# NEWSLETTER

## OF THE EUROPEAN MATHEMATICAL SOCIETY

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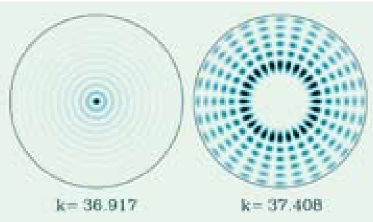


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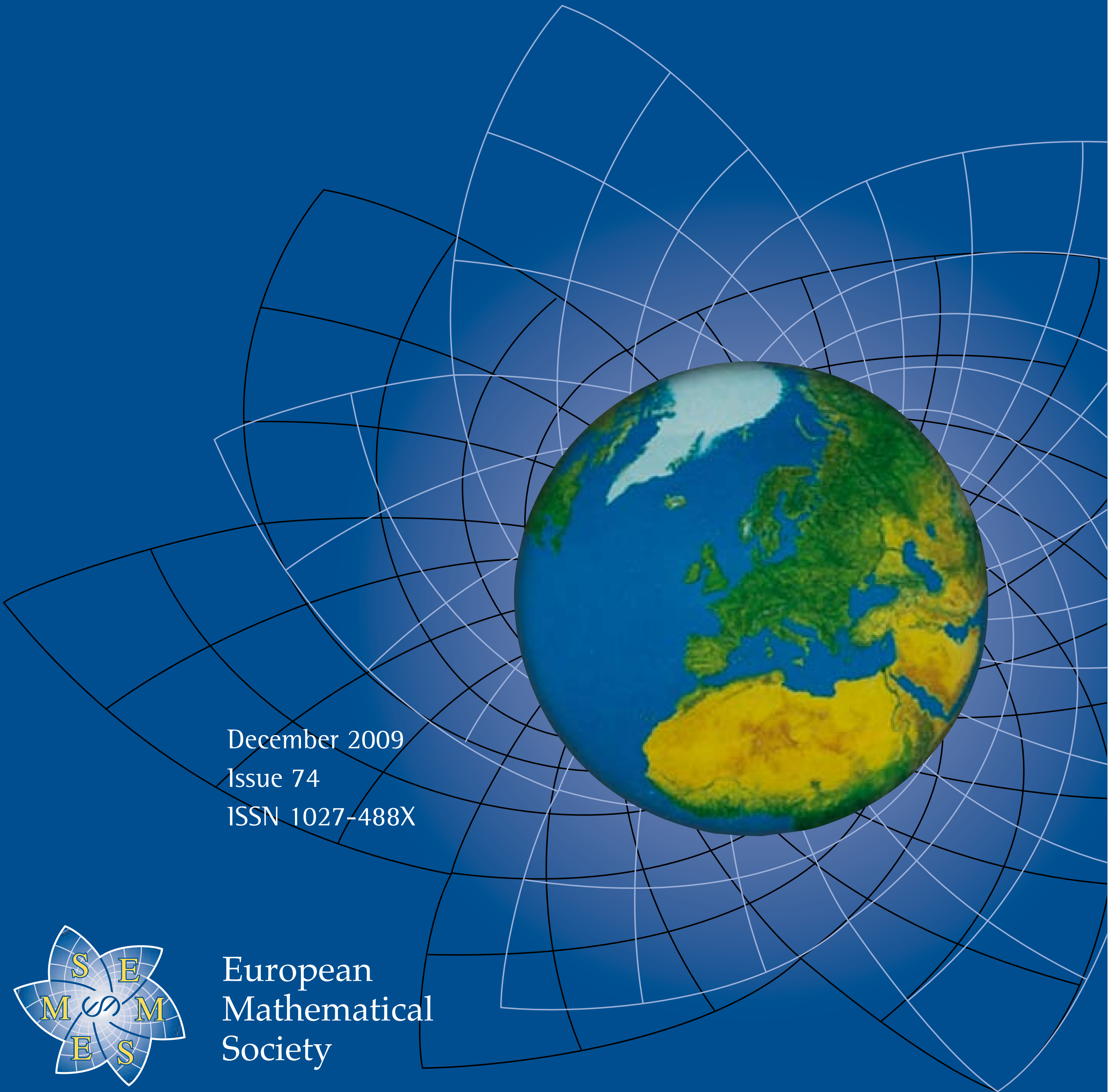

December 2009
Issue 74
ISSN 1027-488X

European
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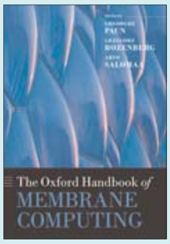


### The Oxford Handbook of Membrane Computing

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Provides both a comprehensive survey of available knowledge and established research topics, and a guide to recent developments in the field, covering the subject from theory to applications.

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**NEW !**


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Dec. 2009 | 376 pp | Paperback | 978-0-19-857063-9 | ~~£34.95~~
Hardback | 978-0-19-857062-2 | ~~£65.00~~

EMS member price: £27.96 / £52.00

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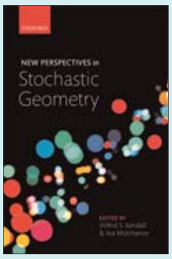


### New Perspectives in Stochastic Geometry

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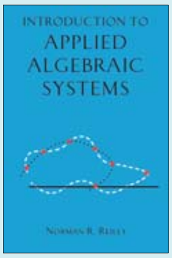


### Introduction to Applied Algebraic Systems

**Norman R Reilly**

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EMS member price: £40.00


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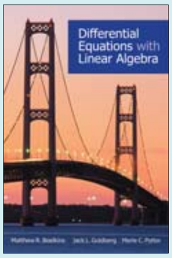
***Oxford Mathematical Monographs***

Oct. 2009 | 432 pp | Hardback |978-0-19-954164-5 | ~~£70.00~~

EMS member price: £56.00




### Differential Equations with Linear Algebra

**Matthew R. Boelkins, Jack L. Goldberg, and Merle C. Potter**

Explores the interplay between linear algebra and differential equations by examining fundamental problems in elementary differential equations.

Oct. 2009 | 464 pp | Hardback | 978-0-19-538586-1 | ~~£52.00~~

EMS member price: £41.60

**FORTHCOMING**

### The Oxford Handbook of Applied Bayesian Analysis

**Edited by Anthony O' Hagan and Mike West**

Explores contemporary Bayesian analysis across a variety of application areas.

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EMS member price: £68.00

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# European Mathematical Society

## Newsletter No. 74, December 2009





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# EMS Agenda

## 2010

**1 February**
Deadline for submission of material for the March issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@mat.ucm.es

**25–28 February**
EUROMATH 2010, Bad Goisern, Austria
http://www.euromath.org

**20–21 March**
EMS EC Meeting (venue to be fixed)
Stephen Huggett: s.huggett@plymouth.ac.uk
Riitta Ulmanen: ems-office@helsinki.fi

**18–19 April**
Meeting of Presidents of European national mathematical societies, Neptun Spa, Romania
Vasile Berinde: vberinde@ubm.ro

**26–27 April**
EMS-ESF Forward Look "Mathematics and Industry"-Consensus Conference (in the framework of the Spanish Presidency of EU), Madrid, Spain

**1 May**
Deadline for submission of material for the June issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@mat.ucm.es

**21–25 June**
SIMAI (Societa Italiana di Matematica Applicata e Industriale) – SEMA (Sociedad Española de Matemática Aplicada) Joint Congress in Applied and Industrial Mathematics, Cagliari, Italy

**2–7 July**
Euroscience Open Forum under the slogan Passion for Science (ESOF2010), Torino, Italy,
http://www.esof2010.org

**10–11 July**
Council Meeting of the European Mathematical Society, Sofia, Bulgaria
Stephen Huggett: s.huggett@plymouth.ac.uk

**11–13 July**
EMS Council Meeting; Conference "Mathematics in Industry"; 20th Anniversary of European Mathematical Society, Sofia, Bulgaria

**26–31 July**
16th Congress of ECMI (European Consortium of Mathematics in Industry), Wuppertal, Germany

**19–27 August**
International Congress of Mathematicians, ICM2010, Hyderabad (India)
http://www.icm2010.org.in

## 2012

**2–7 July**
6th European Mathematical Congress, Kraków (Poland)
http://www.euro-math-soc.eu

# Editorial

The next EMS Council meeting will be organized by the Union of Bulgarian Mathematicians on Saturday–Sunday, 10–11 July 2010 in Sofia in the Metropolitan Hotel http://www.metropolitanhotelsofia.com/en/. The council meeting starts at 13.00 on 10 July and ends at noon on 11 July. Our colleagues in Bulgaria are working hard on the organization of the official part of the meeting and also on the scientific conference on "Mathematics in Industry" that will start on the afternoon of 11 July and end on the evening of 12 July.

This editorial is a presentation of the history of the Union of Bulgarian Mathematicians by its President Stefan Dodunekov.

## Union of Bulgarian Mathematicians

The remarkable history of the Union of Mathematicians in Bulgaria begins at the end of the 19th century with the establishment of the Physical and Mathematical Society (PMS) in Sofia on 14 February 1898 (Gregorian). For more than 110 years this organization fulfilled the mission set by its founders and made a substantial contribution to the development of education and science in Bulgaria.

In the spirit of tradition and the new requirements of social development, the UBM works actively on achieving the objectives enshrined in its statutes. The activities related to promoting the latest scientific achievements in the fields of mathematics and informatics, education and extracurricular work in mathematics and informatics, and the protection of the professional status and interests of teachers, lecturers and researchers in these areas are managed by the following committees:

Committees for Extracurricular Work in Mathematics, Informatics and Mathematical Linguistics.
Committee for Science and Higher Education.
Committee for Secondary Education in Mathematics and Informatics.
Committee on Budget and Business Activities.

We shall briefly review the main activities of the UBM.

### Extracurricular Work in Mathematics, Informatics, Information Technology and Mathematical Linguistics.

The activities in this area are coordinated by the committees for extracurricular work in mathematics, informatics and mathematical linguistics. They are directed towards the scientific, methodological, organizational and financial support of the principal national and regional olympiads and competitions, the training of national teams for Balkan and international olympiads, and workshops for teachers on current problems of extracurricular work. The High School Students' Institute of Mathematics and Informatics (HSSIMI), which was created by the UBM, the Evrika Foundation, the St Cyril and St Methodius International Foundation and the Institute of Mathematics and Informatics at BAS as an initiative to commemorate the year 2000 (declared by UNESCO as World Year of Mathematics), has developed and strengthened its activities in the new millennium. Detailed information about the HSSIMI's history, events, participants, laureates and sponsors can be found at http://www.math.bas.bg/hssi/. One should especially note the active participation in this work of colleagues from the Faculty of Mathematics at the Paisius of Hilendar University in Plovdiv, which led to the establishment of a branch of the HSSIMI in Plovdiv. It is expected that this laudable initiative will be emulated by other university centres – Varna, Burgas, Blagoevgrad and others.

The UBM works successfully with the major foundations that have programmes in fields related to the Union's priorities: St Cyril and St Methodius, Evrika, the American Foundation for Bulgaria and others. Below we shall give some examples illustrating this partnership.

An idea of the scale of the UBM's activities in this direction can be obtained from the list of competitions over the academic year 2009/2010, approved jointly by the EB of the UBM and the Ministry of Education, Youth and Science (MEYS).

#### *Under the auspices of the Ministry of Education and the UBM – 19 events, including:*

Winter contests in mathematics, informatics and mathematical linguistics.
Kangaroo European Mathematical Competition.
Mathematical competition between specialised secondary schools and classes of foreign-language schools, as well as a competition in web-design.
Balkan olympiad in mathematics.
Balkan olympiad in mathematics for junior school students (under 15½ years).
Youth Balkan olympiad in informatics.
International mathematics olympiad.
Balkan olympiad in informatics.
International olympiad in linguistics.
International olympiad in informatics.
National olympiad in mathematics:
municipal, regional and national rounds.
National olympiad in informatics:
municipal, regional and national rounds.
National olympiad in information technologies:
municipal, regional and national rounds.
National olympiad in linguistics:
municipal, regional and national rounds.

#### *Under the auspices of the Regional Education Inspectorates and/or the UBM – 18 events, including:*

Chernorizets Hrabar Mathematics Tournament.
Autumn Mathematics Tournament.
Ivan Salabashev Mathematics Tournament.
Christmas Mathematical Competition.

Hitâr Petâr Mathematical Competition.
Easter Mathematical Competition.
High School Students' Institute of Mathematics and Informatics:
high school students' conference, Plovdiv,
high school students' section at the Spring Conference of the UBM.

The American Foundation for Bulgaria (AFB) develops the programmes *Financing of Extracurricular Activities*, with 20–30 projects every year, and *Scholarships in Secondary Education*, with 50–70 scholarships every year. The projects are evaluated, and applicants for scholarships ranked, by experts from the UBM, and the projects are implemented on site with the active participation of the UBM's sections. Since 2007, the AFB and the UBM have been holding an Autumn Tournament in Mathematics, already acknowledged as one of the most prestigious events of its kind.

A large project of the Evrika Foundation in partnership with the UBM was completed successfully in August 2009. The project *Enhancing the Contribution of Non-Governmental Organizations for a Better Future of the Young Talents in Science and Technology* was funded and implemented under the Operational Programme for Administrative Capacity. Details of the project can be seen at http://www.evrika.org/.

### Science and Higher Education

The Executive Board of UBM and the Committee for Science and Education invest serious effort into intensifying the UBM's activity in this direction.

The UBM's National Colloquium in mathematics is held regularly with the aim of circulating the latest news from the area of mathematics among the Bulgarian mathematical guild through surveys and wide-profile reports by foreign and Bulgarian scientists.

The editorial board of the journal *Mathematica Balkanica* was renewed, and through their efforts it continued being published in this country, now as a journal of the Mathematical Society of South-Eastern Europe (MASSEE). The UBM was entrusted with the organization of MASSEE's First International Mathematical Congress (Borovets, 15–21 September 2003). The UBM also took an active part in the organization and the scientific programme of MASSEE's Second and Third Congresses (Paphos, Cyprus, 31 May–4 June 2006; and Ohrid, FYROM, 16–20 September 2009).

The National Mathematical Olympiad for University Students was established and is held regularly, with the participation of teams from almost all of the universities in the country where mathematics is taught. The university students' olympiad traditionally receives financial support from the Evrika foundation.

The annual Spring Conference of the UBM is the most popular forum in mathematics and informatics in the country. Traditionally, colleagues from abroad also participate. In addition to invited papers and research reports, the scientific programme includes other events – discussions on current scientific and methodological issues, meetings with the editorial boards of journals, etc. The volume of the proceedings of the Spring Conference is a unique collection of new scientific results from different areas of mathematics and informatics and materials related to education and extracurricular work in these sciences.

The UBM has begun issuing a series of scientific monographs in English entitled *Mathematics and Its Applications*. Books of the series *Preparing for Olympiads* and *Kangaroo* are also issued regularly.

The UBM participates as a co-organizer of national and international (thematic and anniversary) conferences in mathematics and informatics in the country.

### Professional Qualification of Teachers of Mathematics and Informatics.

The problems of teaching mathematics, informatics and, as of recently, information technologies are at the focus of the UBM's attention.

A Board of Directors of secondary schools for mathematics and for natural and mathematical sciences, which conducts at least two seminars a year, was formed by the UBM. Its meetings discuss current problems of the legal regulation of secondary education in Bulgaria: the Law on Public Education, final examinations, new curricula, etc. Many of the proposals made by the board have been adopted by the Ministry of Education. The active work of the board has led to the formation of the Association of Secondary Schools for Mathematics and for Natural and Mathematical Sciences as a non-profit legal entity.

The national competitions in mathematics and informatics, as well as the HSSIMI's workshop, proved to be a good opportunity for the country's best teachers to upgrade their skills. The lectures delivered during the competitions and the workshop are received with keen interest. Teachers learn about new state-of-the-art technologies for solving problems, improving their teaching skills and developing the students' creative interests and abilities.

Workshops for advanced training of teachers in mathematics are held at the spring conferences. The teachers who participate in them receive certificates from the Programme Committee and the UBM's Executive Board.

The St Cyril and St Methodius Foundation awards annual prizes to the best Bulgarian teachers of mathematics and informatics. The contest is held by the UBM and the prizes are presented at a ceremony during the Spring Conference of the UBM.

### International Activities

Along with the beneficial international professional contacts of the UBM's members, there are several major events characterising its active international work. The UBM is one of the 27 related organizations that founded the European Mathematical Society (EMS) in Madralin, Poland, on 27 October 1990. Today more than 50 mathematical societies in Europe are partners in the EMS.

Bulgaria is a founding member of the International Mathematical Olympiad and one of three countries that have participated in all its instalments so far.

Bulgaria is the homeland of the International Olympiad in Informatics (IOI). The first IOI was held in 1989 in Pravets, Bulgaria. In recognition of the important role of Bulgarian computer scientists in the consolidation of the IOI and the success of Bulgarian students, Bulgaria was elected, with the aid of the UBM, to host the 21st IOI (Plovdiv, 2009).

Bulgaria also initiated the International Olympiad in Linguistics (IOL) and has been host of two of the seven IOLs so far (the 1st and the 6th, in Borovetz in 2003 and in Sunny Beach in 2008 respectively), on both occasions with the support of the UBM.

Bulgaria is a founding member of:

- The Balkan Mathematical Olympiad, along with 9 other countries from South-Eastern Europe (1984). Four instalments of this Balkan Olympiad have been held in this country (1985 and 1990 in Sofia, 1995 in Plovdiv and 2004 in Pleven).
- The Youth Balkan Mathematical Olympiad (1997). Two instalments of a total of 11 have been held in Bulgaria (1999 in Plovdiv and 2007 in Shumen).

In 2003, MASSEE was established, with the active participation of the UBM, and registered in Athens as a successor to the Balkan Mathematical Union.

The UBM is founder and a member of the European *Kangaroo Without Frontiers* Association, a member of the American Mathematical Society and an associate member of the Canadian Mathematical Society.

A pivotal moment in the UBM's international activity was the implementation of the project's work programme: the initiative *Discovery and Development of Talent – an Important Resource for the Implementation of the EU's Lisbon Strategy*, funded by the Ministry of Foreign Affairs of the Republic of Bulgaria within the Republic of Bulgaria's Communication Strategy for the EU. The aim of this project was to present Bulgarian achievements in identifying students gifted in informatics and mathematics, in developing their talents and attracting them to professional careers in research and innovation. A large team of UBM members worked on carrying out the programme and it was implemented in full and on time.

Here are the main results of the project:

- A website devoted to the Bulgarian system of working with students gifted in mathematics and informatics (in Bulgarian and English) was developed and published on the Internet (http://www.math.bas.bg/talents/).
- A strong impetus was given to the development of international cooperation in working with talented young men and women and the assertion of Bulgaria as one of the European and world leaders in this field. An international conference *Give Talent a Chance* was held, with the participation of professionals from Germany, Italy, Greece, Austria, Romania and the USA. At this conference, representatives of different countries shared their achievements in discovering and developing talents. The Bulgarian system was presented in several reports. As an interesting result of the conference, an agreement was reached for the holding of a peculiar international mathematics competition between teams from Greece, Bulgaria and Romania in Bulgaria in 2008. The aim of the competition was to investigate and demonstrate new approaches to overcoming language barriers between young Europeans by exploring the advantages of mathematics as a universal language.
- The Bulgarian system for working with gifted children was introduced to a group of over 20 experts from Japan (teachers, inspectors and university lecturers), nominated specifically for that purpose by the Japanese Ministry of Education, Culture, Sports, Science and Technology. It emerged that this Ministry has organized an express study of *Scientific and Mathematical Education in Bulgaria* and appointed a special working group.
- The initiative was propagated during the official celebration of the 25th anniversary of the St Cyril and St Methodius International Foundation, held on 20 October 2007 in the National Palace of Culture. The International Conference *Give Talent a Chance* was opened at the end of the celebration in the presence of a wide range of important public figures from Bulgaria and abroad. It should be noted that the conference was partly funded by the Foundation.

In conclusion, we shall list the perspectives that the UBM's Executive Board considers to be the priorities of the organization:

- Expanding the network of sections of the UBM and attracting new members.
- Energetic activities for improving the status of teachers and academic staff.
- Creating new ways to enhance the professional skills of the UBM's members.
- Strengthening the role and place of the UBM in international organizations (EMS, MASSEE, IFIP, ICMI, IMU and so on).
- Deepening the constructive cooperation with the government of Bulgaria and especially with MEYS.
- Effective partnership with non-governmental organizations in Bulgaria for the promotion of education, science and culture.

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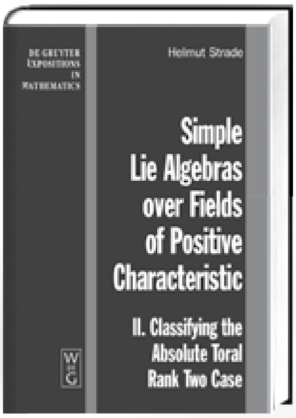
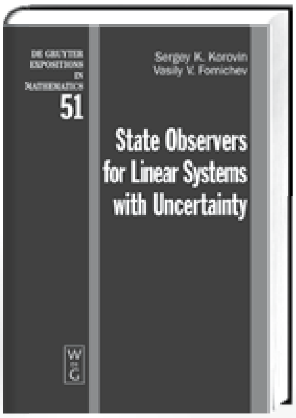
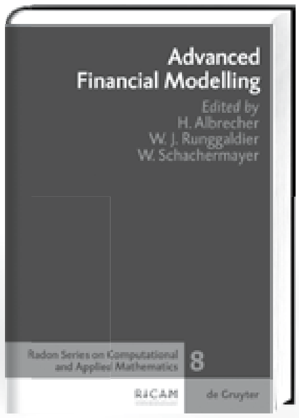
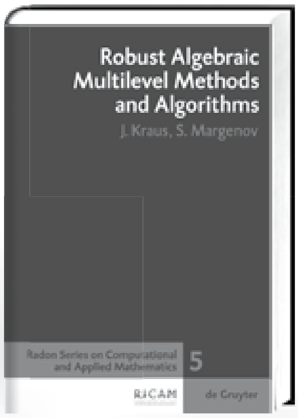
## NEW AT DE GRUYTER




*Helmut Strade*

### SIMPLE LIE ALEGEBRAS OVER FIELDS OF POSITIVE CHARACTERISTIC

II. Classifying the Absolute Toral Rank Two Case

2009. vi, 385 pages. Hardcover.
RRP € [D] 128.00 / *US$ 148.00
ISBN 978-3-11-019701-3
de Gruyter Expositions in Mathematics 42

This is the second volume by the author, presenting the state of the art of the structure and classification of Lie algebras over fields of positive characteristic, an important topic in algebra. The contents is leading to the forefront of current research in this field.

*Sergey K. Korovin / Vasily V. Fomichev*

### STATE OBSERVERS FOR LINEAR SYSTEMS WITH UNCERTAINTY

2009. ix, 242 pages. Hardcover.
RRP € [D] 89.95 / *US$ 126.00
ISBN 978-3-11-021812-1
de Gruyter Expositions in Mathematics 51

This book presents the basic concepts and recent developments of linear control problems with perturbations. The presentation concerns both continuous and discrete dynamical systems. It is self-contained and illustrated by numerous examples.

### ADVANCED FINANCIAL MODELLING

*Ed. by Hansjörg Albrecher / Wolfgang J. Runggaldier / Walter Schachermayer*

2009. viii, 453 pages. Hardcover.
RRP € [D] 109.95 / *US$ 154.00
ISBN 978-3-11-021313-3
Radon Series on Computational and Applied Mathematics 8

This book is a collection of state-of-the-art surveys on various topics in mathematical finance, with an emphasis on recent modelling and computational approaches. The volume is related to a 'Special Semester on Stochastics with Emphasis on Finance' that took place from September to December 2008 at the Johann Radon Institute for Computational and Applied Mathematics of the Austrian Academy of Sciences in Linz, Austria.

*Johannes Kraus / Svetozar Margenov*

### ROBUST ALGEBRAIC MULTILEVEL METHODS AND ALGORITHMS

2009. 24 x 17 cm. x, 246 pages. Hardcover.
RRP € [D] 69.95 / *US$ 89.95
ISBN 978-3-11-019365-7
Radon Series on Computational and Applied Mathematics 5

This book deals with algorithms for the solution of linear systems of algebraic equations with large-scale sparse matrices, with a focus on problems that are obtained after discretization of partial differential equations using finite element methods. The authors provide a systematic presentation of the recent advances in robust algebraic multilevel methods and algorithms, e.g., the preconditioned conjugate gradient method, algebraic multilevel iteration (AMLI) preconditioners, the classical algebraic multigrid (AMG) method and its recent modifications, namely AMG using element interpolation (AMGe) and AMG based on smoothed aggregation.

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# Announcement of the next meeting of the EMS Council

## Sofia (Bulgaria), 10–11 July 2010

The EMS Council meets every second year. The next meeting will be held in Sofia, 10–11 July 2010 in the Metropolitan Hotel. The council meeting starts at 13.00 on 10 July and ends at noon on 11 July.

Delegates to council will be elected by the following categories of members.

### (a) Full Members

Full Members are national mathematical societies, which elect 1, 2, 3 or 4 delegates according to their membership class. The membership class is decided by the council and societies are invited to apply for the new class 4, which was introduced by the 2008 council. However, the number of delegates for the 2010 council is determined by the current membership class of the society.

Each society is responsible for the election of its delegates. Each society should notify the Secretariat of the EMS in Helsinki of the names and addresses of its delegate(s) no later than 28 February 2010. As of October 2009, there were 60 such societies, which could designate a maximum of 91 delegates.

### (b) Institutional Members

Delegates shall be elected for a period of four years. A delegate may be re-elected provided that consecutive service in the same capacity does not exceed eight years. The institutional members, as a group, nominate candidates and elect delegates.

### (c) Individual Members

In October 2009, there were 2380 individual members and, according to our statutes, these members will be represented by (up to) 24 delegates.

Here is a list of the current delegates of individual members who could be re-elected for the 2010 Council or whose terms include 2010:

Anichini, Giuseppe, 2006–2009
Anzellotti, Gabriele, 2004–2007–2011
Bourguignon, Jean-Pierre, 2008–2011
Coti Zelati, Vittorio, 2004–2007–2011
Cotoneschi, Stefania, 2008–2011
Goebel, Kazimierz, 2008–2011
Higgs, Russell, 2004–2007–2011
Kingman, John, 2006–2009
Margolis, Stuart, 2004–2007–2011
Strickland, Elizabetta, 2008–2011
Sodin, Mikhail, 2004–2007–2011
Soifer, Gregory, 2004–2007–2011
Wilson, Robin, 2004–2007–2011

A nomination form for delegates of individual members is attached below. Please note the deadline 28 February 2010 for nominations to arrive in Helsinki. Elections of individual delegates will be organized by the EMS Secretariat by postal ballot among individual members unless the number of nominations does not exceed the number of vacancies.

### Agenda

The Executive Committee is responsible for preparing the matters to be discussed at council meetings. Items for the agenda of this meeting of the council should be sent as soon as possible, and no later than 28 February 2010, to the EMS Secretariat in Helsinki.

### Executive Committee

The council is responsible for electing the President, Vice-Presidents, Secretary, Treasurer and other members of the Executive Committee. The present membership of the Executive Committee, together with their individual terms of office, is as follows.

| | |
|---|---|
| President: | Professor A. Laptev (2007–2010) |
| Vice-Presidents: | Professor P. Exner (2005–2010) |
| | Professor: H. Holden (2007–2010) |
| Secretary: | Dr S. Huggett (2007–2010) |
| Treasurer: | Professor J. Väänänen (2007–2010) |
| Members: | Professor Z. Artstein (2009–2012) |
| | Professor F. Brezzi (2009–2012) |
| | Professor I. Krichever (2009–2012) |
| | Professor M. Martin-Deschamps (2007–2010) |
| | Professor M. Raussen (2009–2012) |

Members of the Executive Committee are elected for a period of four years. The president can only serve one term. Committee members may be re-elected, provided that consecutive service shall not exceed eight years. Six of the ten positions on the Executive Committee will be vacant as of 31 December 2010; elections will arise during council. P. Exner and H. Holden have served on the Executive Committee for the maximum term and cannot be re-elected. Nominations of candidates are invited; there is more information below.

The council may, at its meeting, add to the nominations received and set up a Nominations Committee, disjoint from the Executive Committee, to consider all candidates. After hearing the report by the Chair of the Nominations Committee (if one has been set up), the council will proceed to the elections of the Executive Committee posts.

All of these arrangements are as required in the Statutes and By-Laws. All information and material concerning the council will be made available at www.math.ntnu.no/ems/council10.

*Secretary: Stephen Huggett (s.huggett@plymouth.ac.uk)*
*Secretariat: Riitta Ulmanen (ems-office@helsinki.fi)*

**Nomination Form for Council Delegate**

Name: ..............................................................................

Title: ..............................................................................

Address: ..............................................................................

..............................................................................

Proposed by: ..............................................................................

Seconded: ..............................................................................

I certify that I am an individual member of the EMS and that I am willing to stand for election as a delegate of individual members at Council.

Signature of Candidate: ..............................................................................

Date: ..............................................................................

Completed forms should be sent to:

Riitta Ulmanen
EMS Secretariat
Department of Mathematics and Statistics
P.O.Box 68
FI-00014 University of Helsinki
Finland

to arrive by 28 February 2010. A photocopy of this form is acceptable.

# European Women in Mathematics

Lisbeth Fajstrup

## General meeting in Novi Sad

The 14th general meeting of EWM, European Women in Mathematics, took place this year at the University of Novi Sad, Serbia, 25-28 August 2009. There were 9 invited speakers (see box), including Ingrid Daubechies, the 2009 EMS lecturer, who gave the first three of her lectures during the conference.

**EMS-lecturer Ingrid Daubechies gave three of her EMS-lectures in Novi Sad.**

Of the 71 registered participants from 25 countries, 27 gave contributed talks in parallel sessions; there were also two poster sessions. All speakers and almost all the participants were women.

The enthusiastic and friendly local organizers, headed very efficiently by Dušanka Perišić, did their utmost to make everyone feel welcome. A group of undergraduate mathematics and physics students had volunteered to be our English speaking link to Serbia day and night – literally. All participants were accommodated in a comfortable student residence and the volunteers were there around the clock, eager to help and also to discuss mathematics.

The branding and PR for the conference was handled pro-bono by a professional agency and it was impressive. Newspapers, TV-stations, local politicians – everyone was made very much aware of the fact that a major mathematics conference was taking place. A remarkable event took place at the Novi Sad maternity hospital, where all babies born in the month of the conference were given an all-in-one baby suit decorated with the conference logo. All the main talks were filmed by a professional camera crew and some participants were interviewed on various issues about mathematics. The cameras followed us at lunch breaks, on the conference trip to Sremski Karlovci and even at the delicious fish dinner at a restaurant on the Danube bank. A DVD with talks, walks and interviews is being produced and will be available for download from the EMS website.

Needless to say, an EWM meeting is a high profile mathematics conference – a glance at the list of speakers will confirm this. However, it is much more than this. The joy and sense of relief coming from being among other women who are as passionate about mathematics as oneself is difficult to convey; it is actually quite surprising the first time one attends such a meeting. The Novi Sad meeting was no exception. The plenary speakers all began their talks by introducing themselves and their backgrounds and provided both moving and encouraging tales of the road to becoming female mathematicians.

## Conferences for a general audience

A mathematics conference spanning such a wide range of fields is both a challenge and an opportunity. It is very natural for EWM general meetings to be of this kind. Participants come from many different fields and hence will always comprise a general audience.

To make the most of such a meeting is seen as an interesting and important task by EWM and various ideas have been tested. At some EWM meetings, "planted idiots" have been appointed for each lecture by the organizers. The role of the "idiots" is to ask "dumb" questions during the lecture in order to ensure that the lecturer explains concepts that may not be known to everyone in the audience; perhaps it is just the idiot who doesn't understand but no one will know! At some meetings, the lecturers have even discussed their talks beforehand with the idiots. Some lecturers find this interaction fruitful, while others certainly do not. Some meetings have had only a few themes and lectures have been organized as mini-courses in order actually to teach the participants some new mathematics.

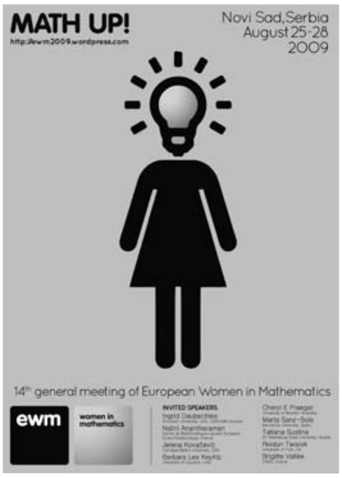
**Some of the participants at Novi Sad. Sitting in front are the volunteers.**

Another traditional conference event, the poster session, has been discussed and experimented with at EWM meetings. Various strategies that have been adopted in the past include:

- All participants presenting posters.
- Posters being grouped according to field.
- Authors of the posters in each field producing a poster explaining the general field and how the various posters are connected.
- All posters having a picture of the author, so that she can be found without reading nametags.

