NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

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Features
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Prime Numbers

Interview
J.-P. Bourguignon

Societies
European Physical Society

Research Centres
ESI Vienna

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EMS Agenda

2013

17 December
Diderot Mathematical Forum 2013, Berlin (Germany), Exeter (UK) and Zagreb (Croatia)
http://www.mathematics-in-europe.eu/da/nyheder/10-frontpage/news/1011-diderot-mathematical-forum-2013

2014

21–22 March
Annual meeting of ERCOM, INdAM, Rome, Italy
Scientific Secretary: Inna Capdeboscq, i.capdeboscq@warwick.ac.uk
INdAM Coordinator: Elisabetta Strickland, strickla@mat.uniroma2.it

12 April
Meeting of Presidents
Rectorate Conference Hall, Boğaziçi University, Istanbul, Turkey

25–26 April
Annual Meeting of the Committee for Developing Countries, Berlin, Germany
http://euro-math-soc.eu/EMS-CDC/
Andreas Griewank: griewank@math.hu-berlin.de

30–31 May
Ethics Committee meeting, Mittag-Leffler Institute, Djursholm, Sweden
Arne Jensen: matarne@math.aau.dk

28–29 June
EMS Council Meeting, San Sebastian, Spain

30 June–4 July
First Joint International Meeting RSME-SCM-SEMA-SIMAI-UMI
Bilbao, Spain
http://www.ehu.es/en/web/fjim2014/

28 July–1 August
EMS-IAMP Summer School on Mathematical Relativity
http://homepage.univie.ac.at/piotr.chrusciel/Summer-School2014/index.html

2015

11–14 June
AMS-EMS-PMS Congress, Porto, Portugal
http://www.euro-math-soc.eu/
http://www.ams.org/meetings/international/internmtgs

2016

18–22 July
7th European Congress of Mathematics, Berlin, Germany

Editorial: Meetings of Presidents

Stephen Huggett (University of Plymouth, UK)

Istanbul, 12 April 2014

The European Mathematical Society is extremely grateful to the Turkish Mathematical Society for its invitation to hold the next Meeting of Presidents in Istanbul. The meeting will take place in the Rectorate Conference Hall of Boğaziçi University. Details will be available on the web site http://www.euro-math-soc.eu

These meetings bring together Presidents of Mathematical Societies in Europe which are corporate members of the EMS in order to strengthen our networking and collaboration. This will be the sixth such meeting, so I have been asked to write an account of them. This will be a personal view, not an official history! I have picked highlights from the lengthy reports that I wrote after each one, which I hope will give a flavour of our discussions.

Luminy, 26 and 27 April 2008

Ari Laptev, President of the European Mathematical Society, welcomed everybody to the meeting. It was the first of its kind, and a very significant step. He expressed his thanks to Stéphane Jaffard and the Société Mathématique de France for their invitation to hold the meeting in Luminy, and everybody present enthusiastically agreed with him.

There were 22 Presidents at the meeting, who each gave a brief introduction to the work of their Society. Altogether, this took some time, but it gave a fascinating overview of mathematics in Europe. The variety in the Societies was intriguing, but several were described as having recently gone through a period of renewal. Highlights included the description of the The Moscow Mathematical Society as "old, large, and poor", the extremely successful "year of Mathematics" in Germany, and the fact that the Portuguese mathematicians have been banned from doing research into mathematics education!

The discussion revealed some common challenges and experiences, such as:

- the falling standards of students entering university to study mathematics,
- the great damage that can be done to mathematics by the thoughtless use of "impact factors",
- the danger that politicians would respond to the promotion of the applications of mathematics by dropping support for pure mathematics altogether,
- the importance of maintaining pressure to preserve standards in the face of the opposite pressure to "dumb down".

Pavel Exner, Vice-President of the European Mathematical Society, briefed the meeting about the European Research Council. It had a total budget of 7.5 billion euros. The annual budget was planned to rise linearly, reaching about 1.7 billion in 2013.

Alexei Sossinsky described the Dubna Summer Schools. These have been very successful in providing a relaxed and creative atmosphere in which students (either just before or immediately after they enter university) can have contact with very distinguished research mathematicians. The intention was to show the students the edge of current research in a few carefully chosen fields. The organizers wanted to make these schools more international.

Warsaw, 9 and 10 May 2009

Ari Laptev welcomed the 28 Presidents to the meeting, and everybody expressed their thanks to Stanisław Janeczko and the Banach Centre for their invitation to hold the meeting in Warsaw. In the opening discussion, a point which stuck in my mind was the growing contradiction between the increasing value of mathematics in subjects such as engineering, chemistry, and biology and the decreasing amount of mathematics taught to students of these subjects.

Tsou Sheung Tsun, Chair of the EMS Committee for Developing Countries, presented its work. She explained that as it only had access to small amounts of money it concentrated on the coordination of efforts funded from elsewhere. A new project for the CDC was the concept of "emerging regional centres", which she described. Then Tsou Sheung Tsun gave a brief introduction to the work of CIMPA, noting that it would in future be supported by the Spanish government as well as the French, and that indeed even broader European support was hoped for.

Stéphane Jaffard described how the evaluation of mathematics, and the procedures for the distribution of research support, had changed in France over the previous two years. Firstly, under the heading of "increased autonomy" for universities, each university now had far greater power in deciding how to allocate research funding. Consequently, many universities, especially the smaller ones, are using entirely inappropriate metrics to evaluate research. Secondly, overall funding is decreasing, and the new agency ANR preferred to award large grants to groups of universities, for short term projects, on currently fashionable themes. All of these changes acted against mathematics. He proposed that the EMS work towards some sort of collective response to these problems. Brian Davies said that sadly the UK was "ahead" of France in many of these developments. The research council was increasingly choosing to award grants in larger amounts, on currently fashionable themes, and the total amount available for foundational work in mathematics had dropped dramatically. Also, the support through university funding suffers from the same difficulties as those described in France: the universities decide how to distribute their research money. Zvi Artstein reported that in Israel the politicians argue to the academics that “the UK is doing this so it must be right”, which reinforced the need for a collective response from the EMS.

Bucharest, 17 and 18 April 2010

The eruption of the Eyjafjallajökull volcano forced the cancellation of this meeting at the very last minute. I was in a weirdly deserted Terminal 5 at Heathrow while emails were whizzing back and forth and it became clear that so many of us were stranded all over northern Europe.

Of course this was a bitter disappointment for the local organisers, to whom the EMS is very grateful for all their work. In fact several Presidents had already arrived in Bucharest, and they held what I believe was a very useful informal meeting.

Bilbao, 7 and 8 May 2011

Marta Sanz-Solé, President of the European Mathematical Society, thanked the RSME and the University of the Basque Country for hosting the meeting, and the Guggenheim Museum for its wonderful tour that morning. She then welcomed the 33 Presidents who had come to the meeting, and spoke briefly about the importance of these meetings in providing support and feedback to the European Mathematical Society, and in strengthening networking among mathematicians in Europe.

One of the opening discussions addressed the idea put forward by Jouko Väänänen that the EMS could provide a sort of jobs management system, a little like that available from the AMS. This provoked the response that there is too much variation from nation to nation in Europe in the procedures governing job applications. However, the view was expressed that a more limited service consisting of an online marketplace of cvs and vacancies might be very valuable.

Mats Gyllenberg reported on very recent developments affecting the future of the European Science Foundation. A group of funding organizations called EUROHORCS started negotiations on a possible merger with the ESF, forming a new organization in Brussels restricted to strategy and lobbying, and not including learned societies. This left the ESF in an extremely difficult position, and its future was highly uncertain.

In contrast, Michael Drmota reported that the Erwin Schrödinger Institute’s future was now secure, at least for the next five years, under the roof of the University of Vienna.

The proposed closure of the geometry section in the VU Amsterdam was discussed in detail. The meeting was unanimous in deciding to write a letter expressing strong opposition to this proposal.

Prague, 31 March and 1 April 2012

Marta Sanz-Solé welcomed the 30 Presidents who had come to the meeting and thanked the organisers very much for the invitation to Prague during the celebrations of the 150th anniversary of the Union of Czech Mathematicians and Physicists. She also thanked the Czech Academy of Sciences for the use of Villa Lanna.

We had a detailed discussion on publishing, including the implications of the boycott of Elsevier. In the longer term this might lead to a growth in the publishing activities of the EMS, and in the short term the Executive Committee had agreed to explore possible collaborations with other learned society publishers in mathematics.

Pavel Exner gave a report on the European Research Council. The ERC was five years old, and had awarded over 100 million euros in more than 140 grants for mathematics. There were two new programmes, the “proof of concept” and the “synergy grants”. The starting and advanced grants would continue as before, but the former was growing so rapidly that it would be split into two, by age, called “starting” and “consolidating”. Under Horizon 2020 the planned growth for the ERC was much slower than originally planned (and achieved) under FP7.

Aarhus, 6 April 2013

Marta Sanz-Solé thanked the organisers very much for the invitation to Aarhus during the Mathematical Weekend being held there, and welcomed the 28 Presidents who had come to the meeting. She went on to announce the EMS Council in Donostia (San Sebastián), June 28 and 29, 2014, noting in particular that there will be vacancies on the Executive Committee to be filled at the Council. In response the comment was made that it was not a good idea that the three main officers (President, Treasurer, and Secretary) will be replaced all at the same time. I recall acknowledging this, and explaining that we will be careful about the hand-over.

Günter Törner, Chair of the EMS Education Committee, gave a presentation introducing the subject of mathematics curricula in secondary school. In the discussion afterwards the following points were made.

- There is no organisation at the European level influencing curricula within all European countries, and nor are there European standards of competencies or of knowledge. Indeed standardization is not even something to aim for.
- A lot of trainee mathematics teachers try to avoid “hard” mathematics, and a lot of practising mathematics teachers know very little mathematics, especially if they come from faculties of education. We need teachers to be well educated and passionate about their subject. They should have high status and salary.

