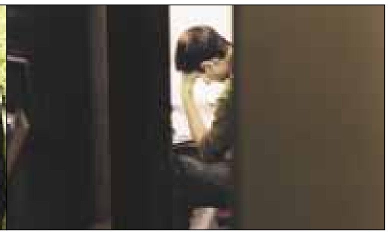
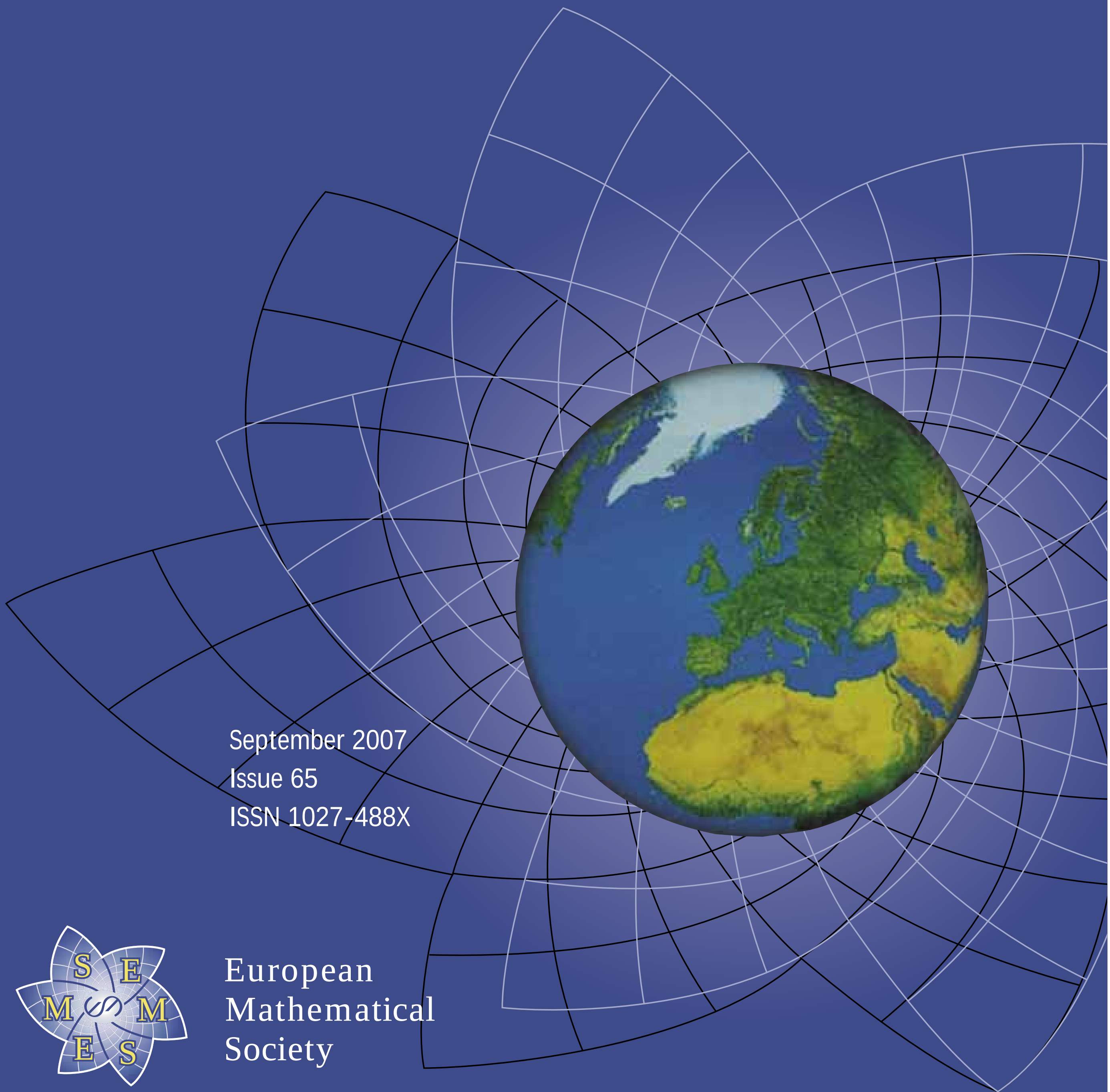
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY





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European
Mathematical
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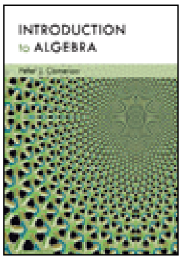

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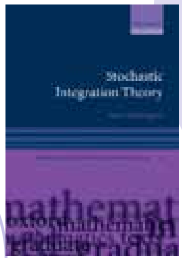

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Oxford Graduate Texts in Mathematics No. 14
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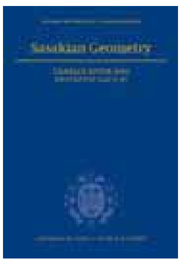

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Charles Boyer and Krzysztof Galicki

This book offers an extensive modern treatment of Sasakian geometry, which is of importance in many different fields in geometry and physics.

Oxford Mathematical Monographs
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978-0-19-856495-9 | £75.00

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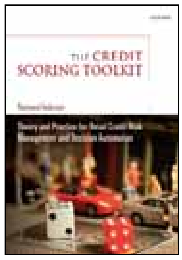
Theory and Practice for Retail Credit Risk Management and Decision Automation



Raymond Anderson

An up-to-date, comprehensive overview of credit scoring, covering the scorecard development process, all aspects of its use and how it affects businesses and consumers, from micro-lending through to large unlisted companies.

August 2007 | 792 pp | Hardback | 978-0-19-922640-5 | £80.00

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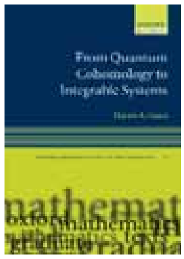

Martin A. Guest

This text focuses on the extraordinary success of quantum cohomology and its connections with many existing areas of traditional mathematics and new areas such as mirror symmetry. Aimed at graduate students in mathematics as well as theoretical physicists, the text assumes basic amiliarity with differential equations and cohomology.

Oxford Graduate Texts in Mathematics No. 15
304 pp | Hardback | 978-0-19-856599-4 | £45 .00

Kelvin: Life, Labours and Legacy

Raymond Flood, Mark McCartney and Andrew Whitaker

A collection of chapters, authored by leading experts, which cover the life and wide-ranging scientific contributions made by William Thomson, Lord Kelvin (1824-1907).

352pp | Hardback | 978-0-19-923125-6

Partial Differential Equations in General Relativity

Alan Rendall

A text that will bring together PDE theory, general relativity and astrophysics to deliver an overview of theory of partial differential equations for general relativity. The text will include numerous examples and provide a unique resource for graduate students in mathematics and physics, numerical relativity and cosmology.

Oxford Graduate Texts in Mathematics
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EMS Calendar

2007

27–28 October
EMS Executive Committee Meeting at the invitation
of the Cyprus Mathematical Society, Nicosia (Cyprus)
Stephen Huggett: s.huggett@plymouth.ac.uk

1 November
Deadline for submission of nominations of candidates for
EMS prizes
tijdeman@math.leidenuniv.nl; http://www.5ecm.nl/ecmpc.pdf

1 November
Deadline for submission of material for the December issue
of the EMS Newsletter
Martin Raussen: raussen@math.aau.dk

2008

1 January
Deadline for the proposal of satellite conferences to 5ECM
top@math.rug.nl; www.5ecm.nl/satellites.html

1 February
Deadline for submission of nominations of candidates
for the Felix Klein prize
ulmanen@cc.helsinki.fi; www.emis.de/tmp2.pdf

1–2 March
Joint Mathematical Weekend EMS-Danish Mathematical
Society, Copenhagen (Denmark)

3 March
EMS Executive Committee Meeting at the invitation of the
Danish Mathematical Society, Copenhagen (Denmark)
Stephen Huggett: s.huggett@plymouth.ac.uk

14–18 July
5th European Mathematical Congress, Amsterdam
(The Netherlands)
www.5ecm.nl

16–31 August
EMS-SMI Summer School at Cortona (Italy)
Mathematical and numerical methods for the cardiovascular system
dipartimento@matapp.unimib.it

8–19 September
EMS Summer School at Montecatini (Italy)
Mathematical models in the manufacturing of glass,
polymers and textiles

28 September – 8 October
EMS Summer School at Będlewo (Poland)
Risk theory and related topics
www.impan.gov.pl/EMSsummerSchool/

The Leonhard Euler Tercentenary The problem of support for young mathematicians

Victor M. Buchstaber (Moscow, Russia and Manchester, UK)

The great mathematician Leonhard Euler was born on 15 April 1707 in Basel, Switzerland. He became a scientist in Basel under the mentorship of Johann Bernoulli, one of the best mathematicians of the time.

The level of scientific achievement in mathematics and physics was already very high at the beginning of the 18^{th} century. Great scientists were renowned and respected people. For instance, when the Russian Tsar Peter I decided to found the Russian Academy of Sciences, he asked Leibniz for advice on this question, which was then regarded as the state priority. Following the order of Peter I, the Russian Academy of Sciences was founded by a decree of the governing senate in 1724.

In June 1724 Leonhard Euler got his first scientific degree of magister. Soon he applied for the position of Professor in Physics at the University of Basel but was not even included on the shortlist of candidates.

At the same time Daniel and Nicolas Bernoulli, friends and colleagues of Euler who also failed to obtain professorships at their home universities, received invitations from the recently founded Russian Academy of Sciences in St. Petersburg. At the beginning of the winter of 1726 Euler received a letter from St. Petersburg: following a recommendation from the Bernoulli brothers he was invited to a research position. As there were no vacant positions in mathematics, he was offered a position in physiology. Euler took the offer and on 5 April 1727 he left Switzerland for good. Soon he was able to secure a mathematical position in St. Petersburg.

The reasons that drove the young mathematician Euler to Russia were exactly the same reasons that forced many mathematicians from the former USSR to move to the West at the end of the 20th century. Euler worked fruitfully in Russia from 1727 to 1741 and from 1766 to 1783, and in Germany from 1741 to 1766. He had a brilliant career, becoming a member of the Russian Academy of Sciences in 1731, carrying the title of founder of the Russian School of Mathematics and being elected an honorary member of the scientific academy in eight different countries.

Euler died on 18 September 1783 in St. Petersburg, Russia, and was buried in the Lutheran cemetery. In the autumn of 1956 his remains were carried over to the Necropolis of Alexander Nevsky Lavra in St. Petersburg, where his tomb is now located.

The 300th anniversary of Leonhard Euler's birth was celebrated in Russia with a special congress comprising a series of scientific events. The events included the Euler Festival (10–12 June 2007) and a series of nine satellite conferences (June–July 2007) on the main fields of Euler's tremendous scientific activity in mathematics, mechanics and physics. Following an application by the Russian Academy of Sciences, the congress obtained the support of the Government of the Russian Federation and the St. Petersburg City Administration. It was also substantially supported by the Russian Academy of Sciences and the Russian Foundation for Basic Research.

The congress was superbly organized thanks to the efforts of the local committee, the staff of the St. Petersburg branch of the Steklov Mathematical Institute and the Euler International Mathematical Institute.

During the opening ceremony on 10 June 2007, the Euler Gold Medal of the Russian Academy of Sciences was awarded. Since 1997, this prize has been awarded for outstanding achievements in mathematics and physics. This year the jubilee medal went to Academician V.V. Kozlov. On the same day a monument to Euler was opened in front of the Euler International Mathematical Institute.

On the occasion of the tercentenary of the great scientist, the Euler Foundation was established in 2007. The funds were used to organize a contest of mathematical articles in three categories: undergraduate students, graduate students and young scientists. The winners were awarded during the opening ceremony of the Euler Festival.

During the three-day festival, several well-known scientists delivered one-hour plenary talks on their research activity (the programme can be found on the home page of the St. Petersburg Mathematical Society). The President of the International Mathematical Union Professor L. Lovasz, the President of the European Mathematical Society Professor Ari Laptev and representatives of the academies and mathematical societies of several European countries spoke during the reception under the chairmanship of the Head of the Organising Committee Academician L.D. Faddeev.

Now let us turn to the 19^{th} century and recall the fate of Niels Henrik Abel (1802–1829), whose bicentenary was celebrated in Oslo in 2002. The young Norwegian genius was unable to get a professorship and therefore did not have means to survive as a mathematician in his home university. His country had recently gained independence and did not have enough funds to open new positions at the university. Following an application from the scientific council of the university acknowledging the mathematical achievements of Abel, the government of Norway awarded him with a fellowship in 1825, which allowed Abel to present his work at the universities of Germany and France. Unfortunately, Abel was not able to secure financial support in Europe. He returned home, became seriously ill and died suddenly at the age of 26. A letter from August Crelle arrived two days after Abel's death, informing him that he had been offered a professorship at the University of Berlin.

A century later, in the 1920s, the USSR experienced a remarkably active development of mathematics. At first it took place in a close interaction with Western science. Young Soviet scientists had many chances to visit the leading scientific centres in the West and establish fruitful collaborations with their foreign colleagues. The Moscow topology congress of 1935 is remarkable evidence of this collaboration. The iron curtain closed soon after but a great potential had already accumulated. By the 1960s the Soviet mathematical school had gained worldwide recognition. It was then represented by outstanding scientists such as A.N. Kolmogorov, I.G. Petrovsky, L.S. Pontrjagin, P.S. Novikov and I.M. Gelfand and continued producing renowned young scientists like V.I. Arnold, D.V. Anosov, Yu.I. Manin, S.P. Novikov, Ya.G. Sinai and L.D. Faddeev. Although research visits to Western countries were made impossible for most Soviet scientists, many Western researchers visited the USSR. There were many large conferences with strong Western participation in Moscow, Leningrad, Novosibirsk, Kiev, Tbilisi and other research centres around the USSR. The 1966 Mathematical Congress in Moscow became the most important scientific event of that time.

The mathematical profession was highly prestigious and it lured many talented young people into mathematics. The presence of actively working, scientific mentors and advisors guaranteed their fast ascent to the heights of mathematical knowledge. It is important to emphasise the role of the mathematical societies and especially the Moscow Mathematical Society, which helped to maintain contacts and the interchange of ideas between mathematicians regardless of their affiliations.

By the time of perestroika many world class scientists in the USSR were unable to get professorships in their country. Perestroika opened the way to the West for them. The flow of scientists from the USSR became particularly strong at the beginning of the 1990s. Political and economic instability in the country together with inferior salaries forced many high class scientists to take positions in the West.

Nowadays the Russian Mathematical School has largely crossed the geographical borders of Russia; we may find its representatives in all the major universities of the world. Their remarkable achievements have been recognised by the mathematical community. So European investment in Russian science personified by Euler turned out to be a highly profitable business.

The future of science depends on striking the balance between maintaining the flow of young people and keeping actively working, mature scientists who carry out scientific mentoring. The events of the last 20 years have broken this balance in Russia but this problem is currently being addressed. One of the important steps in this direction was the foundation of the Scientific Education Centre on the base of the Steklov Mathematical Institute of the Russian Academy of Sciences. Well-known scientists give courses of lectures on important fields of modern mathematics. These lectures are attended by students from many different universities and institutions. The tuition is free; moreover, the students get financial support for every passed examination. The lecture courses organized by the Independent University of Moscow are also of great importance. Some of these lectures are given by Russian scientists who hold permanent positions in the West and foreign students are very welcome to attend these courses. For example, the NSF financially supports American students willing to come to Moscow and attend the lectures.

In 1997, the Moebius Contest Foundation for young scientists was established. The August Moebius contest is aimed at revealing the best student-written, scientific works in mathematics and supporting the authors financially on the condition that they continue their research work in Russia. The Pierre Deligne Contest was established in 2005 to support the most active young mathematicians working in Russia, Ukraine and Belorussia. The contest winner is awarded a three-year research grant. The aim of the contest is to help young mathematicians carry out scientific research while staying in their home countries. This contest is organized by Pierre Deligne; the funds come from the E. Balzan prize awarded to P. Deligne in 2004. From 2006 the Dynasty Foundation of Dmitry Zimin has organized a contest for young mathematicians similar to the Pierre Deligne contest in its aims and rules. The Euler contest is open from 2007.

All these contests have enjoyed strong participation from young scientists. The winners are usually young mathematicians from various cities who have presented world class research papers. The financial support for the winners and the widespread information about the contests definitely draws new talent into mathematics.

Summer and winter mathematical schools are popular all over the world. The Dubna summer schools deserve particular mention as they attract many outstanding scientists who give lectures to the students. This year the European mathematical community has supported the initiative to give the Dubna Schools an international status. The president of the EMS has sent a letter to the presidents of the national societies with information about the Dubna summer school. It is pleasing that this letter has attracted such an encouraging response and that many European students are already coming to Dubna this year.

Victor M. Buchstaber [buchstab@mi.ras.ru and Victor.Buchstaber@manchester.ac.uk] is a corresponding member of the Russian Academy of Sciences (RAS), a principal researcher at the Steklov Mathematical Institute RAS, a professor at Moscow State University, Russia, a professor at Manchester University, UK, and a member of the executive committee of the European Mathematical Society.

Fifth European Congress of Mathematics in Amsterdam, July 14–18, 2008

The Fifth European Congress of Mathematics (5ECM) will be organized in Amsterdam, from 14–18 July, 2008, under the auspices of the European Mathematical Society. This congress is the fifth in a series of successful four-yearly European congresses that cover the whole range of the mathematical sciences, from pure to applied. The series started in Paris (1992), followed by meetings in Budapest (1996), Barcelona (2000), and Stockholm (2004. The ECM congresses alternate with the IMU world congresses.

Next year's ECM congress will be organized under the special patronage of the Koninklijk Wiskundig Genootschap (Royal Dutch Mathematical Society, KWG), and will include the yearly meeting of the members of KWG. The 5ECM Local Organizing Committee consists of André Ran (Free University Amsterdam, chairman), Herman te Riele (CWI Amsterdam, secretary), and Jan Wiegerinck (University of Amsterdam, treasurer).

A Scientific Committee with representatives from all over Europe, chaired by Lex Schrijver (CWI and University of Amsterdam), has composed an interesting scientific program consisting of ten Plenary lectures, three (also plenary) Science lectures, about thirty (parallel) invited lectures, and twenty-one (parallel) Minisymposia. In addition, ten Prize lectures will be presented by outstanding young European mathematicians, selected by a Prize Committee chaired by Rob Tijdeman (Leiden University).

The ten Plenary lectures will be presented by

- Luigi Ambrosio (Scuola Normale Superiore di Pisa),
- Christine Bernardi (Université Paris VI),
- Jean Bourgain (IAS Princeton),
- Jean-François Le Gall (ENS & Université Paris VI),
- François Loeser (ENS Paris),
- László Lovász (Eötvös Loránd University, Budapest),
- Matilde Marcolli (Max Planck Institut Bonn),
- Felix Otto (Universitát Bonn),
- Nicolai Reshetikhin (Univ. of California, Berkeley), and
- Richard Taylor (Harvard University, Cambridge)

and the three Science lectures by

- Ignacio Cirac (Max-Planck-Institut fur Quantenoptik, Garching, Germany), on *Quantum Information Theory*,
- Tim Palmer (ECMWF Reading, UK), on *Climate Change*, and
- Jonathan Sherrat (Heriot-Watt University, Edinburgh, UK), on *Mathematical Biology*.

The topics and the organizers of the Minisymposia are:

- *Advances in Variational Evolution* (Alexander Mielke, Ulisse Stefanelli)
- *Algebra in Optimization* (Jan Draisma, Monique Laurent)
- *Applications of Noncommutative Geometry* (Gunther Cornelissen, Klaas Landsman)
- *Applied Algebraic Topology* (Michael Farber)
- *Combinatorics of Hard Problems* (Josep Diaz, Oriol Serra, Jaroslav Nesetril)
- *Coupled Cell Networks* (Peter Ashwin, Ana Dias, Jeroen Lamb)
- *Discrete Structures in Geometry and Topology* (Dmitry Feichtner-Kozlov)
- *Galois Theory and Explicit Methods* (Bart de Smit)
- *Global Attractors in Hyperbolic Hamiltonian Systems* (Andrew Comech, Alexander Komech)
- *Graphs and Matroids* (Bert Gerards, Hein van der Holst, Rudi Pendavingh)
- *Hypoellipticity, Analysis on Groups and Functional Inequalities* (W. Hebisch, B. Zegarlinski)
- *Mathematical Challenges in Cellular Systems* (Frank Bruggeman, Mark Peletier)
- *Mathematical Logic* (Peter Koepke, Benedikt Löwe, Jaap van Oosten)
- *Mathematical Finance* (Hans Schumacher, Peter Spreij)
- *Mathematics of Cryptology* (Ronald Cramer)
- *Representation Theoretical Methods and Quantization* (Stefaan Caenepeel, Jürgen Fuchs, Alexander Stolin, Christoph Schweigert, Freddy van Oystaeyen)
- *Rough Path Theory* (Peter K. Friz)
- *Singular Structures in Variational PDEs* (Matthias Roeger, Mark Peletier)
- *Spectral Problems and Hilbert Spaces of Entire Functions* (Joaquim Bruna, Hakan Hedenmalm, Kristian Seip, Mikhail Sodin)
- *Spectral Theory* (E.B. Davies, T. Weidl, F.Klopp, T. Hoffmann-Ostenhof)
- *Weak Approximations of Stochastic Differential Equations* (Dan Crisan)

Special activities, organized by the KWG, are the Brouwer medal ceremony (an event organized every three years in memory of the Dutch mathematician L.E.J. Brouwer, consisting of a laudatio, a lecture and a medal presentation, followed by a reception), a historical lecture on Brouwer's life and work (by Dirk van Dalen), and the so-called Beeger lecture (an event organized every two years in memory of the Dutch high-school teacher and mathematician N.G.W.H. Beeger, with a talk on algorithmic and/or computational number theory).

The names of the Brouwer and Beeger lecturers will be announced later.

For more information on the conference, such as grants, up-to-date information on the program, and for registration, please visit our website at www.5ecm.nl .

The organizers are proud that the EMS has selected Amsterdam to be the host city for its fifth congress, and we look forward to meeting you all next year in Amsterdam. Do not miss this opportunity to learn about the latest developments in mathematics, to meet old friends, and make new acquaintances, while enjoying a charming city with many 'do-not-miss-this' sights!

The 5ECM Local Organizing Committee

Call for Nominations of Candidates for Ten EMS Prizes
Fifth European Congress of Mathematics

Principal Guidelines

Any European mathematician who has not reached his/her 35th birthday on 30 June 2008, and who has not previously received the prize, is eligible for an EMS Prize at 5ecm. A total of 10 prizes will be awarded. The maximum age may be increased by up to three years in the case of an individual with a 'broken career pattern'.

Mathematicians are defined to be 'European' if they are of European nationality or their normal place of work is within Europe. 'Europe' is defined to be the union of any country or part of a country which is geographically within Europe or that has a corporate member of the EMS based in that country. Prizes are to be awarded for work published before 31 December 2007.

Nominations of the Award

The Prize Committee is responsible for solicitation and evaluation of nominations.

Nominations can be made by anyone, including members of the Prize Committee and candidates themselves. It is the responsibility of the nominator to provide all relevant information to the Prize Committee, including a résumé and documentation.

The nomination for each award must be accompanied by a written justification and a citation of about 100 words that can be read at the award ceremony.

The prizes cannot be shared.

Description of the Award

The award comprises a certificate including the citation and a cash prize of 5000 Euro.

Award Presentation

The prizes will be presented at the Fifth European Congress of Mathematics by the President of the European Mathematical Society. The recipients will be invited to present their work at the congress (see www.5ecm.nl).

Prize Fund

The money for the Prize Fund is offered by the Foundation Compositio Mathematica.

Deadline for Submission

Nominations for the prize must reach the chairman of the Prize Committee at the following address, not later than 1 November 2007:

5ECM Prize Committee, Prof. R. Tijdeman,
Mathematical Institute, Leiden University,
Postbus 9512, 2300 RA Leiden, The Netherlands.

e-mail: tijdeman@math.leidenuniv.nl
fax: +31715277101, phone: +31715277138

Call for Nominations for the Felix Klein Prize

Principal Guidelines

The prize, established in 1999 by the EMS and the endowing organisation, the Institute for Industrial Mathematics in Kaiserslautern, is awarded to a young scientist or a small group of young scientists (normally under the age of 38) for using sophisticated mathematical methods to solve a concrete and difficult industrial problem.

Nomination for the Award

There are no restrictions on eligibility other than those specified in the Principal Guidelines. The Prize Committee is responsible for solicitation and evaluation of nominations.

Nominations may be made by anyone, including members of the Prize Committee. It is the responsibility of the nominator to provide all relevant information to the Prize Committee, including a resume and documentation of the benefit to industry and the mathematical methods used.

The Prize Committee will report its nomination to the EMS President at least three months before the date of the award. The prize is awarded to a single person or to a small group and cannot be split.

Description of the Award

The award comprises a certificate containing the citation and a cash prize, of EUR 5000.

Award Presentation

The prize is presented every four years at the European Congress of Mathematics. The President of the EMS presents the award. The recipient is invited to present his or her work at the conference.

Prize History

The first prize has been awarded to David C. Dobson (USA) in the year 2000 during 3ECM in Barcelona. The prize was not awarded in 2004.

Prize Fund

The endowing Institute for Industrial Mathematics in Kaiserslautern is responsible for managing the prize fund as well as its administration.

Deadline for Submission

Nominations for the prize must reach the Helsinki office of the EMS at the e-mail address ulmanen@cc.helsinki.fi no later than 1st February 2008. Please use the text "Felix Klein Prize" in the subject field of the e-mail. The complete nomination must be submitted in pdf format.

EMS-EC meeting at EIMI, St. Petersburg, 9th June 2007

Martin Raussen (Aalborg, Denmark)

EC in the garden of EIMI, St. Petersburg

Only three months after its last meeting at Amsterdam, the EMS executive committee (EC) reconvened at the Euler International Mathematics Institute (EIMI) at St. Petersburg on the invitation of the Russian Academy of Sciences. Most of the members stayed for the Euler festival that was held at the University during the following three days.

Events

The situation in Uppsala, where two professors had been forced to resign from their positions at the Mathematics department under very concerning circumstances, is developing; see the report in the previous Newsletter and www.emis.de/press.html . The EMS has contributed to make these events public. Hopefully, this publicity will result in more caution and care by decision makers in the future.

The Mathematical Societies of Turkey and of Montenegro have applied for membership in the EMS. These applications will be brought to vote at the council meeting next year.

Plans for a web site of the EMS have been discussed at a meeting in Stockholm recently, and a framework should be ready very soon.

Business

The financial statement for 2006, ending with a minor surplus, was signed by the EC-members. It was agreed to build up a new system for online payment of membership fees by credit card. It is extremely important to establish and maintain a reliable and up-to-date membership data base as quickly as possible.

Conferences, Meetings, Summer Schools

Details about the 5th European Congress (5ECM) to be held in Amsterdam, July 14–18, 2008, have now been fixed. The list of plenary speakers is now online at www.5ecm.nl. Furthermore, the scientific committee has recently agreed on a list of invited speakers, and it has selected a total of 21 minisymposia to be held during the conference. The EC expressed its concern that a fee of around 300 might result in low attendance. It was suggested to make it possible to attend without receiving the proceedings at a lower fee. In order to attract participants of satellite conferences, it was suggested to ask organisers to offer all 5ECM participants a reduced conference fee. It was agreed to offer publishers of Mathematical Societies to have booths at a low charge. The next EMS council meeting will be held in the Netherlands in the weekend before or after the congress.

A subcommittee of the EC will investigate the three bids (from Krakov, Prague, Vienna) to hold the 6th European Congress; their report will be discussed at the next EC-meeting.

The EC approved plans for a Joint Mathematical Weekend to be organised by the Danish Mathematical Society in Copenhagen on March 1–2, 2008. It suggested adding a session on PDEs. Vice-president Helge Holden agreed to help with the organisation of this session and to act as liaison between the EC and the Danish organisers.

The president reported on early plans for a European mathematical conference with a "bottom-up" structure of embedded conferences, as with most of the AMS conferences. A possible place would be Bruxelles, and the first possible date would be 2009. Two enthusiastic Belgian organisers have already been found.

Mireille Martin-Deschamps reported on an invitation of the French Mathematical Society, who offered its conference centre at Luminy for a brainstorm and strategy meeting with participation of presidents or their representatives from the national mathematical societies and the EC during a weekend in 2008.

The Applied Mathematics Committee has selected two summer schools for support from the EU-framework, both to be held in 2008. Their titles are: *Mathematical models in the manufacturing of glass, polymers and textiles* (Montecatini, Italy) and *Risk Theory and Related Topics* (Będlewo, Poland). The Dubna summer school this year will be attended by some students from Finland, Germany and Hungary.

Mathematical Prizes

The Felix Klein Prize that had not been awarded at the last European Congress will be reinstated. A call for

Cathedral of St. Peter and Paul, St. Petersburg

nominations is included in this Newsletter issue, together with a call for nominations for the European Mathematical Society Prizes.

The EC continued a discussion for a further prize and agreed upon to suggest Springer-Verlag to sponsor a prize in the area *History of Mathematics* under the name *Otto Neugebauer Prize*.

EMS publishing house

The President gave a short report on the meeting of the European Mathematical Foundation, which was held the day before. The finances were healthy. In particular, the Journal of the European Mathematical Society has had a tremendous development; it is widely seen as one of the most important mathematical journals worldwide. Unfortunately, subscription is still quite low, and it was agreed to make publicity efforts, e.g. through an article in the Newsletter.

European Research Council (ERC)

Vice president Pavel Exner, who is a member of ERCs Scientific Council, reported that the first call (to young researchers) had received a very large response. There had been 9167 applications, among them 472 in mathematics. Extra evaluators had been appointed; mathematics can expect to receive around 15 grants.

Final Matters

The President thanked the Russian Academy of Sciences for its hospitality. The next EC-meetings will be held in Cyprus (October 27–28, 2007) on the invitation of the Cyprus Mathematical Society and in Copenhagen (March 3, 2008) on the invitation of the Danish Mathematical Society.

New member of the editorial team

The newsletter's Associate Editor Vasile Berinde (Baia Mare, Romania) has been providing the list of forthcoming conferences for the newsletter for many years. Last year, he was appointed as the society's publicity officer, a position that demands a lot of his time. As a consequence, he has asked to be released from the responsibility for the conference list; he will continue his work as an associate editor. The newsletter's editorial board is happy to welcome his replacement, Ms. Mădălina Păcurar, who presents herself below:

Mădălina Păcurar

Mădălina Păcurar is currently a teaching assistant in the Faculty of Economic Sciences and Business Administration (Department of Statistics, Forecast and Mathematics) at the "Babeș-Bolyai" University of Cluj-Napoca. In the last four years she has been teaching *Mathematics for Economists, Mathematics Applied in Economics and Financial and Actuarial Mathematics* to first year students, in both German and Romanian.

She graduated from the Faculty of Mathematics and Computer Science of the same university in 2003 with a paper on *Algebraic methods in cryptography*, continuing her studies with a masters course on applied mathematics that ended with a paper on *Mathematical models in biomathematics*. She is currently one of the PhD students of the fixed point research school in Cluj-Napoca in the Department of Applied Mathematics, preparing a thesis on *Iterative methods for the approximation of fixed points*, under the supervision of Professor I.A. Rus.

Besides fixed points she is also interested in the history and didactics of mathematics, being a member of the Romanian team that prepared the national presentation at the 10^{th} ICM held in 2004 in Copenhagen, Denmark, and unofficially trying to show everyone, beginning with her students, the beauty of mathematics.

Her husband works in programming, being a graduate of the same mathematics-computer science specialization. A child is expected to increment the number of their family members this autumn.

ERCIM

Stephen Roy (London, UK)

Introduction

The European Research Consortium for Informatics and Mathematics (ERCIM) was established in 1989 by Alain Bensoussan, Cor Baayen and Gerhard Seegmueller. It has eighteen institutional members including INRIA (France) and CWI (The Netherlands). For further information on membership the website can be visited at www.ercim.org. ERCIM has a board of directors that meets twice a year. Keith Jeffery (Rutherford Appleton Laboratory, UK) is the current president (2007–2009). Members of the organisation attended Information Society Technology (IST) 2006, held in Helsinki last November. This conference and exhibition involved the European Commission and the presentation of its 7th framework programme (FP7) discussed in Issue 63 of the EMS newsletter with reference to the newly formed European research council (ERC). At the most recent EMS council meeting it was stressed that the ERCOM subcommittee of the EMS was keen to monitor the development of links between EMS and ERCIM. The main purpose of this article is to provide some descriptions of content within *ERCIM News*, a quarterly that provides a great deal of research related features and details of current affairs.

ERCIM News

Summary

Each edition has a circulation of 10,500 copies and consists of articles mainly concerned with information technology (IT) or informatics. The respective ratio of informatics to mathematics is usually at about 2:1. Articles include news from W3C (world wide web consortium); commentary on research related to a special theme which takes up the majority of each issue, and a section on technology transfer that usually presents the special theme on a marketable platform. Reviews of related events and descriptions of forthcoming conferences are also included.

Special themes

Since subscribing to ERCIM News last summer, I've read through three editions. The first of these focused on embedded intelligence related to robotics. The latest issue entitled *The Digital Patient* is concerned with the interface between medical imaging and computer. At first it's often difficult to extract any mathematical content from the articles – most of which are written in prose style. However the subtlety of the majority of the underlying mathematics, with a great deal of emphasis on discrete maths, starts to dissolve on prolonged reading. Selected articles on special themes are now described in more detail.

1) Embedded Intelligence
1A. System-level design of fault-tolerant embedded systems
Embedded systems contain embedded intelligence that perform simple control and support tasks on command in a pre-programmed manner. Distributed, networked electronic control units, *e.g.* electronics responsible for robotic applications, are often classified as embedded intelligence. In this article, A. Girault (INRIA) presents the use of algorithm and architecture graphs suitable for allowing the operation of systems containing sensors and actuators.

1B. New tool to design the behaviour of embedded systems
Complex behaviour often displayed by embedded systems can be tackled by construction of effective behavioural models. For example, computer processors allow photocopiers to order their own paper, but it's possible that the copier might order twice the required amount owing to message duplication. This sort of problem is currently being investigated at Eindhoven University, with collaboration from CWI. Mathematical methodology based on process algebra is accompanied with tools to model and analyse discrete behaviour.

