NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

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Issue 67
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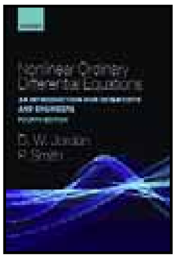

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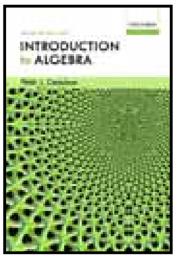

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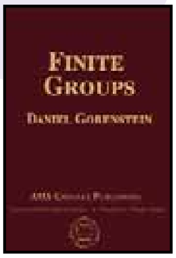
American Mathematical Society



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required reading for anyone who wishes to study the subject.

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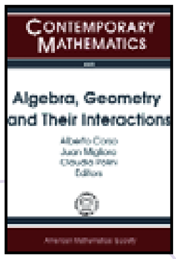

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European Mathematical Society

Newsletter No. 67, March 2008

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EMS Calendar

2008

29 February–2 March
Joint Mathematical Weekend EMS-Danish Mathematical Society, Copenhagen (Denmark)
www.math.ku.dk/english/research/conferences/emsweekend/

3 March
EMS Executive Committee Meeting at the invitation of the Danish Mathematical Society, Copenhagen (Denmark)
Stephen Huggett: s.huggett@plymouth.ac.uk

7–11 April
LMS and EMS invited lecture by Andrei Okounkov, Imperial College, London,UK; http://www.ma.ic.ac.uk/~rpwt/LMS.html

26–27 April
Meeting of presidents of Mathematical Societies in Europe, at the invitation of the French Mathematical Society SMF, CIRM Luminy (France)

1 May
Deadline for submission of material for the June issue of the EMS Newsletter
Martin Raussen: raussen@math.aau.dk

30 June–4 July
The European Consortium For Mathematics In Industry (ECMI), University College London (UK); www.ecmi2008.org/

11 July
EMS Executive Committee Meeting, Utrecht (The Netherlands)
Stephen Huggett: s.huggett@plymouth.ac.uk

12–13 July
EMS Council Meeting, Utrecht (The Netherlands)
Stephen Huggett: s.huggett@plymouth.ac.uk;
Riitta Ulmanen: ems-office@cc.helsinki.fi
http://www.math.ntnu.no/ems/council08/

13 July
Joint EWM/EMS Workshop, Amsterdam (The Netherlands)
http://womenandmath.wordpress.com/joint-ewmems-workshop-amsterdam-july-13th-2008/

14–18 July
5th European Mathematical Congress, Amsterdam
www.5ecm.nl

3–9 August
Junior Mathematical Congress 8, Jena (Germany)
www.jmc2008.org/

16–31 August
EMS-SMI Summer School at Cortona (Italy)
Mathematical and numerical methods for the cardiovascular system; dipartimento@matapp.unimib.it

8–19 September
EMS Summer School at Montecatini (Italy)
Mathematical models in the manufacturing of glass, polymers and textiles
web.math.unifi.it/users/cime/

28 September – 8 October
EMS Summer School at Będlewo (Poland)
Risk theory and related topics
www.impan.gov.pl/EMSsummerSchool/

Breaking the glass ceiling to reach the sky

Frances Kirwan (Oxford, UK) and Sylvie Paycha (Clermont-Ferrand, France)[1]


Women, who according to a Chinese expression hold up half the sky, make up much less than half of the European mathematical academic community. In spite of some progress to be observed when comparing recent statistics with data gathered a decade earlier, we are still far from a gender balanced mathematical academic community throughout Europe on all scales of the hierarchy.

According to the European Commission,[2] in 2003 only 31.6% of the PhD graduates in mathematics and statistics within the European Union were women, the rates in different countries ranging from 16% in Sweden to 58.3% in Portugal and rising up to 75% in Lithuania. High rates in southern Europe contrast with traditionally low ones in northern European countries, although the latter have increased in past years. Whereas in 1993 the percentage[3] of women among academic mathematicians in Scandinavia ranged from 2.4% in Finland to 8.7% in Norway, in 2005 the extremes were 9.7% in Denmark and 14.5% in Finland. The corresponding rates in southern Europe, where the increase is less significant than in the north, ranged from 15.6% in Spain to 45% in Portugal in 1993, whereas in 2005 they went from 26.3% in Spain to 47.6% in Portugal.

A young woman mathematician glancing at this rather rough overview of the European map for women in mathematics[4] might be inclined to move south or east for better job opportunities! She would probably be disappointed to find out that, generally speaking, the status of academic positions – as well as the working conditions and salary that go with it – tends to sink where the percentage of women occupying them increases; put the other way around, women are fewer where the positions are more prestigious.

Other criteria come into play in job-hunting, which might make this young woman mathematician accept a position even if it does not quite meet all her aspirations. Childcare, which is an important criterion to take into consideration for anyone who hopes to raise children and have an academic career, might frighten her away from countries like Germany where places in kindergartens are still relatively rare, where leaving a child in a kindergarten all day is traditionally frowned upon and where even for older children, school finishes as early as 2 or 3 pm. France, which has a long tradition of childcare and where children start school at the age of three and go to school from 8 a.m. to 4.30 or 5 p.m. when they are older, might seem tempting at first glance. Indeed, its percentages by gender (24.3% female PhD graduates in mathematics and statistics in 2003 according to Eurostat, and 23% women among the mathematicians in France according to the Oxford Brookes survey) seem to reflect its intermediate geographical position – neither too far north nor too far south! But this young woman mathematician might find it disconcerting to hear about the decline or at best the stagnation of the percentage of women among mathematicians in France in recent years:[5] 20.8% in 1998 against 20.4% in 2005, and for 19% of women among researchers with a pure research position (CNRS) in 1989, there were only 16% in 2005. The figure of almost 25% women among assistant professors in pure mathematics (Maître de Conférences) at French universities in 1996 drops down to just over 21% in 2006, while almost 9% women among university professors in pure mathematics in France in 1996 drops to just over 6% in 2006. A partial explanation for this decline is the fusion in 1987 of the French elite schools for higher education (Ecoles normales supérieures) which used to be either only for women (Sèvres, Fontenay) or only for men (Ulm, Saint Cloud). Many senior women researchers in France were trained in these elite schools, which now produce many more male researchers than they do women researchers.

Another factor which might influence this young woman's choice as to where to apply for a position is whether a job is permanent or temporary; in a temporary position she might find it difficult to raise children and simultaneously manage to be sufficiently scientifically productive to hope to compete successfully later for a permanent position. Few European countries offer a permanent position early in one's professional life; the fact that France does probably contributes to the relatively high percentage rates of women in French mathematics.

Once this young woman mathematician has finally found a position and settled down somewhere in Europe,

[1]We thank Catherine Hobbs, Marjatta Naatanen and Dusanka Perisic for their useful comments on a preliminary version of this text.

[2]Eurostat Education Statistics EUR22049, She figures 2006, Women and Science, Statistics and indicators. Luxembourg: Office for Official Publications of the European Commission, 1–114.

[3]The data in this paragraph was gathered by Catherine Hobbs and Esmyr Koomen (Oxford Brookes University) whose statistical study was funded by the UK Royal Society Athena Awards.

[4]Comparative maps for 1993 and 2005 presenting the statistical data gathered by C. Hobbs and E. Koomencan be found on the webblog of the EMS Committee for Women in Mathematics: http://womenandmath.wordpress.com/past-activities/statistics-on-women-in-mathematics.

[5]This statistical data was presented by Laurence Broze, University of Lille, at the twentieth anniversary of the French Organization Femmes et Mathématiques in Paris in 2007, and can be found on the webblog http://womenandmath.wordpress.com.

she is of course likely to want a promotion at some stage of her career, but she might come across a few obstacles on the way to that goal. Indeed, in 2003 for example,[6] whereas 34% of the graduate students in science and engineering within the EU were women, only 9% of the academic staff in science and engineering in the highest grades/posts were women. Sadly this goes with a low representation of women in decision-making positions in the scientific academic community, from which they might hope to have some influence on these rather dismal figures.

Before reaching the sky, women mathematicians first need to break the glass ceiling!

[6]Eurostat data (see above).

Frances Kirwan *[kirwan@maths.ox.ac.uk]*
is presently convenor of EWM.
Sylvie Paycha *[Sylvie.Paycha@math.univ-bpclermont.fr]*
is presently chairing the EMS committee for women and mathematics.

EMS Committee on Women and Mathematics

Sylvie Paycha (Clermont-Ferrand, France)[1]

According to the official terms of reference, this committee has the purpose to work as a fact-finding unit exposing the problems and supporting the recognition of achievements of women in mathematics. It is directed to take such actions as it deems appropriate to encourage more women to study mathematics at school level, at university level, and at research level, and to support women mathematicians in the academic positions. For example, this may involve the holding of special meetings, competitions, publicity, summer schools, and the organization of mentor rings directed to women interested in pursuing careers in mathematics, as well as actively supporting such activities initiated by other mathematical societies such as European Women in Mathematics (EWM) and Association *Femmes et Mathématiques*, and closely liaising with them.

It is directed to work towards the goal of achieving a sound (arguably equal) balance between the numbers of men and women involved in mathematics at all levels, with mutual respect towards each other as mathematicians, and with equal career opportunities in mathematics.

Let us now describe some of its concrete activities, both past and present.

Past activities

Questionnaire

Under Emilia Mezzetti's coordination (2002–2005), a questionnaire on mathematicians' careers was distributed in the European mathematical community and then analyzed. The questionnaire contained questions about progression in the career (age, age of Ph.D., age of first permanent position, number of temporary positions…), about family (parents' and partner's jobs and, number of children,…), about scientific activity (scientifically most productive age, possible gaps in mathematical production and reasons for these gaps). The aim was to check in how far there are differences between men and women CVs, and in particular whether women's scientific careers generally evolve slower than that of men. The conclusions drawn from this questionnaire can be found on the web site http://womenandmath.wordpress.com/past-activities.

Mentoring scheme

Catherine Hobbs, who is a member of the committee since 2003, has made an important job, organising the EWM web-based mentoring scheme. The EWM web-based mentoring scheme, set up in 2001 using EU funding, won the British Computer Society prize in the 2003 UK Royal Society Athena Awards. The prize was awarded to the scheme for the best use of information technology in the achievement of the aim of advancing women's careers in science, engineering and technology in higher education and research.

Recent activities and short term projects

Webblog

A new and yet already very lively webblog was started by Dusanka Perisic at

http: //womenandmath.wordpress.com

[1] The author is the present coordinator of the EMS Committee on Women and Mathematics. She is very grateful to Catherine Hobbs, Frances Kirwan, Dusanka Perisic and Sheung Tsun Tsou for their comments and suggestions while writing this text.

which as well as some statistical data and reports on past activities of the committee, also presents a gallery of portraits of living women mathematicians, each of whom is introduced by one of her women colleagues.

This webblog is still at an initial stage and suggestions to improve this web log are very welcome.

Joint EMS/EWM meeting

A one-day joint EMS/EWM meeting will take place on July 13th at the University of Amsterdam. It is organised around four introductory talks by Hélène Esnault (Düsseldorf), Alina Vdovina (Newcastle), Christine Bernardi (Paris) and Francesca Rapetti (Nice). This one-day meeting on the eve of the European Congress of Mathematics will provide an opportunity to get acquainted with some of the areas of research represented at the ECM and to meet other women mathematicians present at the ECM.

Ongoing projects

This joint EMS/EWM meeting is a first step towards strengthening the bonds between EWM and the EMS committee for women in mathematics. Our general aim is to combine EWM's long lasting with the EMS committee's more recent experiences to further develop each of these organizations, making the most of their specific assets but nevertheless respecting their own identities. Here are some of these assets, starting with EWM:

- EWM has a long experience of organizing meetings with all women speakers, which mix scientific talks on a variety of topics and discussions and round tables on issues related with women in mathematics. In spite of their high scientific level, the atmosphere in these meetings is very relaxed and uncompetitive thereby generating many interactions both on a scientific and on a more personal level.
- EWM has set up a mentoring scheme for young women mathematicians around Europe as well as a good e-mail network (now reinforced by the EMS committee Web log) which spreads out useful information to women mathematicians around Europe and helps them keep in contact with each other. This helps break up the scientific isolation some of these women suffer from.
- EWM has acquired some valuable experience in encouraging dialogue between women experts in different fields, in particular via some workshops it has organized in the past on specific but yet interdisciplinary topics such as Renormalisation (Paris), Moduli spaces (Oxford), Group theory (Bulgaria) and String Theory (Oberwolfach).

Let us point out that in spite of numerous activities which helped EWM forge itself a strong identity over its some twenty years of existence, EWM has been functioning in the back stage and is lacking scientific recognition from the European scientific community. In contrast, whereas the EMS committee for Women and Mathematics is still in the process of forging its own identity, it benefits from the scientific aura of the EMS and thereby from an open stage onto the mathematical community. Closer ties between EWM and the EMS committee should help EWM to get better scientific recognition and the EMS comittee to build up a more definite identity.

Here are a few suggestions that arose since Frances Kirwan is coordinating EWM and which should contribute to making new bonds:

1. We are in the process of creating a scientific committee of around 12 prominent women mathematicians either of European origin or Europe based, who could be consulted on scientific issues concerning women in mathematics, which could range from giving their opinion on the scientific program of an EWM meeting to making concrete suggestions to male or female colleagues in search of prominent women mathematicians in some specific field.
   The composition of the committee will be submitted to the EMS for approval.
2. We hope to combine the lectures delivered by women speakers chosen for an EMS lectureship with EWM general meetings which take place every other year, organizing the EWM meeting at a host institution of the lectureship and around the set of lectures delivered that year. The scientific committee mentioned above could take part in the selection of the lecturer. We hope that the next EWM general meeting, to take place in Novi Sad, Serbia in 2009, will include lectures by the 2009 EMS lecturer.

Let us hope that these yet very modest actions can contribute to a better balance between the percentage of men and women mathematicians at all levels of the academic hierarchy!

Sylvie Paycha [Sylvie.Paycha@math.univ-bpclermont.fr], professor at the University of Clermont-Ferrand, presently coordinator of the EMS committee for women and mathematics, chaired the French organisation Femmes et Mathématiques from 1992 to 1995 and was convenor of European Women in Mathematics from 1995 to 1997. Her present mathematical topics of interest are renormalisation procedures in mathematics and physics using pseudodifferential analysis.

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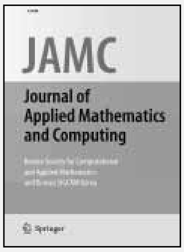

Journal of Applied Mathematics and Computing

Korean Society of Computational and Applied Mathematics

Editor-in-Chief:
Chin-Hong Park, Korea

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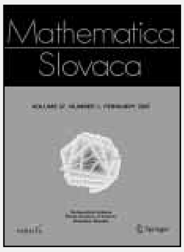
Journal no. 12190



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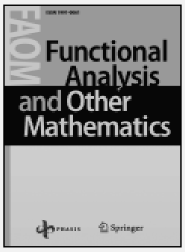

Functional Analysis and Other Mathematics

PHASIS GmbH and Springer-Verlag

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Vladimir I. Arnold, Moscow, Russia

As its name implies, the journal **Functional Analysis and Other Mathematics** publishes papers not only on functional analysis, but in related areas as well. The editorial board is especially interested in work at the interface of functional analysis and other branches of mathematics, in areas that use ideas and methods of functional analysis and that expand its borders. The similarity between the geometry of finite-dimensional spaces and properties of function spaces (such as between the theory of Fourier series and the study of Cartesian orthogonal coordinates) was developed to the point where, at the beginning of the twentieth century, the study of these areas came to be called functional analysis. Today, this enormous branch of mathematics and physics (which has, for example, created the basis of quantum mechanics, constructed the geometry of function spaces, and described the states of physical systems) has become one of the most important domains both in fundamental mathematical research and in applications to the natural sciences.

ISSN 1991-0061 (print version)
ISSN 1863-7914 (electronic version)
Journal no. 11853

Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg

The first issue of the **Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg** was published in the year 1921. This international mathematical journal has since then provided a forum for significant research contributions.
The journal covers all central areas of pure mathematics, such as algebra, complex analysis and geometry, differential geometry and global analysis, graph theory and discrete mathematics, Lie theory, number theory, and topology.

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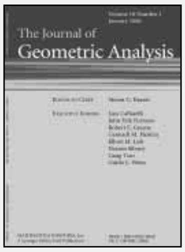

Journal of Geometric Analysis

Managing Editor:
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EMS Executive Committee meeting Nicosia, Cyprus, 27–28 October 2007

Vasile Berinde, EMS Publicity Officer

The archbishop's palace in Nicosia

The last 2007 EMS Executive Committee meeting was held at Hilton Park Hotel in Nicosia, on the invitation of the Cyprus Mathematical Society. Present were: Ari Laptev (*President*, in the Chair), Pavel Exner and Helge Holden (*Vice-Presidents*), Stephen Huggett (*Secretary*), Jouko Väänänen (*Treasurer*), Olga Gil-Medrano, Mireille Martin-Deschamps, Sir John Kingman, Vasile Berinde, Mario Primicerio, Martin Raussen and Riitta Ulmanen. The President expressed the appreciation of the EC for all the work which had been done by the local organizing team in preparation for the meeting and invited its head, Gregory Makrides, president of Cyprus Mathematical Society, to give a brief presentation on the work of that society.[1]

Officers' reports

The President reported on the discussions about a prize in the History of Mathematics with Joachim Heinze from Springer, and described his letter to the Presidents of national mathematical societies which proposed, amongst many other things, a biannual meeting with them. In a positive reaction to the letter, more than 24 Presidents have agreed to attend the meeting in Luminy, France, on the 25th and 26th of April 2008. The EMS Treasurer, Jouko Väänänen, then gave a brief review of the financial position, which was healthy and within the budget. Some details were questioned, and it was agreed that some recommendations will be made to the Council about membership fees.

The Publicity Officer passed around the brand new EMS leaflet. The EC proposes to increase EMS publicity, by sending a poster and an issue of the Newsletter to all mathematics departments in Europe, and from five proposed designs for the EMS poster, one was chosen, with some small changes to the text.

Membership

The EC welcomed a letter from the Serbian Mathematical Society indicating a desire to join; there is now the pleasant prospect of Turkey, Montenegro, and Serbia being elected to membership at the Council in Utrecht. There was then a wide-ranging discussion on the overall membership structure. It was agreed that it was as important as it has ever been to retain both individual members and member societies, but that the development of the new web page opened up significant new opportunities for EMS. In particular, it may well be much better to allow individual members (from any part of the world) the possibility of joining independently of their national society, for a fee only slightly higher than it would be if they chose to join through their national society. It was suggested that for their first year, research students should have free EMS membership. It was also agreed to proceed with the credit card membership payment system as soon as possible.

EMS Lectures and Summer Schools

It was agreed to set aside 2,000 euros in order to pay for the costs of filming (and putting on the EMS web site) the lectures to be given by the 2008 LMS Invited Lecturer Andrei Okounkov (who has agreed that his lectures may also be designated as EMS Lectures) in Imperial College London from the 7th to the 11th of April 2008. The EC then turned to the proposed European lectureship for distinguished women mathematicians, and agreed that it could be part of the EMS Lecturer scheme: we are seeking a lecturer for 2009, and it would be reasonable to expect that this lecturer would very often (perhaps every two years) be a woman.

The two reports of the Dubna Summer School were welcomed, and it was agreed that the report from the student should if possible be published in the Newsletter.[2] The very successful Cyprus Summer Schools were

[1] A report appeared in the Newsletter's issue 66, pp. 41–43
[2] Published in Issue 66, pp. 11–12

described by Gregory Makrides, who had been invited by the EC to report under this item. These summer schools are held for secondary school pupils in July and August, and are in Greek, but it may be possible to have an English version. It was agreed that the EMS would help the organizers to contact the Junior Mathematical Congress and the Dubna Summer Schools, so that other possibilities may be explored. Support from the EMS for the European Student Conference was sought. This is for secondary school pupils, and will be from 5–8 February 2009 in Cyprus.

European Congresses of Mathematics

The Committee thanked Jouko Väänänen for his careful work with the 5ecm local organizers, which was agreed to be a very significant step forward in agreeing the congress fees, among other things. It was also agreed to prepare a document for the Executive Committee to use with future ECMs in order that the rules would be very clear at the outset. It was agreed that a round table discussion on industrial mathematics would be organized at 5ECM. It was also agreed that the EMS leaflets should be included in the 5ECM conference pack. The Committee discussed the reports from the members of the subcommittee set up to investigate the three bids (Krakow, Prague, Vienna) for the organization of the 6th European Congress of Mathematics to be held in 2012. It was agreed that a single report of the site visit would be written, based on the personal reports. A draft would be circulated to the Committee for approval, and once approved it would be sent to all three bidders. Then the three bidders would be invited to submit their revised bids in due time for the Copenhagen EC meeting. These revised bids would be made available to all three bidders, allowing them one final revision for the Council meeting itself which will take a decision.

Mathematical Weekends and Publishing

The plans for the Copenhagen Mathematical Weekend were welcomed by the Committee, and it was agreed to award a grant of 3,500 euros for it. The Junior Congress of Mathematics was discussed, and it was agreed to help to publicize it in several ways.

Martin Raussen's report as EMS Newsletter Editor in Chief was welcomed by the EC, and in particular the proposals for finding a new Editor in Chief were agreed. The current state and future developments at Zentralblatt were discussed. There will be a meeting in Valencia in May 2008 of representatives of funding agencies of European governments. It was agreed to try to present the strong case to this meeting for funding at a high level for Zentralblatt. There was a progress report on the membership database and the new web site. The plan is for a single membership database, with access to individual members, member societies, the Helsinki office, and the Publishing House. The first draft of the web site was welcomed. It was agreed that EMS should accept the offer of the use of a server in Bremen.

Standing Committees

Reports from the following committees were welcomed by the EC: 1) *Applied Mathematics*: the proposed guidelines for EMS Summer Schools in Applied Mathematics were accepted; 2) *Developing Countries*: it was agreed that Le Dung Trang should be a member for 2008–2012. It was also confirmed that Tsou Sheung Tsun would be Chair from 2006–2010; 3) *Eastern Europe*: the question of which countries were eligible for support was considered; 4) *Electronic Publishing*: Pavel Exner reminded the EC that this Committee keeps an eye on various electronic databases of mathematical literature, such as Mathematics Online, Digital Mathematics Library, and Numdam. A new project proposal for the DML would be prepared by Thierry Bouche; 5) *ERCOM*: RICAM Linz was approved as a new ERCOM member; 6) *Group of Relations with European Institutions*: it was agreed to keep this valuable group; 7) *Raising Public Awareness*: it was agreed to encourage the RPA Committee to organize a round table discussion at 5ECM, and, if possible to arrange for the traveling exhibition "Experiencing Mathematics" to visit 5ECM; 8) *Women and Mathematics*: it was agreed that the proposed European lectureship for distinguished women mathematicians could be part of the EMS Lecturer Scheme and to advertise the WM webblog in the Newsletter; 9) *Meetings Committee*: it was agreed that it should serve as the scientific committee for the Brussels meeting, to coordinate the proposed minisymposia. It would be very important to have a member of the Applied Mathematics Committee on the Meetings Committee, and other suggestions are also sought. It was noted that there has been no report from the *Education Committee* since a very brief report just before the EC meeting in March.

Closing matters

The next EC meeting will be in Copenhagen on the 3rd of March 2008, in conjunction with the Joint Mathematical Weekend between the EMS and the Danish Mathematical Society. The second 2008 EC meeting will be in Utrecht (Netherlands) on 11 July 2008 in the opening of the EMS Council Meeting, which will be held in the Academiegebouw of Utrecht University, July 12 and 13, 2008, just before the 5th European Congress of Mathematics, to be held in Amsterdam, 14–18 July, 2008. The last 2008 EMS EC meeting will be in Valencia in October or November 2008.

The President closed the meeting by expressing the gratitude of all present to Gregory Makrides for the excellent arrangements which Cyprus Mathematical Society had made for the EMS EC meeting in Nicosia.

5^{th} European Congress of Mathematics Amsterdam, July 14–18, 2008

Early bird registration deadline of April 15, 2008 is coming up! EMS, KWG members and Satellite Conference visitors enjoy extra fee reduction!

Do not miss this excellent opportunity to enjoy plenary lectures by, and meet, ten European top mathematicians (Luigi Ambrosio, Christine Bernardi, Jean Bourgain, Jean-François Le Gall, François Loeser, László Lovasz, Matilde Marcolli, Felix Otto, Nicolai Reshetikhin, Richard Taylor), enjoy Science lectures on Quantum Information Theory (by Juan Ignacio Cirac), Climate Change (by Tim Palmer) and Mathematical Biology (by Jonathan Sherrat), and make your choice from thirty-two invited presentations by outstanding mathematicians and twenty-two Minisymposia covering almost the entire spectrum of mathematics.

Visit the website www.5ecm.nl for detailed information and registration. The cheapest fee is 220 euros, for EMS and KWG members and for attendants of 5ECM Satellite Conferences, *provided that they register before April 15, 2008*. Want to become an EMS member? Consult your national mathematical society or the EMS web site at http://www.emis.de/individuals/membership.html. Want to become a KWG member? Consult the secretary of the KWG (Rob.van.der.Mei@cwi.nl). The registration fee for students is 120 euros.

5ECM is organized under the auspices of the European Mathematical Society, funded (a.o.) by the Dutch Science Foundation NWO, the Royal Dutch Mathematical Society (KWG), and by the Foundation Compositio Mathematica.

The 2008 Wolf Foundation Prize in Mathematics

The Prize Committee for Mathematics has unanimously decided that the 2008 Wolf Prize will be jointly awarded to:

- **Pierre R. Deligne** (Institute for Advanced Study, Princeton, New Jersey, USA) for his work on mixed Hodge theory; the Weil conjectures; the Riemann-Hilbert correspondence; and for his contributions to arithmetic.
- **Phillip A. Griffiths** (Institute for Advanced Study, Princeton, New Jersey, USA) for his work on variations of Hodge structures; the theory of periods of abelian integrals; and for his contributions to complex differential geometry.
- **David B. Mumford** (Brown University, Providence, Rhode Island, USA) for his work on algebraic surfaces; on geometric invariant theory; and for laying the foundations of the modern algebraic theory of moduli of curves and theta functions.

More information: http://www.wolffund.org.il/cat.asp?id=23&cat_title=MATHEMATICS

Crafoord Prize in Mathematics and Astronomy 2008

The Royal Swedish Academy of Sciences has decided to award the Crafoord Prize in Mathematics and Astronomy 2008 with one half (mathematics) jointly to Maxim Kontsevich and to Edward Witten
"for their important contributions to mathematics inspired by modern theoretical physics"

and the other half (astronomy) to Rashid Alievich Sunyaev (IKI, RAS, Moscow, Russia and MPI Astrophysics, Garching, Germany)
"for his decisive contributions to high-energy astrophysics and cosmology, in particular processes and dynamics around black holes and neutron stars and demonstration of the diagnostic power of structures in the background radiation".

The prize amount is USD 500,000. Kontsevich and Witten are awarded one half and Sunyaev the other half.

From the Prize citation:
The laureates in mathematics, the mathematician **Maxim Kontsevich** and the theoretical physicist **Edward Witten**, have used the methodology of physics to develop a revolutionary new mathematics intended for the study of various types of geometrical objects. Their work is not only of great interest in the discipline of mathematics but may also find applications in totally different areas. Its results are of considerable value for physics and research into the fundamental laws of nature. According to string theory, which is an ambitious attempt to formulate a theory for all the natural forces, the smallest particles of which the Universe is composed are vibrating strings. This theory predicts the existence of additional dimensions and requires very advanced mathematics. The laureates have resolved several important mathematical problems related to string theory and have in this way paved the way for its further development.

Maxim Kontsevich, Russian citizen. Born 1964 in Khimki, Russia. Ph.D. in mathematics 1992 at University of Bonn, Germany. Professor at Institut des Hautes Études Scientifiques (IHÉS), Bures-sur-Yvette, France.
www.ihes.fr/IHES-A/People/pers-scienA.html
Edward Witten, American citizen. Born 1951 in Baltimore, MD, USA. Ph.D. in physics 1976 at Princeton University, NJ, USA. Charles Simonyi Professor at School of Natural Sciences, Institute for Advanced Study, Princeton, NJ, USA.
www.sns.ias.edu/~witten

from a press release by the Swedish academy of Sciences. More information: www.crafoordprize.se, www.kva.se

Argentinian mathematician wins the Ramanujan Prize

Jorge Lauret (38) of the Universidad Nacional de Córdoba in Argentina is the winner of the 2007 Srinivasa Ramanujan Prize. The Prize carries a $10,000 cash award donated by the Niels Henrik Abel Memorial Fund. The prize was awarded on the 3rd of December, 2007, at the International Centre for Theoretical Physics in Trieste, Italy.

The prize, set up by the International Centre for Theoretical Physics (ICTP) and the International Mathematical Union, is designed to honour researchers under 45 years of age who have conducted outstanding research in developing countries.
"*Jorge Lauret has been awarded the prize in recognition of his outstanding contributions to differential geometry and group representation*", says the prize selection committee. Jorge Lauret works in the field of Riemannian geometry, especially on questions related to geometry and symmetry.

More information: www.abelprisen.no

European Science Foundation and mathematics

Ari Laptev (London, UK)

The European Science Foundation (ESF) was founded in 1974 in order to promote research in Europe through conferences, communication and the funding of research programmes. Its main office is situated in Strasbourg but it also has offices in Brussels.

The ESF is now an association of 76 major national funding agencies devoted to scientific research in 30 countries. The ESF represents all disciplines via its committees: physical and engineering sciences, life and environmental sciences, medical sciences, humanities and social sciences. Each committee consists of representatives of relevant national research foundations, which provide major funding for various ESF activities.

The most relevant committee for mathematics is the Physical and Engineering Sciences Committee (PESC). Many of the members of this committee are heads of

ESF Mathematics Conference in Poland in partnership with MSHE and the Institute of Mathematics (PAN)

Operator Theory, Analysis and Mathematical Physics

Mathematical Research and Conference Center, Będlewo, Poland; 15–22 June 2008

It is hard to overestimate the importance of spectral analysis of self-adjoint operators due to important applications in quantum mechanics. Recent developments in nano-physics opened a completely new chapter in this area. Important developments are related with random operators and differential operators on families of connected manifolds. Surveys of the state-of-art in the above fields are planned.

The scope of this conference is rather vast and covers such fields as: random and quasi-periodic differential operators, orthogonal polynomials, Jacobi and CMV matrices, and quantum graphs. In particular spectral properties of operators from various domains of mathematical physics will be discussed.

Recent scientific achievements of speakers and diversity of participants are considered as the highlights of the conference. Special sessions for young scientists presenting their recent results will be organized.

Application Deadline: 30 March 2008

Further Information and Online Application: www.esf.org/conferences/08279 national research foundations. It is unusual that a mathematician has such a post at national level and as a result mathematics has not had its own voice at the PESC until recently. Thanks to the efforts of two former EMS presidents, Jean Pierre Bourguignon and Rolf Jeltsch, it became possible for an EMS president to be an observer at the committee meetings. Moreover, since 2006, mathematics has been represented by Professor Manuel de Leon from Madrid, who is a member of the core group of the PESC.