## European Level Organizations for Women Mathematicians

There are several organizations and committees supporting women mathematicians in Europe. The first is the membership organization *European Women in Mathematics* (EWM), http://www.math.helsinki.fi/EWM/. Founded in 1986, EWM has several hundred members in over 30 European countries. Its executive consists of a convenor, currently Frances Kirwan, Oxford, UK (from January 2010 it will be Marie-Françoise Coste Roy, Rennes, France) and a standing committee. EWM holds a biennial scientific meeting and a general conference. Recent meetings have taken place in Cambridge, UK (2007) and Novi Sad as described above and the next will be at CRM Barcelona, Spain (2011). EWM also sponsors or co-organizes various interim meetings and other activities, for example the ICM satellite meeting ICWM 2010 in Hyderabad, http://www.icm2010.org.in/docs/ICWM2010.pdf.




**The logo for the conference was on the poster, on T-shirts, bags, pens, baby suits etc.. It was designed and donated to EWM by a local PR-company.**

EWM is independent from, but has links to, its sister organization, the *Association for Women in Mathematics* (AWM) based in the United States. It also has close links to various national level organizations, for example *Femmes et Mathématiques* in France.

The second organization is the *EMS's Women and Mathematics Committee*, http://www.euro-math-soc.eu/comm-women.html, currently chaired by Sylvie Paycha, Clermont-Ferrand, France (from January 2010 it will be chaired by Dušanka Perišić, Novi Sad, Serbia). The function of this committee is to undertake actions that will help or promote women mathematicians in Europe. Past activities have included gathering statistics on the numbers of women mathematicians in different countries and setting up a blog, see http://womenandmath.wordpress.com/. Currently the committee is exploring possible further collaboration with both EWM and the *European Research Centres on Mathematics* (ERCOM).

Finally, in 2008, EWM and the EMS Women and Mathematics Committee jointly set up the *EWM/EMS Scientific Committee*, http://womenandmath.wordpress.com/emsewm-scientific-committee/. Its members are twelve distinguished women mathematicians, including Dusa McDuff, Nina Uraltseva and Michèle Vergne, and it is currently chaired by Ulrike Tillmann (Oxford, UK). The main function of this committee is to advise on scientific programmes and speakers for EWM and the EMS.

**Ingrid Daubechies** (Princeton), *i) Harmonic analysis with applications to image processing, ii) Analog to digital conversion, iii) Case study: independent component analysis as used in fMRI (functional Magnetic Resonance Imaging) of the human brain.*

**Nalini Anantharaman** (Centre de Mathématiques Laurent Schwartz, France), *Entropy of eigenfunctions on locally symmetric spaces*.

**Jelena Kovačević** (Carnegie Mellon University, USA), *Problems in biological imaging: opportunities for mathematical signal processing.*

**Marta Sanz-Solé** (Barcelona University, Spain), *Hitting probabilities for random fields: some criteria with applications to systems of stochastic partial differential equations.*

**Tatiana Suslina** (St. Petersburg State University, Russia), *Spectral approach to homogenization of periodic differential operators*.

**Reidun Twarock** (University of York, UK), *Applications of group theory in virology: affine extension of nonchrystallographic groups predicting virus architecture.*

**Brigitte Vallée** (Université de Caen, France), *On the non-randomness of modular arithmetic progressions: a solution to a problem by V. I. Arnold.*

**Barbara Lee Keyfitz** (University of Houston, USA), *Where the wild things might be: functional spaces for multidimensional conservation laws*.

**Cheryl Praeger** (University of Western Australia). *The normal quotient philosophy for studying edge transitive graphs.*

*Lisbeth Fajstrup [fajstrup@math.aau.dk] is an Associate Professor at University of Aalborg, Denmark. She got her PhD from the University of Aarhus in 92 in algebraic topology. Lisbeth is one of the founders of the growing area directed topology, which is still her research area. She runs a blog http://numb3rs.math.aau.dk (in Danish), on the mathematics in the TV-series Numb3rs. Lisbeth enjoys walking her dog and to her own amazement actually quite enjoys running.*

# The Year of Mathematics in Germany

Ehrhard Behrends

The year 2008 was the "Year of Mathematics" in Germany. Generally it is con-sidered as a great success. Several hundred people contributed with an abundance of original ideas, one cannot aim at a complete description of all the activities which have been realized.

## The tradition of the "years of science" in Germany

Since the beginning of this century each year is a "year of science" in Germany.

Here is the complete list of the fields presented so far: 2000: physics, 2001: life sciences, 2002: geography, 2003: chemistry, 2004: technics, 2005: Einstein's year, 2006: computer science, 2007: humanities, and 2008: mathematics. (Since 2009 one has abandoned this concept. Rather than to present a particular science one chooses a "motto". 2009, for example, is the year "Forschungsexpedition Deutschland".)

There are certain events which had always been repeated: Big opening and closing ceremonies in different cities, the "Wissenschaftssommer" where in some city for one week special events are presented, the "Wissenschaftsschiff", a ship which serves as a travelling museum and which visits several dozens of ports all over Germany etc.

## Why a "Year of Mathematics"?

It is a commonplace all over the world that the public understanding of mathematics is at a very unsatisfactory level. Most people think that "everything is known since centuries" and that mathematics is not related with really interesting problems, and there are very few who think of their mathematical schooldays with pleasure.

The result is a decrease in the number of students in the "hard" sciences, and in the next future the number of those who leave the university with a degree in these areas will not suffice to fill the vacant positions.

To change this was the pragmatic aspect of a "Year of Mathematics". But the goal was more ambitious: the mathematicians aimed at presenting their field as

an indispensible part of human culture which plays a fundamental role in economy and various sciences.

## Partners and sponsors

The mathematicians applied rather late to have the year 2008 as the "Year of Mathematics". From the beginning on it was clear that many members in the community would collaborate and that it would be a joint effort of all of the German mathematical societies (DMV, GAMM, MNU and GDM). It was surely also very important that a large foundation, the "Deutsche Telekom Stiftung", decided to act as a generous sponsor of such a year.

After the positive decision a very fruitful and effective collaboration began. The government was represented by the ministry of science and education, the mathematicians mainly by the DMV under their president Günter Ziegler from the Technical University in Berlin (see the picture).

The organizers had a budget of several million Euro at their disposal. A considerable part was used to hire the renowned advertising agency Scholz and Friends to guarantee a professional presentation of the activities. In fact, an abundance of creative suggestions were presented, e.g. the motto ("Du kannst mehr Mathe als Du denkst"[1]) of the year and the logo ("Mathe – alles, was zählt"[2]).

Wissenschaftsjahr 2008
Mathematik
Alles, was zählt

## Some figures

Here are some figures in order to illustrate the extent of the activities:

- "Mathemacher": There were more than 1300 partners who prepared various acitivities like talks, exhibitions, special events, articles in newspapers, …
- School children: 34.000 schools received special information on mathematics.
- Exhibitions: Four great exhibitions attracted the attention of more than 500.000 visitors ("Zahlenbitte", "Mathema", "Imaginary", the science ship).

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[1] "You know more of mathematics than you think".
[2] "Mathematics – everything that counts".

- More than 30 mathematical competitions of different levels were organized.
- The mathematical films of the “film festival” were shown in more than 100 cities.
- 3500 articles in journals and 2500 contributions in online media were concerned with mathematics.
- There were presented 500 broadcastings centering around mathematics in the TV and in the radio.

### Examples of the activities

In the sequel we describe some of the activities in the German “Year of Mathematics”.

#### *Posters*

Already in January 2008 one could see large mathematical posters at many places: in the streets, at the railway stations, at the airports, etc. They were thought of to transfer the message “Mathematics can be found everywhere”. Here are two examples which concern knot theory and data compression.



#### *Special events and competitions for school children*

At nearly every German school there were organized special events for school children. Of particular importance was the “Mathekoffer” (the math trunk), a present sponsored by the Deutsche Telekom foundation. It contained hands-ons to visualize certain mathematical concepts like space and chance.


#### *The science ship*

The “science ship” is an ordinary transport ship which from September to March carries potatoes, charcoal etc. and which in summertime transfoms to a museum. For the 2008 exhibition the mathematical institutions from universities, from the Fraunhofer Gesellschaft and the Helmholtz Gesellschaft were asked to propose exhibits. An impressing collection was prepared which was presented on roughly 1000 square meters: films, hands-ons, exhibits, lectures, special information for school teachers etc. Never before this ship has had such a large number of visitors.

#### *A special book for high-school graduates*

Springer Verlag has had the idea to prepare a special book for high-school graduates. This *Kaleidoskop der Mathematik* was edited by E. Behrends (Berlin), P. Gritzmann (Munich) and G. Ziegler (Berlin). It contains a variety of popular articles on mathematics from the books of the German mathematical editors.

In each of the 3500 German high-schools the best graduate in mathematics has got this “Kaleidoskop” as a present (together with a free membership in the DMV for one year).



### *A mathematical advent calendar*

The "mathematical advent calendar" is a competition for school children in the German speaking European countries. Each day between December 1st and December 24th there is presented on a special internet page a mathematical problem, the solutions are submitted electronically. The winners are invited to come to a prize ceremony in the Berlin Urania.

This calendar was launched several years ago. It was repeated every year with increasing success, now there are thousands of participants from all over the world.

### *The science summer in Leipzig*

The summer in 2008 was really hot, and during the hottest period the science summer took place in Leipzig. It consisted of numerous activities, the most important part of the science summer was surely a large exhibition on the central square at the opera house. Here mathematical departments from universities as well as mathematical research institutes presented applications of contemporary mathematics. The science summer was a big success, many school children with their teachers but also families with their children visited the presentations.

## A media prize

It is now a common place that good relations with the media are of fundamental importance. Already in 2002 the DMV has invited journalists from the important

German newspapers to a "working dinner" in a nice restaurant to discuss the problems with the presentation of mathematics in the journals. Also since several years a "media prize" is awarded for the best article on mathematics. In 2008 an additional prize was created: the prize for the best mathematical cartoon.

### *Mathematical exhibitions*

Many mathematical exhibitions have been prepared for the mathematical year, some of them have been shown at several places. Also parts of the Mathematicum – a mathematical museum in Gießen – were presented in many cities.

The largest among these exhibitions is "Mathema" which is shown between November 2008 and August 2009 in the Technical Museum of Berlin (see www.mathema-ausstellung.de). The organizers aimed at presenting mathemat-ics as an indispensible part of human culture.


## The success of the "Year of Mathematics"

Was the "Year of Mathematics" a success? The answer is a clear "yes" if one compares with competing years: the largest number of articles in the media, the largest number of visitors on the science ship etc. It is, however, too early to decide whether the public understanding of mathematics really has changed considerably to the positive. It will surely be necessary to continue the efforts into this direction.

*Ehrhard Behrends [behrends@math.fu-berlin.de] is Professor for mathematics at the Free University of Berlin. His research is concerned with functional analysis and probability. Also he has realized several projects in order to popularize mathematics. For example, he has created the popular webpage www.mathematik.de, and he is the author of several popular books (e.g. "Fünf Minuten Mathematik" which has been translated into several other languages). For more than 10 years he was the secretary of the German mathematical society. At present he is the chair of the committee "raising the public awareness of mathematics" of the EMS.*

The paper appeared in: *Gazette des mathématiciens (SMF)*, 121, 2009, 101–106. Reprinted with permission.

Avec les conclusions du rapport de la commission d'enquête sur *les évènements survenus en République du Tchad du 28 janvier au 8 février 2008 et leurs conséquences*, les sociétés savantes de mathématiques (SFdS, SMAI, SMF) ont la douleur d'annoncer leur quasi-certitude que

**Ibni Oumar Mahamat Saleh**

**est mort en détention**

**dès le début du mois de février 2008**

**Ibni Oumar Mahamat Saleh, professeur de mathématiques à l'université de N'Djamena et ancien ministre, était une des figures majeures de l'opposition démocratique au parti au pouvoir au Tchad. Il avait été enlevé à N'Djamena à son domicile le 3 février, au lendemain du départ des troupes rebelles, par des forces de l'armée nationale tchadienne.**

***Ibni Oumar Mahamat Saleh avait fait toutes ses études supérieures à Orléans. Il était docteur de l'université d'Orléans. Il était à l'initiative d'échanges inter-universitaires entre la France et le Tchad. Il était aimé et respecté de tous ses collègues et amis. Sa mort est une perte immense.***

*Pour plus d'informations,*
*consultez le site de la pétition lancée par la SFdS, la SMAI et la SMF pour*

**demander la vérité**
**sur le sort d'Ibni Oumar Mahamat Saleh :**

**http://smf.emath.fr/PetitionSaleh/**

The Price "Ibni Oumar Mahamat Saleh" is created in memory of our colleague and to continue its commitment to Mathematics in Africa. The price will be awarded annually to a young mathematician from subsaharian Africa by a scientific committee set up by ICPAM. It will fund a scientific visit.

There are more than 3000 of us who have signed the Petition on "Ibni Oumar Mahamat Saleh". We hope that there will be many of us contributing to the international subscription for the Price "Ibni Oumar Mahamat Saleh".

You can contribute to the subscription at the web site http://smf.emath.fr/en/SouscriptionSaleh/

You can also disseminate the information about the price, the call for candidates is in http://smf.emath.fr/en/SouscriptionSaleh/candidatures.html

*Aline Bonami, Alain Godinot, Marie-Francoise Roy*

FOUNDATION COMPOSITIO MATHEMATICA

As the winner of the first Compositio Prize has been chosen the paper

**D. Maulik, N. Nekrasov, A. Okounkov, R. Pandharipande: Gromov-Witten theory and Donaldson–Thomas theory. II. Compositio Mathematica 142 (2006), no. 5, 1286–1304**

The Compositio Prize is a prize awarded every third year by the Foundation Compositio Mathematica in recognition of an outstanding piece of mathematical research that is published in the journal Compositio Mathematica during a three year period. The 2009 Compositio Prize is the first one to be awarded.

The actual prize consists of a model of the Cayley surface and will be handed over to the authors during a festive colloquium featuring talks by the authors in the spring of 2010.

**http://www.compositio.nl/prijs.html**

# The measure of a great idea: 50 years on from the creation of the International Mathematical Olympiad

Vasile Berinde (Baia Mare) and Mădălina Păcurar (Cluj-Napoca)

*The International Mathematical Olympiad (IMO) is nowadays the most important and prestigious mathematical competition for high school students. Its 50th edition was held this year in Bremen, Germany (see [6] for details and also [11] for several data related to the previous 49 editions). Although on the front page of [11] it is mentioned that: "The first IMO was held in 1959 in Romania, with 7 countries participating. It has gradually expanded to over 100 countries from 5 continents," little is known about the concrete context in which this competition was created. It is the main aim of this note to give, on the occasion of its 50th anniversary, a brief report on the origins and first two years in the life of the IMO, which were essential for its subsequent evolution. For more details, see the booklets [2]–[3].*

## The tradition of mathematical competitions in Romania

To understand the mathematical and political context of the period in which the first two editions of the IMO were organized, let us summarize the most significant chronological reference points about the tradition of mathematical contests in Romania.

**1895, *September 15***: The first issue of the monthly *Gazeta Matematică* is published. This is an extremely important year in the history of Romanian mathematical education. The aims of *Gazeta Matematică*, as stated in its first issue, were: "1) to publish original papers in mathematics; 2) to develop the appetite for the study of mathematics and for doing original research." The journal has been continuously published over the last 115 years and has successfully accomplished its aims over more than a century.

**1897:** The Minister of Education Spiru Haret (who got a brilliant PhD in Mathematics at Sorbonne, Paris, in 1878) introduces a new education law, in this way establishing a modern mathematical education system of French inspiration in Romania. The basic principles of Haret's law can be easily detected to a significant extent in the framework of the Romanian mathematical education system during the whole of the 20th century.

**1902:** Under the auspices of *Gazeta Matematică*, the first mathematics competition (by mail) at a national level is organized for the correspondents of the journal. In the beginning, awards are given to the best solutions received by the Editorial Board.

**1905–1908:** A new competition system is introduced: the contestants are examined in certain competition centres (there were 11 centres in 1905) after a preliminary registration. The contestants are selected from among the best correspondents of *Gazeta Matematică* – students at civil and military high schools.

**1909:** Written and oral examinations are introduced in the Annual Contest *Gazeta Matematică*. They are organized in a unique centre, Bucharest, a rule which is preserved until 1916.

**1945, March 6:** The coming to power of the communist party, with the strong support of the USSR army, opens a period of almost half a century of communist rule in Romania. The jubilee of the journal *Gazeta Matematică* is celebrated.

**1948:** A new education law, of Soviet inspiration, structurally changes the education system in Romania. After this reform, mathematics gets a very important position in both secondary and high school curricula. The Annual Contest *Gazeta Matematică*, which survived the Second World War, will not survive under this name to the instauration of communism in Romania, mainly because it was a "capitalist" business.

**1949:** *Societatea de Ştiinţe Matematice şi Fizice (The Society of Mathematical and Physical Sciences, SSMF)* is established by unifying the *Romanian Society* for Sciences (founded in 1897) and the Society *Gazeta Matematică* (founded in 1910). SSMF inherits the publications and tradition of mathematics competitions of both parent societies, while the organization system is inherited from the former. Instead of the "Annual Contest *Gazeta Matematică*", *SSMF* organizes, in a different manner, the ***National Mathematical Olympiad (NMO).*** Being financially supported by the state, this soon becomes a wider competition. With the new three-round system (local round, county round and final round), *NMO* continues to grow over the subsequent years. The contest problems are proposed by the editors of *Gazeta Matematică* and the lists of all prize-winners are usually published in a special column of *Gazeta Matematică* (issued under the name *Gazeta Matematică şi Fizică* during the period 1949–1963).

**1953:** As an attempt to revive the tradition of the old Annual Contest *Gazeta Matematică*, a mail competition is opened for students in secondary schools (11–14 years old), with special problem columns published in *Gazeta Matematică*. This new competition soon significantly raises the number of participants in the *NMO* (after graduating secondary school). Stalin dies. This opens a short fresh air period in the history of some Eastern European countries.

## The 4th Congress of the Romanian Mathematicians

Under these new political circumstances, The 4th Congress of the Romanian Mathematicians was held between 27 May and 4 June 1956. Its main organizers, the Romanian Academy and the SSMF, had the full approval and strong financial support of the communist leaders, who intended to show in this way the achievements of communist rule to participants from abroad and especially to those coming from Western countries. This was an impressive scientific event for those times and, surely, it was one of the most important international congress of mathematics ever organized in Romania before 1989, if we count the number of total foreign participants (74 participants from 18 countries, with most participants from France and Germany). Moreover, most foreign participants were leading mathematicians of the moment. We mention a few of them here (see [12] for the full list): J. Hadamard (who was 92 years old!), E. Hille, A. H. Stone, A. Denjoy, K. Kuratowski, S. Eilenberg, W. Engel, E. Keller, P. Erdős, B. Segre, O. Varga, M. Villa, P. Turán, V. Turán, T. Wazewski, J. Favard and G. de Rham. The scientific program of the congress, see [12], included not only presentations on higher mathematics but also elementary mathematics and mathematics education. Note that most of the leading Romanian mathematicians at that moment, including many participants and also some of the main organizers of the congress, were former participants or even prize winners of the Annual Contest *Gazeta Matematică*: N. Teodorescu, Gr. C. Moisil, M. Nicolescu, C. Iacob etc. It is not surprising then that, in the enthusiastic atmosphere of the congress, a very exciting idea was launched: to organize an ***International Mathematical Olympiad***. The idea was warmly up-held by several mathematicians and, essentially, it was adopted and supported by the Minister of Education Ilie Murgulescu, a chemist and member of the Romanian Academy, who was involved in the organization of the congress and also gave an introductory speech at the opening ceremony.

**Jaques Hadamard (center), Grigore C. Moisil (left), and Nicolae Teodorescu (right) during the 4th Congress of Romanian Mathematicians (1956)**

## The first IMO

In 1959, at the end of July (23–31), the ***First International Mathematical Olympiad*** was organized in Braşov and Bucharest, under the auspices of the Romanian Society for Mathematics and Physics (SSMF), the forerunner of the current Romanian Mathematical Society (SSMR). The president of the SSMF was at that time Grigore C. Moisil (1906–1973), a mathematician and member of the Romanian Academy. The general secretary of the SSMF was the mathematician Tiberiu Roman (1916–2004), who is credited with the very idea of organizing the IMO (see [7]). It was clear from its first edition's minutes [9] that, at least in the beginning, the IMO was thought to be a "soviet matter" – *SSMF* did not invite any of the non-soviet obeying countries amongst the ones represented at the 4th Congress of the Romanian Mathematicians in 1956: Austria, Belgium, Bulgaria, China, Czechoslovakia, England, France, Germany, Hungary, Israel, Italy, Japan, Yugoslavia, Norway, Poland, Switzerland, the USA and the USSR. Nevertheless, only six countries that were under Soviet influence at that time accepted Romania's invitation and attended the first edition: Bulgaria, Czechoslovakia, Hungary, Poland, East Germany (DDR) and the USSR (Albania was the only invited country that declined the invitation).

It would be interesting to quote here the IMO's aims stated in the official invitation, as mentioned in [9]. Essentially, most of them are still extant: 1) to establish personal contacts and friendship between youngsters of the same age and having common preoccupations from the *friend and neighbor socialist countries* (at that time Yugoslavia had some conflicting matters with the USSR); 2) to establish a basement for a future scientific collaboration between promising youngsters who will form the new generation of researchers in mathematics; 3) to give the opportunity for an exchange of ideas between teachers of mathematics in the participating countries with respect to the evolution of mathematical education in high schools; 4) to create the background for similar national mathematical olympiads in the participating countries which did not have such contests so far; 5) to offer the participants the chance to know more about Romania.

At the first IMO, each team was formed of eight students accompanied by a teacher (delegate) except for the USSR, which brought only four students because the entrance examination to university was being organized at the end of July in this country. Therefore, 52 contestants from 7 participating countries attended the first IMO. Compare this to the 565 contestants from 104 countries that attended the 50th IMO in 2009 to arrive at an impressive conclusion: the competition grew 10 times in terms of number of contestants and 14 times in terms of participating countries in its 50 years of existence!

It is interesting to note that, due to the consistent Romanian experience in organizing mathematical contests, the format of the first IMO was well designed from the very beginning, being basically preserved until today. It consisted of two written papers, held on two consecutive days, each paper requiring the solution of three problems. The two written papers were scheduled on the second and third days, respectively, of the contest, on 24 and 25 July 1959, in Brasov (at that time temporarily named "Stalin town"), while the first day, 23 July, was allocated to arrivals and the opening ceremony. The delegates of the participating countries selected six problems to be proposed to the contestants from a long list of more than 70 problems. More precisely, for the first paper two selected problems were proposed by the Czechoslovakian delegation (arithmetic and space geometry) and one problem by Poland (plane geometry), while the problems given on the second day were proposed by the Romanian delegation (algebra and plane geometry) and the Hungarian delegation (trigonometry). It is also interesting to note the difficulties faced by the team leaders in selecting the problems in order to meet the particular curricula from each participating country. Afterwards, the contest curricula had to be clearly stated in the IMO regulations, see [13]. The papers were graded on 27 and 28 July. The prize ceremony and closing dinner were scheduled on 28 July and took place in Bucharest, being chaired by the president of the SSMF, Professor Grigore C. Moisil. The remaining days were allocated to various tourist and social events for both teachers and students.

**Gr. C. Moisil (1906–1973), President of SSMF at the time the first IMO was organized**

It is worth mentioning that the first IMO also included many other scientific events. For example, during the written papers time, the team leaders met to discuss in detail the mathematical education system in their countries and exchanged ideas about curricula and the organization of mathematical contests. A mutual exchange of mathematical publications was also discussed. Because no such mathematical contests existed in their countries, the delegates from Bulgaria and East Germany declared that they would try to get approval and financial support for organizing such competitions.

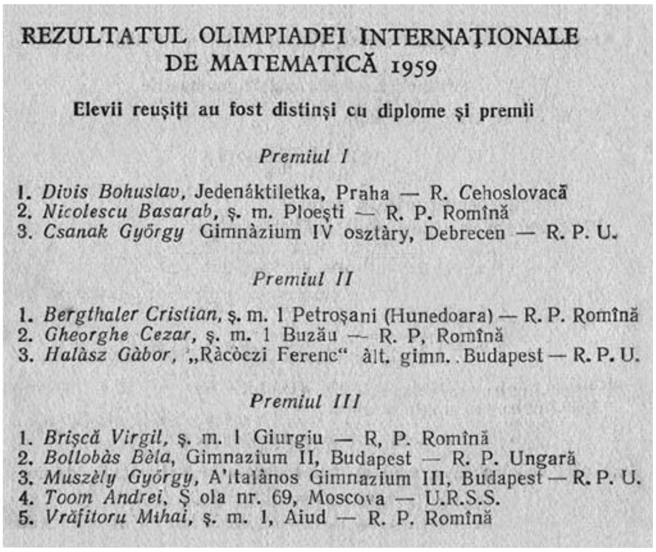
The second IMO took place in almost the same format in Bucharest and Sinaia the following year (1960) but only five countries were represented (Poland and the USSR missed the contest). Yet during the first IMO it appeared practically difficult or even impossible for only one teacher from each delegation to cope with both scientific and organizational matters. Therefore two improved regulations were adopted starting with the second IMO: 1) instead of one, there were two teachers accompanying the students of each participating country; 2) the selection procedure of the proposed problems would be scheduled a few days prior to the students' arrival. These regulations have in fact been preserved until today: each participating country has a team leader and a deputy team leader.

In the absence of the USSR at the second and third editions of the IMO, it appeared to some extent that the competition would eventually die. Fortunately, the contest continued to be organized in the other participating countries (Hungary-1961, Czechoslovakia-1962, Poland-1963, the USSR-1964 and so on) and started soon to grow, step by step, until the impressive number of participants recorded at its 50th edition.

## Conclusions

Looking back in time, the idea of organizing the IMO appears today as an extraordinary and visionary one. Its impact was immediate and, step by step, many countries started to organize their own national mathematical competitions following the IMO format. Moreover, this idea was soon adopted by other related sciences, including physics and chemistry. So, the first International Physics Olympiad was organized in 1967 (Warsaw, Poland), with its 40$^{th}$ edition in 2009. The first International Chemistry Olympiad was organized in 1968 (Prague, Czechoslovakia), having in 2009 its 41$^{st}$ edition. The International Olympiad of Informatics had its first edition 30 years later than the IMO in 1989 (Pravets, Bulgaria) and its 21$^{st}$ edition was held in 2009. In recognition of the IMO's importance, let us quote W. Gorzkowski [5]: "The possibility of organizing the International Physics Olympiads was suggested before 1967. It was clear that the International Physics Olympiad should be an annual event like the International Mathematics Olympiads. The success of the International Mathematics Olympiads, and the positive experience gained from its organization, greatly stimulated physicists involved in physics education and interested in comparison of knowledge of the best students from different countries."

The IMO in particular and the mathematical competitions in general are recognized to have a significant role in identifying, training and directing towards mathematical careers many leading research mathematicians of the day. Six of them: Béla Bollobás, Timothy Growers, László Lovasz, Stanislav Smirnov, Terence Tao and Jean-Cristophe Yoccoz, were invited to Bremen for the 50$^{th}$ anniversary of the IMO [6]. As a creator and promoter of the IMO, Romania's scientific benefits are significant. If



REZULTATUL OLIMPIADEI INTERNAŢIONALE DE MATEMATICĂ 1959

Elevii reuşiţi au fost distinşi cu diplome şi premii

Premiul I

1. *Divis Bohuslav*, Jedenáktiletka, Praha — R. Cehoslovacă
2. *Nicolescu Basarab*, ş. m. Ploeşti — R. P. Romînă
3. *Csanak György* Gimnàzium IV osztàry, Debrecen — R. P. U.

Premiul II

1. *Bergthaler Cristian*, ş. m. 1 Petroşani (Hunedoara) — R. P. Romînă
2. *Gheorghe Cezar*, ş. m. 1 Buzău — R. P. Romînă
3. *Halàsz Gàbor*, „Ràcòczi Ferenc" ált. gimn. Budapest — R. P. U.

Premiul III

1. *Brişcă Virgil*, ş. m. 1 Giurgiu — R. P. Romînă
2. *Bollobàs Bèla*, Gimnazium II, Budapest — R. P. Ungară
3. *Muszèly György*, A'ltalànos Gimnazium III, Budapest — R. P. U.
4. *Toom Andrei*, Ş ola nr. 69, Moscova — U.R.S.S.
5. *Vrăjitoru Mihai*, ş. m. 1, Aiud — R. P. Romînă

**The prizes I, II and III awarded to the first IMO, as published in Gazeta Matematică și Fizică (1959)**

we use the data collected in [3] for the period 1959-2003, more than two thirds of former Romanian IMO contestants are or were involved in academia or research, in Romania or abroad. We mention here a few names, accompanied by the year when they have first competed in the IMO: V. Barbu (1959), S. Strătilă, C. Năstăsescu and T. Zamfirescu (1960), G. Lusztig and L. Bădescu (1961), L. Zsido (1963), D. Voiculescu and E. Popa (1965), D. Ralescu (1967), Al. Dimca and R. Gologan (1970), D. Timotin (1971), M. Pimsner (1972), A. Ocneanu (1973), M. Colțoiu and D. Vuza (1974), Al. Zaharescu and V. Nistor (1978), M. Mitrea (1981), L. Funar (1983), P. Mironescu and D. Tătaru (1984), A. Moroianu and A. Vasiu (1987), F. Belgun and T. Bănică (1988), S. Moroianu, M. Crainic and D. Iftimie (1990).

Due to the advantage taken from the first two editions organized, Romania is so far the only country to have hosted five IMOs: 1959, 1960, 1969, 1978 and 1999. Note also that Romania together with Bulgaria and both the Czech Republic and Slovakia (the former Czechoslovakia) are the only countries in the world that have attended all the 50 editions of the IMO organized from 1959 to 2009 (1980 was the only year when the IMO was not organized).

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# Chaotic vibrations and strong scars

Nalini Anantharaman and Stéphane Nonnenmacher

## 1 Introduction

What relates an earthquake, a drum, a 2-dimensional mesoscopic cavity, a microwave oven and an optical fibre? The equations governing wave propagation in these systems (seismic, acoustic, electronic, microwave and optical) are linear. As a result, the solutions of the wave equations can be decomposed as a sum over vibrating eigenmodes (or stationary modes). The discrete (or "quantum") nature of this eigenmode decomposition is due to the compact geometry of the above-mentioned "cavities". Mathematically, the latter are modelled by compact, $d$-dimensional Riemannian manifolds $(X,g)$ with or without boundaries.

To simplify the presentation, we will restrict ourselves to scalar waves, described by a real wavefunction $\psi(x,t)$. The eigenmodes $(\psi_n(x))_{n\geq 0}$ then satisfy Helmholtz's equation

$$\Delta\psi_n + k_n^2\,\psi_n = 0\,, \qquad (1.1)$$

where $\Delta : H^2 \to L^2$ is the Laplace-Beltrami operator on $X$ and $k_n \geq 0$ is the vibration frequency of the mode $\psi_n$. If the manifold has a boundary (as is the case for an acoustic drum or for electromagnetic cavities), the wavefunction must generally satisfy specific boundary conditions, dictated by the physics of the system: the simplest ones are the Dirichlet ($\psi_{|\partial X} = 0$) and Neumann ($\partial_\nu \psi_{|\partial X} = 0$) boundary conditions, where $\partial_\nu$ is the normal derivative at the boundary.

Our goal is to describe the eigenmodes, in particular the high-frequency eigenmodes ($k_n \gg 1$). Specifically, we would like to predict the localization properties of the modes $\psi_n$, from our knowledge of the geometry of the manifold $(X,g)$.

Consider, for instance, the case of a bounded domain in the Euclidean plane, which we will call a *billiard*. For some very particular billiard shapes (e.g. a rectangle, a circle or an ellipse), there exists a choice of coordinates allowing one to separate the variables in Helmholtz's equation (1.1), thereby reducing it to a one-dimensional eigenvalue problem (of the Sturm-Liouville type). In the high-frequency limit, the latter can be solved to arbitrarily high precision through WKB[1] methods, or sometimes even exactly (see Figure 1). The high-energy eigenmodes of such domains are hence very well-understood.

This separation of variables can be interpreted as a particular symmetry of the *classical dynamics* of the billiard (the motion of a point particle rolling frictionless across the billiard and bouncing on its boundary). This dynamics is *Liouville-integrable*, which means that there is a conserved quantity in addition to the kinetic energy. For instance, in a circular billiard the angular momentum of the particle is conserved. The classical trajectories are then very "regular" (see Figure 1). The same regularity is observed in the eigenfunctions and can be explained by the existence of a non-trivial differential operator commuting with the Laplacian.

