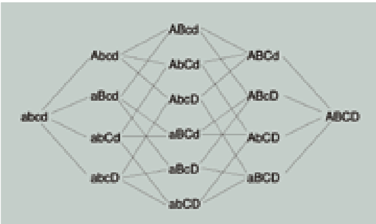
NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY



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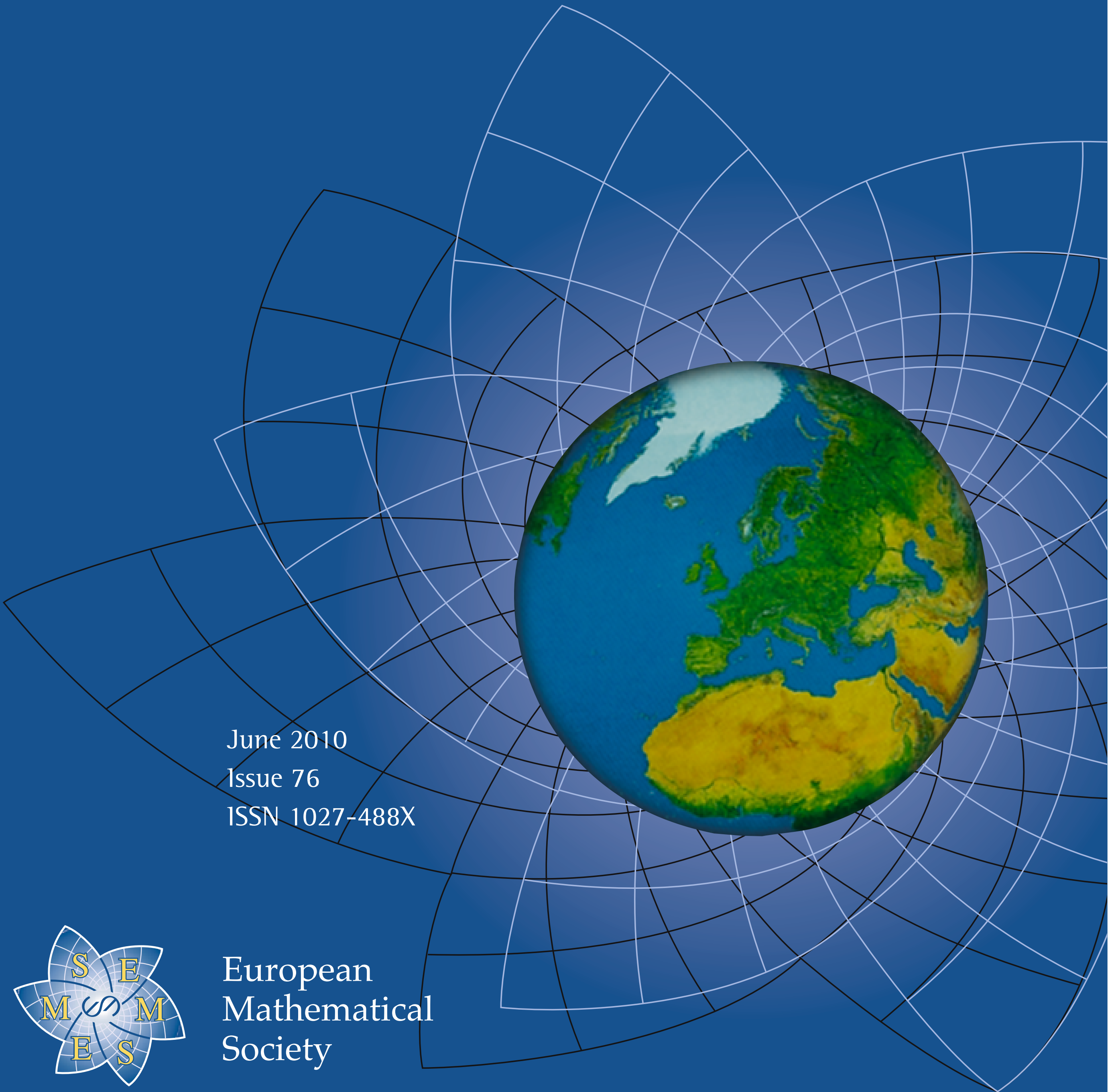

June 2010
Issue 76
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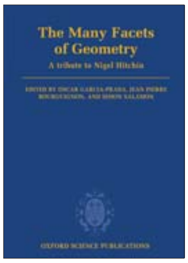

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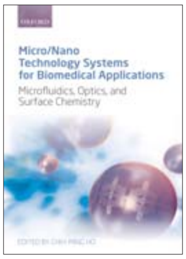

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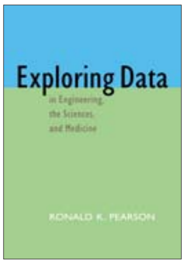

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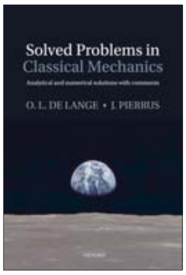

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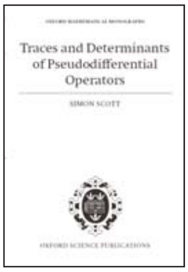

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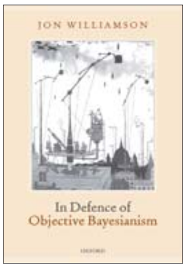

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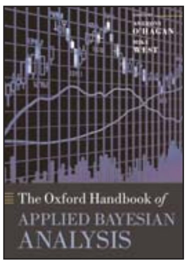

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European Mathematical Society

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EMS Agenda

2010

9–11 July
EMS EC Meeting; EMS Council Meeting; 20th Anniversary of European Mathematical Society, Sofia, Bulgaria;
http://www.math.ntnu.no/ems/council10/

11–14 July
Conference “Mathematics in Industry”, Sofia, Bulgaria
http://www.math.bas.bg/MathInIndustry/index.php

26–31 July
16th Congress of ECMI (European Consortium of Mathematics in Industry), Wuppertal, Germany
http://www.ecmi2010.eu/

1 August
Deadline for submission of material for the September issue of the EMS Newsletter
Vicente Muñoz: vicente.munoz@mat.ucm.es

22–29 August
EMS-ESMTB Summer School on Mathematical Ecology and Evolution, Helsinki
http://wiki.helsinki.fi/display/huippu/mathbio2010

6–11 September
EMS-CIME summer school Topics in mathematical fluid-mechanics, Cetraro, Italy
Contact: cime@math.unifi.it

12–17 September
ESF-EMS/ERCOM Conference on “Highly Oscillatory Problems: From Theory to Applications”
Cambridge, United Kingdom
http://www.newton.ac.uk/programmes/HOP/

10–18 October
EMS-IMPAN School in Applied Mathematics, Będlewo, Poland

2 December
Final Conference of the Forward Look “Mathematics and Industry”, Bruxelles

2011

18–22 July
ICIAM 2011 Congress, Vancouver, Canada

2012

2–7 July
6th European Mathematical Congress, Kraków (Poland)
http://www.euro-math-soc.eu

Editorial

The 20th Anniversary of the European Mathematical Society

Vasile Berinde, EMS Publicity Officer

At the end of this year the European Mathematical Society will celebrate the 20th anniversary of its foundation, on 28 October 1990 in Madralin, Poland. This important event will be marked a little in advance during the EMS council meeting in Sofia, 10–11 July 2010, by means of some brief historical presentations and also by organizing a Round Table of former EMS presidents.

As mentioned in an editorial by A. Pelczar published in Issue 33 (September 1999) of the newsletter to mark the first ten years of the existence of the EMS, such an anniversary is a good opportunity to evaluate its main achievements and point out possible less successful actions, if any, but with a clear and optimistic look to the future.

At first glance, this could appear an easy task but when going into details, in view of the current impressive format of the society, it turns out to be an extremely difficult work to try to summarise the highlights and breakthroughs achieved by the EMS in the past 20 years. We will attempt to do this by looking first at some relevant extracts from Statutes, Article 2:

> *"1. The purpose of the Society is to promote the development of all aspects of mathematics in the countries of Europe, with particular emphasis on those which are best handled on an international level. The Society will concentrate on those activities which transcend national frontiers and it will in no way seek to interfere with the national activities of the member societies. In particular, the Society will, in the European context, aim to promote mathematical research (pure and applied), assist and advise on problems of mathematical education, concern itself with the broader relations of mathematics to society, foster the interaction between mathematicians of different countries, establish a sense of identity amongst European mathematicians, and represent the mathematical community in supranational institutions."*

By all accounts, the dominant conclusion of a detailed inspection of the archived documents related to its varied actions over these 20 years is that the EMS has fully achieved its main goals of becoming a true, distinct and authoritative voice of European mathematicians. In this rather short period of time, the EMS has grown into a fully-fledged pan-European society that is recognized by the Commission of the European Union as a trusted partner on mathematics matters. This is exemplified by the following diverse activities.

European Congresses of Mathematics. Since its inception in 1990, the society has actively promoted scientific meetings. A four year cycle of European Congresses of Mathematics (ECM) began in Paris in 1992, followed by Budapest in 1996, Barcelona in 2000, Stockholm in 2004 and Amsterdam in 2008. The next European Congress of Mathematics will be held in Krakow, 2–7 July 2012 (see http://www.6ecm.pl//), while a call for bids for the 7ECM in 2016 will be launched during the council meeting in Sofia this year.

EMS Prizes. At each European Congress of Mathematics, ten EMS Prizes are awarded to young mathematicians. To these has been added in 1999 the EMS Felix Klein Prize, established by the EMS and the endowing organisation, the Institute for Industrial Mathematics in Kaiserslautern, which is awarded to a young scientist or a small group of young scientists (normally under the age of 38) for outstanding applications of mathematics. This year, a new EMS prize was established, with Springer as the endowing organization: the Neugebauer Prize in the History of Mathematics. The prize will consist of 5,000 Euros and will be awarded for a specific piece of work, which could be a book or an article, and so it could be split between co-authors. This makes it distinct from other prizes in this field. The first awarding of the Neugebauer Prize will be at the 6ECM in Krakow. The influence of the EMS nowadays in Europe is also proved by the fact that it is asked to nominate representatives to the board of various research institutes and prize committees, like the Abel Prize, which is awarded annually by the Norwegian Academy of Science and Letters.

EMS Publishing House. In 2000, the EC of the EMS approved the foundation of its own publishing house (see http://www.ems-ph.org), which started its activity in 2002. The EMS Publishing House (PH) is a not-for-profit organization dedicated to the publication of high quality, peer reviewed journals and books, including e-books, on all academic levels and in all fields of pure and applied mathematics. The EMS PH current series of books consists of the following: *EMS Monographs in Mathematics*; *EMS Series of Congress Reports; EMS Series of Lectures in Mathematics*; *EMS Textbooks in Mathematics*; *EMS Tracts in Mathematics*; *ESI Lectures in Mathematics and Physics*; *Heritage of European Mathematics*; *IRMA Lectures in Mathematics and Theoretical Physics*; *Zurich Lectures in Advanced Mathematics;* and also several titles published outside the above series.

Let us also mention the journals published by the EMS PH: *Journal of the European Mathematical Society* (the society's own young but very prestigious research journal); *Commentarii Mathematici Helvetici*; *Elemente der Mathematik*; *Groups, Geometry, and Dynamics*; *Interfaces and Free Boundaries*; *Journal of Noncommutative Geometry*; *Portugaliae Mathematica*; *Publications of the Research Institute for Mathematical Sciences*; *Quantum Topology*; *Rendiconti Lincei – Matematica e Applicazioni*; *Zeitschrift für Analysis und ihre Anwendungen*; *Oberwolfach Reports*; and the recently launched *Journal of Spectral Theory.*

EMS Newsletter. Of all the EMS PH journals and EMS journals, the EMS Newsletter, which is the society's quarterly journal of record, is certainly the most popular and well known publication, due to its editorial profile and also to the fact that it is posted to all individual members. The first issue of the Newsletter appeared in September 1991, almost a year after the foundation of the society itself, and evolved into the current extremely rich and diverse format, mainly due to the substantial contributions of its devoted successive Editors-in-Chief but also to many collaborators. The Editors-in-Chief have been: David Singerman and Ivan Netuka (1991–1995), Roy Bradley and Martin Speller (1996–1998); Robin Wilson (January 1999–September 2003), Martin Raussen (December 2003–March 2009) and Vicente Muñoz (June 2008–present). As can be seen, Martin and Vicente were co-editors for four issues.

From the initial standard format, including informative material like agendas, reports on the activities of the EMS or related to the EMS, announcements of forthcoming conferences, interviews, presentations of member societies, book reviews, etc., the *Newsletter* has become a publication that now also includes genuine mathematics and therefore, not surprisingly, is reviewed by Zentralblatt MATH on a regular basis since 2003 (presently 64 reviews). From Issue 75 (March 2010) of the *Newsletter*, more editorial space has become available for mathematics articles by removing the columns *Forthcoming Conferences* (which is now hosted by the society's website) and *Recent Books* (which was reduced in size to the new column *Book Reviews*).

Membership. Except for Albania and Moldova, which are not yet members of the EMS but are expected to apply and be admitted as institutional members of the EMS at the council meeting in Sofia this year, all other European countries are represented in the EMS by one or more member societies. So, from this point of view, the geographical coverage of Europe appears to be complete.

Other Activities. The EMS is running annually a series of summer schools in pure and applied mathematics, with a focus on helping young mathematicians. For example, three such events are scheduled for the second part of this year: EMS-ESMTB Summer School on *Mathematical Ecology and Evolution*, Helsinki (22–29 August), EMS-CIME Summer School *Topics in Mathematical Fluid-mechanics*, Cetraro, Italy (6–11 September) and *EMS-IMPAN School in Applied Mathematics, Będlewo*, Poland (10–18 October). Most of the summer schools have been supported by the European Commission and UNESCO-ROSTE. The society has also held joint meetings with SIAM (in Berlin and Los Angeles) and with the French Mathematical Societies (in Nice). A regular series of joint mathematical weekends with its member societies started in Lisbon in 2003. The subsequent meetings took place in Prague (2004), Barcelona (2005), Nantes (2006) and Copenhagen (2008).

The EMS has also sponsored several lecture series and Diderot Forums, the latter being public events held simultaneously in several locations, using electronic conferencing.

These are, of course, not all the activities run by or under the umbrella of the EMS during the 20 years of its existence that deserve a mention but we restrict ourselves to the ones already presented.

Despite the above mentioned achievements of the European Mathematical Society, it will be productive to consider new and flexible mechanisms to facilitate growth of the individual EMS membership and also to ensure an increase of the involvement in the activities directly related to mathematics education in Europe. The two most notably less successful activities of the EMS in these 20 years are related: firstly the rather low individual membership attracted until now (currently steady at somewhere around 2000) and secondly the extent to which the EMS has been able to assist and advise on problems of mathematical education in Europe.

Last but not least, let's mention the names of former (and current) EMS Presidents who directly contributed to the growth and reinforcement of the society in these 20 years. As the term for the EMS President is restricted by the Statutes to a period of four years, the EMS has so far had five presidents: Fritz Hirzebruch (1990–1994); Jean-Pierre Bourguignon (1995–1998); Rolf Jeltsch (1999–2002); John Kingmann (2003–2006) and the current president, Ari Laptev (2007–2010). The 6th EMS President will be elected by the council in Sofia this year and will have the mission of continuing the trend of this impressive development of the European Mathematical Society over its first 20 years of life.

The EMS has opened Internet access to the EMS member database on the webpage

http://www.euro-math-soc.eu/members

If you do not have an account yet, click the "Create a new account" button on the webpage. The new page that opens will ask for your member ID, which is on the mailing label of your EMS Newsletter. After putting in the member ID, you will be sent an email containing a temporary password. Then you can log on to the above webpage and change your password. On the member database page you will see your address data and your payment status. If you are late in your payments, there is a button for making an Internet payment. You can also edit your address data and, if you wish, make it invisible to others.

New member of the Editorial Board

Olaf Teschke is a new member of the Editorial Board of the Newsletter of the EMS.

Olaf Teschke was born in 1972 on the Island of Rügen, then East Germany, as the son of a fisherman and a bank assistant. He finished his studies at the Institut of Pure Mathematics of Humboldt Universität, Berlin, under the supervision of Herbert Kurke, in the field of algebraic geometry.

Since 1999, he has held several research and assistant positions at Humboldt Universität and Brandenburg Technical University in Cottbus. Besides his continuing work in algebraic geometry, especially for moduli of vector bundles and invariants of affine structures, he has extended his focus to such rather diverse areas as arithmetic geometry and mathematical education.

While still a researcher at Humboldt, he was appointed section editor of Zentralblatt MATH for algebraic geometry in 2004 and then switched to a full editorial position in 2008. Since 2009, he has served as the Managing Editor of ZBMATH.

When getting in touch with mathematics as a child, he developed the obviously naive desire for reading everything about the subject. While trying to handle more than 120,000 new publications added to Zentralblatt MATH in his first year, he realized that some dreams may come true with a twist.

EMS Executive Committee meeting in Edinburgh, 20–21 March 2010

Vasile Berinde, EMS Publicity Officer

Venue and attendance

The first 2010 EMS Executive Meeting took place in Edinburgh on Saturday 20 March, from 9:00 to 13:00 and 14:00 to 18.00, and on Sunday 21 March, from 9:00 to 13:00, at Edinburgh University Informatics Forum, at the invitation of the Edinburgh Mathematical Society. Present were: Ari Laptev (*President and Chair*), Pavel Exner and Helge Holden (*Vice-presidents*), Stephen Huggett (*Secretary*), Jouko Väänänen (*Treasurer*), Zvi Artstein, Franco Brezzi, Igor Krichever and Martin Raussen (*EC members*) and, by invitation, Vasile Berinde, Terhi Hautala, Vicente Muñoz (Editor-in-Chief of the EMS Newsletter), Fred van Oystaeyen (*Chair of the EMS Meetings Committee*), Marta Sanz-Solé, Günter Törner (*Chair of the EMS Committee for Education*) and Riitta Ulmanen (*Helsinki EMS Secretariat*).

Officers' reports

The president first gave brief reports on several issues such as: his visit to Beijing in December 2009, where he had taken part in judging a very impressive research competition for high school students; the NSF "visitors" meeting in Washington he attended; and the recipients of the Clay Prize (G. Perelman), for which he suggested someone should write a newsletter article, and the Wolf Prize (S.-T. Yau and D. Sullivan). He then reported in detail on the MATHEI project proposal and the very low marks it got after evaluation. The discussion that followed highlighted a number of points that could explain the failure of the application and also what can be learned from that experience.

A very healthy financial situation of the society was presented by the treasurer, who focused on the 2009 figures. It was agreed that the 2011/12 budget should be circulated by email well before the coming council meeting and would be discussed with full knowledge of the case at the next EC meeting in Sofia.

The vice-president Helge Holden then reported that the EMS website was now hosted and maintained in Helsinki and that this is working extremely well such that new features could be easily incorporated in the further development of the site.

The publicity officer reported that EMS flyers had been made available for various meetings and conferences and would be on the Publications stand at the ICM in India, in close cooperation with the EMS Publishing House. It was agreed that both the EMS flyer and the poster should be re-designed to include the new EMS website. He also informed everyone about the number of registered participants for the EMS meeting of Presidents to be held

From left to right: Terhi Hautala, Vicente Muñoz, Günter Törner, Riitta Ulmanen, Fred van Oystaeyen, Franco Brezzi, Igor Krichever, Ari Laptev, Martin Raussen, Stephen Huggett, Marta Sanz-Solé, Helge Holden, Pavel Exner, Jouko Väänänen and Zvi Artstein.

in Bucharest. Under a separate item, the EC agreed the main points for the agenda of this meeting.

Membership and Meetings

Under the membership item, the issues that emerged in presentations and discussions included: the possibility of admitting the Department of Mathematics at the University of Vlora in Albania as an Academic Institutional Member of the EMS; inviting the Albanian Mathematical Society to consider joining the EMS; the letter of intention of the Kosovar Mathematical Society to join the EMS; a possible lifetime membership scheme; and the new individual membership database that would be available online soon after the meeting, presented by the treasurer, who has been warmly congratulated by the EC for the impressive work he has done in this respect.

The EC considered the suggested membership of the Programme Committee and the current membership of the Prizes Committee for 6ECM, the last one being approved. It also agreed that the PC should be reduced a little in size and that the EMS Meetings Committee would have the task of giving guidance on the overall structure of 6ECM. The EC also agreed that Jeremy Gray would be the Chair of the Prize Committee of the new Neugebauer Prize in the History of Mathematics, funded by Springer, which will first be awarded during 6ECM in Krakow.

The council meeting in Sofia, 10–11 July 2010, was then the dominant issue on the agenda. We summarize here some of the main items that were addressed by the EC: a) Several possible nominations for the EC; b) One change to the Statutes and By-Laws in order to allow for lifetime membership; c) Scrutinizing the list of nominations of delegates of individual members; d) Confirmation of the election of three delegates of Institutional Members made by ERCOM; e) Voting procedures at the council meeting; f) The status of practical arrangements for the council meeting; g) Financial support of 7,000 Euros to local organizers.

The agenda for the council meeting was also discussed and it was agreed that it would include, amongst the common issues, a Round Table of Former Presidents, the 20th anniversary of the EMS, a presentation of 6ECM, a call for bids for 7ECM, splinter groups on the work of the EMS committees and a discussion on the appropriate voting procedures at council. It was also agreed that, as in Utrecht, all the Chairs of the EMS committees would be invited to meet the EC on the morning of 10 July.

At the end of the first day's agenda Penny Davies, President of the Edinburgh Mathematical Society, gave a very nice, concise and clear 20 minute presentation of this society, under the inspired title "EMS (Edinburgh Mathematical Society) welcome EMS (European Mathematical Society)". A relaxing and well deserved dinner at Blonde Restaurant then closed a long, tiring but very fruitful first meeting day.

Standing Committees and Publishing

As usually, a major focus of the EC agenda was the reports received from or presented by the Chairs of the EMS standing committees or by the EC members responsible for the respective committees. Two Chairs, G. Törner (Education Committee), who was then confirmed for a 2010-2012 term, and Fred van Oystaeyen (Meetings Committee), had the opportunity to directly present their detailed reports. Reports were also presented by the following: H. Holden (Applied Mathematics) on behalf of the Chair, P. Exner (Electronic Publishing) and M. Raussen (Raising Public Awareness). No report had been received from the Chair of the Women and Mathematics Committee. Reports received from the Chairs of the other committees were approved by the EC. I. Krichever, who is the EC member responsible for the Eastern Europe Committee, opened a discussion that ended in the conclusion that this should perhaps be renamed the "Solidarity Committee". The EC then paid much attention to the newly established Ethics Committee and appointed I. Krichever as the EC member responsible for this committee too. The EC agreed that the Ethics Committee will focus on unethical behaviour in mathematical publications. This includes, for example, plagiarism, duplicate publication, inadequate citations, inflated self citations, dishonest refereeing and other violations of the professional code. The committee will be mainly responsible for the following three tasks:

1. To raise the awareness of the problem by preparing a code of practice.
2. To encourage journals and publishers to respond to allegations of unethical behaviour in a conscientious way.
3. To provide a mechanism whereby researchers can ask the committee to help them pursue claims of unethical behaviour. The committee may take up other relevant questions related to ethics, in connection with its work on the above points.

Under the publishing item, several issues were also addressed. We only mention here the presentation by the president of the new *Journal of Spectral Theory* and the appointing of Olaf Teschke to the Editorial Board of the EMS Newsletter, at the proposal of the Editor-in-Chief Vicente Muñoz.

Closing matters

The president expressed the EC's gratitude to the Edinburgh Mathematical Society and especially to Penny Davies and Sandy Davie for hosting such a successful and pleasant meeting in Edinburgh. The next EC meeting will be in Sofia on 9 July, preceding the EMS Council Meeting (10–11 July 2010). The last EC meeting in 2010 was also scheduled for 13–14 November but its venue has not yet been decided.

The CIRM offers a new mode of financing for conferences

Created in 1981 under the auspices of the Mathematical Society of France, with the support of the CNRS and based on the MFO model in Germany, the CIRM (International Centre for Mathematical Meetings) has kept on developing. Every year, almost 3,000 researchers focus on several activities spread out over 50 weeks, including conferences, small workgroups, research in pairs, workshops and thematic sessions. The CIRM combines agreeable hosting with first-class scientific equipment, thus laying the ground for high-level mathematical meetings that are both fruitful and convivial. The conference auditorium holds 95 people. Other meeting halls, as well as computer halls and work halls, are available. The mathematics library is one of France's largest (with 38,000 books and 700 review titles in hardback and paperback). The hotel holds 84 people and the much appreciated restaurant offers high-quality French cuisine.

The CIRM's location is ideal: it is close to Marseille's downtown (half an hour by bus) but only a 20 minute walk from the seashore. The centre's large park is next to the "Calanques" natural reserve, one of the "French Riviera's" most beautiful and well-preserved zones. Walks may be organized.

And the CIRM now offers particularly interesting financial conditions: **starting in 2010, the centre offers free accommodation for small workgroups and "research in pairs" modules. Starting in 2011, it will also take charge of accommodation fees for 40 participants per conference.** This amount may be increased upon the organizer's request and following the scientific council's approval.

Young researchers are welcome to participate and all research encounters are promoted. Discussions over coffee, at ping-pong tables or under the shade of centennial trees allow the researchers to compare their work and draw new development ideas.

For almost 30 years, the CIRM has been organizing mathematical encounters, promoting new collaborations and thus contributing to mathematical research advances. This significant improvement of the financing of scientific activities will make it easier for mathematicians of all nationalities – conference organization committees may be exclusively made up of foreign researchers – to participate and will significantly improve the CIRM's international reputation.

For more information, visit http://www.cirm.univ-mrs.fr.

Ferran Sunyer i Balaguer Prize 2010

The Ferran Sunyer i Balaguer Prize 2010 winner is:

Carlo Mantegazza (Scuola Normale Superiore di Pisa) **for his monograph "Lecture Notes on Mean Curvature Flow"**

Abstract: This monograph is an introductory text to the mean curvature flow at a high level. It is an excellent book for Ph.D. students and for researchers that desires introduce themselves in the research on this topic. It fills a gap in the literature, because, in spite that it obviously overlaps with material from other books, it organises very good an important amount of material on the mean curvature flow. The monograph is very well written, with a precise and clear style.

This monograph will be published by Birkhäuser Verlag in the series *Progress in Mathematics*.

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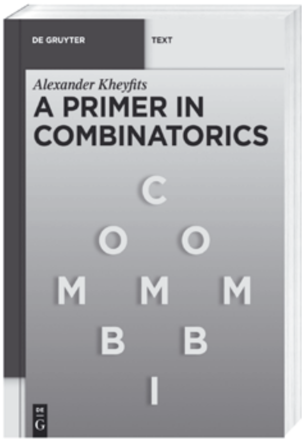

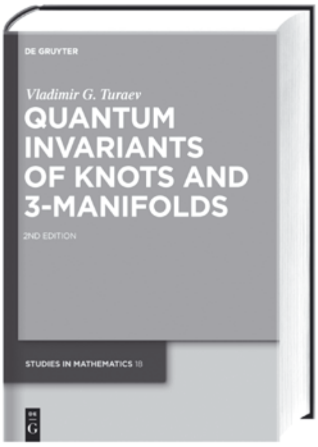

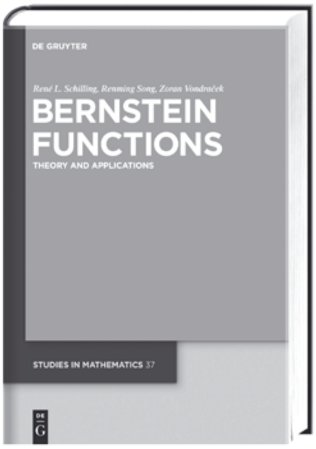

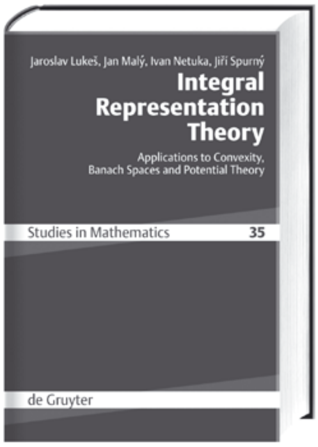

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Introducing EuDML The European Digital Mathematics Library

Thierry Bouche

In the light of mathematicians' reliance on their discipline's rich published heritage and the key role of mathematics in enabling other scientific disciplines, the mathematical community has spent a huge effort over more than a decade toward making the significant corpus of published mathematics scholarship available online, in the form of an authoritative and enduring distributed digital collection.

While the initial idea of a centralised, worldwide initiative never approached consensus on governance and funding, many national projects led to the digitisation of large quantities of mathematical literature, especially in Europe. Regarding supranational integration of these collections, a breakthrough was achieved last year when the EuDML project obtained support from the European Commission in the framework of the Information and Communications Technologies Policy Support Programme.

EuDML will design and build a collaborative digital library service that will collate the mathematical content brought by its partners and make it accessible from a single platform, tightly integrated with relevant infrastructures such as the Zentralblatt.

Introduction

Mathematicians and users of mathematics rely crucially on lasting access to original, validated articles, monographs and textbooks. The mathematical corpus, however, is more than just a collection of works; it is a complex network of interconnected items, some centuries old but still valid and relevant, each referring or related to, dependent upon or supporting each other. Whether a researcher needs to follow a subtle pyramid of reasoning through an obscure chain of articles or an engineer needs to find results related to a particular concept or a university student needs to follow the history of a specific mathematical issue, there is a common need for an integrated, interconnected gateway to the body of preserved mathematical literature.

It has long been recognised by the mathematical community that one of the prominent tools needed in order to perform mathematics-based research in the digital age is a trusted repository of mathematical knowledge that is one click away from researchers.

At a national level, especially in Europe, digitisation programmes have devoted considerable effort over the last decade to constructing impressive digital repositories of the mathematical literature published worldwide. Many more repositories of mathematical knowledge are in operation at various local levels. However, this corpus is not yet as accessible and usable as it should be. This is mainly due to a lack of coordination among stakeholders, so that existing repositories are not interoperable or interlinked. References in an article in one repository cannot usually be followed to find the target article in a different repository. Search facilities are of varying capabilities and many items are lacking the metadata essential to be able to find and exploit them. An action plan for coordinated long-term preservation of the digital mathematical corpus is also much needed.

Context and Background

Mathematics is unique among the hard sciences in its dependence on its scholarly literature. As mathematics is the mother of all sciences, which need reliable foundations, published mathematical results should be carefully checked, and the checked versions should be stored indefinitely. The storage should be carefully organized, with a clean and detailed catalogue, so that any one of those items can be referred to at any time later on, with no ambiguity. The reference graph should be constructed as well, so that new material partly based on old material can still be trusted. Because users of mathematics do not necessarily rely on current mathematical output, it should also be easily accessible over long periods of time.

