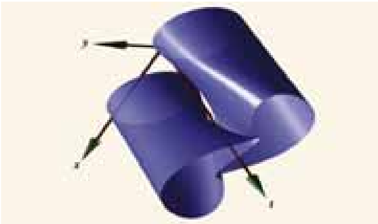
# NEWSLETTER

## OF THE EUROPEAN MATHEMATICAL SOCIETY

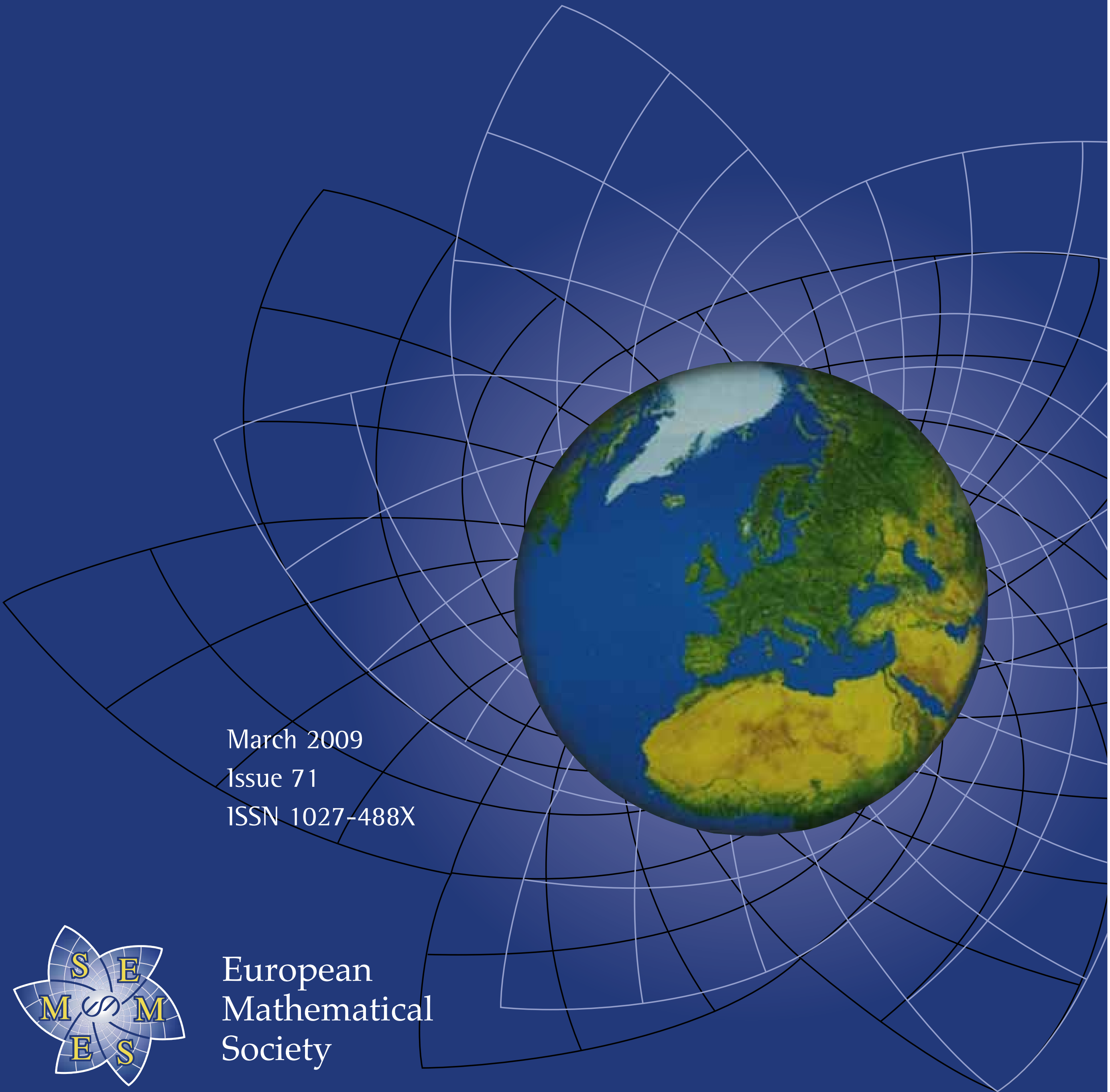


**Feature**
Contour dynamics
p. 25

**History**
Stefan Banach
p. 29

**Interview**
John Ball
p. 35

**ERCOM**
ICTP
p. 41



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# European Mathematical Society

## Newsletter No. 71, March 2009



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# EMS Agenda

## 2009

**4–8 March**
4th World Conference on 21st Century Mathematics,
Lahore (Pakistan), wc2009.sms.edu.pk

**21–22 March**
EMS Executive Committee Meeting, Athens, Greece
Stephen Huggett: s.huggett@plymouth.ac.uk

**1 May**
Deadline for submission of material for the June issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@imaff.cfmac.csic.es

**9–10 May**
Meeting of Presidents of national mathematical Societies,
Banach Center, Bedlewo, Poland
Ari Laptev: laptev@kth.se

**21 May**
International Council for Industrial and Applied Mathematics
(ICIAM) Board meeting, Oslo (Norway)

**8–11 June**
25$^{th}$ Nordic and 1st British-Nordic Congress of Mathematicians
University of Oslo, Norway
http://www.math.uio.no/2009/

## 2010

**19–27 August**
International Congress of Mathematicians, ICM2010,
Hyderabad (India), www.icm2010.org.in

## 2012

**2–7 July**
6$^{th}$ European Mathematical Congress, Kraków (Poland)
www.euro-math-soc.eu

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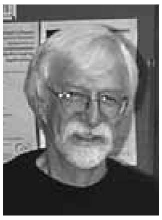
# Editorial

Klaus Schmidt



I was a member of the Executive Committee (EC) of the EMS from 2004 to 2008. Due to conflicting demands on my time from other commitments I was unable to extend my membership of that committee beyond 2008 and with regret I left the EC in December 2008.

My time on the EC was both interesting and rewarding. The two presidents of the EMS I served under were Sir John Kingman and Ari Laptev, both of whom impressed me very much with their dedication to the cause of European Mathematics and their energy in pursuing this cause. The regular meetings of the EC were always a pleasure to attend (not only because they were usually held in nice locations) and I enjoyed the friendly atmosphere, the quality and sincerity of the discussions and the insights they provided into the challenges facing both the EMS and European mathematics.

Before turning to some of these challenges I should recall a little bit of the history of the EMS.

The EMS was founded in 1990 but its origins go back to attempts by the European Science Foundation in 1976 to improve cooperation between European mathematical societies. At the International Congress of Mathematicians in 1978 a *European Mathematical Council* (EMC) was created to facilitate this cooperation but efforts in this direction were hampered by the political divisions of Europe at that time.

The political changes in 1989 raised hopes and offered opportunities for new developments, and in October 1990 a meeting was hosted by the Polish Academy of Sciences, under the auspices of the EMC, to draw up the statutes of a *European Mathematical Society*.

The cultural and scientific diversity of Europe, from which this continent draws much of its strength and creativity, was something of a mixed blessing for attempts to forge a union of national mathematical societies but after some lively discussions the statutes of the EMS, as a society composed of individual members (rather than a federation of national mathematical societies), were accepted. The mathematical societies participating in that meeting joined the newly established EMS as full members and the precursor of the EMS, the EMC, was formally dissolved.

The establishment of the EMS as a mathematical society separate from the national societies has financial implications. Practically all the money of the EMS comes from individual and institutional membership fees but the percentage of members of individual mathematical societies who have joined the EMS varies enormously, largely due to the varying degrees of encouragement by these national societies. Taken across Europe, the percentage of mathematicians who have joined the EMS is still woefully low. The resulting financial tightness prevents the EMS from taking on some responsibilities that cannot be borne by national societies but which would be of the utmost importance for European mathematics.

Some of the activities of the EMS are very successful in spite of its operation on a very restricted budget (examples are the coordination of conferences, in particular the European Congress of Mathematics, and the EMS Publishing House is another).

In other respects the success of the EMS has been more modest. Let me mention an example which is indicative both of the success and the limitations of mathematical cooperation in Europe.

In 1995, the IHÉS in Bures-sur-Yvette, the Isaac Newton Institute in Cambridge and the Max-Planck-Institut für Mathematik in Bonn founded the *European Post-Doctoral Institute for Mathematical Sciences* (EPDI) to *encourage mobility of young scientists on a European scale*. The EPDI was subsequently enlarged with the inclusion of the Max-Planck-Institut für Mathematik in den Naturwissenschaften in Leipzig, the Mittag-Leffler Institute in Djursholm, the Erwin Schrödinger Institute in Vienna, the Banach Center in Warsaw, the Centre de Recerca Matemàtica in Barcelona, the Forschungsinstitut für Mathematik at the ETH Zürich and the Mathematisches Forschungsinstitut Oberwolfach. All these institutes are members of ERCOM (European Research Centres on Mathematics), one of the committees of the EMS which has the purpose of liaison and cooperation between these centres.

Each year the EPDI offers 7 two-year positions to outstanding European post-docs to give them the opportunity to work at EPDI Institutes of their choice for periods of 6 to 12 months at each selected institution.

Although this scheme is quite limited in terms of numbers and funding, it attracts excellent applicants, whose number exceeds the number of available positions by a factor of five. However, many of the best selected candidates for the EPDI are lost every year to competing offers by leading US universities.

The reason for this is not only the excellence of these US universities but also the attractiveness of the huge US market for post-doc and tenure-track positions, the outstanding working environments at some of its scientific institutions and the great diversity of funding sources available in the US.

A small, and perhaps symbolic, measure to assist young mathematicians seeking positions in Europe (and perhaps help programmes like the EPDI in the process) would be an expansion of the EMS database of mathematical post-doc and longer term academic positions at http://www.euro-math-soc.eu/jobs.html, modelled on the AMS services 'EIMS' and 'Mathjobs' available at http://www.ams.org/eims/ and http://www.mathjobs.org/jobs respectively. The provision of a comprehensive database of mathematical job offers would give easy access to employment opportunities for mathematicians across Europe and would require only a moderate amount of money, resources and manpower.

More substantial progress towards improving the prospects of mathematical research in Europe will be possible only through EU funding. Thanks to efforts by many people (notably Ari Laptev, President of the EMS), ERCOM has now been recognized as one of the European Research Infrastructures for the next EU call in September 2009. This is significant progress that underlines the important role of the EMS in the scientific development of the European Union.

The EMS not only helps to get the voice of mathematics heard by European politicians and civil servants, it also brings direct benefits to its individual members. For example, *EMS members will soon be provided with free access to the Zentralblatt MATH database.*

**If mathematicians want to enable the EMS to successfully pursue its aim of benefiting European mathematics, they need to join it and to strengthen its role as an organization representing the needs and interests of mathematical researchers across Europe.**

# New members of the editorial board of the EMS Newsletter

*Starting in 2009, three new members will join the editorial board of the Newsletter of the EMS: Jorge Buescu, Dmitry Feichtner-Kozlov and Erhard Scholz. Thanks are due to Giuseppe Anichini, Ana Bela Cruzeiro and Walter Purkert, who all left the editorial board at the end of 2008, after many years of work in the Newsletter.*

**Jorge Buescu** is a professor of mathematics in the faculty of science of the University of Lisbon. He was a student of Ian Stewart at the University of Warwick and is an active research mathematician with interests in dynamical systems and differential equations, the theory of computation and functional analysis. Besides research papers, he has published two mathematics books in Birkhäuser: *Exotic attractors – from Liapunov stability to riddled basins* (1997) and *Bifurcation, symmetry and patterns* (2002) as editor.

He is also strongly committed to the dissemination of mathematical knowledge to the general public, writing articles and giving general talks at universities, schools and other bodies. He has had three books (in Portuguese) devoted to the popularisation of mathematics published at *Gradiva*. He is currently the editor-in-chief of *Gazeta de Matemática*, a publication of the Portuguese Mathematical Society.

**Dmitry Feichtner-Kozlov** works at the interface of discrete mathematics, algebraic topology and theoretical computer science. He is the author of over 40 research and survey articles and has recently published a book *Combinatorial Algebraic Topology* (Springer Verlag) on the topic.

He has been a recipient of the Wallenberg Prize of the Swedish Mathematics Society (2003), the Gustafsson Prize of the Goran Gustafsson Foundation (2004) and the European Prize in Combinatorics (2005).

He obtained his doctorate from the Royal Institute of Technology, Stockholm, in 1996. After longer stays at the Mathematical Sciences Research Institute at Berkeley, the Massachusetts Institute of Technology, the Institute for Advanced Study at Princeton, the University of Washington, Seattle, and Bern University, he became a senior lecturer at the Royal Institute of Technology, Stockholm, and an assistant professor at the ETH Zurich.

Currently he holds the Chair of Algebra and Geometry at the University of Bremen, Germany.

He is married to a professor of mathematics, who works at the same department, and has a 2-year-old daughter.

**Erhard Scholz** was a student of mathematics and physics at the universities of Bonn (Germany) and Warwick (UK), obtaining a Diplom at Bonn in 1975 and completing his PhD in 1979 at Bonn, with a thesis on the history of the concept of manifold.

He then obtained an assistantship at the University of Wuppertal, completing his Habilitation in 1986 on case studies relating theoretical mathematics and applications in the 19$^{th}$ century. After being a lecturer at the Universities of Wuppertal and Bonn in 1984, he became a professor of the history of mathematics at the University of Wuppertal in the latter part of 1989. During the summer of 1993, he obtained a guest professorship at the Institute for History of Science at the University of Göttingen.

Erhard Scholz is coeditor, with E. Brieskorn, F. Hirzebruch, W. Purkert and R. Remmert, of the *Collected Works of Felix Hausdorff* published by Springer in 1996. He has also been editor, with E. Knobloch, of Science Network – Historical Studies, Birkhäuser, since 2001. He was a cofounding member of the "Interdisciplinary Center for Science and Technology Studies", University of Wuppertal, with F. Steinle (history of science) and G. Schiemann (philosophy of science) in the summer of 2005.

# New members of the EMS executive committee

*At its meeting in Utretch (Holland) in July 2008 on the weekend preceding the European Congress, the EMS council elected four new members of the executive committee of the society; Zvi Artstein, Franco Brezzi, Igor Krichever and Martin Raussen. The thanks of the society are due to Victor Buchstaber, Olga Gil-Medrano, Carlo Sbordone and Klaus Schmidt, who all left the executive committee at the end of 2008 after many years of service to the society.*

**Zvi Artstein** received his PhD from the Hebrew University of Jerusalem under the supervision of Robert J. Aumann. Following a post-doctoral position at Brown University he joined the Weizmann Institute of Science where, on top of his research, he held several administrative positions. His current research interests are in dynamical systems and control (in particular variational limits, relaxation, singular perturbations and hybrid systems) and also in decision theory, evolution and mathematical education. He believes that mathematics should not be separated from culture, technology and the other sciences.

**Franco Brezzi** was born in Vimercate (Milan) on 29 April 1945. He graduated in mathematics at the University of Pavia in 1967 and became a full professor (of mathematical analysis) in Turin (Politecnico) in 1975. He moved back to Pavia in 1977 and in 2006 he moved to IUSS (Istituto Universitario di Studi Superiori, always in Pavia).

He is presently the Director of the IMATI (Istituto di Matematica Applicata e Tecnologie Informatiche) of the Italian National Research Council (CNR) and the President of the Italian Mathematical Union.

His scientific interests are mainly in scientific computing and mostly in numerical analysis of partial differential equations. From the point of view of applications, he has worked in several fields, including elasticity, elastoplasticity, plate and shell bending, semiconductor device simulation, fluid mechanics and electromagnetism. Recently he has been working on more methodological aspects, including the so-called Discontinuous Galerkin methods and cochain discretizations of differential forms.

He is a member of the Istituto Lombardo, Accademia di Scienze e Lettere (Milan), and the Accademia Nazonale dei Lincei (Roma). He received the Gauss-Newton Medal from the IACM (International Association for Computational Mechanics) in 2004 and he has been an ISI – Highly Cited Researcher since 2002.

**Igor Krichever** is a leading researcher at the Landau Institute for Theoretical Physics of the Russian Academy of Science and Chair of the Department of Mathematics at Columbia University, New York. He obtained his PhD from Moscow State University in 1975 and his Doctor of Science Degree in 1983. He is renowned for pioneering works on applications of algebraic geometry to the theory of classical and quantum integrable systems and the spectral theory of periodic operators. He is a member of the Council of the Moscow Mathematical Society.

**Martin Raussen** was born, raised and educated in Germany. He studied mathematics and computer science at the universities of Saarbrücken and Göttingen, where he obtained his PhD in 1981 with a thesis on the edge of differential and algebraic topology. During his time as a graduate, he spent nine months in Paris, where he met his future Danish wife. As a result, he finally moved to Denmark and established a family (with three daughters, meanwhile grown-up). After several post-doctorate stays at two universities, he was in 1984 appointed to his present job as associate professor at Aalborg University in North Jutland, Denmark – 100 km from the Northern tip of the peninsula.

In recent years, he has participated in the efforts of a growing community attempting to make methods and ideas from algebraic topology useful for applications in engineering and computer science. His research efforts have mainly centred on "Directed Algebraic Topology", a new field motivated from certain models in concurrency theory from theoretical computer science. He teaches and supervises students at Aalborg University from freshman to PhD-level. For several years, he was on the editorial board of the Danish Mathematical Society's Newsletter "Matilde". Between 2003 and 2008, he was the editor-in-chief of the Newsletter of the European Mathematical Society; he will continue to work for the newsletter, on a minor scale as an associate editor.

One of the central tasks of the EMS is to facilitate cooperation and joint efforts between mathematicians and their societies in Europe at various levels. As a member of the executive committee, Martin intends to support, in particular, the Raising Public Awareness Committee and the Meetings Committee.

# EMS Executive Committee meeting Valencia, Spain, 8–9 November 2008

Vasile Berinde, EMS Publicity Officer

The last EC meeting of 2008 was held in Valencia, a beautiful and impressive city. The Executive Committee had to work hard to get through its long agenda in a day and a half, but succeeded. It was very pleasant to walk through the old town between the hotel and the Colegii Mayor Rector Peset of Universitat de Valencia, which hosted the meeting.

Present at the meeting were: Ari Laptev (Chair), Pavel Exner, Helge Holden, Stephen Huggett, Jouko Väänänen, Olga Gil-Medrano, Mireille Martin-Deschamps, Riitta Ulmanen, Sir John Kingman, Luc Lemaire, Mario Primicerio, Vicente Munoz, Andrzej Pelczar, Vasile Berinde and the new elected EC members, Martin Raussen, Zvi Artstein, Franco Brezzi and Igor Krichever, whose term started on January 1, 2009. Except for Olga Gil-Medrano, our gracious and hospitable hostess in Valencia, none of the EC members whose term ended on December 31, 2008, that is, Carlo Sbordone, Victor Buchstaber and Klaus Schmidt, could attend. They were successfully replaced by the newcomers.

From the dense and varied business of this pleasant and efficient meeting, I will glean just a few matters:

**President's report**
This included the 5ECM in Amsterdam and its finances; applications for the ESF-EMS-ERCOM meetings; attendance at a European Round Table in Brussels with representatives from industry; and the suggestion that the Executive Committee should set up an Ethics Committee. Following Martin Raussen's proposal, the President would start to form a search committee for the future President and Vice-Presidents: this would also be on the agenda of the next EC meeting.

**Treasurer's report**
A detailed report, including the budget for 2009–2010, was tabled and discussed.

**Secretary's report**
The EC agreed the proposal to design the future EC agendas in such a way that at least one open-ended discussion is included in each meeting.

**Publicity Officer's report**
The EC agreed to set up a link from the EMS web page to the web site containing all the photographs of EMS meetings, thus making it public.

**Membership matters**
The Eastern Europe Committee of EMS would help to contact mathematicians in Albania and Moldova, which are not members of EMS.

**EMS Web Site**
Helge Holden described work on the new site in order to continue to improve it.

**6$^{th}$ European Congress of Mathematics**
Andrzej Pelczar presented the progress so far in planning the congress; the EC will set up the Programme and Prizes Committees.

**EMS committees**
New EC members responsible for EMS committees were appointed: Igor Krichever, for Eastern Europe Committee; Franco Brezzi, for Education Committee (who was asked to start completely re-forming this committee); Martin Raussen, for Raising Public Awareness; and Zvi Artstein for Women and Mathematics Committee. As an EMS Meetings Committee does not exist yet, the President proposed Martin Raussen and Zvi Artstein to help set this committee up properly.

**Publishing**
A brief account on the activity of EMS Publishing House was given by John Kingman; Vicente Munoz, the new Editor-in-Chief, reported on various changes in the editorial policy of the Newsletter as well as on the changes to the Editorial Board; the new contract with FIZ-Karlsruhe regarding Zentralblatt was described by Jouko Väänänen.

**Closing matters**
Ari Laptev expressed the EC thanks to the Real Sociedad Matematica Espanola and especially to Olga Gil-Medrano for the wonderful hospitality in Valencia.

The EC agreed that the next meeting would be all day on the 21$^{st}$ of March 2009, and in the morning of the 22$^{nd}$. The venue was not decided during the meeting, although several suggestions were made. [Since the meeting, the venue has been agreed to be Athens.]

# 2011 ICPAM-CIMPA research schools call for projects

The aim of the International Centre for Pure and Applied Mathematics ICPAM-CIMPA is to promote international cooperation in higher education and research in mathematics and their interactions, as well as related subjects, particularly computer science, for the benefit of developing countries. Our action concentrates at the places where mathematics emerges and develops, and where a research project is possible.

ICPAM-CIMPA is a UNESCO centre located in Nice, with financial support from the French *Ministère de l'enseignement supérieur et de la recherche*, the Université de Nice Sophia Antipolis, and UNESCO.

We organize research schools of about two weeks in developing countries. The purpose of these schools is to contribute to the research training of the new generation of mathematicians, women and men.

Once selected by the Scientific committee and the Governing board of ICPAM-CIMPA, research schools are organized locally with the help of ICPAM-CIMPA. ICPAM-CIMPA's financial contribution is essentially for young mathematicians from neighbouring countries to be able to attend the research school. ICPAM-CIMPA can help with obtaining founds from other sources. Additional information can be found in the *instruction sheet* (available on the web site of ICPAM-CIMPA). You can also write to ICPAM-CIMPA.

**Research schools call for projects begins on March 1st, 2009.**

The deadline for a pre-proposal is June 15, 2009. The complete proposal is due October 1st, 2009.

**Centre International de Mathématiques Pures et Appliquées**
**International Centre for Pure and Applied Mathematics**

The application form can be found on ICPAM-CIMPA website, you can also write to cimpa@unice.fr
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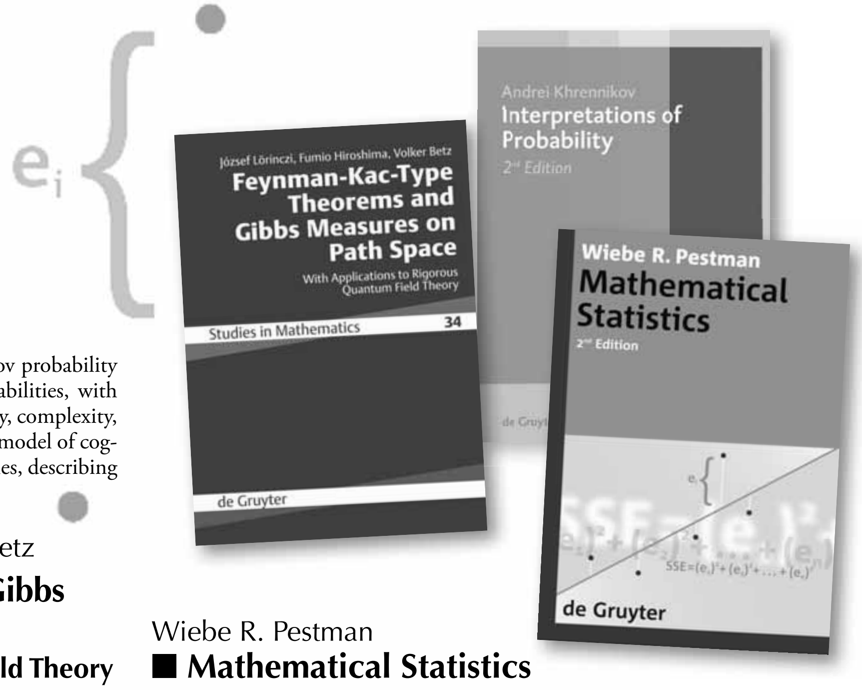


Andrei Khrennikov

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# Oded Schramm (1961–2008)

Wendelin Werner

Oded Schramm, one of the greatest probabilists of our time, died last summer in a hiking/climbing accident in the mountains near Seattle. He radiated so much life and intelligence that it seems today impossible that we will no longer see his smile, read his emails or listen to his clear and calm mathematical explanations. It is unreal to write such a text on him and to thereby accept his passage from life to history.

The Bulletin of the EMS is probably not the appropriate place to express personal things. But before describing some of his mathematics, I want to say a few words on Oded's personality. All those who met him know that he was a particularly nice person, very calm, with an ability to really listen to what others tried to express. Friends of Oded have created a memorial blog[1], where it is possible to read texts and memories about Oded. He was truly an outstanding man.

Mathematicians have the privilege of being able to directly perceive his scientific legacy. Oded was particularly creative. He was able to construct harmonious and original approaches to the most difficult questions using only elementary building blocks. His track could at first appear surprising and difficult to the "specialist" but it usually turned out to be the right and elegant one. In mathematics as in life, he was able to simply choose the good options, without being influenced by the environing fashion.

Oded Schramm received many prizes for his work: the Erdős Prize, the Salem Prize, the Henri Poincaré Prize, the Clay Research award, the Loève Prize, the Polyà Prize and the Ostrowski Prize. He gave a plenary lecture at the 2006 International Congress in Madrid (his talk is available on the ICM's website). But Oded considered the mathematics to be much more important than awards and honours. An appropriate way to pay tribute to him here therefore seems to be to describe some of his results. This is very enjoyable because, even when the proofs are difficult, many of these statements are simple and do not require pages of background material. Let us now browse through some of them chronologically.

## PhD and complex analysis

Oded Schramm obtained a master's degree at the Hebrew University in Jerusalem, where he worked under the supervision of Gil Kalai; his thesis gave rise to two publications. He then went to Princeton to write his PhD under the supervision of William Thurston. He defended his thesis in 1990 with the title "Packing bodies with prescribed combinatorics and applications to the construction of conformal and quasiconformal mappings". It deals with combinatorial and discrete approximations of complex analysis and contains no probability theory; he generalizes (via a different proof) Koebe-Andreev-Thurston's Circle Packing Theorem to the situation where circles are replaced by other convex shapes. Let us just recall the (classical) Circle Packing Theorem, which states that it is possible to view any planar graph as a circle packing by choosing the centres of the circles to be the sites of the graph and calling two sites adjacent when the corresponding (well-chosen) circles are adjacent. Oded then used this new result, together with the relation between circle packings and conformal maps initiated by Thurston and elaborated by Rodin and Sullivan, to establish the following generalization of Koebe's Uniformization Theorem.

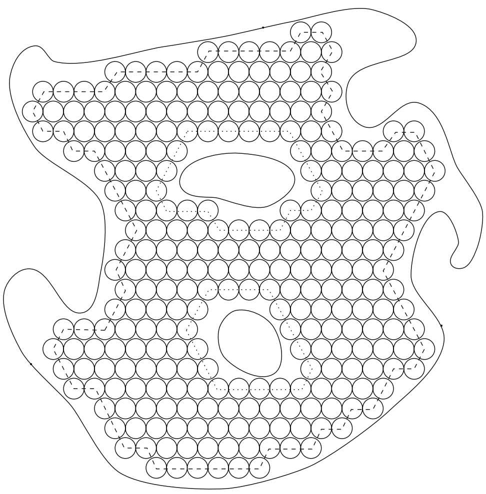

**Figure 1. Image from Oded Schramm's PhD**

**Theorem 1** (PhD thesis [1]). *Let $G$ denote a planar open set of the type $G = H \setminus \bigcup_{j=1}^{n} F_j$, where $H$ is simply connected and $F_1, \dots, F_n$ are $n$ disjoint compact simply connected subsets of $H$ that are not singletons. Then, for any simply connected domain $H'$ and any compact convex sets $P_1, \dots, P_n$ that are not singletons, it is possible to find disjoint sets $P'_1, \dots, P'_n$ in $H'$ that are respectively homothetic to $P_1, \dots, P_n$, such that there exists a conformal (i.e., angle-preserving) map from $G$ onto $H' \setminus \bigcup_{j=1}^{n} P'_j$ that sends the boundaries of $F_1, \dots, F_n$ respectively onto the boundaries of $P'_1, \dots, P'_n$.*

This thesis, like most of Oded's papers, can be downloaded from the arXiv preprint server. It already possesses Oded's characteristic style: clear, precise, not too lengthy and straight to the point. After his PhD, Oded Schramm went to

San Diego for a post-doc where he continued to work on similar themes with Zheng-Xu He. Together, they wrote eight striking papers containing numerous results that answer natural questions about the existence of certain conformal and quasi-conformal mappings and about the properties and rates of convergence of their discrete approximations. One can, for instance, mention the following (almost general) solution to Koebe's Kreisnormierungsproblem from 1908:

**Theorem 2** ([2])**.** *Any planar connected open set, whose boundary contains at most a countable number of connected components is conformally equivalent to a circular domain (i.e., the Riemann sphere minus disjoint disks).*

Later, Oded gave an elegant alternative proof of this result. Here is another result from that period, which shows that he was also interested in higher dimensions:

**Theorem 3** (How to cage an egg [3])**.** *Let $P$ denote a convex polyhedron and $K$ a strictly convex domain in Euclidean space. Then, there exists a polyhedron $Q$ that is combinatorially equivalent to $P$ such that all edges of $Q$ are tangent to $K$. Furthermore, under certain additional conditions on $\partial K$, the space of such $Q$'s is a 6-dimensional differentiable manifold.*

### First probabilistic papers

In 1992, Oded Schramm moved back to Israel, where he worked at the Weizmann Institute of Science, not far from Tel Aviv. He still continued to work on questions from complex analysis but he also started to think about probabilistic problems. This is where his collaboration with Itai Benjamini began; together, they have written more than 20 papers over these last 15 years that explore (among other things) numerous relations between geometric properties of graphs and the behaviour of probabilistic structures (random walks, percolation, uniform spanning trees/forests) defined on these graphs. On this theme, he also interacted a lot with Yuval Peres and Russ Lyons. Here are some statements to illustrate this large body of results:

Recall that the isoperimetric constant $h(G)$ of an infinite connected graph is the infimum, over all finite subsets $K$ of $G$, of the ratio between the number of points that are at distance exactly 1 from $K$ and the number of points in $K$ (in shorthand, $h(G) = \inf_K \#\partial K/\#K$). The following statement is not probabilistic but its proof involves probability theory:

**Theorem 4** ([4])**.** *If $h(G) > 0$, then one can find a tree $T$ contained in $G$ such that $h(T) > 0$.*

In a series of papers with Itai Benjamini, Russ Lyons, Yuval Peres and Harry Kesten, he explored many relations between "uniform" measures on spanning trees or forests in a graph. These measures can be defined as follows. Consider an increasing family of finite connected subgraphs $G_n$ that converges to $G$ (i.e., $\bigcup G_n = G$) and then look at the limit when $n \to \infty$ of the uniform distribution on the set of subgraphs of $G_n$ that contain only one connected component but no cycle (this is a uniformly chosen spanning tree in $G_n$). In the nineties, nice results on this combinatorially rich model (one can quote the names of Aldous, Pemantle, Wilson and Kenyon) have emerged. It turns out to be closely related to random walks and electric networks. Here is, for instance, a result contained in the important paper of Benjamini, Lyons, Peres and Schramm "Uniform Spanning Forests":

**Theorem 5** ([5])**.** *For every planar recurrent infinite graph with bounded codegree, one can define in an unambiguous way the "harmonic measure seen from infinity".*

This paper, just like many other papers by Oded (such as his contribution to the last ICM's proceedings [7] or the paper "Percolation beyond $Z^d$, many questions and a few answers" [6]), contains a section with numerous stimulating conjectures and open questions.

### SLE processes

It is not surprising that Oded Schramm started to think about the (at the time conjectural) conformal invariance of lattice-based probabilistic models from statistical physics, such as percolation or the Ising model taken at their "critical" values. Indeed, this question seems to combine probability theory on graphs and the approximation of conformally invariant objects by discrete models. It is around 1998-1999, just before moving back to the US to work at Microsoft's "Theory Group" near Seattle, that Oded Schramm discovered/invented the SLE processes (SLE stands for Stochastic Loewner Evolutions, or Schramm-Loewner Evolutions as we call it now) that turned out to be a revolution for the understanding of these phenomena both for mathematicians and physicists. The important basic idea, which appears in detail in his 2000 paper "Scaling limit of loop-erased random walks and uniform spanning trees", is particularly simple and elegant. It ties probability theory with complex analysis at their respective roots, and defines a new, natural, one-dimensional class of stochastic processes via (infinitely divisible) iterations of independent random conformal maps.

Let us give an heuristic description of these processes. Let us first recall that Riemann's mapping theorem allows us to distort (via a smooth bijection) any simply connected subset $\Omega$ of the unit disc onto the unit disc in an angle-preserving way. If $0 \in \Omega$ then it is also possible to specify that 0 is mapped to itself and that the image of some given boundary point of $\Omega$ is 1 (the transformation is then unique).

Let us now use a "kitchen analogy". One is preparing a pastry and buys a round-shaped one, i.e., the unit disc $U$. Then, one takes scissors and cuts it open randomly a little bit starting from the boundary point $\gamma_1(0) = 1$. The new domain is now a "slit" disk $\Omega_1 = U \setminus \gamma_1[0,t]$, where $\gamma_1[0,t]$ is the "trajectory" of the scissors. The point $\gamma_1(t)$ is a boundary point of this new domain.

Then, one distorts the pastry in a "conformal" way back into its initial round shape as described before (0 is mapped to itself and the point $\gamma_1(t)$ is mapped to 1). This transformation

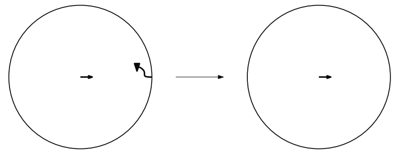

**Figure 2. The map $\Phi_1$**

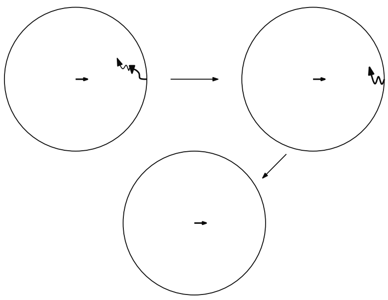

**Figure 3. The map $\Phi_2 \circ \Phi_1$**

(i.e., the conformal map $\Phi_1$) is unique by Riemann's mapping theorem.

Now, one starts again using a second random slit $\gamma_2[0,t]$ that is independent of $\gamma_1[0,t]$ but chosen according to the same distribution. The new slit domain can be distorted back into the original disc via a conformal transformation $\Phi_2$. Hence, $\Phi_1$ and $\Phi_2$ are clearly independent identically distributed random conformal maps. Furthermore, it makes sense to consider the composition $\Phi_2 \circ \Phi_1$: it is the (normalized) conformal map from the slit disc $\Omega_1 \setminus \Phi_1^{-1}(\gamma_2[0,t])$ onto the unit disc. The slit is now "longer" than $\gamma_1[0,t]$ because one has also removed the preimage of $\gamma_2[0,t]$.

