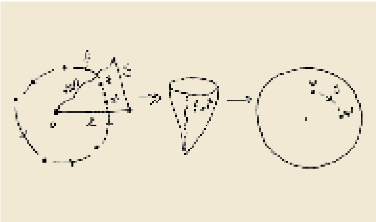
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

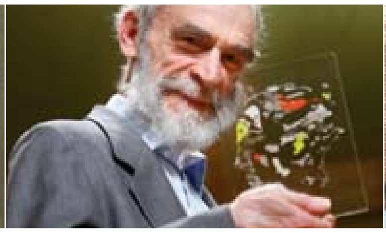


Feature
Hedgehogs

p. 11

History
Richard von Mises

p. 31



Interview
Mikhail Gromov

p. 19

ERCOM
IHP

p. 35

September 2009
Issue 73
ISSN 1027-488X

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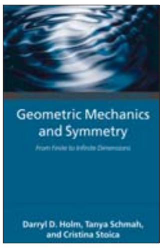
Geometric Mechanics and Symmetry
From Finite to Infinite Dimensions

Darryl D. Holm, Tanya Schmah, and Cristina Stoica



A graduate level text based partly on lectures in geometry, mechanics, and symmetry given at Imperial College London, this book links traditional classical mechanics texts and advanced modern mathematical treatments of the subject.

2009 | 460 pp
Paperback | 978-0-19-921291-0 | £29.50
Hardback | 978-0-19-921290-3 | £65.00

FORTHCOMING

Thermoelasticity with Finite Wave Speeds

Józef Ignaczak and Martin Ostoja-Starzewski

Extensively covers the mathematics of two leading theories of hyperbolic thermoelasticity: the Lord-Shulman theory, and the Green-Lindsay theory.

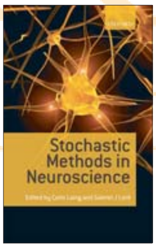
Oxford Mathematical Monographs

October 2009 | 432 pp
Hardback | 978-0-19-954164-5 | £70.00

NEW!

Stochastic Methods in Neuroscience

Edited by Carlo Laing and Gabriel J. Lord



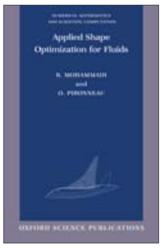
This text concentrates on the intersection between stochastic dynamics and neuroscience, presenting a series of self-contained chapters on major aspects of noise and neuroscience, each written by an expert in their particular field.

September 2009 | 400 pp
Hardback | 978-0-19-923507-0 | £49.95

NEW!

Applied Shape Optimization for Fluids
Second Edition

Bijan Mohammadi and Olivier Pironneau



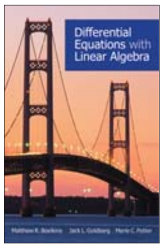
This new edition deals with shape optimization problems for fluids, with the equations needed for their understanding and with the numerical simulation of these problems.

September 2009 | 296 pp
Hardback |978-0-19-954690-9 | £65.00

FORTHCOMING

Differential Equations with Linear Algebra

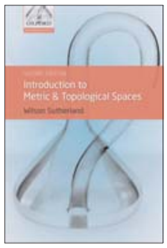
Matthew R. Boelkins, Jack L Goldberg, and Merle C. Potter



Explores the interplaybetween linear algebra and differential equations by examining fundamental problems in elementary differential equations. This text is accessible to students who have completed multivariable calculus and is appropriate for courses in mathematics and engineering that study systems of differential equations.

October 2009 | 464 pp
Hardback | 978-0-19-538586-1 | £52.00

Introduction to Metric and Topological Spaces
Second Edition

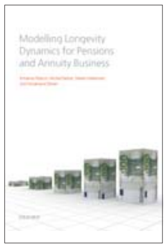


Wilson A. Sutherland

This fully updated new edition of Wilson Sutherland's classic text, *Introduction to Metric and Topological Spaces*, establishes the language of metric and topological spaces with continuity as the motivating concept, before developing its discussion to cover compactness, connectedness, and completeness.

2009 | 224 pp
Paperback | 978-0-19-956308-1 | £19.99

Modelling Longevity Dynamics for Pensions and Annuity Business



Ermanno Pitacco, Michel Denuit, Steven Haberman, and Annamaria Olivieri

Provides a comprehensive and detailed description of methods for forecasting future mortality trends, and an extensive introduction to some important issues concerning longevity risk in the area of life annuities and pension benefits.

2009|416 pp
Hardback | 978-0-19-954727-2 | £55.00

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Editor: Dr. Manfred Karbe

EMS Executive Committee

President

Prof. Ari Laptev
(2007–10)
Department of Mathematics
South Kensington Campus
Imperial College London
SW7 2AZ London, UK
e-mail: a.laptev@imperial.ac.uk
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EMS Secretariat

Ms. Riitta Ulmanen
Department of Mathematics
and Statistics
P.O. Box 68
(Gustaf Hällströmin katu 2b)
FI-00014 University of Helsinki
Finland
Tel: (+358)-9-191 51507
Fax: (+358)-9-191 51400
e-mail: ems-office@helsinki.fi
Web site: http://www.euro-math-soc.eu

EMS Agenda

2009

17–18 October
EMS Executive Committee Meeting, Istanbul, Turkey
Stephen Huggett: s.huggett@plymouth.ac.uk

1 November
Deadline for submission of material for the December issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@imaff.cfmac.csic.es

2010

25–28 February
EUROMATH 2010, Bad Goisern, Austria
http://www.euromath.org

2–7 July
Euroscience Open Forum under the slogan Passion for Science (ESOF2010), Torino, Italy,
http://www.esof2010.org

10–11 July
Council Meeting of the European Mathematical Society, Sofia, Bulgaria
Stephen Huggett: s.huggett@plymouth.ac.uk

11–14 July
EMS Conference "Mathematics in Industry", Sofia, Bulgaria
Stefan Dodunekov: stedo@math.bas.bg;
Stephen Huggett: s.huggett@plymouth.ac.uk

19–27 August
International Congress of Mathematicians, ICM2010, Hyderabad (India)
http://www.icm2010.org.in

October
20th Anniversary of the European Mathematical Society, tentatively planned to be organized in conjunction with a EMS Mathematical Weekend. Waiting for bids
Vasile Berinde: vberinde@ubm.ro

2012

2–7 July
6th European Mathematical Congress, Kraków (Poland)
http://www.euro-math-soc.eu

EMS Publicity Officer

Prof. Vasile Berinde
Department of Mathematics
and Computer Science
North University of Baia Mare
430122 Baia Mare, Romania
e-mail: vberinde@ubm.ro

Editorial

Michèle Artigue

Dear Reader of the EMS Newsletter,

Thanks to the column edited by Mariolina Bartolini Bussi, you are now regularly informed of the activities of the International Commission on Mathematical Instruction (ICMI). However, this editorial gives me an opportunity for emphasising the importance that the ICMI (the commission of the International Mathematical Union in charge of education) attaches to the quality of its relationships with the community of professional mathematicians and thus with associations such as the EMS.

The commission was created a century ago at the 4th International Congress of Mathematicians, which was held in Roma in 1908, and celebrated its centennial last year in the Accademia dei Lincei, the site where it was born. From the first years of its existence, it has been an institution trying to organise productive collaboration and exchanges between all those sharing its engagement for the improvement of mathematics education and its commitment for international solidarity. The origin of the commission attests these values: the idea was initially proposed in 1905 by a US mathematics educator and historian of mathematics, David Eugene Smith, and it was supported at the 1908 ICM by the most famous mathematicians at that time, and especially by the mathematician that would become its first president, Felix Klein.

In the first years of its existence, the commission was impressively productive, with no less than 300 reports accumulated in one decade, but then suffered from the difficulties successively created by the two World Wars before its renaissance in the fifties. The first ICME Congress was held in Lyon in 1969, under the presidency of Hans Freudenthal; the series of ICMI Studies was launched in 1985 under the presidency of Jean-Pierre Kahane; five Affiliated Study Groups were progressively created, the last one being the ASG on Applications of Mathematics and Modelling in 2004; and regional conferences and networks were progressively institutionalised, the first one being the IACME in Latin America under the presidency of Marshall Stone in 1961 and the last two being EMF (Espace Mathématique Francophone) in 2000 and AFRICME in Anglophone Africa in 2005, under the presidency of Hyman Bass.

During its first century, the ICMI was always situated at the interface of mathematics and mathematics education, trying to benefit from the diversity of expertise and perspectives existing in the mathematics community that it represents and trying to make this diversity serve the cause of the improvement of mathematics education all around the world. During the years I have been a member of the ICMI Executive Committee (EC), first as vice-president and then as president, I have progressively understood up to what point this interface makes ICMI unique and potentially useful for the mathematical community. Cultivating the interface has been a high priority for the members of the ICMI EC in the last decade and among other incentives this has led to the Klein project, which I would now like to introduce.

Revisiting the intent of Felix Klein when he wrote *Elementary Mathematics from an Advanced Standpoint*, the aim of the Klein project is to produce a book for secondary teachers that communicates the breadth and vitality of the research discipline of mathematics and connects it to the senior secondary school curriculum. This project, expected to take about four years, is developing in a tight collaboration between the ICMI and the IMU.

The two institutions have constructed an International Design Team made of eight members who met for the first time in the first days of June in Paris. The team has confirmed the production of a 300-page book written to inspire teachers to present to their students a more complete picture of the growing and interconnected field represented by the mathematical sciences in today's world. This book will be simultaneously published in many languages (six at least) and will be backed up by web, print and DVD resources, and will be open to evolution.

Currently, the plan is to have the book structured into twelve chapters, nine being topic chapters and three addressing more general themes such as interactions between mathematical domains, interactions between mathematics, sciences and society, and the way mathematicians work.

As was the case for Klein's book, the ambition is not to be comprehensive but insightful, making clear, in a way accessible to high school teachers, the most important ideas and evolutions from the last century, and to carefully select and develop appropriate examples in order to support the general and synthetic views offered. The actual writing will be done by invited authors of proven experience in expert and inspiring authorship and several "Klein conferences" will be organised around the world in the next three years where participants will be asked to offer feedback on draft ideas and material, and will also be able to offer original contributions.

The Design Team thus welcomes input from all those working in the mathematical sciences, researchers and educators alike. Anyone wishing to be on a mailing list to be kept up-to-date and receive draft material is invited to send an email in the first instance to Bill Barton b.barton@auckland.ac.nz, who leads this project. Moreover, a website will soon be established.

This project is a very challenging one for the ICMI and the IMU, and I hope that many EMS members will contribute to its success.

Michèle Artigue
President of ICMI
artigue@math.jussieu.fr

This section is open to the opinions of readers of the Newsletter. For sending a letter for publication in this section, please contact any member of the Editorial Committee.

Opinions expressed in the section *Letters to the Editor* are those of the authors and do not necessarily reflect the policies and views of the EMS. Letters from readers appearing in the Newsletter are published without change.

Some Remarks on the Loewner Theory

1. In 1921, Charles Loewner presented a new approach to the theory of univalent functions based on families of functions that depend on a time parameter. His theory culminated in deBranges' proof (1984) of the Bieberbach conjecture. Gerald Goodman was a student of Loewner. In his excellent dissertation he introduced optimal control theory into the Loewner theory. In the June 2009 issue of the EMS Newsletter, Gerald Goodman published a letter entitled "Reinventing Loewner's approach to univalent functions", where he expresses strong objections to a recent paper of Bracci, Contreras and Díaz-Madrigal entitled "Evolution families and the Loewner equation I: The unit disc". He writes

> *In Th.1.1 on p.3, they introduce the same normalized families as Pommerenke and Goodman did … This produces a theorem resembling … but its proof is trivial as it reduces the derivation of Loewner's equation to a remark, since the form of the infinitesimal generators is already known. They fail to notice this …*

I think that Goodman overlooked something very important. Loewner himself and most later papers on the Loewner method only considered functions that leave the same point in the unit disk fixed for all values of the time parameter. More recently some interesting papers consider special cases with a fixed boundary point.

It seems to me that Goodman misunderstood the aim of the paper of Bracci, Contreras and Díaz-Madrigal. He obviously assumed that the authors were again considering the classical case of a fixed interior point. This misunderstanding may have been due to an unfortunate phrase in the introduction. The authors present the equations that hold in the classical case and then continue

> *We call such a family an evolution family in the unit disc (see Definition 3.1 for a precise definition).*

A quick reader might now jump to Theorem 1.1 and, misinterpreting this theorem, form the mistaken view that the authors were talking about something already well-known.

However, a few paragraphs later the authors clearly state that they are studying the general case where no assumption is made about keeping any point fixed. Also, Definition 3.1 contains no such assumption; the rather technical third assumption is later used to establish the absolute continuity with respect to time, a question also criticised by Goodman. It turns out that, in the general case, there are many difficulties to overcome that are not present in the classical case.

But there is no doubt that, as the authors recognise, the principal idea is due to the genius of Loewner.

Christian Pommerenke, Prof. emeritus, Technical University Berlin, pommeren@math.tu-berlin.de

2. Recently, we read a distressing article in the Newsletter of the European Mathematical Society, June issue, written by professor emeritus Gerald S. Goodman in the section of Letters to the Editor, where he attacks several recent developments in the Loewner theory claiming that most of them are trivial consequences of old Loewners results. In particular, he explicitly stands out against results by Bracci, Contreras, and Díaz-Madrigal published recently in several prestigious sources, in particular, in Mathematische Annalen. One of the principal differences from the classical Loewners and Pommerenkes results is the fact that the normalization point in the generator for the evolution families (Denjoy-Wolff point for semigroups) depends on the time parameter as a measurable function taking values inside the unit disk and/or on its boundary, which makes the problem difficult and completely different from the case of the constant function inside the unit disk as in the classical case.

In his letter, Prof. Goodman in a very vague way, without any mathematically justifiable reason, attacks not only some persons but also instruments, which supported activities in these directions, in particular, the European Science Foundation and its networking programme 'Harmonic and complex analysis and its applications', in which many European first class researchers participate, and which comprises 10 countries and more than 25 universities. All conferences mentioned in his letter were announced openly in major information sources and on the web, however prof. Goodman being aware of them decided to write public letters instead of entering an open discussion with colleagues in person.

In our letter, we do not enter into a deep mathematical discussion, but just want to inform you that these recent activities in the Loewner theory, in particular, those mentioned in the letter, are supported by many good specialists active in this field and documented in papers published in top rank journals after thorough peer review, such as Advances in Math., Math. Ann., J. Anal. Math., to name few.

We express our strong concern about prof. Goodman's letter and hope that the Editors will take into account our opinion.

Sincerely yours,

Alexander Vasiliev, Professor
Chairman of the Steering Committee
European Science Foundation
Networking Programme HCAA
University of Bergen
Bergen N-5008, Norway

Mathematics and its interfaces with science, technology and society

Ari Laptev

Ari Laptev, President of the EMS

Although the role of mathematical sciences in civilization has been of central importance for many centuries, the current trend towards a global economy and a knowledge society has made information and innovation technologies increasingly dependent on scientific research driven by mathematics. In the present day context, mathematical sciences, including statistics and computing, are accepted as an integral part of technological progress.

Mathematics provides the context for communication and discovery in many scientific disciplines and modern industry. It is the language of innovation, which is vital for society and industry. As an analogy, it is impossible to argue with the fact that the development of speech has had an enormous impact on the development of the human race and has added to the richness of human culture.

Mathematics, as the oldest of sciences, has also contributed substantially to the development of our civilization and began its advance at the moment when it became necessary to count and when counting became a part of every language. It is interesting, however, that the written symbols, namely figures, by which numbers are expressed, appeared only in Arabic and Roman characters.

Different historical eras required different types of mathematics. Its development was accelerated by the tasks that naturally arose at different stages of social and technological progress. Nowadays research in mathematics is responsible not only for continuing to produce a language that would universally allow us to understand a variety of deep phenomena but also to provide a link between pure mathematics and its applications that would serve the community.

Two thousand years ago mathematics was very close to philosophy. Plato and Aristotle used relatively advanced mathematics to solve practical problems and also as a means of understanding life.

Starting with Newton, mathematics became a tool in the explanation of physical processes. Basic equations describing classical mechanics were set up and could be considered as a starting point of the incredible technological development that we observe today.

So, what is the mathematics of today? Many people often wonder if there is anything left to discover and are surprised that mathematicians are still involved in mathematical research.

It is true that many new results in mathematics do not have immediate applications but, eventually, most of them do become applicable. Let me mention a few recent examples where the use of mathematics has proved crucial:

- Integral geometry, dealing with so-called inverse problems, has provided a methodology used in: medical imaging for identifying tumours, weather radars, the search for oil fields, astronomy, etc.
- The creation of modern fibre optic cables would not be possible without the discovery of special solutions of non-linear equations called solitons.
- The arrival of the Internet made people fear that the world would be drowned in vast amounts of information. This problem has been successfully resolved by Google, which invariably delivers, instantly, the information sought. It seems like magic but the searching algorithm of Google was in fact provided by mathematicians.
- The theory of wavelets has been enormously important in telecommunications. It allows us to transmit information in a most compact way and ultimately gives us the possibility of all sorts of wireless connections.
- Credit card security is only possible thanks to cryptology, which uses a branch of number theory.
- Mathematicians are involved in improving the understanding of fundamental problems in genomics research, cell signalling, systems physiology, infection and immunity, developmental biology, the spreading of disease and ecology.

- Together with theoretical physicists, mathematicians are working on the unified physical theory that involves the latest developments in algebraic geometry.

The mathematical theories used in these examples were not originally developed with any particular application in mind but purely as a result of the curiosity of scientists.

Nowadays, funding agencies are taking different initiatives in defining (top-down) prioritized research areas within mathematics. These areas often have an interdisciplinary nature and usually financial support of such initiatives is provided by making cuts in the funding of core activities in mathematical sciences. Because the nature of mathematical research is often far from being directly applied, mathematicians are not always considered as major contributors to such projects. For example, within the EU Frame Programme 7 (FP7), networking projects are required to have links with industry. As a result, FP7 has awarded almost no networking projects in mathematics during the last few years.

Although influenced by tasks driven by applications, the progress of mathematical research is also measured by its own natural development motivated by the curiosity of mathematicians. This fact is recognised less and less by funding agencies in many European countries, where funding of fundamental research has dramatically declined. For example, during the last five years EPSRC in the UK has reduced its funding of mathematics from £21 million to £12 million. The vital role that research in mathematics plays in the general progress of technology is not sufficiently acknowledged.

North America still remains a very attractive place for many European mathematicians because, thanks to the US Natural Science Foundation (NSF), mathematical sciences are very well established.

European mathematicians feel positive towards the creation of the European Research Council (ERC), which provides support for fundamental sciences. However, the small number of Starting and Advance Grants within PE1 Mathematical Foundations, distributed by the ERC, have not, so far, had a serious impact on the level of mathematical research in Europe.

Traditionally, most research in mathematics takes place at mathematics departments of universities. Besides "proving theorems", mathematicians are usually involved in teaching. During recent decades the number of students at European universities has increased substantially whereas the number of professors and lecturers has remained almost the same. Professors and lecturers are overloaded with the teaching of basic courses and are left with very little time for research. Indeed, at many universities their research is considered a personal hobby.

It is not only the large number of students that makes teaching mathematics difficult.

The majority of first year students we receive at our universities are poorly prepared for the study of this subject. Modern schools often focus on weaker pupils and do not provide adequate support for those who are more talented. One of my former PhD students, for example, really suffered at school. Mathematics classes were boring for him mostly because the teacher did not allow him to solve mathematical problems as fast as he could, believing this would psychologically damage some of his classmates who were not as quick as he was.

Such methods of education contribute to the fact that students arrive at our universities unprepared and university teachers are forced to lower the standards of their courses. Ultimately, this affects the technological development of our countries since European industry is unable to find enough engineers who are sufficiently educated by our universities.

Recently, on 2 October 2008, the European Round Table of Industrialists (ERT) organised a meeting in Brussels on how to harness the potential of mathematics, science and technology to drive innovation and competitiveness in Europe. The main concern at this meeting was the poor level of mathematical education and its consequences regarding European economy and society. This important initiative was intended to unite industrial commitments with a systematic, large-scale approach that would support education and research. At this meeting the need for a strong collaboration between schools, universities, businesses and governments, concerning the promotion of mathematics, science and technology, was unanimously recognised. The main reason for such an initiative was the dissatisfaction of European industry with the quality of specialists emerging from European universities.

About two years ago the European mathematical community discovered that, when defining European research infrastructures, Brussels had decided not to consider mathematical science as a separate infrastructure but instead include it under the title "Computer and data treatment". Fortunately, this upsetting error was later rectified and we are now looking forward to Brussels' call for a project on "Infrastructures for mathematics and its interfaces with science, technology and society at large". This is a small step in the right direction.

The level of competence of European mathematicians is still high but they desperately need appropriate support and official recognition of the enormous importance of their research.

In conclusion, I would like to point out that mathematics is one of the core subjects in almost any science or engineering education. Research mathematicians provide a culture of logical thinking, develop a language that explains many complicated processes around us and secure the essence of technological development. I find it ironic, dismaying and deeply saddening that mathematics, a subject that over the last 2000 years has been largely responsible for human progress, still has to fight for its existence and true acknowledgement.

Ari Laptev
President of the European Mathematical Society
a.laptev@imperial.ac.uk

This article appeared first in *Public Service Review: Science and Technology 3*

The Chern Medal Award

The International Mathematical Union (IMU) and the Chern Medal Foundation (CMF) jointly launch a new mathematical prize, the

Chern Medal Award,

in memory of the outstanding mathematician Shiing-Shen Chern.

The Award is to be given to an individual whose lifelong remarkable achievements in the field of mathematics warrant the highest level of recognition. It consists of a medal and a monetary award of US$ 500,000. Half of the amount shall be donated to organizations of the recipient's choice to support research, education, outreach, or other activities to promote mathematics.

For more information see the Press Release at

http://www.mathunion.org/fileadmin/IMU/Prizes/Chern/Chern_MedalPress_Release_090601.pdf

and the Chern Medal Program Guidelines at

http://www.mathunion.org/fileadmin/IMU/Prizes/Chern/Chern_Medal_Program_Guidelines.pdf

The Chern Medal Program Guidelines also inform about the nomination procedure. The Chern Medal will be awarded for the first time at the opening ceremony of ICM 2010 in Hyderabad, India on August 19, 2010.

Please forward this information to your friends and colleagues, to journalists and mathematical newsletters, and please consider making a nomination for the Chern Medal.

Ferran Sunyer i Balaguer Prize 2009

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Institvt d'Estvdis Catalans

The Ferran Sunyer i Balaguer Prize 2009 winner has been

Tim Browning (Bristol University),

for his monograph "Quantitative Arithmetic of Projective Varieties".

Abstract: This book is about computing asymptotic estimates for the number of rational points in algebraic varieties defined over Q that have infinitely many rational points. This is done using methods coming from analytic number theory (like the Hardy–Littlewood circle method) and some techniques coming from arithmetic algebraic geometry (the universal torsor, for instance). The book contains a lot of examples and exercises, as well as a detailed discussion of the conjectures in the subject, like Manin conjectures and the dimension growth conjecture. The book will be published in the Birkhäuser series *Progress in Mathematics*.

The 50th International Mathematical Olympiad

Bremen, Germany, 10–22 July 2009

Hans-Dietrich Gronau, Rostock / Dierk Schleicher, Bremen

The International Mathematical Olympiad is the most important annual mathematics competition for high school students. The 50th Anniversary Olympiad took place in July 2009 in Bremen/Germany. A particular highlight was the anniversary ceremony in the presence of some of the leading research mathematicians that had, in their youths, participated at IMOs themselves.

The International Mathematical Olympiad (IMO) is an annual event for highly talented international high school students. Since it started in 1959 in Romania with 7 participating countries, it has continued to grow. This year, the IMO celebrated its 50th anniversary, and also the first time with at least 100 countries participating: we welcomed 565 contestants from 104 countries, up to six from each country. These six contestants are usually highly selected in their home countries; for instance, the German contestants are selected from the German Mathematical Olympiad with more than 200.000 annual participants, as well as the German Federal Mathematics Competition "Bundeswettbewerb Mathematik".

The IMO contest took place in the largest exhibition hall in Bremen. At each of the two contest days, the 565 contestants had 4.5 hours to solve 3 problems.

The international jury consists of the team leaders of the 104 participating countries. They met 10–16 July in the port city of Bremerhaven near Bremen and selected the six contest problems from a shortlist assembled by the Problem Selection Committee, based on suggestions by the participating countries.

The contestants arrived on 13 July in Bremen. The Opening Ceremony (where the jury had to be strictly separated from the contestants) was on 14 July and featured a video message from the German Chancellor Angela Merkel, herself a former participant at German Mathematical Olympiads. A core element of this ceremony is the traditional flag parade in which the 104 participating teams march across stage. At our anniversary IMO, the countries were ordered by the year of first IMO participation: starting with Romania, the host of the first two IMOs, and ending with the 4 countries that were welcomed to their very first IMO ever.

The mathematical contest took place on 15 and 16 July in Bremen's largest exhibition hall. On both days, the students had 4.5 hours to solve 3 problems each. During the subsequent two days, the solutions were graded by the international jury (meanwhile based in Bremen on the same campus as the contestants) in cooperation with 70 coordinators who ensured fair and consistent grading. During this time, the contestants enjoyed an excursion program to destinations such as the cities of Hamburg and Bremerhaven, the German test center of the magnetic levitation train "Transrapid", or the shipyard "Meyer Werft Papenburg" that builds the world's largest ocean cruise ships (some 40 km inland!). There is a traditional "joint excursion"; this year, it took place on 20 July and went to the small island of Wangerooge in the newly declared Unesco World Natural Heritage area of the "German/Dutch Wadden Sea". It was organized so that everyone could meet at the same place and relax together.

A particular highlight of the 50th IMO was the anniversary celebration on 19 July in the presence of six leading research mathematicians who were former IMO participants: Béla Bollobás, Timothy Gowers, László Lovász, Stanislav Smirnov, Terence Tao, and Jean-Christophe Yoccoz (who jointly obtained 10 gold, 3 silver, and 2 bronze medals at the IMOs 1959–1988). They offered most interesting and very different presentations on their IMO experience, on current research mathematics, and on the relations and differences between these. Most of them were also present at the closing ceremony on 21 July, in the presence of the German Federal Minister of Education and Research, Dr. Annette Schavan, and her colleague from the State of Bremen, Renate Jürgens-Pieper. They all helped award the medals to the contestants. The six problems from the IMO 2009 can be found at www.imo-official.org. This year's problem 6 was considered the second-hardest ever; it was solved only by the three most successful contestants of this IMO.

Problem 6. Let $a_1, a_2, \ldots, a_n$ be distinct positive integers and let M be a set of $n-1$ positive integers not containing $s = a_1 + a_2 + \ldots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths $a_1, a_2, \ldots, a_n$ in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M.

The 50th IMO anniversary celebration featured presentations by Terence Tao “Structure and Randomness in the Primes”, Béla Bollobás “The Christian and the King”, Timothy Gowers “How do IMO Problems Compare with Research Problems”, Stanislav Smirnov “How do Research Problems Compare with IMO Problems”, Jean-Christophe Yoccoz “Small divisors: Number Theory in Dynamical Systems”, and László Lovász “The Role of Graph Theory in 1963 (my First Olympiad) and Today”.

The Prize Award Ceremony of the 50th IMO. The 565 contestants were awarded 49 Gold medals (for those who obtained at least 32 out of 42 points), 98 Silver medals (for those who obtained between 24 and 31 points), and 135 Bronze medals (for those who obtained between 14 and 23 points). In addition, 96 Honorable Mentions were awarded to those who solved at least one problem completely. This picture shows the three most successful contestants Makoto Soejima (Japan), Lisa Sauermann (Germany), and Dongyi Wei (PR China) with their Gold Medals. In the background: Hans-Dietrich Gronau, Chair of the Jury (left), and IMO anniversary guests László Lóvasz and Terence Tao. (Pictures courtesy of Stephanie Schiemann)

This 50th IMO was organized by the association “Bildung und Begabung” that runs and coordinates many programs for highly talented high school students in Germany, including the German Mathematical Olympiads, the German Federal Mathematics Competition, and the annual training of the German team to the IMOs. The organization was done in cooperation with Jacobs University, a small private English-language university in Bremen that attracts some of the most talented students from around the world and that has a particularly strong international program in mathematics. The IMO was based on this university campus where during the year some 1000 people from 100 countries study and live together. This had the advantage that the IMO community could use the sports and recreation facilities of the university, and all contestants could stay at the same location (including the jury, after the contest had taken place): it was particularly important for us that the entire IMO community could live together at the same location and people from all countries could meet and interact freely, in the hope that this would help the international contestants to establish and maintain lasting contact among each other.

Nonlinearizable Holomorphic Dynamics and Hedgehogs

Kingshook Biswas

Let α be an irrational number and $R_\alpha(z) = e^{2\pi i\alpha}z$ the rigid rotation around the origin by angle $2\pi\alpha$. The origin is the unique fixed point of R_α and the orbit of any other point z is aperiodic and dense on the circle passing through z with centre the origin. A classical question is the following: does the stability of orbits persist under nonlinear perturbations? A stronger form of this question is the *linearization* problem: is the perturbed dynamics *linearizable*, i.e. conjugate to the linear dynamics of the rotation?

We consider perturbations of two kinds: holomorphic germs of diffeomorphisms f fixing the origin with differential $Df(0) = R_\alpha$ and analytic circle maps $g : S^1 \to S^1$ with rotation number α. We recall that the rotation number $\rho(g) \in \mathbf{R}/\mathbf{Z}$ of an orientation-preserving homeomorphism g of the circle is a topological conjugacy invariant measuring the "long term average speed" of the dynamics. For a germ f it is also customary to refer to $\rho(f) := \frac{1}{2\pi i}\log f'(0) \in \mathbf{R}/\mathbf{Z}$ as the rotation number of f.

Both germs and circle maps exhibit very similar phenomena, the most striking and well-known being that of *small divisors*: stability of orbits persists when the rotation number is poorly approximable by rationals (of 'Diophantine' type) but may be destroyed if the rotation number has extremely good rational approximations (of 'Liouvillean' type). Stability for analytic maps is in fact equivalent to linearizability. If a germ f has a stable fixed point it is easy to construct a small simply connected invariant domain D containing the origin; any conformal mapping ϕ taking D to the unit disk $\mathbf{D}$ and fixing the origin then conjugates f to an automorphism of the unit disk fixing the origin, which is therefore a rigid rotation. Such a domain D is called a *linearization domain* and the maximal such domain is called the *Siegel disk* of f. Similarly an analytic circle map g with stable dynamics is conjugate to the rotation R_α on a maximal invariant annulus containing the unit circle called the *Herman ring* of g.

The conjugacy equation $h^{-1}fh = R_\alpha$ has a unique formal solution h tangent to the identity; the nth coefficient h_n is given by a polynomial in the coefficients of f and $h_1, \dots, h_{n-1}$, divided by the so-called 'small divisor' $(\lambda^n - \lambda)$ where $\lambda = e^{2\pi i\alpha}$. Linearizability of f is equivalent to convergence of the formal series h, which depends on the arithmetic of α via the presence of small divisors. The continued fraction convergents (p_n/q_n) of α are (in a precise sense) the best rational approximations to α and a natural way to quantify the approximability of α by rationals is via the growth of the sequence (q_n). Hypotheses on the rate of growth allow one to estimate convergence or divergence of the series h leading to the following classical results:

Theorem.
(1) *(Cremer) If* $\operatorname{Sup} \frac{\log q_{n+1}}{q_n} = +\infty$ *then there exists a nonlinearizable germ f.*

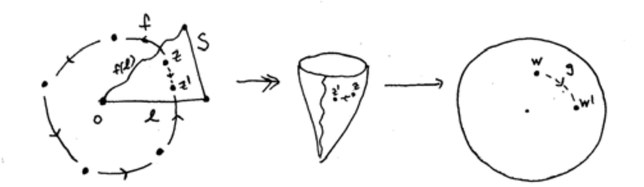

Fig. 1. Sectorial renormalization

(2) *(Siegel) If* $\operatorname{Sup} \frac{\log q_{n+1}}{\log q_n} < +\infty$ *then every germ f with $\rho(f) = \alpha$ is linearizable.*
(3) *(Brjuno) If* $\sum_n \frac{\log q_{n+1}}{q_n} < +\infty$ *then every germ f with $\rho(f) = \alpha$ is linearizable.*

We denote the sets of numbers satisfying conditions (2) and (3) above as $\mathscr{D}$ (for Diophantine) and $\mathscr{B}$ (for Brjuno) respectively. In view of these results it is natural to consider the problem of determining the optimal condition for linearizability. This condition is arithmetic in the following sense:

Theorem (Douady-Ghys). *The set* $\mathscr{O} := \{\alpha \in \mathbf{R} : \rho(f) = \alpha \ (\mathit{mod}\,\mathbf{Z}) \Rightarrow f \text{ is linearizable}\}$ *is invariant under the action of* $SL(2,\mathbf{Z})$.

Since $\mathscr{O}$ is clearly invariant under $x \mapsto x+1$ it is enough to show the invariance under $x \mapsto -1/x$. This is achieved by constructing for any germ f a return map $g = \mathscr{R}(f)$ called the sectorial renormalization of f. Taking a small line segment $l = [0,\varepsilon]$ one forms the sectorial domain S bounded by $l, f(l)$ and the line segment $[\varepsilon, f(\varepsilon)]$ (see figure 1). For l small enough, any z in S has a first return z' to S; the map $z \mapsto z'$ is well-defined on the quotient S/f obtained by gluing l to $f(l)$ by f. The quotient is biholomorphic to a disc and upon uniformization we obtain the desired return map g that satisfies $\rho(g) = -1/\rho(f) (\mathrm{mod}\,\mathbf{Z})$ (to see this consider the case when f is a rotation). The dynamics of f is stable if and only if the dynamics of g is stable, and the invariance of $\mathscr{O}$ follows.

Yoccoz quantified this construction to give a geometric proof of Brjuno's theorem. The partial sums s_n of the sum in Brjuno's condition appear (up to a constant) as minus the logarithm of the radius of the disk around the origin on which the nth return map of f is well-defined. Yoccoz also developed a reverse return map construction incorporating holomorphic surgery whereby given g one obtains an f such that $\mathscr{R}(f) = g$ and moreover f has an extra fixed point near the origin. Iterating this construction gives maps with periodic orbits, which Yoccoz shows are at distance from the origin of the optimal order e^{-s_n}, leading in the limit to:

Theorem (Yoccoz *[Yo]*). *If α does not satisfy the Brjuno condition then there exists a germ f with $\rho(f) = \alpha$, which has a sequence of periodic orbits converging to the origin (in par-*

ticular f is nonlinearizable). There are uncountably many conjugacy classes of such germs f.

Hence the optimal arithmetic condition $\mathscr{O}$ for linearization of germs is in fact the Brjuno condition $\mathscr{B}$.