The ESF provides the management structure and administration for the European Cooperation in the field of Scientific and Technological Research (COST). Among the regular ESF activities are:

- Forward looks (foresight studies),
- Exploratory workshops (scientific workshops on emerging scientific fields and new topics),
- Research Network Programmes – RNPs (4–5 year network projects comprising a series of summer schools (CHECK), conferences and workshops, accompanied by exchange grants and short-term visits for scientists),
- ESF research conferences (as the one in Będlewo, Poland, advertised with this article),
- The annual European Latsis Prize, awarded by the ESF itself.

The PESC usually meets twice a year. Autumn meetings are held in Strasbourg and Spring meetings are held in various European countries. The head of the PESC committee is Michel Mareschal, a physicist from Brussels.

From 2002 to 2007, I was coordinating an ESF Programme in Spectral Theory (SPECT), which included thirteen teams from twelve different European countries. I would now like to share my experience of running such a programme.

To begin with, in 2002, I was really surprised by how easy it was to fulfil all the necessary formalities of submitting an official application. I simply wrote about ten A4 pages of text. About five pages contained a description of the project, including what had already been achieved in my area of mathematics and what kind of problems we would like to study in the future (no 'fancy' language was necessary). The second part (about five pages) was a description of the members of all the teams involved. Naturally, before writing this application, I contacted my colleagues asking for suggestions, which I then included in my application.

Eventually our proposal was accepted. The PESC contacted the national research foundations of countries whose teams were involved in the project for funding, and in May 2002 we started our activities with an annual budget of about 85,000 Euro. Most of our expenditure went on supporting conferences, personal research visits and financing travel expenses for PhD students.

All the formalities relating to reports on how we had used the money were most reasonable and not very time consuming at all. I think that the ESF, especially compared with Brussels, found a very good balance in how the organisation functions in supporting research in Europe. The quality of programmes would be even better if mathematicians would be more active in applying for funding from the PESC. Mathematics in the PESC is really welcomed.

The EMS is trying to be more active in establishing closer contacts with the ESF. The EMS on behalf of ERCOM is about to sign a contract with the ESF that will provide partial funding for 5-6 conferences a year, beginning in 2009, at different ERCOM centres. The first contract is supposed to be for five years. This will be important for national mathematics centres, giving them an additional international profile and additional financial support.

The budget of the ESF is supplied by the national research foundations of European countries. It should be noted that the ESF is planning to apply for funding from Brussels for the financial support of EMS conferences rather than obtaining it from their member organisations. Since the ESF is such an established organisation, it has an excellent chance of succeeding (probably a much better chance than the EMS).

The committee of the ESF Research Conferences Unit sent out a call for proposals for conferences in mathematics to be held in 2008. Initially a maximum of three proposals were to be selected. As it turned out only two proposals were received by the deadline that had even been extended to 20 October 2007. It appears that the mathematical community in general did not know about this possibility. We have therefore agreed with the PESC that in future such advertisements should be given via the EMS, which has much better means of delivering information to the mathematical research community.

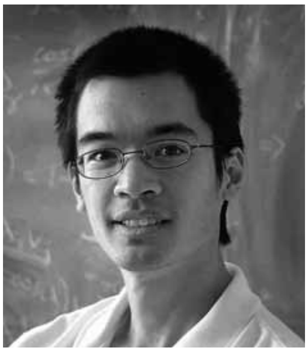
On 28–29 November, the ESF organised a conference on science policy in Europe with the aim of increasing the awareness of politicians regarding the importance of developing science in Europe. The ESF considers itself an important part of the future infrastructure of European research.

Ari Laptev *[a.laptev@imperial.ac.uk] and [laptev@kth.se] is the President of the European Mathematical Society.*

The Theorem of Green–Tao

Valentin Blomer (Toronto, Canada)

"Prime numbers were invented to multiply them, not to add" – Gelfand attributes this famous quote to Russian physicist Lev Landau. Nonetheless, the additive properties of the primes have been a fascinating subject for generations of professional (and lay) mathematicians, and by now there is an ample supply of powerful and flexible methods to attack classical problems.

Ben Green (left) and Terence Tao

A remarkable breakthrough was achieved in the joint work of Ben Green and Fields Medalist Terence Tao. Ben Green completed his thesis in 2002 under the supervision of Tim Gowers and is a professor at Cambridge, UK. Terence Tao was a student of Elias Stein (Princeton) and is currently at the University of California in Los Angelos. He is known for fundamental contributions to an impressive range of mathematical disciplines including harmonic analysis, partial differential equations, signal analysis, operator theory, combinatorial and algebraic geometry and additive number theory. In 2004, Green and Tao announced the following result:

Theorem 1 (Green-Tao [6])**.** *The set of prime numbers contains arbitrarily long arithmetic progressions; in other words, for every integer $k \geq 3$ there is a sequence of primes $p_1, \dots, p_k$ such that $p_2 - p_1 = p_3 - p_2 = \dots = p_k - p_{k-1}$. More precisely, there is a constant $\delta_k > 0$ such that*

$$\#\{(n,d) \in [1,N]^2 \mid n, n+d, \dots, n+(k-1)d \text{ prime}\} \geq \delta_k \frac{N^2}{(\log N)^k}.$$

For example, $7, 37, 67, 97, 127, 157$ is an arithmetic progression of length $k = 6$ consisting only of primes. In 2004, Frind, Underwood and Jobling found a progression of length 23 with first element of size $\approx 5.6 \times 10^{13}$ but of course the existence of such a progression could be a sporadic freak of nature. Notice that the theorem guarantees arbitrarily long progressions of primes but infinitely long progressions do not exist: $n + jd$ is certainly not prime if $j = n$.

This note presents some of the ideas and techniques that go into the proof of the theorem. There are other surveys, for instance [5, 11, 14, 15].

Heuristic approaches

Why could one reasonably expect the theorem of Green-Tao to be true? Prime numbers do not seem to follow any pattern but tend to be distributed rather randomly. This suggests a probabilistic approach: by the prime number theorem

$$\pi(x) := \{p \leq x \mid p \text{ prime}\} \sim \frac{x}{\log x}, \tag{1}$$

the probability that a randomly chosen number n is prime is about $1/\log n$. Let us now draw randomly the first element $n \leq N$ and the difference $d \leq N$ of our progression. Then the probability that the k numbers $n, n+d, n+2d, \dots, n+(k-1)d$ are all prime is about $1/(\log N)^k$, so there ought to be about $N^2/(\log N)^k$ progressions of length k with first element and difference at most N. Here we have tacitly assumed that the events "n prime" and "$n+d$ prime" are independent. This is obviously wrong, as one can see for $d = 1$: if $n > 2$ is prime, then $n+1$ is never prime. Conversely, if $d = 2$ and n is prime, then the (conditional) probability that $n+2$ is prime increases to $2/\log N$, because we know already that $n+2$ is odd. These effects, however, can easily be controlled. They come from obvious divisibility and congruence conditions or, put into more formal language, from p-adic obstructions of the various non-archimedean completions $\mathbb{Q}_p$ of $\mathbb{Q}$. For each prime p we obtain a correction factor so that we arrive at the following conjecture:

Conjecture. For fixed $k \geq 3$ one has

$$\#\{(n,d) \in [1,N]^2 \mid n, n+d, \dots, n+(k-1)d \text{ prime}\} \sim \gamma_k \frac{N^2}{(\log N)^k} \tag{2}$$

for $N \to \infty$, where $\gamma_k = \prod_p \alpha_p(k)$ is an explicitly given, absolutely convergent product over primes satisfying $\gamma_k = \exp((1+o(1))k \log\log k)$ as $k \to \infty$.

If this conjecture was true, then it is obvious that for every fixed $k \geq 3$ there are (in fact, very many) arithmetic progressions of length k if only one takes N sufficiently large, $N = k^k$ say. In particular, up to the constant, the theorem of Green-Tao yields the conjectured order of magnitude for the number of the progressions in question.

In addition to probabilistic considerations, the conjecture is supported by a method of Hardy and Littlewood, the so-called circle method. Developed around 1920, it has been substantially extended and refined and is by now one of the most powerful tools in additive number theory. For $k = 3$, for instance, one could argue as follows. For $\alpha \in \mathbb{R}$, let

$$f(\alpha) := \sum_{\substack{p \leq N \\ p \text{ prime}}} e^{2\pi i p \alpha}$$

be a finite exponential sum over prime numbers.[1] Then one has

$$
\begin{aligned}
I &:= \int_0^1 f(\alpha)^2 f(-2\alpha)\,d\alpha \\
&= \sum_{p_1,p_2,p_3\le N} \int_0^1 \exp\left(2\pi i(p_1+p_2-2p_3)\alpha\right) d\alpha \\
&= \#\left\{(p_1,p_2,p_3)\in[1,N]^3 \mid p_1-p_3=p_3-p_2,\right. \\
&\qquad\qquad\qquad\qquad\qquad \left.\text{all } p_j \text{ prime}\right\},
\end{aligned}
$$

since the integrand vanishes unless $p_1+p_2-2p_3=0$. The right hand side is essentially the quantity that we want to count, so we have to calculate or estimate the integral I. We observe that f can be easily evaluated at $\alpha=0$; one simply has $f(0)=\pi(N)$. Similarly one finds[2]

$$
\begin{aligned}
f(1/3) = 1 &+ e^{2\pi i/3}\#\{p\le N \mid p \text{ prime}, p\equiv 1\,(\mathrm{mod}\,3)\} \\
&+ e^{4\pi i/3}\#\{p\le N \mid p \text{ prime}, p\equiv 2\,(\mathrm{mod}\,3)\}.
\end{aligned}
$$

In general, the integrand of I can be approximated reasonably well at rational numbers p/q with denominator q not too large, if one has some knowledge on the distribution of primes in residue classes modulo q. By continuity one can extend this approximation to a small neighbourhood of these rationals. The integral over this part of the unit interval yields the main term, up to a small error. If one could control the integrand slightly off rational points by other techniques, the remaining portion of the integral can be identified as an error term and (2) is proved. For $k=3$ this is indeed possible and was carried out by van der Corput [1] in 1939.

Another approach to (2) is to detect the primality condition by combinatorial means using the inclusion-exclusion principle, or the sieve of Erathostenes: in order to count primes in a set $\mathscr{S}\subseteq[1,N]$ one subtracts from the total count the multiples of 2, then the multiples of 3 (but adds the doubly subtracted multiples of 6) and so forth up to the largest prime number below $\sqrt{N}$.[3] This yields a sum of $2^{\pi(\sqrt{N})}$ terms with alternating signs that cannot be controlled satisfactorily. Sieve theory provides combinatorial means that in many situations make it possible to carry out a similar procedure ("sieve") at least up to N^δ for some (small but fixed) $\delta>0$. A number $s\in\mathscr{S}$ that is only divisible by primes $p>N^\delta$ has at most $1/\delta$ prime factors; hence we can detect "almost primes" in $\mathscr{S}$ having only a fixed number of prime factors. More generally, a sieve can be applied to k-tuples of numbers, say arithmetic progressions of length k. E. Grosswald [9] showed in 1980 that there are arbitrarily long progressions in almost primes. More precisely, for each k there is a number $A=A(k)$ such that there are (many) arithmetic progressions of length k consisting of numbers with at most A prime factors. The circle method and the sieve method can be coupled: Heath-Brown [10] proved in 1981 that there are infinitely many progressions of four numbers, three of which are prime and one of which has at most two prime factors.

Finally one could hope that no arithmetic properties of the primes are necessary to prove (2) but that the density (1) given by the prime number theorem would suffice to guarantee arbitrarily long arithmetic progressions. This has been a folklore conjecture but it has resisted any proof so far. A famous theorem of van der Waerden [18] (1927) states that for each disjoint partition $\mathbb{N}=\bigcup_{j=1}^n S_j$ of $\mathbb{N}$ into finitely many subsets S_j at least one of them contains arbitrarily long arithmetic progressions. By an ingenious application of the circle method, Fields medalist Klaus Roth [12] showed in 1956 that for some constant $c>0$ every set $\mathscr{S}\subseteq[1,N]$ of cardinality $\#\mathscr{S}\ge cN/\log\log N$ contains an arithmetic progression of length 3. One of the most important theorems in this context is Szemerédi's theorem [13] (1975): for $\mathscr{S}\subseteq\mathbb{N}$ let $d^*(\mathscr{S}) := \limsup \#(\mathscr{S}\cap[1,N])/N$ be the upper density. If $d^*(\mathscr{S})\ge c>0$, then $\mathscr{S}$ contains arbitrarily long progressions. The proof uses, among other things, van der Waerden's theorem. Subsequently, a number of other proofs of Szemerédi's theorem have been found, for example by Furstenberg [2] (1977) and by Gowers [4] (2001) who proved the following stronger result. There are constants $c_k,d_k>0$ such that every subset $\mathscr{S}\subseteq\mathbb{N}$ satisfying $\#(\mathscr{S}\cap[1,N])\ge c_kN(\log\log N)^{-d_k}$ contains an arithmetic progression of length k. Unfortunately, by (1) this is not yet enough for the prime numbers. In 1936, Erdős and Turan conjectured more generally that every subset $\{a_1,a_2,\dots\}\subseteq\mathbb{N}$ contains arbitrarily long progressions providing that $\sum 1/a_j$ diverges (this would be the case for the primes). This conjecture is still open; it is not even known if such a set contains a progression of length 3.

Ideas of the proof

None of the heuristic approaches presented so far can prove (2) but they all play a fundamental role in the proof of the theorem of Green-Tao. The basic idea is to distinguish between sets that behave like random sets (pseudo random sets) and sets that carry a certain structure. For pseudo random sets one can argue probabilistically, whilst otherwise one can exploit the particular structure. One could hope to decompose arbitrary sets into a random and a structured part, and in either case one has a method to detect progressions. Of course, the difficulty is to make this idea precise.

If one searches for arithmetic progressions in the primes, there are obvious effects coming from the distribution of primes modulo "small" numbers; for instance there are no such progressions having difference $d=1$. These effects are responsible for the main term in the circle method and, more or less equivalently, for the correction factor γ_k in the probabilistic approach. For technical reasons, we remove these properties from the primes: while almost all primes are odd, the sequence $(p-1)/2$, $p\ge 3$, is essentially equidistributed modulo 2, since there are about as many primes congruent 1 modulo 4 as primes congruent 3 modulo 4. By further scaling, the sequence of primes can be modified such that it becomes equidistributed in residue classes $p\le P$. Let $W := \prod_{p\le P} p$ and define a number $\tilde p$ to be a "modified prime" if $W\tilde p+1$ is prime. In order to prove (2), two goals need to be achieved: the notion of "structure" versus "pseudo random set" has to be made precise and we need to study the influence of this structure in the case of the primes. Both questions are, in general, very hard.

To get started, let us regard subsets $\mathscr{S}\subseteq[1,N]$ as a subset of $\mathbb{Z}/N\mathbb{Z}$ and let us consider the Fourier coefficients of the characteristic function $\mathbf{1}_{\mathscr{S}}(x)$. Thus we look at exponential sums of the form

$$
\sum_{s\in\mathbb{Z}/N\mathbb{Z}} e^{2\pi i sr/N}\mathbf{1}_{\mathscr{S}}(s) = \sum_{s\in\mathscr{S}} e^{2\pi i sr/N}, \quad r\in\mathbb{Z}/N\mathbb{Z}. \tag{3}
$$

We compare these with the Fourier coefficients of the expected value

$$\sum_{s\in\mathbb{Z}/N\mathbb{Z}} e^{2\pi i s r/N}(\#\mathscr{S}/N), \quad r\in\mathbb{Z}/N\mathbb{Z}. \qquad (4)$$

If the difference of (3) and (4) is small for all $r\in\mathbb{Z}/N\mathbb{Z}$ then this might indicate that $\mathscr{S}$ behaves like a random set. Indeed, this approach suffices for progressions of length 3, which is why van der Corput could successfully complete the case $k=3$. In general, however, the definition of a pseudo random set depends on the objects being considered. If one counts longer progressions, the definition of a pseudo random set becomes much harder. Here one also has to sum the characteristic function of $\mathscr{S}$ against quadratic exponentials such as $e^{2\pi i s^2/N}$, or even more complicated terms of the type $e^{2\pi i s\sqrt{2}\{s\sqrt{3}\}}$.[4] The precise description of the right set of test functions is very hard as is the proof that the characteristic function on the modified primes does not correlate with these test functions. With hindsight, Green and Tao worked this out in [7, 8] for $k=4$. In general, however, this is in some sense harder than the (still unsolved) twin prime problem. Therefore, Green and Tao use a slightly different approach.

By (1), the prime numbers are too thin to apply Szemerédi's theorem, even in Gowers' quantitative form. The idea of Green-Tao is to embed the prime numbers into a suitable, sufficiently manageable and not much larger set $\mathscr{P}'$ with the aim of proving a version of Szemerédi's theorem: every sufficiently large subset of $\mathscr{P}'$ contains arbitrarily long arithmetic progressions. In particular, this would imply the stronger statement that every not too thin subset of the primes contains arbitrarily long arithmetic progressions, for example, the set of all primes $\equiv 1 \pmod 6$. "Sufficiently manageable" means that $\mathscr{P}'$ behaves in some precise sense like a random set. Roughly, one can imagine $\mathscr{P}'$ to be the set of almost primes. More precisely, the set is constructed by applying Selberg's sieve and it is possible (but not easy) to check the required properties of $\mathscr{P}'$ by classical techniques.

This version of Szemerédi's theorem is at the heart of the theorem of Green-Tao. For a rigorous realization a complicated and in part newly developed machinery is necessary. For technical reasons it is convenient to replace sets by their characteristic functions and to work generally with nonnegative functions instead of sets. Thus we can reformulate Szemerédi's theorem as follows. If $f:\mathbb{Z}/N\mathbb{Z}$ is a nonnegative, bounded function satisfying $\sum_{n=1}^{N} f(n)\geq \delta N$ for some $\delta>0$ then

$$\sum_{n\in\mathbb{Z}/N\mathbb{Z}}\ \sum_{d\in\mathbb{Z}/N\mathbb{Z}} f(n)f(n+d)\cdot\ldots\cdot f\big(n+(k-1)d\big) \geq c(k,\delta)N^2+o(N^2) \qquad (5)$$

for some positive constant $c(k,\delta)$. In particular, if f is the characteristic function of a set then the left hand side counts the number of arithmetic progressions. The methods developed in Furstenberg's and Gowers' proof of Szemerédi's theorem now play an important role.

Let us consider the following situation. Let $(X,\mathscr{X}\subseteq\mathscr{P}(X),\mu)$ be a probability space, $T:X\to X$ a measurable map with $\mu(A)=\mu(T^{-1}A)$ for all $A\in\mathscr{X}$, and let $k\geq 1$ be an integer. If $E\in\mathscr{X}$ has positive measure, then Furstenberg [2] showed the existence of $n\in\mathbb{N}$ such that $\bigcap_{j=0}^{k-1}T^{-jn}E$ has positive measure. Moreover, he proved the following combinatorial correspondence. If the upper density of $d^*(A)$ of A is positive then there is a quintuple $(X,\mathscr{X},\mu,T,E)$ as above such that $\mu(E)=d^*(A)$ and

$$d^*\big(A\cap(A+m_1)\cap\ldots\cap(A+m_{k-1})\big) \geq \mu(E\cap T^{-m_1}E\cap\ldots\cap T^{-m_{k-1}}E)$$

for all $m_1,\ldots,m_{k-1}$. Putting $m_j=jn$, we recover Szemerédi's theorem. The important novelty of this proof is the introduction of ergodic theory into the discussion that inspired many ideas of Green and Tao.

Gowers' proof uses harmonic analysis. An important construction is the U^d-norm of a function $f:\mathbb{Z}/N\mathbb{Z}\to\mathbb{C}$, defined by

$$\|f\|_{U^d} := \left(\frac{1}{N^{d+1}}\sum_{x\in\mathbb{Z}/N\mathbb{Z}}\ \sum_{\mathbf{h}\in(\mathbb{Z}/N\mathbb{Z})^d}\ \prod_{\omega\in\{0,1\}^d} f(x+\omega\cdot\mathbf{h})\right)^{1/2^d}$$

with the usual inner product $\omega\cdot\mathbf{h}$ in $\mathbb{R}^d$. These norms are used to measure the distance of a function f from its expected value, i.e. from the constant function $N^{-1}\sum_{n=1}^{N} f(n)$. As in (3) and (4), for the characteristic functions of a pseudo random set $\mathscr{S}$ this distance is small in all norms U^d for $d\leq k-1$.[5]

The proof of the theorem of Green-Tao rests on the following method. The characteristic function on the modified primes is normalized by multiplication with $\log N$, such that the new function f has expected value 1. Using the above mentioned Selberg sieve, one constructs a majorant g that is well-behaved as it comes from a pseudo random set (almost primes). Szemerédi's theorem is not applicable for f, since f is not bounded, but Green and Tao show that f can be decomposed essentially into two parts. The first part is close to its expected value with respect to the Gowers-norm U^{k-1} and hence manageable. The second part is majorized by the corresponding portion of g. Using properties of almost primes, one can show that this part of g is bounded. A fortiori, the considered part of f is bounded and the original version (5) of Szemerédi's theorem completes the proof.

The full proof contains an impressive combination of number theory, ergodic theory, harmonic analysis and combinatorics and the techniques are also applicable in other situations. For example, Tao and Ziegler prove in [17] that more generally primes contain arbitrarily long polynomial progressions: if $q_1,\ldots,q_k\in\mathbb{Z}[x]$ are arbitrary polynomials with $q_1(0)=\ldots=q_k(0)=0$ then there are infinitely many $n,d\in\mathbb{N}$ such that $n+q_1(d),n+q_2(d),\ldots,n+q_k(d)$ are simultaneously prime. Similar results are possible, for example for Gaussian primes [16]. Using the methods of Green and Tao it is proved in [3] that every set $\mathscr{S}\subseteq\mathbb{N}$ with positive upper density contains infinitely many progressions of length 4, such that the common difference is of the form $p+1$ (p prime). It is clear that the methods developed by Green and Tao have opened many new doors for arithmetic combinatorics.

Notes

1. Since f is a 1-periodic function, it can be regarded as a function on S^1. This is where the name circle method comes from.
2. The first term 1 comes from the prime 3.

3. This excludes, of course, the primes between 1 and $\sqrt{N}$ but their number should be small compared to all primes in $\mathscr{S} \cap [1,N]$.
4. As usual, we denote by $\{x\} = x - [x]$ the fractional part of x.
5. Recall that the definition of a pseudo random set depends on k.

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Michel Kervaire: 1927–2007

Shalom Eliahou (Calais, France), Pierre de la Harpe, Jean-Claude Hausmann, and Claude Weber (Geneva, Switzerland)

Michel Kervaire, Geneva 2005. Photo: Teresa Dib

Michel Kervaire died on 19 November 2007 in Geneva. He was born in Poland on 26 April 1927. His secondary school education was in France and he studied mathematics at ETH Zurich, defending his thesis in 1955 under the supervision of Heinz Hopf. He published a second thesis in France in 1965. He was a professor in New York (1959–1971) and in Geneva (1972–2007), with several long-term academic visits to Princeton, Paris, Chicago, MIT, Cambridge (UK) and Bombay. He was very active as editor of *Commentarii Mathematici Helvetici* (1980–2001) and Chief Editor of *L'Enseignement Mathématique* (1978–2007). He has an h.c. doctorate from Neuchâtel (1986).

Kervaire has been an inspiring example that a mathematician can be both a generalist and a specialist at the highest level. As a generalist he knew how to appreciate a large range of subjects and encourage others to progress in their work. Among other things, he organised uncountably many meetings in the small mountain village of Les Plans-sur-Bex, mixing younger students with the best international specialists in all kinds of domains: Brauer groups, foliations, arithmetic, von Neumann algebras, knot theory, representations of Lie groups, the theory of codes, ergodic theory and finite groups (to name but a few). As a specialist, he made his mark in several subjects: he strongly contributed to changing differential topology and homotopy theory from the mid fifties onwards, he created the subject of high-dimensional knots, he formulated a famous conjecture (still open) in abstract group theory and he made deep contributions on problems mixing algebra, number theory and combinatorics.

The Kervaire manifold and the Kervaire-Milnor results

In his 1960 *Commentarii* paper, Kervaire constructed the first example in history of a closed topological manifold (indeed a PL manifold) that does not admit any differentiable structure, not even up to homotopy type. This 10-dimensional manifold is now known as the *Kervaire manifold*. The main tools include an invariant of quadratic forms defined over the field of two elements (the Arf invariant) and deep results on stable homotopy groups of spheres; the closely related *Kervaire invariant* is now important in this subject.

In the mid 50s, Milnor had discovered that, on the sphere of dimension 7, there exists a differentiable structure that is not diffeomorphic to the standard one. In their 1963 *Annals* paper, Kervaire and Milnor showed that the set of differentiable structures on S^n, which for $n \neq 3,4$ is also the set of h-cobordism classes of smooth homotopy n-spheres, is a finite abelian group. In particular, S^7 has exactly 28 differentiable structures.

Thus a topological or a PL manifold can have zero or more than one differentiable structures. This was a revolution in our understanding of regularity conditions in topology. The work of Kervaire and Milnor (first independently and then jointly) is a gem from this flourishing time of the topology of manifolds; it announces most of the capital discoveries of the next period: Browder-Novikov, Wall, Sullivan, Kirby-Siebenmann and so on.

From high-dimensional knots to the Kervaire conjecture and to later work

In his French thesis (*Bull. Soc. Math. France*, 1965), Kervaire created the theory of high-dimensional knots, namely that of smooth embeddings of homotopy n-spheres in the $(n+2)$-sphere, for $n \geq 2$. His first result is a characterisation by three purely group-theoretic conditions of those finitely presented groups that can be fundamental groups of knot complements $S^{n+2} \setminus S^n$, for any $n \geq 3$ (stating results for $n=2$ would require too much space here). The proof involves completely original arguments, using the new technique of surgery. The same paper was also the origin of knot modules and of the cobordism of knots.

A by-product of this was the following conjecture, first stated by Kervaire in 1963/64 during conversations with G. Baumslag: let G be a group presented by generators and relations; if adding one generator and one relation gives rise to the trivial group, then G itself is (conjecturally) trivial.

This has been solved in the torsion-free case by Klyachko (1993) but the full conjecture is still open.

Kervaire knew much more than the topics he published about; he was an expert on many subjects of arithmetic, algebra and combinatorics: class-field theory, quadratic forms, algebraic K-theory, etc. In the last twenty years, he has published nearly 30 papers, most of them on the borderline between algebra and combinatorics, covering subjects such as commutative algebra (the so-called Eliahou-Kervaire resolution for stable monomial ideals in polynomial rings), the Hadamard conjecture (on square matrices having ± 1 entries that are orthogonal up to a factor), the possible lengths of Golay complementary pairs of sequences of ± 1 (the original proof used properties of cyclotomic integers) and vast generalisations of the Cauchy-Davenport theorem from additive number theory (given integers $r,s \geq 1$ and a group G, say abelian here, the question is to compute the minimal size of all sets of sums $a+b$ where a,b range in subsets A,B of cardinality r,s respectively).

Michel Kervaire had a special gift for turning moments of life into celebration: at the blackboard, around a coffee, offering great wines or in memorable meals, sometimes in restaurants and sometimes at home with his wife, painter Aimée Moreau.

A longer version of this notice will appear in French in the next issue of *La Gazette des Mathématiciens*.

The first author [Shalom.Eliahou@lmpa.univ-littoral.fr] was one of the many PhD students of Kervaire and is now professor of mathematics at the Université du Littoral in Calais. The other three authors [Pierre.delaHarpe@math.unige.ch, Jean-Claude.Hausmann@math.unige.ch, Claude.Weber@math.unige.ch] were colleagues of Kervaire in Geneva.

ICMI Study 19: Proof and Proving in Mathematics Education

Announcement and Call for Contributions

The International Commission on Mathematical Instruction (ICMI) announces its next ICMI Study: *Proof and Proving in Mathematics Education*.
The *Study Conference* will be held in Taipei, Taiwan, from May 10 to May 15, 2009.

Participation in the Conference is by invitation to the authors of accepted contributions following a refereeing process. The printed proceedings, available at the conference, will contain the accepted refereed submissions of all participants and will form the basis of the study's scientific work. The Conference will be a working one; every participant will be expected to be active. We therefore hope that the participants will represent a diversity of backgrounds, expertise, experience and nationalities.

Call for contributions

The International Program Committee (IPC) invites individuals or groups to submit original contributions. A submission should represent a significant contribution to knowledge about learning and teaching proof. It may address questions from one or more of the study themes, or further issues relating to these, but it should identify its primary focus. The Study themes are set out in the Discussion Document which is available in the "conference program" section of the ICMI Study 19 website (still under construction but functional) http://jps.library.utoronto.ca/ocs/index.php?cf=8 (or via Google: 'ICMI 19').

Submissions will be a maximum of 6 pages, including references and figures, written in English, the language of the conference. Further technical details about the format of submissions will be available on the Study website.

Important dates:

By 30 June 2008: Potential authors upload their papers to the conference website.

By 15 November 2008: Potential authors receive the result of the refereeing process. Invitations to participate in the conference are sent to authors whose papers are accepted.

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Spengler's Mathematics Considered and A Phoenix Reborn?

Philip J. Davis (Brown Univ., Providence, R.I., USA)

Introduction

Sometime during the first years of my graduate study, I came across Oswald Spengler's *The Decline of the West (Der Untergang des Abendlandes*, July, 1918). I surfed it a bit, considered it pretty heavy and indigestible cake, but made a mental note to look at it more carefully when I had the leisure. But at the time I did notice that Spengler had a chapter on mathematics quite at the beginning of Vol. I of his book: *The Meaning of Numbers (Vom Sinn der Zahlen.)*

More than a half century passed without my reading Spengler, but in the course of a recent group discussion of John Horgan's *The End of Science* and *Fukuyama's The End of History*, and Smolin's *The Trouble With Physics*, Spengler's name came up. Several of the participants had not heard of Spengler (!) and I determined then to take a look at *The Decline* – at least the chapter on mathematics. What I found there was both intriguing and annoying, but I also found a number of notions with which I was in agreement and that therefore I was a Spenglerian in those regards. This discovery prompted the present article.

Since there has arisen a generation of mathematicians who know not Spengler, it will not be amiss to present some general information about Spengler and his book. Later, I will focus my comments on one chapter: *The Meaning of Numbers*. My views derive from my experience as a research mathematician with a lively interest in the history and sociology of mathematics.

Spengler and *Der Untergang*

Oswald Spengler (1880–1936) was the eldest of four children born to a middle class and conservative German family. His father was a white collar worker in the Post Office Department. As a child, Spengler read extensively, and later went to several universities reading in an undirected manner in many fields including mathematics and the natural sciences. Failing an appropriate degree, for a number of years he taught science, mathematics and German history in a number of high schools. In 1911, he came into a small inheritance and for the rest of his lonely life, living in Munich; he devoted himself to his writing. On an examination of Spengler's Journal writings, Gilbert Merlio, a critic of culture, concluded that "Angst (fear and anxiety) was the basso continuo of Spengler's life." Anxiety for anything and anybody.