In this way, it is possible to define iteratively a sequence of independent identically distributed conformal maps and to note that their composition $\Phi_n \circ \ldots \circ \Phi_1$ in fact defines a random slit (just look at $U$ minus the preimage of the unit disc under this transformation).

Let us now define a continuous version of this iterative procedure with the idea that one cuts progressively, in a random and continuous manner. This is encoded by a family $(\psi_t, t \geq 0)$ of conformal maps such that each $\psi_t$ is the normalized map from $U \setminus \gamma[0,t]$ onto $U$. For each $n$, the map $\psi_t$ is obtained by iterating $n$ independent copies of $\psi_{t/n}$.

Note that $a_t = |\psi_t'(0)|$ is an increasing function such that $a(t+s) = a(t)a(s)$ (when one composes conformal maps, the derivatives are multiplied). So (up to a linear time-change), we can take $a(t) = e^t$. If one now defines $(\theta_t, t \geq 0)$ in a continuous manner, in such a way that $\theta_0 = 0$ and

$$\psi_t'(0) = e^t \times \exp(i\theta_t),$$

then the process $\theta$ necessarily has stationary and independent increments. We also know that $\theta$ is continuous and we assume that it is symmetric in law. A classical fact from probability theory shows that the only possibility is that $\theta_t = \beta(\kappa t)$, where $\kappa$ is a non-negative real and $\beta$ is a standard one-dimensional Brownian motion. This leads to the following conclusion:

**Key observation**. *Suppose that a continuous random curve without double points in the unit disc started from $\gamma_0 = 1$ satisfies the following properties:*

- *The curve $\gamma$ is parametrized in such a way that for each $t$, the conformal map $\psi_t$ from $U \setminus \gamma[0,t]$ onto $U$ defined as before (i.e., such that $\psi_t(0) = 0$ and $\psi_t(\gamma_t) = 1$) satisfies $|\psi_t'(0)| = e^t$ (this condition just means that one has chosen a particular time-parametrization for $\gamma$).*
- *For all given $t \geq 0$, the conditional law of the process $(\psi_t(\gamma_{t+s}), s \geq 0)$ given $\gamma[0,t]$ is identical to that of the curve $\gamma$ itself (this is the continuous version of the fact that one iterates identically distributed random cuts).*
- *The law of $\gamma$ is symmetric with respect to the real axis.*

*Then, the argument $(\theta_t, t \geq 0)$ of the process $\psi_t'(0)$ is a one-dimensional Brownian motion $\beta$ running at constant non-negative speed $\kappa$.*

What makes this observation so useful is that it is known due to Charles Loewner in the early 20th century that the function $t \mapsto \theta_t$ fully characterizes the curve $\gamma$ (this is the idea of Loewner's equation, one of the classical tools in complex analysis, used, for instance, to study questions related to the Bieberbach conjecture – it is actually an instrumental tool in its proof by de Branges). Hence, one can go the other way round. First define $\theta$ to be a Brownian motion running at constant speed and then let $\gamma$ be the corresponding curve. This curve is random since the function $\theta$ is random.

Note that there is a subtle point. If the curve $\gamma$ exists, there necessarily exists a corresponding function $\theta$ and two different curves define different functions. However, it is not so clear that every continuous function $\theta$ corresponds to a curve $\gamma$. In fact, a function $\theta$ necessarily defines a so-called Loewner chain, i.e., a certain type of decreasing family of open sets $(\Omega_t)$ that does not always correspond to slit domains. Non-trivial arguments are needed to show that in the case where $\theta$ is a Brownian motion, it indeed corresponds to a random continuous curve (with probability one). More precisely, Steffen Rohde and Oded Schramm showed the following.

**Theorem 6** ([9]). *When $\kappa \leq 4$, this procedure indeed defines a continuous curve $\gamma$ without double points such that $\Omega_t = U \setminus \gamma[0,t]$. When $\kappa > 4$, this procedure still defines a continuous curve $\gamma$, but this time, it has double points and one has to change the previous construction slightly: $\Omega_t$ is now the connected component that contains the origin of the set $U \setminus \gamma[0,t]$.*

Hence, this construction indeed defines a family (indexed by $\kappa$) of random continuous curves. These are the SLE paths.

In many lattice-based models from statistical physics, one can define natural random paths that can be interpreted as interfaces. This is, for instance, the case for the percolation model that we now briefly describe. Each cell of the honeycomb lattice is coloured independently black or white by tossing a fair coin. Large clusters of white cells and large clusters

**Figure 4. Picture of an SLE for $\kappa = 2$**

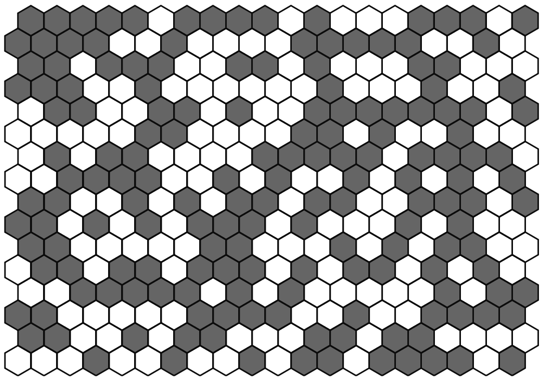

**Figure 5. Percolation in a rectangle**

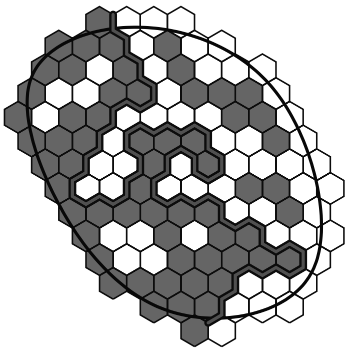

**Figure 6. An interface in a small domain**

of black cells appear, and the outer boundaries of the black clusters are inner boundaries of white clusters and vice-versa.

It is conjectured (and it is now proved in several important cases) that such interfaces behave asymptotically in a conformally invariant way on large scale (we will not detail here what is meant by this). Combining this conjecture with the previous analysis, Oded Schramm concluded that:

**Key conclusion.** *For a large class of models from statistical physics, if they behave in a conformally invariant way in the scaling limit, then the laws of the interfaces converge (in the limit when the mesh of the lattice vanishes) to that of SLE curves. The value of $\kappa$ depends on the studied model.*

### Study of SLE processes and consequences.

The definition of SLE processes makes it possible to compute explicitly the probability of certain events. For example (see Oded's paper "A percolation formula" [10]), the probability that an SLE process in a simply domain $D$ from $A \in \partial D$ to $B \in \partial D$ passes to the right of a given point $Z \in D$ can be expressed easily via a differential equation. More precisely, one first maps the domain $D$ conformally onto the upper half-plane in such a way that $A$ and $B$ are respectively mapped onto 0 and $\infty$, and the image of $Z$ has height 1. The probability in question therefore just depends on the real value $x$ of the image of $Z$. One can then study how this probability $h(x)$ evolves as the SLE starts to grow, and one is led to the differential equation $\kappa h''(x) + 8x/(1+x^2)h'(x) = 0$ that enables one to deduce that $h$ is a hypergeometric function of the type that had appeared in the conformal field theory literature.

More generally, and sometimes with some technical difficulties, it is possible to compute the asymptotic behaviour of certain exceptional events. Rohde and Schramm [9] have, for instance, shown that when $\kappa \leq 8$, the asymptotic probability that an SLE of parameter $\kappa$ passes through the $\varepsilon$-neighbourhood of a given point $z \in D$ decays like $\varepsilon^{1-\kappa/8}$ when $\varepsilon \to 0$. This implies that the Hausdorff dimension $d(\kappa)$ of the curve is not larger than $1+\kappa/8$ (in fact, $d(\kappa)$ is equal to this value, as later proved by Vincent Beffara). Such fractal dimensions are closely related to the critical exponents studied by our friends in the physics community via various techniques including the above-mentioned conformal field theory.

The study of SLE processes has enabled mathematicians to prove results for all the models for which asymptotical conformal invariance is established. This includes simple random walks, loop-erased random walks and uniform spanning trees, the critical percolation model that we have briefly described and the Ising model on the square grid (the conformal invariance of these last two models has been proved recently by Stas Smirnov). Here are two examples extracted from joint papers of Oded with Greg Lawler and myself:

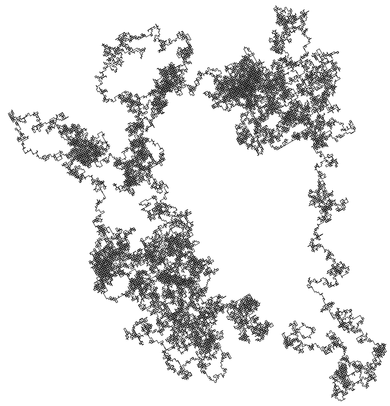

**Figure 7. A planar Brownian motion**

**Theorem 7** ([11]). *Consider a planar Brownian motion $(B_t, t \in [0,1])$ and define its outer boundary $F$ as the boundary of the unbounded connected component of the complement of $B[0,1]$. Then the Hausdorff dimension of $F$ is almost surely equal to $4/3$.*

**Theorem 8** ([12]). *Consider the percolation model when one colours each cell of the honeycomb lattice independently black or white with probability $1/2$. The probability that there exists a white path originating at the origin that reaches distance $R$ from the origin decays asymptotically like $R^{-5/48+o(1)}$ as $R \to \infty$.*

SLE has now become a research area of its own. The following recent (some are still being written up) important results by Oded on the subject are not so easy to state without a long introduction, so that we only describe them heuristically.

- In the spirit of his earlier papers on the subject with Itai Benjamini and Gil Kalai [15], and with Jeff Steif [16], Oded, together with Christophe Garban and Gábor Pete, gives in [17] precise and complete answers to the question of "sensitivity to small perturbations" of critical percolation on a large scale. In another paper, they provide in [18] a detailed description of percolation near its critical point.

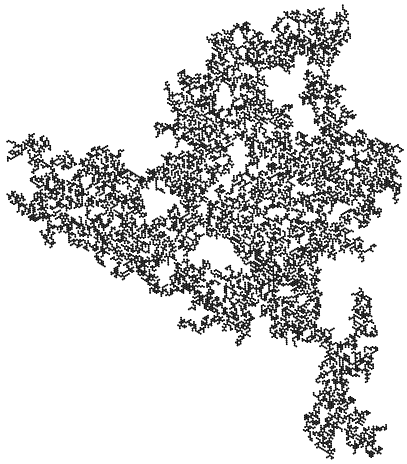

**Figure 8. A percolation cluster**

- In joint work with Scott Sheffield [13, 14], Oded Schramm establishes a direct link between SLE curves and the Gaussian Free Field. Loosely speaking, the SLE curves appear as "level lines" of a (generalized) random surface (see Figure 9).

As we can see, Oded Schramm is a mathematician who has worked a lot in collaboration with others. All his coauthors have experienced his generosity, his inventiveness and his ability to tackle technical challenges. Those of us who have had the privilege to work and interact with him have had our mathematical lives changed enormously.

Oded had many ongoing projects, so that we will have the pleasure to see several new papers coauthored by him in the coming months. More generally, his beautiful mathematical ideas will feed the thoughts of numerous probabilists and continue to live through them.

The references below are those papers by Oded that are directly quoted in this text. A complete biography of his (approximately) 80 papers can easily be found on the web, for example via his former webpage.

## Note

1. The Oded Schramm memorial blog is at http://odedschramm.wordpress.com.

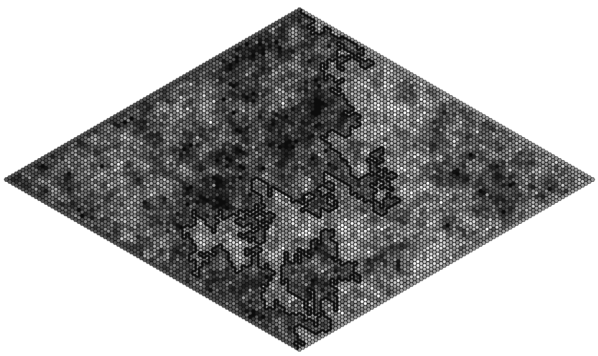

**Figure 9. Picture from a paper of Scott Sheffield and Oded Schramm**

## Bibliography


[1] Oded Schramm, Packing two dimensional bodies with prescribed combinatorics and applications to the construction of conformal and quasiconformal mappings, PhD thesis, Princeton University (1990).

[2] Zheng-Xu He and Oded Schramm, Fixed points, Koebe uniformization and circle packings, Ann. of Math. (2) Vol. 137 No. 2, 369–406 (1993).

[3] Oded Schramm, How to cage an egg, Invent. Math. Vol. 107 No. 3, 543–560 (1992).

[4] Itai Benjamini and Oded Schramm, Every graph with a positive Cheeger constant contains a tree with a positive Cheeger constant, Geom. Funct. Anal. Vol. 7 No. 3, 403–419 (1997).

[5] Itai Benjamini, Russell Lyons, Yuval Peres and Oded Schramm, Uniform spanning forests, Ann. Probab. Vol. 29 No. 1, 1–65 (2001).

[6] Itai Benjamini and Oded Schramm, Percolation beyond $Z^d$, many questions and a few answers, Electron. Comm. Probab. Vol. 1, no. 8, 71–82 (1996).

[7] Oded Schramm, Conformally invariant scaling limits: an overview and a collection of problems, International Congress of Mathematicians. Vol. I, 513–543, Eur. Math. Soc. (2007).

[8] Oded Schramm, Scaling limits of loop-erased random walks and uniform spanning trees, Israel J. Math. Vol. 118, 221–288 (2000).

[9] Steffen Rohde and Oded Schramm, Basic properties of SLE, Ann. of Math. (2) Vol. 161 No. 2, 883–924 (2005).

[10] Oded Schramm, A percolation formula, Electron. Comm. Probab. Vol. 6, 115–120 (electronic) (2001).

[11] Greg Lawler, Oded Schramm and Wendelin Werner, Values of Brownian intersection exponents II: Plane exponents, Acta Math. Vol. 187 No. 2, 275–308 (2001). 8, 221–288 (2000).

[12] Greg Lawler, Oded Schramm and Wendelin Werner, One-arm exponent for critical 2D percolation, Electron. J. Probab. Vol. 7, no. 2, 13 (2002).

[13] Oded Schramm and Scott Sheffield, Contour lines of the two-dimensional discrete Gaussian free field, Preprint (2006).

[14] Oded Schramm and Scott Sheffield, in preparation.

[15] Itai Benjamini, Gil Kalai and Oded Schramm, Noise sensitivity of boolean functions and applications to percolation, Inst. Hautes Etudes Sci. Publ. Math. No. 90, 5–43 (1999).

[16] Oded Schramm and Jeffrey E. Steif, Quantitative noise sensitivity and exceptional times for percolation, Ann. of Math., to appear.

[17] Christophe Garban, Gábor Pete and Oded Schramm, The Fourier Spectrum of Critical Percolation, Preprint (2008).

[18] Christophe Garban, Gábor Pete and Oded Schramm, in preparation.



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# Communicating mathematics: a historical and personal journey

Robin Wilson


**Robin Wilson at his inaugural lecture.**

***This article is adapted from the author's recent inaugural lecture as Professor of Pure Mathematics at the Open University, UK.***

My topic is '*Communicating mathematics*': something we all try to do through our various teaching activities and publications.

I was first directed to think about what this involves by a 1990 examination paper for an Open University history of maths course, which I was studying as a part-time OU student. While sweating it out in the exam room I saw Question 15 that began: *Describe some of the ways in which mathematicians have communicated their results to each other*. My enjoyment in answering this question has led me over the past eighteen years to think about the various ways in which we communicate mathematics – to our students, to our colleagues, and to the general public.

I won't be able to cover every form of communication we use, but here are some ways that maths has been propagated over the centuries. As you can see below, I've divided my presentation into two main parts: *the spoken word*, on everything from lectures and TV broadcasts to casual conversations in the corridor, and *the written word*, on everything from research papers and books to newspaper articles and websites. In each part I'll try to give both a historical and a personal account, with a wide range of examples covering 4000 years.

## Part I: The spoken word

### *Inaugural lectures*

It seems appropriate to begin with *inaugural lectures*, which historically seem to be mainly of two types: those that present and explain original research in a wider context than is usually possible, and others (such as this one) that are more of an expository nature.

An example of the latter was that of Christopher Wren, appointed at age 27 to the Astronomy Chair at Gresham College, who noted that London was particularly favoured *with so general a relish of mathematicks and the liberal philosophia in such measure as is hardly to be found in the academies themselves* and concluded by enthusing that *Mathematical demonstrations being built upon the impregnable Foundations of Geometry and Arithmetick are the* only *truths that can sink into the Mind of Man, void of all Uncertainty; and all other Discourses participate more or less of Truth according as their Subjects are more or less capable of Mathematical Demonstration.*

An example of the former type of inaugural lecture was G. H. Hardy's 1920 presentation as Savilian Professor of Geometry in Oxford. Hardy was interested in an old number theory result of Edward Waring, that *every positive whole number can be written as the sum of at most*

**The spoken word**

| | | |
|---|---|---|
| *inaugural lectures* | *films and plays* | *classroom teaching* |
| *casual conversations* | *TV documentaries* | *tutorials and classes* |
| *university seminars* | *radio programmes* | *video- and audio-tapes* |
| *maths society meetings* | *summer schools* | *podcasts* |
| *talks to high schools* | *competitions* | *conference talks* |

**The written word**

| | | |
|---|---|---|
| *research monographs* | *popular books* | *exams and assignments* |
| *clay tablets and papyri* | *e-mail messages* | *conference proceedings* |
| *correspondence* | *research papers* | *newspaper articles* |
| *manuscripts* | *the internet* | *correspondence texts* |

Bernhard Riemann (1826–1866).

James Joseph Sylvester (1814–1897).

*four perfect squares, nine perfect cubes, and so on*, and much of his inaugural lecture was devoted to it, leading eventually to several celebrated papers on the problem with his colleague J. E. Littlewood of Cambridge.

Some sixty years earlier, in Germany, two inaugural lectures had transformed the world of geometry. At the beginning of the 19th century there'd been essentially only one type of geometry around – *Euclid's geometry*, the one we're all familiar with. But as the century progressed, other types of geometry appeared: *projective geometry* with its origins in perspective painting, and *non-Euclidean geometry* which defied one of Euclid's postulates. In his celebrated inaugural lecture at Göttingen in 1854, Bernhard Riemann explained a new distance-based geometry, now called *Riemannian geometry*, that would be of great importance in Einstein's theory of relativity, long after Riemann's death. Eighteen years later, at his inaugural lecture in Erlangen, Felix Klein presented what is now called the *Erlanger Programm* in which he tried to sort out the mess that geometry had become – *so many geometries: how can we make sense of them*? His answer involved the algebraic idea of a *group*, a subject that's now studied by our undergraduate students.

Connoisseurs of inaugural lectures have a soft spot for the two given by James Joseph Sylvester. At Oxford he was appointed to the Savilian Geometry Chair at the age of 69, and gave an inaugural lecture in 1885 in which he outlined the current state of algebra, with particular reference to his work on invariant theory, and recited a sonnet he'd specially composed on noticing that an expected term in $b^4d$ did not appear in a table of 'reciprocantive protomorphs':

*Lone and discarded one! Divorced by fate,*
*From thy wished-for fellows – whither art thou flown?*
*Where lingerest thou in thy bereaved estate,*
*Like some lost star, or buried meteor stone? …*

Nine years earlier, he'd been appointed the first professor of mathematics at the newly-founded Johns Hopkins University in Baltimore, USA. He based his inaugural lecture there on a supposed link between chemistry and algebra, and later described how he came up with the idea in a wonderful paragraph of the type that we no longer find in mathematical journals. Paragraph 2 of his lengthy research paper on chemistry and algebra consists of just one sentence: *Casting about, as I lay in bed last night, to discover some means of conveying an intelligible conception of the objects of modern algebra to a mixed society, mainly composed of physicists, chemists and biologists, interspersed only with a few mathematicians …* (the sentence rambles on) *I was agreeably surprised to find, all of a sudden, distinctly pictured on my mental retina a chemico-graphical image*, etc., etc. This discovery led to his using the word *graph* for the first time (in the graph theory sense), and to his use of graphs to represent algebraic invariants.

Unfortunately, inaugural lectures don't always take place, for a variety of reasons. In 1827 the newly appointed Savilian Professor at Oxford expected to give his inaugural lecture: this was the Revd Baden Powell, father of the founder of the Boy Scout movement. But mathematics in Oxford was then in low esteem and Powell was advised not to give his lecture as he'd most surely not attract an audience.

### *Other lectures*

Of course, these aren't the only one-off lectures. Gresham College in London has been presenting lectures to the general public since 1597, while mathematicians immediately think of the famous lecture by David Hilbert at the Paris International Congress in 1900, where he set the mathematical agenda for the next century by discussing a number of important problems that he wished to see solved.

Another celebrated lecture took place in Cambridge in 1993, when Andrew Wiles announced that he'd proved Fermat's so-called 'last theorem', a result that had remained unproved for 350 years. Although his announcement turned out to be premature – it took two more years to finish the proof – his lecture caused excitement in the national press, who are not usually very interested in mathematics.

Indeed, many invited plenary lectures at conferences take one of the two forms of lectures I've described: the presentation of recent research by a mathematician who's made a notable discovery, or an overview of a whole area of interest to the audience. Both types of lecture are important and valuable.

Sometimes, a whole series of lectures is remembered years later, such as Karl Pearson's 1893 Gresham College lectures on probability, entitled *Laws of Chance*, in which the familiar terms 'standard deviation' and 'histogram' were used for the first time.

Another example, many years earlier, were the lectures given by Hypatia, probably the most celebrated woman mathematician of the ancient world. Although mainly remembered for the grisly death she suffered at the hands of the Roman soldiers, she was a geometer of some renown, being Head of the Neoplatonic School in Alexandria, and is credited with impressive commentaries on many classic texts, such as Apollonius's *Conics* and Ptolemy's *Almagest*. She was apparently such a fine expositor that people came from miles around to hear her geometry lectures.

Sir Isaac Newton (1642–1727).

However, not all lectures have been so popular. Those of Isaac Newton, the second Lucasian Professor of Mathematics in Cambridge (the post now held by Stephen Hawking) were apparently so abstruse or poorly delivered that, according to his assistant, *so few went to hear Him, & fewer that understood him, that oft-times he did in a manner, for want of hearers, read to the Walls*. And his bitter rival Robert Hooke, probably my most distinguished predecessor as Gresham Professor of Geometry, was very conscientious about his lectures, but frequently failed to attract an audience. As his diary records: No auditory came morning or afternoon so I read not … *No lecture but a rusty old fellow walked in the hall from 2 until almost 3 … Only one came, peeped into the hall but stayed not …*

These days, the lecture is the usual vehicle in universities for transferring material from the notebook of the lecturer to the notebook of the student, sometimes (it is said) without going through the mind of either. It's considered an efficient vehicle for communicating the subject, especially when there are large numbers of students, but has the obvious disadvantage that it forces everyone to move at the same speed. Indeed, this is always our challenge: *how should we design a lecture that's not too fast for the weaker students, or too slow for the stronger ones?* Or, for a general lecture such as this one, *how should I choose material that's accessible to most of the audience, yet also contains something of interest to the specialists?*

In spite of the inherent disadvantages of lectures, I don't propose that we get rid of them. We've all been to lectures that excited and inspired us, and there's no substitute for them.

### Informal contact

Meeting informally, such as happens at summer schools or conferences, is an important means for communicating. Informal chats over lunch, or in the corridor or the bar, are often as productive as the more structured teaching sessions, such as classes or tutorials, just as networking at meetings and conferences can be more important than the organised programme. Indeed, we should never underestimate the casual conversation as a way of communicating maths, whether it's with our colleagues in a corridor, or in a brief discussion with a distinguished mathematician at a conference.

An example occurred in 1871. A well-known problem, solved in the 18th century by Leonhard Euler, concerned *Königsberg*, a city of four land areas linked by seven bridges: the problem was to find a walk that crossed each of the seven bridges just once. Euler proved that no such route exists and produced a recipe for deciding, for any given arrangement of land areas and bridges, whether such a route is possible. But he failed to show *how* to find it, and it wasn't until 130 years later that a young Viennese mathematician named Karl Hierholzer filled in the gap, described his proof to a colleague in a corridor, and then promptly died. Fortunately this colleague, Christian Wiener, had understood the proof and wrote it up for publication.

Mathematicians have gathered together since earliest times. In Plato's Academy men and women met to discuss geometry, number theory, philosophy and other subjects of interest, and over the entrance were the words: *Let no-one ignorant of geometry enter these doors*. Another early example was the Pythagorean brotherhood, while modern gatherings include the European Mathematical Society, the London Mathematical Society and the British Society for the History of Mathematics.

Some interesting gatherings took place in the 1620s when Marin Mersenne initiated regular meetings in Paris at which mathematicians could meet to talk about their latest findings. Mersenne himself described it as: *the most noble academy in the world which has recently been formed in this town. It will no doubt be the most noble for it is wholly mathematical.* Mersenne's meetings led eventually, after his death, to the founding by Louis XIV of the French Royal Academy of Sciences.

In London, the Royal Society, founded at Gresham College in 1660, provided a forum for developing the experimental sciences and presenting practical experiments to the general public, while two hundred years later saw the formation of the London Mathematical Society, with Augustus De Morgan as its first president. Today the LMS presents a range of communal activities, from specialist mathematical talks by world experts to popular lectures for the general public.

In 2002 I was particularly pleased to be able to organise a special LMS meeting to celebrate the 150th anniversary of the *four-colour problem*, which asks you to prove that the countries of every map, however complicated, can be coloured with just four colours so that neighbouring countries are differently coloured. This problem took 124 years to solve, and Appel and Haken (who first solved it) both spoke at the meeting.

### The media

Following the success of his 2005 TV programme on *The Music of the Primes*, which achieved TV audiences of well over a million, Marcus du Sautoy has recently travelled the world filming an exciting series of four one-hour BBC-OU TV documentaries on *The Story of Maths*, broadcast this autumn. The OU's presenting a short course to accompany these programmes, and I'm hoping to make the material available to high schools. I showed two programmes at the Joint Winter meeting of the American mathematics societies in Washington in January.

Other TV programmes have communicated mathematics to the general public – Jakob Bronowski's BBC programmes in the 1950s, for example, and the Royal Institution Christmas lectures, which three times have featured maths – with Christopher Zeeman in 1978, Ian Stewart in 1979, and Marcus du Sautoy in 2006. And on

BBC radio, Melvyn Bragg's programmes *In our Time* regularly deal with mathematical topics: recent examples have included Archimedes, probability, maths and music, symmetry, Newton's *Principia*, Poincaré, and Gödel's incompleteness theorem: such programmes take mathematical ideas to some two million listeners.

There are even the occasional film and dramatic productions. There was a mathematical component to *Good Will Hunting* and some plays, such as the recent Broadway hit *Proof* and Tom Stoppard's *Arcadia*, have featured mathematical topics.

There are many other examples of verbal communication of mathematics which I can't deal with here – from the teaching of scribes in ancient Egypt to the modern school classrooms of today. But I'll end this part with a bit of a moan.

### *Missing the boat*

Over the past 40 years I've given some 1400 one-off lectures (recently over 50 per year) to a wide range of audiences: university seminars, conferences, groups of sixth-formers or undergraduates, and audiences (both general and specialist) of interested adults. I've always regarded a lecture as a performance, and my topics have usually ranged from graph theory to the history of mathematics, both of which lend themselves to a popular treatment.

Several of my lectures have been to audiences who don't normally encounter mathematics, and this provides a good opportunity to spread the word – to 'raise the public awareness of mathematics' in a small way. But this is something that the mathematical community is generally rather bad at doing. There are the occasional exceptions – for example, there's been a great deal of mathematical activity in Germany as they've celebrated 2008 as their 'Year of Mathematics', and there's also a travelling exhibition called *Experiencing Mathematics* which has been touring the world for the past three years attracting crowds numbering hundreds of thousands.

But so often we miss the boat. As a trivial example of a missed opportunity, our trains, buses and planes are full of people enjoying the delights of combinatorial mathematics as they work on their latest sudoku puzzles, but the mathematical world has hardly taken up the opportunity to capitalise on this. There's still a widespread belief, which we've failed to counteract, that sudoku has no connection with maths, apart from the appearance of the numbers 1–9. Indeed, a couple of years ago I was invited by the Danish Mathematical Society to give a sudoku lecture: the Society was in dispute with a national newspaper whose sudoku column assured readers that 'No knowledge of maths is involved – it's all logic', as if there were a difference. The bargain they eventually struck was that the paper would cease printing that sentence if I'd give a sudoku lecture for their readers, and it was duly reported on nationally the next morning.

Another opportunity we missed for communicating our subject was the 300$^{th}$ anniversary last year of Leonhard Euler, the most prolific mathematician of all time, but one who's largely unknown to the general public. The three places he was mainly associated with (Basel, Berlin and St Petersburg) celebrated his anniversary in style, as did the United States, but most countries did very little.

**Charles Lutwidge Dodgson (Lewis Carroll), 1832–1898.**

As I said earlier, in order to spread the word, I sometimes lecture to audiences that don't normally encounter mathematics. I've lectured to philatelists about mathematical stamps, to the University of the Third Age about Isaac Newton, and to the Lewis Carroll Society about the mathematics that pervades his works; I've also written a one-hour play about Carroll's mathematical life, which has been performed to various audiences around the world.

A particularly fruitful area for disseminating mathematical ideas is though music, an idea that goes back to Pythagoras's time. Marcus du Sautoy has given several radio talks on the links between maths and music, and I've given live presentations on the subject as a Royal Institution Friday Evening discourse and at the 2006 Cambridge Music Festival where, due to its popularity, I agreed to lecture at 3 pm as well as the advertised time of 5 pm, and the poster announced, *Due to popular demand, Robin Wilson's lecture will be repeated two hours earlier*. I was also fortunate to be invited to take part in the BBC Radio 3 musicprogramme Private Passions, where I described a number of connections between maths and music to largely non-mathematical listeners.

## Part II: The written word

We now move on briefly to the written word. From earliest times people have communicated their mathematical ideas in writing – indeed, some of the earliest examples of writing are Mesopotamian financial accounts imprinted on clay tablets. Other tablets were used to teach mathematical ideas, such as a 9-times and a square with the length of its diagonal calculated, while in Egypt the famous Rhind mathematical papyrus was probably used for instructing scribes.

### Letters

An important means of written communication was by letter. Well-known examples, still in print, are Euler's famous *Letters to a German Princess*, in which he described scientific and mathematical topics to the Princess of Alhalt-Dessau.

Earlier, around 1600, Mersenne (whom we mentioned earlier) corresponded with all the principal scientists of his day, such as Descartes, Fermat and Pascal, acting as a sort of European clearing house for scientific discoveries. Other celebrated letters include those from Archimedes to Dositheus on geometrical topics, and a well-known note, written by Evariste Galois to his friend Chevalier in 1832 on the night before he was killed in a dual, containing many of his discoveries in what later became known as group theory.

Two letters of particular interest to me relate to combinatorics. The first, sent by Euler in 1750 to his St Petersburg friend and colleague Christian Goldbach, contained the first appearance of the polyhedron formula: *in any polyhedron, the number of vertices plus the number of faces is equal to the number of edges plus 2*. The other was the 1852 letter that initiated the four colour problem, when De Morgan wrote to his friend William Rowan Hamilton in Dublin, about 'a fact, which I did not know was a fact, and do not yet'.

But letters don't always communicate. J. J. Sylvester's writing was notoriously difficult to read, as a letter from him to Felix Klein shows: *Dear Professor Klein: I find some difficulty in making out some of the words in your highly esteemed letter and would take it as a great favour if you could write a little more clearly for my behalf as I am not very familiar with German handwriting.*

These days, letters have largely given way to e-mail and the internet as convenient vehicles for communicating mathematics. These have the advantage of being much quicker, but will cause problems for the historians of the future who won't have access to them.

### Mathematics books

In fact, mathematical writings can take many forms – newspaper articles, scholarly texts, research papers in journals, popular books, and Open University correspondence texts. Like lectures, mathematical books and articles range widely from research works, designed for the specialist, to popular accounts. An example of the former is Newton's *Principia Mathematica*, one of the most important scientific books of all time.

Since the invention of printing in the $15^{\text{th}}$ century, books have been accessible to all at low cost. The earliest maths book from Oxford University Press is the *Compotus* of 1520, from which (among other things) you can learn how to calculate the date of Easter on your fingers.

Early printed texts were increasingly written in the vernacular rather than in Latin, such as those of Robert Recorde, the $450^{\text{th}}$ anniversary of whose death is being commemorated next week. Recorde communicated his mathematical ideas in the form of a dialogue between teacher and pupil, much like those in Plato's writings. The writing of popular texts necessitated the introduction of suitable terminology and notation, and Recorde introduced the term *straight line*, as well as his most famous legacy – the *equals sign*, which first appeared in his 1557 algebra book, *The Whetstone of Witte*.

But undoubtedly the most widely used of all mathematics books has been Euclid's *Elements*, written in Alexandria around 300 BC, and in constant use for the next 2000 years. It's been claimed as the most printed book of all time, after the Bible, with the first printed version appearing in Venice in 1482. In the $19^{\text{th}}$ century alone hundreds of English editions were published, some selling half a million copies or more.

In the last few years there's been an explosion in so-called 'popular' maths books: books on the general nature of mathematics, books of mathematical puzzles, books on history or biography, and novels with a mathematical theme or context, such as the recently filmed *Oxford Murders*. There are also books on specific topics, such as Simon Singh's *Fermat's Last Theorem*, Marcus du Sautoy's *The Music of the Primes* on the Riemann hypothesis, and Ian Stewart's *Nature's Numbers*, and recently there have been four books on symmetry and the classification of simple groups.

I've found that I enjoy writing for such an audience – people who get a kick out of maths – and eight years ago I was invited to write a popular book on the *four-colour problem*. This has a complicated but fascinating history, and I enjoyed the challenge of trying to explain the proof in accessible terms without overloading the reader.

I was once asked why I wrote and edited so many books, and my answer was quite clear: *all the books I produce are ones I want to have on my shelf, and when no-one else seems to be doing them for me, I've had to do them myself*. Several of my books have included contributions by experts from around the world, and it's been a great privilege to work with some of the 'stars' in their respective fields, even if they were somewhat shaken at first when their carefully written contributions received the 'Open University treatment', with as much red ink on the page as what they originally wrote. Fortunately, most have agreed that the final product was better than either of us could have produced separately.

Writing student textbooks can also be an enjoyable but exacting experience. I learned my trade with *Introduction to Graph Theory* in 1972, and there have been three further editions since. As any teacher knows, the first thing when writing or lecturing is to *know your audience*, as far as possible, and this applies just as much to research papers for your peers as to books for the general public. We all write in different ways for different audiences, but this doesn't absolve us from making every effort to be as lucid as possible, with careful choices of terminology and notation, appropriate use of diagrams, and suitable examples to motivate the theory and then illustrate it afterwards: all are crucial when we write for our audiences 'out there'.

This is particularly important for us at the Open University who have to communicate our maths at a distance. The OU has a long and impressive history of producing attractive correspondence material that's a pleasure to learn from. For our first *Graphs, Networks and Design* course, a course team of a dozen mathematicians and technologists worked for three years to produce some 1600 pages of printed text, as well as 16 TV programmes and audio-tapes, which were then studied by many thousands of students over the coming years.

