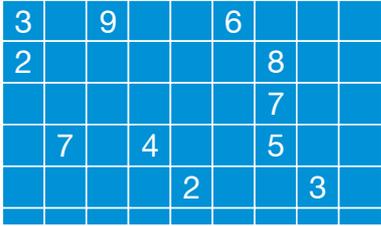


# NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



**Feature**  
Sudoku puzzles

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**History**  
Euler in Berlin

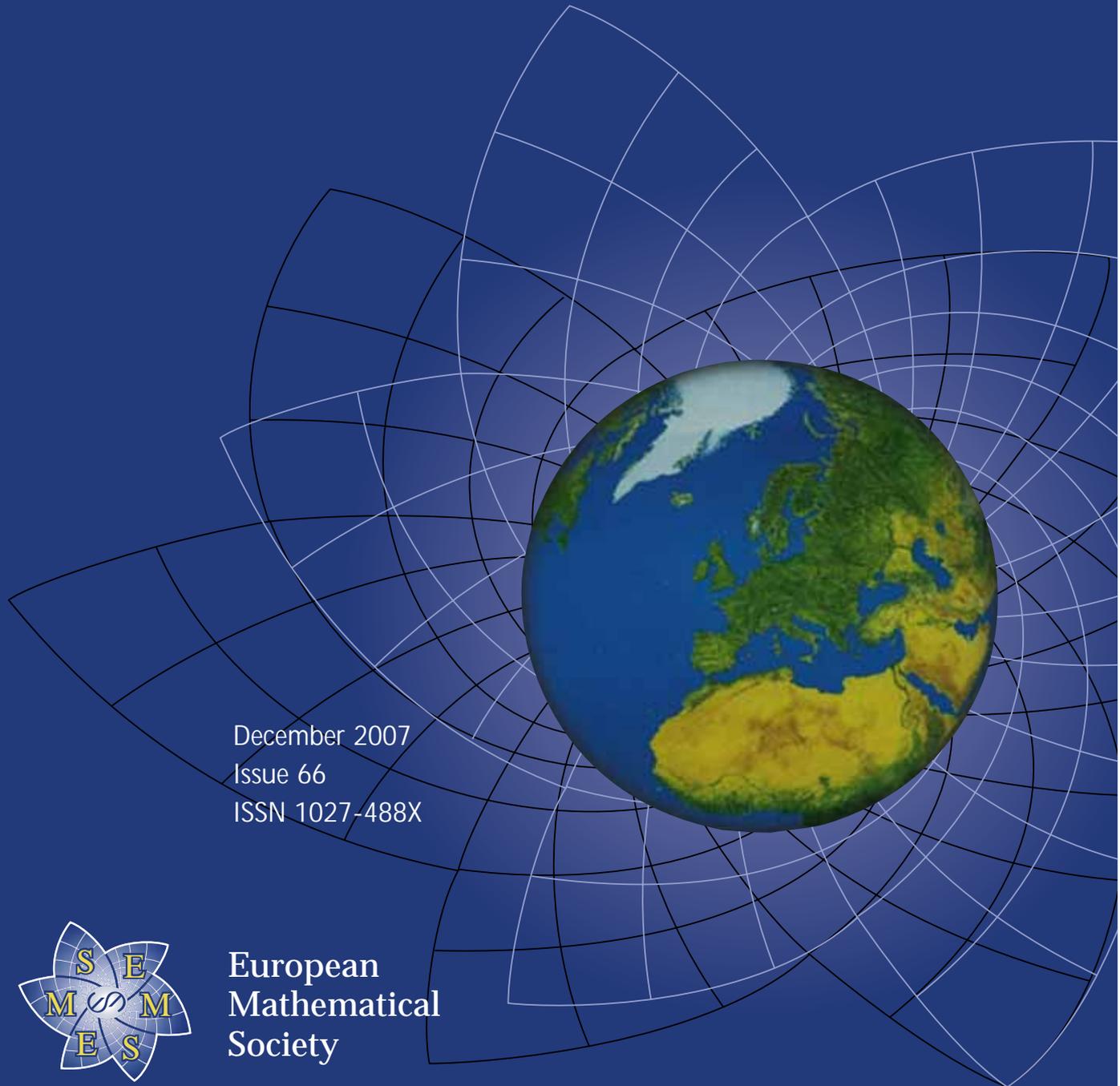
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December 2007  
Issue 66  
ISSN 1027-488X



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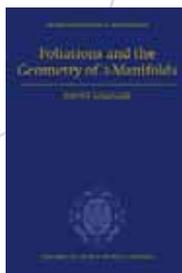
## Foliations and the Geometry of 3-Manifolds

Danny Calegari

This unique reference, aimed at research topologists, gives an exposition of the 'pseudo-Anosov' theory of foliations of 3-manifolds.

OXFORD MATHEMATICAL MONOGRAPHS

May 2007 | 378 pp | Hardback  
978-0-19-857008-0 | £60.00



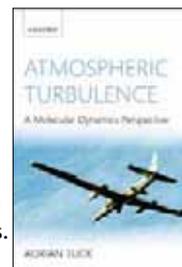
## Forthcoming titles for 2008

### Atmospheric Turbulence A Molecular Dynamics Perspective

Adrian Tuck

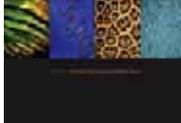
This book focuses on the direct link between molecular dynamics and atmospheric variation, uniting molecular dynamics, turbulence theory, fluid mechanics and non equilibrium statistical mechanics.

January 2008 | 168 pp | Hardback  
978-0-19-923653-4 | £45.00



### Reconstructing Evolution New Mathematical and Computational Advances

Edited by Olivier Gascuel and Mike Steel



Evolution is a complex process, acting at multiple scales, from DNA sequences and proteins to populations of species. This collection of 10 chapters provides a detailed overview of the key topics, from the underlying concepts to the latest results.

June 2007 | 348 pp | Hardback | 978-0-19-920822-7 | £39.50

### Flips for 3-folds and 4-folds

Edited by Alessio Corti

Aimed at graduates and researchers in algebraic geometry, this collection of edited chapters provides a complete and essentially self-contained account of the construction of 3-fold and 4-fold klt flips.

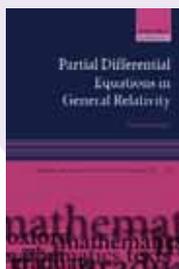
OXFORD LECTURE SERIES IN MATHEMATICS AND ITS APPLICATIONS No. 35

June 2007 | 200 pp | Hardback  
978-0-19-857061-5 | £39.50



### Partial Differential Equations in General Relativity

Alan Rendall



A text that will bring together PDE theory, general relativity and astrophysics to deliver an overview of theory of partial differential equations for general relativity. A unique resource for graduate students in mathematics and physics, numerical relativity and cosmology.

OXFORD GRADUATE TEXTS IN MATHEMATICS

April 2008 | 352 pp | Paperback | 978-0-19-921541-6  
April 2008 | 352 pp | Hardback | 978-0-19-921540-9

### Invitation to Discrete Mathematics 2/e

Jiří Matoušek and Jaroslav Nešetřil

A clear and self-contained introduction to discrete mathematics for undergraduates and early graduates.

May 2008 | 456 pp | Paperback | 978-0-19-857042-4 | £35.00  
May 2008 | 456 pp | Hardback | 978-0-19-857043-1 | £75.00

### Mathematics of Evolution and Phylogeny

Olivier Gascuel



This book of contributed chapters is authored by renowned scientists and covers recent results in the highly topical area of mathematics in evolution and phylogeny.

new in paperback

October 2007 | 442 pp | Paperback | 978-0-19-923134-8 | £24.95  
February 2005 | 442 pp | Hardback | 978-0-19-856610-6 | £51.00

### Advanced Distance Sampling

Estimating abundance of biological populations

Edited by S.T. Buckland, D.R. Anderson, K.P. Burnham, J.L. Laake, D.L. Borchers, and L. Thomas

new in paperback

This advanced text focuses on the uses of distance sampling to estimate the density and abundance of biological populations. It addresses new methodologies, new technologies and recent developments in statistical theory.

Nov 2007 | 434 pp | Paperback | 978-0-19-922587-3 | £24.95  
August 2004 | 434 pp | Hardback | 978-0-19-850783-3 | £59.00

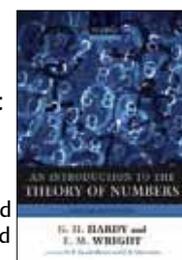


### An Introduction to the Theory of Numbers 6/e

Godfrey H. Hardy and Edward M. Wright

The sixth edition of this classic undergraduate text includes a new chapter on elliptic curves and their role in the proof of Fermat's Last Theorem, a foreword by Andrew Wiles and extensively revised and updated end-of-chapter notes.

June 2008 | 480 pp | Paperback | 978-0-19-921986-5 | £30.00  
June 2008 | 480 pp | Hardback | 978-0-19-921985-8 | £75.00

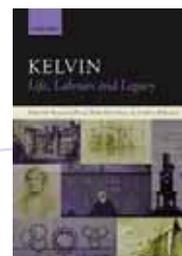


### Kelvin: Life, Labours and Legacy

Raymond Flood, Mark McCartney and Andrew Whitaker

A collection of chapters, authored by leading experts, which cover the life and wide-ranging scientific contributions made by William Thomson, Lord Kelvin (1824-1907). This volume fills a gap in the market for an accessible yet scholarly work on Kelvin.

March 2008 | 352pp | Hardback | 978-0-19-923125-6 | £55.00



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# European Mathematical Society

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Web site: <http://www.emis.de>

# EMS Calendar

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## 2008

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### 1 January

Deadline for the proposal of satellite conferences to 5ECM  
[top@math.rug.nl](mailto:top@math.rug.nl); [www.5ecm.nl/satellites.html](http://www.5ecm.nl/satellites.html)

### 1 February

Deadline for submission of nominations of candidates for the  
Felix Klein prize

[ems-office@cc.helsinki.fi](mailto:ems-office@cc.helsinki.fi); [www.emis.de/tmp2.pdf](http://www.emis.de/tmp2.pdf)

### 1 February

Deadline for submission of material for the March issue of the  
EMS Newsletter

Martin Raussen: [raussen@math.aau.dk](mailto:raussen@math.aau.dk)

### 29 February–2 March

Joint Mathematical Weekend EMS-Danish Mathematical  
Society, Copenhagen (Denmark)

[www.math.ku.dk/english/research/conferences/emswweekend/](http://www.math.ku.dk/english/research/conferences/emswweekend/)

### 3 March

EMS Executive Committee Meeting at the invitation of the  
Danish Mathematical Society, Copenhagen (Denmark)

Stephen Huggett: [s.huggett@plymouth.ac.uk](mailto:s.huggett@plymouth.ac.uk)

### 30 June–4 July

The European Consortium For Mathematics In Industry (ECMI),  
University College London (UK)

[www.ecmi2008.org/](http://www.ecmi2008.org/)

### 11 July

EMS Executive Committee Meeting, Utrecht (The Netherlands)

Stephen Huggett: [s.huggett@plymouth.ac.uk](mailto:s.huggett@plymouth.ac.uk)

### 12–13 July

EMS Council Meeting, Utrecht (The Netherlands)

Stephen Huggett: [s.huggett@plymouth.ac.uk](mailto:s.huggett@plymouth.ac.uk)

Riitta Ulmanen: [ems-office@cc.helsinki.fi](mailto:ems-office@cc.helsinki.fi)

### 13 July

Joint EWM/EMS Workshop, Amsterdam (The Netherlands)

<http://womenandmath.wordpress.com/joint-ewm-members-workshop-amsterdam-july-13th-2008/>

### 14–18 July

5th European Mathematical Congress, Amsterdam  
(The Netherlands)

[www.5ecm.nl](http://www.5ecm.nl)

### 3–9 August

Junior Mathematical Congress 8, Jena (Germany)

[www.jmc2008.org/](http://www.jmc2008.org/)

### 16–31 August

EMS-SMI Summer School at Cortona (Italy)

*Mathematical and numerical methods for the cardiovascular  
system*

[dipartimento@matapp.unimib.it](mailto:dipartimento@matapp.unimib.it)

### 8–19 September

EMS Summer School at Montecatini (Italy)

*Mathematical models in the manufacturing of glass, polymers  
and textiles*

[web.math.unifi.it/users/cime/](http://web.math.unifi.it/users/cime/)

# 5ECM: 14–18 July 2008 in Amsterdam

André Ran, Herman te Riele and Jan Wiegerinck



In a little more than half a year we will be enjoying 5ECM. It is going to be a fantastic event for European mathematics. Over the period 14–18 July 2008 the RAI Convention Center in Amsterdam will be the meeting point of mathematicians from all over Europe.

The Scientific Committee, chaired by Lex Schrijver, has put together a wonderful program. With ten plenary speakers, over 30 invited presentations by outstanding mathematicians on the newest developments in both pure and applied mathematics, three science lectures and over twenty mini-symposia, the programme is already of a high level. In addition there will of course be the ten young winners of the EMS prizes. The programme is an excellent mix of various areas of mathematics, both pure and applied. All in all, this is an event to look forward to. Do come and join us in Amsterdam next year!

The website <http://www.5ecm.nl> is growing rapidly with information on the lectures, the mini-symposia and much more relevant information.

For EMS members the conference fee is a very modest 220 Euro when registering before 01 April 2008 (note that this fee is lower than the registration fee for the ICIAM meeting last year, which amounted to 240 Euro!). The organisers acknowledge generous support from the NWO (the Dutch Science Foundation), the KWG (the Royal Mathematical Society of The Netherlands) and the Foundation Compositio Mathematica. Several Dutch companies have also supported the conference. At present the organisers are still working extremely hard to gather more support for the conference, in particular for funds to support the participants.

## The plenary lecturers

To convince you that it is worthwhile to come to Amsterdam next summer for an excellent conference, we will introduce the ten plenary lecturers below followed by the three science lecturers. We are confident that with such an extraordinary group of main lecturers you will not want to miss the fifth European Congress of Mathematics.

**Luigi Ambrosio** (Scuola Normale Superiore di Pisa)  
Born in Italy in 1963, Ambrosio obtained his PhD in 1985 in Pisa under the guidance of E. De Giorgio. He has previously been a speaker at the ECM in 1996 and at the



ICM in 2002. He was a winner of the Fermat Prize of the University of Toulouse in 2003 and several other prizes of the Unione Matematica Italiana.

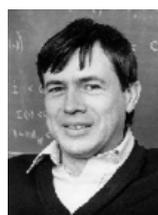
He is one of the leading experts in the field of calculus of variations. He has discovered some important results, including the introduction of SBV spaces, the theory of currents, mass transport, and rectifiable sets.



**Christine Bernardi** (CNRS and Université Pierre et Marie Curie Paris)

Born in France in 1955, Christine Bernardi obtained her PhD in 1979. She was awarded the 1995 Prix Blaise Pascal of the GAMNI-SMAI of the French Academy of Sciences.

She is a member of the top group studying Numerical Analysis in Europe. She and her group have made remarkable contributions to several fields in numerical analysis, from spectral methods to domain decomposition methods, to more recent and interesting techniques such as the reduced-bases method and some new applications of a posteriori analysis.



**Jean Bourgain** (IAS Princeton)

Born in Belgium in 1954, Bourgain obtained his PhD in Brussels in 1977 in Banach space theory. He was winner of the Fields medal in 1994, the Ostrowski Prize in 1991 and of many other awards and distinctions. He was an invited speaker at the ICM in 1983, 1986 and 1994, and at the ECM in 1992.

He has published more than 300 papers in several branches of mathematics, including analysis (Banach spaces, harmonic analysis), analytic number theory, combinatorics, ergodic theory and partial differential equations.



**Jean-François Le Gall**  
(Université Paris-Sud, Orsay)

Born in France in 1959, Le Gall obtained his PhD in 1982 on probability theory under the supervision of Marc Yor. He was a winner of the 1997 Loève Prize in Probability, the 2005 Fermat Prize and the Grand Prix

Sophie Germain de l'Académie des Sciences de Paris in the same year. He was an invited speaker at the ECM in 1992 and at the ICM in 1998. He was also the thesis advisor of the 2006 Fields medallist Wendelin Werner.

His area of research is probability theory, including Brownian motion, Lévy processes, superprocesses and their connections with partial differential equations, the Brownian snake, random trees, branching processes, and coalescence. Particularly interesting is his recent work on the scaling limits of random planar maps. This is in the intersection of probability, combinatorics and statistical physics.



**François Loeser** (ENS Paris)

Born in France in 1958, Loeser obtained his PhD in 1983 under the supervision of B. Teissier. He was a plenary speaker at the 2005 AMS Algebraic Summer School in Seattle. In 2007 he was awarded the Prix Charles-Louis de Saulces de Freycinet of the French Academy of Sciences.

His research areas are algebraic geometry, arithmetic geometry and motivic integration.



**László Lovász**

(Eötvös Loránd University, Budapest)

Born in Hungary in 1948, Lovász received his PhD in 1971 under the supervision of Tibor Gallai. He was a collaborative member of the Microsoft Research Redmond lab. (WA, USA) until 2006. He has received many awards and honours, among them the Wolf Prize and the Knuth Prize, and he was a plenary speaker at the ICM in 1990. Recently he was awarded the Bolyai Prize.

His research focuses on discrete mathematics, optimization, complexity and algorithms. His name is connected to several problems and results in mathematics: in graph theory, there are the Erdős-Faber-Lovász conjecture about the colouring of graphs and the Lovász conjecture concerning Hamiltonian paths in certain classes of graphs; and in the theory of algorithms there is the Lenstra-Lenstra-Lovász lattice basis reduction (LLL) algorithm.



**Matilde Marcolli**

(Max Planck Institut Bonn)

Born in Italy in 1969, Marcolli obtained her PhD in 1997 in Chicago under the supervision of M. Rotherberg. She was a recipient of the Sofia Kowalewskaya Award and the Heinz-Maier-Leibnitz-Preis of the DFG. She is editor of the European Mathematical Society Publishing House's "Journal of Noncommutative Geometry".

Her research is concerned with noncommutative geometry, applications to physics, number theory and differential geometry. Marcolli is one of the most active and original present-day mathematicians of Europe.



**Felix Otto** (Universität Bonn)

Born in Germany in 1966, Otto obtained his PhD in Bonn in 1993 under guidance of S. Luckhaus. After that he spent several years in the USA, at Courant Institute, Carnegie Mellon University and UC Santa Barbara, before returning to Bonn in 1999. He has given invited lectures at several important conferences (SIAM, GAMM, ICM sectional speaker in 2002, ICIAM 2007) and was Stieltjes Visiting Professor in the Netherlands in 2004. He was the recipient of the 1997 A. P. Sloan Research Fellowship, the 2001 Max-Planck Forschungspreis, the 2006 Gottfried-Wilhelm-Leibniz-Preis and the 2007 Collatz Prize.

His research topics are centred around the analysis of pattern formation in models from physics, partial differential equations and multi-scale methods. The methods of analysis used in his group are wide ranging, including numerical simulation, asymptotic analysis and rigorous analysis. Micromagnetism, complex rheology and coarsening of spatial structure in time are just a few topics of recent research in his group.



**Nicolai Reshetikhin**

(University of California, Berkeley)

Born in Russia in 1958, Reshetikhin obtained his PhD in 1984 at LOMI in his home town Leningrad. He was a speaker at the ICM in 1990. Presently he is at UC Berkeley and at Aarhus, where he holds the Niels Bohr professorship for the years 2006–2010.

Reshetikhin is working in the overlapping areas of geometry, topology and mathematical physics. He has had great influence in a number of mathematical disciplines, including representation theory, knot theory and moduli spaces of flat connections. His work with V. Turaev on topological quantum field theories in dimension three has had a huge influence on lots of young researchers – it opened up the subject of what could be called Quantum Topology.

**Richard Taylor**

(Harvard University, Cambridge)

Born in England in 1962, Taylor obtained his PhD in 1988 in Princeton under the supervision of Andrew Wiles. He received the Fermat Prize 2001, the Ostrowski Prize 2001, the Cole Prize 2002 of the American Mathematical Society and the Shaw Prize 2007 in Mathematical Sciences for his work on the Langlands program. He has been a speaker at the ICM meetings in 1994 and 2002.

Taylor is perhaps best known for his contribution to the solution of Fermat's last problem, work he did with Andrew Wiles several years after receiving his PhD. His research interests are algebraic number theory, modular forms and Galois representations. He is considered to be the prime expert on the recent progress on the interaction of automorphic forms and arithmetic geometry, in particular the Sato-Tate conjecture and Serre's modular conjecture.

The three Science lecturers



**Juan Ignacio Cirac** (Garching, Germany) on *Quantum Information Theory*  
 Juan Ignacio Cirac is an internationally renowned leader in quantum information science with an extraordinary broad scope in the field, ranging from theoretical aspects to physical realizations.

He graduated from the Universidad Complutense de Madrid in 1988 and moved to the United States in 1991 to work as a postdoctoral scientist in the Joint Institute for Laboratory Astrophysics in the University of Colorado at Boulder. Later he became a professor at the Institut für Theoretische Physik in Innsbruck, Austria. He has been the Director of the Theoretical Division of the Max Planck Institute for Quantum Optics in Garching, Germany, since 2001.

His research is on the quantum theory of information. He has developed a computational system based on quantum mechanics that is hoped to lead to the design of much faster algorithms in the future. He has contributed to applications that demonstrate the viability of his theories, performing calculations that are impossible with current systems for processing and transmitting information. According to his theories, quantum computing will revolutionise the information society and lead to much more efficient and secure communication of information. Apart from his interest in quantum theory, he has investigated degenerated quantum gases and quantum optics. He is one of the most cited authors in his field of research.



**Tim Palmer** (Reading, UK) on *Climate Change*

Tim Palmer has been working at the European Centre for Medium-Range Weather Forecasts in Reading since the mid 1980s, where he now heads up the Predictability and Seasonal Forecast Division. He is a

Fellow of the Royal Society.

He has contributed significantly to the application of nonlinear mathematical methods to understanding non-trivial aspects of global warming. His team develops

complicated models of the Earth system that are based on the laws of physics and comprise tens of millions of equations. Forecasts from this model are fed to most of the National Meteorological Services around Europe. Palmer has been applying ensemble forecast techniques to longer timescales, leading a team that develops predictions through coupled ocean-atmosphere models. These models can, for example, give risk predictions of the failure of the monsoon rains months in advance. On an even longer timescale, he has been involved with questions concerning global warming. In addition, Palmer is engaged in interdisciplinary research on forecasting rare events like malaria epidemics, river flooding and crop failure (all weather related). The ultimate goal of such research would be to target, for instance, resources to help prevent a malaria epidemic in the regions that are most at risk.



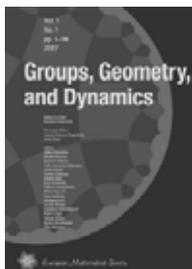
**Jonathan Sherratt** (Edinburgh, UK) on *Mathematical Biology*

Jonathan Sherratt has been a professor of mathematics at Heriot-Watt University since January 1998. He was elected as a Fellow of the Royal Society of Edinburgh in 2001. He has been very prominent since

his first work on wound healing. In 2006 he was awarded the Whitehead Prize of the London Mathematical Society for his contributions to mathematical biology and, in particular, the development and analysis of new mathematical models for complex biological processes.

He has worked on a broad range of topics. Notable among these are wound healing, pattern formation, tumour growth and spatiotemporal chaos. His work is truly multidisciplinary and often collaborative, and is characterized by originality and novelty. He has published widely in mathematical biology journals and the biological literature. His papers are motivated by biological and medical problems, applying modelling and analysis to understand these, and then providing a significant link back to biology.

*André Ran, Herman te Riele and Jan Wiegerinck are the members of the local organising committee of 5ECM.*



**Groups, Geometry, and Dynamics**  
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**Aims and Scope:** *Groups, Geometry, and Dynamics* is devoted to publication of research articles that focus on groups or group actions as well as articles in other areas of mathematics in which groups or group actions are used as a main tool. The journal covers all topics of modern group theory with preference given to geometric, asymptotic and combinatorial group theory, dynamics of group actions.

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# EUROPEAN MATHEMATICAL SOCIETY JOINT MATHEMATICAL WEEKEND

## University of Copenhagen February 29-March 2nd 2008

### PLENARY SPEAKERS

**Xavier Buff**  
(Toulouse)

**Nigel Higson**  
(Pennsylvania State)

**Frank Merle**  
(Cergy-Pontoise)

**Stefan Schwede**  
(Bonn)

### TOPICS

**Algebraic topology**

[Chairs: Jesper Grodal and Ib Madsen]

**Coding theory**

[Chairs: Olav Geil and Tom Høholdt]

**Non-commutative geometry/operator algebra**

[Chairs: Ryszard Nest and Mikael Rørdam]

**Dynamical systems**

[Chairs: Carsten Lunde Petersen and Jörg Schmeling]

**Algebra and representation theory**

[Chairs: Jørn Børling Olsson and Henning Haahr Andersen]

**Partial differential equations**

[Chairs: Gerd Grubb and Helge Holden]

[www.math.ku.dk/ems/](http://www.math.ku.dk/ems/)



European Mathematical Society



Danish Mathematical Society



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ORGANIZED BY: The Danish Mathematical Society & The University of Copenhagen

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# EMS Council Meeting, Utrecht 2008



**Academiegebouw Utrecht.**

The EMS Council meets every second year. The next meeting will be held in Utrecht, the Netherlands, July 12 and 13, 2008 in the Academiegebouw of Utrecht University, before the 5ECM conference in Amsterdam, 14–18 July, 2008. The Council meeting starts at 13.00 on July 12 and ends at noon on July 13.

Delegates to the Council will be elected by the following categories of members.

## (a) Full Members

Full Members are national mathematical societies, which elect 1, 2, or 3 delegates according to their membership class. Each society is responsible for the election of its delegates. Each society should notify the Secretariat of the EMS in Helsinki of the names and addresses of its delegate(s) no later than 28 February 2008. As of 1 July 2007, there were 57 such societies, which could designate a maximum of 86 delegates.

## (b) Institutional Members

Delegates shall be elected for a period of four years. A delegate may be re-elected provided that consecutive service in the same capacity does not exceed eight years. The institutional members, as a group, nominate candidates and elect delegates.

## (c) Individual Members

On 30 June 2007, there were 2306 individual members and, according to our statutes, these members will be represented by (up to) 24 delegates.

The present delegates of individual members are:

Anichinii, Giuseppe, 2006–2009  
 Berinde, Vasile, 2000–2003–2007  
 Anzellotti, Gabriele, 2004–2007–2011  
 Conte, Alberto, 2000–2003–2007  
 Coti Zelati, Vittorio, 2004–2007–2011  
 Guillopé, Laurent, 2000–2003–2007  
 Higgs, Russell, 2004–2007–2011  
 Kingman, John, 2006–2009  
 Margolis, Stuart W, 2004–2007–2011

Pelczar, Andrzej, 2000–2003–2007  
 Sodin, Mikhail, 2004–2007–2011  
 Soifer, Gregory, 2004–2007–2011  
 Tronel, Gerard, 2000–2003–2007  
 Wilson, Robin, 2004–2007–2011  
 Xambo-Descamps, Sebastian, 2002–2005–2009

The mandates of Berinde, Conte, Guillopé, Pelczar, and Tronel end on 31 December 2007. They cannot be re-elected because they have served in this capacity for eight years. A nomination form for proposals of new delegates of individual members is attached below. Please note the deadline 28 February 2008 for nominations to arrive in Helsinki. Elections of individual delegates will be organised by the EMS secretariat by postal ballot among individual members unless the number of nominations does not exceed the number of vacancies.

## Agenda

The Executive Committee is responsible for preparing the matters to be discussed at Council meetings. Items for the agenda of this meeting of the Council should be sent as soon as possible, and no later than 28 February 2008, to the EMS Secretariat in Helsinki.

## Executive Committee

The Council is responsible for electing the President, Vice-Presidents, Secretary, Treasurer and other members of the Executive Committee. The present membership of the Executive Committee, together with their individual terms of office, is as follows.

President: Professor A. Laptev (2007–2010)  
 Vice-Presidents: Professor P. Exner (2005–2008)  
 Professor H. Holden (2007–2010)  
 Secretary: Dr S. Huggett (2007–2010)  
 Treasurer: Professor J. Väänänen (2007–2010)  
 Members: Professor V. Buchstaber (2001–2004–2008)  
 Professor O. Gil-Medrano (2005–2008)  
 Professor M. Martin-Deschamps (2007–2010)  
 Professor C. Sbordone (2005–2008)  
 Professor K. Schmidt (2005–2008)

Members of the Executive Committee are elected for a period of four years. The President can only serve one term. Committee members may be re-elected, provided that consecutive service shall not exceed eight years. One position of vice-president and four positions of members-at-large of the Executive Committee will be vacant as of 31 December 2008; elections will arise during Council. V. Buchstaber has served on the Executive Committee for eight years and cannot be re-elected. Nominations of candidates are invited; more information below.

The Council may, at its meeting, add to the nominations received and set up a Nominations Committee, disjoint from the Executive Committee, to consider all

candidates. After hearing the report by the Chair of the Nominations Committee (if one has been set up), the Council will proceed to the elections to the Executive Committee posts.

All these arrangements are as required in the Statutes and By-Laws. All information and material concerning the Council will be made available at [www.math.ntnu.no/ems/council08](http://www.math.ntnu.no/ems/council08).

Secretary: Stephen Huggett ([s.huggett@plymouth.ac.uk](mailto:s.huggett@plymouth.ac.uk))  
 Secretariat: Riitta Ulmanen ([ems-office@helsinki.fi](mailto:ems-office@helsinki.fi))

**Elections to the Executive Committee: Nominations**

At the end of 2008 there will be vacancies for the position of one Vice-President, as well as of four ordinary members of the Executive Committee.

Nominations for these positions are invited. Any member of the Society may be nominated. It would be convenient if nominations for office in the Executive Committee, duly signed and seconded, could reach the Secretariat by 28 February, 2008. It is strongly recommended that a statement of intention or policy is enclosed with each nomination. If the nomination comes from the floor during the Council meeting there must be a clear declaration of the willingness of the person to serve. It is recommended that statements of policy of the candidates nominated from the floor should be available.

**Nomination Form for Council Delegate**

Name: .....

Title: .....

Address: .....

.....

Proposed by: .....

Seconded: .....

