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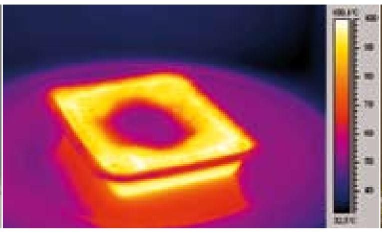

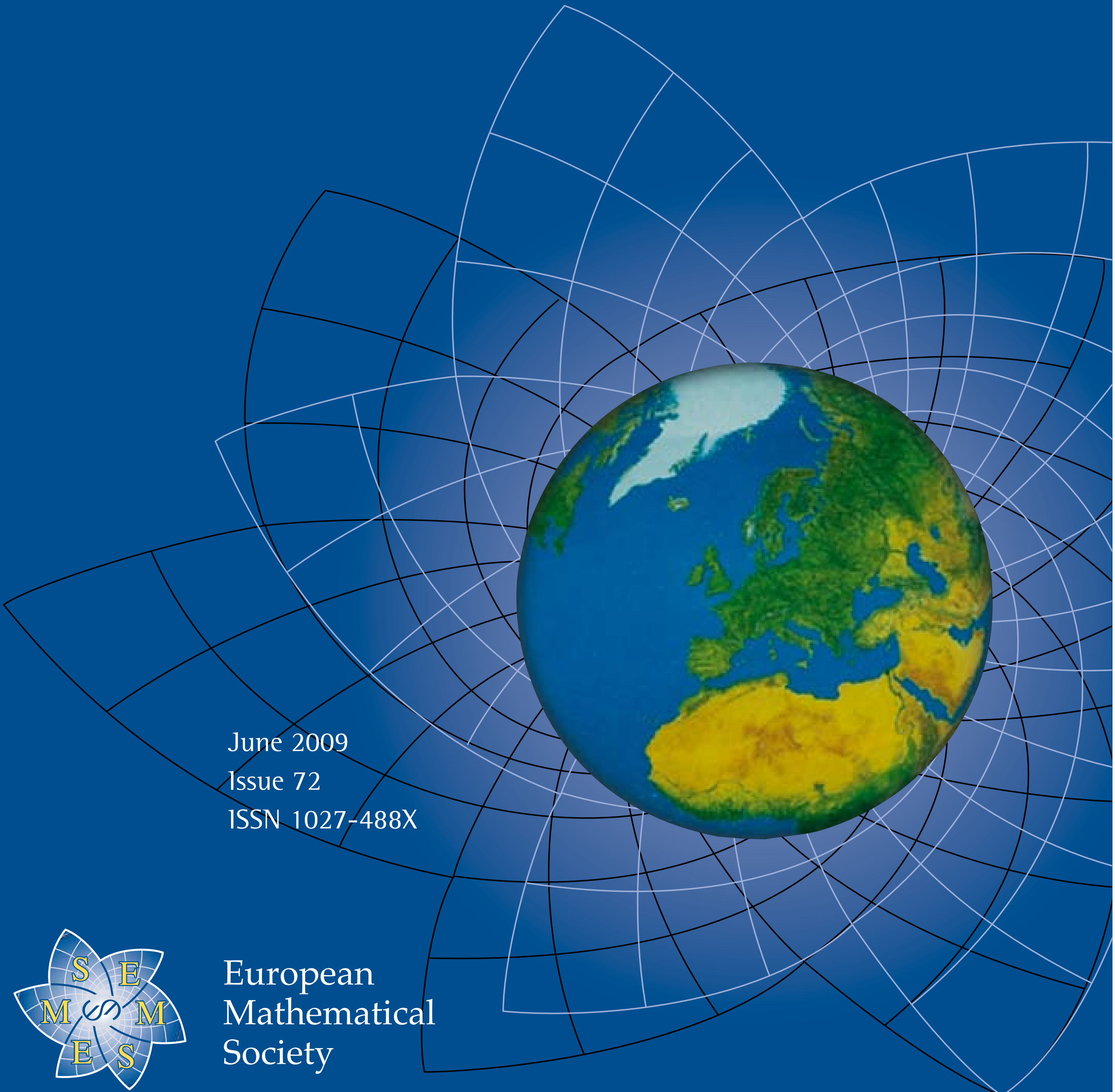

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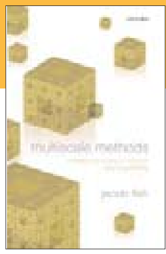

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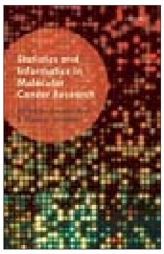

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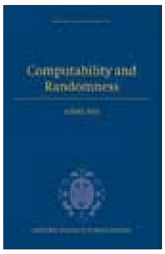
André Nies



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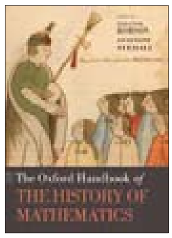

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EMS Agenda

2009

8–11 June
25th Nordic and 1st British-Nordic Congress of Mathematicians, University of Oslo, Norway
http://www.math.uio.no/2009/

21–27 June
Workshop and EMS Summer School in Applied Mathematics, The Mathematical Research and Conference Center, Będlewo, Poland
http://www.impan.pl/BC/Program/conferences/09Waves.html

22–27 June
3rd Nordic EWM Summer School for PhD Students in Mathematics, University of Turku, Finland
http://www.math.utu.fi/projects/ewm/organization.html

1 August
Deadline for submission of material for the September issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@imaff.cfmac.csic.es

3–8 August
XVI International Congress on Mathematical Physics (ICMP 09) Prague, Czech Republic
http://www.icmp09.com/

17–18 October
EMS Executive Committee Meeting
Stephen Huggett: s.huggett@plymouth.ac.uk

1 November
Deadline for submission of material for the December issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@imaff.cfmac.csic.es

2010

25–28 February
EUROMATH 2010, Bad Goisern, Austria
http://www.euromath.org

25–28 March
Meeting of Presidents of European national mathematical societies, Romania.

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Editorial

Laszlo Lovász

Dear Reader,

The next International Congress of Mathematicians will take place 19–27 August 2010 in Hyderabad, India. As the President of the International Mathematical Union, I would like to invite you all to come to India and contribute to the success of the congress.

The ICM is the single most important event in mathematics every four years. Its organisation, from the work of the local organisers to the program committee, the prize committees and the publishers of the proceedings (and many others), is the most important task for our community.

Often one can hear scepticism about having such congresses in general. One of the points people make is the large size of it (at least for a mathematics meeting). Indeed, a single participant will only know a small fraction of the other participants and one can walk down the crowded corridors for a long time without seeing a familiar face. A participant will be able to follow only a small fraction of the section talks. A lot of effort has been made, especially at recent congresses, to make the invited talks, especially the plenary talks, accessible to a general mathematical audience but it is still difficult to follow so many ideas from different parts of mathematics within such a short time.

However, talking to scientists working in physics, computer science or any other branch of science, they are envious of the fact that we mathematicians have such an event. We can listen to carefully chosen speakers describing the latest developments. We learn first hand about the most important prizes in mathematics and hear the recipients describe their work. We have panel discussions about important issues like education, applications, electronic publications, scientometry and other topics that are in the forefront of interest at the time of the congress. And there are interesting exhibitions; over the years, there have been a variety of topics for these exhibitions like the history of the IMU or the history of mathematics in the host country. There is a book fair and there are meetings of special groups, poster sessions, satellite conferences and many other events.

Just before the congress, the general assembly will meet in Bangalore to decide on the next site of the congress, elect the new leadership and discuss and make decisions about many other issues that are important for the mathematical community. The most important decisions will be announced at the opening ceremony of the congress.

The Fields Medal and the Nevanlinna Prize themselves are unique in their scope. Most scientific prizes, like the Nobel Prize, award the highest recognition to older people whose work is known and well recognized. In contrast, these prizes of the IMU go to young people and new results. In this way, the whole community can learn about these breakthrough results and can meet these young mathematicians who are bound to become important personalities of our science in the near future.

Mathematics is a rather lonely science, most of our work happening inside our heads. It is very good to have a big event bringing together our community every four years.

The site of the congress moves around in the world and India, with its long tradition in mathematics and also with its attraction for tourists, is a fascinating congress venue. To have congresses in developing countries as well as in developed ones is important: the impact of such an event on the local community and on its position in the eyes of the general public may be larger here than in a developed country. But of course fewer mathematicians have first hand experience of the site and there are more concerns about the unknown, so people might hesitate attending. Last December I went to India, visiting the site of our 2010 Congress in Hyderabad and attending a "Pre-ICM" conference in Delhi. Some friends wondered about such a trip, mentioning all sorts of dangers from snakes to malaria. If you recall, a few weeks earlier there had been terrorist attacks in Mumbai and indeed quite a few participants of the conference cancelled their trips. Needless to say, the terrorist attacks had no influence on my visit (except for some increased security at public buildings) and with some caution, it was easy to avoid infections (and snakes). India is a country where crime, especially violent crime, is rare.

I strongly recommend visiting India to everyone and hope to see you at ICM 2010 in Hyderabad!

Laszlo Lovász

Opinions expressed in the section "Letters to the Editor" are those of the authors and do not necessarily reflect the policies and views of the EMS. Letters from readers appearing in the Newsletter are published without change.

Reinventing Loewner's Approach to Univalent Functions

Gerald S. Goodman

In his authoritative historical account, Lawson (1992), credits Loewner (1923) with the discovery that regarding holomorphic self-mappings of the unit disk that keep the origin fixed as the elements of a semigroup under composition gives the possibility of generating them, in the tradition of Sophus Lie, by integrating their *infinitesimal generators*.

The latter are defined by considering one-parameter families $t \to f_t, t \geq 0$, of mappings in the semigroup, for which f_0 is the identity, and taking their right-derivative – which is assumed to exist – there. Loewner showed that they have the form $-wp(w)$, where p is holomorphic in w throughout $|w| < 1$ and has $\mathrm{Re}\,p(w) > 0$, with $p(0)$ real.

Such a family f_t need not form a semigroup, but, once the generator is known, it yields, by quadrature, a one-parameter semigroup having itself as generator. Thus, it is logical to assume that the family of mappings f_t themselves form a one-parameter semigroup, and are real-analytic, but that is not part of the definition of an infinitesimal generator, it is a consequence of it.

No doubt, Loewner would have begun his study by first examining these one-parameter semigroups. He saw that the functions that are generated in this way have restricted mapping properties, making them atypical for the semigroup as a whole. He thus rejected them as inadequate and took the decisive step of allowing the infinitesimal generators to vary with t. This gave rise to the non-autonomous differential equation $df/dt = fp(f,t)$, known today as *Loewner's equation*, where t varies over an interval having 0 as its left endpoint, and f reduces to the identity there. The function p has positive real part, and it is holomorphic in the first variable and is assumed to be piecewise continuous in the second, while $p(0,t) > 0$. This was weakened to measurability by Pommerenke (1965), p. 160 (2.3), and by Goodman (1968), p. 217 ff – and applied by Goodman (1969) – who proved, in different ways, that if a two-parameter semigroup, or "evolution," $h_{s,t}$, $0 \leq s \leq t$, of conformal self-mappings of the unit disk that keep the origin fixed is normalized so that $h'_{s,t}(0) = e^{s-t}$, then it is absolutely continuous and is the general solution of Loewner's equation, understood in the Carathéodory sense, with $p(0,t) = 1$ for all t. A detailed account of Pommerenke's method is given in the textbook by Graham & Kohr.

Afterwards, Berkson & Porta (1978) treated one-parameter semigroups of holomorphic self-mappings of the unit disk, or the half-plane, subject to various hypotheses about the location of their fixed points, and showed that they could be generated by an autonomous differential equation, whose right side was an infinitesimal generator of the corresponding semigroup.

In the case of the unit disk with fixed point at the origin, this was precisely what Loewner had done, but the authors were unaware of it. When the fixed point is elsewhere in the interior of the disk, the problem can be reduced to the previous one by conjugating with a Moebius transformation. The half-plane can be treated in a similar way, and Loewner made extensive use of time varying infinitesimal generators in his study of holomorphic self-mappings of the half-plane, Loewner (1957), in connection with his theory of monotone matrix functions.

When seen from the point of view of the iteration theory of holomorphic functions, the work of Berkson & Porta has been hailed as an historical "breakthrough", passing from the discrete parameter case to the "fractional", or continuous parameter, one. This opinion was expressed by Marco Abate in his book Abate (1989), and in his paper Abate (1992).

Abate characterized the infinitesimal generators according to the position of the fixed point, as had done Berkson & Porta, by an analytical argument, as an exercise involving the Schwarz-Pick, or the Wolff, lemmas. Loewner had done similarly in the case of a fixed point at the origin by giving a geometrical one, based on Schwarz's Lemma. Abate makes no mention of Loewner, and was evidently unacquainted with his work.

In Loewner's paper, as in Berkson & Porta's, the standard definition of infinitesimal generator, described by Lawson, is employed. Accordingly, the one-parameter family of mappings that defines it does not have to form a semigroup, but one can assume that it does.

Abate conflates the two and *defines* an infinitesimal generator only in the case in which the family forms a one-parameter semigroup, and thus is differentiable. That assumption is unnecessary, and it has no bearing on the characterization of the infinitesimal generators.

The paper by Bracci et al. (2008) fits into this scheme. What it attempts to do is to extend the Berkson & Porta results from one-parameter semigroups back to two-parameter ones, from whence they came, by using Abate's definition of infinitesimal generator, in place of Loewner's.

In Th. 1.1, on p. 3, they introduce the same normalized families $h_{s,t}$ as Pommerenke and Goodman did, without citing the source, but they then abandon the normalization, and, in order to ensure differentiability, they assume absolute continuity as a hypothesis, whereas, originally, it had been a conclusion.

This alteration produces a theorem resembling the one given by Pommerenke and Goodman, but its proof is trivial, as it reduces the derivation of Loewner's equation to a remark, since the form of the infinitesimal generators is already known. They fail to notice this and supply a lengthy and convoluted argument.

The resulting theorem is unusable in practice, bccause, before it can be applied, the absolute continuity would have to be verified, and there are cases where it fails to hold. To see this, just replace t by its image under a continuous, strictly increasing, singular function.

What the authors miss is that an order-preserving map of the t-interval preserves functoriality, so any two-parameter semigroup is transformed into another, but absolute continuity may be lost. However, normalizing the time-scale induces it.

Loewner's 1923 paper is listed by the authors in their bibliography, as is Pommerenke's 1965 one, but they reserve for themselves full credit for identifying time-varying infinitesimal generators as the source of Loewner's equation, ignoring both Lawson's attribution of that result to Loewner, and Loewner's own 1921 announcement of his discovery, which bears the title "On the Generation of Univalent Conformal Mappings from Infinitesimal Ones."

Apparently, neither the authors, nor their peers at the ESF or INDAM, were familiar enough with the literature to recognize that what they called their "main result," Th. 1.1, was but a flawed version of a theorem already proven by others, and the two agencies were persuaded by the authors to allow them to organize an international conference INDAM (2008) in Rome, where they could give the keynote address, and, thereby, benefit from institutional endorsements of their spurious claims and gain legitimacy for their naive belief that they had originated Loewner's method and put it into its present form.

This feat has been repeated at a recent conference, Braude (2009), sponsored in part by the NSF, who have thus adorned with their own prestige the author's claims.

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Gerald Goodman [gerald.goodman@gmail.com] is an American mathematician who has made his career in Europe. As an undergraduate at Haverford, he was imbued with a sense of the ethical dimension of human actions. He was inspired, both mathematically and spiritually, by the presence there of Mark Kac. Later, he became assistant to Loewner at Stanford and wrote his doctoral thesis on the Carathéodory version of Loewner's equation. He is now retired from a tenured post at the University of Firenze and lives on a Medici estate, where he engages in mathematical research and studies the sociology of science.

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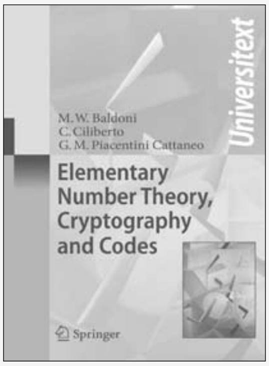

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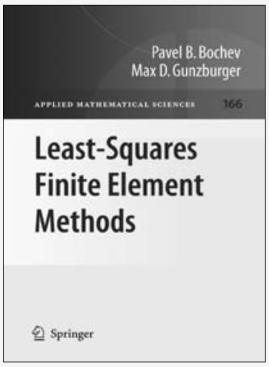

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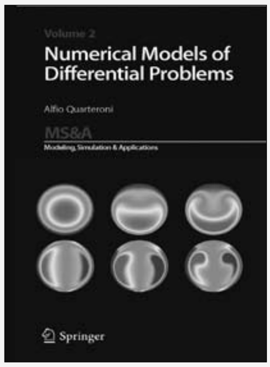

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EMS Executive Committee meeting Athens, Greece, 21–22 March 2009

Vasile Berinde, EMS Publicity Officer

Venue and attendance

The Hellenic Mathematical Society invited the EMS Executive Committee (EC) to meet for its first 2009 "democratic exercise" in the native place of democracy, Athens – an invitation impossible to decline. However, the meeting place was neither the antique Agora nor the famous Acropolis in Athens but the underground conference hall of President Hotel in Greece's capital city.

Present at the meeting were all EC members: Ari Laptev (President, Chair), Zvi Artstein, Franco Brezzi, Pavel Exner (Vice-President), Helge Holden (Vice-President), Stephen Huggett (Secretary), Igor Krichever, Mireille Martin-Deschamps, Martin Raussen, Jouko Väänänen (Treasurer) and, by invitation, Ehrhard Behrends (Chair of the EMS Committee for Raising Public Awareness of Mathematics), Vasile Berinde (Publicity Officer), Vicente Muñoz (Editor-in-Chief of the EMS Newsletter) and Mario Primicerio (Chair of the EMS Committee for Applied Mathematics).

The Chair welcomed all the participants and noted the importance of the meeting and the decisions to be reached over the time allocated. Despite the dense agenda of the meeting, it had been well structured by the secretary and gently chaired in a very flowing way by the president to allow ample time for discussions and debates.

Summary of the main points of the meeting

- ***Ratification of electronic votes*:** Nils Baas, John Barrow, Franka Miriam Brueckler, Krzysztof Ciesielski, Steen Markvorsen, George Szpiro and Betül Tanbay were appointed as members of the EMS Committee for Raising Public Awareness of Mathematics.
- ***Officers' Reports***: the president, secretary, treasurer, vice-president H. Holden and publicity officer presented their reports on several important issues. We mention here only two: the accounts and activity report for 2008 and budget for 2009, presented by the treasurer and approved by the EC, and the proposals presented by H. Holden for an Ethics Committee of the EMS, which were welcomed and also approved by the EC.
- ***Membership***: a proposal was presented by the secretary that Susan Oakes, recently retired from the London Mathematical Society, be invited to work on encouraging members of national societies to become individual members of the EMS; this proposal was unanimously agreed by the EC.
- ***EMS website***: H. Holden presented the latest developments on the webpage and noted that the membership database is now working well.
- ***EMS/Springer Prize on the History of Mathematics***: an ad-hoc committee was set up (M. Raussen, V. Muñoz, S. Huggett, and V. Berinde) to prepare a draft of the regulations for the prize.
- ***Meeting of presidents of national societies***: the next meeting will be in Warsaw (9–10 May 2009); Romania will host the 2010 meeting.
- ***Standing committees***: reports by the chairs of committees were either directly presented by the chairs themselves (E. Behrends and M. Primicerio) or by the EC members responsible for those committees. Some new members were appointed by the EC: Mete Soner (Istanbul) and Miroslav Feistauer (Prague) for

From left to right: Zvi Artstein, Vicente Muñoz, Mireille Martin-Deschamps, Mario Primicerio, Igor Krichever, Martin Raussen, Ari Laptev, Stephen Huggett, Helge Holden, Pavel Exner, Jouko Väänänen, Ehrhard Behrends, Franco Brezzi.

the Applied Mathematics Committee and Anna Fino (Torino) for the Developing Countries Committee. The Meetings Committee report was presented by M. Raussen and Z. Artstein. It was discussed how the meetings could be improved by including the so-called "distinguished EMS lectures" in the programme and A. Laptev suggested that the Meetings Committee should also take responsibility for the EMS Lectures and for Mathematical Weekends.

- ***Discussion***: under this new item in the EC agenda, a fairly brief discussion, introduced by M. Martin-Deschamps, highlighted the evaluation of researchers and university teachers by using various rather inappropriate indicators. Noting the need to distinguish clearly between appropriate and inappropriate indicators for each subject, it was agreed that this should be discussed in more detail at the Meeting of Presidents in Warsaw and that several societies would be asked for successful examples of evaluations.
- ***Future projects***: it was agreed that in October 2010 there should be some special event marking the 20th anniversary of the EMS.
- ***Publishing***: Vicente Muñoz reported that the new editorial board of the newsletter was working well and there was plenty of new material. He would eventually like to publish six issues a year instead of four, which comes down mainly to a question of money. The book reviews and conference announcements will soon move from the printed version in the newsletter to the EMS website, in this way freeing up about 14 pages. The EC thanked Vicente for his excellent work.
- ***Special invitees***: the first day of the EC meeting closed with two invited presentations. Nikolaos Alexandris, President of the Hellenic Mathematical Society, presented an overview of the society; this was seconded by Takis Vlamos. Then, Professor Stefan Dodunekov (Bulgaria), President of MASEE at the time (the current President of MASEE is Doru Ştefănescu), gave a brief presentation of MASSEE (the Mathematical Association of South Eastern Europe), which is the continuation of the former Balkan Mathematical Union.

Closing matters and next meeting

The meeting ended in good time to allow people to have a quick lunch and catch their flights but not before the president Ari Laptev had thanked our hosts N. Alexandris (President of the Hellenic Mathematical Society) and Takis Vlamos (Secretary General of MASEE) for the wonderful hospitality in Athens. The date of the next EC meeting was fixed to be 17–18 October.

ALL SOULS COLLEGE, OXFORD
Senior Research Fellowships

All Souls College proposes to elect up to four Senior Research Fellows with effect from 1st October 2010 (or an agreed later date): in Classical Studies, in Law and in Mathematics (all subjects broadly conceived).
A Senior Research Fellowship is of comparable academic standing to a Professorship in the University of Oxford. Applicants are expected to have a correspondingly distinguished record of achievement in research.
Further particulars, including details of salary and other terms of appointment, may be obtained from the Warden's Secretary, All Souls College, Oxford OX1 4AL; *mary.yoe@all-souls.ox.ac.uk*. See also the College's website: *www.all-souls.ox.ac.uk*. The deadline for applications will be Friday, 25th September 2009.
Potential mathematical candidates may contact *graeme.segal@all-souls.ox.ac.uk* or *dan.segal@all-souls.ox.ac.uk* for more information about the position.

RUPRECHT-KARLS-
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The **Faculty of Mathematics and Informatics**, University of Heidelberg, invites applications for a

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in Scientific Computing

At the Interdisciplinary Center for Scientific Computing (IWR) a W3-Professorship (Full Professor) is to be filled. This Professorship is associated to the Faculty of Mathematics and Informatics. Applicants should have an eminent research record in methodological and software development preferably in the area of complex systems, efficient solution methods, multi-scale methods, parallel computing. They should have experiences in a wider range of application areas. An active role in current as well as planned cooperative research projects at IWR is offered. Further an adequate participation is expected in the teaching of undergraduate and graduate courses within the study programs of the Faculty. The general teaching language is German.

Prerequisites for application are a university degree and (in accordance with Article 47, paragraph 2 of the Higher Education Law of the state of Baden-Württemberg) a Habilitation, a successfully evaluated junior professorship or equivalent qualifications.

The faculty intends to increase the number of women in teaching and research; women are therefore explicitly invited to apply. Handicapped persons with the same qualifications will be given preference.

Applications (hardcopy and electronic) with the usual documents (curriculum vitae, description of scientific interests, list of publications (no reprints), record of teaching activities) should be submitted until June 08, 2009, to: **Dekan der Fakultät für Mathematik und Informatik, Universität Heidelberg, Im Neuenheimer Feld 288, D-69120 Heidelberg, Germany.**

Please send copies, applications cannot be returned.

EUROMATH-2009

European Student Conference in Mathematics, 5–8 February 2009, Cyprus

Vasile Berinde, EMS Publicity Officer

The EUROMATH student conference was introduced in order to give the opportunity for young people to express their talent in working toward a mathematics theme with an investigative approach and presenting it in an organised manner to their classmates and to their teachers. Participation in the conference was also expected to provide a multicultural environment that would allow young people to interact, understand and learn from each other.

The chairman and founder of EUROMATH-2009, Dr Gregory Makrides, President of the Cyprus Mathematical Society, said during the opening ceremony of the conference: "In today's world the precious asset is its young people – their skills and knowledge, their ideas and creativity potential and most of all, their potential to do research".

In this very successful first conference, in which the European Mathematical Society was a collaborator, there were 35 presentations with about 100 students aged between 12 and 18 involved as co-authors and another 100 students or more who came as participants. The countries represented were Bulgaria, Finland, Romania, Italy, the USA, the Czech Republic and Cyprus.

The conference is expected to become an annual event. Students from different countries had the chance to meet each other and begin to cooperate in mathematics projects, which could then be presented jointly at future EUROMATH conferences.

The plan for EUROMATH 2010 is already underway with a very enthusiastic town hosting the next conference. The event is planned for 25-28 February 2010 in the town of Bad Goisern in Austria. The conference is expected to grow and those interested should keep a close eye on the websites www.euromath.org and www.thalescyprus.com

The person behind this promising initiative is Dr Gregory Makrides (President of the Cyprus Mathematical Society, President of the THALES Foundation and Director of Research at the University of Cyprus) who is a pioneer in identifying, motivating and supporting mathematical talent among young people. He has also initiated other activities and events, like the ambitious project MathEU regarding the Identification, Motivation and Support of Mathematical Talents in European Schools (http://www.matheu.eu/), which has been carried out with the support of the European Community within the framework of the Socrates Programme.

Left: Dr Gregory Makrides, Chair of EUROMATH-2009,
Right: Professor Andreas Demetriou, Minister of Education from Cyprus

Professor Vasile Berinde, representing the European Mathematical Society

Kirichenko Dimitri, a EUROMATH 2009 participating student from Finland

A group of students from Cyprus during their presentation at EUROMATH 2009

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Public Lecture on Nonlinear PDEs (Monday 13 July)

Pierre-Louis Lions (Paris)

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- I.2. Differential equations: complex methods, applications
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- II.2. Methods in Clifford- and Cayley-
- III.1. Toeplitz operators and applications
- III.2. Reproducing kernels
- III.3. Integral transforms
- III.4. Spaces of differentiable functions
- III.5. Analytic function spaces
- III.6. Spectral theory
- IV.1. Pseudo-differential operators
- IV.2. Dispersive equations
- IV.3. Control and optimisation of evolutionary systems
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Some aspects of K. Itô's works

Marc Yor (Paris)

K. Itô, one of the most prominent probabilists of the 20th century, passed away in Kyoto on 10 November 2008, aged 93. He had just received, on 06 November 2008, the "Order of Culture" – the highest possible distinction – from the Emperor of Japan.

The first Gauss prize was also given to K. Itô, in recognition of the numerous applications of his works "to the real world", during the International Congress of Mathematicians, held in Madrid in August 2006. K. Itô was an associate member of the Academy of Sciences of Paris from 1989, as well as the National Academy of Sciences of the United States from 1998.

In Japan, K. Itô developed a very active Probability School. Among his former students are Professors N. Ikeda, T. Hida, H. Kunita, M. Fukushima and S. Watanabe, who, in turn, have instructed many students in probability theory.

Here is a succinct presentation of the two main topics developed by K. Itô, topics that made him famous: Itô's stochastic calculus and Itô's excursion theory.

Itô's stochastic calculus, and in particular the famous "Itô formula", constitutes a central pillar of probability theory bearing upon stochastic processes. These works, which were published between 1942 and 1951, took approximately 25 years before being integrated in all graduate courses in probability throughout the world. In 1970, K. Itô published a second revolutionary work in which he develops the theory of excursions of a Markov process, which allowed the taking up and completion of the pioneering works of Paul Lévy on Brownian excursions.

Apart from being the coauthor, with H.P. McKean, of the famous book "Diffusion processes and their sample paths" (1965), K. Itô is also the author of a book about infinite dimensional Markov processes, a subject he was particularly fond of, as shown by his writing in the Preface of his *Selected Papers* (1987): "From a certain point in my life, I started to consider systematically an infinite dimensional viewpoint, even for probabilistic studies which only involved finite dimensional processes". Let us see how this sentence of Itô allows the shedding of light on his theory of the excursions of a Markov process.

To simplify, let us limit the discussion to excursions away from *0*, for example, on the real line, for real-valued Brownian motion. These excursions are (adequately labelled) fragments of the Brownian trajectory considered for time intervals between two successive zeros. Adopting a convenient time scale for this study, K. Itô showed that this family of excursions is a gigantic Poisson process (the usual terminology is Poisson Point Process). This is quite remarkable for several reasons:

The trajectories of (real-valued) Brownian motion are extremely erratic whereas those of the Poisson process are extremely simple. Indeed, they may be described as a staircase, with each step of height one but whose "length" (duration!) is a random variable with exponential law. In fact, Itô's excursion theory consists of constructing an infinity of such Poisson processes, which "count" different types of Brownian excursions and for which the dependence, or independence, properties are well understood. Thus, at the cost of the consideration of infinitely many Poisson processes, Itô has been able to replace the complexity of the Brownian trajectories by infinitely many staircases.

At this point in our presentation of Itô's main works, one may reasonably ask which is the most efficient theory, the (Itô) excursion theory or the (Itô) stochastic calculus? It is difficult to answer this question. In fact, for some topics, the two theories may be applied in conjunction. A good illustration of this is perhaps the Feynman-Kac formula, which involves solutions of Sturm-Liouville equations but which may be understood either from stochastic calculus arguments (involving relevant martingales) or by excursion theory arguments.

For example, Paul Lévy (1939) proved that the time spent positive up to time t by Brownian motion is given by the arc sine law (of parameter t). There are now beautiful proofs of this result using either stochastic calculus or excursion theory. More generally, Itô's excursion theory allows the simple computation of the laws of many Brownian functionals by using differential (or integral) calculus with respect to these Poisson trajectories. This Poisson calculus is much simpler than Itô's stochastic calculus for Brownian motion; the latter is – despite its relative simplicity – a second order calculus for which one is quickly confronted with Sturm-Liouville and Ricatti equations.

Still remaining in the framework of Itô's theory of Brownian excursions, it allows the construction of many processes with stationary independent increments, nowadays usually called Lévy processes. This "unification", which consists of considering Lévy processes within the Brownian framework is, by itself, extremely remarkable and differs sharply from the "classical" teaching and presentation of these processes. Traditionally, the objective was to describe *all* Lévy processes in terms of their Lévy-Khintchine formulae.

Then, within this discussion, it was noticed that, among all the Lévy processes, only the Brownian motions with constant drifts had continuous trajectories. In contrast, with Itô one is led to construct as many Lévy processes as possible, with the help of Brownian trajectories and the inverse of Brownian local time. In 1981, F. Knight on the one hand and S. Kotani-S. Watanabe on the other hand were able to describe precisely all the Lévy processes one may thus obtain.

It is now time to discuss in precise terms the main subject developed by Itô, whose numerous applications made him famous: Itô's stochastic calculus, a term which encompasses at the same time stochastic integration theory with respect to Brownian motion and more generally with respect to a semi-martingale (the sum of a martingale and of a process of bounded variation), as well as the theory of stochastic differential equations.

Here are some of the steps developed by Itô. As a first step, Itô showed that one may integrate processes that depend on the whole past of Brownian motion with respect to the "differential of Brownian motion". In this way he obtained a considerable extension of the Paley-Wiener theory, which only allowed the integration of deterministic functions.

As a second step, Itô used his theory of stochastic integration to define (i.e. give a meaning to the notion of) stochastic differential equations and to solve them. An important difficulty is that one needs to make sure a priori that the unknown process is indeed measurable with respect to the past of the Brownian motion! This was in particular proven by Itô, who showed that Picard's iteration method, which is the key tool for solving ordinary differential equations, admits a stochastic counterpart when the coefficients of the (stochastic) equation are assumed to be Lipschitz.

However, Itô's program goes much further. This study is not "gratuitous" – it has in fact a very precise objective: Picard's iteration method allows the transfer of the independence of increments of Brownian motion into a Markov property of the process that solves the stochastic differential equation.

Thus, starting from his theory of stochastic integration, Itô, by working "directly" at the level of Brownian trajectories, was able to construct many Markov processes, which had so far essentially been studied from an analytic point of view, as Kolmogorov's papers on the subject clearly show.

To summarize, the two masterpieces of Itô that have just been evoked consist of establishing an "interior" calculus bearing upon Brownian trajectories. It succeeds in many cases, in superseding the "exterior" calculus of a more analytic character, which was well developed in the pre-Itô era.

Marc Yor [Yormarc@aol.com] has been a professor at the Laboratoire de Probabilités of Paris VI since September 1981. He was elected as a member of the Académie des Sciences de Paris in November 2003 in the Section de Mathématique. He has also been a senior member of the Institut Universitaire de France since September 2004. Between 1980 and 2006 Marc Yor was editor of the Séminaires de Probabilités, published by Springer and founded by Paul-André Meyer.

The works of Marc Yor in probability theory are centred on the study of Brownian motion and related processes; over the last fifteen years, he has been concerned with applications in mathematical finance, either in order to compute option prices or to model price processes.

