 'That's it, I don't go to any other lecture, just Takesaki's'.

Not a very scientific attitude...

No, and moreover at this time I was fascinated by everything Japanese; it was more at the level of a sensibility to something totally different, that I didn't know at all… If there is a lesson to be drawn, it is that this pulled me completely out of the circle of ideas I was engrossed in at the time. And just then there was another coincidence, so that when I came back I had one more incredible stroke of luck. I had understood a bit of Tomita's theory, a small bit; I wasn't able to do research. But when I came back, I told myself I would go to the seminar in Paris which deals with operator algebras. So I went to the Dixmier seminar and the first time I went there, it was the organizational meeting; the main theme for the year was to be the Araki–Woods work on infinite tensor products. Dixmier was distributing the papers among the participants, a bit at random. There was just one left; I raised my hand. Riding home on the RER [suburban train], I was bored. I looked a bit at the paper I had been given and then I was really knocked over backwards. I realized that in this paper there were formulas that I would have had to be a complete idiot not to see were identical, exactly matching those in Tomita's theory. And these formulas said that a certain vector was an eigenvector for the operator defined by Tomita.

An hour after I arrived at home, I wrote a letter to Dixmier saying, 'Here are the Araki–Woods invariants and here is Tomita's theory'. You can see that one can get the first invariants from the intersection of the spectra of Tomita's operators and I gave him the formulas. And since I had been raised by Choquet, I wrote all this in half a page. Dixmier immediately wrote back, '*What you write is totally incomprehensible, I need details*'. And so I wrote back three pages of details, which wasn't difficult, explaining that one could define an invariant that I called *S*[1]. Dixmier fixed an appointment with me for after his next seminar. I went to see him and all he said then was, "Foncez", which in French is a strong form of "Go ahead!" That was the point of departure. It was really an incredible piece of luck; it wasn't really difficult. Though not exactly written black on white, it was there in the formulas.

It is sure that if I had remained in Paris, if I hadn't moved outside of my circle, I would have continued to work in a narrow direction and I wouldn't have opened up to totally different horizons. I really had this impression at that moment that I got a breath of completely fresh air which allowed me access to a more central part of mathematics. I have often had the impression that there are concentric circles in the mathematical world, that one begins to work in a totally eccentric part and one tries to get gradually closer to the heart.

What is this heart? Is it subjective?

What I mean by the heart of mathematics is that part which is interconnected to essentially all others. A bit like all roads lead to Rome. What I mean is that when the mental picture you get of a mathematical subject becomes more and more precise, you realize in fact that whatever the topic you begin with, if you look at it sufficiently precisely, after a while it converges toward this heart: modular forms, L functions, arithmetic, prime numbers, all sorts of things linked to that. It is not that these things are more difficult and I would hate to follow the wrong example I was discussing before of putting down eccentric topics. What I mean is that if you walk long enough, you are obliged to go towards these domains, you cannot remain outside. If you do, it is a bit out of fear. You can succeed in doing a lot of things by refining techniques in a given topic but unless you keep moving towards this heart you feel you are left outside. It is very strange and surely subjective.

In your research, you have got lightning-like results – you mentioned earlier the discovery of the invariant S. There is also the case of the 2 × 2 cocycles and others which cost you considerable efforts

Of course. This 2 by 2 matrix trick which is of utter simplicity[2] came indeed to me all of a sudden but only after I had spent three months doing horrible computations; I was doing concrete computations of modular automorphisms with almost-periodic states, etc. In fact, before discovering this cocycle property, I had come across it by experience. The 2 by 2 matrix trick came to me by chance in a flash, but because the ground had been prepared by tons and tons of examples – tons and tons of computations.

My impression is that I have never obtained anything at low cost. All my results have been preceded by preparatory ones – setting up work, a very long experimentation – hoping that at the end of this experimentation, an incredibly simple idea occurs which comes and solves the problem. And then you need to go through the checking period, almost intolerable because of the fear you have of being mistaken. I will never let anyone believe that you can wait just like this until results come all by themselves.

I spent the whole summer [of 2006] checking a formula which gives the standard model coupled with gravitation in our joint work with Chamseddine and Marcolli. The computation is monumental: in the standard model, there are four pages of terms with coefficients 1/8, 1/4, of sine or cosine of the Weinberg angle… and if you have not checked everything with all the coefficients, you can't claim that the computation gives the right result. I found different coefficients than those in Veltman's book, which obliged me to do again and again these computations until Matilde Marcolli [with whom I am writing a book] realized that the coefficients we had were the right ones and had already been corrected by Veltman in his second edition! There is always this permanent fear of error which doesn't improve over the years. And there is this part of the brain which is permanently checking and emitting warning signals. I have had haunting fears about this.

For example, some years ago, I visited Joachim Cuntz in Germany and on the return train I looked at a somewhat bizarre example of my work with Henri Moscovici on the local index theorem. I had taken a particular value of the parameter and I convinced myself on the train that the theorem didn't work. I became a wreck – I saw that in the eyes of the people I crossed on the suburban train to go back home. I had the impression that they read such despair in me, they wanted to help... Back home, I tried to eat but I couldn't. At last, taking my courage in both hands, I went to my office and I redid the verifications. And there was a miracle which made the theorem work out in this case… I have had several very distressing episodes like this.

Concerning heuristics: you have written several times that geometry is on the side of intuition. On the other hand, formulas seem to play a leading role in the way you work.

Ah, yes, absolutely. I can think much better about a formula than about a geometrical object because I

[1] the intersection of the spectra of all modular operators

[2] Groupe modulaire d'une algèbre de von Neumann, C. R. Acad. Sci. Paris, Sér. A-B, 274, 1972.

never trust that a geometric picture, a drawing, is sufficiently generic. I don't really have a geometrical mind. When there is some geometry problem and I succeed in translating it into algebra, then it's fine. There are several steps: first the translation, then the purely algebraic thinking. I always try to distinguish between the intuitive side (the geometrical one) and the linguistic one (the algebraic one) in which one manipulates formulas, and I think much better on that side. For me, algebra unfolds in time: I can see a formula live and turn and exist in time, whereas geometry has something instantaneous about it and I have much more difficulty with it. As far as I go, formulas create mental pictures.

You often give the impression you love computations
Absolutely. My mathematical thinking is heavily dependent on computations. But, of course, computing does not suffice. Then, one has to interpret things at the conceptual level. Galois was one of the first to understand that one can deal with a computation even if the latter is not practically feasible. For instance, take an equation of degree 7; the polynomial that Galois associates has degree 7! And one has to factorize it. What Galois says,

«Sauter à pieds joints sur ces calculs; grouper les opérations, les classer suivant leurs difficultés et non suivant leurs formes; telle est selon moi, la mission des géomètres futurs»

is that one should jump *above* the computations, organize them according to their difficulty. One should do them but only like a thought experiment in one's mind, not in a concrete manner. In Galois' example, you can give an explicit function f of the roots of an equation $E = 0$ which takes the n! different values when you permute the roots – you just take a linear form with generic rational coefficients. You can then go ahead and express the roots of $E = 0$ as rational functions of f; this can be done by Euclid's algorithm and by elimination. One can use the computer, and the expression one gets is awfully complicated, even when the starting equation $E = 0$ has degree 4 or 5. If you tried to implement the computations concretely, you would quickly get lost in the complexity of the results. On the contrary, what you have to be able to do is to perform them abstractly and to build mental objects which represent the intermediate steps and results at an idealized level.

I always proceed in the following way. Whatever the complexity of the problem, instead of trying it first on a piece of paper with a pencil, I just go out for a walk and try to have all the ingredients present in my mind, in order to start manipulating them mentally. Only after this exercise am I able to see clearly, think about the various steps and begin to get a mental picture. This is a painful process which consists in gathering in your mind, in your memory, all the elements of the problem, in order to begin manipulating them. It is an exercise that I recommend – well, of course, different people function differently – if one wants to be able not to depend upon paper and pencil. Because with paper and pencil you get tempted to start writing immediately and if you haven't thought long enough before, you will get nowhere. You will get discouraged before having had enough time to create in the linguistic part of the brain specific mental pictures that you can then manipulate, as usual, by zipping them, transforming them into something smaller, and then moving them around.

If you make computations, it is crucial to avoid mistakes. There are ways to check, for instance using different paths to the same result. Also one can see if the result of a computation looks right or not. I remember when I worked with Michel Dubois-Violette, we had a sum of 1440 integrals, each of which was an integral over a period of a rational function of theta functions and their derivatives. We expected the sum to have a simple factorization. Indeed, we found a simple result which was a product of modular forms, elliptic functions, etc. When you find that a huge sum like that gives a product, you feel rather confident that no mistake was made along the way.

Non-commutative geometry

What is non-commutative geometry? In your opinion, is 'non-commutative geometry' simply a better name for operator algebras or is it a close but distinct field?
Yes, it's important to be more precise. First, non-commutative geometry for me is this duality between geometry and algebra, with a striking coincidence between the algebraic rules and the linguistic ones. Ordinary language never uses parentheses inside the words. This means that associativity is taken into account, but not commutativity, which would permit permuting the letters freely. With the commutative rules my name appears 4 times in the cryptic message a friend sent me recently:

«Je suis alenconnais, et non alsacien. Si t'as besoin d'un conseil nana, je t'attends au coin annales. Qui suis-je ?»

So somehow commutativity blurs things. In the non-commutative world, which shows up in physics at the level of microscopic systems, the simplifications coming from commutativity are no longer allowed. This is the difference between non-commutative geometry and ordinary geometry, in which coordinates commute. There is something intriguing in the fact that the rules for writing words coincide with the natural rules of algebraic manipulation, namely associativity but not commutativity. Secondly, for me, the passage to non-commutative is exactly the passage from a completely static space in which points do not talk to each other, to a non-commutative space, in which points start being related to each other, as isomorphic objects of a category. When some points are related to each other, they will be represented by matrices on the algebraic side, exactly in the same way as Heisenberg discovered the matrix mechanics of microscopic systems.

One does not go very far if one remains at this strictly algebraic level, with letter manipulations… and the real point of departure of non-commutative geometry is von Neumann algebras. What really convinced me that operator algebras is a very fertile field is when I realized – because of this 2 by 2 matrix trick – that a non-commutative operator algebra evolves with time! It admits a *canonical* flow of outer automorphisms and in particular it has "periods"! Once you understand this, you realize that the non-commutative world instead of being only a pale reflection, a meaningless generalization of the commutative case, admits totally new and unexpected features, such as this generation of the flow of time from non-commutativity.

However, I don't identify non-commutative geometry with operator algebras; this field has a life of its own. New phenomena are discovered and it is very important to study operator algebras per se – I have spent a large part of my life doing that. But on the other hand, operator algebras only capture certain aspects of a non-commutative space, and the "only" commutative von Neumann algebra is $L^\infty[0;1]$! To be more specific, von Neumann algebras only capture the measure theory, and Gelfand's C*-algebras the topology. And there are many more aspects in a geometric space: the differential structure and crucially the metric.

Non-commutative geometry can be organized according to what qualitative feature you look at when you analyze a space. But, of course, as a living body you cannot isolate any of these aspects from the others without destroying its integrity. One aspect on which I worked with greatest intensity in recent times is a shift of paradigm which is almost forced on you by non-commutativity: it bears on the metric aspect, the measurement of distances. This is where the Dirac operator plays a key role. Instead of measuring distances effectively by taking the shortest path from one point to another, you are led to a dual point of view, forced upon you when you are doing non-commutative geometry: the only way of measuring distances in the non-commutative world is spectral. It simply consists of sending a wave from a point a to a point b and then measuring the phase shift of the wave. Amusingly this shift of paradigm already took place in the metric system, when in the sixties the definition of the unit of length, which used to be a concrete metal bar, was replaced by the wavelength of an atomic spectral line. So the shift which is forced upon you by non-commutative geometry already happened in physics. This is a typical example where the non-commutative generalization corresponds to an abrupt change even in the commutative case.

I realized recently that the only information we have on the very distant universe is spectral. I hadn't understood that the 'red shift' is not a frequency shift but a scaling of frequencies. If you look far enough back in the universe, frequencies are divided by a factor of up to a 1000. This is amazing. And you see it purely in a spectral way. This spectral point of view is the one which appears from experiments when you study the universe; this is no fantasy. And this is a compulsory point of view when you look at a geometrical space from the perspective of non-commutative geometry. From this point of view one is led very naturally to the spectral action principle which allows one to encode geometrically in a nutshell the tremendous complexity of the standard model coupled with gravity. What happens is simply that space-time admits a fine structure, a bit like atomic spectra, and is neither a continuum nor a discrete space but a subtle mixture of the two.

In the book I am writing with Matilde Marcolli, we develop the first three hundred pages on physics: the standard model and renormalization – linked to motives and Galois groups, and the last three hundred pages on the zeta function: its spectral realization and the spontaneous symmetry breaking of arithmetic systems. We are reaching the end of the write-up of the book and we are finding out that quite surprisingly there is a deep relation between the two a-priori disconnected pieces of the book. In fact there is an analogy, a conversion table, between the formalism of spontaneous symmetry breaking which is used for arithmetic systems, zeta functions, dual systems, etc., and a formalism which seems extremely tempting to people who are trying to quantize gravity.

While establishing this dictionary, we found out in the literature that the notion of KMS state, which plays a fundamental role in our work on symmetry breaking for arithmetic systems, also plays a role in the electroweak symmetry breaking which gives masses to particles in the standard model. This allows us to go further in the analogy and it suggests that the people who are trying to develop quantum gravity in a fixed space are on the wrong track. We know that the universe has cooled down; well, it suggests that when the universe was hotter than, say, at the Planck's temperature, there was no geometry at all, and that only after the phase transition was there a spontaneous symmetry breaking which selected a particular geometry and therefore the particular universe in which we are. This is something we would never have thought of – we would never have had this idea – if our book was not written with the two parallel texts. Of course there is no point where one part really uses, or relies, on the other – but you can see an analogy emerging between the two parts.

As André Weil pointed out, this type of mysterious similarity is one of the most fertile things in mathematics. The human mind is still ahead of the computer, for the moment and for a long time to come I hope, in detecting the structural analogies between theories which look quite different in content but in which the same kind of phenomena appear. Translation will never be a literal one and there will always be two texts written in two different languages and there will never be a one to one correspondence between the words of one language and the words of the other. But there will be these strange hints which may well evaporate if you try to rush and write them down too precisely. There are boxes that are very well understood on one side – and not understood at all on the other. Even if it doesn't provide a key to open something, it binds us; it forces us to think from the other side.

It's true that the name 'non-commutative geometry' is a bit unfortunate because there is this 'non', the negation. What is important is to think of it as 'non necessarily commutative', so that it includes the commutative part. We could have given it 36 other names. A name that would have been better in the Riemannian part is 'spectral geometry'. What this geometry shows so well is that all the things we perceive are spectral, that seeing them from the set-theoretic point of view is not the right standpoint. We could have used different names, though certainly not 'quantum'.

Why?

Because in the word 'quantum' there is a perversion, i.e. people don't understand that the word 'quantum', from the beginning, is not so much 'non-commutative' but rather 'integer'. In the word 'quantum' there is really this discovery by Planck of the formula for blackbody radiation, from which he understood that energy had to be quantized in quanta of $\hbar\nu$.

There is a terrible confusion, created by people doing deformation theory who let one believe that quantizing an algebra just means deforming it to a non-commutative one. They take a commutative space and since they deform the product into a non-commutative algebra, they believe they are quantizing. But this is completely wrong. You succeed in quantizing a space only if you give a deformation into a very specific algebra: the algebra of compact operators. And then, there is an integrality, the integrality of the Fredholm index. The use of the wrong vocabulary creates confusion and does not help at all in understanding. That's why I am so reluctant to use the word 'quantum' – this looks more flashy, perhaps, but the truth is that you are doing something quantum only in very particular cases, otherwise you are doing something non-commutative, that's all. Then this may be less fashionable at the linguistic level, but never mind – it is much closer to reality.

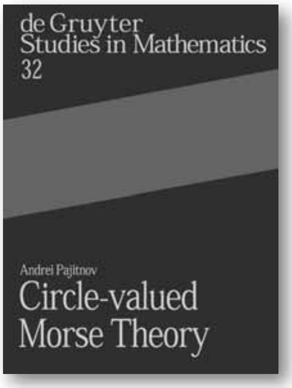
What is more important for you in your mathematical work: unity or evolution?

It's difficult to decide. Every mathematician has a kind of Ariadne's thread which he (or she) follows from his (or her) starting point and that he (or she) should absolutely try not to break. So there is a unity, a kind of trajectory, which makes you start from a place, and because you have started there, in a slightly bizarre and special place, you have a certain originality, a certain perspective, different from that of the others. And this is essential, otherwise you put everybody in the same mould – everybody would have the same reactions to the same questions. This is not what we want; we want different people who have their own approaches, their own methods. So there is a unity in the trajectory, which is not at all the unity of mathematics. The unity of mathematics, you discover bit by bit, when you realize that extremely different trajectories, of extremely different people, get closer to the same vibrant heart of mathematics. But what I have felt above all is the unity, the fidelity to a trajectory.

Part II of the interview is to appear in the next issue of the Newsletter.

Alain Connes is a Professor at Collège de France, IHES and Vanderbilt University. Among his awards are a Fields Medal in 1982, the Crafoord Prize in 2001 and the CNRS Gold Medal in 2004.

The interviewers, Catherine Goldstein [cgolds@math.jussieu.fr] (Directeur de Recherches – CNRS) and Georges Skandalis [skandal@math.jussieu.fr] (Professor at Université Paris Diderot – Paris 7) are both members of the Institut de Mathématiques de Jussieu.