For circle maps, by a classical theorem of Denjoy, any C^2 (in particular analytic) circle map g with irrational rotation number α is topologically conjugate on the circle to the rotation R_α, the conjugacy h being uniquely determined up to composition with a rotation. The map g is linearizable if and only if the conjugacy h is analytic. Arnold and Herman proved the following local and global linearization results respectively, while Yoccoz used similar renormalization constructions for circle maps to find the optimal arithmetic condition $\mathscr{H}$ for linearizability (called the Herman condition):

Theorem.
(1) *(Arnold)* [Ar] *For every $\alpha \in \mathscr{B}$, there exists $\Delta = \Delta(\alpha) > 0$ such that if $\rho(g) = \alpha$ and g is analytic on the annulus $\{e^{-2\pi\Delta} < |z| < e^{2\pi\Delta}\}$ then g is linearizable.*
(2) *(Herman)* [He] *If $\alpha \in \mathscr{D}$ and $\rho(g) = \alpha$ then g is linearizable.*
(3) *(Yoccoz)* [Yo] *If $\alpha \in \mathscr{H}$ and $\rho(g) = \alpha$ then g is linearizable. If $\alpha \notin \mathscr{H}$ then there exists a nonlinearizable g with $\rho(g) = \alpha$.*

The Herman condition is a bit involved to describe explicitly but we remark that $\mathscr{D} \subset \mathscr{H} \subset \mathscr{B}$.

Invariant continua and the correspondence between germs and circle maps

The preceding results resolve the problem of stability but leave open the following natural question: are there nontrivial invariant continua (i.e. connected compacts) near the fixed point of a nonlinearizable germ f (or near the unit circle in the case of circle maps)? The following result of Perez-Marco asserts that indeed there are always invariant continua that extend up to the boundary of any domain on which the dynamics is univalent:

Theorem (Perez-Marco). *Let f be a germ with irrational rotational number and U a Jordan domain with C^1 boundary containing the origin such that f and f^{-1} are univalent on a neighbourhood of $\overline{U}$. Then there is a unique full continuum $K = K(U)$ such that $0 \in K \subset \overline{U}, K \cap \partial U \neq \emptyset$ and $f(K) = f^{-1}(K) = K$.*

We will refer to domains U satisfying the above hypotheses as admissible domains for f. Here 'full' means that the complement $\mathbf{C} - K$ is connected. Note that even for linearizable germs (which clearly have local invariant full continua) the above statement is nontrivial. The proof relies on a fundamental construction due to Perez-Marco linking the problems of germs and circle maps. Given a germ f and a totally invariant full continuum K, let ϕ be a conformal mapping of the simply connected domain $\Omega_K = \overline{\mathbf{C}} - K$ onto the complement of the unit disk $\overline{\mathbf{C}} - \overline{\mathbf{D}}$ normalized so that $\phi(\infty) = \infty$ and $\phi'(\infty) > 0$. The conjugate $g = g(f, K) = \phi f \phi^{-1}$ is univalent in an annulus in $\overline{\mathbf{C}} - \overline{\mathbf{D}}$ one of whose boundary components is the unit circle. In fact g extends continuously to a map preserving the unit circle (and hence by Schwarz reflection to an analytic circle map univalent in an annulus containing S^1). To see this one uses Caratheodory's theory of *prime ends* (see [Po]): ϕ extends to the prime-end compactification $\Omega_K \cup \mathscr{E}$ taking the prime end boundary $\mathscr{E}$ homeomorphically onto S^1. Since f is uniformly continuous in a neighbourhood of K and fixes K, it induces an action on prime ends that corresponds under ϕ to the extension of g to S^1.

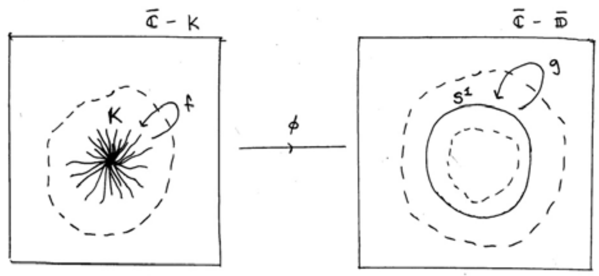

Fig. 2. The fundamental construction

The most important feature of this construction is that it preserves rotation numbers: $\rho(g) = \rho(f)$. Note that f has a linearization domain containing K if and only if g is linearizable. Consider a germ f with a Diophantine rotation number and an admissible domain U; f is linearizable, and the above construction applied to the closure $K = \overline{D}$ of any linearization domain D compactly contained in U always gives a linearizable circle map g (since $\rho(g) = \rho(f)$ is Diophantine) and hence a strictly larger linearization domain $D' \supset D$ for f. Thus the Siegel disk cannot be compactly contained in U and there is a unique linearization domain $D \subset U$ such that $\overline{D} \cap \partial U \neq \emptyset$. This proves the existence theorem for germs with Diophantine rotation numbers; for general f, one perturbs f by composing with small rotations to get maps f_n converging to f with Diophantine rotation numbers, and then passes to a Hausdorff limit of invariant continua of the f_n's to get an invariant continuum for f (the uniqueness of the invariant continuum K is a more delicate issue relying on estimates for the dynamics near K).

A full invariant continuum K for f is called a *hedgehog* if f is nonlinearizable or a *linearizable hedgehog* if f is linearizable but K is not contained in the closure of the Siegel disk. As we shall see, while the topology of hedgehogs is quite pathological, the dynamics on hedgehogs is in many ways reminiscent of an irrational rotation; informally speaking, it seems natural to think of them as "degenerate linearization domains".

Dynamics on hedgehogs

The results of this section unless otherwise stated are due to Perez-Marco. It turns out that the key to understanding the dynamics of a germ f on a hedgehog K is by controlling the dynamics near K. This is in turn achieved by means of estimates for the analytic circle map g given by the fundamental construction. These are given by estimates for renormalization of analytic circle maps due to Yoccoz. The estimates are conformally invariant in the following sense: they give closed curves surrounding S^1 and converging to S^1 that are almost invariant under high iterates of g with respect to the Poincaré metric on

$\mathbf{C} - \mathbf{D}$. Pulling these curves back gives curves γ_n near K that are quasi-invariant under f with respect to the Poincaré metric on $\mathbf{C} - \mathbf{K}$ (which is much larger than the Euclidean metric near K).

Perez-Marco uses these quasi-invariant curves to prove the uniqueness of the hedgehog $K = K(U)$ associated to an admissible domain U. The hedgehog $K(U)$ can be described as the connected component containing the origin of the set of points whose full orbit is contained in $\overline{U}$. It follows that if we take a continuous monotone family $(U_t)_{0<t<1}$ of admissible domains with $U_t \downarrow \{0\}$ as $t \downarrow 0$ and $U_t \uparrow U$ as $t \uparrow 1$ then we get a corresponding filtration $(K_t = K(U_t))_{0<t<1}$ of the hedgehog $K = K(U)$ by sub-hedgehogs. Indeed the family (K_t) is precisely the collection of hedgehogs contained in U and we have the following:

Theorem ([PM2]). *The family $\mathscr{K}$ of all hedgehogs of a nonlinearizable germ f is a monotone one-parameter family $(K_t)_{t>0}$. The dependence of K_t on t is continuous with respect to the Hausdorff topology.*

This is similar to the family of linearization domains of a linearizable germ. Smaller hedgehogs K_s are "hidden" from the exterior of larger hedgehogs K_t $(s < t)$ in both a geometric and a potential-theoretic sense: no point of K_s is conically accessible from $\Omega_t = \mathbf{C} - K_t$ and the harmonic measure of K_s as a subset of $\partial\Omega_t$ is zero.

The filtration of a hedgehog K by sub-hedgehogs $(K_t)_{0<t<1}$ can be used to show that (as in the case of linearization domains):

Theorem ([PM2]). *There are no periodic orbits on K except for the fixed point at the origin.*

A periodic orbit for f would give, for t close enough to 1, periodic orbits for the circle maps $g_t = g(f, K_t)$ and therefore in the limit a periodic orbit for $g = g(f, K)$, implying that $\rho(f) = \rho(g)$ must be rational.

Every orbit of an irrational rotation is recurrent, with the denominators q_n giving the times of closest recurrences. A similar statement holds for hedgehogs:

Theorem ([PM2]). *The q_n-th iterates of a nonlinearizable germ f converge uniformly to the identity on any hedgehog.*

The proof uses the estimates for the quasi-invariant curves: f^{q_n} is uniformly close to the identity on γ_n and hence by the maximum principle also on the hedgehog. In fact the convergence of f^{q_n} to the identity is fast enough so that, for any sequence $\varepsilon_n \in \{-1, 1\}$, the iterates $f^{\varepsilon_1 q_1} f^{\varepsilon_2 q_2} \dots f^{\varepsilon_n q_n}$ converge to a homeomorphism of the hedgehog. A hedgehog therefore always has a Cantor set of topological symmetries generated by the dynamics.

We note that recurrence of the dynamics and the absence of periodic orbits imply that a hedgehog has empty interior; otherwise some large iterate f^{q_n} would have to fix a simply connected component D of the interior (which does not contain the origin) and therefore have a fixed point in $\overline{D}$. Now in the fundamental construction, the harmonic measure of the domain $\Omega_K = \mathbf{C} - K$ on $\partial\Omega_K = K$ is transported by the conformal map ϕ to Lebesgue measure on the circle. By a result of Herman and Katok, the dynamics of the associated circle map is ergodic with respect to Lebesgue measure. This translates into the following result for the germ f:

Theorem ([PM1]). *The orbit of almost every point of the hedgehog (with respect to the harmonic measure of the complement) is dense in the hedgehog. In particular, a germ is nonlinearizable if and only if it has an orbit accumulating at the fixed point.*

Commuting linearizable germs are linearized by the same map and hence share the same linearization domains. Replacing linearization domains by hedgehogs, the same holds in the nonlinearizable case: commuting germs share a common family of hedgehogs. As in the linearizable case, the converse is also valid:

Theorem ([Bi1]). *If two nonlinearizable germs share a common hedgehog then they commute.*

This is an easy consequence of the following rigidity result: if a germ preserves a hedgehog and is tangent to the identity then it is the identity. This rigidity result implies that the dynamics is completely determined by the hedgehogs and the rotation number in the following sense:

Theorem ([Bi1]). *Two nonlinearizable germs f_1, f_2 with the same rotation number are conjugate by a germ ϕ if and only if ϕ takes hedgehogs of f_1 to hedgehogs of f_2.*

Indeed, if K_i is a hedgehog of f_i, $i = 1, 2$, and $\phi(K_1) = K_2$ then $f_2^* = \phi f_1 \phi^{-1}$ preserves K_2 and has the same rotation number as f_2, so $f_2^{-1} f_2^*$ is tangent to the identity and hence equal to the identity.

Topology of hedgehogs

In contrast to the nice properties of the dynamics on hedgehogs, their topology is very complicated and seems to be very difficult to understand in general. Perez-Marco proves however that they must always have some pathological properties.

Again these follow via consideration of the associated circle map g. By Denjoy's theorem, finitely many translates under g of any small interval cover the circle. Therefore, for any simple arc in $\mathbf{C} - \mathbf{D}$ with distinct endpoints on S^1 (a "crosscut" of $\mathbf{C} - \mathbf{D}$), finitely many translates under g of the domain bounded by the arc and the circle cover an annulus with S^1 as one boundary component. Using the conformal representation, we see that for any cross-cut γ of $\mathbf{C} - K$, finitely many translates of the domain $D(\gamma)$ bounded γ and K cover an annulus with K as one boundary component. So, for any cross-cut γ the domain $D(\gamma)$ must accumulate the origin. This has the following immediate consequences:

Theorem ([PM3]).
(1) *A hedgehog K is not locally connected at any point $z_0 \in K - \{0\}$.*
(2) *If $x, y \in K$ are points accessible from the complement of K and $C \subset K$ is a continuum containing x, y then $0 \in C$.*
(3) *A hedgehog always contains points inaccessible from the complement of the hedgehog.*
(4) *The 'punctured' hedgehog $K - \{0\}$ has an uncountable number of constituents, i.e. connected components by continua.*

Indeed, if a hedgehog K was locally connected at a point z_0 distinct from the origin, then there would be a small compact, simply connected neighbourhood U of z_0 not containing the origin such that $U \cap K$ was connected; but then one could find a cross-cut γ contained in U such that the domain $D(\gamma)$ was also contained in U. Similarly the second assertion above follows by constructing a cross-cut γ landing at x and y; if the continuum C avoided the origin then so would the closure of the domain $D(\gamma)$. The third statement follows from the second, since at most one point of a continuum contained in $K - \{0\}$ can be accessible from the complement of K. By a classical result of Fatou, almost all points (with respect to the harmonic measure of the complement) of K are accessible from the complement; by (2), each of these is contained in a distinct constituent of K, and statement (4) follows.

Tube-log Riemann surfaces and construction of examples

While the results above indicate that the topology of a hedgehog is necessarily complicated, we still do not have an explicit description of their topology even in special cases. The only positive results so far in this direction have been obtained via techniques for constructing germs due to Perez-Marco. These generalize the renormalization and surgery techniques of Yoccoz by introducing in the surgery procedure a class of Riemann surfaces called *tube-log Riemann surfaces*. These were first used by Perez-Marco to construct example of germs without periodic orbits satisfying the optimal arithmetic condition in this situation:

Theorem ([PM4]).
(1) *If f has no periodic orbits in a neighbourhood of the origin and $\alpha = \rho(f)$ satisfies $\sum \frac{\log\log q_{n+1}}{q_n} < +\infty$ then f is linearizable.*
(2) *If α satisfies $\sum \frac{\log\log q_{n+1}}{q_n} = +\infty$ then there exists a nonlinearizable germ f with no periodic orbits.*

Tube-log Riemann surfaces are surfaces given by pasting together isometrically flat complex cylinders $\mathbf{C}/\mathbf{Z}$, half-cylinders $\mathbf{H}/\mathbf{Z}$ and complex planes $\mathbf{C}$. For example, the tube-log Riemann surface $\mathscr{S}_0$ intervening in the construction of the germ in (2) above is obtained by pasting one cylinder and an infinite family of planes as shown in figure 3.

The connection with dynamics is as follows. We can view the flow of rotations $\mathscr{F} = (R_\alpha : z \mapsto e^{2\pi i\alpha} z)$ on the punctured plane $\mathbf{C}^*$ as the flow of translations $(T_\alpha : w \mapsto w + \alpha)$ on the cylinder $\mathbf{C}/\mathbf{Z}$ via the logarithm $w = \frac{1}{2\pi i}\log z$ (the fixed point at the origin corresponds to the top end $+i\infty$ of the cylinder). Slitting this cylinder and pasting planes to it, we get a flow of translations $\tilde{\mathscr{F}}$ on the surface $\mathscr{S}_0$. Note that, now, orbits of points below the branch point in the cylinder are no longer bounded but flow out into the neighbouring planes and escape to infinity. The surface $\mathscr{S}_0$ is biholomorphic to $\mathbf{C}^*$ and by conjugating $\tilde{\mathscr{F}}$ by the uniformizing map $K_0 : \mathscr{S}_0 \to \mathbf{C}^*, K(+i\infty) = 0$ we get a new flow $\mathscr{F}'$ on $\mathbf{C}^*$.

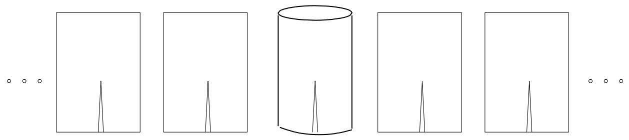

Fig. 3. The tube-log Riemann surface $\mathscr{S}_0$

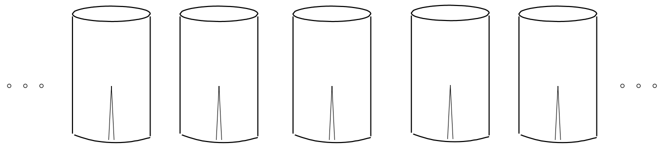

Fig. 4. The tube-log Riemann surface $\mathscr{S}_1$

The above procedure is more general; we could start with any flow $\mathscr{F}$ of maps fixing the origin, lift to a flow $\tilde{\mathscr{F}}$ on $\mathscr{S}_0$ and then conjugate by K_0 to get a new flow $\mathscr{F}'$. We can think of the pair $(\mathscr{S}_0, K_0)$ as a machine for 'folding' dynamics; in the passage from $\mathscr{F}$ to $\mathscr{F}'$ nonlinear behaviour is injected into the dynamics, namely unbounded orbits. To paste different kinds of nonlinearities we can vary the tube-log Riemann surface used. For example, using the the surface $\mathscr{S}_1$ shown in figure 4 one pastes periodic orbits to the dynamics:

Iterated foldings give in the limit nonlinearities accumulating the origin. Some care must be taken since each folding introduces singularities in the dynamics but Perez-Marco ensures that, for a Cantor set of rotation numbers all very well-approximated by rationals, the singularities of the corresponding elements are bounded away from the origin:

Theorem ([PM5]). *There exists a Cantor set $C \subset \mathbf{R}/\mathbf{Z}$ and a commuting family of nonlinearizable holomorphic germs $(f_t)_{t\in C}$ univalent on a common neighbourhood of the origin.*

In the above theorem, by different choices of tube-log Riemann surface, the family $(f_t)_{t\in C}$ can be made to have either a common sequence of periodic orbits accumulating the origin or to have no periodic orbits in a neighbourhood of the origin. It is also possible to control the geometry of the hedgehogs obtained in the limit. At each finite stage n the dynamics is linearizable with Siegel disk given by the image of a stable half-cylinder in the nth tube-log Riemann surface under the composition of n uniformizations, and explicit formulae for the uniformizations allow one to estimate the shape of the Siegel disks and hence of the limiting hedgehog. The author proves for example the following results:

Theorem ([Bi2], [Bi3]). *There exists a Cantor set $C \subset \mathbf{R}/\mathbf{Z}$ and a commuting family of nonlinearizable holomorphic germs $(f_t)_{t\in C}$ with a common hedgehog K such that*
(1) *K contains a "smooth comb", i.e. a homeomorphic image of the product of a Cantor set and an interval, given by a union of C^∞ curves containing a dense set of analytic arcs.*
(2) *K has Hausdorff dimension one.*

Open problems and questions

There are several natural questions that the results described above suggest. While they demonstrate for example that the topology of hedgehogs is necessarily complicated, it seems a very difficult problem to describe their topology explicitly. A first step in this direction would be to give a topological model for the class of hedgehogs constructed using tube-log Riemann surfaces. One expects to be able to construct hedgehogs homeomorphic to a union of Jordan arcs $K = \cup_{\theta\in\mathbf{R}/\mathbf{Z}}\gamma_\theta$, where γ_θ reduces to $\{0\}$ except for a measure zero set of angles, and the dynamics f takes γ_θ to $\gamma_{\theta+\alpha}$ where $\alpha = \rho(f)$.

In the above model the hedgehogs $(K_t)t > 0$ associated to the dynamics are all homeomorphic to each other. It is natural to ask whether this holds in general. In the case of linearizable dynamics, the linearization domains are naturally parametrized by their conformal radii; the parametrization is natural in the sense that it is invariant under change of variables. A natural question is whether such a conformally invariant parametrization exists for the family of hedgehogs of a nonlinearizable germ, in some sense a “degenerate conformal radius”.

More generally, if one thinks of hedgehogs as “degenerate linearization domains” then it is very tempting to ask whether there exist associated “degenerate linearizations" conjugating the dynamics on the hedgehogs to that of an irrational rotation. The theory that seems most likely to provide an answer in this direction is that of Borel monogenic functions [Bo]. These are quasi-analytic functions whose domains of definition are not necessarily open sets and whose singularities are not necessarily isolated. The hope then is that nonlinearizable germs admit monogenic linearizations defined on the hedgehogs that determine the conjugacy class of the dynamics.

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Kingshook Biswas [kingshook@rkmvu.ac.in] was a student of Ricardo Perez-Marco and got his PhD from the Universite Paris XIII in 2005. After working in post-doctoral positions at Stony Brook University, Universite Toulouse III and the Universita di Pisa, he returned to India in 2008 and is currently an Assistant Professor at the Vivekananda University in Kolkata. His research interests include holomorphic dynamics, Riemann surfaces and Kleinian groups.

Abel Prize 2009 to Mikhail Gromov

The Russian-French mathematician Mikhail Leonidovich Gromov, Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France received the 2009 Abel Prize from His Majesty King Harald of Norway at an award ceremony in Oslo, 19 May. The Abel Prize that carries a cash award of NOK 6,000,000 (close to Euro 700,000, US$ 950,000), was awarded to him by the Norwegian Academy of Science and Letters "for his revolutionary contributions to geometry".

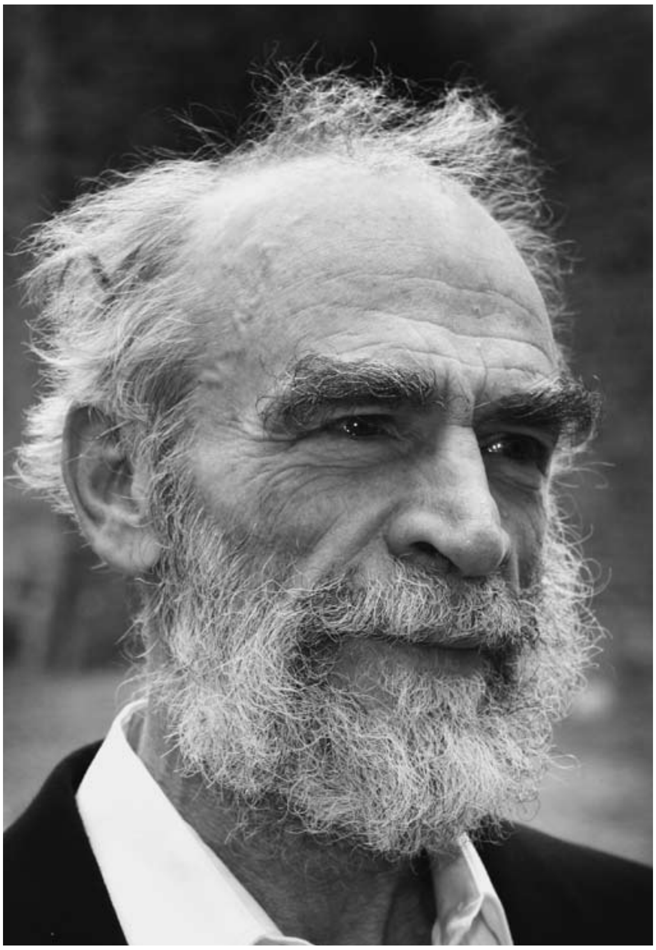
From the citation: *Geometry is one of the oldest fields of mathematics; it has engaged the attention of great mathematicians through the centuries, but has undergone revolutionary change during the last 50 years. Mikhail Gromov has led some of the most important developments, producing profoundly original general ideas, which have resulted in new perspectives on geometry and other areas of mathematics.*

Riemannian geometry developed from the study of curved surfaces and their higher dimensional analogues and has found applications, for instance, in the theory of general relativity. Gromov played a decisive role in the creation of modern global Riemannian geometry. His solutions of important problems in global geometry relied on new general concepts, such as the convergence of Riemannian manifolds and a compactness principle, which now bear his name.

Gromov is one of the founders of the field of global symplectic geometry. Holomorphic curves were known to be an important tool in the geometry of complex manifolds. However, the environment of integrable complex structures was too rigid. In a famous paper in 1985, he extended the concept of holomorphic curves to J-holomorphic curves on symplectic manifolds. This led to the theory of Gromov-Witten invariants, which is now an extremely active subject linked to modern quantum field theory. It also led to the creation of symplectic topology and gradually penetrated and transformed many other areas of mathematics.

Gromov's work on groups of polynomial growth introduced ideas that forever changed the way in which a discrete infinite group is viewed. Gromov discovered the geometry of discrete groups and solved several outstanding problems. His geometrical approach rendered complicated combinatorial arguments much more natural and powerful.

Mikhail Gromov is always in pursuit of new questions and is constantly thinking of new ideas for solutions of old problems. He has produced deep and original work throughout his career and remains remarkably creative. The work of Gromov will continue to be a source of inspiration for many future mathematical discoveries.



Abel Laureate Mikhail Gromov. (Photo: Knut Falch Scanpix)

Abel lectures and science lecture

The day after the prize ceremony, a series of scientific lectures connected to the laureate's work were presented at the University of Oslo. The laureate himself gave the first lecture entitled *Power spaces and bulk experiments* available as a video at http://www.abelprisen.no/en/multimedia/2009/. Particular aspects of his work were then explained by Jeff Cheeger (Courant Institute, New York University; *How does he do it?*) and by Martin Bridson (Oxford University; *Geometry everywhere: Fiat lux!*) who focused particularly on Geometric Group Theory. The final Science Lecture was given by Guillermo Sapiro (University of Minnesota, Minneapolis MN, USA; *One small step for Gromov, one giant leap for shape analysis*) and described how shape recognition and matching benefits from applications of the concept Gromov-Hausdorff distance.

Interview with Abel laureate Mikhail Gromov

Martin Raussen (Aalborg, Denmark) and Christian Skau (Trondheim, Norway), Oslo, 18 May 2009

A Russian education

Raussen & Skau: First of all, we would like to congratulate you warmly for having been selected as the 2009 Abel Prize winner. We would like to start with some questions about your early years and your early career. You were born towards the end of World War II in a small town called Boksitogorsk, 245 km east of St. Petersburg (at that time Leningrad).

Gromov: My mother was a medical doctor in the fighting army – and to give birth at that time, she had to move a little away from the frontline.

Could you tell us about your background, your early education and who or what made you interested in mathematics?

My first encounter with mathematics besides school was a book my mother bought me called "Numbers and Figures" by Rademacher and Toeplitz, which had a big influence on me. I could not understand most of what I was reading but I was excited all the same. I still retain that excitement by all the mysteries that you cannot understand but which make you curious.

Did you know you would go into mathematics while at high school?

In my middle and later years at high school I was more interested in chemistry than in mathematics. But then I was hooked. There were some very good books in Russia on mathematical problems for youngsters. I was going through them and I immersed myself in all this for a year. In my last year of high school I was attending a so-called mathematics circle, something for youngsters at the university, run by two people Vasia Malozemov and Serezha Maslov (Maslov became a logician; coincidentally, he was the one who suggested Hilbert's tenth problem to Matiasevic). They were running an extremely good group for young children that I attended. This was in St. Petersburg in 1959, the year before I started at university, and it was the major reason for my decision to study mathematics.

You started studying mathematics at Leningrad University. Please tell us about the environment there, how you were brought up mathematically and about the teachers that were important for you.

I think it was a pleasant environment despite the political surroundings, which were rather unpleasant. There was an extremely high spirit in the mathematical community and among professors. I remember my first teachers, including Professor Isidor Pavlovich Natanson, and also I

Abel interview, from left to right: Christian Skau, Martin Raussen, Mikhail Gromov. (Photo: Heiko Junge / Scanpix)

attended a class run by Boris Mikhailovich Makarov. You could see the high intensity of these people and their devotion to science. That had a very strong impact on me, as well as the interactions with the senior students. Let me mention one, the young algebraist Tolia Yakovlev who projected this image of absolute dedication to mathematics. On the other hand, there was a general trend in Leningrad of relating mathematics to science. This was influenced, I think, by Kolmogorov and Gelfand from Moscow. Kolmogorov made fundamental contributions to hydrodynamics and Gelfand was working in biology and also in physics. Basically, there was an idea of the universality of knowledge, with mathematics being kind of the focus of intellectual ideas and developments. And that, of course, shaped everybody who was there, myself included. And I learned very much of the Moscow style of mathematics from Dima Kazhdan, with whom we were meeting from time to time.

Can you remember when and how you became aware of your exceptional mathematical talent?

I do not think I am exceptional. Accidentally, things happened and I have qualities that you can appreciate. I guess I never thought in those terms.

At least towards the end of your studies, your academic teacher was Vladimir Rokhlin. Do you still sense his influence in the way you do mathematics today?

You see, Rokhlin himself was educated in Moscow and the Moscow mathematical way of thinking was very different from that in Leningrad. They had a different kind of school that was much more oriented towards Western mathematics. Leningrad was more closed and focused on classical problems; Moscow was more open to new developments. And that is what he brought to Leningrad. Another person with the same attitude was Boris Venkov, an algebraic geometer. From him and from Rokhlin, I got a much broader view and perception of mathematics than what I could have got from the traditional school in Leningrad. On the other hand, the traditional school was also very strong; for instance, the geometry school of Aleksandr Danilovich Alexandrov. There were people like Zalgaller and Burago from whom I learned most of my geometry. Burago was my first teacher in geometry.

You were very successful at Leningrad University at the beginning of the 1970s. Still, you left Leningrad and the Soviet Union shortly after in 1974. What was the background for your desire to leave?
This is very simple. I always say, if someone tells you you should not do something, you try to do exactly that. You know what happened when God prohibited Eve eating the apple. This is human nature. It was said that you cannot leave the country; it is just impossible, it is wrong, it is horrible. It is like in scientific work: if it is impossible, you try to do it anyway.

It was probably not that easy to get out of the Soviet Union at that time?
For me it was relatively easy. I was very lucky. But in general it was difficult and risky. I had to apply, I waited for several months and then I got permission.

Russian mathematics

Jacques Tits, one of the Abel prize winners last year, praised Russian mathematical education and Russian schools for the strong personalities and the strong ties between motivations, applications and the mathematical apparatus, as well as the lively seminars and discussions sometimes lasting for many hours. What is your perception: what is special about the Russian mathematical style and school?
Like I said, it was somewhat different in Leningrad compared to Moscow. What Tits was probably referring to was Gelfand's seminars in Moscow. I attended this seminar in Moscow only once when I was invited to give a talk, so my recollection might not be typical. But when I came, it took about two hours before the seminar could start because Gelfand was discussing various matters with the audience. Another seminar was run by Piatetsky-Shapiro and that was very rigorous. When something was presented on the blackboard and the audience asked questions, then Shapiro would express his attitude, which was very strong and a bit aggressive: on what students should know and should not know, the idea that they should learn this and this and that… Extremely powerful indications of his personality!

Do you still feel that there is a specific Russian mathematical background that you build your work upon?
Yes, definitely. There was a very strong romantic attitude towards science and mathematics: the idea that the subject is remarkable and that it is worth dedicating your life to. I do not know whether that is also true in other countries because I was not elsewhere at that time of my education. But that is an attitude that I and many other mathematicians coming from Russia have inherited.

Is there still a big difference between Russian mathematics and, say, Western mathematics in our days? Or is this difference about to disappear, due to the fact that so many Russians are working in the West?
This I cannot tell given there are so many Russians working in the West. I do not know much about mathematical life in Russia nowadays; certainly, things have changed tremendously. In my time in Russia, this intensity was partly a reaction to the outside world. Academic life was a peaceful garden of beauty where you could leave a rather ugly political world outside. When this all changed, this sharp concentration went down. It might be so. I don't know. This is only a conjecture.

Do you still have a lot of contact with Russian mathematicians? Do you go there once in a while?
I have been there twice since I left the country. You still feel the intensity of life there but things go down, partially because so many gifted people are leaving. They are drawn to larger centres where they can learn more.

Can you tell us about other Russian mathematicians that have influenced you, like Linnik?
Yes. Yuri Linnik was a great scientist, professor and academician in Leningrad. He was running educational seminars in algebraic geometry one year. A remarkable thing was that he always admitted his complete ignorance. He never pretended to know more than he did, rather the contrary. And secondly, there was always a complete equality between him and his students. I remember one time I was supposed to give a talk there but I overslept and arrived one hour late. But he was just laughing at it – not annoyed at all. And that, I think, exhibits some of his spirit in mathematics – the atmosphere of how we were all in the same boat, regardless of who you were.

How would you compare him with Rokhlin as a person?
Rokhlin was a more closed person as he had gone through a very complicated life. He was a prisoner in the Second World War. He was Jewish but he somehow managed to conceal it. He had an extremely strong personality. After he was liberated, he was sent to a prison in Russia, a labour camp, because it was considered that he hadn't finished his military service. Being a prisoner of war didn't count as military service! After some work he came to Moscow. It was difficult to say what he thought. He was very closed and tried to keep high standards on everything but he was not so relaxed and open as Linnik was. It was at first unclear what it was but then you realised that he was shaped by those horrible experiences.

Was Linnik also Jewish?
I think Linnik was half Jewish but he did not participate in the war. He had a different kind of life. He was bet-

ter positioned in his career as a member of the Academy and so on. Rokhlin was always discriminated against by the authorities, for reasons I don't know. I heard some rumours that he was getting into conflict with some officials in Moscow.

For some time he was a secretary for Pontryagin because Pontryagin was blind and, as an academician, needed a secretary. Rokhlin had this position until he had defended his second thesis. Then he was kicked out of Moscow because he was over-qualified. A. D. Alexandrov, then the rector at Leningrad University, made a great effort to bring him to Leningrad in 1960. That had a very strong influence on the development of mathematics in Leningrad. The whole school of topology grew out of his ideas. Rokhlin was a very good teacher and organiser.

Is it true that Pontryagin was anti-Semitic?
I believe he became anti-Semitic after his second marriage He was blind and it is unclear how independent his perception of the world was. In his later years he became anti-Semitic and he also wrote pamphlets that sounded absolutely silly. It is unclear what or who influenced him to get those ideas.

History of Geometry

You are the first Abel laureate to receive the prize explicitly for your "revolutionary contribution to geometry". From Euclid`s time geometry was, so to say, the "face" of mathematics and a paradigm of how to write and to teach mathematics. Since the work of Gauss, Bolyai and Lobachevsky from the beginning of the nineteenth century, geometry has expanded enormously. Can you give us your thoughts on some of the highlights since then within geometry?
I can only give a partial answer and my personal point of view. It is very difficult to find out about how people thought about the subject in ancient times. Seen from today, geometry as a mathematical subject was triggered from observations you make in the world; Euclid gave a certain shape of how to organise observations and made an axiomatic approach to mathematics and what followed from those. It happened that it worked very badly beyond the point that it was designed for. In particular, there was a problem with the parallel postulate and people tried to prove it.

There was a mixture: on one hand they believed that the way you see the world was the only way for you to see it and they tried to justify that axiomatically. But it did not work. Eventually, mathematicians realised that they had to break out of the naïve way of thinking about axioms. The axioms happened to be very useful but only useful in a limited way. Eventually, you had to deny them. This is how they served.

From this point on, mathematics started to move in different directions. In particular, Abel was one of the people who turned mathematics from just observing and formalizing what you see to formalizing what you cannot see directly – what you can only see in a very opaque way. Modern mathematics was shaped in the beginning of the nineteenth century. Then it became more and more structural. Mathematics not only deals with what you see with your eye but what you see in the structure of things. At a more fundamental level, I would say. If you formulate the problem in modern language, the mathematicians at the time faced trying to understand the limitations of Euclidean geometry; it is completely obvious. But it took centuries to develop this language. This work was started by Lobachevsky, Bolyai and Gauss, and in a different domain by Abel and Galois.

The laureate's research in geometry

It is said that you revolutionized Riemannian geometry in the late 1970s. Could you explain to us what your novel and original idea consisted of, the idea that turned out to be so groundbreaking?
I cannot explain that since I never thought of them as groundbreaking or original. This happens to any mathematician. When you do something new, you don't realise it is something new. You believe everybody knows it, that it is kind of immediate and that other people just have not expressed it. This happens in fact with many mathematical proofs; the ideas are almost never spoken out. Some believe they are obvious and others are not aware of them. People come from different backgrounds and perceive different things…

A hallmark of your work has been described as the softening of geometry, whereby equations are replaced by inequalities or approximate or asymptotic equations. Examples include the "coarse viewpoint" on Riemannian geometry, which considers all Riemannian structures at once. This is very original. Nobody had thought about that before. Is not that true?
That is probably true. But again, I am not certain whether somebody else had had this idea before. For me it was clear from the very beginning and I actually never articulated it for a long time believing everybody knew it. I believe that some people knew about it but they never had an occasion to say it aloud. In the end, I formulated it because I gave a course in France.