Twenty-five years of CRM (1984–2009)

The Centre de Recerca Matemàtica (CRM), Barcelona, celebrates its 25th anniversary with a social event on 27 April and a scientific meeting on 27 November 2009. Since its creation in 1984, the centre has hosted almost 2,000 research visitors, including 78 postdoctoral fellows, and has organised more than 120 scientific events such as conferences, workshops and advanced courses.

Since 2001, the CRM has been a consortium run jointly by the Catalan Government and the Institute of Catalan Studies. It is part of Catalonia's CERCA network, which currently comprises 38 research institutes in various disciplines. About one third of the CRM's budget is covered by the Catalan Government through a contract programme, while the rest comes from applications submitted to several funding bodies.

The CRM has been a member of the ERCOM committee of the EMS since its origin in 1997 and it has also been a member of the European Post-Doctoral Institute (EPDI) since 2000. It is currently a node of the Spanish research project Ingenio Mathematica (2006–2011).

The current CRM Director is Joaquim Bruna. He was elected in 2007, taking over from Manuel Castellet (now Honorary Director) who founded the CRM in 1984 and steered it with energy and insight during the first 23-year period of its existence. Joaquim Bruna is assisted by a team of directors consisting of Carles Casacuberta, Marta Sanz-Solé and Joan Solà-Morales. Marta Sanz-Solé, who was a member of the EMS Executive Committee in the period 1997-2004, is currently the liaison between the CRM and European entities, including ERCOM. She was the coordinator of a scientific session organised jointly by the EMS and the CRM within the EuroScience Open Forum (ESOF) in July 2008 in Barcelona. She also participated in the recent and successful EMS-ERCOM application for mathematics conferences organised by the European Science Foundation, one of which will take place in the CRM in June 2009.

The largest amount of activity at the CRM is concentrated on its research programmes. These are periods of intensive research in specific areas of mathematics, bringing together researchers from all over the world to work on open problems and to analyse the state and perspectives of the area. The duration of a CRM Research Programme is between three months and a full academic year. An annual call is made in the last quarter of each year, seeking for proposals two years in advance of their starting date. The CRM Scientific Advisory Board is responsible for the review process of the proposals and their final evaluation.

CRM Research Programmes (2008–2010)

Homotopy Theory and Higher Categories (2007–2008)
Geometric Flows and Equivariant Problems in Symplectic Geometry (2008)
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Mathematical Biology: Modelling and Differential Equations (2009)
Harmonic Analysis, Geometric Measure Theory and Quasiconformal Mappings (2009)
Probabilistic Techniques in Computer Science (2009)
Arithmetic Geometry (2009–2010)
The Infinity Project (2009–2011)
Foliations (2010)
The Cuntz Semigroup and the Classification of C*-Algebras (2010)

Besides research programmes, the CRM organises other activities, either on its own initiative or by accepting proposals submitted by research teams. The CRM has permanent open calls for proposals of conferences, workshops, summer/winter advanced courses and thematic days. At present, the centre is conducting a monthly seminar cycle on computational and systems neuroscience and another one on financial engineering in collaboration with the Barcelona Stock Exchange and Barcelona's European Finance Centre (BCFE).

Although research stays at the CRM are normally restricted to participants of research programmes, the CRM maintains a permanent programme of collaboration research stays for small groups of people working together and offers several different opportunities for post-doctoral grants. These include hosting EPDI and Marie Curie fellows but also holders of grants from the Spanish Government, the Catalan Government and post-doctoral fellows funded by the CRM itself. There were fifteen in 2008 altogether.

CRM Auditorium

Young participants of a research programme

Since 2004, the CRM has also offered student grants for starting research on emerging topics. This is one among many actions undertaken by the CRM within its Strategic Areas Incentive Programme (IAE). Indeed, the CRM is highly committed to promoting new research directions, especially those aiming at collaborative work with agents from other scientific disciplines or from industry. With this purpose, and following the guidelines of its new contract programme with the Catalan Government, the CRM started to offer in 2007 long-term appointments to researchers working in specialties with a potential impact on society and not sufficiently well implemented in the neighbouring universities.

The CRM is located in the science faculty building of the Autonomous University of Barcelona, in the town of Bellaterra. Its current floor space is 1,225 m^2, which, besides lecture rooms and common rooms, allows up to 30 researchers to be hosted at any one time alongside the administration staff and the director. A 900 m^2 expansion is planned during 2009.

The website www.crm.cat contains information about past, current and future CRM activities, together with a detailed description of the centre, annual reports and guidelines for open calls.

Carles Casacuberta
Team of Directors

Mathematics for the people

Nuno Crato

The Portuguese mathematician Nuno Crato was awarded in March 2008 the second prize as Communicator of the Year at the European Science Awards, a European Commission Prize that is partly sponsored by the EMS. The first prize went to the physicist Jean-Pierre Luminet. There was no third prize.

Professor Nuno Crato's main research areas at the Technical University of Lisbon are in stochastic modelling and the statistical description of random events and their behaviour in time. He has made significant contributions in these fields in areas such as volatility persistence in financial markets and his applied stochastic models have found application in as diverse end-uses as computer science and fishery forecasting. He served as the President of the International Symposium on Forecasting in 2000 and has served as the President of the Portuguese Mathematical Society since 2004.

Besides his scientific and institutional work, Nuno Crato is a regular contributor of articles to newspapers, magazines, radio and TV programmes. Through these channels he brings a regular scientific perspective on a wide variety of current news and events to a vast audience. He is also the author of best-selling popular science books and takes a leading role in bringing mathematics to a wider audience. He particularly values this prize since "science popularization has been repeatedly recognized by the European Commission as an important means to bring science to the public stage; it's important that popularization of mathematics, 'the queen of sciences', is publicly recognized as well".

The European Mathematics Society's Raising Public Awareness Prize had already been awarded to him in 2003 for a three-part article that is typical of the "Crato approach". Primarily it is easy to read but it is also informative and scientifically sound. The article described mathematical cryptography in the context of credit cards and, specifically, the growth of internet shopping. It addressed a topic of major public interest and dealt with it with humour, intelligence and a distinct journalistic style. Crato's writings have appeared in many diverse publications from in-flight magazines to historical journals. In total he has written over 500 popular science pieces.

In addition he is the author of many books on scientific issues. These are, like his articles, always topical. For example, his book on solar eclipses was published just prior to the 1999 Solar Eclipse and his very successful co-authored book on science in the Da Vinci Code novel *A Espiral Dourada* (*The Golden Spiral*) coincided with the international release of the film of the book. He has also contributed regularly to radio and television.

As President of the Portuguese Mathematical Society, Crato has helped foster dozens of events for children and students. These include Mathematics Saturday Afternoons, competitions and a mathematics fair, which runs as a "drop-in" type event during the summer and includes many hands-on activities from trigonometry to probability in addition to music and other cultural activities.

Asked to comment briefly on this prize for the Newsletter of the EMS, Nuno Crato stated: "I gladly thank the EMS, the RPA committee, his past chair, Robin Wilson, and all members, namely José Francisco Rodrigues, for having enthusiastically proposed and supported my nomination. It seems to me this is, above all, a prize for the EMS. It's the EMS that advanced my nomination for the prize".

Jorge Buescu [jbuescu@gmail.com] is professor of mathematics in the faculty of science of the University of Lisbon. He is member of the editorial board of the EMS Newsletter, in charge of the Book Review section. He is also editor-in-chief of Gazeta de Matemática*, a publication of the Portuguese Mathematical Society.*

Groups, Geometry, and Dynamics

European Mathematical Society

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André Lichnerowicz prize in Poisson geometry

The André Lichnerowicz prize in Poisson geometry was established in 2008. It will be awarded for notable contributions to Poisson geometry, every two years at the "International Conference on Poisson Geometry in Mathematics and Physics", to researchers who completed their doctorates at most eight years before the year of the Conference.

H. Bursztyn and M. Crainic, at the Poisson 2008 Conference, Lausanne (Switzerland).

The prize was named in memory of André Lichnerowicz (1915–1998) whose work was fundamental in establishing Poisson geometry as a branch of mathematics. It is awarded by a jury composed of the members of the scientific committee of the Conference, who may invite members of the organizing committee to participate in their deliberation and vote. In 2008, the prize amount was 500 euros for each recipient and the funds have been provided by the host institution of the Conference, the Centre Interfacultaire Bernoulli at the École Polytechnique Fédérale de Lausanne.

The prize for the year 2008 was awarded to
Henrique Bursztyn and Marius Crainic
on July 7, 2008 at the EPFL in Lausanne.

Henrique Bursztyn holds a Ph.D. in mathematics which he completed in 2001 at the University of California at Berkeley under the direction of Alan Weinstein. After post-doctoral positions at the Mathematical Sciences Research Institute (MSRI) in Berkeley, the University of Toronto and the Fields Institute, he was appointed associate researcher in the Arminio Fraga chair at the Instituto Nacional de Matematica Pura e Aplicada (IMPA) in Rio de Janeiro in 2005. His numerous publications range from the theory of deformation quantization to Morita equivalence in the categories of Poisson manifolds and symplectic groupoids. His work in Dirac geometry not only advanced the subject, it also was the source of inspiration for many further developments.

Marius Crainic completed his Ph.D. in mathematics in 2000 at the University of Utrecht under the direction of Ieke Moerdijk. Since then he has held prestigious research fellowships at the University of California at Berkeley and at the University of Utrecht, where he is presently teaching. His work is an important contribution to the theory of Lie groupoids with applications to noncommutative geometry, to foliation theory, Lie algebroid cohomology, momentum map theories and questions of rigidity and stability in Poisson geometry. Together with Rui Loja Fernandes he solved the deep question of generalizing Sophus Lie's third theorem from the setting of Lie groups to that of Lie groupoids and he developed applications of this result to Poisson geometry.

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Laying foundations

Stanisław Janeczko

In education – which is one of the three attributes of modern Europe, together with research and innovation – training in fundamental understanding plays a crucial role. Among basic exact sciences, mathematics occupies a distinguished place. It requires special abilities from researchers and a multicomponent aesthetic and rational motivation to reach the highest intellectual creativity. Interest in mathematics is a very fragile phenomenon, while a mathematical achievement is a result of a subtle interplay between flashes of insight, intuition and perseverance. It always goes beyond the current limits of understanding and often substantially extends the existing research methods.

Mathematical questions, problems and conjectures are frequently taken from descriptive models of real world and science. After being transformed by the genius of a mathematician, they return to the real world through the natural process of feedback, resulting in an authentic progress of a knowledge-based society. This type of mathematical activity, involving the cooperation of the abstract and the concrete, is the area of applied mathematics. It should not be confused with making use of ready-made mathematical results in science, technology and culture, which is the domain of applications of mathematics. The latter is nowadays a huge field of powerful and successful methods of universal science. Also, it has become obvious that the progress of most sciences strictly depends on the progress of basic mathematical research.

Information technology, finance, new medical technology, high and nanotechnology, cryptographic code transmission, global navigation and control, macro-economics and the modelling of bio-collective phenomena are just a few of the large variety of areas showing the practical and unavoidable impact of pure mathematical ideas on the enormously rapid social changes of the last 20 years.

In spite of that, in short-sighted projects, because of the demand for fast innovative results, basic research in mathematics is frequently ignored and treated as useless. Therefore, it is indispensable to build a new form of interdisciplinary approach with mathematicians working as part of a team.

Investing in the power of pure mathematical intellect should be one of Europe's priorities. To create substantial mathematical results, we absolutely need to ensure 'incubator' conditions at all three stages of an individual mathematical career. First, at the stage of education, talents should be recognised and supported by successful researchers and individual, advanced study programs. The research-driven education should then pass smoothly to the stage of unconstrained creativity in an atmosphere of liberty, freedom of thought and originality. At this stage, one needs a highly research-oriented environment, as well as access to global intellectual resources and the support of the world scientific community. The third stage naturally adds the master's duties of fostering young talents, thus completing the cycle and bringing a generational advance.

In Poland, these goals are pursued by the Institute of Mathematics and Stefan Banach International Mathematical Centre in Warsaw, which came into being in the beautiful tradition of the Polish Mathematical School. The two institutions were founded as a unique flexible frame to produce innovative mathematical ideas and original theories, and to support mathematicians at all stages of their careers. Today, besides participating in intellectual globalization, the institute has a nationwide character and continues to be one of the leading scientific communities in Poland. It provides a supportive and enriching environment for researchers, with a central mathematical library, a publishing house, conference facilities and branches in most active academic communities. To encourage a collaborative exchange of ideas between the most creative mathematicians of the world, the Stefan Banach International Mathematical Centre hosts events gathering almost 2,000 researchers from nearly 50 countries each year. The institute also provides graduate and post-doctoral training opportunities. It develops strong partnerships with leading academic centres in Poland and is now building a platform, the Mathematical Centre for Science and Technology, to encourage the transfer of mathematical methods to modern technology. The current quest and challenge is to create new forms of work and strengthen academic excellence in pure mathematics.

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Stanisław Janeczko [janeczko@mini.pw.edu.pl] (b. 1953) is a professor of mathematics at the Institute of Mathematics Polish Academy of Sciences, the director of this institute and director of the Stefan Banach International Mathematical Centre. He is also a professor of analysis and singularity theory at the Warsaw University of Technology and from 1993 to 1999 he was a dean of the Faculty of Applied Physics and Mathematics in this university. He graduated from Warsaw University and obtained his PhD in 1981 studying under Krzysztof Maurin and Stanisław Łojasiewicz. His main mathematical interests include differential topology, symplectic geometry and singularity theory with applications to natural sciences.

Confessions of an industrial mathematician

Chris Budd (FIMA, CMATH, University of Bath)

Some general thoughts

What is *industrial mathematics*? Or indeed more generally what is *applied mathematics*? One view, commonly held amongst many 'pure' mathematicians is that applied maths is what you do if you can't do 'real' mathematics and that industrial mathematics is more (or indeed less) of the same only in this case you do it for money. My own view is very different from this (including the money aspect). Applied mathematics is essentially a two way process. It is the business of applying really good mathematics to problems that arise in the real world (or as close an approximation to the real world as you think it is possible to get). It constantly amazes me that this process works at all, and yet it does. Abstract ideas developed for their own sake turn out to have immensely important applications, which is one (but not the only) reason for strongly supporting abstract research. However, just as importantly, applied mathematics is the process of learning *new maths* from problems *motivated by applications*. Anyone who doubts this should ponder how many hugely important mathematical ideas have come from studying applications, varying from calculus and Fourier analysis to nonlinear dynamics and cryptography. It is certainly true that nature has a way of fighting back whenever you try to use maths to understand it, and the better the problem the more that it fights back at you! To solve even seemingly innocuous problems in the real world can often take (and lead to the creation) of very powerful mathematical ideas. Calculus is the perfect example of this. The beauty of the whole business is the way that these same ideas can take on a life of their own and find applications in fields very different from the one that stimulated them in the first place. This process of applying mathematics in as many ways as possible can change the world. A wonderful example of this is the discovery of radio waves by Maxwell. Here a piece of essentially pure mathematics led to a whole technology which has totally transformed the world in which we live. Imagine a world without TV, radio, mobile phones and the Internet. But that is where we would be without mathematics.

So, how does industrial maths fit into the above? It is still hard to define exactly what industrial maths means, but as far as I'm concerned it's the maths that I do when I work with organisations that are not universities. This certainly includes 'traditional' industry, but (splendidly) it also includes the Met Office, sport, air traffic control, the forensic service, zoos, hospitals, Air Sea rescue organisations, broadcasting companies and local education authorities. I will use the word *industry* to mean all of these and more. Even traditional industry contains a huge variety of different areas ranging from textiles to telecommunications, space to food and from power generation to financial products. What is exciting is that all of these organisations have interesting problems and that a huge number of these problems can be attacked by using a mathematical approach. Indeed, applying the basic principles I described above, not only can the same mathematics be used in many different industries (for example the mathematics of heat transfer is also highly relevant to the finance industry) but by tackling these problems we can learn lots of new maths in the process. It is certainly true that many industrial problems involve routine mathematics and, yes, money is often involved, but this is not the reason that I enjoy working with industry. Much more importantly, tackling industrial problems requires you to think *out of the box* and to take on challenges far removed from traditional topics taught in applied mathematics courses. The result is (hopefully) new mathematics. I think it is fair to say that significant areas of my own research have followed directly from tackling industrial problems. An example of this is my interest in discontinuous dynamics: the study of dynamical systems in which the (usual) smoothness assumption for these systems is removed. Discontinuous dynamics is an immensely rich area of study with many deep mathematical structures such as new types of bifurcation (grazing, border-collisions, corner-collisions), chaotic behaviour and novel routes to chaos such as period-adding. It also has many applications to problems as diverse as impact, friction, switching, rattling, earthquakes, the firing of neurons and the behaviour of crowds of people (see illustration). Studying both the theory and the applications has kept me, my PhD students and numerous collaborators and colleagues busy for many years. However, for me at least, and for many others, the way into discontinuous dynamics came through an industrial application. In my case it was trying to understand the rattling behaviour of loosely fitting boiler tubes. Trying to solve this problem by using the 'usual' theory of smooth dynamical systems quickly ran into trouble as it became apparent that the phenomena that were being observed were quite different from those predicted by the text books. Trying to address this problem immediately forced us to look at non-smooth dynamics (inspired, I should say, by some brilliant theoretical work by the industrialist who was interested in the problem and was delighted to find an academic they could talk to). However the way into this fascinating field could equally well have been a problem in power transmission in a car or the motion of buildings in an earthquake. The point is that an interesting industrial problem, far from just being an excuse to use cheap and dirty maths to make money, has in contrast led to some very exciting new mathematical ideas with many novel applications.

I must confess that I find this constant need to rise to a mathematical challenge to solve an industrial problem intensely stimulating and it continues to act as a driver for much of my research (although I don't neglect the 'pure' aspects of my research as both are needed to be able to do good mathematics). Presently I'll develop this a bit further through a couple of case studies.

How does industrial mathematics work?

Working at the interface of academia and industry is, and continues to be, a constant conflict of interest. Industry, quite rightly, has to concentrate on short term results, obtained against deadlines and may well want the second best answer tomorrow, rather than the best answer in a year (or indeed never). The picture I described in the previous section, of the beautiful development of a mathematical theory stimulated by industrial research, may cut little ice to the manager that needs an answer *tomorrow*. (Indeed, to be quite honest with you, whilst I now know vastly more about discontinuous dynamics than when I first started to look at the problem, I still cannot solve the original question of the boiler tube which turns out to be immensely difficult!) . In contrast, the average academic is under intense pressure to publish scholarly research in leading journals, to develop long term research projects and to work with PhD students who often require a lengthy period of training before they are up to (mathematical) speed. This seems, at first sight, to be the exact opposite of the requirements of industry. It can, in fact, be very hard to persuade some of our colleagues on grant review panels that it is worth investing in industrial maths at all; the argument being that if it were any good then industry would pay for it, and if it is not any good then it doesn't deserve a grant! Indeed it gets worse, whilst the life blood of academics is publishing results in the open literature; industry is often constrained (quite reasonably) by the need for strict confidentiality. At first there would appear to be no middle way in which both parties are satisfied. Fortunately however, there are a number of ways forward in which it is possible to satisfy both parties. Perhaps the best of these are the celebrated *Study Groups with Industry* founded in the 1960's by Alan Tayler CBE and John Ockendon FRS. The format of a typical study group is rather like a cross between a learned conference and a paintballing tournament. It lasts a week. On the first day around eight industrial problems are presented by industrialists themselves. Academics then work in teams for a week to try to solve the problems. On the last day the results of their work are presented to an audience of the industrialists and the academics on the other teams. Does this method always lead to a problem being cracked? Sometimes the answer is yes, but this usually means that the problem has limited scientific value. Much more value to both sides are problems that lead both to new ideas and new, and long lasting collaborations, taking both academics and industrialists in completely new directions. The discontinuous dynamics example above was in fact brought to just such a meeting in Edinburgh in 1988. Another vital aspect of the Study

A scramble crossing in Japan. The behaviour of the pedestrians in this crossing can be analysed by using the theory of discontinuous dynamical systems.

Groups is the training that it gives to PhD students (not to mention older academics) in the skills of mathematical modelling, working in teams and of developing effective computer code under severe time pressure. It is remarkable what can be done in the hot house atmosphere of the Study Groups. As a way of stimulating progress in industrial applied mathematics, the Study Groups have no equal. The model which started in Oxford has now been copied all over the world. I have personally been attending such Study Groups (again all round the world) since 1984 and have worked on a remarkable variety of problems including: microwave cooking, land mine detection, overheating fish tanks, fluorescent light tubes, fridges, aircraft fuel tanks, electric arcs, air traffic control, air sea rescue, weather forecasting, boiler tubes and image processing, to name just a few. Of course one week is not long enough to establish a viable collaboration with industry and it is worth considering some other effective ways of linking academia to industry. One of my favourite mechanisms is the use of MSc projects. For seven years at Bath we have been running an MSc programme in *Modern Applications of Mathematics* which has been designed to have very close links with industry. All of the students on this course do a short three month project which is often linked to industry, with both an academic and an industrial supervisor. The nice feature about this system is that everyone wins. First and foremost, the students have an interesting project to work on. Secondly we are able to work together with industry on a project with something more closely approximating an industrial timescale and with little real risk of anything seriously going wrong. Thirdly (and to my mind perhaps most importantly) the MSc project can easily lead on to a much more substantial project, such as a PhD project, with the student hitting the ground running at the start. Ideas for such MSc projects may well come from previous Study Groups, from the Industrial Advisory Board of the MSc or (in a recent development) from the splendid Knowledge Transfer Partnership (KTP) in Industrial Applied Mathematics coordinated by the Smith Institute. The KTP is partly funded by the DTI and part by EPSRC and exists to establish, and maintain, links between academia and industry. Similar organisations such as MITACS in Canada or MACSI in the Republic of Ireland, have closely related missions.

Where is industrial mathematics going (or leading us)?

I think it fair to say that some of the great driving forces of 20th Century mathematics have been physics, engineering and latterly biology. This has been a great stimulus for mathematical developments in partial differential equations, dynamical systems, operator theory, functional analysis, numerical analysis, fluid mechanics, solid mechanics, reaction diffusion systems, signal processing and inverse theory to name just a few areas. Many of the problems that have arisen in these fields, especially the 'traditional' industrial mathematics problems (and certainly anything involving fluids or solids) are *continuum problems* described by *deterministic differential equations*. Amongst the variety of techniques that have been used to solve such equations (that is to find out what the answer looks like as opposed to just proving existence and uniqueness) are simple analytical methods such as separation of variables, approximate and formal asymptotic approaches, phase plane analysis, numerical methods for ODEs and PDEs such as finite element or finite volume methods, the calculus of variations and transform methods such as the Fourier and Laplace transforms. (It is worth noting that formal asymptotic methods have long regarded as somewhat second rate methods to be used to find rough answers rather then wait till 'proper' maths did the job correctly. However, stimulated in part by the need to address very challenging applied problems, asymptotic methods have become an area of intense mathematical study, especially the area of *exponential asymptotics* which looks at problems in which classical asymptotic expansions have to be continued to all orders so that exponentially small – but still very significant – effects can be resolved.) One of the consequences of concentrating on continuous problems described by PDES is that applied mathematics has, for some considerable time, been almost synonymous with fluid or solid mechanics. Whilst these are great subjects of extreme importance, and are central to 'traditional' industries involving problems with heat and mass transfer, they only represent a fraction of the areas that mathematics can be applied to. Here I believe we may see industry driving the agenda of many of possible developments of 21st Century mathematics in a very positive and exciting way. At the risk of making a fool of myself and gazing into the future with too much abandon, I think that the key drivers of mathematics will be problems dealing with *information* (such as genetics, bio-informatics and, of course the growth of the Internet and related systems), problems involving *complexity* in some form (such as problems on many scales with many connecting components and with some form of network describing how the components interact with each other), and problems centred not so much in the traditional industries but in areas such as retail and commerce. To address such problems we must move away from 'traditional' applied mathematics and instead look at the mathematics of discrete systems, systems with huge complexity and systems which are very likely to have a large stochastic component. We will also have to deal with the very difficult issues of how to optimise such systems and to deal with the increasingly large computations that will have to be done on them. One reason that I believe this is that I have seen it happening before my eyes. I have had the privilege (or have been foolish enough) to organise three Study Groups. The first of these, in 1992, had every problem (with one exception) posed in terms of partial differential equations. In contrast the Study Group I organised in Bath in 2006 had ten problems. Of these precisely one involved partial differential equations, one other involved ordinary differential equations. All the rest were a mixture of (discrete) optimisation, complexity, network theory, discrete geometry, statistics and neural networks. I have seen similar trends in other Study Groups around the world. At a discussion forum at the Bath Study Group the general feeling was one of excitement (mixed with apprehension it must be said) that industry was prepared to bring such challenging problems to be looked at by mathematicians. Personally I greatly welcome this challenge. I also welcome the fact that much of the mathematics that needs to be used and developed to solve these sort of problems is mathematics that has often been thought of as very pure. Two obvious examples are number theory which comes into its own when we have to deal with discrete information (witness the major application of number theory to cryptography) and graph theory which lies at the heart of our understanding of networks. As someone that calls themselves a mathematician (rather than a pure mathematician or an applied mathematician) I strongly welcome these developments. Of course, the new directions that industry is taking applied mathematics pose interesting, and equally challenging, questions about how we should train the 'applied mathematicians' of the future. It is clear to me (at least) that any such training should certainly include discrete mathematics, large scale computing and methods for stochastic problems. How we do this is of course another matter.

Some case studies

I thought that it would now be appropriate to flesh out the rather general comments above, by looking at a couple of examples. The first is (mainly) an example of a continuum problem whilst the second has a more discrete flavour to it.

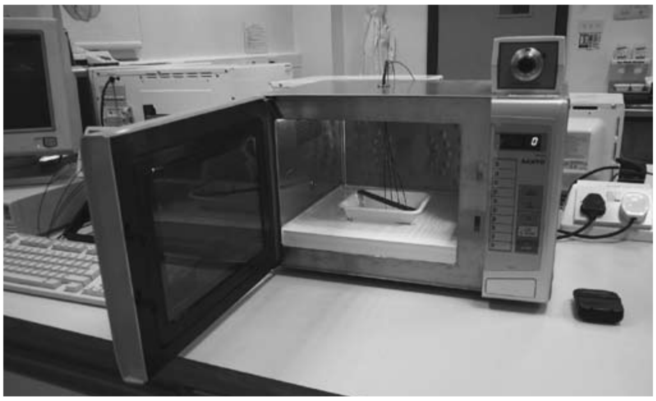
Case Study one: Maths can help you to eat

One of the nicest (well certainly it tastes nice) applications of mathematics (well in my opinion at least) arises in the food industry. The food industry takes food from farm to fork, after that it's up to you. Food has to be grown, stored, frozen, defrosted, boiled, transported (possibly when frozen), manufactured, packaged, sorted, marketed, sold to the customer, tested for freshness, cooked, heated, eaten, melted in the mouth (in the case of chocolate) and digested. Nearly all of these stages must be handled very carefully if the food is going to be safe, nutritious and cheap for the customer to eat. It is very easy to think of working on food as a rather trivial application of mathematics, but we must remember that not only is the food industry one of the biggest sources of income to the UK, but also that we all eat food, it affects all of our lives and mistakes in producing food can very quickly make a lot of people very ill. Trivial it is not. Food is also a source of some wonderful mathematical problems, and (a point I take very seriously) the application of maths to food is a splendid way of enthusing young people into the importance of maths in general (especially if you bring free samples along with you!) Some of this maths is very 'traditional' applied maths much of the issues in dealing with food involve classical problems of fluid flow (usually non-Newtonian), solid mechanics (both elastic and visco-elastic), heat transfer, two phase flow, population dynamics (such as fish populations) and free boundary problems. Chocolate manufacture for example, involves very delicate heat flow calculations when manufacturing such delicate items as soft centre chocolates However problems involving the packaging, marketing, distribution and sale of food lie more properly in the realms of optimisation and discrete mathematics. It is clear that the food industry will act as a source of excellent mathematical problems for a very long time to come.

There is lots of maths in chocolate, and it tastes good too!

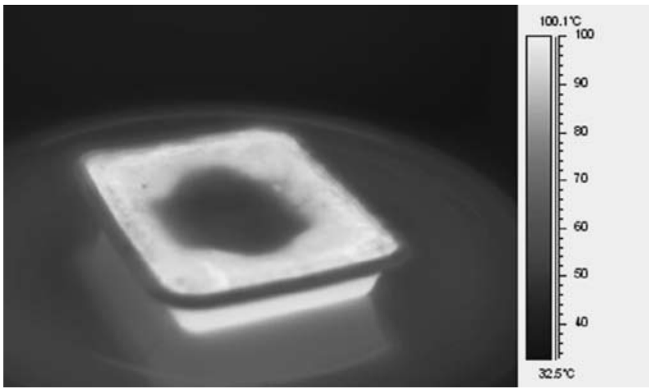
Two problems, in particular, that I have worked on concern the micro-wave cooking and the digestion of food. For the sake of the readers sensitivities I shall only describe the former in any detail, although it is worth saying that modelling digestion is a fascinating exercise in calculating the (chaotic) mixing of nutrients in a highly viscous Non-Newtonian flow driven by pressure gradients and peristaltic motion and with uncertain boundary conditions. As any student knows, a popular way of cooking (or at least of heating) food is to use a micro-wave cooker. In such a cooker, microwaves are generated by a Magnetron (also used in Radar sets) and enter the oven cavity via a waveguide or an antenna. An electric field is then set up inside the oven which irradiates any food placed there. The microwaves pen etrate the food and change the orientation of the dipoles in the moist part of the food leading to heating (via friction) of the foodstuff and consequent phase changes.

A problem with this process is that the field can have standing wave patterns, which can result in localised 'cold-spots' where the field is relatively weak. If the food is placed in a cold-spot then its temperature may be



A domestic mode-stirred microwave oven with four temperature probes used to test the predictions of the model.

lower there and it will be poorly cooked (see figure). To try to avoid this problem the food can either be rotated through the field on a turntable, or the field itself can be 'stirred' by using a rotating metal fan to break up the field patterns.



Thermal camera image of food in a domestic turntable oven, showing a distinct 'cold spot' in the centre caused by a local minimum in the radiant electro-magnetic field.

An interesting 'industrial mathematics' problem is to model the process by which the food is heated in the oven and to compare the effectiveness of the turntable and mode-stirred designs of the micro-wave oven in heating a moist foodstuff. This problem came to me through the KTN and a Study Group and was 'sub-contracted' to a PhD CASE student Andrew Hill. One way to approach it is to do a full three dimensional field simulation by solving Maxwell's equations, and to then use this to find the temperature by solving the porous medium equations for a two-phase material. The problems with this approach are (i) the computations take a very long time, making it difficult to see the effects of varying the parameters in the problem (ii) it gives little direct insight into the process and the way that it depends upon the parameters and (iii) micro-wave cooking (especially the field distribution) is very sensitive to small changes in the geometry of the cavity, the shape and type of the food and even the humidity of the air. This means that any one calculation may not necessarily give an accurate representation of the electric field of any particular micro-wave oven on any particular day. What is more useful is a representative calculation of the average behaviour of a broad class of (domestic) micro-wave ovens in which the effects of varying the various parameters is more transparent. Here a combination of both an analytical and a numerical calculation proved effective. In this calculation we used a formal asymptotic theory both to calculate an averaged field and to determine how it penetrated inside a moist foodstuff. (Note, contrary to popular myth, microwaves do not cook food from the inside. Instead they penetrate from the outside, and if the food is too large then the interior can receive almost no micro-wave energy, and as a result little direct heating. This is why the manufacturers of micro-wave cookable foods generally insist that, after a period of heating in the oven, the food is stirred to ensure that it is all at a similar temperature.) The temperature T of the food satisfies the equation $u_t = k\Delta^2 T + P(x,y,z,t)$ where u is the enthalpy and P is the power transferred from the microwave field to the food. As remarked above, finding P exactly was very hard, however a good approximation could be found asymptotically (in particular by using the WKBJ method) for ovens with either a mode-stirrer or a turntable. This approximation showed that the overall the field decayed exponentially as it penetrated the food but that on top of this decay was superimposed an oscillatory contribution (due to reflexions of the radiation within the food) the size of which depended upon the dimensions of the food. A relatively simple calculation showed that these oscillations were small provided that the smallest dimension of the food was larger than about 2 cm. Fortunately, most foodstuffs satisfy this condition. As a result it was possible to use a much simpler description of the electric field in the enthalpy equation than that given by a full solution of Maxwell's equations, and numerical approximations to the solution of these simplified equations were found very quickly on a desktop PC. When compared against experimental values of both the temperature and the moisture content these solutions were surprisingly accurate, given the approximations that are made, and gave confidence in the use of the model for further design calculations. It was the combination of mathematics, numerical methods, physical modelling and the careful use of experimental data that made this whole approach successful and is typical of the mix of ideas that have to be combined to do effective industrial mathematics.