- Industrialists are worried about the lack of mathematical knowledge and reasoning skills among their employees, and call for training courses. Perhaps we should work to support the continuing education of teachers, via a European summer school for teachers.

Bernard Teissier gave a report on the first meeting of the Publications Committee, and introduced a discussion on open access. The meeting wondered how open access would influence the way people behave, and noted that the French declaration "Open Access: a warning on the inherent flaws of the author pays model" http://smf.emath.fr/sites/smf.emath.fr/files/open_access_3_soc-trans.pdf identifies good practice. It was argued that peer review is crucially important, but in danger, because the assessment of the quality of a paper will be tainted by financial considerations. In addition, there are too many journals, especially those of lower quality. Our concern about prices should not divert us from the importance of the quality of the publications.

Miguel Abreu showed the preliminary English version of the first episode of the collection of short films "Isto e Matematica". I thought it was superb. The first series of thirteen episodes will be available in English at http://www.mathematics-in-europe.eu/

Postscript

Let me emphasize again that this was only my personal choice of highlights: I have missed out many other important discussions. It is no surprise that in spite of the wonderful cultural diversity in Europe, mathematicians share many of the same aspirations and face similar challenges. These meetings are playing a small but very important part in helping European mathematicians to overcome these challenges together.

A New Cover for the Newsletter

The Editorial Board

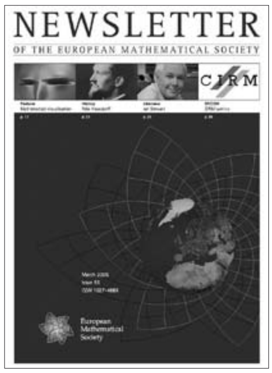
You will certainly have noticed that the newsletter has a new cover!

The previous cover was introduced in March 2005, when the newsletter was first published by the EMS Publishing House. As Thomas Hintermann and Martin Raussen pointed out at the time, "the new cover design communicates the European Mathematical Society's focus on mathematics in Europe, while being open to the whole world at the same time" (Newsletter 55, page 3). This is a crucial concern for us, as it surely has been for the previous editorial boards. Changing the cover has been a challenging undertaking.

For the new cover, we wanted the chance to highlight a topic of the newsletter, to keep the rhythm of the changes that the European mathematical community faces, by choosing an emblematic picture every three months. It will focus on a person, an event, a place or a mathematical object; it will illustrate the content or be a subject on its own. This is not a minor change, even if we have chosen to maintain some continuity in the theme and the colours, which are strongly reminiscent of Europe, and to put the accent on the fact that the newsletter is, first of all, a magazine which brings professional and scientific news to the readership.

We encourage readers and authors of the newsletter to make suggestions for future cover stories.

Cover 1998–2004



Cover 2005–2013

Announcement of the Next Meeting of the EMS Council San Sebastián, June 28 and 29, 2014

Stephen Huggett (University of Plymouth, UK)

The EMS Council meets every second year. The next meeting will be held in San Sebastián, June 28 and 29, 2014, in the Auditorio Antonio Beristain, Campus de Gipuzkoa. The Council meeting starts at 14.00 on June 28 and ends at lunch time on June 29.

Delegates

Delegates to the Council shall be elected for a period of four years. A delegate may be re-elected provided that consecutive service in the same capacity does not exceed eight years. Delegates will be elected by the following categories of members.

(a) Full Members

Full Members are national mathematical societies, which elect 1, 2, 3, or 4 delegates according to their membership class. The membership class is decided by Council, and societies are invited to apply for the new class 4, which was introduced in the 2008 Council. However, the number of delegates for the 2014 Council is determined by the current membership class of the society.

Each society is responsible for the election of its delegates. Each society should notify the Secretariat of the EMS in Helsinki of the names and addresses of its delegate(s) no later than 7 April 2014.

(b) Associate Members

Delegates representing associate members shall be elected by a ballot organized by the Executive Committee from a list of candidates who have been nominated and seconded by associate members, and have agreed to serve. According to our statutes, these members may be represented by (up to) one delegate. There is currently no delegate of these members. A nomination form for delegates of associate members will be provided to these members by the EMS Secretariat.

(c) Institutional Members

Delegates representing institutional members shall be elected by a ballot organized by the Executive Committee from a list of candidates who have been nominated and seconded by institutional members, and have agreed to serve. A nomination form for delegates of institutional members will be provided to these members by the EMS Secretariat. In October 2013, there were 39 institutional members and, according to our statutes, these members may be represented by (up to) 4 delegates.

The only current delegate whose term includes 2014 is Sverre Olaf Smalo. The delegates who can be re-elected are Joaquim Bruna, Gert-Martin Greuel, and Mina Teicher.

(d) Individual Members

In October 2013, there were 2457 individual members and, according to our statutes, these members may be represented by (up to) 25 delegates. However, this number may have increased by the time we call the election (if any) for individual members.

Here is a list of the current delegates of individual members whose terms include 2014:

Thierry Bouche
Jean-Pierre Bourguignon
Mireille Chaleyat-Maurel
Krzysztof Ciesielski
Mirna Džamonja
Pavel Exner
Vincent Heuveline
Arne Jensen
Paul Kettler
Ari Laptev
José Francisco Rodrigues
Marie-Francoise Roy
Stepan Agop Tersian
Robin Wilson

Here is a list of the delegates of individual members who could be re-elected for the 2014 Council:

Ehrhard Behrends
Vasile Berinde
Jean-Marc Deshouillers
Rolf Jeltsch
Christian Kassel
Oriol Serra
Angela Slavova

Elections of individual delegates will be organised by the EMS secretariat by a ballot among individual members unless the number of nominations does not exceed the number of vacancies. A nomination form for delegates of individual members will be provided by the EMS Secretariat.

The deadline for nominations for delegates of individual members to arrive in Helsinki is 28 February 2014.

Agenda

The Executive Committee is responsible for preparing the matters to be discussed at Council meetings. Items for the agenda of this meeting of the Council should be sent as soon as possible, and no later than 7 April 2014, to the EMS Secretariat in Helsinki.

Executive Committee

The Council is responsible for electing the President, Vice-Presidents, Secretary, Treasurer and other members of the Executive Committee. The present membership of the Executive Committee, together with their individual terms of office, is as follows.

President: Marta Sanz-Solé (2011–2014)

Vice-Presidents: Franco Brezzi (2013–2016)
Martin Raussen (2011–2014)

Secretary: Stephen Huggett (2007–2014)

Treasurer: Jouko Väänänen (2007–2014)

Members: Alice Fialowski (2013–2016)
Gert-Martin Greuel (2013–2016)
Laurence Halpern (2013–2016)
Volker Mehrmann (2011–2014)
Armen Sergeev (2013–2016)

Members of the Executive Committee are elected for a period of four years. The President can only serve one term. Committee members may be re-elected, provided that consecutive service shall not exceed eight years.

The Council may, at its meeting, add to the nominations received and set up a Nominations Committee, disjoint from the Executive Committee, to consider all candidates. After hearing the report by the Chair of the Nominations Committee (if one has been set up), the Council will proceed to the elections to the Executive Committee posts.

All these arrangements are as required in the Statutes and By-Laws, which can be found here, together with the web page for the Council:

http://www.euro-math-soc.eu/governance.html

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Call for Applications EMS Schools in Applied Mathematics (ESSAM)

Currently, four series of summer schools in applied mathematics take place every year under the EMS banner. Their existence and organisation are part of the activities of the EMS Applied Mathematics Committee. The EMS considers as a priority the goal that the schools keep a high scientific level and focus on topics of relevant impact.

The EMS helps these schools in fundraising and, in particular, contributes towards the participation of young researchers from European and Mediterranean countries.

Initially, there was a school organised by IMPAN in Bedlewo and a school organised by CIME taking place every year. In 2010, a summer school in biomathematics joined ESSAM. It is organised every year by the ESMTB (European Society for Mathematical and Theoretical Biology). The last ESSAM school was created in 2011: a summer school in mathematical finance. It is organised by the Institut Louis Bachelier.

The programme committees of the organising institutions incorporate a representative of the EMS Applied Mathematics Committee for the planning of future schools and to certify their scientific level and the fulfilment of EMS requirements.

If you want to create such a series or you are in charge of an existing one and you want to become part of the essam schools, please send a message to Helge Holden at holden@math.ntnu.no.

List of ESSAM 2013 schools:

1. IMPAN-EMS Bedlewo School: "EMS School on Computational Aspects of Gene Regulation" (supported by Cost).
2. CIME-EMS Summer School in Applied Mathematics: "Vector-valued Partial Differential Equations and Applications" (supported by the EMS).
3. EMS-ESMTB Summer School: "Multiscale models in the life sciences".
4. Sixth European Summer School in Financial Mathematics.

You can find more information about these schools and their guidelines at the EMS Applied Committee's webpage: http://www.euro-math-soc.eu/EMS-AMC.

First Joint International Meeting AMS-EMS-SPM, 10–13 June 2015, Porto

Greetings from the Organizers

Each year the American Mathematical Society liaises with a host society from another country to organize a meeting, linking together members of the mathematical community from the US and abroad. The 2015 meeting will bring together not only AMS and the hosting Portuguese Mathematical Society but also the European Mathematical Society in the UNESCO world heritage city of Porto, from which Portugal derives its name. Perched high above the Douro River, the venue is close at hand to local cultural sites and a variety of accommodations. The scientific scope of this meeting ranges from plenary talks of general interest to special sessions focusing on current research in specific areas. This offers opportunities to broaden and establish new research contacts.