First of all, you had this new perspective. The basic ideas are perhaps very simple but you were the first to get any deep results in that direction.
Well, there were predecessors. This trend in Riemannian geometry started with the work of Jeff Cheeger. Earlier, up to some point, people were thinking about manifolds in very abstract terms. There were many indices and you could not take the subject into your hand. I think that one of the first works in which Riemannian geometry was turned into something simple was by John Nash. Actually, he had a tremendous influence on me. He was just taking manifolds in his hands and putting them in space, just playing with them. From this I first learned about this very concrete geometry. Simple things, but you had to project it to very high dimensions. And then there was the work by Jeff Cheeger, formally a very different subject but with the same attitude, realising that things got

quite simple when formalized, if that was done properly. So I was just following in the steps of these people.

This means that you read Nash's work and were impressed by it very early?
Yes, I read it very carefully. And I still believe I am the only person who read his papers from the beginning to the end. By judging what people have written about it afterwards, I do not think they have read it.

Why not?
At first, I looked at one of Nash's papers and thought it was just nonsense. But professor Rokhlin said: "No, no. You must read it". I still thought it was nonsense; it could not be true. But then I read it and it was incredible. It could not be true but it was true. There were three papers; the two more difficult ones, on embeddings, they looked nonsensical. Then you look at the way it is done and you also think that it looks nonsensical. After understanding the idea you try to do it better; many people tried to do it in a better way. But when you look at how they were doing it, and also what I tried, and then come back to Nash, you have to admit that he had done it in a better way. He had a tremendous analytic power combined with geometric intuition. This was a fantastic discovery for me: how the world may be different from what you think!

John Nash received the Nobel Prize in economics and he was also the person behind the "Beautiful Mind" movie. Many people think he should have gotten the Fields Medal for his efforts. Do you subscribe to this idea?
Yes. When you think about this guy and his achievements in science, forgetting about medals, the discoveries he made were fantastic. He was a person thinking in a most unusual way. At least, his work in geometry was contrary to what everybody would expect, concerning the results, the techniques and the ideas he used. He did various matters in an extremely simple way, so that everybody could see it but nobody would believe it could work. He also had a tremendous power of implementing it, with a dramatic analytic power. What he has done in geometry is, from my point of view, incomparably greater than what he has done in economics, by many orders of magnitude. It was an incredible change in attitude of how you think about manifolds. You can take them in your bare hands and what you do may be much more powerful than what you can do by traditional means.

So you admit that he had an important influence upon you and your work.
Yes, absolutely. All over, his work and the work of Smale, which was explained to me by Sergei Novikov at a summer school in the early 60s, have had the most important influence on me.

You introduced the h-principle, where "h" stands for homotopy, in order to study a class of partial differential equations that arises in differential geometry rather than in physical science; it has proved to be a very powerful tool. Could you explain the h-principle and your ideas behind introducing the concept?

This was exactly motivated by the work of Smale and Nash. And I realised then that they dealt more or less with the same topic – which had not been clear at all. In particular, if you use Nash's techniques you immediately get all the results of immersion theory. You do not have to go deep. The first lemma in Nash proves all immersion theorems in topology! I was thinking about this for several years, trying to understand the mechanism behind it. I realised there was a simple general mechanism, which was rather formal but incorporated the ideas of Nash and Smale by combining them. This applies to a wide class of equations because you interpolate between rather remote topics and then you cover a very large ground.

You proved a celebrated theorem, precursors of which were theorems by Milnor-Wolf and Tits. It tells us that if a finitely generated group has polynomial growth then it contains a nilpotent subgroup of finite index. A particularly remarkable aspect of your proof is that you actually use Hilbert's Fifth Problem, which was proved by Gleason, Montgomery and Zippin. And this is the first time, apparently, that this result has been used in a significant way. Can you explain and expand on this?
I thought previously about applying this theorem in Riemannian geometry, though in a different context, inspired by Margulis' 1967 paper on 3-dimensional Anosov flows and by his 1970 rendition of Mostow's rigidity theorem, where Margulis introduced and exploited quasi-isometries. I wanted to prove something that happened to be wrong. I tried to apply a version of the Shub-Franks construction in topological dynamics. It didn't work either. Also, there was a paper by Hirsch concerning exactly this question about polynomial growth – a special case of this problem – where he tried to apply the classification of topological groups; and again it didn't work. So I believed it couldn't be applied. It was kind of clear to us that it was close but it didn't seem to work. But when I was formalizing the idea of limits of manifolds, I tried to think in those terms and then I saw that it might work. This was kind of a surprise to me.

It must have been a very nice experience when you realised that this would work out?
Well, it was not really a sudden insight. I realised what was needed was just a slight change in conceptions. Then it is not difficult to do it. The proof is extremely simple in a way. You take an obvious concept of a limit and then, by the power of analysis, you can go to the limit many times, which creates structures that you have not seen before. You think you have not done anything but, amazingly, you have achieved something. That was a surprise to me.

You introduced the idea of looking at a group from infinity, which is an apt description of looking at the limit of a sequence of metric spaces associated to the group in the so-called Gromov-Hausdorff metric. You

have used this technique with impressive effect. Please give us some comments.

After proving the theorem about polynomial growth using the limit and looking from infinity, there was a paper by Van den Dries and Wilkie giving a much better presentation of this using ultrafilters. Then I took it up again and I realised it applied to a much wider class of situations where the limits do not exist but you still have the ultralimits, and it gives you a very good view on many mathematical objects including groups. But it is still not tremendously powerful.

In the context of groups, I was influenced by a survey of the small cancellation theory by Paul Schupp in the book *Word Problems* (1973) where he said – and I think this was a very honest and very useful remark – that "people don't understand what small cancellation groups are". And I felt very comfortable because I didn't understand it either. I started thinking about what they could be and then I came up with this concept of hyperbolicity. This was rather pleasing to me but there were some technical points I could not handle for some time, such as the rough version of the Cartan-Hadamard theorem, before I could write an article about it.

When did you introduce the concept of a hyperbolic group?

My first input on the geometry of groups came from Dima Kazhdan who explained to me in the middle of the 60s the topological proof of the Kurosh subgroup theorem. Later on I read, in the same 1971 issue of *Inventiones*, the paper by Griffiths on complex hyperbolicity and the paper by Klingenberg on manifolds of hyperbolic type. The latter contained the idea of rough hyperbolicity, albeit the main theorem in this paper was incorrect. And, as I said, I had read the paper by Schupp.

I presented the first definition of hyperbolicity during the '78 meeting at Stony Brook under the name of Is(2)-groups as they satisfy the linear isoperimetric inequality in dimension two. The article appeared three years later. Also, I recall, I spoke about it at the Arbeitstagung in 1977. I tried for about ten years to prove that every hyperbolic group is realisable by a space of negative curvature, which I couldn't do, and this is still unknown. Then Steve Gersten convinced me to write what I already knew about it and I wrote that but I was very dissatisfied because I couldn't decide if you needed the theory of such groups. If they were "geometric", the way I said, we would not need hyperbolicity theory and we would have much better theorems.

You said that almost all groups are hyperbolic?

Right. That was actually the point. When I realised that we could see hyperbolicity in certain generic constructions better without an appeal to curvature then I accepted it as a worthwhile notion. In my first article I suggested a rather technical definition and terminology. I believed it was a preliminary concept. But then I realised eventually that it probably was the right concept, regardless of whether the geometrization theorem I was trying to prove was true or not. Also, I was encouraged by talking to Ilia Rips in the early 80s who, by that time, had developed hyperbolic group theory in a combinatorial framework, well beyond what I knew at the time, by the ongoing development of Thurston's 3D-theory and by Cannon's solution of Thurston's rationality conjecture.

We move to a different area, symplectic geometry, that you have also made a revolutionary contribution to. You introduced methods from complex analysis, notably pseudo-holomorphic curves. Could you expand on this and explain how you got the idea for this novel approach? And also on the Gromov-Witten invariant, which is relevant for string theory and which came up in this connection.

Yes, I remember very vividly this amazing discovery I made there. I was reading a book by Pogorelov about rigidity of convex surfaces. He was using the so-called quasi-analytic functions developed by Bers and Vekua. He talked about some differential equations and said that the solutions were quasi-analytic functions. I couldn't understand what the two had in common. I was looking in his books and in articles of these people but I couldn't understand a single word; and I still don't.

I was extremely unhappy about this but then I thought about it in geometric terms. And then you immediately see there is an almost complex structure there and the solutions are just holomorphic curves for this almost complex structure. It is nothing special because any elliptic system in two variables has this property. It has the same principal symbol as the Cauchy-Riemann equation. The theorem he was using is obvious once you say it this way. You didn't have to use any theory; it is obvious because complex numbers have a forced orientation. That's all you use!

You say obvious but not many mathematicians were aware of this?

Yes, exactly. They were proving theorems but they never looked at this. If you look at this in certain terms, it becomes obvious because you have experience with algebraic geometry. Once you know algebraic geometry you observe it as the same. We have this big science of complex analysis and algebraic geometry with a well-established theory; you know what these things are and you see there is no difference. You use only some part of this but in higher generality.

Then, I must admit that for some time I was trying to use it to recapture Donaldson theory but I couldn't do it because there were some technical points that did not work. Actually, it was similar to the obstruction of being Kähler in dimension four. I spoke with Pierre Deligne and asked him whether there was an example of a complex surface that was not Kählerian and that would have certain unpleasant properties. He said, yes, and showed me such examples. I turned then to the symplectic case and I realised that it worked very well. And once again, things were very simple, once you knew where to go. It was so simple that I had difficulty believing it could work because there was only one precedent, due to Donaldson. It was Donaldson's theory that said that such mathematics can give you that kind of conclusion. It had never happened before Donaldson and that was very encouraging. Otherwise I probably wouldn't have believed it would work if not for Donaldson's discovery. Besides, I was prepared by Arnold's conjectures, which I learned from Dima Fuks in the late 60s, by the symplectic rigidity ideas of Yasha Eliashberg developed by him in the 70s and by the Conley-Zehnder theorem.

Could you say something about the proof by Perelman and Hamilton of the Poincaré conjecture? Did they use some of your results?
No. If at all, then just some very simple things. That is a completely different mathematics. There are interactions with the geometry I know but they are minor. It is essentially a quite different sort of mathematics, which I understand only superficially, I must admit. But I must say that it is a domain that is basically unexplored compared to what we know about Cauchy-Riemann equations in a generalized sense, or Yang-Mills, Donaldson or Seiberg-Witten equations. Here, it is one theorem and it is still somewhat isolated. There is no broader knowledge around it and we have to wait and see what comes. We certainly expect great developments from this yet to come.

Do you have any interaction with Alain Connes?
Oh yes, certainly. We have interacted quite a bit though we think in very different ways. He understands one half and I understand the other half, with only a tiny intersection of the two parts; amazingly, the outcome turns out to be valid sometimes. I have had two joint papers with him and Moscovici, proving particular cases of the Novikov conjecture.

You came up with an example of some expanders on some groups and thus produced a counter-example to the Baum-Connes conjecture.
This counter-example is due to Higson, Lafforgues and Skandalis, where they used the construction of random groups.

Is there one particular theorem or result you are the most proud of?
Yes. It is my introduction of pseudo-holomorphic curves, unquestionably. Everything else was just understanding what was already known and to make it look like a new kind of discovery.

You are very modest!

Mathematical biology

We have been told that you have been interested in questions and problems in mathematical biology recently. Can you describe your involvement and how your mathematical and geometric insights can be useful for problems in biology?
I can explain how I got involved in that. Back in Russia, everybody was excited by ideas of René Thom on applying mathematics to biology. My later motivation started from a mathematical angle, from hyperbolic groups. I realised that hyperbolic Markov partitions were vaguely similar to what happens in the process of cell-division. So I looked in the literature and spoke to people and I learned that there were so-called Lindenmayer systems. Many biologists think that they represent a very good way of describing the growth of plants by patterns of substitution and cell division.

Then, at the base of that, we had a meeting at the IHES in Bures on pattern formation, in particular in biology. I got interested and I wanted to learn more about biology. Soon, I realised that there had been a huge development in molecular biology in the 80s, after the discoveries of genetic engineering and of PCR (polymerase chain reaction). It was really mathematical procedures applied to living cells. Mathematicians could invent PCR. It didn't happen but mathematicians could have invented PCR. It was one of the major discoveries of the century. It changed molecular biology completely. I started to learn about these mathematical procedures and to realise that it led to fantastic mathematical questions. But it was hard to say exactly what it is; I just cannot formulate it. Of course there are very particular domains like sequencing and there are specific algorithms used there. But this is not new mathematics; it is old mathematics applied to this domain. I believe there is mathematics out there still unknown to us that is yet to be discovered. It will serve as a general framework, just like differential equations give a framework for classical mechanics. It will be rather abstract and formal but it should embed our basic knowledge of biology and maybe accumulate results that we still do not know. I still think about this but I do not know the answer.

Would you please explain the term PCR?
It means polymerase chain reaction and you can see it as follows. You come to a planet that is populated by rats and they all look the same. In your lab, you also have rats that are very similar. They look absolutely identical but they are of a different species. Now, one of the female rats escapes. One year later you want to decide whether it has survived or not. There are billions of those rats, so you cannot check all of them, so what do you do? Here is the idea. You throw in several billions of male rats and if the escaped rat is still there then you will find a certain population of your rats. Then you wait a little bit and the number of them will grow into billions. You take a sample and check if it contains your rat.

This is how a polymerase chain reaction works but instead of rats you use DNA. There are billions of different DNAs of various kinds and if you want to know if a particular one of them is out there then there is a way to do that with a given molecule that amplifies exponentially. If one had been out there, you would have billions of them after several cycles. This incredible idea is very simple and powerful. One fundamental thing happening in biology is amplification; it is specific for biology. Mathematics should be useful for biologists. We cannot make it yet but I believe it can be done. It will have impact on problems in genetic engineering and identifying gene functions but it has not been developed yet. It will be very different from other kinds of mathematics.

Mediation between mathematics and science

Is it your impression that biologists recognise and appreciate your work and the work of other mathematicians?
I have not done anything. I just communicated with biologists. But I think many of them were quite satisfied talking to me, as well as to other mathematicians. Not because we know something but because we ask many questions. Sometimes they cannot answer but that makes them think. That is about it but this is not so little in my opinion. In this way, mathematicians can be useful by being very good listeners.

It happens very rarely that something is done by mathematicians in science. One of the most remarkable examples happened here in Norway in the middle of the nineteenth century. In collaboration, the mathematician Guldberg and the chemist Waage invented chemical kinetics. I do not know of any other situation since then where mathematicians have contributed to experimental science at this level. This shows that it is possible but it happened through a very close collaboration and in a special situation. I think something like that may happen in biology sometime but it cannot come so easily.

You came across Guldberg and Waage in connection with your interest in chemistry?
Yes. This is kind of the fundamental equation in chemistry and also in molecular biology, always on the background of things. Mathematicians can have their word but it is not so easy. You cannot program it. You have to be involved. Sometimes, very rarely, something unexpected happens, with a very strong impact!

To our amazement, we realised that one of the Abel lectures in connection with the prize, the science lecture, was given on computer graphics. It is said that computer graphics or computer vision, and shape analysis in particular, benefits from your invention: the Gromov-Hausdorff distance. Can you explain where this notion comes in and how it is used?
When you have to compare images, the question is how you compare them. Amazingly – for a geometer it looks unbelievable – the early work on computer vision was based on matching images with another, taking differences in intensity – which is certainly completely contrary to what your eyes do!

Actually, the idea of how eyes operate with images goes back to Poincaré. In his famous book called *Science and Hypothesis* he thinks, in particular, about how the human mind can construct Euclidean geometry from the experience we have. He gives an almost mathematical proof that it would be impossible if your eyes could not move. So, what you actually reconstruct, the way your brain records visual information, is based on the movement of your eyes and not so much on what you see. Roughly, the eye does this. It does not add images. It moves images. And it has to move them in the right category, which is roughly the category that appears in Riemannian geometry, with Hausdorff convergence or whatever, using small distortions and matchings of that.

For a mathematician who has read Poincaré, this is obvious. But for the people in computer science, following different traditions from linear analysis, it was not obvious at all. And then, apparently, they brought these ideas from geometry to their domain… Actually, several times I attended lectures by Sapiro since I became interested in vision. He is someone who has thought for a long time about how you analyse images.

It seems that there is not enough mediation between science and mathematics.
Absolutely, I completely agree. To say "not enough" is an understatement. It is close to zero. The communities have become very segregated due to technical reasons and far too little communication. A happy exception is the Courant Institute. We still have many people interacting and it happens that mathematicians fall in love with science. To see these young people at Courant is extremely encouraging because you don't see this kind of applied mathematicians anywhere else. But they are well aware of the body of pure mathematics where they can borrow ideas and then apply them. Typically, applied mathematicians are separated from the pure ones. They, kind of, don't quite like each other. That's absurd. This has to be changed because we have the same goals. We just understand the world from different sides.

Do you have any ideas of how to improve this situation?
No. But I think in any subject where you have this kind of problem, the only suggestion is that you have to start by studying the problem. I don't know enough about this; I just have isolated examples. We have to look at where it works, where it doesn't work and just try to organise things in a new way. But it has to be done very gently because you cannot force mathematicians to do what they don't like. The obvious way to do it is to design good combined educations in mathematics and science.

Actually, there is a very good initiative by François Taddei in Paris who organises classes with lectures on biology for non-biologists – for young people in mathematics and physics. He is extremely influential and full of enthusiasm. I attended some of those classes and it was fantastic. He was teaching biology at École Normale for mathematicians and physicists and he manages to make those ideas accessible for everybody. That is what I think should be done at the first stage. We have to have this special kind of education that is not in any curriculum; you cannot formalize it. Only people who have enough enthusiasm and knowledge can project this knowledge to young people. An institutionalized system is much harder to design, and it is very dangerous to make it in any way canonical, because it may just misfire. Forcing mathematics on non-mathematicians only makes them unhappy.

We have already talked about your affiliation to the Courant Institute in New York but for a much longer time you have been affiliated with the Institut des Hautes Etudes Scientifique (IHES) at Bures-sur-Yvette, close to Paris. Can you explain the role of this institution for your research – and for your daily life, as well?

It is a remarkable place. I knew about it before I came there; it was a legendary place because of Grothendieck. He was kind of a god in mathematics. I had met Dennis Sullivan already at Stony Brook but then met him again at IHES, where I learned a lot of mathematics talking to him. I think he was instrumental bringing me there because he liked what I was doing and we interacted a lot. Dennis interacted with many people. He had a fantastic ability of getting involved in any idea – absorbing and helping to develop an idea. Another great man there was René Thom but he was already into philosophy apart from doing mathematics. Pierre Deligne was also there. From Pierre I learned some stuff rather punctually; on several occasions, I got fantastic answers when I asked him questions. He would take an idea from your mind and turn it in another direction.

Basically, the whole atmosphere created at this institution was very particular. You are almost completely free of anything except for doing research and talking to people – a remarkable place. I think my best memories go back to when I was there as a first year visitor. Then I was really free. When I became a part of it there were some obligations. Not much, but still. It is ideal for visitors to come for half a year and just relax but being there permanently was also not so bad.

Did you get your best results when you were at Bures?
Yes. When I was between 35 and 39, I would say. That's when I was the most productive.

Computers for mathematicians and for mathematics

It is clear that the use of computers has changed the everyday life of mathematicians a lot. Everybody uses computers to communicate and editing is done with computer tools by almost everybody. But other people use computers also as essential research equipment. What are your own experiences? Do you use computers?
No, unfortunately not. I am not adept with computers. I can only write my articles on a computer and even that I learned rather recently. I do believe that some mathematics, particular related to biology, will be inseparable from computers. It will be different mathematics when you, indeed, have to combine your thinking with computer experiments. We have to learn how to manipulate large amounts of data without truly understanding everything about it, only having general guidelines. This is, of course, what is happening but it is not happening fast enough. In biology, time is the major factor because we want to discover cures or at least learn about human diseases. And the faster we do it, the better it is. Mathematicians are usually timeless. You are never in a hurry. But here you are in a hurry and mathematicians can accelerate the process. And there, computers are absolutely a part of that. In this way, I believe computers are playing and will play a crucial role.

And that will change the way mathematics is done in the long run, say within the next fifty years?
I think that within 50 years there will be a radical change in computers. Programming develops very fast and I also believe mathematicians may contribute to the development in a tremendous way. If this happens, we will have very different computers in 50 years. Actually, nobody has been able to predict the development of computers. Just look at how Isaac Asimov imagined robots and computers thirty years ago when he was projecting into the 21st century how they looked like in the 70s. We probably cannot imagine what will happen within 50 years. The only thing one can say is that they will be very different from now; technology moves at a very fast speed.

What do you think about quantum computing?
Well, I am not an expert to say anything about that. You have to ask physicists but they have very different opinions about it. My impression is that the experimental physicists believe we can do it and theorists say: "No, no, we cannot do it". That is the overall impression I have but I cannot say for myself because I don't understand either of the aspects of it.

Mathematical work style

You have been described as a mathematician who introduces a profoundly original viewpoint to any subject you work on. Do you have an underlying philosophy of how one should do mathematics and, specifically, how one should go about attacking problems?
The only thing I can say is that you have to work hard and that's what we do. You work and work, and think and think. There is no other recipe for that. The only general thing I can say is that when you have a problem then – as mathematicians in the past have known – one has to keep the balance between how much you think yourself and how much you learn from others. Everybody has to find the right balance according to his or her abilities. That is different for different people so you cannot give any general advice.

Are you a problem solver more than a theory builder? Would you describe yourself in any of those terms?
It depends upon the mood you are in. Sometimes you only want to solve one problem. Of course, with age, you become more and more theoretical. Partly because you get wiser but you can also say it is because you get weaker. I suppose it depends on how you look at it.

Concerning your mathematical work style, do you think about mathematics all the time?
Yes, except when I have some problems of a personal nature; if there is something else that disturbs me then I cannot think. But if everything is ok and, at least, if there is nothing else to do at the moment, I immerse myself in mathematics, or other subjects, like biology, but in a mathematical way, so to say.

How many hours per day do you work with mathematics?
Not as much as I used to. When I was young I could go on all day, sometimes from nine in the morning to eleven

at night. Nothing could distract me. Of course, now I cannot do that any longer. I can only do five, six hours a day without getting tired.

When you were younger, you had more energy but now you are a lot wiser, right?
You can say you become more experienced and wiser when you get older. But you also lose your mental powers and you become weaker. You certainly just have to accept that. Whether you become wiser is questionable. But it is obvious that you become weaker.

John von Neumann once said that you do the most important things in mathematics before you are thirty. When he himself turned thirty he added that you get wiser as you get older. Do you think that the best mathematics is done before you are thirty?
I can say about myself that I think my best work was done when I was between thirty and forty years old. When I started, I didn't have any perspective and was just doing whatever was coming first. As I was learning more, I kept changing my attitude all the time. Now, if I had to start anew, I would do something completely different, wrongly or rightly, I cannot judge.

On the other hand, I must say that everything I think about now, I had already thought of forty years ago. Ideas were germinating in me for a long time. Well, some people probably create radically new work late in life but basically you develop certain feelings very early. Like your abilities to talk, right? You learn to talk when you are three years old but it doesn't mean you say the same things when you are thirty as when you are three. That's how it works.

We are surprised that you are so modest by playing down your own achievements. Maybe your ideas are naïve, as you yourself say; but to get results from these ideas, that requires some ingenuity, doesn't it?
It is not that I am terribly modest. I don't think I am a complete idiot. Typically when you do mathematics you don't think about yourself. A friend of mine was complaining that anytime he had a good idea he became so excited about how smart he was that he could not work afterwards. So naturally, I try not to think about it.

Having worked so hard as you say, have you ever suffered from depression because you have overexerted yourself?
No. Sometimes some outside unhappy things have distracted my work. Of course, sometimes you get very tired and you are glad that someone interrupts your work but other times you cannot stop. You work and work, like an alcoholic, so then it is good to get some rest.

Abel and the Abel Prize

You once complained that the mathematical community only has digested a minor part of your work, rather the technical details than the underlying big ideas and vistas. Do you think that being awarded the Abel Prize may change that situation?
First about this complaint: it was kind of a half-joke. There were some pieces of work where there happened to be ideas that could not be developed, unlike more successful ones, and I was unhappy about that. It depends on how you look at it; either the ideas were no good or people were not paying attention. You just never know. I wished something I was saying could be developed further but this was not happening. And that was my complaint, or rather the motivation for my complaint. It has nothing to do with the Abel Prize.

What do you think about prizes in general and, in particular, about the Abel Prize?
Objectively, I don't think we need these prizes for mathematicians who have already achieved much. We need more to encourage young people at all levels and we must put more effort into that. On the other hand, it is very pleasant to receive this prize. I enjoy it, and it may have some overall positive effect on the perception of the mathematical community in the eyes of the general public. That may be just self-justification because I like it, of course, for appreciation of my work by my friends and by receiving this prize. But as the general scientific concern, the far more serious issue is projecting a much greater effort in getting funds for educating and motivating young people to embrace mathematics. What I have seen here in Oslo, at the high school I visited earlier today – with these young people – I was tremendously impressed. I want to see this kind of event everywhere in the world. Of course, mathematicians are not so ascetic that they don't like prizes but in the long run it is not prizes that shape our future.

Coming back to Abel, do you admire him as a mathematician?
Yes, absolutely. As I said, he was one of the major figures, if not the major figure, in changing the course of mathematics from what could be visualized and immediately experienced to the next level, a level of deeper and more fundamental structures.

There is a posthumous paper by Abel where he writes about the theory of equations, which later became Galois theory, and in the introduction he says something very interesting. He says something like: "A problem that seems insurmountable is just seemingly so because we have not asked the right question. You should always ask the right question and then you can solve the problem".
Absolutely. He changed the perspective on how we ask questions. I do not know enough about the history of mathematics but it is obvious that the work of Abel and his way of thinking about spaces and functions has changed mathematics. I do not know enough history to say exactly when this happened but the concept of underlying symmetries of structures comes very much from his work. We still follow that development. It is not exhausted yet. This continued with Galois theory and in the development of Lie group theory, due to Lie, and, in modern times, it was done at a higher level, in particular by Grothendieck. This will continue and we have to go through all that to see where it brings us before we go on to the next stage. It is the basis of all we do now in mathematics.

Mikhail Gromov is received in audience by King Harald and Queen Sonja. (Photo: Erlend Aas/Scanpix)

Future of mathematics

After this excursion into the history of mathematics, may we speculate a little about the future of mathematics? You once compared the whole building of mathematics with a tree, Hilbert's tree, with a metric structure expressing closeness or nearness between different areas and results. We know from Kurt Gödel that there are parts of that tree we will never reach. On the other hand, we have a grasp of a certain part of the tree but we don't know how big this part is. Do you think we know a reasonable part of Hilbert's tree? Is human mind built for grasping bigger parts of it or will there stay areas left uncharted forever?
Actually, I am thinking about that now. I don't know the answer but I have a programme of how we can approach it. It is a rather long discussion. There are certain basic operations by which we can perceive the structure. We can list some of them and apparently they bring you to certain parts of this tree. They are not axioms. They are quite different from axioms. But eventually you cannot study the outcome with your hands and you have to use computers. With computers you come to some conclusions without knowing the intermediate steps. The computational size will be too huge for you. You have to formalize this approach to arrive at certain schemes of computations. This is what I think about now but I don't know the answer. There are indirect indications that it is possible but those are of a non-mathematical nature, rather biological.

If you try to look into the future, 50 or 100 years from now…
50 and 100 is very different. We know more or less about the next 50 years. We shall continue in the way we go. But in 50 years from now, the Earth will run out of the basic resources and we cannot predict what will happen after that. We will run out of water, air, soil, rare metals, not to mention oil. Everything will essentially come to an end within 50 years. What will happen after that? I am scared. It may be okay if we find solutions but if we don't then everything may come to an end very quickly!

Mathematics may help to solve the problem but if we are not successful, there will not be any mathematics left, I am afraid!

Are you pessimistic?
I don't know. It depends on what we do. If we continue to move blindly into the future, there will be a disaster within 100 years and it will start to be very critical in 50 years already. Well, 50 is just an estimate. It may be 40 or it may be 70 but the problem will definitely come. If we are ready for the problems and manage to solve them, it will be fantastic. I think there is potential to solve them but this potential should be used and this potential is education. It will not be solved by God. People must have ideas and they must prepare now. In two generations people must be educated. Teachers must be educated now, and then the teachers will educate a new generation. Then there will be sufficiently many people who will be able to face the difficulties. I am sure this will give a result. If not, it will be a disaster. It is an exponential process. If we run along an exponential process, it will explode. That is a very simple computation. For example, there will be no soil. The soil is being exhausted everywhere in the world. It is not being said often enough. Not to mention water. It is not an insurmountable problem but it requires solutions on a scale we have never faced before, both socially and intellectually.

Education systems for the future

Education is apparently a key factor. You have earlier expressed your distress realising that the minds of gifted youths are not developed effectively enough. Any ideas about how education should change to get better adapted to very different minds?
Again I think you have to study it. There are no absolutes. Look at the number of people like Abel who were born 200 years ago. Now there are no more Abels. On the other hand, the number of educated people has grown tremendously. It means that they have not been educated properly because where are those people like Abel? It means that they have been destroyed. The education destroys these potential geniuses – we do not have them! This means that education does not serve this particular function. The crucial point is that you have to treat everybody in a different way. That is not happening today. We don't have more great people now than we had 100, 200 or 500 years ago, starting from the renaissance, in spite of a much larger population. This is probably due to education. This is maybe not the most serious problem with education. Many people believe in very strange things and accordingly make very strange decisions. As you know, in the UK, in some of the universities, there are faculties of homeopathy that are supported by the government. They are tremendously successful in terms of numbers of students. And anybody can learn that nonsense. It is very unfortunate.

You point out that we don't have anybody of Abel's stature today, or at least very few of them. Is that because we, in our educational system, are not clever

enough to take care of those that are exceptionally gifted because they may have strange ideas, remote from mainstream?
The question of education is not obvious. There are some experiments on animals which indicate that the way you teach an animal is not the way you think it happens. The learning mechanism of the brain is very different from how we think it works: like in physics, there are hidden mechanisms. We superimpose our view from everyday experience, which may be completely distorted. Because of that, we can distort the potentially exceptional abilities of some children.

There are two opposite goals education is supposed to achieve: firstly, to teach people to conform to the society they live in; on the other hand, to give them freedom to develop in the best possible way. These are opposite purposes and they are always in collision with each other. This creates the result that some people get suppressed in the process of adapting them to society. You cannot avoid this kind of collision of goals but we have to find a balance between the two and that is not easy, on all levels of education.

There are very interesting experiments performed with Chimpanzee and Bonobo apes and under which conditions they learn, or even how you teach a parrot to talk. How do you do that? The major factor is that it should not see the teacher. You put a mirror between you and the parrot and then you speak behind the mirror. The parrot then sees a bird – it talks to a bird. But if it sees you, it will learn very badly.

That is not an obvious thing. The very presence of a teacher, an authority, moves students in a particular direction and not at all the direction the teacher wants them to move. With all this accumulated evidence, you cannot make any simple decision. If you say "do this and this", you are wrong for sure. Solutions are not obvious; they can only come after analysing deeply what is actually known and by studying the possibilities. I think the answers will be unexpected. What children can learn and what they cannot learn, we don't know because we don't know how to conduct experiments to be ethical and instructive at the same time. It is a very non-trivial issue, which has not been studied much. With animals we have results but not very much with people.

Let us come back to mathematics and to mathematics education. It seems that many people stop dealing with mathematics as soon as they have left high school. But as mathematicians we know that mathematics is everywhere, though often hidden: as the workhorse in science and technology but also as a pillar in human culture, emphasising rigour and organised thinking. Do you have any ideas on how we can make this double role perceived and appreciated by society and how to make decision makers realise that mathematics needs support?
It is a very difficult question because we have to project mathematical ideas to people who work very far from mathematics – to people that make decisions in society. The way we think is very different from the way they operate.

I don't know but I think that within our mathematical society we can make some steps towards education, like creating good mathematical sources for children. Today we have the Internet so we should try to make Internet presentations. Actually, in France there are some people trying to organise extra-curricula activities for younger children on a small scale. We should try to do something like that on a big scale: big centres of stimulating creativity in all directions. I would not only focus on mathematics but on science and art and whatever can promote creative activity in young people. When this develops, we may have some influence but not before that. Being inside our ivory tower, what can we say? We are inside this ivory tower and we are very comfortable there. But we cannot really say much because we don't see the world well enough either. We have to go out but that is not so easy.

You mentioned that you first got interested in mathematics after reading the book "Numbers and Figures" by Rademacher and Toeplitz. We could also mention the book "What is mathematics?" by Courant and Robbins. Should we encourage pupils in high school who show an interest in mathematics to read books like that?
Yes. We have to produce more such books. Already there are some well-written books, by Martin Gardner, by Yakov Perelman (*Mathematics can be fun*), by Yaglom and co-authors – very remarkable books. Other mathematicians can contribute by writing such books and combine this with the possibilities of the Internet, in particular visualization.

It is relatively simple to write just one page of interesting mathematics. This should be done so that many different subjects in mathematics become easily available. As a community we should go out and create such structures on the Internet. That is relatively easy. The next level is more complicated; writing a book is not easy. Within the community we should try to encourage people to do that. It is a very honourable kind of activity. All too often mathematicians say: "Just vulgarization, not serious". But that is not true; it is very difficult to write books with a wide appeal and very few mathematicians are actually able to do that. You have to know things very well and understand them very deeply to present them in the most evident way.

This could be a way to get more young people to take up mathematics?
You will attract more young people. Moreover, the political figures will sense it on a much larger scale because it will have a much wider appeal than what we do internally.

Poetry

You have mentioned that you like poetry. What kind of poetry do you like?
Of course, most of what I know is Russian poetry – the so-called Silver Age of Russian Poetry at the turn of the twentieth century. There were some poets but you, probably, do not know them. They are untranslatable, I guess. People in the West know Akhmatova but she was not the greatest poet. The three great poets were Tsvetaeva (also a woman), Blok and Mandelstam.

What about Pushkin?
You see, with Pushkin, the problem is as follows. He was taught at school and that has a tremendously negative impact. But 40 years later I rediscovered Pushkin and found him fantastic – when I had forgotten what I had learned in school.

What about modern poetry and English poetry?
I have read some English poetry. I know some pieces but I don't know it on a larger scale. It is difficult. Even with modern Russian poetry, e.g. Brodsky, I find it difficult to absorb a new style. To absorb a poet is non-trivial. For English poetry, there are a few particular pieces that I learned and appreciate. Some of them are easy to deal with; some have Russian translations. A remarkable one is Edgar Allan Poe. He is kind of simple in a way. But many other English poets are more remote from Russian style. I know a little bit of French poetry, like François Villon; I can appreciate him in French. But modern poetry is very difficult for me.

To finish the interview, we would like to thank you very much on behalf of the Norwegian, the Danish and the European Mathematical Societies.