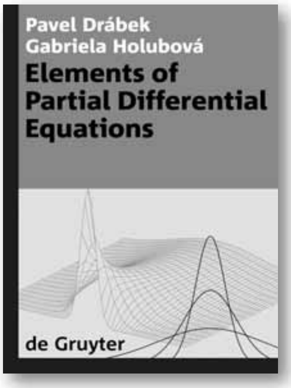


Andrei Pajitnov

■ Circle-valued Morse Theory

2006. ix, 454 pages. Cloth. € [D] 98.00 / sFr 157.00 / *US$ 128.00
ISBN 978-3-11-015807-6
(de Gruyter Studies in Mathematics 32)

In 1927 M. Morse discovered that the number of critical points of a smooth function on a manifold is closely related to the topology of the manifold. This became a starting point of the Morse theory which is now one of the basic parts of differential topology. It is a large and actively developing domain of differential topology, with applications and connections to many geometrical problems. The aim of the present book is to give a systematic treatment of the geometric foundations of a subfield of that topic, the circle-valued Morse functions, a subfield of Morse theory.

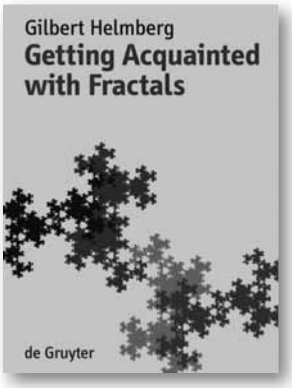


Pavel Drábek / Gabriela Holubová

■ Elements of Partial Differential Equations

2007. Approx. 290 pages. Paperback. € [D] 34.95 / sFr 56.00 / *US$ 45.00
ISBN 978-3-11-019124-0
(de Gruyter Textbook)

This textbook presents a first introduction to PDEs on an elementary level, enabling the reader to understand what partial differential equations are, where they come from and how they can be solved. The intention is that the reader understands the basic principles which are valid for particular types of PDEs, and to acquire some classical methods to solve them, thus the authors restrict their considerations to fundamental types of equations and basic methods. Only basic facts from calculus and linear ordinary differential equations of first and second order are needed as a prerequisite.



Gilbert Helmberg

■ Getting Acquainted with Fractals

2007. Approx. 192 pages. Cloth. € [D] 78.00 / sFr 125.00 / *US$ 98.00
ISBN 978-3-11-019092-2

This well-written book provides a mathematically oriented introduction to fractals, with a focus upon three types of fractals: fractals of curves, attractors for iterative function systems in the plane, and Julia sets. The presentation is on an undergraduate level, with an ample presentation of the corresponding mathematical background, e.g., linear algebra, calculus, algebra, geometry, topology, measure theory and complex analysis. The book contains over 100 color illustrations.

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An Interview with Rolf Jeltsch

Prior to ICIAM07, July 2007

During the ICM congress in Madrid the former president of the International Council on Industrial and Applied Mathematics (ICIAM), Olavi Nevanlinna, who served from 1999 until 2003, had an interview with the President-Elect of the ICIAM, Rolf Jeltsch, who will serve as president from 2007 until 2011. The ICIAM consists of about two dozens member societies and associated members of which the EMS is one. One can find more information on the ICIAM as a society at http://www.iciam.org/. Rolf Jeltsch is also the Congress Director of the ICIAM07 congress, which will take place in Zurich, 16th-20th July 2007 (see http://www.iciam07.ch/ and the footnote at the end of the interview).

Rolf, you did an interview with me seven years ago when I had started as President of ICIAM. It seems to me that all we need to do is to look at the mirror and let it reverse our roles. But, unfortunately, these guys look seven years older. So, maybe we could see which problems are still there, where there has been progress and so on.

Let me start with the progress. I think ICIAM has seen an incredible growth since 1999. At the first board meeting I attended in Paris in 2000, ten societies were members or associated members. By now, there are more than two dozen associated or regular member societies.

Finances have improved through the introduction of a licence fee for the ICIAM congresses. In addition, the organisers of ICIAM 2003 donated a substantial sum. This made it possible to start a program to support industrial and applied mathematics in developing countries. The support is given to conferences to hand out travel grants to delegates from developing countries. This year the International Congress on Applications of Mathematics in Santiago de Chile and a CIMPA summer school in Spain have received a contribution. Next year the ICIAM Society will support with $15,000 persons from developing countries to attend ICIAM07. The ICIAM07 organizers have already received more than 300 applications for such support.

I also think that ICIAM has received a lot more recognition. In particular its relation with the International Mathematical Union has improved. ICIAM has Andreas Griewank as a representative in the IMU Developing Countries Strategy Group.

What is on the down side? We still have no members in Africa. Brazil is the only member from Latin America and the Caribbean, despite the fact that excellent applied mathematicians work in other countries of this area. In the Middle East and in the Far East, there are still large areas where ICIAM is not represented. Up to now, our activities basically consisted of organizing our congress every four years and handing out the ICIAM prizes. Now it is time to extend these activities.

You were, Rolf, last year elected as President of ICIAM for the term 2007–2011. Additionally that includes working as President-Elect 2005–2007 and Past President during 2011–2013. Remembering that you also are the director of ICIAM 2007 congress in Zurich, that shall sum up to a great amount of voluntary work for the world applied mathematics community. Before going on, please give us your way of pronouncing ICIAM so that the readers can taste it while reading.

Right from the beginning I always used the soft ‘isiam’ pronunciation. I do admit that some English speakers were able to convince me to use ‘ikiam’, because the ‘c’ is pronounced ‘k’ in congress and in council, but now I am back at the smooth ‚isiam’. It just sounds better!

You have also worked in EMS, GAMM, SMG (Swiss Mathematical Society), CEIC of IMU, related to science, publishing, congresses and so on. What is closer to your heart?

In each society the objectives are different and hence I have no preference. For me it is always the work directly at hand that has the highest priority and is closest to my heart. I did however notice that I had problems being for one year simultaneously president of EMS (my last year) and SMG (my first year). The reason was not a conflict of interest but the mental switching from my long time working for EMS to finding a vision on what are the important issues for SMG. I do not foresee the same problem with my switching from my GAMM presidency, which ends in 2007, to my presidency in ICIAM. The overlap is very short and being congress director for ICIAM07 I have already a heavy involvement in ICIAM.

My involvement in CEIC of IMU was on a very low level. The CEIC does important work on many fields: copyright issues, digitisation of mathematical literature, best practice for publishing and there are many more. I became a member because, as president of the EMS, I pushed hard for digitising old mathematical literature and was the coordinator of an application for a six framework project, which unfortunately was not funded. I was in CEIC just to represent a 'normal' mathematician and not as a specialist on any of the above issues dealt with in CEIC.

I do admit that the EMS publishing house is very close to my heart. I think it is a true success story and it's nice to be part of it.

I never felt that I had conflict of interests when working for different societies. The reason is that I always try to figure out what would be best for the individual mathematician.

You have organized many congresses in the past and ICIAM 2007 is coming. How does the stress curve look? When does it start, when does it peek and when is it completely over?

Of course one wants to do the best job possible. I want ICIAM07 to attract as many people as possible and to provide a space for them to interact, learn from lectures, enjoy the congress and the city.

Hence I am slightly worried at the moment whether enough mini-symposia proposals will be submitted by the end of this month. People are now on vacation and might have not enough time to prepare a proposal. Moreover at the last day of the submission, the abstract handling system could break down.[1]

Of course there have been moments of stress, for example when ETH decided that part of the north portion of the main building would be renovated during the congress. By now, I have achieved that the renovation will be stopped during this week and we have found more lecture rooms for the almost fifty parallel sessions. Other moments of stress have been when the abstract handling system was not delivered in time, and when it was delivered it was full of flaws. Unfortunately not all deficiencies could be corrected and hence our assistants working on the helpline were covered with large numbers of e-mails. I apologise to all users of the system and to our students for the system's deficiencies. Clearly there will be some other moments of pressure but usually these can not be planned; things happen and one has to react. But it is fun to work together with an excellent team and see how all your work will finally result in a whole huge event. It is like making a puzzle with one thousand pieces. You start with putting together the pieces and in the end you a have the complete picture.

I do not think about the congress as stress. I more see the positive sides. It is great that I could help creating the Olga Taussky Todd lecture with AWM, the Association of Women in Mathematics, and EWM, the Society of European Women in Mathematics, which I hope will become a regular event at ICIAM congresses. It was fantastic to see how people helped when Walter Gautschi, who will give the Euler Lecture, needed a paper translated from Latin to German. The secretary of the Euler Archive in Basel, M. Mattmüller, did it within a few weeks.

I promised my wife to go for vacations in the mountains a few days after the end of the congress. Refreshed, I will have to look at the finances and the proceedings still have to be done. Hence I am sure this will occupy me until the end of 2007.

We both have just been in Santiago de Compostela at the IMU General Assembly. You asked me seven years ago about the relations between IMU and ICIAM. How would you answer the same question now?

As mentioned earlier, ICIAM is now recognised much more than it was in 1999. I think ICIAM became stronger. On the other hand IMU became more aware of ICIAM and this is mainly due to two presidents: Jacob Palis and John Ball, who is just stepping down. As a result of this recognition we have always a representative of the executive committee of IMU present at our board meetings. I am convinced that the relation with IMU will become even better with its new president László Lovász and the new secretary Martin Groetschel.

Presently we are on a break during the ICM 2006 in Madrid. I have heard people here say things like, "not much applied math around". What do you think about dividing mathematics into "pure" and "applied"?

I do not like the distinction between "pure" and "applied" mathematics. It is all mathematics. I feel it is more a question of culture and of the attitude of why you do mathematics. I call a mathematician "applied" when he or she wants to solve problems which originate from outside mathematics. The person must be willing to create new concepts to formulate the problem and then use mathematical techniques to solve it.

If I have to compare the volume of "pure" versus "applied" mathematics, one version is to just count the persons in mathematics departments. By now I feel that between 1/3 and up to almost 1/2 of the people in a department are applied. I never compared the volume of produced articles in journals. If I look at the lectures presented at this ICM 2006 it is obvious that it does not reflect the population distribution in departments just mentioned a minute ago.

Do you see areas where IMU and ICIAM should cooperate?

I think this can be easily done on all issues common to all mathematicians. I see at least two immediate ways of cooperating.

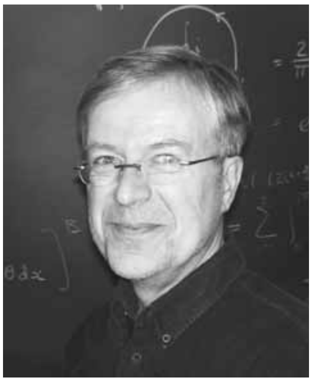
One is on the level of supporting mathematics in developing countries. As I said, we have already Andreas Griewank as our representative in the IMU Developing Countries Strategy Group.

All issues which are discussed in CEIC are issues common to all mathematicians: best practices, copyright, digitization, popularisation of mathematics. Hence ICIAM should participate in this committee and ICIAM should give its support to results from this committee.

ICIAM is organized through societies, of which many are international while IMU has countries as members. Do you see this as a natural situation, orthogonality allowing overlap?

I think this is just one aspect of the differences between


Rolf Jeltsch


Olavi Nevanlinna

IMU and ICIAM. Let me first respond to this particular question. In IMU mathematicians living in countries which are not members are not represented and there is no way that they can be represented. Even in countries with a representation, a mathematician belonging to a politically suppressed group may not be properly represented as the representation is government controlled. On the other hand IMU can influence a government at least a little bit as they feel responsible for all mathematicians in a country.

In ICIAM any mathematician can join one of the member societies and therefore in principle be represented. Even if one is a mathematician in a developing country one can be represented as some of our member societies have special low rates or even waive the fees for persons in economically difficult situations.

Another aspect is the mathematical subjects represented in the organisation. I mentioned above that in IMU at least in the "old" days, industrial and applied mathematics was under-represented in the ICMs as well as in the executive committee. This was one of the major reasons why the ICIAM congresses and therefore our organisation has been created. As mentioned before there has been a slow opening towards our subjects in recent years and we shall have to see how it develops.

Clearly there is also a financial difference. In most countries the governments pay for the fee. For governments even high fees are in a sense peanuts. ICIAM however has to ask for rather low fees as our member societies usually pay these out of their low income. Hence the ICIAM budget is one order of magnitude lower as the one of IMU.

I do think that it is good to have both organisations. As you indicate, in a sense they complement each other.

ICIAM has started to give out prizes at ICIAM congresses. Does the world applied mathematics community need them? I chaired last time the search for winners but felt a little bit ashamed as the cheques in the brown envelopes just covered the air tickets for spouses. When the ICIAM society was founded four prizes had been donated: the Pioneer Prize by SIAM, the Collatz Prize by GAMM, the Maxwell Prize by IMA and the Lagrange Prize by SMAI, SEMA and SIMAI. These prizes are always handed out during the congresses; the first time was 1999. For the next year CSIAM has created the ICIAM Su Buchin Prize.

Clearly the monetary value of the prize is extremely low, \$1000 multiplied by the number of supporting societies. Up to now all prize winners have been excellent choices and I feel this will raise the stature. In addition it is crucial to have an excellent selection procedure. This procedure is already very good, nevertheless we shall further work on it, for example by defining more precisely how to handle conflict of interests of committee members. Up to now the winners have been kept secret until the opening ceremony of the congress, similar to how it is done with the Fields Medals. This time the winners will be announced on 18th September, well ahead of ICIAM07, see http://www.iciam.org/prizes2007.html. It is not clear whether this is a good way of announcing the prizes. The idea is that one obtains publicity twice, first due to the early announcement and then we expect another publicity push when the prizes will actually be handed over during ICIAM07. All this will add to the prestige of the prizes.[2] I also think that the monetary value should be increased in the long run.

At ICIAM 2007 we have the additional feature of the Olga Taussky Todd lecture. This is a lecture created by the AWM together with the EWM. This honour is to be conferred to a woman who has made outstanding contributions in applied mathematics and/or scientific computation. The name of this lecture pays tribute to the memory of Olga Taussky Todd, whose scientific legacy is in both theoretical and applied mathematics and whose work exemplifies the qualities to be recognized. Formally this is, at the moment, not an ICIAM event but I do hope we shall make it a regular lecture at our congresses.

What is your experience in raising money from private sources for congresses? Do you see trends?

Currently we are trying in Zurich very hard to raise funds for the congress. A few years ago companies would donate amounts between 1000 and 3000 Swiss francs. Nowadays many companies have a sponsoring concept put well into place. They will sponsor sports events and art events but not congresses in mathematics. Hence very potent companies will not contribute at all. Another difficulty is to get support from an international company. They usually have in each country a local company which has a very local viewpoint. We have been able to attract some money through the industry days and companies where mathematics plays a role. Overall it is a tough business.

Do you want to comment on publishing of mathematics? In short, publishing new and digitizing old.

As you know, EMS created its own publishing house and this has been a great success. I think generally we should push for all ideas to make publishing, and here I mean hard copies as well as electronically based systems, and distribution of mathematical results more independent of the expensive commercially oriented companies. One way is that societies create their own publishing houses or join together to do this. Another is to create free manuscript depositories such as the mathematics ArXiv.

When it comes to digitisation, copyrights and so on, I think we should join efforts with IMU.

Impact factors annoy mathematicians but the outside community needs something like that. Should we do it better for us mathematicians or for the whole scientific community or should we just suffer?

I am very much against any bibliometric measurements of the quality of a colleague. Impact factors have been introduced by commercial enterprises who want to hook us to their journals. It is the contents of an article which is important, not some number. Here in Madrid we have seen the appreciation of Grigori Perelman. His important papers have appeared in ArXiv and by the definition of the impact factor would supposedly have no impact at all! I feel one can not measure a human by a number even if the number is designed as well as possible. Hence we should not try to do this. Look for example in tennis; we have the world ranking. But it happens almost daily that a higher ranked player loses against a lower ranked one. Moreover the ranking is time dependent. For me trying to assign a number to a mathematician is just playing into the hands of administrators and even mathematics department heads who are afraid of making decisions. When one wants to hire a person there are many more aspects to account for than just a number. The same applies when you want to give a pay rise.

Globalization due to affordable travel, e-mail, and the internet changes the way we operate in the world. What shall be the role of small national societies in the future as science and mathematics in particular is international in its very nature? Do the societies disappear? Who will be the winners?

Societies can have several tasks and obligations and I distinguish here 'local' ones and 'non-local' ones. As we know, political systems such as countries and provinces are 'local'. Hence university education, but also high school education and of course the education of teachers on all levels, is local. The same may be true for research funding. In addition the language is in most cases very local. Hence it is a natural task for country or province based societies to care for these issues. Talking to politicians is also local. Hence as long as there are countries with different educational systems, different languages and different research funding we need the country based societies and these will not disappear. However societies can offer also a professional support by publishing journals, books and organising conferences. This professional help can also be provided by commercial enterprises, e.g. a commercial publisher will produce and sell books and journals. A university or even a private person might organise a conference to earn money. Some of the societies have been very successful also in this commercial enterprise and there is a danger that they push the small societies out of this market. I personally feel that societies should in this aspect cooperate. I am happy to say that I have seen such cooperation. Unfortunately, I have seen some societies disappear. At the moment this is happening especially in Russia as the financial situation of colleagues worsens and the government has increased administrative fees for such organisations making it impossible for them to exist. This might actually be an issue for ICIAM together with IMU to talk to such a government.

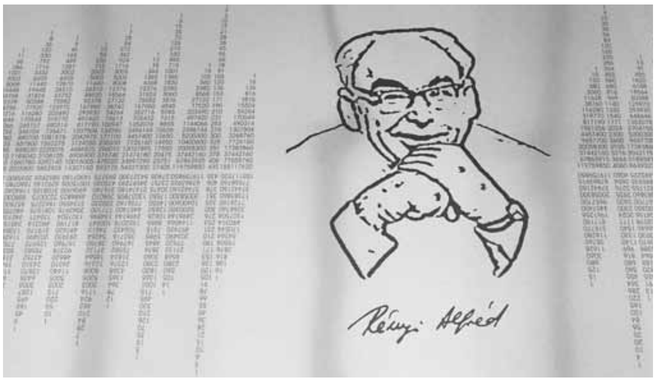
Rolf Jeltsch [jeltsch@math.ethz.ch] is a full professor at the Department of Mathematics at ETH in Zurich and heads the Seminar for Applied Mathematics. His major research interests are in numerical analysis. In the 1970's his work centred on ordinary differential equations. Since the 1980's, he has focused on hyperbolic partial differential equations, especially systems of conservation laws with applications. In addition he conducts large-scale computing in science and engineering. He is the Congress Director of ICIAM07 and President-Elect of the ICIAM. He was a president of the EMS, 1999–2002. Currently he is President of GAMM, 2005–2007.

