NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

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European
Mathematical
Society

AMERICAN MATHEMATICAL SOCIETY

AN EPSILON OF ROOM, I: REAL ANALYSIS

pages from year three of a mathematical blog

Terence Tao, *University of California*

Graduate Studies in Mathematics, Vol. 117

Mar 2011 333pp
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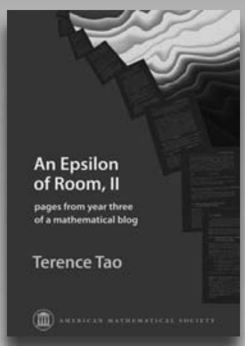
AN EPSILON OF ROOM, II

pages from year three of a mathematical blog

Terence Tao, *University of California*



In 2007 Terry Tao began a mathematical blog to cover a variety of topics, ranging from his own research and other recent developments in mathematics, to lecture notes for his classes, to nontechnical puzzles and expository articles. The first two years of the blog have already been published by the American Mathematical Society. The posts from the third year are being published in two volumes.
The first volume consists of a second course in real analysis, together with related material from the blog.
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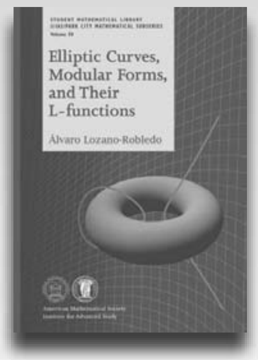
Álvaro Lozano-Robledo, *University of Connecticut*



Many problems in number theory have simple statements, but their solutions require a deep understanding of algebra, algebraic geometry, complex analysis, group representations, or a combination of all four. The original simply stated problem can be obscured in the depth of the theory developed to understand it. This book is an introduction to some of these problems, and an overview of the theories used nowadays to attack them, presented so that the number theory is always at the forefront of the discussion.

Student Mathematical Library, Vol. 58

Mar 2011 195pp
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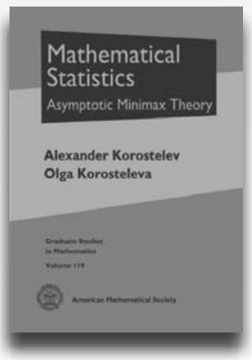
MATHEMATICAL STATISTICS

Asymptotic Minimax Theory

Alexander Korostelev, *Wayne State University &* Olga Korosteleva, *California State University*



This book is designed to bridge the gap between traditional textbooks in statistics and more advanced books that include the sophisticated nonparametric techniques. It covers topics in parametric and nonparametric large-sample estimation theory. The exposition is based on a collection of relatively simple statistical models. It gives a thorough mathematical analysis for each of them with all the rigorous proofs and explanations. The book also includes a number of helpful exercises.

Graduate Studies in Mathematics, Vol. 119

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Editorial Team

EMS Executive Committee

EMS Agenda

2011

30 March–3 April
EUROMATH-European Student Conference in Mathematics
Athens, Greece
www.euromath.org

7–8 May
EMS Meeting of Presidents of Mathematical Societies in Europe, Bilbao, Spain
Stephen Huggett: s.huggett@plymouth.ac.uk

24 May
Abel Prize Award Ceremony, Oslo, Norway
www.abelprisen.no/en/

27 June–2 July
CIME-EMS Summer Course on "Current challenges in stability issues for numerical differential equations", Cetraro, Italy
php.math.unifi.it/users/cime/

3–8 July
Third European Set Theory Conference,
ESF-EMS-ERCOM Conference, Edinburgh, United Kingdom
www.esf.org/activities/esf-conferences/details/2011/confdetail368.html

3–8 July
Completely Integrable Systems and Applications,
ESF-EMS-ERCOM Conference, Erwin Schrödinger Institute, Vienna, Austria
www.esf.org/conferences/11369

18–22 July
ICIAM 2011 Congress, Vancouver, Canada
www.iciam2011.com

24–30 July
ESMTB/EMS summer school "Dynamical Models in Life Sciences" Evora, Portugal
c3.glocos.org/ssmtb/

5–9 September
15th General Meeting of European Women in Mathematics, CRM, Barcelona
www.crm.cat/ewm/

2012

30 June–1 July
Council Meeting of European Mathematical Society, Kraków, Poland
www.euro-math-soc.eu

2–7 July
6th European Mathematical Congress, Kraków, Poland
http://www.6ecm.pl

Editorial

Marta Sanz-Solé, EMS President

It is a great pleasure for me to address all EMS members, and other possible readers, at the beginning of my responsibility as president. With a history of 20 years, our society has set up solid roots, reached its maturity and achieved a significant level of influence. All this could not have been possible without the wise vision, the strong leadership and the intensive efforts of all my honourable predecessors. And I should not forget the very many people who have devoted part of their time working on the different committees and cooperating on numerous activities. Thanks to their generous collaboration, the EMS has established itself as the leading society for the promotion, development and dissemination of mathematics within Europe.

I could not start talking about projects for the future without mentioning and taking into consideration the last message from the previous president Ari Laptev. His extensive account of the state of the society makes us all aware of the great diversity and extent of the objectives achieved during his term of office, the suitability of the strategies in the approaches, and the variety and importance of the ongoing projects.

The standing EMS committees are working with a thoroughly designed roadmap and are covering a wide range of topics of interest. There are, however, some important questions which have been less considered and that, in my view, deserve our more intensive attention. For example: mathematical publications in general, some crucial aspects of mathematical education and the coordination between mathematical research and industrial mathematics.

Let me be more explicit. The editorial activity developed by the EMS Publishing House has reached a significant level of expansion and consolidation. This has placed us in a good position to be partners in the discussion, elaboration and adoption of good practices under the leadership of the IMU. Moreover, according to the mission statement of the EMS Publishing House, we should endeavour to make its publications available, under sustainable conditions, to less favoured communities of mathematicians.

In the field of mathematical education, the EMS must support and be a partner in experiences and initiatives aimed at promoting the mathematical talent of young students, building on their fascination with the subject and attracting them to the discipline. We must not forget to convey the message of utility to other sciences and its crucial role in very many scientific and technological innovations. It is my belief that a strong involvement of the EMS could give to such existing activities a very beneficial trans-national dimension and a broader scope. Increasing attraction for mathematics and improving its education is crucial to maintaining European scientific leadership and the scientific excellence of mathematics in Europe.

The final document of the ESF-EMS Forward Look project Mathematics and Industry, which has been published recently, assesses the role of mathematics as an essential factor in industrial creation and, therefore, as one of the ingredients needed to increase Europe's competitiveness. Among the strategic objectives mentioned in this document are: to foster a European network in applied mathematics towards a smart economy, to facilitate the mobility and to encourage the exchange of knowledge between industry and academia, and to promote and improve the career path in industrial mathematics. I would like the EMS to play a very active part in the design of actions to achieve objectives along these lines. These three issues – publications, education and coordination with industrial mathematics – require us to set up suitable structures to facilitate their analysis and also to make possible their coordination with committees.

One of the foundational objectives of the EMS was to set itself up as the European partner in mathematics for the EU, and this has already been achieved. Nowadays, the EMS is representing mathematics at European political and scientific forums. However, the organisation chart for scientific policy decision making and its further implementation has become more and more complex with the evolution and enlargement of the European Union. In order not to miss opportunities of communication and lobbying, we have just appointed a Group of Relations with European Institutions consisting of Jean-Pierre Bourguignon, Pavel Exner, Ari Laptev, Luc Lemaire, Mario Primicerio and myself. The aim of the group is to keep our community informed of developments and opportunities, to open doors within the EU and to design efficient strategies which could eventually capitalise on the leadership of the EMS to obtain large-scale funding from the EU. Our immediate task will be to elaborate a position paper for the 8th Framework Programme with the objective of convincing policymakers to acknowledge and specifically mention mathematics in the calls of this next Programme.

With about 60 member societies in Europe and more than 2500 individual members, the EMS has established itself as the leading society for the promotion, development and dissemination of mathematics within Europe, and for service to European mathematicians. This is all good news. But we must actively pursue the recruitment of new members and this is one of my important goals.

With the political evolution of Europe, we are witnessing emergent mathematical societies, led by a young generation of mathematicians with ambitious and solid plans. These communities need our support to develop soundly. The EMS will make every possible effort to help them with the process of reaching their maturity.

I would also like to increase cooperation with more learned and professional societies, both European and non-European, in order to reinforce the relationship between different scientific communities and work on projects of common interest. I have already started very promising contacts with the Bernoulli Society for Mathematical Statistics and Probability and also with the Latin American Society UMALCA, with the aim of establishing reciprocity agreements.

We must not neglect actions towards keeping the very positive trend on the increasing number of individual members. For this, we plan to focus on an active policy of dissemination of the EMS objectives, its ongoing projects, its available infrastructure and its achievements. We shall design publicity mechanisms and establish a more direct and personal contact with European mathematicians. We also plan to analyse the membership map of the society to detect the communities where our presence is still scarce and eventually to improve the situation. Communication between the EMS and its full members has benefited a lot from the annual meetings with presidents of national mathematical societies. I shall continue with this tradition and I can already announce that the next meeting will be held on 7–8 May 2011, hosted by the Royal Spanish Mathematical Society. National societies are the natural partners for extending the visibility of the EMS and for recruiting new members. Finally, we would like to give a more visible role and specific tasks to the whole body of individual members and the delegates of societies at the council.

There is a part of the mathematical community that deserves special attention, i.e. young mathematicians. We should help them to start their professional lives and to find the most appropriate jobs. There is a specific project that we would like to develop. This is a European Math-Jobs-Site, a virtual fair where employers and potential employees could meet. This could also be complemented by a Math-Jobs fair being held at large scientific events, like the European Congresses of Mathematics. We are already seeking funds for this project and if we are successful, this will provide a useful service not only to the young community of mathematicians but also to mathematics departments, institutes and research centres.

Let me end this letter by saying that it is a great honour for me to devote my efforts to these and other related challenges, and that I am most grateful for the confidence you have expressed by giving me your vote. I also feel very privileged to have the support of a team consisting of such distinguished mathematicians and devoted people with deep vision and stature. I am indeed fortunate to be able to count on their continuing support and collaboration as an essential foundation as I begin to preside over the EMS for the next four years.

Ari Laptev has conducted the EMS with an extremely positive, integrating, creative and lucid style, calling the tune firmly and vigorously and heading ambitious goals. By taking over from him, I am very well aware of the impossibility of matching his incommensurable activity, devotion and energy. But, as if we were in a relay race, I take the baton from him, with deep indebtedness, great enthusiasm and the strong commitment to work for the mathematical community and to keep the EMS at the service of the best interests of our discipline.

Funding crisis for scientific institutions in Bulgaria

Bulgarian science is mainly concentrated at the Bulgarian Academy of Sciences and at Sofia University. The government has announced that it plans to transfer many of the institutes of the academy to other institutions and, at the same time, to cut down its support to Bulgarian Science in a very substantial manner. This has drastic negative effects also for the Bulgarian mathematical community. The EMS is very concerned, and EMS-president Marta Sanz-Solé has written a letter supporting the case of Bulgarian mathematics to the Bulgarian Prime Minister and to the Chair of the Bulgarian Parliament. She has asked the presidents of mathematical societies all over Europe to take similar action.

More information about the development in Bulgaria (and a much more positive development for Romanian scientists) can be found in two recent articles in *Nature* (469, p. 131–132, Different Strokes, www.nature.com/nature/journal/v469/n7329/full/469131b.html, and p. 142–143, Science fortunes of Balkan neighbours diverge, www.nature.com/news/2011/110112/full/469142a.html).
Bulgarian colleagues have started an internet petition to their support asking for signatures at www.science.nauka2010.com/

Meetings of Presidents of National Member Societies of the EMS

The Royal Spanish Mathematical Society (RSME) commemorates the centenary of its founding this year. In this regard, the society has invited the presidents of the national member societies of the EMS to a meeting on the weekend 7-8 May 2011 in Bilbao. This is the fourth meeting of a very informative and successful series of meetings, previously held in Warsaw (2008), Luminy (2009) and Bucharest (2010). The latter had to be officially cancelled because of the volcanic ashes disrupting air traffic; nevertheless, several of the presidents gathered for an informal meeting.

New members of EC of the EMS

Rui Loja Fernandes is the Head of the Department of Mathematics at Instituto Superior Técnico, Lisbon. In 1994, he obtained his PhD in mathematics at the University of Minnesota, under the direction of Peter Olver, and his current interests include Poisson geometry, integrable systems and Lie theory. Together with Marius Crainic, he found a solution of the long-standing problem of integrability of Lie algebroids to Lie groupoids.

He was a recipient of the Best Thesis Award of the University of Minnesota for his PhD thesis and of the Gulbenkian Foundation prize for basic sciences (2003). In 2006, he became a corresponding member of the Lisbon Academy of Sciences. He is currently the Editor-in-Chief of *Portugaliae Mathematica*, a journal published by the EMS Publishing House.

He is married to a lawyer and has three children.

Volker Mehrmann was born in Detmold, Germany, on 24 April 1955. He studied mathematics and physics at Universität Bielefeld, where he completed his Diploma in 1979, the teacher examination in 1981 and a PhD in 1982, after spending a year as a graduate student at Kent State University, Ohio. After a year as visiting professor at the University of Wisconsin in Madison 1984/85 he did his habilitation again in Bielefeld and then spent a year at the IBM Research Center in Heidelberg. From 1990 to 1992 he held a full professor position at RWTH Aachen and then in 1993 he became full professor for numerical algebra at TU Chemnitz. In 2000, he accepted an offer to become full professor for numerical mathematics at TU Berlin. In 2002, he was one of the founding members of the DFG Research Center Matheon and the deputy chair until 2008, when he followed Martin Grötschel as chair.

His research interests are in numerical mathematics, control theory and matrix theory, as well as scientific computing. In recent years, he has focused on the development and analysis of numerical methods for nonlinear eigenvalue problems and differential-algebraic systems with applications in many fields such as mechanical systems, electronic circuit simulation and acoustic field computations.

He is a member of acatech, the German academy of engineering, President of GAMM (the International Associations for Applied Mathematics and Mechanics) and co-Editor in Chief of the journal *Linear Algebra and its Applications*. He believes that mathematics has become a central prerequisite for the societal development of the 21st century and that mathematical methods play a key role in the modelling, simulation, control and optimization of all areas of technological development but that the role of mathematics (pure and applied) is highly underestimated and has to be made more visible and, in particular, the position of mathematics in the European research landscape has to be improved.

EMS Executive Committee Meeting in Lausanne, November 13–14, 2010

Stephen Huggett, EMS Secretary

This was an unusually large meeting and, as will be seen from this report, there were several extra guests, including member-elect Rui Loja Fernandes and president-elect Marta Sanz-Solé.

Reports

Ari Laptev gave a brief report as president, starting with the meeting in Berne celebrating the 100th anniversary of the Swiss Mathematical Society. There were some extremely fruitful discussions over the future of *Commentarii Mathematici Helvetici* and a new contract was agreed with the EMS Publishing House.

He also noted that he had attended the Digital Mathematical Library meeting in Prague and, among other things, was pleased to be able to report that all of the Proceedings of the European Congresses of Mathematics would be digitized.

Pavel Exner reported that the imminent danger to the Erwin Schrödinger Institute had been averted but that it was still at risk in the longer term, so it remained important to maintain the campaign in its support.

Vasile Berinde gave his last report to the EC as Publicity Officer. He described some recent activity, such as the page in the ICM Intelligencer and the new leaflets and posters which had been sent to the ICM and to 2,400 departments throughout Europe. The EC discussed how to gauge the effectiveness of these methods and agreed that we should try to be present at as many large meetings as possible.

The president thanked Vasile for all his hard work and said that he had also been an invaluable link to the mathematical communities in south-east Europe. Vasile expressed his regret at having to resign but pressure of other work had made it unavoidable. The Executive Committee then agreed to appoint Dmitry Feichtner-Kozlov as Publicity Officer.

Helge Holden demonstrated the beta version of the Book Reviews part of the website. The intention was that it would also incorporate a blog. The online membership database was also discussed briefly and it was noted that a quarter of the members had now set up their own individual accounts.

Stefan Jackowski gave a presentation describing the latest preparations for the 6th European Congress of Mathematics, emphasising that for him the most important thing was to attract people to the conference.

Mireille Martin-Deschamps reported on a new series of summer schools for good pre-university students, following the Dubna model. The Scientific Committee is chaired by Etienne Ghys and the first school will be in Germany in 2011. After that it will alternate between Germany and France. The talks will be in English and the schools will be open to all nationalities.

Committees

Mario Primicerio, the Chair of the Applied Mathematics Committee, reported that the meeting in Sofia immediately following the council had been very successful and he expressed his gratitude to the mathematicians in Bulgaria for organising it. Since then, the committee had met in Paris and, among other things, was exploring the possibility of forming a European network of laboratories in industrial mathematics.

Tsou Sheung Tsun, the Chair of the Committee for Developing Countries (CDC), presented her report, noting in particular that the workshop on information retrieval arranged in Addis Ababa by the university there, in collaboration with the committee, had been extremely positive. She then described in detail the work of the CDC on the proposal for Emerging Regional Centres of Excellence, with which the Executive Committee (EC) was happy to agree. Also, the EC agreed to elect Ramadas Ramakrishnan to the committee and to re-elect Anders Wandahl.

At the Rolex Learning Center, Lausanne

Marta Sanz-Solé proposed that the EC appoint a new "Group of Relations with European Institutions", chaired by herself from January 2011, consisting of Jean-Pierre Bourguignon, Pavel Exner, Ari Laptev, Luc Lemaire and Mario Primicerio. Its remit would be to:

- Write a position paper for the 8th Framework Programme.
- Open doors within the EU.
- Keep our community informed of developments and opportunities.

The Executive Committee agreed with this proposal.

Zvi Artstein described the working practices of the Meetings Committee in some detail, and the EC agreed to delegate to the committee responsibility for four specific activities:

- EMS Lectures.
- EMS distinguished speakers.
- Mathematical Weekends.
- The Mathematics Day.

Arne Jensen gave a full report as Chair of the new Ethics Committee. The committee met in Oberwolfach in September and was already working on cases of plagiarism and a code of practice. There were open questions over whether the committee should address the problems of (i) the influence of citations, and (ii) funding being tied to very specific projects, which can lead to unethical behaviour.

Publishing

Ari Laptev reported that the Board of Trustees of the EMS Publishing House had agreed to enlarge its membership by two, in order to broaden representation from mathematicians across Europe.

European Research Council

Pavel Exner reported that the third round of European Research Council starting grants finished recently, with 25 grants to mathematics. The third call for advanced grants would be published soon. The total support (starting and advanced) for mathematics now stands at 100 million euros, for 99 grants. It is important to work on the role of the ERC in the 8th Framework Programme and, as always, the support of the community is vital.

Thanks

The president expressed the deep gratitude of the entire Executive Committee to the Swiss Mathematical Society and the EPFL for their extreme generosity and warm hospitality.

Otto Neugebauer Prize in the History of Mathematics

Jeremy Gray

The European Mathematical Society has decided to award a prize for work in the history of mathematics. It is to be named after the distinguished historian of mathematics Otto Neugebauer and it carries a cash award of €5,000 generously provided by Springer-Verlag. The prize will be awarded every four years by the EMS President during the prize ceremony of the European Congress of Mathematics, and will be given for the first time at the 6ECM in Kraków (Poland) in 2012.

For the purposes of the prize, history of mathematics is to be understood in a very broad sense. It reaches from the study of mathematics in ancient civilisations to the development of modern branches of mathematical research, and it embraces mathematics wherever it has been studied in the world. In terms of the Mathematics Subject Classification it covers the whole spectrum of item 01Axx (History of mathematics and mathematicians). Similarly, there are no geographical restrictions on the origin or place of work of the prize recipient. All methodological approaches to the subject are acceptable.

The prize is to be awarded for highly original and influential work in the field of history of mathematics that enhances our understanding of either the development of mathematics or a particular mathematical subject in any period and in any geographical region. The prize may be shared by two or more researchers if the work justifying it is the fruit of collaboration between them.

The European Mathematical Society wishes, through this prize, to strengthen its links with the history of mathematics community. The history of mathematics has enjoyed something of a resurgence in recent years across all fields, places and periods. In the field of ancient Mesopotamian and Egyptian mathematics and astronomy – topics perhaps closest to Neugebauer's own interests – there have been major studies reviving and deepening our understanding of how the mathematics was done and how the ancient practice of mathematics can be illuminated by the close study of archaeological records. The study of Indian mathematics, which is to be quarried from particularly difficult sources, has been opened up by the persistent work of several scholars. The mathematics and science of ancient Islam has even progressed from scholarly works of many kinds to popular books and television programmes. There is now an abundance of translations and studies of mathematics in China and much more is known than ever before of the indigenous mathematical practices of many regions of Africa and of pre-Columbian America, as well as the Pacific.

The 17th and 19th centuries in Europe have been particularly well served, with a series of works on Descartes, Leibniz and Newton among others, as well as coverage of almost every field of algebra, analysis and geometry of the period. More remains to be done on the 20th century, of course, and a number of important topics in the last several centuries remain to be treated, most notably the development of ordinary and partial differential equations. Despite a number of initiatives from both sides over the years, more needs to be done to build up links with the field of history of science, which has tended to move away from detailed studies of the sciences to social and contextual analyses. However, many national mathematical societies, as well as the organisers of the International Congress of Mathematicians, continue to support the history of mathematics through their journals and in other ways. The establishing of the Otto Neugebauer Prize comes at an important time for the discipline of history of mathematics, given the number of institutional changes that have affected the funding of research in many European countries, and it is to be hoped it will prove a valuable stimulus to research.

The prize honours the work of Otto Neugebauer. Neugebauer was born in Innsbruck, Austria, on 26 May 1899, and after the First World War he entered the University of Graz intending to study physics. He switched to Munich in 1921, where he studied under Arnold Sommerfeld, on whose advice he moved the next year to Göttingen. There he met and became close friends with the new director of the mathematics institute, Richard Courant. He became an assistant at the institute in 1923 and Courant's special assistant the next year. Then, he spent a year in Copenhagen, where he wrote a paper with Harald Bohr on differential equations involving almost periodic functions. Bohr also became a close friend but by this time Neugebauer had found what became his lifelong interest: the history of ancient mathematics. He obtained the necessary qualifications and became a Privatdozent in the autumn of 1927 and began lecturing on mathematics and the history of ancient mathematics. In 1929, he founded, with O. Toeplitz and J. Stenzel as co-editors, the Springer series *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*. The journal was devoted to the history of the mathematical sciences and, as its title indicated, published sources and studies.

By then Neugebauer had learned Akkadian and had begun to write on the mathematics of ancient Mesopotamia. This work culminated in the three-volume *Mathematische Keilschrift-Texte* (1935–37), the work which showed definitively that the range and richness of ancient Mesopotamian mathematics far exceeded what could have been guessed from Egyptian or Greek sources.

When the Nazis came to power in Germany, Neugebauer was swiftly dismissed. For a time he worked in Copenhagen on astronomy in Mesopotamia and at the start of the Second World War he moved to America where he obtained a position at Brown University, where there was an excellent library on the history of mathematics established by R. C. Archibald. The move to Brown University was assisted by R. G. D. Richardson, who was the secretary of the American Mathematical Society and dean of the graduate school at Brown University and was very supportive of another of Neugebauer's activities: the establishing of *Mathematical Reviews,* which appeared for the first time in 1940. Neugebauer had previously become the founding editor of the review journal *Zentralblatt für Mathematik und ihre Grenzgebiete* in 1931 but refused to submit to the restrictions the Nazi regime had sought to impose.

Neugebauer's achievements after the war include the publication *Mathematical Cuneiform Texts* (1945), which for many years was the standard account of Mesopotamian mathematics, his most popular book *The Exact Sciences in Antiquity* (1951, 2nd ed. 1957) and the three-volume *Astronomical Cuneiform Text*s, which was finally published in 1955. He then worked on yet another definitive three-volume project *A History of Ancient Mathematical Astronomy* (1975), which was the first publication in Springer's *Sources and Studies in the History of Mathematics and Physical Sciences*. He was also able to create at Brown the leading research centre into ancient science and mathematics, where Abraham Sachs, Richard Parker, Gerald Toomer and David Pingree also worked for many years.

Neugebauer was the recipient of numerous honorary degrees and academic awards. He died on 19 February 1990.

For the first award the prize committee will be chaired by Jeremy Gray (Open University, England) and will consist of Lennart Berggren (Simon Fraser University, Canada), Jesper Lützen (University of Copenhagen, Denmark), Jeanne Peiffer (CNRS, France) and Catriona Byrne of Springer.

The right to nominate one or several laureates is open to anyone. Nominations are confidential; a nomination should not be made known to the nominee(s). Self-nominations are not acceptable.

A nomination letter should be addressed to the EMS office at Helsinki and received by the office during the year before the European Congress of Mathematics. It should contain a CV and a description of the candidate's work motivating the nomination, together with names of specialists who may be contacted. A call for nominations will appear in the Newsletter and on the webpage of the EMS no later than a year before the congress.

Note

Information about Otto Neugebauer was drawn from *Otto E. Neugebauer May 26, 1899-February 19, 1990* by N. M. Swerdlow (http://www.mat.ufrgs.br/~portosil/neugebau.html), which in turn was based on his article in the *Proceedings of the American Philosophical Society* (1993, vol. 137, 139–65).

The workshop on "Raising the Public Awareness of Mathematics" in Óbidos (Portugal)

Erhard Berhends (Berlin)

Óbidos is a charming town situated one hour by car to the north of Lisbon, Portugal. In the "International Year of Mathematics 2010" (WMY2000) it was the site of the creation of the international exhibition "Beyond the Third Dimension" (http://alem3d.obidos.org/en/) and of a meeting of the EMS WMY2000 Committee. This committee launched the CD-ROM "Raising Public Awareness of Mathematics" in the framework of an EC project.

This year Óbidos again hosted an international workshop: "Raising the Public Awareness of Mathematics" (organisers: E. Behrends, Berlin; N. Crato, Lisbon; J. F. Rodrigues, Lisbon; see http://c2.glocos.org/index.php/RPAM/rpam2010).

The opening was on 26 September 2010; it took place in connection with a "mathematical afternoon" organised by the Portuguese Mathematical Society (SPM) in cooperation with the town of Óbidos. At this event mathematical films and lectures for the general public were presented. One of these lectures was given by G.-M. Greuel, the current president of ERCOM (the EMS committee of the European Research Centres on Mathematics) and the other was given by H. Leitão about mathematics in the Age of Discoveries.

Later, one could participate in a reception for an itinerant mathematical exhibition ("Medir o Tempo, o Mundo, o Mar") on the use of geometry to measure the universe and help astronomical navigation, jointly organised by the SPM and the Museum of Science of the University of Lisbon. The exhibition and reception took place at a local art gallery.

Also, on the occasion of this public awareness event, the website www.mathematics-in-europe.eu of the EMS was "officially" launched. And the fact that many members of the EMS/RPA committee were present in Óbidos was used to discuss in a separate meeting the next steps in connection with the realisation of this website.

"Raising the Public Awareness of Mathematics" was a joint initiative of the CIM (Centro Internacional de Matemática, Portugal) and the rpa ("raising public awareness") committee of the EMS. About 40 participants from Europe and the USA attended the workshop. In more than 30 lectures, information concerning various rpa activities was presented. Four aspects were of particular importance:

1. **National experiences**
   In and after the "International Year of Mathematics 2000" several European countries have realised extensive rpa projects that were described in a number of lectures. As examples we mention the talks of Reinhard Laubenbacher (USA, "Mathematics and the public in the USA") and Thomas Vogt (Germany, "Public Understanding of Math – Communicating Mathematics to Society at Large in Germany").
2. **Exhibitions / Mathematical Museums**
   Over the last few years there have been realised a number of (temporary or permanent) mathematical exhibitions. The experiences of the organisers were presented, e.g. the very successful project "Imaginary" was described by several participants and the structure of some large exhibitions in Germany ("Mathema", "Mathematicum"), Italy ("Giardino di Archimede") and Portugal ("Atractor") was explained by the organisers.
3. **Popularisation activities**
   The large variety of rpa projects that have been realised in the various countries was really impressive: films, popular websites, rpa using computer games or the history of mathematics. Surprisingly most of these activities were unknown to the majority of participants until this workshop.
4. **Popularisation: why and how?**
   A number of talks were of a more "fundamental" character. The contribution of Maria Dedó, for example, mirrored the experiences of the majority of participants: "To be rigorous – when is it appropriate and when is it only pedantry?" There is a rather narrow area of "exactness" between accessibility for a general audience and a precision that meets the professional standards

It should be noted that the results of this workshop will be published in a book: *Raising Public Awareness of Mathematics* (Springer, 2011). Everyone who wants to realise rpa projects in the future is invited to profit from the experience of the experts who met in Óbidos.

