NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

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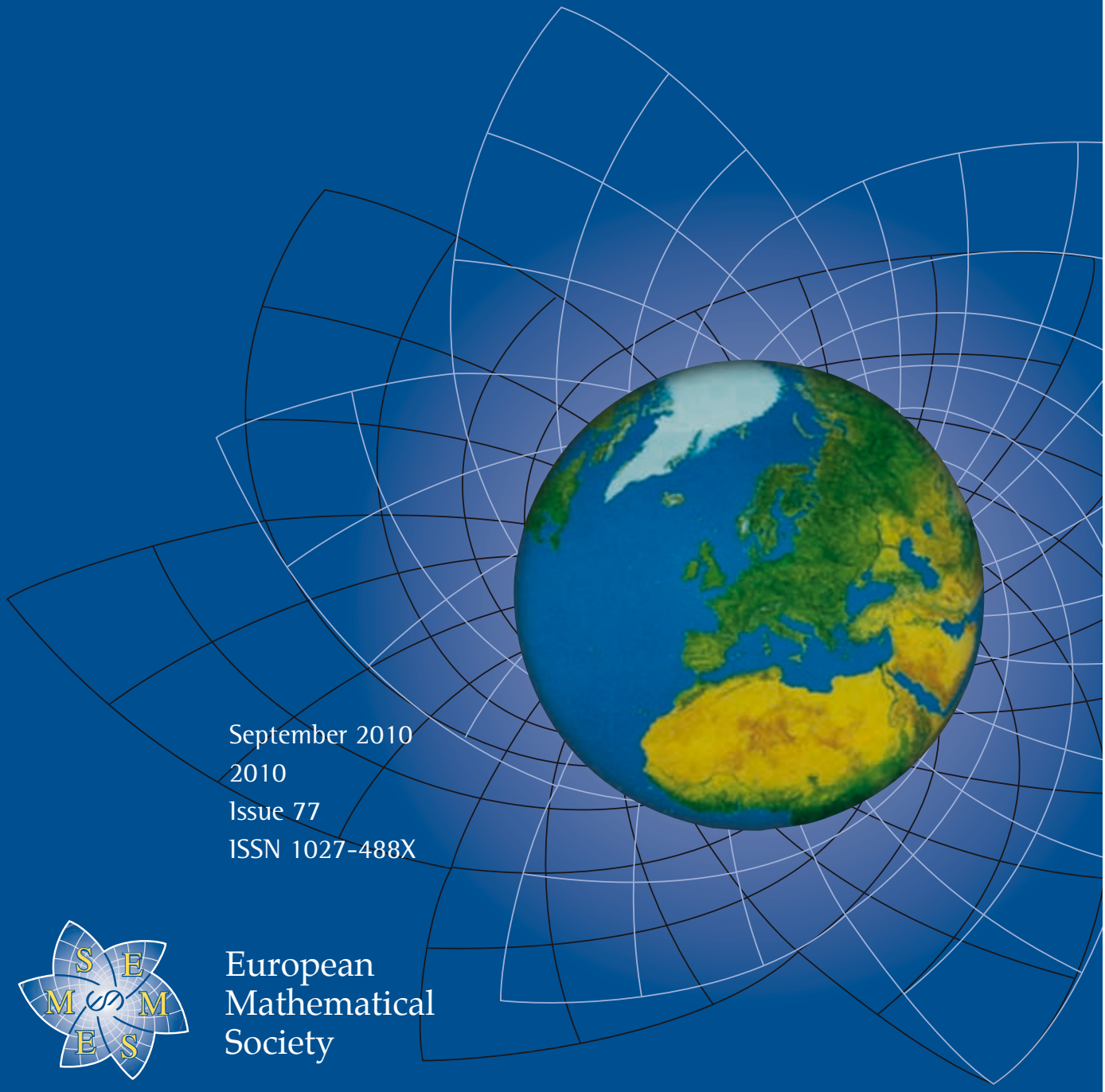

September 2010
2010
Issue 77
ISSN 1027-488X

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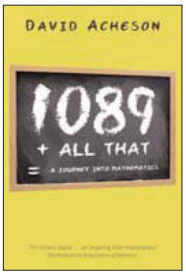

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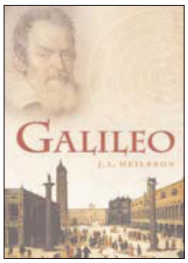

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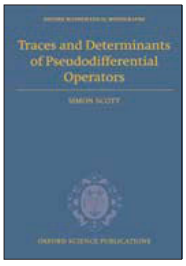

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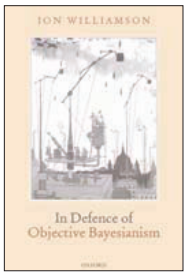

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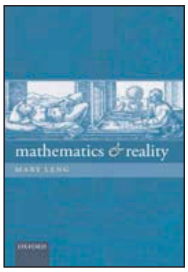

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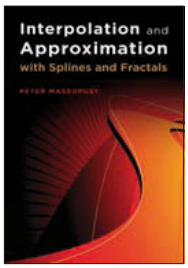

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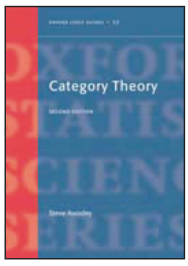

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- EMS Newsletter

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European Mathematical Society

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The views expressed in this Newsletter are those of the authors and do not necessarily represent those of the EMS or the Editorial Team.

ISSN 1027-488X

© 2010 European Mathematical Society

Published by the
EMS Publishing House
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CH-8092 Zürich, Switzerland.
homepage: www.ems-ph.org

For advertisements contact: newsletter@ems-ph.org

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EMS Agenda

2010

6–11 September
EMS-CIME summer school
Topics in mathematical fluid-mechanics, Cetraro, Italy
Contact: cime@math.unifi.it

12–17 September
ESF-EMS/ERCOM Conference on
"Highly Oscillatory Problems: From Theory to Applications"
Cambridge, United Kingdom
www.esf.org/index.php?id=6532

10–18 October
EMS-IMPAN School in Applied Mathematics, Będlewo, Poland
www.impan.pl/BC/Program/conferences/10Industrial.html

1 November
Deadline for submission of material for the December issue of the EMS Newsletter
Vicente Munoz: vicente.munoz@mat.ucm.es

13–14 November
EC Meeting, Lausanne, Switzerland
Stephen Huggett: s.huggett@plymouth.ac.uk

2 December
Final Conference of the Forward Look
"Mathematics and Industry", Bruxelles

2011

1 February
Deadline for submission of material for the March issue of the EMS Newsletter
Vicente Munoz: vicente.munoz@mat.ucm.es

18–22 July
ICIAM 2011 Congress, Vancouver, Canada
www.iciam2011.com

2012

2–7 July
6th European Mathematical Congress, Kraków (Poland)
www.euro-math-soc.eu

mathematics-in-europe.eu

Ehrhard Behrends, Chair of the rpa committee

Ehrhard Behrends

Public awareness

A large number of editorials have been published in the Newsletter of the EMS in which the importance of public awareness activities has been stressed. There is no need to repeat those arguments here.

A subcommittee of the EMS – the raising public awareness (rpa) committee – was founded several years ago to improve public understanding of mathematics and a number of projects have been realised since then.

The Diderot forums have been particularly successful, taking place simultaneously in a number of European cities and including activities such as lectures, panel discussions and exhibitions. Each Diderot forum was devoted to a specific aspect of mathematics (e.g. "Mathematics and Music").

In addition, an attempt was made to organise competitions to identify the best popular mathematical articles in the media. But despite Herculean efforts to encourage colleagues and journalists to submit contributions, the committee received only a few.

A popular mathematical webpage

A new idea arose at the beginning of 2009: the rpa committee was asked by the Executive Committee to launch a popular mathematical webpage under the auspices of the EMS – www.mathematics-in-europe.eu.

It was hoped that this domain would address everyone with an interest in mathematics. It would also make available rpa material developed in European countries and provide a forum for the exchange of ideas.

Initial suggestions for the structure of such a domain were discussed at a meeting of the committee in Brussels (May 2009). The basic language would be English but the majority of the articles would be presented in other European languages as well.

After negotiations in the summer, a sponsoring agreement with the Munich RE (a large German reinsurance company) was signed at the end of 2009. (For more details see the author's article in Newsletter 75, March 2010.)

Since then, much work towards realisation of www.mathematics-in-europe.eu has been undertaken.

One of the first important decisions concerned the design. The committee accepted the proposal of the graphics agency Iser&Schmidt (Bonn, Germany). The key idea is to have pictures at the top of the screen that play with the motifs "circles in nature" and "the digits of Pi". Each day, another picture, chosen at random, will be presented.

The graphic design has already been transformed into a real webpage with a preliminary address - it will serve as our working platform in the coming months.

A considerable quantity of material has already been collected. These articles will find their way to the various subpages of www.mathematics-in-europe.eu: information, popularisation activities, competitions, mathematical help, mathematics as a profession and miscellaneous.

The webpage will be advertised at the ICM in Hyderabad. A large poster has been prepared that will be presented at the booth of the IMU. The actual state of www.mathematics-in-europe.eu on the Web, however, will be rather sketchy at that time. The official launch is scheduled for the opening of an international rpa conference in Óbidos (Portugal) at the end of September 2010 (see www.cim.pt/RPAM).

The dream …

Those of us who are working for the realisation of www.mathematics-in-europe.eu have a vision. Our ambitious aim is to create a webpage that provides interesting information about mathematics for many groups.

For example: schoolchildren will find help if they fail to understand a particular mathematical topic in class; students can prepare for visiting a university where mathematics is spoken in a different language; teachers will find suggestions on how to present special aspects of mathematics in an attractive way; and from colleagues who want to be active in the popularisation of mathematics we will collect ideas for projects that have been realised successfully in other countries.

We hope that the domain will have extensive name recognition in Europe rather soon and that this will increase the visibility of the EMS and attract additional sponsors to support our work.

The realisation

An Italian proverb says "Tra il dire e il fare c'è di mezzo il mare": there is often a huge gap between first ideas and a satisfactory realisation. In order to bridge this gap as quickly as possible, the help of many will be necessary.

Indeed, we are asking *you* who are reading this editorial to collaborate. "What can I do?" you may ask. You could send us one (or some) of your popular articles or help us with the translation of the menus into your language or with the creation of a database of rpa activities in European countries.

While this is a rather general desire, there is also a more concrete one. On the subpage "Mathematical help" we will provide a mathematical dictionary, where one can

Munich RE

find basic mathematical terms in all the European languages. Here one can learn not only, for example, that the German "Körper" is equivalent to the English "field" but also how expressions such as a/b and n! are correctly pronounced.

The webpage will need a large matrix containing the terms and translations to fulfil this task. The English column of mathematical items has already been prepared and some further columns (Croatian, French, German, Italian, Polish and Turkish) have been supplemented by members of the rpa committee. However, not all European languages are represented and thus we would be grateful if you could provide a column in your language corresponding to the column with the English terms.

Please send me an email to behrends@mi.fu-berlin.de if you are willing to assist us!

European Congress of Mathematics – Call for Bids for the 7ECM

Outline bids from mathematicians to organize the 2016 Congress are now invited, and should reach the EMS Secretariat by 28 February 2011. The address of the Secretariat is Ms Terhi Hautala, Department of Mathematics & Statistics, P.O. Box 68 (Gustaf Hällströmink. 2b, 00014 University Helsinki, Finland (Phone: +358 9 1915 1507, Fax: +358 9 1915 1400; Email: terhi.hautala@helsinki.fi).

The information below may be helpful to possible organizers. Informal discussions are welcomed, and may be addressed to the Secretary Stephen Huggett (e-mail: s.huggett@plymouth.ac.uk) or to the President of the EMS.

General information about ECMs

European Congresses of Mathematics are organized every four years. The first Congress was held in Paris in 1992, and since then they have been held in Budapest, Barcelona, Stockholm and Amsterdam. In 2012 the Congress will take place in Kraków.

Experience of previous Congresses suggests that around a thousand people may attend. The duration has so far been five days. Ten EMS Prizes are awarded to outstanding young European mathematicians at the opening ceremony, together with the Felix Klein and Neugebauer Prizes. The Congress programme should aim to present various new aspects of pure and applied mathematics to a wide audience, to offer a forum for discussion of the relationship between mathematics and society in Europe, and to enhance cooperation among mathematicians from all European countries. The standard format of previous ECMs has been:

- about 10 plenary lectures;
- section lectures for a more specialized audience, normally with several held simultaneously;
- mini-symposia;
- film and mathematical software sessions;
- poster sessions;
- round tables.

An exhibition space for mathematical societies, booksellers, and so on is required. No official language is specified and no interpretation is needed. Proceedings of the Congress are published by EMS Publishing House (http://www.ems-ph.org/).

Decision process for 7ECM

(i) Bids are invited via this notice in the EMS Newsletter, and via letters to the EMS member societies; the deadline for bids is 28 February 2011. These bids need only be outline bids giving a clear idea of the proposal and possible sources of financial and local support.

(ii) The Executive Committee (EC) of the EMS will consider the bids received. It will invite one or more of the bids to be set out in greater detail so that it can decide which bids are sufficiently serious options to be considered further. The deadline for such "worked up" bids, which will include a draft budget and a commitment to accept the conditions imposed by the Executive Committee, is 30 July 2011.

(iii) The EC will then create a short-list of sites that appear to offer the best possibilities for a successful Congress, and appoint a Site Committee to visit the short-listed sites between September and December 2011 to check a range of items in connection with the development of the Congress. For example:
- Size and number of auditoriums; location and equipment
- Room for exhibitors
- Hotel rooms and dormitories; location, prices, number in different categories and transportation to lectures
- Restaurants close to Congress site, number and prices
- Accessibility and cost of travel from various parts of Europe
- Financing of the Congress; support to participants from less favoured countries, in particular
- Financing for the EMS Prizes
- Experience in organizing large conferences
- Timing of the Congress
- Social events
- Plans to use the occasion of the Congress as publicity for mathematics

The EC may make a recommendation to the Council based on the report of the Site Committee.

(iv) In 2012, at the EMS Council prior to 6ECM, a decision will be reached.

Relations between the EC and the Organizing Committee of 7ECM

The actual Congress organization is the responsibility of the local organizers. At least two committees must be appointed: the Scientific Committee and the Prize Committee. The Scientific Committee is charged with the responsibility for conceiving the scientific programme and

selecting the speakers. The Prize Committee is charged with the responsibility of nominating the EMS Prizewinners. For each of these committees the Chairs are are appointed by the EC. The members of the committees are suggested by the Chairs and approved by the EC. The EC must be kept informed by the Chair of each committee about the progress of their work.

The local organizers are responsible for seeking financial support for the Congress and for the meetings of its committees. However, the EMS will provide some financial support for the travel of Eastern European mathematicians to the ECM, and will also assist and advise in seeking sources of funding.

The level of the registration fee is of great importance to the success of an ECM, and the EC must be involved before a final decision on the level of fees is made; members of the EMS normally receive a reduction of some 20% on the registration fees. The EC would be pleased to offer advice to the local organizers on matters such as the scientific programme, the budget, registration, accommodation, publications, web site, and so on, but in any case the EC must be kept informed of progress at its regular meetings. Publicity for the ECM via the EMS Newsletter and the EMS site, http://www.euro-math-soc.eu/ should appear regularly.

Ari Laptev and Stephen Huggett

Report on a cancelled EMS meeting

or The effect of the volcanic ash cloud on mathematics

Vasile Berinde, EMS Publicity Officer

On the occasion of the centenary of the Society "Gazeta Matematică" – one of the forerunners of the Romanian Mathematical Society (RMS) – celebrated as the centenary of the RMS itself, that was held in Bucharest on 17 April 2010 (see the EMS Newsletter article) the EMS planned to organise a meeting of the presidents of the (national) mathematical societies in Europe that are (or will soon be) institutional members of the EMS.

This series of meetings was initiated in 2007 by the current EMS President Ari Laptev. The first meeting in the series was hosted by CIRM (Centre International pour Rencontres Mathématiques), Luminy, France, 26–27 April 2008, while the second was held at the Banach Center in Warsaw, 9–10 May 2009. So, the Bucharest meeting, planned to be hosted on the Campus of University "Politehnica", would have been the third one in this series, if it had been held. But, unfortunately, it wasn't – at least in the originally planned format.

Fifty-five presidents, vice-presidents, secretaries and other representatives of 40 mathematical societies in Europe, from 33 European countries, confirmed their participation at the EMS meeting of presidents in Bucharest. The countries represented were Albania, Austria, Azerbaijan, Belgium, Bosnia-Herzegovina, Bulgaria, Croatia, Czech Republic, Cyprus, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Latvia, Lithuania, Macedonia, Moldova, Montenegro, Norwey, Netherlands, Poland, Portugal, Serbia, Slovakia, Slovenija, Spain, Sweden, Turkey and United Kingdom.

The EMS itself was to be represented at the highest level by: Ari Laptev (*President*), Pavel Exner and Helge Holden (*Vice-presidents*), Stephen Huggett (*Secretary*), Jouko Vaananen (*Treasurer*), Terhi Tuulia Hautala (*from the Helsinki EMS Secretariat*) and Susan Oakes.

Due to the almost closed air traffic in Europe at the time preceding the meeting, only 20 of the total 55 foreign registered participants eventually arrived in Bucharest. They represented the following 17 national mathematical societies: Albania, Azerbaijan, Belgium, Bosnia-Herzegovina, Bulgaria, Czech Republic, Cyprus, Israel, Latvia, Macedonia, Moldova, Montenegro, Poland, Portugal, Romania, Slovakia and United Kingdom.

For most of the rest of the registered participants it was impossible to fly to Bucharest and some few others (e.g. the representatives of Greece, Turkey and MASSEE) cancelled their trip soon after they got the circular message from Ari Laptev, sent to all registered participants on Friday 16 April at 11:00, local hour in Bucharest, to announce the cancellation of the meeting itself.

A special case was with Harry Dim, President of the Israel Mathematical Society, who, immediately after he met the RMS president Radu Gologan at the Bucharest airport and found out that the meeting was already cancelled, decided instantly to go back to Israel with the same aircraft, a few hours before the airport was totally closed!

Under these unexpected circumstances, the local organisers of the meeting in Bucharest proceeded as initially scheduled and so, on Friday 16 April, 16.00–19.00, the planned Round Table "Current trends in mathematics education in Europe" attracted about 40 people.

Because, after the anniversary ceremony of the RMS held at the Romanian Academy on the morning of Saturday 17 April, about 30 people (including a few Romanian participants) were prepared to attend the EMS meeting

Group photograph

Professor Muharem Avdispahic introducing the Bosnian Mathematical Society

of presidents, the local organisers decided to also have an informal meeting of the presidents in the afternoon, from 16.00 to 19.00. Of the initial agenda of the meeting, we went first to Section B: Introductions, which lasted exactly two hours. The following briefly presented their societies: Ervisa Kosova (Albanian Mathematical Society, Secretary); Fikret Aliev (Azerbaijan MS, Vice-president); Stefaan Caenepeel (Belgian MS, President); Muharem Avdispahic (Bosnian MS, P); Stefan Dodunekov (Bulgarian MU, P); Andrejs Reinfelds (Latvian MS, P); Vesna Manova-Erakovik (Macedonian Union of Math, P); Mitrofan Choban (Moldova MS, P); Stefan Jackowski (Polish MS, P); Miguel Abreu (Portuguese MS, VP); Roman Nedela (Slovak MS, P); Angus MacIntyre (London MS, P); and Doru Stefanescu (MASSEE, P). Gregory Makrides, President of the Cyprus Mathematical Society, who attended the Round Table on the previous day where he had a presentation, missed this informal meeting in his desperate attempt to catch a last flight leaving from Bucharest airport before it totally closed! Note that the RMS was not more presented in this session because there were long stories about its history during the ceremony in the morning on the same day.

Under Section E of the agenda, Stefan Dodunekov also made a presentation about the website, registration facilities and other details related to the Council Meeting in Sofia, 10–11 July, and the conference Mathematics and Industry following it. Professor Vasile Brânzănescu, Director of the Institute of Mathematics of the Romanian Academy, gave a short presentation on the main activities of this important research institution.

In the final part of the meeting, some advertising presentations on the EMS were given in order to convince the presidents and vice-presidents present in Bucharest to encourage the members of their societies to join the EMS. In this context, the Albanian MS and the Moldova MS were also invited to apply for EMS membership.

In the last 30 minutes (19.00-19.30) before the start of the Official Dinner, a joint meeting of the Executive Committees of the Romanian MS and the Moldova MS was organised in order to discuss some mutual collaboration matters.

On the morning of Sunday 18 April, instead of the originally scheduled second part of the meeting of presidents, a visit in Bucharest was organised for those participants who did not leave by train (some few did!) to their countries and therefore had to spend some extra days until the re-opening of airports in Europe.

This is the brief story of one of the dramatic effects of a volcanic ash cloud on mathematics…

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The Centenary of the Romanian Mathematical Society

Vasile Berinde (Baia Mare, Romania)

On 17 April 2010, the centenary of the society "Gazeta Matematică" – one of the forerunners of the *Societatea de Științe Matematice din România* (Romanian Mathematical Society, RMS) – has been celebrated as the centennial of the RMS itself (the other forerunner, the society "Friends of Mathematical Sciences" was founded earlier in 1894). In conjunction with this event, an EMS meeting of the presidents of mathematical societies in Europe has been scheduled (see the article *Report on an EMS cancelled meeting or The effect of the volcanic ash cloud on mathematics* published in this issue of the Newsletter).

The ceremony was organised at the Romanian Academy Aula and was attended by several important Romanian mathematicians, representatives and members of the regional branches of the RMS, the Presidents of the Romanian Physical Society and the Romanian Chemistry Society and 20 representatives of the mathematical societies in Europe, who succeeded in landing in Bucharest for the EMS meeting of presidents despite the volcanic ash cloud.

The first part of the ceremony was devoted to a brief presentation of the most important moments in the history of the RMS and also for accepting the acknowledgements from other societies, while the second part was allocated to the awarding of centennial medals and diplomas to several mathematicians who essentially contributed to the organisational activity of the RMS or the development of mathematical research and mathematical education in Romania in the past decades.

Considering the two articles on the old and middle history of the RMS that have been published in the EMS Newsletter (see [2], [4]), we shall give here a brief account on the current activity of the RMS.

Goals of the society and membership

The main current goal of the RMS is to help maintain a high level of mathematics education and mathematical research in Romania to continue the good work done over the 20th century. The RMS is a non-profit organisation created under Romanian law in 1910 and re-established, after the breakdown of communist rule, in 1990. It consists of individual members only. Over its 100 years of existence the RMS has grown up to a society with more than 5000 members, university professors, teachers, other professionals from research institutes and companies, PhD students, etc., all linked by their interest in mathematics and/or mathematics education.

Structure and activities of the society

The Council of the RMS, which consists of 91 members, is elected every four years during the General Assembly and establishes the general policy of the society. It meets every year, usually in December, and also elects the members of the Executive Committee (President, Prime Vice-president, two Vice-presidents, General Secretary, Secretary and nine Ordinary Members), to which the activity of the society between the council meetings is entrusted. The current President or the RMS is Professor Radu Gologan from "Politechnica" University of Bucharest and the Mathematics Institute of the Romanian Academy. The RMS is also organised on a territorial basis, its units being the local and regional branches (Filiale) of the RMS (there are 54 branches).

Meetings, publications and other activities

The RMS organises an annual National Olympiad (in collaboration with the Ministry of Education; the 61st edition took place in 2010), the RMS Annual Conference, the Summer Contest "Gazeta Matematica" for the best problem solvers and the summer school for teachers of mathematics, as well as other meetings, workshops and conferences.

The following three journals are published by the RMS: 1) *Bulletin Mathematique de la Societe des Sciences Mathematiques de Roumanie*, which is a quarterly research journal (4 issues a year). It was founded in 1896 and the current volume is **53** (or **101** – old series) in 2010; 2) *Gazeta Matematica, seria B*, which is a monthly journal (12 issues/volume) devoted to elementary mathematics (primary, secondary and high school students and teachers). It was founded in 1895 and has been published continuously in this format. The current volume is 115 (2010); 3) *Gazeta Matematica, seria* A, which is a quarterly journal (4 issues/volume) devoted to teachers of mathematics. The current volume is 107 (or **28** – new series) in 2010.

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4. V. Berinde and M. Pacurar, The measure of a great idea: 50 years on from the creation of the International Mathematical Olympiad, *Eur. Math. Soc. Newsl.* 74 (2009), 15–18.
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An Appeal to Colleagues from the Committee for Developing Countries

The Committee for the Developing Countries (CDC) is one of 11 committees of the European Mathematical Society.

In the course of our work in developing countries in Africa, Asia and Latin America we have come across a number of talented and promising young mathematicians. Partly under supervision of our programs they have successfully completed their Master of Science degrees and should now proceed to doctoral studies. Through the CDC they are now looking for host institutions willing to enrol them in their PhD programs and possibly help them to find at least partial financing.

At the same time there are some talented young students who have been able to complete PhD degrees at their home universities. In our experience these students, in spite of their ambitions and enthusiasm, are not yet ready to be thesis advisors for younger colleagues. They need to complete their initiation to research, to mature as mathematicians and also to have a chance to broaden their vision of mathematics in general. This brings us to our second appeal to the mathematical community: creating possibilities of postdoc positions for young researchers from developing countries.

If your department would be interested in assisting at least one of these young mathematicians from developing countries, please contact Tsou Sheung Tsun (University of Oxford) tsou@maths.ox.ac.uk, Michel Waldschmidt (Université Paris VI) miw@math.jussieu.fr, Paul Vaderlind (Stockholm University) paul@math.su.se or any other CDC member.

We take the opportunity here to give a short presentation of the CDC. More information can be found on http://www.euro-math-soc.eu/comm-develop.html.

The committee consists of about ten members, most of whom are engaged in development work in their capacity as heads of various projects, e.g. CIMPA (France), ISP (Sweden) and IWR (Germany), and are also active in other committees for developing countries around the world, notably within the IMU, the ICIAM, the ICTP, the LMS and a number of national mathematical societies and academies.

The main objective of the CDC is to assist developing countries at the following levels:

- Mathematics curriculum development for schools and for universities.
- Cooperation with local staff in conducting MSc and PhD programs and holding special courses in various areas of mathematics in which there is no local expertise.
- Helping to build up libraries through donations from colleagues in developed countries, supplying mathematical literature on request from institutions and/or individual researchers in developing countries and negotiating with publishers on special book rates for developing countries.
- Helping to build up regional centres and networks of excellence – these are centres directly attached or connected in part to universities that provide expertise in areas and on levels in which regional universities are in need.
- Provide information about where students from developing countries (who already have an MSc) can do their PhD, and what possibilities for PhD grants exist. At the same time, in order to avoid a brain drain, as much as possible, the CDC will support efforts to build up PhD programmes in developing countries according to international standards (regional centres of excellence could serve this purpose).
- Provide information on postdoc positions for young researchers from developing countries and promote the creation of such positions.
- Mobilise funds for junior and senior researchers to attend conferences in developing countries and also help (both on an academic and financial level) organising conferences in developing countries.

On behalf of the EMS-CDC,
Tsou Sheung Tsun
Michel Waldschmidt
Paul Vaderlind

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Report from the Committee for Mathematics Education

Günter Törner (Duisburg-Essen, Germany)

The European Mathematical Society[1] (EMS), consisting of about 50 mathematical societies in Europe, 20 academic institutional members and many individual members, has a number of standing committees,[2] among them the Education Committee. This committee is working to bridge the gap between mathematics as a field of research and application and its teaching and learning in society.

A new Education Committee[3] was initiated by the EMS last autumn. Konrad Krainer (Austria) led it temporarily and organised a first meeting, Günter Törner (Germany) took over as chair from 01 January 2010, Despina Potari (Athens) was appointed as vice-chair by the Executive Committee of the EMS and the correspondent within the EMS Executive Board is Franco Brezzi[4] (Italy). It is worth noting that the committee has decided that it cannot take a particular stance with regard to specific policy issues since there are many differences in tradition, conditions, emphases and priorities in European countries. Positions and recommendations in favour of one approach over another would be inappropriate, as it would make the committee a partisan agent in a highly complex terrain.

The members of the committee are: Törner, Günter (Germany) (chair); Arzarello, Ferdinando (Italy); Dreyfus, Tommy (Israel); Gueudet, Ghislaine (France); Hoyles, Celia (Great Britain); Krainer, Konrad (Austria); Niss, Mogens (Denmark); Novotná, Jarmila (Czech Republic); Oikonnen, Jaho (Finland); Planas, Núria (Spain); Potari, Despina (Greece) (vice-chair); Sullivan, Peter (Australia) and Verschaffel, Lieven (Belgium).

The new committee wishes to encourage all mathematicians, mathematics related institutions and mathematics educators to communicate with the committee on issues of interest to European mathematicians and educators. The committee is interested in becoming a platform for the exchange of information, experience and views and to explore issues and problems of significance to European mathematics education and promote its development everywhere. The committee intends to be present at all major European and international conferences on mathematics education, e.g. at the next PME Conference, 18–23 July, Belo Horizonte, Brazil. We will also inform everyone about the committee and some of its first actions at the next CERME Conference in Rzeszów (Poland) in February 2011.

The committee had a very encouraging meeting at Athens, 11–14 June, which was organised by Despina Potari; every participant brought their expertise from their own culture. The first results will be presented at CERME 7. The committee has decided on an initiative to support the dialogue between mathematicians and mathematics educators, namely to identify a number of solid, well-established findings in mathematics education. Discussing this topic at our Athens meeting, we became aware that it is far from trivial to identify and rank findings from the last 50 years. Nevertheless, we are open for any message[5] offering us proposals for specific candidates. We expect to disseminate descriptions of these solid findings in appropriate publications that are read by the community of European mathematicians.

1 http://www.euro-math-soc.eu/.
2 http://www.euro-math-soc.eu/committees.html.
3 http://www.euro-math-soc.eu/comm-education.html.
4 brezzi@imati.cnr.it.
5 guenter.toerner@uni-due.de.

New book from the European Mathematical Society

European Mathematical Society Publishing House
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Hans Triebel (Friedrich-Schiller-Universität Jena, Germany)
Bases in Function Spaces, Sampling, Discrepancy, Numerical Integration
(EMS Tracts in Mathematics Vol. 11)

ISBN 978-3-03719-085-2. 2010. 305 pages. Hardcover. 17 x 24 cm. 58.00 Euro

The first chapters of this book deal with Haar bases, Faber bases and some spline bases for function spaces in Euclidean *n*-space and *n*-cubes. This is used in the subsequent chapters to study sampling and numerical integration preferably in spaces with dominating mixed smoothness. The subject of the last chapter is the symbiotic relationship between numerical integration and discrepancy, measuring the deviation of sets of points from uniformity.