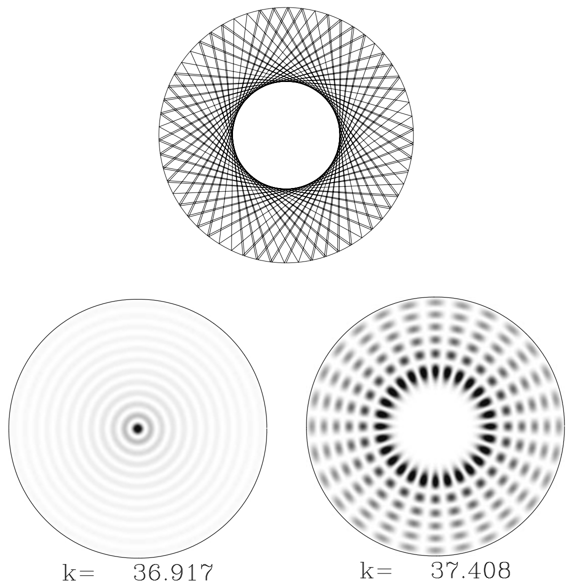


**Figure 1. Top: one orbit of the circular billiard. Bottom left and right: two eigenmodes of that billiard, with their respective frequencies**

As soon as an integrable billiard is slightly deformed, the symmetry is broken: the geodesic flow is no longer integrable; it becomes *chaotic* in some regions of phase space. We do not have any approximate formula at hand to describe the eigenmodes. The extreme situation consists of *fully chaotic* billiards, like the "stadium" displayed in Figure 2 (the word "chaotic" is a fuzzy notion; the results we present below will always rely on precise mathematical assumptions).

We mention that the most recent numerical methods (the boundary operator and the "scaling method") allow one to compute a few tens of thousands of eigenmodes for 2-dimensional billiards, at most a few thousands in 3 dimensions and much less if the metric is not Euclidean. The difficulty stems from the fact that a mode of frequency $k_n \gg 1$ oscillates on a scale $\sim 1/k_n$ (the wavelength); one thus needs a finer and finer mesh when increasing the frequency[2]. On the other hand, the analytical methods and results we present below are especially fitted to describe these high-frequency modes.

### Semiclassical methods

In the general case of a Riemannian manifold, the classical dynamics (away from the boundaries) consists of the Hamiltonian flow $g^t$ on the cotangent bundle[3] $T^*X$, generated by the free motion Hamiltonian

$$H(x,\xi) = \frac{|\xi|^2}{2}, \quad (x,\xi) \in T^*X. \qquad (1.2)$$

The flow on the energy layer $H^{-1}(1/2) = S^*X = \{(x,\xi) : |\xi| = 1\}$ is simply the *geodesic flow* on the manifold (with reflections on the boundary in the case $\partial X \neq 0$).

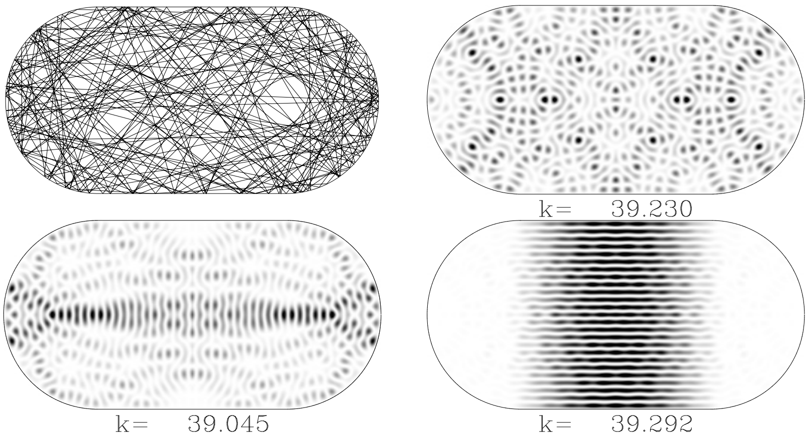


**Figure 2. Top left: one typical "ergodic" orbit of the "stadium": it equidistributes across the whole billiard. The three other plots feature eigenmodes of frequencies $k_n \approx 39$. Bottom left: a "scar" on the (unstable) horizontal periodic orbit. Bottom right: a "bouncing ball" mode.**

The high-frequency regime allows us to use the tools of semiclassical analysis. Indeed, the Helmholtz equation (1.1) can be interpreted as a stationary Schrödinger equation: taking $\hbar_n = k_n^{-1}$ as an "effective Planck's constant", the eigenmode $\psi_n$ satisfies

$$-\frac{\hbar_n^2 \Delta}{2}\,\psi_n = \frac{1}{2}\,\psi_n. \qquad (1.3)$$

The operator $-\frac{\hbar^2\Delta}{2}$ on the left-hand side is the quantum Hamiltonian governing the dynamics of a particle moving freely inside the cavity; it is the *quantization* of the classical Hamiltonian (1.2). The above equation describes a quantum particle in a stationary state of energy $E = 1/2$ (in this formalism, the energy is fixed but Planck's "constant" is the running variable). The high-frequency limit $k_n \to \infty$ exactly corresponds to the semiclassical regime $\hbar = \hbar_n \to 0$. In the following, the eigenmode will be denoted by $\psi_n$ or $\psi_\hbar$.

The *correspondence principle* provides a connection between the *Schrödinger propagator*, namely the unitary flow $U^t = e^{it\hbar\frac{\Delta}{2}}$ acting on $L^2(X)$ and the geodesic flow $g^t$ acting on the phase space $T^*X$. The former "converges" towards the latter in the semiclassical limit $\hbar \to 0$, in a sense made explicit below. The aim of semiclassical analysis is to exploit this correspondence and use our understanding of the geodesic flow in order to extract properties of the Schrödinger flow.

To analyse the eigenmodes we need to *observe* them by using *quantum observables*. For us, an observable is a real function $A \in C^\infty(T^*X)$ that will be used as a test function to measure the phase space localization of a wavefunction. One can associate to this function a quantum observable $\mathrm{Op}_\hbar(A)$, which is a selfadjoint operator on $L^2(X)$ obtained from $A$ through a certain ($\hbar$-dependent) quantization procedure. For instance, on $X = \mathbb{R}^d$ a possible procedure is the Weyl quantization

$$\mathrm{Op}_\hbar^W(A)f(x) = \frac{1}{(2\pi\hbar)^d}\int A\left(\frac{x+y}{2},\xi\right) e^{\frac{i}{\hbar}\xi\cdot(x-y)} f(y)\,dy\,d\xi. \qquad (1.4)$$

The simplest case consists of functions $A(x,\xi) = A(x)$ independent of the momentum; $\mathrm{Op}_\hbar^W(A)$ is then the operator of multiplication by $A(x)$. If $A = A(\xi)$ is a polynomial in the variable $\xi$ then $\mathrm{Op}_\hbar^W(A)$ is the differential operator $A\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)$. The role of the parameter $\hbar$ in the definition of $\mathrm{Op}_\hbar^W(A)$ is to adapt that operator to the study of functions oscillating on a spatial scale $\sim \hbar$. On a general smooth manifold $X$, one can define a quantization $\mathrm{Op}_\hbar(A)$ by using the formula (1.4) in local charts and then glue together the charts using a smooth partition of unity.

A mathematical version of the correspondence principle takes the form of an *Egorov theorem*. It states that quantization (approximately) commutes with evolution for observables:

$$\|e^{-it\hbar\frac{\Delta}{2}}\,\mathrm{Op}_\hbar(A)e^{it\hbar\frac{\Delta}{2}} - \mathrm{Op}_\hbar(A\circ g^t)\|_{\mathscr{L}(L^2)} = \mathscr{O}_{A,t}(\hbar). \quad \text{(Egorov)} \quad (1.5)$$

### Semiclassical measures

In quantum mechanics, the function $|\psi(x)|^2$ describes the probability (density) of finding the particle at the position $x \in X$. A measuring device will only be able to measure the probability integrated over a small region (a "pixel") $\int_B |\psi(x)|^2\,dx$, which can be expressed as a diagonal matrix element $\langle \psi, \mathbb{1}_B\psi\rangle$. Here $\mathbb{1}_B$ is the multiplication operator (on $L^2(X)$) by the characteristic function on $B$.

More generally, for a nontrivial observable $A(x,\xi)$ supported in a small *phase space* region, the matrix element $\langle \psi, \mathrm{Op}_\hbar(A)\psi\rangle$ provides information on the probability of the particle lying in this region. From the linearity of the quantization scheme $A \mapsto \mathrm{Op}_\hbar(A)$, this matrix element defines a distribution $\mu_\psi$ on $T^*X$:

$$\mu_\psi(A) \stackrel{\text{def}}{=} \langle \psi, \mathrm{Op}_\hbar(A)\psi\rangle, \qquad \forall A \in C_0^\infty(T^*X).$$

This distribution (which depends on the state $\psi$ but also on the scale $\hbar$) is called the *Wigner measure of the state* $\psi$. The projection of $\mu_\psi$ on $X$ is equal to the probability measure $|\psi(x)|^2\,dx$; for this reason, $\mu_\psi$ is also called a *microlocal lift* of that measure. Still, $\mu_\psi$ contains more information: it takes the *phase* of $\psi$ into account and thereby also describes the local momentum of the particle (measured at the scale $\hbar$).

In order to study the localization properties of the eigenmodes $\psi_n$, we will consider their Wigner measures $\mu_{\psi_n} = \mu_n$ (constructed with the adapted scales $\hbar_n$). It is difficult to state anything rigorous about the Wigner measures of individual eigenmodes so we will only aim to understand the limits of (subsequences of) the family $(\mu_n)_{n\geq 0}$ in the weak topology on distributions. Such a limit $\mu$ is called a **semiclassical measure** of the manifold $X$. Basic properties of the quantization scheme imply that:

- $\mu$ is a probability measure supported on the energy shell $S^*X$.
- $\mu$ is *invariant* through the geodesic flow: $\mu = (g^t)_*(\mu)$, $\forall t \in \mathbb{R}$.
- The collection of semiclassical measures $\mu$ does not depend on the choices of local symplectic coordinates involved in the definition of the quantization scheme $A \mapsto \mathrm{Op}_\hbar(A)$.

The second property is a direct consequence of the Egorov theorem (1.5).

Starting from the family of quantum stationary modes $(\psi_n)_n$, we have constructed one or several probability measures $\mu$ on $S^*X$, invariant through the classical flow. Each of them describes the asymptotical localization properties of the modes in some subsequence $(\psi_{n_j})_{j\geq 1}$.

### Is any invariant measure a semiclassical measure?

On a general Riemannian manifold $X$, the geodesic flow admits many different invariant probability measures. The Liouville measure, defined as the disintegration on the energy shell $S^*X$ of the symplectic volume $dxd\xi$, is a "natural" invariant measure on $S^*X$. We will denote it by $L$ in the following. Furthermore, each periodic geodesic carries a unique invariant probability measure. The chaotic flows we will consider admit a countable set of periodic geodesics, the union of which fills $S^*X$ densely.

Given a manifold $(X,g)$, we are led to the following question: Among all $g^t$-invariant probability measures on $S^*X$, which ones do actually appear as semiclassical measures? Equivalently, to which invariant measures can the Wigner measures $(\mu_n)$ converge to in the high-frequency limit?

At the moment, the answer to this question for a general manifold $X$ is unknown. We will henceforth be less ambitious and restrict ourselves to geodesic flows satisfying well-controlled dynamical properties: the *strongly chaotic systems*.

## 2 Chaotic geodesic flows

The word "chaotic" is quite vague so we will need to provide more precise dynamical assumptions. All chaotic flows we will consider are *ergodic* with respect to the Liouville measure. This means that $S^*X$ cannot be split into two invariant subsets of positive measures. A more "physical" definition is the following: the trajectory starting on a *typical* point $\rho \in S^*X$ will cover $S^*X$ in a uniform way at long times (see Figure 2) so that "time average equals spatial average".

The "stadium" billiard (see Figure 2) enjoys a stronger chaoticity: *mixing*, meaning that any (small) ball $B \subset S^*X$ evolved through the flow will spread uniformly throughout $S^*X$ for large times. The strongest form of chaos is reached by the geodesic flow on a manifold of negative curvature; such a flow is *uniformly hyperbolic* or, equivalently, it has the Anosov property [3]. All trajectories are then unstable with respect to small variations of the initial conditions. Paradoxically, this strong instability leads to a good mathematical control on the long time properties of the flow. Such a flow is fast mixing with respect to $L$.

Numerical computations of eigenmodes are easier to perform for Euclidean billiards than on curved manifolds; on the other hand, the semiclassical analysis is more efficient in the case of boundary-free compact manifolds so most rigorous results below concern the latter.

### Quantum ergodicity

Ergodicity alone already strongly constrains the structure of the high-frequency eigenmodes: *almost all* of these eigenmodes are equidistributed on $S^*X$.

**Theorem 2.1** (**Quantum ergodicity**). *[22, 24, 9] Assume the geodesic flow on $(X,g)$ is ergodic with respect to the Liouville measure on $S^*X$.*

*Then, there exists a subsequence $(n_j) \subset \mathbb{N}$ of density 1 such that the Wigner measures of the corresponding eigenmodes satisfy*

$$\mu_{n_j} \xrightarrow[j\to\infty]{} L.$$

The phrase "of density 1" means that $\frac{\#\{n_j \leq N\}}{N} \stackrel{N\to\infty}{\to} 1$. Therefore, if there exists a subsequence of eigenmodes converging towards a semiclassical measure $\mu \neq L$, this subsequence must be sparse and consist of **exceptional eigenmodes**.

### "Scars" and exceptional semiclassical measures

Numerical computations of eigenmodes of some chaotic billiards have revealed interesting structures. In 1984, Heller [15] observed that some eigenmodes of the "stadium" billiard (the ergodicity of which had been demonstrated by Bunimovich) are "enhanced" along some periodic geodesics. He called such an enhancement a "scar" of the periodic geodesic upon the eigenmode (see Figure 2). Although it is well-understood that an eigenmode can be concentrated along a *stable* periodic geodesic, the observed localization along *unstable* geodesics is more difficult to justify. The enhancement observed by Heller was mostly "visual"; the more quantitative studies that followed Heller's paper (e.g. [5]) seem to exclude the possibility of a positive probability weight remaining in arbitrary small neighbourhoods of the corresponding geodesic. Such a positive weight would have indicated that the corresponding semiclassical measures "charge" the unstable orbit (a phenomenon referred to as "strong scar" in [21]). Contrary to the case of Euclidean billiards, numerical studies on surfaces of constant negative curvature have not shown the presence of "scars" [4].

On the mathematical level, the most precise results on the localization of eigenmodes are obtained in the case of certain surfaces of constant negative curvature enjoying specific *arithmetic symmetries*, called "congruence surfaces". A famous example is the modular surface (which is not compact). For these surfaces, there exists a commutative algebra of self-adjoint operators on $L^2(X)$ (called Hecke operators), which also commute with the Laplacian. It is then reasonable to focus on the orthonormal bases formed of joint eigenmodes of these operators (called Hecke eigenmodes). Rudnick and Sarnak have shown [21] that semiclassical measures associated with such bases cannot charge any periodic geodesic ("no strong scar" on congruence surfaces). This result, as well as the numerical studies mentioned above, suggested to them the following

**Conjecture 2.2 (Quantum unique ergodicity).** *Let $(X,g)$ be a compact Riemannian manifold of negative curvature. For any orthonormal eigenbasis of the Laplacian, the sequence of Wigner measures $(\mu_n)_{n\geq 0}$ admits a unique limit (in the weak topology), namely the Liouville measure.*

This conjecture goes far beyond the non-existence of "strong scars". It also excludes all the "fractal" invariant measures.

This conjecture has been proved by E. Lindenstrauss in the case of compact congruence surfaces, provided one only considers *Hecke eigenbases* [18]. The first part of the proof [7] (which relies heavily on the Hecke algebra) consists of estimating from below the *entropies* of the ergodic components of a semiclassical measure. We will see below that the entropy is also at the heart of our results.

### The role of multiplicity?

*A priori*, there can be multiple eigenvalues in the spectrum of the Laplacian, in which case one can make various choices of orthonormal eigenbases. On a negatively curved surface, it is known [6] that the eigenvalue $k_n^2$ has multiplicity $\mathscr{O}\left(\frac{k_n}{\log k_n}\right)$ but this is far from what people expect, namely a uniformly bounded multiplicity. One could modify Conjecture 2.2 so that the statement holds for a given basis (e.g. a Hecke eigenbasis in the case of a congruence surface) but may be false for another basis.

In parallel with the study of chaotic geodesic flows, people have also considered toy models of discrete time symplectic transformations on some compact phase spaces. The most famous example of such transformations is better known as "Arnold's cat map" on the 2-dimensional torus. It consists of a linear transformation $(x,\xi)\to M(x,\xi)$, where the unimodular matrix $M\in SL(2,\mathbb{Z})$ is hyperbolic, i.e. it satisfies $|\mathrm{tr}M|>2$. The "Anosov property" then results from the fact that no eigenvalue of $M$ has modulus 1. That transformation can be *quantized* to produce a family of unitary propagators, depending on a mock Planck parameter $\hbar_N=(2\pi N)^{-1}$, where $N$ is an integer [14]. Such propagators have been named "quantum maps" and have served as a "laboratory" for the study of quantum chaotic systems, both on the numerical and analytical sides.

Concerning the classification of semiclassical measures, the "quantized cat map" has exhibited unexpectedly rich features. On the one hand, this system enjoys arithmetic symmetries, allowing one to define "Hecke eigenbases" and prove the quantum unique ergodicity for such eigenbases [16]. On the other hand, for some (scarce) values of $N$ the spectrum of the quantum propagator contains large degeneracies. This fact has been exploited in [10] to construct sequences of eigenfunctions violating quantum unique ergodicity: the corresponding Wigner measures $\mu_N$ converge to the semiclassical measure

$$\mu=\frac{1}{2}\delta_O+\frac{1}{2}L, \tag{2.1}$$

where $L=dx\,d\xi$ is now the symplectic volume measure and $\delta_0$ is the $M$-invariant probability measure supported on a periodic orbit of $M$.

This result shows that the quantum unique ergodicity conjecture can be *wrong* when extended to chaotic systems more general than geodesic flows. More precisely, for the "cat map" the conjecture holds true for a certain eigenbasis but is wrong for another one.

Another result concerning the "cat map" is the following: the weight $1/2$ carried by the scar in (2.1) is *maximal* [11]. In particular, no semiclassical measure can be supported on a countable union of periodic orbits. In the next section, dealing with our more recent results on Anosov geodesic flows, we will see this factor $1/2$ reappear.

## 3 Entropic bounds on semiclassical measures

In this section, we consider the case of a compact manifold (without boundary) of negative sectional curvature. As mentioned earlier, the corresponding geodesic flow has many invariant probability measures. The **Kolmogorov-Sinai entropy** associated with an invariant measure is a number $h_{KS}(\mu)\geq 0$, defined below. We stress a few important properties:

- A measure supported by a periodic trajectory has zero entropy.
- The maximal entropy $h_{\max}$ is reached for a unique invariant measure, called the Bowen-Margulis measure, of support $S^*X$.
- According to the Ruelle-Pesin inequality,
$$h_{KS}(\mu)\leq\int\sum_{k=1}^{d-1}\lambda_k^+\,d\mu, \tag{3.1}$$
where the functions $\lambda_1^+(\rho)\geq\lambda_2^+(\rho)\geq\cdots\geq\lambda_{d-1}(\rho)>0$, defined $\mu$-almost everywhere, are the positive Lyapunov exponents of the flow. Equality in (3.1) is reached only if $\mu$ is the Liouville measure [17].
- On a manifold of constant curvature $-1$, the inequality reads $h_{KS}(\mu)\leq d-1$. The Bowen-Margulis measure is then equal to the Liouville measure.
- The functional $h_{KS}$ is affine on the convex set of invariant probability measures.

These properties show that the entropy provides a quantitative indication of the *localization* of an invariant measure. For instance, a positive lower bound on the entropy of a measure implies that it cannot be supported by a countable union of periodic geodesics. This is precisely the content of our first result.

**Theorem 3.1.** *(1)* [1] *Let $X$ be a compact Riemannian manifold such that the geodesic flow has the Anosov property. Then every semiclassical measure $\mu$ on $S^*X$ satisfies*

$$h_{KS}(\mu)>0.$$

*(2)* [2] *Under the same assumptions, let $\lambda_j^+(\rho)$ be the positive Lyapunov exponents and $\lambda_{\max}=\lim_{t\to\infty}\frac{1}{t}\log\sup_{\rho\in S^*X}||dg_\rho^t||$ be the maximal expansion rate of the geodesic flow. Then the entropy of $\mu$ satisfies*

$$h_{KS}(\mu)\geq\int\sum_{k=1}^{d-1}\lambda_k^+\,d\mu-\frac{d-1}{2}\lambda_{\max}. \tag{3.2}$$

*In constant curvature $-1$, this bound reads $h_{KS}(\mu)\geq\frac{d-1}{2}$.*

**Corollary 1.** [1] Let $X$ be a compact manifold of dimension $d$ and constant sectional curvature $-1$. Then, for any semiclassical measure $\mu$, the support of $\mu$ has Hausdorff dimension $\geq d$.

In constant negative curvature, the lower bound $h_{KS}(\mu) \geq \frac{d-1}{2}$ implies that at most $1/2$ of the mass of $\mu$ can consist of a scar on a periodic orbit. This is in perfect agreement with the similar result proved for "Arnold's cat map" (see §2).

The right-hand side of (3.2) can be negative if the curvature varies a lot, which unfortunately makes the result trivial. A more natural lower bound to hope for would be

$$h_{KS}(\mu) \geq \frac{1}{2} \int \sum_{k=1}^{d-1} \lambda_k^+ d\mu. \tag{3.3}$$

This lower bound has been obtained recently by G. Rivière for surfaces ($d = 2$) of **nonpositive** curvature [20]. B. Gutkin has proved an analogous result for certain quantum maps with a variable Lyapunov exponent [12]; he also constructed eigenstates for which the lower bound is attained.

From the Ruelle–Pesin inequality (3.1), we notice that proving the quantum unique ergodicity conjecture in the case of Anosov geodesic flows would amount to getting rid of the factor $1/2$ in (3.3).

We finally provide a definition of the entropy and a short comparison between the entropy bound of Bourgain-Lindenstrauss [7] and ours.

**Definition 1.** The shortest definition of the entropy results from a theorem due to Brin and Katok [8]. For any time $T > 0$, introduce a distance on $S^*X$,

$$d_T(\rho, \rho') = \max_{t \in [-T/2, T/2]} d(g^t \rho, g^t \rho'),$$

where $d$ is the distance built from the Riemannian metric. For $\varepsilon > 0$, denote by $B_T(\rho, \varepsilon)$ the ball of centre $\rho$ and radius $\varepsilon$ for the distance $d_T$. When $\varepsilon$ is fixed and $T$ goes to infinity, it looks like a thinner and thinner tubular neighbourhood of the geodesic segment $[g^{-\varepsilon}\rho, g^{+\varepsilon}\rho]$ (this tubular neighbourhood is of radius $e^{-T/2}$ if the curvature of $X$ is constant and equal to $-1$).

Let $\mu$ be a $g^t$–invariant probability measure on $S^*X$. Then, for $\mu$-almost every $\rho$, the limit

$$\lim_{\varepsilon \longrightarrow 0} \liminf_{T \longrightarrow +\infty} -\frac{1}{T} \log \mu\big(B_T(\rho, \varepsilon)\big)$$
$$= \lim_{\varepsilon \longrightarrow 0} \limsup_{T \longrightarrow +\infty} -\frac{1}{T} \log \mu\big(B_T(\rho, \varepsilon)\big) \stackrel{\text{def}}{=} h_{KS}(\mu, \rho)$$

exists and it is called the local entropy of the measure $\mu$ at the point $\rho$ (it is independent of $\rho$ if $\mu$ is ergodic). The Kolmogorov-Sinai entropy is the average of the local entropies:

$$h_{KS}(\mu) = \int h_{KS}(\mu, \rho) d\mu(\rho).$$

**Remark 3.2.** In the case of congruence surfaces, Bourgain and Lindenstrauss [7] proved the following bound on the microlocal lifts of Hecke eigenbases: for any $\rho$, and all $\varepsilon > 0$ small enough,

$$\mu_n\big(B_T(\rho, \varepsilon)\big) \leq C e^{-T/9},$$

where the constant $C$ does not depend on $\rho$ or $n$. This immediately yields that any semiclassical measure associated with these eigenmodes satisfies $\mu(B_T(\rho, \varepsilon)) \leq C e^{-T/9}$, which implies that *any ergodic component of* $\mu$ *has entropy* $\geq \frac{1}{9}$.

In [2], we work with a different, but equivalent, definition of the entropy. On a manifold of dimension $d$ and constant curvature $-1$, the bound we prove can be (at least intuitively) interpreted as

$$\mu_n\big(B_T(\rho, \varepsilon)\big) \leq C k_n^{\frac{d-1}{2}} e^{-\frac{(d-1)T}{2}}, \tag{3.4}$$

where $k_n^2$ is the eigenvalue of the Laplacian associated with $\psi_n$. This bound only becomes non-trivial for times $T > \log k_n$. For this reason, we cannot directly deduce bounds on the weights $\mu(B_T(\rho, \varepsilon))$; the link between (3.4) and the entropic bounds of Theorem 3.1 is less direct and uses some specific features of quantum mechanics.

## Notes

1. Wentzel-Kramers-Brillouin.
2. The code used to compute the stadium eigenmodes featured in this article was written and provided by Eduardo Vergini [23].
3. This bundle is often called "phase space". It consists of the pairs $(x, \xi)$, where $x \in X$ and $\xi \in \mathbb{R}^d$ is the coordinate of a covector based at $x$, representing the momentum of the particle.

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# Pythagoras, Facts and Legends*

António Machiavelo

**Abstract.** The result of the so called Pythagorean theorem was already known well before Pythagoras was born. Why, then, is it attributed to Pythagoras? And how was it discovered?

The image we all have of Pythagoras is of one of the first great mathematicians, that lived immediately after Thales of Miletus, of which he supposedly was a student, and that he was the discoverer that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. It is therefore highly probable that the reader will be rather shocked with the announcement that a great deal of what he or she knows about Pythagoras was fabricated, to some extent deliberately!

As one can read in the page on Pythagoras of the *Stanford Encyclopedia of Philosophy*[1], the oldest known evidence shows that Pythagoras' fame, during his lifetime and stretching as far as up to 150 years later, at the time of Plato and Aristotle, had nothing to do with Mathematics. Rather, Pythagoras was then famous as a specialist on the soul and religious ritual, as well as a wonderworker[2], and as the founder of a rigorous and disciplined way of life[3].

The most detailed and lengthy accounts on the life and thought of Pythagoras, which are responsible for the image we have of him today, were written about 800 years after he lived, and in works whose aim was not quite to report, objectively and rigorously, historical facts. As Jean-François Mattei remarks[4]:

*All these traditional texts are composed in a literary genre well known in the hellenic epoch that idealizes, with edifying purposes, the moral portrait of the Sage or of the religious orders...*

Even worse[5]:

*These […] accounts of Pythagoras were in turn based on earlier sources, which are now lost. Some of these earlier sources were heavily contaminated by the Neopythagorean view of Pythagoras as the source of all true philosophy, whose ideas Plato, Aristotle and all later Greek philosophers plagiarized.*

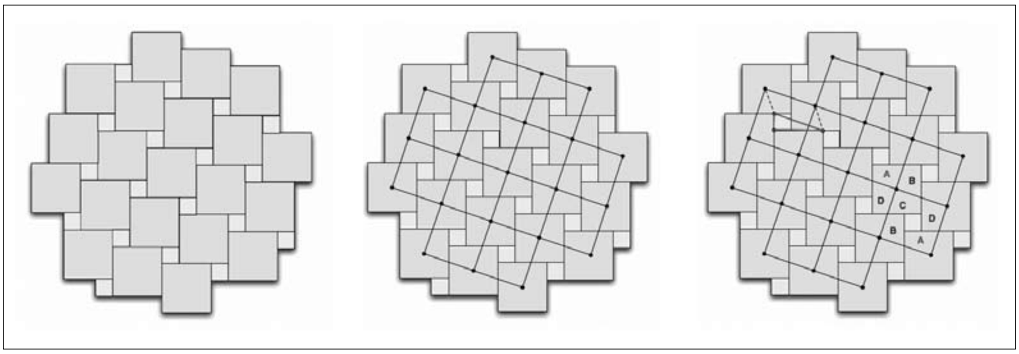
That is, more or less fanatic disciples distorted accounts and invented stories, going as far as forging documents, so as to display Pythagoras superiority with respect to subsequent philosophers.

The philosopher and theologian Eduard Zeller[6] (1814–1908), author of the scholarly *Die Philosophie der Griechen*[7], had already noted that the further removed from the period in which Pythagoras lived, the more detailed are the accounts on pythagorism and its founder, an expansion that reflects dogmatic preconceptions and ideological interests with propagandistic goals. As a result, it is not at all easy to find out, in particular, if Pythagoras had or not anything to do with Mathematics. To get a good grasp on how hard is to separate fact from legend and pure fabrication, nothing better than to read section 5 of the already mentioned document of the *Stanford Encyclopedia of Philosophy*, as well as the review to two recent books on Pythagoras by Myles Burnyeat[8], entitled "*Other Lives*"[9], published in the *London Review of Books* (vol. 29, nº 4, 2007). Better yet it would be to read, if possible, the works cited in those two documents.

On the other hand, it is quite well known that the "Pythagorean" theorem was, in some sense, known in ancient Egypt, in ancient Mesopotamia[10], India and China, hundreds of years, even millennia, before Pythagoras was born[11]. It is possible that this result was discovered more than once, independently, although there are some that argue for a common origin, of which there is no record, only some clues that are far from conclusive. But how was it discovered? On this question the historical record is completely silent.

* This article is a translation, slightly revised, of an article published in Gazeta de Matemática of the Portuguese Mathematical Society.

1 Available at http://plato.stanford.edu/entries/pythagoras
2 The "historical" sources through which we "know" Pythagoras contain descriptions of miracles and prophecies that are supposed to show the supernatural powers of this "god-man".
3 With strict rules, one of the most famous (whether true or not) being the interdiction to eat fava beans. On this topic, see the curious paper "*Favism – from the ‚avoid fava beans' of Pythagoras to the present*", in the journal of the Hellenic *Haema*tology Society, Haema (2004, vol. 7, pp. 17–21), as well as the curious note by George Discombe on the correspondence section of the *British Medical Journal* of August 5, 1961, p. 385, both available online.
4 In the book "*Pythagore et les Pythagoriciens*", Presses Universitaires de France, collection "*Que sais-je*?", no. 2732, 1993, p. 8.
5 See section 2.2 of the Stanford *Encyclopedia of Philosophy* page on Pythagoras.
6 Quoted by Jean-François Mattei, in the book mentioned above (p. 7), and by Walter Burkert in "Lore and Science in Ancient Pythagorism", Harvard University Press, 1972 (the german original is from 1962), pp. 2–3.
7 Written between 1844 and 1852, and continuously enlarged and amended up to its last edition in 1902. Available, free, at http://books.google.com
8 Laurence Professor of Ancient Philosophy at Cambridge University from 1984 to 1996.
9 Available at http://www.lrb.co.uk/v29/n04/burn02_.html
10 Several greek authors state that Thales travelled to Egypt, from where he brought the science of Geometry to Greece, and that Pythagoras travelled and received teachings from Egypt and Mesopotamian sages. True or not, these stories show the high regard that ancient greeks had for those two civilizations.
11 See the books by Bartel van der Waerden "*Science Awakening*", originally published in dutch in 1950 and translated into english in 1954, and "*Geometry and Algebra in Ancient Civilizations*" (Springer, 1983), as well as the paper by Abraham Seidenberg "*The Origin of Mathematics*", Archive for History of Exact Sciences 18 (1978) 301–342.



Tiling containing a proof of the "Pythagorean Theorem"

There are only a few speculations, some inventions and lots of nonsense that one can even read in some textbooks whose authors ought to be a little more careful.

One of those speculations, a little unfortunate and somewhat poor from an epistemological point of view, is due to Moritz Cantor (1829–1920) who puts it forward in his monumental treatise *Vorlesungen über Geschichte der Mathematik*[12], rightly considered the founding work of the History of Mathematics as a rigorous discipline, and a truly remarkable book. Cantor conjectures (vol. 1, pp. 105–106) that the ancient egyptians could have known the right triangle with sides 3, 4 and 5, that they could have used to make right angles with ropes on which a certain unit of length would be marked off at equally spaced intervals with knots. This, Cantor conjectured, would help them on the various impressive constructions that survived for millennia. There is, however, not a shred of evidence to back Cantor's hypothesis, that was harshly criticized by Bartel van der Waerden in the preface to his book "*Science Awakening*". Note that it remains to be explain how the Egyptians would have discovered that peculiarity of the triangle with sides 3, 4, and 5. Regrettably, because Cantor's work was (quite deservedly, it should be emphasized!) so influential, and because some people have a craving for certitude were none can be had, this conjecture became fact, repeated to exhaustion in other works and uncountable textbooks as something certain and known.