2) Traffic Planning and Logistics
2A. Railyard shunting: a challenge for combinatorial optimisation
In this piece, M. Aronsson and P. Kreuger (SICS, Sweden) discuss logistics associated with cargo transport trains in rail networks. The evident mathematical slant involves

use of impressive computer generated Gantt charts and resource histograms to represent specific train schedules. These charts contain time axes with blocks that are used to indicate time periods corresponding to the operation under scrutiny.

2B. Fluid-dynamic approach to traffic flow problems
The road network of the city of Salerno in Italy has been examined in this special theme article, with a new simulation algorithm based on fluid dynamics. It computes numerical solutions to traffic flow problems on road networks. The reason why macroscopic fluid-dynamic models have been picked to replicate road traffic is because they're capable of representing phenomena such as the creation of shocks and their backward propagation along roads. The algorithm is based on approximation methods such as the Godunov scheme, and kinetic arrangements with suitable boundary conditions at junctions. This report is supplemented with computer generated images of urban networks such as bottlenecks, traffic circles and intersections.

3) The Digital Patient
3A. Modelling the pathophysiological human brain function
The most recent edition of *ERCIM News* includes spectacular computer generated images of human biological systems such as ventricles, knee joints and tumours. Discrete maths again features quite regularly throughout the issue. One group of researchers based at ICS-FORTH in Greece have used graph theory to visualise cognitive brain functions and model both local and long-range brain interactions.

3B. From Riemannian geometry to computational anatomy of the brain
Computational anatomy, which is at the interface of geometry, statistics and image analysis, forms the basis of this report. The individual anatomy of the brain is hard to understand owing to the complicated nature of inherent shapes. Geometric features in brain research usually belong to curved manifolds rather than Euclidean spaces, which eliminates the use of classical linear statistics. To overcome this difficulty, scientists from INRIA in France have made use of Riemannian manifolds with covariance matrices able to be extrapolated to the whole brain.

Subscription to ERCIM News by contacting the editorial office (or visiting http://ercim-news.ercim.org/) is currently free of charge. Back issues are also available online.

Stephen Roy [roy.stephen@yahoo.co.uk] obtained his PhD in electrochemistry at St Andrews University in 1994. His current research interest is based around the history of Riemann's zeta function.

Editorial board of K-Theory resigns

Open letter

Dear fellow mathematicians,

The Editorial Board of 'K-Theory' has resigned. A new journal titled *Journal of K-theory* has been formed, with essentially the same Board of Editors. The members are A. Bak, P. Balmer, S.J. Bloch, G.E. Carlsson, A. Connes, E. Friedlander, M. Hopkins, B. Kahn, M. Karoubi, G.G. Kasparov, A.S. Merkurjev, A. Neeman, T. Porter, D. Quillen, J. Rosenberg, A.A. Suslin, G. Tang, B. Totaro, V. Voevodsky, C. Weibel, and Guoliang Yu.
The new journal is to be distributed by Cambridge University Press. The price is 380 British pounds, which is significantly less than half that of the old journal. Publication will begin in January 2008. We ask for your continued support, in particular at the current time. Your submissions are welcome and may be sent to any of the editors.

Board of Editors *Journal of K-theory*

Christiaan Huygens and Contact Geometry

Hansjörg Geiges (Cologne, Germany)

" – Oui, voilà le géomètre! Et ne crois pas que les géomètres n'aient pas à s'occuper des femmes!" (Jean Giraudoux, *La guerre de Troie n'aura pas lieu*)

For me, the most evocative painting in the Mauritshuis in Den Haag has always been *Het meisje met de parel*, even before a novel and a film turned the girl into something of a pop icon. However, that museum is the home to another portrait that cannot fail to attract the attention of any scientifically interested visitor, and one where the identity of (some of) the portrayed, like in Vermeer's famous painting, is shrouded in mystery. I am speaking of Adriaan Hanneman's *Portret van Constantijn Huygens en zijn kinderen* (figure 1). This family portrait depicts C. Huygens (1596–1687) – "the most versatile and the last of the true Dutch Renaissance virtuosos" (Encyclopaedia Britannica), whose most notable contributions lay in the fields of diplomacy and poetry, – together with his five children. Among them is Christiaan Huygens (1629–1695), who would go on to become one of the most famous mathematical scientists of his time, later to be characterised as "ein Junggeselle von hervorragendem Charakter und außergewöhnlicher Intelligenz" [11]. While I expound some of the mathematical themes of Christiaan Huygens' life and hint at their relation to modern contact geometry, I leave the reader to ponder the question just which of the four boys in the family portrait shows that intellectual promise, a question to which I shall return at the end of this article.

An inaugural lecture is not only an opportunity to present one's field of research to a wider public, it also allows one to reflect on the standing of mathematics within the general intellectual discourse. On an earlier occasion of this kind [5] I have not been overly optimistic in this respect, and I have no reason to qualify anything I said there. Still, it is worth remembering that there have been even more precarious times for mathematics. In [4] we read that "The new Savilian professor [Baden Powell, Savilian professor of geometry at the University of Oxford 1827–1860] was shocked and dismayed by the low esteem accorded to mathematics in the University. He had been advised not to give an inaugural lecture on arrival, as he would almost surely not attract an audience."

Disclaimer

A foreigner, even one who has lived in the Netherlands for several years, is obviously carrying tulips to Amsterdam (or whatever the appropriate turn of phrase might be) when writing about Christiaan Huygens in a Dutch journal. Then again, from a visit to the Huygensmuseum Hofwijck in Voorburg near Den Haag I gathered that in the Netherlands the fame of Constantijn Huygens tends to outshine that of his second-

Figure 1. Adriaan Hanneman's *Portret van Constantijn Huygens en zijn kinderen, ©Mauritshuis, Den Haag*. Among them is Christiaan (see last page of the article).

eldest son. Be that as it may, this article is intended merely as a relatively faithful record of my inaugural lecture (with some mathematical details added) and entirely devoid of scholarly aspirations.

The best slide for twins

Imagine that you are trying to connect two points A, B in a vertical plane by a slide along which a point mass M will move, solely under the influence of gravitation, in shortest time from A to B (see figure 2). This is the famous brachistochrone problem (from Greek βράχιστος = shortest, χρόνος = time), posed by Johann Bernoulli in 1696 in rather more erudite language: "Datis in plano verticali duobus punctis A & B assignare Mobili M viam AMB, per quam gravitate sua descendens & moveri incipiens a puncto A, brevissimo tempore perveniat ad alterum punctum B." (*Problema novum ad cujus solutionem Mathematici invitantur*, Joh. Op. XXX (pars), [3], p. 212).

A related problem is to find the slide connecting the points A and B in such a way that one will reach the end-point B of

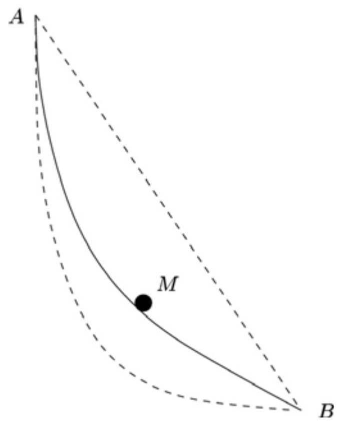

Figure 2. Imagine that you are trying to connect two points A,B in a vertical plane by a slide along which a point mass M will move, solely under the influence of gravitation, in shortest time from A to B.

the slide in the same amount of time, no matter where on the slide one starts. This is known as the tautochrone problem.

Rather surprisingly, it turns out that the solution to either question is one and the same curve, the so-called *cycloid*. This is obviously the best slide a doting uncle can build for his twin nephews: not only will their slide be faster than anybody else's; if both of them start at the same time at any two points of the slide, they will reach the bottom of the slide simultaneously. This gives them the chance and the time to fight over other things.

In 1697 Jacob Bernoulli responded to the challenge set by his brother concerning the brachistochrone with a paper bearing the beautiful title *Solutio Problematum Fraternorum, una cum Propositione reciproca aliorum*, Jac. Op. LXXV ([3], p. 271–282). Johann's own solution appeared the same year (Joh. Op. XXXVII, [3], p. 263–270). The tautochrone problem had been solved by Christiaan Huygens as early as 1657, but the solution was not published until 1673 in his famous *Horologium Oscillatorium* [9], cf. [16].

The cycloid

The cycloid is the locus traced out by a point on the rim of a circle as that circle rolls along a straight line (figure 3). Choose cartesian coordinates in the plane such that the circle rolls along the x-axis, with the point on the rim initially lying at the origin $(0,0)$. Let a be the radius of the circle. When the circle has turned through an angle t, its centre lies at the point (at,a), and so a parametric description of the cycloid is given by

$$x(t) = a(t - \sin t), \ \ y(t) = a(1 - \cos t).$$

The cycloidal slide is obtained by turning this curve upside down. It is convenient to effect this by reversing the direction of the y-coordinate, while keeping the parametric equations unchanged. Given two points $A = (0,0)$ and $B = (b_1,b_2)$ with $b_1 > 0$, $b_2 \geq 0$ in the xy-plane, there is a unique radius a and angle $t_B \in (0,2\pi]$ such that $A = (x(0),y(0))$ and $B = (x(t_B),y(t_B))$. There are various ways to see this, the following is due to Isaac Newton, (cf. [3], p. 43): Draw any cycloid starting at A, and let Q be its intersection with the straight line segment AB. Then expand the cycloid by a factor AB/AQ. (Here and below I use the same symbol AB to denote a curve or line segment between two points A,B, as well as the length of that segment, provided the meaning is clear from the context.)

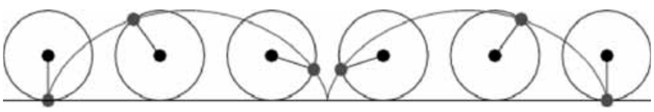

Figure 3. The cycloid

For some of the reasonings below I shall assume implicitly that $t_B \leq \pi$, so that the cycloidal segment connecting A and B does not have any upward slope; this is equivalent to requiring $b_2 \geq 2b_1/\pi$.

The brachistochrone and tautochrone problems were two of the most challenging geometric questions of 17th century mathematics, attracting the attention of the most famous (and cantankerous) mathematicians of that time, including the Marquis de L'Hospital, Leibniz, and Newton. As a result, these problems were the source of acrimonious battles over priority – the publications of the Bernoulli brothers on this topic have even been published in a collection bearing the title *Streitschriften* [3]. This was not the only occasion when the cycloid was the object of desire in a mathematical quarrel, and so this curve has often been dubbed the 'Helen of Geometers'.

The following allusion to the tautochronous property of the cycloid in Herman Melville's *Moby Dick* ([13], Chapter 96, The Try-Works) shows that there were happy times when the beauty of mathematics had to some degree entered popular consciousness: "[The try-pot] [a pot for trying oil from blubber -HG] is a place also for profound mathematical meditation. It was in the left hand try-pot of the *Pequod* [Captain Ahab's ship, named after an Indian people -HG], with the soapstone diligently circling around me, that I was first indirectly struck by the remarkable fact, that in geometry all bodies gliding along the cycloid, my soapstone for example, will descend from any point in precisely the same time."

The cycloidal pendulum

Besides the discovery of the true shape of Saturn's rings and one of its moons, namely Titan, Christiaan Huygens' most important scientific contributions are his theory of light, based on what has become known as Huygens' principle (discussed in the next section), and his development of a pendulum clock starting from his proof of the tautochronous property of the cycloid.

At the time of Huygens, pendulum clocks were built (as they usually are today) with a simple circular pendulum. The problem with such a pendulum is that its frequency depends on the amplitude of the oscillation. With regard to the pendulum clock in your living room this is no cause for concern, since there the amplitude stays practically constant. But arguably the most outstanding problem of applied mathematics at that time was to build a clock that was also reliable in more adverse conditions, say on a ship sailing through gale force winds. Why are such accurate clocks important?

As is wryly remarked in the introduction to the lavishly illustrated proceedings of the *Longitude Symposium* [2], "Travelling overseas, we now complain when delayed for an hour: we have forgotten that once there were problems finding con-

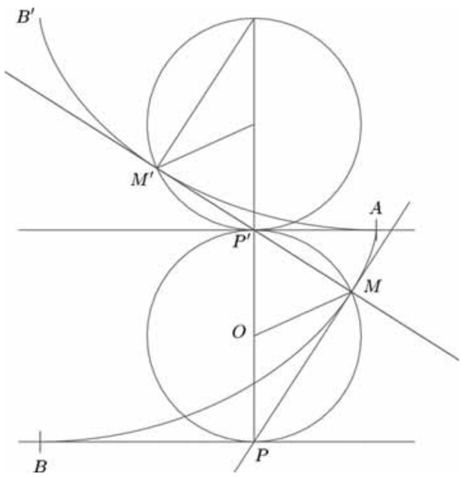

Figure 4. The construction of Huygens' amplitude invariant pendulum. The curve AB is (half) a cycloid, along which the pendulum mass M, attached to the string $B'M$, is supposed to move.

tinents". Indeed, how was it possible to determine your exact position at sea (or anywhere else, for that matter), prior to the days of satellite-based Global Positioning Systems? Mathematically the answer is simple (at least on a sunny day): Observe when the sun reaches its highest elevation. This will be noon local time. Moreover, the angle α of elevation will give you the latitude: If the axis of the earth's rotation were orthogonal to the plane in which the earth moves around the sun, that latitude would simply be $90^\circ - \alpha$. In order to take the tilting of the earth's axis by 23° into account, one needs to adjust this by an angle that depends on the date, varying between 0° at the equinoxes and $\pm 23^\circ$ at the solstices.

The longitude, on the other hand, cannot be determined from this observational data alone. Indeed, the actual value of the longitude at any given point is a matter of convention. The fact that the zero meridian passes through Greenwich is a consequence of the scientific achievements and geopolitical power of the British, not astronomy. However, if you keep a clock with you that shows accurate Greenwich time, and you bear in mind that the earth rotates by a full 360° in 24 hours, then multiplying the difference between your local time and that shown on the clock by $15^\circ/h$ will determine your longitude relative to that of Greenwich.

All the practical problems involved in building such an accurate clock were first solved by John Harrison in 1759: cf. [2] and the thrilling account of Harrison's life in [14].

From a mathematical point of view, the question addressed by Huygens concerned the most interesting aspect of these practical problems: Is it possible to devise a pendulum whose frequency does not depend on the amplitude of the pendular motion? The hardest part of this question is to find the tautochronous curve, along which the pendulum mass should be forced to move. This Huygens established to be the cycloid. He further observed that one could make the pendulum move along a cycloid by restricting the swinging motion of the pendulum between appropriately shaped plates.

Take a look at figure 4 (kindly provided by Manfred Lehn). Here AB is (half) a cycloid, along which the pendulum mass M, attached to the string $B'M$, is supposed to move. This means that we require this string to be tangent to the curve $B'A$ at the point M', and the length $B'M$ to equal $B'A$, the length of the pendulum. In other words, the cycloid AB is given by tightly unrolling (whence the title of [16]) a string from the curve $B'A$. If the pendulum is forced to swing between two plates shaped like $B'A$, then the pendulum mass will move along the cycloid, as desired.

Such a curve AB obtained by unrolling a string from a curve $B'A$ is called the *involute* of $B'A$ (and $B'A$ the *evolute* of AB). So the second question faced by Huygens was: Which curve has the cycloid as its involute? Rather miraculously, the answer is again: the cycloid. Here is the geometric proof: Let AB be the cycloid traced out by the point M as the lower circle in figure 4 rolls to the left along the horizontal line between the two circles (with $M = A$ at $t = 0$), and $B'A$ the cycloid traced out by the point M' as the upper circle rolls to the right along a horizontal line through B' (with $M' = B'$ at $t = 0$). With the defining equations for the cycloids as in the previous section, the situation shown in the figure corresponds to $t = t_0$ for some $t_0 \in [0, \pi]$ for the lower circle and $t = \pi - t_0$ for the upper circle.

The velocity (with respect to the parameter t) of the point M can be split into two vector components of length a: one in horizontal direction, corresponding to the speed of the centre of the circle, and one in the direction tangent to the circle, corresponding to the angular speed of the rolling circle. An elementary consideration shows that the line MP bisects the angle between these two directions, and so this line constitutes the tangent line to the cycloid at M. Analogously, the line $M'P'$ is the tangent line to the cycloid $B'A$ at M'. By symmetry of the construction, the line $M'P'$ passes through M. In order to conclude that AB is the involute of $B'A$ it therefore suffices to show that the length of the cycloidal segment $M'A$ equals the length of the line segment $M'M$. Also observe that, by the theorem of Thales, the line $M'M$ is orthogonal to the tangent line MP at M ; this is a general phenomenon for an involute.

The angle $\angle MOP'$ equals t_0, hence $\angle MPP' = t_0/2$. This yields

$$M'M = 2P'M = 4a \sin \frac{t_0}{2}.$$

On the other hand, from the defining equations of the cycloid we have

$$\dot{x}^2 + \dot{y}^2 = a^2(1 - \cos t)^2 + a^2 \sin^2 t = 4a^2 \sin^2 \frac{t}{2},$$

whence

$$M'A = \int_{\pi - t_0}^{\pi} 2a \sin \frac{t}{2}\, dt = 4a \cos \frac{\pi - t_0}{2} = 4a \sin \frac{t_0}{2},$$

that is, $M'M = M'A$, which was to be shown.

Huygens did not stop at these theoretical considerations, but proceeded to construct an actual pendulum clock with cycloidal plates. The construction plan from Huygens' *Horologium Oscillatorium*, with the cycloidal plates indicated by 'FIG. II', is shown in figure 5. A replica of this clock can be seen in the Huygensmuseum Hofwijck.

Geometric optics

Either of the following fundamental principles can be used to explain the propagation of light:

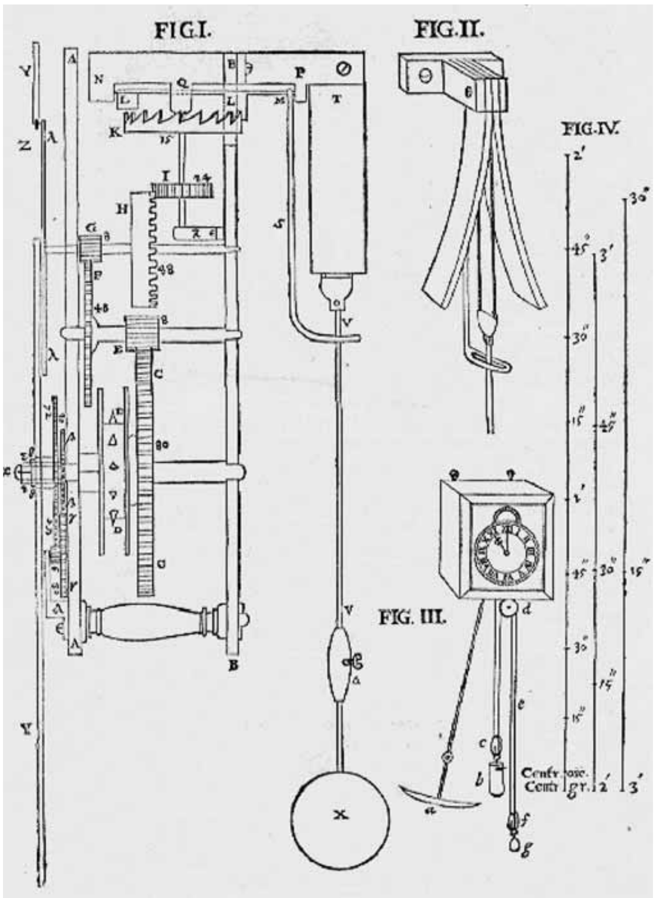

Figure 5. The construction plan from Huygens' *Horologium Oscillatorium*, with the cycloidal plates indicated by 'FIG. II'

Fermat's Principle. Any ray of light follows the path of shortest time.

Huygens' Principle. Every point of a wave front is the source of an elementary wave. The wave front at a later time is given as the envelope of these elementary waves [10].

The simplest possible example is the propagation of light in a homogeneous and isotropic medium. Here we expect the rays of light to be straight lines. Figure 6 illustrates that this is indeed what the two principles predict. We merely need to observe that, in a homogeneous and isotropic medium, the curves of shortest time are the same as geometrically shortest curves, i.e., straight lines, and elementary waves are circular waves around their centre.

Whereas Fermat's principle can only be justified as an instance of nature's parsimony, cf. [8], Huygens' principle can be explained mechanistically from a particle theory of light, see figure 7.

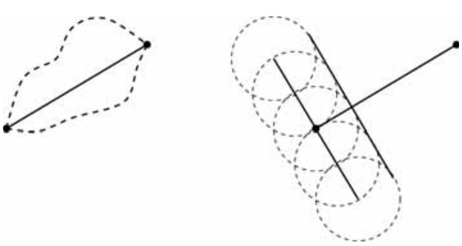

Figure 6. Fermat's principle versus Huygens' principle

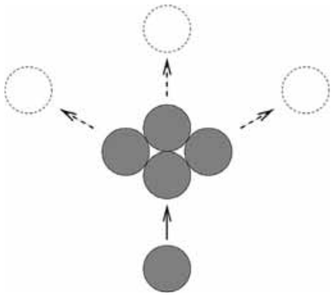

Figure 7. Explanation of Huygens' principle from the particle theory of light

To illustrate the power of these principles, here are two further examples. The first is the law of reflection, which states that the angle of incidence equals the angle of reflection. Figure 8 shows how this follows from Fermat's principle: The path connecting A and B has the same length as the corresponding one connecting A and the mirror image B' of B, and for the latter the shortest (and hence quickest) path is given by the straight line.

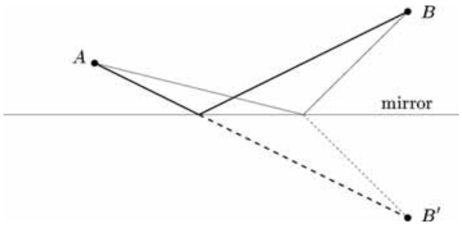

Figure 8. The law of reflection, by the Fermat principle

The explanation of the law of reflection from Huygens' principle is illustrated in figure 9.

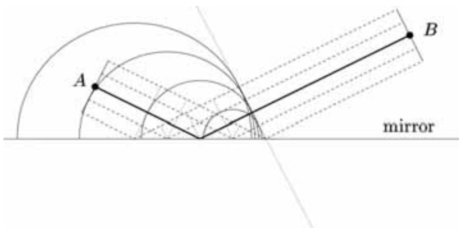

Figure 9. The law of reflection, by the Huygens principle

As a final application of the two principles, we turn to the law of refraction, also known as Snell's law after the Dutch astronomer and mathematician Willebrord van Roijen Snell (1580–1628), whose latinised name Snellius now adorns the Mathematical Institute of the Universiteit Leiden. Snell discovered this law in 1621; in print it appears for instance in Huygens' *Traité de la lumière*, with proofs based on either of the two principles. The law states that as a ray of light crosses the boundary between two (homogeneous and isotropic) optical media, the angle of incidence α_1 (measured relative to a line perpendicular to the separating surface) and the angle of refraction α_2 (see figure 10) are related by

$$\frac{\sin\alpha_1}{v_1} = \frac{\sin\alpha_2}{v_2},$$

where v_1 and v_2 denote the speed of light in the respective medium.

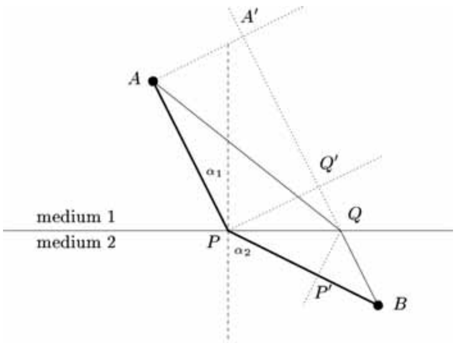

Figure 10. Snell's law, derived from Fermat's principle

Figure 10 shows how to derive Snell's law from Fermat's principle. The path from A to B via P (drawn in bold) is supposed to be the one satisfying Snell's law. We need to show that it takes longer to travel along any other broken path from A to B via some Q different from P. We compute

$$\frac{PP'}{v_2} = \frac{PQ\sin\alpha_2}{v_2} = \frac{PQ\sin\alpha_1}{v_1} = \frac{QQ'}{v_1},$$

that is, $t(PP') = t(QQ')$, where $t(\cdot)$ denotes the amount of time it takes to travel along a certain line segment in the corresponding medium. Therefore

$$t(AQ)+t(QB) > t(A'Q)+t(P'B) = t(A'Q')+t(Q'Q)+t(P'B)$$
$$= t(AP)+t(PP')+t(P'B) = t(AP)+t(PB).$$

Figure 11 indicates how Snell's law is implied by Huygens' principle.

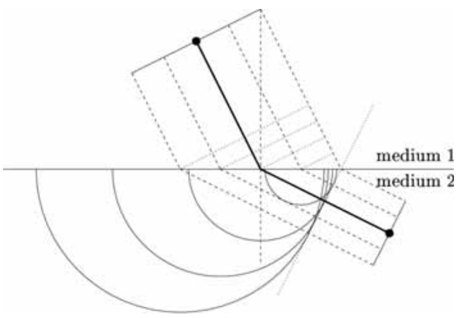

Figure 11. Snell's law, derived from Huygens' principle

Johann Bernoulli's solution of the brachistochrone problem

Jacob Bernoulli, in his response (cited above) to the fraternal challenge, developed a general method for dealing with problems of this kind, nowadays known as the calculus of variations. In the present section we shall be concerned with Johann's own solution, which nicely relates to the concepts of geometric optics discussed above.

When the mass M has reached a point (x,y) on the slide from $A = (0,0)$ to B, with the y-coordinate oriented downwards, its speed has reached, under the influence of gravitation, the value $v = \sqrt{2gy}$, where $g = 9.81m/s^2$ denotes the gravitational acceleration near the surface of the Earth. In order to determine which path the point M should follow so as to take the shortest time from A to B, we discretise the problem. Imagine that the region between A and B is layered into finitely many horizontal slices, in each of which the speed of M stays constant. In particular, M should follow a straight line in each layer. As M passes from the ith to the $(i+1)$st layer, the angle α_i of incidence and α_{i+1} of 'refraction' should be related to the respective speeds v_i, v_{i+1} by Snell's law

$$\frac{v_{i+1}}{\sin\alpha_{i+1}} = \frac{v_i}{\sin\alpha_i},$$

for the fact that Snell's law is an instance of Fermat's principle guarantees this to yield the quickest path (figure 12).

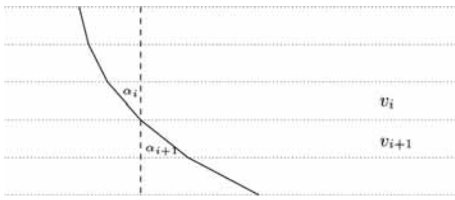

Figure 12. A region layered into finitely many horizontal slices

As we let the number of slices tend to infinity, the equation describing the brachistochrone becomes

$$\frac{v}{\sin\alpha} = c$$

for some constant c, see figure 13. Bravely computing with infinitesimals, we have $\sin\alpha = dx/\sqrt{dx^2+dy^2}$, whence

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot\sqrt{2gy} = c.$$

This can be written as

$$\frac{dx}{dy} = \sqrt{\frac{y}{2a-y}}$$

with $a = c^2/4g$. Substitute $y(t) = 2a\sin^2\frac{t}{2} = a(1-\cos t)$. Then

$$\frac{dx}{dt} = \frac{dx}{dy}\cdot\frac{dy}{dt} = \sqrt{\frac{1-\cos t}{1+\cos t}}\cdot a\sin t = 2a\sin^2\frac{t}{2} = y,$$

hence (with $x(0) = 0$) $x(t) = a(t-\sin t)$ "ex qua concludo: curvam *Brachystochronam* esse *Cycloidem vulgarem*" ([3], p. 266).

This is as good a point as any to recommend the wonderful textbook [7]. It contains an extensive discussion of both the brachistochrone and tautochrone problem in their historical context, and many other historical gems that so sadly are missing from our usual introductory courses on analysis, which tend to suffer from the dictate of efficiency and the haste to 'cover material'.

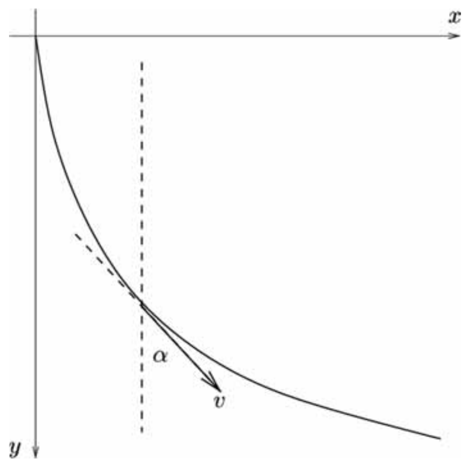

Figure 13. The limit case of figure 12, as the number of slices tends to infinity

Elementary contact geometry

Here at last we come to the second part of this article's title. My modest aim is to convey a couple of basic notions of contact geometry and to show how they relate to some of the ideas discussed above. In doing so, I am aware of W. Thurston's warning that "one person's clear mental image is another person's intimidation" [15].

One of the fundamental notions of contact geometry is the so-called *space of (oriented) contact elements* of a given manifold. Let us first consider a concrete example (see figure 14).

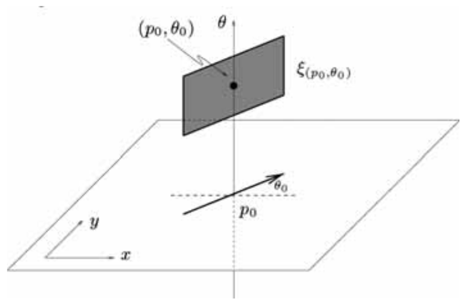

Figure 14. An oriented contact element to the 2-plane $\mathbf{R}^2$

An oriented contact element to the 2-plane $\mathbf{R}^2$ at some point $p_0 \in \mathbf{R}^2$ is simply an oriented line passing through the point p_0. Such a line is uniquely determined by p_0 and an angle θ_0. We can think of this angle θ_0 as an element of the unit circle S^1, so the space of all contact elements of $\mathbf{R}^2$ can be identified with the product $\mathbf{R}^2 \times S^1$.

Let ∂_x, ∂_y denote the unit vectors in the coordinate directions of $\mathbf{R}^2$ at any given point $(x_0, y_0, \theta_0) \in \mathbf{R}^2 \times S^1$. They can be thought of as the velocity vectors of the curves

$$x \longmapsto (x_0 + x, y_0, \theta_0),$$
$$y \longmapsto (x_0, y_0 + y, \theta_0).$$

Similarly, we can speak of the velocity vector ∂_θ of the curve

$$\theta \longmapsto (x_0, y_0, \theta_0 + \theta).$$

In the local picture of figure 14, where θ is measured along a real axis, this is once again simply the unit vector in the direction of the θ-coordinate.

We now specify a 2-plane $\xi_{(p_0,\theta_0)}$ at any point $(p_0, \theta_0) \in \mathbf{R}^2 \times S^1$ as the plane spanned by the vectors

$$\partial_\theta \quad \text{and} \quad \cos\theta_0\, \partial_x + \sin\theta_0\, \partial_y.$$

Alternatively, this plane is determined by the condition that it contain ∂_θ and that it project to the contact element at p_0 defined by θ_0. The collection of all these 2-planes is called the *natural contact structure* on the space of contact elements of $\mathbf{R}^2$.

This probably sounds esoteric or banal, depending on your education. It is unavoidable that at this point I shall have to assume a certain level of mathematical literacy. But I make no apology for continuing, whenever possible, also to address those who are beginning to feel just a little intimidated. The space of contact elements of $\mathbf{R}^2$ in fact has a very natural interpretation as a space of physical configurations. If you want to describe the position of a wheel of your bicycle, say, you should describe its position p and its direction, given by θ. Moreover, instantaneously the wheel can only roll in the direction in which it points at any given moment, so the motion of the wheel, interpreted as a curve in the 3-dimensional space of contact elements, will be tangent to the natural contact structure.

The next concept we want to introduce is that of a *contact transformation*. Such transformations play an important role in the geometric theory of differential equations. Most physicists first encounter them in their special incarnation as so-called Legendre transformations. For our purposes, we can define a contact transformation as a diffeomorphism ϕ of the space of contact elements $\mathbf{R}^2 \times S^1$ with the property that if a curve w passes through a point (p, θ) and is tangent to the 2-plane $\xi_{(p,\theta)}$ at that point, then the image curve $\phi \circ w$ will be tangent to $\xi_{\phi(p,\theta)}$ at $\phi(p, \theta)$.

Here is an example of a whole family of contact transformations: For $t \in \mathbf{R}$, define

$$\phi_t : \mathbf{R}^2 \times S^1 \longrightarrow \mathbf{R}^2 \times S^1$$
$$(x, y, \theta) \longmapsto (x - t\sin\theta, y + t\cos\theta, \theta).$$

In order to verify that these are indeed contact transformations, consider a parametrised curve

$$s \longmapsto w(s) = (x(s), y(s), \theta(s)) \in \mathbf{R}^2 \times S^1,$$

$s \in (-\varepsilon, \varepsilon)$, for some small $\varepsilon > 0$ say, with tangent vector

$$w'(0) = (x'(0), y'(0), \theta'(0))$$

assumed to lie in $\xi_{w(0)}$. With $\pi\colon\ \mathbf{R}^2 \times S^1 \to \mathbf{R}^2$ denoting the natural projection, this is equivalent to saying that the tangent vector $(x'(0), y'(0))$ of the projected curve $\pi \circ w$ at the point $(x(0), y(0))$ lies in the line determined by $\theta(0)$, i.e., is a multiple of $(\cos\theta(0), \sin\theta(0))$.

The transformed curve is

$$\phi_t \circ w(s) = (x(s) - t\sin\theta(s), y(s) + t\cos\theta(s), \theta(s)).$$

Notice that the θ-coordinate remains unchanged under ϕ_t. We compute

$$\frac{d}{ds}(\phi_t \circ w)(s) = (x'(s) - t\theta'(s)\cos\theta(s),$$
$$y'(s) - t\theta'(s)\sin\theta(s), \theta'(s))$$

and observe that the $\mathbf{R}^2$-component of this vector at $s = 0$ does again lie in the line determined by $\theta(0)$.