Participants of the workshop "Public Awareness in Mathematics" in Óbidos

Here is the complete list of lectures:

1 Ehrhard Behrends (Freie-Universität Berlin, Germany): MATHEMA (http://www.mathema-ausstellung.de/en.html).
2 Albrecht Beutelspacher (Universität Giessen, Germany): Mathematikum in Giessen, Germany – The success story of a mathematical science center.
3 Jean Pierre Bourguignon (Institut des Hautes Études Scientifiques, France): Raising Public Awareness – Discussing some Recent Initiatives in France.
4 F. Thomas Bruss (Université Libre de Bruxelles, Belgium): Public Awareness of Mathematics – Image, Influence and Importance.
5 Franka Brückler (University of Zagreb, Croatia): The maxmin principle in popularisation of mathematics – maximum effect with minimum costs.
6 Jorge Buescu (Universidade de Lisboa, Portugal): The Importance of Useful Mathematics.
7 Mireille Chaleyat-Maurel (Université de Paris 5, France): World Mathematical Year 2000 – ten years on…
8 Manuel Arala Chaves (Universidade do Porto, Portugal): Atractor ¬ examples of «interactive mathematics».
9 Krzysztof Ciesielski (Jagiellonian University, Kraków, Poland): The beauty of topology and a magic room.
10 Barry Cipra (Northfield, Minnesota, USA): WWMD?
11 Nuno Crato (ISEG-Universidade Técnica de Lisboa, Portugal), Renata Ramalho (Sociedade Portuguesa de Matemática): Balancing math popularization with public intervention: A mathematical society continued effort for raising public awareness in mathematics and youth mathematical education.
12 Maria Dedo (Università degli Studi di Milano, Italia): To be rigorous – when is it appropriate and when is it only pedantry?
13 Jean-Paul Delahaye (Université de Lille, France): From (some) computer games to mathematics.
14 Ana M Eiró (Universidade de Lisboa, Portugal): Engaging the public in Mathematics in a historical environment.
15 João Fernandes (Universidade de Coimbra, Portu-

Meeting of the EMS/RPA committee at the occasion of this workshop

gal): Raising the Public Awareness in Mathematics: how Astronomy can be useful?
16 Carlos Fiolhais (Universidade de Coimbra, Portugal): Physics and Mathematics outreach – do they need to split in science communications?
17 Enrico Giusti (Università degli Studi di Firenze, Italia): Playing with mathematics: "Il Giardino di Archimede".
18 Gert-Martin Greuel: (Universiy of Kaiserslautern and MFOberwolfach) IMAGINARY – Mathematical Creations and Experiences.
19 Vagn Lundsgaard Hansen (Technical University of Denmark): Keeping Mathematical Awareness Alive.
20 Wolfram Koepf (Universität Kassel, Germany): The German website www.mathematik.de.
21 Reinhard Laubenbacher (Virginia Tech, Blacksburg, USA): Mathematics and the public in the U.S.A.
22 Thibaut Lery (European Science Foundation, France): Bringing the mathematical communities forward, a European perspective.
23 António Machiavelo (Universidade do Porto, Portugal): The Importance of Useless Mathematics.
24 Steen Markvorsen (Technical University of Denmark): From PA(X) to RPAM(X)
25 Andreas Daniel Matt (Mathematisches Forschungsinstitut Oberwolfach, Germany): IMAGINARY – How to set up a traveling math exhibition.
26 Yasser Omar (ISEG-Universidade Técnica de Lisboa, Portugal): Short presentation of NGO SiW – Scientists in the World.
27 Konrad Polthier (Freie Universität Berlin, Germany): A Mathematical Picturebook.
28 José Francisco Rodrigues (Centro Internacional de Matemática, Portugal): RPAM in research centers and institutes?
29 Jorge Nuno Silva (Universidade de Lisboa, Portugal): Matemática em Acção.
30 Carlota Simões (Universidade de Coimbra, Portugal): The project "Playing with Science": Mathematics and interdisciplinarity.
31 John M Sullivan (Technische Universität Berlin, Germany): Mathematical pictures: visualization, art and outreach.
32 Raul Ibañez Torres (Universidad del País Vasco, Spain): Butterfly effect and Popularization of Mathematics: Spanish case.
33 Thomas Vogt (Technische Universität Berlin, Germany): Public Understanding of Math – Communicating Mathematics to Society at Large in Germany.
34 Robin Wilson (Open University, UK): RPA in Britain – three case studies.
35 Sebastià Xambó (Universidad Politécnica Cataluña, Spain): RSME – IMAGINARY

Call for Applications/Expression of Interest Emerging Regional Centres of Excellence (EMS-ERCE)

With the proliferation of emerging economies worldwide there are among developing countries varying degrees of development, just as among the developed world. In order to benefit from this situation our strategy of cooperation and help has to be adapted to the different levels of development.

Very good centres exist in emerging economies where students from the least developed regions can be trained to the master's level or higher; after the master's degree, such a student could be given the option of coming to Europe to do a Ph.D. This is much more cost-effective than sending such a student directly to Europe. In fact, EMS-CDC in collaboration with CIMPA, has already educated a Cambodian student in Vietnam to master level, with two more doing the course right now.

It is in this spirit that the Committee for Developing Countries of the European Mathematical Society (EMS-CDC) wishes to propose a scheme of Emerging Regional Centres of Excellence (EMS-ERCE). The idea is for EMS to select, endorse and help a number of such centres to offer training to M.Sc. level to students from less developed countries in their region. With the above encouraging example, provided there are institutions in the emerging economies who are interested in participating, and with the backing of the EMS, our committee is confident that such a scheme will work well.

We have spoken of this idea to a number of mathematicians and the reaction has been really positive, from Europe, South America, South Africa and Asia.

The advantages of such a procedure are threefold:

- It is cheaper in general to send a student to a nearby country or region.
- The student will be less disoriented, and in some cases, they may not need a higher European degree.
- The educating institution will gain experience and prestige.

First, to test practicability and to make sure we can handle properly the applications we get, we propose as a **Pilot Scheme** to stagger the regions and start our first call with **Southern Asia, including the Indian Subcontinent** and shall concentrate on other regions in subsequent calls.

Secondly, as we know, there are already a number of prestigious institutions in emerging regions of international renown. They are of course welcome to apply, if the scheme interests them. In that case, they would add lustre to the scheme.

The criteria for eligibility are:

- The centre is of good scientific standing in the region and neighbouring regions.
- It has a good track record both in research and in pedagogy.
- The centre has a fairly international outlook.
- The centre has a long-term prospective with reasonable guarantee for such.
- The centre is willing to admit and educate graduate students from less developed regions. It should have the infrastructure to do so, e.g. the language of instruction should preferably be in one of the main European languages (English, French or Spanish).
- The degree aimed at is M.Sc., and Ph.D. in exceptional cases.
- The centre is willing to welcome well-established foreign visiting mathematicians for collaboration in research and for teaching the graduate courses.

If selected, the centre will be labelled EMS-ERCE, initially for four years, but renewable thereafter subject to mutual agreement.

The advantages for the centre are:

- The label can add prestige and visibility to the centre, which will most probably attract more and better students.
- Often this will in turn attract funding from local and regional sources.
- The members of CDC will be there to give support and advice whenever needed. Since this will be considered part of our direct mission, they will get priority of our time and resources.
- CDC will be on hand to help those of the students who might wish to and who are capable of continuing their studies after their M.Sc.
- CDC will try to send experienced lecturers to give short or medium courses, e.g. by involving the Voluntary Lecturers Scheme, run by the IMU.
- CDC will seek European hosts for researchers from these centres for visits or collaborations, or both.

- CDC will make available small grants for members of the centres to attend conferences, when appropriate.

If this scheme succeeds, these EMS-ERCE will provide education to other, less developed, regions and get in exchange help to further develop themselves. We think it will be a very advantageous scheme for all. At the same time, with much less expenditure a larger number of students can receive their first graduate education, in a culture not too removed from their own. This will be a practical and efficient way for mathematicians to help other mathematicians.

The members of the ERCE subcommittee of EMS-CDC are:

Georg Bock (Heidelberg)
Giulia Di Nunno (Oslo)
Anna Fino (Torino)
Michel Jambu (Nice)
Mikael Passare (Stockholm), chair
Michel Thera (Limoges)
Ramadas Ramakrishnan Trivandrum (ICTP)
Tsou Sheung Tsun (Oxford)
Begoña Victoriano (Madrid)
Paul Vaderlind (Stockholm)
Michel Waldschmidt (Paris)

Application or Expression of Interest

European Mathematical Society Emerging Regional Centres of Excellence

In this **Pilot Scheme**, we welcome applications from

Southern Asia, including the Indian Subcontinent

Each interested institute is asked to send us a brief description, its activities, and its suitability, together with a covering letter and supporting material if any, to:

Mikael Passare: passare@math.su.se
Michel Waldschmidt: miw@math.jussieu.fr
Tsou Sheung Tsun: tsou@maths.ox.ac.uk

Institutes are welcome to discuss informally with any member of the ERCE subcommittee (named above) before sending their applications.

The preliminary deadline for application or expression of interest is **30 April 2011**, which is expected to be extended if there is sufficient interest.

Please note that calls for other emerging regions will soon follow this first call.
http://www.euro-math-soc.eu/comm-develop.html

About the final event of the ESF-EMS Forward Look Project on Mathematics and Industry: A new road for industrial mathematics in Europe is now open

Maria J. Esteban, Member of the FLMI SOC

FLMI
Forward Look on Mathematics & Industry

On 2 December 2010, in Brussels, the final event of Forward Look on Mathematics and Industry (FLMI) took place, a project financed by the European Science Foundation (ESF) and scientifically managed by the Applied Mathematics Committee of the EMS.

This was not a scientific conference but rather a presentation of the final results of the project to representatives of the European political and scientific world.

Unfortunately, a strong snowstorm prevented some participants from reaching Brussels. The high commissioner for industry, Mr. Antonio Tajani, and the president of the ESF, Professor Ian Halliday, were due to be present but could not come. But we were honoured by the presence of the vice-president of the European parliament, Silvana Koch-Mehrin, and some officials of the ESF and some other European agencies, such as the French CNRS, the institute which launched the idea of this Forward Look project within the ESF. The president of the EMS, Ari Laptev, and the president-elect, Marta Sanz-Solé, also attended the conference.

After the addresses of some of the officials present, the results of this project were presented by the members of its SOC (Scientific and Organisational Committee): Mario Primicerio and Maria J. Esteban gave a general overview of the scope and the results of the project, and the scientists in charge of the three working groups of Forward Look, Magnus Fontes, Yvon Maday and Volker Mehrmann, described in more detail the main recommendations made about training and education, the academia-industry interface, and the opportunities and challenges existing in this area. The scientific part of the event ended with a presentation by Thibaut Lery, ESF scientific officer, on a collection of (European) success stories, covering the interaction of mathematicians with companies, which were gathered during the project and are to be published soon.

The aim of this project, which has been functioning for the last two years, was to reflect on the interface between mathematics and industry in Europe and to propose scenarios and solutions to address its present shortcomings. Studies of this kind have been carried out by different entities in various countries and also at the international level by the OCDE. This organisation launched a committee to do the same job, at the global level, and this committee's conclusions appeared in two reports, released in 2008 and 2009. Of course, the group working on the FLMI project was well acquainted with these previous studies and aimed to study the particular situation in Europe, where the variety of situations adds a new difficulty to the writing of a list of recommendations. The main idea was to analyse the existing situation and propose solutions to the very visible fragmentation found in Europe, where some countries have already established rules and centres to foster smooth industrial collaborations and other countries have done little or nothing. Some countries have already reflected on adapted mechanisms to support and further develop industrial mathematics. Others have some activities but they are not well structured. Others have no practice or no organisation at all.

It has been noted by many actors in the scientific world, and also in society at large, that innovation is instrumental for the development of the European economy and society and that mathematics can and has to play an important role at this level. But for mathematics to play this role well, and at its maximum, action has to be taken.

We need interconnection of activities and centres involved in industrial mathematics at the European level, so that experiences and expertise are shared quickly and efficiently to provide everyone with answers to industrial needs.

We need to improve the training of students and young researchers in order to make them capable of working on industrial problems. The necessities of dealing with complex projects and interacting with actors from different communities (industry, other sciences, etc.) imply the need for inclusion in training programmes of not only good mathematics but also skills which facilitate industrial work, like numerical programming, writing codes and algorithms, communicating with different people and interacting with soft developers, etc.

We need better tools for the evaluation of industrial mathematics in the academic world. We also need to develop contacts with industry in countries in which this activity is unknown. This can be done through training but must be in collaboration with local mathematicians who are willing to change their mentality and adapt to the needs of the new endeavour.

And we need much more than this! In this direction, the main recommendations of the FLMI are the following:

Recommendation 1: Policymakers and funding organisations should join their efforts to fund mathematics activities through a European Institute of Mathematics for Innovation via the coordination of a virtual research infrastructure.

Recommendation 2: In order to overcome geographic and scientific fragmentation, academic institutions and industry must share and disseminate best practice across Europe and disciplines via networks and digital means.

From left to right, M. Primicerio, S. Koch-Mehrin, A. Laptev and M.J. Esteban

Recommendation 3: Mathematical societies and academic institutions should create common curricula and educational programmes in mathematics at a European level, taking into account local expertise and specialities.

The Brussels event finished with an interesting panel discussion about the results of the project and about general issues related to the mathematics-industry interface. The pleasant lunch which followed allowed many informal discussions between participants about the project and about its future. In particular, there were discussions about a design study for the creation of a European Infrastructure for Industrial Mathematics in Europe that has been applied for recently, in the framework of FP7, by the SOC of the FLMI. The creation of this infrastructure is one of the main recommendations of this Forward Look, where it has been called EIMI (European Institute for Mathematics and Innovation). A Forward Look project has to be continued, in order that as many of its recommendations as possible are implemented in the coming years!

After this event, we invite all mathematicians and members of the EMS to review the final report of this project, which contains an analysis of the present situation and a long list of recommendations aimed at the European Commission, policymakers, funding agencies, the mathematical community and European industry, together with some detailed roadmaps to achieve them. Its final version can be downloaded at the website of the FLMI, where other information about this project is also available. In particular, on the same page are the success stories on industrial collaboration of mathematicians which have been collected all over Europe and that will be published soon in book form. The website is at http://www.ceremade.dauphine.fr/FLMI/.

Short Analysis of a Survey on the Use of Bibliometric Indicators in French Universities

Société Mathématique de France

About two years ago, several French mathematicians expressed very strong concerns about a phenomenon that appeared for funding or promotion in some academic French institutes: judging the scientific quality of mathematics only through bibliometric indicators such as impact factors for journals and either g-index, h-index or citation counts for authors. They asked the French Mathematical Society (SMF)[1] to make a study on the extent of this phenomenon and start to reflect on its potential threat to the quality of scientific research. Note that this phenomenon is not particular to France: discussions that took place during a meeting of the presidents of European mathematical societies almost two years ago showed that it is spreading fast across Europe. A recent report of the International Mathematical Union reveals the important bias introduced by bibliometric indicators, their lack of reliability and the disastrous effect for judging mathematics of some kinds of indicators developed for other fields. The SMF started by evaluating precisely the importance of this phenomenon; for that purpose, a list of questions was sent in March 2009 to the local corresponding members of the society. Over half of them answered. The diversity of sizes and geographic situations of the 30 universities they belong to allows us to draw an initial set of conclusions. In order to refine the short analysis given below, it would be pertinent to extend the survey to other countries and to renew it regularly.

1 List of questions

1. Did your university or department adopt some written regulation concerning bibliometric measures in evaluation procedures? If so, we would appreciate that you answer the following questions:
   At which level have they been discussed (mathematics or sciences department, scientific or executive board of the university, …)? For which use (individual or collective evaluation, grants, basic funding, exceptional donation, hiring, promotion, …)?
   Which data basis has been used as reference, which kind of indicators have been considered? Who have they been asked to (department heads, faculty, researchers, specialized agency, …)?

[1]Société Mathématique de France

2. Did you personally provide one or several bibliometric indicators in an activity report (within the department, for a grant, for a promotion, …)? If so, which kind of information have you transmitted (number of publications, impact factor of journal, h-index, …)? Was it compulsory? Otherwise, did some of your colleagues provide such information?
3. Has some bibliometry been used for judging or funding your department, or yourself? If so, at which level (local, national, international)? Did it increase or decrease the funding?
4. Have discussions been organized on the way of using these indicators (between mathematicians, in your department, at a higher level in your university, with some specialized agency, …)?
5. Do you personally think that using these indicators changes the way of publishing? You are welcome to express your opinion on them.

2 Global analysis of the answers

Which indicators are considered? Mainly journal impact factors, number of ranked A publications and number of publishing members of the department. The latter number is produced according to the AERES[2] criteria,[3] under which a member of a mathematics department is a publishing member if he has published at least two papers within the last four years.

What for and how? In some universities, no discussion at all on bibliometry has ever been organised, neither in the department nor at the university level. In others, more or less official discussions have occurred in the scientific or executive boards, leading for some of them to the effective use of bibliometric indicators. Generally speaking, discussions mainly took place in the large universities. Precisely:

- In 48% of the universities, an official discussion has taken place in the scientific board about the appropriateness of using indicators in judging individuals or departments.
- In 31% of the universities, the executive board has tried, with or without success, to use these indicators for some evaluation.
- In 17% of the universities, bibliometric indicators have effectively been used for funding.
- In one university, these indicators have been used for a promotion.

Consequences for faculty members. Even if they generally only suggest it, some colleagues have already changed their behaviour, sometimes before any bibliometric indicator has been used in their university.

Personal opinions of the corresponding members. Unanimously, indicators are judged as very bad because they measure only a part of the information and deform the estimation of scientific quality.

Most mathematicians are pessimistic on the use of these indicators to compare mathematics with other disciplines within the same university. They feel the comparison will always be at the disadvantage of mathematicians. Examples do confirm this feeling, even if two counter-examples have been given.

A strong concern also appears about the long-term negative consequences of the sole use of bibliometric tools for funding and promotion: scientists will tend to optimise these criteria instead of making the best possible research. This phenomenon may even be insidious; the scientists themselves, far from being cynical, could be unaware of this slow shift of their priorities.

Let us end this short analysis by an example sent by one of the corresponding members. Some years ago, a South American mathematician having stayed in France for a while but going back to his country had to count the citations of his papers in order to negotiate his future salary. He was a renowned researcher who had not published a lot of papers. Yet, the number of citations was very high. Indeed, he once published a paper with a wrong proof of a statement that has become an important conjecture in the domain. Logically, all papers on the conjecture include the citation of that paper. It helped him negotiate a good salary, on the level of his talent. Wrong proofs are known to be useful in mathematics but this example nevertheless shows how bibliometric tools can lead to inverse ranking and should be used only with great care.

3 Bibliography

Definition of bibliometric indicators and examples of their use or misuse can be found in the following short bibliography and the references therein.

- Que mesurent les indicateurs bibliométriques? Document d'analyse de la commission d'évaluation de l'Institut National de Recherche en Informatique et Automatique, A.-M. Kermarrec, E. Faou, J.-P. Merlet, P. Robert, L. Segoufin, (2007). http://www.inria.fr/inria/organigramme/documents/ce indicateurs.pdf.
- Citation Statistics – a report from the International Mathematical Union in cooperation with the International Council of Industrial and Applied Mathematics and the Institute of Mathematical Statistics, Robert Adler, John Ewing, Peter Taylor (2008). http://www.mathunion.org/fileadmin/IMU/Report/CitationStatistics.pdf.
- Evaluation et indicateurs Image des mathématiques, F. Sauvageot (2009). http://images.math.cnrs.fr/Evaluation.html.
- Integrity Under Attack: The State of Scholarly Publishing, SIAM News, D. N. Arnold (2010). http://www.siam.org/news/news.php?id=1663.

[2] The Agence d'Evaluation de la Recherche de l'Enseignement Supérieur is the national agency that evaluates the quality of research in mathematics departments at a group level (i.e. not individuals) and supplies a grade that is used for funding the department.

[3] Critères d'identification des chercheurs et enseignants-chercheurs "publiants" http://www.aeres-evaluation.fr/IMG/pdf/Criteres Identification Publiants.pdf. <

International Mathematical Union issues Best Practice document on Journals

At its General Assembly held August 16–17, 2010 in Bangalore, India, the International Mathematical Union (IMU) endorsed a new document giving best practice guidelines for the running of mathematical journals (see http://www.mathunion.org/fileadmin/CEIC/bestpractice/bpfinal.pdf). The document deals with the rights and responsibilities of authors, referees, editors and publishers, and makes recommendations for the good running of such journals based on principles of transparency, integrity and professionalism.

The document was written by the IMU Committee on Electronic Information and Communication (CEIC) in collaboration with Professor Douglas Arnold (University of Minnesota), President of the Society for Industrial and Applied Mathematics, who has recently made a study* of unethical practices such as impact factor manipulation in mathematics. Sir John Ball (University of Oxford), the Chair of CEIC, said "*It is important that everyone involved in the publication process has full information on how papers are handled and on what basis they are accepted or rejected. For example, we are uncomfortable with the routine use of confidential parts of referee reports that are not transmitted to authors*."

The IMU President Professor László Lovász (Eötvös Loránd University, Budapest) commented "*Well run journals play a vital role in the scientific process. Although the document is concerned with mathematics journals, we hope that those in other fields will find it interesting and useful*."

Contact details:
Professor László Lovász (lovasz@cs.elte.hu)
Sir John Ball (ball@maths.ox.ac.uk),
Professor Douglas Arnold (arnold@umn.edu).

* Integrity Under Attack: The State of Scholarly Publishing http://ima.umn.edu/~arnold/siam-columns/integrity-under-attack.pdf

Plea to publishers and authors: Please help blind Mathematicians

Emmanuel Giroux (CNRS and ENS-Lyon, France)

For more than 25 years, TeX/LaTeX has imposed itself as the most efficient software for editing mathematical texts and its use by publishers is nowadays standard. A marginal but notable consequence of this general use of TeX/LaTeX is that the whole of present-day mathematical production is in principle accessible to blind people. Indeed, TeX/LaTeX typesetting is based on source files consisting only of ASCII characters and each of these characters has a Braille translation, so every TeX/LaTeX source file can be read directly by a blind person using a Braille display connected to a computer. Of course, the readability of source files is sometimes questionable and strongly depends on the carefulness of authors but it can easily be improved with very little effort (in particular by removing all TeX/LaTeX commands not needed for understanding the content and the structure of the text); it is much better to have these files, which contain all the information, rather than text without formulas (as one can sometimes get using converters from PDF to TXT) or just nothing. On the other hand, as for writing mathematics, it is remarkable and commendable that TeX/LaTeX puts blind people exactly on the same footing as sighted people.

All this is really great. The only problem is that TeX/LaTeX source files, though they do exist, are most often not available. Subscriptions to electronic versions of journals only give access to PDF files, in which mathematical notations and formulas are no longer encoded in ASCII characters, and cannot therefore be faithfully translated into Braille. Similarly, articles and books that can be found on professional webpages of their authors are available only in PDF or PS format. There is actually one important exception: the mathematical ArXiV – where the TeX/LaTeX source files are (almost) systematically available – and hence electronic journals such as Geometry and Topology, which post the papers they publish on ArXiV. This is something for which I am personally thankful every day.

As a conclusion, here is my plea to publishers and authors: please find a way of making the TeX/LaTeX source files of your publications available. Remove from them, if you wish, all the editing parameters which are necessary to print them out but not to understand the text – the files will be even more readable. But please be aware that for a (small but nonzero) number of mathematicians TeX/LaTeX is the only accessible document format.

Hans Duistermaat (1942–2010)

Richard Cushman

Hans Duistermaat (2007)[1]

Johannes Jisse (Hans) Duistermaat died unexpectedly on 19 March 2010 from a renewed bout of lymphoma, which his doctors thought they had brought under control. Hans was born on 20 December 1942 in the Hague, the Netherlands, and was raised in Indonesia. He got his undergraduate degree from the University of Utrecht and went on to write his PhD thesis on thermodynamics [2] officially under the direction of Hans Freudenthal but actually under Günther K. Braun. He obtained his PhD in 1968. He married Saskia in 1969 and had two daughters Kim and Maaike. In 1972 he was appointed full professor at the Catholic University in Nijmegen and in 1974 to the Chair at the University of Utrecht previously held by Hans Freudenthal, which he kept for the rest of his life. In the course of his career Hans supervised 23 PhD students, published 50 scientific papers and wrote 11 books [1].

Hans' scientific papers deal with many areas of geometric analysis: partial differential equations, Lie groups, symplectic geometry and classical mechanics. As I have little expertise in partial differential equations or Lie groups, I will concentrate on his work on classical mechanics and symplectic geometry.

Classical mechanics

In [3], Hans studied the persistence of short period periodic solutions near an equilibrium point of a two degree of freedom Hamiltonian system that is in resonance. In this situation the well-known theorem of Lyapunov on the persistence of periodic solutions fails. Years later Hans returned to this subject in [8]. Here, using the theory of singularities of mappings that are invariant under a circle action that fixes the origin, he proved a stability result for the set of these short period periodic orbits. In particular, he showed that this set is diffeomorphic to the set of critical points of rank one of the energy momentum mapping of this resonant Hamiltonian system. Here the energy is the Hamiltonian of the Birkhoff normal form of the original resonant Hamiltonian truncated at some finite order. The momentum of the circle action is the quadratic terms of this normal form. For me this result is the definitive generalisation of Lyapunov's theorem.

One of Hans' most important contributions to the geometric study of Hamiltonian systems is his discovery of monodromy in [6]. To describe what monodromy is we look at a two degree of freedom Hamiltonian system on four dimensional phase space, which we assume is Euclidean space. We suppose that this Hamiltonian system is completely integrable, that is, there is another function, called an integral, which is constant on the motions of the original Hamiltonian system. Such an integrable system has an integral map, which is given by assigning to each point in phase space the value of the Hamiltonian and the extra integral. If we assume that the integral map is proper and each preimage of a point is connected, then the action-angle theorem shows that the preimage of a suitably small open 2-disc in the set of regular values of the integral map is symplectically diffeomorphic to a product of a 2-torus and a 2-disc. In [6], Hans showed that this local theorem need not hold globally on the set of regular values. In particular, if we have a smooth closed, nonintersecting curve in the set of regular values of the integral map then the preimage of this curve in phase space under the integral map is the total space of a 2-torus bundle. In general this bundle is twisted, that is, it need not be diffeomorphic to a product of the closed curve and a 2-torus. To understand this global twisting, we note that a 2-torus bundle over a circle may be constructed from a product bundle over a closed interval with a typical fiber a 2-torus by identifying each of its two end 2-tori, which are Euclidean 2-space modulo the lattice of points with integer coordinates, by an integer 2×2 matrix with determinant 1. The monodromy of a 2-torus bundle constructed in this way is just this integer matrix. If the monodromy is not the identity matrix then the 2-torus bundle is not a product bundle. In [6], Hans gave a list of geometric and analytic obstructions for local action-angle coordinates to be global. Monodromy is the simplest obstruction. Monodromy would not be interesting if there were no two degree of freedom integrable Hamiltonian systems having it. When Hans was starting to write [6], he asked me to find an example of such a system. The next day I told him that the spherical pendulum, which was studied by Christiaan Huygens in 1612, has monodromy and gave him a proof. When writing up the paper Hans found a much simpler geometric argument to show that the spherical pendulum had monodromy. In the early days, showing that a particular integrable system had monodromy was not easy. In [9], Hans did this for the normal form of the Hamiltonian Hopf bifurcation.

Eight years after the discovery of monodromy in classical mechanics came its discovery in quantum mechanics in [10]. In particular, Hans and I showed that the quantum analogue of monodromy appears in the joint spectrum of the energy and angular momentum operators of the quantised spherical pendulum as the failure of the local lattice structure of this spectrum to be a global lattice. This discovery has now been recognised as fundamental by chemists who study the spectra of molecules and has led to a very active area of scientific research.

In the mid 1990s Hans became interested in nonholonomically constrained systems such as the disc or a dynamically symmetric sphere with its centre of mass not at its geometric centre. Both systems are assumed to roll without slipping on a horizontal plane under the influence of a constant vertical gravitational force. This interest gave rise to [11] in which a theorem is proved showing when a completely integrable nonholonomically constrained system of two degrees of freedom has monodromy. In this paper [11], Hans shows that an oblate ellipsoid of revolution rolling without slipping on a horizontal plane under the influence of a constant vertical gravitational force has a cycle of heteroclinic hyperbolic equilibria whose local monodromies add up to the identity. Consequently, this energy preserving, time reversible, nonholonomic system cannot be made into a Hamiltonian system. The book [12] contains a complete qualitative study of the motion of the rolling disc. This gives an idea of some of Hans' contributions to the geometric study of nonholonomically constrained systems.

Symplectic geometry

One of Hans' early uses of symplectic geometry was to prove a generalisation of the Morse index theorem [4]. In Riemannian geometry, the Morse index theorem states that the number of focal points along a geodesic, whose end points are fixed and nonfocal, is equal to the number of negative eigenvalues of the Hessian of the energy function evaluated along the geodesic. To explain Hans' argument we need some concepts from symplectic geometry. Consider the pair (V, σ) where V is a finite dimensional real vector space with a nondegenerate skew symmetric bilinear form σ. A real linear map $\Phi : V \to V$ is an automorphism of (V, σ) if and only if $\Phi^*\sigma = \sigma$. The automorphism group $\mathrm{Sp}(V, \sigma)$ of (V, σ) is a Lie group called the real symplectic group. Its Lie algebra $\mathrm{sp}(V, \sigma)$ is the set of all real linear maps $A : V \to V$ such that $\sigma(Av, w) + \sigma(v, Aw) = 0$ for every $v, w \in V$. A linear subspace α of V is a Lagrange plane if $\sigma|\alpha = 0$ and $\dim \alpha = \frac{1}{2} \dim V = n$. To illustrate the above concepts consider the symplectic vector space (V, σ). Then the vector space $V \times V$ with symplectic form $\tau = \pi_1^*\sigma - \pi_2^*\sigma$, where $\pi_i : V \times V \to V$ is the projection map on the i^{th} factor, is a symplectic vector space $(V \times V, \tau)$. If $\Phi \in \mathrm{Sp}(V, \sigma)$ then $\mathrm{graph}\,\Phi = \{(v, \Phi v) \in V \times V \,\big|\, v \in V\}$ is a Lagrange plane in $(V \times V, \tau)$. Returning to defining other concepts in symplectic geometry, let $\Lambda(V, \sigma)$ be the set of all Lagrange planes in (V, σ). Then $\Lambda(V, \sigma)$ is a smooth connected manifold of dimension $\frac{1}{2}n(n+1)$ called the Lagrangian Grassmannian. Let β be a given Lagrange plane in (V, σ). Then the set $\Sigma_\beta(V, \sigma)$ of all Lagrange planes α in V such that $\alpha \cap \beta \neq \{0\}$ is called the Maslov cycle. $\Sigma_\beta(V, \sigma)$ is an algebraic subvariety of $\Lambda(V, \sigma)$, whose set of nonsingular points is the set $\Lambda^1(V, \sigma)$ of Lagrange planes α in V such that $\dim \alpha \cap \beta = 1$. It can be shown that $\Lambda^1(V, \sigma)$ is a connected smooth orientable codimension one submanifold of $\Lambda(V, \sigma)$ whose first homology group is $\mathbb{Z}$. The subvariety of singular points of $\Sigma_\beta(V, \sigma)$ has codimension three in $\Lambda(V, \sigma)$. In addition, the complement of $\Sigma_\beta(V, \sigma)$ in $\Lambda(V, \sigma)$ is diffeomorphic to $\mathbb{R}^{\frac{1}{2}n(n+1)}$. This geometric information allows us to define an intersection number with the Maslov cycle $\Sigma_\beta(V, \sigma)$ for a smooth curve $\gamma : [a, b] \to \Lambda(V, \sigma)$ of Lagrange planes whose end points do not lie in $\Sigma_\beta(V, \sigma)$. This intersection number is an integer, which is homotopy invariant as long as the end points of the curves in the homotopy do not lie in $\Sigma_\beta(V, \sigma)$. Hans found that if one added a certain correction term, depending only on $\gamma(a)$, $\gamma(b)$ and β, to the intersection number of γ with the Maslov cycle $\Sigma_\beta(V, \sigma)$ then one obtained an integer $\mathrm{ind}\,\gamma$, called the index of γ, which depended only on $\gamma(a)$ and $\gamma(b)$.