Fads and trends go: the criteria for eligibility in such an archive should not be the popularity of an author or a subject at some point in time but the conformance to rigorous standards of production and validation. Each new result with an original proof that has been carefully checked by independent experts can become a crucial reference for unexpected developments and find tremendous applications in other scientific as well as technological areas.

These facts together explain why mathematicians have always taken great care of their libraries, which are the central infrastructure of all mathematics departments. The ideal library should be exhaustive, acquire new publications in real time and enjoy long opening hours and low administrative barriers to occasional visitors from other locations or disciplines. Thanks to the stubbornness of the mathematical community, there are many (paper) libraries approximating the ideal situation and they are almost evenly distributed throughout the developed countries. However, each laboratory has an idiosyncratic bias toward some subjects and a limited budget, so that no one among these lab libraries provides a full reference to the mathematical corpus. Luckily, though, interlibrary loaning assembles these dispersed libraries into one virtual global resource, more or less providing the desired capability, as long as paper remains the archival format of mathematical literature.

It should be stressed that the value of this reference library system does not only reside in the opportunity for researchers to have fast access to the resources that they most need in their daily work. On the contrary, original mathematical research has a small audience, and is seldom consulted, but basic sciences could not be performed without the reliable foundations provided by the mathematical corpus as a whole.

The birth of electronic communication at the end of 20th century, which has now become a ubiquitous, almost exclusive means of disseminating knowledge, did not dramatically change science's needs. It has opened new opportunities for easier, faster dissemination and more powerful discovery of scientific results. Unfortunately, it has also fostered such a level of competition and disorganization among digital content providers that many scientists now face increasing difficulty in accessing the published references needed to achieve their mission. For example:

- Budget pressure on libraries together with electronic "big deals" yield an ever decreasing proportion of the paper corpus physically available at the local library.
- Local libraries store volatile material (e.g. print-on-demand and photocopies) while the master digital files are not curated as they should be and risk disappearing in a few years.
- The electronic offer has quickly expanded to the level where virtually all current journals have an electronic edition and a substantial part of the core journals of the past have been retrodigitised. Books, theses, proceedings and other useful components of a mathematics research library are increasingly available digitally as well. But these are dispersed among a myriad of providers, each with specific policies on licensing and accessing their digital content. Moreover, these providers, their services and their locations are very volatile; entire collections move or disappear when publishing companies are sold, merge or become bankrupt.
- Some important resources are mostly unavailable to scientists because they are not registered at one of the services they use to dig into the literature.

Previous Work

After pioneering generalist digitisation projects launched in the 1990s in France (Gallica), in the USA (JSTOR) and in the EU (DIEPER), the seminal JFM/ERAM project (funded by DFG over 1998–2002) was the first systematic attempt to digitise a relevant corpus of mathematics, using the reviewing journal *Jahrbuch über die Fortschritte der Mathematik* as a guide to the literature of its period (1868–1942).

Philip Tondeur, the then director of the Division of Mathematical Sciences at the National Science Foundation (USA), asked John Ewing (at that time executive director of the American Mathematical Society, and a member of the CEIC of the IMU) to write a white paper on the concept of "a virtual library containing much of the past literature – a library that could eventually grow into a World Mathematics Library". This led in 2001 to "Twenty centuries of mathematics: digitizing and disseminating the past mathematical literature" (later published in [1]).

While new mathematics oriented projects were successfully launched (Project Euclid in USA, NUMDAM in France, etc.), the NSF funded a one year (2002–2003) planning project in order to study further the feasibility of the Digital Mathematics Library (DML). Its steering committee, including three European members in a total of five, met in Washington DC (USA), Baltimore (Maryland), Grenoble (France) and Göttingen (Germany). In October 2004, the final report was delivered to the NSF, which declined to support the planned project any further; important changes had occurred at NSF during that year. The report contained the output of the working groups on: content selection, technical standards, metadata, rights and licences, archiving and the economic model.

The International Mathematical Union (IMU) proposed to assume further coordination, under the flag "World Digital Mathematics Library" (WDML), with supervision by the Committee on Electronic Information and Communication (CEIC). This worldwide activity culminated in 2006 with the General Assembly of the IMU adopting in Santiago de Compostela on 20 August 2006 the communication "Digital Mathematical Library: a Vision for the Future" that had been prepared by the CEIC.

However, no actual coordination emerged among interested parties, whose number was steadily growing. The ultimate WDML goal being endorsed by virtually all mathematicians, new initiatives appeared worldwide, from a wide range of organizations with sometimes different views on content selection, cooperation and the business model.

In parallel, the EMANI project was launched in 2002 as a cooperation between Bertelsmann's Springer/Birkhäuser mathematics division and libraries in Cornell (USA), Göttingen (Germany), Tsinghua (China) and Orsay/Grenoble (France). The goal was to set up a distributed archive for the publisher's back and recent content. As such, it was the first supranational project aligned with the WDML objectives trying to overcome the organizational dead end by dealing with one distinguished commercial partner. Unfortunately, new orientations of the Springer group after the merger with Kluwer stalled the activity.

The European Mathematical Society (EMS), which had been involved from the beginning through some of its members participating in almost all the activities mentioned above, organized brainstorming meetings in Berlingen (Switzerland) in 2002 and 2003, which yielded an "expression of interest" letter sent to the European Commission in 2003, and a proposal to FP5 for an infrastructure called DML-EU, which was not selected.

While organizing the ongoing efforts under supranational governance was never achieved, the number of attempts in this direction and the number of meetings and conferences organized since 2003 underline the urgent desire of the global scientific community. The main ones have taken place in Stockholm, Minneapolis (2004), Berkeley (2005), Aveiro, Minneapolis (2006), Prague, Birmingham (2008), Santiago de Compostela and Grand Bend (2009).

Some of these allowed participants to keep track of the ongoing activities and the emerging best practices, while others focused on the technical challenges related to the creation of a "mathematical semantic web".

Our Vision

Taking into account the needs of the mathematicians and of science at large and the fact that the paper library is slowly declining into a dead archive, we infer that there is a need for a new infrastructure providing the facility of the reference mathematical library in the digital paradigm. As a lot of work has already been done to transfer the (past and current) mathematical content into digital files, we feel that the stress should now be on integrating this dispersed content into one distributed digital mathematics library (DML).

The main outcome of the envisioned library service would be to set up a network of institutions where the digital items would be physically archived. Each *local* institution would take care of selecting, acquiring, developing, maintaining, cataloguing and indexing, as well as preserving its own collections according to clear policies: it should have the role of a reference memory institution for a well-defined part of the mathematical corpus.

The network of institutions as a whole would make it possible to assemble a global, virtual library providing a one-stop gateway to the distributed content through user-friendly retrieval interfaces. Moreover, published standards of interoperability would allow this virtual library to serve as an infrastructure layer for any component of the scientist's environment where reference to a published mathematical result is necessary, turning a mere intellectual reference into an actual link to the result's statement and proof.

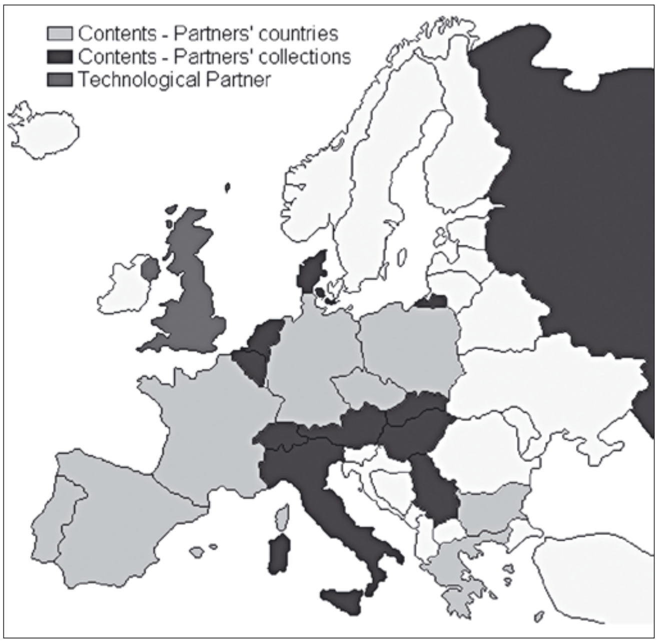
The EuDML Project

As no consensus has yet been established among all stakeholders worldwide (mathematicians as authors and mathematicians as users, as well as other scientists, publishers of any kind, librarians, information specialists, science policy makers, etc.), the EuDML project aims to set up a pilot implementation of this vision on a medium scale. It will take advantage of the number of active public DML projects across Europe aligning with its objectives, as well as a diversity of publishers willing to cooperate. It is estimated that a critical mass will be reached rapidly thanks to the quality of the content contributed by the partners. This will help promote our model toward those who did not want to partner since Day 1, thus supporting our further growth.

The EuDML project obtained support from the European Commission in the framework of the Information and Communications Technologies Policy Support Programme (CIP ICT PSP, "Open access to scientific information", project no. 250503). It was formally begun on February 1st 2010 for 36 months with a foreseen global budget of 3.2 M€. The European funding will cover up to 1.6 M€.

The EuDML Consortium

The general project coordinator is José Borbinha from the Instituto Superior Técnico (Lisbon, Portugal), who was associated with the digitisation project of the Portuguese journal *Portugaliae Mathematica* and brings general expertise in the area of digital libraries. The scientific coordinator is Thierry Bouche. The EuDML consortium consists of 14 European partners (see the full list in Table 1), who bring together the main collections of digital mathematics that have been assembled by research institutions in Europe, an impressive diversity of technical skills from digital library and e-publishing services to leading edge mathematical knowledge management, and an already wide spectrum of publishers cooperating with our innovative reference digital library framework. The consortium is gaining more strength with its two associated partners, who won't request European funding: the European Mathematical Society is associated with the project as the relevant organization for setting the goals and assessing the usefulness of the project's outcome – it will chair the Scientific Advisory Board; and the University Library of Göttingen will contribute journals digitised in the ERAM and RusDML projects, as well as the largest collection of digitised mathematical books.



The European dimension of EuDML.

Partners' collections to be integrated in EuDML.

Table 1: The EuDML partners

Partner	Location	Contributed content	Technical expertise
Instituto Superior Técnico: Computer Science Department	Lisbon, Portugal	*Portugaliae Mathematica*	Digital libraries
Université Joseph-Fourier & CNRS: Cellule MathDoc	Grenoble, France	NUMDAM, CEDRAM	Mathematics metadata, journal publishing
University of Birmingham: Computer Science Department	UK		Mathematics accessibility
FIZ Karlsruhe, Zentralblatt MATH	Berlin, Germany	Zentralblatt, EMIS ELibM	Mathematics metadata, journal hosting
Masarykova univerzita: Faculty of Computer Science	Brno, Czech Republic	Czech mathematics journals	Mathematics metadata, electronic publishing
University of Warsaw: Interdisciplinary Centre for Mathematical and Computational Modelling	Poland	DML-PL	Access platform & interoperability
CSIC: Instituto de Estudios Documentales sobre Ciencia y Tecnología	Madrid, Spain	DML-E	Electronic scientific literature
Édition Diffusion Presse Sciences	Paris, France	SMAI/EDPS journals	Journal publishing & interoperability
Universidade de Santiago de Compostela: Instituto de Matemáticas	Spain		Mathematics
Bulgarian Academy of Science: Institute of Mathematics and Informatics	Sofia, Bulgaria	BulDML	Digitisation and digital libraries
Institute of Mathematics of the Academy of Sciences	Prague, Czech Republic	DML-CZ	Mathematics, journal publishing
Ionian University	Corfu, Greece	HDML	Mathematics and education metadata
Made Media Ltd	Birmingham, UK		User interface

The State of the Art

Many findings of the NSF DML planning project publicised in 2004 are still relevant. While some of them led to a dead end because no agreement was reached worldwide on some important issues, such as content selection, copyright and the business model, others led to a rather general consensus and have indeed been endorsed by the IMU in 2002 (free navigation and metadata, and eventual open access [2]) and 2006 (best practice for retro-digitisation [4] and DML vision [3]). On the other hand, a lot of activities have taken place in the meantime that have yielded a very different landscape and have raised different problems to be tackled. The available digital mathematical content is now huge, especially in Europe, where national and smaller sized projects have assembled a significant part of the corpus in digital form. Commercial publishers, who were reluctant to invest in digitisation, have now done so. It is the right time to invite these parties to coordinate their efforts and deliver the best infrastructure out of their recent investment.

The European landscape is rich enough to be the proper context where we can design the ultimate worldwide goal, while it is still manageable in size, and we can expect support from national and European bodies to provide incentives for cooperation of all stakeholders to reach a consensus in finite time.

While local digital libraries have been set up, an active research community, based primarily in Europe, has emerged under the flag of MKM (Mathematical Knowledge Management). It aims to develop tools for dealing gracefully with mathematical knowledge in digital format, making a bridge between formal mathematics and machine reasoning, and the more textual, loosely structured texts humans write. Software for optical mathematical formula recognition, and automatic metadata extraction or inference, is beginning to be usable. Together with matching algorithms turning references into links, this paves the way toward cost-effective workflows for upgrading collections with insufficient metadata and yet more unexpected developments in the field of mathematical information retrieval, especially across discipline boundaries.

Long-term preservation of the mathematical corpus is an important issue. It is probable that bad decisions in this regard have already affected some of the items published in the early electronic era, possibly up to very recently. We hope that investigating these shortcomings now can help us rapidly identify the endangered items and rescue them before their obsolescence.

Table 2: Estimation of existing DML content. An *item* is an original mathematical text such as a book, a journal article or a dissertation. Depending on the providers listed below, it is not always possible to get a reasonable estimate for these figures. A lower bound for the available digital items is given by the 1.2 million DOIs registered by both *Math. Reviews* and *Zentralblatt*.

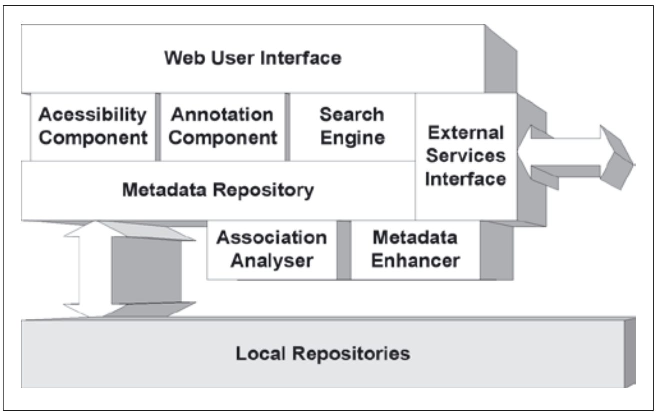
America: JSTOR (235,000 items) Project Euclid (100,000) Canadian Math. Society (4,000)
Asia: DML-JP (30,000 items)
Europe: **EuDML** partners, *friends* and future (205,000 items) Bulgaria: **BulDML** (2,500 items) Czech Rep.: **DML-CZ** (26,000 items) France: *Gallica-Math*, *Tel*, **NUMDAM**, **CEDRAM** (50,000 items) Germany: **ElibM**, *Mathematica*, *ERAM/JFM* (85,000 items) Greece: **HDML** (15,000 items) Poland: **DML-PL** (13,000 items) Portugal: **SPM/BNP** (2,000 items) Russia: *RusDML* (13,000 items) Serbia: No formalised project (3,700 items) Spain: **DML-E** (5,000 items) Switzerland: SwissDML (5,000 items)
Commercial: 700,000 items? Springer: 14 journals in *GDZ*, 1 in **NUMDAM**, 120 in Online Archives, 179 alive (300,000 items) Elsevier: 4 journals in **NUMDAM**, 63 in Backfiles, 100 alive (320,000 items) Small/medium: Cambridge University Press: 20 journals; Oxford University Press: 30; Hindawi: 18; Walter de Gruyter: 13; Wiley: 42; Taylor & Francis: 58…

The EuDML Action Plan

In order to make the distributed content more exploitable, we shall aggregate a central metadata repository and endow each item with a persistent identifier and an associated resolution system pointing back to the original resource.

All the added-value services will thus be built on top of this basic platform. We will first define a common metadata schema and export from each participating catalogue to this format. The services exploiting this metadata will then be designed and built. These activities will have a second round after assessment of the implemented prototype and will have achieved their principal task when all content partners will be able to export their catalogue to the agreed format, which will be exploitable in the central system. We expect to have a working demo website up and running around summer 2011 and a fully working system at the end of 2012. We will call the mathematical community to torture-test our system when these important milestones are met.

In parallel, we will invite all stakeholders (project partners, users, publishers as prospective content providers, policymakers, etc.) to discuss long-term policies and organizational matters toward acceptance and sustainability



EuDML System Architecture

of the service beyond the project's lifetime. This activity will take advantage of the variety of operative models among our consortium's partners, the ultimate goal being to define a reasonable archiving policy securing eventual open access to mathematical scholarly content.

For availability and convenience, EuDML will be accessed via a web interface for human users but also through a web service interface for tools and systems. For better community involvement and enhanced interactivity, we will add the Web 2.0 features of user's sharable annotation and customization.

To make our content even more accessible, we will provide explicit support, using the latest technologies, for visually impaired and dyslexic users, as well as automatic language translation support.

Since the existing metadata is heterogeneous, often sparse and monolingual, we want to go beyond the mere sum of the existing services by enhancing it with all the existing technologies at hand. We expect to augment the metadata to a minimal level of quality among all integrated collections. This will be an iterative, ongoing process using matching techniques to get some more metadata from available sources, OCR when applicable, exploitation of mathematical content, identification of citations, and generation of internal links between components of the project.

To make our content more usable, we want to exploit the fact that these collections' content is heavily mathematical in nature, so that mathematical knowledge management techniques can be applied to overcome language barriers and connect various items related by their subject, supporting a new paradigm of mathematical literature search and discovery that surpasses the pure text based and "Google style" search paradigm prevalent today.

Conclusion

Mathematics is a basic science for a wide range of scientific disciplines. There are fundamental applications of mathematical knowledge in almost every area of the natural and social sciences and the humanities. New technological developments and innovations are often based on mathematical results years or decades old. While mathematics is probably the most affordable science in the sense that it doesn't need expensive research infrastructures in order to be developed at the highest level, it is entirely depend-

Table 3: URLs of EuDML related services.

EuDML partners: digital libraries	
DML-CZ: Czech Digital Mathematics Library	http://dml.cz/
NUMDAM: French Serials Digital Mathematics Library	http://www.numdam.org/?lang=en
HDML: Hellenic Digital Mathematics Library	http://dspace.eap.gr/dspace/handle/123456789/46
DML-PL: Polish Digital Mathematics Library	http://matwbn.icm.edu.pl/
SPM/BNP: Digitised *Portugaliae Mathematica*	http://purl.pt/index/pmath/PT/index.html
DML-E: Spanish Digital Mathematics Library	http://dmle.cindoc.csic.es/en/portada_en.php
EuDML partners: publishing platforms	
CEDRAM: Centre for dissemination of academic mathematics journals	http://www.cedram.org/?lang=en
ELibM: Mathematical journals on the European Mathematical Information Service	http://www.emis.de/journals/
EDP Sciences: Mathematical journals	http://www.edpsciences.org/
EuDML partner: reviewing database	
ZMATH: Zentralblatt MATH	http://www.zentralblatt-math.org/
EuDML associated partners and collections	
Gallica-Math: Mathematical content from Gallica	http://math-doc.ujf-grenoble.fr/GALLICA/
TEL: French electronic theses	http://tel.archives-ouvertes.fr/
GDZ: Mathematica and RusDML collections at SUB Göttingen and DigiZeitschriften	http://gdz.sub.uni-goettingen.de/ http://www.digizeitschriften.de/
EuDML future partners?	
Serbia: eLibrary of Mathematical Institute of the Serbian Academy of Sciences and Arts	http://elib.mi.sanu.ac.rs/
Switzerland: SwissDML	http://retro.seals.ch/
EuDML website	http://www.eudml.eu/

ent upon accessibility to the published, validated literature, which sets standards and records trusted results. Making our mathematical heritage easily available from everywhere will considerably augment its impact and the competitiveness of scholars from all over Europe and beyond.

Beyond the professional environment of the working mathematician, remote users of mathematics face great difficulties in identifying or searching for a mathematical result they need. It is very unlikely that Google-type serendipity will help you find previous work involving a mathematical structure or formula if you don't know the name they had 70 years ago (which might be when an active research team published all its known properties) or in a discipline different from yours. This kind of frustration calls for a "semantic mathematical Web", whose feasibility will be investigated, and the foundations settled, in this project. Any advance in this area obtained in the narrow context of mathematical research papers would impact accessibility to all technical and scientific documentation.

Publishers produce new material that needs to be archived safely over the long-term and made more visible, usable and interoperable with the legacy corpus on which it settles. EuDML invites them to join together with leading technology providers in constructing the Europe-wide interconnections between their collections to create a document network as integrated and transnational as the discipline of mathematics itself.

Finally, EuDML aims to generate consensus on a sound model for satisfying the demand for reliable and long-term availability of mathematical research output, which is cared for locally by a network of institutions but is visible and usable globally.

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This article owes a lot to my colleagues in the EuDML consortium.

Promoting Mathematics[1]

Ronnie Brown

Why promote mathematics?

The following was an Introduction to a booklet produced at Bangor in World Mathematical Year 2000 as part of an EC Contract led by Mireille Chaleyat-Maurel, and which was distributed at the European Mathematical Congress, 2000, with a CDRom of John Robinson sculptures.

"Raising Public Awareness of Mathematics is probably the most important goal originally set for the World Mathematics Year 2000. And there are good reasons for that. The role of mathematics in society is subtle and not generally recognised in the needs of people in everyday life and most often it remains totally hidden in scientific and technological advancements. The old saying "The one who lives hidden lives best" is not true in present day society. If a subject becomes invisible, it may soon be forgotten and eventually it may even disappear. Mathematics has such a prominent place in school curricula all over the world that probably nobody can imagine such a fate for this subject. But if we do not constantly care about the image of mathematics, we will see continuing pressures to lower the amount of mathematics at primary schools, secondary schools and at the university level. Mathematics is exciting to many people but at the same time is considered difficult and somewhat inaccessible by many more. Since mathematics is the fundamental cornerstone in many diverse areas of society, it is important for civilisation as a whole that mathematicians do their utmost to help explaining and clarifying the role of mathematics."

Vagn Lundsgaard Hansen,
Chairman of the World Mathematics Year 2000
Committee of the European Mathematical Society.

Promoting mathematics to students?

In this article I discuss some roles of mathematics, and explore how trying to use the mathematics degree to promote mathematics to students might affect the emphasis of courses.

There are over 10,000 students doing honours degrees in mathematics in the UK. It would help to promote mathematics if they acquire at University the tools of language, background and preparation to act as ambassadors for the subject, to be able to argue the case for mathematics, and even for particular courses. They should be able to describe the role of a course, and to explain why mathematics is important not just for its applications, but for itself; and how the investigations by many, the development of technique, the following through of concepts, ideas and explanation in mathematics have led to the opening of new worlds and in the end to many applications. This means students being assessed not only on the technical skills which we expect from a mathematics degree, but also on having some idea of what judgement in mathematics entails, what is professionalism, what is the context. This is perhaps analogous to what in music is called 'musicality'.

In the UK there are 60 Royal Institution Mathematics Masterclasses with over 3,000 participants which get young people excited about mathematics, the ideas, free discussion, and co-operation rather than competition. Every effort is made by presenters to get over the key ideas, and if the children are bored, or unhappy, as evidenced by questionnaires, it is regarded as the fault of the presenter. The approach is non authoritative. Will those who, inspired by these courses, come to study mathematics for a degree expect these features to be continued, especially at a 'top' University?

Dr Brian Stewart of Oxford, an ex Chairman of the London Mathematical Society Education Committee, wrote to me: "The theme that interests me most is: 'how do we educate them while they are with us'. My own experience is that mathematicians do almost always discuss this in terms of *content*; how much can we pack in. (There's a 19th C Punch verse: Ram it in, Cram it in, Children's heads are hollow! which I like to quote.) This is very odd, because it doesn't match how we speak to each other about how we learn and develop new ideas/understandings."

An emphasis on content and assessment may lead to difficult and inaccessible courses, to 'mind-forg'd manacles' (Blake), from which some students emerge scarred, and which may give students the impression that the highest achievement for a mathematician is to write many neat answers to examination questions.

But what is such a training for? (see [12,10,4])

Training for research?

There is the old question: "How much do you need to know to do mathematical research?" and the old answer: "Everything, or nothing"!

Is the standard degree and assessment structure the best possible preparation for research which in any area requires independence and creativity, and the developing of expansive accounts of areas? Writing a thesis will

[1] This is an updated and modified version of an article published in MSOR Connections 7 no 2 July 2007, 21–25. Republished with permission.

surely involve finding and formulating problems, and solving some of them, but the key is determining the overall aims, and developing and evaluating the background to those aims, on a 'need to know' basis. Research students at Bangor have found that writing up this background has proved very helpful to the progress of their research.

Training for employment?

Most graduates go on to some form of employment. The 1974 McLone Report [12] wrote:

"A description of the employers' view of the average Mathematics graduate might be summarized thus: Good at solving problems, not so good at formulating them, the graduate has a reasonable knowledge of mathematical literature and technique; he has some ingenuity and is capable of seeking out further knowledge. On the other hand the graduate is not particularly good at planning his work, nor at making a critical evaluation of it when completed; and in any event he has to keep his work to himself as he has apparently little idea of how to communicate it to others."

Perhaps the situation has changed since 1974. Can the education and assessment in mathematics degrees be designed and to train in matters such as 'formulating problems'? 'communicating to others'? And then evaluated in quality assurance crtiteria?

On the other hand, one of our graduates went into a software firm, and I asked him when he visited us what course was most useful. To my surprise he said: 'Your course in analysis, as it gave me an idea of rigour.'

A Bangor PhD in Pure Mathematics, Keith Dakin, wrote to us in 1976 from Marconi: "We can get as many computer scientists as we want, but a mathematician who can see what mathematics is relevant to the problem at hand, is worth his weight in gold."

Are such skills developed in a mathematics degree? Compare [10].

Mathematics in context?

Conveying to students something about professionalism in mathematics was part of the idea behind a course in 'Mathematics in Context' at Bangor, where the frank discussions were stimulating and led to some great work, [5]. One student was a poor examinee, but wanted to be a teacher. He decided as a Maths in Context Project to write an assessment of the first year linear algebra course, comparing the syllabus, the lectures, the example classes, the text and the examinations! It was written in a very mature way, and he wrote: "Doing this project enabled me to come to terms with my own attitudes towards mathematics." I know many others have experimented similarly, particularly in the USA, and the debate on this needs extending, to tackle the questions raised in [4]. There is a tendency in the UK to evaluate the 'quality' of a degree course in terms of the proportion of top grade degrees; and it can then be that for the bottom 40% 'the devil take the hindmost'.

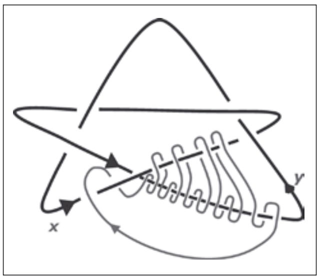
An old debating society tag is: "Text without context is merely pretext." What about "maths without context"?

My argument is that the dough of content needs leavening with the yeast of context to form digestible and flavoursome bread. The notion of 'context project' allows for a wide range of topics and a wide range of interest and abilities.

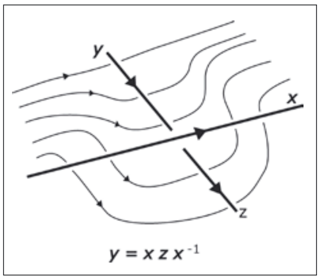
A relevant aspect is the experience of communication, in which the student wholly owns the result by determining the topic, the information, and the mode of presentation.

Communication of the essence of mathematical ideas

Graduates in mathematics will need to communicate their subject and ideas to lay people. Hassler Whitney remarked that a major problem with mathematics teaching is to make mathematical objects real. One of our demonstrations in the spirit of this and of 'advanced mathematics from an elementary viewpoint' relates to the general themes of: (i) local-to-global problems, (ii) non commutativity.



Looping the pentoil



Relation at a crossing

As shown in the picture on the left, we tie string onto a pentoil knot according to the left hand side of the rule

$$xyxyxy^{-1}x^{-1}y^{-1}x^{-1}y^{-1} = 1,$$

Amazingly, the string can be slipped off the knot, showing how algebra can structure space! The pentoil is defined by the global interrelationship of the five crossings, which are each local, at a place.

The above global rule is deduced from five relations, one at each crossing of the type shown on the right (for more details see [3]). (There is a bit of a cheat here since the mathematics of the fundamental group is about loops which can pass through each other, unlike the string!)

Notice here that if we had commutativity, i.e. $xy = yx$, the above formula degenerates. So to study knots we need a form of non commutative algebra.

We can see a further problem from this demonstration: how do you classify the ways of taking the loop off the knot? This 'higher dimensional problem' requires extending group theory as usually understood to a theory extending to all dimensions, not just dimension 1 as for loops, [3]. In some sense, this theory is 'more non-commutative' than group theory! The intuition for this is the start of what is called higher dimensional algebra, which combines many themes sketched in [1], namely 'commutative to non-commutative', 'local-to-global', 'higher dimensions', 'unifying mathematics', and could become a grand area of mathematics for the 21st century.

The above demonstration and others have been given to a wide range of audiences, including [7]. After that talk a senior neuroscientist came up to me and said: 'That was the first time I have heard a lecture by a mathematician which made any sense.' That this was 'the first time' illustrates the serious problems there are in promoting mathematics to other scientists! It also suggests that scientists in general are very interested in what new ideas and concepts have recently come into mathematcs.

Such demonstrations were developed over many years including for Royal Institution Mathematics Masterclasses for Young People in North West Wales, which have been running supported by Anglesey Aluminium since 1985.

Our exhibition 'Mathematics and Knots' was developed over four years for the 1989 Pop Maths Roadshow at Leeds University. In its development, it was very easy to say: 'I think we should use this picture or graphics', but then another member of the team will say: 'What are you trying to say about mathematics, and what is the relation of this picture to all the other pictures?' In this way we developed a philosophy of the exhibition, [6], and also learned a lot about presentation from four graphic designers!

So when one asks 'For whom does one promote mathematics?' a partial answer has to be: 'For oneself, as a mathematician and scientist.'

Applications of mathematics

A major role of mathematics is its wide range of applications. However there is a puzzlement, put to me by a young woman, Bree, at a party in Montana in 2004, where I was giving a seminar to the Department of Computational Biology. She wanted to know **why** mathematics has lots of applications. I replied: 'Mathematics is a developing language for expression, description, deduction, verification and calculation.' Bree seemed very happy with this.

Students, and Governments, need to appreciate what have been the contributions of mathematics to society and that some of these contributions are long term, unpredictable, possibly of tortuous route. Many advances have come from trying to improve understanding and exposition, following the lead of Euclid, Galileo, Euler, Faraday, Klein, Poincaré, Einstein, Hilbert, Feynman, and many others.