Olavi Nevanlinna [Olavi.Nevanlinna@tkk.fi] is a professor of mathematics at Helsinki University of Technology TKK. His research interests range from numerical analysis to function and operator theory. In the 1970s and the early 1980s he worked a lot together with Rolf Jeltsch.

1 By the time the revised interview has been submitted to the editor, 24th January 2007, almost 300 minisymposia with a total of almost 1700 lectures and more than 1100 contributed talks have been submitted. Overall there will be approximately 3000 talks and poster presentations. 3895 persons have pre-registered indicating that they intend to participate. As some of these will not be able to attend due to financial reasons a participation of 3500 is expected.

2 By the time the revised interview has been submitted o the editor, 24th January 2007 we know that this way of handling prizes has been a great success. The officers of ICIAM could concentrate on the press release and its distribution. The result was a lot of publicity; institutions of winners made press releases; interviews with winners had been made.

ERCOM: Alfréd Rényi Institute, Budapest, Hungary



Alfréd Rényi (1921–1970)

Short history

The institute was founded in 1950 as the Institute for Applied Mathematics of the Hungarian Academy of Sciences. The first director was the brilliant 29-year-old mathematician Alfréd Rényi. Besides applied mathematics, more and more theoretical research characterized the achievements of the institute, and this was reflected when it changed its name to the Mathematical Research Institute of the Hungarian Academy of Sciences in 1955. Paul Erdős was a member of the institute. When he spent time in Hungary he received a salary from the institute and frequently stayed in the academy's guest house on Castle Hill in Buda. Many of his papers were written with co-authors working at this institute, for example the famous paper with Rényi on the evolution of random graphs. After the untimely death of Rényi the institute has continued to flourish under the directorship of László Fejes Tóth (1970–1982), András Hajnal (1983–1992), Domokos Szász (1993–1995), Gyula O. H. Katona (1996–2005) and Péter P. Pálfy (since 2006). In 1999 the institute was named after its first director, now called the Alfréd Rényi Institute of Mathematics. It still belongs to the network of research institutes of the Hungarian Academy of Sciences and the main part of its budget comes from the state. However, an increasing portion of the institute's finances are covered by various international projects. In 2000 the Rényi Institute was awarded the prestigious title "Centre of Excellence" by the European Commission.

Activities

The mathematical research carried out in the Rényi Institute covers a wide spectrum of fields; there are research groups in algebra, algebraic geometry, analysis, differential topology, discrete mathematics, geometry, information theory, logic, number theory, set theory, stochastics and topology. Moreover, some applied topics such as bioinformatics and cryptography are also represented. According to a survey the Institute's members produce about one third of the mathematics papers written by Hungarian authors. There are 63 permanent members of the institute. At present eleven of them are on leave, working at various places, e.g. the University of Chicago, Rutgers University, University College London, etc. The institute also offers some postdoctoral positions for up to three years. Currently eleven postdoctoral fellows work at the institute. The lecture rooms have a busy schedule. There are ten weekly seminars of the various research groups and the institute has a series of monthly colloquium lectures. Mathematicians from universities and other institutions in Budapest and sometimes from other university towns (Szeged, Debrecen) regularly attend these seminars. A large proportion of the members of the institute regularly teach at various universities, most of them at Eötvös University and at Budapest University of Technology and Economics. The Rényi Institute runs a joint PhD program in collaboration with the Mathematics Department of the Central European University in Budapest. In addition, quite a number of doctoral theses at Eötvös University are supervised by members of the institute. Many members give courses at Budapest Semesters in Mathematics, a program aimed at visiting North American undergraduates. The journal Studia Scientiarum Mathematicarum Hungarica is edited by leading members of the institute.

Visitors

The Rényi Institute runs a number of projects (mainly European programs) that finance visits of foreign mathematicians to Budapest, as well as financing members of the institute to travel abroad. In addition there are several bilateral exchange agreements that make visits by mathematicians possible, mainly from Eastern European countries and from the former Soviet Union. In 2006 the institute had 21 visitors for 2–4 months and about 200 for shorter periods.

Conferences

The Rényi Institute has excellent facilities to hold conferences, workshops and training courses. Over the past few years, let us mention the Clay Mathematics Institute Summer School on Floer homology, gauge theory, and low dimensional topology (6th–26th June 2004), Quanti-

Reading room in the library

The main lecture hall

The Rényi Institute's building in the centre of Budapest

tative and mathematical finance (sponsored by Morgan Stanley, 20th–21st October 2005), and Large-scale random graph methods for modelling mesoscopic behaviour in biological and physical systems (sponsored by NSF, 28th August–4th September 2006).

In 2007 the following conferences will be held at the Rényi Institute: Quasi-random structure, regularity lemma and their applications (educational workshop in the framework of phenomena in high dimensions, Marie Curie Research Training Network, 22nd–26th January); Workshop on p-adic methods on rational points (18th-20th May); Extremal combinatorics (4th–8th June); Geometry fest (11th–15th June).

Location and facilities

The institute is located in the downtown of Pest, Reáltanoda utca (street) 13–15. The building was erected in the 1880's for Baron Béla Rédl. From 1907 the palace housed the Hungarian Engineers' and Architects' Association and its present day appearance is due to the famous architect Alajos Hauszmann, who was the president of the Association at the time of the acquisition of the edifice. The Mathematical Institute moved into the building in 1958. The Rényi Institute has the largest mathematical library in Hungary, with about fifty thousand books and more than 300 journals. Quite a number of journals arrive in exchange for Studia Sci. Math. Hung. The institute provides high standard computer facilities for members and for visitors. It has a large lecture hall seating 100 people and two additional seminar rooms.

Website

For further information please visit our website www.renyi.hu.

Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

The inequalities problems included in this issue are chosen from among a number of problems proposed by research mathematicians and university educators. One could also look at them as a step or link to mathematical research at all levels. Each one of the proposed problems is chosen for the simplicity of its statement, its depth, beauty and usefulness. We hope that mathematics educators will find these problems useful in training their students.

Sir Michael F. Atiyah writes in the preface of the book *Mathematics: Frontiers and Perspectives*,[1] "… some problems open doors, some problems close doors, and some remain curiosities, but all sharpen our wits and act as a challenge and a test of our ingenuity and techniques."

As already stated above, the proposed and open problems refer to inequalities. I wish to mention here three of the most influential books on inequalities:

1. G. H. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, Cambridge University Press, Cambridge, 1934.
2. E. F. Beckenbach and R. Bellman, *An Introduction to Inequalities*, Random House, New York, 1961.
3. D. S. Mitrinović (with P. M. Vasić), *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.

It is generally acknowledged that the classic book *Inequalities* by G. H. Hardy, J. E. Littlewood and G. Pólya transformed the field of inequalities from a collection of isolated formulas into a systematic discipline. The modern theory of inequalities, as well as the continuing and growing interest in this field, undoubtedly stems from that work.

Richard Bellman said during the Second International Conference on General Inequalities (Oberwolfach, 30th July–5th August 1978), "There are three reasons for the study of inequalities: practical, theoretical, and aesthetic". On the aesthetic aspects he said: "As has been pointed out, beauty is in the eyes of the beholder. However, it is generally agreed that certain pieces of music, art, or mathematics are beautiful. There is an elegance to inequalities that makes them very attractive."

After the classic book by Hardy, Littlewood and Pólya, the book of Mitrinović is the next most cited book in the field of inequalities. Mitrinović often used to say: "There are no equalities, even in the human life the inequalities are always present".

The book of G. V. Milovanović, D. S. Mitrinović, and Th. M. Rassias, *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*,[2] presents a vast subject of polynomial inequalities and approximation theory in an integrated and self-contained fashion. Some 1200 references have been cited in it.

I. Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

10. Let G be the set of all functions $g \in C^1(0,\infty)$ such that

$$g(x) \ge 0,\ g'(x) \le 0,\ \text{for all } x \in (0,\infty),\ \text{and} \int_0^\infty g(x)dx < \infty.$$

Find $A < 1$ so that

$$\int_0^\infty g(x)\sin x dx \le A \int_0^\infty g(x)dx$$

for all $g \in G$. (A. M. Fink, Iowa State University, USA)

11. Let $(H;\langle\cdot,\cdot\rangle)$ be a complex Hilbert space with norm $\|\cdot\|$. For any $X \in H^n$ with $X = (x_1,\dots,x_n)$, define

$$\|X\|_e := \sup_{(\lambda_1,\dots,\lambda_n)\in \mathbf{B}_n} \left\| \sum_{j=1}^n \lambda_j x_j \right\| \quad \text{and} \quad \|X\|_2 := \left(\sum_{j=1}^n \|x_j\|^2 \right)^{\frac{1}{2}},$$

where $\mathbf{B}_n := \{\lambda = (\lambda_1,\dots,\lambda_n) \in \mathbf{C}^n : \sum_{k=1}^n |\lambda_k|^2 \le 1\}$. Show that

$$\|X\|_2 \ge \|X\|_e \ge \frac{1}{\sqrt{n}} \|X\|_2 .$$

(S. S. Dragomir, Victoria University, Australia)

12. Let $P_n(x) = \sum_{k=0}^n a_k (1-x)^k (1+x)^{n-k}$ $(\not\equiv 0)$, with $a_k \ge 0$, $k = 0,1,\dots,n$. Prove that for every $x \in [-1,1]$, the following inequality

$$(1-x^2)\big(P_n'(x)^2 - P_n''(x)P_n(x)\big) \le nP_n(x)^2 - 2xP_n(x)P_n'(x)$$

holds. (G. V. Milovanović, University of Niš, Serbia)

13. If $f,g \in L^2(0,\infty)$ prove that

$$I := \int_0^\infty \int_0^\infty \frac{f(x)g(y)}{1+xy} dx dy \le \pi \left(\int_0^\infty f^2(x)dx \int_0^\infty g^2(x)dx \right)^{\frac{1}{2}},$$

where the constant factor π is the best possible.

(B. C. Yang, Guangdong, P. R. China)

14. If $0 < \sum_{n=1}^\infty a_n^2 < \infty$ and $0 < \sum_{n=1}^\infty b_n^2 < \infty$, prove that

$$\sum_{n=1}^\infty \sum_{m=1}^\infty \frac{|\ln(\frac{m}{n})| a_m b_n}{\max\{m,n\}} < 8 \left(\sum_{n=1}^\infty a_n^2 \sum_{n=1}^\infty b_n^2 \right)^{\frac{1}{2}},$$

where the constant factor 8 is the best possible.

(B. C. Yang, Guangdong, P. R. China)

15. Let L^1_∞ be the set of functions $x : \mathbf{R} \to \mathbf{R}$ which are locally absolutely continuous and such that x' is essentially bounded (i.e. $\|x'\| < \infty$ where $\|y\| = \operatorname{ess\,sup}\{|y(t)| : t \in \mathbf{R}\}$). For a function $x \in L^1_\infty$, derivatives of order α, $0 < \alpha < 1$, in Marchaud sense are defined as

$$(\mathbf{D}^\alpha_\pm x)(u) := \frac{\alpha}{\Gamma(1-\alpha)} \int_0^\infty \frac{x(u) - x(u \mp t)}{t^{1+\alpha}} dt.$$

For any $\delta > 0$ find

$$\sup_{\substack{x \in L^1_\infty \\ \|x\| \le \delta, \|x'\| \le 1}} \|\mathbf{D}^\alpha_\pm x\|.$$

(V. F. Babenko, M. S. Churiliva, Dnepropetrovsk State University, Ukraine)

II. Two new open problems

16*. Let $\zeta > 0$, $\sin(2\zeta) < 0$, $N = [\zeta/\pi]$, where $[t]$ denotes the integer part of t. If

$$b_\nu = (N-\nu+1)\pi/\zeta, \quad \nu = 1,\dots,N$$

and

$$I_k = (-1)^N \int_{-1}^{1} \left(\prod_{\nu=1}^{k} (t^2 - b_\nu^2)\right) t \sin\zeta t \, dt,$$

then $I_k > 0$ for all $k = 1,\dots,N$.

(A conjecture of G. V. Milovanović, A. S. Cvetković, and M. P. Stanić, Serbia)

17*. Examine whether the following inequality holds

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{|\ln(\frac{m}{n})| a_m b_n}{m+n} \le k_0 \left(\sum_{n=1}^{\infty} a_n^2 \sum_{n=1}^{\infty} b_n^2\right)^{\frac{1}{2}}$$

and decide if the sum of the series $\sum_{k=0}^{\infty} \frac{8(-1)^k}{(2k+1)^2}$ is the best possible value for the constant k_0.

(B. C. Yang, P. R. China)

III. Solutions

1. Determine all C^2 functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the functional equation

$$f(x+y)f(x-y) = f(x)^2 + f(y)^2 - 1,$$

for all $x, y \in \mathbb{R}$.

(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

Solution by the proposer. Call the given equation (E). Differentiation of (E) with respect to y gives

$$f'(x+y)f(x-y) - f(x+y)f'(x-y) = 2f(y)f'(y), \tag{1}$$

and a further differentiation of (E) with respect to x yields

$$f''(x+y)f(x-y) - f(x+y)f''(x-y) = 0. \tag{2}$$

Setting $x = y = 0$ in (1) and (2), we obtain

$$f(0)^2 = 2f(0)^2 - 1$$
$$f(0)f'(0) = 0.$$

Thus

$$f'(0) = 0, \qquad f(0) = \pm 1. \tag{3}$$

Next, setting $x = y$ in (2), we have

$$f''(2x)f(0) - f(2x)f''(0) = 0.$$

Setting $t = 2x$ implies

$$f''(t)f(0) - f(t)f''(0) = 0,$$

or

$$f''(t) - \frac{f''(0)}{f(0)} f(t) = 0.$$

Let $k = |f''(0)|^{1/2}$. Then

$$f''(t) \pm k^2 f(t) = 0.$$

Case 1: $f''(t) + k^2 f(t) = 0$, $k \neq 0$.

Solving this equation and taking into account the initial conditions (3), one obtains

$$f(t) = \pm\cos(kt), \quad t \in \mathbf{R}.$$

Case 2: $f''(t) - k^2 f(t) = 0$, $k \neq 0$.

Solving this equation and using the initial conditions (3), we have

$$f(t) = \pm\cosh(kt), \quad t \in \mathbf{R}.$$

Case 3: $k = 0$.

Then $f''(t) = 0$. Taking into account the initial conditions (3), we get

$$f(t) = \pm 1, \quad \text{for all } t \in \mathbf{R}.$$

◊

Also solved by Anton Deitmar (Germany), Wolfgang Fensch (Germany), Erich N. Gulliver (Germany), Sin-Man Lam (student, University of Hong Kong), Guannan Lou (student, University of Hong Kong), Stevo Stević (Serbia), Dandan Zhou (PhD student, University of Hong Kong).

2. Determine all functions $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying the functional equation

$$f(ux - vy, uy - vx) = f(x,y) + f(u,v) + f(x,y)f(u,v) \tag{4}$$

for all $x, y, u, v \in \mathbb{R}$.

(Prasanna K. Sahoo, University of Louisville, USA)

Solution by the proposer. The solutions of the functional equation (4) are

$$f(x,y) = 0 \tag{5}$$
$$f(x,y) = -1 \tag{6}$$
$$f(x,y) = M(x^2 - y^2) - 1, \tag{7}$$

where $M : \mathbb{R} \to \mathbb{R}$ is a multiplicative function that is not identically constant. Note that a function $M : \mathbb{R} \to \mathbb{R}$ is a multiplicative function if and only if it satisfies $M(xy) = M(x)M(y)$ for all $x, y \in \mathbb{R}$.

It is easy to check that the solutions (5)–(7) satisfy the functional equation (4). Next, we show that (5)–(7) are the only solutions of (4).

Suppose f is identically constant, say $f \equiv c$. Then from (4) we have $c^2 + c = 0$, which implies $c = 0$ or $c = -1$. Hence the identically constant solutions of (4) are $f(x,y) = 0$ and $f(x,y) = -1$ for all $x, y \in \mathbb{R}$.

From now on we assume that f is not identically constant, that is $f \not\equiv c$, where c is a constant. We define a function $F : \mathbb{R}^2 \to \mathbb{R}$ by

$$F(x,y) = f\left(\frac{x+y}{2}, \frac{x-y}{2}\right) + 1 \tag{8}$$

for all $x, y \in \mathbb{R}$. Next, using (8) in (4), we obtain

$$F\big((x+y)(u-v), (x-y)(u+v)\big) = F(x+y, x-y)F(u+v, u-v) \tag{9}$$

for all $x, y, u, v \in \mathbb{R}$. Substituting $x_1 = x+y$, $y_1 = x-y$, $x_2 = u+v$ and $y_2 = u-v$ in (9), we have

$$F(x_1y_2, x_2y_1) = F(x_1,y_1)F(x_2,y_2) \tag{10}$$

for all $x_1, y_1, x_2, y_2 \in \mathbb{R}$.