I certify that I am an individual member of the EMS and that I am willing to stand for election as a delegate of individual members at Council.

Signature of Candidate: .....

Date: .....

Completed forms should be sent to:  
 Riitta Ulmanen  
 EMS Secretariat  
 Department of Mathematics and Statistics  
 P.O.Box 68  
 FI-00014 University of Helsinki  
 Finland

to arrive by 28 February, 2008. A photocopy of this form is acceptable.

Journal  
**Stochastic Processes and their Applications**  
*alternative subscription*



Following negotiations during 2007 involving the Bernoulli Society, the Editorial Board of the journal SPA and Elsevier, advantageous changes regarding publishing and pricing policy have been agreed.

Effective as of January 2008, the following will apply:

All ScienceDirect subscribers to the journal - including the large numbers who access through Elsevier's collection agreements - will have access back to 1995, addressing some of the concerns about accessing missing volumes and issues. This change will happen automatically in January 2008.

The reduced "alternative subscription" to the Journal will give the same ScienceDirect access as the "standard subscription", including the new access back to 1995.

All papers published from 2008 will be made freely available to all readers, whether subscribers or not, four years after publication, via ScienceDirect -- delayed Access. Members of Bernoulli will continue to have the low cost personal print membership subscription and/or a complimentary electronic subscription, which gives access back to Volume 1, Issue 1 - a real benefit of Bernoulli membership

Please inform the librarian of your institution about the above changes. Libraries interested in changing to the alternative model should contact their agent or the Publisher.

More information can be obtained at the website of the Journal: <http://www.elsevier.com/locate/spa>

# EMS-CDC – Alive and kicking

Some of the readers of the EMS Newsletter might have got the impression during the last couple of years that the EMS Committee for Developing Countries (EMS-CDC for short) has faded away. Don't worry, EMS-CDC is alive and spreading its wings in line with its policy statement: see <http://www.emis.de/committees.html#dc>

There was a short period in 2004/05 when our most important activity so far – our book donation programme – came to a temporary halt because of a shortage of funds needed to cover the shipping expenses of scientific literature from the donors to the recipients. In fact, ICTP which had generously supported this programme could no longer do so. EMS-EC bailed us out, and then the current EMS-CDC chair, in her capacity as one of the editors of the *Encyclopaedia of Mathematical Physics* (Elsevier 2006), called upon the authors to donate their honorariums to EMS-CDC, which many of them did. Here we want to say once more 'Thank you' to these authors. As a consequence, our Book Donation Programme is running smoothly ever since. This also because we have joined forces with AMS in this direction. Moreover, we have enlarged the scope of the book donation programme by producing CDs of lecture notes in English, for the exclusive use by colleagues in developing countries. It is intended to make CDs of lecture notes in French and Spanish to make it easier for students and colleagues of francophone developing countries and of Latin American countries, respectively.

We had several requests for financial support of conferences (notably ICM 2006 in Madrid, and a conference in Mongolia), and also workshops, which we were able to fulfil. However, we can do this only in exceptional cases precisely because of our relative shortage of finances. We are also supporting a Cambodian student to do a master's degree in Vietnam. We note that the situation in Cambodia is extremely poor for mathematics: there is only one Ph.D. in mathematics in the whole country. The IMU and CIMPA are organizing series of lectures there and we intend to join forces with them.

Moreover, we are currently interacting with the University of KwaZulu Natal (Durban/Westville campus) in developing a proof oriented M.Sc. programme in mathematics. This programme is ultimately intended to become a regional programme for students of southern if not sub-Saharan Africa. This programme is based on the experience gathered in developing countries that programmes in mathematical modelling are very often focusing too much on teaching techniques of how to solve this or that problem and (necessarily) neglecting at the same time to prove basic mathematical results. On the other hand, universities in many developing countries are facing a permanent shortage of academic staff. Thus, proof oriented M.Sc. programmes facilitate the training of academic staff who have a solid training in proving mathematical results in various mathematical areas.

Several of our EMS-CDC members are heads of development organizations and are directly involved in various programmes in developing countries. Very often such programmes are independent of the existence of EMS-CDC. This makes it even easier for us to collaborate and to assist such programmes – partly financially, partly by manpower. Thus we have enlarged our scope of activities also in this direction, by supervising students both at M.Sc. and Ph.D. level.

As before, we are closely cooperating with various mathematical organizations, some of which (like CIMPA, but also various national mathematical societies) are represented directly or indirectly by members of EMS-CDC. This results in useful information flowing in both directions, but also support of EMS-activities such as the book donation programme.

There seems to be a growing awareness among mathematicians in developed countries that their help is needed in developing countries. Taking this growing readiness to assist maths departments in developing countries, it was proposed at one of our annual meetings that universities in developed countries could "adopt" a university in a developing country and pay, for example, for the subscription of *Zentralblatt* or at least its online version, or for MathSciNet (both *Zentralblatt* and *Math Reviews* have special rates for developing countries – in many cases the rate is only 10% of the list price). This would help considerably those maths departments in developing countries where some research capacities are already existing or evolving. As a matter of fact, one of the main obstacles for developing countries to access scientific literature on a broader scale, are unfavourable exchange rates. In many cases, however, local currencies cannot be converted into hard currencies, and that makes matters even worse for universities in such countries. In one instance so far, EMS-CDC was able to pay for the subscription of *Zentralblatt* using some of the honorariums which various reviewers of *Zentralblatt* donate to us. Unfortunately, our funds are too small to act in this direction on a regular basis.

As you can see from the above, in widening our scope of activities we are also faced with the problem of funding these activities. We fully depend on financial assistance as mentioned above. In addition, some money came in directly from national mathematical societies where individual members donated to EMS-CDC when paying their membership fees (see the accompanying letter to Presidents of national mathematical societies below). If this could happen on a more regular basis, this would be of great help to us.

At our last annual meeting in April, this year, we had a regular change of leadership. Tsou Sheung Tsun (Oxford) took over from Herbert Fleischner the chair of EMS-CDC (he had been chairman since 2002, and Tsou Sheung Tsun was the vice-chair). The vice-chair is now

Leif Abrahamsson of Uppsala University who has vast experience related to assisting universities in developing countries. Herbert Fleischner remains member of EMS-CDC and will focus his work on assisting in developing M.Sc. programmes.

However, we agreed with EMS-EC that changes in the leadership of EMS-CDC, but also changes in membership should be done gradually since sudden changes may interrupt activities. We are thus calling upon colleagues who are interested in participating in EMS-CDC activities, to contact its leadership. Apart from regular members we also have some EMS-CDC associates who

are not members as such, but cooperate with EMS-CDC on a more or less regular basis. Such associates might become regular EMS-CDC members if they wish to.

More information about our membership, links, collaborations, activities can be found in a website being built: <http://www.maths.ox.ac.uk/~tsou/ems/>.

*Tsou Sheung Tsun [tsou@maths.ox.ac.uk], Chair  
Leif Abrahamsson [leifab@math.uu.se], Vice-Chair  
Herbert Fleischner, [fleisch@dbai.tuwien.ac.at], Programme Director.*

## To the Presidents of National Mathematical Societies

Dear Colleagues,

We are writing in the hope of enlisting your help in a scheme to promote aid to the mathematicians in the developing world. There is no question that the progress in mathematics in those less privileged parts of the world is of utmost importance, not only for mathematicians there but also for mathematicians everywhere. And it is also important that as many mathematicians as possible get engaged in this process.

To this end, a first step is to ask your members to donate a small amount of money, at the time they renew their membership in your Society, to enable the Committee for Developing Countries of the European Mathematical Society to carry on with and expand their work. One efficient way we can think of is to provide a space on your annual renewal form for your members to add an appropriate amount, for example 10 EUR, to their remittance or cheque for this purpose. Practically speaking, you could ask them to tick a box, or even better, one of two boxes, with one marked 10 EUR (or local equivalent) and the other blank for them to fill in their desired amount. The total collected can then be transferred to the EMS-CDC account in Finland, say once a year. We hope this will not add a significant amount of administrative work to your staff. We are sure that a large number of your members are happy to make such a donation, if so facilitated by their own Society, as it has already been shown to be the case in some European national societies already. If yours is one of them, we thank you most heartily and hope you will continue the good work.

We are planning further appeals to involve more mathematicians in our development work, and not just financially, because we believe that what works and what counts ultimately is mathematicians helping mathematicians. After all, it's mathematicians who care most about mathematics, anywhere in the world.

For information, we are attaching a short description of this committee, which you may wish to use in initiating the above scheme if you approve it.

Best wishes,

*Ari Laptev*  
(President, European Mathematical Society)

*Tsou Sheung Tsun*  
(Chair, Committee for Developing Countries, EMS)

# Summer School in Contemporary Mathematics: Dubna, Russia, 2007

Michael Eickenberg (Luxembourg)

*Since 2001, the Russian Academy of Sciences (mathematical section), the Steklov Mathematical Institute, the Moscow Department for Education and the Moscow Center for Continuous Mathematical Education organize a summer school at Dubna (125 km north of Moscow) that is unique in its choice of professors and participants. During two weeks around a hundred students (from the last two years of high-school or from the first two years of university) take part in 70-80 lectures and seminars. In 2007, the School accommodated foreign students for the first time; contacts were made through the EMS. The Newsletter asked one of the participants to report on his experiences. Additional information on the summer school including the list of speakers and the titles of lecture series can be found on the web site <http://www.mccme.ru/dubna/eng/>.*

## Description of the event

This year's summer school took place 19-30 July 2007 in an installation for education camps in Ratmino, a village close to the science town Dubna. Classes were attended in the facilities on site (featuring a grand auditorium in the main building, a common room in each of the two dormitories and the library) on every day except the first. There were usually four lectures of 74 minutes each day, two as a block in the morning from 9:00 and two in the afternoon from 14:30, with a break of around 30 minutes between two lectures of a block. Occasionally there would be three or five lectures depending on necessity. In a given time slot there were either up to four, smaller lectures in parallel or a single plenary lecture. Some lectures were stand-alone one-time events whilst other, more elaborate ones continued over several days in the same time slot and place.



Lecture at Dubna

A generous break was given for lunch and recreational activities, which could consist of swimming in the river Dubna, playing sports such as football, volleyball, basketball, table tennis, badminton and chess, or resting, depending upon personal preference.

Breakfast, lunch and dinner were served in the canteen, which was also situated in the main building. The evenings were free; the sports mentioned above were offered and tournaments were organized, which culminated in finals on the last day with a little prize and mention during the closing ceremony (the author managing to claim first place in badminton :)). Furthermore there were numerous cultural activities, such as a presentation of the works of Tchaikovsky, movies, a poetry evening and singing along to (or in my case listening to) guitar music. Talks on non-mathematical subjects such as genetically modified organisms were given and the events of the coup d'état attempt of August 1991 that led to the fall of the soviet union were recalled by Alexander Muzykantsky, who was vice-chairman of the Moscow government at the time. In addition to this, during one afternoon there was a boat ride along the Volga (as luck would have it, this was the only day it rained but it was not hard enough to spoil the fun).

The classes were held by some of the most renowned Russian and international mathematicians to an audience of around 100 participants whose education did not exceed second year university level, most of them having achieved high ranks in mathematical competitions. The subjects covered many fields of contemporary and classical mathematics, from algebra and analysis to geometry, number theory, game theory and logic. For instance, four proofs of Gödel's incompleteness theorem were discussed, as well as the theory of auctions, arithmetic progressions in prime numbers, polyhedrons and polytopes, and the likes of harmonic functions defined on fractals and discrete holomorphic functions.

Outside of lectures, the professors were always approachable for any questions or conversation. During free time one could often spot students and teachers engaged in discussion.

## The school from an international participant's point of view

In total there were five international participants, from Hungary, Finland, Norway and in my case Germany. We were warmly welcomed into the community and its friendly, open and down-to-earth atmosphere after having been guided through the visa application process



Discussion after a lecture

and the arrival by the office of the MCCME in Moscow. Virtually everybody in the camp had a very high level of English so communication was no problem. From the third day on there was one lecture per day given in English, organized especially for us. In the other lectures, including the plenary ones, there was a sufficient number of the organising team or participants capable of giving a running commentary in English. This helped to capture the essence of what was being conveyed, yet not every detail – and when a joke needs explaining (provided it can be explained) it is usually too late ... Naturally, not being able to read exactly what the professor has written two blackboards before makes it very difficult to catch up again once you lose the thread.

### Personal note

After the remarkable feat of managing to organize my arrival within a week of the departure date, due to the quick responses of Vitaly Arnold and Nadezhda Shikova of the MCCME to my emails, I knew I was in good hands. My decision to participate came so late because the stu-



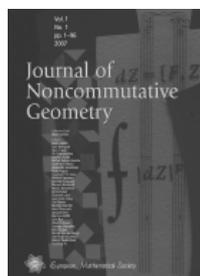
Outdoors coaching

dent nominated to go decided not to so I was able to take her place.

The summer school has shown me the vastness and also the beauty and ‘music’ of contemporary and classical mathematics. And the fact that it has invoked profound changes in the way I think is underlined by stating that because of my participation in this school I have decided to change my studies from physics to mathematics at the University of Luxembourg. My thanks go out to all the organizers, teachers and participants for a great ten days in the world of mathematics and to the mathematical society of Luxembourg for sending me there.



*Michael Eickenberg [m.eickenberg@web.de] is a third year student of mathematics and physics at the University of Luxembourg. Currently, he is taking a term at University Henri Poincaré I in Nancy, France. He expects to write a thesis for a bachelor degree in 2008. Apart from his studies he is an eager reader, is interested in languages and plays Ultimate Frisbee and the guitar.*



### Journal of Noncommutative Geometry

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**Editors:** Paul F. Baum, (USA), Jean Bellissard (USA), Alan Carey (Australia), Ali Chamseddine (Lebanon), Joachim Cuntz (Germany), Michel Dubois-Violette (France), Israel M. Gelfand (USA), Alexander Goncharov (USA), Nigel Higson (USA), Vaughan F. R. Jones (USA), Mikhail Kapranov (USA), Gennadi Kasparov (USA), Masoud Khalkhali (Canada), Maxim Kontsevich (France), Dirk Kreimer (France and USA), Giovanni Landi (Italy), Jean-Louis Loday (France), Yuri Manin (Germany), Matilde Marcolli (Germany), Henri Moscovici (USA), Ryszard Nest (Denmark), Marc A. Rieffel (USA), John Roe (USA), Albert Schwarz (USA), Georges Skandalis (France), Boris Tsygan (USA), Michel Van den Bergh (Belgium), Dan-Virgil Voiculescu (USA), Raimar Wulkenhaar (Germany), Guoliang Yu (USA).



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# Sudoku puzzles and how to solve them

Andries E. Brouwer (Eindhoven, the Netherlands)

3	9			6				
2					8			
					7			
	7		4			5		
				2			3	
	1							
			8	5				
6								9
			7					

		2		9		1	7	
	3	8	6					
4								
					5			
		9		1		3		
			4					
								4
					7	9	2	
8	6		3		7			

Two puzzles – the second one is difficult

## Sudoku

A Sudoku puzzle (of ‘classical type’) consists of a 9-by-9 matrix partitioned into nine 3-by-3 submatrices (‘boxes’). Some of the entries are given, and the puzzle is to find the remaining entries, under the condition that the nine rows, the nine columns, and the nine boxes all contain a permutation of the symbols of some given alphabet of size 9, usually the digits 1-9, or the letters A-I.

Some mathematicians will claim that since this is a finite problem, it is trivial. The time needed to solve a Sudoku puzzle is  $O(1)$  – indeed, one can always try the  $9^{81}$  possible ways of filling the grid. But one can still ask for efficient ways of finding a solution. Or, if one knows the solution already, one can ask for a sequence of logical arguments one can use to convince someone else of the fact that this really is the unique solution.

### Backtrack and elegance

It is very easy to write an efficient computer solver. Straightforward backtrack search suffices, and Knuth’s ‘dancing links’ formulation of the backtrack search for an exact covering problem takes a few microseconds per puzzle on common hardware today.

For a human solver, backtrack is the last resort. If all attempts at further progress fail, one can always select an open square, preferably with only a few possibilities, and try these possibilities one by one – maybe using pencil and eraser, or maybe copying the partially filled diagram to several auxiliary sheets of paper and trying each possibility on a separate sheet of paper.

For very difficult Sudoku puzzles, this is the fastest way to solve them, also for humans.

However, one solves puzzles not because the answer is needed, but for fun, in order to exercise one’s capabilities in logical reasoning. And solving by backtrack is dull, boring, mindless, no thinking required, better left to a computer, no fun at all.

So, Backtrack, or Trial & Error, is taboo. And if it cannot be avoided one prefers some limited form. Maybe whatever can be done entirely in one’s head.

### Grading

Most Sudoku puzzles one meets are computer-produced, and it is necessary to have a reasonable estimate of the difficulty of these puzzles. To this end one needs computer solvers that mimic human solvers. Thus, one would also implement the solving steps described below in a Sudoku solving program, not in order to find the solution as quickly as possible, but in order to judge the difficulty of the puzzle, or in order to be able to give hints to a human player. Such AI-type Sudoku solving programs tend to be a thousand to a million times slower than straightforward backtrack.

### Generating

Given the backtrack solver, generating Sudoku puzzles is easy: start with an empty grid, and each time the backtrack solver says that the solution is not unique throw in one more digit. (If now there is no solution anymore, try a different digit in the same place.) To generate a puzzle in this way requires maybe thirty calls to the backtrack solver, less than a millisecond. One can polish the puzzle a little by checking that none of the givens is superfluous.

Afterwards one feeds the puzzles that were generated to a grader. Maybe half will turn out to be very easy, and most will be rather easy (‘humanly solvable’). It is very difficult to generate very difficult puzzles, puzzles that are too difficult even for very experienced humans.

### Solving

Below we sketch a possible approach for a human solver. The goal is to be efficient. In particular, the boring and time-consuming action of writing all possibilities in every empty

square is postponed as much as possible. On the other hand, some form of markup helps.

Baby steps

When eight digits in some row or column or box are known, one can find the last missing digit.

1	9	6	3		5	4	8	2
4	2	7		1	8	3	5	6
5	3	2		4		7	9	
6	1	2	7	4		8	3	5
9	4	8	5		1			
7	5		8	2		4	1	
5			2	6	7	9		
7	8	9	1	5		2	6	
6	4	8	9	7	5	1		

Exercise: (i) Solve this puzzle using baby steps only. (ii) Show that if a puzzle can be solved using baby steps only, it has at most 21 open squares.

Singles

When there is only one place for a given digit in a given row or column or box, write it there. If there is only one digit that can go in a given square, write it there.

	1							
			1					
								1

5						2	3	
			6					
								6
			6					

Left: The 1 in the middle right box must be in the colored square. Right: The 6 in the top row must be in the colored square.

Baby steps are particularly easy cases of singles. Checking for singles requires 324 steps. Knuth's 'dancing links' back-tracker will take 324 steps if and only if the puzzle can be solved by singles only. It is unknown how many open squares a puzzle can have and be solvable by singles only. There are examples with 17 givens. It is unknown whether any Sudoku puzzles exist with 16 givens and a unique solution.

7	8				5			
				1				4
			2			9		
1							5	
3		4						
		6						
							1	3
2		8		5				

5	2				4			
			7	1				3
					4	6		
	7		2					
	1							
6				2				
4				3			1	

Left: Solve using singles only Right: 16 clues and 2 solutions

Pair markup

If one checks where a given digit can go in some row or column or box and finds that there are precisely two possibilities, then it helps to note this down. That is efficient, one does not do the same argument over and over again, and helps in further reasoning.

1	2							
								7
3	4							
5	6				3			
8	9				4	5		

1	2	7						
								7
3	4							
5	6						3	7
8	9						4	5

Thus, a small stroke labeled with a digit and joining two squares indicates that one of the two squares must have that digit.

Given two pairs for the same digit straddling the same two rows or columns, the digit involved cannot occur elsewhere in those rows or columns.

			3					
			5					


Given two pairs between the same two squares, one concludes that these two squares only contain the two digits involved and no other digit.



Matchings

A sudoku defines 36 matchings (1-1 correspondences) of size 9: between positions and digits given a single row or column or box, and between rows and columns given a single digit. For each of these 1-1 correspondences between sets  $X$  and  $Y$  of size 9, if we have identified subsets  $A \subset X$  and  $B \subset Y$  of the same size  $n$ ,  $1 \leq n \leq 9$ , such that we know that the partners of every element of  $B$  must be in  $A$ , then  $A$  and  $B$  are matched, and nothing else has a partner in  $A$ . More explicitly: (A) If for some set of  $n$  positions in a single row, column, or box there are  $n$  digits that can be only at these positions, then these positions do contain these digits (and no other digits).

For example,

		7	2	1	5	8	6
		8			7		
						2	1
		5					
			5				
				1			
							2

		7	2	1	5	8	6
		8			7		
						2	1
		5					
			5				
				1			
							2

The three digits 3, 4, 9 in the second row must be in columns 4, 6, 9, so the digit 5 cannot be there.

(B) If for some set of  $n$  digits there are  $n$  positions in a single row, column, or box, that cannot contain any digits other than these, then these digits must be at those positions (and not elsewhere in the same row, column, or box).

For example,



Here all possibilities for the fields in column 4 are given. Note the four colored fields: together, they only have the four possibilities 6, 7, 8, 9. So, these colored fields contain 6, 7, 8, 9 in some order, and we can remove 6, 7, 8, 9 from the possibilities of the other fields in that column.

(C) Pick a digit  $d$ . If for some set of  $n$  rows  $R$  there is a set of  $n$  columns  $C$  such that all occurrences of  $d$  in these rows must be in one of the columns in  $C$ , then the digit  $d$  does not occur in a column in  $C$  in a row not in  $R$  (and the same with rows and columns interchanged).

(This argument is called *X-wing* for  $n = 2$ , *Swordfish* for  $n = 3$ , *Jellyfish* for  $n = 4$ .)

For example,

				5	1	4
*		*		2	8	3
8		4	5	9		
		2	*	7	*	
	3	1				
			3	6	5	
6				7		
		1	4			
*		3	7		*	

				5	1	4	
				9	2	8	3
8		4	5	9			
		2	7				
	3	1					
			3	6	5		
6				7			
		1	4				
		3	7				

Consider the three (colored) columns 1, 5, 7. The only possibilities (indicated by an asterisk) for a digit 1 in these columns occur in rows 2, 4, 9. Therefore, digit 1 cannot occur outside these columns in these rows.

**Exercise** Complete this Sudoku. The ‘Sue de Coq’ argument will be useful.

**The subset principle**

Let  $S$  be a subset of the set of cells of a partially filled Sudoku diagram, and let for each digit  $d$  the number of occurrences of  $d$  in  $S$  be at most  $n_d$ . If  $\sum n_d = |S|$ , then the situation is tight: each digit  $d$  must occur precisely  $n_d$  times in  $S$ . In this case we can eliminate a digit  $d$  from the candidates of any cell  $C$  such that the presence of a  $d$  in  $C$  would force the number of  $d$ 's in  $S$  to be less than  $n_d$ .

For example,

6				8	3
2	5				1
			1	6	
	7	6			
8		4	5	7	3
					8
	2			1	9
5			8		
4				7	

6				8	3
2	5				1
			1	6	
	7	6			
8		4	5	7	3
					8
	2			1	9
5			8		
4				7	

In the five colored squares, the five digits 2,3,4,5,9 each occur at most once (since all occurrences of 3,4,5 are in a single box and all occurrences of 2,5,9 in a single column). Since the situation is tight, digits 3,4,5 do not occur elsewhere in this box, and digits 2,5,9 do not occur elsewhere in this column.

More generally, one may remove a candidate for a cell outside  $S$  if its presence would force  $\sum n_d < |S|$ .



**Hinge**

The previous subsection used that each cell contains at least one digit. Conversely, each digit is in at least one cell in any given row, column or box. For example,

4	2	8			1	6		
8	<i>b</i>		2	<i>c</i>	9	5	4	
			4		2	8		
9	4	1		2	8	3	5	
2			3	8	4	6	9	1
6	8	3		9		4	7	2
7	<i>a</i>	8				9	2	4
3	2	4	9		8	7	5	6
			4	7	2	3		8

Consider the above diagram. If cell  $a$  has a 1, then cell  $b$  does

not have a 1, and then the 1 in row 2 must be in cell  $c$ . But then the colored area cannot contain a 1, impossible. (So, cell  $a$  has a 5.)

### Forcing Chains

Consider propositions  $(i, j)d$  'cell  $(i, j)$  has value  $d$ ' and  $(i, j)!d$  'cell  $(i, j)$  has a value different from  $d$ '. By observing the grid one finds implications among such propositions.

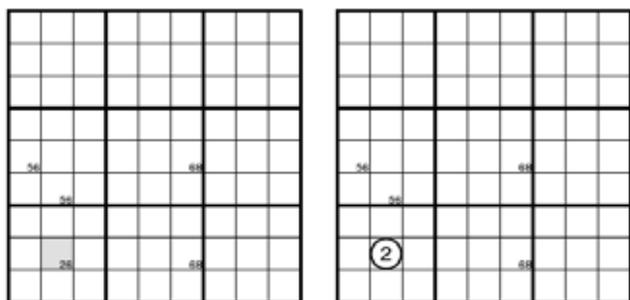
There are at least three obvious types of such implications. Let us say that two cells are 'adjacent' (or, 'see each other') when they lie in the same row, column or box, so that they must contain different digits. This gives the first type: If  $(i, j)$  is adjacent to  $(k, l)$  then  $(i, j)d > (k, l)!d$  where  $>$  denotes implication.

In case  $(i, j)$  is adjacent to  $(k, l)$  and  $(k, l)$  only has the two possibilities  $d$  and  $e$ , then  $(i, j)d > (k, l)e$ . This is the second type of implication.

Finally, if some row, column or box has only two possible positions  $(i, j)$  and  $(k, l)$  for some digit  $d$ , then  $(i, j)!d > (k, l)d$ . This is the third type.

Consider chains of implications. If  $(i, j)d > \dots > (i, j)!d$  then  $(i, j)!d$ .

For example



Here  $(8, 2)6 > (8, 6)8 > (5, 6)6 > (5, 1)5 > (6, 2)6 > (8, 2)2$  was used to conclude  $(8, 2)2$ .

For example



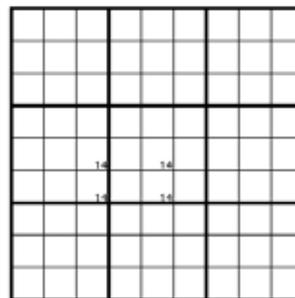
Here  $(8, 2)1 > (6, 2)!1 > (6, 9)1 > (4, 7)!1 > (8, 7)1 > (8, 2)!1$  was used to conclude  $(8, 2)!1$ . For such chains that involve a single digit only, and where the implication types alternate between I and III, one often uses a simplified notation like  $1 : (8, 2) - (6, 2) = (6, 9) - (4, 7) = (8, 7) - (8, 2)$  where  $-$  denotes that at most one is true and  $=$  that at least one is true.

Finding useful chains may be nontrivial, and there are various techniques such as 'colouring' that help.

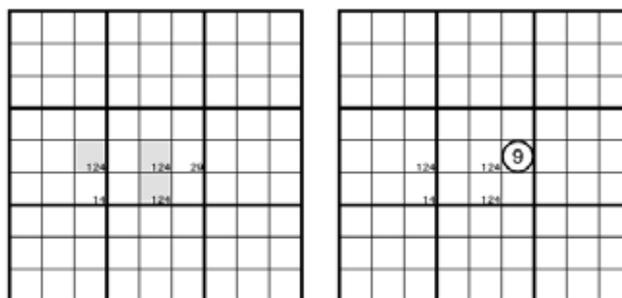
### Uniqueness

A properly formulated Sudoku puzzle has a unique solution. One can assume that a given puzzle actually is properly formulated, and use that in the reasoning, to exclude branches that would not lead to a unique solution.

For example, a grid

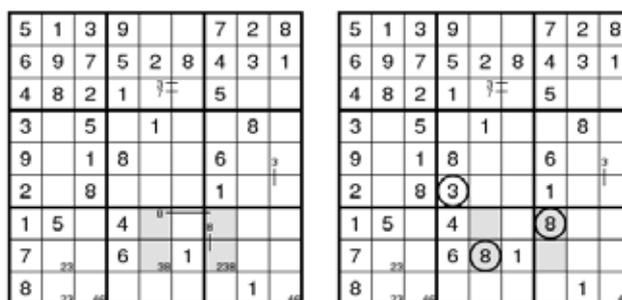


can be completed in at least two ways, violating the uniqueness assumption. That means that this must be avoided, so that



At least one of the corners of the rectangle is 2.

For example,



Look at the colored rectangle. If  $(7, 7)3$ , then  $(7, 5)8$  and  $(8, 7)8$  so that  $(8, 5)3$  and we have a forbidden rectangle with pattern 83-38. So,  $(7, 7)!3$ , which means that we have an X-wing: digit 3 in columns 2,7 can only be in rows 8,9 and does not occur elsewhere in these rows. In particular  $(9, 4)!3$  so that  $(6, 4)3$ , and  $(8, 5)!3$  so that  $(8, 5)8$ .

More generally:

**Theorem.** Suppose one writes some (more than 0) candidate numbers in some of the initially open cells of a given Sudoku diagram, 0 or 2 in each cell, such that each value occurs 0 or 2 times in any row, column, or box. Then this Sudoku diagram has an even number of completions that agree with at least one of the candidates in each cell where candidates were

given. In particular, if the Sudoku diagram has a unique solution, then that unique solution differs from both candidates in at least one cell.

### Digit patterns and jigsaw puzzles

A more global approach was described by Myth Jellies. Solve a puzzle until no further progress is made. Then, for each of the nine digits, write down all possible solution patterns for that digit. One hopes to find not more than a few dozen patterns in all. Now the actual solution has one pattern for each digit, where these 9 patterns partition the grid. Regard each digit pattern ('jigsaw piece') as a boolean formula ('this pattern occurs in the solution'). Write down the formulas that express that for each of the nine digits exactly one pattern occurs, and that overlapping pieces cannot both be true. Solve the resulting system of propositional formulas.

This approach allows one to solve some otherwise unapproachable puzzles.

### Bibliography

- [1] There is an enormous amount of literature on Sudoku on the web. This note is a condensed version of <http://homepages.cwi.nl/~aeb/games/sudoku/>.