The urge to understand is often a motive for studying mathematics. Writing and rewriting mathematics to make things clear has been for me a stimulus to new ideas and approaches. If you write something out five times, you may see that it can be expressed a little differently, and then a bit more differently, and so on. You may even get the insight: What is really going on is …! This can be the glimmerings of a new concept. The rigour and aura of certainty in mathematics comes from the structure of interlocking concepts and proofs, each of which has been tested by thousands of people. Mathematics is by no means 'absolute truth' as some have suggested; that would not give room for development. Again, to suggest mathematics reduces to logic is like trying to describe a path to the station in terms of the paving stones. But the interlocking concepts give a landscape in which to find new paths, even repair old ones, and this landscape changes over time.

Often advances have been made by the less clever simply in order to make things clear. Indeed it is a lot of fun to take a standard subject and attempt to rewrite it in one's own, or someone else's, language or vision, formulating one's own problems. Whatever else, one will be sure to learn a lot.

My efforts to clarify to myself and obtain a nice exposition in the first edition of [2] led me to entirely new research areas, [3,7], though some high-ups have called all these 'nonsense' or 'completely irrelevant to the mainstream'. However, as Grothendieck wrote to me in 1982: 'The introduction of the cipher 0 or the group concept was general nonsense too, and mathematics was more or less stagnating for thousands of years because nobody was around to take such childish steps…', [11].

Mathematics and famous problems

An impression is sometimes given by mathematicians that the most important aspect is tackling famous problems, for example the 'million dollar problems'. Yet often their solutions, in the common view, or even in the view of scientific colleagues, are not of great moment. Mathematics is not only about doing difficult things, but also providing the framework to make difficult things easy (thus giving new opportunities for difficult tasks!).

Certainly such problems have been a challenge and have led to the development of great new methods in mathematics. Often this has involved developing notation and concepts which explain why things are true.

As G.-C. Rota writes in [13, p.48]:

"What can you prove with exterior algebra that you cannot prove without it?" Whenever you hear this question raised about some new piece of mathematics, be assured that you are likely to be in the presence of something important. In my time, I have heard it repeated for random variables, Laurent Schwartz' theory of distributions, ideles and Grothendieck's schemes, to mention only a few. A proper retort might be: "You are right. There is nothing in yesterday's mathematics that could not also be proved without it. Exterior algebra is not meant to prove old facts, it is meant to disclose a new world. Disclosing new worlds is as worthwhile a mathematical enterprise as proving old conjectures."

I met S. Ulam in Syracuse, Sicily, in1964, and he told me: 'A young person may think the most ambitious thing to do is to tackle some famous problem; but that may be a distraction from developing the mathematics most appropriate to him or her.' It was interesting to me that this should be said by someone as good as Ulam!

Mathematics and concepts

Mathematics deals with and defines concepts, by combining them into mathematical structures. These structures, these patterns, show the relations between concepts and their structural behaviour. The objects of study of mathematics are patterns and structures. These patterns and structures are abstract; the power of abstraction is that it allows for far reaching analogies.

An advantage to this theme is the democratisation of mathematics. There is a tendency in the social structure of mathematics to assume that the only interesting mathematics is that done by acknowledged geniuses, and the rest is a kind of 'fringe mathematics'. A (once!) young researcher told me the advice he was given by a top topologist was to look at what the top people do, and then find some little thing they have not done. This is the 'crumbs from the table' approach. But what young person with gumption wants to do that? In what way will that attract young people to the subject?

A related question is that of historiography, the 'history of history'. There is a considerable literature on this, and a lot on the historiography of science (the discussion in Wikipedia on these topics is informative). In contrast, the 'historiography of mathematics' is quite limited, and the 'history of mathematics' is largely concerned with the works of 'great men' (and a few women). Can this really give a fair assessment of the contributions to the progress of mathematics of the tens of thousands who have worked in the subject? What should be the methodology of making such an assessment?

For me the interest is not so much in problems already formulated, but in the ideas and intuitions, the aims, the concepts behind these formulations. It is these intuitions and concepts, the development of language, which fuel the next generation of discoveries and which, it can be argued, have been the major contribution of mathematics to science, technology and culture over the last two and a half millennia.

The theme of concepts is confirmed by the famous physicist, E. Wigner [14]

"Mathematics is the science of skilful operations with concepts and rules invented just for this purpose. [this purpose being the skilful operation …]
The principal emphasis is on the invention of concepts.
The depth of thought which goes into the formation of mathematical concepts is later justified by the skill with which these concepts are used."

Just some of the great concepts to which mathematics has given rigorous treatment are: number, length, area, volume, rate of change, randomness, proof, computation and computability, symmetry, motion, force, energy, curvature, space, continuity, infinity, deduction.

Often the route to solving problems is, in the words of a master of new concepts, Alexander Grothendieck, "to bring new concepts out of the dark", [11]. Is it possible to help students to see for themselves, in even a small way, how this comes about, and how concepts come to be invented both for applications and to disclose new worlds?

Conclusion

Einstein wrote in 1916 [9] with regard to the theory of knowledge:

"…the following questions must burningly interest me as a disciple of science: What goal will be reached by the science to which I am dedicating myself? To what extent are its general results 'true'? What is essential and what is based only on the accidents of development?… Concepts which have proved useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts. … … It is therefore not just an idle game to exercise our ability to analyse familiar concepts, and to demonstrate the conditions on which their justification and usefulness depend, and the way in which these developed, little by little. [my emphasis]"

I hope this article will provoke discussion, for example in the pages of the EMS Newsletter, of the challenge to departments to help to promote their subject through wide activities; and also by helping and encouraging students to explain and clarify to themselves and others the justification and usefulness of mathematics, as an important part of an education in mathematics.

Acknowledgements
I am grateful to R.L. Surowiec and Caroline Davis for helpful comments on a draft of this paper.

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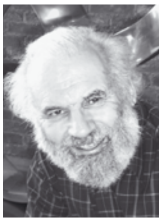


R. Brown [ronnie.profbrown@btinternet.com] was born in 1935. His undergraduate and postgraduate work was at Oxford, and he held University posts successively at Liverpool, Hull and Bangor, where he was appointed Professor of Pure Mathematics in 1970. He has currently 119 items on MathSciNet, and this has involved much national and international collaboration. One theme of his research since the 1970s has been the use of groupoids and higher dimensional versions of groupoids in mathematics, leading to his coining in 1987 the term 'higher dimensional algebra' for the study and application of algebraic systems with partial operations defined under geometric conditions. He has also written extensively on teaching and popularisation, and has given many presentations at various levels and to various audiences on knots and on the Symbolic Sculptures of John Robinson.

Clay Millenium Prize

The Clay Mathematics Institute (CMI) announces that

Dr. Grigoriy Perelman
(St. Petersburg, Russia)

is the recipient of the Millennium Prize for resolution of the Poincaré conjecture.

For more, see www.claymath.org/millennium/

International Centre for Mathematical Sciences (Edinburgh, UK)

New Scientific Director

The Board of ICMS is pleased to announce that

Professor Keith Ball

of University College London has been appointed to the post of Scientific Director of ICMS.

Later this year he will succeed John Toland who was appointed in 2002.

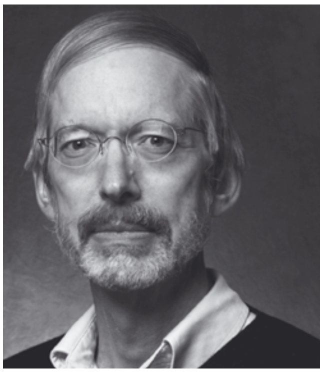
The 2010 Wolf Foundation Prize in Mathematics

The Prize Committee for Mathematics has unanimously decided that the 2010 Wolf Prize will be jointly awarded to:

Shing-Tung Yau (Harvard University, USA)
for his work in geometric analysis that has had a profound and dramatic impact on many areas of geometry and physics; together with

Dennis Sullivan (Stony Brook University and CUNY Graduate School and University Center, USA)
for his innovative contributions to algebraic topology and conformal dynamics.

Heinz Hopf Prize 2009 to Robert MacPherson

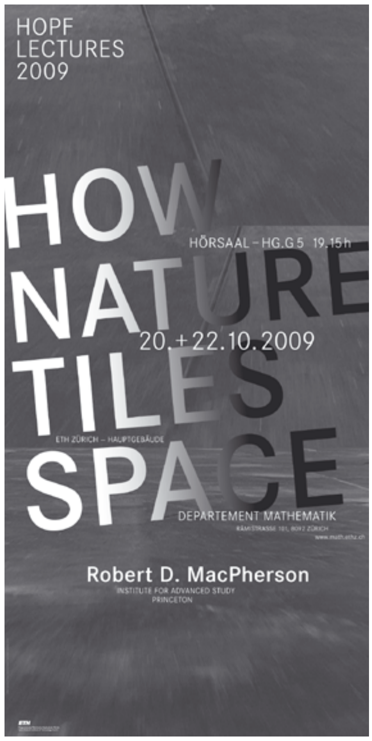


The Heinz-Hopf Laureate, Prof. Robert MacPherson.

Heinz Hopf, after whom the prize was named.

The winner of the first Heinz Hopf Prize is Robert D. MacPherson from the Institute for Advanced Study in Princeton. He received the prize in the amount of 30'000 Swiss francs on October 20, 2009, at ETH Zurich for his exceptionally broad scientific achievement. Robert MacPherson has made fundamental contributions to many branches of mathematics. His best known result is probably the joint work with Mark Goresky which developed the theory of intersection homology of singular spaces. "Robert MacPherson combines geometric vision with algebraic rigidity. If you study his work you see that it is glowing with elegance and profound in its depth", said Gisbert Wüstholz, chairman of the prize committee and professor for mathematics at ETH Zurich.

The Heinz Hopf Prize was established on the basis of a donation by a former student of Heinz Hopf. The prize will be awarded every two years to an outstanding mathematician in any field of pure mathematics. The ceremony takes place on the occasion of the Heinz Hopf Lectures, which are an established tradition at ETH Zurich; the lectures are now given by the prize winner. Former presenters of the prestigious Heinz-Hopf Lectures include celebrated mathematicians from a wide range of fields, like Curtis T. McMullen from Harvard, Helmut Hofer from the IAS and Don Zagier from the Collège de France and the Max Planck Institute.



Poster announcing the lectures connected to the award.

Laudatio (delivered by G. Wüstholz)

Just recently, in connection with the new accelerator at CERN in Geneva, black holes ripped through Swiss and European newspapers. People got very worried and speculated about their potential danger. They saw miniature black holes flying around Switzerland, prophesying the end of the world in these scientific experiments. They even brought in the courts of law to prevent the scientists from starting their experiments.

But what have black holes to do with Robert D. MacPherson? Not much, it seems at first sight. Black holes are objects of the real world of physics and astrophysics, yet we know and celebrate MacPherson as a mathematician, even a pure mathematician … maybe even a very pure mathematician.

Although he grew up in a family that was very much involved with physics and engineering, he decided to study mathematics, not particularly pleasing his father.

But at least his most successful paper, from the pragmatic point of view of his father, written as an undergraduate during an internship at Oak Ridge National Laboratory, dealt with random numbers and a question that comes up in nuclear physics. The paper was so successful and fundamental that a leading computer scientist Donald Knuth devoted many pages in his famous book on the Art of Computer Programming to explaining it.

But then Robert MacPherson turned into a real mathematician, becoming a leading world expert in singularities. To explain singularities and their exquisitely intricate nature to somebody who is not a mathematician is very difficult, yet they are essential objects not only in the real world but also in the mathematical world.

And these objects are exactly that which relates MacPherson with black holes – black holes are singularities!

Indeed, it turns out that nature produces singularities everywhere. Just watch a small stream and you will see how the current curls around and sinks, or simply watch in your bath the water disappearing down the drain, or perhaps observe at the beach how the waves crest at a certain critical moment.

If you try to describe these mostly harmless looking phenomena in nature in a mathematical language, in the

MacPherson receives the prize from the ETH president.

effort to compute and to predict reality, you very quickly run into serious problems. The mathematics just does not behave as you would expect. But you need not be a mathematician to experience what can happen. Just swim into such a turbulence in the stream or into a wave in the sea and see how you get yourself into problems.

Singularities can be studied in different ways using analysis, or they can be regarded as geometric phenomena. For the latter, their study demands a deep geometric intuition and profound geometric insight; this is what MacPherson masters in a most striking and extremely artful way.

He combines geometric visions with algebraic rigidity. If you study his work you see that it is glowing with elegance and profound in depth. He was consistently ahead of his time, developing new ideas and new approaches – ones often not shaped by the main streams of mathematical thought of the day but rather characterized by great vision. Repeatedly, the mathematical community came to embrace, extend and apply his ideas and results as they caught up with that vision.

MacPherson very often collaborated with quite distinguished co-authors, amplifying the spectrum of his mathematical charisma. These collaborations include:

- Fulton-MacPherson (1976–1995) Intersection theory, characteristic classes, enumerative geometry, singular spaces.
- Goresky-MacPherson (since 1977) Intersection homology, stratified Morse theory.
- Borho-MacPherson (1981–1989) Nilpotent orbits, characteristic classes, resolution of singularities, representation theory of Weyl groups.

MacPherson's connections with the Swiss mathematical community date back to 1983 when he participated in and significantly contributed to the famous Borel seminar, a joint seminar organized by several Swiss Universities, including ETH Zurich, Lausanne, Geneva, Bern and Basel. The seminar was initiated by Armand Borel, one of the most distinguished Swiss mathematicians of the last century. The topic of the seminar was the Goresky-MacPherson intersection homology and its use for the cohomology of arithmetic groups, one of the main research areas of Armand Borel. In this connection, mention should be made of MacPherson's papers written jointly with Harder, which have a similar mathematical context.

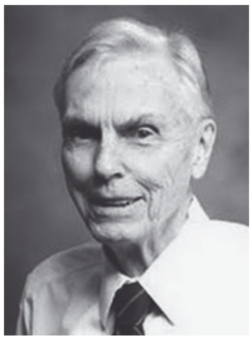
This illustrates only a small proportion of the work of MacPherson, for it spans a wide spectrum of contributions in a wide range of different areas:

- Algebraic geometry and topology.
- Algebraic groups, group actions and representation theory.
- Enumerative geometry and combinatorics.
- Locally symmetric spaces, L2-cohomology, arithmetic groups and the Langlands program.

This list is certainly not complete and is only a representative selection of his contribution to mathematics. He influenced a whole generation of mathematicians by giving them new tools to attack difficult problems and teaching them novel geometrical, topological and algebraic ways of thinking.

It is no surprise that MacPherson was and still is an admirer of music and in particular of Johann Sebastian Bach. A strong commonality exists between the work of Bach and the work of MacPherson: simplicity, elegance and beauty. Some people have said that he studied parts of Bach's work more intensively and carefully than mathematical papers – another example of the very intimate relationship between mathematics and music!

Abel Prize 2010



The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2010 to

John Torrence Tate
(University of Texas at Austin)

for his vast and lasting impact on the theory of numbers. The President of the Norwegian Academy of Science and Letters, Nils Christian Stenseth, announced the name of the 2010 Abel Laureate at the Academy in Oslo on March 24th. John Tate will receive the Abel Prize from His Majesty King Harald at an award ceremony in Oslo, Norway, May 25th. The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of NOK 6,000,000 (close to 730,000 Euros or US$ 1 mill.)

For more information consult the Abel Prize website www.abelprisen.no/

Israel Moiseevich Gelfand

Vladimir Retakh

Israel Gelfand, a mathematician compared by Henri Cartan to Poincare and Hilbert, died in New Brunswick, New Jersey, USA, on 5 October 2009.

Israel Moiseevich Gelfand was born on 2 September 1913 in the small town of Okny (later Red Okny) near Odessa in the Ukraine. There was only one school in town but Gelfand was lucky enough to have a good and encouraging mathematics teacher (one of his classmates David Milman also became a mathematician). In 1923, the family moved and Gelfand entered a vocational school for laboratory technicians. However, he was expelled in the ninth grade as a son of a "bourgeois element" – his father was a mill manager.

In his early years, Gelfand lived in total mathematical isolation. The only books available to him were secondary school texts and several community college textbooks. Through them, he deepened his understanding of mathematics, jumping over centuries of development. Like Ramanujan, he was experimenting a lot. He was not, however, simply interested in solving separate problems but also trying to understand how these problems related.

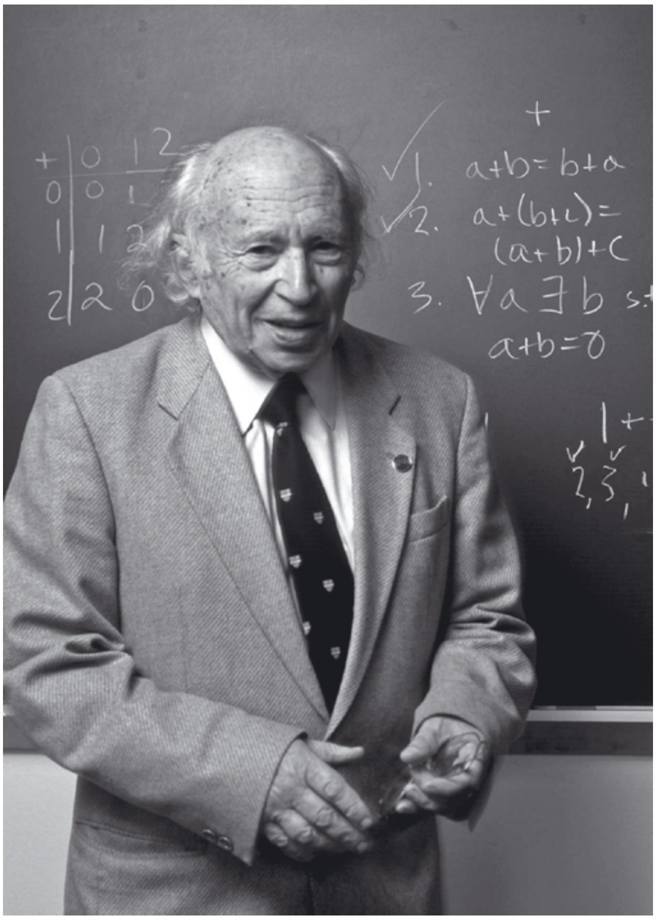
From this period came his Mozartean style and his belief in the unity and harmony of mathematics – the unity determined not by rigid and loudly proclaimed programs but rather by invisible and sometimes hidden ties connecting seemingly different areas. Gelfand described his school years and mathematical studies in an interview published in "Quantum", a science magazine for high school students [1]. In this interview he often repeats "Loser takes all" (the title of a Graham Greene novel). In his own words:

> It is my deep conviction that mathematical ability in most future professional mathematicians appears … when they are 13 to 16 years old … This period formed my style of doing mathematics. I studied different subjects but the artistic form of mathematics that took root at this time became the basis of my taste in choosing problems that continue to attract me to this day. Without understanding this motivation, I think it is impossible to make heads or tails of the seeming illogicality of my ways in working and the choice of themes in my work. Because of this motivating force, however, they actually come together sequentially and logically.

At the age of 15 Gelfand learned of a series for calculating the sine. He described this moment in the "Quantum" interview:

> Before this I thought there were two types of mathematics, algebraic and geometric … When I discovered that the sine can be expressed algebraically as a series, the barriers came tumbling down, and mathematics became one. To this day I see various branches of mathematics, together with mathematical physics, as parts of a united whole.

In 1930, sixteen and a half years old, Gelfand left his parents and moved to Moscow to live with distant relatives. For a while he did not have steady work and lived on earnings from occasional odd jobs. At some point he had the good fortune to work at the check-out counter at the Lenin Library. This gave



Israel Moiseevich Gelfand (Courtesy of Rutgers University)

Gelfand a rare opportunity to talk with mathematics students from Moscow University. From these contacts he learned that none of his discoveries were new. Neither this nor other circumstances of his life deterred him and his interest in mathematics grew. He continued his intensive mathematical studies and began attending seminars at Moscow University.

Just as in his abrupt expulsion from school, the next twist in Gelfand's fate was also one of many paradoxes of life in the Soviet Union. On the one hand, as the son of a "bourgeois element" he could not become a university student. On the other hand, at 18 he was able to obtain a teaching position at one of many newly created technical colleges and, at just 19, to enter the PhD program at Moscow University. The reasons are simple: the Soviet state needed knowledgeable instructors to educate its future engineers and scientists with the proper "proletarian origins". Furthermore, the system was not rigid enough to purge or even strictly regulate graduate schools. Therefore, Gelfand could enter a PhD program without a college or even a high school diploma.

Gelfand was influenced by several Moscow mathematicians, especially his thesis adviser A. N. Kolmogorov. In the "Quantum" interview Gelfand said that from Kolmogorov he learned "that a true mathematician must be a philosopher of nature". Another influence was the brilliant L. G. Shnirelman. In 1935, Gelfand defended his "candidate" (PhD) thesis and in 1940 obtained the higher degree of Doctor of Science. In 1933, he began teaching at Moscow University, where he became a full professor in 1943. He lost this position temporarily in 1952 but was allowed to continue his famous seminar. Gelfand also worked at the Steklov Institute and for many years at the Institute for Applied Mathematics. There he took part in the secret program related to the Soviet version of the Manhattan project and its extensions. Andrey Sakharov mentioned his work with Gelfand in [2].

In 1953, Gelfand was elected a Corresponding Member of the Academy of Science (an important title in the Soviet Hierarchy). This happened right after Stalin's death and the end of his anti-Semitic campaign. The political uncertainty of the times made his election possible. Later the situation in the USSR stabilized and Gelfand became a full member of the Soviet Academy only in 1984 after being elected to many leading foreign academies.

In 1989, Gelfand moved to the USA. After spending some time at Harvard and MIT he became a professor at Rutgers University, where he worked until his death.

A famous joke claims that a mathematician proves theorems until the age of forty and then spends ten years writing textbooks for graduate students and the next ten years for undergraduates. Nothing could be further from Gelfand's life in mathematics; about every decade he established a new area of research.

In one of his early papers he introduced the idea of considering maximal ideals of Banach algebras (commutative normed rings) as points of a topological space – an approach that had an enormous impact in analysis and algebraic geometry. The modern procedure of replacing algebraic manifolds by categories of certain sheaves owes much to Gelfand.

In his fundamental paper with D. Raikov, Gelfand showed the existence of "sufficiently many" unitary representations for locally compact groups. Together with M. Naimark, he founded the new area of noncommutative C^* algebras.

In the mid-1940s, Gelfand began another cycle of papers in representation theory that, in a way, continued for more than 50 years. A leading American expert in this field once said that after reading all definitions in Gelfand's papers he could easily prove all his theorems. This 'easiness' was based on long computations and a thorough consideration of carefully selected examples. A final paper looked very different from the intermediate versions. The following quip is sometimes attributed to Moscow mathematician A. Ya. Povzner: "Gelfand cannot prove hard theorems. He just turns any theorem into an easy one." Gelfand introduced several basic concepts in representations. For example, the eponymous notion of "Gelfand pairs" is based on the "involution method" discovered by Gelfand.

Gelfand always closely watched von Neumann, whom he considered one of his main competitors. After von Neumann's work on factors it became clear that infinite discrete groups do not have a good theory of representations. The same was expected of noncompact Lie groups. Contrary to this belief, Gelfand, M. Naimark and M. Graev constructed a rich and beautiful representation theory for such groups.

Gelfand and Naimark introduced other famous tools in representation theory, such as the construction of irreducible representations of reductive groups by induction from a Borel subgroup and defined characters of infinite-dimensional representations, and they were the first to use what is now known as the Bruhat decomposition for classical groups.

Gelfand-Tsetlin bases are now a household name in representation theory and combinatorics. Surprisingly, this original and fundamental work was at first ignored by mathematicians but heavily used by physicists.

In the late 1950s, Gelfand and Graev began the study of representations of reductive groups over finite and p-adic fields k. Among their achievements is the Gelfand-Graev model and the construction of irreducible representations of the group $SL(2,k)$. This work was later amplified by R. Langlands.

The category O and the celebrated BGG-resolution of I. Bernstein, I. Gelfand and S. Gelfand are cornerstones of geometric representation theory, an area of great current interest.

Gelfand's long-term study of the Plancherel measure and its role in representation theory led to an explicit construction of such measures for semisimple Lie groups. By developing these ideas Gelfand, together with S. Gindikin and M. Graev, developed integral geometry, one of his favourite subjects. It is now used to convert data from MRI and CAT scans into three-dimensional images.

In 1972, Gabriel discovered that a quiver has finitely many indecomposable representations if and only if it is of ADE type. A year later J. Bernstein, Gelfand and V. Ponomarev explained that indecomposable representations of any ADE quiver are parameterized by positive roots. This provided a conceptual proof of the Gabriel theorem using root systems and Weyl groups. Later, Gelfand and Ponomarev introduced the preprojective algebra of a quiver. The relation between quiver representations and Lie groups was further explored by C. Ringel, G. Lusztig and H. Nakajima and this influenced the development of geometric representation theory of quantum groups.

Representation theory also stimulated the joint work of Gelfand and his former student A. Kirillov on algebraic growth. It led to the introduction of another basic notion: the Gelfand-Kirillov dimension.

Gelfand also made important contributions to differential equations. In joint work with Levitan, Gelfand developed a procedure for reconstructing a potential $q(x)$ from the spectral data of the operator $d^2/dx^2 + q(x)$. This procedure is a kind of non-linear Fourier transform. In the 70s, it became clear that this transform is a Korteweg-de-Vries analogue of the usual Fourier transform. This led Gelfand and L. Dickey to their theory of formal variational calculus and Gelfand-Dickey algebras.

In topology, Gelfand produced a beautiful series of papers on combinatorial computations of characteristic classes with A. Gabrielov and M. Losik. Gelfand and D. Fuchs originated the study of cohomology groups of Lie algebras of vector fields on smooth manifolds. Contrary to the common belief that such groups must be trivial or too big, they constructed a rich theory with numerous connections to conformal field theory, string theory and foliations.

In the 1980s, Gelfand turned to the classical subjects of hypergeometric functions and determinants. At his Moscow seminar he proclaimed that the next century will be a century of analysis and combinatorics. His theory of hypergeometric functions as solutions of a holonomic system of differential equations with M. Graev, M. Kapranov and A. Zelevinsky (GKZ-system) found numerous applications in toric varieties, mirror symmetry and, yes, combinatorics.

Nobody expected many new results in the good old theory of determinants. However, a study of multidimensional determinants and resultants with M. Kapranov and A. Zelevinsky and noncommutative determinants (I was lucky to participate in this enterprise from the very beginning) was the favourite project of Gelfand in the last period of his life. Once more, he did not follow the mainstream and once more he was right: the project led to a flow of new results and ideas in algebra, geometry and combinatorics.

One cannot write about Gelfand without mentioning his legendary seminar, which started in 1943 and continued until his death. Gelfand considered the seminar as one of his most important creations.

It is hard to describe the seminar in a few words: it was a "mathematical stock exchange", a breeding ground for young scientists, a demonstration of how to think about mathematics, a one-man show and much more. It was not about functional analysis or geometry – it was about mathematics. Some would come to the seminar just to hear Gelfand's jokes and paradoxes (here is a mild one: "present your talks in a quiet voice if you want a Fields Medal") and some to find out about fresh preprints coming from the inaccessible West. But most participants were attracted by the power and originality of Gelfand's approach to mathematics. He used the simplest examples but turned them around in totally unexpected ways. He could show the world in a drop of water.

Gelfand always paid special attention to students, who formed more than half of the audience. From time to time he would repeat his favourite paradox: "My seminar is for high school students, decent undergraduates, bright graduates and outstanding professors." The best description of the seminar was given by S. Gindikin [3] (see also [4] and [5]).

The seminar also served as a constant supply of Gelfand's collaborators, who were already familiar with Gelfand's style and his way of thinking. Their roles were quite different. Sometimes they would discuss specific examples, sometimes very vague ideas and sometimes bring their own suggestions that would be ridiculed, torn apart, turned upside down and then transformed into something exquisite. Few could bear the task but the pool of mathematicians in Moscow was enormous.

The seminar was a reflection of Gelfand's passion to teach, as he tried to teach everyone and everywhere. Among his former students are F. Berezin, J. Bernstein, E. Dynkin, A. Goncharov, D. Kazhdan, A. Kirillov, M. Kontsevich and A. Zelevinsky. All these people are totally different in style and reflect the multi-faceted face of Gelfand's school. The number of his informal students is hard to estimate.

Out of this passion grew his famous Correspondence School for middle and high school students. Gelfand founded, ran and wrote several textbooks for the school. His university textbooks "Linear Algebra" and "Calculus of Variations" (written with S. Fomin) also bear an imprint of his style and personality.

Gelfand's interests spread far beyond pure and applied mathematics. He left a number of papers in biology, physiology and other fields.

Gelfand was the first to obtain the Wolf Prize in 1978 (together with C. L. Siegel) and received many other awards including the Kyoto Prize (1989) and the McArthur Fellowship (1994). He was elected to all leading academies and received honorary degrees from many universities.

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Vladimir Retakh [vretakh@math.rutgers.edu] was born in 1948 in Kishinev, Moldova (former USSR). In 1965, he became a student of the Moscow Pedagogical Institute and started to attend the Gelfand Seminar. He defended his PhD in 1973. Until 1989, he worked in industrial institutions and then joined one of Gelfand's groups in the Soviet Academy of Science. In 1993, he emigrated to the US. Since 1999, he has been a professor at Rutgers University (New Jersey, USA). He has written more than 30 papers with Gelfand (the first paper appeared in 1988). His personal recollections can be found on his website [4].

Mathematics of evolution

José A. Cuesta

A mathematician is a blind man in a dark room looking for a black cat which isn't there.
Charles Darwin

It is somehow strange to read an article on Darwin and evolution in a mathematical journal. Both are always associated, obviously, with biology. However, evolutionary theory, like every other theory deserving that name, admits quantitative formulations of many of its aspects. Darwin himself, not very skilled in mathematics but nevertheless an educated scientist, acknowledged that "every new body of discovery is mathematical in form, because there is no other guidance we can have". For Darwin, mathematicians were people with a sixth sense that allowed them to "see" in places of the mind where everybody else is "blind". Hence his quotation at the beginning of this article which, funny as it may seem, expresses Darwin's admiration for mathematicians.[1]

Darwin's great contribution, the theory of evolution by natural selection [3], received a fundamental input when the laws of inheritance, discovered by Gregor Mendel in 1865 [15] and rediscovered by de Vries, Correns and von Tschermak in 1900, were incorporated into the theory. From that moment on, and in a way that later became known as *population genetics,* a group of mathematicians, amongst whom Fisher, Haldane, Wright and later Kimura are prominent examples, laid the foundations of the mathematical theory of evolution. Nowadays, this theory has a status of its own within the field of applied mathematics, and it has developed and diversified, allowing us to understand the subtle mechanisms that evolution operates with, not just in biology but in many other disciplines sharing similar principles, like linguistics, economics, sociology and computer science.

