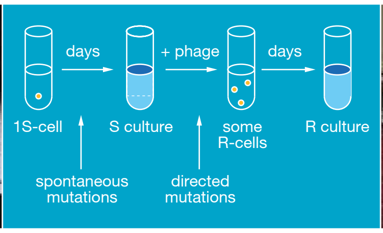
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



September 2008
Issue 69
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European
Mathematical
Society

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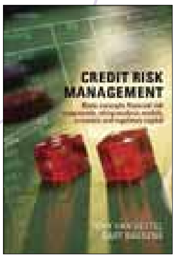
Credit Risk Management

Basic Concepts: Financial Risk Components, Rating Analysis, Models, Economic and Regulatory Capital

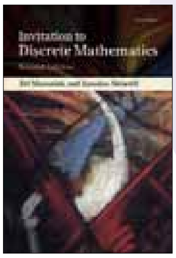


Tony Van Gestel and Bart Baesens

This first of three volumes on credit risk management, providing a thorough introduction to financial risk management and modelling. Contains comprehensive glossary of terms and extensive cross-referencing.

August 2008 | 504 pp
Hardback | 978-0-19-954511-7 | £75 .00

Invitation to Discrete Mathematics 2/e



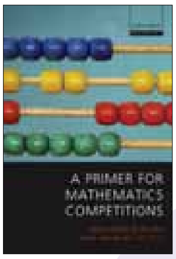
Jirí Matoušek and Jaroslav Nešetril

'A far-from-traditional textbook and...a joy to read. The text is lucid and sprinkled with small jokes and background stories.'

The Times Higher Education Supplement
(review from previous edition)

August 2008 | 456 pp
Paperback | 978-0-19-857042-4 | £35.00
Hardback | 978-0-19-857043-1 | £75.00

A Primer for Mathematics Competitions

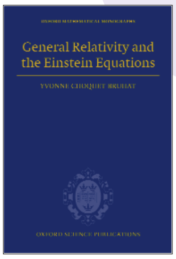


Alexander Zawaira and Gavin Hitchcock

A comprehensive resource containing an entertaining selection of problems in mathematics. Including numerous exercises, illustrations, hints, and solutions, it is aimed at students of mathematics looking for an introduction to problem solving in mathematics, as well as Mathematical Olympiad competitors and other recreational mathematicians.

September 2008 | 352 pp
Paperback | 978-0-19-953988-8 | £22.50
Hardback | 978-0-19-953987-1 | £50 .00

General Relativity and the Einstein Equations



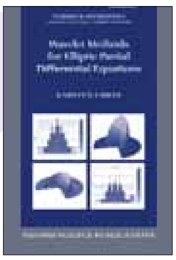
Yvonne Choquet-Bruhat

Aimed at researchers in mathematics and physics, this monograph, in which the author overviews the basic ideas in General Relativity, introduces the necessary mathematics and discusses some of the key open questions in the field.

Oxford Mathematical Monographs

September 2008 | 600 pp
Hardback | 978-0-19-923072-3 | £65.00

Wavelet Methods for Elliptic Partial Differential Equations



Karsten Urban

A text based on the author's course that introduces graduates to the basics of wavelet methods for partial differential equations and describes the construction and analysis of adaptive wavelet methods.

Numerical Mathematics and Scientific Computation

October 2008 | 488 pp
Hardback | 978-0-19-852605-6 | £65.00

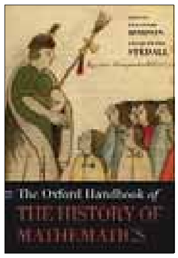
Modelling Longevity Dynamics for Pensions and Annuity Business

Ermanno Pitacco, Michael Denuit, Steven Haberman, and Annamaria Olivieri

Provides a comprehensive and detailed description of statistical methods for projecting mortality, and an extensive discussion of some important issues concerning the longevity risk in the area of life annuities and pension benefits. Based on authors' personal experience in research and teaching and includes an extensive list of references and detailed suggestions for further reading.

November 2008 | 400 pp
Hardback | 978-0-19-954727-2 | £55.00

Oxford Handbook of the History of Mathematics



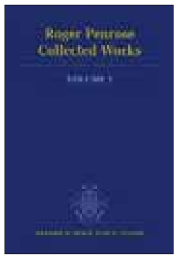
Edited by Eleanor Robson and Jacqueline Stedall

Explores the history of mathematics, addressing what mathematics has been and what it has meant to practice it. Thirty-six self-contained chapters, each written by a specialist, provide a fascinating overview of 5000 years of mathematics and its key cultures for academics in mathematics, historians of science, and general historians.

December 2008 | 800 pp
Hardback | 978-0-19-921312-2 | £85.00

Roger Penrose

Collected Works Six Volume Set*



Roger Penrose

Professor Sir Roger Penrose is one of the truly original thinkers of our time and one of the eminent mathematicians of the 20th Century. Here his works, spanning 50 years of science and including his previously unpublished theses, have been collected and arranged chronologically over six volumes, each with an introduction from the author.

December 2008 | 3,900 pp
Pack | 978-0-19-921944-5 | £600.00
***volumes available individually**

Computability and Randomness

André Nies

The interplay between computability and randomness has been an active area of research in recent years. Covering the basics as well as recent research results, this book provides a very readable introduction to the exciting interface of computability and randomness for graduates and researchers in computability theory, theoretical computer science, and measure theory.

January 2009 | 420 pp
Hardback | 978-0-19-923076-1 | £55.00

Statistical Modelling in R

Murray Aitkin, Brian Francis, John Hinde, and Ross Darnell

A comprehensive treatment of the theory of statistical modelling in R with an emphasis on applications to practical problems and an expanded discussion of statistical theory. Includes a wide range of case studies and an expanded discussion of statistical theory.

Oxford Statistical Science Series

January 2009 | 568pp
Paperback | 978-0-19-921913-1 | £34.50
Hardback | 978-0-19-921914-8 | £70.00

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EMS Calendar

2008

8–19 September
EMS Summer School at Montecatini (Italy) University College
Mathematical models in the manufacturing of glass, polymers and textiles
web.math.unifi.it/users/cime/

22 September
Zentralblatt Scientific Users Committee meeting, Oberwolfach (Germany)
Stephan Klaus: klaus@mfo.de

24 September
Meeting of the Applied Mathematics Committee, Frankfurt (Germany)
Helge Holden: holden@math.ntnu.no

28 September–8 October
EMS Summer School at Będlewo (Poland)
Risk theory and related topics
www.impan.gov.pl/EMSsummerSchool/

1 November
Deadline for submission of material for the December issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@imaff.cfmac.csic.es

8–9 November
EMS Executive Committee Meeting, Valencia (Spain)
Stephen Huggett: s.huggett@plymouth.ac.uk

2009

5–8 February
European Student Conference in Mathematics
EUROMATH 2009, Cyprus
www.euromath.org

4–8 March
4th World Conference on 21st Century Mathematics, Lahore (Pakistan)
wc2009.sms.edu.pk

2012

2–7 July
6th European Mathematical Congress, Kraków (Poland)

Editorial

Vicente Muñoz (Madrid, Spain)

Dear Newsletter readers,

It is an honour to introduce myself as the new Editor-in-Chief of the Newsletter of the European Mathematical Society. I have been on the editorial committee looking after the book review section of the newsletter since 2004 and I must confess that I have thoroughly enjoyed this task. All the reviewers that I have approached have reacted positively and have produced nice, thoughtful reviews. Publishers have also been very kind to provide complimentary copies of their books for the reviewers in all cases.

When the EMS asked me to take this job I was flattered but also overwhelmed with the huge task ahead. Martin Raussen, who has been editor-in-chief over the last five years, has kept the newsletter at a very high standard and it will not be easy to maintain the level to which the readers have become accustomed. One of our goals in the long run is to popularize the newsletter so that all European mathematicians know it as well as the newsletters of their national societies. We even hope to reach a broader public beyond the European frontiers.

This is the first issue for which I take the main responsibility of production. Firstly, I want to thank Martin Raussen for his dedication in explaining to me the internal functioning of the newsletter. His help has been invaluable and he is responsible for the smooth transition in the duties of editor-in-chief. Having just started in this position, I have already had the opportunity to see that all the editors of the editorial committee are very committed to their work on the newsletter. They dedicate much time and effort to make it possible to produce interesting material on time. Most of the previous editors will stay on the editorial committee, in most cases carrying out the same responsibility as they did previously. Some of the editors will be stepping down at the end of 2008 and new editors will be joining the committee. I want to thank them all for their contributions, time and dedication, without which the newsletter would not have been possible. Of course, suggestions for new editors are very welcome. We especially hope to increase the representation of Eastern European countries on the editorial board.

It can often be difficult to get good articles for the newsletter. Some of the most important are the feature articles. This is an article on a topic of interest to the mathematical community. It may be an article explaining developments in a particular area, a survey treating a theme of recent interest that is attracting special attention, or a discussion with several points of view. Also of importance are the interviews, which could be about the achievements of some mathematician or a discussion on some specific topic. Some of the articles that appear in the newsletter are reprinted from journals of national mathematical societies around Europe. Other articles are written specifically for the newsletter and sometimes these are reprinted later elsewhere. We feel very proud when this happens. All editors work hard to produce articles and interviews and to convince possible authors to provide them. I would like to take this opportunity to encourage all readers of the newsletter to contact any of the editors if they have any idea for a possible feature article or an interview that may be of interest. Also contact any editor if you or someone you know wishes to write an article.

The newsletter is one of the main tools for increasing the awareness of European mathematicians to events and news in our community. For instance, the current issue contains all the details of the 5th European Congress of Mathematics, which was held in Amsterdam, 10–13 July. The European Congress of Mathematics happens every four years and it is the event where the EMS Prizes are given. This distinction is given to mathematicians working in Europe or of European origin (under 35 years of age) to recognize a very promising career. The EMS is proud of the fact that most of the European Fields medallists have previously been awarded the EMS prize.

It is very important to have up-to-date information in the news section. Please have a look at the personal column and the forthcoming conferences section. If you come across any information that may be of interest for these sections, please contact the relevant editor since they collect the information provided by the general public.

The European Mathematical Society will soon have a new web site (http://www.euro-math-soc.eu) on which some information from the newsletter will appear (such as the forthcoming conferences and recent books). This will increase the availability of more up-to-date information. In addition, the web site will have enhanced properties like a searchable database, links, etc. In the future, this will leave more space in the newsletter for feature articles.

Nowadays, mathematics is very much based on the cooperation between researchers worldwide. To assist in this, the mathematics community needs international bodies on many different levels: the IMU worldwide, the EMS on the European scene and also more specific bodies for specific branches of mathematics (e.g. mathematical physics and statistics). The EMS promotes specific European cooperation, strengthening the national societies in joint initiatives while retaining an important presence at the European political level. To support these actions, the EMS is trying to increase its number of individual members. If you are reading these words and are not yet affiliated to the EMS, we encourage you to consider doing so. You will receive a printed copy of the newsletter (although the EMS generously distributes the newsletter for free through the web page http://www.ems-ph.org/journals/journal.php?jrn=news). Also tell any colleagues who may want to join the EMS. Every member of a national society will receive a discount and you may easily become a member through the new web pages. If you need more reasons to become a member of the EMS, please have a look at http://www.emis.de/reasons/12reasons.html. Do not be a math-euro-sceptic!

Abel Prize dinner speech

Ari Laptev, President of the EMS

Ari Laptev, EMS President

Your majesties, Abel Prize laureates, ladies and gentlemen.

On behalf of the European Mathematical Society I would like to congratulate Professors John Griggs Thompson and Jacques Tits on receiving the Abel Prize Award 2008 for their outstanding contribution to group theory. This year we are celebrating their profound discoveries that have defined the development of this area for many generations of mathematicians.

Group theory is a theory devoted to the beauty of symmetries. We all appreciate many symmetries around us, created both by nature and by humans, for example a bee's nest, a flower, a leaf, a crystal, buildings and bridges. Indeed, this evening we are privileged to be seated in this castle, which is itself a wonderful example of the perfection of symmetry. Symmetries are an integral part of harmony in our lives.

By establishing the Abel Prize, Norway has made a fantastic contribution to mathematics. Every year many distinguished mathematicians come to this country in order to present their results and celebrate some of the greatest achievements in mathematics.

Mathematics is a subject that we all know from the time when we are schoolchildren. Some of us like this subject, some dislike it and some of us are not interested at all.

I do not believe that we are all intended to enjoy mathematics and I find this fact very positive. It reminds us of the value of diversity in our society.

However, it is vital that the general public should be aware of the importance of mathematics and its relevance to everyday life.

The Norwegian Abel Prize plays a crucial role in doing just this.

About three weeks ago there was a meeting in Luminy of all the Presidents of European National Mathematical Societies, where we enjoyed the hospitality of the French Mathematical Society. The meeting was very fruitful. We discussed many problems which we all have. However, no matter what problem we started to discuss we ended up with the problem of education of mathematics. This is obviously a universal problem for European countries.

Therefore I should also like to take this opportunity to say a few words about the teaching of mathematics in today's school education.

About 30 years ago, many Western countries adopted new school policies that included a significant shift away from previously accepted values.

For example, I am constantly hearing complaints about the trend of teachers who often choose not to correct students' mistakes for fear of creating psychological problems and lack of self-confidence in students.

The implications of such practices are very serious.

Some time ago when our daughter returned from school with uncorrected texts, my wife, who is a musician, was horrified. If this method were to be applied in music, a child would continue to play or sing wrong notes forever.

Another alarming example is the text on the home page of a small university (I don't want to mention any names!) whose main task is the education of future teachers. There one can read the following:

"We are not interested in a teacher acquiring a deeper competence in any particular subject. Instead, we would like a teacher to have a general competence that would be useful for everybody in our community".

Nowadays a "good" teacher is often a teacher who is not fully competent in the subject he or she is supposed to teach but has, instead, a so-called "general competence". This would mean, for example, that it is not of prime importance for a teacher of mathematics to be competent in the subject of mathematics.

Clearly, with such teaching policies we shall never achieve good results in schools however many resources we invest into the school system.

As a result, students arrive at universities unprepared and we (university professors in mathematics) are forced to lower the standards of our courses.

Ultimately, this affects the technological development of our countries since European industry is unable to find enough engineers who are sufficiently well-educated by our universities.

Of course we should not argue against the need for self-confidence in our students.

However, it is important to remember that recent, and not so recent history, is full of terrifying examples of people in power who have made decisions based only on self-confidence and ignoring knowledge and experience.

The Abel Prize ceremony that we have attended today is a celebration of deep knowledge, discovered by our outstanding laureates. Let us be consistent. Let us make it clear that deep knowledge and professionalism, at every level, is what our society needs today.

Report on 5ECM Amsterdam, 14–18 July

Winners of EMS prizes and Chairman Prof. André Ran

About 1000 mathematicians gathered in Amsterdam, the Netherlands, in the week of 14–18 July to discuss the latest developments in mathematics. On Monday morning the congress was officially opened by Professor Robbert Dijkgraaf, President of the Royal Netherlands Academy of Sciences. For those who did not attend the congress, his opening speech can be viewed on the web site of the congress *http://www.5ecm.nl*.

A very important facet of the ECM are the prizes awarded to young and promising mathematicians. This year's prizes again demonstrate an impressive range of young talent in the field of mathematics. The EMS Prize Committee was chaired by Professor Rob Tijdeman and the ten EMS Prizes were funded by the Foundation Compositio Mathematica. The Felix Klein Prize was funded by the Fraunhofer Institute for Industrial Mathematics and the prize committee for that award was chaired by Yvon Maday. After the official opening of 5ECM, the laudatios were read by Rob Tijdeman and the prizes were handed out jointly by Professor Jozef Steenbrink (Chairman of the Foundation Composito Mathematica) and Professor Ari Laptev (President of the EMS). The winners of the EMS Prizes and the titles of their lectures are:

Artur Avila (Brazil):
Dynamics of quasiperiodic cocycles and the spectrum of the almost Mathieu operator.

Alexei Borodin (USA)
Random surfaces in dimensions two, three, and four.

Ben Green (England)
Patterns of primes.

Olga Holtz (USA)
Complexity and stability of linear problems.

Bo'az Klartag (USA)
High-dimensional distributions with convexity properties.

Alexander Kuznetsov (Russia)
Derived categories and rationality of cubic fourfolds.

Assaf Naor (USA)
The story of the sparsest cut problem.

Laure Saint-Raymond (France)
Some results about the sixth problem of Hilbert.

Agata Smoktunowicz (Poland)
On some open questions in noncommutative ring theory.

Cédric Villani (France)
Optimal transport and Riemannian geometry – Monge meets Riemann.

The Felix Klein Prize winner and the title of his lecture are:

Josselin Garnier (France)
Passive sensor imaging using cross correlations of noisy signals.

To get the congress under way, the first of the ten plenary lectures was delivered by Professor Richard Taylor. He did a wonderful job (as did all the plenary lecturers) of explaining his work to a general mathematical audience. Another item on the program of this ECM were three science lectures. These lectures outlined applications of mathematics in other sciences. For example, the crucial role that mathematical modelling plays in predicting climate change was stressed by Tim Palmer (European Centre for Medium Range Weather Forecasts). He also outlined what would be necessary to improve on the current state of affairs in mathematical modelling to obtain predictions on a finer scale than is presently possible. Ignacio Cirac (Max Planck Institute für Quantenoptik) discussed quantum information theory and the challenges in that area. The third science lecture was given by Jonathan Sherratt (Heriot-Watt University) who talked about the latest developments in mathematical modelling for population dynamics.

There were 22 minisymposia, which were spread over a whole range of mathematics. These minisymposia played a role in attracting people to the ECM meeting who may not have normally come to such a broad mathematics congress.

The congress attracted close to 1000 participants, distributed over the five days. A congress of this size is impossible to organize without generous financial support from the government, businesses and industry, and the local mathematical community. A list of those who supported 5ECM is available on the web site. Although all subsidy-providers and sponsors are equally appreciated, the list is too long to name them all here so we shall single out the most important ones. The largest single subsidy was provided by NWO, the Netherlands Organization for Scientific Research. The biggest sponsors were the Foundatio Compositio Mathematica and the ING corporation. It should perhaps be stressed that the registration fee paid by the participants only covered about one-third of the actual cost per participant, a ratio that is unthinkable in any other field of the sciences, humanities, social sciences and medicine. The fact that the congress closed with some financial losses must be attributed to this. Only the generous guarantee subsidies from several universities, the Royal Netherlands Academy of Arts and Sciences and the government, as well as from a private individual, made it possible to have the congress at all.

At the closing of the congress it was announced that the Sixth European Congress of Mathematics will be held in Krakow in 2012. We wish the organizers the best of luck and hope to see all the participants again at 6ECM.

The local organizers
André Ran
Herman te Riele
Jan Wiegerinck

Details about the prize winners' work can be found at www.5ecm.nl/prizewinnersbook.pdf"

Supporting Mathematics in the developing world

Offer of seed grants by the Committee for Developing Countries

A number of initiatives are currently underway to address the challenge of supporting mathematics in the developing world.

In particular, our committee EMS-CDC is impressed by a scheme of the International Science Program ISP (*http://www.isp.uu.se/*) at Uppsala University, initiated by one of our members Anders Wandahl, whereby a department in the developed world "twins" with a department in the developing world to help them in various ways, including paying for subscriptions to mathematical databases.

What has been done so far by our Swedish colleagues can be found on the site "e-Math for Africa" (*http://math.golonka.se/*).

Another effort towards helping developing countries access literature and sharpening links with colleagues from institutions in developed countries has been initiated by the French Mathematical Society SMF (http://smf.emath.fr/en/Adhesions/ParrainagePED/). Under this program, laboratories or institutes with enough funds will support active mathematical centres with less financial support by offering them a registration as an institutional member of the SMF, and the SMF will offer a subscription to one of its journals. The idea is that such a process is part of a cooperation between the two institutions and that eventually, after a few years, the institution from the south should be in a position to subscribe itself to SMF.

We wish to support this scheme, which we call "twinning", in every possible way because we think it directly involves mathematicians on a one-to-one basis, and once links are forged they can be easily sustainable in the long term at no great initial cost.

Although the CDC does not have much funding, we want to launch this project by offering, during an initial period, a kind of "seed grant" of up to 500 euros to any department or institution in the developed world that wishes to participate in such a twinning scheme and is willing to provide at least the same amount in support. We hope that such an initial grant will introduce mathematicians to some very worthwhile work that they will find satisfying to continue afterwards.

Interested departments or individuals in the developed world should address any of us below in the first instance.

As of now, such twinning schemes have been established between the following institutions:

A University of Alger, Algeria, and Grenoble, France

B University of Bangui, Central African Republic, and Uppsala University, Sweden
University of Burundi, Burundi, and Linkoping University, Sweden

C University of Changsha, China, and LMAM, Vannes, France
Université de Cocody, Ivory Coast, and Stockholm University, Sweden

D University of Dakar, Senegal, and Strasbourg, France

H University of Havana, Cuba, and Humboldt-Universität, Berlin, Germany
University of Hanoi, Vietman, and Toulouse, France,
University of Hanoi, Vietnam, and Paris XIII, France
University of Natural Science in Ho Chi Minh, Vietnam, and Institut de Mathématiques de Jussieu, Paris, France

K University of Kenitra, Morocco, and Nancy, France
Jomo Kenyatta University of Agriculture and Technology, Kenya, and Delta College, USA
University of Khartoum, Sudan, and Umea University, Sweden
Université de Kinshasa, DR Congo, and Lund University, Sweden
Kwame Nkrumah University of Science and Technology, Ghana, and University of Leicester, UK
University of Kerouan, Tunisia, and University of Cergy-Pontoise, France

M University of Malawi, Malawi, and the Royal Inst. of Technology, Sweden
University of Malawi Polytechnic, Malawi, and the Royal Inst. of Technology, Sweden
Université des Sciences et Techniques de Masuku, Gabon, and Stockholm University, Sweden
Université Marien Ngouabi, Congo Rep., and Lund University, Sweden
Moi University, Kenya, and Delta College, USA
Université de Monastir, Tunisia, and Marne la Vallée, France

N Université Abdou Moumouni de Niamey, Niger, and Chalmers/Göteborg University, Sweden

This list is not very long but it is a beginning! And we hope colleagues from other European countries will join in the scheme.

We would certainly like to see this scheme broadened both in scope and in the geographical areas involved. The activities we have in mind for the twins to share could include:

(1) Help with journal and database subscriptions, as detailed above.
(2) Donation of books, journals and equipment.
(3) Supplying guest lecturers and supporting visits from the less developed department.
(4) Mentoring (see also the London Mathematical So-

ciety scheme: http://www.lms/ac.uk/grants/nuffield_scheme.html).
(5) Support for attending major conferences: this we find very important for the career of individual mathematicians in the developing world.

There is no doubt that as the scheme matures, more activities will be found beneficial. The ultimate aim would be that the elder twin would take care of the younger twin in the real sense of "twins", i.e. it has the latter's mathematical wellbeing at heart and would come to its help whenever necessary. Thus the younger twin will have somebody to "turn to" in case of need, ideally until no such needs are required as a result of global progress!

On behalf of the EMS Committee for Developing Countries:

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ERCIM NEWS
online edition

The April 2008 issue, No. 73, of ERCIM-News was dedicated to 'Maths for Everyday Life'. It included an editorial contributed by Jouko Väänänen (member of the EMS Executive Committee) and Ulrich Trottenberg.

More information and issues of ERCIM-News are available at http://ercim-news.ercim.org/.

International Mathematical Olympiad, Madrid, 10–22 July 2008

Marco Castrillón (Madrid, Spain), Vicente Muñoz (Madrid, Spain)

IMO 2008 MADRID
49th INTERNATIONAL MATHEMATICAL OLYMPIAD

The International Mathematical Olympiad (IMO) is certainly the queen of all mathematical competitions. Since the first competition in 1959, the IMO has challenged generations of brilliant youngsters from all around the world and has gifted the mathematical community with beautiful problems in geometry, combinatorics, algebra and number theory. In July of this year, the 49th IMO took place in Madrid, organized by the Spanish Ministry of Education and the Real Sociedad Matemática Española. It is the first time that Spain has hosted an IMO since its first participation in 1981. This perfect square edition has gathered contestants from 97 participating countries together with three observer countries (Syria, Benin and Mauritania), which will certainly be future participating countries, thus representing an historic record. The 537 students did theirs exams, as usual in the IMO, in two sessions of 4 hours and 30 minutes each, which took place over 16-17 July. Each exam consists of three problems, graded in increasing difficulty. The students have the first half hour to make questions to the Jury.

The Jury is the governing body of the IMO. It is composed of a leader from each participating country. The Jury was accommodated in La Granja de San Ildefonso, a peaceful and splendorous town founded in the 18th century by King Felipe V in the times of the Illustration. The question and answer sessions were thus performed by Internet connection. In the days before the contest, the Jury chose the six problems from a list provided by a Problem Selection Committee and then translated them

At the exam.

Group photograph of all participants with Prince Felipe and Princess Letizia.

into the 52 different languages of the students, from Albanian to Vietnamese.

After the exam, the contestants enjoyed an exciting social programme including a gymkhana and a visit to the Prado Museum, as well as visits to the ancient city of Toledo and the Royal Monastery of San Lorenzo del Escorial. During these days, the marking sessions were carried out. This process consists of an agreement between the leader of each country, who marks the problems of the students of his/her country from 0 to 7 points, and a board of coordinators. The 75 coordinators were organized in teams for marking each problem and their task was to ensure that the same amount of work from different students is given the same marking. This agreement process is called coordination for obvious reasons.

According to IMO regulations, no more than half of the participants can be awarded a medal. The final meeting of the Jury decides the medals. The awards of this year's IMO were as follows:

- Gold medals: from 31 to 42 points, 43 recipients.
- Silver medals: from 22 to 30 points, 100 recipients.
- Bronze: from 15 to 21 points, 126 recipients.
- Honourable mentions: given to students who solve at least one problem complete, 103 recipients.

Three contestants (Xiaosheng Mu, Dongyi Wei, both from PR of China, and Alex Zhai, from the USA) obtained the perfect score of 42 points. In a classification by country, the first five were: China, Russia, the USA, Korea and Iran. This classification is made by adding up the points of all the six students in the team instead of by counting the number of medals as is customary in olympiads in sport (for instance, China obtained five golds and a silver, whereas Russia obtained six golds but were second).

The host country Spain finished in 43rd position, with three bronzes and three honourable mentions.

Medals were awarded in the closing ceremony, which took place on 21 July and was presided over by the Crown Prince Felipe and Princess Letizia, accompanied by the Minister of Education and the President of the Region of Madrid. After the exciting ceremony, the contestants had the opportunity to join the prince and the princess in an informal meeting in which they took pictures and had a conversation with their highnesses.

The event finally closed with a farewell dinner. Contestants, leaders, deputy leaders, guides, volunteers, coordinators and organizers enjoyed a Spanish summer night in which last pictures, exchange of addresses and best wishes for the future organizers were the main courses. In July 2009, the 50th IMO will take place in Bremen, Germany. The hosting countries for the following IMOs are Kazakhstan (2010), the Netherlands (2011) and Argentina (2012).

As an example, here is the hardest problem of the first day:

Prove that there exist infinitely many positive integers n *such that* n^2+1 *has a prime divisor greater than* $2n+\sqrt{2n}$.

Have some fun with it and remember that you do not need deep results from number theory but ingenuity.

For the other problems (in any of the 52 translated languages) as well as pictures and other information, please check the web site http://www.imo-2008.es. General information on the IMOs, like rankings of countries over the years, can be found at http://www.imo-official.org.

Marco Castrillón López [mcastri@mat.ucm.es] is an associate professor at Universidad Complutense de Madrid, Spain. His main fields of research are geometric mechanics, symmetries in field theories and homogeneous structures in Riemannian manifolds. He is a member and the treasurer of the Mathematical Olympiad Committee IMO2008, which depends upon the Real Sociedad Matemática Española.

Vicente Muñoz [vicente.munoz@imaff.cfmac.csic.es] is the Editor-in-Chief of the Newsletter of the EMS. He has been Chairman of the Problem Selection Committee of the International Mathematical Olympiad 2008. He has also been a member of the coordination team of the Olympiad.

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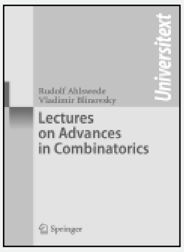

Lectures on Advances in Combinatorics

R. Ahlswede, University of Bielefeld, Germany; **V. Blinovsky**, Russian Academy of Sciences, Moscow, Russua

This volume features lectures that focus on basis extremal problems and inequalities – two sides of the same coin. Additionally, the lectures help prepare approaches and methods useful and applicable in a broader mathematical context. Highlights of the book include a solution to the famous 4m-conjecture of Erdös/Ko/Rado 1938, one of the oldest problems in combinatorial extremal theory, an answer to a question of Erdös (1962) in combinatorial number theory "What is the maximal cardinality of a set of numbers smaller than n with no k+1 of its members pair wise relatively prime?", and the discovery that the AD-inequality implies more general and sharper number theoretical inequalities than for instance Behrend's inequality.

2008. XIII, 314 p. 3 illus. (Universitext) Softcover
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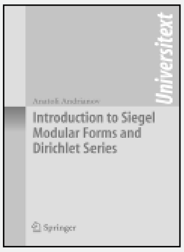

Introduction to Siegel Modular Forms and Dirichlet Series

A. Andrianov, Russian Academy of the Sciences, Petersburg Department of the Steklov Institute of Mathematics, St. Petersburg, Russia

This book is intended for a graduate course on Siegel modular forms, Hecke operators, and related zeta functions.

Topics include ► analytical properties of radial Dirichlet series attached to modular forms of genuses 1 and 2 ► the abstract theory of Hecke–Shimura rings for symplectic and related groups ► action of Hecke operators on Siegel modular forms; applications of Hecke operators to a study of multiplicative properties of Fourier coefficients of modular forms ► Hecke zeta functions of modular forms in one variable and to spinor (or Andrianov) zeta functions of Siegel modular forms of genus two ► the proof of analytical continuation and functional equation (under certain assumptions) for Euler products associated with modular forms of genus two.