Case Study Two: Maths Can Save Your Life

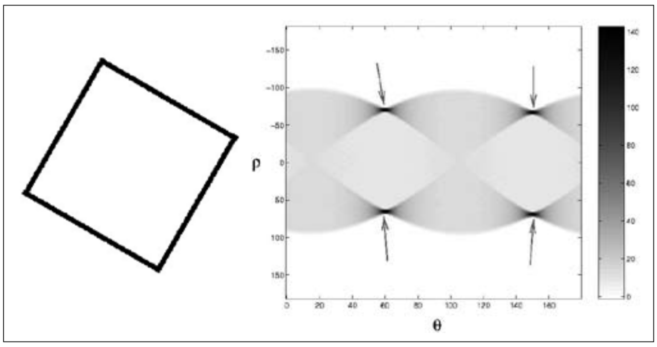
One of my favourite 'industrial mathematics' problems came up in a recent study group, and is an example of an application of signal processing and information theory which can potentially save peoples lives. One of the nastiest aspects of the modern world is the existence of anti-personnel land mines. These unpleasant devices, when detonated, jump up into the air and kill anyone close by. They are typically triggered by trip-wires which are attached to the detonators. If someone catches their foot on a trip wire then the mine is detonated and the person dies. To make things a lot worse, the land mines are typically hidden in dense foliage and thin nylon fishing line is used to make the trip wires almost invisible. One way to detect the land mines is to look for the trip wires them selves. However, the foliage either hides the trip-wires, or leaf stems can even resemble a trip wire. Any detection algorithm must work quickly, detect trip-wires when they exist and not get confused by finding leaves. An example of the problem that such an algorithm has to face is given in the figure below in which some trip-wires are hidden in an artificial jungle.

Three trip-wires are hidden in this image. Can you find them?

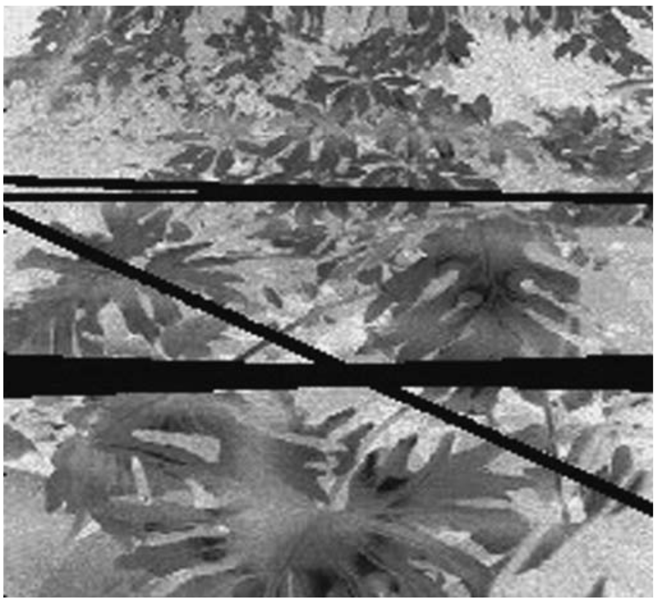
In order to detect the trip wires we must find a way of finding partly obscured straight lines in an image. Fortunately, just such a method exists; it is the Radon Transform (or its various discrete versions). In this transform, the line integral $R(\theta,\rho)$ of an image with intensity $u(x,y)$ is computed along a line at an angle θ and a distance ρ from the centre of the image so that

$$R(\theta,\rho)=\int u\,(\rho\cos(\theta)-s\sin(\theta),\ \rho\sin(\theta)+s\cos(\theta))\mathrm{d}s.$$

This transformation lies at the heart of the CAT scanners used in medical image processing and other applications as it is closely related to the formula for the attenuation of an X-Ray (which is long and straight, like a trip-wire) as it passes through a medium of variable density (such as a human body). Indeed, finding (quickly) the inverse to the Radon Transform of a (potentially noisy) image is one of the key problems of modern image processing. It has countless applications, from detecting tumours in the brain of a patient (and hence saving your life that way), to finding out what (or who) killed King Tutankhamen. For the problem of finding the trip-wires we don't need to find the inverse, instead we can apply the Radon transform directly to the image. In the two figures below we see on the left a square and on the right its Radon Transform.



The key point to note in these two images is that the four straight lines making up the sides of the square show up as points of high intensity (arrowed) in the Radon Transform and we can easily read off their orientations. Basically the Radon Transform is good at finding straight lines which is just what we need to detect the trip-wires. Of course life isn't quite as simple as this for real images of trip-wires and some extra work has to be done to detect them. In order to apply the Radon transform the image must first be pre-processed (using a Laplacian filter and an edge detector) to enhance any edges. Following the application of the transform to the enhanced image a threshold must then applied to the resulting values to distinguish between true straight lines caused by trip wires (corresponding to large values of R) and false lines caused by short leaf stems (for which R is not quite as large). However, following a sequence of calibration calculations and analytical estimates with a number of different images, it was possible to derive a fast algorithm which detected the trip-wires by first filtering the image, then applying the Radon Transform, then applying a threshold and then applying the inverse Radon Transform. (The beauty of this is that most of these algorithms are present in the MATLAB Signal Processing Toolbox. Indeed, I consider MATLAB to be one of the greatest tools available to the industrial mathematician.) The result of applying this method to the previous image is given below in which the three detected trip-wires are highlighted.



Note how the method has not only detected the trip-wires, but, from the width of the lines, an indication is given of the reliability of the calculation. All in all this problem is a very nice combination of analysis and computation.

Signal processing problems of this form are not typically taught in a typical applied mathematics undergraduate course. This is not only a shame, but denies the students on those courses the opportunity to see a major application of mathematics to modern technology.

Conclusions

I hope that I have managed to convey some of the flavour of industrial mathematics as I see it. Far from being a subject of limited academic value, only done for money, industrial maths presents a vibrant intellectual challenge with limitless opportunities for growth and development. This poses significant challenges for the future, not least in the way that we train the next generation of students to prepare them for the very exciting ways that maths will be applied in the future, and the new maths that we will learn from these applications.

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This article originally appeared in Maths Today*, the Journal of the UK Institute of Mathematics and its Applications.*

Is Reuben Hersh 'Out there'?

Martin Gardner

Brian Davies, in his paper "Let Platonism Die[1]" defines mathematical Platonism as the belief that mathematical entities exist 'in the mathematical realm outside the confines of space and time.' This is not what I or, I think, most mathematical Platonists believe. Aristotle, a mathematical realist, grabbed Plato's universals (redness, cowness, two-ness and so on) from a transcendental realm, and attached them to objects in space and time. The geometrical shape of a vase, for example, is 'out there', on the vase, not something floating outside Plato's cave.

Consider pebbles. On the assumption that every pebble is a model of the number 1, obviously all the theorems of arithmetic can be proved by manipulating pebbles. You even in principle can prove that any integer, no matter how large, is either a prime or composite.

Reuben Hersh, my old adversary, in a paper 'On Platonism[2]' says this:

My view of Platonism – always referring to the common, every-day Platonism of the typical working mathematician – is that it expresses a correct recognition that there are mathematical facts and entities, that these are not subject to the will or whim of the individual mathematician but are forced on him as objective facts and entities which he must learn about and whose independent existence and qualities he seeks to recognize and discover.

Welcome, Professor Hersh, to the Plato club! All Platonists would agree completely with your remark. But then Hersh goes on to make an incredible statement '*The fallacy of Platonism is the misinterpretation of this objective reality, putting it outside of human culture and consciousness'*.

The theorem and objects of mathematics, Hersh continues, like 'many other cultural realities' are external, objective *from the viewpoint of any individual* [italics his], but internal, historical, socially conditioned *from the viewpoint of the society or that culture as a whole* [italics still his]'.

So Hersh is not a Platonist after all! Does he really think that manipulating pebbles to prove, say, that 17 ia a prime is not a process going on out there, unconditioned by a given culture? Of course manipulating pebbles is culturally conditioned in the trivial sense that everything humans do are so conditioned. But that is not the deeper question. The primality of 17, in an obvious way, is out there in the behaviour of pebbles in much the same way that the elliptical orbit of Mars is out there, or the spiral of our galaxy.

Hersh is so addicted to squeezing math inside the folkways that in his book *What is mathematics, Really?* (Oxford University Press, 1997) he writes, so help me, that 8 plus 5 is not necessarily 13 because some skyscrapers have no floor 13. So if you go up 8 floors in an elevator, then go 5 more floors, you find yourself on floor 14. Is Hersh suggesting that in the subculture of some skyscrapers the laws of arithmetic are constantly violated?

Need I point out that since geometry was arithmetized by Descartes, it too can, in principle, be modeled with pebbles. Indeed, the universe is saturated with models of nearly all of mathematics. Even a topologist can prove that bisecting a Klein bottle produces two Moebius bands of opposite handedness by making a crude model out of an envelope, then cutting it in half.[3]

Complex numbers and derivatives may not have material models, but they also are embedded throughout the universe. Newton and Leibniz in an obvious sense invented calculus, but in another sense they discovered fundamental ways the universe behaves. The Mandelbrot set is not outside space and time. It exists on computer screens. Does an anti-realist believe that a mathematician exploring properties of the Mandelbrot set is really exploring a structure inside his brain because his eyes and brain are seeing the screen, or that he is exploring part of his culture because his culture built the computer?

Such statements are the same kind of language distortion as claiming that astronomers are not studying patterns, 'out there' because telescopes are part of culture, not to mention all of astronomy as well. It is not far from insisting that the universe exists because human cultures observe it, rather than we exist because the universe fabricated us.

Cantors' alephs may not be out there, but who knows? They could be hiding somewhere in the cosmos. Like physicists, mathematicians often discover things by investigating material models. Frank Morley, to consider a classic instance, discovered 'Morleys theorem' by investigating the angles of a paper models of arbitrary triangles – models as much out there as stones and stars. In no way can it be said Morley invented his theorem, or found it inside his skull or as part of his culture.

In his paper Hersh correctly calls me a theist. He adds that I believe in the efficacy of prayer. Hersh, an atheist, considers this an insult. Well, it all depends on what 'efficacy' means. I don't believe that if someone prays for a football team to win a game or that a loved one will have a remission of cancer, that God will insert a hand into the universe and change it accordingly. I suppose it is possible for God to alter probabilities on the quantum level, now a popular conjecture by theists, but I'm inclined to doubt it.

I do think prayers for forgiveness are justified, or prayers for wisdom to make right decisions. Gilbert Chesterton somewhere says that it is a sad day for atheists when something wonderful happens to them and they have nobody to thank.

Hersh also writes that I once accused him of Stalinism. I can't imagine I could do such a thing. If I did, I

1 EMS Newsletter, June 2007
2 EMS Newsletter, June 2008
3 The dubious reader is referred to chapter two in my Sixth Book of Mathematical Games from Scientific American (W.H.Freeman, 1971) where instructions are given on how to construct a Klein bottle out of an envelope.

apologize. Perhaps I once reminded him of that harrowing scene in Orwell's *1984* when an official manages to torture a prisoner into believing that when two fingers are added to two fingers a fifth finger appears.

Hersh also claims that I once accused him of being a solipsist. Again, I'm at a loss to know what he has in mind. It is possible I described his anti-realism as a vague kind of social or collective solipsism. Hersh is a great admirer of a paper by anthropologist Leslie White titled 'The Locus of Mathematical Reality'. The locus, White argues, is not in the outside world but in human cultures. Mathematical theorems are similar to such things as traffic regulations, fashions in clothes, art, music, and so on.

This of course isn't solipsism in the usual sense. No one outside a mental hospital is a true solipsist. But the anti-realism of White and Hersh is tinged with social solipsism in the sense that if human culture were to disappear, all of mathematics would also vanish. True, the universe would persist, but there would be nobody around to do mathematics unless there are mathematicians on other planets. Presumably Hersh would agree that what we call mathematical structures and events would still exist, but if there were no sentient beings to study them there would be nothing in the universe deserving the name of mathematics.

Here once more the question arises of what is the best, least confusing language to use. I think it best to say that if all sentient creatures vanished, 2 plus 2 would still be 4, the circumference of the moon's disk divided by its diameter would still be close to π and Euclidean triangles would still have interior angles that added to a straight angle. I assume that Hersh would prefer to say that none of these assertions would still hold because there would be no cultures around in which such statements could be made. To think otherwise would, God forbid, turn Hersh into a Platonist.

Like Paul Dirac, and thousands of other eminent mathematicians, I believe there is a God who is a superb mathematician with a knowledge of math enormously superior to ours. Whether infinite or not, how could I know? God surely does not know the last decimal digit of π because there is no last digit. Even if I were an atheist I would still consider it monstrous hubris to suppose that mathematics has no reality apart from the little minds of intelligent monkeys.

Martin Gardner should need no introduction. He authored the legendary 'Mathematical Games' column in the Scientific American between 1956 and 1981 (columns which are being edited and reissued by Cambridge University Press). He started out in philosophy a graduate of the University of Chicago, has been a very active debunker of pseudo-science and also been active as a magician, an excellent qualification for the former. He has published over fifty books during his long life, and has in recent years retired to his native Oklahoma.

Photo of Martin Gardner taken by Gilbert Jain. Courtesy of AMS.

Some Recent Articles about Platonism

E.B. Davies

This article is one of a series on Platonism in the EMS Newsletter, the first being my own article 'Let Platonism Die'[1]. There are many things that I would like to comment on in the earlier articles, but space forces me to make choices.

The first paragraph of David Mumford's article 'Why I am a Platonist'[2] describes what he means by Platonism. As I wrote in my original article, there are many flavours of Platonism[3], including that of Roger Penrose, who has stated that 'the natural numbers were there before there were human beings, or indeed any other creature here on earth'; by 'there' he meant in the Platonic world of mathematical forms[4]. He also believes that our perception of the Platonic realm is mediated in some way by microtubules in the neurons. Mumford objects to my notion of Platonism as involving the existence of mathematical objects in a Platonic realm outside space and time. My notion, which I will call Platonism$_{D}$, is close to that described by Thom, Gödel, Penrose and Davis-Hersh[5]. Mumford states that Platonism$_{D}$ makes an ontological assumption that is not directly present in his version, Platonism$_{M}$, namely

[Platonism$_{D}$ is] the belief that there is a body of *mathematical objects, relations and facts about them* that is independent of and unaffected by human endeavors to discover them.

I do not generally like detailed discussions of people's phraseology, but it seems unavoidable on this occasion. I agree that Platonism$_{M}$ does not explicitly refer to mathematical objects as existing outside space and time, but in Mumford's second paragraph he wrote that 'mathematics seems to me to be universal and unchanging, [its impact on people being] invariant across time and space[6]. The only significant difference between the published version and my 'mathematics exists outside space and time' is the replacement of 'outside' by 'invariant across'. I used the admittedly inadequate word 'outside' to refer to the belief of many Platonists that mathematical entities would

[1] EMS Newsletter 64 (2007) 24–25.
[2] EMS Newsletter 70 (2008) 27–30.
[3] See M Balaguer, 'Platonism and Anti-Platonism in Mathematics', Oxford Univ. Press, 1998.
[4] R Penrose, 'Shadows of the Mind', Oxford Univ. Press, 1994, pp. 412, 423.
[5] See Chap. 7 of PJ Davis and R Hersh, 'The Mathematical Experience', Birkhäuser, 2003.
[6] Mumford added the phrase in brackets after private correspondence with the author.

still exist and theorems still be true even if the universe had never been created. It appears that Mumford dislikes the notion of a Platonic realm, which for me is just the name of the class of all mathematical entities, *provided* they are assumed to exist independently of human society. Plato, of course, meant more than this, because he believed that people were born with faint memories of an earlier existence in the Platonic realm.

Another possibility is that Mumford's statement is analogous to 'the speed of light is universal and unchanging, invariant across time and space'. If so he may be taking an Aristotelian position rather than a Platonic one. Aristotle argued that mathematical entities did not exist in their own right, but arose by the process of idealizing certain features of the physical world, which was the ultimate reality.

Existence, truth and reality are linked concepts. Paul Cohen contrasted two views of set theory – that it refers to an objective reality or that it is a purely formal extrapolation of our intuitions with finite objects – and rejected the Platonist view:

There is almost a continuum of beliefs about the extended world of set theory… Through the years I have sided more firmly with the formalist position. This view is tempered with a sense of reverence for all mathematics which has used set theory as a basis, and in no way do I attack the work which has been done in set theory. However, when axiom systems involving large cardinals or determinacy are used, I feel a loss of reality, even though the research is ingenious and coherent.

Much of the passion relating to Platonism in mathematics arose in connection with Gödel's theorems and the status of set theory, in which Cohen was a world leader. A key issue is whether there is an important logical distinction between the truth of a mathematical statement and the existence of a proof of it; believing in the truth of an unprovable mathematical statement forces one towards Platonism and the belief that mathematical statements are about something independent of human existence. One of several anti-Platonistic responses is that saying that a theorem is true *means* that it has a proof, and having a proof means that it has been written down somewhere or that experts agree that it could be written down with little effort; the number of true theorems in this sense is increasing as time passes.

My original article was short, possibly too short, and was written at the request of Ari Laptev to stimulate debate; it was clearly successful in that respect. There are many philosophical positions that one can take towards mathematics beyond a simplistic opposition between Platonism and materialism – Platonism is not the same as taking abstraction seriously. My own position is *not* materialistic, but a blend of ideas due to Aristotle, Kant, Poincaré and Popper. More about this below. I support much of Reuben Hersh's article 'On Platonism[7], but need to clarify one misunderstanding. I do not think that very ill-defined forms of Platonism can be disproved by references to research on brain processes. Sufficiently vague and/or theistic world-views can never be disproved – they can only be rendered steadily less plausible by presenting a wide variety of contrary evidence, combined with a credible alternative. Unfortunately beliefs with little justification do not always wither over time; even young earth creationism, that most implausible of beliefs if one thinks that scientific knowledge has any substance at all, is active and thriving. On the other hand if a version of Platonism is strong enough to have some real consequences, it faces the prospect of being tested against other knowledge that we have about the world. Most experts would say that Penrose's views about the role that microtubules play in the brain are unfounded, but his suggestions had some interest because they might have led research in a new and productive direction.

The article 'Why I am a (moderate) social constructivist' of Philip Davis provides an account of the history of negative numbers[8]. This is highly relevant, because it shows how different it was from, let us say, the discovery of America or the moons of Jupiter, whose existence was accepted relatively quickly. There is little in this history to support the contention that mathematicians over several hundred years had a direct perception of the Platonic realm, whether this is interpreted metaphorically or literally. After the event one can of course forget the history and claim that the concept of a negative number is universal and eternal, even self-evident, but it was not always like that. Mumford comments briefly on this issue but admits that his interpretation follows from his commitment to Platonism. Davis mentions some of the endless variety of number systems, and I would further emphasize that finitists, who reject some aspect of the 'received' system of natural numbers, are doing perfectly sound mathematics.

Platonism is relatively harmless, but no form does anything to explain *why* the orbits of the planets correspond so closely to the solutions of Newton's law of gravitation. Saying that the equations control the motion is vacuous unless one can at least begin to explain how this might happen, and I do not know of any significant attempt to do this. My own approach is to admit that we do not know why the world exhibits so much regularity, but to regard this as a problem about the world rather than about mathematics. Mathematics is simply our way of describing the regularity. As Michael Atiyah wrote on the occasion of being awarded the Abel Prize:

Mathematics is an evolution from the human brain, which is responding to outside influences, creating the machinery with which it then attacks the outside world. It is our way of trying to reduce complexity into simplicity, beauty and elegance. It is really very fundamental, simplicity is in the nature of scientific inquiry – we do not look for complicated things. I tend to think that science and mathematics are ways the human mind looks and experiences – you cannot divorce the human mind from it. Mathematics is part of the human mind[9].

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7 EMS Newsletter 68 (2008) 17–19.
8 EMS Newsletter, 70 (2008) 30–31.
9 See http://www.abelprisen.no/en/prisvinnere/2004/

Mumford's statement 'Brian Davies argues that we should study fMRI's of our brains when [we] think about 5, about Gregory's formula or about Archimedes' proof and that these scans will provide a scientific test of Platonism' does not represent my views. I cannot imagine *any* test of the weak version of Platonism that he is interested in. The brain is amazingly complex and it is very unlikely that fMRIs will provide any direct insights into our thoughts about Gregory's formula or Archimedes' proof. However, *several different types* of investigation about what is going on when we think about the number 5 show that simple arithmetic depends on particular circuits in the brain, which are different from those involved in processing language. In my first article I made reference to experimental work that Brian Butterworth and many others are doing to try to build up an understanding of what is happening in our brains when we do arithmetic; this is a tiny part of mathematics, but science proceeds step by step, and what is being revealed is so different from what one might intuitively have expected that it might well influence our thinking about the philosophical issues involved.

Mumford comments that the cortex of the human brain is astonishingly uniform and suggests that our mathematical ability is the result of the huge expansion of memory in homo sapiens. This is what one would conclude if the anatomy of the cortex were the only input to the discussion, but fMRI, experiments with babies and the very specific effects of brain traumas such as strokes show that the picture is far more complex than this. Everyone in the field accepts that conscious thought relies on many specific subcortical structures, each of which processes a particular type of information, and that mathematics is not a function of general intelligence. Experimental studies show that dyscalculia is specific and neurological in nature. Some people of otherwise normal intelligence simply cannot process arithmetic data in the way that the rest of us can; they find it almost impossible to say whether 6 or 9 is larger, but some can devise strategies to minimize the effects of their disabilities. There are also people who can count extremely well although they do not understand addition or subtraction.

In 'Mathematical Platonism and its opposites[10], Barry Mazur argues that there is no connection between how our brains allow us to do mathematics and what mathematics itself is about; Mumford expresses a similar viewpoint when he refers to 'two orthogonal sorts of existence'. I am in full agreement that mathematics exists at an abstract level, but not that this implies Platonism. I have already described the Aristotelian alternative. Kant argued that mathematics provides synthetic a priori knowledge, which in modern times became the statement that human beings have innate abilities, including a disposition to interpret the world in terms of cause and effect, and that we can only interpret the world by means of these. This insight is not invalidated by his belief that Euclidean geometry was a part of this knowledge; our visual system is certainly innate, and and it is active by the time we are born, even if it develops further after that[11]. Popper, an admirer of Kant, assigned mathematics to his World 3, which he defined as 'the world of products of the human brain'; these are abstract and time-dependent, but completely real because they can influence how people and hence other physical objects behave. All of these are serious positions, discussed in my forthcoming book 'Why Beliefs Matter'. The objective and constant truth of theorems after their discovery does not force one to be a Platonist. On the other hand, the statement that a theorem was already true before the human species came into existence has no content, or none that anyone has attempted to explain. Thus, when we assert that diplodocus had four legs rather than six, we necessarily do so using our present concepts, just as when we refer to non-numerical facts about the distant past.

For Mazur the issue behind Platonism is one of invention versus discovery. One problem with this is that many things can be viewed both ways. For example, were stone axes invented or was it accidentally discovered that a stone that was hit a certain way acquired a sharp edge that was useful? Does the history matter or should we be discussing what axehood itself is about? Many invented entities turn out to have unexpected properties, which are later discovered. For example, chess was certainly invented, but it was later discovered that one cannot mate one's opponent if one has a knight and king against his king. More relevantly, perhaps, was language invented or discovered? I would suggest neither: research shows that it is an innate capacity of the human mind that is not possessed by any other species. Perhaps some aspects of mathematics are similar.

My article asked people to acknowledge that mathematical Platonism is the last remnant of an ancient religion. Mazur goes even further by calling it a 'fully-fledged theistic position' and stating that

> the only way one can hope to persuade others of its truth is by abandoning the arsenal of rationality, and relying on the resources of the prophets.

Nowhere in his article does he reveal whether or not he is an adherent to the belief. Indeed he seems to have given a description of Platonism and anti-Platonism in which no sensible choice between the two possibilities could possibly be made. Something must be wrong with his approach! I do not agree that Platonism is 'fully-fledged' because I consider that as a theistic position it has little to offer.

Mumford seems to like Mazur's approach to Platonism, but does not mention the theistic side of Mazur's article. Instead he argues that there are universal truths in fields other than mathematics and cites Jefferson's phrase 'all men are created equal'. I am fully committed to the principle of equal rights, think that the world would be a better place if everyone acted on it, and try, imperfectly, to live my life according to it. However, declaring it to be

[10] EMS Newsletter 68 (2008) 19–21

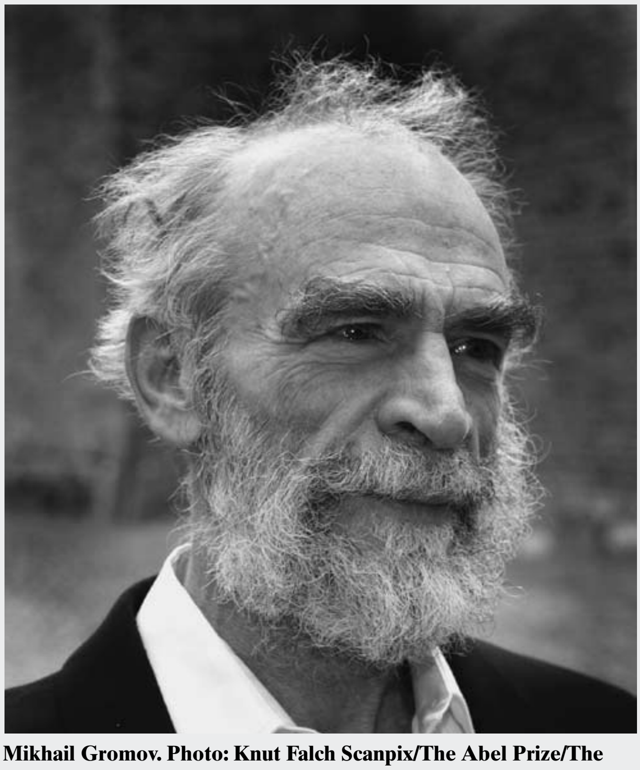
[11] I am adopting the 'evolutionary Kantian position', which owes a lot to Poincaré, writing at the start of the twentieth century.

an ethical universal is wishful thinking. Many different religions and sects deny that women can ever have the right to officiate in religious ceremonies, and many societies assign them a lower status in a wide r ange of not very subtle ways. Racial discrimination is commonplace and it may even be innate; I hope not, but my hopes are not relevant. Many cultures around the world, involving billions of people, do not accept Jefferson's phrase, which is indeed contingent on a particular cultural outlook. I hope that this outlook will eventually triumph, but there is no certainty about it.

Plato would have approved of Mumford's belief in universals of many types, but it is a belief imposed on facts rather than deduced from them. The steady progress of science has depended on people being willing to question beliefs whose main merit is that they match our intuitive feelings. Why should our attitude towards the nature of mathematics be different?

Brian Davies [E.Brian.Davies@kcl.ac.uk] is a professor at King's College London who has specialized in spectral theory; his research over the last ten years has focused particularly on non-self-adjoint operators. He is currently President of the London Mathematical Society and is the author of "Science in the Looking Glass", published by CUP, as well as of several research papers in the philosophy of mathematics.

Mikhail Gromov received the Abel Prize 2009



Mikhail Gromov. Photo: Knut Falch Scanpix/The Abel Prize/The Norwegian Academy of Science and Letters

The Russian-French mathematician Mikhail Leonidovich Gromov (Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France) received the 2009 Abel Prize from His Majesty King Harald at an award ceremony in Oslo, 19 May. The Abel Prize that carries a cash award of NOK 6,000,000 (close to EUR 700,000, US$ 950,000), is awarded by the Norwegian Academy of Science and Letters.

The 2009 Abel Prize is awarded to Mikhail Gromov for his revolutionary contributions to geometry. Kristian Seip, the chairman of the Abel Committee, elaborated on this in his speech at the award ceremony: "Mikhail Gromov is a remarkably creative mathematician. He is always in pursuit of new questions and is constantly thinking of new ideas for solutions of long-standing problems. The work of Gromov will for a long time to come continue to be a source of inspiration for many important mathematical discoveries."

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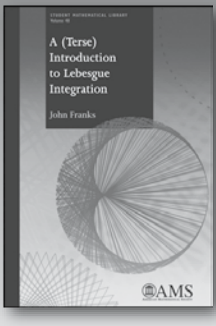

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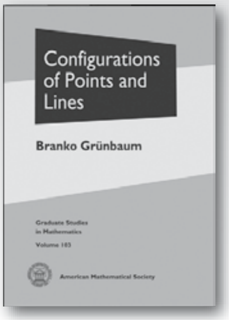

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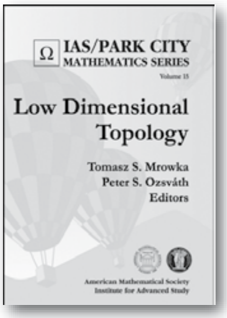

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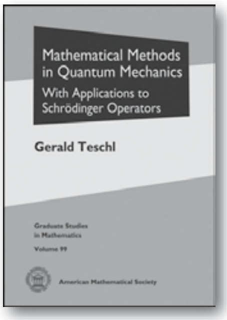

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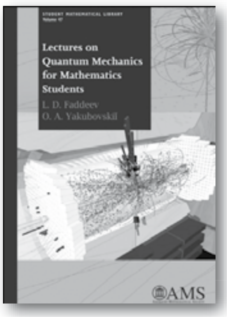

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Berlin in the 19th century

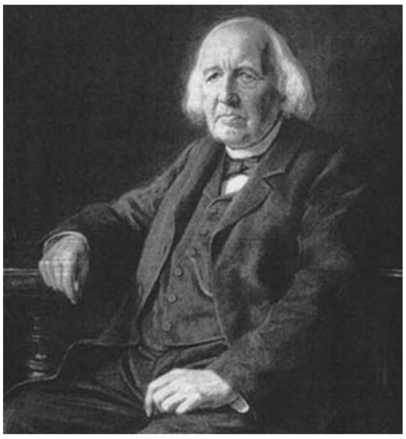
Jeremy Gray

Abstract

The triumvirate of Kronecker, Kummer, and Weierstrass dominated mathematics at Berlin in the second half of the 19th Century and made it for a time the leading place for mathematics in the world. Weierstrass in particular built up the new subject of analytic function theory into one the great success stories of the period. Yet the result of all their efforts was that Berlin was overtaken and Weierstrass' ideas as often marginalised as accepted. This article analyses how this disappointing outcome came about.

1. Weierstrass arrives in Berlin

In 1854 Karl Weierstrass, then a 39-year-old Gymnasium teacher in East Prussia, published a paper that caused such a stir that within two years he was brought to Berlin and was found a position in the new Industrial Institute (the Gewerbe Institut) while further steps were then taken to move him to the University of Berlin, where by late 1856 he was installed as a Professor Extraordinarius.



Karl Weierstrass (1815–1897)

The topic of his remarkably successful paper (W 1854) and the follow-up he published two years later (W 1856) was hyper-elliptic functions. It was an attempt to obtain functions from integrals of the form $\int dz/w$, where w^2 is a polynomial in x with no repeated factors. Weierstrass' method was analogous to the successful way Abel and Jacobi had shown how to obtain elliptic functions from elliptic integrals. But Jacobi had strongly suggested that the way forward with hyper-elliptic integrals was to consider them a number at a time as defining that number of functions in that number of variables, where this number was determined by the polynomial in z. Progress had been very slow and Weierstrass' production of meaningful power series for these functions was a great advance.

The university he joined had been founded in 1810 by Wilhelm von Humboldt and built up as part of the Prussian response to the shock of being walked over by the Napoleonic armies on their way to defeat at the gates of Moscow. After joining with the British to defeat Napoleon on the battlefield of Waterloo the Prussians set about modernising their state. When it came to the role of mathematics in higher education their view was that the old, narrow focus on applications, coupled with some fairly tedious literal-minded pure mathematics, had been part of the reason for the weakness of the Prussian army. Instead they adopted a view that subjects should be studied for their own sake, and applications would emerge naturally from the richness that would emerge. When Weierstrass' name was mentioned in academic circles in Berlin, opinion was largely favourable. Only Dirichlet held out in early 1855 for more and better arguments from the new man – thus starting a phenomenon of the modern university world in which everyone is impressed by the new brilliant figure except for the most demanding of the older generation. It helped everyone who supported Weierstrass that in 1855 Gauss died, at the age of 77. This meant that a distinguished chair was vacant at the older Prussian university of Göttingen and it was filled in 1856 by Dirichlet, who moved from Berlin. His position there was now occupied by Kummer (on Dirichlet's recommendation).

Kummer was best known as a number theorist, one who had responded to Gauss' call that important results and inter-connections were to be found in number theory and complex function theory (Dirichlet was another, as were Eisenstein and Jacobi). But he also worked on differential equations, notably the hypergeometric equation, and increasingly on the study of configurations of lines and their connection to optics, from which came his discovery in the 1860s of the Kummer quartic surface with its 16 nodal points.

Kummer's best student was Leopold Kronecker, who was another person greatly drawn to the study of number theory and complex function theory. But Kronecker had given up mathematics for a while to run the family banking business, only to return to publish a series of deep papers that earned him election to the Prussian Academy of Sciences in 1860. This allowed him to lecture at the University of Berlin and not surprisingly he chose to lecture to small groups of students on advanced topics.

2. Weierstrass' lecture courses

Weierstrass, on the other hand, saw his task as building up a discipline and training those who would then spread the word to other universities in Prussia and beyond. But it took him some years to work out what this would involve. His inaugural lecture at the Berlin Academy of Sciences (to which he was also elected in 1857) was on the importance of elliptic functions and applications of mathematics and his first lecture course at the University of Berlin (WS 1856/57) was likewise on topics in mathematical physics. The summer semester course of 1857 was on analytic functions and convergent series: Lazarus Fuchs and Leo Koenigsberger were among the six students in the audience and it made a powerful impression on them both for its content and Weierstrass' simple, unvarnished means of expression. But Weierstrass still lectured at the Industrial Institute. In 1861, for example, he lectured on differential and integral calculus; Schwarz was in the audience and seems not to have minded the rather sloppy treatment of limits that Weierstrass gave. Around this time, Weierstrass developed his own theory of what the real numbers are, later superseded by the approaches of Dedekind and Heine and Cantor, and included it at the start of the lecture courses on complex functions.