Last year we celebrated Darwin's bicentennial as well as the sesquicentennial of the publication of *The Origin of Species,* and these anniversaries provide an appropriate excuse to revise evolutionary theory from a mathematical point of view. This is the goal of this article. It should be clarified at this point that mathematical contributions to evolutionary theory are so many and so diverse that only a few of them can be sampled here. Besides, the divulgatory aim of this article discourages any attempt at deeply reviewing them, so the interested reader is referred to the excellent texts on the matter that are referenced in this article [5, 7, 16]. Another clarification is also needed. There are two kinds of reproductive mechanisms in living beings: asexual, by which an organism can replicate by itself alone, and sexual, by which the intervention of more than one organism (almost always two) is necessary for reproduction. The former is typical (but not exclusive) of simple organisms, like viruses, and is the subject matter of most mathematical models; the latter implies combining genetic material coming from at least two parents, which leads to particular complications that are also the subject matter of more elaborate models. This article will almost exclusively deal with asexual reproduction (which is already complicated enough), although sexual reproduction will occasionally be mentioned at specific points.

1 Fundamental mechanisms of evolution

Upon reflection of what is necessary, at an abstract level, for an evolutionary process to occur, no matter what the context is, one realizes that the necessary condition is the concurrence of three fundamental mechanisms: *replication,* the mechanism by which entities can create copies of themselves; *mutation,* the mechanism that generates small variations within those copies; and *selection,* the mechanism by which the "best" copies are able to eliminate all the others generation after generation. Let us consider these three mechanisms one by one.

Replication

A typical bacteria divides every 20 minutes, generating two copies of itself; 20 minutes later there will be four bacteria; after one hour there will be eight… Bacterial population in a generation n_t is related to that of the previous generation by the simple equation $n_t = 2n_{t-1}$, whose solution, assuming $n_0 = 1$, is $n_t = 2^t$. A replicative process like this one leads to exponential growth. It was Malthus who first proposed this law of growth in his book *An Essay on the Principle of Population* [13]. According to this law, assuming a situation in which generations are intertwined, if a population $n(t)$ reproduces at a constant replication rate per individual r, i.e. $\dot{n} = rn$, then $n(t) = n(0)e^{rt}$. This is Malthus' model for human population growth and the continuum version of the bacterial reproduction law that we have just obtained.

Malthus was one of the most important influences on Darwin's thoughts because of what has been referred to as "Malthusian catastrophe". It can be illustrated with the example of bacteria. According to the law we have obtained, after only

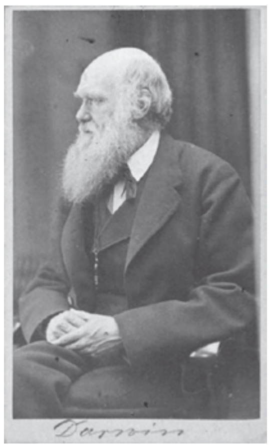

Figure 1. A photograph of Charles Darwin, circa 1871, by Oscar Gustave Rejlander (1813–1875). Source: Wikimedia Commons.

two days (144 divisions) we will have $2^{144} \approx 2 \times 10^{43}$ bacteria. More precisely, if the bacterial diameter is $\sim 1\,\mu$m and the bacterial density is that of water ($1\,\mathrm{g\,cm^{-3}}$) then 2×10^{43} bacteria have a mass of approximately 2×10^{28} kg, i.e. about 3000 times the mass of the Earth! Obviously such a growth cannot be sustained and should drive most individuals to extinction. In fact, the lack of resources can be effectively added to Malthus' law by replacing the constant replication rate per individual with a decreasing function of the population that vanishes at the point where the population reaches its sustainability limit. In its simplest form this leads to the *logistic* law

$$\dot{n} = r\left(1 - \frac{n}{K}\right)n, \tag{1}$$

whose solution

$$n(t) = \frac{Kn_0e^{rt}}{K + n_0(e^{rt} - 1)}, \qquad \lim_{t\to\infty} n(t) = K \tag{2}$$

is a curve that increases monotonically in time (when $n_0 < K$) up to a saturation value K, referred to as the "carrying capacity" of the environment. Therefore we can consider that populations undergoing such a growth process reach an equilibrium with their environment that keeps the population constant.[2] In practice, the population will fluctuate around that value due to stochastic effects that the kind of laws we are considering simply neglect.

Selection

In a situation where resources are scarce, as we have just described, individuals struggle to obtain them, survive and reproduce. Competition with like individuals yields the saturation predicted by the logistic law but when there are individuals of different types (species), their differences, no matter how small, play a crucial role in deciding who survives and who perishes. The key evolutionary parameter here is *fitness,* defined as the mean number of adults that an individual yields in the next generation. In case of Malthusian populations, fitness is measured by the parameter r. If logistic, the fitness, $f = r(1 - n/K)$, will depend on the total population. In the general case, f will be a function of the total population.

Suppose the population is made up of n different species with fitness f_i, $i = 1, \dots, n$. By definition, their respective populations grow according to $\dot{n}_i = f_i n_i$. The total population $N = \sum_{i=1}^n n_i$ will thus grow as

$$\dot{N} = \sum_{i=1}^n \dot{n}_i = \sum_{i=1}^n f_i n_i = N\phi, \qquad \phi = \sum_{i=1}^n f_i x_i, \tag{3}$$

$x_i = n_i/N$ being the fraction of the population corresponding to species i and ϕ being the population *mean fitness.* The total population is thus Malthusian with replication rate ϕ (remember that f_i can depend on N). We can now obtain a growth law for x_i,

$$\dot{x}_i = \frac{\dot{n}_i}{N} - x_i\frac{\dot{N}}{N} = x_i(f_i - \phi). \tag{4}$$

This evolutionary law is commonly known as the *replicator equation* [16] and not only describes the evolution of biological systems but also plays a prominent role in game theory [8]. The law followed by the population fractions is similar to Malthus' law, only that now the replication rate is measured with respect to its mean over the population. This means that the population fraction of a species will increase only if its fitness is above that mean and will otherwise decrease. We can envisage here the principle of "survival of the fittest". In fact, it is very easy to derive this principle from equation (3). Let us assume that species k is fitter than the rest of them, i.e. $f_k > f_i\ \forall i \neq k$. The evolution equation for x_k can be rewritten as

$$\begin{aligned}\dot{x}_k &= x_k\left(f_k - \sum_{i=1}^n f_i x_i\right) = x_k \sum_{i=1}^n (f_k - f_i)x_i \\ &= x_k \sum_{i\neq k,\, i=1}^n (f_k - f_i)x_i,\end{aligned} \tag{5}$$

where we have used $\sum_{i=1}^n x_i = 1$. It follows from (5) that the sum on the right side will be positive as long as there is at least one species $i \neq k$ with $x_i > 0$, and in that case x_k will increase in time at the expense of the population fractions of the remaining species. In other words,

$$\lim_{t\to\infty} x_k(t) = 1, \qquad \lim_{t\to\infty} x_i(t) = 0, \quad \forall i \neq k, \tag{6}$$

which expresses mathematically the principle of survival of the fittest.

In the case where the fitness is constant we have a result, due to Fisher, referred to as the *fundamental theorem of natural selection* [5]. Its derivation amounts to computing

$$\begin{aligned}\dot{\phi} &= \sum_{i=1}^n f_i\dot{x}_i = \sum_{i=1}^n f_i x_i(f_i - \phi) = \sum_{i=1}^n x_i(f_i - \phi)^2 \\ &= \sigma_f^2 \geq 0.\end{aligned} \tag{7}$$

Put in a different way, the mean fitness never decreases in time and its growth rate is the fitness variance over the population. Thus it will increase as long as there is variability in the population and will do so by increasing the population of the fittest.

Mutation

Replication is not error-free. In general, replication errors lead to non-viable individuals that cannot survive. These errors can be accounted for by adjusting the replication rate. Occasionally, however, a mutation can produce an offspring of a different and viable type. Thus mutation can be regarded as a stochastic process by which individuals of species i produce individuals of species j with a probability $q_{ij}\,(\ll 1)$. This mechanism introduces variability in an otherwise homogeneous population. The replicator equation (3) must be modified to account for this new process:

$$\dot{x}_i = x_i\left(f_i - f_i\sum_{j\neq i} q_{ij} + \sum_{j\neq i} f_j q_{ji} - \phi\right), \tag{8}$$

where the first new term accounts for mutations transforming individuals of species i into individuals of any other species and the second one accounts for mutations transforming individuals of any other species into individuals of species i. Defining $q_{ii} = 1 - \sum_{j\neq i} q_{ij}\,(\geq 0)$ and introducing the stochastic matrix $Q = (q_{ij})$ (which we will refer to as the *mutation matrix),* the equation above can be rewritten in vector form as

$$\dot{\mathbf{x}} = \mathbf{x}FQ - \phi\mathbf{x}, \tag{9}$$

where $\mathbf{x} = (x_1, \dots, x_n)$ and $F = \mathrm{diag}(f_1, \dots, f_n)$. This equation is termed the *replicator-mutator equation* [16]. The stochastic character of Q can be expressed in matrix form as

$Q\mathbf{1}^\mathsf{T} = \mathbf{1}^\mathsf{T}$, where $\mathbf{1} = (1, \ldots, 1)$. Because of this, as $\phi = \mathbf{x}F\mathbf{1}^\mathsf{T} = \mathbf{x}FQ\mathbf{1}^\mathsf{T}$, it follows immediately that $\mathbf{x}\mathbf{1}^\mathsf{T} = 1$ is a constraint preserved by equation (9).

Since FQ is a non-negative matrix, Perron-Frobenius theory [18] tells us that the rest points of the dynamical system (9) are all given by the left-eigenvectors of matrix FQ corresponding to its largest eigenvalue, all of whose components are non-negative. Furthermore, if Q is irreducible then there is only one such eigenvector. Every irreducible matrix has at least one non-diagonal element in each row, so the equilibrium vector necessarily has more than one non-zero component. Therefore, when all species can mutate, the equilibrium population cannot be homogeneous. This is an interesting observation because it implies that ϕ is not at its maximum in equilibrium, as it was when there were no mutations; there is competition between selection, which pushes ϕ towards its maximum value, and mutation, which tends to decrease it. We can try to see what happens to the fundamental theorem in this case. A similar reasoning to the one leading to equation (7) yields

$$\dot{\phi} = \mathbf{x}(FQ - \phi I)^2\mathbf{1}^\mathsf{T}. \tag{10}$$

Despite its similarity to σ_f^2, the non-negativity of this term is no longer guaranteed.

2 The problem of reversion

Despite Mendel publishing his work almost simultaneously with Darwin, it seems that Darwin was never aware of its existence. This put Darwin in serious trouble. Reading *The Origin of Species* one realizes how Darwin stumbles once and again over it: the inheritance theory in sexually reproducing species that was accepted in his time (formulated by Galton [2] in 1875) led to the phenomenon of reversion. According to this theory, every progenitor contributes fifty-fifty to a given trait, e.g. height. We can then formulate a simple stochastic law for the quantitative value of the trait of an offspring given those of its parents: $X_{n+1} = \frac{1}{2}(X_n^{(1)} + X_n^{(2)}) + Z_n$, where $X_n^{(1)}$ and $X_n^{(2)}$ are two stochastic variables, identically distributed according to $P_n(x) = \Pr(X_n \leq x)$, representing the parents' traits in generation n, and where Z_n is a noise, which we will assume to be normally distributed $N(0, \sigma)$ for all n. If $F_n(q) = \int_{-\infty}^{\infty} e^{iqx}\, dP_n(x)$ denotes the characteristic function of distribution P_n then $F_{n+1}(q) = F_n(q/2)^2 e^{-\sigma^2 q^2/2}$. The solution to this equation is

$$\log F_n(q) = 2^n \log F_0(2^{-n}q) - \frac{\sigma^2}{2} q^2 \sum_{k=0}^{n-1} 2^{-k} \tag{11}$$

and therefore

$$\lim_{n\to\infty} F_n(q) = e^{qF_0'(0) - \sigma^2 q^2}. \tag{12}$$

Since $-iF_0'(0) = \mu$, the mean value of the initial distribution $P_0(x)$, we conclude that $P_n(x)$ approaches the normal distribution $N(\mu, \sqrt{2}\sigma)$ as $n \to \infty$. So, regardless of the initial distribution (provided it has a finite mean), the distribution of the trait approaches a normal distribution with the same mean value as the initial distribution. This means that, even if the latter was bimodal, in the end all the population ends up being of homogeneous type (up to some noise). The difficulty this poses to Darwin's theory is to suppress the variability introduced by mutations. This is the reason why Darwin often resorted to the argument that mutant populations must remain isolated for quite some time in order for a new species to emerge.

3 Mendel, or the "quantum" theory of inheritance

Mendel's crucial contribution was to discover that traits are transmitted in "quanta" of inheritance (what we nowadays call *genes)* that do not admit gradations.[3] Traits such as height, which seem to violate this principle, are but complex traits resulting from the combined effect of several simple traits, all of which are "quantum" (either they are present or absent). Every sexed individual carries two of those quanta per trait, one from its father and another from its mother, and in its turn transmits one of them (at random) to each of its offspring.

Consider a trait (e.g. the red colour of a rose) determined by a variant (an *allele* in genetic parlance) A of the corresponding gene. Suppose that this allele has a muted variant a, which does not produce colour. According to Galton's inheritance model, hybrid descendants should show a pink colour grading, generation after generation, until its eventual return to the original red colour. According to Mendel's laws, if the parent generation is made of p A-alleles and q a-alleles, if mating is random and if the population is sufficiently large then there will be p^2 AA-individuals, $2pq$ Aa-individuals and q^2 aa-individuals in the next generation. Furthermore, the distribution will remain stable in successive generations. This result is known in genetics as the Hardy-Weinberg law [5]. It implies that there is no reversion to the wild type: pure aa mutants remain in the population, generation after generation, in a ratio which depends on the initial amount of mutants. Mendelian genetics therefore is responsible for the maintenance of the variability in the populations introduced by mutations.

4 The fourth element: genetic drift

In section 1, we mentioned that replication, selection and mutation form the basic triad of evolutionary dynamics. Although strictly speaking this is true, there is a fourth ingredient that is unnecessary in principle and yet becomes crucial in understanding the mechanisms of adaptation and speciation undergone by evolving entities: *genetic drift.* All the previous discussion presumes that evolving populations are infinitely large; hence the deterministic dynamics we have introduced so far and the validity of principles like the Hardy-Weinberg law. But when it comes to finite populations some noise appears due to the statistical sampling inherent to replicative processes. This noise is what we refer to as genetic drift. Its effects can be dramatic in small populations; that is why it becomes crucial when populations traverse an evolutionary "bottleneck", i.e. a situation in which the population gets strongly reduced, because of epidemics, climate change, geographical isolation, etc.

There are two basic mathematical models of genetic drift: Fisher-Wright and Moran [5]. Both assume a constant population, limited by the carrying capacity of the environment. The first one describes situations in which every generation replaces the previous one (such as in stational plants); the second one describes the case in which generations can overlap.

Fisher-Wright model

Assume that we deal with a population of N individuals and focus on a particular trait (e.g. the red colour of a rose again). Suppose that this character is determined by the presence of an allele A, of which there is a mutant a producing a different colour (e.g. white). Assume further that the population reproduces asexually (i.e. A begets A and a begets a) and that mutations A $\to$ a occur with probability μ and mutations a $\to$ A occur with probability ν. If initially there are k A-individuals and there is no selective difference between both alleles (both have the same fitness) then the probability that an individual in the next generation is of type A will be $\psi_k = (k/N)(1-\mu) + (1-k/N)\nu$. Imagine that every individual yields a large number of offspring to a pool and that we randomly extract N individuals from that pool to form the next generation. If $X_t \in \{0, 1, \ldots, N\}$ is a random variable describing the number of A-alleles in generation t then the Fisher-Wright model is defined as a Markov chain with transition probability

$$P_{kj} = \Pr(X_{t+1} = j \mid X_t = k) = \binom{N}{j}\psi_k^j(1-\psi_k)^{N-j}. \qquad (13)$$

The chain is ergodic for all $0 < \mu, \nu < 1$ because $P_{kj} > 0$ $\forall k, j \in \{0, 1, \ldots, N\}$, thus there is a unique stationary probability distribution $\mathbf{w} = (w_0, w_1, \ldots, w_N)$ that can be obtained as the solution to the equation $\mathbf{w} = \mathbf{w}P$ [10, 18]. There is no analytic expression for $\mathbf{w}$ except for a very particular case: $\mu + \nu = 1$ (of no biological interest because mutation rates are too large). In this case $\psi_k = \nu = 1 - \mu$ and

$$(\mathbf{w}P)_j = \binom{N}{j}\nu^j(1-\nu)^{N-j} \qquad (14)$$

for every $\mathbf{w}$, so the right side of the previous equation describes the stationary distribution.

What can be calculated in general is the mean value of $\mathbf{w}$. To obtain it let us denote $X_\infty = \lim_{t\to\infty} X_t$ and define the vector $\boldsymbol{\xi} = (0, 1, \ldots, N)$. We can then write $E(X_\infty) = \mathbf{w}\boldsymbol{\xi}^\mathsf{T} = \mathbf{w}P\boldsymbol{\xi}^\mathsf{T}$. Now,

$$(P\boldsymbol{\xi}^\mathsf{T})_k = \sum_{j=0}^{N} j\binom{N}{j}\psi_k^j(1-\psi_k)^{N-j} = N\psi_k$$
$$= k(1-\mu) + (N-k)\nu, \qquad (15)$$

so $E(X_\infty) = (1-\mu)E(X_\infty) + [N - E(X_\infty)]\nu$; hence $E(X_\infty) = N\nu/(\mu+\nu)$. By a similar, albeit more tedious, procedure we can obtain the variance $\sigma^2 = N^2\mu\nu/[(\mu+\nu)^2(1+2N\mu+2N\nu)] + \epsilon$, where ϵ contains terms of smaller order (for instance, if $\mu, \nu = O(N^{-1})$ then $\epsilon = O(N)$).

The most interesting case to be considered is that in which there is only genetic drift ($\mu = \nu = 0$, thus $\psi_k = k/N$). The Markov chain is not ergodic anymore because it has two absorbing states: $k = N$ (all individuals are of type A) and $k = 0$ (all individuals are of type a). This means that, regardless of the initial population and in spite of the lack of selective factors, eventually one type invades the whole population. The interesting magnitude now is the probability $\pi_j = \Pr(X_\infty = N \mid X_0 = j)$ that the population ends up being of type A given that there were initially j individuals of that type. Denoting $\boldsymbol{\pi} = (\pi_0, \pi_1, \ldots, \pi_N)$, it can be shown that $\boldsymbol{\pi}^\mathsf{T} = P\boldsymbol{\pi}^\mathsf{T}$ and that $\pi_0 = 0$ and $\pi_N = 1$. However, the simplest way to find $\boldsymbol{\pi}$ is by showing that this Markov chain is a martingale, i.e. that $E(X_t \mid X_{t-1}) = X_{t-1}$ (the proof is simple: it is the mean value of a binomial distribution). This means that $E(X_\infty) = j$. But this mean value can also be obtained as $E(X_\infty) = N\pi_j + 0(1-\pi_j)$, whereby $\pi_j = j/N$. An interesting by-product of this result is the probability that a single mutant invades the population: $\pi_1 = 1/N$. This probability is small in large populations but non-negligible during evolutionary bottlenecks, so the fixation of a new allele is something that surely has occurred more than once in the past.

Moran model

The Moran model is more interesting from a theoretical point of view because it is more amenable to analytic treatment than the Fisher-Wright model. It describes the same situation: a population of N individuals, k of which are A-alleles and $(N-k)$ a-alleles. The difference is that now individuals reproduce at a constant rate in time τ. The offspring will be a mutant with the same probabilities μ and ν as the Fisher-Wright model. After the reproduction event the newborn will replace a random individual of the population (even its parent!) chosen with uniform probability. The Markov process is specified by the conditional probabilities

$$\Pr(X(t+dt) = j \mid X(t) = k) = \tau T_{kj}\,dt, \qquad \forall j \neq k, \qquad (16)$$

where $X(t)$ represents the population of A individuals at time t. The magnitude of interest in this stationary process is $P_{ij}(t) = \Pr(X(t+s) = j \mid X(s) = i)$, the solution to either the equation resulting from multiplying (16) by $P_{ik}(t)$ and summing for $0 \le k \le N$

$$\dot{P}_{ij}(t) = \tau\sum_{k=0}^{N}[P_{ik}(t)T_{kj} - P_{ij}(t)T_{jk}], \qquad (17)$$

or the equation resulting from multiplying (16) by $P_{jl}(t)$ and summing for $0 \le j \le N$ (with an appropriate change of indices),

$$\dot{P}_{ij}(t) = \tau\sum_{k=0}^{N}[T_{ik}P_{kj}(t) - T_{ik}P_{ij}(t)]. \qquad (18)$$

In both cases the initial condition is, of course, $P_{ij}(0) = \delta_{ij}$. Equations (17) and (18) are, respectively, the forward and backward forms of the *master equation* of the process.

In any infinitesimal time interval $(t, t+dt)$ there can be at most one reproduction event in a Moran process, so if the state at time t is k, at time $t+dt$ it can be k, $k+1$ or $k-1$. Denoting $\tau T_{k\,k+1} = \lambda_k$ and $\tau T_{k\,k-1} = \mu_k$, equation (17) becomes the *forward Kolmogorov equation*

$$\dot{P}_{ij}(t) = \mu_{j+1}P_{i\,j+1}(t) - (\mu_j + \lambda_j)P_{ij}(t) + \lambda_{j-1}P_{i\,j-1}(t),$$
$$0 \le j \le N, \qquad (19)$$

and equation (18) the *backward Kolmogorov equation*

$$\dot{P}_{ij}(t) = \lambda_i P_{i+1\,j}(t) - (\mu_i + \lambda_i)P_{ij}(t) + \mu_i P_{i-1\,j}(t),$$
$$0 \le j \le N, \qquad (20)$$

where $P_{i\,-1}(t) = P_{-1i}(t) = P_{i\,N+1}(t) = P_{N+1\,i}(t) = \mu_0 = \lambda_N = 0$. This kind of Markov process is referred to as a *birth-death process* [10]. For the Moran model $\lambda_k = \tau\psi_k(1-k/N)$ (the probability that an A-allele is created and replaces an a-allele) and $\mu_k = \tau(1-\psi_k)k/N$ (the probability that an a-allele is created and replaces an A-allele), with ψ_k as in the Fisher-Wright model.

In the steady state $w_j = \lim_{t\to\infty} P_{ij}(t)$ can be obtained by setting $\dot{P}_{ij}(t) = 0$ in (19), which yields $\mu_{j+1}w_{j+1} - \lambda_j w_j = \mu_j w_j - \lambda_{j-1}w_{j-1}$ for all $1 < j < N$. For $j = 0$ we have $\mu_1 w_1 - \lambda_0 w_0 = 0$, thus $\mu_j w_j - \lambda_{j-1}w_{j-1} = 0$, a simple difference equation whose solution is

$$w_j = w_0 \frac{\lambda_0\lambda_1\cdots\lambda_{j-1}}{\mu_1\mu_2\cdots\mu_j}, \qquad 0 < j \le N, \tag{21}$$

or alternatively

$$w_j = w_N \frac{\mu_{j+1}\mu_{j+2}\cdots\mu_N}{\lambda_j\lambda_{j+1}\cdots\lambda_{N-1}}, \qquad 0 \le j < N. \tag{22}$$

The values w_0 or w_N are determined through the normalization $\sum_{j=0}^N w_j = 1$.

For the Moran model with mutations a closed expression for $\mathbf{w}$ can be obtained in the limit $N\to\infty$, $k\to\infty$ with $k/N = x \in [0,1]$, $N\mu \to \gamma$ and $N\nu \to \kappa$ [10]. For that we write $\lambda_j = A(N-j)j[1+a/j]$ and $\mu_j = A(N-j)j[1+b/(N-j)]$, where $A = \tau(1-\mu-\nu)/N^2$, $a = N\nu/(1-\mu-\nu)$ and $b = N\mu/(1-\mu-\nu)$. Then,

$$w_k = \frac{w_0 N a}{k(N-k)[1+b/(N-k)]}\prod_{j=1}^{k-1}\frac{1+a/j}{1+b/(N-j)}. \tag{23}$$

Now, using the Taylor expansion for $\log(1+x)$, the asymptotic behaviour $\sum_{j=1}^k j^{-1} \sim \log k$ when $k\to\infty$ and the fact that $\sum_{j=1}^\infty j^{-p} < \infty$ for all integers $p > 1$,

$$\begin{aligned}\sum_{j=1}^{k-1}\log\left(1+\frac{a}{j}\right) &\sim \log(k^a) + c,\\ \sum_{j=1}^{k-1}\log\left(1+\frac{b}{N-j}\right) &\sim \log\left(\frac{N^b}{(N-k)^b}\right) + d\end{aligned} \tag{24}$$

when $k\to\infty$, for certain constants c and d. Therefore (considering that in this limit $b/(N-k)\to 0$),

$$w_k \sim C k^{\kappa-1}\left(1-\frac{k}{N}\right)^{\gamma-1}, \tag{25}$$

C being a normalization constant. In this limit

$$\begin{aligned}\sum_{k=0}^N w_k = CN^\kappa \sum_{k=0}^N \frac{1}{N}\left(\frac{k}{N}\right)^{\kappa-1}\left(1-\frac{k}{N}\right)^{\gamma-1}\\ \sim CN^\kappa\int_0^1 x^{\kappa-1}(1-x)^{\gamma-1}\,dx,\end{aligned} \tag{26}$$

therefore the stationary probability distribution $w_k \sim x^{\kappa-1}(1-x)^{\gamma-1}\,dx/B(\kappa,\gamma)$ has the shape of a beta distribution with parameters κ and γ.

When there are no mutations ($\mu = \nu = 0$ and $\psi_k = k/N$), $\lambda_0 = \mu_N = 0$. Then (21) and (22) lead, respectively, to the two vectors $\mathbf{w} = (1,0,\ldots,0)$ and $\mathbf{w} = (0,\ldots,0,1)$, thus expressing the fact that, with probability 1, the system ends up absorbed either in state $k = 0$ or in state $k = N$. The magnitude that characterizes this absorption is $\pi_k = \lim_{t\to\infty} P_{kN}(t)$, i.e. the probability that if the process starts with k A-alleles it ends up with N A-alleles. The equation for $\boldsymbol{\pi}$ can be obtained by setting $\dot{P}_{ij}(t) = 0$ in (20). The resulting equation can be written as $\lambda_k(\pi_{k+1}-\pi_k) = \mu_k(\pi_k - \pi_{k-1})$, where $0 < k < N$. Taking into account that $\pi_0 = 0$, this equation implies that $\pi_k - \pi_{k-1} = q_{k-1}\pi_1$, for all $0 < k \le N$, where

$$q_0 = 1, \qquad q_k = \frac{\mu_1\mu_2\cdots\mu_k}{\lambda_1\lambda_2\cdots\lambda_k}. \tag{27}$$

Thus $\pi_k = \pi_1\sum_{j=0}^{k-1} q_j$ for all $0 < k \le N$ and π_1 follows from the condition $\pi_N = 1$. The shape of π_k for the Moran process is very simple because $\mu_j = \lambda_j$ for all $0 \le j \le N$. Therefore $\pi_k = k/N$, just as in the Fisher-Wright process.

Karlin and McGregor proved [9] that the solution to (20) can be expressed in the form

$$P_{ij}(t) = \frac{w_j}{w_0}\int_0^\infty e^{-xt}R_i(x)R_j(x)\,d\varphi(x), \tag{28}$$

where w_j is given by (21), $R_j(x)$ is the (finite, because $\lambda_N = 0$) system of polynomials defined by the three-term recurrence

$$-xR_j(x) = \lambda_j R_{j+1}(x) - (\lambda_j+\mu_j)R_j(x) + \mu_j R_{j-1}(x), \qquad 0 \le j < N, \tag{29}$$

with $R_{-1}(x) = 0$ and $R_0(x) = 1$, and $\varphi(x)$ is a unique measure with unit mass and with increments in $N+1$ points, with respect to which the family of polynomials is orthogonal.

And a bonus of the Moran model is that it permits the inclusion of selection. If f_{A} and f_{a} denote the fitness of the two alleles, the probability that, given a reproduction event, the individual that reproduces is of type x will be proportional to f_{x}. This yields a new expression for ψ_k, namely

$$\psi_k = \frac{kf_{\mathsf{A}}(1-\mu) + (N-k)f_{\mathsf{a}}\nu}{kf_{\mathsf{A}} + (N-k)f_{\mathsf{a}}}. \tag{30}$$

Similar arguments to those employed, in the limit of large populations and small mutations, to find the stationary distribution in the absence of selection now lead to

$$w_k \sim (N\log r^{-1})^{\kappa/r}x^{\kappa/r-1}r^{Nx}\,dx/\Gamma(\kappa/r) \text{ if } r < 1$$

and

$$w_k \sim (N\log r)^{\gamma r}(1-x)^{\gamma r-1}r^{N(x-1)}\,dx/\Gamma(\gamma r) \text{ if } r > 1,$$

where $r = f_{\mathsf{A}}/f_{\mathsf{a}}$ is the fitness of the A-alleles relative to that of the a-alleles.

In the absence of mutations $\mu_j/\lambda_j = r^{-1}$, thus $\pi_k = (1-r^{-k})/(1-r^{-N})$. Therefore, the fixation probabilities of a mutant allele ($\rho_{\mathsf{A}} = \pi_1$ or $\rho_{\mathsf{a}} = 1-\pi_{N-1}$) become

$$\rho_{\mathsf{A}} = \frac{1-r^{-1}}{1-r^{-N}}, \qquad \rho_{\mathsf{a}} = \frac{1-r}{1-r^N}. \tag{31}$$

As expected, if $r > 1$ then the probability to fix a mutant A-allele increases and the probability to fix a mutant a-allele decreases, and vice versa if $r < 1$.

Diffusion approximation

In the limit $N\to\infty$, $i,j\to\infty$ with $i/N\to y$, $j/N\to x$, if $NP_{ij}(t)\to f(x,t\mid y,0)$, $\lambda_j-\mu_j\to m(x)$ and $\lambda_j+\mu_j\to s(x)$ then equations (19) and (20) become, respectively,

$$\frac{\partial}{\partial t}f(x,t\mid y,0) = -\frac{\partial}{\partial x}[m(x)f(x,t\mid y,0)] + \frac{1}{2N}\frac{\partial^2}{\partial x^2}[s(x)f(x,t\mid y,0)], \tag{32}$$

$$\frac{\partial}{\partial t}f(x,t\mid y,0) = -m(y)\frac{\partial}{\partial y}f(x,t\mid y,0) + \frac{s(y)}{2N}\frac{\partial^2}{\partial y^2}f(x,t\mid y,0). \tag{33}$$

In both cases the initial condition is $f(x, 0 \mid y, 0) = \delta(x - y)$. These are two diffusion equations that describe the Moran process in this special regime. Notice that the diffusion term is proportional to N^{-1}, which means that the deterministic approximation is valid when populations are large. Thus, the finiteness of real populations adds a "noise" to the deterministic behaviour.