The Programme Committee includes:
Jorge Almeida, University of Porto; Georgia Benkart (Chair), AMS Associate secretary, Professor Emerita of University of Wisconsin – Madison; Nils Dencker, Lund University; Gustavo Granja, Instituto Superior Técnico, Lisbon; Alexey Parshin, Steklov Institute, Moscow; Carlos Rocha, University of Lisbon; Jean Taylor, Courant Institute of New York University, Professor Emerita of Rutgers University; Sussana Terracini, University of Torino; Luís Nunes Vicente, University of Coimbra.

Jean-Pierre Bourguignon: New President of the European Research Council

Jean-Pierre Bourguignon, the second president of the European Mathematical Society (1995–98), will be the next president of the European Research Council (ERC). Professor Bourguignon was nominated for this position by the EMS to a search committee set up by the European Commission.

Since 2007, the ERC has funded top European researchers through grants (ERC starting grants, consolidator grants, advanced grants, and proof of concept and synergy grants) given on the basis of applications evaluated by expert panels. In Horizon 2020, the ERC appears as a crucial component in the EU long-term research strategy to support the most talented and creative scientists in blue-sky research.

As a mathematician, Jean-Pierre Bourguignon is well known for his contributions to modern differential geometry. He was the president of the French Mathematical Society (SMF), 1990–1992. Since 1994, and until his retirement in August 2013, he has served mathematicians and physicists all over the world as director of the prestigious research centre Institut des Hautes Études Scientifiques at Bures-sur-Yvette, close to Paris.

Through his remarkable activity, Jean-Pierre Bourguignon has promoted collaboration between mathematicians and researchers from other sciences. He has also been enthusiastically involved in activities raising the public awareness of mathematics, like films and exhibitions.

The EMS congratulates him very warmly for this achievement and looks forward with great pleasure to the great benefits expected for the ERC and for science in Europe under his leadership.

Peter Scholze to Receive 2013 Sastra Ramanujan Prize

Krishnaswami Alladi (University of Florida, Gainesville, USA)

The 2013 SASTRA Ramanujan Prize will be awarded to Professor Peter Scholze of the University of Bonn, Germany. The SASTRA Ramanujan Prize was established in 2005 and is awarded annually for outstanding contributions by young mathematicians to areas influenced by the genius Srinivasa Ramanujan. The age limit for the prize has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded in late December at the International Conference on Number Theory and Galois Representations at SASTRA University in Kumbakonam (Ramanujan's hometown) where the prize has been given annually.

Professor Scholze has made revolutionary contributions to several areas at the interface of arithmetic algebraic geometry and the theory of automorphic forms, and especially in the area of Galois representations. Already in his Master's thesis at the University of Bonn, Scholze has given a new proof of the Local Langlands Conjecture for general linear groups. There were two previous approaches to this problem, one by Langlands–Kottwitz and another by Harris and Taylor. Scholze's new approach was striking for its efficiency and simplicity. Scholze's proof is based on a novel approach to calculating the zeta function of certain Shimura varieties. This work completed in 2010 and appeared in two papers in Inventiones Mathematicae in 2013. Scholze has generalised his methods, partly in collaboration with Sug Woo Shin, to determine the l-adic Galois representations defined by a class of Shimura varieties. These results are contained in two papers published in 2013 in the *Journal of the American Mathematical Society.*

While this work for his Master's was groundbreaking, his PhD thesis written under the direction of Professor Michael Rapoport at the University of Bonn was a more marvellous breakthrough and a step up in terms of originality and insight. In his thesis, he developed a new p-adic machine called perfectoid spaces and used it brilliantly to prove a significant part of the weight monodromy conjecture due to the Fields Medallist Pierre Deligne, thereby breaking an impasse of more than 30 years. This work was presented in a massive paper that appeared in *Publications Mathématiques de l'IHES* in 2012. In a paper that appeared in *Forum of Mathematics* π in 2013, Scholze extended his theory of perfectoid spaces to develop a p-adic Hodge theory for rigid analytic spaces over p-adic ground fields, generalising a theory due to Fields Medallist Gerd Faltings for algebraic varieties. As a consequence, he could answer a question on spectral sequences that Abel Prize Winner John Tate had raised four decades earlier. One key ingredient of Scholze's approach is his construction of a *pro-étale site*; this has led to a new foundation of étale cohomology, which he is investigating with Bhargav Bhatt.

Yet another seminal work of Scholze is his collaboration with Jared Weinstein extending earlier results of Rapoport-Zink on moduli spaces of p-divisible groups. Scholze-Weinstein show that at an infinite level, these carry a structure of a perfectoid space. One significant consequence of this is that it yields a very simple description of p-divisible groups over the ring of integers of an algebraically closed extension of the p-adic rationals which is analogous to Riemann's description of Abelian varieties over the complex numbers.

Scholze's most recent work establishes the existence of Galois representations associated with the mod p cohomology of locally symmetric spaces for linear groups over a totally real or *CM* field. It has startling implications on the Betti cohomology of locally symmetric spaces. The Betti cohomology of a modular curve is torsion free but the Betti cohomology of locally symmetric spaces may have torsion and hence cannot be computed in terms of automorphic forms. Numerical evidence going back to the 1970s suggested that some analogue of the Langlands programme should apply to these torsion classes. Scholze's breakthrough is the first progress in this direction in 40 years.

Peter Scholze was born in Dresden in December 1987 — at the time of the Ramanujan Centennial. At the age of 25, he is now one of the most influential mathematicians in the world. As a student he won three gold medals and one silver medal at the International Mathematics Olympiads. He finished his bachelor's degree in three semesters and his Master's courses in two semesters. He was made a full professor soon after his PhD. His work has been estimated by experts to possess the quality of the timeless classics and is expected to have a major impact in the progress of mathematics in the coming decades.

Peter Scholze was the unanimous choice by the Prize Committee to receive the award this year. The international panel of experts who formed the 2013 SASTRA Ramanujan Prize Committee were Professors Krishnaswami Alladi – Chair (University of Florida), Kathrin Bringmann (University of Cologne), Roger Heath-Brown (Oxford University), David Masser (University of Basel), Barry Mazur (Harvard University), Ken Ribet (University of California, Berkeley) and Ole Warnaar (University of Queensland).

Previous winners of the SASTRA Ramanujan Prize are Manjul Bhargava and Kannan Soundararajan in 2005 (two full prizes), Terence Tao in 2006, Ben Green in 2007, Akshay Venkatesh in 2008, Kathrin Bringmann in 2009, Wei Zhang in 2010, Roman Holowinsky in 2011 and Zhiwei Yun in 2012. By awarding the 2013 prize to Peter Scholze, the SASTRA Ramanujan Prize continues its great tradition of recognising the most exceptional work by a young mathematician.

Conference on Number Theory and Galois Representations

After receiving the prize on Ramanujan's birthday on 22 December, Professor Scholze will deliver the Ramanujan Commemoration Lecture. Other confirmed senior speakers at the conference include Professors Michael Rapoport (University of Bonn, Germany), M. Ram Murty (Queens University, Canada), V. Kumar Murty (University of Toronto, Canada), Sinai Robins (Nanyang University, Singapore), N. Saradha (Tata Institute, India) and Sanoli Gun (Institute of Mathematical Sciences, India). In the spirit of Ramanujan, the conference will also have invited talks by talented graduate students and post-docs. Three such younger speakers from Europe are Michael Th. Rassias (doctoral student, ETH, Zurich), Rene Olivetto (doctoral student, University of Cologne) and Larry Rolen (post-doc, University of Cologne).

Krishnaswami Alladi, Chair
SASTRA Ramanujan Prize Committee

Krishnaswami Alladi is a professor of mathematics at the University of Florida where he was Department Chairman, 1998-2008. He received his PhD from UCLA in 1978. His area of research is number theory. He is the founder and Editor-in-Chief of the Ramanujan Journal published by Springer. He helped create the SASTRA Ramanujan Prize and has chaired the prize committee since its inception.

The Stephen Smale Prize – Call for Nominations of Candidates

The second Stephen Smale Prize will be awarded at the meeting Foundations of Computational Mathematics (FoCM) in Montevideo in December 2014, http://www.fing.edu.uy/~jana/www2/focm_2014.html.

We hereby call for nominations of candidates.

Nominations should be sent to FoCM secretary Antonella Zanna at: Antonella.Zanna@math.uib.no.
Deadline: 24:00 (GMT), 10 March 2014.

Background

The Society for the Foundations of Computational Mathematics was created in the summer of 1995, following a month-long meeting in Park City, Utah, which was principally organised by Steve Smale. The Park City meeting aimed, in Smale's words from the preliminary announcement, "to strengthen the unity of mathematics and numerical analysis, and to narrow the gap between pure and applied mathematics". Smale's vision has been the society's inspiration for all these years. The journal Foundations of Computational Mathematics was created, several colloquia and research semesters were organised and international conferences have been held every three years. After 15 years of existence, with an established and recognised position in the scientific community, the society has created the "Stephen Smale Prize", with the objective to recognise the work of a young mathematician in the areas at the heart of the society's interests and to help promote their integration among the leaders of the scientific community. The first Stephen Smale Prize was awarded at the Budapest meeting in 2011 to Snorre H. Christiansen.

Summary of prize rules
(see http://focm-society.org/smale_prize.php for full details):

- The goal of the Smale Prize is to recognise major achievements in furthering the understanding of the connections between mathematics and computation, including the interfaces between pure and applied mathematics, numerical analysis and computer science.
- To be eligible for the prize a candidate must be in their early to mid career, meaning, typically, removed by at most 10 years from their (first) doctoral degree by the first day of the FoCM meeting (11 Dec 2014). Allowances might be made for atypical life circumstances.
- Eligible candidates should be nominated (self-nominations excluded) by sending an email to the secretary of FoCM, Antonella.Zanna@math.uib.no, no later than 10 March 2014. Each nomination should be accompanied by a brief case for support.
- There is no compulsory format for the nomination but it should include at least three letters of recommendation.
- The recipient of the prize will be expected to give a lecture at the meeting. A written version of this lecture (tagged as the Smale Prize Lecture) will be included in the volume of plenary talks.