Weierstrass worked hard and as a result his health collapsed in 1861/62. Starting in 1863 he shifted over to what became the canonical two-year lecture cycle: one semester on complex function theory, the next on elliptic function theory, the third and most difficult on Abelian function theory and the fourth either on applications or the calculus of variations. The applications of the fourth semester could all be found in books written by Legendre at the start of the century: the mathematics may have moved on but the topics had not – and the calculus of variations was improved by Weierstrass but still with much left to be done.

The remaining topics are more interesting. Weierstrass, as noted above, had made his name by making decisive advances in the difficult subject of hyper-elliptic integrals and their associated functions. This still left the whole topic of the general integral of an algebraic function largely unresolved. In 1857 Riemann had opened the topic up much more (R 1857) and Weierstrass held the view that the topic of Abelian functions was central in mathematical research and his personal mission. In line with all previous investigators, he saw this as a topic in the theory of functions of several complex variables. However, in the 1850s and 1860s, the available techniques in that subject were impoverished. It made good sense for him to look at the much better, but still nascent, single variable theory. Cauchy, and Riemann, had started with the definition of complex differentiability that leads at once to the Cauchy-Riemann equations. However, the generalisations of these equations to several variables have more equations than unknown functions so it is hard to see that they can be of any use (the system is overdetermined). The key theorem that takes you from the Cauchy-Riemann definition of holomorphic to the power series expansions of a complex function is the Cauchy integral theorem and the Cauchy residue theorem that follows from it. However, the Cauchy integral theorem is a feeble thing in several variables because it gets the dimension of the boundary wrong. That only leaves the theory of convergent power series and that is therefore what Weierstrass insisted upon.

Weierstrass therefore tried to build up a theory of complex functions in one variable that would generalise to several variables. Concepts and techniques were preferred that work well in any number of variables or which might at least hint at a suitable generalisation. The extent to which Weierstrass went to avoid those aspects of the single variable theory that do not generalise is remarkable. He rejected the use of the Cauchy-Riemann equations on the grounds that not enough was known about partial differential equations to make them an acceptable basis for a theory. He rejected even the integral, on obscure grounds that had something to do with the fact that one can change the values of a function in many ways and not change the integral, a topic that in his lifetime was very poorly understood.

But on the other hand he did clarify a number of crucial topics. In the early 1860s he clarified the distinction between finite poles and essential singularities. His contemporaries were reasonably comfortable with functions that go to infinity like $1/z^n$ as $z \to 0$ because their reciprocals tend to zero.

But the behaviour of e^z at infinity or of $e^{1/z}$ at the origin confused them. Inspired by the behaviour of rational functions and elliptic functions that take every value equally often, mathematicians tried to argue that all functions, including such examples as e^z and $e^{1/z}$, do the same. Weierstrass' arguments and the (Casorati)-Weierstrass theorem that he proved more or less put a stop to such talk Weierstrass had a perfectly reasonable starting point for his theory of complex functions: power series convergent on a disc and their analytic continuations out to their natural boundaries. A great success for him was characterising the zero sets of complex functions. He did a lot to improve the theory of elliptic functions by bringing in his new sigma and $\wp$ functions. Less successfully, he tried to answer the question of how to characterise elliptic functions along highly algebraic lines. His answer was that, among all the functions that can be defined from integrals, only the elliptic functions allow certain changes of variable and only they, therefore, satisfy an algebraic addition theorem.

Meanwhile, he was largely blocked in his search for a good theory of complex functions in several variables and for the properties of Abelian functions. This was perhaps a personal misfortune but it provoked a much more important matter: Weierstrass' limited interest in publishing his ideas. The situation is very strange, even by the gentle standards for publication that prevailed in the 19th century. There is only one book in Weierstrass' name: the *Formeln und Lehrsätze zum Gebrauche der elliptischen Functionen* (*Formulae and Theorems in the Use of Elliptic Functions*). There are actually very few papers. The seven volumes of his *Werke* testify to his contemporary importance but they are filled with editions of his lectures, with the republication of early papers and only a modest number of original works published in his lifetime. He even disdained attending the Berlin Academy of Sciences and presenting his works there. Among the discoveries treated that way was the continuous nowhere differentiable function that he had discovered when trying to understand the concept of a natural boundary and Riemann's work on elliptic functions.

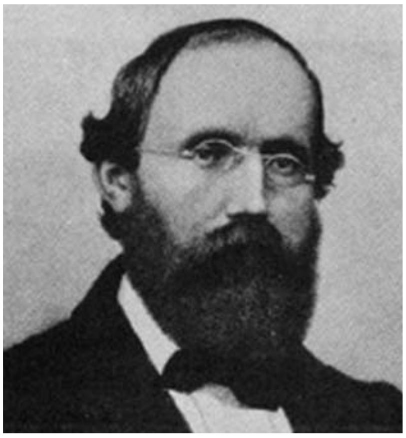
His preferred forum, outside his study, was the lecture theatre. As his two-year semester cycle drew to a close he would ask his best students for copies of the lecture notes and use them as a basis for the next revision. These lecture notes, and several versions survive often taken by mathematicians who went on to distinguished careers, e.g. Hurwitz, document his increasingly systematic approach to the function theory of one and several variables.

There are intriguing stories about the number of people who came to these lectures. In the German system of that time students could study anywhere and staff with an Habilitation (a post-doctoral qualification) could teach anywhere; pay was not guaranteed. It was quite usual for good, ambitious undergraduates and, more usually, graduates working for a PhD to be registered at one institution but to study for a semester or more at another. For anyone wishing to learn complex function theory the natural place to go was Berlin and the Weierstrass lecture cycle. But while we hear that Dirichlet's lectures in the mid-1850s at Göttingen drew an audience of about 30, we hear of 107 attending Weierstrass' first lecture in his 1869 course on Abelian functions. Apparently only seven stuck with it and of those only a few gave the impression they understood what was going on. Of course, the Franco-Prussian War affected attendance the next year (1870) but by 1878 Weierstrass was claiming an audience of 102 listeners and in 1881 Mittag-Leffler wrote to Kovalevskaya that 250 students were attending Weierstrass' lectures, and this figure is often quoted. Probably no-one in the administration was looking at retention figures. By the way, it should also be said that it is not clear what a student on a course actually did. English textbooks of the period are unique in offering exercises. Weierstrass' lectures are all theory with no work sheets and no exercises. There must have been some because of course there were exams and because every competent mathematician of the day was a good, accurate and often quick manipulator of the formulae and techniques that bulk out any branch of mathematics. Be that as it may, many students came to Berlin but perhaps few stayed. And perhaps that was not a problem; the system comfortably sorted students into mathematicians, physicists, engineers and teachers with a surprising degree of personal freedom.

3. Weierstrass, his students and Riemann

The detrimental side of Weierstrass' approach begins with his attitude to Riemann's work. Weierstrass' contributions to hyper-elliptic functions in 1854 and 1856 had been overshadowed by Riemann's contribution the next year. In response, Weierstrass withdrew his own paper on Abelian functions – to make a proper comparison of his work with Riemann's, as he put it – and did not return to the subject publicly until he lectured on it in 1869. Throughout his life Weierstrass disagreed fundamentally with Riemann's whole approach to complex function theory, allowing only that a researcher has the right to find any path to his discoveries. This disagreement shaped a significant part of Weierstrass' research priorities.

The student who responded most energetically to this side of Weierstrass' activities was Hermann Amandus Schwarz. In 1863, Schwarz responded to a comment by a



Bernhard Riemann (1826–1866)

fellow student in a graduate seminar about the Riemann mapping theorem. This had been claimed by Riemann in his doctoral thesis of 1851 and it says that any two homeomorphic simply connected domains whose boundaries are circles can be mapped onto each other by complex analytic maps. The student remarked to Schwarz that the theorem was all very well but not a single interesting example was known. Schwarz decided to square the circle – to map a circular disc analytically onto a square – and he succeeded with a lovely application of elliptic functions in a short paper where he also introduced (but fudged the proof of) the Schwarz reflection principle. From this work flowed numerous explicit constructions for the conformal representation of a planar polygon on a disc, known collectively as the Schwarz-Christoffel formulae.

At the same time, it emerged that Riemann had found a way of introducing complex function theory into the construction of minimal surfaces (surfaces of least area). Every (orientable) surface has what is called a Gauss map, associating to each point on a sphere the image of the unit normal at that point. This map is central to the definition of (Gaussian) curvature. Riemann noted that the map is holomorphic if and only if the surface is a minimal surface. Riemann's ideas remained unpublished until his death; in the meantime Weierstrass had come to the same realisation and Schwarz set to work to claim this problem too for the Weierstrassian way of doing things.

Also at around this time Lazarus Fuchs came to Berlin as a student. Nominally he was a student of Kummer's but he gravitated to the orbit of Weierstrass. His first important work was on another Riemannian theme, in this case Riemann's work on the hypergeometric equation, a second-order linear ordinary differential equation with many important properties. Fuchs generalised this equation, which has three singular points, to higher order linear ordinary differential equations with an arbitrary number of singular points and characterised the class that have the property that none of the solutions have essential singularities. His early papers were quite Riemannian in style but he too became more Weierstrassian as the years went by.

Weierstrass' influence was increased by the collapse of Riemann's health in 1862 from a bad attack of pleurisy. Riemann spent the last few years of his life in Italy, where he died in 1866 aged only 39. Weierstrass' strong emphasis on algebraic methods, and his hostility to the integral, to the interplay (in one variable) between complex functions and harmonic functions and to geometry, now had no countervailing force in Germany. It pushed his students and all who came under his influence away from Riemann's work and indeed from the work of the French school who, naturally, had taken their lead from Cauchy. This effect was most marked in the lecture courses, which did not direct students to the growing body of existing literature and made no openings towards it. Anyone who studied complex function theory in Berlin ran a serious risk of finding all other approaches to the subject literally unreadable. The few good enough to overcome that barrier most likely would remain in Weierstrass' orbit, where such work was discouraged and algebraic reformulations and, on occasion, actual refutations of Riemann's views were preferred.

4. Weierstrass and Kronecker

The famous topic of Weierstrass' relations with Kronecker fits in here. Originally, they were very close and Kronecker was happy to halt some of his investigations believing that Weierstrass was going to deal with the same topic. Kronecker had learned from Kummer the message that Gauss had taught: many subjects, such as number theory and elliptic function theory, meet beneath the surface and he brought to this a strong algebraic preference. Gradually this hardened into a philosophical view about the existence or otherwise of mathematical objects. Officially, so to speak, he was of the view that rigorous mathematics is about finite algebraic extensions of the rational numbers and the fields of rational functions in finitely many indeterminates over such fields. Talk about anything else, π for example, was just that – talk: a convenient shorthand for a limiting process that could be conducted to any arbitrary level of accuracy but never concluded. There was something about π that distinguished it from really existing objects like 3: which leads to his aphorism that "God made the natural numbers, all else is the work of man" (W 1891, 19). It followed from this, in his view, that mathematics, pure mathematics, was algebra and number theory. Geometry required an extraneous concept, that of space, and mechanics that of time, and so they were not really pure mathematics – not really mathematics.

Ironically, in his work he was much more willing to use the Cauchy integral theorem than Weierstrass was. Indeed he praised it in his lectures as one of the great results of the century, as surely anyone involved in analytic number theory would have to. There is a curious ambiguity in Kronecker's work every time it comes to the complex numbers; did he, or even could he, adhere to his algebraic philosophy and do his best work?

But it was Kronecker's growing hostility to the real numbers that appalled Weierstrass. Weierstrass became very worried that when he retired Kronecker, who was eight years younger than him, would take over the department and undo all his work in analysis out of a severely misplaced view about mathematical existence. This hostility polarised the department. Schwarz, who

Leopold Kronecker (1823–1891)

in this matter as in every other was a Weierstrass loyalist, on one occasion in 1885 organised a birthday party for Weierstrass to which he invited Kronecker in these terms: "He who does not honour the Smaller, is not worthy of the Greater" (see B 2008, 507). He intended it, apparently, as a jocular reference to the respective sizes of Weierstrass and Kronecker – Weierstrass was much bigger – but Kronecker took great offence, interpreting as a reference to their respective mathematical abilities and contributions. Whether Schwarz was quite so naive as to miss this interpretation of his joke, whether indeed there was not a hint of anti-Semitism here (Kronecker was a Jew) – on these matters history is silent.

In the event, Kronecker's wife died in 1890 and he was much affected by this loss and died the next year. So Weierstrass, whose health had been declining for many years, could move to retirement secure in the knowledge that Berlin University would continue to agree that the real numbers existed and that sets of numbers with an upper bound have a least upper bound.

5. An assessment

Weierstrass had come to Berlin in 1856 with the reputation of having solved a very difficult problem. He then set himself the task of building up complex function theory in one and several variables so that there could be a theory of Abelian functions in whatever relation to a general theory of complex function in several variables that the facts would permit. Despite his enduring reputation, and despite many genuine accomplishments, the verdict on his time at Berlin must be mixed.

When he arrived, it was the prestigious university in Prussia, the one with the ear of government and the only one with a large mathematics department. When he left, it was not only overshadowed by Göttingen but it was going down as Göttingen was coming up. By 1892 the Göttingen mathematics department had Klein, the great mathematical empire builder, at its head and significantly he had made the Göttingen journal, *Mathematische Annalen*, into a truly international journal whereas the Berlin journal, the *Journal für die reine und angewandte Mathematik*, was very much a Berlin journal, and it was beginning to be obvious that even Berlin could not take on the rest of the world and win. And Klein, famously, made sure that he got the star of the next generation: David Hilbert.

What is more, the succession in Berlin was flawed. Kummer, Kronecker and Weierstrass re-placed themselves with Schwarz, Fuchs and Frobenius. Schwarz arrived in 1892 with his Collected Works already published, indicative not only of his vanity but of the amount of original work he was to go on to do. Fuchs, careless, kind but ineffectual, was not a leader of men. Frobenius was tougher but even he could not overcome what had become Berlin's fundamental problem: it was too narrow.

Weierstrass had built up a loyal group of followers. Perhaps he could do nothing about the fact that none of them were truly first-rate but perhaps his own narrowness was part of that. He saw mathematics as complex function theory first, including, he always hoped, the theory of Abelian functions, with real analysis second, and he himself lectured on almost nothing else after a time: no geometry, not even any number theory. There were some tired applications to mechanics and he offered a brave attempt to make sense of the muddled world of the calculus of variations. Nothing, however, on, for example, Fourier series.

Weierstrass' complex function theory was, as we have seen, deliberately narrow. Its foundations excluded the integral; pedagogically he made little or no reference to the literature on the subject and therefore made that literature difficult to read. His inability to publish extended to discouraging his students from publishing copies of their notes of his lectures, although lithographed versions did circulate and Schwarz eventually persuaded Weierstrass to write up an account of elliptic function theory. Some of his best discoveries were read to the Berlin Academy but not published in their proceedings. Nor was Weierstrass' approach mere insularity. He increasingly disliked alternative approaches and encouraged Schwarz to rework every discovery of Riemann's that seemed eligible for such a treatment.

Conversely, his reluctance to publish made his ideas difficult to pick up outside Berlin. French textbooks, as they emerged, naturally started with the final years of Cauchy's theory of complex functions. Germans were divided between a dwindling band of Riemannians and people who had attended Weierstrass' lectures. But before Weierstrass' death there was only the pirate edition by Biermann (a work dishonestly claiming to have Weierstrass' approval – Weierstrass was outraged) and a short work by Thomae, who said in the preface that he thought it was unreasonable that students should have to work out the details of Weierstrass' work for themselves. Oddly enough Weierstrass' approach did better in the 1890s in books in other languages, such as English, some of which aimed at giving all three approaches and some of which valued Weierstrass' for starting at an elementary position and proceeding systematically.

Probably most people, as the years go by, find themselves working in just one part of the forest, making a virtue of their limitations, bothered or not by excitements elsewhere. Most would be happy to believe that they had opened a small part of their subject up to further exploration and shown there was more to it than mere puzzles. But some have the opportunity, and perhaps even the burden thrust upon them, of leading a field. Weierstrass was one of these and he recognised and accepted that responsibility. It is true that he turned 60 in 1875, some years before Bismarck set a retiring age of 70 and when old age and disability went more closely together than they do today. But at 60 Cauchy merely upped the pace at which he published. A very good way to decide, finally, to seize the day is to realise that you do not have many days left. And Weierstrass was in the leading university for mathematics of its time, with no shortage of opportunities to publish, to bring in the best students, to promote his vision of the subject. Instead, he retreated, taking his supporters with him. His, and their, achievements are many but a model for how to play a winning hand he is not.

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The informal character of this essay will be evident and a full-length collection of references is inappropriate. Major sources are indicated below and all the relevant information will appear in the forthcoming book *Hidden Harmony – Geometric Fantasy: The Rise of Complex Function Theory*, Umberto Bottazzini and Jeremy Gray. I would like to thank Umberto Bottazzini for his help and guidance during this work.

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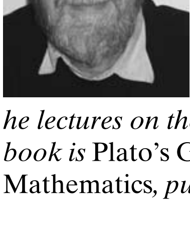

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Interview with Jacques Dixmier

Martin Raussen (Aalborg, Denmark)

Jacques Dixmier

Education

Can we start with your early years? You were born in 1924. Where did you grow up and what was school education like in the years between the two world wars?
I was born in Saint-Étienne, not far from Lyon. Since my parents were teachers, both of them, we travelled around France, so I am not from a particular place. I lived in Rouen then Lille and then in Versailles, from when I was nine years old. That was the most important place for me.

Your school education was mostly in Versailles?
Yes, in Versailles from 6th degree to mathématiques spéciales (2 years after the baccalauréat[1]) except, due to the war, one year in St.-Brieuc, which is a small town in Bretagne. Even there I had a very good teacher in mathematics.

And that was probably important?
That was extremely important. I had very good teachers in mathematics for my whole period in school. Only during the 6th grade, the teacher was absolutely zero. But after that, during almost 10 years, each year I had a wonderful professor of mathematics. That is probably one of the main reasons for my vocation.

Can you remember a particular instance that made you think: mathematics, that is something for me?
I always liked mathematics but originally I did not believe that I was able to do research. I realised that I liked teaching and I thought I would be a good professor of mathematics. That was my main reason for continuing to study mathematics – and then to enter the École Normale Supérieure.

Do you have any family with background in science or mathematics?
Not that I know of. My father would have preferred me to become an engineer. When he saw I was good at mathematics, he thought that I should enter École Polytechnique. He himself was a teacher – in French, Latin and Greek. He made no serious opposition when I said that I would prefer the École Normale. My mother always accepted all my plans.

Before entering the ENS you had to go to École préparatoire?
Yes. After the baccalauréat, some people in France go to a so-called Classe préparatoire. In my times, that was called mathématiques supérieures and then mathématiques spéciales.

Was that in Paris?
No. It was in Versailles.

You entered the ENS at age 18 in 1942?
I was at the ENS from 1942 to 1945 or 46; the last year is a little bit dubious. Before 1945, ENS education in mathematics lasted three years. At the end of the war, a 4th year was added. But I worked at the CNRS[2] – still quite new – at the same time. In fact I wrote my thesis during this year.

Can you tell me about your reminiscences from your student time at the ENS? How was it like to study in these difficult times, under German occupation?
Many people in France had not enough to eat but that was not the case for me. Food was not good but it was enough; we had heating as well.

You probably want to hear about my attitude towards resistance[3]. I joined the resistance but only at the end. Only very little was known about that because students in the resistance would not tell you about it; it was too dangerous! In fact, the resistance became much more important when we were forced to go to Germany to work in factories; the so-called "service du travail obligatoire (STO)"[4] was introduced in 1942. I was not concerned with that except in the last month because I was still too young. But many of my friends were old enough. Many of them left the ENS

[1] Graduation from High School.
[2] Centre national de la recherche scientifique
[3] French resistance movement
[4] Compulsory labour service

and the country to avoid the STO. In fact, one of my closest friends went first to Spain, then Portugal, then England and he studied in order to become a pilot. He just finished his studies on 8 May 1945, the last day of the war.

What did it mean for you to be a member of the resistance?
I suppose if I had been arrested I would have been imprisoned. I never had an arm. I knew nothing about arms. I participated in some very small meetings.

And then, liberation came in August 1944 in Paris.
Yes. In fact the Gestapo came to the École normale supérieure a few days before that. Finally they arrested Georges Bruhat[5] who was the director at the time.

What happened to him?
He was transferred to Germany and he died there in a camp. He was already old; I do not know much about the details. The general secretary of the ENS, Baillou, who was also taken to Germany, survived and returned to Paris.

In fact, they came during the holidays in August and they arrested several pupils, including me. There is something which concerned me personally that I still do not understand. I was at the ENS doing something that was probably illegal. I heard some noise but I did not know what happened. So, I went to the entry of the ENS. There were two officers of the Gestapo. They cried out and I was taken completely by surprise. They fired and I don't understand how I was not wounded. They took me and I was sent to detention in an office. Finally, instead of taking several pupils, they took the director and Mr Baillou, who probably proposed themselves in place of the pupils.

Did school teaching still go on as usual, as before and after the war?
There is a very big difference between now and then but that happened gradually. There was no big rupture in the form of teaching between 1944 and 1946, for instance. During my stay at Ecole Normale, I had courses by Cartan[6] and de Broglie[7]. We understood nothing of de Broglie's courses; we were there, there was a blackboard, de Broglie would only talk and write nothing at all. We had great difficulties in understanding Cartan but we tried. With de Broglie, we did not even try.

Which of your teachers at the time were important for you and your future career?
My main teachers were Cartan and Julia[8]. I had Cartan already as a teacher for two years at École normale supérieure. More importantly, after that I attended Séminaire Cartan and later on, I was a colleague of Cartan at Bourbaki. So, the importance of Cartan for me is immense but this dates mainly from the time after the two years at École normale.

Julia was important for a different reason. I followed his course in 1943 during my first year at École normale. Julia gave a course every year in advanced mathematical analysis. This sort of course, which was also given by Montel[9], Denjoy[10] and others, was too difficult for us students. But we were very anxious to have a diploma as soon as possible because of the war; you did not know what could happen in a few months! Julia was an extremely good teacher and we understood him. In fact the level of his course about Hilbert space was not very high but we understood almost everything. As a result, Hilbert space became for me as familiar as ordinary space. That was certainly the reason for my research orientation later on. It is very important to be completely familiar with the basics of your subject.

Thesis under Gaston Julia

I would like to ask you a bit more about your supervisor, your directeur de thèse, Gaston Julia. Nowadays he is famous because he had already as a young man studied the iteration of rational functions in the complex plane. Many laymen have seen pictures of Julia sets, etc. Was he well-known at the time? What sort of a person was he?
First of all, I think he was a very good mathematician. When he was about 40 years old, he became more interested in academic questions than in real mathematics. In particular, I don't think he was very much aware of the recent developments concerning Hilbert space – fortunately for me because that made it easier for me to understand what he was talking about! Another important point about him is that he was very severely wounded during World War I; he lost his nose and wore a mask and he suffered very much. He wrote his thesis, which consists of two big papers, while he was at the hospital. I don't think he had a patron de thèse in the usual sense.

Oh, I wanted to ask you about this! I remarked that the American Mathematics Genealogy Project does not indicate a directeur de thèse for Julia?
I tried to solve this problem a few years ago. A young girl who was a student of Mathieu[11] knew that Mathieu was a student of Duflo[12], that Duflo was a student of me and that I was a student of Julia. So she came to me and asked me, could I tell her who was the directeur de thèse of Julia. I did not know and so I looked at his first two big papers. There is no hint of a thesis director, not even anybody who inspired him to write these papers. I asked his grandchild, who is a mathematician as well, and he does not know either. One would have to look through the archives of the Sorbonne. I still don't know.

Do you know who inspired him to look at these questions?
Probably Picard[13]. I was recently informed by Michèle Audin (Strasbourg) that the jury de thèse (assessment

[5] Gorges Bruhat (1887–1945)
[6] Henri Cartan (1904–2008)
[7] Louis de Broglie (1892–1987)
[8] Gaston Julia (1893–1978)
[9] Paul Montel (1876–1975)
[10] Arnaud Denjoy (1884–1974)
[11] Olivier Mathieu
[12] Michel Duflo
[13] Charles É. Picard (1856–1941)

committee) for Julia consisted of Picard, Humbert[14] and Lebesgue[15]. Unfortunately, I am not competent concerning Julia's work on functions of a complex variable. In his courses about Hilbert space, he tried to make an analogy between these two topics. I think this analogy was far-fetched but maybe I missed some important point. One cannot be sure.

How was he as your director de thèse? How was the collaboration like?
He was not an easy person. As you probably know, he was politically at the extreme right and hoped for a German victory during the war. He and my father were fellow students at École normale for two years: my father in literature and he in mathematics. So they knew each other. Julia had lived in Versailles and my father was a professor at the grammar school in Versailles. That made things easier for me. Julia had six sons. My mother, who was a teacher in elementary school, taught several of them. So there were links between the two families.

Did he propose a subject for your thesis?
No, but I was certainly inspired by some of the things he had told us – generally speaking, on Hilbert space. In my thesis, I studied some particular subspaces, non-closed but not arbitrary – very special subspaces; my entire thesis is concerned with this class of subspaces. Julia had talked about that but I don't remember any details. As you know, when you are completely embedded in a subject, this subject becomes your own. But I am sure he played a role in this choice.

Did you consult him during your thesis work?
Yes. Finally, I was summoned at the end; it was probably in April 1947. He was rather critical in his appreciation. But the main point was that he finally said that this would be a thesis.

Career

You had already started to work at the CNRS while you wrote your thesis?
As I said, during 1945-46 I was at the same time pupil at the École normale supérieure and employed at the CNRS. I did the main part of my thesis during that year; after that, during the following year, I was only at the CNRS. In 1947, I was nominated, due to Julia's influence I am sure, as maître de conférences[16] in Toulouse. So, I was a thèsard (PhD student) only for two years.

How long did you stay in Toulouse? And how were the conditions?
I was in Toulouse for two years. It was tiring for me because I lived in Paris. My wife was a professor of mathematics at grammar schools in several towns in France. The lodging question was extremely difficult. We had a one room apartment in Paris. We travelled a lot and travelling by train from Paris to Toulouse was not easy at all in 1947. The bridge over the Loire at Orléans had not been completely repaired.

After the two years in Toulouse, there was a new position in Dijon in 1949. Things were very easy for a young mathematician at that time because the number of students was exploding. The government created new positions so I had no difficulty at all. Compared with young mathematicians now, I was very privileged. Dijon was much closer to Paris. I was there for six years and after that, in 1955, I was nominated for a position in Paris at the Institut Henri Poincaré.

The number of students was enormous. I held my courses in a place called Conservatoire des Arts et Métiers; there was a big auditorium. During the first 2 or 3 years, I had 500 students. Jussieu[17], my later workplace, was only created several years later.

At that time it was just the University of Paris?
Yes. You must make a distinction between the creation of Jussieu and the creation of the different universities of Paris. After 1968, instead of one Université de Paris, some 13 or 14 new universities were founded. I stayed at Jussieu.

I took my retirement when I was 60 years old, for personal reasons. A new law had just made that possible.

Research

Can we continue with your research work? Can you give me an account of the research areas that you have been most active in? To begin with operator algebras – coming from Hilbert space that you knew so well?
Being familiar with Hilbert space, I was able to read easily a lot of papers but mainly the work of von Neumann[18] – a lot of papers of von Neumann and in particular his papers, some of them with Murray[19], about operator algebras.

Which were motivated by quantum physics at the time?
He explained that in the introduction of the first paper (there are five or six big papers). But also by the theory of unitary representations! For me that was the main point because I never really understood quantum mechanics. But I understood unitary representations. These papers were prophetic, which was not evident in 1936, the year of their first paper on operator algebras. It became evident ten years later when I wrote my first papers in 1946, immediately after my thesis. I was helped by a friend Godement[20], who was also a student at École normale. I did not know him at École normale; he was two years older than me. He explained to me a lot of things about unitary representations. I even had the privilege of reading some papers by Godement that he never published; he was reluctant to publish.

So I studied what I called von Neumann algebras.

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[14] Marie Georges Humbert (1859–1921)
[15] Henri Lebesgue (1875–1941)
[16] Associate professor
[17] For many years the home of the Universities Paris-6 and Paris-7.
[18] John von Neumann (1903–1957)
[19] Francis Joseph Murray (1911–1996)
[20] Roger Godement (* 1921)

Is it true that you coined the notion von Neumann algebra?

No, that was an idea of Dieudonné[21]; he proposed this name during a discussion at Bourbaki. I thought that this was self-evident; I should have thought of it myself! I then used this notion in my book on operator algebras[22] that appeared in 1957. After that, I still studied operator algebras, but more from a C*-algebra point of view, and at the same time unitary representation theory. Godement had played a big role for me but I was also influenced by all the papers of Harish-Chandra[23]. Then I began to work on enveloping algebras[24] (of course, connected to my previous work). I wrote a book about C*-algebras in 1964 and then my book about enveloping algebras[25] in 1974. Then I got interested in invariant theory. There is a connection between these two things. It is a little technical, so I will not explain it here.

After my retirement, I went to the IHES[26]. I was much freer in my work. I did a lot of work about invariant theory; moreover, there is also a connection with the theory of partitions. I did a lot of work but I realised that this work was less good than what I had done previously. I am not a great mathematician but I am a good mathematician. My papers from this period are technically difficult but less important than what I had done before. I think this is due to age; this is the only reason I can see, since I had the best possible facilities to work in.

Probably partition theory is also a very difficult topic?

Yes, but no more than operator algebras. There are a lot of people who say: ooh, operator algebras!

Are there any of your results that you are most proud of – if you could single out a few?

I have to think, I was never asked about that: maybe the paper where I prove that real algebraic groups are of type I; and also some papers about enveloping algebras; and also a paper, which is not well-known because it was completely absorbed by Harish-Chandra later on, about the De Sitter group; and finally, a paper about quasi-unitary algebras.

When you think about how you achieved your results, was that usually very systematic or would it sometimes occur that you had a sudden jump of inspiration?

Both! At least for every important paper, I needed a lot of time and a lot of work – and of course, at some points the ideas come. A particular idea came when I was travelling in a bus.

Oh, that reminds me of a similar comment by Poincaré[27]!

Yes, and Serre[28] told me about having an important idea on a train journey[29]. That may be one of the reasons why ordinary people judge us as abstracted people. I don't know how my face looked like in the bus when I suddenly had this idea. I was certainly not interested in what was going on around me.

Coming back to von Neumann, did you ever meet him personally?

Yes, very little, the first time in 1947. He came for the first time after the war from the United States. He stayed at Hotel Lutetia, which is a very big hotel in Paris, and many people came to him. I was extremely flattered that he proposed that I should come and see him.

He knew about your papers?

I had not written anything about operator algebras at the time. I had only my first two papers to give to him. He was, I think, extremely kind because I realised later that they were not good papers.

You were very young at that time?

Yes, I was 23 by then. Later, I wrote to him. I think I saw him one more time at the Amsterdam Congress in 1954. I heard a little later that he was very ill. I wrote to him several times. He did not answer immediately but he answered. He was not working on algebras any more but he took pains to answer me.

Are there other mathematicians that you have a particular admiration for?

I have to limit myself to a few whose work I understand: certainly Gelfand[30] and Harish-Chandra whom I already mentioned. I did not collaborate with them but I met them. I also received them at home. I met them at some congresses, Gelfand at the ICM in Moscow in 1966 and later on in Hungary and several other places. I met Harish-Chandra also in Moscow where he was an invited speaker and several times in Paris. But I think I never met him in the United States where he lived up to his death. As you know he died relatively young.

Did you travel a lot in general?

I did not travel a lot – of course, more than an ordinary Frenchman but less than an ordinary mathematician. The reason was that I did not want to leave my family. Anyway, since I was a member of Bourbaki, I was absent from my family for one month every year, and I found that was already very much. I refused most of the invitations with one exception: I lived with my family for one academic year in New Orleans. In fact, I often went with my family to the summer congress of Bourbaki. So I was not always separated.

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[21] Jean Dieudonné (1906–1992)
[22] *Les algèbres d'opérateurs dans l'espace hilbertien (Algèbres de von Neumann)*, Gauthier-Villars, Paris, 1957
[23] Harish-Chandra (1923–1983)
[24] *Les C*-algèbres et leurs représentations*, Gauthier-Villars, Paris, 1964
[25] *Algèbres enveloppantes*, Gauthier-Villars, Paris, 1974
[26] Institut des Hautes Études Scientifiques, Bures-sur-Yvette
[27] Henri Poincaré (1854–1912)
[28] Jean-Pierre Serre (* 1926)
[29] Also reported in: M. Raussen, Chr. Skau, Interview with Jean-Pierre Serre, *Newsletter of the European Mathematical Society* 49, 18–20
[30] I. Gelfand (* 1913)

Bourbaki

You became involved in the work with the Bourbaki group already as a young man?
Yes, but not younger than most of the others from that generation: Godement, Koszul[31], Samuel[32], Serre and then a little bit later Borel[33], Cartier[34], Bruhat[35], Douady[36]. We were very young all of us when we became members.

More or less at the same age?
Yes, more or less around 25 years old.

Were you asked to join?
That was due to Serre and Samuel. Serre has probably explained to you that he almost forced himself into Bourbaki. There was a Bourbaki congress at École normale supérieure. During the first years after the war, it was difficult to meet at a hotel, so Bourbaki held meetings at the ENS. Serre was a pupil there and he just asked to listen. From his questions they quickly understood that he would be a very valuable member.

Pierre Samuel is a Jew. He was a pupil at École normale supérieure but he had to remain hidden during the war. He and Cartan knew each other, probably dating back to the entrance competition to the ENS. Just after the war he went to America, where he was a pupil of Chevalley[37]. Anyway, he became a very young member of Bourbaki, one or two years after the war. One day in 1949, Serre and Samuel, who both knew me, approached me and asked whether I would accept to become a member of Bourbaki. At the time, this was extremely flattering; I would have jumped! It turned out to be very important for me because otherwise I would probably have worked on Hilbert space all my life. Bourbaki forced me to learn a lot of other subjects.