I will now explain what the above geometry has to do with the Morse index theorem. Consider a classical variational problem with Lagrangian (equal to the energy in the Riemannian case), which satisfies the Legendre condition together with some boundary conditions such as having fixed end points or being periodic. Linearising along a solution of the Euler-Lagrange equation associated to this variational problem gives rise to a Sturm-Liouville problem with boundary conditions imposed at two given finite times. Translating the associated eigenvalue problem into symplectic geometric terms, we obtain a two parameter family $(-1, 0] \times [0, T] \to \mathrm{sp}(V, \sigma) : (\lambda, t) \mapsto A(\lambda, t)$ such that the solutions of the Sturm-Liouville equations are solutions of the linear Hamiltonian system $\dot{v} = A(\lambda, t)v$ on (V, σ). Here the eigenvalue λ is thought of as a parameter. Integrating the preceding differential equation gives a two parameter family $\gamma : (-1, 0] \times [0, T] \to \mathrm{Sp}(V, \sigma) : (\lambda, t) \mapsto \Phi_{(\lambda,t)}$ of linear symplectic mappings of (V, σ) into itself such that $t \mapsto \Phi_{(0,t)}$ is the flow of the differential equation with $\lambda = 0$. Because the boundary conditions are imposed at two times, they give rise to a Lagrange plane δ in $(V \times V, \tau)$. The generalised Morse index theorem states that up to a correction term (which Hans computed explicitly) the sum μ of the intersection numbers of the eigenvalue curve $\Gamma^{(\,,T)} : \lambda \mapsto \mathrm{graph}\,\gamma(\lambda, T)$ with the Maslov cycle $\Sigma_\delta(V \times V, \tau)$ is equal to the sum $\widetilde{\mu}$ of the intersection numbers of the flow $\Gamma^{(0,\,)} : t \mapsto \mathrm{graph}\,\gamma(0, t)$. The number μ counts the number of negative eigenvalues of the Sturm-Liouville problem with multiplicity. μ is finite because of the Legendre condition. The number $\widetilde{\mu}$ counts with multiplicity the number of solutions of the Sturm-Liouville equations that satisfy the boundary conditions. When the boundary conditions are periodic the Morse index theorem reads $\mu = \mathrm{ind}\,\Gamma^{(0,\,)} - \frac{1}{2}\dim V$.

Now suppose that the curve $\mathbb{R} \to \mathrm{sp}(V, \sigma) : t \mapsto A(0, t)$ is periodic of period T. Let $\Phi_1 : [0, T] \to \mathrm{Sp}(V, \sigma) : t \mapsto \Phi_{0,t}$ be the corresponding flow. For $k \in \mathbb{Z}_{\geq 1}$ we say that the curve $\Phi_k : [0, kT] \to \mathrm{Sp}(V, \sigma) : t \mapsto \Phi_{(0,t)}$ is the k^{th} iterate of the curve Φ_1. Let μ_k be the Morse index of Φ_k. In [5], Hans gave an explicit formula for μ_k.

We now turn to describing Hans' most important contribution to symplectic geometry – the Duistermaat-Heckman theorem. Let (M, σ) be a symplectic manifold and let T be a torus (with Lie algebra $\mathfrak{t}$) acting on M in a Hamiltonian way. Let $J : M \to \mathfrak{t}^*$ be the momentum map of the torus action, which we assume to be proper. Let $\xi \in \mathfrak{t}$ be a regular value of J. Then $J^{-1}(\xi)$ is a T invariant compact submanifold of M on which T acts in a locally free way. For simplicity we shall assume that this T-action is free. The T-orbit space $M_\xi = J^{-1}(\xi)/T$ is a smooth symplectic manifold, called the reduced phase space, with reduced symplectic form σ_ξ, which is the push down by the T-orbit map, of $\sigma|J^{-1}(\xi)$. Fix a regular value ξ_0 of J and let ξ vary in an open convex neighbourhood U of ξ_0. There is a canonical identification of $\mathrm{H}^*(M_\xi; \mathbb{R})$ with $\mathrm{H}^*(M_{\xi_0}; \mathbb{R})$. We want to compare the various cohomology classes $[\sigma_\xi] \in \mathrm{H}^2(M_\xi; \mathbb{R}) \cong \mathrm{H}^2(M_{\xi_0}; \mathbb{R})$ when $\xi \in U$. The main result is $[\sigma_\xi] = [\sigma_{\xi_0}] + \langle c, \xi - \xi_0 \rangle$, where $c \in \mathrm{H}^2(M_{\xi_0}; \Lambda_{\xi_0})$. Here Λ_{ξ_0} is the lattice $\{\eta \in \mathfrak{t} \,\big|\, \exp \eta = e \in T\}$, since we have assumed that the action of T on $J^{-1}(\xi)$ is free.

As a corollary, Duistermaat and Heckman show that the push forward $J_*(\mathrm{d}m)$ of the canonical measure on M by the momentum mapping J has a piecewise polynomial density if J has at least one regular point in each connected component of M. If M is compact, this result can be used to deduce an explicit formula for the oscillatory integral $\int_M e^{i\langle J(m),\xi\rangle}\,\mathrm{d}m$, for $\xi \in \mathfrak{t}$, when the locally free action of T has fixed points. This formula reduces to one of Harish-Chandra when T is a maximal torus of a compact connected, simple Lie group G acting on a G coadjoint orbit in the dual $\mathfrak{g}^*$ of its Lie algebra.

Working with Hans and collaborating on our joint publications is at the core of my mathematical career. It is hard for me to comprehend that he is not here any more.

Note

1. Pictures in this article are a courtesy of Emil Horozov. These pictures were taken in 2007 in occasion of Richard Cushman's 65 years conference in Utrecht.

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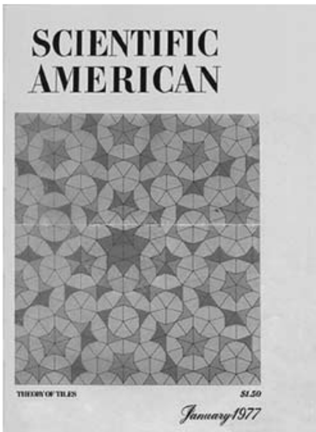
Richard Cushman (rcushman@ucalgary.ca). Before my retirement as an associate professor at the Mathematical Institute of the University of Utrecht in 2007, I had worked as a colleague of Hans for the preceding thirty years. We collaborated in areas of mutual mathematical interest such as symplectic geometry and Hamiltonian systems. The core of my mathematical career was this collaboration, which resulted in ten joint scientific articles and one mathematical monograph. This is a substantial fraction of my total scientific work of over ninety papers and two books.

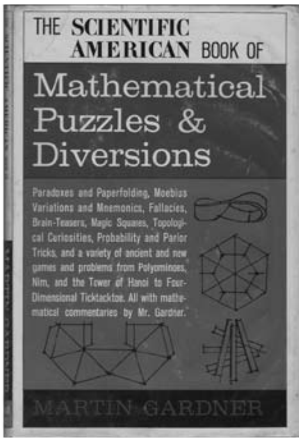
Martin Gardner (1914–2010)

Jorge-Nuno Silva

Gardner is the model and inspiration for everybody who enjoys recreational mathematics. He is clearly the greatest mathematical popularizer that ever lived. As Richard Guy wrote, *Gardner brought more math to more millions than anyone else*.

From 1956 to 1981 he published the column "Mathematical Games" in the prestigious *Scientific American (SA)*. His section quickly became very popular and influential and his articles inspired several covers of the magazine.



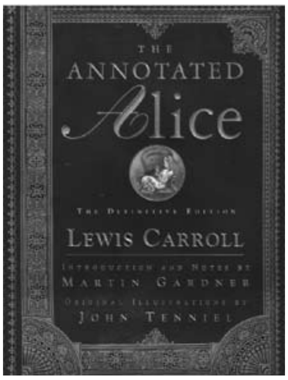


Roger Penrose's work on the cover of SA (left)

Martin Gardner got a college degree in philosophy and worked for most of his life as a journalist and a writer. He never took a single mathematics course in college. However, he managed to learn difficult mathematical theories that he then exposed with unsurpassed clarity and enthusiasm.

Gardner brought to a wide audience several hard theorems and mathematical constructions, making names like John Conway *(The Game of Life, Surreal Numbers, Tilings,…)*, Raymond Smullyan (*Logic*), Roger Penrose (*Tilings*), Escher (*Visual Art*) and Mandelbrot (*Fractals*) very familiar.

His works were collected in several books that achieved wide circulation.



He wrote about two hundred books on several subjects, like recreational mathematics, philosophy and magic. He even wrote novels. The monumental edition of Lewis Carroll's *Alice in Wonderland* annotated by Martin Gardner was one of his best-sellers.

The lay public first heard of some of the most important mathematical results and their authors in Gardner's works, who delivered them in a humanized and clear way. We will mention some examples from the pages of *Scientific American*.

John Napier (1550–1617), the creator of logarithms, popularized a singular calculation device, *Napier's Bones*. Gardner gave them life again.

Raymond Smullyan, the great American logician born in 1919, who was a professional magician in his youth, created several puzzles that looked innocent but were theoretically relevant. Sometimes he used the familiar chess set. In this diagram, Smullyan asks where the white King goes (it just fell to the ground) and what the last two moves of this legitimate chess game were.

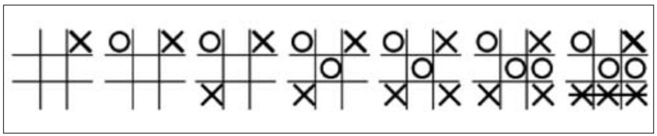
John Horton Conway, one of the most creative mathematicians that ever lived, invented the *Game of Life*, to which Martin Gardner dedicated several columns of *Scientific American* in the 1970s.

Who but Martin Gardner could turn Benoît Mandelbrot's *fractals* into a common subject of conversation?

And so many, many others!

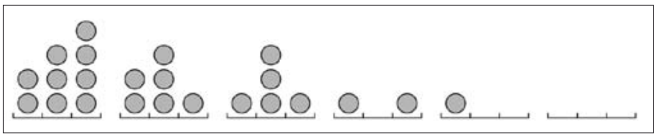
Martin Gardner's column in *Scientific American* was entitled "Mathematical Games". Lots of mathematical games were popularized during the 25 years of the column's existence. Let us recall just a few.

Even games that we all know (or think we know) very well but whose analysis can surprise us, like Tic Tac Toe, were given space in *SA*'s pages.

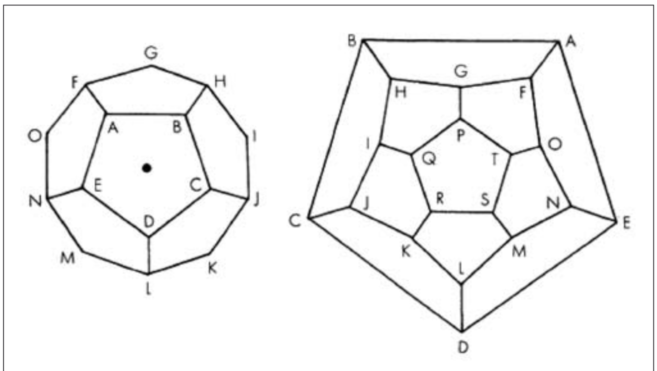


Some other pencil-and-paper games, like Hackenbush (each player's turn consists of cutting an edge; only the edges connected to the ground survive this action), belong to an extremely complex field: Combinatorial Game Theory.

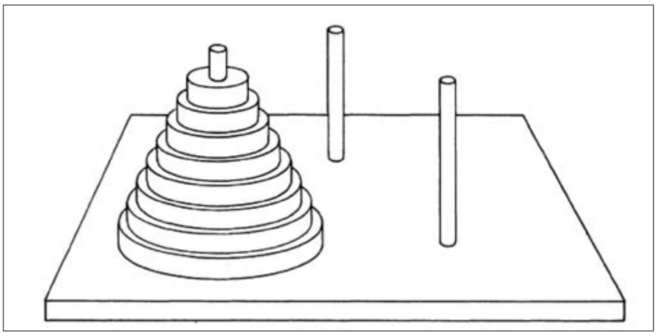
And there is also the classic *NIM*, which was the first game to be solved mathematically in a research article. NIM can be played with piles of beans, two players alternately choosing a pile and decreasing its number of beans. The winner is the last player moving (who takes the last bean).



Some puzzles appeared, like the *Icosian*, created by the Irish mathematician Hamilton, and the *Hanoi Towers*, invented by Edouard Lucas, both in the 19th century, and whose reciprocal relations Gardner explained.

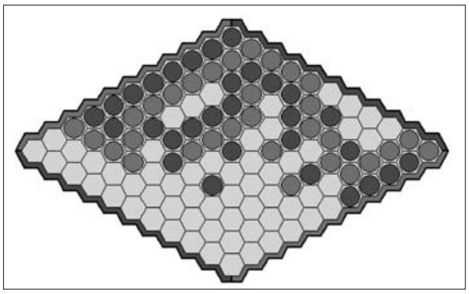


Icosian game: visit every vertex once



Hanoi Towers: move the pile of discs, one by one, never landing a disc on a smaller one

Board games also captured Gardner's attention. He recognised that several of these games have far-reaching mathematical content, like *Hex*, invented independently by Piet Hein and John Nash.



Hex: a connection game

Martin Gardner described and praised some card games, such as *Eleusis* by Robert Abbott – a very special case. It is a game that emulates the process of scientific discovery, in which the players try to find certain rules of the very game they are playing…

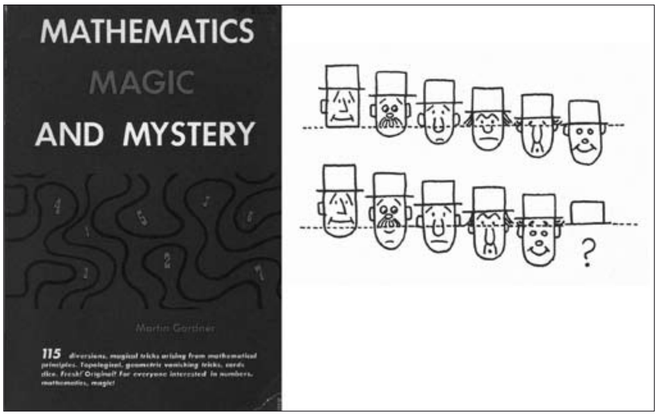
The visual artist Escher (1898–1972) produced very mathematical work; his creations are filled with references to advanced mathematical concepts, from self-reference to hyperbolic geometry. Gardner appreciated his art very much, being very proud of an original that Escher offered him.

Escher's work on the cover of *Scientific American*

Martin Gardner got interested in magic very early in life, when his father showed him a card trick. Martin became an expert in this field, contributing with original material to magic journals. Gardner, even if not a professional, was one of them, an element of the community of magicians. This included, as it still does, some leading mathematicians.

His most impressive work in this field, the *Encyclopedia of Impromptu Magic*, contains almost 600 large pages filled with magic tricks that use only everyday objects. The mathematics behind these activities is always presented and explained by Martin Gardner in his clear and enthusiastic style. Gardner dedicated a major part of his time and effort to exposing scientific hoaxes.

His interest in these activities was triggered by his changing of opinion about a book he read and approved



of in his youth. This book denied the theory of evolution. *The New Geology*, by George McCready Price, contained good arguments against Darwin's theory. However, later, at college, he became a critic of this text. Science and its methods guided Gardner for the rest of his life.

His knowledge of magic turned out to be important in the process of demystifying charlatans. A major part of surprising and "supernatural" effects can easily be explained by trained magicians. In 1950, his *Fads and Fallacies in the Name of Science* brought him recognition in the field.

Other works followed, as Martin Gardner dedicated himself to this subject with his usual determination.

He was one of the founding fathers of *Skeptical Enquirer*, where he published his articles regularly. He sent by mail his last contribution a few days before passing away. These works gave rise to several books: *New Age: Notes of a Fringe Watcher (1988), On the Wild Side* (1992), *Weird Water and Fuzzy Logic* (1996), *Did Adam and Eve Have Navels* (2000) and *Are Universes Thicker than Blackberries* (2003).

The goal of the foundation *Gathering for Gardner* is to promote the lucid exposition and discussion of new ideas in recreational mathematics, magic, puzzles and philosophy.

Through its support, biannual conferences are held in honour of Martin Gardner, encouraging the work of amateurs and young people by bringing them in contact with professional scholars, world-class expositors and innovative performers in venues that promote the cross-discipline fertilization of ideas.

The first meeting G4G1 happened in 1993. Elwyn Berlekamp promoted the idea among mathematicians, Setteducati did the same among magicians and Tom Rogers approached the puzzles community. Eight more meetings followed. The ninth and most recent G4G9 took place in March 2010.

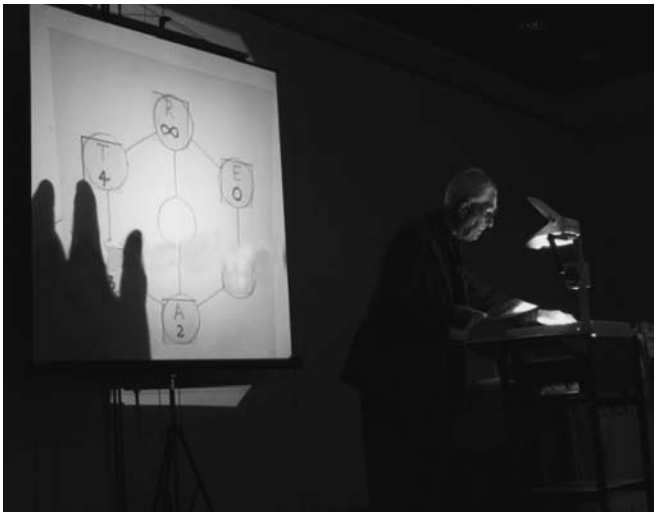
In these workshops, which have been absolutely unique events (and more like parties!), hundreds of mathematicians mingle with as many magicians, puzzle experts and others.

Social activities at G4G

Social activities include a dinner at the house of one of the organisers, where some mathematical concepts can be flavoured in original ways…

As the G4G meetings happen in the USA, some European enthusiasts organise, on odd numbered years,[1] a similar event on this side of the ocean.

The first one *Recreational Mathematics Colloquium I* was hosted by the Universidade de Évora, Portugal, in 2009.



Richard Guy opening RMC I

We will honour Martin Gardner's as he would like: having lots of fun!

Jorge-Nuno Silva [jnsilva@cal.berkeley.edu] is a professor at the Department of History and Philosophy of Science, University of Lisbon. He received his PhD in mathematics in 1994 (UC Berkeley). His interests include mathematics (mainly recreational!), history of mathematics, board games and their history, and magic. He is the leader of the group Matemágicos Silva *(they perform mathematical tricks, with playing cards and other materials) and president of* Associação Ludus. *He is married to Galina and has two children, Laura and Manuel.*

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Mathematicians' self-confidence and responsibility

F. Thomas Bruss (Bruxelles)

Abstract: This article is an essay written for mathematicians. Its objective is to stimulate discussion. The essay deals with specific prejudices against mathematics and mathematicians. Should mathematicians feel responsible for defending mathematics against indifference or attacks? If so, when and why should we take our responsibility? Can we also speak of a mission for mathematicians? Are there suggestions of how to assume the task of a mission? Examples of rather typical situations in everyday life will try to convince the reader that such questions are important for us and our field.

1. Mathematicians and their reputation

Some people think that mathematicians are insecure, even very timid. Others, on the contrary, see them as condescending. Perhaps, as so often, the truth is somewhere in between but I think it would be unfair to leave it without trying a better analysis. Can we say something more definite about shyness or self-assertion of mathematicians?

My former student Yvik Swan asked me not long ago to tell the difference between an introverted mathematician and a self-assured mathematician. I was not sure what to answer but I certainly enjoyed Yvik's description: "The latter would look on your shoes when he is talking to you."

We know that this is in general not true. Even more, as many independent statistical analyses show, good marks in mathematics are the best statistical predictor of being good in all other school subjects. And if I look around among the bright mathematics students of our university I typically see young, open-minded and seemingly self-assured people. But isn't it most interesting? Even though we know that, as we have pointed out, the prejudice is in general not at all true, we mathematicians still enjoy stereotype descriptions of our breed by others, as exemplified by the joke. Is it only because we really know better?

Perhaps. Still, many people, including mathematicians, do think we should be more open, should try to communicate more of what we are doing and defend it adequately. Communication of what a mathematician is doing is sometimes difficult, as we know best ourselves. Nevertheless, we should probably be more willing to point out what we think whenever we believe there is a good reason to do so. It seems worthwhile discussing a few examples.

2. Confidence in mathematics

When a medical doctor whom we meet for the first time tells us after two minutes that they were always a zero in mathematics, I think we should not say that we have heard this from others. Still worse would be adding that we know others who were very bad in mathematics and succeeded in their life. I am sure that a better answer is: "Oh, this is very sad! This is likely to impair all of your decisions that involve elementary logic or a sound feeling for the order of magnitude of numbers." Elementary logic is a must for medical doctors, and elementary logic is one of the typical and far-reaching failures of doctors who say that they were very bad in mathematics (and are okay with that). The immediate reaction of the surprised doctor on our alert is to respond that they are of course "very logical", simply trying to say that they could not do the other stuff with 'x's and 'f(x)'s and so on. Realising that you doubt their "logical abilities", annoyance will show up on their face by now. However, we should firmly show how much we worry about what we just heard and should not compromise. As politely as we can, we should insist on our sadness by telling the further truth: "Unfortunately, the lack of knowing a minimum of how to handle 'x's or 'f(x)'s or so on at the end of high school is known to be highly correlated throughout life with failures in elementary logic and a bad feeling for orders of magnitude."

A physician who cannot negate correctly a statement containing one or two implications or one or two and/or attributes can do harm to a patient. The same holds for the failure to see or to use (our beloved) equivalence of «A implies B» and «not-B implies not-A». Mathematicians use the latter of course mostly in the convenient form «A and not-B implies a contradiction» for a proof by contradiction. This appealing form is however equally important for everyday decisions just to clean up one's mind. Indeed, many wrong statements, worldwide and in many domains, could probably be avoided by applying this equivalence on a regular basis. There are several other examples of elementary reasoning of similar importance. Unfortunately, the importance of elementary logic is seemingly not that clear to everybody. Otherwise, textbooks of general mathematics for medicine, biology and pharmacy students for instance would all have a dominant introduction to elementary reasoning and its importance; this is far from the truth.

Another typical and sometimes dangerous error lies on the side of elementary probabilistic or statistical reasoning, in particular reasoning involving conditional probabilities, the regression effect, Simpson's paradox and so on. The example of 82% wrong answers of medical doctors and students in a test of Bayes' formula at Harvard Medical School – arguably one of the most prestigious and famous medical schools in the world – shows how serious the problem can be (see [1]). The largest block of wrong answers were even drastically wrong with estimating the probability of the incidence of the tested disease to be 95% whereas the correct answer was close to 2% (see Casscells et al., [1], 1978). This is some time ago but we should not be mistaken; things have not necessarily improved (see for instance [2], 2009). According to my own modest experience, it is also linked with a lack of interest. When discussing more recently the results of a multi-targeted blood test I dared to remark that if a test goes over a sufficiently large number of factors, which are seen as relatively independent of each other, then it becomes "not-normal" to have everywhere "normal" values.

It's no problem of course that the physician does not see immediately what we mean. The problem begins when we see they are not interested in understanding what we may have meant, possibly (?) worth being taken into account. Similarly, pretending to analyse data of a test and proposing medication having no idea whether the order of magnitude of the standard deviation is 0.1 or rather 1 or even 5 and not being keen to look it up – this is more than a faux pas; it is a lack of responsibility. We should not accept this; we should react and take our responsibility. If we leave the doctor's cabinet politely on the basis of such a conversation, he or she will remember and possibly for good. This is probably a wise choice for us; in the same vein, it may be for the benefit of society and undoubtedly helpful for the appreciation or recognition of our field. In summary, I dare to affirm my conviction that no doctor who advertises his mathematical incompetence, however competent they may be for certain problems, should be trusted in medical decisions involving weighing alternatives, and that it is our and *mainly our* responsibility to point this out.

It would be unfair to exemplify medical doctors without mentioning other examples. So I remember a round-table discussion in Germany with a member of the European Commission as the main speaker. This was shortly before the first introduction of the Euro currency (around 2000). He spoke about the laws and recommended rules that would be implemented to make the European Central Bank an institution at least as reliable as the Deutsche Bundesbank, and hence the Euro at least as strong a currency as the Deutsche Mark. He argued: "If you increase the size of an Institution by a factor c you must increase control by factor c to maintain the same level of safety," assuring everybody in the audience that the European Union has (of course) the financial means to do this. I was not sure how an increase of control was supposed to be measured, quite apart from the important fact that now member states with national interests would all have their word to say in this new central bank. In particular I wondered why he could assume such a simple linear (sub-linear) relationship between control and safety. I must admit that my question was hardly helpful for the later discussion but I am still glad that I had tried to draw attention to conclusions that seemed naive.

Why is it that people have difficulty in listening to us even if we just try to say elementary (useful) things? The problem seems to be rather widespread. Brian Vidakovic (Atlanta) commented: "… are the typical bad guys but we get it also from the unlikely sources that should appreciate mathematics better: engineers, biologists etc." Is it some impression we may leave on others? Too modest, insecure, no humour, too proud, condescending? Let us have a short look at this.

3. Modesty, condescension and contradictions

Can mathematicians be proud, even condescending? Well, we can imagine that John Littlewood would have said that there is hope. As far as I remember the following examples, among my preferred ones, can be found in *Littlewood's Miscellany* [3]:

> *Two dons from the Mathematics Department of Oxford or Cambridge at the high table: "Did you hear already that our colleague such and such is to become the new Minister of Defence?"*
> *– "Really? Oh, what a come-down!"*
>
> *"What did God do before creation?"*
> *– "A thousand pages must have been written on this; but he was doing Pure Mathematics and thought it would be a nice change to do some Applied."*

There are certainly many mathematicians who have superb humour. The eminent Carl Friedrich Gauss, known for so many wonderful things but hardly cited for his humour, is no exception (see for instance the introduction in [4]). And we should be proud of this witty side of so many mathematicians. A few mathematicians, like anybody else in any arbitrary field, may of course be condescending in some way or other. But I think we hardly see a problem that would be specific for mathematicians. In particular we do not know the notion of a "school" in mathematics as understood in many other fields. This helps to keep standards of honesty and prevent what one may call artificial condescension. Even the beloved pecking order pure-applied is for us more a cultivated, recurrent amusement than a problem.

Sometimes it is almost the contrary of pride or condescension that can be the problem. Even the most brilliant of our breed, who would have all reason to stand up in the world, to stand up for mathematics, do not always do so or do it in an inadequate form. I remember a reader of the Newsletter of the EMS of some years ago who complained in his letter that one of the awardees of one of our most distinguished prizes in mathematics showed up at an official event dressed very "unconventionally" and behaving accordingly. Fortunately, I did not see this and don't know to what extent it was true in the sense that many of our colleagues would have felt the same. However, if it was like that, we should be grateful for the complaint and encourage similar letters in similar cases. Perhaps these letters would inspire a second thought. Inadequate dress or behaviour is likely to harm our field. If the celebrity cannot be suspected of the slightest bad intention then this is a different matter but the effect still stays the same. We cannot prove mathematically that the lack of respect for conventions is likely to harm but we may attempt an analogy. Let us visualise in our mind the portraits of Gauss, Euler, Newton, Leibniz, the Bernoullis or others of our celebrities, just as we know them. They appear before us dressed in the style a person in their position would dress at their time for a portrait or an official event. Would we like to see them differently? Even if we are indifferent, would we expect our former heroes to not have been as brilliant as the new one to dare to break conventions? (NB: Heroes, like everybody else, are allowed to break conventions but should be consistent and do the thinking before. – Respect for Perelman.)

Now to another problem, much smaller but still of some concern. This is the exaggerated display of modesty. Mathematics is, I believe, no place for the strategy of "stoop to conquer". If you name your textbook "Elements of …", it is arguably not a good idea to turn off the reader with a first page that most people do not understand! Similarly, if you have already excused yourself twice in your introduction that your presentation will lack mathematical rigour, how consistent is it then to refer to Bourbaki twice on the first page? One should keep in mind that the reader of the book may have bought it just because it was named "Elements of …" and because they saw no alternative. They will now feel badly served.

Eminent mathematicians should take particular care if they want to serve or help others to serve our field. Exaggerated modesty "thrown" at the public seems awkward. If a Fields Medallist, say, expresses amazement that he or she is invited to speak on aspects of mathematics at a most prestigious occasion, underlining repeatedly that many others would be more competent to do so, then this illustrates both sides of the coin. It may be alienating rather than polite. Come on, Professor, the audience may think, you got the Fields Medal in mathematics; you are not invited because you won second prize in raising white rabbits. Don't you mathematicians say yourselves that this medal is the highest distinction for young mathematicians and comparable to the Nobel Prize? – Hic Rhodos, hic salta! Also, we all, eminent or not, make errors in judgment, and errors in prediction. This needs no explanation. But we try to do our best saying things we can prove, and if not, in what we believe. Is it then adequate to advertise that one will *probably* change one's point of view in five years completely? With all respect, one should see the implication: a listener may think that this means that their decision to come here today was *probably* a completely bad investment.

To summarise, mathematics is for many people a mysterious subject, possibly a suspicious subject. However, it is undoubtedly seen as a difficult discipline and hence a priori not seen as something possibly ridiculous. Any self-respecting public with a minimum of interest in our field expects to be treated correspondingly. In accordance with this awareness the audience has a right: they are entitled to admire but equally entitled to be treated with respect. This is why mathematicians, whatever their level, whatever their fame, have a responsibility towards the field of mathematics, be it in front of an audience or somewhere else. If necessary, they have a mission.

4. Responsibility and Mission

The challenge of responsibility may arise anywhere on any day. The last example is intended to show that the charge or challenge of a mission may have to be assumed already on a very basic level.

You happen to sit with four lawyers, or lobbyists having studied law, on the same reception table, and you are put to the test (such a constellation is, by the way, not unlikely in certain towns, such as Brussels). It is sad enough if two of these not only praise their complete incompetence in mathematics but also try to ridicule our subject. It can become worse. The third man of law says he cannot imagine that mathematics has any impact on real life decisions. The fourth shows no support either. Grateful enough, you sense a hint of wit in his only remark: "ius non calculat."

The evening threatens to become rather short but the mission begins. We may try to change the subject. Why not law? We show interest; we ask questions. With four layers on the table we are likely to hear from one of them, early enough, that lawyers are superior in logical thinking to anybody else, *simply by the great importance the study of law attaches to consistency and the frequent use of logical arguments in general*. But we may confidently take the risk: "Why do you believe this? I am convinced that none of you would have a chance competing in logic with an average colleague of mine, *simply by the permanent requirement of all this in mathematics.*" (It is true that some people in law are really very good, and I beg their pardon.) Sure enough we can live for half an hour with two miffed faces on our table annoyed by our unexpected arrogance. There is a good chance, however, that we have still made a point for mathematics. We may win over one on the table, perhaps even two, at least in the sense that they admit that we also may have a point, somewhere…, somehow…. Now, having a lawyer half on our side, things become easier. The others may follow to some extent and the evening may turn out not too bad after all.