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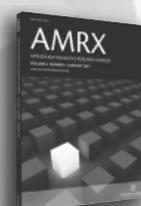
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# Paulette Libermann, 1919–2007

Michèle Audin (Strasbourg, France)



**Paulette Libermann**  
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Born on 14 November 1919, Paulette Libermann died on 10 July 2007.

In 1938, she joined the *École Normale Supérieure de Jeunes Filles*, also called the *École de Sèvres*. Until then, this school had only prepared its students to pass the (feminine) 'agrégation', which would allow them to become teachers in secondary schools (for girls). The director Eugénie Cotton (both a physicist and a left-wing militant) decided to raise the level of her students to make them attain the same level as the men of the *École Normale Supérieure*. Among the professors who taught these young ladies were Élie Cartan and two younger mathematicians Lichnerowicz and Jacqueline Ferrand.

At this point, Paulette Libermann's story meets History. In the fall of 1940, she was beginning to prepare for the agrégation when the so-called French State, anticipating the desire of German occupying forces, established a series of laws called the 'status of Jews'. These laws forbid the people they defined as Jews (among them Paulette Libermann) from practising a certain number of professions, one of which was teaching. Therefore she could not pass the aggregation ... and this led to Élie Cartan suggesting that she start research instead.

Later, she said that the anti-Semitic laws were a bit of luck for her. But at the time, the threat to French Jews was becoming more and more serious and in 1942 Paulette Libermann's family left Paris for Lyon to live a semi-clandestine life until the liberation.

She then went back to the *École de Sèvres* to pass the agrégation. As this was the case for all young French mathematicians (both men and women) she was sent to teach in a secondary school. But now she knew that she wanted to do mathematical research. At this point Élie Cartan gave her a second piece of good advice, namely to start a thesis with Ehresmann.

## Student of Ehresmann

It is hard to imagine what Ehresmann's school of differential geometry and topology was at that time. Charles Ehresmann himself, a member of Bourbaki, passed a thesis with Élie Cartan in 1934. He had many students. The first one, Jacques Feldbau, proved in 1939 that a bundle over a simplex is a product before inventing, jointly with Ehresmann, the notion of an associate bundle ... and the exact homotopy sequence of a fibration. Regrettably, Feldbau was less lucky than Paulette Libermann; the anti-Semitic policy sent him to his death in a concentration camp. Georges Reeb (1920–1993), Wu Wen-Tsun, Paulette Libermann, André Haefliger, Valentin Poenaru, are among the best known of Ehresmann's students.

## Equivalence problems

Paulette Libermann published numerous papers on differential

geometry. She defended her thesis, *Sur le problème d'équivalence de certaines structures infinitésimales*, in 1953. The equivalence problem is a general problem and has been investigated by many including Élie Cartan. Roughly speaking, the question is to classify, up to local isomorphism, structures on manifolds. For instance, all the manifolds of the same dimension are equivalent (this is a local question). But this is not true anymore if the manifolds are endowed with Riemannian metrics: a (curved) sphere is not locally isometric to a (flat) plane.

The metric can be replaced by a lot of structures, for instance a family of 1-forms (a Pfaff system), a notion on which Paulette Libermann worked a lot.

## Symplectic geometry

Her results on symplectic geometry have become classics in the 70s and the 80s (when symplectic geometry became fashionable). The symplectic equivalence problem is solved by the Darboux theorem: all symplectic manifolds of the same dimension are locally isomorphic. There is no local invariant in symplectic geometry!

So:

- (1) Look for global invariants. One of the most powerful tools is Gromov's theory of pseudo-holomorphic curves (1985). This is based on the notion of almost complex structures on symplectic manifolds, one of the notions Paulette Libermann investigated in her thesis.
- (2) Make the structure more rigid, for instance by considering two transverse Lagrangian foliations on the manifold. This problem, that she investigated in one of her papers, lies at the basis of the theory of integrable systems.

She will also be remembered as the author, jointly with Charles-Michel Marle, of one of the very first textbooks on symplectic geometry.

After her thesis, she became a professor in Rennes and then in Paris. She was very helpful to young mathematicians.

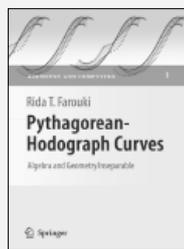
Tiny, energetic, smiling, chatty, sometimes caustic, she was also a memory of the mathematical community. She liked to speak of those who helped her, either personally or professionally: Cartan's family, Jacqueline Ferrand, Ehresmann, anonymous others ... and she also liked to speak of those who did *not* help her. She participated, almost until the end, to conferences all around the world. The last time we met, in April, I was leaving for Vietnam. I cannot go, she told me, too tiring, I am getting old. She was 87.

We shall miss her.



*Michèle Audin [maudin@math.u-strasbg.fr] is a professor of mathematics at the Institut de Recherche Mathématique Avancée, Strasbourg, France.*

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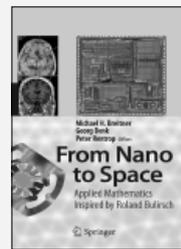
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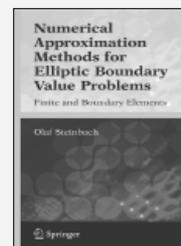
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# Leonhard Euler in Berlin

Jochen Brüning (Humboldt-Universität Berlin, Germany)

## 1. Introduction

Three hundred years ago, on 15 April 1707, Leonhard Euler was born in Basel. Thus, 2007 became a veritable Euler year (even without being named so by an international agency) and his achievements have been exposed, scrutinised and celebrated all around the world. Euler's work in mathematics is very likely the most voluminous and the most influential of any mathematician, not only for the depth and the variety of his results but also for his influence on how mathematics is written and taught. As an indirect proof of this statement we can regard the fact that a comprehensive scientific biography of Leonhard Euler is still missing, in spite of the enormous amount of work devoted to special aspects of his life and his production.

The remarkable stability of his living conditions makes it easy to give a rough sketch of Euler's biography. Even though he was apparently not a prodigy, his enormous talents showed early. The son of a protestant priest, he took up university studies in his home town Basel at the age of 13, enrolling in various faculties. Euler studied theology, philology and history, only afterwards turning to his real favourites, the sciences and above all mathematics, finishing off even with some physiology. In 1727 he wrote his dissertation in physics, presenting a theory of sound on the basis of which he applied for an open physics professorship in Basel. Since this application was unsuccessful, the young Euler decided to follow an invitation to the Petersburg academy in the same year, solicited by the two sons of Johann Bernoulli with whom he had studied and who were already there. Euler began to work in St. Petersburg with the medical department, gradually shifting his subject towards astronomy, geography and mathematics, while improving his standing, and his salary, in the academy. Eventually, he became a professor of mathematics in 1737. Two important private events must be mentioned: in 1733, Euler married his Swiss compatriot Katharina Gsell, the daughter of a well-known painter and in 1738 he lost his right eye, probably as the consequence of an infection.

After the death of Peter the Great the political circumstances in Russia became more and more unstable, with increasingly unsafe conditions and imminent danger to the existence of the academy. Euler and his wife were very disquieted by this. Thus he followed, in 1741, the prestigious offer by Friedrich II, the new king of Prussia, to come to Berlin and help construct a new Royal Academy of Sciences. He arrived in Berlin on 25 July 1741 and he was to stay until 1766. In this year, after getting more and more disappointed with the king's handling of the academy and Euler himself, he followed a second call to Russia, this time from Katharina II; he returned



**Fig. 1: Euler at the age of 30, the only existing portrait showing him with full eyesight; after an original by Brucker.**  
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to St. Petersburg and stayed there until his death on 18 September 1783.

We devote these pages to the Berlin period in Euler's life, trying to highlight not only, and not even primarily, his scientific work but also his many other activities. We hope that in this way a more complete picture of Euler may arise and, perhaps, a better appreciation of how much we owe to him.

## 2. People

Even for a person with the mental powers of Euler, projects and activities are always connected with people. In fact, Euler was not shy or even introvert; on the contrary, he enjoyed company for conversations, games and music, and he loved to appreciate and being appreciated.

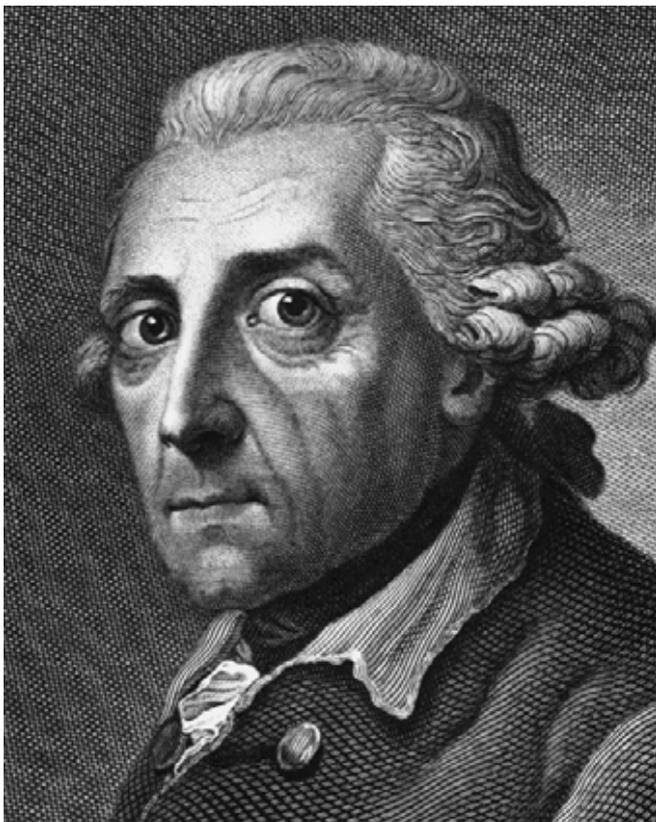


Fig. 2: Friedrich II, king of Prussia, contemporary engraving; Universitätsbibliothek Basel. In: EO IV, 6, p. 276

Hence it is helpful to take a closer look at the two persons who were, without any doubt, most important to Euler's everyday life in Berlin, even though he met them in person only rarely: the king Friedrich II and the president of the academy Pierre-Louis Moreau de Maupertuis.

### *Friedrich II*

Friedrich II was certainly the most intellectually gifted monarch of his time but he was also a very complicated character. All his life he felt attached to philosophy, music and poetry, and above all to French culture, while he disliked military exercises and quite possibly the military as such. His father, Friedrich Wilhelm I, had equipped Prussia with too large an army but a solid financial basis, at the expense of a stern and parsimonious regime. Friedrich suffered a lot in this atmosphere and tried to flee the country, together with his friend Hans Hermann von Katte; he was caught, his friend was executed and he was kept as a prisoner by his father for two years. In 1740, at the age of 28, Friedrich inherited the throne and was suddenly confronted with the realities of power, without much preparation; in fact, he had spent the previous years in the somewhat remote castle of Rheinsberg, with a circle of friends enjoying French culture and the arts.

Two decisions of his first weeks in power highlight the very different aims he now wanted to unite: firstly he started an unjustified war for the rich province of Silesia belonging to the Habsburg Empire and secondly he tried to rebuild the *Société des Sciences*, the precursor of the Prussian Academy of Sciences, which had been founded

by Leibniz but had deteriorated very much during the reign of Friedrich Wilhelm I. The latter decision is important for our context because it brought Euler to Berlin in 1741. The first decision was important too as it would keep Friedrich busy until 1763, the end of the Seven Years War (incidentally the first war that was really global in the sense that it was fought on several continents). Necessarily, the extended warfare crucially limited the king's potential of caring for the sciences and the arts, with the exception of a short period between 1745 and 1753. This is also the period during which Euler felt most happy being in Berlin, while after 1763 he felt more and more alienated and thought about going back to Russia, an idea that became a reality in 1766.

Euler liked Petersburg and the people he met there very much and he managed to keep very close relations with his colleagues in the academy and with the Russian government, even after he had left for Berlin. That he left at all to accept the offer by Friedrich to come to Berlin and to reorganise the old *Société des Sciences* by winning other prominent scientists and mathematicians to Berlin is probably due to the political instabilities and the ensuing uncertainty of daily life after the death of Peter I, which was especially oppressive for Euler's wife Katharina. But once he had accepted the call to Berlin, he was enthusiastic about the possibilities to organise a new academy that, according to the goal set by Friedrich, should be soon among the leading institutions of this kind in the world. Euler was apparently under the impression that Friedrich would entrust him with the whole architecture of this new academy and also with shaping its scientific, organisational and administrative details.

This, however, was not so: even before calling on Euler, Friedrich had invited Pierre-Louis Moreau de Maupertuis to become president of the new academy. He promised to equip the new president with full powers, limited only by the king himself, and Maupertuis had immediately reacted positively to this offer, though he only took office in 1746. Even then and until his resignation in 1756, Maupertuis was not continuously present in Berlin; his stays were interrupted by long periods of absence. Thus Euler was in fact running the academy in all practical terms while formally he was only director of the mathematical class. But he did not even become president when Maupertuis resigned, which came as a bitter surprise to him. He had apparently misjudged from the beginning his relationship with the king, regarding himself as leading scientific counsellor. For the king, however, there was a clear difference between a philosophical approach to government, which he claimed for himself, and the involvement of scientific advice and judgment in political decisions. Though Friedrich was fully aware of the role of technology for the welfare of states and their political standing, he did not regard scientists in general as being able to speak out in political matters and to discuss politics; he certainly appreciated their usefulness but he wanted to keep them at a distance from the centre of decisions (he once wrote to Voltaire that the king should keep a scientific academy as a rustic squire keeps a pack of hounds, a statement of somewhat exaggerated poign-

ancy that nevertheless seems to reveal a deep conviction often found in politics even today).

The philosopher Voltaire, a leading protagonist of French enlightenment, was in contact with Friedrich some time before his enthronement, at the occasion of which he rejoiced that the sciences and the arts had come to power. He visited Berlin in 1740 and stayed there from 1750 to 1753, shaping to a large extent the style of conversation cultivated in Friedrich's environment. Leonhard Euler certainly did not fit the ensuing ideal of a courtier: his French was not brilliant, his language was free of irony and cynical jokes and his way of arguing showed the influence of Calvinist sermons and of mathematical reasoning. Besides, unfair as it may seem, Friedrich disliked his one-eyed appearance, calling him a 'Cyclops' in private conversations. Euler certainly felt this fundamental difference but tried, in the tradition of the enlightenment, to convince Friedrich of his abilities and his efficiency by showing that mathematical thinking and its practical applications provided the best means to further all interests of the state and the government. It seems that he worked hard to reach this goal. For example, immediately after arrival in Berlin he wrote an essay entitled "Commentatio de matheseos sublimioris utilitate"<sup>1</sup>, which was unfortunately not published until 1847. A second example is his book on artillery on which we will comment below.

This approach conformed very well with Euler's firm Calvinist faith, since he believed that God reveals some of the secrets of the creation through mathematics, allowing in this way for man-made improvements of the human condition. But propagating this, Euler confronted the anti-religious tendency of the French enlightenment and thus of the Prussian court. Only when he finally understood that he was not able to bridge this discrepancy, in spite of all achievements harvested through his singular abilities, he left Prussia for St. Petersburg.

#### *Pierre-Louis Moreau de Maupertuis*

Pierre-Louis Moreau de Maupertuis was a celebrity when he was asked to become president of Friedrich's academy. He had a solid mathematical education that he had acquired in England and in Basel with the Bernoullis. He was among the first scientists on the continent who adopted and popularised Newton's theories but he quickly got involved in the attempts to clarify Newton's prediction about the flattening of the Earth near the poles, quite contrary to the view the Cartesians held. After having been employed as a geometer by the French Academy in 1731, Maupertuis worked hard for securing funds to measure a meridian in Lapland, thus being able to prove or disprove Newton's assertions. He was successful in 1736 and he returned from this strenuous expedition with convincing data that established Newton's victory over Descartes and made Maupertuis the most famous scientist of his time. But his health had suffered greatly from the strains of the expedition, another reason for his many absences from meetings of the academy, and his frequent voyages to France, especially during the German winter, which was very hard on him.



**Fig. 3: Pierre-Louis Moreau de Maupertuis, engraving after the famous polar expedition of 1736; Universitätsbibliothek Basel. In: EO IV, 6, p. 29**

Even though Maupertuis' career as a productive scientist had already been terminated in 1732, he started his presidency in Berlin with a spectacular result, namely his "principle of least action", which was published in 1746. In this work, he asserted in vague terms, and without making any use of the new infinitesimal calculus, that the quantity of "action" he had introduced was minimised in any dynamical transition of physical states. He based this very general assertion on metaphysical reasoning, which should explain the effectiveness, if not parsimony, of the Creator.

Euler could have claimed priority in this matter since he had published extensively on a general theory of curves enjoying maximising or minimising properties with respect to certain functionals given by integrals. This work began in 1736 and culminated in the book entitled "Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes"<sup>2</sup> published in 1744. This book presented the first systematic treatment of the calculus of variations and was described by Constantin Carathéodory as "the most beautiful mathematical book ever written". Quite obviously, Euler's mathematical treatment was far superior to Maupertuis' reasoning and, in

<sup>1</sup> "Commentary on the usefulness of higher mathematics", see E 790. Here and below we refer in this way to the numbers in the bibliography of *Gustaf Eneström: Verzeichnis der Schriften Leonhard Eulers in Jahresbericht der Deutschen Mathematiker-Vereinigung, 1. Ergänzung zum 4. Band 1910*. Likewise we will refer to *Euler's Collected Works* which appear with Birkhäuser, Basel, in 4 series with a total of 72 volumes, as e.g. EO IV, 6 for "Volume 6 of the fourth series".

<sup>2</sup> "A method to invent curved lines with minimal or maximal properties", see E 65.

particular, he had emphasised from the beginning that it is not always a question of minimising but that maxima may occur in nature (which are described in the same way). However, he did not object to Maupertuis' statement that his all embracing principle of least action had been nicely illustrated by Euler's work. Euler even explained at some length that the principle of Maupertuis was not a mathematical but a philosophical statement and hence had no place in the world of mathematics.

This statement was certainly compatible with Euler's basic convictions but it also showed a certain respect for the powers that be. After all, Friedrich had bestowed upon the president of the academy the full power to decide everything whatsoever and Euler always respected this fundamental rule. Only this careful respect for the president's position made it possible for him to deal with each and every matter and to carry out in effect what Maupertuis ordered or might have ordered. The rather substantial, and extant, exchange of letters between Maupertuis and Euler, which is explained by the frequent absence of the president from the academy shows that both men were, if not outright friends, at least very smoothly cooperating partners entertaining a style of mutual understanding and respect. Maupertuis was a politician and a courtier (where Euler was not) but he was also a gifted and educated scientist who could understand what Euler thought and intended. The subtle balance in their relationship was quite vital for the very positive development of the Berlin academy from 1741 to 1752.

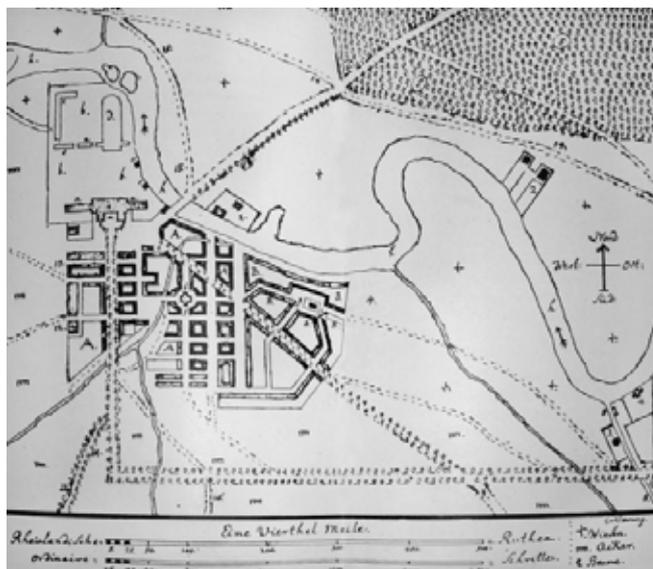
#### *Family and friends*

Saying that Euler was definitely not a courtier does not mean at all that he was not a person who loved company and was not able to brilliantly entertain his guests. Nor does it imply that Euler was unable to keep company or even friendship with members of the higher society. To the contrary, his big house in what is today Behrenstraße not only had enough room for his large family but regularly housed visitors from abroad. Euler was surely a family man who has been reported to work at his desk amidst his many children<sup>3</sup>, without any sign of impatience. Whatever he did he could interrupt at any time, taking up a completely different matter and returning, seemingly uninterrupted, to his concentrated work afterwards.

Besides his nice house, Euler owned an orchard and a farm (which was recently identified by Wolfgang Knobloch, see figure 4); on the farm his mother lived as a widow until her death.

Among the many friends of Leonhard Euler was the Swiss medallist Johann Carl Hedlinger (1691–1771). The two compatriots had met in Russia, where Hedlinger had done some work for Tsareva Elisabeth and her court. He was then employed by the Swedish king and generally known as one of the most brilliant medallists in the world.

<sup>3</sup> Euler had 13 children with Katharina of whom only 5 reached adulthood, and only his 3 sons Johann Albrecht (1734–1800), Karl Johann (1740–1790), and Christoph (1743–1808) survived him.



**Fig. 4: Euler's farm in Lietzow now Charlottenburg, a part of Berlin; the Euler possession is marked with the letter "B".**

**In: W. Gundlach: Geschichte der Stadt Charlottenburg, 1905, Beilage XII**



**Fig. 5: Price medal of the Société des Sciences et des Belles Lettres, Medailleur Johann Carl Hedlinger, Berlin 1747/48, Ø 67 mm.**

**© Staatliche Museen zu Berlin-Preußischer Kulturbesitz, Münzkabinett**

His fame had also reached the Prussian court and Friedrich's counsellor in all matters of architecture and art Georg Wenzelslaus von Knobelsdorff had inquired under what conditions Hedlinger would be willing to come to Berlin. But this letter remained unanswered since Hedlinger was in Switzerland and Euler stepped in and invited him and his wife to his home for an extended visit. From all we know, Hedlinger's stay in Euler's house must have been very pleasant except for the fact that Friedrich II did not care at all for him, apparently angered by the negligence shown by Hedlinger in reaction to his generous offer. After Hedlinger had already spent six months in Berlin, Friedrich wrote a letter to Euler saying that he would like to employ Hedlinger, either permanently or for a couple of years, at any salary Hedlinger might request.

Now, however, it was too late. Hedlinger was already resolved to go back to Sweden from where impatient signals were coming asking for his return. But as much as Hedlinger hated the style of conversation at the Prussian court, like his friend Euler he admired Friedrich as the

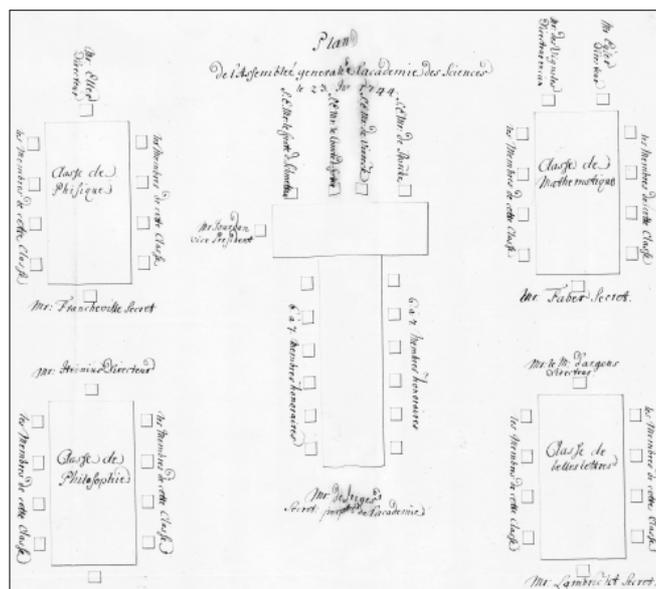
brilliant monarch of the enlightenment and he had long wanted to produce an image of him. Thus he had made preparations during his stay but only much later did the work come to perfection. In 1748 he produced a medal for the academy (see figure 5), based on discussions with both Euler and Maupertuis, who were so impressed that they elevated Hedlinger to the state of honorary member of the academy. In 1750 he completed the medal for Friedrich II. The king was delighted with this work and wanted to buy the stamp at any cost, as he wrote to Euler, but soon afterwards he did not remember that he wanted to pay anything to Hedlinger; in spite of all his talents and all his brilliance, Friedrich II remained the son of his father in trying to save money wherever he could. Many projects were damaged or even prevented by his miserly attitude.

### 3. Projects

#### *The Academy reform*

As mentioned before, the academy in Berlin was founded by Gottfried Wilhelm Leibniz in the year 1700 under the name *Société des Sciences*. After a slow development in the first decade of its existence the academy was finally opened officially on 19 January 1711. It resided in a building Unter den Linden, known as “Der neue Marstall” where the older royal academy of arts had found its quarters in the year 1700. This cohabitation of scholars and artists and their usually quite different institutions was unique in Europe at that time. The alimentation of the academy was effected through the so-called “Kalendar-Privileg”, that is the monopoly to produce calendars and related publications throughout Prussia. In this way a reliable though narrow financial basis was provided, which nevertheless did not allow the realisation of the ambitious dreams of Leibniz. In addition, the main burden caused by the bread-and-butter work of calendar-making fell upon the mathematicians and astronomers in the academy, a fact that regularly gave rise to some disturbances. In addition, during the reign of Friedrich Wilhelm I (1713–1740) the *Société des Sciences* deteriorated under the total neglect if not contempt shown by the king, who installed his jester as its president. Nevertheless, some outstanding scholars were working in Berlin even then, like the philologist and botanist Johann Leonhard Frisch.

As mentioned above, Friedrich II was prepared to change things as soon as he came to power, an intention that was realised with the calls he sent out to some of the most prominent scientists of Europe preferring, of course, those conforming with his ideal, the philosophical attitude of French persuasion. When Friedrich II called Euler to Berlin he probably did not know of his enormous energies extending to everything in his neighbourhood that offered possibilities for an effective treatment. Thus Euler started immediately to reorganise the academy, being instructed by the envoys of the king that he was aiming at an institution only to be rivalled by the Paris and London academies. Euler’s patience was cer-



**Fig. 6** Sketch of the seating arrangement for the opening ceremony of the new academy, January 24, 1744.

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tainly severely challenged by the first two Silesian wars, which absorbed all the energy of the government and left little room for matters concerning the academy. But Euler worked relentlessly, relying on his Swiss tenacity, and certainly not without effect.

The long interrupted series of academy publications came to new life, the first volume being full of publications authored by Euler, like many others to come. He insisted that meetings of the academy would happen under a regular schedule and with substantial protocols. He brought new people to Berlin and constantly developed his communication network, which included most of the significant scientists of his time. Fortunately he was not alone and some influential people in the neighbourhood of the king also wanted progress in academy matters, even though their emphasis was more on arts and literature. In 1743, a *Société de Belles Lettres* was founded as a further precursor of the promised new academy. Honouring these attempts, Leonhard Euler worked on a unification of the two societies and was successful on 24 January 1744 when the new *Académie Royale des Sciences et des Belles Lettres* was founded in the Berliner Stadtschloss.

The organisation was entirely Euler’s, from the conception of the statutes to the details of the opening ceremony, and it shows some remarkable aspects. Notably, among the four classes representing the sciences and the humanities, the *Classe de Philosophies* was unique in Europe, attesting to the important role this academy would play soon in the philosophical disputes and quarrels of the 18<sup>th</sup> century. As we can see from figure 6, Euler already appears as director of the mathematical class, in which function he was officially installed only on 03 March 1746. Thus, in spite of all shortcomings and frustrations, Euler quickly built a sound basis for what was to become probably the most fruitful period of his life.



**Fig. 7: Academy building Unter den Linden, its location from 1752 to 1904; drawing by Calau, engraving by Lauréns and Thiele.**  
 © Archiv der Berlin-Brandenburgischen Akademie der Wissenschaften, Grafiksammlung, P/BON – 1135

In 1752, after a fire had destroyed the “Neue Marstall”, both academies moved into a new and quite representative building, which had to make room for the new Prussian state library only in 1903 (figure 7).

#### *Practical mathematics*

As we have noted already, Euler was resolved to prove the power of pure mathematics through practical applications, not only because he wanted to convince his king of the possibilities of his own field, and hence of his own abilities, but also as a matter of principle, since it was his deepest conviction that it is through mathematics that we can get insight into the work and even the intentions of the Creator. Naturally, the academy had to serve the king and the state in many respects, in engineering projects, in questions of time measurement and calendar calculation, and regularly in the evaluation of technological innovations, acting very much like a modern patent office. Euler was involved in these activities a lot, including some greater projects. The first among these was the rebuilding of the Finow canal, a waterway that provided a direct connection between Berlin and the river Oder but which had deteriorated over the years. Euler was a member of the commission that had to visit the canal, his task being the exact measurement, the levelling, on which corrections had to be based. Fortunately, not all his missions expanded into such strenuous expeditions.

Another well-known story concerns the fountain of Sanssouci and the dream of the king to have one of tremendous height such as to impress any visitor. It is usually told that Euler gave detailed advice that failed to comply with reality and that Friedrich never saw a fountain higher than 30 centimetres. This outcome in turn strengthened Friedrich’s contempt of mathematics and mathematicians. The latter statement is true but some of the facts have to be corrected, as was carefully shown

by M. Eckert<sup>4</sup>. His detailed account shows that Euler’s analysis was quite appropriate and that his calculations would have given the desired result if only the stinginess of king Friedrich had not interfered again. Thus he twice selected completely inexperienced but cheap craftsmen and ordered them to use wood for the water conducting tubes, a material which simply could not resist the necessary pressure, while Euler had repeatedly insisted upon tubes made from lead.

Other tasks of Euler were closer to mathematics, for example when the king enquired about the correct setup of a lottery in order to fill the notoriously empty coffers of the state. The first such lottery had been established in Berlin in 1740 but it did not live up to expectations. Euler was also asked to think about the basis of a life insurance system and a (restricted) pension system, ideas which attest to the modernity of Friedrich’s thinking.