This book is addressed to graduate students and mathematicians having a working knowledge of basic elements of function spaces and approximation theory, and who are interested in the subtle interplay between function spaces, complexity theory and number theory (discrepancy).

Andrzej Pelczar (1937–2010)

Andrzej Pelczar

The 6th European Congress of Mathematics will be held in Kraków in 2012. The idea of organising the congress in Kraków was due to Andrzej Pelczar, the Chairman of the organising committee of the congress. Sadly, the congress will now take place without him, as he died suddenly of a heart attack on 18 May 2010. He was in good shape and no health problems were indicated.

He worked actively in many areas. On the day before he died, a Fields Medallist, Shing-Tung Yau, visited Kraków and delivered a special lecture – the first *Łojasiewicz lecture*. Over supper after the lecture, Pelczar and Yau talked a lot. This turned out to be one of his last mathematical conversations.

It is impossible to describe all his achievements in this short note. We outline just the most important facts.

Andrzej Pelczar was born in 1937 and spent his childhood in Gdańsk. His father Marian Pelczar, an historian, received his PhD degree from the Jagiellonian University in Kraków before the Second World War; after the war, he worked in Gdańsk. Andrzej Pelczar's life and work were strongly connected with the Jagiellonian University from 1954, when he began mathematical studies. He received his PhD from there in 1964 under the supervision of Tadeusz Ważewski and in 1980 he was promoted to professor at the mathematics institute. On many occasions he emphasised how grateful he was to Ważewski.

The activities of Andrzej Pelczar may be divided into three main parts.

Firstly, he discovered many important results in the theory of differential equations and dynamical systems. Of particular importance are: his adaptation of the Ważewski retract method to the case of partial differential equations; the wonderful unification of stability theory in general dynamical systems; the creation of new methods of successive approximations and their applications; and some results on sufficiency for jets. Recently, he became interested in the history of mathematics. A couple of years' research resulted in a fully documented monograph on a 17th century Polish mathematician, Jan Brożek. Fourteen mathematicians wrote PhD dissertations under Pelczar's supervision.

He authored more than 80 scientific papers, several books (in particular, the two volumes of the monograph on differential equations, the first part written with Jacek Szarski), a popular book *Time and dynamics* and many articles about mathematics and academic problems for a general academic audience.

He was a very active teacher. At the Jagiellonian University, he gave lectures and classes on a wide range of topics. His lectures ranged from very advanced topics to elementary courses, including those for first-year students.

We must also mention the seminar he led for more than 35 years. During this seminar he was always a most careful listener and his accurate questions and comments gave new ideas on the results presented and inspired further research. He skilfully encouraged young researchers to pursue further work. His ability to find the true centre of a problem was admirable.

During the 1970s, Kraków lost several leading figures from its differential equations group. Ważewski, Opial and Szarski died, while Olech and Lasota moved to other cities. Andrzej Pelczar was the only person who could undertake the duty of leading the group and may now be considered as the creator of the Kraków group of dynamical systems.

Secondly, we write about his work for the Jagiellonian University and the entire Polish academic community.

Andrzej Pelczar with some of his former PhD students. From the left: M.Sobański, J. Tabor, A. Pelczar, J. Kłapyta, A. Marciński, R. Srzednicki, K. Ciesielski, J. Ombach.

From 1981 to 1984 he was Chairman of the Mathematical Institute at the Jagiellonian University and from 1984 to 1987 he was Vice-Rector, responsible for teaching and student problems. In 1987, he was elected Rector of the University. At that time Poland was ruled by the communist regime and the Rector's election had to be approved by the Government. Andrzej Pelczar was known to be an independent person and the Minister did not agree. In 1989, after the political changes, the universities were free to elect their Rectors. Pelczar was elected Rector in 1990 and held this position from 1990 to 1993. It was a difficult period for the university since after independence several new problems appeared.

Later on, from 1993 to 1996, he was Vice-Head of the Polish Council for Higher Education, and from 1996 to 2002 he was Head of this council. In those years the council perfectly represented the academic community in Poland and many enterprises were undertaken.

Several years ago he was elected a member of the Polish Academy of Arts and Sciences and took part in many of its activities.

In April 2010, he was awarded the title of Honorary Professor of the Jagiellonian University. This was a great honour, awarded to very few scientists.

Finally, we emphasise his work for the mathematical community in general. He played a great role in the Polish Mathematical Society. From 1975 to 1977 he was President of the Kraków Branch of the Society. In 1985, he was elected Vice-President of the parent society and from 1987 to 1991 he was President. From then until his death he was elected a member of the council of the society, being re-elected continuously year after year.

Andrzej Pelczar was also actively involved in the creation of the European Mathematical Society in the early 1990s. In 1996 he was the main organiser of the International Conference 'Topological Methods in Differential Equations and Dynamical Systems', a satellite conference of the Second European Congress of Mathematics in Budapest, celebrating the 100th anniversary of the birth of Tadeusz Ważewski. From 1997 to 2000 he was Vice-President of the society and he was the main organiser of the Kraków EMS Diderot Forum in 1998.

His wife Janina was also a mathematician. They studied together and Andrzej asserted several times that his wife was a better student than he was. She decided to devote her abilities to teaching at school. She died tragically in the Tatra mountains in 1995. They had two daughters; the younger one, Anna, is also a mathematician and works at the Jagiellonian University, specialising in functional analysis.

Andrzej Pelczar had a great personality. He was a man with a deep mathematical and general knowledge. He had a great sense of humour. He told stories in a very interesting way and many people enjoyed these stories. He was a fascinating companion, both at meetings and in private conversations. He had the ability to deliver speeches at important occasions but he was also an extremely good, helpful and honest person. One could always count on his help and support when this support was needed but it was also possible to differ with him on a topic and to discuss it without quarrelling.

All three of us were his pupils, writing our PhD theses under his supervision. As the years passed, our contact with him moved to another level, as he became not only our teacher but also a colleague and an older friend. Nevertheless, all this time we continued to have a great respect for him.

We still cannot believe that Andrzej has passed away and that we will have no opportunity to talk to him and work with him in the future.

Krzysztof Ciesielski
Jerzy Ombach
Roman Srzednicki

Letter of condolence

On behalf of the European Mathematical Society we would like to express our deep sadness on learning of the death of Professor Andrzej Pelczar. It is not only a tragic loss for his family and friends but also for the mathematical community. We all remember Andrzej for his generosity of spirit, human warmth and his unfailing interest in promoting the EMS. He was a remarkable person and a truly devoted member of the European Mathematical Society.

The EMS was founded in Mądralin, Poland, in 1990 and, as the President of the Polish Mathematical Society, Andrzej was actively involved in its creation. For many years Andrzej worked enthusiastically for the EMS, bringing his experience and balanced views to the service of the society. He was elected as an ordinary member of the EMS Executive Committee for the period 1993-1996 and from 1997 until 2000 Andrzej held the post of EMS Vice-President.

At the last EMS Council meeting in Amsterdam, the Council approved Professor Pelczar's proposal to organise the 6th European Congress of Mathematics in Kraków in 2012. The EMS congress in Krakow was his dream and he has worked passionately to ensure its success. We now all share the responsibility of making Andrzej's dream come true.

Andrzej will be greatly missed by everyone who knew him, particularly those who, like us, had the good fortune to work with him.

With our deepest sympathy,

Jean-Pierre Bourguignon	EMS President 1995–1998
Rolf Jeltsch	EMS President 1999–2002
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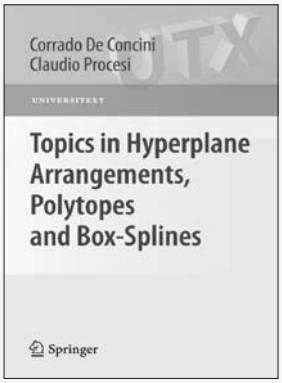

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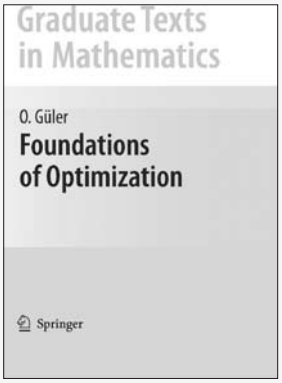

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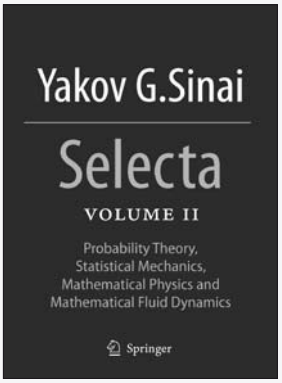

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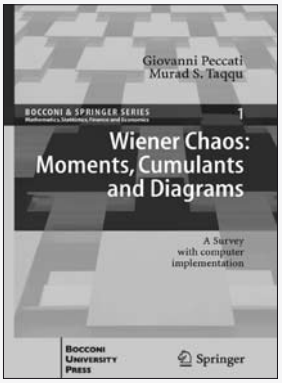

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ERC: Achievements and Challenges

Pavel Exner

It has been two years since I summarised the progress made in building the *European Research Council* and with all modesty I can say that it has not been a waste of time. In the summer of 2008, I was able to report the results of the first *Starting Grant* call; now we have successfully passed two *Starting Grant* calls and two *Advanced Grant* calls and have entered into a regular switching pattern between the two – the results of this year's *Starting Grant* should be announced by the time this issue is published (there has been a slight delay – nature has demonstrated her strength and some panel meetings had to be postponed due to the Eyjafjallajökull eruption).

The organisation keeps growing at the prescribed pace. Recently the ERC celebrated awarding its 1000th grant and next year the budget will for the first time surpass the billion euro threshold. For mathematics, in particular, in the first four calls 74 grants were awarded – the list of grantees in the last three calls is given below – which brought to our field about 74.8 million euros; the present *Starting Grant* will push this sum to the 90 million euro limit.

While money is – in the words of Robert Merle – the only freedom that is not purely theoretical, the ERC is much more than just a pecuniary establishment. In spite of its short history, ERC grants have won wide recognition. The grantees testify that their success in this competition gave them substantial career boosts in their own environments. And, needless to say, the European mathematical community is proud of them and wishes them success with their projects.

After the Scientific Council adjusted the rules following the flood of applications in the first call, we have witnessed a certain stabilisation. The number of submitted projects ranges between two and three thousand and the success rate is around 15%, which is probably the right number for grants that are supposed to be highly prestigious and competitive.

The success of the ERC in its first three years comes from the hard work of the many people involved. First of all, the topic panels should be mentioned because a high quality peer review is the crucial component of the ERC granting process. One set of panels served the Starting Grants and two others served the *Advanced Grant*s in a switching pattern. The mathematics panels were chaired by Jean-Pierre Bourguignon, Enrique Zuazua and Rolf Jeltsch; thanks go to them as well as to all the panel members.

Neither the Scientific Council nor the panels could achieve these results alone. An operation of this size needs a solid administrative background. In our case it was supplied by the ERC Executive Agency, which grew fast from a provisional structure into an efficient organisation with many competent and devoted people. True, there are complaints that the process has bureaucratically difficult moments but we work in a real Brussels environment and it is, to a large extent, the work of the people of the ERCEA that ensures that the ERC functions much better than other EU programmes.

While it is pleasing to see the achievements, it is more important to look at the challenges that lay ahead. Some of them resulted from a midterm evaluation that was previewed in the constituting act of the ERC. It was performed last year by a commission headed by the former Latvian president Vayra Vike-Freyberg. The commission greatly appreciated the work done but it also proposed numerous improvements of the system. One suggestion says that the Executive Agency should be headed by an accomplished scientist with a robust administrative experience in analogy with the practice of the NSF, the NIH and similar organisations. The choice is underway and the new director will assume the post in the autumn.

In the long run, the main challenge is the fate of the ERC after 2013 when the present Framework Programme expires. There is no doubt that anything other than growing gradually into a permanent, rock-solid structure would be regarded as a failure. Such a development, however, is by no means automatic, in particular because of the present financial crisis, as well as the existence of other programmes competing for the same resources, some of them with strong political support.

To keep the momentum the ERC has to strive for further progress. An important thing is to further develop funding instruments adding new schemes to the two existing grants. The work is ongoing but it is only likely to be finished by the next Scientific Council because this body, too, will soon be up for renewal, the mandate of the present council expiring in February 2011.

Let me finish on the same note as I did two years ago. The crucial element of the success of the ERC has been the fact that the scientific community has shown a strong and permanent support to it. I am well aware that this is not automatic and even if people find the ERC useful they may get tired of manifesting it. I nevertheless encourage you not to give up; without explicitly expressed community support the future of this wonderful project is uncertain.

Pavel Exner
EMS Vice-President
ERC Scientific Council Member

Starting Grants 2009 – 15 grants

Noam Berger (Hebrew University of Jerusalem): Limit theorems for processes in random media

Stefano Bianchini (Scuola Internazionale Superiore di Studi Avanzati): Hyperbolic Systems of Conservation Laws: singular limits, properties of solutions and control problems

Ugo Boscain (Centre National de la Recherche Scientifique (CNRS)): Geometric control methods for heat and Schroedinger equations

Frédéric Bourgeois (Université Libre de Bruxelles): Legendrian contact homology and generating families

Tom Coates (Imperial College of Science, Technology and Medicine): Gromov-Witten Theory: Mirror Symmetry, Modular Forms, and Integrable Systems

Jukka Ilmari Corander (Åbo Akademi): Intelligent Stochastic Computation Methods for Complex Statistical Model Learning

Radek Erban (Chancellor, Masters and Scholars of the University of Oxford): Stochastic and Multiscale Modelling in Biology

Francis Filbet (Universite Claude Bernard Lyon 1): Numerical simulations and analysis of kinetic models – applications to plasma physics

Alessandro Giuliani (Università degli Studi Roma Tre): Collective phenomena in quantum and classical many body systems

Oleksandr Gorodnyk (University of Bristol): Dynamics of Large Group Actions, Rigidity, and Diophantine Geometry

Peter Keevash (Queen Mary and Westfield College, University of London): Extremal Combinatorics

Omri Moshe Sarig (Weizmann Institute of Science): Ergodic theory on non-compact spaces

Benjamin Schlein (Chancellor, Masters and Scholars of the University of Cambridge): Mathematical Aspects of Quantum Dynamics

Chrysoula Tsogka (Foundation for Research and Technology): Algorithmic Development and Analysis of Pioneer Techniques for Imaging with waves

Nathalie Wahl (Københavns Universitet): Surfaces, 3-manifolds and automorphism groups

Advanced Grants 2008 – 20 grants

Remi Abgrall (Institut National de Recherche en Informatique et en Automatique): Adaptive Schemes for Deterministic and Stochastic Flow Problems

Noga Alon (Tel Aviv University): Discrete Mathematics: methods, challenges and applications

Boris Dubrovin (International School for Advanced Studies): Frobenius Manifolds and Hamiltonian Partial Differential Equations

Hélène Esnault (Universität Duisburg-Essen): Fundamental groups, etale and motivic, local systems, Hodge theory and rational points

Olivier Dominique Faugeras (Institut National de Recherche en Informatique et Automatique): From single neurons to visual perception

Nicola Fusco (Università di Napoli 'Federico II'): Analytic Techniques for Geometric and Functional Inequalities

Ludmil Katzarkov (University of Vienna): Generalized Homological Mirror Symmetry and Applications

Antti Jukka Kupiainen (Helsingin yliopisto): Mathematical Physics of Out-of-Equilibrium Systems

Roberto Longo (Università degli Studi di Roma Tor Vergata): Operator Algebras and Conformal Field Theory

László Lovász (Eötvös Loránd University): HU From discrete to continuous: understanding discrete structures through continuous approximation

Alexander Lubotzky (The Hebrew University of Jerusalem): Expander Graphs in Pure and Applied Mathematics

Ib Madsen (Københavns Universitet): Topology of Moduli Spaces and Strings

Fabio Martinelli (Universita degli Studi Roma Tre-Dipartimento di Matematica): S Phase transitions in random evolutions of large-scale structures

Philippe Michel (Ecole Politechnique Fédérale de Lausanne): Equidistribution in number theory

Janos Pintz (MTA Renyi Alfred Kutatointezet – Alfred Renyi Institute of Mathematics, Hung): Gaps between primes and almost primes. Patterns in primes and almost primes. Approximations to the twin prime and Goldbach conjectures

Alfio Quarteroni (Ecole Politechnique Fédérale de Lausanne): Mathematical Modelling and Simulation of the Cardiovascular System

Stanislav Smirnov (Université de Genève): Conformal fractals in analysis, dynamics, physics

Halil Mete Soner (Sabanci University): Mathematical Methods for Financial Risk Management

Andrew Stuart (University of Warwick): Problems at the Applied Mathematics-Statistics Interface

Bernt Øksendal (Universitetet i Oslo): Innovations in stochastic analysis and applications with emphasis on stochastic control and information

Advanced Grants 2009 – 16 grants

Luigi Ambrosio (Scuola Normale Superiore di Pisa): Geometric Measure Theory in Non-Euclidean spaces

Christoph Buchs-Schwab (Eidgenössische Technische Hochschule Zürich): Sparse Tensor Approximations of High-Dimensional and Stochastic Partial Differential Equations

Demetrios Christodoulou (Eidgenössische Technische Hochschule Zürich): Partial Differential Equations of Classical Physics

Simon Kirwan Donaldson (Imperial College of Science, Technology and Medicine): Geometric analysis, complex geometry and gauge theory

Uffe Valentin Haagerup (Københavns Universitet): Operator Algebras, Free Probability, and Groups

David Kazhdan (The Hebrew University of Jerusalem): Langlands correspondence and its variants
François Labourie (Université Paris-Sud XI): Higher Teichmüller-Thurston Theory: Representations of Surface Groups in PSL(n,R).
Carlangelo Liverani (Università degli Studi di Roma Tor Vergata): Macroscopic Laws and Dynamical Systems
François Loeser (Ecole Normale Supérieure): New Methods in Non-Archimedean Geometry
Grigory Mikhalkin (Université de Genève): Tropical Geometry
Jean-Michel Morel (Ecole Normale Supérieure de Cachan): Twelve Labours of Image Processing
Walter Schachermayer (Universität Wien): Risk and Valuation of Financial Assets: A Robust Approach
Aner Shalev (The Hebrew University of Jerusalem): Words and Waring type problems
Alain-Sol Sznitman (Eidgenössische Technische Hochschule Zürich): Random Walks, Percolation and Random Interlacements
Günter M. Ziegler (Technische Universität Berlin): Structured Discrete Models as a Basis for Studies in Geometry, Numerical Analysis, Topology, and Visualization
Enrique Zuazua (BCAM – Basque Center for Applied Mathematics): New analytical and numerical methods in wave propagation

The project "Mathematics and Industry"

Mario Primicerio

The project "Mathematics and Industry" was initiated in spring 2009 by the EMS and by its Applied Mathematics Committee; it is supported by the European Science Foundation through the scheme "Forward Looks".

The purpose and objectives of the project are:
- To explore ways of stimulating and/or intensifying the collaboration between mathematics and industry.
- To identify common issues, questions and "good practices" between mathematics and industry in order to envisage strategies for a stronger interaction of mathematicians with large companies and SME aimed at technological advancement.
- To indicate strategic objectives and formulate recommendations addressed to academia, industry, governments and European institutions, and national and international funding agencies.

These topics had already been the subject of several previous reports and obviously their contents were all taken into account (and indeed one of the starting points of the work was to draw up and circulate extended summaries of them). Nevertheless, with respect to them, this project had several advantages. First of all, it was focused on the European situation and was a timely response to the implementation of the "Lisbon strategy" adopted by the European Union. But an even more important aspect is that it did not involve just a restricted group of experts but instead pursued the involvement of the European mathematical community. The help of the EMS and of the national mathematical societies was of course instrumental in this respect. An online survey was also carried out; almost 1000 answers came from academia and from companies.

The project produced a first draft of a report ("Green Paper") containing recommendations to policymakers, funding agencies, academia and industry (including banks, administration and services) and a brochure collating around 100 "success stories" of cooperation.

These two documents were presented and discussed during a "Consensus Conference" that took place in Madrid, 24–25 April. The meeting also included invited presentations: Pierre Louis Lions and Alfredo Bermudez from the academic side and Adel Abbas (Airbus), Philippe Ricoux (Total), Jan Sijbrand (NIBC), Andreas Schuppert (Bayer) and Javier Bullòn (Ferroatlantica) from the side of companies.

Venue of the "Consensus Conference", Madrid, 24–25 April 2010

The conference was attended by more than 100 participants from all over Europe; it was included in the Official Agenda of the Spanish presidency of the EU and gave important suggestions for the final version of the report.

It is probably worth pointing out that the report, from its very beginning, asserts that "…*development and progress of mathematics have always been driven both*

by internal forces (to cross the boundary) and by external forces (the need of solving problems arising outside the discipline). (…) The term 'industrial mathematics' (is used) to designate all the research that is oriented at the solution of problems posed by industrial applications. It cannot be considered as a field disjoint from the counterpart of a so-called 'pure mathematics', since on one hand it uses potentially all fields of mathematics and, on the other hand, it is a continuous source of challenges to the fundamental research on structure and methods of mathematics because it has often to deal with completely new mathematical problems."

Then it points out some basic facts:
- Europe aims to promote a *knowledge-based economy*. Indeed, the challenge posed to European industry by global competition nowadays is as simple as it is dramatic: *Innovate or perish*!
- Mathematics provides a universal framework and language for *innovation*; mathematicians are natural candidates to coordinate *multidisciplinary* teams aiming at modelling, controlling and optimising industrial processes. Mathematics is a necessary instrument for innovation, and thus for achieving significant competitive advantages: *mathematics truly gives industry the edge*.
- The interaction between mathematics and industry in Europe is *far from optimal.* Fragmentation is another characteristic. On the other hand the complexity of the situation is such that a strong interconnected community and a *vision* for Europe are needed more than ever.

The report develops these concepts through detailed arguments, examples and comparisons with the situations in Asia, Australia and America and concludes with three main *recommendations* (each of them is accompanied by a "roadmap" for its implementation):

1 Policymakers and funding organisations should unite their efforts and fund mathematics activities through a *European Institute of Mathematics for Innovation*.
2 In order to *overcome geographical and scientific fragmentation*, academic institutions and industry must share and disseminate best practices across Europe and disciplines via networks and digital means.
3 Mathematical societies and academic institutions must harmonise *the curriculum and educational programmes in industrial mathematics* at the European level.

The final report ("White Paper") will be presented in Brussels in the presence of the European Commissioner for Industry on 02 December 2010. The event is included in the Official Agenda of the Belgian Presidency of the EU.

On the webpage http://www.ceremade.dauphine.fr/FLMI it is possible to:
- Access the online questionnaire.
- Send comments/remarks/criticism on the draft of the final document of the project.
- Contribute a "success story" to the brochure dedicated to collaboration with companies (*deadline 15 September 2010*).

7th International Congress on Industrial and Applied Mathematics 18–22 July 2011 Vancouver, British Columbia, Canada

ICIAM 2011 will highlight the most recent advances in the discipline and demonstrate their applicability to science, engineering, and industry. In addition to the traditional, strong focus on applied mathematics, the Congress will emphasize industrial applications and computational science. An integrated program will highlight the many outstanding contributions of applied mathematics. ICIAM 2011 will also be an opportunity to demonstrate to young researchers and graduate students the vast potential of mathematics.

The Congress is being planned by the Canadian Applied and Industrial Mathematics Society (CAIMS), the Mathematics of Information Technology and Complex Systems (MITACS), and the Society for Industrial and Applied Mathematics (SIAM).

Scientific Program Committee (Co-Chairs)
Ivar Ekeland, Professor, Mathematics, University of British Columbia
Jerrold Marsden, Professor, Control and Dynamical Systems, California Institute of Technology

Invited Speakers
Dr. John Bell (*Lawrence Berkeley National Laboratory*)
Dr. Andrea L. Bertozzi (*University of California, Los Angeles*)
Dr. Vicent Caselles (*Universitat Pompeu Fabra*)
Dr. Sergio Conti (*Universität Bonn*)
Dr. Michael C. Ferris (*University of Wisconsin Madison*)
Dr. Gilles Francfort (*Université Paris-Nord, Villetaneuse*)

Dr. Peter Fritzson (*Linköpings Universitet*)
Dr. Zhi Geng (*Peking University*)
Dr. Clovis Gonzaga (*Federal University of Santa Caterina*)
Dr. Matthew Hastings (*University of California at Santa Barbara*)
Dr. John Hogan (*University of Bristol*)
Dr. Philip Holmes (*Princeton University*)
Dr. Yoh Iwasa (*Kyushu University*)
Dr. Ron Kimmel (*Israel Institute of Technology*)
Dr. Shigeo Kusuoka (*University of Tokyo*)
Dr. Kerry Landman (*University of Melbourne*)
Dr. Mark Lewis (*University of Alberta*)
Dr. Christian Lubich (*Universität Tübingen*)
Dr. Alexander McNeil (*Heriot-Watt University*)
Dr. Benoit Perthame (*Université Pierre et Marie Curie*)
Dr. Benedetto Piccoli (*Consiglio Nazionale delle Ricerche*)
Dr. Pierre Sagaut (*Université Pierre et Marie Curie*)
Dr. Andrzej Świerniak (*Silesian Technical University – Politechnika Śląska*)
Dr. Rudiger Westermann (*Technische Universität München*)
Dr. Bin Yu (*University of California, Berkeley*)
Dr. Pingwen Zhang (*Peking University*)
Dr. Francis Zwiers (*Canadian Centre for Climate Modelling and Analysis*)

Deadlines to remember

- **18 August 2010**
  Early deadline for minisymposium proposals
  Online registration opens
- **17 September 2010**
  Early decisions announced for minisymposium proposals
- **4 October 2010**
  Final submission deadline for minisymposium proposals
- **29 October 2010**
  Final decisions announced for minisymposium proposals
- **15 December 2010**
  Submission deadline – accepted minisymposia abstracts
  Submission deadline – contributed paper abstracts
- **14 January 2011**
  Notifications sent to accepted contributed speakers
- **February/March 2011**
  ICIAM 2011 program posted online

Special Focus Themes

- Computational and Modeling Challenges in Industry
- Control Theory and Applications
- Design and Verification of Complex Systems
- Dynamical Systems and its Applications
- Economics and Finance
- Environmental Science, including Ocean, Atmosphere, and Climate
- Fluid Mechanics
- Graphics, Visualization, and Computation
- Image and Signal Processing
- Materials Science
- Mathematical Biology
- Mathematical Medicine and Physiology
- Mathematical Physics
- Mathematical Programming and Industrial Applications
- Molecular Simulation: Quantum and Classical
- Numerical Analysis
- ODE, PDE, and Applications
- Scientific Computing
- Solid Mechanics
- Statistical Sciences

Fine arts to the service of Mathematics

C. P. Bruter

Mathematics rarely receives positive media coverage. It almost feels like a concerted rejection by mainstream culture.

The reason for this rejection is partly a consequence of mathematicians' behaviour and the unique character of their cultural environment. Many in the profession live in a sheltered world, work uninterrupted on their favourite project and are subjected only to the pressure of their peers and the administrative structure that delivers advancement and recognition. The processes by which their work is developed and publicised obey rules that are replete with professional, well thought out virtues, yet the stiff formalism they generate shapes psychological attitudes.

If those rules developed upon years of practice are well adapted to the environment in which they are expressed, it is far from certain that they are suitable for the broader audience they could or should address. One has only to look at the existing student population to notice a serious educational shortcoming.

The failure to connect with younger generations is compounded by a failure to communicate and a lack of understanding between mathematicians and the public at large – the same public, who at a younger age, were often turned away from the mathematical world.

This serious issue is also affecting the majority – if not the totality – of the decision-makers in today's political, economic and cultural world.

It would be judicious and wise to re-establish contact and resume the dialog with Youth and the larger adult audience, based on trust, understanding and mutual intellectual respect.

But by what means can this be accomplished?

Among the many tools used in communication, fine art stands out for its consistency and effectiveness. Throughout history, architecture, sculpture, visual expression, poetry and music have been a vehicle for a larger aesthetic and existential discourse. It is through work of this nature that mankind has expressed itself again and again, with the most compelling depth, relevance and conviction.

The beauty of an aesthetic statement conveys an attractive power one contemplates, admires, understands and listens to, generation after generation. It reaches out to many. Its appreciation does not require an a priori intellectual undertaking, yet the message spreads so extensively and it subtly connects with its audience. Eventually it is assimilated to the core meaning of the display.

Visual expression, bound by the penetrating power of light, has always occupied a dominant place in communication. Could this approach be applied to the dialogue between mathematicians and the public? This question is all the more relevant because a fundamental part of mathematics is the description and representation of space – a field explored by many artists as well.

In the light of this opportunity, collaboration between mathematics and art could lead to a lowering of the psychological barrier that separates the mathematical community from the public at large.

The abundance of forms represented by mathematical objects in understated or more explicit artistic representations embodies mathematical science in many ways. It allows the artist to familiarise the public with its dissemination.

The abstract character of a mathematical demonstration may recede into the background. It may open to a more direct, tangible form of assertion that reaches to both the senses and the mind. Reticence toward mathematics may wane and the obstacles impeding mathematical discussion and exchange may be lowered.

Many professionals have already contributed to this approach as exemplified in the status of the 20 year old ARPAM (Association pour la Réalisation du Parc d'Activités Mathématiques), the homepage of the American Mathematical Society and the recent lecture of IHP director Cedric Villani at the Francois Mitterand National Library.

At the beginning of the year, mathematicians from several European countries joined together to coordinate attempts made in that direction and created a European Society for Mathematics and Art (ESMA*).

ESMA is the latest development of an effort that originated at the Maubeuge conference of 2000 and that has been sustained over the last 10 years by the ARPAM association. Its purpose is to promote communication between the scientific, mathematical and computer science community and the public at large through artistic means. The society facilitates the exchange of information and ideas through its website, publications, seminars and conferences, both locally and internationally.

The European Society for Mathematics and Art held its first conference at the Poincaré Institute in Paris July 19–22. A brief descriptive of the event is available in the August issue of the ESMA newsletter. (http://mathart.eu/)

Following its annual meeting, the Society set up several committees to develop further its activities in specific area of interest. The next Mathematic and Art conference will be organized and hosted by Dr. Konrad Polthier in Berlin in 2012.