This family ϕ_t of transformations is called the *geodesic flow* of $\mathbf{R}^2$. Here is why: In a general Riemannian manifold, geodesics are locally shortest curves. In $\mathbf{R}^2$ (with its euclidean metric), therefore, geodesics are simply the straight lines. Given a point $p \in \mathbf{R}^2$ and a direction $\theta \in S^1$ defining a contact element, let $\ell_{p,\theta}$ be the unique oriented line in $\mathbf{R}^2$ passing through the point p and positively orthogonal to the contact element θ. This line is parametrised by

$$t \longmapsto p + t(-\sin\theta, \cos\theta), \ \ t \in \mathbf{R}.$$

Lo and behold, this is the same as $t \mapsto \pi \circ \phi_t(p, \theta)$. The θ-component of $\phi_t(p, \theta)$ encodes the direction orthogonal to this geodesic; in our case this component stays constant.

Great, I hear you say, but what does all that have to do with Huygens? Well, it turns out that we are but one simple step away from proving, with the help of contact geometry, the equivalence of the principles of Fermat and Huygens.

Let $\overline{f}$ be a wave front in $\mathbf{R}^2$, thought of as a parametrised curve $s \mapsto (x(s), y(s))$, $s \in (-\varepsilon, \varepsilon)$. For simplicity, we assume this to be regular, i.e.,

$$\overline{f}'(s) = (x'(s), y'(s)) \neq (0,0)$$

for all $s \in (-\varepsilon, \varepsilon)$. Such a wave front lifts to a unique curve

$$s \longmapsto f(s) = (x(s), y(s), \theta(s))$$

in the space of contact elements subject to the requirement that $(x'(s), y'(s))$ be a positive multiple of $(\cos\theta(s), \sin\theta(s))$; this lift will be tangent to the natural contact structure. *Fermat's principle* says that light propagates along the geodesic rays (i.e., straight lines) orthogonal to the wave front $\overline{f}$, which translates into saying that the wave front at some later time t is given by $\pi \circ \phi_t \circ f$.

Next consider the curve

$$h\colon\ \theta \longmapsto (x(0), y(0), \theta).$$

This is simply the circle worth of all contact elements at the point $\pi \circ h \equiv (x(0), y(0))$. Under the geodesic flow and projected to $\mathbf{R}^2$, this becomes an elementary wave in the sense of Huygens: for each fixed $t \in \mathbf{R}$ the curve

$$\theta \longmapsto \pi \circ \phi_t \circ h(\theta) = (x(0), y(0)) + t(-\sin\theta, \cos\theta)$$

is a circle of radius t centred at $(x(0), y(0)$.
The curves h and f are both tangent to $\xi_{f(0)}$ at the point $f(0) = h(\theta(0))$. Since ϕ_t is a contact transformation, the transformed curves $\phi_t \circ h$ and $\phi_t \circ f$ will be tangent to $\xi_{\phi_t \circ f(0)}$ at $\phi_t \circ f(0)$. Then, by the definition of the natural contact structure, the transformed wave front $\pi \circ \phi_t \circ f$ and the elementary wave $\pi \circ \phi_t \circ h$ will be tangent to each other at the point $\pi \circ \phi_t \circ f(0)$ – this is *Huygens' principle*.

The general argument is entirely analogous: A contact element on a Riemannian manifold is a (cooriented) tangent hyperplane field. The space of all these contact elements once again carries a natural contact structure. A geodesic is uniquely determined by an initial point and a direction positively orthogonal to a contact element at that point. Like in the special case of $\mathbf{R}^2$ one can show that the geodesic flow preserves the natural contact structure on the space of contact elements, and this translates into the equivalence of the two principles of geometric optics. A quick proof of this general case is given in [6]; full details of that proof are meant to appear in a forthcoming book on contact topology.

The family portrait

It remains to identify the young Christiaan Huygens in Hanneman's family portrait. In the biography [1] (from an aptly named publishing company!), a whole chapter is devoted to this question, so we seem to be in muddy waters.

Since Christiaan was the second-eldest son, there is actually only a choice between the two boys at the top. My first guess was that Christiaan is the one on the left, who has arguably the most striking face. This intuitive feeling is confirmed by the catalogue of the Mauritshuis ([12], p. 67) and by the afterword in [10]. Alas, it is wrong.

It appears that the confusion was started by an engraving of the printing carried out for a late 19th century edition of the collected works of Christiaan Huygens. Here Christiaan's name is placed at the upper left, contradicting an earlier engraving; the original painting does not associate names with the four boys. However, family iconography of the time demanded that the eldest son be placed to his father's right, i.e., on the left side of the portrait. This identification of the eldest brother Constantijn as the boy on the upper left, and thus Christiaan as the one on the right, seems to be confirmed by a comparison of the painting with other portraits from the same period.

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This article had originally appeared in Nieuw Archief voor Wiskunde *(5) 6 (2005), 117–123, and the Newsletter thanks the editor for the permission to reproduce it. It is based on the inaugural lecture given by Hansjörg Geiges [geiges@math.uni-koeln.de] on taking up his post as Professor of Mathematics at the Universität zu Köln, Germany, in 2002. Before then, he held the chair in geometry and topology at the Universiteit Leiden, the Netherlands. This personal connection and his own research in contact geometry explain the fascination of the author with Huygens.*

Global Analytic Geometry

Jérôme Poineau (Université de Rennes 1, France)

The theory of complex analytic functions of one variable was born in the 19^{th} *century and has been extensively studied by a number of great mathematicians up until the present day. As time has passed, it has grown closer to the theory of Riemann surfaces and complex analytic geometry in general. For instance, if you are looking for some properties of a polynomial P with coefficients in* **C***, it may be of some use to think about it as a function over the space* **C** *or even as a ramified cover of the space* **C** *by itself:*

$$\begin{array}{ccc} \mathbf{C} & \to & \mathbf{C} \\ z & \mapsto & P(z) \end{array}.$$

Complex analytic geometry is very useful in this context but is it possible to adapt the same techniques to deal with objects other than complex analytic functions? For example, could we handle integers this way?

Looking for a space

The first problem that emerges is that it is hard to think of a space on which integers should be functions. However, Grothendieck's theory of schemes provides an answer: the set of all prime numbers, denoted Spec($\mathbf{Z}$). If n is an integer and x_p a point of Spec($\mathbf{Z}$) associated to a prime number p, we set

$$n(x_p) = n \mod p \text{ in } \mathbf{F}_p.$$

We have built a function out of the integer n. Let us point out that the value of the function lies in a field that depends on the point where it is evaluated!

However, if we want to have a faithful analogy between complex analytic functions and numbers, we have to go further and build a sort of projective space. What kind of properties would we like to have? Let us recall the following result. Let f be a non-zero meromorphic function over $\mathbf{P}^1(\mathbf{C})$. For z in $\mathbf{P}^1(\mathbf{C})$, let us denote by $v_z(f) \in \mathbf{Z}$ the order of vanishing of the function f at the point z. Then we have

$$\sum_{z \in \mathbf{P}^1(\mathbf{C})} v_z(f) = 0.$$

How does this formula translate in terms of numbers? Let p be a prime number. We define a valuation v_p on $\mathbf{Q}$ in the following way. Let k be a non-zero rational number. There exists a unique triple (n,a,b) in $\mathbf{Z}^3$, with $(p,ab) = (a,b) = 1$ such that

$$k = p^n \frac{a}{b}.$$

We set $v_p(k) = n$. Unfortunately, if we now compute the sum of all the integers $v_p(k)$, with p describing the set of all prime numbers, we are not going to find anything interesting. However, if we add some coefficients, we get the so-called *product formula*:

$$\sum_{p \in \mathrm{Spec}(\mathbf{Z})} \log(p) v_p(k) - \log(|k|_\infty) = 0,$$

where $|.|_\infty$ denotes the usual absolute value. It seems that in the space Spec($\mathbf{Z}$) there is a point missing and that it corresponds to the usual absolute value. This point of view is used in Arakelov geometry (see [4]).

Berkovich spaces

We are now going to present another way of defining an analytic space on which numbers are functions. At the end of the eighties, Vladimir G. Berkovich defined analytic spaces over any valued field (see [1] and [2]). The spaces he constructed share some remarkable topological properties with complex analytic spaces such as separatedness, local compactness and local arcwise connexity. For our purpose, it is important to note that the points of the Berkovich spaces are defined in terms of absolute values! They soon became of great interest and have now been used in various domains of mathematics: arithmetical geometry, dynamical systems, resolution of singularities, etc. We advise the reader who desires a thorough introduction to the subject to look at [3].

As Berkovich himself pointed out, the construction he gave made sense for the ring of integers $\mathbf{Z}$. It also makes sense for the ring of integers of any number field and everything works the same way, so we are not going to say anything more about them. Let us now give the definition. The **analytic spectrum** of $\mathbf{Z}$ is the set of multiplicative semi-norms on $\mathbf{Z}$, *i.e.* the set of all the applications of the form $|.| : \mathbf{Z} \to \mathbf{R}_+$ satisfying the following properties:

(i) $|0| = 0$ and $|1| = 1$.

(ii) $\forall f, g \in \mathbf{Z}$, $|f+g| \le |f| + |g|$.

(iii) $\forall f, g \in \mathbf{Z}$, $|fg| = |f|\,|g|$.

The analytic spectrum $\mathcal{M}(\mathbf{Z})$

By Ostrowski's theorem, we can give an explicit description of the points of the set $\mathcal{M}(\mathbf{Z})$. We have

1. The trivial absolute value $|.|_0$ ($|0|_0 = 0$ and $|f|_0 = 1$ if $f \neq 0$).
2. The Archimedean absolute values $|.|_\infty^\varepsilon$, for $\varepsilon \in \,]0,1]$.
3. For every prime number p, the p-adic absolute values $|.|_p^\varepsilon = p^{-\varepsilon v_p(.)}$, for $\varepsilon \in \,]0,+\infty[$.
4. For every prime number p, the semi-norm $|.|_p^{+\infty}$ induced by the trivial absolute value on $\mathbf{F}_p$ ($|f|_0 = 0$ if $f = 0 \mod p$ and $|f|_0 = 1$ if $f \neq 0 \mod p$).

By definition, you get a base of the topology of $\mathcal{M}(\mathbf{Z})$ by taking the following sets:

$$\bigcap_{1\le i\le u} \left\{|.|_x \in \mathcal{M}(\mathbf{Z}) \,\middle|\, |f_i|_x < r_i\right\} \cap \bigcap_{1\le j\le v} \left\{|.|_x \in \mathcal{M}(\mathbf{Z}) \,\middle|\, |g_j|_x > s_j\right\},$$

with $u,v \in \mathbf{N}$, $f_1,\dots,f_u,g_1,\dots,g_v \in \mathbf{Z}$ and $r_1,\dots,r_u,s_1,\dots,s_v \in \mathbf{R}$. A little thinking shows that the space $\mathcal{M}(\mathbf{Z})$ is composed of branches (one for each prime number and one for the usual absolute value) homeomorphic to $[0,1]$ that meet at one point. The neighbourhoods of this central point are a little tricky: they contain entirely all the branches except a finite number of them. To convince yourself, try drawing the following open set

$$\left\{|.|_x \in \mathcal{M}(\mathbf{Z}) \,\middle|\, |2|_x > \frac{1}{2}\right\}.$$

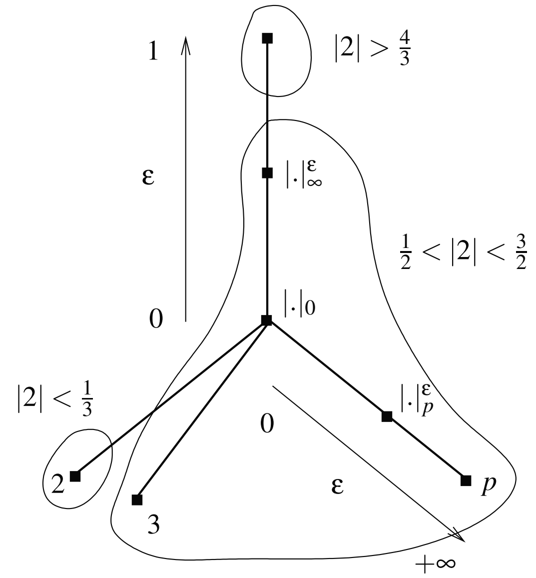

Figure 1. Open subsets of $\mathcal{M}(\mathbf{Z})$

Functions over $\mathcal{M}(\mathbf{Z})$

We have now defined the topological space $\mathcal{M}(\mathbf{Z})$. It is clear what the *absolute value* of an integer is at any point of this space but we still have to define what its *value* is. Let x be a point of $\mathcal{M}(\mathbf{Z})$ and $|.|_x$ the associated multiplicative semi-norm on $\mathbf{Z}$. The set

$$\mathfrak{p}_x = \left\{f \in \mathbf{Z} \,\middle|\, |f|_x = 0\right\}$$

is a prime ideal of the ring $\mathbf{Z}$. The quotient $\mathbf{Z}/\mathfrak{p}_x$ is a domain on which the semi-norm $|.|_x$ induces an absolute value. We define the field $\mathcal{H}(x)$ to be the completion of the field of fractions of $\mathbf{Z}/\mathfrak{p}_x$ with regard to the absolute value $|.|_x$. By construction, we have a morphism

$$\mathbf{Z} \to \mathcal{H}(x),$$

which we call the **evaluation morphism**. Again, as in the case of schemes, the value of an integer lies in a field that depends on the point of evaluation.

Now that we have defined what the value of a function at a point is, we can define analytic functions. Let U be an open subset of $\mathcal{M}(\mathbf{Z})$. An **analytic function over** U is an application

$$U \to \bigsqcup_{x\in U} \mathcal{H}(x),$$

which is locally a uniform limit of rational functions (*i.e.* elements of $\mathbf{Q}$) with no poles.

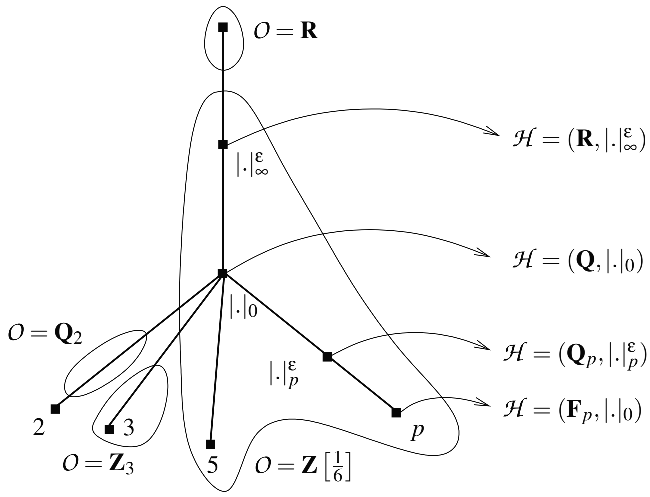

Figure 2. Analytic functions over $\mathcal{M}(\mathbf{Z})$

The analytic line $\mathbf{A}^{1,\mathrm{an}}_{\mathbf{Z}}$

Let us now mention that it is also possible to construct an analytic line over $\mathbf{Z}$. The definition is due to V. Berkovich and can be obtained from the former by changing every $\mathbf{Z}$ into a polynomial ring $\mathbf{Z}[T]$. We invite the reader interested in the details and applications to consult [5].

The topological space underlying $\mathbf{A}^{1,\mathrm{an}}_{\mathbf{Z}}$ is rather intricate and we are not going to describe it. However, one can prove that it is Hausdorff, locally compact, locally arcwise connected and of topological dimension 3. Let us say a few words about the analytic functions. Typical examples are *convergent arithmetic power series*, *i.e.* elements of

$$\mathbf{Z}\left[\frac{1}{N}\right][\![T]\!],$$

for $N \in \mathbf{N}^*$, with positive radius of convergence with reference to the usual absolute value $|.|_\infty$ and the p-adic absolute value $|.|_p$, for any prime number p. In the same way that complex analytic geometry is useful to study complex analytic functions, we claim that the analytic line $\mathbf{A}^{1,\mathrm{an}}_{\mathbf{Z}}$ over $\mathbf{Z}$ carries much information about convergent arithmetic power series. To conclude, let us give an example of such an application from the author's thesis. We denote by D the open unit disc in $\mathbf{C}$.

Theorem. *Let E and F be two disjoint, locally finite subsets of D that do not contain* 0. *Let $(n_a)_{a\in E}$ be a family of positive integers and $(P_b)_{b\in F}$ be a family of polynomials with no constant term. We assume that*

(i) For any $a \in E$, $\bar{a} \in E$ and $n_{\bar{a}} = n_a$.

(ii) For any $b \in F$, $\bar{b} \in F$ and $P_{\bar{b}} = \overline{P_b}$.

Then there exist two series g and h in $\mathbf{Z}[\![T]\!]$ that satisfy the following properties:

(a) *The function* $f = g/h$ *is meromorphic over* D *and holomorphic over* $D \setminus F$.

(b) *For any* $a \in E$, *the function* f *vanishes at* a *with order at least* n_a.

(c) *For any* $b \in E$, *the function* $f(z) - P_b$ *is holomorphic in the neighbourhood of* b.

(d) *The function* f *is holomorphic in the neighbourhood of* 0 *and its Taylor expansion has coefficients in* $\mathbf{Z}$.

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Jérôme Poineau [jerome.poineau@univ-rennes1.fr] studied mathematics at École polytechnique, Paris. He continued his work as a graduate student at Université de Rennes (France) with a PhD on Berkovich spaces over the integers.

Atle Selberg (1917–2007)

Nils A. Baas (Trondheim, Norway)

Atle Selberg, 7th of June 2007

The renowned Norwegian mathematician Atle Selberg died on 6 August 2007 at his home in Princeton, New Jersey, USA. He was 90 years old.

Atle Selberg was one of the greatest mathematicians of the 20th century. His contributions to mathematics are so deep and original that his name will always be an important part of the history of mathematics. His special field in mathematics was number theory in a broad sense.

Selberg was born on 14 June 1917 in Langesund, Norway. He grew up near Bergen and went to high school at Gjøvik. His father was a high school teacher with a doctoral degree in mathematics, and two of his older brothers – Henrik and Sigmund – became professors of mathematics in Norway. Already when he was 12 years old he studied mathematics at the university level. When he was 15 he published a little note in *Norsk Matematisk Tidsskrift*.

He studied at the University of Oslo where he obtained the cand. real. degree in 1939 and in the fall of 1943 he defended his thesis, which was about the Riemann Hypothesis. At that time there was little numerical evidence supporting the Riemann Hypothesis. By his new techniques Selberg proved that a positive fraction of the zeros must lie on the critical line. This result led to great international recognition.

After the war Carl Ludwig Siegel, who had stayed in the USA, asked Harald Bohr what had happened in mathematics in Europe during the war. Bohr answered with one word: "Selberg".

In 1947 Selberg went to the Institute for Advanced Study in Princeton, USA. At this time he had developed his famous sieve method. An application of this resulted in Selberg's Fundamental Formula, which in 1948 led to an elementary proof of the Prime Number Theorem. This was a sensation since even the possibility of an elementary proof had been questioned by Hardy and other mathematicians.

For these results he was awarded the Fields Medal in 1950, at the time the highest award in mathematics. He became a permanent member of the Institute for Advanced Study in 1949 and a professor in 1951, a position he held until he retired in 1987.

In the early 1950s Selberg again produced a new and very deep result, namely what is now called the Selberg Trace Formula. This result has had many important implications in mathematics and theoretical physics. Here Selberg combines many mathematical areas like automorphic forms, group representations, spectral theory and harmonic analysis in an intricate and profound manner. Selberg's Trace Formula is considered by many mathematicians as one of the most important mathematical result of the 20th century.

In 2003 Selberg was asked whether he thought the Riemann Hypothesis was correct. His response was: "If anything at all in our universe is correct, it has to be the Riemann Hypothesis, if for no other reasons, so for purely aesthetical reasons."

In addition to the Fields Medal (1950) Selberg received the Wolf Prize in 1986 and an honorary Abel Prize in 2002 prior to the regular awards. He was also a member of numerous academies. Atle Selberg was highly respected in the international mathematical community. He possessed a natural and impressive authority that made everyone listen to him with the greatest attention.

He loved his home country Norway and always spoke dearly of Norwegian nature, language and literature. In 1987 he was named Commander with Star of the Norwegian St. Olav Order.

Norway has lost one of her greatest scientists through all times.

Nils A. Baas *[baas@math.ntnu.no] is a professor of mathematics at the Department of Mathematical Sciences at the Norwegian University of Science and Technology in Trondheim. Fields of interest: algebraic topology and complex systems. He has visited IAS in Princeton several times; recently, in the spring of 2007, as a Member. He was a close friend of Atle Selberg for many years. In November 2005, Baas and his colleague C. Skau interviewed Selberg at length (twelve hours, tape and video) about his life and mathematics. The interview is in Norwegian and it will be translated into English.*

Leonhardi Euleri Opera Omnia: a centenary project

Andreas Kleinert (Martin-Luther-Universität Halle-Wittenberg, Germany) and Martin Mattmüller (Euler-Archiv Basel, Switzerland)

During his long career as a mathematician and scientist, Leonhard Euler published more than 500 research papers and about two dozen books. When he died in 1783, he left behind about 300 more articles in manuscript form; most of these were haphazardly printed during the following eighty years. Several 19th century projects for the publication of Euler's collected works failed; only in 1907, at the celebrations of Euler's 200th birthday, an initiative taken by Ferdinand Rudio led to the establishment of the Euler Edition under the wing of the Swiss Naturalist Society (now the Swiss Academy of Sciences). The current Euler Tercentenary Year therefore sees the 100th birthday of the edition of Euler's Collected Works, which is nearly completed.

Leonhard Euler. Engraving by Friedrich Weber (1851) after Emanuel Handmann's 1756 portrait in the Old Aula of Basel University (courtesy of Emil A. Fellmann, Basel)

Euler's life and writings

Leonhard Euler was born in Basel on 15 April 1707. At the age of 13 he attended Basel University where he progressed rapidly in mathematics and physics. In 1727 he was appointed to the newly founded Russian Academy at St. Petersburg. In 1741 he moved to the Berlin Academy, which was revived by Frederick II, and in 1766 he returned to Petersburg, where he died on 18 September 1783.

Euler was a tremendously productive scientist: the works published during his lifetime comprise more than 500 research papers, published mainly in the journals of the most prestigious scientific academies throughout Europe. Although he never had regular teaching obligations, he authored influential textbooks on a great variety of subjects including differential and integral calculus, mechanics, ballistics and acoustics, astronomy, the theory of music and ship-building, as well as the *Letters to a German Princess*, a three-volume compendium of his century's views on natural science. The flow of Euler's creativity was not even curbed by the almost complete loss of his eyesight in 1771.

There is no doubt that Leonhard Euler belongs among the great scientists of all time. His work exhibits a unique combination of broad interests and brilliant insights. It displays original ways of tackling challenges and incredible persistence in the pursuit of his ideas and it shows a profound yet sympathetic appreciation of his predecessors' and colleagues' achievements. Euler is chiefly remembered as the leading mathematician of his time but his works also comprise ground-breaking contributions to physics, astronomy and engineering. Moreover, his vast correspondence yields fascinating insights into the development of his ideas and into the scientific community of the 18th century.

Shortly before his death, Euler predicted that the Petersburg Academy would take at least twenty years to publish all the manuscripts he was leaving behind. It turned out that this prediction was too optimistic: only in 1830, nearly fifty years after Euler's death, was the stock of unpublished Euler manuscripts exhausted. In 1844, Euler's great-grandson Paul-Heinrich Fuss found another 60 manuscripts in his attic, which he published in 1862 in two volumes under the title *Opera postuma*.

In the early 20th century, the Swedish mathematician Gustav Eneström compiled the standard inventory of Euler's writings. This inventory, which is generally referred to as the Eneström Index, was published between 1910 and 1913. The 866 publications listed by Eneström include a certain number of correspondences published in the 19th and early 20th century that were not really printed publications of Euler. When we omit these, the number of Euler's publications – not counting second editions, translations and the like – amounts to about 850. Since the publication of the Eneström Index, only one more printed publication by Euler has been identified: an anonymously published paper on gravity that had escaped the attention of the Swedish mathematician.

First attempts at editing Euler's works

The first attempts to publish Euler's complete works go back to the 1830s. Two such initiatives were launched simultaneously. One of them was started by Paul-Heinrich Fuss, who was the permanent secretary of the Petersburg Academy of Science. Although Fuss was encouraged by many prominent mathematicians, including Carl Gustav Jacobi, the project was finally abandoned when it turned out that it would surpass the financial capacities of the academy's budget. The only result of Fuss's and Jacobi's initiative was the publication, in 1849, of two volumes of *Commentationes arithmeticae*, which include 94 articles that had already been published and five unpublished manuscripts.

At the same time, a group of Belgian mathematicians was undertaking a similar project, and they were more successful than the Russians in that five volumes of this edition were indeed printed (*Œuvres complètes de L. Euler*, 5 vol., Bruxelles 1838–1839). However, this edition has been sharply criticized, in particular by the Belgian historian of mathematics Henri Bosmans who qualified it as a very poor piece of workmanship[1]. With the intention of making Euler's works accessible to a large section of the public, the editors had arbitrarily manipulated the original texts, even where the original was already written in French.

All these would-be editors of Euler's works had one thing in common: their main aim was to make Euler's work accessible to contemporary scientists and in particular to mathematicians. They believed that Euler's writings would still stimulate mathematical research and that mathematicians should study his works with unremitting intensity, according to Laplace's famous invitation, "Read Euler, read Euler, he is the master of us all."

At the beginning of the 20th century, with the approach of the bicentenary of Euler's birth, the Russian Academy of Science launched a new initiative for the publication of Euler's complete works. Bearing in mind the failure of previous attempts, the Russians looked for allies with whom they could share work and expense; of course the institution that came to mind with regard to Euler was the Prussian Academy of Science in Berlin, where Euler had served for 25 years. Initially the Berlin academicians were quite enthusiastic about this plan but when it turned out that the Russians wanted to divide the task so that they would publish the mathematical works, while leaving the physical writings to the Germans, the Berlin Academy asked the most distinguished physicist among its members, Max Planck, for advice. In a famous statement, Planck said that it might be true that mathematicians still get inspiration from Euler's writings but that was no longer the case with the physicists. He argued that the publication of Euler's physical writings was "not in the interest of physics as a science of our time" and as a result of this statement, the Prussian Academy declined to participate in funding the project. Since a complete edition was too expensive for the Russian Academy, this initiative also ended in failure.


Vollſtändige
Anleitung
zur
Algebra
von
Hrn. Leonhard Euler.
Zweyter Theil.
Von Auflöſung algebraiſcher Gleichungen
und der unbeſtimmten Analytic.
St. Petersburg.
gedruckt bey der Kayſ. Acad. der Wiſſenſchaften 1770.

Title page of Euler's German algebra textbook (1770), Part 2 (Euler-Archiv Basel)

The Euler Committee and the beginnings of the *Opera Omnia* edition

Meanwhile, a professor of mathematics at the Zürich Polytechnicum, Ferdinand Rudio, had untiringly lobbied for an initiative that ultimately turned out to be successful. On every occasion, and most particularly at the First International Congress of Mathematicians, which was held at Zürich in 1897, Rudio had urged the worldwide community to honour its obligations toward the great scientist by undertaking the edition of Euler's complete works. When the city of Basel commemorated Euler's 200th birthday in 1907, Rudio delivered a thrilling speech in which he appealed to Swiss patriotism and to international solidarity in favour of an edition of Euler's works: "Switzerland will always be grateful to the academies of Berlin and St. Petersburg for having given our Euler, to whom his native country was too narrow,

[1] Henri Bosmans: *Sur une tentative d'édition des œuvres complètes de L. Euler faite à Bruxelles en 1839*, Louvain 1909.

the opportunity to perform his outstanding work”[2]. He addressed his speech in particular to the representatives of the Swiss Naturalist Society (Schweizerische Naturforschende Gesellschaft, SNG, now the Swiss Academy of Science, SCNAT) and to the representatives of the academies of Berlin and St. Petersburg who assisted at the ceremony.

These were the right words on the right occasion. The Naturalist Society decided that the publishing of Euler’s work was a duty of honour (“Ehrenpflicht”) for them and they installed a committee (Euler-Kommission) that was in charge of realizing the project. One year later, their plan was strongly encouraged by the 4th International Congress of Mathematicians in Rome. In an official resolution, the congress declared that it would be of the greatest importance for pure and applied mathematics to make Euler’s works available in a new and complete edition.

The first step taken by the Euler Committee was a fund-raising effort. They printed so-called *Zeichnungsscheine* (subscription bills) that were sent to public institutions, enterprises, companies and individuals. The addressees of these *Zeichnungsscheine* were requested to indicate the amount that they would be prepared to contribute and they were informed that the society’s decision to launch the project would critically depend upon the total amount of funds promised by the donors.

The response to this campaign was noteworthy: 93,500 Swiss Francs were offered by donators in Switzerland and 31,500 Francs from other countries. A great number of individuals subscribed to the edition in advance and each of the three academies of Berlin, Paris and St. Petersburg signed a subscription for 40 copies. The total amount promised by the subscribers was nearly three times as much as the donations, i.e. about 300,000 francs. So the finances were assured at least for a mid-range perspective.

Money was not the only precondition for realizing such a project; a certain number of qualified people able and willing to do the work would also be needed. In this respect, the Euler Committee was equally successful: twenty mathematicians of international reputation spontaneously agreed to serve as editors of one or more volumes, including Jacques Hadamard from Paris, Gustaf Eneström from Stockholm, Tullio Levi-Civita from Padova, Gerhard Kowalewski from Prague and Heinrich Weber from Strasbourg (he was to be the editor of the first volume published in 1911). A Redaction Committee was established, composed of the scholars who were working on the individual volumes.

When work on the edition started in 1908, the committee optimistically envisaged that there would be 43 volumes altogether and that a single volume would cost no more than 25 Francs. However, a few years later, after the publication of the Eneström Index, it turned out that they had considerably underestimated the size of Euler’s written legacy. In 1913, the estimated number of volumes was increased to 66 and within the following years the number of volumes needed was raised to 72.

The edition was divided into three series:

I. Mathematics (29 volumes)
II. Mechanics and astronomy (31 volumes)
III. Physics and miscellaneous (12 volumes)

The first volume was published in 1911: it comprised Euler’s *Vollständige Einleitung zur Algebra*, with Lagrange’s supplements to this work, and the *Eulogy of Euler* by Nicolaus Fuss. Before the outbreak of World War I, twelve volumes were published.

Completing the Collected Works series

Inevitably, the impulse the huge enterprise had been given by the enthusiasm of its founders did not carry it over all the obstacles presented by the 20th century. The first generation of editors passed the work on, to name just a few of the most prominent contributors, to Georg Faber, Henri Dulac, Rudolf Fueter and Constantin Carathéodory in mathematics, and Charles Blanc and Clifford Truesdell in physics. Andreas Speiser (in 1928), then Walter Habicht (1965) and Hans-Christoph Im Hof (1985) succeeded Rudio as “General Redactors”.

The continuous publication of the Euler volumes was not only burdened and slowed down by the effects of two wars: in 1931, the Christ-Paravicini bank, where the Euler Committee had deposited its funds, crashed and the committee lost 80,000 Francs. For political and financial reasons, the publisher also had to be changed several times. Until 1935, the publisher Teubner in Leipzig was in charge of the edition. The volumes printed between 1935 and 1950 were published jointly by Teubner and Orell Füssli (Zürich). From 1952 to 1974, Orell Füssli was the sole publisher and in 1975, the edition went on to Birkhäuser (Basel).

With time it became more and more difficult to find qualified editors: beginning in the 1950s, mathematicians

Shelf with *Opera Omnia* at Euler-Archiv Basel

[2] “Die Schweiz wird der Petersburger und der Berliner Akademie stets das Gefühl der Dankbarkeit bewahren, dass sie unserm Euler, für den das eigene Vaterland zu klein war, ein grösseres geboten und ihm die Möglichkeit bereitet haben, in ungetrübter Schaffensfreudigkeit sein grosses Lebenswerk zu vollenden.” In *Vierteljahrsschrift der Naturforschenden Gesellschaft zu Zürich*. 52 (1907), p. 541.

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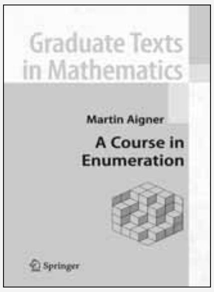

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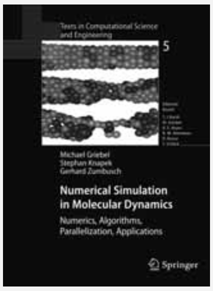

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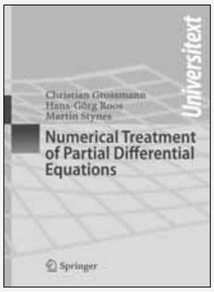

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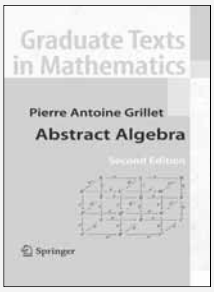

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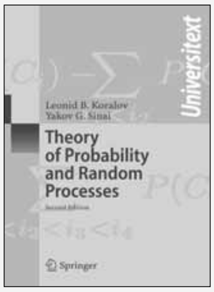

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who could read Latin were rapidly becoming a threatened species. This led to the consequence that for the volumes published after 1960, mathematicians and physicists were increasingly replaced as editors by professional historians of science like Emil Fellmann, Otto Fleckenstein, Clifford Truesdell, David Speiser, Eric Aiton, Patricia Radelet-de Grave and Karine Chemla.

With the background of the editors the scope of the edition itself also underwent a change. In the beginning of the edition its promoters had mainly intended to make Euler's works accessible to scientists and in particular to mathematicians, but for several reasons this justification has lost much of its importance.

On the other hand, the Euler edition has increasingly become a valuable reference tool for professional historians of science who have, in a certain way, taken over from mathematicians as readers as well as in the role of editors. As a consequence of this change, the more recent volumes are characterized by more thorough introductions and more extensive footnotes and commentaries.