I do not believe that there is an in-depth English language biography of Spengler: there is probably very little of interest other than his intellectual life (But see both Hughes and Koktanek). On the other hand, with the appearance of *Der Untergang*, with its high sales, its critical approval and condemnation, its influence direct and indirect, positive and negative, on all sorts of thinkers, e.g., the mathematician and logician Ludwig Wittgenstein, the amount of copy produced discussing Spengler's notions has been immense.

The Decline of the West is located among the apocalyptic works of which, over the centuries, there has been a vast and steady production. The apocalyptic vision in the New Testament's *Book of Revelation* comes to mind. Continental and American apocalypses combined with warnings of degeneracy of various sorts clog the literature of the 19th Century and still clog it (See Herman). H.G.Wells' *The War of the Worlds* came out in 1898, and though Spengler was mainly influenced by Goethe and Nietzsche, he may very well have read in translation Wells' vision of the future.

At the turn of the 20th Century, Henry Adams and his brother Brooks Adams, American historians, wrote that the country and the world were going down the tube. "Declinists" latch on to a wide variety of symptoms that prompt their pessimism. Today's comparisons of the present state of things with the decline of the Roman Empire are so frequent that though troubling, they've become tedious. The production of "post-apocalyptic" fiction and movies seems endless. However, these works are considered as entertainment while serious signs of degeneracy such as the environment or the infrastructures of civilization, are given only courtesy attention. I would estimate that the mood today is only 20% Spenglerian (pessimistic) and 20% Macaulayan (optimistic), and 60% *après nous la deluge*-ian. I will explain Macaulay later in this essay.

What, very briefly, does Spengler assert in *The Decline*? History has patterns that are not linear but are cyclical. And just as an individual experiences birth, growth, maturity, decay, all great creations and forms in art, religion, politics, economy, science, fulfill themselves and then peter out. Man does not improve and contemporary man is on the downswing. Mathematics is no exception to the general rule of emergence, growth and decay. At the very end of his chapter on number, Spengler writes *"and with this culmination (the theory of groups) our western mathematic, having exhausted every inward possibility and fulfilled its destiny as the* copy and purest expression of the idea of the Faustian soul*, loses its development in the same way as the mathematic of the Classical Culture concluded in the third century. … The time of the* great *mathematicians is past. Our tasks today are those of preserving, rounding off, refining, and selection – in place of big dynamic creation. The same clever detail-work which characterized the Alexandrian mathematic of late Hellenism." p. 90.*[1]

Spengler distinguishes between Apollonian Man and Faustian Man. Apollonian Man is harmonious, measured, ordered, reasoned, limited, balanced in character[2]. Faustian Man strives beyond what is possible and to arrive at infinity. The classic Greek/Roman civilization was Apollonian, epitomized by statuary of great beauty; our Western civilization, beginning, say, in the 14th century is Faustian. Recall: in the Faust legend, a 16th Century tale of hubris and nemesis, Faust sold his soul to the Devil to acquire *all knowledge* (and one night with Helen of Troy.) In between the Apollonian and the Faustian, Spengler interpolates the "Magian" man or culture. By this expression he means the culture associated with what he calls 'Magian group of religions' i.e., Judaism, ancient Chaldean religion, early Christianity, and Islam. By Magian mathematics, he means that of the Arabs (9th to the 15th Centuries.) For reasons that will emerge later, I find the adjective "Faustian" both appropriate and suggestive; I am indifferent to the terms "Apollonian" and "Magian."

The reception of *The Decline* was immediate and completely polarized. It became a best seller in post-WWI Germany and sold well in the many languages into which it was translated. Its advocates were mesmerized, stimulated, locked into its pessimism, and went on to write their own elaborations. Thus, Arnold J. Toynbee produced *A Study of History* in 12 volumes, describing the rise and fall of twenty two civilizations. More recently a whole spate of "gloom and doom" books have been published.

Its detractors gave *The Decline* the cold shoulder. They pointed to Spengler's factual mistakes, his lack of rational reasoning, his pompous posturing and prophesizing, his disturbing pessimism, his ignorance of this and that aspect of human history. Besides, they said that Spengler's main thesis was old hat.

Predicting the future from the past, whether or not the forecasts are pessimistic or optimistic, is known in historiographic circles as "historicism." Generically, and apart from Spengler's variety of historicism, philosopher Karl Popper decried this methodology. Yet, despite the strong polarization of opinion, one critic wrote that "It's easy to criticize Spengler. It's not so easy to get rid of him." In a letter of January, 1920, to Max Born, Einstein said that one goes to bed convinced of Spengler's ideas and wakes up aware of all that is wrong with them.

The Meaning of Numbers and its Reception

I come now to Spengler's Second Chapter in Volume I: *The Meaning of Numbers*[3]. But before discussing its ideas, I would like to point out that this chapter has been met with an almost absolute silence on the part of the Mathematical Establishment. True, as reported in detail by Paul Forman, in the trauma of defeat in WWI, there was an initial manifestation of "Spenglerism" among some German/Austrian mathematicians. Eminent names such as Richard von Mises[4], Gustav Doetsch and others, reflected either directly or by inference the "failure of nerve, …, of the esprit, the self confidence which we expect…" as described by Spengler. But their outbursts were deemed offensive, ultimately rejected, and silence ensued.

In the "standard" English language histories of mathematics you will not find a single reference to Spengler. I have found a few short articles here and there. What explains this Great Silence, this *Todesstille*, as Spengler himself might have called it? Paradoxically, the "voice of silence" may point to what is perceived as deeply disturbing.

I can come up with many reasons. The thought is that Spengler is simply wrong in his interpretation of the history of mathematics. He is uniformed and not up to date as to the fullness and richness of contemporary mathematical achievements. Einstein scoffed at Spengler's ignorance calling it "*Monomanie aus der Schullehrer-Mathematik…*" He is weak on or ignores the mathematics of places like India, China, Central America or the Polynesian Islands. Yet, writing in 1914 or so, Spengler does take note of Cantor's set theory which dates from the 1870s. Spengler expresses little interest in the applications of mathematics or how the physical world can elicit new mathematics. However Spengler takes note of Minkowski's work on relativity which dates from just a few years before he took pen in hand.

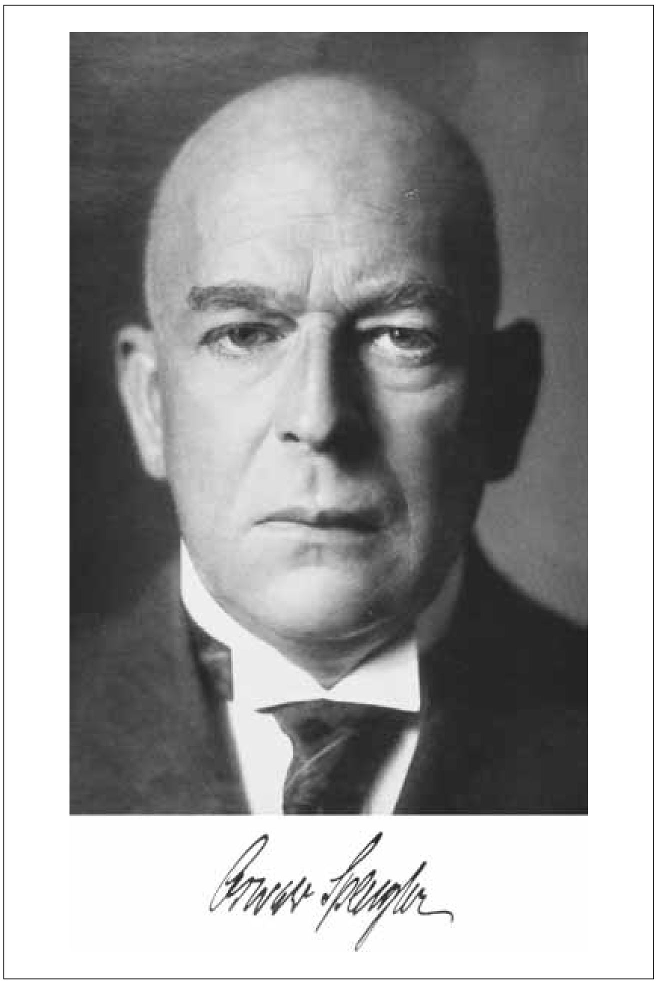
More complaints were voiced: Spengler was a high school teacher and not a part of the mathematical establishment. He embedded his view of mathematics in a much wider and (often fuzzy) and problematic view of human history. He was not a Platonist and most practicing math-

[1] Pages numbers refer to the translation by Atkinson. I maintain the italics and the capitalization. The word "mathematic" (sic) is used in the translation to indicate the mathematics of separate cultures. Occasionally I use this unusual term so as to connect with Spengler.

[2] The terms Apollonian and Dionysian derive from Nietzsche's *The Birth of Tragedy* where they designate two distinct aspects of Classical Greek culture. The Dionysian is the intuitive, the orgiastic, etc. As regards classical mathematics, Spengler downplays the Dionysian.

[3] Spengler's major discussion of mathematics occurs here. However there are less developed notices of mathematics later in Volume 1: e.g., destiny and the principle of causality in Chapter 3, mathematical symbolisms and conceptions of space in Chapter 4.

[4] The conflict in Mises' mind is shown by the statement in his Inaugural Address in 1920 that the age of technology was on its way out; and yet, over the following years he published major works on hydrodynamics and aerodynamics.



ematicians are Platonists. His pessimism was pervasive, melodramatic and insufferable. None of us enjoy thinking of ourselves as living during the decaying end of a historic era. On the contrary, contemporary mathematicians think of the mathematical accomplishments of the past several centuries and especially of today, as alive and vibrant; a new efflorescence, full of new ideas and striking accomplishments contributing to and formatting life as we now live it to a degree that the mathematicians of no previous era had accomplished. Consult Arnold et al. or Engquist and Schmid for descriptions of such accomplishments. And yet, in an interview with Lennart Carleson, brilliant prize-winning mathematician and past president of the International Mathematical Union, reported in Engquist and Schmid, Carleson is quoted as saying "In all probability, we have been living in the golden age of mathematics." Does this imply that we have maxed out?

There may also be political reasons for the silence. Spengler hated freedom and democracy; he saw the necessity (and even the advisability) of constant warfare. He welcomed WWI as the means through which Germany would be revitalized and made dominant through a kind of militaristic socialism. He backed the Nazis initially as a fulfillment of his laws of development and they welcomed his views. Later he balked at the Nazi racial laws and his books were proscribed.

Yet, the silence was not absolutely total. One of the major English language notices of Spengler's chapter *The Meaning of Numbers* was from the pen of James R. Newman, an informed mathematical aficionado. Newman, a lawyer by training and practice, a social activist, a co-author of the Columbia mathematician Edward Kasner and of the logician Ernest Nagel. In 1956 his anthology *The World of Mathematics* appeared in four volumes. These volumes contained a collection of original articles by mathematicians together with brief introductions by Newman. Despite a huge sale, these boxed volumes became "coffee table books," displayed but largely unread. After all, original mathematical papers are difficult; their study and contemplation lies in the domain of professional historians of mathematics; such works are not required reading for mathematics majors and graduate students and a fortiori are beyond the purview of the non-specialist.

In Volume 4 of *The World of Mathematics*, Newman reprints the whole of Spengler's The Meaning of Numbers, together with his appraisal. Spengler's chapter, Newman writes, is

> *"one of the most remarkable discussions in* The Decline. *It is unnecessary to agree with Spengler's thesis to be stimulated by this performance. No one else has made even a comparable attempt to cast a synoptic eye over the evolving concept of number. A good deal of what Spengler has to say on this subject strikes one as far-fetched and misty… [but] his ideas cannot be dismissed as hollow.. This is a disturbing and exciting essay. "*

Historian and social activist, H. Stuart Hughes, writing a few years before Newman, considered this chapter as "perhaps the most provocative section of the whole *Decline*." I agree that it is impossible to dismiss it as unworthy of attention.

Clips from *The Meaning of Numbers*

It is now time for me to take up the contents of this "notorious" chapter which, in Atkinson's translation, runs to about fifty pages. The chapter is packed with everything that Spengler has ever thought about the history of mathematics and its relation to art, architecture, music, poetry, aesthetics, religious feelings and dogmas, mysticism, metaphysics. There is in it both redundancy and vagueness. In view of the length, density, and often (to me) impenetrability of the chapter, I can, alas, present only a few of Spengler's assertions – ones that raised my eyebrows and occasionally my hackles.

Taking the concepts of number and of space and its interpretations as two basic indicators and characterizations of a civilization, Spengler compares and contrasts the number concept of the classical Greek/Roman period with that of the Western period. The former, as I have already noted, he dubs Apollonian, the latter, Faustian. I shall clip and comment principally on Spengler's observations on the concept of number.

In the Classical period Spengler asserts that number relates to magnitude, while in the Western period it relates increasingly to relations.

While Pythagoras (c. 550 BCE) said that number is the essence of all things; Spengler modifies this to: the Classical formulation would be: "Number is the essence of all things *perceptible to the senses*." p. 63

The Classical number concept is limited in its scope and in its overtones:

> *"Classical number is a thought-process dealing not with spatial relations but with visibly limitable and tangible units, and it follows naturally that the Classical knows only the ‚natural' (positive and whole) numbers which on the contrary play in our Western mathematics a quite undistinguished part in the midst of complex, hypercomplex, non-Archimedean and other number systems…*
> *On this account, the idea of irrational numbers - the unending decimal fractions of our notation - was unrealizable within the Greek spirit.… in considering the relation, say, between diagonal and side in a square, the Greek would be brought up suddenly a quite another sort of number, which was fundamentally alien to the Classical soul, and was consequently feared as a secret of its proper existence too dangerous to be unveiled."* p. 64, 65

Among Classical numbers, there is no infinity, no infinitesimals, and no limiting processes.

> *"Classical mathematic of small things deals with the concrete* individual instance *and produces a once-for-all construction, while of mathematic of the infinite handles whole* classes *of formal possibilities,* groups *of functions, operations, equations, curves…"*

As regards numbers, they have become increasingly demagnitudeized:

> *"the development of the new mathematic consists of a long, secret, and finally victorious battle against the notion of magnitude."* p. 76
>
> *"The history of Western knowledge is thus one of* progressive emancipation *from Classical thought…"*
>
> *"And so for the last two centuries… there has been growing up the idea of a general morphology of mathematical operations, which we are justified in regarding as the real meaning of modern mathematics as a whole.* p. 85

Considering the various features of Classical and Western mathematics:

> *"For the first time, it is possible to comprehend in full the elemental opposition of the Classical and Western souls. In the whole panorama of history, innumerable and intense as historical relations are, we find no two things so fundamentally alien to one another as these."*

Apollonian man invented geometry, Magian man invented algebra, Faustian man invented the calculus and, with Descartes, geometry became increasingly separated from the visual experience. Spengler, who thought about art intensely, likes to make analogies that express his view of cultural parallelisms:

> *"The idea of Euclidean geometry is actualized in the earliest forms of Classical ornament, and that of infinitesimal calculus in the earliest forms of Gothic architecture, centuries before the first learned mathematicians of the respective Cultures were born."*
>
> *"The modern mathematic, though 'true' only for the Western spirit, is undeniably a master-work of that spirit; and yet to Plato it would have seemed a ridiculous and painful aberration from the path leading to the ‚true' – to wit the Classical – mathematic."* p. 67

Predicting the twilight of Western mathematics, Spengler writes:

> *"Classical math was fulfilled by the end of the 2nd century AD."*
>
> *"With this culmination, our Western mathematic having exhausted every inward possibility and fulfilled its destiny as the* copy and purest expression of the idea of the Faustian soul*, closes its development in the same way as the mathematic of the Classical Culture concluded in the third century."* p. 90

The divergence of spirit, of inner meaning, between the classical and the modern, leads Spengler to assert that

> "There is not and cannot be number as such. *There are several number-worlds as there are several Cultures. We find an Indian, an Arabian, a Classical, a Western type of mathematical thought and, corresponding with each, a type of number – each type fundamentally peculiar and unique, an expression of a specific world feeling, a symbol having a specific validity which is even capable of scientific definition, a principle of ordering the Become which reflects the central essence of one and only one soul, viz., the soul of that particular Culture.*
>
> *Consequently there are more mathematics than one. For indubitably, the inner structure of Euclidean geometry is something quite different from that of the Cartesian , the analysis of Archimedes is something other than the analysis of Gauss, and not merely in matters of form, intuition and method, but above all in essence, in the intrinsic and obligatory meaning of number which they respectively develop and set forth. … The style of any mathematic which comes into being, then, depends wholly on the Culture in which its is rooted, the sort of mankind it is that ponders it. The soul can bring its inherent possibilities to scientific development, can manage them practically, can attain the highest levels in its treatment of them - but is quite impotent to alter them."* p. 59

The incommensurability of the different cultures is one of Spengler's major points and thus he concludes,

> *"There are no eternal verities in this most abstract and disembodied intellectual activity."*

Some personal reactions

Having given my readers a selection of quotes, I leave to the historians of science and mathematics whether the naked facts as asserted by Spengler are correct. For purposes of my reactions, I take them at face value. In the chapter under review, Spengler takes the long view in which a unit of time is perhaps 500 years. Contemporary mathematicians engaged primarily in teaching or in research, probably consider and need only the work of the past century, if that long a period. Their belief is that later work embraces and incorporates past work even as it refines and strengthens it; and though we are often advised to "consider the works of the masters", we rarely do because we rarely need to. Furthermore, Spengler looks only at what he considers to be major tendencies and trends, perhaps a half dozen in all, often associated with the "great giants in the field." On the contrary, I would assert that there is an integrated corpus of mathematics that includes "the rounding off, the refinement," the further development, the reformulations and simplifications, the applications, without which the major trends, standing in isolation, would be unidentifiable, incomprehensible, and ultimately meaningless.

Number, which is the principal focus of the chapter serves many functions. As cardinals they answer the question: how many. As ordinals they answer the question: how far along in a sequence. As tags they answer the question; which of many. Numbers are also the carriers of the magic, the metaphoric, the iconic.

The movement of number away from magnitude and into relations, as emphasized by Spengler, is clear. One cannot put the complex numbers or n-tuples of numbers into a linear ordering according to size. Numbers are seen now as instantiations of certain abstract structures obeying certain interrelationships. In the 1960s work of Grothendieck, the discrete (i.e., numbers) and the continuous (i.e., the mise en scène of the calculus) are combined into an arithmetic geometry, elevating the non-visual conception of space to a still higher level than that of linear algebra or Banach spaces.

It is very likely the case, that in an age of digital computer communication, by far the major use of number now occurs not to express magnitudes, but to transmit and then to decode and reinterpret trillions of long sequences of 0's and 1's as sentences in natural or technical languages, as graphical images, or as some sort of mechanical or physical action.

The movement away from visualized space is clear. See the chapter *The Decline and Resurgence of the Visual in Mathematics* in Davis [9]. Does the current resurgence vis-à-vis computer graphics fit in with Spengler's theory of cycles?

Spengler sees the mathematics of major cultures as being essentially autonomous; if mutual influences existed, they did not break the barrier of diverse inner expressions. He wishes to determine what those inner expressions are, to tell the story in it own terms without the intervention of contemporary knowledge and of our own contemporary spirit and understanding. In this regard, although he does not mention the name of the historian Leopold von Ranke, he is close to Ranke's famous instruction to write history "wie es eigentlich gewesen," to tell it in its own terms; how it really resided in and engaged the inner life of those who experienced it.

It is a very difficult task to get into the mind and spirit of an individual. It is, e.g., no easy matter to comprehend the original writings of Isaac Newton. One must not only master its Latin, its difficult geometric approach to what is now better expressed by differential equations. One must learn to appreciate how Newton's concern with Biblical exegesis enters into the background and to understand why John Maynard Keynes called Newton "the last of the magicians:"

> *"Newton was not the first of the age of reason. He was the last of the magicians, the last of the Babylonians and Sumerians, the last great mind which looked out on the visible and intellectual world with the same eyes as those who began to build our intellectual inheritance rather less than 10,000 years ago."*

I have said that Spengler's chapter is full of material which I have largely ignored in displaying certain of his statements. The complete text points to a particular mind set which I can only partially understand. By looking hard at Spengler's life, his milieu, and his other writings, I might be able to get just a bit closer. Now if this is the case for one individual, how difficult it is to understand the mind-set (*Weltanschauung*) of a whole civilization in the fullness of its individual lives. Cuneiformist Eleanor Robson has written

> *"It will never be possible to comprehend this [Mesopotamian] mathematics as it was meant to be read , for we cannot entirely escape our own twenty-first century lives and and brains and training, however hard we try…."*

Whig History of Mathematics

In opposition to Rankean historiography, there is what is now often called "Whig History." This term, exemplified early by Macaulay's *The History of England* (1849–1861), interprets history as a story of a linear and inevitable progression toward the present. Whig history is optimistic, politically liberal, and materialistic. The past is prologue and things are getting better and better.

For the most part, comprehensive histories of mathematics are Whiggish. When a piece of ancient mathematics is explicated by translating its statements into current mathematical terminology and current symbols, admittedly this makes for easier comprehension, but the historian is overlaying on what is an ancient point of view with what is a contemporary interpretation in all its overtones.

While all history is selective, Whig history of mathematics, in its rational, Enlightenment point of view, has its own selectivity. As an example, it is well known that over the centuries, mathematics has had a special relation to theology. Whig history, in an act of intellectual cleansing to rid mathematics of what it considers as the irrational, gives short shrift to the relations between theology and mathematics. There has been constant self-censoring to keep mathematics defined by what is at the moment considered to be its true and pure essence: axiomatization, deduction, proof, etc., and this applied to approved material.

Spengler's Introduction to Vol. 1 contains a sharp repudiation of Whig History:

> *"It is a quite indefensible method of presenting world-history to begin by giving rein to one's own religious, political or social conventions and endowing the sacrosanct three-phase system [child, youth, and man] with tendencies that will bring it exactly to one's own standpoint. This is, in effect, making of some formula. a criterion whereby to judge the whole millennia of history. And so we judge that they [i.e., the past] were ignorant of the 'true path', or that they failed to follow it, when the fact is that their will and purposes were not the same as ours."*

Spengler's views are close to the recent emphasis and integrity given to "ethnomathematics" (in e.g., Ascher) a subject that has now taken on political colorations and is given short shrift within western-oriented histories. Today, mathematical expositors tend to view all these various "mathematics", taken together, as inter-interpretable, and as a more or less linear and inevitable development to the present. The modern is regarded as the fulfillment of the classical Apollonian and the Magian just as in theological statements the *New Testament* is regarded as the inevitable fulfillment of the *Old Testament*. Spengler made no such claim; and those who assert dogmatically "the unity of mathematics" would scorn Spengler's denial[5].

Spengler's views are also close to what is now called "social constructivism" (see, e.g., Berger and Luckman) to which I am a partial adherent believing that a good fraction of what is set forth as knowledge derives from and is validated by social interactions. Paul Ernest put it this way:

> *"The social constructivist thesis is that mathematics is a social construction, a cultural product, fallible like any other branch of knowledge."*

Eleanor Robson agrees:

> *"To study mathematics, then, as only mathematics and not as the product of a person's body, brain, and culture, we willfully ignore a historical source of unparalleled richness that has the potential to help us understand the interconnections between mathematics and other aspects of culture and society…"* Robson [27]

A classic instance of social constructivism (or social parallelism) occurs in a major article by Paul Forman wherein Forman claims that the acausality embodied in quantum theory is directly related to and influenced by a general acausal intellectual *Weltanschauung* prevalent in the post WWI Weimar Republic of Germany. Forman shows how widely read Spengler was among Weimar mathematicians and physicists and how much their view of their fields was affected by the spirit and even by the letter of Spengler's book. This thesis was challenged by John Hendry with the counterclaim that acausality in quantum theory was determined largely by considerations internal to the physical evidence and to the formulation of the associated mathematical theory. Both points of view appear to have validity.

The End of Mathematics?

Spengler was by no means the first to cry the "End of Mathematics." In 1751, the skeptical mathematician and encyclopedist Jean le Ronde D'Alembert saw some limitations to the applications of mathematics:

> *"Some [mathematicians] have tried to reduce even the art of curing to calculations, and the human body, that most complicated machine, has been treated by our algebraic doctors as if it were the simplest or the easiest one to reduce to its component parts."*

Yet, today, mathematics has entered biology and medical practice in many different ways. (See the quote in the next section from Freeman Dyson who conjectures that biology will soon be upstaging physics.)

Consider next the 1808 words of Jean-Baptiste Delambre (1749–1822), distinguished mathematician and astronomer:

> *"It would be difficult and rash to analyze the chances which the future offers to the advancement of mathematics; in almost all its branches one is blocked by insurmountable difficulties; perfection of detail seems to be the only thing which remains to be done. All these difficulties appear to announce that the power of our analysis is practically exhausted."*

Consider also the views of the Scottish essayist and historian Thomas Carlyle. Carlyle (1795–1881) knew a bit of mathematics, taught it for a while and in 1821 undertook to translate Legendre's *Eléments de Géométrie* which ran to many editions. But he soon ran into the stone wall of non-comprehension (which all of us do) and abandoned the field. In his 1829 essay *Signs of the Times* Carlyle wrote

> *"The science of the age is … in all shapes mechanical. Our favourite Mathematics, the highly prized exponent*

[5] But consider: "We know of many confirmed instances of transmission of mathematical ideas from one of these cultures to Europe or from one of these cultures to another, but there are numerous instances where, although there is circumstantial evidence of transmission, there is no definitive documentary evidence. Whether such will be found as more translations are made and more documents uncovered in libraries and other institutions around the world, is a question for the future to answer." – Introduction in Katz [22]

of these other sciences, has also become more and more mechanical. Excellence in what is called its higher departments depends less on natural genius than on acquired expertness in wielding its machinery. Without undervaluing the wonderful results which a Lagrange or Laplace educes by means of it, we may remark that their calculus, differential and integral, is little else than a cunningly-constructed arithmetical mill; where the factors, being put in, are, as it were, ground into the true product, under cover, and without other effort on our part than steady turning of the handle. We have more Mathematics than ever; but less Mathesis. Archimedes and Plato could not have read the Mécanique Celeste: but neither would the whole French Institute see aught in that saying "God geometrises" but a sentimental rodomontade."

Moving slightly away from mathematics, Ludwig Wittgenstein lamented that rigidity was choking philosophy:

The nimbus of philosophy has been lost. For we now have a method of doing philosophy, and can speak of skilful *philosophers… But once a method has been found the opportunities for the expression of personality are correspondingly restricted. The tendency of our age is to restrict such opportunities; this is characteristic of an age of declining culture or without culture.*

The choking goes on not just in philosophy. Lee Smolin's book describes the restriction of ideational opportunities within current physics.

The End of Mathematics – meaning the end of mathematical creativity? How can that be? Current practitioners would be outraged at such a thought. Just consider all the developments, the new mathematical ideas that have been promulgated since Spengler wrote; the famous, long standing problems that have been solved, the work of the new crop of prize-winning mathematicians which is of the greatest depth and imagination; or consider the applications that now reach into every aspect of our lives and that promise an even wider reach. But how much more genuinely new mathematics can there be? Do we need another Rembrandt? Do we need or can we have any more Bach or Milton or more Newton or Euler or Riemann? Or is this question absurd?

Is There A Spenglerian „Mathematic" in the Future ?

In consonance with Spengler's assertion of cyclicity, we may view his thesis as an optimistic prediction that a new "mathematic" will arise distinct from our present Faustian mathematics. What would be the hallmarks of a new "mathematic" in the Spenglerian sense? We can answer this by considering the distinctions he made – admittedly problematic and disputed – between the Apollonian, Magian, and the Faustian. These three mathematics are distinct in virtue of great separation of time, space, and in the mindsets of the mathematicians as influenced by the conditions of life in their respective societies. Distinct mindsets lend more than formal interpretations to the lines of naked symbols. In view of modern communications and travel that have greatly reduced both space and time, and computers that have magnified our so-called intelligence and relieved the burdens of formal manipulation, might we anticipate a new Spenglerian mathematic to arise in the not too distant future. Can a mathematical Phoenix for whom Spengler recited the *Nunc Dimittis* emerge from its Faustian ashes?

What harbingers of such a rebirth can I detect? A spotty selection of the major theories or trends of the 20th Century yields, among numerous others, logic/set theory, Bourbakism, catastrophe theory, wavelets, category theory, fractals. Of these, I think that fractals, born of the computer and with its initial deduction-free outlook, comes closest to presaging a new mathematic. Ignoring the embarrassing question of whether the mathematics of the past century or so poses nothing essentially new – "having [in Delambre and Spengler's sense] exhausted every inward possibility," let us instead concentrate on how mathematics is now changing.

The change is rapid and profound: mathematics is "on a roll." The computer is changing the mode of discovery, operation and conceptualization of mathematics. It has overcome some of the "insurmountable difficulties" mentioned by Delambre. It is changing notions of what mathematical developments are interesting, applicable and even what mathematics is. It is my belief that the old paradigms and emphases, especially the current tremendous stress on proof, will become dated, and while they last, they will be considered as exercises in ritualistic nostalgia. Yet, all this is still totally Faustian in Spengler's sense.

The philosopher Alfred North Whitehead remarked somewhere that all later philosophy consists of nibbling on the rump of Plato. Western mathematics has all along, but more substantially since the early 1800's, been nibbling on the rump of Euclid. Some years back [8], I suggested the possibility of accepting into the mathematical canon "visual theorems" generated with an assist from the computer.

[A computer generated visual image] "is a gestalt, complete in itself, self-vindicating, rejoicing in its uniqueness, the carrier for an unlimited number of 'theorems of perceived type' that are grasped or intuited and do not even have to be stated.*"*

This opinion was met with the unraised eyebrows of indifference.

Jonathan Borwein, with parallel but deeper views, emphasizes the role of the experimental mathematics via the computer, and this presents a substantial outline of a possible new mathematic. Borwein predicts (or at least advocates) a mathematics that emphasizes induction, empirical discipline, and finding rather than proving. He writes:

"I hope to have made convincing arguments that the traditional deductive accounting of Mathematics is a largely ahistorical caricature – Euclid's millennial sway not withstanding.".

Paradoxically, the numerous interesting and supporting examples that Borwein and I have brought forth in support of our position are firmly lodged in what might be called neo-Eulerian mathematics. (I refer here to the derivation of numerous special function identities and not to Euler's position on proof which was favorable.) Thus, this kind of development would not of itself be a future mathematic in the Spenglerian sense.

Reading a recent article in the field of Computer AI (Artificial Intelligence), I found that I was immediately thrust into a different mathematical world than the one I grew up in. I found different modes of exposition, different goals, different kinds of argumentation, and different criteria of validity, targeted to a different clientele. And yet, the influence of my generation of mathematics is clear: axiomatization, logics, topological considerations, etc. Despite this "new world" feeling, is it enough to constitute a new mathematic? I think not.