More recently, Paulus Gerdes, from the Universidade Pedagógica in Maputo, Mozambique, has put forward[13] a much more plausible conjecture, that leaves less to be explained. After pointing out the widespread use of spirals in egyptian ornamentation, Gerdes suggests that in order to draw certain rather complex spiral motives, the artisans could have used grids composed with two squares of different sizes to help with the task. These grids make up tilings as the one illustrated on the left image of figure 1. By joining, with straight lines, the centers of the bigger squares, one obtains a figure (central image of fig. 1) that contains, simultaneously, the statement and the proof of "Pythagoras' theorem"! Indeed, as suggested in the image on the right of fig. 1, the sides of the original squares are the legs of a right triangle whose hypotenuse is precisely the side of the squares obtained when one joins the centers just mentioned (see the left upper corner of the image on the right in fig. 1). On the other hand, each one of these latter squares contains one of the smaller squares of the original tiling, as well as four pieces that exactly make up one of the bigger squares (see the right lower corner of the image on the right in fig. 1). The Pythagorean theorem immediately follows. As Gerdes points out, egyptian artisans had to make similar constructions for millennia, which makes it very probable that during that whole time someone, or some of them, could have observed that they contained something interesting: a square containing two squares in a rather curious manner...

Throughout his book "*African Pythagoras*", as well as in "*Geometry from Africa: mathematical and educational explorations*"[14], Gerdes gives lots of examples of situations in which the work of an artisan leads to, more or less naturally, the statement and a proof of the theorem that certainly is not from Pythagoras!

But why, it could now be asked, do the legends and untrue stories on Pythagoras persist? I do not know the exactly right answer to this question, but I guess it must have something to do with the fact that it is so much easier to repeat simplistic legends than to tell complicated truths. And also with hero worship, the deification of leaders, leading to the excessive importance that historical accounts tend to ascribe to individuals, that are glorified. In this way more attention is given to "who" rather than "how". It is wholly ironic that Pythagoras, who founded a community in which goods and knowledge were common property, is credited with all the intellectual production of that community, of some people that lived after he died, and even some that lived long before!

12 "*Lessons on the History of Mathematics*", freely available online at http://www.archive.org (searching for "Moritz Cantor", for example).

13 In his book "*African Pythagoras: a study in culture and mathematics education*", Instituto Superior Pedagógico, Maputo/ Beira, Moçambique, 1994 (first published in portuguese in 1992). Gerdes had already formulated alternative hypothesis to the one by Moritz Cantor in "*Cultura e o Despertar do Pensamento Geométrico*" (Culture and the Awakening of Geometrical Thinking), Instituto Superior Pedagógico, 1991, which is an abridged version of his 1985 Ph.D. thesis.

14 Published by *The Mathematical Association of America*, in 1999.

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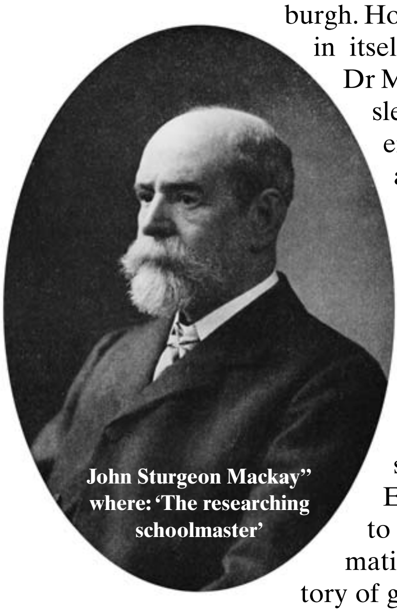
# Death of a Schoolmaster

Marit Hartveit

*The Edinburgh Mathematical Society has, since its foundation in 1883, provided its members with lectures on advanced mathematics and, since 1884, the publication 'Proceedings of the Edinburgh Mathematical Society'. The society is today a research society mainly for academics. It started out rather differently and the path towards the current state was not necessarily the obvious route. This article will sketch how the society was shaped into the society it is today.*

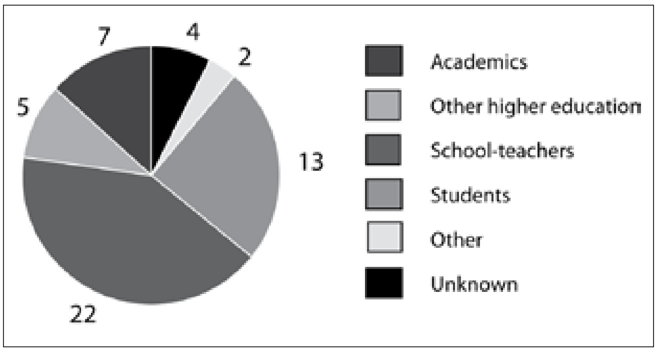
## The researching schoolmaster

Dr John Sturgeon Mackay would have been a rather unusual man by today's standards. He was the head mathematical master at Edinburgh Academy, which at the time was the most prestigious school for boys in Edinburgh. Holding that position was in itself an achievement but Dr Mackay had more up his sleeve. Few schoolteachers of today can boast an honorary doctorate from the University of St Andrews, nor an election to the Royal Society of Edinburgh, yet Dr Mackay could. Such honours were easily justified by his substantial contribution to Euclidean geometry and to the history of mathematics, in particular the history of geometry. He had quite the scholarly knowledge of Greek and Latin and spent years meticulously editing what could have been the first complete edition of Pappus' *Collections.* This Magnum Opus of his never saw the light of day, as Friedrich Hultsch beat him to the finishing line. Undeterred, he kept working and put vast amounts of time and effort into improving mathematical teaching in secondary schools. Amongst the fruits of his labour were several textbooks, the most important being *Elements of Euclid* (1884) and *Plane Geometry* (1905) [G].[1]



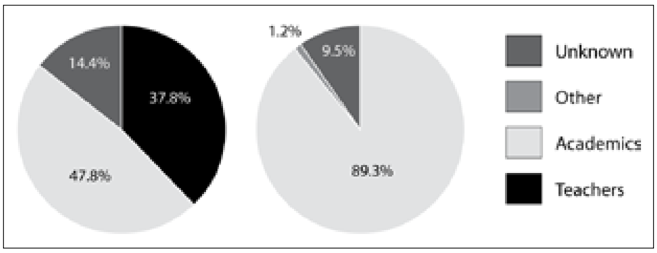
John Sturgeon Mackay" where: 'The researching schoolmaster'

As useful as these two textbooks may have been at the time, he would perhaps have been entirely forgotten today had it not been for his work in the Edinburgh Mathematical Society. He was involved with this society from the very beginning, when he became its first president. He was not, however, one of the founding fathers; the initiative had in fact come from two other schoolmasters, Alexander Yule Fraser and Andrews J G Barclay, both working at George Watson's College in Edinburgh. They felt there was very little provision in Scotland for those who wished to pursue their studies of mathematics after university. The mathematical foundation they had from university was far from ideal. The mathematics taught in the M.A., which was the degree most students took, left something to be desired. The subjects were more or less fixed, so the mathematical courses had to cater for everyone, even students with little love or understanding for the subject. This unavoidably kept the level of difficulty to a minimum. Fraser and Barclay believed a society with strong ties to the university would solve the problem and so they joined forces with Cargill Gilston Knott. Dr Knott had all the necessary contacts, being as he was the assistant of Professor P. G. Tait, who held the Chair of Natural Philosophy at Edinburgh University.[2]





**Occupation of the founding members**





**Occupation of authors, 1890–95 and 1930–35**

The trio composed a letter, stating their aims and goals, which they proceeded to send out to everyone they thought might take an interest. The noble goal of the 'mutual improvement of [the society's] members in the mathematical sciences, pure and applied' [EMS83] attracted no less than 53 men to the first meeting on 2 February 1883. As one might expect, considering the profession of the founding fathers, teachers were heavily represented

1 For more information on Mackay, see the MacTutor History of Mathematics http://www.gap-system.org/~history/Mathematicians/Mackay_J_S.html.

2 Natural philosophy would eventually evolve into physics.

with 22 of the 53. It should be noted that the society was not aimed specifically at teachers. It was open to everyone who wished to improve themselves in the mathematical sciences, provided they had the necessary background, and the reason for the large proportion of teachers was simply the job market at the time. There were precious few job opportunities for a graduate who wished to do research or even keep in touch with current mathematics. Scotland had four universities, each with one professor of mathematics and one or two assistants. Even when including the natural philosophers, the number of positions was only between 15 and 20 and these positions were very rarely available. There were other institutions for higher education but the vast majority of the graduates had to look elsewhere for work and many highly skilled mathematicians chose a teaching career out of necessity rather than anything else. Many of these then joined the society and, although most of them participated as audience only, a fair few took a more active role by giving talks and writing papers. Dr Mackay was certainly the most prolific of the lot, with well over 30 papers published. Other teachers deserve a mention too, such as R.F. Muirhead and the rather more famous Thomas Muir (later Sir), who was working at Glasgow High School at the time. He would eventually take up positions in South Africa, where he finished his *History of Determinants* [A].[3]

This predominance of teachers was not to last and by the 1930s things had changed radically. Although the actual number of teachers had increased, it had not increased as much as one might expect, considering how the number of teachers in Scotland had soared [Cr]. The number of academic members had increased a lot more and so the percentage of teacher members was now almost halved. However, the real change between the late 19th century and the 1930s does not become apparent until we consider how active the groups were. Between 1890 and 1895, teachers were responsible for almost 40% of the papers in the proceedings, albeit most of them being written by three authors. Between 1930 and 1935, the corresponding figure was 0%.

The teachers were slowly and steadily retreating from society life. This process had been going on for quite some time and the reasons for this can, although fairly complex, be narrowed down to two major events. The first thing to happen was the arrival of Edmund Taylor Whittaker (later Sir) in 1912, when he took up the Chair of Mathematics at Edinburgh University [Ma].

### New times

Professor Whittaker was one of the leading figures of his day. Not only was he a first-class researcher, who contributed greatly to bringing British mathematics up-to-date, but he was also unusually skilled at communicating new research to others [Mc].[4] He spurred the society forwards, turning the focus to current, more advanced mathematics. This had an unfortunate effect on the average teacher member who found it very difficult to keep up. The talks would no longer explain the underlying theory but rather assume a certain level of knowledge in the audience. This would not have been as great a problem in the early days, as the teachers and academics had a common meeting-ground in Euclidean geometry, which formed a major part of the curricula for both groups. By 1930, the universities hardly taught any Euclidean geometry at all but the schools still did and so the common interest had almost disappeared.

**Sir Edmund Taylor Whittaker**

This leads on to the second event, which was the exodus of the researching teacher, aptly symbolised by the death of Dr Mackay in 1914. Dr Mackay and his like-minded colleagues would most likely have been able to keep up with the new mathematics, as their own research interests covered more than just Euclidean geometry. The difference was that people like Mackay no longer went into school teaching. For one, they did not have to; the Scottish universities combined had more than tripled their mathematical staff, and Edinburgh and Glasgow had quadrupled theirs. This would efficiently have skimmed the top off a student body that would otherwise have been forced into school teaching.

One would perhaps expect that the increased student body would counter this and leave approximately the same percentage of excellent students out of academia. However, there appears to have been a change of attitude amongst the students, in particular amongst those considering the teaching profession. There was now quite a different breed of teachers. One aspect of this would be teacher training. Although it was not a novel idea, it was fairly new that everyone had to go through it [Cr]. All the new secondary school mathematics teachers in the 1930s would have an honours degree in mathematics with an added year of teacher training. Although this system was excellent for securing a minimum level of education and teaching skills, it was criticised for not encouraging further studies [I]. One interpretation of this could be that

[3] For more information, see their biographies at the MacTutor History of Mathematics:
Muirhead – http://www.gap-system.org/~history/Mathematicians/Muirhead.html;
Muir – http://www.gap-system.org/~history/Mathematicians/Muir.html.

[4] For more information on Whittaker, see MacTutor: http://www.gap-system.org/~history/Biographies/Whittaker.html.

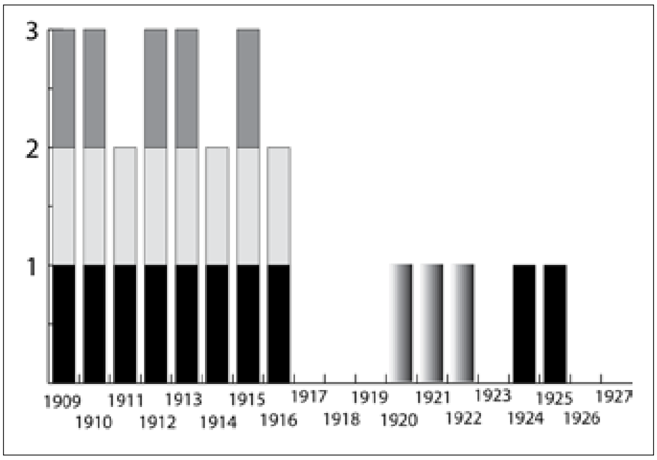
where the teachers from the 19th century had chosen subject first and then profession, the new teachers chose profession first and then subject. This could in turn make the average teacher less interested in new mathematical developments. There were certainly voices within the society that felt the teachers were losing ground because of their indifference.

This indifference could have been an effect of the social conditions at the time. Interwar Europe was arguably not in the most stable of positions, especially not with an ongoing depression. This could lead students to hunger more for a steady job rather than academic excellence.[5] There would, quite simply, be fewer students aspiring to research and the academic world than before.

### A journal for teachers

These factors do not fully explain why the teachers were leaving. No society wants to lose half of its members and the Edinburgh Mathematical Society certainly did not. They might have conquered these obstacles and managed to keep a high percentage of both professions and, in fact, they made a very decent attempt. The problem with the increasing level of mathematics was present even before Whittaker's arrival, though on a smaller scale. In an attempt to please the teachers, the society decided to establish a teacher-friendly journal. In 1909, the first issue of *Mathematical Notes: A Review of Elementary Mathematics and Science appeared*. When it ultimately failed to bridge the ever-increasing gap between the teachers and the academics, most teachers must have felt there was nothing left for them in the society. The circumstances regarding the journal's ultimate demise are therefore crucial to understanding how and why the teachers disappeared.

The Notes was off to a very decent start, enjoying moderate success for several years, before reaching an abrupt stop in 1916. It is perhaps understandable that matters were a little out of the ordinary around that time but it is less clear why the Notes never picked up the pace again after that; no proper issue was then published until 1924. Three batches of notes appeared between 1920 and 1922 but these were simply stuck in at the back of the Proceedings. All in all, the Notes quite suddenly changed into a journal of no consequence to anyone and was, as a committee member put it, 'for all practical purposes, dead' [Co].




**Issues of the *Mathematical Notes* 1909–1927**

### The long struggle

As the minute books from the committee meetings show, this was not taken lightly by several of the committee members, especially not by Professor Thomas M. MacRobert of Glasgow University [EMS]. He, and others with him, argued that the society owed its very existence to the teachers and they felt obliged to provide something for them. In early 1927, a long and arduous debate began that would not end for the next four years. It originated with a discussion on publishing policy. The aforementioned gap had just been expanded even further by the new custom of accepting papers for the Proceedings that had not been given to the society as talks. This allowed the papers to become even more technical, which made the teachers lose ground faster than ever before. The committee agreed that something had to be done and decided that the way to go was to revive the Notes.

**Thomas Murray MacRobert**

This, however, turned out to be a lot more difficult than they had anticipated.

In the autumn of 1927, Professor MacRobert suggested replacing the Notes with a new journal. He proposed to call it the *Journal of the Edinburgh Mathematical Society* and after much debate, the committee agreed. In May the following year they decided to start issuing the journal in its new form. All seemed settled but something must have happened behind the scenes. When the next issue appeared in 1929, it was under the name of the Notes and not the Journal. The committee never rescinded their decision and there is no explanation for why the Journal did not get to see the light of day.

Despite the setback, Professor MacRobert did not consider the battle lost. He re-launched the idea in 1930, during a process of updating the society's rules. This time

[5] This, at least, was the case with the biology students at St Andrews, as Professor D'Arcy Thompson wrote in a letter to G.T. Bennett at Cambridge, on 8 February 1940 [St Andrews University Library, ms26240].

he met firm opposition. Professor Whittaker raised objections to the name, arguing that it would conflict with *the Journal of the London Mathematical Society*, which was a rather different type of journal. The subsequent committee meetings must have been a trying experience for everyone involved, as hardly anything is reported from them. On the surface, it looks as if they spent the next few months arguing about something as trivial as a name but that was not the core of the struggle. The discussion was really on the future of the society and whether or not it should take an even sharper turn towards pure research. Although he did not say so explicitly at the time, Whittaker believed the time had come for two societies, one for teachers and one for researchers. He did not confess to holding such sentiments until later, when it had all ended in the academic version of an uproar.

The four-year-long debate culminated with MacRobert's resignation from the committee, closely followed by the resignation of the editor of the *Notes*, Dr William Arthur, also a Glasgow academic. Whittaker, possibly having waited for such an occasion, immediately began pushing for two societies. Interestingly enough, this seems to have been the only time that Whittaker actually lost the argument, as the rest of the committee disagreed. The following session saw a complete revision of the intended syllabus and the new version provided only talks of general interest. The committee hoped that this would counter for the failure of the *Notes* and were quite prepared to keep it up if it worked. It did not, and gradually the number of general talks fell, until they struggled to keep it at one a year.

As for the *Notes*, the committee eventually came to see matters from Whittaker's point of view and they made a few attempts to find another organisation that could take over responsibility for publication. They failed at this and grudgingly agreed to keep publishing. The journal limped along, with approximately one issue every second year, before it finally collapsed in 1961.

### Why the teachers left

The main question that remains to be answered is then: exactly why did the Notes fail? If one were to ask the society itself, in 1927, it would have said it was because of a lack of material. This was broadly speaking true but that was a consequence and not a cause. It is hardly surprising that few would consider submitting their articles to a journal that was hardly ever issued. Another explanation is therefore required and it turns out to be this: shortly after the First World War, the cost of printing rose considerably and the society was forced to prioritise. By 1920, the situation had become so severe that the society had to send out an appeal to various businesses around Scotland in order to pay their bills. Incidentally, 1920 was the same year that the *Notes* appeared inside the *Proceedings* for the first time, presumably done as an attempt to save money. By 1927, the society was dependent on financial aid from outside in order to publish anything at all.

This long break created the problem of lack of material. This would remain a problem but it was not the only factor at work. When Whittaker argued for two societies, he had financial aspects in mind. He knew that the financial grants the society had come to rely on required that the society's main activity was novel research. The *Notes* did in general not contain novel research and so, allowing it to become too great an expenditure could make them lose funding. The society did later have the opportunity to publish larger issues of the *Notes* but the offer was rejected for this particular reason.

It would be very unfair to give Whittaker the blame for the departure of the teachers. It is true that it was under his influence that the society turned towards current research and under his guidance that they prioritised the Proceedings over the *Notes* in times of financial crisis but it was all done in the spirit of the founding fathers. The society was not founded to educate the schoolteachers; it was founded to promote higher mathematics. In a sense, the society took over where the university education stopped. When the universities changed their courses and taught more advanced mathematics, the society followed suit. Had Whittaker not steered the ship, the society could easily have swung the other way and become a true teacher organisation. In my opinion, this would have been a much bigger breach with the old ways of the society than the turn towards research. This way, the aim of the society remained the same.

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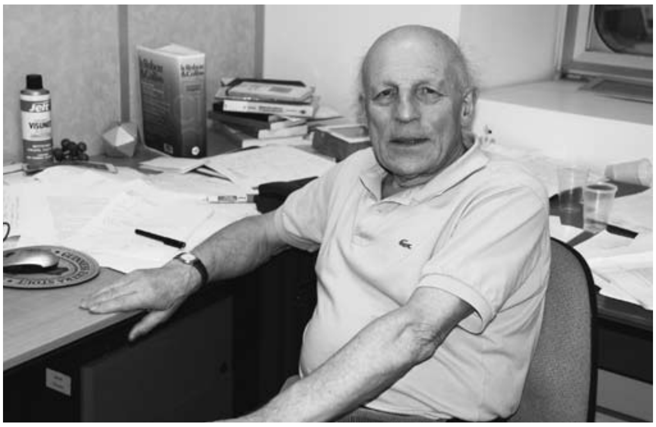
# The Castle of Groups Interview with Pierre Cartier*

Javier Fresán



**Pierre Cartier**

"I have just met a wonderful person. He understands everything, has an insatiable curiosity and an outstanding openness of mind. Indeed, he has a sporty look and comes to Bures by bike." These are the words with which Alain Connes described to his wife his first impression of Pierre Cartier 30 years ago, the same as those he still gives today to the mathematicians who attend his lectures on topics as diverse as operad theory, algebraic groups, multizeta functions and functional integration. A former alumnus of the École Normale Supérieure, where he wrote his thesis with Henri Cartan, Pierre Cartier was deeply involved in Bourbaki's group from 1955 to 1980. In Algebraic Geometry, the duality between abelian varieties and formal groups, as well as the divisors defined by an open cover and a family of rational functions all carry his name.

After finishing his thesis, he stayed for two years at the Institute for Advanced Studies. He then taught at the University of Strasbourg before becoming a professor at the Institut des Hautes Études Scientifiques. A Renaissance man, Cartier is passionate about music, philosophy and literature, as is illustrated by his choice of the title *A Mad Day's Work* for an article exploring the evolution of the idea of space. At the beginning of this interview he goes back to his first *souvenirs d'apprentissage* and talks about Grothendieck, who was a very close friend of his for a long time. In the second part, we approach the not so easy relationship between physics and mathematics and the mysterious "cosmic Galois group" which he named.

## School years

***After your first education at Sedan and the Lycée Saint-Louis, you entered the École Normale in 1950, when Henri Cartan was a professor there. What are your memories of the discovery of real mathematics?***

During my youth, Sedan was destroyed by the war, it was really hard to get food and the school survived in very precarious conditions. When I entered it in 1942, it operated more or less normally but there were very few books. It was a brother of my grandmother and a teacher in the elementary school who taught me the very beginning of algebra and elementary geometry. Very soon after, I was reading all the books I could lay hands on. I remember that I had great difficulty understanding a very bad textbook which explained relativity without any mathematics. It was a completely self-taught work. For instance, I skipped through the encyclopedias looking for articles about mathematics. I remember the entry "Abelian: There are three types of Abelian integrals: first kind, second kind, third kind". But what was an Abelian integral?

At 16, I won the first prize of the *concours général* and a friend of my parents decided to make me a present. He took me to a bookstore in the Latin Quarter, he whispered the amount he would pay for me to the owner and then he left. When he came back two hours later, I had chosen a book on tensor calculus by André Lichnerowicz and the first volume of Bourbaki's topology, which was hard for me to read because I was not used to the notation of set theory. When I arrived in Paris, the education we received at the Lycée Saint-Louis was quite solid but old-fashioned. However, I had good teachers and in my spare time I could read Chevalley's book on Lie groups and the works of Hermann Weyl in German. So when I entered the École Normale, I already had an idea of what the mathematics of the 20th century was about.

## Entering Bourbaki

***At the end of your first year at the École Normale you were invited to a Bourbaki meeting. What was the group like when you joined it for the first time?***

During my first year at the École Normale, I attended all the courses I was able to. Samuel Eilenberg had been invited for the whole year by Henri Cartan, with whom

* This is an extract from the original interview *Le château des groupes. Entretien avec Pierre Cartier*, Prépublications IHÉS, M/09/41. The author would like to acknowledge the generosity of Pierre Cartier, Michael Eickenberg, Cécile Gourgues, David Krumm, Vicente Muñoz and Guillermo Rey Ley during the preparation of the text. He is also grateful to Gerd Fischer and the Archives of the Mathematisches Forschungsinstitut Oberwolfach for the permission to reproduce the photograph with Cartier, Dieudonné and Leray.

**Jean Leray (second from left), Jean Dieudonné (left) and Pierre Cartier (right), 1985. Photograph by Gerd Fischer. Archives of the Mathematisches Forschungsinstitut Oberwolfach.**

he was writing *Homological Algebra*, so I learned a lot from him. But I also took physics and philosophy courses and seminars. At the end of the year I had to choose: Althusser, the Marxist, advised me that it was better to take the mathematics rather than the philosophy exams, Yves Rocard proposed to me that I help build the French atomic bomb and Henri Cartan invited me to a Bourbaki meeting.

It was the beginning of the best years of Bourbaki, when each book added something new and people waited for its release. The initial project ended in 1940 with the publication of the first volumes about Set Theory and Topology. Then the war came and many of the Bourbaki members had to escape from the Nazis. In spite of the very difficult circumstances, the group managed to continue working during those years. In 1950 a new generation, whose natural leader was Serre, had taken control. In that period we had huge ambitions; Bourbaki really wanted to write down all of mathematics. For me it was a dazzling experience; I learned so many things during the week I spent with them. According to Bourbaki's method, we studied reports on the topics which were to be treated in the series. Moreover, it was there where I met André Weil, to whom I kept very close during the rest of his life.

***In fact, two of the founding fathers of Bourbaki, Henri Cartan and André Weil, later become your PhD advisors.***

Officially my advisor was Roger Godement, who was at Nancy, but I was more inspired by the ideas of Cartan and Weil, so I decided to change subject. André Weil had a position in the United States but he came back every summer and I always took the opportunity to explain to him the advances on my thesis. In the winter of 1952, he had given a course on adeles and ideles. I took notes on the lectures and I helped to simplify some proofs and to add some supplementary notions. This allowed me to really get in touch with André Weil.

In addition, I had been reading his book *Foundations of Algebraic Geometry* word by word until the revolution of Serre, after which Weil told me: "I see that my *Foundations* are outdated". The best result of my thesis solved a problem posed by Weil in his book on Abelian varieties and algebraic groups. One day I had an inspiration: I was aware of what Dieudonné was doing with formal groups, I also had in mind the question posed by Weil and I said to myself: "This is linked". I saw it immediately but it took a very long time to prove because it lies on the crystalline cohomology of schemes (to be developed only after 1965).

I have never abandoned Group Theory; for me it is still the central point of everything on which I have worked. In my opinion, a compulsory reading was *Group Theory and Quantum Mechanics* by Hermann Weyl, a text which I still read today with the same interest. Group Theory is crucial in physics, geometry and arithmetic. For me it is like having a fortress: the castle of groups. I can go from here to there and maybe ring another doorbell but I always come back to my castle.

***You were the secretary of Bourbaki from 1970 to 1983, after Dieudonné, Samuel and Dixmier. What was the working method?***

The way of working was well-established: there were one week meetings in spring and in autumn and a two week meeting in the summer. For twenty-five years, I devoted a third of my mathematical career to Bourbaki. It is a lot! We published two new volumes every year and each book was rewritten several times. To start the process of writing, we asked the member who best mastered the topic to make a report on the most important theorems. From there, we planned what we wanted to explain and the relations to what we had already published and what we were going to present in the future. After that, somebody wrote a first draft, which we never liked.

Sometimes the process could take up to eight years, with plenty of changes, until the moment when Dieudonné knocked on the table: "It is finished, it has to be published". Then he took all the documents of the process and in the course of two months he did the synthesis, added all the exercises, sent the book to the printer and corrected the proofs. Dieudonné, with this amazing capacity for work, was the secret of Bourbaki. He said: "I do not work too much. I only write five pages a day between five and eight in the morning". But every day, every week, every year over sixty years, that makes a total of 110,000 pages. In fact, when he left Bourbaki, everything became harder.

## The divorce between physics and mathematics

***To people who talk about the destructive influence of Bourbaki's books, what would you say is the unquestionable heritage of the group?***

The ambition of Bourbaki was to provide an encyclopedia which showed that there is only one mathematics and not several branches. That was the reason why we wanted to write *Élements de mathématique*, in the singular. According to Thomas Kuhn's theory on the structure of scientific revolutions, there always exist two periods: the scientific revolution itself, in which new questions are posed and new methods are invented, and the periods of normalization, when a paradigm is created which lasts until the next revolution. Bourbaki contributed to this stage of normalization, after the great conceptual revolution of mathematics in which Set Theory, Topology, Operator Theory and Modern Algebra were created.

However, when consolidating a theory, sometimes new ideas also appear. For instance, Bourbaki deeply changed Ricci's tensor calculus. In Commutative Algebra, in spite of the fact that 400 pages in the line of Zariski-Samuel were almost in print, Bourbaki restarted everything following the ideas of Serre and Grothendieck on localization, the spectrum of a ring, the filtration and topologies in Commutative Algebra and so on. It is also commonly accepted that the books on Lie theory anticipated the development of p-adic and algebraic groups, which is a landmark of success.

***What about education?***
I am much more critical in that respect. Bourbaki is an encyclopedia, not a textbook. In a religion the founder can be a great man but disciples do not always match up to the master. In the 70s, many extremist disciples of Bourbaki, who were not generally creative mathematicians, wanted to found an education scheme which began, starting even from kindergarten, from the most rigorous basis. The result was the five-year-old girl who went back home saying: "I do not want to be a set". Another consequence of the evident excess of abstraction in pedagogy is that nowadays the balance has moved to the other side and the idea of what a proof means is not considered important. In mathematics there are facts and proofs and both are equally worthy.

***You tried to introduce mathematical physics, a subject in which you have been interested since you were young, to the Séminaire Bourbaki. How do you explain the refusal of many of the members?***
It is really hard for me to understand André Weil's contempt for physics, since he was in Göttingen the year that Quantum Mechanics was born and he admired the work of Riemann, Gauss, Euler and Fermat, who were as much physicists as they were mathematicians. This may be due to the supremacy of experimental over theoretical physics in France, where Mechanics disappeared completely from universities with a reform around 1950. In the *Séminaire*, given my position in the group, I could choose any topic which I wanted, but every time that I lectured on physics I felt some kind of resistance.

In Grothendieck's case it was mostly an ideological reason. For him, who was extremely anarchistic, physics formed part of the military-industrial complex and there was an evident equation 'physics = atomic bomb'. In fact, when he left the IHÉS, in a very rude manner, it was under the pretext of military funding which his institute was receiving. It could have been very easy to keep the peace but Grothendieck did not want it. I remember that I found myself once between him and the police chief of Nize; he wanted to go to prison and the chief refused to arrest Grothendieck!

## Grothendieck

***What was the atmosphere at the IHÉS like during the years in which the* Eléments *and the* Séminaires de Géométrie Algébrique *were written?***
In my opinion there are two miracles which explain Grothendieck's success in Algebraic Geometry. As David Mumford told me humorously, there was on the one hand the Zariski school in the United States, which had obtained many results using the method of resolution of singularities but which had already reached the limit. "We were a group which had problems without methods, and on the other side Grothendieck had methods without problems." So Zariski had the enormous generosity of sending all his students to learn Grothendieck's ideas and the IHÉS became an annex of Harvard and Princeton. The second miracle consisted of a completely improbable collaboration between three very different people: Grothendieck, a prophet who was more interested in general ideas than in the details, Serre, an extremely logical spirit, precise, no nonsense, and Dieudonné, with his extraordinary capacity for work and understanding. The *Éléments* and the *Séminaires de Géométrie Algébrique* are the result of this trio, on which no one would have bet.

***What was the ontological status of categories for Grothendieck?***
For him, categories are the language of mathematics and there is no other: we could not even state the problems solved by Grothendieck without talking about categories and functors. His aim was to prove the Weil conjectures by constructing a homology theory satisfying some categorical properties. Next he invented a concept of which he was prouder: topos, which he considered the last truth about space. In my opinion, the idea of topos is very important but it does not exhaust the conception of space. For instance, Connes' noncommutative geometry is another very good illumination.

Ontological, ontological? Nowadays, one of the most interesting points in mathematics is that, although all categorical reasonings are formally contradictory, we use them and we never make a mistake. Grothendieck provided a partial solution in terms of universes but a revolution of the foundations similar to what Cauchy and Weierstrass did for analysis is still to arrive. In this respect, he was pragmatic: categories are useful and they give results so we do not have to look at subtle set theoretic questions if there is no need. Is today the moment to think about these problems? Maybe… I have just proposed a possible program to give a solid basis to categories at a meeting in Oberwolfach.

***It might not seem so but Grothendieck's biography is much more similar to Simone Weil's than to the life of her brother André.***
It is true. In spite of the fact that Grothendieck had always lived in unbearable material conditions, while Simone Weil came from a wealthy family, many analogies can be drawn between them. Their anarchistic con-

victions and their preoccupation for being close to the poorest is what linked them. Once Grothendieck told me very proudly that his father had been a political prisoner under 17 different regimes. On the other hand, it was a common joke that Trotsky's 4th International had been founded in Simone Weil's apartment.

In the second part of his life, Grothendieck was dragged by a self-destructive behaviour very similar to Simone Weil's, who died at 34 years of age due to badly cured tuberculosis and a refusal to eat. Both had an excessive way of looking at things and lived a totally personal religious experience: in Simone Weil's case it is a blend of Indian religion and Christianity as seen through the Greek prism, and it is close to Buddhism in Grothendieck's.

### In search of the cosmic Galois group

***In your works on the concept of space you have conjectured the existence of a "cosmic Galois group", an idea which Grothendieck would have probably liked. What is the meaning of this group?***

In science you sometimes have to find a word that strikes, such as "catastrophe", "fractal" or "Noncommutative Geometry". They are words which do not express a precise definition but a program worthy of being developed. What allowed me to formulate the notion of "cosmic Galois group" was having kept a very close eye on the advances in mathematical physics throughout all my career. On the one hand, by introducing a new group Connes and Kreimer achieved a completely new reformulation of the problem of regularizing a family of integrals in such a way that the algebraic relations expressing different physical phenomena were preserved. On the other hand, in some works in which I had collaborated, we were interested in series and integrals representing numbers which generalize the powers of $\pi$ and zeta values. When studying relations between these numbers, one encounters the motivic Galois group, which provides a kind of transcendental Galois theory. Inspired by the analogy between the two constructions, I suggested that they are actually the same group. Why are they equal? When explicit calculations are made in physics, the constants which appear are the same which number theory treats. In the Standard Model there is a table of about twenty constants, on which the history of our universe depends in a very precise manner. My dream is a total fusion of these ideas which would permit an interpretation of the Connes-Kreimer group as a symmetry group of these fundamental constants.