2008. Approx. 210 p. (Universitext) Softcover
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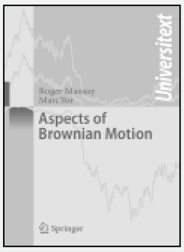


Aspects of Brownian Motion

R. Mansuy, **M. Yor**, Université Paris VI, Paris, France

Stochastic calculus and excursion theory are very efficient tools to obtain either exact or asymptotic results about Brownian motion and related processes. The emphasis of this book is on special classes of such Brownian functionals as:
► Gaussian subspaces of the Gaussian space of Brownian motion; ► Brownian quadratic funtionals; ► Brownian local times, ► Exponential functionals of Brownian motion with drift;
► Winding number of one or several Brownian motions around one or several points or a straight line, or curves; ► Time spent by Brownian motion below a multiple of its one-sided supremum.
Besides its obvious audience of students and lecturers the book also addresses the interests of researchers from core probability theory out to applied fields such as polymer physics and mathematical finance.

2008. Approx. 210 p. (Universitext) Softcover
ISBN 978-3-540-22347-4 ► **€ 39,95 | £30.50**

Malliavin Calculus for Lévy Processes with Applications to Finance

G. Di Nunno, **B. Oksendal**, **F. Proske**, University of Oslo, Norway

The book is an introduction to Malliavin calculus as a generalization of the classical non-anticipating Itô calculus to an anticipating setting, most valuable for graduate students and lecturers in stochastic analysis and applications. The calculus is presented both for the Brownian noise and for Lévy type of noise.
While the original works on Malliavin calculus aimed to study the smoothness of densities of solutions to stochastic differential equations mainly within the Brownian frame, this book has another goal: it presents the development of the theory and its use in new fields of application such as stochastic integral representations and stochastic control. The applications presented in the environment of mathematical finance include: hedging in complete and incomplete markets, optimization in the presence of asymmetric information and also pricing and sensitivity analysis.

2008. Approx. 410 p. (Universitext) Softcover
ISBN 978-3-540-78571-2 ► **€ 39,95 | £30.50**

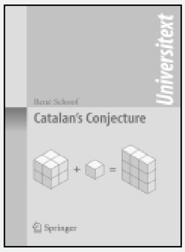
Proof Theory

The First Step into Impredicativity

W. Pohlers, Westfälische Wilhelms-University, Münster, Germany

This book verifies with compelling evidence the author's inclination to "write a book on proof theory which needs no previous knowledge of proof theory". Avoiding the cryptic terminology of proof as far as possible, the book starts at an elementary level and displays the connections between infinitary proof theory and generalized recursion theory, especially the theory of inductive definitions. As a "warm up" the classical analysis of Gentzen is presented in a more modern terminology to proceed with explaining and proving the famous result by Feferman and Schütte on the limits of predicativity. The author, too, provides an introduction to ordinal arithmetic, introduces the Veblen hierarchy and employs these functions to design an ordinal notation system for the ordinals below Epsilon 0 and Gamma 0, while emphasizing the first step into impredicativity, i.e., the first step beyond Gamma 0.

1st ed. 1989. 2nd printing 2008. Approx. 385 p. (Universitext) Softcover
ISBN 978-3-540-69318-5 ► **€ 39,95 | £30.50**



Catalan's Conjecture

R. Schoof, Università di Roma, Italia

Eugène Charles Catalan made his famous conjecture – that 8 and 9 are the only two consecutive perfect powers of natural numbers – in 1844 in a letter to the editor of Crelle's mathematical journal. One hundred and fifty-eight years later, Preda Mihailescu proved it.
Catalan's Conjecture presents this spectacular result in a way that is accessible to the advanced undergraduate. The author dissects both Mihailescu's proof and the earlier work it made use of, taking great care to select streamlined and transparent versions of the arguments and to keep the text self-contained. Only in the proof of Thaine's theorem is a little class field theory used; it is hoped that this application will motivate the interested reader to study the theory further. Beautifully clear and concise, this book will appeal not only to specialists in number theory but to anyone interested in seeing the application of the ideas of algebraic number theory to a famous mathematical problem.

2008. Approx. 160 p. 10 illus. (Universitext) Softcover
ISBN 978-1-84800-184-8 ► **€ 39,95 | £27.00**

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One of the most influential mathematicians of our time, **Henri Cartan**, has passed away at age 104. His contribution to modern Mathematics during his long life has been enormous and invaluable. He was a remarkable teacher and one of the founders and long term member of the Bourbaki group. He was also passionately committed to a united Europe and Human Rights. Henri Cartan will be remembered with great respect and gratitude by many mathematicians of all generations. On behalf of the European Mathematical Society I would like to express our deepest condolences to his family.

Ari Laptev (President of the EMS)

Marcus du Sautoy tells The Story of Maths

In the autumn of 2008 Marcus du Sautoy will present a landmark series for the BBC on the history of mathematics. Called '*The Story of Maths*', the four one-hour television programmes will take the viewer from the pyramids of Egypt to the deserts of Arizona and from the backwaters of Kerala to the suburbs of St Petersburg, in pursuit of where and how mathematics evolved over the last seven millennia.

Programme one covers the mathematics of the ancient world: Egypt, Mesopotamia and Greece, and shows how the Egyptians used early ideas of calculus to calculate the volume of pyramids. Programme two takes the viewer on a journey through the mathematics of the East from China to India, and we discover that the Keralean school of mathematicians knew Leibniz's infinite series for π some centuries before its discovery in the West. Programme three presents the mathematics of Europe from Descartes via Euler through to Riemann. Programme four encompasses the mathematics of the modern era from Hilbert and Cantor through to Perelman's proof of the Poincaré Conjecture.

The series is partly funded by the Open University and will be shown initially on BBC4. It forms part of Marcus's work as a Senior Media Fellow for EPSRC. Combining stunning graphics with colourful locations, the series will bring to life the intellectual journey that has taken mathematicians from fractions to fractals and from the circle to the hypersphere.

Marcus and film crew at the pyramids in Cairo.

The Story of The Story of Maths

Marcus du Sautoy

I didn't really know a lot about the history of my subject: I always believed that what matters most is the mathematics. If you know the theorems and the proofs, is it really important who created them, or in what circumstances? Certainly the way we are taught mathematics both in school and at university reinforces this anti-historical message. So you might think that, with such a mentality, I would not be the ideal candidate to present a landmark series on the history of maths for the BBC.

But in some ways I think that it has worked in my favour – the series has become a real journey of discovery for me. Uncovering quite how much the Ancient Egyptians and Mesopotamians knew about mathematics before the Ancient Greeks has been a revelation for someone brought up on the myth that it all started with Pythagoras. I was amazed to discover quite how much the Indian mathematicians of the medieval period knew about infinite series and pre-calculus. And visiting the places where Descartes, Fermat, Euler and Cantor grew up brought these characters alive for me in a way that I hope will come over on the screen.

The programmes pick up on this intellectual journey and mirror it with a real physical journey: the hope was to make something that looked like a cross between Michael Palin and *The Ascent of Man*. The programmes open with the story of the mathematics of Ancient Egypt and Babylon; Cairo and the pyramids provide an exotic location for the former. But, unfortunately, health and the safety restrictions at the BBC stopped us from braving war-torn Iraq for the sake of mathematics. So Damascus, an outpost of the Babylonian empire, became our backdrop to talk about the mathematics hidden inside the clay tablets that have survived.

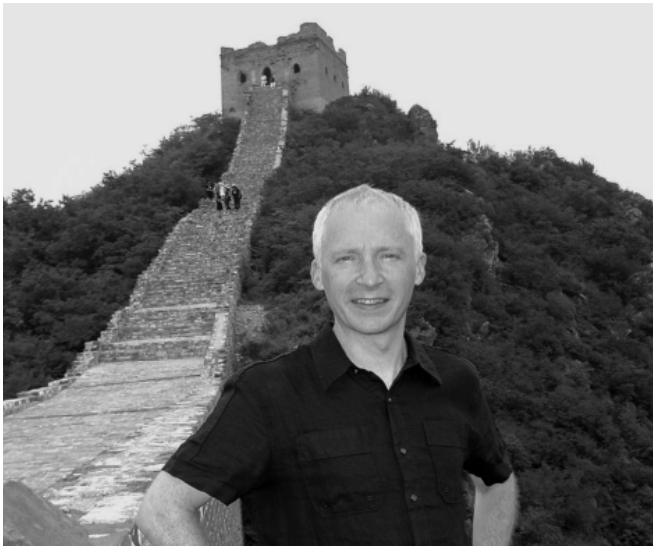
The second programme took us to the East and an exploration of Chinese and Indian mathematics. One of the highlights for me was the pilgrimage to Gwalior to see a tiny little temple hanging off the side of a mountain fort, just big enough to fit the presenter and a cameraman inside. We scoured the inscriptions on the walls for the earliest known example of the number zero, one of the greatest and most revolutionary inventions made in India.

The mathematics of India found its way to Europe, via the spice routes through central Asia. Again health and safety denied us a trip to Iran to recreate the adventures of Omar Khayyam (the British sailors had not long before been released from captivity). So Morocco became our central Asian backdrop where we found some fantastic horses to ride across the Atlas mountains in my reincarnation of the great Persian poet and mathematician (my director informed me afterwards that he'd decided to leave that reckless afternoon out of the health and safety report).

Programmes three and four took us to the colder climes of Europe and then on to the US: a town called Descartes; Fermat day in Beaumont-de-Lomagne (his home town); St Petersburg for the mighty Euler and the elusive Perelman; Göttingen for Gauss, Riemann and Hilbert; the Nervenklinik in Halle for the unsettled Cantor; the Paris café where Bourbaki began (now a fast-food burger joint); and the Arizona desert to look for Julia Robinson's childhood haunts.

But if I had to pick out one location that excited me more than any other, it has to be our one-day trip from St Petersburg to the grey city of Kaliningrad. This is the modern name for Königsberg, the home of the seven bridges that some see as the beginning of modern topology. The city was bombed heavily during the Second World War and today only three of the original bridges are left standing, while two of the others have been rebuilt – they now carry a huge dual carriageway through the centre of the town. Despite the ugly nature of this modern city, I felt I was in a mathematical Disneyland. To be able to make the journey over the bridges to see if there is a path filled me with a childish excitement that my crew just could not understand. Of course, with just the five existing bridges it is in fact possible to make the journey today, unlike the seven bridges that faced the earlier inhabitants of Königsberg.

My crew were only too pleased to leave behind the grim skyline of Kaliningrad but for me it was one of the days out of the months of filming that I will always treasure. For me it encapsulated what this whole series is about – bringing alive the stories behind the amazing intellectual journey that mathematicians have made over the last seven millennia.



Marcus at the Great Wall of China.

These articles first appeared in the Oxford University Mathematical Institute Newsletter, and appear with permission.

Special issue on "Citation Statistics"

A report analyzing impact factors and similar statistics based on citations

This is the first special issue of IMU-Net, which can be found as IMU-Net 29b, June 2008.

The IMU has released an important document called "Citation Statistics", which we want to bring to your attention.

IMU-Net 24 (July 2007) announced the creation of a committee on "Quantitative assessment of research", which was asked to investigate various aspects of impact factors and similar statistics based on citations. The committee was appointed jointly by the executive committees of the International Mathematical Union (IMU), the International Council for Industrial and Applied Mathematics (ICIAM) and the Institute of Mathematical Statistics (IMS). It consists of:

- John Ewing (Providence, USA), Chair, appointed by IMU.
- Robert Adler (Haifa, Israel), appointed by IMS.
- Peter Taylor (Melbourne, Australia), appointed by ICIAM.

The terms of reference given to the committee can be found at
http://www.mathunion.org/Publications/2007/Charge-ComOnQuantAssessmRes070521.pdf.

The committee has addressed this charge by reviewing and discussing current practices as well as the extensive literature on the use of citations to evaluate research. Its report, written from the perspective of mathematical scientists, was submitted to the executive committees of the IMU, the ICIAM and the IMS, and all three endorsed the report. The three organizations are making the report "Citation Statistics" public today.

The report can be found at:
http://www.mathunion.org/Publications/Report/CitationStatistics.

A press release that was mailed out today to journalists is at:
http://www.mathunion.org/Publications/PressRelease/2008-06-11/CitationStatistics.

This effort was triggered by numerous requests from IMU member countries, mathematical societies, important mathematical institutions and individuals who reported the increasing use (and misuse) of impact factors and other citation-based indicators to measure the quality of the research of individuals, departments and whole institutions.

The IMU suggests that the readers of IMU-Net not only read the report but also distribute it to administrators and decision-makers who are involved in the assessment of research quality, in order to give them a mathematical science perspective. The IMU, the ICIAM and the IMS have agreed, in order to ensure as wide a distribution as possible, that journals, newsletters and similar publications that are interested in publishing this report will have the non-exclusive right to publish it in one of their issues.

Please contact the newsletters/journals you are connected with and suggest publication of the report "Citation Statistics".

All three organizations, representing the world community of pure, applied, and industrial mathematics and statistics, hope that the careful analysis and recommendations in this report will be considered by decision-makers who are making use of citation data in research assessment.

Nominations for IMU prizes 2010

The executive committee of the International Mathematical Union has appointed the selection committees for the IMU prizes to be awarded in August 2010 during the opening ceremony of ICM 2010 in Hyderabad, India. The Chairs of the prize committees are as follows:

Fields Medals:
László Lovász
Eötvös Loránd Tudományegyetem, Hungary
e-mail: lovasz@cs.elte.hu

Rolf Nevanlinna Prize:
Ravindran Kannan
Microsoft Research Labs, India
e-mail: kannan@microsoft.com

Carl Friedrich Gauss Prize:
Wolfgang Dahmen, Germany
e-mail: dahmen@igpm.rwth-aachen.de

Information about the three prizes and the type of work for which they are given can be found on the IMU web site at http://www.mathunion.org/general/prizes/.

The guidelines for nominations are specified at the web page http://www.mathunion.org/general/prizes/nomination-guidelines/.

Nominations should ideally be sent by 15 December 2008 to the prize committee Chairs.

Please distribute this information within your mathematical communities so that as many good candidates as possible are nominated for these important prizes.

Seminar Sophus Lie

Alice Fialowski (Eötvös Loránd University, Budapest, Hungary) and
Ágnes Szilárd (Alfréd Rényi Institute of Mathematics, Budapest, Hungary)

Marius Sophus Lie (1842–1899)

About Sophus Lie

"My life is actually quite incomprehensible for me. As a young man, I had no idea that I was blessed with originality. Then, as a 26-year-old, I suddenly realized that I could create. I read a little and began to produce. In these years, 1869–1874, I had a lot of ideas which, in the course of time, I had developed only very imperfectly. In particular, it was group theory and its great importance for the differential equations which interested me." – S. Lie.

http://www.math.uit.no/seminar/Lie_biog.htm.

Marius Sophus Lie (1842–1899) was born in Norway. He was taught mathematics by Ludwig Sylow at the University of Christiania. A turning point came when he read papers on geometry by Poncelet and Plucker. In 1869, Lie went to Berlin where he and Felix Klein became friends. The two started working on transformation groups. Because of the French-German war Lie decided to return to Christiania (now Oslo) where he obtained his PhD. Klein suggested that his student Friedrich Engel study under Lie and assist him in the preparation of his large unpublished theory. Lie collaborated with Engel for nine years. They jointly published "Theorie der Transformationsgruppen" in three volumes in 1893. This was Lie's major work on continuous groups of transformations.

In 1886 Lie succeeded Klein in the Chair of Mathematics at Leipzig with Engel as his assistant. Because of his growing illness, in 1898 he returned to Christiania where he died the next year.

Brief History of the Seminar

Seminar Sophus Lie is now an international seminar of mathematicians interested in the theory of Lie groups and their wider horizons. It was founded in Germany in 1989–90, just after the wall separating East and West Berlin came down. Since the end of World War II, mathematical research and education in East and West Germany, while arising from the same tradition, had been separate. The communist regimes had a phobia against contacts between eastern and western scholars. As a further impediment they erected the Berlin Wall in 1961. After that there was essentially no interaction. Each side knew little of the scholarly activities (or mathematicians) of the other. When the wall came down, communication could finally begin again.

The first order of business in the autumn of 1989 was to reconnect scientists and scholars from both sides. The Seminar Sophus Lie was one of those attempts. It was supported by the Deutsche Forschungsgemeinschaft (DFG) and also by the local university administrations from all four universities involved: Darmstadt and Erlangen from the west and Leipzig and Greifswald from the east (historically, Leipzig was particularly important

Seminar Sophus Lie XXXV, Budapest, Hungary

in Lie's life). The founders of the seminar were *Helmut Boseck* (Greifswald), *Karl Heinrich Hofmann* (Darmstadt), *Konrad Schmüdgen* (Leipzig), and *Karl Strambach* (Erlangen).

The first meeting was in Leipzig (coincidentally the meeting started the day the first Gulf War began). The speakers were:

K. Schmüdgen (Leipzig)
K.H. Hofmann (Darmstadt)
G. Czichowski (Greifswald)
N. Dörr (Darmstadt)
H. Schlosser (Greifswald)
U. Zimmermann (Darmstadt)
P. Plaumann, K. Strambach (Erlangen)
R. Matthes (Chemnitz)

For more details see http://math-www.upb.de/user/hilgert/static/Seminare/jan91.html.

Meetings then rotated around these four universities. Participants got to know each other and as time went on, cooperations began to firm up. As one might guess from the historical background, the first steps were not all that easy.

The organizers had two main goals:

1. Retain the idea of a peripatetic seminar each semester – if possible.
2. Try to document the seminars by making regular seminar reports. These were originally called "Seminar Sophus Lie Reports".

There was financial support for the "Reports" from DFG and from the Technische Hochschule Darmstadt (as it was then called). The university was willing to offer support from time to time but not to commit itself to longer term plans.

The original efforts gave birth to a wide range of long-term effects. Seminar Sophus Lie still meets once every semester (typically for a Friday-Saturday). Its geographic range gradually expanded. The seminar enjoyed an astonishing welcome by both younger and older mathematicians. Without this acceptance, flourishing of the seminar in a period of 18 years could not have happened.

Over the years it had participants from more and more countries, such as Austria, Belgium, Bulgaria, the Czech Republic, Croatia, Denmark, England, Finland, France, Holland, Hungary, Italy, Norway, Poland, Romania, Sweden and Switzerland. Meetings have so far been held in:

Leipzig (1991 and 1992)
Metz, France (1999, 2004)
Darmstadt (1991, 1993, 1994, 1997, 2002, 2005, 2007)
Stuttgart (1999)
Erlangen (1991, 1993, 1996, 2002)
Bedlewo, Poland (2000)
Greifswald (1992, 1994, 1997, 2001)
Berlin (2001)
Clausthal (1995, 1998)
Paderborn (2004)
Bielefeld (1995, 1998, 2003, 2007)
Nancy, France (2005)
Vienna, Austria (1996, 2000, 2003, 2006)
Budapest, Hungary (2008)
Cluj-Napoca, Romania (2008).

The next meeting is to be held in Paderborn (January 2009).

Since 1993 the seminars have received no regular support; only the actual host institute tries to get some local funding. In spite of this fact, more and more people attend the meetings.

The founders of the seminar should be pleased with the fruits of their efforts.

Journal of Lie Theory

One of the most important and lasting effects is the development of *Journal of Lie Theory*. While there was an early agreement that "Seminar Sophus Lie" and the "Journal of Lie Theory" were separate entities and concepts, the founders of the Seminar see them linked in their genesis more than most of us. There were several Oberwolfach Meetings on Lie Theory. Although these meetings were independent of Seminar Sophus Lie, there was a substantial overlap of participants. The meetings were devoted to the same topics that were active in the first meetings of Seminar Sophus Lie. They were documented by books published by DeGruyter Verlag, Berlin.

Oberwolfach was an excellent international platform to discuss the foundation of a new international journal on Lie theory. Ernest Borisovich Vinberg made the suggestion to call it the "Journal of Lie Theory" (JOLT). After a year or two, JOLT evolved from the Seminar Sophus Lie Reports, published by Norbert Heldermann Verlag. The journal has been a real success – internationally as well.

Originally, at every seminar meeting there was a report on the development and well-being of the journal; since both the seminar and the journal are thriving, this is no longer necessary.

JOLT is electronically available via its homepage at the European Mathematical Information System (see http://www.emis.de/journals/JLT/ for more details about the journal).

Seminar Sophus Lie XXXV, Budapest, Hungary

Seminar Sophus Lie XXXV was held 27–29 March 2008 at the *Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences*, in Budapest, Hungary. The organizers were *Alice Fialowski* and *Ágnes Szilárd*. The meeting was partially supported by the Hungarian Academy of Sciences. It had about 40 participants with a number of PhD students from different countries. The speakers at the meeting were:

Faouzi Ammar (Sfax)
Dietrich Burde (Vienna)
Ágota Figula (Debrecen)
Helge Gloeckner (Paderborn)
Johannes Huebschmann (Lille)
Andrey Lazarev (Leicester)

Abdenacer Makhlouf (Mulhouse)
Martin Markl (Prague)
Karl-Hermann Neeb (Darmstadt)
Michael Penkava (Eau Claire)
Claude Roger (Lyon)
Troels Roussau Johansen (Kiel)
Sergei Silvestrov (Lund)
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We very much hope that this successful European event continues to operate with regular financial support and more and more mathematicians and students getting involved.

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The Luria-Delbrück experiment: are mutations spontaneous or directed?

Ellen Baake (Bielefeld, Germany)

Where do mutations come from and what do they have to do with mathematics? The answer is 'a lot', as this article will demonstrate. Indeed, it was an historical milestone of mathematical biology that revealed some fundamental insight into the nature of mutations during the 1940s. An unresolved question in those days was: 'Are mutations due to directed adaptation to environmental change or do they occur spontaneously, in a random way?'. Today, the answer is basic knowledge in genetics and many textbooks briefly describe the crucial experiment, along with some plausibility arguments. It is less well-known, however, that it was a fascinating interaction of theory and experiment that made this breakthrough possible. This is what will be described here.

1. The problem and some conceptual considerations

The initial observation

Before we formulate the problem in a precise way, let us describe the experimental observation that solicited the question in the early 1940s (see figure 1). The microbiologist S. Luria analyzed how resistance against certain bacteriophages originated in *E. coli* bacteria. Bacteriophages are viruses that attack bacteria, multiply within their cells and eventually destroy them – provided the bacterium is *sensitive* to this phage, i.e. if it is not resistant. To this end, Luria raised *E. coli* cultures in a suitable medium, starting with a single sensitive cell every time (or a minimal number of them, but let us idealize a bit here). The growth of the bacterial population is visible through the increasing turbidity of the medium, caused by the scattering of light by bacteria (which distribute themselves all over the medium). After a few days, when the population is fairly dense, one adds a certain amount of phages. As a consequence, the bacteria are destroyed, the dead cell fragments sediment and the culture clears. But after a few days, the medium turns turbid again, meaning that a new bacterial population is growing. So, what has happened?

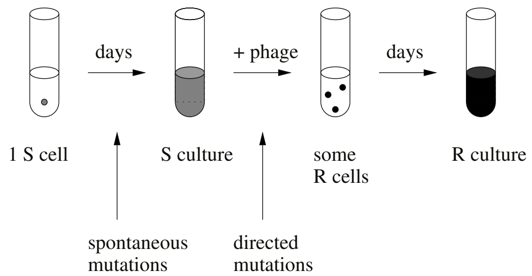

Figure 1. The origin of resistance in a bacterial culture. 'S' stands for 'sensitive' and 'R' stands for 'resistant'.

Two hypotheses

Clearly, a few cells have survived that are happy in the presence of the phage, so they must be *resistant*. This resistance persists over many further generations in culture, so it must be *heritable*. But all our bacteria go back to a single sensitive cell so a *mutation* must have happened along the way. The crucial question is when. Figure 2 shows the two principal possibilities.

(SM) A few bacteria are already resistant before the phage arrives; mutations thus happen *spontaneously* while the culture is growing, independent of the selection to be applied later by the phage.

(DM) A few bacteria acquire resistence when they are exposed to the phage; that is, mutations are *directed* (in the sense of a specific response to the selection applied; they would not occur in the absence of the phage).

Today, it is basic biological knowledge that (SM) is true: mutations occur in a spontaneous way. Beneficial mutations (these tend to be very rare!) may later be filtered out (selected) by the environment (in our case, only resistant cells survive the phage) but they are not directed by selection.

Luria and Delbrück managed to prove this in 1943 [1], at a time when one could not even imagine the powerful genetic methods that are standard in modern labs. The molecular basis of inheritance, and even more so that of mutations, was

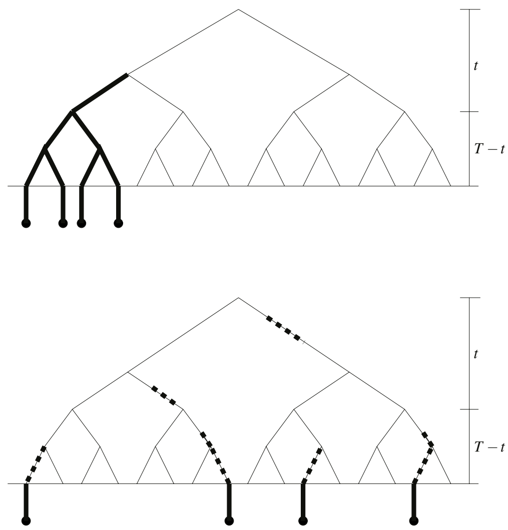

Figure 2. Genealogy (discrete generations) of a bacterial culture with spontaneous (top) and directed (bottom) mutations. Thin lines indicate sensitive bacteria, fat lines mean resistant ones. Dashed lines indicate cells that are predisposed to become resistant when they come in contact with the phage. The phage arrives in generation T; only resistant, or predisposed, cells survive it. If mutations arise spontaneously, we observe resistant clones; if they are directed, the resistant cells are independent.

still in the dark (it had to wait until 1952, when Hershey and Chase discovered that genetic information is carried by DNA, and 1953, when Watson and Crick unravelled its famous double helix structure). Indeed, Luria and Delbrück's decisive experiment boiled down to nothing more than counting cells and the evaluation of their data required only elementary probability. However, the story is fascinating precisely because of this simplicity, in combination with the underlying idea and the stringency of the argument.

2. The model

In what follows, we will even simplify the original considerations a bit further. This will make the essential idea still clearer; we hope that specialists will forgive us. The hypotheses (SM) and (DM) lead to the following fundamental consideration (see figure 2).

After T rounds of division (or generations) starting from the initial cell, the bacteria come into contact with the phage. In case (SM), the mutants have originated before this; one will assume that every sensitive (S) bacterium becomes resistant (R) in every generation $t < T$ with some small probability p. Resistance is heritable, that is, it is passed on to the offspring. In generation T, we will therefore have resistant *clones*, the offspring of the primary mutants.

In case (DM), resistance solely originates in generation T. Now, every bacterium mutates independently of all others, with probability $\tilde{p}$, and survives; the others fall prey to the phage. It is actually irrelevant whether every individual cell truly has the same chance $\tilde{p}$; it may also be conceived that the result is determined by small physiological differences between the cells (e.g. size or nutritional status, which may vary within certain limits). In this case, one would assume that cells with certain favourable properties are predisposed in the sense that they will become (stably) resistant if they meet a phage. As long as these physiological dispositions themselves are not heritable, i.e. as long as they are not passed on to the offspring over several generations, the situation still boils down to the described random experiment (provided the experimentalist cannot, or does not, distinguish between the different cell variants).

The essential property of (DM), relative to (SM), lies in the fact that physiological dispositions are defined by their temporal occurrence – if they were heritable, we would be in the setting (SM). This is because it is irrelevant to the result whether the cell is already resistant before T or whether it has a heritable disposition that leads to resistance as soon as it comes across the phage.

How could Luria und Delbrück distinguish between these possibilities? The only method available to them was counting resistant cells in generation T (we will describe below exactly how this is done). The question is therefore how we can decide, by counting resistant cells, whether they are clones or independent individuals (in the sense of a random sample from the population).

Clearly, this is impossible to decide on the basis of the number of R cells in a single experiment (see figure 2) and the average over many experiments is equally uninformative. However, the stochastic fluctuations between experiments yield the desired distinction. To analyze these, we will now formulate an idealization (a model) of the scenarios (SM) and (DM) and make our assumptions more precise.

The model must describe both cell division (i.e. population growth) and the mutation process. *Population growth* will be modelled *deterministically*, owing to the fact that it only takes a few generations until the population is very large. For simplicity, we assume that cells divide into two daughter cells in a synchronous way (that is, time is discrete). Starting with a single cell then results in a population of $n(t) = 2^t$ cells in generation t. We set $n(T) = 2^T =: N$, the number of cells in generation T.

In contrast, the *mutation process* must be described in a stochastic manner, as mutations are rare events ($p, \tilde{p} \ll 1$). The quantity to be described is Z, the number of resistant cells in generation T; it is a random variable. In this section, we will calculate the expection $\mathbb{E}(Z)$ and the variance $\mathbb{V}(Z)$ under the hypotheses (SM) and (DM); on the basis of these quantities, Luria and Delbrück solved the problem in 1943. In the next section, we will characterize the distribution of Z in more detail.

In the case (DM), no further assumptions are required. Each of the N cells in generation T turns resistant with probability $\tilde{p}$, hence Z has a binomial distribution with parameters N and $\tilde{p}$; we write $Z \sim \mathscr{B}(N, \tilde{p})$. Therefore,

$$\mathbb{E}(Z) = N\tilde{p}, \quad \mathbb{V}(Z) = N\tilde{p}(1-\tilde{p}). \tag{1}$$

In experiments, $\tilde{p} \approx 10^{-8}$ and $N = 10^8 \ldots 10^{10}$; hence, expectation and variance are equal up to a tiny error term (this error term is just the deviation that occurs if the binomial is approximated by a Poisson distribution with parameter $\lambda = N\tilde{p}$).