Look across the campus and ask where the action is. Physicist Freeman Dyson recently wrote:

> *It has become part of the accepted wisdom to say that the twentieth century was the century of physics and the twenty-first century will be the century of biology. .. Biology is now bigger than physics, as measured by the size of budgets, by the size of the workforce, or by the output of major discoveries; and biology is likely to remain the biggest part of science through the twenty-first century. Biology is also more important than physics, as measured by its economic consequences, its ethical implications, or by its effects on human welfare.*

If physics is winding down as both Smolin and Dyson suggest, can the new biology, including biotechnology and biomathematics, only now beginning to feel their strength, give rise to a new mathematic? Will a new mathematic emerge from the work of such groups as the Center for Computational Molecular Biology at Brown University? Stay tuned on this question. Such a presumptive mathematic should be applicable to a different set of human concerns, should constitute an upstaging of what is now around, should raise the question as to what mathematics really is, and raise the cry among Old Believers that it is not mathematics at all. It should create a new relationship between it and society. It should fit in with and reflect a new *Zeitgeist* which I cannot yet detect. If anything, I see a reversion away from rationality in which the future carries us back to mediaeval mysticism, theosophy, and similar esoteric doctrines and these may indeed be reflected in the future mathematic.

Mathematics and Society

Mathematics underlies our whole civilization in a way that would have astounded, pleased, but confused Pythagoras. Build a bridge, conduct an election, study the galaxies and, in some way you will engage mathematics. Make an investment and mathematics plays a role. Consider DNA profiling and you enter the burgeoning fields of mathematical genetics and bioinformatics.

The world is being mathematized, computerized, chipified at an increasing rate and the public is hardly aware that mathematics is its basis.[6] Mathematics is a method and a language employed in increasing amounts to give order and to format our social, economic and political lives. It is a method and an attitude that has diffused into biology, medicine, cognitive science, war, entertainment, art, aesthetics, law, sports; an attitude that has created schools of philosophy, and has given support to views of cosmology, mysticism, and theology.

I combine the high marks I give to mathematics with a measure of skeptical caution. I believe that mathematics and its applications lie between common sense and the irrelevance of common sense, between what is intuitive and what is counter intuitive, between the obvious and the esoteric. Mathematics creates the infinitely large and the infinitely small. It sets forth ideals that cannot be achieved by human actions.

Judging from the articles and books about the future of mathematics produced at the turn of the millennium (2000), e.g., in Engquist and Schmid, the sky's the limit with little thought for an Icarian nemesis. Every technological innovation has its down side. This is the message of the Myth of Prometheus who stole fire from the gods and gave it to the humans. Mathematics and its applications have a downside and Spengler was aware of this. Mathematics frequently transforms subjective opinions into so-called objective conclusions bearing the cachet of absolute truth. In the name of logic, mathematics can create seeming impossibilities and nonsense. It the name of social ideals, conceived as logical truths, it can persuade and seduce individuals (thankfully, rarely) to commit anti-social acts.

Natural languages are symbolic systems that have raised humans from the level of Caliban brutes, and the same is true of mathematics which carries with it ethical implications. It is a language that has transformed our lives for good but it can go hog wild when it becomes the handmaiden of new and unprecedented dimensions of human cruelty. The ethical issues raised by science are in the daily papers. The ethical issues involved in mathematical thinking should also be recognized and pondered so as – in the words of Bertrand Russell – "to tip the balance on the side of hope against vast forces."

We are, as asserted by Spengler, in the Faustian Age and I believe this will persist for the foreseeable future. The spirit of Googolization now wafts over all enterprises, and like that of Faust, wants to embrace all, capture the whole of human culture and, indeed, the whole of the cosmos, in one vast system provided with search engines and other peripherals. Giant businesses are swallowed up by behemothian businesses; theories of everything (TOE's) are promulgated in physics; and in mathemat-

[6] In a recently appeared book, *Technology in Postwar America*, Carroll Pursell, Columbia Univ. Press, 2007, hardly a word is said about mathematics as the infrastructure that underlies much contemporary technology. The book mentions computers but not that they serve mathematical functions.

ics, generalizations, abstractions and "foundations" are sought that purport to wrap up more and more with less and less. Even Hilbert's program of proposing to formalize *all* of mathematics, of discovering a method for deciding the truth of *any* mathematical statement, may be viewed as Faustian. (Of course this program crashed on the rocks of Gödel's Incompleteness Theorem.)

More than Faustian: if one gives credence to the predictions emerging from the futuristic think tanks, one would conclude that the computer is moving us inevitably and with juggernaut force towards a super- or trans-Faustian civilization. Thus, Hank Bostrom, the Director of the Future of Humanity Institute at Oxford, in a scenario right out of sci-fi, envisions the creation of computers with more brain power than all the humans in the world put together, and the consequent emergence of "post-humans" who play around with virtual ancestors, etc., and where the distinction between the real and the virtual has totally disappeared. This rewarms the ancient notion of "the world as illusion" by giving it a technological boost.

But "The End of Mathematics"? The language of mathematics is inextricably embedded in natural languages and hence is a vital part of communication. Can one imagine an end to language? But whether or not the languages of the future, natural or mathematical, will bring forth expressions that will recall and match the Homers, the Archimedes, the Shakespeares, the Newtons, the Mozarts or the Riemanns, is beyond our vision but not our hope.

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Two books, The Mathematical Experience and Descartes' Dream, written jointly with Reuben Hersh of the University of New Mexico, explore certain questions in the philosophy of mathematics, and the role of mathematics in society.The Mathematical Experience won an American Book Award for 1983.

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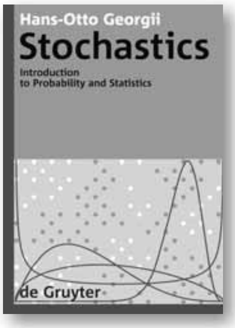

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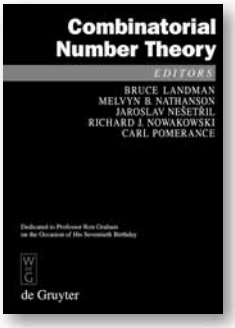

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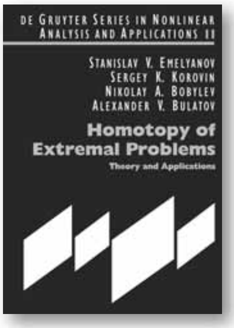

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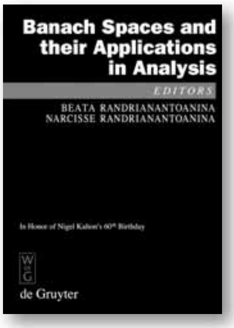

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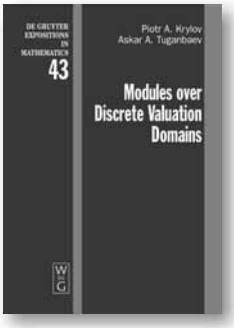

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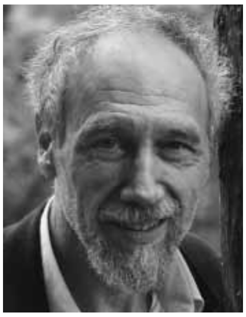
An interview with Alain Connes, part II

The interview was conducted by Catherine Goldstein and George Skandalis, Paris.[1]
The first part appeared in issue 63, pp. 25–31 of the Newsletter.



Alain Connes

Among the results you have obtained, is there one you are most proud of?

Being a scientist is (as far as I am concerned) a pretty humbling activity and I am not keen on showing any pride for any result. I tend to be suspicious of arrogant people. In fact what really matters to me is the pleasure of the discovery as opposed to the appreciation of the result by the community. The amount of joy one gets, the "kick" is of course quite variable and for instance, just to try to answer your question, the link between renormalization and the Birkhoff decomposition, that we found in 99 in our joint work with Dirk Kreimer gave me a great kick that lasted over a full week. I used to behave in a proud manner as a kid till I reached the age of ten, when I was sent to the scouts by my parents. I landed among a tough group and they taught me, one day, by a "group mockery" of myself that they did not buy my proud attitude. Since then I have, like the bulls in the corrida after the session with the picadors, always stood with a slightly bent back.

Following your mathematical quest since the Seventies, one has the impression that you have always been fascinated by physics – and the zeta function.

Absolutely. My fascination for the Riemann zeta function comes from reading Weil's work on his reformulation in terms of idèles of the explicit formulas of Riemann which relate the zeros of the zeta function with the distribution of primes. There is a striking analogy between the "prime number" side of this formula and the fixed point contributions in a Lefschetz formula and the first problem is to find a space X on which the idèles are acting so that the Riemann-Weil explicit formula becomes a trace formula. At some point, after reading a paper of Victor Guillemin on foliations and the Selberg trace formula I realized that the space X should be a space of leaves of a foliation and hence a noncommutative space. I remained fascinated by this idea for ten years until after going to a conference in Seattle on the Riemann zeta function, I realized that the space X was already present in my work on quantum statistical mechanics with Bost, and is simply the adèle class space: the quotient of the space of adèles by the action of the multiplicative group of the field. This gives the interpretation of the Riemann-Weil explicit formulas of number theory as a trace formula and the spectral realization of the zeros as an absorption spectrum.

[1] Thanks to Jim Ritter for his linguistic help.

It is still quite far from giving the relevant information on the location of the zeros but it gives a geometric framework in which one can start to transpose the proof of Weil for the case of global fields of positive characteristic. In our joint work with Katia Consani and Matilde Marcolli we have now shown how to understand the spectral realization from a cohomological point of view, compatible with Galois theory. What emerges in particular is that while, as I explained in the first part of the interview, noncommutative spaces generate their own time, this new dynamical feature enables one to cool them down and obtain in this way, when the temperature goes to zero, a set of classical points. Moreover one can refine this thermodynamical procedure and get the analogue of the points over the algebraic extensions of the residue field, and these are organised in the same way as the points of a curve under the action of the Frobenius when dealing with the case of positive characteristic. It is a great challenge for noncommutative geometry now to develop the general conceptual tools allowing to transpose Weil's proof from algebraic geometry to our analytic framework.

My fascination for physics comes from quantum mechanics which, with the discovery of Heisenberg, is at the origin of noncommutative geometry. I have always admired the sophisticated computations physicists do, more specifically those which are motivated by experiment. It is a great motivation to discover that, hidden behind these recipes that physicists are finding from their physics motivation, there are marvellous mathematics. In these recent years the earlier work with Kreimer on renormalization and the Birkhoff decomposition has been pursued further in my collaboration with Marcolli. We discovered a universal group, obtained from a Riemann-Hilbert correspondence, which plays the role of the "cosmic Galois group" that Pierre Cartier had conjectured a few years ago. Indeed it is a universal symmetry group of all renormalizable quantum field theories. It contains the renormalization group of physicists as a one parameter subgroup but has a much richer structure. We have not been able to understand completely its relation with the motivic Galois groups, and in that sense it does not yet fully implement the dream of Cartier but deep work of Bloch, Esnault and Kreimer will surely shed more light on that aspect.

For the Standard Model, this work started a few years ago with Ali Chamseddine and it has been pushed further on in my recent collaboration with Chamseddine and Marcolli. It turns out that the incredibly complicated Lagrangian of gravity coupled with the Standard Model is obtained just as pure gravity (of the simplest form, just counting the eigenvalues of the line element) for a spacetime which has a fine structure. Namely it is described not as ordinary 4-dimensional continuum but as the product of an ordinary continuum by a finite noncommutative space of the simplest kind whose effect is to correct the dimension modulo 8 coming from K-theory. It is clear that these are interesting ideas but, so far, they have not passed the experimental test and thus still belong to the realm of pure mathematics.

You have spoken of the relation between mathematics and physics. Could you say something about the relationship between mathematicians and physicists, which is not the same thing?

Yes. It is normal for the true physicist not to worry too much about mathematical rigor. And why? Because one will have a test at the end of the day which is the confrontation with experiment. This does not mean that sloppiness is admissible: an experimentalist once told me that they check their computations ten times more than the theoreticians! However it's normal not to be too formalist. This goes with a certain attitude of physicists towards mathematics: loosely speaking, they treat mathematics as a kind of prostitute. They use it in an absolutely free and shameless manner, taking any subject or part of a subject, without having the attitude of the mathematician who will only use something after some real understanding.

After the heroic period that culminated in the elaboration of the Standard Model, and renormalization of gauge theories, an entire generation of physicists drifted away from the contact with experimental physics in search for a theory that would not only "explain" the Standard Model but also unify it with gravity. And pursuing the idea called string theory, these physicists became mathematicians and had a great impact on mathematics. The objects they manipulate are Riemann surfaces, Calabi-Yau manifolds: and they do mathematics, real sophisticated mathematics. But so far there is no physical test showing any relation between these ideas and the real world. Moreover, because of their origin from physics, the way they proceed is totally different from that of mathematicians.

This is true in particular at the sociological level: they work in huge groups and the amount of time they spend on a given topic is quite short. At a given time t, most of them are going to be working on the same problem, and the preprints which will appear on the web are going to have more or less the same introduction. There is a given theme, and a large number of articles are variations on that theme, but it does not last long. This happened in particular in the relation between string theory and noncommutative geometry. A herd of people tried to do field theory on a noncommutative space at the beginning of the years 2000, and after a relatively short time, they concluded that field theory on a noncommutative space was not renormalizable, because of the phenomenon of mixing between infrared and ultraviolet frequencies.

This conclusion remained in force for two or three years, but after the pack had moved to another topic, a completely different, very small group of people showed that in fact, the theory was renormalizable, provided one added a missing term in the Lagrangian. This required tremendous insight on the part of the main actors Wulkenhaar and Grosse, and then with Rivasseau, Vignes-Tourneret, Gurau etc… they developed the general theory which is now in a remarkable state, closing on the first effective construction in 4 dimensions. The pack never came back, and continued to move on from one topic to the next.

The sociology of science was deeply traumatized by the disappearance of the Soviet Union and of the scientific counterweight that it created with respect to the overwhelming power of the US. What I have observed during the last two decades since the fall of the USSR and the emigration of their scientific elite to the States is that there is no longer a counterweight. At this point, if you take young physicists in the US, they know that, at some point, they will need a recommendation written by one of the big shots in the country, and this means that if one of them wants to work outside string theory he (or she) won't find a job. In this way there is just one dominant theory and it attracts all the best students.

I heard some string theorists say: "if some other theory works we will call it string theory", which shows they have won the sociological war. The ridiculous recent episode of the "exceptionally simple theory of everything" has shown that there is no credibility in the opponents of string theory in the US. Earlier with the Soviet Union, there was resistance. If Europe were stronger, it could resist. Unfortunately there is a latent herd instinct of Europeans, particularly in theoretical physics. Many European universities, at least in France or England, instead of developing original domains as opposed to those dominant in the United States, simply want to follow and call the big shots in the US to decide whom to hire. It is not by lack of original minds such as my friend and collaborator Dirk Kreimer. But it is a lack of self-confidence of Europe, which means that we are not capable of doing what needs to be done, of resisting and safeguarding this diversity at any price. I don't think that we see similar things in mathematics, so there is a fundamental sociological difference between mathematics and physics. Mathematicians seem very resistant to loosing their identity and following fashion.

In your conversations with Changeux you discussed mathematics and reality. Have you advanced in your thinking about this?

I have no doubt that mathematical reality is something which exists, that it exists independently of my own brain trying to see it, and has exactly the same properties of resistance as external reality. When you want to prove something, or when you examine if a proof is correct or not, you feel the same anguish, the same external resistance as you do with external reality. Some people will tell you that this reality does not exist because it is not "localized" somewhere in space and time. I just find this absurd and I adopt a diametrically opposed point of view: for me even a human being is better described by an abstract scheme than by a material collection of cells – which in any case are totally renewed and replaced over a relatively short period of time and hence possess less meaning or permanence than the scheme itself, which might eventually be reproduced in several identical copies…

If one wants to reduce everything to "matter localized somewhere" one soon meets a wall which comes from quantum mechanics and one finds that this reduction of the outer reality to matter is an illusion that only makes sense at intermediate scales but by no means at a fundamental level. Thus I have no doubt on the subtleness and existence of a reality which can be neither reduced to "matter" nor "localized". Now the question of whether mathematical reality is something created or something pre-existing is much easier to discuss if one uses the distinction which appears in Gödel's theorem between "truth" and "provability" of a mathematical statement. I discussed this in details in my book "Triangle of thoughts" with Lichnerowicz and Schultzenberger and I refer to that book for the detailed argument which is rather involved. I was a bit frustrated after the book "Matière à Pensée" with Changeux, by the lack of an effective communication, and I made a point of writing another book where I could explain better the input coming from Gödel's theorem. There is a fundamental mathematical reality out there, and the mathematician creates tools to understand it.

The relation between the deductions of the mathematician (which – great recent discovery – take place in his brain) and that reality is similar to the relation between the deductions performed in a court as opposed to what actually happens in the real world. It hinges on a fine grammatical distinction between mathematical statements at the level of quantifiers– some are provable if they are true etc… This analogy with the court hall as opposed to external world is perfectly explained in the book of J.Y. Girard on Gödel's theorem. It allows one, after some real work, to get a clear mental picture of the distinction between the role of the mathematician (creating tools to uncover a piece of this reality) and the reality itself.

You have mentioned originality and fashion in mathematicians. Do you have an example?

I had just arrived as a newcomer in IHES [Institut des hautes études scientifiques, in Bures-sur-Yvette, near Paris] in 1976. The first people I met were talking about stuff I just didn't know. I was in the cafeteria and they would discuss "étale cohomology", all kinds of things like that, which, with my culture coming from functional analysis and operator algebras, I didn't know at all.

Fortunately, I soon ran into Dennis Sullivan who, as long as he was in Bures, used to go up to any newcomers, whatever their field or personality, and ask them questions. He asked questions that you could, superficially, think off as idiotic. But when you started thinking about them, you would soon realize that your answers showed you did not really understand what you were talking about. He has a kind of Socratic power which would push people into a corner, in order to try to understand what they were doing, and so unmask the misunderstandings everyone has. Because everyone talks about things without necessarily having cleaned out all the hidden corners. He has another remarkable quality; he can explain things you don't know in an incredibly clear and lucid manner. It's by discussing with Dennis that I learnt many of the concepts of differential geometry. He explained them by gestures, without a single formula. I was tremendously lucky to meet him, it forced me to realize that the field I was working in was limited, at least when you see it as tightly closed off. These discussions with Dennis pushed me outside my field, through a visual, oral dialogue. And not at all through reading texts.

You have talked about the importance of diversity, that people should have different backgrounds. But do you have some ideas about what sort of mathematical common ground everyone should share?

It is a bit subtle. I mentioned the vibrant heart of mathematics. You could say: Why not teach this to everyone? But this would result in a disaster! Because people would end up knowing Riemann surfaces, modular forms, etc, but they would be ignorant of large parts of mathematics, like Hopf algebras or other subjects that might look more esoteric. So, I don't know. I have the impression there should be a minimal common background - fundamental notions of differential and algebraic geometry, algebraic structures, real and complex analysis. Topology, basic number theory … are all needed. You can't avoid it. People must know that much.

After that, when you want to enter into more elaborate subjects, diversity should be the rule. We have to cultivate original people, as I explained in the first part of the interview, who are able to provide students with a totally original background with respect to this common knowledge. This will give young mathematicians keys, completely personal keys, which will allow them to open their own worlds. If they are lucky, they will be interested by many different things, because it is important for them to be able to flip from one thing to another for a while at the very beginning, until they find a subject that will really inspire them. I think it is important not to go beyond a certain limit for this common background. Then you should find and follow your own line, with an advisor who will allow you to strengthen your own originality. But of course there is no general recipe.

But would you really recommend that a young mathematician learns a lot of mathematics without being a specialist in something?

For a young mathematician, it is absolutely crucial to prove first that he or she is a mathematician. And that means to become a specialist in a topic and prove that you are able to do something very difficult. And this is not compatible with the dream of learning a bit about everything at the same time. Thus after finding the topic that you find enticing it is mandatory to concentrate, perhaps for a number of years, till you make a real dent. Afterwards, of course, once you've succeeded, once you have your passport to do mathematics, it's wonderful if you succeed in enlarging your spectrum to avoid remaining a specialist of a narrow discipline for the rest of your life. But it's very difficult to be a generalist. Because there is the danger of not doing real stuff in mathematics any more.

Do you have some ideas about the way mathematics should be taught?

We must absolutely train very young people to do mathematical exercises, in particular geometry exercises — this is very good training. I find it awful when I see that, in school, kids are taught recipes, just recipes, and aren't encouraged to think. When I was at school, I remember that we were given problems of solid (spatial) geometry. We went to a lot of trouble to solve them. It wasn't baby geometry. These were difficult things, with subtle proofs. And two years earlier, we were doing problems of planar geometry. We used to spend all night doing these problems. And now if you gave the same problems in an exam (the experiment was performed recently) you would be called a murderer! This is no progress. Problems in geometry are easily set, and then you have to go to a lot of trouble to find a proof.

It's a shame we don't do it anymore. I saw recent high-school problems, in which you define groups of rotations, rotations being equivalence classes… staying at a prehistorical level of sophistication just because of the heavy weight of the "formalism"… This is dreadful… Because geometry involves drawing figures, it should be directly accessible. Unfortunately, it's not impossible that this exaggerated use of mathematical formalism was inadvertently inherited from Bourbaki - who does not define real numbers until chapter 9 of Topology, long after defining uniform structures…

You mention Bourbaki. How do you judge now Bourbaki's role?

Bourbaki played a phenomenal role. You can't deny he transformed many subjects – in which the deepest obscurity reigned – into fields of an incredible clarity. There are some marvellous books by Bourbaki: Algebra Chapter III and all the volumes on Lie groups, you can only be dumfounded with admiration. Now, once all this has been done, it's done. There are still fields where something of the sort could have been done and was not done. But I don't think that doing more of it would make a big difference. All in all, Bourbaki had such a great influence in giving us a concern for clarity and rigor that the beneficial effect has already occurred. If Bourbaki hadn't been there, mathematics would have drifted towards lots of results that you could not rely on.

Do you think that it would be possible now to launch such an ambitious and unselfish project?

Unselfish to that degree now is not an obvious thing, since everybody is so busy with all sorts of "things to do". There was a marvellous spirit in the beginning of the Bourbaki group, an idea of unselfish service to the community. I participated for a short period at the end of the seventies. I wrote some drafts but what stopped me to continue was when I realized that, in a room in Ecole Normale, there were hundreds of manuscripts, 100 to 150 pages each, which would never see daylight. I found that depressing. Of course there were partial duplicates… but there was such a demand for perfection before the content would be published, that finally it was as if these texts didn't exist. Time passed, and as time passed, they became obsolete. There is this incredible dedication of Bourbaki members in writing drafts. When a manuscript is finished, it is true that you have learned a lot, you understand things better, but if the text never appears you get a real feeling of frustration. For a very long time Dieudonné was playing a key role to ensure that things would converge at some point, but after he left a lot of the efficiency left with him somehow.

What are you working on now?

Just at this time I am working on hard analysis which has to do with the spectral axioms of noncommutative geometry. This is the content of my class in College de France this year and it is a lot of technical work but also a welcome diversion. Just before this diversion I had reached, after we handed out the manuscript of our book[2] with Matilde Marcolli, an obsessive mental state due to the inevitable risk of some mistake in such a large body of work. Of course one can check things and try to view them from all sorts of different angles, but for instance as soon as it touches physics the difficulties pile up since the accuracy of the calculations one does is not enough to ensure that they will have any "meaning" for the real world and pass the reality test. In that respect, I try to share the attitude of the great physicist Pierre-Gilles de Gennes when he said:

> *"Le vrai point d'honneur n'est pas d'être toujours dans le vrai. Il est d'oser, de proposer des idées neuves, et ensuite de les vérifier. Il est aussi, bien sûr, de savoir reconnaître publiquement ses erreurs. L'honneur du scientifique est absolument à l'opposé de l'honneur de Don Diègue. Quand on a commis une erreur, il faut accepter de perdre la face."*

What certainly matters, in what we do, is to try to constantly put one's ideas to the test and see what happens. Nothing better than waking up in the middle of the night in that respect. And one should not be afraid. Here is what Alexandre Grothendieck writes in his unpublished book *Récoltes et Semailles* about this:

> *"Craindre l'erreur et craindre la vérité est une seule et même chose. Celui qui craint de se tromper est impuissant à découvrir. C'est quand nous craignons de nous tromper que l'erreur qui est en nous se fait immuable comme un roc. Car dans notre peur, nous nous accrochons à ce que nous avons décrété "vrai" un jour, ou à ce qui depuis toujours nous a été présenté comme tel. Quand nous sommes mûs, non par la peur de voir s'évanouir une illusoire sécurité, mais par une soif de connaître, alors l'erreur, comme la souffrance ou la tristesse, nous traverse sans se figer jamais, et la trace de son passage est une connaissance renouvelée."*

How do you read mathematics?

The only way I manage to read mathematics is extremely slow because I read a statement and then I try to think about it. I can't understand a proof if I haven't tried to prove it myself before. Once I've been stumped a long time on a result, I can understand it in seconds while scanning the proof; I see the one place where something happens and which I couldn't guess before. The problem is that this method of reading is very slow, I need an enormous amount of time to make myself familiar with the result. I am almost unable to read a mathematical book linearly. A discussion or a talk, on the contrary, allow me to go faster. But I am aware that other mathematicians function in a very different way.

Is it the same with physics?

No, it's totally different. In physics I adore reading; I spent about fifteen years studying the book of Schwinger, *Selected Papers on Quantum Electrodynamics*. He collected all the crucial articles, by Dirac, Feynman, Schwinger himself, Bethe, Lamb, Fermi, all the fundamental papers on quantum field theory, those of Heisenberg too, of course. This has been my bedside book for years and years. Because I have always been fascinated by the subject and I wanted to understand it. And that took a very long time to understand. Not so much to understand the detail of the articles, but to understand what they meant, what mathematics were behind them. In physics, then, I have a totally different reaction. I have not at all this inability to read. It's strange. I think there is a possible reason: in mathematics I need to protect myself more, in some ways. In physics, I don't feel this need.

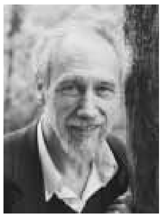
And outside science? Would you like to speak about something else, music, art?

These last two years, I no longer had time because I had to work harder, but before I used to take lessons in drawing and in piano. What struck me in music was to see how some composers had reached an incalculable level of perfection in their art. In studying some scores, I was struck to realise that you learn as much as in reading some mathematical papers. Simply because of the level of sophistication. This is not a question of analogy between mathematics and music. Some composers reached, by an hallucinatory work of precision, a level of perfection close to that of some of Riemann's work.

And faced with this level of perfection I react in the same way, a feeling of admiration - but an admiration which creates motion, something which is not at all static: beauty plus perfection puts thought into motion, it forces you to think. This perfection in the form of a work of art is of course very rare. To take an example, this time in literature, there is a striking difference of "form" between [Flaubert's] *Madame Bovary* and [Balzac's] *Le lys dans la vallée*. *Madame Bovary* is absolute perfection, a marvel of precision which is the outcome of a phenomenal amount of work, while the other is a bit botched. *Le lys dans la vallée* contains also marvellous stuff but there's an obvious difference in appearance.

I often have this impression when I look at mathematical papers or art works, that I feel intensely this distinction. Some pieces stand out way above the others, one gets the feeling that the author, instead of stopping at time t and saying, "fine, that will do, I'll hand in my stuff" (Balzac was forced to do that, he had a knife at his throat, he had no choice) just kept working until reaching something which is close to absolute perfection.

This is mainly what I feel about art. These works, the ones with this absolute perfection, give you momentum. They give you something which is not only a feeling; they give you an extraordinary power, a force, which allows you to carry on further. It passes something on to you. I have this impression with some papers in mathematics or in physics. Riemann's paper on zeta, Einstein's article on relativity for instance… There are few of them, very few. They put the level of writing standards so high. It's marvellous. You see something and you really understand. This is an extraordinary instrument for understanding and, beyond the clarity, you feel something which puts you in motion. It tells you: Go on.



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Catherine Goldstein [cgolds@math.jussieu.fr] is Directrice de recherches at the Institut de mathématiques de Jussieu. Her research projects lie in the history of mathematics, in particular of number theory. She recently coedited 'The Shaping of Arithmetic after C. F. Gauss's Disquisitiones Arithmeticae' and currently works on the impact of World War I on mathematical sciences.

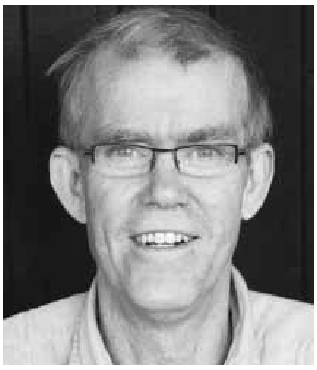
Georges Skandalis [skandal@math.jussieu.fr] is Professor at Université Paris Diderot – Paris 7 and the Institut de mathématiques de Jussieu. His main research subject is non commutative geometry. He currently studies singular foliations and the associated index theory. He is Alain Connes' former student.

Publishing Mathematical Journals

An interview with Gerard van der Geer (Amsterdam) and Ulrike Tillmann (Oxford) conducted by Ulf Persson (Göteborg)


Gerard van der Geer


Ulrike Tillmann (© Marc Atkins)


Ulf Persson

'Publish or Perish' is something most scientists live or die by. The result is of course that there is a tremendous amount of scientific publication, much of it written under pressure to serve ulterior motives, such as justifying support in promotion, and not only to communicate exciting and important results. Mathematics is but a small section of the general scientific community but even here we do witness an avalanche of papers, overwhelming any individual trying to stay abreast, even in a comparatively narrow area. Is this necessary? Would it not be a good idea to try and constrict the number of publications? Would that not make it easier to have an overview?

UT: 'Publish or perish' is the existential alternative that was coined in the US tenure system. It has long come across the pond and taken new dimensions as for example in the form of the RAE (Research Assessment Exercise) in the UK. Thus for most mathematicians there is a necessity to publish and as publication lists are often reviewed by non-experts, it also matters where.

And who would be the all-powerful judges in any scheme trying to restrict the number of publications? I prefer the free market of different journals aiming at attracting high quality papers. I also don't see how any such schemes should help with "keeping up with the literature". Good survey articles, a (loose) hierarchy in the quality of journals and search engines help at present to cope with the increasing number of research papers.

GvdG: A well-accessible well-organized mathematical literature is a main research tool for mathematicians. In order to be able to use the literature efficiently the selection and refereeing process is absolutely necessary. Quality journals form the core of this but in my view the math.arXiv is also part of the process. By putting a paper on the server a wider audience can get to know the paper and it can be discussed and checked. I consider this a very helpful step before submitting a paper to a good journal. Mistakes can be detected early on and comments and criticism can be taken into account. It assures you of a rather wide audience for the paper. It has the additional advantage that also mathematicians outside the main centres get an immediate access to new ideas. The disadvantage that there is no refereeing was valid also for the preprints of the old days and could only become an obstacle if the arXiv would get too many low quality postings. Maybe the differences in the use that various disciplines make of math.arXiv could be a measure of the dynamics in that area.

UT: We all have heard the arguments against this kind of pressure to publish many times. But on balance there are many good reasons for writing down one's thoughts and work. Arguments get clarified and new ideas come up often during the writing process. The intent to publish will focus the author's mind and impose a certain discipline on the writing. Publication itself is a means of communication, making one's work known to a wider audience, present and future. Finally, publication in a respectable journal will bring with it a refereeing and revision process.

GvdG: Besides journal articles books are also important. They filter the literature by distilling the ingredients that really form the content and usually they polish and simplify the ideas and results in research articles. They are a main tool for the training and education of young mathematicians. And finally, for the author books are usually a more secure way to earn some renown than articles in journals. The generation of top mathematicians after the second world war (e.g. Atiyah, Borel, Hirzebruch, Milnor, Serre and later Manin and Mumford) took the time and energy to produce books that set a standard. Is it still the case that the leading mathematicians take the time to write such books?