A full announcement of the prize can be found at http://focm-society.org/smale_prize.php.

European Level Organisations for Women Mathematicians

Caroline Series (University of Warwick, Coventry, UK)

There are several organisations and committees supporting women mathematicians in Europe. The first and best known is the membership organisation European Women in Mathematics (EWM), europeanwomeninmaths.org. Founded in 1986, EWM has several hundred members and coordinators in over 30 European countries. Its executive consists of a convenor, currently Susanna Terracini (Torino, Italy), and a standing committee. Every other year, EWM has a general meeting which this year took place at the Hausdorff Center, Bonn. It has an email network and publishes an electronic newsletter. It also runs a biennial summer school; the 2013 school was held at ICTP Trieste as part of the Mathematics for Planet Earth programme. EWM also organises or supports other women in maths activities; in France there will be a meeting `Women in Numbers' in Luminy this October and in November a forum for young women in mathematics in Lyon. EWM has produced several videos which can be found on its website; watching them is a good way to get a taste of the unique atmosphere of EWM meetings. Details of how to join EWM can be found on the website.

EWM is independent from, but has links to, its sister organisation the Association for Women in Mathematics (AWM) based in the United States. It also works closely with various national level organisations, for example the LMS Women in Mathematics Committee and *femmes et mathématiques* in France. A variety of links can be found on the EWM website.

The second organisation for women mathematicians in Europe is the European Mathematical Society's Women and Mathematics Committee, www.euro-math-soc.eu/comm-women2.html, currently chaired by Caroline Series (Warwick). The committee's remit is to address issues relating to the involvement and progression of women in mathematics and to support and promote the recognition of the achievements of women mathematicians. It acts largely as a liaison body between the EMS, EWM and other similar organisations on these issues. Past activities have included gathering statistics on the numbers of women mathematicians in different European countries, setting up a blog (now incorporated into the EWM website) and organising panel discussions at European congresses.

Finally, in 2008, EWM and the EMS Women in Maths Committee jointly set up the EWM/EMS Scientific Committee, www.euro-math-soc.eu/comm-women2.html. Its members are 12 distinguished women mathematicians, among them Dusa McDuff, Ursula Hamenstaedt and Ragni Piene. Currently chaired by Cornelia Drutu (Oxford), the main function of this committee is to advise the EMS and EWM on scientific programmes and speakers for events aimed at emphasising women's scientific contributions, such as those organised separately or jointly by EWM and the EMS. Together, the three groups have recently been instrumental in setting up a majority female summer school on Apollonian Packings to be held at the Institut Mittag Leffler in June 2014, www.math.ucsd.edu/~alina/ewm/.

There is currently no international organisation for women mathematicians. However, under the direction of Ingrid Daubechies, current President of the IMU, progress towards further international coordination is being made. Following the first International Conference for Women Mathematicians (ICWM), which took place just before the 2010 ICM in Hyderabad, a second ICWM will be held in Seoul immediately before the ICM in August 2014, sites.google.com/site/icwm2014/. The ICWM is an opportunity to meet other ICM participants in a relaxed atmosphere and hear talks by prominent female mathematicians and we encourage any interested readers to consider taking part.

Caroline Series
Chair, EMS Women in Maths Committee
October 2013

The Newsletter thanks the London Mathematical Society Newsletter and C. Series for the permission to republish this article, originally appeared in the *LMS Newsletter* No. 430, November 2013.

Caroline Series is currently Chair of the EMS Women in Mathematics Committee. She has had a long involvement in initiatives for women mathematicians, having been a founding member of EWM. She works in dynamical systems and geometry and is a professor at the University of Warwick where she has been for most of her career. She is a co-author of the well-known book Indra's Pearls, a reader friendly journey through the fascinating world of limit sets of Kleinian groups.

Forty Years of the Epimorphism Theorem

Peter Russell (McGill University, Montreal, QC, CA) and Avinash Sathaye (University of Kentucky, Lexington, KY, US)

1 Introduction

Let k be a field. Let us think of the affine plane $\mathbb{A}^2$ as the algebraic variety with coordinate ring $k[Y, X]$ and the affine line $\mathbb{A}^1$ as the algebraic variety with coordinate ring $k[Z]$. (If k is algebraically closed it is safe to think k^2 and k.) Then the zero-set C_1 of the polynomial Y is a closed subvariety of $\mathbb{A}^2$ isomorphic to $\mathbb{A}^1$, and so is the zero-set C_2 of the (non-linear) polynomial $Y + X^2$. Here we obviously have an automorphism of $\mathbb{A}^2$ that sends C_2 to C_1, and it is natural to ask whether this is always the case. Along the way it is natural to enquire about the nature of the group of all automorphisms of $\mathbb{A}^2$.

The paper "Embeddings of the line in the plane" by S. S. Abhyankar and T.-T. Moh [AM1] deals with these questions. For reasons that will become clear, it is usually called the "Epimorphism Paper". The earlier paper "Newton-Puiseux expansion and generalized Tschirnhausen transformation" [AM2] will be termed the "Expansions Paper". It gives a comprehensive treatment, which was very novel at the time, of "plane curves with one place at infinity". It is an essential predecessor to [AM1] and has found other far reaching applications, such as the Jacobian problem, for instance. Both papers were very influential.

Many of the topics of this review were intensely discussed in Abhyankar's circle during the Séminaire de Mathématiques Supérieures of 1970 at the Université de Montréal. The Séminaire is vividly remembered by the participants for the great intellectual ferment surrounding it, mathematical and otherwise. The lecturers were Shreeram Abhyankar, Michael Artin, Alexandre Grothendieck and Masayoshi Nagata. This was a time of great abstraction in algebraic geometry but also a time of newly heightened interest in concrete and seemingly elementary problems, some very famous for being as hard to solve as they are easy to formulate.

Both the "Jacobian Problem" and the "Lines in the Plane Problem" belong in that class. For the lines, it was accepted that the case of "one characteristic pair" (relatively prime X- and Y-degrees) was doable, and Oscar Zariski had suggested to Abhyankar that the case of bi-degrees having a prime GCD, or, a bit more generally, those with "two characteristic pairs", be seriously considered. The brilliant full solution came soon after, not only settling a fundamental mathematical problem but also leading to a veritable explosion in related research activity. The result clearly fascinated a large number of mathematicians. Many additional proofs were published with techniques from topology, algebraic surface theory, complex variables, alternate arrangements bypassing or modifying the Newton-Puiseux expansions of [AM2], and so on. Some of these are mentioned below when needed as a reference but we do not try to present a full list. Of course, all alternate proofs are valuable and may lead to new developments. The interested reader can easily locate variant proofs through a simple database search.

Shreeram Abhyankar was our friend, guru, mentor and teacher. We dedicate this review to his inspiring mathematical career.

2 The Epimorphism Paper

Let k be a field of characteristic π and let X, Y, Z be indeterminates over k.

Main Theorem. *Let u and v be non-constant polynomials of degree m and n in Z with coefficients in k. Assume that $k[Z] = k[u, v]$. Assume also that either m or n is not divisible by π. Then either m divides n or n divides m.*

We will also describe the conclusion of the theorem by the phrase "(m, n) is principal".

Using the theorem, we can deduce that if $m \geq n$, there is a $c \in k$ so that $u' = u - cv^{m/n} \in k[Z]$ with $\deg(u') < \deg(u)$. Clearly $k[Z] = k[u', v]$. If $m < n$ then we find c so that $k[Z] = k[u, v'], v' = v - cu^{n/m}, \deg(v') < \deg(v)$. By induction on $\deg(u) + \deg(v)$, this gives the Epimorphism Theorem.

Epimorphism Theorem. *Let $\gamma : k[Y, X] \to k[Z]$ be the k-epimorphism with $\gamma(X) = 0$ and $\gamma(Y) = Z$. Let $\alpha : k[Y, X] \to k[Z]$ be any k-epimorphism such that at least one of* $\deg_Z \alpha(X)$ *and* $\deg_Z \alpha(Y)$ *is not divisible by π. Then there exists an automorphism $\delta : k[Y, X] \to k[Y, X]$ such that $\gamma = \alpha\delta$.*

Call an automorphism $\tau : k[Y, X] \to k[Y, X]$ *elementary* if $\tau(Y, X) = (X, Y)$ or $\tau(Y, X) = (bY + f(X), aX)$ where $f(X) \in k[X]$ and $a, b \in k$ are non-zero. Call τ *tame* if it is a composite of elementary automorphisms.

It is then deduced that:

Addendum to the Epimorphism Theorem. *δ can be chosen to be a tame automorphism.*

The following lemma is crucial in linking the epimorphism problem to the expansion techniques of the earlier paper [AM2]. In [AM1] it is given a very elementary treatment in the spirit of high school algebra. It will reappear later in a more sophisticated form in a general discussion of plane curves with one place at infinity.

We will call a polynomial $F(Y, X)$ pre-monic in Y of degree n if $F(Y, X) = a_0 Y^n + a_1 Y^{n-1} + \cdots \in k[X][Y]$ with $0 \neq a_0 \in k$.

Lemma. *Let $u \in k[Z]$ be of Z-degree $n > 0$. Let $v \in k[Z]$ be such that $k[Z] = k[u, v]$. Let $\alpha : k[Y, X] \to k[Z]$ be the k homomorphism with $\alpha(Y) = v$ and $\alpha(X) = u$. Then a generator $F(Y, X)$ of $Ker(\alpha)$ is pre-monic of degree n in Y and irreducible in $k((X^{-1}))[Y]$. This is also described as $F(Y, X)$ is a curve with one rational place at infinity.*

We can, of course, choose F to be monic in Y but let us note that, unless $v = 0$ and hence $\deg(u) = 1$, F is also pre-monic in X of some degree $m \geq 0$.