ESMA organized an exhibit of mathematical art in conjunction with the conference. Numerous works from over 35 participants were shown on the ground-floor and in the library of the Poincare Institute. The exhibition, initially scheduled from June 7 to July 25 was extended to September 15 at the invitation of the Institute. The catalogue can be found on the ESMA site at: http://www.mathart.eu/ihp10/.

We hope that with your encouragement and support the European Society of Mathematics and Art will meet its objective and be of further service to the mathematical community.

Claude Paul Bruter [cpbruter@mathart.eu] is now a retired professor of mathematics from Paris 12 University. In his late mathematical work, he gave a generalisation for any dimension of the so-called fundamental theorem of algebra. He has written around ten books on various subjects, including mathematics and the history of mathematics. Involved in popularising mathematics through art, he founded the French association ARPAM and recently the European association ESMA.

* More information on the events calendar & schedule can be found in the newsletter that appears on the ESMA website: http://www.mathart.eu.

The Clay Public Lecture and Conference on the Poincaré Conjecture,

Paris, 7–9 June 2010

Eva Miranda

When in 1904 Henri Poincaré wrote in [P] the sentence "mais cette question nous entraînerait trop loin", as he referred to the problem of knowing whether S^3 could be the only (up to homeomorphism) closed simply connected manifold in dimension 3, he did not know that it was going to take almost 100 years to solve this problem and that the history of its resolution would involve the striving of different research groups in topology and geometry.

The Poincaré conjecture is one of the "one million dollar prize" problems in Clay's list of Millennium Prizes. This list was announced on 24 May 2000 at the Collège de France. The other problems (still unsolved) in this list are the Birch and Swinnerton-Dyer Conjecture, the Hodge Conjecture, the Navier-Stokes problem, the P versus NP problem, the Riemann Hypothesis and the Quantum Yang-Mills Theorem. The fact that the Poincaré Conjecture was one the Millennium Prizes probably incentivised many mathematicians to attack this problem but it also gave a social value and put a symbolic price to a continuous and longstanding effort to prove or disprove the conjecture. When in 2003 Grigoriy Perelman posted his 7-page paper on the arxiv "Finite extinction time for the solutions to the Ricci flow on certain three-manifolds", the experts understood that the proof of the technical "lemma" about finite extinction time could really close a cycle of almost one century in search of a proof. Many different teams of researchers all around the world struggled to verify the proofs and check whether the arguments were complete.

The story of the resolution of this conjecture has led to the development of top research groups all around the world but it was a Russian Mathematician, Grigoriy Perelman, who solved the Poincaré Conjecture, following Hamilton's programme to the extreme and including the study of collapse and singularities into the scene. On the way, he also proved the ambitious Geometrization Conjecture of Thurston's programme of which the Poincaré Conjecture is a special case. What is more surprising in this progression to the proof is that Grigoriy Perelman solved a problem with a sheer topological statement using hard methods in differential geometry and geometric analysis (including the Ricci flow techniques). Therefore, the resolution of the Poincaré Conjecture was really an example of the principle "les maths ne sont qu'une histoire des groupes" [mathematics is nothing but the story of groups].

Public awareness of the proof of the Poincaré Conjecture arrived at the International Conference of Mathematicians in 2006 in Madrid when Grigoriy Perelman was awarded the Fields Medal. The mathematical community and even the general public were shocked when it was understood that Grigoriy Perelman would not accept this prestigious prize. Four years later, on 18 March 2010, James Carlson (President of the Scientific Advisory Board of the Clay Mathematics Institute, CMI) announced that the Clay Prize was awarded to Grigoriy Perelman. The CMI Scientific Advisory Board (James Carlson, Simon Donaldson, Gregory Margulis, Richard Melrose, YumTong Siu and Andrew Wiles) appointed a Special Advisory Committee, formed of Simon Donaldson, David Gabai, Mikhail Gromov, Terence Tao and Andrew Wiles, to consider the correctness and attribution of the proof of the conjecture. After this announcement and out of the blue, the world was staring back at the mathematical community and was wondering whether Grigoriy Perelman would accept this prize. The media informed everyone that some institutions in Russia were urging Grigoriy Perelman to accept it.

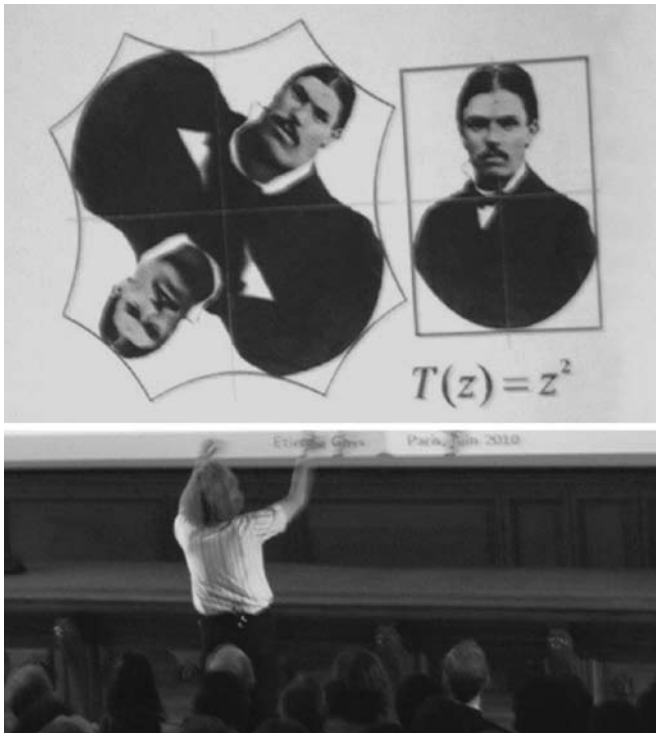
The conference

Also on 18 March 2010, James Carlson announced that the 2010 Clay Research Conference would take place in Paris at the Institut Henri Poincaré. The official ceremony for the prize would take place during this conference.

One could not imagine a better place to do that than the Institut Henri Poincaré, IHP, in Rue Pierre et Marie Curie in the beautiful 5ème arrondisement of the city of lights. The IHP is located close to ENS-Ulm where many talented mathematicians have been scientifically born. These streets have registered in their pavement the steps of hesitation for many conjectures but this summer the resolution of a big conjecture was going to be celebrated there. It was the end of an era and the Institut Henri Poincaré was ready for that.

The Clay Institute, with the collaboration of the Institut Henri Poincaré, had overseen two big events to celebrate the Poincaré Conjecture. The first was a Clay Public Lecture and the second a Clay Research Conference on the resolution of the Poincaré Conjecture. The Clay Public Lecture was delivered by Étienne Ghys on 7 June at the Institute Océanographique. The Clay Research Conference took place on 8–9 June at the Institut Océanographique and the Institut Henri Poincaré (salle Darboux and salle Hermite).

The conference of Étienne Ghys was a grand public lecture beautifully organised in the historical Salle of the Institut Océanographique. According to Florence Lajoinie, who was responsible for the practical organisation of this event at the IHP, 450 people registered for the event. Among the participants, we recognised the faces of many well-known mathematicians but also many young "mathematicians-to-be", together with the Animath students.



Étienne Ghys looking at Henri Poincaré through a Fuchsian perspective

Étienne Ghys' talk[1] was focused on the research of Henri Poincaré concerning Fuchsian groups. The talk, nicely prepared and energetically delivered, ended with the big conclusion that, in parallel to the individual efforts, mathematics is a story of groups. The title of his talk "Les maths ne sont qu'un histoire des groupes" was declared as a general principle in the work in mathematics at the end of the talk. The motto had the double meaning of the story of Fuchsian groups and the common striving in mathematics work.

The speakers on 8–9 June were certainly some of the composers of the key scores that led to the proof of this conjecture. They wrote several pages in the history of the Poincaré Conjecture and led, in one way or another, to the big endeavour of the proof.

The talks on 8 June were still taking place at the Institute Océanographique and were devoted to a general mathematical audience who wanted to know about the proof of Perelman and the history and influence of this result. After the opening by James Carlson and the director of the IHP Cédric Villani, there were talks by Sir Michael Atiyah and John Morgan about the history of the Poincaré Conjecture and related open problems. These talks were followed by the impressive talk of Curtis McMullen on "The evolution of geometric structures on 3-manifolds". The early afternoon was conducted by two of the fathers of the main streams in the history of the proof William Thurston and Steven Smale. The talk by William Thurston was about "*The mystery of 3-manifolds*" and contained some pedagogical scenifications of the eight Thurston's geometries. Steven Smale, who had proven

James Carlson, President of the Clay Institute, at the opening ceremony of the Clay Institute Conference at the Institut Océanographique

John Morgan on the blackboard. John Morgan together with Gang Tian wrote a book on the proof of the Poincaré Conjecture entitled "Ricci Flow and the Poincaré Conjecture".

McMullen and his vision on the wilderness of 3-geometry

the Poincaré conjecture in dimension 5 and higher, talked about topological problems post-Perelman. The last talk of 8 June was delivered by Simon Donaldson who spoke about invariants of manifolds and the classification problems with a special emphasis on manifolds with additional structures like symplectic manifolds in dimension 4. After the talks a reception took place at ENS-Ulm.

The second day of the conference was more specialised and tackled different aspects of the proof of Perelman. Speakers in the morning session were David Gabai (*Volumes of hyperbolic 3-manifolds*) and Mikhail Gromov, who gave a wonderful talk that had been announced with great simplicity under the title of "*What is a manifold?*". After Gromov came the talk of Bruce Kleiner on "*Collapsing with lower curvature bounds*". (Kleiner together with Lott maintain a webpage with material on the Poincaré Conjecture and the Geometrization Conjecture and its proof by Perelman [KL].) The speakers of the afternoon were Gérard Besson and Gang Tian. Gérard Besson talked about an alternative approach for the last step in Perelman's proof of Thurston's Geometrization Conjecture (joint work with L. Bessières, M. Boileau, S. Maillot and J. Porti). Gang Tian talked about counterparts of Perelman's techniques and the Ricci flow in dimension 4.

[1] The video of the conference is available under the link: http://www.poincare.fr/evenements/item/18-les-maths-ne-sont-qu-une-histoire-de-groupes.html"

The ceremony

The ceremony took place immediately after noon on 8 June. It was now the time for the award of the prize and the "laudatio" for Grigoriy Perelman.[2] Andrew Wiles' laudation described the importance of the problem and solution of the Poincaré Conjecture and thanked Perelman as well as his predecessors, especially Richard Hamilton, for that. Michael Atiyah used the parallelism of mathematics and landscape and pointed out that "*Grigory Perelman is the mountaineer who reached this pinnacle of the 3-dimensional world*". William Thurston explained the mathematical importance of Perelman's contribution and ended up with a touching paragraph about Perelman's attitude to what he denominated "*public spectacle*", closing his speech with the mindful sentence: "*Perhaps we should also pause to reflect on ourselves and learn from Perelman's attitude toward life.*" Simon Donaldson described the work of Perelman and his originality as a cut of a Gordian knot in the problem and ended up his laudation with the following meaningful sentence: "*Perelman's achievement is a testament to the continued power of the individual human mind in bringing about the most fundamental advances in mathematics.*" Mikhail Gromov used a metaphor to describe the work in three manifolds as a sea expedition to discover new lands. "*It will probably take a decade or so for the mathematical community to build up new edifices on the land discovered by Perelman*," he pointed out at the end of his speech.

After Laudatio, it was time for the awarding of the prize. It had been well-known that Grigoriy Perelman would probably not show up to the Clay Conference in Paris. But when Cédric Villani walked slowly out of the room at that precise moment, the idea that Perelman could walk into the majestic room of the Institut Océanographique to pick up the prize really crossed the mind of some of us. But Perelman did not show up. Needless to say, one of the highlights of the ceremony was when Landon Clay offered the award symbolising the Millennium Prize looking at "infinity" in the big room of Océanographique and relaxed the formal atmosphere of the act by pronouncing the words "*to award this prize* – silence – *to whoever takes it*" before pronouncing the formal attribution: "*The Clay Mathematics Institute hereby awards the Millennium Prize for resolution of the Poincaré Conjecture to Grigoriy Perelman.*"

Probably one of the most emotive moments of the conference took place when Lavinia Clay stood up and walked to the centre of the main room at Océanographique with François Poincaré, one of the grandsons of Henri Poincaré. François Poincaré was holding an honorary plaque for the resolution of the Poincaré Conjecture (which will remain at the IHP) as he thanked the mathematical community for spreading the ideas of his grandfather and pointed out that he had been involved with his work as a linguist since he translated many of the letters of Felix Klein to Henri Poincaré.

[2] The transcript of laudations is available at the webpage of the CMI [C] under the link http://www.claymath.org/poincare/laudations.html.

Mikhail Gromov on the laudatio

One of the grandsons of Poincaré, François Poincaré, in the ceremony.

Conclusions

Clay Research Conference was a big event marking the end of an era but also the beginning of a new one. The Poincaré Conjecture is now Perelman's theorem but another six conjectures still remain unsolved in the Millennium problems list.

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Figures about the Studies of Mathematics in European Countries and the USA – A Nontrivial Research Analysis

Miriam Dieter and Günter Törner (Duisburg-Essen, Germany)[1]

We are often confronted with questions like: *How many students are involved in a mathematical field of study? How many freshmen and graduates are involved in mathematics?*

2008 was the year of mathematics in Germany. During this time both the authors were involved in a project named "Numbers around Mathematics" and looked into these questions (see Dieter et al. (2008a,b,c,d)). Furthermore, they compiled and evaluated data concerning the German labour market (see Dieter & Törner (2009a,b)), gathering information from Eurostat and comparing German and other European figures. A country specific comparison seems necessary and in the following the first statistical evaluations of this work are presented.

Subsequent to writing this article, the authors became aware of a pipeline project (http://www.mathunion.org/icmi/other-activities/pipeline-project/), initiated by the IMU through the ICMI, with the objective "to look into the supply and demand of mathematics students and personnel in educational institutions and the workspace and to provide findings that will be useful for decision-making in different countries as well as providing a better understanding of the situation internationally" (see Holton et al. (2009)). In its current stage, data is being collected from Australia, Finland, France, Korea, New Zealand, Portugal, the UK and the USA. At this point, no comment will be made on the pipeline project; this will be picked up in an upcoming article.

The approach taken here is to present figures based on official data that originates from a single source, created for comparisons just like those being made here. Because of this the figures between the distinct countries become comparable. In contrast, the pipeline project collects data from many different authorities and there are difficulties in overcoming these inconsistencies. Nevertheless, both approaches have one important fact in common. International figures about mathematics have only just begun to be identified and it will take time to develop a better understanding of the international situation.

Eurostat – Terminology and offer of information

Eurostat is the statistical office of the European Union and is based in Luxembourg. It provides information and statistics for the European Union at a European level. Eurostat has statistics available that cover all issues of economy and society. These issues are divided into nine topics: general and regional statistics; economy and finance; population and social conditions; industry, trade and services; agriculture and fisheries; foreign trade; transport; environment and energy; and science and technology.

The national statistical offices of the EU member states, as well as the EFTA countries (Iceland, Liechtenstein, Norway and Switzerland), transmit their data to Eurostat. To make the data comparable they are harmonised by Eurostat. This gives an overview of the European Union as a whole, with insights into the situations of several countries or regions and, furthermore, comparisons are possible with other states (outside of the EU).

The majority of the data can be downloaded for free from the portal, which is located on the Eurostat homepage (http://ec.europa.eu/eurostat/). The following material is from this site and it will enable international comparisons to be made.

In Germany the four *fields of study*: mathematics, statistics, technomathematics and business mathematics, are subsumed into the '*area of studies*: mathematics' (for further information see Dieter et al. (2008a)). The equivalent at Eurostat to the German '*area of studies*: mathematics' is the field ef46 Mathematics/Statistics, which consists of mathematics, operations research, numerical analysis, actuarial science, statistics and other related fields.

There are different programmes that are relevant for mathematics. These programmes are:

- Diploma (university and university of applied sciences).
- Bachelors.
- Masters.
- Teacher training.
- Doctorate.

To make the data between distinct countries comparable, these programmes are structured by the *International Standard Classification of Education (isced)*. According to UNESCO (2006) "ISCED is designed to serve as an instrument suitable for assembling, compiling and presenting statistics of education both within individual countries and internationally". However, the systematics of Eurostat assigns the programmes that are important for this article on the basis of isced to two levels that are composed in the following way (see Eurostat (2003)).

- The *isced level 5* contains the first stage of tertiary education. These programmes normally have an edu-

[1] The authors thank Deutsche Telekom Stiftung for a partial financial support while doing this analysis.

cational content but do not lead to the award of an advanced research qualification. These programmes must have a cumulative duration of at least two years. The *level isced5* is constituted of the relevant sub-categories *isced5a_d1* (bachelors, diploma and teacher training) and *isced5a_d2* (Masters and postgraduate studies).

- The *isced level 6* contains the second stage of tertiary education. These courses of study lead to the award of an advanced research qualification. The programmes are therefore devoted to advanced study and original research and are not based on coursework only. They typically require the submission of a thesis or dissertation of publishable quality that is the product of original research and represents a significant contribution to knowledge. In Germany the doctorate program belongs to the *level isced6*.

However, this classification of programmes as given by Eurostat is deficient. In the *level isced5a_d1*, diploma students and teacher trainees are not considered separately. Limitations also arise from the fact that the data of Eurostat does not allow the fields of study of the *level Mathematics/Statistics* to be viewed individually. As can be seen in Section 2, this procedure can distort results.

There are data from 37 countries present but some of these countries can be excluded from this comparison because there are no data available for the whole inquiry period from 1998 to 2007. The data from the following countries will be presented here: Germany, France, England, Sweden, Spain, Portugal, Italy, Turkey, Poland, Romania and the USA.

In the following sections the total figures of students, graduates and doctorates of these countries will be used.

Total figures of students

On the basis of the Eurostat data it is possible to calculate the share of the level *isced5a* for the field *ef46 Mathematics/Statistics*. The numbers of all mathematics and statistics students of the countries under consideration were compared with the total figures of all students of these countries. The absolute figures and the resulting quotas are shown in Table 1.

Comparing the data of the 11 countries over the inquiry period from 1998 to 2007 there are some astonishing facts. Germany is the *front runner* based on the proportion of all students that are *level Mathematics/Statistics*! In Germany, this share has always been between 2.2% and 2.6% over the last 15 years. Furthermore, since 2000 this quota has been increasing. Italy and Poland – countries that have just as many students as Germany – have quotas of less than 1%. The registered increase of the total number of students in Germany in the period from 1998 to 2007 is also noticeable in England, Sweden, Poland, Romania and Turkey.

A completely different tendency is recognisable in the states in the south of Europe. In Italy, the total number of students has increased by 8.8% from 1,823,210 to 1,983,005 but the figure of those who study mathemat-

		1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	*Total*	1,785,938	1,767,978	1,742,234	1,766,734	1,835,558	1,902,408	1,981,373	1,927,299	1,953,504	1,950,468
	Math/Stat	42,569	40,404	38,982	39,974	44,212	48,604	53,194	51,499	54,708	56,608
	Quota	2.4	2.3	2.2	2.3	2.4	2.6	2.7	2.7	2.8	2.9
France	*Total*									1,595,742	1,567,977
	Math/Stat									32,787	28,904
	Quota									2.1	1.8
England	*Total*	1,279,679	1,362,395	1,337,734	1,347,501	1,444,685	1,454,085	1,645,232	1,678,686	1,730,046	1,747,197
	Math/Stat	19,802	17,188	17,511	19,607	19,855	22,073	26,901	27,266	28,190	28,755
	Quota	1.6	1.3	1.3	1.5	1.4	1.5	1.6	1.6	1.6	1.6
Sweden	*Total*	263,760	302,985	312,287	323,840	348,100	378,570	389,390	386,656	380,846	371,307
	Math/Stat	16,927	7,529	7,564	7,007	6,529	7,069	7,117	6,683	6,773	5,636
	Quota	6.4	2.5	2.4	2.2	1.9	1.9	1.8	1.7	1.8	1.5
Spain	*Total*	1,596,644	1,602,653	1,603,743	1,571,639	1,541,743	1,519,599	1,507,520	1,484,962	1,472,127	1,468,942
	Math/Stat	24,368	23,451	21,670	19,213	16,764	15,020	13,439	12,132	10,734	10,134
	Quota	1.5	1.5	1.4	1.2	1.1	1.0	0.9	0.8	0.7	0.7
Portugal	*Total*	259,544		343,352	364,024	373,774	379,488	372,521	357,639	342,567	345,120
	Math/Stat	5,716		4,821	4,945	5,437	4,840	4,612	3,773	3,099	2,367
	Quota	2.2		1.4	1.4	1.5	1.3	1.2	1.1	0.9	0.9
Italy	*Total*	1,823,210	1,754,601	1,729,887	1,747,654	1,805,315	1,862,545	1,926,956	1,955,988	1,976,850	1,983,005
	Math/Stat	29,015	24,852	23,857	22,372	20,788	19,720	18,592	17,820	16,848	17,175
	Quota	1.6	1.4	1.4	1.3	1.2	1.1	1.0	0.9	0.9	0.9
Turkey	*Total*			777,726	830,174	869,780	898,328	1,385,094	1,463,424	1,625,337	1,702,182
	Math/Stat			23,391	25,044	26,010	27,188	28,723	30,924	33,330	34,475
	Quota			3.0	3.0	3.0	3.0	2.1	2.1	2.1	2.0
Poland	*Total*	1,157,935	1,362,269	1,539,312	1,731,554	1,858,502	1,931,543	1,989,889	2,062,612	2,089,762	2,092,162
	Math/Stat	10,341	9,836	14,778	16,591	15,937	17,342	17,106	16,219	16,862	16,798
	Quota	0.9	0.7	1.0	1.0	0.9	0.9	0.9	0.8	0.8	0.8
Romania	*Total*	339,569	380,476	416,593	484,072	527,151	565,664	621,501	673,266	785,403	887,526
	Math/Stat	8,632	9,367	11,336	11,440	12,651	11,951	12,174	12,482	18,112	21,286
	Quota	2.5	2.5	2.7	2.4	2.4	2.1	2.0	1.9	2.3	2.4
USA	*Total*								13,251,603	13,427,655	13,636,967
	Math/Stat								92,476	93,647	95,110
	Quota								0.7	0.7	0.7

Table 1: Proportion of the total number of students that are *level Mathematics/Statistics.*

ics has decreased by 40.8% from 29,015 to 17,175. Something similar can be observed in Portugal, too. Here there was an increase of the total number of students by 33% (from 259,544 to 345,120) and in the same period (1998-2007) a decrease of the mathematics students by 58.6 % (from 5,716 to 2,367). Probably the most serious change took place in Spain. Here, there was a decrease of the number of students by 8% and a simultaneous decrease of the mathematics students by 58.4%.

What led to such drastic changes in these countries? Is it the lack of mobilisation in school education or can reasons be identified in the job market? Only the figures from 2005 to 2007 are available for the USA but comparing these figures to the other results it can be seen that the USA, with a share of only 0.7%, has the lowest quota of all these countries.

It is important to pay particular attention to the *quota of women*, i.e. the proportion of all students of the *level Mathematics/Statistics* that are women. The necessary data set is available and the quotas of women of the 11 states over the period from 1998 to 2007 have been calculated. There are difficulties, however. It should be underlined that Eurostat has combined in the level isced5a the two quite differently structured programmes: diploma and *teacher training*. From previous studies (see Dieter et al. (2008a,b,c)) it can be seen that teacher training is preferred by women in Germany. This applies exceedingly in the area of primary education and there occurs, compared to the *diploma students*, a substantially higher quota of women. Furthermore, it is important to ask the question of whether these tendencies exist in all states or if it is a country-specific feature. Therefore, any conclusions should be drawn with care.

Table 2 shows the proportion of mathematics students of the particular countries that are women. Germany is about average in terms of the quota of women (note again that teacher training disciplines and prospective expert scientists are mixed). Since 1998, the proportion of women has increased continuously from 39.4% to 49.4% in 2007. For Sweden, over the same period, there was an increase from 35.5% in 1998 to 49.9% in 2007). England (average 38.5%), France (average 38%) and Turkey (average 41.9%) have the lowest quotas. The definite front runners are Portugal (average 60.2%), Italy (average 57.3%) and Poland (average 63.6%). It is reasonable to assume that the high quotas of women in these countries are due to a disproportionally high participation of women in the area of teacher training while the actual mathematics sector is poorly developed.

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	39.4	40.2	41.8	43.7	45.4	46.4	47.1	48.7	48.8	49.4
France									37.4	38.6
England	38.7	37.9	38.1	39.2	37.5	39.2	39.0	38.5	38.1	38.4
Sweden	35.8	42.2	44.1	45.6	44.5	46.2	46.2	46.8	49.0	49.9
Spain	52.0	52.2	51.7	50.9	51.7	51.6	51.8	50.3	50.9	50.6
Portugal	60.2		56.7	56.4	62.6	63.2	61.5	61.1	60.6	59.7
Italy	60.6	57.9	59.7	58.7	57.8	57.4	55.9	55.3	55.1	54.5
Turkey			42.1	42.1	41.3	41.3	42.0	41.9	41.8	43.0
Poland	62.5	60.0	66.7	65.2	64.2	64.8	63.4	63.4	63.5	62.6
Romania	50.7	49.1	52.8	47.7	46.6	48.1	48.0	45.5	44.4	39.9
USA								46.8	47.0	47.0

Table 2: Proportion of *level Mathematics/Statistics* that are women.

Figures of graduates

Thanks to Eurostat, beside the total figures of students there are *figures of graduates* available, too. These will be examined in the level *isced5a_d1* and afterwards in the level *isced6*.

Note that the *level isced5a_d1* of Eurostat subsumes the numbers for diploma (university and university of applied sciences), bachelors and teacher training. The important differentiation between teachers and specialist mathematicians is not possible because of the categorisation of Eurostat. Therefore conclusions are again made with care. The presented data does not permit answers to questions such as: *Is the reason behind the great number of graduates that many teachers passed their exams? Do the specialised mathematicians prevail? Is the ratio of both groups balanced?*

Concerning the data of Germany one thing is conspicuous: in 2006, the number of graduates is twice as high as in the previous year. As the value of 7,832 for 2006 cannot be confirmed with the values from the German Federal Statistical Office and as the figure seems to be far too high, it will be disregarded (and likewise for the 2007 value). If the figures of German graduates from 1998 to 2005 are compared with the figures of the remaining countries, it is noticeable that the numbers for Germany are again about average.

It has already been suggested that the number of women among the German graduates is so high because many of these women pursue teaching certification. However, can this conclusion be drawn for the other ten countries, too? It would be foolish to conclude this. Nevertheless, differences can be determined. Poland and

		1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	*Total*	3,951	3,750	3,335	2,932	2,953	3,027	3,349	3,904	7,832	8,688
	Female	1,722	1,683	1,503	1,402	1,443	1,574	1,887	2,237	5,364	5,825
	Quota	43.6	44.9	45.1	47.8	48.9	52.0	56.4	57.3	68.5	67.0
France	*Total*		8,610	9,392	7,606		8,077		5,206	5,802	6,033
	Female		3,828	4,286	3,360		3,573		2,089	2,323	2,483
	Quota		44.5	45.6	44.2		44.2		40.1	40.0	41.2
England	*Total*	4,599	4,865	4,569	4,799	4,575	5,266	5,422	5,271	5,500	5,645
	Female	1,273	1,320	1,824	1,988	1,858	2,209	2,322	2,195	2,264	2,302
	Quota	27.7	27.1	39.9	41.4	40.6	42.0	42.8	41.6	41.2	40.8
Sweden	*Total*	1,129	152	149	157	135	146	224	233	242	255
	Female	287	58	54	73	50	51	103	87	89	102
	Quota	25.4	38.2	36.2	46.5	37.0	34.9	46.0	37.3	36.8	40.0
Spain	*Total*	2,625	3,020	2,864	2,672	2,445	2,027	1,977	1,762	1,436	1,280
	Female	1,321	1,617	1,633	1,497	1,367	1,160	1,171	989	847	705
	Quota	50.3	53.5	57.0	56.0	55.9	57.2	59.2	56.1	59.0	55.1
Portugal	*Total*	313	593	583	725	676	706	1,028	902	866	627
	Female	186	430	425	529	467	504	723	635	634	437
	Quota	59.4	72.5	72.9	73.0	69.1	71.4	70.3	70.4	73.2	69.7
Italy	*Total*	2,986	3,123	2,806	2,972	3,343	3,401	3,281	2,895	2,496	1,936
	Female	1,976	2,015	1,770	1,883	2,014	2,091	2,003	1,731	1,426	1,094
	Quota	65.9	64.5	63.1	63.4	60.3	61.5	61.1	59.8	57.1	56.5
Turkey	*Total*		3,486	3,503	4,112	4,096	4,100	3,928	4,265	4,615	5,203
	Female		1,541	1,636	1,879	1,892	1,878	1,878	1,874	2,182	2,434
	Quota		44.2	46.7	45.7	46.2	45.8	47.8	43.9	47.3	46.8
Poland	*Total*	1,308	1,644	2,170	2,210	2,251	2,235	2,104	2,179	2,809	3,065
	Female	912	1,214	1,584	1,595	1,620	1,534	1,420	1,515	1,932	2,057
	Quota	69.7	73.8	73.0	72.2	72.0	68.6	67.5	69.5	68.8	67.1
Romania	*Total*	1,711	1,616	1,779	1,754	1,906	1,985	2,089	2,251	2,499	2,654
	Female	898	819	983	976	1,044	1,064	1,161	1,230	1,335	1,214
	Quota	52.5	50.7	55.3	55.6	54.7	53.6	55.6	54.6	53.4	45.7
USA	*Total*	12,820	12,328	12,070	11,674	12,395	12,863	13,327	14,351	14,770	14,954
	Female	5,912	5,732	5,688	5,567	5,787	5,817	6,124	6,414	6,655	6,594
	Quota	46.1	46.5	47.1	47.7	46.7	45.2	46.0	44.7	45.1	44.1

Table 3: Proportion of graduates (*isced5a*) that are women.

Portugal are unchallenged leaders with a share of women at an average of 70%. In Italy and Spain, the share of women is consistently high and amounts to more than 50%. Sweden, England, France, the USA and Turkey at no time achieve these values.