The diffusion approximation can also be obtained starting from the Fisher-Wright process. The result is similar but the time-scale is N times faster than in the Moran process. This approximation allows calculations that cannot be achieved for the discrete processes, and it can be easily generalized to situations in which there are multiple alleles or individuals have more than just one gene. Exploring all its possibilities would lead us too far from the scope of this article, so the interested reader is urged to consult the extensive literature on this topic (see, e.g. Refs. [1, 5, 11]).

5 The role of genetic drift in evolution

Genetic drift is present in the first studies on population genetics as an additional mechanism that accounts for the "noise" that small populations introduce in evolutionary dynamics. This noise acquires a special relevance in those times in which species go through evolutionary bottlenecks. However, the breakthroughs achieved in the field of molecular biology have put forward the very relevant role that this mechanism plays even in ordinary situations, so much so that we are at the onset of a change of paradigm: neutral evolution by pure genetic drift seems to be not only the most common way in which life forms evolve but also the mechanism that *de facto* allows them to adapt and speciate. Therefore genetic drift is one of the keys to the origin of species.

But in order to investigate the true implications of neutral evolution we first need to take a small walk through biology.

Biology is built on sequences and networks

At its most fundamental level, life is written in a DNA molecule consisting of a long sequence of four kinds of bases: adenine (A), thymine (T), guanine (G) and cytosine (C). We could say that this sequence of bases is a coding of all information on the building of a cell, and eventually of a life being.[4]

DNA is made of introns (pieces of the chain that do not code for proteins) and exons (the coding elements). In the first stages of transcription DNA transfers its information into RNA molecules (replacing thymine with a new base: uracil, U) in which exons get isolated through a splicing mechanism of the chain and are combined to form genes. These genes are the true pieces of code that translate into proteins in the cell rhybosomes (kind of reading-translating "machines"). At this level the DNA chains that build up chromosomes can be regarded as sequences of genes.

RNA gets transcribed into proteins by translating *codons* (sequences of three consecutive base pairs) into amino acids. This translation forms a universal code[5] known as *genetic code*. Transcription produces a new kind of sequence – proteins, this time made of 20 different types of amino acid. As a consequence of the interaction between amino acids, proteins get folded into three-dimensional structures, sometimes rigid and sometimes including mobile elements, just as if they were a sort of small machines. This three-dimensional structure determines its function insofar as a change of its conformation can make the protein lose its biological function or acquire a new one. The set of proteins of a cell (its proteome) forms a complex network; proteins interact with each other and with genes in very varied ways, activating or inhibiting the production of other proteins, catalyzing reactions, etc. The result forms a metabolic and regulatory network of interactions, a kind of protein ecosystem, whose result is cell activity.

We can still go up the scale and consider multicellular organisms as a new complex network of different types of cells that interact with each other. And in their turn, these organisms (animals, plants, etc.) entangle their life activity, competing for resources, eating each other, cooperating, etc., to give rise to the top biological scale: ecosystems.

All these biological organizations – sequences or networks – share a common property: they are made of a well-defined set of elements whose modification in any way (by changes, eliminations or additions) can induce drastic changes in the whole organization. At the most basic level – DNA chains – these modifications are commonly known as mutations. Mutations can be just replacements of one base by another, or addition or removal of some bases, or more drastic changes like inversions of whole pieces of the DNA sequence, duplications, etc. Drastic mutations are normally lethal: the resulting organism is not viable anymore (imagine, for instance, the effect of removing a single base if we take into account that protein transcription occurs through codon reading). Nonetheless many other mutations can be innocuous, and some of them can even give rise to viable modifications; the genetic code contains 64 different codons that only codify for 20 amino acids – plus a stop sequence; the redundancy is such that there are some amino acids that are coded by up to six different codons. This implies that there will be many base substitutions that have no effect whatsoever on the transcription into proteins. They are therefore innocuous.

Regarding proteins, we have already mentioned that they can fold into three-dimensional (tertiary) structures because of amino acid interactions, and these structures determine their functions. It turns out that most amino acids of the chain have little or no influence in the tertiary structure because the folding is determined by a small set of them, placed at strategic positions. Substituting one of these key amino acids will modify the tertiary structure, hence the protein function. However, substituting any of the other amino acids will either not change the tertiary structure or change it only slightly, so little that the protein maintains its function. This means that even mutations that lead to amino acid substitutions may have no biological effect, enormously enhancing the redundancy already existing in the genetic code.

Also, it often happens that adding, eliminating or replacing a protein in the metabolic and regulatory network of a cell has little influence on the global dynamics of the system. Thus, even at this level some changes are innocuous, hence not subject to selection. And not just at this level – a similar thing happens at the ecological level with the species forming an ecosystem. In summary, many changes can occur at all scales of a biosystem producing little or no effect at all. Selection is thus blind to these changes. But, which fraction of the set of possible changes do they represent?

Most evolution is neutral

In 1968, Kimura surprised the scientific community with the idea that most genome mutations are neutral [12]. His argument goes as follows. Comparative studies of some proteins lead to the conclusion that chains 100 amino acids long undergo one substitution every 28 million years. The length of DNA chains in the two chromosome sets of mammals is about 4×10^9 base pairs. Every three base pairs code for one amino acid and, due to the existing redundancy, only 80 % of base pair substitutions lead to an amino acid substitution. Therefore there are 16 million substitutions in the whole genome every 28 million years, i.e. almost one substitution every 2 years! Kimura's conclusion is that organisms can only afford such a mutational load provided the great majority of mutations are neutral.

Recent studies on RNA molecules reach similar conclusions [6]. RNA molecules fold as a result of the interaction between the bases forming their sequences. This folding can be regarded as the molecule phenotype because it determines the function of these chains. Thus natural selection acts directly on it being blind to the actual sequences. The number of different sequences folding into the same structure is huge. This implies, once more, that a large number of mutations in the chain leave the phenotype intact, thus avoiding selection.

Neutrality seems the rule rather than the exception, at least at the molecular level. The consequences of this fact are far reaching but to see how much, we need to introduce a new concept: *adaptive landscapes,* and resort again to mathematics.

6 Adaptive landscapes

Perhaps Sewall Wright's most relevant contribution to evolutionary theory is his metaphor of *adaptive landscape* [19]. From a formal viewpoint, an adaptive landscape is a mapping $f : \mathcal{X} \to \mathbb{R}$, where $\mathcal{X}$ denotes a configuration space equipped with some notion of adjacency, proximity, distance or accessibility, and whose image is the fitness associated to a particular configuration in $\mathcal{X}$. This structure, together with the fact that the set of all adaptive landscapes forms the vector space $\mathbb{R}^{|\mathcal{X}|}$, allows the development of a rich theory of combinatorial landscapes that covers not only the adaptive landscapes of biology but also the energy landscapes of physics or the combinatorial optimization problems arising in computer science [17].

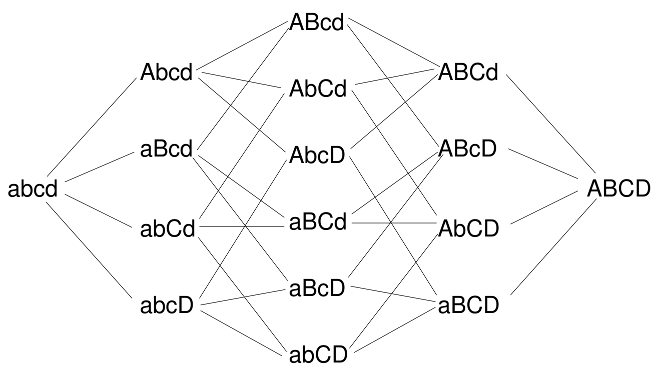

Figure 2. Graph for a sequence of length 4 and 2 alleles per locus

In our particular case, $\mathcal{X}$ is made of the set of underlying sequences or networks. When talking about sequences we will generically use loci to refer to particular genetic traits and alleles to describe the different variants of that trait. Depending on the context, by allele we may refer to bases (DNA), amino acids (proteins) or genes (chromosomes). Therefore, for a DNA chain of length L, $\mathcal{X} = \{\text{A, T, C, G}\}^L$, for a protein of the same length, $\mathcal{X} = \{\text{Phe, Leu, Ile,}\dots, \text{Gly}\}^L$ and for a chromosome $\mathcal{X} = \{A_1, \dots, A_{n_a}\} \times \{B_1, \dots, B_{n_b}\} \times \cdots$, where letters represent the different alleles of a given locus. On $\mathcal{X}$ we can introduce Hamming distance, $d_{\mathrm{H}}(x, y)$, which represents the number of different loci of sequences $x, y \in \mathcal{X}$.

The structure of $\mathcal{X}$ is determined by the allowed transitions between its sequences, which we will refer to generically as mutations. We will speak about point mutations to refer to changes at a given locus of the chain, i.e. substitutions of a base pair in a DNA chain, of an amino acid in a protein or of a different allele in a chromosome. If all mutations are of this type we can build the graph $\mathcal{G} = \{\mathcal{X}, \mathcal{L}\}$, where the set of links $\mathcal{L}$ is made of all pairs of sequences $x, y \in \mathcal{X}$ with $d_{\mathrm{H}}(x, y) = 1$ (Figure 2 illustrates the case for $L = 4$ and two alleles per locus). We can also consider mutations that amount to adding or eliminating a given locus of the sequence. If $\mathcal{X}_L$ denotes the set of sequences of length L then $\mathcal{X} = \bigcup_L \mathcal{X}_L$, and the graph will contain links between sequences of different length.

The probability that a mutation of those defining $\mathcal{G}$ occurs need not be uniform. The general way of describing the evolution of a given sequence is by introducing a transition probability matrix T whose element T_{xy} is the probability to mutate from one sequence $x \in \mathcal{X}$ to another $y \in \mathcal{X}$. The zeros of the adjacency matrix of the graph $\mathcal{G}$ are also the zeros of matrix T. The evolution of the sequence is therefore the random walk across $\mathcal{X}$ described by the Markov process associated to T.

But we began this section talking about a metaphor. And indeed, beyond the formal description and mathematical treatment of adaptive landscapes, it is the mental picture they provide that leads our intuitions. As a matter of fact, in the development of population genetics there are three metaphors that have been widely used: Fisher's Fujiyama landscape, Wright's rugged landscape and Kimura's flat landscape. Let us examine these three models in more detail.

Fujiyama landscape: quasispecies and the error catastrophe

Fisher imagined that species were in a situation of optimal adaptation to their environment. Hence each species should sit at one of the many tops of the adaptive landscape. According to this picture, a sequence would be maximally adapted and as sequences get away from it in Hamming distance, their fitness should decrease. Fisher did not have in mind sequences when he elaborated this metaphor because molecular biology was in its early stages. It was Eigen [4] who used it to elaborate his theory of *quasispecies* and discover the *error catastrophe.* If the Markov process defined on the set of sequences $\mathcal{X}$ is ergodic, there will be a stationary probability distribution. When the landscape is of Fujiyama type, this distribution will be localized around the optimally adapted (or master) sequence. In spite of this being the most probable sequence, close to it (i.e. a few mutations away) there will be a "cloud" of less adapted sequences coexisting with the master sequence. Quasispecies is the term that Eigen introduced to describe this ensemble of sequences.

In order to study the behaviour of a quasispecies more closely we will resort to equation (9). For simplicity we shall assume that sequences have finite length $L \gg 1$, that there are just two alleles per locus, that the master sequence has fitness $f_1 = f > 1$ and that all other sequences have fitness $f_2 = \cdots = f_{2^L} = 1$. We shall also assume that the probability that a point mutation occurs is $\mu \ll 1$, independent of the sequence. Let $x_1 = x$ denote the population fraction of the master sequence; thus $x_2 + \cdots + x_{2^L} = 1 - x$ and $\phi = fx + 1 - x$. Equation (9) then becomes

$$\dot{x} = x[f(1-\mu)^L - 1 - (f-1)x] + O(\mu). \qquad (34)$$

The term $O(\mu)$ accounts for the transitions from the L nearest neighbour sequences of the master sequence that revert to the master sequence. Neglecting these terms and approximating $(1-\mu)^L \approx e^{-L\mu}$ we can see that if $fe^{-L\mu} > 1$ then x asymptotically approaches $x^* = (e^{-L\mu} f - 1)/(f-1)$, whereas if $fe^{-L\mu} < 1$ the bracket in equation (34) becomes negative and therefore $x = O(\mu)$. The threshold $\mu_{\text{err}} = \log f/L$ defines the error catastrophe. When $\mu < \mu_{\text{err}}$ the quasispecies is well-defined because the master sequence is the most probable one. However, when $\mu > \mu_{\text{err}}$ the identity of this master sequence gets lost in the cloud of mutants and the quasispecies disappears as such.

Experimental studies performed in the '90s seem to confirm [16] that indeed the length of the genome of different species – ranging from virus to Homo sapiens – and the mutation rate per base are related as $\mu L \leq O(1)$. Hence an increase in the mutation rate is a mechanism that this theory puts forward to fight viral infections. We will come back to this point later.

Rugged landscape

Although locally the adaptive landscape can be well described by the Fujiyama model, Wright visualized it as a rugged landscape, full of high peaks separated by deep valleys. The reason is that mutations that change the sequence minimally may induce large variations in the fitness of individuals. In addition, there exists the well known phenomenon of *epistasis,* according to which some genes interact, either constructively or destructively, amplifying these large variations in response to small changes in the sequences.

According to the rugged landscape metaphor, species evolve by climbing peaks and sitting on the summits. Different peaks correspond to different species with different fitnesses. This picture seems to fit well with our idea of evolution by natural selection. However, it has a serious drawback: species that are at a summit can only move to a higher one by going through an unfit valley. In the most favourable case this valley will consist of a single intermediate state. Formula (31) tells us that if a population is small, it is not impossible that an unfit allele replaces a fitter one. Nevertheless, the probability that this happens is very small, i.e. adaptation times should be very large. And this is only the most favourable case. The high speed of adaptation to rapidly changing environments that viruses exhibit seriously challenges this model. What is then wrong in our picture of adaptive landscapes?

Holey landscape: neutral networks

Let us review the most extreme case of a rugged landscape: the random landscape. In this case every sequence of $\mathcal{X}$ has a random fitness, independent of the other sequences. In general, rugged landscapes are not that extreme because there is some degree of correlation between the fitness of neighbouring sequences. However, beyond the correlation length, fitness values become uncorrelated. The random landscape is the extreme case in which the correlation length is smaller than 1. Suppose now that the length of the sequences is large, and that every locus can host A independent alleles. The degree of graph $\mathcal{G}$ will thus be $g = (A-1)L$ and its size $|\mathcal{X}| = A^L$. With $L = 100$ and $A = 2$ (a rather modest choice), $g = 100$ and $|\mathcal{X}| = 2^{100} \approx 10^{30}$. To all purposes such a graph can be locally approximated by a tree, the more so the larger the degree (see Figure 3). Imagine an extreme assignment of fitness: 1 if the sequence is viable and 0 if it is not. Let p be the fraction of viable sequences. Evolution can only proceed by jumping between consecutive viable nodes. According to Figure 3, which illustrates what this landscape looks like locally in a particular graph, it becomes clear that if p is small, the number of viable nodes a distance d apart from the initial node is well approximated by a branching process where, except for the first generation, the number of offspring (viable nodes) is given by $p_k = \binom{g-1}{k} p^k (1-p)^{g-1-k}$, with an expected value of $(g-1)p$. The theory of branching processes [10] tells us that, with a finite probability, the process never ends provided $(g-1)p > 1$. Translated to our graphs this implies that whenever $p \gtrsim 1/g$ (with $g \gg 1$) there is a connected subgraph of viable nodes containing a finite fraction of all nodes of $\mathcal{G}$. This kind of subgraph is called a *neutral network* [7].

If we consider a more general model in which $\mathcal{P}(f)$ describes the probability density that a node fitness is between f and $f + df$, if $\int_{f_1}^{f_2} \mathcal{P}(f)\,df \gtrsim 1/g$ then there will be a quasineutral network whose node fitnesses will all lie in the interval (f_1, f_2). As g is usually very large (proportional to the sequence length), the existence of neutral networks becomes

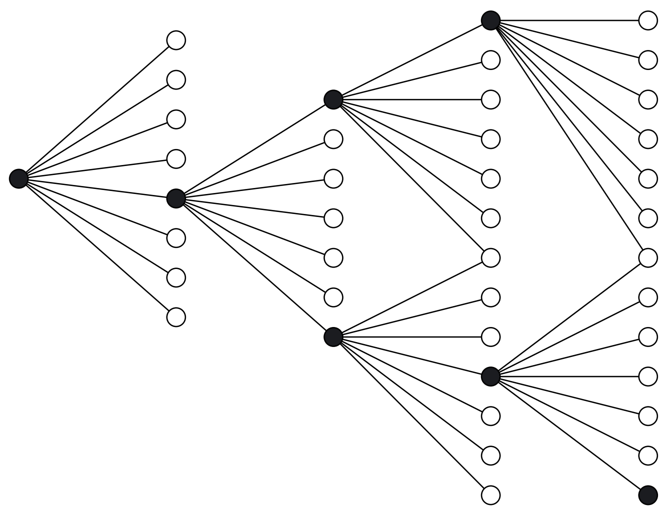

Figure 3. Local section of a configuration graph with $A = 2$ and $L = 8$. Black nodes are viable, whereas white nodes are not viable.

the rule rather than the exception. Wright's metaphor of a landscape full of mountains and valleys becomes utterly inappropriate to describe the existence of these neutral networks in rugged adaptive landscapes. A more appropriate metaphor would be that of a flat landscape (*á la* Kimura) with holes. Evolution moves sequences across this neutral network, transforming them into completely different sequences without ever decreasing their fitness. Undoubtedly, this mechanism dramatically speeds up not only adaptation of species to the environment but even speciation.

The fact that the fitness depends on phenotype and not on genotype favours the appearance of neutral networks. This is what is observed in RNA [6]. The properties of these networks have an enormous influence in evolutionary dynamics, an influence that we are now only beginning to understand. Just as an example, if we reconsider Eigen's model under the viewpoint of this new metaphor, we will realize that its main hypothesis, namely that locally the landscape is Fujiyama, is completely wrong. There is no such a thing as a master sequence. Instead there is a master network (or phenotype) that contains a huge number of sequences. Accordingly, the probability that a mutation recovers the optimal fitness is much larger than what Eigen's theory assumes because it can be recovered by hitting any of the sequences of the network, not necessarily the initial one. When this probability is not negligible the error catastrophe goes away [14].

7 Conclusion

This article tries to provide an overview of the contributions of mathematics to evolutionary theory. From population genetics to the theory of complex networks, going through the theory of stochastic processes, many relevant results about the evolutionary mechanisms driving life have been obtained thanks to their mathematical descriptions. We still cannot say that the theory of evolution is a fully mathematically formulated scientific theory, like Darwin would have liked it to be, but it is unquestionable that we are getting closer and closer to such an achievement. Nowadays we could say that the theoretical studies of evolutionary processes are at least as important as the experimental ones and that, as the opening sentence by Darwin states, it is those that shed light in the darkness.

Given the divulgatory nature of this article many interesting topics have been left out. Some of them provide new insights into evolutionary mechanisms and some illustrate further contributions of mathematics to evolutionary theory. Among them we can mention the infinite allele model [5], which is currently employed in analysing evolutionary divergence of DNA or protein sequences, or the coalescent process [5], which is an interesting and practical backward formulation of genetic drift. We have not mentioned the important contributions of game theory to evolution either. This theory is currently being used to deal with the problem of the evolutionary emergence of cooperation [16]. Instead, the focus has been on the subject of adaptive landscapes and neutral networks because, in the author's opinion, this is the area where a new reformulation of the evolutionary paradigm can emerge in the following years. Understanding evolution requires, against all expectations, an understanding of the role of genetic drift on neutral networks. And due to the complexity of this problem, it is one of the topics in which mathematics can be our eyes in the dark room that help us find the black cat that is not there.

I want to acknowledge Susanna Manrubia for her critical reading and her valuable remarks. This work is supported by projects MOSAICO (Ministerio de Educación y Ciencia, Spain) and MODELICO-CM (Comunidad de Madrid, Spain).

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Notes

1. Admiration not unrelated to Darwin's care for his progeny: Sir George Howard Darwin, the fifth of Darwin's children, became an astronomer and mathematician.
2. Human beings have so far not reached this equilibrium with the environment because this environment is the whole planet. In spite of that, the law provides a reasonable description of isolated populations in low resource environments. The permanent famine suffered by many African countries provides an illustration of what our situation will be when we reach that equilibrium with the resources of the planet.
3. It is most remarkable that de Vries rediscovered Mendel's laws for the scientific world in 1900, the same year that Planck proposed the quantum hypothesis for the first time.
4. This classic dogma is not quite true because in cell division every daughter cell inherits not only an exact copy of the parent cell's DNA but also half of its cytoplasm. This makes the two daughter cells slightly different in composition and this difference induces different gene expressions. This lies at the heart of cell differentiation and becomes the core of what is currently known as *epigenetics*.
5. In 1979 it was discovered that this code is not quite universal: mitochondria, as well as some bacteria and yeasts, use codes that differ slightly from the "universal" one.

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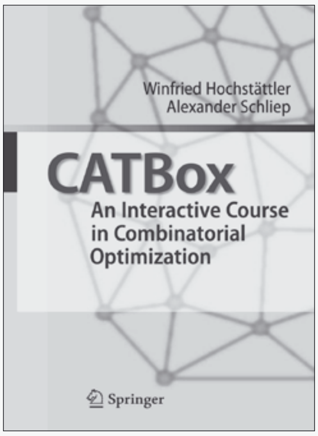

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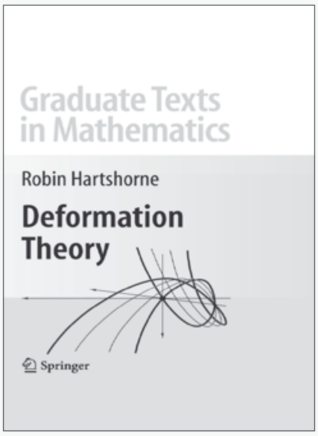

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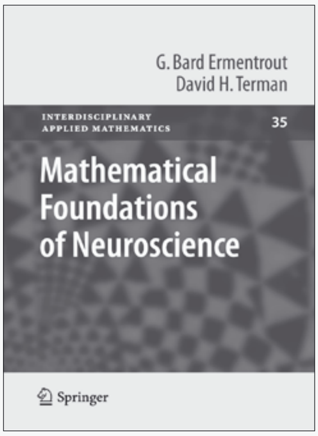

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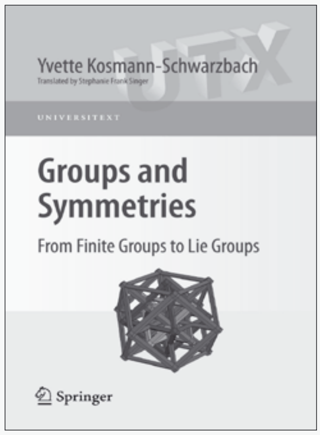

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It began with Pestalozzi and Schleiermacher Reflections on the polymath Hermann Graßmann (1809–1877)

Hans-Joachim Petsche

"I therefore decided to make the presentation, extension, and application of this analysis my life task."

Graßmann, in the preface to his *Extension Theory* of 1844

His life's destiny: Graßmann's Ausdehnungslehre, a story of disappointment and success

In 1853, the brilliant Irish mathematician and physicist W. R. Hamilton (1805–1865) stumbled upon Graßmann's *Extension Theory* of 1844 (hereafter, A1) while doing historical research for the introduction to his *Lectures on Quaternions* [19]. Graßmann's book impressed Hamilton greatly. In a letter to his friend De Morgen, dated 31 January 1853, he wrote:

William Rowan Hamilton (1805–1865).

> *"I have recently been reading […] more than a hundred pages of* Graßmann's Ausdehnungslehre*, with great admiration and interest. […] If I could hope to be put in rivalship with Des Cartes on the one hand, and with Graßmann on the other, my scientific ambition would be fulfilled!" [18, 441]*

Hamilton's enthusiasm contrasted greatly with the reactions that Graßmann's main mathematical work had provoked up to that point. Later, Moritz Cantor (1829–1920) would dryly remark:

> *"In 1844, O. Wigand published the book in Leipzig. Nobody reviewed it, nobody bought it, and almost all of the copies of that edition were destroyed." [3,569]*

Die

lineale Ausdehnungslehre

ein

neuer Zweig der Mathematik

dargestellt

und

durch Anwendungen auf die übrigen Zweige der Mathematik,

wie auch

auf die Statik, Mechanik, die Lehre vom Magnetismus und die Krystallonomie erläutert

von

Hermann Grassmann

Lehrer an der Friedrich-Wilhelms-Schule zu Stettin.

Mit 1 Tafel.

Leipzig, 1844.

Verlag von Otto Wigand.

Conscious of the fact that he was providing the basis for a completely new approach, Graßmann had relied on *conceptual construction* in order to create an algebraic theory of *n-dimensional* manifolds in his *Extension Theory*. These manifolds were completely disconnected from geometrical *sense-perception.* By doing so, his theory provided its own *philosophical* foundation; it began by presenting an abstract *general theory of forms* of conjunctions, underlying the entire structure of mathematics, and continued to develop its theoretical object – linear and multilinear algebra of n-dimensional space – by closely connecting mathematical and conceptual constructions to *philosophical and heuristic* reflections.

By taking a look at Wilhelm Traugott Krug's *General Handbook of the Philosophical Sciences* of 1827, we can fathom to what point Graßmann's approach contradicted scientific zeitgeist at the time. For the term "mathematics", we find the following entry:

> *"Mathematics […] only [deals with] magnitudes which appear in time and space […] A philosopher should familiarize himself with mathematics and a mathematician with philosophy, as far as their talent, interests, time and surroundings will permit. But one should not confuse and throw into one pot what the progress of scientific knowledge has separated, and rightly so. […] mathematical philosophy and philosophical mathematics – in the commonly accepted sense of the terms, namely as a mixture of both – are scientific or, rather, unscientific monsters." [21]*

When Graßmann's contemporaries, the mathematicians Gauß (1777–1855), Apelt (1812–1859), Grunert (1797–1872) and Balzer (1818–1887), rejected A1, they were following this Kantian view [see 27, ix sqq.].

However, Graßmann's philosophical work had not compromised the autonomous integrity of the concept of mathematics. Rather, by disconnecting it from sense-perception, he had contributed to the establishment of *pure mathematics*.

In 1862, Graßmann presented a completely revised version of his *Extension Theory* [9] (hereafter, A2). Reluctantly, he had removed all philosophical considerations from the text. Now the book approached its object by developing a string of formulas, making it just as readable to us as any contemporary mathematical treatise. Nevertheless, at the time, the majority of the mathematical readership did not accept it.

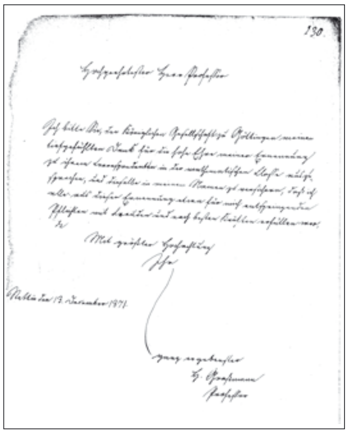
Ernst Abbe (1840–1905).

It took a new generation of mathematicians, Hermann Hankel (born 1839), Ernst Abbe (born 1840), Josiah Willard Gibbs (born 1839) and Felix Klein (born 1849), before Graßmann's ideas found approval in different contexts.

As a young man, Ernst Abbe (1840–1905) had attended Riemann's lectures. Hermann Hankel (1839–1873) had relied on Abbe's manuscripts from the lectures [see 1, xxiii]. Abbe, whose work in optics contributed to the enormous entrepreneurial success of the Carl Zeiss Company for optical instruments, had encountered A1 in 1861 and was enthusiastic about the book's "heuristic form" of presentation. In June 1862, informing his friend Harald Schütz (1840–1915) about his findings in Graßmann's texts, he wrote:[1]

> *"I must say that this is very enjoyable work. This man has certainly produced a brilliant piece of work, and I find it hard to believe that it has remained completely unknown. My satisfaction grows larger with every page I read. The more I manage to penetrate and understand this utterly unique and in fact quite un-mathematical way of looking at things, the more I realize how adequate and justified it really is. […] But this is not merely about shedding light on the philosophical background of mathematics. Rather, the author convincingly demonstrates that there really are things to be gained. He applies the principles of his 'new analysis' to a large number of actual problems – be they geometrical, mechanical or purely arithmetical – and thereby uncovers completely new methods everywhere, which produce new theorems in an incredibly efficient and transparent way. And even if we should already be familiar with these theorems, our procedures of generating them presuppose a long and complicated list of concepts and formulas." [1, 214–215]*

Unfortunately, Graßmann never learned of Abbe's positive reaction to his work. Despite the fact that a university professorship would have meant a certain loss of income, Graßmann applied more than once for such a position – in vain. Graßmann would always remain a teacher in Stettin. Disappointed by mathematics, he turned to studies of philology, which he had once considered mere recreation from his mathematical efforts. His works on the ancient Indian hymns of the *Rig-Veda* [10], [11] are important references even today, and Graßmann almost immediately received attention from philologers of Sanskrit.



Graßmann's letter of thanks upon becoming a member of the Göttingen Society for the Sciences.

It was late in his life when Graßmann finally received the acclaim he had longed for. On 2 December 1871, thanks to the initiative of Alfred Clebsch (1833–1872), he was made a member of the Göttingen Society for the Sciences.

A second edition of A1, which Graßmann was still able to prepare, was published in 1878, a year after his death. Beginning in 1876/77, Ernst Abbe started giving lectures on Graßmann's calculus [see 30, 200]. Rudolf Mehmke (1857–1944) lectured on *Extension Theory* in 1881/82 and the following years [see 29]. As a young man, in 1888, Alfred North Whitehead (1861–1947) also began his career as a university lecturer by addressing Graßmann's *Extension Theory*. In 1898, Whitehead published his *Universal Algebra*. In the introduction, he wrote:

Alfred North Whitehead (1861–1947).

> *"The greatness of my obligations in this volume to Grassmann will be understood by those who have mastered his two Ausdehnungslehres. The technical development of the subject is inspired chiefly by his work of 1862, but the underlying ideas follow the work of 1844." [39, x]*

Inspired by Josiah Willard Gibbs (1839–1903), Felix Klein (1849–1925) began to demand an edition of Graßmann's works in mathematics and the natural sciences. Edited by Friedrich Engel (1861–1941), the five volumes of Graßmann's collected works appeared between 1894 and 1911 (volume 6 was reserved for Friedrich Engel's biography of Graßmann [6]).