To treat case (SM), we must first make our assumption about the mutation mechanism more precise. We will use the following *idealization*.

(A0) The initial cell is sensitive.

(A1) Mutations (S → R) occur on the occasion of cell divisions. With probability p, one of the daughter cells is a mutant (here we have in mind a mechanism of division where the 'original' remains intact, whereas the 'new copy' may contain an error).

(A2) As long as the phage is absent, S and R cells divide in the same way, i.e. in every generation.

(A3) The number of resistant cells is, at any time, negligible relative to $n(t)$ (this is justified since $p \ll 1$), so the number of S cells may be described by $n(t)$.

(A4) Back mutations (R → S) are negligible. (The probability for any single R cell to mutate back is of a similar magnitude to p. Therefore, given (A3), the number of events is tiny indeed.)

In order to calculate $\mathbb{E}(Z)$ and $\mathbb{V}(Z)$, we will now proceed generation-wise. We will denote by $X(t)$ the number of *mutation events* that occur at the t-th cell division (that is, the division that leads from generation $t-1$ to generation t, where the initial cell is generation 0). Let $Y_T(t)$ then be the number of *mutants* in generation T that go back to a *mutation event* in generation t. With a *mutation event*, we mean every transition $S \to R$, whereas a *mutant* is any R cell, whether it just originated by a 'primary' mutation event or is the offspring of an already resistant cell.

Due to (A2), every mutation event that occurred in generation t will produce a resistant clone of size 2^{T-t} until generation T. Therefore,

$$Y_T(t) = 2^{T-t} X(t), \quad 1 \le t \le T. \tag{2}$$

For ease of notation, we will, in what follows, abbreviate $Y_T(t)$ by $Y(t)$; but keep in mind the dependence on T.

Finally, the number of resistant cells in generation T is

$$Z = \sum_{t=1}^{T} Y(t). \tag{3}$$

Let us now calculate the expectation and variance for $X(t)$, $Y(t)$ and Z. Due to (A1), (A3) and (A4), we have $X(t) \sim \mathscr{B}(n(t), p)$ and therefore

$$\begin{aligned} \mathbb{E}\big(X(t)\big) &= n(t)p, \\ \mathbb{V}\big(X(t)\big) &= n(t)p(1-p) = (1-p)\mathbb{E}\big(X(t)\big). \end{aligned} \tag{4}$$

In contrast to $X(t)$, $Y(t)$ is not binomially distributed (the members of a clone are not independent). Actually, it has none of the standard distributions but its expectation and variance follow directly from (2) and (4) (as well as $2^T = N$):

$$\mathbb{E}\big(Y(t)\big) = 2^{T-t}\mathbb{E}\big(X(t)\big) = Np \tag{5}$$

$$\mathbb{V}\big(Y(t)\big) = 2^{2(T-t)}\mathbb{V}\big(X(t)\big) = 2^{T-t}(1-p)\mathbb{E}\big(Y(t)\big). \tag{6}$$

These relationships are simple but illuminating. Equation (5) shows that, on average, every generation eventually produces the same number of mutants: for small t, there are few mutation events but those that do occur have large offspring, and vice versa, so that these effects just compensate each other. Equation (6) contains an important hint towards the idea of the Luria-Delbrück experiment; the variance of $Y(t)$ is increased by a factor of $2^{T-t}(1-p)$ (i.e. close to clone size) relative to the expectation.

Finally, (3), (5) and (6) jointly give

$$\mathbb{E}(Z) = \sum_{t=1}^{T} \mathbb{E}\big(Y(t)\big) = TNp \tag{7}$$

and

$$\begin{aligned} \mathbb{V}(Z) &= \sum_{t=1}^{T} \mathbb{V}\big(Y(t)\big) \\ &= 2^T Np(1-p) \sum_{t=1}^{T} \Big(\frac{1}{2}\Big)^t \\ &= 2^T \Big(1 - \frac{1}{2^T}\Big) Np(1-p). \end{aligned} \tag{8}$$

In the first step of (8), we have used the fact that the $Y(t)$ are independent of each other (one has dependence *within* clones but none *between* clones – the latter being due to (A3)).

Comparing (1), (7) and (8) now yields the crucial difference between spontaneous and directed mutations, which we summarize as:

Fact 1. *Under assumptions (A0)–(A4), the ratio of variance and expectation is*

$$\frac{\mathbb{V}(Z)}{\mathbb{E}(Z)} = \begin{cases} 1 - \tilde{p} \approx 1 & \textit{for directed mutations,} \\ (2^T - 1)(1-p)/T \gg 1 & \textit{for spontaneous mutations.} \end{cases} \tag{9}$$

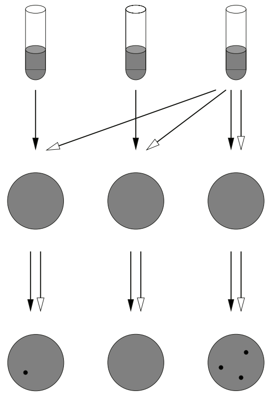

Figure 3. The Luria–Delbrück experiment (filled arrowheads) and its control experiment (hollow arrowheads).

3. Experiment and control

Equation (9) suggests how to distinguish between (SM) and (DM). Grow a large number of parallel cultures (each from a single cell), add the phage, count the surviving cells and compare the mean m_z to the empirical variance s_z^2 (as estimates of $\mathbb{E}(Z)$ and $\mathbb{V}(Z)$). What is still missing is the counting method. This is illustrated in figure 3 and works as follows. Every culture is transferred to a separate agar plate that has already been covered amply with a suspension of phages (filled arrowheads in figure 3). The sensitive cells die and the (few) resistant ones continue to divide. Every single one of them forms a colony, which may be discerned with the unaided eye. Proceeding this way with 50–100 parallel cultures and counting the resistant cells in each of them, Luria and Delbrück [1] obtained values for s_z^2/m_z in the range of 4 to 620, with a typical value of 225 as in the example in figure 4. This figure shows the empirical distribution of Z in one particular set of parallel cultures and compares it with the corresponding distribution that would be expected under the directed mutation hypothesis. The observed histogram deviates from the expected one in a striking way, clearly demonstrating the 'jackpot effect' that results from the rare early mutations with their large progeny.

Although the results look very convincing, it must be mentioned that a factor as large as $(2^T - 1)(1-p)/T$ (as predicted by (9) for spontaneous mutations) is never observed in any such set of experiments. Indeed, it is not expected to be observed in any real experiment due to a sampling effect. By (6), the largest contribution to the variance comes from the very early generations; here, however, cells are so few, and mutation events so rare, that they are practically never observed in any given (*finite*) number of parallel experiments. To correct for this, Luria and Delbrück replaced T by $\hat{T}$, where $\hat{T}$ is (essentially) the expected age of the oldest mutation, taken over the given number of parallel experiments. With this (heuristic) correction for sampling, the predicted factor is even smaller than the observed one in all but one set of experiments [1].

Clearly, therefore, the observations do point towards spontaneous rather than directed mutations. As we have seen, this has been decided on the basis of the underlying source(s) of randomness, for which there are two possibilities. The fluctuations caused by the fact that some cells become resistant while others do not is present in either case; it leads to a bi-

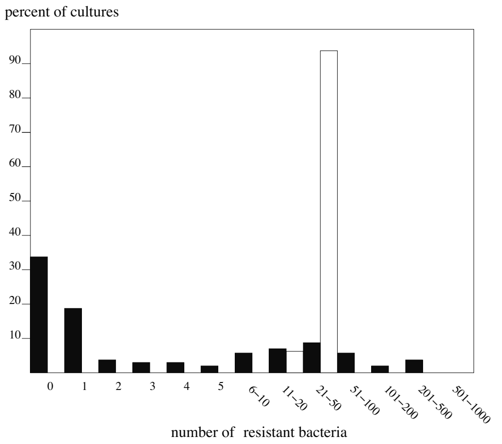

Figure 4. Histogram of the number of resistant bacteria, as observed in 87 parallel cultures (black bars; set of experiments no. 23 in [1]), and the corresponding distribution expected under directed mutation (white bars). The latter is a binomial distribution determined by the observed total number of cells per culture, $N = 2.4 \times 10^8$, and the observed mean number of resistant cells, $N\bar{p} = 28.6$; this is indistinguishable from the Poisson distribution with mean 28.6, $\mathscr{P}(28.6)$. Note the increasing class sizes on the horizontal axis. In this set of experiments, $s_z^2/m_z = 225$.

nomial distribution. The additional (and much larger) fluctuations caused by the variability of the *time* at which a mutation event occurs are only present if mutations are spontaneous. Since early mutations with very large progeny are rare, they cause the jackpot distribution with its abnormally high variance.

This latter insight occurred to Delbrück (the theoretician in the team) while watching a (possibly illegal) slot machine in a country club in Bloomington, which, in true slot machine manner, spits out a little money fairly frequently and a large return only very rarely.

Luria (the experimentalist among the two) suspected something different. Performing a large number of experiments over an extended period of time, he worried about the large fluctuations of his cell counts from day to day, and first blamed the counting method (incubation on selective medium and colony formation) as being unreliable and bringing about the fluctuations – until he performed the decisive *control* experiment, which we will now describe. Here, a large number of plates (again with phage suspension) is inoculated with samples from *the same* bacterial culture (figure 3, hollow arrowheads). If the counting method works correctly, the number of bacteria per plate should now be distributed according to $\mathscr{B}(m, \bar{p})$, if m is the number of bacteria per plate and $\bar{p}$ the proportion of resistant cells in the particular culture used, so one would expect $s_z^2/m_z = 1 - \bar{p}$, i.e. a value very close to one. This is indeed what is observed: the average and empirical variance of Z now turn out approximately equal. This proves that the extra fluctuations observed in the experiment proper are inherent in the original cultures, rather than being an artifact of plating and counting.

4. Afterthoughts

The insight gained by Luria and Delbrück was the beginning of our understanding of mutations. Of course, the analysis may be (and has been) improved (statistically and otherwise) but the essence remains unchanged: large fluctuations point to spontaneous mutations. It should be noted that Luria and Delbrück did not 'only' *answer* the question about the nature of mutations – in fact, they were the first to clearly *pose* it, formulate the alternatives and put up the correct conceptual framework. Their original paper is illuminating to read because of the clarity of the argument.

Their historical experiment has lost nothing of its relevance even today. Known under the name of 'fluctuation test', it belongs to the standard repertoire of many genetics practicals, for the simple reason that it always works, in a foolproof way, independently of the selection pressure applied. In particular, resistance to antibiotics is readily bred by applying antibiotics – an ardent problem these days.

For Luria and Delbrück, their 1943 paper was only the start of further fundamental research in the genetics of both bacteria and phages; together with A. Hershey, they received the Nobel Prize for physiology and medicine in 1969.

Spontaneous, directed and induced mutations

To avoid misunderstandings, let us briefly revisit the notions concerning the mutation mechanism. The attentive reader may have noticed that, following standard terminology, we have formulated the alternative mutation mechansims as 'spontaneous' versus 'directed', in the sense of 'arising independently of selection' versus 'arising as a response to selection'. We have, so far, not considered the alternative that mutations may be *induced* by the environment but in an undirected way. Of course, it is common knowledge today that UV radiation, mutagenic substances, etc. can drastically increase an organism's mutation rate; and these mutations are independent of whether they are advantageous or deleterious for the organism under the given circumstances. In the conceptual framework used here, they would simply be spontaneous mutations. Luckily, the T1 phages used by Luria and Delbrück were not mutagenic (phages are rarely mutagenic). If the phage had been (highly) mutagenic, the counts of R cells would have been dominated by the mutations induced by the phage at the moment of its appearance, which would have pointed to directed mutations. Indeed, as Luria and Delbrück correctly note, their experiment tells us *when* the mutations arise but not *why*.

Acknowledgement. It is my pleasure to thank Evelyn Herrholz for designing the figures.

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Four colours are not enough

Tomasz Bartnicki (Zielona Góra, Poland) and Jarosław Grytczuk (Kraków, Poland)

First of all we suppress the idea that the present district map of Poland is not a counterexample to the celebrated Four Colour Theorem (we checked that). Nevertheless, through a funny series of coincidental events, the title of this note is not completely unjustified. However, let us postpone its explanation till the end of this story …

In 1980, Steven Brams proposed the following game. Alice and Bob colour regions of a plane map alternately with a fixed set of colours. Both players may use any colour from the set and both are not allowed to act improperly, that is to colour a region with a colour that is already present on a neighbouring region. Alice starts the game and her goal is to colour the whole map properly; only in this case is she a winner. But Bob is perfidious and wants to prevent this from happening; his goal is to cause a situation in which some uncoloured region will be surrounded by all available colours. Then the proper colouring of the whole map will not be possible and he will become the winner. The question is naturally how many colours are needed to guarantee a win for Alice?

Brams' game was popularized in 1981 by Martin Gardner in his column "Mathematical Games" in *Scientific American* [7] (and [8]). First it was noticed that four colours are no longer enough. Indeed, consider a six region map corresponding to the faces of the cube. Suppose that four colours $\{1,2,3,4\}$ are available and let Alice colour the bottom face with colour 1. Bob answers by colouring the opposite face with colour 2 (see figure 1). This perfidious response eliminates both colours 1 and 2 from further use, so Alice has to play with a new colour, say 3. Then Bob responds with colour 4 on the opposite face and the game is over. This example is due to Bob High, a future president of the American Go Association. Actually, it was found after Lloyd Shapley provided an ingenious argument that even five colours are not enough. Indeed, consider a twelve region map corresponding to the faces of the regular dodecahedron.

Suppose that five colours $\{1,2,3,4,5\}$ are available and let Alice colour the bottom face with colour 1. This time Bob answers with the same colour on the opposite face (see Figure 1). This eliminates colour 1 from further use, so Alice has to play with a new colour, say 2. Then Bob responds with colour 2 on the opposite face. Playing in this way he successively eliminates colours and wins in the fifth round. After Bob High found a (now complicated) counterexample to the "six colour conjecture", an anxiety started to grow that perhaps there is no finite bound at all.

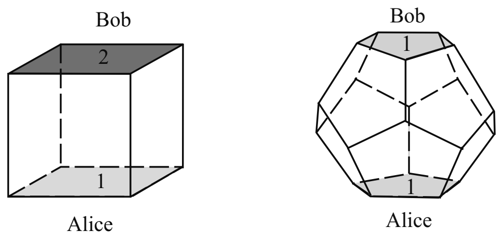

Figure 1. First two moves on the cube and the dodecahedron

The problem remained unnoticed by the mathematical community until ten years later when Hans Bodlaender [3] reinvented the game in a more general setting of graphs. He formally defined the *game chromatic number* $\chi_g(G)$ of a graph G as the minimum number of colours guaranteeing a win for Alice and made a conjecture, without proposing any concrete number, that it is bounded for planar graphs. Since then things started to run faster. First it was established that four colours are sufficient but also necessary for trees. However, the method used was not sufficient for the general case. A major breakthrough came when Hal Kierstead and Tom Trotter [10] applied a more powerful technique based on *arrangeability* – a parameter related to specific vertex orderings, used earlier in graph Ramsey theory. They proved that if a graph is a-arrangeable (whatever this means) and k-colourable (in the usual sense) then $\chi_g(G) \leq 2ka+1$, . Since it was known at the time that planar graphs are 761-arrangeable, the danger of infinity was averted. Actually, by drastically improving existing bounds, they proved that $\chi_g(G) \leq 33$ for any planar graph G. The method of Kierstead and Trotter also worked for graphs embeddable on surfaces of every arbitrary genus.

The next achievement was made after a few years by Thomas Dinski and Xuding Zhu [5]. Not only did they improve the previous bound with a much simpler argument but they also found a striking relation to another important graph parameter. The *acyclic colouring* of a graph G is a proper colouring of the vertices of G such that no two colour classes span a cycle. In 1979, Oleg Borodin [4], solving a problem posed by Branko Grünbaum [9], proved that every planar graph has an acyclic colouring using at most five colours, which is the best possible. The result of Dinski and Zhu asserts that $\chi_g(G) \leq k(k+1)$ for any acyclically k-colourable graph G. By the result of Borodin this gives a bound of 30 for planar graphs.

Recently, Kierstead found a proof from the book of this fact that goes as follows (see [2]). Fix a partition of the vertex set into k subsets $V_1,\ldots,V_k$ corresponding to colour classes in any acyclic colouring of G. So, the edges between any pair V_i, V_j form a collection of trees. Fix a root in each tree and orient the edges towards it. Notice that every vertex has at most one out-neighbour in a set V_i. Now assume we have k disjoint sets of colours $C_1,\ldots,C_k$, each set C_i consisting of $k+1$ different *shades* of colour i. A shade of colour i is a *correct* colour for a vertex from the set V_i. We also say that a vertex $v \in V_i$ is *dangerous* if it is yet uncoloured but some vertex u dominating v has been coloured by a shade of colour i. Notice that correct colouring cannot create new dangerous vertices, while incorrect colouring of a vertex may create at most one new dangerous vertex. Alice's strategy is now really simple.

If there are no dangerous vertices she colours any vertex with a correct colour. If Bob creates a dangerous vertex, say $v \in V_i$, she kills it immediately in her next move by colouring v with a correct colour. This is always possible since there are $k+1$ shades of colour i available, while at most k neighbours of v coloured with a shade of i. Indeed, there are at most $k-1$ out-neighbours of v in total, and only one in-neighbour that has just been incorrectly coloured by Bob. This completes the proof.

Further progress was made by modifying the game so that it became even more difficult for Alice. Suppose Alice is a full daltonist and she just cannot distinguish any colours. While playing the game she only points on a vertex v but Bob colours it in her name. The rules for his own moves remain unchanged. Surprisingly enough Zhu [12] proved that Alice can win on planar graphs if only 19 colours are available. Soon it was reduced to 18 by Kierstead [11] but the current record of 17 went back to Zhu [13]. All of these bounds were obtained by a sophisticated technique called *activation strategy*. Meanwhile, a lower bound in the original version of the game was lifted to 8, and that is all that is known so far.

Let us say a few words about the origin of the map colouring game. Since Brams is a political scientist, we suspected for a while that perhaps his personal motivation came from political games. Indeed, it often happens (at least recently in Poland) that one of the parties in a coalition pretends cooperation but at the same time perfidiously tends to a fiasco of a joint undertaking. The map colouring game could have served, as we speculated, as an abstract model of such situations. However, this idea was entirely fallacious – as we learned from Brams, his primary aim was to create a new approach to the original map colouring problem, with the hope of finding a computer-free proof of the Four Colour Theorem. Heuristically, if we could somehow determine for how many more colours Bob is responsible, then perhaps we could deduce something for the "one player" case. Here one should mention that there are graphs for which the game chromatic number is arbitrarily far from the usual chromatic number.

Lost in translation

This note is based on the article that previously appeared in a Polish popular science magazine *Delta* [1]. The names of the players (Jacek and Placek) were borrowed from a popular children's novel by Kornel Makuszyński. In 1962, a movie was made, where the main characters were played by two young amateur twins. In 2005, these twins became well-known as major political players. We are not quite sure they are aware of the map colouring game but the fact is that shortly after our publication, Jacek won presidential elections, while a party led by Placek won parliamentary elections. Soon Placek became Prime Minister of the Polish Government but lost this position in 2007 after precipitate elections.

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Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

L. Lovász (current president of the IMU) and K. Vesztergombi, *Discrete Mathematics, Yale University, 1999*, have written: "There are many success stories of applied mathematics outside calculus. A recent hot topic is mathematical cryptography, which is based on number theory (the study of positive integers $1,2,3,\dots$), and is widely applied, among others, in computer security and electronic banking. Other important areas in applied mathematics include linear programming, coding theory, theory of computing. The mathematics in these applications is collectively called *Discrete Mathematics*." ('Discrete' here is used as the opposite of 'continuous'; it is also often used in the more restrictive sense of 'finite'.)

In addition to the text above, we could add to the content of discrete mathematics with algorithms, network theory, enumerative combinatorics, algebraic combinatorics, combinatorial optimization, discrete algebra and geometry, and discrete probability theory. Subjects of discrete mathematics are fundamental in solving problems involving computer algorithms and programming languages. For example, graph theory has been applied for the determination of the 'near' optimal way to route and schedule airplanes.

Terence Tao (Fields Medallist, 2006) has asked: "What is the best way to separate structure from randomness in combinatorial problems?" and has claimed: "The combinatorial approach to number theory promises to offer a rather new set of tools (including ergodic theory!) to the other methods we currently have in analytic number theory".

Over the past few years, we have been witnessing an increasing interest in the interplay between probabilistic ideas and deterministic ones, which has opened new avenues of theory and applications.

It should be noted that discrete mathematics has been fuelled by the computer revolution and rekindled by an ongoing interest in number theory, most notably the Riemann Hypothesis, as well as the Goldbach Conjecture.

In all the above, let's not forget that the mathematical logician Alan Turing is by many considered to be the father of modern computer science and the most important inventor of the modern computer, which as we know has revolutionized society. Thus, the predominant theoretical branch in mathematics, mathematical logic, resulted in the most applied tool of our civilization.

The digital world is still ahead of us.

I. Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

35. Determine a sequence of positive integers $(a_n)_{n\geq 1}$ for which

$$a_n^2 - a_{n-1}\cdot a_{n+1} = 1$$

and $a_4 = 4$.

(V. Copil and L. Panaitopol, University of Bucharest, Romania)

36. Let p_n be the n-th prime number, $\alpha > 1$ and $c > 0$. If the sum $\sum'$ refers to the indices n for which

$$p_{n+1} - p_n > c\log^{\alpha} p_n\,,$$

prove that $\sum' \frac{1}{p_n}$ converges.

(V. Copil and L. Panaitopol, University of Bucharest, Romania)

37. Show that, for any $n \in \mathbb{N}$ and any sequence of (decimal) digits $x_0, x_1, \dots, x_{n-1}$ in $\{0,1,\dots,9\}$, there exists an $m \in \mathbb{N}$ such that the first n decimal digits of the power 2^m are, from left to right, $x_{n-1}x_{n-2}\dots x_1x_0$. As an example, the power of 2 beginning with the digits 409 is $2^{12} = 4096$.

(Proposed by K. Drakakis, University College Dublin, Ireland)

38. Consider the sequences $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}, (c_n)_{n\geq 1}, (d_n)_{n\geq 1}$ defined by $a_1 = 0, b_1 = 1, c_1 = 1, d_1 = 0$ and $a_{n+1} = 2b_n + 3c_n, b_{n+1} = a_n + 3d_n, c_{n+1} = a_n + 2d_n, d_{n+1} = b_n + c_n,\ n \geq 1$. Find a closed formula for the general term of these sequences.

(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania)

39. Let $f : \mathbb{N}\times\mathbb{N} \to \mathbb{N}$ be a function such that $f(1,1) = 2$,

$$f(m+1,n) = f(m,n) + m \quad \text{and} \quad f(m,n+1) = f(m,n) - n$$

for all $m,n \in \mathbb{N}$, where $\mathbb{N} = \{1,2,3,\dots\}$.
Find all pairs (p,q) such that $f(p,q) = 2001$.

(Titu Andreescu, University of Texas, Dallas, USA)

40. (a) For any non-negative double sequences $\{a_{mn}\}_{m,n=1}^{\infty}$ and $\{b_{mn}\}_{m,n=1}^{\infty}$, and for any $c \geq 0$, if

$$a_{mn} \leq c + \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij}a_{ij} \quad \text{for all } m,n,$$

prove that

$$a_{mn} \leq c\cdot\exp B_{mn},$$

where

$$B_{mn} := \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij}.$$

(b) As usual, we denote

$$\Delta_1 z_{mn} = z_{m+1,n} - z_{mn},$$
$$\Delta_2 z_{mn} = z_{m,n+1} - z_{mn},$$

and consider the boundary value problem

$$\begin{cases}\Delta_{12} z_{mn} = F(m,n,z_{mn}) \\ z_{m,1} = a_m; z_{1,n} = b_n; a_1 = b_1 = 0,\end{cases} \qquad (*)$$

where F is any real-valued function. If

$$\left|F(m,n,v_1) - F(m,n,v_2)\right| \leq b_{mn}|v_1 - v_2|$$

for all $m,n \geq 1$ and all $v_1, v_2 \in \mathbb{R}$, show that $(*)$ has at most one positive solution $\{z_{mn}\}$.

(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

II. Two New Open Problems

41*. Let $a_n \in \mathbb{Z}, n \geq 0$ be a sequence of integers. The formal power series $G_a(z) = \sum_{n\geq 0} a_n z^n$ is the generating series of a_n. If $C \in \mathbb{Z}$ is a constant, we write

$$G(z) >> C \quad \Leftrightarrow \quad a_n \geq C, \quad \forall n \geq 0$$
$$G(z) << C \quad \Leftrightarrow \quad a_n \leq C, \quad \forall n \geq 0.$$

Let a_n be a sequence of integers with $G_a(z) >> 0$. Prove or disprove the following statements:
1. If there is a constant $C > 0$ with $G_a(z)^m << C$ for all $m > 0$, then $G_a(z) = z^k$ for some $k \geq 0$.
2. If there are constants C, D such that

$$G_a(z) << C \quad \text{and} \quad G_a(z)^m << D,$$

where $m > 0$, then there are infinitely many indices $n_i \in \mathbb{N}, i \geq 0$, such that $a_{n_i} = 0$ for all $i \geq 0$.
3. If $G_a(z) << 1$ and $G_a^2(z) << C$ then there are infinitely many indices $n_i \in \mathbb{N}, i \geq 0$, such that $a_{n_i} = 0$ for all $i \geq 0$.
4^+. Let $G_a(z) = \sum_{n=1}^{\infty} z^{n^2}$. Is there a constant C such that $G_a^2(z) << C$?
5^{++}. Let $G_a(z)$ be like in question 3 and

$$Z_a(N) = \{n \leq N : a_n = 0\} \quad Z(N) = |Z_a(N)|,$$
$$U_a(N) = \{n \leq N : a_n = 1\} \quad U(N) = |U_a(N)|.$$

Investigate best estimates for the asymptotic orders of magnitude $O(U_a(N)), O(Z_a(N))$ when $N \to \infty$.

Note: question 1 has a quite simple solution and question 2 is related to question 3, which is a research problem proposed in chapter 7 of R. Graham, D. Knuth and O. Patashnik, *Concrete Mathematics*, Addison-Wesley (1989). Questions 4^+ and 5^{++} are deeper open problems related to question 3 and have no solution known to the proposer.

(Preda Mihăilescu, University of Göttingen, Germany)

42*. Let $f : [n] \to [n]$, where $[n] := \{1, 2, \ldots, n\}$, be a permutation with the extra property that

$$\forall i, j, k \text{ such that } 1 \leq i, j, i+k, j+k \leq n:$$
$$f(i+k) - f(i) = f(j+k) - f(j) \Rightarrow i = j \text{ or } k = 0.$$

For example, for $n = 4$, the permutation 1243 has this property. Prove or disprove the statement that such permutations exist for all $n \in \mathbb{N}$, $\mathbb{N} = \{1, 2, 3, \ldots\}$.

(Proposed by K. Drakakis, University College Dublin, Ireland)

III. SOLUTIONS

26. Determine all positive integers n such that all integers consisting of $n-1$ digits of 1 and 1 digit of 7 are prime.

(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

Solution by the proposer. Suppose N is such an integer. Then we may write

$$N = A_n + 6 \times 10^k,$$

where A_n is the n-digit number consisting of all 1s and $0 \leq k \leq n$.

Observe first that if $3|n$ then $3|A_n$ and so $3|N$. Thus assume 3 does not divide n. Write

$$A_n \equiv r \pmod 7.$$

It is easy to check that

$$10^0 \equiv 1, \quad 10^1 \equiv 3, \quad 10^2 \equiv 2,$$
$$10^3 \equiv 6, \quad 10^4 \equiv 4, \quad 10^5 \equiv 5 \pmod 7$$

and thus

$$A_1 \equiv 1, A_2 \equiv 4, A_3 \equiv 6, A_4 \equiv 5, A_5 \equiv 2, A_6 \equiv 0 \pmod 7.$$

Hence $A_n \equiv 0$ only when $6|n$. But then since 3 does not divide n, we have

$$A_n \equiv r \not\equiv 0 \pmod 7.$$

For $n \geq 6$, it is clear that there exists an integer k with $0 \leq k \leq 5$ such that

$$6 \times 10^k \equiv -10^k \equiv 7 - r \pmod 7,$$

thus

$$N = A_n + 6 \times 10^k \equiv r + (7 - r) \equiv 0 \pmod 7.$$

Therefore, the only possible values of n would be

$$n = 1, 2, 4, 5.$$

When $n = 1$, the only possible integer N is $N = 7$, which is a prime number.
When $n = 2$, $N = 17$ or 71, both prime numbers.
When $n = 4$, $1711 = 29 \times 59$, which is not prime.
When $n = 5$, $11711 = 7 \times 1673$, which is not prime.
Thus $n = 1, 2$. □

Also solved by Wolfgang Fensch (Karlsruhe, Germany), Reinhold Kainhofer (Vienna University of Technology, Austria), Maohua Le (Zhanjiang Normal College, P.R. China) and Yaming Yu (University of California, Irvine, USA)

27. Find all triples (x, y, z) of strictly positive integers such that

$$x^3 + y^3 + z^3 - 3xyz = p,$$

where p is a prime number greater than 3.
(Titu Andreescu, University of Texas, Dallas, USA, and Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania)

Solution by the proposers. The equation is equivalent to

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx) = p.$$

Since $x+y+z > 1$, we must have $x+y+z = p$ and $x^2+y^2+z^2-xy-yz-zx = 1$. The last equation is equivalent to $(x-y)^2+(y-z)^2+(z-x)^2 = 2$. Without loss of generality, we may assume that $x \geq y \geq z$. If $x > y > z$, we have $x-y \geq 1$, $y-z \geq 1$ and $x-z \geq 2$, implying $(x-y)^2+(y-z)^2+(z-x)^2 \geq 6 > 2$.