The following is a more geometric, and equivalent, version of the Epimorphism Theorem. It explains the title of the Epimorphism Paper.

Embedding Theorem. *Let C be a closed curve in the affine plane $\mathbb{A}^2_k = \mathrm{Spec}(k[Y,X])$. Assume that C is biregularly isomorphic to the affine line $\mathbb{A}^1_k = \mathrm{Spec}(k[Z])$. Let F be a generator of its ideal in $k[Y,X]$ and assume that either $\deg_Y F$ or $\deg_X F$ is not divisible by π. Then F is a* variable *in $k[Y,X]$, i.e., there exists $G \in k[Y,X]$ such that $k[Y,X] = k[F,G]$. Moreover, the automorphism $\delta : k[Y,X] \to k[Y,X]$ defined by $\delta(Y) = F, \delta(X) = G$ is tame.*

The first counterexample to the above theorems in the case where $\pi > 0$ and the gcd of $\deg_Y F$ and $\deg_X F$ is divisible by π was, to our knowledge, given by B. Segre in [Se]. Abhyankar and Moh give a family of counterexamples of the same type. Let us describe the simplest one.

Consider $u = Z - Z^{r\pi}$ and $v = Z^{\pi^2}$ in $k[Z]$. Then $Z = u + (u^\pi + v^r)^r$. Let $\alpha : k[Y,X] \to k[Z]$ be the k homomorphism with $\alpha(Y) = v$ and $\alpha(X) = u$. Then $F(Y,X) = Y^{r\pi} - Y + X^{\pi^2}$ is a generator of $Ker(\alpha)$, which is monic in Y. Taking $r > 1$ and prime to π we have a counterexample to the Main Theorem.

Interest in the automorphism group of $k[Y,X]$ is older than the above results. We have the following theorem, proved by H. W. E. Jung in 1942 [J] for $\pi = 0$ and by W. van der Kulk in general in 1953 [VdK]. It can now be deduced from the embedding theorem when $\pi = 0$.

Automorphism Theorem. *Any automorphism of $k[Y,X]$ is tame.*

Van der Kulk also proves a uniqueness theorem for the decomposition of a given automorphism into elementary ones. This was further elaborated by M. Nagata in [Na2].

In a letter dated 25 April 1971, at the time the news of the solution of the epimorphism problem first spread, Abhyankar was informed by G. M. Bergman that as a consequence of the commutative result the epimorphism theorem also holds in the non-commutative case. We have:

Non-Commutative Epimorphism Theorem. *Let $k\langle X,Y\rangle$ and $k\langle Z\rangle = k[Z]$ be the free associative algebras. Then the Epimorphism Theorem holds with $k\langle X,Y\rangle$ and $k\langle Z\rangle$ replacing $k[Y,X]$ and $k[Z]$.*

3 The Expansions Paper

Curves with One Place at Infinity

A place at infinity of an affine irreducible curve with coordinate ring A and function field K is a valuation ring V of K/k which does not contain A.

We say that A has one place at infinity if there is exactly one place at infinity and, moreover, the residue field of the valuation ring coincides with k; in other words, the valuation is "residually rational". Important examples are what Abhyankar called **polynomial curves**, i.e. curves parametrised by polynomials in one variable.

For a curve defined by a polynomial $F(Y,X) \in k[Y,X]$, a place at infinity is concretely described by what Abhyankar, tongue-in-cheek, called Newton's theorem on Puiseux expansions, or "meromorphic branches" of $F(Y,X)$. We will call them Newton-Puiseux (NP) expansions. These are substitutions $(X,Y) = (\tau^{-n}, \eta(\tau))$, where $\eta(\tau) \in k^*((\tau))$ and (i) $F(\eta(\tau), \tau^{-n}) = 0$, (ii) k^* is a finite algebraic extension of k, (iii) n is not divisible by the characteristic and (iv) the gcd of n and the support of $\eta(\tau)$ is 1. This defines the valuation ring V consisting of all rational functions $g(Y,X)$ for which $\mathrm{ord}_\tau g(\eta(\tau), \tau^{-n}) \geq 0$.

For a plane curve defined by $F(Y,X)$ with, say, $\deg_Y F(Y,X)$ not divisible by the characteristic, it can be deduced that it has one rational place at infinity (sometimes shortened to just "F is a one place curve") if and only if it has an NP expansion with $k^* = k$ and $n = \deg_Y F(Y,X)$.

Note that a polynomial F with one place at infinity is irreducible in $k[Y,X]$ and irreducible as well in the larger ring $k((X^{-1}))[Y]$. It can be shown that F is pre-monic in any choice of variables (which are not absent from F). In particular, the above equivalence is not dependent on our choice of coordinates, up to possibly switching X and Y. *For simplicity we will assume hereafter that $F(Y,X)$ is monic in Y and $\pi \nmid n = \deg_Y F$.*

It is a classic procedure to associate certain "characteristic sequences" to elements of $k[[\tau]]$. The basic tool in [AM2] is a certain clever reorganisation invented by Abhyankar of such sequences associated to any *meromorphic power series* $\eta(\tau) \in k((\tau))$. Abhyankar used to recount that he recorded several ideas about plane curves in his personal notes of Zariski's lectures on curves but always presumed that he had simply learned them in Zariski's course. He discovered that they were his own inventions only when Zariski was not aware of them. His idea of introducing the q-sequence, for instance, made the complicated formulas involved in manipulating Newton-Puiseux expansions into much simpler statements of invariance. He wrote two separate papers entitled Inversion and Invariance of Characteristic Pairs which explore the power of these techniques.

Following [A] 6.4, we give a brief outline. Let $\eta(\tau) \in k((\tau))$ and $n \in \mathbb{N}$ be given. Let J be the support of $\eta(\tau)$ and assume that the gcd of J with n is 1.

Start with $m_1 = \min(J)$ and $d_1 = n$. Set $d_2 = \gcd(m_1, d_1)$. Define m_2 as the first element of J not divisible by d_2. Set $d_3 = \gcd(m_2, d_2)$. Continue until $d_{h+1} = 1$ (since then no m_{h+1} can be found).

This **m-sequence** marks the places of gcd-drops in J and the **d-sequence** gives the successive gcd's.

A sequence equivalent to the m-sequence, but more useful, is the **q-sequence** defined by $q_1 = m_1$ and $q_{i+1} = m_{i+1} - m_i$ thereafter.

We define the **s-sequence** and **r-sequence** by $s_i = \sum_{j=1}^{i} q_j d_j$ and $r_i = s_i / d_i$. These play a crucial role in computing intersection numbers, as exemplified by the following kind of computation which plays an important role in Abhyankar–Moh theory.

Consider **a deformed initial part** of the series $\eta(\tau)$, namely $u(\tau) = c_1\tau^{m_1} + \cdots + Z\tau^{m_i}$, agreeing with $\eta(\tau)$ in all terms preceding τ^{m_i} and with Z an indeterminate. Then it is easy to calculate that the initial term of the product $\prod_\omega (u(\tau) - \eta(\omega\tau))$, where ω ranges over the n-th roots of unity, is of the form $c(Z)\tau^{s_i}$, where $c(Z)$ is a non-zero polynomial in Z.

Where necessary, we will indicate the dependence on η and n in these definitions by a suitable notation.

Note that each of the m-, q- and r-sequences have the same associated d-sequence as a sequence of successive gcd's.

One Place and Expansions

Let $F(Y, X)$ be a one place curve with NP expansion $(X, Y) = (\tau^{-n}, \eta(\tau))$. We get the well-known induced factorisation $F(Y, \tau^{-n}) = \prod_{j=1}^{n}(Y - \eta(\omega^j \tau))$, where ω is a primitive n-th root of unity. In particular, all roots of $F(Y, \tau^{-n})$ have the same support and the characteristic sequences we defined depend only on F. We note that $m_1(-n, \eta(\tau)) = -\deg_X F(Y, X)$.

We mention two essential ingredients of Abhyankar–Moh theory which are responsible for most of its successes.

1. **The Irreducibility Criterion of Abhyankar and Moh.** Assume that $n = \deg_Y F(Y, X) \not\equiv 0 \mod \pi$ and $F(Y, X)$ is monic in Y. Then $F(Y, X)$ has one place at infinity iff there is a "test series" $u(\tau) \in k((\tau))$ such that $\mathrm{ord}_\tau(F(u(\tau), \tau^{-n})) > s_h(-n, u(\tau))$.
   Moreover, given any series $u(\tau)$ passing this test, there is a "root" $y(\tau)$ satisfying $F(y(\tau), \tau^{-n}) = 0$ and $\mathrm{ord}_\tau(y(\tau) - u(\tau)) > m_h(-n, u(\tau))$.
   This Lemma, originally in [AM2], was later re-proved by Abhyankar in greater detail in [A2].

2. **The Innovation of the Approximate Roots.** Let $F = F(Y, X)$ have one place at infinity, arranged to be monic in Y. Consider the characteristic sequences as described above for an NP expansion $(X, Y) = (\tau^{-n}, \eta(\tau))$. Then for each d_i, $i = 1, 2, \ldots, h$, we get *approximate roots* $G_i(Y, X)$ defined by
   - For $i = 1$, $G_1 = Y$ and for $i > 1$, $G_i(Y, X)$ is monic in Y of degree n/d_i and
   - $\deg_Y(F - G_i^{d_i}) < n - n/d_i$.