***When one looks at your papers, there is an admirable variety of topics and research domains. How do you choose the problems which you will work on? Do you usually think about different questions at the same time?***

It is said that Feynman once explained the following: "It is very easy to be a genius: you have ten problems in mind, you think constantly about them. For each thing that you come across, you wonder *Can this help me?* After some time there will be a box full of theorems". My method is my character: I am curious by nature and everything attracts my attention. Since I have acquired a *savoir faire* in very different directions, when I study a problem I always have several techniques in mind. I am also very interested in questions concerning history and the philosophy of mathematics. André Weil taught me to learn from the mathematicians of the past as if they were our contemporaries. For me the big question is always the same: what guarantees that mathematics tells the truth? How does it tell it? Does it always tell it?

My background, characterised by the contrast between the Alsatian common sense of my grandmother and the somewhat delirious imagination of my father, gave me a huge curiosity for people, countries and books. My 13-year-old grandson once told me: "When I am older I want to be like you. I hope to be as strong as you, travel as much as you and have as many friends as you".

*Javier Fresán [http://jfresan.wordpress.com] began his undergraduate studies at Universidad Complutense de Madrid. He then moved to Paris, where he is now an M2 student at Paris XIII. He is the author of a monograph on Gödel's life and work. He loves literature and has travelled to more than 15 countries.*

# Max Planck Institute for Mathematics in the Sciences, Leipzig

## History

The Max Planck Society (MPG) is a scientific organization that runs 76 Max Planck Institutes (MPIs), mostly in Germany. It covers all fields from natural sciences to life sciences to humanities. The MPG originally developed from the 'Kaiser-Wilhelm-Gesellschaft' (1911–1945). After the Second World War, in February 1948, the society was re-opened by Max Planck under the present name.

Recently, Germany has celebrated the 20th anniversary of its reunification. This event reshaped the scientific landscape in the eastern region. Former Academy institutes were closed down or brought in under other umbrella organizations. Independently of this development, the MPG decided to found about 20 new MPIs in the new region of Germany. Thanks to the personal engagement of the founding director Eberhard Zeidler, who referred to the long tradition of Leipzig in mathematics and physics connected with names such as Felix Klein, Sophus Lie, Leon Lichtenberg, Werner Heisenberg, Erich Kähler, Bartel van der Waerden and others, the 'Max Planck Institute (MPI) for Mathematics in the Sciences' was founded on 01 March 1996. The building is situated close to the centre of Leipzig (Saxony). It is the second MPI in the field of mathematics (the first is the 'MPI for Mathematics' in Bonn, which started in 1980).

Outside view of the department

## Mission and activities

In general, the Max Planck Society (MPG) supports fundamental research. This must be seen in contrast to other organizations like the Fraunhofer Society, which runs institutes with close connections to industry. Correspondingly, the MPI in Leipzig is devoted to fundamental mathematical research. As the name indicates, mathematical areas are of interest that are connected with the natural sciences (this term is understood in a broader sense: besides physics and chemistry, engineering and life sciences are also included).

Having described a huge field, a restriction is given by the size of the MPI. The institute has four working groups (departments) each headed by a director (at the moment not all of the director positions are filled). These working groups comprise postdocs, doctorate students and possibly undergraduate students. In addition, there are guests staying for short or long visits. The administration of the guest program is an important part of our institute. Here, the availability of a guest house nearby is helpful.

Since the MPI cannot confer doctoral degrees by itself, cooperation with the University of Leipzig is important. All directors are also members of the university. In particular, there is the IMPRS (International Max Planck Research School), a graduate school financed by the MPG in cooperation with the University of Leipzig. Although lecturing is not part of the duties at the MPI, courses are offered, in particular for the IMPRS students but also within the university programme.

Besides the four working groups (departments) there are independent Max Planck Research Groups, i.e. young researchers with projects guaranteed with their own budget and personnel. Such positions are either advertised by the MPI or in a more general framework by the MPG.

A dense programme of talks takes place during the week: guest lectures, seminar talks, internal discussions, etc. Furthermore, workshops and conference series are organized at the MPI. The conference room takes up to eighty participants and the institute has a well-equipped library.

**The reading room of the library**

**A lecture hall during a seminar**

### How to make use of the facilities

For the different working groups see http://www.mis.mpg.de/research/groups.html. Information about the actual activities can be found there.

For applications to the guest programme or for postdoc and PhD positions see http://www.mis.mpg.de/institute/application/open-positions.html. In general, PhD applications should be directed to the IMPRS http://www.imprs-mis.mpg.de). Of course, we are also open for scientists with their own financial support who are looking for a host institution.

The publications related to the people of the MPI can be seen at http://www.mis.mpg.de/publications/completelist.html.

A lot more information is to be found on the general website http://www.mis.mpg.de.

# ICMI column, The KLEIN Project

Mariolina Bartolini Bussi

In 2008 the IMU (International Mathematical Union) and the ICMI (International Commission on Mathematical Instruction) commissioned a project to revisit the intent of Felix Klein when he wrote Elementary Mathematics from an Advanced Standpoint, i.e. produce a book for secondary school teachers that communicates the breadth and vitality of the research discipline of mathematics and connects it to the senior secondary school curriculum. It is intended as a stimulus for mathematics teachers, to help them make connections between the mathematics they teach, or can be asked to teach, and the field of mathematics, while taking into account the evolution of this field over the last century. The project is managed by an international Design Team consisting of:

**Michèle Artigue**, Université Paris Diderot, France, President of the ICMI until December 2009.
**Ferdinando Arzarello**, Università degli Studi di Torino, Italy.
**Bill Barton (Chair)**, University of Auckland, New Zealand, President of the ICMI from January 2010.
**Graeme Cohen**, University of Technology, Sydney, Australia.
**William (Bill) McCallum**, University of Arizona, USA.
**Tomas Recio**, Universidad de Cantabria, Spain.
**Christiane Rousseau**, Université de Montréal, Canada.
**Hans-Georg Weigand**, Universität Würzburg, Germany.

The international Design Team for the project met for the first time in June 2009 in Paris. The next meeting of the Design Team will be held in Auckland (NZ) in April 2010. The team confirmed the production of a 300-page book written to inspire teachers to present to their students a more complete picture of the growing and interconnected field represented by the mathematical sciences in today‘s world. We expect that this will be backed up by web, print and DVD resources. The project is expected to take about four years. The book will be neither comprehensive nor definitive of the field. Whatever chapter structure is chosen, the text will emphasize links between branches of the field and generic themes (such as the impact of computing). Insights from mathematics education will not be addressed specifically but will be implicit in many places. The Design Team seeks input from all those working in the mathematical sciences, researchers and educators alike. We welcome written communications but will also be holding several "Klein conferences" around the world where feedback on draft ideas and material can be given and original contributions offered. The actual writing will be done by invited authors of proven experience in expert and inspiring authorship. Anyone wishing to be on a mailing list to be kept up-to-date and receive draft material is invited to send an email in the first instance to the Chair of the Design Team Bill Barton (b.barton@auckland.ac.nz). A website is in the process of being established.

- Comments are invited on the choice of chapter titles:
- Introduction,
- Topic Chapters,
- Arithmetic,
- Logic,
- Algebra & Structures, Geometry,
- Functions & Analysis,
- Discrete & Algorithmic Mathematics,
- Mathematics of Computation,
- Probability & Statistics,
- Theme Chapters,
- Intradisciplinarity (i.e. internal connections),
- Mathematics as a Living Discipline Inside Science and Society, How Mathematicians Work.
- The first associated „Klein Conference" was held in Madeira at the beginning of October 2009 (http://glocos.org/index.php/dm-md/dm-md2009).

## Short News

The Executive Committee of the ICMI (2010–2012).

The last meeting of the present EC was held in Saint Petersburg (RU) at the beginning of September 2009. In January 2010, the new EC, elected on the occasion of ICME11 in 2008 (Monterrey, MX), is appointed. The members are:

**President**:
William (Bill) Barton (New Zealand)

**Secretary-General**:
Jaime Carvalho e Silva (Portugal)

**Vice-Presidents:**
Mina Teicher (Israel)
Angel Ruiz (Costa Rica)

**Members at large**:
Mariolina Bartolini Bussi (Italy)
Sung Je Cho (Korea)
Roger Howe (USA)
Renuka Vithal (South Africa)
Zhang Yingbo (China)

**Ex-officio Members will be:**
Michele Artigue (former President of the ICMI)
László Lovász (President of the IMU)
Martin Grötschel (Secretary of the IMU)
The first meeting of the new EC will be held in Auckland (NZ) next April (2010).

## The ICMI Digital Library Project

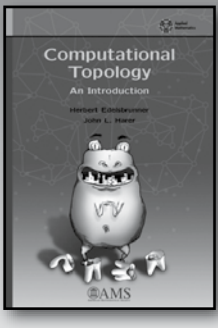
The ICMI Digital Library Project aims to provide open access to ICMI publications. The Commission has been contemplating for a long time making freely available online all the documents produced by the ICMI or on the occasion of the various activities organized under its auspices. This long-term initiative received a major impetus in 2007 when, in the context of the Digitisation Programme of the International Mathematical Union, the ICMI was offered the support of the IMU Committee on Electronic Information and Communication (CEIC) by the IMU Executive Committee, in particular regarding the digitisation of past ICMI publications.

The material to be made accessible via the ICMI Digital Library includes, for instance, the issues of the ICMI Bulletin, the proceedings of all the ICME congresses and the volumes resulting from the ICMI Studies. But it also refers to other publications related to activities supported or sponsored by the ICMI, such as the proceedings of ICMI regional conferences. Access to the ICMI Digital Library is at http://www.mathunion.org/index.php?id=815.

In the next issue of this newsletter, a presentation of ICMI Study 17 on *Mathematics Education and Technology – Rethinking the Terrain* is announced.

*http://www.mathunion.org/icmi*
*Mariolina Bartolini Bussi (member of the Executive Committee of the International Commission on Mathematical Instruction).*

**AMERICAN MATHEMATICAL SOCIETY**




## COMPUTATIONAL TOPOLOGY

### An Introduction

Herbert Edelsbrunner, *Duke University and Geomagic* &
John L. Harer, *Duke University*

Combining concepts from topology and algorithms, this book delivers what its title promises: an introduction to the field of computational topology. It covers motivating problems in both mathematics and computer science.

Jan 2010 241pp
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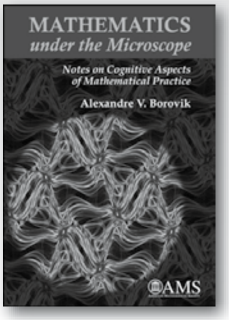


## MATHEMATICS UNDER THE MICROSCOPE

### Notes on Cognitive Aspects of Mathematical Practice

Alexandre V. Borovik, *University of Manchester*

Discusses, from a working mathematician's point of view, the mystery of mathematical intuition: Why are certain mathematical concepts more intuitive than others? And to what extent does the 'small scale' structure of mathematical concepts and algorithms reflect the workings of the human brain?

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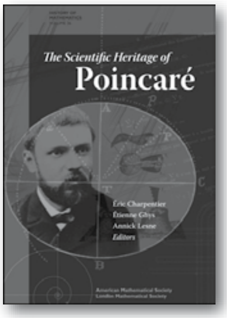


## THE SCIENTIFIC LEGACY OF POINCARÉ

Edited by Éric Charpentier, *Université Bordeaux,* Étienne Ghys, *École Normale Supérieure de Lyon* & Annick Lesne, *Université Pierre et Marie Curie*

Henri Poincaré (1854-1912) was one of the greatest scientists of his time, perhaps the last one to have mastered and expanded almost all areas in mathematics and theoretical physics. In this book, twenty world experts present one part of Poincaré's extraordinary work.

History of Mathematics, Volume 36
Jan 2010 396pp
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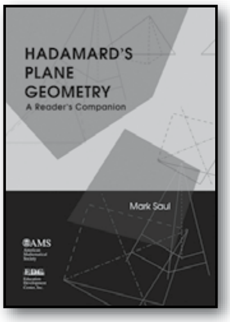


## HADAMARD'S PLANE GEOMETRY

### A Reader's Companion

Mark Saul, *Education Development Center, Newton*

Jacques Hadamard, among the greatest mathematicians of the twentieth century, made contributions to a number of fields. This title offers solutions to the problems in the first part of Hadamard's work (*Lessons in Geometry I Plane Geometry*).

Jan 2010 353pp
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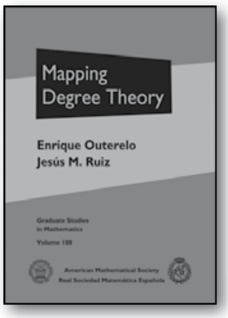


## MAPPING DEGREE THEORY

Enrique Outerelo & Jesus M. Ruiz, *Universidad Complutense de Madrid*

Treats the classical parts of mapping degree theory, with a detailed account of its history traced back to the first half of the 18th century. This title includes 180 exercises and problems of different scope and difficulty.

Graduate Studies in Mathematics, Volume 108
Jan 2010 244pp
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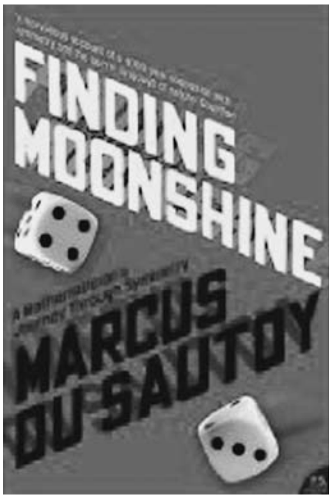
# Book Review

Ulf Persson




Marcus du Sautoy

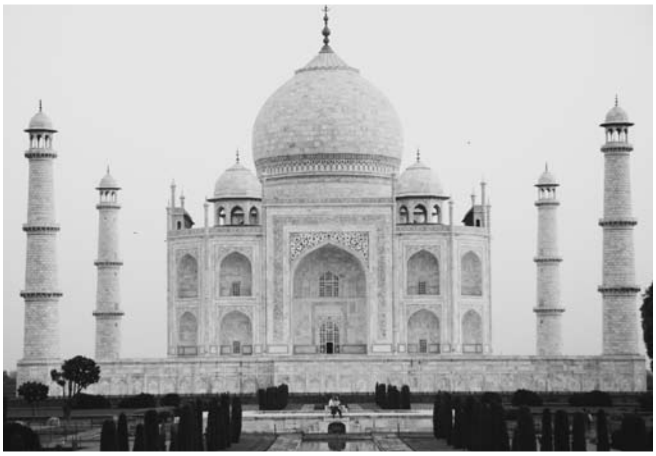
**Finding Moonshine (Mathematicians, Monsters and the Mysteries of Symmetry)**

Harper Perennial
ISBN 978-0-00-721462-4

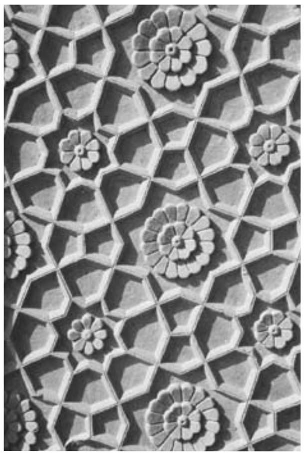
Marcus du Sautoy is sweating. True, he is in the middle of the desert and it is 40 °C but more to the point he is turning 40 years old. If you have not received the Fields medal by then, you will never get it and you are over the hill as a mathematician. But maybe there are other mathematics rewards and du Sautoy is a very successful popularizer of mathematics (which is maybe a distinction more unusual than being a Fields medalist). Not only does he write acclaimed books, such as the one under discussion, but he has also, in classical David Attenborough style, hosted a series on the history of mathematics that was aired on the BBC (with the possibility of similar assignments pending). The latter is far from the experience of the ordinary academic mathematician, who might look upon it with a mixture of haughty disdain and burning envy, but of course du Sautoy is no ordinary mathematician. His career as a popularizer is not even a full-time one; basically, he is a research mathematician like the rest of us, doing popularization on the side. In fact, he is the successor of Richard Dawkins as the Professor of Public Understanding of Science at Oxford. A hard act to follow one might surmise, as mathematics is surely in the public eye rather marginal compared to evolution and religion. On the other hand, we as mathematicians may underestimate the potential interest among the public and it is surely his mission to mine those hidden seams. In *Searching for Moonshine*, his ambition is, as is indicated by the initial lines, to show the public what it really means to be a mathematician – what makes one tick. For that purpose, he has decided to focus on symmetry. An excellent choice! And the fact that his own professional work is intimately related to it also makes it an appropriate one. The notion of number is supposedly an abstraction of cardinality, a step we all take instinctively. Thus the number five can be manifested by any collection of five objects, none of them constituting the notion of five. Similarly, behind every symmetry there is an abstract group and thus groups, like numbers, are 'out there', having a bewildering number of different tangible manifestations. The author does not stoop to give a formal definition of a group and this might irritate the mathematically minded reader but the book is not written for such a reader. Instead, it addresses the general public encouraged to bring it along to the beach. And besides, can you really define the notion of numbers to someone who does not already instinctively know what they are? Why should groups be any different?

Now the subject of symmetries enables the author to explore a number of topics: whether symmetries are beautiful; the connection between mathematics and art; the social structure of mathematicians, in this case illustrated by the quest for the complete classification of simple groups; and, more titillatingly, the peculiar eccentricities of individual mathematicians, exemplified by players such as John Conway and Simon Norton. Tempting as the latter subjects may be, the reviewer would nevertheless prefer to focus on the first, given that space is not unlimited.

Why are symmetries beautiful? Or are they really? In principle, to take the special case of symmetries of the plane, we can pick any fundamental domain and randomly paint or decorate it and then let it be translated by the underlying group. Muslim artists forbidden to explore the animate world instead turned to abstract designs and incidentally discovered the complete list of the seventeen wallpaper patterns. Such abstract designs are to be found all over the world but the 'Mecca' is of course Alhambra, where the author takes his family to explore the various artworks with the specific intention of identifying the underlying symmetry groups, a task, which the author has discovered, that can be quite challenging even to a professional. Is it beautiful? Is it art? Can the abstract beauty of mathematics be made manifest to the non-mathematician by being projected onto visual designs on the walls? Is the randomness within the fundamental domain completely irrelevant? Is it only the abstract symmetry to which we respond? Yet, challenging as it may be to identify the hidden symmetries, once it is done, it is done and the mind moves on, eager for new challenges. The process is really not one of visual appreciation per se but more in the nature of figuring out a rebus or doing a sudoku. Consequently, such work may be admired for the skill in craftsmanship it displays and for a general pleasing appearance but it is not considered great art because, after all, it could in principle, given the basic assumption of the irrelevance of the fundamental domain, be churned out automatically. In short, it is ultimately boring. For a similar reason, the work of Escher, which is so intriguing to the mathematician, is looked down upon by the larger art world. And the brief period of so-called Op-Art in the 60s left no lasting legacy. Surely there is beauty to mathematics but it is too subtle to be so easily manifested in visual terms. There is usually no connection claimed between mathematical ability and visual art. In fact, the



**Taj Mahal**



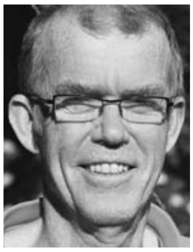
**Detail from Akbars tomb, Sikandra outside Agra**

last time the two really intersected was in the Middle Ages when the laws of perspective were developed and applied. Strictly speaking, perspective is just a technical device to aid in mimesis by tricking the brain yet nobody can deny its lasting influence.

When people think of art and mathematics, music is usually what they have in mind. According to Leibniz, music is unconscious counting. There is an inner logic to music – how else could the young Mozart memorize Gregori Allegri's 'Misere' on one hearing and then write it down flawlessly in defiance of the papal decree that it should on pain of excommunication be kept secret – if he did not perceive its 'invisible' patterns, consciously or unconsciously? What are those invisible patterns of music? Symmetry? The music of Bach abounds in symmetries, which in principle should be faithfully reflected by the musical score, such as palindromic sequences and the like. Once again, one is tempted to speculate that given a random interval of sound, it could be used as a fundamental interval for symmetry to act on. Or more generally, music should be generated mathematically, given a limited input and some basic mathematical principles. Maybe symmetry not so easily appreciated by the eye will find a more congenial reception by the ear? Maybe there is something after all to the hackneyed saying that music and mathematics are intimately related? Maybe mathematics can be manifested by sound alone, music in its linear development over time being a more exacting art form than visual display, which invites the randomly walking gaze. There is supposedly an industry in generating music algorithmically using computers. The schemes for a Bach fugue are (almost) rigid, just like verse meters, and in principle everyone should be able to generate their own Bachian music to be piped at will. But, once again, mathematical beauty is too subtle to be manifested within such straight-jackets, just as musical beauty is too elusive. The mathematical strictures a given piece might voluntarily submit itself to may admittedly contribute to its aesthetics, just as the rhythmic scheme of a meter both challenges the imagination of the poet and enhances the diction and flow; but, as all mathematicians know, necessity is not the same thing as sufficiency. It does matter what you put into the fundamental domain. A poem is not only about sound; it is also, one would like to think, about meaning. The meaning of music – this highly abstract art form, which unlike the visual arts never had any ambition of mere representation (and here we might find at least a similarity with mathematics, at least on the metaphorical level) – is far from obvious (as in mathematics). But even in music symmetry might be too much, at least in a perfect sense. Even the author, carried away by symmetry, admits that he has never been able to really appreciate the Goldberg variations, at least not its rigid symmetry. As Leonard Bernstein once remarked about a piece by Mozart, it is almost symmetrical, it generates the expectation of symmetry, only to disappoint them. And by ultimately disappointing the predictable it becomes exciting.

Maybe the difference between art and mathematics is that the former is sensual. Were it only a matter of manifesting abstract symmetries, the medium would not be important. The symmetry of the musical piece would be exhibited well enough by the score itself and the actual performance would be unnecessary. Somehow the timbre of the sounds produced or the glow of the colours makes a definite difference. Few of us can read a musical score and 'hear' the music come alive in the way that the narration of a book leaps off the page when we scan the letters in a similar way; and definitely no one can appreciate a picture by having the grey scales of each pixel read off linearly.

Maybe mathematics is its own art form and any attempts to represent it sensually by sound or colour is as hopeless as to paint Bach or play Michelangelo.



*Ulf Persson [ulfp@chalmers.se] is Professor of Mathematics at Chalmers University of Technology. He is an algebraic geometer, but is also interested in philosophical aspects of mathematics. Persson is chief editor of the newsletter of the Swedish Math Society (SMS) and the journal 'Normat' of popular mathematics, and also one of the editors of EMS Newsletter.*

There is an extended review of the book avalaible at www.math.chalmers.se/~ulfp/Review/moonshine.pdf

# Personal column

Please send information on mathematical awards and deaths to Dmitry Feichtner-Kozlov (dfk@math.uni-bremen.de).

## Awards

**Ian Stewart** of the University of Warwick has received the Christopher Zeeman Medal for the promotion of mathematics to the public.

The Henri Poincaré Prize 2009 was awarded to **Jürg Fröhlich** (ETH), **Robert Seiringer** (Princeton), **Yakov G. Sinai** (Princeton) and **Cédric Villani** (U. Lyon).

**Kathrin Bringmann**, a number theorist at the University of Minnesota and the University of Cologne, has been awarded the one million Euro prize, for a five-year period, by the Alfried Krupp von Bohlen und Halbach Foundation; furthermore, she was awarded the SASTRA Ramanujan prize for 2009.

**Jean-Loup Waldspurger** (UPMC, France) has received one of the 2009 Clay Research Awards for his work in p-adic harmonic analysis, particularly his contributions to the transfer conjecture and the fundamental lemma.

**Idun Reiten** at the Norwegian University of Science and Technology (NTNU) was recognized with the national Fridtjof Nansen Prize for outstanding research in science and medicine.

The annual Shaw Prize in Mathematical Sciences was awarded to **Simon K. Donaldson** (Imperial College, London) and **Clifford H. Taubes** (Harvard University) "for their many brilliant contributions to geometry in three and four dimensions". The prize carries a cash award of USD 1 million.

**Roger Heath-Brown**, of the University of Oxford, received the Polya prize of the LMS for his many contributions to analytic number theory and his dynamic application of analytic methods in wide-ranging investigations of problems spanning number theory and arithmetic geometry.

**Vladimir Mazya**, of the University of Liverpool, received the Senior Whitehead Prize from the LMS in recognition of his contributions to the theory of differential equations.

**Philip Maini**, of the University of Oxford, was awarded the Naylor Prize and Lectureship in Applied Mathematics from the LMS in recognition of his contributions to, and influence on, the field of mathematical biology.

**Joseph Chuang**, of City University London, and Dr. **Radha Kessar**, of the University of Aberdeen, received the Berwick Prize from the LMS in recognition of their joint paper *Symmetric Groups, Wreath Products, Morita Equivalences and Broué's Abelian Defect Conjecture*, which featured in the Bulletin of the London Mathematical Society.

**Andrzej Schinzel** (Warsaw) and **Józef Siciak** (Kraków) were awarded honorary membership of the Polish Mathematical Society.

**Tadeusz Januszkiewicz** (Wrocław) was awarded the Main Banach Prize of the Polish Mathematical Society for his work on geometric group theory.

**Roman Duda** (Wrocław) was awarded the Main Dickstein Prize of the Polish Mathematical Society for his work on the history of mathematics.

**Alice Guionnet** of the École Normale Supérieure de Lyon has been awarded the 2009 Line and Michel Loêve International Prize in Probability.

**Roberto Car** (Princeton University) and **Michele Parrinello** (Swiss Federal Institute of Technology, ETH-Zürich) have been jointly awarded the 2009 Dirac Medal by the Abdus Salam International Centre for Theoretical Physics (ICTP).

**Guy Brousseau** of the University Institute for Teacher Education (IUFM), Aquitaine, and the University of Montreal has been awarded the first Felix Klein Medal of the International Commission on Mathematical Instruction (ICMI).

**Vladimir Manuilov** (Moscow State University) and **Klaus Thomsen** (Aarhus University) are the winners of the 2009 Canadian Mathematical Society G. de B. Robinson Award.

**Elon Lindenstrauss** (Princeton University) and **Cédric Villani** (ENS de Lyon) have been awarded le prix Fermat 2009 de Recherche de Mathématiques.

**The LMS Whitehead Prizes were awarded to:**
**Mihalis Dafermos**, of the University of Cambridge, for his work on the rigorous analysis of hyperbolic partial differential equations in general relativity.
**Cornelia Drutu**, of the University of Oxford, for her work in geometric group theory.
**Robert Marsh**, of the University of Leeds, for his work on Representation Theory and especially for his research on cluster categories and cluster algebras.
**Markus Owen**, of the University of Nottingham, for his contributions to the development of multiscale modelling approaches in systems medicine and biology.

### Deaths

We regret to announce the deaths of:

**Ignacio Garijo Amilburu** (Spain, 19 September 2009)
**Klaus Dieter Bierstedt** (Germany, 23 May 2009)
**Kathleen Collard** (UK, 2 June 2009)
**Susanne Dierolf** (Germany, 24 April 2009)
**Alberto Dou** (Spain, 18 April 2009)
**Lazar Dragos** (Romania, 2 April 2009)
**Marijke van Ganz** (UK, 21 April 2009)
**Israel Gelfand** (USA, 5 October 2009)
**Israel Gohberg** (Israel, 12 October 2009)
**Frederick de Jong** (USA, 8 April 2009)
**Georg Jähnig** (Germany, 1 April 2009)
**Liliana Pavel** (Romania, 7 February 2009)
**Peter Rado** (UK, 25 June 2009)
**Pierre Samuel** (France, 23 August 2009)
**Ilya Piatetski-Shapiro** (Israel, 21 February 2009)
**Sergey Vinnichenko** (Russia, 29 March 2009)
**James Wiegold** (UK, 4 August 2009)

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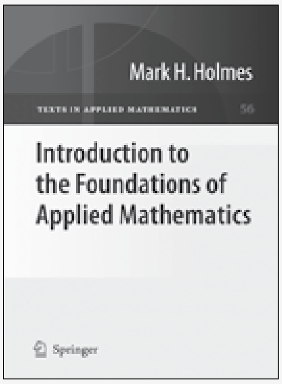
# Highlights in Springer's eBook Collection




The objective of this textbook is the construction, analysis, and interpretation of mathematical models to help us understand the world we live in. Students and researchers interested in mathematical modelling in mathematics, physics, engineering and the applied sciences will find this text useful.

2009. XIV, 470 p. (Texts in Applied Mathematics, Volume 56) Hardcover
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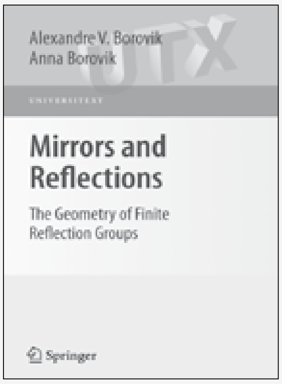


Starting with basic principles, this book provides a comprehensive classification of the various types of finite reflection groups and describes their underlying geometric properties. Numerous exercises at various levels of difficulty are included.

2010. Approx. 185 p. 74 illus. (Universitext) Softcover
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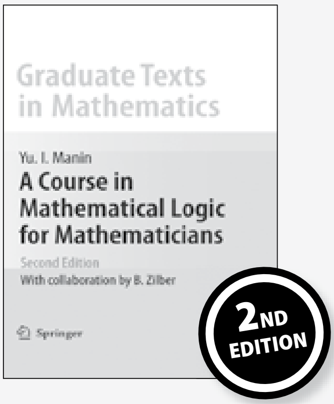


This new edition of the straightforward introduction to mathematical logic retains its appeal to the intuition of working mathematicians, yet along with material from the first edition, it has fresh chapters, one of which deals with Model Theory.