Therefore we must have $x = y = z+1$ or $x-1 = y = z$. The prime number p has one of the forms $3k+1$ or $3k+2$. In the first case the solutions are

$$\left(\frac{p+2}{3}, \frac{p-1}{3}, \frac{p-1}{3}\right)$$

and the corresponding permutations. In the second case the solutions are

$$\left(\frac{p+1}{3}, \frac{p+1}{3}, \frac{p-2}{3}\right)$$

and the corresponding permutations. □

Remark. For $p = 3$ the equation does not have solutions in the positive integers.

Also solved by Mihai Cipu (Institute of Mathematics, Romanian Academy), Con Amore Problem Group (Copenhagen, Denmark), Wolfgang Fensch (Karlsruhe, Germany), Reinhold Kainhofer (Vienna University of Technology, Austria), Maohua Le (Zhanjiang Normal College, P.R. China), Yaming Yu (University of California, Irvine, USA) and K. Zelator and O. Furdui (University of Toledo, USA).

28. Let n be a positive integer greater than 3 such that $k^{n-1} \equiv 1 \pmod{n}$ for $k = 3, 4, \ldots, n-1$. Prove that

$$(n-1)! + 1 \equiv (-1)^{n-1} \frac{(n-1)(n-2)}{2} (2^{n-1} - 1) \pmod{n}.$$

(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania)

Solution by Reinhold Kainhofer (Vienna University of Technology, Austria). We will show that the assumptions of the problem can only be satisfied by prime numbers and then prove the assertion in that case.

Firstly, observe that $n = 4$ is not possible because $3^3 = 27 \equiv 3 \pmod{4}$. Also, n cannot be an even number greater than 4 because 4^{n-1} is even and thus not congruent to 1 modulo the even n.

Secondly, the case where n is an odd compound number is also not possible: let $p \in \mathbb{P}$, where $\mathbb{P}$ denotes the set of prime numbers with $3 \le p \le n-1$ and $p|n$. »From the assumption we know that $p^{n-1} \equiv 1 \pmod{n}$, i.e. $p^{n-1} = 1 + mn$. Since $p|p^{n-1}$ and $p|n$, we obtain the contradiction that $p|1$.

Finally, notice that any prime $n \in \mathbb{P}$ fulfils the conditions $k^{n-1} \equiv 1 \pmod{n}$ for $k = 3, \ldots, n-1$ due to Fermat's little theorem.

Due to Wilson's theorem, which states that $(p-1)! \equiv -1 \pmod{p}$ iff $p \in \mathbb{P}$, the LHS reduces to

$$(n-1)! + 1 \equiv -1 + 1 \equiv 0 \pmod{n}$$

Since the RHS also reduces to

$$\underbrace{(-1)^{n-1}}_{\substack{\equiv 1,\text{ since}\\ n \text{ is odd}}} \frac{n-1}{2} \underbrace{(n-2)}_{\equiv -2} \left(2^{n-1} - 1\right)$$

$$\equiv \underbrace{\frac{1-n}{2} \cdot 2}_{\equiv 1-n \equiv 1} \underbrace{\left(2^{n-1} - 1\right)}_{\substack{\equiv 0 \text{ due to}\\ \text{Euler-Fermat}}} \equiv 0 \pmod{n}$$

the problem is proved. □

Also solved by Mihai Cipu (Institute of Mathematics, Romanian Academy), Wolfgang Fensch (Karlsruhe, Germany) and the proposer.

29. Let A_n be the number of positive integers N having the property that all digits in N are chosen from the collection $\{1, 3, 4\}$ and the sum of digits equals n. Show that A_{2n} is a complete square.

(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

Solution by the proposer. Write N in decimal representation as $N = a_1 a_2 \cdots a_k$, where

$$a_i \in \{1, 3, 4\}, \quad i = 1, \ldots, k \quad \text{and} \quad \sum_{i=1}^{k} a_i = n.$$

Observe that for $n > 4$

$$\sum_{i=2}^{k} a_i = n-1 \quad \text{or} \quad n-3 \quad \text{or} \quad n-4.$$

Thus

$$A_n = A_{n-1} + A_{n-3} + A_{n-4} \quad \forall\, n > 4.$$

By elementary consideration, it is easily seen that

$$A_2 = 1, \quad A_4 = 2^2, \quad A_6 = 3^2.$$

We claim that for any $n \ge 3$

$$A_{2n} = \left(\sqrt{A_{2n-2}} + \sqrt{A_{2n-4}}\right)^2. \qquad (*)$$

It is clear that $(*)$ holds when $n = 3$. Suppose $(*)$ holds for all $n \le k$. Let $n = k+1$. Then

$$\begin{aligned}
A_{2n} &= A_{2k+2} \\
&= A_{2k+1} + A_{2k-1} + A_{2k-2} \\
&= \left(A_{2k} + A_{2k-2} + A_{2k-3}\right) + A_{2k-1} + A_{2k-2} \\
&= A_{2k} + 2A_{2k-2} + \left(A_{2k-1} + A_{2k-3} + A_{2k-4}\right) - A_{2k-4} \\
&= 2\left(A_{2k} + A_{2k-2}\right) - A_{2k-4} \\
&= 2\left[\left(\sqrt{A_{2k-2}} + \sqrt{A_{2k-4}}\right)^2 + A_{2k-2}\right] - A_{2k-4} \\
&= 4A_{2k-2} + A_{2k-4} + 4\sqrt{A_{2k-2}}\sqrt{A_{2k-4}} \\
&= \left(2\sqrt{A_{2k-2}} + \sqrt{A_{2k-4}}\right)^2 \\
&= \left[\left(\sqrt{A_{2k-2}} + \sqrt{A_{2k-4}}\right) + \sqrt{A_{2k-2}}\right]^2 \\
&= \left(\sqrt{A_{2k}} + \sqrt{A_{2k-2}}\right)^2.
\end{aligned}$$

Hence $(*)$ holds for all $n \ge 3$. □

Also solved by Reinhold Kainhofer (Vienna University of Technology, Austria) and Yaming Yu (University of California, Irvine, USA).

Remark. Yaming Yu (University of California, Irvine, USA) provided another solution by using generating functions and Reinhold Kainhofer (Vienna University of Technology, Austria) showed that the number A_{2n} is the square of the $(n+1)$-th Fibonacci number.

30. Let x, y, z be integers such that

$$\frac{x^2 - 1}{y(y+1)} = z^2. \qquad (1)$$

Prove that either $(2y+1)|x$ or $(2y+1)|z$.

(L. Panaitopol, University of Bucharest, Romania)

Solution by the proposer. The given relation implies $y \in \mathbb{Z} \setminus \{-1, 0\}$. We may then write equation (1) in the form $x^2 - y(y+1)z^2 = 1$. Let us denote $d = y(y+1)$. Obviously, d is not a perfect square. We therefore face the Pell equation $x^2 - dz^2 = 1$.

As is well-known, this Pell equation always has non-trivial solutions given by $(\pm x_n, \pm z_n)$, where $x_n + z_n\sqrt{d} = (x_0 + z_0\sqrt{d})^n$, $n \in \mathbb{N}$, (x_0, z_0) being the solution in positive integers with minimum $z_0 > 0$.

The minimal solution of our equation is clearly $x_1 = 2y+1$, $z_1 = 2$. Since $x_{n+1} + z_{n+1}\sqrt{d} = (x_0 + z_0\sqrt{d})(x_n + z_n\sqrt{d})$, the general solution will verify relations of the type

$$\begin{aligned}
x_{n+1} &= \pm(2y+1)x_n \pm 2dz_n, \;\; n \ge 1, \\
z_{n+1} &= \pm(2y+1)z_n \pm 2x_n, \;\; n \ge 1.
\end{aligned}$$

We derive $(2y+1)|z_2, (2y+1)|x_3$ and so on. □

Also solved by Mihai Cipu (Institute of Mathematics, Romanian Academy), Wolfgang Fensch (Karlsruhe, Germany), Maohua Le (Zhanjiang Normal College, P.R.China) and Yaming Yu (University of California, Irvine, USA).

31. Find the values $a \in \mathbb{N}$ for which there exist $n \in \mathbb{N}$, $n \ge 232$, such that a^n has $2n+1$ digits.

(L. Panaitopol, University of Bucharest, Romania)

Solution by the proposer. For a as in the statement, we have the inequalities $10^{2n} \leq a^n \leq 10^{2n+1}$. Thus $a \geq 100$. Supposing $a \geq 101$, we derive $10^{2n+1} > 101^n$, so

$$n < 1/(log_{10}101 - 2) < 231.5 < 232\,,$$

a contradiction. Therefore $a < 101$ and hence $a = 100$, which obviously satisfies the required property for every $n \in \mathbb{N}$. □

Remark. Reinhold Kainhofer (Vienna University of Technology, Austria) as well as K. Zelator and O. Furdui (University of Toledo, USA) observed that if $n = 231$ is allowed, then $a = 101$, $n = 231$ is also a solution (101^{231} has exactly 463 digits). Similarly, by lowering n even further, the number of possible solutions increases until at $n = 1$ any three-digit integer a is allowed.

Also solved by Mihai Cipu (Institute of Mathematics, Romanian Academy), Wolfgang Fensch (Karlsruhe, Germany), Reinhold Kainhofer (Vienna University of Technology, Austria), Maohua Le (Zhanjiang Normal College, P.R.China) and K. Zelator and O. Furdui (University of Toledo, USA).

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will also be devoted to *discrete mathematics*.

The ERC in its second year

About a year and a half ago, I described in these pages the aims and principles of the newly created *European Research Council*. Time is flying fast and it is appropriate to recall briefly what has been achieved during the opening period of the ERC and what the challenges and dangers are ahead.

The first Starting Grant call showed that the need for such an all-European basic research agency was strong. To be frank, the flood of applications – in total we received more than 9000 of them – almost crushed the system and we have to thank many distinguished colleagues who were able, at rather short notice, to come to the rescue and help us with the evaluations. In mathematics alone there were 472 applications, scores of them of a really exceptional quality.

Being well aware that the budget does not allow us to accept many good projects, the panels selected about 500 "semi-finalists" who were invited to Brussels for interviews. Everyone who saw the parade of bright young men and women probably got more than a small dose of optimism about the future of European science. About 300 of them were finally awarded the first *Starting Grants* and at present they are signing the grant agreements. The heavy thoughts about financial restrictions are relieved a bit by generous offers by some national science foundations to step in and support those left "under the line"; more on that can found at the ERC web page *http://erc.europa.eu/.*

Twenty-three young mathematicians won the *Starting Grants*; their names are mentioned elsewhere in this issue. It is not necessary to stress that the European mathematical community is proud of them and wishes them great results.

At the same time we progress with our main programme of *Advanced Grants*, which targets great ideas without any age or other restriction; our budget for the first AdG call is slightly over €500 million. This time the deadlines were spread between February and April for the three topical domains. We are naturally learning on the way and with last year's experience in mind we made the project requirements stricter; as a result we have received a smaller number of applications, in total something over 2000. Of these, 135 came from mathematicians, the largest number among all the disciplines.

The panels have already done most of their work and the results will be announced soon; the information I have shows that the level of the projects likely to be chosen in most panels, including mathematics, is again very good.

Comparing to the *Starting Grants*, the maximum size of a grant was enlarged to 7th €2.5 million over five years, in exceptional cases to €3.5 million. It applies, however, to projects with costly experimental equipment; mathematical grants are expected to be closer to the lower bound of an annual €100 thousand. It is also worth mentioning that 13% of the budget for the *Advanced Grants* is reserved for interdisciplinary projects.

All this is achieved through the dedicated efforts of many people. This includes panel members, who play the most responsible role by evaluating the projects, as well as the external referees. No less appreciated is the work of our administrative staff, who we hope will be the flesh and blood of the crystallizing ERC Executive Agency. In short, everybody who contributes to making the ERC a well functioning organism deserves hearty thanks.

Unfortunately, there are also things that inspire a less optimistic point of view. The transformation of our temporary administrative structure functioning under the aegis of the European Commission into a fully-fledged executive agency (supposed to be completed by the time of the midterm evaluation prescribed by the 7th Framework Programme in about a year from now) does not proceed satisfactorily.

It includes numerous administrative acts that have to be processed by the Commission apparatus. The agency needs a new director and staff, naturally more numerous than the provisional one we have had so far – common sense suggests using people who have already proven themselves to be dedicated and efficient. In addition, the facilities we use and our software equipment need to be upgraded. The pace at which these transformations are proceeding at the moment does not guarantee that we will be able to fulfil our next goal of running both the Starting and Advanced Grant calls in 2009.

Moral support from our colleagues is now more important than ever. If you agree that our programmes are going in the needed direction, please do not stay silent. European politicians must hear loud and clear from the community that proper functioning of the ERC is vital for a future flourishing of European science.

Pavel Exner
EMS Vice President and an
ERC Scientific Council Member

The first ERC Starting Grants awarded

The European Research Council has awarded its first Starting Grants targeting young researchers, 2–9 years after a PhD. In the field of mathematics the grants were awarded to:

1. Professor Dimitris Achlioptas (EL, Research Academic Computer Technology Institute): Rigorous Mathematical Connections between the Theory of Computations and Statistical Physics
http://www.cse.ucsc.edu/~optas/
2. Professor Massimiliano Berti (IT, Dipartimento di Matematica e Applicazioni "R. Cacciappoli"): Hamiltonian Partial Differential Equations – new connections between dynamical systems and PDEs with small divisors phenomena.
http://www.dma.unina.it/~berti/
3. Dr Emmanuel Breuillard (FR, Ecole Polytechnique): Group Actions: Interactions between Dynamical Systems and Arithmetic
http://www.math.polytechnique.fr/~breuilla/
4. Dr Annalisa Buffa (IT, Consiglio Nazionale delle Ricerche): Innovative compatible discretization techniques for Partial Differential Equations
http://www.imati.cnr.it/~annalisa/
5. Dr Diego Cordoba (ES, Consejo Superior de Investigaciones Cientificas): Contour dynamics and singularities in incompressible flows
http://www.mat.csic.es/fichapersonal.php?id=15
6. Dr Mihalis Dafermos (UK, The Chancellor Master and Scholars of the University of Cambridge): Mathematical Problems in General Relativity
http://www.damtp.cam.ac.uk/user/gr/about/members/dafermos.html
7. Dr Javier Fernandez de Bobadilla (ES, Consejo Superior de Investigaciones Cientificas): Topological, Geometric and Analytical Study of Singularities
http://www.mat.csic.es/fichapersonal.php?id=34
8. Dr Tsachik Gelander (IL, The Hebrew University of Jerusalem): Independence of Group Elements and Geometric Rigidity
http://www.ma.huji.ac.il/~gelander/
9. Dr David Holcman (FR, Ecole Normale Superieure): Modeling cytoplasmic trafficking and molecular delivery in cellular microdomains
http://www.wisdom.weizmann.ac.il/math/profile04/scientists/holcman-prof04.html
10. Dr Alan George Beattie Lauder (UK, The Chancellor Master and Scholars of the University of Cambridge): Effective methods in rigid and crystalline cohomology
http://www.genealogy.math.ndsu.nodak.edu/id.php?id=83123
11. Professor Shahar Mendelson (IL, Technion – Israel Institute of Technology): Asymptotic Geometric Analysis and Learning Theory
http://wwwmaths.anu.edu.au/~mendelso/
12. Professor Giuseppe Mingione (IT, Universita degli Studi di Parma): Vectorial Elliptic, Parabolic and Variational Problems – Singularities and Regularity
http://cvgmt.sns.it/people/mingione/
13. Dr Gilles Schaeffer (FR, Centre National de la Recherche Scientifique): Combinatorial methods, from enumerative topology to random discrete structures and compact data representations
http://www.lix.polytechnique.fr/~schaeffe/
14. Dr Ivan Smith (UK, The Chancellor Master and Scholars of the University of Cambridge): Towards symplectic Teichmueller theory
http://www.dpmms.cam.ac.uk/site2002/People/smith_i.html
15. Professor Stefaan Vaes (BE, Katolieke Universiteit Leuven): Von Neumann algebras, group actions and discrete quantum groups
http://wis.kuleuven.be/analyse/stefaan/
16. Professor Ingrid Van Keilegom (BE, Universite Catholique de Louvain): M- and Z-estimation in semiparametric statistics: applications in various fields
http://www.stat.ucl.ac.be/ISpersonnel/vankeile/vankeile.html
17. Professor Otmar Venjakob (DE, Ruprecht-Karls-Universitaet Heidelberg): Iwasawa theory of p-adic Lie extensions
http://www.mathematics.uni-bonn.de/faculty/Venjakob/
18. Dr Johan Hoffman (SE, Kungliga Tekniska Hogskolan): New Adaptive Computational Methods for Fluid-Structure Interaction using a Unified Continuum Formulation with Applications in Biology, Medicine and Industry
http://www.nada.kth.se/~jhoffman/
19. Dr Anna Marciniak-Czochra (DE, Ruprecht-Karls-Universitaet Heidelberg): Multiscale mathematical modelling of dynamics of structure formation in cell systems
http://www.iwr.uni-heidelberg.de/groups/amj/People/Anna.Marciniak/research.html

20. Dr Ulisse Stefanelli (IT, Consiglio Nazionale delle Ricerche): Mathematics for Shape Memory Technologies in Biomechanics
http://www.imati.cnr.it/~ulisse/
21. Professor Katrin Wendland (DE, University of Augsburg): The geometry of topological quantum field theories
http://www.math.uni-augsburg.de/de/prof/geo/mitarbeiter/wendland/
22. Professor Martin Zerner (DE, Eberhard Karls Universitaet Tuebingen): Non-classical interacting random walks
http://www.mathematik.uni-tuebingen.de/~zerner/
23. Dr Soeren Fournais (DK, Aarhus Universiet): Mathematical Problems in Superconductivity and Bose-Einstein Condensation
http://www.math.u-psud.fr/~fournais/

The EMS President Ari Laptev sends personal congratulations to each of the grant winners.

Interview with ERC Starting Grant recipient Javier Fernández de Bobadilla

Conducted by Vicente Muñoz (Madrid, Spain)
Madrid, 22 July 2008

Javier Fernández de Bobadilla in Obergurgl (Austria).

Javier Fernández de Bobadilla is one of the candidates who obtained an ERC Starting Grant. He is working at the Consejo Superior de Investigaciones Científicas, Madrid, Spain, in the recently created Instituto de Ciencias Matemáticas. His area of research is singularity theory.

Muñoz: First of all, I would like to congratulate you and the other candidates who have obtained an ERC Starting grant. How do you feel about this?
Fernandez de Bobadilla: Being awarded with one of these grants motivates you to work even harder, to keep up with the high expectations that are put on you. On the other hand, it changes your way of working. From now on, I will be able to form a group with post-docs and students. This gives a fantastic opportunity to achieve better results.

On the practical side, this grant allows you to have a competitive salary, mainly in countries like Spain, where the salaries of researchers and professors are fixed by law and are not very high.

In general, applications for grants are a very complicated and lengthy process. This has the effect that many good candidates do not apply in the end. How is it that you decided to apply for this grant?
Actually this situation is absolutely not the case with the ERC Starting Grants. It is easy to apply for these grants. Perhaps this encouraged many candidates to apply for them, which had the effect that there was a very competitive selection. The opportunity was very open to all researchers willing to work in Europe – in particular mathematicians.

Could you explain the process in a little bit more detail?
The ERC grants work with a two stage process, and this makes it easier for candidates to apply for. In the first stage, candidates are required to write a short application. Moreover, the main bulk of work in the application consisted of writing the scientific project, and the bureaucratic part of the application is very small. And I would like to point out that the Internet system through which we had to do the application worked really smoothly.

After passing to the second stage, we were required to write a longer scientific project, giving more details. In my case, I initially wrote a project, which turned out to be very long for the initial application, so I had to cut it down. When I had to submit the second stage application, my initial project was already suitable for this.

I suppose all this process takes some time.
Unfortunately, the evaluation process is very long. From the open call until signing the contract, it takes almost a year and a half.

Numbers are impressive; less than 10% pass from the first to the second stage, and then around half of the candidates in the second stage obtain the grant. This probably guarantees prestige for a researcher with an awarded ERC grant.
Yes, I have noticed the prestige that candidates who have obtained the grant have gained. Actually, I have been

surprised by the high impact of this, which I was not expecting as much. I suppose this is also prestige for the ERC grant system itself.

But also candidates passing from the first to the second stage have got a lot of recognition, even without being successful in the end. Probably most of them will have a better opportunity when applying for grants from their national funding institutions.

What makes this grant special? Is there any deep difference between this grant and other ones? You may compare with grants that you have enjoyed previously.
The main difference is the tremendous flexibility of the ERC grants. I received a VENI grant from the NWO as a post-doc in the Netherlands after completing my PhD, which allowed me to use a good amount of money for my own research but did not allow me to make contracts for post-docs or give grants to pre-doc students. Later I returned to Spain with a Ramón y Cajal contract. This goes with a grant with a very small extra allowance. However, ERC grants give a good deal of money, which allows you to make post-doc contracts. These grants give the maximum flexibility, not putting any restrictions except those imposed by the national institutions through which the grant is paid to the awarded candidates. The idea is that with an ERC grant one should be able to settle and lead one's own research group.

You have mentioned that the ERC grant is very generous and I am getting rather curious. If I may ask, how much are you actually getting?
In pure mathematics the amount of money received is obviously not as high as in applied disciplines where laboratories are necessary, which requires much more funding. Just to satisfy your curiosity, I receive over 600 thousand euros for the five years of the duration of the grant. This money is being used for two post-doc three-year contracts and to buy all the computer equipment required for the team, and the rest will be used for trips, conference attendance, inviting collaborators for short stays in Madrid and the organization of seminars.

Are there any aspects that you feel could improve the functioning of ERC grants?
It is a pity that after all the flexibility allowed by Brussels for the use of the money of the ERC grants, the occasionally strict bureaucratic systems that the different national institutions impose for the use of this money diminish this flexibility. This sometimes puts difficulties on the use of the resources and therefore slows down the research undertaken. One wonders whether it is possible that the system could be a little bit more independent of national institutions.

And what about your future?
Now I want to enjoy doing maths and working on the project for which I have been awarded this grant.

Good luck!
Thank you.

Interview with the winners of the Abel Prize 2008: John G. Thompson and Jacques Tits

Martin Raussen (Aalborg, Denmark) and Christian Skau (Trondheim, Norway), Oslo, 19 May 2008

From left to right: Christian Skau, Martin Raussen, Jacques Tits, John G. Thompson

Early experiences

Raussen & Skau: On behalf of the Norwegian, Danish and European Mathematical Societies we want to congratulate you for having been selected as Abel Prize winners for 2008. In our first question we would like to ask you when you first got interested in mathematics. Were there any mathematical results or theorems that made a special impression on you in your childhood or early youth? Did you make any mathematical discoveries during that time that you still remember?

Tits: I learned the rudiments of arithmetic very early; I was able to count as a small child – less than four years, I believe. At the age of thirteen, I was reading mathematical books that I found in my father's library and shortly after I started tutoring youngsters five years older than me who were preparing for the entrance examination at the École Polytechnique in Brussels. That is my first recollection.

At that time I was interested in analysis but later on I became a geometer. Concerning my work in those early years, I certainly cannot talk about great discoveries but I think that some of the results I obtained then are not without interest.

My starting subject in mathematical research has been the study of strictly triple transitive groups; that was the subject essentially given to me by my professor (Paul Libois). The problem was this: we knew axiomatic projective geometry in dimension greater than one. For the one-dimensional case, nobody had given an axiomatic definition. The one-dimensional case corresponds to $PSL(2)$. My teacher gave me the problem of formulating axiomatics for these groups. The idea was to take triple transitivity as the first axiom. So I started with this kind of problem: giving axiomatics of projective geometry based on triple transitivity. Of course, I was then led naturally to consider quadruple and quintuple transitivity. That is how I rediscovered all the Mathieu groups, except, strangely enough, the biggest one, the quintuple transitive. I had to rediscover that one in the literature!

So you didn't know about the Mathieu groups when you did this work?

Tits: No, I didn't.

How old were you at that time?

Tits: 18 years old, I suppose. In fact, I first found all strictly quadruple transitive groups. They were actually known by Camille Jordan. But I didn't know the work of Camille Jordan at the time. I rediscovered that.

You must have been much younger than your fellow students at the time. Was it a problem to adjust in an environment where you were the youngest by far?

Tits: I am very grateful to my fellow students and also to my family. Because I was what is sometimes called a little genius. I was much quicker than all the others. But nobody picked up on that; they just let it go. My father was a little bit afraid that I would go too fast. My mother knew that this was exceptional but she never boasted about it. In fact, a female neighbour said to my mother: "If I had a son like that, I would go around and boast about it". My mother found that silly. I was not at all put on a pedestal.

Hardy once said that mathematics is a young man's game. Do you agree?

Tits: I think that it is true to a certain extent. But there are people who do very deep things at a later age. After all, Chevalley's most important work was done when he was more than 40 years old and perhaps even later. It is not an absolute rule. People like to state such rules. I don't like them really.

Thompson: Well, it is true that you don't have any childhood geniuses in politics. But in chess, music and mathematics, there is room for childhood exceptionalism to come forth. This is certainly very obvious in the case of music and chess and to some extent in mathematics. That might sort of skew the books in a certain direction.

As far as Hardy's remark is concerned, I don't know what he was feeling about himself at the time he made that remark. It could be a way for a person to say: "I am checking out now; I have reached the age where I don't want to carry on". I don't know what the sociologists and psychologists say; I leave it to them. I have seen mathematicians reach the age of 50 and still be incredibly lively. I don't see it as a hard and fast rule. But then Tits and I are really in no position to talk given our age.

John von Neumann said, exaggerating a little, that whatever you do in mathematics beyond 30 is not worth anything, at least not compared to what you had done before 30. But when he himself reached the age of 30, he pushed this limit. Experience comes in, etc…

Thompson: But he was a prodigy. So he knows the childhood side of it.

Tits: We all have known very young and bright mathematicians. The point is that to find deep mathematics, it is not necessary to have all the techniques. They can find results that are deep without having all of those techniques at hand.

What about your memories on early mathematical experiences, Professor Thompson?

Thompson: I don't have any particularly strong memories. I have an older brother, three years older than me, who was very good at mathematics. He was instrumental, I guess, in interesting me in very elementary things. He was obviously more advanced than I was.

We also played cards in our family. I liked the combinatorics in card play. At that time, I was 10 or 12 years old. I also liked playing chess. I never got any good at it but I liked it. When my brother went to the university, he learned about calculus and he tried to explain it to me. I found it totally incomprehensible but it intrigued me, though. I did get books out of the library myself. But I didn't make too much progress without him.

Early group theory

You have received this year's Abel Prize for your achievements in group theory. Can we start with a short historical introduction to the subject? We would like to ask you to tell us how the notion of a group came up and how it was developed during the 19th century. In fact, Norwegian mathematicians played quite an important role in that game, didn't they?

Tits: Well, when you talk about groups it is natural to talk about Galois. I think Abel did not use groups in his theory – do you know?

Thompson: At least implicitly. I think the equation of the fifth degree comes in there. It was a great eye opener. I myself looked at a very well-known paper of Lagrange, I think around 1770 – before the French revolution. He examined equations and he also said something about equations of degree five. He was definitely getting close to the notion of a group. I don't know about the actual formal definition. I guess we have to attribute it to Galois. Anyway, it was certainly he that came up with the notion of a normal subgroup – I am pretty sure that was Galois' idea. He came up with the idea of a normal subgroup which is really basic.

Tits: But the theorem on the equation of degree five was discovered first by Abel, I think. Of course Galois had a technique which helped with many equations of different types that Abel did not have. Galois was really basically an algebraist, whereas Abel was also an analyst. When we now talk about Abelian functions – these ideas go back to Abel.

Can you explain why simple groups are so important for the classification of finite groups in general? That realization came about, we guess, with Camille Jordan and his decomposition theorem. Is that correct?

Tits: You see, I think that one of the dreams of these people was always to describe *all* groups. And if you want to describe all groups you decompose them. The factors are then simple. I think that was one of the aims of what they were doing. But of course they didn't go that far. It is only very recently that one could find all finite simple groups, a solution to the problem to which John Thompson contributed in a major way.

What about Sylow and Lie in the beginning of group theory?

Thompson: Those are two other Norwegians.

Tits: Lie played an important role in my career. In fact, practically from the beginning, the main subject of my work has centred around the so-called exceptional Lie groups. So the work of Lie is fundamental in what I have done.

Could you comment on the work of Frobenius and Burnside?

Thompson: Of course. After the last half of the 19th century Frobenius among other things put the theory of group characters on a solid basis. He proved the orthogonality relations and talked about the transfer map. Burnside eventually got on the wagon there. And eventually he proved his p^aq^b-theorem, the two prime theorem, using character theory, namely that groups of these orders are solvable. That was a very nice step forward, I feel. It showed the power of character theory, which Frobenius had already done. Frobenius also studied the character theory of the symmetric groups and multiply transitive permutation groups. I don't know how much he thought of the Mathieu groups. But they were pretty curious objects that had been discovered before character theory. For a while there was quite a bit of interest in multiply transitive permutation groups – quite complicated combinatorial arguments. Burnside and Frobenius were very much in the thick of things at that stage.

Tits: When I was a young mathematician, I was very ignorant of the literature. For instance, I rediscovered a lot of results that were known about multiply transitive groups, in particular on the strictly 4-fold and 5-fold transitive groups. Fortunately, I did this with other methods than the ones that were used before. So these results were in fact new in a certain sense.

Was it a disappointment to discover that these results had been discovered earlier?
Tits: Not too much.