For the founders of the Euler edition and for the first generation of editors, the main purpose of the edition had been to make the original text widely available; commentaries were kept to a strict minimum. In one paragraph of the Editorial Outline of 1910, it was clearly said the annotations should not degenerate into long historical treatises. This sound principle was increasingly abandoned as historians of science replaced scientists as editors. Some of them used this occasion as an opportunity for presenting all their knowledge and erudition, and there is even one 435-page volume (II/11.1 by Truesdell) that does not include a single line by Euler: instead it contains just an, admittedly important, historical treatise on the history of elastic bodies between 1639 and 1788.

The distribution of the volumes published over the years is given in the following table:

1911–1912:	12 volumes
1915–1919:	2 volumes
1920–1927:	8 volumes
1928–1931:	–
1932–1940:	4 volumes
1941–1946:	4 volumes
1947–1960:	21 volumes
1961–1979:	16 volumes
1980–1990:	–
1990–2004:	3 volumes
2008/09(?):	2 volumes
Total	**72 volumes**

In the present year, which sees both the Tercentenary of Leonhard Euler's birth and the Centenary of the Euler Committee, the edition of Euler's printed works in the *Opera Omnia* is nearly finished. The missing volumes 26 and 27 of series II, comprising Euler's papers on perturbation theory in astronomy, are currently being prepared for the press. Hopefully the editor Andreas Verdun will finish his manuscript this year and the volumes will be out in 2008/09.

Editing Euler's correspondence and manuscripts

In the first project of 1910, it was mentioned that Euler's scientific correspondence should be included in the publication of the *Opera Omnia*. But the plan did not specify what should be defined as "scientific" correspondence and it was decided to postpone the issue to a later date. Priority was given to the publication of Euler's printed works.

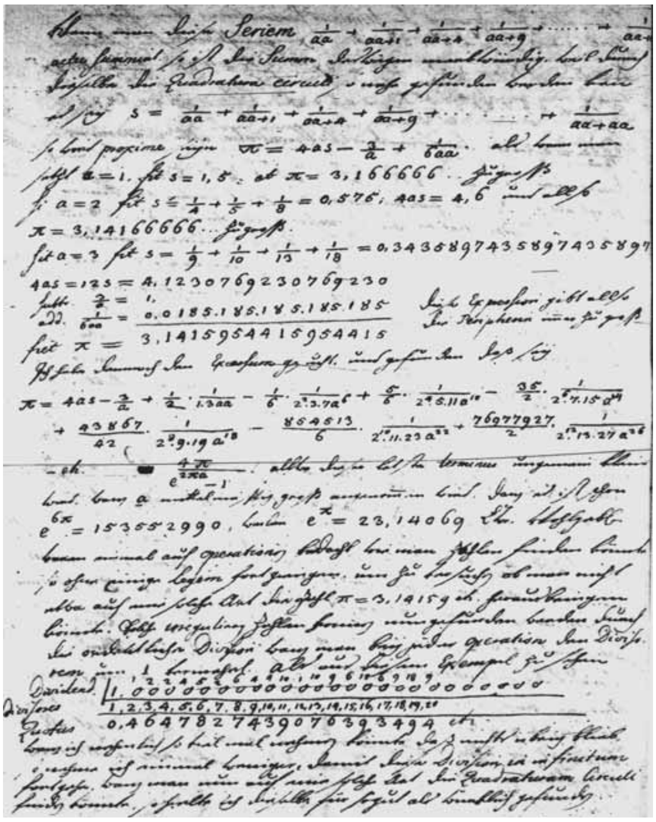
Euler's extant correspondence contains approximately 3100 letters exchanged with nearly 300 correspondents; about 1000 letters are by Euler. Most of the letters are from the time when Euler lived in Berlin (1741–1766).

The people he corresponded with most include (the numbers in parentheses are the letters written by/addressed to Euler): Daniel Bernoulli (19/81), Christian Goldbach (102/94), Pierre Louis Moreau de Maupertuis (124/5), Gerhard Friedrich Müller (111/101), Johann Daniel Schumacher (176/131) and Johann Andreas Segner (0/159). Only three of these are of interest for historians of mathematics: Bernoulli, Goldbach, and Segner. The remaining three mostly talk about academy business matters; Schumacher and Müller were officials of the Petersburg Academy and Maupertuis was the president of the Prussian academy.

The first 20th century initiative for publishing part of Euler's correspondence started in the 1960s. It was not related to the *Opera Omnia* edition but resulted from a cooperation between the Leningrad Section of the Soviet Academy of Science and the Academy of Science of the German Democratic Republic, which considered itself the successor to the Prussian Academy. On the occasion of Euler's 250th anniversary, these two academies decided to publish all those letters of Euler that were related to their cooperation in the 18th century. The resulting three volumes, with more than 600 letters, were published by Adolf P. Juškevič and Eduard Winter between 1959 and 1976 under the title "Die Berliner und die Petersburger Akademie der Wissenschaften im Briefwechsel Leonhard Eulers".

In 1965, the same editors republished the correspondence between Euler and Christian Goldbach, which had already been printed by Fuss in the 19th century. Furthermore the Soviet Academy of Science published three other volumes concerning Euler's correspondence: *Pis'ma k ucenym* (edited by T. Klado et al. in 1963: 49 letters addressed by Euler to 19 scientists); *Relations scientifiques russo-françaises* (edited by A.P. Juškevič et al. in 1968: including Euler's correspondence with Delisle); and *Perepiska* (edited by A.P. Juškevič et al. in 1967: an inventory with summaries of all the correspondence at the Leningrad Archive).

Encouraged by these Soviet and East German activities, the Swiss Euler Committee finally decided in 1967 to start an additional series of the *Opera Omnia*, which was



Manuscript page from a letter by Euler to Christian Goldbach (Euler-Archiv Basel)

to contain Euler's correspondence and manuscript heritage. It was planned that this series IV should be divided into two sub-series: IVA for the correspondence and IVB for the unpublished manuscripts.

Since most of the original letters addressed to Euler were preserved in the Leningrad Archive of the Soviet Academy of Science, and there was a considerable number of Euler experts living in the Soviet Union, the new Series IV was set up as a joint project of the Swiss and the Soviet Academies. A second redaction committee was established, composed of four members from the USSR and four from Switzerland. This committee, which was to be exclusively responsible for Series IV, was chaired by Walter Habicht and then in 1985 by Emil Fellmann, who was also the director of the Euler Archive in Basel.

The first decision of this committee was to postpone the publication of the manuscript sub-series IVB and to focus on the correspondence. The following guidelines for the publication of Euler's correspondence were set up:

1. The correspondence is **not** published in a general chronological order; instead every volume will include the exchange of letters with one or more correspondents.
2. Earlier decisions, in particular concerning the scientific or non-scientific character of some letter, have been revised several times; therefore **all** the letters to and from a certain correspondent will be edited if a correspondence is published.
3. For each volume, there will be a "working language" for the introduction, footnotes and commentaries. As a general rule, this will be the language of most of the letters in the respective volume. Consequently, German was chosen as the working language for volumes 2, 3 and 8 and French for volumes 5, 6 and 7.
4. The text of the letters will be published in full (including the phrases of civility at the beginning and at the end, which had often been omitted in former editions) and in the original language. Only letters in Latin will additionally be translated into the working language of the volume.

An exception was made for the Goldbach correspondence. The redaction committee and the editors were convinced that this correspondence includes so many ideas and suggestions that are of interest for modern mathematicians (in particular in number theory) that it ought to be accessible to a worldwide community of scientists and not only to historians of science. Hence it was decided to choose English as the working language for this volume (volume IVA/4) and to translate all the letters into English, in addition to the original text, which is written in a curious mixture of German, Latin and French.

The first volume of series IVA was published in 1975: it is an inventory of all Euler's correspondence known at that time. For each letter, it gives a short summary and some information about date, language, existing copies, where the original is located and whether it has already been published.

Five years later, the first "proper" correspondence volume appeared: volume IVA/5 includes Euler's correspondence with Clairaut, d'Alembert and Lagrange, edited by René Taton and A.P. Juškevič. In 1986, Pierre Costabel, Eduard Winter, A.T. Grigorjan and A.P. Juškevič published Euler's correspondence with Maupertuis and Frederick II (volume IVA/6) and in 1998 volume IVA/2 presented Euler's correspondence with Johann and Nicolaus Bernoulli, edited by Emil Fellmann and G.K. Mikhajlov.

Due to problems with funding and the recruitment of qualified editors, no further volume of series IVA has been published since 1998. The funds gathered at the beginning of the 20th century, fed from time to time by donations and by revenue from the sale of the printed volumes, were originally **not** intended for salaries. Their purpose was to cover the expense for collecting the sources and for the actual printing of the volumes. But nobody was paid at that time for their time spent in collaborating: the editors of series I–III were mostly mathematicians who had stable positions as researchers or university professors and who considered it an honour to contribute to the Euler Edition.

When the correspondence series started, it became more and more difficult to find fully qualified editors: there are simply not so many people around who possess the special skills required for this work. They should be familiar with the mathematics, physics and/or astronomy of the 18th century, have a solid knowledge of Latin, French and German and be able to read 18th century handwriting, which is often a challenge in itself.

So it turned out that all of the editors and most of the collaborators of the correspondence volumes were retired university professors; some of them began cooperating with the Euler Committee shortly before their retirement, hoping that they would soon be free from other obligations and could concentrate on this work. In principle, this is a good concept: such people have long experience with these matters and are financially independent. The major disadvantage of this principle, however, is that the Euler Committee has virtually no leverage in motivating them to finish their work within a reasonable amount of time and regrettably many of them pass away before the work is done.

A typical example is afforded by the history of volume IVA/7, which was delayed by a long series of misfortunes. About twenty years ago, one of the authors (A.K.) was asked by the editors A.P. Juškevič and René Taton to take care of Euler's correspondence with the Geneva physicist Georges-Louis Lesage (9 letters) to be published in this volume. He submitted his manuscript in 1992. At that time, Pierre Speziali, a retired mathematician and historian of mathematics at the University of Geneva, was working on the correspondence with Gabriel and Philibert Cramer. In 1993 Juškevič died, and on the request of René Taton, Andreas Kleinert was made co-editor of the whole volume. Speziali died in 1995 and Taton, who was taking care of several correspondences of that volume, passed away in 2005, not to mention other deceased collaborators like Mirko Grmek, Roselyne Rey and Pierre Costabel, who all left piles of unfinished manuscripts behind. Last year, the Euler Committee finally decided to charge a young scholar, Siegfried Bodenmann, with the task, giving him a half-time paid position in Basel. He is a native speaker of French and grew up near Geneva and we hope he will bring the work to an end in 2007 or 2008. Acquiring the money for this half-time position was, however, not an easy task.

For the work to be done on the other outstanding volumes, the committee has obtained funds for two more part-time positions: Martin Mattmüller, besides serving as the secretary of the committee, participates in editing the Euler-Goldbach correspondence together with Günther Frei, a retired professor of the University of Québec and a specialist in number theory. The other paid editor Thomas Steiner is working on the Segner correspondence.

In view of the vast amount of letters remaining to be published, the perspectives for the future are not very promising. Funding is now secured for volumes IVA/3 (Daniel Bernoulli), IVA/4 (Goldbach), IVA/7 (various correspondence in French) and IVA/8 (Euler's correspondence with Segner and other scientists from Halle). All these volumes will hopefully come out before 2012.

But the Swiss National Science Foundation, which is now paying for much of the editorial work, will probably not continue to finance collaborators after 2012. This is regrettable because some of the unpublished correspondence is a real treasure.

As an example, let us mention the Euler-Knutzen correspondence, which could easily fill a whole volume of series IVA. Martin Knutzen (1713–1751) died at a young age and apart from his publications we have very few original sources or documents about him. However, as a professor of philosophy at the University of Königsberg, Knutzen was one of the most influential academic teachers of Immanuel Kant and, as far as we know, it was through Knutzen that Kant became familiar with Newtonian physics and with the philosophy of Leibniz. The topics of his letters to Euler include physics, astronomy, philosophy and also details about Knutzen's private life and various events at the university of Königsberg. The correspondence consists of 72 letters from Knutzen and two letters from Euler.

Thus there are important and interesting parts of Euler's correspondence that we will probably not be able to publish on the same basis as the four volumes already in print and the four that are currently being prepared. Moreover, the series IVB comprising the rest of Euler's manuscript heritage cannot at present be tackled with confidence. It is the fate of large-scale projects like the edition of Euler's collected works that their aims, the standards in realizing them and the means available are constantly changing.

So will we have to leave the project outlined in 1907 unfinished? Yes and no. Unless some new way of financing the editorial work and the actual printing is found, no more volumes will be added to the imposing row of *Leonhardi Euleri Opera Omnia* after 2012. However, the new possibilities of electronic publishing give us a chance to realize the principal goal set by Ferdinand Rudio and his colleagues, i.e. making all of Euler's writings available to the scientific community.

The Euler Committee is currently working on a project to present this important heritage in an electronic database of scans on the Internet, made accessible by a reliable inventory. Hopefully the task that the mathematical community undertook a century ago will be completed in a few years, for the benefit of 21st century mathematicians and historians of science but also as a monument to the lasting glory of Leonhard Euler.

Andreas Kleinert *[kleinert@physik.uni-halle.de], born in 1940, earned the degree of Diplom-Physiker at the Technische Hochschule Aachen and a PhD in history of science at the University of Stuttgart. From 1980 to 1995, he was professor of history of science at the University of Hamburg. In 1995 he obtained the newly created chair of history of science at the Martin-Luther-Universität Halle-Wittenberg in Halle. After his retirement in 2006, he became editor in chief of the Euler correspondence (series IVA of the Euler edition).*

Martin Mattmüller *[euler-archiv@unibas.ch], born in 1957, studied mathematics at Basel. Since 1987, he has been working on editing projects in 17th- and 18th-century science. He is currently the secretary of the Euler Committee and the Euler Archive at Basel and co-editor of vol. IVA/4 of the Opera Omnia, which comprises Euler's correspondence with Christian Goldbach.*

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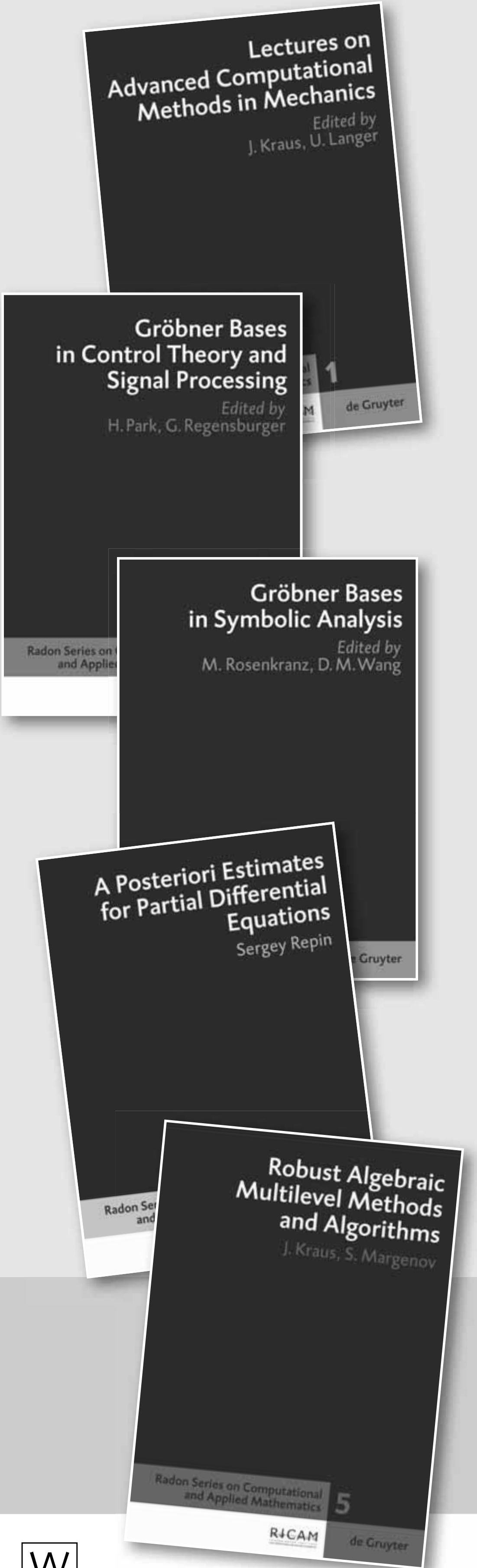

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Interview with Abel Prize recipient Srinivasa Varadhan

Conducted by Martin Raussen (Aalborg, Denmark) and Christian Skau (Trondheim, Norway)
Oslo, May 21, 2007

Professor Varadhan, first of all I would like to congratulate you for having been awarded the Abel Prize this year. By extension, my congratulations go to the field of probability and statistics since you are the first recipient from this area. Incidentally, last year at the International Congress of Mathematicians in Madrid, Fields medals were given to mathematicians with expertise in this area for the first time, as well.

How come that it took so long time before probability and statistics were recognized so prestigiously, at the International Congress of Mathematicians last year and with the Abel Prize this year? Is it pure coincidence that this happens two years in a row? Could you add some comments on the development of the relations between probability and statistics on the one hand and the rest of mathematics on the other hand?

Probability became a branch of mathematics very recently in the 1930s after Kolmogorov wrote his book. Until then it was not really considered as a proper branch of mathematics. In that sense it has taken some time for the mathematical community to feel comfortable with probability the way they are comfortable with number theory and geometry. Perhaps that is one of the reasons why it took a lot of time.

In recent years probability has been used in many areas. Mathematical finance for example uses a lot of probability. These days, probability has a lot of exposure and connections with other branches of mathematics have come up. The most recent example has to do with conformal invariance for which the Fields medal was given last year. These connections have brought probability to the attention of the mathematics community, and the awards are perhaps a reflection of that.

Career

The next question is about your career. You were born in Chennai, the capital of Tamil Nadu, on the South-East coast of India, in 1940. You went to school there and then to the Presidency College at Madras University. I would like to ask you about these formative years: What was the first reason for your interest in mathematical problems? Did that happen under the influence of your father, who was a teacher of mathematics? Were there other people, or were there specific problems that made you first interested in mathematics?

My father was in fact a teacher of science, not so much mathematics. In my early school days I was good in mathematics, which just meant that I could add, subtract and multiply without making mistakes. Anyway I had no difficulty with mathematics. At high school I had an excellent mathematics teacher who asked some of his better students to come to his house during weekends, Saturday or Sunday, and give them extra problems to work on. We thought of these problems just as intellectual games that we played, it was not like an exam; it was more for enjoyment. That gave me the idea that mathematics is something that you can enjoy doing like playing chess or solving puzzles. That attitude made mathematics a much more friendly subject, not something to be afraid of, and that played a role in why I got interested in mathematics.

After that I went to college for five years. I had excellent teachers throughout. By the time I graduated with a master degree in statistics, I had three years of solid grounding in pure mathematics. My background was strong when I graduated from College.

Was there a specific reason that you graduated in statistics rather than in other branches of mathematics?

The option at that time was either to go into mathematics or into statistics. There was not that much difference between these two. If you went into mathematics, you studied pure and applied mathematics; if you went into statistics, you studied pure mathematics and statistics. You replaced applied mathematics with statistics; that was the only difference between the two programs. Looking back, part of the reason for going into statistics rather then mathematics, was the perception that if you went into statistics your job opportunities were better; you could be employed in the industry and so on. I f you went into mathematics, you would end up as a school teacher. There was that perception; I do not know how real it was.

With your degree in statistics it seemed quite natural that you continued at the Indian Statistical Institute at Kolkata. There you found yourself quite soon in a group of bright students that, seemingly without too much influence from their teachers, started to study new areas of fundamental mathematics and then applied those to problems coming from probability theory; with a lot of success: You were able to extend certain limit theorems for stochastic processes to higher dimensional spaces; problems that other mathematicians from outside India had been working on for several years without so much success. Could you tell us a bit about this development and whom you collaborated with?

The Indian system at that time was very like much the British system: If you decided to study for a doctoral degree, there were no courses; you were supposed to do research and to produce a thesis. You could ask your advisor questions and he would answer you, but there was no formal guidance as is the case in the USA for example. When I went there I had the idea that I would be looking for a job within some industry. I was told that I should work on statistical quality control, so I spent close to 6 or 8 months studying statistical quality control; in the end, that left me totally unsatisfied.

Then I met Varadarajan, Parthasarathy and Ranga Rao, who worked in probability from a totally mathematical point of view. They convinced me that I was not spending my time usefully, and that I better learn some mathematics if I wanted to do anything at all. I got interested, and I think in the second year I was there, we said to ourselves: let us work on a problem. We picked a problem concerning probability distributions on groups. That got us started; we eventually solved the problem and in the process also learned the tools that were needed for it.

It was a lot of fun: the three of us constantly exchanged ideas starting at seven o'clock in the morning. We were all bachelors, living in the same dormitory. The work day lasted from 7 am to 9 pm; it was a terrific time to learn. In fact, the second paper we wrote had Abel in its title, because it has something to do with locally compact abelian groups.

From what you tell us, it seems that your work can serve as an example for the fact that the combination of motivations and insights from real world problems on the one hand and of fundamental abstract mathematical tools on the other hand has shown to be extremely fruitful. This brings me to a question about the distinction between pure and applied mathematics that some people focus on. Is it meaningful at all - in your own case and maybe in general?

I think that distinction, in my case at least, is not really present. I usually look at mathematics in the following way: There is a specific problem that needs to be solved. The problem is a mathematical problem, but the origin of the problem could be physics, statistics or could be another application, an economic application perhaps. But the model is there, and it is clear what mathematical problem you have to solve. But of course, if the problem came from physics or some application, there is an intuition that helps you to reason what the possible answer could be. The challenge is how to translate this intuition into rigorous mathematics. That requires tools, and sometimes the tools may not be around and you may have to invent these tools and that is where the challenge and the excitement of doing mathematics is, as far as I am concerned. That is the reason why I have been doing it.

India and the 3rd world

May I come back to your Indian background? You are the first Abel Prize recipient with an education from a 3rd world country. In 1963, you left Kolkata and went to the Courant Institute of Mathematical Sciences in New York, where you still are working. I wonder whether you still strongly feel your Indian background - in mathematics, in training, your life style, your religion and philosophy?

Srinivasa S.R. Varadhan received the Abel Prize from King Harald (Photo: Terje Bendiksby/Scanpix)

For 23 years, I grew up in India, and I think that part of your life always stays with you. I am still very much an Indian in the way I live. I prefer Indian food to anything else, and I have some religious feelings about Hinduism and I am a practising Hindu. So my religious beliefs are based on my real life, and my lifestyle is very much Indian. But when you are living in the United States you learn to adjust a little bit, you perhaps have a combination of the two that you are comfortable with.

My training in India has been mainly in classical analysis. No matter what you do, even if you do the most abstract mathematics, you use it as a tool. At crucial points, I think you need to go back to your classical roots and do some tough estimates here and there; I think the classical training definitely helps there. The abstract mathematical tools then help you to put some results in perspective. You can see what the larger impact of what you have done is. To assess that, modern training gives you some help.

The best known Indian mathematician of the past, at least here in the West, is certainly Srinivasa Ramanujan. He is known both for his very untraditional methods and results, and his note books are still studied by a lot of mathematicians around the world. He is certainly also known for his tragically fate and his untimely death. Has he played a specific role in your life as a role model? Is that still true for many Indian mathematicians?

I think the name of Ramanujan has been familiar to most Indians today. Maybe, when I was growing up, it was more familiar to people from the South than from the North, because he came from the southern part of India, but we definitely knew of him as a great mathematician. At that age, I did not really know the details of his work. Even now, I have only a vague idea of what it is about. People still do not seem to know how exactly he arrived at those results. He seemed to have a mental process that led him to these things, which he has not fully explained in his work. In spite of spending years with Hardy, the West was not able to penetrate the barrier and understand how his mind worked. I do not think we can do anything about it now.

Mathematical Institutions

You spent the last years of your life in India at the Indian Statistical Institute (ISI) at Kolkata. There is another well-known research institute in India, the Tata Institute. I know that there has been some competition between these two institutions although they are specialising in different fields. Can you comment on this competition, the ongoing relations between the two institutes and their respective strengths?

I do not know when the competition started. The Indian Statistical Institute was founded by Mahalanobis in 1931; the Tata Institute was founded by Bhabha in 1945. They were both great friends of Jawaharlal Nehru, the prime minister at the time, he encouraged them both. Maybe, there are some rivalries at that level, the institutional level. The mathematics division of the Indian Statistical Institute had Dr. C.R. Rao, who was my advisor, as its scientific director, and the mathematics division of the Tata Institute was headed by Dr. Chandrasekharan; he was the moving force behind the mathematics school of Tata Institute. Maybe, there is some competition there.

I know many of the faculty of the Tata Institute; in fact many of them were from the same region in the South and they went to the same university, the same college, perhaps even to the same high school. So the relationships between the two faculties have always been friendly.

It is true, the emphasis is very different. At Tata, they have concentrated on number theory and algebraic geometry and certain parts of abstract mathematics. The Indian Statistical Institute on the other hand has concentrated more on probability and statistics. Although there has been some overlap, it is really not that much.

We have heard that you still entertain close relations to India and to Chennai and its Mathematical Institute, in particular. And in general, you are interested in the academic development of 3rd World countries, in particular through the Third World Academy of Sciences. Please tell us about your connections and your activities there?

I go to Chennai maybe once a year now. Earlier it used to be twice a year, when my parents were alive. I use to go and spend a month or two in Chennai, and I visit the two mathematical institutions in Chennai: There is the Chennai Mathematics Institute, and there is also the Institute of Mathematical Sciences in Chennai. I have visited both of them at different times; I have close contacts with their leadership and their faculty.

In earlier times, I visited the Bangalore centre of the Tata Institute: The Tata Institute in Mumbai has a Centre for Applicable Mathematics in Bangalore. I spent some time visiting them, and we have had students from there coming to the Courant Institute to take their degrees and so on. To the extent possible, I try to go back and keep in touch. Nowadays, with e-mail, they can ask me for advice, and I try to help out as much I can. The next couple of years, I have some plans to spend part of my sabbatical in Chennai lecturing at Chennai Mathematics Institute.

You are already the second Abel Prize winner working at the Courant Institute of Mathematical Sciences in New York, after Peter Lax. At least in the world of applied mathematics, the Courant Institute seems to play a very special role. Could you explain how this worked out? What makes the Courant Institute to such a special place?

We are back to the 1930s, when the Courant Institute was started. There was no applied mathematics in the United States. Richard Courant came and he started this mathematics institute with the emphasis on applied mathematics. His view of applied mathematics was broad enough so that it included pure mathematics. I mean, he did not see the distinction between pure and applied mathematics. He needed to apply mathematics, and he developed the tools, he needed to do it. And from that point of view, I think analysis played an important role.

The Courant Institute has always been very strong in applied mathematics and analysis. And in the 1960s, there was a grant from the Sloan foundation to develop probability and statistics at the Courant Institute. They started it, and probability was successful, I think. Statistics did not quite work out, so we still do not have really much statistics at the Courant Institute. We have a lot of probability, analysis, and applied mathematics, and in recent years some differential geometry as well. In these areas we are very strong.

The Courant Institute has always been successful in hiring the best faculty. The emphasis has mostly been on analysis and applied mathematics. Perhaps that reflects why we have had two Abel prize winners out of the first five.

Mathematical Research: Process and Results

You have given deep and seminal contributions to the area of mathematics which is called probability theory. What attracted you to probability theory in the first place?

When I joined my undergraduate program in statistics, probability was part of statistics; so you had to learn some probability. I realised that I had some intuition for probability in the sense that I could sense what one was trying to do, more than just calculating some number. I cannot explain it, I just had some feeling for it. That helped a lot; that motivated me to go deeper into it.

Modern probability theory, as you mentioned earlier, started with Kolmogorov in the 1930's. You had an interesting encounter with Kolmogorov: He wrote from Moscow about your doctoral thesis at the Indian Statistical Institute, that you finished when you were 22 years: "This is not the work of a student, but of a ma-

From left to right: Christian Skau, Martin Raussen, Srinivasa Varadhan

ture master". Did you ever meet Kolmogorov? Did you have any interaction with him mathematically later?

Yes, I have met him; he came to India in 1962. I had just submitted my thesis, and he was one of the examiners of the thesis, but he was going to take the thesis back to Moscow and then to write a report; so the report was not coming at that time. He spent a month in India, and some of us graduate students spent most of our time travelling with him all over India. There was a period where we would meet him every day. There were some reports about it mentioned in the Indian press recently, which were not quite accurate.

But there is one incident that I remember very well. I was giving a lecture on my thesis work with Kolmogorov in the audience. The lecture was supposed to last for an hour, but in my enthusiasm it lasted an hour and a half. He was not protesting, but some members in the audience were getting restless. When the lecture ended, he got up to make some comments and people started leaving the lecture hall before he could say something, and he got very angry. He threw the chalk down with great force and stormed out of the room. My immediate reaction was: There goes my PhD! A group of students ran after him to where he was staying, and I apologized for taking too much time. He said: No no; in Russia, our seminars last three hours. I am not angry at you, but those people in the audience, when Kolmogorov stands up to speak, they should wait and listen.

That is a nice story! Among your many research contributions, the one which is associated with so-called large deviations must rank as one of the most important. Can you tell us first what large deviations are and why the study of these is so important; and what are the applications?

The subject of large deviations goes back to the early thirties. It in fact started in Scandinavia, with actuaries working for the insurance industry. The pioneer who started that subject was named Esscher[1]. He was interested in a situation where too many claims could be made against the insurance company, he was worried about the total claim amount exceeding the reserve fund set aside for paying these claims, and he wanted to calculate the probability of this. Those days the standard model was that each individual claim is a random variable, you assume some distribution for it, and the total claim is then the sum of a large number of independent random variables. And what you are really interested in is the probability that the sum of a large number of independent random variables exceeds a certain amount. You are interested in estimating the tail probabilities of sums of independent random variables.

People knew the central limit theorem at the time, which tells you that the distribution of sums of independent random variables has a Gaussian approximation. If you do the Gaussian approximation, the answer you get is not correct. It is not correct in the sense that the Gaussian approximation is still valid, but the error is measured in terms of difference. Both these numbers are very small, therefore the difference between them is small, so the central limit theorem is valid. But you are interested in how small it is, you are interested in the ratio of these two things, not just the difference of these small numbers.

The idea is: how do you shift your focus so that you can look at the ratio rather then just at the difference. Esscher came up with this idea, that is called Esscher's tilt; it is a little technical. It is a way of changing the measure that you use in a very special manner. And from this point of view, what was originally a tail event, now becomes a central event. So you can estimate it much more accurately and then go from this estimate to what you want, usually by a factor which is much more manageable. This way of estimation is very successful in calculating the exact asymptotics of these tail probabilities. That is the origin of large deviations. What you are really interested in is estimating the probabilities of certain events. It does not matter how they occur; they arise in some way. These are events with very small probability, but you would like to have some idea of how small it is. You would like to measure it in logarithmic scale, "e to the minus how big". That is the sense in which it is used and formulated these days.

Large deviations have lots of applications, not the least in finance; is that correct?

I think in finance or other areas, what the theory actually tells you is not just what the probability is, but it also tells you if an event with such a small probability occurred, how it occurred. You can trace back the history of it and explain how it occurred and what else would have occurred. So you are concerned of analysing entire circumstances. In Esscher's method, there is the tilt that produced it; then that tilt could have produced other things, too; they would all happen if this event happened; it gives you more information than just how small the probability is. This has become useful in mathematical finance because you write an option which means: if something happens at a certain time, then you promise to pay somebody something. But what you pay may depend on

[1] F. Esscher, On the probability function in the collective theory of risk. *Skandinavisk Actuarietidskrift* 15 (1932), 175–195.

not just what happened at that time, it may depend on the history. So you would like to know if something happened at this time, what was the history that produced it? Large deviation theory is able to predict this.

Together with Donsker you reduced the general large deviation principle to a powerful variational principle. Specifically, you introduced the so-called Donsker-Varadhan rate function and studied its behaviour. Could you elaborate a little how you proceeded, and what type of rate functions you could handle and analyse?

If you go back to the Esscher theory of large deviations for sums of random variables, that requires the calculation of the moment generating function. Since they are independent random variables, the moment generating functions are products of the individual ones; if they are all the same, you get just the n-th power of one moment generating function. What really controls the large deviation is the logarithm of the moment generating function. The logarithm of the n-th power is just a multiple of the logarithm of the original moment generating function, which now controls your large deviation. On the other hand, if your random variables are not independent, but dependent like in a Markov chain or something like that, then there is no longer just one moment generating function. It is important to know how the moment generating function of the sum grows; it does not grow like a product but it grows in some way. This is related by the Feynman-Kac formula to the principal eigenvalue of the generator of the Markov process involved. There is a connection between the rate function and the so-called principal eigenvalue. This is what our theory used considerably. The rate function is constructed as the Legendre transform or the convex conjugate of the logarithm of the principal eigenvalue.

Before we leave the subject of the large deviation principle, could you please comment on the so-called Varadhan integral lemma which is used in many areas. Why is that?

I do not think Varadhan's lemma is used that much, probably large deviation theory is used more. The reason why I called it a lemma is that I did not want to call it a theorem. It is really a very simple thing that tells you that if probabilities behave in a certain way, then certain integrals behave in a certain way. The proof just requires approximating the integral by a sum and doing the most elementary estimate. What is important there is just a point of view and not so much the actual estimates in the work involved; this is quite minimal.

But it pops up apparently in many different areas; is that correct?

The basic idea in this is very simple: if you take two positive numbers a and b and raise them to a very high power and you look at the sum, the sum is just like the power of the larger one; the smaller one is insignificant, you can replace the logarithm of the sum by just a maximum. The logarithm of the sum of the exponential behaves just like the maximum. That is the idea, when you have just a finite number of exponentials, then in some sense integrating is not different from summation if you have the right estimates. That was how I looked at it, and I think this arises in many different contexts. One can use the idea in many different places, but the idea itself is not very complicated.

That is often the case with important results in mathematics. They go back to a simple idea, but to come up with that idea, that is essential!

You realized that Mark Kac's old formula for the first eigenvalue of the Schrödinger operator can be interpreted in terms of large deviations of a certain Brownian motion. Could you tell us how you came to this realization?