But times change. From the very beginning the Open University has embraced the computer. Our teaching and assessment increasingly embrace the electronic, with internet teaching, podcasts and whatever. I submitted my assignments electronically for the OU course I studied this year, and the course I've been writing has been delivered as pdfs over the web. At Gresham College all our lectures are webcast and watched round the world. It's an exciting and challenging time, and if our future is to be as bright as our past, we need encouragement to experiment and innovate, even if some of our bolder experiments fail from time to time.

I'd like to conclude with a further conversation I had in the corridor at the Open University. In the 1970s my former colleague Norman Gowar, later Principal of Royal Holloway College, London, said to me something I never forgot: *It doesn't matter what you do, as long as: it's worthwhile, you do it well, and all the jobs that need to be done are done.* We all have different talents and in general the Open University is better than many at recognising these.

At a time when so much of our funding is tied to narrow research activity (even though most pure maths research papers are read by only a handful of people), it's regrettable that so little official recognition is given by the powers-that-be for communicating our work to the wider world. Research is important, but it's not the only thing we do. In some recent words of Ian Stewart: *It is becoming increasingly necessary, and important, for mathematicians to engage with the general public … Our subject is widely misunderstood, and its vital role in today's society goes mostly unobserved ... Many mathematicians are now convinced that writing about mathematics is at least as valuable as writing new mathematics ... In fact, many of us feel that it is pointless to invent new theorems unless the public gets to hear of them.*

The various styles of communication I've described here require from us different, but no less valuable, talents – and when it comes down to it, we're all in the same game.

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# Tribute to André Lichnerowicz (1915–1998)

Yvette Kosmann-Schwarzbach

**André Lichnerowicz (1915–1998)**

Thirty years ago: *Annals of Physics*, **111**. Deformation quantization was born. This sequence of two articles [BBFLS 1978] had been preceded by several papers written by Daniel Sternheimer, who has just addressed you[1], by Moshé Flato whose memory Daniel has just recalled and whose personality and work in mathematics and physics were so remarkable, and by André Lichnerowicz.

Who was Lichnerowicz? To his students, he was "Lichné". To his friends, he was "André". As a mathematician, mathematical physicist, reformer of the French educational system and its mathematical curriculum, and as a philosopher, he was known to the public as "Lichnerowicz", a man of vast culture, an affable person, a great scientist.

There are many ways to approach André Lichnerowicz. One would be to read all his more than three hundred and sixty articles and books that have been reviewed in *Mathematical Reviews* (MathSciNet), or only read his personal choice among them, those that were published by the Éditions Hermann in 1982 as a 633-page book, *Choix d'œuvres mathématiques* [L 1982]. You can read the summaries of his work up to 1986 that were published by Yvonne Choquet-Bruhat, Marcel Berger and Charles-Michel Marle in the proceedings of a conference held in his honor on the occasion of his seventieth birthday [PQG 1988], or his portrait and an interview which appeared in a handsome, illustrated volume describing the careers of twenty-eight of the most important French scientists of the twentieth century, *Hommes de Science* [HdeS 1990]. Finally, and sadly, scientific obituaries appeared in the *Gazette des Mathématiciens* [Gazette 1999] and they were quickly translated for the *Notices of the American Mathematical Society* [Notices 1999] because Lichnerowicz was as famous outside France as he was within it. Other obituaries appeared in the *Journal of Geometry and Physics* of which he had been one of the founders, and in many other journals.

Lichnerowicz had an exceptional career. He was a student from 1933 to 1936 at the École Normale Supérieure in Paris where he studied under Élie Cartan, who had a lasting influence on his mathematics. He completed his thesis, written under the direction of Georges Darmois, in 1939 and was named professor of mechanics at the University of Strasbourg in 1941. Because of the war, the faculty of the University of Strasbourg had already moved to Clermont-Ferrand in order to avoid functioning under the German occupation. However in 1943, the Germans occupied Clermont-Ferrand as well, and there was a wave of arrests in which Lichnerowicz was taken, but he was fortunate enough, or daring enough, to escape. In those days, he did what he could to help those who were in mortal danger, in particular Jewish colleagues[2]. After the Liberation, the University of Strasbourg returned to Strasbourg. In 1949, he was named professor at the University of Paris, and then in 1952 he was elected to a chair at the Collège de France, the most prestigious position in French higher education[3].

When Lichnerowicz was elected to the Académie des Sciences de Paris – he was only 48, exceptionally young for a member of the Académie in those days – his students, as was customary, collected money to offer him his Academician's sword. (The sword is the only part of the Academician's very elaborate uniform which reflects his or her personality and accomplishments.) But two years later, for his fiftieth birthday, they contributed nearly as much to offer him something more to his taste, ... a pipe! Indeed we could not imagine him without his pipe at any time ... except during his lectures when he would fill the blackboard with equations in his dense handwriting, equations almost always comprising many tensorial indices. It is a fact that he can be seen in every photograph ... with his pipe.

Lichnerowicz's work in general relativity began with his thesis in which he gave the first global treatment of Einstein's theory of general relativity and determined necessary and sufficient conditions for a metric of hyperbolic signature on a differentiable manifold to be a global solution for the Einstein equations. He proved that there cannot exist any gravitational solitons; he established the "Lichnerowicz equation", an elliptic semi-linear equation used in the solution of the constraint equations to be satisfied by the inital conditions for Einstein's equations. He pursued this area of study throughout his research career. "Differential geometry and global analysis on manifolds", "the relations between mathematics and physics", "the mathematical treatment of Einstein's theory of gravitation", this is how he, himself, described his main interests and achievement in the interview published in *Hommes de Science* [HdeS 1990].

His work in Riemannian geometry remains particularly important. He was among the first geometers to establish a relation between the spectrum of the Laplacian and the curvature of the metric; he proved the now classical equivalence of the various definitions of Kähler manifolds; he showed, together with Armand Borel, that the restricted holonomy group of a Riemannian manifold is compact, and of course many other important results.

In the early sixties, Lichnerowicz established Cartan's and Weyl's theory of spinors in a rigorous differential geometric framework, on a pseudo-Riemannian manifold with a hyperbolic (Lorentzian) metric. Using this geometric approach in his courses at the Collège de France in 1962–1964, he developed Dirac's theory of the electron and that of Rarita-Schwinger for spin $\frac{3}{2}$, and then the Petiau-Duffin-Kemmer theory as well as the theory of the CPT transformations, while also in 1963 he published the landmark 'Note aux *Comptes rendus*' of the Académie on harmonic spinors [L 1963], in which he proved that, for any spinor field, $\psi$,

$$\Delta\psi = -\nabla^{\rho}\nabla_{\rho}\psi + \frac{1}{4}R\psi ,$$

where $\Delta = P^2$ is the Laplacian on spinors, the square of the Dirac operator $P$, $\nabla$ is the covariant derivative, and $R$ is the scalar curvature. And he continued working on spinors to his last days.

At the beginning of the seventies, Lichnerowicz's interest turned to the geometric theory of dynamical systems. Symplectic geometry had been studied for some time[4]. In January 1973, a conference, "Geometria simplettica e fisica matematica", was held in Rome that was, I believe, the first international meeting on this topic. As a young researcher I attended the congress, and heard and met many of the founders of symplectic geometry among whom were Jean Leray, Irving Segal, Bertram Kostant, Shlomo Sternberg, Włodzimierz Tulczyjew, Jean-Marie Souriau as well as the young Alan Weinstein and Jerry Marsden. And Lichnerowicz was one of the organizers of the meeting and delivered the opening lecture.

The main reason that we pay tribute to Lichnerowicz's memory here, today, at this conference on Poisson geometry, is that he founded it. This was a few years before the publication of the deformation quantification paper I recalled at the beginning of this tribute.

His son, Jérôme Lichnerowicz, speaking of his father's collaboration with Moshé Flato, said: "There was no master and no student but an incredible synergy between friends. I saw Moshé encourage André when, ageing, he doubted his own strength," and he added: "I heard Moshé tell me: 'It is unbelievable, he [Lichnerowicz] had an arid period, but now he is back doing mathematics as before!'." [CMF 2000] Starting in 1974, working with Moshé Flato and Daniel Sternheimer, Lichnerowicz formulated the definition of a Poisson manifold in terms of a bivector, *i.e.*, the contravariant 2-tensor advocated by Lie, Carathéodory and Tulczyjew, which is the counterpart of the 2-form of symplectic geometry. In his article published in *Topics in Differential Geometry* [L 1976] he defined the canonical manifolds and one can already find in that paper a formula for the bracket of 1-forms associated to a Poisson bracket of functions, although still only for exact forms,

$$[df, dg] = d\{f, g\} .$$

(Later he showed that the canonical manifolds are those Poisson manifolds whose symplectic foliation is everywhere of co-dimension one.) In his 1977 article in the *Journal of Differential Geometry*, "Les variétés de Poisson et leurs algèbres de Lie associées" [L 1977], Lichnerowicz introduced the cohomology operator that is now called the "Poisson cohomology operator" but really should be called the "Lichnerowicz-Poisson cohomology operator", a profound discovery. We read there as well as in [BFFLS, 1978] that, in the particular case of a symplectic manifold,

$$\mu([G,A]) = d\mu(A) ,$$

where $A$ is a field of multivectors. (The notations are $G$ for the Poisson bivector and $\mu$ for the prolongation to multivectors of the isomorphism from the tangent bundle to the cotangent bundle defined by the symplectic form, the bracket is the Schouten-Nijenhuis bracket, and $d$ is the de Rham differential.) This formula, which we rewrite in a more familiar notation,

$$\omega^{\flat}([\pi,A]) = d(\omega^{\flat}(A)) \qquad \text{or} \qquad \pi^{\sharp}(d\alpha) = d_{\pi}(\pi^{\sharp}\alpha)$$

(here G is replaced by $\pi$, and $\mu$ by $\omega^{\flat}$, with inverse $\pi^{\sharp}$, while $\alpha$ is a differntial form and $d_{\pi}$ is the Lichnerowicz-Poisson differential, $d_{\pi} = [\pi, \cdot]$, acting on multivectors), is the precursor of the chain map property of the Poisson map, mapping the de Rham complex to the Lichnerowicz-Poisson complex, and, more generally, this article is the point of departure for the great development of Poisson geometry that we have witnessed and in which we are participating here. Together with his earlier articles written jointly with Flato and Sternheimer [FLS 1974, 1975, 1976] and with the article in the *Annals of Physics* [BBFLS 1978], solving quantization problems by a deformation of the commutative multiplication of the classical observables when given a Poisson structure, this article established the foundation of what has become a vast field of mathematical research.

It was a privilege for Lichnerowicz's many doctoral students, of whom I was one in the late 1960s, to be received by him in his small office under the roof of the Collège de France, or in the study of his apartment on the Avenue Paul Appell, on the southern edge of Paris. Surrounded by collections of journals and piles of papers, Lichné, with his pipe, would offer encouragement and invaluable hints as to how to make progress on a difficult research problem. I knew then, we all knew, that we were talking to a great mathematician. But I did not even guess that I was talking to the creator of a theory which would develop into a field in its own right, one with ramifications in a very large number of areas of mathematics and physics.

## Notes

1. This tribute was delivered at the Conference "Poisson 2008, Poisson Geometry in Mathematics and Physics" at the École Polytechnique Fédérale, Lausanne, in July 2008. It was preceded by tributes to Stanisław Zakrzewski, founder of the series of international conferences on Poisson geometry, who died in April 1998 (by Alan Weinstein), to Paulette Libermann on the first anniversary of her death (by Charles-Michel Marle), and to Moshé Flato, co-author and friend of Lichnerowicz, who died in November 1998 (by Daniel Sternheimer). This tribute was followed by the award of the "André Lichnerowicz prize in Poisson Geometry" to two prominent young mathematicians. An announcement of the prize appeared in the *Newsletter of the EMS*, 70, December 2008, p. 50.
2. In his autobiography [S 1997, p. 200], Laurent Schwartz tells that one day Lichnerowicz contrived to be in a police station and, while an officer

was inattentive, he managed to borrow a stamp and apply it to a false identity card which he, himself, did not need, but which could save the life of a colleague or a student. To survive the war years in France with some honor was itself a great achievement.

3. A chair in mathematics was established by François I at the foundation of the Collège in 1531!
4. As Hermann Weyl explained at the beginning of Chapter VI of his *The Classical Groups* [W 1939], he had coined the adjective "symplectic" after the Greek as an alternative to the adjective "complex".

## References


[BBFLS 1978] François Bayen, Moshé Flato, Chris Fronsdal, André Lichnerowicz, Daniel Sternheimer, Deformation theory and quantization, *Annals of Physics*, **111** (1978).
I. Deformations of symplectic structures, 61-110.
II. Physical applications, 111-151.

[L 1982] André Lichnerowicz, *Choix d'œuvres mathématiques*, Hermann, Paris, 1982.

[PQG 1988] *Physique quantique et géométrie, Colloque Géométrie et Physique de 1986 en l'honneur de André Lichnerowicz*, Daniel Bernard and Yvonne Choquet-Bruhat, eds., Hermann, Paris, 1988.
– Yvonne Choquet-Bruhat, *L'œuvre de André Lichnerowicz en relativité générale*, pp. 1–10.
– Marcel Berger, *L'œuvre d'André Lichnerowicz en géométrie riemannienne*, pp. 11–24.
– Charles-Michel Marle, *L'œuvre d'André Lichnerowicz en géométrie symplectique*, pp. 25–42.

[HdeS 1990] *Hommes de Science*, 28 portraits, Entretiens et photographie de Marian Schmidt, Hermann, Paris, 1990.

[Gazette 1999] *Gazette des Mathématiciens*, **81** (1999), 94–95 and **82** (1999), 90–108.

[Notices 1999] *Notices Amer. Math. Soc.*, **46** (11) (1999), 1387–1396.

[S 1997] Laurent Schwartz, *Un Mathématicien aux prises avec le siècle*, Odile Jacob, Paris, 1997.

[L 1963] André Lichnerowicz, Spineurs harmoniques, *C. R. Acad. Sci. Paris*, **257** (1963), 7–9.

[W 1939] Hermann Weyl, *The Classical Groups*, Princeton University Press, Princeton, NJ, 1939; second edition, 1946.

[CMF 2000] Jérôme Lichnerowicz, in *Conférence Moshé Flato 1999*, vol. I, Giuseppe Dito and Daniel Sternheimer, eds., Kluwer, Dordrecht, 2000, pp. 7–8.

[L 1976] André Lichnerowicz, Variétés symplectiques, variétés canoniques et systèmes dynamiques, in *Topics in Differential Geometry, in memory of E. T. Davies*, Academic Press, New York, 1976, pp. 57–85.

[L 1977] André Lichnerowicz, Les variétés de Poisson et leurs algèbres de Lie associées, *J. Differential Geometry*, **12** (1977), 253–300.

[FLS 1974] Moshé Flato, André Lichnerowicz, Daniel Sternheimer, Déformations 1-différentiables d'algèbres de Lie attachées à une variété symplectique ou de contact, *C. R. Acad. Sci. Paris* Sér. A, **279** (1974), 877–881.

[FLS 1975] Moshé Flato, André Lichnerowicz, Daniel Sternheimer, Algèbres de Lie attachées à une variété canonique, *J. Math. Pures Appl.* (9), **54** (1975), 445–480.

[FLS 1976] Moshé Flato, André Lichnerowicz, Daniel Sternheimer, Deformations of Poisson brackets, Dirac brackets and applications, *J. Mathematical Phys.*, **17** (1976), 1754–1762.


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*The early work of Yvette Kosmann-Schwarzbach [yks@math.polytechnique.fr], beginning with her Doctorat d'État in mathematics (University of Paris, 1970) in which she defined the Lie derivative of spinors and proved the conformal covariance of Dirac's equation on spin manifolds, was directly inspired by the work of her advisor, André Lichnerowicz. She is the author of some 70 research articles in differential geometry, algebra and mathematical physics, has published a historical study on* The Noether Theorems *(in French, soon to appear in English), and co-edited several books on integrable systems. She has been a professor of mathematics at the University of Lille, at Brooklyn College of the City University of New York, and most recently at the École Polytechnique (France).*

# Contour dynamics for 2D active scalars

Diego Córdoba and Francisco Gancedo

## Introduction

One line of research in the mathematical analysis of fluid mechanics is focused on solving problems that involve the possible formation and propagation of singularities. In these scenarios it becomes crucial to understand the role played by the singularities in the formation of patterns. For this purpose we present two physical models that are of interest from this mathematical point of view as well as for their applications in physics. It is an enormous challenge to approach these problems, which require a combination of analytic techniques, asymptotics, numerics and modelling. With the more sophisticated numerical tools now available, the subject has gained considerable momentum. Recently, numerical simulations indicate a possible singularity formation in the free boundary of a fluid domain, which is a weak solution to a family of incompressible equations.

Successful analysis of singularities in incompressible flows would solve a major problem of mathematics and would establish a new method for addressing blow up formation in non-linear partial differential equations. A fluid dynamic understanding of these singularities could lead to important insights on the structure of turbulence, one of the major open scientific problems of classical physics.

On the search for singularities we study the simplest possible models that capture the non-local structure of an incompressible flow: active scalars. These remarkable examples (see [5]) are solutions $\rho = \rho(x,t)$ to the following non-linear equation:

$$(\partial_t + u\cdot\nabla)\rho = 0 \tag{1}$$

with $(x,t) \in \mathbb{R}^2 \times \mathbb{R}^+$ and $u = (u_1, u_2)$ the fluid velocity. This transport equation reveals that the quantity $\rho$ moves with the fluid flow and is conserved along trajectories. The velocity field is divergence free, providing the incompressibility condition, and is determined by the active scalar by singular integral operators as follows:

$$u_i(x,t) = P.V.\int_{\mathbb{R}^2} K_i(x-y)\rho(y,t)dy \tag{2}$$

where $P.V.$ denotes principal value and $K_i$ is a classical Calderon-Zymund kernel (see [16]). The above identity indicates the non-local structure of the equation and that the velocity is at the same level as the scalar $\rho$: $\|u\|_{L^p}(t) \le C\|\rho\|_{L^p}(t)$ for $1 < p < \infty$.

The incompressibility of the flow yields the system to be conservative in such a way that the $L^p$ norms of $\rho$ are constants for all time: $\|\rho\|_{L^p}(t) = \|\rho\|_{L^p}(0)$ for $1 \le p \le \infty$.

A fundamental property of the active scalars is that the level sets move with the flow, i.e. there is no transfer of flow along a level set. Then, a natural solution, with finite energy, can be given by

$$\rho(x,t) = \begin{cases} \rho^1 & \text{if } x \in \Omega^1(t) \\ \rho^2 & \text{if } x \in \Omega^2(t) = \mathbb{R}^2 \backslash \Omega^1(t) \end{cases} \tag{3}$$

for $\Omega^i(t)$ connected regions. This represents an evolution problem for a fluid with different characteristics $\rho^i$ that remain constant inside each domain $\Omega^i(t)$. These solutions start initially with a jump on the boundary of $\Omega^i(t)$ and the equation of the contour dynamics is highly singular.

Below, we consider solutions of type (3) for the following different physical scenarios: one given by the quasi-geostrophic equation and the other modelled by Darcy's law. These systems satisfy equations (1) and (2) in a weak sense with completely different outcomes regarding well-posedness and regularity issues.

## The 2D surface quasi-geostrophic equation

The 2D surface quasi-geostrophic equation that we will address below has the property that the velocity $u$ in the form (2) is given by

$$u = (-R_2\rho, R_1\rho)$$

where the scalar $\rho$ represents the temperature of the fluid and $R_j$ are the Riesz transforms:

$$R_j\rho(x,t) = \frac{1}{2\pi}P.V.\int_{\mathbb{R}^2} \frac{(x_j - y_j)\cdot\rho(y,t)}{|x-y|^3}dy.$$

The symbols of these operators in the Fourier side are $\widehat{R_j} = i\frac{\xi_j}{|\xi|}$, therefore the energy of the system is conserved due to the fact that

$$\|u\|_{L^2} = \|(-R_2(\rho-\rho_\infty), R_1(\rho-\rho_\infty))\|_{L^2} = \|\rho-\rho_\infty\|_{L^2},$$

for $\rho_\infty$ the constant value of $\rho$ at infinity.

This equation, which we will denote by QG, has applications in meteorology and oceanography and is a special case of the more general 3D quasi-geostrophic equation. There has been high scientific interest in understanding the behaviour of the QG equation because it is a plausible model to explain the formation of fronts of hot and cold air. In a different direction, Constantin, Majda and Tabak [4] proposed this system as a 2D model for 3D vorticity intensification and they showed that there is a geometric and analytic analogy with 3D incompressible Euler equations.

An interesting approach is to study the dynamics of the $\alpha - patches$ (see [7]), which are weak solutions of a family of equations that interpolate 2D incompressible Euler and QG. An $\alpha - patch$ $(0 < \alpha < 1)$ consists of a 2D region $\Omega(t)$ (bounded and connected) that moves with velocity given by

$$u(z(\gamma,t),t) = \frac{C_\alpha}{2\pi}\int_{-\pi}^{\pi} \frac{\partial_\gamma z(\eta,t)}{|z(\gamma,t) - z(\eta,t)|^\alpha}d\eta \tag{4}$$

where $z(\gamma,t)$ is the position of the boundary of the domain $\Omega(t)$ parametrized for $\gamma \in [-\pi,\pi]$ and $C_\alpha$ depends on $\alpha$, $\rho^1$ and $\rho^2$. The evolution of its boundary satisfies

$$z_t(\gamma,t) = u(z(\gamma,t),t) \tag{5}$$

and they are weak solutions of (1) and the following equation

$$u = -\nabla^\perp (-\Delta)^{\frac{\alpha}{2}-1}\rho, \tag{6}$$

where the symbol of $(-\Delta)^\beta$ is $|2\pi\xi|^{2\beta}$ and $x^\perp = (-x_2,x_1)$.

In the limiting case $\alpha = 0$, the identity (6) becomes the Biot-Savart law and we can therefore recover the 2D incompressible Euler equation. With this notation the scalar $\rho$ represents the vorticity of the two dimensional flow. This case has been studied analytically with success by Chemin and Bertozzi-Constantin in [3] and [2] where they show global existence and therefore no singularity formation.

In the case $\alpha = 1$, for a temperature $\rho$ satisfying (4), the velocity blows up logarithmically at the free boundary $z(\gamma,t)$. Thus, there is difficulty in even deriving the evolution problem. This issue can be solved since the normal component of the velocity is well-defined for a regular contour. For this type of free boundary problem the tangential component of the velocity does not modify the shape of the interface (in this case it blows up) and the dynamics are given by the normal component.

Rodrigo in [13] gave a closed system for the *patch* problem for QG where the front is represented by a function. He proved local existence for a periodic and infinitely differentiable contour by using the Nash-Moser iteration. This tool was applied for infinitely differentiable initial data because the operator involved loses two derivatives.

In the case for which the free boundary is not parametrized as a function, the fact that the interface collapses leads to a singularity in the fluid. Then it becomes crucial to get control of the evolution of the following quantity:

$$\mathscr{F}(z)(\gamma,\eta,t) = \frac{|\eta|}{|z(\gamma,t)-z(\gamma-\eta,t)|} \quad \forall \gamma,\eta \in [-\pi,\pi],$$

with

$$\mathscr{F}(z)(\gamma,0,t) = \frac{1}{|\partial_\gamma z(\gamma,t)|},$$

which measures the arc-chord condition of the curve. Let us point out that the operators involved in the equations are ill-defined otherwise. Also, one could modify the contour equation (4) as follows:

$$z_t(\gamma,t) = \frac{C_1}{2\pi}\int_{\mathbb{T}} \frac{\partial_\gamma z(\gamma,t)-\partial_\gamma z(\eta,t)}{|z(\gamma,t)-z(\eta,t)|}d\eta + c(\gamma,t)\partial_\gamma z(\gamma,t). \tag{7}$$

A curve that satisfies this new equation also yields a solution of the *patch* problem for QG: the terms introduced in the evolution system are tangential and therefore they change the parametrization of the interface but not the shape. Choosing $c(\gamma,t)$ in a wise fashion enables the length of the tangent vector to $z(\gamma,t)$ to be a function in the variable $t$ only:

$$A(t) = |\partial_\gamma z(\gamma,t)|^2.$$

With this property it is easy to obtain the following two identities:

$$\partial_\gamma^2 z(\gamma,t)\cdot\partial_\gamma z(\gamma,t) = 0,$$
$$\partial_\gamma^3 z(\gamma,t)\cdot\partial_\gamma z(\gamma,t) = -|\partial_\gamma^2 z(\gamma,t)|^2.$$

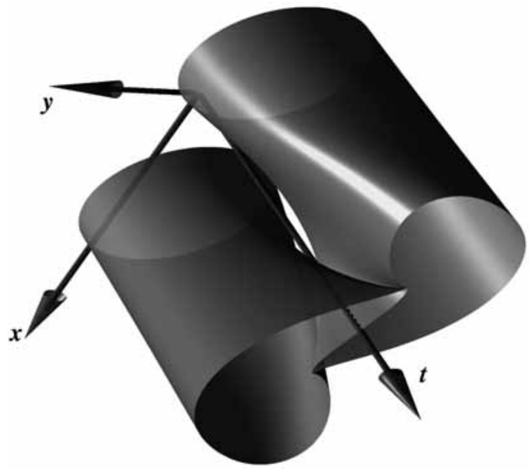


**Figure 1. The evolution of two *patches* with $\alpha = 1$**

The first equality gives extra cancellation in the system (7) and the second one is a kind of ad hoc integration by parts. Both are used in [10] to obtain a proof of local-existence for a patch convected by the QG equation within the chain of Sobolev spaces. Similar results follow for the $\alpha - patch$.

In [7] numerically possible candidates are shown that lead to a singularity for the family of equations (6). For the particular case $\alpha = 1$ there is a formation of a corner that develops a high increase on the curvature at the same point where it reaches the minimum distance between two *patches* (see Figure 1).

Furthermore, by re-scaling the spatial variable in the following form:

$$z = (t_0 - t)^\delta y,$$

where $\delta = \frac{1}{\alpha}$, and introducing a new variable $\tau = -log(t_0 - t)$, the contour dynamic equations (4) and (5) become

$$\frac{\partial y}{\partial \tau} - \delta y = \frac{C_\alpha}{2\pi}\int_{-\pi}^{\pi} \frac{\partial_\gamma y(\gamma,t) - \partial_\gamma y(\eta,t)}{|y(\gamma,t)-y(\eta,t)|^\alpha} d\eta. \tag{8}$$

Solutions of (8) independent of $\tau$ represent solutions of (6) with the property that the maximum curvature grows as

$$\kappa = \frac{1}{R} \sim \frac{C}{(t_0-t)^{\frac{1}{\alpha}}} \quad \text{when} \quad t \to t_0$$

and the minimum distance of the two *patches* satisfies

$$d \sim C(t_0-t)^{\frac{1}{\alpha}} \quad \text{when} \quad t \to t_0.$$

These singularities are self similar and stable, occurring in one single point where the curvature blows up at the same time as the two level sets collapse (see Figure 2).

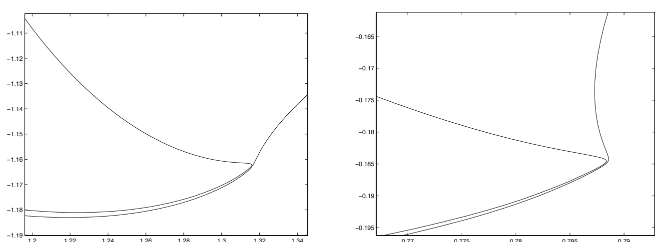

**Figure 2. Detail of the corner region at t=16.515 for $\alpha = 0.5$ (left) and t=4.464 for $\alpha = 1$ (right). Observe that the singularity is point-like in both cases.**

### Darcy's law

The evolution of fluids in porous media is an important topic in fluid mechanics encountered in engineering, physics and mathematics. This phenomena has been described using the experimental Darcy's law that, in two dimensions, is given by the following momentum equation:

$$\frac{\mu}{\kappa}u = -\nabla p - (0, g\rho).$$

Here $u$ is the incompressible velocity, $p$ is the pressure, $\mu$ is the dynamic viscosity, $\kappa$ is the permeability of the isotropic medium, $\rho$ is the liquid density and g is the acceleration due to gravity.

The Muskat problem [11] models the evolution of an interface between two fluids with different viscosities and densities in porous media by using Darcy's law. This problem has been considered extensively without surface tension, in which case the pressures of the fluids are equal on the interface. Saffman and Taylor [14] made the observation that the one phase version (one of the fluids has zero viscosity) was also known as the Hele-Shaw cell equation, which, in turn, is the zero-specific heat case of the classical one-phase Stefan problem.

The problem considers fluids with different constant viscosities $\mu^1$, $\mu^2$ and densities $\rho^2$, $\rho^1$. Therefore, using Darcy's law, we find that the vorticity is concentrated on the free boundary $z(\gamma,t)$ and is given by a Dirac distribution as follows:

$$\omega(x,t) = \varpi(\gamma,t)\delta\big(x - z(\gamma,t)\big),$$

with $\varpi(\gamma,t)$ the vorticity strength. Then $z(\gamma,t)$ evolves with an incompressible velocity field coming from the Biot-Savart law:

$$u(x,t) = \nabla^{\perp}\Delta^{-1}\omega(x,t). \tag{9}$$

It can be explicitly computed on the contour $z(\gamma,t)$ and is given by the Birkhoff-Rott integral of the amplitude $\varpi$ along the interface curve:

$$BR(z,\varpi)(\gamma,t) = \frac{1}{2\pi}P.V.\int \frac{\big(z(\gamma,t)-z(\eta,t)\big)^{\perp}}{|z(\gamma,t)-z(\eta,t)|^2}\varpi(\eta,t)d\eta. \tag{10}$$

Using Darcy's law, we close the system with the following formula:

$$\varpi(\gamma,t) = (I + A_\mu T)^{-1}\big(-2g\kappa\frac{\rho^2-\rho^1}{\mu^2+\mu^1}\partial_\gamma z_2\big)(\gamma,t), \tag{11}$$

where

$$T(\varpi) = 2BR(z,\varpi)\cdot\partial_\gamma z, \quad A_\mu = \frac{\mu^2-\mu^1}{\mu^2+\mu^1}.$$

Baker, Meiron and Orszag [1] have shown that the adjoint operator $T^*$, acting on $\varpi$, is described in terms of the Cauchy integral of $\varpi$ along the curve $z(\gamma,t)$ and has real eigenvalues that have absolute values strictly less than one. This yields that the operator $(I + A_\mu T)$ is invertible so that equation (11) gives an appropriate contour dynamics problem.

The first important question to be asked is whether local-existence is guaranteed. However, such a result turns out to be false for general initial data. Rayleigh [12] and Saffman-Taylor [14] gave a condition that must be satisfied in order to have a solution locally in time, namely that the normal component of the pressure gradient jump at the interface has to have a distinguished sign. This is known as the Rayleigh-Taylor condition. Siegel, Caflish and Howison [15] proved ill-posedness in a 2D case when this condition is not satisfied (unstable case and same densities). On the other hand, they showed global-in-time solutions when the initial data are nearly planar and the Rayleigh-Taylor condition holds initially.

Recently in [6], we have obtained local existence in the 2D case when the fluid has different densities and viscosities. In our proof it is crucial to get control of the norm of the inverse operators $(I + A_\mu T)^{-1}$. The arguments rely upon the boundedness properties of the Hilbert transforms associated to $C^{1,\delta}$ curves, for which we need precise estimates obtained with arguments involving conformal mappings, the Hopf maximum principle and Harnack inequalities. We then provide bounds in the Sobolev spaces $H^k$ for $\varpi$ obtaining

$$\begin{aligned}\frac{d}{dt}(\|z\|^2_{H^k} + \|\mathscr{F}(z)\|^2_{L^\infty})(t) \\ \le -K\int_{\mathbb{T}} \frac{\sigma(\gamma)}{|\partial_\gamma z(\gamma)|^2}\partial^k_\gamma z(\gamma)\cdot\Lambda(\partial^k_\gamma z)(\gamma)d\gamma \\ + \exp C(\|z\|^2_{H^k} + \|\mathscr{F}(z)\|^2_{L^\infty})(t),\end{aligned} \tag{12}$$

where $K = -\kappa/(2\pi(\mu_1+\mu_2))$, $\sigma(\gamma,t)$ is the difference of the gradients of the pressure in the normal direction (Rayleigh-Taylor condition) and the operator $\Lambda$ is the square root of the Laplacian. When $\sigma(\gamma,t)$ is positive, there is a kind of heat equation in the above inequality but with the operator $\Lambda$ in place of the Laplacian. Then, the most singular terms in the evolution equation depend on the Rayleigh-Taylor condition. In order to integrate the system we study the evolution of

$$m(t) = \min_{\gamma\in\mathbb{T}} \sigma(\gamma,t),$$

which satisfies the following bound:

$$|m'(t)| \le \exp C(\|\mathscr{F}(z)\|^2_{L^\infty} + \|z\|^2_{H^k})(t).$$

Using the pointwise estimate $f\Lambda(f) \ge \frac{1}{2}\Lambda(f^2)$ in estimate (12), we obtain

$$\frac{d}{dt}E_{RT}(t) \le C\exp CE_{RT}(t),$$

where $E_{RT}$ is the energy of the system given by

$$E_{RT}(t) = \|z\|^2_{H^k}(t) + \|\mathscr{F}(z)\|^2_{L^\infty}(t) + \big(m(t)\big)^{-1}.$$

Here we point out that it is completely necessary to consider the evolution of the Rayleigh-Taylor condition to obtain bona fide energy estimates.

In the case where the viscosities are the same, the free boundary is given by a fluid with different densities. In order to simplify the notation, one could take $\mu/\kappa = g = 1$ in Darcy's law and then apply the rotational operator to obtain the vorticity given by $\omega = -\partial_{x_1}\rho$. The Biot-Savart law (9) yields the velocity field in terms of the density as follows:

$$u(x,t) = P.V.\int_{\mathbb{R}^2} H(x-y)\rho(y,t)dy - \frac{1}{2}\big(0,\rho(x,t)\big),$$

where the Calderon-Zygmund kernel $H(\cdot)$ is defined by

$$H(x) = \frac{1}{2\pi}\left(-2\frac{x_1x_2}{|x|^4}, \frac{x_1^2-x_2^2}{|x|^4}\right).$$

By means of Darcy's law, we can find the following formula for the difference of the gradients of the pressure in the normal direction: $\sigma(\gamma,t) = g(\rho^2 - \rho^1)\partial_\gamma z_1(\gamma,t)$. A wise choice of parametrizing the curve is that for which we have $\partial_\gamma z_1(\gamma,t) = 1$ (for more details see [8]). This yields the denser fluid below the less dense fluid if $\rho^2 > \rho^1$ and therefore the Rayleigh-Taylor condition holds for all time. An additional advantage is that we avoid a kind of singularity in the fluid when the interface collapses due to the fact that we can take $z(\gamma,t) = (\gamma, f(\gamma,t))$, which implies $\mathscr{F}(z)(\gamma,\eta) \leq 1$, obtaining the arc-chord condition for all time. Then the character of the interface as the graph of a function is preserved and in [8] this fact has been used to show local-existence in the stable case ($\rho^2 > \rho^1$), together with ill-posedness in the unstable situation ($\rho^2 < \rho^1$).

Currently we are studying the long-time behaviour of the stable case for which we can show that the $L^\infty$ norm of any interface decays and numerical simulations of the dynamics of the contour develop a regularity effect (see [9]).

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## References

[1] G. Baker, D. Meiron and S. Orszag. Generalized vortex methods for free-surface flow problems. *J. Fluid Mech.* 123 477–501, 1982.

[2] A. L. Bertozzi and P. Constantin. Global regularity for vortex patches. *Comm. Math. Phys.* 152 (1), 19–28, 1993.