Figures of doctorates

Concerning the completed doctorates, the proportion of women is of peculiar interest. Unfortunately, Eurostat does not have figures from Poland and Romania available, leaving the nine remaining states. The proportion of women in the *level isced6* can be seen in Table 4.

		1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Germany	*Total*	466	547	523	473	465	588	429	474	508	460
	Female	103	121	120	98	101	164	120	130	126	122
	Quota	22.1	22.1	22.9	20.7	21.7	27.9	28.0	27.4	24.8	26.5
France	*Total*	424	322	334	334		272		307	344	338
	Female	85	77	84	84		66		75	89	83
	Quota	20.1	23.9	25.2	25.2		24.3		24.4	25.9	24.6
England	*Total*	306	385	347	388	393	373	415	413	450	471
	Female	59	78	75	93	89	90	99	106	121	142
	Quota	19.3	20.1	21.6	24.0	22.7	24.1	23.9	25.7	26.9	30.1
Sweden	*Total*	222	81	85	91	87	106	133	60	116	120
	Female	37	9	16	16	14	17	41	10	27	28
	Quota	16.7	11.1	18.8	17.6	16.1	16.0	30.8	16.7	23.3	23.3
Spain	*Total*	153	158	191	165	189	228	176	149	162	142
	Female	64	73	74	65	70	92	69	64	71	55
	Quota	41.8	46.2	38.7	39.4	37.0	40.4	39.2	43.0	43.8	38.7
Portugal	*Total*	124	103	106	109	144	144	221	247	308	257
	Female	57	57	59	63	85	84	127	155	183	166
	Quota	46.0	55.3	55.7	57.8	59.0	58.3	57.5	62.8	59.4	64.6
Italy	*Total*	676	704			165	217	272	275	288	301
	Female	326	323			86	92	112	126	127	132
	Quota	48.2	45.9			52.1	42.4	41.2	45.8	44.1	43.9
Turkey	*Total*		55	42	43	77	91	71	77	73	99
	Female		23	17	19	28	26	22	29	29	34
	Quota		41.8	40.5	44.2	36.4	28.6	31.0	37.7	39.7	34.3
USA	*Total*	1,174	1,259	1,106	1,024	958	1,020	1,060	1,176	1,293	1,351
	Female	283	323	276	295	278	275	298	335	382	402
	Quota	24.1	25.7	25.0	28.8	29.0	27.0	28.1	28.5	29.5	29.8

Table 4: Proportion of postgraduates (*isced6*) that are women.

On closer inspection of the data, some interesting facts emerge. Taking the Alps as a natural border in Europe, north of the Alps the proportion of women is stable at a maximum of 25%. The average values for the relevant countries are: Germany 24.4%, France 24.2%, England 23.8% and Sweden 19%. The positive aspect in England is that the quota has been growing since 2002, rising to 30.1% in 2007. Outside of Europe, the USA is about the same with an average of 27.6%. However, looking at the states south of the Alps there are totally different values. In Turkey on average 37.1% of the doctorates are assigned to women. In Spain and Italy, the average values are 40.8% and 45.5%. However, the unchallenged front runner is Portugal with an average of 57.6% of the doctorates assigned to women.

There is a remarkable regional difference in these results. The following questions suggest themselves: *What is done in the south European countries to motivate the women into doctorates? Are there special promotional programmes that do not exist in the latitudinal lines of Germany? How else can this tremendous difference be explained? What have we missed in Germany so far?* There is evidence that these tendencies are conditioned by history.

Moving away from the proportions of women in the *level isced6* and looking at the absolute figures in Table

4, there are some more interesting facts. Germany is the country with most doctorates in the *level Mathematics/ Statistics* throughout Europe. Over recent years (1998-2007), the figures of doctorates were at an annual average of 500. There is a completely different situation in Turkey. Indeed, the figures of doctorates were constant here, too, but over the period from 1999 to 2007 there were only an average of 70 doctorates per year. While the figures of doctorates were constant in most of the states that are being used as the basis of this comparison over the inquiry period (1998-2007), two countries are exceptions. In Italy, there was a fall of graduate numbers of almost 55% to the year 1998. In contrast, the opposite took place in Portugal. In 1998, there were only 124 graduates and in 2006 there were 308.

Conclusions and further research

Because of the underlying data, no explanations can be offered for most of the observations outside of Germany, although it is believed that cultural influences explain some of the differences. Because of this, some of the questions have to remain unanswered and necessitate a more detailed country-specific and thereby more complex evaluation.

Eurostat mingles in its data "real" mathematicians and teachers but these two groups are not homogeneous. It is not known how many teachers are contained in the figures of the distinct countries. Surveys only make sense if the considered populations are comparable.

During this research, it was found that the figures of Eurostat rely on political agreements of the EU member states with the objective of drawing comparisons between these countries. As a matter of fact, a more detailed analysis based on these figures remains impossible. Despite this, this research portrays a first overview of figures concerning the studying of mathematics at universities. As a consequence of this, the mathematical associations may be encouraged to expand on this preliminary approach of collecting international figures.

The reader is invited to confer with us if questions arise. Further suggestions are also welcomed.

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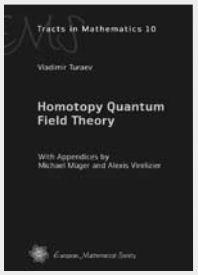

Vladimir Turaev (Indiana University, Bloomington, USA)
Homotopy Quantum Field Theory
(EMS Tracts in Mathematics Vol. 10)

ISBN 978-3-03719-086-9. 2010. 290 pages. Hardcover. 17 x 24 cm. 58.00 Euro

This book is the first systematic exposition of homotopy quantum field theory. It starts with a formal definition of an HQFT and provides examples of HQFTs in all dimensions. The main body of the text is focused on 2-dimensional and 3-dimensional HQFTs. A study of these HQFTs leads to new algebraic objects: crossed Frobenius group-algebras, crossed ribbon group-categories, and Hopf group-coalgebras. These notions and their connections with HQFTs are discussed in detail. The text ends with several appendices including an outline of recent developments and a list of open problems. Three appendices by M. Müger and A. Virelizier summarize their work in this area.

The book is addressed to mathematicians, theoretical physicists, and graduate students interested in topological aspects of quantum field theory. The exposition is self-contained and well suited for a one-semester graduate course. Prerequisites include only basics of algebra and topology.

Memories of Paulette Libermann

Marc Chaperon

This article is a free translation of the article by the same author entitled "Souvenirs de Paulette Libermann", which appeared online at "Images des Mathématiques" (http://images.math.cnrs.fr/Souvenirs-de-Paulette-Libermann.html).

Mademoiselle Libermann, as everyone called her, left us on 10 July 2007, after an exceptionally long and fruitful mathematical life. For example, till the end she actively participated in our seminar of Hamiltonian geometry, organised by her friend and collaborator Charles-Michel Marle; during the ensuing lunches, we benefited from her remarkable knowledge of mathematics and mathematicians but also from her stimulating vision of the world.

I do not think she had changed substantially nor lost in vivacity since her admission in 1938 to the École Normale Supérieure de Jeunes Filles. In those days, the "École de Sèvres", or "École Normale Supérieure de Jeunes Filles", was exclusively for women students and prepared them for the competition of the *agrégation* so that they could become teachers or, more rarely, researchers.

The picture represents a group of "Sévriennes"[1] around their professor, the mathematician André Lichnerowicz. Paulette Libermann is the small girl on the extreme right.

All her life, she had retained the spirit of childhood – with more merit than others as her studies at Sèvres and her life in France during the German occupation had been spoiled by the persecution of Jews. Already in October 1940, the anti-Semitic laws of the Vichy government denied them access to the civil service and therefore to teaching. That year, Paulette Libermann was beginning to train for the *agrégation*, a competition that she could pass only after the liberation of France; in the meantime she survived by giving private lessons in Lyon.

As Agatha Christie's famous "little lady", she hid an intrepid temperament under a reassuring and somewhat outmoded appearance. The first *Sévrienne* to defend a thesis in mathematics and obtain a chair at the university, she began as a student of Élie Cartan, who supported and helped her during the occupation, suggesting to her to launch into research since it was forbidden for her to teach. Her thesis, written under the supervision of Charles Ehresmann, was defended at the University of Strasbourg in 1953.

Her masters Élie Cartan and Charles Ehresmann were two of the boldest designers of differential geometry. The thesis of Paulette (as we called her … when she was not around) contained notions and results that were often "rediscovered" 20 or 30 years later, on almost complex structures as well as contact or symplectic geometry; to the latter, she and Marle devoted a book [1],[2] which is still a reference in the subject 20 years after its publication.

She was quietly daring but also very faithful: to Élie Cartan of course, whose vision of the world was perpetuated by her seminar with Yvette Kosmann, during a period where his viewpoint was a bit out of fashion;[3] and

[1] Students of the École de Sèvres.

[2] Its style is somewhat in opposition to that of the great Russian mathematician Vladimir Arnol'd, who himself had reacted against his rather formalist education by a mathematician father.

[3] Due to glorious successes in the topological understanding of manifolds, spaces that are extremely simple locally.

Students of the École de Sèvres (Paulette Libermann's family archives). Paulette Libermann is standing on the extreme right on the first row.

even more significantly to Ehresmann, gradually marginalised[4] after having occupied a central focus in the mathematics of his time. At the Kosmann-Libermann seminar, the *a fortiori* marginalised students of this great mind could express themselves. Paulette Libermann was in a good position to understand the meaning of both their work and exclusion. Perhaps due to the help that she had received from Élie Cartan as a beginner, she was always ready to welcome and encourage previously unknown young mathematicians, including Michèle Audin, Daniel Bennequin, Jean-Pierre Françoise and the author of this article.

Her work was centred on what Élie Cartan called the "equivalence problem". The idea is easy to grasp: even though coordinates are needed to *compute*, e.g. trajectories or positions, it is important to determine the notions that do not depend on the choice of the frame providing the coordinates. A phenomenon depending on such a particular choice would not have any "physical" meaning.[5] This is why, under the influence of general relativity among other things, scientists became gradually convinced of the significance of spaces called *differentiable manifolds*, whose very definition implies that they are independent from the choice of local coordinates enabling computations.

Exceptionally gifted, Élie Cartan did not need a formal definition to know such spaces and even explore them in his moving frames. Paulette Libermann entered mathematics at the time where, having finally defined manifolds, mathematicians were forging a whole arsenal of concepts enabling computations independent of the choice of local coordinates, now called calculus on manifolds. Being in the forge of Master Ehresmann and benefiting from Élie Cartan's advice, she was in the best possible position to contribute to these developments but this required much talent.

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See also the obituary for Paulette Libermann by Michèle Audin in number 66 (2007).

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[4] Historians will have to determine his responsibilities and those of the mathematical community in the process.

[5] Thus, "singularities of space-time" were sometimes discovered that were just singularities of the coordinate system, comparable to the horizon line, which exists only in the frame associated to each one of us.

André Lichnerowicz Prize in Poisson geometry – 2010

The André Lichnerowicz Prize was established in 2008 to be awarded for notable contributions to Poisson geometry. The prize is to be awarded every two years at the "International Conference on Poisson Geometry in Mathematics and Physics" to researchers who have completed their doctorates at most eight years before the year of the conference.

The prize for 2010 was awarded to **Marco Gualtieri** (University of Toronto) and **Xiang Tang** (Washington University in Saint Louis).

Marco Gualtieri

received his doctorate at Oxford University in 2004 under the direction of Nigel Hitchin. His pioneering work on generalised complex geometry has been the source of inspiration for many related studies. More recently, he has studied D-branes in generalised complex manifolds and their relation to noncommutative geometry, as well as further generalisations of classical geometries.

Xiang Tang

completed his PhD in mathematics at the University of California at Berkeley in 2004 under the direction of Alan Weinstein. Among his important contributions are a new proof of the Atiyah-Weinstein conjecture on the index of Fourier integral operators and the relative index of CR structures, the study of non-commutative Poisson structures on orbifolds, the study of various Hopf-like structures and the index theory on orbifolds.

Report on the fundamental lemma

P.-H. Chaudouard, M. Harris and G. Laumon

The statement known as the fundamental lemma, a collection of identities in harmonic analysis on reductive groups over local fields, evolved in the course of Langlands' attempt to use the Arthur-Selberg trace formula to establish important special cases of his *functoriality conjecture*, the heart of his conjectural reorganization of number theory, representation theory and automorphic forms under a single heading. Thanks to the work of many mathematicians (see Section 8), and especially Bao Châu Ngô, the fundamental lemma is now a theorem. In this article, we explain Langlands' approach to the trace formula and the central role of the fundamental lemma in completing what might be considered the first stage of the Langlands program. The completion of this stage will unquestionably be rich in applications to algebraic number theory and arithmetic geometry; some of the applications obtained thus far will be described briefly here. The article concludes with a rapid sketch of Ngô's proof of the fundamental lemma, a remarkable synthesis of automorphic methods with techniques of geometric representation theory (the affine Hecke algebra) and algebraic geometry (the Hitchin fibration in the theory of moduli of Higgs bundles).

1 The Selberg trace formula

Let G be a locally compact group that admits a Haar measure $\mathrm{d}g$, e.g. $G = \mathrm{SL}(2,\mathbb{R})$.

Let Γ be a discrete subgroup of G, e.g. $\Gamma = \mathrm{SL}(2,\mathbb{Z})$. Then $\underline{X} = \Gamma\backslash G$ can be equipped with the right G-invariant measure $\mathrm{d}\underline{g}$ that is induced by $\mathrm{d}g$. For simplicity, let us assume that X is *compact*.

The group G acts by right translations on X and thus on the space $L^2(X)$ of square integrable complex functions on X. For every $f \in C_c^\infty(G)$ one has the operator

$$\rho(f) : \varphi \mapsto \rho(f)(\varphi)(x) = \int_G \varphi(x\cdot g) f(g)\mathrm{d}g$$

on $L^2(X)$.

Let $\widehat{G}$ be a system of representatives of the equivalence classes of irreducible continuous representations of G in Hilbert spaces.

The representation ρ of G on $L^2(X)$ can be split into a discrete Hilbert sum of the representations $\pi \in \widehat{G}$ with finite multiplicities $m(\pi)$ and one has

$$\mathrm{tr}\rho(f) = \sum_{\pi\in\hat{G}} m(\pi)\mathrm{tr}\pi(f).$$

For any $\gamma \in \Gamma$ let

$$G_\gamma = \{g \in G \mid g^{-1}\gamma g = \gamma\}$$

be its centralizer in G. It also admits a Haar measure $\mathrm{d}g_\gamma$. For every $f \in C_c^\infty(G)$, one has the *orbital integral*

$$\mathrm{O}_\gamma(f, \mathrm{d}g_\gamma) = \int_{G_\gamma\backslash G} f(g^{-1}\gamma g)\frac{\mathrm{d}g}{\mathrm{d}g_\gamma},$$

and the volume

$$v(\gamma, \mathrm{d}g_\gamma) = \mathrm{vol}((\Gamma\cap G_\gamma)\backslash G_\gamma, \mathrm{d}g_\gamma).$$

Theorem 1 (Selberg)**.** *Let $\widetilde{\Gamma}$ be a system of representatives of the conjugacy classes in Γ. Then the following equality*

$$\sum_{\gamma\in\widetilde{\Gamma}} v(\gamma, \mathrm{d}g_\gamma)\mathrm{O}_\gamma(f, \mathrm{d}g_\gamma) = \sum_{\pi\in\widehat{G}} m(\pi)\mathrm{tr}\pi(f)$$

holds.

If G is finite and $\Gamma = \{1\}$, this is nothing else than the Frobenius reciprocity law.

In general X is not compact but in the interesting cases it is of finite volume, and then a more general trace formula holds. It is due to Selberg for SL(2) and to Arthur in general. It is impossible to explain the details without introducing a vast quantity of notation but the structure of the formula is the same as in the compact case: the right (*spectral*) side contains the multiplicities $m(\pi)$ – the information of interest; the left (*geometric*) side is expressed in terms of data that can, in principle, be computed.

2 The Langlands program

Langlands has elaborated a comprehensive program for classifying automorphic representations. In particular he has conjectured the *principle of functoriality* linking automorphic representations of different groups.

One of the goals of the Langlands functoriality program is to relate the multiplicities $m(\pi)$ for automorphic representations of a pair of groups G, G' related by functoriality. Although the geometric sides of the trace formulas for the two groups can rarely be calculated explicitly, they can often be compared and this leads to comparisons of the multiplicities.

To carry out this comparison one first needs to *stabilize* the Arthur-Selberg trace formula for both groups. This highly sophisticated process involves a series of combinatorial identities of orbital integrals, all of which can be deduced from a basic collection of identities that form the so-called *fundamental lemma*.

The stabilization of the trace formula and thus the fundamental lemma are also required for arithmetic applications, such as the computation of the Hasse-Weil zeta functions of Shimura varieties.

3 *p*-adic orbital integrals

Let p be a prime number and let $F = \mathbb{Q}_p$ be the field of p-adic numbers or, more generally, a finite extension of $\mathbb{Q}_p$.

The field F comes with the p-adic topology for which $O = \mathbb{Z}_p \subset \mathbb{Q}_p$ (or more generally the integral closure O of $\mathbb{Z}_p$ in F) is an open compact subring.

If $\mathcal{G}$ is a semisimple linear algebraic group ($\mathcal{G} = \mathrm{SL}(n)$, $\mathrm{SO}(n)$, $\mathrm{Sp}(2n)$, ...), the p-adic topology on F induces a topology on $G = \mathcal{G}(F)$. For this topology, G is a locally compact group and $K = \mathcal{G}(O)$ is an open compact subgroup of G that is maximal for these properties.

Regular semisimple elements of G are the elements that have "distinct eigenvalues". Orbital integrals at regular semisimple elements of locally constant complex functions on G with compact supports can be defined as before. The most basic ones are those for the characteristic function 1_K of K in G,

$$\mathrm{O}_\gamma(1_K) = \int_{G_\gamma\backslash G} 1_K(g^{-1}\gamma g)\frac{\mathrm{d}g}{\mathrm{d}g_\gamma},$$

where $\gamma \in G$ is any regular semisimple element and the Haar measure $\mathrm{d}g$ is normalized by $\mathrm{vol}(K, \mathrm{d}g) = 1$. These are the integrals that are compared in the statement of the fundamental lemma.

It is easy to check that the above integral is a finite sum,

$$\mathrm{O}_\gamma(1_K, \mathrm{d}g_\gamma) = \sum_g \frac{1}{\mathrm{vol}(G_\gamma \cap gKg^{-1}, \mathrm{d}g_\gamma)},$$

where g runs through a system of representatives of the double classes in the finite set

$$G_\gamma\backslash\{g \in G \mid g^{-1}\gamma g \in K\}/K.$$

4 Stable conjugacy

Let k be a field and $\overline{k}$ be an algebraic closure of k. Following Langlands, one says that two regular semisimple elements $\gamma, \gamma' \in \mathcal{G}(k)$ are *stably conjugate* if there exists $\overline{g} \in \mathcal{G}(\overline{k})$ such that $\gamma' = \overline{g}\gamma\overline{g}^{-1}$.

If $\mathcal{G} = \mathrm{GL}(n)$, two regular semisimple elements $\gamma, \gamma' \in G$ that are stably conjugate are automatically conjugate: one can find $g \in \mathcal{G}(k)$ such that $\gamma' = g\gamma g^{-1}$.

This is no longer true if $\mathcal{G}$ is $\mathrm{SL}(n)$, $\mathrm{SO}(n)$ or $\mathrm{Sp}(2n)$. For most reductive groups $\mathcal{G}$, two regular semisimple elements in $\mathcal{G}(k)$ can be stably conjugate without being conjugate. The simplest example is given by $k = \mathbb{R}$, $\mathcal{G} = \mathrm{SL}(2)$ and the two matrices

$$\gamma = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \text{and} \quad \gamma' = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

which are conjugated by $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \in \mathrm{SL}(2, \mathbb{C})$ but which are not conjugate in $\mathrm{SL}(2, \mathbb{R})$.

5 κ-integral orbitals

Let F be as before a p-adic field and let $\gamma \in \mathcal{G}(F)$ be regular semisimple.

The stable conjugacy class of γ is a finite union of ordinary conjugacy classes. This finite union can be indexed by a finite abelian group $R(\gamma)$ in a unique way.[1] Let us denote by $(\gamma_r)_{r\in R(\gamma)}$ a system of representatives of the conjugacy classes inside the stable conjugacy class of γ. The Haar measure dg_γ induces Haar measures dg_{γ_r} on G_{γ_r} for every r.

For every character $\kappa : R(\gamma) \to \mathbb{C}^\times$, one has the *$\kappa$-orbital integral*

$$\mathrm{O}_\gamma^{G,\kappa}(1_K, dg_\gamma) = \sum_{r\in R(\gamma)} \kappa(\gamma_r)\mathrm{O}_{\gamma_r}(1_K, dg_{\gamma_r}).$$

For $\kappa \equiv 1$, the κ-orbital integral is also called the *stable orbital integral*

$$\mathrm{SO}_\gamma^G(1_K, dg_\gamma) = \sum_{r\in R(\gamma)} \mathrm{O}_{\gamma_r}(1_K, dg_{\gamma_r}).$$

6 Endoscopic groups and the fundamental lemma

Semisimple (and more generally reductive) algebraic groups are classified by their root systems $(X, \Phi, X^\vee, \Phi^\vee)$, where X and $X^\vee$ are dual free abelian groups of finite rank, $\Phi \subset X$ is a set of roots and $\Phi^\vee \subset X^\vee$ is a set of co-roots satisfying the usual axioms of a root system.

The *Langlands dual* of a semisimple algebraic group $G(F)$ as before is the complex semisimple (or reductive) group whose root system $(X^\vee, \Phi^\vee, X, \Phi)$ is the dual of the root system $(X, \Phi, X^\vee, \Phi^\vee)$ of $\mathcal{G}$. For example the Langlands dual of $\mathrm{SL}(n, F)$ is $\mathrm{PGL}(n, \mathbb{C})$ and the Langlands dual of $\mathrm{SO}(2n+1, F)$ is $\mathrm{Sp}(2n, \mathbb{C})$ and vice-versa. The *L-group* ${}^L G$ of $G(F)$ is the semidirect product of the Langlands dual with the absolute Galois group (or Weil group) of F with respect to a certain natural action of the latter on the Langlands dual.

Using his duality, Langlands has attached to the group $G(F)$ other semisimple algebraic groups $H(F)$, which are called the *endoscopic groups*. For example $H(F) = \mathrm{SO}(2n_1+1, F)\times\mathrm{SO}(2n_2+1, F)$ is an endoscopic group of $\mathcal{G} = \mathrm{SO}(2n+1, F)$ for any non trivial partition $n = n_1 + n_2$.

The *Langlands-Shelstad fundamental lemma* – more precisely the Langlands-Shelstad fundamental lemma for endoscopy – is a series of identities

$$\mathrm{O}_\gamma^{G,\kappa}(1_K, dg_\gamma) = \Delta_H^G(\gamma, \delta)\mathrm{SO}_\delta^H(1_{K^H}, dh_\delta). \qquad \text{(FL)}$$

Here, $\mathcal{H}$ is an endoscopic group of $\mathcal{G}$ determined by κ and $K^H = \mathcal{H}(\mathcal{O})$, $\delta \in H = \mathcal{H}(F)$ and $\gamma \in G$ are regular semisimple elements that match, i.e. "they have the same characteristic polynomial". The Haar measure dh_δ on H_δ is induced by dg_γ.

Finally $\Delta_H^G(\gamma, \delta)$ is the so-called *transfer factor*: it is a power of p, easy to define, multiplied by a constant, typically a root of unity, whose definition is explicit but quite intricate. In particular, the constant naturally incorporates the group $R(\gamma)$ mentioned in the previous section. Thanks to this property of the transfer factor, the κ-orbital integrals, as κ varies, can be viewed as a Fourier transform over the conjugacy classes in a stable conjugacy class.

7 Example

Let us give the simplest possible non-trivial example of the fundamental lemma for Lie algebras in equal characteristic. Let $\mathcal{G}$ be $\mathrm{SL}(2)$ over $F = \mathbb{F}_p((t))$. Let $a \in \mathbb{F}_p$ that is not a square. Then the two elements

$$\gamma = \begin{pmatrix} 0 & at^2 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \gamma' = \begin{pmatrix} 0 & at \\ t & 0 \end{pmatrix}$$

in $\mathrm{SL}(2, F)$ are stably conjugate but not conjugate. The union of their two conjugacy classes is the stable conjugacy class of γ, which is the Kostant representative (it is a companion matrix – see note 1).

Then one checks that for suitable normalization of the Haar measures, one has

$$\mathrm{O}_\gamma(1_K) = p + 1 \text{ and } \mathrm{O}_{\gamma'}(1_K) = 1,$$

so that,

$$\mathrm{O}_\gamma^{G,\kappa}(1_K) = (p+1) - 1 = p.$$

In this particular case, the transfer factor is $\Delta_H^G(\gamma, \delta) = p$ and the endoscopic stable orbital integral is $\mathrm{SO}_\delta^H(1_{K^H}) = 1$.

8 History of the subject

The first occurrence of the fundamental lemma for endoscopy is in a 1979 paper by Labesse and Langlands on automorphic forms for SL(2). At a certain point the authors found they needed a rather technical lemma. This lemma, verified by direct computation, plays a fundamental role in the paper, hence the terminology attached to the conjecture of Langlands and Shelstad, which was first formulated in complete generality in 1987.

Kottwitz (1992) and Rogawski (1990) proved the fundamental lemma for U(3). In 1991, Waldspurger proved the fundamental lemma for SL(n). Hales (1997) and Weissauer (1993) proved the fundamental lemma for Sp(4).

In 1997, Waldspurger introduced a variant of the fundamental lemma for Lie algebras that is easier to formulate and showed that this variant implies the version for the group. In 2005, Waldspurger showed that the obvious analogue of the fundamental lemma, in which the p-adic field F is replaced by a finite extension of $\mathbb{F}_p((t))$, implies the fundamental lemma over a p-adic field.

These last two results of Waldspurger provide the starting point for any geometric approach to the fundamental lemma.

Results obtained by geometric methods

Using the ℓ-adic equivariant cohomology of affine Springer fibers, Goresky, Kottwitz and MacPherson proved the Langlands-Shelstad fundamental lemma for unramified elements γ, all of whose eigenvalues (roots) have the same valuation.

Using the ℓ-adic equivariant cohomology of the Hitchin fibration, Laumon and Ngô completely proved the Langlands-Shelstad fundamental lemma for unitary groups U(n) assuming that $p > n$.

Finally, again using the ℓ-adic cohomology of the Hitchin fibration, Ngô has proved the Langlands-Shelstad fundamental lemma for general reductive groups provided that p does not divide the order of the Weyl group of $\mathcal{G}$. We say a few words about this proof in the final section.

9 Applications to the Langlands program

Some years ago, Arthur showed how to *stabilize* the Arthur-Selberg trace formula for any reductive group G over a number field, assuming the Langlands-Shelstad fundamental lemma and a variant known as the *weighted fundamental lemma*. The latter has now been proved by Chaudouard and Laumon, building on Ngô's work on the Hitchin fibration. Thus Arthur's stabilization is complete and it is possible to compare stable trace formulas for related groups in a number of situations.

Let G and H be two reductive groups over a local or global field F. An *L-homomorphism* is a homomorphism ${}^{\mathrm{L}}H \to {}^{\mathrm{L}}G$ that commutes with the natural projection to the Galois group. Langlands' functoriality conjectures predict that any such L-homomorphism gives rise to a *functorial transfer* of representations from H to G. If F is local, the transfer takes irreducible representations of $H(F)$ to irreducible representations of $G(F)$. The transfer is not in general defined on individual representations but rather takes a finite collection of representations, called an *L-packet*, for $H(F)$ to an L-packet on $G(F)$. If F is a global field then the transfer takes an L-packet of automorphic representations of H to an L-packet on $G(F)$.

Let $G = GL(n)$; then the Langlands dual of G is $GL(n, \mathbb{C})$ and each L-packet of G, local or global, is in fact a singleton. If now H is a classical group, there is a natural L-homomorphism ${}^{\mathrm{L}}H \to {}^{\mathrm{L}}GL(N)$ for appropriate N. For example, the Langlands dual of $H = SO(2n + 1)$ is $Sp(2n, \mathbb{C})$ and we take $N = 2n$.

By comparing the stable trace formula for a classical group H with the stabilized *twisted trace formula* (see below) for the corresponding $GL(N)$, Arthur has constructed a version of the expected Langlands functorial transfer from H to $GL(N)$. More precisely, he is able to classify irreducible representations of $H(F)$ when F is a p-adic field, and discrete automorphic representations of H when F is a number field, in terms of the corresponding objects for $GL(N)$. He is thus able to derive the Langlands parametrization for classical groups over p-adic fields from that for $GL(N)$, proved just over ten years ago by Harris-Taylor and Henniart. In a similar way, he can classify discrete automorphic representations of a classical group H in terms of cuspidal automorphic representations of varying $GL(n)$s.

The twisted trace formula is a variant of the Arthur-Selberg trace formula for a disconnected reductive group, in this case the semi-direct product of $GL(N)$ by the non-trivial outer automorphism. Its stabilization is expected to be possible along the lines of that carried out by Arthur in the standard case. Twisted analogues of most of the relevant fundamental lemmas have been established by Waldspurger and Ngô.

10 Applications to number theory

Following a strategy developed by Langlands and Kottwitz, the fundamental lemma is used in two separate versions – the Langlands-Shelstad version for endoscopy and a second twisted version – to compute the Hasse-Weil zeta functions of Shimura varieties. When the Shimura varieties can be identified with the moduli spaces for certain families of algebraic varieties with additional structure, as is known to be the case for most Shimura varieties attached to classical groups, the numbers of points on these varieties over finite fields can be expressed in terms of explicit orbital integrals, and the resulting expression can be stabilized just like the Arthur-Selberg trace formula.

Now that the relevant fundamental lemmas have been established, the computations explained by Kottwitz over 20 years ago can be completed. S. Morel has carried out this program completely for the Shimura varieties of PEL type attached to unitary groups. These are moduli spaces for polarized abelian varieties with additional endomorphisms respecting the polarization.

The methods used to calculate the Hasse-Weil zeta functions for Shimura varieties attached to unitary groups also permit the construction of ℓ-adic representations of Galois groups of number fields attached to certain classes of automorphic forms, generalizing the classical theory of Eichler and Shimura to dimension > 2 and to number fields other than $\mathbb{Q}$. This is the starting point for the generalization of the methods introduced by Wiles, in his proof of Fermat's Last Theorem, to arbitrary dimension. The proof of the Sato-Tate

conjecture for modular forms of arbitrary weight is a typical consequence of these results, none of which could have been proved without the new understanding of the cohomology of Shimura varieties provided by the fundamental lemma.