   Such polynomials are easily seen to be uniquely defined by F for any factors of n *but, for the d_i chosen from the characteristic sequence, it is shown that each $G_i(Y, X)$ is a curve with one place at infinity and that $G_i(Y, X)$, taken mod $F(Y, X)$, has value r_i in the valuation at infinity of F.* This implies that an NP expansion of G_i matches that of F up to m_i. The idea that this should happen for $i = 2$ was first put forward by Moh and perfected to the above form by the genius of Abhyankar.

The following result is now recognised as describing a fundamental property of one place curves.

The One Place Theorem for Translates of a One Place Curve. *If $\pi = 0$ and F has one place at infinity then $F + \lambda$ also has one place at infinity for any $\lambda \in k$. Moreover, all translates have NP expansions that match through the last characteristic term. In geometric language, this means that F and $F + \lambda$ go through each other at infinity at all the singular points in a sequence of quadratic transforms.*

This is deduced from the irreducibility lemma and the explicit calculation of the initial forms in terms of the approximate roots. Over $\mathbb{C}$ it implies that the fibration defined by F is topologically trivial in a neighbourhood of infinity.

The Value Semigroup of a One Place Curve, Standard Basis

Let $F(Y, X)$ have one place at infinity with NP expansion $(X, Y) = (\tau^{-n}, \eta(\tau))$ and coordinate ring $A = k[Y, X]/(F(Y, X))$. Let $\alpha : k[Y, X] \to k[y, x] = A$ be the canonical homomorphism with $\alpha(X) = x, \alpha(Y) = y$. The valuation V at infinity on the quotient field of A is defined by $V(h(y, x)) = \mathrm{ord}_\tau h(\eta(\tau), \tau^{-n})$. We define the **value semigroup of** F as $\Gamma_F = \{V(h) \mid 0 \neq h \in A\}$. Define $g_0 = x$ and put $r_0 = V(x) = -n$. For $i = 1, \ldots, n$ let $g_i = \alpha(G_i)$, where the G_i are the approximate roots introduced above and, as we said, $V(g_i) = r_i$. Let $\mathfrak{A}$ be the set of $(h+1)$-tuples $a = (a_0, a_1, \ldots, a_h)$ of integers with $a_0 \geq 0$ and $0 \leq a_i < d_i/d_{i+1}$ for $i = 1, 2, \ldots, h$. The following results are the technical core of Abhyankar–Moh theory:

- $\Gamma_F = \{a_0 r_0 + a_1 r_1 + \cdots + a_h r_h | (a_0, \ldots, a_h) \in \mathfrak{A}\}$. These "restricted expansions" are unique.
- The "standard monomials" $\{g^a = \prod_{i=0}^{h} g_i^{a_i} | a \in \mathfrak{A}\}$ provide a k-basis of A. Note that in each linear combination over k of standard monomials there is a unique term whose valuation at infinity is the valuation of the sum.

It can be deduced from the above that *if d_2 belongs to Γ_F then (r_0, r_1) is principal and hence one of m, n divides the other*. We obtain a *proof of the Epimorphism Theorem* since in that case Γ_F is $\{0, 1, 2, \ldots\}$.

We remark that expansion techniques can also be applied to irreducible elements $F(U, V)$ of the power series ring $k[[U, V]]$, in particular when it is the completion of the local ring at the point at infinity in $\mathbb{P}^2$ of a one-place curve. If one does not insist that the expansion be of NP type (with one of the variables a power of the parameter τ), the basic definitions can be made without explicit reference to the characteristic π. This is useful in some applications (see [Ru3]).

4 Suzuki's Proof

A proof of the Embedding Theorem contemporary with that of Abhyankar and Moh was given by M. Suzuki [Su]. It is very different in spirit and the Embedding Theorem nowadays is often cited as the **AMS-Theorem.** Suzuki's paper was also very influential. It uses methods of complex analysis, in particular the theory of pluri-subharmonic functions, to study polynomial maps $F : \mathbb{C}^2 \to \mathbb{C}$, where F is an irreducible polynomial. A key result, now usually referred to as Suzuki's formula, is that the topological Euler characteristic of any singular (special) fiber is at least as big as that of a regular (general) fiber. (This generalises a fact well known in the case of proper maps.) In the case $F_0 = F^{-1}(0) \simeq \mathbb{C}$, Suzuki then goes on to show that F_0 is in fact a regular fiber. His methods apply to morphisms $\Phi : X \to \mathbb{C}$ for surfaces more general than $\mathbb{C}^2$, in particular all smooth affine surfaces. His results have been extended and sharpened by M. Zaidenberg [Za] and a proof of Suzuki's formula relying on geometric methods rather than complex analysis (or, let us say, more accessible to algebraic geometers) has been given by R. Gurjar [Gu2].

5 Positive Characteristic

The counterexamples to the Embedding Theorem in positive characteristic π cited above left open the question of whether at least the "One Place Theorem" (see 3.2) still holds. This question was raised by Abhyankar in [A]. A partially positive, partially negative answer was given by R. Ganong [Ga1]. For simplicity we assume that k is algebraically closed.

The Generic One Place Theorem. *Let $F \in k[Y, X]$ have one place at infinity. Then the generic member $F - t$, t transcendental over k, of the pencil $F - \lambda, \lambda \in k$, has one place at infinity with residue field purely inseparable over $k(t)$. Moreover, for almost all $\lambda \in k$, $F - \lambda$ has one place at infinity with multiplicity sequence at infinity the same as that of $F - t$ over the algebraic closure of $k(t)$.*

This is a "best possible" result; Ganong also gives examples of one place curves F with a special member having more than one place at infinity, or with general member having a multiplicity sequence at infinity different from that of F. Following the lead of M. Nagata [Na1] and M. Miyanishi [Mi1], Ganong investigates special properties of the pencil obtained by eliminating the base points at infinity of the pencil $F - \lambda$. Here the fiber at infinity is simply connected (it is a tree of curves isomorphic to the projective line) and a key result is a positive characteristic version of a lemma of Kodaira [Kod] on the global multiplicity of such fibers. We remark that in the case where the place at infinity of $F - t$ is rational over $k(t)$ (i.e. the residue field is $k(t)$, e.g. if $\pi = 0$) then all $F - \lambda$ have the same multiplicity sequence and the same infinitely near multiple points at infinity. In particular, the Embedding Theorem follows if F is a line.

Lines in the plane in positive characteristic are still poorly understood. Some contributions to the question have been made by [Da2, Da3, Ga1, Mo2]. The following closely related conjectures seem to have been made by several researchers. An overview of relevant results is given in [Ga3] (see also [Mi2]).

Lines Conjectures in Characteristic π.
(i) If $F \in k[Y, X]$ is a line then all $F - \lambda, \lambda \in k$, are lines.
(ii) If $F \in k[Y, X]$ is a line then the relative Frobenius w.r.t. F is a plane, i.e. $k[X^\pi, Y^\pi, F]$ is a polynomial ring.

6 Further Developments from the Expansions Paper

The Jacobian Problem

Inspired by their new machinery, Abhyankar and Moh independently produced several papers attacking the famous *Jacobian Problem*. In dimension two, it asks if polynomials f_1, f_2 in the polynomial ring $k[X, Y]$ over a field k of characteristic zero with Jacobian determinant 1 generate $k[X, Y]$.

Indeed, this problem was rejuvenated and popularised by Abhyankar along with several other problems in affine geometry of two and three dimensions as a way to attract new students to important but accessible problems in algebraic geometry.

Abhyankar and Moh propose considering f_1, f_2 as elements of $k(X)[Y]$, that is, as defining a *polynomial curve* over $k(X)$ with Y serving as parameter (see [A] for details). They quickly translated the Jacobian condition into conditions on the resulting NP expansion and produced the following striking result, among others:

The Two Point Theorem. *The Jacobian condition implies that f_1, f_2 have at most two points at infinity, i.e. their top degree forms in X, Y have at most two non-associate factors. Moreover,* if it can be deduced *that the Jacobian condition implies that f_1, f_2 have at most one point at infinity then the Jacobian problem has an affirmative answer.*

For brevity we stop here, but the problem has a long colourful history and many results (even in higher dimensions) are available at the touch of a key stroke!

Finiteness of Embeddings of One Place Curves

One way of stating the Epimorphism Theorem is to say that there is only one equivalence class of embeddings of an affine line in the affine plane up to automorphisms of the plane. Abhyankar raised the corresponding question for general one place curves. Suppose that α, β are two epimorphisms from $k[Y, X]$ onto the coordinate ring A of a plane curve with one rational place at infinity. Does it follow that α and β are equivalent? If not, is it at least true that the number of equivalence classes is finite?

Let F be a generator of $\ker(\alpha)$, say. We can then arrange by an automorphism of $k[X, Y]$ that $(r_0, r_1) = (-\deg_Y F, -\deg_X F)$ is non-principal. This gives that $d_2 = \gcd(r_0, r_1)$ is a number not in the value semigroup, by the non principal condition (see 3.3). In [ASi] Abhyankar and Singh prove the following striking result. *Two embeddings are equivalent if and only if the corresponding d_2's are equal.* This, combined with the fact that there are only finitely many negative numbers not in the value-semigroup, gives the finiteness of embeddings with a very explicit bound on the number. (We have tacitly assumed $\pi = 0$. Otherwise even the line has infinitely many inequivalent embeddings [Ga1].)

Planar Semigroups

The properties of the *characteristic r-sequence* in Abhyankar–Moh theory can be codified abstractly and semigroups generated by an r-sequence as in 3.3 have been called *planar semigroups* by Sathaye. (Sathaye actually preferred to work with the negative of an r-sequence.) It was announced in [Sa2] and shown in [SS] that every planar semi-group is the value semigroup of a one place curve (see [A2] as well). The irreducibility criterion 3.2.1 plays a significant role here.