[3] J. Y. Chemin. Persistence of geometric structures in two-dimensional incompressible fluids. *Ann. Sci. Ecole. Norm. Sup.* 26 (4), 517–542, 1993.

[4] P. Constantin, A. J. Majda, and E. Tabak. Formation of strong fronts in the 2-D quasigeostrophic thermal active scalar. *Nonlinearity*, 7, 1495–1533, 1994.

[5] P. Constantin. Scaling exponents for active scalars. *Journal of Statistical Physics*, vol 90, 3–4, 571–595, 1998.

[6] A. Córdoba, D. Córdoba and F. Gancedo. Interface evolution: the Hele-Shaw and Muskat problems. Preprint 2008, arXiv: 0806.2258.

[7] D. Córdoba, M. A. Fontelos, A. M. Mancho and J. L. Rodrigo. Evidence of singularities for a family of contour dynamics equations. *Proc. Natl. Acad. Sci. USA* 102, 5949–5952, 2005.

[8] D. Córdoba and F. Gancedo. Contour dynamics of incompressible 3-D fluids in a porous medium with different densities. *Comm. Math. Phys.* 273, 2, 445–471 (2007).

[9] D. Córdoba, F. Gancedo and R. Orive. A note on the interface dynamics for convection in porous media. *Physica D*, 237, 1488–1497, 2008.

[10] F. Gancedo. Existence for the $\alpha$-patch model and the QG sharp front in Sobolev spaces. *Adv. Math.*, 217, 2569–2598, 2008.

[11] M. Muskat. The flow of homogeneous fluids through porous media. *New York*, Springer 1982.

[12] Lord Rayleigh (J. W. Strutt), On the instability of jets. *Proc. Lond. Math. Soc.* 10, 413, 1879.

[13] J. L. Rodrigo. On the Evolution of Sharp Fronts for the Quasi-Geostrophic Equation. *Comm. Pure and Appl. Math.*, 58, 0821–0866, 2005.

[14] P. G. Saffman and Taylor. The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid. *Proc. R. Soc. London, Ser. A* 245, 312–329, 1958.

[15] M. Siegel, R. Caflisch and S. Howison. Global Existence, Singular Solutions, and Ill-Posedness for the Muskat Problem. *Comm. Pure and Appl. Math.*, 57, 1374–1411, 2004.

[16] E. Stein. Singular Integrals and Differentiability Properties of Function. *Princeton University Press.* Princeton, NJ, 1970.

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# Facts and Myths about Stefan Banach[1]

Roman Duda

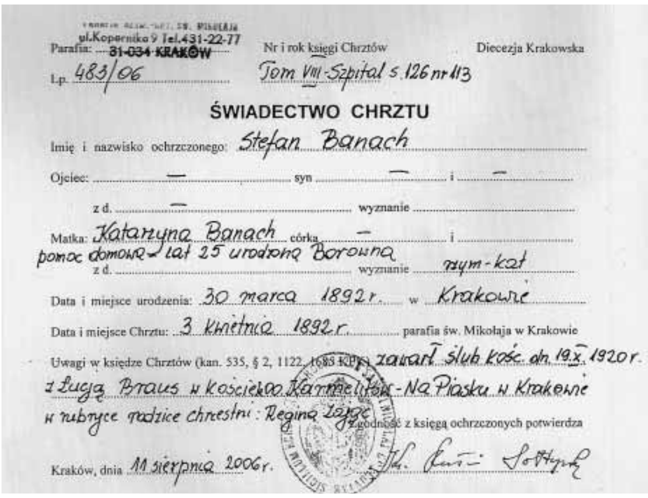
Vitali Milman writes[2], as others have quoted[3], that an older lady with the maiden name "Banach", told him the following anecdote: her grandmother, Netl Banach (married to cousin Mosze Banach), was said to have a younger brother, who later became a prominent mathematician in Lvov. Milman writes that the young man abandoned Jewish traditions and around the age of fifteen became Catholic. His family rejected him and no other information on this topic has been found.

This story is unbelievable. We neither know when and where Netl Banach lived, nor do we know her family, nor the surroundings in which she was brought up, nor where this story originated. What is certain regarding this mysterious rumor is that no further information is available.

Historically, abandoning Judaism is not uncommon, but usually occurs at a mature age, and not by a rebellious fifteen-year old. Accepting baptism in the Roman Catholic Church, if it is not done soon after a child's birth by his parents, requires proper religious instruction, and such a baptism would be recorded at the associated parish. Milman does not find any evidence to support the story mentioned above.

The circumstances surrounding the birth of Stefan Banach could be the basis for this unlikely story. These circumstances have been unclear for a long time[4], but thanks to K. Ciesielski[5] and E. Jakimowicz[6], enough information has been unearthed that today we can reconstruct Banach's historical background. As opposed to the work of Milman, this new narrative will be supported with primary and secondary sources, such as documents, photographs and oral histories of people who knew Banach or knew of him.

Stefan Banach was born on March 30, 1892 in the Lazarus' National Public Hospital in Cracow and was bap-



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ŚWIADECTWO CHRZTU

Imię i nazwisko ochrzczonego: Stefan Banach
Ojciec: — syn — i —
z d. — wyznanie
Matka: Katarzyna Banach córka — i
pomoc domowa – lat 25 urodzona Borowna
z d. wyznanie rzym-kat
Data i miejsce urodzenia: 30 marca 1892 r. w Krakowie
Data i miejsce Chrztu: 3 kwietnia 1892 r. parafia św. Mikołaja w Krakowie
Uwagi w księdze Chrztów (kan. 535, § 2, 1122, 1688 KPK) zawarł ślub kośc. dn. 19.X. 1920 r. z Łucją Braus w kościele OO. Karmelitów-Na Piasku w Krakowie
w rubryce rodzice chrzestni: Regina Zając
Zgodność z księgą ochrzczonych potwierdza
Kraków, dnia 11 sierpnia 2006 r.

Fig. 1. Parish register of the baptism of Stefan Banach.

tized on April 3, 1892 in the hospital chapel belonging to the St. Nicolaus parish. Banach's baptismal certificate (Fig. 1) does not mention a father, but the mother is listed as: "Katarzyna Banach, housekeeper, twenty-five years of age, born in Borowna." The mother gave the boy her surname.

There is not much information on the mother. The village of Borowna is within the parish of Lipnica Murowana (15 km south west of Bochnia) and the parish records of Borowna show that Katarzyna Banach was born in 1864. In Cracow, she was a maid for an Austrian officer. Furthermore, as told by Banach's foster family, she also worked at Franciszka Płowa's laundry service.

We know more about the child's father from his own words. Throughout his son's life, he came to Stefan's help, financially supporting him and inviting his son to his house. When Stefan Greczek disclosed Banach's origin, he did so without a benefit motive.

Stefan Banach's father was Stefan Greczek (1867–1967), a highlander from the region of Nowy Targ, who served in the Austrian Army (Fig. 2) as a batman to the same officer for whom Katarzyna worked at the time of Banach's birth. Some years later, he wrote about the circumstances of Banach's birth (Fig. 3):

*When you were born I was 24 years old and 4 months old. I was serving in the army. Without permission from the military authorities I was not allowed to marry. Permission was only granted to those who were able to document and show that the marriage would improve their means and circumstances. Your mother, who was a maid by trade, was paid only 5 Fl. per month. It was out of the question to even think about getting permission to marry. With my income neither could I provide for your mother. After a month we decided to give you up to be raised in the country and I pledged to pay for you, which I did every month*[7].

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[1] Polish version of this article is to appear in "Wiadom. Mat." The editors of that journal kindly agreed to publish this English version.

[2] V.D. Mil'man, *A look at movements of people and ideas in XX-century mathematics*, in: Yu.S. Osipov and others (eds.), Mathematical events in XX century, Moscow 2003, pp. 279–303 [Russian].

[3] Cf. A. Pietsch, *History of Banach Spaces and Linear Operators*, Birkhäuser, Boston-Basel-Berlin 2003, pp. 638–639. The author, however, is careful not to take a definite stand. pp. 638–639.

[4] Some were unknown even to the author of a thorough biography: R. Kałuża, *The Life of Stefan Banach*, Trans. and edited by A. Kostant and W. Woyczyński, Birkhäuser, Boston 1996.

[5] K. Ciesielski, *On some details of Stefan Banach's life*, Opuscula Math. 13 (1993), pp. 71–74; *On Stefan Banach and some of his results*, Banach Journal of Mathematical Analysis 1 (2007), pp. 1–10.

[6] E. Jakimowicz, *A remarkable life*, in: E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach. Remarkable Life, Brillant Mathematics*, Gdańsk-Poznań 2007, pp. 1–23.


**Fig. 2. Stefan Greczek in uniform.**

Prof. Monika Waksmundzka-Hajnos, Stefan Greczek's granddaughter (daughter of Antonina), sheds light on Stefan Greczek's personality and origins:

*Stefan Greczek was born in November 1867, in the small village of Ostrowsko, situated in the Dunajec River valley near the Tatra Mountains in southern Poland. He was the first-born son of Joseph and Antonina, who were poor farmers. He had four younger brothers and sisters. As a young boy he helped graze cattle and sheep and helped to look after his parents' small farm. He attended elementary schools in Ostrowsko and in the nearby small town of Nowy Targ, but was unable to continue his education there because at that time Nowy Targ did not have a grammar school. His family did not have financial means to send him for further education. Consequently, as a mere youth barely in his teens, he set off in search of work in Budapest, which at that time was a favourite and common destination for many young people from the same largely agricultural area of Poland. Some went to escape difficult conditions, and all went looking to improve their economic prospects. His journey to Budapest (a distance of more than 250 kilometres) was entirely on foot walking the whole way in the company of several other young teenagers from his village. Since Stefan Greczek already had good reading and writing skills at that time, and as he was talented, he soon learned German. He was thus able to obtain a position working in an office. Commenting upon that later, one of his group, who had journeyed with him to Budapest, is noted to have said "all the rest of us to the shovel, Stefan to an office!"*

*His office position did not last long, however, because he was soon conscripted into the Austrian army. Once in the army, and as a young soldier, he met a young girl, a countrywoman from the same area of Poland that he was from, Katarzyna Banach. Although he himself never spoke much about her, it has always been understood by the family that she was employed as a maid servant to the same officer whom Stefan was assigned to as orderly. They fell in love. The fruit of their love was born on 30 March 1892. Stefan was then 24 years old. The young father would have no hope or realistic chance at that time for a better future other than to remain in the army. At his rank he was forbidden to marry without permission. (…)*

*Suffice to say the young couple decided to separate. For a time Katarzyna went to live in Kraków. She subsequently married a railway worker and is known to have later left Kraków. Her whereabouts after that time are unknown. Before they parted the young father made two promises to Katarzyna – that he would look after and take care of the child, whom they christened Stefan, until he was of full age, and that he would never reveal that she was his mother. He kept those promises. And, while it have been easier to commit the child to an orphanage, he assumed full care and responsibility of him.*

*There is information that at that point the child was taken to Ostrowsko to be looked after there by Stefan Sr's mother, Antonina. This was not just some "older lady", as some authors have stated, but in fact the now elderly matriarch, Stefan Banach's grandmother, who lived with family members in a very small cottage, typical of many others in that region – often called chicken huts. Unfortunately, some time after that she became ill. Now she herself and small Stefan both needed to be cared for. It is noteworthy at that point that, even although she had several other children, Antonina preferred to go and live with her son Stefan, now in Kraków, and she spent the remainder of her years there with him. Because of his mother poor health, Stefan Sr (with the full knowledge of Katarzyna Banach) decided to place the little boy Banach with the owner of a laundry business in Kraków, a well-to-do lady, not a washerwoman as she appeared in some articles about Stefan Banach. Even though she was prosperous Stefan Sr paid her regularly for the upkeep of his son. This continued for an extended period until Banach was of an age to be able to leave to go to Lvov to study*[8].

In the words of Stefan Greczek:

*Your mother took work as a nanny for which she received 8 Fl. per month. After several months when a woman brought you to Cracow to show (your mother) that you were alive and well, fate decreed that your mother took her to the house of your (future) foster mother because she (your mother) worked close by. Your foster mother declared that she would be willing to take you in, to bring you up and care for you, without any payment, to which your birth mother agreed. And so she took you from the woman and handed you to your new guardian.*

[7] An excerpt from the letter of Stefan Greczek to Stefan Banach, dated 30 October 1943. For the full text see E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach. Remarkable Life…* , op. cit., pp. 45–50.

[8] Quoted after E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach …*, op. cit., p. 60–62; transl. by John Greczek.

*So now with nothing in your mother's way she met a certain young man, who was serving his third year in the army and, after being discharged and obtaining employment in the railways, he married her. From that time on your mother no longer worried about you and she later had other children*[9].

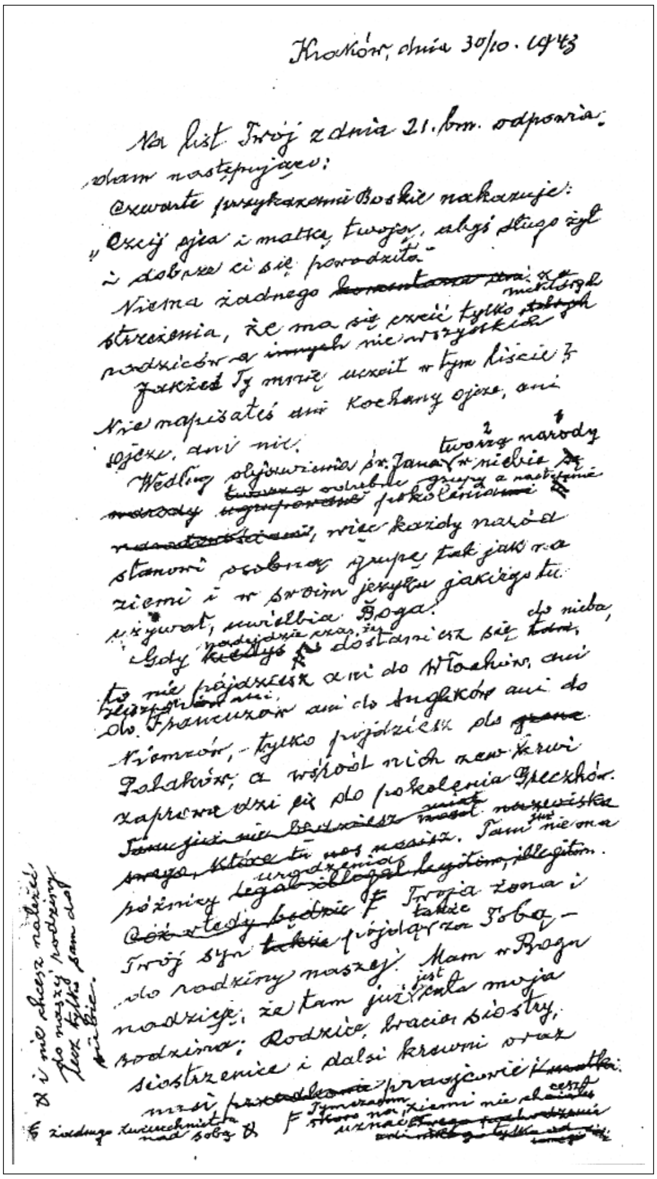
Here end the traces of Stefan Banach's mother. E. Jakimowicz' further investigation into the archives of the Cracow diocese brought no results. Later in an extract of a letter from Stefan Greczek, it is written:

*Your foster mother, a noble and good woman, who had no children of Her own, raised you and loved you like her own child. With a no-good husband, who she knew would not provide for her when she became elderly and could no longer work, she did not want to become a burden on society. That is why she took in a niece and you to bring up believing that one of you would give her shelter in her old age. However, she was never a burden to anyone because she was taken early by God.*

*Your foster mother was proud of you. You were a healthy and attractive child. Mien, the photographer, took pictures of you in different poses and these served for his advertisements. People would stop at his exhibition and admire the beautiful child. As a result of that Mien became famous around time. All the ladies came to him to be photographed and to have their children photographed. In the meantime you were growing up and after 6 years you went to school. You were a good student and every year advanced to a higher level unit until you completed grammar school. Your mother, with chest puffed out, was delighted that her Stefek was such a good student. As I watched your upbringing I saw no reason to interfere and give your mother advice, because everything was progressing as it should. You were provided an excellent living standard because your mother took such good care of you. Your clothes, shoes, underwear, were always of the best quality. Textbooks you also had. If your mother had neglected you, did not take care of you and did not send you to school, be assured, I would have taken you to my house and sent you to attend school. But your mother did that on my behalf. Such was the will of God Almighty and He would most certainly have credited her with this good deed.*

*At the time of your high school graduation (your) mother said to me: you have to provide Stefek with a nice suit for this graduation. And so, you and I went to see a tailor, you chose the material, the tailor measured you and in a week's time the suit was ready. I paid a sum of 100 crowns for the suit. That was a lot of money at the time. I could have taken you to a Jewish tailor and bought a ready-made suit for 20 crowns, because that is what ready-made suit cost. But I did not so, because honours would not allow that.*[10]

The secret of Banach's origin, described in the written fragments from Stefan Greczek's letter, was made clearer to the family during War World II when Stefan Greczek moved to Lvov with other members of the family. It was then that Antonina, Stefan Banach's stepsister, got


Kraków, dnia 30/10. 1943

Na list Twój z dnia 21. bm. odpowiadam następująco:

Czwarte przykazanie Boskie nakazuje:
„Czcij ojca i matkę twoją, abyś długo żył i dobrze ci się powodziło"

**Fig. 3. Facsimile of the rough draft (initial page only) of the letter from Stefan Greczek to Stefan Banach.**

to know his family. A letter from Stefan Banach to Stefan Greczek on October 21, 1943, which has been lost, showed that Stefan Banach had a falling out with his father. We can sense the pain in their relationship within the bitter reply from Stefan Greczek on October 30, 1943, which begins as follows (see Fig. 3):

*In answer to your letter of the 21st of this month:*
*The fourth commandment of God is: "Honour your father and your mother, that your days may be long and that you may prosper."*
*This is not limited to honour only some but all parents. How did you honour me in this letter? You did not write dearest father, or father, or anything. (…)*
*The matter of your birth is as follows:*[11]

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[9] From the letter of 30 October 1943, quoted after E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach* …, op. cit., p. 47.
[10] E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach* …, op. cit., p. 48.
[11] Ibidem, p. 45 and 47

Now follows a detailed explanation of the environment into which Stefan Banach was born. Psychologically, the resentment of the son towards his father appears reasonable. Banach frequented the house of Stefan Greczek, met the family and the other members of the household. Banach knew, as did other members of the family, that they were siblings, but it was not openly acknowledged in the house. He had to be troubled by this secretiveness, and he knew nothing about his mother. The explanations he received about his upbringing were dissatisfying as it took almost half a century for everything to be disclosed. Banach learned of the details of his early life after he turned fifty, after which he had only a year and a half to live. Stefan Banach had reason to be unhappy.

However, Banach did not feel any antagonism towards his stepsiblings. In professor M. Waksmundzka-Hajnos' memoirs we read:

> *Antonina Greczek, daughter of Stefan Greczek, had knowledge of Banach, but only as being a family friend. She was 25 years younger than him. That he was in fact her brother, her beloved father's son, she only found out in 1939. At the outbreak of World War II Antonina, her husband and father Stefan, set off from Cracow in panic and confusion and headed in the direction of Nowy Korczyn. There they became separated and Antonina and her father continued on their own towards Lvov. Once there, with aunts and uncles from her mother's side of the family, Antonina met Stefan Banach for the first time. It was only then she discovered that he was her half-brother. This was a shock and she did not understand how it was she had not known, and why her father had not told her.He calmed her down and explained the circumstances and the difficulties that existed at the time of the Austro-Hungarian rule if someone wanted to register a child born out of wedlock as his own, and how his father realised there were very few good options available to him, and that there was not much he could do. Antonina and Stefan Banach quickly became friends, and confided in and loved each other as brother and sister.*[12]

**Fig. 4. Childhood years of Stefan Banach (photos by J. Mien).**

**Fig 5. The last photograph of Stefan Banach, ca. 1943.**

Let us now fill in the details of Stefan Banach's early life with information gathered from the descendents of Banach's adoptive family in Cracow. Banach's foster mother was Franciszka Płowa (d. 1927), owner of a successful laundry service. Her marriage was childless and ended in divorce. Płowa's niece was Maria Puchalska, a sheltered girl who played the role of a sister to Stefan Banach. Neighbor Jules Mien (1842–1905), a Frenchman in origin and a photographer by profession, was a frequent guest at Płowa's house. Furthermore, he played the role of a good uncle for Stefan Banach and over time became very fond of Stefan, going so far as to teach him French[13]. Mien also liked to take pictures of Banach, many of which document the development of the young Stefan Banach. This foster family surrounding Banach offered him love and made evident their pride in him. Banach felt comfortable in these surroundings and treated his foster mother, Franciszka Płowa, as his mother.

There is more to discuss regarding Banach's father. Stefan Greczek later married twice. In the first, unsuccessful marriage to Helen, (née Alfus), he had a son Wilhelm (1897–1971). Greczek's second marriage was to Iwona Albina Adamska, with whom he had four children: Kazimierz, Tadeusz, Bolesław and Antonina. The Greczek family lived in Cracow. Young Stefan was a frequent visitor to Stefan Greczek's house, but there was little sense of belonging and only some members knew of Banach's ties to the family.

The examples above make it possible to reconstruct Stefan Banach's birth and approximately the first twenty years of his life. Banach was an illegitimate child of Stefan Greczek and Katarzyna Banach. They could not marry,

[12] Quoted after E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach* …, op. cit., p. 72.

[13] Jules Mien's French tutoring was of great importance to the development of Banach's career, since French was an important language in communicating mathematics throughout Banach's life. (translator's commentary)

partly because of Greczek's military service, so they decided to separate. When he was a month old, Banach was sent to Ostrowsko, where Stefan Greczek's mother lived. It was she who most likely took care of Stefan Banach during his time in Ostrowsko. After a few months, Stefan Banach was handed over to the care of Franciszka Plowa. Banach grew up until 1920 in Franciszka's home, except for a period from 1910–1914 that he spent at the Lvov Polytechnic University. He considered Franciszka like his mother and she in return loved him like a mother until her death in 1927. Living with Stefan Banach in the same household were Maria Puchalska, whom he treated like a sister, and the frequent visitor to the house, Jules Mien, whom Banach treated as his uncle. The relationship Stefan Banach had with this household is illustrated by his return from Lvov in 1914, his marriage in 1920 in the Church of Order of the Brothers of Our Lady of Mount Carmel in Cracow and also by the numerous photographs of Banach taken by Jules Mien. Banach spent time at Stefan Greczek's house, where he got to know Greczek's children; some he befriended, but it was an embarrassing secret to know that they were family members.

After explaining Stefan Banach's origins, let us briefly describe his story, focusing on family relationships. Even as a student, Banach wanted to ease the burden on his foster mother by earning money as a private tutor to high school students. He finished high school in 1910. It is documented that he received his matura[14]. His matura certificate has not been preserved, which without a doubt was due to his enrollment at the Lvov Technical University. He was interested in mathematics, but in secondary school he developed the idea that new discoveries in mathematics could not be found. This led him to engineering school, where he could apply mathematics. At the Lvov Technical University he attained a half-diploma. After the start of World War I, because of the imminent danger of a Russian offensive and the closing of the Lvov Technical University, Banach left Lwow and returned to his foster family in Cracow. There, he supported himself through private tutoring and continued his mathematical studies on his own with a group of friends. In 1916, in the Planty Park in Cracow, he was discovered by Hugo Steinhaus, who later joked that this encounter was his "greatest scientific discovery."[15] Hugo Steinhaus qualified for a professorship at the University of Lvov in 1917, but did not lecture there for long. War World I was ending and the city was a scene of fierce fighting, first during the Polish-Ukrainian war, and later during the Polish-Bolshevik war. The situation became quieter only after 1920, and it was then that Steinhaus was called to a professorship at the Jan Kazimierz University. He immediately invited Banach to Lvov, and helped him start his academic career by making him an assistant to Prof. Antoni Łomnicki at the Lvov Polytechnic. Even without a full university diploma, Stefan Banach completed his doctorate at the Jan Kazimierz University in a few months in 1920, and in 1922 became a professor there. Thus, in two years from 1920–1922, Banach rose to the position of professor at the Jan Kazimierz University. Steinhaus was a professor and head of Department B in 1923 and Banach was a professor and head of Department C in 1927. Both became leaders of the dynamic and important Lvov School of Mathematics[16].

**Fig. 6. Stefan Banach with his son on a street in Lvov.**

In 1920, Stefan Banach married Łucja Braus, whom he met through Hugo Steinhaus. They had a son, Stefan (1922–1999), who became a neurosurgeon at the Medical Academy of Warsaw. Stefan Banach liked to take walks with his son (Figure 6), go to the movies and watch the local soccer matches of Pogoń. Stefan Banach Jr.'s wife, children and grandchildren all survive him[17].

The best years of Stefan Banach's life were during the interwar period of 1920–1939, when he could dedicate himself, without interruption, to mathematics. As a university professor, he earned 1040 zloty a month, which was significant at the time. To supplement his salary he wrote textbooks with Wacław Sierpiński and Włodzimierz Stożek. He enjoyed his social life, and was frequently surrounded by a group of talented young mathematicians, often working at the Scottish Café, which became legendary within the mathematical community[18]. During the Soviet occupation of Lvov from 1939 to 1941, Banach was honored by the Soviet authorities – he became a corresponding member of the Ukrainian Academy of Sciences and a dean of faculty at a Ukranian university, the only Pole to have such a title. At the outbreak of World War II, Stefan Greczek and his family fled the Nazis from Cracow to Lvov, where they found shelter in the home of his wife's family. It was then that Greczek revealed the truth about Stefan Banach's birth, upbringing and fam-

[14] Matura is a series of final examinations given to high school students in their final year of studies. (translator's commentary).

[15] H. Steinhaus, *Recollections and Notes*, ed. By A. Zgorzelska, Wrocław 2002, p. 97. [Polish]

[16] R. Duda, *The Lvov School of Mathematics*, Wrocław 2006. [Polish]

[17] Exact data on them can be found in E. Jakimowicz, A. Miranowicz, *Stefan Banach …*, op. cit.

[18] See R.D. Mauldin (ed.), *The Scottish Book. Mathematics from the Scottish Café*, Birkhäuser, Boston-Basel-Stuttgart, 1981.

**Fig. 7. In front of Stefan Banach's grave. From left to right: Prof. W. Lance (alumni of the Lvov school, who remained in Lvov), Prof. T. Riedl (descendant of the owners of the family tomb; after the war he settled in Gdańsk) and Prof. Ya. Prytula (Dean of the Mechanics-Mathematics Department at the University of Iwan Franko in Lvov).**

ily life. Greczek shared this information with Banach's family and with his children from his second marriage. Greczek's stay in Lvov did not last long. Greczek and his family returned to Cracow, where he spent the rest of the war and his life. Learning about the secrets of his biological mother and other details about his life shook Stefan Banach. He could not hide his feelings of anger towards his father who had hidden this information for such a long time. However, a close relationship continued between Banach and his father, and Stefan Banach Jr. visited his grandfather's house after his leave from Lvov. During the Nazi German occupation from 1941 to 1944, Stefan Banach was employed as a lice feeder in the Weigel Institute, which was working on the creation of a typhus vaccine for the Wehrmacht. After the return of the Soviets to Lvov, all of Banach's positions and honors were returned to him, but he was already very ill, and on August 31, 1945 he passed away. The funeral was Roman Catholic, which was a courageous act at the time. He was buried in the Riedl's family tomb (the family with whom he and his family stayed during his last year) at the Łyczakowski cemetery (Fig. 7). There he lies to this day, remembered by Ukrainians and Poles who visit Lvov.

Łucja Banach, after leaving Lvov in the spring of 1946, settled in Wrocław[19]. This city was also a new home to a large group of professors from the Lvov School of Mathematics, including Hugo Steinhaus. Łucja Banach died from a heart attack on December 29, 1954 in Wrocław and was buried at the cemetery of St. Lawrence, next to Bujwid Street.

It is important to note the errors in Stefan Banach's biography that occurred earlier. For some time, the *Encyclopedia Britannica* listed him as a Soviet mathematician, most likely due to the political boundaries at the time,

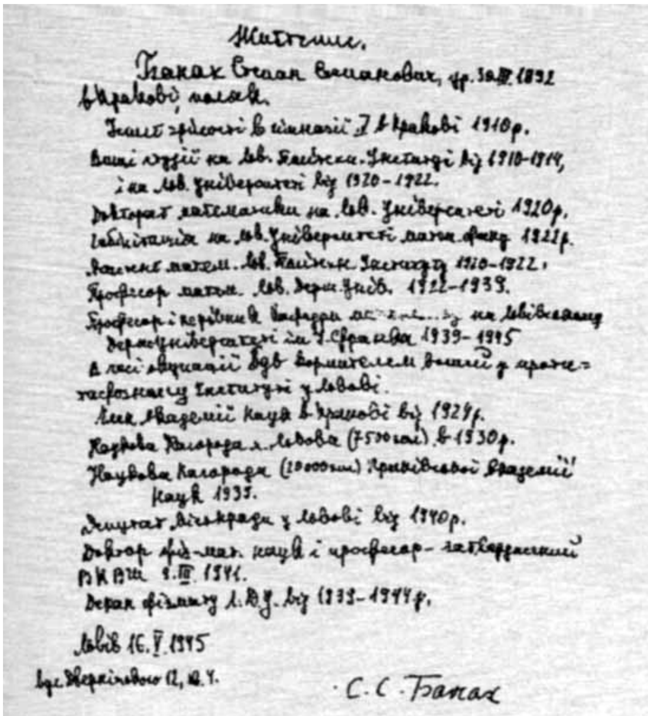

**Fig. 8. Stefan Banach's autobiographical memoir written three months before his death.**

and elsewhere he was classified as a Ukranian mathematician. The speculation is corrected by the personal memoirs of Banach, in which it is written in Ukranian, under Soviet Lvov, that he is Polish (Fig. 8).


### Acknowledgements

The author is obliged to Mrs. E. Jakimowicz for supplying a copy of the parish register (Fig. 1) and for the kind permission to reproduce that copy and some photos (Figs. 2, 4–6) from the book: E. Jakimowicz, A. Miranowicz, *Stefan Banach…*, op. cit. Photo of Banach's tomb (Fig. 7) was kindly offered by Prof. Ya. Prytula from Lvov. Banach's autobiography (Fig. 8) was reproduced by "East Journal of Approximations", vol. 7, No. 2 (2001), p. 254. The text has been generously translated by Joseph Pomianowski, except for letters which were translated in the quoted book.



*Roman Duda (b. 1935) is Professor Emeritus at Wrocław Uniwersity. In 1995–1999 he was the Rector of this university, in 1987–1989 the vice-President of the Polish Mathematical Society, and in 1989–1991 a member of the higher chamber of Polish Parliament (Senate), elected in the first free election after the Second World War. His main mathematical interest includes topology, history of mathematics and philosophy of mathematics. He is an author of many research papers, a 2-volume monograph "Introduction to topology" (in Polish) and a book about Lvov School of mathematics. In 1993–2007 he was the Editor-in-Chief of "Wiadomości matematyczne", the journal of the Polish Mathematical Society. His hobbies include tourism and canoe trips.*


[19] The move from Lvov was similar to a deportation. After World War II, country boundaries were moved and Lvov became Ukranian and no longer Polish. Wroclaw (Breslau) became Polish and no longer German. (translator's commentary)

# Interview with John Ball

by Tom Körner (Cambridge, UK)

**John Ball**

*Sir John M. Ball, FRS [ball@maths.ox.ac.uk] is Sedleian Professor of Natural Philosophy at the University of Oxford, and a fellow of the Queen's College, Oxford. He was the President of the International Mathematical Union from 2003–06. Prior to taking up his Oxford post, he was a professor of mathematics at Heriot-Watt University in Edinburgh. John Ball's research interests include the calculus of variations, nonlinear partial differential equations, infinite-dimensional dynamical systems and their applications to solid mechanics, materials science and liquid crystals.*

## Mathematical Education

***How did you become a mathematician?***
A set of happy accidents (though some seemed less happy at the time). My father was an engineer and my elder brother followed in his footsteps. I was good at science and mathematics and decided that I needed to do something different to engineering. I suppose I could have chosen to study physics but, in the end, I went to Cambridge to study mathematics.

Although I came from a good school and, as I now see, was well taught, Cambridge came as a big shock. I was amazed by how clever everybody was and how fast the course went. I don't think I ever caught up.

***Would it have been better if the course had been less fast?***
No, I don't think so. Of course, some of the teaching was dreadful but some was splendid. The best feature of Cambridge mathematics was the other undergraduates. I was in a very good year of mathematicians at St John's College.

***So what were your experiences as a research student?***
I didn't do well enough to continue at Cambridge. I had an offer from Oxford to do algebra but continuing beyond the first year depended on passing some more examinations. I decided that I wasn't very good at exams so I went to Sussex to do a DPhil in the applied sciences department.

My first year at Sussex was difficult (my research supervisor eventually left mathematics altogether) and I had to fend for myself. Fortunately at that time David Edmunds was organizing a programme in partial differential equations in the mathematics department, during which several distinguished experts such as Felix Browder, Robert Finn, Jim Serrin and Guido Stampacchia gave lecture courses. Another research student Mike Alford, who had been at Sussex a year longer and had the same supervisor, went with me to these courses. I remember that we went to the one given by Stampacchia and realized that we had missed the first lecture. Stampacchia, a very nice man who had never met us before, overheard us discussing this and offered to give the first lecture again for us! Of course we didn't let him. And then there was the lecture of Felix Browder, which went on without a break for three hours, punctuated optimistically by the clinking of the tea trolley arriving in the corridor outside after an hour, and depressingly by more clinking an hour later as it was removed unused!

The programme brought another piece of good fortune, in that Robin Knops (who was later to give me a job at Heriot–Watt) introduced me at a conference to Stuart Antman, having told him that I was searching for a research project. Stuart told me to look at a paper by Wayne Dickey – what was in it and what I could do to it. I was so ignorant about how research was done that it was a revelation that anyone could understand such things about papers at the frontiers of research. So now I had a research project but no mentor. Then, without telling me, Mike Alford went to see the dean to say that my situation was unacceptable, following which I transferred supervisors to David Edmunds. After this, everything went well, though I remained in the applied sciences department and ended up with a DPhil in Mechanical Engineering. Without these acts of kindness I doubt very much that I would have had a successful research career.

## Mathematical Career

***What happened next?***
My thesis used parts of the theory of infinite-dimensional dynamical systems pioneered at Brown University by Constantine Dafermos, Jack Hale, Joe LaSalle and Marshall Slemrod, and so when I was awarded a two-year

post-doctoral research fellowship by the UK Science Research Council, I decided to spend six months of it at Brown, which turned out to be a marvellously productive period.

I spent the rest of the fellowship at Heriot–Watt and at the end they offered me a full-time position. Once again I was extremely lucky. I joined a department, which, under the leadership of Robin Knops, was in the process of becoming a first class research unit but which remained small enough to give its members real freedom. And I had some wonderful students and post-docs.

It was a very difficult decision for me and my family to leave, 23 years later, for Oxford but I haven't regretted it. Like Edinburgh, Oxford is a beautiful place to live and I have been very happy there.