Setting $y_1 = x_2 = 1$ in (10), we see that

$$F(x_1y_2, 1) = F(x_1, 1)F(1, y_2) \tag{11}$$

for all $x_1, y_2 \in \mathbb{R}$. Interchanging x_1 with y_2 in (11) and comparing the resulting equation with (11) we have

$$F(x_1,1)F(1,y_2) = F(y_2,1)F(1,x_1) \tag{12}$$

for all $x_1, y_2 \in \mathbb{R}$. Since f is non-constant, there exists an $x_0 \in \mathbb{R}$ such that $F(x_0,1) \neq 0$ and letting $x_1 = x_0$ in (12), we obtain

$$F(1,y_2) = \alpha F(y_2,1) \tag{13}$$

where α is an arbitrary constant. We claim that α is nonzero. If $\alpha = 0$, then letting $x_1 = x_2 = 1$ in (10) and using (13) we get f to be identically constant contrary to the assumption that f is not identically constant. Hence $\alpha \neq 0$. Using (13) in (11), we get

$$F(x_1y_2,1) = \alpha F(x_1,1)F(y_2,1) \tag{14}$$

for all $x_1, y_2 \in \mathbb{R}$. Defining $M : \mathbb{R} \to \mathbb{R}$ by

$$M(x) = \alpha F(x,1) \tag{15}$$

for all $x \in \mathbb{R}$, we see that (14) reduces to

$$M(x_1x_2) = M(x_1)M(x_2) \tag{16}$$

for all $x_1, x_2 \in \mathbb{R}$. Hence $M : \mathbb{R} \to \mathbb{R}$ is a multiplicative map that is not identically constant.

Now letting $y_1 = 1 = y_2$ in (10), we obtain

$$F(x_1,x_2) = F(x_1,1)F(x_2,1) \tag{17}$$

for all $x_1, x_2 \in \mathbb{R}$, which by (15) yields

$$F(x_1,x_2) = kM(x_1)M(x_2) \tag{18}$$

where $k = 1/\alpha^2$. Using (18) in (10), we see that $k = 1$ (since $k = 0$ yields a constant function f). Thus from (18), (8) and the fact that $k = 1$, we have

$$\begin{aligned} f(x,y) &= F(x+y, x-y) - 1 = M(x+y)M(x-y) - 1 \\ &= M(x^2 - y^2) - 1 \end{aligned} \tag{19}$$

for all $x, y \in \mathbb{R}$, which is the solution (7).

◊

Also solved by Kelly Houston (USA) and Djordje Baralić (student, Serbia and Montenegro), who observed that if f is continuous, then the only nonconstant solutions of (1) are

$$f(x,y) = |x^2 - y^2|^c - 1$$

and

$$f(x,y) = \begin{cases} |x^2-y^2|^c - 1, & \text{if } x^2 - y^2 \geq 0 \\ -|x^2-y^2|^c - 1, & \text{if } x^2 - y^2 < 0 \end{cases}$$

for $c \in \mathbf{R} - \{0\}$.

3. Suppose $f : \mathbf{R^n} \to \mathbf{R^n} (n \geq 2)$ satisfies $\|f(x) - f(y)\|_2 = 1$ whenever $\|x - y\|_2 = 1$, and $\|f(x) - f(y)\|_2 = 2$ whenever $\|x - y\|_2 = 2$, where $x, y \in \mathbf{R^n}$. Is it true that $\|f(x) - f(y)\|_2 = \|x - y\|_2$ for all $x, y \in \mathbf{R^n}$?
(For $a = (a_1, a_2, \ldots, a_n) \in \mathbf{R^n}$, $\|a\|_2$ denotes the Euclidean norm, namely $\|a\|_2^2 = \sum_{i=1}^n a_i^2$).

(Shuhuang Xiang, Central South University, P. R. China)

Solution by the proposer. We claim that $\|f(x) - f(y)\|_2 = \|x - y\|_2$ for all $x, y \in \mathbf{R^n}$, $n \geq 2$. Let $x, y \in \mathbf{R^n}$ with $\|x - y\|_2 = 3$. Set $x_1 = x + \frac{1}{3}(y - x)$ and $x_2 = x + \frac{2}{3}(y - x)$. Then

$$\|x_1 - x\|_2 = \|x_1 - x_2\|_2 = \|y - x_2\|_2 = 1,$$
$$\|y - x_1\|_2 = \|x_2 - x\|_2 = 2.$$

It follows that

$$\|f(x_1) - f(x)\|_2 = \|f(x_1) - f(x_2)\|_2 = \|f(y) - f(x_2)\|_2 = 1,$$

and

$$\|f(x_2) - f(x)\|_2 = \|f(x_1) - f(x)\|_2 + \|f(x_2) - f(x_1)\|_2 = 2,$$
$$\|f(y) - f(x_1)\|_2 = \|f(x_2) - f(x_1)\|_2 + \|f(y) - f(x_2)\|_2 = 2.$$

Hence we get

$$f(x_1) = f(x) + \frac{1}{2}\big(f(x_2) - f(x)\big),$$
$$f(x_2) = f(x_1) + \frac{1}{2}\big(f(y) - f(x_1)\big).$$

Hence

$$f(x) = 2f(x_1) - f(x_2), \quad f(y) = 2f(x_2) - f(x_1),$$
$$\|f(y) - f(x)\|_2 = 3.$$

Similarly, let $x, y \in \mathbf{R^n}$ with $\|x - y\|_2 = 4$. Set $x_1 = x + \frac{1}{2}(y - x)$, $x_2 = x + \frac{3}{4}(y - x)$. Then

$$\|x_1 - x\|_2 = \|y - x_1\|_2 = 2, \|x_2 - x\|_2 = 3, \|y - x_2\|_2 = \|x_1 - x_2\|_2 = 1.$$

Then

$$\|f(x_2) - f(x_1)\|_2 = \|f(x_1) - f(x)\|_2 + \|f(x_2) - f(x_1)\|_2 = 3$$

and

$$\|f(y) - f(x_1)\|_2 = \|f(x_2) - f(x_1)\|_2 + \|f(y) - f(x_2)\|_2 = 2.$$

Thus

$$f(x_1) = f(x) + \frac{2}{3}\big(f(x_2) - f(x)\big),$$
$$f(x_2) = f(x_1) + \frac{1}{2}\big(f(y) - f(x_1)\big)$$

and

$$\|f(y) - f(x)\|_2 = 4.$$

By induction, it follows that for all $x, y \in \mathbf{R^n}$,

$$\|f(x) - f(y)\|_2 = k \text{ whenever } \|x - y\|_2 = k,$$

where k is an arbitrary positive integer.

Let $x, y \in \mathbf{R^n}$ with $\|x - y\|_2 = \frac{1}{k}$, where k is a given positive integer. Select $z, z_1, z_2 \in \mathbf{R^n}$ such that

$$\|z - x\|_2 = \|z - y\|_2 = \|z_1 - z_2\|_2 = 1,$$
$$\|z - z_1\|_2 = \|z - z_2\|_2 = k$$

and

$$x = z + \frac{1}{k}(z_1 - z), \quad y = z + \frac{1}{k}(z_2 - z).$$

We obtain that

$$f(x) = f(z) + \frac{1}{k}\big(f(z_1) - f(z)\big), \quad f(y) = f(z) + \frac{1}{k}\big(f(z_2) - f(z)\big),$$
$$\|f(x) - f(y)\|_2 = \frac{1}{k}.$$

Let $x_0, y \in \mathbf{R^n}$ with $\|x_0 - y\|_2 = \frac{m}{k}$, where m is a given positive integer less than or equal to k. Set

$$z = x_0 + \frac{y - x_0}{\|y - x_0\|_2}, \; x_i = x + \frac{i}{k}(z - x_0) \text{ for } i = 1, 2, \ldots, k,$$

then

$$y = x_m, \ \ z = x_k, \ \ \|f(x_i) - f(x_{i-1})\|_2 = \frac{1}{k}$$

and

$$\|f(z) - f(x_0)\|_2 = \sum_{i=1}^{k} \|f(x_i) - f(x_{i-1})\|_2.$$

Therefore

$$f(x_i) = f(x_0) + \frac{i}{k}\left(f(z) - f(x_0)\right), \ \ i = 1, 2, \ldots, k$$

and

$$\|f(y) - f(x_0)\|_2 = \frac{m}{k}.$$

By the arbitrariness of k, we obtain that for all $x, y \in \mathbf{R^n}$ with $\|x-y\|_2 = r$, where r is an arbitrary positive rational number less than or equal to 1, $\|f(x) - f(y)\|_2 = r$. Let $x, y \in \mathbf{R^n}$ with $\|x-y\|_2 = k + r$. Set

$$z = x + \frac{k+1}{\|y-x\|_2}(y-x), \ x_i = x + \frac{i}{k+1}(z-x) \ \text{ for } i = 1, 2, \ldots, k,$$

then $y = x_k + r(z - x_k)$ and

$$\|f(x) - f(z)\|_2 = \|f(x_1) - f(x)\|_2 + \cdots + \|f(z) - f(x_k)\|_2,$$
$$\|f(z) - f(x_k)\|_2 = \|f(y) - f(x_k)\|_2 + \|f(z) - f(y)\|_2.$$

Hence

$$f(y) = f(x) + \frac{k+r}{\|z-x\|_2}\|f(z) - f(x)\|_2$$

and

$$\|f(y) - f(x)\|_2 = k + r.$$

That is to say, for an arbitrary positive rational number r and for all $x, y \in \mathbf{R^n}$ with

$$\|x-y\|_2 = r, \quad \|f(x) - f(y)\|_2 = r.$$

In the following, we will show that

$$\|f(x) - f(y)\|_2 \le \|x-y\|_2, \ \text{ for all } \ x, y \in \mathbf{R^n}. \tag{20}$$

If $\|x-y\|_2$ is a rational number, (20) is satisfied. If $\|x-y\|_2$ is an irrational number, we can choose $\ell := \|x-y\|_2 + \varepsilon$ to be a rational number for an arbitrarily small positive number ε and select $z \in \mathbf{R^n}$ such that $\|z-x\|_2 = \|z-y\|_2 = \ell/2$. Note that $\ell/2$ is a rational number. Then x, y, z form a triangle, and

$$\|f(x) - f(y)\|_2 \le \|f(x) - f(z)\|_2 + \|f(y) - f(z)\|_2$$
$$= \ell \le \|x-y\|_2 + \varepsilon.$$

Thus $\|f(x) - f(y)\|_2 \le \|x-y\|_2$. Next we will show that

$$\|f(x) - f(y)\|_2 = \|x-y\|_2 \text{ for all } x, y \in \mathbf{R^n}. \tag{21}$$

It is only necessary to show that $\|f(x) - f(y)\|_2 = \|x-y\|_2$ for all $x, y \in \mathbf{R^n}$ when $\|x-y\|_2$ is an irrational number. Suppose for some irrational number

$$\|x-y\|_2 = \ell_0, \ \ \|f(x) - f(y)\|_2 < \|x-y\|_2.$$

Choose $z = x + \frac{[\ell_0]+1}{\ell_0}(y-x)$, where $[\ell_0]$ denotes the maximum integer less than or equal to ℓ_0. Then

$$[\ell_0] + 1 = \|f(z) - f(x)\|_2 \le \|f(x) - f(y)\|_2 + \|f(y) - f(z)\|_2$$
$$< \|x-y\|_2 + \|y-z\|_2$$
$$= [\ell_0] + 1,$$

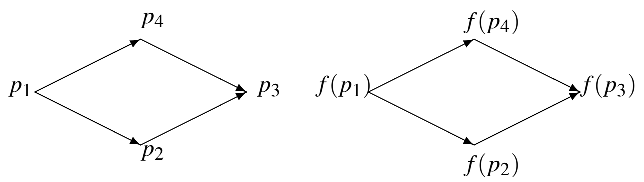
which is a contradiction. ♢

4. Suppose $f : \mathbf{R^n} \to \mathbf{R^n} (n \ge 2)$ satisfies $\|f(x) - f(y)\|_2 = 1$ whenever $\|x-y\|_2 = 1$, and $\|f(x) - f(y)\|_2 = \sqrt{3}$ whenever $\|x-y\|_2 = \sqrt{3}$, where $x, y \in \mathbf{R^n}$. Is it true that $\|f(x) - f(y)\|_2 = \|x-y\|_2$ for all $x, y \in \mathbf{R^n}$?
(For $a = (a_1, a_2, \ldots, a_n) \in \mathbf{R^n}$, $\|a\|_2$ denotes the Euclidean norm, namely $\|a\|_2^2 = \sum_{i=1}^n a_i^2$).

(Shuhuang Xiang, Central South University, P.R. China)

Solution by the proposer. We claim that $\|f(x) - f(y)\|_2 = \|x-y\|_2$ for all $x, y \in \mathbf{R^n}$, $n \ge 2$.

(i) Suppose that p_1, p_2, p_3, p_4 in $\mathbf{R^n}$ are vertices of a rhombus of unit sidelength with $\|p_1 - p_3\| = \sqrt{3}$ and $\|p_2 - p_4\| = 1$.



Then by the parallelogram law, the points $f(p_1), f(p_2), f(p_3), f(p_4)$ are also vertices of a rhombus of unit sidelength with $\|f(p_1) - f(p_3)\| = \sqrt{3}$ and $\|f(p_2) - f(p_4)\| = 1$.

Set $x = f(p_2) - f(p_1)$, $y = f(p_4) - f(p_1)$ and $z = f(p_3) - f(p_1)$. Then

$$\|x\| = \|y\| = \|x-y\| = \|z-x\| = \|z-y\| = 1 \quad \text{and} \ \ \|z\| = \sqrt{3}.$$

Since

$$\|x-y\|_2^2 + \|x+y\|_2^2 = 2(\|x\|_2^2 + \|y\|_2^2),$$

then $f(p_1), f(p_2), f(p_3), f(p_4)$ are vertices of a rhombus of unit sidelength with

$$\|f(p_3) - f(p_1)\| = \sqrt{3} \quad \text{and} \ \ \|f(p_4) - f(p_2)\| = 1.$$

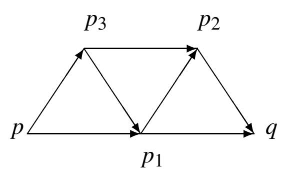
In fact, $f(p_3)$ is in the span of

$$f(p_1), f(p_2), f(p_4), f(p_3) - f(p_1)$$
$$= \big(f(p_2) - f(p_1)\big) + \big(f(p_4) - f(p_1)\big).$$

(ii) Let $p, q \in \mathbf{R^n}$ with $\|p-q\| = 2$ and $p_1 = \frac{p+q}{2}$. We can select p_2, p_3 in $\mathbf{R^n}$ such that $pp_1p_2p_3$ and $p_1p_3p_2q$ form two rhombi of unit sidelength with

$$\|p_2 - p\| = \|p_3 - q\| = \sqrt{3}$$

as follows:



By step (i), $f(p)f(p_1)f(p_2)f(p_3)$ and $f(p_1)f(p_3)f(p_2)f(q)$ are also two rhombi of unit sidelength with $\|f(p_2) - f(p)\| = \|f(p_3) - f(q)\| = \sqrt{3}$.

Set $x = f(p_1) - f(p)$ and $y = f(p_3) - f(p)$. Then $x - y = f(p_1) - f(p_3)$, $f(p_2) - f(p) = x + y$ and $f(q) - f(p_3) = \big(f(p_1) - f(p_3)\big) + \big(f(p_2) - f(p_3)\big)$. Hence $f(q) - f(p) = 2x = 2\big(f(p_1) - f(p)\big)$ and $\|f(q) - f(p)\| = 2$. Thus f preserves distance 2. By Problem 3, $\|f(x) - f(y)\|_2 = \|x-y\|_2$ for all $x, y \in \mathbf{R^n}$. ♢

Note for problems 3 and 4. Erich N. Gulliver (Germany) remarked that, if one omits the second assumption from their statements, there remains the well-known Beckmann–Quarles theorem (1953) of which there are various proofs (see for example the list in *Proc. AMS*, 123 (1995), p. 2859).

8*. Examine if there exists a mapping $T : \mathbf{R^2} \to \mathbf{R^3}$ that preserves unit distance but is not an isometry.

(Th. M. Rassias, Greece)

9*. Let X and Y be normed linear spaces and $T : X \to Y$ be a continuous and/or surjective mapping which preserves unit distance. Is T necessarily an isometry?

(Th. M. Rassias, Greece)

Remark. For the two open problems 8* and 9* the reader is referred to the following papers for a number of related results.

K. Ciesielski and Th.M. Rassias, *On some properties of isometric mappings*, Facta Universitatis. Series: Mathematics and Informatics 7 (1992), 107–115.

Th.M. Rassias, *On the A.D. Aleksandrov problem of conservative distances and the Mazur-Ulam theorem*, Nonlinear Analysis – Theory, Methods & Applications 47 (2001), 2597–2608.

We wait to receive your solutions to problems 10–15 of this issue and ideas on the open problems 8*, 9*, 16*, 17*. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit new problems with their solutions for the next "Solved and Unsolved Problems" column, which will be devoted to *Number Theory*.

Notes

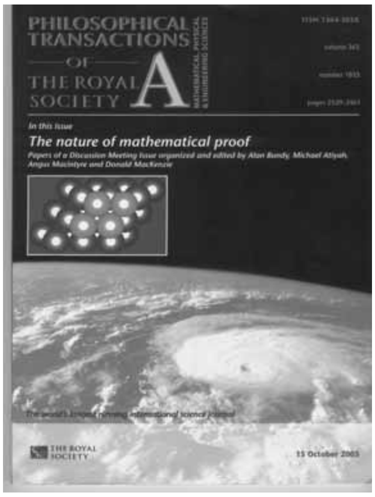
1. eds. V. I. Arnold, M. F. Atiyah, P. D. Lax and B. Mazur, International Mathematical Union and American Mathematical Society, 2000
2. World Scientific Publ. Co., Singapore, New Jersey, London, 1994

Book Review

Antonio Córdoba (UA Madrid, Spain)

The nature of mathematical proof

Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.
363 (2005), no. 1835
Pages 2331–2461
ISSN 1364-503X (Paper) 1471-2962 (Online)



Published in this issue of *Philosophical Transactions* are the results of a two-day debate held by a group of mathematicians, computer scientists and philosophers, organized in October 2004 by the Royal Society. It contains the talks given at that meeting together with discussions, questions and comments of the participants. Here is a sample of its contents:

- Computing and the culture of proving, by D. MacKenzie
- The challenge of computer mathematics, by H. Barendregt and F. Wiedijk
- What is a proof?, by A. Bundy, M. Jamnik and A. Fugard
- Highly complex proofs and implications of such proofs, by M. Aschbacher
- Skolem and pessimism about proof in mathematics, by P.J. Cohen
- The mathematical significance of proof theory, by A. Macintyre
- The justification of mathematical statements, by P. Swinnerton-Dyer
- Pluralism in mathematics, by E.B. Davis.