2nd ed. 2010. XVIII, 384 p. 12 illus. (Graduate Texts in Mathematics, Volume 53) Hardcover
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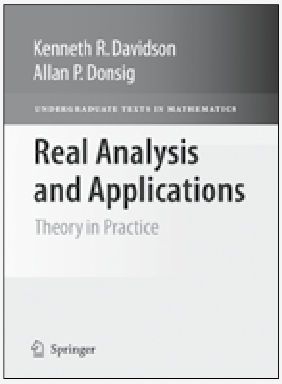


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# Forthcoming conferences

compiled by Mădălina Păcurar (Cluj-Napoca, Romania)

*This is the last issue of the Newsletter of the EMS containing the Forthcoming conferences section. We would like to take the opportunity to thank all those who have contributed to this column by sending their announcements to this section. Information on conferences can still be found on the EMS webpage http://www.euro-math-soc.eu and we encourage everyone to send their announcements of conferences there.*

## December 2009

**4: The Paris-London Analysis Seminar, Paris, France**
*Information:* http://people.math.jussieu.fr/~lerner/index.plans.html

**8–11: Operators and Operator Algebras in Edinburgh**, Edinburgh, Scotland, UK
*Information:* http://www.maths.gla.ac.uk/~saw/ooae/

**9–12: Advanced Course on Algebraic Cycles, Modular Forms, and Rational Points on Elliptic Curves**, CRM, Barcelona, Spain
*Information:* http://www.crm.cat/accycles

**14–18: Meeting on Mathematical Statistics**, CIRM Luminy, Marseille, France
*Information:* colloque@cirm.univ-mrs.fr; http://www.cirm.univ-mrs.fr

**14–18: Workshop on Cycles and Special Values of L-series**, CRM, Barcelona, Spain
*Information:* http://www.crm.cat/wklseries

## January 2010

**1–June 30: Thematic Program on Quantitative Finance: Foundations and Applications**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/09-10/finance/

**11–15: Workshop on Foundations of Mathematical Finance**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/09-10/finance/foundations

**11–16: Topology, Geometry, and Dynamics: Rokhlin Memorial**, Saint Petersburg, Russia
*Information:* http://www.pdmi.ras.ru/EIMI/2010/tgd/

**16–23: 38th Winter School in Abstract Analysis 2010**, Klencí pod Cerchovem, Czech Republic
*Information:* http://www.karlin.mff.cuni.cz/~lhota/

**24–26: International Conference on Analysis and Applications**, Muscat, Oman
*Information:* http://www.squ.edu.om/Portals/87/Conference/ICAA10/Conference2010/ICAA10.html

**25–29: 3rd International Conference on the Anthropological Theory of the Didactic**, CRM, Barcelona, Spain
*Information:* http://www.crm.cat/cdidactic/

**25–30: International School on Combinatorics "Pilar Pisón-Casares"**, Sevilla, Spain
*Information:* http://congreso.us.es/iscpp2010/

**27–28: International Conference on Computer Education, Management Technology & Application (CoEMTA 2010)**, Kathmandu, Nepal
*Information:* http://www.coemta.org/

**28–30: Workshop on Symplectic Geometry, Contact Geometry and Interactions**, Paris, France
*Information:* http://www.math.polytechnique.fr/~viterbo/Workshop2010.html

**30–February 6: Winter School in Abstract Analysis, Section – Topology**, Hejnice, Czech Republic
*Information:* http://www.winterschool.eu/

## February 2010

**8–10: 23rd International Conference of the Jangjeon Mathematical**, Ahvaz, Iran
*Information:* http://en.jmscu.ir/page.aspx?id=29

**12–14: Workshop on General Algebra 79**, Olomouc, Czech Republic
*Information:* http://aaa79.inf.upol.cz/

**15–19: Young Set Theory Workshop**, Seminarzentrum Raach near Vienna, Austria
*Information:* http://www.math.uni-bonn.de/people/logic/events/young-set-theory-2010/

**22–March 5: Advanced Course on Arithmetic Geometry for Function Fields of Positive Characteristic**, CRM, Barcelona, Spain
*Information:* http://www.crm.cat/acarithff/

**25–28: EUROMATH 2010**, Bad Goisern, Austria
*Information:* http://www.euromath.org/index.php?id=21

## March 2010

**17–26: Second International School on Geometry and Physics: Geometric Langlands and Gauge Theory**, CRM, Barcelona, Spain
*Information:* http://www.crm.cat/aclanglands/

**18–20: Workshop on Categorical Topology**, Ponta Delgada, Azores, Portugal
*Information:* http://www.mat.uc.pt/~catop/

**22–24: Workshop on Computational Methods in Finance**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/09-10/finance/computational

## April 2010

**1–2: Workshop on the Theory of Belief Functions**, Brest, France
*Information:* http://www.ensieta.fr/belief2010/

**7–9: Mathematical and Statistical Methods for Actuarial Sciences and Finance (MAF2010)**, Ravello (Amalfi Coast), Salerno, Italy
*Information:* http://maf2010.unisa.it/

**11–18: Algebraic Combinatorics and Applications (ALCOMA10)**, Thurnau, Bayreuth, Germany
*Information:* http://www.algorithm.uni-bayreuth.de/en/projects/ALCOMA10/

**14–18: International Conference on Fundamental Structures of Algebra**, Constanta, Romania
*Information:* http://www.univ-ovidius.ro/math/conference/70/index.htm

**23–24: Workshop on Financial Econometrics**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/09-10/finance/econometrics

## May 2010

**3–6: Statistical Complexity in Classical and Quantum Systems**, Sousse, Tunisia
*Information:* http://rilopezruiz2.spaces.live.com/

**3–7: Advanced Course on Foliations: Dynamics-Geometry-Topology**, CRM, Barcelona, Spain
*Information:* http://www.crm.cat/acfoli/

**23–29: Spring School on Harmonic Analysis**, Paseky nad Jizerou, Czech Republic
*Information*: http://www.karlin.mff.cuni.cz/katedry/kma/ss/may10/

**24–28: Workshop on Financial Derivatives and Risk Management**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/09-10/finance/derivatives

**24–29: 5th International Conference – "Inverse Problems: Modeling and Simulation"**, Antalya, Turkey
*Information:* http://www.ipms-conference.org/index.htm

## June 2010

**7–11: International Functional Analysis Meeting in Valencia on the Occasion of the 80th Birthday of Professor Manuel Valdivia, Valencia**, Spain
*Information*: http://www.adeit.uv.es/fav2010/

**10–12: Geometric and Probabilistic Aspects of General Relativity**, Strasbourg, France
*Information:* http://www-irma.u-strasbg.fr/article874.html

**14–17: 4th Annual International Conference on Mathematics & Statistics**, Athens, Greece
*Information:* http://www.atiner.gr/docs/Mathematics.htm

**17–19: Coimbra Meeting on 0–1 Matrix Theory and Related Topics**, University of Coimbra, Portugal
*Information:* cmf@mat.uc.pt;
http://www.mat.uc.pt/~cmf/01MatrixTheory

**20–25: Analysis, Topology and Applications 2010 (ATA2010)**, Vrnjacka Banja, Serbia
*Information:* http://www.tfc.kg.ac.rs/ata2010

**21–26: "Alexandru Myller" Mathematical Seminar Centennial Conference**, Iasi, Romania
*Information:* http://www.math.uaic.ro/~Myller2010/

**26–30: 2010 International Conference on Topology and its Applications**, Nafpaktos, Greece
*Information:* http://www.math.upatras.gr/~nafpaktos/

**28–July 3: Teichmüller Theory and its Interactions in Mathematics and Physics**, CRM, Barcelona, Spain
*Information:* http://www.esf.org/index.php?id=6305

**30–July 2: The 2010 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, UK
*Information:* http://www.iaeng.org/WCE2010/ICAEM2010.html

**30–July 3: 8th International Conference on Mathematical Problems in Engineering, Aerospace and Sciences (ICNPAA 2010)**, Sao Jose dos Campos (SP), Brazil
*Information:* http://icnpaa.com/index.php/icnpaa/2010

## July 2010

**1–August 30: Thematic Program on the Mathematics of Drug Resistance in Infectious Diseases**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/10-11/drugresistance/index.html

**1–December 31: Thematic Program on Asymptotic Geometric Analysis**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/10-11/asymptotic/index.html

**4–7: 7th Conference on Lattice Path Combinatorics and Applications**, Siena, Italy
*Information:* latticepath@unisi.it;
http://www.unisi.it/eventi/lattice_path_2010

**19–23: Algebra Meets Topology – Advances and Applications**, Barcelona, Spain
*Information:* http://departamento.fisica.unav.es/algebrameetstopology/

**27–31: International Conference on Trends and Perspectives in Linear Statistical Inference (LinStat2010)**, Tomar, Portugal
*Information:* http://www.linstat2010.ipt.pt/?script=1

## August 2010

**2–6: Formal Power Series and Algebraic Combinatorics 2010**, San Francisco, USA
*Information:* http://math.sfsu.edu/fpsac/

**4–7: Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics**, Santa Marta, Colombia
*Information:* http://ima.usergioarboleda.edu.co/SJCH/JCHS2010.htm

**8–11: ICM Satellite Conference on Functional Analysis and Operator Theory**, Indian Statistical Institute, Bangalore, India
*Information:* http://www.isibang.ac.in/~statmath/conferences/icmfasat/icm.htm

**9–13: Workshop on Fluid Motion Driven by Immersed Structures**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/10-11/fluid_motion

September 2010

**7–10: 7th Italian-Spanish Conference on General Topology and its Applications**, Badajoz, Spain
*Information:* http://ites2010.unex.es/

**13–17: Conference on Asymptotic Geometric Analysis and Convexity**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/10-11/asymptotic

**19–23: European Multi-Grid Conference (EMG 2010)**, Isola d'Ischia, Italy
*Information:* http://www.emg2010.unisannio.it/

October 2010

**4–8: Workshop on Concentration Phenomenon, Transformation Groups and Ramsey Theory**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/10-11/asymptotic

November 2010

**1–5: Workshop on Geometric Probability and Optimal Transportation**, Fields Institute, Toronto, Canada
*Information:* www.fields.utoronto.ca/programs/scientific/10-11/asymptotic

# Recent Books

edited by Ivan Netuka and Vladimír Souček (Prague)

*A final word:*
*The first issue of the European Mathematical Society Newsletter was published in September 1991. One year later, in June 1992, a column Brief reviews (called Recent Books later on) was introduced in the journal. In the period 1992–2009, totally 2764 books have been reviewed in the Newsletter. Almost all reviews were written by colleagues from the Faculty of Mathematics and Physics, Charles University in Prague, from the Mathematical Institute of the Czech Academy of Sciences and from several other Czech universities and institutions.*

*This is the last issue where the Recent Books column is published. We would like to take this opportunity to express our sincere thanks to all our colleagues for their contribution to the success of the column. We also greatly appreciate a friendly excellent collaboration with Editorial Teams and all Editors-in-Chief of the Newsletter during the last nineteen years.*

*Ivan Netuka and Vladimir Souček*

***D. V. Alekseevsky, H. Baum, Eds.: Recent Developments in Pseudo-Riemannian Geometry**, ESI Lectures in Mathematics and Physics, European Mathematical Society, Zürich, 2008, 539 pp., EUR 58, ISBN 978-3-03719-051-7*
In Riemannian geometry, important progress has been made over the past thirty years in understanding relations between the local and global structure of Riemannian manifolds. Many classification results for different classes of Riemannian manifolds have been obtained: manifolds with additional geometric structure, manifolds satisfying curvature conditions, symmetric and homogeneous Riemannian spaces, etc. Similar results for pseudo-Riemannian manifolds are rare and many problems are still open. Sometimes, one can use a special 'Ansatz' or 'Wick-rotations' to transform problems of pseudo-Riemannian geometry into questions of Riemannian geometry. But in many aspects, pseudo-Riemannian and Riemannian geometry differ essentially. This book is something like a ‚proceedings' of the scientific program *Geometry of Pseudo-Riemannian Manifolds with Applications in Physics*, which was held in Vienna at the Erwin Schrödinger International Institute for Mathematical Physics between September and December of 2005. The book is addressed to advanced students as well as to researchers in differential geometry, global analysis, general relativity and string theory. It shows essential differences between the geometry on manifolds with positive definite metrics and on those with indefinite metrics, and highlights interesting new geometric phenomena arising naturally in the indefinite metric case. The reader can find a description of the present state of the art in the field, as well as open problems, which can stimulate further research. (zuvl)

***A. Arhangel'skii, M. Tkachenko: Topological Groups and Related Structures**, Atlantis Studies in Mathematics, World Scientific, New Jersey, 2008, 781 pp., USD 183, ISBN 978-90-78677-06-2*
This book has almost 800 pages with 10 chapters, 3 indexes and 549 items in the bibliography. There are many deep results on generalizations of topological groups (like right, semitopological, quasitopological or paratopological ones, as well as semigroups) and even more and deeper results on special classes of topological groups (e.g. compact, free and R-factorizable). The book can be interesting both for beginners (the explanation starts slowly with elementary facts) and for experts in the field. All classical and recent results are included with proofs (which are sometimes new). In addition to classical information, there are chapters on cardinal invariants on topological groups, Moscow topological groups, completions of groups, free topological groups, R-factorizable groups and actions of topological groups on topological spaces. About 90 sections contain exercises and problems, many sections contain open problems and every chapter contains historical remarks. It is a very useful and valuable book. (mihus)

***L. Barreira, Y. Pesin: Nonuniform Hyperbolicity. Dynamics of Systems with Nonzero Lyapunov Exponents****, Encyclopaedia of Mathematics and its Applications 115, Cambridge University Press, Cambridge, 2007, 513 pp., GBP 65, ISBN 978-0-521-83258-8*

This book is a comprehensive account of modern smooth ergodic theory and a representative survey on the theory of dynamical systems with nonzero Lyapunov exponents. In popular terms, this is a book on the mathematical theory of "deterministic chaos". The theory of nonuniformly hyperbolic systems emerged as an independent discipline at the beginning of the 70s. It has a lot of applications in physics, biology, etc. Despite an enormous amount of research in the last few decades, there have been relatively few comprehensible texts covering the whole field until now. This extensive monograph fills the gap. Basic concepts (Lyapunov exponents, cocycles, multiplicative ergodic theorems and methods of estimating the exponents) are defined in the first part. Part II starts with classical, important examples (such as horseshoes) and it then develops nonlinear theory (such as stable and unstable manifolds). Part III contains a discussion of the ergodic theory of smooth and SRB (Sinai-Ruelle-Bowen) measures. Part IV deals with entropy, dimension and other topological properties of hyperbolic measures. The book covers many developments in this important branch of the theory of dynamical systems, mostly with complete proofs and starting from the very beginning. Of course, even in a book of such scope, some important subjects had to be omitted (such as random dynamical systems and chaotic billiards). Historical remarks are added in each section and the bibliography has 250 items. The presented encyclopedia of (nonuniformly) hyperbolic systems will be indispensable for any mathematically inclined reader with a serious interest in the subject. (mzah)

***B. Bekka, P. de la Harpe, A. Valette: Kazhdan's Property (T)****, New Mathematical Monographs 11, Cambridge University Press, Cambridge, 2008, 472 pp., GBP 50, ISBN 978-0-521-88720-5*

The Kazdan 'property (T)' is a certain property of locally compact groups based on the existence of invariant vectors in their continuous unitary representations. The property has been used as a tool to demonstrate that a large class of its lattices is finitely generated. Chapter 1 gives an introduction to the subject, Chapter 2 concentrates on a parallel (and for many groups equivalent) property related to fixed points of the action of the group by affine isometries on a Hilbert space. Chapters 3 and 4 are focused on examples of non-compact groups with 'property (T)'. Chapter 5 is an account of a spectral criterion for 'property (T)'. Chapter 6 contains a small sample of applications (including a construction of expanders and an estimation of spectral gaps of operators, interesting from the point of view of ergodic theory). Chapter 7 is a short collection of open problems. (pso)

***S. Berhanu, P.D. Cordaro, J. Hounie: An Introduction to Involutive Structures****, New Mathematical Monographs 6, Cambridge University Press, Cambridge, 2008, 392 pp., GBP 55, USD 110, ISBN 978-0-521-87857-9*

An involutive sub-bundle $V$ of the complexified tangent bundle $T^cM$ on a manifold $M$ is called locally integrable if its annihilator in the complex cotangent bundle is locally generated by exact differentials. Many problems for locally integrable structures (including local and microlocal regularity and sets of unicity) can be solved using the approximation theorem proved by M.S. Baouendi and F. Treves. This book contains a systematic description of similar results, starting with basic notions of the theory of involutive and locally integrable structures (Chapter 1). The Baouendi–Treves approximation theorem for various function spaces is treated in Chapter 2. The unique continuation property and approximate solutions are discussed in Chapter 3. Properties of locally solvable vector fields are described in Chapter 4. A description and applications of the FBI transform are contained in Chapter 5. Boundary properties of solutions of locally integrable vector fields are studied in Chapter 6. A description of the differential complex associated to an involutive structure and its homological properties is given in the last two chapters. The book ends with an epilogue containing various recent results in the field. The whole book is carefully organized and the reader is expected to have a basic knowledge of real and complex analysis, distribution theory and its use in partial differential equations, and basic facts from several complex variables. The book will be very useful for students wanting to learn the subject and it also introduces interesting recent results for specialists. (vs)

***S.R. Blackburn, P.M. Neumann, G. Venkataraman: Enumeration of Finite Groups****, Cambridge Tracts in Mathematics 173, Cambridge University Press, Cambridge, 2007, 281 pp., GBP 50, ISBN 978-0-521-88217-0*

This mostly self-contained volume is a welcome and well-written addition to the theory of finite groups. The authors present in a unified style, which is accessible to graduate students, an up-to-date account of research concerning the question "how many groups of order n are there?". The book opens with a brief detour into enumeration of semigroups and loops that clearly demonstrates the power of the combination of the associative law and inverses. An investigation of the function $f(n)$ that counts the number of groups of order n up to isomorphism starts in earnest in Part II of the book.

The authors first present Higman's proof of $f(p^m) \geq p^{(2/27)(m-6))m^2}$ and then move onto Sims, Newman and Seeley's upper bound f(pm) $\leq p^{(2/27)m^3+O(m^{(5/2)})}$, where $p$ is a prime. The highlight of the book is the 80 page treatment of Pyber's theorem on the enumeration of solvable and general groups. It is shown that the number of solvable groups of order n with fixed Sylow subgroups $P_1, \ldots, P_k$ does not exceed $n^{8\mu(n)+278833}$, where $\mu(n)$ is the highest power to which any prime divides $n$. From this estimate it is then not hard to conclude that the number of solvable $A$-groups (that is, solvable groups whose nilpotent subgroups are Abelian) of order $n$ does not exceed $n^{8\mu(n)+278834}$. The current proof of the general case of Pyber's theorem depends on the classification of finite simple groups. With this caveat, it is shown that the number of groups of order $n$ with fixed Sylow subgroups $P_1,\ldots,P_k$ does not exceed $n^{(97/4)\mu(n)+278852}$. From this it then follows that $f(n) \leq n^{(2/27)\mu(n)^2+O(\mu(n)^{(3/2)})}$ and that there are at most $n^{(97/4)\mu(n)+278853}$ $A$-groups of order $n$.

The book concludes with estimates on the number of groups in certain varieties. Enumeration of Abelian groups (necessarily combinatorial in nature due to the Fundamental theorem for finitely generated Abelian groups) is followed by enumeration in small varieties of $A$-groups, $d$-generator groups, groups with few non-Abelian composition factors, groups of nilpotency class 3 and other results. The array of techniques needed at various points in the book is impressive and the authors are to be commended for presenting them in a concise yet clear manner. The effort to elucidate ideas behind some of the proofs is impressive, as is the occasional heuristic explanation of results (for instance, why 2/27 appears in the exponent of $f(n)$). Several graduate courses can be designed based on this book, despite the principled decision to avoid exercises. (pvoj)

***A.I. Bobenko, Y.B. Suris: Discrete Differential Geometry. Integrable Structure****, Graduate Studies in Mathematics, vol. 98, American Mathematical Society, Providence, 2008, 404 pp., USD 69, ISBN 978-0-8218-4700-8*

Discrete differential geometry is situated between differential geometry and discrete geometry. Its aim is not only to study smooth objects using discrete methods but also to look, first of all, for discrete analogues of notions and results of differential geometry. Having a discrete object, one naturally tries to pass to a limit of refinement of this object, and to find a return to differential geometry. But what is much more interesting is the influence of discrete differential geometry upon differential geometry itself. Many results of differential geometry can be much better understood, and their proofs can be simplified, when using ideas of discrete differential geometry. The role of integrable systems in differential geometry is well-known. This connection is then even clearer, relating discrete differential geometry and the theory of discrete integrable systems. Discrete differential geometry is a very young branch of geometry and this book covers many results from the last decade. It can serve as a very good introduction into contemporary research and it seems to be the first book devoted to the topic. The authors mention that they wrote this textbook for three categories of readers. The first category comprises graduate students. (The book has already been used for a one semester graduate course. It is interesting that students are not necessarily assumed to have some knowledge of differential geometry.) The second category comprises specialists in geometry and mathematical physics. The third category comprises specialists in geometry processing, computer graphics, architectural design, numerical simulations and animations. The book is well and clearly written. At the end of every chapter are exercises (solutions of some of them can be found in the appendix) and bibliographical *notes*. (jiva)

***D.M. Bressoud: A Radical Approach to Lebesgue's Theory of Integration****, MAA Textbooks, Cambridge University Press, Cambridge, 2008, 329 pp., GBP 24.99, ISBN 978-0-521-88474-7*

This book presents a not-quite-standard approach to measure theory and the theory of the Lebesgue integral. Historical questions that lead to the development of these theories are the motivation for preliminary considerations and form into a starting point of the presentation. In particular, problems of whether the Fourier expansion of a function converges to the function, the relationship between integration and differentiation, and the relationship between continuity and differentiation are discussed at the very beginning of the book. The Riemann integral is presented as the first concept of integration, not only to show its construction but also to show how broadly one could define a function that is still integrable. The story then continues with a description of certain properties of the real axis and problems with the Fundamental Theorem of Calculus and term-by-term integration, finishing with measure theory and the Lebesgue integral, and Fourier series. The way that facts are presented makes the book accessible for graduate or advanced undergraduate students as an alternative to the standard approach of teaching real analysis. The book will be interesting for teachers as well. (mrok)

***O. Brinon: Représentations p-adiques cristallines de de Rham dans le cas relatif****, Mémoires de la Société Mathématique de France, no. 112, Société Mathématique de France,Paris, 2008, 158 pp., EUR 37, ISBN 978-2-85629-250-1*

In this booklet, the author develops a theory of the Fontaine functors $D_{cris}$ and $D_{dR}$ for smooth $p$-adic sheaves over a suitable affine $p$-adic base. The bulk of this work consists of a detailed study of various period rings (BHT, $B_{cris}$ and $B_{dR}$) in this setting. The main result states that – in the good reduction case – the functor $D_{cris}$ is fully faithful. Its essential image is described only for one-dimensional crystalline representations. (jnek)

***T. Camps, V. Rebel, G. Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie****, Berliner Studienreihe zur Mathematik, band 19, Heldermann, Lemgo, 2008, 298 pp., EUR 36, ISBN 978-3-88538-119-8*

This textbook is based on a course on combinatorial and geometric group theory given by the authors at the University of Dortmund. The book is aimed at students of mathematics and computer science of an undergraduate level. The first seven chapters present the basics. It focuses on the Dehn problems and discusses the Nielsen and Reidemeister–Schreier rewriting methods, the Todd–Coxeter coset enumeration method, considering basic constructions as free products and free products with amalgamation, HNN-extensions and groups with one defining relation. This part culminates with the Magnus theorem that every group with one defining relation has a solvable word problem. The following twelve chapters present foundations of geometric group theory. It starts with Lyndon–van-Kampen diagrams and small cancellation theory, followed by the Dehn algorithm in small cancellation groups and the conjugation problem in these groups. These foundations are followed by six chapters on hyperbolic groups. The final short chapter introduces automatic groups. (jtu)

***D. Christodoulou: The Formation of Black Holes in General Relativity****, EMS Monographs in Mathematics, European Mathematical Society, Zürich, 2009, 589 pp., EUR 98, ISBN 978-3-03719-068-5*

Shortly after Einstein's discovery of the gravitation laws, Schwarzschild discovered their particular solution. The Schwarzschild solution, a Lorentzian metric on a four-manifold, depends on one parameter (the so-called mass) and is characterized as the unique spherically symmetric vacuum solution. After a suitable "changes of coordinates", one can remove the so-called non-true singularity of this metric. It was realized that this non-true singularity is actually an event horizon, i.e. a boundary of such a space-time region, the points of which can be linked to "the infinity" via a causal geodesic. In the 60s, R. Penrose formalized the concept of the infinity, introduced the notion of a trapped surface and was able to define

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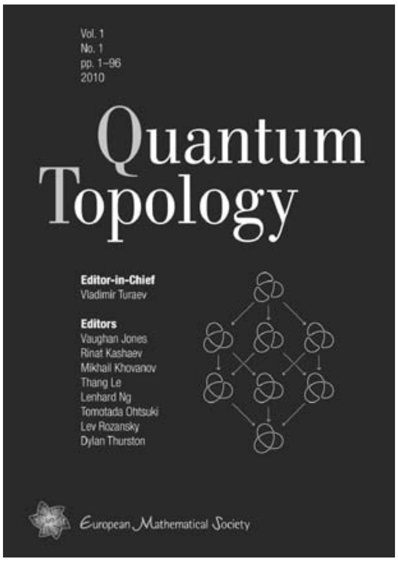


## Quantum Topology

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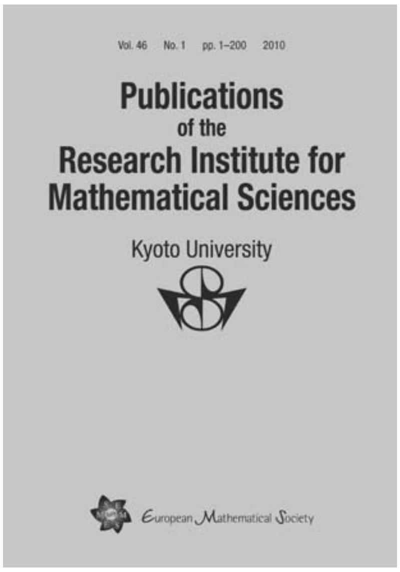


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future event horizons in a proper way. Moreover, he proved his Incompleteness Theorem shortly after. The presence of a trapped surface implies the existence of a so-called black hole in the studied universe. A possible forming of trapped surfaces is investigated in the book using an analysis of the dynamics of gravitational collapse. Formation of trapped surfaces is established using focusing of gravitational waves. The main tool used in this case is based on the short pulse method for general hyperbolic Euler-Lagrange systems. The monograph is written in a fairly technical way, theorems are stated carefully and computations are presented almost in their full length. The book is recommended for physicists or mathematicians working in general relativity, as well as for mathematicians interested in long time range asymptotics for partial differential equations on manifolds. (skr)

***P.G. Ciarlet, T.-T. Li: Differential Geometry – Theory and Applications****, Series in Contemporary Applied Mathematics, vol. 9, World Scientific, New Jersey, 2008, 292 pp., USD 95, ISBN 978-981-277-146-9*

This book presents first of all the theory of linearly and nonlinearly elastic shells. It originated in the ISFMA-CIMPA School on "Differential Geometry: Theory and Applications", which was held in August 2006 in the Chinese-French Institute for Applied Mathematics (ISFMA). The school was jointly organized by the ISFMA and the CIMPA (International Centre for Pure and Applied Mathematics), Nice, France. The book consists of four articles. The first article "An Introduction to Differential Geometry in $R^3$" by P.G. Ciarlet develops systematically, and from the very beginning, differential geometry of three-dimensional Euclidean space. The second article "An Introduction to Shell Theory" by P.G. Ciarlet and C. Mardare presents three-dimensional theory of elastic bodies and then two-dimensional theory of elastic shells. The fundamental differential equations are derived here and their properties are studied. The last two articles, "Some New Results and Current Challenges in the Finite Element Analysis of Shells" by D. Chapelle and "A Differential Geometry Approach to Mesh Generation" by P. Frey, deal with numerical methods suitable for solutions of the above mentioned fundamental equations. We find here the notion of a finite shell element within the framework of finite element methods and mesh generation for the finite element method. Though the exposition starts from the very beginning it leads to the most recent procedures. The book is very important for specialists in the field. On the other hand, because the presentation is very systematic and rather self-contained, it is also convenient for graduate students. (jiva)

***A. Connes, M. Marcolli: Noncommutative Geometry, Quantum Fields and Motives****, Colloquium Publications, vol. 55, American Mathematical Society, Providence, 2008, xxii+785 pp., USD 99, ISBN 978-0-8218-4210-2*

The purpose of this book is to explain the relevance of the subject of non-commutative geometry in dealing with two problems – the first, motivated by physics, is a subject of quantum gravity and the second, coming from mathematics, is the Riemann hypothesis. It is perhaps more aimed at mathematicians than physicists and the material is presented as much as possible in a self-contained way. The book consists of four chapters. The first chapter reviews all the standard aspects of quantum field theories, culminating in the construction of the standard model via non-commutative geometry and the geometric structure hidden behind renormalization. The second chapter describes a spectral realization of the zeros of the Riemann zeta function via non-commutative geometry. The third chapter introduces a realization of the geometry of adèle class space inside a quantum mechanical statistical system as an example of the non-commutative adèlic quotient. Based on the interplay of geometry, thermodynamics and the cohomology of motives, the last chapter clarifies in more detail the previously mentioned spectral realization of zeros of L-functions. (pso)

***G. Cortiñas et al., Eds.: K-Theory and Noncommutative Geometry****, EMS Series of Congress Reports, European Mathematical Society, Zürich, 2008, 440 pp., EUR 88, ISBN 978-3-03719-060-9*

This book contains the proceedings of a satellite ICM 2006 meeting on K-theory and noncommutative geometry held in Valladolid, Spain, in 2006. During the meeting, there were four courses (three lectures each), given by J. Cuntz, B. Keller, B. Tsygan and Ch. Weibel. These courses were complemented by invited talks and short communications. Written versions of 14 of them are included in the book. They cover many aspects of the field, including applications to other branches of mathematics. Three of them are devoted to isomorphism conjectures (R. Meyer; A. Bartels, S. Echterhoff and W. Lück; and H. Emerson and R. Meyer). The paper by F. Muro and A. Tonks discusses K-theory of the Waldhausen category. M. Karoubi has written a review paper on twisted K-theory (including new results on Thom isomorphism and new cohomology operations). Equivariant cyclic homology for quantum groups is discussed in the paper by C. Voigt. Index theory on singular spaces is studied by P. Carrillo Rouse, C*-algebra of a certain type is described by J. Cuntz, C*-algebra methods of infinite dimensional geometry are presented by W. Werner, and an extension of the Pontrjagin duality is described in the paper by U. Bunke, T. Schick, M. Spitzweck and A. Thom. The 2-groupoid of deformations of a germ of a $C^\infty$ manifold is treated in the paper by P. Bressler, A. Gorokhovsky, R. Nest and B. Tsygan. Torsion classes of finite type are discussed by G. Garkusha and M. Prest. The last two papers are devoted to the Parshin conjecture (a paper written by T. Geisser) and axioms for the norm residue isomorphism (written by Ch. Weibel). The book offers a nice overview of a broad range of topics studied in the field. (vs)

***G.P. Csicsery: Julia Robinson and Hilbert's Tenth Problem****, DVD, A.K. Peters, Wellesley, 2008, USD 29.95, ISBN 978-1-56881-428-5*

This movie contains a portrait of Julia Robinson, a mathematician widely recognized for her contribution to the solution of Hilbert's tenth problem. Together with her life story, it explains the history and principles of this outstanding result. It is a chronological biography of Julia Robinson recounting her ways and an unquestionable predestination to becoming a mathematician. After explaining her early results, the movie focuses on her main contribution, the work on Hilbert's tenth problem, which gradually gains its own part in the story. The beginning of the film is mainly narrated by her sister Constance Reid (who is a writer and the author of Julia Robinson's biography) and the rest includes contributions from her colleagues and young professional mathematicians. They explain the basics of Hilbert's tenth problem and the history of its solution culminating in Robinson's hypothesis, leaving the final step to a young Russian mathematician Yuri Matiyasevich, who concluded the solution of the problem in 1970. His solution started an unusual friendship between Robinson and Matiyasevich, which overcame both the age gap and political barriers and led to further collaboration. Julia Robinson's results were finally honoured. She was offered a full professorship at Berkeley and was, as the first woman, elected to the American Academy of Arts and Sciences and the presidency of the American Mathematical Society. (pru)

***J. F. Dars, A. Lesne, A. Papillault, Eds.: The Unravelers. Mathematical Snapshots***, *A.K. Peters, Wellesley, 2008, 208 pp., USD 34.95, ISBN 978-1-56881-441-4*
The Institut des Hautes Études Scientifiques in France is an institute of advanced research in mathematics and theoretical physics. This book contains snapshots of its scientists and visitors in lecture halls, chalk or pencil in hand, engrossed in dialogues. Every one of them contributed a little essay on subjects from mathematics to philosophy. Altogether in the book, there are snapshots of fifty people. There are photographs of M. Atiyah, Y. Choquet-Bruhat, J.-P. Bourguignon, A. Connes, P. Cartier, P. Deligne, J. Fröhlich, M. Gromov, V. Kac, M. Kontsevitvh, D. Sullivan and J. Tits, to mention just a few. The book reflects the enthusiasm of mathematicians and scientists and can serve as a recreational read for anyone. (jl)

***M. Deza, M. D. Sikirić: Geometry of Chemical Graphs. Polycycles and Two-faced Maps***, *Encyclopedia of Mathematics and Its Applications 119, Cambridge University Press, Cambridge, 2008, 306 pp., GBP 55, ISBN 978-0-521-87307-9*
The authors present graph theory as a tool used in the natural sciences. Graphs and their embeddings arise as natural structures in chemistry and crystalography. Graphs can be viewed as models of large molecules and their (abstract) properties can describe characteristics of chemical substances. This book covers two main topics – polycycles and two-faced maps. The main notions are introduced in the first two chapters and the remaining chapters can be read almost independently. Polycycles are 2-connected plane graphs with restricted combinatorial type of interior faces and the same degree q for interior vertices, while degree is at most q for boundary vertices. The book explains a general notion of $(r,q)$-polycycles and describes their classification when it is possible. Two-faced maps are maps (planar, thoroidal, etc.) having at most two types of faces and the same degree of all vertices. Most classical examples of two-faced maps are fullerens (cubic graphs whose faces are pentagons and hexagons). The classification of other types of two-faced maps is presented with respect to face-regularity and symmetries. (opang)

***D. Delbourgo: Elliptic Curves and Big Galois Representations,*** *London Mathematical Society Lecture Note Series 356, Cambridge University Press, Cambridge, 2008, 281 pp., GBP 35, ISBN 978-0-521-72866-9*
There are many deep conjectures relating special values of $L$-functions with arithmetic invariants. This book is concerned with special values of $L$-functions attached to modular forms. After a brief review of elliptic curves and modular forms, the Perrin–Riou and Kato work on the theory of Euler systems for modular forms is introduced. Then corresponding $p$-adic $L$-functions are constructed via modular symbols attached to Euler systems. The technical heart of the book contains the theory of 2-variable Euler systems realized in terms of the lambda-deformation of the space of modular symbols. The construction is compatible with the analytic theory of Greenberg–Stevens. The rest of the book is devoted to a study of the arithmetic of $p$-ordinary families of modular forms. For example, the association of Selmer groups over a one-variable deformation ring is discussed and the $p$-part of the Tate–Shafarevich group of an elliptic curve is computed. (pso)

***M. Disertori, W. Kirsch, A. Klein, F. Klopp, V. Rivasseau: Random Schrödinger Operators, Panoramas and Syntheses***, *no. 24, Société Mathématique de France, Paris, 2008, 213 pp., EUR 48, ISBN 978-2-85629-254-9*
This is a book on a subject intensively developed over the last 50 years by theoretical physicists and also, in recent decades, by researchers stressing a full mathematical rigour in such a study, several prominent mathematical physicists among them. A lot of deep results have been accumulated over the last 30 years (and many problems remain to be rigorously solved). This book aims to give a self-contained introduction and review of the subject. There are three articles in the book: "An invitation to random Schrödinger operators" (by W. Kirsch, with an appendix on the Aizenman-Molchanov method written by F. Klopp); "Multiscale Analysis and Localization of random operators" (by A. Klein); and "Random matrices and the Anderson Model" (by M. Disertori and V. Rivasseau).