How was collaboration in Bourbaki? How did you progress from an exchange of ideas to drafts and to the final volumes of* Éléments de Mathématique*?
Well, most of this is well-known by now[38]. Usually, there were five or six drafts on the same subject written consecutively by different people.

The first draft for the volume *Algèbres de Lie* was written by Koszul when I was already a member of Bourbaki. I wrote the second or the third draft on the subject and, I think, also the final one. In theory, a version is not accepted by Bourbaki unless everybody agrees. In reality, after having discussed five or six consecutive drafts, some people might still not be completely satisfied but they have then become too tired to disagree. Well, for *Algèbres de Lie*, there were no serious disagreements.

I think this book was the standard source for a long time to come!
If you take into account not only the first chapter, which is about the general theory of Lie algebras, but also the chapter about root systems as part of the subject and then the chapter about semi-simple Lie algebras which were written a little later. I think the idea of an independent theory of root systems is due to Cartier, who explained that to Bourbaki, as I remember, at the blackboard. We were convinced and decided to make a separate study of root systems, in general. Cartier found the axiomatics and also wrote the first of several drafts on the subject. I was involved in the second or third draft; at the time, I had to use Coxeter's book about polytopes[39]; at certain points, it was not very satisfying. The book is very beautiful but it uses sometimes ad-hoc methods building on geometric intuition – not really suitable for Bourbaki. It was only later that Borel found algebraic proofs of everything and that was beautiful, too. I wrote one of the last drafts. I was good at writing; I had practically no original ideas for Bourbaki but I think I was instrumental for the writing-up process.

At the "congrés oecuménique". From left to right: Jacques Dixmier, Jean Dieudonné, Pierre Samuel, André Weil and Jean Delsarte. (Copyright: Tous droits réservés. Archives de l'Association Nicolas Bourbaki).

How could all the different personalities work together at Bourbaki? I was often quite puzzled when reading about the Bourbaki meetings and congresses. They seemed to have an almost anarchistic character; but at the end, they resulted in these very rigid volumes!
That is an interesting remark; you are probably right. Bourbaki was also a big machine. And the final versions which were sent to publisher were almost always typewritten by Dieudonné so they were very much influenced by Dieudonné's personal style – but everyone accepted that.

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[31] Jean-Louis Koszul (* 1921)
[32] Pierre Samuel (* 1921)
[33] Armand Borel (1923–2003)
[34] Pierre Cartier (* 1932)
[35] François Bruhat (1929–2007)
[36] Adrien Douady (1935–2006)
[37] Claude Chevalley (1909–1984)
[38] More about this topic in A. Borel, *Twenty-five years with Nikolas Bourbaki* (1949–1973), *Notices Amer. Math. Soc.* 45 (3): 373–380; M. Mashaal, *Bourbaki: une société secrète de mathématiciens*, Pour la Science, 2002; English translation: AMS. 2006
[39] H.S.M. Coxeter, *Regular Polyopes*, Macmillian, 1963

Was André Weil[40] still an important figure in your times?
Certainly, but mainly by correspondence since he was in America when I started to work with Bourbaki in 1949. I certainly did not see Weil during my first year at Bourbaki. Of course, his influence was enormous, in particular during the first years of the Bourbaki enterprise before the war. After that, his influence was mainly felt through what he wrote. He participated in some but not all of the meetings; I would say he was there about half of the time – this can be checked in the archives. He could even have been influent for a longer time but he had decided himself that the age limit was 50.

Was this age limit generally accepted?
Yes. Weil was invited to the congress shortly after he had become 50 years old. It was held as usual but when the last day ended with a resuming meeting for the congress and decisions for the next one, Serre and Borel asked Weil to stay away from this meeting. They thought that if you are no longer a member of Bourbaki, you should no longer have any influence. Weil was certainly not happy but he did not object. Many of the people present said nothing and would have preferred him to stay. But if somebody objected, we all did. Those were the rules of the game.

How about Dieudonné; did he stay after becoming 50 years old?
No, but he was still often invited to the congresses. I do not remember whether he still wrote some final versions of the Bourbaki books.

Just a few days ago, we had the celebration of the 60 years of Séminaire Bourbaki[41] at Institut Henri Poincaré; Jean-Pierre Serre gave the lecture number 1000. You have given several lectures for this famous seminar.
Not that many, five or six perhaps; far fewer than Grothendieck[42] or Serre.

I am not angry but I think Bourbaki is no longer Bourbaki. The seminar is all very well but the reason for Bourbaki was to publish books.

It no longer has the influence it had back in the 50s and 60s?
I suppose there are still meetings at Bourbaki but what do they do? They are perfectly able to write books! For some of the subjects, we had written drafts that have never been published and that is a pity, I think. These drafts were of course just the beginning; they should have worked more on them. For a long time, nothing appeared and about 10 years ago, a single ninth chapter appeared in the volume on commutative algebra, I think. It is a pity that nothing has happened ever since.

I remember that Demazure[43], who worked very actively with Bourbaki, expressed that you often may have the feeling that if you sit down, think about a topic for enough time and then write down your findings, then after some time you have found everything of interest; he said that this feeling is wrong. He was right because mathematics changes all the time. Maybe it is true that the story had to stop. But not so soon, I think!

In the drawers, there are drafts for chapters on, for instance, complex semi-simple Lie groups, on class-field theory. Also, several more chapters on spectral theory had been written.

Research Education, PhD students

My next questions are concerned with your PhD students. You have had several PhD students?
Yes. I was a directeur de thèse (PhD supervisor) for twenty students. Moreover, there was a number of students who wrote their thèse de 3ième cycle with me.

The best of my PhD students was certainly Alain Connes[44]. He appeared in my seminar. Usually, I knew the participants of my seminar beforehand because they had followed one of my courses. I was stuck – who is this man? A few months after that, he offered me a four page paper. I understood immediately that his result was very important and that he had given a very simple proof – absolutely amazing!

Connes said several times that Choquet[45] had been very important for him; he also mentioned me very generously.

I assume you had more collaboration with him and your other students than you had yourself with your supervisor Julia?
Yes, but after all it is difficult to compare. For my 20 students, supervision was very different from one to another. In fact, Connes worked mainly on his own. Roughly speaking, after a few years, he brought me his thesis. I read it and asked him to change a few commas. So with him, in a sense, it worked out the same way as between me and Julia.

Let me also mention Michel Duflo and Michèle Vergne. With Duflo, it was essentially very much as with Connes; he worked independently. Michèle Vergne wrote several papers. We also wrote a paper in collaboration, after her thesis. During her thesis work, I read everything she wrote and gave some advice but that was not very important. With some other students, I had to contribute in a more substantial way. I have collaborated with some of my students in joint papers: with Bernat, Duflo, Vergne, Maréchal, Berline and Brion.

Are you still in contact with your former PhD students?
With several of them; my first PhD student Alain Guichardet and his wife are very good friends. I also see some of the others from time to time – and Connes is a friend; I meet him and his wife occasionally.

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[40] André Weil (1906–1998)
[41] Three times a year, with five lectures nowadays
[42] Alexandre Grothendieck (*1928)
[43] Michel Demazure
[44] Alain Connes (*1947)
[45] Gustave Choquet (1915–2006)

Books. Teaching.

You are the author of a series of textbooks.
Not really many. I wrote two books for the first two years[46], called first cycle at the time. They were in fact quite important for me, since they sold very well and thus they were a source of income – not big money, of course! Otherwise, I wrote a textbook on general topology[47], which was not a very big success. Then there was a book about Lebesgue integration[48]. And then, I have my research books[49] but that is another story.

Did you have a particular philosophy of how to teach and how to write? Was it close to the Bourbaki style?
That would probably be true for the written text. For oral presentations, I have certainly a philosophy that differs from most of my colleagues. In order to make good talks, I prepared them word for word. Most of my colleagues think that this is a far too formal attitude, that you have to rely on inspiration. I do not believe that at all. I have often heard the opinion that it is good to get stuck; otherwise students do not understand that there is a real difficulty. I must say, I do not agree at all. I was usually very careful to write in big letters, to avoid talking into the blackboard and so on.

I really wanted to be different from Denjoy; it is said that, during a lecture, he thought *a*, he said *b*, he wrote c and d would have been correct! In fact, when I was a student at the ENS, during my second year, I tried to follow a course given by Denjoy. After the second lecture, I left; it was hopeless for me. Well, not for everyone; Choquet was a pupil of Denjoy. I know that the written work of Denjoy is very good. But as a teacher, he was terrible. For written text, I must admit that I like the Bourbaki style!

Probably it depends on how you use it?
Yes, of course. You can be extremely Bourbaki or not at all; I would place myself somewhere in the middle. But I like the theorem-proof structure. This is probably an effect of what I suffered from during my education at the ENS. During that, my fellow students and I read the books of Goursat[50], Picard[51] and Darboux[52]. To understand what was going on, then you would sometimes have to guess and jump 40 pages back and forth. I did not like that and, probably as a reaction, I loved a very formal presentation, which of course, may be very effective at the same time. I like brevity, although I can see disadvantages as well. For presentations, it is probably best to mix a formal part with motivations, applications and so on. But I do not know how to do that in the best possible way.

And of course it depends on who listens; students are different!
Well, I think one cannot talk about an "ordinary student".

Some people like geometric intuition; others despise it…
For instance, Grothendieck told us at Bourbaki about the time when he was a student in Montpellier. He said that he did not understand the theory of functions of a complex variable at all. The professor would just draw loops and so on; for Grothendieck, that was not mathematics. I had such a course with Valiron[53], who was a very boring teacher but his course was very good. He made drawings and I accepted that. I did not insist on perfect rigour at the time.

Mathematics after retirement

Let me ask you about mathematics after your retirement. You explained that you stayed at the IHES for five years?
Yes, in an informal manner. I was not invited but I came to live in Bures-sur-Yvette. Some friends from the institute kindly offered me an office. I came to the IHES on a daily basis; it was an extraordinary environment. I had never had that much stimulation except at Bourbaki.

And you participated in lots of discussions and collaboration at the time?
Not a lot but much more than in Jussieu. That, in the end, was dominated by administration, meetings, boards, etc. I had in fact succeeded in avoiding much of the administration. At the IHES, there was nothing of that kind. And I could ask Connes, Jones[54] and others.

Did you follow the development that came from Jones' work, the interplay with knot theory for example?
A little, certainly! The few things I know about knot theory come from the acquaintance with Vaughan Jones at the IHES but I never used them in my own work. On the other hand, Jones asked me questions about enveloping algebras and I was once able to provide a counterexample.

Do you still write research papers?
I stopped working regularly when I was 68. I was at the IHES from age 60 to 65. Then I wrote to the director, Marcel Berger, that it was not correct to use an office because I was too old and I came too little. After I had stopped, at two occasions I could not resist pursuing certain ideas. This work resulted in two papers. I wrote the last one at age 82; it is about triangle geometry.[55] Well, not ordinary triangle geometry. We associate to a triangle a certain point and define in this way a nowhere differentiable function of two variables; we only use methods of classical analysis. I think I would be unable to work on

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[46] J. Dixmier, *Cours de mathématiques, 1ère année, 2ème année*, Dunod
[47] J. Dixmier, *Topologie générale*, PUF, 1981
[48] J. Dixmier, *L' intégrale de Lebesgue*, Les Cours de Sorbonne, 1962
[49] J. Dixmier, *Les C*-algèbres et leurs représentations, Les algèbres d'opérateurs dans l'espace Hilbertien, Algèbres enveloppantes*, Jacques Gabay
[50] E. Goursat, *Cours d'analyse mathématique*
[51] É. Picard, *Traité d'Analyse*
[52] G. Darboux, *Leçons sur la théorie générale des surfaces et applications géométriques du calcul infinitésimal*
[53] Georges Valiron (1884–1955
[54] Vaughan Jones (*1952)

a really modern subject. For instance, I had a very hard time trying to read Alain Connes' new book[56] which he presented to me.

Do you still come to seminars?
A few times! For instance, I did not come to the last Séminaire Bourbaki because I feared not understanding the lectures. I was a little in doubt because Serre is such a good lecturer.

Do you use a computer? Do you read and write emails?
No, I have no computer and do not use email. If I were ten years younger, I would have been forced to use them. Computers began to become useful for a mathematician when I was around 65 years old; too late for me! And everywhere people of my age told and tell me that they have difficulties with their computers. I did not want to bother!

A secret life has advantages?!
In essence, yes.

Other interests

Can you tell me about your interests apart from mathematics?
I wrote two science fiction books – well, short stories, not big novels.

Was that a long time ago?
Not so much. I was between 60 and 65 for the first and a little older for the second. These are two small groups of short stories.

They were published?
The first one sold 600 copies. For the second one, I do not even know. After that I wrote a detective novel and sent it to the same publisher but he said that my books were too difficult to sell.

But you enjoyed writing them?
Yes. But I do not do it anymore. I lack new ideas!

You are a big reader, it seems!
You see only a little part of my library. But you are right, one of my main occupations now is reading.

I would like to thank you very much for this interview!

Martin Raussen [raussen@math.aau.dk] is an associate professor of mathematics at Aalborg University in Denmark. Back in 1980, he followed a course on Lie algebras given by Jacques Dixmier in Paris. During the period 2003–2008 he served as the Editor-in-Chief of the Newsletter. Currently, he is an associated editor of the Newsletter and a member of the EMS Executive Committee. His research is concerned with "Directed Algebraic Topology", a recent field mainly motivated by certain models in concurrency theory within theoretical computer science.

[55] J. Dixmier, J.-P. Kahane, J.-L. Nicolas, Un exemple de non-dérivabilité en géométrie du triangle, *Enseign. Math.* (2) 53, no. 3-4, 369–428

[56] A. Connes, M. Marcolli, Noncommutative Geometry, Quantum Fields and Motives, *Colloquium Publications* 55, AMS, 2008

The 15th ICMI Study on The Professional Education and Development of Teachers of Mathematics

Mariolina Bartolini Bussi

At the beginning of 2009, the volume *The Professional Education and Development of Teachers of Mathematics* was published, edited by Ruhama Even and Deborah Loewenberg Ball, as the outcome of the equivalently named ICMI Study. As the concern for teacher education was one of the leading forces for the birth of the ICMI in 1908, it might be considered that this study is quite late in coming, being from the fifteenth in the ICMI series.

The premise of the 15th ICMI Study is that teachers are key to the opportunities that students have to learn mathematics. What teachers of mathematics know, care about and do is a product of their experiences and socialisation, together with the impact of their professional education. The Professional Education and Development of Teachers of Mathematics assembles important new international work – development, research, theory and practice – concerning the professional education of teachers of mathematics. As it examines critical areas to reveal what is known and what significant questions and problems warrant collective attention, the volume also contributes to the strengthening of the international community of mathematics educators.

Preparing and maintaining a high-quality, professional teaching force that can teach mathematics effectively and prepare youth for a future of social responsibility is a worldwide challenge. As such, systems of teacher education, both initial and continuing, are essential to the success of mathematics educators and their students. More international exchange of knowledge and information about professional development would certainly benefit both groups and this volume seeks to address that need.

In the conclusions, the editors emphasise three main problems that could profit from stronger and more systematic international connections (e.g. international conferences and international research journals) focused on improving the education and professional development of teachers.

First is the need *to focus the teachers' education on practice*. One crucial objective for the future is that, for a focus on practice to be more consistently possible, teachers need robust examples with which to work, either from their own classrooms or collected systematically from others. Another challenge is the need to develop approaches to teaching practice, i.e. theoretical frameworks that help to focus, analyse and design teaching practices.

A second significant issue is the identification and *development of teachers' developers*. In different countries, a wide range of professionals are responsible for supporting the learning of teachers (e.g. university mathematicians and cognitive scientists). Little is known about this issue at the research level.

Third, a growing need exists for valid *and reliable assessments of the learning of teachers*.

The conference study was held in May 2005 in Brazil with the participation of around 150 researchers and practitioners from around the world. Many came from Europe and witnessed some common features of the education of mathematics teachers after the Bologna Declaration on a European space for higher education: in Europe the initial education of mathematics teachers for both primary and secondary schools is realised within universities. This is not the case in other parts of the world, where the education of primary school teachers is done within secondary schools. In that case, usually, a stronger and systematic in-service program for teacher training is realised.

The Professional Education and Development of Teachers of Mathematics offers a cross-cultural conversation about the education of mathematics teachers around the world with attention given to research, theory, practice and policy.

Table of contents

References

Even, R., Loewenberg Ball, D. (Eds.), (2009), The Professional Education and Development of Teachers of Mathematics: The 15th ICMI Study, Springer; 280 p., Hardcover. ISBN: 978-0-387-09600-1. 90,43€.

News

In issue 70 of this Newsletter (December 2008), the ICMI-ICIAM Study: Educational interfaces between mathematics and industry (2008–2011) was briefly introduced by the co-chairs Alain Damlamian and Rudolf Sträßer. Now the discussion document is available at the website http://www.cim.pt/eimi/.

Deadline

Submissions for participation in the study should be uploaded to the website by 15 September 2009.

Subscription to ICMI-News (free)

To be informed about ICMI activities, send an email to icmi-news-request@mathunion.org with the Subject: subscribe. Previous issues can be seen at http://www.mathunion.org/pipermail/icmi-news.

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The new ICMI website is online and can be accessed via http://www.mathunion.org/ICMI

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Recent and future activities of the European society for Research in Mathematics Education (ERME)

Ferdinando Arzarello
New President of the European society for Research in Mathematics Education

In a previous issue of this newsletter, the previous ERME president Barbara Jaworski wrote about the history and aims of the *European Society for Research in Mathematics Education* (ERME) and about the preparation for CERME 6. The conference was held at the University of Lyon in France from 28 Jan to 1 Feb 2009. 486 people from more then 30 different European and non-European countries attended the meeting. It was a big success not only for the high number of participants but above all for the scientific quality of the two plenaries (by L. Radford and P. Valero) and their reactors (M. Cesar and H. Steinbring), the panel (T. Dreyfus, A. Bikner and J. Monaghan), the invited lecture (by E. Ghys) and the 12-hour activities in the 15 thematic working groups. At the time of writing, the International Program Committee and the Local Organisers of the Conference were collecting the papers for the proceedings, which as usual will be in electronic format. Hopefully more information about the proceedings will be available for the next issue of the newsletter.

During the general assembly of ERME the new president Ferdinando Arzarello was elected and two retiring members of the ERME Board were replaced through election. The current board comprises:

Ferdinando Arzarello (President)	Italy
Nad'a Stelhikova	Czech Rep.
Viviane Durrand–Guerrier	France
Janet Ainley (Treasurer)	UK
Christer Bergsten	Sweden
Constantinos Christou	Cyprus
Maria Alessandra Mariotti	Italy
Heinz Steinbring (Vice-President)	Germany
Dina Tirosh (Responsible for the YERME summer schools)	Israel

Among the policy statements made by the new president in the nomination document he wrote as a candidate, one can read the following:

"If elected, I will pursue such goals and particularly I shall try to promote a team work with the new Board according to the following guidelines:

1 <u>CERME</u>: I believe this form of conference really enables us to work together inclusive of all nations and of young researchers. Possibly the Conference can be an occasion for deepening common research themes among researchers from different parts of Europe, and for improving the collaboration among the colleagues from different countries.
2 <u>Cooperation among the European Societies in the field of education</u>. This issue has been widely discussed (in CERME 6 there is a slot dedicated to this issue). I think that ERME can engage as an active partner in this cooperation.
3 <u>Editorial policy</u>. The issue of a journal of ERME has been discussed in these years. I think that it is opportune that the Board engages in taking a decision, after an overall evaluation of pros and contras.
4 YERME. I do believe in supporting and encouraging young researchers in Math Education; they are our future. So I will strongly support and try to improve all YERME activities, in particular the summer schools that alternate between conferences.
5 Relationships with the professional mathematicians. I think that the interaction with mathematicians on a parity base is an important issue for our community. ERME has a natural partner in this respect, the European Mathematical Society. I will continue the efforts of Barbara in this direction (see her paper in ESN Newsletter n. 70 pag. 43-44) launching a policy of attention and possibly of collaboration with EMS.
6 Relationships with the whole world of mathematics education. I think that ERME, with the richness of different cultures that it embodies, can play an important role within the landscape of mathematics education that is now growing up in the world. I will involve the Board in an analysis of this issue and in considering the opportunity of a 'policy of attention' with respect to ICMI, which officially represents such planetary instances."

During the meeting after the conference, the board decided to accept the proposal for hosting the next conference in 2011 (CERME 7) in Poland at Rzesz_w. The board also nominated the chair and the members of the International Programme Committee (IPC) for CERME 7; they were all accepted. The IPC for CERME 7 comprises:

Tim Rowland (Chair)	United Kingdom
Marku Hannula	Finland
Ivy Kidron	Israel
Susanne Prediger	Germany

Viviane Durand-Guerrier (from the ERME Board)	France
Maria Alessandra Mariotti (from the ERME Board)	Italy
Florence Ligozat	Switzerland
Ewa Swoboda	Poland
José Carrillo	Spain
Carl Winslow	Denmark
Demetra Pitta	Cyprus

The IPC is now starting the work for planning CERME 7. Its first task will be the elaboration of the questionnaires filled out by the participants of CERME 6. The questionnaire asked people to evaluate quantitatively and qualitatively many aspects of the conference: its scientific and logistic organisation, the working group activities and their scientific quality, the interest of the plenary lectures, the possibility of sharing ideas during the conference, etc.

A specific issue in the questionnaire asked for comments and evaluation of the balance between *scientific quality* and *inclusion* in the working groups. This is a crucial issue in ERME activities, which are committed to the three Cs: *Communication*, *Cooperation* and *Collaboration* in research in mathematics education. The CERME spirit, which is deeply inspired by these principles, faces a crucial dilemma when the issue of acceptance of papers for the conference is considered. From an inclusive point of view, all papers should be welcome and all those wishing to participate should have a place. However, from a scientific point of view, papers should be reviewed according to scientific criteria and only those that are of a suitable scientific quality (according to the group leaders) should be accepted and others rejected. In practice this means that authors of rejected papers may not be able to attend the congress since funding depends on an accepted paper. The practice seems to go against the principles of inclusion. The ERME board and the programme committees of CERME conferences have been aware of these issues and have addressed them by creating a two stage review process. For presentation of papers at the congress, a much more open attitude should be taken to the criteria, aiming to include as many participants as possible. At this stage, feedback to prospective participants should detail what is required for a paper to be acceptable for the scientific proceedings following the congress. Papers not meeting these requirements would not be accepted for the proceedings. Of course, it is then up to the group leaders to determine how to make the necessary decisions: what is acceptable for presentation and what are the more strict criteria for publication? They also have to decide how to conduct the work of the group in an inclusive way. Similarly, those organising YERME events have to decide how to ensure both quality and inclusion in practice. The board has therefore agreed to survey participants in CERME 6 and seek views on the processes and issue that are involved. The participants have had the opportunity to comment on this issue in the evaluation questionnaire; moreover, group leaders, both present and past, have been asked to make explicit how they have made decisions and what difficulties (if any) they have met.

A final decision has also been taken about the fifth YERME Summer School (YESS-5): it will be held 18-25 August 2010 in Terrasini, on the mountains near Palermo, Italy. As usual, young researchers can apply for participation at the school, whose activities will be organised in thematic working groups led by an expert in the field. The applicants will be offered the following thematic working groups (with the associated expert who has accepted the role of coaching the group in parentheses):

I) Advanced Mathematics (M. A. Mariotti);
II) Teacher Education (B. Jaworski);
III) Information Technologies (J. B. Lagrange);
IV) Theory and Practice (Günter Törner);
V) Theoretical Perspectives – this could also include linguistic aspects and modelling (J. Ponte).

Each expert is asked to provide the participants with suggestions and remarks about their research plans, questions, etc. within the activity in the working group and to give them an example of research from his/her personal research work (a plenary presentation and discussion).

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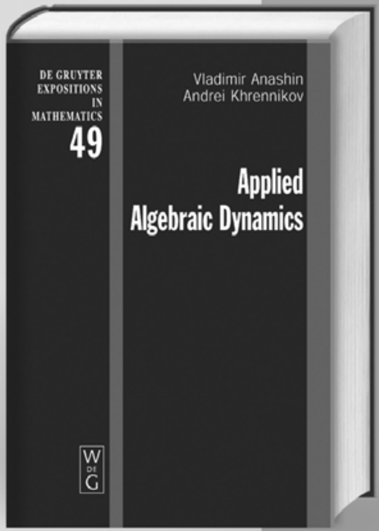
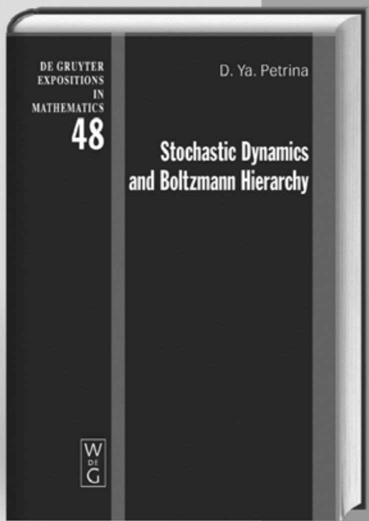


Vladimir Anashin and Andrei Khrennikov

■ Applied Algebraic Dynamics

2009. Approx. 400 pages. Hardcover.
RRP € 99.95 [D] / *US$ 140.00.
ISBN 978-3-11-020300-4
(de Gruyter Expositions in Mathematics 49)
to be published June 2009

This monograph presents recent developments of the theory of algebraic dynamical systems and their applications to computer sciences, cryptography, cognitive sciences, psychology, image analysis, and numerical simulations. The most important mathematical results presented in this book are in the fields of ergodicity, p-adic numbers, and noncommutative groups.

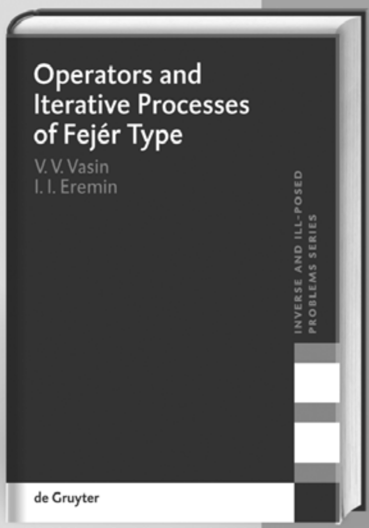


D. Ya. Petrina

■ Stochastic Dynamics and Boltzmann Hierarchy

2009. Approx. 240 pages. Hardcover.
RRP € 99.95 [D] / *US$ 140.00.
ISBN 978-3-11-020804-7
(de Gruyter Expositions in Mathematics 48)
to be published June 2009

This monograph is devoted to the stochastic Boltzmann hierarchy, a special type of initial-boundary value problems for first order systems of partial differential systems. The considered topics have many applications, e.g., the derivation of the classical Boltzmann equation, one of the most important partial differential equations.



Vladimir V. Vasin and Ivan I. Eremin

■ Operators and Iterative Processes of Fejér Type

Theory and Applications

2009. Approx. 160 pages. Hardcover.
RRP € 99.95 [D] / *US$ 140.00.
ISBN 978-3-11-021818-3
(Inverse and III-Posed Problems Series)
to be published June 2009

This book deals with classes of iterative methods for the approximate solution of fixed points equations for operators satisfying a special contractivity condition, the Fejér property. The book is elementary, self-contained and uses methods from functional analysis, with a special focus on the construction of iterative schemes. Applications to parallelization, randomization and linear programming are also considered.

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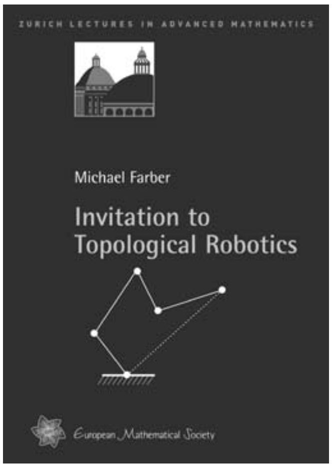
Book Review

Martin Raussen (Aalborg, Denmark)



Michael Farber
Invitation to Topological Robotics

Zurich Lectures in Advanced Mathematics
European Mathematical Society, 2008
viii+134 pages
ISBN 978-3-03719-054-8

Not too long ago, algebraic topology had the reputation of being a difficult mathematical discipline, certainly far removed from practical applications. This has changed quite dramatically in recent years; topological methodology is important in string theory (a topic still remote from applications) but also in the more qualitative aspects of several areas in computer science and engineering. Those aspects that cannot be described quantitatively by a few equations have been investigated using topological tools and "ideology". One of these areas is robotics. Topological robotics is a fairly new mathematical discipline studying topological problems inspired by robotics and engineering, as well as problems of practical robotics viewed through a topological looking glass. It is part of the broader research area named "computational topology".

This book deals with four areas that the author covered in a one-semester lecture course at the ETH Zürich in 2006. The first three areas analyze configuration spaces of mechanical systems applying and developing existing topological machinery. The last area looks at a new topological invariant of topological spaces motivated by applications in robotics and engineering.

The first chapter studies *configuration spaces* of plane mechanical linkages consisting of a number n of bars of fixed length (given by an n-dimensional length vector) and forming a closed polygonal chain. Generically, such a configuration space is a closed manifold of dimension $n–3$; well-understood singularities occur for non-generic length tuples. It is shown that the integral homology of these configuration spaces is free Abelian and a formula for the Betti numbers (depending on the length vector) is derived. The analysis is based on a Morse-theoretical study of a related configuration manifold (with boundary) equipped with an involution.

How much information about a linkage is hidden in the (co)homology of the associated configuration space? Proving a conjecture from 1985 going back to an undergraduate thesis by Walker, it is shown that the *relative sizes* of bars of a linkage are determined, up to a certain equivalence, by the cohomology algebra of the linkage configuration space.

The second chapter describes and proves a result of Gal from 2001 on the Euler characteristics of the configuration spaces $F(X,n)$ of n distinct particles moving in a polyhedron X for all $n > 0$. The main result expresses the power series with coefficients the Euler characteristics of the quotients $B(X,n)$ – under the natural action of the symmetric group – as a rational function given explicitly in terms of local topological properties of the polyhedron X, i.e. the links of its cells. Explicit examples are given in the cases where X is a manifold or a graph. The proof relies on a thorough analysis of the effects of "cut and paste surgeries" on that power series.

Chapter 3 deals with the "*knot theory*" of a planar robot arm. The object of study is again the space of all admissible configurations of a robot arm with a fixed length vector; self-intersections are forbidden. The main result is an *unknotting* theorem for planar robot arms, proven by Connelly, Demaine and Rote in 2003. This configuration space is path-connected and the factor space obtained by dividing out the action of the group of orientation preserving planar isometries is even contractible.

The final chapter deals with *motion planning algorithms* on a given space X. Such an algorithm is a function that to a pair of states associates a motion from one to the other, i.e. a section of the natural path fibration $\pi: PX \to X\times X$ from path space PX. It is easy enough to see that such a section cannot be continuous unless X is contractible. The author defines the topological complexity $\mathbf{TC}(X)$ of X as the minimal number of disjoint subsets of X covering X such that there is an algorithm (section) that is continuous on each of the pieces. The topological complexity of a sphere S^n is at most 3 and is 2 for n odd. More generally, $\mathbf{TC}(X) < \frac{2\dim(X)+1}{r+1}+1$ for an r-connected polyhedron X. This complexity invariant is shown to equal the Schwarz genus of the path fibration, itself a generalization of the well-known Lusternik–Schnirelman category **cat** of a topological space. For a connected Lie group G, $\mathbf{TC}(G) = \mathbf{cat}(G)$; in particular, for the important configuration space SO(3) of a rigid body in space with a fixed point, $\mathbf{TC}(SO(3)) = 4$.

In greater generality, it is shown that the topological complexity is bounded below by the cup-length of zero-divisors in the cohomology ring of $X\times X$. Further information about the invariants can be derived from investigations involving stable cohomology operations; this allows the computation of the topological complexity of lens spaces in many cases. For applications, the calculation of **TC** for the configuration spaces $F(\mathbf{R}^m,n)$ – concerning simultaneous control of multiple objects avoiding collisions – is probably more important. The topological complexity of real projective spaces $\mathbf{RP}^n$ deals with algorithms sweeping one line into another; it is shown that, for $n \neq 1,3,7$, $\mathbf{TC}(\mathbf{RP}^n)$ equals the smallest k such that there exists an immersion from $\mathbf{RP}^n$ into $\mathbf{R}^{k-1}$. The connection between n and k for this problem has been studied intensively over the years; exact results can be found in the literature for $n<24$. Given an immersion, the author sketches the construction of motion planning algorithms on k subsets of projective space.

The booklet concludes with an extensive and very useful bibliography comprising more than 100 titles.

The reviewer is not convinced that engineers in robotics would actually use the results from this book in practice but reading it is certainly a pleasure.

For the note about the author of the review, see page 41.