Clearly, I have presented here only bad-case or worst-case situations, though no invented scenarios. Good doctors, competent economists and well-educated lawyers will forgive me because they know that we know that there are many really good ones. Some others may go so far as to say: "no need to excuse." Other readers may have experiences of a different kind. Whatever they may be, if they challenge mathematics, if they require us to take our responsibility, let us stand up!

5. Conclusion

The reader may find that I am somewhat too sensitive with respect to the outside world view of mathematics and its implications. They may be right. It may also have become stronger over the years, possibly favoured by challenges of the entourage of a town like Brussels. Am I opinionated? I am and should be, as much as those colleagues who confidently play on the other side, that is, who assume the role of apostles preaching that mathematics in all its supremacy *simply does not have to care*, that no one in mathematics should be obliged to show interest in public image or even to show "courage civil" to defend it. I respect their feelings but it is clear from what I have said that I think they are wrong.

This does not imply that I am right. But would you write such an article if you were not convinced that you are most probably right? Hence it may be a good idea to discuss it. And those of us who are engaged in some work of public awareness may want to examine whether our efforts to raise the public awareness and the image of mathematics are sufficiently effective. Self-assertion, whenever justified, respect for the public, whenever expected, and courage civil, whenever needed, might, as I argue, add a lot to their effectiveness.

Regardless of the style presented here many colleagues may agree with the essence of what is said in this article. Hence it may offer nothing really new to them. If so, it would be no surprise if they advised me to do more mathematics and not to spend time with such an article. I must admit I should really try my very best to follow their first advice.

Acknowledgement

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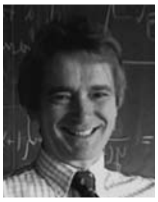
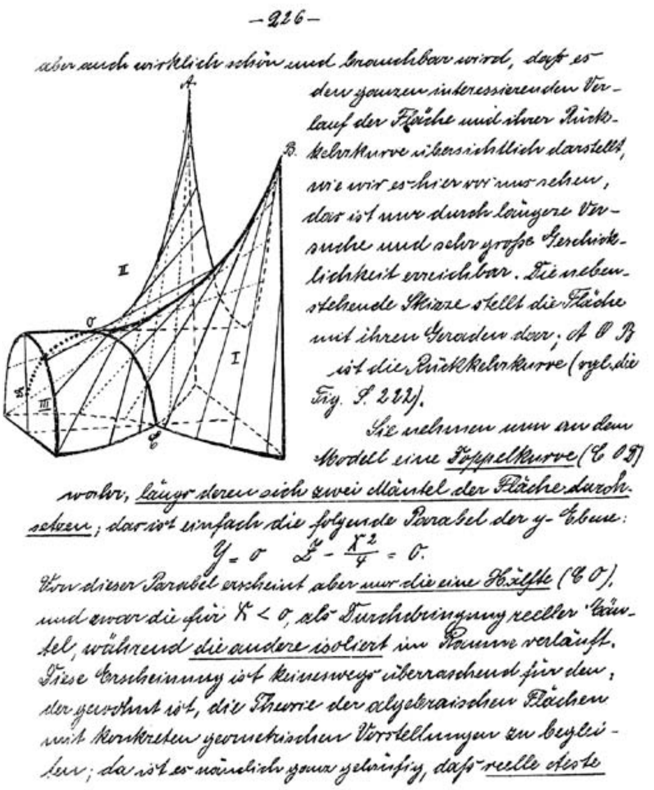


F. Thomas Bruss [tbruss@ulb.ac.be] is a professor of mathematics at the Université Libre de Bruxelles, Director of Service Mathématiques Générales and Co-director of Probability. He is also a member of the Committee Raising Public Awareness of Mathematics of the European Mathematical Society.

Resurfaced discriminant surfaces

Jaap Top and Erik Weitenberg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aber auch wirklich schön und brauchbar wird, daß es den ganzen interessierenden Verlauf der Fläche und ihrer Rückkehrkurve übersichtlich darstellt, wie wir es hier vor uns sehen, das ist nur durch längere Versuche und sehr große Geschicklichkeit erreichbar. Die nebenstehende Skizze stellt die Fläche mit ihren Geraden dar; A O B ist die Rückkehrkurve (vgl. die Fig. S. 222). Sie nehmen nun an dem Modell eine Doppelkurve (C O) wahr, längs deren sich zwei Mäntel der Fläche durchsetzen; das ist einfach die folgende Parabel der y-Ebene: $y = 0 \quad z - \frac{x^2}{4} = 0$. Von dieser Parabel erscheint aber nur die eine Hälfte (C O), und zwar die für $x < 0$, als Durchdringung reeller Mäntel, während die andere isoliert im Raume verläuft. Diese Erscheinung ist keineswegs überraschend für den, der gewohnt ist, die Theorie der algebraischen Flächen mit konkreten geometrischen Vorstellungen zu begleiten; da ist es nämlich ganz geläufig, daß reelle Aeste

The text shown here is from lectures by Felix Klein (1849–1925). The beautiful handwriting belongs to Ernst David Hellinger (1883–1950), who was asked to write down Klein's course while he worked in Göttingen as an assistant between 1907 and 1909. As Klein writes in the preface,

> Man wolle dabei von der Arbeit, die Herr Dr. Hellinger zu erledigen hatte, nicht gering denken. Denn es ist auch so noch ein weiter Weg von der durch allerlei zufällige Umstände bedingten mündlichen Darlegung des Dozenten zu der schriftlichen, hinterher noch wesentlich abgeglichenen, lesbaren Darstellung.
> [One should not underestimate the work that Dr. Hellinger had to accomplish. It is a long way from the by various coincidences influenced oral exposition by the lecturer, to the written and afterwards substantially adjusted, readable exposition.]

Hellinger's notes resulted in the textbook [Kl08] (1908). An English translation [Kl32] based on the third German edition was made by E. R. Hedrick and C. A. Noble and appeared in 1932. Although the original is more than a century old, the text is surprisingly modern and readable. Reprints and new editions have appeared quite regularly, one as recent as 2007.

The page shown here is taken from Klein's exposition concerning equations of degree 4:

$$t^4 + xt^2 + yt + z = 0.$$

Here x, y, z are real parameters, and Klein discusses the (simple) problem: how many real roots does the equation have? His approach may strike us as very geometric: every equation as given here corresponds to a point (x, y, z) in $\mathbb{R}^3$. In fact this geometric approach is not new; it is already present in the paper [Sy64] by J. J. Sylvester published in 1864.

The picture shows points (x, y, z) for which the corresponding equation has a root with multiplicity at least 2, i.e. the discriminant surface. This is by definition the set of points where the discriminant of the quartic polynomial vanishes. Klein proves in detail that the real points of this surface subdivide $\mathbb{R}^3$ into three connected parts, depending on the number of real roots of the equation being 0, 2 or 4. The picture shows more, namely that the discriminant locus contains a lot of straight lines. Indeed, it is a special kind of ruled surface. This is not a new discovery, nor was it new when Klein presented his lectures just over a century ago. For example, the same example with the same illustration also appears in 1892 in a text [Ke92] by G. Kerschensteiner, and again as §78 in H. Weber's *Lehrbuch der Algebra* (1895); see [We95]. An even older publication [MBCZ] from 1877 by the civil engineers V. Malthe Bruun and C. Crone (with an appendix by the geometer H. G. Zeuthen) includes cardboard models of the type of surface considered here. Its geometric properties are discussed in Volume 1 of É. Picard's *Traité d'analyse* (1895; pp. 290 ff.); see [Pi91].

In this note we discuss some of the fascinating history and classical theory of discriminant surfaces. In particular we focus on models of such surfaces that were designed in the period 1892–1906. Although it will not be treated here, we note that surfaces like these and in particular their singularities are still studied in algebraic geometry, differential geometry and dynamical systems (catastrophe theory, bifurcation theory, singularity theory, . . .).

1 String models

The mathematics department of the Johann Bernoulli Institute in Groningen (the Netherlands) owns a large collection of models of geometrical figures. Most of these models originate in Germany, where first L. Brill in Darmstadt between 1880 and 1899 and after him M. Schilling (at first in Halle a.d. Saale, later in Leipzig) between 1899 and 1935 produced hundreds of different models. In fact, the obituary [Yo28] mentions that at the end of his life, Klein proudly asserted that

> thanks to his efforts and those of his friends, the Brills and Dyck, among others, no German university was without a proper collection of mathematical models.

To some extent, the truth of this statement can still be verified: an Internet search for the phrase 'Sammlung mathematischer Modelle' finds websites of such collections in Dresden, Freiberg, Göttingen, Halle-Wittenberg, Regensburg and Vienna (Wien); moreover it mentions similar such collections at a number of other universities. The books [Fi86] and the

PhD thesis [P07] contain information about some of these models. Groningen owes its large collection mainly to the geometer Pieter Hendrik Schoute (1846–1913).

In identifying the models, catalogues such as M. Schilling's [S11] play a crucial role. We will discuss here three string models from the Groningen collection. It turns out that these models are not contained in the Schilling Catalogue, although they are closely related to Schilling's Series XXXIII. Fifteen years before the appearance of this series, they were designed by Schoute in Groningen and presented by him in Amsterdam on 27 May 1893 at the monthly physics meeting of the royal Dutch academy of sciences KNAW.

Series XXXIII of Schilling's Catalogue also consists of three string models. Two of these were designed by Roderich Hartenstein, a student of Klein, as part of his "Staatsexamen" in Göttingen in 1905/06. In fact, Klein's discussion shown above reviews Hartenstein's work. The third model in Series XXXIII was designed by Mary Emily Sinclair, a student of Oskar Bolza (who himself was also a student of Klein) as part of her Master's thesis at the University of Chicago in 1903. We will elaborate on the relation between Schoute's models and the models in Schilling's Series XXXIII below.

Schoute

Biographical notes concerning the Groningen geometer Pieter Hendrik Schoute (1846–1913) are presented at [Sch1]. Schoute's explanation of his models appeared in the minutes, written in Dutch, of the KNAW meetings [KNAW, p. 8–12] in 1894 and also, written by him in German, in the supplement to Dyck's Catalogue [Dy93] published in 1893. The text in the KNAW minutes has as title *Drie draadmodellen van ontwikkelbare oppervlakken, die met hoogere-machtsvergelijkingen in verband staan.* [Three string models of developable surfaces related to equations of higher degree.]

He considers three families of equations, namely

$$t^3 + xt^2 + yt + z = 0$$

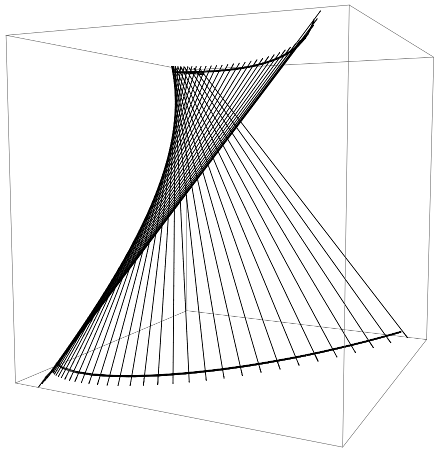
and

$$t^4 + xt^2 + yt + z = 0$$

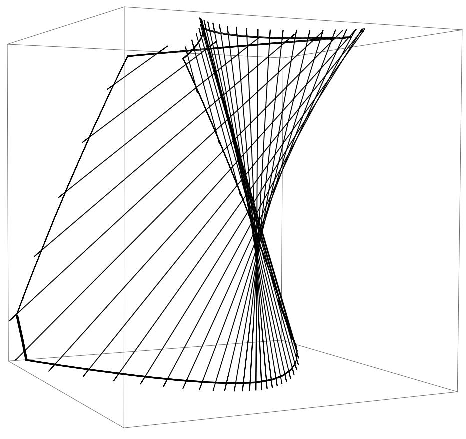
and

$$t^6 - 15t^4 + xt^2 + yt + z = 0.$$

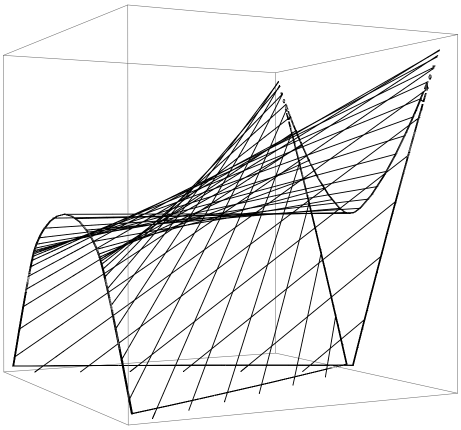
In each case, any real point $(x, y, z) \in \mathbb{R}^3$ corresponds to a polynomial of the given shape. The models show the real points for which the corresponding polynomial has a multiple zero; these points form a (ruled) surface. Moreover, in his description Schoute exhibits the singular points of this surface, for all three cases, and he explains in which intervals he takes x, y, z and how he scales these parameters, in order to obtain the actual models. The precise information given in this way makes it possible to verify that indeed the three models that are still present in the mathematics institute of the university of Groningen are Schoute's models from 1893. Below we show a picture of each of the models together with a mathematica plot based on Schoute's description.



Cubic polynomials: Schoute's model and a plot



Bi-quadratic polynomials: Schoute's model and a plot



Sextic polynomials: Schoute's model and a plot

Schoute also explains why he considers polynomials of degree 3, 4 and 6: the points in space not on the surface correspond to real polynomials with only simple zeros. For degree 3, there are two possibilities for the number of real zeros, so the surface partitions space in two parts depending on this number of zeros. In the case of degree 4, there are three possibilities for the number of real zeros (which obviously all occur in the given family of polynomials). Schoute does not consider degree 5 since that also gives three possibilities hence a somewhat similar partitioning of space. In degree 6, however, there are four possibilities for the number of real zeros. Schoute chooses the particular family of sextics since all four possibilities occur in this family.

Finally, we find in the minutes of the 1893 KNAW meeting motivation for why Schoute designed such models. He explains that it was inspired by the last sentence of Klein's text [Dy93, p. 3–15] in Dyck's Catalogue. Here Klein writes concerning the visualisation of discriminant surfaces

> Es wird sehr dankenswert sein, wenn jemand die Herstellung solcher Modelle in die Hand nehmen wollte.
> [We will be very indebted if someone takes up the construction of such models.]

This is exactly what Schoute did. He writes in the minutes that after his models were ready he noted that pp. 168–173 of Dyck's Catalogue contain a discussion of discriminant surfaces, written by 'Gymnasiallehrer' (teacher at a grammar school) G. Kerschensteiner. This account, published in 1892, even contains sketches of such surfaces:

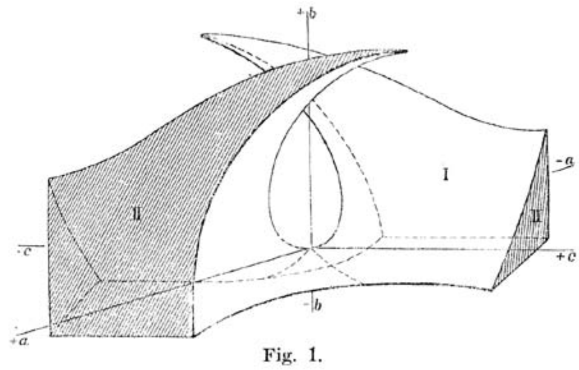


Fig. 1.

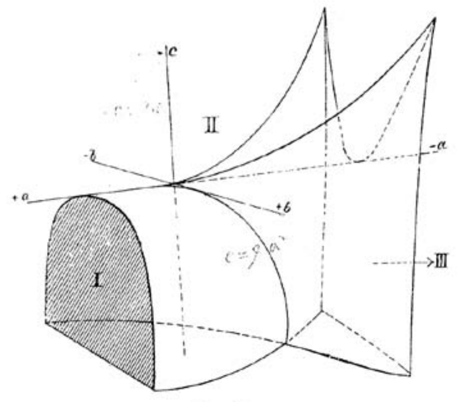


Fig. 2.

G. Kerschensteiner's drawings of discriminant surfaces (1892)

Schoute finds no indication in Kerschensteiner's text that, apart from drawing, he also constructed models. So Schoute continues and presents his models. However, during the next KNAW meeting (Saturday 24 June 1893), Schoute informs his colleagues that

> Dr. Kerschensteiner in Schweinfurt indeed constructed models of discriminant surfaces, but not string models. Rather than content himself therefore with only drawings, Dr. Kerschensteiner made tinplate cross sections which he moulded together using a kneadable substance. (See [KNAW, p. 44] for the original text in Dutch.)

The evident correspondence between Schoute and Kerschensteiner probably inspired the latter to continue his research on discriminant surfaces. This resulted in a text [Ke93] published in the supplement of Dyck's Catalogue [Dy93]. Here, knowing that Schoute had already discussed polynomials of degree 3, 4 and 6, Kerschensteiner treats the family of degree 5 polynomials

$$t^5 + xt^2 + yt + z.$$

One of his results is that if x, y, z are real numbers such that the discriminant of the polynomial is nonzero then the number of real zeros is either 1 or 3. The discriminant surface in this case partitions real 3-space into two parts, (probably disappointingly) much like what also happens for polynomials of degree 3. Kerschensteiner finishes his text by noting that Schoute had shown that discriminant surfaces are unions of straight lines (in fact, developable surfaces) and that this allows one to construct string models of such surfaces. He did this for his family of quintic polynomials.

According to Kerschensteiner, Schoute's presentation at the KNAW on Saturday 27 May 1893 took place in the "Accademie in Groningen". This is very unlikely: KNAW meetings took (and still take) place in Amsterdam. One may expect that on the particular Saturday, Schoute, carrying his three models each of size $8.5 \times 8.5 \times 8.5$ inches, spent several hours on the steam train between Groningen and Amsterdam.

The KNAW minutes record that Schoute's presentation "is followed by a short discussion with Grinwis and Korteweg". The latter is the Amsterdam mathematician famous for the partial differential equation having his name (KdV = Korteweg–de Vries equation). C. H. C. Grinwis (1831–1899) was a mathematical physicist who worked in Delft and in Utrecht.

Dr G. Kerschensteiner (1854–1932) later became well-known as a school reformer and pedagogist working in Munich. In particular his ideas concerning vocational schools have had a large influence on the German school system and have brought him worldwide recognition. Although he taught in his early years mathematics and physics in grammar schools (in Schweinfurt from 1890 till 1893), and he also published some mathematical texts such as the one described here, in mathematics Kerschensteiner seems to be mostly forgotten. In physics however, Germany still has a 'Kerschensteiner Medal' honouring particularly good physics teachers.

Hartenstein

The fact that Roderich Hartenstein did his 'Staatsexamen' with Klein in Göttingen in 1905/06 indicates that he probably became a high school teacher. His thesis work on the discriminant surface corresponding to

$$t^4 + xt^2 + yt + z$$

was not just recalled by Klein in his celebrated lecture notes [Kl08], [Kl32]; two of the string models in Schilling's Series XXXIII were based on it, and as a supplement to these models Schilling published the thesis in 1909 [Ha09]. This was noticed on several occasions. For example, in 1910 the Bulletin of the AMS mentions it [BAMS10, p. 503]. It is also referred to in a paper [Em35] of Arnold Emch in 1935.

To a large extent, the results of Hartenstein coincide with what Schoute already wrote concerning the polynomials of degree 4. Hartenstein discusses one novel issue. Remarkably, this is not reviewed by Klein in his treatment in [Kl08], [Kl32]. The problem is, given two real bounds L, B with $L < B$, how to find geometrically using the discriminant surface all polynomials $t^4 + xt^2 + yt + z$ having a zero τ such that $L \leq \tau \leq B$. One of the two Hartenstein models in Series XXXIII simply shows the discriminant surface corresponding to these biquadratic polynomials. The other model illustrates his solution to the problem with the two bounds.

In a 'Vorbemerkung' (preceding remark) [Ha09, p. 5], Hartenstein writes that he had already created his models in 1905/06, inspired by Klein. From footnotes on p. 10 and p. 12 one can see that he is well aware of the fact that, in the meantime, Klein used his results in his lectures and text [Kl08]. Hartenstein also gives credit to Schoute and to Kerschensteiner (footnote on p. 15) by observing that they also represented discriminant surfaces. He includes the precise places [Ke92], [Sch93] in Dyck's Catalogue [Dy93] where they describe their work.

His two models in Schilling's Series XXXIII can be admired on the web.[1] Note that M. Schilling's firm was located in the same town Halle for a while before it moved to nearby Leipzig. So it is not surprising that here one finds a large collection of models.

Sinclair

Some biographical notes concerning Mary Emily Sinclair (1878–1955) can be found in [GL08]. Her Master's thesis [Si03] completed at the University of Chicago in 1903 (supervised by Klein's former student Bolza, who returned to Germany (Freiburg) in 1910) is closely related to the work of Schoute and Kerschensteiner discussed above.

THESIS

Presented to the Faculties of Arts, Literature, and Science of the University of Chicago, in candidacy for the degree of Master of Arts, by Mary Emily Sinclair.

Subject:

Concerning the Discriminantal Surface for the Quintic in the Normal Form:

$$u^5 + 10xu^3 + 5yu + z = 0$$

March 1903.

Sinclair considers the family

$$t^5 + xt^3 + yt + z.$$

This appears only slightly different from the quintics considered ten years earlier by Kerschensteiner in [Ke93] but there are two differences. The first and important one is that in Sinclair's family real x, y, z exist such that the discriminant is nonzero and the number of real zeros of the polynomial is any of the possibilities 1, 3, 5. So her discriminant surface partitions real 3-space into three parts, which was not the case for Kerschensteiner's surface. The second difference is a computational one: Sinclair's family is chosen in such a way that the zeros of the derivative are quite easily expressible in the coefficients x, y, z. This is an advantage exploited throughout her thesis.

In the last sentence of the Introduction, Sinclair states "A model of the surface accompanies this investigation." This model seems to have disappeared. A sketch of the surface can be found in her thesis (p. 36). Five years after this Master's thesis was completed, Schilling reproduced Sinclair's model as Nr. 1 in the Series XXXIII. Accompanying this new model, an eight page summary Sinclair wrote about her Master's work was published [Si08]. Searching through the collections available at various German universities, one finds Schilling's reproduction of Sinclair's model (Series XXXIII Nr. 1) in, for example, the Mathematics Department of the Martin Luther Universität of Halle-Wittenberg.[2]

In 2003, the American sculptor Helaman Ferguson made a stone model [Fe03] based on Sinclair's thesis.

2 Discriminant surfaces

Here we discuss some of the classical geometry of discriminant surfaces. We do this using the language of algebraic geometry. Of course, for the actual classical models one was interested in the surfaces over the real numbers $\mathbb{R}$ (and to some extent, over the complex numbers $\mathbb{C}$). Here we work over an arbitrary field k. We fix a separable closure of k, which will be denoted $\overline{k}$. All classical examples fit in the following framework.

Consider two polynomials $f, g \in k[t]$ such that f is monic and $\deg(f) > \deg(g) > 1$. Put

$$P(x,y,z,t) := f(t) + xg(t) + yt + z \in k[x,y,z,t].$$

Replacing z by $z' = f(0) + xg(0) + z$ and y by $y' = \frac{df}{dt}(0) + x\frac{dg}{dt}(0) + y$ and x by $x' = \frac{d^2f}{dt^2}(0) + x$, one can moreover assume that $g(t) \equiv 0 \bmod t^2$ and $f(t) \equiv 0 \bmod t^3$. Also, multiplying x by the leading coefficient of g allows one to assume that g is monic.

The examples treated above are indeed of this kind:

Designer	f	g
Schoute	t^3	t^2
	t^4	t^2
	$t^6 - 15t^4$	t^2
Kerschensteiner	t^4	t^2
	t^5	t^2
Sinclair	t^5	t^3
Hartenstein	t^4	t^2

There is one more condition, which we assume only in Proposition 2.4 below: the polynomials f, g should be chosen in such a way that the derivative of f and the second derivative of g are not identically zero. Since we also assume $\deg(f) > \deg(g) > 1$, this is automatic in the case $\mathrm{char}(k) = 0$. However, in positive characteristic it poses an extra condition. For example, the cases treated by Sinclair satisfy $\frac{d^2g}{dt^2} \neq 0$ and $\frac{df}{dt} \neq 0$ precisely when $\mathrm{char}(k) \neq 2, \neq 3, \neq 5$.

We now review some geometric properties of the set

$$S := \Big\{(\alpha,\beta,\gamma) \in \overline{k}^3 \mid P(\alpha,\beta,\gamma,t) \text{ has a zero with multiplicity } \geq 2\Big\}.$$

Obviously S is an algebraic set: it consists of all points where the discriminant $\Delta(x,y,z) \in k[x,y,z]$ of P (regarded as polynomial in the variable t) vanishes. In more detail, put $K := k(x,y,z)$ the field of fractions of $k[x,y,z]$. Then $P \in K[t]$. Write $P' = \frac{dP}{dt}$.

By definition, S consists of all (α,β,γ) such that $p := P(\alpha,\beta,\gamma,t)$ and $p' := P'(\alpha,\beta,\gamma,t)$ have a zero in common. This condition implies in particular that no polynomials $r, s \in \overline{k}[t]$ exist such that $rp + sp' = 1$. The converse also holds: if 1 is not a linear combination of p and p' then p, p' have a common factor, so p has a multiple zero. This simple observation leads to the following.

For $d > 0$, consider

$$\mathcal{P}_d := \{r \in K[t] \mid \deg(r) < d\}.$$

This is a finite dimensional vector space over K. Put $d := \deg(P) = \deg(f)$ and write $P = z + yt + (\sum_{m=2}^{d-1} a_m t^m) + t^d$. The K-linear map

$$\varphi \;:\; \mathcal{P}_{d-1} \oplus \mathcal{P}_d \to \mathcal{P}_{2d-1} \;:\; (r,s) \mapsto rP + sP'$$

is invertible precisely when 1 is in the image, which is the case precisely when $\gcd(P, P') = 1$. Choose the basis

$$\left\{t^{2d-2}, t^{2d-3}, \ldots, t^2, t, 1\right\}$$

for $\mathcal{P}_{2d-1}$ and

$$\left\{(t^{d-2},0), (t^{d-3},0), \ldots, (1,0), (0,t^{d-1}), (0,t^{d-2}), \ldots, (0,t), (0,1)\right\}$$

for $\mathcal{P}_{d-1} \oplus \mathcal{P}_d$. With respect to these bases, φ is given by the matrix

$$\begin{pmatrix}
1 & 0 & \ldots & 0 & d & 0 & \ldots & 0 \\
a_{d-1} & 1 & & 0 & (d-1)a_{d-1} & d & \ldots & 0 \\
\vdots & \ddots & & & & & \ddots & \vdots \\
a_2 & a_3 & & 1 & 2a_2 & 3a_3 & & 0 \\
y & a_2 & & a_{d-1} & y & 2a_2 & & d \\
z & y & & & 0 & y & & (d-1)a_{d-1} \\
0 & z & & & 0 & 0 & & \\
\vdots & \ddots & & & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & z & 0 & 0 & \cdots & y
\end{pmatrix}.$$

The determinant of this matrix is called the resultant of P and P'; this is (up to a sign) the discriminant $\Delta(x,y,z)$ of P. This is a polynomial in x, y, z; from the description as a determinant given here one concludes that its degree with respect to z equals $d-1$, unless $\mathrm{char}(K)$ divides d, in which case the degree is strictly less than $d-1$.

So as an algebraic set,

$$S := \Big\{(\alpha,\beta,\gamma) \in \overline{k}^3 \mid \Delta(\alpha,\beta,\gamma) = 0\Big\}.$$

An immediate consequence of this is that S is an (affine) algebraic surface. To prove more properties of S, observe that S consists of all points (α,β,γ) such that $\tau \in \overline{k}$ exists with $P(\alpha,\beta,\gamma,\tau) = P'(\alpha,\beta,\gamma,\tau) = 0$. In other words, S is the image under the projection map $(\alpha,\beta,\gamma,\tau) \mapsto (\alpha,\beta,\gamma)$ of the algebraic set $\tilde{S} \subset \overline{k}^4$ defined by $P = P' = 0$.

Proposition 2.1. *S is an irreducible algebraic set.*

Proof. This assertion means that S is not the union of two proper nonempty algebraic subsets. Equivalently, the ideal $I(S) \subset \overline{k}[x,y,z]$ consisting of all polynomials that vanish on every point of S, should be a prime ideal. To show that this is indeed true, consider the ideal $I(\tilde{S}) \subset \overline{k}[x, y, z, t]$ consisting of all polynomials vanishing on every point of $\tilde{S}$. Since S is the projection of $\tilde{S}$ it follows that $I(S) = I(\tilde{S}) \cap \overline{k}[x, y, z]$. Hence it suffices to prove that $I(\tilde{S})$ is a prime ideal.

Consider $I := \overline{k}[x, y, z, t] \cdot P + \overline{k}[x, y, z, t] \cdot P'$. By definition, $\tilde{S}$ is the algebraic set defined by the vanishing of all polynomials in I. By Hilbert's Nullstellensatz, this implies that $I(\tilde{S})$ is the radical of the ideal I. However, I is a prime ideal because

$$\overline{k}[x, y, z, t]/I \cong \overline{k}[x, y, t]/\overline{k}[x, y, t] \cdot P' \cong \overline{k}[x, t]$$

is a domain. Hence $I = \mathrm{rad}(I) = I(\tilde{S})$, showing that $I(\tilde{S})$ and therefore also $I(S)$ is a prime ideal. □

Since $I(S)$ and Δ both define the irreducible algebraic set S, it follows that $I(S) = \mathrm{rad}(\overline{k}[x, y, z]\cdot\Delta)$. This means that $I(S)$ is a principal ideal generated by an irreducible $Q \in \overline{k}[x, y, z]$ such that $\Delta = uQ^m$ for some nonzero u in $\overline{k}$ and an integer $m > 0$. Because $I(S)$ is generated by polynomials defined over k (namely, those in $(k[x, y, z, t] \cdot P + k[x, y, z, t] \cdot P') \cap k[x, y, z]$), we can even take $Q \in k[x, y, z]$ (and therefore also $u \in k$) here. In the classical examples one has $m = 1$, so Δ is irreducible. However, this is not true in general, as the case $t^{2p} + xt^p + yt + z$ in characteristic $p > 0$ shows (see Examples 2.5 below).

Proposition 2.2. *S is a ruled surface, i.e. it is a union of straight lines.*

Proof. Indeed, $(\alpha, \beta, \gamma) \in S$ precisely when τ exists such that $P(\alpha, \beta, \gamma, \tau) = P'(\alpha, \beta, \gamma, \tau) = 0$. This can be written as

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ -f'(\tau) \\ \tau f'(\tau) - f(\tau) \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -g'(\tau) \\ \tau g'(\tau) - g(\tau) \end{pmatrix}.$$

Clearly this defines a union of straight lines. □

Note that the above result shows that the straight lines are described by the following property: the points (α, β, γ) on any such line correspond to the polynomials $P(\alpha, \beta, \gamma, t)$ that have a common multiple zero.