The first article is an introductory course on the subject for the case of the discrete Laplacian. It covers all the basic aspects of the theory. The next article explains the method of multiscale analysis (which is also applicable to other classes of random Hamiltonians, like waves in random media) in the continuum. The third article describes the approach of constructive quantum theory when applied to the subject of random Schrödinger operators. It concentrates on the delocalized regime and also uses (in contrast to previous sections, which mostly have an operator theoretic or probabilistic character) tools like Grassman variables, superintegrals and other methods of field theory and random matrix theory. The authors have been successful in combining the choice of the essential themes of this theory with an accessible presentation of its basic methods, often adding remarks enlightening the motivation, the history of the subject and relations of methods used here to other parts of mathematical physics. The book will serve as a useful, self-contained source of information on this important chapter of contemporary mathematical physics. (mzahr)

***A. Djament: Foncteurs en Grassmanniennes filtration de Krull et cohomologie des foncteurs***, *Mémoires de la Société Mathématique de France, no. 111, Société Mathématique de France, Paris, 2007, 213 pp., EUR 37, ISBN 978-2-85629-248-8*
Let $V$ denote a vector space over a finite field $k$ and let $Gr(V)$ be the Grassmannian of subspaces in $V$. Basic objects of study in this book are various categories of functors (called grassmannian functor categories) from the category of couples $(V,W)$, where $W$ belongs to $Gr(V)$, to the category $F$ of vector spaces over $k$.

The book contains a study of finite objects in these categories and their homological properties. General vanishing properties are proved, together with an application of grassmannian functor categories to the Krull filtration of the category of functors of the category *F*. Special attention is given to the case of the basic field *k* with two elements (in this case it is possible to prove Noetherian properties of studied functors). (vs)

***R. Elman, N. Karpenko, A. Merkurjev: The Algebraic and Geometric Theory of Quadratic Forms****, AMS Colloquium Publications, vol. 56, American Mathematical Society, Providence, 2008, 435 pp., USD 79, ISBN 978-0-8218-4329-1*
The contents of this book is divided into three parts. The first part treats classical theory of symmetric bilinear and quadratic forms, including a study of forms under field extensions and construction of basic algebraic invariants of quadratic forms (e.g. the Witt ring, the u-invariant and norm residue homomorphisms). In the second part of the book, basic parts of algebraic geometry are developed (including Chow groups of algebraic cycles modulo rational equivalences on an algebraic scheme). Methods of the second chapter are then applied in the last part to a description of algebraic cycles on quadrics and their powers. Some basic combinatorial objects associated to quadrics (e.g. shell triangles and diagrams of cycles) are also introduced, leading to a simplified visualization of algebraic cycles and operation on them. The book is self-contained; almost no prerequisites are needed. Its point of view is characterized by a neat interplay of geometric approach (given by properties of a quadric and its function field) and algebraic approach (given by properties of quadratic maps over a field). (pso)

***L.D. Faddeev, O.A. Yakubovskii: Lecture on Quantum Mechanics for Mathematics Students****, Student Mathematical Library, American Mathematical Society, Providence, 2009, 234 pp., USD 39, ISBN 978-0-8218-4699-5*
This booklet contains *notes* from a course that was developed by L.D. Faddeev at the beginning of the 70s and taught for many years by O.A. Yakubovskii to students of Leningrad University. As indicated by the title, lectures are prepared mainly for mathematically oriented students requiring a higher level of rigour than is usual in courses given by physicists. The main structure of quantum mechanics is explained gradually starting with a thorough discussion of states and their evolution in classical mechanics, followed by a notion of a state in quantum mechanics and a description of relations between quantum and classical mechanics. After a discussion of one-dimensional examples, several three-dimensional problems are treated using tools from representation theory of the group of rotations. The last part of the book is devoted to scattering theory, a discussion of the spin of particles, multi-electron atoms and the Mendeleev periodic system of elements. The book is very well-written and can be easily understood by students only having knowledge of basic undergraduate mathematics. (vs)

***E. Feireisl, A. Novotný: Singular Limits in Thermodynamics of Viscous Fluids****, Advances in Mathematical Fluid Mechanics, Birkhäuser, Boston, 2009, 382 pp., EUR 79,90, ISBN 978-3-7643-8842-3*
This book is a very interesting contribution to the mathematical theory of partial differential equations describing the flow of compressible heat conducting fluids together with their singular limits. The main aim is to provide mathematically rigorous arguments of how to get from the compressible Navier-Stokes-Fourier system several less complex systems of partial differential equations, used, for example, in meteorology or astrophysics. However, the book also contains a detailed introduction to the modelling in mechanics and thermodynamics of fluids from the viewpoint of continuum mechanics. It also includes the proof of the existence of a weak solution to the full Navier-Stokes-Fourier system, which has not been presented in such a detailed form before. All limits are considered for these solutions whose existence is ensured for large data (i.e. without any smallness assumptions on the data or length of the time interval). Rewriting the full system to the dimensionless form, limits when the Mach, Froude or Péclet numbers tend to zero are studied. According to the rate of convergence, either the Oberbeck-Boussinesq or unelasting approximations are obtained, in bounded domains for thermally insulated domains with either slip or no-slip boundary conditions for velocity. In the case of unbounded domain, these results are obtained independently of boundary conditions. The main difficulty is connected with acoustic waves, which appear due to the compressibility of the fluid. The book is intended for specialists in the field; however, it can also be used for doctoral students and young researchers who want to start to work with the mathematical theory of compressible fluids and their asymptotic limits. (mpok)

***G. Gallavotti, W.L. Reiter, J. Yngvason, Eds.: Boltzmann's Legacy****, ESI Lectures in Mathematics and Physics, European Mathematical Society, Zürich, 2008, 276 pp., EUR 58, ISBN 978-3-03719-057-9*
This collection of papers and essays is the output of the symposium held in ESI, Vienna, June 2006, which commemorated the 100th anniversary of Boltzmann's death by collecting some of the most prominent experts in contemporary mathematical physics at this event. From all the great physicists and mathematicians of the 19th century, Boltzmann's legacy is one of the most influential and inspiring today. His achievements in statistical mechanics, the explanation how macroscopic phenomena can be understood from the laws of microscopic (classical) physics, proved to be a real triumph and a culmination of the 'mechanistic' worldview, the theory whose crucial importance in physics, mathematics and other sciences is even more obvious today. However, Boltzmann's legacy should not be viewed as restricted to classical physics, argues E. Lieb in one of the first chapters "What if Boltzmann had known about quantum mechanics". In comparison to the spectacular and everlasting success of equilibrium statistical physics, even the very foundations of nonequilibrium theory are still a hot discussion topic today. Contributions written by leading scientists reflect this (G. Gallavotti, J. Lebowitz, D. Ruelle, D. Ornstein, C. Cercignani, C. Villani, H. Spohn, E. Cohen, Ya. Sinai and others). There is also a chapter on computer simulations (Ch. Dellago and H. Posch) and on biology (P. Schuster), reflecting Boltzmann's deep interest in evolution theory. The book is framed by introductory and closing essays discussing Boltzmann's philosophical views (J. Renn) and his personality and life (W. Reiter). In summary, this is a valuable book that vividly and from many sides explains, on the occasion of the anniversary of a great scientific personality, one of the most important paradigms of clas-

sical and modern science. (mzahr)

***D.J.H. Garling: Inequalities – A Journey into Linear Analysis**, Cambridge University Press, Cambridge, 2007, 335 pp., GBP 23.99, ISBN 978-0-521-69973-0*

As is well-known, all animals are equal (but some are more equal). The character in George Orwell's timeless book who said (and wrote) this was a crook. So perhaps, in fact, the opposite is true: there is a certain kind of inequality between any two animals (and, pretty much, between any two objects one can think of). Inequalities constitute a basic and fundamental concept in mathematics – and not only in mathematics. A need to compare two parameters of a certain phenomenon is everyday business of almost everybody. There exist several mathematical textbooks dedicated to inequalities, whether they be classical or modern or old or new, but there can never be enough of such books.

This book contains a wealth of inequalities, again both classical and contemporary, complemented with detailed recipes on how to use them. It has several quite favourable features. First, the content is not restricted solely to an array of inequalities. It also involves a broad variety of applications such as Lebesgue decomposition and density theorems and the martingale convergence theorem, as well as a rather detailed treatment of diverse topics such as singular integrals, the Hahn–Banach theorem and eigenvalues of distributions. Each chapter is well equipped with a collection of interesting and revealing *notes* and remarks and, in some cases, also with a number of nontrivial exercises. Many inequalities considered are of course classical but others are not generally known and it is good that they appear in a textbook (such as the Bonami inequality – it is pleasant and refreshing to meet an inequality that is called after a lady for a change). The author also brings back Muirhead's maximal function, which is usually treated as a misnomer quoted to other authors. This book is a compulsory item on every teacher's bookshelf and it should be strongly recommended to students. If nothing else, it is an endless source of very good problems for students' theses of all levels. (lp)

***F. Gesztesy, H. Holden, J. Michor, G. Teschl: Soliton Equations and Their Algebro-Geometric Solutions, vol. II**, Cambridge Studies in Advanced Mathematics 114, Cambridge University Press, Cambridge, 2008, 438 pp., GBP 75, ISBN 978-0-521-75308-1*

This second part of the monograph on soliton equations contains a continuation of a systematic description of properties of solutions for hierarchies of soliton equations using methods from algebraic geometry. This volume is devoted to equations with one continuous (time) variable and one discrete (space) variable. There are three main examples treated in the book: the Toda hierarchy, the Kac–van Moerbeke hierarchy and the Ablowitz–Ladik hierarchy. The appendices collect together some preliminary material on algebraic curves (hyperelliptic, in particular), spectral parameter expansions and Lagrange interpolation. As with the first part, the book is very well-written and carefully organized and it is a pleasure to read it. It can be recommended to all readers interested in the topic. (vs)

***P. M. Gruber: Convex and Discrete Geometry**, Grundlehren der mathematischen Wissenschaften, vol. 336, Springer, Berlin, 2007, 578 pp., EUR 89.95, ISBN 978-3-540-71132-2*

This almost 600 page monograph introduces the reader to the present state of the theory of convex geometry in its many facets. There is a carefully written account of the development of different notions and their roots; the list of references contains more than a thousand items. The book consists of four parts: Convex functions, Convex bodies, Convex polytopes and Geometry of numbers and aspects of discrete geometry. The first part consists of two chapters devoted to convex functions of one or several variables. Even in this part (which is covered in many books and which serves as preparatory material for the book), the author shows his experience and presents an excellent exposition. All proofs are carefully chosen and presented with all details. The second part of the book deals with the theory of convex bodies. It consists of 11 chapters and it covers many topics, including mixed volumes, the Brunn–Minkowski inequality, isoperimetric inequalities, symmetrization and approximation of convex bodies and the space of convex bodies. In the third part (comprising seven chapters), the author studies combinatorial properties of convex polytopes, Hilbert's third problem, isoperimetric problems for polytopes and lattice polytopes. The last chapter is devoted to linear optimization. The most voluminous fourth part (14 chapters) includes a chapter on the Minkowski–Hlawka theorem and a chapter on problems from the geometry of numbers; a great deal of this part is devoted to packing, covering and tiling problems.

The book will be appreciated by specialists in the field but not only by them. The author relates the material to other areas of mathematics and shows numerous applications of studied theories, which makes the reading of the book useful for a much broader section of the mathematical community. The book can be used as a base for some special courses at universities and for further preparation of PhD students. It is a book that should be available in any mathematically oriented library. There are many reasons why a reader should appreciate the book. I would highlight three of them. The style of exposition reflects the author's experience and expertise as one of the leading specialists in the field and this makes the book extremely readable. The material is chosen in such a way that it illuminates the key role of convexity in many areas of mathematics. Finally, it shows that convexity is a fundamental concept with a long history, a strong impact on other mathematical disciplines and an interior beauty, whilst still offering nontrivial open problems. (jive)

***G.H. Hardy: A Course of Pure Mathematics Centenary Edition**, Cambridge University Press, Cambridge, 2008, 509 pp., GBP 24.99, ISBN 978-0-521-72055-7*

This book was first published in 1908 by Cambridge University Press and was intended "for the first-year calculus students, whose abilities reach or approach the scholarship standard". The book came as a third text in a series of Cambridge books that defined a revolution in the teaching of calculus (or rather mathematical analysis), after Whittaker's *A Course of Modern Analysis* in 1902 and Hobson's *The theory of functions of a real variable* in 1907 (all three books are still in print today). Ever since Leibniz and Newton started playing with fluxes and other derivative-like things, mathematicians have been trying to put calculus, including the basic concept of the definition of real numbers, on solid grounds. This immensely difficult task was essentially finished, thanks to great minds such as Bolzano, Weierstrass, Cauchy, Abel, Dirichlet, Riemann, Cantor and Peano, by the end of the 19th century. Naturally, by the outbreak of the 20th century, it was time to establish, for the first time ever, a rigorous university course in mathematical analysis. Such a course was defined by Hardy's book and had an immense affect on British, and later continental, teaching and understanding of the principles of analysis. At the beginning, the audience that could follow the book must have been, naturally, quite small. However, the book became a primer source of analysis and great inspiration to entire generations of mathematicians and it is very much so even today.

Hardy, a giant after whom fundamental mathematical notions are called – such as the very important inequality for sums and integrals, the celebrated maximal operator (a true blessing in real analysis) and function spaces (indispensable in harmonic analysis) – writes in a vigorous and enthusiastic and yet still precise style, with a lot of comments on how the stuff, brand new at the time, should be viewed by the reader. Often, there is a variety of points of view from which certain topics could be regarded (see, for instance, Section 16 on the definition of real numbers). The reader feels safe and well-led. Moreover, Hardy's missionary style is an amusing read. If only contemporary authors had time and courage to write analysis textbooks in a similar style. If I may quote the wonderful foreword to the new edition by T. W. Körner (from which I have been borrowing heavily anyway), the fact that the Cambridge University Press published in 2008 a centenary edition of this book, is not an act of piety. The reason is that, in a hundred years, the book has lost none of its power. It is still a great reading and a unique inspiration. May the generations of young mathematicians for which Hardy's book will be the gate to analysis continue forever. (lp)

***B. Hasselblatt, Ed.: Dynamics, Ergodic Theory, and Geometry****, Mathematical Sciences Research Institute Publications 54, Cambridge University Press, Cambridge, 2007, 324 pp., GBP 50, ISBN 978-0-521-87541-7*

This book is a collection of papers loosely associated with the 'Workshop on recent progress in dynamics' held at MSRI, Berkeley, in 2004 (videos of the lectures presented there are still available on the webpage of MSRI). The book starts with survey papers on the role of symplectic capacities in symplectic geometry (K. Cieliebak, H. Hofer, J. Latschev and F. Schlenk) and on local rigidity of group actions (D. Fisher). There are eight other papers on various topics in hyperbolic, parabolic and symbolic dynamics and ergodic theory. A special feature of the book is the last paper (organized by B. Hasselblatt) collecting together on 50 pages many problems from various fields (not necessarily connected with papers appearing in the book). (vs)

***J. Havil: Impossible? Surprising Solutions to Counterintuitive Conundrums****, Princeton University Press, Princeton, 2008, 235 pp., USD 27.95, ISBN 978-0-691-13131-3*

The second popular-mathematics book of the author is in some sense a free sequel to the first one *Nonplussed*. This book, just like its predecessor, contains a collection of baffling paradoxes and mathematical occurrences that are in sharp conflict with a common, or even scientific, intuition. The book shows how deceiving the intuition can occur and where it can lead us. The author illustrates this unsettling thought on several examples of truly mind-boggling and delightfully amusing paradoxes. After a warm-up of "common-knowledge" classical puzzles and paradoxes based on elementary logic, the serious business starts. The reader will find, among many other topics, Simpson's famous paradox, a nightmare of statisticians and a destructive weapon of demagogues, Braess' paradox, disguised in a less common form with a rather surprising positive effect of a temporary closure of a road on the smoothness of traffic in a big city, and finally, at the end of the book, the king of all paradoxes, the Banach–Tarski theorem. This paradox, for instance, is a deep and serious result that lies on the crossroads of measure theory, the theory of sets and mathematical logic, a consequence of the existence of sets that have no volume at all, combined with the highly disputed axiom of choice. It states that a three-dimensional ball can be cut into just five pieces from which one can assemble two such balls. This result, whilst true, is so highly counterintuitive that the author himself comments on it: 'If nothing else in this book was considered Impossible by the reader, it is hoped that this result might just have saved the author's day.' (lp)

***S. Helgason: Geometric Analysis on Symmetric Spaces****, Mathematical Surveys and Monographs, vol. 39, second edition, American Mathematical Society, Providence, 2008, 637 pp., USD 89, ISBN 978-0-8218-4530-1*

The principal subject of this monograph is the analysis of Riemannian symmetric spaces of non-compact type. These are spaces of the form $X = G/K$, with $G$ a real semisimple Lie group without compact factors; $G$ is the group of isometries and $K$ is the stabilizing subgroup of a fixed origin. The author's 1978 monograph *Differential Geometry, Lie groups and Symmetric Spaces* gave an exposition of the basic geometric structure of these spaces and the connection with Lie theory. Helgason's 1984 monograph *Groups and Geometric Analysis* dealt with the harmonic analysis of left $K$-invariant functions on $X$. The present monograph deals with the extension to harmonic analysis on $X$ without assuming this left $K$-invariance. The theory is dominated by two integral transforms: on the one hand the Fourier transform, in terms of eigenfunctions on $X$ for invariant differential operators, and on the other hand the horocycle transform. The latter transform is systematically viewed as a generalization of the classical Radon transform to the context of a so-called group invariant double fibration (an idea introduced by S.-S. Chern). The author systematically develops the theory of these transforms, building on Harish-Chandra's spherical Plancherel formula and on the spherical Paley–Wiener theorem, two principal subjects of the 1984 monograph.

In the monograph, the general inversion and Paley–Wiener theorems for both the Fourier and Radon transform are discussed, as well as their connection and the relations with eigenspace representations and with the representations of the spherical principal series for $G$. Application to solvability of invariant differential equations is given. The treatment of all of these subjects relies on research contributions by the author throughout his career. The second edition differs only mildly from the first. The principal addition is the treatment of the multi-temporal wave equation, a remarkable hyperbolic system of equations first introduced by Semenov. The exposition, which emphasizes the geometric and analytic side of the subject, is self-contained and very clear. It is good that this standard in the field, with its wealth of material, has become available again through a second edition. (vdb)

***A. Kechris et al., Eds.: Games, Scales, and Suslin Cardinals. The Cabal Seminar, vol. I****, Lecture Notes in Logic 31, Cambridge University Press, Cambridge, 2008, 445 pp., GBP 45, ISBN 978-0-521-89951-2*

This material, presented at the famous Cabal seminar, appeared in four volumes of Springer Lecture Notes in Mathematics over the years 1978–1988. For those interested in set theory with the axiom of determinacy instead of the axiom of choice, it was the ultimate source of knowledge for decades. Now, edited by A.S. Kechris, B. Löwe and J. R. Steel, the Cabal seminar appears once more in a beautifully typed book, which is quite different from a mere reprint of the previous Lecture Notes. Instead, the material is carefully organized so that it may serve as an advanced textbook equally well as a monograph on descriptive set theory. The book is divided into two parts; both parts have long informative introductions, written by John R. Steel for 'Games and scales' and by Steve Jackson for 'Suslin cardinals'. Seven papers are new and 12 are reprinted from the previous "Proceedings of Cabal Seminar". (psim)

***M. Khalkhali, M. Marcolli, Eds.: An Invitation to Noncommutative Geometry****, World Scientific, New Jersey, 2008, 506 pp., USD 118, ISBN 978-981-270-616-4*

This book is a collection of written versions of lecture series presented at the 'Workshop on non-commutative geometry' held in Teheran in 2005. The two longest ones are general review papers on non-commutative geometry. The first one, written by A. Connes and M. Marcolli, starts with a description of a general strategy of how to deal with problems in non-commutative geometry, followed by a long list of important examples of the theory. The second one (by M. Khalkhali) treats operator algebras, topological K-theory and cyclic cohomology. A lecture by B. Noohi on derived and triangulated categories shows why the language of categories is important for non-commutative geometry. Renormalization of non-commutative QFT is discussed by H. Grosse and R. Wulkenhaar, complex tori and spherical manifolds are described in lectures by J. Plazas and non-commutative algebraic geometry is treated by S. Mahanta. The paper by R.J. Szabo explains a role of noncommutative field theories in the theory of D-branes in a type II superstring context. There is also a paper by G. Landi and W.D. Suilekom on non-commutative bundles and instantons. The book offers a very valuable collection of material on an important and quickly developing field of mathematics. (vs)

***E. Kunz: Residues and Duality for Projective Algebraic Varieties****, University Lecture Series, vol. 47, American Mathematical Society, Providence, 2008, 158 pp., USD 39, ISBN 978-0-8218-4760-2*

The main objective of these lecture notes is to describe local and global duality, primarily focusing on irreducible algebraic varieties over an algebraically closed field. Both local and global duality theorems are based on two linear operators - we have the residue map defined on the top local cohomology of the canonical sheaf, and the integral as a linear form on the top global sheaf cohomology of algebraic variety. There is then a comparison given by the residue theorem relating the integral operator to a sum of residues or, when specializing to projective algebraic curves, yielding the Serre duality theorem expressed in terms of differentials and their residues. Possible applications include the generalization of residue calculus to toric residues. (pso)

***F. Lesieur: Measured Quantum Groupoids****, Mémoires de la Société Mathématique de France, no. 109, Société Mathématique de France, Paris, 2007, 158 pp., EUR 27, ISBN 978-2-85629-233-4*

This volume of 'Mémoires de la SMF' is an extended and revised version of the doctoral thesis of the author defended in 2003 at the University of Orleans. The author proposes a definition of a "measured quantum groupoid", with an aim to generalize previous work on quantum groups and quantum groupoids, including a duality theorem. The defining axioms given in the volume were later simplified by M. Enock. The presentation is rather technical and the reader is expected to have a solid background in the field. The second part of the book contains a lot of varied examples (including the cases of groupoids, quantum groups, quantum space, quantum groupoids and inclusions of von Neumann algebras). (sh)

***X. Ma, W. Zhang: Bergman Kernels and Symplectic Reduction****, Astérisque, no. 318, Société Mathématique de France, Paris, 2008, 154 pp., EUR 37, ISBN 978-285629-255-6*

The authors generalize several recent results concerning asymptotic expansions of Bergman kernels to the framework of geometric quantization and establish an asymptotic symplectic identification property. For a positive line bundle $L$ and an Hermitian vector bundle $E$ over a symplectic manifold $X$, one can construct the so called spin$^c$ Dirac operator $D_p$ acting on the space of holomorphic forms with values in $L^p \otimes E$. The Bergmann kernel is the smooth kernel of the orthogonal projection onto the kernel of the spin$^c$ Dirac operator. The authors consider Hamiltonian action of a compact connected Lie group $G$ on a compact symplectic manifold $X$ and relate the asymptotic expansion for $p \to \infty$ of the $G$-invariant Bergmann kernel of the spinc Dirac operator with the Bergman kernel on the corresponding Marsden–Weinstein reduction $X_G$. A method for computation of the coefficients in the expansions is presented and the first few of them are explicitly calculated. In particular, the authors obtain the scalar curvature of the reduction space from the $G$-invariant Bergman kernel on the total space. To prove the main result of the book, the authors use the analytic localization techniques of Bismut and Lebeau – the key observation is that the $G$-invariant Bergman kernel is the kernel of the orthogonal projection to the zero space of a deformation of $D_p{}^2$ by the Casimir operator. To localize the problem, the spectral gap property and the finite propagation speed of solutions of hyperbolic equations play essential roles. As an application, the authors establish some properties of Toeplitz operators on $X_G$. Moreover, the non-equivariant asymptotic expansion played a crucial role in the recent work of Donaldson on the stability of projective manifolds. (vtu)

***M.A.H. MacCallum, A. V. Mikhailov (Eds.): Algebraic Theory of Differential Equations****, London Mathematical Society Lecture Note Series, No. 357, Cambridge University Press, 2009, ISBN 978-0-521-72008-3*

This book consists of seven chapters, each containing a written version of one lecture series of the School held in Edinburgh in 2006. The first part (82 pages) presents lectures given by Michael F. Singer, containing a description of Galois theory for linear differential equations. It starts with an introduction to differential modules and their connections to linear differential equations. After a short review of the classical Galois theory for polynomials, the Galois group of a differential module is defined to be the group of differential isomorphisms. An analogue of Galois correspondence for differential equations is stated and proved and several examples and applications are discussed, including the notion of the monodromy group of a differential system, factorization of differential equations and connections to transcendental numbers. Finally, an algorithm for computing the differential Galois group of an equation is outlined and it is shown, with examples, how it depends on a parameter in the equation. The techniques introduced use Picard-Vessiot theory and its generalizations.

Further chapters are shorter and are mostly based on Galois differential theory. The second part (written by F. Ulmer) is about methods of how to obtain solutions of particular ordinary differential equations in a closed form. Most of the article consists of examples, computations and MATLAB algorithms. The third part (by S. Tsarev) is an exposition of the theory of factorization of ordinary differential equations and partial differential equations. Techniques used here include Gröbner bases and generalized Laplace and Dini transformations. The next part (by A. Leykin) is an introduction to D-modules. It describes D-modules over Weyl algebra and holonomic D-modules, the Bernstein-Sato polynomial, hypergeometric differential equations and local cohomology. It contains many examples computed using software for research in algebraic geometry.

The fifth part (written by Mikhailov, Novikov and Wang) contains a definition of symbolic representations of the ring of differential polynomials and classification of integrable homogenous evolutionary equations (which are symmetries of nonlinear partial differential equations of orders 2, 3 and 5). The sixth part (by J. Hietarinta) searches for integrable systems of (partial) differential equations by algebraic methods. There are several definitions of integrable systems (Liouville, super- and quantum integrability) and a discussion of two examples (connected with two dimensional point-particle dynamics and the soliton equation), both leading to an overdetermined set of equations. The last part (written by A. Pillay) deals with model theory and its connection with Galois theory, and with a nonlinear generalization of the Grothendieck conjecture. The book may be useful for graduate mathematicians working in differential systems and their invariants. The text covers a large area of research on relatively few pages and contains many examples. The reader is assumed to have a basic knowledge of classical theories (e.g. partial differential equations, ordinary differential equations, integrable systems and classical Galois theory). (pf)

***A. Marden: Outer Circles. An Introduction to Hyperbolic 3-Manifolds****, Cambridge University Press, Cambridge, 2007, 427 pp., GBP 40, ISBN 978-0-521-83974-7*

Three-dimensional geometry is a fascinating topic, which has gone through an enormous evolution over the last 40 years. There are already several books describing this topic (e.g. by W. Thurston, R. Benedetti and C. Petronio and J.G. Ratcliffe). There are also books covering the topic of Kleinian groups. This book offers an overview of both fields (coming from the side of Kleinian groups). The book has two parts. The first part covers the fundamental facts of the theory. The first four chapters cover basic facts on hyperbolic space, Riemann surfaces, discrete groups, basic properties of hyperbolic manifolds and 3-manifold topology. The last two chapters are devoted to specific topics (line geometry and hyperbolic trigonometry). This part of the book is mostly written in the style of a textbook. The second part (chapters 5 and 6) has a different character. It is an excellent survey of topics connected with the proof of three main conjectures in the field (the tameness conjecture, the density conjecture and the ending lamination conjecture). A special feature of the book is the 'exercises and explorations' section added at the end of each chapter, introducing many other complementary topics connected with the main theme of the book. The book contains a lot of material and can be very valuable for getting an overview of this very broad and important field of mathematics. (vs)

***J. Meier: Groups, Graphs and Trees. An Introduction to the Geometry of Infinite Groups****, London Mathematical Society Student Texts 73, Cambridge University Press, Cambridge, 2008, 231 pp., GBP 19.99, ISBN 978-0-521-71977-3*

This book provides an excellent introduction to geometric group theory. The exposition is divided into eleven chapters; those with odd numbers present general theory and those with even numbers analyse classical examples of infinite groups: groups generated by reflections, the Baumslag–Solitar group and the Thompson group. Apart from these, an example of a finitely generated infinite torsion group is constructed. The chapters providing general techniques are organized as follows. The first chapter summarizes results on group actions. The "drawing trick" is explained in order to figure out some Cayley graphs. Finally, for groups acting on connected graphs, the notion of a fundamental domain is introduced. The third chapter presents free groups and free products from a geometric point of view. For example, it is shown that a group is free if and only if it acts freely on a tree. The Nielsen-Schreier theorem is then immediate. The fifth chapter gives information about Dehn's word problem and explains its connection to Cayley graphs. Chapter 7 demonstrates a use of automata and regular languages in group theory, including an interesting proof of the Howson theorem. In chapter 9, a finitely generated group with a fixed, finite set of generators is given the structure of a metric space. It is shown that any finitely generated group has a faithful representation as a group of isometries of a metric space. Various properties studied in previous chapters are intertwined with properties described by the metric. For example, it is proved that any almost convex group has a solvable word problem. The last chapter presents several properties of a Cayley graph of a finitely generated group that are independent of the choice of the finite set of generators. The book offers an interesting introductory course on the topic. Carefully chosen examples are an essential part of the exposition and they really help to understand general constructions. Apart from very elementary group theory, there are no other prerequisites. On the other hand, there are many suggestions for recent papers that use information contained in the book. (ppr)

***P. W. Michor: Topics in Differential Geometry**, Graduate Studies in Mathematics, vol. 93, American Mathematical Society, Providence, 2008, 494 pp., USD 75, ISBN 978-0-8218-2003-2*

This book is an excellent introduction to the field of differential geometry with a strong emphasis on Lie groups. It stresses naturality, functoriality and a coordinate-free approach. Coordinate formulas are, however, always derived for extra information. The first chapter introduces basic notions of differentiable manifolds, vector fields and flows. The next 80 pages are devoted to Lie groups and their actions, ending with invariant theory for polynomials (the Hilbert-Nagata theorem) and smooth functions (the Schwartz theorem). The third chapter builds up the theory of vector bundles, integration on manifolds and de Rham cohomology, along with Poincaré duality. The fourth chapter on general bundles and connections starts with a thorough treatment of the Frölicher-Nijenhuis bracket, which is used to express any kind of curvature and second Bianchi identity, even for fiber bundles (without structure groups). The author proves that every fiber bundle admits the so-called complete connection, which allows one to define parallel transport along any curve, and treats their holonomy groups. The principal bundles, associated bundles and principal and induced connections are dealt with in detail. The chapter finishes with characteristic classes and jets.