I agree that writing down your thoughts is an essential ingredient in doing mathematics, so should we instead ask referees to be far more restrictive? The problem is that refereeing a paper is very hard work and often a referee cannot vouch for the correctness and thus be tempted to give the benefit of the doubt.

UT: I don't really agree with the somewhat cynical view that most papers are accepted by the benefit of the doubt. We cannot expect referees to vouch for the correctness of a paper. The responsibility for this lies with the author(s).

However, a referee's duty should be to do due diligence: checking whether the results are sensible and that the arguments are convincing and written in a clear and well-structured manner. I have been involved with five different journals of somewhat different standards. If the referee(s) and the editor(s) have any doubts with regards to the correctness or whether the paper adds something worthwhile and new to the literature, it is unlikely that it gets accepted for publication.

We all know that refereeing a mathematical paper usually involves a substantial amount of work, taking several days or more, all done anonymously and without any further recognition or payment. So not only the author but often the referee and occasionally the editors put a lot of work into a paper. This all leads to a sense of collective ownership of the published literature that goes beyond the simple ownership by an individual author.

GvdG: The refereeing process for the quality research journals is not perfect but it seems difficult to come up with a better procedure. The long time that it often takes is certainly the biggest drawback of the system. A referee has to invest time and energy but is often rewarded by better insights during the reviewing process. Of course, the authors remain responsible for the correctness of the paper. Moreover, besides checking the correctness, evaluating the relevance of the paper is sometimes an even more important duty of the referee. I personally prefer a partially incorrect but very relevant paper over a totally correct but not so relevant one.

UT: In general, I am also not ruling out the existence of a system different from the present journal system that could function equally well. But we first have to come up with one.

As mentioned before, mathematicians value correctness highly as we need to build on each other's results. Nearly as important is good presentation of the arguments and proofs for it is often inside a proof that new ideas enter the subject, ideas which can be highly influential for the further development of the subject. If an article is published in a good journal, led by responsible editors, then one expects that the paper has been read by an expert referee (sometimes more than one) addressing both these points.

One rather peculiar aspect of mathematical publishing compared to that of Big Science is that the shelf-life of good articles (and especially books) is very long indeed. I recall a figure that 80% of all the references are to sources more than five years old, something that would be unheard of in more applied science but familiar to most humanists. What could be the reasons for this? That mathematical papers tend to be rather complete, while in science in general they are far more tentative. And what are the special consequences?

UT: I wouldn't quite say that mathematical papers tend to be "complete". It is just that we tend to take an idea or method as far as we can to make the most of it. Further progress on the same problem often then requires a new idea, which can take a long time and require new developments in other areas first.

The fact that mathematicians refer back to literature several years, sometimes decades old, makes it evident that archiving is very important for the subject. It also makes it clear that notation and language has to be as lucid as possible so that it can survive the test of time. Papers also have to stand on their own: no recent lecture or discussion might have made the contents and notation of the article familiar to its reader. Similarly, proofs have to be given with enough detail so that an interested future reader, who may no longer be equipped with the common knowledge of experts in the given field at the time of writing, can follow and understand them.

GvdG: Indeed, mathematical literature differs from literature in other sciences in the long term that the literature remains relevant. I think it is wonderful that after one and a half centuries one can still enjoy Riemann's paper on the zeta function. The long shelf-life is a very good thing. A drawback could be that it slows the free development of ideas. For example, in string theory ideas develop quickly as people jot down and communicate half-ripe ideas almost immediately after incubation. But in the end, the reliability counts.

Most of us like books. They are superior when it comes to reading and handling, and an old-fashioned collection of books and journals gives a very tangible and homely sense of accumulated knowledge. But maybe we are too sentimental; in the future should not everything be electronic? Are there any drawbacks to this system or does it just come with advantages?

UT: I am all in favour of electronic publication. One has to distinguish here the publication on the author's homepage (and papers might be put up at a variety of different stages: pre-preprint, preprint or post-print), on the arXiv or in a journal. There is no need here to distinguish between electronic-only journals and more conventional ones, as most of the latter publish articles electronically as well, often long before the paper version appears.

GvdG: Standard university libraries as we know them now will probably disappear and electronic access only to journals is no problem; in fact, general access to the literature is now much higher than it used to be. In contrast, I see a much longer viable future for books. We should ensure that we can keep the libraries for that purpose.

UT: Archiving is important for mathematics. In the past there have been worries about storing electronic files reliably for posterity. These problems, as well as any associated legal issues, can probably be dealt with if not now then soon. The production of paper copies is likely to become a matter of taste and unnecessary in the future.

I would like to question this optimism about electronic archival storage. The Babylonian clay tablets are still readable but will our electronic files be readable even a hundred years from now? There are two problems, the volubility of electronic storage (such as its sensitivity to magnetic fields and even moderate heat) and the rapid rate at which software is becoming obsolete. Even if the physical form survives, will we be able to interpret it?

GvdG: Yes, mathematics chiselled in stone is certainly more perennial than mathematics on paper or in electronic archives. But even for the historians of mathematics that want to study Babylonian clay tablets, a pdf version on the Internet makes it a lot more accessible than one original in a museum in the Middle East.

The ubiquitous presence of digital copies seems to me a much better guarantee for continued availability than a carefully guarded original. In this respect I am quite optimistic; the availability and accessibility of mathematical sources has never been better than it is now. Even if the present technology (like our pdf format) is hopelessly out of date in thirty years, the fact that there is such an amount of data available in that format will make the investment in an intelligent machine that can translate it into the newest format certainly worth it and I trust that our learned societies (like the AMS, the EMS, the LMS, the DMV, etc.) will take care that these investments are made.

Much of actual publishing of mathematics, especially books, is through commercial publishers. This was originally quite natural but in recent years it seems to be a rather costly option for consumers (especially institutions). Then there is a rather widespread resentment that other people reap the material profits from what we mathematicians as writers and referees contribute for free. Would not electronic publication short-circuit the process and make the 'middle-man' superfluous, besides making the process independent of massive financial resources?

GvdG: In view of the importance of the literature for research in mathematics, mathematicians should take care that their main tools for publication of the mathematical literature remain healthy. It seems that not so long ago such things could safely be left to the publishers and there were mutually beneficial relations between the research mathematicians and the publishers. Unfortunately, one may doubt that this is still the case. Many publishers have become part of larger publishing companies and have to produce good profits on a very regular basis. This has resulted in books and journals becoming unreasonably expensive. Mathematicians should therefore organize themselves what the market cannot or does not offer.

Learned societies like the AMS, the LMS, the EMS and the SMF can and do certainly play an important role. But also foundations that own journals can play a beneficial role. One example is *Compositio Mathematica*, owned by a foundation of the same name. The profit that comes from publishing the journal is cycled back into mathematics by supporting conferences or for example the prizes that the EMS is going to award this year at ECM 2008 in Amsterdam.

Other initiatives like *Documenta Mathematica*, the *Journal of Topology*, seem to support the trend away from the commercial publishers. Long term accessibility of the sources is an important feature, and that is why I think that the societies like the AMS, the LMS, the EMS and the SMF are important. I hope that we shall see a similar trend for the books. Many publishers now are not interested in a really wide distribution of a book but make a calculation of the price needed for a good profit based on the sales to the standard libraries that also have a subscription to their journals. The result is books priced so highly that individuals can hardly afford to buy these.

But one should not think that publishing of our research can be gotten for free or almost for free. The distribution and long term availability will require resources but these should remain reasonable in comparison. Maybe the new initiatives mentioned above will lead to new relations with commercial publishers that keep an eye on the long term and are willing to invest in a long term mutually beneficial relationship with the mathematical research community. Given the fluidity of the publishing market, small companies could find new niches and develop new forms of maths publishing. At least, I hope so.

UT: There has been a lot of dissatisfaction in the scientific community with large publishing houses related to pricing policies and so-called bundling of journals. Let me try to address the question why, in my opinion, this is in particular true for the mathematical community.

Mathematical papers tend to have a well defined and relatively small readership, a market mathematicians probably know best. Thus the advantages a big commercial publisher can offer in terms of marketing are limited. The vast majority of mathematicians also write their papers using some form of TeX, producing a professionally typeset manuscript. Again, the added value a publisher can offer is quite small here as well. Given all this, it should not be surprising that mathematicians feel their journals should be reasonably priced and think of alternatives if this is not the case.

What are the alternatives? It would be naive to think that publishing, even electronic-only publishing, of journals can be free. Though editors and referees most often offer their services without remuneration, databases have to be set up and maintained and access to articles has to be ensured for the future. Few of us would think of this as rewarding work. It is also difficult to sustain any such enterprise without any profit margin unless one can rely on another source of income such as a foundation or regular donations.

Thus it is a matter of how much profit and for whose pocket. I am an editor of the newly founded Journal of Topology, which is owned by the London Mathematical Society. In this case any profits are fed back into the mathematical community in terms of travel grants for visitors, conferences, and other research initiatives. Most of the editors are also active members of the LMS so that the editorial board can effectively inform and influence any decisions on price and publishing policy.

I personally would not like to see commercial publishers completely divorced from mathematical journal publishing. It is good to have variety. But publishers must be sensitive to our needs and expectations.

I would like to ask Gerard about his initiative concerning Compositio, which goes back a few years and which moved it away from a commercial publisher. What made you conceive of such a move and was it difficult to implement? Has it so far been successful?

GvdG: The story was this. In the 90s when I was managing editor of *Compositio* I became seriously worried about the regular price increases imposed by Kluwer Academic Publishers, at that time the publisher of *Compositio*. The journal had been founded in the 1930s by Brouwer when he was thrown off the editorial board of *Mathematische Annalen* and in reaction started his own journal. During the war the publication was halted and was resumed in 1951 only. It seems that Brouwer's quarrelsome character had led others to start a foundation *Compositio Mathematica* that owned the name of the journal. This foundation was for various decades a rather sleepy institution but its mere existence allowed us in the 1990s to look around for another publisher for *Compositio*.

I contacted the American Mathematical Society to see whether they would be interested. John Ewing answered immediately that that was the case. I also contacted other parties, like the LMS, the EMS and Springer. It was very difficult to choose between the very attractive offers from the AMS and the LMS. In the end we chose the LMS for geographical reasons (the EMS Publishing House was still too much in its infancy to be a serious competitor). The contract with Kluwer expired in December 2003 and in January 2004 we moved to the LMS.

It is now published in a joint enterprise with the LMS and distributed by CUP. I think it is a great success. Firstly, prices have fallen by a third. Secondly, any surplus money is shared between the LMS and the Foundation Compositio and is fed back into mathematics. The foundation supports mathematical conferences and, for example, subsidizes the ten prizes that the EMS will award during the European Congress for Mathematicians this summer in Amsterdam. I hope that this example of *Compositio* will be followed by other initiatives.

Gerard van der Geer[geer@science.uva.nl] is an algebraic geometer from the famous Dutch algebraic geometry school and is interested in moduli spaces, especially of curves and abelian varieties. He did his PhD in Leiden in 1977. He has been a professor at the University of Amsterdam since 1987. For more than ten years he was the managing editor of Compositio Mathematica.

Ulrike Tillmann [tillmann@maths.ox.ac.uk] grew up in Vreden, Germany (on the Dutch border) where she graduated from high-school in 1982. She studied at Brandeis (B.A. 1985) and Stanford (PhD 1990). She got her Habilitation at Bonn in 1996 but has been employed at Oxford since 1992. Her work on the topological approach to the Mumford conjecture has gained her various distinctions such as the Whitehead prize (2004) and the Bessel Preis (2007). She was an invited speaker at the ICM 2002 in Beijing.

Ulf Persson [ulfp@math.chalmers.se] is a professor of mathematics in Göteborg, Sweden. He is on the editorial board of the EMS newsletter; a short presentation appeared in issue 57 (2005), page 4.

Book review

Victor Rotger Cerdá (Barcelona, Spain)

Rational Points on Modular Elliptic Curves
Henri Darmon, McGill University, Montreal, QC, Canada
CBMS Regional Conference Series in Mathematics 2004.
Volume 101
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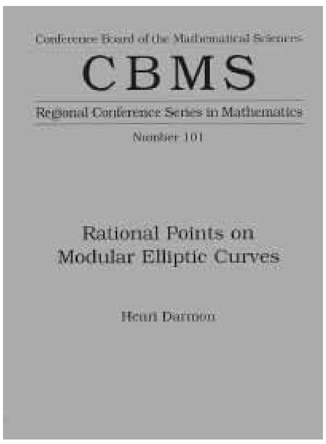

This delightful book by Henri Darmon provides an updated summary of many of the recent developments in the arithmetic of elliptic curves and introduces the reader to the author's new striking contributions to the subject.

The ultimate goal of the material presented in this book is to understand the structure of the group of rational points of elliptic curves E over number fields F, which is known to be abelian and finitely-generated by a classical theorem of Mordell-Weil.

The torsion subgroup of E(F) is already fairly well understood. Thanks to the remarkable results of Mazur, Merel and Parent, there exist methods for explicitly computing it and its order is uniformly bounded in terms of the degree of F.

On the other hand, the rank of E(F) has proved to be a much less feasible invariant of the curve; its behaviour and the conjectural relationship that it should bear with subtle arithmetic and analytic objects attached to E still remain an intriguing open problem. Important progress has been made in this direction in the last few decades and it is the aim of this book to report these developments.

The Shimura-Taniyama-Weil Conjecture, now a theorem due to the fundamental breakthrough of Wiles et al., asserts that every elliptic curve E over the field $\mathbf{Q}$ of rational numbers is modular. This amounts to saying that the L-function $L(E,\mathbf{Q},s)$ of E equals the L-function $L(f, s)$ of a normalized newform f of weight 2 and level N, the conductor of E. In turn, this allows one to prove that $L(E, \mathbf{Q}, s)$, which converges for $Re(s) > 3/2$, admits an analytic continuation and satisfies a functional equation of the form $L^*(E, \mathbf{Q}, s) = \mathrm{sign}(E, \mathbf{Q})\cdot L^*(E, \mathbf{Q}, 2\text{-}s)$, where $L^*(E, \mathbf{Q}, s) = (2\pi)^{-s}\,\Gamma(s)\,N^{s/2}\,L(E, \mathbf{Q}, s)$ and $\mathrm{sign}(E, \mathbf{Q}) = \pm 1$. This material is quickly reviewed in chapters 1 and 2.

The analytic continuation and functional equation of L(E, F, s) is conjectured to hold for all elliptic curves E over any number field F. With a suitable notion of modularity, the ideas of Wiles lead to the proof of this fact when F is totally real and under certain restrictions on E.

The weak conjecture of Birch and Swinnerton-Dyer predicts that the rank of E(F) equals the order of vanishing of L(E, F, s) at s = 1. The strong version of the conjecture provides in addition a conjectural expression of the value of the leading term of L(E, F, s) at s = 1 in terms of the order of the (conjecturally finite) Tate-Shafarevic group, the regulator and the local Tamagawa numbers of E.

As explained in detail in Darmon's book, the best theoretical evidence so far of the Birch and Swinnerton-Dyer Conjecture are the theorems of Gross-Zagier and Kolyvagin for elliptic curves E over **Q**, which prove the weak Birch and Swinnerton-Dyer Conjecture for E and the finiteness of the Tate-Shafarevic group provided that $\mathrm{ord}_{s=1} L(E, \mathbf{Q}, s) \leq 1$. As discussed in chapter 7, Zhang has generalised these results to elliptic curves over totally real number fields.

The main ingredient of the proof of the theorems of Gross-Zagier and Kolyvagin is the existence of a supply of rational points on E over a tower of abelian extensions of an imaginary quadratic extension K of **Q**, a so-called non-trivial Heegner system attached to (E, K).

In order to construct such a Heegner system on E, one first manufactures it by means of the theory of complex multiplication on a suitable modular curve or a Shimura curve X attached to a quaternion algebra, and then projects it onto E through a modular parametrisation $\phi: X \to E$. As explained in this monograph, such a parametrisation may be complex (as classically) or p-adic rigid analytic (within the theory of Mumford curves, thanks to the theorem of Cerednik-Drinfeld).

This construction is reported in its various flavours through chapters 3 to 6. Chapter 7 is devoted to the extension of this circle of ideas to elliptic curves over totally real number fields F and Heegner systems attached to purely imaginary quadratic extensions K of F.

It is a crucial observation that there exist Heegner systems attached to an imaginary quadratic field K if and only if sign(E, K) = –1. For an arbitrary quadratic extension K/F of number fields, define S(E, K) to be the set of archimedean places of K and finite places of K at which E acquires split multiplicative reduction. When F = **Q** and E has semistable reduction, one has sign(E, K) = $(-1)^{\#S(E, K)}$ and the same is expected to hold for any K/F.

A refinement of the Birch and Swinnerton-Dyer conjecture leads to the prediction that there exists a non-trivial Heegner system for a semistable elliptic curve E over F and a quadratic extension K/F if and only if #S(E, K) is odd. Although this conjecture does not mention modularity at all, any attack to this problem should probably involve the connection between E and automorphic forms of some sort. Note for instance that no classical result sheds light on this question when F is not totally real or K admits some real archimedean place.

Chapter 10, which is somewhat independent of the rest of the book, describes the proof of Kolyvagin's theorem, which shows the finiteness of the Tate-Shafarevic group whenever the elliptic curve is equipped with a suitable non-trivial Heegner system.

Chapters 8 and 9 contain the main new contributions of the author, which are alluded to at the end of chapters 3 and 7. For a modular elliptic curve E over a totally real number field F, these concern the (conjectural) construction of Heegner systems on E over abelian extensions of certain quadratic extensions K of F. More precisely, chapter 8 attacks the problem when K is an almost totally real (ATR) extension of F, i.e., a quadratic extension of F which is complex at a single archimedean place of F and real at the remaining places. In this case, the conjectural Heegner system on E is constructed by means of a suitable substitute of the classical uniformization of E by Poincaré's upper half plane. Chapter 9 deals with the case of a real quadratic extension K of **Q**, provided sign(E, K) = –1. Under this assumption, there exists at least one prime factor p of the conductor N of E which does not split in K. Darmon's construction of a conjectural Heegner system attached to (E, K) is based on the same principles of the previous chapter, where now rigid analytic uniformization at *p* plays the role that complex uniformization did before.

This rather short monograph is written in a style that allows the reader to understand the guidelines of an exciting and involved subject, gathering in a unified context a good number of deep results that may seem heterogeneous to a non-specialist. This is of course at the cost of omitting many details, which are often relegated to the references or the list of exercises.

Victor Rotger Cerdá [vrotger@ma4.upc.edu] got his PhD in 2003 at the Universitat de Barcelona under the supervision of Pilar Bayer. Since 2007 he has been an associate professor at the Department of Applied Mathematics IV of the Universitat Politecnica de Catalunya. His research interests are the arithmetic and geometry of abelian varieties and Shimura varieties.

ERCOM: Institut des Hautes Études Scientifiques

In 2008, the Institut des Hautes Études Scientifiques (IHÉS) celebrates its 50th anniversary. This is an opportunity to reflect on the institute's development since its creation, while at the same time giving some perspective on its future direction in the coming years.

IHÉS was founded in 1958 by Léon Motchane, an industrialist with a passion for mathematics, whose ambition was to create a research centre in Europe that would be a counterpart to the Institute for Advanced Study (IAS), Princeton, USA.

Since its creation, the institute has developed in accordance with the vision of its founder: it has grown into a focal point for the international scientific community through significant achievements in mathematics and theoretical physics. The atmosphere of free exchanges between scientists, whatever their specialities and origins, has become a key element of its attraction.

Both the talent and the great creativity of the scientific personalities who have been a part of the history of IHÉS have paved the way for fresh approaches and radical new thinking in mathematics and theoretical physics. The freedom and intellectual stimulation with which the institute provides its researchers undeniably represents a key asset in this respect, an asset that is carefully nurtured.

Since its creation, IHÉS has welcomed some 5000 researchers from 60 countries. Each year, it hosts about 250 visitors. Its present faculty consists of five permanent professors, the Léon Motchane professor and five long-term CNRS visitors (see box for full list). While visiting researchers come and meet the permanent professors, the latter also greatly benefit from interactions they have with visitors. Unplanned interactions between visitors are also part of the routine surprises that scientists encounter while at the institute.

Research work at IHÉS is based upon freedom (of the field studied and of the working methods adopted) and interaction between the different disciplines explored by the scientists (e.g. mathematics, physics, biology and computer science). This model draws on the spontaneity of exchanges and generates tremendous cross-fertilisation. Visiting researchers stay for any length of time between one week and two years. The average is regularly above two months.

The selection on the criterion of the quality of the work produced by applicants is made by the scientific council, which meets twice a year. For the last couple of years, the number of applications has continued to increase, making the selection much tougher. The scientific council invites up to one third of the visitors on its own initiative.

The visiting researchers are, as often as possible, housed at the institute's residence, *L'Ormaille*, which is close to the institute and which has 47 houses and apartments available.

Over the years, IHÉS has developed various partnerships, most of the time aimed at younger researchers, such as the European Post-Doctoral Institute (a programme organised with nine other European institutions that offers young researchers post-doctoral stays following a personalised itinerary) and the first programme of the Franco-Asian summer school (with the Department of Mathematics of Université Paris-Sud). Other partnerships are to be reviewed in that same spirit.

Its present status is that of a foundation "d'intérêt public", independent of any academic institution. It is managed under the responsibility of an international board of directors. Its resources are by now very diverse (some 27 sources of funding in the last few years). The main source (about half of the whole budget) remains a grant from the French government, re-evaluated every four years after a site visit by an international scientific review committee. Some ten countries contribute to the funding through various agencies. IHÉS presently holds the only grant given to a mathematical institution in the framework of the European large research infrastructures but this grant ends in June 2008. Some ten companies and foundations finance the institute on a regular basis. A small endowment, resulting mainly from the products of the first international fundraising campaign run by the institute from 2001 to 2004, has helped to improve the budget over the last few years.

Scientific discussion

IHÉS Foyer

For its 50th anniversary, IHÉS has a comprehensive programme of celebrations planned: a series of scientific conferences will be organised at the institute (on mathematics in May, on theoretical physics in June and on the interface with biology in September), a conference open to the general public will be held at the "Musée du Quai Branly" in Paris in September, a book on visitors to the institute will be published in French, in English and probably also in Japanese, and an exhibition of portraits will be shown in France, the US, Japan and China.

This year will also see the launch of the 50th anniversary international fundraising campaign. Its goal is to raise a minimum of 20 million Euros to more than double the institute's present endowment, allowing it to develop new activities at new frontiers, secure operations in the long-term (typically the funding of permanent professors) and improve accommodation.

Permanent professors since the institute was founded:
Louis MICHEL†, physicist, from particle physics to solid state physics,
Alexander GROTHENDIECK, mathematician, algebraic geometry (now retired),
René THOM†, mathematician and "philosophy apprentice", topology, morphogenesis, philosophy,
Jean DIEUDONNÉ†, mathematician, algebraic geometer and functional analyst,
David RUELLE, physicist, statistical mechanics and dynamical systems (now Emeritus),
Pierre DELIGNE, mathematician, algebraic geometry (now at the Institute for Advanced Study, Princeton, USA),
Dennis SULLIVAN, mathematician, topology (now at the State University of New York at Stony Brook, USA),
Jean BOURGAIN, mathematician, functional analysis and harmonic analysis geometry (now at the Institute for Advanced Study, Princeton, USA),
Jürg FRÖHLICH, physicist, field theory (now at the Eidgenössische Technische Hochschule, Zurich, Switzerland),
Oscar E. LANFORD III, mathematician, dynamical systems (now Emeritus from the Eidgenössische Technische Hochschule, Zurich, Switzerland),
Mikhail GROMOV, mathematician, geometry at large and interface with biology,
Thibault DAMOUR, physicist, gravitational theories and cosmology,
Laurent LAFFORGUE, mathematician, algebraic geometry and representation theory,
Maxim KONTSEVITCH, mathematician, deformation theory and algebra at large,
Nikita NEKRASOV, physicist, string theory and M-theory.

Léon Motchane Chair:
Alain CONNES, mathematician, operator algebras and noncommutative geometry
(also professor at Collège de France).

Directors since the Institute's foundation:
Léon MOTCHANE†
Nicolaas KUIPER†
Marcel BERGER
Jean-Pierre BOURGUIGNON

Long Term CNRS Visitors:
Henri EPSTEIN, physicist, field theories (now Emeritus),
Pierre CARTIER, mathematician, algebraic geometry and functional analysis (now Emeritus),
Krzysztof GAWEDZKI, physicist, conformal field theories (now at École Normale Supérieure de Lyon),
Jean-Benoît BOST, mathematician, algebraic geometry and dynamical systems, (now professor at Université Paris-sud)
Offer GABBER, mathematician, algebra,
Christophe SOULÉ, mathematician, algebraic K-theory and interface with biology,
Dirk KREIMER, physicist, quantum field theory and its mathematical interfaces,
Christophe BREUIL, mathematician, p-adic representations and number theory,
Claire VOISIN, mathematician, algebraic geometry.

Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

Celebrated by Carl F. J. Gauß (1777–1855) as "the queen of Mathematics", number theory might as well be considered to be as remote from physical *reality* as one may think. This suspicion may be strengthened by considering problems like the famous *Fermat's Last Theorem*, which stimulated the development of this branch of mathematics over more than a century until its solution by Andrew Wiles in the mid nineties.

It may be less known that number theory played a certain role in the development of theories in physics such as stability theory. The theory originated in the question of deciding whether the planetary system will keep the same form in the distant future or if incidents like collisions or planets leaving the system may occur. The question has interested mathematicians and astronomers since the eighteenth century and names like Joseph Louis Lagrange (1736–1813) and Pierre Simon Laplace (1749–1827) are related to early attempts to prove stability. In Germany, the respected number theorist Peter Gustav Lejeune Dirichlet (1805–1859) made important contributions to mathematical physics and is known for having communicated to Leopold Kronecker (1823–1891), shortly before his death, that he had a totally new general method for solving mechanical problems. Possibly under the influence of this remark Karl Weierstrass (1815–1897) developed a perturbation theory using generalised Fourier Series of the form

$$\sum_{n\geq 0} A_n \cos\left[(j_1\omega_1 + j_2\omega_2 + \ldots + j_s\omega_s)t\right] + B_n \sin\left[(j_1\omega_1 + j_2\omega_2 + \ldots + j_s\omega_s)t\right],$$

with frequencies $\omega_1, \ldots, \omega_s$. This led to the study of *quasi periodic functions*, where the *Diophantine approximation*, a branch of number theory, plays an important role. In fact, Ludwig Siegel worked intensively in this field and is considered to be one of the predecessors of modern KAM stability theory, which is named after A. N. Kolmogorov, V. Arnold and J. Moser. The reader may enjoy Moser's beautiful exposition in [1].[1]

Talking about connections between number theory and physics one cannot avoid mentioning the mysteriously striking correlations between the distribution of spaces between successive zeroes of the Riemann zeta-function on the critical line and the distribution of spaces between eigenvalues of random Hermitian matrices. Knowing that the latter play an important role in quantum dynamical systems, the correlation is indeed mysterious. The fact was first observed by H. Montgomery and F. Dyson in the seventies and has had a considerable impact on analytic number theory ever since. Captivating details can be found for instance in [2] and [3].

[1] J. Moser, *Is the Solar System Stable?*, The Mathematical Intelligencer, **1**, No. 1 (1978), pp. 65–71.
[2] M. de Sautoy, *The Music of Primes (searching to solve the greatest mystery in mathematics)*, Harper Collins (2000).
[3] J. Derbyshire, *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics* (2003).

The development of computers brought a series of important applications of number theory to computer science, which may have definitively erased the aura of the "queen of Mathematics" not being able to *make her hands dirty with facts of real life*. One of the early and possibly most famous applications is the RSA cryptosystem. The folklore idea behind this system is the fact that it is easy to multiply two large prime numbers on a computer but it is *very* hard to factor the result back into the original primes once these are forgotten or concealed. It takes much number theoretic insight to trust that the factoring problem is indeed so hard and the security of most Internet communication and electronic banking systems relies on this. The system was developed and published by R. Rivest, A. Shamir and L. Adleman (the letters in RSA) in 1977. It is interesting that the idea was natural for mathematically trained persons, who considered the question of distributed information security in the early days of computers. This became clear in this century, when a member of the British Communications – Electronics Security Group revealed having developed essentially the same algorithm in the early seventies for his company, thus more than five years before the RSA paper was even conceived. The confidentiality period of thirty years had then expired and his statement was verified by experts of the community – some interesting notes from this process can be found on Burt Kaliski's home page on the RSA Laboratories web site [4].

[4] Burton Kaliski's home page, http://www.rsa.com/rsalabs/node.asp?id=2146
[5] Henri Cohen, *A Course in Computational Number Theory*, (Third printing). Springer Graduate Texts in Mathematics, **138** (1996).
[6] François Koeune's page on literature on the LLL algorithm. http://www.dice.ucl.ac.be/~fkoeune/LLL.html

In 1984, René Schoof discovered an algorithm for counting points on elliptic curves over finite fields that could run in *polynomial time* and was thus practical for implementation on a computer. This brought the relatively young domain of étale cohomology and algebraic geometry over fields of positive characteristic, grown upon ideas of André Weil and Alexander Grothendieck (Fields Medal, Moscow, 1966) in the sixties, into the world of computer science. Applications to cryptography did not have to wait long, since Niel Koblitz proposed in 1987 an elliptic curve based cryptosystem that is intensively used today, especially in contexts where *communication bandwidth*, that is the volume of bits that can be transmitted in a unit time, is critical. Compared to RSA, elliptic cryptosystems nowadays require roughly one sixth of the bandwidth. The door was open for more applications of algebraic geometry and the research area is highly active today. On the application side one may mention the activity in the Institute for Experimental Mathematics led by Gerhard Frey in Essen; among others, the use of hypergeometric curves for cryptography has been fostered by their research.

A further major breakthrough in computational number theory was the discovery of a fast lattice reduction algorithm by A. K. Lenstra, H. W. Lenstra Jr. and L. Lovàsz in 1982. A lattice is a regular grid in n-dimensional real space, the set of all integer linear combinations of some independent vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$:

$$L = \left\{ \sum_{i=1}^{n} a_i v_i \ : \ a_i \in \mathbb{Z} \right\}.$$

Certainly, the lattice has numerous bases and the closer the base-vectors are to orthogonality, the smaller they are in size: this is because the determinant of the matrix built up of lattice base vectors only depends on the lattice. Many applications require bases with possibly short vectors. The problem had already been considered by Charles Hermite (1822–1901) who gave an algorithm for finding optimal bases; like many algorithms born before the advent of computer, Hermite's was also impractical for problems with large inputs. The contribution of Lovàsz and the Lenstra brothers was to show that by relaxing the minimality condition one could find in polynomial time (and thus fast on a computer) a base of vectors that are reasonably close to the shortest possible. The impact of the LLL algorithm (named after the discoverers) is formidable and ranges from cryptography – mostly, but not only, cryptanalysis, the branch specializing in analysing the vulnerability of cryptosystems – and of course computational number theory to integer optimisation.

I. Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

26. Determine all positive integers n such that all integers consisting of $n-1$ digits of 1 and 1 digit of 7 are prime.
(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

27. Find all triples (x,y,z) of positive integers such that

$$x^3+y^3+z^3-3xyz=p,$$

where p is a prime greater than 3.
(Titu Andreescu, University of Texas at Dallas, USA and Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania)

28. Let n be a positive integer greater than 3 such that $k^{n-1}\equiv 1 \pmod n$ for $k=3,4,\dots,n-1$. Prove that

$$(n-1)!+1\equiv(-1)^{n-1}\frac{(n-1)(n-2)}{2}(2^{n-1}-1) \pmod n.$$

(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania)

29. Let A_n be the number of positive integers N having the property that all digits in N are chosen from the collection $\{1,3,4\}$ and the sum of digits equals n. Show that A_{2n} is a complete square.
(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

30. Let x,y,z be integers such that

$$\frac{x^2-1}{y(y+1)}=z^2.$$

Prove that either $(2y+1)|x$ or $(2y+1)|z$.
(L. Panaitopol, University of Bucharest, Romania)

31. Find the values $a\in\mathbb{N}$ for which there exist $n\in\mathbb{N}$, $n\geq 232$, such that a^n has $2n+1$ digits.
(L. Panaitopol, University of Bucharest, Romania)

II. Three New Open Problems

32*. Does there exist an infinite sequence of positive integers such that the product of any number of consecutive elements of the sequence (starting from any point) is divisible by their sum?
(Sergei Konyagin, Moscow State University, Russia)

33*. Find the sum

$$\sum_{n=1}^{\infty}(\zeta(n+1)-1)^2,$$

where ζ denotes the Riemann-zeta function.
Remark. Since $\lim_{x\to\infty}2^x(\zeta(x)-1)=1$, the limit test shows that the series converges.
(Ovidiu Furdui, University of Toledo, OH, USA)

34*. Let p_n be the n^{th} prime number and let us consider the sequence $(A_n)_{n\geq 1}$ defined by $A_n=\sqrt{p_{n+1}}-\sqrt{p_n}$. For the sequence $(A_n)_{n\geq 1}$ we mention the following open problem, which is called Andrica's Conjecture. *The following inequality holds for any positive integer n*

$$A_n<1$$

(see for instance E. W. Weisstein's, "Andrica's Conjecture" from MathWorld – A Wolfram Web Resource, http://mathworld.wolfram.com/AndricasConjecture/html).
Remark. One of the longstanding conjectures in mathematics is the Riemann Hypothesis:
The nontrivial zeroes of the Riemann zeta function

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s},$$

have real part equal to $\frac{1}{2}$.
There are several arguments which seem to indicate that the following implication holds:
Open problem: Andrica's Conjecture ⇒ Riemann Hypothesis.
(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania).

III. SOLUTIONS

18. Does the equation

$$x^2+x+1=y^2 \tag{1}$$

have any non-trivial integer solutions for x,y? Consider the same question for

$$x^4+x^3+x^2+x+1=y^2. \tag{2}$$

(Preda Mihăilescu, University of Göttingen, Germany)

Solution by Konstantine Zelator and Ovidiu Furdui (USA). We show that the solutions of the first equation are $(0,\pm1)$, $(-1,\pm1)$. The equation is equivalent to the *quadratic* equation $x^2+x+1-y^2=0$, whose *discriminant* is $\Delta=4y^2-3$. Thus, for the preceding equation to have integer solutions, we have to have that $\Delta=k^2$ for some integer k. It follows that $4y^2-3=k^2$, and hence $(2y-k)(2y+k)=3$. Straightforward calculations show that the integer solutions of this equation are $(1,1)$, $(1,-1)$, $(-1,1)$ and $(-1,-1)$. Thus, when $y=\pm1$ we get that $x=0$ or $x=-1$, and the first part of the problem is solved.

Now, we consider the equation

$$x^4+x^3+x^2+x+1=y^2. \tag{3}$$

We prove that the solutions of (3) are $(0,\pm1)$, $(-1,\pm1)$ and $(3,\pm11)$. A simple calculation shows that for all real values of x, the following inequality holds

$$\left(x^2+\frac{1}{2}x\right)^2<x^4+x^3+x^2+x+1\leq\left(x^2+\frac{1}{2}x+1\right)^2. \tag{4}$$

We distinguish two cases according to whether y is a positive or a negative integer.
(1) Let y be a positive integer. It follows based on (4) that

$$x^2+\frac{1}{2}x<y\leq x^2+\frac{1}{2}x+1. \qquad (5)$$

a. Let $x=2k$. Using (5) we get that $4k^2+k<y\leq 4k^2+k+1$ and hence, since y is an integer, we get that $y=4k^2+k+1$. In this case $x=2k$ and $y=4k^2+k+1$. We get, using (3), that $k^2=0$ and hence $x=0$ and $y=1$.
b. Let $x=2k+1$. We have that

$$(2k+1)^2+k+\frac{1}{2}<y\leq (2k+1)^2+k+1+\frac{1}{2}$$

,
and hence, since y is an integer, we get that $y=(2k+1)^2+k+1$. It follows, based on equation (3), that $k^2=1$ and hence $k=\pm 1$. Thus, when $k=1$ we get that $x=3$ and $y=11$ and when $k=-1$ we get that $x=-1$ and $y=1$.

(2) If y is a negative integer, we let $y=-z$, where z is a positive integer. Equation (3) is equivalent to $x^4+x^3+x^2+x+1=z^2$. However, this case reduces to the previous one, which generates the solutions $(0,-1)$, $(-1,-1)$ and $(3,-11)$. □

Also solved by the Con Amore Problem Group (Denmark), Wolfgang Fensch (Germany), József Sándor (Romania) and the proposer.

19. Let p be a prime, $\mathbb{F}_p$ the field with p elements and $\mathbb{K}\supset\mathbb{F}_p$ a finite extension thereof. Let α,β be roots of the distinct irreducible polynomials $f(X),\ g(X)\in\mathbb{K}[X]$ and $\mathbb{L}=\mathbb{K}[\alpha,\beta]\supset\mathbb{K}$ be the extension generated by these two numbers.
(i) Are there values $c\in\mathbb{K}$ for which $\mathbb{L}\neq\mathbb{K}[\alpha+c\cdot\beta]$ for some $c\in\mathbb{K}$?
(ii) If yes, can one give an upper bound for the number of exception-values $c\in\mathbb{K}$?

(Preda Mihăilescu, University of Göttingen, Germany)

Solution by the proposer. The answer to question (i) is *yes*, as illustrated by the following simple example. Consider $a,b\in\mathbb{K}\setminus\{0\}$, so that a is a non quadratic residue; let $\alpha=b-\sqrt{a}$ and $\beta=\sqrt{a}$. The minimal polynomials are $f(X)=(X-b)^2-a$ and $g(X)=X^2-a$ with $f(X)\neq g(X)$. But $\alpha+\beta=b\in\mathbb{K}$ and thus

$$\mathbb{K}[\alpha+\beta]=\mathbb{K}\neq\mathbb{L}=\mathbb{K}[\sqrt{a}]$$

, a quadratic extension by the choice of a. Thus

$$\mathbb{K}[\alpha,\beta]\neq\mathbb{K}[\alpha+c\beta]$$

for $c=1$.
Remark *The question of whether for arbitrary α,β there is a linear combination with $\mathbb{K}[\alpha+c\beta]\neq\mathbb{L}$ is an open one.*

For question (ii), let α,β have the respective orders m,n, $d=(m,n)$ and $D=(m\cdot n)/d$. Thus $[\mathbb{L}:\mathbb{K}]=D$ and $K=\mathbb{K}[\alpha]\cap\mathbb{K}[\beta]$ has degree d over $\mathbb{K}$. We define $\mathbb{L}(c)=\mathbb{K}[\alpha+c\beta]$ and claim that for distinct $c,c'\in\mathbb{K}\setminus\{0\}$ one has $\mathbb{L}(c)\cdot\mathbb{L}(c')=\mathbb{L}$. Indeed, by the definition of the fields $(\alpha+c\beta)-(\alpha+c'\beta)\in\mathbb{L}(c)\cdot\mathbb{L}(c')$ so $(c-c')\beta\in\mathbb{L}(c)\cdot\mathbb{L}(c')$ and eventually also $\alpha,\beta\in\mathbb{L}(c)\cdot\mathbb{L}(c')$.

Let $\Lambda\subset\mathbb{L}$ be the field of degree $[\Lambda:\mathbb{K}]=h=D/d$. Then $\Lambda[\alpha]=\Lambda[\beta]=\mathbb{L}$ (this follows by comparing degrees). The extension $\Lambda[\alpha]:\Lambda$ has degree $r=m/(h,m)=d$. The same holds for $\Lambda[\beta]$. Thus the field $\mathbb{L}'$ of degree d over Λ contains α,β and $\mathbb{L}'\supset\mathbb{L}$. The converse follows by comparing degrees too. For any field K with the property that $K[\alpha]=K[\beta]=\mathbb{L}$ we have $\Lambda\subset K$. In particular, $\Lambda\subset\mathbb{L}(c)\subset\mathbb{L}$ for all $c\in\mathbb{K}\setminus\{0\}$.

Let $d=\prod_{i=1}^{t}q_i$, with q_i being powers of distinct primes. We claim that for each i there is at most a $c=c_i\in\mathbb{K}\setminus\{0\}$ such that $q_i|[\mathbb{L}:\mathbb{L}(c_i)]$. If this was not the case, there is a prime power $q|d$ and a pair $c,d\in\mathbb{K}\setminus\{0\}$ such that $q|[\mathbb{L}:\mathbb{L}(c)]$ and $q|[\mathbb{L}:\mathbb{L}(d)]$. But then, since q is a maximal prime power dividing d, we also have $q|[\mathbb{L}:(\mathbb{L}(c)\cdot\mathbb{L}(d))]$, in contradiction with the fact that $\mathbb{L}=\mathbb{L}(c)\cdot\mathbb{L}(d)$. Thus there can be at most $t=\omega(d)$ distinct values of c for which $\mathbb{L}(c)\neq\mathbb{L}$; here $\omega(x)$ is the number of distinct primes dividing x. □

20. Prove that the Diophantine equations of the form

$$x^a+y^b=(2p+1)z^c \qquad (6)$$

do not accept integer solutions except the trivial one $x=0$, $y=0$, $z=0$ when a, b, c are even positive integers and p is an odd integer ($abc\neq 0$).

(Elias Karakitsos, Sparta, Greece)

Solution by József Sándor (Romania). Let $a=2A$, $b=2B$, $c=2C$ ($A,B,C\geq 1$) be even integers. Put $p=2k+1=$ odd integer. Then the given equation becomes

$$X^2+Y^2=(4k+3)Z^2 \qquad (1)$$

where $X=x^A$, $Y=y^B$, $Z=z^C$.

We will show that (1) has only trivial solutions. If $X,Y\geq 1$, then $Z\geq 1$. It is well-known that the equation

$$X^2+Y^2=n \qquad (2)$$

in positive integers is possible if and only if in the prime factorization of n there is not a prime factor of the form p^e, where p is a prime of the form $4m+3$ and e is an odd integer.

In our case $n=(4k+3)Z^2$. Clearly each prime factor of Z^2 has even powers. For $4k+3$, however, we must have at least a prime divisor at an odd power. Indeed if a prime number has the form $q=4s+1$ then q^e has this form too, for any e; if $q=4s+3$ then q^r for r even has the form $4k+1$, as $(4a+3)(4b+3)=4c+1$ for any a,b,c integer numbers. □

Also solved by Konstantine Zelator (USA) and the proposer.

21. If a, $b>1$ prove that

$$\sum_{k=1}^{\infty}\frac{1}{\sqrt{k^ak^b(k^a+k^b)}}\leq\frac{1}{4}\big(\zeta(a)+\zeta(b)+\zeta(a+b)\big), \qquad (7)$$

where ζ denotes the Riemann zeta function.

(Mihály Bencze, Brasov, Romania)

Solution by József Sándor (Romania). It is sufficient to prove the inequality

$$\frac{1}{\sqrt{k^ak^b(k^a+k^b)}}\leq\frac{1}{4}\left(\frac{1}{k^a}+\frac{1}{k^b}+\frac{1}{k^{a+b}}\right)=\frac{k^a+k^b+1}{4k^{a+b}} \qquad (1)$$

or equivalently

$$4\sqrt{k^ak^b}\leq\sqrt{k^a+k^b}(k^a+k^b+1). \qquad (2)$$

But $k^a+k^b+1\geq 2\sqrt{k^a+k^b}$ and (2) follows by $k^a+k^b\geq 2\sqrt{k^ak^b}$, as an application of the arithmetic-geometric inequality

$$x+y\geq 2\sqrt{xy}$$

for positive real numbers x,y. □

Also solved by the proposer and Ovidiu Furdui (USA), who in addition has shown that the following inequality also holds

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^a k^b \left(k^a + k^b\right)}} \le \frac{1}{\sqrt{2}} \zeta\left(\frac{3(a+b)}{4}\right) \le \frac{1}{4}\big(\zeta(a) + \zeta(b) + \zeta(a+b)\big).. \quad (8)$$

22. Let $p \ge 3$ be a prime number and $f : N^* \to N^*$, where $N^* = N - \{0\}$ and $f(n)$ denotes the number of perfect k powers in the interval $[n^k, pn^k]$, $k \ge 2$. Prove that f is a monotone increasing function. Is f surjective? (Mihály Bencze, Brasov, Romania)

Solution by Ovidiu Furdui (USA). We prove that $f(n) = \lfloor \sqrt[k]{p}n \rfloor - n + 1$, where $\lfloor a \rfloor$ denotes the *integer part* of a. Let x be a natural number and let x^k be a *k perfect power* in the interval $[n^k, pn^k]$. It follows that $n^k \le x^k \le pn^k$ and hence $n \le x \le \sqrt[k]{p}n$. However, the number of all natural numbers x that belong to the interval $[n, \sqrt[k]{p}n]$ is given by $\lfloor \sqrt[k]{p}n \rfloor - n + 1$.

To prove that f is a *monotonically increasing* function, we need to show that for all $n \in N$ the following inequality holds $\lfloor \sqrt[k]{p}(n+1) \rfloor - (n+1) + 1 \ge \lfloor \sqrt[k]{p}n \rfloor - n + 1$. Equivalently $\lfloor \sqrt[k]{p}(n+1) \rfloor - \lfloor \sqrt[k]{p}n \rfloor \ge 1$. However, the last inequality holds since $\sqrt[k]{p}(n+1) - \sqrt[k]{p}n = \sqrt[k]{p} > 1$.

We prove that, in general, the function $f(n) = \lfloor \sqrt[k]{p}n \rfloor - n + 1$ is *not a surjective* function. By way of contradiction, we assume that f is a surjective function. It follows that for $y \in N$ the equation $\lfloor \sqrt[k]{p}n \rfloor - n + 1 = y$ has at least one solution n. This implies that $\lfloor \sqrt[k]{p}n \rfloor = y + n - 1$ and hence $y + n - 1 \le \sqrt[k]{p}n < y + n$. Thus

$$\frac{y-1}{\sqrt[k]{p}-1} \le n < \frac{y}{\sqrt[k]{p}-1}. \quad (9)$$

It follows that the equation $\lfloor \sqrt[k]{p}n \rfloor - n + 1 = y$ has a solution n if and only if inequality (9) holds. However, if $y = 11$, $k = 2$ and $p = 5$ there is no natural number n such that (9) would hold, since this would imply that $8.09 \le n < 8.89$. Thus f is not surjective.

Remark. We prove that, under the additional hypothesis that $p < 2^k$, the function $f(n) = \lfloor \sqrt[k]{p}n \rfloor - n + 1$ is a surjective function. It suffices to show that for a fixed natural number y we can find a natural number n such that the inequality (9) is satisfied. The value of n that satisfies (9) is given by $\left\lfloor \frac{y}{\sqrt[k]{p}-1} \right\rfloor$. We have that

$$\left\lfloor \frac{y}{\sqrt[k]{p}-1} \right\rfloor < \frac{y}{\sqrt[k]{p}-1},$$

and hence the right hand side of (9) holds. On the other hand,

$$\left\lfloor \frac{y}{\sqrt[k]{p}-1} \right\rfloor > \frac{y}{\sqrt[k]{p}-1} - 1 > \frac{y-1}{\sqrt[k]{p}-1}.$$

The last inequality holds since $\frac{1}{\sqrt[k]{p}-1} > 1$. Thus, the left hand side of (9) holds and the problem is solved. □

Also solved by Con Amore Problem Group (Denmark), József Sándor (Romania) and the proposer.

23. Show that the equation $q = 2p^2 + 1$, where p and q are primes, admits the unique solution $q = 19$, $p = 3$. (K. Drakakis, University College Dublin, Ireland)

Solution by Ovidiu Furdui (USA). We distinguish here three cases. If $p \equiv 1 \pmod 3$, i.e. $p = 3k+1$, a calculation shows that $q = 3(6k^2 + 4k + 1)$. We get, since q is a prime number, that $q = 3$, which in turns implies that $p = 1$, which is definitely not a solution of the Diophantine equation. If $p \equiv 2 \pmod 3$, i.e. $p = 3k+2$, another calculation shows that $q = 3(6k^2 + 8k + 3)$ and hence $q = 3$ and $p = 1$. The only case left is when $p \equiv 0 \pmod 3$, i.e. $p = 3$, which shows that $q = 19$ and the problem is solved. □

Also solved by J.F. Bradley (UK), the Con Amore Problem Group (Denmark), Wolfgang Fensch (Germany), József Sándor (Romania) and the proposer.

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will be devoted to *Discrete Mathematics*.

Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the addresses mpacurar@econ.ubbcluj.ro or madalina_pacurar@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files).

March 2008

2–7: IX International Conference "Approximation and Optimization in the Caribbean", San Andres Island, Colombia
Information: appopt2008@univalle.edu.co;
http://matematicas.univalle.edu.co/~appopt2008/

3–5: International Technology, Education and Development Conference (INTED2008), Valencia, Spain
Information: inted2008@iated.org; http://www.iated.org/inted2008

3–6: The Third International Conference On Mathematical Sciences (ICM2008), UAE University – Al-Ain, United Arab Emirates
Information: ICM2008@uaeu.ac.ae; http://icm.uaeu.ac.ae

4–7: 8th German Open Conference on Probability and Statistics, Aachen, Germany
Information: gocps2008@stochastik.rwth-aachen.de;
http://gocps2008.rwth-aachen.de

5–8: The First Century of the International Commission on Mathematical Instruction, Accademia dei Lincei, Rome, Italy
Information: http://www.unige.ch/math/EnsMath/Rome2008/

6–April 4: Cours "Méthodes variationnelles et parcimonieuses en traitement des signaux et des images", Paris, France
Information: gabriel.peyre@ceremade.dauphine.fr; http://www.ceremade.dauphine.fr/~peyre/cours-ihp-2008/

9–12: LUMS 2nd International Conference on Mathematics and its Applications in Information Technology 2008 (in collaboration with SMS, Lahore), Lahore, Pakistan
Information: http://www.lums.edu.pk/licm08

10–14: ALEA meeting, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

17–21: GL (2), CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–21: The IAENG International Conference on Scientific Computing 2008, Regal Kowloon Hotel, Kowloon, Hong Kong
Information: publication@iaeng.org;
http://www.iaeng.org/IMECS2008/ICSC2008.html

25–28: Taiwan-France joint conference on nonlinear partial differential equations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

25–28: 3rd International Conference "Functional spaces. Differential operators. General topology. Problems of mathematical education", dedicated to the 85th anniversary of L.D. Kudryavtsev, Moscow, Russia
Information: srozanova@mail.ru, Victor_lazarev@mail.ru;
http://foroff.phys.msu.ru/math/kudryavtsev/eng/

31–April 4: Mathematical Computer Science School for Young Researchers, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

April 2008

1–4: Workshop on Ergodic Theory and Geometry, Manchester, UK
Information: sharp@maths.man.ac.uk;
http://www.maths.manchester.ac.uk/~sharp/etg.html

6–9: Mathematical Education of Engineers, Mathematics Education Centre, Loughborough University, Loughborough, UK
Information: mee2008@lboro.ac.uk; http://mee2008.lboro.ac.uk/

7–11: Graph decomposition, theory, logics and algorithms, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

14–18: Dynamical quantum groups and fusion categories, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

21–25: Multiscale methods for fluid and plasma turbulence: Applications to magnetically confined plasmas in fusion devices, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

28–May 2: Recent progress in operator and function theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

May 2008

5–9: Statistical models for images, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

12–16: New challenges in scheduling theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

15–17: Twelfth International Conference Devoted to the Memory of Academician Mykhailo Kravchuk (Krawtchouk) (1892≈1942), National Tech. Univ. of Ukraine, Kyiv, Ukraine
Information: kravchukconf@yandex.ru

19–21: ManyVal '08 – Applications of Topological Dualities to Measure Theory in Algebraic Many-Valued Logic, Milan, Italy
Information: http://manyval.dsi.unimi.it/

19–23: Topological & Geometric Graph Theory, Paris, France
Information: tggt2008@ehess.fr; http://tggt.cams.ehess.fr

19–23: Vorticity, rotation and symmetry – Stabilizing and destabilizing fluid motion, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–25: Perspectives in Analysis, Geometry, and Topology, Stockholm, Sweden
Information: pagt@math.su.se;
http://www.math.su.se/pagt/index.html

26–30: Spring school in nonlinear partial differential equations, Louvain-La-Neuve, Belgium
Information: http://www.uclouvain.be/math-spring-school-pde-2008.html

26–30: The Fourth International Conference "Inverse Problems: Modeling and Simulation", Oludeniz-Fethiye, Turkey
Information: ahasanov@kou.edu.tr; http://www.ipms-conference.org/

26–30: Discrete Groups and Geometric Structures, with Applications III, K. U. Leuven Campus Kortrijk, Belgium
Information: Paul.Igodt@kuleuven-kortrijk.be;
http://www.kuleuven-kortrijk.be/workshop

26–30: High dimensional probability, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

26–30: Congrès à la mémoire d'Adrien Douady, Institut Henri Poincaré, Paris
Information: http://www.picard.ups-tlse.fr/adrien2008/

26–30: Journées Numération, Prague, Czech Republic
Information: jn08@kmwww.fjfi.cvut.cz;
http://kmwww.fjfi.cvut.cz/jn08/

26–June 7: Analysis and Topology, Lviv-2008, Lviv, Ukraine
Information: lvivconf08@yahoo.com;
http://www.franko.lviv.ua/faculty/mechmat/Departments/conf/conf2008.html

27–June 5: 5th Linear Algebra Workshop, Kranjska Gora, Slovenia
Information: damjana.kokol@fmf.uni-lj.si; http://www.law05.si

28–31: History of Mathematics & Teaching of Mathematics, Targu-Mures, Romania
Information: matkp@uni-miskolc.hu

29–31: Brownian motion and random walks in mathematics and in physics, Institut de Recherche Mathématique Avancée (Université Louis Pasteur), Strasbourg, France
Information: franchi@math.u-strasbg.fr,
papadop@math.u-strasbg.fr;
http://www-irma.u-strasbg.fr/article545.html

30–June 4: Analysis, Topology and Applications 2008 (ATA2008), Vrnjacka Banja, Serbia
Information: lkocinac@ptt.yu; http://www.tfc.kg.ac.yu/ata2008

June 2008

1–7: Applications of Ultrafilters and Ultraproducts in Mathematics (ULTRAMATH 2008), Pisa, Italy
Information: ultramath@dm.unipi.it;
http://www.dm.unipi.it/~ultramath

2–6: International Conference on Random Matrices (ICRAM), Sousse, Tunisia
Information: abdelhamid.hassairi@fss.rnu.tn;
http://www.tunss.net/accueil.php?id=ICRAM

2–6: Thompson's groups: new developments and interfaces, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

3–6: Chaotic Modeling and Simulation International Conference (CHAOS2008), Chania, Crete, Greece
Information: skiadas@asmda.net;
http://www.asmda.net/chaos2008

5–6: 12th Galway Topology Colloquium, Galway, Ireland
Information: aisling.mccluskey@nuigalway.ie;
http://www.maths.nuigalway.ie/conferences/topology08.html

6–11: Tenth International Conference on Geometry, Integrability and Quantization, Sts. Constantine and Elena resort, Varna, Bulgaria
Information: mladenov@bio21.bas.bg;
http://www.bio21.bas.bg/conference/

8–14: Mathematical Inequalities and Applications 2008, Trogir – Split, Croatia
Information: mia2008@math.hr; http://mia2008.ele-math.com/

8–14: 34th International Conference "Applications of Mathematics in Engineering and Economics", resort of Sozopol, Bulgaria
Information: mtod@tu-sofia.bg;
http://www.tu-sofia.bg/fpmi/amee/index.html

9–11: Net-Works 2008, Pamplona, Spain
Information: networks2008@unav.es;
http://fisica.unav.es/networks2008/

9–13: Geometric Applications of Microlocal Analysis, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

9–13: Conference on Algebraic and Geometric Topology, Gdansk, Poland
Information: cagt@math.univ.gda.pl;
http://math.univ.gda.pl/cagt/

9–19: Advances in Set-Theoretic Topology: Conference in Honour of Tsugunori Nogura on his 60th Birthday, Erice, Sicily, Italy
Information: erice@dmitri.math.sci.ehime-u.ac.jp;
http://www.math.sci.ehime-u.ac.jp/erice/

15–22 Operator Theory, Analysis and Mathematical Physics, ESF Mathematics Conference, Będlewo, Poland
Information: www.esf.org/conferences/08279

15–26: CIMPA-School Nonlinear analysis and Geometric PDE, Tsaghkadzor, Armenia
Information: imprs@mis.mpg.de;
http://www.imprs-mis.mpg.de/schools.html

16–17: Fifth European PKI Workshop, Trondheim, Norway
Information: sjouke.mauw@uni.lu;
http://www.item.ntnu.no/europki08/

16–19: 2nd International Conference on Mathematics & Statistics, Athens, Greece
Information: atiner@atiner.gr;
http://www.atiner.gr/docs/Mathematics.htm

16–20: Workshop on population dynamics and mathematical biology, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

16–20: Homotopical Group Theory and Homological Algebraic Geometry Workshop, Copenhagen, Denmark
Information: http://www.math.ku.dk/~jg/homotopical 2008/

16–20: Fourth Conference on Numerical Analysis and Applications, Lozenetz, Bulgaria
Information: http://www.ru.acad.bg/naa08/

16–20: Conference on vector bundles in honour of S. Ramanan (on the occasion of his 70th birthday), Miraflores de la Sierra (Madrid), Spain
Information: oscar.garcia-prada@uam.es;
http://www.mat.csic.es/webpages/moduli2008/ramanan/

16–20: 15-th Conference of the International Linear Algebra Society (ILAS 2008), Cancun, Mexico
Information: ilas08@star.izt.uam.mx; http://star.izt.uam.mx/ILAS08/

16–21: International Scientific Conference "Differential Equations, Theory of Functions and their Applications" dedicated to the 70th birthday of academician of NAS of Ukraine A. M. Samoilenko, Melitopol, Ukraine
Information: conf2008@imath.kiev.ua;
http://www.imath.kiev.ua/conf2008/

17–20: Structural Dynamical Systems: Computational Aspects, Capitolo-Monopoli, Bari, Italy
Information: sds08@dm.uniba.it;
http://www.dm.uniba.it/~delbuono/sds2008.htm

17–22: International conference "Differential Equations and Topology" dedicated to the Centennial Anniversary of Lev Semenovich Pontryagin, Moscow, Russia
Information: pont2008@cs.msu.ru; http://pont2008.cs.msu.ru

22–28: Combinatorics 2008, Costermano (VR), Italy
Information: combinatorics@ing.unibs.it;
http://combinatorics.ing.unibs.it

23–27: Homotopical Group Theory and Topological Algebraic Geometry, Max Planck Institute for Mathematics, Bonn, Germany
Information: admin@mpim-bonn.mpg.de;
http://www.ruhr-uni-bochum.de/topologie/conf08/

23–27: Hermitian symmetric spaces, Jordan algebras and related problems, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

23–27: Conference on Differential and Difference Equations and Applications 2008 (CDDEA 2008), Strečno, Slovak Republic
Information: cddea@fpv.uniza.sk; http://www.fpv.uniza.sk/cddea/

23–27: Workshop on Geometric Analysis, Elasticity and PDEs, on the 60th Birthday of John Ball, Heriot Watt University, Edinburgh, UK
Information: morag.burton@icms.org.uk;
http://www.icms.org.uk/workshops/pde

25–28: VII Iberoamerican Conference on Topology and its Applications, Valencia, Spain
Information: cita@mat.upv.es; http://cita.webs.upv.es

30–July 3: Analysis, PDEs and Applications, Roma, Italy
Information: mazya08@mat.uniroma1.it;
http://www.mat.uniroma1.it/~mazya08/

30–July 4: Joint ICMI/IASE Study; Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education, Monterrey, Mexico
Information: batanero@ugr.es;
http://www.ugr.es/~icmi/iase_study/

30–July 4: Geometry of complex manifolds, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

30–July 4: The European Consortium for Mathematics in Industry (ECMI2008), University College, London, UK
Information: lucy.nye@ima.org.uk; http://www.ecmi2008.org/

July 2008

1–4: VIth Geometry Symposium, Bursa, Turkey
Information: arslan@uludag.edu.tr;
http://www20.uludag.edu.tr/~geomsymp/index.htm

2–4: The 2008 International Conference of Applied and Engineering Mathematics, London, UK
Information: wce@iaeng.org;
http://www.iaeng.org/WCE2008/ICAEM2008.html

2–11: Soria Summer School on Computational Mathematics: "Algebraic Coding Theory" (S3CM), Universidad de Valladolid, Soria, Spain
Information: edgar@maf.uva.es; http://www.ma.uva.es/~s3cm/

3–8: 22nd International Conference on Operator Theory, West University of Timisoara, Timisoara, Romania
Information: ot@theta.ro; http://www.imar.ro/~ot/

7–10: The Tenth International Conference on Integral Methods in Science and Engineering (IMSE 2008), University of Cantabria, Santander, Spain
Information: imse08@unican.es, meperez@unican.es;
http://www.imse08.unican.es/

7–10: International Workshop on Applied Probability (IWAP 2008), Compiegne, France
Information: nikolaos.limnios@utc.fr,joseph.glaz@uconn.edu;
http://www.lmac.utc.fr/IWAP2008/

7–11: VIII International Colloquium on Differential Geometry (E. Vidal Abascal Centennial Congress), Santiago de Compostela, Spain
Information: icdg2008@usc.es; http://xtsunxet.usc.es/icdg2008

7–11: Spectral and Scattering Theory for Quantum Magnetic Systems, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

7–12: New Horizons in Toric Topology, Manchester, UK
Information: Helen.Kirkbright@manchester.ac.uk;
http://www.mims.manchester.ac.uk/events/workshops/NHTT08/

7–12: International Conference on Modules and Representation Theory, Cluj-Napoca, Romania
Information: aga_team@math.ubbcluj.ro,
aga.team.cluj@gmail.com;
http://math.ubbcluj.ro/~aga_team/AlgebraConference-Cluj2008.html