At this point, the reception of Graßmann's *Extension Theory* was at a peak. Graßmann's book inspired an incredibly large number of scientists. Those that come to mind are: in mathematics – Klein, Mehmke, Peano, Veronese, Gibbs, Heaviside and Whitehead; in philosophy – Natorp, Cassirer, Kuntze, Husserl, Carus and many others; and in psychology – most notably Preyer and Wundt.

The third International Congress of Philosophy took place in 1908. On the evening preceding the 100th anniversary of Graßmann's birth, Friedrich Kuntze (1881–1929) held a lecture on the "significance of Hermann Graßmann's *Extension Theory* for transcendental philosophy". He concluded his talk with the following words:

> *"While Aristotle was the first to isolate pure forms of thinking,* Graßmann was the first to isolate the pure forms of sense-perception. *[…] So if Graßmann's* achievements *are in the vicinity of Aristotle, why not also place his* name *next to Aristotle's? And herewith I respectfully and affectionately dedicate these words to the memory of a great man, who remained unrecognized during his days on earth and who has all the right to expect from history the kind of justice that life denied him." [22, 437]*

Roots of Graßmann's creativity

Whenever we encounter a brilliant scientist like Hermann Graßmann, we automatically pose questions that aim to understand the general aspects of these unique creative eruptions in the history of science. But, at first sight, Graßmann seems like a difficult case:

- Young Hamilton, whose theory of quaternions was closely linked to Graßmann's external algebra, was a complete prodigy. Hamilton could boast that at age 13 he had mastered 13 languages (among them Arabic

[1] Letter from Ernst Abbe to Harald Schütz, 21 June 1862.

and Sanskrit), while young Graßmann was a dreamer, easily distracted and quite sluggish. Apparently, Graßmann's father often assured his son that he would bear it stoically should the boy's career remain limited to garden work.
- Bernhard Riemann (1826–1866), who – like Graßmann – introduced the concept of n-dimensional manifolds to geometry, had teachers as prominent as Moritz Stern (1807–1894), Johann Benedict Listing (1808–1882), Carl Friedrich Gauß (1777–1855), Peter Gustav Dirichlet (1805–1859), Carl Gustav Jakob Jacobi (1804–1851) and Gotthold Eisenstein (1823–1852). Gauß "provoked" Riemann to choose a groundbreaking topic for the lecture he gave for his habilitation. Graßmann was a different case altogether. During his studies of theology in Berlin, he did not attend a single course in mathematics. His personal library contained only a small number of mathematical tracts. Graßmann mostly learned about mathematics from his father's works.
- Graßmann was never part of the university world. Apart from a one-year episode at the Berlin School of Commerce, Graßmann worked as a secondary school teacher in Stettin, separated and isolated from the centres of mathematical research. It was here that he wrote his masterpiece, the *Extension Theory* of 1844. Communicating sporadically with August Ferdinand Möbius (1790–1863) and collaborating intensely with his brother Robert, Graßmann continued to build his mathematical theories in the following years.

So, if at first glance Graßmann's achievements seem disconnected from the scientific ambitions of his time, a second glance will show us that this is not at all the case.

Graßmann's personality was the focal point (despite being on the sidelines of contemporary mathematical research) in which the most diverse influences and predispositions amalgamated, interacted in different ways and propelled him towards conclusions that were decades ahead of his time.

We can quite easily identify a set of conditions that must have transformed apparent disadvantages into mathematical creative energy. In an attempt to summarize these elusive factors in a few points, we might say:

1. In the first half of the 19th century, Stettin went through a phase of provincial and petit-bourgeois prosperity. This heightened the local academics' awareness of their personal creative powers, an insight that also drove Hermann Graßmann.
2. At the time, Prussia was reforming its systems of elementary schools and schools for the poor (Pestalozzi/Schleiermacher, see [24], [26]). This created pedagogical impulses aimed at revising the foundations of mathematics, which affected both Hermann Graßmann and his father.
3. Pestalozzi's collaborator Joseph Schmid (1785–1851) had created a theory of forms for elementary school purposes [33]. It was this project that provided the initial impulse for many of the ideas underlying and supporting *Extension Theory* (Graßmann algebra, the general theory of forms).
4. Schleiermacher's *Dialectic* [31] is largely responsible for the structure and mechanisms that turned a theoretical vision into a mathematical theory.

Graßmann reinvents himself

Young Graßmann was astonishingly open in the official curricula he wrote, describing his inner struggle against "phlegm", which he was aiming to "eradicate completely" in order to attain personal perfection. He attempted to change his way of life, overdid it and failed, finally developing a more moderate view of himself that turned weaknesses into strength: "Thus the Phlegmatic must not aspire to force the inefficiency of his thoughts into a bold flight; for the wing upon which he strives to soar to the sun is not proper to himself; and he will, like Icarus, soon plunge back to earth. Rather, he must seek to give his train of thought clarity, and in clarity depth."[2] [26, 146] For a long time, Graßmann was undecided about whether to become a theologian or a teacher. The ideas forming the point of departure of his *Extension Theory* only came to him in his *Theory of Tides*, which he wrote for his second round of exams on the way to becoming a teacher. He would dedicate the rest of his life to developing these ideas. When he described himself as a phlegmatic person in a letter to his fiancée in November 1848, Graßmann linked phlegm to persistency:

Hermann Graßmann (pencil drawing by a pupil).

> *"He finishes what he begins. If he doesn't succeed on the first try, he begins anew and perseveres until he has finished the task. He focuses on what is immediately before him. He subordinates all of his deeds and goals to the overall view of the whole, to the single idea which he has identified as his life's task and inner motivation. Therefore his entire life and work form a whole, and everything around him becomes a part of it. His will is strong, his endeavours well-planned. He pursues his goals patiently and remains absolutely true to his life's task." [quoted from 6, 150]*

Even after Graßmann had to face the failure of his dream that Extension Theory might immediately influence the further course of mathematics, at more than 50 years of age he nevertheless mustered the energy that had once turned an insecure young man into a scholar. For years, Graßmann sat down every day to do the painstakingly precise work of composing his monumental dictionary of the ancient Indian *Rig-Veda* [10] – apart from teaching in school for 18 to 20 hours a week.

Graßmann and Stettin

Following the War of Liberation of 1813/14, the town of Stettin experienced a phase of petit-bourgeois and

[2] Life history of Hermann Graßmann (23 March 1833) on the occasion of his theological examination. [26, 136–149]

The fortified town of Stettin around 1847.

provincial prosperity, which came to an end in the 1850s, when the railroad connection to Berlin was completed and the bourgeois revolution of 1848 had taken its toll.

During these four decades, romanticism, religiosity and German nationalism thrived within the city walls of Stettin. The petit-bourgeois quest for knowledge and education was underway and the Freemasons experienced an unprecedented upswing in membership. The Stettin secondary school, or "Gymnasium", represented the city's scientific and cultural centre [see 38]. It was home to a faculty of professors – some of them brilliant scientists, others narrow-minded locals – who did not accept any scientific authority as long as it did not live up to their own standards. The only thing that united this highly diverse group of academics was the mindset of romanticism. This was a micro-climate that relied on the creative powers of individuals and that provided the soil for Hermann Graßmann's gradually growing self-confidence. It was here that he started to believe that he could completely revise the structure of mathematics, a project that was to begin with a completely new branch of mathematics: extension theory. Shortly after Hermann's return to Stettin from his one-year stint at the Berlin School of Commerce, he told his brother Robert:

Lodge Garden in Stettin.

"I have now already been a month and a half in Stettin, and I cannot tell you how well it goes with me here, especially in comparison with Berlin. […] The stimuli here are not so stormy but all the more heartfelt; the circle of activity in the calling not so great, but for all that more beneficial for those to whom it is directed, as for me myself; and it is true that for myself

Hermann Graßmann's life

1809–15 April:	Graßmann is born in Stettin.
1827	Hermann Graßmann begins his studies of theology at the University of Berlin.
1834–36	Graßmann obtains a position at the Berlin School of Commerce. After 14 months he gives up and returns to Stettin. He becomes a teacher at the "Ottoschule". He will never leave Stettin again.
1838/39	Graßmann signs up for the second examination in theology and asks the examination commission in Berlin for a second round of examinations in physics and mathematics.
1840	Graßmann submits the examination thesis on low and high tides to the commission in Berlin. *Extension Theory* is born.
1840	Hermann and Robert Graßmann study Schleiermacher's *Dialectic*.
1843	In the autumn Graßmann finishes work on the first volume of *Extension Theory* (A1). Hamilton discovers quaternions.
1846	Graßmann is awarded a prize honouring Leibniz for his *Geometrical Analysis*. Ideas from Extension Theory receive public recognition for the first time.
1847	Graßmann attempts to get a professorship in mathematics at a university. Kummer's assessment destroys Graßmann's hopes.
1852	Graßmann's father dies. In July, Graßmann becomes his father's successor at the Stettin "Gymnasium".
1860	Graßmann publishes his *Teaching Manual of Arithmetic*. That same year, he publishes his first article on philology.
1861	The completely revised and restructured version of *Extension Theory* is published.
1862	For the second time, Hermann Graßmann officially applies for a professorship in mathematics. The project fails. He feels disappointed with mathematics and focuses exclusively on Sanskrit and *Rig-Veda*.
1866	Graßmann begins an exchange of letters with Hankel.
1869	Klein discovers Graßmann by reading Hankel.
1869	Graßmann's oldest son sets out to study mathematics in Göttingen. He brings Clebsch and Stern a copy of Extension Theory.
1871	On Clebsch's initiative Graßmann becomes a member of the Göttingen Science Society.
1876	Graßmann is made a member of the American Oriental Society. On Roth's initiative, the University of Tübingen accords Graßmann an honorary doctorate for his philological work.
1877	Hermann Günther Graßmann dies on 26 September.
1894–1911	Publication of Graßmann's collected works.

the resources here do not flow as extensively, but I am all the more in a position to use them." [26,161–163]

Graßmann's personality arose from a peculiar combination of local convictions and global zeitgeist, linked to provincial resistance to the rapid and self-centred development of modern science. Walking a thin line between provincialism and German nationalism, on the one hand, and scientific creativity and brilliance, on the other, Graßmann mirrored the academic milieu surrounding him (though his brother Robert Grassmann (1815–1901) was even more affected by his surroundings).

Felix Klein was already aware of the fact that a certain degree of distance from institutionalized scientific research could produce considerable potential for theoretical force and depth:

"We academics grow in strong competition with each other, like a tree in the middle of the forest which must stay slender and rise above the others simply to exist and conquer its portion of light and air. But he who stands alone, like Grassmann, can grow on all sides, can harmoniously develop and finish his nature and work. Of course such versatility as Grassmann embodied must inevitably be accompanied by a certain amount of dilettantism…" [20, 161]

Graßmann's father

The introduction to A1 is the only instance in which Hermann Graßmann pointed to the inspiration he had received from his father Justus Graßmann (1779–1852).

Justus Graßmann (1779–1852).

"While I was pursuing the concept of the product *in geometry as it had been established by my father, I concluded that not only rectangles but also parallelograms in general may be regarded as products of an adjacent pair of their sides […]" [7, 9; emphasis H.–J. P.]*

Hermann Graßmann was referring to ideas from textbooks for elementary schools and lower grades in secondary schools. Most notably, the *Geometry* ("Raumlehre") of 1817 tells us:

"The concept of the present book was born with a number of schools for the poor, which I helped establish […] with no pay whatsoever and for the sake of doing good deeds." [13, iii]

Following Friedrich Schleiermacher (1768–1834) and Wilhelm von Humboldt (1767–1835), who considered the rebirth of the school system a prerequisite for the national rebirth of Germany after the Napoleonic occupation, Justus Graßmann collaborated with his brother Friedrich Heinrich Gotthilf Grassmann (1784–1866) and schools councillor Georg Wilhelm Bartholdy (1765–1815) – a close friend of Schleiermacher – on textbooks that were inspired by the pedagogical methods of Johann Heinrich Pestalozzi (1746–1827). These textbooks were meant to serve as free material for teachers "lacking a scientific education in the strict sense of the word". As it turned out, the three collaborators were highly successful in their project.

This is how Wilhelm Dilthey (1833–1911) put it:

Detail of the illustration "Pestalozzi in Stanz" (1845). The publisher and critic Diesterweg took note of Pestalozzi's followers in the right column, among them Friedrich Heinrich Gotthilf Grassmann.

"The collaboration between the excellent school councilor Bartholdy, the teacher Graßmann and his brother had created a lively pedagogical movement in Stettin. Without making any noise about it, they transformed a school for the poor into a model for other institutions…" [5, 479]

Justus Graßmann's discussions with Bartholdy did not remain confined to Pestalozzi's pedagogical methods (Schleiermacher remarked that he was totally in accord with Bartholdy's reception of Pestalozzi's ideas[3]). Bartholdy and Justus Graßmann probably also discussed Schleiermacher's first publications on the theory of science[4] while the latter was working on his schoolbooks on geometry, which relied on the theory of forms developed by a follower of Pestalozzi, Joseph Schmid. In these schoolbooks, he developed the outlines of a "geometrical theory of combinations". In 1829 [12], he applied it to crystallography and presented his theory to the scientifically educated public. By thinking about an adequate way to make geometry accessible to children, Justus Graßmann started to think about ways of presenting geometry that deviated from the methods of Euclid. This, in turn, led him to reflect upon the foundations of mathematics.

"Methodological work on the pedagogical subjects in question" had to entail "clarity as to which elements make up these pedagogical subjects," Justus Graßmann wrote in a textbook of 1817. "Only by establishing the primordial

3 Schleiermacher about Bartholdy: "In Berlin, he told me about his plans for a seminar, which made me very happy, and from which I conclude that I share his view of Pestalozzi's idea and its essential importance." Letter from Schleiermacher to Joachim Christian Gaß (1766–1831), May 1805. [32, 23]

4 Copies of the first lectures of Schleiermacher in Halle began to circulate amongst interested readers. Bartholdy also received a copy. He studied Schleiermacher's text carefully with Gaß, copied it down for himself and spoke very positively about Schleiermacher's ideas. According to a letter from Gaß to Schleiermacher from July 1805, the lectures on philosophical ethics contained a detailed explanation of Schleiermacher's transcendental postulates. Here he developed his own position as an alternative to Schelling's. (Letter from J. Chr. Gaß to Schleiermacher, 13 July 1805 [32, 25sq.], and also [27, 244sqq].)

Hermann Graßmann as a teacher

As Max Ludewig – who graduated from Stettin "Gymnasium" in 1865 – remembered him, Hermann Graßmann's behaviour as a teacher was not overly scholarly. But "concerning the demands of everyday life, he seemed like a benevolent child. His eyes were hidden behind spectacles, but generous and friendly. He left it to his favourite pupil to maintain order in class. He never reprimanded us. Usually, he stood by the first row of desks. We smeared chalk on the buttons of his vest, without him noticing. If and when he noticed something worthy of his disapproval, he merely murmured 'now, now!' But we still learned a lot from him because his thinking was full of scientific clarity."

(M. Ludewig, *Erinnerungen eines alten Stettiners*. Stettin 1918)


Raumlehre
für Volksschulen.
Von
J. G. Graßmann,
Subrector am Gymnasio zu Stettin.
Erster Theil.
Ebene räumliche Verbindungslehre.
Mit 5 Kupfertafeln.
Berlin, 1817.
In der Realschulbuchhandlung.

prerequisites of the subject in question, from a scientific or a classroom perspective, are we able to establish the true and lasting importance of what we are dealing with…" [13, viii]

While he was writing his textbooks of geometry, which received favourable reviews from Friedrich Adolf Wilhelm Diesterweg (1790–1866) [4: vol.1, 195–197, 264, 525; vol. 3, 224], Justus Graßmann was already aware of the fact that the geometric theory of combinations was a *new mathematical discipline*. It is here that we find the first reflections concerning the essence of mathematical synthesis, the dialectic structure of mathematics and the organic integrity of theory-building that supposedly made theory "analogous to a work of art". In his treatise *On physical crystallonomy and the geometric theory of combinations* [12], Justus Graßmann developed his mathematical theory. This is how Erhard Scholz described it:

> *"In modern terminology, Justus Grassmann introduced into his 'calculus of complexions' a three-dimensional free Z-module, whose elements […] represented directed lines of Euclidean space in an algebraic symbolism. He thus introduced with great clarity a three-dimensional vectorial calculus with integer coefficients." [34, 41]*

The ideas of father and son were closely connected and we might say that Hermann Graßmann completed and perfected what his father had begun. Other publications offer a detailed account of this relationship.

Hermann Graßmann had referred to his father's *concept of the product* in his *Extension Theory*. But he had understated the case when he wrote that the influence remained confined to the surfaces of rectangles or parallelograms as products of the adjacent sides. In a footnote to his textbook on trigonometry (1835), to which Hermann also referred [7, 9], Justus Graßmann wrote:

> *"If one takes the concept of product in its purest and most general meaning, then – in mathematics – the concept designates the result of a synthesis. This synthesis uses the entities produced by a preceding synthesis, replaces these entities, but maintains the rules governing the initial entities. […] In arithmetic, this entity is an element […]. In geometry, this element is a point." [14, 10 footnote]*

This way of putting it makes it easy to discern Graßmann's abstract approach to mathematical conjunctions, an approach that also expressed itself in the general theory of forms, preceding the main corpus of A1. As a genetic and constructive program, this led to the first *abstract* algebraic theory. In L. G. Biryukova and B. V. Biryukov's words: "In such of Hermann Grassmann's works as the Ausdehnungslehre is to be found the definition of an abstract group (ten years before Cayley's work on groups), and the concept of ring is developed, yielding both left and right rings." [2, 137]

Schleiermacher

The hints concerning the influence of the theologian and philosopher Friedrich Schleiermacher are just as elusive as those concerning Graßmann's father. In the curriculum vitae that he submitted for an examination in theology at the University of Berlin in March 1833, Graßmann wrote:


Friedrich Schleiermacher (1768–1834).

> *"Yet only in the last year did Schleiermacher attract me completely; and although by that time I was more concerned with philology, still only then did I realize what one can learn from Schleiermacher for every science, since he did not so much provide positive answers, as he made one skilled in attacking every investigation from the correct side and continuing independently, and thus to stand in a position to find the positive answer oneself." [26, 145–146]*

Robert Grassmann's writings support this view. In an obituary for his brother, published only recently, he wrote that Schleiermacher's "ingenious dialectical method exerted the greatest influence on the young Grassmann." [26, 203sq.]

One of Hermann's sons Justus Graßmann (1851–1909) also wrote a biographical summary of his father's life. This recently discovered document also reveals information on Schleiermacher's influence. Justus wrote that, as a student, his father "seems to have been drawn especially close to

Schleiermacher, on whose lectures and works many commentaries were found among the papers he left behind." [25] Unfortunately, this collection of papers on questions concerning the humanities and philosophy remains lost, like the papers on mathematics and the natural sciences. The two brothers read and discussed Schleiermacher's *Dialectic* (1840), his *Aesthetic* (1845) and, finally, his *Theory of the State* (1847).

In the *Edifice of Knowledge*, Robert Grassmann repeatedly emphasised how important he considered Schleiermacher to be, saying that Schleiermacher was the "*most important […] critic we have seen in recent times.*" [15, 82] "*Schleiermacher's great merit*," he explained, "*is that he was the first to truly grasp and introduce into science a theory of scientific discovery or speculation as the highest branch of the science of logic. He has this merit even though his idea remained stuck in a theoretical stage and even though Schleiermacher did not yet know how to use it to reorganize the sciences.*" [15, 82–83]

And Robert went on to say:

> *"According to his 'Dialectic' Berlin 1839, only two academic fields can show us the idea of knowledge. Both of these fields deal with the idea of knowledge, that is to say, the mutual relationships between thinking and existence. Dialectics, which deals with the oppositions within unity, does so in the conceptual frame of the general, whereas mathematics, which only deals with equal and unequal magnitudes, does so in the conceptual frame of the particular. According to him, all true thinking is scientific thinking depending on to what extent dialectics and mathematics are a part of it (§§344–346). Mathematics is closer to the empirical, dialectics closer to the speculative form. The empirical process always precedes the speculative process, contextualizing it. Schleiermacher is completely on the mark in these theorems; but, as he remarked himself, he lacked knowledge of mathematics." [15, 83]*

In September 2009, historian of mathematics Ivor Grattan-Guinness broke new ground by pointing out the fact that Schleiermacher's Dialectic had provided the foundational structure for Robert Grassmann's life's work – the *Edifice of Knowledge* [see 17]. Albert C. Lewis [23] and H.-J. Petsche [27, 244–248; 28] have shown that just this approach serves to unfold the new mathematical theories of Graßmann's *Extension Theory* of 1844 [see 16].[5]

[5] That is to say that Gert Schubring's view, which he stated in 1996 [36] and renewed in 2009 [37], that the philosopher Jakob Friedrich Fries (1773–1843) influenced Hermann Grassmann through his brother Robert, cannot be confirmed. On the contrary, there can be no doubt that Hermann communicated the thinking of Friedrich Schleiermacher to Robert Grassmann and that the methodological vision of Robert's thousands of pages of the "Edifice of Knowledge" bore its imprint, as Ivor Grattan-Guinness pointed out [17]. Fries is not mentioned in any of Hermann or Robert Grassmann's writings. Robert Graßmann also composed a 120-page History of Philosophy [15], in which he did not mention Fries. Schleiermacher, though, receives much acclaim and is hailed as the "most important […] critic we have seen in recent times". [15, 82]

Graßmann's *Extension Theory* is one of the rare and precious cases in which a philosophical theory puts its heuristic potential into action and spawns new theoretical insights. On the other hand, it was unfortunate for Graßmann that mathematicians at the time could not accept A1 due to their reservations concerning the philosophical discussions that were underway.

Graßmann, the polymath

Even though the two versions of *Extension Theory* are clearly the creative core of Graßmann's work, his total intellectual output was far more diverse. His T*extbook of Arithmetic* [8] was an important impulse for the development of axiomatic mathematics, inspiring Peano, Frege and many others. Graßmann was also noted for his discoveries in the theory of electricity (where his work had some points of contact with Clausius), and the theories of colour and of vowels. Graßmann made amendments to Helmholtz's theory and extended it. He published works on the didactics of experiments in the teaching of chemistry and on crystallography. He was also a pioneer of comparative philology and Vedaic research. A sixth, revised edition of Graßmann's dictionary of the *Rig-Veda*, a collection of pre-Buddhist divine hymns from India (12^{th}–6^{th} centuries BC) was published in 1996 [10]. Graßmann's translation of *Rig-Veda* is the authoritative translation to this day.

> *"Indeed I well know," Graßmann wrote in the last lines of his introduction to the second version of the* Extension Theory *of 1862, "that the form I have given the science is, and must be, imperfect. But I also know and must declare, even at the risk of sounding presumptuous, – I know, that […] a time will come when it will be drawn forth from the dust of oblivion and the ideas laid down here will bear fruit." [9, xvii]*

Celebrating the occasion of Graßmann's birth 200 years ago, an international conference took place in Potsdam and Sczcecin in September 2009. For four days, over 75 scientists from four continents – among them a Chinese crystallographer, an Austrian robotics-engineer, an Australian software-specialist, a Japanese philologer, a Finnish philosopher, a Russian mathematician, a German hardware-architect, a Polish historian and many others – engaged in a transdisciplinary dialogue, following the traces of Graßmann's achievements and renewing their importance for scientific research today.

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The joy of doing Mathematics Interview with Professor Ioan A. Rus

Mădălina Păcurar (Cluj-Napoca, Romania) and Vasile Berinde (Baia Mare, Romania)

Short Biographical Note

Professor Ioan A. Rus was born in Ianoșda village, Bihor County, Romania, on 28 August 1936. He graduated from "Babeș-Bolyai" University of Cluj-Napoca, Faculty of Mathematics, in 1960 and got his PhD in mathematics in 1968 with a thesis on the Dirichlet problem for strong elliptic systems of second order, under the supervision of Professor Emeritus D. V. Ionescu. He held all possible teaching positions at "Babeș-Bolyai" University of Cluj-Napoca: Teaching Assistant (1960–1967); Assistant Professor (1967–1972); Associate Professor (1972–1977); Full Professor (1977–2006) and, after his retirement in 2006, Consulting Professor. The administrative positions he held were: vice-dean, Faculty of Mathematics (1973–1976); vice-rector (1976–1984 and 1992–1996); Head, Chair of Differential Equations (1985–2002); and Head, Department of Applied Mathematics (1998–2002).

Ioan A. Rus has published about 150 original works and more than ten books (monographs and textbooks), mainly in the field of *fixed point theory* and differential equations, which have more than 1300 citations. Amongst the most important monographs (co)-authored by Ioan A. Rus, we mention: *Fixed Point Theory* (Cluj Univ. Press, 2008), *Fixed Point Structure Theory* (Cluj Univ. Press, 2006), *Generalized Contractions and Applications* (Cluj Univ. Press, 2001) and *Principles and Applications of Fixed Point Theory* (in Romanian, Dacia, 1979).

He is the creator of two important research directions: the techniques of Picard and weakly Picard operators, and fixed point structures theory.

He founded and is currently the Editor-in-Chief of *Fixed Point Theory*, the first journal in the world on this topic. He has been on the editorial boards of several international journals: *Mathematica (Cluj)*; *Studia Univ. Babes-Bolyai. Mathematica*; *Carpathian J. Mathematics*; *Gazette des Mathematiciens (Paris)*; *Pure Mathematics Manuscripts (Calcutta)*; *The Global J. Math. & Math. Sciences*; *Scientiae Mathematicae Japonicae*; and *Notices from the ISMS*.

Professor Rus has received the degrees of Honorary Doctor (*Doctor Honoris Causa*) from the North University of Baia Mare and Technical University of Cluj-Napoca. He has 26 PhD students and 29 descendants.

In an attempt to explain the achievements of his impressive career, one can say that the main force in Professor Rus' life is without any doubt his enthusiasm and joy for doing mathematics and his inexhaustible intellectual and physical energy. Overall, he is a very kind, open and warm person, a pleasant conversation partner and an extremely subtle and sharp mind.

When did you decide that you would study mathematics and for what reasons?

During primary school (grades 1–4, in my birthplace village), I was a student good in all subjects, without a specific preference for any of them. During secondary school (grades 5–7, in the same location) I had a single mathematics teacher, Ion Marin Nistorescu. At the very first mathematics class, he brought with him the class catalogue, the mathematics textbook and some recent issues of the journal *Gazeta Matematică*. From the very beginning he warned us: those who aim at the maximal marks must also solve problems from *Gazeta Matematică*.

Through his particular way of teaching, he systematically cultivated our interest in mathematics, an interest which, for some of us, has turned into a passion. We liked to solve problems which seemed difficult at first glance. It was then when I first realized that, apart from material goods, humans possess a wealth to cherish: reasoning. When I graduated from secondary school, professor I. M. Nistorescu persuaded my parents to send me for further studies at the best high school in the region, the *Emanuil Gojdu* High School in Oradea, the capital city of Bihor County. The extra-curricular activities concerning mathematics were very well organized in this high school. There was a weekly *Student Mathematics Circle* and the gifted students were encouraged by their teachers to solve problems from *Gazeta Matematică*. There was also a monthly *Students Mathematics Circle*, organized at a city level by the County School Inspectorate. On top of these activities, there was the *National Mathematics Olympiad*, organized by the Ministry of Education at local, regional and national levels.

During my high school years it was the school and the activity at the *Gazeta Matematică* that mainly contributed to my training. I had no a priori plans. I enjoyed consciously attending all the mathematics activities held for students, while also fulfilling my curricular obligations. It was also during this period that I began reading mathematics books. For example, I purchased an antique version of Euclid's *The Elements*, translated into Romanian by Professor Victor Marian, and *Geometrische Untersuchungen zur Theorie der Parellellinien* by N. Lobatchevski. Among my main achievements during my high school studies I would mention that in 1954 and 1955 I was awarded the 1st prize in the national contest for problem solvers, organized by the journal *Gazeta Matematică*, and that I finished my graduation examination paper in mathematics within 30 minutes, of the 3 hours allocated. Of the teachers that have guided me during this period I would mention Beniczky Ladislau, Ștefan Musta and Paul Tăvălug.

Please mention the key moments and facts related to your beginnings in scientific research.
From 1955 to 1960 I was a student in the mathematics faculty at "Babeș-Bolyai" University of Cluj-Napoca. I enthusiastically participated in all the curricular programs, while systematically observing the scientific life within the faculty. During this period, each department used to organize a weekly *Scientific Seminar*. I took part in the *Seminar of Analysis*, led by Professor Tiberiu Popoviciu (a member of the Romanian Academy), and the *Seminar of Differential Equations*, led by Professor Emeritus Dumitru V. Ionescu.

Within the course of differential equations, I was intrigued by Sturm's theorem on the separation of the zeros of the solutions of a second order linear homogeneous differential equation. I gave simpler proofs to some of the generalizations of this theorem. It was in this manner that I wrote my first scientific article, which was later published.

In my third year at university, Professor D. V. Ionescu taught the course of *Partial differential equations* (*The equations of mathematical physics*). This course brought to my attention the maximum principles and so I wrote my dissertation on maximum principles for parabolic equations.

Upon graduation from university (1960), I was hired as teaching assistant in D. V. Ionescu's Chair of Differential Equations. My PhD thesis, written under his supervision, was entitled *Contributions to study of the uniqueness of the solution of the Dirichlet problem related to second order strong elliptic systems*. The doctoral evaluation committee consisted of Gh. Călugăreanu from Cluj, C. Iacob from Bucharest, both members of the Romanian Academy, and Professor Emeritus A. Haimovici from Iași.

How was the scientific scene during that period?
Within the faculty, the research activity was both intensive and highly organized. It would be enough to mention a few aspects of this organization: the weekly research seminars of the departments, the monthly conferences of the faculty (which regularly included three parts: a didactical conference, a didactical lecture and a student lecture), the contractual research activity, the research programs of the research groups, the periodic organization of national and international scientific events, and the editorial activity for two publications: *Mathematica* (*Cluj*) and *Studia Univ. Babeș Bolyai, Mathematica* series. Regarded from this point of view, the atmosphere was highly positive. And I was mainly interested from this point of view.

Which one of your professors was mostly influential for you? Did you have a spiritual mentor?
Among the university professors which mostly influenced my training and my formation, I would mention the mathematicians D. V. Ionescu, Gh. Călugăreanu, T. Popoviciu, Gh. Pic and C. Kalic, the physicist V. Marian, the historian C. Daicoviciu and the philosopher D. D. Roșca. Each one of these personalities had specific particularities. To answer your second question, I have to confess I have never believed in the concept of a role-model for the youngsters. The freshness brought by the youngsters within the research teams is far above any models and it is vital that leaders of research teams take this into account. The young professional has to distill the input received (from lectures, seminars, scientific events, private talks, etc.) and select what matches his/ her own skills and interests.