The next property shows that in fact S is a developable surface: not only is it a union of straight lines; the points on any such line all have a common tangent plane, which in fact is provided with an easy equation.

Proposition 2.3. *Suppose char$(k) \neq 2$. Let $(\alpha, \beta, \gamma) \in S$ correspond to a zero τ of $P(\alpha, \beta, \gamma, t)$ with multiplicity 2. Then the plane with equation*

$$f(\tau) + xg(\tau) + y\tau + z = 0$$

is tangent to S in (α, β, γ).

Proof. First, observe that by assumption indeed (α, β, γ) is a point on the given plane. The multiplicity of τ being 2 implies, since $\mathrm{char}(k) \neq 2$, that

$$f''(\tau) + \alpha g''(\tau) \neq 0.$$

Our strategy will be to determine the tangent plane Π to S in (α, β, γ) by first computing the tangent plane $\tilde{\Pi}$ to $\tilde{S}$ in $v := (\alpha, \beta, \gamma, \tau)$ and then project this to the x, y, z-space.

By definition, $\tilde{\Pi}$ consists of all points $v + w$ (for $w = (w_1, w_2, w_3, w_4)$) such that

$$q(v + \lambda w) \equiv 0 \bmod \lambda^2,$$

for all $q \in I(\tilde{S})$. We know that the ideal $I(\tilde{S})$ is generated by P and P' hence, instead of using all q, it suffices to consider only these two polynomials. Using that $(\alpha, \beta, \gamma, \tau) \in \tilde{S}$, the condition $v + w \in \tilde{\Pi}$ then reads

$$\begin{cases} g(\tau)w_1 + \tau w_2 + w_3 = 0; \\ g'(\tau)w_1 + w_2 + (f''(\tau) + \alpha g''(\tau))w_4 = 0. \end{cases}$$

Note that since $f''(\tau) + \alpha g''(\tau) \neq 0$, the second equation uniquely determines w_4 in terms of the other coordinates. This implies that the projection Π of $\tilde{\Pi}$ to the x, y, z-space consists of all $(\alpha, \beta, \gamma) + (w_1, w_2, w_3)$ satisfying the equation

$$g(\tau)w_1 + \tau w_2 + w_3 = 0.$$

Alternatively, this can be written as $P(w_1, w_2, w_3, \tau) = f(\tau)$. Since $P(\alpha, \beta, \gamma, \tau) + P(w_1, w_2, w_3, \tau) = f(\tau) + P(\alpha + w_1, \beta + w_2, \gamma + w_3, \tau)$, one concludes that the tangent plane consists of all (x, y, z) satisfying $P(x, y, z, \tau) = 0$, which is what we wanted to prove. □

From now on we will exploit the additional assumption that $g''(t)$ is nonzero. Let s be a new variable and put

$$w := \begin{pmatrix} -f(s) \\ -f'(s) \\ -f''(s) \end{pmatrix}.$$

The condition says that

$$A := \begin{pmatrix} g(s) & s & 1 \\ g'(s) & 1 & 0 \\ g''(s) & 0 & 0 \end{pmatrix}$$

is invertible over $k(s)$. In particular, there exists a unique solution

$$v := \begin{pmatrix} x(s) \\ y(s) \\ z(s) \end{pmatrix}$$

to the linear equation $Av = w$. This system says that $t = s$ is a triple zero of the polynomial

$$p(t) := f(t) + x(s)g(t) + y(s)t + z(s) \in k(s)[t].$$

Hence $p(t)$ can be written as

$$p(t) = (t - s)^3 q(t)$$

for some $q(t) \in k(s)[t]$. Taking the derivative with respect to s, one concludes that $t = s$ is a zero of $\frac{\partial p}{\partial s} = x'(s)g(t) + y'(s)t + z'(s)$ of multiplicity at least 2. So for fixed $s_0 \in \overline{k}$ such that $g''(s_0) \neq 0$, and for every $\lambda \in \overline{k}$, the polynomial

$$f(t) + (x(s_0) + \lambda x'(s_0))g(t) + (y(s_0) + \lambda y'(s_0))t + z(s_0) + \lambda z'(s_0)$$

in $\overline{k}[t]$ has a zero $t = s_0$ of multiplicity ≥ 2. In other words,

$$(x(s_0), y(s_0), z(s_0)) + \lambda\,(x'(s_0), y'(s_0), z'(s_0))$$

is in the discriminant surface S.

Proposition 2.4. *Assume that g'' and f' are not identically zero.*

Then $s \mapsto (x(s), y(s), z(s))$ parametrizes a curve in S corresponding to the polynomials $f(t) + xg(t) + yt + z$ with a triple zero.

For general $s_0 \in \overline{k}$, the tangent line to this curve in the point corresponding to s_0 is contained in S, and S is the Zariski closure of the union of all these tangent lines.

Proof. As before, let s be a variable over k. Define $(x(s), y(s), z(s))$ as above. We claim that the point $(x'(s), y'(s), z'(s)) \in S$ is not the point $(0, 0, 0)$: if it were, then $(0, 0, 0)$ is a point on the line in S corresponding to all polynomials with a double zero at $t = s$. The parametrization of this line given in Proposition 2.2 now implies $f'(s) = 0$ (and also $f(s) - sf'(s) = 0$). This contradicts the assumption that $f' \neq 0$.

Since $(x'(s), y'(s), z'(s)) \neq (0, 0, 0)$, it follows that $(x(s), y(s), z(s))$ indeed parametrizes a curve in S. The Zariski closure of the set of tangent lines to this curve is an irreducible subvariety V of S. By Proposition 2.1, S is irreducible, so V equals S unless it has dimension strictly smaller than 2. This can only happen if the closure of the image of $s \mapsto (x(s), y(s), z(s))$ is a line ℓ in S. However, that is impossible: if $s_0 \in \overline{k}$ is taken such that the derivative $(x'(s_0), y'(s_0), z'(s_0))$ is nonzero then we have seen that the tangent line to the parametrized curve corresponding to s_0 is the line in S coming from all polynomials having s_0 as a multiple zero. On the other hand, this tangent line is obviously equal to ℓ. This is absurd, since it would imply that the polynomials corresponding to the points of ℓ would have infinitely many multiple zeros. This finishes the proof. □

Examples 2.5.

1. Suppose p is a prime number. Consider $f(t) = t^{2p}$ and $g(t) = t^p$ over a field k of characteristic p. Obviously, the conditions $g''(t) \neq 0$ and $f'(t) \neq 0$ used in Proposition 2.4 are not satisfied.
   The discriminant surface S is given by the equation $y = 0$. Every point in S corresponds to a polynomial of which the roots have multiplicity a multiple of p. This implies that also the conditions used in Proposition 2.3 are not satisfied. Clearly, S has no singular points. The discriminant is a constant times y^{p+1}. The tangent planes to S in points of S are not of the form described in Proposition 2.3 in this case.
2. Suppose $p > 3$ is a prime number. Take $f(t) = t^{p+3}$ and $g(t) = t^{p+1}$ over a field k of characteristic p. Then $f' \neq 0$ but $g'' = 0$. In this example, $f(t) + xg(t) + yt + z$ has a zero of multiplicity > 2 if and only if $z = y = 0$. These equations define a line in the surface S, namely the one corresponding to the polynomials with $t = 0$ as a multiple zero.
3. Suppose $p > 2$ is a prime number. We now consider an example in which $g''(t)$ is nonzero but $f'(t)$ is identically zero: put $f(t) = t^p$ and $g(t) = t^2$ over a field k of characteristic p. Here the curve in S corresponding to polynomials with a zero of multiplicity > 2 is parametrized by $s \mapsto (0, 0, -s^p)$. It is a line in (the singular locus of) S but not a line obtained by demanding that the polynomial has some fixed multiple zero.

3 An "appendix"

One may wonder why M. Schilling decided to reproduce the models by Sinclair and Hartenstein, and not the older ones by Schoute. According to the catalogue [S11, p. 158], the models in Series XXXIII were inspired by Bolza and by Klein. Since Bolza was a former graduate student of Klein, and Klein's interest in models was well-known, probably Bolza informed him of the work of his Master's student Sinclair. It is only natural that in some contact with Schilling, Klein mentioned the models of his student Hartenstein and the related model of Sinclair, leading to Series XXXIII.

Nevertheless, Klein was quite likely aware of the existence of Schoute's models. They are described in the well-known 'Nachtrag' (supplement) of Dyck's Catalogue [Dy93] and Hartenstein even refers to this. Moreover, mathematically the results of Schoute and Sinclair nicely complement each other: together they provide 3-parameter families of polynomials of degree 3, 4, 5 and 6, such that in each family all possible numbers of real zeros occur. Incidentally, there is no evidence that Sinclair was aware of Schoute's work: she very carefully mentions in her thesis where to find relevant results she uses and this does not include a reference to Schoute.

One mathematical reason to take a model by Hartenstein is that he describes the set of polynomials with a given number of (real) zeros in a prescribed interval. This topic is not considered by Schoute or Sinclair. The model XXXIII Nr. 3 illustrates Hartenstein's description, providing an argument for using this model in the series.

However, there may be a different mathematical reason. As we have seen, the lines (strings) in the models correspond to real polynomials with a fixed (real) multiple zero. Now it is possible that a real polynomial does not have a real multiple zero but instead it has a pair of complex conjugate ones. In the example $t^4 + xt^2 + yt + z$ this happens precisely for two double zeros $t = \pm\sqrt{-b}$ and $b > 0$, so for the polynomials $(t^2 + b)^2$. It defines one half of a parabola $(2b, 0, b^2)$ with $b > 0$, which is part of the discriminant surface but no real lines (strings) pass through this part. Although Schoute describes this phenomenon, his actual models consist of strings only and ignore this "appendix". Hartenstein did include it in his models by means of a brass wire. And exactly at the place in his text [Ha09] where he describes this wire ("Messingdraht"), he places a footnote referring to older models by Schoute and by Kerschensteiner. So this makes a case in favour of Hartenstein's models over one by Schoute. There is one problem with this reasoning: the same phenomenon also appears in Sinclair's family $t^5 + xt^3 + yt + z$, namely, the polynomials $t(t^2 + b)^2$ with $b > 0$ give the points $(2b, b^2, 0)$ in the corresponding discriminant surface. For $b > 0$, there are no real lines in the surface passing through such a point. Sinclair describes this in detail; however, when she discusses how to construct her model, no mention of these special points is made. Contrary to Hartenstein's examples, it also appears that this "appendix" is missing in Schilling Model XXXIII Nr. 1. So maybe the choice for Hartenstein and Sinclair was not motivated by the "appendix".

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Notes

1. See, for example, http://did.mathematik.uni-halle.de/modell/modell.php?Nr=Dj-002 and http://did.mathematik.uni-halle.de/modell/modell.php?Nr=Dj-003 (both at the Martin Luther Universität in Halle-Wittenberg, Germany)
2. See http://did.mathematik.uni-halle.de/modell/modell.php?Nr=Dj-001

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Erik Weitenberg [e.r.a.weitenberg@gmail.com] was born in Drachten, the Netherlands, in 1987 and attained his Bachelor of Mathematics in 2010 at the University of Groningen. His bachelor's thesis treats the discriminantal hypersurfaces of polynomials. His main interests are algebra, cryptology and teaching.

Jaap Top [j.top@rug.nl] was born in 1960. He attained his PhD in Utrecht in 1989 with Frans Oort as supervisor. Since 1992 he works at the University of Groningen where he became Professor of number theory and algebraic geometry in 2005.

ZeTeM, Zentrum für Technomathematik, Universität Bremen, Germany

In 1995, the University of Bremen founded several centres of excellence in order to highlight and strengthen its key areas of research and its R&D activities in technology transfer. Based on the recognition that mathematical research, modelling and simulation is a major driving force in modern industrial development, a centre of industrial mathematics (ZeTeM, Zentrum für Technomathematik) was founded at the core of this programme.

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Mathematical research at ZeTeM is focused on partial differential equations (modelling, analysis and numerical methods) and inverse problems. We are proud that our research activities have resulted in ten professorships for our former PhD students and PostDocs.

Moreover, our collaborations with researchers from the engineering sciences have become a major part of our activities. Projects on multi-scale problems and phase-transition models for distortion engineering and laser welding, as well as parameter identification problems for high precision surface finishing, bridge the gap between mathematical research and the engineering sciences.

At present, we are part of several major research initiatives in mathematics and engineering, such as collaborative research centres (SFB, Sonderforschungsbereich) and priority programmes (SPP, Schwerpunktprogramm) of the German Science Foundation DFG: SFB 570 'Distortion Engineering', SFB 747 'Micro Cold Forming', SFB 637 'Autonomous Cooperating Logistic Processes', SPP 1180 'Prediction and Manipulation of Interactions between Structure and Process', SPP 1480 'Modelling, Simulation and Compensation of Thermal Effects for Complex Machining Processes', SPP 1324 'Mathematical Methods for Extracting Quantifiable Information from Complex Systems' and SPP 1253 'Optimization with Partial Differential Equations'.

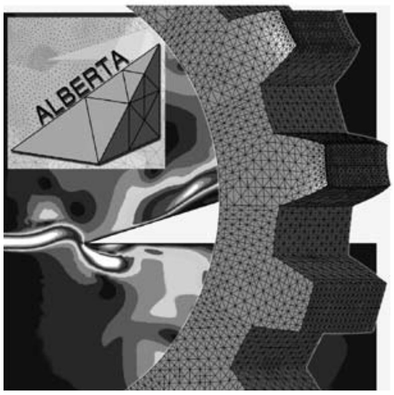
Our direct collaborations with industry concentrate on optimization and control, with a focus on applications in aerospace technology, as well as data analysis, where most applications address problems of monitoring industrial processes.

AG Technomathematik (led by Prof. Dr. Peter Maass)

The research areas of this group are inverse problems and wavelet analysis for applications in signal and image processing. A major field of application is in monitoring industrial production processes, particularly for the automotive industry and for applications in high precision surface finishing.

The mathematical research is focused on sparsity constrained regularization for non-linear inverse problems such as parameter identification problems for partial differential equations.

Peter Maass was the initiator and coordinator of a national priority programme on 'Mathematical methods for data analysis and image processing' (2002–2008). He is presently heading an EU consortium in the Future Emerging Technology (FET) programme on data analysis.



High performance algorithms for technical problems (fluid dynamics, materials sciences)

Research Group Modelling & Partial Differential Equations (led by Prof. Dr. Michael Boehm)

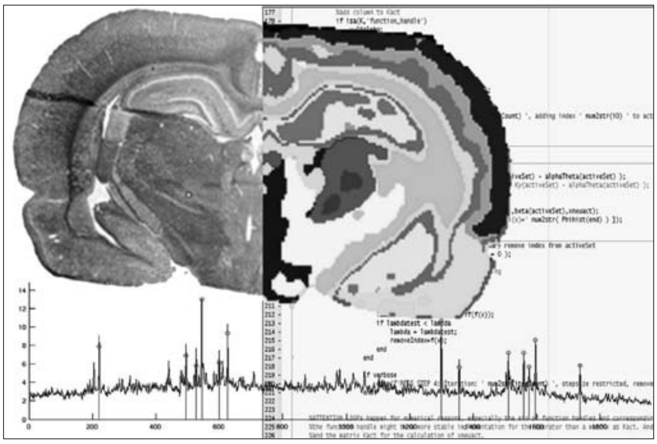
This team specialises on modelling in materials sciences, with a focus on multi-scale models incorporating complex micro structures in physicochemical models. The mathematical analysis is based on homogenization techniques for micro-macro models of reaction diffusion problems.

The material behaviour of steel distortion is a prototypical example, which is investigated in close collaboration with different engineering institutes in Bremen.

Research Group Numerical Linear Algebra (led by Prof. Dr. Angelika Bunse-Gerstner)

Model reduction and efficient numerical simulation of large-scale problems in electrical engineering and climate modelling are major research fields of the numerical linear algebra group. Numerical methods which retain characteristic structures of the considered problems are of special interest.

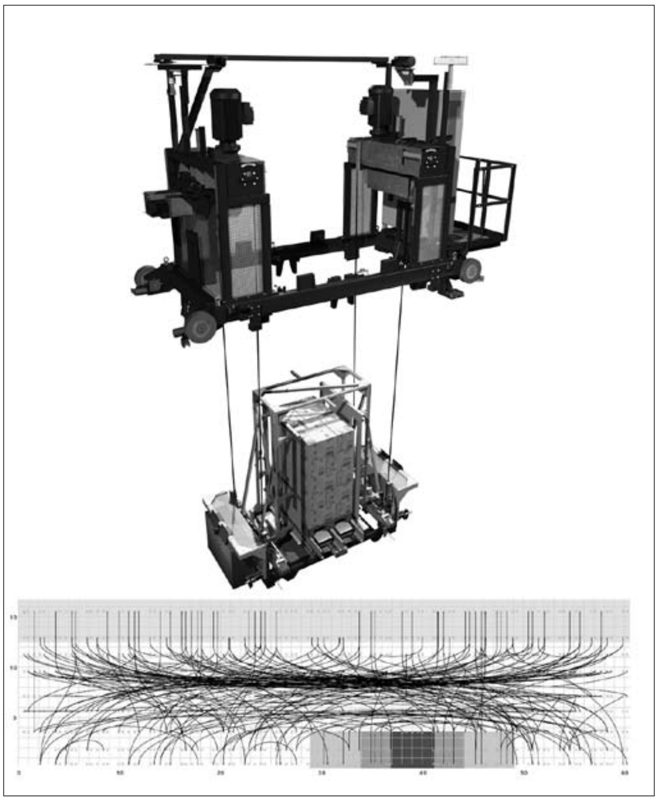
Angelika Bunse-Gerstner has been the Vice-President of Bremen University (2006–2008) and is presently a member of the SIAM Council.



MALDI imaging recovers the metabolic structure of an organism as a basis for individualized therapy planning and biomarker detection

AG Optimization and Optimal Control (led by Prof. Dr. Christof Bueskens)

Optimal control of technical, natural scientific or economic processes and systems is the field of expertise of this group. Typical applications arise from robotics, aerospace engineering and vehicle dynamics. The team specialises in high-performance numerical algorithms for real time applications, for which the software code NUDOCCCS has been developed. Together with several European partners, the package WORHP is under construction to create an NLP solver that will rival and exceed the capabilities of established sparse NLP solvers. WORHP will serve as the central tool in various large-scale optimization problems.



Real time optimization of high-dimensional complex models for industrial applications

The group collaborates with several internationally leading companies and with the European Space Agency on optimal control problems. Christof Bueskens and his team received the Loehn Award (2008) as well as the Bernd-Artin-Wessels Award (2009) for outstanding results in collaboration with industry.

AG Numerical Methods for Partial Differential Equations (led by Prof. Dr. Alfred Schmidt)

The research team of Alfred Schmidt specialises in adaptive finite element methods. The group covers different aspects of scientific computing like numerical analysis, aspects of implementation, applications and modelling. Current projects include macro- and meso-scale models in materials science, mostly involving phase changes. These applications are developed in close cooperation with production engineers and materials scientists.

Together with Prof. Dr. Wolfgang Hiller from the Alfred Wegener Institute for Polar and Marine Research, Alfred Schmidt is head of BremHLR, the competence centre for high performance computing. BremHLR at ZeTeM is the entrance portal for all scientists in Bremen who require access to the national high performance computing centre 'Höchstleistungs-rechnen Nord (HLRN)'.

AG Scientific Computing in Life Sciences (led by Dr. Theodore Alexandrov)

This group is oriented towards development of cutting-edge computational methods for mass spectrometry, in particular for imaging mass spectrometry. The group closely cooperates with leading mass spec. industrial companies as well as with research centres in Europe and the United States with prominent applications in proteomics, biology, medicine and pharmacy. Theodore Alexandrov has a joint affiliation with the University of California, San Diego.

Teaching and Promotion of Young Scientists

Establishing mathematics as a key technology for industrial research and development requires people who have a solid training in modelling and in mathematical theory as well as knowledge in engineering. ZeTeM has designed a curriculum (BSc, MSc) for 'technomathematik' which combines a classical programme in applied mathematics with courses in physics, geosciences and mechanical and electrical engineering.

This idea of combining mathematics with engineering at a research level is continued with the PhD programme SCiE (Scientific Computing in Engineering), where two PhD students (one mathematician, one engineer) work in tandem on problems of modelling and numerical approximation. This programme has been designed in particular for international students and more than ten nationalities have been represented on the programme over the past five years.

In addition, ZeTeM has exchange programmes with IIT Kharagpur and Clemson University (South Caroli-

Summer course on industrial mathematics as part of our exchange programme with IIT Kharagpur and Clemson University.

na). Every year, several students from these institutions attend a summer course on industrial mathematics and modelling at ZeTeM.

Future Developments

We aim to strengthen ZeTeM at both ends of its spectrum of activities. An additional assistant professorship, which is designed as a pure research position, will bring new fields of research to the inverse problems team. We will also intensify our collaboration with our colleagues in the mathematics department, where a professorship on applied analysis will be filled. This professor should establish a close link between ZeTeM and the mathematics department on the research side.

On the other side of the spectrum, ZeTeM has founded four Steinbeis Centres for mathematical transfer to specific fields of application (optimization, aerospace industry, life science and industrial data analysis). We are currently in the process of restructuring the transfer activities of ZeTeM.

Peter Maass [pmaass@math.uni-bremen.de] is a professor of mathematics at the University of Bremen, Germany, and director of the centre of industrial mathematics (ZeTeM). Since 2009 he has been an adjunct professor at Clemson University. In 1988, he received his PhD from the Technical University of Berlin after studying mathematics in Cambridge, UK, and Heidelberg. In 1997, he received a national award from Gerhard Schröder for his achievements in industrial data analysis.

ICMI and UNESCO: a long-term collaboration

Michèle Artigue

ICMI and UNESCO have a long tradition of collaboration with roots back in the early 60s at least. In this column, we briefly review this history before focusing on its last episode: the publication of a document on the challenges of basic mathematics education (UNESCO, 2011).

Reviewing past collaborations:

ICMI history shows that a first important step for this collaboration was the organisation of an Inter-American Conference on Mathematics Education in Bogota in 1961, instigated by Marshall Stone, president of the ICMI at the time. Occurring every four years, these conferences led to the first regional ICMI network: IACME, today an Affiliate Group to the ICMI, celebrating its 50th anniversary in June 2011. UNESCO then made a major contribution to the organisation of the first ICME, which took place in Lyon in 1969, and also supported the creation of the journal *Educational Studies in Mathematics*. At that time, ICMI and UNESCO also tightly collaborated in the preparation and publication of the four volumes of UNESCO's series: *New Trends in Mathematics Teaching*. Throughout the 70s, collaboration with UNESCO was very active, UNESCO sponsoring not only ICMEs but also various conferences and projects. For instance, ICMI and UNESCO cooperated on the organisation of two meetings which were held in Africa, one in Nairobi in 1974 on Language and Mathematics Teaching and one in Khartoum in 1978 on Developing Mathematics in Third World Countries. Bent Christiansen, involved in UNESCO activities since the 50s and vice-president of the ICMI from 1975 to 1986, played a major role in this collaboration.

During the 80s, this support of UNESCO extended to a new and important component of ICMI activities: the ICMI Studies. Moreover, an updated version of the first study carried out in 1985 and entitled *The influence of computers and informatics on mathematics and its teaching* was prepared with the support of UNESCO and published in its *Science and Technology Education* series (Cornu & Ralston 1992). At that time Edward Jacobsen at UNESCO played a major role in the success of such collaborations. When he retired without being replaced, the collaboration decreased. It started again in the early 2000s. The ICMI was then asked to join the emerging project of an international travelling exhibition devoted to mathematics piloted by Minella Alarcon, from the Division of Basic and Engineering Sciences at UNESCO, and Mireille Chaleyat-Maurel, who had been president of the UNESCO-IMU Committee WMY2000 and then in charge of Raising Public Awareness at the EMS. As vice-president of the ICMI, I joined the committee in charge of the preparation of this exhibition called "*Experiencing Mathematics*!", which was presented for the first time at ICME-10 in Copenhagen in July 2004. Since then, three sets have been travelling all around the world making this project a tremendous success (www.mathex.org). The renewal of relationships with UNESCO has not been limited to the exhibition project and, since 2002, UNESCO has been sponsoring several ICMI actions in the developing world. More recently, the ICMI was involved in the preparation of a policy document on basic mathematics education.

The challenges of basic mathematics education

In 2009, as ICMI president, I was asked to take part in a meeting of experts about science and mathematics education policies, organised by UNESCO in Paris, and then on the basis of the conclusions of this meeting to prepare a document on the challenges of basic mathematics education, to be published and disseminated by UNESCO together with a parallel document addressing the challenges of scientific education. The two documents have just been published. In line with the declaration from the World Conference in Science held in Budapest in 1999, they consider that anyone should have access to a scientific education of quality and also that such an education has a decisive contribution to offer to the realisation of the Millennium Goals adopted in 2000. Mathematics education is here considered an essential component of scientific education at large and the connections to be established between education in mathematics and in science are emphasised in the two documents, which share a common introduction and structure.

The mathematics document presents first the agreed vision of the committee on the significance of a mathematics education of quality, emphasising that it should be faithful to mathematics, both in its content and practices, raise students' interest towards the discipline and develop students' confidence in their mathematical capacities. It should thus enable students to understand that mathematics is part of a long history combined with the history of humanity, that mathematics is not a fixed corpus of knowledge but, on the contrary, a living and expanding science, whose development nourishes that of other scientific fields and is nourished by them in return. It should enable students to see mathematics as a science that can and must contribute to solving the main problems faced by the world today. Such an education should make it possible to experience mathematics in both an individual and a collective experience, and to perceive the value of communication and exchange with others. It should be stimulated by challenges while cultivating values of solidarity. It should also reflect a school open to the world and thus be in phase with the scientific and social mathematics practices that exist outside school. In particular it should rely in a relevant way on the technological tools used in these practices.

The document then considers a diversity of challenges that have to be taken up to make quality mathematics education for all more than just a "slogan". Among these, the first challenge is of course that of accessibility: access to education is still denied to more than 75 million children, and even in countries where access to education is ensured (as in Europe), quality mathematics education is far from being a reality for all students. How to cope with the increasing and changing demands of mathematical literacy? How to overcome the tension often experienced between quality education and education for all? How to combine the coherence required by progression in mathematical content with the development of more transversal mathematical competences? How to organise the necessary evolution towards more effective and stimulating teaching practices? How to make evaluation practices more coherent with educational values? How to provide teachers with an adequate initial preparation and how to efficiently support their professional development? How to face issues of teacher recruitment and retention? How to adequately combine the affordability of formal and informal education? How to pilot and regulate curricular changes? How to make more efficient use of technological advances? How to address social, cultural, linguistic and gender diversity? How to make efficient use of educational research? How to organise the necessary synergy between the different communities involved? These are some of the challenging issues that a mathematics education of quality for all faces and that the text discusses.

Cover of the French version

The document also aims to show that for taking up these challenges we are not deprived of resources. Over the last few decades, knowledge has been accumulating through research, innovation and a multiplicity of experiences. It is one of the ambitions of the ICMI to identify this knowledge and to make it serve to improve mathematics education worldwide. Beyond the list of references, 14 appendices presenting insightful case studies complement the core part of the document, thanks to the collaboration of the ICMI community at large.

The current published version of the document is in French. An English translation has been prepared by the ICMI Executive Committee and should soon be published by UNESCO, which also takes charge of the preparation of a Spanish version. An Italian translation is being prepared and will be published by the Italian Mathematical Union. UNESCO is open to translation in any other language and the different versions will be made accessible on the ICMI website. The ICMI sincerely hopes that the publication of this document and its dissemination will productively support reflection and action among all those who can contribute to reaching the goal of quality mathematics education for all, and that the EMS community will contribute.

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Teaching general problem solving does not lead to mathematical skills or knowledge

John Sweller, Richard E. Clark and Paul A. Kirschner

Problem solving is central to mathematics. Yet problem-solving skill is not what it seems. Indeed, the field of problem solving has recently undergone a surge in research interest and insight but many of the results of this research are both counterintuitive and contrary to many widely held views. For example, many educators assume that general problem-solving strategies are not only learnable and teachable but are a critical adjunct to mathematical knowledge. The best known exposition of this view was provided by Pólya (1957). He discussed a range of general problem-solving strategies, such as encouraging mathematics students to think of a related problem and then solve the current problem by analogy, or to think of a simpler problem and then extrapolate to the current problem. The examples Pólya used to demonstrate his problem-solving strategies are fascinating and his influence can probably be sourced, at least in part, to those examples. Nevertheless, in over half a century, no systematic body of evidence demonstrating the effectiveness of any general problem-solving strategies has emerged. It is possible to teach learners to use general strategies such as those suggested by Pólya (Schoenfeld, 1985) but that is insufficient. There is no body of research based on randomised, controlled experiments indicating that such teaching leads to better problem solving.

Recent "reform" curricula both ignore the absence of supporting data and completely misunderstand the role of problem solving in cognition. If, the argument goes, we are not really teaching people mathematics but are teaching them some form of general problem solving then mathematical content can be reduced in importance. According to this argument, we can teach students how to solve problems in general and that will make them good mathematicians able to discover novel solutions irrespective of the content.

We believe this argument ignores all the empirical evidence about mathematics learning. Although some mathematicians, in the absence of adequate instruction, may have learned to solve mathematics problems by discovering solutions without explicit guidance, this approach was never the most effective or efficient way to learn mathematics.

The alternative route to acquiring problem-solving skill in mathematics derives from the work of a Dutch psychologist, De Groot (1946, 1965), investigating the source of skill in chess. Researching why chess masters always defeated weekend players, De Groot managed to find only one difference. He showed masters and weekend players a board configuration from a real game, removed it after five seconds and asked them to reproduce the board. Masters could do so with an accuracy rate of about 70% compared with 30% for weekend players. Chase and Simon (1973) replicated these results and additionally demonstrated that when the experiment was repeated with random configurations rather than real-game configurations, masters and weekend players had equal accuracy (roughly 30%). Masters were superior only for configurations taken from real games.

Chess is a problem-solving game whose rules can be learned in about thirty minutes. Yet it takes at least ten years to become a chess master. What occurs during this period? When studying previous games, chess masters learn to recognise tens of thousands of board configurations and the best moves associated with each configuration (Simon & Gilmartin, 1973). The superiority of chess masters comes not from having acquired clever, sophisticated, general problem-solving strategies but rather from having stored innumerable configurations and the best moves associated with each in long-term memory.