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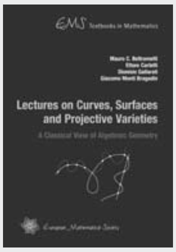


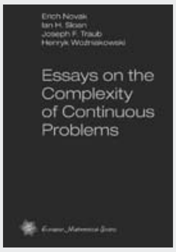
Mauro C. Beltrametti, Ettore Carletti,
Dionisio Gallarati, Giacomo Monti Bragadin
(Università degli Studi di Genova, Italy)
Lectures on Curves, Surfaces and Projective Varieties. A Classical View of Algebraic Geometry (EMS Textbooks in Mathematics)

ISBN 978-3-03719-064-7. 2009. 506 pages. Hardcover. 17.0 x 24.0 cm. 58.00 Euro

This book offers a wide-ranging introduction to algebraic geometry along classical lines. It consists of lectures on topics in classical algebraic geometry, including the basic properties of projective algebraic varieties, linear systems of hypersurfaces, algebraic curves (with special emphasis on rational curves), linear series on algebraic curves, Cremona transformations, rational surfaces, and notable examples of special varieties like the Segre, Grassmann, and Veronese varieties. An integral part and special feature of the presentation is the inclusion of many exercises, not easy to find in the literature and almost all with complete solutions.

The text is aimed at students of the last two years of an undergraduate program in mathematics. It contains some rather advanced topics suitable for specialized courses on the advanced undergraduate or beginning graduate level, as well as interesting topics for a senior thesis. The prerequisites have been deliberately limited to basic elements of projective geometry and abstract algebra. Thus, for example, some knowledge of the geometry of subspaces and properties of fields is assumed.

The book will be welcomed by teachers and students of algebraic geometry who are seeking a clear and panoramic path leading from the basic facts about linear subspaces, conics and quadrics to a systematic discussion of classical algebraic varieties and the tools needed to study them. The text provides a solid foundation for approaching more advanced and abstract literature.



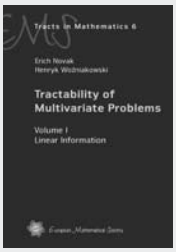
Erich Novak (University of Jena, Germany), Joseph F. Traub (Columbia University, USA), Ian H. Sloan (University of New South Wales, Sydney, Australia), Henryk Woźniakowski (Columbia University USA and Warsaw University, Poland)
Essays on the Complexity of Continuous Problems

ISBN 978-3-03719-069-2. 2009. 106 pages. Hardcover. 17.0 x 24.0 cm. 20.00 Euro

This book contains five essays on the complexity of continuous problems, written for a wider audience.

- Henryk Woźniakowski and the complexity of continuous problems
- Complexity as a new challenge for mathematicians
- A brief history of information-based complexity
- How high is high-dimensional?
- What is information-based complexity?

The first four essays are based on talks presented in 2008 when Henryk Woźniakowski received an honorary doctoral degree of the Friedrich Schiller University of Jena. The focus is on introduction and history of the complexity of continuous problems, as well as on recent progress concerning the complexity of high-dimensional numerical problems. The last essay provides a brief and informal introduction to the basic notions and concepts of information-based complexity addressed to a general readership.



Erich Novak (University of Jena, Germany)
Henryk Woźniakowski (Columbia University, New York, USA, and University of Warsaw, Poland)
Tractability of Multivariate Problems. Volume 1: Linear Information (EMS Tracts in Mathematics Vol. 6)

ISBN 978-3-03719-026-5. 2009. 395 pages. Hardcover. 17.0 x 24.0 cm. 68.00 Euro

Multivariate problems occur in many applications. These problems are defined on spaces of d-variate functions and d can be huge – in the hundreds or even in the thousands. Some high-dimensional problems can be solved efficiently to within ε, i.e., the cost increases polynomially in ε^{-1} and d. However, there are many multivariate problems for which even the minimal cost increases exponentially in d. This exponential dependence on d is called intractability or the curse of dimensionality.

This is the first of a three-volume set comprising a comprehensive study of the tractability of multivariate problems. It is devoted to algorithms using linear information consisting of arbitrary linear functionals. The theory for multivariate problems is developed in various settings: worst case, average case, randomized and probabilistic. A problem is tractable if its minimal cost is *not* exponential in ε^{-1} and d. There are various notions of tractability, depending on how we measure the lack of exponential dependence. For example, a problem is polynomially tractable if its minimal cost is polynomial in ε^{-1} and d. The study of tractability was initiated about 15 years ago. This is the first research monograph on this subject.

Many multivariate problems suffer from the curse of dimensionality when they are defined over classical (unweighted) spaces. But many practically important problems are solved today for huge d in a reasonable time. One of the most intriguing challenges of theory is to understand why this is possible. Multivariate problems may become tractable if they are defined over weighted spaces with properly decaying weights. In this case, all variables and groups of variables are moderated by weights. The main purpose of this book is to study weighted spaces and to obtain conditions on the weights that are necessary and sufficient to achieve various notions of tractability.

The book is of interest for researchers working in computational mathematics, especially in approximation of high-dimensional problems. It may be also suitable for graduate courses and seminars. The text concludes with a list of thirty open problems that can be good candidates for future tractability research.

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Personal column

Please send information on mathematical awards and deaths to Dmitry Feichtner-Kozlov (dfk@math.uni-bremen.de).

Awards

The 2008 Srinivasa Ramanujan Prize will be awarded to **Enrique R. Pujals**, Instituto Nacional de Matematica Pura e Aplicada (IMPA), Brazil. The prize is in recognition of "his outstanding contributions to dynamical systems, especially the characterization of robust dynamics for flows and transformations and the development of a theory of generic systems".

The 2008 Autumn Prize of MSJ was awarded to **Masanao Ozawa**, Professor of Nagoya University, for his outstanding contribution to the mathematical foundation of quantum information.

The Analysis Prize has been awarded to **Ken-iti Sato**, Professor Emeritus of Nagoya University, for his contributions to developments in the theory of Levy processes, to **Hideo Tamura** of Okayama University for his contribution to the asymptotic analysis of the spectrum arising from quantum physics and to **Nakao Hayahi** of Osaka University for many works on various nonlinear dispersive equations.

The AMS Steele Prize for Mathematical Exposition was awarded to **I. G. MacDonald** for his book *Symmetric Functions and Hall Polynomials* (second edition, Clarendon Press, Oxford University Press, 1995).

Laure Saint-Raymond of the Ecole Normale Superieure in Paris has received the 2009 AMS Ruth Lyttle Satter Prize. Awarded every two years, the prize recognises an outstanding contribution to mathematics research by a woman in the previous five years.

Jeremy J. Gray of the Open University and the University of Warwick has received the 2009 AMS Albert Leon Whiteman Memorial Prize. Presented every three years by the American Mathematical Society, the prize honours a notable exposition that centres on the history of mathematics and that reflects exceptional mathematical scholarship.

Ismat Beg (Lahore University of Management Sciences) has received the 2008 Gold Medal in the Field of Mathematics awarded by the Pakistan Academy of Sciences.

The Ferran Sunyer Prize 2009 was awarded to **Tim Browning** (Bristol University) for his monograph *Quantitative Arithmetic of Projective Varieties*.

Burkhard Wilking (University of Munster) and **Martin Zirnbauer** (University of Cologne) have received the 2009 Gottfried-Wilhelm-Leibniz prize. The award has been given in recognition of groundbreaking work in mathematical physics (Zirnbauer) and differential geometry (Wilking).

Eric Demaine was awarded a 2007–2008 International Francqui Chair by the Francqui Foundation (http://www.francquifoundation.be). He is an associate professor of computer science at the Massachusetts Institute of Technology.

Assaf Naor (Courant Institute of Mathematical Sciences) and **Boaz Klartag** (Tel Aviv University) have been awarded the 2008 Salem Prize for their contributions to the structural theory of metric spaces and their applications to computer science.

Wojciech Lubawski (Kraków), **Marcin Preisner** (Wrocław) and **Krzysztof Putyra** (Kraków) have been awarded the first Marcinkiewicz Prizes of the Polish Mathematical Society for students' research papers.

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2009 to the Russian-French mathematician **Mikhail Leonidovich Gromov** (IHÉS, Paris) for "his revolutionary contributions to geometry".

Deaths

We regret to announce the deaths of:

Maurice Austin (UK, 31 October 2008)
Beno Eckmann (Switzerland, 25 November 2008)
Heinz Gumin (Germany, 24 November 2008)
Edmund Hlawka (Austria, 19 February 2009)
Benno Klotzek (Germany, 13 June 2008)
Ilppo Simo Louhivaara (Germany, 19 December 2008)
Ronald Maude (UK, 15 March 2008)
Claus Müller (Germany, 6 February 2008)
Douglas Munn (UK, 26 October 2008
Peter Pleasants (UK, 20 April 2008)
Bodo Renschuch (Germany, 30 January 2009)
André Revuz (France, 27 October 2009)

Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the addresses madalina.pacurar@econ.ubbcluj.ro or madalina_ pacurar@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files).

By the end of 2009, the Forthcoming Conferences section will be removed from the EMS Newsletter. An electronic announcement of upcoming events is already set up and fully operative on the EMS web site www.euro-math-soc.eu. Everyone is encouraged to submit their events to that site.

June 2009

1–5: School on Combinatorics, Automata and Number Theory (CANT'09), Liege, Belgium
Information: M.Rigo@ulg.ac.be;
http://www.cant.ulg.ac.be/cant2009/

1–5: Geometry and Topology 2009, Münster, Germany
Information: sfb478mi@math.uni-muenster.de;
http://www.math.ku.dk/~erik/muenster/

1–5: From Braid Group to Teichmuller Spaces, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

1–5: Hypercyclicity and Chaos for Linear Operators and Semigroups, Valencia, Spain
Information: http://www.upv.es/entidades/HOLC09/

2–5: Holomorphically Symplectic Varieties and Moduli Spaces, Lille, France
Information: http://math.univ-lille1.fr/~markushe/MOD2009/

3–12: Four Advanced Courses on Quasiconformal Mappings, PDE and Geometric Measure Theory, CRM, Barcelona, Spain
Information: http://www.crm.cat/acmappings/

8–9: Journées ANR de Théorie Géométrique des Groupes, Strasbourg, France
Information: http://www-irma.u-strasbg.fr/article773.html

8–11: 25th Nordic and 1st British-Nordic Congress of Mathematicians, Oslo, Norway
Information: http://www.math.uio.no/2009/

8–12: International Conference AutoMathA: from Mathematics to Applications, Liege, Belgium
Information: M.Rigo@ulg.ac.be;
http://www.cant.ulg.ac.be/automatha/

8–12: Geometrie Algebrique en Liberte XVII, Leiden, Netherlands
Information: deul@strw.LeidenUniv.nl; http://www.lorentzcenter.nl/lc/web/2009/326/description.php3?wsid=326

8–12: Noncommutative Lp Spaces, Operator Spaces and Applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

9–12: MAFELAP 2009 – 13th Conference on the Mathematics of Finite Elements and Applications, Brunel University, England
Information: Carolyn.sellers@brunel.ac.uk;
http://people.brunel.ac.uk/~icsrsss/bicom/mafelap2009

11–13: Representation Theory in Mathematics and in Physics, Strasbourg, France
Information: http://www-irma.u-strasbg.fr/article717.html

11–13: XXèmes Rencontres Arithmétiques de Caen: p-adic Cohomologies, Caen, France
Information: http://www.math.unicaen.fr/~rencarit/2009/

14–20: Stochastic Analysis and Random Dynamical Systems, Lviv, Ukraine
Information: sard@imath.kiev.ua;
http://www.imath.kiev.ua/~sard/

14–20: Geometric Group Theory, Będlewo, Poland
Information: topics09@math.uni.wroc.pl;
http://www.math.uni.wroc.pl/ggt/

14–20: Harmonic Analysis, Geometric Measure Theory and Quasiconformal Mappings, CRM, Barcelona, Spain
Information: http://www.esf.org/activities/esf-conferences/details/2009/confdetail308.html?conf=308&year=2009

14–27: ESI Workshop on Large Cardinals and Descriptive Set Theory, Vienna, Austria
Information: esi2009@logic.univie.ac.at;
http://www.logic.univie.ac.at/conferences/2009_esi/

15–18: 5th International Conference on Dynamical Systems and Applications, Constanta, Romania
Information: cgherghina@gmail.com, cristinatoncu@canals.ro;
http://www.univ-ovidius.ro/faculties/civil_eng/conferinta%20iunie%202009/Home.html

15–18: 3rd International Conference on Mathematics and Statistics, Athens, Greece
Information: atiner@atiner.gr;
http://www.atiner.gr/docs/Mathematics.htm

15–19: Waves 2009, Pau, France
Information: helene.barucq@inria.fr, julien.diaz@inria.fr;
https://waves-2009.bordeaux.inria.fr/

15–19: The Analytic Theory of Automorphic Forms (at the 65th Birthday of Roelof Bruggeman), Woudschoten, The Netherlands
Information: rb65@math.uu.nl; http://www.math.uu.nl/rb65.html

15–19: Dynamics and Complex Geometry, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

15–19: International Summer School on Operator Algebras and Applications, Lisbon, Portugal
Information: http://oaa.ist.utl.pt

15–20: Strobl09 Conference on Time-Frequency, Salzburg, Austria
Information: strobl09.mathematik@univie.ac.at; http://nuhag.eu/strobl09

16–19: Kähler and Sasakian Geometry in Rome, INdAM Workshop, Roma, Italy
Information: ksgr@mat.uniroma1.it; http://www.mat.uniroma1.it/~ksgr/

16–22: 6th International Workshop on Optimal Codes and Related Topics (OC 2009), Varna, Bulgaria
Information: oc2009@math.bas.bg; http://www.moi.math.bas.bg/oc2009/oc2009.html

16–25: International Workshop on Resonance Oscillations and Stability of Nonsmooth Systems, London, UK
Information: http://www.ma.ic.ac.uk/~omakaren/rosns2009/index.html

17–20: Fourth Statistical Days at the University of Luxembourg, University of Luxembourg, Luxembourg
Information: http://sma.uni.lu/stat4/

17–21: Modern Complex Analysis and Operator Theory and Applications IV, El Escorial (Madrid), Spain
Information: http://www.uam.es/dragan.vukotic/conf-mecano09.html

21–25: Second African International Conference on Cryptology, (AfricaCrypt 2009), Gammarth, Tunisia
Information: sami.ghazali@certification.tn; http://www.africacrypt2009.tn/en/index.php?id=116

22–26: Topology of Algebraic Varieties: a Conference in Honour of the 60th Birthday of Anatoly Libgober (LIB60BER), Huesca, Spain
Information: jaca2009@math.uic.edu; http://www.math.uic.edu/~jaca2009/

22–26: The Poetry of Analysis (in Honour of Antonio Córdoba on the Occasion of his 60th Birthday), Madrid, Spain
Information: cordobaconf@gmail.com; http://www.uam.es/gruposinv/ntatuam/cordoba/index.html

22–26: Dynamics and Geometry in the Teichmuller Space, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

22–27: Geometry "In Large", Topology and Applications (Devoted to the 90th Birthday of Alexey Vasilievich Pogorelov), Kharkiv, Ukraine
Information: pmc2009@ilt.kharkov.ua; http://www.ilt.kharkov.ua/pmc2009/index_engl.html

22–27: First Conference of the Euro-American Consortium for Promoting the Application of Mathematics in Technical and Natural Sciences, Sozopol, Bulgaria
Information: conference@eac4amitans.org; http://www.eac4amitans.org/

22–27: Sage Days Barcelona, UPC, Barcelona, Spain
Information:http://www.uam.es/personal_pdi/ciencias/engonz/sagedays16

22–27: 3rd Nordic EWM Summer School for PhD Students in Mathematics, University of Turku, Finland
Information: http://www.math.utu.fi/projects/ewm/organization.html

23–26: 9th Central European Conference on Cryptography, Trebic, Czech Republic
Information: cecc09@fme.vutbr.cz; http://conf.fme.vutbr.cz/cecc09/

25–27: Current Geometry: The X Edition of the International Conference on Problems and Trends of Contemporary Geometry, Naples, Italy
Information: http://www.levi-civita.org/CG2009.html

27–July 3: XXVIII Workshop on Geometric Methods in Physics, Białowieża, Poland
Information: http://wgmp.uwb.edu.pl/index.html

28–July 2: 20th International Workshop on Combinatorial Algorithms (IWOCA 2009), Hradec nad Moravici, Czech Republic
Information: iwoca09@iwoca.org; http://www.iwoca.org/iwoca09

28–July 3: Affine Isometric Actions of Discrete Groups, Ascona, Switzerland
Information: http://www.unige.ch/math/folks/arjantse/ascona09/

28–July 4: 6th St. Petersburg Workshop on Simulation, Saint Petersburg, Russia
Information: pws2009@statmod.ru; http://pws.math.spbu.ru

29–July 3: 4th International Conference on Logic, Computability and Randomness, Taranto, Italy
Information: http://www.scienzetaranto.uniba.it/index.php/notizie-di-rilievo.html

29–July 3: Evolution Equations and Mathematical Models in the Applied Sciences, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

30–July 4: Barcelona Financial Engineering Summer School: the Practice of Derivatives Modelling, CRM, Barcelona, Spain
Information: http://www.crm.cat/bfess09

July 2009

1–3: The 2009 International Conference of Applied and Engineering Mathematics, London, UK
Information: WCE@iaeng.org; http://www.iaeng.org/WCE2009/ICAEM2009.html

1–3: International Conference on Design Theory and Applications, Galway, Ireland
Information: http://larmor.nuigalway.ie/~detinko/DeSign.htm

2–4: 6th International Conference on Applied Financial Economics, Samos, Greece
Information: afe@ineag.gr; http://www.ineag.gr/AFE/index.html

3–7: International Conference on Mathematical Control Theory and Mechanics, Suzdal, Russia
Information: http://mctm2009.vlsu.ru

5–8: Algebra and Analysis Around the Stone-Cech Compactification, Cambridge, UK
Information: garth@maths.leeds.ac.uk, stferri@uniandes.edu.co; http://matematicas.uniandes.edu.co/~stferri/donaconference.html

5–10: 22nd British Combinatorial Conference, St Andrews, UK
Information: bcc2009@mcs.st-and.ac.uk; http://bcc2009.mcs.st-and.ac.uk

5–10: The Second European Set Theory Meeting: in Honour of Ronald Jensen, Będlewo, Poland
Information: http://www.esf.org/index.php?id=5749

6–8: International Conference on Education and New Learning Technologies (EDULEARN09), Barcelona, Spain
Information: http://www.iated.org/edulearn09

6–10: 26th Journées Arithmétiques, Saint-Etienne, France
Information: ja2009@univ-st-etienne.fr; http://ja2009.univ-st-etienne.fr

6–10: Equivariant Gromov-Witten Theory and Symplectic Vortices, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

6**–11: Conference on Algebraic Topology (CAT'09)**, Warsaw, Poland
Information: cat09@mimuw.edu.pl; http://www.mimuw.edu.pl/~cat09/

6–11: International Conference on Topology and its Applications, Ankara, Turkey
Information: icta@hacettepe.edu.tr; http://www.icta.hacettepe.edu.tr

7–10: VII Geometry Symposium, Kirsehir, Turkey
Information: geometry@ahievran.edu.tr; http://fef.ahievran.edu.tr/sempozyum/AnaSayfa.htm

8–11: 3rd International Conference on Experiments/Process/System Modelling/Simulation/Optimization, Athens, Greece
Information: info2009@epsmso.gr; http://www.epsmso.gr/2009/

8–11: 5th International Meeting on Lorentzian Geometry (GeLoBa2009), Taranto, Italy
Information: http://www.dm.uniba.it/geloba2009/

9–12: XVIIIth Oporto Meeting on Geometry, Topology and Physics, Porto, Portugal
Information: http://www.fc.up.pt/cmup/omgtp/2009/

13–17: International Conference on Complex Analysis, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

13–18: 7th International ISAAC Congress, London, UK
Information: info@isaac2009.org; http://www.isaac2009.org

14–17: 24th Summer Conference on Topology and its Applications, Brno, Czech Republic
Information: slapal@fme.vutbr.cz, Eva.Tomaskova@law.muni.cz; http://www.umat.feec.vutbr.cz/~kovar/webs/sumtopo

14–24: Banach Algebras 2009, Będlewo, Poland
Information: asoltys@amu.edu.pl; http://www.siue.edu/MATH/BA2009/

16–31: XII Edition of the Italian Summer School, Santo Stefano del Sole, Avellino, Italy
Information: school09it@diffiety.ac.ru; http://school.diffiety.org/page3/page0/page93/page93.html

20–24: 21st International Conference on Formal Power Series and Algebraic Combinatorics, RISC Linz, Schlosz Hagenberg, Austria
Information: ppaule@risc.uni-linz.ac.at; http://www.risc.jku.at/about/conferences/fpsac2009/

20–24: 6th International Conference on Positivity and its Applications, Madrid, Spain
Information: positivity6@mat.ucm.es; http://www.mat.ucm.es/~cpositi6/

20–24: Advanced Course on Optimization: Theory, Methods and Applications, CRM, Barcelona, Spain
Information: http://www.crm.cat/OPT2009

20–24: EQUAdIFF 12, Brno, Czech Republic
Information: http://www.math.muni.cz/~equadiff/

20–August 28: CEMRACS, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

25–29: 6th International Conference on Lévy Processes: Theory and Applications, Dresden, Germany
Information: http://www.math.tu-dresden.de/levy2010

26–31: Geometry, Field Theory and Solitons, Leeds, UK
Information: http://www.maths.leeds.ac.uk/pure/geometry/solitons

27–31: Stochastic Processes and their Applications, Berlin, Germany
Information: kongresse@tu-servicegmbh.de; http://www.math.tu-berlin/SPA2009

August 2009

1–15: Groups St Andrews 2009 in Bath, Bath, England
Information: gps2009@mcs.st-and.ac.uk; http://www.groupsstandrews.org

2–8: p-Adic Geometry and Homotopy Theory, Loen, Norway
Information: http://folk.uio.no/rognes/yff/alexandra.html

3–7: Logic and Mathematics Conference, York, UK
Information: http://maths.york.ac.uk/www/York2009

3–7: Chern-Simons Gauge Theory: 20 Years After, Bonn, Germany
Information: http://www.hausdorff-center.uni-bonn.de/event/2009/gauge_theory

3–8: 16th International Congress of Mathematical Physics, Prague, Czech Republic
Information: icmp09@ujf.cas.cz; http://www.icmp09.com/

4–10: International Conference of Mathematical Sciences (ICMS 2009), Istanbul, Turkey
Information: other@maltepe.edu.tr; http://mathsciencesconf.maltepe.edu.tr/

9–14: Model Theory, Będlewo, Poland
Information: http://www.esf.org/index.php?id=5675

10–14: Nonlinear Problems for p-Laplace and Laplace, Linköping, Sweden
Information: conf-p-laplace@mai.liu.se; http://www.mai.liu.se/TM/conf09/

10–20: International Conference and Young Scientists School "Theory and Computational Methods for Inverse and Ill-posed Problems", Akademgorodok, Novosibirsk, Russia
Information: http://math.nsc.ru/conference/onz09/engl.html

13–17: 7th International Algebraic Conference in Ukraine, Kharkov, Ukraine
Information: iaconu2009@univer.kharkov.ua; http://iaconu2009.univer.kharkov.ua

17–21: The 12th Romanian-Finnish Seminar – International Conference on Complex Analysis and Related Topics, Turku, Finland
Information: romfin2009@lists.utu.fi; http://www.math.utu.fi/projects/romfin2009/

24–26: International Symposium on Analysis and Theory of Functions, Istanbul, Turkey
Information: http://fen-edebiyat.iku.edu.tr/atf/

24–28: Summer School on Geometry and Rigidity of Groups, Münster, Germany
Information: http://www.math.uni-muenster.de/u/sauerr/conferences/groups-09.html

25–28: 14th General Meeting of European Women in Mathematics, Novi Sad, Serbia
Information: romfin2009@lists.utu.fi; ewm2009@im.ns.ac.yu

30–September 4: Algebraic Groups and Invariant Theory, Monte Veritá, Ascona, Switzerland
Information: baur@math.ethz.ch, donna.testerman@epfl.ch; http://www.math.ethz.ch/~baur/AGIT/

31–September 4: Vlasovia, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

31–September 4: International Conference on Generalized Functions (GF2009), Vienna, Austria
Information: http://www.mat.univie.ac.at/~gf2009/

31–September 6: International School on Geometry and Quantization, University of Luxembourg, Luxembourg
Information: geoquant@uni.lu; http://math.uni.lu/geoquant/school

September 2009

2–4: New Trends in Model Coupling: Theory, Numerics & Applications (NTMC'09), Paris, France
Information: mcparis09@ann.jussieu.fr; http://www.ann.jussieu.fr/mcparis09/

2–4: Workshop in Nonlinear Elliptic PDEs, A Celebration of Jean-Pierre Gossez's 65th Birthday, Bruxelles, Belgium
Information: wnpde09@ulb.ac.be; http://wnpde09.ulb.ac.be

3–5: Complex and Harmonic Analysis 2009, Archanes, Crete, Greece
Information: http://fourier.math.uoc.gr/ch2009

3–6: International Conference on Theory and Applications of Mathematics and Informatics (ICTAMI 2009), Alba-Iulia, Romania
Information: www.uab.ro/ictami

4–9: 2nd Dolomites Workshop on Constructive Approximation and Applications, Alba di Canazei (Trento), Italy
Information: http://www.math.unipd.it/~dwcaa09

7–9: 13th IMA Conference on the Mathematics of Surfaces, York, UK
Information: ralph@cs.cf.ac.uk; http://ralph.cs.cf.ac.uk/MOSXIIIcall.html

7–11: 3rd International Conference on Geometry and Quantization (GEOQUANT), University of Luxembourg, Luxembourg
Information: geoquant@uni.lu; http://math.uni.lu/geoquant

7–11: History of Mathematical Periodics: Problems and Methods, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

7–12: Advanced School on Homotopy Theory and Algebraic Geometry, Sevilla, Spain
Information: http://congreso.us.es/htag09/php/index.php

8–12: 4th International Course on Mathematical Analysis in Andalucia, Jerez de la Frontera, Spain
Information: http://cidama.uca.es

10–12: Quantum Topology and Chern-Simons Theory, Strasbourg, France
Information: http://www-irma.u-strasbg.fr/article744.html

14–16: Complex Data Modelling and Computationally Intensive Methods for Estimation and Prediction (S.Co.2009), Milan, Italy
Information: http://mox.polimi.it/sco2009/

14–18: Quiver Varieties, Donaldson-Thomas Invariants and Instantons, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

14–18: Conference on Probabilistic Techniques in Computer Science, CRM, Barcelona, Spain
Information: http://www.crm.cat/ccomputer

16–20: MASSEE International Congress on Mathematics (MICOM 2009), Ohrid, Macedonia
Information: http://micom2009.smm.org.mk

17–20: 17th Conference on Applied and Industrial Mathematics (CAIM 2009), Constanta, Romania
Information: caim2009@anmb.ro; www.anmb.ro

18–22: International Conference of Numerical Analysis and Applied Mathematics 2009 (ICNAAM 2009), Crete, Greece
Information: tsimos.conf@gmail.com; http://www.icnaam.org/

21–25 September: Galois and Arithmetical Theory of Differential Equations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

24–30: 6th International Conference on Functional Analysis and Approximation Theory (FAAT 2009), Acquafredda di Maratea, Italy
Information: faat2009@dm.uniba.it;
http://www.dm.uniba.it/faat2009

28–October 2: Approximation and Extrapolation of Sequences and Convergent and Divergent Series, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

October 2009

4–8: Kolmogorov Readings. General Control Problems and their Applications (GCP-2009)", Tambov, Russia
Information: http://www.tambovopu2009.narod.ru/

5–9: Loss of Compactness in Nonlinear Problems: New Trends and Applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

12–14: Workshop on Computational Optimization (WCO-2009), Mragowo, Poland
Information: http://www.imcsit.org/pg/227/181

12–16: Algebra, Geometry and Mathematical Physics, Będlewo, Poland
Information: tralle@matman.uwm.edu.pl;
http://www.agmf.astralgo.eu/bdl09/

19–23: Fall School in Weak Kam Theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–24: Advanced Course on Shimura Varieties and L-functions, CRM, Barcelona, Spain
Information: http://www.crm.cat/acshimura

22–24: Partial Differential Equations and Applications: International Workshop for the 60th birthday of Michel Pierre, Vittel, France
Information: Colloques@loria.fr;
http://edpa2009.iecn.u-nancy.fr/

November 2009

9–13: Geometry, Dynamics and Group Representations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

16–20: ANGD Mathrice, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; www.cirm.univ-mrs.fr

23–25: MoMaS Scientific Days, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; www.cirm.univ-mrs.fr

23–27: Mathematics and Astronomy: a Joint Long Journey, Madrid, Spain
Information: www.astromath2009.com

30–December 4: Number Theory and Applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; www.cirm.univ-mrs.fr

December 2009

8–11: Operators and operator algebras in Edinburgh, Edinburgh, Scotland, U.K.
Information: s.white@maths.gla.ac.uk
http://www.maths.gla.ac.uk/~saw/ooae/

9–12: Advanced Course on Algebraic Cycles, Modular Forms and Rational Points on Elliptic Curves, CRM, Barcelona, Spain
Information: http://www.crm.cat/accycles

14–18: Meeting on Mathematical Statistics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

14–18: Workshop on Cycles and Special Values of L-series, CRM, Barcelona, Spain
Information: http://www.crm.cat/wklseries

April 2010

11–18: Algebraic Combinatorics – Designs and Codes (ALCOMA10), Thurnau – Bayreuth, Germany
Information: http://www.mat.uc.pt/~cmf/01MatrixTheory

June 2010

17–19: Coimbra Meeting on 0-1 Matrix Theory and Related Topics, University of Coimbra, Portugal
Information: cmf@mat.uc.pt;
http://www.mat.uc.pt/~cmf/01MatrixTheory

21–26: "Alexandru Myller" Mathematical Seminar Centennial Conference, Iasi, Romania
Information: http://www.math.uaic.ro/~Myller2010/

26–30: 2010 International Conference on Topology and its Applications, Nafpaktos, Greece
Information: http://www.math.upatras.gr/~nafpaktos/

July 2010

4–7: 7th Conference on Lattice Path Combinatorics and Applications, Siena, Italy
Information: latticepath@unisi.it;
http://www.unisi.it/eventi/lattice_path_2010

Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

By the end of 2009, the Recent Books section will be removed from the EMS Newsletter. Please do not submit books for review to the editors any longer.

S. Albeverio et al., Eds.: Traces in Number Theory, Geometry and Quantum Fields, *Aspects of Mathematics E 38, Vieweg, Wiesbaden, 2007, 223 pp., EUR 54.90, ISBN 978-3-8348-0371-9*
This book presents an overview of different ongoing research directions centred around the main theme: traces, determinants and their zeta functions. The collection of papers in the book arises from an activity held at the Max-Planck Institute for Mathematics in Bonn in the autumn of 2005. The authors discuss many topics, including traces in number theory, traces in dynamical systems (based on the Ruelle zeta function and its generalisation), traces in non-commutative geometry (ranging from spectral triples for III-factors and non-commutative geometry on trees and buildings to quantum groups), traces on pseudo-differential operators and associated invariants of underlying manifolds (e.g. invariants of CR manifolds produced by non-commutative residue or defect formulas and zeta values for boundary valued problems) and gauge fields and quantum field theory (in particular analysis of the Schwinger-Dyson equation in perturbative quantum field theory or various applications in QCD and interacting particle systems). (pso)

A. Andretta, K. Kearnes, D. Zambella, Eds.: Logic Colloquium 2004, *Lecture Notes in Logic 29, Cambridge University Press, Cambridge, 2008, 220 pp., GBP 40, ISBN 978-0-521-88424-2*
This book contains invited surveys, tutorials and selected research papers from the 2004 Annual European Summer Meeting of the Association for Symbolic Logic held in Torino, Italy, covering recent reflection of the work in the fields of model theory, foundations and universal algebra. More specifically: J. Baldwin surveys recent development in the study of abstract elementary classes and raises a number of interesting questions; T. Bartoszynski and S. Shellah use forcing to prove that density of Hausdorff ultrafilters in Rudin-Keisler order is independent of ZFC; and A. Berarducci presents various results on 0-groups and o-minimal structures. On the foundational note, A. Cantini and L. Corosilla present an interesting extension of constructive set theory, A. Setzer starts a series of papers on a proof of theoretically strong extensions of Martin-Löf type theory and G. Tamburrini with E. Datteri revive Turing's and von Neumann's early ideas on computational modelling of intelligent behaviours. R. Cluckers discusses the theory of parametrized p-adic integrals, B. Larose and L. Haddad investigate algorithmic complexity of strong colouring problems in hypergraphs of permutation groups and P. Lipparni introduces a new tolerance identity for algebras. F. Stephan constructs in an oracle-relative world a set whose weak truth-table degree's Hausdorff-dimension is between ¼ and ½. The short note by I. Tomašić clarifies aspects of invariants of the theory of constructible sheaves and cohomologies, which are definable in the language of rings. The proceedings ends with the R. Willard tutorial introducing some deep contemporary results in universal algebra to non-specialists. (ppaj)

D. Z. Arov, H. Dym: J-Contractive Matrix Valued Functions and Related Topics, *Encyclopedia of Mathematics and Its Applications 116, Cambridge University Press, Cambridge, 2008, 575 pp., GBP 70, ISBN 978-0-521-88300-9*
The purpose of this book is to describe the theory of matrix valued functions (mvf's). The main topics treated in the book are J-contractive and *J*-inner mvf's. The book also deals with the theory of meromorphic functions applied to mvf's and with generalisation of certain theorems from functional analysis. After an introduction, the second chapter introduces a substantial number of definitions and transformations concerning various classes of complex matrices. The considered classes are constructed on the base of a self-adjoint and unitary matrix *J* (which is called a signature matrix). The Potapov–Ginsburg transform and various linear fractional transformations are described here for various classes of complex matrices. The first part of the third chapter presents a rich overview of definitions and basic properties of the classes of meromorphic scalar and matrix vector functions. The classes associated with the names of Nevanlinna, Smirnov, Schur, Carathéodory and Hardy are considered. The authors study, as is usual in algebra, representations and factorization of these classes and the concepts of minimal common multiple and maximal common divisor of a given set of mvf's. The notions of denominators and scalar denominators are developed in this part.