An important question, originally raised by Abhyankar himself, is to characterise the semi-groups at infinity, or equivalently the degree semigroups, of *plane polynomial curves.* This remains unsolved to date. For further calculations and conjectures about these, see [SS], [SFY] and [M-L]. An interesting special case is the *Lin–Zaidenberg Theorem* [LZ] which asserts that there is only one class for polynomial curves with only unibranch singularities. (The theorem proves more, namely that such curves have only one quasi-homogeneous singularity and that $d_2 = 1$.)

7 A Sampling of Further Related Results

The Epimorphism Theorem inspired a large amount of research on related questions. We can only give a brief sample here. Coefficient rings more general than fields have been considered in [Ba], [RS] and [Ve]. More general closed embeddings of affine m-space $\mathbb{A}^m$ in affine n-space $\mathbb{A}^n$ have been studied in [Kal1] and [Sr]. Abhyankar, in particular, advocated the study of embeddings of lines and planes in affine 3-space. For some results on lines see [A3], [AS], [Cr], [BR] and [Sh] and for planes see [Sa1], [Ru2], [Wr] and [San]. These papers depend in a crucial way on the Epimorphism Theorem. An unexpected generalisation of the Epimorphism Theorem was developed by Sathaye in [Sa3], [Sa4] and [Sa5]. It became an important tool in several studies of $\mathbb{A}^2$-fibrations over curves. The Epimorphism Theorem is also the essential ingredient in the proof of a special case of the linearisation conjecture for $\mathbb{C}^*$-actions on $\mathbb{C}^3$ (see [KKM-LR]).

A suggestion coming out of Abhyankar's Purdue seminar was to consider closed curves in the affine plane with several places at infinity. A once punctured affine line, $\mathbb{C}^*$ when $k = \mathbb{C}$, is the obvious first candidate (see [BZ], [C-NKR], [Kal2] and [Ko2] for results in this case). Another suggestion was to investigate "field generators" (instead of "ring generators" as in the Epimorphism Theorem), that is, *polynomials F that together with a complementary rational function G generate the field $k(X, Y)$* (see [Ja], [NN], [MS], [Ru1] and [Da1]).

The Epimorphism Theorem gave a strong boost to *affine algebraic geometry,* the study of algebraic varieties closely related to affine spaces, in particular affine rational surfaces. The study of affine lines on such surfaces became an important part of their classification via logarithmic Kodaira dimension [It], [GMMR], [GM1] and [KK]. In turn, the classification theory has been used to prove the Epimorphism Theorem [GM2], [Gu1] and [Ko1].

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Prime numbers: Emergence and victories of bilinear forms decomposition

Olivier Ramaré (CNRS, Lille, France)

1 Towards a proper question: Before 1800

Historical papers on primes often start with a line like: "*The quest for the primes has a long history that begins in ancient Greece with Euclid at least 2300 years ago*." This is fundamentally true ... or is there a catch?

Let us change the lens to see better: there *are* primes in the ring $\mathbb{Q}[X]$ of polynomials in one variable over $\mathbb{Q}$, though no one asks whether one "*knows*" them. They are, they exist and we have a definition and sound algorithms to recognise them. When pushed further, we may answer: "*Yes. There are many of them, infinitely many in fact*." And a question emerges: can you find an irreducible polynomial for any given degree?

This is a completely different problem! This question mixes multiplicative properties together with some size questions! It is not about primes as such but about their sizes. And the problem gets even more entangled in the case of integers, for the size structure is closely linked with addition.

The reader can now understand why questions about primes are often difficult: they are couched in a simple language that hides their difficulty. Take some of the early "*observations*" of the period:[1]

1742: Exchanges initiated by Christian Goldbach with Leonhard Euler led, on 7 June 1742, to the statement: every even integer ≥ 4 is a sum of two primes.[2]

1752: C. Goldbach tells the same L. Euler that every odd integer can be written in the form $p + 2a^2$ where p is a prime number and a an integer.[3]

1775: L. Euler writes that every arithmetic progression starting with 1 contains infinitely many primes.

1792: Carl Friedrich Gauss gives an argument showing that there are approximately $x/\log x$ prime numbers below x, when x becomes large.[4]

1839: Johann Dirichlet proves (in today's terminology!) that every arithmetic progression without any constant factor has infinitely many primes.

1845: Joseph Bertand announces that, for each integer $n > 1$, there exists at least one prime p that satisfies $n < p < 2n$.[5]

1849: Alphonse de Polignac announces in an equally vague manner that every even integer h is the difference of two primes. The case $h = 2$ is known (since Paul Stäckel) as the "prime twin conjecture", according to Heinrich Tietze in 1959 [56].

As the reader can check, all these questions mix both the additive structure and the multiplicative structure. Individually, we understand each structure perfectly well but how do they interact? An obvious interaction is given by distributivity: $2a + 2b = 2(a + b)$, which means that if you sum two even numbers (and this latter property belongs to the multiplicative realm), you still get something that has a multiplicative property: it is ... even! The question 20th-century mathematicians endeavoured to settle is: 'Is this the *only* relation that exists?'[6]

There are many other, confusingly simple-looking questions, as well as far too many false proofs that keep roaming the web every year, some by genuine beginners who just missed a step and some by well-known difficult cases[7] (some of whom sadly occupy academic positions). I hope this paper will help the beginners with some mathematical background to understand where the difficulties lie and where the field is open. The list above contains old questions but modern work has shown deep ties between modular forms in various senses and more classical problems, in particular via the use of Kloosterman sums. Now, classical problems include the evaluation of $\sum_{p\leq X}\lambda(p)$ for cusps forms in the modular case, or the Maass case, or the automorphic case, as well as that of $\sum_{p\leq X}\lambda(p)e^{2i\pi p\alpha}$ for any $\alpha \in \mathbb{R}/\mathbb{Z}$.

Recently, many impressive results concerning primes or the Moebius function have been proved and the second aim of this paper is to present a main tool to attack these problems. Indeed, these achievements are, of course, due to the work of some tenacious individuals but have emerged after a long toiling of a large community. As an outcome, a general and flexible tool has been created, whose history will now be recounted. If this tool is now fairly common knowledge to specialists, this does not imply (by far!) that all the questions above have been answered. This tool is, however, a good weapon, whose conception has reached an evolved enough stage that it should be presented to a more general audience. Some of the ideas here may be useful in other contexts and other fields may also contribute. Such a crossing of borders has, for instance, led ergodic theorists to add their own input, including the impressive work of Ben Green, Terence Tao, Peter Sarnak, Jean Bourgain and many others.

To be complete and before embarking on the storytelling, it should be noted that several other tools have been invented. Here we concentrate on the one that is the most specific to prime numbers.

Now that we have underlined the difficulty of the diverse questions asked, let us turn toward the strategy that has been developed to tackle them. We start at the very beginning of this trade: how to handle prime numbers? We make here the first decision: instead of studying the *set* of prime numbers $\mathcal{P}$, we study its *characteristic function* $\mathbb{1}_{\mathcal{P}}$. We further assume a positive (large) real number X be given and study $\mathbb{1}_{X<\mathcal{P}\leq 2X}$ which takes value 1 on prime numbers p that are such that

$X < p \leq 2X$, and 0 otherwise. Studying a set and its characteristic function are of course equivalent but we are now ready to express $\mathbb{1}_{X<\mathcal{P}\leq 2X}$ as a linear combination of functions that do not have any special geometrical interpretation.

Historically, there have been two main lines of approach, which converged in 1968:

– The first branch of this story can be nicknamed *combinatorial* and gave birth to *the sieve*.
– The second branch, which I call *Eulerian*, goes through what is nowadays known as *Dirichlet series*.

In the rest of this paper, we shall present the characteristics of both approaches, try to point out how they interacted in history and see how they mingled to create the modern theory.

2 Dirichlet and Riemann: The early history

The Eulerian approach was really started by Bernhard Riemann in his 1859 memoir [53] and came to the fore in 1896; this is the path taken by Jacques Hadamard and à Charles de la Vallée-Poussin [11] to prove the prime number theorem. The next crucial moment was in 1968 when this method hybridised with the combinatorial approach (but this is for later!). The idea of Leonhard Euler was to consider the decomposition

$$\zeta(s) = \sum_{n\geq 1} \frac{1}{n^s} = \prod_{p\geq 2}\Big(1 - \frac{1}{p^s}\Big)^{-1}, \tag{1}$$

where the variable p ranges over the prime numbers. On the right side, one finds the primes, while on the left side, one find only integers: we potentially have a machine to extract information on the primes from information on the integers! L. Euler used it in 1737 to prove that there are infinitely many primes and, in 1796, C. F. Gauss refined the analysis to guess the prime number theorem. It was only in 1837–39 that serious proofs started with G. Dirichlet (followed by B. Riemann in 1859). In fact, L. Euler restricted the variable s above to integer values (he even considered the case $s = 1$ and, in some roundabout way, the case of negative values of s as well!). G. Dirichlet applied the logarithm of both sides of the equation above to handle the product and considered $s > 1$ a real number. Shortly after this work, B. Riemann simplified that in his eight page, epoch defining memoir [53] and took the logarithmic derivative of both members:

$$\begin{aligned} -\frac{\zeta'(s)}{\zeta(s)} &= \sum_{p\geq 2}\Big(\frac{\operatorname{Log} p}{p^s} + \frac{\operatorname{Log} p}{p^{2s}} + \frac{\operatorname{Log} p}{p^{3s}} + \cdots\Big) \\ &= \sum_{n\geq 2} \frac{\Lambda(n)}{n^s} \end{aligned} \tag{2}$$

where $\Lambda(n)$ would – 50 years later! – be called the van Mangoldt function. This formula still has the property of the Euler formula: it is potentially a machine to extract knowledge on the primes from information on the integers. The reader may worry that the Λ-function does not only detect primes but also their powers (but the latter are in negligible quantity).