13: Joint EWM/EMS Workshop, Amsterdam (The Netherlands)
Information: http://womenandmath.wordpress.com/joint-ewmems-worskhop-amsterdam-july-13th-2007/

14–18: Fifth European Congress of Mathematics (5ECM), Amsterdam, Netherlands
Information: http://www.5ecm.nl

14–18: Efficient Monte Carlo: From Variance Reduction to Combinatorial Optimization. A Conference on the Occasion of R.Y. Rubinstein's 70th Birthday, Sandbjerg Estate, Sønderborg, Denmark
Information: oddbjorg@imf.au.dk;
http://www.thiele.au.dk/Rubinstein/

15–18: Mathematics of program construction, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

15–19: The 5th World Congress of the Bachelier Finance Society, London, UK
Information: mark@chartfield.org; http://www.bfs2008.com

17–19: 7th International Conference on Retrial Queues (7th WRQ), Athens, Greece
Information: aeconom@math.uoa.gr;
http://users.uoa.gr/~aeconom/7thWRQ_Initial.html

20–23: International Symposium on Symbolic and Algebraic Computation, Hagenberg, Austria
Information: franz.winkler@risc.uni-linz.ac.at;
http://www.risc.uni-linz.ac.at/about/conferences/issac2008/

21–24: SIAM Conference on Nonlinear Waves and Coherent Structures, Rome, Italy
Information: meetings@siam.org; http://www.siam.org/meetings/nw08/

21–25: Operator Structures and Dynamical Systems, Leiden, the Netherlands
Information: mdejeu@math.leidenuniv.nl;
http://www.lorentzcenter.nl/lc/web/2008/288/info.php3?wsid=288

21–25: Summer School 'PDE from Geometry', Cologne, Germany
Information: http://www.mi.uni-koeln.de/~gk/school08

21–August 29: CEMRACS, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

24–26: Workshop on Current Trends and Challenges in Model Selection and Related Areas, University of Vienna, Vienna, Austria
Information: hannes.leeb@yale.edu;
http://www.univie.ac.at/workshop_modelselection/

August 2008

3–9: Junior Mathematical Congress, Jena, Germany
Information: info@jmc2008.org; http://www.jmc2008.org/

13–19: XXVII International Colloquium on Group Theoretical Methods in Physics (Group27), Yerevan, Armenia
Information: pogosyan@ysu.am; http://theor.jinr.ru/~group27

16–31: EMS-SMI Summer School: Mathematical and numerical methods for the cardiovascular system, Cortona, Italy
Information: dipartimento@matapp.unimib.it

18–22: International conference on ring and module theory, Ankara, Turkey
Information: http://www.algebra2008.hacettepe.edu.tr

19–22: Duality and Involutions in Representation Theory, National University of Ireland, Maynooth, Ireland
Information: involutions@maths.nuim.ie;
http://www.maths.nuim.ie/conference/

21–23: International Congress of 20th Jangjeon Mathematical Society, Bursa, Turkey
Information: cangul@uludag.edu.tr, hozden@uludag.edu.tr, inam@uludag.edu.tr;
http://www20.uludag.edu.tr/~icjms20/

September 2008

1–5: Representation of surface groups, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

1–5: Conference on Numerical Analysis (NumAn 2008), Kalamata, Greece
Information: numan2008@math.upatras.gr;
http://www.math.upatras.gr/numan2008/

2–5: X Spanish Meeting on Cryptology and Information Security, Salamanca, Spain
Information: delrey@usal.es;
http://www.usal.es/xrecsi/english/main.htm

8–12: Chinese-French meeting in probability and analysis, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

8–12: International Workshop on Orthogonal Polynomials and Approximation Theory 2008. Conference in honor of professor Guillermo López Lagomasino at his 60th Anniversary, Universidad Carlos III de Madrid, Leganés, Spain
Information: iwopa08@gmail.com;
http://www.uc3m.es/iwopa08

8–19: EMS Summer School: Mathematical models in the manufacturing of glass, polymers and textiles, Montecatini, Italy
Information: cime@math.unifi.it;
http://web.math.unifi.it/users/cime//

10–12: Nonlinear Differential Equations (A Tribute to the work of Patrick Habets and Jean Mawhin on the occasion of their 65th birthdays), Brussels, Belgium
Information: node2008@uclouvain.be;
http://www.uclouvain.be/node2008.html

14–18: 7th Euromech Fluid Mechanics Conference, Manchester, UK
Information: http://www.mims.manchester.ac.uk/events/workshops/EFMC7/

15–19: Geometry and Integrability in Mathematical Physics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

16–20: International Conference of Numerical Analysis and Applied Mathematics 2008 (ICNAAM 2008), Psalidi, Kos, Greece
Information: tsimos@mail.ariadne-t.gr; http://www.icnaam.org/

18–21: 6th International Conference on Applied Mathematics (ICAM6), Baia Mare, Romania
Information: vberinde@ubm.ro; http://www.icam.ubm.ro

19–26: International Conference on Harmonic Analysis and Approximations IV, Tsaghkadzor, Armenia
Information: mathconf@ysu.am;
http://math.sci.am/conference/sept2008/conf.html

21–24: The 8th International FLINS Conference on Computational Intelligence in Decision and Control (FLINS 2008), Madrid, Spain
Information: flins2008@mat.ucm.es;
http://www.mat.ucm.es/congresos/flins2008

22–25: Symposium on Trends in Applications of Mathematics to Mechanics (STAMM 2008), Levico, Italy
Information: stamm08@gmail.com;
http://mate.unipv.it/pier/stamm08.html

22–26: 10th International workshop in set theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

22–28: 4th International Kyiv Conference on Analytic Number Theory and Spatial Tessellations, Jointly with 5th Annual International Conference on Voronoi Diagrams in Science and Engineering (dedicated to centenary memory of Georgiy Voronoi), Kyiv, Ukraine
Information: voronoi@imath.kiev.ua;
http://www.imath.kiev.ua/~voronoi

29–October 3: Commutative algebra and its interactions with algebraic geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

29–October 8: EMS Summer School: Risk theory and related topics, Będlewo, Poland
Information: stettner@iman.gov.pl;
www.impan.gov.pl/EMSsummerSchool/

October 2008

6–10: Partial differential equations and differential Galois theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

9–12: The XVI-th Conference on Applied and Industrial Mathematics (CAIM 2008), Oradea, Romania
Information: serban_e_vlad@yahoo.com; http://www.romai.ro

13–17: Hecke algebras, groups and geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

20–24: Symbolic computation days, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

22–24: International Conference on Modeling, Simulation and Control 2008, San Francisco, USA
Information: wcecs@iaeng.org;
http://www.iaeng.org/WCECS2008/ICMSC2008.html

26–28: 10th WSEAS Int. Conf. on Mathematical Methods and Computational Techniques in Electrical Engineering (MMACTEE 8), Corfu, Greece
Information: info@wseas.org;
http://www.wseas.org/conferences/2008/corfu/mmactee/

26–28: 7th WSEAS Int. Conf. on Non-Linear Analysis, Non-Linear Systems and Chaos (NOLASC 8), Corfu, Greece
Information: info@wseas.org;
http://www.wseas.org/conferences/2008/corfu/nolasc/

26–31: New trends for modeling laser-matter interaction, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

27–31: New trends for modeling laser-matter interaction, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

November 2008

3–7: Harmonic analysis, operator algebras and representations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

5–7: Fractional Differentiation and its Applications, Ankara, Turkey
Information: dumitru@cankaya.edu.tr; http://www.cankaya.edu.tr/fda08/

10–14: The 6th Euro-Maghreb workshop on semigroup theory, evolution equations and applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

17–21: Geometry and topology in low dimension, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

24–28: Approximation, geometric modelling and applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

December 2008

1–5: Homology of algebras: structures and applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

8–12: Latent variables and mixture models, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

15–19: Meeting on mathematical statistics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

16–18: Eighth IMA International Conference on Mathematics in Signal Processing, Cirencester, UK
Information: pam.bye@ima.org.uk; http://www.ima.org.uk/Conferences/signal_processing/signal_processing08.html

July 2009

6–10: 26th Journées arithmétiques, Saint-Etienne, France
Information: ja2009@univ-st-etienne.fr; http://ja2009.univ-st-etienne.fr

Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to: Ivan Netuka, MÚUK, Sokolovská, 83, 186 75 Praha 8, Czech Republic.

***J. M. Alongi, G.S. Nelson: Recurrence and Topology**, Graduate Studies in Mathematics, vol. 85, American Mathematical Society, Providence, 2007, 221 pp., USD 45, ISBN 978-0-8218-4234-8*
The main topic of this book is recurrence of dynamical systems on compact metric spaces (the authors consider continuous flows). The book starts with an introduction to flows, their special points and corresponding invariant sets (from fixed points and sets via periodic points and recurrent points to nonwandering and chain recurrent points). The next chapter deals with irreducible sets of flows (minimal sets, transitive sets, chain-transitive sets, attracting sets and repelling sets). The last chapter describes certain test functions (the potential function, the Hamiltonian function, the Morse function and several kinds of Lyapunov functions). The chapter ends with the fundamental theorem of dynamical systems: every continuous flow on a compact metric space has a complete Lyapunov function.

Every chapter contains exercises. Three short appendices contain a discussion of discrete dynamical systems, recurrence of circle rotation, and completeness and compactness of Hausdorff hyperspaces. In 'Afterwords', the main ideas of the book and relations to hyperbolic systems are briefly described, and suggestions for further reading are given. The specific feature of the book is that concepts (and many results) are demonstrated on flows of explicit systems of differential equations. (mihus)

***V. Apostolov, A. Dancer, N. Hitchin, M. Wang, Eds.: Perspectives in Riemannian Geometry**, CRM Proceedings and Lecture Notes, vol. 40, American Mathematical Society, Providence, 2006, 248 pp., USD 85, ISBN 978-0-8218-3852-5*
The papers in this volume are written by some of the participants of the Short Program in Riemannian Geometry held at the Centre de Recherche Mathématiques, Montreal, 2004. The main topics of the proceedings are covered by three papers based on lectures given by M. Anderson, K. Grove and N. J. Hitchin. The first paper is Anderson's comprehensive survey of recent results on the existence of Einstein metrics on open manifolds with a certain structure at infinity. The second paper (complementing the topic) is the Biquard survey of Einstein metrics with asymptotic structure at infinity modelled on the complex hyperbolic space and their applications to CR geometry. The Grove lecture notes survey comparison geometry, mainly in the framework of lower bounds of sectional curvatures. Hitchin addresses the question of special geometry in dimensions 6, 7 and 8 and their relation to the geometry of open orbits of Lie groups.

Another two papers in the volume are concerned with special geometric structures. R. Bryant's paper provides a systematic study of a class of special Lagrangian submanifolds in complex domains, and A. Dancer together with M. Wang explain their Hamiltonian approach to cohomogeneity one Einstein metrics. Three of the papers are devoted to the interaction between Riemannian and complex geometry (the paper by C. Boyer and K. Galicki is a survey of new Sasaki-Einstein metrics build out of Kähler-Einstein orbifolds; L. David and P. Gauduchon's paper provides a thorough study of Bochner-flat Kähler orbifolds from the point of view of the CR-geometry of the standard sphere; and C. LeBrun's paper studies the stability of complex curves with boundary in twistor spaces appearing in the new approach to Zoll manifolds). The last paper by A. Nabutovsky contains a survey of the relationship between the space of Riemannian structures on closed manifolds, computability theory and algorithmic information theory. (pso)

***A. Banner: The Calculus Lifesaver. All the Tools You Need To Excel at Calculus**, A Princeton Lifesaver Study Guide, Princeton University Press, Princeton, 2007, 728 pp., USD 24.95, ISBN 978-0-691-13088-0*
On more than 700 pages the author explains standard calculus material for real functions of one real variable (from limits of functions to ordinary differential equations). The explanation is based not only on some theorems but mainly on carefully selected examples and their elaboration. It takes more time and more pages but it is surely convenient for students to understand methods used in this part of calculus both from theoretical and practical points of view. Care is devoted to the introductory chapters dealing with basic properties of functions, as can

be seen from the distribution of pages in the book. The first 300 pages are devoted to functions, their limits and derivatives, graphs of functions, optimization and linearization. Almost 200 pages deal with integration and its applications (volumes, lengths and surfaces), while the rest is devoted to sequences and series (about 100 pages, including Taylor series) and differential equations (about 25 pages). Proofs of some theorems are contained in the last 40 pages of the book. Videotapes of the whole course are available for free on the Internet. (mihus)

D. Bertrand, P. Debes, Eds.: Groupes de Galois arithmétique et différentials*, Séminaires & Congrès, no. 13, Société Mathématique de France, Paris, 391 pp., EUR 76, ISBN 978-2-85629-222-8*
This collection presents the proceedings of the conference on arithmetic and differential Galois groups. On almost 400 pages there are 15 contributions delivered at the conference showing vivid interactions between two highly developed theories: Galois theory of algebraic functions and the theory of differential operators. The range of topics covers expositions on many facets of the field. The reader will find here a wealth of interesting material: on arithmetic differential algebra and its applications to the invariant theory of correspondences; on the geometry behind the sixth Panlevé differential equation and construction of some its algebraic solutions; and on the Abhyankar conjecture, explicit computations or the realisation of Galois groups. Also contained in this collection is how construction problems of algebraic varieties with various topological behaviour motivate the search for methods of doing mathematics in a machine checked way. The collection of papers will certainly help orient anybody interested in the subject and will provide them with a stimulating impetus for further reading or motivation for future research. (spor)

D. Bressoud: A Radical Approach to Real Analysis*, second edition, Classroom Resource Materials, Mathematical Association of America, Washington, 2007, 323 pp., GBP 27.99, ISBN 978-0-88385-747-2*
Every teacher of mathematical analysis at any university knows the painful dilemma of how to properly teach this subject. The standard syllabus starts with real numbers and then goes on to sequences, real functions, series of numbers, continuity, differentiation, integration, functions of several variables, and finally to sequences and series of functions. Such organisation of the course is technically perfect; it contains carefully prepared definitions followed by brilliant theorems with polished proofs and intriguing examples, however it brings very little motivation for studying the subject. The motivation actually stems from the topics with which the course usually culminates, that is, the application of infinite series of functions to solving partial differential equations that describe phenomena from the real world.

This remarkable book offers a different way of teaching university analysis. It begins with the discussion of the crisis in mathematics in the outbreak of the 19th century connected with the first appearance of Fourier series. Then it takes a journey through analysis keeping in mind the important historical background, taking the reader through the wonderful mathematical landscape full of dangerous, subtle pitfalls into which even the greatest mathematical minds such as Lagrange, Cauchy and even Weierstrass would fall, culminating in a return to Fourier series and Dirichlet's proof of their convergence. As the author claims in the foreword, this is not a book on the history of analysis but an attempt to follow Poincaré's injunction to let history inform pedagogy. This is a unique book, quite different from any analysis textbook I have seen before. It is immensely interesting, enlightening and extraordinarily historically knowledgeable. It may be disputable whether it should be used as the only source of analysis for a student. But, beyond any doubt, it is excellent collateral reading, especially for those who have been through a traditional course. And, for an instructor, it is absolutely indispensable. (lp)

G. Brightwell, I. Leader, A. Scott, A. Thomason, Eds.: Combinatorics and Probability*, Cambridge University Press, Cambridge, 2007, 633 pp., GBP 60, ISBN 978-0-521-87207-2*
This volume contains 31 papers presented at the conference "Combinatorics in Cambridge", held in Cambridge, 4–7 August 2003 on the occasion of the 60th birthday of Béla Bollobás. Therefore the authors were chosen with the aim of representing the many areas of mathematics that Béla Bollobás has influenced. The papers are written by excellent scientists in their research areas. Most of the papers are devoted to recent research in graph theory and its applications and connections to other fields of mathematics. Probability theory is used as a tool and new results are presented in many papers. Research in fields like number theory and convex geometry is also contained in some papers of this proceedings. All the papers have also appeared in the journal Combinatorics, Probability and Computing over the last two years. (dh)

J. M. F. Castillo, W. B. Johnson, Eds.: Methods in Banach Space Theory*, London Mathematical Society Lecture Note Series 337, Cambridge University Press, Cambridge, 2006, 357 pp., GBP 40, ISBN 978-0-521-68568-9*
The 2004 conference on 'Methods in Banach space theory' was held in Spain. The conference brought together 86 participants from 14 countries, including leading specialists in the field, and was organized, like this proceedings volume, in five sections reflecting a wide spectrum of modern Banach space theory: geometrical methods, homological methods, topological methods, operator theory methods and function space methods. There are sixteen papers written by specialists in these fields. They represent surveys of the state of the art in individual fields and also contain some of the latest and most important results of infinite dimensional Banach space theory. The book is warmly recommended to researchers and graduate students in Banach space theory and functional analysis. (jl)

A. Cattaneo, B. Keller, C. Torossian. A. Bruguieres: Déformation, quantification, théorie de Lie*, Panoramas et Syntheses, no. 20, Société Mathématique de France, Paris, 2005, 186 pp., EUR 26, ISBN 2-85629-183-X*
In 1997, M. Kontsevich proved that every Poisson manifold admits a formal quantization, canonical up to a suitable equivalence. He thus solved a longstanding problem in mathematical physics. His proof also opened up new horizons for many areas of mathematics like Lie theory, quantum group theory, deformation theory, operads, and their links to knot theory, number theory and the theory of motives. This volume is divided into three parts and one appendix devoted to the geometry of configuration spaces. The first part presents the main result on deformation quanti-

zation and includes a description of the Tamarkin approach to it. The second part shows the relevance of the Kontsevich theorem to Lie theory. The third part explains ideas from topological string theory that inspired the Kontsevich proof. (pso)

I. Chajda, R. Halaš, J. Kühr: Semilattice Structures*, Research and Exposition in Mathematics, vol. 30, Heldermann, Lemgo, 2007, 228 pp., EUR 28, ISBN 978-3-88538-230-0*

This book contains an algebraic study of semilattice structures. The choice of semilattice structures is motivated by their connections to logic (i.e. the origins and the motivation of axioms for various types of semilattice structures can be found in logic). The authors concentrate on algebraic properties of these structures and do not discuss the impact of their results to logic. They use the usual tools of general algebra. They investigate congruences of algebras, free objects in the corresponding varieties, the subdirectly irreducible algebras in the varieties, etc. After two chapters on algebraic preliminaries, the authors investigate pseudo-complemented semilattices, relatively pseudo-complemented semilattices, implication algebras, sectionally pseudo-complemented semilattices and residuated semilattices. The last chapter on weak systems investigates structures that are not semilattices but where there is still some order in the structure. The topics covered include Hilbert algebras, BCC-algebras and non-associative MV-algebras. (jtu)

M. Chebotar et al., Eds.: Rings and Nearrings. In Memory of Kostia Beidar*, Walter de Gruyter, Berlin, 2007, 167 pp., EUR 128, ISBN 978-3-11-019952-9*

This is the proceedings of the "International Conference of Algebra" held in memory of the distinguished Russian mathematician Kostia Beidar in Tainan in March 2005. The volume consists of seven survey papers written by leading experts in the area (and Beidar's co-authors) that present the current status of the branches of algebra and logic in which Beidar made fundamental contributions. These include the theory of functional identities and its applications to Lie maps and the solution of the Hernstein Conjectures, the theory of orthogonal completions and its applications to semiprime rings, and the theory of prime rings and strongly prime modules. Also presented are the deep results of Beidar on near-rings and radical theory, Hopf algebras, and the (still open) separativity problem for von Neumann regular rings. Finally, a list of all Beidar's publications (divided into sections by subject) is presented. All the survey papers are written very carefully and will thus be of great help both for researchers and graduate students working in the related areas of algebra and logic. (jtrl)

J. Déserti, D. Cerveau: Feuilletages et actions de groupes sur les espaces projectifs, *Mémoires Société Mathématique de France, no. 103, Paris, 2005, 124 pp., EUR 26, ISBN 2-85629-182-1*

The main topic of this book is a study of invariants with respect to an action of a complex Lie group G on a compact complex manifold M. The action of G defines a decomposition *F* of M to G-orbits. The decomposition *F* is called L-foliation if *F* coincides generically with a holomorphic foliation of M. Most attention is given to a study of L-foliations of codimension one in projective spaces. Basic properties of L-foliations are studied in chapter 1. Chapter 2 contains a lot of different examples of L-foliations of various actions on (complex) projective spaces. L-foliations of a low degree are studied in chapter 3. A particular case of L-foliations in dimension 3 is discussed in chapter 4. Quadratic L-foliations form a topic of chapter 5. The final chapter treats a particular case: L-foliations of degree 3 in dimension 4. The book contains some classification results in low dimensions. (vs)

L. Hogben, Ed.: Handbook of Linear Algebra, Discrete Mathematics and Its Applications*, Chapman & Hall/CRC, Boca Raton, 2006, 1400 pp., USD 119.95, ISBN 1-58488-510-6*

The editor has put together an impressive list of authors. This handbook covers all the major topics of linear algebra at both graduate and undergraduate level, as well as their applications and related software packages. It starts with elementary aspects of the subject and then takes the reader to the frontiers of current research. The book is organized in five main parts. The first part, consisting of 26 chapters, covers linear algebra. The next 10 chapters deal with combinatorial linear algebra and are followed by 13 chapters on numerical linear algebra. Applications of linear algebra both inside and outside of mathematics are dealt with over 21 chapters. Finally, there are 7 chapters on software packages useful for linear algebra computations.

Each chapter is written by a different team of authors. Individual chapters are divided into sections and each section is organized into a uniform format consisting of definitions, facts and examples. The facts (usually called theorems, lemmas, etc.) are presented in a list format without proofs but with detailed references. The examples illustrate the definitions and facts. The handbook will be without doubt a valuable resource for anyone using linear algebra, i.e. basically for any mathematician. (jtu)

D. Huybrechts: Fourier-Mukai Transforms in Algebraic Geometry*, Oxford Mathematical Monographs, Clarendon Press, Oxford, 2006, 307 pp., GBP 50, ISBN 0-19-929686-3*

This book is based on a course given by the author at the Institut de Mathématiques de Jussieu in 2004–2005. The topic treated in the book is the derived category of coherent sheaves on a smooth projective variety. In the last couple of years, the Kontsevich homological mirror symmetry has revived interest in these questions by proposing an equivalence of the derived category of coherent sheaves of certain projective varieties with the Fukaya category associated to the symplectic geometry of the mirror variety. It is this point of view that motivates and in some sense explains many of the central results as well as the open problems in this area. The most prominent example of this equivalence was observed by Mukai in the very first paper on the subject. He showed that the Poincaré bundle induces an equivalence between the derived category of an Abelian variety A and the derived category of its dual Ă, although Ă is generally not isomorphic to A. These results naturally led to the question, 'under what conditions do two smooth projective varieties give rise to equivalent derived categories?', which is the central theme of the book. (pso)

L. A. Hügel, D. Happel, H. Krause, Eds.: Handbook of Tilting Theory*, London Mathematical Society Lecture Note Series 332, Cambridge University Press, Cambridge, 2007, 472 pp., 978 fig., GBP 43, ISBN 978-0-521-68045-5*

This is the long awaited handbook of tilting theory whose publication was first suggested at the Twenty Years of Tilting Theory Colloquium at Fraueninsel in November 2002. In a series of 15 survey chapters written by leading experts in the area, the handbook presents all major aspects and applications of tilting theory starting from the 'pre-history' in the 1973 works of Bernstein, Gelfand and Ponomarev up to the very recent cluster tilting approach. The key point of classical tilting theory is the close relationship between the module category of the original algebra and of the tilted (endomorphism) algebra End(T) where T is a tilting module. This goes back to the works of Brenner-Butler (1979) and Happel-Ringel (1982). The relationship is in terms of a pair of category equivalences generalizing Morita equivalence, and it is particularly strong in the case when the original algebra is hereditary.

This theory and its fundamental applications to algebras of finite representation type are presented in chapters 2 and 3. A further major step was Ricard's 1989 work making tilting theory part of Morita theory for derived categories. This approach is explained in chapters 4 and 5, while its recent applications to modular representation theory of finite groups is covered in chapter 14. Chapter 7 surveys the recent use of derived categories in non-commutative algebraic geometry, while Morita theory for ring spectra and its role in algebraic topology are presented in chapter 15. Happel's theorem characterizing hereditary categories with a tilting object and its applications appear in chapter 6. The simplicial complex of tilting modules is presented in chapter 10.

In 1991 Auslander and Reiten started to use tilting theory to study homologically finite subcategories of modules. These results and their applications to quasi-hereditary algebras and algebraic groups are surveyed in chapters 8 and 9. Infinite dimensional tilting theory and its relations to finitistic dimension conjectures are presented in chapters 11 and 12. The dual notions of infinite dimensional cotilting modules and the corresponding generalizations of Morita duality theory appear in chapter 13. Of particular interest is the appendix written by C. M. Ringel: it both ties up the individual chapters of the handbook and serves as an introduction to the new field of cluster tilting. The handbook presents a key and very active part of contemporary representation theory in a concise but complete way. It will be indispensable for a wide audience, from graduate students to active researchers in algebra, geometry and topology. (jtrl)

H. Iwaniec, W. Narkiewicz, J. Urbanowicz, Eds.: Andrzej Schinzel – Selecta, vol. I, II*, Heritage of European Mathematics, European Mathematical Society, Zürich, 2007, 1417 pp., EUR 168, ISBN 978-3-03719-038-8*

Andrzej Schinzel is one of the leading figures and a living legend of contemporary mathematics. This selection of 100 of his papers shows the extraordinary variety of his mathematical interests. He wrote his first paper at the age of 17 and now the list contains more than 200 items. The central focus of his work is the arithmetic and algebraic properties of polynomials in one or several variables but as mentioned, this is only the tip of the iceberg, as shown by the fact that the selection is divided into 13 major sections, each devoted to a central theme and commented on by an expert. They include Diophantine equations and integral forms (10 papers, R. Tijdeman), continued fractions and integral forms (3 papers, E. Dubois), algebraic number theory (10 papers, D. W. Boyd and D. J. Lewis), polynomials in one variable (17 papers, M. Filaseta), polynomials in several variables (10 papers, U. Zannier), the Hilbert Irreducibility Theorem (3 papers, U. Zannier), arithmetic functions (6 papers, K. Ford), divisibility and congruences (11 papers, H.W. Lenstra Jr.), primitive divisors (6 papers, C. L. Stewart), prime numbers (5 papers, J. Kaczorowski), analytic number theory (4 papers, J. Kaczorowski), geometry of numbers (4 papers, W. M. Schmidt), and other papers (5 papers, S. Kwapien).

The selection ends with a chapter devoted to unsolved problems and unproved conjectures proposed by Schinzel up until 2006, which contains 56 items. The first, still open conjecture appeared in his 5th paper written in 1955 and concerns the decomposition of rationals into 3 unit (Egyptian) fractions. One of the most famous, the conjecture H, has the reference number 4. This extremely valuable collection of the most important papers of Andrzej Schinzel should certainly be on the shelf in every library. (spor)

S. Katok: p-adic Analysis Compared with Real*, Student Mathematical Library, vol. 37, American Mathematical Society, Providence, 2007, 152 pp., USD 23, ISBN 978-0-8218-4220-1*

The courses of p-adic (non-Archimedean) analysis seldom find a place in the undergraduate curriculum and often run separately from the classical (Archimedean) one with sporadic references to mutual differences. This book tries to introduce the reader to the first one making use of his/her familiarity with the classical counterpart and an intuitive experience from number theory and topology. The book starts with the construction of p-adic numbers based on the completion process of the field of rationals by replacing the Euclidean distance by a p-adic one. Then it continues with a study of their basic arithmetic and algebraic properties and a comparison of the topology of the field p-adic numbers, where p is a prime, with the field of real numbers (e.g. the strong triangle inequality, that balls are clopen, and the condition for the convergence of series).

The analytic highlights of the book are formed by elements of the analysis in the field of p-adic numbers and the basics of the calculus of p-adic functions. The reader will also find here some non-standard topics for undergraduate courses, including totally disconnected spaces, the Baire Category Theorem, and isometries of compact metric spaces. There are a large number of useful exercises and this makes the book more readable for advanced undergraduate or postgraduate students who want to learn something about p-adic numbers. In fact, many of them extend or complement the presented theory and some of them complete the proofs of results appearing in the text. Due to the role of p-adic numbers in many branches of mathematics and physics the book gives a good impetus to students to study the "p-adic worlds" more deeply. This role of the book is not only supported by carefully selected material but also by the fact that it is written in a very lively and lucid style. (spor)

Y. Katznelson, Y.R. Katznelson: A (Terse) Introduction to Linear Algebra*, Student Mathematical Library, vol. 44, American Mathematical Society, Providence, 2008, 215 pp., USD 35, ISBN 978-0-8218-4419-9*

This small booklet published in the series Student Mathematical Library of the AMS is devoted to a basic course on linear algebra as taught in standard undergraduate courses on the topic. It covers (in an economical way) all the necessary topics to be expected (vector spaces, basis and dimension, systems of linear equations, linear maps and their relations to matrices, determinants, linear functionals and adjoint maps, inner-product spaces with a discussion of normal, unitary and orthogonal operators, nilpotent operators and the Jordan canonical form, and characteristic and minimal polynomials of a map). The last chapter also covers the basics of quadratic forms, the Perron-Frobenius theory, stochastic matrices and representations of finite groups in a reduced form. The appendix contains a review of the necessary prerequisites (in particular concerning polynomials). The book is written in an elegant, condensed way. It contains many exercises, mostly of theoretical character. The main advantage (in particular for teachers and talented students) is that basic ideas are carefully isolated and presented in a simple, minimal and understandable way. It is a very good complement to many other books containing calculus, specific examples, and applications of linear algebra. (vs)

L. Keen, N. Lakic: Hyperbolic Geometry from a Local Viewpoint*, London Mathematical Society Student Texts 68, Cambridge University Press, Cambridge, 2007, 271 pp., GBP 23.99, ISBN 978-0-521-68224-4*

This book contains a careful presentation of hyperbolic geometry based on an infinitesimal approach to the notion of a distance between two points. The principal notion here is the notion of a density. It makes it possible to measure lengths of curves, hence also the distances between points. After the motivation coming from Euclidean geometry, the hyperbolic metric on the unit disc in the complex plane is studied in detail. In the next two chapters, necessary notions from topology (covering spaces and the uniformization theorem) and from complex function theory (the Riemann mapping theorem and the Schwarz reflection principle) are introduced. After a discussion of the symmetry groups of the Euclidean and hyperbolic planes, densities for hyperbolic geometry of a domain, their generalizations (the Kobayashi metric and the Carathéodory pseudo-metric) and relations between holomorphic mappings and the hyperbolic metric are discussed. The next three chapters contain applications (iterated function systems and their limiting behaviour). The last chapters contain various estimates on hyperbolic metrics using inclusion mappings. Parts of the book can be used for lecture series of various lengths. (vs)

F. Kirwan, J. Woolf: An Introduction to Intersection Homology Theory*, second edition, Chapman & Hall/CRC, Boca Raton, 2006, 229 pp., USD 69.95, ISBN 1-58488-184-4*

This book represents the second edition of the book of the same title but only one author, namely F. Kirwan. It appeared as volume 187 in the Pitman Research Notes in Mathematical Sciences (Longman Scientific & Technical, Harlow) in 1988. For this second edition another author was asked to cooperate with F. Kirwan and the text has been substantially extended. The original idea of the first author was to explain intersection homology and provide the reader with the proof (or at least a sketch of the proof) of the Kazhdan-Lusztig conjecture. The book in the present form can serve as a relatively easy introduction to intersection homology, showing its relations to other homology theories and presenting various applications. Not many prerequisites are required.