De Groot's results have been replicated in a variety of educationally relevant fields, including mathematics (Sweller & Cooper, 1985). They tell us that long-term memory, a critical component of human cognitive architecture, is not used to store random, isolated facts but rather to store huge complexes of closely integrated information that results in problem-solving skill. That skill is knowledge domain-specific, not domain-general. An experienced problem solver in any domain has constructed and stored huge numbers of schemas in long-term memory that allow problems in that domain to be categorised according to their solution moves. In short, the research suggests that we can teach aspiring mathematicians to be effective problem solvers only by helping them memorise a large store of domain-specific schemas. Mathematical problem-solving skill is acquired through a large number of specific mathematical problem-solving strategies relevant to particular problems. There are no separate, general problem-solving strategies that can be learned.

How do people solve problems that they have not previously encountered? Most employ a version of means-ends analysis in which differences between a current problem-state and goal-state are identified and problem-solving operators are found to reduce those differences. There is no evidence that this strategy is teachable or learnable because we use it automatically.

But domain-specific mathematical problem-solving skills can be taught. How? One simple answer is by emphasising worked examples of problem-solution strategies. A worked example provides problem-solving steps and a solution for students (Van Merriënboer & Kirschner, 2007). There is now a large body of evidence showing that studying worked examples is a more effective and efficient way of learning to solve problems than simply practising problem-solving without reference to worked examples (Paas & van Gog, 2006). Studying worked examples interleaved with practice solving the type of problem described in the example reduces unnecessary working memory load that prevents the transfer of knowledge to long-term memory. The improvement in subsequent problem-solving performance after studying worked examples rather than solving problems is known as the worked example effect (Paas & van Gog).

Whereas a lack of empirical evidence supporting the teaching of general problem-solving strategies in mathematics is telling, there is ample empirical evidence of the validity of the worked-example effect. A large number of randomised controlled experiments demonstrate this effect (e.g. Schwonke et al., 2009; Sweller & Cooper, 1985). For novice mathematics learners, the evidence is overwhelming that studying worked examples rather than solving the equivalent problems facilitates learning. Studying worked examples is a form of direct, explicit instruction that is vital in all curriculum areas, especially areas that many students find difficult and that are critical to modern societies. Mathematics is such a discipline. Minimal instructional guidance in mathematics leads to minimal learning (Kirschner, Sweller & Clark, 2006).

Reformers' zeal to improve mathematics teaching and increase students' mathematical problem-solving is laudable. But instead of continuing to waste time devising "reform" curricula based on faulty ideas, mathematicians and mathematics educators should work together to develop a sound K-12 curriculum that builds students' mathematical knowledge through carefully selected and sequenced worked examples.

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John Sweller [j.sweller@unsw.edu.au] is an emeritus professor of education at the University of New South Wales and a Fellow of the Academy of Social Sciences in Australia. His research is associated with cognitive load theory. The theory is a contributor to both research and debate on issues associated with human cognition, its links to evolution by natural selection and the instructional design consequences that follow.

Richard Edward Clark [clark@usc.edu] is a professor of educational psychology, clinical research professor of surgery and Director of the Center for Cognitive Technology at the University of Southern California in Los Angeles. He is the author of over 250 articles and book chapters, as well as three recent books on the design of instruction for complex learning and the use of technology to teach science and mathematics.

Paul A. Kirschner [paul.kirschner@ou.nl] is a professor of educational psychology, Director of the Learning and Cognition Programme at the Centre for Learning Sciences and Technologies, and Director of the Cognitive Education Programme at the Netherlands Laboratory for Lifelong Learning at the Open University of the Netherlands. He is the author of the highly acclaimed book Ten steps to complex learning and is the current President of the International Society of the Learning Sciences.

This article is adapted from "Teaching General Problem-Solving Skills Is Not a Substitute for, or a Viable Addition to, Teaching Mathematics", which appeared in the November 2010 issue of the *Notices of the American Mathematical Society*. It appeared also in the Winter 2010–2011 issue of *American Educator*, the quarterly journal of the American Federation of Teachers, AFL-CIO. Reprinted with permission.

Author profiles at Zentralblatt MATH

Olaf Teschke and Bernd Wegner

High and low profiles

Two years ago[1] we reported on the first version of the author identification at Zentralblatt. Since then, its new release within the new MathML interface[2] has been further improved, both by advances in the software and, more importantly, by the overwhelming feedback from the authors themselves. When the regular tests reported a stably high level of proper assignment, it was time to go one step further and display the further author-related information which can be derived from the database. This is what is usually known as an *author profile*.

Obviously, proper identification is a necessary prerequisite because erroneously attached or missing publications give rise to multiplied errors in the derived information. However, even with correct assignments the word *profile* has to be taken cum grano salis: it is limited by the reliably available content and the scope of the database. In the following, we will outline what can reasonably be derived from the ZBMATH author profiles and what the limitations are.

First of all, there is information directly derived from the author identification, namely various name spellings. Then, there are a number of publications and their structure. The number is obtained by direct counting (no split count for joint articles, which is used by some bibliometric studies but uncommon for mathematics publications). *Publication structure* means the type of publication, where we currently distinguish articles in peer-reviewed journals, books and book articles (the latter include proceedings articles and book chapters). This distinction is quite stable for the last few decades but not fully reliable for historical items since it is obviously not possible to impose the recent understanding of publication precisely to every item from the 19th century.

Maybe the most important information is the area where the author is active. This is clustered by main MSC 2010[3] areas and ranked by count. Naturally, this has worked well since the establishment of MSC but also works for items before the 70s, which have mostly been retro-classified at least to the top MSC level. On the other hand, mathematics is dynamic and imposing a classification from the 21st century on 19th century publications provides a rich source of uncertainty.

Secondly, given the advanced specialisation in mathematics, there is the natural wish to have a profile on a more detailed scale, e.g. by including second or even third MSC level. However, this would usually give wrong impressions and false certainties. Precise classification often depends on the taste of the reviewers and editors and, above all, is very unstable over the decades. With every MSC revision, third level classifications die, split or are newly introduced. Since the publication life of the average mathematician is typically longer than a decade, it is almost impossible to derive stable information from detailed MSC. For most authors, subject information split to third-level MSC provides sparse entries which cannot be reasonably used.

Additional information comes from matching the author to the journal database, giving a list of journals in which the author's articles have been published. Since the journal attachment is quite advanced for the majority of Zentralblatt entries, the information is reliable back to the 30s, having in mind that a journal might change its identity on a longer timescale (name/ISSN changes, etc.). It becomes less precise for Jahrbuch data, where the journals changed their identity (as defined by current standards) much more frequently. The data are reasonably distributed among the journals for most authors but one may also identify cases of quite extreme publication behaviour.

Furthermore, a list of co-authors is given. This part is most sensitive to author identification, since errors may occur on both sides. Again, the list is ordered by count, and in alphabetical order for equal numbers.

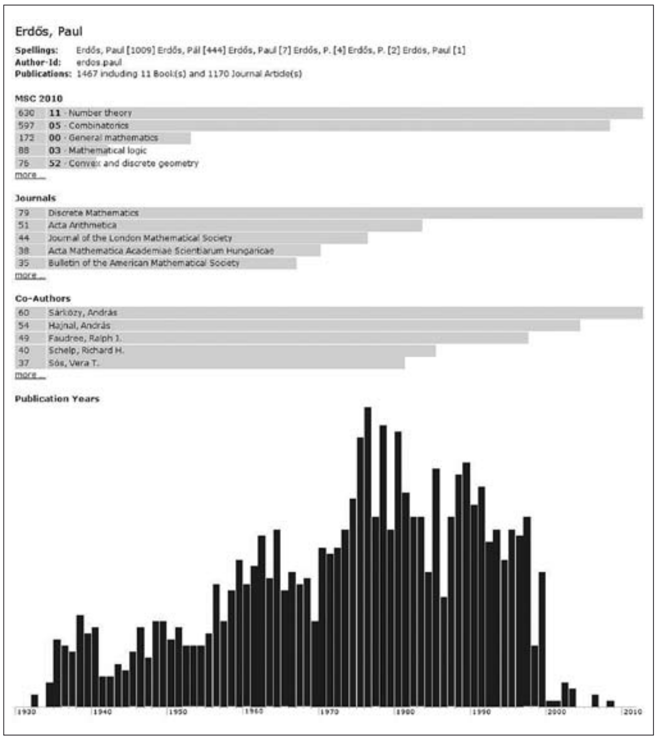
The last two diagrams display the publication timeline – first at a relative scale (from the first to the last year of activity) and then on the absolute scale (limited by the beginning and the end of the overall available database entries). Though both scales seem quite redundant, they visualise different aspects of the publication timeframe and exist in their own right – so far, there has been no clear preference that would allow one of them to be dropped. Perhaps the publication year belongs to the most reliable data but, again, one has to keep in mind distortions inherent in the publication system – that an article may appear quite delayed in a journal or that due to new editions and reprints the publication timeframe may considerably extend the lifetime of an author.

The following screenshot (see top of next page) should give an impression of how this information is organised in Zentralblatt author profiles. For convenience, lists are cut after the Top 5 entries, with the opportunity of quick one-click full extension. Currently, the author database and profiles are freely available and feedback to authorid@zentralblatt-math.org is appreciated.

Good and bad usage

As is the case with most services produced by data mining, there are problems with wrong expectations concerning the validity of the results. The impact factor available from the Science Citation Index is a prominent example for being used for purposes it had not been designed

[1] On authors and entities, Zentralblatt Corner, EMS Newsletter 71, March 2009.
[2] http://www.zentralblatt-math.org/zbmath/authors/.
[3] http://msc2010.org/.



An (actually, the largest) author profile in ZBMATH

for and for leading to conclusions which are not fair or even damaging to the reputation of scientists or scientific journals. Services like the author profiles offered by ZBMATH (and also by MSN[4]) should try to avoid these problems or at least give a precise idea of what could be expected from them. Our profiles are based on the information stored at ZBMATH. Hence they refer to peer-reviewed publications in mathematics and its applications appearing in accessible sources. We do not follow any ideas to please special authors by picking up additional publications to make their profiles more complete. We try to handle every author the same and that can only be done by machine-work.

In many cases a profile of a scientist as an author may look quite different from what can be seen from his mathematical segment provided by ZBMATH. A first internal obstruction is the lack of 100% precision in the author identification. As can be seen by a thorough investigation of the offers from MSN and ZBMATH, it is an illusion that 100% can be reached, though efforts are undertaken to get near to this mark. Mixing up authors or splitting them clearly gives a wrong picture of their profiles. The second author of this article has subjects and co-authors in his profile he had never known before. Taking care of this by hand at ZBMATH does not solve the general problem or improve the wrong information at MSN. We have to take into account that these problems cannot be avoided. Taking care of individual cases by hand can only be a small-scale, temporary solution.

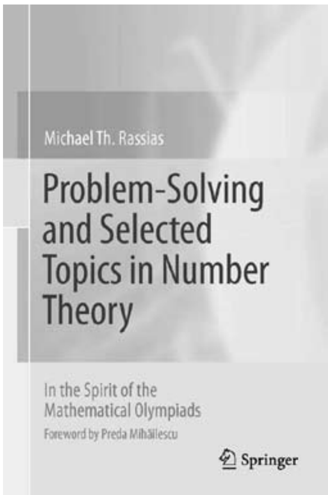
Another gap will be papers of a scientist that are not considered as mathematics. In particular, authors in the applications of mathematics will have a relevant part of their author profile left out because the papers are in applied areas. In general, most authors will have occasional papers out of the scope of ZBMATH and MSN, and they may be disappointed about the incompleteness of their profile. The same refers to published writings of authors which are not formally considered as mathematical publications (introductions, essays, book reviews, etc.). What an author wants to find in his publication list is a matter of taste. Some even list their reviews written for ZBMATH or MSN.

Finally, even relevant mathematical publications may have been left out by MSN or ZBMATH because they appeared in a journal not covered by our services. This will not be the case with journals where the majority of articles deal with mathematics. But if only one or two relevant items in mathematics appear among a set of 100, for example, we cannot afford to look up the journal to find these two. This small gap may be filled with automatic alerts by digital versions of journals but at present we can only react to complaints brought to our notice by the authors.

In spite of these problems we consider author profiles an important service. But the restrictions resulting from the nature of the mathematical reference databases should be kept in mind.

[4] http://www.ams.org/mathscinet/searchauthors.html.

Book Reviews



Michael Th. Rassias

Problem-Solving and Selected Topics in Number Theory
In the Spirit of the Mathematical Olympiads

Springer, 2011, 324 pp.
ISBN: 978-1-4419-0494-2

Reviewer: Preda Mihăilescu (Göttingen, Germany)

In the last two decades, the International Mathematics Olympiad (IMO) has become an international institution with an impact in most countries throughout the world, fostering young mathematical talent and promoting a certain approach to complex, yet basic, mathematics. It lays the ground for an open, unspecialized understanding of the field to those dedicated to this ancient art.

Accordingly, quite a few collections of Olympiad problems have been published by various major publishing houses. These collections include problems from past Olympic competitions or problems proposed by various participating countries. Through their variety and required detail of solution, the problems offer valuable training for young students and a captivating source of challenges for the mathematically interested adult.

In the so-called Hall of Fame of the IMO, which includes numerous presently famous mathematicians and several Fields medallists, one finds a list of the participants and results of former mathematical Olympiads (see [HF]). We find in the list of the participants for Greece, in the year 2003, the name of Michael Th. Rassias. At the age of 15 at that time, he won a silver medal and achieved the highest score on the Greek team. He was the first Greek of such a young age to receive a silver medal in over a decade. He is the author of the book under review: one more book of Olympiad problems among other similar, beautiful books.

Every single collection adds its own accent and focus. This book has a few particular characteristics which make it unique among similar problem books. While most of these books have been written by experienced mathematicians after several decades of practising their skills as a profession, Michael wrote this book during his undergraduate years in the Department of Electrical and Computer Engineering of the National Technical University of Athens. It is composed of some number theory fundamentals and also includes some problems that he undertook while training for the Olympiads. He focused on problems of number theory, which was the field of mathematics that began to capture his passion. It appears like a confession of a young mathematician to students of his age, revealing to them some of his preferred topics in number theory based on solutions of particular problems – most of which also appear in this collection. Michael does not limit himself to those particular problems. He also deals with topics in classical number theory and provides extensive proofs of the results, which read like "*all the details a beginner would have liked to find in a book*" but are often omitted.

In this spirit, the book treats Legendre symbols and quadratic reciprocity, the Bertrand Postulate, the Riemann ζ-function, the Prime Number Theorem, arithmetic functions, diophantine equations and more. It offers pleasant reading for young people who are interested in mathematics. They will be guided to an easy comprehension of some of the jewels of number theory. The problems will offer them the chance of sharpening their skills and applying the theory.

After an introduction of the principles, including Euclid's proof of the infinity of the set of prime numbers, there follows a presentation of the extended Euclidean algorithm in a simple matricial form known as the Blankinship method. Unique factorization in the integers is presented in full detail, thus giving the basics necessary for the proof of the same fact in principal ideal domains. The next chapter deals with rational and irrational numbers and supplies elegant, comprehensive proofs of the irrationality of e and π, which are a first taste of Rassias' way of breaking down proofs in explicit, extended steps.

The chapter on arithmetic functions presents, along with the definition of the Möbius μ and Euler ϕ functions, the various sums of divisors

$$\sigma_a(n) = \sum_{d|n} d^a ,$$

as well as nice proofs and applications that involve the Möbius inversion formula. We find an historical note on Möbius, which is the first of a sequence of such notes by which the author adds a temporal and historical frame to the mathematical material.

The third chapter is devoted to algebraic aspects, perfect numbers, and Mersenne and Fermat numbers, as well as an introduction to some open questions related to these. The fourth chapter deals with congruences, the Chinese Remainder Theorem and some results on the rings $\mathbb{Z}/n\cdot\mathbb{Z}$ in terms of congruences. These results open the door to a large number of problems contained in the second part of the book.

Chapter 5 treats the symbols of Legendre and Jacobi and gives Gauss' first geometric proof of the law of quadratic reciprocity. The algorithm of Solovay and Strassen – which was the seminal work leading to a probabilistic perspective of fundamental notions of number theory, such as primality – is described as an application of the Jacobi symbol. The next chapters are analytic, introducing the ζ and Dirichlet series. They lead to a proof of the Prime Number Theorem, which is completed in the ninth chapter. The tenth and eleventh chapters are not only a smooth transition to the problem part of the book, containing numerous examples of solved problems, but they also lead up to some theorems. In the last two subsections of the appendix, Michael discusses special cases of Fermat's Last Theorem and Catalan's conjecture.

I could close this introduction with the presentation of *my favourite problem* but instead I shall present and briefly discuss another short problem which is included in the book. It is a *conjecture* that Michael Th. Rassias conceived of at the age of 14 and tested intensively on the computer before realising its intimate connection with other deep conjectures of analytic number theory. These conjectures are still considered intractable today.

Rassias Conjecture. *For any prime p with $p > 2$ there are two primes p_1, p_2, with $p_1 < p_2$, such that*

$$p = \frac{p_1 + p_2 + 1}{p_1}. \tag{1}$$

The conjecture was verified empirically on a computer and was published along with a series of problems from international Olympiads (see [A]). The purpose of this short note is to put this conjecture in its mathematical context and relate it to other known conjectures.

At first glance, expression 1 is utterly surprising and it could stand for some unknown category of problems concerning representation of primes. Let us, though, develop the fraction in 1:

$$(p-1)p_1 = p_2 + 1.$$

Since p is an odd prime, we obtain the following slightly more general conjecture: *For all $a \in \mathbb{N}$ there are two primes p, q such that:*

$$2ap = q + 1. \tag{2}$$

Of course, if 2 admits a solution for any $a \in \mathbb{N}$, then a fortiori 1 admits a solution. Thus, the Rassias conjecture is true. The new question has the particularity that it only requires proof of the existence of a single solution. We note, however, that this question is related to some famous problems, in which one is required more generally to show that there is an infinity of primes verifying certain conditions.

For instance, the question of whether there is an infinity of Sophie Germain primes p, i.e. primes such that $2p + 1$ is also a prime, has a similar structure. While in version 2 of the Rassias conjecture we have a free parameter a and search for a pair (p, q), in the Sophie-Germain problem we may consider p itself as a parameter subject to the constraint that $2p + 1$ is prime, too. The fact that there is an infinity of Sophie Germain primes is an accepted conjecture, and one expects the density of such primes to be $O(x/\ln^2(x))$ [Du]. We obtain from this the modified Rassias conjecture by introducing a constant a as factor of 2 and replacing $+1$ by -1. Thus $q = 2p + 1$ becomes $q = 2ap - 1$, which is 2. Since a is a parameter, in this case we do not know whether there are single solutions for each a. When a is fixed, this may of course be verified on a computer or symbolically.

A further related problem is the one of Cunningham chains. Given two coprime integers m, n, a Cunningham chain is a sequence $p_1, p_2, \ldots, p_k$ of primes such that $p_{i+1} = mp_i + n$ for $i > 1$. There are competitions for finding the longest Cunningham chains but we find no relevant conjectures related to either length or frequencies of such chains. In relation to 2, one would rather consider the Cunningham chains of fixed length 2 with $m = 2a$ and $n = -1$. So the question 2 reduces to the statement: *there are Cunningham chains of length two with parameters $2a, -1$, for any $a \in \mathbb{N}$.*

By usual heuristic arguments, one should expect that 2 has an infinity of solutions for every fixed a. The solutions are determined by one of p or q via 2. Therefore, we may define

$$S_x = \{p < ax \ : \ p \text{ is prime and verifies 2}\}$$

and the counting function $\pi_r(x) = |S_x|$. There are $O(\ln(x))$ primes $p < x$, and $2ap - 1$ is an odd integer belonging to the class -1 modulo $2a$. Assuming that the primes are equidistributed in the residue classes modulo $2a$, we obtain the expected estimate:

$$\pi_r(x) \sim x/\ln^2(x) \tag{3}$$

for the density of solutions to the extended conjecture 2 of Rassias.

Probably the most general conjecture on distribution of prime constellations is Schinzel's *Conjecture H*:

Conjecture H. *Consider s polynomials $f_i(x) \in \mathbb{Z}[X], i = 1, 2, \ldots, s$ with positive leading coefficients and such that the product $F(X) = \prod_{i=1}^{s} f_i(x)$ is not divisible, as a polynomial, by any integer different from ± 1. Then there is at least one integer x for which all the polynomials $f_i(x)$ take prime values.*

Of course, the Rassias conjecture follows for $s = 2$ with $f_1(x) = x$ and $f_2(x) = 2ax - 1$. Let us finally consider the initial problem. Can one prove that 2 has at least one solution in primes p, q, for arbitrary a? In [SW], Schinzel and Sierpiński show that Conjecture H can be stated for one value of x or for infinitely many values of x, since the two statements are equivalent. Therefore, solving the conjecture of Rassias is as difficult as showing that there are infinitely many prime pairs verifying 2. Of course, this does not exclude the possibility that the conjecture could be proved more easily for certain particular families of values of the parameter a.

The book is self-contained and rigorously presented. Various aspects of it should be of interest to graduate and undergraduate students in number theory, and high school students and the teachers who train them for the Putnam Mathematics Competition and Mathematical Olympiads, as well as, naturally, scholars who enjoy learning more about number theory.

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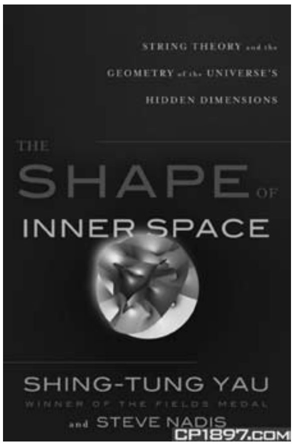
Preda Mihăilescu [preda@uni-math.gwdg.de] was born in Bucharest, 1955. He studied mathematics and computer science in Zürich receiving his Ph.D. from ETH Zürich. He was active during 15 years in the industry, as a numerical analyst and cryptography specialist. He moved back to academia in 2002, after solving a long-standing conjecture in number theory, formulated by Catalan. The result is known as Mihăilescu's Theorem. He is curently a professor at the Institute of Mathematics of the University of Göttingen.



Shing-Tung Yau and
Steve Nadis

The Shape of Inner Space: String Theory and the Geometry of the Universe's Hidden Dimensions

Basic Books, 2010, 400 p.
ISBN 978-0-465-02023-2

Reviewer: Gustavo Granja

There is a long tradition of leading physicists writing accounts of recent developments in their field for the general public. In "The Shape of Inner Space", Shing-Tung Yau, one of the world's most influential mathematicians, and Steve Nadis, an experienced science writer, attempt the arguably much more difficult task of describing recent progress in Geometry. They succeed admirably.

As Yau writes in the preface, this book "describes [Yau's] explorations in the field of mathematics, focusing on one discovery in particular that has helped some scientists build models of the universe". The discovery being referred to is Yau's proof of the Calabi conjecture in Kähler geometry. This theorem established the existence of certain Kähler manifolds later dubbed Calabi-Yau by physicists, who found them to be essential for the development of string theory.

One overarching theme of the book is the central role played by geometry in attempts at describing nature throughout history. The authors give a very compelling account of the symbiotic relation between mathematics and theoretical physics. This is a task to which Yau is ideally suited, having been one of the main facilitators of this interaction throughout his career. As the subtitle might suggest, the book also provides an excellent popular account of certain aspects of string theory but this is done with much more mathematical depth than usual and, more often than not, with a view towards the relationship with mathematics.

In addition, the book gives a remarkably vivid and honest depiction of the practice of mathematical research, including its social aspects, which will resonate with many readers of this newsletter and inform the interested layperson.

As Nadis says in the preface, this book is an unusual blend of the two authors' different backgrounds and perspectives. Parts of it are written in the first person and read like an autobiography, while the remainder is written in the third person and is based on interviews of physicists and mathematicians (many quotes are included), as well as on other research conducted by Nadis. He proposes that the main character of the book is not Yau but the class of Kähler manifolds whose existence Yau established. It is amusing that this difference in perspectives is already manifest in the preface, where Yau refers to his *discovery* of Calabi-Yau manifolds and states that "to a mathematician, mathematics is reality", while Nadis talks about Yau's *invention* of Calabi-Yau shapes.

I will try to give a sense of the book by summarising its contents. The first chapter, entitled "A universe in the margins", sets the stage by suggesting that there may be hidden dimensions in the universe that are crucial to understanding Physics and that some physicists think the extra dimensions have the shape of a Calabi-Yau manifold.

The following four chapters, roughly one third of the book, build up to the proof of the Calabi conjecture. In the second chapter, there is a description of the development of geometry, which will be familiar to most readers of this newsletter, from Pythagoras and Euclid to Descartes, Gauss and Riemann, ending with Einstein's recognition that "the mathematics of Riemann aligns with the physics of gravity". The great impact of Einstein's theory on Yau as a beginning graduate student at Berkeley then leads to an engaging autobiographical sketch describing a trajectory that was "as likely to have taken [Yau] to the poultry trade as to geometry".

The third chapter, entitled "A new kind of hammer", describes the emergence of geometric analysis in the 70s, in which Yau played a leading role. It includes an account of his proof with Schoen of the positive mass conjecture in general relativity. The authors then describe what Yau considers some of the most important achievements in geometric analysis to date: firstly, Donaldson's work on the Yang-Mills equations and its spectacular applications to four dimensional topology and then the work of Hamilton and Perelman on the Ricci flow leading to a proof of the Poincaré and geometrization conjectures. Yau's involvement with Hamilton's work is described but no mention is made of the controversy involving Yau during the efforts by the mathematical community to absorb

Perelman's papers on the proof of the geometrization conjecture. We are told that "it is widely acknowledged that the program pioneered by Hamilton and carried through by Perelman has solved the geometrization conjecture" and "if that consensus is correct" it represents a great triumph for mathematics.

The next chapter, entitled "Too good to be true", explains the statement of the relevant portion of the Calabi conjecture: a Kähler manifold with vanishing first Chern class admits a metric with vanishing Ricci curvature. The extent to which a book containing almost no formulas is able to provide some understanding of this statement is quite surprising. In just over 20 pages we go over complex numbers, Riemann surfaces then leading to complex geometry and Kähler manifolds where "space stays close to being complex Euclidean when you move away from a point", the explanation of which involves a discussion of Riemannian and Hermitian metrics. We then move on to parallel transport and the characterization of Kähler manifolds through holonomy, onwards to a discussion of the first Chern class through the Euler characteristic and singularities of vector fields on surfaces and, finally, Ricci curvature. Chapter 5 describes Yau's actual work on the proof (including his initial attempts to disprove the conjecture) and his eventual success.

The next five chapters, a bit over a third of the book, give a first-hand account of the impact of Yau's theorem on string theory and string theory's subsequent impact on the development of algebraic and differential geometry.

Chapter 6 describes the "first string revolution" and how Strominger, Candelas and Horowitz and independently Witten arrived in the mid 80s at the realisation that Calabi-Yau manifolds provided models for the extra dimensions of space in string theory allowing for (relatively) realistic physics. The next chapter, entitled "Through the looking glass", describes the emergence of mirror symmetry and its extraordinary impact on algebraic geometry and, in particular, enumerative geometry. It includes a discussion of the controversy regarding the proof of the mirror conjecture by Givental and Yau "and colleagues", ending with a detailed discussion of the Strominger-Yau-Zaslow conjecture, which attempts to give a geometric interpretation of mirror symmetry.

Chapter 8 tells the story of string theory's success in providing an explanation for the entropy of black holes and Chapter 9, entitled "Back to the real world", describes in more detail attempts at recreating the Standard Model of particle physics in string theory, highlighting the role played by the Donaldson-Uhlenbeck-Yau theorem. The importance of recent work by Tian, Yao, Luo and Donaldson in Kähler geometry, concerning the approximation of Kähler metrics by embeddings in projective space, for the possible prediction of masses of particles in string theory models, is also discussed. The following chapter, entitled "Beyond Calabi-Yau", describes the moduli problem that threatens "to topple this whole glorious enterprise" of describing the universe using the geometry of Calabi-Yau manifolds. To Yau it suggests the need to better understand these manifolds, which leads to a discussion of the current understanding of their topological classification as well as "Reid's fantasy" that the components of Calabi-Yau moduli space may all be connected by conifold transitions and to current work of Yau and collaborators on the Strominger system.

Chapters 11 and 12, arguably the weakest in the book, describe extremely speculative physical theories regarding the future of the universe and the search for experimental proof of string theory. Chapter 13, entitled "Truth, beauty and mathematics", gives an engaging discussion of the evolving relation between mathematics and physics. It gives familiar arguments in support of string theory, discussing the role of elegance in the search for truth in physical theories and the importance of mathematical consistency in checking the validity of a physical theory. Giving ample examples, the authors tell us that "while other physical theories have informed mathematics in the past, the influence of string theory has penetrated much deeper into the internal structure of mathematics".

The final chapter, entitled "The end of geometry?", argues that the fundamental incompatibility between quantum theory and general relativity suggests that "a new kind of geometry" will be needed to describe nature. This leads to a wonderful discussion of progress in mathematics, and geometry in particular, through increasing generalisation. It is suggested that a promising avenue of research is to look for situations in string theory where geometry behaves non-classically such as during conifold and flop transitions. It is said that "creating a theory of quantum geometry surely stands as one of the greatest challenges facing the field of geometry" and that this field will not be created without input from physics as, to be "truly fruitful, geometry must describe nature at some basic level".

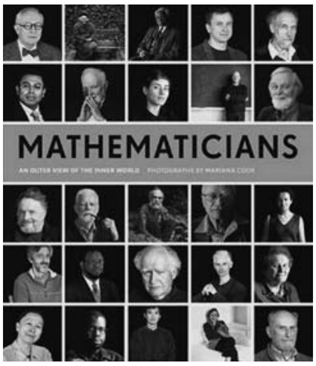
I hope the above quotes from the text hint at the lively and engaging style with which it is written. The book covers an enormous amount of material in its 300 pages, to which the summary above does not do justice. This is not light reading by any means, and it is fair to ask how much a layperson (or even a mathematician in an area far removed from geometry) will be able to grasp of this fantastic story. To be honest, I found myself rereading the sections on material I was not familiar with several times but that effort was rewarded with new understanding of the relation between string theory and geometry. I think an interested reader, even one with little background in mathematics, will be able to gather much new knowledge of, and appreciation for, both mathematics and physics from the elegant analogies and beautiful illustrations in this book.

No book is perfect and this one too has its misleading statements, confusing typos and puzzling phrasings. For instance, it is hard to agree with the explanation that one needs 10 rather than 16 coefficients for a Riemannian metric in four dimensions "because gravity is inherently symmetrical" (p.10) and the discussion of Yau's first paper generalising Preissman's Theorem in Riemannian geometry contains the following unreasonable statement: "I had to make use of some mathematics that had not previously been linked to topology or differential geometry:

group theory" (p. 41). But in the end, the book's virtues far outnumber its flaws. The book gives insight into the mind of one of the world's greatest mathematicians and will provide intellectual stimulation to interested readers with any kind of background. As Witten writes in the book's jacket, even specialists are likely to enjoy Yau's reminiscing on his education and work.