Another project with practical implications was closer to Euler’s mathematical thinking than the aforementioned ones and seems to have played a particular role in his attempts to convince the king of the usefulness of mathematics. Finding the Prussian army involved in extended warfare, Euler must have thought hard about an application of mathematics in this context, especially one that would improve the performance of the Prussian weapons in an undisputable manner. Knowing very well that successful applications on a larger scale are impossible without calibrating experiments, it must have come like a heavenly gift to Euler when he discovered the book “New Principles of Gunnery”, which was printed in London in 1742. The author Benjamin Robins was not unknown to Euler since he owed to him a very negative criticism of his first book on mechanics written in St. Petersburg. However, like in the case of Maupertuis’ priority, Euler cared very little about such questions and did not retaliate in his treatment of Robins’ book, even though he found many mistakes that had to be corrected. In this way, he improved the work greatly and elevated Robins to a fame that he would never have achieved otherwise. At any rate, what Euler was interested in were the results of Robins’ experiment; he had found a rather ingenious way to measure the true velocity of cannon balls and the force of gunpowder.

With this data, Euler was able to apply infinitesimal calculus, thus creating the foundations of modern ballistics. Much to his satisfaction he could improve tremendously the results and predictions of Robins, correcting along the way a formula for the air resistance given by Newton by a factor of two. The new book by Euler finally consisted of a translation of Robins work enriched with extensive comments, theoretical developments and calculations, which resulted in a volume five times the size of the original, with the appropriate baroque title “Neue Grundsätze der Artillerie enthaltend die Bestimmung der Gewalt des Pulvers nebst einer Untersuchung über den Unterscheid des Widerstands der Luft in schnellen und langsamen Bewegungen aus dem Englischen des

<sup>4</sup> M. Eckert: Euler and the fountains of Sanssouci. *Archive for the History of the Exact Sciences* **56** (2002), 451–468.

Herrn Benjamin Robins übersetzt und mit den nöthigen Erläuterungen und vielen Anmerkungen versehen von Leonhard Euler königl. Professor in Berlin. Berlin bey A. Haude königl. und der Academie der Wissenschaften privil. Buchhändler. 1745"<sup>5</sup>.

Euler certainly met the success he intended, at least outside Prussia, since his work was immediately introduced to military schools, notably in France from where he received significant praise and a sizeable remuneration. Napoleon, whose love for mathematics is well-known, had to study Euler's book as a young officer. However, we do not have any proof of a similar reaction from the Prussian court. We know about a letter that Euler sent to Friedrich II some time in 1744 wherein he announces his plan to translate and extend Robins' book and asks for permission to devote his time to this project but we don't know of any answer. We also know of the letter of dedication to the king accompanying the completed book. This letter is dated 20 April 1745 and seems to be hitherto unpublished.

Euler makes it quite clear, in spite of the formal modesty of his writing, that he regards this piece of work as the desired proof of the all embracing power of higher mathematics. He describes the relationship between what he calls here "Theoretical Mathematics" and its applications, which the king seems to have asked for, as adding experimental data to determine the constants of integration to the equations of motion that arise from general principles like his calculus of variations. It seems that the idea of equivalent but different theories describing the same phenomena was still alien to Euler, his religious convictions leading him to believe in "true equations". Alas, we do not know of any comments or signs of grace from the side of Friedrich in reaction to this remarkable achievement of Euler's.

#### Private Matters

As should be expected, Euler also handled his family business with great care and great effectiveness. Thus, the Euler family could be called wealthy since Leonhard not only enjoyed a rather exceptional salary but also earned considerable money from many academy prizes he won, among which we find the prize of the French academy at least twelve times. Besides, he had revenues from his farmland, which must have also been significant. At least we know that during the Russian-Saxonian occupation of 1760 Euler lost, by the official record, 2 horses, 13 cattle, 7 pigs and 12 sheep (see figure 8). For this damage he was reimbursed by the occupation forces and received, on top of this, a very generous compensation from Katharina the Great. This Russian generosity had, unfortunately, no parallel in Prussia, which made it eventually even easier for Euler to go back to St. Petersburg. There have probably been many other sources of occasional monetary

<sup>5</sup> New principles of gunnery with determination of the true force of the gunpowder and an investigation of the different air resistance for fast and slow motions. From the English of Mr. Benjamin Robins translated and where necessary commented on and amended by Leonhard Euler, etc.

No.	Art.	Stück	Wert	Art.	Stück	Wert	Art.	Stück	Wert
1	Christ. Pferd	8	1	70	2	4	1	3	
2	2 Gf. des Aufhoppers	10	4	12	100	3	7	1100	
3	2 Gf. Weiff	400	1	30	2	5	8		
4	2 Gf. Pfeffer-Ceder	1	1	6	30	2	13	7	12
5	2 Gf. v. Fuchlmeise	1	1	40	2	6	2	44	
6	2 Gf. Nothe	30	10	12	40	4	2	8	
7	2 Gf. Nothweiss Koll	37	10	10	30	3	9	8	
8	2 Gf. Nothweiss	20	1	6	6	2	4		
9	2 Gf. Nothweiss	10	20	40					
10	2 Gf. Nothweiss	10	1	10	4	3	3		
11	2 Gf. Nothweiss	30							27
12	2 Gf. Nothweiss	60	2	30	3	8	3		
13	2 Gf. Nothweiss	13							
14	2 Gf. Nothweiss	4	3	24	1	3	2		
15	2 Gf. Nothweiss	1	2	10	30				
Summa des Lictzen		612	4	26	4000	3	72	57	216
Summa des Charlotten		1209	12	42	6	69	13	241	26
Summa des Lictzen		1209	12	42	6	69	13	241	26
Summa des Charlotten		1209	12	42	6	69	13	241	26

Handwritten signatures: *Ad. Müller*, *M. Müller*, *M. Müller*

Fig. 8: Official list of the damages caused by plunderings in the village Lietzow 1760.

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Fig. 9: Emanuel Handmann, Portrait of the mathematician Leonhard Euler; pastel on paper, 57 x 44 cm, 1753.

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1744A	<i>Methodus inveniendi</i> Euler's groundbreaking foundation of the calculus of variations; E 65
1744B	<i>Theoria motuum planetarum et cometarum</i> Improved method for the calculating the orbits of planets and comets; E 66
1745	<i>Neue Grundsätze der Artillerie</i> Translated and enormously enlarged edition of <i>New Principles of Gunnery</i> by B. Robins; E 77
(not before) 1745	<i>Anleitung zur Natur-Lehre</i> <i>Introduction to the Natural Sciences</i> ; published posthumously in <i>Opera posthuma</i> II, 1862; E 842
1746	<i>Gedancken von den Elementen der Körper</i> <i>Thoughts about the Elements of Bodies</i> ; gives objections to Leibniz' theory of monads based on arguments from physics and theology; E 81
1747	<i>Rettung der göttlichen Offenbarung gegen die Einwürfe der Freygeister</i> [anonymous] <i>Rescue of Divine Revelation from the Objections of the Freethinkers</i> ; argues against atheistic tendencies of the enlightenment; E 92
1748	<i>Introductio in analysin infinitorum</i> <i>Elements of Analysis</i> ; first part of the Analytic Trilogy; E 101,102
1749	<i>Scientia navalis</i> Encyclopedic work on shipbuilding and navigation, prepared in St. Petersburg 1738; E 110-111
1753	<i>Theoria motus lunae</i> So called first lunar theory; E 187
1755	<i>Institutiones calculi differentialis</i> <i>Introduction to the Differential Calculus</i> , prepared around 1748; E 212
1765A	<i>Theoria motus corporum</i> His foundational work on the mechanics of solids; E 289
1765B	<i>Théorie général de la dioptrique</i> <i>General Theory of Lenses</i> , prepared in Berlin, published in 1862; E 844
1768A	<i>Lettres à une princesse d'Allemagne</i> Popular letters on science and philosophy, prepared in Berlin, but published in St. Petersburg, with unusual success: at least 12 French, 9 English, 7 German, and 4 Russian editions; E 343, 344, 417
1768B	<i>Institutiones calculi integralis</i> <i>Introduction to the Integral Calculus</i> ; Euler's exhaustive book on the integral calculus was written in 1763 and appeared in 1768-1770; E 342, 366, 385, 660

**Fig. 10: List of books which Leonhard Euler has written or published while he was in Berlin, 1741–1766.**

gains, e.g. a lottery prize, and a careful look at the famous Handmann portrait (figure 9) reveals that Euler is clad in silk that was produced from the Berlin academy's own mulberry plantation.

#### 4. Scientific work

As mentioned above, a comprehensive treatment of Euler's scientific oeuvre is missing in the literature whereas there is an abundance of detailed discussions of specific aspects of his work. In the framework of a single essay any in-depth study is ruled out if one tries to describe some extended period of Euler's life, as we do here. However, it is easy enough to collect some statistics of what he was doing during his 25 years in Berlin. Thus we can say that he prepared roughly 380 articles or books in this time of which 275 were also published in this period<sup>6</sup>. For this, we rely on the very careful catalogue of Euler's writings compiled by Eneström (see footnote 1), who comes up with a total of 866 (by also counting letters and prefaces and the like but not reprints or translations). There is no way of briefly surveying this massive production but the printed books provide a fairly accurate guide to the areas of interest to which Euler devoted a substantial portion of his time in Berlin; a complete list is given in figure 10. From the rich menu that Euler serves us here, I want to select two important topics for a somewhat closer scrutiny.

#### *The birth of analysis*

Euler's continuous and widespread flow of work developed from various sources: from questions that had become famous because they had withstood the attempts of many illustrious colleagues, like the Basel problem; derived from personal interests like the building of ships<sup>7</sup>; questions about optics, certainly fostered by the injuries to his eyes; and problems arising from accidental impulses, like his work on the foundation of ballistics. Besides, Leonhard Euler was very familiar with the work of the giants of the past and eager to develop their work further or even correct them or prove outstanding conjectures<sup>8</sup>. More importantly, in spite of being constantly absorbed by considerations directed at the solution of specific problems, it seems that he always kept in mind the theoretical framework he was working in and that he was able to adapt the whole architecture of the relevant theory according to the new notions and arguments that protruded from a special study. Whenever he felt the ideas to have reached a certain maturity, he put them together in a book and those books usually remained highly influential for a long time.

The process of development did not stop, though, once he had written a book and improvements or modifications he introduced later on in an article often did not reach the public for quite some time; his educational style of writing was simply too convincing.

Let us try to exemplify this sketch of Euler's working habits with his "analytic trilogy" consisting of the books marked 1748 A, 1755, and 1768 B in the list of figure 10. It is a well-known fact that Euler built the foundation for what we call "Analysis", that is the theory of real functions, the various limit processes that build them, and the differential and integral calculus from which most of the interesting functions can be derived. What is maybe less known is the fact that in Euler's work for the first time analysis arises on a foundation that is independent of geometry (while elementary algebra is presupposed and used as an appropriate substitute). It fits this picture that Euler, like most other early analysts, was called a "geometer", but an even more striking illustration is provided by the total absence of diagrams, which have accompanied mathematical texts since the days of Euclid, and a closer look reveals that this effect is by no means compensated by an abundance of formulae; in Euler's texts, the written language dominates by far.

If we look at the table of contents of the "Introductio in analysin infinitorum", we see that the book is divided into two parts, the first one establishing the foundations of analysis, the second one building what we would call linear algebra. That the two parts arise in this order is

<sup>6</sup> The publication of scientific articles by Euler was not finished before 1862 (with Eneström's number E 856), almost 80 years after his death.

<sup>7</sup> It seems to be unknown why Euler was so much attracted to ships which he could hardly be familiar with.

<sup>8</sup> That we talk about "Fermat's Last Theorem" today is due to the fact that Euler had proved, one by one, all other conjectures left behind by Fermat.

another proof of the above statement that, with Euler, analysis arises independently of geometry. The main goal of the first part is the study of functions with real arguments and real values, though occasionally also complex numbers are considered. Euler does not give a motivation for this since he could assume that his readers already knew enough examples for the overwhelming importance of functions for all applications of mathematics and had some experience with polynomials and rational functions. He insists, however (in §4), that a function must be given by analytic expressions under which notion he covers the basic algebraic formulae and the specific limit expressions that are the main topic of the whole book. Then he proceeds to introduce these limit expressions, notably infinite series and, in particular, power series and infinite products; the last chapter is devoted to continuous fractions.

He then makes systematic use of these operations to introduce the elementary transcendental functions and, as a particular highlight, he presents the trigonometric functions as power series, without any recourse to their geometric definition. Along the way he shows what powerful conclusions can be reached by relating infinite series and infinite product representations of a function to each other. Transferring the decomposition of polynomials in linear factors to entire functions he solves the “Basel problem” mentioned above, which asks for the precise value of the sum of inverse squares of the positive integers; in fact, he computes the value of Riemann’s (perhaps more correctly Euler’s) zeta function at all positive even integers.

It is fair to say that Euler prefigures here the modern (physics) concept of a partition function. His arguments, as is well-known, are not entirely satisfying with respect to their rigour and some of his conclusions are extremely bold indeed. It seems, however, that Euler was completely convinced of the truth of his assertions but did not want to spoil the flow of the presentation by leaving out a beautiful result. In addition, he was probably too impatient to postpone the publication to a later date when he would have arrived at more convincing proofs. And this he did in many cases, especially in the case of the Basel problem for which he provided many more proofs later and for which he had numerical corroborations when he first published it.

The second volume in the trilogy concerns differential calculus (volume “1755” in the list of figure 10), which builds explicitly on the “Introductio” just discussed. The preface is a marvellous piece to read (especially in Latin) and offers some striking features. First of all, Euler immediately develops a very general notion of function that could easily be identified with our modern view, freeing the definition completely from any requirement of analytic expressions. This change has been little noticed, as can be seen from the fact that the modern definition of function is usually attributed to Dirichlet and thus placed a hundred years later. Next, Euler deals at length with the problem of “evanescent quantities”, to make it perfectly clear that a quotient of quantities tending to zero may have a finite limit. He also indicates here that the

task of determining these limits is of vital importance for applications since the real problems posed by nature are only understood by solving differential equations. These equations are derived, constituted and solved by infinitesimal procedures only; we would not be surprised if Euler had added here that “nature is simple only infinitesimally”, a remark due to Einstein.

A further interesting feature of this preface is the fact that Euler illustrates the reasoning of this introduction by just one example, namely the firing of cannon balls (which he had dealt with at length, as we know). Moreover, he calls the differential quotient in this connection the “ratio ultima”, which must have had an ironic connotation for his contemporaries who knew that Friedrich II had written “Ultima Ratio Regis”, the last resort of the king, on his cannons, following the example of Richelieu. This nice pun may explain why Euler did not illustrate the goals of the calculus by explaining Newton’s brilliant derivation of Kepler’s laws.

The fact that the ensuing chapters present the material of higher differential analysis very much in the way it is presented in most calculus courses nowadays may come as a surprise for some but is easily explained by the fact that Euler’s work was copied ever after, at least indirectly, notwithstanding many refinements and extensions. This is also true of the third volume in the analytic trilogy, which is devoted to the art of integration and will always remain its true and lasting foundation.

Euler’s architecture of analysis does convince by its methodological consequence but not so much by its rigour. It furnishes a very impressive proof of Euler’s analytic instinct that his edifice lived through many logical crises only to become reinforced and augmented, without significantly changing its structure or losing its beauty.

### *Educational writings*

In the thinking of Leonhard Euler we find a remarkable emphasis on the presentation of the many insights he had. He always tried to make his thinking clear to other people and not only to those of comparable insight, as if the truth of a thought could only be established by communicating it successfully; here again it seems that we meet the influence of the Calvinist sermon. Be this as it may, Euler engaged himself in mathematical education early on and this occupation accompanied him all his life. As early as 1735 he wrote a very successful schoolbook on arithmetic (E 17: *Einleitung zur Rechen-Kunst, zum Gebrauch des Gymnasii bey der Kayserlichen Academie der Wissenschaften in St. Petersburg. Gedruckt in der Academischen Buchdruckerey* – 1738, 1740).

In Berlin he conceived his educational masterpiece, a collection of 234 letters to a German princess on questions of physics and philosophy. These letters were written to the daughter of the margrave of Brandenburg-Schwedt with whom Euler kept very friendly relations. Thus he was asked to give private lessons to the margrave’s daughter, by then 16 years old, in science and philosophy. When, in the course of the Seven Years War, the Prussian court left Berlin temporarily, anticipating the short occupation of Berlin by Russian and Saxonian

troops in 1760, Euler was forced to continue his lessons by letter. Apparently he was already resolved to combine these letters into a book, which appeared only much later, when he was already back in St. Petersburg (E 343, 344, 417 *Lettres à une princesse d'Allemagne sur divers sujets de physique et de philosophie Tome première A Saint Pétersbourg de l'imprimerie de l'académie impériale des sciences*. 1768, 1769, 1770).

Reading this book is a pleasure even today and one cannot but admire the clarity and the ease with which Euler explains very difficult matters, like the constitution of the solar system according to Newton or the six possibilities to measure longitude at sea that were in use or under examination in Euler's days. We note in passing that Euler contributed substantially to the eventual solution of the problem and received a (small) gratification from the English parliament that had offered a sum of up to £100,000 for any method that would allow the measurement of longitude with enough precision; larger portions of this money were allocated to the clockmaker Thomas Harris and to the mathematician and geographer Tobias Mayer who, in turn, had made substantial use of Euler's theory in compiling his lunar tables. Other remarkable parts of Euler's letters concern an exposition of elementary logic, where we meet what is now known as *Venn diagrams*, Euler's own theory of light and sound, which to some extent predated the optical theories of the 19th century, and his review of the major philosophical positions of the 18th century and their origins. That Euler was understandable when writing about such complicated matters is proved best by the fact that these letters were among the most economically successful books of the 18th century and that they have been reprinted and translated many times in almost 250 years that have passed since their first appearance; even a modern reader will enjoy Euler's remarkable clarity of exposition even if some parts have inevitably become obsolete with time.

Another project of pedagogical as well as scientific nature concerned an atlas designed for the schoolchildren in Prussia. This enterprise, initiated by the king and ordered by the president of the academy showed Euler in all his capacities in an exemplary way. As we would expect, Maupertuis entrusted Euler with the details of the project, which comprised the selection, construction and design of the maps to be shown, the selection, contracting and paying of the craftsmen who should work on the project, and finally the printing and the distribution of the completed work. Scientifically, the resulting book showed a few innovations, like unusual projections or careful depictions of the lines of magnetic aberration.

For the practical use of schoolchildren a much smaller format was chosen differing greatly from the usual atlas formats, an example setting a new standard quickly. The book appeared in 1753, followed by a second edition in 1760.

The final masterpiece in Euler's educational work is the *Vollständige Anleitung zur Algebra von Hrn. Leonhard Euler. 1. Theil. Von den verschiedenen Rechnungs-Arten, Verhältnissen und Proportionen*. St. Petersburg, gedruckt bey der Kays. Acad. der Wissenschaften 1770 (E 387, 388). Euler's pedagogical fame is underlined by the well-known story (or maybe legend) that the servant to whom Euler, then completely blind, dictated the manuscript in St. Petersburg was afterwards quite competent in algebra although he had never experienced any formal education. Even though the book was produced in St. Petersburg, there is little doubt that the main body of the material had already been outlined in Berlin.

Leonhard Euler was an exceptional human being and a singular scientist and mathematician, to be compared only to Archimedes, Newton, and Gauss. His habit of relentless work, carried out with the greatest care and indefatigable energy, was based on the firm belief that the world can be understood and changed for the better by applying the scientific method of rational explanation. He applied his seemingly unlimited intellectual powers to all questions he was confronted with, always looking for new concepts and fruitful ideas even before he had a definite method to handle them. His only major limitation arose from his religious faith, which was at the same time the major source of his strength, deeply rooted in his Swiss origin and his Calvinist family background.



*Jochen Brüning [bruening@mathematik.hu-berlin.de] (born 1947) was a mathematics professor in München, Duisburg and Augsburg. Since 1995, he has been the Chair for Mathematical Analysis at Humboldt-Universität zu Berlin. From 1990 to 1995 he was acting director of the Institute for European Cultural History (Augsburg). Since 1999, he has been acting director of the Hermann von Helmholtz-Zentrum für Kulturtechnik (HU Berlin) and since 2002 he has been a member of the Berlin-Brandenburgische Akademie der Wissenschaften. His research and publications are in cultural history and mathematics (especially in geometric analysis and spectral theory).*

# An Interview with Beno Eckmann

Conducted by Martin Raussen (Aalborg, Denmark) and Alain Valette (Neuchâtel, Switzerland) in Zurich on 10 January 2007.

## Education

**Professor Eckmann, you were born on 31 March 1917 in Bern, Switzerland, and you are approaching your 90<sup>th</sup> birthday now. Could you please tell us a bit about your school education, in particular who and what aroused your interest in mathematics?**

I went to school in Bern. I will mainly talk about high school, which is called “gymnasium”. I did the classical gymnasium – that means with Greek and Latin and languages. Everything was very good. Except mathematics; mathematics was very weak! I don’t regret that I studied Greek and Latin. And I still know Latin well.

I really don’t know why I decided to study mathematics. It is not that I no longer remember. I just don’t know! I was thinking about German languages or other languages, or something else – all kind of things! All of a sudden, I said: “I want to study mathematics” – here in Zurich, at the ETH (Swiss Federal Institute of Technology). Someone told me: “Don’t study mathematics. It’s a very old science. Everything is known. There’s nothing to get interested in.” Nevertheless, I went to Zurich and studied maths!

**How old were you when you started?**

Eighteen years.

**What was your student time like in Zurich and who were your most important teachers and supervisors?**

In the first year we had *Michel Plancherel* (1885–1967), *Ferdinand Gonseth* (1890–1975) and as a supervising assistant *Eduard Stiefel* (1909–1978). Plancherel was very old-fashioned, extremely old-fashioned; but in fact he was not bad! Since I was not really properly prepared, linear algebra and analysis were quite difficult for me. However, everything we learned was a revelation and I realised that mathematics was indeed something I had expected in my dreams.

The second and later years brought even more interesting teachers: *Heinz Hopf* (1894–1971), *George Polya* (1887–1985) and *Wolfgang Pauli* (1900–1958). As we understood, Hopf was working in a new field: algebraic topology, a higher type of geometry. I decided early that later on, I would try to work with him. Hopf was very modern, he taught in the style of van der Waerden’s “Moderne Algebra” or later Bourbaki. In fact, at a very early stage, I started to read *B.L. van der Waerdens’* book, “Modern Algebra” (later called “Algebra”). Mathematical objects were sets provided with additional structure fulfilling certain axioms. This was exactly what made the definitions of Hopf very clear and transparent (groups, spaces, etc.).

Polya was a very good teacher. But he was always far too slow in the beginning, and in the end the courses



Beno Eckmann during the interview (photos: Indira Lara Chatterji)

were too difficult. Moreover, his definitions were often not that clear. His books with Szegö were very interesting. One could learn a lot from the problems when he followed them chapter by chapter.

As for Pauli, he gave a course in theoretical physics. Even though I was not really involved in physics, I realised that in his thinking all types of mathematics were involved – we had the possibility to get acquainted with many highly interesting aspects.

**How many students were you altogether at the time?**

Six students. We were twelve in the whole group: six mathematicians and six physicists. We were practically always together; there was not much difference between us except that the physicists had to go to the laboratories more often.

**You graduated from the ETH at the dawn of World War II.**

Yes, I got the diploma in 1939; this corresponds to a Masters Thesis today. I did my diploma thesis in topology under Hopf’s supervision. He was really nice and a good man.

**Please tell us about your doctoral thesis work!**

After the diploma, I became an assistant to Professor *Walter Saxer* (1896–1974). There were not many assistants at the time because there were not many engineering students who needed assistants. Saxer was an analyst, not worldwide renowned but a good professor; he needed



**Beno Eckmann with interviewers Alain Valette (left) and Martin Raussen (right) on top of the ETH building**

assistants because he became rector at the ETH. As his assistant I replaced him at times and taught the problem sessions for him. That was a very good training.

Simultaneously I could start working on my PhD with Hopf. He asked: “Do you want to work on something else?” But I started immediately on the theme he gave me, which was homotopy groups. Nobody else had worked on homotopy groups, except *Witold Hurewicz* (1904–1956) in his famous and absolutely wonderful notes.

## Career

### ***Having finished your thesis, you were appointed to Lausanne.***

Indeed, right after the PhD, in 1942. While I was in Lausanne, I remained lecturing at the ETH. At that time, I concentrated on combinatorial problems; I do not know why! At Lausanne, I became acquainted with *Georges de Rham* (1903–1990). He lived in Lausanne and he was a professor at both Lausanne and Geneva. My main mathematical contacts at the time were with him and with Hopf.

### ***It was wartime in Europe and you had to serve military service at that time. What did you have to do?***

Of course, I went to the Army, serving in the mountain artillery. We had to stay in the mountains, normally in summer, for two weeks at a time. Then I could go back for two weeks to give all my lectures in Lausanne, and so on.

### ***But there was no communication with abroad during the war?***

Very little. There was some before France was completely occupied. There was the “free zone” in the south. *Charles Ehresmann* (1905–1979) was in the free zone. He came to Switzerland; we had vacations together.

### ***How did this situation change after the war?***

In 1947, I went to the Institute for Advanced Studies (IAS) in Princeton for an academic year. I had to get myself to learn English at first, since I was supposed to give lectures in English, like everybody! Very few of my colleagues had

a good knowledge of English; it was not part of the school curriculum everywhere. It was particularly important for my mathematical development that I had the opportunity to meet people like *Solomon Lefschetz* (1884–1972) and *Norman Steenrod* (1910–1971) at the IAS.

One year after that year in Princeton, in 1948, I was appointed at ETH. Soon after, Robert Oppenheimer (1904–1967) became the director of IAS. He invited me to spend another year at IAS, from 1951 to 1952. Again, I met interesting people, among them *Raoul Bott* (1926–2005) who was then a beginner with fascinating ideas. I had the possibility to discuss many different topics with *Albert Einstein* (1879–1955); he was happy to talk German and to remember his old experiences from Switzerland.

### ***You travelled a lot to the United States and to other countries.***

I went regularly to the US. Not for the full academic year but for summer vacation or shorter periods. MSRI at Berkeley was established and I went there when it was still very young and talked a great deal to *Shiing-Chen Chern* (1911–2004). He explained that they planned to have a specific topic for every year and they would invite people for that year. But it never worked that way: people would come for some period and then they would perhaps come two years later, and so on.

## Scientific work

### ***Under the influence of Heinz Hopf, you started to work in homotopy theory at a time when algebraic topology was hardly established. Please give us some reminiscences on the development. You must be one of the few left who can witness that algebraic topology has not always been associated with commutative diagrams, exact sequences, spectral sequences and so on.***

Yes, indeed – exact sequences, commutative diagrams. When I wrote my first paper they did not exist at all. Not even a map was denoted as we do it today, with an arrow from its domain to its codomain. It’s unbelievable! It was much more difficult to express things and to compute the exactness of a sequence. At each stage you had to show explicitly what you needed. And then, as soon as maps were denoted just by two letters with an arrow in-between and with this a suitable notation for exact sequences and diagrams, everything became simple and clear, and you could use them for clear statements and easy proofs. So many things that we used a lot of energy on in the past are almost obvious today!

### ***And then you had to bring in homological algebra...***

You see, if a topological space  $X$  is acyclic and has fundamental group  $G$ , then it follows from Hurewicz theory that the homology of  $X$  depends on  $G$  alone. Thus we were looking for an algebraic description of the homology depending only on  $G$ . It makes use of the group algebra of  $G$ , and thus the homology of a group algebra was introduced. These were problems that many people were

dealing with but it seems not to be widely known that Heinz Hopf was the first person to construct a free resolution over a group ring. People don't know that; they talk about *Eilenberg-McLane*<sup>1</sup> but it was Heinz Hopf who invented free resolutions. He also phrased precisely what it means that two free resolutions are equivalent. I carried this line of thought further on.

***Let us talk about your many other contributions to mathematics. Apart from algebraic topology, your name is associated with results in differential geometry, group theory and more recently  $L^2$ -invariants, at the boundary between topology and analysis. How would you describe the common thread through your work?***

Topology, in the spirit of Hopf, was always to be applied to geometry. That was the idea. It was not just something abstract. One of the geometries was differential geometry, manifolds. So, at an early stage I went through complex manifolds, Kähler<sup>2</sup> manifolds, with my student *Heinrich Guggenheimer*. We even created the name Kähler manifold!

***Ah, that was your invention!***

Indeed. The reason was that we used the operator of Kähler on differential forms. Of course we used the book by *William Hodge* (1903–1975). As I explained, the topology of the classifying space of a group depends only on the group. So at the same time, we had to develop the formalism to work directly with the homology of groups, and then the homology of algebras because groups lead to group algebras; so we went to algebras. Then I went on to dualize every map, considering a map in the other direction as well. From this point of view, one obtains new theorems. Pursuing this direction further on, I got interested in groups by themselves: in applying geometry to groups, topology to groups. And this then led to Poincaré duality, Poincaré duality groups and duality groups. It is a much more general setting that I developed in collaboration with *Robert Bieri*. Together with many other people and after a long development I could prove that a Poincaré duality group of cohomological dimension 2 is the group of a Riemann surface. That was actually a conjecture of *Jean-Pierre Serre*. “You have to prove it!” he had always insisted.

In my earlier papers in topology, I had used cell complexes, chains (which are linear combinations of cells) and harmonic chains (which are cycles and cocycles at the same time). There seemed to be something hidden, which is typical for operator analysis. It was Jean-Pierre Serre who always insisted: “There must be something much deeper!” I did not know what it was for a long time.

Finally  $L^2$ -theory came up, with idealized chains. Now you can have harmonic chains inside that space of  $L^2$ -chains. And then you get so many things from earlier considerations that guided me through  $L^2$ -theory: topol-

<sup>1</sup> Samuel Eilenberg (1913–1998), Saunders Mac Lane (1909–2005).

<sup>2</sup> Erich Kähler (1906–2000).

<sup>3</sup> Held 11–12 April 2007 at the ETH Zurich

## Feier zum 90. Geburtstag von Prof. Beno Eckmann

Departement Mathematik, Forschungsinstitut für Mathematik

ETH Hauptgebäude, Auditorium Maximum, HG F 30

Mittwoch, 11. April 2007

15:00 Uhr (bis ca. 16:30 Uhr)

Eröffnung durch den Rektor der ETH Zürich, Professor Konrad Osterwalder. Anschliessend:

Eine Collage aus Literatur, Musik und Mathematik unter Mitwirkung von Peter Arens (Sprecher) und Mitgliedern der Akademischen Orchester (Leitung: François Theis).