11 Remark on the proof of the fundamental lemma

Thanks to the work of Waldspurger, it suffices to prove the fundamental lemma for Lie algebras over a local field F of characteristic $p > 0$. Such a local field can be viewed as the completion at a closed point of a smooth projective curve C over a finite field $\mathbb{F}$.

In the process of stabilization of the trace formula, the fundamental lemma implies identities between linear combinations of global orbital integrals (over the function field of C) that form some sort of *global fundamental lemma*. Conversely the global fundamental lemma implies the local one by a classical global-to-local argument.

The *Hitchin moduli stack* $\mathcal{M}$ is an algebraic stack over $\mathbb{F}$ classifying G-bundles E over C endowed with a *Higgs field* θ, which is roughly a section of the vector bundle $\mathrm{Ad}(E)$ over C (induced by E via the adjoint representation) with poles along a given effective divisor D. The *Hitchin fibration* is the map from $\mathcal{M}$ to an affine space $\mathbb{A}$ that associate to the pair (E, θ) a finite collection of invariants of θ generalizing the coefficients of the characteristic polynomial of an endomorphism.

In 2003, using Weil's adelic description of G-bundles, Ngô Bao Châu remarked that global orbital integrals naturally occur when one counts the number of points of the fibers of the Hitchin map. Moreover, appealing to the Grothendieck dictionary between functions and sheaves, and making systematic use of the decomposition theorem of Beilinson, Bernstein, Deligne and Gabber, he showed that the global fundamental lemma is a consequence of a very particular property of the relative ℓ-adic cohomology of the Hitchin map.

This property can be summarized in the following way: despite the very high dimension of the Hitchin fibers, the Hitchin map cohomologically behaves in some sense as if the fibers were all finite (at least over the elliptic locus).

In 2008, Ngô proved this cohomological property as a consequence of: a geometric property of the Hitchin map that is reminiscent of a theorem for plane curves due to Severi (in the projective space of plane curves of degree d, and thus arithmetic genus $q = (d-1)(d-2)/2$, the curves of geometric genus $g = q - \delta$ form a locally closed subset of codimension δ); and a cohomological property of a variety equipped with a free action of an abelian variety (the cohomology of the abelian variety acts freely on the cohomology of the variety).

Note

1. The origin corresponds to the so-called *Kostant ordinary conjugacy class* inside the stable conjugacy class. Kostant's construction generalizes the section of the map from matrices to their characteristic polynomials, which, to a monic polynomial of degree n, associates its companion matrix. The Kostant representative plays a crucial role in the study of the Hitchin fibration.

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The History of Mathematics in the Czech Republic

Martina Bečvářová[1]

In the Czech lands, there is a long and fruitful tradition of research and study of the history of mathematics that began in the second half of the 19th century. The most important papers and books were written by J. Smolík, F. J. Studnička, J. Úlehla, K. Rychlík and Q. Vetter. But from the 1950s to the 1980s only a few professionals from the Institute of History of the Academy of Sciences of the Czech Republic, along with a few university professors, devoted their attention to the history of mathematics[2]. For example, we can mention two historians of mathematics J. Folta and L. Nový, whose papers and activities became well-known (also) in Europe. But on account of various professional and political events, no new generation of historians of mathematics was ever raised. The first step to the new development of research in the history of mathematics was made in the 1980s when the special *commission on history of mathematics* was created at the Faculty of Mathematics and Physics of Charles University, thanks to activities of J. Bečvář, I. Netuka and J. Veselý.

The situation only changed after 1990 when the new discipline of postgraduate studies, *History and Didactics of Mathematics and Computer Science*, was accredited at the Faculty of Mathematics and Physics of Charles University in Prague and at the Faculty of Natural Sciences of Masaryk University in Brno. In Prague, PhD studies were opened in the academic year 1992/1993. An individual study plan is prepared for each student. It contains a large background and deeper study in the chosen area, as well as a section directly connected with the proposed thesis topic. At present, there are 29 PhD students and 32 already defended dissertations, most of them specialising in the history of mathematics.2

A further important stimulation is the possibility of reporting on one's work and presenting one's results at special events, such as the regular *International Conference on the History of Mathematics* (this year it will held for the 31st time), the Seminar in the History of Mathematics, which takes place in Prague and Brno,[3] and the special methodological seminars focused on specific problems that appear in the course of work on the history of mathematics. Another seminar, dealing with the history of mathematics, computer science and astronomy, is organised at the Czech Technical University.[4] The conference was established in 1980 by the *School on the History of Mathematics* and it was predominantly focused on professors from universities preparing future teachers of mathematics for secondary schools and high schools. Its main aim was to give them enough material and to supplement and increase their knowledge to enable them to teach the newly created subject *Philosophical Problems of Mathematics* (later on titled History of Mathematics), which became a compulsory part of the curriculum for the updated education of future teachers. In the 1990s, the conference noticeably changed its character because of completely new participants (undergraduate students, PhD students and teachers from universities, as well as professional historians of mathematics) who participated at the conference to present their investigations, studies, results and works. Since 2003, the conference has been titled *International Conference on the History of Mathematics* and every year more than 60 participants from the Czech Republic, Slovakia and Poland, and sometimes also from Germany, Russia, Ukraine and Italy, attend the conference and present their research or give invited plenary lectures on the development of some discipline of mathematics or some mathematical problem from an historical perspective. In order to document the contents of the conference, the proceedings, containing extended versions of the individual contributions, is published every year (since 2006) thanks to the financial support of the Faculty of Mathematics and Physics of Charles University in Prague.[5]

A great encouragement for a young, incipient researcher is the possibility of having their results published. In the case of more extensive work in the history of mathematics, this is a problem. Thanks to *Jindřich Bečvář* from the Faculty of Mathematics and Physics of Charles University and to *Eduard Fuchs* from the Faculty of Natural Sciences of Masaryk University, a publication series entitled The *History of Mathematics* was established in 1994. This series makes it possible to publish both shorter and longer works, as well as entire monographs and textbooks on the history of mathematics in the Czech, English and Slovak languages.

The editorial board is composed of Czech and Slovak mathematicians and historians of mathematics: *Jindřich Bečvář* and *Ivan Netuka* (Faculty of Mathematics and Physics of Charles University in Prague), *Martina Bečvářová, Magdalena Hykšová* and *Miroslav*

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2 For more information see http://www.karlin.mff.cuni.cz/~becvar/pgs/pgs.htm.

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4 See http://mat.fsv.cvut.cz/SEDM.

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At present, the series has 43 volumes (37 in Czech or Slovak and 6 in English), which can loosely be divided into seven groups. The first group consists of seven monographs devoted to the evaluation of the scientific and pedagogical work of leading Czech mathematicians of the second half of the 19th century and the first half of the 20th century (František Josef Studnička, Emil Weyr, Eduard Weyr, Jan Vilém Pexider, Karel Rychlík, Vladimír Kořínek and Ladislav Svante Rieger). The second group demonstrates several interesting topics from the history of mathematics (mathematics in Egypt, Mesopotamia and old China, Greek mathematics, mathematics in the Middle Ages and Renaissance in Europe, and European mathematics in the 16th and 17th centuries). The volumes from the third group analyse the development of certain mathematical disciplines (e.g. integral calculus, graph theory, probability theory, number theory, product integration, the geometry of curves, discrete optimisation, lattice theory and linear algebra). The fourth group describes the development of mathematical research, schools and mathematical education, teaching methods and textbooks, and the establishment and evolution of some communities and associations in the past in the Czech lands (mathematics at the Jesuit Clementinum in the years 1600–1740, the birth and the first decade of the Union of Czech Mathematicians, the development of the German Technical University in Brno, the Czech mathematical community from 1848 to 1918, and the role of some Czech mathematicians in the development of mathematics in the Balkans). The fifth group contains the commented transcriptions of unknown or forgotten mathematical manuscripts (Czech versions of Euclid's Elements from the 1880s and the first decade of the 20th century, and Jarník's notebook of the Göttingen mathematical lecture course given by P. S. Aleksandrov in the academic year 1927/1928). The sixth group presents the Czech translations of classical mathematical works (the old Egyptian hieratic mathematical texts and the Mathematics in Nine Chapters) and some unique personal memories (Em. Weyr's diary describing his studying stay and mathematical work in Italy and his contacts with Italian mathematicians around 1870). The seventh group offers proceedings of some national conferences on the history of mathematics showing various relations and connections between mathematics and art, architecture, geography, techniques, etc.

It should be mentioned that some volumes are extended versions of PhD or habilitation theses and others contain results of many research projects from the last twenty years. Most are unique contributions to our understanding of the development of mathematics and would be of interest not only to mathematicians but also to historians, linguists and anyone who wants to learn about mathematics and mathematical thinking in the past.

The series has a non-commercial character. It was and is supported in part by projects financed by the Czech Ministry of Education, the Czech Science Foundation, the Grant Agency of the Czech Academy of Science, the Grant Agency for the Development of Czech Universities, the Research Center for the History of Sciences and Humanities,[6] the Faculty of Mathematics and Physics of Charles University, the Faculty of Transportation Sciences of the Czech Technical University in Prague and the Czech Mathematical Society.[7]

5 For more information see http://www.fd.cvut.cz/personal/becvamar/konference/hlavnindex.html.

6 The *Research Center for the History of Sciences and Humanities* was founded by the Institute for Contemporary History of the Academy of Sciences of the Czech Republic and Charles University in Prague in 2000. It existed until 2004 and had four scientific modules: the module for the history of natural sciences, the module for the history of social sciences and humanities, the module for the history of universities and the module for the history of scientific institutions. The first module also included a big group consisting of historians of mathematics and mathematicians interested in topics such as the history of mathematics from the Middle Ages to the present and the evaluation of scientific results of some prominent Czech mathematicians. For more information see http://www.vcdv.cas.cz/endepartments.php.

7 For more information on the series *History of Mathematics* (for example the contents and the front pages of volumes) see http://www.fd.cvut.cz/personal/becvamar/Edice/Edice.htm. The books can be ordered and bought in the bookshop *Knihkupectví Karolinum*, Celetná 18, 116 36 Praha 1, Czech Republic, and the *Department of Mathematics Education*, Faculty of Mathematics and Physics of Charles University, Sokolovská 83, 186 75 Praha 8, Czech Republic.

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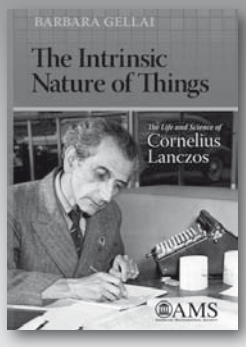

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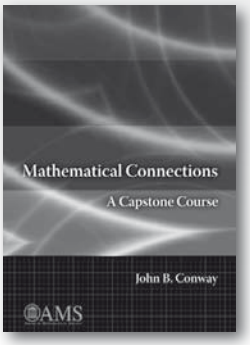

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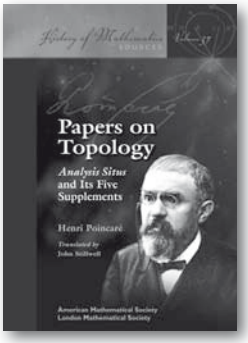

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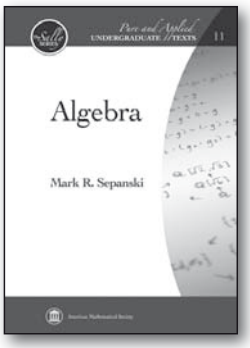

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Interview[1] with Abel laureate John Tate

Martin Raussen (Aalborg, Denmark) and Christian Skau (Trondheim, Norway)

Education

Professor Tate, you have been selected as this year's Abel Prize Laureate for your decisive and lasting impact on number theory. Before we start to ask you questions we would like to congratulate you warmly on this achievement.

You were born in 1925 in Minneapolis, USA. Your father was a professor of physics at the University of Minnesota. We guess he had some influence on your attraction to the natural sciences and mathematics. Is that correct?

It certainly is. He never pushed me in any way, but on a few occasions he simply explained something to me. I remember once he told me how one could estimate the height of a bridge over a river with a stopwatch, by dropping a rock, explaining that the height in feet is approximately 16 times the square of the number of seconds it takes until the rock hits the water. Another time he explained Cartesian coordinates, and how one could graph an equation and, in particular, how the solution to two simultaneous linear equations is the point where two lines meet. Very rarely, but beautifully, he just explained something to me. He did not have to explain negative numbers – I learned about them from the temperature in the Minnesota winters.

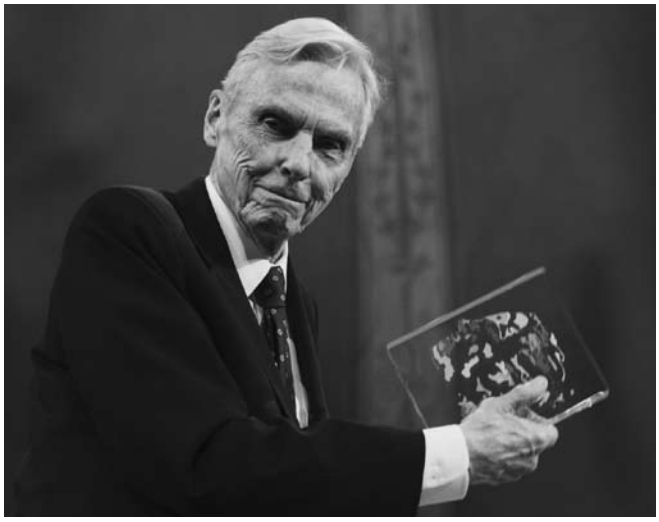
But I have always, in any case, been interested in puzzles and trying to find the answers to questions. My father had several puzzle books. I liked reading them and trying to solve the puzzles. I enjoyed thinking about them, even though I did not often find a solution.

Are there other people that have had an influence on your choice of fields of interest during your youth?

No. I think my interest is more innate. My father certainly helped, but I think I would have done something like physics or mathematics anyway.

You started to study physics at Harvard University. This was probably during the Second World War?

I was in my last year of secondary school in December 1941 when Pearl Harbour was bombed. Because of the war Harvard began holding classes in the summer, and I started there the following June. A year later I volunteered for a Naval Officer Training Program in order to avoid being drafted into the army. Some of us were later sent to MIT to learn meteorology, but by the time we finished that training and Midshipman School it was VE day.[2] Our campaign in the Pacific had been so successful that more meteorologists were not needed, and I was sent to do minesweeping research. I was in the Navy for three years and never aboard a ship! It was frustrating.



John Tate having received the Abel prize from King Harald. (Photo: Scanpix)

Study conditions in those times must have been quite different from conditions today. Did you have classes regularly?

Yes, for the first year, except that it was accelerated. But then in the Navy I had specific classes to attend, along with a few others of my choice I could manage to squeeze in. It was a good program, but it was not the normal one.

It was not the normal college social life either, with parties and such. We had to be in bed or in a study hall by ten and were roused at 6:30 AM by a recording of reveille, to start the day with calisthenics and running.

Then you graduated in 1946 and went to Princeton?

Yes, that's true. Harvard had a very generous policy of giving credit for military activities that might qualify – for instance some of my navy training. This and the wartime acceleration enabled me to finish the work for my undergraduate degree in 1945. On my discharge in 1946, I went straight from the Navy to graduate school in Princeton.

When you went to Princeton University, it was still with the intention of becoming a physicist?

That's correct. Although my degree from Harvard was in mathematics, I entered Princeton graduate school in physics. It was rather silly and I have told the story many times: I had read the book, "Men of Mathematics" by Eric Temple Bell. That book was about the lives of the greatest mathematicians in history, people like Abel. I knew I wasn't in their league and I thought that unless I was, I wouldn't really be able to do much in mathematics. I

1 This is a slightly edited version of an interview taken on the morning preceding the prize ceremony: 25 May 2010, at Oslo.

2 Victory in Europe day: 8 May 1945.

didn't realize that a less talented person could still contribute effectively. Since my father was a physicist that field seemed more human and accessible to me, and I thought that was a safer way to go, where I might contribute more. But after one term it became obvious that my interest was really in mathematics. A deeper interest, which should have been clear anyway, but I just was too afraid and thought I never would be able to do much research if I went into mathematics.

Were you particularly interested in number theory from the very beginning?
Yes. Since I was a teenager I had an interest in number theory. Fortunately, I came across a good number theory book by L.E. Dickson so I knew a little number theory. Also I had been reading Bell's histories of people like Gauss. I liked number theory. It's natural, in a way, because many wonderful problems and theorems in number theory can be explained to any interested high-school student. Number theory is easier to get into in that sense. But of course it depends on one's intuition and taste also.

Many important questions are easy to explain, but answers are often very tough to find.
Yes. In number theory that is certainly true, but finding good questions is also an important part of the game.

Teachers and fellows

When you started your career at Princeton you very quickly met Emil Artin who became your supervisor. Emil Artin was born in Austria and became a professor in mathematics at the University of Hamburg. He had to leave Germany in 1937 and came to the US. Can you tell us more about his background? Why did he leave his chair and how did he adjust when he came to the States?
His wife was half Jewish and he eventually lost his position in Germany. The family left in '37, but at that time there weren't so many open jobs in the US. He took a position at the University of Notre Dame in spite of unpleasant memories of discipline at a Catholic school he had attended in his youth. After a year or two he accepted an offer from Indiana University, and stayed there until 1946. He and his wife enjoyed Bloomington, Indiana, very much. He told me it wasn't even clear that he would have accepted Princeton's offer in 1946 except that President H.B. Wells of Indiana University, an educational visionary, was on a world tour, and somehow Indiana didn't respond very well to Princeton's offer. Artin went to Princeton the same year I did.

Artin apparently had a very special personality. First of all, he was an eminent number theorist, but also a very intriguing person – a special character. Could you please tell us a bit more about him?
I think he would have made a great actor. His lectures were polished; he would finish at the right moment and march off the scene. A very lively individual with many interests: music, astronomy, chemistry, history … He loved to teach. I had a feeling that he loved to teach anybody anything. Being his student was a wonderful experience; I couldn't have had a better start to my mathematical career. It was a remarkable accident. My favourite theorem, which I had first learned from Bell's book, was Gauss' law of quadratic reciprocity and there, entirely by chance, I found myself at the same university as the man who had discovered the ultimate law of reciprocity. It was just amazing.

What a coincidence!
Yes, it really was.

You wrote your thesis with Artin, and we will certainly come back to it. After that you organised a seminar together with Artin on Class Field Theory. Could you comment on this seminar? What was the framework and how did it develop?
During his first two years in Princeton Artin gave seminars in Algebraic Number Theory, followed by Class Field Theory. I did not attend the former, but one of the first things I heard about Artin concerned an incident in it. A young British student, Douglas Northcott, who had been captured when the Japanese trapped the British army in Singapore, and barely survived in the Japanese prison camp, was in Princeton on a Commonwealth Fellowship after the war. Though his thesis was in analysis under G.H. Hardy, he attended Artin's seminar, and when one of the first speakers mentioned the characteristic of a field, Northcott raised his hand and asked what that meant. His question begot laughter from several students, whereupon Artin delivered a short lecture on the fact that one could be a fine mathematician without knowing what the characteristic of a field was. And indeed, it turned out that Northcott was the most gifted student in that seminar.

But I'm not answering your question. I attended the second year, in which Class Field Theory was treated, with Chevalley's non-analytic proof of the second inequality, but not much cohomology. This was the seminar at the end of which Wang discovered that both published proofs of Grunwald's theorem, and in fact the theorem itself, were not correct at the prime 2.

At about that time, Gerhard Hochschild and Tadasi Nakayama were introducing cohomological methods in Class Field Theory, and used them to prove the main theorems, including the existence of the global fundamental class which A. Weil had recently discovered. In 1951-52 Artin and I ran another seminar giving a complete treatment of Class Field Theory incorporating these new ideas. That is the seminar you are asking about. Serge Lang took notes, and thanks to his efforts they were eventually published, first as informal mimeographed notes, and in 1968 commercially, under the title "Class Field Theory". A new edition (2008) is available from AMS-Chelsea.

Serge Lang was also a student of Emil Artin and became a famous number theorist. He is probably best known as the author of many textbooks; almost every graduate student in mathematics has read a textbook by Serge Lang. He is also quite known for his intense temper, and he got into a lot of arguments with people.

What can you tell us about Serge Lang? What are your impressions?

He was indeed a memorable person. The memories of Lang in the May 2006 issue of the Notices of the AMS, written by about twenty of his many friends, give a good picture of him. He started Princeton graduate school in philosophy, a year after I started in physics, but he too soon switched to math. He was a bit younger than I and had served a year and a half in the US army in Europe after the war, where he had a clerical position in which he learned to type at incredible speed, an ability which served him well in his later book writing.

He had many interests and talents. I think his undergraduate degree from Caltech was in physics. He knew a lot of history and he played the piano brilliantly.

He didn't have the volatile personality you refer to until he got his degree. It seemed to me that he changed. It was almost a discontinuity; as soon as he got his PhD he became more authoritative and asserted himself more.

It has been noted that there are many mathematical notions linked to my name. I think that's largely due to Lang's drive to make information accessible. He wrote voluminously. I didn't write easily and didn't get around to publishing; I was always interested in thinking about the next problem. To promote access, Serge published some of my stuff and, in reference, called things "Tate this" and "Tate that" in a way I would not have done had I been the author.

Throughout his life, Serge addressed great energy to disseminating information; to sharing where he felt it was important. We remained friends over the years.

Research contributions

This brings us to the next topic: your PhD thesis from 1950, when you were 25 years old. It has been extensively cited in the literature under the sobriquet "Tate's thesis". Several mathematicians have described your thesis as unsurpassable in conciseness and lucidity, and as representing a watershed in the study of number fields. Could you tell us what was so novel and fruitful in your thesis?

Well, first of all, it was not a new result, except perhaps for some local aspects. The big global theorem had been proved around 1920 by the great German mathematician Erich Hecke, namely the fact that all L-functions of number fields, abelian L-functions, generalizations of Dirichlet's L-functions, have an analytic continuation throughout the plane with a functional equation of the expected type. In the course of proving it Hecke saw that his proof even applied to a new kind of L-function, the so-called L-functions with Grössencharacter. Artin suggested to me that one might prove Hecke's theorem using abstract harmonic analysis on what is now called the adele ring, treating all places of the field equally, instead of using classical Fourier analysis at the archimedean places, and finite Fourier analysis with congruences at the p-adic places as Hecke had done. I think I did a good job – it might even have been lucid and concise! – but in a way it was just a wonderful exercise to carry out this idea. And it was also in the air. So often there is a time in mathematics for something to be done. My thesis is an example. Iwasawa would have done it had I not.

Abel interview, from left to right: Martin Raussen, Christian Skau, John Tate (Photo: Eirik F. Baardsen)

What do you think of the fact that, after your thesis, all places of number fields are treated on an equal footing in analytic number theory, whereas the situation is very different in the classical study of zeta functions; in fact, gamma factors are very different to non-Archimedean local factors.

Of course there is a big difference between archimedean and non-archimedean places, in particular as regards the local factors, but that is no reason to discriminate. Treating them equally, using adeles and ideles, is the simplest way to proceed, bringing the local – global relationship into clear focus.

The title of your thesis was "Fourier analysis in number fields and Hecke's zeta-functions". Atle Selberg said in an interview five years ago that he preferred – and was most inspired by – Erich Hecke's approach to algebraic number theory, modular forms and L-functions. Do you share that sentiment?

Hecke and Artin were both at Hamburg University for a long time before Artin left. I think Artin came to number theory more from an algebraic side, whereas Hecke and Selberg came more from an analytic side. Their basic intuition was more analytic and Artin's was more algebraic. Mine was also more algebraic, so the more I learned of Hecke's work, the more I appreciated it, but somehow I did not instinctively follow him, especially as to modular forms. I didn't know much about them when I was young.

I have told the story before, but it is ironic that being at the same university, Artin had discovered a new type of L-series and Hecke, in trying to figure out what kind of modular forms of weight one there were, said they should correspond to some kind of L-function. The L-functions Hecke sought were among those that Artin had defined, but they never made contact – it took almost 40 years until this connection was guessed and ten more before it was proved, by Langlands. Hecke was older than Artin by about ten years, but I think the main reason they did not make contact was their difference in mathematical taste. Moral: be open to all approaches to a subject.

You mentioned that Serge Lang had named several concepts after you, but there are lots of further concepts and conjectures bearing your name. Just to mention a few: Tate module, Tate curve, Tate cohomology group, Shafarevich–Tate group, Tate conjecture, Sato–Tate conjecture, etc. Good definitions and fruitful concepts, as well as good problems, are perhaps as important as theorems in mathematics. You excel in all these categories. Did all or most of these concepts grow out of your thesis?
No, I wouldn't say that. In fact, I would say that almost none of them grew out of my thesis. Some of them, like the Tate curve grew out of my interest in p-adic fields which were also very central in my thesis, but they didn't grow out of my thesis. They came from different directions. The Tate cohomology came from my understanding the cohomology of Class Field Theory in the seminar that we discussed. The Shafarevich–Tate group came from applying that cohomology to elliptic curves and abelian varieties. In general, my conjectures came from an optimistic outlook, generalizing from special cases.

Although concepts, definitions and conjectures are certainly important, the bottom line is to prove a theorem. But you do have to know what to prove, or what to try to prove.

In the introduction to your delightful book "Rational points on elliptic curves" that you co-authored with your earlier PhD student Joseph Silverman, you say, citing Serge Lang, that it is possible to write endlessly on elliptic curves. Can you comment on why the theory of elliptic curves is so rich and how it interacts and makes contact with so many different branches of mathematics?
For one thing, they are very concrete objects. An elliptic curve is described by a cubic polynomial in two variables so they are very easy to experiment with. On the other hand, elliptic curves illustrate very deep notions. They are the first non-trivial examples of abelian varieties. An elliptic curve is an abelian variety of dimension one, so you can get into this more advanced subject very easily by thinking about elliptic curves. On the other hand, they are algebraic *curves*. They are curves of genus one, the first example of a curve which isn't birationally equivalent to a projective line. The analytic and algebraic relations which occur in the theory of elliptic curves and elliptic functions are beautiful and unbelievably fascinating. The modularity theorem stating that every elliptic curve over the rational field can be found in the Jacobian variety of the curve which parametrizes elliptic curves with level structure its conductor is mindboggling.

By the way, by my count about one quarter of Abel's published work is devoted to elliptic functions.

Among the Abel-prize laureates so far, you are probably the one whose contributions would have been closest to Abel's own interests. Could we challenge you to make an historical sweep, to put Abel's work in some perspective and to compare it to your research?
In modern parlance, Abel studied the multiplication-by-n map for elliptic equal parts, and studied the algebraic equations that arose. He also studied complex multiplication and showed that, in this case, it gave rise to a commutative Galois-group. These are very central concepts and observations, aren't they?
Yes, absolutely, yes. Well, there's no comparison between Abel's work and mine. I am in awe of what I know of it. His understanding of algebraic equations, and of elliptic integrals and the more general, abelian integrals, at that time in history is just amazing. Even more for a person so isolated. I guess he could read works of Legendre, and other great predecessors, but he went far beyond. I don't really know enough to say more. Abel was a great analyst and a great algebraist. His work contains the germs of many important modern developments.

Could you comment on how the concept of "good reduction" for an elliptic curve is so crucial, and how it arose?
If one has an equation with integer coefficients it is completely natural, at least since Gauss, to consider the equation mod p for a prime p, which is an equation over the finite field F_p with p elements.

If the original equation is the equation of an elliptic curve E over the rational number field then the reduced equation may or may not define an elliptic curve over F_p. If it does, we say E has "good reduction at p". This happens for all but a finite set of "bad primes for E", those dividing the discriminant of E.

The Hasse Principle in the study of Diophantine equations says, roughly speaking: if an equation has a solution in p-adic numbers then it can be solved in the rational numbers. It does not hold in general. There is an example for this failure given by the Norwegian mathematician Ernst Selmer…
Yes. The equation $3x^3 + 4y^3 + 5z^3 = 0$.

Exactly! The extent of the failure of the Hasse Principle for curves of genus 1 is quantified by the Shafarevich–Tate group. The so-called Selmer groups are related groups, which are known to be finite, but as far as we know the Shafarevich-Tate group is not known to be finite. It is only a conjecture that it is always finite.
What is the status concerning this conjecture?
The conjecture that the Shafarevich group Sha is finite should be viewed as part of the conjecture of Birch and Swinnerton–Dyer. That conjecture, BSD for short, involves the L-function of the elliptic curve, which is a function of a complex variable s. Over the rational number field, $L(s)$ is known to be defined near $s = 1$, thanks to the modularity theorem of A. Wiles, R. Taylor, et al. If $L(s)$ either does not vanish or has a simple zero at $s = 1$, then Sha is finite and BSD is true, thanks to the joint work of B. Gross and D. Zagier on Heegner points, and the work of Kolyvagin on Euler systems. So, by three big results which are the work of many people, we know a very special circumstance in which Sha is finite.

If $L(s)$ has a higher order zero at $s = 1$, we know nothing, even over the field of rational numbers. Over an imaginary quadratic field we know nothing, period.

Do you think that this group is finite?
Yes. I firmly believe the conjecture is correct. But who knows? The curves of higher rank, or whose L-functions have a higher order zero – BSD says the order of the zero is the rank of the curve – one knows nothing about.

What is the origin of the Tate Conjecture?
Early on I somehow had the idea that the special case about endomorphisms of abelian varieties over finite fields might be true. A bit later I realized that a generalization fit perfectly with the function field version of the Birch and Swinnerton–Dyer conjecture. Also it was true in various particular examples which I looked at, and gave a heuristic reason for the Sato–Tate distribution. So it seemed a reasonable conjecture.

In the arithmetic theory of elliptic curves, there have been major breakthroughs like the Mordell–Weil theorem, Faltings' proof of the Mordell conjecture, using the known reduction to a case of the Tate conjecture. Then we have Wiles' breakthrough proving the Shimura–Taniyama–Weil conjecture. Do you hope the next big breakthrough will come with the Birch and Swinnerton–Dyer conjecture? Or the Tate conjecture, maybe?
Who knows what the next big breakthrough will be, but certainly the Birch and Swinnerton–Dyer conjecture is a big challenge, and also the modularity, i.e. the Shimura–Taniyama–Weil idea, which is now seen as part of the Langlands program. If the number field is not totally real we don't know much about either of these problems. There has been great progress in the last thirty years, but it is just the very beginning. Proving these things for all number fields and for all orders of vanishing, to say nothing of doing it for abelian varieties of higher dimension, will require much deeper insight than we have now.