Burnside was also interesting because he posed problems and conjectures that you and others worked on later, right?
Thompson: Right. Well, I sort of got started on working on the Frobenius conjecture, which was still open. I think it was Reinhold Baer or maybe Marshall Hall who told me about the Frobenius conjecture: the Frobenius kernel of the Frobenius group was conjectured to be nilpotent. I liked that conjecture for the following reason: if you take the group of proper motions of the Euclidean plane, it is a geometric fact that every proper motion is either a translation or a rotation. I hope kids are still learning that. It is a curious phenomenon. And the translations form a normal subgroup. So that is something you could actually trace back to antiquity.

No doubt Frobenius knew that. So when he proved his theorem about the existence of the normal complement, that was a link back to very old things to be traced in geometry, I feel. That was one of the appeals. And then the attempt to use Sylow's theorems and a bit of character theory, whatever really, to deal with that problem. That is how I first got really gripped by pure mathematics.

Mathieu discovered the first sporadic simple groups, the Mathieu groups, in the 1860s and 1870s. Why do you think we had to wait 100 years before the next sporadic group was found by Janko, after your paper with Feit? Why did it take so long a time?
Thompson: Part of the answer would be the flow of history. The attention of the mathematical community was drawn in other directions. I wouldn't say that group theory, certainly not finite group theory, was really at the centre of mathematical development in the 19th century. For one thing, Riemann came along; topology gained and exerted tremendous influence and, as Jacques has mentioned, analysis was very deep and attracted highly gifted mathematicians. It is true, as you mentioned earlier, that Frobenius was there and Burnside; so group theory wasn't completely in the shadows. But there wasn't a lot going on.

Now, of course, there is a tremendous amount going on, both within pure and applied mathematics. There are many things that can attract people, really. So why there was this gap between these groups that Mathieu found and then the rather rapid development in the last half of the 20th century of the simple groups, including the sporadic groups – I have to leave that to the historians. But I don't find it all that mysterious, really. You know, mathematics is a very big subject.

The Feit-Thompson theorem

The renowned Feit-Thompson theorem – finite groups of odd order are solvable – that you proved in the early 1960s, that was originally a conjecture by Burnside, right?
Thompson: Burnside had something about it, yes. And he actually looked at some particular integers and proved that groups of that order were solvable. So he made a start.

When you and Feit started on this project, were there any particular results preceding your attack on the Burnside conjecture that made you optimistic about being able to prove it?
Thompson: Sure. A wonderful result of Michio Suzuki, the so-called CA theorem. Absolutely basic! Suzuki came to adulthood just at the end of the Second World War. He was raised in Japan. Fortunately, he came to the University of Illinois. I think it was in 1952 that he published this paper on the CA groups of odd order and proved they were solvable by using exceptional character theory. It is not a very long paper. But it is incredibly ingenious, it seems to me. I still really like that paper. I asked him later how he came about it, and he said he thought about it for two years, working quite hard. He finally got it there. That was the opening wedge for Feit and me, really. The wedge got wider and wider.
Tits: Could you tell me what a CA group is?
Thompson: A CA group is a group in which the centralizer of every non-identity element is Abelian. So we can see Abel coming in again. Abelian centralizer – that is what the A means.

The proof that was eventually written down by Feit and you was 255 pages long, and it took one full issue of the Pacific journal to publish.
Thompson: It was long, yes.

It is such a long proof and there were so many threads to connect. Were you nervous that there was a gap in this proof?
Thompson: I guess so, right. It sort of unfolded in what seemed to us a fairly natural way: part group theory, part character theory and this funny little number-theoretic thing at the end. It all seemed to fit together. But we could have made a mistake, really. It has been looked at by a few people since then. I don't lose sleep about it.

It seems that, in particular in finite group theory, there did not exist that many connections to other fields of mathematics like analysis, at least at the time. This required you to develop tools more or less from scratch, using ingenious arguments. Is that one of the reasons why the proofs are so long?
Thompson: That might be. It could also be that proofs can become shorter. I don't know whether that will be the case. I certainly can't see that the existing proofs will become tremendously shorter in my lifetime. These are delicate things that need to be explored.
Tits: You see, there are results that are intrinsically difficult. I would say that this is the case for the Feit-Thompson result. I personally don't believe that the proof will be reduced to scratch.
Thompson: I don't know whether it will or not. I don't think mathematics has reached the end of its tether, really.

Tits: It may of course happen that one can go around these very fine proofs, like John's proof, using big machinery like functional analysis. That one suddenly gets a big machine which crushes the result. That is not completely impossible. But the question is whether it is worth the investment.

The theory of buildings

Professor Tits, you already mentioned Lie groups as a point of departure. Simple Lie groups had already been classified to a large extent at the end of the 19th century, first by Killing and then by Élie Cartan, giving rise to a series of matrix groups and the five exceptional simple Lie groups. For that purpose, the theory of Lie algebras had to be developed. When you started to work on linear algebraic groups, there were not many tools available. Chevalley had done some pioneering work but the picture first became clear when you put it in the framework of buildings, this time associating geometric objects to groups. Could you explain to us the idea of buildings – consisting of apartments, chambers, all of these suggestive words – how it was conceived, what it achieved and why it has proven to be so fruitful?

Tits: First of all, I should say that the terminology like buildings, apartments and so on is not mine. I discovered these things but it was Bourbaki who gave them these names. They wrote about my work and found that my terminology was a shambles. They put it in some order and this is how the notions like apartments and so on arose.

I studied these objects because I wanted to understand these exceptional Lie groups geometrically. In fact, I came to mathematics through projective geometry; what I knew about was projective geometry. In projective geometry you have points, lines and so on. When I started studying exceptional groups I sort of looked for objects of the same sort. For instance, I discovered – or somebody else discovered, actually – that the group E_6, E_7, E_8 is the colineation group of the octonion projective plane. And a little bit later, I found some automatic way of proving such results, starting from the group to reconstruct the projective plane. I could use this procedure to give geometric interpretations of the other exceptional groups, e.g. E_6, E_7, E_8. That was really my starting point.

Then I tried to make an abstract construction of these geometries. In this construction I used terms like skeletons, for instance, and what became apartments were called skeletons at the time. In fact, in one of the volumes of Bourbaki, many of the exercises are based on my early work.

An additional question about buildings. This concept has been so fruitful and made connections to many areas of mathematics that maybe you didn't think of at the time, like rigidity theory for instance?

Tits: For me it was really the geometric interpretations of these mysterious groups, the exceptional groups, that triggered everything. Other people have then used these buildings for their own work. For instance, some analysts have used them. But in the beginning I didn't know about these applications.

Jacques Tits receives the Abel Prize from King Harald. John Griggs Thompson to the left with the prize. To the right: Kristian Seip, chairman of the Abel committee. (Photo: Heiko Junge/Scanpix)

You asked some minutes ago about CA groups. Maybe we can ask you about BN-pairs: what are they and how do they come in when you construct buildings?

Tits: Again, you see, I had an axiomatic approach towards these groups. The BN-pairs were an axiomatic way to prove some general theorems about simple algebraic groups. A BN-pair is a pair of two groups B and N with some simple properties. I noticed that these properties were sufficient to prove, I wouldn't say deep but far-reaching results – for instance, proving the simplicity property. If you have a group with a BN-pair you have simple subgroups free of charge. The notion of BN-pairs arises naturally in the study of split simple Lie groups. Such groups have a distinguished conjugacy class of subgroups, namely the Borel subgroups. These are the Bs of a distinguished class of BN-pairs.

The classification of finite simple groups

We want to ask you, Professor Thompson, about the classification project – the attempt to classify all finite simple groups. Again, the paper by Feit and you in 1962 developed some techniques. Is it fair to say that without that paper the project would not have been doable or even realistic?

Thompson: That I can't say.

Tits: I would say yes.

Thompson: Maybe, but the history has bifurcations so we don't know what could have happened.

The classification theorem for finite simple groups was probably the most monumental collaborative effort done in mathematics, and it was pursued over a long period of time. Many people have been involved; the final proof had 10,000 pages, at least originally. A group of people, originally led by Gorenstein, are still working on making the proof more accessible.

We had an interview here five years ago with the first Abel Prize recipient Jean-Pierre Serre. At that time, he told us that there had been a gap in the proof, which was only about to be filled in at the time of the inter-

view with him. Before, it would have been premature to say that one actually had the proof. The quasi-thin case was left.

How is the situation today? Can we really trust that this theorem finally has been proved?

Thompson: At least that quasi-thin paper has been published now. It is quite a massive work itself (by Michael Aschbacher and Stephen Smith) and quite long, well over 1000 pages. Several of the sporadic simple groups come up. They characterize them because they are needed in quasi-thin groups. I forget which ones come up but the Rudvalis group certainly is among them. It is excruciatingly detailed.

It seems to me that they did an honest piece of work. Whether one can really believe these things is hard to say. It is such a long proof that there might be some basic mistakes. But I sort of see the sweep of it, really. It makes sense to me now. In some way it rounded itself off. I can sort of see why there are probably no more sporadic simple groups, but not really conceptually. There is no conceptual reason that is really satisfactory.

But that's the way the world seems to be put together. So we carry on. I hope people will look at these papers and see what the arguments are and see how they fit together. Gradually this massive piece of work will take its place in the accepted canon of mathematical theorems.

Tits: There are two types of group theorists. Those who are like St. Thomas, they don't believe because they have not seen every detail of the proof. I am not like them and I believe in the final result although I don't know anything about it. The people who work on or who have worked on the classification theorem may of course have forgotten some little detail somewhere. But I don't believe these details will be very important. And I am pretty sure that the final result is correct.

May we ask about the groups that are associated with your names? You have a group that's called the* Thompson *group among the sporadic simple groups. How did it pop up? How were you involved in finding it?

Thompson: That is, in fact, a spin-off from the Monster Group. The so-called Thompson group is essentially the centralizer of an element of order three in the Monster. Conway and Norton and several others were beavering away – this was before Griess constructed the Monster – working on the internal structure where this group came up, along with the Harada–Norton group and the Baby Monster. We were all working trying to get the characters.

The Monster itself was too big. I don't think it can be done by hand. Livingstone got the character table, the ordinary complex irreducible characters of the Monster. But I think he made very heavy use of a computing machine. And I don't think that has been eliminated. That's how the figure 196883 came about, the degree of the smallest faithful complex representation of the Monster Group. It is just too big to be done by hand. But we can do these smaller subgroups.

The Tits group was found by hand, wasn't it? And what is it all about?

Tits: Yes, it was really sort of a triviality. One expects that there would be a group there except that one must take a subgroup of index two so that it becomes simple. And that is what I know about this.

Professor Tits, there is a startling connection between the Monster Group, the biggest of these sporadic groups, and elliptic function theory or elliptic curves via the j-function. Are there some connections with other exceptional groups, for instance in geometry?

Tits: I am not a specialist regarding these connections between the Monster Group, for instance, and modular functions. I don't really know about these things, I am ashamed to say. I think it is not only the Monster that is related to modular forms but also several other sporadic groups. But the case of the Monster is especially satisfactory because the relations are very simple in that case. Somehow, smaller groups give more complicated results. In the case of the Monster, things sort of round up perfectly.

The inverse Galois problem

May we ask you, Professor* Thompson, *about your work on the inverse Galois problem? Can you explain first of all what the problem is all about? And what is the status right now?

Thompson: The inverse Galois problem probably goes back already to Galois. He associated a group to an equation, particularly to equations in one variable with integer coefficients. He then associated to this equation a well-defined group now called the Galois group, which is a finite group. It captures quite a bit of the nature of the roots, the zeros, of this equation. Once one has the notion of a field, the field generated by the roots of an equation has certain automorphisms and these automorphisms give us Galois groups.

The inverse problem is: start with a given finite group. Is there always an equation, a polynomial with one indeterminate with integer coefficients, whose Galois group is that particular group? As far as I know it is completely open whether or not this is true. And as a test case if you start with a given finite simple group, does it occur in this way? Is there an equation waiting for it? If there is one equation there would be infinitely many of them. So we wouldn't know how to choose a standard canonical equation associated to this group. Even in the case of simple groups, the inverse problem of Galois Theory is not solved. For most general finite groups, I leave it to the algebraic geometers or whoever else has good ideas whether this problem is amenable. A lot of us have worked on it and played around with it, but I think we have just been nibbling at the surface.

For example the Monster is a Galois group over the rationals. You can't say that about all sporadic groups. The reason that the Monster is a Galois group over the rationals comes from character theory. It is just given to you.

Tits: This is very surprising; you have this big object and the experts can tell you that it is a Galois group. In fact, I would like to see an equation.

Is there anything known about an equation?
Thompson: It would have to be of degree of at least 1020. I found it impressive, when looking a little bit at the j-function literature before the days of computers, that people like Fricke and others could do these calculations. If you look at the coefficients of the j-functions, they grow very rapidly into the tens and hundreds of millions. They had been computed in Fricke's book. It is really pleasant to see these numbers out there before computers were around – numbers of size 123 million. And the numbers had to be done by hand, really. And they got it right.
Tits: It is really fantastic what they have done.

Could there be results in these old papers by Fricke and others that people are not aware of?
Thompson: No. People have gone through them; they have combed through them.
Tits: Specialists do study these papers.

The E_8-story

There is another collaborative effort that has been done recently, the so-called E_8-story: a group of mathematicians has worked out the representations of E_8. In fact, they calculated the complete character table for E_8. The result was publicized last year in several American newspapers under the heading "A calculation the size of Manhattan" or something like that.
Thompson: It was a little bit garbled maybe. I did see the article.

Can you explain why we should all be interested in such a result, be it as a group theorist or as a general mathematician or even as a man on the street?
Thompson: It is interesting in many ways. It may be that physicists have something to do with the newspapers. Physicists – they are absolutely fearless as a group. Any mathematical thing they can make use of they will gobble right up and put in a context that they can make use of, which is good. In that sense mathematics is a handmaiden for other things. And the physicists have definitely gotten interested in exceptional Lie groups. And E_8 is out there, really. It is one of the great things.

Is there any reason to believe that some of these exceptional groups or sporadic groups tell us something very important – in mathematics or in nature?
Thompson: I am not a physicist. But I know physicists are thinking about such things, really.
Tits: It is perhaps naive to say this but I feel that mathematical structures that are so beautiful like the Monster must have something to do with nature.

Mathematical work

Are there any particular results that you are most proud of?
Thompson: Well, of course one of the high points of my mathematical life was the long working relationship I had with Walter Feit. We enjoyed being together and enjoyed the work that we did, and, of course, the fusion of ideas. I feel lucky to have had that contact and proud that I was in the game there.
Tits: I had a very fruitful contact for much of my career with François Bruhat and it was very pleasant to work together. It was really working together like you did it, I suppose, with Walter Feit.

Was not Armand Borel also very important for your work?
Tits: Yes, I also had much collaboration with Borel. But in a sense that was different. But that was different in the following sense: When I worked with Borel, I had, very often, the impression that we both had found the same thing. We just put the results together in order not to duplicate. We wrote our papers practically on results that we had both found separately. Whereas with Bruhat, it was really joint work, complementary work.

Have either of you had the lightning flash experience described by Poincaré – seeing all of a sudden the solution to a problem you have struggled with for a long time?
Tits: I think this happens pretty often in mathematical research, that one suddenly finds that something is working. But I cannot recall a specific instance. I know that it has happened to me and it has happened to John, certainly. So certainly some of the ideas one has work out, but it sort of disappears in a fog.
Thompson: I think my wife will vouch for the fact that when I wake in the morning I am ready to get out there and get moving right away. So my own naïve thinking is that while I am asleep there are still things going on. And you wake up and say: "Let's get out there and do it". And that is a wonderful feeling.

You have both worked as professors of mathematics in several countries. Could you comment on the different working environments in these places and people you worked with and had the best cooperation with?
Tits: I think the country which has the best way of working with young people is Russia. Of course, the French have a long tradition and they have very good, very young people but I think Russian mathematics is in a sense more lively than French mathematics. French mathematics is too immediately precise. I would say that these are the two countries where the future of mathematics is the clearest. But of course Germany has had such a history of mathematics that they will continue. And nowadays, the United States have in a sense become the centre of mathematics because they have so much money – that they can...

…buy the best researchers?
Tits: That's too negative a way of putting it. Certainly many young people go the United States because they cannot earn enough money in their own country.

And of course the catastrophe that happened in Europe in the 1930s with Nazism. A lot of people went to the United States.

What about you, Professor Thompson? You were in England for a long time. How was that experience compared to work at an American university?

Thompson: Well, I am more or less used to holding my own role. People didn't harass me very much any place. I have very nice memories of all the places I have visited, mainly in the United States. But I have visited several other countries, too, for shorter periods, including Russia, Germany and France. Mathematically, I feel pretty much comfortable everywhere I am. I just carry on. I have not really been involved in higher educational decision making. So in that sense I am not really qualified to judge what is going on at an international basis.

Thoughts on the development of mathematics

You have lived in a period with a rapid development of mathematics (in particular in your own areas) including your own contributions. Some time ago Lennart Carleson, who received the Abel Prize two years ago, said in an interview that the 20th century had possibly been the Golden Age of Mathematics and that it would be difficult to imagine a development as rapid as we have witnessed it.

What do you think? Have we already had the Golden Age of Mathematics or will development continue even faster?

Tits: I think it will continue at its natural speed, which is fast – faster than it used to be.

Thompson: I remember reading a quote attributed to Laplace. He said that mathematics might become so deep, that we have to dig down so deep, that we will not be able to get down there in the future. That's a rather scary image, really. It is true that prerequisites are substantial but people are ingenious. Pedagogical techniques might change. Foundations of what people learn might alter. But mathematics is a dynamic thing. I hope it doesn't stop.

Tits: I am confident that it continues to grow.

Traditionally, mathematics has been mainly linked to physics. Lots of motivations come from there and many of the applications are towards physics. In recent years, biology, for example with the Human Genome Project, economics with its financial mathematics, computer science and computing have been around as well. How do you judge these new relations? Will they become as important as physics for mathematicians in the future?

Tits: I would say that mathematics coming from physics is of high quality. Some of the best results we have in mathematics have been discovered by physicists. I am less sure about sociology and human science. I think biology is a very important subject but I don't know whether it has suggested very deep problems in mathematics. But perhaps I am wrong. For instance, I know of Gromov, who is a first class mathematician and who is interested in biology now. I think that this is a case where mathematics, really highbrow mathematics, goes along with biology. What I said before about sociology, for instance, is not true for biology. Some biologists are also very good mathematicians.

Thompson: I accept that there are very clever people across the intellectual world. If they need mathematics they come up with mathematics. Either they tell mathematicians about it or they cook it up themselves.

Thoughts on the teaching of mathematics

How should mathematics be taught to young people? How would you encourage young people to get interested in mathematics?

Thompson: I always give a plug for Gamow's book *One, Two, Three … Infinity* and Courant and Robbins' *What is Mathematics?* and some of the expository work that you can get from the libraries. It is a wonderful thing to stimulate curiosity. If we had recipes, they would be out there by now. Some children are excited and others are just not responsive, really. You have the same phenomenon in music. Some children are very responsive to music; others just don't respond. We don't know why.

Tits: I don't know what to say. I have had little contact with very young people. I have had very good students but always advanced students. I am sure it must be fascinating to see how young people think about these things. But I have not had the experience.

Jean-Pierre Serre once said in an interview that one should not encourage young people to do mathematics. Instead, one should discourage them. But the ones that, after this discouragement, are still eager to do mathematics, you should really take care of them.

Thompson: That's a bit punitive. But I can see the point. You try to hold them back and if they strain at the leash then eventually you let them go. There is something to it. But I don't think Serre would actually lock up his library and not let the kids look at it.

Maybe he wants to stress that research mathematics is not for everyone.

Thompson: Could be, yeah.

Tits: But I would say that, though mathematics is for everyone, not everyone can do it with success. Certainly it is not good to encourage young people who have no gift to try to do something because that will result in sort of a disaster.

Personal interests

In our final question we would like to ask you both about your private interests besides mathematics. What are you doing in you spare time? What else are you interested in?

Tits: I am especially interested in music and, actually, also history. My wife is an historian; therefore I am always very interested in history.

What type of music? Which composers?

Tits: Oh, rather ancient composers.

And in history, is that old or modern history?

Tits: Certainly not contemporary history but modern and

medieval history. All related to my wife's speciality.
Thompson: I would mention some of the same interests. I like music. I still play the piano a bit. I like to read. I like biographies and history – general reading, both contemporary and older authors. My wife is a scholar. I am interested in her scholarly achievements. Nineteenth century Russian literature; this was a time of tremendous achievements. Very interesting things! I also follow the growth of my grandchildren.
Tits: I should also say that I am very interested in languages, Russian for instance.

Do you speak Russian?
Tits: I don't speak Russian but I have been able to read some Tolstoy in Russian. I have forgotten a little. I have read quite a lot. I have learned some Chinese. In the course of years I used to spend one hour every Sunday morning studying Chinese. But I started a little bit too old so I forgot what I learned.

Are there any particular authors that you like?
Tits: I would say all good authors.
Thompson: I guess we are both readers. Endless.

Let us finally thank you very much for this pleasant interview on behalf of the Norwegian, the Danish and the European Mathematical Societies. Thank you very much.

Thompson: Thank you.
Tits: Thank you for the interview. You gave us many interesting topics to talk about!

ICMI – ICME 11 Monterrey (Mexico)

Mariolina Bartolini Bussi

The 11th International Congress on Mathematical Education (ICME) was held in Monterrey, Mexico, 6–13 July 2008, in the main campus of the UANL (Universidad Autónoma de Nuevo León). More than 2000 participants from all the continents took part in the conference. It is impossible to report on the many activities of the program, which can be seen at the web site http://icme11.org/. In the following, I shall try to briefly convey the meaning of this conference as an event of the 100-year-old ICMI, the official commission for mathematical instruction of the International Mathematical Union (IMU).

The General Assembly

The general assembly of the ICMI took place the day before the opening session. Every four years the assembly brings together the representatives of the ICMI member countries (of which there are currently 72). The 2008 general assembly also had a special meaning in the history of the ICMI. Until 2006 the Executive Committee (EC) of the ICMI had been elected by the general assembly of the IMU. However, it was approved in the 2006 general assembly of the IMU held in Spain that, as from 2008, the EC of the ICMI was to be elected by the general assembly of the ICMI itself[1]. This represents a milestone in the history of the ICMI and bears witness to the good relationship existing today between the IMU and the ICMI. The next EC will be elected by the general assembly of the ICMI in 2012.

The Participants

There were more than 2000 participants of the conference, a similar number to the previous ICMEs held in Denmark (ICME10, 2004) and Japan (ICME9, 2000). As usual, the participation of the geographic area close to the venue was higher (nearly 400 from Latin America and more than 400 from the US). Yet there were also many Europeans present (around 630 – see the table attached). Africa was still a less represented continent with around 70 participants, mostly from South Africa. Australia and New Zealand were represented, as usual, by about 100 participants. What is re-

[1] The elected members are:
President: William (Bill) Barton (New Zealand).
Secretary-General: Jaime Carvalho e Silva (Portugal).
Vice-Presidents: Mina Teicher (Israel), Angel Ruiz (Costa Rica).
Members at large: Mariolina Bartolini Bussi (Italy), Sung Je Cho (Korea), Roger Howe (USA), Renuka Vithal (South Africa), Zhang Yingbo (China).
Michèle Artigue (France), present president of the ICMI, will be an ex-officio member of this EC. Also, the president and vice-president of the IMU will be ex-officio members of the EC. The term of this EC will be from 01 January 2010 to 31 December 2012.

ally new is the increase of participants from East Asia, with strong delegations from China (around 160), Japan (more than 70), Korea (around 40) and Singapore (nearly 30). These increased numbers seem to be connected to: the increasing relationships between China and the so-called 'Western' world; the excellent results of Chinese students in international assessment studies; the involvement of the Korean community who are in charge of organizing the next ICME in 2012; and the involvement of Japan in many collaborative studies with Latin America. In particular China offered a very well organized and crowded national presentation, where the interesting and challenging features of the Chinese education system for mathematics were reported. The increasing importance of the countries outside Europe and North America is mirrored by the composition of the next EC.

The Program

Only some snapshots from the plenary activities are reported here. All the plenaries have been broadcast on the web and a cdrom is available from the organizers. In the opening session the Klein and Freudenthal Medals for 2005 and 2007 were awarded (see the June issue of this newsletter) to Ubiratan D'Ambrosio (Klein, 2005), Jeremy Kilptrick (Klein, 2007), Paul Cobb (Freudenthal, 2005) and Anna Sfard (Freudenthal, 2007).

The first plenary was given by Michèle Artigue (the ICMI President) and Jeremy Kilpatrick, in the form of a dialogue on *What do we know that we did not know ten years ago? And how do we know it?* This dialogue declared the focus of this conference, where most plenary activities were panels addressing issues interesting for practitioners (teachers, teacher educators, policy makers and so on). Some other plenary panels focused on: *What do we need to know? Does research in mathematics education address the concerns of practitioners and policy makers?; Equal access to quality mathematics education; Knowledge for teaching mathematics; The impact of research findings in mathematics education on students' learning of mathematics; and Representations of mathematical concepts, objects and processes in mathematics teaching and learning.* Another panel reported on the *History of the development of mathematics education in Latin American countries.* Finally, two plenary speechs were given by Celia Hoyles (*Transforming the mathematical practices of learners and teachers through digital technology*) and by José Antonio de la Peña (*Current trends in mathematics*).

The beautiful logo of the conference: a Möbius band with Aztec hieroglyphs.

Besides the plenary activities, about 60 one-hour lectures were given in twelve parallel sessions and a rich program of 38 parallel topic study groups, 28 parallel discussion groups, workshops, posters, presentations and so on was offered to the participants. The large Latin American community attended the plenary activities in a different auditorium with simultaneous translation into Spanish and enjoyed a special parallel program for part of the conference.

The ICMI Booth

An ICMI booth was present in the exhibition area to provide information about recent and forthcoming publications and events. In particular, information was available about the proceedings of ICME10 (see http://www.icme10.dk/ to download the form) and the proceedings of the *Symposium on the Occasion of the 100th Anniversary of ICMI*, which has already been reported in June (see http://www.unige.ch/math/EnsMath/Rome2008/AnProc08.pdf to download the form). Other information was given about the volumes of the ICMI studies in press (within the New ICMI Studies Series – http://www.springer.com/series/6351) and about the ICMI studies and other ICMI projects to come. Information about these future activities will be given in the following issues of this newsletter.

Austria	9	Lithuania	2
Belgium	8	Macedonia	1
Bulgaria	6	Netherlands	19
Croatia	3	Norway	20
Cyprus	9	Poland	3
Czech rep.	3	Portugal	51
Denmark	35	Romania	4
Estonia	1	Russia	18
Finland	16	Serbia	1
France	53	Slovakia	6
Germany	44	Slovenia	3
Greece	7	Spain	45
Hungary	7	Sweden	64
Iceland	5	Switzerland	1
Ireland	7	Turkey	8
Israel	22	UK	101
Italy	41	Ukraina	1
Latvia	5	**Total**	**629**

The European participation at ICME 11.

ERCOM: National Institute for Advanced Mathematics "Francesco Severi" (INdAM)

INdAM is the Italian Mathematics Research Institute; it is a self-governing, state research institute, similar to the National Research Council (CNR) and the National Institute for Nuclear Physics (INFN), legally constituted and supervised by the Ministry responsible for Education University and Research (MiUR).

Founded in 1939 by the mathematician Francesco Severi, it was subsequently given in 1992 the current legal status (Decree of the President of the Republic no. 153).

Aims (Mission Statement)

To promote the training of researchers in mathematics at national, international and European community level, in particular to supplement the training potential of the Italian universities.

To develop research in pure and applied mathematics, especially in the emerging branches, and to foster the transfer of knowledge to technological applications.

To further close and sustained contact between Italian and international mathematical research, in particular by participating in programs within the framework of the European Union.

Membership

INdAM is a member of ERCOM.
INdAM is a member of International Mathematical Sciences Institutes (IMSI), the international pool of research institutes in the mathematical sciences that run thematic research programs and have large visitor programs.

Governing Body

The governing body of INdAM is organized as follows:

This is the location of the Institute

The President – elected by the Scientific Council (Comitato Direttivo).
The Scientific Council (Comitato Direttivo) – composed of eleven members; seven of them (including the president) are elected for four-year terms from the community of university mathematicians. The remaining four are the Directors of the National Research Groups.
The Board of Administration – composed of the president, the two vice-presidents and four experts nominated by the competent Ministries.
The Board of Auditors of Accounts.

Scientific Council

The Scientific Council is composed of:
- Vincenzo Ancona (President).
- Angelo Alvino (Vice-president).
- Alfredo Bellen.
- Italo Capuzzo Dolcetta.
- Vittorio Coti Zelati.
- Nicola Fusco.
- Giorgio Patrizio.
- Mario Pulvirenti.
- Tommaso Ruggeri.
- Elisabetta Strickland (Vice-president).
- Sandro Verra.

Internal Evaluation Committee

In 2001, INdAM established the Internal Evaluation Committee with the mandate of assessing the state of INdAM's activities and to advise both INdAM itself and the competent authorities on its future orientations.

The Internal Evaluation Committee is presently composed of:
- Enrico Bombieri (Chairman), Institute for Advanced Study.
- Francis Clarke, Université de Lyon I.
- Gianluigi Galeotti, Università di Roma La Sapienza.
- Francesco Guerra, Università di Roma La Sapienza.
- Victor Kac, Massachusetts Institute of Technology.

Committee for Evaluation of Research – CIVR

The Committee for Evaluation of Research (CIVR) is a body nominated by the Italian government. The fundamental task of the CIVR is to promote research evaluation activities for optimal utilization of national scientific and technological research resources.

To the scientific production of INdAM, CIVR has conferred a rating of 94/100, the highest rating obtained by any Italian Research Public Institution.

The role of INdAM in research: the national research groups

INdAM has no permanent research staff. Most of the research activities of INdAM are carried out by the four national research groups, which draw together researchers, including both early stage and advanced fellows from universities, public and private research centres and the INdAM advanced fellows. The groups carry out the following activities:

1. Invitation of foreign scientists (visiting professors) to develop research and for consultation and high-level training.
2. Payment of missions abroad for their members.
3. Organization of and participation in meetings prepared by their members.
4. Financing of travel abroad for research or meetings of doctorate and research fellows.
5. Research projects.In 2007 the research groups have supported 105 visiting professors from various countries, organized or co-organized 38 conferences and financed 43 research projects.