Jean-Pierre Bourguignon: Mikhail Gromov's role at IHÉS

R&S: Abel Prize winner Mikhail Gromov has been affiliated to the Institut des Hautes Études Scientifiques (IHÉS) in Bures-sur-Yvette since 1982. Professor Bourguignon, could you please tell us about his role at the institute and about his influence on its development?
Bourguignon: Misha is one of the five permanent professors at IHÉS. He is definitely one of the attractions of the place. Many people want to come and talk with him and he likes to interact with people quite a lot. He often suggests visitors with whom he wants to interact. He is not the kind of person who conducts regular seminars but rather works through direct personal contacts.

About ten years ago, starting from his interest in the theory of complexity, he became interested in biological systems. Since then, he has very consistently invited biologists to come to the institute and talk with him. The way he does it is not at all by imposing his own theories on them but rather by listening to them and having them explain how they conduct their experiments, how they work, how they proceed with their ideas. He tries to go to the bottom of things and to force his partners to make everything explicit. These discussions are very intense and can be quite tough; more often than one would think, they result in new ideas for designing experiments.

Misha has written a few pamphlets about the interaction between mathematics and biology. Not all of them were well received by the community since they were quite sharp; he pointed out that several assumptions made by people were not consistent.

These days, Misha is involved in a project that is more a medical than a biological project. He got involved in it through contacts with a Russian physicist who is now a medical doctor in a hospital in Amsterdam. It deals with methods to get new types of information about dysfunctions of the heart. The main idea is to consider the heart as a black box and to study its electrophysiological geometry. This is a priori a very hard mathematical problem because of its really three-dimensional nature. Also, the number of measurements you can make, the number of electrodes you can place, is strictly limited. So you need at the same time a very robust and sophisticated mathematical method to recover this internal geometry, not the mechanical one but the one that represents the way waves propagate in the heart. This is a very risky project in which Misha is very much involved, and the institute is accompanying him. I think we have found some very good people to participate in the project. To get it started some experiments had to be performed on animals in order to check some of the fundamental assumptions. This was done in a laboratory in the United States.

Jean-Pierre Bourguignon. Director of the IHÉS.

For IHÉS, Misha Gromov is a great asset – because he attracts many scientists to the institute to interact with him but also thanks to his ideas to move into completely new fields. He was the driving force that led the institute to develop a new activity at the interface between mathematics and biology.

Richard von Mises (1883–1953): a pioneer of applied mathematics in four countries[1]

Reinhard Siegmund-Schultze

Richard von Mises (1883–1953) can be seen as a symbolic figure for the evolution and convolutions of international mathematics in the first part of the 20th century [1]. He was born in the Austro-Hungarian Empire, he taught in four different languages (German, French, Turkish, and English) during his career, spent decisive years (1909–1914/18) under difficult political circumstances in the centrally located European city of Strasbourg (then German occupied Straßburg).

Today, von Mises' crucial and pioneering effort in applied mathematics is still appreciated, however – for various external and personal reasons – somewhat obscured. He worked in the pre-computer age so that some of his work in numerical analysis could not immediately be implemented, while his function-theoretic approach to aerodynamics is partly obsolete in modern computational fluid dynamics. Trained in Vienna as a mechanical engineer with a strong mathematical background, von Mises was "too mathematical" for many aerodynamicists. Unlike them he had no experimental facilities (wind tunnels) at his disposal, which were crucial for progress in the field at that time. His efforts, however, to implement the standpoint of applied mathematics in the foundations of probability theory, in particular his theory of 'collectives', ran into logical problems and into a clash with the pure mathematicians. While von Mises was not a dominating figure in any of the three fields mentioned (numerical analysis, aerodynamics, foundations of probability), he left traces in all of them. However, of much more consequence was von Mises' work in mathematical statistics and in the theory of plasticity. Maybe of even more importance was von Mises' organizational and conceptual work, the founding of the *Zeitschrift für Angewandte Mathematik und Mechanik* (*ZAMM* 1921) and the "Gesellschaft" (Society) of the same name (GAMM), the running of the institute of applied mathematics at Berlin, and, not least, his programmatic publications on the goals and values of applied mathematics as a discipline. Finally one should not forget that von Mises was expelled from Berlin and Germany as a Jew in 1933, at the height of his intellectual and organizational powers.

Today, the mathematicians of Germany, and of Berlin in particular, recognize von Mises' importance and show, at the same time, their political awareness of the dark sides of Germany's past by awarding a yearly von Mises prize (awarded by GAMM) and organizing "Richard von Mises – Lectures" (Humboldt University Berlin) each June.

1. Von Mises' program

Richard von Mises (1883–1953) as professor in Berlin and as a pilot during his time in Straßburg [Both pictures taken from: R. von Mises: Selected Papers, 2 volumes, AMS, Rhode Island, volume 1, page 151]

"Tasks and Goals of Applied Mathematics" (1921)

In 1921, in the first issue of his new journal *ZAMM*, von Mises – now professor at the new institute for applied mathematics at the University of Berlin – published his programmatic article "On the tasks and goals of applied mathematics". In it he emphasized that logical rigor was as important to applied mathematics as to its "purer" domains:

> *"The truth of the matter is that the precise notions of mathematics … imply simplifications and idealizations, which we cannot do without, in view of the limitations of our mental capacities, and which the engineer can miss even less, who can only spend a limited amount of his time and his effort on mathematical studies." ([2], p. 3)*

Von Mises principally proposed a very broad and nuanced notion of "applied mathematics," as far as the potential fields of application were concerned. However, due to pragmatic considerations, not least due to the interests of the readers of *ZAMM*, von Mises restricted the main topics of the new journal to mathematics in engineering, such as civil engineering, to plasticity, hydromechanics, aerodynamics, together with several fields of applied mathematics in a more explicit sense, such as graphical and numerical methods. Statistics and mathematical physics, however, were largely excluded from *ZAMM*.

[1] Parts of this article have been published before in German in *Rundbrief GAMM* 2008, no. 1, pp. 6–12.

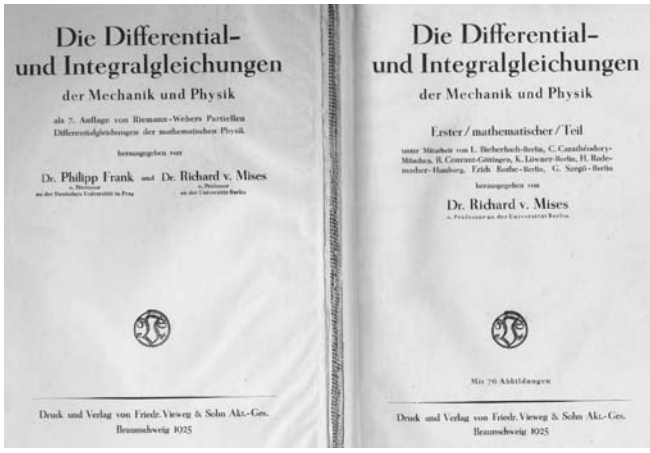
2. Richard von Mises: the most important biographic dates

1883	Born April 19, in Lemberg, Galicia (today Ukrainian Lvív) in the Austro-Hungarian Empire.
1901–1905	Studied mechanical engineering at the Technische Hochschule in Vienna, first publication 1905 (mathematical), Doctoral-engineer 1908
1906–1909	Assistant for mechanics and construction at the German Technical University in Brünn (today Czech Brno), 1908 Habilitation
1909–1914	Associate professor for applied mathematics at the University of Straßburg
1914–1918	Engineer-officer in the Austrian Airforce. Construction of a "huge airplane" ["Großflugzeug"] 1916, which never went into service
1919	Short employments in Frankfurt (Main) and Dresden
1920–1933	First director of the Institute for Applied Mathematics of the University of Berlin, Co-founder of *ZAMM* (1921) and GAMM (1922). Hilda Geiringer (1893–1973) was his assistant from 1921.
1933	As a Jew von Mises saw the untenability of the situation and fled to Istanbul, where Hilda Geiringer followed him in 1934.
1939	Felt renewed threat and emigrated further to the USA (Harvard University), where only in 1945 he obtained a regular professorship.
1943	Marriage with Hilda Geiringer
1953	Von Mises died July 14 from cancer in Boston.

3. Von Mises' versatility and his accomplishments outside applied mathematics

Given von Mises' restless organizational effort for applied mathematics during the 1920s, his versatility in research – which reached beyond the more narrow field of applied mathematics as described in *ZAMM* – is even more astounding.

- Researchers in the foundations of probability theory still know von Mises' notion of probability (1919), based on the limit of the relative frequency of the occurrence of events and using a still rudimentary notion of randomness. In the 1970s von Mises' approach witnessed a certain renaissance in the context of A. N. Kolmogorov's theory of algorithmic complexity ([4], [5]).
- Statisticians are familiar with several tests and distributions (such as the 'circular' one) named after von Mises. Many have heard about fundamental new creations such as von Mises' "statistical functions," which have been used in robust statistics [6].
- To the physicists of his time von Mises was well known, not least due to the two-volume handbook *Die Differential- und Integralgleichungen der Mechanik und Physik* (1925/1927), which he edited together with his friend, the physicist of Prague, Philipp Frank. For several decades the "Frank-Mises" remained an invaluable tool in the daily work of physicists [7].


Die Differential-
und Integralgleichungen
der Mechanik und Physik

Dr. Philipp Frank und Dr. Richard v. Mises

Druck und Verlag von Friedr. Vieweg & Sohn Akt.-Ges.
Braunschweig 1925

Die Differential-
und Integralgleichungen
der Mechanik und Physik

Erster / mathematischer / Teil

Dr. Richard v. Mises

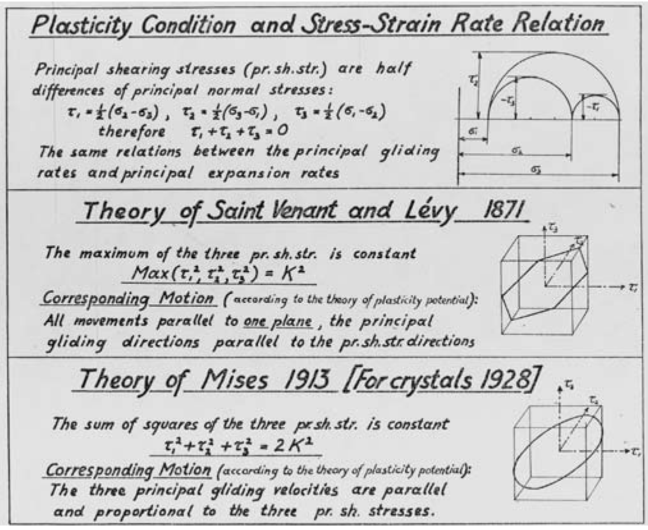
Druck und Verlag von Friedr. Vieweg & Sohn Akt.-Ges.
Braunschweig 1925

4. Three accomplishments by von Mises in applied mathematics

None of the three contributions by von Mises just mentioned above can be discussed here, this has partly been done elsewhere. Instead we want to say a little about von Mises' research, teaching and propaganda for "applied mathematics" in the (somewhat) more narrow sense of *ZAMM* of the 1920s.

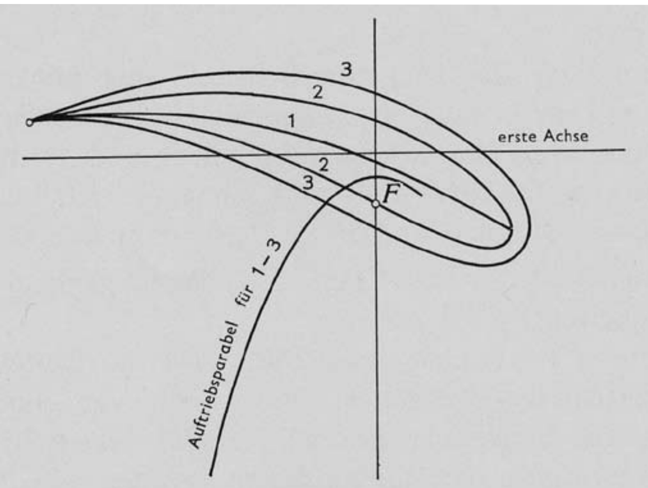
In 1913, as associate professor for applied mathematics in Straßburg, von Mises published an important article [8], communicated by the first full professor for applied mathematics in Germany, Carl Runge in Göttingen (who had been appointed in 1904). The article was about von Mises' (later) famous yield criterion. The latter suggests that the yielding of materials begins when the second deviatoric stress invariant reaches a critical value. It is part of a plasticity theory that applies best to ductile materials, such as metals. Prior to yield, material response is assumed to be elastic. This criterion was generally found to provide slightly better agreement with experiment than the criterion given by the father of plasticity theory, Henri Tresca (1814–1885). Most work on the applications, particularly in steel construction, today uses the form found by von Mises. It is also referred to as belonging to the Maxwell–Huber–Hencky–von Mises theory. In material science and engineering the von Mises yield criterion can be also formulated in terms of the von Mises stress, a scalar stress value that can be computed from the stress tensor. [see figure on next page]

During World War I, von Mises was an officer in the Austrian Airforce. His "Fluglehre" (*Theory of Flight*) of 1918, which went through many editions and is still available in print in an extended English version today, testifies to von Mises' activity in this field as well as several particular results. In 1920 von Mises showed with function-theoretic methods within the two-dimensional theory of airfoils (ignoring drag at the airfoil tips) the existence of what today (mostly without mentioning von Mises' name



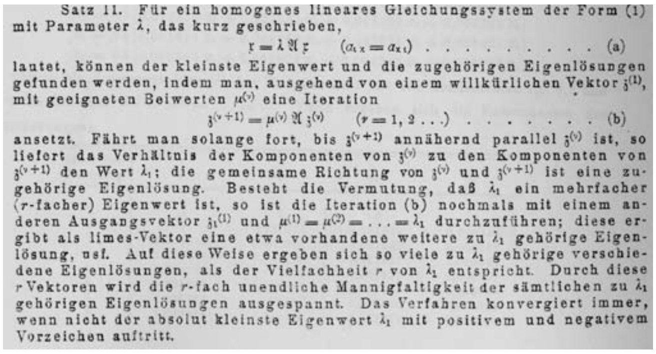
Von Mises' historical overview of plasticity theory, apparently drawn by himself for lectures at Harvard University after his emigration to the USA in 1939 (Copy from the Richard von Mises Papers, Harvard University Archives, HUG 4574.24, reproduced by courtesy of the Archives).

and contribution) is called the aerodynamic centre of an airfoil. The theory showed that there exists a point F (not necessarily lying within the section of the airfoil) for an airfoil of any shape, with respect to which the moment of the lift is independent of the angle of attack of the airfoil (the picture is from his original publication [9], p. 72).



"Jedes Profil besitzt einen Punkt, für den das Moment des Auftriebs vom Anstellwinkel unabhängig ist; die Einhüllende aller Lagen der Auftriebsresultanten ist im allgemeinen eine Parabel, die sich auf ein Strahlbüschel reduzieren kann; in diesem Grenzfall erhält man Profile mit festem Druckmittelpunkt, die sonach durch eine einzige Bedingungsgleichung aus der Gesamtheit aller Profilformen herausgehoben werden." (German orginal [9], p. 68)

Finally it should be mentioned that von Mises produced several contributions in the more specific and methodical area of applied mathematics. Together with his assistant in Berlin and future wife during emigration, Hilda Geiringer (1893–1973), and based on his lectures, von Mises published in 1929 a proof of the convergence of the Gauß-Seidel procedure (iteration in single steps) for the solutions of systems of linear equations. In the same paper [10] one finds on page 154 as theorem 11 what one calls today the "von Mises-Iteration" or the "power method" for the calculation of the biggest eigenvalue of matrices (note that the authors consider only symmetric matrices and that their notion of the eigenvalue is reciprocal to today's notion):


Satz 11. Für ein homogenes lineares Gleichungssystem der Form (1) mit Parameter λ, das kurz geschrieben,

$$\mathfrak{x} = \lambda \mathfrak{A} \mathfrak{x} \qquad (a_{\iota\varkappa} = a_{\varkappa\iota}) \quad \dots \dots \quad (a)$$

lautet, können der kleinste Eigenwert und die zugehörigen Eigenlösungen gefunden werden, indem man, ausgehend von einem willkürlichen Vektor $\mathfrak{z}^{(1)}$, mit geeigneten Beiwerten $\mu^{(\nu)}$ eine Iteration

$$\mathfrak{z}^{(\nu+1)} = \mu^{(\nu)} \mathfrak{A} \mathfrak{z}^{(\nu)} \qquad (\nu = 1, 2 \dots) \quad \dots \dots \quad (b)$$

ansetzt. Fährt man solange fort, bis $\mathfrak{z}^{(\nu+1)}$ annähernd parallel $\mathfrak{z}^{(\nu)}$ ist, so liefert das Verhältnis der Komponenten von $\mathfrak{z}^{(\nu)}$ zu den Komponenten von $\mathfrak{z}^{(\nu+1)}$ den Wert λ_1; die gemeinsame Richtung von $\mathfrak{z}^{(\nu)}$ und $\mathfrak{z}^{(\nu+1)}$ ist eine zugehörige Eigenlösung. Besteht die Vermutung, daß λ_1 ein mehrfacher (r-facher) Eigenwert ist, so ist die Iteration (b) nochmals mit einem anderen Ausgangsvektor $\mathfrak{z}_1^{(1)}$ und $\mu^{(1)} = \mu^{(2)} = \dots = \lambda_1$ durchzuführen; diese ergibt als limes-Vektor eine etwa vorhandene weitere zu λ_1 gehörige Eigenlösung, usf. Auf diese Weise ergeben sich so viele zu λ_1 gehörige verschiedene Eigenlösungen, als der Vielfachheit r von λ_1 entspricht. Durch diese r Vektoren wird die r-fach unendliche Mannigfaltigkeit der sämtlichen zu λ_1 gehörigen Eigenlösungen ausgespannt. Das Verfahren konvergiert immer, wenn nicht der absolut kleinste Eigenwert λ_1 mit positivem und negativem Vorzeichen auftritt.

5. Von Mises' teaching and his organizational effort in the 1920s in Berlin

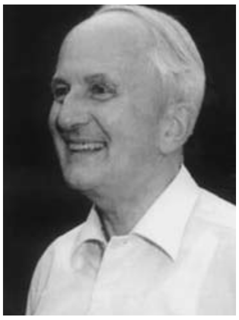
Von Mises was restless in his effort for his discipline of applied mathematics and its position within the German university system. He did not shrink from polemics either in this context. He was critical, at times contemptuous, of those "pure" mathematicians who – as he wrote 1927 in a discussion with another applied mathematician, Göttingen's Richard Courant (1888–1972) –

> *"belong to the overwhelming majority of our university professors who declare with more or less pride, at least however with full justification that they are unable to perform the smallest numerical calculation or geometrical construction." [11]*


Hilda Geiringer (courtesy Ms. Magda Tisza, Boston)

In Berlin von Mises had to be constantly concerned about the reputation of his subject area in the eyes of the representatives of traditional mathematics. The procedure for habilitation of Hilda Geiringer (1893–1973) was opposed by his colleagues from "pure" mathematics. Her teaching permit in 1927 was explicitly restricted to "applied mathematics", while the "pure" mathematicians claimed as a matter of course a general responsibility for the whole of mathematics. There were apparently non-mathematical reasons involved in Geiringer's problems, such as anti-Semitism and anti-feminist motives. Hilda Geiringer became finally an internationally recognized specialist in statistics and the theory of plasticity [12].

Von Mises remained active in Berlin until the Nazi regime made further work impossible for him and Hilda Geiringer. As late as the first days of November 1933, when von Mises was about to depart for Istanbul, he conducted the state examination of his student Lothar Collatz (later a pioneer for methods of functional analysis in applied mathematics) and gave him advice for his future.



Lothar Collatz (1910–1990)
One of the most important students of von Mises in a letter to the author, dated 10 November 1987:
"I enrolled in Berlin in 1930 … Prof. Dr. Richard von Mises had explained in his excellent, very clear and stimulating lectures on practical analysis … that it would be desirable to develop difference methods of higher exactness. …
I took the state exam in November 1933; Prof. von Mises examined me on the day before his departure. The same day he talked to me for about one hour, giving advice for my further research … I met Prof. v. Mises again only several years after the war."

6. Von Mises' death in American exile

Von Mises went late, in 1939, to the United States, when he felt no longer safe in his Turkish exile [14]. Starting at Harvard University, he had first to use his personal savings. He was, however, highly appreciated, above all for his work in aerodynamics but also for his continued creative work in asymptotic statistics (statistical functions). He obtained a professorship at Harvard in 1945 which was extended beyond retirement age until his death from cancer in 1953.

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Reinhard Siegmund-Schultze
[reinhard.siegmund-schultze@uia.no]
Born in Halle (Saale) 1953, mathematics diploma 1975, dissertation on the history of functional analysis 1979. Assistant at Humboldt University in Berlin until 1990, habilitation 1987. Feodor-Lynen-fellow of the Alexander-von-Humboldt Foundation in the USA 1991–1994. Projects of the German Research Association (including, among others, on Richard von Mises) 1998–2000. From 2000 professor for the history of mathematics at the University of Agder in Kristiansand (Norway). Publication of a book (in German) on refugee mathematicians from Nazi Germany, 1998; extended English version, 2009.

Address of the author:
Prof. Dr. Reinhard Siegmund-Schultze
University of Agder, Faculty for Engineering and Sciences
Gimlemoen, Serviceboks 422
4604 Kristiansand S, Norway

The first decade of the Institut Henri Poincaré, in particular the role of the Rockefeller Foundation

Reinhard Siegmund-Schultze, University of Agder, Norway

In order to understand the policies underlying the foundation of the Institut Henri Poincaré (IHP) in 1928, one has to take a broader look at the requirements for modernising French mathematics at the time. One has to relate the latter to the general political, economic and scientific situation in French mathematics after World War I and consider the influence exerted by foreigners, particularly by the Americans, who gave most of the money for the IHP and who also pursued interests of their own.

In the early 1920s, it was generally understood that in theoretical physics and in several areas of mathematics France had fallen behind German developments by about a generation[1]. Historians agree on the relative decline of geometry in France (with the exception of Élie Cartan). There was also an almost total absence of algebra, number theory and applied mathematics, and considerable weaknesses in mathematical physics. In addition to the war losses, French science and mathematics were particularly weakened by limited international contacts, where language problems and a lack of travel allowances were contributing factors. A third difficulty lay in the general lack of financial support, especially in the first few years after the war. As late as 1926, American philanthropists considered the French government as "nearly bankrupt".[2]

Émile Borel (1871–1956)

The three most influential French mathematicians of the time were Émile Borel, Jacques Hadamard and Émile Picard, all of whom combined political engagement with exceptional mathematical competence.

Henri Poincaré had been a figure of undisputed genius and universal scope in French mathematics around 1900. Poincaré, who died shortly before the First World War, was thus an appropriate figure for French post-war aspirations to regain international stature both in mathematics and physics.

Although better known for his seminal results in mathematical physics, topology, differential equations and function theory, Poincaré also had influential ideas on the foundations of the theory of probability, at that time a rather unsettled topic. He published an early textbook on probability theory in 1896. His so-called "method of arbitrary functions" and work on the movement of mechanical systems influenced researchers like Borel and disciplines such as ergodic theory (particularly in the work of G. D. Birkhoff, who was to play a key role in the American effort to help the French).

Henri Poincaré (1854–1912)

At the same time, in the 1920s, American philanthropists were concerned with establishing American science on the international stage and spreading "American values" in Europe in the aftermath of World War I. The most important scientific agencies with international impact were related to the American Rockefeller family. Rockefeller's International Education Board (IEB), founded in 1923, which was one of several Rockefeller philanthropies, finally funded both the stipends for young European mathematicians to go abroad (including several future Bourbaki members) and the construction of the Institut Henri Poincaré in Paris.

Given the wartime alliance between the US and France, it was no surprise that the European office of the IEB was created in Paris in 1925. From the outset, the work of the office was heavily influenced by cultural and political considerations on the part of the Americans.

With respect to mathematics, the philanthropies were advised by the American George David Birkhoff, who had come to Europe as a Rockefeller envoy for a stay of six months in early 1926. Birkhoff had never visited Europe before but considered Poincaré his spiritual

[1] Harry W. Paul, *From Knowledge to Power. The Rise of the Scientific Empire in France 1860–1939*, (Cambridge: Cambridge University Press 1985), 347.

[2] Rockefeller Archives (New York) as quoted in Reinhard Siegmund-Schultze: *Rockefeller and the Internationalization of Mathematics Between the Two World Wars. Documents and Studies for the Social History of Mathematics in the 20th century* (Basel, etc.: Birkhäuser, 2001), pp. 180–81, henceforth quoted as RSS (2001).

George D. Birkhoff (1884–1944)

teacher; indeed, he had become known in Europe by solving Poincaré's "last geometric theorem" in 1913 and his admiration for Borel and Hadamard further strengthened the French connection.

From the first, Borel and, as it seems, the IEB functionaries in Paris, as well as Birkhoff, had been much more interested in the scientific aspects of the project and the ensuing encouragement of international communication than the mere construction of a building. In an IEB memorandum from May 1926 it can be read that:

> *"Borel was glad to learn that the project would not have to be built up with a building dominating the whole situation, which was evidently what he thought when he came. He thinks that a small and very modest building on the site of the new University centre here in Paris would be advisable, something with one lecture room, library space for a technical library in Mathematics and Physics and a few offices for professors. … Roughly that would call for a modest building as stated above and foundation of three or four new chairs in Mathematical Physics, Applied Mathematics, etc.[3]"*

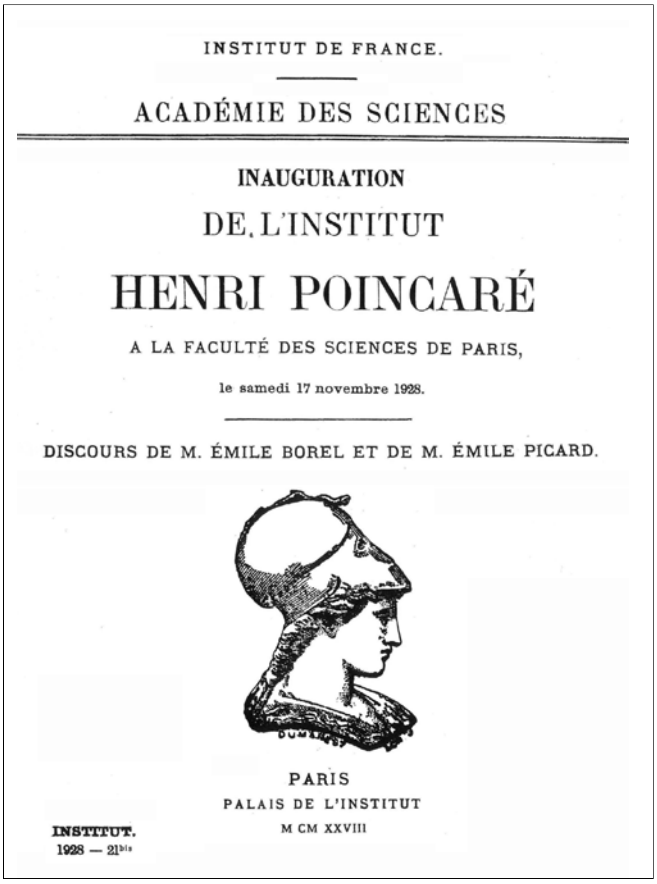
On 19 November 1926, the IEB officially announced the donation for the institute in Paris and for another in Göttingen[4]. The institute in Paris received \$100,000, subject to the condition that a source was found for a matching grant of \$25,000. The additional amount soon materialised in the form of a donation from Baron Edmond de Rothschild. It was only left for the French government (via the Sorbonne) to provide the site for the construction of the institute.

The "Institut Henri Poincaré" opened officially on 17 November 1928, with addresses by its founding director Borel and by Émile Picard and others, with Prime Minister Raymond Poincaré, a cousin of the deceased mathematician, in attendance.[5]

The Institut Henri Poincaré would become the real centre of mathematical research and, in part, for teaching in Paris, and consequently in all of France.

It took some time before the Rockefeller agencies recognised the particular strength of the IHP, i.e. probability. Borel's Chair had existed at the Sorbonne since 1896 under the name "Calcul des probabilités et physique mathématique". Borel had occupied this position from 1920

[3] Rockefeller Archives as quoted in RSS (2001), 253–255.
[4] RSS (2001), 161ff.
[5] See *Inauguration de l'Institut Henri Poincaré a la Faculté des Sciences de Paris, le samedi 17 novembre 1928. Discours de M. Émile Borel et de M. Émile Picard*, (Paris: Académie des Sciences, 1928), 11pp.



INSTITUT DE FRANCE.

ACADÉMIE DES SCIENCES

INAUGURATION

DE L'INSTITUT

HENRI POINCARÉ

A LA FACULTÉ DES SCIENCES DE PARIS,

le samedi 17 novembre 1928.

DISCOURS DE M. ÉMILE BOREL ET DE M. ÉMILE PICARD.

PARIS

PALAIS DE L'INSTITUT

M CM XXVIII

INSTITUT.
1928 — 21bis

Opening of Institut Henri Poincaré 1928

Entrance of the IHP (2009).
Copyright Pierre Kitmacher

onwards and it was to provide the inspiration and source of many of the activities of the Institut Henri Poincaré. Probability remained Borel's main interest, although he himself was no longer a leading scholar in stochastical research. Three out of the four lecturers at the IHP were "stochasticians" proper: Borel and Maurice Fréchet, who were both more concerned with probability, and George Darmois, who leaned more towards mathematical statistics. No description of research in stochastics in Paris in the 1930s can ignore the work of Paul Lévy, who like Fréchet was one of Hadamard's students. He was arguably the most important French research worker in this field of the decade, though not directly affiliated with the institute. Lévy pioneered in the general theory of stochastic processes (martingales, etc.). As a professor at the École Polytechnique Lévy, he occasionally gave lectures in the IHP series.

The Institut Henri Poincaré played a part in supporting refugees from Nazi Germany as well[6]. Among the most gifted students of Fréchet and Lévy was Wolfgang Doeblin (1915–1940)[7]. Doeblin was the son of the famous German-Jewish writer Alfred Döblin. In 1933, at the age of 18, he had been expelled from Berlin with his father and other family members. In the mid-30s, Doeblin had made good use of the new liberal policies of the mathematical library of the IHP. After the occupation of France in 1940, Doeblin became a French soldier and subsequently committed suicide rather than submit to capture.

In 1938, another student of Fréchet, Jean Ville, introduced the name "martingale" in the general theory of stochastic processes. With the appearance of this group of talented students in stochastics toward the end of the 1930s, the peculiar scientific orientation of the Institut Henri Poincaré finally demonstrated its international worth and was recognised by the Rockefeller officials as a respectable object of patronage.

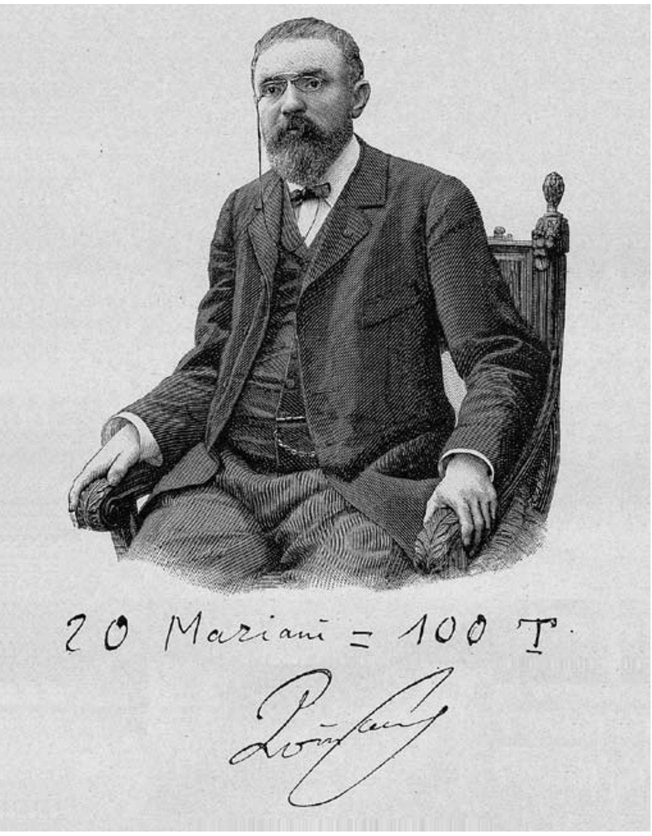
At the same time, in a period of increasing governmental constraints, the Americans were able to view the IHP and its internationalising role with some satisfaction. French scientists and, to some extent, the French government had at last come to see the desirability of international contacts. French science was breaking out of the self-satisfied isolation in which it had been languishing.

[6] This applies in particular to E. J. Gumbel, F. Bloch, F. Pollaczek, H. Reichenbach, R. von Mises and M. Born. See RSS (2001).

[7] See the recent popular book by Marc Petit *L'équation de Kolmogoroff*, Paris: Éditions Ramsay, 2003.

Institut Henri Poincaré

Alain Comtet and Cédric Villani



Portrait of Henri Poincaré.

In his contribution to the present volume, Reinhard Siegmund-Schultze tells us about the glorious birth of the Institut Henri Poincaré and how, by the end of the 30s, it was already playing its role in helping French science "breaking out of the self-satisfied isolation in which it had been languishing".

The story could have ended up with "And then the Institute lived happily ever after" … but that was not the case. In 1968, the University of Paris was torn apart and most mathematicians in Paris departed to Jussieu or Orsay. In 1978, the Rectorate of Paris (the new administrator of the IHP) decided to allocate most of the building to institutions whose missions were unrelated to the primary purpose of the institute. Confusion set in and soon prevailed. This was the end of the IHP as several generations of mathematicians and physicists had known it.

Around 1980, the French scientific community started to rethink the role of the IHP and make plans for the future. Jean-Pierre Aubin, and then Bernard Julia, Nicole El Karoui and Bernard Teissier, took the lead in this reflection. In 1988, the Demazure report summarised the projects; two years later a decree was signed by the government authorities. Thus the Institut Henri Poincaré was reborn, on 28 February 1990!

Technically, the new IHP has a dual status: firstly, it is a component – or "internal school" – of the University Pierre et Marie Curie (UPMC, formerly University Paris

VI Jussieu); and secondly, it is a joint service unit of the CNRS and the UPMC. But, simply stated, the IHP is just "**the house of mathematicians and theoretical physicists**" (here, theoretical should be understood in a very broad sense).

The main activity of the IHP is to host thematic trimesters (three per year), where hundreds of senior and junior scientists gather from all around the world under the supervision of a few organisers. The administrative entity inside IHP in charge of the organisation of these trimesters is called the Centre Émile Borel (CEB), in recognition of Borel's enormous contribution to the original project. For these trimesters, the IHP benefits from the support of the UPMC and the CNRS, and also the University Paris Diderot (formerly Paris VII) and the University Paris-Sud (formerly Paris XI Orsay). In addition, the Fondation Sciences Mathématiques de Paris contributes via invited positions or reimbursements. In this way, through the IHP, visitors are actually invited by a large complex of academic institutions.