Mathematical proofs rank among the highest peaks of human thought but their complexity and variety have increased continuously along the years. Until the middle of the last century it was commonplace to say that a rigorous mathematical proof consisted of a chain of logical steps whose correctness could be checked, in due time, by a person in possession of the appropriate training. Euclid´s Elements contains numerous examples and time, as was correctly stated by Hardy, has not been able to add a single wrinkle to the freshness of their beauty and precision. But the nature of proof has not been static and mathematicians have created new and more powerful strategies, involving new concepts and tools, which provide us with greater freedom and power of reasoning.

Some of these proofs involve long chains of thoughts in an indirect and complicated manner. An example is given by Carleson's theorem about the almost everywhere convergence of the Fourier series of a square integrable function. Another is the proof obtained by A. Wiles of Fermat's last theorem. More recently we have the solution of Poincaré's conjecture by G. Perelmann. They are good examples of rigorous proofs that need very many ingenious steps, and which are also deep, elegant and beautiful. But they are so complex that it is doubtful that there exists a single mathematician who would be able to verify the three of them in a reasonable amount of time.

A special treatment is deserved by the classification of finite simple groups, the proof of which is scattered in more than 10,000 pages, divided into hundred of papers written by a hundred different mathematicians. Since the probability of finding a mistake on a very long mathematical text is not negligible, we may legitimately ask about the necessity of such long and complex proofs and their reliability, particularly when taking into account the next turn of the screw: the famous birth of the so-called computer-assisted, or computed-based, proofs of the "four colour problem" and the solution of "Kepler's conjecture".

- Will computers in the future be able to formulate interesting conjectures and prove theorems?
- Are mathematicians an endangered species?
- Will the mathematics of tomorrow be full of proofs that depend upon calculations that can only me performed by very powerful computers?
- Has the existence of the modern computer changed mathematics into an experimental science?
- Will computers help mathematicians to successfully treat the more complex models of nature?

These are just a sample of the interesting questions that were addressed by the participants in the debate, giving ideas and opinions that are collected in this special issue of *Philosophical Transactions*, whose very clarifying and stimulating reading we strongly recommend.

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[antonio.cordoba@uam.es] is a professor in the Department of Mathematics of the Universidad Autónoma de Madrid (Spain). His areas of research are harmonic analysis, number theory, differential equations and mathematical physics. He obtained his PhD at the University of Chicago in 1974 under the supervision of Professor Charles Fefferman. He is one of the founders of Revista Matemática Iberoamericana.

Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses vberinde@ubm.ro or vasile_berinde@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

March 2007

6–11: SEEMOUS (South-Eastern European Mathematical Olympiad for University Students), Cyprus
Information: makrides.greg@usa.net, cms@cms.org.cy

21–25: MAT-TRIAD 07, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–30: Structured Perturbations and Distance Problems in Matrix Computations, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

26–30: Workshop: Homotopy theory of schemes, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

27–April 4: MATHEU (Identification, Motivation and Support of Mathematical Talents in European Schools) Training Course (Comenius 2.2), Bulgaria
Information: makrides.greg@usa.net; www.matheu.org; http://ec.europa.eu/education/trainingdatabas/

29–31: Commutative Algebra and Related Topics. Honoring the 60th birthday of Professor Dorin Popescu, Constanta, Romania
Information: http://www.univ-ovidius.ro/math/conference/60/index.htm

April 2007

4–18: TAMTAM'07, Tendances dans les Applications Mathématiques en Tunisie, Algerie, Maroc; Algiers, Algeria
Information: http://tamtam07alger.ifrance.com/

10–14: Workshop on Control Theory & Finance, Lisbon, Portugal
Information: wmctf@iseg.utl.pt; http://srv-ceoc.mat.ua.pt/conf/wmctf2007/

11–12: Colloquium on the occasion of the 90th birthday of Prof. Beno Eckmann, ETH Zürich, Switzerland
Information: http://www.fim.math.ethz.ch/eckmann90.pdf

18–20: SMAI Conference of Optimization and Decision Making CODE 2007, Paris, France
Information: code2007@math.jussieu.fr; http://www.ann.jussieu.fr/~plc/code2007.html

23–27: Dynamics in Perturbations. On the occasion of the 60th birthday of Freddy Dumortier, Hasselt University (campus Diepenbeek), Belgium
Information: patrick.bonckaert@uhasselt.be; http://www.uhasselt.be/dysy/dynper/

25–29: III Workshop on Coverings, Selections and Games in Topology, Serbia
Information: lkocinac@ptt.yu; http://www.pmf.ni.ac.yu/spm2007

30–May 6: Advances in Mathematics of Finance, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

May 2007

6–12: Semstat 2007, Statistics for Stochastic Differential Equations Models, La Manga del Mar Menor, Cartagena, Spain
Information: mathieu.kessler@upct.es; http://www.dmae.upct.es/semstat2007

14–18: Workshop: Stacks in Geometry and Topology, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

14–18: Conference on Cryptography and Digital Content Security, Centre de Recerca Matematica, Barcelona, Spain
Information: ContentSecurity@crm.es; http://www.crm.cat/ContentSecurity

18–20: Workshop on *p*-adic Methods and Rational Points, Alfréd Rényi Institute, Budapest, Hungary
Information: http://www.renyi.hu/conferences/padic/

20–26: Convex and Fractal Geometry, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

21–25: International Workshop on Applied Evolution Equations, Budapest, Hungary
Information: KovecsiA@ceu.hu; http://www.ceu.hu/math/Workshop_07/Workshop_07.html

27–June 1: Stochastic Networks and Related Topics, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

27–June 2: Spring School on Analysis: Function Spaces, Inequalities and Interpolation, Paseky nad Jizerou, Czech Republic
Information: pasejune@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/katedry/kma/ss/jun07/

28–June 1: Complex Analysis and Geometry – XVIII, Levico Terme (Trento), Italy
Information: michelet@science.unitn.it; http://www.science.unitn.it/cirm/

28–June 2: Advanced Course on Group-Based Cryptography, Centre de Recerca Matematica, Barcelona, Spain
Information: ACGroupBased@crm.es; http://www.crm.cat/ACGroupBased

28–June 2: Workshop on Finsler Geometry and its Applications, Balatonfuldvar, Hungary
Information: kozma@math.klte.hu; http://www.math.klte.hu/finsler2007

29–June 1: XIIth Applied Stochastic Models and Data Analysis International Conference (ASMDA2007), Chania, Crete, Greece
Information: skiadas@ermes.tuc.gr;
http://www.asmda.com/id7.html

30–June 6: MATHEU (Identification, Motivation and Support of Mathematical Talents in European Schools) Training Course (Comenius 2.2), Cyprus
Information: makrides.greg@usa.net; www.matheu.org;
http://ec.europa.eu/education/trainingdatabas/

June 2007

1–30: Geometric Applications of Homotopy Theory, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/ index.html

3–10: Geometric Analysis and Nonlinear Partial Differential Equations, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

4–9: Algebraic Geometry in Higher Dimensions, Levico Terme (Trento), Italy
Information: aghdtn@gmail.com;
http://www.science.unitn.it/~occhetta/aghd/index.html

8–13: The Ninth International Conference on Geometry, Integrability and Quantization, Sts. Constantine and Elena resort, Varna, Bulgaria
Information: mladenov@obzor.bio21.bas.bg;
http://www.bio21.bas.bg/conference/

9–13: Workshop Higher Categories and their Applications, Fields Institute, Toronto, Canada
Information: jardine@uwo.ca; http://www.fields.utoronto.ca/programs/scientific/06-07/homotopy/index.html

11–14: Ergodic Theory and Limit Theorems. 9th Rencontres Mathématiques de Rouen, Rouen, France
Information: RMR2007@univ-rouen.fr;
http://www.univ-rouen.fr/LMRS/RMR07/rmr07_eng.html

11–15: Barcelona Conference on C*-Algebras and Their Invariants, Centre de Recerca Matematica, Barcelona, Spain
Information: OAlgebras @crm.es;
http://www.crm.cat/OAlgebras

11–15 Geometry Fest in celebration of Ted Bisztriczky's 60th birthday, Alfréd Rényi Institute, Budapest, Hungary
Information: http://www.renyi.hu/conferences/geofest/

12–16: Complex Function Theory and Geometry, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

16–18: Fourth International Conference on Computability and Complexity in Analysis, Siena, Italy
Information: http://cca-net.de/cca2007/

16–22: Fifth International Workshop on Optimal Codes and Related Topics (OC 2007). Dedicated to the 60th anniversary of the Institute of Mathematics and Informatics, Hotel White Lagoon, Balchik, Bulgaria
Information: oc2007@moi.math.bas.bg;
http://www.moi.math.bas.bg/oc2007/oc2007.html

17–23: Skorokhod Space Conference. 50 Years On Skorokhod Space, Kyiv, Ukraine
Information: skor_space@imath.kiev.ua;
http://www.imath.kiev.ua/~skor_space/

18-23: CiE 2007 (Computability in Europe 2007 - Computation and Logic in the Real World), University of Siena, Italy
Information: http://www.amsta.leeds.ac.uk/~pmt6sbc/cie07.html

18–24: Algebraic Topology: Old and New (M. M. Postnikov Memorial Conference), Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

19–20: Fifth Computer Algebra in Mathematics Education Symposium (CAME-5), Pecs, Hungary
Information: http://matserv.pmmf.hu/cadgme/

21–23: First Central and Eastern European Conference on Computer Algebra and Dynamic Geometry Systems in Mathematics Education, Pecs, Hungary
Information: http://matserv.pmmf.hu/cadgme/

24–29: Fifth School on Analysis and Geometry in Metric Spaces, Levico Terme (Trento), Italy
Information: michelet@science.unitn.it;
http://www.science.unitn.it/cirm/MeSpa07.html

24–30: Nonlocal and Abstract Parabolic Equations and their Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

24–30: Lyapunov Memorial Conference. International Conference on the occasion of the 150th birthday of Aleksandr Lyapunov, Kharkiv, Ukraine
Information: lmc07@ilt.kharkov.ua;
http://www.ilt.kharkov.ua/lmc07/

24–30: Seventh International Conference "Symmetry in Nonlinear Mathematical Physics", Kiev, Ukraine
Information: appmath@imath.kiev.ua;
http://www.imath.kiev.ua/~appmath/conf.html

25–26: Mathematical Modelling in Sport, Manchester, UK
Information: http://www.ima.org.uk/Conferences/conferences.htm

25–29: Conference on Enumeration and Probabilistic Methods in Combinatorics, Centre de Recerca Matematica, Barcelona, Spain
Information: Enumeration @crm.es; http://www.crm.cat/Enumeration

25–30: Topics in Geometric Group Theory, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

25–30: ERLOGOL-2007. Intermediate problems of Model theory and Universal algebra, Novosibirsk-Altai-Novosibirsk, Russia
Information: algebra@nstu.ru

26–29: Biennial Conferences on Numerical Analysis, Dundee, Scotland, UK

26–29: ITES2007 – Sixth Italian-Spanish Conference on General Topology and Applications, Bressanone (Bolzano), Italy
Information: topology@math.unipd.it;
http://www.math.unipd.it/topology/

26–30: 3rd International Conference Computational Methods in Applied Mathematics (CMAM-3), Minsk, Belarus
Information: http://www.cmam.info/conferences

27–29: Fifth Italian Latinoamerican Conference on Industrial and Applied Mathematics, Trieste, Italy

28–July 4: 6th Congress of Romanian Mathematicians, Bucharest, Romania
Information: congmatro@imar.ro;
http://www.imar.ro/~purice/announcements.html

July 2007

1: Summer Conference on Topology and its Applications 2007, Castellon, Spain
Information: http://www.sumtop07.uji.es

1–7: Groups and Their Actions, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

2–4: The 2007 International Conference of Applied and Engineering Mathematics, Imperial College London, U.K.
Information: williamyoung@iaeng.org;
http://www.iaeng.org/worldeng2007/ICAEM2007.html

2–4: Algebraic Biology 2007, RISC, Castle of Hagenberg, Austria
Information: http://www.risc.uni-linz.ac.at/about/conferences/ab2007/

2–6: 2nd European Conference for Aerospace Sciences, Brussels, Belgium
Information: http://www.vki.ac.be/eucass2007/

4–6: Third Spain, Italy, Netherlands Meeting on Game Theory (SING3), Madrid, Spain
Information: http://www.mat.ucm.es/congresos/sing3

4–8: International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA-2007), Cluj-Napoca, Romania
Information: nodeacj@math.ubbcluj.ro;
http://www.math.ubbcluj.ro/~mserban/confan.html

8–11: EURO XXII, 22nd European Conference on Operations Research, Prague, Czech Republic
Information: http://euro2007.vse.cz/

9–11: MCP 2007. 5th international conference on multiple comparison procedures, Vienna, Austria
Information: http://www.mcp-conference.org

9–11: Eccomas Thematic Conference on Meshless Methods, University of Porto, Portugal
Information: http://paginas.fe.up.pt/~meshless/

9–12: International Conference on Preconditioning Technique, Toulouse, France
Information: http://www.precond07.enseeiht.fr/

9–13: The First European Set Theory Meeting, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

9–13: Dynamics Days Europe, Loughborough University, UK
Information: http://www.lboro.ac.uk/dynamicsdays07

9–13: SciCADE'07 International Conference on SCIentific Computation And Differential Equations, Saint-Malo, France
Information: http://scicade07.irisa.fr/

9–13: 9th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, Lindau, Germany
Information: http://www.fully3d.org/2007/

11–13: International Conference on Approximation Methods and Numerical Modelling in Environment and Natural Resources, Granada, Spain
Information: http://www.ugr.es/local/mamern07

15–August 10: 6th Annual Summer School of the Atlantic Association for Research in the Mathematical Sciences (AARMS), Dalhousie University, Halifax, Nova Scotia, Canada
Information: keast@mathstat.dal.ca;
http://www.aarms.math.ca/summer/2007

16–20: 6th International Congress on Industrial and Applied Mathematics (ICIAM 07), Zürich, Switzerland
Information: http://www.iciam07.ch

22–25: OPTIMIZATION 2007, University of Porto, Porto, Portugal.
Information: opti2007@fep.up.pt; http://www.fep.up.pt/opti2007/

22–28: Topological Theory of Fixed and Periodic Points (TTFPP 2007), Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

23–December 21: Strong Fields, Integrability and Strings, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/SIS/index.html

23–27: 23rd IFIP TC 7 Conference on System Modelling and Optimization, Cracow, Poland
Information: http://ifip2007.agh.edu.pl/

23–27: Waves 2007. The 8th International Conference on Mathematical and Numerical Aspects of Waves, Reading, UK
Information: http://www.waves2007.org/

23–29: CIEAEM 59. For the memory of Tamás Varga, Dobogóko, Hungary
Information: http://www.tofk.elte.hu/cieaem/

24–27: 22nd Summer Conference on Topology and Its Applications, Castellon, Spain
Information: sumtop07@uji.es; http://www.sumtop07.uji.es

31–August 3: First Joint International Meeting between the AMS and the PTM, Warsaw, Poland
Information: http://www.impan.gov.pl/ptm/ams/

August 2007

14–19: Workshops Loops '07, Prague, Czech Republic
Information: loops07@karlin.mff.cuni.cz;
http://www.karlin.mff.cuni.cz/~loops07/workshops.html

19–25: Loops '07, Prague, Czech Republic
Information: loops07@karlin.mff.cuni.cz;
http://www.karlin.mff.cuni.cz/~loops07

19-26: XXIInd International Workshop on Differential Geometric, Methods in Theoretical Mechanics, Banach Center, Będlewo, Poland. *Information*: http://www.impan.gov.pl/BC/

20–24: Geometric Aspects of Analysis and Mechanics. A Conference in Honor of the 65th Birthday of Hans Duistermaat, Utrecht, The Netherlands
Information: kolk@math.uu.nl; http://www.math.uu.nl/people/kolk/Duistermaat65/duistermaat65.html

27–September 1: Workshop on Combinatorics and Commutative Algebra, The Aristotle University of Thessaloniki, Greece
Information: http://users.auth.gr/~hara/combcom07/webpage.htm

30–September 2: International Conference on Theory and Applications in Mathematics and Informatics (ICTAMI), Alba Iulia, Romania
Information: dbreaz@uab.ro; web site: www.uab.ro/ictami

September 2007

2–8: Linear and Non-Linear Theory of Generalized Functions and its Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

3–6: European Women in Mathematics 07, Fitzwilliam College, Oxford, UK
Information: www.maths.cam.ac.uk/ewm

3–December 21: **Phylogenetics**, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/PLG/index.html

5–10: Conference on Homological and Combinatorial Aspects in Commutative Algebra, Busteni, Romania
Information: http://www.univ-ovidius.ro/math/workshop/2007/index.htm

9–12: Grid Applications and Middleware Workshop 2007 (GAMW'2007), Gdansk, Poland; in conjunction with **PPAM 2007**

9–12: 7th International Conference on Parallel Processing and Applied Mathematics (PPAM 2007), Gdansk, Poland
Information: gamw@man.poznan.pl; http://ppam.pcz.pl

9–15: Measure Theory-Edward Marczewski Centennial Conference, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

10–14: Fif[th] Symposium on Nonlinear Analysis, Torun, Poland
Information: sna2007@mat.uni.torun.pl;
http://www-users.mat.uni.torun.pl/~sna2007/index.html

24–29: 18[th] Congress of Unione Matematica Italiana, Bari, Italia
Information: segreteria@congressoumi2007.it;
http://www.congressoumi2007.it/

December 2007

27–30: The Second International Conference on Mathematics: Trends and Developments (ICMTD07), Cairo, Egypt
Information: conf07@etms-web.org;
http://www.etms-web.org/conf07/

July 2008

14–18: Fif[th] European Congress of Mathematics (5ECM), Amsterdam, The Netherlands
Information: www.5ecm.nl

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Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to: Ivan Netuka, MÚUK, Sokolovská, 83, 186 75 Praha 8, Czech Republic.