***Which of your results are you proudest of?***

The existence theorem for energy minimizers in 3D nonlinear elasticity that I proved while at Brown and which was the first such result under physically realistic hypotheses on the material response. For some years after that I laboured under the delusion that these hypotheses should apply to all elastic solids. But Jerry Ericksen was wiser and persuaded me of Hilbert's point of view that every variational problem should have a minimizer, provided 'minimizer' was suitably interpreted. Some years later Dick James was visiting Heriot–Watt and asked what might happen in elasticity if the energy minimum was not attained. I drew some picture, which Dick said reminded him of something, and the next day he came in saying that it was an austenite-martensite interface, an important feature of the microstructure of alloys and thus interpretable as a generalized minimizer. This was the start of a very satisfying collaboration that brought me much closer to experiments than before.

## The International Mathematical Union (IMU)

***How does one become president of the IMU?***

It used to be that the choice of president was the recommendation of the current IMU Executive Committee, which was an awkward arrangement since members of the Executive Committee are potential candidates. One of the actions of the 2002-06 Executive Committee was to initiate a change to this system through the introduction of a Nominating Committee, itself chosen by a partially random algorithm. In my case the conflict of interest wasn't present since I was not a member of the previous Executive Committee. It was a complete surprise when I was asked, though I had had some previous experience of the IMU through being a delegate at the General Assembly and a member of the Fields Medal and Program Committees.

***Even national societies like the AMS and LMS feel rather distant from their members. It must be even worse for the IMU.***

Yes, but the IMU serves different purposes. To start with, the members of the IMU are the national mathematical societies or academies, not individual mathematicians. In turn, the IMU is a member of the ICSU (the International Council for Science), which thus exists at an even higher level of abstraction.

If the IMU did not exist it would have to be invented since the various national societies have common interests which would otherwise need to be the subject of a large and unnecessary accumulation of bilateral agreements.

Of course it is desirable, even if difficult, to establish better links between the IMU and the 'mathematician in the street'. The new IMU newsletter IMU-Net (see http://www.mathunion.org/imu-net) is an effort in this direction which seems to have been quite successful.

***Can the IMU really speak for all mathematicians in the world?***

One thing that surprised me when I became president was to learn of the research hurdles that IMU imposed in order for countries to become members. Thus the IMU cannot really claim to represent all mathematicians in the world, just the research mathematicians. Some people feel strongly that this should remain the case but I have always preferred a more open posture that recognizes the importance of mathematics for all countries. There are currently 68 member countries out of the 193 recognized by the United Nations, a figure typical for scientific unions. Of course these members cover a large proportion of the world's population but they exclude, for example, several members of the IMU's own commission on mathematical education (ICMI). But it's fair to say that there isn't a queue of rejected countries clamouring to become members – for one thing member countries need an adhering organization that the IMU can liaise with and which can afford to pay the dues.

Those countries outside the magic circle of mathematically developed nations often lack the means to construct strong national mathematical communities and without such communities it is hard to make progress in mathematical teaching and research. Recently the IMU introduced a new category of associate membership, without dues or voting rights, designed to encourage such mathematical development and to pave the way for future full membership. I think that this is a good step forward.

***Most people identify the IMU with the International Congress of Mathematicians and the Fields Medals. Can you point to some important roles played by the IMU?***

I already mentioned the ICMI, which is an important player in mathematical education. Two other key roles are in supporting mathematics in developing countries (see http://www.mathunion.org/activities/developingcountries) and in electronic information and communication (see http://www.ceic. math.ca/), where a number of best practice documents have been influential.

***Talking of prizes, what did you think of Perelman's case against them?***

When he talked to me, Perelman was not against prizes in general. Incidentally, I was both surprised and pleased with the sympathetic approach of the media to his refusal of the Fields Medal.

The cases for and against prizes are obvious. On the one hand, they can make people jealous and upset. On the other hand, I think most mathematicians like giving concrete recognition to outstanding achievements. Most importantly, such prizes attract a lot of favourable publicity to mathematics.

***Once an organisation starts talking to governments the problem arises that not all governments are good governments. On the whole, the mathematical community is against boycotts but surely some line must be drawn.***

I am not against all boycotts but I am against academic boycotts. I think we must try and maintain intellectual contacts between all countries, and especially those between which there are bad relations. At least that way someone is talking to someone. If people of reason cannot talk together then who can?

For most mathematicians, intellectual isolation is a disaster. The more repressive a government is, the more its people need outside contact. The same applies to war-torn countries. Mathematicians in such countries can be effectively abandoned by our community because no-one will visit them.

***Does the IMU have a role as a kind of mathematicians' trade union?***

Not really. The IMU does occasionally take up the cause of individuals but only in humanitarian cases. It simply doesn't have the resources to investigate individual cases of mathematicians who are in dispute with universities or others. But it can usefully make occasional representations when there are issues that affect the community at a larger scale, such as major governmental reductions in support for mathematics or closures of departments.

## Supporting Mathematical Education

***Isn't there an inherent problem with any association speaking for a particular subject? Everybody knows that the Association for Underwater Basket-Weaving will always say that there is a dangerous lack of underwater basket-weavers and it must be a national priority to train more underwater basket-weavers.***

But mathematics has a stronger case than basket-weaving. For example, psychology is an important subject and I would like to see at least a little of it taught to all schoolchildren. But does it make sense economically or culturally that there are twice as many UK university undergraduates studying psychology than mathematics? And more undergraduates do media studies than mathematics – I wish that they took compulsory mathematics and science courses!

In the UK we have a vicious circle. There is a shortage of mathematicians, with a wide variety of well paid outlets for our students. Teaching is neither particularly well paid nor particularly respected so our students, on the whole, do not go into school teaching, so many schoolchildren get uninspiring teaching, so they do not choose to go on with mathematics, so there is a shortage of mathematicians … Of course, similar situations affect other key subjects, such as chemistry. When universities began to close chemistry departments the UK government washed its hands of the matter, leaving their fate in the hands of the university funding councils and individual universities. Some things are too important to be left to market forces.

Although you can be a good and inspiring teacher with weak qualifications, I think that, on average, a less well-qualified teaching force will deliver less good teaching.

***Isn't this a reflection of a similar trend in Europe and the US?***

It is very difficult and expensive to make reliable comparisons. The ICMI is currently undertaking a study comparing trends in eight selected countries. You can look at surveys which administer the same test to children in different countries, though such surveys differ in quality and relevance. But I think it is clear that the quality of teachers is a key issue.

## Final comments

***What did you like most and least about being President of the IMU?***

Of course in any big job there are frustrations but overall it was a great experience and privilege.

***And what have you liked most and least about being a mathematician?***

The least – petty disputes about what does and doesn't constitute applied mathematics. The most – those rare special moments of discovery and understanding after periods of confusion and the pleasure of belonging to a truly worldwide community linked by a common language.

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# ICMI Affiliated Study Groups

Mariolina Bartolini Bussi

The International Commission on Mathematical Instruction (ICMI) organisational outreach includes five permanent study groups that have obtained affiliation to the ICMI, each focusing on a specific field of interest and study in mathematics education consistent with the aims of the commission. The study groups affiliated to the ICMI are independent from the commission, being neither appointed by the ICMI nor operating on behalf or under the control of the ICMI, and they are self-financed but they collaborate with the ICMI on specific activities (e.g. the scientific organization of the ICME, the International Conference on Mathematics Education held every four years, and the management of ICMI Studies). Each of the affiliated study groups holds separate meetings on a more or less regular basis. Some additional information is given below, with special attention given to the future meetings to be held in Europe. Further information is available on the relevant websites.

The current study groups affiliated to the ICMI, with their year of affiliation, are:
- The International Study Group on the Relations between the History and Pedagogy of Mathematics (1976; HPM: http://www.clab.edc.uoc.gr/HPM/);
- The International Group for the Psychology of Mathematics Education (1976; PME: www.igpme.org);
- The International Organization of Women and Mathematics Education (1987; IOWME: http://extra.shu.ac.uk/iowme/)
- The World Federation of National Mathematics Competitions (1994; WFNMC: http://www.amt.edu.au/wfnmc.html);
- The International Study Group for Mathematical Modelling and Application (2003; ICTMA: http://www.ictma.net/);

The *International Study Group on the Relations between the History and Pedagogy of Mathematics* (HPM) was founded in 1976. By combining the history of mathematics with the teaching and learning of mathematics, HPM is the link between the past and the future of mathematics. Therefore, the group aims to stress the conception of mathematics as a living science, a science with a long history, a vivid present and an, as yet, unforeseen future. An ICMI Study on the role of the history of mathematics in the teaching and learning of mathematics was realized a few years ago: the volume was published in the New ICMI Study Series (Springer) in 2003 with the title *History in Mathematics Education* (eds. J. Fauvel & J.A. van Maanen). The last meeting of HPM was held in 2008 as a satellite meeting of ICME-11 (Mexico). The next meeting is not yet announced.

The *International Group for the Psychology of Mathematics Education* (IGPME) was founded in 1976. The major goals of PME are: to promote international contacts and the exchange of scientific information in the field of mathematics education; to promote and stimulate interdisciplinary research in the aforementioned area; to further a deeper and more correct understanding of the psychological and other aspects of teaching and learning mathematics and the implications thereof. The last meeting of the IGPME was held in 2008 as a satellite meeting of ICME-11 (Mexico). The next meeting (PME33) will be held in Greece (Thessaloniki, July 2009). The next year (PME34: 2010) the conference will be held in Brazil; after that it will again be in Europe (Turkey, PME35: 2011).

The *International Organization of Women and Mathematics Education* (IOWME) started in 1976 as an international network of individuals and groups who share a commitment to achieving equity in education and who are interested in the links between gender and the teaching and learning of mathematics. In 1976, a special meeting was arranged during the course of the Third International Congress on Mathematical Education (ICME-3) to discuss the issue of "Women and Mathematics". IOWME was formed as a result of that discussion and the group had its inaugural meeting at ICME-4. IOWME became an affiliated study group of the ICMI in 1987 and continues as such today. An ICMI Study on this issue was organized in the early 90s: the volume with the title Gender and Mathematics Education was published in the New ICMI Study Series (Springer) in 1993 (ed. G. Hanna).

The *World Federation of National Mathematics Competitions* (WFNMC) was founded in 1984 during ICME-5, held in Adelaide, Australia. In 1994 the federation became the fourth affiliated study group of the ICMI. Both events were inspired by the late Peter O'Halloran (1931–1994), the founding president of the federation. The goal of the federation is to promote excellence in mathematics education and to provide those persons interested in promoting mathematics education through mathematics contests with an opportunity of meeting and exchanging information. The activity on mathematics competitions was one of the foci of the ICMI Study no. 16 on *Challenging Mathematics in and beyond the Classroom*, whose volume was published in the New ICMI Study Series (Springer) in 2009 (eds. E.J. Barbeau & P. J. Taylor). The discussion document of the study was presented in issue no. 55 (March 2005, pp. 14–16) of this newsletter.

The *International Study Group for Mathematical Modelling and Application* was founded in 2003. ICTMA emerged from the International Community of Teachers of Mathematical Modelling and Applications (hence the acronym ICTMA), a community of scholars and researchers supporting the series of International Conferences on the Teaching of Mathematical Modelling and Applications and which has been concerned with research, teaching and practice of mathematical modelling since 1983. The aim of the group is to foster the teaching and researching of the teaching and learning of mathematical modelling and the ability to apply mathematics to genuine real world problems in primary, secondary and tertiary educational environments including teacher education, together with professional and workplace environments. Modelling and applications in mathematics education was the focus of the ICMI Study no. 14, whose volume with the same title was published in the New ICMI Study Series (Springer) in 2007 (eds. W. Blum, W.P. L. Galbraith, H.-W. Henn et al.). The next meeting of the ICTMA (ICTMA14) will be in Hamburg (Germany) in 2009 (http://www.ictma14.de/).

# A decade of ERME and a sixth conference

Barbara Jaworski (President of ERME) and Ferdinando Arzarello (Chair of CERME 6)

In the previous issue of this newsletter, ERME President Barbara Jaworski wrote about the history and aims of the *European Society for Research in Mathematics Education* (ERME). The first CERME (the biannual Congress of ERME) was held in 1999 and now, in 2009, we hold our sixth congress CERME 6 at the University of Lyon in France. This celebrates more than a decade of activity in ERME. Our local organising team is busy with the final arrangements and we meet in Lyon on 28 January 2009. A record number of 450 people are registered for the conference.

By the time you read this account the conference will be over but our deadline for completing this writing comes before the end of the conference. So here we will talk about our preparations for the conference and in the next newsletter we will report on its outcomes.

Planning for CERME 6 began immediately after CERME 5 ended. The ERME Board had to nominate and invite a Chair of the International Programme Committee (IPC) for CERME 6 and, together with this person, select and invite members of the IPC. Ferdinando Arzarello from the University of Turin, Italy, was nominated and invited and he accepted the invitation. An IPC of twelve members from twelve European countries was also invited and all invitees accepted. The IPC then had to start work planning CERME 6. It was expected that the group structure of CERME would be maintained so the IPC had to propose the number of groups and their focuses, taking into account groups that would wish to continue from CERME 5. This is a complex process, since leading scholars are needed to coordinate and lead group work and the communication process is not simple. Since communication is one of the main strands of ERME philosophy, this process of creating the groups is a very important one and rather time consuming. Fifteen groups were agreed, as shown below. Since CERME is a congress designed to foster a communicative spirit in European mathematics education according to the three Cs of ERME: *communication*, *cooperation* and *collaboration*, it deliberately moves away from research presentations by individuals towards collaborative group work. In these thematic working groups, members will work together in a common research area.

The work of groups is based on the following two concerns:

We need to know more about the research that has been done and is ongoing and about the research groups and research interests in different European countries.

We need to provide opportunities for cooperation in research areas and for inter-European collaboration between researchers in joint research projects.

The structure of each working group is essentially the same and each group has more than 12 hours to discuss its topic. A final session will be held in which, in parallel, each working group will present its work to interested participants. This will happen twice to allow delegates to gain insight into two groups of their choice.

### The CERME 6 Thematic Working Groups

**Group 1: Affect and mathematical thinking** – This includes the role of beliefs, emotions and other affective factors.
*Chair: Markku Hannula (Finland)*
**Group 2: Argumentation and proof** – This includes epistemological and historical studies, learning issues and classroom situations.
*Chair: Maria Alessandra Mariotti (Italy)*
**Group 3: Stochastic thinking** – This includes epistemological and educational issues, pupils' cognitive processes and difficulties, and curriculum issues.
*Chair: Andreas Eichler (Germany)*
**Group 4: Algebraic thinking** – This includes epistemological and educational issues, pupils' cognitive processes and difficulties, and curriculum issues.
*Chair: Giorgio Bagni (Italy)*

**Group 5: Geometrical thinking** – This includes epistemological and educational issues, pupils' cognitive processes and difficulties, and curriculum issues.
*Chair: Alain Kuzniak (France)*
**Group 6: Mathematics and language** – This includes semiotics and communication in classrooms, social processes in learning and teaching mathematics.
*Chair: Candia Morgan (UK)*
**Group 7: Technologies and resources in mathematical education** – This includes teaching and learning environments.
*Chair: Ghislaine Gueudet (France)*
**Group 8: Cultural diversity and mathematics education** – This includes students' diverse backgrounds and identities, social and cultural processes, political issues in education and school policies.
*Chair: Guida de Abreu (UK)*
**Group 9: Different theoretical perspectives/approaches in research in mathematics education** – This includes ways of linking theory and practice and paradigms of research in ME.
*Chair: Susanne Prediger (Germany)*
**Group 10: From a study of teaching practices to issues in teacher education** – This includes teachers' beliefs and the role of the teacher in the classroom, as well as strategies for teacher education and links between: theory and practice, research and teaching and teacher education, and collaborative research.
*Chair: Leonor Santos (Portugal)*
**Group 11: Applications and modelling** – This includes theoretical and empirical-based reflections on: the modelling process and necessary competencies, adequate applications and modelling examples, epistemological and curricular aspects, beliefs and attitudes, assessment and the role of technology.
*Chair: Morten Blomhoej (Denmark)*
**Group 12: Advanced mathematical thinking** – This includes conceptual attainment, proof techniques, problem-solving and processes of abstraction at the upper secondary and tertiary educational level.
*Chair: Roza Leikin (Israel)*
**Group 13: Comparative studies in mathematics education** – This includes questions surrounding mathematics teaching and learning in the classroom, learners' and teachers' experiences and identities, and policy issues in different cultures and/or countries.
*Chair: Eva Jablonka (Sweden)*
**Group 14: Early years mathematics** – This working group deals with the research domain of mathematics learning and mathematics education in the early years (age 3 to 7). In the last few decades interest in this topic has increased immensely.
*Chair: Götz Krummheuer (Germany)*
**Group 15: Theory and research on the role of history in mathematics education** – The integration of the history of mathematics in mathematics education is a subject that has received increasing attention over the last few decades.
*Chair: Fulvia Furinghetti (Italy)*

In addition to the working group sessions, the conference includes two plenary lectures of 75 minutes in which each plenary speaker has a reactor: they each have 60 minutes for their presentation and then there will be 15 minutes for questions from the floor. The two plenaries at CERME 6 are as follows:

- Luis Radford (Université Laurentienne, Ontario, Canada)
  *Signs, gestures, meanings: algebraic thinking from a cultural semiotic perspective*.
  *Reactor*: Heinz Steinbring (Duisburg-Essen University)
- Paola Valero (Aalborg University, Denmark)
  *Attending to social changes in Europe: challenges for mathematics education research in the 21st century*
  *Reactor*: Margarida Alexandra da Piedade Silva Cesar (Lisbon University)

A third plenary event involves a panel whose aim is to discuss a topic emerging from the previous CERME, analysing it from different standpoints and giving people the possibility of a wide debate.
Special plenary panel: Ways of working with different theoretical approaches in mathematics education research
Speakers: Angelika Bikner-Ahsbahs (Bremen University, Germany)
John Monaghan (University of Leeds, United Kingdom)
Chair: Tommy Dreyfus (Tel Aviv University, Israel)
There will be two parallel one-hour sessions where the participants will have the opportunity of debating with the plenarists. Moreover the interested people will have the opportunity to meet the plenarists in an informal meeting on another day.

In addition, there will be a poster session in which posters submitted by members of all groups will be displayed and delegates can discuss their research with poster presenters.

A particular feature of this congress will be an invited lecture from an eminent French mathematician Professor Etienne GHYS (Ecole Normale Supérieure de Lyon, France) on "Creating a mathematical video: an exciting personal experience".

The congress will include a General Assembly for ERME and opportunities for CERME participants to present their views to the ERME Board regarding current practices and future directions in ERME. It will also include a meeting of the learned societies in mathematics education throughout Europe to inform each other and discuss common issues and concerns. A potentially exciting new venture concerns a proposal to introduce a new European journal associated with ERME. This proposal will be discussed in an open meeting and views sought widely on possibilities and practicalities.

At the biannual CERME events, retiring members of the ERME Board are replaced by election. Nominations are posted on the website in advance of the conference and elections are held at CERME. This year is the time for retirement of the current president Barbara Jaworski (UK) and two other members of the board. So a new president will be voted in at the meeting.

The newly elected President of ERME for three terms, 2009 to 2013, is Ferdinando Arzarello (University of Turin, Italy).

# Abdus Salam International Centre for Theoretical Physics

**The heads of the maths section. Left to right: Le Dung Trang, M.S. Narasimhan, A. Verjovski, J. Eells.**

The mathematics section of the Abdus Salam International Centre for Theoretical Physics (ICTP) was created in 1986 with the help of Professor James Eells. Under the supervision of Jim Eells, the section was run by Professor Alberto Verjovsky.

Like the other sections of the ICTP the mathematics section is devoted to the development of its subject in developing countries. It hosts visitors, including post-doctoral fellows, for one to twelve months and organizes mathematical schools and workshops.

Schools and workshops on mathematics were held before the creation of the section but the increase in the number of schools and workshops and their success among scientists from developing countries shows the necessity of having a mathematical section within the centre.

The topics of the schools and workshops cover different areas of mathematics, from the most fundamental to the most applied. The choice of topic is usually made by the members of the section but external proposals are always welcome. Usually there are three schools/workshops every year and each of them lasts three weeks. The organizers are renowned specialists in the topic of the school/workshop, with the majority coming from developed countries. The visitors to these schools/workshops make up the majority of the visitors to the mathematics section.

The proceedings of most of these schools and workshops have been published by the World Scientific Publishing Corporation. One can also find Lecture Notes Series for several activities on the Web.

There are about ten to twelve post-doctoral fellows and seventy visitors every year. Altogether this makes around 400 visitors to the mathematics section of the ICTP every year.

Among the other programmes of the ICTP are the associateships and the federation agreements, which bring more visitors to the mathematics section.

Associateships are given to scientists from developing countries for a duration of six years. They offer three round-trips and altogether 270 days of stay at the ICTP. This allows the scientists to use the facilities of the ICTP: the library, the computers and an office where they can pursue their research. The federation agreement is signed with chosen institutions of developing countries. The travel for the scientists is paid by the home institution but the stay is sponsored by the ICTP. Associateships and federation agreements add about fifty visitors to the mathematics section. Associates and scientists coming through federation agreements can interact with each other and also with resident scientists.

As mentioned above, J. Eells was the first director of the section. In practice, he was helped by Alberto Verjovsky and Sharon Laurenti, the secretary of the section. The leadership of the section was renewed in 1992; Professor M. S. Narasimhan became the new director. Le Dung Trang became the director of the section at the end of 2002.

The section has been very strong in geometry, initially with algebraic geometry, with Lothar Göttsche and Ramadas Ramakrishnan. Charles Chidume is a specialist of functional analysis. Li Jiayu, who arrived in 2004, works in differential geometry, especially in the theory of geometric flows. Recently the mathematics section has recruited Stefano Luzzatto from Imperial College. He is a specialist in dynamical systems.

The mathematics section tries to extend study in the areas of probability, statistics and applied mathematics. The hiring of Luzzatto aids in this attempt to enlarge the mathematical expertise of the section.

Since 2004, the programme committee of the ICTP has decided to fund a school outside the premises of the ICTP in a developing country, each time on a different continent.

The mathematics section also benefits from different collaboration schemes with developing countries that are more advanced. The Brazilian CNPq is helping the ICTP through several channels. The schools on dynamical systems usually receive extra money from the CNPq. Over the years, the CNPq has funded travels and even stays of Brazilian researchers. Since 2005, there has been a common agreement between the ICTP and the CNPq to finance schools in mathematics and physics in Latin America, especially Brazil. Several research institutions such as the DST in India or the NSFC in China or the CONICET in Argentina have programmes to cover travel for their researchers who are invited by the ICTP. The mathematics section also hosts post-doctoral fellows or researchers who are sent by research institutions from developing countries and also developed countries. The facilities of the ICTP are obviously an attractor for these visitors.

Summer School and Conference on Geometry and Topology of 3-Manifolds, 6–24 June 2005. Confirmation of solution of Poincaré conjecture. Left to right: A. Verjovski, S. Maillot, L. Göttsche, M. Boileau, Le Dung Trang, S.M. Narasimhan, L. Bessières, G. Besson, J. Souto, B. Zimmermann

John Nash, ICTP 40th Anniversary Conference, 2004

The ICTP has a very good library, which contains a large collection of books and journals. In recent years subscriptions online have increased. The visitors will usually be able to print articles and parts of books that they need for their work. All visitors have access to computers and printers. The Visa section, the Housing Office, the Operation Unit and the mail services are used to help the numerous visitors to the ICTP.

The existence of good administration and a good library makes the ICTP attractive, in particular during the periods when mathematical schools/workshops are held. However, visitors who are seeking a quiet working place are always welcome to the ICTP.

Since the ICTP receives visitors from all over the world, basic regulations are necessary. Of course, these regulations are imposed by the existence of the right visa to come to Italy but for obvious reasons of security, precise requirements are requested from the visitors. Only people with PhDs are allowed to the ICTP. A proof of the PhD, either the original document or an original certification, is requested. Any visitors have to be recommended by renowned mathematicians. A CV is also requested.

By its nature, the ICTP essentially invites scientists from developing countries but scientists from developed countries are also welcome to visit. Exceptional top scientists are sometimes invited. Scientists of developed countries are only invited if they satisfy the same conditions as above. Usually scientists from developed countries do not receive financial support from the ICTP. Only if the ICTP receives a contribution from a developed country aimed at supporting their scientists can it also financially support these scientists.

The ICTP depends mainly on the financial contribution of Italy. The International Atomic Energy Agency (IAEA) was the first international agency to sponsor the ICTP and this is why the ICTP is regulated by a tripartite committee depending on Italy, the IAEA and UNESCO. However the scientific personnel of the ICTP depend on UNESCO. Whenever there is an opening, the rules of recruitment follow the rules of UNESCO. This makes the ICTP an administration that is different from the usual academic institutions. On the other hand, the exceptional existence of the ICTP ensures that its personnel are scientifically highly qualified. These two requirements make the choice of good personnel for ICTP very difficult. A particular sensibility to developing countries is also requested. Many good scientists are not sensitive to the problems of developing countries and for this reason they cannot be suitable for such a position. Many scientists of the ICTP, especially the ones of the mathematics section, come from developing countries. However, scientists from outside the ICTP do not always clearly understand these requirements.

Since 2005, the mathematics section has been associated to the International Mathematical Union (IMU), which granted half a secretary devoted to the Commission on Development and Exchanges (CDE) of the IMU. The reason the IMU has been interested in the ICTP is essentially the existence of a huge database of the mathematicians of developing countries and a good knowledge of their different needs.

Since the times of M. S. Narasimhan, the mathematics section has belonged to the ERCOM, a committee of the European Mathematical Society (EMS). In 2007 it hosted its annual meeting on the premises of the ICTP.

Since 2004, the mathematics section of the ICTP has become a member of the EMS. As a consequence, the head of the section became a member of the Committee of Developing Countries (CDC) of the EMS and hosted, in 2008, the annual meeting of the CDC.

The ICTP is a European institution paid for by Italy; it does not require too much money but it could be considered to be costly. European countries have to view this institution as an important bridge with the scientists of developing countries and the fact that it does not receive financial support from other European countries, or at least from the European Union, is not a good sign. This is especially the case firstly because students from European countries have difficulty attending the schools and workshops because they are not financially supported but also because the financial burden that it represents is not shared by European countries, which would allow more scientists from developing countries to come.

We hope that this article will give a clearer view of the mathematics section of the ICTP and will contribute to a better understanding of one of the institutes belonging to the ERCOM.

# Zentralblatt Corner

Olaf Teschke

Zentralblatt MATH

EMS and Zentralblatt MATH have a long-term partnership, resulting in many joint activities ranging from electronic publishing to acquisitions of new EMS members. For instance, in 2008 a free individual ZMATH account was distributed to each of the more than 2000 EMS members for an initial period of two years (with the positive side-effect of enhancing the EMS membership database significantly by collecting and confirming about 1500 email addresses). This column aims to illustrate several aspects of the ZMATH database and to improve access to the information stored in the almost 2.8 million items.

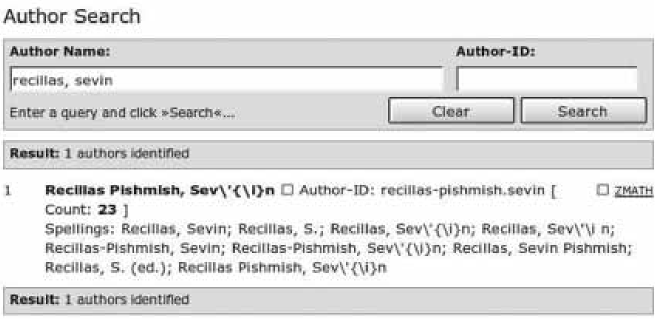
The first note gives a brief introduction to the author identification, which was implemented in ZMATH in 2008.

## On authors and entities

*What's in a name?*
(Romeo and Juliet, II,2)

*Name ist Schall und Rauch.*
(Faust I, 3457)

***What's the problem?***
More than half of the requests in the ZMATH database include names and 28% of the queries are author-only. Mathematicians typically look for publication lists, fields of interest of a person and related papers of the same author. However, a name typed into the "author" field is *per se* just a character string, without any additional information. Even for standard Western names, the pure string search may provide misleading results, caused by variations due to abbreviations, existing/non-existing middle names, hyphens and accents. The first name of the highly identifiable Grothendieck may be written Alexandre instead of Alexander. My late colleague Sevin Recillas Pishmish managed to obtain nine different spellings in just 23 publications.




Naturally, things get worse with different transcriptions. The variations of Russian names are notorious (exercise: try to find all the syntactic versions of the Chebyshev inequality or Lyapounov stability). Typographic errors and name changes should be taken into account as well. But probably the hardest task is to distinguish between different authors who bear the same name (at least, in the Roman syntax) – a problem which has existed for a long time due to some extremely widespread surnames but which has become particularly urgent due to the immense activity of Chinese authors.

***Where to start?***
When the printed version of Zentralblatt was dominant, index volumes were produced on a regular basis, including an author index reflecting the content of many thousand clip cards. However, the value of this information is limited because the abbreviation of first names was standard up to the mid-80s. One remainder of these data are the *name variations* that were integrated into the name search in ZMATH but failed to match the full complexity of the problems.

Hence, an author database had to be created mainly from the existing bibliographical data – together with an automated system handling more than 110,000 new items every year. This is obviously a gargantuan task, which one cannot expect to be completed 100% error-free. The first version, intended to solve the problem with maximal precision by exploiting some sophisticated graph theory relations of clustered authors, turned out to be simply too intelligent - the matching algorithms took far longer than 24 hours, thus breaking the daily update.

***What has been done?***
Therefore, we had to go back to an extremely simplified approach: forming entities of authors with identical spellings and similar profiles, and joining them later automatically. Comparing two entities and joining close matches is by far easier than splitting existing ones. It turns out that it is most efficient to create the first entities just from the spellings and the MSC classification. Other data, like affiliation and period of activity are useful to compare two entities but don't contribute much to the initial step.

Naturally, this generates a vast variety of clusters. The crucial task is to find a suitable function to match them without merging different authors. First, one has to compare different spellings. There are well-known measures like the Damerau-Levenshtein distance that can be adapted to specifics of authors' names in ZMATH. MSC distance turns out to be highly important, while affiliation and time-scale data are often misleading and therefore incorporated with a much smaller weight. Much more information is provided by the co-author (or, at this stage, co-entity) graph. Even the simple observation that a joint publication ensures that two entities are *not* identical helps greatly but its main use is, of course, to join closely related entities who frequently collaborate with the same entities. This process starts slowly (because at the beginning even the co-authors are usually scattered) but gains efficiency after several iterations.

By far the most difficult task is to calibrate the weights for these ingredients. When the matching program started to identify name changes by marriages without producing bulks of conglomerates we decided that this should be sufficient to go online. Still, only about 80% of the identification can be done purely automatically; the remaining entries require human judgment and even external information from the authors. By now, about 70% of the publications has been attached to more than 450,000 identified authors. The latter number will both increase (with the incorporation of more publications) and decrease (with collapsing entities).

### *How to use?*

The author database of ZMATH is accessible via the "Author Search" link. One can look using names (which can be truncated with a *), usually obtaining a number of identified authors. The author ID is just a version of the most common, or most informative, spelling together with a number (if necessary for distinction). There is a direct link to the publication database. On the other hand, author IDs can be used directly when browsing and searching through ZMATH. Clicking on the name of an identified author will provide the full list of attached publications. Especially useful is the free combination of IDs with queries. While (au: erdös, p* & cc:05) (or the respective search when using the advanced mask) just matches names to MSC, (ai: erdoes.paul & cc:05) will provide all the papers of the author in the subject. Users should be aware of the fact that the au: search also incorporates biographical references (like birthdays, obituaries and festschriften) while ai: is cleanly restricted to authored publications.

### *What remains to be done?*

At the moment, about one million documents are not assigned to an identified author, most of them from the pre-1985 era where the abbreviated first names often do not provide sufficient information to generate an entity. The number becomes less frightening if one looks at the distribution of publications. Judging from newer data, more than 20% of the authors publish just a single paper in their lifetime. At the other end of the scale, a large part of the publications belongs to the most productive authors, which can be handled by an adapted program.

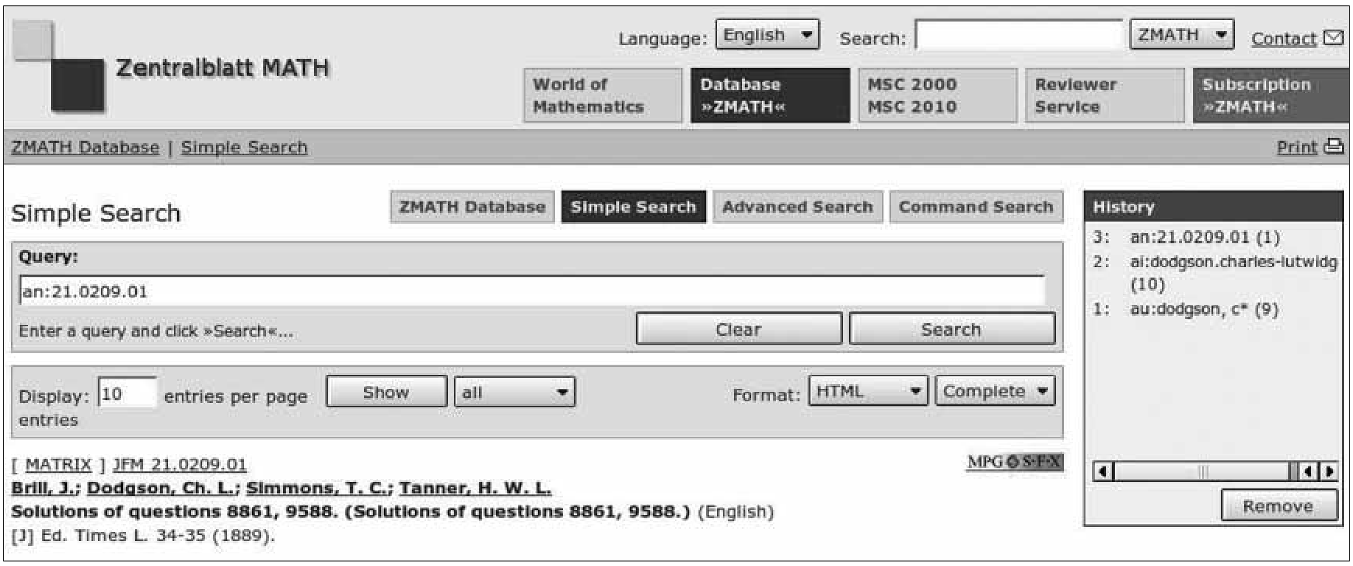
Joining entities, as well as splitting them, is an ongoing activity. The latter will be crucial when incorporating identities for Chinese authors effectively. While the Roman transcription of the names simply doesn't provide sufficient information, our editorial unit at the Academia Sinica stores data in Chinese and Roman characters, thus allowing a significantly finer distinction. Matching this information with ZMATH would improve the quality of the identification significantly. This would clear the way to integrate in a second step the cardbox IDs from Beijing.

### *Side effects*

With a fully established identification, one can set up the usual gadgets like collaboration distance tools. Thanks to the inclusion of the Jahrbuch data in ZMATH, the co-author graph can be extended back to the time of Riemann. When testing the first version, Paul Erdős earned an honourable "Alice Number" of 10, being connected to Lewis Carroll (naturally, the only mathematician with an Alice number of 1) via the remarkably nice chain of Ivan Niven (Zbl 0061.12903), Samuel Eilenberg (Zbl 0063.01228), Saunders MacLane (Zbl 0061.40601), J. H. C. Whitehead (Zbl 0035.39001), Solomon Lefschetz (Zbl 0006.37006), Virgil Snyder (Zbl 0213.47101), James McMahon (JFM 29.0229.01) and a certain J. Brill (JFM 21.0688.03), who happened to be one of the co-authors of JFM 21.0209.01.