Is there any particular work from your hand that you are most proud of, that you think is your most important contribution?
I don't feel that any one of my results stands out as most important. I certainly enjoyed working out the proofs in my thesis. I enjoyed very much proving a very special case of the so-called Tate conjecture, the result about endomorphisms of abelian varieties over finite fields. It was great to be able to prove at least one non-trivial case and not have only a conjecture! That's a case that is useful in cryptography, especially elliptic curves over finite fields. Over number fields, even finitely generated fields, that case of my conjecture was proved by Faltings, building on work of Zarhin over function fields, as the first step in his proof of the Mordell conjecture. I enjoyed very much the paper which I dedicated to Jean-Pierre Serre on the K_2 groups of number fields. I also had fun with a paper on residues of differentials on curves giving a new definition of residue and a new proof that the sum of the residues is zero, even though I failed to see a more important aspect of the construction.

Applied number theory

Number theory stretches from the mysteries of the prime numbers to the way we save, transmit and secure information on modern computers. Can you comment on the amazing fact that number theory, in particular the arithmetic of elliptic curves, has been put to use in practical applications?
It certainly is amazing to me. When I first studied and worked on elliptic curves I had no idea that they ever would be of any practical use. I did not foresee that. It is the high speed computers which made the applications possible, but of course many new ideas were needed also.

And now it's an industry: elliptic curves, cryptography, intelligence and communication!
It's quite remarkable. It often happens that things which are discovered just for their own interest and beauty later turn out to be useful in practical affairs.

We interviewed Jacques Tits a couple of years ago. His comment was that the Monster group, the biggest of all the sporadic simple groups, is so beautiful that it has to have some application in physics or whatever.
That would be interesting!

Collaboration and teaching

You have been one of the few non-French members of the Bourbaki group, the group of mathematicians that had the endeavour of putting all existing mathematics into a rigid format. Can you explain what this was all about and how you got involved?
I would not say it was about putting mathematics in a rigid format. I view Bourbaki as a modern Euclid. His aim was to write a coherent series of books which would contain the fundamental definitions and results of all mathematics as of mid-twentieth century. I think he succeeded pretty well, though the books are somewhat unbalanced – weak in classical analysis and heavy on Lie Theory. Bourbaki did a very useful service for a large part of the mathematics community just by establishing some standard notations and conventions.

The presentation is axiomatic and severe, with no motivation except for the logic and beauty of the development itself. I was always a fan of Bourbaki. That I was invited to collaborate may have been at Serge Lang's suggestion, or perhaps Jean-Pierre Serre's also. As I mentioned, I am not a very prolific writer. I usually write a few pages and then tear them up, and start over, so I never was able to contribute much to the writing. Perhaps I helped somewhat in the discussion of the material. The conferences were enjoyable, all over France, in the Alps and even on Corsica. It was a lot of fun.

You mentioned Jean-Pierre Serre, who was the first Abel Prize laureate. He was one of the driving forces in the Bourbaki project after the Second World War. We were told that he was – as Serge Lang – instrumental

in getting some of your results published in the form of lecture notes and text books. Do you have an ongoing personal relation with Jean-Pierre Serre?
Yes. I'm looking forward to meeting him next week when we will both be at Harvard for a conference in honour of Dick Gross on his 60th birthday. Gross was one of my PhD students.

I think Serre was a perfect choice for the first Abel Prize laureate.

Another possible choice would have been Alexander Grothendieck. But he went into reclusion. Did you meet him while you were in Paris or maybe at Harvard?
I met him in Paris. I had a wonderful year. Harvard had the enlightened policy of giving a tenure track professor a year's sabbatical leave. I went to Paris for the academic year '57–'58 and it was a great experience. I met Serre, I met Grothendieck and I was free from any duty. I could think and I could learn. Later, they both visited Harvard several times so I saw them there too. It's great good fortune to be able to know such people.

Did you follow Grothendieck's program reconstructing the foundations of algebraic geometry closely?
Well, yes, to the extent I could. I felt "ah, at last, we have a good foundation for algebraic geometry". It just seemed to me to be the right thing. Earlier I was always puzzled, do we have affine varieties, projective varieties? But it wasn't a category. Grothendieck's schemes, however, did form a category. And breaking away from a ground field to a ground ring, or even a ground scheme, so that the foundations could handle not only polynomial equations, but also Diophantine equations and reduction mod p, was just what number theorists needed.

We have a question of a more general and philosophical nature. A great mathematician once mentioned that it is essential to possess a certain naivety in order to be able to create something really new in mathematics. One can do impressive things requiring complicated techniques, but one rarely makes original discoveries without being a bit naive. In the same vein, André Weil claimed that breakthroughs in mathematics are typically not done by people with long experience and lots of knowledge. New ideas often come without that baggage. Do you agree?
I think it's quite true. Most mathematicians do their best work when they are young and don't have a lot of baggage. They haven't worn grooves in their brains that they follow. Their brains are fresher, and certainly it's important to think for oneself rather than just learning what others have done. Of course, you have to build on what has been done before or else it's hopeless; you can't rediscover everything. But one should not be prejudiced by the past work. I agree with the point of view you describe.

Did you read the masters of number theory early in your career?
I've never been such a good reader. My instincts have been to err on the side of trying to be independent and trying to do things myself. But as I said, I was very fortunate to be in contact with brilliant people, and I learned very much from personal conversations. I never was a great reader of the classics. I enjoyed that more as I got older.

You have had some outstanding students who have made important contributions to mathematics. How did you attract these students in the first place, and how did you interact with them, both as students and later?
I think we were all simply interested in the same kind of mathematics. You know, with such gifted students there is usually no problem; after getting to know them and their interests you suggest things to read and think about, then just hear about progress and problems, offering support and encouragement as they find their way.

Did you give them problems to work on or did they find the problems themselves?
It varies. Several found their own problem. With others I made somewhat more specific suggestions. I urged Dick Gross to think about a problem which I had been trying unsuccessfully to solve, but very sensibly he wrote a thesis on a quite different subject of his own choosing. I was fortunate to have such able students. I continued to see many of them later and many are good friends.

You have taught mathematics for more than 60 years, both at Harvard and at Austin, Texas. How much did you appreciate this aspect of your professional duties? Is there a particular way of teaching mathematics that you prefer?
I always enjoyed teaching at all levels. Teaching a subject is one of the best ways to learn it thoroughly. A few times, I've been led to a good new idea in preparing a lecture for an advanced course. That was how I found my definition of Neron's height, for example.

Work style

Would you consider yourself mainly as a theory builder or as a problem solver?
I suppose I'm a theory builder or maybe a conjecture maker. I'm not a conjecture prover very much, but I don't know. It's true that I'm not good at solving problems. For example, I would never be good in the Math Olympiad. There speed counts and I am certainly not a speedy worker. That's one pleasant thing in mathematics: it doesn't matter how long it takes if the end result is a good theorem. Speed is an advantage, but it is not essential.

But you are persistent. You have the energy to stay with a problem.
At least, I did at one time.

May we ask you a question that we, in various ways, have asked almost everybody in previous interviews? Looking back on how you came up with new concepts, or made a breakthrough in an area you had been working on for some time, did that usually happen when you were concentrated and worked intensely on the problem

or did it happen in a more relaxed situation? Do you have concrete examples?
The first thing I did after my thesis was the determination of the higher-dimensional cohomology groups in class field theory. I had been working on that for several months, off and on. This was at the time of the seminar after my thesis at Princeton. One evening I went to a party and had a few drinks. I came home after midnight and thought I would think a little about the problem. About one or two in the morning I saw how to do it!

So this was a "Poincaré-moment"?
In a way. I think that, like him, I had put the work aside for a longer time when this happened. I remember what it was: I had been invited to give some talks at MIT on class field theory and I thought "what am I going to say?" So it was after a party, motivated by needing something to say at MIT that this idea struck me. It was very fortunate.

But it varies. Sometimes I've had an idea after talking to someone, and had the impression the person I was talking to had the idea and told me about it. The PhD thesis of my student Jonathan Lubin was on what should be called the Lubin groups. They somehow have been called the Lubin-Tate groups. Incidentally, I think it's useful in math that theorems or ideas have two names so you can identify them. If I say Serre's Theorem, my God, that doesn't say too much. But anyway, they are called Lubin-Tate groups and it occurred to me, just out of the blue, that they might be useful in class field theory. And then we worked it out and indeed they were. One gets ideas in different ways and it's a wonderful feeling for a few minutes, but then there is a let-down, after you get used to the idea.

Group cohomology had been studied in various guises, long before the notion of group cohomology was formulated in the 1940s. You invented what is called Tate cohomology groups, which are widely used in class field theory, for instance. Could you elaborate?
In connection with class field theory it suddenly dawned on me that if the group is finite – the operating group G – then one could view the homology theory of that group as negative dimensional cohomology. Usually the homology and the cohomology are defined in non-negative dimensions, but suddenly it became clear to me that for a finite group you could glue the two theories together. The i'th homology group can be viewed as a (1-i)'th cohomology group and then you can glue these two sequences together so that the cohomology goes off to plus infinity and the homology goes off, with renumbering, to minus infinity and you fiddle a little with the joining point and then you have one theory going from negative infinity to plus infinity.

Was this a flash of insight?
Perhaps. There was a clue from the finite cyclic case, where there is periodicity; a periodicity of length two. For example, H^3 is isomorphic to H^1, the H^5 is isomorphic to H^3, etc., and it's obvious that you could go on to infinity in both directions. Somehow it occurred to me that one could do that for an arbitrary finite group. I don't remember exactly how it happened in my head.

The roles of mathematics

Can we speculate a little about the future development of mathematics?
When the Clay Millennium Prizes for solving outstanding problems in mathematics were established back in the year 2000, you presented three of these problems to the mathematical public. Not necessarily restricting to those, would you venture a guess about new trends in mathematics: the twenty-first century compared to the twentieth century? Are there trends that are entirely new? What developments can we expect in mathematics and particularly in our own field, number theory?
We certainly have plenty of problems to work on. One big difference in mathematics generally is the advent of high-speed computers. Even for pure math, that will increase the experimental possibilities enormously. It has been said that number theory is an experimental science and until recently that meant experimenting by looking at examples by hand and discovering patterns that way. Now we have a zillion-fold more powerful way to do that, which may very well lead to new ideas even in pure math, but certainly also for applications.

Mathematics somehow swings between the development of new abstract theories and the application of these to more concrete problems and from concrete problems to theories needed to solve them. The pendulum swings. When I was young better foundations were being developed, things were becoming more functorial, if you will, and a very abstract point of view led to much progress. But then the pendulum swung the other way to more concrete things in the 1970s and 1980s. There were modular forms and the Langlands program, the proof of the Mordell conjecture, and of Fermat's last theorem. In the first half of my career, theoretical physics and mathematics were not so close. There was the time when the development of mathematics went in the abstract direction, and the physicists were stuck. But now in the last thirty years they have come together. It is hard to tell whether string theory is math or physics. And non-commutative geometry has both sides.

Who knows what the future will be? I don't think I can contribute much in answering that question. Maybe a younger person would have a better idea.

Are you just as interested in mathematics now as you were when you were young?
Well, not as intensely. I'm certainly still very much interested, but I don't have the energy to really go so deeply into things.

But you try to follow what is happening in your field?
Yes, I try. I'm in awe of what people are doing today.

Your teacher Emil Artin, when asked about whether mathematics was a science would rather say: "No. It's an art." On the other hand, mathematics is connected to the natural sciences, to computing and so on. Perhaps it has become more important in other fields than ever; the mutual interaction between science and engineering on

one side and mathematics on the other has become more visible. Is mathematics an art, is it more to be applied in science or is it both?
It's both, for heaven's sake! I think Artin simply was trying to make a point that there certainly is an artistic aspect to mathematics. It's just beautiful. Unfortunately it's only beautiful to the initiated, to the people who do it. It can't really be understood or appreciated much on a popular level the way music can. You don't have to be a composer to enjoy music, but in mathematics you do. That's a really big drawback of the profession. A non-mathematician has to make a big effort to appreciate our work; it's almost impossible.

Yes, it's both. Mathematics is an art, but there are stricter rules than in other arts. Theorems must be proved as well as formulated; words must have precise meanings. The happy thing is that mathematics does have applications which enable us to earn a good living doing what we would do even if we weren't paid for it. We are paid mainly to teach the useful stuff.

Public Awareness of Mathematics

Have you tried to popularize mathematics yourself?
When I was young I tried to share my enthusiasm with friends, but I soon realized that's almost impossible.

We all feel the difficulty communicating with the general audience. This interview is one of the rare occasions providing public attention for mathematics![3] Do you have any ideas about how mathematicians can make themselves and what they do more well-known? How can we increase the esteem of mathematics among the general public and among politicians?
Well, I think prizes like this do some good in that respect. And the Clay Prizes likewise. They give publicity to mathematics. At least people are aware. I think the appreciation of science in general and mathematics in particular varies with the country. What fraction of the people in Norway would you say have an idea about Abel?

Almost everyone in Norway knows about Abel, but they do not know anything about Lie. And not necessarily anything about Abel's work, either. They may know about the quintic.
I see. And how about Sylow?

He is not known either. Abel's portrait has appeared on stamps and also on bills, but neither Lie's nor Sylow's.
I think in Japan, people are more aware. I once was in Japan and eating alone. A Japanese couple came and wanted to practise their English. They asked me what I did. I said I was a mathematician, but could not get the idea across until I said: "Like Hironaka." Wow! It's as though in America I'd said "Like Babe Ruth", or Michael Jordan, or Tiger Woods. Perhaps Hironaka's name is, like Abel's, the only one known, but in America I don't think any mathematician's name would get any response.

Private interests

Our last question: what other interests do you have in life? What are you occupied with when you are not thinking about mathematics? Certainly that happens once in a while, as well?
I'm certainly not a Renaissance man. I don't have wide knowledge or interests. I have enjoyed very much the outdoors, hiking and also sports. Basketball was my favourite sport. I played on the Southeast Methodist church team as a teenager and we won the Minneapolis church league championship one year. There were several of us who went to church three out of four Sundays during a certain period in the winter, in order to play on the team. In the Navy I coached a team from the minesweeping research base which beat Coca-cola for the Panama City league championship. Anyway, I have enjoyed sport and the outdoors.

I like to read a reasonable amount and I enjoy music, but I don't have a really deep or serious hobby. I think I'm more concentrated in mathematics than many people. My feeling is that to do some mathematics I just have to concentrate. I don't have the kind of mind that absorbs things very easily.

We would like to thank you very much for this interview, as well as on behalf of the Norwegian, Danish and European mathematical societies. Thank you very much!
Well, thank you for not asking more difficult questions! I have enjoyed talking with you.

Abel Lecture and Science Lecture 2010

11.00–12.00	Welcome Speech + Abel Laureate John Torrence Tate; "The arithmetic of elliptic curves"
12.00–13.30	Lunch
13.30–14.15	Abel Lecture Richard Taylor; "The Tate Conjecture"
14.30–15.15	Science Lecture Andreas Enge; "The queen of mathematics in communication security"

[3] The interview was broadcast on Norwegian television. See http://www.abelprisen.no/en/multimedia/2010/.

Istanbul Center for Mathematical Sciences – IMBM

IMBM istanbul matematiksel bilimler merkezi
istanbul center for mathematical sciences

Betül Tanbay (Istanbul, Turkey)

The IMBM

When Martin Raussen asked me to introduce the Istanbul Centre for Mathematical Sciences to the readers of the EMS Newsletter, the first sentence that came to my mind was: "It all started with a mistake!" After all, every serious piece of information I shall eventually give in this article is accessible through the address http://www.imbm.org.tr/, so why not start with a few informal lines about the beginning?

Some history

It was early in 2003. My colleague Cem Yalçın Yıldırım was running in the hallways of the Boğaziçi University Mathematics Department, incredulous. With his collaborator Dan Goldston on the other side of the world, in San Jose, they seemed to have an important result on small gaps between twin primes. Good old times when a result had to be checked for years were over! Rumours ran faster than proofs and in no time, due to an announcement by Montgomery in Oberwolfach, Yalçın found himself on the first page of *The New York Times*. He would not have cared less, were it not for the fact that a sudden interest in mathematics in the Turkish media had brought local paparazzi to our department doors. Yalçın ran away and the Chair (me in those days) had to deal with the most profound (!) questions they were insistently asking. Rather strange interest. News was not scarce in those days: despite millions protesting in all the big cities of the world, the US and their European partners had entered Iraq. While chasing journalists away, I was trying to convince Yalçın to give a talk on the new result at the department's Wednesday colloquium. The proofs were not full-proof, so Yalçın resisted but media and academic pressure rose and finally we posted a discrete announcement! It was to be expected that our regular seminar room wasn't going to host all the curious minds. We moved to a big amphitheatre. Luckily the university president happened to be a *New York Times* reader and also joined the colloquium. At the tea service that followed, it was an excellent moment to tell him that we badly needed a location to build a mathematical research centre. We agreed to meet in the following days to discuss a possible spot. A little path in the woods of the campus led to a small shack overlooking the Bosphorus. A small hidden paradise just in the middle of the campus! The president said: "If you can find the money to turn this into a centre, go ahead!" It took a few days for *The New York Times* to announce that the proof was fallacious but we had already found a generous mathematics fan who happened to chair ENKA, the country's biggest construction company. The university executive committee accepted the plan. A lucky visit to the university from David Mumford for a talk on "Vision" gave us the confidence we needed. He visited the shack, listened to our intentions and had the trust (vision?) to accept being the first member of the scientific advisory committee, which over the years came to include Victor Kac, Selman Akbulut, Edriss Titi and Gilles Pisier. Another fruitful visit during the planning stage was made by K. R. Sreenivasan, then the director of ICTP. We talked about collaboration possibilities. A long visit to IHES, where Jean-Pierre Bourguignon offered generous advice and precious support, allowed us to watch our steps carefully in this still modest but already serious project. We constituted the scientific steering committee with five members of the Turkish Academy of Sciences (Attila Aşkar, Rahmi Güven, Mete Soner, Tosun Terzioğlu and Ali Ülger) and the management committee (Alp Eden, Mete Soner, Betül Tanbay, Ali Ülger and Yalçın Yıldırım) was chosen from the faculties of Boğaziçi, Koç and Sabancı Universities, which promised their support

The view from the IMBM

Ben Green's special talk at IMBM, 14 October 2009

to the centre. To symbolise the cooperation of several universities, two co-directors were chosen from Sabancı and Boğaziçi Universities: Mete Soner and Betül Tanbay. Mete has become a professor at ETH since then but he is still on our team.

In October 2006, mathematicians of the city were all present at the opening of the Istanbul Centre for Mathematical Sciences (IMBM) where Tosun Terzioğlu (then the Rector of the Sabancı University and the President of the Turkish Mathematical Society) presented Carathéodory's life, which started in an old neighbourhood of Istanbul – a promising opening talk for the IMBM!

Do not be too sorry for the lost theorem! The first workshop that followed the opening was held by the Goldston-Pinz-Yıldırım trio with a much stronger result on small gaps between twin primes than the first fallacious one! Furthermore, the fallacious approach was to be cited by Fefferman at the 2006-ICM in Madrid when explaining the Green-Tao theorem. As ever, mistakes were as precious as theorems in mathematicians' lives.

Some financial support

Already a framework of support was developing. The Boğaziçi University was taking care of all the infrastructural needs, from electricity and heating to cleaning and the Internet. The collaborating universities were funding many activities. Thanks to the visits and efforts of Jennifer Chayes and Peter Sarnak, the US National Committee of Mathematics had decided to support Goldston's participation in the first workshop and thereafter to fully support the visit to the IMBM of two mathematicians from the US every year. Let us not forget the first precious gift: John Pym, professor emeritus at Sheffield, donated his beautiful collection of mathematics books.

As a committee of mathematicians, we started to work with the Scientific and Technological Research Council of Turkey, TÜBİTAK, in order to create a fund to support mathematical collaboration and the creation of a network in which the IMBM had an essential role to play. After a long negotiation period and bureaucratic difficulties, a support of four years, between 2008 and 2012, was obtained, allowing a serious number of activities in a network comprising more than 60 researchers in 15 universities of Istanbul, organised under a project director (Betül Tanbay) backed by an executive committee (Tosun Terzioğlu and Ali Ülger) and six group leaders (Tekin Dereli, Alp Eden, Ismail Güloğlu, Ali Nesin, Henning Stichtenoth and Yalçın Yıldırım) conducting research in the subjects of geometry-topology, differential equations, ring theory, foundations, arithmetic and finite fields, analysis and analytical number theory.

Some legitimacy

The TÜBİTAK grant was naturally bound to give a certain emphasis to the listed areas but in its mission the IMBM had not actually specialised in any particular branch of mathematics. The first priority had been to have an accessible home for researchers in the city and the possibility for them to invite their colleagues from other cities and countries. David Mumford was the first to point out that given the geographical location of the centre and the attractiveness of Istanbul (see AMS-Notices May 2005 issue) the IMBM had the potential to become an international centre.

In October 2009, the visit to Istanbul of the European Mathematical Society Executive Board, presided over by Ari Laptev, was certainly a confirmation of this foresight. The IMBM became one of the 28 research centres in the proposal for the MATHE-I project, which aimed to provide a mathematical European Infrastructure under a call from the European Commission.

Some results

Let us now try to describe the kind of activities at the IMBM. If we were to look at the activities in the first three years, they can be grouped as follows:

- Year long **seminars** attended by a fixed group of researchers joined by temporary visitors, such as the Geometry-Topology-Physics Seminars, the Partial Differential Equations Seminars and the Discrete Mathematics Seminars. These seminars have weekly or monthly meetings.
- One or two week long **workshops** have become one of the "specialities of the house", such as the workshops for prime numbers, the Kadison-Singer Conjecture, non-linear dispersive equations, non-commutative geometry, supergravity, contact topology, Banach Algebras, and singularity. The IMBM is located in a very calm and beautiful location and, within a minute's walk, all the facilities of one of the best universities of Turkey are at the disposal of the visitors. It has guestrooms and, as needed, uses the university residences. The offices allow small teams to work and discuss. A small kitchen allows decent coffee breaks. The seminar room, where all the standard technology is installed, allows up to 50 listeners. Again, for bigger events, the Boğaziçi University Mathematics Department offers extra space. This environment also suits teams of 5-6 researchers, who may reserve the centre for a week or more to intensely discuss their common topic in the framework of a **mini-workshop**.
- Besides seminars and workshops, more than 100 **special lectures** were given at the IMBM, such as those by P.L. Lions, G. Pisier, P. Sarnak, E. Titi, S.R.S Varadhan

and E. Zuazua, to name just a few. Cooperation with the Gökova Geometry-Topology Conference, led by Selman Akbulut and Turgut Önder, allowed us special talks by Yum-Tong Siu and Gang Tian. Similarly, thanks to the collaboration with the Middle East Technical University-Cahit Arf Lecture Series, David Mumford, Günther Harder and Ben Green were able to visit the IMBM. We have also been able to give a few talks to a more general public, one of them being the talk on the Four-Colour Theorem by Robert Wilson.

Some hopes?

Research possibilities have developed enormously thanks to the development of technology. A mathematician sitting in her office in Beijing can watch a conference given in Sao Paolo! It just needs a cc to a few more people to have an article published together!

Mathematics certainly has a very peculiar relationship to the "time" of humanity. Sometimes it is much slower, sometimes much faster than the rest of the world. Sometimes ahead, sometimes behind, sometimes even beside. The open problems in mathematics can sometimes be explained to a child and sometimes there is a need for weeks of high level lectures.

Research methods also follow this strange pattern. Indeed, mathematicians cooperate in very important research projects without even having met. But can any one of us deny the importance of a simple silent room with a few comfortable armchairs, a blackboard and some chalk, to see, to hear and to discuss with one another?

I am writing these lines at a time when the IMU secretary Martin Groetschel is warning us about the difficulties of visa problems for delegates participating in the 2010 General Assembly. Many of us know how difficult it is to enter the USA, the UK, Germany and so many other countries where mathematics has seen the most important achievements of the last 300 years. How long can we insist on the importance of a theorem, faced with the ridiculously humiliating or the humiliatingly ridiculous visa processes? Mathematicians are mostly people who want to keep a clear distance from politics. But we find ourselves in a world where politics does not want to keep at a distance from anything.

Before we get too depressed, let us come back to the IMBM! Turkey only asks for visas (to save honour) from countries that require it from its own citizens – and issues it right at the arrival at the airport. Istanbul is one of the most interesting cities in the world. And the IMBM has armchairs and chalk, and a lot more to offer to all mathematicians.

Betül Tanbay [tanbay@boun.edu.tr] obtained her PhD at UC Berkeley in operator algebras and joined the faculty of the Boğaziçi University Mathematics Department. She has initiated the IMBM during her chairwomanship and is one of the co-directors.

ICMI column

Bill Barton and Louise Sheryn (Auckland, New Zealand)

In this issue, the ICMI column, authored by Bill Barton (President of the ICMI) and Louise Sheryn, concerns the Pipeline Project. The Pipeline Project is a study about issues associated with the supply and demand of mathematics students and personnel in educational institutions and the workplace. The study is coordinated by the ICMI at the request of its mother organisation, the International Mathematical Union (IMU). It also builds on, and is an extension of, the work of the Survey Team for ICME-11 on the topic of "Recruitment, entrance and retention of students to university mathematical studies in different countries", chaired by Derek Holton. The study aims at providing data for decision making in different countries as well as promoting better understanding of the situation internationally. At the moment, the following countries are participating in the study: Australia, UK, Finland, France, Korea, New Zealand, Portugal and the USA.

Similar problems are addressed, mainly for European countries and with more detail, in the paper by Miriam Dieter & Günter Törner published in this issue.

The Pipeline Project

Preliminary Summary

The aim of the Pipeline Project has been to collect data, on an international basis, on the number of mathematical science students passing through four transition points:

1. School to undergraduate courses.
2. Undergraduate courses to postgraduate courses.
3. University into employment.
4. University into teaching.

After distributing an initial questionnaire to establish the availability of data, the Pipeline Project began collecting data from ten countries: Australia, Finland, France, Hong Kong, New Zealand, Portugal, Scotland, Taiwan, UK and USA. There were great fluctuations between countries as to what data was available.

Overall, the perception that there are declining numbers of students seems to be unfounded. There are many

fluctuations within the data for each country but generally the findings for each transition point are:

1 The number of school leavers with mathematical science is increasing although Australia and France are exceptions to this. The number of bachelors graduates is increasing although the number in the USA is currently decreasing.
2 Generally there is an overall increase in the number of Masters graduates, although currently Australia seems to be an exception. For all the countries for which we have data, there is an increase in the number of mathematical science PhD graduates.
3 There are small increases in the numbers of mathematical science bachelors graduates who are in full-time employment six months after graduation. There are corresponding small decreases in the numbers of students entering further full-time study.
4 The data for the industries the mathematical science graduates have entered is very complex. It is difficult to make any judgments on this.
5 The number of mathematical science graduates entering teaching is generally decreasing. Many of the fluctuations in the availability of the data, as well as fluctuations in the data, are due to reasons unknown. However there are several fluctuations that can be attributed to curricula changes, governmental policy and the financial climate.

To generalise, the number of qualified mathematical science graduates from school and university is increasing. However the number of mathematical science bachelors graduates as a proportion of total bachelors graduates seems to be decreasing for the three countries for which we have data. Although more mathematical science graduates are entering employment six months after graduation they are not entering full-time study in the numbers they have previously. More specifically the graduates are not entering into teaching, which does not bode well for the future.

The Pipeline Project concludes that, in general, school leavers are turning away from mathematical sciences at university and being drawn into an increasing number of alternative choices of degree subjects. This in turn produces a shortfall of graduates entering the workforce who are sufficiently qualified in mathematical sciences.

The Pipeline Project recommends the development of a website, under the auspices of the ICMI, to maintain data collection and add future data as supplied by the different countries. A further recommendation is a continuation project investigating the value of a bachelors degree in mathematical science: 'Has the value changed over time?'

Bill Barton & Louise Sheryn
University of Auckland

News from ERME: CERME 7

Ferdinando Arzarello

The 7th Congress of the European Society for Research in Mathematics Education (CERME) will be held in Rzeszów, Poland, 9–13 February 2011.

The conference has been designed scientifically by an International Program Committee (composed of Tim Rowland, Chair – United Kingdom, José Carrillo – Spain, Viviane Durand-Guerrier – France, Markku Hannula – Finland, Ivy Kidron – Israel, Florence Ligozat – Switzerland, Maria Alessandra Mariotti – Italy, Demetra Pitta-Pantazi – Cyprus, Susanne Prediger – Germany, Ewa Swoboda – Poland, Carl Winslow – Denmark) and locally organised by Ewa Swoboda (Congress Secretariat) with the support of a Local Organising Committee, chaired by Aleksander Bobko, the National Organising Committee, with representatives of the Universities of Warsaw and Wrocław, and the Pedagogical University of Kraków.

CERME is organised every two years by ERME with the aim of promoting communication, cooperation and collaboration in research in mathematics education in Europe. In fact, its community needs to know more about the research that has been done and is ongoing and the research groups and research interests in different European countries. For this reason ERME provides opportunities for cooperation in research areas and for pan-European collaboration between researchers in joint research projects.

Previous CERME Congresses have been held at: Osnabrueck, Germany (1999); Marianske Lazne, Czech Republic (2001); Bellaria, Italy (2003); Sant Feliu de Guíxols, Spain (2005); Larnaca, Cyprus (2007); and Lyon, France (2009). The proceedings of these conferences can be found on ERME's official website http://www.erme.unito.it/index.php.

CERME is designed to foster a communicative spirit. It deliberately and distinctively moves away from research presentations by individuals towards collaborative group work. Its main feature is a number of thematic working groups (WG), whose members work together in a common research area. The topics of the groups evolve over the years and are defined by the Program Committee; they always concern a wide spectrum of themes taken from the rich diversity of European research.

Each WG has a chair and 2–3 co-chairs, who organise the call for papers, the reviews of the submitted papers, the discussions during the working sessions, the final presentation of the WG's work at the congress and the final reviewing of the papers accepted for publication in the Proceedings of the Congress after the conference.

Here is a list of the thematic working groups of CERME 7:

Group 1: Argumentation and proof; chair:
Viviane Durrand-Guerrier (France) vdurand@math.univ-montp2.fr.