L. V. Ahlfors: Lectures on Quasiconformal Mappings, second edition*, University Lecture Series, vol. 38, American Mathematical Society, Providence, 2006, 162 pp., USD 35,00, ISBN 0-8218-3644-7*

These lectures develop the theory of planar quasiconformal mapping. Each linear mapping L of the plane onto itself maps the unit circle to an ellipse. The ratio of its axes (major/minor) is called the dilatation of L. Roughly, a quasiconformal mapping is a homeomorphism such that the dilatation of its derivative (computed pointwise) is bounded. A more technical definition is needed for non-differentiable maps: either the differentiability is relaxed to absolute continuity on almost all lines (analytic definition) or the differential properties are replaced by global conditions of geometric nature. The comparison of these two approaches is one of the first achievements of the lectures. Further results and topics include estimates of moduli of special condensers with an excursion to elliptic and modular functions, the sharp Hölder estimate (Mori's theorem), analysis of mappings of quadruplets of points, boundary behaviour, quasiconformal reflection, the solution of the Beltrami equation and the Calderón-Zygmund inequality. The lectures conclude with a treatment of Teichmüller spaces (including the Bers embedding and the Teichmüller curve).

The book is based on Ahlfors' course that was given at Harvard University in 1964. The first edition appeared in 1966. The new second edition contains three supplementary chapters demonstrating the efficiency of methods of quasiconformal mappings in various branches of analysis and geometry. A supplementary chapter is written by Earle and Kra. They begin with a brief survey of the theory of quasiconformal mappings with emphasis on issues mentioned in the lectures and their new developments. Most of the chapter is, however, devoted to the theory of Teichmüller spaces and their connections to Kleinian groups. A chapter by Shishikura presents applications of quasiconformal theory in complex dynamics. The third appendix, by Hubbard, shows how tools like quasiconformal mappings, the measurable Riemann mapping theorem and quasi-Fuchsian groups have been combined with some new methods to obtain Thurston's very deep theory on hyperbolization of irreducible 3-manifolds. The lectures and supplements constitute a very efficient way of learning some complicated theories with numerous applications. (jama)

F. Albiac, N. J. Kalton: Topics in Banach Space Theory*, Graduate Texts in Mathematics, vol. 233, Springer, Berlin, 2006, 373 pp., EUR 56,95, ISBN 0-387-28141-X*

This book provides a sequel treatise on classical and modern Banach space theory. It is mainly focused on the study of classical Lebesgue spaces L_p, sequence spaces l_p, and Banach spaces of continuous functions. The early chapters use bases and basic sequences as a tool for understanding the isomorphic structure of Banach spaces. The next few chapters deal with $C(K)$-spaces (including Miljutin's theorem) and $L_1(\mu)$-spaces. A chapter discussing the basic theory of L_p-spaces includes notions of type and cotype and the next one presents the Maurey-Nikishin factorization theory. This leads to the Grothendieck theory of absolutely summing operators. Other topics treated include perfectly homogeneous bases, the Ramsey theory, Rosenthal's l_1 theorem, Tsirelson space, finite representability of l_p spaces and an introduction to the local theory of Banach spaces (the John ellipsoid, Dvoretzky's theorem and the complemented subspace problem). The final chapter covers important examples of Banach spaces (and also a generalization of James space and constructions of spaces via trees).

Each chapter ends with many exercises and problems of varying difficulty giving further applications and extensions of the theory. There is a comprehensive bibliography (225 items). The book is understandable and requires only a basic knowledge of functional analysis (all the prerequisites assumed in the book are collected without proofs in the appendices). It can be warmly recommended to a broad spectrum of readers – to graduate students, young researchers and also to specialists in the field. (jl)

M. A. Al-Gwaiz, S. A. Elsanousi: Elements of Real Analysis*, Pure and Applied Mathematics, vol. 284, Chapman & Hall/CRC, Boca Raton, 2006, 436 pp., USD 89,95, ISBN 1-58488-661-7*

This book is an introduction to mathematical analysis. It offers a self-contained treatment of some elementary and some advanced topics. After preliminaries concerning sets and functions, the authors describe real numbers (in an axiomatic way), sequences (Cauchy criterion, upper and lower limits, open and closed sets), infinite series, limits of a function, continuity (including compactness), differentiation (including Taylor's theorem), the Riemann integral (the fundamental theorem of calculus and improper integrals), sequences and series of functions (uniform convergence and power series), the Lebesgue measure and Lebesgue integration (including convergence theorems). The course is clearly written; it starts with elementary topics and it finishes with the complicated theory of integration. The book contains a lot of solved examples and exercises. It can be used as an introductory course at the senior undergraduate level. (pp)

W. Ballmann: Lectures on Kähler Manifolds*, ESI Lectures in Mathematics and Physics, European Mathematical Society, Zürich, 2006, 172 pp., EUR 38, ISBN 3-03719-025-6*

This book offers a systematic description of the basic properties of Kähler manifolds together with a few more advanced topics chosen from differential geometry and global analysis. The first three chapters review basic facts about the Laplace and the Hodge operators on differential forms, vector fields and forms on complex manifolds, and the Dolbeault cohomology and connections on holomorphic vector bundles. Kähler manifolds and their cohomology are studied in the next two chapters (including the corresponding Levi-Civita connection, its curvature, the Ricci tensor, Killing fields, the Lefschetz theorem, the Hodge-Riemann bilinear relations, the Hodge index theorem and the Kodaira vanishing theorem). The next four chapters contain some advanced topics: a few relations between the behaviour of the Ricci tensor and global properties of the manifold, the

Calabi conjecture (together with substantial parts of the proof), the Gromov results on Kähler hyperbolic spaces and the Kodaira embedding theorem.

The book is written in a systematic, precise and understandable style. The background needed includes parts of differential geometry (vector bundles and connections, curvature and holonomy) and global analysis (Sobolev embedding theorems, regularity and the Hodge theory for elliptic partial differential equations). The Chern-Weil theory of characteristic classes and basic facts on symmetric spaces are comfortably summarized in appendices. The book will be very useful both for mathematicians and theoretical physicists who need Kähler manifolds in their research. (vs)

M. Benedicks, P. W. Jones, S. Smirnov, Eds.: Perspectives in Analysis – Essays in Honor of Lennart Carleson's 75th Birthday*, Mathematical Physics Studies, vol. 27, Springer, Berlin, 2005, 376 pp., EUR 69,95, ISBN 3-540-30432-0*

The conference on "Perspectives in Analysis", organised on the occasion of the 75th birthday of Lennart Carleson, was held in 2003 in Stockholm. Essays on relations between physical intuition and mathematical analysis written by leading specialists in the fields are collected in this book. The purpose of the variety of excellent contributions by the authors is to consider the future of analysis and related areas of physics. The book ends with a more philosophical and extensive essay by Lars Gårding consisting of five dialogues on "Encounters with science". A DVD with videos of the talks by the authors is included. (jl)

J. Bewersdorff: Galois Theory for Beginners – A Historical Perspective*, Student Mathematical Library, vol. 35, American Mathematical Society, Providence, 2006, 180 pp., USD 35, ISBN 0-8218-3817-2, ISBN 978-0-8218-3817-4*

This book is the English translation (by D. Kramer) of the second edition of the Bewersdorff German book *Algebra für Einsteiger: Von der Gleichungsauflösung zur Galois-Theorie* (2004). The main aim of the book is to present Galois theory as the culmination of centuries-long investigations of solving algebraic equations by radicals. The book carefully describes results made in the first half of the 19th century involved in a long and complicated historical evolution, and the difficult transformation of classical methods used for algebraic equations solved by radicals into modern mathematical abstractions. Each chapter of the book begins with rhetorical questions or simple exercises illustrating important points of what lies ahead. Then the historical roots of investigations, their motivations, solutions, results and their applications are shown step by step.

Results are formulated first in an elementary way, then in their modern form (with the help of ideas and properties of groups and fields). The author describes solutions of cubic and biquadratic equations with their geometrical applications, including ancient problems, the birth of complex numbers, the discovery and proofs of the fundamental theorem of algebra, an attempt to solve equations of higher degrees and results of these attempts made by Paolo Ruffini, Etienne Bézout, Ehrenfried Walther, the Count of Tschirnhaus, Erland Samuel Bring, George Biích Jerrard, Niels Henrik Abel, Évariste Galois, Emil Artin, etc. The book can be recommended to undergraduate and graduate students. (mbec)

N. Bourbaki: Séminaire Bourbaki, volume 2003/2004, exposés 924-937*, Astérisque 299, Société Mathématique de France, Paris, 2005, 350 pp., EUR 66, ISBN 2-85629-173-2*

The proceedings of the Séminaire Bourbaki appear regularly in Astérisque. This book contains fourteen written versions of seminars held in the academic year 2003/2004. They are taken from diverse branches of mathematics. There are contributions from Y. André (finite-dimensional motives), A. Beauville (the Green conjecture), E. Peyre (obstructions for the Hasse principle), S. Alinhac (geometric methods for the Einstein equation), I. Gallagher (the Euler equations and the incompressible limit), P. Gérard (mean field dynamics for the quantum N-body problem), N. Tzvetkov (long-time behaviour of KdV-type equations), K. Belabas (parametrization of algebraic structures), J. Bertoin (Schramm-Loewner evolution and conformal invariance), R. Krikorian (deviations of ergodic averages for area preserving flows), B. Maurey (geometric inequalities), H. Pajot (analytic capacity and the Painlevé problem), J.-P. Serre (complete reducibility in group theory) and S. Vaes (free quasi-free states and type III factors). (vs)

J. Cannon, B. Shivamoggi, Eds.: Mathematical and Physical Theory of Turbulence*, A Series of Lecture Notes in Pure and Applied Mathematics, vol. 250, Chapman & Hall/CRC, Boca Raton, 2006, 193 pp., USD 139,95, ISBN 0-8247-2323-6*

This book presents the proceedings of an international turbulence workshop held at the University of Central Florida in 2003. Over a dozen contributions, written by leading experts of this highly interdisciplinary research field, cover all aspects of what is meant by turbulence according to the present state of knowledge. Among others, the contributions deal with two-dimensional turbulence, statistical as well as wavelet type analysis, the relation to the problem of regularity of Navier-Stokes flow, numerical experiments, geophysical flows, magneto-hydrodynamics effects, to name just a few. The book is primarily aimed at experts in the field but applied mathematicians and engineers will find it interesting as well. (dpr)

R. Carlson: A Concrete Introduction to Real Analysis*, Pure and Applied Mathematics, vol. 280, Chapman & Hall/CRC, Boca Raton, 2006, 296 pp., USD 89,95, ISBN 1-58488-654-4*

This book is an introduction to mathematical analysis. The main feature of the book is a focus on the understanding of basic concepts. It is a book that tries to switch the reader's point of view from "calculus" to "mathematical analysis", i.e. from an application of rules to explanations and proofs. The book can be used as an introductory two-semester course. The necessary background is reviewed (including proof by induction and some elementary logic). The topics treated are discrete calculus (proofs by induction, calculus of sums and differences, sums of powers), selected area computations (areas under power function graphs, the computation of π, natural logarithms, the Stirling formula), limits and the Taylor theorem (limits of infinite sequences, series representations, Taylor series), infinite series (positive series, general series, grouping and rearrangement), logic (mathematical philosophy, propositional logic, predicates and quantifiers, proofs), real numbers (field axioms, order axioms, completeness axioms, subsequences and compact intervals, products and fractions), functions (limits and continuity, derivatives), integrals (integrable functions, properties of integrals, numerical computation of integrals, improper integrals, integrals with parameters). Each chapter provides a series of problems for the reader. (pp)

***J.-Y. Chemin, B. Desjardins, I. Gallagher, E. Grenier: Mathematical Geophysics – An Introduction to Rotating Fluids and the Navier-Stokes Equations**, Oxford Lecture Series in Mathematics and its Applications 32, Clarendon Press, Oxford, 2006, 250 pp., GBP 45,00,ISBN 0-19-857133-X*

Making the subtitle more specific, this book provides a transparent introduction to the mathematical analysis of rotating incompressible fluids described by the evolutionary Navier-Stokes equations (NSEs) with dominant Coriolis forces. The authors first discuss physical aspects related to the investigated geophysical model. Then, after recalling necessary facts on function spaces, the theory of weak solutions related to the three-dimensional Navier-Stokes equations is provided. Next, NSEs with significant Coriolis forces are treated in various geometries: in the whole space, in domains with periodic boundary conditions and in domains bounded by two parallel plates. The final part focuses on the boundary layer phenomena related to the investigated systems and to some other relevant problems. This well organized and nicely presented book can be recommended to everybody interested in mathematical issues related to the Navier-Stokes equations and to geophysical fluid mechanics, as well as to the stability theory for dynamical systems described by systems of partial differential equations. (jmal)

***C. Ciliberto, A.V. Geramita, B. Harbourne, R.M. Miró-Roig, K. Ranestad, Eds.: Projective Varieties with Unexpected Properties**, Walter de Gruyter, Berlin, 2005, 392 pp., EUR 148, ISBN 3-11-018160-6*

A conference with the same title as this book was held in Siena in June 2004. The book contains research papers related to the topic of the meeting. There are twenty-three contributions in the proceedings on various topics, written by more than forty authors. They include classical themes (Grassmann and Veronese varieties, Fano threefolds, Chow and Segre varieties, Hilbert schemes), discussions of secant and defective varieties, various questions of birational geometry, moduli spaces, topology of real and complex algebraic varieties, phylogenetic algebraic geometry and many others. The book also contains complete information concerning the conference (the schedule, the list of participants and the list of contributors). (vs)

***A. C. C. Coolen: The Mathematical Theory of Minority Games. Statistical Mechanics of Interacting Agents**, Oxford University Press, Oxford, 2005, 324 pp., GBP 50, ISBN 0-19-852080-8*

Minority games are statistical mechanical models where individuals, "agents in the market", try to optimize their strategy in an economical game. They change their behaviour on the basis of knowledge of the previous history of (the global state of) the market. The goal of the game is to become a member of the "minority which wins". There is a random noise ("irrationality") in the decision of each individual acting in the game and no direct interaction between individual participants of the market is assumed. Thus the situation resembles physical models studied in (nonequilibrium) statistical mechanics, in systems of the mean field type and in spin glasses. Phenomena like phase transitions (i.e. abrupt changes of the "macroscopic" characteristics of the system) have to be expected in such models and the aim is to mathematically analyze their nature. Of course, this is usually a very difficult problem (of nonequilibrium statistical mechanics of disordered models). So, in most cases, only numerical simulations can give some insight into the behaviour of such complicated systems. The book aims to give an introduction to the relevant mathematics so that the reader need not consult specialized texts on statistical mechanics. In short, this book offers a self-contained introduction to the applications of powerful methods, originally developed by physicists, to socio-economic systems. (mzahr)

***M. Demuth, M. Krishna: Determining Spectra in Quantum Theory**, Progress in Mathematical Physics, vol. 44, Birkhäuser, Boston, 2005, 219 pp., EUR 98,00, ISBN 0-8176-4366-4*

This is a book on an important branch of contemporary mathematics: the spectral theory of selfadjoint (Schrödinger) operators. Many extremely interesting classes of such operators appear in quantum mechanics and statistical physics, and progress in detailed understanding of the nature of their spectra is one of the great achievements of this mathematical theory in recent decades. The book aims to give both an introduction (starting with very basic mathematical prerequisites) and, at the same time, an overview of some of the important methods and results in the theory, both classical (to be found in books by Kato, Reed-Simon and others) and very recent.

Some of the keywords of the contents are: the notions of a measure, the Fourier transform, wavelet transforms, the Borel transform, operators on Hilbert space and the sesquilinear forms associated to them, the spectral theorem, a decomposition of the spectrum, the scattering theory, the wave operator, the absolutely continuous spectrum, Laplacians and the related potential theory, and perturbations (by deterministic, random, singular potentials) of Laplacian operators. Applications to random potentials are treated in more detail in the last part of the book. The authors have taken an effort to explain the necessary prerequisites with many details so an interested reader will find here an accessible introduction to the advances in this important branch of mathematical physics, which has so many important applications. (mzahr)

***J.-P. Dufour, Nguyen Tien Zung: Poisson Structures and Their Normal Forms**, Progress in Mathematics, vol. 242, Birkhäuser, Basel, 2005, 321 pp., EUR 48, ISBN 3-7643-7334-2*

This book is a very good introduction to contemporary research in the field of Poisson structures. Its first chapter serves as an introduction to the theory for beginners. The subsequent chapters then introduce the reader to deeper parts of the theory. Four chapters are devoted to the subject mentioned explicitly in the title, namely the normal forms of Poisson structures, including the Poisson cohomology, the Levi decomposition of Poisson structures, the linearization of Poisson structures, and multiplicative and quadratic Poisson structures (including Poisson-Lie groups and their relations to r-matrices). Then we find a very interesting chapter about Nambu structures. The last two chapters are devoted to Lie groupoids and Lie algebroids. There is also an appendix where various notions are explained, which should help the reader to understand the main text of the book.

The book also contains quite recent results, even some results which are so far not published. The presentation is re-

ally very good and even the beginner will find not just a good introduction into the subject but also further reading, which will bring him smoothly to areas of present research. There are many examples that help in the understanding of the theory. The reader can test his or her knowledge by solving the many exercises contained in the book. The book fills a gap in the literature and is indispensable for specialists in the field. In short, it is very well written and can be strongly recommended. (jiva)

D. S. Freed, D. R. Morrison, I. Singer, Eds.: Quantum Field Theory, Supersymmetry, and Enumerative Geometry*, IAS/Park City Mathematics Series, vol. 11, American Mathematical Society, Providence, 2006, 285 pp., USD 55, ISBN 0-8218-3431-2*
Every summer, the IAS/Park City Mathematics Institute organizes a graduate summer school, with the main topics varying from year to year. In 2001, the school was devoted to quantum field theory, supersymmetry and enumerative geometry. The courses at the School were centred on supersymmetry and supermanifolds, general relativity, enumerative geometry, and mirror symmetry, and also included introductions to quantum field theory and string theory. The book contains lecture notes on several topics discussed at the school. There are two contributions (by W. Fulton and by A. Bertram) treating enumerative geometry. The first one describes Schubert calculus for Grassmann manifolds, its quantum version, and Gromow-Witten invariants. The second one shows how to compute Gromow-Witten invariants and their generating function by localization techniques.