### *Help us!*

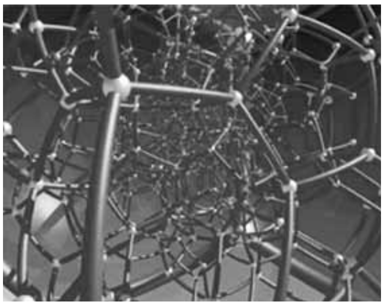
We would like to encourage you to send hints and corrections with respect to author identification to authorid@zentralblatt-math.org.




**JFM 21.0209.01, the only joint publication of a certain Charles Lutwidge Dodgson in the Jahrbuch Database.**

# Book Review

Frederic Paugam (Paris, France)



**Dimensions: a walk through mathematics**
A mathematical film by Jos Leys, Etienne Ghys and Aurélien Alvarez

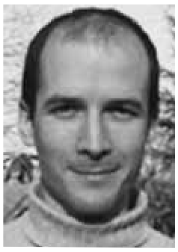
Buy the DVD at
http://www.dimensions-math.org/
Licenced under the creative commons licence
(free use anywhere) and translated into many languages

This film is a presentation of various mathematical notions for a very wide audience. The story and mathematics are due to Etienne Ghys and this beautiful work shows Ghys‘ well-known and exceptional skills for vulgarization already remarked on by the audience of his 2006 ICM talk in Madrid, which used part of this work.

The amazing drawings in three and more dimensions also play a large part in the magic of this film. It contains a lot of animated sequences showing the beauty of geometry and can be used at various levels of mathematical teaching. It contains nine chapters - two hours of visual mathematics - that take you gradually up to the fourth dimension. Mathematical vertigo is guaranteed! The contents of the film are as follows.

In chapter 1, intitled ‘Dimension two’, Hipparchus shows us how to describe the position of any point on Earth with two numbers and explains stereographic projection: how to draw a map of the world. In chapter 2, intitled ‘Dimension three’, M. C. Escher talks about the adventures of two-dimensional creatures trying to imagine what three-dimensional objects look like. In chapters 3 and 4, intitled ‘The fourth dimension’, Ludwig Schläfli talks about objects that live in the fourth dimension and shows a parade of four-dimensional polytopes, strange objects with 24, 120 and even 600 faces! Chapters 5 and 6 deal with complex numbers, explained by Adrien Douady - the square root of negative numbers made easy! Transforming the plane, deforming images, creating fractal images... In chapters 7 and 8, entitled ‘Fibration’, Heinz Hopf explains his ‘fibration’. Using complex numbers he constructs pretty patterns of circles in space. Circles, tori ... everything rotating in four-dimensional space. In chapter 9, Bernhard Riemann explains the importance of proofs in mathematics. He proves a theorem concerning the stereographic projection.

It is quite hard for everyday man (and also for students in mathematics) to understand geometrically what the fourth dimension is visually. This film gives an exceptional view of various properties of four-dimensional space, using aesthetically appealing schematics and eye candy movies. It can be used at various levels of education in mathematics and the technical points are clear enough to be understandable by anyone in the street. The only drawback of this beautiful piece of work is that one would like to have more. But this will be rapidly rectified since the authors are preparing a second opus.



*Frederic Paugam (frederic.paugam@math.jussieu.fr) is maître de conférence at the University of Paris 6. His thesis was written in Rennes in 2002 and was about the arithmetic of Abelian varieties. He had post-doctoral positions at the University of Regensburg, MPIM-Bonn and IHES before he got his permanent job in Paris. He now works on arithmetic geometry and mathematical physics, with main interests in global analytic geometry and functional equations of L-functions on the one side and formalization of standard physics on the other side.*

# Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

Terence Tao (Fields Medallist, 2006) in an interview after the opening ceremony of the International Congress of Mathematicians in Madrid on 23 August 2006 (see the *Daily News* of the Congress) stated: "I think the most important thing for developing an interest in mathematics is to have the ability and the freedom to play with mathematics - to set little challenges for oneself, to devise little games, and so on".

He further states: "I'm also a great fan of interdisciplinary research – taking ideas and insights from one field and applying them to another".

I wish to mention here the following references:


[1] V. Arnold, M. Atiyah, P. Lax and B. Mazur (eds.), *Mathematics: Frontiers and Perspectives,* International Mathematical Union, American Mathematical Society, Providence, Rhode Island, 2000.
[2] A. Baker, B. Bollobás and A. Hajnal (eds.), *A Tribute to Paul Erdős,* Cambridge University Press, Cambridge, New York, 1990.
[3] M. W. Hirsch, J. E. Marsden and M. Shub (eds.), *From Topology to Computation: Proceedings of the Smalefest*, Springer-Verlag, New York, Berlin, Heidelberg, 1993.
[4] Th. M. Rassias, *Major trends in Mathematics*, Newsletter, European Mathematical Society, Issue 62, Dec. 2006, pp.13-14.
[5] B. H. Yandell, *The Honors Class: Hilbert's Problems and their Solvers,* A. K. Peters, Natick, Massachusetts, 2002.


## I. Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

**43**. Let $(a_n)_{n\geq 1}$, $(b_n)_{n\geq 1}$, $(c_n)_{n\geq 1}$ be sequences of positive integers defined by

$$(1+\sqrt[3]{2}+\sqrt[3]{4})^n = a_n + b_n\sqrt[3]{2} + c_n\sqrt[3]{4}, \text{ where } n\geq 1.$$

Prove that

$$2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k = \begin{cases} a_n, & \text{if } n\equiv 0 \pmod 3\\ b_n\sqrt[3]{2}, & \text{if } n\equiv 2 \pmod 3\\ c_n\sqrt[3]{4}, & \text{if } n\equiv 1 \pmod 3\end{cases}$$

and find similar relations for $(b_n)_{n\geq 1}$ and $(c_n)_{n\geq 1}$.
(Titu Andreescu, University of Texas at Dallas, USA, and Dorin Andrica, "Babeş-Bolyai" University of Cluj-Napoca, Romania)

**44**. Define $S(n,p) = \sum_{i=1}^{n}(n+1-2i)^{2p}$ for all positive integers $n$ and $p$. Prove that for all positive real numbers $a_i$, $i=\overline{1,n}$, the following inequality holds

$$\min_{1\leq i<j\leq n}(a_i-a_j)^{2p} \leq \frac{4^p}{S(n,p)}\sum_{i=1}^{n}a_i^{2p}.$$

(Dorin Andrica, "Babeş-Bolyai" University of Cluj-Napoca, Romania)

**45**. As part of developing a filing system for our collection of $n$ DVDs, we need to label each DVD by a number from 1 to $n$. In order to form this number, we are going to use digit stickers: for example, the number 123 will be formed by the three stickers 1, 2, and 3 side by side (we do not want to add zeros in the beginning, such as 00123, as this would be a terrible waste).
These stickers are sold in sets of 10, and each decimal digit $\{0,1,2,\dots,9\}$ appears exactly once in the set. How many sets of stickers do we need to buy? As an example, for $n=21$ DVDs, digit 1 appears 13 times (in numbers 1, 10–19 and 21 – note that it appears twice in 11!), 2 appears 4 times (2, 12, 20 and 21) and every other digit from 3 to 9 appears exactly twice, so we would need 13 sets.
(K. Drakakis, University College Dublin, Ireland)

**46**. (How to connect two binary words avoiding prohibited patterns ?)
There is a finite family of binary strings $f_1,\dots,f_k$, each of length at most $m$, and two strings $s$ and $t$ of length $m$ each. Is it possible to decide within polynomial time (in the input: the strings $s,t,f_1,\dots,f_k$) if there is a binary string starting with $s$, ending with $t$ and containing no $f_i$ as a substring?
(Vladimir Protasov, Moscow State University, Russia)

**47**. For $h\in\mathbb{Z}$, $r\in\mathbb{Z}_{\geq 0}$ and $k=k_1+k_2$ with $k_1,k_2\in 2\mathbb{Z}_+$ set $h^* := |h-1/2|-1/2$ and

$$A_h^k(r) := \frac{(-1)^{k/2}}{r!}\frac{\Gamma(h-k/2+r)}{\Gamma(h-k/2)}\frac{\Gamma(h+k/2)}{\Gamma(h+k/2-r)}.$$

For $s\in\mathbb{C}$, $y>0$, $l\in\mathbb{Z}_+$ and $u,v\in\mathbb{Z}$ with $u^*<k_1/2, v^*<k_2/2$ set

$$\theta_k(s) = \pi^{-s}\Gamma(s+k/2)\zeta(2s),$$
$$e_k(0;y,u) = \theta_k(u)y^u + \theta_k(1-u)y^{1-u},$$
$$e_k(l;y,u) = \frac{\sigma_{2u-1}(l)}{l^u}e^{-2\pi ly}\sum_{r=0}^{u^*+k/2}\mathscr{A}_u^k(r)(4\pi ly)^{-r+k/2}$$

and

$$S_1(y) := y^{-k/2}e_{k_1}(0;y,u)e_{k_2}(l;y,v) + y^{-k/2}e_{k_1}(l;y,u)e_{k_2}(0;y,v),$$

with $\sigma_s(n)$ denoting the sum of the $s$-th powers of the positive divisors of $n$. Prove that

$$\pi^{k/2-1}\int_0^\infty S_1(y)e^{-2\pi ly}y^{k-2}\,dy$$

is rational and compute it.

(N. Diamantis, University of Nottingham, UK, and C. O'Sullivan, CUNY, USA)

**48**. Find all possible decimal digits $a$ such that, for a given $n$, the decimal expansions of $2^n$ and $5^n$ both begin by $a$, and give a necessary and sufficient condition to determine all such integers $n$.
(K. Drakakis, University College Dublin, Ireland)

## II. Two New Open Problems

**49***. (Local minima of functions on a graph).
A function is defined at nodes of a given connected non-directed graph $G$ with $n$ nodes. The values of $f$ are all different and unknown. One can compute the value $f(x)$ at any node $x \in G$ and this computation takes a unit of time. The problem is to find at least one local minimum of $f$ (i.e. to find a node, at which the value of $f$ is smaller than at all the adjacent nodes).
For which graphs are there algorithms taking time $N(G) = Cn^{\alpha}$ ($\alpha < 1$) or $N(G) = \log n$ independently of the function $f$?

*Comments by the proposer.* This problem originates from convex programming – derivative-free minimization of convex functions (V. Protasov, Algorithms for approximate calculation of the minimum of a convex function by its values, *Math. Notes, 59 (1996), No. 1, 69–74*). For clicks $N(G) = n$. For graphs with successively connected nodes (homeomorphic to a segment) and graphs homeomorphic to a circle $N(G) = C \log n$. In these two cases precise values of $N(G)$ are known for all $n$ and the optimal algorithm involves Fibonacci numbers. The latter case (graphs homeomorphic to a circle) was given as a problem in the Russian Soros Olympiad in 1997 (Problem 11-II-6 "A King and his 199 corrupted ministers sitting around the table"). For other graphs the answer of the problem is unknown.

(Vladimir Protasov, Moscow State University, Russia)

**50***. *Conjecture:* We let $K_{r,s}$ denote the complete bipartite graph, i.e., the graph formed by joining all vertices of a set of $r$ vertices to every vertex of a different set of $s$ vertices and allowing no other edges. Suppose that $G$ is a $K_{r,s}$-minor-free graph, i.e., a graph with the property that no isomorphic copy of $K_{r,s}$ can be obtained from $G$ by contracting edges and deleting edges and vertices. Then, it is possible to colour the vertices of $G$ with $r+s-1$ colours so that no pair of vertices joined by an edge share the same colour.

(D. R. Woodall, University of Nottingham, UK)

## III. SOLUTIONS

**35**. Determine a sequence of positive integers $(a_n)_{n\geq 1}$ for which

$$a_n^2 - a_{n-1} \cdot a_{n+1} = 1 \qquad (1)$$

and $a_4 = 4$.

(V. Copil and L. Panaitopol,[1] University of Bucharest, Romania)

*Solution by the proposers.* Let $a_1 = 1$ and $a_2 = b$ with $a, b \in \mathbb{N}$.

We set $n = 2$ in (1) and get

$$a_3 = \frac{b^2-1}{a}.$$

Next, we set $n = 3$ in (1) and use the fact that $a_4 = 4$. It follows that

$$\frac{(b^2-1)^2}{4b+1} = a^2. \qquad (2)$$

Therefore $4b+1$ must divide $(b^2-1)^2$, in order to have $a^2 \in \mathbb{N}$.

We have

$$256 \cdot \frac{(b^2-1)^2}{4b+1} = 64b^3 - 16b^2 - 124b + 31 + \frac{225}{4b+1}.$$

Consequently it follows that $(4b+1)|225 = 3^2 \cdot 5^2$. Since $4b+1 > 0$ it results that $4b+1 \in \{5, 9, 25, 45, 225\}$. Taking into account (2), $4b+1$ has to be a square and therefore $b \in \{2, 6, 56\}$.

If $b = 2$ it follows from (2) that $a = 1$ and using an induction it results that $a_n = n$ for $n \geq 1$.

If $b = 6$, then $a = 7$ and it follows using (1) several times that $a_9 = -1$, which is no longer positive.

If $b = 56$, then $a = 209$ and as in the previous case it follows that $a_7 = -1$.

In conclusion $a_n = n$ for $n \geq 1$. □

*Also solved by Alberto Bersani (University of Rome "La Sapienza", Italy), Con Amore Problem Group (Copenhagen, Denmark), Tim Cross (UK), T. Delatolas (National Technical University of Athens, Greece), Alberto Facchini (Universita di Padova, Italy), Gerald A. Heuer (Concordia College, USA) and Nicusor Minculete ("Dimitrie Cantemir" University, Brasov, Romania).*

**36**. Let $p_n$ be the $n$-th prime number, $\alpha > 1$ and $c > 0$. If the sum $\sum'$ refers to the indices $n$ for which

$$p_{n+1} - p_n > c \log^{\alpha} p_n,$$

prove that $\sum' \frac{1}{p_n}$ converges.

(V. Copil and L. Panaitopol, University of Bucharest, Romania)

*Solution by the proposers.* Let

$$S_n = \sum_{k=2}^{n} \frac{p_{k+1} - p_k}{k \log^{\beta} k}.$$

Then

$$S_n = \frac{p_{n+1}}{n \log^{\beta} n} + \sum_{k=2}^{n} p_k \left( \frac{1}{(k-1)\log^{\beta}(k-1)} - \frac{1}{k \log^{\beta} k} \right) + O(1).$$

We have

$$p_k \left( \frac{1}{(k-1)\log^{\beta}(k-1)} - \frac{1}{k \log^{\beta} k} \right)$$

$$\sim k \log k \cdot \frac{\log^{\beta} k + \beta \log^{\beta-1} k}{k^2 \log^{2\beta} k} \sim \frac{1}{k \log^{\beta-1} k}.$$

For $\beta > 2$ the series

$$\sum \frac{1}{k \log^{\beta-1} k}$$

converges and therefore taking $\beta = \alpha + 1$,

$$\sum_{k=2}^{\infty} \frac{p_{k+1} - p_k}{k \log^{\beta} k}$$

converges as well.

Since

$$\sum{}' \frac{p_{n+1} - p_n}{n \log^{\alpha+1} n} > \sum{}' c \frac{log^{\alpha} p_n}{(n \log n) \log^{\alpha} n} > M \sum{}' \frac{1}{p_n}$$

for some constant $M > 0$, it follows that $\sum' \frac{1}{p_n}$ is also convergent. □

**37**. Show that for any $n \in \mathbb{N}$ and any sequence of (decimal) digits $x_0, x_1, \ldots, x_{n-1}$ in the set $\{0, 1, \ldots, 9\}$, there exists an $m \in \mathbb{N}$ such that the first $n$ decimal digits of the power $2^m$ are, from left to right, $x_{n-1}x_{n-2} \ldots x_1 x_0$. As an example, the power of 2 beginning with the digits 409 is $2^{12} = 4096$.

(K. Drakakis, University College Dublin, Ireland)

*Solution by the proposer.* Since the number $\log_{10} 2$ is irrational, the sequence $\{n \log_{10} 2 \bmod 1,\ n \in \mathbb{N}\}$ is dense in $[0,1]$. This implies that, for every closed interval $I$ of the real line, there exist infinitely many pairs $(m,l) \in \mathbb{N}^2$ such that $m\log_{10}(2) - l$ lies in $I$. Let us now choose $n \in \mathbb{N}$, a sequence of decimal digits $x_0, x_1, \ldots, x_{n-1}$, form the number $x = \sum_{k=0} n - 1 x_k 10^k$, set $x' = \log_{10}(x)$ and finally choose a small $\varepsilon' > 0$ (to be further specified later). It follows that infinitely many numbers of the form $m\log_{10}(2) - l$ lie in $I = [x', x' + \varepsilon']$, hence we may choose one of them. It follows that

$$x' \leq m\log_{10}(2) - l \leq x' + \varepsilon \Leftrightarrow x \leq \frac{2^m}{10^l} \leq x 10^{\varepsilon'}.$$

Setting $\varepsilon' = x\log_{10}(1+\varepsilon)$, we obtain

$$x \leq \frac{2^m}{10^l} \leq x(1+\varepsilon) < x+1 \Leftrightarrow 0 \leq 2^m - 10^l x < 10^l$$

for $\varepsilon < \frac{1}{x}$. This, however, implies that $2^m$ has at least $n+l$ decimal digits and that at most the $l$ least significant ones differ from the decimal digits of $10^l x$; therefore, at least the $n$ most significant digits of $2^m$ and $10^l x$ are in agreement, which is exactly what we need to prove. □

*Also solved by Erich N. Gulliver (Germany), Gerald A. Heuer (Concordia College, USA) and B. Sury (Indian Statistical Institute, Bangalore, India).*

*Remark.* Gerald A. Heuer remarked that the following theorem was proved in 1950 by Leo Moser and Nathaniel Macon in the paper: *On the distribution of first digits of powers*, Scripta Math. 16 (1950), 290–292, and again by him in a more elementary manner in: *Rational numbers generated by two integers*, Amer. Math. Monthly 78 (1971) 996–997.

If $a$ is a positive integer, which is not a power of 10, and $d_1, d_2, \cdots, d_n$ is any finite sequence of decimal digits, then some integral power of $a$ has $d_1 d_2 \cdots d_n$ as its initial sequence of digits.

**38.** Consider the sequences $(a_n)_{n\geq 1}$, $(b_n)_{n\geq 1}$, $(c_n)_{n\geq 1}$, $(d_n)_{n\geq 1}$ defined by $a_1 = 0$, $b_1 = 1$, $c_1 = 1$, $d_1 = 0$ and $a_{n+1} = 2b_n + 3c_n$, $b_{n+1} = a_n + 3d_n$, $c_{n+1} = a_n + 2d_n$, $d_{n+1} = b_n + c_n$, $n \geq 1$. Find a closed formula for the general term of these sequences.

(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania)

*Solution by the proposer.* First note that $(\sqrt{2}+\sqrt{3})^n = a_n + b_n\sqrt{2} + c_n\sqrt{3} + d_n\sqrt{6}$, $n \geq 1$.

Let $n = 2k$ and

$$x_k = \frac{1}{2}\left[(5+2\sqrt{6})^k + (5-2\sqrt{6})^k\right],$$
$$y_k = \frac{1}{2\sqrt{6}}\left[(5+2\sqrt{6})^k - (5-2\sqrt{6})^k\right].$$

Observe that the numbers $x_k, y_k$ for $k \geq 1$ are rational numbers.

Then

$$(\sqrt{2}+\sqrt{3})^{2k} = x_k + y_k\sqrt{6}, \text{ where } k \geq 1,$$

hence

$$a_n = x_{\frac{n}{2}},\ b_n = c_n = 0 \text{ and } d_n = y_{\frac{n}{2}}.$$

Let $n = 2k+1$. Then

$$\begin{aligned}(\sqrt{2}+\sqrt{3})^{2k+1} &= (\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})^{2k}\\ &= (x_k + 3y_k)\sqrt{2} + (x_k + 2y_k)\sqrt{3},\end{aligned}$$

for all $k \geq 1$. Hence

$$a_n = 0,\ b_n = x_{\frac{n-1}{2}} + 3y_{\frac{n-1}{2}},$$
$$c_n = x_{\frac{n-1}{2}} + 2y_{\frac{n-1}{2}},\ d_n = 0.$$

□

*Also solved by Alberto Bersani (University of Rome "La Sapienza", Italy), Con Amore Problem Group (Copenhagen, Denmark), Tim Cross (UK), T. Delatolas (National Technical University of Athens, Greece), Gerald A. Heuer (Concordia College, USA) and Nicusor Minculete ("Dimitrie Cantemir" University, Brasov, Romania).*

**39.** Let $f : \mathbb{N}\times\mathbb{N} \to \mathbb{N}$ be a function such that $f(1,1) = 2$,

$$f(m+1,n) = f(m,n) + m \text{ and } f(m,n+1) = f(m,n) - n$$

for all $m,n \in \mathbb{N}$, where $\mathbb{N} = \{1,2,3,\ldots\}$.
Find all pairs $(p,q)$ such that $f(p,q) = 2001$.

(Titu Andreescu, University of Texas at Dallas, USA)

*Solution by the proposer.* We have

$$\begin{aligned}f(p,q) &= f(p-1,q) + p - 1\\ &= f(p-2,q) + (p-2) + (p-1)\\ &= \ldots\\ &= f(1,q) + \frac{p(p-1)}{2}\\ &= f(1,q-1) - (q-1) + \frac{p(p-1)}{2}\\ &= \ldots\\ &= f(1,1) - \frac{q(q-1)}{2} + \frac{p(p-1)}{2}\\ &= 2001.\end{aligned}$$

Therefore

$$\frac{p(p-1)}{2} - \frac{q(q-1)}{2} = 1999$$
$$(p-q)(p+q-1) = 2\cdot 1999.$$

Note that 1999 is a prime number and that $p-q < p+q-1$ for $p,q \in \mathbb{N}$. We have the following two cases:
(a) $p-q = 1$ and $p+q-1 = 3998$. Hence $p = 2000$ and $q = 1999$.
(b) $p-q = 2$ and $p+q-1 = 1999$. Hence $p = 1001$ and $q = 999$.
Therefore $(p,q) = (2000,1999)$ or $(1001,999)$. □

*Also solved by Erich N. Gulliver (Germany), Con Amore Problem Group (Copenhagen, Denmark), T. Delatolas (National Technical University of Athens, Greece) and Nicusor Minculete ("Dimitrie Cantemir" University, Brasov, Romania).*

**40.** **(a)** For any non-negative double sequences $\{a_{mn}\}_{m,n=1}^{\infty}$ and $\{b_{mn}\}_{m,n=1}^{\infty}$, and for any $c \geq 0$, if

$$a_{mn} \leq c + \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij}a_{ij} \quad \text{for all } m,n,$$

prove that

$$a_{mn} \leq c \cdot \exp B_{mn},$$

where

$$B_{mn} := \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij},$$

**(b)** As usual, we denote

$$\Delta_1 z_{mn} = z_{m+1,n} - z_{mn},$$
$$\Delta_2 z_{mn} = z_{m,n+1} - z_{mn},$$

and consider the boundary value problem

$$\begin{cases} \Delta_{12} z_{mn} = F(m,n,z_{mn}) \\ z_{m,1} = a_m \,; z_{1,n} = b_n \,; a_1 = b_1 = 0, \end{cases} \qquad (*)$$

where $F$ is any real-valued function. If

$$\left|F(m,n,v_1) - F(m,n,v_2)\right| \leq b_{mn}|v_1 - v_2|$$

for all $m,n \geq 1$ and all $v_1, v_2 \in \mathbb{R}$, show that $(*)$ has at most one positive solution $\{z_{mn}\}$.

(Wing-Sum Cheung, University of Hong Kong, Hong Kong)

*Solution by the proposer* **(a)** It suffices to consider the case $c > 0$. The case for $c = 0$ is then obtainable by continuity arguments. Let

$$g_{mn} := c + \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij}a_{ij} > 0.$$

Then $a_{mn} \leq g_{mn}$ for all $m,n$ and so

$$\begin{aligned} \Delta_1 g_{mn} &:= g_{m+1,n} - g_{mn} \\ &= \sum_{j=1}^{n-1} b_{mj}a_{mj} \\ &\leq \sum_{j=1}^{n-1} b_{mj}g_{mj} \\ &\leq g_{m,n-1}\sum_{j=1}^{n-1} b_{mj}. \end{aligned}$$

Therefore,

$$\begin{aligned} \Delta_1 \ln g_{mn} &= \ln g_{m+1,n} - \ln g_{mn} \\ &= \int_{g_{mn}}^{g_{m+1,n}} \frac{ds}{s} \\ &= \frac{1}{\xi}\Delta_1 g_{mn} \\ &\leq \frac{1}{g_{mn}}\Delta_1 g_{mn} \\ &\leq \frac{g_{m,n-1}}{g_{mn}}\sum_{j=1}^{n-1} b_{mj} \\ &\leq \sum_{j=1}^{n-1} b_{mj} \end{aligned}$$

for some $g_{mn} < \xi < g_{m+1,n}$ by the mean-value theorem. Thus

$$\begin{aligned} &\ln g_{mn} - \ln g_{1n} \\ &= \sum_{i=1}^{m-1} \Delta_1 \ln g_{in} \\ &\leq \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij} \\ &= B_{mn} \end{aligned}$$

and so

$$\ln g_{mn} \leq \ln g_{1n} + B_{mn} = \ln c + B_{mn}.$$

Therefore,

$$a_{mn} \leq g_{mn} \leq c \cdot \exp B_{mn}.$$

**(b)** Observe that $\{z_{mn}\}$ and $\{\bar{z}_{mn}\}$ solve $(*)$ if and only if they satisfy respectively

$$z_{mn} = a_m + b_n + \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} F(i,j,z_{ij})$$

and

$$\bar{z}_{mn} = a_m + b_n + \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} F(i,j,\bar{z}_{ij}).$$

So

$$\begin{aligned} \left|z_{mn} - \bar{z}_{mn}\right| &\leq \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} \left|F(i,j,z_{ij}) - F(i,j,\bar{z}_{ij})\right| \\ &\leq \sum_{i=1}^{m-1}\sum_{j=1}^{n-1} b_{ij}\left|z_{ij} - \bar{z}_{ij}\right|. \end{aligned}$$

An application of (a) to the double sequence $\left|z_{ij} - \bar{z}_{ij}\right|$ yields

$$\left|z_{mn} - \bar{z}_{mn}\right| \leq 0 \cdot \exp B_{mn} = 0.$$

Thus $z_{mn} = \bar{z}_{mn}$. □

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will also be devoted to *Real Analysis*.

## Note

1. Professor Laurentiu Panaitopol passed away in November 2008.

# Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

*Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the addresses madalina.pacurar@econ.ubbcluj.ro or madalina_ pacurar@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files).*

## March 2009

**4–8: 4th World Conference on 21st Century Mathematics**, Lahore, Pakistan
*Information*: wc2009@sms.edu.pk; http://wc2009.sms.edu.pk/

**10–14: ALEA meeting**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**9–11: International Technology, Education and Development Conference (INTED2009)**, Valencia, Spain
*Information*: inted2009@iated.org;
http://www.iated.org/inted2009

**9–13: Harmonic Analysis and PDE: Fluid Mechanics and Kato's Problem**, CRM, Barcelona, Spain
*Information*: http://www.crm.cat/harmonicpde/

**15–20: ALGORITMY 2009 – Conference on Scientific Computing**, High Tatra Mountains, Podbanske, Slovakia
*Information*: algoritm@math.sk; http://www.math.sk/alg2009

**18–20: IAENG International Conference on Operations Research (ICOR'09)**, Hong Kong
*Information*: imecs@iaeng.org;
http://www.iaeng.org/IMECS2009/ICOR2009.html

**20–22: 77th Workshop on General Algebra**, Potsdam, Germany
*Information*: kdenecke@rz.uni-potsdam.de;
http://www.math.uni-potsdam.de/~denecke/Aaa77.htm

**23–25: Workshop on CR and Sasakian Geometry**, University of Luxembourg, Luxembourg
*Information*: http://math.uni.lu/CRSasaki

**23–27: Evolution Equations and Dynamical Systems**, Hammamet, Tunisia
*Information*: ma.jendoubi@fsb.rnu.tn;
http://poncelet.sciences.univ-metz.fr/~harauxconference/

**23–27: Numeration – Mathematics and Computer Science**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**25–28: Taiwan-France Joint Conference on Nonlinear Partial Differential Equations**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**26–27: 2nd Conference of Physics Students**, La Laguna, Canary Islands, Spain
*Information*: info@coeffis.es;
http://www.coeffis.es/index_08.html

**30–April 2: Variational Problems in Differential Geometry**, Leeds, UK
*Information*: vpdg@maths.leeds.ac.uk;
http://www.maths.leeds.ac.uk/pure/geometry/vpdg/

**30–April 3: Adaptivity, Robustness and Complexity of Multiscale Algorithms**, Edinburgh, UK
*Information*: enquiries@icms.org.uk;
http://www.icms.org.uk/workshop.php?id=74

## April 2009

**1–3: 100 Years of Queueing – The Erlang Centennial**, Copenhagen, Denmark
*Information*: vbiv@fotonik.dtu.dk; http://www.erlang100.dk/

**6–9: Joint Meeting of the 61st British Mathematical Colloquium and the 22nd Annual Meeting of the Irish Mathematical Society**, Galway, Ireland
*Information*: http://www.maths.nuigalway.ie/bmc2009/bmc09.html

**7–11: Graph Decomposition, Theory, logics and algorithms,** CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**14–18: Young Researchers in Set Theory Workshop 2009**, CRM, Barcelona, Spain
*Information*: http://www.crm.cat/wkset_theory/

**21–23: 5th International Conference on Multimedia and ICT in Education (m-ICTE2009)**, Lisbon, Portugal
*Information*: micte2009@formatex.org;
http://www.formatex.org/micte2009/

**22–26: 6th Mediterranean Conference on Mathematics Education (MEDCONF 2009)**, Plovdiv, Bulgaria
*Information*: http://medconf2009.fmi-plovdiv.org;
http://www.medconf2009.com

**27–May 2: Master-class in Geometry (Surface Group Representations, Hyperbolic Geometry and Teichmüller Theory)**, Strasbourg, France
*Information*: http://www-irma.u-strasbg.fr/article770.html

**28–May 2: Recent Progress in Operator and Function Theory**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

## May 2009

**1–June 20: INdAM Intensive Period on Geometric Properties of Nonlinear Local and Nonlocal Problems**, Milan and Pavia, Italy
*Information*: ugopietro.gianazza@unipv.it;
http://www.imati.cnr.it/gianazza/bimestre

**4–8: Multilinear Harmonic Analysis and Weights**, CRM, Barcelona, Spain
*Information*: http://www.crm.cat/multilinearharmonic/

**7–9: Algebra and Probability in Many-Valued Logics**, Darmstadt, Germany
*Information*: apmvl@mathematik.tu-darmstadt.de; http://www.mathematik.tu-darmstadt.de/fbereiche/logik/events/apmvl/

**8–9: Irish Geometry Conference 2009**, Cork, Ireland
*Information*: b.mckay@ucc.ie; http://euclid.ucc.ie/pages/staff/Mckay/conferences/irish-geometry-2009/

**9–17: 51st Workshop on Variational Analysis and Applications (In Memory of Ennio De Giorgi)**, Erice, Italy
*Information*: maugeri@dmi.unict.it; http://www.ccsem.infn.it/

**10–15: Leopoldina-Symposium in Algebraic and Arithmetic Algebraic Geometry**, Monte-Verita, Ascona, Switzerland
*Information*: Clemens.Fuchs@math.ethz.ch; http://www.math.ethz.ch/leo09/

**11–15: A Conference in Ergodic Theory – Dynamical Systems and Randomness**, Paris, France
*Information*: http://ergodic2009.math.cnrs.fr/

**11–15: Workshop and Advanced Course on Deterministic and Stochastic Modeling in Computational Neuroscience and Other Biological Topics**, CRM, Barcelona, Spain
*Information*: http://www.crm.cat/wkmodeling/

**23–26: International Conference on Interdisciplinary Mathematical and Statistical Techniques (IMST 2009)**, Plzen, Czech Republic
*Information*: pgirg@kma.zcu.cz; http://home.zcu.cz/~pgirg/IMST2009/

**25–29: 6th European Conference on Elliptic and Parabolic Problems**, Gaeta, Italy
*Information*: gaeta@math.uzh.ch; http://www.math.uzh.ch/gaeta2009

**26–30: High Dimensional Probability**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

**27–30: Dynamical trends in Analysis**, Stockholm, Sweden
*Information*: dynamictrends@ath.kth.se; http://www.math.kth.se/dynamictrends/

**27–June 1: Infinite-Dimensional Analysis and Topology**, Yaremche, Ivano-Frankivsk, Ukraine
*Information*: idat@pu.if.ua; http://www.idat.frankivsk.org

**31–June 6: Spring School on Analysis – Function Spaces, Inequalities and Interpolation**, Paseky nad Jizerou, Czech Republic
*Information*: pasejune@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/katedry/kma/ss/jun09/

## June 2009

**1–5: School on Combinatorics, Automata and Number Theory (CANT'09)**, Liege, Belgium
*Information*: M.Rigo@ulg.ac.be; http://www.cant.ulg.ac.be/cant2009/

**1–5: Geometry and Topology 2009**, Münster, Germany
*Information*: sfb478mi@math.uni-muenster.de; http://www.math.ku.dk/~erik/muenster/

**2–6: Thompson's groups – New Developments and Interfaces**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

**3–12: Four Advanced Courses on Quasiconformal Mappings, PDE and Geometric Measure Theory**, CRM, Barcelona, Spain
*Information*: http://www.crm.cat/acmappings/

**8–12: International Conference AutoMathA: from Mathematics to Applications**, Liege, Belgium
*Information*: M.Rigo@ulg.ac.be; http://www.cant.ulg.ac.be/automatha/

**8–12: Geometrie Algebrique en Liberte XVII**, Leiden, Netherlands
*Information*: deul@strw.LeidenUniv. NL; http://www.lorentzcenter.nl/lc/web/2009/326/description.php3?wsid=326

**9–12: MAFELAP 2009 – 13th Conference on the Mathematics of Finite Elements and Applications**, Brunel University, England
*Information*: Carolyn.sellers@brunel.ac.uk; http://people.brunel.ac.uk/~icsrsss/bicom/mafelap2009

**9–13: Geometric Applications of Microlocal Analysis**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

**11–13: Representation Theory in Mathematics and in Physics**, Strasbourg, France
*Information*: http://www-irma.u-strasbg.fr/article717.html

**14–20: Stochastic Analysis and Random Dynamical Systems**, Lviv, Ukraine
*Information*: sard@imath.kiev.ua; http://www.imath.kiev.ua/~sard/

**14–20: Geometric Group Theory**, Bedlewo, Poland
*Information*: topics09@math.uni.wroc.pl; http://www.math.uni.wroc.pl/ggt/

**14–27: ESI Workshop on Large Cardinals and Descriptive Set Theory,** Vienna, Austria
*Information*: esi2009@logic.univie.ac.at; http://www.logic.univie.ac.at/conferences/2009_esi/

**15–18: 5th International Conference on Dynamical Systems and Applications**, Constanta, Romania
*Information*: cgherghina@gmail.com, cristinatoncu@canals.ro; http://www.univ-ovidius.ro/faculties/civil_eng/conferinta%20iunie%202009/Home.html

**15–18: 3rd International Conference on Mathematics and Statistics,** Athens, Greece
*Information*: atiner@atiner.gr; http://www.atiner.gr/docs/Mathematics.htm

**15–19: Waves 2009**, Pau, France
*Information*: helene.barucq@inria.fr, julien.diaz@inria.fr; https://waves-2009.bordeaux.inria.fr/