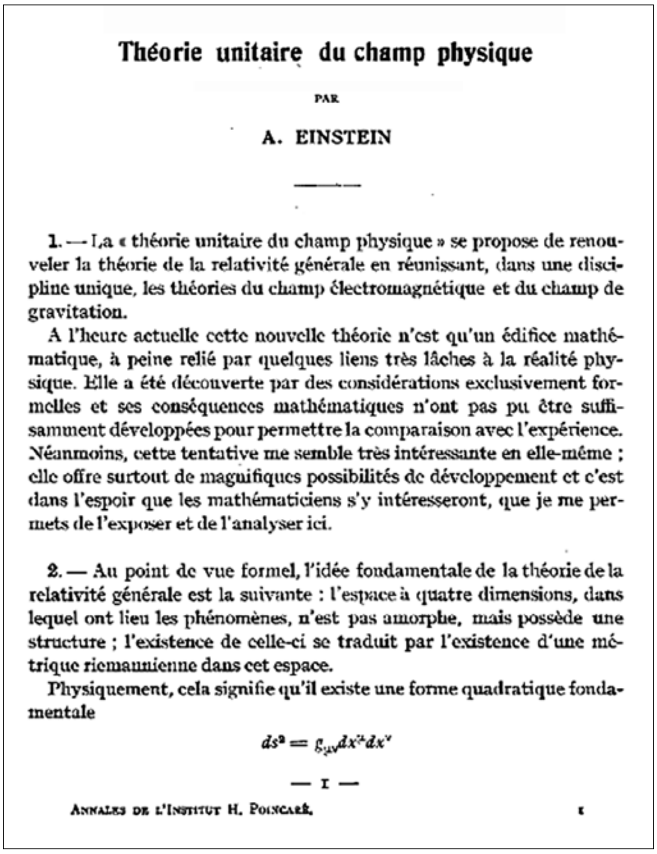
While a slight majority of these programmes are organised by mathematicians, a lot of them are also organised by physicists, or jointly by physicists and mathematicians. Recently the IHP has been opening its range of themes to biology, and computer and information sciences.

The programmes at the IHP are mainly theoretical but interaction with experimentalists is encouraged. As a distinctive feature, a programme should have some interdisciplinary touch; a mathematical programme at the IHP is not complete without some dose of physics, biology or some other field. The programmes should also include some courses, giving the opportunity for non-experts to get acquainted with a field of research. Students are encouraged to attend these courses, which are a great opportunity to learn directly from contact with international research. Everybody is also encouraged to submit projects of programmes; this will be made even easier when the current renovation of the Web system is completed. Participants of the programmes enjoy not only the scientific activities of the meeting but also an office at the heart of a fantastically active and vivid scientific campus where researchers from everywhere can be encountered. The IHP has offered the same hospitality to a small number of postdocs and visitors outside the framework of thematic trimesters.

A seminar from the trimester "Dispersive equations", organized by Frank Merle and Fabrice Planchon, Spring 2009. (Copyright Pierre Kitmacher)

The IHP houses a number of seminars, organised by universities in Paris or elsewhere, and thematic workshops of any kind. Some of the most prestigious mathematics seminars in France take place at the IHP: the Bourbaki seminar, talks by the laureates of awards of the French Academy of Science, etc. Each year, several thousands of visitors participate in all this activity, contributing in making the IHP one of the most lively meeting points for mathematicians and physicists in the world.

Since its birth, the IHP has been responsible for the publication of the Annales de l'Institut Henri Poincaré (AIHP), which has played an important role in spreading science. In 1964, the AIHP was split into two distinct journals: the AIHP (A) devoted to theoretical physics and the AIHP (B) devoted to probability and statistics. In 1983, the AIHP (C) appeared, specialising in nonlinear analysis and especially partial differential equations. More recently, the AIHP (A) has merged with the Swiss journal *Acta Physica Helvetica* and changed its name to the *Annales Henri Poincaré*. All these journals are now well established. A prize is attributed annually to the authors of the most remarkable paper published in each of the AIHP over the year.



Théorie unitaire du champ physique

PAR

A. EINSTEIN

1. — La « théorie unitaire du champ physique » se propose de renouveler la théorie de la relativité générale en réunissant, dans une discipline unique, les théories du champ électromagnétique et du champ de gravitation.

A l'heure actuelle cette nouvelle théorie n'est qu'un édifice mathématique, à peine relié par quelques liens très lâches à la réalité physique. Elle a été découverte par des considérations exclusivement formelles et ses conséquences mathématiques n'ont pas pu être suffisamment développées pour permettre la comparaison avec l'expérience. Néanmoins, cette tentative me semble très intéressante en elle-même ; elle offre surtout de magnifiques possibilités de développement et c'est dans l'espoir que les mathématiciens s'y intéresseront, que je me permets de l'exposer et de l'analyser ici.

2. — Au point de vue formel, l'idée fondamentale de la théorie de la relativité générale est la suivante : l'espace à quatre dimensions, dans lequel ont lieu les phénomènes, n'est pas amorphe, mais possède une structure ; l'existence de celle-ci se traduit par l'existence d'une métrique riemannienne dans cet espace.

Physiquement, cela signifie qu'il existe une forme quadratique fondamentale

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

— I —

ANNALES DE L'INSTITUT H. POINCARÉ. I

The first article of the AIHP: conferences given at the IHP by Albert Einstein in 1929, written by A. Proca. (Picture from the Web site NUMDAM)

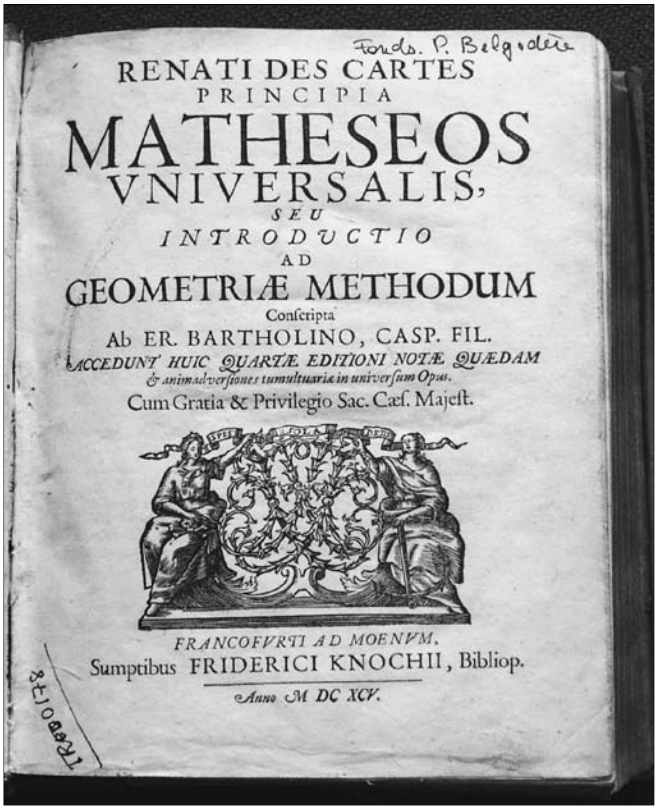
The IHP also houses the offices of a number of scientific organisations that promote mathematics and physics (the Société Mathématique de France, the Fondation Sciences Mathématiques de Paris and many others). Soon the renovated website will provide a common access for all these organisations.

Last but not least, the IHP hosts a quiet and convenient library, usually regarded as one the best in France, with no less than 35,000 books and 255 periodicals. It is remarkable for its wide electronic access (made possible through several networks including that of the University Pierre et Marie Curie) and for its historical fund, inherited from the library of the Faculty of Sciences of the University of Paris, as can be seen from the inventory that has been kept since 1900! Some of the books go back to the 17th century. The library also has hundreds of mathematical models, in particular geometric figures for pedagogic purposes (made out of plaster, wire, wood and string), most of which were acquired from Schilling in Leipzig between 1900 and 1920, and some of which were designed and made by Caron, a professor of descriptive geometry, between 1912 and 1915. These objects have in their time inspired some of the leading artists from the surrealist group and were famously photographed by Man Ray himself: another example of the role of the IHP in breaking barriers between intellectual fields and between nations.

3rd degree surface, mathematical model from the collection of the IHP. (Copyright Dominique Dartron)

Elliptic function, model from the IHP collection. (Copyright Dominique Dartron)

The library of the IHP. (Copyright Étienne Gouin-Lamourette)



Fonds. P. Belgodère

RENATI DES CARTES
PRINCIPIA
MATHESEOS
VNIVERSALIS,
SEU
INTRODVCTIO
AD
GEOMETRIÆ METHODUM
Conscripta
Ab ER. BARTHOLINO, CASP. FIL.
ACCEDUNT HUIC QUARTÆ EDITIONI NOTÆ QUÆDAM
& animadversiones tumultuariæ in universum Opus.
Cum Gratia & Privilegio Sac. Cæs. Majest.

FRANCOFVRTI AD MOENVM,
Sumptibus FRIDERICI KNOCHII, Bibliop.
Anno M DC XCV.

Principia Matheseos Universalis, by Rene Descartes, 1695 edition, from the ancient book collection at IHP.

The 16th ICMI study on Challenging mathematics in and beyond the classroom

Mariolina Bartolini Bussi

At the beginning of 2009, the volume *Challenging mathematics in and beyond the classroom* was published, edited by Edward J. Barbeau and Peter J. Taylor, as the product of the 16th ICMI study. This study is in some way connected with one of the earliest studies commissioned by the ICMI concerning the popularisation of mathematics (Howson & Kahane, 1990).

The topic of the 16th study was chosen in 2002, and the co-chairs (E. J. Barbeau and P. J. Taylor) and the International Program Committee were appointed. They met for the first time in November 2003 in Modena, Italy, at the Laboratory of Mathematical Machines, to draft the discussion document (see Issue 55, March 2005, of this newsletter). About fifty people were invited to attend the study conference, which was held in Trondheim, Norway, in July 2006, at the Norwegian Center for Mathematics Education. Both sites were chosen in order to offer to either the IPC members or all the participants the experience of places where mathematics challenges for young people are carried out regularly. In particular, on the occasion of the Trondheim conference, the KappAbel[1] competitions were also being held at the Norwegian Center for Mathematics Education.

The aim of this study was to present the state of the art initiatives developed in different parts of the world relating to the idea of challenge in mathematics in the perspective of life-long education.

The back cover of the volume reads:

"The last two decades have seen significant innovation both in classroom teaching and in the public presentation of mathematics. Much of this has centered on the use of games, puzzles and investigations designed to capture interest, challenge the intellect and encourage a more robust understanding of mathematical ideas and processes. ICMI Study 16 was commissioned to review these developments and describe experiences around the globe in different contexts, systematize the area, examine the effectiveness of the use of challenges and set the stage for future study and development. A prestigious group of international researchers, with collective experience with national and international contests, classroom and general contests and in finding a place for mathematics in the public arena, contributed to this effort. The resulting volume deals with challenges for both gifted as regular students, and with building public interest in appreciation of mathematics."

The list of contents is impressive:

Challenging Problems: Mathematical Content and Sources; Presentation of Challenges Beyond the Classroom: Organizational Issues; Challenging Mathematics Beyond the Classroom Enhanced by Technological Environments; Challenging Tasks and Mathematical Learning: Role of the Student; Mathematics in Context: Focusing on Students in Challenging Environments; Teacher Development and Mathematical Challenge; Challenging Mathematics Classroom Practices; Curriculum and Assessment: Use of Challenges.

It covers not only activity in mathematics classrooms but also activities in other "homes" of mathematics education (competitions, journals, books and other resources, conferences and projects, exhibitions, playgrounds, mathematics and science centres, clubs, camps, lectures, websites, summer schools, family programs, community fairs and events, and so on).

It covers not only challenging tasks (e.g. problem solving) but also challenging contexts (more suitable to long-term activities) and the relationships between challenging problems and the school curriculum. It covers not only new technologies but also classical technologies that have been available in the past.

In the volume, the eight comprehensive chapters, coauthored by groups of the participants in the spirit of ICMI study volumes, are not proceedings of a conference but rather a collective product coming out of a long process. Pre-conference individual contributions are available at http://www.amt.edu.au/icmis16.html.

References

Howson, A. G. & Kahane, J.-P. (Eds.) (1990). *The popularization of mathematics*. Cambridge, Cambridge University Press, ICMI Study series.

Barbeau E. J. & Taylor P. J. (Eds.) (2009). *Challenging Mathematics in and Beyond the Classroom: the 16th ICMI Study*. Springer; 325 p. 5 illus., Hardcover. ISBN: 978-0-387-09602-5. 90,43 Euro.

News

The ICMI inaugurated its *Digital Library* at the beginning of July. The project of an ICMI Digital Library, where eventually "all" publications related to ICMI and its activities will be made freely available online, has been under discussion for a long time. Thanks to the support received from the International Mathematical Union, and especially the IMU Committee on Electronic Information and Communication (CEIC), much progress has been made recently on this project and in particular the digitisation of past ICMI material.

The first document posted online was the proceedings of the symposium organised in 2000 on the occasion of the centennial of L'Enseignement Mathématique, the official production of the ICMI. More material will be made accessible progressively, including all the issues of the ICMI Bulletin, the volumes resulting from the ICMI Studies and the proceedings of the ICME congresses.

The Digital Library can be directly accessed via the ICMI website "http://www.mathunion.org/ICMI/"http://www.mathunion.org/ICMI/.

Subscription to ICMI-News (free)

To be kept informed on ICMI activities, send an email to icmi-news-request@mathunion.org with the Subject: subscribe.

Previous issues can be seen at http://www.mathunion.org/pipermail/icmi-news.

KappAbel is a Nordic mathematics competition for pupils in their 9th year of schooling (pupils in the 8th grade in Denmark, Finland and Sweden, and in the 9th grade in Iceland and Norway). It is a competition based on collaborative project work in school classes: when a class has registered, the whole class counts as ONE participant.

A discount of 60% from the list price on this volume is available for personal use only. The procedure for obtaining the ICMI society discount is as follows.

i) Orders must be placed personally, on the Springer website, through the NISS series homepage http://www.springeronline.com/series/6351.
ii) In order to obtain the society discount granted to the ICMI, individuals should enter the respective tokens when asked to during the ordering process, under "SpringerToken/promocode".
iii) The following Token Numbers have been issued by Springer for use by the members of the ICMI community:

Token for hardbound: zqwqqJQFWzQ8WqY.
Token for softbound: hAjkrckgWFaEYQA.

ERME activities for young researchers

Ferdinando Arzarello, President of ERME / Dina Tirosh, ERME, responsible for YESS

In previous issues of this newsletter, some information was given about the history and aims of the European Society for Research in Mathematics Education (ERME) and about its last conference CERME 6 (Lyon, 28 Jan–1 Feb 09).

This article reports on the initiatives of the ERME for young researchers in mathematical education. Essentially, there are two main scientific events, each occurring every two years: the YERME day and the YERME Summer School (YESS). The former is a special two half-day meeting attached to the ERME conference and the latter is a one week summer school organised in years when no ERME conference is scheduled (as illustrated in previous articles, the ERME conferences occur every two years).

The main idea of both initiatives is to support young researchers in their first years of work, particularly during and immediately after their PhDs, according to the spirit expressed in the ERME manifesto, namely:

- to let people from different countries meet and establish a friendly and co-operative style of work in the field of mathematics education research;
- to let people compare and integrate their preparation in the field of mathematics education research in a peer discussion climate and with the help of highly qualified and differently oriented experts;
- to let people present their research ideas, theoretical difficulties, methodological problems, and preliminary research results, in order to get suggestions (from experts and from other participants) about possible developments, different perspectives, etc. and open the way to possible connections with nearby research projects and co-operation with researchers in other countries.

A new initiative for young researchers started last year at YESS4. The idea is to constitute a YERME-committee, consisting of a group of young researchers and a representative of the YERME from each country (for their names see http://yerme.eu/). This initiative will continue at YESS5 next year.

The YERME Day

Typically, a YERME day starts the afternoon before a CERME (Conference of the European Society for Research in Mathematics Education). Describing briefly the last YERME day may give an idea of the nature of YERME days. CERME 6 in Lyon started on 28 Jan in the afternoon. The related YERME day started in the afternoon of 27 Jan and ended at 13:00 on 28 Jan. Sixty participants from 15 countries (mostly PhD students and researchers who have recently finished their PhD work) participated in the meeting. The young researchers at tended a lecture by a young researcher on the theme "There is a life after a PhD Thesis" and took part in 3-hour working groups and 3-hour discussion groups. Each group was guided by an expert researcher and required active participation. Examples of topics were: *Reading a research report or an article* (DG)*; Designing, conducting and analyzing interviews in mathematics education* (DG)*; Planning, analyzing and reporting an experimental activity* (WG)*; Designing and analyzing tasks in mathematics education* (WG)*; and Pre-service teacher knowledge at work in task design* (WG).

YESS – the ERME Summer School

The ERME has so far organised four successful summer schools: YESS1 (2002) at Klagenfurt (Austria), YESS2 (2004) at Podĕbrady (Czech Republic), YESS3 (2006) at Jyväskylä (Finland) and YESS4 (2008) at Trabzon (Turkey). YESS5 will take place 18-25 August 2010 near Palermo (Sicily, Italy).

Participants from many countries apply to these summer schools and there are usually more applications than available places (about 50–60); the applicants are chosen by the scientific committee. Each participant pays a fee, which partially covers the expenses for the school. The remaining expenses are covered by ERME, using a budget that ERME locates for this important activity, as well as local financial support.

The main scientific activity of the summer school consists of the *Thematic Working Groups* sessions. They are organised around several research themes, including Advanced Mathematics, Teacher Education, Information Technologies, Cognitive and Affective Factors in Learning and Teaching Mathematics, and Theoretical Perspectives (this could also include linguistic aspects and modelling).

Each participant chooses a specific theme, according to their ongoing research. They are asked to circulate (before the school) a short paper concerning the research they are doing among the other participants of their group. An expert in the specific theme is responsible for each group. They guide the group of students that participate in the theme group and provide the participants with suggestions, remarks and support regarding their research plans, questions, etc. During the summer school, each thematic group meets ten times (each meeting lasts two hours) and, with the guidance of the expert, discusses the participants' papers and addresses related "leading questions" concerning, for example, how to choose research problems and research questions, how to refer to and use existing literature and to frame one's own research, how to identify, present and interpret research results, and how to position theory in research conceptualisation and analysis. Specific attention is paid to each participant's own needs, regarding their own work.

Besides this main activity, the summer school offers several important, complementary activities, including:

- Five 45-minute lectures given by each expert, illustrating their own current research. These lectures are followed by 30-minute small-group discussions. Each expert circulates a summary of their talk before the summer school, pointing out relevant issues to be discussed.
- Two 90 minute plenary discussion group sessions: these discussions are devoted to transverse topics, possibly including the "leading questions" mentioned above.
- Two 90 minute informal discussion group sessions: these sessions are devoted to issues that are relevant to individual YERME participants' needs and are organised by the participants themselves.

Each YESS has an Organising Programme Committee, which consists of a scientific coordinator, two ERME board representatives, three YERME representatives and two Local Group Team representatives. In addition, each YESS has a Local Organising Team.

The first official announcement for YESS5 will be on the website of YERME (http://yerme.eu/) and on http://math.unipa.it/~grim/ no later than 15 September 2009. The deadline for applications to the school will be 15 January 2010. The deadline for paying the conference fee and the deposit for room reservation will be 30 March 2010 (the whole cost for registration, six days accommodation and meals will be kept below 500 euros). All the information will be published both on the YERME and the ERME websites (http://ermeweb.free.fr/).

For organisational questions about YESS5, contact Giorgio Santi (grpsanti@gmail.com), Maria Lo Cicero or Benedetto Di Paola (grimconf@math.unipa.it).

For scientific questions about YESS5, please contact Professor Paolo Boero (boero@dima.unige.it).

For general questions about ERME activities for young researchers, contact Professor Dina Tirosh (Dina@post.tau.ac.il).

Jahrbuch über die Fortschritte der Mathematik and *Zentralblatt MATH* – reporting on more than 140 years of mathematics

Silke Göbel

Jahrbuch
über die
Fortschritte der Mathematik
im Verein mit anderen Mathematikern
herausgegeben
von
Dr. Carl Ohrtmann und Dr. Felix Müller.
Erster Band.
Jahrgang 1868.
Berlin.
Druck und Verlag von Georg Reimer.
1871.

ZENTRALBLATT FÜR
MATHEMATIK
UND IHRE GRENZGEBIETE
(REINE UND ANGEWANDTE MATHEMATIK
THEORETISCHE PHYSIK · ASTROPHYSIK · GEOPHYSIK)
HERAUSGEGEBEN VON
P. ALEXANDROFF J. BARTELS W. BLASCHKE R. COURANT H. HAHN
G. H. HARDY F. HUND G. JULIA O. KELLOGG H. KIENLE
T. LEVI-CIVITA R. NEVANLINNA H. THIRRING B. L. VAN DER WAERDEN
SCHRIFTLEITUNG:
O. NEUGEBAUER-GÖTTINGEN
1. BAND
BERLIN
VERLAG VON JULIUS SPRINGER
1931

A Short History

The *Jahrbuch über die Fortschritte der Mathematik* is the world's oldest reviewing journal in mathematics. It was founded in 1868 by the Berlin mathematicians Carl Ohrtmann and Felix Müller with the aim of providing a complete survey of European publications in mathematics. For all published books and research articles, the bibliographical data were collected together with a short review summarising the content. The reviews were written by scientists from all over the world. The first volume contained 889 references to articles from 78 European scientific journals. The *Jahrbuch* continued until 1945 but ceased to exist after the war. Already in the decades before, the *Jahrbuch* principle of first collecting and classifying all publications of a specific calendar year and then to publish in a complete volume, caused considerable delays. As a more up-to-date alternative, the *Zentralblatt für Mathematik* was founded by Springer-Verlag in 1931. While the *Jahrbuch* contained only reviews in German, *Zentralblatt* also permitted texts in English, French and Italian, which were published without delay, hence providing a more recent and international coverage. With a short interruption during and after World War II, *Zentralblatt* was continued in 1947.

In the course of the development of databases, Internet and new opportunities of digitization, the *Jahrbuch* data have been digitized within the project "Electronic Research Archive for Mathematics (ERAM)", which started in 1998. The project was supported by the "Deutsche Forschungsgemeinschaft". Since 2003, complete information from the Jahrbuch covering the years 1868-1942 is available in the Internet via ZMATH database, thereby providing a unique source of mathematical information from 1868 to today (www.emis.de/ZMATH/), and as a separate database JFM (www.emis.de/MATH/JFM/) allowing for free access.

In 2004, the timeframe was extended even further by adding the bibliographical data from the articles of the "Journal für die Reine und Angewandte Mathematik" from its beginning in 1826. The *Crelle-Journal*, named after its first publisher August Leopold Crelle (1780–1855), was the first German journal solely devoted to mathematical publications. The famous results of Niels Henrik Abel (1802–1829) were published in the first issues. The volumes 1–493 have been digitized by the Staats- und Universitätsbibliothek Göttingen and are freely accessible, and the respective links have been integrated into the database.

Improvements of ZMATH and JFM for accessing classical mathematical literature

With the introduction of a new interface in 2010, new software, design and webpages are available. The links to full texts provided by the digital libraries at Göttingen ("Göttinger Digitalisierungszentrum"), Cornell ("Cornell Historic Math Book Collection"), Michigan ("University of Michigan Historical Math Collection"), Paris (GALLICA, "Bibliothèque nationale de France") and Grenoble (NUMDAM, "Numérisation de documents anciens mathématiques") have been updated and extended. New DOIs and other stable links are added permanently. An ongoing process is the integration of links to publications of the "Preußische Akademie der Wissenschaften" (currently digitized up to 1900).

Author identification, as implemented for the ZMATH database (see this column, EMS Newsletter 71, March 09), will be further improved for the historical *Jahrbuch* data. This is a rather sophisticated task, since almost all forenames in the *Jahrbuch* are abbreviated (or sometimes even missing).

Internal links are updated continuously. This includes cross-linking for the period 1931–1942, when the *Jahrbuch* and the *Zentralblatt* were published in parallel. The two reviews often provide interesting viewpoints by different reviewers (though it happened during war-time that the same review was printed in both journals). Citations of other reviews of the *Jahrbuch* are often only implicit (like "cf. the previous review") and have been made explicit by adding the JFM access number in question.

A special problem is the journal identification for the Jahrbuch. Abbreviations were not uniform; for instance, the *Mathematische Annalen* were first abbreviated as "Clebsch Ann." (named after its founder Alfred Clebsch, 1833–1872) then "Klein Ann." (named after its founder Felix Klein, 1849–1925) and, finally, "Math. Ann.". Hitherto, a separate journal database for JFM has been provided but now the journal identification process will provide a joint journal database of ZMATH and JFM and the matching of the historical and modern abbreviations according to international standards.

New Historic Information Pages

Several new webpages of the re-launched site provide sources for the history of both the *Jahrbuch* and the *Zentralblatt*:

1) A chronology of *Zentralblatt* history.
2) A chronology of *Jahrbuch* history.
3) Sheets of publication data of the *Jahrbuch* volumes and issues together with their year of publication and editors.
4) A bibliography.
5) A list of the *Jahrbuch* editors and staff (as far as is known).

The list of the *Jahrbuch* staff, which is not contained in the printed volumes, appears to be quite interesting. From 1927, the *Jahrbuch* has been published by the "Preußische Akademie der Wissenschaften". Therefore, several documents have been preserved in the archive of the "Berlin-Brandenburgische Akademie". Annual reports on the progress of the editorial work, accounts and correspondence have been discovered. Unfortunately, the files are incomplete due to the war. Before the times of the academy, the *Jahrbuch* was supported by the publisher (first Georg Reimer then de Gruyter), the "Notgemeinschaft der Wissenschaften" (from 1920) and donators (Max Henoch). Reviewers earned 1 RM (Reichsmark) per review at that time, postgraduates 2 RM per day and scientific assistants 100–350 RM per month.

The list of *Jahrbuch* editors ("Herausgeber" or "Schriftleiter"), according to the volumes, consists of Carl Orthmann, Felix Müller, Albert Wangerin, Emil Lampe, Erich Salkowski, Max Henoch, Georg Wallenberg, Arthur Korn, Leon Lichtenstein, Georg Feigl, Helmut Grunsky und Harald Geppert.

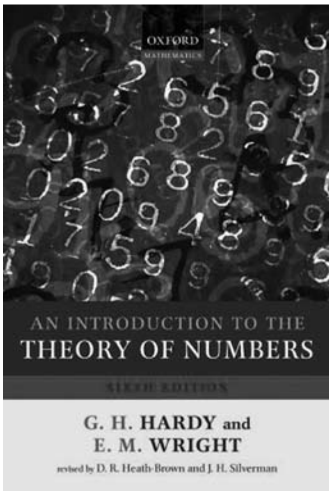
In the academy files, the following long-term staff are listed: Erika Pannwitz (15 years), Hans Pietsch (9 years), Max Zacharias (5 years), Edmund Scholz (5 years), Wolfgang Hahn (4 years), Willi Rinow (4 years), Fritz Dueball (3 years) and Rudolph Kochendörffer (3 years). Many mathematicians of later fame can be found among the postgraduates who worked as scientific assistants: Hans Freudenthal, Lilly Buchhorn (Görke), Maximilian Pinl, Herbert Jehle, Hanna v. Caemmerer (Neumann), Alfred Stöhr, Helmut Wielandt and Wolfgang Gröbner. During World War II, two French mathematicians also worked for the Jahrbuch: Christian Pauc und Frédéric Roger. They came to Berlin as prisoners of war but were soon released and stayed till the end of 1943 as staff of the academy. When it became too dangerous to stay in Berlin, C. Pauc became an assistant of O. Haupt in Erlangen and F. Roger went to W. Süß at Freiburg.

Many of these mathematician maintained relations to *Zentralblatt* as staff or reviewers in reestablishing the reviewing journal after the war. E. Pannwitz, H. Pietsch and F. Dueball resumed work in 1946 for the academy and E. Pannwitz became editor-in-chief of *Zentralblatt* from 1956 to 1967. A number of the former postgraduates became prolific reviewers and supported *Zentralblatt*, such as H. Freudenthal, M. Pinl, H. and B. Neumann, W. Rinow, R. Kochendörffer and C. Pauc.

Silke Göbel [goebel@zentralblatt-math.org] is a member of the staff of Zentralblatt MATH. She studied at the Freie Universität and the Humboldt Universität in Berlin, and wrote her thesis about structures in real Banach spaces. Later, her interests extended to history of mathematics, and she became involved in the Jahrbuch Project. Electronic Research Archiv of Mathematics (ERAM).

Book Review

Jorge Jiménez



G. H. Hardy and
E. M. Wright
(revised by
D. R. Heath-Brown and
J. H. Silverman)

An introduction to the theory of numbers
Sixth edition

Oxford University Press
Great Britain, 2008
ISBN: 978-0-19-921985-8
ISBN: 978-0-19-921986-5 (Pbk)

This is not a review of a book. This is a review of the sixth edition of one of the great classics of number theory. I think it is safe to say that number theory is one of the most attractive branches of mathematics at all levels, and maybe it is the most appealing to beginners. This might well be because of the wide variety of problems it offers, which are often very easy to state in simple terms but which have solutions that have frequently revealed rich and diverse mathematical techniques. Many books and notes introducing number theory, in one way or another, appear every year trying to cover this demand. But still this book reappears, this time in its sixth edition, even though modern technologies make it very easy to reproduce many other existing manuscripts or even previous editions of the very same one! Why is this?

The book does not assume any advanced mathematics, just previous knowledge of introductory calculus and algebra. In principle, this allows basically anybody with an interest in studying number theory to start reading the book. However, not everyone who has passed such basic courses in calculus and algebra is prepared to understand the content of the book. The reader will also need some mathematical intuition and what might be the most important prerequisite: they must be willing to spend some time and effort working on the concepts and meanings of the stated results and their proofs (things that are needed anyway not only for learning number theory but mathematics in general).

It is clear that the two educated mathematicians, Hardy and Wright, wrote this book to try to help the reader to learn number theory. You can see their intention in different aspects. Firstly, there is a good choice of subjects to cover. As they themselves say, the book does not have a definite plan other than to show beautiful theorems, either theoretical or practical (such as the proof of $2^{p-1} = 1 \bmod p^2$ for $p = 1093$), or just games, and to show them in a beautiful way. All this is perhaps the secret of their success. Secondly, the style in which the book is written is excellent. They present one result after another to complete a total of 460 theorems in 548 pages, including theorems on the theory of primes, diophantine equations, the geometry of numbers, quadratic fields, irrationality, approximation, arithmetic functions, partitions and representation of integers by particular forms. Many times, one chapter is not connected to the next one in a logical way but, in fact, it is not particularly important to read the book from the first to the last page, one after the next. Rather, the book is intended to serve as the first step in introducing the interested reader to each particular subject listed above. This phenomenon is well explained in the foreword of this edition by A. Wiles, describing his own experience with it. Finally, the authors also present the theorems in a very particular way. Sometimes they give several proofs of the same result, either to introduce new concepts or simply because of the beauty of the proofs. But in any event, they know that there is no way to learn mathematics if one does not involve oneself actively in the process and the proofs of the theorems are given in a way that forces the reader to become part of it. Indeed, although proofs are complete, little details, which could be logically redundant or could be deduced with appropriate thinking, are omitted, so when the reader finds his way to the end of the proof, somehow they have also given the proof by themselves, which helps them appreciate the content of the theorem.

But on the other hand, if there is an aspect or concept that might escape, maybe because it has a nature different from the principal path discussed, the authors quickly give a hint so it is not overlooked by the reader. Let me give a short example. Once I heard a joke that the last thing an expert in analytic number theory says before dying is "log log log log log…". Hardy and Wright know the importance of this function and in the first chapter (of a book introducing number theory), dedicate a section to the logarithm. It is obvious that anybody with a basic course in calculus would already know this function but the authors, being very interested that the reader not only knows it but understands it, exhibit an instance of how slow the function $\log x$ grows to infinity with x: if $x = 10^9 = 1{,}000{,}000{,}000$ then $\log x = 20.72$ and if $x = 10^{1000}$ then $\log \log \log x$ is a little greater than 2. Let me emphasise that in order to give these examples, one first has to spend some time in choosing the number of logs one wants to use and then the number x so the result is enlightening.

Saying why this is a great book, which is a common statement in any of the reviews of the previous editions of the book, does not prove why a new edition is necessary. There are no changes in the bulk of the 24 chapters of the fifth edition of the book. In fact, it seems that the remarks made in the review for the fifth edition from MathSciNet were not considered and most would probably have been interesting improvements. However, it is mentioned there that the inclusion of complex variables, for example, might ease the proof of the prime number theorem. In this point the reviewers perhaps made the right decision by keeping it unchanged. The elementary proof shown in Chapter 22 of the book, with integrals, avoids the obscurity of the sums of the original proof of Selberg without losing the spirit. It might be more difficult to follow than the proof that uses complex analysis but it is definitely more illuminating once the idea is taken out. Keeping this proof is certainly a wise move.

In any event, the changes made by Heath-Brown and Silverman in this new edition are essential. Firstly, you see immediately that the book has been re-typeset and no longer looks so dense. The counterpart to this change is that a large number of typographical errors have been introduced into the text. But these with no doubt will be removed for future printings. Secondly, the notes at the end of each chapter have been updated in a very careful manner. Some have been improved with more historically accurate explanations, some with added references and most of them including the progress made since the fifth edition[1], the last printing of which came out in 2000. It is interesting to note that only five of the 24 chapters did not need any update in the notes. These are Chapters 3, 9, 10, 17 and 23. This reveals just how much progress has been made in many classical questions, fundamental for modern number theory. But still not one of the unsolved problems concerning primes from the original list proposed by Hardy and Wright in the second chapter of the first edition have been completely settled in the past 70 years. It is true, however, that the enormous effort made to solve them has given three of the greatest achievements of mathematics of the last decade, namely representation of primes by the forms $a^2 + b^4$ and $a^3 + 2b^3$, the distance between consecutive primes and the existence of long arithmetic progressions among the prime numbers, and this edition includes in its appendix a page dedicated to them.

The most significant change of this new edition is the inclusion of a final chapter about elliptic curves. This is neither a good idea nor a bad one. It is simply a necessity. In the last few years, number theory has experienced a big revolution around the theory of elliptic curves. More than 50 percent of the 2097 entries on MathSciNet with the words ‘elliptic curves’ in the title were made in the last ten years and two thirds of them after Wiles’ proof of Fermat’s Last Theorem in 1995. And this is without taking into account its direct relation with modular forms or Abelian varieties of higher dimension.

The style is completely different from that of the previous chapters. In 40 pages, plus two pages of notes, the chapter gives an impressive general view of some of the most important achievements of the last century. It includes, apart from basic definitions (a little bit confusing when talking about fields of definition) and group structure, the theorems of Nagell-Lutz, Mordell-Weil and Mazur, heights, Siegel’s theorem, L-functions and the Gross-Zagier theorem, modular curves and finally a quick explanation of Fermat’s Last Theorem. No wonder half of the theorems are stated without proof. Even though it includes nice and complete results, such as the description of points of finite order with modular curves or the structure of the group of rational points in certain cases by previous results in Diophantine equations, the price to pay of including such a massive amount of material (some of which needs completely new theories envisioned and developed in the last century) is that, in general, the chapter is more descriptive than constructive. And sometimes, as is the case in the last section, the words do not give too much light on the huge subject behind them. In my opinion, the main deficiency is in Section 25.6 about points modulo p. It is less than one page and has just one result: Hasse’s inequality, stated without proof. For example, the inclusion of points modulo p in the torsion of the curve over the global field would have made a nice connection with points of finite order. But, overall, this section would have been an excellent spot to include some easy and nice results related to cryptography. There is, for example, a direct relation between counting points modulo p and factoring RSA moduli. It is a fact that it is not only Fermat’s Last Theorem that has inspired the development of the theory of elliptic curves but also, and in a great manner, its cryptographical applications. The inclusion of such results would have also been coherent with the spirit of the remaining chapters of the book. The reviewers, however, have added some comments on counting points and relations with cryptography in the notes at the end of the chapter.