A classical background needed for discussions between mathematicians and theoretical physicists is covered in the other three contributions. A long contribution (written by D. Freed) is devoted to classical field theory and its supersymmetric version (including basic facts on their quantization) and is complemented by a short lecture (by J. Morgan) on supermanifolds and super Lie groups. The last lecture notes (by V. Johnson) describe in detail the physical principles behind general relativity, along with the associated field equations and variational principles. The book collects together some useful material for anybody hoping to better understand recent important ideas coming to mathematics from theoretical physics. It can be, in particular, strongly recommended to postgraduate students and young mathematicians interested in modern streams of mathematics. (vs)

L. Gasifski, N.S. Papageorgiou: Nonlinear Analysis*, Series in Mathematical Analysis and Applications, vol. 9, Chapman & Hall/CRC, Boca Raton, 2005, 971 pp., USD 99,95, ISBN 1-58488-484-3*
This book deals with several important topics in nonlinear analysis, presented both on their own and as a basic tool for solving a broad class of nonlinear problems. It includes problems arising in the theory of partial differential equations, in particular the theories of boundary value problems, control theory, and calculus of variations. The book is written as a self-contained textbook. The reader will be pleased to find, in a rather large appendix, all the basic facts on topology, measure theory and functional analysis. But even when reading the book from the beginning, the reader will find that the book can serve as a well-written textbook, providing the basic knowledge and containing material of a deeper level.

Successively, one learns about Hausdorff measures and capacity, covering theorems, Dini derivatives, area formulas, Lebesgue-Bochner and Sobolev spaces, vector valued integration, evolution triples needed for PDE theory, together with the standard inequalities and embedding theorems that are the core of the theory. The modern concepts of nonlinear operators and Young measures, also in the context of the Nemytskii operators, and the theory of superposed convergences are dealt with. The book will be valuable both for postgraduate students beginning their professional career in the field and for experts who are looking for a well-written and comprehensive handbook on the field. (mrok)

N. K. Govil, H. N. Mhaskar, R. N. Mohapatra, Z. Nashed, J. Szabados: Frontiers in Interpolation and Approximation*, Pure and Applied Mathematics, vol. 282, Chapman & Hall/CRC, Boca Raton, 2006, 431 pp., USD 169,95, ISBN 1-58488-636-6*
This monograph is a collection of nineteen papers created in memory of Professor Ambikeshwar Sharma, who passed away in December, 2003. Topics covered include the theory of multivariate polynomial approximation, inequalities for multivariate polynomials, exponential sums, linear combinations of Gaussians, orthogonal polynomials and their zeros, uncertainty principles in wavelet analysis, approximation on the sphere, interpolation in the complex domain, weighted approximation on an infinite interval, and abstract approximation theory. Containing both original research and comprehensive survey papers, the book is valuable for research and graduate students as it brings together many important results of interpolation and approximation. (knaj)

D. V. Griffiths, I. M. Smith: Numerical Methods for Engineers*, second edition, Chapman & Hall/CRC, Boca Raton, 2006, 479 pp., USD 79,95, ISBN 1-58488-401-0*
This textbook provides an introduction to numerical methods, incorporating theory with specific computing exercises and programmed examples of presented techniques. Chapters 2-8 of the book describe 49 programs written in Fortran 95 covering a wide range of numerical applications. Many of the programs discussed use a sub-program library called nm _ lib that contains 23 subroutines and functions. In addition, there is a precision module that controls the precision of calculations. Chapter 2 deals with numerical solutions of systems of linear algebraic equations, while chapter 3 considers the roots of a single nonlinear equation and systems of nonlinear equations. In chapter 4, eigenvalue equations are considered, while chapter 5 deals with interpolation and curve fitting. Chapter 6 is devoted to numerical quadrature and chapter 7 introduces the solving of ordinary differential equations by numerical tools. Chapter 8 shows how to solve partial differential equations using finite difference and finite element approaches.

In all chapters, mathematical ideas and definitions are introduced as they occur and most numerical aspects are illustrated with computer programs. The library routines are described and listed in appendices A and B, respectively. All software described in the text can be downloaded from the web site, *www.mines.edu/~vgriffit/NM*. The textbook shows many different contexts of numerical analysis and offers an excellent introduction to more comprehensive subroutine libraries such as the numerical algorithm group (NAG). The book may be of interest to graduate students and software developers in engineering and mathematics. (knaj)

B. Heidergott, G.J. Olsder, J. van der Woude: Max Plus at Work – Modelling and Analysis of Synchronized Systems – A Course on Max-Plus Algebra and Its Applications*, Princeton Series in Applied Mathematics, Princeton University Press, Princeton, 2006, 213 pp., USD 50, ISBN 0-691-11763-2*

This is the first textbook on max-plus algebra, which represents a convenient tool for the description and analysis of discrete event systems like traffic systems, computer communication systems, production lines and flows in networks. The book is divided into three main parts and an introductory chapter, which illustrates main ideas in an informal way. The first part provides the foundations of max-plus algebra viewed as a mutation of conventional algebra, where instead of addition and multiplication the central role is played by the operations maximization and addition, respectively. It starts with the definitions of fundamental concepts (max-plus algebra and semiring, vectors and matrices over max-plus algebra) and an investigation of their properties. Then the spectral theory of matrices over a max-plus semiring is built, followed by a study of linear systems in max-plus algebra and their behaviour in terms of throughput, growth rate and periodicity.

The first part of the book ends with two chapters dealing with numerical procedures for the calculation of eigenvalues and eigenmodes. The second part starts with an introduction to Petri nets and their subclass, event graphs that are shown to be a suitable modelling aid for constructions of max-plus linear systems. Real-life applications related to timetable design for railway networks are discussed. It covers construction of large-scale systems, the throughput and periodicity of such systems, delay propagation, stability measures for railway networks and optimal allocation of trains and their ordering. The last part deals with various extensions (stochastic extensions, min-max-plus systems that also contain a minimization operation and thus enable modelling of a larger class of problems, and continuous flows on networks viewed as the continuous counterpart of discrete events on networks). The whole text ends with a bibliography, a list of frequently used symbols and an index. The book can be warmly recommended to final-year undergraduate students of mathematics, as well as to all interested applied mathematicians, operations researchers, econometricians and civil, electrical and mechanical engineers with quantitative backgrounds. (mhyk)

C. Heil, D. F. Walnut, Eds.: Fundamental Papers in Wavelet Theory*, Princeton University Press, Princeton, 2006, 878 pp., USD 49,50, ISBN 0-691-12705-0*

This book traces the development of modern wavelet theory by collecting many of the fundamental papers in signal processing, physics and mathematics stimulating the rise of wavelet theory, together with many important papers from its further development. These papers were published in a variety of journals from different disciplines, making it difficult to obtain a complete view of wavelet theory and its origin. Additionally, some of the most significant papers have been available only in French or German. Heil and Walmut bring together these documents into a book allowing the reader a complete view of the origins and development of wavelet theory. The volume is an excellent book and a first-class reference for the history of wavelets. By collecting all these various papers together in one volume, the editors and the publisher are offering a wonderful gift to graduate students as well as to researchers in engineering and mathematics. (knaj)

H. Hida: Hilbert Modular Forms and Iwasawa Theory*, Oxford Mathematical Monographs, Clarendon Press, Oxford, 2006, 402 pp., GBP 65, ISBN 0-19-857102-X*

Haruzo Hida is the inventor of (and the main contributor to) the theory of p-ordinary modular and automorphic forms. His new book presents several new results on the arithmetic of p-ordinary Hilbert modular forms, as well as an exposition on the relevant technical background. The book has five chapters. Chapter 1 is an extended introduction, in which the main objects appear (two-dimensional pseudo-representations, Greenberg's Selmer groups, deformation rings and adjoint L-invariants). Chapter 2 discusses the necessary background on automorphic forms on quaternionic algebras over totally real number fields (basic definitions, Hecke operators, the relation to representation theory of $GL(2)$ over local fields, an overview of the Jacquet-Langlands correspondence and of Galois representations associated to Hecke eigenforms).

In chapter 3, which is the core of the book, the author first proves a special case of Fujiwara's "$R=T$" theorem, from which he deduces several applications: an integral version of Jacquet-Langlands correspondence (the case of classical modular forms is treated earlier in chapter 2), the Iwasawa-theoretical version of "$R=T$" (for nearly p-ordinary Hilbert modular forms) and a formula for the adjoint of the L-invariant of a nearly p-ordinary Hilbert modular form. Chapter 4 sums up algebra-differential theory of classical and Hilbert modular forms in the spirit of the author's earlier book, 'p-adic automorphic forms on Shimura varieties' (including the theory of Igusa varieties and control theorems for nearly p-ordinary Hecke algebras). Exceptional zeros of the adjoint p-adic L-function are interpreted in terms of extensions of p-adic automorphic representations. Chapter 5 treats deformation rings along the cyclotomic Zp-extension of a totally real number field, with applications to adjoint Selmer groups.

The author's style, which is a mixture of expository treatment and new research, will not be to everybody's liking. The book itself has a fair share of confusing or incorrect statements and definitions, which makes it ideal for graduate students, who will learn a lot by trying to correct the abundant inaccuracies in the text. (jnek)

J***. Horváth, Ed.: A Panorama of Hungarian Mathematics in the Twentieth Century, I****, Bolyai Society Mathematical Studies, vol. 14, Springer, Berlin, 2006, 639 pp., EUR 129,95, ISBN 3-540-28945-3*

This highly interesting book is more than a report on a glorious period of Hungarian mathematics. It is a survey of twentieth century mathematics describing the development of the fields covered in the book in a broad context. Contributions of Hungarian mathematicians are analyzed in detail and there is a lot of biographical material. The present volume deals with topology, constructive function theory (orthogonal series and polynomials, interpolation, extremal properties of polynomials), harmonic analysis (commutative and non-commutative), real and functional analysis, differential equations, holomorphic functions, differential, discrete and convex geometry, mathematical statistics and probability, information theory and game theory. There is an article on the works of Kornél Lánczos on the theory of relativity. At the end of the book, a short guide to the history of Hungary in the 20th century and a short paper on education and research in Hungary are attached. Readers will surely appreciate brief biographies of 145 Hungarian mathematicians. The book is a valuable contribution to the history of 20th century mathematics. I can recommend it not only to historians of mathematics but to working mathematicians as well. (in)

A. Hulpke et al., Eds.: Finite Geometries, Groups, and Computation*, Walter de Gruyter, Berlin, 2006, 278 pp., EUR 128, ISBN 3-11-018220-3*

This book contains the proceedings of the conference of the same title held at the Colorado State University in September 2004, coinciding with the 60th birthday of William Kantor. The proceedings contain three survey papers: one by W. Kantor on finite semifields (division algebras that need not be associative), one by E. A. O'Brian about the search for effective algorithms for linear groups, and one by T. Pentilla on applications of computer algebra to finite geometry. Besides these survey papers, there are fifteen research papers exploring deeper interplay between the three main topics of the conference. Among the themes of the papers, one can find Hadamard designs, generalized quadrangles, symplectic translation planes, reduction algorithms for matrix groups, efficient presentations for the Mathieu simple group M_{22} and its cover, finite primitive permutation groups, etc. (jtu)

S. Katz: Enumerative Geometry and String Theory*, Student Mathematical Library, vol. 32, American Mathematical Society, Providence, 2006, 206 pp., USD 35, ISBN 0-8218-3687-0*

This book is based on lectures given at a summer school for undergraduate students at the Park City Mathematical Institute in 2001. The aim of the lectures was quite ambitious – to explain to (a group of selected) undergraduate students classical problems of enumerative algebraic geometry, together with the recent approach to these problems based on Gromov-Witten invariants and quantum cohomology and to describe how these notions were created using ideas coming from string theory! It seems impossible and it was necessary to adopt quite an unusual style of writing to reach the goal. The author had to introduce a long list of concepts (at least definitions and main examples) starting from basic ones (categories of topological spaces and continuous maps, groups and their homomorphisms) to more advanced ones (topological, smooth and complex manifolds, differential forms and the de Rham cohomology, various types of homological and cohomology groups and their relations, Poincaré duality and cohomology rings, smooth and holomorphic vector bundles) as well as pieces of modern physics education (classical and quantum mechanics, path integrals, supersymmetry, a pair of dual models in string theory).

He provides precise definitions as much as he can and adds basic examples and comments, helping to develop the intuition of the reader, and a lot of references for additional study. The book contains a lot of extra material that was not included in the original fifteen lectures. It is a nicely and intuitively written remarkable little booklet (only about 200 pages of A5 format) covering a huge amount of interesting material describing a beautiful area, where modern mathematics and theoretical physics meet. It can give inspiration to teachers for a lecture series on the topic as well as a chance for self-study by students. (vs)

R. E. Klima, N. P. Sigmon, E. L. Stitzinger: Applications of Abstract Algebra with MAPLE and MATLAB*, second edition, + CD, Discrete Mathematics and Its Applications, Chapman & Hall/CRC, Boca Raton, 2006, 505 pp., USD 89,95, ISBN 1-58488-610-2*

This book provides a comprehensive introduction to real-world applications based on abstract algebra. In each chapter, theoretical study is immediately complemented with the use of software programs Maple and MATLAB for the solution of problems from the fields of cryptography, coding theory and combinatorics. Incorporation of these programs makes it possible to eliminate extensive computations and thus further illuminate particular topics. The book is divided into twelve chapters supplemented with two appendices containing the user-written Maple and MATLAB functions used in previous parts, a bibliography, hints or answers for selected exercises, and an index. The book also contains a CD-ROM with all the programs and codes that are used in the text.

The first chapter provides a comprehensive and concise review of all the prerequisite advanced mathematics necessary for understanding the book. The second chapter is devoted to block designs. The next three chapters deal with coding theory, namely with error correcting codes, including Hadamard codes, Reed-Muller codes, Hamming codes, BCH codes and Reed-Solomon codes. Chapters 6 through 10 contain a presentation of interesting topics from cryptography. Specific parts are devoted to algebraic cryptography, including shift, affine and Hill ciphers, Vigenère ciphers, the popular RSA cryptosystem and related topics, the ElGamal cryptosystem and the incorporation of elliptic curves into it, and the Advanced Encryption Standard. Chapter 11 deals with Pólya counting techniques, including Burnside's and Pólya's theorems. The last chapter shows the application of Pólya theory to the problem of counting undirected graphs. The book can be directly used in a senior-level course on the applications of algebra. It represents a beneficial tool for students as well as researchers in the areas of mathematics and computer science. (mhyk)

T. Lawson: Topology – A Geometric Approach*, Oxford Graduate Texts in Mathematics 9, Oxford University Press, Oxford, 2006, 388 pp., GBP 29,95, ISBN 0-19-920248-6*

This is the soft cover edition of the book, which was first published by Oxford University Press in 2003. Its review was published in the EMS Newsletter, issue 53, Sept 2004. (vs)

Kai Liu: Stability of Infinite Dimensional Stochastic Differential Equations with Applications*, Monographs and Surveys in Pure and Applied Mathematics, vol. 135, Chapman & Hall/CRC, Boca Raton, 2005, 298 pp., USD 89,95, ISBN 1-58488-598-X*

This book can be recommended to everybody interested in an advanced theory of stochastic differential equations and, in particular, in the stability problem. Reading it will be easier with some preliminary knowledge of infinite dimensional stochastic differential equations (SDE), nevertheless all necessary probability background is briefly reviewed in the book. The book is readable and systematically written. It starts with a chapter devoted to the theory of SDEs in infinite dimensions. Chapters 2–4 constitute the essential part of the monograph, with a detailed study of stability properties. Different notions of stability are introduced in chapter 1 (stability in probability, stability in p-th moment, asymptotical stability, almost sure stability, exponential stability, and many others). In chapter 2, stability is studied for stochastic linear evolution equations. The nonlinear equations are studied in chapter 3 and this part is the core of the book. Stochastic functional differential equations and their stability is treated as a more specific topic in chapter 4. The last part (chapter 5) contains applications and some related topics of stability. The book can be recommended to specialists in stochastic analysis but it can also be useful for researchers in the area of deterministic differential equations as a stochastic counterpart to this branch of mathematics. Advanced students of probability, researchers applying the stochastic systems in their work, and many others can also profit from this monograph. (dh)

L. Lorenzi, M. Bertoldi: Analytical Methods for Markov Semigroups*, Pure and Applied Mathematics, vol. 283, Chapman & Hall/CRC, Boca Raton, 2006, 526 pp., USD 99,95, ISBN 1-58488-659-5*

This book describes past and present results on Markov semigroups. It begins with the existence of solutions for elliptic and parabolic equations with unbounded coefficients on the whole of R^n . Then it continues with uniqueness and nonuniqueness results and regularity properties (e.g. compactness, uniform and pointwise estimates of the derivatives) of the associated semigroup, both on the space of bounded continuous functions and on L_p with invariant measure. One chapter is devoted to the Ornstein-Uhlenbeck operator as a prototype of an elliptic operator with unbounded coefficients. The second part of the book is devoted to elliptic and parabolic problems on open unbounded domains in R^n with Dirichlet and Neumann boundary conditions. The third part deals with degenerate problems. The monograph contains a very well-arranged collection of the results on Markov semigroups. It will be mainly appreciated by experts on Markov semigroups as well as researchers working in related topics. But not only by them, since the results are presented in a way suitable for applications. The book does not contain many examples but they are not necessary since the text is suitably understandable. (tba)

J. McCleary: A First Course in Topology – Continuity and Dimension*, Student Mathematical Library, vol. 31, American Mathematical Society, Providence, 2006, 211 pp., USD 35, ISBN 0-8218-3884-9*

This is a very nicely written elementary book on topology, which is (by the author's words) "suitable for a semester-long course on topology for students who have studied real analysis and linear algebra. It is also a good choice for a capstone course, senior seminar, or independent study". The book contains a lot of material for a one semester course. A leading concept (sometimes rather hidden) is that of dimension. To explain basic topics of general topology, the authors use the Peano curve (with a construction and a proof) and Brouwer's fixed point theorem in the plane with its proof based on the Sperner lemma (which is also proved). After introducing homotopy groups, the fundamental group of circle is computed and, as a corollary, Brouwer's fixed point theorem for disks is proved with its usual corollaries. The Jordan curve theorem is proved using the notion of an index and gratings. The remaining two chapters contain basics of simplicial homology theory with the proof of the generalisation of Brouwer's fixed point theorem and the Euler-Poincaré characteristic of complexes. (mih)

R. A. Mollin: Codes: The Guide to Secrecy from Ancient to Modern Times*, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2005, 679 pp., USD 79,95, ISBN 1-58488-470-3*

This book starts with the history of security and privacy for laymen and it finishes with a rather mathematical treatment of error-correcting codes. All this is followed by about one hundred pages of mathematical appendices. The first two chapters (altogether about one hundred pages) comprise a brief history of ciphers from the times of Ancient Egypt to the end of World War II. The author describes in detail some important developments, while the description of others (like the breaking of the German Enigma cipher by Polish mathematicians in 1932 and its continuation in the British cryptoanalytical centre in Bletchley Park through World War II) are rather sketchy. Then the author devotes almost three hundred pages to a more detailed description of the modern "computer era" cryptology over the last sixty years. He also touches on various ethical issues of privacy, cyber-crime, etc. The book concludes with a rather technical treatment of Shannon's theory of information and error-correcting codes. The book has a very broad range aimed at various groups of readers. Anyone interested in the area can probably find something useful to them in it. (jtu)

A. D. Polyanin, A. V. Manzhirov: Handbook of Mathematics for Engineers and Scientists*, Chapman & Hall/CRC, Boca Raton, 2007, 1509 pp., ISBN 978-1-58488-502-3, 1-58488-502-5*

This book covers the main fields of mathematics and focuses on the methods used for obtaining solutions of various classes of mathematical equations that underline the mathematical modelling of numerous phenomena and processes in science and technology. To accommodate different mathematical backgrounds, the pre-eminent authors outline the material in a simplified, schematic manner, avoiding special terminology wherever possible. Organized in ascending order of complexity, the material is divided into two parts.