Donnerstag, 12. April 2007

Mathematische Vorträge

10:00 Uhr

Wolfgang Lück, Universität Münster  
“The K-theoretic Farrell-Jones conjecture for hyperbolic groups”

11:30 Uhr

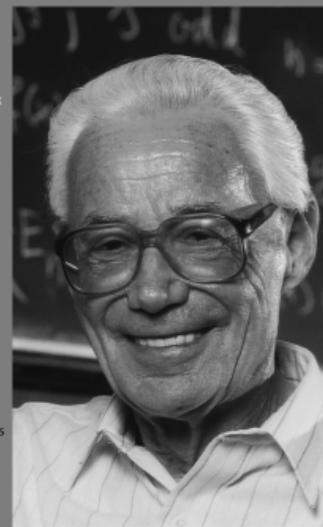
John Milnor, SUNY at Stony Brook  
“50 years ago: Topology in the 50's and 60's”

14:30 Uhr

Nicolas Monod, Université de Genève  
“A dialogue between the cohomology of groups and their geometry”

16:00 Uhr

Don Zagier, MPI Bonn / Collège de France  
“From topology to number theory to physics”



Der Anlass wird in grosszügiger Weise unterstützt durch die Bank Julius Bär.

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ogy with Hilbert spaces instead of vector spaces, operators – these were the things I worked on, until I more or less stopped recently. Well, maybe not quite ... I can still read, for example I read what *Wolfgang Lück* has done. You see, when Lück was very young, I got a paper from him where he proved something I had just announced also having proved.

***There will be a meeting<sup>3</sup> next April for your 90th birthday. Clearly you are still active. What is the driving force that pushes you to continue doing mathematics?***

The same force that was there at the beginning; because I like it. I like it and I find it fascinating. I try to follow a little bit of what the young people are doing, to understand a little bit of what directions they go and how they use the old stuff.

***Coming back to your own contributions: is there one single result that you view as your most important?***

One single result - that is difficult! But if I would single out something, it is probably what I did in the beginning. It is so elementary today that it belongs to the first semester of topology: homotopy groups, the exact sequence of a fibration, calculating the homotopy groups for the orthogonal groups and so on. There was nothing like that before! And then I used the same homotopy methods to prove that on a sphere of dimension  $4k+1$ , you can only have one tangent unit field, not two that are linearly independent. It was the beginning of the whole theory of

vector fields on spheres, which was developed by others later on. It was very difficult in higher dimensions until the famous papers by *John Milnor* and *Michel Kervaire* (1927-2007). Milnor was not my student but I took him from Princeton to Zurich, when he was a student. Kervaire was my student but he wrote his thesis with Hopf.

At this stage, I should at least mention various geometric and algebraic techniques introduced in algebraic topology during my most active years, like spectral sequences, cohomology operations, general homology theories and so on. But in this conversation, I think we should concentrate on comparatively elementary aspects.

Another direction was Eckmann-Hilton duality, which went on for many years. There was even a section in *Mathematical Reviews* under that name, with many contributions.

Still another important area is Poincaré duality for groups, invented by Robert Bieri and myself. They behave like manifolds: homology, cohomology, you see, in complementary dimensions, but with another dualizing module. Many groups that are interesting in algebraic geometry, group theory or other areas are such duality groups. I had a draft of a paper about these topics with me in Princeton and Jean-Pierre Serre was there at the same time. He could not come to my lecture but the day after he wrote to me that he wanted to publish it in the *Inventiones*! It was not ready to be published yet – two months were still necessary.

I don't know what is very important, what is less important. What you do is always interesting; it is more difficult to judge importance! And then, I had so many students! I gave many ideas and interests to my PhD students and they then published work that I could not have thought about myself.

## Students

***You mentioned your many PhD students; indeed according to the Mathematics Genealogy Project, you had 60 PhD students and more than 600 descendants. How did you manage?***

That is a good question! I don't know! The first of the students was an assistant who wanted to write a PhD with me. I told him to write down, in one or two weeks, what he really wanted to do, an abstract. I told him to read a little bit and after a long time, maybe half a year or even more, he should come and tell me what he really wanted to do. And once he had his topic, we would see each other, for one or two hours, and discuss things in detail, and start to write down first results. Then I had the next student, and the next, and the next ... more and more.

***Is there a particular reason why you attracted so many students?***

I don't know! I mean, it's probably because they liked my style. In fact, I gave many lectures. At that time we gave more lectures than today. To teach, you must make it very clear in your mind what you want to lecture about, how to present it and what to say first, and then you head towards a result, a theorem.

My lecturing style is very old-fashioned, and probably young people do not agree with me; that's normal! When lecturing, I always used blackboard and chalk, developing the ideas gradually further and further, saying exactly where I wanted to go. Sometimes I had to lecture with overhead projectors. But then I wrote maybe four or five lines and I would cover all the rest, except the one line that I would have written on the blackboard. So it's really old-fashioned but I know there are many mathematicians who still organise their lectures in that way. My students seemed to like it because they followed my courses. I did not allow any script, mimeographed notes or anything. I said: "You have to think here and I go with you step by step." When the course is finished, you must get the book and read it, and you will find other similar things.

Today many students just use the mimeographed notes. They have their colour pens and they underline this and that; I don't think it's the same. But it works as well! Today also, my colleagues find good students and they have good PhDs. It's just different!

## Collaboration

***Among your many collaborators, Peter Hilton clearly plays a privileged role. Can you say some words about the way you did your joint work?***

I met *Peter Hilton* when he was a graduate student with *Henry Whitehead* (1904–1960) at Oxford. I went in 1947 from the IAS to Oxford to meet Whitehead. Peter was very shy then and he asked me whether he could contact me at Zurich later on. I agreed and so he did. In 1955 he came to Zurich and he stayed here for the whole year. I could guide him a little bit and explain many things to him about homological algebra, and then he got more and more into that idea of dualizing lots of our mathematics. And this became Eckmann-Hilton duality, which was quite well followed for a while: in geometry, topology and algebra. When he left Zurich, we continued of course by correspondence. Sometimes I went to England, or he came to Zurich and that was alright. After a long time, he changed direction and I always had the wish to do more concrete mathematics, more geometry, more group theory; so we took different routes. Our minds were a bit different and that was alright: we remained very good friends but we did not collaborate after that.

***In your long career, you met quite a number of famous mathematicians. Is there anybody whom you would like to mention in terms of influence, or friendship?***

I already mentioned Peter Hilton and Robert Bieri. Then there is *Guido Mislin*; we have joint papers on Chern classes of group representations. This work again combines topology with group theory and with number theory because the Chern class gives really interesting limitations related to Bernoulli numbers and so on; it's an interesting topic!

These were collaborators with whom I wrote joint papers. Georges de Rham was very important for me in Lausanne, and also afterwards; I went to see him from time to time. But then I got of course a lot of very lucky influence very early on from *Henri Cartan*<sup>4</sup>, who is already more than one hundred years old, and later on from Jean-Pierre Serre. Actually Jean-Pierre Serre is younger than I am; he has followed my first papers very carefully and that was really an interesting advantage. I would always have wonderful contact with him; he asked important questions. Unfortunately, I could not follow him any longer when he went into number theory.

## Forschungsinstitut für Mathematik

***I would like to ask you about FIM, the Forschungsinstitut für Mathematik, which you founded at the ETH, and of which you were the first director for 20 years. What was the prime idea for creating this institute? How did it develop? Are you satisfied with its present activities?***

Indeed, I founded it because I thought it was necessary to have an organization to welcome visitors and to do everything so that they can work here together with faculty members. The institute was not to have permanent members, except for the director who had to be one of the faculty members.

Something like that did not exist previously. The Institute for Advanced Study was separated from Princeton University and was not linked to it. It was essential for me that our institute was to be linked with the department here so that every member of the department could invite visitors to work with or to learn from. And the institute should care for these visitors in every respect. That was an idea that people found strange at the time and many did not agree. I went with this idea to the ETH president *Hans Pallmann*. I argued that we needed such an institute because otherwise our professors do not have enough interaction with the world outside. He said: "We do not have the money, but you get it! Just start right away!" Soon afterwards, I could invite *K. Chandrasekharan* and *Lipman Bers* (1914–1993). Many others followed.

I remained the director for twenty years. At the end we had a huge number of visitors. My successor was *Jürgen Moser* (1928–1999). He had a different style but he worked towards the same objectives. He was followed by *Alain-Sol Sznitman* and the present director is *Marc Burger*. I think it will continue in the same spirit, although many new features have been added, for example the Nachdiplomvorlesungen (post-diploma lectures): we invite people to the institute to give very high level graduate courses<sup>5</sup>.

We have two or three such courses every semester. Nowadays, they organise workshops as well. All this

<sup>4</sup> Born in 1904.

<sup>5</sup> In 1999, one of the interviewers, A.V., gave such a Nachdiplomvorlesung on the Baum-Connes conjecture. This led to a very stimulating interaction with Profs Eckmann and Mislin.

changed the size of the institute, of course; it has grown. But the institute still takes care of apartments for visitors and for their offices within the ETH.

The director Marc Burger has an excellent knowledge of mathematics and of mathematicians all over the world, so he attracts good visitors. Moreover, with all our later appointments of high level mathematicians to the department, people expressed interest in the institute during negotiations: "Can I invite people to work with my PhD-students?" It is quite important and I am very pleased.

Nowadays, almost every university has such an institute but at that time, in 1964, there was not a single one – nowhere!

## Publishing mathematics

***Can we talk about your involvement in the publishing of mathematics? For many years, you have been an editor of the series Grundlehren der Mathematischen Wissenschaften and also of Lecture Notes in Mathematics. I was asked to join the managing board of the Grundlehren because they needed people. Wolfgang Schmidt who was there wanted to retire and van der Waerden said that he no longer wanted to do that much.***

***At what time did you join Grundlehren?***

That was in 1966. Every volume was refereed before being accepted and this was heavy work.

*Konrad Springer*, 4<sup>th</sup> generation of Springer, was a biologist who studied in Zurich and I talked with him about the publishing of lecture notes. The institute published lots of lecture notes. Who could be a commercial publisher of such notes? Springer-Verlag, of course! I talked to him and said: "That is something I have wished for a long time, so if you help me..." He tried to convince his father and they finally liked the idea. They made photocopies of the typescript and published it! You could send the typescript directly to Springer who provided the copies. It was immediately a great success. Since I could not take both series myself, I asked *Albrecht Dold* to take over the Lecture Notes; the first volume is under him. But he did not want to do the work alone. He argued that I knew all the Springer people and convinced me that we should both be editors.

Over the years it became easier, with computers and the internet; it certainly takes less time. Now they receive a computer-processed manuscript, only one or two referees need to accept it and it runs very quickly. The contact with Springer was always very interesting; we had long discussions over many years.

## IMU

***You were also very much involved in national and international mathematical societies. You have been the President of the Swiss Mathematical Society for a two year period...***

That was almost compulsory; I had to do that...

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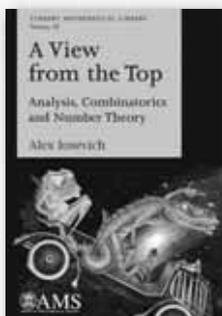
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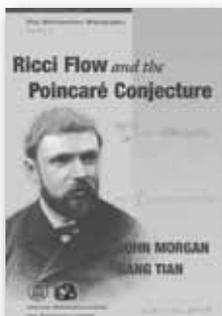
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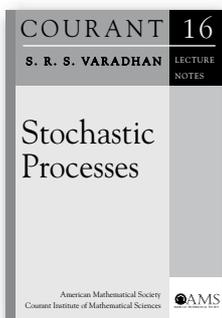
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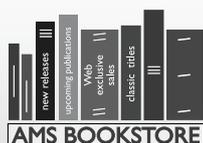
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*...and secretary of the IMU...*

Well, that was not compulsory. Heinz Hopf was IMU's president and he asked me to become the secretary of the international union. I said: "Yes, if I can have a helping secretary, because I do not have a secretary here!" This is how I got a secretary to do the typing and mailing for me. It was a very interesting period, 1956–1961.

*What were the important issues at the time, during the cold war?*

Two important goals were achieved:

Many countries (some of them very large) that had not adhered to the IMU became members. One can imagine that many difficulties had to be overcome, difficulties of personal, political and financial character. This was heavy but gratifying work for the secretary.

The International Congress of Mathematicians had to become a task of the IMU. The last congress organised solely by a single country was the Congress in Edinburgh in 1958, organised by the UK. With the increasing number of

research areas and of participants, this became too heavy a burden for a national mathematical society. The local organisation is of course still taken care of by the organising country but the scientific plans are made by the union.

**Private Interests**

What are your other main private interests – apart from mathematics?

Through my entire mathematical life I was always able to find time for other activities (sometimes combined with mathematical work): I spent interesting periods with my wife and my big family, on weekends, during vacations, with music and art, and with school and student problems. Love and happiness are important inside and outside mathematics.

*Thank you very much for this interesting conversation!*

# A Survey of ICMI Activities

Maria G. (Mariolina) Bartolini Bussi (Modena, Italy)



*Maria G. (Mariolina) Bartolini Bussi is a member of the editorial board of this newsletter and serves as a member of the executive committee of the International Commission on Mathematical Instruction from 01 January 2007 until 31 December 2009. In this column, she will periodically provide news from the ICMI.*

The information below is taken from the official website of the eleventh International Congress on Mathematical Education (ICME11), which is to be held in Monterrey, Mexico, 6–13 July 2008. The interested reader is welcome to visit the website (<http://icme11.org/>), where the second announcement will appear over the next few weeks. Below is a summary of the session types given. Most activities (**topic study groups, discussion groups, workshops, sharing experiences groups, a poster exhibition and round tables**) welcome contributors. Each activity will have its own deadline, but not later than 20 January 2008.

The organizers expect to gather between 3000 and 4000 professionals from 100 countries in the mathematics education area, including researchers, educators and teachers.

The International Congress on Mathematical Education (ICME) aims to:

- Show what is happening in mathematics education worldwide, in terms of research as well as teaching practices.
- Inform about the problems of mathematics education around the world.
- Learn and benefit from recent advances in mathematics as a discipline.

ICME consists of several different *session types*.

**Plenary Activities**

Lectures or panels on themes of current actuality and relevance to the practice of the international community of mathematics educators.

**National Presentations**

It is customary to select a small number of countries so that the international mathematics community may gain a closer knowledge on the state and trends of mathematics education in those countries.

National representatives of those countries are asked to make the presentations.

### Survey Teams

ICME 11 survey teams, first created in ICME 10, are groups entrusted to carry out a survey of the latest developments regarding a certain theme or issue of mathematics education. Emphasis is placed on pinpointing new knowledge, new perspectives and emerging challenges. The teams' work will be presented in a lecture at the congress. Survey teams ensure we are made aware of developments in the field addressed since the time of the previous ICME, thus giving continuity to the ICME.

- ST 1: Recruitment, entrance and retention of students to university mathematics studies in different countries.
- ST 2: Challenges to mathematics education research faced by developing countries.
- ST 3: The impact of research findings in mathematics education on students' learning of mathematics.
- ST 4: Representations of mathematical concepts, objects and processes in mathematics teaching and learning.
- ST 5: Mathematics education in multicultural and multilingual environments.
- ST 6: Societal challenges to mathematics education in different countries.
- ST 7: The notion and role of theory in mathematics education research.

### Regular Lectures

These lectures will be presented by experts invited by the International Program Committee (IPC).

### Topic Study Groups (TSGs)

The purpose of a TSG is to gather participants interested in a certain topic of mathematics education. The organising team of each TSG will review, select and organise contributions, some by invitation and some submitted by interested participants, that account for advances, new trends and important work done in the last few years on the topic the TSG addresses. The contribution selected will be made available in one or more of the following forms: as a download from the web page of the TSG on the ICME11 web site; as a printed hand-out prior to a TSG session during the congress; or by oral presentation during any of the four TSG sessions. There will be some discussion during the sessions but emphasis is on presentation (in contrast to discussion groups).

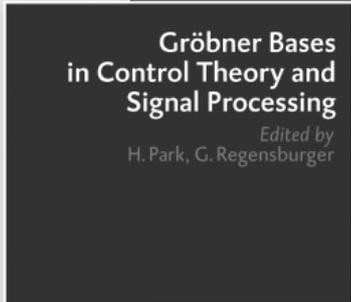
- TSG 1: New developments and trends in mathematics education at preschool level.
- TSG 2: New developments and trends in mathematics education at primary level.
- TSG 3: New developments and trends in mathematics education at lower secondary level.
- TSG 4: New developments and trends in mathematics education at upper secondary level.
- TSG 5: New developments and trends in mathematics education at tertiary level.

- TSG 6: Activities and programs for gifted students.
- TSG 7: Activities and programs for students with special needs.
- TSG 8: Adult mathematics education.
- TSG 9: Mathematics education in and for work.
- TSG 10: Research and development in the teaching and learning of number systems and arithmetic.
- TSG 11: Research and development in the teaching and learning of algebra.
- TSG 12: Research and development in the teaching and learning of geometry.
- TSG 13: Research and development in the teaching and learning of probability.
- TSG 14: Research and development in the teaching and learning of statistics.
- TSG 15: Research and development in the teaching and learning of discrete mathematics.
- TSG 16: Research and development in the teaching and learning of calculus.
- TSG 17: Research and development in the teaching and learning of advanced mathematical topics.
- TSG 18: Reasoning, proof and proving in mathematics education.
- TSG 19: Research and development in problem solving in mathematics education.
- TSG 20: Visualization in the teaching and learning of mathematics.
- TSG 21: Mathematical applications and modeling in the teaching and learning of mathematics.
- TSG 22: New technologies in the teaching and learning of mathematics.
- TSG 23: The role of history of mathematics in mathematics education.
- TSG 24: Research on classroom practice.
- TSG 25: The role of mathematics in the overall curriculum.
- TSG 26: Learning and cognition in mathematics - students' formation of mathematical conceptions, notions, strategies and beliefs.
- TSG 27: Mathematical knowledge for teaching.
- TSG 28: In-service education, professional life and development of mathematics teachers.
- TSG 29: The pre-service mathematical education of teachers.
- TSG 30: Students' motivation and attitudes towards mathematics and its teaching.
- TSG 31: Language and communication in mathematics education.
- TSG 32: Gender and mathematics education.
- TSG 33: Mathematics education in a multilingual and multicultural environment.
- TSG 34: Research and development in task design and analysis.
- TSG 35: Research on mathematics curriculum development.
- TSG 36: Research and development in assessment and testing in mathematics education.
- TSG 37: New trends in mathematics education research.
- TSG 38: The history of the teaching and learning of mathematics.

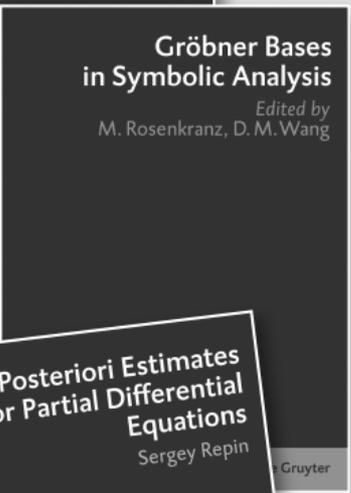
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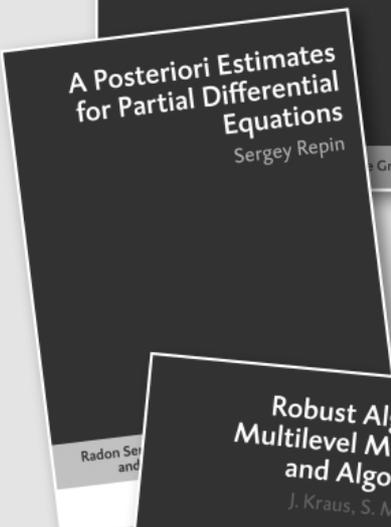
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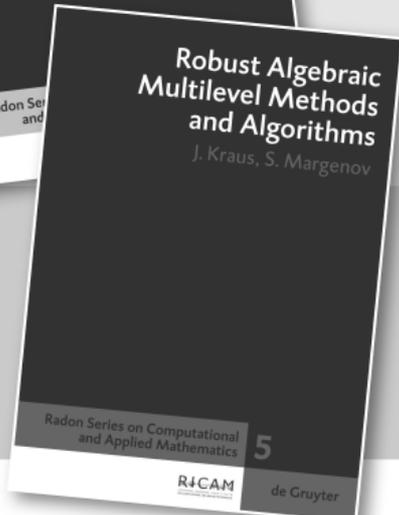
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### Discussion Groups (DGs)

DGs are meant to gather congress participants who wish to actively discuss, in a genuinely interactive way, certain challenging or controversial issues and dilemmas of a substantial, non-rhetorical nature pertaining to the theme of the DG. During the year from now up to the congress, the discussion group will post contributions on their page on the ICME11 web site that define, limit, and/or present basic premises, viewpoints, theoretical considerations, research findings and facts that should be accounted for if a fruitful discussion is to be attained.

- DG 1: Curriculum reform – movements, processes and policies.
- DG 2: The relationship between research and practice in mathematics education.
- DG 3: Mathematics education – for what and why?
- DG 4: Re-conceptualizing the mathematics curriculum.
- DG 5: The role of philosophy in mathematics education.
- DG 6: The nature and roles of international cooperation in mathematics education.
- DG 7: Dilemmas and controversies in the education of mathematics teachers.
- DG 8: The role of mathematics in access to tertiary education.
- DG 9: Promoting creativity for all students in mathematics education.
- DG 10: Public perceptions and understanding of mathematics and mathematics education.
- DG 11: Quality and relevance in mathematics education research.
- DG 12: Rethinking doctoral programs in mathematics education.
- DG 13: Challenges posed by different perspectives, positions and approaches in mathematics education research.
- DG 14: International comparisons in mathematics education.
- DG 15: The shaping of mathematics education through assessment and testing.
- DG 16: The evaluation of mathematics teachers and curricula within educational systems.
- DG 17: The changing nature and roles of mathematics textbooks - form, use, access.
- DG 18: The role of ethno-mathematics in mathematics education.
- DG 19: The role of mathematical competitions and other challenging contexts in the teaching and learning of mathematics.
- DG 20: Current problems and challenges in primary mathematics education.

- DG 21: Current problems and challenges in lower secondary mathematics education.
- DG 22: Current problems and challenges in upper secondary mathematics education.
- DG 23: Current problems and challenges in non-university tertiary mathematics education.
- DG 24: Current problems and challenges in university mathematics education.
- DG 25: Current problems and challenges in distance teaching and learning.
- DG 26: Current problems and challenges in the conditions and practice of mathematics teachers.
- DG 27: How is technology challenging us to re-think the fundamentals of mathematics education?
- DG 28: The role of professional associations in mathematics education - locally, regionally and globally.

### Workshops

Workshops will provide hands-on experience to delegates wishing to learn new skills. These workshops are created via proposals to the IPC.

### Sharing Experiences Groups (SEGs)

SEGs are small and intimate groups designed to exchange and discuss experiences pertaining to research and/or teaching. SEGs are formed via proposals to the IPC.

### Poster Exhibition and Round Tables

Participants are invited to submit proposals for the display and presentation of posters. Round tables will address groups of posters developed on the same theme.

### Latin America: Perspectives on development through collaboration

Concurrent with other ICME11 activities, we will organize meetings that will address the issue of Latin American development and collaboration. In spite of their differences, Latin American countries share cultural roots, ethnic diversity and a sense of identity. We wish to provide a forum where participants will explore the possibilities, advantages and perils of development through collaboration, not only within Latin America but also with other regions.



*Mariolina Bartolini Bussi [bartolini@unimo.it] is the Newsletter Editor within Mathematics Education. A short biography can be found in issue 55, page 4.*

# Cyprus Mathematical Society



1983–2008, 25 years of contribution and development to mathematical science and education in Cyprus and Europe

Gregory Makrides (Nicosia, Cyprus)

Attempts to found the Cyprus Mathematical Society started long before 1983. However, in 1983 a small group of mathematicians took the initiative and at a meeting of the Higher Technical Institute of Cyprus decided to establish the Cyprus Mathematical Society. On 27 October 1983, the Cyprus Mathematical Society (CMS) was born. In the founding statement it was reported that “the objectives of CMS are the promotion and upgrading of mathematical science and education”. The CMS decided to use as its emblem the Cypriot Philosopher *Zenon of Kition* (333 BC–264 BC) from Citium of Cyprus.

The CMS began with limited funds. Characteristically, for initial international obligations we could not send a complete team because of lack of resources. Today, the CMS owns two offices, employs a secretary, offers scholarships and meets without particular difficulty its economic obligations, with an ultimate goal of establishing a large Mathematics Centre of Excellence and a library.

The first president of the CMS was the inspector of mathematics Mr Panagiotis Michael. The second president Mr Glafkos Antoniadis, who was also an inspector of mathematics, was elected in 1990, and the current president Dr Gregory Makrides, who is the Director of Research and International Relations at the University of Cyprus, was elected in 1998.

## Main activities of the Cyprus Mathematical Society

The activities of the Cyprus Mathematical Society have progressively increased over the years both in quantity and quality. In the presentation below we will focus on the end product, which is the comprehensive situation over the last decade.

### 1. Local competition

#### 1.1 National competition of mathematics

The Cyprus Mathematical Society organises many different regional and national mathematics competitions. There is one process of competitions that progressively converges to the selection of the national teams of Cyprus that participate in different International Olympiads. The sequence starts in November each year and continues until May, at which time the national teams for different International Olympiads are selected.

#### 1.2 Cyprus Mathematical Olympiad

On 08 January 2000, because the year 2000 was announced by UNESCO as the “year of mathematics”, the CMS organised a new type of competition, called the

“Cyprus Mathematical Olympiad (CMO)”, using multiple choice tests and which, for the first time in Cyprus history, was open to students from 4<sup>th</sup> Grade up to 12<sup>th</sup> Grade (ages 9 to 18). The attendance and participation of students exceeded expectations. Over 4000 students participated in this contest and because of this unexpected interest it was decided to make it an annual event. Since then the number of contestants has nearly reached 12000 students, which is about 15% of the national student body. The award ceremony for this Olympiad attracts more than 3000 people. The CMO as an activity appeared in the European Commission’s report in 2001, which reported five activities that took place in countries in Europe that contributed to the advancement of mathematics.

#### 1.3 *Mathimatiki Skytalodromia (Mathematical Relay Race)*

In 2007 the Cyprus Mathematical Society decided to celebrate the “Day of Learning” with the introduction of a new competition, which would include some gaming activity. This was a competition between schools and groups of students with the aim of promoting cooperative learning and team competition. The competition was trialled in 2007 in one district of Cyprus between 13 gymnasiums (grades 7 to 9). The national competition, a “Mathematics Relay Race” will run for the first time on 1 February 2008.

## 2. International competitions

### 2.1 *Balkan Mathematical Olympiad (BMO)*

In recent years, Cyprus has become active in hosting many international events relating to mathematics, with some of them initiated by the CMS. The CMS has organised four Balkan Mathematical Olympiads: the 5<sup>th</sup> in 1988, the 10<sup>th</sup> in 1993, the 15<sup>th</sup> in 1998 and the 23<sup>rd</sup> in 2006.

### 2.2 *Junior Balkan Mathematical Olympiads for students under the age of 15 years (JBMO)*

Since the first JBMO in 1997, the CMS has participated every year. It organised the 5th JBMO in 2001 in Cyprus, which was a great success. The 2007 JBMO was held in Shumen, Bulgaria. It is expected that Cyprus will organise the 2009 JBMO.

### 2.3 *International Mathematical Olympiad (IMO)*

From the first year of operation of the CMS in 1984, Cyprus was invited to the IMO and participated with a full team. Since then Cyprus, via the CMS, has participated in almost all Mathematical Olympiads. Our first participa-

tion was at the 25<sup>th</sup> IMO that was organised in Prague in 1984 and we have participated in all IMOs since including the 48<sup>th</sup> in 2007 in Hanoi, Vietnam. The CMS is making plans to host a future IMO in Cyprus.

*2.4 International Mathematical Olympiad – Primary*  
Since 2001, Cyprus has participated in a special Olympiad for primary school students. In 2007 we participated in an “International Youth Mathematics Contest” with a section called EMIC: Elementary Mathematics International Contest. This was hosted in Hong Kong in late July 2007.

*2.5 SEEMOUS: Mathematical Olympiad for University Students*

Through its active membership in the Mathematical Society of South Eastern Europe (MASSEE), for which the CMS holds one of the vice-president positions, CMS proposed the development of a mathematical Olympiad for first and second year university students. The proposal materialised as SEEMOUS (South Eastern European Mathematical Olympiad for University Students) with international participation. The first event SEEMOUS 2007 was hosted in Cyprus in March 2007. The results appear on [www.seemous.org](http://www.seemous.org).

### 3. Conferences

*3.1 Cyprus national conference on mathematical science and education*

This conference was initiated by the CMS and in 2008 we will organise the 10<sup>th</sup> national conference. This conference has grown in size to 350 participants in 2007 and includes sessions covering all levels of education.

*3.2 Mediterranean conferences on mathematics education*

This is another series of conferences that was established by the Cyprus Mathematical Society. The first and second took place in Cyprus, in 1997 and 2000 respectively, the third was hosted in Athens by the Hellenic Math Society in 2003, the fourth was hosted by the University of Palermo in Italy in 2005 and the fifth was hosted by the University of the Aegean in Greece in 2007. It is expected that the sixth will be hosted in Bulgaria in 2009. The conferences have grown with the participation of some 200 mathematicians from around the world.

*3.3 National student conference*

The first Student Conference in Mathematics was organised in February 2005 with the participation of about 90 students from grades 7 to 12 with 40 presentations given. The event has become very popular and in 2007 there were 350 students with 95 presentations given. Students present mathematical projects and studies of all kinds. The Cyprus Mathematical Society has decided to attempt the organisation of a European Student Conference in Mathematics in February 2009. The announcement of this conference is expected to be circulated throughout Europe in January 2008.



### 4. Seminars and training courses for adults

*4.1 Seminars*

The CMS organises European Grundtvig courses under the title, “Preparation of Full Proposals and Management of European Education Projects”. The courses appear on the Europa course database [ec.europa.eu/education/trainingdatabase](http://ec.europa.eu/education/trainingdatabase).