Group 2: Teaching and learning of number systems and arithmetic;
chair: Susanne Prediger (Germany) prediger@math.uni-dortmund.de.

Group 3: Algebraic thinking;
chair: Jeremy Hodgen (UK) jeremy.hodgen@kcl.ac.uk.

Group 4: Geometry teaching and learning;
chair:Alain Kuzniak (France) kuzniak@math.jussieu.fr.

Group 5: Stochastic thinking;
chair: Dave Pratt (UK) d.pratt@ioe.ac.uk.

Group 6: Applications and modelling;
chair: Gabriele Kaiser (Germany) gabriele.kaiser@uni-hamburg.de.

Group 7: Mathematical potential, creativity and talent;
chair: Roza Leikin (Israel) rozal@construct.haifa.ac.il.

Group 8: Affect and mathematical thinking;
chair: Marilena Pantziara (Cyprus) marilena.p@cytanet.com.cy.

Group 9: Mathematics and language;
chair: Maria Luiza Cestari (Norway) maria.l.cestari@uia.no.

Group 10: Cultural diversity and mathematics education;
chair: Paola Valero (Denmark) paola@learning.aau.dk.

Group 11: Comparative studies in mathematics education;
chair: Eva Jablonka (Sweden) Eva.Jablonka@ltu.se.

Group 12: History in mathematics education;
chair: Uffe Thomas Jankvist (Denmark) utj@ruc.dk.

Group 13: Early years mathematics;
chair: Götz Krummheuer (Germany) goetzkrummheuer@mac.com.

Group 14: University mathematics education;
chair: Elena Nardi (UK) e.nardi@uea.ac.uk.

Group 15: Technologies and resources in mathematics education;
chair: Jana Trgalova (France) jana.trgalova@inrp.fr.

Group 16: Different theoretical perspectives and approaches in research in mathematics education;
chair: Ivy Kidron (Israel) ivy@jct.ac.il.

Group 17: From a study of teaching practices to issues in teacher education;
chair: Leonor Santos (Portugal) leonordsantos@sapo.pt.

Researchers wishing to present a paper at the congress are asked to submit their paper to one of these groups. The deadline for submitting a paper is 15 September 2010.

In addition to the thematic working groups, at CERME 7 there will be: plenary presentations by invited speakers Anna Sierpinska (CDN), Markku Hannula (FIN) and Maria Alessandra Mariotti (I); plenary sessions in which the work of each group can be communicated to other participants; poster sessions for new researchers, and others, to communicate their work and gain feedback; and policy and purpose sessions to negotiate the work and directions of ERME.

As usual, CERME is preceded by the YERME Meeting (Young European Researchers in Mathematics Education) on 8-9 February 2011. YERME is an aggregation of young researchers (including researchers, postdoctoral and graduate students).The main aims of YERME are to let young people from different countries meet and establish a friendly and cooperative style of work in the field of mathematics education research, to open a way of establishing possible connections with research projects and cooperation with researchers in other countries and to support the development of professional preparation for careers in the field of mathematics education. One of the main activities of YERME is the YERME days (the other one is the Summer School, about which we have written in a previous EMS-News: see http://math.unipa.it/~grim/YESS-5/Home_YESS-5.html).

The aim of the YERME days is to give concrete support to young researchers who are just starting their careers. Some experts are charged to present and discuss a topic that can be useful for such purposes. YERME will have three Discussion Groups and three Working groups, as follows:

DG 1: Writing an article, or: From theoretical framework, to methodology, to analyses and cogently argued conclusions (Paolo Boero – I).

DG 2: Designing and analysing tasks in mathematics teacher education (Pessia Tsamir – IL).

DG 3: The role of theories in mathematics education (Dina Tirosh – IL).

WG1: Design of a research study: what are the component parts, how are they related to each other and how might they be discussed in a thesis? (Barbara Jaworski – GB).

WG 2: Interpretative data analysis (João Pedro da Ponte – P).

WG 3: Elaborating theoretical constructs as efficient research concepts in mathematics education (Heinz Steinbring – D).

Further information on CERME 7 and YERME days can be found at:
http://www.cerme7.univ.rzeszow.pl/index.php.

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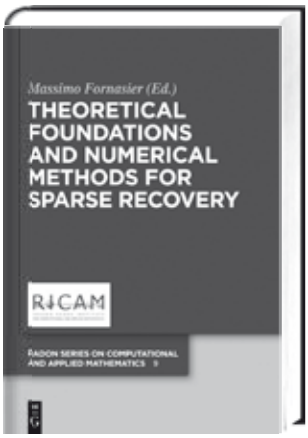

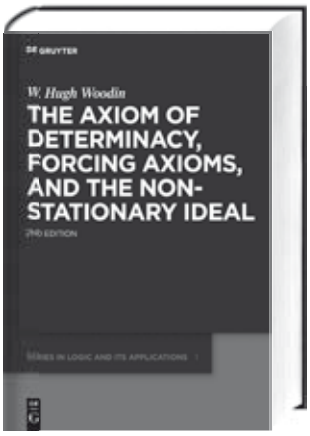

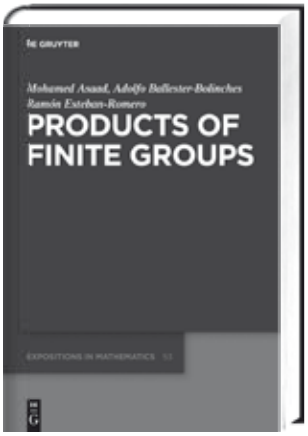

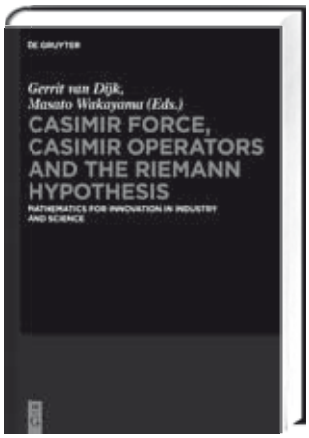

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Mohamed Asaad/Adolfo Ballester-Bolinches/ Ramon Esteban-Romero

PRODUCTS OF FINITE GROUPS

To be published October 2010. 320 pages.
Hardcover RRP € [D] 109.95/*US$ 154.00. ISBN 978-3-11-020417-9
eBook RRP € 109.95/*US$ 154.00. ISBN 978-3-11-022061-2
De Gruyter Expositions in Mathematics 53

The study of finite groups factorised as a product of two or more subgroups has become a subject of great interest during the last years with applications not only in group theory, but also in other areas like cryptography and coding theory. The aim of this book is to gather, order, and examine part of this material, including the latest advances made, give some new approach to some topics, and present some new subjects of research in the theory of finite factorised groups.

CASIMIR FORCE, CASIMIR OPERATORS AND THE RIEMANN HYPOTHESIS

Mathematics for Innovation in Industry and Science

Ed. by Gerrit van Dijk/Masato Wakayama

To be published October 2010. Approx. 290 pages.
Hardcover RRP € [D] 139.95/*US$ 196.00. ISBN 978-3-11-022612-6
eBook RRP € 139.95/*US$ 196.00. ISBN 978-3-11-022613-3
[De Gruyter Proceedings in Mathematics]

This volume contains the proceedings of the conference "Casimir Force, Casimir Operators and the Riemann Hypothesis – Mathematics for Innovation in Industry and Science" held in November 2009 in Fukuoka (Japan). The motive for the conference was the celebration of the 100th birthday of Casimir and the 150th birthday of the Riemann hypothesis.

*for orders placed in North America. Prices are subject to change. Prices do not include postage and handling. eBooks currently only available for libraries/institutions.

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MathEduc – the reference database for teaching and learning mathematics

Beate Ruffer-Henn and Bernd Wegner

About 50 years ago, didactics in several scientific disciplines was established for university curricula as an essential ingredient for improving education in that discipline. At that time the documentation service "Zentralblatt für Didaktik der Mathematik" was founded, collecting information about publications on didactics in mathematics and publishing smaller essays on the subject. When most reference services changed their offer from a printed service to a reference database the didactics in mathematics was offered in the form of the database MathDi. Shortly after that time the EMS joined the supervising body of MathDi as a member of its coordinating committee. Some years ago the subject of MathDi was broadened to cover more publications of interest for mathematics education. This is MathEduc[1] now. The following will give some idea about the aims and mission and the facilities available with MathEduc.

1. Mathematics and society

People claiming to be educated are often ashamed if they are unable to discuss politics, society, literature, music or the arts. They have, however, no problem admitting: "I have always had bad marks in maths." On a simple educational level, this is reduced to the comparison between writing and reading abilities, which, of course, should have top priority in school, and acquiring basic mathematical skills. Neglecting the latter, after graduating from school, is more likely to be excused, despite the fact that mathematical skills are an integral part of our everyday life. They are required when checking a restaurant bill, estimating the price of goods in the shopping basket, comparing the conditions offered by insurances or creditors or when making strategic decisions in games like poker or bridge. Mathematicians know a lot more examples: assessing the risk of loss in gambling (which is inevitable in the long run); diagrams showing the development of stock market prices in the business section of a newspaper; satellite navigation systems; optimisation of time schedules; correct image transmission; ballot-rigging through adequate voting rules, etc.

There are also many ways of applying mathematics in science and technology. In most cases, only specialists or people interested in science are able to understand them. Thus, understanding mathematical relations is an indispensable part of general education. Last but not least, improving general mathematical knowledge and the training of logical problem-solving methods connected with it could help keeping the widespread charlatanism involving numbers and allegedly "mathematical" explanations at bay. Everyone has mathematical abilities, even if these may differ from one person to the other. It is not true that women are less gifted than men. However, as a result of a prejudiced education and maths lessons of inferior quality, individuals may develop reservations against mathematics, consider themselves untalented and settle for this alleged inability, as described above.

2. Why MathEduc?

Most likely this can be solved by improving mathematical education, publishing materials motivating and enabling people to study maths on their own and offering maths teachers' training meeting today's requirements. Future maths teachers not only need to be taught elementary mathematics at a higher level but also have to be given the necessary skills to understand teaching and learning processes and to apply their expertise in a modern teaching environment.

Many surveys and publications are dedicated to these problems. Long before TIMSS and the PISA study became an issue, schools and universities recognised the need to reform the way of teaching mathematics. One classical example that springs to mind is the highly controversial discussion about "New Maths". This discussion was held at a time when mathematical education was a hot topic of interest at teacher training institutions. At the same time, at the end of the 1960s, "Zentralblatt für Didaktik der Mathematik" was founded. It contained an articles section on mathematical education and a documentation section with reviews of relevant literature in the field. Using modern information technology, this document section evolved into the online reference database MathEduc.

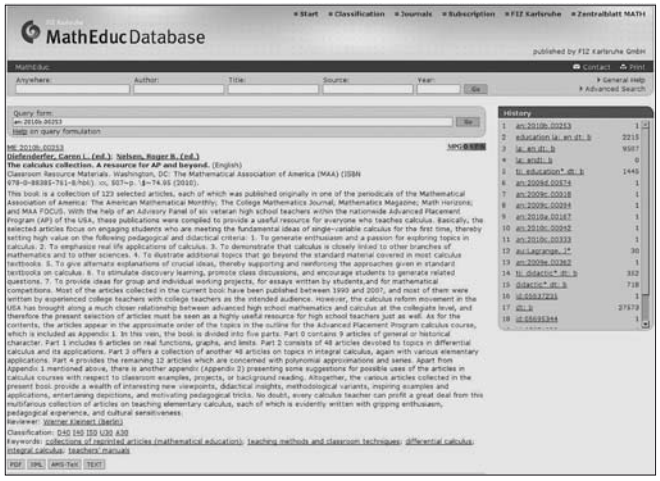
Its task is to evaluate the available literature on mathematical education, as well as publications dedicated to conveying mathematical skills at school, university and to the interested public. The features of this database are similar to that of the database ZBMATH (Zentralblatt MATH). Topics range from pre-school to all school levels and school types and from vocational training, teacher training and tertiary education to popular science presentations for anyone interested. MathEduc provides bibliographic references to relevant topics such as learning, educational psychology, teaching methods and lesson planning, interdisciplinary and application-oriented approaches, etc. In addition to subject-specific literature,

1 MathEduc is available via www.emis.de or directly via http://www.zentralblatt-math.org/matheduc/ with trial access for non-subscribers.

publications from educational science, psychology, sociology and other basic sciences dealing with teaching and learning are reviewed. Appropriate indexing allows the user to find information for the educational level they are interested in.

3. MathEduc – the cooperational network

In the foundational period "Zentralblatt für Didaktik der Mathematik" very much focused on Germany. Of course there were, like in mathematical research, some journals on mathematical education targeted towards an international audience. But the different school systems in different countries and different ways of mathematical reasoning led to a system of local, regional and national publications, which are of interest for looking beyond the borderlines but will not be recognised from the German point of view. Approaches towards overcoming problems in mathematical education may be of global interest, even if the systems may vary from country to country. Hence, the decision was made to broaden the scope of the literature to be covered accordingly.



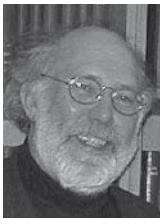
MathEduc online database

For MathEduc, this meant that there is the option to include the description of the content in its original language while English metadata are obligatory. This internationalisation of MathEduc is an ongoing process. In the meantime, much progress has been made. But it always depends on support from institutions or individuals who dedicate some part of their time to caring about this kind of national input. For example, thanks to a long-term partnership with the American information service ERIC, relevant publications from North America are well represented in MathEduc. In other countries, groups of experts have formed with the necessary expertise to decide whether or not a regional publication is to be included in MathEduc and who then provides the required input. Thanks to this cooperation, the database has a sound coverage of literature from France, Italy, Spain, Portugal, Serbia and the Czech Republic. An excellent first step will be to have all member societies of the EMS in the team.

Not to be underestimated is the interest people in the developing countries have in MathEduc content. It is true that apart from some few mathematicians who return there after receiving their training and doctorate in the industrialised countries (mainly the USA), not many people in these countries require current information on mathematical research results. Burning issues, however, are methods of mathematical training and the lack of textbooks and qualified teachers. Here, information like that provided in MathEduc finds grateful but mostly penniless recipients. In this field MathEduc is turning publicly funded editorial work into development assistance. At present institutions from developing countries are entitled to free access to MathEduc.

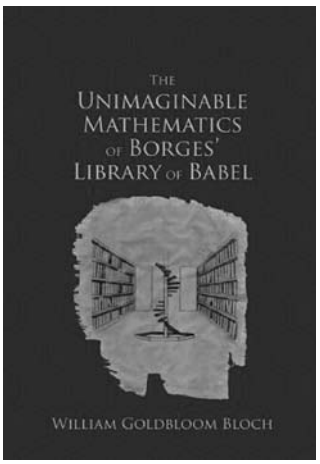
Another parameter, which will require further enhancement of the input activities for MathEduc, is the numerous online offers on mathematical education. This is an important trend with respect to promoting mathematics and improving the dissemination of mathematical information. The growth of these offers must not be underestimated. At the same time, they create new problems; they are the result of more or less spontaneous initiatives. They are hard to find. It is not clear whether they will be available on a permanent basis. Not all of them have been checked for quality and if they have, the applied criteria are not clear. In most cases, they are not intended for public use, although they are of sound quality and could be of interest to a large public audience. To deal with these problems needs a project on the European level and has to be delegated to a more representative group than just the editorial team of MathEduc.

Beate Ruffer-Henn [beate.ruffer-henn@fiz-karlsruhe.de] studied mathematics and economics at the University Mannheim, Germany. She has been a member of the MathEduc editorial staff since 1983.



Bernd Wegner [wegner@math.tu-berlin.de] has been a professor at TU Berlin since 1974 and is Editor-in-Chief of Zentralblatt MATH and MathEduc. He has participated in several electronic library and mathematics knowledge management projects. He has also been involved in the organisation of meetings on these subjects. He has been a member of the electronic publishing committee of the EMS for a longer period.

Book reviews



The Unimaginable Mathematics of Borges' Library of Babel

William Goldbloom Bloch
Oxford University Press, 2008
ISBN 0195334574

Reviewer: Solomon Marcus (Bucharest, Romania)

What a pleasure to read and to comment on such a book!

William Goldbloom Bloch tells us that the ideal reader of his book would be the famous semiotician and writer Umberto Eco. Indeed, it is difficult to find, among contemporary writers, one who is so much on the same wavelength with Borges as the editor of "Variaciones Borges". In "The Name of the Rose", Eco adopts as basic starting metaphors the library and the labyrinth, exactly like Borges does in "The Library of Babel" (LB). Accepting Bloch's challenge, we will take Eco as the point of departure for this review.

In one of his first books ("Opera aperta", Milano, Bompiani, 1962), Eco develops the idea that a literary text is open to infinitely many ways of being read. It is the reading process that creates the meaning, so the meaning of a literary work is the result of the interaction between the given work and its readers: there are infinitely many possible meanings of the same work and the same reader may change his reading at different moments of his life.

One can illustrate this situation by a simple mathematical exercise, showing that Eco's assumption also works in mathematics. Given the finite sequence 1, 2, 3, we can read it in various ways: it can be the beginning of the sequence of natural numbers, of a Fibonacci sequence, of prime numbers, of the periodic infinite sequence of period 1, 2, 3, etc. So, there are infinitely many readings of the given sequence. We observe that: a) any way to give a meaning to a finite sequence leads to an infinite one; b) there are infinitely many sequences having the given finite sequence as a prefix.

Something similar happens with any text over a finite non-empty alphabet and, obviously, Borges' "Library of Babel" (LB) is such a text over the English alphabet (because we refer to its English translation). Bloch's reading of LB focuses on the mathematical potential of various terms, ideas and representations in LB. Borges avoids using mathematical symbolism beyond that of secondary school level. Bloch says (p. xvii): "I am a tour guide through a labyrinth." But the way he represents this labyrinth and the interpretation of the guidance process are created by the Bloch-Borges interaction. Maybe some readers could believe that the inventory of the mathematical ideas occurring in LB does not leave room for subjective choices. They would be wrong. We will indicate the way Bloch makes this inventory, in order to show that, despite his very interesting approach, a lot of room remains for alternative choices.

Bloch begins by referring to elementary combinatorics (in order to determine the number of books in the library, a number ultimately considered unimaginable). But for the author of this review it would be equally interesting to refer to the hot field of current mathematical computer science called "Words". A word is a string over a finite non-empty alphabet and it can be finite or infinite. Started by Axel Thue 100 years ago, the study of infinite words was stimulated in the 30s and 40s of the previous century by the work of Marston Morse. Then a few decades later problems of complexity of an infinite word became important, stimulated by questions coming from molecular biology and from music. On the other hand, within theoretical computer science a theory of text was developed, alternative to a similar theory in linguistics and semiotics. The text-metaphor for the world became important.

The next mathematical field considered by Bloch, legitimised by the need for cataloguing the collections, is Shannon's Information Theory (although none of its technical aspects are involved). But Bloch refers to the possibility that the author of all books could be "an algorithm embodied in a very short computer program that would, given time and resources, generate all possible variations of 25 ortographic symbols in strings of length 1, 312, 000". Therefore, one could introduce in debate the Kolmogorov-Chaitin algorithmic-information theory, coping with individual strings, in contrast with Shannon's approach, related to global aspects of information. With respect to the difficulty of having a total catalogue, Berry's paradox of the library should be considered.

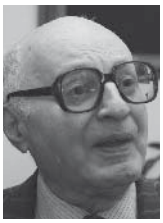
A third mathematical field involved in Bloch's analysis is real analysis, intelligently defined as that part of mathematics that explores, among other ideas, the nuances of the arbitrarily small. This is required by the attention paid in LB to pages that are infinitely thin. Reference is made to the whole history of the problem, from Zeno's paradox to Robinson's Nonstandard Analysis (NSA). Bloch considers NSA to be not so attractive but he should pay attention to the successful field of Nonstandard Exchange Economy (Gerard Debreu, Nobel Prize in Economics). In the same order of ideas, Raymond Smullyan and Martin Gardner deserve attention for their interesting paradoxes and games.

Topology and cosmology are considered by Bloch with respect to the structure of the Universe ("which others call the Library"). With respect to speculations on Borges' claim that the Library is a sphere whose exact centre is any

hexagon and whose circumference is unattainable – moreover, it is limitless and periodic – Bloch concludes that the 3-spherical Library satisfies "both the classic dictum and the librarian's cherished hopes". But there is room enough for alternative representations, for instance involving non-Euclidean geometries. Bloch prepares the way by referring to Moebius' band and to the Klein bottle.

The fifth field involved in Bloch's analysis is "Geometry and Graph Theory", in order to point out the way in which "LB inspired many artists and architects to provide a graphic or atmospheric rendition of the interior". Here is the moment to ask for Borges' preference for "hexagonal galleries". Here is Bloch's Conjecture of Extreme Disconsolation: "There are unimaginably vast numbers of pairs of adjacent hexagons such that the span of our combined lives would not suffice to travel from the one to the other." On page 116, Bloch observes that "as such, the books of this infinite Library are maximally disordered and yet this ultimate disorder forms a unique overarching Order of all orderings: the Grand pattern". This fact is reflected in the mathematical concept of randomness, proposed by Kolmogorov and Chaitin, according to which a finite string s is random if none of its algorithmic descriptions is shorter than s, while an infinite random word is such that each of its prefixes is random. The surprising facts are that any infinite, random string over the alphabet A includes all finite strings over A and no instantiation of a finite random string is possible, despite the fact that most finite strings are random. So, mathematically, each random, infinite word includes much order. The maximum disorder is made by much order. Remember the slogan of Ramsey theory: total disorder is not possible. Borges' intuition was right. His idea of total library is mathematically coherent.

Bloch's merits are best visible in the last parts of his book, where he is able to bridge mathematics, literature, cognitive science, brain studies, computer science and semiotics. The references to Turing and to Floyd Merrell's book concerning Borges are essential, while the citation from John Searle is very pertinent. Moreover, Bloch leaves many interesting questions unanswered, related to holography, to alternative logics, to the perfect knowledge problem, to the role of circularity and of endless recurrence, and to the attenuation of the form-content distinction in Borges' writings. One cannot discuss LB by ignoring the whole context of Borges' writings. Bloch has the rare quality of knowing how to transgress artificial borders among disciplines.



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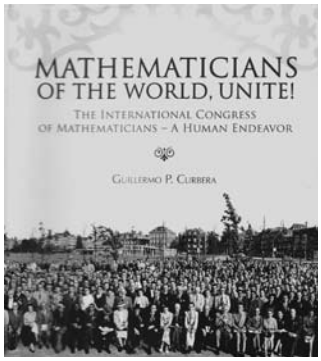

Guillermo P. Curbera

Mathematicians of the world, unite!

A. K. Peters Ltd. Wellesley, Massachusetts
ISBN 978-1-56881-330-1

Reviewer: Ioan A. Rus (Cluj-Napoca, Romania)

The International Congress of Mathematicians and associated publications (Verhandlungen, Comptes rendus, Proceedings, Atti, Trudny, Actes, etc.) have had and still have a relevant impact on the development of mathematics (see for example [1]–[6] and the references therein). This book is focused on the International Congress of Mathematicians from the 1st in Zürich (1897) to the 25th in Madrid (2006). The author wrote the book by "reading the proceedings of all the congresses" and solving "the problem of finding the images that would support the script" with a focus on the human aspect.

The book begins with a foreword by Lennart Carleson, which ends: "I welcome this book and hope it will inspire international cooperation without politics and promote curiosity and interest in the whole area of mathematics."

The structure of the book is as follows:

- Part I: Early Times (Zürich (1897), Paris (1900), Heidelberg (1904), Rome (1908), Cambridge (1912).
- Interlude: Images of the ICM.
- Part II: Crisis in the Interwar Period (Strasbourg (1920), Toronto (1924), Bologna (1928), Zürich (1932), Oslo (1936)).
- Interlude: Awards of the ICM.
- Part III: The Golden Era (Cambridge (MA) (1950), Amsterdam (1954), Edinburgh (1958), Stockholm (1962).
- Interlude: Buildings of the ICM.
- Part IV: On the Road (Moscow (1966), Nice (1970), Vancouver (1974), Helsinki (1978), Warsaw (1983), Berkeley (1986)).

- Interlude: Social Life at the ICM.
- Part V: In a Global World (Kyoto (1990), Zürich (1994), Berlin (1998), Beijing (2002), Madrid (2006)).
- Coda: International Mathematical Union.

For each ICM the author pays attention to the following parameters: the plenary lectures, the sections of the congress, the attendance and the scientific level.

In 1897 there were four plenary lectures, whilst in 2006 there were 20.

In 1897 there were four sections: Arithmetic and Algebra; Analysis and Function Theory; Geometry, Mechanics and Mathematical Physics; and History and Bibliography. In 2002 there were 19 sections: Logic, Algebra, Number Theory, Differential Geometry, Topology, Algebraic and Complex Geometry, Lie Groups and Representation Theory, Real and Complex Analysis, Operator Algebras and Functional Analysis, Probability and Statistics, Partial Differential Equations, Ordinary Differential Equations and Dynamical Systems, Mathematical Physics, Combinatorics, Mathematical Aspects of Computer Science, Numerical Analysis and Scientific Computing, Applications of Mathematics in the Sciences, Mathematics Education and Popularisation of Mathematics, and History of Mathematics.

In 1897 there were 208 participants from 16 countries, whilst in 2006 there were 3,425 from 108 countries.

The book ends with many references (congress proceedings, books, articles), which may be helpful to gain more understanding of the impact of the ICM on the development of mathematics.

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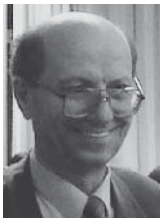

Ioan A. Rus [iarus@math.ubbcluj.ro] (born 1936) is a professor in the Department of Applied Mathematics, "Babeș-Bolyai" University of Cluj-Napoca, Romania. He has published about 150 original papers and more than ten books (monographs and textbooks), mainly in the field of fixed point theory and differential equations, which have more than 1300 citations. He is the creator of two important research directions: the techniques of Picard and weakly Picard operators and fixed point structures theory. He founded and is currently the Editor-in-Chief of Fixed Point Theory, the first journal in the world on this topic.

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Journal of Spectral Theory

New in 2011

ISSN 1664-039X
2011. Vol 1, 4 issues
Approx. 400 pages. 17 x 24 cm
Price of subscription, including electronic edition:
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Other subscriptions available on request.

Aims and Scope: The *Journal of Spectral Theory* is devoted to the publication of research articles that focus on spectral theory and its many areas of application. Articles of all lengths including surveys of parts of the subject are very welcome. The following list includes several aspects of spectral theory and also fields which feature substantial applications of (or to) spectral theory: Schrödinger operators, scattering theory and resonances; eigenvalues: perturbation theory, asymptotics and inequalities; quantum graphs, graph Laplacians; pseudo-differential operators and semi-classical analysis; random matrix theory; the Anderson model and other random media; non-self-adjoint matrices and operators, including Toeplitz operators; spectral geometry, including manifolds and automorphic forms; linear and nonlinear differential operators, especially those arising in geometry and physics; orthogonal polynomials; inverse problems.

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Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

You are never sure whether or not a problem is good unless you actually solve it.
Mikhail Gromov (Abel Prize, 2009)

I Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

67. A function $f : \mathbb{N} \to \mathbb{N}$ is defined as follows: writing a number $x \in \mathbb{N}$ in its decimal expansion and replacing each digit by its square we obtain the decimal expansion of the number $f(x)$. For example, $f(2) = 4$, $f(35) = 925$, $f(708) = 49064$.
(a) Solve the equation $f(x) = 29x$;
(b) Solve the equation $f(x) = 2011x$.
(Vladimir Protasov, Moscow State University, Russia)

68. Solve the equation

$$\left\lfloor x^2 - 3x + 2 \right\rfloor = 3x - 7\,, x \in \mathbb{R}\,.$$

(Elias Karakitsos, Sparta, Greece)

69. What is the largest positive integer m with the property that, for any positive integer n, m divides $n^{241} - n$? What is the new value of m if n is restricted to be odd?
(Konstantinos Drakakis, University College Dublin, Ireland)

70. Find the minimum of the product xyz over all triples of positive integers x, y, z for which 2010 divides $x^2 + y^2 + z^2 - xy - yz - zx$.
(Titu Andreescu, The University of Texas at Dallas, USA)

71. Prove that for any positive integer k with $k \geq 1$ the equation

$$x_1^3 + x_2^3 + \cdots + x_k^3 + x_{k+1}^2 = x_{k+2}^4$$

has infinitely many solutions in positive integers such that

$$x_1 < x_2 < \cdots < x_{k+1}\,.$$

(Dorin Andrica, "Babes-Bolyai" University, Cluj-Napoca, Romania

72. Let $p \geq 3$ be a prime number. For $j = 1, 2, \cdots, p-1$, divide the integer $(j^{p-1} - 1)/p$ by p and get the remainder r_j . Prove that

$$r_1 + 2r_2 + \cdots + (p-1)r_{p-1} \equiv \frac{p+1}{2} (mod\ p).$$

(Dorin Andrica, "Babes-Bolyai" University, Cluj-Napoca, Romania

II Two new open problems

Proposed by Preda Mihăilescu (Mathematisches Institut der Universität Göttingen, Germany)

The unimodular $n \times n$ matrices over $\mathbb{Z}$ are the invertible matrices in $GL(\mathbb{Z}, n)$. This problem invites an investigation of unimodular matrices over the polynomial ring $\mathbb{Z}[X]$. It raises questions of different levels of difficulty, concerning the *density* of unimodular $\mathbb{Z}[X]$ matrices among all the $\mathbb{Z}[X]$-matrices. *It is an open research problem*, see also [Co], [GMV].

73*. Let $R = \mathbb{Z}[X]$ and $R_m = \{f \in \mathbb{Z}[X] : \deg(f) \leq m\}$. If $S \in \{R, R_m\}$, we define $\mathbf{R}_n(S) = GL(S, n)$, the ring of invertible $n \times n$ matrices over S. Note that the integers n, m are uncorrelated. Let $Z(S) = S^{n^2}$ be the space of n^2-dimensional vectors in S. We may identify a matrix $M \in M(S, n \times n)$ with a vector $v \in Z(S)$, for instance by appending its column vectors. In this way we may identify $\mathbf{R}_n(S)$ with a subset of $Z(S)$ via

$$U(S) = \left\{ (f_{i+jn})_{i,j=1}^{n} \in Z(S) : (f_{I \bmod n, I \div n})_{I=1}^{n^2} \in GL(n, S) \right\} \subset Z(S),$$

where the vector f_{i+jn} is converted into the appropriate matrix; here $a \div b$ is the Euclidean quotient of the integers a, b.