The structure of the research groups.

Each group is governed by a scientific committee composed of four elected members and three members appointed by INdAM. The director of each group is chosen by the scientific council from among the four elected members. They report their activities and expenditures to INdAM on an annual basis, INdAM having given mandatory guidelines regulating a number of issues (e.g. visiting professors and sabbatical years). Each group is subdivided into a number of thematic sections and composed of research units at the level of the various universities.

About the groups

The four national groups comprise a total of about 2500 members as follows:

- GNAMPA: National group of mathematical analysis, probability and its applications – 920 members.
- GNSAGA: National group of algebraic, geometric structures and their applications – 663 members.
- GNFM: National group of mathematical physics – 549 members.
- GNCS: National group of computer science (with two principal applications: medical engineering and technology for the knowledge society) – 448 members.

Fellowships

To further the attainment of its objectives in training researchers, INdAM promotes fellowship programmes for undergraduate students (Licence (L) or Master (M) in the new LMD System), graduate students, postgraduates and experienced researchers; some of these programmes are intended for foreign researchers.

Since 1992 the institute has awarded more than 1000 fellowships, 270 of which were for experienced researchers. At present there are 150 fellows of the institute.

The scientific policy of INdAM about fellowships matches the most important features of the "European charter for researchers":

1) The applicants for fellowships can freely choose the national host institution (or the international host institution in the case of outgoing fellowships) where they will carry out their programme.
2) They can freely choose their research program, provided it is concerned with mathematics and/or its applications.
3) They can freely choose their research directors or supervisors.
4) They can receive additional scientific and financial support from the INdAM national research groups and the INdAM research units located in the main Italian universities.

INdAM policy for fellowships can be summarized in the following statement:

"We support researchers so that they can choose their host institutions, their supervisors, their programs; we do NOT support institutions or senior researchers so that they can choose their fellows".

University grants

These bursaries are awarded to students at the Licence (Bachelor's) level of study in the new LMD System, with the aim of nurturing vocations for mathematics among the young.

This program is co-financed by the Scientific Degree Project of the Ministry of Education University and Research ("Progetto Lauree Scientifiche" of the MIUR).

The amount of the grant is €4000 per year. Currently there are 107 students.

Doctoral fellowships. Senior study grants

For Italian students doing graduate work abroad, these are given on an ad hoc basis for variable durations.

School for the Applications of Mathematics in Industry

Typically these incorporate 12 to 15 courses of thirty hours each and include seminars given by people from industry. The school is now co-financed by the Cariplo bank and is being run for three years (2006–2008) in collaboration with the University of Milano-Bicocca.

Foreign doctoral fellowships

INdAM offers some fellowships to foreign graduates to undertake Italian doctorate (PhD level) studies.

In 2005, INdAM launched a further program at the post-doctoral level, in which young Italian researchers receive grants for visits of up to six months to foreign research centres.

Summer school

INdAM participates in the Mathematics Summer Schools of Perugia and Cortona organized by the SMI (Scuola Matematica Interuniversitaria). Perugia runs for five weeks and targets Masters students intending to prepare a doctorate in Italy or abroad; Cortona is intended for a higher pre- or post-doctoral level of student.

In 2004, INdAM formed a consortium with the SNS (Scuola Normale Superiore di Pisa), the International School for Advanced Studies, SISSA (Trieste area) and the University of Perugia, which has also become the centre for educational activities at an undergraduate level.

The President of the Institute, Vincenzo Ancona, while signing the cooperation agreement between CNRS and INdAM in front of S.E.M. Jean-Marc de La Sablière, Embassador of France.

The administration of the two SMI Summer Schools is based at Perugia University.

Visiting professors for doctorate courses

The visiting professors stay at least two months, during which time they do 24 hours of teaching in different universities. The aim is to foster the internationalization of scientific research.

Post-doctoral support

There have traditionally been a number of post-doctoral grants awarded by INdAM each year. Since 2004, a new program of prestigious ‘Francesco Severi post-doctoral fellowships’ has been launched, the aim being to compete with the Marie Curie fellowships. These are renewable once or twice and have involved an annual salary of up to €40,000.

INdAM Day

A new initiative was launched on 18 June 2004, when the first INdAM Day took place in Rome. There were four high-level expository lectures.

In 2005, INdAM Day took place in Napoli, in 2006 it was in Milano, in 2007 it was in Pisa and in 2008 it was in Padova.

INdAM’s workshops

Each workshop is organized by two directors and consists of lectures and short lecture series on clearly defined topics. The duration is 4 to 5 days and involves 20 to 25 invited participants comprising established specialists and promising young researchers. Since 2005, eleven such workshops have been held.

The workshops to be held in 2008 are:
- *“Analysis, PDEs and Applications”.*
- *“Geometria delle varietà proiettive”.*
- *“Holomorphic Iteration, Semigroups and Loewner chains”.*
- *“Sobolev spaces and continuity”.*
- *“Recent Advances in Geometry and Topology of Submanifolds”.*

Meetings

INdAM organizes periods of one week of study and research on a specific topic with the participation of about twenty invited researchers, at least ten of them from abroad. Other researchers may be admitted on request, in particular young researchers.

Cycles of lectures are given, mainly by invited persons but the results of other participants are also presented; seminars and workshops are organized, culminating in a final meeting. The proceedings may be published in special issues of the Rendiconti di Matematica (Sapienza University, Rome).

The meetings to be held in 2008 are:
- *“Liouville Theorems and Detours”.*
- *“Analysis and Topology in Interaction”*
- *“Symmetries in Mathematics and Physics”*
- *“Structured Linear Algebra Problems: Analysis, Algorithms and Applications”*
- *“Equazioni a derivate parziali, teoria dei semigruppi, problemi inversi e teoria del controllo”*

Intensive research periods

Each year, INdAM supports two or three intensive research periods to be held at Italian universities or research centres.

In 2008 the following INdAM intensive period is scheduled:
- *“Groups in Algebraic Geometry”*

Research Units

INdAM has entered into conventions with Italian universities to constitute research units (see listing for research locations) of the institute inside these universities. The units are coordinated by the institute.

Their purpose is to organize activities on a local level (e.g. administration, international relations). The research units are listed below:

1) University of L’Aquila. Director: Carlo Scoppola.
2) University of Bari. Director: Francesca Mazzia.
3) University of Basilicata. Director: Vito Antonio Cimmelli.
4) University of Cagliari. Director: Sebastiano Seatzu.
5) University of Camerino. Director: Stefano Isola.
6) University of Catania. Director: Mario Marino.
7) University of Chieti-Pescara. Director: Cristina Caroli Costantini.
8) University of Ferrara. Director: Luisa Zanghirati.
9) University of Firenze. Director: Luigi Brugnano.
10) University of Genova. DIMA Director: Mario Bertero. DISI Director: Giancarlo Mauceri.
11) IAC (Istituto per le Applicazioni del Calcolo “Mauro Picone”). Director: Benedetto Piccoli.
12) University of Messina. Director: Domenico Fusco.
13) University of Milano I. Director: Alfredo Lorenzi.
14) Polytechnic of Milano. Director: Vittorino Pata.
15) University of Modena e Reggio Emilia. Director: Emanuele Galligani.
16) University of Napoli I. Director: Paolo de Lucia.

From the left: Elisabetta Esposito (Council Secretary); Scientific Council: Giorgio Patrizio, Angelo Alvino (Vice-President) Alfredo Bellen, Italo Capuzzo Dolcetta, Elisabetta Strickland (Vice-President), Tommaso Ruggeri, Vincenzo Ancona (INdAM President), Nicola Fusco, Sandro Verra, Mario Pulvirenti.

17) University of Padova. Director: Wolfgang Runggaldier.
18) University of Parma. Director: Gianmario Tessitore.
19) University of Pavia. Director: Piero Colli Franzone.
20) University of Perugia. Director: Giulianella Coletti.
21) University of Pisa. Director: Ferruccio Colombini.
22) University of Roma Tor Vergata. Director: Carlangelo Liverani.
23) University of Roma Tre. Director: Fabio Martinelli.
24) University of Salerno. Director: Maria Transirico.
25) University of Siena. Director: Paolo Costantini.
26) University of Torino. Director: Luigi Rodino.
27) Polytechnic of Torino. Director: Anita Tabacco.
28) University of Trento. Director: Marco Andreatta.
29) University of Trieste. Director: Alfredo Bellen.
30) SISSA – Trieste. Director: Antonio De Simone.

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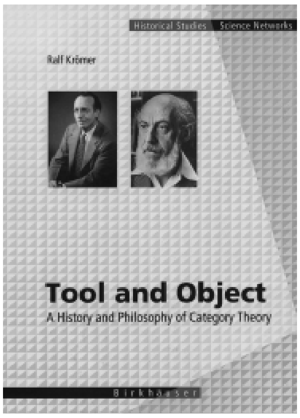
Book Review

Narciso Martí-Oliet (Madrid, Spain)



Ralf Krömer
**Tool and Object.
A History and Philosophy of Category Theory**

Science Networks – Historical Studies. Volume 32, Birkhäuser
Basel · Boston · Berlin, 2007
ISBN-13: 978-3-7643-7523-2

This book presents a history of the essential aspects of Category Theory (CT) since its inception in the mid 1940s until the beginning of the 1970s. But is it more than that, because the author thinks that "the related conceptual innovations challenged formerly well-established epistemological positions" and is persuaded that "mathematical uses of the tool CT and epistemological considerations having CT as their object cannot be separated, neither historically nor philosophically". Thus, as the title conveys, the history goes together with the philosophy.

The text is an adaptation into English of the author's doctoral dissertation originally written in German. It is an adaptation instead of a simple translation because the author decided to take advantage of the occasion to update some parts with more recent information and, at the same time, to eliminate other parts not so directly related to his main argument. In particular, sections dedicated to universal mappings, direct and inverse limits, groupoids, and categories and Bourbaki have already or will be published as separate works, probably papers. In a related way, the author does not consider his book as a complete or standard reference in any way, and prefers to see it as an addition complementing the existing literature on the subject. For this reason, throughout the book, there are numerous references to a book by Corry [1], which was published some years ago in the same series, and to some papers by McLarty, among others. On the other hand, for some reason no mention is made of a book by Mac Lane and Moerdijk [2]; it is not an historical book but it is a book by one of the main characters in this story on a subject that is important in the context of at least a couple of chapters.

Let us review the contents of the book, chapter by chapter.

Chapter 1 is a prelude of philosophical character, explaining the basis for the author's pragmatist epistemological position. Although there is some advice stating that "the reader who is more interested in historical than epistemological matters may skip this chapter", I myself, with a limited philosophical formation, decided to consider this part of the book as a challenge to learn more on those matters, such as Poincaré's opinion on the task of the philosophers of science or the reductionist epistemology of mathematics and its critiques due to Peirce and Wittgenstein.

Chapter 2, on CT in algebraic topology, begins summarizing the conditions of the study of homology theory in the 1930s that gave rise to CT, including the study of mappings and the search for a homology theory for general topological spaces. It continues with the collaboration of Eilenberg and Mac Lane on the paper *Group extensions and homology*, which paved the way for their 1945 seminal paper *General theory of natural equivalences*, and how this paper was received in the community (with comments ranging from the *most trivial* to the *most influential*). It goes on to consider the importance of the then recently introduced categorial language in Eilenberg and Steenrod's book *Foundations of Algebraic Topology*, and continues with a paper by Mac Lane on duality for groups and the works by Kan on simplicial sets and adjoint functors, which showed the importance of the latter concept.

Chapter 3, on CT in homological algebra, begins reviewing the role of categorial concepts in Cartan and Eilenberg's book *Homological Algebra* and some achievements of Buchsbaum's dissertation concerning Abelian categories and duality. It continues summarizing the work on sheaves by Leray, Cartan and Serre, before concentrating in great detail on Grothendieck's 1957 "Tôhoku paper" entitled *Sur quelques points d'algèbre homologique* . Finally, the author methodically analyses the use of categorial and set-theoretical language, and also discusses the usual judgements according to which Grothendieck's work transformed CT from a mere language into a tool and then into an autonomous discipline.

Chapter 4, on CT in algebraic geometry, is not as detailed as the previous two chapters. The first part summarizes Grothendieck's innovations: from the concept of variety to the concept of scheme, from the Zariski topology to sites and Grothendieck topologies, and then to Grothendieck toposes. The second part summarizes the role of categorial methods in Grothendieck's work on the Weil conjectures. The author acknowledges that the subject matter has become too complex at this point to be able to follow it from the original writings and to expose it with some detail in the context of this work. Thus, his goal is simply to provide an overview relying more on the appropriate literature. In my opinion the result is more than satisfying and provides a good summary of the importance of CT in Grothendieck's achievements.

Chapter 5 goes back, leaving the application of CT in different areas, and instead studies how these applications, as recounted in the previous chapters, have led to the development of the most essential concepts in CT, thus paying more attention to the interaction between historical and philosophical aspects promised early in the book. The chapter begins with the transformation of some concepts such as homology, complexes, coefficients for homology and cohomology, and sheaves, leading to the concept of Grothendieck topos. The chapter continues with the functor concept, from hom-functors to adjoint functors, and the object concept; here is a good place to compare objects with structured sets, with the conclusion that "it is not convincing to describe category theory as a mere theory of structured sets and structure-preserving mappings". Instead, objects have to be studied through the algebra of arrow composition and their identification criterion must be equality up to isomorphism. The final concept is that of categories, which themselves become objects when considering categories of categories.

Chapter 6 provides a detailed study of the problems encountered when trying to give CT a set-theoretical foundation, without the need to consider the category of all categories but simply considering the category of all sets or of all groups, for example. These problems, together with some of the proposed solutions, are considered from the beginning, with Eilenberg and Mac Lane's paper from 1945, going through a paper by Mac Lane on *Locally small categories and the foundations for set theory*, up to the work by Grothendieck on universes, originating as part of a Bourbaki discussion on the matter of this chapter. The study of the significance of this proposal and its relationship to inaccessible cardinals is followed by comments on some other proposals, so that the final impression is that this matter is not yet completely settled. This idea is reinforced by chapter 7 on categorial foundations, which continues studying foundational problems but now from a different point of view: CT as an alternative mathematical foundation. This is exemplified mainly in the work done during the 1960s and the beginning of 1970s by Lawvere, first by his attempts to axiomatize the category of sets and the category of categories and later by the essential concept of an elementary topos, a generalization of Grothendieck toposes, which can be considered as providing alternative "set theories." Here I would have liked more information on all the work by Lawvere on categorial logic and the importance there of adjoints. Instead, the author goes on to discuss a more recent paper by Bénabou on categorial foundations based on fibred categories.

The book concludes with the relatively short chapter 8, which closes the circle by going back to the philosophical matters discussed in chapter 1. In my opinion this final chapter is a bit disappointing. While I liked all the philosophical discussion interspersed in the previous chapters or made more explicit in chapter 5, for example, I was unable to get the interest of the comments on realism versus pragmatism discussed in this final chapter. Probably I was just expecting a different kind of grand finale.

The quality of the presentation is excellent overall. The author has devoted great attention to details, such as using different typefaces to distinguish the different uses of the word object that, of course, appears hundreds of times throughout the book. There are lots of footnotes (550), many of them quotations in the original language, and a useful table of contents, structured up to four levels inside each chapter. I found myself going again and again to check this table of contents due to the huge number of internal cross-references. The concept index is also appropriate.

I recommend this book to any mathematical reader with an algebraic background, not necessarily too advanced (not even in category theory itself), who is interested in knowing more of the history behind an important part of the mathematics developed in the middle of the 20th century. Moreover, this book is an excellent illustration of the fact that together with some mathematics and some history, one can also learn some philosophy and thus the gap between working mathematicians and philosophers of mathematics may not be so big after all.

Let me finish with a comment on the cover, which consists of pictures of Mac Lane and Eilenberg. Given the emphasis by the author on Grothendieck's work, it is clear to me that his picture also deserved to be there on the cover. Since, for whatever reason, this is not the case, one can see several pictures of Grothendieck at the web page http://www.grothendieckcircle.org and also in the articles [3] about his life.

References

[1] Leo Corry. *Modern Algebra and the Rise of Mathematical Structures*. Birkhäuser, 1996.

[2] Saunders Mac Lane and Ieke Moerdijk. *Sheaves in Geometry and Logic. A First Introduction to Topos Theory*. Springer, 1992.

[3] Allyn Jackson, Comme appelé du néant – As if summoned from the void: The life of Alexandre Grothendieck, *Notices of the American Mathematical Society* 51(9): 1038–1056, October 2004, and 51(10): 1196–1212, November 2004.

Narciso Marti-Oliet [narciso@esi.ucm.es] is a full professor in the Departamento de Sistemas Informáticos y Computación of the Universidad Complutense de Madrid, Spain. He got his PhD in mathematics in 1991 from this university, with a thesis supervised by J. Meseguer on the categorical semantics of linear logic and order-sorted algebra. Since then his research has focused on the subjects of declarative multiparadigm programming, and algebraic and logical methods for software specification, design and verification. He currently leads the research group on Formal Analysis and Design of Software Systems.

Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

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September 2008

1–5: Representation of surface groups, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

1–5: Conference on Numerical Analysis (NumAn 2008), Kalamata, Greece
Information: numan2008@math.upatras.gr;
http://www.math.upatras.gr/numan2008/

1–6: School (and Workshop) on the Geometry of Algebraic Stacks, Trento, Italy
Information: michelet@science.unitn.it;
http://www.science.unitn.it/cirm/

1–12: School on Algebraic Topics of Automata, Lisbon, Portugal
Information: patricia@cii.fc.ul.pt;
http://caul.cii.fc.ul.pt/SATA2008/

2–5: X Spanish Meeting on Cryptology and Information Security, Salamanca, Spain
Information: delrey@usal.es;
http://www.usal.es/xrecsi/english/main.htm

2–7: International Conference on Geometry, Dynamics, Integrable Systems, Belgrade, Serbia
Information: gdis08@mi.sanu.ac.yu;
http://www.mi.sanu.ac.yu/~gdis08/

6–14: First European Summer School on Mathematical Finance, Dourdan near Paris, France
Information: euroschoolmathfi@cmap.polytechnique.fr;
http://www.ceremade.dauphine.fr/~bouchard/ESCMF/

8–12: Chinese-French meeting on probability and analysis, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

8–10: Calculus of Variations and its Applications from Engineering to Economy, Universidade Nova de Lisboa, Caparica, Portugal
Information: cva2008@fct.unl.pt;
http://ferrari.dmat.fct.unl.pt/cva2008/

8–12: International Workshop on Orthogonal Polynomials and Approximation Theory 2008. Conference in honour of professor Guillermo López Lagomasino on his 60th Anniversary, Universidad Carlos III de Madrid, Leganés, Spain
Information: iwopa08@gmail.com;
http://www.uc3m.es/iwopa08

8–19: EMS Summer School: Mathematical models in the manufacturing of glass, polymers and textiles, Montecatini, Italy
Information: cime@math.unifi.it;
http://web.math.unifi.it/users/cime//

9–12: INDAM Workshop on Holomorphic Iteration, Semigroups and Loewner Chains, Rome, Italy
Information: iterates@mat.uniroma2.it;
http://www.congreso.us.es/holomorphic/

10–12: Nonlinear Differential Equations (A tribute to the work of Patrick Habets and Jean Mawhin on the occasion of their 65th birthdays), Brussels, Belgium
Information: node2008@uclouvain.be;
http://www.uclouvain.be/node2008.html

10–12: Colloquium Logicum 2008, Darmstadt, Germany
Information: klingenburg@mathematik.tu-darmstadt.de;
http://www.mathematik.tu-darmstadt.de/fbereiche/logik/events/collogicum/

12–15: Mathematical Analysis, Differential Equations and their Applications, Famagusta, North Cyprus
Information: fabdul@mersin.edu.tr;
http://madd2008.emu.edu.tr/contact.php

14–18: 7th Euromech Fluid Mechanics Conference, Manchester, UK
Information: http://www.mims.manchester.ac.uk/events/workshops/EFMC7/

15–19: Geometry and Integrability in Mathematical Physics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

15–19: International Conference on K-Theory and Homotopy Theory, Santiago de Compostela, Spain
Information: regaca@usc.es;
http://www.usc.es/regaca/ktht/

15–19: 9th SIMAI Congress, Rome, Italy
Information: simai2008@iac.rm.cnr.it;
http://www.simai.eu/

16–19: Rings and Modules, in honour of Patrick F. Smith's 65th birthday, Lisboa, Portugal
Information: pfs2008@cii.fc.ul.pt;
http://pfs2008.cii.fc.ul.pt/

16–20: International Conference of Numerical Analysis and Applied Mathematics 2008 (ICNAAM 2008), Psalidi, Kos, Greece
Information: tsimos@mail.ariadne-t.gr;
http://www.icnaam.org/

17–20: Braids in Paris, Paris, France
Information: tresses08@math.jussieu.fr;
http://tresses08.institut.math.jussieu.fr/index-en.html

18–21: 6th International Conference on Applied Mathematics (ICAM6), Baia Mare, Romania
Information: vberinde@ubm.ro;
http://www.icam.ubm.ro

18–29: Crimean Autumn Mathematical School-Symposium, Batiliman, Ukraine
Information: pavelstarkov@list.ru

19–26: International Conference on Harmonic Analysis and Approximations IV, Tsaghkadzor, Armenia
Information: mathconf@ysu.am;
http://math.sci.am/conference/sept2008/conf.html

21–24: The 8th International FLINS Conference on Computational Intelligence in Decision and Control (FLINS 2008), Madrid, Spain
Information: flins2008@mat.ucm.es;
http://www.mat.ucm.es/congresos/flins2008

22–25: Symposium on Trends in Applications of Mathematics to Mechanics (STAMM 2008), Levico, Italy
Information: stamm08@gmail.com;
http://mate.unipv.it/pier/stamm08.html

22–26: 10th International workshop on set theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

22–28: 4th International Kyiv Conference on Analytic Number Theory and Spatial Tessellations jointly with 5th Annual International Conference on Voronoi Diagrams in Science and Engineering (dedicated to the centenary memory of Georgiy Voronoi), Kyiv, Ukraine
Information: voronoi@imath.kiev.ua;
http://www.imath.kiev.ua/~voronoi

29–October 3: Commutative algebra and its interactions with algebraic geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

29–October 8: EMS Summer School – Risk theory and related topics, Będlewo, Poland
Information: stettner@iman.gov.pl;
www.impan.gov.pl/EMSsummerSchool/

October 2008

3–5: II Iberian Mathematical Meeting, Badajoz, Spain
Information: imm2@unex.es;
http://imm2.unex.es

5–12: International Conference "Differential Equations. Function Spaces. Approximation Theory" dedicated to the 100th anniversary of the birthday of S.L. Sobolev, Novosibirsk, Russia
Information: sobolev100@math.nsc.ru;
http://www.math.nsc.ru/conference/sobolev100/english

6–10: Partial differential equations and differential Galois theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

9–11: Algebra, Geometry and Mathematical Physics, Tartu, Estonia
Information: agmf@astralgo.eu;
http://www.agmf.astralgo.eu/tartu08/

9–12: The XVI-th Conference on Applied and Industrial Mathematics (CAIM 2008), Oradea, Romania
Information: serban_e_vlad@yahoo.com;
http://www.romai.ro

13–17: Hecke algebras, groups and geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–22: IV Congress of Mathematicians of the Republic of Macedonia, Struga, Macedonia
Information: smmk@iunona.pmf.ukim.edu.mk;
http://smmk.pmf.ukim.edu.mk

20–24: Symbolic computation days, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

22–24: International Conference on Modeling, Simulation and Control 2008, San Francisco, USA
Information: wcecs@iaeng.org;
http://www.iaeng.org/WCECS2008/ICMSC2008.html

26–28: 10th WSEAS Int. Conf. on Mathematical Methods and Computational Techniques in Electrical Engineering (MMACTEE 8), Corfu, Greece
Information: info@wseas.org;
http://www.wseas.org/conferences/2008/corfu/mmactee/

26–28: 7th WSEAS Int. Conf. on Non–Linear Analysis, Non-Linear Systems and Chaos (NOLASC 8)
Corfu, Greece
Information: info@wseas.org;
http://www.wseas.org/conferences/2008/corfu/nolasc/

27–31: New trends for modeling laser-matter interaction, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

November 2008

3–7: Harmonic analysis, operator algebras and representations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

5–7: Fractional Differentiation and its Applications, Ankara, Turkey
Information: dumitru@cankaya.edu.tr;
http://www.cankaya.edu.tr/fda08/

5–7: Modern Problems of Differential Geometry and General Algebra, Saratov City, Russia
Information: vagner2008@bk.ru;
http://mexmat.sgu.ru/vagner2008.php?lang=en

10–14: The 6th Euro-Maghreb workshop on semigroup theory, evolution equations and applications, CIRM Luminy, Marseille, France

Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

12–15: First Mathematical Meeting Spanish-Moroccan. Mathematics as Interdisplineary Science for Development and Cooperation, Casablanca, Morocco
Information: abdellatifroc@hotmail.com, marisa.fernandez@ehu.es;
http://titanium.univh2m.ac.ma/sedy/accueil.html

17–21: Geometry and topology in low dimension, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

21–22: International Conference on Nolinear Analysis and Applied Mathematics, Targoviste, Romania
Information: dteodorescu2003@yahoo.com;
http://icnaam.valahia.ro/

24–28: Approximation, geometric modelling and applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

December 2008

1–5: Homology of algebra: structures and applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

8–12: Latent variables and mixture models, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

15–19: Meeting on mathematical statistics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

16–18: Eighth IMA International Conference on Mathematics in Signal Processing, Cirencester, UK
Information: pam.bye@ima.org.uk;
http://www.ima.org.uk/Conferences/signal_processing/signal_processing08.html

January 2009

12–16: Algebraic Geometry and Complex Geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–23: Resonances in Mathematical Physics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

26–30: Quantum Gravity meets Non Commutative Geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

26–30: Winter School on Quantum Chaos, Talence, France
Information: qchaos2009org@math.u-bordeaux1.fr;
http://www.math.u-bordeaux1.fr/qchaos2009/

28–Feb 1: CERME 6 - Sixth Conference of European Research in Mathematics Education, Lyon, France
Information: cerme6@univ-lyon1.fr
http://cerme6.univ-lyon1.fr/

February 2009

2–March 6: Thematic month – Scientific computing and Partial Differential Equations on nonlinear partial differential equations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

March 2009

15–20: ALGORITMY 2009 – Conference on Scientific Computing, High Tatra Mountains, Podbanske, Slovakia
Information: algoritm@math.sk;
http://www.math.sk/alg2009

23–27: Numeration – Mathematics and Computer Science, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

30–April 3: Arithmetic, geometry, cryptography and coding theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

April 2009

6–10: Group actions in sympletic and algebraic geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

May 2009

14–16: Algebra and Probability in Many-Valued Logics, Darmstadt, Germany
Information: apmvl@mathematik.tu-darmstadt.de;
http://www.mathematik.tu-darmstadt.de/fbereiche/logik/events/apmvl/

25–29: 6th European Conference on Elliptic and Parabolic Problems, Gaeta, Italy
Information: gaeta@math.uzh.ch;
http://www.math.uzh.ch/gaeta2009

27–June 1: Infinite-Dimensional Analysis and Topology, Yaremche, Ivano-Frankivsk, Ukraine
Information: idat@pu.if.ua;
http://www.idat.frankivsk.org

July 2009

6–10: 26th Journées arithmétiques, Saint-Etienne, France
Information: ja2009@univ-st-etienne.fr;
http://ja2009.univ-st-etienne.fr

27–31: Stochastic Processes and their Applications, Berlin, Germany
Information: kongresse@tu-servicegmbh.de;
http://www.math.tu-berlin/SPA2009

Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to: Ivan Netuka, MÚUK, Sokolovská, 83, 186 75 Praha 8, Czech Republic.

D. L. Applegate, R. E. Bixby, V. Chvátal, W. J. Cook: The Traveling Salesman Problem. A Computational Study*, Princeton Series in Applied Mathematics, Princeton University Press, Princeton, 2007, 593 pp., USD 45, ISBN 978-0-691-12993-8*

The traveling salesman problem is probably one of the best known problems in combinatorial optimization: given a collection of cities, find the shortest round-trip route that visits each of the cities exactly once. Though simple to state, finding an optimal solution is far from trivial. The authors of the book have led investigations of the problem for almost two decades and this book presents their findings. The main aim of the book is to explain the theory and algorithms utilized in a computer code (called Concorde), which has successfully solved a number of large scale instances of the problem. Having said that, it must be stressed that programming skills are not prerequisites for reading the book and that the book is written in a readable style.