It was in 1973, I just came back from India after a sabbatical, and I was in Donsker's office. We always spent a lot of time talking about various problems. He wanted to look at the largest eigenvalue which controls the asymptotic behaviour of a Kac integral: I think people knew at that time that if you take the logarithm of the expectation of a Kac type exponential function, its asymptotic growth rate is the first eigenvalue. The first eigenvalue is given by a variational formula; that is classical. We knew that if we do large deviations and calculate asymptotically the integrals, you get a variational formula, too. So, he wanted to know if the two variational formulas have anything to do with each other: Is there a large deviation interpretation for this variational formula?

I was visiting Duke University, I remember, some time later that fall, and I was waiting in the library at Duke University for my talk which was to start in half an hour or so. Then it suddenly occurred to me what the solution to this problem was: It is very simple, in the Rayleigh-Ritz variational formula; there are two objects that compete. One is the integral of the potential multiplied by the square of a function; the other one is the Dirichlet form of the function. If you replace the square of the function and call it a new function, then the Dirichlet form becomes the Dirichlet form of the square root of that function. It is as simple as that. And then the large deviation rate function is nothing but the Dirichlet form of the square root of the density. Once you interpret it that way, it is clear what the formula is; and once you know what the formula is, it is not that difficult to prove it.

This brings me naturally to the next question: If you occasionally had a sudden flash of insight, where you in an instant saw the solution to a problem that you had struggled with, as the one you described right now: Do these flashes depend on hard and sustained preparatory thinking about the problem in question?

Yes, they do: What happens is, once you have a problem you want to solve, you have some idea of how to approach it. You try to work it out, and if you can solve it the way you thought you could, it is done, and it is not interesting. You have done it, but it does not give you a thrill at all. On the other hand, if it is a problem, in which everything falls in to place, except for one thing you cannot do; if only you could do that one thing, then you would have the whole building, but this foundation is missing. So you struggle and struggle with it, sometimes for months, sometimes for years and sometimes for a life-time! And eventually, suddenly one day you see how to fix that small piece. And then the whole structure is complete. That was the missing piece. Then that is a real revelation, and you enjoy a satisfaction which you cannot describe.

How long does the euphoria last when you have this experience?
It lasts until you write it up and submit it for publication; and then you go on to the next problem!

Your cooperation with Daniel Stroock on the theory of diffusions led to several landmark papers. The semigroup approach by Kolmogorov and Feller had serious restrictions, I understand, and Paul Levy suggested that a diffusion process should be represented as a stochastic differential equation. Îto also had some very important contribution. Could you explain how you and Stroock managed to turn this into a martingale problem?
I have to step back a little bit: Mark Kac used to be at Rockefeller University. Between New York University and Rockefeller University, we used to have a joint seminar; we would meet one week here and one week there and we would drive back and forth. I remember once going to Rockefeller University for a seminar and then coming back in a taxi to NYU. Somebody mentioned a result of Ciesielski, a Polish probabilist who was visiting Marc Kac at that time: You can look at the fundamental solution of a heat equation, for the whole space, and look at the fundamental solution with Dirichlet boundary data in a region. The fundamental solution for the Dirichlet boundary data is smaller, by the maximum principle, than the other one. If you look at the ratio of the two fundamental solutions, then it is always less than or equal to one. The question is: As t, the time variable in the fundamental solution, goes to zero, when does this ratio go to 1 for all points x and y in the region? The answer turns out to be: if and only if the region is convex!

Of course, there are some technical aspects, about sets of capacity zero and so on. Intuitively, the reason it is happening is that the Brownian path, if it goes from x to y, in time t, as time t goes to zero, it would have to go in a straight line. Because its mean value remains the same as that of the Brownian bridge, which is always linear, and thus a line connecting the two points. The variance goes to zero, if you do not give it much time. That means it follows a straight line.

That suggests that, if your space were not flat but curved, then it should probably go along the geodesics. One would expect therefore that the fundamental solution of the heat equation with variable coefficients should look like e to the minus the square of the geodesic distance divided by $2t$; just like the heat equation does with the Euclidean distance.

This occurred to me on the taxi ride back. That became the paper on the behaviour of the fundamental solution for small time. In fact, I think that was the paper that the PDE people at Courant liked, and that gave me a job. At that time, I was still a postdoc.

Anyway, at that point, the actual proof of it used only certain martingale properties of this process. It did not really use so much PDE, it just used certain martingales. Stroock was a graduate student at Rockefeller University at that time; we used to talk a lot. I remember, that spring, before he finished, we would discuss it. We thought: If it is true that we could prove this by just the martingale properties, then those martingale properties perhaps are enough to define it. Then we looked at it and asked ourselves: Can you define all diffusion processes by just martingale properties?

It looked like it unified different points of view: Kolmogorov and Feller through the PDE have one point of view, stochastic differential people have another point of view, semigroup theory has still another point of view. But the martingale point of view unifies them. It is clear that it is much more useful; and it turned out, after investigation, that the martingale formulation is sort of the weakest formulation one can have; that is why everything implies it. Being the weakest formulation, it became clear that the hardest thing would be to prove uniqueness.

Then we were able to show that whenever any of the other methods work, you could prove uniqueness for this. We wanted to extend it and prove uniqueness for a class which had not been done before, and that eluded us for nearly one and a half year until one day the idea came, and we saw how to do it and everything fell into place.

That was another flash of inspiration?
That was another flash; that meant that we could do a lot of things for the next four to five years that kept us busy.

Before we leave your mathematical research, I would like to ask you about your contribution to the theory of hydrodynamic limits that is describing the macroscopic behaviour of very large systems of interacting particles. Your work in this area has been described as viewing the environment from the travelling particle. Could you describe what this means?
I will try to explain it. The subject of hydrodynamic scaling as it is called, or hydrodynamic limits is a subject that did not really start in probability. It started from classical mechanics, Hamiltonian equations, and it is the problem of deriving Euler equations of fluid flow directly as a consequence of Hamiltonian motion. After all, we can think of a fluid as a lot of individual particles and the particles interact, ignoring quantum effects, according to Newtonian rules. We should be able to describe how every particle should move. But this requires solving a 10 to the 68 -dimensional ODE, and only then you are in good shape. Instead we replace this huge system of ODEs by PDEs, a small system of nonlinear PDEs, and these nonlinear PDEs describe the motion of conserved quantities.

If there are no conserved quantities, then things reach equilibrium very fast, and nothing really moves. But if there are conserved quantities, then they change very slowly locally, and so you have to speed up time to a different scale. Then you can observe change of these things. Mass is conserved, that means, density is one of the variables; momenta are conserved, so fluid velocity is one of the variables; the energy is conserved, so temperature becomes one of the variables. For these conserved quantities, you obtain PDEs. When you solve your partial differential equations, you get a solution that is supposed to describe the macroscopic properties of particles in that location. And given these parameters, there is a unique equilibrium for these fixed values of the parameters which are the average values.

In a Hamiltonian scheme, that would be a fixed surface with prescribed energy and momentum etc. On that surface the motion is supposed to be ergodic, so that there is a single invariant measure. This invariant measure describes how locally the particles are behaving over time. That is only described in statistical terms; you cannot really pin down which particle is where; and even if you could, you do not really care.

This program, although it seems reasonable in a physical sense, it has not been carried out in a mathematical sense. The closest thing that one has come to is the result by Oscar Lanford who has shown, that for a very small time scale, you can start from the Hamiltonian system and derive the Boltzmann equations. Then to go from Boltzmann to Euler requires certain scales to be large, it is not clear if the earlier results work in this regime. The mathematical level of these things is not where it should be.

On the other hand, if you put a little noise in your system, so that you look at not a deterministic Hamiltonian set of equations, but stochastic differential equations, with particles that move and jump randomly, then life becomes much easier. The problem is the ergodic theory. The ergodic theory of dynamical systems is very hard. But the ergodic theory of Markov processes is a lot easier. With a little bit of noise, it is much easier to keep these things in equilibrium. And then you can go through this program and actually prove mathematical results.

Now coming to the history: We were at a conference in Marseille at Luminy, which is the Oberwolfach of the French Mathematical Society. My colleague George Papanicolaou, who I think should be here in Oslo later today, and I, we were taking a walk down to the calanques. And on the way back, he was describing this problem. He was interested in interacting particles, Brownian motion interacting under some potential. He wanted to prove the hydrodynamic scaling limit. I thought the solution should be easy; it seemed natural somehow. When I came back and looked at it, I got stuck regardless how much I tried. There were two critical steps, I figured out, needed to be done; one step I could do, the second step I could not do. For the time being, I just left it at that.

Then, a year later, we had a visitor at the Courant Institute, Josef Fritz from Hungary. He gave a talk on hydrodynamic limits; he had a slightly different model. By using different techniques, he could prove the theorem for that model. Then I realized that the second step on which I got stuck in the original model, I could do it easily in this model. So we wrote a paper with George Papanicolaou and one of his students Guo; that was my first paper on hydrodynamic limits. This work was more for a field than for an actual particle system which was what got me interested in the subject.

Terry Lyons, Ofer Zeitouni, Srinivasa Varadhan and George Papanicolaou at the University of Oslo in connection with the Abel Lectures. (Photo: Terje Bendiksby/Scanpix)

When you look at particles, you can ask two different questions. You can ask what is happening to the whole system of particles, you do not identify them; you just think of it as a cloud of particles. Then you can develop how the density of particles changes over time. But it does not tell you which particle moves where. Imagine particles have two different colours. Now you have two different densities, one for each colour. You have the equation of motion for the sum of the two densities, but you do not have an equation of motion for each one separately. Because to do each one separately, you would have to tag the particles and to keep track of them! It becomes important to keep track of the motion of a single particle in the sea of particles.

A way to analyse it that I found useful was to make the particle that you want to tag the centre of the Universe. You change your coordinate scheme along with that particle. Then this particle does not move at all; it stays where it is, and the entire Universe revolves around it. So you have a Markov process in the space of universes. This is of course an infinite dimensional Markov process, but if you can analyse it and prove ergodic theorems for it, then you can translate back and see how the tagged particle would move; because in some sense how much the Universe moves around it or it moves around the Universe is sort of the same thing. I found this method to be very useful, and this is the system looked from the point of view of the moving particle.

Work Style

Very interesting! A different question: Can you describe your work style? Do you think in geometric pictures or rather in formulas? Or is there an analytic way of thinking?

I like to think physically in some sense. I like to build my intuition as a physicist would do: What is really happening, understanding the mechanism which produces it, and then I try to translate it into analysis. I do not like to think formally, starting with an equation and manipulating and then see what happens. That is the way I like to work: I let my intuition guide me to the type of analysis that needs to be done.

Your work in mathematics has been described by a fellow mathematician of yours as "Like a Bach fugue, precise yet beautiful". Can you describe the feeling of beauty and aesthetics that you experience when you do mathematics?

I think the quotation you are referring to can be traced back to the review of my book with Stroock by David Williams. I think mathematics is a beautiful subject because it explains complicated behaviour by simple means. I think of mathematics as simplifying, giving a simple explanation for much complex behaviour. It helps you to understand why things behave in a certain manner. The underlying reasons why things happen are usually quite simple. Finding this simple explanation of complex behaviour, that appeals most to me in mathematics. I find beauty in the simplicity through mathematics.

Public awareness

May we now touch upon the public awareness of mathematics: There appears to be a paradox: Mathematics is everywhere in our life, as you have already witnessed from your perspective: in technical gadgets, in descriptions and calculations of what happens on the financial markets, and so on. But this is not very visible for the public. It seems to be quite difficult for the mathematical community to convince the man on the street and the politicians of its importance.

Another aspect is that it is not easy nowadays to enrol new bright students in mathematics. As to graduate students, in the United States more than half of the PhD students come from overseas. Do you have any suggestions what the mathematical community could do to enhance its image in the public, and how we might succeed to enrol more students into this interesting and beautiful subject?

Tough questions! People are still trying to find the answer. I do not think it can be done by one group alone. For a lot of reasons, probably because of the nature of their work, most mathematicians are very introverted by nature. In order to convince the public, you need a kind of personality that goes out and preaches. Most research mathematicians take it as an intrusion on their time to do research. It is very difficult to be successful, although there are a few examples.

The question then becomes: How do you educate politicians and other powerful circles that can do something about it about the importance of education? I think that happened once before when the Russians sent the Sputnik in 1957, I do not know how long it will take to convince people today. But I think it is possible to make an effort and to convince people that mathematics is important to society. And I think that signs are there, because one of the powerful forces of the society today are the financial interests, and the financial interests are beginning to realize that mathematics is important for them. There will perhaps be pressure from their side to improve mathematics education and the general level of mathematics in the country; and that might in the long run prove beneficial; at least we hope so.

In connection with the Abel Prize, there are also other competitions and prizes, like the Niels Henrik Abel competition and the Kapp Abel for pupils, the Holmboe prize to a mathematics teacher, and furthermore the Ramanujan prize for an outstanding 3rd world mathematician. How do you judge these activities?

I think these are very useful. They raise the awareness of the public. Hopefully, all of this together will have very beneficial effect in the not too distant future. I think it is wonderful what Norway is doing.

Private Interests

In my very last question, I would like to leave mathematics behind and ask you about your interests and other aspects of life that you are particularly fond of. What would that be?

I like to travel. I like the pleasure and experience of visiting new places, seeing new things and having new experiences. In our profession, you get the opportunity to travel, and I always take advantage of it.

I like music, both classical Indian and a little bit of classical Western music. I like to go to concerts if I have time; I like the theatre, and New York is a wonderful place for theatre. I like to go to movies.

I like reading Tamil literature, which I enjoy. Not many people in the world are familiar with Tamil as a language. It is a language which is 2000 years old, almost as old as Sanskrit. It is perhaps the only language which today is not very different from the way it was 2000 years ago. So, I can take a book of poetry written 2000 years ago, and I will still be able to read it. To the extent I can, I do that.

At the end, I would like to thank you very much for this interesting interview. These thanks come also on behalf of the Norwegian, the Danish and the European mathematical society. Thank you very much.

Thank you very much. I have enjoyed this interview, too.

Errata

The editor wishes to apologize for several errors in the interview with Prof. Marius Iosifescu in the last issue 64 of the Newsletter:

- Page 39, 1st col., after line 15 insert: "Director of the latter. Since 2002, after an expansion of the Centre, he has been"
- Page 39, 1st col., line 17: „Mathematics." should read "Mathematics (IMSAM)."
- Page 39, 1st col., line 24: delete „America and"
- Page 42, 1st col., line 22: "(ISMMA)" should read „(IMSAM)"
- Page 43, 1st col., lines 14 & 19: "immigration" should read "emigration"

A Survey of ICMI Activities

Maria G. (Mariolina) Bartolini Bussi (Modena, Italy)

The author is a member of the editorial board of this newsletter and serves as a member of the Executive Committee of the International Commission on Mathematical Instruction (ICMI) from 1 January 2007 until 31 December 2009. In this column, she will periodically present news from ICMI. Most of the information will be taken from the official website of ICMI, http://www.mathunion.org/Organization/ICMI/index.html.

The International Commission on Mathematical Instruction (ICMI) was first established at the International Congress of Mathematicians held in Rome in 1908 on the suggestion of the American mathematician and historian of mathematics David Eugene Smith. The first president of ICMI was Felix Klein and the first secretary-general was Henri Fehr. After an interruption of activity between the two world wars, ICMI was reconstituted in 1952 (at a time when the international mathematical community was being reorganized) as an official commission of the International Mathematical Union, IMU, a formal position that ICMI still holds today.

The ICMI Executive Committee 2007-2009 was elected for a three-year term by the 2006 General Assembly of the International Mathematical Union (IMU). It consists of ten voting members: four officers, i.e. the president (Michèle Artigue), two vice-presidents (Jill Adler and Bill Barton) and the secretary-general (Bernard Hodgson); and six members-at-large (Mariolina Bartolini Bussi, Jaime Carvalho e Silva, Celia Hoyles, S. Kumaresan, Frederick Koon-Shing Leung and Alexei L. Semenov). It also has three ex-officio members: Hyman Bass (ex-president of ICMI), László Lovász (president of the IMU) and Martin Grötschel (secretary of the IMU).

In June 2007 the first meeting of the new Executive Committee took place in London, with all the ten voting members plus Hyman Bass and László Lovász attending. Here follows a brief synopsis of some of ICMI's activities.

Symposium on the occasion of the 100th anniversary of ICMI (Rome, 5–8 March 2008)

A symposium will be held in Rome to celebrate the 100th anniversary of ICMI. It will involve a selected group of participants (150–200 people) under the title of *The First Century of the International Commission on Mathematical Instruction (1908-2008): Reflecting and Shaping the World of Mathematics Education*. Starting out from an historical analysis of the principal themes regarding the activities of ICMI (e.g. reforms in the teaching of the sciences, teacher training, and relations with mathematicians and research), discussions will focus on identifying future directions of research in mathematics education and possible actions to be taken to improve the level of scientific culture in various countries. Participants have been invited by the organizers of various activities (working groups and short talks). Further information is available at http://www.unige.ch/math/EnsMath/Rome2008/.

ICME 11 international conference (2008)

A major event in the life of the international mathematics education community is the quadrennial International Congress on Mathematical Education (ICME) held under the auspices of ICMI. The next ICME will be held in Monterrey (Mexico), 6–13 July 2008. A great number of activities have already been planned for the program: *plenary activities, survey teams, regular lectures, topic study groups, discussion groups, national presentations, posters and round tables*. The first announcement for this conference has been posted in July at http://www.icme11.org.mx/icme11/ . In the following issues more details about the program will be summarised.

ICMI studies

Since the mid-80s, ICMI has found it important to involve itself directly in the identification and investigation of issues or topics of particular significance to the theory or practice of contemporary mathematics education and to invest an effort in mounting specific studies on these themes. The main emphasis of a given study may be on analytical or action-oriented aspects but some analytical component will always be present. Built around an international seminar, each study is directed toward the preparation of a published volume intended to promote and assist discussion and action at the international, national, regional or institutional level.

An ICMI study is composed of a series of steps. Once a theme has been decided upon by the Executive Committee (EC) of ICMI, the EC appoints an International Program Committee (IPC), which on behalf of ICMI is responsible for conducting the study. Usually a country that is willing to host the corresponding study conference has been identified concurrently with the appointment of the IPC. The first task of the IPC is to produce a discussion document in which a number of key issues and sub-themes related to the theme of the study are identified, presented and described in a preliminary way. The discussion document is circulated internationally as widely as possible in journals, magazines, newsletters, etc. Readers are invited to respond in writing to the IPC, which then organizes an international ICMI study conference with a limited number (50–100) of invited participants. This conference forms a working forum for investigating the theme of the study. Particular emphasis is given to bringing together both experts in the field and newcomers with interesting ideas or promising work in progress, as well as to gathering representatives with a variety of backgrounds from different regions, traditions and cultures.

The final outcome of an ICMI study is a study volume, i.e. a carefully structured and edited book, not a conference proceedings. The most recent studies are mentioned below (the complete list of studies is given at http://www.mathunion.org/Organization/ICMI/ICMIstudies_past.html):

Study 14. Applications and Modelling in Mathematics Education.
Study conference held in Dortmund, Germany, February 2004.
Study volume published by Springer, 2007: *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, eds: Werner Blum, Peter L. Galbraith, Hans-Wolfgang Henn and Mogens Niss.

Study 15. The Professional Education and Development of Teachers of Mathematics.
Study conference held in Aguas de Lindóia, Brazil, May 2005. Study volume to be published by Springer in 2008.

Study 16. Challenging Mathematics in and Beyond the Classroom.
(information about this study has been published in Newsletter 55, March 2005). Study conference held in Trondheim, Norway, June 2006. Study volume to be published by Springer in 2008.

Study 17. Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain.
Study conference held in Hanoi, Vietnam, December 2006. Study volume to be published by Springer in 2009.

The following studies are currently under way:

Study 18. Statistics Education in School Mathematics: Challenges for Teaching and Teacher Education.
This study is organized by ICMI and the International Association for Statistical Education. The chair of this joint ICMI/IASE study is Carmen Batanero (batanero@ugr.es), University of Granada, Spain. The discussion document is now available at http://www.ugr.es/~icmi/iase_study/ and the deadline for submission of preliminary papers is 1 October 2007. The study conference will be held at the Instituto Tecnológico y de Estudios Superiores de Monterrey from 30 June to 4 July 2008.

Study 19. The Role of Mathematical Reasoning and Proving in Mathematics Education.
The two co-chairs for this study are Gila Hanna (ghanna@oise.utoronto.ca), University of Toronto, Canada, and Michael de Villiers (profmd@mweb.co.za), University of KwaZulu-Natal, South Africa. The discussion document should be available in 2008 and the study conference is planned to take place in 2009.

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[bartolini@unimo.it] is the Newsletter editor within Mathematics Education. A short biography can be found in issue 55, p. 4

ERCOM: The Centrum voor Wiskunde en Informatica turns 60

Bennie Mols (Amsterdam, the Netherlands)

Recently, the Centrum voor Wiskunde en Informatica (CWI) in Amsterdam celebrated its 60th anniversary. For six decades the institute has had a mission to conduct research in mathematics and computer science and transfer the acquired knowledge to society.

Celebrating a 60th anniversary is a natural time to reflect on past highlights. General director Jan Karel Lenstra mentions a few: "After the flood in the province of Zeeland in 1953, our statisticians calculated how high the dikes should be to prevent this from ever happening again. More than 50 years later, the Netherlands remains highly-regarded for this class of extreme-value statistics. In the same period, our numerical analysts calculated the shape of the wing of the Fokker F27 Friendship. Last year this aircraft was elected Best Dutch Design of the 20th century. In the sixties our computer scientists stood at the forefront of new programming languages. Adriaan van Wijngaarden and Edsger Dijkstra made major contributions to ALGOL-60 and Van Wijngaarden played a founding role for ALGOL-68. In the eighties, our institute formed the first Internet exchange in Europe. For years, all email traffic between the US and Europe ran through CWI's 'MCVAX' computers, controlled by Piet Beertema. He came up with the .nl extension for identifying Dutch hosts, and our *cwi.nl* was the first national domain name anywhere in the world."

The memories of the early Internet lead Lenstra to a cold war anecdote from that period: "In the eighties, many others also worked with VAX-machines. On 1st April (April Fools' Day) Beertema posted a message on the net, allegedly coming from KREMVAX, the machine of the Kremlin. In broken English, the communists conveyed the happy news that they were on the Internet now. This created much commotion at the American Department of Defense."

CWI acts as a meeting point for researchers around the globe. Picture: Wim Klerkx.

Room for talent. A new wing will be ready in 2009, with new physical areas for meeting and discussion. Picture: Wim Klerkx.

Apart from its contributions to solving concrete problems, CWI is also important for the more fundamental development of mathematics and computer science in the Netherlands. "Over the past 60 years, the institute has produced over 170 full professors," states Lenstra. "At present, 25 members of our staff have professorial appointments."

Instrument of post-war reconstruction

The institute started in 1946 under the name of Mathematisch Centrum (MC). It was established as an instrument of post-war reconstruction to contribute to applications of mathematics. "In those days, the Netherlands lacked a knowledge transfer mechanism that allowed mathematics to help address societal problems," says Lenstra. "As an extension of developments in the US and Great Britain, the Mathematisch Centrum was established. Our mission was to conduct research that could serve the direct needs of society, where the primary mission of universities was to teach, to educate people."

In the early years, the institute was divided into three departments: pure mathematics, applied mathematics and computation. The computation department built the first open computers in the Netherlands (apart form the closed systems at Philips Natlab and the PTT) and it developed programming languages for general use. One of the first employees of the Mathematisch Centrum, Adriaan van Wijngaarden, was immediately sent to the US to study how their recently built computers were used for numerical computing. The computation department also investigated computational accuracy, including the tackling of rounding errors.

"At the MC, computer science naturally emerged from numerical mathematics," says Lenstra. "Statistics was later separated from the department of applied mathematics. And in the early seventies, the computational division gave birth to our departments of numerical mathematics and computer science."

Societal themes

In the early eighties, the Dutch government created an impulse for computer science, which, in 1983, led to the change of the name of the institute. Mathematisch Centrum became Centrum voor Wiskunde en Informatica (CWI), the national research centre for mathematics and computer science in the Netherlands.

A decade later, a switch in research sections followed. The existing research departments – based on scientific disciplines – were arranged in interdisciplinary themes, aimed at addressing societal problems. Nowadays CWI still retains four research clusters: *Modelling, Analysis and Simulation (MAS), Probability, Networks and Algorithms (PNA), Software Engineering (SEN) and Information Systems* (INS). Lenstra does not want a tight division between mathematics and computer science: "Our strength is operating along the borderline of mathematics and computer science."

"The heart of our work", says the director of CWI, "lies in algorithmic mathematics and fundamental computer science. Due to the development of computer science in the seventies, mathematics discovered algorithmics: mathematicians started calculating. Naturally, this development had an impact on our research." All current clusters include three to five groups, with important research areas like planning and logistics, biological and physical systems, multimedia, complex software systems, security and machine learning.

CWI employs over 210 people, of which 160 are researchers (including 40 postdocs and 70 PhD students) and about 50 employees in support. It has an extensive library and a communication department, which supports the organization of conferences. On 14–18 July 2008, the Fifth European Congress of Mathematics will be organized in Amsterdam (see www.5ecm.nl). CWI is currently expanding the programme for visiting researchers. About two thirds of its annual budget is covered by the Netherlands Organisation for Scientific Research (NWO), with the balance coming from national and international research programmes and assignments from industry. For 2006 the total budget amounted to 16.2 million euros.

Future strategy

Recently, CWI formulated a new strategic plan for research in the coming years. "Contemplating our future strategy," says Lenstra, "we allowed ourselves to consider matters like the expertise of the institute and the socially important themes for the coming years. This brought us to four main themes: Earth and Life Sciences, the Data Explosion, Societal Logistics and Software as Service."

Bio-scientists are more and more confronted with complex problems demanding advanced methods from mathematics and computer science for their modelling and simulation. System biologists want to model living organisms completely in physical and chemical terms and they try to simulate how large quantities of molecules cooperate in time and space to make biological processes run – an enormous computational job. Earth scientists encounter complex questions about changes in gulf streams or improvement of climate models.

The data explosion, the second new theme, arose because researchers collect data much more easily and in much larger quantities than before the ICT revolution. From all the data, they have to collect useful information. In the following years CWI wants to make a major contribution to the development of new techniques to quickly extract the right information from gigantic databases.

Our postdocs and PhD students come from more than 25 countries world-wide. Picture: Wim Klerkx.

Jan Karel Lenstra, director of CWI, is currently the chairman of ERCOM. Picture: Wim Klerkx

Also, current society is confronted with an increasing number of logistical problems, for instance in the transport business (think of the queuing problem), industry and health care. The question is always how to calculate a social and affordable optimum. The third new theme will focus on that.

Where, until recently, software was an independent component on a computer, modern software has increasingly developed towards a complex service collaborating with other software components. Anyone who buys a book using the Internet starts a process of activities, from finding, downloading and sending the book to electronic privacy protection. CWI also wants to contribute to this fourth important societal development.

"There is a rough similarity between those four themes and the existing four research clusters," says Lenstra, "but it is my ambition to highlight the newly formulated themes more prominently. Of course that does not mean that, from now on, our researchers only have to think about societal applications. An institute like ours cannot be driven by product development. We're not a cookie factory. We are driven by challenging research themes and exceptional persons. And that's something that hasn't changed in sixty years."

CWI, Centrum voor Wiskunde en Informatica, is an institute of NWO, the Netherlands Organisation for Scientific Research. If you wish to know more, please contact us:

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A more detailed version (in Dutch) of this article appeared in STAtOR, a publication of the Dutch Statistical Society (VVS).
Cover photo CWI: Wim Klerkx

Book review

Christophe Letellier (Rouen, France)

Physical and Numerical Models in Knot Theory – Including Applications to the Life Sciences
Editors:
J.A. Calvo, K.C. Millett, E.J. Rawdon and A. Stasiak
628 pages, 127 US-$
Series on Knots and Everything –Vol. 36, 2005
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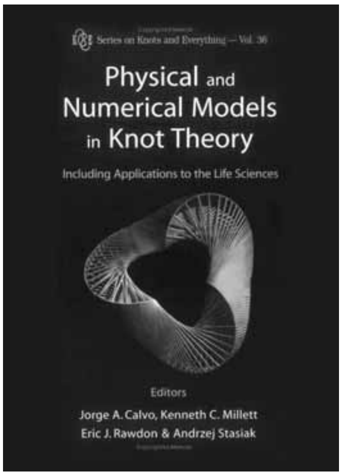

This book, the 36th in the *Knots and Everything* series, ably illustrates the wide range of research involving knots. It provides a rich path through the four 'countries' of the knot world: physics, life sciences, numerical simulations and mathematics. There are different countries for various tastes but all are visited with the theory of knots as a companion. It can thus be seen that particle physics returns to knots – as Thomson did by the end of the 19th century with his atomic model – which can be associated with electric flux tubes used for classifying some hadronic states. In plasma physics, it appears that knotted magnetic flux tubes are much more stable than unknotted single loops. Physics is also involved in modelling polymers using knot energies.

But it is in life sciences that most of the applications in this book are proposed. DNA knots and protein folds are discussed using various approaches such as Monte-Carlo simulations, microscopy, topological concepts and thermodynamics. One of the most amazing examples is illustrated by the pictures of the uncommon formation of incredibly knotted umbilical cords. Using atomic force microscopy, DNA knots can be observed. It is also known that more complex (i.e. more compact) knots migrate in a gel faster than simple knots, thus explaining why gel electrophoresis can separate knotted DNA according to knot type. On the other hand, knot type can be used to measure the effect of chemical composition on polymer shape. In fact, many applications are devoted to polymers: topological constraints can be related to anomalous osmotic pressure, entanglements to elastic properties, etc. Polymers appears as one of the preferred subjects for stimulating numerical simulations of knot configurations and developing algorithms to investigate properties such as ropelength, i.e. the quotient of the knot length by its thickness.

The book is stimulating because it provides many different types of analysis; analytic computations, numerical simulations and new concepts are discussed with different backgrounds. The diversity of the domains covered occasionally makes the book quite difficult since there are undoubtedly fields that the reader will encounter in which they are not specialized. For instance, there are large gaps between quantum chromodynamics, topological constraints, minimal flat knotted ribbons and gel-electrophoresis. Once this difficulty is left behind (it is always possible to consult the bibliography to obtain more details), the main interest of the book is to provide a wide range of approaches to investigating knots and, consequently, to familiarize the reader with topological invariants, numerical techniques and other statistical techniques according to their own taste.

Since it is devoted to a large number of approaches and applications, this book is recommended to anyone interested in using knots in applied science. Many techniques and concepts are discussed and there is likely to be one for opening a "breach" in the reader's problem. Among others, there is an interesting open problem concerning the application of knot theory to open strings. Such investigations could have many implications in applied science where closed loops are often hard to identify. The book already provides stimulating approaches to this problem, leading to the concept of knottedness and minimal flat knotted ribbons.

It is definitely not an introductory book but provides a nice opportunity to be introduced to advanced research and applications in knot theory. One of its main interests is the intermingling of different research areas such as solid state physics, statistical physics and biophysics. Polymer chains are simply disordered knots that can be viewed on a lattice investigated using statistical properties. Thus thermodynamics and topology can be used together to help understand the complexity of underlying knots and links. What are the possible knots matching a given constraint? Numerical methods can be used to produce various knot types and compute some properties (e.g. ropelength, topological entropic force, average crossing number and probability of knotting). It is interesting to see that crossing numbers can be related to complexity measure and that knots can also inspire statistical approaches. In order to do that, special attention is paid to knots with a large number of crossings. The book offers a nice bridge between various topics too often considered as isolated islands.

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Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

Number theory in European culture began with Diophantus of Alexandria ($\sim$ 200–284 A.D.) and the preoccupation for solving equations in integer numbers. It is in this vein that many outstanding problems grew with the reputation of being very simple to state and understand but resisting all attempts at a solution for decades and even centuries.

Some problems that have been known for several hundreds of years have been solved recently. Most famous among them is *Fermat's Last Theorem*, which states that the equation

$$x^n + y^n = z^n \quad (1)$$

has no solutions in coprime integers x, y, z and $n > 2$. The problem was noted by Pierre de Fermat (1601–1665) on the page margin of a book together with the claim that he had a proof that unfortunately did not fit into the space of that page. After more than 350 years of attempts leading to only partial results, a new direction for a possible solution was indicated by G. Frey and Y. Hellegouarch. They considered elliptic curves associated to presumed solutions of (1) and noticed that, if some recent conjectures on elliptic curves were true, these curves had properties that made their existence impossible. Based on work of J.-P. Serre, B. Mazur and K. Ribet, A. Wiles succeeded in 1993 in proving Fermat's Last Theorem up to a gap in the proof, an error that he corrected in 1995 with R. Taylor. The proof and correction were published in the following papers:

[1] A. Wiles: *Modular elliptic curves and Fermat's Last Theorem*, Annals of Math 2, **141** (1995), No. 3, pp. 443–551.
[2] A. Wiles and R. Taylor: *Ring theoretic properties of certain Hecke algebras*, Annals of Math 2, **141** (1995), No. 3, pp. 553–572.

Another longstanding problem was raised by Eugéne Charles Catalan (1814–1894) in 1844, asking whether the equation

$$x^m - y^n = 1 \quad (2)$$

has any other non trivial integer solutions except $3^2 - 2^3 = 1$. The first general result on this equation was obtained by J. Cassels who proved in 1960 that if (2) has an integer solution with prime exponents m, n, then $m|y$ and $n|x$. This was used by R. Tijdeman, who applied a recent work in which A. Baker had given a sharpening of his earlier results on linear forms in logarithms, to prove that (2) accepts at most finitely many integer solutions. The method of linear forms in logarithms was successfully improved by many authors in the period since Tijdeman's breakthrough, mainly by M. Mignotte.

When the Catalan conjecture was eventually solved in 2002 by P. Mihăilescu, his new algebraic insights allowed him to reduce the analytic apparatus involved in his proof, which appeared in the following papers:

[3] P. Mihăilescu: *Primary cyclotomic units and a proof of Catalan's conjecture*, J. Reine Angew. Math. **572** (2004), pp. 167–195.
[4] P. Mihăilescu: *On the class group of cyclotomic extensions in presence of a solution to Catalan's equation*, J. Number Theory **118** (2006), pp. 123–144.