To conclude, this book deserves its fame. It is an excellent starting point for anybody with an interest in learning number theory and, with additions such as the ones provided by Heath-Brown and Silverman, it will certainly remain in the most visible place on the bookshelf.

Jorge Jiménez Urroz [jjimenez@ma4.upc.edu] is an associate professor at UPC, Barcelona. He obtained his PhD in Madrid in 1995 in elementary number theory and, afterwards, he did several post-docs in the USA learning about elliptic curves and modular forms. His current interest is in number theory, with special attention to sieves, distribution of primes, elliptic curves and the invariants of their reduction modulo primes, and applications of modular forms and L-functions. But, for him, any simple problem (simple enough to be solved) is more than welcome.

[1] The footnote on page 245 is, and may always be, one of the most complicated sentences to ever be included in a text.

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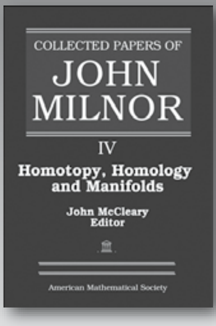

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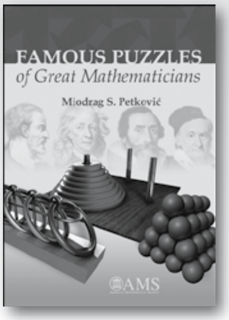

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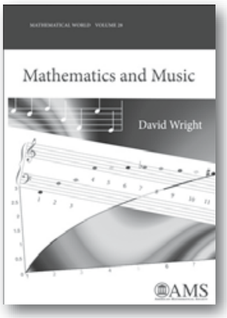

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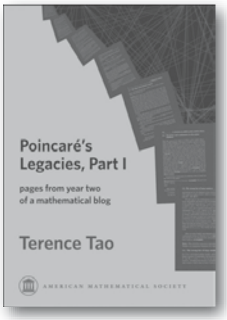

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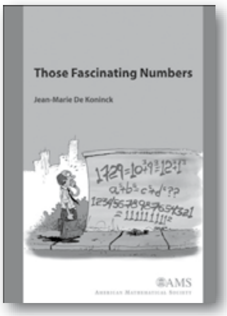

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Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

When Sir Michael Atiyah (Fields Medallist, 1966, and Abel Prize winner, 2004) was asked during an interview (*Math. Intelligencer* 6 (1984), 9–19) how he selects mathematical problems to investigate, he replied as follows: "Some people may sit back and say 'I want to solve this problem' and they sit down and say 'How do I solve this problem?'. Personally, this is not my approach. I just move around in the mathematical waters, thinking about things, being curious, interested, talking to people, stirring up ideas; things emerge and I follow them up. Or I see something which connects up with something else I know about, and I try to put them together and things develop. I have practically never started off with any idea of what I'm going to be doing or where it's going to go. I'm interested in mathematics; I talk, I learn, I discuss and then interesting questions simply emerge. I have never started off with a particular goal, except the goal of understanding mathematics."

I wish to mention the following references:

[1] M. Aigner and G. M. Ziegler, *Proofs from THE BOOK*, Springer-Verlag, Berlin, Heidelberg, 1998.
[2] D. Djukic, V. Jankovic, I. Matic and N. Pertovic, *The IMO Compendium. A Collection of Problems Suggested for the International Mathematical Olympiads: 1959–2004*, Springer Science, New York, 2006.
[3] G. H. Hardy and E. M. Wright (revised by D. R. Heath-Brown and J. H. Silverman), *An Introduction to the Theory of Numbers*, sixth edition, Oxford University Press, 2008.
[4] G. Pólya and G. Szegő, *Problems and Theorems in Analysis I*, Springer-Verlag, Berlin, Heidelberg, 1978.

I. Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

51. Find all continuous functions $f:\mathbb{R}\to[0,\infty)$ such that

$$f^2(x+y)-f^2(x-y)=4f(x)f(y)$$

for all real numbers x,y.

(Titu Andreescu, University of Texas at Dallas, USA)

52. Let $(a_n)_{n\geq 1}$ be a convergent sequence. Evaluate

$$\lim_{n\to\infty}\left(\frac{a_1}{n+1}+\frac{a_2}{n+2}+\cdots=+\frac{a_n}{2n}\right).$$

(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania, and Mihai Piticari, "Dragoş-Vodă" National College, Câmpulung Moldovenesc, Romania)

53. Find all nonconstant functions $f:\mathbb{R}_+\to\mathbb{R}$ and $g,h:\mathbb{R}\to\mathbb{R}$ that satisfy the functional equation

$$g(x)\,h(y)=f\left(\sqrt{x^2+y^2}\right)$$

for all $x,y\in\mathbb{R}$.

(Prasanna K. Sahoo, University of Louisville, USA)

54. Let $f:[a,b]\to\mathbb{R}$ be a continuous function on the closed interval $[a,b]$ and differentiable on the open interval (a,b). Let $p\in(0,\infty)$ and assume for a given $x\in(a,b)$ that

$$M_p(x):=\sup_{u\in(a,b)}\left\{|x-u|^{1-p}\left|f'(u)\right|\right\}<\infty.$$

Prove that the following inequality holds

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)\,dt\right| \leq \frac{1}{p(p+1)(b-a)}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]M_p(x).$$

(Sever S. Dragomir, Victoria University, Melbourne, Australia)

55. Find all nonnegative solutions of the following system of equations:

$$\left(x_1+\cdots+x_k\right)\left(x_k+\cdots+x_{2009}\right) = 1, \quad \text{where } k=1,\ldots,2009.$$

Author's comment. This problem gives two wrong impressions at first glance. Firstly, it seems to be simple and routine, which is not the case at all (this is seen from the answer). Secondly, it seems that the problem is algebraic, which is not the case either. Its solution (the only solution I know) combines real analysis and plane geometry.

(Vladimir Protasov, Moscow State University, Russia)

56. Consider the set of logarithmic derivatives of all algebraic polynomials that have no real roots:

$$S=\left\{\frac{P'}{P}\ \middle|\ P \text{ is a polynomial without real roots}\right\}.$$

Clearly, this set is in the space $L_p(\mathbb{R})$ for any $p\in(1,+\infty)$. Is S dense in that space ? In other words, examine whether any function from $L_p(\mathbb{R})$ can be approximated by elements of S?

Author's comment. By the Weierstrass theorem, algebraic polynomials are dense in the space L_p on a finite segment. For the real line this is not true, since polynomials are not in $L_p(\mathbb{R})$. Nevertheless, some approximation bases for $L_p(\mathbb{R})$ can be made of polynomials, e.g. some classes of rational functions (Padé approximations, etc.). In this context, the class S is a natural pretender to constitute an approximation basis in $L_p(\mathbb{R})$. This class, for instance, is dense in L_p on a segment (this follows from the same Weierstrass theorem) in the space $C_0(\mathbb{R})$ of continuous functions vanishing at infinity, etc.

(Vladimir Protasov, Moscow State University, Russia)

II. Two New Open Problems

57*. Find all functions $f:(0,1)^2 \to \mathbb{R}$ that satisfy the functional equation

$$f(xu,yv)+f(xv,yu)=f(x,y)f(u,v)$$

for all $x,y,u,v \in (0,1)$.

(Prasanna K. Sahoo, University of Louisville, USA)

58*. Does there exist an harmonic homeomorphism of the open unit ball B in $\mathbb{R}^n$ onto $\mathbb{R}^n$, where $n=3,4,\dots$, i.e., do there exist harmonic functions $f_1,f_2,\dots,f_n$, defined in B such that the mapping $f=(f_1,f_2,\dots,f_n)$ is a homeomorphism of B onto all of $\mathbb{R}^n$?

(Themistocles M. Rassias, National Technical University of Athens, Greece)

III. SOLUTIONS

43. Let $(a_n)_{n\geq 1}$, $(b_n)_{n\geq 1}$, $(c_n)_{n\geq 1}$ be sequences of positive integers defined by

$$(1+\sqrt[3]{2}+\sqrt[3]{4})^n = a_n + b_n\sqrt[3]{2}+c_n\sqrt[3]{4}, \quad \text{where } n\geq 1.$$

Prove that

$$2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k = \begin{cases} a_n, & \text{if } n\equiv 0 \pmod 3\\ b_n\sqrt[3]{2}, & \text{if } n\equiv 2 \pmod 3\\ c_n\sqrt[3]{4}, & \text{if } n\equiv 1 \pmod 3\end{cases}$$

and find similar relations for $(b_n)_{n\geq 1}$ and $(c_n)_{n\geq 1}$.

(Titu Andreescu, University of Texas at Dallas, USA, and Dorin Andrica, "Babeş-Bolyai" University of Cluj-Napoca, Romania)

Solution by the proposers. We have

$$\begin{aligned}a_n+b_n\sqrt[3]{2}+c_n\sqrt[3]{4} &= (1+\sqrt[3]{2}+\sqrt[3]{4})^n\\ &= \frac{(\sqrt[3]{2}(1+\sqrt[3]{2}+\sqrt[3]{4}))^n}{(\sqrt[3]{2})^n}\\ &= 2^{-\frac{n}{3}}(\sqrt[3]{2}+\sqrt[3]{4}+2)^n\\ &= 2^{-\frac{n}{3}}(1+(1+\sqrt[3]{2}+\sqrt[3]{4}))^n\\ &= 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}(1+\sqrt[3]{2}+\sqrt[3]{4})^k\\ &= 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}(a_k+b_k\sqrt[3]{2}+c_k\sqrt[3]{4}),\end{aligned}$$

hence

$$\begin{aligned}&a_n+b_n\sqrt[3]{2}+c_n\sqrt[3]{4}\\ &\quad= 2^{-\frac{n}{3}}\sum_{k=1}^{n}\binom{n}{k}a_k+\left(2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}=b_k\right)\sqrt[3]{2}\\ &\qquad+\left(2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k\right)\sqrt[3]{4}. \qquad (1)\end{aligned}$$

We study three cases.

(i) If $n\equiv 0 \pmod 3$ then $2^{-\frac{n}{3}}\in\mathbb{Q}$, hence

$$\begin{aligned}2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k &= a_n,\\ 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k &= b_n,\\ 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k &= c_n. \qquad \text{(I)}\end{aligned}$$

(ii) If $n\equiv 2 \pmod 3$ then $2^{\frac{-n+2}{3}}\in\mathbb{Q}$. Multiplying the relation (1) by $2^{\frac{2}{3}}=\sqrt[3]{4}$, we obtain

$$\begin{aligned}&a_n\sqrt[3]{4}+2b_n+2\sqrt[3]{2}c_n\\ &\quad= 2^{\frac{-n+2}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k+\left(2^{\frac{-n+2}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k\right)\sqrt[3]{2}\\ &\qquad+\left(2^{\frac{-n+2}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k\right)\sqrt[3]{4}, \qquad (2)\end{aligned}$$

then

$$\begin{aligned}2^{\frac{-n+2}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k &= 2b_n,\\ 2^{\frac{-n+2}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k &= 2c_n,\\ 2^{\frac{-n+2}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k &= 2a_n.\end{aligned}$$

Hence

$$\begin{aligned}2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k &= b_n\sqrt[3]{2},\\ 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k &= c_n\sqrt[3]{2},\\ 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k &= \frac{a_n}{\sqrt[3]{4}}. \qquad \text{(II)}\end{aligned}$$

(iii) If $n\equiv 1 \pmod 3$ then $2^{\frac{-n+1}{3}}\in\mathbb{Q}$. By multiplying the relation (1) by $2^{\frac{1}{3}}=\sqrt[3]{2}$, we obtain

$$\begin{aligned}&a_n\sqrt[3]{2}+b_n\sqrt[3]{4}+2c_n =\\ &\quad 2^{\frac{-n+1}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k+\left(2^{\frac{-n+1}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k\right)\sqrt[3]{2}\\ &\qquad+\left(2^{\frac{-n+1}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k\right)\sqrt[3]{4}, \qquad (3)\end{aligned}$$

then

$$\begin{aligned}2^{\frac{-n+1}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k &= 2c_n,\\ 2^{\frac{-n+1}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k &= a_n,\\ 2^{\frac{-n+1}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k &= b_n.\end{aligned}$$

Hence

$$\begin{aligned}2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k &= c_n\sqrt[3]{4},\\ 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k &= \frac{a_n}{\sqrt[3]{2}},\\ 2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k &= \frac{b_n}{\sqrt[3]{2}}. \qquad \text{(III)}\end{aligned}$$

Relations (I), (II) and (III) imply

$$2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}a_k=\begin{cases} a_n, & n\equiv 0 \pmod 3\\ b_n\sqrt[3]{2}, & n\equiv 2 \pmod 3\\ c_n\sqrt[3]{4}, & n\equiv 1 \pmod 3\end{cases}$$

$$2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}b_k=\begin{cases} b_n, & n\equiv 0 \pmod 3\\ c_n\sqrt[3]{2}, & n\equiv 0 \pmod 3\\ \frac{a_n}{\sqrt[3]{2}}, & n\equiv 1 \pmod 3\end{cases}$$

and

$$2^{-\frac{n}{3}}\sum_{k=0}^{n}\binom{n}{k}c_k=\begin{cases} c_n, & n\equiv 0 \pmod 3\\ \frac{a_n}{\sqrt[3]{4}}, & n\equiv 0 \pmod 3\\ \frac{b_n}{\sqrt[3]{2}}, & n\equiv 1 \pmod 3\end{cases}$$

□

Also solved by M. Bencze (Brasov, Romania), Con Amore Problem Group (Copenhagen, Denmark) and S. E. Louridas (Athens, Greece)

44. Define $S(n,p)=\sum_{i=1}^{n}(n+1-2i)^{2p}$ for all positive integers n and p. Prove that for all positive real numbers a_i, $i=\overline{1,n}$, the following inequality holds

$$\min_{1\le i<j\le n}(a_i-a_j)^{2p}\le\frac{4^p}{S(n,p)}\sum_{i=1}^{n}a_i^{2p}.$$

(Dorin Andrica, "Babeş-Bolyai" University of Cluj-Napoca, Romania)

Solution by the proposer. By multiplying the numbers $a_1,a_2,\dots,a_n$ by a suitable factor

$$\mu=\frac{1}{\sum_{i=1}^{n}a_i^{2p}}$$

we may reduce the problem to the case when $\sum_{i=1}^{n}a_i^{2p}=1$.

Assume without loss of generality that

$$a=a_1\le a_2\le\cdots\le a_n.$$

Let $\alpha=\frac{2}{\sqrt[2p]{S(n,p)}}$ and suppose by way of contradiction that

$$\min\{a_2-a_1,a_3-a_2,\dots,a_n-a_{n-1}\}>\alpha.$$

Then

$$a_i-a=(a_i-a_{i-1})+(a_{i-1}-a_{i-2})+\cdots+(a_2-a_1)>(i-1)\alpha,$$

hence

$$\sum_{i=1}^{n}a_i^{2p}>\sum_{i=1}^{n}[a+\alpha(i-1)]^{2p}.$$

Consider the function $\varphi:\mathbb{R}\to(0,\infty)$,

$$\varphi(x)=\sum_{i=1}^{n}[x+\alpha(i-1)]^{2p}.$$

Then

$$\varphi'(x)=2p\sum_{i=1}^{n}[x+\alpha(i-1)]^{2p-1}$$

and

$$\varphi''(x)=2p(2p-1)\sum_{i=1}^{n}[x+\alpha(i-1)]^{2p-2}>0.$$

Because of the fact that $\varphi'(x)$ is a polynomial of odd degree and $\varphi''(x)>0$ for all real values of x, it follows that φ' has a unique real zero:

$$x_0=\frac{1-n}{2}\alpha.$$

The number x_0 is also a minimum point of the function φ, thus

$$\sum_{i=1}^{n}a_i^{2p}>\varphi(x_0)=\sum_{i=1}^{n}\left[\frac{1-n}{2}\alpha+\alpha(i-1)\right]^{2p}=\frac{\alpha^{2p}}{4^p}S(n,p)=1,$$

a contradiction.

Hence

$$\min_{1\le i<j\le n}(a_i-a_j)^{2p}\le\frac{4^p}{S(n,p)}\sum_{i=1}^{n}a_i^{2p},$$

as desired.

Remark. For $p=1$ we obtain the Mitrinović's inequality

$$\min_{1\le i<j\le n}(a_i-a_j)^{2}\le\frac{12}{n(n^2-1)}\sum_{i=1}^{n}a_i^{2}.$$

□

Also solved by M. Bencze (Brasov, Romania) and T. Delatolas (National Technical University of Athens, Greece)

45. As part of developing a filing system for our collection of n DVDs, we need to label each DVD by a number from 1 to n. In order to form this number, we are going to use digit stickers: for example, the number 123 will be formed by the three stickers 1, 2, and 3 side by side (we do not want to add zeros in the beginning, such as 00123, as this would be a terrible waste).
These stickers are sold in sets of 10, and each decimal digit $\{0,1,2,\dots,9\}$ appears exactly once in the set. How many sets of stickers do we need to buy? As an example, for $n=21$ DVDs, digit 1 appears 13 times (in numbers 1, 10–19 and 21 – note that it appears twice in 11!), 2 appears 4 times (2, 12, 20 and 21) and every other digit from 3 to 9 appears exactly twice, so we would need 13 sets.

(K. Drakakis, University College Dublin, Ireland)

Solution by the proposer. Assume n has m digits, so that it can be written in the form:

$$n=\sum_{k=0}^{m-1}n_k10^k, n_k\in\{0,1,\dots,9\},\ k\in\{0,1,\dots,m-1\},\ n_{m-1}\ne 0.$$

Observe that, since unnecessary 0s are not used in the beginning of numbers, the first time a particular digit becomes active it becomes equal to 1 and therefore, for any n, no digit appears more times than 1. Since each set contains exactly one sticker of each digit, the number of sets needed will equal the number of 1s needed. To determine the latter, we will count the number of times each digit becomes equal to 1 and then sum. Let us work with the general kth digit.

$n_k=0$: the kth digit goes through a complete cycle

$$\sum_{l=k+1}^{m-1}n_l10^l$$

times and each cycle contains 1 once; it follows that the number of 1s obtained is

$$\sum_{l=k+1}^{m-1}n_l10^{l-1}.$$

Note that $k=m-1$ is not allowed in this case.

$n_k=1$: in addition to the number of 1s counted in the previous case, there are some extras due to the numbers whose kth digit is actually one. These are precisely

$$1+\sum_{l=0}^{k-1}n_l10^l,$$

where we add 1 to account for the number whose kth digit is 1 and all less significant digits are 0.

$n_k > 1$: here, in addition to the number counted in the first case, all of the numbers whose jth digit is n_j, $j > k$, and whose kth digit is 1 appear as well. These are clearly 10^k.

To conclude, the number of times the kth digit is 1 can be written as:

$$\sum_{l=k+1}^{m-1} n_l 10^{l-1} + [n_k = 1]\left(1 + \sum_{l=0}^{k-1} n_l 10^l\right) + [n_k > 1]10^k$$

and the total number of times the digit 1 appears is written as

$$S(n) = \sum_{k=0}^{m-1}\left(\sum_{l=k+1}^{m-1} n_l 10^{l-1} + [n_k = 1]\left(1 + \sum_{l=0}^{k-1} n_l 10^l\right) + [n_k > 1]10^k\right).$$

The bracket used is the logical bracket:

$$[P] = \begin{cases} 1, & P \text{ true} \\ 0, & P \text{ false}. \end{cases}$$

□

Also solved by Con Amore Problem Group (Copenhagen, Denmark)

46. (How to connect two binary words avoiding prohibited patterns ?)
There is a finite family of binary strings $f_1, \ldots, f_k$, each of length at most m, and two strings s and t of length m each. Is it possible to decide within polynomial time (in the input: the strings $s, t, f_1, \ldots, f_k$) if there is a binary string starting with s, ending with t and containing no f_i as a substring?
(Vladimir Protasov, Moscow State University, Russia)

Solution by the proposer. The answer is positive. The algorithm is the following. Let G be a directed graph whose vertices are all prefixes of all strings f_i, s and t, including s and t themselves but not including any f_i. If some f_i is a prefix of another string, it does not appear. The empty string is also a vertex. Note that even if some prefix of a string is a suffix of a prefix of another then they both appear as distinct vertices. From each vertex we have three edges, labeled 0 or 1. The edge labelled $\alpha \in \{0,1\}$ from a vertex corresponding to a prefix g is leading to the longest prefix in our list that is a suffix of the concatenation $g\alpha$, and there is no edge if a suffix of $g\alpha$ is some f_i. The question is now simply to decide if there is a directed path in this graph starting from s and ending at t. This problem is obviously polynomially solvable.
Author's comments. I put forward this problem in 2004 when working with my co-authors on a paper [1]. That paper deals with a problem of capacity of binary codes avoiding given prohibited difference patterns. The problem came from electronic engineering. The problem appeared as an auxiliary result to recognise codes of zero capacity. Neither me nor my co-authors could solve it and I sent it to some of my colleagues who specialise in algorithms and complexity. The solution above came from N. Alon and A. Razborov (IAS, Princeton). Later on my co-author R. Jungers (UCL, Belgium) found a different solution based on the so-called *Aho-Corasick* automaton that checks whether a given text contains as a subsequence a pattern out of a given set [2]. That solution appeared to be more efficient but not as elementary as the one of Alon and Razborov. □

[1] V. Blondel, R. Jungers, V. Protasov, "On the complexity of computing the capacity of codes that avoid forbidden difference patterns", *IEEE Transactions on the Information Theory, Vol. 52 (2006), No. 11, pp.5122–5127.*

[2] A. V. Aho, Algorithms for finding patterns in strings. *Handbook of Theoretical Computer Science (Vol. A): Algorithms and Complexity, pp.255–300. MET Press, Cambridge, MA, USA, 1990.*

47. For $h \in \mathbb{Z}$, $r \in \mathbb{Z}_{\geq 0}$ and $k = k_1 + k_2$ with $k_1, k_2 \in 2\mathbb{Z}_+$ set $h^* := |h - 1/2| - 1/2$ and

$$A_h^k(r) := \frac{(-1)^{k/2}}{r!} \frac{\Gamma(h - k/2 + r)}{\Gamma(h - k/2)} \frac{\Gamma(h + k/2)}{\Gamma(h + k/2 - r)}.$$

For $s \in \mathbb{C}$, $y > 0$, $l \in \mathbb{Z}_+$ and $u, v \in \mathbb{Z}$ with $u^* < k_1/2, v^* < k_2/2$ set

$$\theta_k(s) = \pi^{-s}\Gamma(s + k/2)\zeta(2s),$$
$$e_k(0; y, u) = \theta_k(u)y^u + \theta_k(1-u)y^{1-u},$$
$$e_k(l; y, u) = \frac{\sigma_{2u-1}(l)}{l^u} e^{-2\pi l y} \sum_{r=0}^{u^*+k/2} A_u^k(r)(4\pi l y)^{-r+k/2}$$

and

$$S_1(y) := y^{-k/2} e_{k_1}(0; y, u) e_{k_2}(l; y, v) + y^{-k/2} e_{k_1}(l; y, u) e_{k_2}(0; y, v),$$

with $\sigma_s(n)$ denoting the sum of the s-th powers of the positive divisors of n. Prove that

$$\pi^{k/2-1} \int_0^\infty S_1(y) e^{-2\pi l y} y^{k-2}\, dy$$

is rational and compute it.

(N. Diamantis, University of Nottingham, UK, and C. O'Sullivan, CUNY, USA)

Solution by the proposers. Set

$$F_{k_1,k_2}(l; u, v) := \int_0^\infty \theta_{k_1}(u) y^u \left(\frac{\sigma_{2v-1}(l)}{l^v} e^{-2\pi l y} \sum_{r=0}^{k_2/2-1-v^*} A_v^{k_2}(r)(4\pi l y)^{-r+k_2/2}\right) e^{-2\pi l y} y^{k/2-2}\, dy$$

$$= \theta_{k_1}(u) \frac{\sigma_{2v-1}(l)}{l^v} \sum_{r=0}^{k_2/2-1-v^*} \frac{A_v^{k_2}(r)}{(4\pi l)^{u+k/2-1}} \int_0^\infty (4\pi l y)^{u+k/2-1} (4\pi l y)^{-r+k_2/2} e^{-4\pi l y} \frac{dy}{y}$$

$$= \frac{(4\pi)^v \theta_{k_1}(u) \sigma_{2v-1}(l)}{(4\pi l)^{k/2-1+u+v}} \sum_{r=0}^{k_2/2-1-v^*} A_v^{k_2}(r) \Gamma(k/2 + k_2/2 - 1 + u - r).$$

We have

$$A_v^{k_2}(r) = (-1)^{k_2/2+r} r! \binom{k_2/2 - v}{r} \binom{k_2/2 - 1 + v}{r}$$

so that

$$\sum_{r=0}^{k_2/2-1-v^*} A_v^{k_2}(r)\Gamma(k/2+k_2/2-1+u-r)$$
$$= (-1)^{k_2/2}\sum_{r=0}^{k_2/2-1-v^*}(-1)^r\binom{k_2/2-v}{r}\binom{k_2/2-1+v}{r}$$
$$r!(k/2+k_2/2-2+u-r)!$$
$$= (-1)^{k_2/2}(v+k_2/2-1)!(u-v+k/2-1)!$$
$$\sum_{r=0}^{k_2/2-1-v^*}(-1)^r\binom{k_2/2-v}{k_2/2-v-r}\binom{k/2+k_2/2-2+u-r}{u-v+k/2-1}$$
$$= (-1)^v(v+k_2/2-1)!(u-v+k/2-1)!$$
$$\sum_t(-1)^t\binom{k_2/2-v}{t}\binom{(k/2-2+u+v)+t}{u-v+k/2-1}.$$

Using the identity

$$\sum_t(-1)^t\binom{a}{t}\binom{b+t}{c} = (-1)^a\binom{b}{c-a}$$

we obtain

$$F_{k_1,k_2}(l;u,v) = \frac{(-1)^{k_2/2}4^v\pi^{v-u}\zeta(2u)\sigma_{2v-1}(l)}{(4\pi l)^{k/2-1+u+v}}$$
$$(k/2-1+u-v)!(k/2-2+u+v)!$$
$$= \frac{(-1)^{k_2/2}\rho(2u)\sigma_{2v-1}(l)}{2(4\pi l)^{k/2-1}l^{u+v}}\Gamma(s)\Gamma(w) \qquad (1)$$

where (with B_{2n} denoting the $2n$-th Bernoulli number)

$$\rho(2n) := \begin{cases}(-1)^{n+1}B_{2n}/(2n)! & n\geq 0\\ 0 & n<0\end{cases}$$

and $s = u-v+k/2$, $\quad w = u+v+k/2-1$. Clearly

$$\int_0^\infty S_1(y)e^{-2\pi ly}y^{k-2}\,dy$$
$$= F_{k_1,k_2}(l;u,v)+F_{k_1,k_2}(l;1-u,v)+$$
$$F_{k_2,k_1}(l;v,u)+F_{k_2,k_1}(l;1-v,u)$$

and the Proposition follows. □

48. Find all possible decimal digits a such that, for a given n, the decimal expansions of 2^n and 5^n both begin by a, and give a necessary and sufficient condition to determine all such integers n.

(K. Drakakis, University College Dublin, Ireland)

Solution by the proposer. Let us set

$$2^n = 10^{m(n)}(a(n)+a'(n)),\ 5^n = 10^{l(n)}(b(n)+b'(n))$$

so that $m,l\in\mathbb{N}$, $a,b\in\{1,\dots,9\}$ and $a',b'\in[0,1)$. We observe now that

$$10^n = 5^n2^n \Leftrightarrow (a+a')(b+b') = 10^{n-m-l}.$$

But $1\leq(a+a')(b+b')<100$, which forces the two possibilities $n-m-l=0$ or 1 only. The former case leads to

$$(a+a')(b+b') = 1\Leftrightarrow a=b=1,\ a'=b'=0\Leftrightarrow 2^n=5^n=1\Leftrightarrow n=0,$$

while the latter case, setting $b=a$, leads to

$$a^2\leq(a+a')(a+b') = 10<(a+1)^2\Leftrightarrow a=3.$$

This is indeed a possibility, as, for example, $2^5=32,\ 5^5=3125$. But which values of n lead to such a coincidence?

For such an n let us rewrite:

$$2^n = 10^{m(n)}(3+a'(n)) = 10^{m(n)}(\sqrt{10}+3+a'(n)-\sqrt{10})$$
$$= 10^{m(n)+\frac12}(1+u(n)),\ u(n) := \frac{3+a'(n)}{\sqrt{10}}-1$$

and similarly

$$5^n = 10^{l(n)+\frac12}(1+v(n)),\ v(n) := \frac{3+b'(n)}{\sqrt{10}}-1.$$

Of course, u and v are not independent:

$$10^n = 5^n2^n = 10^{m+l+1}(1+u)(1+v)\Leftrightarrow$$
$$(1+u)(1+v) = 1\Leftrightarrow v = -\frac{u}{1+u}$$

and, furthermore,

$$\frac{3}{\sqrt{10}}-1\leq u,v<\frac{4}{\sqrt{10}}-1\Leftrightarrow\frac{3}{\sqrt{10}}-1\leq u,-\frac{u}{1+u}<\frac{4}{\sqrt{10}}-1.$$

We have thus obtained two inequalities for u. From the first one we get

$$2^n = 10^{m+\frac12}(1+u)\Leftrightarrow n\log 2$$
$$= m+\frac12+\log(1+u)\Leftrightarrow\log 3-\frac12\leq n\log 2-m-\frac12<\log 4-\frac12,$$

which implies that

$$\left[n\log 2-m-\frac12\right] = 0\Leftrightarrow m = \left[n\log 2-\frac12\right] = \lfloor n\log 2\rfloor,$$

where the square brackets denote rounding to the nearest integer, while the L-shaped brackets denote the floor function. This is obtained because

$$-1<\log 3-\frac12<0<\log 4-\frac12<1$$

and it implies that

$$\log 3\leq n\log 2-\left[n\log 2-\frac12\right]<\log 4.$$

From the second inequality we get

$$\frac{3}{\sqrt{10}}-1\leq-\frac{u}{1+u}<\frac{4}{\sqrt{10}}-1$$
$$\Leftrightarrow\frac{3}{\sqrt{10}}\leq\frac{1}{1+u}<\frac{4}{\sqrt{10}}$$
$$\Leftrightarrow\frac{\sqrt{10}}{4}-1<u\leq\frac{\sqrt{10}}{3}-1.$$

Combining the two inequalities gives

$$\frac{3}{\sqrt{10}}\leq 1+u = 2^n10^{-\frac12-\left[n\log 2-\frac12\right]}\leq\frac{\sqrt{10}}{3},$$

which, by taking logarithms, yields

$$\log 3-\frac12\leq n\log 2-\left[n\log 2-\frac12\right]-\frac12<\frac12-\log 3$$
$$\Leftrightarrow\log 3\leq n\log 2-\left[n\log 2-\frac12\right]<1-\log 3.$$

Finally, then, we need

$$|s-[s]| < \frac{1}{2} - \log 3, \ s := n\log 2 - \frac{1}{2} \approx 0.0228787453,$$

as a necessary and sufficient condition for the 2^n and 5^n to have the same first decimal digit in their decimal expansions. We can find the first few such values of n using a computer:

$$n = 5, 15, 78, 88, 98, 108, 118, 181, 191, 201, \ldots$$

□

Also solved by M. Bencze (Brasov, Romania), Con Amore Problem Group (Copenhagen, Denmark) and S. E. Louridas (Athens, Greece)

Remark. The problems 35, 36, 38 and 39 were also solved by B. Sury, Indian Statistical Institute, India.

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will be devoted to *Real Analysis*.

Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the addresses madalina.pacurar@econ.ubbcluj.ro or madalina_pacurar@yahoo.com. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files).

By the end of 2009, the Forthcoming Conferences section will be removed from the EMS Newsletter. An electronic announcement of upcoming events is already set up and fully operative on the EMS web site www.euro-math-soc.eu. Everyone is encouraged to submit their events to that site.