It opens with an overview of the rich history of the problem, complemented with many illustrations. Chapter 2 contains a description of the main applications of the traveling salesman problem. The next two chapters survey development in the 50s on the traveling salesman problem by Dantzig, Fulkerson and Johnson and later by Grötschel and Padberg. Development on the problem was closely bound up with development in linear programming, and the cutting plane method for linear programming is at the core of chapters 3 and 4. In chapters 5-11, the authors describe their own techniques for finding cutting planes for the problem; these techniques form the core of the Concorde code. This part of the book requires a solid background in linear programming but reading it carefully will pay off. The subject of chapters 12 and 13 is how to manage and solve the large linear programs derived by the techniques described in previous chapters. Chapters 14 and 15 deal with enumeration schemes and with heuristics complementing the cutting plane method. Finally, chapter 16 provides results of the computational tests with Concorde and chapter 17 concludes the book with a sketch of possible future research directions. To summarize, the book provides a comprehensive treatment of the travelling salesman problem and I highly recommend it not only to specialists in the area but to anyone interested in combinatorial optimization. (pkol)

Argos Seminar on Intersections of Modular Correspondences*, Astérisque, no. 312, Société Mathématique de France, Paris, 2007, 208 pp., EUR 47, ISBN 978-2-85629-231-0*

This book, which is a result of a collective effort of the Bonn seminar on arithmetic geometry 2003/04, gives a thorough account of the work of Gross and Keating on (proper) triple intersections of arithmetic Hecke correspondences on the arithmetic threefold Spec **Z**[j,j'] and their relation to Fourier coefficients of the central derivative of a suitable Siegel-Eisenstein series (a special instance of "Kudla's programme"). This is a very pleasant introduction to an important topic in contemporary arithmetic geometry, which also covers some useful background material on supersingular elliptic curves, deformation theory of one-dimensional formal groups and quadratic forms over p-adic integers. (jnek)

S. Asmussen, P.W. Glynn: Stochastic Simulation – Algorithms and Analysis, Stochastic Modelling and Applied Probability*, vol. 57, Springer, Berlin, 2007, 476 pp., EUR 49.95, ISBN 978-0-387-30679-7*

This is a very interesting book for all who are interested in stochastic simulations. The book is divided into two parts. While the first part focuses on general methods, such as generating random variables, output analysis, steady state simulations, variance reduction methods, rare events simulations, derivative estimation and stochastic optimization, the second half discusses model specific algorithms, such as numerical integration via simulations, stochastic differential equations, generation of Gaussian and Levy processes and MCMC. The level of mathematical discussion is different from that presented elsewhere. A quite broad knowledge of probability and statistics is expected. According to the authors, the book is designed as a potential teaching and learning tool for use in a wide variety of courses. I am afraid that it is intended rather for PhD students than for Masters level due to the fact that many topics are only touched upon and a considerable effort is often needed to move from the ideas raised in the book to the well and efficiently working algorithms. I had a feeling in some parts of the book that it is a series of remarks intended more for the authors than for the readers. However, I am convinced that it is a book that should be on the bookshelf of everybody who is seriously interested in stochastic simulations. (jant)

D. H. Bailey, J. M. Borwein, N. J. Calkin, R. Girgenson, D. R. Luke, V. H. Moll: Experimental Mathematics in Action*, A.K. Peters, Wellesley, 2007, 322 pp., USD 49, ISBN 978-1-56881-271-7*

This book contains material originating from a course on Experimental Mathematics in Action, organized by Jonathan Borwein in San Antonio in 2006. One of aims of the book is to defend an assertion that the statement "the real mathematicians don't compute" is no longer valid with a new generation of mathematicians. Computer-aided research, taking advantage of modern computational packages (such as *Maple* or *Mathematica*) has its importance nowadays. The book presents several examples and methodological ways to make clear what computational or experimental mathematics is or should be. Starting with some remarks of philosophical character, the authors present many algorithms for experimental mathematics, such as high-precision arithmetic, integer relation detection, prime number computations and finding the roots of polynomials. Moreover, their aim is also to present (the slightly controversial) idea that "mathematics is done more like physics in that you come about things experimentally". This does not mean that a mathematician should give up proving theorems; this is just a statement about how some of the mathematical facts can be discovered. The book is nicely written, with a special touch of mathematical poetry and beauty-behind-the-computation opinion. It will be appreciated not only by number theoreticians but also by anyone who does not prevent computers from entering the pure garden of mathematical delights. (mr)

S.-M. Belcastro, C. Yackel: Making Mathematics with Needlework*, A.K. Peters, Wellesley, 2008, 184 pp., USD 30, ISBN 978-1-56881-331-8*

Both these authors aim to demonstrate in this book that there is a close relationship between mathematics and the fibrous arts (including knitting, crocheting, cross-stitch and quilting). Every chapter of the book is divided into four parts. The first part is a brief description of the mathematics involved and of the corresponding needlework at a level understandable to both mathematicians and needleworkers. The next part is more technical discussion of the mathematical topics, the third contains teaching ideas and the last part consists of detailed instructions for completing the needlework project. Each chapter presents a different mathematical theme, such as the Quilted Möbius Band (making a möbius quilt), Diophantine Equations (making a bi-directional hat), Sierpinski Variations (making a Sierpinski shawl), Algebraic Structure (making algebraic socks), The Graph Theory of Blackwork Embroidery (embroidering a Holbeinian Graph), making Hyperbolic Pants and so on. The book contains interesting pictures and photographs. Anyone interested in mathematics or needlework will find new, interesting topics and inspiration in the book. (jahr)

O. Bogopolski: Introduction to Group Theory*, EMS Textbooks in Mathematics, European Mathematical Society, Zürich, 2008, 177 pp., EUR 38, ISBN 978-3-03719-041-8*

There are many introductions to group theory. This one tries to be different. Firstly, it skips a lot of standard material to make space for several dives into quite advanced topics and, secondly, it gives preference to combinatorial group theory over finite and matrix groups. The book is divided into three parts of uneven length. The first part starts with basic notions followed by Sylow's theorem. The author then starts to discuss finite simple groups. He begins with automorphisms of the icosahedron, proves the simplicity of A5 and then moves to a representation of the Mathieu group M22 by automorphisms of ovals and lines in an extension of P2(4). This is followed by a brief discussion of extensions and by a description of the Higman-Sims group. The first part ends with a diagram of sporadic groups. The rest of the book is concerned with combinatorial group theory. The second part presents most of the standard topics (Tietze transformations, Reidemeister-Schreier, amalgamated free products, HNN, Kurosh, van Kampen, coverings and Hopfian groups). The exposition relies upon an efficient formalism that may be a bit surprising for the beginner. The third part is aimed at proving the theorem of Bestvina and Handel that describes irreducible outer automorphisms of a free group by means of a train track map. This part relies heavily on the Perron-Frobenius eigenvalue. There are some (but not many) exercises in the book and detailed solutions are provided for more than twenty of them. (ad)

E. Bombieri, W. Gubler: Heights in Diophantine Geometry*, New Mathematical Monographs 4, Cambridge University Press, Cambridge, 2007, 652 pp., GBP 35, ISBN 928-0-521-71229-3*

This remarkable book is an introduction to most of the key areas of diophantine geometry and diophantine approximations (with a notable exception of Baker's method, which would merit a book on its own). The authors do not strive for utmost generality; they give a clear and thorough account of the principal results but they also include a wide range of additional material, some of which has not appeared in book form before. The first seven chapters develop the classical theory of heights and its applications to Diophantine approximations (the unit equation, Roth's theorem and Schmidt's subspace theorem) and to diophantine properties of subvarieties of tori. Chapters 8 to 11 treat geometry and arithmetic of Abelian varieties and Jacobians; they culminate in Bombieri's version of Vojta's proof of Faltings' theorem (the Mordell conjecture). Chapter 12 discusses the abc-conjecture in its various forms and its consequences. Nevanlinna's theory is introduced in chapter 13 as a prelude to Vojta's conjectures in the concluding chapter 14. Three appendices contain, respectively, a thorough summary of algebraic geometry used in the main text (in the language of varieties), ramification theory for number fields and curves, and geometry of numbers. (jnek)

A. Bonato: A Course on the Web Graph*, Graduate Studies in Mathematics, vol. 89, American Mathematical Society, Providence, 2008, 184 pp., USD 45, ISBN 978-0-8218-4467-0*

The web graph is a directed graph whose vertices are web pages and whose oriented edges are links between the pages. The book is a readable and up-to-date exposition of the mathematical theory of web graphs and related real-world self-organizing networks. After some preliminaries, the classical Erdös-Rényi theory of random graphs is briefly introduced in chapter 3, together with the main probabilistic techniques used later on. Chapter 4 presents several stochastic models of the development of a web graph. The models usually start with an initial seed graph, and new vertices and edges are added according to some probabilistic rules. The objective is to capture statistical properties of the web graph, namely the power law degree distribution and the small world property. Chapter 5 exposes the mathematics behind search engines, e.g. the Google ranking algorithms PageRank and the Stochastic Approach for Link Structure Analysis (SALSA). The theory is based on Markov chains and associated matrices. Computational aspects are involved since graphs are huge. Chapter 6 describes an interaction between infinite graph theory and the web graph models. Chapter 7 presents three topics for web graph research: spectra of power law graphs, modelling viruses on the Web and domination in web graph models. (pk)

S. J. Brams: The Presidential Election Game*, A.K. Peters, Wellesley, 2007, 194 pp., USD 29, ISBN 978-1-56881-348-6*

This monograph is devoted to the presidential election system in the United States, and to voting theory in general. In the first three chapters, the author offers mathematical models related to the three stages of the presidential elections in the USA (primaries, party conventions and the general election) and discusses the disadvantages of the Electoral College system. The other chapters deal with coalition politics, the Watergate scandal and a general comparison of voting systems (negative voting and approval voting). The models presented in the book employ only elementary probability theory and game theory. Most results are stated without proof but an extensive list of references is included. There is a wealth of examples from past US presidential elections. (asl)

***V. Burd: Method of Averaging for Differential Equations on an Infinite Interval. Theory and Applications**, Lecture Notes in Pure and Applied Mathematics, vol. 255, Chapman & Hall/CRC, Boca Raton, 2007, 343 pp., USD 152.96, ISBN 978-1-58488-874-1*
This is a very readable book introducing the reader to the method of averaging for systems of ordinary differential equations with almost periodic right hand sides and a small parameter. The existence of almost periodic solutions and stability of equilibria are investigated in detail. Abstract results are applied to concrete equations (such as a general pendulum, van der Pohl and Duffing's equation and the Hopf bifurcation). Equations having slow and fast time are also studied. The book is divided into two parts; the first one describes averaging for linear systems and the second one is devoted to nonlinear systems. There are also three short appendices on almost periodic functions, the Lyapunov first and second methods and basic facts of functional analysis. This clearly written book can be highly recommended to students with interests in ordinary differential equations. Non-experts and researchers in natural sciences will also find interesting methods that are useful in applications. (jmil)

***R. H. Chan, Ch. Greif, D. P. O'Leary: Milestones in Matrix Computation. The Selected Works of Gene H. Golub with Commentaries**, Oxford Science Publications, Oxford University Press, Oxford, 2007, 565 pp., GBP 65, ISBN 978-0-19-920681-0*
At the meeting in Moscow in June 2005, Gil Strang suggested that there should be a collection of Gene Golub's work to highlight his many important contributions to numerical analysis. Three mathematicians: Raymond H. Chan, Chen Greif and Dianne P. O'Leary, were honoured to take this pleasant task, aimed for February 2007, the 75th anniversary of Gene's birth. Twenty-one papers (chosen by Gene Golub) included in the book reveal a lot about his working style, creativity and quickness. His papers are divided into five groups (a majority of the papers are well-known so we will not mention names of co-authors).

"Gene Golub has been a driving force in development and analysis of iterative methods for solving large sparse linear systems," begins Anne Greenbaum in her commentary to the first group of four papers (Iterative methods for linear systems). Two of the included papers deal with Chebyshev semi-iterative methods, SOR and Hermitian and Skew-Hermitian splittings methods. The other two papers deal with the generalized conjugate gradient method for non-symmetric systems and for numerical solutions of elliptic partial differential equations.

Five papers, collected in the second group, are related to various least square problems, including singular value decomposition, numerical methods for solving least squares problems and an analysis of the TLS problem. The commentary to the second part (Solution of least squares problems) has been written by Åke Björck.

Five papers in the third group (Matrix factorizations and applications) illustrate several different facets of the matrix factorization paradigm. The papers deal with calculating singular values, the simplex method using LU decomposition, methods for modifying matrix factorizations and computing angles between linear subspaces. The three papers of the fourth group (Orthogonal polynomials and quadrature) have, according to Walter Gautschi, a distinctly interdisciplinary character in the sense that classical analysis problems are recast in terms of, and successfully solved by, techniques of linear algebra and, vice versa, problems that have a linear algebra flavour are approached and solved using tools of classical analysis. The last group (Eigenvalue problems) is introduced by G. W. Stewart. The papers solve specific problems: a modified matrix eigenvalue problem, an ill-conditioned eigensystem and a computation of the Jordan canonical form, the block Lanczos method and a numerically stable reconstruction of a Jacobi matrix from spectral data. A list of publications, major awards, students of Gene Golub and his fascinating biography are included at the beginning of the book. (jzi)

***P.-L. Chow: Stochastic Partial Differential Equations**, Applied Mathematics and Nonlinear Science Series, Chapman & Hall/CRC, Boca Raton, 2007, 281 pp., USD 79.95, ISBN 978-1-58488-443-9*
Deterministic partial differential equations (PDEs) originated from mathematical models for physics and biology. They gradually developed from specific problems (such as heat conduction) to a deep and complex mathematical theory. Stochastic partial equations have been studied since the 60s to extend the range of applicability both of ordinary stochastic equations (Itô equations) and of the classical PDEs. Among notable applications we may list turbulent flows in fluid mechanics and diffusions in random media. The book is exclusively devoted to a study of stochastic PDEs for a random evolution in Hilbert and Banach spaces. Recall that the first existence result harks back to V. V. Baklan (1963). The book summarizes basic facts about stochastic analysis and ordinary stochastic differential equations. It employs eigenfunction expansions, Green functions and Fourier transforms in a study of stochastic transport and heat-wave equations. Specific examples provide a motivation for the investigation of stochastic evolution equations in Hilbert space; general theorems both on the existence and uniqueness and asymptotic behaviour of solutions are proved. The text may be characterized as an excellent guide to current research topics that opens possibilities for further developments in the field. (jste)

***C. J. Colbourn, J. H. Dinitz: Handbook of Combinatorial Designs**, second edition, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2006, 984 pp., USD 129.95, ISBN 1-58488-506-1*
This is a comprehensive reference book on combinatorial designs. It covers all of the most important results in the field as well as the basic ones. These include not only constructions of designs, their properties and existence results but also many examples and applications of combinatorial designs (in cryptography, communication, networking, statistics, testing and many more). The core of the book is in five parts, four of them being devoted to four main classes of combinatorial designs: balanced incomplete block designs, orthogonal arrays and latin squares, pairwise balanced designs and the Hadamard and orthogonal designs. The fifth part contains sixty-five chapters that concern other classes of designs. Each chapter makes references to related topics, many of them naturally crossing the sections. There are two more parts. The first one is introductory, providing an overview of the field and an historical perspective. The last part gives the necessary mathematical and computational background. It is not expected that the book will be read in a

sequential way. Hence, to have direct access to relevant topics information is given in an easy-to-access way, often captured in a table indexed by parameters of designs. Moreover, the book is equipped with a large index providing even faster navigation through pages. An extensive bibliography with more than 2200 items can serve as an excellent source of information itself. The book has more than 900 pages (prepared by more than 100 contributors). You will find here an answer to any of your questions about combinatorial designs. (os)

S. V. Emelyanov, S. K. Korovin, N. A. Bobylev, A. V. Bulatov: Homotopy of Extremal Problems. Theory and Applications*, de Gruyter Series in Nonlinear Analysis and Applications, vol. 11, Walter de Gruyter, Berlin, 2007, 303 pp., EUR 128, ISBN 978-3-11-018942-1*

This book is entirely devoted to the continuation (i.e. homotopy) method for solving the equation $f(x) = 0$. According to this method, the function f is embedded in a one-parameter family of equations $f(x,\lambda) = 0$, where $f(.) = f(.,1)$ and a solution of $f(x,0) = 0$ is known. The first chapter is introductory and covers the necessary facts from functional analysis. In chapter 2, properties of general and special (e.g. linear) deformations of f are studied in the finite-dimensional case. The converse problem, i.e. the existence of a convenient deformation, is also investigated. In chapter 3, some results of the second chapter are extended to infinite dimensions. The Conley index is introduced in chapter 4 and its homotopy invariance is proved. The last chapter contains applications of the homotopy method to proofs of various classical inequalities, properties of extremals in nonlinear programming and calculus of variations, optimal control and bifurcation of critical points. The book is carefully written and it can be read by graduate students. Physicists and engineers who use variational methods will also find here a good source of information. (jmil)

T. G. Faticoni: Direct Sum Decompositions of Torsion-Free Finite Rank Groups*, Pure and Applied Mathematics, vol. 285, Chapman & Hall/CRC, Boca Raton, 2007, 315 pp., USD 89.96, ISBN 978-1-58488-726-3*

This monograph is devoted to direct sum decompositions of reduced torsion free finite rank (rtffr) Abelian groups. Any Abelian group G of finite rank has an indecomposable decomposition, hence one can study questions related to the uniqueness of indecomposable decompositions of G. The book offers a technique that passes these problems to study the factor of End(G) modulo its nilradical N(End(G)). Let us denote this ring by $E(G)$. The book provides a lot of interesting results not included in other books. The first chapter contains some preliminaries. The second chapter explains some motivation. The Krull-Schmidt-Azumaya and the Baer-Kulikov-Kaplansky theorems are explained as examples of good behaviour. On the other hand, the Corner result and other constructions are mentioned to show almost arbitrarily bad behaviour of direct-sum decompositions of rtffr Abelian groups. The notion of quasi-isomorphism and local isomorphism is introduced and the Jónsson theorem shows that one gets much better behaviour when considering this problem up to a quasi-isomorphism. Chapter 3 explains how the local isomorphism classes of finitely generated projective modules over a semiprime ring having its additive group rtffr are translated into isomorphism classes of finitely generated projectives over a different ring. The next chapter gives results when some commutativity conditions in End(G) are satisfied.

The fifth chapter investigates what can be said about the number of isomorphism classes of groups locally isomorphic to a strongly indecomposable rtffr group with E(G) being a commutative domain. Chapter 6 studies the Baer splitting property. In chapter 8, the author studies Gabriel filters, in particular the filter of divisibility and its relation to the quasi-splitting of some exact sequences. The last chapter returns to E-properties and possible values of homological dimensions of G over End(G) are discussed. The reader of this book is supposed to be rather advanced in Abelian groups. The exposition is almost self-contained, with just a few results going far beyond the scope of the book (for example results from analytic number theory) and which are stated without proofs. A lot of examples illustrate the theory. The author provides a number of exercises and suggestions for further research at the end of each chapter. (ppr)

G. Feldman: Functional Equations and Characterization Problems on Locally Compact Abelian Groups*, Tracts in Mathematics, vol. 5, European Mathematical Society, Zürich, 2008, 256 pp., EUR 58, ISBN 978-3-03719-045-6*

Characterization theorems in mathematical statistics started with the celebrated Kac-Bernstein characterization result, which dates back to 1939 and 1949, stating that the independence of the sum $X_1 + X_2$ and of the difference $X_1 + X_2$ of independent identically distributed L_2-random variables X_1 and X_2 implies that the variables X_j' are Gaussian. The statement was the first among those saying (under some restrictions) that independent linear forms of independent random variables X_j may exist only for Gaussian X_j'. Skitovich and Darmois (1953) solved the problem completely in dimension one and the multidimensional extension was provided by Ghurye and Olkin in 1962. The monograph Characterization theorems in mathematical statistics by A. M. Kagan, Yu. V. Linnik and C. R. Rao (J. Wiley, 1973) provides a full account of achievements in Rd settings. In recent years, the above topic has been extensively studied for random variables with values in more general algebraic structures (such as locally compact Abelian groups, Lie and quantum groups and symmetric spaces). This book contains generalizations of the Kac-Bernstein and Skitovich-Darmois theorems to locally compact Abelian groups and, in particular, the characterization of Gaussian and idempotent probability distributions. Solutions to these problems are transformed to solutions of some functional equations in the set of positive definite functions on the corresponding character group. The self-contained and well-written monograph is aimed at mathematicians interested in probability on algebraic structures and abstract harmonic analysis. Future research may be stimulated by a collection of unsolved problems at the end of the book. (jste)

J. B. Garnett: Bounded Analytic Functions*, Graduate Texts in Mathematics, vol. 236, Springer, Berlin, 2007, 459 pp., EUR 54.95, ISBN 978-0-387-33621-3*

This edition of *Bounded Analytic Functions* is the same as the first edition (Academic Press, New York, 1981) except for the correction of several mathematical and typographical errors. The book contains information about Hardy spaces, conjugate functions, bounded mean oscillation and duality, interpolating sequences, corona construction, Douglas algebras and many other topics. The reader is supposed to have a background in complex analysis and function algebras. The author has selected a wide range of beautiful topics and some of them are very deep. This book is not intended as a starting place for the novice but it is a gold mine for researchers and experts. (shen)

***H.-O. Georgii: Stochastics. Introduction to Probability and Statistics**, de Gruyter Textbook, Walter de Gruyter, Berlin, 2008, 370 pp., EUR 39.95, ISBN 978-3-11-019145-5*

This is a translation of the third edition of the German textbook "Stochastik". It presents fundamental ideas and results of both probability theory and statistics. The areas covered from probability are: principles of modelling chance, stochastic standard models, conditional probabilities and independence, expectation and variance, the law of large numbers and the central limit theorem and Markov chains. The book covers the following areas of mathematical statistics: estimation, confidence regions, the normal distribution, hypothesis testing, asymptotic tests and rank tests, regression models and the analysis of variance. The basic notions and theorems are accompanied by interesting illustrative examples. At the end of each chapter there is a collection of problems offering applications, additions and supplements to the text. The book is primarily aimed at students of mathematics but also to scientists with an interest in the mathematical side of stochastics. Knowledge of abstract mathematics including elements of measure theory is assumed. The author gives a recommendation of how to successfully study the book. The book is well-written and mathematically oriented students and researchers will certainly find that it provides a high level introduction to probability theory and mathematical statistics. (mahu)

***S. Grabarchuk, P. Grabarchuk, S. Grabarchuk Jr.: The Simple Book of Not-So-Simple Puzzles**, A.K. Peters, Wellesley, 2008, 105 pp., USD 19.95, ISBN 978-1-56881-418-6*

This book contains more than 100 original, ingenious and creative "mini-puzzles" of different kinds: grids and patterns, assembling, dividing, number, word, mathematical, matchstick, coin, dot-connecting, dissection, logical, visual, spatial brainteasers, etc. They have a wide range of difficulty levels and the reader will find a mix of easy, moderate and very difficult challenges. Each puzzle is carefully formulated and described, solved, explained and supplemented with further pictures and diagrams. Solving the puzzles could improve mathematical skills, logical thinking, spatial imagination, visual perception, creative working and self-confident approaches to challenges and studies. This collection of puzzles will provide many interesting, fun and enjoyable moments to anyone who solves them. It can be recommended to all readers who like solving brainteasers. (mbec)

K. Gürlebeck, K. Habetha, W. Sprössig: Holomorphic Functions in the Plane and n-dimensional Space (+ CD-Rom), Birkhäuser, Basel, 2008, +394 pp., EUR 34.90, ISBN 978-3-7643-8271-1

This textbook introduces both classical complex analysis and its higher dimensional generalization, Clifford analysis. In higher dimensions, algebra of complex numbers is replaced by non commutative algebra of (real) quaternions or with a Clifford algebra. After developing standard results and notions from classical complex analysis, a discussion of its higher dimensional counterpart follows. In the first chapter complex numbers, quaternions and Clifford numbers are introduced and their algebraic and geometric properties are studied. In chapter 2, holomorphic functions in the plane are defined. After a discussion of possible higher dimensional generalizations, the definition of holomorphic functions is extended to functions in n-dimensional space with values in the corresponding Clifford algebra. Then, 'simple' holomorphic functions in the plane and n-dimensional space are dealt with, including powers and Möbius transformations.

The next chapter presents integral formulae for holomorphic functions in the plane and their higher dimensional analogues, namely the Cauchy integral formula, the Borel-Pompeiu formula and the Plemelj-Sokhotski formula. The last chapter deals with Taylor and Laurent series, isolated singularities of holomorphic functions and the residue theorem, and elementary functions and special functions, including the Gamma function, the Zeta function and automorphic functions. The book is very well written. Each chapter contains historical remarks, examples and exercises illustrating the topics treated. Moreover, the enclosed CD contains an up-to-date literature database and a Maple package solving some of the problems discussed in the text. This book can be recommended as a text for a basic course on complex analysis, as well as a readable introduction to Clifford analysis accessible to students of mathematics and physics. (rl)

***G. Harman: Prime-Detecting Sieves**, London Mathematical Society Monographs, vol. 33, Princeton University Press, Princeton, 2007, 362 pp., USD 65, ISBN 978-0-691-12437-7*

The sieve method has undergone a rapid development in the last few decades and has become an important tool in analytic and combinatorial number theory. This is also reflected in the fact that several very good books devoted to this topic appeared in the last newsletter. This evokes an impression that all the important facets of development in the surrounding theory have already been covered in monograph form. But sometimes this is not the most important aspect. The approach used to describe the main ingredients of the idea makes the point. All this can be found in the book.

The author writes in the preface: "This book charts my own mathematical journey and the adventures of the others in using sieve methods to generate primes". These words illustrate the contents of the book. The author describes very carefully the main ingredients and motivations behind the development of ideas and their impact on solutions of presented problems. The prime-detecting aspects of modern sieve methods are demonstrated in the way they were used to handle such important themes as primes in short intervals, the greatest prime factor of the sequence of shifted primes, Goldbach numbers in short intervals, the distribution of Gaussian primes (including the recent work of Friedlander and Iwaniec on primes that are a sum of a square and a fourth power) and Heath-Brown's work on primes represented as a cube plus twice a cube. The book is written in a very accessible style for a wide spectrum of readers from graduate students to researchers wishing to learn deep ideas of modern sieve methods. Besides mathematical ideas, the presentation also contains many important historical comments, which make the book useful for a general mathematical audience trying to orient themselves in the evolution of the main techniques applied in sieve methods. (spor)

I. Kleiner: A History of Abstract Algebra*, Birkhäuser, Boston, 2007, 168 pp., EUR 39.90, ISBN 978-0-8176-4684-4*

This book gives an overview of the origin and development of the basic ideas of modern abstract algebra. The author shows how abstract algebra has arisen from the study of solutions of polynomial equations to a theory of abstract algebraic structures and axiomatic systems, such as groups, rings and fields. He briefly describes the transition from "classical" to "modern" theory, which happened in the 19th century, i.e. the process of creation and development of modern structural algebra. He demonstrates that abstract algebra became an independent, interesting and flourishing subject of mathematics in the early decades of the twentieth century thanks to the pioneering work of Emmy Noether. In the first short chapter, the author deals with the prehistory and history of classical algebra. He starts with the early roots of algebra and he goes through Greek algebra, Islamic algebraic accomplishments, the discovery of solutions of cubic and quartic equations, and the Fundamental Theorem of Algebra to the birth of symbolic algebra.

In the second chapter, the book indicates the roots of group theory (for example some of the origins of group theory can be found in linear algebra, number theory, geometry and analysis), illustrates the development of specialized theories of groups (for example permutation groups, Abelian groups and transformation groups) and explains the role of abstraction and consolidation of the abstract group concept and further development and accomplishment of modern group theory. The third chapter deals with the creation and development of ring theory. It starts with non-commutative ring theory (hypercomplex number systems, their classification and their structure) and moves on to commutative ring theory, which is analysed from a mathematical and an historical point of view (the most important events and results on algebraic number theory, algebraic geometry and invariant theory are presented). At the end of the chapter, the abstract definition of a ring is covered and interesting results of Emmy Noether and Emil Artin are offered to the reader in a simple, understandable way.

The fourth chapter is divided into nine parts concentrating on the historical aspects of the birth and development of Galois theory, algebraic number theory (Dedekind's and Kronecker's ideas, Hensel's p-adic numbers, etc.), algebraic geometry (fields of algebraic or rational functions), the role of symbolic algebra and the abstract definition of a field. The fifth chapter contains a description of the evolution of linear algebra and its connections with group theory, ring theory and field theory. Only an overview of the fundamental developing aspects of linear algebra is given (for example problems of solving linear equations, the birth and application of determinants, matrices and linear transformations, and the role of linear independence, basis, dimension and vector spaces in the development of algebra). The sixth chapter is devoted to an analysis of Noether's fundamental algebraic works on invariant theory, commutative algebra, non-commutative algebra and representation theory. Applications of Noether's results on non-commutative to commutative algebra are also explained. The seventh chapter offers very interesting suggestions to instructors on how the history of abstract algebra could be integrated into their teaching. In the eighth chapter, there are six biographies of major contributors to the development of modern algebra (A. Cayley, R. Dedekind, E. Galois, C. F. Gauss, W. R. Hamilton and E. Noether) describing their lives and works, which are written as readable mini-essays.

The book is a far from exhaustive account of the history of abstract algebra but for readers who want to pursue the subject in more detail, the author indicates where additional material can be found. In each chapter, the author makes extensive references to relevant literature. The book can be recommended to mathematicians, teachers of mathematics (especially of algebra), historians of the sciences and students, who can find many useful references and ideas for their research, teaching or studies. The book may also serve as a supplementary text for courses on the history of modern mathematics or abstract algebra. (mbec)

A. Knoebel, R. Laubenbacher, J. Lodder, D. Pengelley: Mathematical Masterpieces. Further Chronicles by the Explorers*, Undergraduate Texts in Mathematics, Springer, Berlin, 2007, 333 pp., EUR 32.95, ISBN 978-0-387-33061-7*

This book is closely related to courses of mathematics held for students at New Mexico State University and parts of its preliminary versions were used in courses on the history of mathematics. Four main subjects treated in the reproduced texts are: (1) The bridge between continuous and discrete, describing the transition from figural numbers, sums of powers, the Euler-Maclaurin formula, etc., to Euler's solution of the Basel problem; (2) Solving equations numerically: finding our roots, from which we learn how the problem of finding roots of functions was tackled through centuries, at first for polynomials and later for more general functions; (3) Curvature and the notion of space, covering the story of Riemann's inaugural lecture (the subject was chosen since it was a particular interest to Gauss) and later describing studies of Huygens, Newton and Gauss, culminating with the notion of higher-dimensional space; (4) Patterns in prime numbers: the quadratic reciprocity law, containing contributions by Fermat, Legendre, Euler and others, in particular Eisenstein and Cayley.