Open problems generalizing the two major solved conjectures mentioned above concern the following Diophantine equations:

$$x^m + y^m = z^n, \quad (x,y,z) = 1, \quad m,n > 2$$

and

$$x^p + y^q = z^r, \quad (x,y,z) = 1 \quad \text{and} \quad p,q,r \geq 2,\ 1/p + 1/q + 1/r < 1.$$

A more recent, deep conjecture that in particular implies the fact that all the above mentioned equations have no solutions – except for Catalan's equation – is the ABC conjecture. This was proposed in 1985 by D. Masser and J. Oesterlé based on an analogy to a similar fact which holds in function fields. It claims that if an equality $A + B = C$ holds between three positive integers A, B, C, then for every $\varepsilon > 0$ there is some constant k_ε such that

$$|\, A \cdot B \cdot C \,| < k_\varepsilon \cdot (ABC)^\varepsilon,$$

where the radical of an integer n, which we denote by (n), is the product of all the primes dividing n, see for instance the lively exposition given in:

[5] A. Granville and T. Tucker: *It's Easy as abc*, Notices of the AMS, **49**, Nr. 10.

In analytic number theory, many problems relate to the distribution of primes and the analytic properties of functions related to algebraic integers, and one may also encounter old, unsettled conjectures that are easy to formulate. An example is Goldbach's conjecture, which states that each even integer n greater than two can be represented as a sum of two primes. It takes more premises to formulate *Riemann's hypothesis*, which has crucial consequences in several domains of mathematics and is one of the seven million-dollar problems of the Clay Foundation.

The pursuit of solutions to longstanding conjectures, and certainly the independent thriving of research, has fostered the development of some important branches of mathematics. Number theory interacts with domains like algebra, analysis, and algebraic and arithmetic geometry. Recently, number theory has found applications in cryptography and coding theory, which have themselves become part of the curriculum of applied mathematics and engineering departments. Algorithms related to number theory are also treated in computational number theory.

Carl F. J. Gauß (1777–1855) is reputed to have said that "Mathematics is the queen of sciences and number theory is the queen of mathematics". Even today number theory continues to connect various disciplines of mathematics by the problems it raises.

I. Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

18. Does the equation

$$x^2 + x + 1 = y^2 \quad (3)$$

have any non-trivial integer solutions for x, y? Consider the same question for

$$x^4 + x^3 + x^2 + x + 1 = y^2. \quad (4)$$

(Preda Mihăilescu, University of Göttingen, Germany)

19. Let p be a prime, $\mathbb{F}_p$ the field with p elements and $\mathbb{K} \supset \mathbb{F}_p$ a finite extension thereof. Let α, β be roots of the distinct irreducible polynomials $f(X),\ g(X) \in \mathbb{K}[X]$ and $\mathbb{L} = \mathbb{K}[\alpha, \beta] \supset \mathbb{K}$ be the extension generated by these two numbers.

(i) Are there values $c \in \mathbb{K}$ for which $\mathbb{L} \neq \mathbb{K}[\alpha + c \cdot \beta]$ for some $c \in \mathbb{K}$?

(ii) If yes, can one give an upper bound for the number of exception-values $c \in \mathbb{K}$?

(Preda Mihăilescu, University of Göttingen, Germany)

20. Prove that the Diophantine equations of the form

$$x^a + y^b = (2p+1)z^c \quad (5)$$

do not accept integer solutions except the trivial one $x = 0$, $y = 0$, $z = 0$ when a, b, c are even positive integers and p is an odd integer.

(Elias Karakitsos, Sparta, Greece)

21. If $a, b > 1$ prove that

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^a k^b (k^a + k^b)}} \le \frac{1}{4}(\zeta(a) + \zeta(b) + \zeta(a+b)), \quad (6)$$

where ζ denotes the Riemann zeta function.

(Mihály Bencze, Brasov, Romania)

22. Let $p \ge 3$ be a prime number and $f : N^* \to N^*$, where $N^* = N - \{0\}$ and $f(n)$ denotes the number of perfect k powers in the interval $[n^k, pn^k]$, $k \ge 2$. Prove that f is a monotone increasing and surjective function. (Mihály Bencze, Brasov, Romania)

23. Show that the equation $q = 2p^2 + 1$, where p and q are primes, admits the unique solution $q = 19$, $p = 3$.

(K. Drakakis, University College Dublin, Ireland)

II. Two New Open Problems

24*. Let p be a prime, $\mathbb{F}_p$ the field with p elements and $\mathbb{K} \supset \mathbb{F}_p$ a finite extension thereof. Let α, β be roots of the distinct irreducible polynomials $f(X),\ g(X) \in \mathbb{K}[X]$ and $\mathbb{L} = \mathbb{K}[\alpha, \beta] \supset \mathbb{K}$ be the extension generated by these two numbers.

(i) Can one find instances for $\mathbb{K}; \alpha, \beta$ for which

$$\mathbb{K}[\alpha + c\beta] \ne \mathbb{K}[\alpha, \beta], \quad (7)$$

for more than one value $c \in \mathbb{K} \setminus \{0\}$?

(i) Is there an upper bound for the number of distinct values of c for which (7) holds?

(i) If yes, can one find examples in which the upper bound in point (ii) is reached?

(Preda Mihăilescu, University of Göttingen, Germany)

25*. For every prime number p greater than two there are two primes p_1, p_2 $(p_1 < p_2)$ such that

$$p = \frac{p_1 + p_2 + 1}{p_1}.$$

The conjecture can also be stated in the following way:
For every prime number p greater than two there are two primes p_1, p_2 $(p_1 < p_2)$ such that the numbers $(p-1)p_1, p_2$ are consecutive integers.

(A Conjecture of Michael Th. Rassias, undergraduate student, National Technical University of Athens, Greece)

III. SOLUTIONS

10. Let G be the set of all functions $g \in C^1(0,\infty)$ such that

$$g(x) \ge 0,\ g'(x) \le 0,\ \text{for all } x \in (0,\infty),\ \text{and } \int_0^\infty g(x)dx < \infty.$$

Find $A < 1$ so that

$$\int_0^\infty g(x)\sin x dx \le A \int_0^\infty g(x)dx,$$

for all $g \in G$. (A.M. Fink, Iowa State University, USA)

Solution by Kee-Wai Lau (Hong Kong, China). We show that

$$\int_0^\infty g(x) sinxdx \le \left(\frac{1}{2} + \frac{1}{\pi}\right) \int_0^\infty g(x)dx. \quad (8)$$

For $x \ge 0$, let $G(x) = \int_0^x g(t)dt$. Integrating by parts we have

$$\begin{aligned}
\int_0^\pi g(x) sinxdx &= -\int_0^\pi G(x) cosxdx \\
&= -\int_0^{\pi/2} G(x) cosxdx - \int_{\pi/2}^\pi G(x) cosxdx \\
&= -\int_0^{\pi/2} G(x) cosxdx + \int_0^{\pi/2} G(\pi - x) cosxdx \\
&= \int_0^\pi g(x)dx \\
&\quad - \int_0^{\pi/2} (G(x) + G(\pi) - G(\pi - x)) cosxdx.
\end{aligned}$$

For $0 < \varepsilon < 1$, let

$$I = \int_\varepsilon^{\pi/2} (G(x) + G(\pi) - G(\pi - x)) cosxdx.$$

Since $g(x) \ge 0$ and $g'(x) \le 0$ it follows that

$$\begin{aligned}
G(x) + G(\pi) - G(\pi - x) &\ge \frac{1}{n}\left(\sum_{k=1}^{n} \int_{(k-1)x}^{kx} g(t)dt + \int_{\pi - x}^{\pi} g(t)dt\right) \\
&\ge \frac{1}{n}\int_0^\pi g(t)dt,
\end{aligned}$$

where n is the integral part of π/x. Hence

$$I \ge \frac{1}{\pi}\left(\int_0^\pi g(t)dt\right)\left(\int_\varepsilon^{\pi/2} xcosxdx\right),$$

which approaches $\frac{\pi-2}{2\pi}\int_0^\pi g(t)dt$ as ε tends to 0. It follows that

$$\int_0^\pi g(x) sinxdx \le \left(\frac{1}{2} + \frac{1}{\pi}\right)\int_0^\pi g(x)dx.$$

Clearly,

$$\int_\pi^{2\pi} g(x) sinxdx \le \left(\frac{1}{2} + \frac{1}{\pi}\right)\int_\pi^{2\pi} g(x)dx,$$

so that

$$\int_0^{2\pi} g(x) sinxdx \le \left(\frac{1}{2} + \frac{1}{\pi}\right)\int_0^{2\pi} g(x)dx$$

as well. In a similar manner one can prove that

$$\int_{2\kappa\pi}^{2(\kappa+1)\pi} g(x) sinxdx \le \left(\frac{1}{2} + \frac{1}{\pi}\right)\int_{2\kappa\pi}^{2(\kappa+1)\pi} g(x)dx$$

for any positive integer k. This finishes the solution. ◇

Remark: The number $\frac{1}{2} + \frac{1}{\pi}$ is not necessarily the best possible value for A.

Also solved by the proposer A.M. Fink, USA and G. Kouvaras, Greece.

11. Let $(H;\langle\cdot,\cdot\rangle)$ be a complex Hilbert space with norm $\|\cdot\|$. For any $X\in H^n$ with $X=(x_1,\dots,x_n)$, define

$$\|X\|_e := \sup_{(\lambda_1,\dots,\lambda_n)\in\mathbf{B}_n}\left\|\sum_{j=1}^n \lambda_j x_j\right\| \quad\text{and}\quad \|X\|_2 := \left(\sum_{j=1}^n \|x_j\|^2\right)^{1/2}, \tag{9}$$

where $\mathbf{B}_n := \{\lambda=(\lambda_1,\dots,\lambda_n)\in\mathbf{C}^n : \sum_{k=1}^n |\lambda_k|^2\le 1\}$. Show that

$$\|X\|_2 \ge \|X\|_e \ge \frac{1}{\sqrt{n}}\|X\|_2. \tag{10}$$

(S.S. Dragomir, Victoria University, Australia)

Solution by the proposer. By the Cauchy-Schwarz-Bunyakovsky inequality we can state that

$$\left\|\sum_{j=1}^n \lambda_j x_j\right\| \le \left(\sum_{j=1}^n |\lambda_j|^2\right)^{1/2}\left(\sum_{j=1}^n \|x_j\|^2\right)^{1/2} \tag{11}$$

for any $(\lambda_1,\dots,\lambda_n)\in\mathbf{C}^n$. Taking the supremum over $(\lambda_1,\dots,\lambda_n)\in\mathbf{B}_n$ in (11) we obtain the first inequality in (10).

If by σ we denote the rotation-invariant normalised positive Borel measure on the unit sphere $\vartheta\mathbf{B}_n$, $\vartheta\mathbf{B}_n := \{(\lambda_1,\dots,\lambda_n)\in\mathbf{C}^n \mid \sum_{i=1}^n |\lambda_i|^2 = 1\}$, whose existence and properties have been pointed out in W. Rudin, *Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer Verlag, New York, Berlin, 1980, then one has

$$\int_{\partial\mathbf{B}_n} |\lambda_k|^2 d\sigma(\lambda) = \frac{1}{n} \tag{12}$$

and

$$\int_{\partial\mathbf{B}_n} \lambda_k\overline{\lambda_j}\, d\sigma(\lambda) = 0 \text{ if } k\ne j,\ k,j=1,\dots,n. \tag{13}$$

Utilizing these properties we have

$$\begin{aligned}
\|X\|_e^2 &= \sup_{(\lambda_1,\dots,\lambda_n)\in\mathbf{B}_n}\left\|\sum_{k=1}^n \lambda_k x_k\right\|^2 \\
&= \sup_{(\lambda_1,\dots,\lambda_n)\in\mathbf{B}_n}\left[\sum_{k,j=1}^n \lambda_k\overline{\lambda_j}\langle x_k,x_j\rangle\right] \\
&\ge \int_{\partial\mathbf{B}_n}\left[\sum_{k,j=1}^n \lambda_k\overline{\lambda_j}\langle x_k,x_j\rangle\right] d\sigma(\lambda) \\
&= \sum_{k,j=1}^n \int_{\partial\mathbf{B}_n}[\lambda_k\overline{\lambda_j}\langle x_k,x_j\rangle] d\sigma(\lambda) \\
&= \frac{1}{n}\sum_{k=1}^n \|x_k\|^2 = \frac{1}{n}\|X\|_2^2,
\end{aligned}$$

from where we deduce the second inequality in (10). ◊

Also solved by Abbas Najati (Iran), who, in addition, provided the following generalization:

Let $1/p+1/q=1$ and let $(H;\langle,\rangle)$ be a complex Hilbert space with norm $\|.\|$. For any $X\in H^n$ with $X=(x_1,\dots,x_n)$, define

$$\|X\|_{eq} := \sup_{(\lambda_1,\dots,\lambda_n)\in\mathbf{B}_n}\left\|\sum_{j=1}^n \lambda_j x_j\right\|$$

and

$$\|X\|_p := \left(\sum_{j=1}^n \|x_j\|^p\right)^{1/p},$$

where $\mathbf{B}_n := \{(\lambda_1,\dots,\lambda_n)\in\mathbf{C}^n : \sum_{k=1}^n |\lambda_k|^q \le 1\}$. Then one has:

$$\|X\|_p \ge \|X\|_{eq} \ge \frac{1}{n^{1/p}}\|X\|_p.$$

◊

12. Let $P_n(x) = \sum_{k=0}^n a_k(1-x)^k(1+x)^{n-k}$ $(\not\equiv 0)$, with $a_k\ge 0$, $k=0,1,\dots,n$. Prove that, for every $x\in[-1,1]$, the inequality

$$(1-x^2)\big(P_n'(x)^2 - P_n''(x)P_n(x)\big) \le nP_n(x)^2 - 2xP_n(x)P_n'(x) \tag{14}$$

holds. (G.V. Milovanović, University of Niš, Serbia)

Solution by the proposer. Define $Q_n(t)=\sum_{k=0}^n a_k t^k$, $a_k\ge 0$, $k=0,1,\dots,n$. Putting $t=(1-x)/(1+x)$, the interval $[-1,1]$ maps to $[0,+\infty)$ and we have

$$\begin{aligned}
Q_n(t) &= \sum_{k=0}^n a_k\left(\frac{1-x}{1+x}\right)^k \\
&= \frac{1}{(1+x)^n}\sum_{k=0}^n a_k(1-x)^k(1+x)^{n-k} \\
&= \frac{1}{(1+x)^n}P_n(x),
\end{aligned}$$

i.e.,

$$(1+x)^n Q_n(t) = P_n(x). \tag{15}$$

Differentiating (15) with respect to x, we get

$$n(1+x)^{n-1}Q_n(t) + (1+x)^n Q_n'(t)\frac{dt}{dx} = P_n'(x),$$

i.e.,

$$2(1+x)^{n-1}Q_n'(t) = nP_n(x) - (1+x)P_n'(x). \tag{16}$$

Similarly, we find

$$\begin{aligned}
&4(1+x)^{n-2}Q_n''(t) \\
&\quad = n(n-1)P_n(x) - 2(n-1)(1+x)P_n'(x) + (1+x)^2P_n''(x).
\end{aligned} \tag{17}$$

On the other hand, using the Cauchy-Schwarz inequality

$$\Big|\sum_{k=0}^n x_k y_k\Big|^2 \le \Big(\sum_{k=0}^n |x_k|^2\Big)\Big(\sum_{k=0}^n |y_k|^2\Big)$$

for $x_k = a_k^{1/2}t^{k/2}$ and $y_k = ka_k^{1/2}t^{k/2}$ $(t\ge 0)$, we obtain

$$\Big(\sum_{k=0}^n ka_kt^k\Big)^2 \le \Big(\sum_{k=0}^n a_kt^k\Big)\Big(\sum_{k=0}^n k^2a_kt^k\Big).$$

Since

$$tQ_n'(t) = \sum_{k=0}^n ka_kt^k$$

and

$$t^2Q_n''(t) = \sum_{k=0}^n k^2a_kt^k - \sum_{k=0}^n ka_kt^k,$$

the previous inequality becomes

$$tQ_n'(t)^2 \le tQ_n(t)Q_n''(t) + Q_n(t)Q_n'(t),$$

which is equivalent to

$$t\left(Q_n'(t)^2 - Q_n(t)Q_n''(t)\right) \le Q_n(t)Q_n'(t), \quad t\ge 0.$$

Finally, substituting $Q_n(t)$, $Q_n'(t)$, and $Q_n''(t)$ from (15), (16), and (17), respectively, into the last inequality, one obtains (14).

Remark. Inequality (14) can be represented in the form

$$\frac{d}{dx}\left\{(x^2-1)\frac{P_n'(x)}{P_n(x)}\right\} \le n.$$

◊

13. If $f, g \in L^2(0,\infty)$ prove that

$$I := \int_0^\infty \int_0^\infty \frac{f(x)g(y)}{1+xy} dxdy \le \pi \left(\int_0^\infty f^2(x)dx \int_0^\infty g^2(x)dx \right)^{\frac{1}{2}}. \quad (18)$$

Is π the best possible constant?

(B. C. Yang, Guangdong, P. R. China)

Solution by the proposer. For fixed t, setting $u = ts$, we obtain

$$\omega(t) := \int_0^\infty \frac{1}{1+st}\left(\frac{t}{s}\right)^{\frac{1}{2}} ds = \int_0^\infty \frac{1}{1+u}\left(\frac{1}{u}\right)^{\frac{1}{2}} du = \pi. \quad (19)$$

By Cauchy's inequality with weight, we have

$$\begin{aligned} I &= \int_0^\infty \int_0^\infty \frac{1}{1+xy}\left[\left(\frac{x}{y}\right)^{\frac{1}{4}} f(x)\right]\left[\left(\frac{y}{x}\right)^{\frac{1}{4}} g(y)\right] dxdy \\ &\le \left\{ \int_0^\infty \left[\int_0^\infty \frac{1}{1+xy}\left(\frac{x}{y}\right)^{\frac{1}{2}} dy\right] f^2(x)dx \right. \\ &\quad \left. \times \int_0^\infty \left[\int_0^\infty \frac{1}{1+xy}\left(\frac{y}{x}\right)^{\frac{1}{2}} dx\right] g^2(y)dy \right\}^{\frac{1}{2}} \\ &= \left(\int_0^\infty \omega(x) f^2(x)dx \int_0^\infty \omega(y) g^2(y)dy \right)^{\frac{1}{2}}. \quad (20) \end{aligned}$$

Then by (20) and (19), we have (18).

For any $\varepsilon \in (0, \frac{1}{2})$, putting $\tilde{f}, \tilde{g}$ as: $\tilde{f}(x) = x^{\frac{\varepsilon-1}{2}}$, $x \in (0,1)$; $\tilde{f}(x) = 0$, $x \in [1,\infty)$, $\tilde{g}(x) = 0$, $x \in (0,1)$; $\tilde{g}(x) = x^{-\frac{1+\varepsilon}{2}}$, $x \in [1,\infty)$, and setting $u = \frac{1}{xy}$, we find

$$\begin{aligned} \tilde{I} &:= \int_0^\infty \int_0^\infty \frac{\tilde{f}(x)\tilde{g}(y)}{1+xy} dxdy \\ &= \int_1^\infty \left[\int_0^1 \frac{x^{\frac{\varepsilon-1}{2}} y^{-\frac{1+\varepsilon}{2}}}{1+xy} dx\right] dy \\ &= \int_1^\infty y^{-1-\varepsilon} \left[\int_{\frac{1}{y}}^\infty \frac{u^{\frac{1-\varepsilon}{2}-1}}{1+u} du\right] dy \\ &\ge \int_1^\infty y^{-1-\varepsilon} \left[\int_0^\infty \frac{u^{\frac{1-\varepsilon}{2}-1}}{1+u} du\right] dy - \int_1^\infty y^{-1} \left[\int_0^{\frac{1}{y}} u^{\frac{1-\varepsilon}{2}-1} du\right] dy \\ &= \frac{1}{\varepsilon} \int_0^\infty \frac{u^{\frac{1-\varepsilon}{2}-1}}{1+u} du - \frac{4}{(1-\varepsilon)^2}. \quad (21) \end{aligned}$$

If the constant $k (\le \pi)$ in (18) is the best possible, then by (21) and (18), we have

$$\int_0^\infty \frac{u^{\frac{1-\varepsilon}{2}-1}}{1+u} du - \frac{4\varepsilon}{(1-\varepsilon)^2} \le \varepsilon \tilde{I} \le \varepsilon k \left(\int_0^\infty \tilde{f}^2(x)dx \int_0^\infty \tilde{g}^2(x)dx \right)^{\frac{1}{2}} = k,$$

and then $\pi \le k$ ($\varepsilon \to 0^+$). In view of the fact that $k \le \pi$, we have $k = \pi$. ◊

Also solved by S. E. Louridas (Greece).

14. If $0 < \sum_{n=1}^\infty a_n^2 < \infty$ and $0 < \sum_{n=1}^\infty b_n^2 < \infty$, prove that

$$\sum_{n=1}^\infty \sum_{m=1}^\infty \frac{|\ln(\frac{m}{n})| a_m b_n}{\max\{m,n\}} < 8 \left(\sum_{n=1}^\infty a_n^2 \sum_{n=1}^\infty b_n^2 \right)^{\frac{1}{2}}, \quad (22)$$

where the constant factor 8 is the best possible.

(B. C. Yang, P. R. China)

Solution by the proposer. We have the following two inequalities:

$$\sum_{n=1}^\infty \sum_{m=1}^\infty \frac{|\ln(\frac{m}{n})| a_m b_n}{\max\{m,n\}} \le 2 \sum_{n=1}^\infty \sum_{m=1}^\infty \frac{a_m b_n}{\max\{m,n\}}; \quad (23)$$

$$\sum_{n=1}^\infty \sum_{m=1}^\infty \frac{a_m b_n}{\max\{m,n\}} < 4 (\sum_{n=1}^\infty a_n^2 \sum_{n=1}^\infty b_n^2)^{\frac{1}{2}} \quad (24)$$

(see G. H. Hardy et al., "Inequalities", Th. 355, Th. 341). Then, (22) follows.

For any $\varepsilon \in (0, \frac{1}{2})$, set $\tilde{a}_n$ as: $\tilde{a}_n = n^{-\frac{1+\varepsilon}{2}}, n \in N$. Since for $x \ge 4$, we find $[\frac{\ln x}{x^{(3+\varepsilon)/2}}]' = \frac{1-\frac{1}{2}(3+\varepsilon)\ln x}{x^{(5+\varepsilon)/2}} < 0$, then for $n \ge 4$, $f(x) = \frac{\ln x}{x^{(3+\varepsilon)/2}}, x \in [n,\infty)$ is decreasing and so is $g(x) = \frac{\ln(n/x)}{x^{(1+\varepsilon)/2}}, x \in [3,n]$. Hence

$$\begin{aligned} \sum_{m=3}^\infty \frac{|\ln(\frac{m}{n})|}{\max\{m,n\} m^{\frac{1+\varepsilon}{2}}} &= \frac{1}{n} \sum_{m=3}^{n-1} \frac{\ln(\frac{n}{m})}{m^{\frac{1+\varepsilon}{2}}} + \sum_{m=n}^\infty \frac{\ln(\frac{m}{n})}{m^{\frac{3+\varepsilon}{2}}} \\ &= \frac{1}{n} \sum_{m=3}^{n-1} \frac{\ln(\frac{n}{m})}{m^{\frac{1+\varepsilon}{2}}} + \sum_{m=n}^\infty \frac{\ln m}{m^{\frac{3+\varepsilon}{2}}} - \frac{\ln n}{n^{\frac{3+\varepsilon}{2}}} - \sum_{m=n+1}^\infty \frac{\ln n}{m^{\frac{3+\varepsilon}{2}}} \\ &\ge \frac{1}{n} \int_3^n \frac{\ln(\frac{n}{x})}{x^{\frac{1+\varepsilon}{2}}} dx + \int_n^\infty \frac{\ln x}{x^{\frac{3+\varepsilon}{2}}} dx - \frac{\ln n}{n^{\frac{3+\varepsilon}{2}}} - \int_n^\infty \frac{\ln n}{x^{\frac{3+\varepsilon}{2}}} dx \\ &= \frac{1}{n^{\frac{1+\varepsilon}{2}}} \left[\frac{4}{(1-\varepsilon)^2} + \frac{4}{(1+\varepsilon)^2}\right] + \left[\frac{2\ln(\frac{3}{n}) 3^{\frac{1-\varepsilon}{2}}}{(1-\varepsilon)n} - \frac{4 \cdot 3^{\frac{1-\varepsilon}{2}}}{(1-\varepsilon)^2 n} - \frac{\ln n}{n^{\frac{3+\varepsilon}{2}}}\right] \\ &= \frac{1}{n^{\frac{1+\varepsilon}{2}}} \left[\frac{4}{(1-\varepsilon)^2} + \frac{4}{(1+\varepsilon)^2}\right] + \frac{1}{n^{\frac{3}{4}}} O(1). \quad (25) \end{aligned}$$

If the constant $k (\le 8)$ in (22) is the best possible, then by (25) and (22), we have

$$\begin{aligned} &\left[\frac{4}{(1-\varepsilon)^2} + \frac{4}{(1+\varepsilon)^2}\right] \sum_{n=1}^\infty \frac{1}{n^{1+\varepsilon}} + O(1) \sum_{n=1}^\infty \frac{1}{n^{\frac{5+2\varepsilon}{4}}} \\ &\le \sum_{n=1}^\infty \frac{1}{n^{\frac{1+\varepsilon}{2}}} \sum_{m=3}^\infty \frac{|\ln(\frac{m}{n})|}{\max\{m,n\} m^{\frac{1+\varepsilon}{2}}} \\ &\le \sum_{n=1}^\infty \sum_{m=1}^\infty \frac{|\ln(\frac{m}{n})| \tilde{a}_m \tilde{a}_n}{\max\{m,n\}} \le k \sum_{n=1}^\infty \tilde{a}_n^2 \\ &= k \sum_{n=1}^\infty \frac{1}{n^{1+\varepsilon}}. \end{aligned}$$

Hence we find

$$\left[\frac{4}{(1-\varepsilon)^2} + \frac{4}{(1+\varepsilon)^2}\right] + \left[O(1) \sum_{n=1}^\infty \frac{1}{n^{\frac{5+2\varepsilon}{4}}}\right] \left[\sum_{n=1}^\infty \frac{1}{n^{1+\varepsilon}}\right]^{-1} \le k,$$

and then $8 \le k$ ($\varepsilon \to 0^+$). In view of $k \le 8$, we have $k = 8$. ◊

15. Let L_∞^1 be the set of functions $x : R \to R$ that are locally absolutely continuous such that x' is essentially bounded (i.e. $\|x'\| < \infty$ where $\|y\| = \operatorname{ess\,sup}\{|y(t)| : t \in R\}$). For a function $x \in L_\infty^1$, derivatives of order α, $0 < \alpha < 1$, in the Marchaud sense are defined as

$$(D_\pm^\alpha x)(u) := \frac{\alpha}{\Gamma(1-\alpha)} \int_0^\infty \frac{x(u) - x(u \mp t)}{t^{1+\alpha}} dt.$$

For any $\delta > 0$ find

$$\sup_{\substack{x \in L_\infty^1 \\ \|x\| \le \delta, \|x'\| \le 1}} \|D_\pm^\alpha x\|.$$

(V.F.Babenko, M.S.Churiliva, Ukraine)

Solution by the proposers. Given $\varepsilon > 0$, consider truncated derivatives in the Marchaud sense

$$(D^{\alpha}_{\pm x,\varepsilon})(u) := \frac{\alpha}{\Gamma(1-\alpha)} \int\limits_{\varepsilon}^{\infty} \frac{x(u) - x(u \mp t)}{t^{1+\alpha}} dt.$$

For any $x \in L^1_\infty$ and any $u \in R$ we have

$$|(D^{\alpha}_{\pm x})(u)| \le |(D^{\alpha}_{\pm x})(u) - (D^{\alpha}_{\pm x,\varepsilon})(u)| + |(D^{\alpha}_{\pm x,\varepsilon})(u)|.$$

Further

$$\begin{aligned} &|(D^{\alpha}_{\pm x})(u) - (D^{\alpha}_{\pm x,\varepsilon})(u)| \\ &\qquad \le \frac{\alpha}{\Gamma(1-\alpha)} \int\limits_{0}^{\varepsilon} \frac{|x(u) - x(u \mp t)|}{t^{1+\alpha}} dt \\ &\qquad \le \frac{\alpha \|x'\|}{\Gamma(1-\alpha)} \int\limits_{0}^{\varepsilon} \frac{t}{t^{1+\alpha}} dt = \frac{\alpha \varepsilon^{1-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \|x'\| \end{aligned}$$

and

$$|(D^{\alpha}_{\pm x,\varepsilon})(u)| \le \frac{2\alpha \|x\|}{\Gamma(1-\alpha)} \int\limits_{\varepsilon}^{\infty} \frac{1}{t^{1+\alpha}} dt = \frac{2}{\Gamma(1-\alpha)} \|x\|.$$

Thus

$$\|D^{\alpha}_{\pm} x\| \le \frac{\alpha \varepsilon^{1-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \|x'\| + \frac{2}{\Gamma(1-\alpha)\varepsilon^{\alpha}} \|x\|.$$

Minimizing the right hand part of this inequality over $\varepsilon > 0$ we obtain

$$\|D^{\alpha}_{\pm} x\| \le \frac{2^{1-\alpha}}{(1-\alpha)\Gamma(1-\alpha)} \|x\|^{1-\alpha} \|x'\|^{\alpha}.$$

Note that this inequality becomes an equality for the function $x(u)$ that is defined as: $x(u) = |u| - 1/2$ if $|u| \le 1$ and $x(u) = 1/2$ if $|u| \ge 1$.

From the last inequality we obtain the following solution of the proposed problem:

$$\sup_{\substack{x \in L^1_\infty \\ \|x\| \le \delta, \|x'\| \le 1}} \|D^{\alpha}_{\pm} x\| = \frac{(2\delta)^{1-\alpha}}{(1-\alpha)\Gamma(1-\alpha)}.$$

◊

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which *will also be devoted to Number theory*.

Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the addresses mpacurar@econ.ubbcluj.ro or madalina_pacurar@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files).