The CMS was also an associate partner in the well known MATHEU project, “Identification, Motivation and Support of Mathematical Talents in European Schools”. The project ran from 2003 to 2006 and now the CMS helps in the organisation of the in-service training courses for teachers. The website of the project is [www.matheu.eu](http://www.matheu.eu) and information on the courses can be found at [ec.europa.eu/education/trainingdatabase](http://ec.europa.eu/education/trainingdatabase).

### 5. Publications

5.1 One of the most popular student periodicals has been published by the Cyprus Mathematical Society since 1984. Its name is “**Mathematiko Vima**” and it is published annually. During the last two years the periodical has been divided into two volumes A and B by separating its content so that volume A is of interest to all students and volume B is more aimed at those who have a special interest in mathematics.

5.2 An important scientific step was taken by the Cyprus Mathematical Society when it decided to develop a scientific journal on mathematics education called the “**Mediterranean Journal for Research Mathematics Education**”. The publication began in 2003 and two issues are published per year. It is becoming well-known and is gaining an international reputation.



#### 5.1 Other publications of CMS include:

- The proceedings of all the annual national conferences in mathematics education.
- The proceedings of all the annual student conferences.
- The proceedings of the five Mediterranean conferences on mathematics education.
- "Mathematics for Competitions", published in English in 2006.

### 6. Other Activities

#### 6.1 Summer mathematics school

Since the summer of 1991, the CMS has been organising a summer mathematics school with the aim of increasing the interest of the student body in mathematics and to show them the fun that can be had while studying mathematics. The summer school accepts students from grades 7 up to 10 only. The summer school has grown from about 100 students in 1991 to 1200 in 2007 with applications of more than 2500.

#### 6.2 Prize to the best graduating university student in mathematics

The CMS has established an annual award of 850 Euro for the best graduating mathematics student from the University of Cyprus.

#### 6.3 Television education

One of the activities organised by the CMS in the past was a series of television programmes on mathematics offering lectures to help students prepare for the national university entrance examinations. This programme was sponsored by the national television station and it ran for two years, 1995 and 1996.

#### 6.4 National career day

The CMS participates every year in the career day, which is organised by the Association of Career Guidance teachers in Cyprus. We have our own booth and we distribute a leaflet explaining what kinds of jobs a mathematician can do and the importance of mathematics in all studies.

#### 6.5 The CMS knows how to honour friends and collaborators

The CMS rewards, on an annual basis, retiring mathematics teachers. We also honour government officials, especially from the Ministry of Education and Culture, who for years have been cooperating in supporting the activities of the society. We have also established the "Zenon Award", which is given to a Cypriot mathematician who has made a special contribution to the advancement of mathematics. This award has only been given once so far, to the Cypriot professor Dimitris Christodoulou at Princeton University, USA, for his contribution to mathematical science. Other awards include the honorary presidency offered to the first two presidents of the Cyprus Mathematical Society.

#### 6.6 Member of the European Mathematical Society

The Cyprus Mathematical Society is a member of the European Mathematical Society (EMS). Its president participates in the Education Committee of the EMS. Relations between the two societies are expected to grow as EMS has accepted an invitation to collaborate on the new European Student Conference on Mathematics as well as on summer mathematics schools.

It should be noted that the Cyprus Mathematical Society is a non-profit organisation, and has regular members, special members and reciprocity members. It is currently managed by a 15 person management council and its current mixed membership is close to 800.



*Dr Gregory Makrides [makrides.g@ucy.ac.cy] has been the president of the Cyprus Mathematical Society since 1998.*

*The address of the Cyprus Mathematical Society is 36 Stasinou street, Office 102, Strovolos 2003, Nicosia, Cyprus. Tel: +357-22378101. Fax: +357-22379122. www.cms.org.cy, cms@cms.org.cy*

# Personal column

Please send information on mathematical awards and deaths to the editor.

## Awards

**Claire Voisin** (CNRS Jussieu, Paris) has been awarded the 2007 Ruth Lyttle Satter prize of the American Mathematical Society “for her deep contributions to algebraic geometry”.

The Fermat Prize 2007 for Mathematics Research has been awarded to **Chandrashekar Khare** (Univ. of Utah) “for his proof in collaboration with Jean-Pierre Wintenberger of Serre’s modularity conjecture in number theory”.

The 2006 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, has been awarded to **Martin Grötschel** (Technical University of Berlin), **László Lovász** (Eötvös Loránd University, Budapest) and **Alexander Schrijver** (University of Amsterdam and the CWI) “for their fundamental ground-breaking work in combinatorial optimization”.

**Hô Hai Phùng** (University of Duisburg-Essen, Germany) has been awarded the von Kaven Prize in Mathematics “in recognition of his outstanding work on quantum groups”.

The 2007 Spring Prize of the Mathematical Society of Japan (MSJ) has been awarded to **Kenji Nakanishi** (Kyoto University) for his distinguished and fundamental contributions to the study of nonlinear dispersive equations.

The 2007 Seki-Takakazu Prize has been awarded to the **Institut des Hautes Études Scientifiques** (IHES, Bures-sur-Yvette, France) for its outstanding contribution to establishing strong relationships between mathematicians in Japan and France by offering invaluable research-exchange opportunities for the development of mathematics since 1958.

The 2007 MSJ Algebra Prize was awarded to **Eiichi Bannai** (Kyushu University) for his contribution to the study of algebraic combinatorics and to **Kouta Yoshioka** of Kobe University for his contribution to the theory of moduli spaces of vector bundles.

**Remco van der Hofstad** (Eindhoven University of Technology, Netherlands) has been awarded the 2007 Rollo Davidson Prize. Van der Hofstad was honoured for his work in probability and statistical mechanics.

**Mikołaj Bojańczyk** (Warsaw) was awarded the Kuratowski Prize.

**Łukasz Kosiński** (Kraków) was awarded the first Marcinkiewicz Prizes of the Polish Mathematical Society for students’ research papers.

**Steven Rudich** (Carnegie Mellon University) and **Alexander A. Razborov** (Steklov Mathematical Institute, Moscow) were

named recipients of the Gödel Prize of the Association for Computing Machinery (ACM). They were recognized for their work on the P versus NP problem.

**Bryan Birch** (University of Oxford) has been awarded the De Morgan Medal of the London Mathematical Society (LMS) in recognition of his influential contributions to modern number theory.

**Béla Bollobás** (University of Cambridge) has been awarded the Senior Whitehead Prize from the LMS for his fundamental contributions to almost every aspect of combinatorics.

**Michael Green** (University of Cambridge) received the Naylor Prize and Lectureship in Applied Mathematics in recognition of his founding work in superstring theory.

Four Whitehead Prizes were awarded to: **Nikolay Nikolov** (University of Oxford and Imperial College, London; group theory), **Oliver Riordan** (University of Cambridge; graph theory and combinatorics), **Ivan Smith** (University of Cambridge; symplectic topology) and **Catharina Stroppel** (University of Glasgow; representation theory).

**Paul Fearnhead** (Lancaster University, UK) has been awarded the 2007 Adams Prize from the University of Cambridge for major contributions to several areas of computational statistics and population genetics.

**Ian Stewart** (University of Warwick, UK) has been awarded the 2006 Premio Peano from the Associazione Subalpina Mathesis in Turin, Italy, for the Italian translation of his book Letters to a Young Mathematician.

## Deaths

We regret to announce the deaths of:

Graham R. Allan (UK, 9.8.2007)  
 Jakow Baris (Belarus, 26.7.2007)  
 Joachim Bergmann (Germany, 14.3.2007)  
 Harry Burkill (UK, 9.4.2007)  
 Fokko du Cloux (France, 10.11.2006)  
 Paul Joseph Cohen (UK, 23.3.2007)  
 Carl Geiger (Germany, 15.6.2007)  
 Gisbert Hasenjäger (Germany, 2.9.2006)  
 Anthony Horsley (UK, 26.5.2006)  
 Ahmet Hayri Körezlioğlu (Turkey, 26.6.2007)  
 Paulette Libermann (France, 10.7.2007)  
 Wladyslaw E. Lyantse (Ukraine, 29.3.2007)  
 Dietrich Morgenstern (Germany, 27.6.2007)  
 Gert H. Müller (Germany, 9.9.2006)  
 Wolfgang Müller (Germany, 19.12.2006)  
 Morris Newman (UK, 4.1.2007)  
 Mircea Puta (Romania, 26.7.2007)  
 Gareth Roberts (UK, 6.2.2007)  
 Felice L. Ronga (Switzerland, 22.5.2007)  
 Johann Schröder (Germany, 3.1.2007)  
 Atle Selberg (Norway, 6.8.2007)  
 John Todd (UK, 21.6.2007)  
 Klaus Wohlfahrt (Germany, 3.7.2007)

# Book review

Xavier Gràcia (Barcelona, Spain)

## Music and mathematics.

### From Pythagoras to fractals.

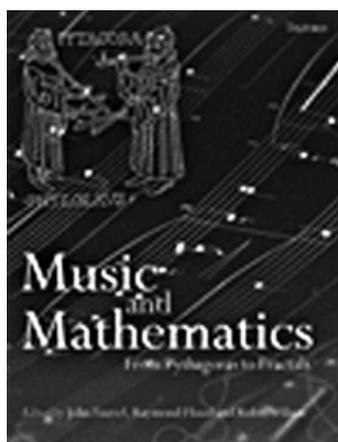
Edited by John Fauvel,

Raymond Flood and Robin Wilson.

200 pages, £18 (Paperback edition).

Oxford University Press, Oxford, 2003 and 2006.

ISBN: 0-19-851187-6 and 978-0-19-929893-8.



Leibniz described music as a secret exercise in arithmetic of the soul unaware of its act of counting. The relationship between music and mathematics is much older, having been in existence since at least the time of Pythagoras. Nevertheless, there is a need to remind people from both sides about this relationship. That seems to be the purpose of this

book, which aims to demonstrate and analyse “the continued vitality and vigour of the traditions arising from the ancient beliefs that music and mathematics are fundamentally sister sciences”, as stated in the preface. The book is a collection of articles so let us begin by describing their contents, paying special attention to the mathematical aspects.

The book begins with an introductory article, “Music and mathematics: an overview”, by Susan Wollenberg. It contains some notes describing how the relationship between music and mathematics has been perceived and discussed through history and more particularly since the seventeenth century.

Part I: Music and mathematics through history, contains two articles. The first one is “Tuning and temperament: closing the spiral”, by Neil Bibby. It explains one of the most basic facts about the mathematical structure of musical scales. As is well-known, the interval between two notes is given by the ratio of their frequencies. The most basic ratio is 2:1, the octave. Two notes an octave apart are heard as equivalent so to construct a scale other intervals are needed. The next basic interval is the perfect fifth, with frequency ratio 3:2 – a specially pleasing interval. The Pythagorean scale is constructed by adding fifths. One of the fundamental problems in music theory has always been how many fifths one should add and how those then can be modified conveniently. This resulted in the division of the octave into 12 equal semitones that has pervaded Western music since the 19th century.

The other article is “Musical cosmology: Kepler and his readers”, by Judith V. Field. Seemingly Johannes Kepler believed that geometry and musical harmony could explain the structure of the Universe. This idea, drawn from the ancient tradition of the music of the spheres, was discussed shortly after by Marin Mersenne and Athanasius Kircher.

Part II: The mathematics of musical sound, contains three articles. The first one, “The science of musical sound”, by Charles Taylor, gives a qualitative account of some properties of sound, its perception and its production by musical instruments.

The second article in this part, “Faggot’s fretful fiasco”, by Ian Stewart, describes a historical accident. In the 18th century, a craftsman Daniel Strähle devised a simple way to determine the position of the frets on a guitar, which in mathematical terms amounts to approximating the 12th root of 2. However, this was dismissed by a mathematician Jacob Faggot due to a regrettable mistake in his calculations. The article discusses Strähle’s proposal in terms of fractional approximations and continued fractions.

Finally, “Helmholtz: combinational tones and consonance”, by David Fowler, describes two of the many contributions of Hermann Helmholtz to the science of sound. One is of a psychological nature: combinational tones, that is tones that are sometimes heard due to the nonlinearity of the ear. The other one is his explanation of the consonance associated with frequency ratios of small integers in terms that are very close to our present theories.

Part III: Mathematical structure in music, also consists of three articles. The first one, “The geometry of music”, by Wilfrid Hodges, studies music from a geometric perspective. The basic dimensions of time and pitch constitute a 2-dimensional space, subject to transformations like translations and rotations. A piece of music may contain motifs, each motif being considered as a subset of the musical space. Motifs are analysed according to their symmetry groups. In the same way, there is a classification of friezes. Many musical examples are given to illustrate all these possibilities.

The second paper in this part is “Ringing the changes: bells and mathematics”, by Dermot Roaf and Arthur White. Here, permutation groups and graph theory are applied to solve the old problem of change-ringing, that is to ring a set of bells in all possible orders, without repetition.

“Composing with numbers: sets, rows and magic squares”, by Jonathan Cross, describes various usages of numbers by some 20th century composers, from Arnold Schoenberg to Iannis Xenakis. The mathematical models described here are new sources of musical material and, as the author reminds us, the results may be as inspired or as mechanical as in any other musical system.

In Part IV: The composer speaks, two articles can be found. “Microtones and projective planes”, by Carlton Gamer and Robin Wilson, deals with microtonal music, based on  $n$  equal divisions of the octave. More particularly, they are interested in the case where  $n = k^2 - k + 1$ , which

is the number of points (and lines) in a finite projective plane. Then there is a connection between the musical inversion and the duality of projective geometry.

Finally, "Composing with fractals", by Robert Sherlaw Johnson, describes a computer program that generates musical patterns based on fractals.

The essays contained in the book concern very different subjects and have different mathematical density and depth. In this sense, the book lacks some coherence and elegance. Nevertheless, this allows the readers to choose their own ways to enjoy some of the connections between music and mathematics. Of course, many other connections are almost absent: the theory of dissonance, spectra of musical instruments, combinatorics of chords and scales, rhythmic patterns, etc. But, in less than 200 pages, the book conveys the idea that both disciplines have been closely related through history and that this relationship is going to remain and even to increase.

Music searches for new ways of expression and mathematics grows unceasingly. Therefore more and more connections will show up between these disciplines. The recent launch of a Journal of Mathematics and Music,

and the appearance of books like Dave Benson's 'Music: a mathematical offering' is a clear signal in this direction. With respect to this, I would dare say that there lacks an appropriate place in the Mathematics Subject Classification for the many existing and forthcoming literature on mathematics and music. Now that a revision of the classification scheme is in progress, a three-digit section near the end of the list could accommodate the many mathematical faces of music.



*Xavier Gràcia [xgracia@ma4.upc.edu] has degrees in physics and mathematics and obtained a PhD in physics in 1991 from the University of Barcelona. He has been working since 1988 at the Technical University of Catalonia in the Department of Applied Mathematics IV. His research interests are differential geometry and its applications, especially to mathematical physics. He is also an amateur musician and teaches a course on Music and Mathematics.*

## Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

*Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the addresses [mpacurar@econ.ubbcluj.ro](mailto:mpacurar@econ.ubbcluj.ro) or [madalina\\_pacurar@yahoo.com](mailto:madalina_pacurar@yahoo.com). Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files).*

December 2007

**3-7: Arithmetic geometry and rational varieties**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**7-8: The fifth conference on nonlinear analysis and applied mathematics (CNAAM 2007)**, University Valahia, Targoviste, Romania  
*Information:* [cmortici@valahia.ro](mailto:cmortici@valahia.ro)

**10-14: l-adic cohomology and number fields**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**17-21: Nonlinear equations and complex analysis**, Ufa, Russia  
*Information:* [bannoe07@matem.anrb.ru](mailto:bannoe07@matem.anrb.ru); <http://matem.anrb.ru/bannoe07>

**17-21: Meeting on mathematical statistics**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr); <http://www.cirm.univ-mrs.fr>

**17-22: International Conference on Transformation Groups**, Moscow, Russia  
*Information:* [tg2007@mccme.ru](mailto:tg2007@mccme.ru); <http://www.mccme.ru/tg2007/>

**27-30: The Second International Conference on Mathematics: Trends and developments**, Cairo, Egypt  
*Information:* [conf07@etms-web.org](mailto:conf07@etms-web.org);  
<http://www.etms-web.org/conf07/>

January 2008

**6-13: 1st Odense Winter School on Geometry and Theoretical Physics**, University of Southern Denmark, Odense, Denmark  
*Information:* [swann@imada.sdu.dk](mailto:swann@imada.sdu.dk);  
<http://www.imada.sdu.dk/~swann/Winter-2008/>

**7-11: Small group: D'Alembert's Opuscles and mathematical correspondences**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**14-18: Algebraic Geometry and Complex Geometry**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

21–24: Conference in Geometric Analysis and its Applications, Bern, Switzerland

*Information:* geoan@math.unibe.ch; <http://www.geoan.unibe.ch>

21–25: International Conference on Uniform Distribution, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

28–31: VII International Conference “System Identification and Control Problems” SICPRO ‘08, Moscow, Russia

*Information:* sicpro@ipu.rssi.ru;

[http://www.sicpro.org/sicpro08/code/e08\\_01.htm](http://www.sicpro.org/sicpro08/code/e08_01.htm)

28–February 1st: Holomorphic partial differential equations, small divisors and summability, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

29–31: Workshop on integrability in the AdS/CFT correspondence, Utrecht University, Utrecht, Netherlands

*Information:* l.k.hoevenaars@math.uu.nl;

<http://www.math.uu.nl/adscft>

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February 2008

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1–12: VIII Edition of the Russian Winter School, Kostroma, Russia

*Information:* cdipietr@unisa.it, school08ru@diffiety.ac.ru;

<http://school.diffiety.org/page3/page0/page63/page63.html>

4–8: 2nd GNAMPA School on Harmonic Analysis and Evolution Equations, Università di Parma, Parma, Italy

*Information:* alessandra.lunardi@unipr.it, mauceri@dima.unige.it;

<http://www.unipr.it/arpa/dipmat/rivista/HAEE/HAEE.html>

4–March 7: GREFI-MEFI, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

5–9: Second Winter School in Complex Analysis and Operator Theory, Sevilla, Spain

*Information:* contreras@us.es; <http://www.congreso.us.es/ws-caot/>

11–15: Fifth International Symposium on Foundations of Information and Knowledge Systems, Pisa, Italy

*Information:* contact@foiks.org; <http://2008.foiks.org/>

12–16: Foundations of Lattice-Valued Mathematics with Applications to Algebra and Topology, Linz, Austria

*Information:* ep.klement@jku.at;

<http://www.fll.jku.at/research/linz2008>

19–22: International Conference on Mathematics and Continuum Mechanics, Porto, Portugal

*Information:* ferreira@fe.up.pt;

<http://paginas.fe.up.pt/~cim2008/index.html>

29–2 March: Joint Mathematical Weekend EMS-Danish Mathematical Society, Copenhagen, Denmark

*Information:* [www.math.ku.dk/english/research/conferences/emsweekend/](http://www.math.ku.dk/english/research/conferences/emsweekend/)

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March 2008

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2–7: IX International Conference “Approximation and Optimization in the Caribbean”, San Andres Island, Colombia

*Information:* appopt2008@univalle.edu.co;

<http://matematicas.univalle.edu.co/~appopt2008/>

3–5: International Technology, Education and Development Conference (INTED2008), Valencia, Spain

*Information:* inted2008@iated.org; <http://www.iated.org/inted2008>

3–6: The Third International Conference On Mathematical Sciences (ICM2008), UAE University – Al-Ain, United Arab Emirates

*Information:* ICM2008@uaeu.ac.ae; <http://icm.uaeu.ac.ae>

4–7: 8th German Open Conference on Probability and Statistics, Aachen, Germany

*Information:* gocps2008@stochastik.rwth-aachen.de;

<http://gocps2008.rwth-aachen.de>

5–8: The First Century of the International Commission on Mathematical Instruction, Accademia dei Lincei, Rome, Italy

*Information:* <http://www.unige.ch/math/EnsMath/Rome2008/>

6–April 4: Cours “Méthodes variationnelles et parcimonieuses en traitement des signaux et des images”, Paris, France

*Information:* gabriel.peyre@ceremade.dauphine.fr; <http://www.ceremade.dauphine.fr/~peyre/cours-ihp-2008/>

9–12: LUMS 2nd International Conference on Mathematics and its Applications in Information Technology 2008 (in collaboration with SMS, Lahore), Lahore, Pakistan

*Information:* <http://www.lums.edu.pk/licm08>

10–14: ALEA meeting, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

17–21: GL (2), CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

19–21: The IAENG International Conference on Scientific Computing 2008, Regal Kowloon Hotel, Kowloon, Hong Kong

*Information:* publication@iaeng.org;

<http://www.iaeng.org/IMECS2008/ICSC2008.html>

25–28: Taiwan-France joint conference on nonlinear partial differential equations, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

# Does your research add up?

The Cambridge Journals mathematics list is growing all the time...with 5 new journals in 2008, here are a few from our selection:

## *Pure mathematics*

### **New for 2008**

#### **Journal of the Australian Mathematical Society**

*Published for the Australian Mathematical Society*

#### **Bulletin of the Australian Mathematical Society**

*Published for the Australian Mathematical Society*

#### **Journal of K-Theory**

*Published for the Independent Scholarly Online and Print Publishing (ISOPP)*

#### **Glasgow Mathematical Journal**

*Published for the Glasgow Mathematical Journal Trust*

## *Applied mathematics*

### **New for 2008**

#### **The ANZIAM Journal**

*Published for the Australian Mathematical Society*

#### **Review of Symbolic Logic**

*Published for the Association of Symbolic Logic*

#### **Compositio Mathematica**

*Produced, marketed and distributed for the London Mathematical Society for the Foundation Compositio Mathematica*

#### **Proceedings of the Edinburgh Mathematical Society**

*Published for the Edinburgh Mathematical Society*

#### **Mathematical Proceedings of the Cambridge Philosophical Society**

*Published for the Cambridge Philosophical Society*

#### **European Journal of Applied Mathematics**

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**31–April 4: Mathematical Computer Science School for Young Researchers**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

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April 2008

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**6–9: Mathematical Education of Engineers**, Mathematics Education Centre, Loughborough University, Loughborough, UK

*Information:* [mee2008@lboro.ac.uk](mailto:mee2008@lboro.ac.uk); <http://mee2008.lboro.ac.uk/>

**7–11: Graph decomposition, theory, logics and algorithms**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**14–18: Dynamical quantum groups and fusion categories**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**21–25: Multiscale methods for fluid and plasma turbulence: Applications to magnetically confined plasmas in fusion devices**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**28–May 2: Recent progress in operator and function theory**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

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May 2008

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**5–9: Statistical models for images**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**12–16: New challenges in scheduling theory**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**19–21: ManyVal '08 – Applications of Topological Dualities to Measure Theory in Algebraic Many-Valued Logic**, Milan, Italy

*Information:* <http://manyval.dsi.unimi.it/>

**19–23: Topological & Geometric Graph Theory**, Paris, France

*Information:* [tggt2008@ehess.fr](mailto:tggt2008@ehess.fr); <http://tggt.cams.ehess.fr>

**19–23: Vorticity, rotation and symmetry – Stabilizing and destabilizing fluid motion**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**26–30: Spring school in nonlinear partial differential equations**, Louvain-La-Neuve, Belgium

*Information:*

<http://www.uclouvain.be/math-spring-school-pde-2008.html>

**26–30: The Fourth International Conference “Inverse Problems: Modeling and Simulation”**, Oludeniz-Fethiye, Turkey

*Information:* [ahasanov@kou.edu.tr](mailto:ahasanov@kou.edu.tr); <http://www.ipms-conference.org/>

**26–30: Discrete Groups and Geometric Structures, with Applications III**, K.U. Leuven Campus Kortrijk, Belgium

*Information:* [Paul.Igodt@kuleuven-kortrijk.be](mailto:Paul.Igodt@kuleuven-kortrijk.be);

<http://www.kuleuven-kortrijk.be/workshop>

**26–30: High dimensional probability**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**27–June 5: 5th Linear Algebra Workshop**, Kranjska Gora, Slovenia

*Information:* [damjana.kokol@fmf.uni-lj.si](mailto:damjana.kokol@fmf.uni-lj.si); <http://www.law05.si>

**28–31: History of Mathematics & Teaching of Mathematics**, Targu-Mures, Romania

*Information:* [matkp@uni-miskolc.hu](mailto:matkp@uni-miskolc.hu)

**29–31: Brownian motion and random walks in mathematics and in physics**, Institut de Recherche Mathématique Avancée (Université Louis Pasteur), Strasbourg, France

*Information:* [franchi@math.u-strasbg.fr](mailto:franchi@math.u-strasbg.fr), [papadop@math.u-strasbg.fr](mailto:papadop@math.u-strasbg.fr); <http://www-irma.u-strasbg.fr/article545.html>

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June 2008

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**2–6: International Conference on Random Matrices (ICRAM)**, Sousse, Tunisia

*Information:* [abdelhamid.hassairi@fss.rnu.tn](mailto:abdelhamid.hassairi@fss.rnu.tn);

<http://www.tunss.net/accueil.php?id=ICRAM>

**2–6: Thompson's groups: new developments and interfaces**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**8–14: Mathematical Inequalities and Applications 2008**, Trogir - Split, Croatia

*Information:* [mia2008@math.hr](mailto:mia2008@math.hr); <http://mia2008.ele-math.com/>

**9–13: Geometric Applications of Microlocal Analysis**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);

<http://www.cirm.univ-mrs.fr>

**9–19: Advances in Set-Theoretic Topology: Conference in Honour of Tsugunori Nogura on his 60th Birthday**, Erice, Sicily, Italy

*Information:* [erice@dmitri.math.sci.ehime-u.ac.jp](mailto:erice@dmitri.math.sci.ehime-u.ac.jp);

<http://www.math.sci.ehime-u.ac.jp/erice/>

**16–20: Workshop on population dynamics and mathematical biology**, CIRM Luminy, Marseille, France

*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**16–20: Homotopical Group Theory and Homological Algebraic Geometry Workshop**, Copenhagen, Denmark  
*Information:* <http://www.math.ku.dk/~jg/homotopical2008/>

**17–20: Structural Dynamical Systems: Computational Aspects**, Capitulo-Monopoli, Bari, Italy  
*Information:* [sds08@dm.uniba.it](mailto:sds08@dm.uniba.it);  
<http://www.dm.uniba.it/~delbuono/sds2008.htm>

**17–22: International conference “Differential Equations and Topology”** dedicated to the Centennial Anniversary of Lev Semenovich Pontryagin, Moscow, Russia  
*Information:* [pont2008@cs.msu.ru](mailto:pont2008@cs.msu.ru); <http://pont2008.cs.msu.ru>

**22–28: Combinatorics 2008**, Costermano (VR), Italy  
*Information:* [combinatorics@ing.unibs.it](mailto:combinatorics@ing.unibs.it);  
<http://combinatorics.ing.unibs.it>

**23–27: Homotopical Group Theory and Topological Algebraic Geometry**, Max Planck Institute for Mathematics Bonn, Germany  
*Information:* [admin@mpim-bonn.mpg.de](mailto:admin@mpim-bonn.mpg.de);  
<http://www.ruhr-uni-bochum.de/topologie/conf08/>

**23–27: Hermitian symmetric spaces, Jordan algebras and related problems**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**25–28: VII Iberoamerican Conference on Topology and its Applications**, Valencia, Spain  
*Information:* [cita@mat.upv.es](mailto:cita@mat.upv.es); <http://cita.webs.upv.es>

**30–July 3: Analysis, PDEs and Applications**, Roma, Italy  
*Information:* [mazya08@mat.uniroma1.it](mailto:mazya08@mat.uniroma1.it);  
<http://www.mat.uniroma1.it/~mazya08/>

**30–July 4: Joint ICMI/IASE Study; Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education**, Monterrey, Mexico  
*Information:* [batanero@ugr.es](mailto:batanero@ugr.es);  
<http://www.ugr.es/~icmi/iasestudy/>

**30–July 4: Geometry of complex manifolds**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**30–July 4: The European Consortium for Mathematics in Industry (ECMI2008)**, University College, London, UK  
*Information:* [lucy.nye@ima.org.uk](mailto:lucy.nye@ima.org.uk); <http://www.ecmi2008.org/>

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July 2008

**2–4: The 2008 International Conference of Applied and Engineering Mathematics**, London, UK

*Information:* [wce@iaeng.org](mailto:wce@iaeng.org);  
<http://www.iaeng.org/WCE2008/ICAEM2008.html>

**3–8: 22nd International Conference on Operator Theory**, West University of Timisoara, Timisoara, Romania  
*Information:* [ot@theta.ro](mailto:ot@theta.ro); <http://www.imar.ro/~ot/>

**7–10: The Tenth International Conference on Integral Methods in Science and Engineering (IMSE 2008)**, University of Cantabria, Santander, Spain  
*Information:* [imse08@unican.es](mailto:imse08@unican.es), [meperez@unican.es](mailto:meperez@unican.es);  
<http://www.imse08.unican.es/>

**7–11: VIII International Colloquium on Differential Geometry (E. Vidal Abascal Centennial Congress)**, Santiago de Compostela, Spain  
*Information:* [icdg2008@usc.es](mailto:icdg2008@usc.es); <http://xtsunxet.usc.es/icdg2008>

**7–11: Spectral and Scattering Theory for Quantum Magnetic Systems**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**13 : Joint EWM/EMS Workshop**, Amsterdam, Netherlands  
*Information:* <http://womenandmath.wordpress.com/joint-ew-mems-worskhop-amsterdam-july-13th-2008/>

**14–18: Fifth European Congress of Mathematics (5ECM)**, Amsterdam, Netherlands  
*Information:* [www.5ecm.nl](http://www.5ecm.nl)

**15–18: Mathematics of program construction**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**15–19: The 5th World Congress of the Bachelier Finance Society**, London, UK  
*Information:* [mark@chartfield.org](mailto:mark@chartfield.org); <http://www.bfs2008.com>

**17–19: 7th International Conference on Retrial Queues (7th WRQ)**, Athens, Greece  
*Information:* [aeconom@math.uoa.gr](mailto:aeconom@math.uoa.gr);  
[http://users.uoa.gr/~aeconom/7thWRQ\\_Initial.html](http://users.uoa.gr/~aeconom/7thWRQ_Initial.html)

**20–23: International Symposium on Symbolic and Algebraic Computation**, Hagenberg, Austria  
*Information:* [franz.winkler@risc.uni-linz.ac.at](mailto:franz.winkler@risc.uni-linz.ac.at);  
<http://www.risc.uni-linz.ac.at/about/conferences/issac2008/>

**21–24: SIAM Conference on Nonlinear Waves and Coherent Structures**, Rome, Italy  
*Information:* [meetings@siam.org](mailto:meetings@siam.org);  
<http://www.siam.org/meetings/nw08/>

**21–August 29: CEMRACS**, CIRM Luminy, Marseille, France  
*Information:* [colloque@cirm.univ-mrs.fr](mailto:colloque@cirm.univ-mrs.fr);  
<http://www.cirm.univ-mrs.fr>

**24–26: Workshop on Current Trends and Challenges in Model Selection and Related Areas**, University of Vienna, Vienna, Austria

*Information:* hannes.leeb@yale.edu;

[http://www.univie.ac.at/workshop\\_modelselection/](http://www.univie.ac.at/workshop_modelselection/)

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August 2008

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**3–9: Junior Mathematical Congress**, Jena, Germany

*Information:* info@jmc2008.org; <http://www.jmc2008.org/>

**16–31: EMS-SMI Summer School: Mathematical and numerical methods for the cardiovascular system**, Cortona, Italy

*Information:* dipartimento@matapp.unimib.it

**19–22: Duality and Involutions in Representation Theory**, National University of Ireland, Maynooth, Ireland

*Information:* involutions@maths.nuim.ie;

<http://www.maths.nuim.ie/conference/>

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September 2008

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**1–5: Representation of surface groups**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**2–5: X Spanish Meeting on Cryptology and Information Security**, Salamanca, Spain

*Information:* delrey@usal.es;

<http://www.usal.es/xrecesi/english/main.htm>

**8–12: Chinese-French meeting in probability and analysis**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**8–19: EMS Summer School: Mathematical models in the manufacturing of glass, polymers and textiles**, Montecatini, Italy

*Information:* cime@math.unifi.it;

<http://web.math.unifi.it/users/cime/>

**15–19: Geometry and Integrability in Mathematical Physics**

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**21–24: The 8th International FLINS Conference on Computational Intelligence in Decision and Control (FLINS 2008)**, Madrid, Spain

*Information:* flins2008@mat.ucm.es;

<http://www.mat.ucm.es/congresos/flins2008>

**22–26: 10th International workshop in set theory**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**29–October 3: Commutative algebra and its interactions with algebraic geometry**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**29–October 8: EMS Summer School: Risk theory and related topics**,

Będlewo, Poland

*Information:* stettner@iman.gov.pl;

[www.impan.gov.pl/EMSsummerSchool/](http://www.impan.gov.pl/EMSsummerSchool/)

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October 2008

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**6–10: Partial differential equations and differential Galois theory**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**13–17: Hecke algebras, groups and geometry**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**20–24: Symbolic computation days**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**27–31: New trends for modeling laser-matter interaction**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

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November 2008

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**3–7: Harmonic analysis, operator algebras and representations**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**5–7: Fractional Differentiation and its Applications**, Ankara, Turkey

*Information:* dimitru@cankaya.edu.tr;

<http://www.cankaya.edu.tr/fda08/>

**17–21: Geometry and topology in low dimension**, CIRM Luminy, Marseille, France

*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>

**24–28: Approximation, geometric modelling and applications**, CIRM Luminy, Marseille, France

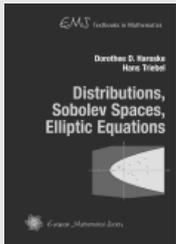
*Information:* colloque@cirm.univ-mrs.fr;

<http://www.cirm.univ-mrs.fr>



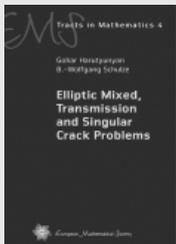
Guus Balkema (University of Amsterdam, The Netherlands), Paul Embrechts (ETH Zurich, Switzerland)  
**High Risk Scenarios and Extremes – A Geometric Approach** (Zurich Lectures in Advanced Mathematics)  
ISBN 978-3-03719-035-7. 2007. 388 pages. Softcover. 17.0 cm x 24.0 cm. 48.00 Euro

Quantitative Risk Management (QRM) has become a field of research of considerable importance to numerous areas of application, including insurance, banking, energy, medicine, reliability. Mainly motivated by examples from insurance and finance, the authors develop a theory for handling multivariate extremes. The approach borrows ideas from portfolio theory and aims at an intuitive approach in the spirit of the Peaks over Thresholds method. The book is based on a graduate course on point processes and extremes and primarily aimed at students in statistics and finance as well as professionals involved in risk analysis.