In the preface, Yau mentions that describing mathematics as merely a language does not do the subject justice, as it may suggest "the whole business has been pretty well sorted out". It is hard for me to conceive of a more effective tool than this book for dispelling such misconceptions.

Gustavo Granja [ggranja@math.ist.utl.pt] is Assistant Professor of Mathematics at Instituto Superior Técnico in Lisbon. He received his PhD from MIT in 1999 and works in Algebraic and Symplectic Topology.



Mariana Cook

Mathematics: an outer view of the inner world

208 pages

Princeton University Press, 2009, 208 p.
ISBN: 0691139512

Reviewer: Tomás Gómez

In this book by photographer Mariana Cook there are black and white portraits of 92 mathematicians. Each portrait is accompanied by a one-page autobiographical essay. Some of them describe how they chose a career in mathematics or why they find mathematics interesting. As can be expected, most of them had an early talent and interest for mathematics, and they had the good fortune of finding teachers who inspired and guided them to become mathematicians. But it is also interesting to find that some of them did not think about a career in mathematics to start with but only later at university, almost by chance, they found that this was what they wanted to do.

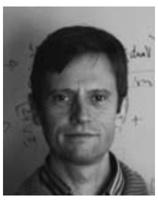
These portraits and texts give us a glimpse into the personal side of these mathematicians. Only some of them explain, very briefly, the kind of work that they do. This makes the book interesting to all: non-mathematicians will not get lost in technical descriptions and they will get to know something about how mathematicians are, what they do and why they find it interesting. For a mathematician, it is a pleasure to read this book, enjoying the beautifully constructed portraits and having a glimpse into a more personal side of the mathematicians whose work we study. In fact, sometimes the short essays left me wanting to know more.

As a curiosity, in the book we find two father-and-daughter pairs (Heisuke and Eriko Hironaka, and George and Kate Adebolo Okikiolu) as well as two examples of siblings (Charles and Robert Fefferman, and William, Felix and Andrew Browder).

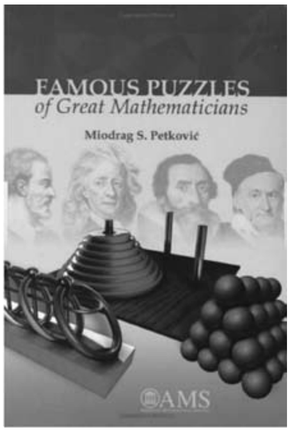
The author Mariana Cook, after doing a thesis in Yale on the American photographer Alfred Stieglitz, learnt the craft of photography from the renowned landscape photographer Ansel Adams, under whose direction she worked for about five years.

Some of her photographs are now in the collections of the Metropolitan Museum of Art and the Museum of Modern Art in New York, the Victoria and Albert Museum in London, and the Bibliothèque Nationale de France among others. She has published several books: "Fathers and daughters", "Mothers and Sons", "Generations of Women", "Couples", "Close at Hand" and "Faces of Sciences". After this last book (containing portraits of famous scientists) was published, it was suggested to her to do another project with mathematicians and the result is the book under review.

From a photographic point of view, some of the portraits are quite interesting in terms of composition or other elements that appear in the photograph besides the main subject. She has a very personal style but in many of the portraits one can see the influence of her mentor Ansel Adams, one of the founders of the f/64 group, with a characteristic sharp focus and attention to detail, as well as a rich rendering of all shades of gray. She only uses natural light, which can force the photographer to need long exposures in which the object has to remain still. As an amusing consequence of this, in the portrait of Kenneth Ribet we see a turtle in the foreground, which was moving and hence appears blurred in the photograph.



Tomás Gómez [tomas.gomez@icmat.es] obtained his PhD at Princeton in 1997, under the supervision of R. Friedman. After working in the Tata Institute of Fundamental Research (Mumbai, India) for four years, he returned Madrid, where he is currently a member of the Instituto de Ciencias Matemáticas (CSIC-UAM-UC3M-UCM). His research interest is in moduli spaces of bundles from the point of view of algebraic geometry.



Miodrag S. Petković

Famous puzzles of great mathematicians

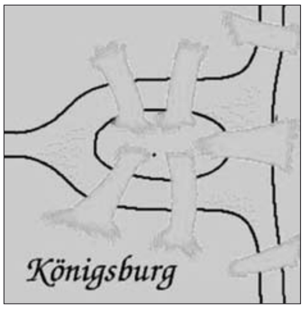
American Mathematical Society, 2009
ISBN 978-0-8218-4814-2

Reviewer: Adhemar Bultheel

There are already numerous books on mathematical puzzles and recreational mathematics. So what is so special about this one? M. S. Petković is a professor of mathematics at Niš (Serbia). Besides his work on interval arithmetic and polynomial root-finding, he has developed a great interest in recreational mathematics. His first book was called *Mathematics and Chess* and was published by Dover Publications in 1997. In the book under review, however, he collects a number of "classic" puzzles and connects them with their history. So what emerges is a collection of mostly well-known puzzles, their solutions and some historical facts about the mathematicians that "invented" them, or at least were involved in finding the solutions (but most well-known biographical facts are skipped; just the witty bits are given). Some variations on these puzzles are included as brain-teasers for the reader. The solutions of these are only found at the end of each chapter. After an introductory chapter placing recreational mathematics in a broader mathematical and historical perspective (there is a lot of name-dropping here for problems discussed in subsequent chapters), the book consists of eight chapters arranging the problems around mathematical disciplines (arithmetic, number theory, geometry, tiling and packing, physics, combinatorics, probability and graphs) and a last chapter on chess problems, which seem to attract the author's special interest. Some miscellaneous teaser-problems appear in the short chapter 11. The book ends with appendices giving some mathematical background and a list of short biographies of the important mathematicians featuring in the book.

River crossing problem. Illustration from *Introduction to the Design and Analysis of Algorithms* by Anany Levitin.

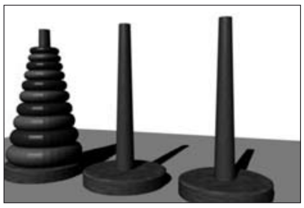
The puzzles are almost all classical, e.g. the river-crossing problem where a man has to take a goat, a cabbage and a wolf across the river. This is attributed to Alcuin of York (735–804) and can be solved by finding a shortest path in a digraph. The proof by Leonhard Euler in 1736 that there was no solution to the problem of the Königsberg bridges was in fact the start of graph theory. This is one of the statements that Petković makes: what may have started as recreational mathematics, has turned later sometimes into a fully fledged mathematical discipline, so that they are more than just a gratuitous time-passing.



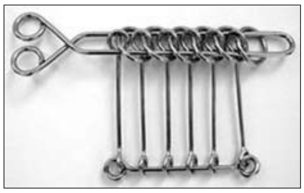
Bridges of Königsberg

It will come as no surprise that probability found its origin in problems that were posed by gamblers. Antoine Gombaud, chevalier de Méré, who was a notorious gambler, asked his friend Blaise Pascal (1601–1665) to design a good strategy for his gambling. Pascal discussed the problem with Pierre de Fermat (1601–1665) and this became the embryo of probability.

Combinatorics is of course a rewarding subject to design puzzles. So this chapter is well stuffed. You may learn here that the formula for the combinatorial numbers $C^n_k = K!/(k!(n–k)!)$ is probably from the Indian mathematician *Mahāvira* (ca. 800–ca. 870). A very well-known puzzle from this chapter is the "Tower of Hanoi" problem: take the pile of rings from one stack to another while at no point placing a larger ring on top of a smaller one and using only one auxiliary stack. This seems to originate from Edouard Lucas (1842–1891), who formulated it in 1883 under the alias M. Claus (an anagram of his name). Also, the Chinese rings puzzle is a familiar toy that is sold even today. The solution is similar to the solution of the "Tower of Hanoi". Its origin is less certain. It could be that it was used by French countrymen as a lock for chests or it could have been invented by a Chinese general Hung Ming (181–234) to amuse his wife while he was at war. In any case, Geroldamo Cardano (1501–1576) seems to be the first to describe it in 1550.



Tower of Hanoi

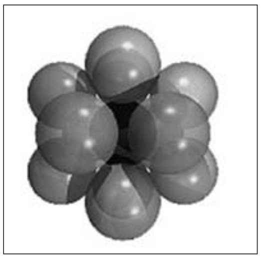


Chinese ring puzzle

The physics chapter has many problems about motions, which often result in summing infinite series. Everyone is probably familiar with Zeno's paradox on the race of Achilles and the tortoise. There is also the problem of a girl and a pigeon both starting in the direction of a wall that is 500 feet away. The girl walks at 5 feet per second and the pigeon flies back and forth between the girl and the wall at 15 feet per second. The question is: what distance does the pigeon travel before the girl reaches the wall?

The hard way is to compute the subsequent distances between the girl and the wall when the pigeon reaches the girl and sum all these distances. However, the much simpler solution is to observe that the girl reaches the wall after 100 seconds and that the pigeon has travelled 100 times 15 feet in that time. The legend says that John von Neumann (1903–1957) immediately came up with an answer and his teacher said, “Ah, you noticed the trick,” whereupon von Neumann answered, “What trick? It was an easy series”.

Problems related to tiling and packing have also been a recurrent subject of recreational mathematics. The drawings of M. C. Escher are well-known but the names of mathematicians such as Roger Penrose (b. 1931), who discovered nonperiodic tilings of the plane in 1974, and John Horton Conway (b. 1937) are immediately linked to this field. Conway was also the inventor of “the game of life” which gave rise to the study of cellular automata. Donald Knuth (b. 1938) also participated in this and, besides being the designer of TEX and metafont and the author of the four volumes of *The art of computer programming*, he also invented many problems with polyominoes (any figure of n edge-connected unit squares). But some tiling and packing problems are much older. The problem of “kissing spheres” came about as the result of a discussion between David Gregory (1659–1708) and Isaac Newton (1643–1727) about how many unit spheres can simultaneously touch a given unit sphere. According to Newton there were 12 and according to Gregory there were 13, but neither of them had a proof. That proof was only given by K. Schüttle and B. L. van der Waerden in 1953: there are at most 12.



Kissing spheres

There are numerous puzzles that relate to geometric problems. For example, the problem of finding the diameter of a sphere using only a ruler and compasses is ascribed to Tābit ibn Corra (826–901). There is also the so-called Dido problem: among all closed curves of a given length, which is the one enclosing the largest region. This naming refers to the Princess Dido, who negotiated to buy the land that could be enclosed by a bull’s skin. She cut it into narrow strips and acquired the land on which Carthage was later founded.

From number theory, there is the stamp problem of James Joseph Sylvester (1814–1897) and a similar problem formulated with coins by Ferdinad Georg Frobenius (1849–1917): what is the largest amount that cannot be made up with an unlimited number of stamps of 5 and 17 cents? The answer is 63. Of course there are many generalisations of this problem.

A typical arithmetic problem as posed by Isaac Newton reads: in four weeks, 12 oxen will consume 10/3 acres of pasture. In nine weeks 21 oxen will consume 10 acres of pasture. How many oxen will it take to consume 24 acres in 18 weeks? In his biographical note Petković says that Newton was from Jewish origin, which is remarkable since all sources I consulted seem to deny this. A slip of the pen perhaps.

The most typical chess problems are the “knight’s circles” (a knight has to visit all squares exactly once), which has attracted the attention of many mathematicians, and the “eight queens” problem (how many ways can eight queens be placed on the chess board so they do not attack one another).

There is a lot of trivia, with many funny quotes and one-liners by historical figures. I have only shown here a few sips from a plentiful bowl of facts. Thus, even for puzzle fanatics, I think there is a lot of material in this carefully compiled book that will be new to them. The name index at the end shows the long list of people that appear in the text; it would also have been interesting to have a subject index so that one could look up the history of some puzzles.

Adhemar Bultheel [Adhemar.Bultheel@cs.kuleuven.be] is an emeritus professor at the Computer Science Department of the K. U. Leuven, Belgium. He got a PhD in mathematics in 1979. His professional interest is in numerical analysis, rational approximation, orthogonal functions and structured linear algebra problems. He has been Vice-President (1999–2002) and President (2002–2005) of the Belgian Mathematical Society.

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Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

God created the natural numbers. The rest is the work of man.
Leopold Kronecker (1823–1891)

I Six new problems – solutions solicited

Solutions will appear in a subsequent issue.

75. Prove that for any integer $k \geq 1$ the equation

$$x_1^2 + x_2^2 + \cdots + x_{2k+1}^2 + 1 = x_{2k+2}^2$$

has infinitely many solutions in positive integers.

(Dorin Andrica, "Babes-Bolyai" University, Cluj-Napoca, Romania)

76. Solve the equation

$$\lfloor 3x - 2 \rfloor - \lfloor 2x - 1 \rfloor = 2x - 6\,,\ x \in \mathbb{R}\,.$$

(Elias Karakitsos, Sparta, Greece)

77. Find all positive integers n with the following property: there are two divisors a and b of the number n such that $a^2 + b^2 + 1$ is a multiple of n.

(Vladimir Protasov, Department of Mechanics and Mathematics, Moscow State University, Russia)

78. Let n be a nonnegative integer. Find the closed form of the sums

$$S_1(n) = \sum_{k=0}^{n} \left\lfloor \frac{k^2}{12} \right\rfloor \quad \text{and} \quad S_2(n) = \sum_{k=0}^{n} \left[\frac{k^2}{12} \right],$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x and $[x]$ denotes the nearest integer to x, i.e. $[x] = \left\lfloor x + \frac{1}{2} \right\rfloor$.

(Mircea Merca, "Constantin Istrati" Technical College, Câmpina, Romania)

79. Let $m, s \geq 2$ be even integers. Compute

$$\prod_{\substack{k = 1 \\ k \not\equiv 0 (mod\ m)}}^{sm-1} \cos \frac{k\pi}{sm}.$$

(Dorin Andrica, "Babes-Bolyai" University, Cluj-Napoca, Romania)

80. Let the Riemann zeta function $\zeta(s)$ and the Hurwitz (or generalized) zeta function $\zeta(s, a)$ be defined (for $\Re(s) > 1$) by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad \text{and}$$

$$\zeta(s, a) = \sum_{n=0}^{\infty} \frac{1}{(n + a)^s} \qquad (a \neq 0, -1, -2, \cdots), \qquad (1)$$

respectively, and (for $\Re(s) \leq 1;\ s \neq 1$) by their meromorphic continuations.
In the usual notation, let $B_n(x)$ and $E_n(x)$ denote, respectively, the classical Bernoulli and Euler polynomials of degree n in x, defined by the following generating functions:

$$\frac{te^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \frac{t^n}{n!} \qquad (|t| < 2\pi)$$

and

$$\frac{2e^{xt}}{e^t + 1} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!} \qquad (|t| < \pi).$$

It is proposed to show that the values of the Bernoulli polynomials $B_n(\mathrm{x})$ at rational arguments are given, in terms of the Hurwitz zeta function $\zeta(s, a)$, by

$$B_{2n-1}\left(\frac{p}{q}\right) = (-1)^n \frac{2(2n-1)!}{(2\pi q)^{2n-1}} \sum_{j=1}^{q} \zeta\left(2n - 1, \frac{j}{q}\right) \sin\left(\frac{2j\pi p}{q}\right)$$

$$(n \in \mathbb{N} \setminus \{1\}; \mathbb{N} := \{1, 2, 3, \cdots\};\ p \in \mathbb{N}_0 := \mathbb{N} \cup \{0\};\ q \in \mathbb{N};\ 0 \leq p \leq q)$$

and

$$B_{2n}\left(\frac{p}{q}\right) = (-1)^{n-1} \frac{2(2n)!}{(2\pi q)^{2n}} \sum_{j=1}^{q} \zeta\left(2n, \frac{j}{q}\right) \cos\left(\frac{2j\pi p}{q}\right)$$

$$(n \in \mathbb{N};\ p \in \mathbb{N}_0;\ q \in \mathbb{N};\ 0 \leq p \leq q)\,.$$

Similarly, the values of the Euler polynomials $E_n(\mathrm{x})$ at rational arguments are given, in terms of the Hurwitz zeta function $\zeta(s, a)$, by

$$E_{2n-1}\left(\frac{p}{q}\right) = (-1)^n \frac{4(2n-1)!}{(2\pi q)^{2n}} \sum_{j=1}^{q} \zeta\left(2n, \frac{2j-1}{2q}\right) \cos\left(\frac{(2j-1)\pi p}{q}\right)$$

$$(n \in \mathbb{N};\ p \in \mathbb{N}_0;\ q \in \mathbb{N};\ 0 \leq p \leq q)$$

and

$$E_{2n}\left(\frac{p}{q}\right) = (-1)^n \frac{4(2n)!}{(2\pi q)^{2n+1}} \sum_{j=1}^{q} \zeta\left(2n + 1, \frac{2j-1}{2q}\right) \sin\left(\frac{(2j-1)\pi p}{q}\right)$$

$$(n \in \mathbb{N};\ p \in \mathbb{N}_0;\ q \in \mathbb{N};\ 0 \leq p \leq q)\,.$$

(Djurdje Cvijović, Atomic Physics Laboratory, Belgrade, Republic of Serbia, and H. M. Srivastava, Department of Mathematics and Statistics, University of Victoria, Canada)

II Two new open problems

Proposed by Djurdje Cvijović (Atomic Physics Laboratory, Belgrade, Republic of Serbia) and H. M. Srivastava (Department of Mathematics and Statistics, University of Victoria, Canada)

81*. Let the Riemann zeta function $\zeta(s)$ and the Hurwitz (or generalized) zeta function $\zeta(s,a)$ be defined (for $\Re(s) > 1$) by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad \text{and}$$

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s} \qquad (a \neq 0, -1, -2, \cdots), \quad (1)$$

respectively, and (for $\Re(s) \leq 1;\ s \neq 1$) by their meromorphic continuations (see, for details, any standard book on the subject including [3], [5] and [8]).

Determinants Associated with the Continued-Fraction Expansion of the Riemann Zeta Function. Suppose that r is a non-negative integer and m and n are positive integers. For any fixed r, m and n, we define $A_m^{(r)}(n)$ as the determinant of an $m \times m$ square matrix in the form:

$$A_m^{(r)}(n) = \det \left\| \frac{(-1)^{i+j+r}}{(r+i+j-1)^n} \right\|_{1 \leq i,j \leq m}. \quad (2)$$

Cvijović and Klinowski [2, Eqs. (5a), (5b) and (5c)] showed that the kth partial numerators a_k $(k \in \mathbb{N})$ of the following continued-fraction expansion of the Riemann zeta function $\zeta(n)$ $(n \in \mathbb{N} \setminus \{1\})$:

$$\zeta(n) = \frac{1}{1-2^{1-n}} \cdot \frac{a_1}{1+}\,\frac{a_2}{1+}\,\frac{a_3}{1+}\,\frac{a_4}{1+} \cdots = \frac{1}{1-2^{1-n}} \cdot \cfrac{a_1}{1+\cfrac{a_2}{1+\cfrac{a_3}{1+\cfrac{a_4}{1+\cdots}}}} \quad (3)$$

$(n \in \mathbb{N} \setminus \{1\})$ are given in terms of $A_m^{(0)}(n)$ and $A_m^{(1)}(n)$.
It is well-known that $A_m^{(r)}(1)$ (that is, the determinant of the so-called *generalized Hilbert matrix* [4]) can be evaluated in closed form as follows ([6, pp. 98–99 and 300] and [4]):

$$\det \left\| \frac{(-1)^{i+j+r}}{r+i+j-1} \right\|_{1 \leq i,j \leq m} = \prod_{k=0}^{m-1} \frac{(k!)^2}{(k+r+1)_m}, \quad (4)$$

where $(\lambda)_n$ is the Pochhammer symbol given by

$$(\lambda)_0 := 1 \quad \text{and} \quad (\lambda)_n := \lambda(\lambda+1)\cdots(\lambda+n-1) \qquad (n \in \mathbb{N}).$$

The obvious *open problem* is whether it is possible to evaluate $A_m^{(r)}(n)$ or, at least, to evaluate $A_m^{(0)}(n)$ and $A_m^{(1)}(n)$, in closed form, for $n \in \mathbb{N} \setminus \{1\}$. This could make it possible to explicitly determine the partial numerators a_k in (2) and thus obtain the infinite continued-fraction expansion of the Riemann zeta function $\zeta(n)$ $(n \in \mathbb{N} \setminus \{1\})$.

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Proposed by Vladimir Protasov (Department of Mechanics and Mathematics, Moscow State University, Russia)

82*. (I) Under the assumptions of Problem 67 (*Problem Corner*, Newsletter 77 (2010), page 60), find all natural numbers k for which the equation $f(x) = k\,x$ has a solution $x \in \mathbb{N}$.
(II) (*Generalized binary partition function*). Let $\mathbb{Z}_+$ be the set of nonnegative integers. For a given finite subset $D \subset \mathbb{Z}_+$, we define the generalized binary partition function $b(k) = b(D,k) : \mathbb{Z}_+ \to \mathbb{Z}_+$ as the total number of different binary expansions

$$k = \sum_{j=0}^{\infty} d_j 2^j,$$

where the "digits" d_j take values from D.

(a) For which D do we have $b(k) > 0$, whenever k is large enough? Find sufficient or necessary conditions for the set D.
(b) For a given D characterize the asymptotic growth of $b(k)$ as $k \to \infty$.

Comments. Several special cases of problem (b) are well-known. For $D = \{0, 1\}$, obviously,

$$b(k) \equiv 1\,.$$

For $D = \mathbb{Z}_+$, we obtain the Euler binary partition function, whose asymptotic was studied by L. Euler, K. Mahler, N. G. de Bruijn, D. E. Knuth, etc. The case $D = \{0, \ldots, d-1\}$, $d \geq 3$, was studied by B. Reznick [1] and V. Protasov [2,3]. There are certain relations of this problem to the theory of refinement equations and subdivision algorithms [4–5].

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III Solutions

67. A function $f : \mathbb{N} \to \mathbb{N}$ is defined as follows: writing a number $x \in \mathbb{N}$ in its decimal expansion and replacing each digit by its square we obtain the decimal expansion of the number $f(x)$. For example, $f(2) = 4$, $f(35) = 925$, $f(708) = 49064$.
(a) Solve the equation $f(x) = 29x$;
(b) Solve the equation $f(x) = 2011x$.
(Vladimir Protasov, Moscow State University, Russia)

Solution by the proposer. Answer. (a) $x = 325 \cdot 10^m$, $m \geq 0$. (b) No solutions.

(a). Let $x = d_1 \dots d_n$ be the decimal expansion of x. We shall find successively all the digits from d_n to d_1. If $d_n = 0$ then we divide x by 10, and the number $f(x)$ will be divided by 10, so the relation $f(x) = 29x$ remains true.

Therefore, it suffices to consider the case $d_n \neq 0$, and then multiply all the solutions by $10^m, m \geq 0$. Thus, $d_n \neq 0$. Since the last digit of $f(x)$ is the last digit of $29x$, we obtain $d_n^2 \equiv 9d_n \pmod{10}$. Therefore $d_n = 4$, 5 or 9. Consider each case.

$\mathbf{d_n = 4.}$ Since $4^2 = 16$ and $4 \times 29 = 116$, we obtain the following equation on the digit d_{n-1} : $1 + 9d_{n-1} \equiv 1 \pmod{10}$. This equation has a unique solution on the set of digits: $d_{n-1} = 0$. Then we obtain the equation on d_{n-2}: $1+9d_{n-1} \equiv 0 \pmod{10}$, whose unique solution is $d_{n-2} = 1$. At every step we get the next digit in a unique way: $d_{n-3} = 2, d_{n-4} = 2, d_{n-5} = 2, \dots$. We see that the sequence cycles, so this process will never terminate, which means that there is no solution.

$\mathbf{d_n = 5}$. Computing the digits successively, we get $d_{n-1} = 2, d_{n-2} = 3$, and the process terminates. Thus, $x = 325$ is the only solution in this case.

$\mathbf{d_n = 9}$. Computing successively the last four digits: $x = \dots 9189$, we see that three of these digits have two-digit squares, therefore the number $f(x)$ has, at least, three digits more than x, and hence $f(x) > 100x$. Thus, there is no solution in this case.

(b). Arguing as in part (a) we show that there is no solution.

Also solved by S. E. Louridas (Athens, Greece)

68. Solve the equation

$$\lfloor x^2 - 3x + 2 \rfloor = 3x - 7 , x \in \mathbb{R} .$$

(Elias Karakitsos, Sparta, Greece)

Solution by the proposer. Set $\lfloor x^2 - 3x + 2 \rfloor = a$, where $a \in \mathbb{Z}$. Then, it follows that there exists ϑ, with $0 \leq \vartheta < 1$, such that

$$x^2 - 3x + 2 - \vartheta = a .$$

Thus

$$\vartheta = x^2 - 3x + 2 - a . \tag{1}$$

Furthermore

$$3x - 7 = a$$

or

$$x = \frac{a+7}{3} . \tag{2}$$

By (1) and (2), it follows that

$$\vartheta = \left(\frac{a+7}{3}\right)^2 - 3\left(\frac{a+7}{3}\right) + 2 - a$$

or

$$\vartheta = \frac{a^2 - 4a + 4}{9} .$$

However, since $0 \leq \vartheta < 1$, we get

$$0 \leq \frac{a^2 - 4a + 4}{9} < 1 .$$

Thus, it is evident that

$$0 \leq a^2 - 4a + 4$$

and

$$a^2 - 4a - 5 < 0 .$$

Therefore, in order to determine the integer values of a, it suffices to solve the following system:

$$a^2 - 4a + 4 \geq 0,$$
$$a^2 - 4a - 5 < 0.$$

However, $a^2 - 4a + 4 \geq 0$ always holds true, since $(a - 2)^2 \geq 0$, for every real value of a. Furthermore, by $a^2 - 4a - 5 < 0$ it follows that

$$-1 < a < 5 ,$$

and hence $a = 0$ or 1 or 2 or 3 or 4. Hence, the solutions of the equation can be derived by calculating the value of $x = (a + 7)/3$ for $a = 0, 1, 2, 3, 4$.

– For $a = 0$, we have $x = \frac{0+7}{3} = \frac{7}{3}$.
– For $a = 1$, we have $x = \frac{1+7}{3} = \frac{8}{3}$.
– For $a = 2$, we have $x = \frac{2+7}{3} = \frac{9}{3} = 3$.
– For $a = 3$, we have $x = \frac{3+7}{3} = \frac{10}{3}$.
– For $a = 4$, we have $x = \frac{4+7}{3} = \frac{11}{3}$.

Therefore, the real solutions of the equation are the numbers

$$\frac{7}{3}, \frac{8}{3}, 3, \frac{10}{3} \text{ and } \frac{11}{3}.$$

□

Also solved by Gerald A. Heuer (Concordia College, USA), Reinhold Kainhofer (Vienna University of Technology, Austria), Martin Lukarevski (Leibniz Univerisität, Hannover, Germany).

69. What is the largest positive integer m with the property that, for any positive integer n, m divides $n^{241} - n$? What is the new value of m if n is restricted to be odd?
(Konstantinos Drakakis, University College Dublin, Ireland)

Solution by the proposer. (We will use the notation $m|n$ and $m \nmid n$ to denote the fact that an integer m does or does not divide the integer n, respectively, and the notation $m^k \| n$ to denote the fact that $m^k|n$ but $m^{k+1} \nmid n$.)

Observe that $n^{241} - n = n(n^{240} - 1)$ and that $240 = 2^4 \cdot 3 \cdot 5$. Recall now that Fermat's Little Theorem guarantees that $p|n^s - n$ for all n, where p is a prime, as long as $p - 1|s - 1$, and that, assuming this condition holds, in general $p^2 \nmid n^s - n$ (although this may be true for some n it is certainly not true for all n: a counterexample is $n = p$ for $s \geq 2$). Testing exhaustively all integers of the form $2^i \cdot 3^j \cdot 5^k + 1$ for primality, where $0 \leq i \leq 4$, $0 \leq j, k \leq 1$, the following list of primes is obtained: 2, 3, 5, 7, 11, 13, 17, 31, 41, 61, 241.

It follows that m is the least common multiple of the list of primes p such that $p - 1|240$ found above, and hence simply their product:

$$m = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 31 \cdot 41 \cdot 61 \cdot 241$$
$$= 9,538,864,545,210.$$

Now, let n be odd. Using repeatedly the identities $x^2 - 1 = (x - 1)(x + 1)$ and $x^{2k+1} + 1 = (x + 1) \sum_{i=0}^{2k} (-x)^i$, $n^{241} - n$ is eventually factorized into irreducible polynomials of the form $\sum_{i=0}^{l} a_i n^i$ where $a_i \in \{0, 1, -1\}$. Clearly, any such polynomial with an odd number of nonzero coefficients will take an odd value for odd n (to see this, just use $n \equiv 1 \bmod 2$ and $-1 \equiv 1 \bmod 2$ to reduce such a polynomial into the sum of an odd number of 1s, which is obviously equivalent to 1 modulo 2), hence we may write

$$n^{241} - n = (n-1)(n+1)(n^2+1)(n^4+1)(n^8+1)F(n),$$

where $F(n)$ is some polynomial taking odd values for odd n. Based on the two facts that (a) $n \equiv 1 \bmod 2 \Rightarrow n^2 \equiv 1 \bmod 4$ and (b) $n \equiv 1 \bmod 2 \Rightarrow 8 \| n^2 - 1$, which are easy to verify, it follows that $(n-1)(n+1)$ has three factors of 2 and the remaining three binomials one factor of 2 each, hence $2^6 = 64 \| n^{241} - n$, whence the new value of m is

$$\begin{aligned}\operatorname{lcm}(64; 9,538,864,545,210) &= 9,538,864,545,210 \cdot 32 \\ &= 305,243,665,446,720.\end{aligned}$$

□

Also solved by John N. Lillington (Wareham, UK)

70. Find the minimum of the product xyz over all triples of positive integers x, y, z for which 2010 divides $x^2+y^2+z^2-xy-yz-zx$.

(Titu Andreescu, The University of Texas at Dallas, USA)

Solution by the proposer. Without loss of generality assume that $x > y > z$ and write

$$x^2+y^2+z^2-xy-yz-zx = \frac{1}{2}[(x-y)^2+(y-z)^2+(z-x)^2].$$

Let $x-y=a$ and $y-z=b$. Then

$$x^2+y^2+z^2-xy-yz-zx = \frac{1}{2}[a^2+b^2+(a+b)^2] = a^2+ab+b^2$$

and the condition in the problem becomes $2010 \mid a^2+ab+b^2$. It is clear that if a prime $p \equiv 2 \pmod 3$ divides a^2+ab+b^2 then p divides a and p divides b. Hence 2 and 5 divide both a and b and so $a = 10u$ and $b = 10v$ for some positive integers u and v. It follows that $u^2+uv+v^2 = 201k$ for some positive integer k. Because we are seeking the minimum of $xyz = (z+10u+10v)(z+10v)z$, we must have $z = 1$ and $v \le u$. For $k = 1$, the only pair (u, v) of positive integers with $v \le u$ satisfying this equation is $(11, 5)$. We obtain $xyz = 161 \cdot 51$.