In the introduction the reader will find information about the homology and cohomology of manifolds and is instructed that these nice results (e.g. Poincaré duality) are no more valid if we admit manifolds with singularities. It is explained from the very beginning that if we want to extend the relevant results to manifolds with singularities then we must use intersection homology. The next two chapters still have an introductory character. We find here more details about classical simplicial and singular homology and cohomology, the theory of sheaves and sheaf cohomology, and the theory of derived categories. The fourth chapter brings a relatively elementary definition of intersection homology. Many of its basic properties are studied here. The next chapter is then devoted to the application of intersection homology to special topological pseudomanifolds called Witt spaces. Then there is a chapter on the relation between intersection cohomology and L^2-cohomology, and a chapter on sheaf-theoretic intersection homology (this interpretation allows one to prove the topological invariance of intersection homology).

After a chapter devoted to perverse sheaves, there is a chapter where intersection cohomology is applied to toric varieties associated with fans. Chapter 10 is devoted to the Weil conjectures and is oriented towards the Weil conjectures for singular varieties. The next chapter introduces D-modules and the Riemann-Hilbert correspondence. Having this material available, the authors then pass on, in the last chapter, to the Kazhdan-Lusztig conjecture. The authors declare that their aim was not to write a fundamental treatise on intersection homology but rather to give propaganda for this new homology. Therefore they give, quite systematically after each chapter, recommendations for further reading. The book can be used as a first reading on intersection homology and its applications. The authors present many examples (and exercises) so that the presentation has quite a concrete character. (jiva)

K. Kodaira: Complex Analysis*, Cambridge Studies in Advanced Mathematics 107, Cambridge University Press, Cambridge, 2007, 406 pp., GBP 40, ISBN 978-052-1-80937-5*

This book of the well-known Japanese laureate and winner of the Fields medal appeared for the first time in Japanese (1977, 1978) in three volumes. The present version contains all three volumes and it appears in English for the first time in a such form (the first two volumes were published in English under the title "Introduction to Complex Analysis" in 1984). The book starts on a basic level and many theorems (which are quite often considered as prerequisites of such texts) are included. The approach can be characterized as geometrical: it was the intention of the author to avoid a detailed study of topological properties of the complex plane.

The first part of the book contains classical material on holomorphic functions, the Cauchy theorem and conformal mapping, followed by chapters on analytic continuation and the Riemann mapping theorem. More than the last third of the book is devoted to Riemann surfaces. It is divided into three chapters: Riemann surfaces, The structure of Riemann surfaces and Analytic functions on a closed Riemann surface. The last chapter includes the Riemann-Roch and Abel theorems. While most of the material included in the first part could be used in a basic course on complex analysis, the whole book could serve as a text for an advanced course on Riemann surfaces. The book contains many pictures (helping to build geometric intuition) and problems (elementary and advanced). The book could be very helpful for students as well as for experts in the field. (jive)

T. Mochizuki: Kobayashi-Hitchin Correspondence for Tame Harmonic Bundles and an Application, *Astérisque, no. 309, Société Mathématique de France, Paris, 2006, 117 pp., EUR 27, ISBN 978-2-85629-226-6*
The classical Kobayashi correspondence relates polystability of holomorphic bundles on Kähler manifolds and the existence of the Hermitean-Einstein metric. Hitchin considered some additional structures (Higgs fields) and proved the correspondence between the corresponding stability and the existence of Hermitean-Einstein metrics for Higgs bundles on a compact Riemann surface. The work of C. Simpson generalized it to smooth irreducible projective varieties. This book contains a discussion of the generalization of these results to the quasiprojective case. Results include an inequality for μ_L-stable parabolic Higgs bundles of the Bogomolov-Gieseker type, a deformation of any local system on a smooth quasiprojective variety to a variation of polarized Hodge structure, and a discussion of possible split quotients of the fundamental group of a smooth irreducible quasiprojective variety. (vs)

H. R. Miller, D. C. Ravenel, Eds.: Elliptic Cohomology – Geometry, Applications, and Higher Chromatic Analogues*, London Mathematical Society Lecture Note Series 342, Cambridge University Press, Cambridge, 2007, 364 pp., GBP 38, ISBN 978-0-521-70040-5*
This book contains material connected with the Isaac Newton Institute's activity on elliptic cohomology and its higher chromatic analogues organized in Cambridge in December, 2002. The book contains 17 contributions on many different topics. It includes papers on possibilities of geometric descriptions of elliptic cohomology (G. Segal, and N. Kitchloo and J. Morava), on equivariant cohomology theories (J. Greenlees, M. Ando and Ch. French, and I. Gronowski), on modular properties of rational vertex operator algebras (G. Mason), on motivic Thom isomorphism (J. Morava), on a relation between M-theory and E8 gauge theory in dimensions 10, 11 and 12 (E. Diaconescu, G. Moore, and D. Freed), on the endomorphism ring of a formal group and numerical polynomials (K. Johnson) and several papers on higher analogues of elliptic cohomology (J. Devoto, D. Ravenel, H.-W. Henn, N. Minami, and a review article by M. Hovey). (vs)

M. Mukerji: Marvelous Modular Origami*, A. K. Peters, Wellesley, 2007, 75 pp., USD 14.95, ISBN 978-1-56881-316-5*
This book is the first book by Meenakshi Mukerji, who was awarded the 2005 Florence Temko Award by OrigamiUSA for her contributions to origami. The book contains detailed instructions for construction of 30 original modular origami models. Every instruction is supported by many schematic coloured pictures. The author describes basic folding techniques required to construct the "modules" that are used to construct complex ornamental models. In the book, it is possible to find various types of origami like sonobes, floral balls and patterned dodecahedra. The reader can also find here information about types of material suitable for origami, recommended colour combinations and the mathematics behind the models. The book is intended for intermediate but beginners are also encouraged to try. (jhr)

P. J. Nahin: Chases and Escapes. The Mathematics of Pursuit and Evasion*, Princeton University Press, Princeton, 2007, 253 pp., USD 24.95, ISBN 978-0-691-12514-5*
In what could be called an advanced popular mathematics book, the author discusses about a dozen mostly classical problems (as well as some of their more intriguing variants) related to mathematical modelling of pursuit and evasion. Detailed description, historical background, practical applications and a number of illustrations accompany each problem, making the text easily readable. The mathematical tools used are mostly elementary calculus; some problems require simple probabilistic and game-theoretic considerations. Hence, the book is easily accessible to any first year university student, or even possibly to an advanced high-school student. (dpr)

O. O'Shea, U. Dudley: The Magic Numbers of the Professor*, Spectrum Series, Mathematical Association of America, Washington, 2007, 168 pp., GBP 21.99, ISBN 978-0-88385-557-7*
The title, an obvious gentle tribute to Martin Gardner's *The Magic Numbers of Dr. Matrix*, gives the reader a hint as to what type of recreational mathematics one should expect in this book. However, the book is quite unique. At the beginning, one of the authors Owen O'Shea meets an imaginary person, a most amazing man called Richard Stein, nicknamed The Professor. Together they travel through Ireland visiting places such as Mallow or Carrowkeel for various peculiar reasons while, on the way, the professor informs O'Shea about remarkable numerical coincidences that he seems to be able to find literally everywhere. The size of The Professor's collection of such things is unbelievable. His curious numerical findings range from ancient history through the year 1776 and the 1915 sinking of Lusitania, to the 9/11 atrocities and the Iraq war.

The reader, who might think he has already heard everything about the Number of the Beast or all possible coincidences between Lincoln and Kennedy, will be surprised by a broad variety of surprising new facts that have not been published before. Do you know what temperature it is if it was zero degrees yesterday and today it is twice as cold? Any idea how many different ways a US dollar can be changed? Can you guess how big a deck of cards is needed for its number of rearrangements to equal the age of the universe in seconds? Do you want to know some intriguing facts about the number 23? Easy. Just consult The Professor! The book is fascinating and uniquely entertaining though it should not be read as a novel. The author warns the reader in the introduction that not too many of the items in the book should be consumed at the same time. That is good news; the joy brought to the reader by the book then lasts longer. (lp)

R. Piziak, P.L. Odell: Matrix Theory. From Generalized Inverses to Jordan Form*, Pure and Applied Mathematics, vol. 288, Chapman & Hall/CRC, Boca Raton, 2007, 548 pp., USD 89.95, ISBN 978-1-58488-625-9*
This book is designed for a "second" course in linear algebra and matrix theory taught at the senior undergraduate and early postgraduate level. It presupposes that the reader has already taken a one-semester course on the elements of linear algebra. The necessary prerequisites are summarized in four appendices. The text is divided into twelve chapters. Chapter 1 is on solutions of systems of linear equations with an emphasis on invertible matrices, and it contains a treatment of the Henderson-Searle formula for the inverse of a sum of matrices and its generalizations. Chapter 2 introduces LU factorization and the Frame algorithm for computing the coefficients of the char-

acteristic polynomial leading to the Cayley-Hamilton theorem. Chapter 3 is on Sylvester's rank formula and its many consequences. The chapter culminates with the characterization of nilpotent matrices. Left and right inverses are also introduced.

Chapter 4 introduces the main theme of the book: the Moore-Penrose inverse. It is followed in chapter 5 by generalized inverses. A short chapter 6 is about norms followed by chapter 7 on inner products, in particular on the QR factorization and algorithms to find it. The minimum norm and the least square solutions and its connection to the Moore-Penrose inverse are also presented. Chapter 8 discusses orthogonal projections and a connection between the Moore-Penrose inverse and the orthogonal projections on the fundamental subspaces of a matrix. Chapter 9 covers the spectral theorem and chapter 10 covers the primary decomposition theorem, Schur's triangularization theorem and singular value decomposition. The book then culminates with the Jordan canonical form theorem in chapter 11 and a brief introduction to multilinear algebra in chapter 12. The book contains numerous exercises and homework problems as well as suggestions for further reading. Many concepts are demonstrated with the help of MATLAB. (jtu)

L. H. Rowen: Graduate Algebra – Commutative View*, Graduate Studies in Mathematics, vol. 73, American Mathematical Society, Providence, 2006, 438 pp., USD 65, ISBN 0-8218-0570-3*

This first volume of Rowen's textbook is aimed mainly towards the commutative-algebraic framework for algebraic geometry (the second volume, Graduate algebra: Non-Commutative view, will be motivated by representations of groups). It consists of three major parts. The first one (Modules) describes a structure of finitely generated modules and composition series. The second one (Affine algebras and Noetherian rings) includes Galois theory and the Krull dimension theory. In the third part (Applications to geometry and number theory), the reader finds an introduction to algebraic geometry and other theoretical applications of commutative algebra. The book is consistently organized in several layers (main text, supplements, appendices), which makes it a valuable source for readers of various levels, from graduate students to researchers. Though the textbook is (in theory) self-contained, the reader has to have a good background in undergraduate algebra. As a result of the author's lifelong teaching experience and several years of maturation of the text, this is a valuable and readable textbook on modern commutative algebra. (dst)

L. Sabinin, L. Sbitneva, I. Shestakov, Eds.: Non-Associative Algebra and Its Applications*, Lecture Notes in Pure and Applied Mathematics, vol. 246, Chapman & Hall/CRC, Boca Raton, 2006, 516 pp., USD 169.95, ISBN 0-8247-2669-3*

This book contains the proceedings of the 5th international conference on nonassociative algebra and its applications held in the summer of 2003 in Mexico. The book is dedicated to the memory of one of its editors, Lev Sabinin, who passed away after the meeting when the proceedings were being edited. The theory of nonassociative algebras has expanded a lot over the past two decades and has now reached a stage of maturity and great depth in fields as diverse as Lie algebras, nonassociative rings and algebras, quasigroups and loops, and applications of all these to physics and other natural sciences. The present volume contains essays and paper in all these areas (33 in total) giving thus a picture of the current state of work in nonassociative algebras.

The book includes papers on the theory of free Lie algebras (L. Bokut and E.S. Chibrikov), an overview of Kikkawa spaces in the context of the theory of smooth loops and smooth modules (R. Carrillo-Catalán), a survey on approximation of locally compact groups by finite groups, semigroups and quasigroups (L.Y. Glebsky, E.I. Gordon and C.J. Rubio), an up-to-date overview of nonassociative geometry, discrete models of spacetime, their continuous limits and the associated causal structure (A.I. Nesterov), a reformulation of the Sabinin principle in operadic terms (E. Paal) and a survey (Sabinin and L.V. Sbitneva) on recent research in algebras (quasigroups and loops, the geometry of Lie groups, Lie algebras and homogeneous spaces with applications) and mathematical physics (the loopuscular approach to discrete spacetime). (tk)

H. Salzmann, T. Grundhöfer, H. Hähl, R. Löwen: The Classical Fields. Structural Features of the Real and Rational Numbers*, Encyclopedia of Mathematics and Its Applications 112, Cambridge University Press, Cambridge, 2007, 401 pp., GBP 60, ISBN 978-0-521-86516-6*

This book studies sets of rational and real numbers from the algebraic, ordered and topological points of view, both separately and with interrelations. The core of the book is in chapter 1, covering all the basic and deep results related to the reals. It treats R as an additive or multiplicative group, as an ordered set, a topological space (with its topological characterizations, groups of homeomorphisms, continuous images of R or of the Cantor set, and p-adic metrics), a field, an ordered group, a topological group, a measure space, an ordered field, and finally as a topological field. The next chapter defines non-standard rationals and reals using ultraproducts. Then comes a study of the rationals done in a similar way to the study of reals in chapter 1, and completions of ordered groups and fields, of topological groups, and of topological rings and fields. One chapter is devoted to p-adic numbers, again from the point of view of groups and fields, both with and without topologies. The appendix contains the basics of ordinal and cardinal numbers, of topological groups and of Pontryagin duality.

Most of the sections have exercises (with hints or solutions at the end of the book). The book ends with an extensive bibliography (about 350 items) and not too large an index. The book is certainly useful for readers with an interest in the interrelations of algebra and topology not based on abstract objects but on concrete ones, here reals and rationals. (mihus)

P. Sarnak, F. Shahidi, Eds.: Automorphic Forms and Applications*, IAS/Park City Mathematics Series, vol. 12, American Mathematical Society, Providence, 2007, 427 pp., USD 75, ISBN 978-0-8218-2873-1*

The 2002 IAS/ParkCity Mathematics Institute conference on "Automorphic form and their applications" took place in Park City, Utah. The lectures were divided into six general areas. Lectures by A. Borel and J. Bernstein on "Basic theory of Eisenstein series" addressed general aspects of the theory of automorphic forms on reductive groups over adèles (the lectures by Bernstein are not included in the proceedings). Lectures on "Converse Theorems and Langlands-Shahidi method" were delivered by J. Cogdell and F. Shahidi. The lectures by L. Clozel, W. Li and A. Valette covered the topic of the (now proven) Ramanujan conjectures for definite quaternion algebras and their applications to

Ramanujan graphs and spectral gaps. The lectures by P. Michel on analytic number theory were devoted to the technique of studying averages over "families of automorphic L-functions".

The lectures by A. Terras focused on a topic of current interest called arithmetic quantum chaos. For example, the geodesic flow on the unit sphere tangent bundle of an arithmetic hyperbolic surface leads to an interesting area for various conjectures in number theory. The last lecture series by A. Eshin described a very powerful tool that has emerged in recent years in connection with problems of equi-distribution of measures associated with arithmetic and spectral geometry in ergodic theory, e.g. the Ratner theorem. (pso)

A. H. Schoenfeld, Ed.: Assessing Mathematical Proficiency*, Mathematical Sciences Research Institute Publications 53, Cambridge University Press, Cambridge, 2007, 391 pp., GBP 17.99, ISBN 978-0-521-69766-8*

This book is the product of the 2004 MSRI (Mathematical Sciences Research Institute, Berkeley) conference "Assessing Students´ Mathematics Learning: Issues, Costs and Benefits". The conference articulated different purposes of assessment of student performance in mathematics because the assessing of student learning supports instructional improvement. The book focuses on ethical issues related to assessment, including how assessment interacts with concerns for equity, sensitivity to culture, and the severe pressures on urban and high-poverty schools. Different frameworks are introduced, as well as tools and methods for assessment, comparing the kinds of information they offer about students' mathematical proficiency. The book describes the complexities of assessment when English is not a student's native language: if a student is not fluent, is his or her failure to solve a problem a result of not understanding the problem or of not understanding the mathematics? The book highlights the kinds of information that different assessment can offer, including many examples of some of the best mathematics assessments worldwide. (ood)

W. Stein: Modular Forms, a Computational Approach*, Graduate Studies in Mathematics, vol. 79, American Mathematical Society, Providence, 2007, 268 pp., USD 55, ISBN 978-0-8218-3960-7*

The theory of modular forms is an important tool of contemporary mathematics, in particular in number theory and arithmetic geometry of elliptic curves (see for instance the spectacular proof of Fermat's last theorem). The books devoted to this part of mathematics are characterized by deeply developed theories seemingly far from concrete computations. This book is highly non-traditional in this respect. Besides developing the theory of modular forms, the author brings the reader down to earth by addressing the computational aspects of the theory. Various theoretical aspects are transformed into algorithms and practical programs. To formulate them, the author uses his free open source computer algebra system SAGE (Software for Algebra and Geometry Experimentation).

The main body of the book is formed of eleven chapters, complemented by an appendix "Computing in Higher Rank" of about 50 pages and written by P. E. Gunnells. Each chapter contains numerous examples and exercises. The solutions to most of the exercises can be found in the final chapter. The introductory first chapter starts with basic material about congruence subgroups and modular forms on the upper-half plane, and lists some main applications of modular forms throughout mathematics. The remaining chapters then cover a variety of important aspects of the theory. The exposition is very clear and vivid showing the author's mastery in the subject. It contains illustrating examples, many methodological comments and references to the relevant research literature for further reading. Since the prerequisites are moderate, the highly original approach makes the book suitable for advanced undergraduates in related algebraic number theory or arithmetic geometry, and the wealth of explicit algorithms and the unique approach makes the book almost indispensable for mathematicians interested in the field. (spor)

I. Stewart: How to Cut a Cake*, Oxford University Press, Oxford, 2006, 231 pp., GBP 9.99, ISBN 0-19-920590-6*

There is an easy way of how to make a fair division of a cake between two persons: one cuts and the other then chooses. If the first one does not cut fairly, then the second one can take the bigger (or the better) half. The analogous task becomes a good deal more difficult when the cake is to be divided between three or even more people. This puzzle, having certain obvious practical applications, leads to a serious mathematical problem mathematicians have been grappling with for more than 50 years. This beautiful book, written by one of the most famous writers of mathematics, gives an amusing description of this problem, its history, false solutions and the correct solution.

Cake-cutting algorithms however occupy just the first chapter of the book and there are nineteen more chapters. Their topics range from sardine tins to chess games and from quasicrystals to the Sierpinski Gaskets. The book also explains what the portioning of the Moon has got in common with electric circuits. In twenty chapters, the author takes the reader for an amazing journey through a diverse world of mathematics and its applications, pointing out mind-boggling conundrums and mysteries, some with deadly serious applications in practice. As he says in the introduction, this is a book for the fans, the math enthusiasts, the people who actively like mathematics. Being one of those, I can confirm that, for those who do, this book is another must. Make a space for it in your bookcase. (lp)

C.-L. Terng, Ed.: Integrable Systems, Geometry, and Topology*, AMS/IP Studies in Advanced Mathematics, vol. 36, American Mathematical Society, Providence, 2006, 256 pp., USD 59, ISBN 978-0-8218-4048-1*

This book contains a written version of (some of) the lecture series given in the program on integrable systems and differential geometry organized in Taiwan in 1999. In the book, the reader can find review papers in five different fields. The first paper (F. Burstall) describes the classical theory of isothermic surfaces in Rn from the point of view of integrable systems. Classical geometric transformations are constructed using a loop group action. The second contribution (M. Guest) introduces the reader to quantum cohomology. In particular, quantum cohomology is computed for the flag manifold and for Hirzebruch surfaces. This part also contains a description of the Givental quantum differential equation and a construction of the Dubrovin connection. The fourth article (E. Heintze, X. Liu and C. Olmos) describes a generalization of the Chevalley restriction theorem to the case of an isometric submanifold. The last two articles are devoted to relations between Yang-Mills fields and harmonic maps (M. Mukai-Hidano and Y. Ohnita) and to Schrödinger flow on Grassmannians (K. Uhlenbeck). (vs)

J. Tilouine, H. Carayol, M. Harris, M.-F. Vignéras, Eds.: Formes automorphes I*, Astérisque 298, Société Mathématique de France, Paris, 2005, 410 pp., EUR 86, ISBN 2-85629-172-4*

This is the second volume of the proceedings of the Automorphic Semester held at the Centre Emile Borel in Paris in 2000. It contains five important articles on various arithmetic aspects of automorphic forms on *GSp(4)*: G. Laumon extends his earlier results on the cohomology of Siegel 3-folds to the case of non-trivial coefficient systems; R. Weissauer constructs ℓ-adic Galois representations attached to discrete series automorphic forms *GSp(4)* and establishes some of their properties; E. Urban discusses the local behaviour of these representations at ℓ; A. Genestier and J. Tilouine prove a modularity lifting theorem for 4-dimensional symplectic Galois representations; D. Whitehouse proves a weighted twisted fundamental lemma necessary for establishing a functorial transfer of packets from *GSp(4)* to *GL(4)*. The book will be of interest to researchers and graduate students interested in arithmetic applications of automorphic forms. (jnek)

F. R. Villegas: Experimental Number Theory*, Oxford Graduate Texts in Mathematics 13, Oxford University Press, Oxford, 2007, 214 pp., GBP 29.50, ISBN 978-0-19-822730-3*

This booklet is intended for graduate students in pure mathematics with a number-theory orientation, and for those who incline to learn mathematics through experiments. Its main goal is neither to give an introduction to number theory, nor to learn a programming language, but to show how the computer can be used for numerical experimentations in number theory in the spirit of J. W. S. Cassels' motto "Number theory is an experimental science". Sample programs are written in the PARI-GP language and are available to download from the author's website.

The book is written in an easily readable style, giving the reader basic information on the computed issues, as well as a sufficient amount of facts for further orientation in the surrounding theory. The examples draw from the theory of the quadratic reciprocity law, special sequences (like Bernoulli numbers, trinomial ones, etc.), sums of squares or binary quadratic forms, zeta functions of varieties over finite fields, elementary class field theory, modular forms, p-adic analysis, to mention just a few. The book can be warmly recommended not only for those with a preference for number theory but even for the general readership in pure mathematics or supervisors looking for creative examples for further motivation of scientific computations. (spor)

W. D. Wallis: A Beginner's Guide to Graph Theory*, Birkhäuser, Boston, 2000, 230 pp., DM 78, ISBN 0-8176-4176-9*

The style of this book is very friendly and the book is obviously well equipped to serve its main purpose, i.e. the exposition to undergraduates of the main kinds of combinatorial designs. As one can expect, the topics include balanced incomplete block designs, their development by means of difference sets, Latin squares, one-factorization, tournaments, Steiner triple systems and their large sets, Hadamard matrices and Room squares. There is a section on the Bruck-Ryser-Chowla theorem (in fact, there are two different proofs). Interactions with other parts of mathematics are limited.

Of course, one needs to develop the basic notions of affine and projective geometries (there is a section on ovals) and one needs to be able to work with matrices and quadratic residues. Besides matrices, all other necessary notions are explained in the book. That is usually done at the first point where the notion is needed, which makes it possible for the student to get quickly to the objects he or she is interested in. The book is equipped with exercises, there are quite a few historical remarks, and nearly all claims are stated with proof. Information about unsolved problems is limited, which perhaps makes the book a little less exciting than it could be. (adr)

P. Winkler: Mathematical Mind-Benders*, A. K. Peters, Wellesley, 2007, 148 pp., USD 18.95, ISBN 978-1-56881-336-3*

This book is a new collection of mathematical puzzles, a collection of interesting and challenging problems from diverse sources and mathematical disciplines. The puzzles in the book are difficult but they are elegant, entertaining and they do not require advanced mathematics to be appreciated or solved. Most of them are supposed to confound the intuition and stimulate the brain and mathematical thinking. The puzzles are divided into chapters for convenience and they are classified loosely by mathematical topics. Formulation of the puzzles and problems is followed by their solutions, together with information on their background presented at the end of each chapter. This structure offers a good opportunity for the reader to think at least a little about each puzzle. The book can be recommended to students, teachers and authors of problems, as well as to lovers of mathematics, puzzles and challenges. (mbec)

Xiong Bin, Lee Peng Yee, Eds.: Mathematical Olympiad in China*, World Scientific, Singapore, 2007, 251 pp., USD 25, ISBN 978-981-270-789-5*

This book begins by offering basic information on the organization of mathematical competitions and olympiads in China as well as a brief overview of the International Mathematical Olympiad and its winners. The book is a collection of problems and solutions of major mathematical competitions in China, showing how the Chinese national team is selected, formed, prepared and trained. The competition problems and solutions from the China Mathematical Competition 2002–2005, China Mathematical Competition – Extra Test 2002–2005, China Mathematical Olympiad 2003–2006, China Girls' Mathematical Olympiad 2003–2005, China Western Mathematical Olympiad 2002–2005 and International Olympiad 2003–2006 are presented. The book will be quite useful for students, teachers, heads of national teams and authors of problems, as well as to people who are interested in mathematics and solving difficult problems. (mbec)

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The Maximum Principle

Pucci, P., Università degli Studi di Perugia, Italy / **Serrin, J.**, University of Minnesota, MN, USA

Maximum principles are bedrock results in the theory of second order elliptic equations. This principle, simple enough in essence, lends itself to a quite remarkable number of subtle uses when combined appropriately with other notions. Intended for a wide audience, the book provides a clear and comprehensive explanation of the various maximum principles available in elliptic theory, from their beginning for linear equations to recent work on nonlinear and singular equations.

2007. X, 234 p. Hardcover
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PNLDE — Progress in Nonlinear Differential Equations and Their Applications, Vol. 73

The Bartle-Dunford-Schwartz Integral

Panchapagesan, T.V., Bangalore, India

This volume is a thorough and comprehensive treatise on vector measures. The functions to be integrated can be either [0,infinity]- or real- or complex-valued and the vector measure can take its values in arbitrary locally convex Hausdorff spaces. Moreover, the domain of the vector measure does not have to be a sigma-algebra: it can also be a delta-ring. The book contains not only a large amount of new material but also corrects various errors in well-known results available in the literature. It will appeal to a wide audience of mathematical analysts.

2008. Approx. 350 p. Hardcover
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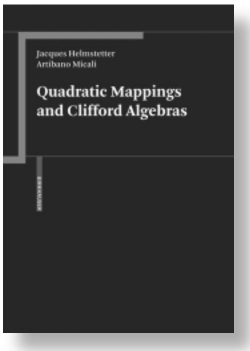

Quadratic Mappings and Clifford Algebras

Helmstetter, J., Université de Grenoble, France / **Micali, A.**, Université de Montpellier II, France

After general properties of quadratic mappings over rings, the authors more intensely study quadratic forms, and especially their Clifford algebras. To this purpose they review the required part of commutative algebra, and they present a significant part of the theory of graded Azumaya algebras. Interior multiplications and deformations of Clifford algebras are treated with the most efficient methods. The connection between orthogonal transformations and Clifford algebras is established in a new way, by means of Lipschitz monoids. Lipschitz monoids also allow a more efficient study of hyperbolic spaces.

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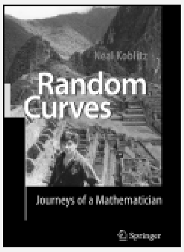

Random Curves

Journeys of a Mathematician

N. Koblitz, University of Washington, Seattle, WA, USA

Neal Koblitz is a co-inventor of one of the two most popular forms of encryption and digital signature, and his autobiographical memoirs are collected in this volume. Besides his own personal career in mathematics and cryptography, Koblitz details his travels to the Soviet Union, Latin America, Vietnam and elsewhere; political activism; and academic controversies relating to math education, the C. P. Snow "two-culture" problem, and mistreatment of women in academia.

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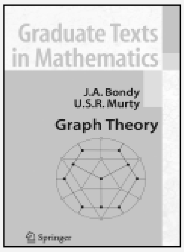

Graph Theory

A. Bondy, Université Claude-Bernard, Lyon, France; **U. Murty**, University of Waterloo, ON, Canada

The primary aim of this book is to present a coherent introduction to graph theory, suitable as a textbook for advanced undergraduate and beginning graduate students in mathematics and computer science. It provides a systematic treatment of the theory of graphs without sacrificing its intuitive and aesthetic appeal. Commonly used proof techniques are described and illustrated.

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W. Schwarz, Universität Potsdam, Germany

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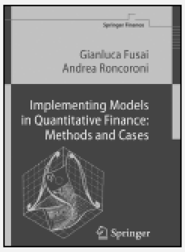

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G. Fusai, Università del Piemonte Orientale, Novara, Italy; **A. Roncoroni**, ESSEC, Cergy Pontoise, France

This book puts numerical methods in action for the purpose of solving practical problems in quantitative finance. The first part develops a toolkit in numerical methods for finance. The second part proposes twenty self-contained cases covering model simulation, asset pricing and hedging, risk management, statistical estimation and model calibration. Each case develops a detailed solution to a concrete problem arising in applied financial management and guides the user towards a computer implementation.

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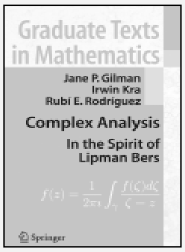

Complex Analysis

In the Spirit of Lipman Bers

J. P. Gilman, Rutgers University, Newark, NJ, USA; **I. Kra**, Math for America, New York, NY, USA; **R. E. Rodriguez**, Pontificia Università Catolica de Chile, Santiago, Chile

The authors' aim here is to present a precise and concise treatment of those parts of complex analysis that should be familiar to every research mathematician. They follow a path in the tradition of Ahlfors and Bers by dedicating the book to a very precise goal: the statement and proof of the Fundamental Theorem for functions of one complex variable. They discuss the many equivalent ways of understanding the concept of analyticity, and offer a leisure exploration of interesting consequences and applications. Readers should have had undergraduate courses in advanced calculus, linear algebra, and some abstract algebra. No background in complex analysis is required.

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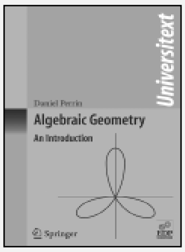

Algebraic Geometry

An Introduction

D. Perrin, Université Paris-Sud, France

Aimed primarily at graduate students and beginning researchers, this book provides an introduction to algebraic geometry that is particularly suitable for those with no previous contact with the subject; it assumes only the standard background of undergraduate algebra. The book starts with easily-formulated problems with non-trivial solutions and uses these problems to introduce the fundamental tools of modern algebraic geometry: dimension; singularities; sheaves; varieties; and cohomology.

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Reprint of the 1st ed. Berlin Heidelberg New York 1987 2008. XIV, 516 p. 22 illus. (Classics in Mathematics) Softcover
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