A chapter on Riemannian geometry starts with a careful treatment of connections to geodesic structures to sprays to connectors and back to connections, going via the second and third tangent bundles. The Jacobi flow on the second tangent bundle is a new aspect coming from this point of view. Isometric immersions and Riemannian submersions are treated in analogy to each other. The sixth chapter treats homogeneous Riemann manifolds, the beginning of symmetric space theory and polar actions. The final chapter covers symplectic geometry and classical mechanics, completely integrable Hamiltonian systems and Poisson manifolds. The emphasis is on group actions, momentum mappings and reductions. The careful reader will gain a working knowledge in a wide range of topics of modern coordinate-free differential geometry. A prerequisite for using the book is a good knowledge of undergraduate analysis and linear algebra. The book grew out of the author's three decades of experience in the field and meets the highest standards. One could hardly find a better book for graduate students and researchers interested in modern differential geometry. (vtu)

***C. C. Mountfield: Synthetic CDOs. Modelling, Valuation and Risk Management, Mathematics, Finance and Risk**, Cambridge University Press, Cambridge, 2008, 369 pp., GBP 45, ISBN 978-0-521-89788-4*

This book covers three topics: modelling, valuation and risk management of a rather complex financial derivative, i.e. collateralised debt obligations (CDOs). It is a financial product that takes targeted risk for the purpose of achieving targeting returns. CDOs fall into the class of credit risk derivatives. The first five chapters give an introduction to debt obligations, their mathematical modelling and valuation. Credit indices as well as valuation of default baskets are treated here. Valuation and the mechanism of synthetic CDOs together with market considerations are discoursed in Chapters 6 and 7. The following chapters (8 to 14) deal with risk quantification, extensions of the standard market model, exotic CDOs, portfolio analysis and simulation methods for hedging strategies. Some of the methods are well illustrated on cases from the real financial world. Despite the complexity of the financial instrument in question, the mathematics used for modelling and analysing the phenomena is of college level and therefore understandable to a wide community of potential readers. The book can be highly recommended for financial mathematicians and financial analysts. (jh)

***M. Nakahara, R. Rahimi, A. Saitoh, Eds.: Mathematical Aspects of Quantum Computing 2007**, Kinki University Series on Quantum Computing, vol. 1, World Scientific, New Jersey, 2008, 222 pp., USD 89, ISBN 978-981-281-447-0*

This volume offers contributions of the summer school held at Kinki University in Osaka in 2007. It contains six lectures of invited speakers and six poster summaries. The lecture notes cover basic aspects of the fields of quantum computing, the phenomenon of entanglement and quantum error-correcting codes. It also contains a discussion of some more specialized topics connected with topological quantum computing, holonomic quantum computing and playing games in quantum mechanical settings. All contributions are written in a way that allows readers with only a small amount of experience in the field to comfortably grasp the basics of the presented topics. (šh)

***E. Novak, H. Woźniakowski: Tractability of Multivariate Problems, vol. I**, Tracts in Mathematics, vol. 6, European Mathematical Society, Zurich, 2008, 384 pp., EUR 68, ISBN 978-3-03719-026-5*

This is a book – the first one of three planned volumes – on recent research in the field of computational mathematics, dealing with high dimensional multivariate problems. Problems of information complexity discussed in the book appear in several areas of mathematics and physics: computation of path integrals (Feynman–Kac formulae), global optimization of functions of very many variables (like in image processing – often using simulated annealing algorithms), when solving the many body Schrödinger equation and many other situations. The exponential dependence of the "cost" of the computation on d (where d denotes the number of variables) means the "intractability" (or curse of dimensionality) of the problem. A simple example of intractability (in $d$) is integration of a smooth function of $d$ variables (to a given precision $\varepsilon$). Depending on d and $\varepsilon$, several types of (in)tractability can be defined and the book offers a careful analysis of various situations. The problems discussed in the book range from computer science to abstract functional analysis and many interesting open problems are presented here. (mzah)

***P.J. Rippon, G. M. Stallard, Eds.: Transcendental Dynamics and Complex Analysis**, London Mathematical Society Lecture Note Series 348, Cambridge University Press, Cambridge, 2008, 451 pp., GBP 40, ISBN 978-0-521-68372-2*

Transcendental dynamics is a research subject on the borderline between the theory of dynamical systems and classical analysis in the complex domain. Coming from the pioneering works of Fatou and Julia almost a hundred years ago, and having obtained a strong impetus since the advent of modern computers in the last quarter of the previous century, it remains an active research area. This book, aimed primarily at specialists, gathers together 16 contributions covering the up-to-date state of knowledge of the topic. The volume is dedicated to Noel Baker (1932–2001), who was one of the leading figures of the theory over many decades. (dpr)

***M. Rosenkranz, D. Wang, Eds.: Gröbner Bases in Symbolic Analysis****, Radon Series on Computational and Applied Mathematics, vol. 2, Walter de Gruyter, Berlin, 2007, 349 pp., EUR 98, ISBN 978-3-11-019323-7*

Gröbner bases have become a key tool in many elimination techniques and in computer calculations. They were introduced in the thesis of B. Buchberger, who was one of two people responsible for a special semester devoted to Gröbner bases and related methods at the Radon Institute of Computational and Applied Mathematics in Linz. This book brings together 11 survey papers based on lectures presented in workshops organized during the semester and they cover a broad territory. Use of Gröbner bases in algebraic analysis (including applications to control theory) is treated in the paper by J.-F. Pommaret. Applications of Gröbner bases to linear partial differential and difference equations are described by U. Oberst and F. Pauer and applications to computations of the strength of systems of differential equations are covered in the paper by A. Levin. Differential Gröbner bases are discussed in the contribution by G. Carrà Ferro and differential elimination and its applications in biological modelling are described by F. Boulier. The Janet algorithm, involutive bases and generalized Hilbert series are treated by D. Robertz. The paper by W.M. Seiler is devoted to the Spencer cohomology, homological formulation of the Cartan test for involutivity of a system and its dual version, involutive bases (in particular the Pommaret bases) and applications to general differential equations. Local Lie groups of transformations have an infinite-dimensional analogue called Lie pseudo-groups. Their description can be found in the paper by P. Olver and J. Pohjanpelto, including a generalization of the method of moving frames developed first in a finite dimensional situation. They also describe algorithms for the computation of algebra of differential invariants. Constructive techniques in invariant theory for rings of differential operators are studied in the contribution by W.N. Traves (including a computation of the ring of invariants for the case of the Grassmannian G2,4). The paper by K. Krupchyk and J. Tuomela discusses compatibility complexes for overdetermined boundary problems. The last contribution (F.L. Pritchard and W.Y. Sit) treats initial value problems for ordinary differential-algebraic equations. (vs)

***Séminaire Bourbaki, Volume 2005/2006, Exposés 952–966****, Astérisque, no. 311, Société Mathématique de France, Paris, 2007, 401 pp., EUR 88, ISBN 978-2-85629-230-3*

This volume contains 15 articles based on survey talks at the Bourbaki seminar in Paris in the academic year 2005/06. They cover a wide range of topics: Julia sets of positive measure, (non)-regularity of solutions of partial differential equations, singular integral operators, rigidity of Bernoulli actions, non-projective Kähler manifolds, $p$-adic analytic geometry, compactifications of moduli spaces, classification of $p$-divisible groups, Arakelov geometry, the Serre modularity conjecture, prime number gaps, the perfect graph theorem and representations of Lie superalgebras. (jnek)

***Séminaire Bourbaki, Volume 2006/2007, Exposés 967–981****, Astérisque, no. 317, Société Mathématique de France, Paris, 2008, 535 pp., EUR 90, ISBN 978-2-85629-253-2*

In this volume of the seminar, there are 15 review lectures on topics of current interest. The volume contains seminars on Nakajima varieties (O. Schiffmann) and on the geometry of the moduli space of K3 surfaces (C. Voisin). There is a paper on Tate-Sato conjectures by H. Carayol. The lecture by O. Debarre is devoted to pluricanonical systems on varieties and the lecture by D. Harari treats rational points of subvarieties of Abelian varieties (over a function field). The theme of algebraic independence in non-zero characteristics is discussed by F. Pellarin, ordinary differential equations with BV coefficients are studied by C. De Lellis and counting the number of solutions of polynomial equations in finite fields is described by A. Chambert-Loir. There are three lectures on differential geometry: algebraization of webs (J. V. Pereira), semi-classical measures on Riemannian manifolds (Y. Colin de Verdièr) and closed contact manifolds (V. Colin). There are also two lectures on group theory: the Kashiwara-Vergne conjecture (Ch. Torossian) and groups generated by automata (A. Zuk). A paper on the solutions of the Hamilton-Jacobi equations was prepared by J.-M.Roquejoffre and the subspace theorem in the theory of Diophantine approximation is described in the contribution by Yuri F. Bilu. (pso)

***W.T. Shaw: Complex Analysis with Mathematica + CD****, Cambridge University Press, Cambridge, 2006, 571 pp., GBP 45, ISBN 978-0-521-83626-5*

The first version (2006) of the book was announced in EMS Newsletter June 2006, No. 60, p. 8, and praised there by Roger Penrose and then voted one of *Choice* magazine's Outstanding Academic Titles for 2007. The current reprinted edition 2008 includes some corrections and a CD updated for the newer Version 6 of *Mathematica*®. The book presents complex numbers and complex analysis with many applications in a state-of-the-art computational environment. It offers enrichment of the standard complex analysis course: Newton–Raphson iterations and complex fractals, the Mandelbrot set, complex chaos and bifurcations, Fourier and Laplace transforms, fluid dynamics, discrete Fourier and Laplace transforms, Schwarz–Christoffel mapping, mathematical art and tiling of the hyperbolic planes and physics in three and four dimensions (e.g. Minkowski space and twistors). Each chapter uses *Mathematica*® as a powerful and flexible tool to develop the reader's geometric intuition and enthusiasm with perfect illustrations and checks of classical calculations (equations, residue, contour integrals, summation of series, mappings and transforms and their inverses). The sophisticated *Mathematica*® codes – included both in the text and on the CD – enable the user to run computer experiments. The book is far more than a standard course of complex analysis or a guide to *Mathematica*® tools. (zvl)

***G.I. Shishkin, L.P. Shishkina: Difference Methods for Singular Perturbation Problems****, Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, vol. 140, CRC Press, Boca Raton, 2008, 393 pp., USD 119.95, ISBN 978-1-58488-459-0*

This book focuses on the development of robust difference schemes for wide classes of boundary value problems. It justifies the ε-uniform convergence of these schemes and surveys the latest approaches important for further progress in numerical methods. Part I (Grid approximations of singular perturbation partial differential equations) begins with an introduction containing a short history of the field and its main ideas, the principles and the main problems encountered in the construction of the special schemes in the book. In further chapters of Part 1, the following problems are considered: BVP for elliptic reaction-diffusion equations in domains with smooth and piecewise-smooth boundaries, some generalisations, parabolic reaction-diffusion equations, elliptic convection-diffusion equations and parabolic convection-diffusion equations. Part II (Advanced trends in ε-uniformly convergent difference methods) contains material mainly published in the last four years. This includes problems with boundary layers and additional singularities generated by nonsmooth data, unboundedness of the domain and also by the presence of the perturbation vector parameter. Another aspect considered in this part is that both the solution and its derivatives are found with errors that are independent of the perturbation parameters. The book can be of use for scientists and researchers, both for students and for professionals in the field of developing numerical methods for singularly perturbed problems and also for anybody interested in mathematical modelling or in the fields where the problems with boundary and interior layers arise naturally. (oj)

***V. Shoup: A Computational Introduction to Number Theory and Algebra**, second edition, Cambridge University Press, Cambridge, 2008, 580 pp., GBP 35, ISBN 978-0-521-51644-0*

In general, this book alternates between theory and applications, i.e. one or two chapters of purely mathematical concepts are followed by one or two chapters on algorithms and applications. The mathematics covered here includes basic number theory (unique factorization, congruences, distribution of primes and quadratic reciprocity) and abstract algebra (groups, rings, fields and vector spaces). It also contains an introduction to discrete probability theory. The choice of topics is motivated primarily by their applicability to computing and communications, especially to specific areas of cryptography, coding theory, automata and complexity theory. The material beyond basic calculus presented in the book is developed from scratch. On general grounds, the book could serve as a course of discrete mathematics for computer science students. (pso)

***K.P. Shum et al., Eds.: Advances in Algebra and Combinatorics**, World Scientific, New Jersey, 2008, 371 pp., USD 145, ISBN 978-981-279-000-2*

This volume consists of a selection of lectures presented at the Second International Congress in Algebra and Combinatorics held in July 2007 in Guangzhou, Beijing, and Xian, China. There are 22 papers in the volume. It is not possible to list all of them here but let us mention papers on *Quantum polynomials* (V. A. Artamonov), *Gröbner-Shirshov bases* (L. A. Bokut and Y. Q. Chen) and the application of *Gröbner-Shirshov bases to Normal Forms of Coxeter groups* (D. Lee). There are also papers on *Paper-folding*, *polygons*, *complete symbols and the Euler quotient function* (P. Hilton, J. Pedersen and B. Walden), *Irreducible subalgebras of Matrix Weyl Algebras* (P. S. Kolesnikov) and *Conformal Field Theory and Modular Forms* (K. Ueno). M. Jambu contributed a paper on *Koszul Algebra and Hyperplane Arrangements* and J. Fountain and V. Gould presented a paper on *Stability of the Theory of Existentially Closed S-Acts over a Right Coherent Monoid S*. There are also papers on hyperidentites and hypersubstitutions, various aspects of group theory and so on. (jtu)

***A. M. Sinclair, R. R. Smith: Finite von Neumann Algebras and Masas**, London Mathematical Society Lecture Note Series 351, Cambridge University Press, Cambridge, 2008, 400 pp., GBP 40, ISBN 978-0-521-71919-3*

The main aim of this book is to present a systematic study of parts of the theory of von Neumann subalgebras of finite von Neumann algebras and, in particular, the theory of maximal Abelian self-adjoint subalgebras of separable $II_1$ factors (usually called masas). The main part of the book contains a description of perturbations of von Neumann subalgebras and general results on masas, including a detailed study of singular masas in $II_1$ factors. There is a special chapter devoted to the Pukánszky invariant of a masa. Prerequisites for reading the book are basic facts on von Neumann algebras, which are summarized at the beginning of the book. The book is suitable for graduate students wanting to learn this part of mathematics. (vs)

***S. Szabó, A. D. Sands: Factoring Groups into Subsets**, Lecture Notes in Pure and Applied Mathematics, vol. 257, CRC Press, Boca Raton, 2009, 269 pp., USD 161, 96, ISBN 978-1-4200-9046-8*

A factorization of an Abelian group $G$ is a decomposition of $G$ into the sum $G = A + B$ of two subsets $A$ and $B$ of $G$. The fundamental theorem on finite Abelian groups says that each finite Abelian group can be factored into a direct sum of cyclic subgroups. This book deals with general non-subgroup factorizations focusing mainly on cyclic groups. It starts with various constructions that produce new factorizations from old ones. Then it discusses periodic and non-periodic factorizations, quasiperiodicity and factorizations of periodic subsets. The authors deal with factorizations of infinite Abelian groups and their applications in combinatorics (Ramsey numbers, Latin squares and Hadamard matrices). They also investigate connections between factorizations and codes, including error correcting codes, variable length codes and integer codes. (jtu)

***I.A. Taimanov: Lectures on Differential Geometry**, EMS Series of Lectures in Mathematics, European Mathematical Society, Zürich, 2008, 211 pp., EUR 34, ISBN 978-3-03719-050-0*

This book is a translation of the Russian original that is based on lecture notes from a one-semester course for undergraduate students given by I. A. Taimanov in 1998. It consists of three parts. The first part is devoted to a study of curves and surfaces in Euclidean 2- and 3-space. The second part covers basics of smooth manifolds and Riemannian geometry, including basic properties of curvature, sectional curvature, the Levi–Cività connection and geodesics. Many interesting examples are given, including a trivial proof that a smooth manifold may be embedded to $R^n$ for large $n$. Finally, the Lobachevski plane is described as a Riemannian manifold, whereas the Lie group $PSL(2,R)$ is recognized as being the group of Lobachevski transformations. The third part covers more advanced materials. It begins with a proof that any two two-dimensional surfaces are locally conformally equivalent, followed by a description of a relation between umbilic points and the Hopf differential. The proof of the Hopf theorem (any topological sphere with constant mean curvature is isometric to the standard sphere) is given. Further topics included are: minimal surfaces, basics of Lie group theory and Lie algebra theory, basics of representation theory, including the proof of the Peter–Weyl theorem, and the definition of the Fourier transform on a group. In the last chapter, fundamental facts on Poisson and symplectic geometry are presented, leading to integrable Hamilton systems and Hamilton's variation principle. The book is aimed at undergraduate students, as well as anyone who wants to learn basic facts from differential geometry. (pf)

***L.A. Takhtajan: Quantum Mechanics for Mathematicians****, Graduate Studies in Mathematics, vol. 95, American Mathematical Society, Providence, 2008, 387 pp., USD 69, ISBN 978-0-8218-4630-8*

This book is, in a sense, a continuation of the book by L.D. Fadeev and O.A. Yakubovskii, which is also reviewed in this issue of the newsletter. Both books are based on courses given to mathematically oriented students with the aim of describing basic ideas, fact and tools from quantum physics. This book covers a broader array of topics, including more advanced ones. In the first part, the author reviews the main facts of classical mechanics, introduces the basic principles of quantum mechanics and describes the Schrödinger equation (including a description of the hydrogen atom and the first part of a discussion of semi-classical asymptotics). This part ends with a discussion of particles with spin and systems of identical particles. The second part starts with a description of the Feynman path formulation of quantum mechanics (also containing a discussion of regularized determinants of differential operators and a further discussion of semi-classical asymptotics). The next chapter treats Gaussian and Wiener measures and the Gaussian Wiener integral. A description of fermionic systems using Grassmann algebras, graded linear algebra and path integrals for anticommuting variables then leads naturally to an introduction to supermanifolds and supersymmetry. The book is an excellent introduction to quantum physics both for students of mathematics and for mathematicians wanting to learn basic facts from quantum mechanics and quantum field theory in the language of mathematics. (vs)

***T. tom Dieck: Algebraic Topology****, EMS Textbooks in Mathematics, European Mathematical Society, Zürich, 2008, 567 pp., EUR 58, ISBN 978-3-03719-048-7*

This is a textbook on algebraic topology. It covers all topics now considered to be standard knowledge in the field: basic homotopy theory, (co)homology, duality theorems and also characteristic classes and bordism. The author's viewpoint is that homotopy is more basic than homology. He starts by explaining fundamental groups and covering spaces. After the necessary preliminaries (suspension, (co)fibration, etc.), basic properties of homotopy groups are derived, followed by a brief exposition of stable homotopy. Homology is first introduced abstractly via spectra for pointed spaces and then by an explicit construction of singular homology. The theory is applied for cell complexes and manifolds to get results on cellular homology, fundamental class, winding numbers, etc. An exposition of cohomology (the cup product, Thom isomorphism and the Leray–Hirsch theorem) precedes a chapter on Poincaré duality. The chapter on characteristic classes emphasises the role of classifying spaces. An interplay between homotopy and homology (including the Hurewicz theorem, cohomology of Eilenberg–Mac Lane spaces and homotopy groups of spheres) is illustrated briefly. The last chapter is devoted to bordism as a homology theory. The theory is carefully built and the book may serve as an extensive source for references. The mainstream of the exposition is occasionally supplemented by examples going beyond topology. The reader can also find many exercises here. There are only minimal prerequisites: elementary point set topology, algebra and category theory. Many concepts, such as cell complexes, manifolds and bundles, are explained from scratch. (md)

***K. Ueno: Conformal Field Theory with Gauge Symmetry****, Fields Institute Monographs, American Mathematical Society, Providence, 2008, 168 pp., USD 59, ISBN 978-0-8218-4088-7*

The purpose of this book is to give a systematic approach to conformal field theory with gauge symmetry, the so-called Wess–Zumino–Witten–Novikov model, from the viewpoint of complex algebraic geometry. After a concise introduction to the theory of Riemann surfaces and representation theory of affine Lie algebras in Chapters 1 and 2, respectively, the conformal blocks for stable pointed curves with coordinates are constructed in Chapter 3. Chapter 4 presents a sheafified version of conformal blocks coming from the family of stable pointed curves with coordinates. Chapter 5 describes a projectively flat connection on the sheaf of conformal blocks. Finally, Chapter 6 is devoted to the example of a particular family parameterized by a projective line. The book covers basic material needed for construction of the modular functor from conformal field theory to topological quantum field theory. (pso)

***D.C. Ullrich: Complex Made Simple, Graduate Studies in Mathematics****, vol. 97, American Mathematical Society, Providence, 2008, 489 pp., USD 75, ISBN 978-0-8218-4479-3*

This book has three parts. The first part may be thought of as a first year course in complex analysis. In the second part, proofs of some more advanced results are given. The third part is formed of appendices. The book starts with a definition of holomorphic functions and proofs of basic theorems, followed by a proof of the Cauchy theorem, the residue theorem and the open mapping theorem. The next chapter gives an application of what has been done so far; it is about Euler's formula for sin(z). The next two chapters are devoted to inverses of holomorphic and conformal mappings, followed by two chapters covering the Riemann mapping theorem and the relation between the theory of holomorphic and harmonic functions. The following chapters lead to a classification of elements of the automorphism group of the unit disk. The first part of the book ends with an analytic continuation of functions and the proof of the Picard theorems. Part two is devoted to Abel's theorem, a characterization of Dirichlet domains, more advanced versions of the maximal modulus theorem, properties of the Gamma function and its analytic continuation, universal covering spaces and Cauchy's theorem for nonholomorphic functions. The book can be used as a textbook of basics of complex analysis. There are many exercises and the exposition is very nice. (zuvl)

***F. Van Oystaeyen: Virtual Topology and Functor Geometry****, Lecture Notes in Pure and Applied Mathematics, vol. 256, Chapman & Hall/CRC, Boca Raton, 2007, 150 pp., USD 99.95, ISBN 978-1-4200-6056-0*

This book has a special character. Its main theme is to describe development of new branches of non-commutative geometry on a different level of realizations, ranging from areas already fully developed to many different suggestions for possible future investigations. In particular, a lot of attention in the book is concentrated on a formulation of a suitable version of non-commutative topology and sheaves in this situation. A standard version of non-commutative geometry consists of an associative algebra, which is a generalization of the commutative algebra of functions on an ordinary space. It is a pointless geometry, which makes the formulation of a topology seemingly hopeless. The author has earlier developed a version of scheme theory over a non-commutative algebra based on module theory and quasi-coherent sheaves. The language used in the book is that of category theory (summarized briefly in Chapter 1). In the book, the author discusses possibilities of extending it to a more general setting, using a non-commutative version of lattices as a tool. This is contained in Chapter 2, ending with the two representative examples of the theory (the lattice of torsion theories and the lattice of closed linear subspaces of Hilbert space). Chapter 3 includes the use of a general notion of a quotient representation for a description of a relation between non-commutative affine spaces and non-commutative projective spaces. A dynamical version of topology and sheaf theory is introduced in the last chapter. A very particular feature of the book is a formulation of numerous suggestions for research projects throughout the book. Some of them are more accessible but often they are quite advanced. On the whole, the book is very inspiring and worth reading. (vs)

***S. H. Weintraub: Factorization – Unique and Otherwise****, CMS Treatises in Mathematics, A.K. Peters, Wellesley, 2008, 260 pp., USD 49, ISBN 978-1-56881-241-0*

The starting point of this book is the concept of unique factorisation. Using an algebraic approach, the author introduces the reader to basic concepts on the border between algebra and number theory (integral domains, Euclidean rings, principal ideal domains, etc.) and, using them, he opens the door to number theory up to the level of quadratic fields, together with a moderate introduction to algebraic number theory. One of the appendices is devoted to quadratic reciprocity. The exposition contains a lot of details and exercises complement the exposition so that the book can also be used for self-study. The book can also be useful for instructors seeking an algebraically oriented complement for a standard text in elementary number theory. (špr)

***G. Zampieri: Complex Analysis and CR Geometry****, University Lecture Series, vol. 43, American Mathematical Society, Providence, 2008, 200 pp., USD 45, ISBN 978-0-8218-4442-7*

CR geometry is nowadays a very broad subject with its scope spanning from the geometric theory of partial differential equations and microlocal analysis to complex and symplectic geometry and foliation theory. This book by Giuseppe Zampieri does not aim to introduce all topics of current interest in CR geometry. Instead, it attempts to be friendly to the novice by moving in a fairly relaxed way from elements of the theory of holomorphic functions in several complex variables to advanced topics of modern CR geometry. The choice of topics provides a good balance between a first exposure to CR geometry and subjects representing current research.

The first chapter of the book covers classical results in several complex variables (Cauchy formulas in polydiscs, Hartogs' theorems on separability and extendability of holomorphic functions and the logarithmic supermean of the Taylor radius of holomorphic functions). It finishes with the $L^2$ and subelliptic estimates for the $\partial$-bar operator. The second chapter covers real/complex structures and real/complex symplectic spaces. The Frobenius and Darboux theorems are proved. The third chapter, which constitutes the second half of the book, covers CR structures. A particular emphasis is devoted to analysis of the conormal bundle to a real submanifold of $C^n$ under a canonical transformation. The author then describes the theory of analytic discs attached to real submanifolds and their infinitesimal deformations. The problem of construction of lifts and partial lifts of analytic discs is also addressed. Zampieri also deals with Bloom–Graham normal forms and separate real analyticity. The book is written in a very readable style; every section starts with a short summary and there are a lot of *notes* and remarks providing a broader context for discussed questions. The reader is supplied with a lot of exercises (hints are provided) and also with suggested research topics, where the author discusses open problems related to the material of the chapter. (vtu)

## List of reviewers for 2009.

***The editors would like to thank the following for their reviews this year.***

J. Anděl, M. Bečvářová, L. Boček, T. Cipra, M. Doubek, J. Drahoš, A. Drápal, V. Dupač, P. Franěk, J. Haslinger, P. Holický, Š. Holub, J. Hron, J. Hurt, M. Hušek, O. John, Z. Kalas, O. Kalenda, P. Kaplický, A. Karger, J. Kofroň, J. Kratochvíl, S. Krýsl, L. Křižka, M. Kulich, J. Lukeš, M. Mádlík, J. Malý, J. Milota, K. Najzar, J. Nekovář, O. Odvárko, P. Pajas, O. Pangrác, L. Pick, Š. Porubský, D. Pražák, P. Příhoda, P. Pyrih, M. Rokyta, T. Salač, I. Saxl, P. Simon, A. Slavík, P. Somberg, V. Souček, J. Spurný, D. Stanovský, D. Šmíd, J. Štěpán, J. Trlifaj, V. Tuček, J. Tůma, E. Van den Ban, J. Vanžura, J. Veselý, Z. Vlasáková, Z. Vlášek, P. Vojtěchovský, M. Zahradník, M. Zelený, J. Žemlička.

All of the above are on the staff of Charles University, Faculty of Mathematics and Physics, Prague, except J. Vanžura (Mathematical Institute, Czech Academy of Sciences), M. Bečvářová, (Technical University, Prague), Š. Porubský (Institute of Computer Science, Czech Academy of Sciences), J. Nekovář (University Paris VI, France), P. Vojtěchovský (University of Denver, Colorado) and E. Van den Ban (University of Utrecht).

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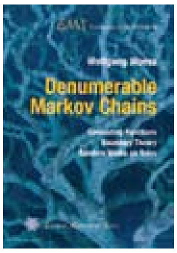



Wolfgang Woess (Graz University of Technology, Austria)
**Denumerable Markov Chains. Generating Functions, Boundary Theory, Random Walks on Trees** (EMS Textbooks in Mathematics)

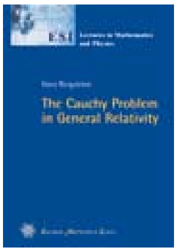
ISBN 978-3-03719-071-5. 2009. 368 pages. Hardcover. 16.5 x 23.5 cm. 48.00 Euro

Markov chains are the first and most important examples of random processes. This book is about time-homogeneous Markov chains that evolve with discrete time steps on a countable state space. Measure theory is not avoided, careful and complete proofs are provided. A specific feature is the systematic use, on a relatively elementary level, of generating functions associated with transition probabilities for analyzing Markov chains. Basic definitions and facts include the construction of the trajectory space and are followed by ample material concerning recurrence and transience, the convergence and ergodic theorems for positive recurrent chains. The level varies from basic to more advanced, addressing an audience from master's degree students to researchers in mathematics, and persons who want to teach the subject on a medium or advanced level. A specific characteristic of the book is the rich source of classroom-tested exercises with solutions.


Mauro C. Beltrametti, Ettore Carletti, Dionisio Gallarati and Giacomo Monti Bragadin (all University of Genova, Italy)
**Lectures on Curves, Surfaces and Projective Varieties. A Classical View of Algebraic Geometry** (EMS Textbooks in Mathematics)

ISBN 978-3-03719-064-7. 2009. 506 pages. Hardcover. 16.5 x 23.5 cm. 58.00 Euro

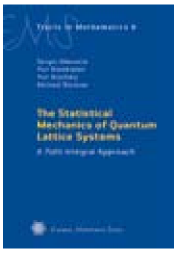
This book offers a wide-ranging introduction to algebraic geometry along classical lines. It consists of lectures on topics in classical algebraic geometry, including the basic properties of projective algebraic varieties, linear systems of hypersurfaces, algebraic curves (with special emphasis on rational curves), linear series on algebraic curves, Cremona transformations, rational surfaces, and notable examples of special varieties like the Segre, Grassmann, and Veronese varieties. An integral part and special feature of the presentation is the inclusion of many exercises, not easy to find in the literature and almost all with complete solutions.
The text is aimed at students of the last two years of an undergraduate program in mathematics. It contains some rather advanced topics suitable for specialized courses on the advanced undergraduate or beginning graduate level, as well as interesting topics for a senior thesis. The prerequisites have been deliberately limited to basic elements of projective geometry and abstract algebra. Thus, for example, some knowledge of the geometry of subspaces and properties of fields is assumed.




Hans Ringström (KTH, Stockholm, Sweden)
**The Cauchy Problem in General Relativity** (ESI Lectures in Mathematics and Physics)

ISBN 978-3-03719-053-1. 2009. 307 pages. Softcover. 17 x 24 cm. 42.00 Euro

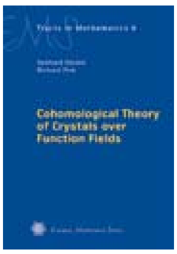
The general theory of relativity is a theory of manifolds equipped with Lorentz metrics and fields which describe the matter content. Einstein's equations equate the Einstein tensor (a curvature quantity associated with the Lorentz metric) with the stress energy tensor (an object constructed using the matter fields). In addition, there are equations describing the evolution of the matter. Using symmetry as a guiding principle, one is naturally led to the Schwarzschild and Friedmann–Lemaître–Robertson–Walker solutions, modelling an isolated system and the entire universe respectively. In a different approach, formulating Einstein's equations as an initial value problem allows a closer study of their solutions. This book first provides a definition of the concept of initial data and a proof of the correspondence between initial data and development. It turns out that some initial data allow non-isometric maximal developments, complicating the uniqueness issue. The second half of the book is concerned with this and related problems, such as strong cosmic censorship.




Sergio Albeverio (University of Bonn, Germany), Yuri Kondratiev (University of Bielefeld, Germany), Yuri Kozitsky (Maria Curie-Skłodowska University, Lublin, Poland) and Michael Röckner (University of Bielefeld, Germany)
**The Statistical Mechanics of Quantum Lattice Systems. A Path Integral Approach** (EMS Tracts in Mathematics Vol. 8)

ISBN 978-3-03719-070-8. 2009. 392 pages. Hardcover. 17 x 24 cm. 62.00 Euro

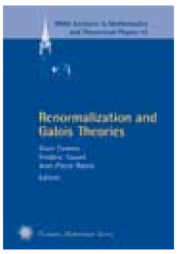
Quantum statistical mechanics plays a major role in many fields such as, for instance, thermodynamics, plasma physics, solid-state physics, and the study of stellar structure. While the theory of quantum harmonic oscillators is relatively simple, the case of anharmonic oscillators, a mathematical model of a localized quantum particle, is more complex and challenging. Moreover, infinite systems of interacting quantum anharmonic oscillators possess interesting ordering properties with respect to quantum stabilization. This book presents a rigorous approach to the statistical mechanics of such systems, in particular with respect to their actions on a crystal lattice. The text is addressed to both mathematicians and physicists, especially those who are concerned with the rigorous mathematical background of their results and the kind of problems that arise in quantum statistical mechanics. The methods developed in the book are also applicable to other problems involving infinitely many variables, for example, in biology and economics.




Gebhard Böckle (University of Duisburg-Essen, Germany) and Richard Pink (ETH Zürich, Switzerland)
**Cohomological Theory of Crystals over Function Fields** (EMS Tracts in Mathematics Vol. 9)

ISBN 978-3-03719-074-6. 2009. 195 pages. Hardcover. 17 x 24 cm. 48.00 Euro

This book develops a new cohomological theory for schemes in positive characteristic $p$ and it applies this theory to give a purely algebraic proof of a conjecture of Goss on the rationality of certain $L$-functions arising in the arithmetic of function fields. These $L$-functions are power series over a certain ring $A$, associated to any family of Drinfeld $A$-modules or, more generally, of $A$-motives on a variety of finite type over the finite field $\mathbb{F}_p$. By analogy to the Weil conjecture, Goss conjectured that these $L$-functions are in fact rational functions. In 1996 Taguchi and Wan gave a first proof of Goss's conjecture by analytic methods à la Dwork. The present text introduces $A$-crystals, which can be viewed as generalizations of families of $A$-motives, and studies their cohomology.
The book is intended for researchers and advanced graduate students interested in the arithmetic of function fields and/or cohomology theories for varieties in positive characteristic. It assumes a good working knowledge in algebraic geometry as well as familiarity with homological algebra and derived categories, as provided by standard textbooks. Beyond that the presentation is largely self-contained.




**Renormalization and Galois Theories** (IRMA Lectures in Mathematics and Theoretical Physics Vol. 15)
Alain Connes (IHÉS, Bures-sur-Yvette, France), Frédéric Fauvet (IRMA, Strasbourg, France ) and Jean-Pierre Ramis (Institut de Mathématiques de Toulouse, France), Editors

ISBN 978-3-03719-073-9. 2009. 279 pages. Softcover. 17 x 24 cm. 42.00 Euro

This volume is the outcome of a CIRM Workshop on Renormalization and Galois Theories held in Luminy, France, in March 2006. The subject of this workshop was the interaction and relationship between four currently very active areas: renormalization in quantum field theory (QFT), differential Galois theory, noncommutative geometry, motives and Galois theory.
The last decade has seen a burst of new techniques to cope with the various mathematical questions involved in QFT, with notably the development of a Hopf-algebraic approach and insights into the classes of numbers and special functions that systematically appear in the calculations of perturbative QFT (pQFT). The analysis of the ambiguities of resummation of the divergent series of pQFT, an old problem, has been renewed, using recent results on Gevrey asymptotics, generalized Borel summation, Stokes phenomenon and resurgent functions.
The purpose of the present book is to highlight, in the context of renormalization, the convergence of these various themes, orchestrated by diverse Galois theories. It contains three lecture courses together with five research articles and will be useful to both reseachers and graduate students in mathematics and physics.

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