If $k > 1$ and $v \ge 5$ then each solution (u, v) will have $u > 11$, implying

$$xyz > 161 \cdot 51.$$

Thus we need to check what happens for $v \le 4$.

If $2 \mid k$ then $2 \mid u$ and $2 \mid v$ and the equation $u^2+uv+v^2 = 201k$ is either not solvable or it reduces to the equation $U^2+UV+V^2 = 201 \cdot \frac{k}{4}$, where $u = 2U$ and $v = 2V$. In the latter situation we obtain the solution $(2 \cdot 11, 2 \cdot 5)$, making xyz greater than $161 \cdot 51$. The same argument works for 5 and all other primes of the form $3k+2$: $11, 17, 23, 29, \ldots$

If $3 \mid k$ then, since $(u-v)^2 + 3uv = 9 \cdot 67\frac{k}{3}$, 3 divides $u - v$, 9 divides $(u-v)^2$ and so 9 divides $3uv$. It follows that 3 divides u or v and, because 3 also divides $u - v$, $u = 3s$ and $v = 3t$ for some positive integers s and t. We obtain the equation $s^2+st+t^2 = 67$, whose only solution (s, t) with $s \ge t$ is $(7, 2)$. Hence $(u, v) = (21, 6)$ and $xyz = 271 \cdot 61 > 161 \cdot 51$.

For $k \le 30$, the only values we have to worry about are $k = 7, 13, 19$. For $k > 30$, $201k \ge 6231$. If $v = 1$, $u(u+1) \ge 6230$, implying $u \ge 79$ and $xyz \ge 801 \cdot 11 > 161 \cdot 51$. Because none of the equations $u(u+1) = 201 \cdot 7 - 1$, $u(u+1) = 201 \cdot 13 - 1$, $u(u+1) = 201 \cdot 19 - 1$ have an integer solution, we are done in the case $v = 1$. If $v = 2$ and $k \le 12$, we only need to check that the equation $u(u+1) = 201 \cdot 7 - 4$ has no solution in positive integers, because $k > 12$ implies $201k - 4 \ge 2609$, so $u(u+1) \ge 2609$ and so, again, $u \ge 79$, implying $xyz \ge 801 \cdot 21 > 161 \cdot 51$. Cases $v = 3$ and $v = 4$ are now easy.

In conclusion, the minimum is $161 \cdot 51$.

□

71. Prove that for any positive integer k with $k \ge 1$ the equation

$$x_1^3 + x_2^3 + \cdots + x_k^3 + x_{k+1}^2 = x_{k+2}^4$$

has infinitely many solutions in positive integers such that

$$x_1 < x_2 < \cdots < x_{k+1}.$$

(Dorin Andrica, "Babes-Bolyai" University, Cluj-Napoca, Romania

Solution by the proposer. For any positive integer n we have the well-known identity:

$$\begin{aligned}1^3 + 2^3 + \cdots + n^3 + (n+1)^3 + \cdots + (n+k)^3 \\ = \left(\frac{(n+k)(n+k+1)}{2}\right)^2,\end{aligned}$$

that is

$$\left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3 + \cdots + (n+k)^3 = \left(\frac{(n+k)(n+k+1)}{2}\right)^2.$$

Consider the positive integers n such that the triangular number

$$t_{n+k} = \frac{(n+k)(n+k+1)}{2}$$

is a perfect square. There are infinitely many such integers since the relation $t_{n+k} = u^2$ is equivalent to Pell's equation $(2n+2k+1)^2 - 2w^2 = 1$, where $w = 2u$. The fundamental solution to Pell's equation is $(3, 2)$, i.e., $2n+2k+1 = 3$ and $w = 2$. Hence all these integers are given by the sequence (n_s), where

$$2n_s + 2k + 1 + w_s\sqrt{2} = (3 + 2\sqrt{2})^s$$

for s big enough such that $n_s \ge 1$.

We can take

$$x_1 = n_s + 1, \cdots, x_k = n_s + k, \quad x_{k+1} = \frac{n_s(n_s+1)}{2}, \quad x_{k+2} = w_s.$$

It is clear that for s big enough we have $n_s \ge 1$ and $\frac{n(n+1)}{2} > n + k$, hence we get an infinite family of solutions. □

Also solved by Knut Dale (Telemark University College, Norway).

72. Let $p \ge 3$ be a prime number. For $j = 1, 2, \cdots, p-1$, divide the integer $(j^{p-1} - 1)/p$ by p and get the remainder r_j. Prove that

$$r_1 + 2r_2 + \cdots + (p-1)r_{p-1} \equiv \frac{p+1}{2} (mod\ p).$$

(Dorin Andrica, "Babes-Bolyai" University, Cluj-Napoca, Romania

Solution by the proposer. For $j = 1, 2, \cdots, p-1$, we have

$$\frac{j^{p-1} - 1}{p} = a_j p + r_j$$

for some integer a_j. It follows that

$$\frac{j^p - j}{p} = j a_j p + j r_j,$$

hence

$$\frac{j^p - j + (p-j)^p - (p-j)}{p} = ja_j p + jr_j + (p-j)a_{p-j}p + (p-j)r_{p-j}.$$

We obtain

$$\frac{j^p + (p-j)^p}{p} = ja_j p + jr_j + (p-j)a_{p-j}p + (p-j)r_{p-j} + 1.$$

Because

$$j^p + (p-j)^p = \binom{p}{0}p^p - \binom{p}{1}p^{p-1}j + \cdots + \binom{p}{p-1}pj^{p-1},$$

we obtain that $p^2 \mid j^p + (p-j)^p$ and we get for all $j = 1, 2, \cdots, p-1$,

$$jr_j + (p-j)r_{p-j} + 1 \equiv (mod\ p).$$

Adding up all these relations it follows that

$$2(r_1 + 2r_2 + \cdots + (p-1)r_{p-1}) \equiv -(p-1)(mod\ p),$$

hence

$$r_1 + 2r_2 + \cdots + (p-1)r_{p-1} \equiv \frac{p+1}{2}(mod\ p).$$

□

Remark: Problem 60 was also solved by W. Fensch (Karlsruhe, Germany).

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR-15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will be devoted to *Real Analysis*.

Letters to the Editor

This section is open to the opinions of readers of the Newsletter. For sending a letter for publication in this section, please contact any member of the Editorial Committee.

Opinions expressed in the section *Letters to the Editor* are those of the authors and do not necessarily reflect the policies and views of the EMS. Letters from readers appearing in the Newsletter are published without change.

An American visa for Jacques Hadamard, ICM 1950 and Henri Cartan

I was quite surprised when I read, in the *Newsletter* of December 2008, in the obituary of Henri Cartan written by Jean-Pierre Bourguignon [1]:

> *The visa application Laurent Schwartz had made to attend the ICM where he was to receive the Fields Medal had been set aside by the U.S. Embassy in Paris. In order to exert maximum pressure, Henri Cartan collected the passports of all the French ICM participants and threatened that there would be no French participation if Schwartz was not allowed to enter the United States.*

I considered the idea of writing to Jean-Pierre to mention that this was erroneous, but it seemed to me that it was so much a well-known history that somebody else would have already told him and that a correction would appear. Then the very same text appeared in the September 2010 of the *Notices of the AMS*, with the same error(s). I thus decided to send a mail to Jean-Pierre, in which I wrote:

> *I was very surprised to read the paper "Cartan, Europe and Human rights" in the September issue of the* Notices. *In the second column, the information on ICM 1950 seems to be erroneous.*
> *It is true that Cartan (who was the president of the SMF this year) collected (not the passports but) the boat tickets of (not all but several of) the French participants. But this was not because Laurent Schwartz could not get his visa (there had been a problem with this visa, but this was solved), but rather because Jacques Hadamard, who was 85, a vice-president of the Congress, and had spent the wartime in the States, was now considered a dangerous communist and could not get a visa.*

Jean-Pierre thanked me for pointing out the error and told me that he had based the story on something Henri Cartan (not very young) told him. I then wrote a letter to the editors of the *Notices* – after all, this is more about American history than about France, or Europe…

Well… So what am I doing here? The aim of this short paper is neither to discuss the fact that "B tells A" is not the same as "A", nor to insist on the fact that "C thinks to remember that B told him A" is also a different issue. This is rather to understand how it was possible that nobody noticed there was a mistake.

Let us go back to the preparation of the ICM in Cambridge in (1949–)1950. The previous International Congress was held in Oslo in 1936 and it was decided there that the next one, in 1940, would take place in the US. Of course there was no International Congress in 1940. Even in 1950, it was quite a challenge to organise a truly international event. Many questions had to be solved. It was not very long ago, only thirty years, that the French had organised an "International Congress" without any German mathematician (in Strasbourg and in 1920) and the disastrous boycott of German mathematicians after the first world war had not been forgotten.

Not very surprisingly, except for a few Yugoslavian mathematicians, no East-European or Soviet mathematicians participated in the Cambridge Congress.

But one of the main problems the American Mathematical Society, in charge of the organisation, had to face was a purely American one, namely the McCarthyism and Witch-Hunt in the States. It is not the place here to discuss the many (awful) effects this policy had on life in America. The point here is that it had some consequences on the organisation of the Congress. Let us come back to the mathematicians mentioned at the beginning of this paper.

Laurent Schwartz. He was 35 and was to be awarded a Fields Medal for having created the theory of distributions. Hence he was a young man, had never travelled to the States, and he was a Trotskyist. Hence he was dangerous. It was not that easy, but he succeeded getting an American visa, a few months before the Congress, after the personal intervention of President Truman himself (according to Schwartz in his Mémoires [4]).

But the real problem was not Laurent Schwartz's visa, but Jacques Hadamard's.

Jacques Hadamard. He was 85 and was invited to be one of the honorary presidents of the Congress. He was a very well-known mathematician. Moreover, he was especially well-known in the States, for example because he had spent some time there during the German Occupation of France (he was threatened by the French anti-Semitic legislation and had to leave France). It seems that he did not take advantage of his presence on the American soil to develop anti-American activities during this period, he did not try to murder the President, but one never knows… the State Department decided not to give him a visa. The reason was that this nice old man[1] was a dangerous communist – he was not a member of the Communist Party but had indeed some sympathy for it, and his daughter Jacqueline was a member.

Unbelievable. Well, this is the point where we should start the discussion mentioned above – about "C believes that B told A". The point is that the idea that old Jacques Hadamard could have been considered dangerous to US security is simply unbelievable.

However, this is the truth. And this is a reason why we should care for history. As unbelievable as these things seem to be, they happened and they can happen again.

Henri Cartan and the visa problem. Now you probably want to know what happened and how it happened. Henri Cartan, who was 46 and was one of the invited plenary speakers of the Congress, was also the President of the Société Mathématique de France (SMF) this year. He was also a lot of other things, but we shall concentrate on that. Under his impulsion, the SMF proposed to threaten to boycott the Congress. This was not very well accepted by the AMS and was not accepted by all the French participants either. But most of the French followed Cartan and the SMF, and the AMS did its best to help. The French people were supposed to all take the same liner of the Cunard Line.

The ultimatum Cartan gave the Americans was to expire on 30 July; this was the extreme limit to confirm the tickets or not. It was only very late on 26 July that the State Department eventually decided to give the visa, so that Cartan was informed by a telegram which arrived on the 27 July. He was on vacation in his family house in Die, in the South of France. He went to the Post Office and sent telegrams, first to the Cunard Line, then to J. R. Kline (who was the Secretary of the Congress and was very active in the US trying to obtain the visa), then to Jacques Hadamard and the twelve other French mathematicians who were waiting to know whether they would go or not.

And then he went back to mathematics and started preparing his talk *Problèmes globaux dans la théorie des fonctions analytiques de plusieurs variables complexes* for the Congress.

The Congress was opened on 30 August by an address delivered by Oswald Veblen, who reminded the audience of what had happened since the Oslo 1936 Congress, including the absorption by the US of a large number of European mathematicians fleeing Nazi Germany (and Europe). He then said:

> *We are holding the Congress in the shadow of another crisis, perhaps even more menacing than that of 1940, but one which at least does allow the attendance of representatives from a large part of the mathematical world. It is true that many of our most valued colleagues have been kept away by political obstacles and that it has taken valiant efforts by the Organizing Committee to make it possible for others to come.*[2]
> *[…]*
> *To our non-mathematical friends we can say that this sort of a meeting, which cuts across all sorts of po-*

[1] On Jacques Hadamard, see [3].

[2] Surprisingly enough, none of the political problems encountered in the preparation of the Congress and to which Veblen alluded to are mentioned in the book [**2**] on the history of the IMU – although there is in this book a long chapter devoted to the political problems of the period 1979–86.

litical, racial, and social differences and focuses on a universal human interest will be an influence for conciliation and peace. But the Congress is, after all, just a meeting of mathematicians. Let us get about our business.

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Michèle Audin
Institut de Recherche Mathématique Avancée
Université de Strasbourg et CNRS
France
Email: Michele.Audin@math.u-strasbg.fr

On the Grossone and the infinity computer

The mass media announced on 3 November 2010 that Yaroslav Sergeyev, a professor of Calabria University and Lobachevsky State University of Nizhny Novgorod, had received the Pythagoras Award. It was mentioned that "the professor constructed and patented a new 'Infinity Computer'" and "he suggested a new mathematical language that enables one to record various infinitely large and infinitely small numbers". This information deserves comment.

Sergeyev's idea is to introduce into arithmetic some infinitely large number, a grossone, consider only the numbers that are less than the grossone and operate exclusively on these numbers using the grossone as the radix. Sergeyev embellishes his idea with metaphysical arguments, emphasising that he does not use Cantor's approach and returns to the Ancient Greeks.

Elliot Mendelson remarked in his review of Sergeyev's book [1] that "the systems he deals with consist of objects which are called extended real numbers, but the descriptions of these objects and their properties are not clear enough to permit any warranted judgments about the assertions made by the author about these systems".

Sergeyev confronts his ideas with the nonstandard analysis of Abraham Robinson, defining his grossone as "the number of elements of the set of natural numbers". In fact, the role of this would-be mysterious entity can happily be performed by the factorial of an arbitrary infinite number, which are abundant in nonstandard analysis. The principal shortcomings of Sergeyev's approach and attempts at implementing calculations with a grossone on a computer were given in [2]. Unfortunately, the series of Sergeyev's publications continues in the various international journals hav-ing little if anything in common with the foundations of analysis. Miraculously, none of Sergeyev's publications on his grossone are in Russian.

Ancient Italian grossones are linguistically close to Sergeyev's grossone but differ in value.

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S. S. Kutateladze
Sobolev Institute of Mathematics
Novosibirsk, Russia
E-mail address: sskut@math.nsc.ru

Response to Professor Haggstrom's review of "In defence of objective Bayesianism"

Olle Haggstrom wrote a detailed but negative review of my recent book "In defence of objective Bayesianism" for the last issue of this newsletter. Fortunately Haggstrom's concerns are rather straightforward to address and I hope that by presenting responses to his concerns here, the reader will have a more balanced view of the book.

The book sets out to defend *objective Bayesian epistemology*, which is a theory about how strongly we should believe the various propositions we can express. According to this theory, the strengths of one's beliefs should be representable by probabilities, should be calibrated with empirical probabilities where known, and should otherwise equivocate between the basic possibilities that one can express. (It is this latter *equivocation* norm that sets objective Bayesianism apart from other versions of Bayesian epistemology.) At least on finite spaces, entropy is a natural measure of the extent to which a probability function equivocates, so the theory is often fleshed out by appealing to the *maximum entropy principle*: the strengths of one's beliefs should be representable by a probability function, from all those that are calibrated with evidence, that has maximum entropy. Objective Bayesianism has been roundly criticised, and is out of fashion amongst philosophers concerned with belief, but variants of the theory have been taken up fairly widely, e.g. in AI, physics and statistics. The book argues that many of the objections can be met and that objective Bayesianism is a promising research programme.

Haggstrom's first criticism concerns a uniform distribution over a large but finite state space:

> *Consider the following image analysis situation. Suppose we have a very fine-grained image with $10^6 \times 10^6$ pixels, each of which can take a value of black or white. The set of possible images then has $2^{10^\wedge 12}$ elements. Suppose that we assign the same probability $1/2^{10^\wedge 12}$ to each element. This is tantamount to assuming that each pixel, independently of all others, is black or white with probability 1/2 each. Standard probability estimates show that with overwhelming probability, the image will, as far as the naked eye can tell, be uniformly grey. In fact, the conviction of uniform greyness is so strong that even if, say, we split the image into four equally sized quadrants and condition on the event that the first three quadrants are pure black, we are still overwhelmingly convinced that the fourth quadrant will turn out grey. In practice, this can hardly be called unbiased or objective. (p.59)*

But this criticism dissolves when one is careful about the interpretation of the probabilities in question. In the total absence of evidence, while objective Bayesianism advocates a particular (uniform) distribution, this is a distribution of *degrees of belief*. It is neither a distribution of empirical probabilities nor of estimates of empirical probabilities. Hence the claim that "Standard probability estimates show that with overwhelming probability, the image will, as far as the naked eye can tell, be uniformly grey" is false because the probability in question is epistemic rather than empirical. (In fact standard probability estimates show that, if one were to sample images from the agent's belief distribution then, with overwhelming probability, images would appear uniformly grey; but of course there is no sampling from the belief distribution.) This confusion between epistemic and empirical probabilities is an elementary but common mistake and, in retrospect, perhaps the book could have been more forthright in warning the reader against such confusions.

Haggstrom's second objection concerns language dependence. According to objective Bayesianism, the strengths of one's beliefs should depend on the range of basic possibilities that one can express in one's language as well as on the explicit evidence available:

> *Williamson is aware of the language dependence problem and devotes Section 9.2 of his book to it. His answer is that one's language has evolved for usefulness in describing the world and may therefore itself constitute evidence for what the world is like. "For example, having dozens of words for snow in one's language says something about the environment in which one lives; if one is going to equivocate about the weather tomorrow, it is better to equivocate between the basic states definable in one's own language than in some arbitrary other language." (Williamson, p. 156–157). This argument is feeble, akin to noting that all sorts of dreams and prejudices we may have are affected by what the world is like, and suggesting that we can therefore happily and unproblematically plug them into the inference machinery. (p.60)*

It is sufficient to point out here that the analogy is a false one: languages are not as ephemeral as dreams or prejudices. The language one uses in a particular operating context is rather tightly constrained by the context itself – whether it is the language of a baker or a carpenter or a molecular biologist. For sure, in fictional contexts we can invent gobbledygook terms that are remotely related to reality but that does not apply to day-to-day languages or scientific languages – our terms in these latter languages generate what we consider to be the basic possibilities. Language dependence, then, is not obviously problematic and it is incumbent on anyone who thinks otherwise to come up with realistic cases that demonstrate otherwise. Note that, while objective Bayesianism has it that the strength to which one should believe a proposition may depend on one's language, there is no claim that recommendations of the scientific community should be sensitive to small changes in language. Just as subjective Bayesians (who hold that strengths of belief are relative to prior beliefs as well as evidence) test conclusions for robustness under natural variations in prior beliefs when trying to convince others of the conclusions they have reached, so too can objective Bayesians test conclusions for robustness under natural variations in language when making recommendations to others.

Subjective Bayesians update degrees of belief by a process called *Bayesian conditionalization*, which is a conservative rule in the sense that it yields new beliefs as similar as possible to old beliefs while accommodating the evidence. This is a plausible way of reconciling one's beliefs to new evidence but it has been criticised as leading to poor updates in certain pathological cases. In apparent contrast, objective Bayesianism is not explicitly conservative with its updates: updating amounts to determining the strengths of one's beliefs afresh with each change of evidence (e.g. by applying the maximum entropy principle over again). However, as I point out in my book, objective Bayesian updating will agree with the results of conditionalization, except in the pathological cases; in these cases, the recommendations of the maximum entropy principle are more plausible than those of conditionalization. Now, Bayesian conditionalization has been justified as a universal updating rule by appealing to a *dynamic Dutch book* argument: apparently, you ought to update your degrees of belief by conditionalization because otherwise if you bet according to the strengths of your beliefs, a cunning bettor can force you to lose money whatever happens. In my book I object to dynamic Dutch book arguments on the grounds that there are scenarios in which one can force someone to lose money *however* they change their degrees of belief in the light of new evidence. One kind of scenario is this: someone is attempting to convince you of some proposition by ascertaining how strongly you believe it currently and presenting evidence that should increase your degree of belief still further. You know this is the protocol, you know that the person in question is competent enough not to present evidence that will decrease your degree of belief and you know that you are sceptical enough that the new evidence will not render the proposition absolutely certain. It turns out that a cunning bettor with the same knowledge can force you to lose money should you change your degree of belief at all in the light of the new evidence. The real conclusion, of course, is not that you should not change your beliefs in the light of new evidence but that it can be more important to change beliefs to reflect new evidence than to avoid a dynamic Dutch book. Dynamic Dutch book arguments lose their force.

Haggstrom's third objection is to this reductio of dynamic Dutch book arguments:

> *as a Bayesian conditionalizer I would never find myself in a situation where I know beforehand in which direction my update will go, because then I would already have adjusted my belief in that direction. So what he's actually referring to is a situation where the Dutch bookmaker has access to evidence that I lack. A typical scenario would be the following. … (p. 60)*

Haggstrom then goes on to provide an example that does not fit the scenario outlined above. So Haggstrom's tactic is to deny my scenario and then present a new one that is less problematic. There should be no need for me to point out that this is a fallacious move: the new scenario is irrelevant to my argument, and in any case Haggstrom assumes without justification that my scenario is inapplicable. It is certainly true that in order to apply Bayesian conditionalization one should never find oneself in a situation where one knows beforehand in which direction the update will go (arguably, this is one of the problems with Bayesian conditionalization). But it *is* possible to conditionalize in my scenario because it is not a foregone conclusion that one's degree of belief will increase in the light of the new evidence – it could stay the same. So there is no reason why my scenario should not be viewed as one in which a conditionalizer might find herself.

Finally, Haggstrom claims that a specific probability function that I appeal to (in an example due to Bacchus, Kyburg and Thalos) is incoherent, i.e. ill-defined. This is simply not true: it is a well-defined probability function (modulo misprints in Haggstrom's review where a large and a small 'E' are printed in exactly the same way leading to confusion on this issue). Haggstrom renders it incoherent by adding further information that was neither present nor required in the original example.

While Haggstrom's review may be instructive for the fallacies it invokes – his objections exhibit, respectively, the fallacies of equivocation, false analogy, red herring and straw man, and would be a nice case study for students of a critical thinking course – it sheds less light on the real challenges facing objective Bayesianism. To be fair, the review does close by mentioning one such challenge, pointing out that my book says little in detail about the problem of induction. This limitation has been partly addressed in my subsequent paper "An objective Bayesian account of confirmation" (in Dennis Dieks, Wenceslao J. Gonzalez, Stephan Hartmann, Thomas Uebel, Marcel Weber (eds), *Explanation, Prediction, and Confirmation. New Trends and Old Ones Reconsidered*, *The philosophy of science in a European perspective* Volume 2, Springer, 2011). But there are other challenges and the curious reader is advised to consult the book itself in order to judge the prospects of the objective Bayesian research programme.

Jon Williamson
University of Kent, UK
Email: j.williamson@kent.ac.uk

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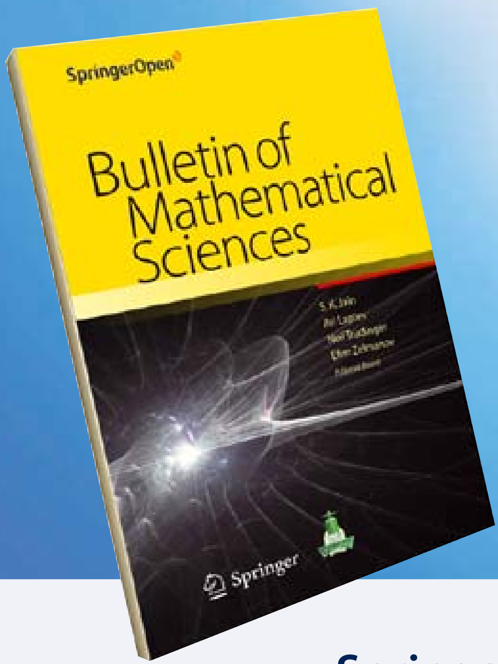

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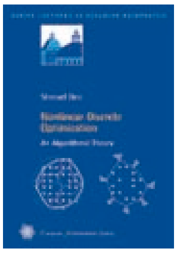

Shmuel Onn (Technion – Israel Institute of Technology, Haifa, Israel)
Nonlinear Discrete Optimization. An Algorithmic Theory (Zurich Lectures in Advanced Mathematics)

ISBN 978-3-03719-093-7. 2010. 147 pages. Softcover. 17 x 24 cm. 32.00 Euro

This monograph develops an algorithmic theory of nonlinear discrete optimization. It introduces a simple and useful setup which enables the polynomial time solution of broad fundamental classes of nonlinear combinatorial optimization and integer programming problems in variable dimension. An important part of this theory is enhanced by recent developments in the algebra of Graver bases. The power of the theory is demonstrated by deriving the first polynomial time algorithms in a variety of application areas within operations research and statistics, including vector partitioning, matroid optimization, experimental design, multicommodity flows, multi-index transportation and privacy in statistical databases.
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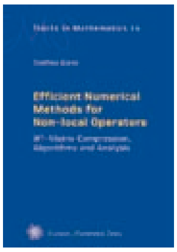
Geometrisation of 3-Manifolds (EMS Tracts in Mathematics Vol. 13)

ISBN 978-3-03719-082-1. 2010. 247 pages. Hardcover. 17 x 24 cm. 48.00 Euro

The Geometrisation Conjecture was proposed by William Thurston in the mid 1970s in order to classify compact 3-manifolds by means of a canonical decomposition along essential, embedded surfaces into pieces that possess geometric structures. It contains the famous Poincaré Conjecture as a special case. In 2002, Grigory Perelman announced a proof of the Geometrisation Conjecture based on Richard Hamilton's Ricci flow approach, and presented it in a series of three celebrated arXiv preprints.
Since then there has been an ongoing effort to understand Perelman's work by giving more detailed and accessible presentations of his ideas or alternative arguments for various parts of the proof. This book is a contribution to this endeavour.



Steffen Börm (Kiel University, Germany)
Efficient Numerical Methods for Non-local Operators. $\mathcal{H}^2$-Matrix Compression, Algorithms and Analysis (EMS Tracts in Mathematics Vol. 14)

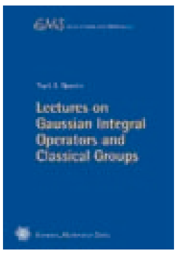
ISBN 978-3-03719-091-3. 2010. 441 pages. Hardcover. 17 x 24 cm. 58.00 Euro

Hierarchical matrices present an efficient way of treating dense matrices that arise in the context of integral equations, elliptic partial differential equations, and control theory. $\mathcal{H}^2$-matrices offer a refinement of hierarchical matrices: using a multilevel representation of submatrices, the efficiency can be significantly improved, particularly for large problems.
This books gives an introduction to the basic concepts and presents a general framework that can be used to analyze the complexity and accuracy of $\mathcal{H}^2$-matrix techniques. Starting from basic ideas of numerical linear algebra and numerical analysis, the theory is developed in a straightforward and systematic way, accessible to advanced students and researchers innumerical mathematics and scientific computing. Special techniques are only required in isolated sections, e.g., for certain classes of model problems.

Koichiro Harada (The Ohio State University, Columbus, OH, USA)
"Moonshine" of Finite Groups (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-090-6. 2010. 83 pages. Softcover. 17 x 24 cm. 24.00 Euro

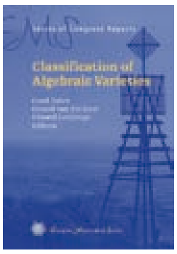
This is an almost verbatim reproduction of the author's lecture notes written in 1983–84 at the Ohio State University, Columbus, Ohio, USA. A substantial update is given in the bibliography. Over the last 20 plus years, there has been an energetic activity in the field of finite simple group theory related to the monster simple group. Most notably, influential works have been produced in the theory of vertex operator algebras whose research was stimulated by the moonshine of the finite groups. Still, we can ask the same questions now just as we did some 30–40 years ago: What is the monster simple group? Is it really related to the theory of the universe as it was vaguely so envisioned? What lays behind the moonshine phenomena of the monster group? It may appear that we have only scratched the surface. These notes are primarily reproduced for the benefit of young readers who wish to start learning about modular functions used in moonshine.



Yurii A. Neretin (University of Vienna, Austria; Institute for Theoretical and Experimental Physics and Moscow State University, Moscow, Russia)
Lectures on Gaussian Integral Operators and Classical Groups (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-080-7. 2011. 571 pages. Softcover. 17 x 24 cm. 58.00 Euro

This book is an elementary self-contained introduction to some constructions of representation theory and related topics of differential geometry and analysis. Topics covered include the theory of various Fourier-like integral operators as Segal–Bargmann transforms, Gaussian integral operators in L^2 and in the Fock space, integral operators with theta-kernels, the geometry of real and p-adic classical groups and symmetric spaces. The heart of the book is the Weil representation of the symplectic group (real and complex realizations, relations with theta-functions and modular forms, p-adic and adelic constructions) and representations in Hilbert spaces of holomorphic functions of several complex variables.
The book is addressed to graduate students and researchers in representation theory, differential geometry, and operator theory. The reader is supposed to be familiar with standard university courses in linear algebra, functional analysis, and complex analysis.



Classification of Algebraic Varieties (EMS Series of Congress Reports)
Carel Faber (Royal Institute of Technology, Stockholm, Sweden), Gerard van der Geer (University of Amsterdam, The Netherlands) and Eduard J.N. Looijenga (University of Utrecht, The Netherlands), Editors

ISBN 978-3-03719-007-4. 2010. 346 pages. Hardcover. 17 x 24 cm. 78.00 Euro

Fascinating and surprising developments are taking place in the classification of algebraic varieties. Work of Hacon and McKernan and many others is causing a wave of breakthroughs in the Minimal Model Program: we now know that for a smooth projective variety the canonical ring is finitely generated. These new results and methods are reshaping the field.
Inspired by this exciting progress, the editors organized a meeting at Schiermonnikoog and invited leading experts to write papers about the recent developments. The result is the present volume, a lively testimony of the sudden advances that originate from these new ideas. It will be of interest to a wide range of pure mathematicians, but will appeal especially to algebraic and analytic geometers.