1. Let $\overline{U(S)} \subseteq Z(S)$ be the linear span of $U(S)$. Does $\overline{U(S)} = Z(S)$ hold for all m and n ? If not, can an upper bound be given for $\dim \overline{(U(S))}$?
2*. We define the height of a polynomial $\|f\|$ as the maximal absolute value of its coefficients in $\mathbb{Z}$ and for $B > 0$ we let $S_B = \{f \in S : \|f\| \leq B\}$. Prove that

$$\lim_{B \to \infty} \frac{U(S_B)}{\overline{U(S_B)}} = 0.$$

3*. Answer the two questions above when $\mathbb{Z}$ is replaced by some normed integral ring I, for instance $I = \mathbb{Z}_p$.

The following problem has simpler first and second questions, while the third one is still open. See also [Sch] for some related questions on multivariate polynomials.

74*. Let R be a factorial ring and $f \in R[X]$ a monic polynomial of degree d.

1. Show that for each $k > 1$ there is a polynomial $g(X) = h(X^k) \in R[X]$ with $h \in R[Y]$ a polynomial of degree d and such that $f(X) | g(X)$.
2. If $F \in R[X]$ has the factorization $F = \prod_{i=1}^{k} p_i^{e_i}(X)$ with irreducible $p_i \in R[X]$, we call $e = (e_1, e_2, \ldots, e_k)$ the factorization pattern of F. If f, g are as in point 1., show that the factorization pattern of f is in general not conserved by g.

3*. Under what additional conditions for f is the factorization pattern conserved for all $k > 1$?

References

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III Solutions

59. Let a, b, c be positive real numbers such that $c \geq a \geq b$ and $a^2 \geq bc$. Prove that the following inequality holds.

$$\frac{\sqrt{a^2b+b^2c}}{a+c}+\frac{\sqrt{b^2c+c^2a}}{a+b}+\frac{\sqrt{c^2a+a^2b}}{b+c}\geq\frac{\sqrt{a}+\sqrt{b}+\sqrt{c}}{2}.$$

(Tuan Le, Fairmont H. S., California, USA)

Solution by the proposer. Applying the Cauchy-Schwarz inequality into $\sqrt{a^2b+b^2c}$ we have:

$$\sqrt{(a^2b+b^2c)(b+c)}\geq b(a+c).$$

Multiplying both sides of this inequality by $\frac{1}{(a+c)\sqrt{b+c}}$, we get:

$$\frac{\sqrt{a^2b+b^2c}}{a+c}\geq\frac{b}{\sqrt{b+c}}.$$

Similarly,we have:

$$\frac{\sqrt{b^2c+c^2a}}{a+b}\geq\frac{c}{\sqrt{c+a}}$$

and

$$\frac{\sqrt{c^2a+a^2b}}{b+c}\geq\frac{a}{\sqrt{a+b}}.$$

Adding those 3 inequalities above side by side, we get:

$$\frac{\sqrt{a^2b+b^2c}}{a+c}+\frac{\sqrt{b^2c+c^2a}}{a+b}+\frac{\sqrt{c^2a+a^2b}}{b+c}\geq\frac{a}{\sqrt{a+b}}+\frac{b}{\sqrt{b+c}}+\frac{c}{\sqrt{c+a}}.$$

Now, it is sufficient to show that:

$$\frac{a}{\sqrt{a+b}}+\frac{b}{\sqrt{b+c}}+\frac{c}{\sqrt{c+a}}\geq\frac{\sqrt{a}+\sqrt{b}+\sqrt{c}}{\sqrt{2}}. \qquad (1)$$

Applying Hölder's inequality, we have:

$$LHS^2 of\ (1)\geq\frac{(a+b+c)^3}{a^2+b^2+c^2+ab+ac+bc}.$$

We only need to prove that:

$$\frac{(a+b+c)^3}{a^2+b^2+c^2+ab+ac+bc}\geq\frac{\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)^2}{2}.$$

Multiplying both sides of this inequality by

$$2(a^2+b^2+c^2+ab+ac+bc)$$

and after doing some algebraic manipulations, the inequality is equivalent to:

$$a^3+b^3+c^3+4ab(a+b)+4bc(b+c)+4ac(a+c)+9abc\geq 2\left(\sqrt{ab}+\sqrt{bc}+\sqrt{ac}\right)\left(a^2+b^2+c^2+ab+bc+ac\right).$$

After subtracting both sides by the *RHS* of this inequality, cancelling common terms and grouping suitably, we need to prove that

$$\sum_{cyc}2\left(\sqrt{a}-\sqrt{b}\right)^2\left(a^2+b^2+c^2+ab+ac+bc\right)-\sum_{cyc}\left(\sqrt{a}-\sqrt{b}\right)^2(a+b+c)\left(\sqrt{a}+\sqrt{b}\right)^2\geq 0.$$

The inequality above is equivalent to:

$$\sum_{cyc}\left(\sqrt{a}-\sqrt{b}\right)^2\left(2a^2+2b^2+2c^2+2ab+2bc+2ac-(a+b+c)(a+b+2\sqrt{ab})\right)\geq 0.$$

Now, we will show that:

$$2a^2+2b^2+2c^2+2ab+2bc+2ac-(a+b+c)\left(a+b+2\sqrt{ab}\right)\geq 0.$$

After cancelling common terms, this inequality is equivalent to:

$$a^2+b^2-2\sqrt{ab}(a+b)+2c^2+c\left(\sqrt{a}-\sqrt{b}\right)^2\geq 0.$$

Since $c \geq a \geq b$ and applying the Cauchy-Schwarz and AM-GM inequalities, we have:

$$2c^2+a^2+b^2\geq\frac{(a+b)^2}{2}+2ab\geq 2\sqrt{ab}(a+b).$$

Hence, the following inequality is true:

$$(\sqrt{a}-\sqrt{b})^2\left(2a^2+2b^2+2c^2+2ab+2bc+2ac-(a+b+c)(a+b+2\sqrt{ab})\right)\geq 0.$$

Similarly, we can also show that:

$$\left(\sqrt{c}-\sqrt{b}\right)^2\left(2(a^2+b^2+c^2+ab+bc+ac)-(a+b+c)(b+c+2\sqrt{bc})\right)\geq 0$$

since

$$a^2\geq bc.$$

Because of the fact

$$\sqrt{c}-\sqrt{b}\geq\sqrt{c}-\sqrt{a}\geq 0$$

and

$$\sum_{cyc}2\left(a^2+ab\right)-(a+b+c)\left(b+c+2\sqrt{bc}\right)\geq 0,$$

we have:

$$\left(\sqrt{c}-\sqrt{b}\right)^2\left(\sum_{cyc}2\left(a^2+ab\right)-\left(b+c+2\sqrt{bc}\right)\sum_{cyc}a\right)\geq\left(\sqrt{c}-\sqrt{a}\right)^2\left(\sum_{cyc}2\left(a^2+ab\right)-\left(\sqrt{b}+\sqrt{c}\right)^2\sum_{cyc}a\right).$$

Hence, we only need to prove:

$$\left(\sqrt{c}-\sqrt{a}\right)^2\left(\sum_{cyc}2\left(a^2+ab\right)-\left(\sqrt{b}+\sqrt{c}\right)^2\sum_{cyc}a+\sum_{cyc}2\left(a^2+ab\right)-\left(\sqrt{a}+\sqrt{c}\right)^2\sum_{cyc}a\right)\geq 0.$$

After expanding and cancelling all of the common terms inside the bracket and grouping these terms suitably, we are left with:

$$3(a^2+b^2+c^2)-2\sqrt{bc}(b+c)-2\sqrt{ac}(a+c)+a(\sqrt{c}-\sqrt{b})^2+b(\sqrt{a}-\sqrt{c})^2\geq 0 \qquad (*)$$

By using the AM-GM inequality, we have:

$$2\sqrt{bc}(b+c)\leq(b+c)^2$$
$$2\sqrt{ac}(a+c)\leq(a+c)^2.$$

Since $3\left(a^2+b^2+c^2\right) \geq (a+c)^2+(b+c)^2$ by using AM-GM inequality for $2a^2$ and $\frac{c^2}{2}$, $2b^2$ and $\frac{c^2}{2}$, we get:

$$3\left(a^2+b^2+c^2\right)-2\sqrt{bc}\,(b+c)-2\sqrt{ac}\,(a+c)\geq 0.$$

Therefore, $(*)$ is true. We infer that (1) is also true, which is all we need to prove. Thus, the original inequality is true. The equality case occurs at $a=b=c$. □

References

[1] Pham Kim Hung, *Secrets in inequality*, GIL Publisher, 2007.
[2] Vasile Cirtoaje, *Algebraic Inequalities*, GIL Publisher, 2007.

Also solved by Mihály Bencze (Brasov, Romania), S. E. Louridas (Athens, Greece).

60. Let a, b, c be positive real numbers. Prove that the following inequality holds.

$$\frac{16}{27}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)^3+\sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}}\geq\frac{5}{2}.$$

(Tuan Le, Fairmont H. S, California, USA)

Solution by the proposer. Using the Nesbit inequality we have:

$$\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)^3\geq\frac{9}{4}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right).$$

Multiplying both sides of this inequality by $\frac{16}{27}$, we have:

$$\frac{16}{27}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)^3\geq\frac{4}{3}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right).$$

Now, it is sufficient to show that:

$$\frac{4}{3}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)+\sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}}\geq\frac{5}{2}.$$

Applying the AM-GM inequality, we have:

$$\frac{abc}{(a+b)(b+c)(c+a)}\leq\frac{1}{8}.$$

Using this result, we will have the following simple inequality:

$$\sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}}\geq\frac{4abc}{(a+b)(b+c)(c+a)}. \qquad (1)$$

We will show that the following inequality is true:

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}+\frac{4abc}{(a+b)(b+c)(c+a)}\geq 2. \qquad (2)$$

If $a+b+c=p$, $ab+bc+ac=q$ and $abc=r$ $(p,q,r>0)$, we have:

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=\frac{p^3-2pq+3r}{pq-r}$$

and

$$\frac{4abc}{(a+b)(b+c)(c+a)}=\frac{4r}{pq-r}.$$

Rewrite the *LHS* of (1) and clearing the denominator, we need to prove:

$$p^3-2pq+7r\geq 2pq-2r$$

since

$$pq-r\geq 8r>0.$$

This inequality is equivalent to:

$$p^3-4pq+9r\geq 0,$$

which is just a *Schur* 3-degree inequality.

Hence, (1) is true.

On the other hand, using the Nesbit inequality again, we obtain:

$$\frac{1}{3}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)\geq\frac{1}{2} \quad (3)$$

Adding (1), (2) and (3) side by side, we get:

$$\frac{4}{3}\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)+\sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}}\geq\frac{5}{2},$$

which is what we need.

We conclude that the original inequality is true.

The equality case occurs at $a=b=c$. □

References

[1] Pham Kim Hung, *Secrets in inequality*, GIL Publisher, 2007.
[2] Vasile Cirtoaje, *Algebraic Inequalities*, GIL Publisher, 2007.

Also solved by Mihály Bencze (Brasov, Romania), S. E. Louridas (Athens, Greece).

61. Find all functions $f:\mathbb{R}\to\mathbb{R}$ that satisfy the functional equation

$$f(x+y+xy)=f(x)+f(y)+f(x)\,f(y) \qquad (1)$$

for all $x, y\in\mathbb{R}$.

(Prasanna K. Sahoo, University of Louisville, USA)

Solution by the proposer. The general solution of (1) is given by

$$f(x)=M(x+1)-1\,,$$

where $M:\mathbb{R}\to\mathbb{R}$ is a multiplicative function. That is, M satisfies

$$M(xy)=M(x)\,M(y) \quad \text{for all} \quad x,y\in\mathbb{R}.$$

It is easy to check that the above solution satisfies (1). Next, we show that it is the only solution of (1). Adding 1 to both sides of (1), we get

$$1+f(x+y+xy)=1+f(x)+f(y)+f(x)f(y)$$

which is

$$1+f(x+y+xy)=(1+f(x))(1+f(y)) \qquad (2)$$

for all $x, y\in\mathbb{R}$. Defining $F:\mathbb{R}\to\mathbb{R}$ as

$$F(x)=1+f(x) \qquad (3)$$

we see that (2) reduces to

$$F(x+y+xy)=F(x)F(y) \qquad (4)$$

for all $x, y\in\mathbb{R}$. Replacing x by $x-1$ and y by $y-1$ in (4), we have

$$F(xy-1)=F(x-1)F(y-1) \qquad (5)$$

for all $x, y\in\mathbb{R}$. Define

$$M(x)=F(x-1). \qquad (6)$$

Then (5) reduces to

$$M(xy)=M(x)M(y) \qquad (7)$$

for all $x, y \in \mathbb{R}$. By (6) and (3), we get

$$\begin{aligned} f(x) &= F(x) - 1 \\ &= M(x+1) - 1. \end{aligned}$$

□

Also solved by Marc Troyanov (École Polytechnique Féderale De Lausanne, Switzerland), Tom De Medts (Ghent University, Belgium), Reinhold Kainhofer (Vienna University, Austria), W. Fensch (Zenthen, Germany).

62. Let $f : \mathbb{R} \to \mathbb{R}$ be a function with Darboux property. If f^2 has limit in any point of $\mathbb{R}$ then f is continuous on $\mathbb{R}$.

(Dorin Andrica, "Babeş-Bolyai" University, Cluj-Napoca, Romania, and Mihai Piticari, "Dragoş-Vodă" National College, Câmpulung Moldovenesc, Romania)

Solution by the proposer. Let $g(x) = f^2(x)$. Then

$$f(x) = \begin{cases} \sqrt{g(x)} & \text{if} \quad x \in A \\ -\sqrt{g(x)} & \text{if} \quad x \in \mathbb{R} \setminus A \end{cases}$$

where $A \subseteq \mathbb{R}$. Consider $x_0 \in \mathbb{R}$. If g has infinitely limit at x_0, then $\lim\limits_{x\to x_0} g(x) = +\infty$. If there is a neighbourhood V of x_0 such that $V \subseteq A$, then $\lim\limits_{x\to x_0} f(x) = +\infty$, which is not possible because f has Darboux property. Similarly, if $\lim\limits_{x\to x_0} f(x) = -\infty$.

Assume that there are sequences $(\alpha_n)_{n\geq 1}, (\beta_n)_{n\geq 1}$ with $\alpha_n < x_0 < \beta_n$ such that for any positive integer n, one can find $x_n, y_n \in (\alpha_n, \beta_n)$ with $x_n \in A$ and $y_n \in \mathbb{R} \setminus A$. It follows that $f(x_n) \to +\infty$ and $f(y_n) \to -\infty$, hence there is $N > 0$ such that for any $n \geq N$ we have $f(x_n) > 0$ and $f(y_n) < 0$. Since the function f has Darboux property, there exists z_n between x_n and y_n such that $f(z_n) = 0$. Hence $f(z_n) \to 0$ and so $g(z_n) \to 0$, which is a contradiction. Therefore $\lim\limits_{x\to x_0} g(x)$ is finite.

If $\lim\limits_{x\to x_0} g(x) = 0$, then $\lim\limits_{x\to x_0} f(x) = 0$. Assume that $\lim\limits_{x\to x_0} g(x) > 0$. If there is a neighbourhood V of x_0 with $V \subseteq A$, then $\lim\limits_{x\to x_0} f(x) = \lim\limits_{x\to x_0} \sqrt{g(x)}$, and therefore $\lim\limits_{x\to x_0} f(x)$ is finite. The same result holds if there is a neighbourhood W of x_0 with $W \subseteq \mathbb{R} \setminus A$. Assume that in any neighbourhood V of x_0 there are points of A and points of $\mathbb{R} \setminus A$. Then there are sequences $(x_n)_{n\geq 1}, (z_n)_{n\geq 1}$ with $x_n < x_0 < z_n$, $x_n \nearrow x_0$, $y_n \searrow x_0$ and $f(z_n) = \sqrt{g(x_n)}$, $f(z_n) = -\sqrt{g(z_n)}$. It follows that $\lim\limits_{n\to\infty} \sqrt{g(x_n)} = \lim\limits_{n\to\infty} -\sqrt{g(z_n)}$, i.e. $\lim\limits_{x\to x_0} \sqrt{g(x)} = 0$, hence $\lim\limits_{x\to x_0} g(x) = 0$, contradicting the assumption $\lim\limits_{x\to x_0} g(x) > 0$. Finally, there is a neighbourhood V of x_0 with $V \subseteq A$ or $V \subseteq \mathbb{R} \setminus A$. It follows that $\lim\limits_{x\to x_0} f(x) = \lim\limits_{x\to x_0} \sqrt{g(x)}$ or $\lim\limits_{x\to x_0} f(x) = -\lim\limits_{x\to x_0} \sqrt{g(x)}$. Hence F has finite limit at any point. But f has Darboux property, so it must be a continuous function on $\mathbb{R}$. □

Also solved by Reinhold Kainhofer (Vienna University, Austria), Mihály Bencze (Brasov, Romania) .

63. Let $\alpha > 0$ be a positive real number and let $\beta \geq 2$ be an even integer. Let $x \in \mathbb{R}$ be a fixed real number and let $f : \mathbb{R} \to \mathbb{R}$ be a bounded function on $\mathbb{R}$ such that $\lim\limits_{t\to x} f(t) = L$. Prove that

$$\lim_{n\to\infty} n^{\frac{\alpha}{\beta}} \int_{-\infty}^{\infty} \frac{f(t)}{1 + n^\alpha (t-x)^\beta} dt = \frac{2\pi L}{\beta \sin\frac{\pi}{\beta}}.$$

(Ovidiu Furdui, Campia Turzii, Cluj, Romania)

Solution by the proposer. First we observe that

$$\int_{-\infty}^{\infty} \frac{dt}{1 + n^\alpha (t-x)^\beta} = \int_{-\infty}^{\infty} \frac{dt}{1 + n^\alpha t^\beta} = 2\int_0^{\infty} \frac{dt}{1 + n^\alpha t^\beta} = \frac{2\pi}{n^{\frac{\alpha}{\beta}} \beta \sin\frac{\pi}{\beta}}.$$

Let $M = \sup\limits_{x\in\mathbb{R}} |f(x)|$ and let $\epsilon > 0$. We have since $\lim\limits_{t\to x} f(t) = L$, that there is $\delta = \delta(x, \epsilon) > 0$ such that for all $|t - x| < \delta$ one has that $|f(t) - L| < \epsilon$. We have,

$$\begin{aligned} n^{\frac{\alpha}{\beta}} \int_{-\infty}^{\infty} \frac{f(t)}{1 + n^\alpha (t-x)^\beta} dt - \frac{2\pi L}{\beta \sin\frac{\pi}{\beta}} &= n^{\frac{\alpha}{\beta}} \int_{-\infty}^{\infty} \frac{(f(t) - L)dt}{1 + n^\alpha (t-x)^\beta} \\ &= n^{\frac{\alpha}{\beta}} \left(\int_{-\infty}^{x-\delta} + \int_{x-\delta}^{x+\delta} + \int_{x+\delta}^{\infty} \right) \frac{(f(t) - L)dt}{1 + n^\alpha (t-x)^\beta} \\ &= A_n + B_n + C_n. \end{aligned} \tag{8}$$

On the other hand,

$$\begin{aligned} |A_n| &= \left| n^{\frac{\alpha}{\beta}} \int_{-\infty}^{x-\delta} \frac{f(t) - L}{1 + n^\alpha (t-x)^\beta} dt \right| \leq n^{\frac{\alpha}{\beta}} \int_{-\infty}^{x-\delta} \frac{|f(t) - L|}{1 + n^\alpha (t-x)^\beta} dt \\ &\leq n^{\frac{\alpha}{\beta}} (M + |L|) \int_{-\infty}^{x-\delta} \frac{dt}{1 + n^\alpha (t-x)^\beta} \overset{t-x=y}{=} n^{\frac{\alpha}{\beta}} (M + |L|) \int_{-\infty}^{-\delta} \frac{dy}{1 + n^\alpha y^\beta} \\ &\overset{\beta \text{ even}}{=} n^{\frac{\alpha}{\beta}} (M + |L|) \int_{\delta}^{\infty} \frac{dy}{1 + n^\alpha y^\beta} = n^{\frac{\alpha}{\beta}} (M + |L|) \left(\int_0^{\infty} - \int_0^{\delta} \right) \frac{dy}{1 + n^\alpha y^\beta} \\ &= n^{\frac{\alpha}{\beta}} (M + |L|) \left(\frac{\pi}{n^{\frac{\alpha}{\beta}} \beta \sin\frac{\pi}{\beta}} - \int_0^{\delta} \frac{dy}{1 + n^\alpha y^\beta} \right) \\ &\overset{n^\alpha y^\beta = z}{=} n^{\frac{\alpha}{\beta}} (M + |L|) \left(\frac{\pi}{n^{\frac{\alpha}{\beta}} \beta \sin\frac{\pi}{\beta}} - \frac{1}{\beta n^{\frac{\alpha}{\beta}}} \int_0^{n^\alpha \delta^\beta} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz \right) \\ &= (M + |L|) \left(\frac{\pi}{\beta \sin\frac{\pi}{\beta}} - \frac{1}{\beta} \int_0^{n^\alpha \delta^\beta} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz \right). \end{aligned} \tag{9}$$

By symmetry we also have that

$$|C_n| \leq (M + |L|) \left(\frac{\pi}{\beta \sin\frac{\pi}{\beta}} - \frac{1}{\beta} \int_0^{n^\alpha \delta^\beta} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz \right). \tag{10}$$

On the other hand,

$$\begin{aligned} |B_n| &= \left| n^{\frac{\alpha}{\beta}} \int_{x-\delta}^{x+\delta} \frac{f(t) - L}{1 + n^\alpha (t-x)^\beta} dt \right| \leq n^{\frac{\alpha}{\beta}} \int_{x-\delta}^{x+\delta} \frac{|f(t) - L|}{1 + n^\alpha (t-x)^\beta} dt \\ &\leq \epsilon n^{\frac{\alpha}{\beta}} \int_{x-\delta}^{x+\delta} \frac{dt}{1 + n^\alpha (t-x)^\beta} \\ &\overset{t-x=u}{=} \epsilon n^{\frac{\alpha}{\beta}} \int_{-\delta}^{\delta} \frac{du}{1 + n^\alpha u^\beta} \overset{\beta \text{ even}}{=} 2\epsilon n^{\frac{\alpha}{\beta}} \int_0^{\delta} \frac{du}{1 + n^\alpha u^\beta} \\ &\overset{n^\alpha u^\beta = z}{=} \frac{2\epsilon}{\beta} \int_0^{n^\alpha \delta^\beta} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz \\ &\leq \frac{2\epsilon}{\beta} \int_0^{\infty} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz = \frac{2\epsilon\pi}{\beta \sin\frac{\pi}{\beta}}. \end{aligned} \tag{11}$$

Combining (8), (9), (10) and (11) we obtain that

$$\left| n^{\frac{\alpha}{\beta}} \int_{-\infty}^{\infty} \frac{f(t)}{1+n^{\alpha}(t-x)^{\beta}} dt - \frac{2\pi L}{\beta \sin\frac{\pi}{\beta}} \right| \leq 2(M+|L|)\left(\frac{\pi}{\beta \sin\frac{\pi}{\beta}} - \frac{1}{\beta} \int_0^{n^{\alpha}\delta^{\beta}} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz \right) + \frac{2\epsilon\pi}{\beta \sin\frac{\pi}{\beta}}.$$

Letting $n \to \infty$ in the preceding inequality we obtain, since

$$\lim_{n\to\infty} \int_0^{n^{\alpha}\delta^{\beta}} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz = \int_0^{\infty} \frac{z^{\frac{1}{\beta}-1}}{1+z} dz = \frac{\pi}{\sin\frac{\pi}{\beta}},$$

that

$$\lim_{n\to\infty} \left| n^{\frac{\alpha}{\beta}} \int_{-\infty}^{\infty} \frac{f(t)}{1+n^{\alpha}(t-x)^{\beta}} dt - \frac{2\pi L}{\beta \sin\frac{\pi}{\beta}} \right| \leq \frac{2\epsilon\pi}{\beta \sin\frac{\pi}{\beta}}.$$

Since $\epsilon > 0$ is arbitrary the desired result follows. □

64. A function $f(x,y)$ is defined on the unit square $[0,1]^2$ and is continuously-differentiable in each variable separately, i.e. $f(\cdot,y) \in C^1[0,1]$ for any $y \in [0,1]$, and $f(x,\cdot) \in C^1[0,1]$ for any $x \in [0,1]$. Is it possible that $\int_0^1 f(x,y)\,dx > 0$ for any y, and $\int_0^1 f(x,y)\,dy < 0$ for any x?

Author's comment. There is an obvious but incorrect solution for this problem. Applying the Fubini-Tonelli theorem, we get

$$0 < \int_0^1 \left(\int_0^1 f(x,y)dx \right) dy = \int_0^1 \left(\int_0^1 f(x,y)dy \right) dx < 0,$$

which is a contradiction. So, the answer is "No". But actually the answer is "Yes". Moreover, there are even infinitely smooth examples of such functions f. The mistake in the reasoning above is that Fubini and Tonelli's theorem may not be applicable for our function. Fubini's theorem assumes that the modulus $|f(x,y)|$ is summable on the square and Tonelli's theorem assumes that both the repeated integrals exist and the function f is bounded at least from one side (either from below or from above). Our function f may not satisfy these assumptions, although it is smooth in each variable.

(Vladimir Protasov, Moscow State University, Russia)

Solution by the proposer. *Answer.* Yes.

Solution. Consider an arbitrary function $\varphi \in C^{\infty}(\mathbb{R})$ supported on the segment $[0, \frac{1}{2}]$ and negative at all interior points of this segment. Let

$$h(t) = \frac{\varphi(t) - \varphi(t-1/2)}{t}$$

for $t \in \mathbb{R} \setminus \{0\}$, and $h(0) = 0$.

Clearly, $h \in C^{\infty}(\mathbb{R})$ and supp $h \subset [0,1]$. It is easy to see that

$$\int_0^1 h(t)dt < 0 \quad \text{and} \quad \int_y^1 th(t)dt \geq 0 \quad \text{for any} \quad y \in [0,1].$$

Multiplying h by a positive constant we may assume that the first integral is -1. Consider now the following function $g(x,y)$:

$$g(x,y) = \begin{cases} x^{-3}h(\frac{y}{x}) & \text{for} \quad x \in (0,1], y \in [0,1]; \\ 0 & \text{for} \quad x = 0, y \in [0,1]. \end{cases}$$

We have

$$\int_0^1 g(x,y)dy = -x^{-2} \leq -1 \text{ for any } x \in (0,1)$$

and

$$\int_0^1 g(x,y)dx = y^{-2} \int_y^1 th(t)dt \geq 0 \text{ for any } y \in [0,1].$$

Now we define the function $f(x,y)$ as follows:

$$f(x,y) \quad = \quad g(x,y) + g(1-x,y) - g(y,x) - g(1-y,x)\,.$$

We have $\int_0^1 f(x,y)dx \geq 1$ for any $y \in [0,1]$ and $\int_0^1 f(x,y)dy \leq -1$ for any $x \in [0,1]$. □

Note. Problem 52 was also solved and generalized by Valery V. Karachik (South-Ural State University, Chelyabinsk, Russia).

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will be devoted to *Number Theory*.

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Shmuel Onn (Technion – Israel Institute of Technology, Haifa, Israel)
Nonlinear Discrete Optimization. An Algorithmic Theory
Zurich Lectures in Advanced Mathematics

ISBN 978-3-03719-093-7
2010. 147 pages. Softcover. 17 x 24 cm
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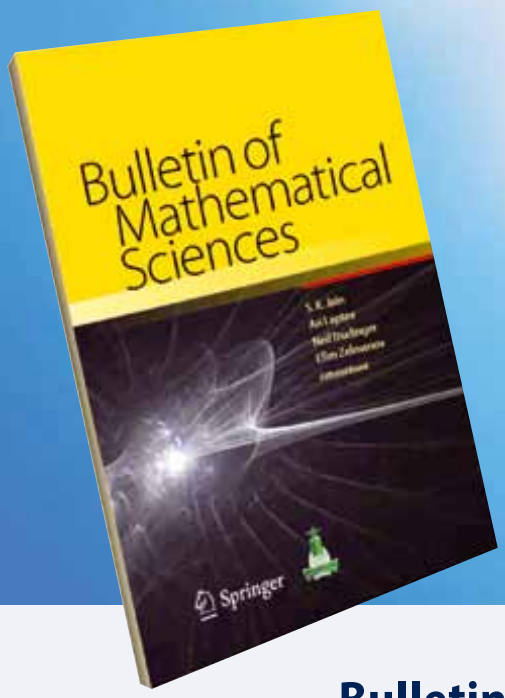

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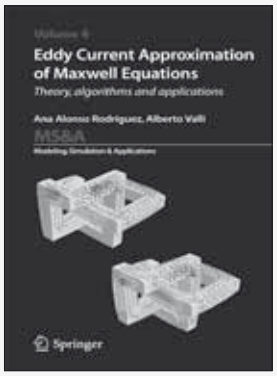

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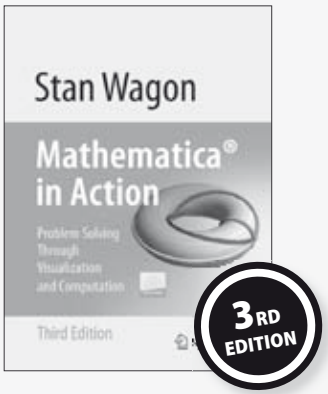

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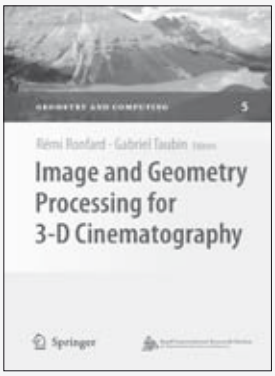

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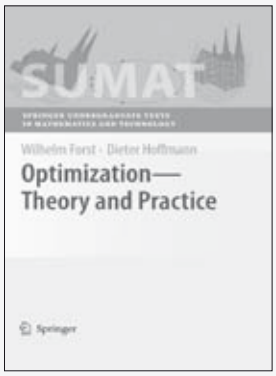

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