September 2009

2–4: New Trends in Model Coupling: Theory, Numerics & Applications (NTMC'09), Paris, France
Information: mcparis09@ann.jussieu.fr; http://www.ann.jussieu.fr/mcparis09/

2–4: Workshop in Nonlinear Elliptic PDEs – a celebration of Jean-Pierre Gossez's 65th birthday, Bruxelles, Belgium
Information: wnpde09@ulb.ac.be; http://wnpde09.ulb.ac.be

3–5: Complex and Harmonic Analysis 2009, Archanes, Crete, Greece
Information: http://fourier.math.uoc.gr/ch2009

3–6: International Conference on Theory and Applications of Mathematics and Informatics (ICTAMI 2009), Alba-Iulia, Romania
Information: www.uab.ro/ictami

4–9: 2nd Dolomites Workshop on Constructive Approximation and Applications, Alba di Canazei (Trento), Italy
Information: http://www.math.unipd.it/~dwcaa09

7–9: 13th IMA Conference on the Mathematics of Surfaces, York, UK
Information: ralph@cs.cf.ac.uk; http://ralph.cs.cf.ac.uk/MOSXIIIcall.html

7–11: 3rd International Conference on Geometry and Quantization (GEOQUANT), University of Luxembourg, Luxembourg
Information: geoquant@uni.lu; http://math.uni.lu/geoquant

7–11: History of Mathematical Periodics – Problems and Methods, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

7–12: Advanced School on Homotopy Theory and Algebraic Geometry, Sevilla, Spain
Information: http://congreso.us.es/htag09/php/index.php

8–12: 4th International Course on Mathematical Analysis in Andalucia, Jerez de la Frontera, Spain
Information: http://cidama.uca.es

10–12: Quantum Topology and Chern-Simons Theory, Strasbourg, France
Information: http://www-irma.u-strasbg.fr/article744.html

14–16: Complex Data Modeling and Computationally Intensive Methods for Estimation and Prediction (S.Co.2009), Milan, Italy
Information: http://mox.polimi.it/sco2009/

14–18: Quiver Varieties, Donaldson-Thomas Invariants and Instantons, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

14–18: Conference on Probabilistic Techniques in Computer Science CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/ccomputer/web-ccomputer/default.asp

16–20: MASSEE International Congress on Mathematics (MICOM 2009), Ohrid, Macedonia
I*nformation*: http://micom2009.smm.org.mk

17–18: 5th International Conference on History of Statistics and Probability, Santiago de Compostela, Spain

Information: http://www.neventia.es/vcongreso/lang1/

17–19: 5th William Rowan Hamilton Geometry and Topology Workshop on Computational and Algorithmic Geometry, Trinity College Dublin, Ireland
Information: http://www.hamilton.tcd.ie/events/gt/gt2009.htm

17–20: 17th Conference on Applied and Industrial Mathematics (CAIM 2009), Constanta, Romania
Information: caim2009@anmb.ro; www.anmb.ro

18–22: International Conference of Numerical Analysis and Applied Mathematics 2009 (ICNAAM 2009), Crete, Greece
Information: tsimos.conf@gmail.com; http://www.icnaam.org/

21–25: Galois and Arithmetical Theory of Differential Equations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

24–30: 6th International Conference on Functional Analysis and Approximation Theory (FAAT 2009), Acquafredda di Maratea, Italy
Information: faat2009@dm.uniba.it; http://www.dm.uniba.it/faat2009

28–October 2: Approximation and Extrapolation of Sequences and Convergent and Divergent Series, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

October 2009

5–9: Kolmogorov Readings. General Control Problems and their Applications (GCP-2009), Tambov, Russia
Information: http://www.tambovopu2009.narod.ru/

5–9: Lack of Compactness in Nonlinear Problems – New Trends and Applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

8–11: International Conference of Differential Geometry and Dynamical Systems (DGDS-2009), University Politehnica of Bucharest, Romania
Information: http://www.mathem.pub.ro/dept/dgds-09/DGDS-09.htm

12–14: Workshop on Computational Optimization (WCO-2009), Mrągowo, Poland
Information: http://www.imcsit.org/pg/227/181

12–16: Algebra, Geometry and Mathematical Physics, Będlewo, Poland
Information: tralle@matman.uwm.edu.pl;
http://www.agmf.astralgo.eu/bdl09/

14–16: Techniques and Challenges from Statistical Physics, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/wkstatisticalphysics/web-wkstatisticalphysics/default.asp

19–23: Fall School in Weak Kam Theory, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

19–23: Advanced Course on Topological Quantum Field Theories, University of Almeria, Spain
Information: http://nevada.ual.es:81/topologia/curso.php

19–24: Advanced Course on Shimura Varieties and L-functions, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/acshimura/web-acshimura/default.asp

22–24: Partial Differential Equations and Applications – International Workshop for the 60th birthday of Michel Pierre, Vittel, France
Information: Colloques@loria.fr; http://edpa2009.iecn.u-nancy.fr/

23–24: 16th Spanish Topology Meeting, University of Almeria, Spain
Information: http://nevada.ual.es:81/topologia/index.php

November 2009

5–7: 12th Symposium of Mathematics and its Applications, Timisoara, Romania
Information: http://www.mat.upt.ro/SimpozionMatematica/index.html

9–13: Geometry, Dynamics and Group Representations, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

16–20: ANGD Mathrice, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

16–18: International Conference of Education, Research and Innovation (ICERI 2009), Madrid, Spain
Information: http://www.iated.org/iceri2009/

19–21: 2nd Meeting on Optimization Modelization and Approximation (MOMA 2009), Oasis-Casablanca, Morocco
Information: http://www-lmpa.univ-littoral.fr/MOMA09/

23–25: MoMas Scientific Days, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

23–27: Mathematics and Astronomy – a Joint Long Journey, Madrid, Spain
Information: http://www.astromath2009.com/

30–December 4: Number Theory and Applications, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr;
http://www.cirm.univ-mrs.fr

30–December 10: 3rd de Brún Workshop in Computational Algebra, National University of Ireland, Galway, Ireland
Information: http://larmor.nuigalway.ie/~detinko/DeBrun3.htm

December 2009

8–11: Operators and Operator Algebras in Edinburgh, Edinburgh, Scotland, UK
Information: http://www.maths.gla.ac.uk/~saw/ooae/

9–12: Advanced Course on Algebraic Cycles, Modular Forms, and Rational Points on Elliptic Curves, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-

2010/accycles/web-accycles/default.asp

14–18: Meeting on Mathematical Statistics, CIRM Luminy, Marseille, France
Information: colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

14–18: Workshop on Cycles and Special Values of L-series, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/wklseries/web-wklseries/default.htm

January 2010

25–29: 3rd International Conference on the Anthropological Theory of the Didactic, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/cdidactic/web-cdidactic/default.asp

25–30: International School on Combinatorics "Pilar Pisón-Casares", Sevilla, Spain
Information: http://congreso.us.es/iscpp2010/

February 2010

12–14: 79th Workshop on General Algebra, Olomouc, Czech Republic
Information: http://aaa79.inf.upol.cz/

22–March 5: Arithmetic Geometry for Function Fields of Positive Characteristic, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/acarithff/web-acarithff/default.asp

April 2010

1–2: Workshop on the Theory on Belief Functions, Brest, France
Information: http://www.ensieta.fr/belief2010/

11–18: Algebraic Combinatorics – Designs and Codes (ALCOMA10), Thurnau, Bayreuth, Germany
Information:http://www.algorithm.uni-bayreuth.de/en/projects/ALCOMA10/

May 2010

3–6: Statistical Complexity in Classical and Quantum Systems, Sousse, Tunisia
Information: http://rilopezruiz2.spaces.live.com/

3–7: Advanced Course on Foliations – Dynamics-Geometry-Topology, CRM, Bellaterra, Spain
Information: http://www.crm.cat/Activitats/Activitats/2009-2010/acfoli/web-acfoli/default.asp

24–29: 5th International Conference – "Inverse Problems: Modeling and Simulation", Antalya, Turkey
Information: http://www.ipms-conference.org/index.htm

June 2010

10–12: Geometric and Probabilistic Aspects of General Relativity, Strasbourg, France
Information: http://www-irma.u-strasbg.fr/article874.html

17–19: Coimbra Meeting on 0-1 Matrix Theory and Related Topics, University of Coimbra, Portugal
Information: cmf@mat.uc.pt; http://www.mat.uc.pt/~cmf/01MatrixTheory

21–26: "Alexandru Myller" Mathematical Seminar Centennial Conference, Iasi, Romania
Information: http://www.math.uaic.ro/~Myller2010/

26–30: 2010 International Conference on Topology and its Applications, Nafpaktos, Greece
Information: http://www.math.upatras.gr/~nafpaktos/

July 2010

4–7: 7th Conference on Lattice Path Combinatorics and Applications, Siena, Italy
Information: latticepath@unisi.it; http://www.unisi.it/eventi/lattice_path_2010

19–23: Algebra Meets Topology – Advances and Applications, Barcelona, Spain
Information: http://departamento.fisica.unav.es/algebrameetstopology/

August 2010

8–11: ICM Satellite Conference on Functional Analysis and Operator Theory,
Indian statistical Institute, Bangalore, India
Information: http://www.isibang.ac.in/~statmath/conferences/icmfasat/icm.htm

September 2010

7–10: 7th Italian-Spanish Conference on General Topology and its Applications, Badajoz, Spain
Information: http://ites2010.unex.es/

Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

By the end of 2009, the Recent Books section will be removed from the EMS Newsletter. Please do not submit books for review to the editors any longer.

S. R. Athreya, V. S. Sunder: Measure and Probability*, CRC Press, Boca Raton, 2009, 221 pp., USD 62.96, ISBN 978-1-4398-0126-0*

This book is designed as an introduction to probabilistic measure theory for MSc and PhD students oriented to stochastics. A standard treatment of the fundamentals of measure theory (measure and its extension, measurability and integration) is presented in chapter 1 and chapter 2. Useful probabilistic specifications, e.g. random variables, their distribution and (conditional) expectation, and independent events and variables, are discussed in chapter 3. Chapter 4 goes as far as Kolmogorov's zero-one law and consistency theorem. Chapter 5 treats characteristic functions calculus, modes of convergence (for R-valued variables) and ends with the central limit theorem and the law of large numbers, the proof of which is based on the ergodic theorem for a stationary sequence of random variables. Chapter 6 focuses on discrete-time Markov chains with a countable state space. The basic notions (aperiodicity, irreducibility, transience, recurrence and stationarity) are treated with a balanced mixture of intuition and mathematical rigour. Chapter 7 completes the text with L_p-spaces, the Radon-Nikodym theorem, differentiation and change of variables rules and, finally, the Riesz representation theorem. The book is neatly written and can be recommended as an introduction to all students who intend to start courses on advanced modern probability. (jstep)

M. Audin: Hamiltonian Systems and Their Integrability*, SMF/AMS Texts and Monographs, vol. 15, American Mathematical Society, Providence, 2008, 149 pp., USD 55, ISBN 978-0-8218-4413-7*

This book serves as an introduction to mathematical aspects of integrable Hamiltonian systems. In the first chapter, the author introduces symplectic manifolds, Poisson brackets on a symplectic manifold, Hamiltonians and integrable Hamiltonian systems (together with basic examples), as well as the classical Darboux theorem and its proof. The second chapter contains a description of symplectic action and the local Arnold-Liouville theorem on action-angle variables, as well as a global version of the theorem. In the third chapter, a relationship of integrability and the differential Galois group is treated. The author proves a strictly stronger version of the Zeglin lemma and the Morales-Ramis theorem, giving a necessary condition for integrability in terms of the Galois group. In the fourth chapter, the author explains a relationship of the Arnold-Liouville theorem and algebraic curves via the Lax-pair approach. The last two chapters are appendices containing basics of differential Galois theory and algebraic geometry (algebraic curves and the Riemann-Roch theorem), which are used in the two previous chapters. The book is a nice introduction to integrability of Hamiltonian systems connecting it with two mentioned branches of mathematics, which were developed independently but which are deeply related to the studied topic. The book contains many nice illustrative examples (including the simple and spherical pendulum, the Euler-Poinsot rigid body, the Lagrange and the Kowalevski top, harmonic and anharmonic oscillators and the Hénon-Heiles system). It also contains many exercises, the solutions of which help the reader to understand the theory. The book offers a nice, short and concise introduction to this attractive topic for mathematicians as well as physicists. For the latter, it also represents a natural opportunity to learn more not only about integrable systems but also about the basics of differential Galois theory and algebraic curves. (skr)

Z. Balanov, W. Krawcewicz, H. Steinlein: Applied Equivariant Degree, Differential Equations and Dynamical Systems*, vol. 1, American Institute of Mathematical Sciences, Springfield, 2006, 552 pp., GBP 36.50, ISBN 978-1-60133-001-7*

The aim of this book is to provide an overview of the technique of equivariant degree in the study of bifurcations of dynamical systems invariant with respect to a group action. The first part summarises a few preliminaries in representation theory, equivariant topology and bordism theory and then develops the theory of equivariant degree, firstly for a general compact Lie group G. Later, a concrete description is developed for groups of the type $\Gamma \times S^1$, where Γ are various subgroups of the group $SO(3)$. The second part shows several examples of the method with the focus on Hopf bifurcations in the ordinary differential and functional differential setting and various kinds of symmetry. The authors also provide an Equivariant Degree Library for Maple and explain its usage in a separate chapter. The book is well organised and can be useful to a wide audience. A reader interested mainly in the applications may find a slight inconvenience in having to wait for more concrete examples until after the general theory is fully developed. An appendix on the classical Brouwer and Leray-Schauder degrees would also be handy for review and comparison. (dš)

H. T. Banks, H. T. Tran: Mathematical and Experimental Modeling of Physical and Biological Processes + CD*, Textbooks in Mathematics, CRC Press, Boca Raton, 2009, 299 pp., USD 71.96, ISBN 978-1-4200-7337-9*

The aim of this book is twofold: to develop some standard models of physical and biological processes (the transport equation, heat conduction, the beam equation, fluid dynamics and structured population models) in mathematical language; and, probably more importantly, to show how and why to design concrete engineering experiments for comparing numerical results of models with specific experimental data. It is necessary to find adequate values of coefficients to do such a comparison. For these purposes, the book contains chapters on statistical aspects of inverse problems and on feedback control systems. A basic calculus course is sufficient for understanding the mathematics used in the text. The book can be recommended to advanced undergraduate students, for whom mathematics is a bit more than just proving theorems. Teachers can also find suggestions of motivations for introductory parts of lectures on ordinary differential equations and partial differential equations. (jmil)

B. Barnes, G. R. Fulford: Mathematical Modelling with Case Studies. A Differential Equations Approach Using Maple and

***MATLAB**, second edition, CRC Press, Boca Raton, 2008, 349 pp., USD 59.95, ISBN 978-1-4200-8348-4*
This book studies growth and decay processes, interacting populations and heating problems. The presented approach describes each problem from the beginning, i.e. starting from a real case, formulating a "word equation", building a suitable model (in general a differential equation), solving the equation both in a classical way and using computer software Maple or Matlab, and explaining results from various points of view. Necessary mathematical material and computer tools are included for the reader's convenience. The chapter list is: Introduction to mathematical modelling, Compartment models, Models of single populations, Numerical solution of differential equations, Interacting population models, Phase-plane analysis, Linearisation analysis, Some extended population models, Formulating basic heat models, Solving time dependent heat problems, Solving heat conduction problems, and Introduction to partial differential equations. The modelling approaches cover empirical, stochastic, simulation, deterministic and statistical models. Readers are expected to have some basic familiarity with differential equations. Supplementary material is in the appendices: Differential equations, Further mathematics, Notes on Maple and MATLAB, and Units and scaling. The book can be useful for students of mathematical modelling. They will find many skills for modelling and solving real problems. Useful sheets for Maple and MATLAB are included for numerical solution. The most important feature of the book is that it contains many real life examples. This shows at the same time how useful and close to real life mathematical methods are. The main examples are solved in detail and the others are left for the reader. This is the best treasury of real case problems seen in a single book. (pp)

***A. Barvinok: Integer Points in Polyhedra**, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008, 189 pp., EUR 34, ISBN 978-3-03719-052-4*
The main goal of this book is to lead the reader towards the most recent advances in the enumeration of integer points in polyhedra. The author develops the structural theory and deals with algorithmic applications. He gives a new perspective on some of the old results, such as the role of the classical construction of continued fractions or the "continuity" property of polynomials enumerating integer points in parametric polytopes seen through the prism of identities in the algebra of polyhedra. Furthermore, the text contains new results, including the remarkable Berline–Vergne local formula. The text is based on the author's lecture notes for a graduate course and it contains numerous figures and various exercises, and thus it is suitable for study. It requires general mathematical maturity (e.g. basic linear algebra and analysis). The combination of new and classical results, theory and algorithmic applications, up-to-date results and an original approach makes the text useful for researchers and relatively easy to read for students. (zhal)

***J. Beck: Combinatorial Games. Tic-Tac-Toe Theory**, Encyclopedia of Mathematics and Its Applications 114, Cambridge University Press, Cambridge, 2008, 732 pp., GBP 85, ISBN 978-0-521-46100-9*
This extensive book contains a close to ultimate review of the topic of combinatorial game theory. However, it is not just a simple review as most of the book contains the results of the author, who is currently one of the greatest pioneers of this branch of game theory. This seems to be the best and most useful treatment of the subject so far. Traditional game theory has been successful at developing strategy in games of incomplete information: when one player knows something that the other does not. But it has little to say about games of complete information, for example tic-tac-toe, solitaire and hex. This is the subject of combinatorial game theory. To analyse a position in such a game, one has to examine available options, then further options available after selecting any option and so on. This leads to "combinatorial chaos", where brute force study is impractical. In this comprehensive volume, József Beck shows readers how to escape from the combinatorial chaos via the "fake probabilistic method", a game-theory adaptation of the probabilistic method in combinatorics.

At the beginning, the book explains the basic concepts: Tic-Tac-Toe-like games, weak win and strong draw, the connection with Ramsey theory, the strategy stealing argument and plenty of game examples and techniques. Then we move on to full use of the powerful potential technique – games-theory first and second moments. The book continues developing these methods in many complex ways. Using them, the author is able to determine exact results about infinite classes of many games, leading to the discovery of some striking new duality principles and showing many new approaches to understanding combinatorial games. At the end, the volume contains an extremely helpful dictionary and a list of challenging open problems. The book is recommended for a broad mathematical audience. Almost all concepts from other parts of mathematics are explained so it is convenient both for the specialist seeking a detailed survey of the topic and for students hoping to learn something new about the subject. The book has a potential to become a milestone in the development of combinatorial game theory. (tval)

***J.-M. Bismut, G. Lebeau: The Hypoelliptic Laplacian and Ray–Singer Metrics**, Annals of Mathematics Studies, no. 167, Princeton University Press, Princeton, 2008, 367 pp., USD 45, ISBN 978-0-691-13732-2*
In this book, analytic theory of the hypoelliptic Laplacian is developed and corresponding results on the associated Ray–Singer torsion are established. The hypoelliptic Laplacian is a second order differential operator defined on the cotangent bundle of a compact Riemannian manifold. It is supposed to interpolate the classical Laplacian and an operator related to a geodesic flow. In this way, it gives a semiclassical version of the fact that the Witten Laplacian on the corresponding loop space should interpolate between the classical Hodge Laplacian and Morse theory for the same energy functional. The authors develop Hodge theory for the studied Laplacian and the local index theory of the associated heat kernel. They adapt the theory of Ray–Singer torsion and the analytic torsion forms of Bismut-Lott and they develop an appropriate pseudodifferential calculus. They show that when the deformation parameter tends to zero, the hypoelliptic Laplacian tends to the usual Hodge Laplacian of the base space by a collapsing argument letting the fibre shrink to a point. They also obtain small time asymptotics for the supertrace of the associated heat kernel. A comparison formula between the elliptic and hypoelliptic analytic torsions is derived, studying an equivariant setting of the Ray–Singer

torsion of the studied operator and the associated Ray–Singer metrics on the determinant of the cohomology ring. To obtain localisation estimates, probabilistic methods related to diffusion processes are used. (skr)

F. Blanchet-Sadri: Algorithmic Combinatorics on Partial Words*, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2007, 385 pp., USD 89.95, ISBN 978-1-4200-6092-8*

Combinatorics of words (often referred to as Stringology) is an area of combinatorics studying combinatorial and algorithmic properties of sequences of symbols. As such, it has become vital for modern computer science, encompassing and developing (among other topics) data compression and data mining techniques. At the same time, it builds on theoretical grounds of algebraic methods as well as on connections to automata theory. Applications in computer science and bioinformatics need the introduction of probabilistic elements in a similar way to the development of logic (fuzzy logic) and other areas of mathematics oriented to applications. This leads to the notion of partial words, i.e. words with incomplete information in terms of wild cards in the place of some characters.

This book is the first reference book on the theory of partial words. In 12 sections, it gradually leads the reader from a gentle introduction to the topic to an advanced theory. The main goal is to compare combinatorics on partial words to the classical theory of full words. The reader will be surprised at how many of the well-known results from classical semigroup theory can be translated to partial words. The topics treated in various sections start with a discussion of the compatibility relation and equations over partial words, and they further include concepts of periodicity, factorisation, primitivity, codes, solution of special equations and unavoidable sets. Methods range from algebraic methods to algorithms to complexity and present a truly complex treatment of the topic. Every section concludes with exercises sorted by level of difficulty (hints to selected exercise being provided at the end of the book), followed by programming exercises, a bibliography and links to established web interfaces. A complex treatment and presentation makes the book an excellent textbook as well as a reference book for interested researchers in this emerging area on the borderline of algebra, combinatorics and computer science. (jkrat)

A. A. Borovkov, K. A. Borovkov: Asymptotic Analysis of Random Walks. Heavy-Tailed Distributions*, Encyclopedia of Mathematics and Its Applications 118, Cambridge University Press, Cambridge, 2008, 625 pp., GBP 95, ISBN 978-0-521-88117-3*

This book deals with the asymptotic behaviour of probabilities of rare events related to large deviations of random walks whose jump distributions do not vanish too fast at infinity and are *regularly* behaved. The monograph presents an up-to-date, unified and systematic exposition of the field. Most of the results presented are appearing in a monograph for the first time and a good proportion of them were obtained by the authors. The key concepts of the monograph have a crucial standing in modern probability: the Random walk is a classical, probabilistic object of great importance in mathematical statistics, risk and queuing theory. Large deviation and rare events are of great interest in many applied areas, since computing the large deviation probabilities enables one to establish, for example, small error probabilities in statistics or ruin probabilities in risk theory. Regular distributions offer an important alternative to the classic case of distributions decaying exponentially fast at infinity, which unfortunately fails in many applied problems. The book presents some beautiful and useful mathematics that may attract a number of probabilists to the *large deviations* topic in probability. (jstep)

N. P. Brown, N. Ozawa: C*-Algebras and Finite-Dimensional Approximations*, Graduate Studies in Mathematics, vol. 88, American Mathematical Society, Providence, 2008, 509 pp., USD 79, ISBN 978-0-8218-4381-9*

This monograph is devoted to the study of C*-algebras with a focus on their finite-dimensional approximations. The book is divided into 17 chapters and accompanied by 6 appendices. The first chapter is an introduction. Chapters 2-10 form Part 1 (Basic Theory), which is devoted, in particular, to the study of nuclear and exact C*-algebras. It contains definitions and basic properties of these classes. It also deals with tensor products and other constructions of C*-algebras (including crossed products, free products and C*-algebras associated to groups) and the behaviour of exactness and nuclearity with respect to these operations is studied, including quasidiagonal C*-algebras and local reflexivity for C*-algebras. Part 2 (Special Topics) comprises chapters 11–14. It deals with simple C*-algebras, approximation properties for groups (including Kazhdan's property (T), Haagerup's property and weak amenability), the weak expectation property, the local lifting property and weakly exact von Neumann algebras. Chapters 15-17 form Part 3 (Applications), containing results on the classification of von Neumann algebras associated to groups, a solution of the Herrero approximation problem and some counterexamples in K-homology and K-theory. (okal)

R. A. Brualdi, D. Cvetković: A Combinatorial Approach to Matrix Theory and Its Applications*, Discrete Mathematics and Its Applications, CRC Press, Boca Raton, 2008, 267 pp., USD 89.95, ISBN 978-1-4200-8223-4*

This book is about expressing and proving basic facts of linear algebra (including some more advanced topics) in the language of oriented (directed) graphs (digraphs). The graphs are weighted, with the edges of zero weight regarded as removed from the base set of all edges. The weights come from elements of matrices. With a matrix $A = [a_{ij}]$, one can always associate a bipartite graph in which the edges go from symbols representing rows to symbols representing columns so that the edge from the row i to the column j gets the weight aij. This representation allows an interpretation of matrix products but for further purposes one has to consider the digraph in which the ith row and ith column are represented by the same symbol. Of course, this presupposes that the matrix A is square. By representing permutations as subgraphs one can define determinants as a sum of weighted paths, where the weight of a path is by default the product of the weights. The standard proofs of determinant properties, Laplace development, inverses and Cramer's formula are then formulated in this setting.

The main idea seems to be to make the basics of linear algebra more understandable and concrete. There are many exercises and the intended audience obviously includes electrical engineers and students of electrical engineering. The basics comprise a bit more than half of the book. They are followed by an interesting proof of the Cayley-Hamilton theorem in this language and also by a proof of the Jordan Canonical Form. The Perron-Frobenius theorem also appears but without proof. Digraphs are used to discuss some of its consequences and some related notions. The last chapter presents applications from three areas. One is concerned with flow graphs (electrical engineering), another with vibrations of membranes (physics) and the last with unsaturated hydrocarbons (chemistry). The book is written very carefully and the speed of exposition is leisurely, in particular at the beginning. The book can serve well as a supplementary material that sheds light upon subjects that are sometimes regarded as too abstract. It can help the reader to grasp such subjects with more confidence and understanding. (ad)

J. P. Buhler, P. Stevenhagen, Eds.: Algorithmic Number Theory. Lattices, Number Fields, Curves and Cryptography*, MSRI Publications 44, Cambridge University Press, Cambridge, 2008, 652 pp., GBP 50, ISBN 978-0-521-80854-5*

This volume consists of twenty survey articles on various topics in algorithmic number theory written by leading experts in the field. It starts with two introductory papers, one on the Pell equation (H. W. Lenstra, Jr.) and the other on basic algorithms in number theory (J. Buhler and S. Wagon). They are followed by eight articles covering the core of the field, including smooth numbers and the quadratic sieve (C. Pomerance), the number field sieve (P. Stevenhagen), primality testing algorithms (R. Schoof), lattices (H. W. Lenstra, Jr.), elliptic curves (B. Poonen), the arithmetic of number rings (P. Stevenhagen), computational number theory (A. Granville) and fast multiplication (D. Bernstein). The remaining ten articles contain surveys of specific topics, including discrete logarithms, cryptography, Arakelov class groups, computational class field theory and the algorithmic theory of zeta functions over finite fields. The book can be warmly recommended to anyone interested in the fascinating area of computational number theory. (jtu)

T. Ceccherini-Silberstein, F. Scarabotti, F. Tolli: Harmonic Analysis on Finite Groups*, Cambridge Studies in Advanced Mathematics 108, Cambridge University Press, Cambridge, 2008, 440 pp., GBP 40, ISBN 978-0-521-88336-8*

There are not many books that can be used both as an elementary textbook and a research monograph with the same ease and success. This one, written by three Italian mathematicians, is a rare example. Its main aim is to introduce some modern tools for studying asymptotic behaviour of Markov chains on certain finite groups. This goal is illustrated in the very beginning on four examples: a random walk on a circle, the Ehrenfest and Bernoulli-Laplace diffusion models and a Markov chain on the group of permutations defined via random transpositions. Asymptotics of some of these models shows a surprising cut-off phenomenon, or a sudden transition from "order" to "chaos" after a certain (large) number of steps. This and other phenomena are studied mostly with the tools of representation theory of finite groups. These are introduced starting from the most elementary level, with complete proofs, many examples and exercises with solutions, so that the book would be perfectly suitable for an introductory course on discrete Fourier transforms and related representation theory. No prerequisites on probability theory and Markov chains are required; everything is explained in detail. From a researcher's point of view, the introduction and detailed study of Gelfand pairs in the context of finite groups is very valuable. These are pairs of a group G and a subgroup K such that the decomposition of the set of functions on the homogeneous space G/K is multiplicity free. The Gelfand pair action on a Markov chain, if available, often simplifies the analysis of its asymptotic behaviour. This is illustrated in detail on the examples introduced at the beginning. There is an extensive bibliography and an appendix on discrete trigonometric transforms. The book can be warmly recommended for anyone interested in the subject and/or looking for interesting applications of representation theory. (dš)

W. Cheney, W. Light: A Course in Approximation Theory*, Graduate Studies in Mathematics, vol. 101, American Mathematical Society, Providence, 2009, 357 pp., USD 69, ISBN 978-0-8218-4798-5*

A large portion of this book focuses on multivariable approximation theory, i.e. the approximation of functions in several variables, as opposed to the classical theory of functions in one variable. Its purpose is to guide the reader in exploring contemporary approximation theory. The textbook has 36 chapters. A central theme of the book is the problem of interpolating data by smooth multivariable functions. Several chapters investigate interesting families of functions that can be employed in this task; among them are polynomials, positive definite functions and radial basis functions. The book then moves on to the consideration of methods for concocting approximations, such as by convolutions, neural nets and interpolation at more and more points. A major departure from the theme of multivariate approximation is found in the two chapters on univariate wavelets, which comprise a significant fraction of the book.

Most of the topics in the book, heretofore accessible only through research papers, are treated from the basis of currently active research, often motivated by practical problems arising in diverse applications such as science, engineering, geophysics, business and economics. Among these topics are projections, interpolation paradigms, positive definite functions, the interpolation theorems of Schoenberg and Micchelli, tomography, artificial neural networks, wavelets, thin-plate splines, box splines, ridge functions and convolution. An important and valuable feature of the book is the bibliography of almost 600 items directing the reader to important books and research papers. There are 438 problems and exercises scattered throughout the book allowing the student reader to get a better understanding of the subject. The book can be used as a text for courses, seminars or even solo study and is designed for graduate students in mathematics, physics, engineering and computer science. (knaj)

D. Christodoulou: Mathematical Problems of General Relativity I*, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008, 147 pp., EUR 28, ISBN 978-3-03719-005-0*

In this book, the mathematics of general relativity and especially questions related to asymptotic flatness, conserved quantities and stability of Minkowski space-time are treated. In the first chapter, the author introduces the Einstein equation of gravitation on four dimensional Lorentzian manifolds. One can find a concise treatment of the Cauchy problem for the Einstein equation as well as the concepts of causal past and future, regular hyperbolicity and maximal development. The chapter devoted to asymptotic flatness contains (besides other notions) a definition of asymptotically flat initial data, total energy and linear and angular momenta. A Noether-type theorem adapted to general relativity and the result of Schoen and Yau on the positivity of the total energy are also presented. In the last chapter, the problem of stability of Minkowski space-time is introduced and a sketch of a proof of this result is given. Roughly speaking, the theorem predicts that under some given conditions, one can construct a geodesically complete solution to the Einstein equation on a strongly asymptotically Euclidean Lorentzian manifold, which tends to Minkowski space-time along any geodesic. The book is based on a lecture course of the author given at ETH Zurich. It is very well-written and gives a balanced description of the subject, balancing the level of full-length proofs and citations of proofs, disturbed neither by a lack of ideas motivated by physics nor by an absence of mathematical proofs that are written concisely and carefully. The book can serve as an introduction to mathematics of general relativity for physicists as well as mathematicians who would like to familiarise themselves with general concepts of this theory and who would like to know some recent applications that give answers to up-to-date problems of gravitation theory. (skr)

***H. Davenport: The Higher Arithmetic**, eighth edition, Cambridge University Press, Cambridge, 2008, 239 pp., GBP 23.99, ISBN 978-0-521-72236-0*

This is already the 8th revised edition of a book that has been very popular since its first edition in 1952. Its popularity is based on a very readable style of exposition. Since its 6th edition, the book has been enlarged with chapter VIII entitled Computers and Number Theory (written by James H. Davenport) containing some known applications of the topics (e.g. factorisation, random number generation and the RSA cryptographic method) presented in the preceding parts of the book. The publisher promises a companion website (www.cambridge.org/davenport) bringing state-of-the-art material on primality testing and the use of computers in number theory with more details on the latest advances and sample codes for important algorithms. An important part of the book consists of some useful historical comments and references making it very readable and useful for anyone interested in number theory. (špr)

***E. de Faria, W. de Melo: Mathematical Tools for One-Dimensional Dynamics**, Cambridge Studies in Advanced Mathematics 115, Cambridge University Press, Cambridge, 2008, 191 pp., GBP 35, ISBN 978-0-521-88861-5*

The title may be a bit misleading; this book is largely focused on complex dynamics, which is one-dimensional from the complex point of view. There is no doubt that bizarre pictures of Mandelbrot and Julia sets attract attention to this field of mathematics. This is the theme for numerous expositions at a popular level. Here, however, the subject is treated as a precise and deep mathematical theory. Let us mention the main goals of the text. The uniformisation theorem states that a subdomain of the Riemann sphere whose complement contains at least three points can be covered by a holomorphic map from the unit disc. The measurable Riemann mapping theorem (Alfors-Bers version) on the solution of the Beltrami equation with measurable coefficients is used to prove Sullivan‘s no-wandering-domains theorem on Fatou sets of rational maps. Finally, the Bers-Royden theorem discusses the existence of an extension of a holomorphic motion. This concerns functions of two variables, holomorphic in one variable and quasiconformal in the other. These theorems are presented with full proofs, applications and related developments. The book also contains a chapter on some topics of real dynamics and appendices on Riemann surfaces and Teichmüller theory. Each chapter is equipped with exercises. The book is intended for advanced students and researchers. It is a successful self-contained exposition of an important part of the theory with indications for further studies and discussion of perspectives, including fundamental open problems. (jama)

***H. G. Diamond, H. Halberstram, W. F . Galway: A Higher-Dimensional Sieve Method – With Procedures for Computing Sieve Functions**, Cambridge Tracts in Mathematics 177, Cambridge University Press, Cambridge, 2008, 266 pp., GBP 45, ISBN 978-0-521-89487-6*

Sieve methods form one of pillars of contemporary number theory. The aim of this monograph is to provide a concise treatment of integer and half integer dimensional sieves, which extends Jurkat and Richert’s original method for dimension 1. The book is written very carefully, giving all the necessary details (or at least hints), as this is a highly technical topic. A very interesting feature of the book is the appendix “Procedures for computing sieve functions“, containing details of programs of the Mathematica software package for sieve-theory calculations, which is accessible on the authors‘ website. The monograph is a very good source of details of techniques used in sieve methods and their applications, and therefore it can be warmly recommended for everyone interested in sieve methods. (špr)

***H. M. Edwards: Higher Arithmetic. An Algorithmic Introduction to Number Theory**, Student Mathematical Library, vol. 45, American Mathematical Society, Providence, 2008, 210 pp., USD 39, ISBN 978-0-8218-4439-7*

This is an introduction to the basic parts of number theory culminating in quadratic extensions. The author uses an algorithmic approach following Gauss and Kroneckers’ dictum that everything defined should be managed by exact computations (manipulations) with positive integers. The book contains many examples (each chapter ends with examples or computer experiments, the answers given at the end of the book). To work preferably with positive integers and to avoid negative numbers is the first unusual element in the exposition. Hypernumbers (which are formal expressions of the form of a quadratic irrational) form the second nonstandard tool used here. The third unexpected feature of the book is that the author gives new names to familiar objects. Nevertheless, the book is written in a lucid and readable style and it can be recommended to students interested in learning classical parts of number theory and some of its applications in a less standard way. (špr)

***M. Erickson, A. Vazzana: Introduction to Number Theory**, Discrete Mathematics and its Applications, Chapman & Hall/CRC, Boca Raton, 2007, 521 pp., USD 89.95, ISBN 978-1-58488-937-3*

This is an introductory text to number theory (with chapters on congruences, quadratic residues, large primes, continuous fractions, Diophantine equations and cryptography) also covering some more advanced topics, e.g. elliptic curves and their applications, the prime number theorem, the Riemann hypothesis, the Dirichlet theorem on primes in arithmetic progressions and the connection of logic to number theory (including a sketch of the negative solution to Hilbert's tenth problem). The reader can find worked examples and programs written in Mathematica and Maple (the book gives tutorials to these languages in the appendices). Moreover, it contains more than 500 exercises with notes at the end of chapters and lots of historical facts. The book is certainly more than a standard introduction to number theory and it can be recommended to everyone looking for a firm foundation of the fundamentals of number theory. (špr)

P. Flajolet, R. Sedgewick: Analytic Combinatorics*, Cambridge University Press, Cambridge, 2009, 810 pp., GBP 45, ISBN 978-0-521-89806-5*

This book provides a thorough and self-contained treatment of analytic combinatorics, centred around the notion of a generating function. In the first part of the book, the authors develop a very general framework for symbolic description of classes of combinatorial structures. This framework allows one to easily express the link between combinatorial classes and their corresponding generating functions, both in the unlabelled and labelled setting. The second part of the book deals with applications of complex analysis in combinatorial enumeration, with a main focus on coefficient asymptotics established by singularity analysis and contour integration. In the third part of the book, the authors explain how to apply analysis of multivariate generating functions in the study of limit laws for distributions of various parameters in random combinatorial structures. The presentation of all these topics is very well organised, starting from basic notions and proceeding towards the most advanced recent developments of the theory. In the appendix, the authors give an overview of basic probability and analysis, as well as other necessary preliminary concepts. The book provides an ample amount of examples and illustrations, as well as a comprehensive bibliography. It is valuable both as a reference work for researchers working in the field and as an accessible introduction suitable for students at an advanced graduate level. (vjel)

M. Gardner: Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi*, Cambridge University Press, Cambridge, 2008, 193 pp., GBP 8.99, ISBN 978-0-521-73525-4*

The name of the author, Martin Gardner, is well-known throughout the world. He has produced more than 60 books and many of them are still in print. The first edition of the present book was published in 1959. Since it is long time ago, the chapters in the new edition are complemented with afterwords, addendums, postscripts and numerous bibliographies. Of course, the solutions of the posed problems are also included. The contents of the book can be characterised as "recreational mathematics". The author explains that he is not a creative mathematician but a journalist who loves math and who enjoys writing about what the real mathematicians discover. And we see that his writing is very successful. The book is divided into short chapters with very different topics. A few of them are mentioned in the title of the book. For example, hexaflexagons are a special class of flexagons which are objects created by folding strips of paper in various ways. Probability paradoxes contain, among others, the birthday paradox and some paradoxes arising from the (mis)use of conditional probability. Further problems concern, for example, the Moebius band, mathematical card tricks, memorising numbers and fallacies. The book demonstrates principles of logic, probability, geometry and other fields of mathematics. I believe that many readers will enjoy the book with great pleasure. (ja)