The introductory section of each chapter gives the reader an overview of material studied and then leads them step-by-step through more than twenty well-chosen parts of historical texts. The book contains many nice pictures including portraits of mathematicians, parts of old texts, drawings and schemes. An important aspect of the book is the numerous exercises, which should help students to gain a deeper insight into the presented material. Numerous references and well-organized indices make the book easy to use. It can be recommended for university libraries and students with an interest in the history of mathematics presented from a modern point of view. (jive)

J. Kraus, U. Langer, Eds.: Lectures on Advanced Computational Methods in Mechanics*, Radon Series on Computational and Applied Mathematics, vol. 1, Walter de Gruyter, Berlin, 2007, 226 pp., EUR 78, ISBN 978-3-11-019556-9*

This book consists of a collection of carefully written lecture notes delivered as part of the special semester on computational mechanics held in 2005 at the Radon Institute for Computational and Applied Mathematics. It provides valuable overviews of recent developments in four topics. The first lecture (by B. Kaltenbacher and M. Kaltenbacher, on modelling and iterative identification of hystersis via Preisach operators in partial differential equations) deals with mathematical modelling of hysteresis effects, occurring in many different areas in mechanics. In the second lecture (entitled 'Multilevel methods for anisotropic elliptic problems'), J. Kraus and S. Margenov present various preconditioners based on optimal complexity multilevel methods suitable for problems with anisotropy in the mesh and in coefficients of a discrete problem. The third lecture (by S. Nepomnyaschikh on 'Domain decomposition methods') presents methods for developing efficient solvers for large scale numerical problems on massively parallel computers using the domain decomposition method and the fictious space method. The last lecture (by S. Repin with the title 'A posteriori error estimation methods for partial differential equations') gives a nice overview of methods for deriving a posteriori error estimates for partial differential equations especially for FEM discretizations. The first part of this lecture summarizes standard approaches to deriving a posteriori error estimates. In the second part, a recent method (free of any mesh-dependent parameters) is presented. (jomal)

B. Le Stum: Rigid Cohomology*, Cambridge Tracts in Mathematics 172, Cambridge University Press, Cambridge, 2007, 319 pp., GBP 50, ISBN 928-0-521-87524-0*

It is true that p-adic cohomology theories for algebraic varieties defined over fields of characteristic p are more difficult, but often more useful, than ℓ-adic (ℓ different from p) étale cohomology. Rigid cohomology was defined by Berthelot as a common generalisation of crystalline cohomology for smooth proper varieties and the Monsky-Washnitzer cohomology for affine varieties. It is defined in terms of overconvergent de Rham cohomology of suitable characteristic zero lifts; this was exploited by Kedlaya and his followers who developed efficient algorithms that are of interest to cryptographers for computing the number of points on (certain) algebraic varieties defined over finite fields. This book, which is based for the most part on Berthelot's unpublished preprints, is the first monograph on the subject. After a user-friendly introduction, the author explains the geometric background of overconvergence (tubes and strict neighbourhoods) in chapters 2 and 3. The following two chapters treat the analytic and sheaf-theoretical aspects of overconvergence. Chapter 6 studies overconvergent de Rham cohomology, which is reinterpreted in chapter 7 in terms of overconvergent isocrystals (i.e. analogues of lisse sheaves in ℓ-adic cohomology). Chapter 8 treats rigid cohomology of algebraic varieties with coefficients in an overconvergent (F-)isocrystal. The final chapter gives an informal overview of several more advanced aspects of p-adic cohomology. The book is well-written, with a mixture of concrete examples and abstract theory. It is accessible to readers familiar with basic concepts of abstract algebraic geometry and p-adic analytic (rigid) geometry. (jnek)

X. Mao: Stochastic Differential Equations and Applications*, second edition, Horwood Publishing, Chichester, 2007, 422 pp., GBP 34, ISBN 978-1-904275-34-3*

Stochastic modelling via stochastic differential equations (SDE) plays an important role in science and applied science. Professor Mao's book reflects the need of those working in mathematical modelling for a deep but understandable presentation not only of the classical Itô SDE theory (functional and backward equations, and equations of neutral type) but also for a summary of new developments, primarily the Carathéodory and the Cauchy-Marayama approximation procedures, in addition to the classical Picard construction. Approximations are used both to prove existence and uniqueness of solutions and to obtain precise numerical solutions needed in applications. The text also offers an analysis of stability in stochastic modelling. The general Lyapunov method and Razumkin calculus are applied in a study of exponential stability and asymptotic bounds in the SDE environment. An illustration of the modelling by means of stochastic differential equations covers stochastic oscillators, stochastic finance and stochastic neural networks. The book makes SDE theory accessible to beginners in the field and to a wide range of scientists and engineers without forcing the reader through a jungle of mathematical details. (jste)

R. Meyer: Local and Analytic Cyclic Homology*, Tracts in Mathematics, vol. 3, European Mathematical Society, Zürich, 2007, 360 pp., EUR 58, ISBN 978-3-03719-039-5*

This monograph is devoted to new variants of periodic cyclic homology, namely to analytic cyclic homology and local cyclic homology. To a great extent, it is based on the author's thesis. The author first presents shortcomings of periodic cyclic homology and shows that it is useful to introduce analytic and local cyclic homology. He also investigates relations of these new cyclic homologies to Alain Connes' entire cyclic homology. A central part of the monograph studies homological properties of these new cyclic homologies, for example invariance of analytic cyclic homology under homotopies of bounded variation and under analytically nilpotent extensions, the excision theorem, and invariance under the passage to isoradial subalgebras. It is also worth mentioning the relation between local and analytic cyclic homology and K-theory (Chern-Connes character and the universal coefficient theorem). Prerequisites for reading the monograph are functional analysis and homological algebra. For this reason, the author included chapter 1 (Bornological vector spaces and inductive systems) and chapter 3 (The spectral radius of bounded subsets and its applications). As the author himself states, these two chapters can be useful even for mathematicians not at all interested in cyclic homology. Necessary notions from algebra are also included in the appendix "Algebraic preliminaries". The monograph is designed primarily for specialists, including postgraduate students. But in any case we must say that it is very well written. The index and the notation and symbols section are also very helpful. (jiva)

M. Miklavčič: Applied Functional Analysis and Partial Differential Equations*, World Scientific, Singapore, 2001, 294 pp., USD 44, ISBN 981-02-3535-6*

This book is an introduction to partial differential equations (PDEs) and the relevant functional analysis tools that PDEs require. The contents of the book is organized in such a way that an undergraduate student, with a basic knowledge of calculus, Lebesgue integrals and the Cauchy theorem of complex analysis, can read it without problems. The other material in the book is self-contained, giving a very nice introduction to functional-analytical tools needed to study the theory of PDEs. The basics of linear operators in Banach and Hilbert spaces are given at the beginning of the book. Then, Sobolev spaces are introduced together with basic properties of distributions and Fourier transforms. Elliptic problems are studied together with the question of regularity of weak solutions to these. In the last three chapters, the reader is led through the theory of semigroups of linear operators, weakly nonlinear elliptic problems and semilinear parabolic equations. The finite difference method and the method of Galerkin approximations (for both parabolic and wave equations) are given as examples. The chapter on semilinear parabolic equations includes studies of the stability of fluid flows and, more generally, of the dynamics generated by dissipative systems, numerical PDEs, elliptic and hyperbolic PDEs, and quantum mechanics. The book is nicely written and is self-contained. Therefore it can be recommended as one of the basic readings for the study of PDEs. (mr)

G. L. Mullen, C. Mummert: Finite Fields and Applications*, Student Mathematical Library, vol. 41, American Mathematical Society, Providence, 2007, 175 pp., USD 35, ISBN 978-0-8218-4418-2*

This book is based on lecture notes for a course which was taught at Pennsylvania State University as part of the Mathematics Advanced Study Semesters (MASS) program. The first chapter is devoted to the theory of finite fields. After covering their elementary properties, the authors discuss the trace and norm functions, bases for extension fields (dual, normal and primitive bases) and polynomials over finite fields (orders of polynomials, counting irreducible polynomials, properties of linearized polynomials and permutation polynomials). The remaining three chapters deal with applications. Chapter 2 explains applications of finite fields to the construction of Latin squares, affine and projective planes, block designs and Hadamard matrices, while chapter 3 is devoted to algebraic coding theory. Constructions of various kinds of linear codes are given and relations between codes, Latin squares and combinatorial designs are presented. The final chapter covers some elementary aspects of cryptography. The RSA cryptosystem and a double-round quadratic system along with key exchange systems are described. The chapter also contains a brief discussion of threshold systems, digital signatures and several cryptosystems based on elliptic curves and Dickson polynomials. Each chapter concludes with notes describing references for further reading and exercises of varying levels of difficulty. Hints and partial solutions for many of the exercises are contained in an appendix. The book provides a brief introduction to the theory of finite fields and to some of their applications. It is accessible for advanced undergraduate students. (lba)

M. D. Neusel: Invariant Theory*, Student Mathematical Library, vol. 36, American Mathematical Society, Providence, 2007, 314 pp., USD 49, ISBN 978-0-8218-4132-7*

This book presents an introduction to invariant theory of finite groups acting on polynomial algebras of characteristic 0. That is, having a vector space V over a field $\boldsymbol{F}$ of characteristic 0, a finite group G and a representation ρ of G in GL(V), we have an action of G on the polynomial algebra F [V]. The main object of study is its subalgebra $\boldsymbol{F}[V]^G$ given by polynomials that are fixed by all elements of G. The book is divided into five parts and one appendix. The first part collects together basic notions of groups, linear representations, associative rings and algebras. The second part presents the so-called Göbel's bound for generators of $\boldsymbol{F}[V]^G$ provided ρ is a permutation representation. This result (taken from a quite recent thesis of M. Göbel) is used in the next part to prove the classical theorem stating that $\boldsymbol{F}[V]^G$ is a finitely generated ring. This theorem is then improved and it is shown how to find generators of $\boldsymbol{F}[V]^G$ as images of polarized elementary symmetric polynomials. Part 4 explains concepts from commutative algebra and module theory and uses them to provide a different attitude to results obtained earlier. The reader can compare the efficiency of these proofs with the algorithmic approach from parts 2 and 3. The last part of the book uses some counting with power series (Poincaré series) to prove the Shephard-Todd-Chevalley theorem, which characterizes finite subgroups of GL(V) generated by pseudoreflections as those finite groups G of GL(V) having $\boldsymbol{F}[V]^G$ isomorphic to a polynomial algebra. An appendix on rational invariants concludes the book.

Although I think that a couple of proofs could be written with more details (for example, in the proof of proposition 6.7, it is shown that the transfers of all variables lie in the image of some restriction of the Noether map but I would need a hint to finish the proof since these transfers do not generate the whole $\boldsymbol{F}[V]^G$), a large part of the book is written in a friendly style and all notions are carefully explained and immediately demonstrated in concrete examples. Each chapter contains a lot of exercises. Moreover, one has to appreciate the many applications of invariant theory to other parts of mathematics, physics and engineering the author has inserted in appropriate places during the exposition. (ppr)

S. Oda, K. Yoshida: Simple Extensions with the Minimum Degree Relations of Integral Domains*, Lecture Notes in Pure and Applied Mathematics, vol. 253, Chapman & Hall/CRC, Boca Raton, 2007, 277 pp., USD 152.96, ISBN 978-1-58488-851-2*

This monograph is devoted to a study of structure and properties of a simple algebraic extension of a Noetherian integral domain. It generalizes classical and naturally motivated questions in commutative algebra about algebraic field extensions but the techniques of study are completely different. The most fruitful tool for solving the problem is an investigation of flatness of an extension as a module over an original ring or over another suitable extension. The authors focus mainly on the case of anti-integral extension and naturally defined and rather common classes of extensions that satisfy stronger conditions (such as super-primitive and ultra-primitive extensions). Suppose that R is a Noetherian domain, K is its quotient field and α is an algebraic element over K. Let $I_{[\alpha]} = \bigcap(R:_R\eta_i)$, where η_i are coefficients of the monic minimal polynomial ϕ_α of α over K, then α is called an anti-integral element of degree d over R if $I_{[\alpha]}\,\phi_\alpha\, R[X]$ is precisely the kernel of the natural projection $R[X]$ onto $R[\alpha]$. Anti-integral elements and the corresponding anti-integral extensions are the central notions of the first two chapters, where their basic properties are proved. Extensions $R<\alpha> = R[\alpha] \cap R[\alpha^{-1}]$ for an anti-integral element α and their generalizations are studied in the large third chapter (note $R<\alpha> = R$ provided α is anti-integral of degree 1), and connections between flatness and excellent elements are shown in the fourth chapter.

The next chapter treats unramified extensions, i.e. extensions S of R for which every prime ideal of S is unramified over

R, and with their differential modules. Chapter 6 is an investigation of subgroups of units in a simple extension. The following two chapters contain descriptions of particular classes of extensions such as super-primitive, exclusive, pure and ultraprimitive. The ninth chapter deepens and extends some previous results on flatness and the last two chapters are devoted to a study of (simple birational) extensions of polynomial rings over Noetherian domains. The monograph sums up results that the authors and their collaborators have obtained during more than ten years of intensive research and publication activity. All topics of the book, which is almost absolutely self-contained, are developed in a clear way and illustrated by many examples. The monograph might be useful for researchers and students interested in methods of modern commutative algebra, particularly for those who need to become familiar with birational ring extensions. (jž)

H. Park, G. Regensburger: Gröbner bases in control theory and signal processing, *Radon Series on Computational and Applied Mathematics 3, Walter de Gruyter, Berlin, 2007, 251 pp., EUR 118, ISBN 978-3-11-019333-6*

This volume grew out of the D3 Workshop on Gröbner bases in control theory and signal processing (18-19 May 2006) held in Linz, Austria, under the auspices of the Special Semester on Gröbner Bases and Related Methods. The special semester was organized in the spring and summer of 2006 by the Johann Radon Institute for Computational and Applied Mathematics (RICAM) in close cooperation with the Research Institute for Symbolic Computation (RISC). The book collects survey articles and original research papers by some leading experts in the area and it offers a review of the state of art in the subject. The topics covered include Gröbner bases in multidimensional systems, the Quillen-Suslin theorem and systems theory, statistical signal processing, parametric problems in control theory, stability of multidimensional input/output systems, wavelets and filter design, synthesis of multidimensional control systems and time-varying linear systems. The volume is a useful reference book for researchers and practitioners. Many interesting new applications of Gröbner bases have been found in multidimensional signal processing. In particular, Gröbner bases proved to be a powerful tool in designing multidimensional filter banks and have shown a good potential for solving partial differential equations. (knaj)

P. Puzzi, J. Serrin: The Maximum Principle, *Progress in Nonlinear Differential Equations and Their Applications, vol. 73, Birkhäuser, Basel, 2007, 234 pp., EUR 49.90, ISBN 978-3-7643-8144-8*

This book deals with a basic notion in the theory of second order elliptic equations, i.e. the maximum principle. Combined with other notions, this principle provides a large amount of information on properties of solutions to, for example, elliptic problems. The authors study, chapter by chapter, the tangency and comparison theorems (starting from the results of Eberhard Hopf), the maximum principles for divergence structure of elliptic differential equations (for more general operators) and the two-point boundary value problems for nonlinear ordinary differential equations. The latter results are a preliminary to strong maximum principles, which are studied in chapter 5. The last chapters deal with maximum principles for the complete quasilinear divergence inequality and with local boundedness and Harnack's inequalities. In the applications chapter we learn about the symmetry for overdetermined boundary value problems, the phenomenon of dead cores and the strong maximum principle for Riemannian manifolds. The book is well written, with a high mathematical standard. The book is meant for a wide audience of all those interested in the theory of partial differential equations, although some experience in the field is necessary (basic knowledge on existence and uniqueness results and an overview of the theory of elliptic partial differential equations), according to the opinion of the reviewer. The book will be appreciated by specialists in the field as well as by PhD students searching for an advanced book on the topic. (mr)

B. Randrianantoanina, N. Randrianantoanina, Eds.: Banach Spaces and their Applications in Analysis, *Walter de Gruyter, Berlin, 2007, 453 pp., EUR 138, ISBN 978-3-11-019449-4*

In recent years there has been a surge of profound new developments in various aspects of analysis by the use of Banach space methods. Many problems, seemingly far from the classical geometry of Banach spaces, have been solved by incorporating Banach space techniques. This book contains papers by participants of the conference "Banach spaces and their applications in analysis" held in May 2006 at Miami University in Oxford, Ohio, in honour of Nigel Kalton's 60th birthday. At this conference, specialists who have been instrumental in these new developments were brought together. The emphasis of the conference was on applications of Banach spaces in the following areas: nonlinear theory (including Lipschitz classification of Banach and metric spaces and linear programming methods), isomorphic theory of Banach spaces (including connections with combinatorics and set theory), algebraic and homological methods in Banach spaces, approximation theory and algorithms in Banach spaces and functional calculus and applications to partial differential equations. The book contains 11 papers by plenary speakers and 16 specialized papers by participants of the conference; the introductory survey of part of the work of N. Kalton is written by G. Godefroy. This article, as well as the rest of the entire volume, illustrates the power of applications of Banach spaces methods and underlying connections between seemingly distant areas of mathematics. (jsp)

C. E. Silva: Invitation to Ergodic Theory, *Student Mathematical Library, vol. 42, American Mathematical Society, Providence, 2008, 262 pp., USD 45, ISBN 978-0-8218-4420-5*

Another introductory book on ergodic theory enters the rich collection that was started by P. R. Halmos and his excellent *Lectures on Ergodic Theory* in 1956 with more recent additions from M.G. Nadkarni, *Basic Ergodic Theory*, Birkhäuser, 1995, and M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge, 2002. This book is meant as a 'special topics' course aimed at students fairly familiar with elements of real analysis, while Lebesgue measure and integration are developed as needed in chapters 1 and 4, to be later on applied in measurable dynamics. Some metric space topology (including the Baire category theorem) is presented in appendix B to support a treatment of topological dynamics. The core of the text is to be found in chapters 2, 5 and 6, where concepts such as recurrence, ergodicity and mixing are treated. The author develops in detail several examples to illustrate concepts of classical ergodic theory (such as the baker's transformation, irrational rotations, the dyadic odometer and the Gauss, Kakutani and Chacon transformations). The text is accompanied by some exercises and open problems formulated with the aim of providing the reader with an orientation to current research in ergodic theory. (jste)

R. J. Swift, S. A. Wirkus: A Course in Ordinary Differential Equations*, Chapman & Hall/CRC, Boca Raton, 2006, 667 pp., USD 94.95, ISBN 978-1-58488-476-7*

This book serves as a guide to a basic course in ordinary differential equations for students that do not have mathematics and differential equations as their main subject. It concentrates on finding solutions to concrete equations. There is no abstract theory (existence and uniqueness); there are just theorems necessary for solving usual types of equations and for understanding the solving algorithms (without proofs). On the other hand, the book contains very detailed explanations of all solving procedures and the theory so it is intelligible for a wide range of students. Moreover, it contains codes for solving equations via Matlab, Maple or Mathematica, which may be of help to many students, professors, scientists and others. The topics dealt with are: first order equations, linear higher order equations, systems of linear equations, the Laplace transform and the power series method. Two chapters are devoted to each of the first three topics. The first of the two chapters contains abstract theory, whilst the second chapter contains algorithms for solving the equations and the codes for Matlab, Maple and Mathematica. The book also contains sections about phase diagrams and bifurcations with appendices introducing Maple, Matlab and Mathematica and some basic linear algebra. The book is self-contained; only knowledge of basic calculus is needed. Each topic is supplemented with many examples, exercises and applications to physical, chemical and biological models. (tba)

S. R. S. Varadhan: Stochastic Processes*, Courant Lecture Notes in Mathematics 16, American Mathematical Society, Providence, 2007, 126 pp., USD 29, ISBN 978-0-8218-4085-6*

This text is a continuation to Varadhan's Probability Theory published by the Courant Institute in 2001 and it is again an excellent introductory text. The book deals with elementary continuous time processes assuming a reader who might be in need of a soft and perhaps more intuitive introduction to the mathematical complexity of the contemporary theory of stochastic processes as presented, for example, in O. Kallenberg's *Foundations of Modern Probability* (Springer 2002). The topics are chosen carefully to cover all essential stochastic dynamic models in dimension one. The main topics are Martingales, processes with independent increments, the Poisson process, point processes, jump Markov processes (semigroups of operators, recurrence and transience, and invariant distributions), Brownian motion, stochastic integrals and calculus (including the Girzanov theorem and the Feynman-Kac formula), stochastic differential equations and 1-dimensional diffusions. The volume covers contents of the author's courses given at the Courant Institute. The reader is assumed to be familiar with the basics of probability and measure theory but a deeper knowledge of mathematical analysis is also crucial. The text is one of those that may be strongly recommended to all young mathematicians as a starter to precede a deeper study of probability and stochastic processes. (jste)

U. Zannier, Ed.: Colloquium De Giorgi 2006*, Edizioni Della Normale, Pisa, 2006, 57 pp., EUR 16, ISBN 978-88-7642-212-6*

In 2001, the Scuola Normale Superiore di Pisa started a tradition of regular colloquia ("Colloquio De Giorgi ") held once a month providing a (non-technical) overview of a certain field of mathematics. Recently, invited speakers have prepared written versions of their talks. This booklet contains, for the first time, written expositions of talks presented in 2006. There is also a list of talks in previous years summarizing the history of the colloquia from its very beginning. In the book, the reader can find seven lectures by Y. F. Bilu (on Diophantine equations with separated variables), C. De Concini (on certain branching rules for quantized enveloping algebras), S. Gindikin (on a role of the Cauchy integral formula in harmonic analysis on symmetric Stein manifolds), D. Goldfeld (on the history of the Gauss class number problem), D. Masser (on polarizations on Abelian varieties), Z. Rudnick (on the statistical distribution of eigenvalues of the Laplacian on compact manifolds) and by L. Szpiro and T. J. Tucker (on dynamics of maps on an algebraic variety). The book offers a very nice collection of (short) reviews of many interesting fields of mathematics. (vs)

S. Zdravkovska, P. L. Duren, Eds.: Golden Years of Moscow Mathematics*, second edition, History of Mathematics, vol. 6, American Mathematical Society, Providence, 2007, 306 pp., USD 59, ISBN 978-0-8218-4261-4*

This book contains articles on the history of mathematics in Moscow throughout the era of the Soviet Union. The articles focus on the development of mathematics, the personal lives and the political events influencing scientific work and life in the Soviet Union. The authors use many documents that were released only after perestroika. The book starts with a description of the Moscow Mathematical School in the 30s, when the attention of Russian mathematicians was focused on function theory, probability theory, applied mathematics, etc. Scientific results of N. N. Luzin, V. I. Smirnov, D. E. Menshov, O. Yu. Schmidt, V. V. Golubev and many others are described here. It is followed by an analysis of mathematics at the Moscow State University in the late 40s and the early 50s. The education systems, the system of scientific degrees, the important research activities, the international collaborations and the lives and professional activities of the most important mathematicians (for example A. N. Kolmogorov, A. A. Markov and P. S. Alexandrov) are discussed.

The book continues with a characterization of mathematical scientific work in the atmosphere of the 50s and 60s. The book finishes with A. B. Sossinsky's personal confessions on his way between two continents and cultures and on his experiences and contacts with several Russian mathematicians. This part gives a description of the atmosphere and events in Russia from the 70s up to 90s. In this second edition, there is a very interesting article written by Tikhomirov giving a general overview of 20th century Moscow mathematics. The book also contains a brief survey of literature on the development of mathematics in the Soviet Union (in English and Russian), which can serve as an introduction to the history of mathematics in the USSR. For a better orientation, the survey is organized in sections with separate references. At the end of the book, there is an index of names and some photos. The book can be recommended to mathematicians, historians and all people who are interested in Soviet mathematical history. (mbec)

BIRKHÄUSER

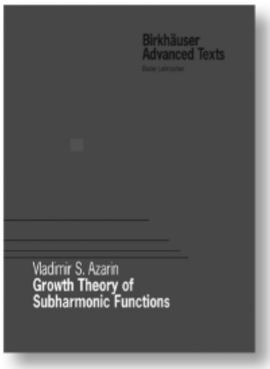
Dimension and Recurrence in Hyperbolic Dynamics

Barreira, L., Universidade Tecnica Lisboa, Portugal

2008. XIV, 300 p. Hardcover
EUR 49.90 / CHF 89.90
ISBN 978-3-7643-8881-2
PM — Progress in Mathematics, Vol. 272

The main objective of this book is to give a broad unified introduction to the study of dimension and recurrence in hyperbolic dynamics. It includes the discussion of the foundations, main results, and main techniques in the rich interplay of four main areas of research: hyperbolic dynamics, dimension theory, multifractal analysis, and quantitative recurrence. It also gives a panorama of several selected topics of current research interest. All the results are included with detailed proofs, many of them simplified or rewritten on purpose for the book. The text is self-contained.

Winner of the Ferran Sunyer i Balaguer Prize 2008



Growth Theory of Subharmonic Functions

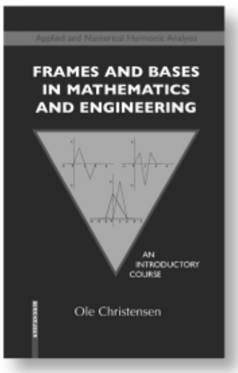
Azarin, V.S., Bar Ilan University, Ramat Gan, Israel

2009. Approx. 340 p. Hardcover
EUR 59.90 / CHF 105.00
ISBN 978-3-7643-8885-0
BAT — Birkhäuser Advanced Texts / Basler Lehrbücher

Due in 10.2008

In this book an account of the growth theory of subharmonic functions is given, which is directed towards its applications to entire functions of one and several complex variables. The presentation aims at converting the noble art of constructing an entire function with prescribed asymptotic behaviour to a handicraft. For this one should only construct the limit set that describes the asymptotic behaviour of the entire function. All necessary material is developed within the book, hence it will be most useful as a reference book for the construction of entire functions.

Contents:
1. Preface.- 2. Auxiliary information. Subharmonic functions.- 3. Asymptotic behavior of subharmonic functions of finite order.- 4. Structure of the limit sets.- 5. Applications to entire functions.- 6. Application to the completeness of exponential systems in a convex domain and the multiplicator problem.- Notation.- List of terms.- References.



Frames and Bases

An Introductory Course

Christensen, O., Technical University of Denmark, Lyngby, Denmark

2008. XVIII, 313 p. 14 illus. Hardcover
EUR 39.90 / CHF 69.90
ISBN 978-0-8176-4677-6
ANHA — Applied and Numerical Harmonic Analysis

Based on a streamlined presentation of the author's previous work, An Introduction to Frames and Riesz Bases, this new textbook fills a gap in the literature, developing frame theory as part of a dialogue between mathematicians and engineers. Newly added sections on applications will help mathematically oriented readers to see where frames are used in practice and engineers to discover the mathematical background for applications in their field. Key features include results presented in a way that is accessible to graduate students, mathematicians, and engineers; a presentation of basic results in finite-dimensional vector spaces, enabling readers with a basic knowledge of linear algebra to understand the idea behind frames without the technical complications in infinite-dimensional spaces; extensive exercises; and a detailed description of frames with full proofs, an examination of the relationship between frames and Riesz bases, and a discussion of various ways to construct frames. Frames and Bases: An Introductory Course will be an excellent textbook for graduate students as well as a good reference for researchers working in pure and applied mathematics, mathematical physics, and engineering. Practitioners working in digital signal processing who wish to understand the theory behind many modern signal processing tools may also find the book a useful self-study resource.

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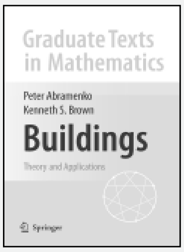


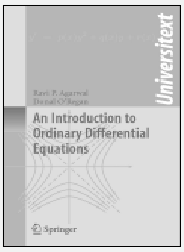
Buildings

Theory and Applications

P. Abramenko, University of Virginia, Charlottesville, VA, USA; **K. S. Brown**, Cornell University, Ithaca, NY, USA

This book treats Jacques Tit's beautiful theory of buildings, making that theory accessible to readers with minimal background. It covers all three approaches to buildings, so that the reader can choose to concentrate on one particular approach. Beginners can use parts of the new book as a friendly introduction to buildings, but the book also contains valuable material for the active researcher. This book is suitable as a textbook, with many exercises, and it may also be used for self-study.

2008. XXII, 754 p. 100 illus. (Graduate Texts in Mathematics, Volume 248) Hardcover
ISBN 978-0-387-78834-0 ► **€ 46,95 | £37.99**



An Introduction to Ordinary Differential Equations

R. P. Agarwal, Florida Institute of Technology, Melbourne, FL, USA; **D. O'Regan**, National University of Ireland, Galway, Ireland

This textbook provides a rigorous and lucid introduction to the theory of ordinary differential equations (ODEs), which serve as mathematical models for many exciting real-world problems in science, engineering, and other disciplines.
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2008. XII, 326 p. 8 illus. (Universitext) Softcover
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Classical Fourier Analysis Second Edition

L. Grafakos, University of Missouri, Columbia, MO, USA

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Naive Lie Theory

J. Stillwell, University of San Francisco, CA, USA

In this new textbook, acclaimed author John Stillwell presents a lucid introduction to Lie theory suitable for junior and senior level undergraduates. In order to achieve this, he focuses on the so-called "classical groups" that capture the symmetries of real, complex, and quaternion spaces. These symmetry groups may be represented by matrices, which allows them to be studied by elementary methods from calculus and linear algebra.
This naive approach to Lie theory is originally due to von Neumann, and it is now possible to streamline it by using standard results of undergraduate mathematics. To compensate for the limitations of the naive approach, end of chapter discussions introduce important results beyond those proved in the book, as part of an informal sketch of Lie theory and its history.

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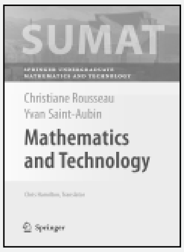

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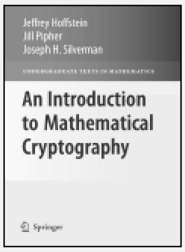

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