Did you benefit from any research periods abroad? If so, how did this influence your way of thinking, method of work and the research topics?
During the academic year 1966–1967 I pursued a specialization research at the University of Lund (Sweden), under the guidance of Jaak Peetre. Following a conference of Lars Hörmandar, Professor Lars Gårding asked me, regarding the research activity at Lund University: "What do you think about this factory of mathematical production?" I have often recalled his question in the years that followed. Yes! It is beyond any doubt that a good mathematics faculty is a "factory" of mathematical production, with a number of specialists and many apprentices. From my point of view, a veritable university is one that promotes good research activity as well as didactical activity based on this research. If I were to choose between a very good teacher who has little to share and a weaker teacher with a lot to share, I would choose the latter.

During my research pursued at the University of Lund, I was supported in all matters by Professor Jaak Peetre. At that time, I was finishing my PhD thesis, which was centred on linear partial differential equations systems. J. Peetre drew my attention towards the complex problems in the nonlinear case, suggesting, among others, the works of F. E. Browder.

Back in Cluj-Napoca I contacted F. E. Browder, asking for reprints of his works. Shortly after that, I received reprints of all his works accomplished to that moment and reading them, I noticed the important role played by the fixed point theorems. So I began a systematic study of the *fixed point theory* (in algebraic structures, functional analysis, general topology, applications in the theory of equations, etc.). This time I had a clearly defined plan: to build a factory for producing fixed point principles.

And so, you also built a "factory" of mathematical production! How would you define a research team, its role and that of the team leader?
A research team is defined by the research topic, the results obtained, the common efforts to ensure a prompt and good impact of them in the mathematical literature, the level of organization and the degree to which it suc-

ceeds in being a seminary for research in a certain field. The genuine group leader is in charge of setting the quality of the thematics of research and of its results, possessing a rich mathematical culture, relevant achievements, but also important managerial skills, supporting the younger collaborators and helping them find answers to questions such as:

How to individually affirm ourselves in a group activity?
How much do we read? How much do we do research?
What is relevant in terms of mathematical innovation?
When and how to present the results of our research?
Where and how to publish a scientific paper?
Which is the right time to write and publish a first monograph?

You have initiated, coordinated and consecrated the Research Seminar "Fixed point theory and its applications", which played an active part in the scientific life of "Babeş-Bolyai" University for more than 40 years. Could you present a few facts about this important research group?

The research seminar began in 1969 and, during the first three years, its activity consisted mainly of my lectures and the discussions generated by them. In the academic year 1969–1970, my lectures were dealing with fixed point theory in algebraic structures, published in a monograph in 1971. The theme of my lectures between 1970 and 1973 was *Fixed Point Theory in Functional Analysis*, which subsequently became a monograph under the same title. During this time, most of the participants in the seminar were final year students. From 1973 onwards, the seminar turned into a traditional research seminar.

Currently, this research group is defined through the quality of its core members: Radu Precup (*Fixed point theory for non-self operators with applications to differential and integral equations), Adrian Petruşel (Fixed point theory for multivalued operators with applications to differential and integral equations*), Adriana Buică (*Coincidence point theory with applications to differential and integral equations), Marcel Adrian Şerban (Fixed point theory for operators on product spaces with applications*), Szilard Andras (*Fixed points and integral equations*) and Veronica Ana Ilea (*Fixed points and functional differential equations*). This research group is also editing the journal *Fixed Point Theory* and periodically organizes the series *International Conference of Nonlinear Operators and Differential Equations (ICNODEA)*.

Tell us how FPT has been founded as a periodical publication associated to the group's activity.

Our first publication directly associated to the research seminar was *Seminar on Fixed Point Theory – Preprint no. 3* (1980–1999), followed by Seminar on *Fixed Point Theory* Cluj-Napoca (2000–2002) and finally, from 2003 onwards, by the present journal, *Fixed Point Theory*, an ISI journal since 2007.

During an impressive career, you have harmoniously blended research and didactical activities, as well as administrative work. How did you manage to balance them all? Please detail as much as possible.

There is nothing special in the statement implied by your question. The activity of a university professor is always threefold: research, teaching and services towards the academic community. In all these three areas, I have constantly selected the essence, seeking to solve problems and not get dragged into bureaucratic activities. It is crucial to consider these components as a whole. For instance, in my teaching activity, I have often used the heuristic approach. I was pursuing a mathematical activity that led to a theorem. The proof was always above the theorem and not following it, as in any heuristic reasoning. In teaching mathematics, it is important to illustrate, as much as time allows it, the path towards the mathematical result. This is the natural milieu for attracting students to mathematical research. Universities should promote research activity and efficient didactical activity based on this research.

Imagine you are a young mathematician today. What opportunities or disadvantages could you define in our days, compared to the conditions you had in the 60s?

In my days, to have access to recent information was extremely difficult. The budget allocated for the mathematics library was insufficient to provide a minimum research basis. But we did not fall into the trap set by the budget myth. On the university level, a collective of professionals was in charge of solving this very hard problem. I was also part of this team, which was led by professor Gh. Pic. We strived to obtain as many journals as possible, by means of an exchange program with the publications of our faculty: *Mathematica* and *Studia*. It was the quality of our publications and the professors' common effort which ensured a solution to the documentation problem. The Romanian mathematicians working abroad were of great help as well. Before 1989, it was also a common procedure to request reprints of papers from their own authors.

Today the access to information is almost complete. Through the financial effort of our university, for example, one can access the most important databases as well as the relevant mathematical journals. We can obtain reference books through our library and through the research grants we are running. Therefore, today, a young Romanian mathematician who possesses the will and the intellectual capacity is able to obtain competitive accomplishments in research.

Could you mention a few of the major mathematical problems you have been interested in?

I would mention two such problems. The first one goes as follows:

Let us consider an operational equation on a set endowed with a convergence structure (metric space, topological space, L-space). Assume that all the solutions of the equation are obtained by successive approximations. Which are the properties of the solutions of such an equation? In order to solve this problem, I have constructed the theory of weakly Picard operators, which has proved

Professor Ioan A. Rus (left) in 2004 as a Honorary Doctor of North University of Baia Mare together with Acad. Petar Kenderov (right) and Vasile Berinde

to be extremely useful in the case of other mathematical problems as well.

Another problem I would like to discuss is dealing with invariant sets:

We have a fixed point theorem T and an operator f, which does not satisfy the conditions of T. Under which conditions does the operator f have an invariant subset Y such that the restriction of f to Y, $f|_Y$*, satisfies the conditions of T?* In order to rigorously formulate this problem, I have introduced the notion of *fixed point structure* on a set and I have solved the problem by constructing the *theory of fixed point structures*.

Which are, in your opinion, the main trends and challenges in this field of research?

A great number of results have been obtained in the field of fixed point theory. I would assume the main issue is to direct these abstract research results towards solving the great problems (constantly open!) of: functional equations, ordinary differential equations, functional-differential equations, differential and integral inclusions, set-differential equations, etc.

What role do you think the history of mathematics plays within mathematical education?

The elements of the history of mathematics often represent a natural path towards mathematical understanding. Observing the evolution of mathematical objects is a key component in teaching mathematics.

You have been constantly involved in promoting mathematical education in Romania. Can you tell us a few words about the current position mathematics holds within Romanian education?

There are a number of factors contributing to the quality and efficiency of an education system but mostly it is the quality of the teachers and the atmosphere created within the school population by the managers that define the attributes. This is how one could explain the quality of the Romanian education system, both before and after 1989.

You have devoted many years and much energy to administrative works, including your membership in the evaluation committees at the Ministry of Education. What do you think about the criteria used in evaluating mathematicians from universities and research institutes? Is there a flawless evaluation grid? What should be, in your opinion, the essential requirements?

Unfortunately, the criteria currently applied in the evaluation of mathematicians are somehow implicit and are based on a kind of arithmetic of research activities: the *number* of ISI papers, the *number* of papers processed in a certain database, the *number* of books, etc. In my opinion, any evaluation system for research activities should be based on principles such as: 1) A clear statement about the aim of the evaluation (keeping the current position, promotion, contest, financing, etc.); 2) A self-evaluation following a flexible, a-priori defined structure; 3) The evaluation of five papers selected by the evaluated mathematician, by a committee of experts in the field outside the home institution of the candidate; 4) Establishing a minimum period T of years for which the evaluation is done ($T \geq 3$ years!).

Generally speaking, excellence cannot be evidenced through an arithmetical evaluation. A genuine evaluation system cannot be applied by a bureaucrat but by highly trained specialists.

In the evaluation of the efficiency of an education system, it is not the results that should prevail but mostly the progress recorded by the students. It is one thing for a student used to getting a 4 mark to achieve a 6, and another thing completely for a 10 mark student to become an 8 mark student (on a scale 1–10). To sum up, I am personally optimistic regarding the future of Romanian mathematical education and research.

Mădălina Păcurar is a lecturer at the Department of Statistics, Forecast and Mathematics, Faculty of Economics and Business Administration, "Babeș-Bolyai" University of Cluj-Napoca. She got her PhD in mathematics at the same university in 2009 with the thesis "Iterative Methods for Fixed Point Approximation" with Professor Ioan A. Rus as her superviser. She is a member of the Editorial Board of the EMS Newsletter, being responsible for Book Reviews.

Vasile Berinde is a professor of mathematics and Director of the Department of Mathematics and Computer Science at the North University of Baia Mare. He is the first PhD student of Professor Ioan A. Rus. He is an Associate Editor of the EMS Newsletter and also the Publicity Officer of the EMS.

The Johann Radon Institute for Computational and Applied Mathematics (RICAM)

The Johann Radon Institute is named after the Austrian mathematician Johann Radon, whose early work in 1917 forms a mathematical basis of computerized tomography and other imaging methods. The Radon transform is named after him. Since RICAM has groups on Inverse Problems and on Imaging, the link to Johann Radon is not just a national one but also mathematical.

Johann Radon (1887–1956)

The institute was founded in 2003, under Professor Heinz W. Engl as Founding Director, by the Austrian Academy of Sciences and is located on the campus of Johannes Kepler Universität Linz, Austria. Its mission statement reads as follows.

The Johann Radon Institute for Computational and Applied Mathematics:

1. does basic research in computational and applied mathematics according to highest international standards;
2. obtains the motivation for its research topics also from challenges in other scientific fields and industry;
3. emphasizes interdisciplinary cooperation between its workgroups and with institutions with similar scope and universities worldwide;
4. cooperates with other disciplines in the framework of special semesters on topics of major current interest;
5. wishes to attract gifted Post-Docs from all over the world and to provide an environment preparing them for international careers in academia or industry;
6. cooperates with universities by involving PhD students into its research projects;
7. promotes, through its work and reports about it, the role of mathematics in science, industry and society.

The institute has about 60 scientific employees, more than half of them funded by a third party (e.g. the Austrian National Science Foundation FWF or the European Union). Its setup and composition are very international; we have Post-Docs and doctoral students from 14 countries.

The scientific work of the institute is performed in the following seven groups:

Computational Methods for Direct Field Problems (led by Professor Ulrich Langer)

This group analyses and implements novel computational methods for complicated field problems usually described by systems of partial differential equations arising in different applications. Specifically, this involves:

- Robust algebraic multigrid and multilevel iteration methods for systems of partial differential equations.
- Domain decomposition and related subspace iteration methods.
- Scalable parallel algorithms including grid-enabled algorithms.

In addition, there are new research topics like:

- Isogeometric analysis.
- Multiscale methods.
- Solvers for non-linear eigenvalue problems arising from partial differential equations for different discretization techniques (FEM, DG-FEM, BEM) and different applications (solid and fluid mechanics, electromagnetics, imaging, life sciences).

Inverse Problems (led by Professor Heinz W. Engl)

The key topic of this group is the further development of theory-based regularization methods for nonlinear inverse problems and various applications. More and more, non-Hilbert space techniques and non-convex regularization functionals are of core interest. Also, since the needs from practice (both in biological and industrial applications) lead to more complex inverse problems, the combination of physics-based and data-driven models is a major topic of research.

Symbolic Computation (led by Professor Josef Schicho)

Linz is the location of the renowned Research Institute for Symbolic Computation (RISC), which is where Professor Schicho graduated. This group specializes in symbolic methods that are beneficial to and benefit from analysis like Gröbner-base methods for the symbolic solution of boundary value problems and using regularization methods to stabilize symbolic methods for root finding of polynomials with noisy coefficients.

Analysis of Partial Differential Equations (co-led by Professor Peter Markowich, Vienna/Cambridge, and Dr Massimo Fornasier)

This group deals with nonlinear kinetic equations for classical mathematical physics models (e.g. Boltzmann

equations) and modelling social interactions and biological processes, nonlinear hyperbolic equations for water wave modelling and traffic flowing networks and variational methods for free-discontinuity and free-boundary problems for image processing. Both analytical and numerical methods are used. Dr Fornasier has recently obtained the START-Prize of the Austrian National Science Foundation, which allows him to build his own research group with major external funding.

Optimization and Optimal Control (led by Professor Karl Kunisch, Graz)

This group deals with analysis and numerics for optimal control problems in connection with partial differential equations. One topic concerns time-optimal control synthesis in the context of the wave equation, using a semi-smooth Newton-method. Other work includes the control of quantum mechanical systems and closed loop control.

Mathematical Imaging (led by Professor Otmar Scherzer, Vienna)

This group specializes in the development of partial differential equation based algorithms for imaging problems in biology and medicine, especially photo-acoustic imaging, for which they also hold a patent. The mathematical basis is regularization theory for non-local and non-convex functionals.

Mathematical Methods in Molecular and Systems Biology (co-leaders Professor Philipp Kügler, Stuttgart, and Professor Christian Schmeiser, Vienna)

This group is located in the Vienna BioCenter, close to the biological research institutions of the university and the Medical University of Vienna and the Austrian Academy of Sciences. Topics addressed are numerical methods for systems biology (with an emphasis on inverse problems) and mathematical methods for cytoskeleton dynamics.

A major reason for establishing RICAM was, beyond the proposed scientific work in the various groups, the close cooperation between these groups. This was based on a ten-year Special Research Area 'Numerical and Symbolic Scientific Computing', funded by FWF. There have been numerous joint publications between members of the various groups and the cooperation between the groups form (nearly) a complete graph.

Another major enterprise of RICAM is the organization of Special Semesters, where a topic within the expertise of the institute is systematically discussed over a period of several months. These special semesters typically have a dozen long-term guests and several groups of thematic workshops with large international participation. The special semesters so far have concerned the following topics:

2005: Special Semester on Computational Mechanics.
2006: Special Semester on Gröbner Bases.
2007: Special Semester on Quantitative Biology Analysed by Mathematical Methods.
2008: Special Semester on Stochastics with Emphasis on Finance.
2009/2010: Mini Special Semester on Computational Methods for Inverse Problems.

From 03 October to 16 December 2011, a 'Special Semester on Multiscale Simulation and Analysis in Energy and the Environment' is planned; further information can be found on the webpage of the institute (see below).

The next step of the development of the institute will be the installation of a Transfer Group, linking RICAM to the Industrial Mathematics Competence Centre (IMCC), which has existed in Linz for a decade and cooperates with industry, developing mathematics-based software for simulation and optimization in fields like steel processing, automotive design, finance and medical imaging.

It is the policy of the institute to (nearly) exclusively offer limited-time contracts for Post-Docs, usually for three years with an extension of another three years based on performance. This strategy has been quite successful, as is shown by the high number of offers of professorships in Europe, America and China for our Post-Docs. The Post-Docs are supposed to apply for external funding for doctoral students (who formally get their degrees at Johannes Kepler Universität, Linz), and this has also been quite successful.

Further information about RICAM and about our publications can be found at www.ricam.oeaw.ac.at.

Professor Heinz W. Engl, Institute Director

ICMI column – http://www.mathunion.org/icmi

Mariolina Bartolini Bussi

In the last few months, there has been much news on the ICMI front. Some of that information shall be given here. For continually updated information, subscribe free to ICMInews. Send an email to icmi-news-request@mathunion.org with the subject-line: Subject: subscribe

The new Executive Committee

On 1 January 2010, there was a changeover from the Executive headed by Michèle Artigue to an Executive with five new members. From the old Executive, Jaime Carvalho and Bill Barton remain as the new Secretary-General and President, respectively, Michèle Artigue remains ex officio as Past President and Mariolina Bartolini Bussi also continues as a member. Among the new members are the new Vice-Presidents Mina Teicher (Israel) and Angel Ruiz (Costa Rica), and new members Sung Je Cho (South Korea), Roger Howe (USA), Renuka Vithal (South Africa) and Zhang Yingbo (China). Two of the new Executive already hold significant international roles in mathematics education: Sung Je Cho is the Convenor for ICME-12 in Seoul in 2012 and Angel Ruiz is currently President of the CIEAM-IACME (Inter-American Committee of Mathematics Education). The Executive of the ICMI also has two other ex officio members: the President and the Secretary-General of the IMU.

ICMI awards

The ICMI awards committee has announced that it has reached a decision concerning the 2009 Felix Klein award and the 2009 Hans Freudenthal Award.

The Felix Klein award goes to Gilah C. Leder, La Trobe University, Bundoora, Victoria, Australia. The official citation of the ICMI Awards Committee is:

It is with great pleasure that the ICMI Awards Committee hereby announces that the Felix Klein Medal for 2009 is given to IAS Distinguished Professor and Professor Emerita Gilah C. Leder, La Trobe University, Bundoora, Victoria, Australia, in recognition of her more than thirty years of sustained, consistent, and outstanding lifetime achievements in mathematics education research and development. With a background as a highly recognised secondary teacher of mathematics, Gilah Leder moved, through a number of steps, into research in mathematics education, with a particular emphasis – from the very beginning of her research career – on gender success and equity in mathematics education, but also more broadly on students' affects, attitudes, beliefs, and self-concepts in relation to mathematics education, at educational levels ranging from school to university. To a very high degree her work has contributed to shaping these areas and made a seminal impact on all subsequent research. Moreover, Gilah Leder has done significant work with regard to assessment in mathematics education, mathematically able students, research methodology, supervision of graduate students, and teacher education. A characteristic feature of Gilah Leder's work – published in almost two hundred scholarly publications – is its application of perspectives and theories from sociology and psychology along with mathematical perspectives. Gilah Leder's achievements include a remarkable amount of work for national, regional, and international mathematics education communities in a leadership role, as well as a committee or board member, an editorial board member for several journals and book series, as a mentor and supervisor of graduate students, as a visiting scholar in several countries, and as an invited key note speaker at numerous conferences in all continents.

The Hans Freudenthal Award goes to Yves Chevallard, IUFM d'Aix-Marseille, France. The official citation of the ICMI Awards Committee is:

It is with great pleasure that the ICMI Awards Committee hereby announces that the Hans Freudenthal Medal for 2009 is given to Professor Yves Chevallard, IUFM d'Aix-Marseille, France, in recognition of his foundation and development over the last two and a half decades of a very original, fruitful and influential research programme in mathematics education. The first part of the programme, developed in the 1980s, was focused on the notion of didactical transposition of mathematical knowledge from outside school to inside the mathematics classroom, a transposition which also transforms the very nature of mathematical knowledge. This idea has been further developed, in the 1990s and beyond, into a more general study of the varying institutional characteristics and cultures within which mathematics is being practised in terms of different praxeologies (combining praxis and logos). This gave rise to the so-called anthropological theory of the didactic (ATD) which offers a tool for modelling and analysing a diversity of human activities in relation to mathematics. On that basis Yves Chevallard has developed an entirely new approach to teacher training focusing on the needs and problems of the profession operating in what he calls "clinics for training" which are also cumulatively establishing "archives for training". It is a characteristic feature of Yves Chevallard's work and impact that he continues to collaborate closely with colleagues in France and Spain and that his work has had a great impact internationally, and not the least so in Latin America. This is reflected in a large number of doctoral dissertations that have been written in *various countries about, or within the framework of, his theory. International conferences on ATD have been held in 2005, 2007, and 2010, each of which has gathered about a hundred researchers from Europe, America, Africa, and Asia. In some countries, including Chile and Mexico, Yves Chevallard's work also has exerted a direct influence on curriculum development and in-service teacher training.*

icmi study 21: mathematics education and language diversity

Around the world, mathematics is learned and taught in situations of language diversity. Whether through historical multilingualism, migration, colonization, globalization or other factors, mathematics classrooms frequently involve multiple language use, learning through second or additional languages, learning through minority or oppressed languages, or through majority or dominant languages. Increasing recognition and awareness of this longstanding reality have led to a growing body of research that seeks to understand the relationship between different facets of language diversity and mathematics learning and teaching. It is time to critically review this work, consider implications for mathematics classroom practices and set an agenda for future research.

The principal aims of ICMI Study 21 "Mathematics education and language diversity" are:

- To gather together a community of researchers who are currently addressing issues of language diversity as they relate to mathematics education.
- To reflect on the current state of research on these issues and propose a research agenda for the future.
- To disseminate findings from research to date and issues for future work to the wider mathematics education research community and to practitioners.

The ICMI Study 21 "Mathematics education and language diversity" is designed to enable researchers and practitioners around the world to share research, theoretical work, project descriptions, experiences and analyses. The Study Conference will be held in São Paulo, Brazil, on 16 - 20 September 2011, the number of participants to be invited being limited to at most 120. It is hoped that the conference will attract not only established researchers but also some "newcomers" to the field and mathematics teachers with interesting and refreshing ideas or promising work in progress, as well as participants from countries usually under-represented in mathematics education research meetings. Participation in the Study Conference is only by invitation, based on a submitted contribution. People with an interest are invited to visit the website http://www.icmi-21.com and download the complete version of the discussion document.

Inquiries on all aspects of the study and suggestions concerning the content of the Study Conference should be sent to both co-chairs: Mamokgethi Setati – setatrm@unisa.ac.za or funkymaths@yahoo.co.uk, and Maria do Carmo Santos Domite – mcdomite@usp.br or mcdomite@gmail.com.

ICMI Study 20 Conference postponed

Given the eruption of the Islandic volcano Eyjafjallajokull, the clouds of ash floating all over Europe and the chaos and almost non-existence of air traffic over Europe, the co-chairs and the chair of the LOC, Alain Damlamian, Rudolf Strässer and José Francisco Rodrigues, respectively, have decided to postpone the Study Conference of ICMI Study 20 "Educational Interfaces between Mathematics and Industry (EIMI)", planned for Lisbon for 19–23 April 2010. The Study Conference will instead take place on 11–15 October 2010 (EIMI website: http://www.cim.pt/eimi/).

European Science Foundation
SETTING SCIENCE AGENDAS FOR EUROPE

ESF-EMS-ERCOM Mathematics Conferences: Call for Proposals

The 2010 Call for Proposals for ESF-EMS-ERCOM Mathematics Conferences to be held in 2012 is open. Details about the Call are given in
http://www.esf.org/activities/esf-conferences/call-for-proposals.html
For on-line submission of proposals, go to
http://www.esf.org/activities/esf-conferences/call-for-proposals/framework-call-for-proposals.html
Submission deadline is September 15, 2010.

Up to four conferences taking place in one of the participating ERCOM Research Centers will be funded.

Zentralblatt MATHMLized

Patrick Baier, Olaf Teschke

1. What for?

MathML[1] (Mathematical Markup Language) is an XML application to describe and display mathematical content on the Web. The current standard 2.0 was defined in 2001. With the release of the first version as a W3C recommendation as early as 1998, it may be fairly considered as a Methusalem by Web measures; but only recent versions of the most common browsers support the MathML standard.

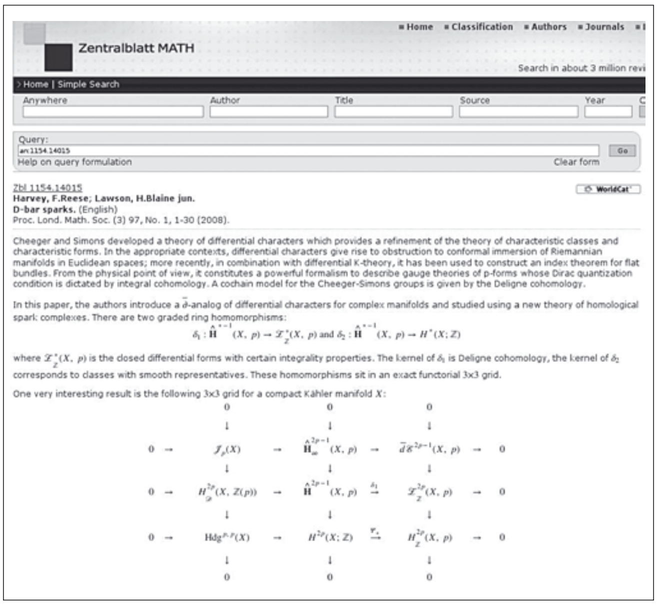
Old Zentralblatt interface: Bulky tex code.

There are several reasons for this rather slow progress. Mathematics content might not have been considered to be commercially interesting by browser developers. Mathematicians were early adopters of the Web, relying on the established TeX/LaTeX formats. Enhanced display, if considered to be necessary, was usually obtained by small solutions like conversion of elements to html if possible (e.g. subscript and superscript) or the integration of small formula images; alternative formats (dvi, ps, pdf) were often provided for convenience. In general, mathematicians were quite satisfied with the language they were used to and didn't care too much about presentation. Moreover, the conversion of the existing TeX/LaTeX corpus to MathML is non-trivial and is an ongoing challenge (probably, no one would like to create the XML directly).

However, with the progress of Web mathematics – and especially its interactive components – the advantage of a common XML standard becomes more evident. Mathematical software is widely used and it is obviously useful to have a common interface with Web presentations. Apart from the enhanced readability, automatic processing is supported. Search and retrieval of mathematics content can be enhanced significantly. As a small but important example, MathML makes mathematics content barrier-free: visually impaired users may access formulas via standard screen-readers.

2. How does one convert (masses of) TeX/LaTeX?

When you set up a mathematics blog today, it is usually not much effort to install MathML via available plugins. Even converting older content can be done with a reasonable effort, given that a single user won't change his (La)TeX environment too frequently. However, a digital library or mathematics database may face difficult problems in the conversion of existing content. There may be a large variety of styles and macros, even a change in TeX/LaTeX standards. Though many approaches exist for the conversion, there is no general optimal solution and one will need to check which existing tool may be the most convenient solution for a specific problem.



New Zentralblatt interface: Direct MathML display.

There is limited data available for the comparison of the existing tools. H. Stamerjohans et al. (2009; Zbl

[1] www.w3.org/Math/

1176.68233)[2] evaluated five converters that produce standard XML+MathML output on a sample of 1000 diverse arXiv documents (there are many more kits but they were excluded since they only translate subsets of TeX/LaTeX or lack documentation, etc.). With mixed results in the assessed categories (documentation, installation, coverage, quality, speed, usability), it becomes clear that the choice of appropriate tools depends very much both on the material and the intentions. For instance, the two converters with arguably the most extensive experience at production level (Tralics,[3] used in NUMDAM and CEDRAM, and LaTeXML,[4] adapted to arXiv content in a 2008 project) have very different features: Tralics is about 30 times faster, quite robust (all documents are transformed into well-formed XML compared with a 10% failure rate of LaTeXML) but has a (formally) very low rate of complete conversion (only 2%), though this figure can be quickly improved by adapting the tool to the corpus.

3. Ready to parse 3,000,000 documents?

It was only in 2009 when the percentage of MathML-supporting browsers employed by Zentralblatt users crossed the 50% mark. Recent versions of Gecko browsers (like Firefox) and Opera support the presentation directly, while Internet Explorer 8 comes along with a handy plugin. It is expected that e.g. Safari or Chrome will follow soon. Anyway, the fraction of MathMLable browsers steadily increases with users switching to more recent updates (by now, the number has reached about 60% – the inclination of many mathematicians to Firefox obviously helps). So last year appeared to be the right time to adapt the W3C standards in ZBMATH and convert the database to MathML.

With almost 3,000,000 documents in the database, there is obviously some work to do. Even a failure rate of 10% (a level no converter is close to reaching ad hoc) would mean that one has to check and debug around 300,000 entries – obviously too much to do by hand. Therefore, adaptation is necessary (fortunately, since the candidates are well-documented, this is also possible). The flexibility of Tralics has turned out to be a major advantage – with relatively few calibrations of inserting appropriate regular expressions into preprocessing we could push the 2% of completely converted documents to well above 50%. Of course, this was possible only because ZBMATH data are quite homogeneous.

The other decision one has to make is whether to generate the XML/MathML data during retrieval ("on-the-fly") or to do the conversion in advance and keep the data stored. The first alternative has several drawbacks. First, one has no good error control. More important is the performance issue: even a few milliseconds conversion time (this may amount to several seconds for some candidates; the javascript-based presentation tool MathJax[5] requires minutes at the moment) may accumulate quickly if the user chooses the popular complete display of 100 items in ZBMATH. Moreover, with many simultaneous users accessing the database, on-the-fly generation is also a waste of resources. Because of that, we decided that in the final version the XML/MathML data will be generated and stored with every daily update whilst the implemented beta version of our MathML display is an on-the-fly generated additional option to the old ZBMATH interface. For test purposes of the conversion this was perfect; moreover, it could be done within the framework of the existing interface, without requiring the pages to be valid XHTML (a singular feature of the new ZBMATH interface). Since the MathML button for the Tralics-supported display was added in spring 2009, it has provided a huge amount of test data, bugs and useful information about real-life performance.

Full implementation into retrieval (e.g. XML/MathML as standard display) required the launch of the new XHTML-valid interface, available since the beginning of this year. In the meantime, Tralics adaptation has been optimized with respect to the Zentralblatt corpus. As a positive side effect of the conversion process, older data generated with early TeX versions could also be adjusted and corrected. Not very surprisingly, the ZBMATH entries resulting from the digitization of the Jahrbuch required most care and could be considerably enhanced this way. In total, the XML data require approximately three times as much disk space as TeX data, which is a reasonable factor.

Quite important for the integration of MathML into daily workflow was the use of timestamps. Even a fast tool like Tralics needs several days to convert the whole database (in the arXMLiv project, LaTeXML needed about a full processor year for about half a million documents), which doesn't fit into the daily update process of automated routines. Fortunately, it is only necessary to convert the new and the corrected documents – both of a magnitude of about 1,000 for a single ZBMATH working day. Even with the inclusion of the necessary debugging routines, this can be handled appropriately.

4. Where will it lead?

With about 3,000,000 entries, Zentralblatt MATH is now one of the largest Internet mathematics resources available in full XML/MathML format. From the experiences of the conversion process, one may draw some conclusions:

a) Converters need adaptation. Therefore, robustness and flexibility is very important. The choice of the right solution requires several tests and depends much on the corpus.
b) Performance is crucial, at least when dealing with large datasets. There exist sophisticated solutions providing nice results that are simply too time-consuming at the moment.