September 2007

2–8: Linear and Non-Linear Theory of Generalized Functions and its Applications, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

3–7: Some Trends in Algebra '07, Czech University of Agriculture, Prague, Czech Republic
Information:zemlicka@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/katedry/ka/sta07.htm

3–7: Mathematical image processing meeting, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

3–7: Conference in Numerical Analysis 2007 (NumAn 2007) – Recent Approaches To Numerical Analysis: Theory, Methods and Applications, Kalamata, Greece
Information: numan2007@math.upatras.gr;
http://www.math.upatras.gr/numan2007/

3–December 21: Phylogenetics, Cambridge, UK
Information: swilkinson@newton.cam.ac.uk;
http://www.newton.cam.ac.uk/programmes/PLG/index.html

4–8: Potential Theory and Stochastics, Albac, Romania
Information: lucian.beznea@imar.ro;
http://www.imar.ro/~purice/conferences/afis-albac.pdf

5–8: Braids, groups and manifolds in Toulouse, Toulouse, France
Information: jttg07@math.ups-tlse.fr;
http://www.picard.ups-tlse.fr/jttg07

5–8: XVI International Fall Workshop on Geometry and Physics, IST, Lisbon, Portugal
Information: xvi-iwgp@math.ist.utl.pt;
http://www.math.ist.utl.pt/IFWGP/

6–8: The 3rd William Rowan Hamilton Geometry and Topology Workshop, The Hamilton Mathematics Institute, Trinity College Dublin, Ireland
Information: bridgem@bc.edu;
http://www.hamilton.tcd.ie/events/gt/gt2007.htm

9–12: Grid Applications and Middleware Workshop 2007 (GAMW'2007), Gdansk, Poland; in conjunction with PPAM 2007

9–12: 7th International Conference on Parallel Processing and Applied Mathematics (PPAM 2007), Gdansk, Poland
Information: gamw@man.poznan.pl; http://ppam.pcz.pl

9–15: Measure Theory – Edward Marczewski Centennial Conference, Banach Center, Będlewo, Poland
Information: http://www.impan.gov.pl/BC/

10–12: Journées Jean-Yves Girard, Conference in Honour of his 60th Birthday, Henri Poincaré Institute, Paris, France
Information: http://www-lipn.univ-paris13.fr/jyg60

10–12: EMATIK 2007 (E-learning in Mathematics, Mathematics in E-learning), Bratislava, Slovak Republic
Information: solcan@fmph.uniba.sk; http://konferencia.ematik.sk/

10–14: Fifth Symposium on Nonlinear Analysis, Torun, Poland
Information: sna2007@mat.uni.torun.pl;
http://www-users.mat.uni.torun.pl/~sna2007/index.html

10–14: Aperiodic Order: new connections and old problems revisited, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

10–14: 11th Workshop on Well-Posedness of Optimization Problems and Related Topics, Alicante, Spain
Information: marco.antonio@ua.es; mgoberna@ua.es;
http://www.eio.ua.es/congreso/index.html

10–15: 10th Quantum Mathematics International Conference – QMath10, Moeciu, Romania
Information: Radu.Purice@imar.ro;
http://www.imar.ro/~purice/QM10/QM10.html

10–15: International Conference "Nonlinear Partial Differential Equations" dedicated to the memory of I.V. Skrypnik, Yalta, Crimea, Ukraine
Information: npde2007@iamm.ac.donetsk.ua,
npde2007@freemail.ru;
http://iamm.ac.donetsk.ua/conferences/npde2007.html

14–17: BALCOR 2007: 8th Balkan Conference on Operational Research, Belgrade-Zlatibor, Serbia
Information: balcor@fon.bg.ac.yu;
http://balcor.fon.bg.ac.yu/

17–21: WORDS 2007. 6th International Conference on Words, CIRM, Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

17–28: Curvature and variational modelling in Physics and Biophysics, Santiago de Compostela, Spain
Information: xtedugr@usc.es;
http://xtsunxet.usc.es/schoolsantiago2007/

18–22: Dynamical Methods and Mathematical Modelling, Valladolid, Spain
Information: dm07@wmatem.eis.uva.es;
http://wmatem.eis.uva.es/~dm07/

18–22: Noncommutative rings and geometry. In honour of Freddy Van Oystaeyen on the occasion of his 60th birthday, Almería, Spain
Information: www.ual.es/Congresos/fred/

20–21: Mathematical Models in Evolution and Ecology, University of Sussex, Brighton, UK
Information: f.j.childs@sussex.ac.uk;
http://www.maths.sussex.ac.uk/MMEE2007

20–22: Finsler geometry (Mathematics and Physics), Institut de Recherche Mathématique Avancée, Strasbourg, France
Information: orphanides@math.u-strasbg.fr;
http://www-irma.u-strasbg.fr/article414.html

24–28: Conference in complex analysis and geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

24–29: 18th Congress of Unione Matematica Italiana, Bari, Italy
Information: segreteria@congressoumi2007.it;
http://www.congressoumi2007.it/

24–29: International Algebraic Conference dedicated to the 100th anniversary of D. K. Faddeev, St. Petersburg, Russia
Information: novikova@pdmi.ras.ru;
http://www.pdmi.ras.ru/EIMI/2007/DKF/

30–October 7: International Conference Dubrovnik VI – Geometric Topology, Dubrovnik, Croatia

October 2007

1–5: Thematic school on mathematics documentation (RNBM), CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

8–10: Young European statisticians Workshop (YES-I), EURANDOM, Eindhoven, The Netherlands
Information: coolen@eurandom.tue.nl;
http://www.eurandom.tue.nl/workshops/2007/YES_1/YES_1_main.htm

8–12: Diophantine approximation: curent trends, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

10–16: Control, Constraints and Quanta, Będlewo, Poland
Information: jkelly@esf.org;
http://www.esf.org/conferences/07250

11–14: Geometric Function Theory and Nonlinear Analysis. On the occasion of the 60th birthday of Tadeusz Iwaniec, Hotel Continental Terme, Ischia, Naples, Italy
Information: tadeusz2007@dma.unina.it;
http://www.dma.unina.it/~tadeusz2007

15–19: Matrix Analysis and Applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

15–19: Diagonally symmetric polynomials and applications, CIEM, Castro-Urdiales, Spain
Information: ebriand@us.es; http://www.congreso.us.es/dsym/

17–23: Algebraic Aspects in Geometry, Mathematical Research and Conference Center, Będlewo, Poland
Information: corefice@esf.org;
http://www.esf.org/conferences/07249

22–26: Workshop on Noncommutative Manifolds II, Trieste, Italy
Information: ncg07@sissa.it; http://www.sissa.it/fm/ncg07.html

22–26: Calabi-Yau algebras and N-Koszul algebras, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

23–24: Journées en l'honneur de Bernard Roynette, Vandoeuvre-les-Nancy, France
Information: vallois@iecn.u-nancy.fr;
http://br07.iecn.u-nancy.fr/

26–28: ICVL 2007-The 2nd International Conference on Virtual Learning, University OVIDIUS Constanta, Romania
Information: vlada@fmi.unibuc.ro; http://www.icvl.eu/2007/

29–November 1: Mathematical models of Traffic Flow, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

November 2007

2–4: 75th Workshop on General Algebra: AAA75+CYA23, Darmstadt, Germany
Information: http://www.math.tu-dresden.de/~aaa/

4–10: Conformal Invariance in Mathematical Physics, Mathematisches Forschungsinstitut Oberwolfach, Germany
Information: mfo@mathematik.uni-kl.de;
http://www.mfo.de/programme/schedule/2007/45a/programme0745a.html

4–10: On Arithmetically Defined Hyperbolic Manifolds, Mathematisches Forschungsinstitut Oberwolfach, Germany
Information: mfo@mathematik.uni-kl.de;
http://www.mfo.de/programme/schedule/2007/45b/programme0745b.html

5–9: Arithmetic, Geometry, Cryptography and Coding Theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

12–16: MODNET workshop on the model theory of fields, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

18–24: Enumerative Combinatorics and Integrable Models of Statistical Mechanics, Mathematisches Forschungsinstitut Oberwolfach, Germany
Information: mfo@mathematik.uni-kl.de;
http://www.mfo.de/programme/schedule/2007/47a/programme0747a.html

18–24: Recent Developments in Conformal Differential Geometry, Mathematisches Forschungsinstitut Oberwolfach, Germany
Information: mfo@mathematik.uni-kl.de;
http://www.mfo.de/programme/schedule/2007/47b/programme0747b.html

19–22: Functional and Harmonic Analysis, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–23: Geometrical Mechanics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

26–30: Einstein manifolds and beyond, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

December 2007

3–7: Arithmetic geometry and rational varieties, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

10–14: *l*-adic cohomology and number fields, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

17–21: Meeting on mathematical statistics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

January 2008

28–31: VII International Conference "System Identification and Control Problems" SICPRO '08, Moscow, Russia
Information: sicpro@ipu.rssi.ru;
http://www.sicpro.org/sicpro08/code/e08_01.htm

February 2008

19–22: International Conference on Mathematics and Continuum Mechanics, Porto, Portugal
Information: ferreira@fe.up.pt;
http://paginas.fe.up.pt/~cim2008/index.html

March 2008

4–7: 8th German Open Conference on Probability and Statistics, Aachen, Germany
Information: gocps2008@stochastik.rwth-aachen.de;
http://gocps2008.rwth-aachen.de

5–8: The First Century of the International Commission on Mathematical Instruction, Accademia dei Lincei, Rome, Italy
Information: http://www.unige.ch/math/EnsMath/Rome2008/

9–12: LUMS 2nd International Conference on Mathematics and its Applications in Information Technology 2008 (in collaboration with SMS, Lahore), Lahore, Pakistan
Information: http://web.lums.edu.pk/licm08

April 2008

6–9: Mathematical Education of Engineers, Mathematics Education Centre, Loughborough University, Loughborough, UK
Information: mee2008@lboro.ac.uk;
http://mee2008.lboro.ac.uk/

May 2008

19–23: Topological & Geometric Graph Theory, Paris, France
Information: tggt2008@ehess.fr; http://tggt.cams.ehess.fr

26–30: Spring school in nonlinear partial differential equations, Louvain-La-Neuve, Belgium
Information:
http://www.uclouvain.be/math-spring-school-pde-2008.html

June 2008

9–19: Advances in Set-Theoretic Topology: Conference in Honour of Tsugunori Nogura on his 60th Birthday, Erice, Sicily, Italy
Information: erice@dmitri.math.sci.ehime-u.ac.jp;
http://www.math.sci.ehime-u.ac.jp/erice/

22–28: Combinatorics 2008, Costermano (VR), Italy
Information: combinatorics@ing.unibs.it;
http://combinatorics.ing.unibs.it

23–27: Homotopical Group Theory and Topological Algebraic Geometry, Max Planck Institute for Mathematics Bonn, Germany
Information: admin@mpim-bonn.mpg.de;
http://www.ruhr-uni-bochum.de/topologie/conf08/

25–28: VII Iberoamerican Conference on Topology and its Applications, Valencia, Spain
Information: cita@mat.upv.es; http://cita.webs.upv.es

July 2008

7–11: VIII International Colloquium on Differential Geometry (E. Vidal Abascal Centennial Congress), Santiago de Compostela, Spain
Information: icdg2008@usc.es; http://xtsunxet.usc.es/icdg2008

14–18: Fifth European Congress of Mathematics (5ECM), Amsterdam, Netherlands
Information: www.5ecm.nl

August 2008

16–31: EMS-SMI Summer School: Mathematical and numerical methods for the cardiovascular system, Cortona, Italy
Information: dipartimento@matapp.unimib.it

September 2008

8–19: EMS Summer School: Mathematical models in the manufacturing of glass, polymers and textiles, Montecatini, Italy

29–October 8: EMS Summer School: Risk theory and related topics, Będlewo (Poland)
Information: www.impan.gov.pl/EMSsummerSchool/

November 2008

5–7: Fractional Differentiation and its Applications, Ankara, Turkey
Information: dumitru@cankaya.edu.tr;
http://www.cankaya.edu.tr/fda08/

Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to: Ivan Netuka, MÚUK, Sokolovská, 83, 186 75 Praha 8, Czech Republic.

***E. del Barrio, P. Deheuvels, S. van de Geer: Lectures on Empirical Processes – Theory and Statistical Applications**, EMS Series of Lectures in Mathematics, European Mathematical Society, Zürich, 2007, 254 pp., EUR 39.50, ISBN 978-3-03719-027-2*
This book is based on three series of lectures delivered at the Summer School on Empirical Processes organised by European Mathematical Society in September 2004. The titles of the lectures are: Empirical and quantile processes in the asymptotic theory of goodness-of-fit tests (Eustasio del Berrio), Topics on empirical processes (Paul Deheuvels) and Oracle inequalities (Sara van de Geer). Techniques developed through probability in a Banach space framework, strong approximations in the Hungarian style and tools from stochastic process theory appear in the book in connection with the main historical applications, as well as with others that are presently in development. The part written by del Barrio focuses on applications of empirical processes to the area of classical as well as recently developed goodness-of-fit tests. The second part, written by Deheuvels, concentrates on empirical distribution functions and empirical quantile functions and their functionals. In the last part, written by van de Geer, the construction of a statistical model is discussed. Attention is paid to penalized M-estimators and oracle inequalities. The book gives an excellent overview of the main techniques and results in the theory of empirical processes and its applications in statistics. It is assumed that the reader is familiar with probability theory and mathematical statistics. It can be used as a text of a course for PhD students (mahus)

***S. Batterson: Pursuit of Genius – Flexner, Einstein and the Early Faculty at the Institute for Advanced Studies**, A.K. Peters, Wellesley, 2006, 301 pp., USD 39, ISBN 1-56881-259-0*
This interesting book describes the history and development of the Institute for Advanced Study in Princeton, New Jersey, which occupies a unique position among institutions of higher learning in the USA. Thanks to many unpublished archive sources the author analyzes the story of the founding of a higher-education institution, a place without teaching commitments, service obligations and financial problems, and a place of deep scientific study and research. He shows the life and scientific activities and contributions of intellectual leaders from the institute (A. Flexner, D. C. Gilman, J. Hopkins, J. von Neumann, A. Einstein, etc.) as well as political, economic and personal situations, conflicts, and intrigues that influenced and determined the future of the institute. The reader may find many unknown events and information concerning the creation and evolution of the institute, the first generation from the faculty of mathematics, international cooperation during the Great Depression and the ascendance of Adolf Hitler, as well as collaboration after the Second World War. At the end of the book, lists of archive sources and acknowledgments, non-archive sources (books and articles) and an index are included. The book can be recommended to anyone who is interested in mathematics and its history. (mbec)

***R. Becker: Convex Cones in Analysis**, Travaux en cours, 67, Hermann, Paris, 2006, 251 pp., EUR 50, ISBN 2-7056-6671-0*
The French original of this book has been reviewed (see R. Becker, Cônes convexes en analyse, Travaux en cours 59, Hermann, Paris, 1999; EMS Newsletter 36, June 2000, p. 34). (jl)

***O. Biquard: Asymptotically Symmetric Einstein Metrics**, SMF/AMS Texts and Monographs, vol. 13, American Mathematical Society, Providence, 2006, 105 pp., USD 39, ISBN 0-8218-3166-6*
The relation between the Riemannian structure of a hyperbolic manifold and the induced conformal structure on its boundary has been intensively studied recently, in particular in connection with the so called AdS/CFT correspondence discovered in theoretical physics. Basic examples of such correspondence are given by suitable homogeneous spaces (and their boundaries) but the whole scheme works as well in a curved situation (where Einstein metrics are just asymptotically symmetric). The correspondence studied in this book is modelled on hyperbolic spaces over basic fields (real, complex, quaternionic and octonionic in dimension two) and their boundaries. Basic homogeneous examples show that the induced conformal metric on the boundary is a Carnot-Carathéodory metric (which is defined, with the exception of the real case, only on a suitable subbundle of the tangent bundle). The main problem addressed in the book is a Dirichlet problem for a nonlinear system of partial differential equations (the Einstein equation with some additional constraints), the boundary data being given by the conformal class of a chosen Carnot-Carathéodory metric. The book contains results both on global solutions and local solutions near infinity. The book brings together new and important results in a modern field of mathematics. (vs)

***B. Chow. P. Lu, L. Ni: Hamilton's Ricci Flow**, Graduate Studies in Mathematics, vol. 77, American Mathematical Society, Providence, 2006, 608 pp., USD 79, ISBN 0-8218-4231-5, ISBN 978-0-8218-4231-7*
The aim of this book is to give an introduction to the subject of Ricci flow on Riemannian manifolds and to the geometrization conjecture, a topic initiated by Hamilton's paper on 3-manifolds with positive Ricci curvature and completed in the recent spectacular developments by G. Perelman. In the introductory chapter, the book reviews from scratch some basic results and facts from Riemannian geometry. The next chapters introduce Ricci flow and discuss the proof of Hamilton's classification of 3-manifolds with positive Ricci curvature together with a presentation of special (e.g. homogeneous, solitonic) solutions. Chapters 5 and 6 start the discussion of various partial differential equations, aspects of the Ricci flow like monotonicity formulas and techniques useful in the analysis of singularities. In chapter 10, the reader can find various differential Harnack estimates allowing the control and comparison of solutions of Ricci flow at different points of the space-time. The last chapter then deals with various implications of Ricci flow towards the existence of special (degenerate, Einstein) metrics.

The book is in fact an introduction to the topic of geometry and topology of 3-manifolds via the Ricci flow technique for graduate students and mathematicians interested in the field. It contains many exercises and open problems throughout the book. (pso)

***P. J. Davis: Mathematics and Common Sense – A Case of Creative Tension**, A.K. Peters, Wellesley, 2006, 242 pp., USD 34.95, ISBN 978-1-56881-270-0*

This book consists of 33 essays trying to show to a nonmathematical community what mathematics and its applications really are, why they are so important and how they influence our day to day life. The essays may be read independently. Thanks to a long experience with mathematics as a researcher and teacher, the author provides many creative discussions and examples, varying from simple to more abstract structures of mathematics. He tries to provide a leitmotif to illustrate the relationship between mathematics and common sense. He writes about more than sixty major topics in mathematics, many of which have significant connections to other branches of knowledge (e.g. cosmology, physics, teaching, logic, philosophy, languages). The reader can find discussions on the nature of logic, numbers, counting and discounting, mathematical thinking, deductions, intuition and creativity, problem solving, conceptions of space, mathematical operations, structures, objects, paradoxes, theorems and proofs, as well as meditations on the influence of the media and wars on the development of mathematics and its position in the society. The author states and answers many interesting questions from many points of view. At the end of each essay the references to material that is both popular and professional are given. The book can be recommended to all who are interested in mathematics and its nature, beauty and role in modern society and science. (mbec)

***L. Debnath, D. Bhatta: Integral Transforms and Their Applications**, second edition, Chapman & Hall/CRC, Boca Raton, 2006, 700 pp., USD 79,95, ISBN 1-58488-575-0*

This is the second edition of a bestseller by the first author, which was published in 1995. The book is aimed as a senior undergraduate or graduate text on integral transforms and has already been widely used for such a purpose. The level of explanation is accessible for a large audience of interested readers; the careful treatment of the basic methods (avoiding excessive abstractness in their description) is accompanied by a great number of selected examples developing the reader's analytical skills, ranging from applications of Laplace transforms in the theory of linear differential equations through to fluid mechanics. The subject treated in the book belongs to the very core of classical analysis and at the same time it is a vivid field with many recent important developments. This is also reflected in the contents of the second edition where new chapters on fractional calculus, Radon transforms and wavelets have been added. A few keywords from the 19 chapters of the book are: Fourier transforms, Laplace transforms, fractional calculus, Hankel, Mellin, Hilbert and Stieltjes transforms, Z transforms, Jacobi and Gegenbauer, Laguere, Hermite transforms, Radon transforms, wavelets, and some special functions. The book is accompanied by tables of transforms of important functions and by hints to selected exercises. Among other existing treatises on integral transforms this book surely deserves attention, especially from the reader who is a novice in the subject looking for an accessible introduction to this vast area of classical and modern analysis. (mzahr)

***P. Drábek, G. Holubová: Elements of Partial Differential Equations**, Walter de Gruyter, Berlin, 2007, 245 pp., EUR 34.95, ISBN 978-3-11-019124-0*

There are plenty of books devoted to partial differential equations and a novice can find many textbooks among them. But these generally have hundreds of pages, e.g. one of the most popular (the Evans PDEs) contains nearly 700 pages. It is surprising that it is still possible to write a readable book in which all of the important basic facts together with their motivation and physical relevance can be found in much fewer pages. This book is such a text, comprising only 245 pages. Evidently, something has had to be omitted. The reader will not be introduced to Sobolev spaces or weak solutions, the classification of equations is rather brief and the Cauchy-Kovalewski theorem is missing. All equations are linear (with a few exceptions in the exercises) and all solutions are classical. On the other hand, the propagation of singularities and the conservation of energy for the three dimensional wave equation are included.

Even though the arrangement of the book mainly follows three basic types of equations (wave, diffusion and Laplace), their common features such as energy estimates, the maximum principle and Fourier's method and integral transforms are explained in one place for all the equations. The text is sufficiently rigorous, with a majority of the theorems being proved. The student will certainly find the illustrative pictures useful (66 figures in all). The book contains 250 exercises demonstrating the main goal of this book, namely to introduce students of mathematics, physics and engineering to partial differential equations as one of the main tools of mathematical modelling. It can be highly recommended for this purpose. (jmil)

***I. Ekerland: The Cat in Numberland**, Carus Publishing, Chicago, 2006, 60 pp., USD 8.95, ISBN 0-8126-2744-X, ISBN 978-0-8126-2744-X*

This small booklet brings the reader to a strange place called Numberland, where all the (integer) numbers live in a big hotel. An experienced reader soon recognizes that the hotel presented here is nothing other than a version of the famous Hilbert hotel, which was constructed as a tool to illustrate the problems in connection with countability. The small size of the book, the style of writing, a (large) number of illustrations and especially the "fairy-tale-like" language all indicates that the booklet is meant for children, probably around or under ten years of age.

And at this point the reviewer was beginning to get a little unsure as to whether this rather difficult piece of mathematics should be presented and explained to children of that age. Maybe, the children should first become reliably accustomed to the notions of "more than", "larger", and even "finitely and infinitely many", and only after that, at a proper age, should they be faced with facts like "there are as many odd integers as there are integers" or "there are as many integer fractions as integers themselves", which are the main "results" of the book. Also, since the concept of uncountability is not at all addressed (which is correct), the child reader can possibly be driven to the misleading realization that all infinite sets are "equally large".

So there remains a small question if the topic is suitable for children of a "fairy-tale" age. However, if the answer to this question is "yes", then nothing can stop the reviewer from claiming that the booklet is written in a very nice way, presenting all the ideas clearly (at least for the adult reader) and in a concise yet comprehensive form. (mrok)

Ya. M. Erusalimsky et al., Eds.: Modern Operator Theory and Applications – The Igor Borisovich Simonenko Anniversary Volume*, Operator Theory - Advances and Applications, vol. 170, Birkhäuser, Basel, 2007, 256 pp., EUR 136.95, ISBN 3-7643-7736-4, ISBN 978-3-7643-7736-6*

This volume of the Birkhäuser "Operator Theory" series is dedicated to I. Simonenko on the occasion of his 70th birthday. The book starts with a brief description of Simonenko's life and his contributions to two important parts of his scientific work, namely to the local method for singular integral operators and the averaging method for nonlinear equations of parabolic type. The list of Simonenko's publications and his students are added. The book contains 13 papers written by colleagues and former students of I. Simonenko. These contributions are devoted to various aspects of operator theory (matrix-valued functions and their factorizations, block operator matrices and the Fredholm theory for them, Toeplitz and Hankel type operators, asymptotics of operator determinants and contraction operators), in particular to convolution operators (e.g. applications of Lévy processes in option theory) and singular operators. Three papers are closely related to the above mentioned Simonenko activity. The book will be interesting for advanced graduate students and researchers in operator theory and its applications to mathematical physics, hydrodynamics and financial mathematics. (jmil)

A. Fedotov, F. Klopp: Weakly Resonant Tunnelling Interactions for Adiabatic Quasi-Periodic Schrödinger Operators*, Mémoires de la Société Mathématique de France, no. 104, Société Mathématique de France, Paris, 2006, 105 pp., EUR 26, ISBN 978-2-85629-188-7*

In this book, the authors collect together the developments they have recently achieved in the Anderson problem, i.e. the question of characterization of nature of spectrum in the theory of Schrödinger operators (on a line). They consider a "slow" adiabatic quasiperiodic perturbation of the one dimensional Schrödinger operator with a periodic potential and they study the corresponding resonance effects affecting the nature of the spectrum (which, in the unperturbed case, consists of spectral "bands", i.e. intervals of absolutely continuous spectra, separated by spectral gaps). They show how the presence of a quasiperiodic perturbation changes the classical behaviour of the spectrum. Namely, it creates resonance effects similar to the ones appearing in the problem of interacting quantum wells. These effects create additional layers, where a singular spectrum appears. The authors investigate the delicate nature of these phenomena and the interplay between the absolutely continuous and singular parts of the spectrum. The book explains in a concise but essentially self-contained way all the needed prerequisites and basic concepts (the monodromy matrix, density of states, Lyapunov exponents, Bloch solutions, etc.) needed for understanding these important and technically complicated phenomena. (mzahr)

G. N. Frederickson: Piano-Hinged Dissections – Time to Fold + CD*, A.K. Peters, Wellesley, 2006, 303 pp., USD 49, ISBN 978-1-56881-299-1*

This interesting book describes piano-hinged dissections, a new type of hinged dissection that generates many challenges. The book starts with mathematical descriptions, definitions and models of piano-hinged dissections (a piano-hinge connects two flat pieces that are side-by-side on the same level and forces one piece to flip on top of the other). In the book the author provides over 150 dissections and outlines methods for discovering them. He has also prepared an excellent CD with video, which provides step-by-step demonstrations for creating new dissections. The author also presents the lost manuscript on geometric dissections written by Ernest Irving Freese, an architect in Los Angeles, who at the end of his life became passionate about dissections. The book gives an overview of Freese's work in its five-part series spread throughout the text. This brilliant book can be recommended to students of geometry and teachers of mathematics, as well as students and all people who are interested in geometric dissections. Every creative reader will find new material for his own discoveries. The reader can easily experiment with the piano-hinge dissections because their mechanism can be simulated by folding a piece of paper without special mathematical knowledge, materials, computer programs, etc. (mbec)

V.A. Galaktionov, S.R. Svirshchevskii: Exact Solutions and Invariant Subspaces of Nonlinear Partial Differential Equations in Mechanics and Physics*, Chapman & Hall/CRC Applied Mathematics and Nonlinear Science Series, Chapman & Hall/CRC, Boca Raton, 2006, 498 pp., USD 89.95, ISBN 1-58488-663-3*

This monograph is devoted to the construction and the properties of exact solutions with the help of finite dimensional invariant subspaces. These subspaces reduce evolution partial differential equations to finite dimensional dynamical systems. In the introduction, the authors explain the merit of invariant subspaces for exact solutions and in the first chapter they present basic examples (there are 41 examples in this chapter). In the second chapter invariant subspaces of maximal dimension for ordinary differential equations are described. The core of the book consists of chapters 3–5 in which invariant subspaces for nonlinear partial differential equations in one dimension are studied. The main attention is paid to thin film, Kuramoto-Sivashinsky, Korteweg-de Vries, Harry Dym, quasilinear and Boussinesq models.

Chapter 6 deals with nonlinear partial differential equations in RN. A more general notion of invariant sets leads to overdetermined dynamical systems. It is shown in chapter 7 that such systems may have solutions in some special cases. Invariant sets are connected with differential constraints and also with sign invariant operators for a given equation. These are studied in chapter 8. The last chapter is devoted to discrete operators. The book contains a considerable number of equations: the index gives 157 different equations. All chapters conclude with bibliographic comments as well as with open problems of a research character. This exhaustive monograph is addressed to advanced graduate students and to researchers in physics and engineering, as well as those who work with evolution partial differential equations, who will surely benefit from a rich and carefully chosen bibliography. (jmil)

***A. Geroldinger, F. Halter-Koch: Non-Unique Factorizations – Algebraic, Combinatorial and Analytic Theory**, Pure and Applied Mathematics, vol. 278, Chapman & Hall/CRC, Boca Raton, 2006, 700 pp., USD 119,95, ISBN 1-58488-576-9*

The investigation of properties of integral domains, in particular rings of integers of algebraic number fields that may not be unique factorization domains, is the original motivation for the study of the phenomena of non-unique factorization. It turns out that the phenomena are of a purely multiplicative nature and therefore we can restrict ourselves to the study of multiplicative monoids of integral domains. This strategy is pursued in this monograph, which is mainly concerned with the non-unique factorization properties of commutative cancellative monoids.

Chapter 1 surveys basic classical notions of the theory of non-unique factorization together with some elementary factorization properties of the rings of integers of algebraic number fields. Various invariants serving to classify the non-unicity of factorizations (sets of lengths, elasticity, catenary degree and tame degree) are introduced. Chapter 2 is an introduction to the theory of non-unique factorization of commutative cancellative monoids. The theory of v-ideals is developed alongside definitions of some auxiliary monoids. At the end of the chapter, results obtained for these monoids are applied to a study of factorization properties of integral domains. Chapter 3 is devoted to a study of arithmetic properties of the auxiliary monoids introduced in the previous chapter and again the results obtained are applied to integral domains. Chapter 4 deals with sets of lengths of factorizations. Under a rather general assumption on a monoid, the structure of its set of length is described.

Chapter 5 is a self-contained introduction to additive group theory. Its results are applied in chapters 6 and 7. Krull monoids with finite class group with the additional property that every class contains a prime are studied. These monoids are of particular interest because they include multiplicative monoids of integers of algebraic number fields and of holomorphy rings in algebraic function fields over finite fields. Chapter 8 is a self-contained introduction to analytic number theory focusing on notions applied in a modern treatment of the analytic theory of non-unique factorization presented in the last chapter. There, some asymptotic formulas for various counting functions are derived and these results are applied in orders in algebraic number fields and in holomorphy rings in algebraic function fields over finite fields. The monograph deals with the phenomena of non-unique factorization naturally appearing in the most fundamental questions of algebra. Combining methods of various branches of mathematics, it brings together a theory from classical results to topics reflecting the recent ideas. It is a nice book written in a precise, readable style. (pruz)

***R. Göbel, J. Trlifaj: Approximations and Endomorphism Algebras of Modules**, de Gruyter Expositions in Mathematics, vol. 41, Walter de Gruyter, Berlin, 2006, 640 pp., EUR 128, ISBN 3-11-011079-2*

This monograph covers two main topics: approximations of modules and realization of R-algebras as endomorphism algebras of R-modules over a commutative ring R. The first introductory chapter surveys notions such as S-completions, pure-injective modules, locally projective modules and slender modules, notions repeatedly used in the rest of the book. The next seven chapters deal with the existence of C-approximations, that is C-(pre)covers and, dually, C-preenvelopes, for a class C of modules. This problem is approached by employing cotorsion theories, a method that is justified by results on complete and perfect cotorsion pairs. In particular, the long-standing Flat Cover Conjecture, which predicts the existence of flat covers in any module category, is solved positively.

Beyond these applications many nice results are obtained: for example, the equality of certain cotorsion pairs characterize Dedekind, Prüfer domains and almost perfect domains. Three chapters are devoted to tilting and cotilting cotorsion theories. First the tilting case is settled. The authors characterize cotorsion pairs induced by n-tilting modules and prove that a class of modules is tilting if and only if it is of finite type. Moreover, they characterize cotorsion pairs induced by a 1-tilting module and study these pairs over Artin algebras, Dedekind, Prüfer and valuation domains. The theory of tilting cotorsion pairs is applied to describe Matlis localizations and, in chapter 7, to attack finistic dimension conjectures. The dual cotilting case is studied in chapter 8. It is proved that cotilting modules are pure injective and, as in the tilting case, cotilting torsion theories are characterized.

The second part of the book begins with an introduction of some set theory prediction principles: the Diamond Principles, requiring additional set theory axioms, and the Black Box Principles, which are proved in ZFC. In the next chapter it is proved that completeness of certain cotorsion pairs is independent on ZFC+GCH. In chapter 11, the lattice of cotorsion pairs of abelian groups is investigated. Next, the Black Box Principles are applied to realize R-algebras as endomorphism algebras of R-modules satisfying various cardinality and structural properties. Chapter 13 is devoted to the construction of $E(R)$-algebras, i.e. R-algebras A that are naturally isomorphic to the endomorphism rings of A as left R-modules. The next chapter settles the problem of realizing R-algebras in case the ring R is not an S-ring for any suitable multiplicative set S. In this case different techniques and set theory principles to the Diamond and the Black Box Principles are needed. The last chapter deals with the realization of some particular algebras, namely Leavitt type algebras, as endomorphism algebras of modules. The complexity of these algebras implies the existence of various pathological decompositions of the modules.

The monograph presents recent achievements in two rapidly developing directions in the theory of modules over associative rings. The reader is introduced to a comprehensive theory based on many nice results and ideas and they are encouraged to participate in its future development. I strongly recommend the monograph to anyone who is interested in the modern theory of modules. (pruz)

***G. Grimmett, C. McDiarmid, Eds.: Combinatorics, Complexity and Chance – A Tribute to Dominic Welsh**, Oxford Lecture Series in Mathematics and its Applications 34, Oxford University Press, Oxford, 2007, 130 pp., GBP 39.50, ISBN 978-0-19-857127-8*

This book is a tribute to Dominic Welsh who formally retired from Oxford University in 2005. Dominic Welsh has been extremely influential in many aspects of discrete mathematics, especially in the theories of graphs, matroids, algorithmic complexity, cryptography and knots, together with discrete physical models and applied probability. His doctoral thesis was written under the supervision of John Hammersley. This was a basis for the important joint publication with Hammersley, which laid the foundations for first-passage percolation and subadditive stochastic processes. Later, Dominic Welsh also laid the foundations of the theory of matroids and wrote the first major text on the subject. The strong influence of Dominic Welsh in these and other areas of mathematics is widely recognised. The topics represented in this volume reflect the wide vision of Dominic Welsh. The contributors include some of his ex-graduate students together with several prominent colleagues. There are articles on graphs, matroids, polynomials, random structures, algorithms and knots. (mloe)

J.L. Gross, J. Yellen: Graph Theory and its Applications, *second edition, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2005, 779 pp., USD 84,95, ISBN 1-58488-505-X*

Graph theory is well-known as a rich source of concepts and algorithms applicable to many different disciplines. This book gives an excellent exposition of graphs, graph algorithms and their applications. It builds the theory from the very basics, so it is easy to understand for people not yet skilled in discrete mathematics, but at the same time it gives deep insight into the topics discussed, which is a virtue rarely seen in books on applications. The first few chapters cover the standard material: graph connectivity, traversals, colourings, flows and graph drawing. The subsequent chapters follow up by explaining several advanced topics like Ramsey theory, extremal graph theory and combinatorial enumeration. The theory is well spiced with examples, figures, algorithms and notes on their implementation. Except for the very beginning, the chapters are independent of each other and each of them is accompanied by a glossary of terms, making the work not only a good textbook but also a handy reference. (mmar)

D. D. Haroske: Envelopes and Sharp Embeddings of Function Spaces*, Chapman & Hall/CRC Research Notes in Mathematics, vol. 437, Chapman & Hall/CRC, Boca Raton, 2006, 227 pp., USD 89,95, ISBN 1-58488-750-8*

The trends in function spaces have always been dictated by the needs in the areas where function spaces are applied, for example partial differential equations, mathematical physics, and probability theory. The most important recent trend in function spaces is the optimality, or sharpness, of the obtained embedding results. Moreover, particular attention is paid to limiting (or critical) cases. It is very important to show that a certain result cannot be made any better unless a new category of objects is introduced. A good choice of optimal function space can often solve a difficult problem. The qualities of a function, studied in the theory of function spaces, vary in dependence on the background application where the particular problem arose.

The book under review presents a thorough investigation of two (and maybe the most) important such qualities of a function, namely its size (or growth) and its continuity (or smoothness). The author presents a new approach to the topic, based on studying the so-called growth envelopes and continuity envelopes. One of the basic ideas involved is to study how big the non-increasing rearrangement of a function can be, given that the function belongs to a certain function space. This task is perhaps similar to the question of finding the smallest Marcinkiewicz space containing the given space. The approach, built upon an impressive series of the author's results, has turned into a worthwhile general theory full of beautiful, deep results, interesting examples and plenty of applications (to name just two examples, let us mention the asymptotic behaviour of the approximation numbers and the compactness of embeddings). All this the reader will find in the text. On top of that, the book is more reader-friendly than the standard; it can be easily read as a 'bed-time fairy tale' (my personal experience). The book comes from one of the world's most famous centres of function space theory, the Friedrich-Schiller University in Jena, which was created and is still led by Professor Hans Triebel. Many wonderful books have already arrived from this University and here we have another one (and certainly not the last one). Truly delightful stuff! (lp)

Andreas K. Heyne, Alice K. Heyne, E. S. Pini: Leonhard Euler. Ein Mann, mit dem man rechnen kann*, Birkhäuser, Basel, 2007, 50 page, USD 34,95, ISBN 978-3-7643-7779-3*

This book, written in German by Andreas K. Heyne and Alice K. Heyne and illustrated by Elena S. Pini, is dedicated to the life and work of Leonhard Euler (1707–1783), who was the pre-eminent mathematician of the 18th century and one of the greatest mathematicians of all time throughout the world. The authors and illustrator describe the most important moments from Euler's life in Switzerland, Russia and Germany. They show his significant discoveries in fields as diverse as analysis, algebra, geometry, topology, mechanics, optics, astronomy and geography. The remarkable book, written like a modern comic, is very readable and can be recommended to children, students and their teachers to attract their interest to mathematics and scientific investigation. It can also be recommended to anyone interested in mathematics, the history of mathematics or science. (mbec)

K. H. Hofmann, S. A. Morris: The Structure of Compact Groups, *de Gruyter Studies in Mathematics 25, Walter de Gruyter, Berlin, 1998, 835 pp., DM 278, ISBN 3-11-015268-1*

This book is an edited and augmented second printing of the first edition of the book, which was reviewed in the EMS Newsletter in June 1999, page 33. New parts of the text mostly present new research published since the last edition and some newly included material. Some typographical errors and misprints have been corrected. The list of references has also been augmented. (vs)

S. Kantorovitz: Introduction to Modern Analysis*, Oxford Graduate Texts in Mathematics 8, Oxford University Press, Oxford, 2006, 434 pp., GBP 29,95, ISBN 0-19-920315-6*

This book presents a nicely written exposition of important topics of measure theory, integration and functional analysis, with applications to probability theory and partial differential equations. The first two chapters deal with abstract Lebesgue integration theory (L^p-spaces, the Lebesgue-Radon-Nikodym theorem, complex measures, construction of measures and product measure). Chapter 3 deals with measure and topology. The Riesz-Markov representation theorem is proved and differentiation of measure in Euclidean spaces is discussed. Chapter 4 is devoted to conjugates of L^p-spaces as well as spaces of continuous functions and the Haar measure. Basic functional analysis is the object of chapter 5 (duality, Hahn-Banach theorems, weak topologies, the Krein-Milman theorem, the Stone-Weierstrass theorem and Marcinkiewicz's interpolation theorem).