In the next chapter, they develop ideas from the second part. It contains a lot of profound facts from the theory of *J*-contractive and *J*-inner mvf's for the later applications. The fifth chapter starts with definitions of a positive kernel and the reproducing kernel Hilbert space (RKHS). The authors describe a number of RKHSs, which together with de Brange space are used in the next chapter for functional models of conservative systems. Interpolation problems are discussed in the seventh chapter. The generalised interpolation problems are discussed in the second part of this chapter. The special generalised Carathéodory interpolation problem is compared with the Krein helical extension problem. Darlington representations and related inverse problems for *J*-inner mvf's, criteria for strong regularity and formulas for entropy functionals and their extremal values form the content of the concluding shorter chapters of the book. The whole book is well written with theoretical results of a high standard. It is necessary for the reader to be proficient in the theory of matrices, meromorphic functions and functional analysis. It can be recommended to scientists, engineers and students preparing their doctoral theses. (jz)

J. Borwein, K. Devlin: The Computer as Crucible. An Introduction to Experimental Mathematics, *A. K. Peters, Wellesley, 2008, 208 pp., USD 29,95, ISBN 978-1-56881-343-1*
This little brochure is aimed at a much broader part of the mathematical community than previously published books on experimental mathematics like *Experimental Mathematics in Action* or *Mathematics by Experiment: Plausible Reasoning in the 21st Century* written by the first author. It can be used at student seminars as a starting point of independent research based on the heavy use of computers. Attention is given to things connected with pi, to recognition of important constants from their decimal expansion or to the zeta function. Results are presented without

proofs and with a maximal effort to make them understandable. The book also contains parts denoted 'Explorations'. They give the reader a chance to try to apply the learned material on some additional problems. It is a pleasant and readable book for any working mathematician who wants to know something more about the use of computers for generating hypotheses on relations among known and less known special numbers. (jive)

J. M. Borwein et al., Eds.: Communicating Mathematics in the Digital Era*, A. K. Peters, Wellesley, 2008, 325 pp., USD 49, ISBN 978-1-56881-410-8*

This book contains 19 talks presented at the conference on the topic described in the title organised at Aveiro in August 2006. It is divided into three parts: Electronic Publishing and Digital Libraries, Technology Enhancements for Disseminating Mathematics, and Educational and Cultural Framework. The first part starts with some historical facts on digital publishing, together with a survey of the effort to convert existing classical sources to a digital form. It also contains examples of drawbacks of electronic publishing and comments on the influence of big specialised companies on the publishing of mathematical journals. It is very interesting to learn the experience from the French NUMDAM project or to read about the present state of the EMIS. There is a discussion of some national activities; all of them are aimed at presenting broad and unlimited access to information relevant for the mathematical community. The book also contains information on new methods of communication and learning in our digital era. I can recommend the book to libraries of universities and institutions that plan to issue or join digitalisation projects. Any such project could be prepared on a better level if it is based on the experience of others. (jive)

R. Brown: Topology and Groupoids, *BookSurge LLC, Charleston, 2006, 512 pp., USD 25.46, ISBN 1-4196-2722-8*

This book is a third edition following the 1968 edition (*Elements of Modern Topology*) and the 1988 edition (*A Geometric Account of General Topology, Homotopy Types and the Fundamental Groupoid*). The words of the author gives the general theme of the book: "a major emphasis of this and previous editions: the modelling of geometry, principally topology, by the algebra of groupoids". A groupoid is a small category where every morphism is an isomorphism. It is a generalisation of groups regarded as a category having one object so that "groupoids can model more of the geometry than groups alone. This leads not only to more powerful theorems with simpler and more natural proofs but also to new theorems and new landscapes." The book starts with basic topological notions (topology on reals, metric spaces, connectedness, compactness and constructions) and continues to objects of more geometric nature (cells, complexes, projective spaces and smash products). After an explanation of the basic categorical concepts, a fundamental groupoid is defined and studied. The next objects of explanation are cofibrations (with fibrations of groupoids), computation of the fundamental groupoid, covering spaces and covering groupoids, orbit spaces and orbit groupoids. At the end of the book, there are five pages of conclusions, an appendix (with basic facts about sets, functions and universal properties of products and sums), a glossary of some terms, a bibliography with almost 400 items and two indexes. Sections end with useful exercises and chapters with notes. It is no surprise that the book has reached its third edition. (mihus)

S. Buyalo, V. Schroeder: Elements of Asymptotic Geometry*, EMS Monographs in Mathematics, European Mathematical Society, Zürich, 2007, 200 pp., EUR 58, ISBN 978-3-03719-036-4*

This book is devoted to a study of the generalisation of classical hyperbolic geometry developed by M. Gromov. It is based on lectures given at St. Petersburg and Zürich. The classical hyperbolic space H^n of dimension n can be realised as a ball in R^n and there is an induced conformal structure on the sphere forming its boundary. Isometries of H^n then induce conformal maps on the boundary. Gromov noticed that there is a specific inequality in terms of mutual distances of four points in H^n, which captures essential asymptotic properties of the hyperbolic space. The inequality is then used as a key tool in a definition of generalised hyperbolic spaces studied in the book. In the first part, the authors describe properties of the Gromov hyperbolic spaces in a close analogy with the homogeneous model H^n. In the second part, more general tools are used and some applications are given (e.g. non-existence theorem for embeddings). Bibliographic notes at the end of each chapter are quite useful. (vs)

H. Casanova, A. Legrand, Y. Robert: Parallel Algorithms, *Chapman & Hall/CRC Numerical Analysis and Scientific Computing, CRC Press, Boca Raton, 2008, 337 pp., USD 79.95, ISBN 978-1-58488-945-8*

This book covers in eight chapters both theoretical and practical knowledge in computing on distributed memory systems. It extracts the main ideas and principles of parallel algorithms developed over the last few decades. The theoretical part presents a rigorous, yet accessible, treatment of theoretical models of parallel computation on two classical examples, namely PRAM and the sorting network. In several chapters, the reader is introduced to fundamental results and common problems in scheduling. Advanced topics cover crucial divisible load scheduling and steady-state scheduling, which are applicable to the master-worker application model. The reader will find important topology and performance discussions about different network models. Subsequent chapters use this knowledge to build parallel algorithms on two popular abstractions: ring and grid logical topologies. But the authors perfectly explain not only homogeneous models (which are everyday problems on clusters of identical nodes) but also load balancing on heterogeneous platforms (connecting different clusters or many different workstations). This book can serve as a very good teaching book or a source of useful material for graduate students and researchers in parallel distributed memory architectures. It contains many schemes, diagrams and pictures for better understanding, including many practical examples, case studies and exercises. (mmad)

J. H. Conway, H. Burgiel, C. Goodman-Strauss: The Symmetries of Things*, A. K. Peters, Wellesley, 2008, 426 pp., USD 69, ISBN 978-1-56881-220-5*

This book is divided into three parts with an increasing level of difficulty. The first part, which needs almost no prerequisites, is devoted to symmetries of repeating planar and spherical patterns and their classification. The authors have chosen an original approach based on the concepts of orbifold and orbifold signature. The key ingredients in the classification process is the so-called magic theorem, together with the classification theorem for topological surfaces (Conway's zip proof is included). The result is the well-known fact that there are exactly seventeen distinct possible types of repeating planar patterns. The second part requires a basic understanding of group theory (somewhat surprisingly, there is no group theory in the first part). Planar and spherical symmetries are now described in a group-theoretic language (using their presentations, i.e. generators and relations). This part also provides an enumeration of coloured planar and spherical patterns. The third part, which is difficult reading even for a professional mathematician, is devoted to repeating patterns in other spaces. The topics discussed in this part include patterns in the hyperbolic plane, Archimedean tilings, generalised Schäfli symbols, three-dimensional crystallographic groups, flat universes and higher dimensional groups. The book contains many new results. Moreover, it is printed on glossy pages with a large number of beautiful full-colour illustrations, which can be enjoyed even by non-mathematicians. (as)

M. Coste, T. Fukui, K. Kurdyka, C. McCrory, A. Parusiński, L. Paunescu: Arc Spaces and Additive Invariants in Real Algebraic and Analytic Geometry*, Panoramas and Syntheses, no. 24, Société Mathématique de France, Paris, 2007, 125 pp., EUR 37, ISBN 978-2-85629-236-5*

This book contains contributions prepared for a meeting in Aussois devoted to various properties and applications of analytic arcs on algebraic varieties. Two papers contain lecture notes of courses presented there. The contribution by M. Coste is devoted to topology of real algebraic sets. Algebraically constructible functions are the main tools used here giving rise to combinatorial topological invariants. The paper by K. Kurdyka and A. Parusiński introduces arc-symmetric sets and arc-analytic mappings. They are used to define a new topology that is stronger than Zariski topology. They show that Z_2-homology classes of compact Nash manifolds can be represented by arc-symmetric semialgebraic sets. There are also two survey papers. The one written by C. McCrory and A. Parusiński describes algebraically constructible functions and conditions for a topological space to be homeomorphic to a real algebraic set. The paper by T. Fukui and L. Paunescu describes germs of maps, which can be transformed to real analytic ones by a locally finite number of blowing-ups. (vs)

J. M. Delgado Sánchez, T. Dominguez Benavides, Eds.: Advanced Courses of Mathematical Analysis III, *Proceedings of the Third International School, World Scientific, New Jersey, 2008, 194 pp., USD 78, ISBN 978-981-281-884-7*

This volume contains an extensive overview of research in different areas of mathematical analysis: An introduction to discrete holomorphic local dynamics in one complex variable (M. Abate); Notes in transference of bilinear multipliers (O. Blasco); Generating functions of Lebesgue spaces by translations (J. Bruna); A new look at compactness via distances to function spaces (C. Angosto and B. Cascales); Spaces of smooth functions (E. Harboure); Domination by positive operators and strict singularity (F. L. Hernandez); The Hahn-Banach Theorem and the sad life of E. Helly (L. Narici and E. Becknstein); The Banach space L_p (E. Odell); Problems on hypercyclic operators (H.N. Salas); Operator spaces: basic theory and applications (B. M. Schreiber); Mathematics and markets: existence and efficiency of competitive equilibrium (A. Villar); and Ideals in F-Algebras (W. Zelazko). The proceedings could be of high interest to graduate students and researchers in the aforementioned areas of mathematical analysis and its applications. (oj)

V. K. Dzyadyk, I. A. Shevchuk: Theory of Uniform Approximation of Functions by Polynomials*, Walter de Gruyter, Berlin, 2008, 480 pp., EUR 68, ISBN 978-3-11-020147-5*

This book contains selected chapters from the monograph *Introduction into the Theory of Uniform Approximation of Functions by Polynomials* (published by the first author in 1977 in Russian). The second author has complemented them with some chapters from his book *Approximation by Polynomials and Traces of Functions Continuous on a Segment* (published in 1992) and has made the final redaction of the text, since the first author passed away in 1998. On 480 pages, the authors present classical material like Tschebyshev theory and Weierstrass theorems including their further development. In other chapters, the authors treat various topics, e.g. smoothness of functions and direct and inverse theorems on the approximation of periodic functions. The last chapter is devoted to approximation by polynomials. The book also contains 40 pages of references and a two page subject index. At the end of each chapter there are interesting additional remarks. Explanations and proofs are clear and detailed. The authors try to trace the origins of theorems, which is of interest in the case of Russian authors (many sources were published in Russian). The book could be of interest for all who work in approximation theory and related fields; it should not be overlooked by university libraries. (jive)

B. Enriquez, Ed.: Quantum Groups*, IRMA Lectures in Mathematics and Theoretical Physics, vol. 12, European Mathematical Society, Zürich, 2008, 133 pp., EUR 38, ISBN 978-3-03719-047-0*

This book contains four contributions on various aspects of quantum group theory. The first one contains lecture notes from the course given by P. Etingof on the structure and classification of tensor categories and is written by D. Calaque. It introduces main definitions, basic results and important examples. It contains a discussion of Ocneanu rigidity for fusion categories, module categories and weak Hopf algebras, module categories and their Morita equivalence, applications to representation theory of groups, the lifting theory, and the theory of Frobenius-Perron dimension and its application for classification of fusion categories.

There are three more research papers in the book. The one written by J. Lieberum discusses Drinfeld associators. They are needed for the Kontsevich construction of a universal Vassiliev invariant. The paper contains an explicit description of a rational even Drinfeld associator in (a completion of) the universal enveloping algebra of the Lie superalgebra gl(1|1) and a discussion of its relation with the Viro generalisation of the multivariable Alexander polynomial. The contribution by A. Odesskii and Vl. Rubtsov contains a description of the relation between integrable systems and elliptic algebras. They construct (classical and quantum) integrable systems on a large class of elliptic algebras. The last paper by N. Andruskiewitsch and F. Dumas contains a computation of the group of automorphisms of the positive part of the quantum enveloping algebra of certain simple (complex) finite dimensional Lie algebras. (vs)

***M. Farber: Invitation to Topological Robotics**, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008, 132 pp., EUR 32, ISBN 978-3-03719-054-8*

Topological robotics is a new mathematical field inspired by recent problems in robotics – for instance the problem of the number of solutions of certain problems, obstacle avoidance and singularity free regions. The quality of a solution of such a problem depends on the topological structure of the configuration space of the considered mechanism. In this book, sophisticated topological methods are used to study topological properties of configuration spaces of planar linkages. The author presents some very interesting results about connections between linkages and their configuration spaces. To illustrate the nature of such results let us cite one: the relative size of bars of a linkage is determined (up to an equivalence) by the cohomology algebra of its configuration space.

The material of the book is based on lectures given by the author on topics of topological robotics at ETH Zürich in 2006. The exposition of material is very simple on the engineering side but requires a good knowledge of algebraic topology and other modern mathematical techniques. It is therefore readable for mathematicians who are interested in applications of topology in other fields of mathematics. Proofs of statements are either given or cited. The author himself says: "The book is intended as an appetizer and will introduce the reader to many fascinating topological problems motivated by engineering". The book is clearly written and gives a good number of examples. The author provides 103 references. This will help the reader to orient himself in the subject. (ak)

***J. Flachsmeyer: Origami und Mathematik. Papier falten – Formen gestalten**, Berliner Studienreihe zur Mathematik, band 20, Heldermann, Lemgo, 2008, 239 pp., EUR 20, ISBN 978-3-88538-120-4*

There are many books dealing with origami, i.e. the art of folding realistic objects (animals) and geometric structures (polygons and polyhedrons) from a single sheet of paper without cutting or pasting. Origami seems to belong to recreational mathematics although it does not need any mathematics. The book under review differs from others on this topic. There are interesting calculations and results like the golden ratio and values of some goniometric functions. The book will appeal to the geometric intuition of the reader, containing a lot of figures and coloured photos of folded structures. It can be strongly recommended to everybody interested in recreational mathematics and elementary geometry. (lboc)

***I. Gallagher, L. Saint-Raymond: Mathematical Study of the Betaplane Model – Equatorial Waves and Convergence Results**, Mémoires de la Société Mathématique de France, no. 107, Société Mathématique de France, Paris, 2006, 116 pp., EUR 26, ISBN 978-2-85629-228-0*

In this very nice book the authors study a model describing the motion of the ocean in the equatorial zone. For the description of the flow, a viscous Saint Venant model for rotating fluid is used. It takes into account gravity, the Coriolis force and depth variation of the fluid $h = H(1+\varepsilon\eta)$ around a mean depth H. The system can be rewritten in a form similar to the Navier-Stokes equations with a singular perturbation of order $1/\varepsilon$. The first chapter of the book is introductory. The second studies the spectral decomposition of the singular perturbation. The third chapter is devoted to the study of well-posedness of the formal limiting problem as ε goes to 0. In the last chapter the limiting process is justified. The book is well written, nicely structured and very clear. Any researcher in the field would enjoy it. The book could also be appreciated by graduate students interested in the study of rotating fluids. (pkap)

***M. Gardner: Origami, Eleusis, and the Soma Cube**, Cambridge University Press, Cambridge, 2008, 234 pp., GBP 25, ISBN 978-0-521-73524-7*

Martin Gardner is a popular American writer specialising in "recreational mathematics". The basis for this book comes from the "Mathematical Games and Recreations" column published in *Scientific American* magazine from 1957 to 1986. The book presents plenty of well-known and some less well-known mathematical problems. Some of the subjects discussed in the book are: the five platonic solids, tetraflexagons, digital roots, the Soma cube, the golden ratio, mazes, magic squares, eleusis, origami and mechanical puzzles. The presentation of problems is completed with many illustrative pictures and photos in every chapter. It is possible to find traditional solutions of every problem and also some newer views, explanations and proofs of solved problems. Every chapter also includes an extensive bibliography. Martin Gardner has already published more than 60 books encouraging readers to play with mathematics and Origami, Eleusis, and the Soma Cube is a well prepared addition to his bibliography. (jhr)

***J. Gasqoui, H. Goldschmidt: Infinitesimal Isospectral Deformations of the Grassmannian of 3-Planes in R^6**, Mémoires de la Société Mathématique de France, no. 108, Société Mathématique de France, Paris, 2007, 92 pp., EUR 27, ISBN 978-2-85629-232-7*

The main topic treated in this book is a further study of (infinitesimal) spectral deformations of Riemannian symmetric spaces of compact type. Both of the authors have treated this topic and its connection to the Radon transform already in other papers and in their previous monograph (published in the Princeton Ann. of Math. Studies). Here they concentrate on the case of real Grassmannian manifolds $G_{n,2n}$ of n dimensional planes in R^{2n} and its reduced version $\breve{G}_{n,2n}$ (the quotient of $G_{n,2n}$ by the action sending an n-plane to its orthogonal complement). It is shown in the book that for n=3 the Grassmannian $\breve{G}_{3,3}$ admits nontrivial isospectral deformations. Another question treated in the book is a study of properties of invariant differential forms on $\breve{G}_{n,2n}$. The authors study necessary and sufficient conditions for exactness of forms in terms of a Radon transform. (vs)

***M. Gyllenberg, D.S. Silvestrov: Quasi-Stationary Phenomena in Nonlinearly Perturbed Stochastic Systems**, de Gruyter Expositions in Mathematics, vol. 44, Walter de Gruyter, Berlin, 2008, 579 pp., EUR 128, ISBN 978-3-11-020437-7*

This book is devoted to studies of quasi- and pseudo-stationary phenomena for nonlinearly perturbed stochastic processes and systems. The methods used are based on exponential asymptotics for the nonlinearly perturbed renewal equation. These methods enable one to get mixed ergodic and large deviation theorems and asymptotics in these theorems for nonlinearly perturbed regenerative processes and then for nonlinearly perturbed semi-

Markov processes and Markov chains. Finally, these results are applied to nonlinearly perturbed queuing systems, population dynamics and epidemic models, as well as nonlinearly perturbed risk processes. The book contains an extended bibliography of works in the area. (vb)

J. Haglund: The q,t-Catalan Numbers and the Space of Diagonal Harmonics – with an Appendix on the Combinatorics of Macdonald Polynomials*, University Lecture Series, vol. 41, American Mathematical Society, Providence, 2008, 167 pp., USD 39, ISBN 978-0-8218-4411-3*

Macdonald polynomials form a distinguished family of orthogonal polynomials in several variables. Properties of those polynomials, combinatorics connected with them and a circle of ideas around them were developed systematically in the last few decades, together with various interesting applications. This is the main topic of the book, which is based on lectures given by the author at the University of Pennsylvania. The book starts with a summary of the properties of q-analogues of counting functions and symmetric functions. Macdonald polynomials and diagonal harmonics, together with an historical background are treated next. The core of the book starts with a discussion of the properties of the q,t-Catalan numbers and q,t-Schröder numbers. The main topic is then the so-called "shuffle" conjecture, which is presented together with its proof in the case of hook shapes. The book contains a lot of exercises (with solutions of many of them in appendix C). (vs)

G. Harder: Lectures on Algebraic Geometry I*, Aspects of Mathematics E 35, Vieweg, Wiesbaden, 2008, 290 pp., EUR 54.90, ISBN 978-3-528-03136-7*

This book series, which grew out of lectures held by the author at Bonn University, is meant to serve as an introduction to modern algebraic geometry. Two thirds of the first of the two volumes contains a detailed introduction to homological algebra, cohomology of groups, cohomology of sheaves and algebraic topology. Only the last chapter describes applications of this machinery to the algebraic geometry of compact Riemann surfaces. It basically covers classical material known from the 19th century, in particular the divisor class group (the Picard group) and the theory of Abelian varieties. The book is aimed at students with a minimal background knowledge of analysis, algebra and set theory, and the material and techniques are developed carefully from very basic notions. (pso)

J. R. Hindley, J. P. Seldin: Lambda-Calculus and Combinators. An Introduction*, Cambridge University Press, Cambridge, 2008, 345 pp., GBP 35, ISBN 978-0-521-89885-0*

This book is a second edition of the *Introduction to Combinators and λ-Calculus* by the same authors. As its name suggests, it is an introduction to two closely related systems: λ-calculus and combinatory logic (CL). The description of these systems goes hand in hand, starting from the basics (β- and weak reductions, the fixed-point theorem and Böhm's theorem, and undecidability results), through proof theory of λβ and CLw and questions of extensionality, to an introduction of several approaches to typing in the two systems. The final chapters of the book introduce the semantics and model theory of λ-calculus and CL, paying special attention to the model D_∞ discovered by Dana S. Scott, touching lightly the relation of weak normalisation, confluence and consistency theorems for the arithmetical version of CL to the consistency of Peano Arithmetic. The book's 345 pages are organised into 16 chapters and five appendices. The text is interleaved with instructive exercises, the harder of which are solved in the last appendix. The book is well-written and offers a broad coverage backed by an extensive list of references. It could serve as an excellent study material for classes on λ-calculus and CL as well as a reference for logicians and computer scientists interested in the formal background for functional programming and related areas. (ppaj)

J.-B. Hiriart-Urruty: Les Mathématiques du Mieux Faire, Vol. 1*, Ellipses, Paris, 2007, 132 pp., EUR 15, ISBN 978-2-7298-3667-2*

Over some 120 pages, the reader is introduced to control theory. In mathematical terms, it describes an investigation of extremes of real functions and functionals. The book is well arranged and offers a systematic approach to the problem, which is classified in detail (e.g. linear programming, convex minimization and differentiable and non-differentiable optimization). Then the conditions of minimization without constraints are treated. Results on existence and uniqueness and the 1st and 2nd order necessary and sufficient conditions of minimization are given. The same topics on minimization under constraints follow. The author looks for simplicity and applicability instead of dry mathematic rigour and maximal generality. However, the proofs of some theorems are original and the contents of the book is rich and inspiring. Accordingly, there are a lot of interesting and useful examples and applications in the book; it is well written and reads well. It covers the topics for students after the first two years of a university mathematics course and contains extended and revised material of the author's book *L'optimization* (1996). As the title shows, it is written in French and is followed by the 2nd volume, which is devoted to extremes of functionals. (jdr)

J.-B. Hiriart-Urruty: Les Mathématiques du Mieux Faire, Vol. 2*, Ellipses, Paris, 2008, 164 pp., EUR 15, ISBN 978-2-7298-3737-2*

The second volume introduces the reader to the theory of optimal control and to the theory of variation, reflected in many mathematical, physical, technical and economical problems. It is designed to improve the education of engineers more than mathematicians, so not all theorems are proven; there is more attention on the approach, the problems and the ideas than on the technique. Being devoted to the problem of minimization of functionals, this volume systematically treats diverse formulations of optimal control, existence of solutions, the necessary conditions, the sufficient conditions and the Pontrjagin minimum principle. Many interesting and useful examples of solutions to diverse minimization problems are discussed. The book is based on lectures taught by the author in a Masters program of variation analysis and control. Like the first volume, this one is skilfully compiled and treated. The whole book is a valuable contribution to the literature in this branch of mathematics. (jdr)

L. Hörmander: Notions of Convexity*, Modern Birkhäuser Classics, Birkhäuser, Boston, 2007, 414 pp., EUR 34.90, ISBN 978-0-8176-4584-7*

This is a reprint of the 1994 edition of the same book devoted to convexity and its applications to analysis. Convexity is meant in rather an abstract sense: a function is convex if it attains its maximum always on the boundary of a given domain, even if another function from some linear space L (of functions satisfying some specific properties, like linearity) is subtracted from it. For L being the space of affine functions, such a notion is equivalent to the usual convexity and the first two chapters of the book develop the classical theory of convex functions and sets. With other choices of L, deep analogies and generalisations of ordinary convexity are obtained. Thus, chapter 3 develops the theory of subharmonic functions (L being, in this case, the linear space of all harmonic functions). Based on this, chapter 4 presents the theory of plurisubharmonic functions and related notions like pseudoconvexity. These concepts have many important applications in partial differential equations, the theory of functions of several complex variables and harmonic analysis.

Chapters 5, 6 and 7 are devoted to more special topics (some of them being systematically developed in a book for the first time) like convexity with respect to a linear group and convexity with respect to differential operators. The book concludes with an exposition of the Trépreau results on microlocal analysis. Undoubtedly, this monograph written by a master of the field will be useful not only for specialists in analysis but also (in particular the first chapters; the rest of the book requires familiarity with distributions, differential geometry and pseudodifferential calculus) to students willing to learn this important, classical but still lively subject where so many prominent mathematicians of the last century made their contribution. (mzah)

J. E. Humphreys: Representations of Semisimple Lie Algebras in the BGG Category O*, Graduate Studies in Mathematics, vol. 94, American Mathematical Society, Providence, 2008, 289 pp., USD 59, ISBN 978-0-8218-4678-0*

This book is devoted mainly to a study of properties of the so-called category O of g-modules, where g is a complex finite dimensional semisimple Lie algebra. The category O was introduced in the 70s by I. N. Bernstein, I. M. Gel'fand and S. I. Gel'fand and study of its properties soon led to the famous Kazhdan-Lusztig conjecture and its beautiful proof by methods of algebraic analysis and algebraic geometry. In the first part of the book, attention is concentrated to the highest weight modules (category O, basic character formulae, BGG reciprocity, the BGG description of composition factors for Verma modules, the BGG resolution of finite-dimensional modules, translation functors, Schubert varieties and the Kazhdan-Lusztig polynomials).

The second part of the book is more advanced. It starts with a treatment of the relative (parabolic) version of the previous theory and then it presents a discussion of projective functors and their connection to translation functors and principal series modules, the role of tilting modules, fusion rules and formal characters, and shuffling, twisting and completion functors and their comparison. The last chapter presents a rapid survey of many recent topics (with many references), including classification of primitive ideals of the universal enveloping algebra, highest weight categories for algebras of other types (e.g. for Kac-Moody algebras, Virasoro algebra and quantised enveloping algebras), quivers and the Kozsul duality. The book is written in a precise and understandable style and it should be very helpful for anybody interested in representation theory. (vs)

I. M. Isaacs: Finite Group Theory*, Graduate Studies in Mathematics, vol. 92, American Mathematical Society, Providence, 2008, xi+350 pp., USD 59, ISBN 978-0-8218-4344-4*

There are many textbooks on group theory. This one is aimed at graduate students who know the basics (however, there is an appendix that covers the introductory topics) and who seek solid knowledge of classical techniques. The book is written with obvious care and the exposition is clear and covers many topics that are not very accessible or not very well explained elsewhere. For example, chapter 1 is essentially about Sylow theorems and consequences but it also covers the Chermak-Delgado measure. Chapter 2 is on subnormality (the Wielandt zipper lemma and the theorems of Baer, Zenkov and Lucchini). Chapter 3 presents the standard material on split extensions (e.g. Hall subgroups and Glaubermann's lemma). Chapter 4, called Commutators, contains, amongst other things, various consequences of the Hall-Witt identity, the theorem of Mann and Thompson's PxQ lemma.

Chapter 5 introduces the technique of transfer, digresses into infinite groups by proving theorems of Schur and Dietzmann, continues with the Burnside theorem on normal p-complements, introduces focal subgroups and finishes with the Frobenius theorem on normal p-complements. The nilpotency of Frobenius kernel is proved in chapter 6. In fact, a part of the proof relies upon the properties of the Thompson subgroup, to which chapter 7 is devoted. This chapter also presents a proof of Burnside's theorem that is independent of linear representation theory (the proof follows arguments of Goldschmidt, Matsuyama and Bender). The ensuing chapter contains standard material on permutation groups, followed by a discussion of the orbital graph and subdegrees, including the proofs of the theorems of Weiss and Manning. The penultimate chapter starts with a definition of the generalised Fitting subgroup and of components and continues with a discussion of their basic properties. It also proves Wielandt's results on automorphism towers, Schenkman's theorem, Thompson's result on corefree maximal subgroups (in connection to the Sims conjecture) and some facts about strong conjugacy. The final chapter returns to the transfer (theorems of Yoshida, Huppert's result on metacyclic groups and connections with the group ring). Each chapter has several subsections (A, B, C, D and sometimes E, F and G) and each subsection finishes with a list of problems, many of which are far from routine. (ad)

B. Kaltenbacher, A. Neubauer, O. Scherzer: Iterative Regularization Methods for Nonlinear Ill-Posed Problems*, Radon Series on Computational and Applied Mathematics, vol. 6, Walter de Gruyter, Berlin, 2008, 194 pp., EUR 78, ISBN 978-3-11-020420-9*

This book deals with regularization techniques for stabilizing ill-posed non-linear inverse problems and their numerical realisation via iterative procedures. After a short introduction, chapter 2 is devoted to the nonlinear Landweber iteration and analysis of its convergence and convergence rates. Chapter 3 deals with modified Landweber methods showing how to get better convergence rates or to get results under weaker assumptions (regularization in Hilbert scales, iteratively regularized Landweber iteration and the Landweber–Kaczmarz method). To make numerical realization more efficient, the authors propose in chapter 4 several variants of Newton type methods (the Levenberg–Marquardt method, the iteratively regularized Gauss–Newton method and its generalisations and Broyden´s method for ill-posed problems). Chapter 5 is devoted to another approach increasing efficiency of numerical realisation of ill-posed problems, namely to multigrid methods. Since many practical problems are related to shape recovery, chapter 6 describes applications of the level set methods and their adaptation for solving inverse problems. Finally, chapter 7 presents two applications: reconstruction of transducer pressure fields from Schlieren tomography and a parameter estimation problem from nonlinear magnetics. The book is intended as a text book for graduate students and for specialists working in the field of inverse problems. (jhas)

S. Khrushchev: Orthogonal Polynomials and Continued Fractions. From Euler's Point of View*, Encyclopedia of Mathematics and Its Applications 122, Cambridge University Press, Cambridge, 2008, 478 pp., GBP 70, ISBN 978-0-521-85419-1*

This book is devoted to two special tools of mathematics: orthogonal polynomials and continued fractions, on the background of historic efforts by Euler (the famous paper "Continued fractions, observation" is included in the book as an appendix). The range of themes covered is very wide including Markoff's theorem on the Lagrange spectrum, Abel's theorem on integration in finite terms and Chebyshev's theory of orthogonal polynomials. Five chapters are devoted to continued fractions (Real numbers, Algebra, Analysis, Euler and Euler's influence). The sixth chapter studies P fractions (constructed by polynomials). The last two chapters contain a study of orthogonal polynomials and orthogonal polynomials on the unit circle. There are exercises at the end of every chapter. (jkof)

C. Lubich: From Quantum to Classical Molecular Dynamics – Reduced Models and Numerical Analysis*, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008, 144 pp., EUR 32, ISBN 978-3-03719-067-8*

To describe dynamics of molecules in classical mechanics, it is necessary to solve Newton's equations. On a quantum level, the corresponding equation is the Schrödinger equation of many-body quantum dynamics. To develop corresponding numerical methods for the latter poses a difficult problem for numerical analysis. This book contains a description of numerical methods for some intermediate models between the classical and quantum cases. The first chapter contains a short review of the necessary notions from quantum mechanics. In chapter 2, time-dependent variational principles are used for a construction of basic intermediate models. Numerical methods for the linear time-dependent Schrödinger equation (in moderate dimensions) are introduced in chapter 3 and the case of nonlinear reduced models is treated in chapter 4. Hagedorn wave packets are used for the time-dependent Schrödinger equation in a semi-classical scaling in the last chapter. The book illustrates the usefulness of mutual interplay between numerical analysis and its applications in mathematical physics. (vs)

E. Miller, V. Reiner, B. Sturmfels, Eds.: Geometric Combinatorics*, IAS/Park City Mathematics Series, vol. 13, American Mathematical Society, Providence, 2007, 691 pp., USD 99, ISBN 978-0-8218-3736-8*

The notion of geometric combinatorics is quickly getting a much broader meaning. At present it covers not only a structure of polytopes and simplicial complexes but many further topics and interesting connections to other fields of mathematics. It is worth looking at the contents of this book, which contains written versions of the lecture series presented at a three-week program organised at the IAS/Park City Mathematics Institute in 2004. Counting of lattice points in polyhedra and connections to computational complexity is discussed in lectures by A. Barvinok. Root systems, generalised associahedra and combinatorics of clusters form the topic of the lectures by S. Fomin and N. Reading. Combinatorial problems inspired by topics from differential topology (Morse theory) and differential geometry (the Hopf conjecture) are studied by R. Forman. M. Haiman and A. Woo treat topics around Catalan numbers and Macdonald polynomials (the positivity conjecture). D.N. Kozlov discusses in his lectures chromatic numbers, morphism complexes and Stiefel-Whitney characteristic classes. Lectures by R. MacPherson cover topics such as equivariant homology, intersection homology, moment graphs and linear graphs and their cohomology. R.P. Stanley discusses topics connected with hyperplane arrangements and M.L. Wachs treats poset topology. The book ends with a contribution by G.M. Ziegler on convex polytopes. The book contains an enormous amount of interesting material (including a substantial numbers of exercises). (vs)

T. Nishiura: Absolute Measurable Spaces*, Encyclopedia of Mathematics and Its Applications 120, Cambridge University Press, Cambridge, 2008, 274 pp., GBP 60, ISBN 978-0-521-87556-1*

A number of results related to the central notion of absolute measurable spaces, or universal measurability, is presented. A separable metrizable space is called absolutely measurable (or absolutely null) if its homeomorphic embedding to any separable metrizable space Y is measurable (null) with respect to every complete continuous finite Borel measure on Y. The relative notions for subsets of a given separable metrizable space are called universally measurable and universally null sets. Let us roughly indicate some of the more concrete main topics covered by the book. Results showing the existence of universally null sets, which are big with respect to cardinality, the Hausdorff dimension and topological dimension appear in chapters 1, 5 and 6. The possibility of testing universal measurability by a smaller family of measures, namely by so-called positive measures, or by the homeomorphic transforms of a particular measure in concrete spaces (e.g. in n-dimensional manifolds) is discussed thoroughly in chapters 2 and 3 and some results are applied in chapter 4. The appendices cover most of the needed preliminaries. (phol)

A. Pietsch: History of Banach Spaces and Linear Operators, *Birkhäuser, Boston, 2007, 855 pp., EUR 109, ISBN 978-0-8176-4367-6*

This comprehensive book aims to cover the history of Banach spaces and operators (and not the history of mathematicians working in the field) and it is mainly focused on the second half of the 20th century. Nevertheless, the book starts with chapters on the birth of Banach spaces and their historical roots and basic results, followed by chapters on topological concepts (centred on weak topologies), including an overview of classical Banach spaces. The heart of the book (300 pages) consists of chapters on basic results from the post-Banach period and on selected topics of modern Banach space theory. The book ends (250 pages) with chapters on victims of politics, a description of scientific schools in Banach space theory, short biographies of some famous mathematicians and short sections including a list of the people working on Banach spaces who have been awarded the Fields Medal or who have been invited to lecture at the ICM and a list of names sorted by country. An extensive bibliography divided into several sections (textbooks, monographs, historical books, collected works, etc.) includes approximately 2600 items. The book can serve as a reference and guide with some historical flavour both for researchers and as an introduction for graduate students learning Banach space theory. As the author has made an enormous effort to cover the main trends of this part of functional analysis, it is quite natural that the content of the book reflects the taste of the author and that some parts of this field are not treated with enough detail. It is really a difficult task to describe in detail the contents of this voluminous book (of 855 pages). A very detailed review by J. Lindenstrauss can be found in Mathematical Reviews MR2300779. (jl)

M.A. Pinsky: Introduction to Fourier Analysis and Wavelets, *Graduate Studies in Mathematics, vol. 102, American Mathematical Society, Providence, 2009, 376 pp., USD 69, ISBN 978-0-8218-4797-8*

This book provides a self-contained treatment of classical Fourier analysis and a concrete introduction to a number of topics in harmonic analysis at the upper undergraduate or beginning graduate level. Necessary prerequisites to using the text are rudiments of the Lebesgue measure and integration on the real line. Frequently, more than one proof is offered for a given theorem to illustrate the multiplicity of approaches. The text contains numerous examples and more than 175 exercises that form an integral part of the text. It can be expected that a careful reader will be able to complete all these exercises. The book contains six chapters. The first chapter provides a reasonably complete introduction to Fourier analysis on the circle and its applications to approximation theory, probability and plane geometry (the isoperimetric theorem). Readers with some sophistication but little previous knowledge of Fourier series can begin with chapter 2 and anticipate a self-contained treatment of the n-dimensional Fourier transform and many of its applications.