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2 MathML-aware article conversion from LaTeX. A comparison study. In: Sojka, Petr (ed.), *DML 2009. Towards digital mathematics library, Grand Bend, Ontario, Canada, 8–9 July 2009. Proceedings*. Brno: Masaryk University. 109–120 (2009; Zbl 1176.68233).

3 http://www-sop.inria.fr/miaou/Tralics/.

4 http://dlmf.nist.gov/LaTeXML/.

5 http://www.mathjax.org/preview/preview-tex-examples/.

The next step, naturally, would be to proceed from presentation to semantics. Content MathML requires further refinement to eliminate ambiguities; at the moment, it appears not yet ready to be implemented on a large scale. On the other hand, it would be very useful, not just for the interaction but also for enhanced retrieval and semantic searches in the database. Time will tell when an article about the change of formula semantics over 150 years of mathematics may appear in this corner.

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Olaf Teschke [teschke@zentralblatt-math.org] is Managing Editor of ZBMATH and the editor of the EMS Newsletter in charge of the Zentralblatt Column. Additional information can be found on page 5.

2012 ICPAM-CIMPA research schools Call for Projects

CIMPA

The aim of the International Centre for Pure and Applied Mathematics ICPAM-CIMPA is to promote international cooperation in higher education and research in mathematics and their interactions, as well as related subjects, particularly computer science, for the benefit of developing countries. Our action concentrates at the places where mathematics emerges and develops, and where a research project is possible.

CPAM-CIMPA is a UNESCO centre located in Nice, with financial support from the French *Ministère de l'enseignement supérieur et de la recherché (France)*, the Université de Nice Sophia Antipolis (France), the Ministerio de Ciencia e Innovacion (MICINN) and UNESCO.

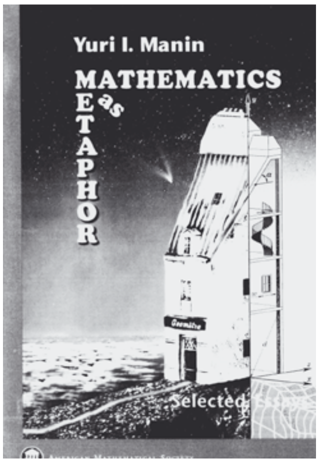
We organize research schools of about two weeks in developing countries. The purpose of these schools is to contribute to the research training of the new generation of mathematicians, women and men.

Once selected by the Scientific committee and the Governing board of ICPAM-CIMPA, research schools are organized locally with the help of ICPAM-CIMPA. ICPAM-CIMPA's financial contribution is essentially for young mathematicians from neighbouring countries to be able to attend the research school. ICPAM-CIMPA can help with obtaining founds from other sources. Additional information can be found in the roadmap (available on the web site of ICPAM-CIMPA). You can also write to ICPAM-CIMPA.

Research schools call for projects begins on March 1st, 2010.
The deadline for proposals is October 1st, 2010.

Book review

Ulf Persson



Yuri Manin

Mathematics as Metaphor
Selected Essays

American Mathematical Society, 2007
ISBN 978-0-8218-4331-4

If you choose to expound philosophical matters you expose yourself to great danger because, unlike in more technical matters such as mathematics, the demarcation lines between the profound and the silly, the potent and the vapid, are hard to draw. One may perhaps think of philosophy as the poetry of science. It is more concerned with evocation than straightforward argument, as it does not generate knowledge in the sense that science does because its insights cannot be objectified and manipulated. Many mathematicians are therefore rather suspicious of the philosophy of mathematics, which is seen as, at best, a harmless after-dinner expose of homilies, having little to do with the actual business of doing mathematics.

Manin, on the other hand, differs rather profoundly from most people expounding the philosophy of mathematics. His presentations are not predictable but go beyond those expected homilies. Part of the reason may be that he is Russian. The somewhat morally ambiguous notion of the 'intelligentsia' was in fact coined by Russian intellectuals of the 19th century. And one associates with the Russian intellectual tradition, more perhaps than with any other, a down-to-earth passionate interest, cutting across many disciplines, into the meaning and purpose of the activities of the mind. Thus one pictures the Russian approach to be poles apart both from the formalistic elegance of a Frenchman and the systematic penetration of a German, something that is borne out by the tenor of mathematical papers produced in the Russian manner. It may well be a sentimental illusion, in addition to being a silly one, but if so there is still with Manin a romantic exception to a mundane truth.

Manin's philosophical interest and acumen became clear to a Western audience, in connection with the translation of his book 'Logic for mathematicians', in the 1970s. But that book only pertained to one segment of his interest, and to get a better idea of the range of his philosophical commentary one could do worse than consult this recently published collection of selected essays, mostly translated from the original Russian. In them he is addressing various meta-issues, such as the formal nature of mathematics, its similarities and differences to languages, especially natural languages, its relation to truth and reality and especially its role in elucidating and inspiring modern physics (although, somewhat cynically, one could claim that physics may recently have been more of an inspiration to mathematics than the other way around). Additionally the author nurtures a deep interest in matters not usually associated to mathematics, such as linguistics, the study of myths and the Jungian concept of the collective unconscious and the various archetypes connected with it, in particular that of the so-called 'Trickster'. Finally, his interest in language also encompasses the paleo-historical origin of language and the emergence of consciousness and its neurological underpinnings, in particular the way consciousness can be restricted and warped (as in autism).

Manin's interest in mathematics is visionary, not so much in the solution of hard problems as in the identification of new uncharted territory and articulation of fundamental research programmes. As an example of the latter he refers in particular to the programme initiated by Cantor and his transfinite arithmetic, which ended up forcing mathematicians to rethink the notion of infinity, mathematical truth and provability, and the role of formalization and computation. Thus this was a programme with many unpredictable consequences, one of the most well known being the incompleteness result of Gödel resulting from a skilled mapping of meta-mathematics into mathematics, simultaneously interpreting a sentence's real meaning as well as its formal meaning. This preoccupation with logic eventually found an outlet in the emerging phenomenon of electronic computation, turning the spirit into flesh and greatly enlarging on the limited human capacity for tedious and mechanical thought recursively generated and thus (for better or for worse) the potential as well as the actual mathematical influence in the world at large.

According to Manin, the influence of mathematics is exhibited at three levels. At the most basic level we have the notion of a mathematical model. A mathematical model is a mathematical approximation of reality and as such an inevitable simplification. Its purpose is to simulate reality and in particular to allow predictions. Depending on the required accuracy of the simulation and its predictions it will need to be suitably modified, and subsequently become more and more involved as more and more features of the real world are taken into account. A classical example of a mathematical model is the Ptolemaic model of celestial motions involving epicycles. The higher the accuracy involved, the more epicycles need to be fitted in. It is a quintessential mathematical model, and if sufficiently elaborated, no doubt capable of any required accuracy. Mathematical models are no longer confined to the physical sciences; they also abound in social sciences, especially in economics. Mathematical models are judged on their efficiency, not their simplicity or beauty, let alone their explanatory power as opposed to their predictive capability. 'Tinkering' is the one word that characterizes mathematical models.

Then there is a higher kind of model, which Manin refers to as belonging to their aristocracy. These are referred to as 'theories' and correspond to the second level. The purpose of a theory is ultimately to explain. Simplicity and beauty are valued aspects of a theory and often indicative of it being 'true'. The notion of truth does not apply to a plebeian model; it is but a convenient representation, something the Catholic Inquisition understood well when it accepted the helio-centric view as merely a convenient mathematical model simplifying computations. With theories we think otherwise; somehow they are supposed to reveal the inner workings of reality. Of course beauty and simplicity are not the only criteria on which the truth of a theory is judged; it also has to be congruent with the so-called facts. However, if the theory is beautiful enough, discrepancies with facts can be thought of as artefacts of reality and not of the theory itself. Dirac had this attitude and he was always ultimately vindicated. Theories are definitely higher in the hierarchy; the very conception of a mathematical model requires some underlying theory. I would personally propose that the Stokes equation is a mathematical model while the Maxwell Equations constitute a theory. Much more comes out of the latter than is put into it. They have a value beyond their predictions. As an example, the invariance group foreshadowed special relativity. To model-builders, mathematics is a convenience, a kind of language, only useful as far as it is applicable. To theory-builders, on the other hand, mathematics does have an independent reality that underlies manifest reality –a Platonic view; it is deeply satisfying to mathematicians, regardless of their interests in physical applications. It is noteworthy that mathematical theories only exist in physics, which may explain that of all sciences, only physics has applications to mathematics, something we will return to below.

Finally, there is an even more rarefied way mathematics can exert its influence and that is through metaphor, hence the very title of this collection. A metaphor should be a stimulation to thought, not a substitute for it. A metaphor by its very nature should never be taken literally, as it then collapses and becomes merely silly. In particular, metaphors are naturally evocative and cannot be manipulated – hence the meaning of the initial comments on philosophy. As an archetypical example of a mathematical metaphor Manin suggests the idea of the brain as a computer. Metaphors are not theories but if they are potent they may inspire the formation of theories. Theories are scientific; metaphors are philosophical.

Physics is another magnet of compelling interest and Manin claims that no mathematical development of the 20th century has matched the revolutionary change of paradigm characterizing the fate of classical physics over the same period. This is a fact that should come as no surprise, he remarks, except possibly to a few 'autistic' mathematicians. Mathematics and physics used to be intimately connected, from the time of Newton until the end of the 19th century, but then there was a split and a divergence, after which neither camp became particularly interested in what the other was preoccupied with, a cultural rift that survives to this day. Paraphrasing the words of Manin, the mathematicians became obsessed with our relation to thought, while the physicists with our relation to reality. The former led to a kind of neurotic introspection and a subsequent hygiene of precision and formal reduction, while the latter led to flights of fancy, at least temporarily unfettered by precision and rigour.

In addition, the intuitions of physicists have recently proved more fertile in solving mathematical problems than the techniques of mathematicians, something the latter, to their credit, have freely acknowledged. In fact, as indicated initially, some parts of physics, e.g. string theory, have had more significant applications to pure mathematics than they have had to physics and its ostensible subject – physical reality. Why this should be is still something of a mystery. Some parts of modern physics, such as relativity theory, have been successfully mathematized and can be, but for the rigid boundaries of academic disciplines, acceptably classified under the mathematical banner. In contrast, quantum theory, which is far more influential and 'useful' than general relativity, is still intellectually marred by internal contradictions and an incompatibility with the latter. This does not seem to really bother the general physicist. The closer we view the material world, the less solid and commonsensical it appears. It is a world that can only be approached and described mathematically, i.e. in the sense of using mathematics as both a language and as a method of manipulation. The reason for this growing effervescence, according to Manin, is that in quantum physics the observer can no longer be separated from what he observes, unlike the classical paradigm in which one could (at least as a thought experiment) isolate pieces of the world and view them as closed systems, in which one was able in principle to control every variable (i.e. its position in an idealized phase-space), as well as its value, without affecting it. It is clear that when it comes to pure adventure, modern physics has provided a far more thrilling ride than modern mathematics.

Language is another one of Manin's preoccupations. Are humans defined by language? In fact, are we simply social creatures to whom individuality is just a consequence? In other words individual consciousness would be impossible without a shared language? Karl Popper claims that sociology is not applied psychology, that it predates psychology and, in fact, is a prerequisite for it. There is a natural evolutionary explanation for this, as various manifestations of social cohesion exist among other mammals. Clearly modern humans evolved from humanoid creatures in which a strong social cohesion was already present. Organisms are not just genes; they are also part of cultural traditions and contingencies. It takes more than genes to make up an organism. The extinction of a species also involves the rupture of a continuous tradition with roots going back to the beginning of life (and beyond), a tradition that cannot be encoded and which 'just happened to be'. In particular it is impossible to resurrect extinct animals simply by reviving their DNA. It is also this sense of language being something autonomous, subjective maybe from the point of view of humanity but objective as far as the individual, which makes sense of

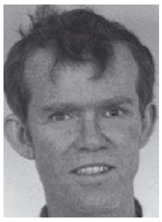
Jung's ideas of the collective unconscious, ideas that are nowadays usually scoffed at by Western intellectuals but which it apparently takes a Russian to sympathetically appreciate.

How are languages learnt? As Chomsky has famously suggested, it is not a question of a regular learning process but instead it is intrinsically hard-wired. The ideas of Chomsky are very charming but so far, I believe, impossible to pin down in any systematic way let alone on a neurological level. This does not mean of course that they are wrong but that they are more of a metaphor, in the sense of Manin, than an actual theory. Languages are products of brains and, ultimately, to understand languages we need to understand brains. We are indeed very far from doing so, probably even further away than we realize. There is, however, one very seductively interesting theory of the brain that has caught the public imagination, including that of Manin himself, namely the split up of the brain into left and right hemispheres, with radically different ways of functioning and the implication that our thinking process is actually an integration of both ways. One feature of human brains (as well as other brains?) is their stability. A computer can go haywire through a single mistake (and set off a nuclear holocaust?), while similar madness seems absent in humans. When madness is manifested it is compartmentalized and does not interfere with the working of the organism as such. The most pervasive form of cerebral dysfunction seems to be advanced Alzheimer's, in which the demented brain is eventually no longer able to direct basic functions. And even in dementia, core properties of the human psyche seem to be intact for a very long time. One explanation is the great plasticity of the brain, its ability to regenerate and allow psychic features to be re-established at alternative locations and, in fact, maybe render the notion of a brain geography moot – hence the attention such an idea has classically commanded. The metaphor of the human brain with the computer seems to be extremely misleading, although it is hard to think of an alternative one, let alone a better one.

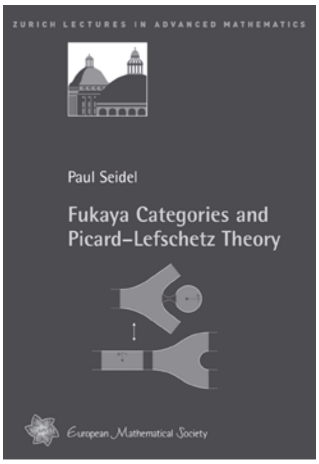
The brain itself, or at least the idea of it, is a creation of the human mind, leading to a loop out of which we seem unable to extricate ourselves. As Penrose notes in the introductory pages to his 'The Road to Reality': Only a tiny part of the brain is concerned with mathematics, only a small part of mathematics is applicable to the physical world, only a small part of the physical world is brain. And the loop goes on. But this is inevitable and we come against the kind of mysteries we may never hope to resolve. Science and rational thinking do not demystify the core of reality; they just point more accurately to where that mystery may be located.

The American Mathematical Society has done the Western mathematical community a great service in publishing a translation of these selected essays. It is a pity, though, that the selection is not complete. I can well imagine that many jewels are still inaccessible to non-Russian readers.

For a fuller version of this review see www.math.chalmers.se/~ulfp/Review/manin.pdf.



Ulf Persson [ulfp@chalmers.se] is a professor of mathematics at Chalmers University of Technology. He is an algebraic geometer but is also interested in philosophical aspects of mathematics. He is Chief Editor of the Newsletter of the Swedish Math Society (SMS) and the journal 'Normat' of popular mathematics, and also one of the editors of the EMS Newsletter.



Paul Seidel

Fukaya Categories and Picard-Lefschetz Theory

Zürich Lectures in Advanced Mathematics, European Mathematical Society (2008)
ISBN 978-3-03719-063-0

Fukaya categories is one of the main ingredients in the statement of Homological Mirror Symmetry (HMS), which is a mathematical formulation of mirror symmetry due to Kontsevich [K]. Mirror symmetry is a phenomenon discovered by string theorists, which, in its geometric incarnations, relates certain symplectic invariants (coming from what physicists call the A-model) to other invariants arising from algebraic or Kaehler geometry (coming from the B-model). A pair of manifolds X and Y, with X symplectic and Y projective, are said to be a mirror pair if the symplectic invariants of X are related to the algebro-geometric invariants of Y via mirror symmetry (this is a very rough definition: at least in some of the proposed mathematical formulations of mirror symmetry, mirror pairs do not consist of manifolds but of families of manifolds degenerating in some particular way; also, in the extension of mirror symmetry to Fano manifolds the projectivity condition is usually relaxed to quasi-projectivity and a so-called Landau-Ginzburg potential is included in the picture).

There have been several different attempts to translate the string theory statement of mirror symmetry into mathematically meaningful terms. Such translation is far from being obvious and even the precise definition of the symplectic and algebro-geometric invariants that are related by mirror symmetry is an extremely deep question, which is only a fraction of the problem of understanding the topological twists of A and B models in mathematical terms.

One of the fascinating aspects of mirror symmetry is that, in any of its mathematical interpretations, it relates invariants, which at first sight are of a completely different nature and which behave very differently; in addition, it quite often allows one to translate difficult computations on one side to routine computations on the other (the interested reader will find in [ABCDG-KMSSW] an updated and extremely useful account on mirror symmetry and its mathematical interpretation).

Perhaps the farthest reaching of the mathematical statements of mirror symmetry is HMS. This statement fits nicely with the point of view of Strominger, Yau and Zaslow [SYZ], as proved by Kontsevich and Soibelman [KS], and it conjecturally implies the isomorphism between the Frobenius manifolds constructed using Gromov-Witten invariants on the symplectic side and variations of Hodge structure on the algebraic-geometry side (see [KKP] for some very interesting progress on the latest question).

According to HMS, the relevant invariants on the symplectic side can be encoded in the so-called Fukaya category F(X) (which is not quite a category in the usual sense but an A_∞ category), whereas the algebro-geometric invariants are given by the derived category of coherent sheaves D^b(Coh Y) (HMS then states that the derived category of F(X) is equivalent to D^b(Coh Y)). Unlike the category D^b(Coh Y), which is a rather classical object in algebraic geometry, the Fukaya category F(X) has only recently been rigorously defined, in what has been one of the major achievements of symplectic geometry of the last few years (see [FOOO]).

The general definition of Fukaya categories is extremely complicated: it involves a great number of results and notions in homotopic algebra, symplectic geometry and geometric partial differential equations. This makes it a really demanding task to assimilate in some detail its definition and properties, a task that cannot be avoided if one wants to understand (not to mention work in) homological mirror symmetry.

This book by Paul Seidel is a wonderful contribution to reducing (as much as this can be done) the difficulty of this task. By considering a setting where the definition simplifies sensibly while still retaining most of its flavour (and encompassing many interesting examples – all affine varieties over C, for example), the author manages to give a very detailed definition of Fukaya categories in roughly 200 pages, assuming a minimum amount of reader knowledge. The remaining 100 pages of the book are devoted to developing new tools to compute the previously defined Fukaya categories using Lefschetz fibrations.

The book is divided into three chapters, the first two giving the definition of Fukaya categories and the third relating them to Lefschetz fibrations.

Chapter I introduces A_∞ categories, gives computational tools such as Hochschild cohomology and introduces two abstract constructions associated to A_∞ categories: twisted complexes (which can be used to extend an A_∞ category to another A_∞ category that is triangulated in an appropriate sense – in particular, it contains mapping cones of morphisms and its cohomology category is a triangulated category in the usual sense) and the A_∞ analogue of Karoubi completion, which extends an A_∞ category to make it split-closed (i.e. containing images of all idempotents). Combining the two constructions one associates to an A_∞ category A another A_∞ category $\prod$(Tw A); this will be applied later to the Fukaya category. The material introduced in this chapter can be addressed at different levels of abstraction, ranging from concrete and down-to-earth definitions to the language of model categories. The author has chosen to follow the first option, which has the great advantage of making the material very accessible to beginners (at the price of some dryness and the lost of the aerial and wider view provided by the most abstract perspectives).

Chapter II is devoted to symplectic geometry and the actual definition of Fukaya categories, using the algebraic notions and tools developed in Chapter I. The author considers only exact symplectic manifolds (i.e. those symplectic manifolds whose symplectic form is equal to $d\theta$ for some 1-form θ, which implies that X is not compact), and defines Fukaya categories by looking only at exact Lagrangian manifolds (i.e. those to which θ restricts to an exact 1-form). The definition and some of the main properties of the moduli space of discs with markings on the boundary are recalled. Gradings of Lagrangian manifolds are also very carefully explained and many details are provided on the definition of higher order products in the Fukaya category using moduli spaces of pseudoholomorphic discs (in particular, the index theory related to gradings of Lagrangian manifolds). Whenever some argument is not explained in detail (this is the case mainly of the analytic and compactness properties of the moduli spaces) a very clear sketch and appropriate references are given, allowing those who wish to understand such arguments in depth to find their way. Finally, the author explains how the Fukaya category of a surface can be described in purely combinatorial terms (this has been known for some time but the author provides many useful details on the description that were not previously in the literature).

Chapter III presents the main new results contained in the book. The setting considered is that of an exact symplectic manifold X with a structure of Lefschetz fibration $\pi: X \to S$. The main problem addressed is that of relating the Fukaya categories of X and a generic fiber X^1 of π when S is the disc. This relation is obtained by constructing a category that interpolates between the two Fukaya categories and is defined by appropriately counting pseudoholomorphic sections of π. A very important result obtained in this context is the identification, conjectured by Kontsevich, of the action of the monodromy around a singular value of π (geometrically, a Dehn twist along the vanishing cycle, which is a Lagrangian sphere) with the twisting construction in the context of A_∞ categories.

The main theorems in Chapter III are proved under the assumption that X is an affine variety and π an algebraic map, subject to some restrictions (in particular, the canonical bundle of X is assumed to be trivial). Theorem A states that $\prod(\mathrm{Tw}\ F(X^1))$ is split generated by the vanishing cycles in X^1 and this is refined by iteration to give an algorithm that eventually provides a combinatorial description of $\prod(\mathrm{Tw}\ F(X^1))$. Theorem B states that a certain category constructed using the vanishing cycles contains a full subcategory quasi equivalent to F(X). Theorem B implies that F(X) is quasi isomorphic to a dg category (i.e. an A_∞ category with vanishing products of order >2) with finite dimensional spaces of homomorphisms. In these results it is assumed that the homs in the relevant categories are vector spaces over a field of characteristic different from 2; this is apparently a technical restriction, imposed by the technique used in the proofs, which relies on an argument on ramified two-coverings.

Despite the highly technical nature of many of the notions appearing in it, the author has managed to write a very clear book, which is very pleasant to read. The reader will find in it, apart from precise statements, many clarifying remarks explaining the philosophy behind the definitions and a great many historical references. Perhaps many readers will miss, after reading the book, finding some more explanations on the applications of mirror symmetry and the computations of Fukaya categories; however, including these explanations in some detail would have made this book much longer and the author provides a long list of references where the reader will be able to find such applications.

The book is definitely recommended to those willing to learn about Fukaya categories.

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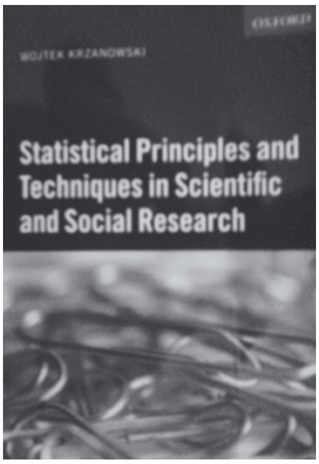

Wojtek J. Krzanowski

Statistical Principles and Techniques in Scientific and Social Research

Oxford University Press, New York, 2007
241 pages
ISBN 978-0-19-921310-8

The book "Statistical Principles and Techniques in Scientific and Social Research" written by Professor Wojtek Krzanowski is an original textbook in statistics. The author aims and succeeds quite well in avoiding the more technical developments, a feature common to many books in statistics, although he covers topics apparently impossible to present without complex mathematical tools. What particularly sets the book apart at this level is the balance between a pragmatic description of the statistical principles and the appropriate mathematical fundamentals of the statistical techniques.

In a compressed, relatively non-technical guide in statistical reasoning and techniques, the author emphasises the usefulness and the arguments behind a collection of statistical techniques, mainly from inferential/Bayesian statistics, linear/generalised linear models and multivariate data analysis. The last chapter briefly reviews some other special topics, such as spatial statistics, survival analysis, analysis of extremes and time series. The book is primarily concerned with the underlying philosophy behind statistical techniques rather than with their rigorous mathematical foundations.

In a text readable even by a novice in statistics, the first three chapters summarise the basic concepts from probability and sampling theory, needed for a good understanding of statistical techniques, including the total probability theorem, Bayes' theorem, random samples and summary statistics. The common probability distributions are described; various possible motivations and real-life situations are indicated for each population model. The information in these chapters is accessible to a large category of investigators including students in any field of the social sciences.

The framework of the frequentist and Bayesian approaches to statistical inference are discussed in two separate chapters. The frequentist approach to statistical inference includes most of the classical topics such as sampling distributions, the point/confidence interval estimation of an unknown parameter, the process of hypothesis testing and the connection between the test and the confidence interval. There is good motivation for the Bayesian approach to statistical inference and a useful parallel between the two approaches. Special attention is given to Bayesian inference, encouraging the readers to get a deeper insight into this topic, which is not as popular as the frequentist approach.

Chapters 6 and 7 cover linear models and their assumptions, also looking at generalizations that allow deviations from these essential assumptions, such as non-linearity of the model, non-constant variance, non-normality and non-independence. The most interesting part is related to generalized linear models, when the values of the response variable are permitted to come from any distribution in the exponential family. There are also some important topics not mentioned here, applied for example in economics and covered by econometrics textbooks.

The most comprehensive chapters, namely Chapters 8 and 9, are concerned with statistical techniques from multivariate data analysis. The emphasis is still on the rationale and the objectives of the techniques rather than on the mathematical details, the author explaining when a particular method should or should not be used. The most popular methods applied to multivariate data are described, without ignoring the necessary mathematical foundation, e.g. latent variable models, factor analysis methodology, principal component analysis, canonical variate analysis, canonical correlation analysis, cluster analysis and discriminant analysis. These chapters are very well-written. The author goes more deeply into the mathematical fundamentals of the methods. As a result, the chapters are a useful guide for researchers and students who want to apply multivariate data analysis in their investigations.

The author devotes a separate chapter to a summary of the basic ideas behind other interesting branches of statistics, such as spatial statistics, survival analysis, analysis of extremes and time series. The reader may find this part too short, having in view the complexity of the statistical methodologies and some recent developments related to these topics. A detailed textbook written in a similar manner, as a non-technical guide concerned with a practical approach of these branches of statistics, would be welcome.

Knowledge of statistical principles is essential not only to those who produce statistics but also to those who receive statistical information, in order to reduce the risk of incorrect understanding. The first four chapters of the book have a high accessibility and can be followed by any reader interested in making correct interpretations of reported statistics. But a complete understanding of the following chapters, which are focused on statistical techniques, requires some basic prerequisites in mathematics, especially in probabilities.

In conclusion, the author has written a valuable guide, useful to many researchers, students, teachers and investigators interested in applying statistics, especially inferential statistics and multivariate data analysis, to their investigations.

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Personal column

Please send information on mathematical awards and deaths to Dmitry Feichtner-Kozlov (dfk@math.uni-bremen.de).

Awards

The 2010 **Oswald Veblen Prize in Geometry** was awarded to **Tobias H. Colding** (MIT) and **William P. Minicozzi II** (John Hopkins University) for their profound work on minimal surfaces and to **Paul Seidel** (MIT) for his fundamental contributions to symplectic geometry and, in particular, for his development of advanced algebraic methods for computation of symplectic invariants.

The 2010 **E. H. Moore Research Article Prize** was awarded to **Sorin Popa** (UCLA) for his article "On the superrigidity of malleable actions with spectral gap", *J. Amer. Math. Soc.* 21 (2008), no. 4, 981–1000.

The 2010 **David P. Robbins Prize** was awarded to **Ileana Streinu** (Smith College and UMass Amherst) for her paper "Pseudo-triangulations, rigidity and motion planning", *Discr. Comp. Geom.* 34 (2005), no. 4, 587–685.

Sergey Tikhonov of the Catalan Institution for Research and Advanced Studies (ICREA) and Centre de Recerca Matemàtica (CRM, Barcelona) has received the **ISAAC award by the International Society for Analysis, its Applications and Computation** for his work on Fourier analysis and approximation theory.

The Norwegian Academy of Science and Letters has decided to award the **Abel Prize** for 2010 to J**ohn Torrence Tate**, University of Texas at Austin, for his vast and lasting impact on the theory of numbers.

The **Klein Award** goes to **Gilah C. Leder** (La Trobe University, Bundoora, Victoria, Australia) in recognition of her more than 30 years of sustained, consistent and outstanding lifetime achievements in mathematics education research and development.

The **Freudenthal Award** goes to **Yves Chevallard** (IUFM, Aix-Marseille, France) in recognition of his foundation and development over the last two and a half decades of a very original, fruitful and influential research programme in mathematics education.

The Clay Mathematics Institute (CMI) announces that **Grigoriy Perelman** of St. Petersburg, Russia, is the recipient of the **Millennium Prize** for resolution of the Poincaré conjecture. For more information, see http://www.claymath.org/millennium/.

The 2010 **Georg-Cantor-Medaille** of the German Mathematical Society was awarded to Professor **Matthias Kreck** (Hausdorff Research Centre, Bonn, Germany) in recognition of his outstanding contributions in topology.

The 2010 **Wolf Foundation Prize in Mathematics** was jointly awarded to **Shing-Tung Yau (Harvard** University), for his work in geometric analysis that has had a profound and dramatic impact on many areas of geometry and physics, together with **Dennis Sullivan** (Stony Brook University), for his innovative contributions to algebraic topology and conformal dynamics.

Tomasz Downarowicz (Wrocław) was awarded the **Banach Main Prize** of the Polish Mathematical Society.

Professor **Themistocles M.Rassias** (Athens) was awarded the Honorary Doctor Degree of the University of Nis (Serbia) on March 5, 2010.

Ryszard Rudnicki (Katowice) was awarded the **Steinhaus Main Prize** of the Polish Mathematical Society.

Michał Kapustka (Kraków) was awarded the Prize of the Polish Mathematical Society for young mathematicians.

Deaths

We regret to announce the deaths of:

M.V. Bodnarescu (Germany, 25 October 2009)
Paul Germain (France, February 2009)
Fritz Grunewald (Germany, 24 March 2010)
John Hall (UK, 24 August 2008)
Bob Hart (UK, 13 December 2009)
Abraham Hochman (Spain, 30 June 2008)
Benjamin Epstein (Israel, 22 December 2004)
Jurgen Hurrelbrink (US, 13 March 2009)
Johannes Jisse Duistermaat (The Netherlands, 19 March 2010)
Witold Nitka (Poland, 1 April 2010)
Andrzej Pelczar* (Poland, 18 May 2010)
Pierre Samuel (France, 23 August 2009)
Benjamin L. Schwartz (Austria, 7 March 1998)
Olof Thorin (Sweden, 14 February 2004)
Eckart Viehweg (Germany, 29 January 2010)
David Edward Williams (UK, 6 December 2009)
Richard Wort (UK, 15 December 2009)
Shaun Wylie (UK, 2 October 2009)

* Just as we went to press, we heard that Andrzej Pelczar died of a heart attack on 18 May. A strong supporter of the EMS, he was a Vice-President of the Society for many years. A fuller appreciation will appear in the next issue.