Chapter 6 is devoted to bounded operators (the uniform boundedness principle, the open mapping and closed graph theorems). Chapter 7 deals with Banach algebras and includes the Gelfand-Naimark-Segal representation theorem. Chapters 8–10 concentrate on Hilbert spaces, integral representation and unbounded operators. More advanced topics include the von Neumann double commutant theorem, spectral representation for normal operators and extension theory for unbounded symmetric operators. The chapter called 'Application I' deals with probability theory and 'Application II' deals with distributions and partial differential equations. The Hörmander-Malgrange theorem is proved and fundamental solutions of linear partial differential equations with variable coefficients are discussed. Some complementary material is included in the exercise sections. The book can be warmly recommended to university students and teachers of mathematics. (in)

R. Kossak, J. Schmerl: The Structure of Models of Peano Arithmetic, *Oxford Logic Guides 50, Oxford University Press, Oxford, 2006, 311 pp., GBP 50, ISBN 0-19-856827-4*

This book is an advanced, comprehensive text covering branches of the problems of models of Peano arithmetic that are not treated in Kayes's book, "Models of Peano arithmetic", a 'bible' of the study of models of arithmetic. It is assumed that the reader is familiar with Kaye's book. Roughly speaking, the two basic themes of the book are the substructure lattices of elementary submodels of a given model of Peano arithmetic and recursive and arithmetic saturation. The first theme is mostly developed in the first four chapters (extensions in chapter 2, minimal and other types in chapter 3 and interstructure lattices in chapter 4). The second main topic can be found in chapters 5–11. Automorphism groups of recursively saturated models are studied in chapters 8 and 9, indiscernible generators are treated in chapter 5, the ω_1-like models in chapter 10 and a classification of reducts in chapter 11. Chapter 6 deals with generics and forcing. 'Twenty questions' is the name of the last chapter. Exercises of various difficulties are included as an integral part of each chapter. The book is a welcome, comprehensive and useful publication. (jmlc)

B. Landman, M. B. Nathanson, J. Nešetřil, R. J. Nowakowski, C. Pomerance: Combinatorial Number Theory*, Walter de Gruyter, Berlin, 2007, 489 pp., EUR 168, ISBN 978-3-11-019029-8*

This is a collection of 34 refereed papers by the participants of the conference held in October 2005 in celebration of Ron Graham's life jubilee. Topics considered include Ramsey theory, automatic proofs of identities, generating functions, combinatorial number theory, juggling patterns, combinatorial game theory, combinatorial enumeration and graph theory. The book contains a survey paper by Ron Graham with a list of open problems on Ramsey theory accompanied with monetary rewards (e.g. $100 for proving that the graph of unit distances in the plane is not four-colourable) and a list of his publications. (mklaz)

H. T. Lau: A Java Library of Graph Algorithms and Optimization *+ CD, Discrete Mathematics and Its Applications, Chapman & Hall/CRC, Boca Raton, 2006, 386 pp., USD 99,95, ISBN 1-58488-718-4*

The series 'Discrete Mathematics and Its Applications' has delivered an unusual fruit this time: a library of algorithms for various combinatorial problems programmed in Java. The book covers all the usual areas of algorithmic graph theory and optimization ranging from the random generation of graphs, connectivity, network flows and graph embedding, to linear and quadratic optimization, each area being discussed in a separate chapter. The chapters share a common structure and each of them lists a bunch of related problems. All problems are first described and then an outline of the algorithm is presented, followed by a full program. The program is accompanied by a description of inputs and outputs and a simple example of its use. The code of the programs is also available on a CD included in the book.

However, serious users of the book will notice several drawbacks: the description of the algorithms sometimes lacks important details such as the time complexity (which is surprising, especially in exponential algorithms for NP-complete problems) and the program code is often hard to understand because the aspiration to make all programs self-contained has led to avoiding all abstractions and repeating the same code patterns in many places. For some problems, faster and more easily implemented algorithms are already known. In short, it is a good book for anybody who needs to solve an isolated problem but more experienced programmers will probably still prefer the existing open-source program libraries like LEDA or Pigale. (mmar)

M. Lübke, A. Teleman: The universal Kobayashi-Hitchin correspondence on Hermitian manifolds*, Memoirs of the AMS, vol. 863, American Mathematical Society, Providence, 2006, 97 pp., USD 58, ISBN 0-8218-3913-6, ISBN 978-0-8218-3913-3*

Over the last few decades, a lot of attention has been paid to a description of many different moduli spaces connected with various holomorphic objects. A typical (classical) problem of that sort can be described as follows. Fix a compact complex manifold X and a smooth vector bundle E on it. We would like to classify all holomorphic structures on E inducing a given holomorphic structure on the determinant line bundle det E on X. To have a Hausdorff moduli space, a condition of stability (semistablity) should be introduced. The book is devoted to a description of a suitable (universal) moduli problem, which includes a significant number of special moduli problems studied recently as special cases. In particular, the authors introduce their universal moduli problem and describe suitable stability (polystability) conditions for it, and they characterize polystable pairs in a geometric way. They also study (using ideas from Donaldson theory) metric properties of a large class of moduli spaces and they apply the results to some interesting cases (Douady Quot spaces, moduli spaces of oriented connections and moduli spaces of non-Abelian monopoles on Gauduchon surfaces). (vs)

D. McMahon, D.M. Topa: A Beginner's Guide to Mathematica*, Chapman & Hall/CRC, Boca Raton, 2006, 725 pp., USD 69,95, ISBN 1-58488-467-3*

This book deals with the *Mathematica* system of computer algebra, or, more generally, with that part of mathematics in which the *Mathematica* software can be used as a helpful tool. After introducing basic concepts of the philosophy of *Mathematica*'s syntax, the reader is led through the questions of data and file structures, the concept of *Mathematica*'s notebook, add-on packages, palettes and the general questions of establishing a communication between the user and the program. Then the basic computation (calculus) and graphics examples are given and the more sophisticated issues are discussed, such as differential equations, integral transforms and special functions. Due to the large command structure and tricky syntax, *Mathematica* is often considered as a computer system difficult to learn in a reasonable time. The first look at this book seems to bear out this impression; while being written for beginners, it runs to almost 700 pages, which can discourage the beginner from learning such a piece of software. On the other hand, if someone decides to learn *Mathematica*, books of this kind are necessary. In this book, the reader is taught step-by-step the basic skills needed to use *Mathematica* in practice. A "non-threatening" style of instruction is used, namely teaching by examples and practical applications. A large index list is also useful. (mrok)

I. Molchanov: Theory of Random Sets, Probability and Its Applications, *Springer, New York, 2005, 488 pp., EUR 96.25, ISBN 1-85233-892-X*

Random sets play an important role in many applications of mathematics and have been studied intensively since the 1970s, when the pioneering book of Georges Matheron was published. This book is an important contribution to the mathematical theory and will surely serve as a valuable textbook for students as well as researchers. It presents a self-contained survey of all the significant results, including the proofs about random sets and capacity functionals, convergence of random sets, expectations, limit laws for Minkowski additions and for unions, and stochastic processes of random closed sets. The theory is illustrated by examples of applications, e.g. in stochastic geometry, stochastic optimisation, belief functions and finance mathematics. A number of open problems are presented and each chapter concludes with a list of bibliographical notes. (jrat)

C. C. Moore: Mathematics at Berkeley – A History*, A.K. Peters, Wellesley, 2007, 341 pp., USD 39, ISBN 978-1-56881-302-8*

This interesting book tells the fascinating story of the evolution of mathematics at the University of California, Berkeley, from 1868 till 2006. The author describes the history of the foundation, development and transformation of the department of mathematics. The major theme is to show how a department in a state university that was, in its early history, devoted to teaching, developed into the major research centre that is among a small group of the very best departments in the USA. The most important mathematicians and their scientific works and activities are carefully presented. The author also includes some interesting and helpful discussions of mathematics instruction in California before the founding of the University of California (chapter 1), as well as discussions on the role of the Mathematical Sciences Research Institute (chapter 19) in the development of mathematical research in the USA. The author shows the development of the mathematics department within the history of the university, the social, political and financial history of the state of California, as well as the history of American mathematics and the American Mathematical Society. The excellent photos, index and bibliography are included at the end of book. The book can be recommended to anybody who is interested in the development of mathematical societies and teaching and research in the 19th and 20th centuries in the USA. (mbec)

R. Müller: Differential Harnack Inequalities and the Ricci Flow*, EMS Series of Lectures in Mathematics, European Mathematical Society, Zürich, 2006, 92 pp., EUR 24, ISBN: 978-3-03719-030-2*

A main core of three topics: Li Yau type differential Harnack inequalities, entropy formulas and space-time geodesics, is presented in this book. Emphasis is put on the connections between these three subjects. Each topic is first explained for the heat equation on a static manifold. The results are then compared with the results for the Ricci flow equation or a heat equation on a manifold evolving by Ricci flow. The presented theory is recent, mainly connected with R. S. Hamilton, P. Li, L. Ni, G. Perelman and S. T. Yau. Applications for three-dimensional manifolds towards the Poincaré conjecture are not considered. The book is comprehensible and well-ordered. The main ideas of proofs are well explained; however, some parts of the book are rather technical. As the author presents the core of the theory in detail, the book is suitable for advanced students or non-experts familiar with the basic concepts and notions of Riemannian geometry. No preliminary knowledge of Ricci flow or Harnack inequalities is required. (pkap)

J.-P. Pier: Mathématiques – entre savoir et connaissance*, Vuibert, Paris, 2006, 211 pp., EUR 28.50, ISBN 2-7117-7181-4*

This book, written in elegant French by a mathematician and historian of mathematics J.-P. Pier, is a special and very interesting contribution to the history of mathematics, the history of science and science itself. The author asks some provocative, creative and motivational questions and he tries to answer them from many points of view (such as mathematics, philosophy, history and sociology). He describes what mathematics means for mathematicians and others and he shows the position of mathematics in our scientific system. He also deals with the influence of mathematics on the historical development of philosophical ideas, daily life, technology and culture, etc. He discusses why we like or dislike mathematics, why we divide mathematics into two parts: pure and applied mathematics or classical and modern mathematics, and how we apply and use mathematics in technological progress, our daily life and work. Each of his ideas is described and illustrated with perfectly chosen subjects or examples from number theory, algebra, geometry and analysis. To underline his ideas, the author gives many interesting quotations and sentences from the works of principal mathematicians, physicists and philosophers. The book is written like a remarkable and easily readable essay and can be recommended to all people who are interested in mathematics, the history of mathematics, philosophy and the history of science. (mbec)

B. Polster: The Shoelace Book*, Mathematical World, vol. 24, American Mathematical Society, Providence, 2006, 125 pp., USD 23, ISBN 0-8218-3933-0*

There are various problems one can have with shoelaces (broken shoelaces, undone shoelaces, missing shoelaces, too long or too short shoelaces and so on). Mathematically minded people however might find interest in yet another set of problems, very different from those mentioned. These include questions like: 'What is the shortest/longest/strongest/weakest way to lace the shoes?' and 'How many ways are there to lace the shoes?'. This beautiful and amusing book attacks all these problems from a mathematical point of view. It all started with an innocent article published by the author in 2002 in the journal *Nature*, which attracted an enormous amount of publicity and great interest in the mathematics of shoelaces from many people in all walks of life. The mathematics of shoelaces is a lovely combination of combinatorics and elementary analysis. It has nice and surprising connections to things like the travelling salesman problem or calculating the area of simply closed planar polygons. The book also has sections on the history of shoelacing, shoelace superstitions, style and fashion, and at the end it tackles the difficult philosophical question: what is the best way to lace the shoes? A very enjoyable book indeed. (lp)

W. Rindler: Relativity – Special, General and Cosmological, *Oxford University Press, Oxford, 2001, 428 pp., GBP 24,95, ISBN 0-19-850836-0, ISBN 0-19-850835-2*
This book is a completely rewritten and substantially extended version of the previous popular book by the same author. It consists of three parts covering the major topics in relativity theory: special relativity (including relativistic optics, relativistic mechanics and electromagnetic fields in vacuum), general relativity (with sections on black holes, plane gravitational waves, de Sitter space and linearized gravity) and cosmology (including Friedman-Robertson-Walker spacetime, its dynamics and light propagation in it). The author continually stresses the main ideas and concepts, and the crucial experiments and observations. The book is based on a sound and rigorous mathematical formalism. Prerequisites include standard differential calculus, standard linear algebra and calculus of tensor fields. Every chapter ends with a set of exercises. This is a very reader-friendly book with many illustrations and diagrams and will be very useful for students as well as for research workers from other branches of the natural sciences who want to understand more about general relativity and its applications. (vs)

I. A. Rus: Fixed Point Structure Theory, *Cluj University Press, Cluj-Napoca, 2006, 215 pp., ISBN 973-610-448-6, ISBN 978-973-610-448-0*
This book is a monograph on the so-called fixed point structures that cover topological and ordered spaces and operators on them possessing fixed points. The book is divided into two parts; the first one is devoted to single-valued operators and the second one to multi-valued operators. Many results are based on the author's research. Even though all the standard conditions for the existence of a fixed point are included, a student may have a problem finding them since they are presented as examples of general structures. The missing subject index is surely a disadvantage of the book. There is an author index but one does not find, for example, the Brouwer and Schauder fixed point theorems under these names. The book will be interesting for researchers in topology studying interconnections between various fixed point properties. Those who are interested in applications will find many references to other texts. (jmil)

V. S. Ryaben'kii, S.V. Tsynkov: A Theoretical Introduction to Numerical Analysis, *Chapman & Hall/CRC, Boca Raton, 2006, 537 pp., USD 79.95, ISBN 1-58488-607-2*
This book introduces the reader to basic ideas of numerical analysis and it covers the main topics in the field. The book is divided into four parts. The introductory part gives the reader fundamental objects of numerical analysis such as errors and basic methods of computation. The first part of the book is devoted to an overview of interpolation (algebraic interpolation and trigonometric interpolation) and quadrature of functions. The second part studies the problem of solving systems of scalar equations (systems of linear algebraic equations; direct methods, iterative methods for solving linear systems and overdetermined linear systems; the method of least squares, numerical solution of nonlinear equations and their systems). The third part studies the method of finite differences for numerical solution of ordinary differential equations, finite difference schemes for partial differential equations, discontinuous solutions and methods of their computation and discrete methods for elliptic problems. The fourth part contains a discussion of the method of boundary equations for the numerical solution of boundary value problems and boundary integral equations and the method of boundary elements, boundary equations with projections and the method of difference potentials.

The basic concepts (discretization, errors, efficiency, complexity, numerical stability, consistency, convergence, etc.) are explained and illustrated in different parts of the book. The prospective reader would be a graduate or senior undergraduate student in mathematics, science or engineering. (jkof)

C. E. Sandifer: The Early Mathematics of Leonhard Euler, *Cambridge University Press, Cambridge, 2007, 391 pp., GBP 25,99, ISBN 978-0-88385-559-1.*
This book is dedicated to the analysis of 49 of Euler's mathematical works, which he wrote between 1725 and 1741, i.e. during his life in St. Petersburg. The author describes Euler's mathematical production year by year. He starts with the work E1 and finishes with E790 (he refers to Euler's individual works by their numbers in the Eneström index). At the beginning of each chapter he gives the full name of Euler's article in its original language and in English, the full quotation of the source (i.e. the name and pages in the original journals and in the *Opera Omnia*). Then he explains the mathematical content and adds a short list of literature. The topics of Euler's articles include series, geometry and curves, calculus of variations, elliptic integrals, differential equations, number theory, the theory of equations, topology and philosophy. The reader can find here some of Euler's important mathematical works, for example the Königsberg bridge problem, the Euler solution to the Basel problem, his first proof of the Euler-Fermat theorem, as well as his contributions to notation, calculus, etc. The book shows how Leonhard Euler grew from a young student of age 18 to a supreme mathematician and scientist of his time. The author creates a portrait of Euler and his mathematical production, which shows a connection with works of other mathematicians and its historical context. The book can be recommended to all people who are interested in mathematics and its history. (mbec)

W. Schwarz, J. Steuding, Eds.: Elementare und Analytische Zahlentheorie (Tagungsband). Proceedings Elaz-Conference May 2004*, Franz Steiner Verlag, Stuttgart, 2006, 348 pp., EUR 48, ISBN 3-515-08757-5, ISBN 978-3-515-08757-5*

ELAZ-Conferences (abbreviated from Elementare und Analytische Zahlentheorie) is a series of conferences devoted to elementary and analytic number theory. The first conference in the series was organized in 2000 by L. Lucht at the Technische Universität Clausthal. This proceedings contains talks and adapted versions of talks presented at the third conference in this series. It was organized by W. Schwarz and J. Steuding (both from Goethe-Universität Frankfurt/Main). At the conference 41 talks were delivered. The proceedings contains 22 papers and one obituary for Thomas Maxsein (1953–2004). The scope of the proceedings is wide and contains contributions ranging from the Goldbach problem, the Waring problem, the zeta function and L-series, arithmetic semigroups, exponential sums, arithmetic functions and the uniform distribution, to probabilistic number theory. Most of the presented papers are research contributions. The volume also contains surveys, for example on estimates of Fourier coefficients of Siegel cusp forms (K. Bringmann), lattice points in large regions and related arithmetic functions (A. Ivić, E. Krätzel, M. Kühleitner and W.G. Nowak), recent results on the Goldbach conjecture (J. Pintz) and ABC for polynomials, dessins d'enfants, and uniformization (J. Wolfart). The book is printed with great care and is worth browsing and reading for anyone interested in elementary and analytic number theory or to people looking for fresh research impetus in this quickly developing branch of mathematics. (špor)

J. D. H. Smith: An Introduction to Quasigroups and Their Representations*, Studies in Advanced Mathematics, Chapman & Hall/CRC, Boca Raton, 2006, 340 pp., USD 99.95, ISBN 1-58488-537-8*

This book is based on a research program that spans over thirty years. During those years the author was consistently trying to develop tools for the theory of quasigroups that would possess a sound categorical basis and would involve the classical representation theory of groups as a special case. The earliest fruits of this effort were the notion of the centralizer ring of a quasigroup and the notion of the universal multiplication group. The former gave rise to the theory of quasigroup characters to which more than one third of the book is devoted. However, this is preceded by the more recent concepts of homogeneous spaces and permutation representations that are based on the usual definition of the multiplication group as the permutation group generated by left and right translations.

Consider the orbits of the permutation group generated by left translations supplied by elements of a subquasigroup. The key notion is that of Markov matrices indexed by these orbits, where for each element of the quasigroup one constructs a matrix in which each row describes the probability distribution of the elements in the orbit indexing the row into all orbits when the right translation is applied. This connection of quasigroup theory and probability is novel and might lead in future to surprising and interesting consequences. While the book certainly introduces the notion of the quasigroup, it cannot be regarded as a general source of knowledge about the development of the theory of quasigroups and loops in recent decades, despite many interesting exercises that often overcome the limits set by the exposition goals. (ad)

T. Tao: Solving Mathematical Problems – A Personal Perspective*, Oxford University Press, Oxford, 2006, 103 pp., GBP 12.99, ISBN 0-19-920560-4, ISBN 0-19-920561-2*

In the six chapters of the book, selected problems at a Mathematical Olympiad level are solved. The first chapter, "Strategies in problem solving", explains in detail the main strategic steps in solving a concrete problem. The following five chapters contain solutions to many problems from number theory, algebra, analysis, and Euclidean and analytic geometry. The sixth chapter, "Sundry examples", is dedicated to interesting (and quite funny) problems that have something in common with combinatorics and game theory. At the end of some solved problems, there are additional exercises whose solutions can be obtained in a similar manner. It is an important feature of the book that problems are not only solved but that various solving strategies and methods are also demonstrated, which make problems easier and help to find the best route towards a solution. Without any exaggeration, I can say that the book gave me enormous pleasure. It not only portrays the author's enthusiasm for the beauty of mathematics but also his ability to explain problems and their solutions to young people who are at the beginning of their mathematical career. And this is one of the reasons why the book will be useful for pupils and students who are interested in mathematics. It can also be recommended to mathematics teachers working with gifted students and will undoubtedly make their lectures more attractive. (ec)

R. Taschner: Numbers at Work – A Cultural Perspective*, A.K. Peters, Ltd., Wellesley, 2007, 209 pp., USD 39, ISBN 978-1-56881-290-8*

This interesting book (originally published in German as "Der Zahlen gigantische Schatten" in 2005) is a fascinating reading on the history and use of numbers. Using a description of lives and works of great mathematicians, musicians, physicists and philosophers (Pythagoras, Bach, Hofmannsthal, Descartes, Leibniz, Laplace, Bohr and Pascal), the author shows connections between number theory and other fields of science (e.g. astronomy, physics and geometry) as well as the role of numbers in painting, architecture, music, religion, politics, economics, thinking, etc. Each of the eight chapters contains many photographs and pictures of high quality (the book contains 163 pictures), as well as many historical and mathematical notes and explanations and a list of references to material that is both popular and professional. A deeper background in mathematics is not necessary in order to read, enjoy and learn from this book. The book can be recommended to all readers interested in the world around us and wanting to understand the importance of numbers in our daily lives. (mbec)

R. R. Thomas: Lectures in Geometric Combinatorics*, Student Mathematical Library, vol. 33, American Mathematical Society, Providence, 2006, 143 pp., USD 29, ISBN 0-8218-4140-8*

This book provides lectures in the geometry of convex polytopes in arbitrary dimension, with its geometric, combinatorial and computational aspects. This course is suitable for graduate students just starting out. The main goal of these lectures is to develop the theory of convex polytopes from a geometric viewpoint to lead up to recent developments centred around secondary and state polytopes arising from point configurations. The geometric viewpoint relies on linear optimization over polytopes. The book starts with the basics of the theory. Schlegel and Gale diagrams are introduced as tools to visualise polytopes in higher dimension. The heart of the book is a treatment of the secondary polytope of a point configuration and its connections to the state polytope of the toric ideal defined by this configuration. There are numerous connections to discrete geometry, classical algebraic geometry and combinatorics. The connections rely on Groebner bases of toric ideals and other methods from commutative algebra. The lectures are self-contained and do not require any background beyond basic linear algebra. (mloe)

J. L. Vázquez: The Porous Medium Equation – Mathematical Theory*, Oxford Mathematical Monographs, Oxford University Press, Oxford, 2006, 624 pp., GBP 65, ISBN 0-19-856903-3*

In this monograph, mathematical issues such as existence, uniqueness, regularity, spatially asymptotic and long-time behaviour, and free boundary evolution that concerns a degenerate scalar nonlinear parabolic equation, called the porous medium equation, are systematically addressed. The porous medium equation appears in a number of physical applications, including gas flow through a porous medium, groundwater flow, heat radiation in plasma, problems in population dynamics, diffusion, and lubrication, to name a few. Each chapter contains a detailed introduction and concludes with commentary notes that include historical remarks, recommended further reading and exercises. The book covers the development of the mathematical theory over the last fifty years. (jomal)

Oberwolfach Seminars November 2007

Conformal Invariance in Mathematical Physics

Date:	November 4th–November 10th, 2007
Deadline for application:	September 15th, 2007
Organisers:	S. Sheffield (New York), St. Smirnov (Geneva), W. Werner (Paris)

On Arithmetically Defined Hyperbolic Manifolds

Date:	November 4th–November 10th, 2007
Deadline for application:	September 15th, 2007
Organisers:	J. Rohlfs (Eichstätt), J. Schwermer (Wien), U. Stuhler (Göttingen)

Enumerative Combinatorics and Integrable Models of Statistical Mechanics

Date:	November 18th–November 24th, 2007
Deadline for application:	October 1st, 2007
Organisers:	Chr. Krattenthaler (Wien), Ph. Di Francesco (Gif-Sur-Yvette)

Recent Developments in Conformal Differential Geometry

Date:	November 18th–November 24th, 2007
Deadline for application:	October 1st, 2007
Organisers:	H. Baum (Berlin), A. Juhl (Uppsala)

The Oberwolfach Seminars are organised by leading experts in the field, and address postdocs and Ph.D. students from all over the world. The aim is to introduce the participants to a particular hot development. The seminars take place at the Mathematisches Forschungsinstitut Oberwolfach (MFO).

The number of participants is restricted to 24. Applications including
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Prof. Dr. Gert-Martin Greuel
Director of the MFO
mfo@mathematik.uni-kl.de

Upon approvement of application, the participants of Oberwolfach Seminars are invited personally by the Director of the Institute. Participation is subject to such an invitation. The Institute covers accommodation and food. Travel expenses cannot be reimbursed. Arrival is on Sunday afternoon. See www.mfo.de for more information (programme etc.).

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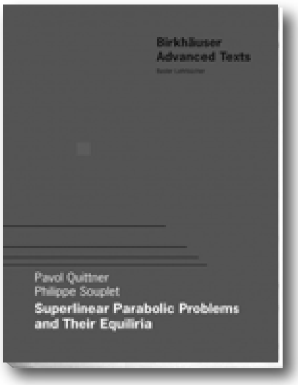

Superlinear Parabolic Problems

Blow-up, Global Existence and Steady States

Quittner, P., University of Bratislava, Slovakia / **Souplet, P.**, Université Paris XIII, France

This book is devoted to the qualitative study of solutions of superlinear elliptic and parabolic partial differential equations and systems. This class of problems contains, in particular, a number of reaction-diffusion systems which arise in various mathematical models, especially in chemistry, physics and biology. The book is self-contained and up-to-date, it has a high didactic quality. It is devoted to problems that are intensively studied but have not been treated so far in depth in the book literature. The intended audience includes graduate and postgraduate students and researchers working in the field of partial differential equations and applied mathematics.

2007. Approx. 595 p. Hardcover
EUR 69.90 / CHF 119.00
ISBN 978-3-7643-8441-8
BAT – Birkhäuser Advanced Texts

L'isomorphisme entre les tours de Lubin-Tate et de Drinfield

Fargues, L. / **Genestier, A.**, both Laboratoire de Mathématiques, Orsay, France / **Lafforgue, V.**, Université Paris 6, France

This book gives a complete and thorough proof of the existence of an equivariant isomorphism between Lubin-Tate and Drinfield towers in infinite level. The result is established in equal and inequal characteristics. Moreover, the book contains as an application the proof of the equality between the equivariant cohomology of both towers, a result that has applications to the local Langlands correspondence. Along the proof background and complements are given on the structure of both moduli spaces, p-divisible formal groups and p-adic rigid analytic geometry.

2007. Approx. 400 p. Hardcover
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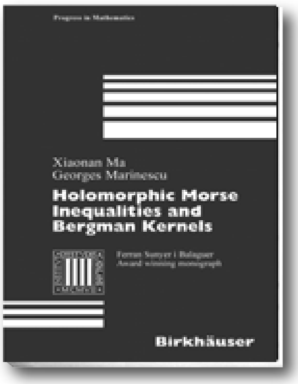
ISBN 978-3-7643-8455-5
PM – Progress in Mathematics, Vol. 262



Holomorphic Morse Inequalities and Bergman Kernels

Ma, X., Ecole Polytechnique, Paris, France / **Marinescu, G.**, University of Köln, Germany

Award-winning monograph of the Ferran Sunyer i Balaguer Prize 2006

This book gives for the first time a self-contained and unified approach to holomorphic Morse inequalities and the asymptotic expansion of the Bergman kernel on manifolds by using the heat kernel, and presents also various applications. The main analytic tool is the analytic localization technique in local index theory developed by Bismut-Lebeau. The book includes the most recent results in the field and therefore opens perspectives on several active areas of research in complex, Kähler and symplectic geometry. A large number of applications are included, e.g., an analytic proof of the Kodaira embedding theorem, a solution of the Grauert-Riemenschneider and Shiffman conjectures, a compactification of complete Kähler manifolds of pinched negative curvature, the Berezin-Toeplitz quantization, weak Lefschetz theorems, and the asymptotics of the Ray-Singer analytic torsion.

2007. Approx. 440 p. Hardcover
EUR 59.90 / CHF 105.– / ISBN 978-3-7643-8096-0
PM – Progress in Mathematics, Vol. 254

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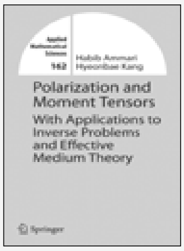
Applied Mathematics in Focus



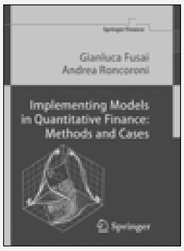
Polarization and Moment Tensors

With Applications to Inverse Problems and Effective Medium Theory

H. Ammari, Ecole Polytechnique, Palaiseau, France; **H. Kang**, Seoul National University, South Korea

This book presents important recent developments in mathematical and computational methods used in impedance imaging and the theory of composite materials. By augmenting the theory with interesting practical examples and numerical illustrations, the exposition brings simplicity to the advanced material. An introductory chapter covers the necessary basics. An extensive bibliography and open problems at the end of each chapter enhance the text. Graduate students and researchers in mathematics will benefit from this book. Researchers in engineering and physics might also find this book helpful.

2007. Approx. 330 p. 25 illus. (Applied Mathematical Sciences, Volume 162) Hardcover
ISBN 978-0-387-71565-0 ► **€ 69,95 | £54.00**

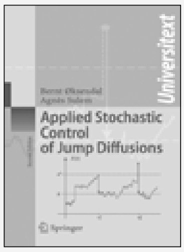


Implementing Models in Quantitative Finance: Methods and Cases

G. Fusai, Università del Piemonte Orientale, Novara, Italy; **A. Roncoroni**, ESSEC, Cergy Pontoise, France

This book puts numerical methods in action for the purpose of solving practical problems in quantitative finance. The first part develops a toolkit in numerical methods for finance. The second part proposes twenty self-contained cases covering model simulation, asset pricing and hedging, risk management, statistical estimation and model calibration. Each case develops a detailed solution to a concrete problem arising in applied financial management and guides the user towards a computer implementation. The appendices contain "crash courses" in VBA and Matlab programming languages. A companion CD provides ready-to-run codes (VBA, MATLAB).

2007. XIX, 686 p. 145 illus., 4 in color. (Springer Finance) Hardcover
ISBN 978-3-540-22348-1 ► **€ 69,95 | £54.00**



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B. Øksendal, University of Oslo, Norway; **A. Sulem**, INRIA Rocquencourt, Le Chesnay, France

Here is a rigorous introduction to the most important and useful solution methods of various types of stochastic control problems for jump diffusions and its applications. Discussion includes the dynamic programming method and the maximum principle method, and their relationship. The text emphasises real-world applications, primarily in finance. Results are illustrated by examples, with end-of-chapter exercises including complete solutions. The 2nd edition adds a chapter on optimal control of stochastic partial differential equations driven by Lévy processes, and a new section on optimal stopping with delayed information.

2nd ed. 2007. XIV, 262 p. 27 illus. (Universitext) Softcover
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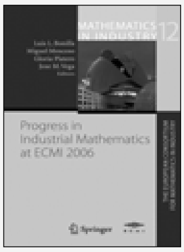
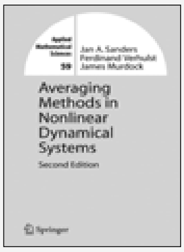


Progress in Industrial Mathematics at ECMI 2006

L. L. Bonilla, **M. Moscoso**, Universidad Carlos III de Madrid, Spain; **G. Platero**, CSIC Madrid , Spain; **J. M. Vega**, Universidad Politécnica de Madrid, Spain (Eds.)

The 14th European Conference for Mathematics in Industry held in Leganés (Madrid) focused on Aerospace, Information and Communications, Materials, Energy and Environment, Imaging, Biology and Biotechnology, Life Sciences, Finances and other topics including Education in Industrial Mathematics and web learning. Attendees came from all over the world. Overall, these proceedings give a lively overview of the importance of mathematical modeling, analysis and numerical methods when addressing and solving problems from today's real world applications.

2007. Approx. 1010 p. (Mathematics in Industry / The European Consortium for Mathematics in Industry, Volume 12) Hardcover
ISBN 978-3-540-71991-5 ► **€ 99,95 | £77.00**



2ND EDITION

Averaging Methods in Nonlinear Dynamical Systems

J. A. Sanders, Free University of Amsterdam, The Netherlands; **F. Verhulst**, State University of Utrecht, The Netherlands; **J. Murdock**, Iowa State University, Ames, IA, USA

Perturbation theory and in particular normal form theory has shown strong growth in recent decades. This book is a drastic revision of the first edition of the averaging book. The updated chapters represent new insights in averaging, in particular its relation with dynamical systems and the theory of normal forms. Also new are survey appendices on invariant manifolds. One of the most striking features of the book is the collection of examples, which range from the very simple to some that are elaborate, realistic, and of considerable practical importance. Most of them are presented in careful detail and are illustrated with illuminating diagrams.

2nd ed. 2007. XXIV, 434 p. (Applied Mathematical Sciences, Volume 59) Hardcover
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D. Jungnickel, University of Augsburg, Germany

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K.Engel, Mathematical Reviews 2002

The third edition of this standard textbook contains additional material: two new application sections and about two dozen further exercises. Moreover, recent developments have been discussed and referenced, in particular for the travelling salesman problem. The presentation has been improved in many places and a number of proofs have been reorganized, making them more precise or more transparent.

3rd ed. 2007. Approx. 665 p. 195 illus. (Algorithms and Computation in Mathematics, Volume 5) Hardcover
ISBN 978-3-540-72779-8 ► **€ 64,95 | £50.00**