The first part is a concise, coherent survey of the most important definitions, formulas, equations, methods and theorems. It covers arithmetic, elementary and analytic geometry, algebra, differential and integral calculus, special functions, calculus of variations, probability theory and much more. Numerous specific examples clarify the methods for solving problems and equations. The second parts provide many in-depth mathematical tables, including those of the exact solutions of various types of equations.

The absence of proofs and the concise presentation has permitted the combination of a substantial amount of reference material in a single volume. The main distinction of this reference book from other general mathematical reference books is a significantly wider and more detailed description of methods for solving equations and obtaining their exact solutions for various classes of mathematical equations (ordinary differential equations, partial differential equations, integral equations, difference equations, etc). In addition to well-known methods, some new methods that have been developing intensively in recent years are described. This book can be viewed as a reasonably comprehensive compendium of mathematical definitions, formulas and theorems intended for researchers, university teachers, engineers and students of various backgrounds in mathematics. Some sections and examples can be used in lectures and practical studies in basic and more mathematically intense courses. (knaj)

R. M. Roth: Introduction to Coding Theory*, Cambridge University Press, Cambridge, 2006, 566 pp., GBP 40, ISBN 0-521-84504-1*

This introductory textbook on error-correcting codes is aimed at senior undergraduates or graduate students of computer science, electrical engineering and mathematics. Starting from the basics of symmetric binary channels and linear codes, the author introduces the first part of necessary material about finite fields and proceeds straight on to Reed-Solomon codes. It is followed by a chapter on finite fields containing material about minimal polynomials and cyclotomic cosets needed for a detailed treatment of cyclic codes and BCH codes. The pillars of the book are three chapters on GRS codes. The last three chapters discuss concatenated codes, graph codes and convolutional codes. There is also a chapter on combinatorial bounds. The material of the first eight chapters (finishing with cyclic codes) can be used as an introductory course on error-correcting codes for students with a background in probability, linear algebra, modern algebra and combinatorics. (jtu)

T. Roubíček: Nonlinear Partial Differential Equations with Applications*, International Series of Numerical Mathematics, vol. 153, Birkhäuser, Basel, 2005, 405 pp., EUR 108, ISBN 3-7643-7293-1*

This book concerns the mathematical analysis of quasilinear partial differential equations (PDEs) where the leading operator is nonlinear, elliptic and in divergence form. Techniques such as monotone operators, pseudomonotone operators, accretive operators, potential operators, variational inequalities and set-valued mappings form the cornerstone basis for the presented analysis. Special attention is also devoted to penalty methods. The author treats both steady-state problems in part I and corresponding evolutionary problems in part II. For time-dependent problems, the Rothe and Faedo-Galerkin methods are incorporated in detail.

Each section has the same structure: a general abstract framework is accompanied by applications of theoretical results to carefully selected examples starting from sample cases up to the cases that have their origin in the physical sciences (thermofluid mechanics, thermoelasticity, reaction-diffusion problems, material science). Frequently the author shows that dealing with a specific (system of) PDE(s) can strengthen the results obtained by abstract methods. Each section is completed with exercises and a representative list of relevant literature. This carefully written book, addressed to graduate and PhD students and researchers in PDEs, applied analysis and mathematical modelling contains on one hand several mathematical approaches developed to understand the basic mathematical properties of nonlinear PDEs and on the other hand many interesting examples. Some are solved, others are complemented by hints and the rest are left for the reader to complete. (jomal)

R. Sivaramakrishnan: Certain Number-Theoretic Episodes in Algebra*, Pure and Applied Mathematics, vol. 286, Chapman & Hall/CRC, Boca Raton, 2006, 632 pp., USD 139,95, ISBN 0-8247-5895-1*

This book is focused on some important aspects of interrelations between number theory and commutative algebra. The book is divided into four parts; each chapter starts with a historical overview and closes with illustrative examples. Part I deals with elements of number theory and algebra. The author discusses many interesting topics in detail, such as Euler's theorem, Fermat's theorem, Lagrange's theorem, integral domains of rational integers, Euclidean domains, rings of polynomials and formal power series, the Chinese remainder theorem, the reciprocity law and finite groups. Part II explains certain aspects of algebraic structures (including ordered fields, fields with valuation, abstract Möbius inversion, generating functions, finite semigroups and convolutions algebras). Part III gives an overview of foundations of algebraic number theory. Noetherian and Dedekind domains, Pell's equations and their solutions, the Dirichlet unit theorem and the ideal class-group are presented in detail. Part IV describes some other interrelations between algebraic number theory and abstract algebra (rings of arithmetic functions, the polynomial analogue of the Goldbach problem, finite dimensional algebras and algebraic structures). Since the reader needs only a rudimentary knowledge of elementary number theory and algebra, the book can be recommended to anyone interested in these domains. (mnem)

J. Stillwell: The Four Pillars of Geometry*, Undergraduate Texts in Mathematics, Springer, Berlin, 2005, 227 pp., 138 fig., EUR 39,95, ISBN 0-387-25530-3*

This book presents a tour on various approaches to a notion of geometry and the relationship between these approaches. It starts with classical Euclidean geometry and its basic questions (axioms, basic constructions, the Thales and Pythagoras theorems). Coordinates and algebra bring new, useful tools and allow the formation of a general notion of a vector space with a given scalar product. Questions connected with perspective drawings lead to a discussion of projective geometry (with a more detailed discussion of the projective plane). The associated transformation group is presented as an example of Klein's approach to geometry and his Erlangen program. The last chapter treats non-Euclidean geometries from the Klein point of view. The book shows clearly how useful it is to use various tools in a description of basic geometrical questions to find the simplest and the most intuitive arguments for different problems. The book is a very useful source of ideas for high school teachers. (vs)

M. Stroppel: Locally Compact Groups*, EMS Textbooks in Mathematics, European Mathematical Society, Zürich, 2006, 302 pp., EUR 52, ISBN 3-03719-016-7*

In addition to theoretical material, this book contains many helpful exercises, suggestions for further reading and a reasonable bibliography. A reader may choose to concentrate on topological groups, locally compact groups or topological fields, rings or semigroups (but a continuous reading may be more convenient from several points of view). After some standard basic material (topology, groups, topological transformation groups), the text continues with the Haar integral and its application to linear representations. A chapter on categories contains representations of compact groups as projective limits of Lie groups (or of finite powers of circles for Abelian groups). The section on locally compact groups deals with Pontryagin duality, approximation of locally compact Abelian groups by Lie groups and a discussion of maximal compact and vector subgroups. It continues with results on automorphism groups of locally compact Abelian groups, results on locally compact rings and fields, and homogeneous topological groups. The short section on locally compact semigroups contains some basic material. The last chapter deals with Hilbert's fifth problem. It contains results on approximation of locally compact groups, their dimension, simplicity, metrizability, connectedness, etc. (mih)

M. E. Taylor: Measure Theory and Integration, *Graduate Studies in Mathematics, vol. 76, American Mathematical Society, Providence, 2006, 319 pp., USD 59, ISBN 0-8218-4180-7*
This book starts with chapters on classical notions of integration and measure theory (the Riemann integral, the Lebesgue measure, the Carathéodory process, product measures, abstract integration, L^p-spaces, Radon and Hausdorff measures, the Vitali, Besicovitch and Rademacher theorems and maximal functions). The next chapters explore connections between measure theory and probability theory (ergodic theory, laws of large numbers, the central limit theorem, the Wiener measure, Brownian motion and martingales). The book has several appendices on topological notions, diffeomorphisms, the Whitney extension theorem, the Marcinkiewicz interpolation theorem, Sard's theorem, integration of differential forms and the Gauss-Green formula. Each chapter ends with many exercises of varying difficulty, which give further applications and extensions of the theory. The book ends with a bibliography, a list of standard symbols and a subject index. The book is very understandable, requiring only a basic knowledge of analysis. It can be warmly recommended to a broad spectrum of readers, to graduate students as well as young researchers. (jl)

J. C. Várilly: An Introduction to Noncommutative Geometry, *EMS Series of Lectures in Mathematics, European Mathematical Society, Zürich, 2006, 113 pp., EUR 28, ISBN 3-03719-024-8*
This book contains lecture notes of a series of lectures given at the EMS summer school in 1997. The lectures were given to a mixed audience of young mathematicians and physicists with the aim of introducing them to a new, important and quickly growing branch of mathematics. The approach to non-commutative geometry used in the book is based on spectral triples, which is a non-commutative generalization of a Riemannian manifold with a given spin structure and its associated Dirac operator. The first two chapters treat the commutative situation, which is then generalized in the next chapters. The author first introduces axioms for (real) spectral triples. Topics treated in other chapters include the geometry of the noncommutative torus, the noncommutative integral (the Dixmier trace and the Wodzicki residue), the Moyal quantization, equivalences among different geometries and action functionals. New developments in the theory over the last few years are described briefly in the last chapter. The book will be useful both for mathematicians and physicists willing to learn more about non-commutative geometry. (vs)

C. Walton: Agency and the Semantic Web, *Oxford University Press, Oxford, 2006, 249 pp., GBP 29,95, ISBN 0-19-929248-5*
The growing complexity and dynamics of the Web have enforced the development and use of ever more intelligent and powerful tools able to look for knowledge rather than simply information on the Web. One intended technique to solve this issue is the Semantic Web. The main aim of this book is to show that a fruitful contribution to the implementation of the Semantic Web can be the application of results of the long existing research of autonomous intelligent agents. The agents must be able to work in the framework of the successful software engineering concept of the Web and Web services. It implies the agents must be able to use related XML–based languages. The book describes ontologies as the central concept of the Semantic Web and discusses the basic properties of the languages for the specification of ontologies as a knowledge description tool on the Web (the RDF and OWL family of web ontology languages).

Several chapters are devoted to various concepts of intelligent agents and their behaviour (reactive agents and several models of planning agents and of deductive and reasoning agents). The remaining chapters (Reasoning on the Web, Agent Communication and Semantic Web service) are devoted to methods that can be used in the implementation of agents as Web services. A majority of chapters contain exercises, which are often supported by sources available on the Web, and suggested reading lists. The book covers a broad collection of topics on agency and on software engineering of the Web. The book presents more than ten languages and several logics. The explanation is intuitive using instructive examples. It usually presents only the main ideas and attitudes. Almost no practical experience with discussed visions, concepts and tools is mentioned. (jkral)

R. J. Williams: Introduction to the Mathematics of Finance, *Graduate Studies in Mathematics, vol. 72, American Mathematical Society, Providence, 2006, 150 pp., SD 39, ISBN 0-8218-3903-9*
This very nice monograph is intended as an introductory course to mathematical finance. It assumes a basic knowledge of probability theory, discrete and continuous time martingales, Brownian motion and stochastic calculus. The textbook consists of five chapters and four appendices. Chapter 1 is an introduction to the basic terms from the financial market, with examples and some assumptions to simplify the presentation. In chapter 2, the Cox-Ross-Rubinstein binomial model is described and the arbitrage free prices for both European and American contingent claims are derived. The idea of European option pricing is extended to a general finite market model in chapter 3, where the first and second fundamental theorems of asset pricing are formulated and proved. In chapter 4, the Black-Scholes model is introduced, for which pricing and hedging of European and American contingent claims are developed, including the famous Black-Scholes option pricing formula. In chapter 5, the multi-dimensional Black-Scholes model is defined, which is considered as a continuous analogue of the discrete finite market model, and the fundamental theorems of asset pricing are extended to a class of continuous models. In the appendices, basic results from conditioning, discrete and continuous time martingales and stochastic calculus for Brownian motion are reviewed. The text is clearly written and well-arranged and most of the results are proved in detail. Each chapter is completed with exercises, which makes the textbook very comprehensive. (zpr)

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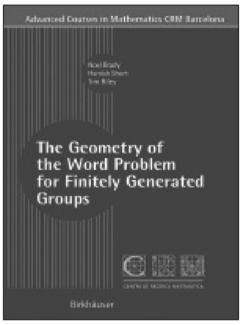

Brady, N., University of Oklahoma, Norman, OK, USA / **Riley, T.**, Cornell University, Ithaca, NY, USA / **Short, H.**, Université de Provence, Marseille, France

The Geometry of the Word Problem for Finitely Generated Groups

2007. VII, 206 p. Softcover
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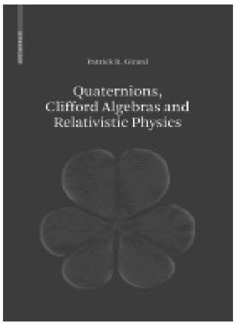
ACM – Advanced Courses in Mathematics - CRM Barcelona

The origins of the word problem are in group theory, decidability and complexity, but, through the vision of M. Gromov and the language of filling functions, the topic now impacts the world of large-scale geometry, including topics such as soap films, isoperimetry, coarse invariants and curvature. The first part introduces van Kampen diagrams in Cayley graphs of finitely generated, infinite groups; it discusses the van Kampen lemma, the isoperimetric functions or Dehn functions, the theory of small cancellation groups and an introduction to hyperbolic groups. One of the main tools in geometric group theory is the study of spaces, in particular geodesic spaces and manifolds, such that the groups act upon. The second part is thus dedicated to Dehn functions, negatively curved groups, in particular, CAT(0) groups, cubings and cubical complexes. In the last part, filling functions are presented from geometric, algebraic and algorithmic points of view; it is discussed how filling functions interact, and applications to nilpotent groups, hyperbolic groups and asymptotic cones are given. Many examples and open problems are included.



Girard, P.R., INSA Lyon, France

Quaternions, Clifford Algebras and Relativistic Physics

2007. XII, 179 p. 2 figs. Softcover
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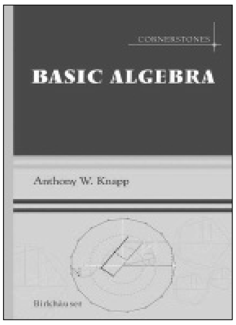

Knapp, A.W., State University of New York, Stony Brook, NY, USA

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2006. XXII, 717 p. 42 illus.
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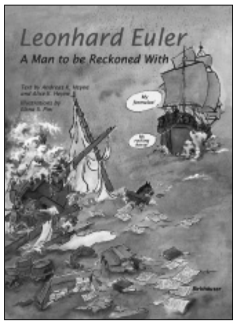

Heyne, A.K. / **Heyne, A.K.** (text) / **Pini, E.** (illustrations)

Leonhard Euler

A Man to Be Reckoned with

2007. 45 p. Hardcover
ISBN 978-3-7643-8332-9

His ideas turned the mathematical world on its head. He calculated the currents of liquids, the moment of inertia, developed the calculus of variations and the modern number theory. As scientist he should be placed on the same level as Newton and Einstein. Engineers all over the world use his formulas every day – whether it be for constructing the hull of the "Alinghi" or for calculating the vibrations of the "Viaduc de Millau", the world's highest motorway bridge.
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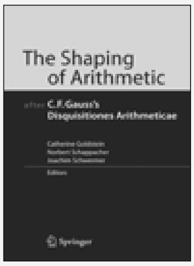

The Shaping of Arithmetic after C.F. Gauss's Disquisitiones Arithmeticae

C. Goldstein, Institut de mathématiques de Jussieu, Paris, France; **N. Schappacher**, Université Louis Pasteur, Strasbourg, France; **J. Schwermer**, University of Vienna, Austria (Eds.)

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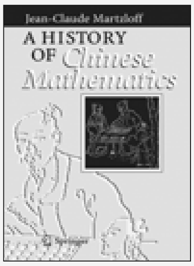

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