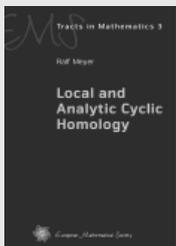
Dorothee D. Haroske (University of Jena, Germany), Hans Triebel (University of Jena, Germany)  
**Distributions, Sobolev spaces, Elliptic equations** (EMS Textbooks in Mathematics)  
ISBN 978-3-03719-042-5. 2007. 303 pages. Hardcover. 16.5 cm x 23.5 cm. 48.00 Euro

It is the main aim of this book to develop at an accessible, moderate level an  $L_2$  theory for elliptic differential operators of second order on bounded smooth domains in Euclidean  $n$ -space, including a priori estimates for boundary-value problems in terms of (fractional) Sobolev spaces on domains and on their boundaries, together with a related spectral theory. The presentation is preceded by an introduction to the classical theory for the Laplace–Poisson equation, and some chapters providing required ingredients such as the theory of distributions, Sobolev spaces and the spectral theory in Hilbert spaces.



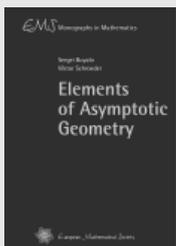
Gohar Harutyunyan (University of Oldenburg, Germany), B.-Wolfgang Schulze (University of Potsdam, Germany)  
**Elliptic Mixed, Transmission and Singular Crack Problems** (EMS Tracts in Mathematics Vol. 4)  
ISBN 978-3-03719-040-1. 2007. 777 pages. Hardcover. 17.0 cm x 24.0 cm. 112.00 Euro

Mixed, transmission, or crack problems belong to the analysis of boundary value problems on manifolds with singularities. The Zaremba problem with a jump between Dirichlet and Neumann conditions along an interface on the boundary is a classical example. The central theme of this book is to study mixed problems in standard Sobolev spaces as well as in weighted edge spaces where the interfaces are interpreted as edges. Parametrices and regularity of solutions are obtained within a systematic calculus of boundary value problems on manifolds with conical or edge singularities.



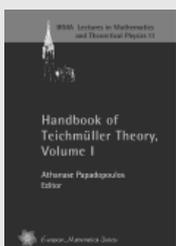
Ralf Meyer (University of Göttingen, Germany)  
**Local and Analytic Cyclic Homology** (EMS Tracts in Mathematics Vol. 3)  
ISBN 978-3-03719-039-5. 2007. 368 pages. Hardcover. 17.0 cm x 24.0 cm. 58.00 Euro

Periodic cyclic homology is a homology theory for non-commutative algebras that plays a similar role in non-commutative geometry as de Rham cohomology for smooth manifolds. While it produces good results for algebras of smooth or polynomial functions, it fails for bigger algebras such as most Banach algebras or  $C^*$ -algebras. Analytic and local cyclic homology are variants of periodic cyclic homology that work better for such algebras. In this book the author develops and compares these theories, emphasising their homological properties.



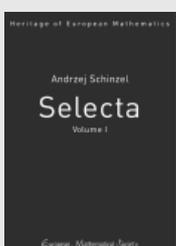
Sergei Buyalo (Steklov Institute of Mathematics, St. Petersburg, Russia), Viktor Schroeder (University of Zurich, Switzerland)  
**Elements of Asymptotic Geometry** (EMS Monographs in Mathematics)  
ISBN 978-3-03719-036-4. 2007. 212 pages. Hardcover. 16.5 cm x 23.5 cm. 58.00 Euro

Asymptotic geometry is the study of metric spaces from a large scale point of view, where the local geometry does not come into play. An important class of model spaces are the hyperbolic spaces (in the sense of Gromov), for which the asymptotic geometry is nicely encoded in the boundary at infinity. In the first part of this book, in analogy with the concepts of classical hyperbolic geometry, the authors provide a systematic account of the basic theory of Gromov hyperbolic spaces. In the second part of the book, various aspects of the asymptotic geometry of arbitrary metric spaces are considered.



**Handbook of Teichmüller Theory, Volume I** (IRMA Lectures in Mathematics and Theoretical Physics)  
Athanase Papadopoulos (IRMA, Strasbourg, France), Editor  
ISBN 978-3-03719-029-6. 2007. 802 pages. Hardcover. 17.0 cm x 24.0 cm. 98.00 Euro

The Teichmüller space of a surface was introduced by O. Teichmüller in the 1930s. It is a basic tool in the study of Riemann's moduli space and of the mapping class group. These objects are fundamental in several fields of mathematics including algebraic geometry, number theory, topology, geometry, and dynamics. The purpose of this handbook is to give a panorama of some of the most important aspects of Teichmüller theory. The handbook will be useful to specialists in the field, to graduate students, and more generally to mathematicians who want to learn about the subject. All the chapters are self-contained and have a pedagogical character. They are written by leading experts in the subject.



**Andrzej Schinzel, Selecta** (Heritage of European Mathematics)  
**Volume I: Diophantine Problems and Polynomials, Volume II: Elementary, Analytic and Geometric Number Theory**  
Henryk Iwaniec (Rutgers University, USA), Władysław Narkiewicz (University of Wrocław, Poland), Jerzy Urbanowicz (IM PAN, Warsaw, Poland), Editors  
ISBN 978-3-03719-038-8. 1417 pages. Hardcover. 17.0 cm x 24.0 cm. 168.00 Euro

These Selecta contain Andrzej Schinzel's most important articles published between 1955 and 2006. The arrangement is by topic, with each major category introduced by an expert's comment.

# Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

**Books submitted for review should be sent to: Ivan Netuka, MÚUK, Sokolovská, 83, 186 75 Praha 8, Czech Republic.**

**S. Alinhac, P. Gérard: *Opérateurs pseudo-différentiels et théorème de Nash-Moser*, Mathématiques, EDP Sciences, Les Ulis, 2000, 188 pp., EUR 35, ISBN 2-7296-0364-6, ISBN 2-222-04535-7**

This book is a course on partial differential equations and pseudodifferential operators. It consists of three parts. The choice of material together with its arrangement shows some non-obvious links between seemingly distant topics of interest. The first part develops the theory of pseudodifferential operators. This generalization of partial differential operators is based on the fact that differentiation acts as polynomial multiplication in the Fourier regime. If we use a (reasonable) non-polynomial function as the multiplier, we obtain a pseudodifferential operator. The multiplier may depend on the initial space variable. Following this the case of variable coefficients is covered. The objective here is to establish the most important formulas of the ensuing calculus.

The second part presents the Littlewood-Paley theory and microlocal analysis (in particular a concept of the wave front set of a distribution in connection with pseudodifferential operators, energy estimates and propagation of singularities). In comparison with the first chapter, the exposition moves towards more specific problems motivated by partial differential equations but still in the pseudodifferential setting.

The third part studies perturbations of problems in partial differential equations. Starting with applications of the implicit function theorem and going through the situation treated by fixed point theorems, the main goal of the chapter is the Nash-Moser theorem. The results describe existence problems and estimates for solutions of perturbed problems. The main difficulty is to handle the loss of derivatives appearing when solving the linearized problem. The Nash theorem on existence of an isometric embedding of a Riemannian manifold is included as a special case. The book is intended as a course for advanced students but it will also be very useful for researchers. The material contained here is deep and very important for the understanding of some issues of the theory of partial differential equations and the more general context of pseudodifferential operators. The presentation is quite compact and the student should be well prepared (in particular, a good knowledge of Fourier calculus is needed). When the authors say “elementary” it should sometimes be read as “short”. The exposition is highly self-contained and the text is complemented by numerous exercises. (jama)

**G. Allaire: *Numerical Analysis and Optimization. An introduction to mathematical modelling and numerical simulation*, Numerical Mathematics and Scientific Computation, Oxford University Press, Oxford, 2007, 455 pp., GBP 30, ISBN 978-0-19-920522-6**

This book represents a modern introduction to the numerical analysis of partial differential equations and to optimization

techniques. The goal is to introduce the reader to the world of mathematical modelling and numerical simulation. It contains finite difference as well as finite element methods for numerical solution of stationary and non-stationary problems. Moreover, the book also treats optimization and operational research techniques. The first chapter introduces the reader to the area of mathematical modelling and numerical simulation. It contains examples of problems leading to partial differential equations of various types and explains some basic questions and obstacles met in their numerical solution. Chapter 2 is concerned with the finite difference method. In chapter 3 the variational formulation of stationary boundary value problems is introduced. Chapter 4 is devoted to the explanation of the concept and basic properties of Sobolev spaces. Chapter 5 presents the qualitative theory of elliptic problems. Chapters 3 to 5 form the basis for the treatment of the finite element method in chapter 6.

Chapter 7 is concerned with eigenvalue problems. In chapter 8, basic qualitative properties and the numerical solution of evolution problems are treated. Chapters 9 to 11 are devoted to optimization techniques. In these chapters, the motivation and examples are given and optimality conditions and basic optimization algorithms are discussed. Finally, methods of operational research are presented. The book contains two appendices: ‘Review of Hilbert spaces’ and ‘Matrix numerical analysis’. The book represents an interesting modern treatment of classical numerical techniques. It contains a number of examples accompanied by results of computations and figures showing solutions of various problems. A positive feature of the book is the fact that it needs only facts from the first few years of university study. Therefore, it can be recommended to students of mathematics and engineering sciences, applied and numerical mathematicians, engineers, physicists and specialists dealing with modelling and numerical simulation of various structures and processes. (mf)

**A.C. Atkinson, A.N. Donev, R.D. Tobias: *Optimum Experimental Designs, with SAS*, Oxford Statistical Science Series 34, Oxford University Press, Oxford, 2007, 511 pp., GBP 35, ISBN 978-0-19-929660-6**

The purpose of this book is to describe a sufficient amount of the theory to make apparent the overall pattern of optimum designs and to provide a thorough background for the use of packages of SAS software in the design of optimum experiments. The book is divided into two parts. The idea of the statistical design of experiments is introduced in the first shorter part. The theory of an optimum experimental design is developed in the second part together with many applications, procedures and examples. The first part (Background) consists of eight chapters. The authors discuss advantages of a statistical approach for the design of experiments and introduce many models and examples, which are applied mainly in the second part of the book. The first part describes stages in experimental research, the choice of model, the least squares method, a design matrix, standard design and analysis of experiments. The packages for data processing and statistical analysis (SAS software) are mentioned here and these packages are used in many examples in the second part (Theory and Applications).

The second part starts with a formulation of the general equivalence theorem. Then follows the criteria of A-, D- and E-

optimality, algorithms for the construction of exact D-optimum design and optimum experimental design with SAS. Chapter 14 is concerned with the design of experiments, where the response depends on both qualitative and quantitative factors. Mixture experiments are described in chapter 15. The comprehensive chapter 17 describes an extension of previous techniques to nonlinear regression models, including those defined by systems of differential equations. The following chapters contain a discussion of Bayesian optimum designs, augmented design, model checking and design of discriminating between models, compound design criteria, generalized linear models, response transformation and structured variances, time-dependent models with correlated observations, design of clinical trials and exercises. The whole book is oriented to the practical mind with many examples and procedures. The examples are mainly drawn from scientific, engineering, pharmaceutical and agriculture practice. The book contains a large list of references. It can be recommended mainly to scientists and to students preparing a Masters or a doctoral thesis. (jzit)

**D. Busneag: *Categories of Algebraic Logic*, Editura Academiei Romane, Bucarest, 2006, 270 pp., ISBN 978-973-27-1381-5**

This monograph summarizes the author's contributions to the study of algebras having their origin in logic. The book is divided into five chapters. The first four chapters (sets and functions, ordered sets, topics in universal algebra and topics in theory of categories) contain basic material, which is needed for the main part (algebras of logic). The corresponding chapter deals with several categories of algebras coming from intuitionistic logic, logic without contraction and fuzzy logic, in particular with Heyting algebras, Hilbert algebras, Hertz algebras, residuated lattices and Wajsberg algebras. The author concentrates on the algebraic and categorical properties of these structures contained in his numerous research articles. Historical background and motivation for the results described are not included; the reader is referred to the literature. (lbar)

**I. Chiswell, W. Hodges: *Mathematical Logic*, Oxford Texts in Logic 3, Oxford University Press, Oxford, 2007, 250 pp., GBP 29.50, ISBN 978-0-19-921562-1**

This book gives a standard first course in propositional and predicate logic. No prior knowledge of the subject is needed to read the book. The basic material is concentrated in chapter 3 (propositional logic), chapter 5 (quantifier-free logic) and chapter 7 (first-order logic, including completeness and compactness). Chapter 8 contains deeper results, in particular Church undecidability, Gödel undecidability and the incompleteness theorems. The proofs are based on Matiyasevich's theorem, formulated in 5.8.6 using the notions of computable and computably enumerable sets stated in 5.8.4 on an intuitive (but acceptable) level. Chapters 1, 2, 4 and 6 contain the motivation and a discussion of formal and psychological aspects of logic considerations in applications. Many examples and exercises are included. Moreover, appendix B provides some information on denotational semantics. The book is written on the foundation of the extensive teaching experience of the authors. They have succeeded in giving a presentation of the subject in a formally correct and intuitively acceptable way. They also include some applications in computer science and linguistics. (jmlc)

**D. Christodoulou: *The Formation of Shocks in 3-Dimensional Fluids*, EMS Monographs in Mathematics, European Mathematical Society, Zürich, 2007, 992 pp., EUR 148, ISBN 978-3-03719-031-9**

Dealing with the relativistic Euler equations for an ideal fluid with an arbitrary equation of state, this book provides an original, mathematically sound and complete theory of the formation of shocks in a three-dimensional spatial setting. More precisely, starting with initial data that coincide outside of a sphere with the data corresponding to a constant state, and assuming a suitable restriction on the size of an initial perturbation of the constant state, theorems describing maximal classical development are established. It is shown, in particular, that the boundary of the maximal classical development includes a singular part, where the inverse density of the wave fronts vanishes, which indicates a formation of shocks. The central concept used in the book is an acoustic spacetime manifold. Methods from differential geometry play an important role in the book. (jmal)

**S. Crovisier, J. Franks, J.-M. Gambaudo, P. Le Calvez: *Dynamiques des difféomorphismes conservatifs des surfaces: un point de vue topologique*, Panoramas et Synthèses, no. 21, Société Mathématique de France, Paris, 2006, 143 pp., EUR 36, ISBN 978-2-85629-220-4**

This book contains a written version of lectures presented at the meeting of the Société Mathématique de France held in the summer of 2004 at Dijon. It consists of four contributions. The first one (by S. Crovisier) is devoted to a description of the space of orbits (and in particular the space of periodic orbits) of a chosen volume preserving diffeomorphism  $f$  of a smooth compact connected surface with a given smooth volume. To avoid pathological cases, the author considers  $C^1$ -generic surface diffeomorphisms. The next contribution (by J. Franks) studies distortion elements in groups of surface diffeomorphisms (and, for comparison, also for circle diffeomorphisms). The results are applied to group actions on surfaces preserving a Borel measure. Three different topics and their relations are treated in the third contribution (by J.-M. Gambaudo): topological invariants associated with flows on three-dimensional manifolds, the space of configurations of an incompressible fluid on an oriented manifold, and a structure of the group of diffeomorphisms (preserving a given area form and isotopic to the identity) of a compact oriented surface. Finally, P. Le Calvez studies in the last contribution various versions of the Brouwer plane translation theorem and its relation to the study of homeomorphisms of surfaces (including a proof of the Conley conjecture in the case of a compact surface of genus greater than zero). (vs)

**W. Ebeling: *Functions of Several Complex Variables and their Singularities*, Graduate Studies in Mathematics, vol. 83, American Mathematical Society, Providence, 2007, 312 pp., USD 59, ISBN 978-0-8218-3319-3**

This book belongs to the AMS series 'Graduate Studies in Mathematics'. Its main topic is a study of isolated singularities of hypersurfaces in several complex variables. The first chapter describes the classical theory of Riemann surfaces (coverings, fundamental groups, branched meromorphic continuation, the Riemann surfaces of an algebraic function and the Puiseux expansion). Basic facts of the theory of functions of several complex variables are explained in chapter 2 (the implicit function

theorem, the Weierstrass preparation theorem and its generalizations, germs of analytic sets and their dimensions and special cases of the Remmert mapping theorem). Chapter 3 includes basic facts from differential geometry (smooth manifolds and their tangent bundles, transversality, homogeneous spaces and complex manifolds) together with a study of isolated critical points of holomorphic functions (the complex Morse lemma, universal unfolding, morsification and the classification of simple singularities).

Chapter 4 contains a description of basic facts from differential topology (the Ehresmann fibration theorem, a holonomy group of a fiber bundle, singular homology groups, the Euler characteristic, intersection and linking numbers, the braid group and the homotopy sequence of a fiber bundle). The last chapter is devoted to a description (partly without proofs) of topological properties of isolated critical points of holomorphic functions (including a discussion of monodromy groups and vanishing cycles, the Picard-Lefschetz theorem, the Milnor fibration, the intersection matrix and the Coxeter-Dynkin diagrams, variation of singularities, action of the braid group and the Arnold classification of unimodal singularities). The book contains a lot of illustrative pictures and diagrams substantially helping the geometric intuition of the reader. (vs)

**E.A. Fellmann: *Leonard Euler*, Birkhäuser, Basel, 2007, 179 pp., EUR 31,99, ISBN 978-3-7643-7538-6**

In spite of the existence of many texts devoted to the description of Euler's life and in spite of the fact that this is "a book about a great mathematician, which is entirely formulae-free", the book brings many facts of interest even for a professional mathematician (as was the intention of the original publisher). The author follows Euler's career in detail. The first chapter contains a brief copy of Euler's curriculum vitae (in German, with English translation, dictated by Euler to his son in 1767) written about 17 years before his death. It also contains a description of Euler's Basel period. Other chapters contain an account of the first Petersburg stay, the Berlin period and the second stay in Petersburg. A reader will learn facts about Euler's parents, his close friends, his children and both his marriages as well as many fields influenced by Euler's mathematical work. The book contains nice pictures, ten pages of bibliographical sources and a lot of notes. The book will be of interest to everybody who wants to know more about the unique phenomenon of "Euler", the most productive mathematician in the history of mathematics. (jiv)

**U. Grenander, M. Miller: *Pattern Theory. From Representation to Inference*, Oxford University Press, Oxford, 2006, 596 pp., GBP 100, ISBN 0-19-850570-1**

This book summarizes Grenander's lifelong efforts to represent empirical knowledge of complex real world processes in a mathematical form. The authors characterize the book's aim as "the formalization of a small set of ideas for constructing the representation of the pattern themselves, which accommodate variability and structure simultaneously". After a short introduction, the basic paradigm of Bayesian setup of the estimation on the posterior distribution is explained in chapter 2. The next four chapters analyze the role of pattern representations in conditioning structure. Chapters 7 and 8 examine groups of geometric transformations suitable for the representation of geometric objects. Chapter 9 is devoted to random processes

and fields on the background spaces – the continuum limits of the finite graphs (e.g. Gaussian processes representing physical processes in the world). Chapters 10 and 11 develop metric space structure of shapes and further extensions of this approach are treated in the next chapter.

Chapters 13 to 15 pass from pure representations of shape to their Bayes estimations and parametric representations. The next two chapters are devoted to infinite dimensional shape covered by a new field of computational anatomy, an emerging discipline introduced by Miller and Grenander at the end of the last century with applications in the biological variability of human anatomy. The last two chapters conclude the inference under the assumption that the posterior distributions describing patterns contain all the information about the underlying regular structure. Markov processes are introduced as models for changes of this structure and random samples are generated via their simulation. The book is designed for a broad public; it contains numerous exercises, examples of applications and an extended bibliography. Appendices outlining some theorems, their proofs and solutions of selected problems are on the website: [www.oup.com/uk/booksites/content/0198505701/appendices](http://www.oup.com/uk/booksites/content/0198505701/appendices). (is)

**A. Ya. Helemskii: *Lectures and Exercises on Functional Analysis*, Translations of Mathematical Monographs, vol. 233, American Mathematical Society, Providence, 2006, 468 pp., USD 129, ISBN 978-0-8218-4098-6**

This book on functional analysis is written with a considerable use of the language of category theory. The introductory chapter is devoted to foundations (metric and topological spaces, categories, morphisms and functors). The next chapters cover basic elements of functional analysis: normed spaces and operators (and the Hahn-Banach theorem and an introduction to quantum functional analysis) and Banach spaces (including tensor products). A special chapter presents the theory of polynormed spaces (basically locally convex spaces), weak topologies and the theory of distributions. Further chapters are devoted to compact and Fredholm operators and spectral theory (among others  $C^*$ -algebras and Borel functional calculus). The final chapters deal with Fourier transforms and fundamental harmonic analysis on groups. The book contains many examples and exercises of different levels of difficulty. It requires only a basic knowledge of linear algebra, elements of real analysis, and metric spaces. It can be recommended to a broad spectrum of readers, to graduate students as well as young researchers. (jl)

**B. Helffer, F. Nier: *Quantitative Analysis of Metastability in Reversible Diffusion Processes Via a Witten Complex Approach – The Case with Boundary*, Mémoires de la Société Mathématique de France, no. 105, Société Mathématique de France, Paris, 2006, 89 pp., EUR 26, ISBN 978-2-85629-218-1**

In the 80s, E. Witten introduced a version of the Laplacian on  $p$ -forms, distorted by a function  $f$ . If a Morse function  $f$  is given on  $M = \mathbb{R}^n$  (or on a compact Riemannian manifold  $M$  without a boundary) and if  $h$  is a positive constant, an analysis of the properties of small eigenvalues of the semiclassical Witten Laplacian  $\Delta(f, h)$  was given by A. Bovier, M. Eckhoff, V. Gayard, B. Helffer, M. Klein and F. Nier. The main theme of the book presented here is to extend the mentioned results to the case with boundary (a domain in  $\mathbb{R}^n$  and its smooth boundary, or a compact connected oriented Riemannian manifold with a

boundary) for 0-forms. An important part of the work consists of a construction of a Witten cohomology complex adapted to the case with boundary. (vs)

**G. Helmberg: *Getting acquainted with fractals*, Walter de Gruyter, Berlin, 2007, 177 pp, EUR 78, ISBN 978-3-11-019092-2**

This book provides an answer to the question of what type of mathematics is behind the famous fractal pictures. One of the main features of fractal objects is their (often non-integral) dimension. The first chapter presents the foundations of the theory of Hausdorff measure and dimension; it shows ways to produce fractal sets by means of “deleting and replacing” and it gives ideas of how to compute the dimension of such objects. The second chapter is related to the second main feature of fractals, namely their self-similarity. Iterative function systems are studied here. Starting with a finite number of similarities, by iteration we obtain a fractal set as the limit image. A rigorous theoretical treatment is complemented by many representative examples showing how a proper choice of iterated mappings with carefully chosen parameters may lead to nice pictures (including island archipelago, tree, Barnsley fern and grass).

In the third chapter, the theory of iteration of complex polynomials, resulting in Julia sets and the Mandelbrot set, is developed. Again, examples of impressive pictures based on precise formulas illustrate the theory. The exposition is written as a rigorous mathematical text with precise definitions, theorems and some proofs. It is readable with skills at the level of an undergraduate student. However, it is not intended as a textbook. In particular, the presentation is not self-contained and difficult proofs are referred to. For readers interested in the full depth of the theory it is recommended to supplement their studies with other sources. The purpose of this book is just to yield a bridge between the fractal pictures and serious mathematics and this is successfully achieved. (jama)

**D.D. Joyce: *Riemannian Holonomy Groups and Calibrated Geometry*, Oxford Graduate Texts in Mathematics 12, Oxford University Press, Oxford, 2007, 303 pp., GBP 34.50, ISBN 978-0-19-921559-1**

The main topic treated in this book is the classical subject of Riemannian holonomy groups, together with a new theme of so-called calibrated submanifolds. The book is designed for graduate students of mathematics (and theoretical physics) and it introduces the reader to modern topics important for understanding new mathematics emerging from string theory.