J. B. Garnett, D. E. Marshall: Harmonic Measure*, New Mathematical Monographs 2, Cambridge University Press, Cambridge, 2008, 571 pp., GBP 30, ISBN 978-0-521-72060-1*

When solving the Laplace equation in a domain, we can look for a harmonic function with prescribed continuous boundary data (at least in a generalised sense). This is the Dirichlet problem. Through the Riesz representation theorem, we can identify the functional that evaluates the solution at a given point with the so-called harmonic measure. This construction works in an arbitrary space dimension. However, the planar case differs in many aspects and leads to a separate rapidly developing theory. This volume builds the planar theory with the aim of reaching several distinguished results with an emphasis on recent achievements. The first four chapters contain the foundations of the subject; the themes include conformal mappings, hyperbolic metrics, potential theory and extremal distance. Chapter V presents Teichmüller's Modulsatz and some results on angular derivatives. Chapters VI and VIII are devoted to the relation between the harmonic measure and the Hausdorff measure (Makarov's theorem) and to Brennan's conjecture about Sobolev regularity of conformal mappings. The topics of chapter VII are conformal images of the unit disc (the relation of their geometry to the properties of the conformal mapping), Bloch functions, Muckenhoupt's weights and quasicircles. Chapter VIII studies domains with an infinite number of components. In chapter IX, a discussion involving the Lusin area function, the Schwarzian derivative and the Jones square is applied to obtain deep results on univalent sums. A nontrivial amount of material, which is regarded as preliminary with respect to the topic of the book, is summarised in the appendices. Each chapter is equipped with numerous exercises. The book can be warmly recommended to students and researchers with a deep interest in analysis. It is an excellent preparation for serious work in complex analysis or potential theory. (jama)

H. Geiges: An Introduction to Contact Topology*, Cambridge Studies in Advanced Mathematics 109, Cambridge University Press, Cambridge, 2008, 440 pp., GBP 45, ISBN 978-0-521-86585-2*

This is a fundamental monograph on the topology of contact structures and its applications. It presents almost all the important results in the field and it simultaneously introduces the reader to contemporary research. There is also a lot about geometry of contact structures. Chapter 1 has an introductory character and represents an invitation to the subject. Chapter 2 is a basis for further reading, containing the Gray stability theorem with the Moser trick and the contact disc theorem. Chapter 3 is devoted to knots in contact three manifolds and surgical constructions of contact manifolds appears here for the first time. Chapter 4 deals with 3-manifolds. It includes the proof of existence of a contact structure on any 3-manifold, the Lutz twist – a topologically trivial Dehn surgery, the Eliashberg classification of overtwisted contact structures, the proof of the Cerf theorem $\Gamma_4 = 0$ and an introduction to convex surface theory in contact 3-manifolds. The remaining four chapters study higher dimensional contact manifolds. The main aim of chapters 5 and 6 is to introduce contact surgery. These results are then applied in chapter 8 to contact 5-manifolds. The final chapter brings together various topological constructions useful in a study of contact manifolds. The book is very well-written. The author mentions that the intended reader is an advanced graduate student. But even a less advanced graduate student, who is really interested, will be attracted by this book. The author has not included exercises in the book but on the other hand there are a lot of examples. There is a list of 246 references, a notation index, an author index and a subject index. All of them are quite helpful. The book can be strongly recommended for graduate students and is indispensable for specialists in the field. (jiva)

L. J. Gerstein: Basic Quadratic Forms*, Graduate Studies in Mathematics, vol. 90, American Mathematical Society, Providence, 2008, 255 pp., USD 55, ISBN 978-0-8218-4465-6*

This book contains an introduction to arithmetics of quadratic forms and applications covering fundamentals of the theory of quadratic spaces and quadratic lattices over principal ideal domains. To read the book, only basic courses in abstract algebra are required; the book is essentially self-contained. As such, it can be used as a preparatory text devoted to deeper applications of quadratic forms. The author's intention is also implied in the number of exercises and notices for further reading spread throughout the text. The part devoted to local-global theory starts with an introduction to valuation theory and p-adic numbers and ends with the Hasse-Minkowski theorem. It is followed by global integral theory, local classification of lattices and the local-global approach to global lattices. The final chapter has some applications of quadratic forms in cryptography and the description of the LLL-algorithm for finding short vectors in a lattice. The appendix, containing characterisations of 35 books on quadratic forms published between 1950 and 2007, is unusual but interesting. The book follows its aims in a straightforward manner omitting lengthy discussions. The goal of the author was to write a textbook appropriate for graduate courses or for independent study. This goal was certainly achieved. (špr)

M. Giaquinta, G. Modica: Mathematical Analysis. An Introduction to Functions of Several Variables*, Birkhäuser, Boston, 2009, 348 pp., EUR 127,33, ISBN 978-0-8176-4509-0*

This book presents the main ideas and results on functions of several variables. Both differential and integral calculus are treated here. The reader is motivated with many examples and exercises. In chapter 1, basic notions of differential calculus of functions of several variables are discussed. In section 1.5, extensions of the material to Banach spaces are given. The Lebesgue integral in several variables is presented in chapter 2. The most relevant results are explained, including the Fubini theorem, the area and co-area theorems and the Gauss-Green formulae. In chapter 3, the authors deal with potentials and integration of differential 1-forms focusing on solenoidal and irrotational fields. Chapter 4 is dedicated to a wide introduction to the theory of holomorphic functions of one complex variable. In chapter 5, the notions of immersed and embedded surfaces in R^n are discussed. The implicit function theorem is given, accompanied by the most important applications. Chapter 6 is devoted to the stability theory of non-linear systems and the Poincaré-Bendixon theorem. This material is discussed in order to show that dynamical systems with one degree of freedom do not present chaos, in contrast to one-dimensional discrete dynamics and higher-dimensional continuous dynamics. The book may be used for advanced undergraduate and graduate students or as a self-study guide. In my opinion, it is one of the best books in the field. This is due, among others reasons, to the considerable pedagogical experience of both authors. (oj)

T. Gowers, J. Barrow-Green, I. Leader, Eds.: The Princeton Companion to Mathematics*, Princeton University Press, Princeton, 2008, 1034 pp., USD 99, ISBN 978-0-691-11880-2*

This book is a unique reference for anyone with a serious interest in mathematics. It is edited by Timothy Gowers, a recipient of the Fields Medal, and it includes entries written by several of the world's leading mathematicians. The entries have many goals, such as to introduce basic mathematical tools and vocabulary, to trace the development of modern mathematics, to explain essential terms and concepts, to describe the achievements of scores of famous mathematicians and to explore the impact of mathematics on other disciplines such as biology, finance and music. The book is divided into eight chapters. The introduction presents basic mathematical language and some fundamental mathematical definitions. The origins of modern mathematics are described in the second chapter. It starts with numbers, geometry and the development of abstract algebra and through algorithms and proofs in mathematics gets to the crisis in the foundations of mathematics. The next chapter is devoted to important mathematical concepts presented in alphabetical order. Thus, one can begin reading with an axiom of choice and end with the Zermelo–Fraenkel axioms. Then the book turns its attention to mathematical branches. From algebraic geometry to stochastic processes, brief descriptions of the main areas of mathematics are provided. Crucial theorems and fundamental open problems occupy the fifth chapter. Then the book presents some achievements of famous mathematicians, starting with Pythagoras and ending with Nicolas Bourbaki. After this, several examples of the influence of mathematics on other branches of science are presented. The companion ends with general thoughts on mathematics and mathematicians. The book contains some valuable surveys of the main branches of mathematics that are written in an accessible style. Hence, it is recommended both to students of mathematics and researchers seeking to understand areas outside their specialties. (jspr)

D. Haskell, E. Hrushovski, D. Macpherson: Stable Domination and Independence in Algebraically Closed Valued Fields*, Lecture Notes in Logic 30, Cambridge University Press, Cambridge, 2008, 182 pp., GBP 35, ISBN 978-0-521-88981-0*

Motivated by some problems of theories of valued fields and algebraically closed valued fields (ACVF), the book presents a model-theory approach combining stability-theory ideas and the o-minimal context because "type spaces work best in stable theories and definable sets and maps in o-minimal theories". The abstract section, developed in Part I, is applied in Part II (in particular to ACVF). In Part I, an extension of stability theory is introduced. Theories that have a stable part are considered and the crucial notion of a stable dominated type is defined. Such a type can be controlled by a very small part, lying in the stable part, analogically to how a power series is controlled with respect to the problem of invertibility, e.g. by its constant coefficients. Moreover, it is shown that there exist o-minimal families of stably dominated types with the property that any type can be seen as a limit of such a family. Furthermore, a notion of metastable theory is defined. In Part II, the theory of ACVF is studied; valued fields are viewed as substructures of models of ACVF. Notions of independence are introduced and applied. For instance, the metastability of ACFV is proved and the stable dominated types are characterised as those invariant types that are orthogonal to the value group. Notice that the key notions and ideas of stability theory are presented in chapter 2 and chapter 7 is an outline of classical results in the model theory of valued fields. The book is comprehensive and stimulating. (jmlc)

P. Henry-Labordere: Analysis, Geometry, and Modeling in Finance. Advanced Methods in Option Pricing*, Chapman & Hall/CRC Financial Mathematics Biology Series, CRC Press, Boca Raton, 2008, 383 pp., USD 79.95, ISBN 978-1-4200-8699-7*
This is an extraordinary monograph, one of the few not to be missed by anybody deeply interested in stochastic financial modelling. It demonstrates in a rather striking manner how concepts and techniques of modern theoretical physics (such as non-Euclidean geometry, supersymmetric quantum mechanics, path integrals and functional derivatives) may be applied to mathematical finance and option pricing theory. Some of the techniques mentioned above have already been known in the financial literature separately under different names. However, it also presents original ideas never before published by researchers in finance. The monograph builds an original bridge to connect analysis, geometry and probability together with stochastic finance, a bridge supported both by very advanced mathematics and imagination. *Mathematica* and C_{++} are used for numerical implementation and many end-of-chapter problems lead the reader to recently published papers. (jstep)

W. Imrich, S. Klavžar, D. F. Rall: Topics in Graph Theory. Graphs and Their Cartesian Product*, A. K. Peters, Wellesley, 2008, 205 pp., USD 75.00, ISBN 978-1-56881-429-2*
This book is focused on the Cartesian product of graphs. This graph operation stems from algebraic and categorical concepts and the prime and simplest example is the hypercube – the Cartesian power of a single edge. With the wealth of well-known results on hypercubes (including error-correcting codes in the Hamming metrics), it is worthwhile to explore the realm of products of graphs. Many classical topics are discussed with respect to the Cartesian product. The first section gives basics about the Cartesian product. The next two sections deal with graph properties like independence, chromatic numbers and crossing numbers. The fourth section discusses distance in the Cartesian product and the last section shows some connections with algebra and algorithms. Reading the book requires only a basic knowledge of graphs. The book is suitable for advanced undergraduates or beginning graduate students interested in graph theory. Every chapter is concluded with many exercises. Hints and solutions to the exercises are provided at the end of the book. The manuscript of the book was used during courses at the University of Maribor, the Mountanuniversität Loeben and Furman University. The book could be used as a textbook for an advanced graph theory course. (bl)

I. M. Isaacs: Algebra. A Graduate Course*, Graduate Studies in Mathematics, vol. 100, American Mathematical Society, Providence, 2009, 516 pp., USD 63. ISBN 978-0-8218-4799-2*
This textbook introduces the reader to methods of modern algebra. It is based on a graduate algebra course but the author‘s aim is more ambitious – the book attempts to present both results and methods of abstract algebra as an exciting and beautiful part of modern mathematics. The fact that this edition is reprinted by the American Mathematical Society from the original that appeared in 1994, which became one of the most popular textbooks on algebra, reflects the fact that the author‘s aim has met with obvious success. The book is divided into two parts. The first of them is devoted to noncommutative algebra (following the author‘s professional interests and his conception of organisation of algebraic topics). The first ten chapters, almost the whole first third of the book, build group theory. The rest of the noncommutative part contains an introduction to the theory of modules and non-commutative rings. Elements of character theory, which forms a natural link between module theory and the theory of groups, are presented in the last chapter of the first part of the book. The commutative algebra part opens with a chapter devoted to polynomial rings, principal ideal rings and unique factorisation domains and is followed by a nine-chapter section covering field theory. The last five chapters present classical topics from commutative algebra and algebraic geometry, including primary decompositions of commutative noetherian rings, Dedekind domains and the Nullstellensatz.

The book covers almost all standard algebraic topics (except homological and categorical algebra) and, moreover, it includes several advanced parts of group theory (e.g. transfer theory). Note that the textbook is almost self-contained since a reader only needs an elementary algebraic background (elements of linear algebra and the basic concept of mathematical structures). Last but not least, a feature of the book that should be mentioned is the number of carefully chosen problems, which are listed at the end of every chapter. The textbook is excellent and an unusually successful combination of the best of traditional introduction to algebra, for which accuracy and correctness are absolutely essential, and a modern text that can be recommended to graduate students of algebra and all those whose interests lie in pure mathematics as well as everyone who wishes or needs to become familiar with the beauty of modern algebra. (jž)

V. Kanovei: Borel Equivalence Relations. Structure and Classification*, University Lecture Series, vol. 44, American Mathematical Society, Providence, 2008, 240 pp., USD 45, ISBN 978-0-8218-4453-3*
This book starts by recalling some facts from descriptive set theory. After a chapter on Borel ideals and an introduction to equivalence relations, the author presents material on countable equivalence relations, hyperfinite equivalence relations, dichotomy theorems, actions of the infinite symmetric group and turbulent group actions. Besides other topics, the author further discusses c_0-equalities, pinned equivalence relations and reduction of Borel equivalence relations to Borel ideals. The book ends with an appendix on Cohen and Gandy–Harrington forcing. Methods of forcing are used in many places as well as methods of effective descriptive set theory. Many recent

results and open problems are included. This is a very interesting book covering many topics of the theory of Borel equivalence relations. (psim)

K. Kato, S. Usui: Classifying Spaces of Degenerating Polarized Hodge Structures*, Annals of Mathematics Studies, no. 169, Princeton University Press, Princeton, 2008, 336 pp., USD 55, ISBN 978-0-691-13821-3*

When P. A. Griffiths (Amer. J. Math., 90 (1968), 568–626) introduced the classifying space D of polarised Hodge structures, he was already aware of the fact that it was quite desirable to add to this space points at infinity. This book is a realisation of Griffiths' idea. Technically, it is not at all easy and the authors describe a whole series of enlargements of the classifying space. The reader is advised to look at the fundamental diagram, where they can find the enlargements and mappings. To accomplish the enlargement idea, Kato and Usui develop a logarithmic Hodge theory. Here they use the logarithmic structures created by J. M. Fontaine and L. Illusie. They divide the exposition into two topics. The first one consists of "Toroidal partial compactifications and moduli of polarised logarithmic Hodge structures"; the second one is "The eight enlargements of D and the fundamentals diagram". The book is a highly specialised monograph, which will be appreciated first of all by specialists in the field. But it is well and carefully written and even a beginner can learn a lot from it. It is recommended for the beginner to skip the overview on a first reading and start directly with chapter 1. At the end, there is list of symbols and an index. Both of them are very good and help substantially with orientation in the book. It is a nice book and can be strongly recommended. (jiva)

A. Katok, V. Climenhaga: Lectures on Surfaces. (Almost) Everything You Wanted to Know about Them*, Student Mathematical Library, vol. 46, American Mathematical Society, Providence, 2008, 286 pp., USD 49, ISBN 978-0-8218-4679-7*

This book is an expanded version of a series of lectures given by one of the authors for a special semester-long program for advanced undergraduates. It provides an introduction to the geometry of surfaces, with a recurring theme of Euler characteristic as a guiding principle. After a warm-up chapter on various ways of constructing a surface, the authors set up for a series of journeys always ending with yet another manifestation of this invariant, including: triangulations on spaces, Betti numbers, Morse functions, the index of a vector field and the Gauss-Bonnet Theorem. The style of the book puts emphasis on motivation rather than clear-cut division into definitions, theorems and proofs. There are a lot of pictures and exercises with hints provided for most of them. The book can be recommended to anyone who wants to get acquainted with the most important ideas of geometry in a quick and elegant way. (dš)

K. Kim, N. Timm: Univariate and Multivariate General Linear Models. Theory and Applications with SAS*, second edition + CD-Rom, Statistics: A Series of Textbooks and Monographs, Chapman & Hall/CRC, Boca Raton, 2006, 549 pp., USD 79.95, ISBN 978-1-58488-634-1*

This book provides a comprehensive overview of many variants of the general linear model, accompanied by examples analysed in SAS. The most significant innovation since the first edition is the relegation of the SAS code and data files to the enclosed CD and the addition of new material. The scope of the book is remarkably wide. The first chapter introduces the general linear model, summarises some basic facts on the multivariate normal distribution, and discusses the assessment of univariate and multivariate normality. Chapter 2 covers unrestricted general linear models, including multiple regression and one-way analysis of variance. Chapter 3 reviews restricted models and applies them to two-way factorial designs, Latin square designs, repeated measures design and the analysis of covariance. Weighted general linear models are introduced in chapter 4 and are applied to the analysis of categorical data and models with heteroscedastic errors. Chapter 5 discusses multivariate general linear models: multivariate regression, multivariate mixed models, and MANOVA and MANCOVA designs. In the next two chapters, the multivariate model is extended to a doubly multivariate model (chapter 6) and a multivariate model with restrictions, with applications to growth curves (chapter 7). Chapter 8 deals with the "seemingly unrelated regression" (SUR) model and the restricted GMANOVA model. Chapters 9 and 10 (new since the first edition) cover simultaneous inference using finite intersection tests and power calculations, respectively. Two-level hierarchical linear models are treated in chapter 11. The last two chapters are devoted to incomplete repeated measurements and structural equation modelling. The authors note that the book "is written for advanced graduate students in the social and behavioral sciences and in applied statistics" and that it is suitable for a one-semester course on linear models. Each chapter is accompanied by numerical examples, discussions of SAS code and interpretations of SAS output. Additional material is available on the authors' webpage. (mkul)

A. Kirillov, Jr.: An Introduction to Lie Groups and Lie Algebras*, Cambridge Studies in Advanced Mathematics 113, Cambridge University Press, Cambridge, 2008, 222 pp., GBP 40, ISBN 978-0-521-88969-8*

The theory of Lie groups and Lie algebras has become a very powerful tool and has found many applications in various branches of mathematics and physics. This book is intended as material for a graduate course on the subject. The author's intention is to keep the book as readable as possible and to avoid the technicalities needed in some proofs. Hence some proofs are only outlined and the details transferred to exercises, and others (e.g. the proofs of the Poincare-Birkhoff-Witt and Engels theorems) are omitted completely. The author instead gives some nice applications of the theory, e.g. a relation between the spectrum of the Laplace operator on the sphere S^2 and representations of $SO(3,R)$.

There are many exercises in the book. The reader is supposed to have some background in differential geometry, algebraic topology and some proofs even require some knowledge of homological algebra. The book starts with basics from the theory of Lie groups and Lie algebras. It contains a description of the usual basic parts of the theory (structure theory of Lie algebras, complex semisimple Lie algebras and their root systems, representations of semisimple Lie algebras, including highest weight representations, Verma modules, Harish-Chadra isomorphism and Bernstein-Gelfand-Gelfand resolution). The last appendix contains a useful detailed sample syllabus for a one-semester graduate course (two lectures a week). (tsal)

***P. Kronheimer, T. Mrowka: Monopoles and Three-Manifolds**, New Mathematical Monographs 10, Cambridge University Press, Cambridge, 2007, 796 pp., GBP 75, ISBN 978-0-521-88022-0*

The goal of this book is to provide a complete foundation, with detailed proofs, for the Seiberg–Witten or monopole Floer homology. The book explains Floer homology theory of 3-dimensional manifolds based on the Seiberg–Witten equations (monopole equations) instead of Floer's construction, which uses the anti-self-dual Yang–Mills equations (instanton equations). The first half of the book is devoted to a detailed study of moduli space (as a Hilbert manifold) and to a definition of monopole Floer homology as Morse theory of the Chern-Simons-Dirac functional. The authors develop analytic properties of the Seiberg–Witten equations. The Floer groups are defined here for any compact, connected, oriented 3-manifold, which is remarkable in a theory known for requiring special conditions to achieve applicability. The next chapter is dedicated to cobordism/topological invariance, exploring implications of the circle-valued Morse theory. Two final chapters are devoted to a calculation of Floer groups and to applications of the theory in topology. The book will be more understandable to readers with experience of gauge theory, Hilbert manifolds and slice theorems. It should be of interest to any mathematician faced with an infinite-dimensional moduli space of some sort. (lkř)

***D. Leemans: Residually Weakly Primitive and Locally Two-Transitive Geometries for Sporadic Groups**, Mémoires de la Classe des Sciences, vol. 11, Académie royale de Belgique, Bruxelles, 2008, 173 pp., EUR 25, ISBN 978-2-8031-0253-2*

This book is a sequel to the *Atlas of residually weakly primitive geometries for small groups* by F. Buekenhout, M. Dehon and D. Leemans, published in 1999. The work deals with group theory, incidence geometries and computational algebra developed to better understand the structure of sporadic groups. The author describes several Magma algorithms that he has applied to classify the geometries in the title for the five Mathieu groups, the first three Janko groups and the Higman-Sims group. Then he proceeds to present some results obtained by studying the list of geometries obtained with the help of these programs. There is also a section presenting full subgroup lattices of the Mathieu groups M22 and M23 and the Janko groups J2 and J3. The book will be of great value for anyone studying finite incidence geometries and the structure of sporadic groups. (jtu)

***J. McKee, C. Smyth: Number Theory and Polynomials**, London Mathematical Society Lecture Note Series 352, Cambridge University Press, Cambridge, 2008, 349 pp., GBP 38, ISBN 978-0-521-71467-9*

This proceedings of a conference devoted to number theory aspects of the theory of polynomials contains 19 papers written by leading specialists in the field. The range of the papers is correspondingly broad and stretches from classical topics (such as Mahler's measure through primitive divisors, greatest common divisors and smooth divisors) to properties of special polynomials. Besides that, we can also find here papers devoted to some more specialised facets of the theory of polynomials. For instance, a paper giving an introduction to Belyi-maps and Grothendieck's dessins, to the Hansen-Mullen primitivity conjecture and a survey of results on algebraic numbers having all Galois conjugates on a conic, etc. The collection is certainly interesting not only for those interested in some selected topic but also for those who like to browse the papers with the aim of extending their knowledge. (šp)

***H. Nagamochi, T. Ibaraki: Algorithmic Aspects of Graph Connectivity**, Encyclopedia of Mathematics and Its Applications 123, Cambridge University Press, Cambridge, 2008, 375 pp., GBP 50, ISBN 978-0-521-87864-7*

This book is devoted to graph connectivity, an important notion in graph theory with numerous practical applications. It gives a rich theoretical background; however, a strong emphasis is put on algorithms and their effectiveness. After the introductory chapter, the next chapters deal with maximum adjacency ordering and its applications (fast maximum flow algorithms or chordality testing; minimum cuts; cut enumeration; cactus representations; extreme vertex sets; edge splitting; connectivity augmentation; source location problems; and submodular and posimodular functions). In each chapter, the authors present a wide overview of algorithms, including the classical ones as well as recent effective methods. The book is comprehensive and detailed. A general insight in the studied topics is useful to the reader, although the first chapter of the book contains a brief introduction and establishes all the used notions. The book can be useful as a reference book for a specialist whose work relates to graph theory but it can also be used as a textbook for advanced courses in discrete mathematics, graph-theory algorithms and optimisation. (ejel)

***S. Repin: A Posteriori Estimates for Partial Differential Equations**, Radon Series on Computational and Applied Mathematics, vol. 4, Walter de Gruyter, Berlin, 2008, 316 pp., EUR 64, ISBN 978-3-11-019153-0*

A posteriori error estimates have attracted much attention of numerical analysts in the last few decades because they allow reliable verification of numerically obtained approximate solutions. This is possible since these estimates enable one to find lower and upper bounds of the distance of a given approximate solution of the studied partial differential equation and the unknown exact solution. These bounds are constructed directly for the given approximated solution. The book starts with a general introduction to the theory of error control. It continues with an overview of classical a posteriori estimate methods developed in the 20th century. The main part of the book deals with new functional a posteriori estimates. The author explains the method on the Poisson equation with Dirichlet boundary conditions. Then he also applies it to the Poisson equation with different boundary conditions, problems arising in linear plasticity and in the theory of viscous fluids, variational inequalities and some other problems. The book contains not only estimates for picked problems but attention is also focused on the methods of how these estimates are derived. This is important, since understanding the method allows one to modify it for different problems of interest. The text requires a moderate background in functional analysis and the theory of partial differential equations. It will be useful for experts in computational mathematics, as well as for students interested in applied mathematics. (pkap)

E. T. Sawyer: Function Theory – Interpolation and Corona Problems*, Fields Institute Monographs, vol. 25, American Mathematical Society, Providence, 2009, 203 pp., USD 65, ISBN 978-0-8218-4734-3*

These lecture notes start with L. Carleson's deep results on interpolation and corona problems in the unit disk and continues with modern analogues in the disk and ball. The reader learns several techniques that provide different proofs of the corona problem. These techniques from classical analysis and operator theory include duality, the Blaschke product, Hilbert space arguments, BMO, best approximations, the Beurling transform, use of trees, the complete Pick property and the Toeplitz corona theorem. The reader is assumed to know basic real and complex analysis and also the theory of the Poisson integral in the unit disk. The book contains a nice and detailed appendix on background material in functional analysis, Sobolev spaces and function theory on the disk. (sh)

A. H. Schoenfeld: Assessing Mathematical Proficiency*, Mathematical Sciences Research Institute Publications 53, Cambridge University Press, Cambridge, 2007, 391 pp., GBP 17.99, ISBN 978-0-521-69766-8*

This book is the product of the 2004 MSRI (Mathematical Sciences Research Institute, Berkeley) conference "Assessing Students' Mathematics Learning: Issues, Costs and Benefits". The conference articulated different purposes of assessment of student performance in mathematics. The book focuses on ethical issues related to assessment, including how assessment interacts with concerns for equity, sensitivity to culture and the severe pressures on urban and high-poverty schools. The book introduces different frameworks, tools and methods for assessment, comparing the kinds of information they offer about a student's mathematical proficiency. The book describes complexities of assessment when English is not a student's native language. If a student is not fluent in English, is their failure to solve a problem a result of not understanding the problem or of not understanding the mathematics? The book highlights the kinds of information that different assessments can offer, including many examples of some of the best mathematics assessments worldwide. (oo)

A. Schürmann: Computational Geometry of Positive Definite Quadratic Forms. Polyhedral Reduction Theories, Algorithms, and Applications*, University Lecture Series, vol. 48, American Mathematical Society, Providence, 2008, 162 pp., USD 39, ISBN 978-0-8218-4735-0*

The aim of this book is to give an introduction to computational geometry of a positive definite quadratic subject. The book can be considered as self-contained, describing on one side classical aspects of the theory (as developed by Minkowski, Voronoi and Delone), while on the other side keeping in touch with modern applications (such as lattice sphere packing and coverings). In the book, new proofs of known results are given in many places. A very good and readable presentation is combined with explicitly formulated algorithms that allow computer assisted experimentations. The book starts with an introduction containing basics of classical theory (the first chapter) and Minkowski's reduction theory (the second chapter). The third and forth chapters are devoted to Voronoi's first and second reduction theory. The fifth chapter "Local analysis of coverings and applications" is devoted to sphere covering and sphere packing-covering problems. The book is written in a lucid style and can be recommended to everybody interested in the theory of quadratic forms and its applications in lattice sphere packing and coverings. (šp)

A. H. W. Schmitt: Geometric Invariant Theory and Decorated Principal Bundles*, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008, 389 pp., EUR 48, ISBN 978-3-03719-065-4*

The first chapter is an introduction to geometric invariant theory, as developed by D. Mumford. Fundamental results by Hilbert and Mumford are explained here, together with more recent topics, such as the instability flag, the finiteness of the number of quotients and the variation of quotients. In the second chapter, geometric invariant theory is applied to solve the classification problem of decorated principal bundles on a compact Riemann surface. The solution is a quasi-projective moduli scheme, which parameterizes those objects that satisfy a semistability condition originating from gauge theory. The moduli space is equipped with a generalised Hitchin map. Via the universal Hitchin-Kobayashi correspondence, these moduli spaces are related to moduli spaces of solutions of certain vortex type equations. Possible applications include a study of representation spaces of the fundamental group of compact Riemann surfaces. The book concludes with a brief discussion of generalisations of these results to higher dimensional base varieties, positive characteristics and parabolic bundles. (lkř)

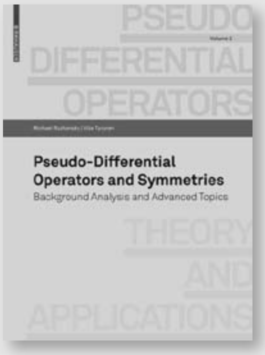
P. Seidel: Fukaya Categories and Picard-Lefschetz Theory*, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008, 326 pp., EUR 46, ISBN 978-3-03719-063-0*

The main topic of this book is a construction of a Fukaya category, an object capturing information on Lagrangian submanifolds of a given symplectic manifold. Fukaya categories are of interest due to the recent formulation of homological mirror symmetry. The first part of the book is a self-contained exposition of A_∞-categories and the underlying homological and homotopical algebra. In the second part, the actual construction of a Fukaya category is presented. The author first presents the main ideas by giving a preliminary construction and then he proceeds in greater generality, though the complete generality already present in recent literature is not reached. The last part treats Lefschetz fibrations and their Fukaya categories and briefly illustrates the theory on the example of A_m-type Milnor fibres. The book is written in an austere style and references for more detailed literature are given whenever needed. The reader is expected to have a certain background in symplectic geometry. (md)

BIRKHÄUSER

Pseudo-Differential Operators

Pseudo-Differential Operators: Theory and Applications is a series of moderately priced graduate-level textbooks and monographs appealing to students and experts alike. Pseudo-differential operators are understood in a very broad sense and include such topics as harmonic analysis, PDE, geometry, mathematical physics, microlocal analysis, time-frequency analysis, imaging and computations. Modern trends and novel applications in mathematics, natural sciences, medicine, scientific computing, and engineering are highlighted.



Pseudo-Differential Operators and Symmetries

Background Analysis and Advanced Topics

Ruzhansky, M., Imperial College, London, UK / **Turunen, V.**, Helsinki University of Technology, Finland

2009. Approx. 720 p. Softcover
EUR 59.95 / CHF 105.00
ISBN 978-3-7643-8513-2
Pseudo-Differential Operators, Vol. 2

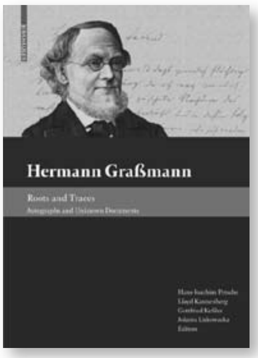
This monograph develops a global quantization theory of pseudo-differential operators on compact Lie groups. Traditionally, the theory of pseudo-differential operators was introduced in the Euclidean setting with the aim of tackling a number of important problems in analysis and in the theory of partial differential equations. This also yields a local theory of pseudo-differential operators on manifolds. The present book takes a different approach by using global symmetries of the space which are often available. First, a particular attention is paid to the theory of periodic operators, which are realized in the form of pseudo-differential and Fourier integral operators on the torus. Then, the cases of the unitary group SU(2) and the 3-sphere are analyzed in extensive detail. Finally, the monograph also develops elements of the theory of pseudo-differential operators on general compact Lie groups and homogeneous spaces.

Quantization and Arithmetic

Unterberger, A., Université de Reims, France

2008. 147 p. Softcover
EUR 39.90 / CHF 69.90
ISBN 978-3-7643-8790-7
Pseudo-Differential Operators,

The primary aim of this book is to create situations in which the zeta function, or other L-functions, will appear in spectral-theoretic questions. A secondary aim is to connect pseudo-differential analysis, or quantization theory, to analytic number theory. Both are attained through the analysis of operators on functions on the line by means of their diagonal matrix elements against families of arithmetic coherent states: these are families of discretely supported measures on the line, transforming in specific ways under the part of the metaplectic representation or, more generally, representations from the discrete series of SL(2,R), lying above an arithmetic group such as SL(2,Z).



Hermann Graßmann – Roots and Traces

Autographs and Unknown Documents

Petsche, H.-J., Potsdam, Germany / **Kannenberg, L.**, Weston, MA, USA / **Keßler, G.**, Potsdam, Germany / **Liskowacka, J.**, Szczecin, Poland (Eds)

2009. XII, 256 p.
EUR 49.95 / CHF 79.90
ISBN 978-3-0346-0154-2

Hermann Günther Graßmann was one of the 19th century's most remarkable scientists, but many aspects of his life have remained in the dark. This book assembles essential, first-hand information on the Graßmann family. It sheds light on the family's struggle for scientific knowledge, progress and education. It puts a face on the protagonists of an exciting development in the history of science. And it highlights the peculiar set of influences which led Hermann Graßmann to brilliant insights in mathematics, philology and physics. This book of sources is meant to complement the biography of Graßmann and the proceedings of the 2009 Graßmann Bicentennial Conference (Birkhäuser 2010). ‚Roots and Traces' will interest all scholars working on Hermann Graßmann and related topics. It offers newly discovered pictures of family members, historical texts documenting life in this exceptional family and an English translation of these previously unpublished papers. Text in German and English.

Hermann Graßmann
Biography
Petsche, H.-J., Universität Potsdam, Germany
2009. XX, 306 p. Hardcover
EUR 69.95 / CHF 125.00
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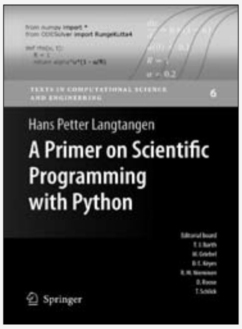

The book serves as a first introduction to computer programming of scientific applications, using the high-level Python language. The exposition is example- and problem-oriented, where the applications are taken from mathematics, numerical calculus, statistics, physics, biology, and finance. The book teaches "Matlab-style" and procedural programming as well as object-oriented programming. Besides learning how to program computers, the reader will also learn how to solve mathematical problems, arising in various branches of science and engineering, with the aid of numerical methods and programming.

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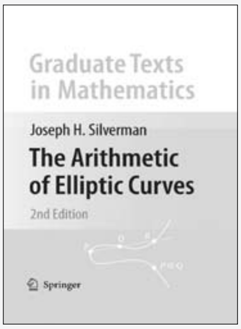

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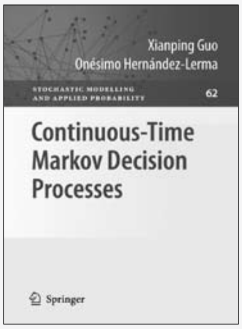

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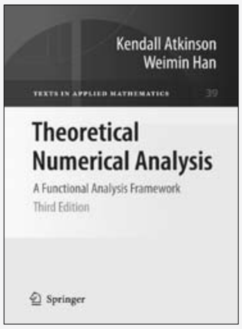

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