Much of modern harmonic analysis is carried out in the L_p spaces for p ≥ 2, which is the subject of chapter 3. The Poisson summation formula treated in this chapter provides an elegant connection between Fourier series on the circle and Fourier transforms on the real line, culminating in the Landau asymptotic formulas for lattice points on a large sphere. This chapter also contains the interpolations theorems of Riesz-Thorin and Marcinkiewicz, which are applied to discuss the boundedness of the Hilbert transform and its application to the L_p convergence of Fourier series and integrals.

In chapter 4 the author merges the subjects of Fourier series and Fourier transforms by means of the Poisson summation formula in one and several dimensions. Chapter 5 explores an application of Fourier methods to probability theory. The central limit theorem, the iterated log theorem and Berry–Esseen theorems are developed using the suitable Fourier-analytic tools. The final chapter furnishes a gentle introduction to wavelet theory, depending only on the L^2 theory of the Fourier transform (the Plancherel theory). The basic notions of scale and location parameters demonstrate the flexibility of the wavelets approach to harmonic analysis. The book can be used as a text for courses, seminars or even solo study and is designed for graduate students in mathematics, physics, engineering and computer science. (knaj)

W. Rautenberg: Messen und Zählen. Eine Einfache Konstruktion der Reellen Zahlen*, Berliner Studienreihe zur Mathematik, band 18, Heldermann, Lemgo, 2007, 188 pp., EUR 22, ISBN 978-3-88538-118-1*

The aim of this book is to provide a rigorous foundation of the real number system. The first step is a treatment of natural numbers and their properties, which are stated as axioms (only the last chapter outlines the possibility of a construction of natural numbers on a set-theoretical basis). The real numbers are then defined as infinite sequences of decimal digits. At this point, it is possible to introduce the ordering of real numbers and prove the supremum property. The operations of addition and multiplication are first defined for numbers with finite decimal expansion (by shifting the decimal point, the problem is reduced to addition or multiplication of natural numbers); by means of a limit process, they are extended to all real numbers. The book also discusses additional interesting topics, in particular the definition of powers with real exponents, exponential and logarithm functions, Egyptian fractions, computer implementation of arithmetic operations and the uniqueness of real numbers up to isomorphism. The text is elementary and contains numerous remarks on the history of the subject. (asl)

D. S. Richeson: Euler's Gem. The Polyhedron Formula and the Birth of Topology*, Princeton University Press, Princeton, 2008, 317 pp., USD 27.95, ISBN 978-0-691-12677-7*

The title of this book "Euler's Gem" refers to the well-known formula stating that the numbers of vertices, edges and faces of a convex polyhedron satisfy the relationship $V-E+F=2$. The author decided not only to trace the history of this simple and elegant theorem but also to present its generalisations and applications in different branches of mathematics, in particular in topology, graph theory and differential geometry. He covers an extremely wide range of topics, including the discovery of the five regular convex polyhedra, Descartes' version of the polyhedron formula, the graph-theoretic version of Euler's formula and its relation to the four colour theorem (with details on Kempe's attempt to prove the theorem), the topological classification of surfaces, knot theory, vector fields on surfaces and the Poincaré-Hopf theorem, the role of Euler's number in global results of the differential geometry of curves and surfaces, n-dimensional manifolds, the Poincaré conjecture and many more. The text requires no knowledge of higher mathematics. There are many proofs or sketches of proofs of simpler theorems and the author tries to provide explanations of more difficult theorems, omitting technical details. This book is a pleasure to read for professional mathematicians, students of mathematics or anyone with a general interest in mathematics. (asl)

L.H. Rowen: Graduate Algebra – Noncommutative View, *Graduate Studies in Mathematics, vol. 91, American Mathematical Society, Providence, 2008, 648 pp., USD 85, ISBN 978-0-8218-4153-2*

This is the second volume (parts IV–VI) of a comprehensive treatment of graduate algebra of which the first volume (parts I–III) dealt with commutative algebra and appeared as Vol. 73

in the same series in 2006. This volume presents a number of classic results from non-commutative algebra but also some of the highlights of much later achievements (e.g. Kemer's solution of Specht's conjecture, Tits' alternative for algebraic groups and Zelmanov's solution to the restricted Burnside problem). Similarly, as polynomials in finitely many variables over a field F form the basic example of a commutative F-algebra, the full matrix algebra $M_n(F)$ is the key example for the non-commutative setting. The author therefore dedicates most of part IV to the classic Wedderburn-Artin structure theory of semisimple rings and modules, the Jacobson density theorem and the Goldie theorems (in chapters 14–16). In chapter 17, he introduces a new point of view: presentations of algebras and groups in terms of generators and relations, the Gelfand-Kirillov dimension and decision problems. Chapter 18 brings more powerful machinery to the scene: that of tensor products of modules and algebras.

Part V is dedicated to representation theory. First, classic results on group algebras and group representations are presented in chapter 19, including a section on representations of the symmetric group and an appendix on the Tits' alternative. Chapter 20 treats classical character theory. Elements of Lie theory (including Cartan's classification) are presented in chapter 21; this chapter ends with a brief overview of alternative and Jordan algebras. Part V ends with an exposition of the theory of Dynkin graphs and Coxeter groups and their applications. Part VI starts with a remarkable chapter on PI-algebras, presenting the main points of the theory up to a rather detailed sketch of Kemer's and Zelmanov's results mentioned above. Chapter 24 deals with central simple algebras and the Brauer group. Chapter 25 presents elements of homological algebra and Morita theory for module categories and the first steps of representation theory of finite dimensional algebras (Gabriel's theorem). The final chapter is an introduction to Hopf algebras, a structure that comprises group algebras, enveloping algebras of Lie algebras and algebraic groups but also has applications in quantum theory.

Each of the three parts of this volume ends with more than 30 pages of exercises, many of which introduce the reader to further research and applications. The volume ends with a 27 page list of major results, a bibliography, a list of names and an index. Thanks to his deep insight in the subject, the author has succeeded in presenting most of classic non-commutative algebra in a unified way together with some of the more recent highlights of the theory. This is a remarkable achievement making the book of interest not just for graduate students in algebra but to graduate students and researchers in a wide spectrum of areas of mathematics and physics. (jtrl)

L. Sadun: Topology of Tiling Spaces, *University Lecture Series, vol. 46, American Mathematical Society, Providence, 2008, 118 pp., USD 29, ISBN 978-0-8218-4727-5*

Topological spaces of tilings (in the same Euclidean space) are studied that are closed under translations and complete in the tiling metric (the metric describes a closeness of two tilings using translations inside a ball). This book starts with an explanation of various constructions of tilings (substitution, cut-and-project, local match and inverse limits). The next step is to study Čech cohomology of the space of tilings (by means of cohomology of inverse limits). A new metric topology is defined on certain sets of tilings, where rotations are also considered, and its cohomology is studied. The so-called pattern-equivariant cohomology is described and its equivalence with Čech cohomology for tilings is shown (the Kellendonk–Putnam theorem). The PE cohomology assigns to every cohomology group a differential form representing the tiling. One chapter is devoted to procedures making work with tilings easier, like proper substitutions, partial collaring and Barge-Diamond collaring. In the last chapter, small motions of tiles are allowed so that tiles can meet in proper parts of edges. The third metric topology adapted to this situation is defined and briefly studied. Exercises appear through the whole text and solutions of selected ones are shown at the end of the book. No index is added. (mihus)

F. Schwarz: Algorithmic Lie Theory for Solving Ordinary Differential Equations*, Pure and Applied Mathematics, vol. 291, Chapman & Hall/CRC, Boca Raton, 2007, 434 pp., USD 89.95, ISBN 978-1-58488-889-5*

When Sophus Lie developed his theory of groups of continuous transformations (nowadays called Lie groups), his aim was to find a systematic way to integrate differential equations. The theory of Lie groups and Lie algebras was systematically developed into a broad field of mathematics, while the progress in Lie's original goal was much slower. A part of the problem was connected with a need for very complicated analytic computations, which can now be done more efficiently using computer algebra software. The aim of this book is to discuss algorithms for solving ordinary differential equations using the Lie approach. The author first introduces basic tools (the Janet algorithm for solving linear partial differential equations, transformation groups, equivalence and invariants of differential equations). The main part of the book is devoted to ordinary differential equations of at most third order, to a study of symmetries of a given equation, its transformation to canonical form and finally to algorithms for solutions. There is a lot of exercises, some of them with solutions. Software for computer calculations is available on a webpage. (vs)

C.H. Skiadas, C. Skiadas: Chaotic Modelling and Simulation. Analysis of Chaotic Models, Attractors and Forms*, CRC Press, Boca Raton, 2008, USA, 349 pp, USD 89,95, ISBN 978-1-4200-7900-5*

This book is intended as a light introduction to the highly fascinating field of chaotic dynamical systems. Keeping the analytic treatment at a minimal level, the authors prefer the "geometric" approach, heavily aided by a great number of computer generated pictures and numerical simulations. The book introduces the reader to all the common (both discrete and continuous) chaotic dynamical systems, including the logistic model, the Hénon model and the Lorenz and Rössler model, as well as a plethora of others together with various modifications and generalisations. Though aimed primarily at a non-specialist audience, an extensive bibliography and a chronology of the most important papers in the field make the book an excellent reference source of general interest. (dpr)

A. Skowroński, Ed.: Trends in Representation Theory of Algebras and Related Topics*, EMS Series of Congress Reports, European Mathematical Society, Zürich, 2008, 710 pp., EUR 98, ISBN 978-3-03719-062-3*

This comprehensive volume contains 15 expository surveys on recent developments and trends in the representation theory of associative algebras, mostly presented at ICRA XII in Torun in August 2007. S. Ariki presents modular representation theory of finite dimensional Hecke algebras (including a reader friendly exposition of quasi-hereditary covers of cyclotomic Hecke algebras in terms of the category O for the rational Cherednik algebra). G. Bobinski, Ch. Riedtmann and A. Skowroński deal with algebras of semi-invariant polynomial functions on the affine varieties of linear representations of quivers of a given dimension vector with respect to conjugate actions of products of general linear groups, and also with the geometry of the sets of common zeros of generating non-constant semi-invariants. The paper by I. Burban and Y. Drozd concerns the category of maximal Cohen-Macaulay modules over surface singularities and its representation type. The focus is on the McKay correspondence for quotient surface singularities and its generalisations.

J.F. Carlson surveys the theory of rank varieties for modules over finite groups and finite group schemes. The article by K. Erdmann and A. Skowroński deals with particular sorts of self-injective algebras (called periodic algebras). Ch. Geiss, B. Leclerc and J. Schröer present the recent connection between representation theory of preprojective algebras of Dynkin type and a class of cluster algebras. The survey by I.A. Gordon is an introduction to Etingov-Ginzburg symplectic reflection algebras. O. Iyama surveys his higher theory of almost split sequences (for which he was awarded the first ICRA Award in 2007). Also, applications to Calabi-Yau triangulated categories and CY-algebras are presented here. Calabi-Yau categories are the major topic of the survey of P. Jørgensen, who studies their links to rational homotopy theory. The main tool here is Auslander-Reiten theory. Calabi-Yau categories are also the subject of the survey by B. Keller, who concentrates on two important classes of examples: the categories arising as orbit categories and those arising as subcategories of derived categories.

The article by S. Kasjan deals with applications of some model theoretic tools in representation theory. S. Koenig's survey provides an elementary introduction to the structure and representation theory of diagram algebras that include Brauer algebras, partition algebras, (affine) Hecke algebras and (affine) Temperly–Lieb algebras. The survey by H. Lenzing and J. A. de la Pena deals with the impact given by the spectrum of the Coxeter transformation of algebras of finite global dimension on the structure and representation theory of some important classes of algebras. M. Reineke's article is an introduction to moduli spaces of representations of quivers, presenting a geometric approach to their classification. The final contribution by A. Skowroński and K. Yamagata surveys representation theory of finite dimensional selfinjective algebras over fields possessing Galois coverings by the repetitive algebras of quasitilted algebras. Each of the fifteen survey papers contains a number of open problems. Thus the volume will not only serve as a remarkable exposition presenting some deep recent ideas in a concise way but also as an important source of problems and directions for research in contemporary representation theory. (jtrl)

S. Szabó: Nahm Transform for Integrable Connections on the Riemann Sphere*, Mémoires de la Société Mathématique de France, no. 110, Société Mathématique de France, Paris, 2007, 114 pp., EUR 27, ISBN 978-2-85629-251-8*

The topic treated in this book fits into the broader program of a study of properties of instantons (i.e. self-dual solutions of Yang-Mills field equations) on 4-manifolds. In this context, the Nahm transform is relating solutions on two different manifolds that are dual to each other. Its properties are in many aspects similar to properties of the standard Fourier transform. Dual manifolds are constructed as the quotient of $X=R^4 / \Lambda$ of R^4 by a close additive subgroup Λ, and as the quotient $X^*=(R^4)^*/ \Lambda^*$, where Λ^* is the dual subgroup. Self-dual solutions of the Yang-Mills equations on R^4 invariant with respect to Λ form the space, which is an analogue of the space of real functions for the standard Fourier transform. The same definition works for the dual version. The Nahm transform then maps solutions on X to solutions on X^*. A lot of special cases have already been studied and are well understood. The author treats here the case where X and X^* are projective spaces of dimension 1. In this situation, it is necessary to allow solutions with singularities; the transform then works under suitable conditions on the singularity behaviour of solutions. (vs)

K. Tent, Ed.: Groups and Analysis*, London Mathematical Society Lecture Note Series 354, Cambridge University Press, Cambridge, 2008, 326 pp., GBP 38, ISBN 978-0-521-71788-5*

This book contains some written versions of survey lectures presented at the conference on the legacy of Hermann Weyl organised in Bielefeld in 2006. Their topics cover a broad area of mathematics that was influenced by the work of H. Weyl. The enormous impact of his work in representation theory and harmonic analysis is illustrated in papers by R. Goodman (algebraic group versions of results in harmonic analysis on compact symmetric spaces and the horospherical Radon transform), E. van den Ban (harmonic analysis on groups and non-compact symmetric spaces), R. Howe, E.-C. Tan and J. F. Willenbring (development of classical invariant theory and its applications to associated pairs of classical symmetric pairs, reciprocity algebras and branching rules) and J. C. Jantzen (generalisations of the Weyl character formula to the setting of semi-simple algebraic groups over algebraically closed fields with prime characteristics, Kac-Moody algebras and quantum groups).

Topics from analysis are discussed by W.N. Everitt and H. Kalf (generalisations of Sturm-Liouville theory and their relations to quantum physics), M.J. Pflaum (the Weyl quantisation, deformations quantisation and the algebraic index theorem), A.M. Hansson and A. Laptev (sharp spectral inequalities for the Heisenberg Laplacian with Dirichlet boundary conditions) and D.W. Stroock (generalisations of the Weyl potential theoretical approach to hypoellipticity proofs). The influence of Weyl's ideas on various themes in number theory can be found in papers by U. Hamendstädt (equidistribution for quadratic differentials, interplay among number theory, geometry and dynamical systems), W. Müller (generalisations of the Weyl law on asymptotic distribution of eigenvalues of the Laplace operator and relations to authomorphic forms on locally symmetric spaces), Ch. Deninger (relations among dynamical systems on foliated manifolds, transversal index theory and arithmetic geometry) and R.M. Weiss (the structure of affine buildings). The book ends with an historical sketch by P. Roquette on relations between E. Noether and H. Weyl in the period 1915–1935. The book offers an interesting overview of the impact of H. Weyl's work on contemporary mathematics. (vs)

***M. Tibar: Polynomials and Vanishing Cycles**, Cambridge Tracts in Mathematics 170, Cambridge University Press, Cambridge, 2007, 253 pp., GBP 45, ISBN 978-0-521-82920-5*

This monograph is devoted to the study of families of hypersurfaces considered as singular fibrations. It is based on the author's own research over the last 12 years. Let us cite here the author's words from the introduction: "The leading idea of this monograph is to bring into new light a bunch of topics – holomorphic germs, polynomial functions, pencils on quasi-projective spaces – conceiving them as aspects of a single theory with vanishing cycles at its core". Really, the notions of vanishing homology and vanishing cycles play a very important role here. The book is divided into three parts. Parts I and II deal with complex polynomial functions and in the very centre of the theory, we find vanishing cycles at infinity. Counting of vanishing cycles is closely related to polar curves, which are investigated in part II. Here we come to important invariants of affine varieties such as CW-complex structure, relative homology, Euler obstruction and the Chern-MacPherson cycle. In part III the author passes to the meromorphic situation and studies the topology of pencils of hypersurfaces. The book is intended for specialists in the field and for graduate students. The exposition is rather self-contained. Nevertheless it requires some preliminary knowledge including some differential and algebraic topology, some algebraic geometry and some familiarity with Milnor's classical book "Singular points of complex hypersurfaces". Each chapter of the book is followed by exercises and sometimes there are hints on how to solve them. (jiva)

***M. Wang, C.-H. Lai: A Concise Introduction to Image Processing Using C++ and CD**, Chapman & Hall/CRC Numerical Analysis and Scientific Computing, CRC Press, Boca Raton, 2008, 252 pp., USD 79.95, ISBN 978-1-58488-897-0*

This book presents a compact overview of the current methods used in modern computer image processing and their applications. The authors start by introducing basic concepts of digital signals and image representations. In the subsequent chapters, they go through image processing methods, image restoration and enhancement methods, a variety of filtering methods including diffusion and partial differential equation based methods, image segmentation algorithms and morphological operations on images. The last chapter is an overview of image compression methods including the wavelet transformation method and methods based on fractal theory. Each chapter includes a brief introduction to necessary mathematical background, a practical presentation of the corresponding algorithms and some examples of typical uses of the given method. All chapters are accompanied by C++ implementation of the method. This book requires only some background in geometry, algebra and calculus and can serve as an excellent starting book for anyone who needs to become familiar with current methods in the field of image processing. (jhr)

***P. M. H. Wilson: Curved Spaces. From Classical Geometries to Elementary Differential Geometry,** Cambridge University Press, Cambridge, 2007, 186 pp., GBP 24.99, ISBN 978-0-521-71390-0*

An introduction to the differential geometry of curves and surfaces is usually contained in the list of basic courses for students of mathematics at undergraduate level. There are excellent books already available that cover the topic well. This book offers, however, a different approach to these standard topics. It starts with the principal examples of classical geometries – Euclidean, spherical, toric and hyperbolic. The author discusses their basic properties, explains their symmetry group and then uses these explicit examples of geometries together with explicit computations in these basic models to introduce many important notions (isometries, curves and their lengths, special forms of the Gauss-Bonnet theorem, triangulations and Riemann surfaces of higher genus). He then introduces Riemannian metrics on open subsets of R^2, defines regularly embedded surfaces in space and discusses geodesics on such surfaces. The book culminates with a definition of abstract surfaces and the Gauss-Bonnet theorem. The book is written in a nice and precise style and explicit computations and proofs make the book easy to understand. A detailed and explicit discussion of the main examples of classical geometries contributes well to a better understanding of later generalisations. A list of examples at the end of each chapter helps as well. It is a very good addition to the literature on the topic and can be very useful for teachers preparing their courses as well as for students. (vs)

***V.G. Zvyagin, D.A. Vorotnikov: Topological Approximation Methods for Evolutionary Problems of Nonlinear Hydrodynamics,** de Gruyter Series in Nonlinear Analysis and Applications, vol. 12, Walter de Gruyter, Berlin, 2008, 230 pp., EUR 98, ISBN 978-3-11-020222-9*

This book is devoted to a study of dynamics of non-Newtonian fluids with complicated rheology, in particular to visco-elastic incompressible fluids with implicit constitutive relations. The first part of the book contains preliminary material: physical background of the models (chapter 1), the necessary function spaces (chapter 2), abstract results concerning evolutionary equations in Banach spaces (chapter 3) and, finally, the theory of dynamical systems and attractors (chapter 4). Most of the presentation is self-contained and can be of independent interest, as is, for example, section 2.2 (vector-valued functions and distributions) and section 4.2 (devoted to the so-called trajectory attractors). The rest of the book is devoted to the analysis of the described models; most of the results are an extension in the spirit of the results that are known to hold for the much simpler Navier-Stokes model. Chapter 5 looks at local existence results of strong solutions, while global existence of weak (possibly non-unique) solutions and their large-time dynamics is studied in chapter 6. The analysis is completed with conditional uniqueness theorems and the existence results for the stationary case. (dpr)

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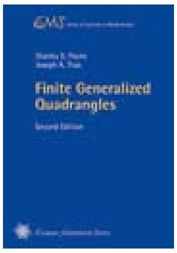
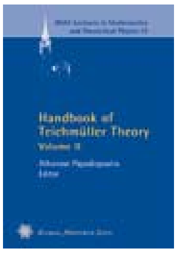


Stanley E. Payne (University of Colorado, Denver, USA), Joseph A. Thas (Ghent University, Belgium)
Finite Generalized Quadrangles. Second Edition (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-061-1. 2009. 300 pages. Softcover. 17.0 x 24.0 cm. 44.00 Euro

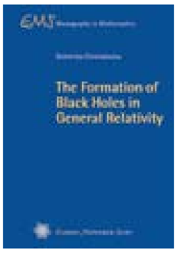
Generalized quadrangles (GQ) were formally introduced by J. Tits in 1959 in order to describe geometric properties of simple groups of Lie type of rank 2. Since its appearance in 1984, Finite Generalized Quadrangles (FGQ) quickly became the standard reference for finite GQ. It presents the whole story of the subject from the very beginning in a book of modest length.
This second edition is essentially a reprint of the first edition. It is a careful rendering into LaTeX of the original, along with an appendix that brings to the attention of the reader those major new results pertaining to GQ, especially in those areas to which the authors of this work have made a contribution. The first edition being out of print for many years, the new edition makes again available this classical reference in the rapidly increasing field of finite geometries.



Handbook of Teichmüller Theory, Volume II (IRMA Lectures in Mathematics and Theoretical Physics)
Athanase Papadopoulos (IRMA, Strasbourg, France), Editor

ISBN 978-3-03719-055-5. 2009. 882 pages. Hardcover. 17.0 x 24.0 cm. 98.00 Euro

This multi-volume set deals with Teichmüller theory in the broadest sense, namely, as the study of moduli space of geometric structures on surfaces, with methods inspired or adapted from those of classical Teichmüller theory. The aim is to give a complete panorama of this generalized Teichmüller theory and of its applications in various fields of mathematics. The present volume has 19 chapters and is divided into four parts: The metric and the analytic theory; the group theory; representation spaces and geometric structures; the Grothendieck–Teichmüller theory.
This handbook is an essential reference for graduate students and researchers interested in Teichmüller theory and its ramifications, in particular for mathematicians working in topology, geometry, algebraic geometry, dynamical systems and complex analysis.



Demetrios Christodoulou (ETH Zürich, Switzerland)
The Formation of Black Holes in General Relativity (EMS Monographs in Mathematics)

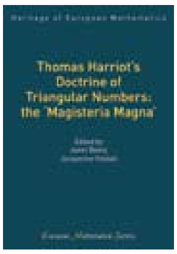
ISBN 978-3-03719-068-5. 2009. 598 pages. Hardcover. 16.5 x 23.5 cm. 98.00 Euro

In 1965 Penrose introduced the fundamental concept of a trapped surface, on the basis of which he proved a theorem which asserts that a spacetime containing such a surface must come to an end. The presence of a trapped surface implies, moreover, that there is a region of spacetime, the black hole, which is inaccessible to observation from infinity. A major challenge since that time has been to find out how trapped surfaces actually form, by analyzing the dynamics of gravitational collapse. The present monograph achieves this aim by establishing the formation of trapped surfaces in pure general relativity through the focusing of gravitational waves. The theorems proved in the present monograph constitute the first foray into the long-time dynamics of general relativity in the large, that is, when the initial data are no longer confined to a suitable neighborhood of trivial data. The main new method, the short pulse method, applies to general systems of Euler–Lagrange equations of hyperbolic type. This monograph will be of interest to people working in general relativity, geometric analysis, and partial differential equations.

6th International Congress on Industrial and Applied Mathematics. Zürich, Switzerland, 16–20 July 2007. Invited Lectures
Rolf Jeltsch (ETH Zürich, Switzerland), Gerhard Wanner (Université de Genève, Switzerland), Editors

ISBN 978-3-03719-056-2. 2009. 524 pages. Hardcover. 16.5 x 23.5 cm. 108.00 Euro

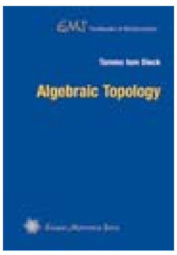
The International Council for Industrial and Applied Mathematics (ICIAM) is the worldwide organisation of societies dedicated primarily or significantly to applied and/or industrial mathematics. The ICIAM Congresses, held every 4 years, are run under the auspices of the Council with the aim to advance the applications of mathematics in all parts of the world. The 6th ICIAM Congress was held in Zürich, Switzerland, 16–20 July 2007, and was attended by more than 3000 scientists from 47 countries.
This volume collects the invited lectures of this Congress, the appreciations of the ICIAM Prize winners' achievements and the Euler Lecture celebrating the 300th anniversary of Euler. The authors of these papers are leading researchers of their fields, rigorously selected by a distinguished international program committee. The book presents an overview of contemporary applications of mathematics, new perspectives and open problems.



Thomas Harriot's Doctrine of Triangular Numbers: the 'Magisteria Magna' (Heritage of European Mathematics)
Janet Beery (University of Redlands, USA), Jacqueline Stedall (University of Oxford, UK), Editors

ISBN 978-3-03719-059-3. 2008. 144 pages. Hardcover. 17 x 24 cm. 64.00 Euro

Thomas Harriot (c.1560–1621) was a mathematician and astronomer, known not only for his work in algebra and geometry, but also for his wide-ranging interests in ballistics, navigation, and optics (he discovered the sine law of refraction now known as Snell's law). By about 1614, Harriot had developed finite difference interpolation methods for navigational tables. In 1618 (or slightly later) he composed a treatise entitled 'De numeris triangularibus et inde de progressionibus arithmeticis, Magisteria magna', in which he derived symbolic interpolation formulae and showed how to use them. This treatise was never published and is here reproduced for the first time. Commentary has been added to help the reader to follow Harriot's beautiful but almost completely nonverbal presentation. The introductory essay preceding the treatise gives an overview of the contents of the 'Magisteria' and describes its influence on Harriot's contemporaries and successors over the next sixty years. Harriot's method was not superseded until Newton, apparently independently, made a similar discovery in the 1660s. The ideas in the 'Magisteria' were spread primarily through personal communication and unpublished manuscripts, and so, quite apart from their intrinsic mathematical interest, their survival in England during the seventeenth century provides an important case study in the dissemination of mathematics through informal networks of friends and acquaintances.



Tammo tom Dieck (University of Göttingen, Germany)
Algebraic Topology (EMS Textbooks in Mathematics)

ISBN 978-3-03719-057-9. 2008. 284 pages. Softcover. 17 x 24 cm. 58.00 Euro

This book is written as a textbook on algebraic topology. The first part covers the material for two introductory courses about homotopy and homology. The second part presents more advanced applications and concepts (duality, characteristic classes, homotopy groups of spheres, bordism). The author recommends to start an introductory course with homotopy theory. For this purpose, classical results are presented with new elementary proofs. Alternatively, one could start more traditionally with singular and axiomatic homology. Additional chapters are devoted to the geometry of manifolds, cell complexes and fibre bundles. A special feature is the rich supply of nearly 500 exercises and problems. Several sections include topics which have not appeared before in textbooks as well as simplified proofs for some important results. Prerequisites are standard point set topology (as recalled in the first chapter), elementary algebraic notions (modules, tensor product), and some terminology from category theory. The aim of the book is to introduce advanced undergraduate and graduate (masters) students to basic tools, concepts and results of algebraic topology. Sufficient background material from geometry and algebra is included.

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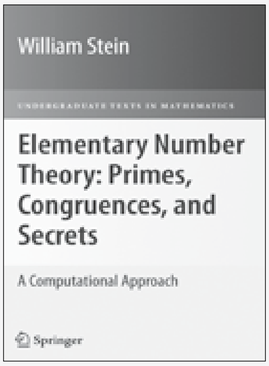

Classical number theory and elliptic curves are examined in this textbook, which moves on from elementary topics such as primes, continued fractions, and quadratic forms, to elliptic curves and their applications to algorithmic and number theory problems.

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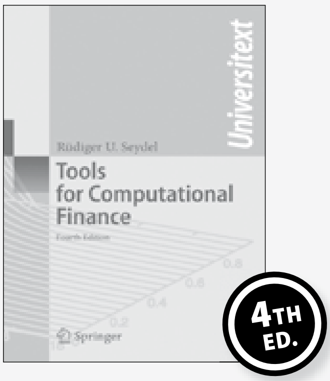

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4th ed. 2009. XXI, 336 p. 85 illus. (Universitext) Softcover
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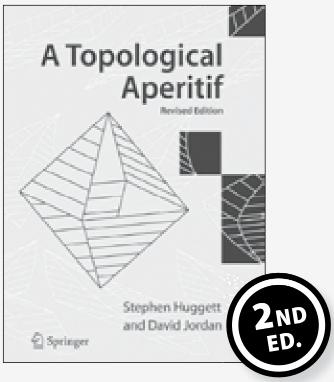

This second edition of Huggett and Jordan's popular work allows geometry to guide calculation. The book starts with a wealth of examples, often subtle, of how to be mathematically certain whether two objects are the same from the point of view of topology.

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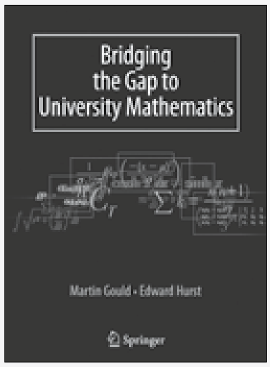

Written by students for students, this book offers a refreshing, new approach to making the transition into undergraduate-level mathematics or a similar numerate degree. The book contains chapters rich with worked examples and exercises.

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