**15–19: The Analytic Theory of Automorphic Forms (at the 65th Birthday of Roelof Bruggeman)**, Woudschoten, The Netherlands
*Information*: rb65@math.uu.nl;
http://www.math.uu.nl/rb65.html

**15–20: Strobl09 Conference on Time-Frequency**, Salzburg, Austria
*Information*: strobl09.mathematik@univie.ac.at;
http://nuhag.eu/strobl09

**16–22: 6th International Workshop on Optimal Codes and Related Topics (OC 2009)**, Varna, Bulgaria
*Information*: oc2009@math.bas.bg;
http://www.moi.math.bas.bg/oc2009/oc2009.html

**16–25: International Workshop on Resonance Oscillations and Stability of Nonsmooth Systems**, London, UK
*Information*: http://www.ma.ic.ac.uk/~omakaren/rosns2009/index.html

**21–25: Second African International Conference on Cryptology, (AfricaCrypt 2009)**, Gammarth, Tunisia
*Information*: sami.ghazali@certification.tn;
http://www.africacrypt2009.tn/en/index.php?id=116

**22–26: Topology of Algebraic Varieties – A Conference in Honour of the 60th Birthday of Anatoly Libgober (LIB-60BER)**, Huesca, Spain
*Information*: jaca2009@math.uic.edu;
http://www.math.uic.edu/~jaca2009/

**22–26: The Poetry of Analysis (in Honour of Antonio Córdoba on the Occasion of his 60th birthday)**, Madrid, Spain
*Information*: cordobaconf@gmail.com;
http://www.uam.es/gruposinv/ntatuam/cordoba/index.html

**22–27: International Conference Geometry "In Large", Topology and Applications (Devoted to the 90th Anniversary of Alexey Vasilievich Pogorelov)**, Kharkiv, Ukraine
*Information*: pmc2009@ilt.kharkov.ua;
http://www.ilt.kharkov.ua/pmc2009/index_engl.html

**22–27: First Conference of the Euro-American Consortium for Promoting the Application of Mathematics in Technical and Natural Sciences**, Sozopol, Bulgaria
*Information*: conference@eac4amitans.org;
http://www.eac4amitans.org/

**23–26: 9th Central European Conference on Cryptography**, Trebic, Czech Republic
*Information*: cecc09@fme.vutbr.cz; http://conf.fme.vutbr.cz/cecc09/

**23–27: Hermitian Symmetric Spaces, Jordan Algebras and Related Problems**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**25–27: Current Geometry: The X Edition of the International Conference on Problems and Trends of Contemporary Geometry**, Naples, Italy
*Information*: http://www.levi-civita.org/CG2009.html

**28–July 2: 20th International Workshop on Combinatorial Algorithms (IWOCA 2009)**, Hradec nad Moravici, Czech Republic
*Information*: iwoca09@iwoca.org; ¨
http://www.iwoca.org/iwoca09

**27–July 3: XXVIII Workshop on Geometric Methods in Physics**, Białowieca, Poland
*Information*: http://wgmp.uwb.edu.pl/index.html

**28–July 4: 6th St. Petersburg Workshop on Simulation**, Saint Petersburg, Russia
*Information*: pws2009@statmod.ru; http://pws.math.spbu.ru

**30–July 4: Geometry of Complex Manifolds**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

## July 2009

**1–3: The 2009 International Conference of Applied and Engineering Mathematics**, London, UK
*Information*: WCE@iaeng.org;
http://www.iaeng.org/WCE2009/ICAEM2009.html

**2–4: 6th International Conference on Applied Financial Economics**, Samos, Greece
*Information*: afe@ineag.gr; http://www.ineag.gr/AFE/index.html

**5–8: Algebra and Analysis Around the Stone-Cech Compactification**, Cambridge, UK
*Information*: garth@maths.leeds.ac.uk, stferri@uniandes.edu.co;
http://matematicas.uniandes.edu.co/~stferri/donaconference.html

**5–10: 22nd British Combinatorial Conference**, St Andrews, UK
*Information*: bcc2009@mcs.st-and.ac.uk;
http://bcc2009.mcs.st-and.ac.uk

**6–10: 26th Journées Arithmétiques**, Saint-Etienne, France
*Information*: ja2009@univ-st-etienne.fr;
http://ja2009.univ-st-etienne.fr

**6–11: Conference on Algebraic Topology (CAT'09)**, Warsaw, Poland
*Information*: cat09@mimuw.edu.pl;
http://www.mimuw.edu.pl/~cat09/

**6–11: International Conference on Topology and its Applications**, Ankara, Turkey
*Information*: icta@hacettepe.edu.tr;
http://www.icta.hacettepe.edu.tr

**8–11: 3rd International Conference on Experiments/Process/System Modelling/Simulation/Optimization**, Athens, Greece
*Information*: info2009@epsmso.gr; http://www.epsmso.gr/2009/

**13–18: 7th International ISAAC Congress**, London, UK
*Information*: info@isaac2009.org;
http://www.isaac2009.org

**14–17: 24th Summer Conference on Topology and its Applications**, Brno, Czech Republic
*Information*: slapal@fme.vutbr.cz, Eva.Tomaskova@law.muni.cz;
http://www.umat.feec.vutbr.cz/~kovar/webs/sumtopo

**14–24: Banach Algebras 2009**, Bedlewo, Poland
*Information*: asoltys@amu.edu.pl; http://www.siue.edu/MATH/BA2009/

**15–18: Mathematics of Program Construction**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**16–31: XII Edition of the Italian Summer School**, Santo Stefano del Sole, Italy
*Information*: school09it@diffiety.ac.ru;
http://school.diffiety.org/page3/page0/page93/page93.html

**20–24: 21st International Conference on Formal Power Series and Algebraic Combinatorics**, RISC Linz, Schlosz Hagenberg, Austria
*Information*: ppaule@risc.uni-linz.ac.at;
http://www.risc.jku.at/about/conferences/fpsac2009/

**20–24: 6th International Conference on Positivity and its Applications**, Madrid, Spain
*Information*: positivity6@mat.ucm.es;
http://www.mat.ucm.es/~cpositi6/

**25–29: 6th International Conference on Lévy Processes: Theory and Applications**, Dresden, Germany
*Information*: http://www.math.tu-dresden.de/levy2010

**27–31: Stochastic Processes and their Applications**, Berlin, Germany
*Information*: kongresse@tu-servicegmbh.de;
http://www.math.tu-berlin/SPA2009

August 2009

**1–15: Groups St Andrews 2009 in Bath**, Bath, England
*Information*: gps2009@mcs.st-and.ac.uk;
http://www.groupsstandrews.org

**3–7: Logic and Mathematics Conference**, York, UK
*Information*: http://maths.york.ac.uk/www/York2009

**3–8: 16th International Congress of Mathematical Physics**, Prague, Czech Republic
*Information*: icmp09@ujf.cas.cz; http://www.icmp09.com/

**4–10: International Conference of Mathematical Sciences (ICMS 2009)**, Istanbul, Turkey
*Information*: other@maltepe.edu.tr;
http://mathsciencesconf.maltepe.edu.tr/

**13–17: 7th International Algebraic Conference in Ukraine**, Kharkov, Ukraine
*Information*: iaconu2009@univer.kharkov.ua;
http://iaconu2009.univer.kharkov.ua

**17–21: The 12th Romanian-Finnish Seminar International Conference on Complex Analysis and Related Topics**, Turku, Finland
*Information*: romfin2009@lists.utu.fi;
http://www.math.utu.fi/projects/romfin2009/

**25–28: 14th General Meeting of European Women in Mathematics**, Novi Sad, Serbia
*Information*: romfin2009@lists.utu.fi; ewm2009@im.ns.ac.yu

**30–September 4: Algebraic Groups and Invariant Theory**, Monte Verita, Ascona, Switzerland
*Information*: baur@math.ethz.ch, donna.testerman@epfl.ch;
http://www.math.ethz.ch/~baur/AGIT/

**31–September 6: International School on Geometry and Quantization**, University of Luxembourg, Luxembourg
*Information*: geoquant@uni.lu;
http://math.uni.lu/geoquant/school

September 2009

**1–5: Representation of Surface Groups**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**2–4: New Trends in Model Coupling – Theory, Numerics & Applications (NTMC'09)**, Paris, France
*Information*: mcparis09@ann.jussieu.fr;
http://www.ann.jussieu.fr/mcparis09/

**2–4: Workshop in Nonlinear Elliptic PDEs, A Celebration of Jean-Pierre Gossez's 65th Birthday**, Bruxelles, Belgium
*Information*: wnpde09@ulb.ac.be; http://wnpde09.ulb.ac.be

**3–6: International Conference on Theory and Applications of Mathematics and Informatics (ICTAMI 2009)**, Alba-Iulia, Romania
*Information*: www.uab.ro/ictami

**7–9: 13th IMA Conference on the Mathematics of Surfaces**, York, UK
*Information*: ralph@cs.cf.ac.uk;
http://ralph.cs.cf.ac.uk/MOSXIIIcall.html

**7–11: Third International Conference on Geometry and Quantization (GEOQUANT)**, University of Luxembourg, Luxembourg
*Information*: geoquant@uni.lu; http://math.uni.lu/geoquant

**8–12: Chinese-French Meeting in Probability and Analysis**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**10–12: Quantum Topology and Chern-Simons Theory**, Strasbourg, France
*Information*: http://www-irma.u-strasbg.fr/article744.html

**14–16: Complex Data Modelling and Computationally Intensive Methods for Estimation and Prediction (S.Co.2009)**, Milan, Italy
*Information*: http://mox.polimi.it/sco2009/

**15–19: Geometry and Integrability in Mathematical Physics**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**17–20: 17th Conference on Applied and Industrial Mathematics (CAIM 2009)**, Constanta, Romania
*Information*: caim2009@anmb.ro; www.anmb.ro

**18–22: International Conference of Numerical Analysis and Applied Mathematics 2009 (ICNAAM 2009)**, Crete, Greece
*Information*: tsimos.conf@gmail.com; http://www.icnaam.org/

**22–26: 10th International Workshop in Set Theory**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**24–30: 6th International Conference on Functional Analysis and Approximation Theory (FAAT 2009)**, Acquafredda di Maratea, Italy
*Information*: faat2009@dm.uniba.it;
http://www.dm.uniba.it/faat2009

**29–October 3: Commutative Algebra and its Interactions with Algebraic Geometry**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

## October 2009

**6–10: Partial Differential Equations and Differential Galois Theory**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**12–14: Workshop on Computational Optimization (WCO-2009)**, Mragowo, Poland
*Information*: http://www.imcsit.org/pg/227/181

**12–16: Algebra, Geometry and Mathematical Physics**, Bedlewo, Poland
*Information*: tralle@matman.uwm.edu.pl;
http://www.agmf.astralgo.eu/bdl09/

**13–17: Hecke Algebras, Groups and Geometry**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**20–24: Symbolic Computation Days**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**22–24: Partial Differential Equations and Applications – International Workshop for the 60th birthday of Michel Pierre**, Vittel, France
*Information*: http://edpa2009.iecn.u-nancy.fr/

## November 2009

**3–7: Harmonic Analysis, Operator Algebras and Representations**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**3–7: Discrete Models of Biological Networks: from Structure to Dynamics**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**10–14: The 6th Euro-Maghreb Workshop on Semigroup Theory, Evolution Equations and Applications**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**17–21: Geometry and Topology in Low Dimension (Dedicated to the 60th birthday of Oleg Viro)**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**24–28: Approximation, Geometric Modelling and Applications**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

## December 2009

**1–5: Homology of Algebra: Structures and Applications**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**8–12: Latent Variables and Mixture Models**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

**15–19: Meeting on Mathematical Statistics**, CIRM Luminy, Marseille, France
*Information*: colloque@cirm.univ-mrs.fr;
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## July 2010

**4–7: 7th Conference on Lattice Path Combinatorics and Applications**, Siena, Italy
*Information*: latticepath@unisi.it;
http://www.unisi.it/eventi/lattice_path_2010

# Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

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***D. J. Albers, G. L. Alexanderson, Eds.: Mathematical People****. Profiles and Interviews, second edition, A.K. Peters, Wellesley, 2008, 386 pp., USD 59, ISBN 978-1-56881-340-0*

This book is the second edition of a unique collection containing 25 interesting interviews with important mathematicians who have influenced mathematics in the twentieth century. The first edition was published in 1985. The editors have updated biographical notes to reflect events and changes that have taken place since the first edition of the book. The well-written classical interviews may influence many young students and scientists beginning their careers to study and do mathematics or its applications. The interviews offer insights into the personal and professional lives of some leading scholars, who answer important questions (e.g. why did they work in mathematics, how did they get into their specializations, what influenced their careers and how did they choose their students and collaborators). The book creates a broad view of the landscape of mathematical personalities, mathematical community and mathematics (in particular in the United States and Canada) in the last few decades. The book can be recommended both for mathematicians and interested readers outside of mathematics. (mbec)

***D. Applebaum: Probability and Information. An Integrated Approach,*** *second edition, Cambridge University Press, Cambridge, 2008, 273 pp., GBP 25.99, ISBN 978-0-521-72788-4*

This book is a nice introductory text for a modern course on basic facts in the theory of probability and information. The author always has in mind that many students, especially those specializing in informatics and/or technical sciences, do not often have a firm background in traditional mathematics. Therefore he attempts to keep the development of material gently paced and user-friendly. In the volume, a systematic development of probability and information is presented, much of which would be suitable for a two-semester course of an undergraduate degree. This would then provide a background for further courses, both from a pure and an applied point of view, in probability and statistics.

Some aspects of the subject that are particularly stressed in the volume are: (i) a strong debate on the foundations of probability, which is polarized between "Bayesians" and "frequentists"; (ii) the classical approach, where equally likely events are automatically assigned equal probabilities and a principle of maximal entropy originating from statistical mechanics; (iii) use of Boolean algebras (quite natural now for all students of informatics) instead of the σ-algebras of Kolmogorov – notice that this restriction to finite additivity is made for purely pedagogical and not ideological reasons; and (iv) use of Bernoulli (alternative) random variables as basic stones, the "generators" of many other important discrete probabilistic distributions. The author presents herein what he sees as the core of probability and information. To prevent the book becoming too large, development of some concepts has been shifted to exercises, in particular when they have a marginal application in other parts of the book. Fully worked solutions of exercises can be found on the author's web page. (jant)

***J. Ayoub: Les six opérations de Grothendieck et le formalisme des cycles évanescentes dans le monde motivique I,*** *Astérisque, no. 314, Société Mathématique de France, Paris, 2007, 464 pp., EUR 88, ISBN 978-2-85629-244-0*

***J. Ayoub: Les six opérations de Grothendieck et le formalisme des cycles évanescentes dans le monde motivique II,*** *Astérisque, no. 315, Société Mathématique de France, Paris, 2007, 362 pp., EUR 78, ISBN 978-2-85629-245-7*

These two volumes, each consisting of two long chapters, represent an important contribution to stable motivic homotopy theory. In chapter 1, the author constructs the Grothendieck four functors $f^*$, $f_*$, $f^!$ and $f_!$ in this context (this result was earlier announced by Voevodsky but the details of his construction are still being awaited). Chapter 2 is devoted to constructability properties of these functors and the Verdier duality. In chapter 3, the author develops a motivic formalism of nearby and vanishing cycles. Chapter 4 sums up a construction of the stable homotopy category over a given scheme. Both volumes are written very clearly and carefully. They also include a substantial amount of abstract background material used by the author (e.g. 2-categories, properties of triangulated categories, derivators and homotopical algebra). (jnek)

***M. Baker, B. Conrad, S. Dasgupta, K. S. Kedlaya, J. Teitelbaum: p-adic Geometry. Lectures from the 2007 Arizona Winter School,*** *University Lectures Series, vol. 45, American Mathematical Society, Providence, 2008, 203 pp., USD 45, ISBN 978-0-8218-4468-7*

This slim but very attractive volume contains a write-up of lectures on several important aspects of $p$-adic analytic geometry and its arithmetic applications. B. Conrad gives an informal introduction into non-Archimedean analytic spaces in their various guises: rigid spaces, their formal models and Berkovich spaces. S. Dasgupta and J. Teitelbaum discuss in great detail the Drinfeld $p$-adic upper half plane and its relation to p-adic representation theory of $GL_2(Q_p)$ and the $L$-invariant of modular forms (the conjecture of Mazur-Tate-Teitelbaum). M. Baker develops potential theory on the Berkovich space attached to the projective line (or more generally to a curve) over a $p$-adic field. K. Kedlaya gives a crash course on algebraic de Rham and rigid cohomology. This book – which also contains historical reminiscences by J. Tate and V. Berkovich – is highly recommended to PhD students and researchers in arithmetic geometry and related fields. (jnek)

***J. Beery, J. Stedall, Eds.: Thomas Harriot's Doctrine of Triangular Numbers: the "Magisteria Magna",*** *Heritage of European Mathematics, European Mathematical Society Publishing House, Zürich, 2009, 135 pp., EUR 64, ISBN 978-3-03719-059-3*

This remarkable book consists of three parts. In the first part, the editors give a very short biography of Thomas Harriot (1560–1621), English mathematician, astronomer and geographer, who developed finite difference interpolation methods for the construction of navigational tables by means of constant differences. Towards the end of his life, he summarized the methods and presented them in his famous treatise entitled "De numeris triangularibus et inde de progressionibus arithmeticis: Magisteria magna" (1618). He explained his mathematical ideas (e.g. triangular numbers and the difference method), expressed the relevant formulae in something very close to modern algebraic notation and described how to use them.

The editors also give a short chronology of the way of the "Magisteria", which was lost for many years, before being rediscovered at the end of the eighteenth century and finally published at the beginning of the twenty-first century. In the second part, the editors give an overview of the contents of the "Magisteria". They explain Harriot's presentations and methods of table construction. They also analyse its influence on Harriot's contemporaries (e.g. N. Torporley, H. Briggs, W. Warner and Ch. Cavendish) and his successors (e.g. J. Pell, N. Mercartor, I. Newton and J. Gregory), who explored his methods before or independently of Newton's rediscovery of them in 1665. Harriot's original pages are published in the third part of this book in facsimile. The editors have added many commentaries helping the reader to follow Harriot's ideas. The book can be recommended to anybody interested in English mathematics of the 17th century. Everybody can find here many stimulating ideas. (mbec)

***V. S. Borkar: Stochastic Approximation. A Dynamical Systems Viewpoint****, Cambridge University Press, Cambridge, 2008, 164 pp., GBP 35, ISBN 978-0-521-51592-4*

This book does not intend to be a comprehensive text on stochastic approximation (SA) but rather a "bite-sized" one. Besides the basic convergence analysis of SA procedures, attention is paid to topics connected with their implementation, such as multiple timescales or asynchronous schemes. A short chapter is devoted to the constant stepsize algorithms. Methodologically, the ODE approach has been adopted, which treats the SA algorithm as a noisy discretization of an ordinary differential equation. An overview of several applications is given. Some less standard mathematical requirements can be found in the appendices. (vdup)

***S. J. Brams: Mathematics and Democracy. Designing Better Voting and Fair-Division Procedures****, Princeton University Press, Princeton, 2007, 373 pp., USD 7.95, ISBN 978-0-691-13321-8*

This book focuses on the use of mathematics to illuminate two essential features of democracy: a) how individual preferences can be aggregated to give an election outcome that reflects interests of the electorate; and b) how public or private goods can be divided in a way that respects the rule of the law. It analyzes procedures, or rules of play, that produce outcomes. The book is divided into two parts.

The first part is on *voting procedures*. The emphasis in this part is on approval balloting, whereby voters can approve as many candidates or alternatives as they like without having to rank them. The author discusses different forms that approval balloting can take. The mathematical analysis mainly uses elementary combinatorics and game theory. The author considers different ways of selecting the election outcome, discusses possibilities of manipulation of the result by various interest groups and stability of the election outcome. The theoretical discussion is supplemented by examples of approval balloting adopted by some organizations. The second part is on fair division procedures. Various procedures of *fair division* applicable to both divisible and indivisible goods are discussed together with their distributional consequences. (jtu)

***H. Brenner, J. Herzog, O. Villamayor: Three Lectures on Commutative Algebra****, University Lecture Series, vol. 42, American Mathematical Society, Providence, 2008, 190 pp., USD 39, ISBN 978-0-8218-4434-2*

This book contains lecture notes from the Winter School on Commutative Algebra and Applications held in 2006. The first lecture (by H. Brenner) contains a discussion of a geometrical approach to problems concerning tight closures. Problems treated here include the question of characterization of the tight closure of an ideal in a Noetherian domain over a field of positive characteristic and the question of whether the tight closures commute with localizations in these rings. Both questions have negative answers and the counter-example concludes this part of the book. However the core of the lecture is the understanding of tight closure under the properties of vector bundles on corresponding projective curves. Next, a positive answer to the first question is given for some particular cases. The second lecture (by J. Herzog) considers relations of various structures and monomial ideals in commutative free algebras naturally given by these structures. These chapters are devoted to various possibilities of shifting operation of simplicial complexes, discrete polymatroids, a variation of Dirac's theorem on chordal graphs and Cohen-Macaulay graphs.

The last lecture (by Orlando Villamayor) gives a quite detailed exposition of two important results in algebraic geometry proved by Hironaka: the desingularization and embedded principalization of ideals (both theorems are in characteristic zero). The original proofs were existence proofs; the proofs presented here are constructive. All the lectures suppose that the reader has some experience in commutative algebra or algebraic geometry. The first one requires knowledge of vector bundles and cohomology and the second one supposes the reader to be familiar with the basic concepts of commutative algebra including the Stanley-Reisner rings. This lecture, in general, seems to be written for readers working in algebraic geometry. The topics of these lectures are quite attractive and the work contains very recent results of the authors. Any reader with a good background in commutative algebra could find this book interesting. (ppr)

***L. Le Bruyn: Noncommutative Geometry and Cayley-smooth Orders****, Pure and Applied Mathematics, vol. 290, Chapman & Hall/CRC, Boca Raton, 2007, 524 pp., USD 98.96, ISBN 978-1-4200-6422-3*

This book is based on a series of lectures on noncommutative geometry given in 1999 in the framework of an EU project. The main theme of the book is the theory of Cayley-smooth orders in central simple algebra over a function field of a variety. The book has several parts.

The first part (chapters 1–4) reviews a lot of material needed in the main parts of the book. In particular, the first two chapters are devoted to Cayley-Hamilton algebras and to a description of orders and their centres from the point of view of invariant theory. The next two chapters deal with étale topology, Asumaya algebras, Luna slices and quivers and their representations. Chapters 5 and 6 form the main core of the book. They are devoted to a study of Cayley-smooth orders, their étale locale structure and a classification of the associated central singularity. Chapter 6 deals with a study of different strata in the Hesselink stratification of the null-cone of quiver representations. The last two chapters contain a study of Quillen-smooth algebras and their representations. The book is very well-ordered and is written in a nice and readable style. It contains a huge amount of interesting material related to many important topics in modern mathematics and mathematical physics. It could be greatly appreciated by talented students and young mathematicians as a well-written introduction to important fields of mathematics. (vs)

***R. Butt: Introduction to Applied Linear Algebra with MATLAB****, Sigma Series in Applied Mathematics, vol. 7, Heldermann, Lemgo, 2008, 517 pp., EUR 68, ISBN 978-3-88538-407-6*

This book discusses four major topics: systems of linear equations, eigenvalues and eigenvectors, the least squares approximation and linear programming. The main emphasis has been given to the presentation of a wide range of algorithms dealing with the problems mentioned above and their implementation on computers. The introductory chapter briefly explains computer representation of numbers (floating-point numbers) and the effect of round-off errors on basic arithmetic operations. The exposition proceeds with direct and indirect methods for solving linear systems (including a discussion of errors and conditioning), algorithms for the computation of eigenvalues and eigenvectors (e.g. the power method, the Jacobi method and QR and LR decomposition methods), the Aitken interpolation method and various forms of the least squares approximation (linear as well as nonlinear). The final chapter is devoted to linear programming, especially to the simplex method and duality. The author assumes no previous knowledge of linear algebra. Most theorems are stated without proof; on the other side, there are many carefully worked-out examples demonstrating all the algorithms in action and additional exercises. The algorithms are implemented in Matlab; the appendix provides a readable introduction to this system. The book could be useful even for readers who do not use Matlab. (jsl)

***D. Calegari: Foliations and the Geometry of 3-Manifolds****, Oxford Mathematical Monographs, Oxford University Press, Oxford, 2007, 363 pp., GBP 60, ISBN 978-0-19-857008-0*

This is a highly specialized monograph dealing with the pseudo-Anosov theory of taut foliations of 3-manifolds. It is intended primarily for specialists and it quickly brings the reader into the very centre of contemporary research. The author explains rather success-fully many notions, procedures and constructions of the theory but the limited extent of the book forces him to be sometimes rather brief. Nevertheless, he always quotes the literature where the relevant facts can be found (there are 256 references). In case some results cannot be easily found in the literature, he presents detailed proofs. Here we want to say that it is probably not possible to read this book without certain knowledge in this specialized subject. The author's idea is to present many examples and we learn relevant notions and general theorems starting with these examples. The main notions studied in the book are the universal circles for taut foliations (as well as for other dynamical objects in 3-manifolds). Generally, the book is full of very interesting results and reading it is a real pleasure. (jiva)

***R. Cerf: On Cramér's Theory in Infinite Dimensions****, Panoramas et Synthèses, no. 23, Société Mathématique de France, Paris, 2007, 159 pp., EUR 37, ISBN 978-2-85629-235-8*

The main topics of the presented book are several versions of the Cramér theorem in various situations (e.g. in $\boldsymbol{R}$ and $\boldsymbol{R}^n$, in weak topology and in a separable Banach space). The main tool used in proofs is the large deviation theory. The large deviation of the empirical mean is controlled by the Cramér transform, which can often be explicitly identified. The main part of the book is devoted to the infinite dimensional case, hence various tools from functional analysis in topological vector spaces are needed here. In the last part of the book, the author discusses relations between the Cramér theorem and the Sanov theorem (showing that the Sanov theorem can be deduced from the version of the Cramér theorem in dual space and the Sanov theorem implies the Cramér theorem in the compact situation). The book concludes with three short appendices. (vs)

***P. Ciatti et al., Eds.: Topics in Mathematical Analysis****, ISAAC Series on Analysis, Applications and Computation, vol. 3, World Scientific, New Jersey, 2008, 447 pp., USD 145, ISBN 978-981-281-105-9*

This volume contains a collection of 13 lectures given over the period 2000–2003 at the University of Padova. The topics of these lectures are from various areas of mathematical analysis: potential theory, differential equations and harmonic analysis. Integral representations in complex, hypercomplex and Clifford analysis are treated in the H. Begehr lectures. O. Martio describes two approaches to Sobolev type spaces on metric spaces and their applications to minimizers of variational integrals. "An introduction to mean curvature flow" is the title of G. Bellettini's five hour course. P. Drábek concentrates on three basic tools used in bifurcation theory: the implicit function theorem, the degree theory and the variational method. The lectures by P. Lindquist deal with the eigenvalue problem for p-Laplacian. The existence of solutions to nonlinear elliptic equations with critical and supercritical exponents is the subject of D. Passaseos' lectures. The lectures of G. Rosenblum are devoted to asymptotics of eigenvalues of singular elliptic operators. The purpose of the lectures of E. Vesentini is to show an approach to semigroups of non-linear operators along the same lines as is followed in the linear case.

The last five lectures deal with the harmonic analysis approach to analysis of geometric objects. The injectivity of Radon's transform is studied in the lectures by M. Agranovsky. Connections with overdetermined boundary problems are also shown. The interplay of Fourier analysis and geometric combinatorics like the Szemeredi-Trotter incidence theorem is described in A. Iosevichs' lectures. C. D. Sogge presents the restriction theorem for Fourier transforms and estimates for the Bochner-Riesz means on Riemann manifolds and he shows how these results can be applied to estimate the eigenfunctions of the Laplacian on a compact Riemann manifold with or without boundary. The contribution "Five lectures on harmonic analysis" written by F. Soria explains how properties of classical equations (heat, Laplace, wave and Schrödinger) can be studied through methods of harmonic analysis (e.g. maximal operators and multipliers). H. Triebel shows that fractals can be interpreted as quasi-metric spaces and therefore the theory of function spaces on them is a decisive instrument, e.g. for the investigation of spectrum of the Laplacian on a fractal structure. All contributions are clearly written, contain motivations and give a large number of references and therefore can be recommended to graduate students. Non-experts can also find a lot of useful information on various aspects of partial differential equations. (jmil)

***E. von Collani, K. Baur: Was zum Teufel ist Qualität?****, Sigma Series in Stochastics, vol. 2, Heldermann, Lemgo, 2007, 205 pp., EUR 32, ISBN 978-3-88538-302-4*

This book is the second volume of the Sigma Series in Stochastics published by Heldermann Verlag (the other series of introductory books is Applied Mathematics and Pure Mathematics). The purpose of the series is "to establish stochastics as an independent, mathematically-based science, the subject of which is chance and uncertainty" (from the preface by the series editors). E. von Collani is a professor at Würzburg University and Karl Baur works in the energy industry (E.ON Kernkraft, Hanover) and has been interested in quality problems for a long time. Perhaps everybody understands the notion of 'quality' that appears in the works of Aristotle, Thomas of Aquinas, Descartes and others but the number of definitions is continually increasing. Five different approaches to quality can be distinguished, namely transcendent, product-based, user-based, manufacturing-based and value-based. The first section, entitled *What is the quality?*, comprises a deep discussion of various approaches from the historical development of the Second World War, with its need for top-quality weapons, to the immediate post-war reaction of quality gurus (W. E. Denning, J. M. Juran, A. W. Feigenbauer, Kaoru Ishikawa and Genichi Taguchi) up to the Six sigma movement of the 80s.

The second section, *Quantification and definition of quality*, contains (after an historical review) an attempt to develop a stochastic model of quality originally proposed by Collani in the first volume of the Sigma Series in Stochastics, *Defining the Science of Stochastics*. The model is then used in the third section, *Solutions of problems*, to discuss problems encountered in the first section. The most important mission of the book is to show that not only physical quantities like temperature and length but also such a dark notion as quality can be quantified. This conclusion also has important consequences relating to recent science and the ways of thinking resulting from it, as well as to human behaviour. This conclusion constitutes the content of the fourth section entitled *Message*. The book can be heartily recommended to all students and scientists understanding that stochastic approaches to the real world can be extremely fertile. (sax)

***J. Cronin: Ordinary Differential Equations. Introduction and Qualitative Theory****, third edition, Pure and Applied Mathematics, vol. 292, Chapman & Hall/CRC, Boca Raton, 2007, 381 pp., USD 89.95, ISBN 978-0-8247-2337-8*

This is the third edition of the classical textbook on qualitative theory of ordinary differential equations (the first edition was published by M. Dekker in 1980). Chapters 1–5 cover all the basic facts: the existence and uniqueness of solutions and their dependence on initial conditions; linear systems with constant, periodic and continuous coefficients; autonomous systems and, for planar systems, their behaviour near an isolated equilibrium and the Poincaré-Bendixson theory; and stability and instability results via linearisation and Lyapunov functions. Chapters 6–8 deal with more advanced topics on the existence of periodic solutions. Topological methods and Poincaré's perturbation method (for nonautonomous as well as autonomous equations) and Hopf's bifurcation theorem are discussed in detail. The last chapter is devoted to a brief discussion of the averaging method. For this third edition the sections on local behaviour near a singular equilibrium and on periodic solutions in small parameter problems have been extended and/or unified. The book is written for advanced undergraduate students with a good knowledge of calculus and linear algebra. Several appendices are included for understanding of the more advanced chapters. The largest chapter discusses degree theory. There are many examples throughout the text serving either as motivations or as illustrations of general results. A lot of them come from biological, chemical and, of course, mechanical models. (jmil)

***T. N. Dac: Compactification des champs de chtoucas et théorie géométrique des invariants****, Astérisque, no. 313, Société Mathématique de France, Paris, 2007, 124 pp., EUR 27, ISBN 978-2-85629-243-3*

One of the key points of L. Lafforgue's proof of the global Langlands correspondence for $GL(r)$ over function fields was a construction of suitable compactifications of moduli stacks of Drinfeld's "shtukas" of rank $r$. In this volume, the author develops a new method, based on geometric invariant theory, for constructing compactifications of such moduli stacks. It is expected that this approach will be useful for establishing Langlands correspondence for more general reductive groups. The compactification obtained by the author includes those constructed earlier by Lafforgue. The same method is then applied to moduli stacks of shtukas with multiple modifications. (jnek)

***C. Davis et al., Eds.: The Shape of Content. Creative Writing in Mathematics and Science****, A.K. Peters, Wellesley, 2008, 194 pp., USD 39, ISBN 978-1-56881-444-5*

This remarkable book collects some interesting creative writing of 21 authors (young poets, writers, artists, mathematicians, geologists and philosophers). At the end, the editors add short biographical notes of the contributors. The contributions are in the form of short stories, poems, essays, dramas, fictions, nonfictions and play excerpts. Each of them has a strong mathematical or scientific content. One of the main aims of the book is to show the beauty of mathematics and the sciences and to reveal some areas where art, science and mathematics come together. Another aim is to present creativity of mathematicians and theoretical scientists and to illustrate their works, results and ideas. The book gives many opportunities to think about and discuss scientific works, their difficulties and their roles in our society, to learn why some people do science, to encourage young students into science, and to criticise the current situation and system. The book can be recommended to readers interested in science and literature. (mbec)

***G. Debs, J. S. Raymond: Borel Liftings of Borel Sets – Some Decidable and Undecidable Statements****, Memoirs of the Amer-*

*ican Mathematical Society, no. 876, American Mathematical Society, Providence, 2007, 118 pp., USD 62, ISBN 978-0-8218-3971-3*
One of the motivations for this book was a topological problem concerning particular classes of mappings between separable metrizable spaces. We recall that a surjective continuous mapping *f* from *X* onto *Y* is perfect if the inverse image of a compact set in *Y* is compact in *X*. We say that f is a compact covering if any compact subset of *Y* is the direct image of a compact set in *X*. The mapping *f* is said to be inductively perfect if there exists a subset *X'* in *X* such that *f*(*X'*) = *Y* and *f* restricted to *X'* is perfect. Obviously, one has that any perfect mapping is inductively perfect and any inductively perfect mapping is a compact covering. A problem raised by Ostrovsky is whether a compact covering image of a Borel space is also Borel. A more general variant of this question is whether any compact covering mapping between two Borel spaces is also inductively perfect. It has turned out that these problems lead to a question on continuous and Borel liftings.