The first part of the book is devoted to an explanation of holonomy groups in a Riemannian setting (including a short review of background material, connections, curvature, holonomy groups, G-structures, Riemannian symmetric spaces, the classification of Riemannian holonomy groups and Kähler manifolds). Special chapters are devoted to the Calabi conjecture and its proof, Calabi-Yau manifolds and their deformations, hyperkähler and quaternionic Kähler manifolds and the exceptional holonomy groups. Most of this material is related to a previous important monograph of the author, ‘Compact manifolds with special holonomy, Oxford University Press, 2000’. Completely new material is contained in chapters on calibrated submanifolds, special Lagrangian geometry, mirror symmetry and the Strominger-Yau-Zaslow conjecture. The book ends with a chapter on special types of calibrated submanifolds of

manifolds with exceptional holonomy. The book is very well-written, it contains a wealth of important material and it should be on the shelf of everybody interested in the topic. (vs)

**M. Junge, Ch. Le Merdy, Q. Xu: *H<sup>∞</sup> Functional Calculus and Square Functions on Noncommutative L<sup>p</sup>-Spaces*, Astérisque, no. 305, Société Mathématique de France, Paris, 2006, 138 pp., EUR 26, ISBN 978-2-85629-189-4**

$H^\infty$  functional calculus, introduced by Alan McIntosh in the 80s and developed in collaboration with M. Cowling, I. Doust and A. Yagi, plays an important role in several branches of operator theory. The main topic treated in the book is natural square functions associated with sectorial operators, or with semigroups, acting on suitable non-commutative  $L^p$ -spaces and their relations with  $H^\infty$  functional calculus. After providing a background on non-commutative  $L^p$ -spaces,  $H^\infty$  functional calculus, sectorial operators and semigroups, the authors introduce square functions and describe their relations to  $H^\infty$  functional calculus. There is a chapter devoted to a noncommutative generalization of the Stein diffusion semigroup. Further topics include multiplication operators, Hamiltonians and the Schur multipliers on Schatten space, semigroups on  $q$ -deformed von Neumann algebras, and the noncommutative Poisson semigroup of a free group. The last chapter briefly treats the non tracial case. (vs)

**P. Kaye, R. Laflamme, M. Mosca: *An Introduction to Quantum Computing*, Oxford University Press, Oxford, 2006, 274 pp., GBP 75, ISBN 0-19-857000-7**

The area of quantum computation and quantum algorithms is developing very quickly. This book describes very carefully and understandably its foundations. The book is intended for a broad spectrum of possible readers from various fields (including physicists and engineers) and it requires only a modest knowledge (basic facts of linear algebra). More advanced topics (e.g. tensor products and spectral properties) are carefully reviewed. The authors treat in a systematic way the main topics in the field. This starts with a review of basic models of computation (going from classical to quantum). After a careful review of notions of linear algebra (and Dirac notation), they explain basic axioms of quantum mechanics and basic models for quantum computations. After a description of basic protocols for quantum information (superdense coding and quantum teleportation), they concentrate on quantum algorithms. They discuss many of these (including the Shor algorithm and the Grover quantum search algorithm). The last two chapters are devoted to quantum computation complexity theory and quantum error corrections. There are a lot of examples throughout the text and the treatment of all topics included in the book is very understandable, clear and systematic. There are no doubts that the book will be very useful for students from various branches of science. (vs)

**D. Khoshnevisan: *Probability*, Graduate Studies in Mathematics, vol. 80, American Mathematical Society, Providence, 2007, 224 pp., USD 45, ISBN 978-0-8218-4215-7**

This book presents a graduate course in measure-theoretic probability designed to cover a one year course at a comfortable pace. The author starts with classical probability (including a neat proof of the de Moivre-Laplace theorem) before moving on to elements of measure theory (measure spaces, integration, modes of convergence

and product spaces). The chapter on independence covers the strong law of large numbers and the Glivenko-Cantalli theorem. The central limit theorem in an independent and identically distributed setting is proved both by the classical harmonic-analytic approach and by the Liapunov-Lindeberg replacement method. The material also covers elements of weak convergence and the Cramer characterization of normal distributions.

Some rich material on conditional expectations and discrete time martingales is offered, the optional stopping theorem and the martingale convergence theorem being the principal results. The applications range from the Lebesgue differentiation theorem to option-pricing procedures. Brownian motion is constructed, its nowhere differentiability and the strong Markov property being the principal achievements. The text terminates with a short introduction to stochastic calculus (the Itô integral and formula, optional stopping,  $L^2$ -martingales and the exit distribution of Brownian motion). The reader might miss a treatment of Markov chains but overall the text is neatly written and presents not only good mathematics but also a heuristic view of modern probability, including numerous exercises and historical notes. (jstep)

**J. M. Lee: *Fredholm operators and Einstein metrics on conformally compact manifolds*, *Memoirs of AMS*, no. 864, American Mathematical Society, Providence, 2006, 83 pp., USD 55, ISBN 978-0-8218-3915-7**

Conformal geometry (in particular in dimension 4) and corresponding geometric partial differential equations have been studied carefully in the last few decades. New tools developed recently by Ch. Fefferman, R. Graham, M. Eastwood, R. Gover and T. Bailey have allowed a much better understanding of conformal invariants (including a complete classification in particular cases). A principal tool here is the ambient metric construction of Fefferman and Graham. The so-called AdS-CFT correspondence recently emerging from string theory is based on the notion of conformal infinity for a (pseudo-)Riemannian metric developed originally by R. Penrose. It caused a dramatic rise of interest in these problems both in mathematics and mathematical physics. The problem treated in the book is to find, for a compact manifold  $M$  with a given conformal structure  $c$  on its boundary  $\partial M$ , a complete asymptotically hyperbolic Einstein metric  $g$  on the interior of  $M$  such that  $c$  coincides with the conformal class of  $g$  on  $\partial M$ .

The book contains a systematic and self-contained description of the perturbation version of the problem (where we are looking for solutions of the problem for  $c$  near to a given  $c_0$  for which the solution exists). An important fact is that the solution has an optimal Hölder regularity up to the boundary (for  $n$  even). Related results (with less boundary regularity but also for the quaternionic and octonionic cases) were described recently in the book by O. Biquard (see EMS Newsletter 65, page 55). Methods used in the book are based on general sharp Fredholm and isomorphism theorems for geometric (degenerate) linear elliptic operators. (vs)

**Q. Liu: *Algebraic Geometry and Arithmetic Curves*, *Oxford Graduate Texts in Mathematics 6*, Oxford University Press, Oxford, 2006, 577 pp., GBP 34.50, ISBN 0-19-920249-4**

The main topics of this book are arithmetic surfaces and reductions of algebraic curves. But these topics are systematically

treated only in chapters 8, 9, and 10. The aim of the first seven chapters is to educate the reader in the theory of schemes. The author devotes the whole first chapter to commutative algebra. Nevertheless, one chapter on algebra is not sufficient for the purposes of algebraic geometry and the author has to make digressions and introduce further algebraic notions and results several times. (It is interesting to mention that the author had the idea to present algebraic aspects in a simpler form; he succeeded in avoiding homological algebra.) Thanks to these algebraic parts, the book is rather self-contained.

Next the theory of schemes is studied. This presentation is excellent indeed. It is not a survey but a systematic and thorough theory of schemes covering many aspects of them. In spite of many necessary details, the text reads very well and moreover, we are not lost in this rich theory and it is possible to keep a certain global overview. The author includes a substantial number of examples, which is very helpful for the reader. Every section is followed by a long series of exercises. They are of two kinds. Some of them are designed to increase understanding of the text, the others extend and complete the theory. These seven chapters can be strongly recommended as a basis for a course on the theory of schemes. I agree with the author that even a good undergraduate student can learn a lot from this book.

The second part uses techniques developed in the first seven chapters. It begins with a description of blowing-ups. Then fibered surfaces over a Dedekind ring and desingularizations of surfaces are studied. The next topic is intersection theory on an arithmetic surface. The last chapter is devoted to the reduction theory of algebraic curves. Special attention is paid to elliptic curves. At the end, stable curves and stable reductions are treated; in particular, the Artin-Winter proof of the stable reduction theorem of Deligne-Mumford. Some concrete computations are also presented. These last chapters introduce the reader to contemporary research. He or she will also find hints for further reading. (jiva)

**G. Maltsiniotis: *La théorie de l'homotopie de Grothendieck*, *Astérisque 301*, Société Mathématique de France, Paris, 2005, 140 pp., EUR 26, ISBN 2-85629-181-3**

**D.-C. Cisinski: *Les préfaisceaux comme modèles des types d'homotopie*, *Astérisque 308*, Société Mathématique de France, Paris, 2006, 390 pp., EUR 78, ISBN 978-2-85629-225-9**

The aim of the book by G. Maltsiniotis is to give an introduction to some of the key ideas of A. Grothendieck's unpublished treatise on abstract homotopy theory via category theory ("Pursuing Stacks"). The fundamental concept is that of a "test category"  $A$ , whose main property is that the localization of the presheaf category  $\tilde{A}$  with respect to weak equivalences is naturally equivalent to the usual homotopy category (a typical example is the category of simplices/cubes, for which  $\tilde{A}$  is the category of simplicial/cubical sets). Furthermore, one can consider more general homotopy categories by replacing the class of weak equivalences by an arbitrary "fundamental localize" (a class of functors that satisfies, among other things, Quillen's Theorem A). The exposition of these concepts and of their basic properties is very clear.

The book by Cisinski develops Grothendieck's theory further. One of its main themes is the question of existence of a well-behaved closed model structure on  $\tilde{A}$  as a characterization of (accessible) test categories  $A$ , as conjectured by Grothendieck. Other topics include a discussion of equivariant homotopy and examples of interesting test categories and homotopy categories. (jnek)

**C. Mazza, V. Voevodsky, C. Weibel: *Lecture Notes on Motivic Cohomology*, Clay Mathematics Monographs, vol. 2, American Mathematical Society, Providence, 2006, 216 pp., USD 45, ISBN 978-0-8218-3847-1**

This book by C. Mazza and Ch. Weibel is based on a series of lectures given by V. Voevodsky in Princeton in the academic year 1999/2000. The idea of these lectures is to relate motivic cohomology to other known invariants of algebraic varieties and rings. The power of motivic cohomology as a tool for proving results in algebra and algebraic geometry lies in the interaction with properties of motivic cohomology itself – its homotopy invariance, Mayer-Vietoris and Gysin long exact sequences, projective bundles, etc. The contents of the book may be divided into two parts, corresponding to the fall and spring terms. The fall term lectures contain the definition of motivic cohomology and proofs for various comparison results (e.g. with the Milnor K-group and the Picard group). The spring term lectures include more advanced results in the theory of sheaves with transfers and a proof of the general comparison result with higher Chow groups of algebraic varieties. (ps0)

**C. Meyer: *Modular Calabi-Yau Threefolds*, Fields Institute Monographs, American Mathematical Society, Providence, 2005, 194 pp., USD 59, ISBN 0-8218-3908-X**

The Taniyama-Shimura conjecture (used in the proof of Fermat's last theorem) relates the number of points on an elliptic curve over a finite field to Fourier coefficients of a modular form of weight two. Calabi-Yau manifolds are generalizations of elliptic curves to higher dimensions and they play a prominent role in the contemporary development of string theory. Their arithmetic properties in dimensions two (i.e. properties of K3 surfaces) have been studied recently. This book is devoted to the case of Calabi-Yau manifolds in dimension three. The dimension of the middle étale cohomology of a Calabi-Yau three-fold  $M$  gives key information on its arithmetic properties. If it equals two, the three-fold is called rigid. There is a precise conjecture on a connection of a rigid three-fold with modular forms, whereas the situation is much more complicated for non-rigid ones. The book contains hundreds of examples of rigid and non-rigid cases. The first chapter reviews basic facts on Calabi-Yau manifolds and their arithmetic. The next four chapters contain a discussion of a considerable number of explicit examples (including many different quintics, double octics and various complete intersections). In the last chapter, the author describes the correspondences between Calabi-Yau three-folds having the same modular form in their  $L$ -series. The book is based on the author's PhD thesis. (vs)

**A. Moroianu: *Lectures on Kähler Geometry*, London Mathematical Society Student Texts 69, Cambridge University Press, Cambridge, 2007, 171 pp., GBP 19.99, ISBN 978-0-521-68897-0**

This book contains a written version of lectures on Kähler geometry prepared for graduate students of mathematics and theoretical physics from a perspective of Riemannian geometry. The first part explains basic notions from differential geometry (manifolds, tensor fields, integration of differential forms, principal and vector bundles, connections, Riemannian manifolds, parallel transport and a concept of holonomy). The second part is devoted to complex and Hermitian geometry (complex manifolds, complex and holomorphic vector bundles,

Hermitian vector bundles, the Chern connection, Kähler metrics, the Kählerian curvature tensor, the Laplace operator on a Kähler manifold, Hodge theory and Dolbeault theory).

Special and more advanced topics are discussed in the last part of the book, including a short description of the first Chern classes of vector bundles, Ricci-flat Kähler manifolds, the Calabi-Yau theorem, the Aubin-Yau theorem on Kähler-Einstein metrics, vanishing theorems on Kähler manifolds, the Hirzebruch-Riemann-Roch formula, hyperkähler manifolds and Calabi-Yau manifolds). Recent intensive collaboration between mathematics and theoretical physics in areas connected with string theory has, as a consequence, created substantially stronger requirements on the mathematical background of young people aiming to work in this broad field. The book covers in a nice, systematic and understandable way basic tools of differential geometry. It can be useful both for student readers and for teachers preparing lectures on the subject. (vs)

**J. Nagel, C. Peters: *Algebraic Cycles and Motives*, vol. 2, London Mathematical Society Lecture Notes Series 344, Cambridge University Press, Cambridge, 2007, 359 pp., ISBN 978-0-521-70175-4**

The second volume of the proceedings of J. Murre's 75th birthday conference (held in Leiden in 2004) contains fifteen research articles on various aspects of motives and algebraic cycles. The two main themes represented here are the conjectures of Bloch and Beilinson (refined by Murre) on the existence of a canonical filtration on Chow groups (articles by Beauville; Bloch and Esnault; Jannsen; Kahn, Murre and Pedrini; and Miller, Müller-Stach, Wortmann, Yang and Zuo) and Hodge-theoretical methods in the study of algebraic cycles (articles by Asakura and S. Saito; Lewis; Nagel; Peters and Steenbrink; and M. Saito). Other topics include: finite-dimensionality of motives – Kimura (and Jannsen); semistable vector bundles – Deninger and Werner, and Brivio and Verra; Mordell-Weil lattices for elliptic K3 surfaces – Shioda; and arithmetic aspects of diffraction patterns – Stienstra. (jnek)

**J. Nekovář: *Selmer Complexes*, Astérisque, no. 310, Société Mathématique de France, Paris, 2006, 559 pp., EUR 86, ISBN 978-2-85629-227-3**

This work gives a unified treatment of commutative Iwasawa theory centered on a small number of sufficiently general principles. It should be regarded as a 'Grothendification' of the subject into a landscape of arithmetic geometry. The contents of each chapter are as follows. The background material from homological algebra is collected together in chapter 1. In chapter 2, the formalism of Grothendieck duality theory over complete local rings is covered. Chapter 3 contains a description of the formalism of continuous cohomology and chapter 4 deals with finiteness results for continuous cohomology of pro-finite groups. Results for big Galois representations from the classical duality results for Galois cohomology of finite Galois modules over local and global fields are deduced in chapter 5.

In chapter 6, the author introduces Selmer complexes in an axiomatic setting and proves a duality theorem for them. A study of generalization of unramified cohomology is contained in chapter 7. The next two chapters contain duality results in Iwasawa theory (deduced from those over number fields) and their applications to classical Iwasawa theory. Various incarnations of generalized Cassels-Tate pairings are constructed and

studied in chapter 10. The last two chapters contain a construction of generalized  $p$ -adic height pairing and applications of ideas developed in chapter 10 to big Galois representations arising from Hida families of Hilbert modular forms, and to anticyclotomic Iwasawa theory of CM points on Shimura curves. (ps)

**S. P. Novikov, I. A. Taimanov: *Modern Geometric Structures and Fields*, Graduate Studies in Mathematics, vol. 71, American Mathematical Society, Providence, 2006, 633 pp., USD 79, ISBN 0-8218-3929-2**

This new volume of the GSM series by the AMS has a much wider scope than the usual textbook on differential geometry. It covers all the basic notions (surfaces in three dimensions and their relations to complex and conformal geometry, smooth manifolds, groups of transformations, including crystallographic groups, tensor algebra and tensor fields, differential forms and their integration, the Stokes theorem and its relation to the de Rham cohomology, connections and their curvature, Riemannian geometry, geodesics and the Gauss-Bonnet formula). The last third of the book treats more advanced topics (conformal geometry, Kähler manifolds, the Morse theory, Hamiltonian formalism, groups of symmetries and conservation laws, Poisson and Lagrange structure on manifolds, and multidimensional calculus of variation). The last chapter describes basic fields of theoretical physics (Einstein's general relativity, Clifford algebras, spinors and the Dirac equation, and Yang-Mills fields). The authors try to avoid unnecessary abstraction; they present many important topics in modern geometry and topology in a unified way. The book is designed for students in mathematics and theoretical physics but it will be very useful for teachers as well. (vs)

**L. Nyssen, Ed.: *Physics and Number Theory*, IRMA Lectures in Mathematics and Theoretical Physics, vol. 10, European Mathematical Society, Zürich, 2006, 265 pp., EUR 39,50, ISBN 978-3-03719-028-9**

The seven articles in this stimulating collection are surveys of a range of contemporary problems arising in physics, number theory and the interface between the two. The topics covered include arithmetic aspects of aperiodic crystals (articles by J.-L. Verger-Gaugry; and J.-P. Gazeau, Z. Masáková and E. Pelantová), phase-locking (both in classical and quantum systems) and number theory (M. Planat), Hopf algebras in renormalization theory (Ch. Bergbauer and D. Kreimer),  $L$ -functions and random matrices (E. Royer), recent applications of Kloosterman sums (P. Michel) and recent progress on the  $p$ -adic Langlands programme (A. Mézard). The book offers interesting material both for mathematicians and theoretical physicists. (jnek)

**Séminaire Bourbaki, Volume 2004/2005, Astérisque 307, Société Mathématique de France Paris, 2006, 520 pp., EUR 86, ISBN 978-285629-224-2**

This volume contains fourteen survey lectures: three on algebraic geometry, two on differential geometry, one on the Poincaré conjecture, one on dynamic systems, one on number theory, one on the fundamentals lemma, one on the André-Oort conjecture, one on quadratic forms, one on algebraic topology, one on mathematical physics, and one on probabilities. As tradition dictates, all of them contain a discussion of important achievements (in this case in the period 2004–2005). For instance, the ar-

ticle devoted to number theory reports on the Green-Tao result on arithmetic progressions in the sequence of primes. The book also contains contributions on the Mumford conjecture (following the work of Madsen and Weiss), a proof of the Parisi formula by Guerra and Talagrand, on 'Formes quadratiques et cycles algébriques' (following Rost, Voevodsky, Vishik, Karpenko and others), and so on. As one of the most important sources of reports on major achievements in contemporary mathematics, the volume will find a prominent place on every shelf. (spor)

**J. Väänänen: *Dependence Logic – A New Approach to Independence Friendly Logic*, London Mathematical Society Student Texts 70, Cambridge University Press, Cambridge, 2007, 225 pp., GBP 23.99, ISBN 978-0-521-70015-3**

This book develops and studies a conservative extension of first order logic called dependency logic, which approaches the study of the phenomena of dependency and independency on a logical basis. Dependency logic introduces a new type of atomic formula, written as  $= (t_1, t_2, \dots, t_n)$ , with an intuitive meaning that the value of the term  $t_n$  depends only on the values of the terms  $t_1, t_2, \dots, t_{n-1}$ . Given proper semantics, the basic language of dependency logic is more expressive than that of first order logic. This is demonstrated on several examples (chapter 4), e.g. by giving a formula with no extralogical symbols whose models are all finite sets with no infinite set. This expressiveness, of course, cannot endure without a cost, which in the case of this logic is the dropping of the law of the excluded middle.

The author acknowledges his inspiration as the Independence Friendly Logic of Hintikka and Sandu, whose basic instrument for expressing (in)dependency is instead the quantifier "there exists  $x_n$  independently of  $x_1, x_2, \dots, x_{n-1}$ ". While these two logics turn out to have the same expressiveness at the level of sentences, the author argues that dependency logic is actually the more expressive of the two at the level of formulae (section 3.6). Model theory for dependency logic is developed by relating the logic to existential second order logic (chapter 6). A brief yet informative chapter 7 deals with the complexity of the decision problem for dependency logic. The last chapter studies a further extension of dependency logic called team logic, which besides the non-classical negation of dependency logic also contains Boolean negation obeying the law of the excluded middle.

The book is based on real teaching experience and it contains many instructive exercises (for which many of the solutions are given in an appendix by Ville Nurmi). In several places, the reader is given the chance to acquire basic skills in a game-theoretic approach to logic and model theory. The book is probably best suited to advanced students of special courses but it remains accessible to all students with a basic knowledge of logic. (ppaj)

**R.E. White: *Elements of Matrix Modeling and Computing with MATLAB*, Chapman & Hall/CRC, Boca Raton, 2006, 402 pp., USD 79,95, ISBN 1-58488-627-7**

An important objective of this textbook is to provide "math-on-line" for undergraduate students of science and engineering. It is expected that students have had at least one semester of calculus. Chapters one and two contain introductory material on complex numbers, 2D and 3D vectors and their products. A connection is established between geometric and algebraic approaches to these topics. This is continued into chapters three, four and five, where higher order algebraic systems are solved

via row operations, inverse matrices and LU decomposition. Linearly independent vectors and subspaces are used to solve underdetermined and overdetermined systems. Chapters six and seven describe first and second order linear equations and introduce eigenvalues and eigenvectors for solutions of linear systems of initial value problems. The last two chapters use transform methods to filter distorted images and signals. The discrete Fourier transform is introduced via continuous versions of the Laplace and Fourier transforms. The discrete Fourier transform properties are derived from the Fourier matrix representation and are used to do image filtering in the frequency domain.

Most sections have some applications, indicating that these topics are really useful. Seven basic applications are developed in various sections of the text, including circuits, trusses, mixing tanks, heat conduction, data modeling and the motion of a mass and image filters. The applications are developed from very simple models to more complex ones. Matlab is used here to do more complicated computations. The strategy of how to use computing tools is given as: firstly learn the math and by-hand calculations; secondly use computing tools to confirm by-hand calculations; thirdly use computing tools to do more complicated calculations and applications. (jant)

**O. Zariski: *The Moduli Problem for Plane Branches*, University Lecture Series, vol. 39, American Mathematical Society, Providence, 2006, 151 pp., USD 35, ISBN 978-0-8218-2983-7**

This book is the English translation of a French version published previously by HERMANN (Éditeurs des Sciences et des Arts, Paris). The classical problem of moduli spaces of curves is generalized here to the problem of a description of the moduli space of curves of the same equisingularity class. The first four chapters are devoted to equisingularity invari-

ants, parametrization of the moduli space and its detailed description (in specific cases). After a review of specific examples, the author studies the generic component of moduli space with a particular characteristic. The last third of the book contains an appendix written by B. Tessier. The reader will find a version of the 'product decomposition' theorem and a natural compactification of the moduli space of a plane branch with a given characteristic. (vs)

#### List of reviewers for 2007.

**The Editor would like to thank the following for their reviews this year:**

J. Anděl, J. Antoch, T. Bárta, L. Barto, M. Bečvářová, L. Boček, E. Calda, A. Drápal, M. Feistauer, S. Henc, D. Hlubinka, Š. Holub, J. Hurt, M. Hušek, M. Hušková, M. Hykšová, P. Kaplický, T. Kepka, M. Klazar, P. Knobloch, J. Kofroň, J. Král, J. Lukeš, J. Málek, J. Malý, M. Mareš, J. Milota, J. Mlček, M. Loebel, K. Najzar, J. Nekovář, I. Netuka, J. Olejníková, P. Pajás, O. Pangrác, L. Pick, Š. Porubský, Z. Prášková, D. Pražák, A. Procházka, P. Pyrih, J. Rataj, M. Rokyta, T. Roubíček, P. Růžička, I. Saxl, A. Slavík, P. Somberg, V. Souček, J. Spurný, J. Stará, J. Štěpán, J. Trlifaj, J. Tůma, J. Vančura, J. Veselý, M. Zahradník, J. Zitko.

All of the above are on the staff of the Charles University, Faculty of Mathematics and Physics, Prague, except: J. Vanžura (Mathematical Institute, Czech Academy of Sciences), M. Bečvářová, M. Hykšová (Technical University, Prague), Š. Porubský (Institute of Computer Science, Czech Academy of Sciences) and J. Nekovář (University Paris VI, France).

## Joint EWM/EMS Workshop

Amsterdam, July 13<sup>th</sup>, 2008

<http://womenandmath.wordpress.com/>



This one day workshop, organized under the auspices of the EMS and EWM, aims at introducing the audience to the topics of the two main women speakers at the European Congress of Mathematics, Christine Bernardi and Matilde

Marcolli. Its program will be organized around three to four introductory talks on their research areas, and will end with a social gathering and informal discussions. The speakers at the workshop will include Christine Bernardi and Alina Vdovina.

We encourage young women mathematicians to attend this one day meeting before the Congress itself. It will provide an opportunity to get acquainted with two areas of research represented by two of the main speakers at the ECM, namely applied mathematics (spectral and variational problems) on the one hand and applications of noncommutative geometry (e.g. to quantum field

theory and number theory) on the other hand.

We draw your attention to the fact that early registration at the ECM brings down the costs ("earlybird" fee) and that you can ask for help from the ECM to find accommodation via the university if you apply before January 1. We encourage those of you who can benefit from a grant delivered by the EMC, to apply for a "one day" extension to be able to attend this meeting.

Hoping to see you in Amsterdam,

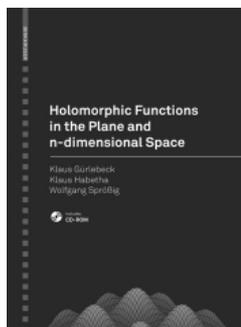
Best wishes,

The organizers,

Colette Guillopé (Femmes et Mathématiques),

Frances Kirwan (convenor of EWM),

Sylvie Paycha (Coordinator of the EMS committee for women in mathematics)



## Holomorphic Functions in the Plane and n-dimensional Space

Gürlbeck, K., Bauhaus University Weimar, Germany / Habetha, K., RWTH Aachen, Germany / Spröbig, W., TU Freiberg, Germany

Complex analysis nowadays has higher-dimensional analogs: the algebra of complex numbers is replaced then by the non-commutative algebra of real quaternions or by Clifford algebras. During the last 30 years the so-called quaternionic and Clifford or hypercomplex analysis successfully developed to a powerful theory with many applications in analysis, engineering and mathematical physics. This textbook introduces both to classical and higher-dimensional results based on a uniform notion of holomorphy. Historical remarks, lots of examples, figures and exercises accompany each chapter. The enclosed CD-ROM contains an extensive literature database and a Maple package with comments and procedures of tools and methods explained in the book.

2008. Approx. 410 pp. With CD-ROM. Softcover  
EUR 34.90 / CHF 59.90  
ISBN 978-3-7643-8271-1



## Determinantal Ideals

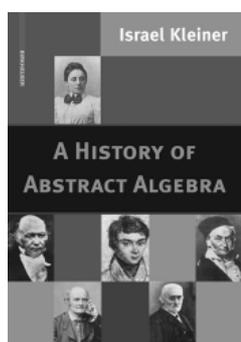
Miró-Roig, R.M., Universitat de Barcelona, Spain



Winner of the Ferran Sunyer i Balaguer Prize 2007.

Determinantal ideals are ideals generated by minors of a homogeneous polynomial matrix. Some classical ideals that can be generated in this way are the ideal of the Veronese varieties, of the Segre varieties, and of the rational normal scrolls. Determinantal ideals are a central topic in both commutative algebra and algebraic geometry, and they also have numerous connections with invariant theory, representation theory, and combinatorics. Due to their important role, their study has attracted many researchers and has received considerable attention in the literature. In this book three crucial problems are addressed: CI-liaison class and G-liaison class of standard determinantal ideals; the multiplicity conjecture for standard determinantal ideals; and unobstructedness and dimension of families of standard determinantal ideals.

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PM — Progress in Mathematics, Vol. 264



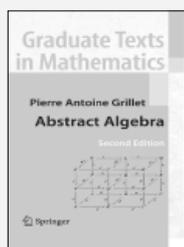
## A History of Abstract Algebra

Kleiner, I., York University, Toronto, ON, Canada

Prior to the nineteenth century, algebra meant the study of the solution of polynomial equations. By the twentieth century it came to encompass the study of abstract, axiomatic systems such as groups, rings, and fields. This presentation provides an account of the history of the basic concepts, results, and theories of abstract algebra. The development of abstract algebra was propelled by the need for new tools to address certain classical problems that appeared unsolvable by classical means. A major theme of the approach in this book is to show how abstract algebra has arisen in attempts to solve some of these classical problems, providing a context from which the reader may gain a deeper appreciation of the mathematics involved. Mathematics instructors, algebraists, and historians of science will find the work a valuable reference. The book may also serve as a supplemental text for courses in abstract algebra or the history of mathematics.

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# New Textbooks from Springer



## Abstract Algebra

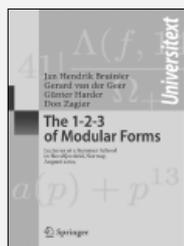
P. A. Grillet, Tulane University, New Orleans, LA, USA

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G. van der Geer, University of Amsterdam, The Netherlands;

G. Harder, University of Bonn, Germany; D. Zagier, Max-Planck-Institut für Mathematik, Bonn, Germany  
K. Ranestad, University of Oslo, Norway (Ed.)

This book grew out of three series of lectures given at the summer school on "Modular Forms and their Applications" at the Sophus Lie Conference Center in Nordfjordeid in June 2004. Each part of the book treats a number of beautiful applications, and together they form a comprehensive survey for the novice and a useful reference for a broad group of mathematicians.

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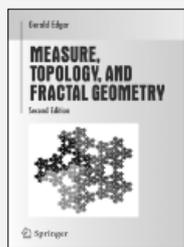
## Probability Theory

### A Comprehensive Course

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Aimed primarily at graduate students and researchers, this text is a comprehensive course in modern probability theory and its measure-theoretical foundations. It covers a wide variety of topics, many of which are not usually found in introductory textbooks. Plenty of figures, computer simulations, biographic details of key mathematicians, and a wealth of examples support and enliven the presentation.

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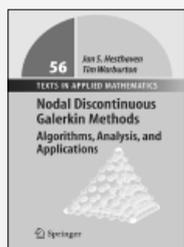
## Measure, Topology, and Fractal Geometry

G. Edgar, The Ohio State University, Columbus, OH, USA

Based on a course given to talented high-school students at Ohio University in 1988, this book is essentially an advanced undergraduate textbook about the mathematics of fractal geometry. It nicely bridges the gap between traditional books on topology/analysis and more specialized treatises on fractal geometry. It contains plenty of examples, exercises, and good illustrations of fractals, including 16 color plates.

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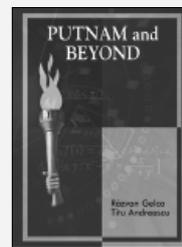
## Nodal Discontinuous Galerkin Methods

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J. S. Hesthaven, Brown University, Providence, RI, USA; T. Warburton, Rice University, Houston, TX, USA

This book introduces the key ideas, basic analysis, and efficient implementation of discontinuous Galerkin finite element methods (DG-FEM) for the solution of partial differential equations. All key theoretical results are either derived or discussed, including an overview of relevant results from approximation theory, convergence theory for numerical PDE's, orthogonal polynomials, and more. Through embedded Matlab codes, the algorithms are discussed and implemented for a number of classic systems of PDEs.

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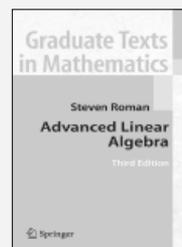
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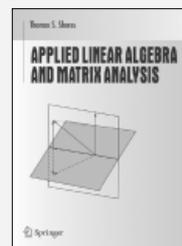


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