The main result of the book shows a close relation between the lifting property and certain set-theory inequalities. Methods of proof based upon a tree and a double-tree representation of Borel sets are presented in the first and the second chapter. A couple of applications are shown in chapter 3, namely Hurewicz type results and Borel separation results. The next part is devoted to a solution of Ostrovsky's problem mentioned above and to a question by H. Friedman on Borel liftings. Chapter 5 presents an application of the main theorem to special aspects of Lusin's problem on constituents of coanalytic non-Borel sets. The last part contains the proof of the main theorem. The book contains deep and powerful ideas that seem likely to have much more interesting applications. Thus it is strongly recommended for everyone interested in descriptive set theory and its applications. (jsp)

***E. Delabaere, M. Loday-Richaud, Eds.: Théories asymptotiques et équations de Painlevé****, Séminares & Congres, no. 14, Société Mathématique de France, Paris, 2006, 363 pp., EUR 76, ISBN 978-2-85629-229-7*
The history of Painlevé differential equations is more than a century long. They are second order nonlinear partial differential equations characterized by the behaviour of the poles of their solutions. Painlevé divided them into six families (P I–PVI) in his classification result. After a quiet period, they have recently become very popular in connection with their applications to important problems in theoretical physics. In the book, there are 14 articles connected with the meeting on Painlevé equations organized in Angers in 2004. A lot of the contributions are devoted to P VI: P. Boalch describes the group of its symmetries and some special solutions; D. Guzzetti discusses its elliptic representations; a long paper by M. Inaba, K. Iwasaki and M.-H. Saito describes its dynamics; and RS-transformations for construction of its algebraic solutions are discussed in the paper by A. V. Kitaev. The paper by P. A. Clarkson is devoted to P II, P III and P IV, while another paper by P. A. Clarkson, N. Joshi and M. Mazzocco discusses the (natural) Lax pair for the modified KdV hierarchy and its relation to isomonodromic problems for P II. The Painlevé property for the Hénon-Heiles Hamiltonians is the topic of the paper by R. Conte, M. Musette and C. Verhoeven. The papers by K. Kajiwara, T. Masuda, M. Nuomi, Y. Ohta and Y. Yamada, and by A. Ramani, B. Grammaticos and T. Tamizhmani are devoted to various aspects of discrete Painlevé equations. Integrability questions for P II are studied in a paper by J. J. Morales-Ruiz. A survey paper by J. Sauloy treats isomonodromy questions for complex, linear q-difference equations. Higher order Painlevé equations (and in particular Nuomi-Yamada systems) are discussed in the paper by Y. Takei. A study of Painlevé equations from the point of view of (infinite dimensional) Galois theory is written by H. Umemura. Asymptotic behaviour of solutions of linear, analytic *q*-difference equations is discussed in the paper by Ch. Zang. (vs)

***E. D. Demaine, J. O'Rourke: Geometric Folding Algorithms. Linkages, Origami, Polyhedra****, Cambridge University Press, Cambridge, 2008, 472 pp., GBP 29.99, ISBN 978-0-521-71522-5*
This book focuses on geometric folding and unfolding problems, which have been studied in mathematical literature since about 1500. But over the past decade, there has been a surge of interest in these problems due to many applications in natural sciences, computer science, engineering, automobile manufacturing and the arts. The book is divided into three parts and it presents hundreds of results and more than 60 unsolved "open problems" to encourage readers to start their own studies and research. In the first part, the authors deal with one-dimensional (1D) objects (so-called linkages). They describe their history and explain algorithms for their construction, reconfiguration and application. In the second part, they describe 2D objects (so-called papers or origami). They explain their mathematical properties, construction, decomposition and approximation. In the third part, they show 3D objects (called polyhedra) and manipulations with them. Although some of the studied problems are described simply and are easily comprehended, they are nevertheless deep and to understand or to solve them, the reader needs high-school geometry, basic discrete mathematics, linear algebra, differential equations, differential geometry, graph theory, basic calculus, some algorithmic techniques and aspects of complexity theory. The authors explain step-by-step interesting solutions of some folding problems. This splendidly illustrated book can be interesting for advanced undergraduate students in mathematics and computer science as well as for geometers and computer specialists who can find many new ideas and impulses there. (mbec)

***B. Edixhoven, G. van der Geer, B. Moonen., Eds.: Modular Forms on Schiermonnikoog****, Cambridge University Press, Cambridge, 2008, 350 pp., GBP 50, ISBN 978-0-521-49354-3*
These are the proceedings of a conference on modular forms from the autumn of 2006. The book contains 17 papers not necessarily related to the lectures at the meeting. The contributions cover a wide range of topics including Siegel modular forms (S. Böcherer, H. Katsurada and R. Schulze-Pillot; B, Heim; T. Ibukiyama; M. H. Weissman), mock theta functions (K. Bringmann), Jacobi forms (N.-P. Skoruppa), geometry of Abelian varieties and Jacobians (R. de Jong; R. Weissauer), non-commutative modular symbols (Y. I. Manin and M. Marcolli), the Petterson trace formula and Hecke eignevalues (A. Knightly and Ch. Li) and Galois representations and their applications to the inverse Galois problem (L. Dieulefait; R. Weissauer; G. Wiese). The book starts with an introductory paper on modular forms (B. Edixhoven, G. van der Geer and B. Moonen). (jnek)

***S. Glaz, J. Growney, Eds.: Strange Attractors. Poems of Love and Mathematics****, A. K. Peters, Wellesley, 2008, 255 pp., USD 39, ISBN 978-1-56881-341-7*

This interesting book collects the best poetry written by mathematicians and poets including Nobel and Pulitzer Prizes winners and Poet Laureates (e.g. R. Alberti, J. Bernoulli, L. Carroll, J. V. Cunningham, R. Dove, R. Forest, J. C. Maxwell and K. O'Brien). This international collection contains about 150 poems with a common theme: love – romantic love, spiritual love, humorous love, love between parents and children, love of mathematics, etc. The editors have gathered poems from the last 3000 years; they include a fragment of The Song of Songs by King Solomon (about 1000 BC), as well as contemporary American poetry. The poems are divided into three parts: romantic love, encircling love and unbounded love. Each poem has a strong link to mathematics (in content, form or imagery) and engages a variety of mathematical topics (for example counting, rings, infinity, Fibonacci numbers, zero, Venn diagrams, functions, distance, calculus, geometry and puzzles). The authors add a "contributions' note" containing biographical notes about contributors and mathematicians appearing in the poems and bibliographical references of sources. These sections give an important impulse for those wishing to find various mathematical concepts appearing in the poems. The book can be recommended to every reader who loves poetry and mathematics and who wants to explore the way that mathematical ideas inhabit poetry. (mbec)

***V. V. Goldberg, Ed.: Maks A. Akivis – Selected Papers,*** *Heldermann, Lemgo, 2008, 596 pp., EUR 100, ISBN 978-3-88538-503-5*

This volume appears in order to honour the 85th birthday and 65 years of mathematical activities of M. A. Akivis. It was published with the support of the International Geometry Center dω, the Foundation "Science" (Odessa, Ukraine) and the International Conference "Geometry in Odessa – 2008" held in Odessa in May 2008. The volume contains an article "On the occasion of his 85th birthday and 65 years of scientific activity" written by A. T. Fomenko, V. V. Goldberg, V. F. Kirichenko, V. V. Lychagin and A. M. Shelekhov. They describe in more detail results of M. A. Akivis in his three main fields of interest, namely conformal differential geometry, web theory and the theory of local differentiable quasigroups, and projective differential geometry. In the volume, there is also a bibliography of the publications of M. A. Akivis (141 items) and a list of PhD theses (28 of them) written under his supervision. The major part of the volume is devoted to 35 papers of M. A. Akivis. They were chosen by the author himself and he has only included research articles. His survey articles, which are very interesting and important, cannot be found here. Twelve of the articles are in Russian and the other papers are in English. (jiva)

***J. Green, J. LaDuke: Pioneering Women in American Mathematics. The Pre-1940 PhD's****, History of Mathematics, Volume 34, American Mathematical Society, London Mathematical Society, 2009, 349 pp., USD 79, ISBN 978-0-8218-4376-5*

This book contains a comprehensive analysis trying to identify all American women studying mathematics and obtaining their PhDs before 1940 (the first woman graduated at the University of Columbia in 1886). Thanks to extensive archival studies, 228 women receiving their PhDs from US institutions or US-born women receiving their degrees abroad have been identified. In the first part of the book (pages 1–118), the authors have presented an introductory essay trying to reconstruct a picture of the women's group in the American mathematical community as completely as possible and to describe their roles in the large scientific and cultural communities. In the second part of the book (pages 119–322), the authors gather biographical and bibliographical information on each of these more than two hundred women. They describe their family backgrounds, education, careers, professional activities and recognition. They also identify each woman's publications, teaching activities, professional presentations and research contributions to mathematics or to industry. At the end of the book, the authors include a list of archives and manuscript collections that were used in the preparation of the biographical entries (pages 325–331). They also add a large biography containing primarily those items that have relevance for the essay (pages 333–337). The book can be recommended to researchers, teachers and students in mathematics, the history of mathematics, the history of science, sociology and gender studies. The material included in the book will stimulate additional studies, analysis and research. (mbec)

***D. Han, K. Kornelson, D. Larson, E. Weber: Frames for Undergraduates****, Student Mathematical Library, vol. 40, American Mathematical Society, Providence, 2007, 295 pp., USD 49, ISBN 978-0-8218-4112-6*

This book is a gentle introduction to the theory of frames. Any element of a vector space can be reconstructed from its (unique) coefficients in a basis of the space. If, moreover, the basis is orthonormal, the coefficients are given by the scalar products with elements of the basis. It is very convenient in practice; they can be computed very easily and quickly. But to use sets of such coefficients is a vulnerable procedure, e.g. if in the considered process they get incomplete. Elements cannot be reconstructed from an incomplete set of its coefficients. On the other hand, any element of a vector space can be reconstructed from the set of its coefficients with respect to a spanning set. Coefficients are no more unique but often it does not matter. It turns out that in Hilbert spaces there are spanning sets that are not bases but which retain the property that the coefficients of any element of the space are its scalar products with the elements of the spanning set. So they keep the convenient property of easy and quick computation. Such spanning sets are called frames.

The book is a very nice introduction into, and survey of, frame theory. Its value is threefold. Firstly, it provides an introduction into the theory of frames. After a terse summary of the necessary items from linear algebra and operator theory (chapters 1 and 2), the reader is acquainted with the main topics in chapters 3–6. Secondly, chapters 7–9 are written at an advanced level. The reader is motivated by interesting open questions and a wide field of applications. Thirdly, the book is an excellent guide for teachers. It is skilfully written at a high mathematical and pedagogical level. Though its title reads "Frames for Undergraduates", it can be very valuable not just for undergraduates but also for professional mathematicians. (jdr)

***L. E. Knop: Linear Algebra. A First Course with Applications,*** *Textbooks in Mathematics, CRC Press, Boca Raton, 2008, 725 pp., USD 99.95, ISBN 978-1-58488-782-9*

This book contains basics of linear algebra (vector spaces, subspaces, bases, linear independence and span, lines and places, solving linear systems, linear transformations and its properties, matrices and operations with them, matrix representations of transformations, determinants and their properties, and eigenvalues and eigenvectors) and it focuses on students starting their studies of linear algebra with almost no experience reading and studying mathematical texts. To encourage them, the book has a specific structure and it does not use the compressed language typical of other mathematical textbooks. Each chapter starts with a short discussion of motivation and solving strategy. Then solutions of some special methodical exercises are given with many explanatory comments. The chapter continues with a typical mathematical text (definition, proposition and proof) and it finishes with various applications and exercises. The author explains step-by-step many solutions of some exercises and integrates a lot of examples of real applications into the theoretical text. He also offers simple instructions for using Maple, MATLAB and TI-83 Plus to give the students the keystrokes to reduce the manual labour associated with many types of exercises, as well as to allow them to focus on the problem at hand and to do more exercises in the same amount of time. The book can be recommended to students beginning their first mathematical course at university, as well as to teachers who want to improve their style of teaching or who are looking for different ways of presenting the basics of linear algebra. (mbec)

***N. Koblitz: Random Curves. Journeys of a Mathematician***, *Springer, Berlin, 2008, 392 pp., 28 fig., EUR 34.95, ISBN 978-3-540-74077-3*

The author of this book is a professor of mathematics at the University of Washington and is well-known for his discoveries in number theory, in particular in hyperelliptic and elliptic curve cryptography. However, only about one sixth of the book is devoted to mathematics and mathematics education, even when the author's opinions are extremely interesting concerning the conflict between the greatest security and greater efficiency in cryptography, as well as his views of undergraduate education and the proportion of computer learning there (the discussions deserve to be completed as articles on the author's personal web page http://www.math.washington.edu/~koblitz/).

The main part of the book is concerned with Koblitz's rebellious student years at Harvard and Princeton, his participation in student leftist activities and primarily with the travels all over the world made together with his wife Ann Hibner Koblitz. She wrote the famous biography of Sofia Kovalevskaia and from the book's royalties founded the Kovalevskaia Prize supporting scientific activities of women from many mathematically less developed countries. The author's interesting impressions and experiences covering the scientific and cultural conditions and mathematics education in the Soviet Union, Vietnam and Middle and South American countries over the last 30 years are extremely attractive reading, especially for older readers able to compare their own knowledge with the experience of the author and his wife. Moreover, many aspects of American academic life are described at length and would certainly spark the reader's interest even in the case of non-mathematicians. (isax)

***T. W. Körner: Naive Decision Making. Mathematics Applied to the Social World***, *Cambridge University Press, Cambridge, 2008, 373 pp., GBP 60, ISBN 978-0-521-51616-7*

This book deals with situations when mathematics can be useful for making decisions when the outcomes are uncertain or when one must respect the interests of others. The author presents various aspects of mathematical decisions using not only realistic examples but also jokes, games, quotations, etc. Chapter 1 shows that some sorts of betting are more sensible than others. Chapter 2 deals with a simple theory of probability (e.g. the weak law of large numbers). In chapter 3, insurance, pensions and a matter of life and death are discussed by means of mathematics of the race-track. Chapter 4 introduces the notion of algorithms (e.g. Euclid's algorithm). Chapter 5 uses the model of a shuffled pack of cards to illustrate various topics (e.g. finding the shortest route from A to B). In chapter 6, the subjects of marrying, voting and preferring are analyzed using e.g. Arrow's theorem. Chapter 7 is devoted to simple games, such as 'Scissors, Paper, Stone'. Chapter 8 uses Nash's arguments for various points of agreement. Chapter 9 is devoted to various forms of duels. Chapter 10 looks at casinos both from the point of view of the customers and from the point of view of the owners. Chapter 11 is called 'Prophecy' and shows that appropriate odds can be found by means of statistics. The 'Final reflections' in the last chapter admits the existence of limits of naive decision making that is discussed in the book. The book can serve as a supplementary text for undergraduate courses in probability, game theory and decision making. Moreover, many exercises are included with solutions available online. (tc)

***P. A. Krylov, A. A. Tuganbaev: Modules over Discrete Valuation Domains***, *de Gruyter Expositions in Mathematics, vol. 43, Walter de Gruyter, Berlin, 2008, 357 pp., EUR 128, ISBN 978-3-11-020053-9*

This book contains a lot of interesting material on modules over discrete valuation domains (DVDs). The first two chapters are introductory and contain an exposition of standard ring and module theory notions (injectivity, purity and completions in the context of modules over a DVD). Chapters 3 and 4 bring some known results on endomorphism rings of modules over a (complete) DVD; in particular the Jacobson radical and the radical factor are calculated for the endomorphism ring of a complete torsion-free module and the Harrison-Matlis equivalence for the endomorphism ring of a divisible primary module is used. The fourth chapter is devoted to the problem of realization of an abstract ring as an endomorphism ring of a module over a DVD; finite topology plays an important role. Such results are used to construct various examples of modules over a DVD with interesting behaviour regarding the direct sum decompositions.

Chapters 5–8 restrict to the case of a commutative DVD. The structure of torsion-free and mixed modules over such rings is studied; some of the techniques resemble those for Abelian groups (passing to the category of quasi-homomorphisms). Among other things, some cardinal invariants providing information for the classification of Warfield modules are defined. Chapter 7 studies when modules over a commutative DVD with isomorphic endomorphism rings are isomorphic. The last chapter is devoted to modules having many endomorphisms or automorphisms. These modules can be seen as a generalization of divisible modules. The book is accessible to any reader, with the only prerequisites being elementary algebra and topology (although some notions like extension groups and inverse limits are referred to other texts). Chapters are concluded by historical remarks, suggestions for further reading, exercises of varying difficulty and open problems. (ppr)

***U. Kulisch: Computer Arithmetic and Validity. Theory, Implementation, and Applications**, de Gruyter Studies in Mathematics, vol. 33, Walter de Gruyter, Berlin, 2008, 409 pp., EUR 78, ISBN 978-3-11-020318-9*

This book deals with the theory of computer arithmetic and it treats an implementation of arithmetic on digital computers. The aim is to improve the accuracy of numerical computing and to control the quality of the computed results. It illustrates how advanced computer arithmetic can be used to compute highly accurate and mathematically verified results. Basic notation and classical results are repeated in order to make the text self-contained. The book has three parts. Part 1 consists of four chapters (First concepts, Ringoids and vectoids, Definition of computer arithmetic, and Interval arithmetic) and it deals with the theory of computer arithmetic. Part 2, also comprising four chapters (Floating-point arithmetic, Implementation of floating-point arithmetic on computer, Hardware support for interval arithmetic, and Scalar products and computer arithmetic) treats implementation of arithmetic on computers. Part 3, comprising only one chapter (Sample applications), illustrates by a few applications how advanced computer arithmetic can be used to compute highly accurate and mathematically verified results. The book can be used as a high-level undergraduate textbook but also as a reference work for research in computer arithmetic and applied mathematics. (knaj)

***S. V. Ngoc: Systemes intégrables semi-classiques – du local au global**, Panoramas et Synthèses, no. 22, Société Mathématique de France, Paris, 2006, 154 pp., EUR 36, ISBN 978-2-85629-221-1*

The main aim of this book is to present a discussion of integrable systems, where both cases – classical systems and quantum systems – are treated together with their mutual relations. The book starts with a nice introduction containing interesting historical comments on classical and quantum integrable systems. The second chapter contains a review of the main properties of (various versions of) the theory of pseudo-differential operators, microlocal analysis and Fourier integral operators. Three representative examples of integrable systems (both on a classical and a quantum level) are treated in the next chapter. The main part of the book is divided into three parts first discussing local analysis of integrable systems (the main tool here being local normal forms), then (very important) semi-local analysis (including a discussion of numerical invariants) and finally treating its global aspects (with results on a globalization of Bohr-Sommerfeld conditions and the Duistermaat-Heckman formulae). (vs)

***M. C. Olsson: Crystalline Cohomology of Algebraic Stacks and Hyodo-Kato Cohomology**, Astérisque, no. 316, Société Mathématique de France, Paris, 2007, 412 pp., EUR 88, ISBN 978-285629-249-5*

This book presents in great detail a stack-theory approach to p-adic Hodge theory, with special emphasis on a construction of a ($\phi$, N, G)-structure on the de Rham cohomology of a proper smooth variety over a p-adic field. The author develops a theory of crystalline cohomology, the de Rham-Witt complex and the Hoydo-Kato isomorphism for certain algebraic stacks. He also reformulates the Kato-Tsuji approach to the semi-stable conjecture ($C_{st}$) in this context and gives a general dictionary between his approach and the "usual" log-geometrical method. As an application of the general theory, he deduces new results on the tameness of the Galois action in certain cases of not necessarily semi-stable reduction. (jnek)

***I. Peeva, Ed.: Syzygies and Hilbert Functions**, Lecture Notes in Pure and Applied Mathematics, vol. 254, Chapman & Hall/CRC, Boca Raton, 2007, 293 pp., USD 152.96, ISBN 978-1-58488-860-4*

This book contains expository chapters on Hilbert functions and resolutions. The chapters point out highlights, conjectures, unsolved problems and helpful examples. There has been a lot of interest and intensive research in this area in recent years with important new applications in many different fields, including algebraic geometry, combinatorics, commutative algebra and computational algebra. In ten chapters written by distinguished experts in the area, the book examines the invariant of Castelnuovo-Mumford regularity, blowup algebras and bigraded rings. Then it outlines the current status of the lex-plus-power conjecture and the multiplicity conjecture. It also reviews results on geometry of Hilbert functions and minimal free resolutions of integral subschemes and of equi-dimensional Cohen-Macaulay subschemes of small degree. It also discusses subspace arrangements and closes with an introduction to multigraded Hilbert functions, mixed multiplicities and joint resolutions. (jtu)

***B. Piccoli, M. Garavello: Traffic Flow on Networks**, AIMS Series on Applied Mathematics, vol. 1, American Institute of Mathematical Sciences, Springfield, 2006, 243 pp., GBP 36.50, ISBN 978-1-60133-000-0*

This book is devoted to a detailed exposition of the nowadays classical approach to modelling traffic flow by first order hyperbolic conservation laws. Networks can be thought of as oriented graphs, where the flow on adjacent edges is related by suitable conditions on the common vertices (= junctions). In this setting, one can model a more complicated traffic setup, including multilanes and junctions with lights or circles. After reviewing the basic theory of conservation laws, the book discusses the background of several models (chapter 3), in particular the Lighthill-Whitham model and the Aw-Rascle model. Some higher order models are mentioned as well. The core of the book (chapters 5–7) is devoted to a study of the dynamics of these models on networks. An interesting case study comparing the regulation using traffic lights with the traffic on circles is presented (chapter 8). The related problem of flow on telecommunications networks is addressed in chapter 9, while numerical results are discussed in the concluding chapter 10. The presentation is clear and readable, accompanied with a lot of graphs and diagrams as well as exercises concluding each chapter. The book is accessible with a mild background in partial differential equations. (dpr)

***S. C. Roy: Complex Numbers. Lattice Simulation and Zeta Function Applications**, Horwood Publishing, Chichester, 2007, 131 pp., GBP 28, ISBN 978-1-904275-25-1*

This book contains a discussion of some interesting topics from applications of complex analysis to a discussion of the Riemann conjecture on a distribution of zeros of the Riemann zeta function and the Ewald lattice summation (used to calculate electrostatic potentials in crystal lattices). The book is clearly designed for non-specialists. It starts with a discussion of the most basic facts from complex analysis (it slowly introduces complex numbers, it describes some elementary functions in the complex domain and it touches on contour integration of complex functions and basic theorems of complex function theory). Many notions in complex analysis are defined in an unusual way (by illustrating them on some particularly simple examples without any attempt to present the precise definitions used in standard mathematical books). The main body of the book contains many tables of values of various functions in chosen finite sets of points as well as lots of graphical presentations of such tables. All of this is mixed with many quotations of various important papers and historical comments. It is a very unusual book. (vs)

***D. Ruelle: The Mathematician's Brain****, Princeton University Press, Princeton, 2007, 160 pp., USD 22.95, ISBN 978-0-691-12982-2*

This book, written by a professor emeritus of mathematical physics, is a bold attempt to describe specific features of mathematical thinking. It presents and elaborates many methodological and philosophical problems connected with this frequently posed and until now unsolved problem. It aroused considerable attention immediately after its publication and critical reviews and discussions are still running. There was a criticism concerning the choice of the word *brain* instead of *mind* in the title but Ruelle's Johann Bernoulli lecture presented in Groningen in April 1999 explains his position, namely the comparison of PC and brain activities. The scope of the book is "to present a view of mathematics and mathematicians that will interest those without training as well as the many who are mathematically literate". An investigation of the activities of a human brain can be found in the Greek and Renaissance periods and the names and results of Leonardo da Vinci, Descartes, Newton and Galileo Galilei are mentioned and discussed in the book. However, Ruelle's attention is primarily focused on personalities that directly influenced mathematics of the 20th century and, consequently, B. Riemann (in particular his conjecture concerning primes), F. Klein (Erlangen program), G. Cantor, D. Hilbert and K. Gödel are frequently recalled throughout the book, and beside the life works also the life stories of A. Turing and A. Grothendieck are described at length.

Ruelle also tries to answer many philosophical problems connected with mathematics and human thinking. However, a serious critique of his conclusions was heard just from philosophical circles and some of the reproaches are worth mentioning. The thesis that "the structure of human science is dependent on the special nature and organization of human brain" is questionable because of our limited knowledge of the brain's activities. Ruelle's conclusion that "mathematics is the unique endeavour where the use of a human language is, in principle, not necessary" is hardly correct. Also his link to previous investigators of the same theme, in particular to H. Weyl (*Philosophy of Mathematics and Natural Science*, 1949) and Saunders Mac Lane (*Mathematics: Form and Function*, 1986) is insufficient. Perhaps also Ruelle's inclination to physics and the way of thinking in physics was an obstacle to attaining more general and correct conclusions. However, all referees praise the book for its ability to present the exceptionally creative ways of mathematical thinking as well as their impact on mathematicians' lives and they recommend it as a daring attempt to elucidate the uniqueness of mathematical thinking in comparison to other scientific activities as well as the similarities in the behaviour of all scientists. (isax)

***J. D. H. Smith: Introduction to Abstract Algebra****, Textbooks in Mathematics, CRC Press, Boca Raton, 2008, 327 pp., USD 89.95, ISBN 978-1-4200-6371-4*

This textbook covers an introductory course in general algebra. The approach is elementary but rather abstract. One can trace the author's research interests to the border between algebra and category theory, which gives the textbook its unique flavour. The abstract approach is balanced by an abundance of illuminating examples and exercises, from the obvious up to difficult study projects. However, the book mostly avoids applications outside algebra and it also avoids some harder parts of the usual algebraic curricula, such as Galois theory. The book uses the traditional concept of groups first and rings next, while many properties are stated on the level of weaker structures such as semigroups and monoids. The contents are as follows. The first three chapters explain the fundamentals: numbers, functions and equivalences, staying on a concrete level. In the middle part, abstract groups and rings and fields, with the standard algebraic technology, are introduced. The final chapters include topics on factorization, modules, group actions and quasigroups. (dsta)

***J. Stedall: Mathematics Emerging. A Sourcebook 1540–1900****, Oxford University Press, Oxford, 2008, 651 pp., GBP 49.50, ISBN 978-0-19-922690-0*

This sourcebook describes and analyses the development of mathematics in Western Europe from the late 16th century to the end of 19th century. It is an important addition to a big family of sourcebooks: A. Speiser, *Klassische Stücke der Mathematik* (1925); H. Wieleitner, *Mathematische Quellenbücher* (1927–1929); D. E. Smith, *A source Book in Mathematics I., II.* (1929, repr. 1959, 1985); J. R. Newman, *The World of Mathematics, I.-IV.* (1956); H. O. Midonick, *The Treasury of Mathematics I., II.* (1965); J. van Heyenoort, *From Frege to Gödel. A Source Book in Mathematics, 1870–1931* (1967); G. Birkhoff, *A Source Book in Classical Analysis* (1973); A. P. Juškević (red.), *Chrestomatija po istorii matematiki, I. Arifmetika i algebra, Teorija ćisel, Geometrija, II. Matematićeskij analiz, Teorija verojatnostej* (1976, 1977); D. J. Struik (ed.), *A source Book in Mathematics 1200–1800* (1969, repr. 1986, 1990); J. Fauvel, J. Gray, *The History of Mathematics. A Reader* (1987); U. Bottazzini, P. Freguglia, L. Toti Rigatelli, *Fonti per la storia della matematica: aritmetica, geometria, algebra, analisi infinitesimale, calcolo delle probabilita, logica* (1992); and V. J. Katz (ed.), *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* (2007), which are very important for a good understanding of the history of mathematics.

The interesting book connects two different ways of approaching the history of mathematics – the way of topics and the way of periods. The book consists of 18 chapters. Each chapter focuses on a particular topic (e.g. Calculus of Newton and Leibniz, Early mechanics, Early number theory, Early probability, Power series, Functions, Calculus, Limits and continuity, Polynomial equations, Algebra, Derivatives and integrals, Convergence and completeness, Complex analysis and Linear algebra). The author describes the development of mathematics using primary source material (provided with explanatory notes), historical comments and modern mathematical interpretations. The main aim of this book is not to present a predigested version of the history of mathematics but to encourage the reader to develop their own critical historical thinking on mathematics and its development. Therefore the book will be a very important original publication related well to contemporary mathematics. At the end of the book, there is a summary of the main mathematicians, institutions and journals mentioned in the text, which provides a quick reference guide to dates and places. The bibliography consists of primary sources, secondary sources, further reading and digital archives as well as an index. The book can be recommended to teachers of mathematics as well as students or researchers in the history of mathematics who are interested in the development of modern mathematics and science. (mbec)

***F. J. Swetz: Legacy of the Luoshu****, A. K. Peters, Wellesley, 2008, 214 pp., USD 35, ISBN 978-1-56881-427-8*

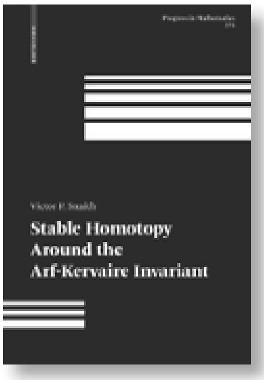
This fascinating book describes the luoshu (i.e. the magic square of order three) from Chinese origins and legends, its role in the development of Chinese mathematical thinking and teaching and its association with the rise of Chinese science and philosophy. For the first time, the author shows the mystical origins of the luoshu, the traditional method of its construction and the earliest references to it. Then he describes the later Chinese variations of the luoshu, constructions of higher order magic squares and later Chinese works on magic number arrangements. The author also reviews a wide scope of relations of the luoshu with sociology, cosmology, numerology, metaphysics, philosophy, religion, mythology and many other topics. He also analyses the process of incorporation of the luoshu into Islamic astrology and alchemy, its way into cabbalistic tradition and its role in the Western European spiritual meaning in numbers. Some miscellanea illustrating the flourishing development of the luoshu in China are included. Many explanatory notes on each chapter, the bibliography, illustration acknowledgments and the index are added at the end of the book. The book is very well illustrated. It can be recommended to anybody interested in the properties of numbers, roots of number theory and the history of mathematics, as well as Chinese mathematical tradition. (mbec)

***M. B. W. Tent: Emmy Noether****, A. K. Peters, Wellesley, 2008, 177 pp., USD 29, ISBN 978-1-56881-430-8*

This book is devoted to the life of one of the most fascinating and influential figures of modern mathematics Emmy Noether (1882-1935), the German Jewish mathematician, who is well-known in particular for her excellent results in modern algebra (the theory of rings and fields), for her teaching activities and her collaboration with many outstanding scientists (e.g. with Albert Einstein). In five chapters, a charming story of Emmy Noether's life is shown, from her childhood, studies, lecturing in Göttingen, mathematical activities and exile to United States up to teaching at Bryn Maws College. Due to the fact that records and information on her childhood and studies are incomplete, some parts contain elements of fiction, while others are based on many archival sources. The author wrote the book primarily for young readers to tell them not only the life story of the most important female mathematician of the twentieth century but also to analyse the creative atmosphere of the German mathematical community, the situation, old traditions and their changes at the University of Göttingen as well as political and social events that were caused by the Nazis when they came to power in Germany in 1933. She also wants to show the reader how young talented people explore mathematics. The book can be useful to anybody interested in mathematics and the history of modern algebra. (mbec)

***X.-P. Wang, C. Zhong, Eds.: Partial Differential Equations and Applications****, Séminares & Congres, no. 15, Société Mathématique de France, Paris, 2007, 331 pp., EUR 68, ISBN 978-2-85629-237-2*

This book contains (updated and expanded) versions of lectures delivered at the school held in Lanzhou (China) in 2004. There are seven contributions on various topics on partial differential equations. B. Grébert describes properties of solutions of Hamiltonian perturbations of integrable systems (the main tool being the Birkhoff normal form and its dynamical consequences – the Birkhoff approach is compared with the Kolmogorov-Arnold-Moser approach). In a long paper, F. Hélein explains many properties of integrable systems on several important examples (sinh-Gordon, Toda, KdV, harmonic maps and ASD Yang-Mills fields). D. Iftimie discusses recent results on large time behaviour of the Euler equations for a perfect incompressible fluid in the plane or half-plane. The role of coherent states in the study of Schrödinger type equations, estimates for their asymptotic solutions and several applications are described in the paper by D. Robert. A short note by W.-M. Wang contains a proof of stability of the bound states for time quasi-periodic perturbations of the quantum harmonic oscillator. A new proof for microlocal resolvent estimates for semi-classical Schrödinger operators with a potential from a special class is presented in the paper by X. P. Wang. Basic methods for a study of standard semi-linear elliptic equations are presented in the last paper by D. Ye. (vs)

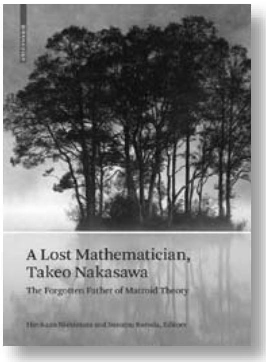



## Stable Homotopy Around the Arf-Kervaire Invariant

*Snaith, Victor P.*

2009, XIV, 240 p., Hardcover
ISBN: 978-3-7643-9903-0
EUR 59,90 / CHF 105,00 / GBP 55.99
A Birkhäuser book

This monograph describes important techniques of stable homotopy theory, both classical and brand new, applying them to the long-standing unsolved problem of the existence of framed manifolds with odd Arf-Kervaire invariant. Opening with an account of the necessary algebraic topology background, it proceeds in a quasi-historical manner to draw from the author's contributions over several decades. A new technique entitled "upper triangular technology" is introduced which enables the author to relate Adams operations to Steenrod operations and thereby to recover most of the important classical Arf-Kervaire invariant results quite simply. The final chapter briefly relates the book to the contemporary motivic stable homotopy theory of Morel-Voevodsky.




## A Lost Mathematician, Takeo Nakasawa

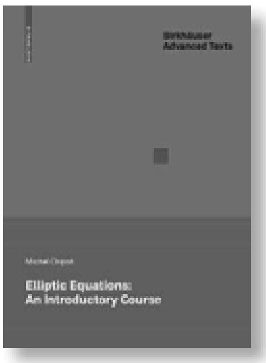
### The Forgotten Father of Matroid Theory

*Nishimura, Hirokazu*
*Kuroda, Susumu*

2009, XII, 234 p. 14 illus., Hardcover
ISBN: 978-3-7643-8572-9
EUR 49,90 / CHF 89,90 / GBP 39.99
A Birkhäuser book

Matroid theory was invented in the middle of the 1930s by two mathematicians independently, namely, Hassler Whitney in the USA and Takeo Nakasawa in Japan. Whitney became famous, but Nakasawa remained anonymous until two decades ago. He left only four papers to the mathematical community, all of them written in the middle of the 1930s. It was a bad time to have lived in a country that had become as eccentric as possible. Just as Nazism became more and more flamboyant in Europe in the 1930s, Japan became more and more esoteric and fanatical in the same time period. This book explains the little that is known about Nakasawa's personal life in a Japan that had, among other failures, lost control over its military. This book contains his four papers in German and their English translations as well as some extended commentary on the history of Japan during those years. The book also contains 14 photos of him or his family. Although the veil of mystery surrounding Nakasawa's life has only been partially lifted, the work presented in this book speaks eloquently of a tragic loss to the mathematical community.

- Gives an exact description of the origin of matroid theory
- First English translation of the four German papers




## Elliptic Equations: An Introductory Course

*Chipot, Michel*

2009, X, 288 p., Hardcover
ISBN: 978-3-7643-9981-8
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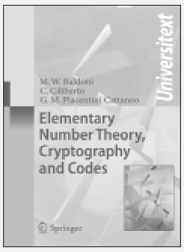


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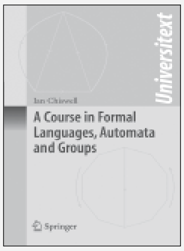


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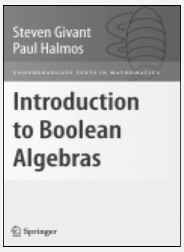


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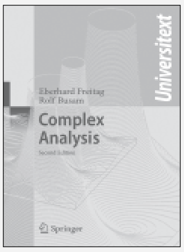


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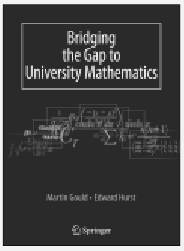


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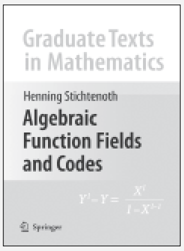


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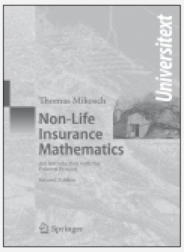


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