B. Riemann considered complex values of s. The reader may easily guess that an inversion formula (akin to the formula that expresses the Fourier coefficients of a function in terms of this function) using complex analysis links $\sum_{n\leq N}\Lambda(n)$ with $-\zeta'/\zeta$ but, to make it work, one needs bounds for this function. The difficulty lies there: the $\zeta(s)$ on the denominator tells us that the *zeroes* of this function are going to give trouble. This is the beginning of the long and yet unfinished chase for these zeroes!

To cut a long story short, when this method applies, it usually gives very precise results. Moreover, it has wide generalisations (to number fields, to curves, be they elliptic or not, to modular forms, etc.). But the weakness of available information on the potential zeroes drastically reduces its range. In modern times, computers have entered the arena and we are now in a position to check numerically that large but finite regions do not contain any zeroes (but this is material for another paper!).

3 Same problem, different tune: The Moebius function

Before introducing the combinatorial approach, let me introduce another player: the Moebius function, named after the German mathematician August Ferdinand Moebius who introduced it in 1832. This player is more discreet than the primes, because it is less geometrical, but it is equally important. Its formal definition reads as follows:

$$\mu(n) = \begin{cases} (-1)^r & \text{when } n = p_1\cdots p_r, (p_i \neq p_j) \\ 0 & \text{else.} \end{cases}$$

This function appears in the inclusion-exclusion formula when applied to the divisor set of an integer as we will see below. It has been noticed, and this was put very formally in theorems by E. Landau in the early 1900s, that studying this function is equivalent to studying the prime numbers. However it is often far from obvious how to translate a property of the Moebius function into a property of the prime numbers: there is no direct dictionary between these two worlds. Note that

$$1/\zeta(s) = \sum_{n\geq 1} \mu(n)/n^s. \tag{3}$$

As stated above, the difficulty in (2) lies in the denominator. With the Moebius function, we directly study this denominator!

4 The combinatorial approach as seen by Legendre

Without further ado, let us embark on the combinatorial approach introduced far above!

The starting point is due to Erathosthenes and stems from the following remark: an integer from the interval $(X, 2X]$ is prime if and only if it has no divisor strictly larger than 1 and below $\sqrt{2X}$ (assuming that $\sqrt{2X} \leq X$, i.e. that X is greater than 2). From this remark, Erathosthenes deduced an efficient algorithm to build tables of all the primes below some limit. The question is how to use this algorithmic efficiency theoretically?

Adrien Marie Legendre put this idea in a *formula* in 1808 but failed to turn it into anything efficient (though this miss would be fruitful!). Let us inspect this approach on the problem of counting the number of primes between X and $2X$. We start with the number of integers between these two bounds, that is $X + O(1)$. From this number, and for each prime

$p \le \sqrt{2X}$, we remove the number of integers that are divisible by p, i.e. we consider

$$X + O(1)$$
$$-\frac{X}{2} - O(1) - \frac{X}{3} - O(1) - \frac{X}{5} - O(1) - \cdots$$

Now we have removed twice the integers that are divisible by a product of two primes, so we have to add

$$\frac{X}{6} + O(1) + \frac{X}{10} + O(1) + \frac{X}{15} + O(1) + \cdots$$

This times integers divisible for instance by $2 \times 3 \times 5$ are removed three times (divisible by 2, by 3 and by 5), then added three times (divisible by 2×3, by 2×5 and by 3×5), so we still need to remove them … The inclusion-exclusion principle is the modern way of rigorously and compactly expressing this idea but we will later need the pedestrian mechanism just described. Indeed, the formula of A.-M. Legendre leads to an enormous difficulty: the number of $O(1)$ that appear is about $2^{\pi(\sqrt{2X})}$, where $\pi(\sqrt{2X})$ is the number of primes below $\sqrt{2X}$. The addition of all these error terms gives rise to a gigantic and final $O(2^{(1+o(1))\sqrt{2X}/\log\sqrt{2X}})$, swallowing the main term with no second thoughts! One has to face a sheer wall: the combinatorial explosion. This phenomenon is often met in complexity; here it spoils the efficiency of the formula.

But the Legendre formula also suffers from a congenital disease: if we ignore the error terms, the supposed main term it gives is incorrect in view of the prime number theorem![8]

5 Two earthquakes: V. Brun, 1919, and I. M. Vinogradov, 1937

The Legendre formula was abandoned for more than 100 years, a period during which the Eulerian approach was developed. In 1910, the young Norwegian mathematician Viggo Brun went to Göttingen, at the time one of the main centres of mathematics in Europe. Edmund Landau was developing and systematising the use of analysis in number theory and, more often than not, in prime number theory. There, the young Brun was introduced to classical problems in this field, such as Goldbach's conjecture (one of E. Landau's 1912 ICM list of problems). V. Brun wanted to use combinatorial methods but E. Landau was convinced such methods would never yield anything of interest. E. Landau waited until 1921 before reading V. Brun's memoir! But this is jumping ahead: here we briefly describe this revolutionary work.

The first idea of V. Brun [5] was to give up hope for an equality in the Legendre formula. Since the error terms accumulate too much, let us stop the process before the end. The pedestrian approach described above gives the principle: after an even number of steps, one has a lower bound and after an odd number of steps, an upper bound is produced! So, if we aim only at inequalities, the process can be made to work! Well, when counting the number of primes, this lower bound is … negative, but the upper bound is much stronger than what was then known!

The Brun method (later called *the Brun sieve*) is exceptionally flexible and this first work was the source of a wealth of activities. The technical side was extremely heavy, and one had to go through page after page of evaluations.

For instance, the Brun sieve [6] gives a sharp upper bound for the number of representations of the even integer N as a sum of two primes. This upper bound is indeed sharp: it is only a multiplicative constant larger than what is expected to be true! This is one of the main ingredients that Lev Šnirel'man used in 1933 [54] to show the existence of a constant C such that every integer is a sum of at most C prime numbers.

The second seismic move occurred in 1937: I. M. Vinogradov [59] proved that every large enough odd integer is a sum of three primes.[9] This achievement relied on a magnificent discovery: Vinogradov found a way to deal with prime numbers! This method is based on the Brun sieve, which is already very intricate, so an anachronical but much clearer exposition is presented here.

When working for my thesis, I realised in 1991 [49], [50] that one could consider that the Brun sieve produces a larger sequence $\mathcal{A}$ that contains the sequence of primes. The sequence $\mathcal{A}$ *envelops* the primes: we lose in size but we gain in control. In functional form, this means that, for any positive function f, we have

$$\sum_{X<p\le 2X} f(p) \le \sum_{\substack{a\in\mathcal{A},\\ X<a\le 2X}} f(a).$$

What is expressed above is that the Brun sieve does not only give an upper bound for a counting function but also provides us with a *local* upper bound! In truth, the situation is somewhat more complicated, since the sequence $\mathcal{A}$ is maybe infinite but only serves as an upper bound for the primes when the variable is between X and $2X$. Attle Selberg in 1947 extended this setting some more: it is enough to find non-negative coefficients $\beta(n)$ such that, for any non-negative function, one has

$$\sum_{X<p\le 2X} f(p) \le \sum_{X<n\le 2X} \beta(n) f(n),$$

where, here and everywhere else, the letter p always denotes a prime variable. A. Selberg provides a construction of such good coefficients $\beta(n)$, which one should think of as a (weighted) sequence. It is easier to see on these coefficients β what has been gained by switching to an upper bound. There exists a parameter $D > 1$ (say, something like $X^{1/4}$; *strictly* less than X anyway) and coefficients λ_d^* such that

$$\beta(n) = \sum_{\substack{d|n,\\ d\le X^{1/4}}} \lambda_d^*. \tag{4}$$

The major feature which renders this expression tractable is that D is small enough. Furthermore, this parameter is at our disposal. I developed fully this idea of an *enveloping sieve* in [52], but let us go back to I. M. Vinogradov. He writes

$$\mathbb{1}_{X<\mathcal{P}\le 2X} = \mathbb{1}_{\mathcal{A}} - \Theta.$$

Nothing has been done so far. I. M. Vinogradov's crucial observation is that Θ has a special shape, namely

$$\Theta(n) = \text{lin. comb. of} \sum_{\ell m = n} a_\ell^{(i)} b_m^{(i)}, \tag{5}$$

where the sequences $(a_\ell^{(i)})$ and $(b_m^{(i)})$ vanish as soon as m or ℓ is either too large or too small; since we have constrained n by $X < n \le 2X$, if $a_\ell^{(i)} = 0$ as soon as $\ell \le L$ then the m's with

$m \geq 2X/L$ do not intervene, so our conditions are somewhat redundant. In practice, we will ensure that both $a_\ell^{(i)}$ and $b_m^{(i)}$ vanish when ℓ (or m) is small. This observation is a major turning point. I. M. Vinogradov termed *type I sums* the sums arising from (4) and *type II sums* the sums arising from (5). Along with many others, I prefer to speak of *bilinear sums* for (5) and, of course, to call (4) a *linear sum*! More will be said later on this bilinear structure.

This step being crucial, let me enunciate a simple lemma that shows the power of this bilinear structure.

Lemma 1 (Toy lemma) Given q, $L \geq q$, $M \geq q^2$ and two sequences $|a_\ell|, |b_m| \leq 1$,

$$\Bigg|\sum_{\substack{\ell \leq L,\\ m \leq M}} a_\ell b_m \, e^{2i\pi \ell m/q}\Bigg| \leq 2LM/\sqrt{q}.$$

The condition $|b_m| \leq 1$ can be relaxed if the upper bound is replaced by $2L\sqrt{M\sum_m |b_m|^2}/\sqrt{q}$. The bound L can be replaced by $L(m) \leq L$ depending on m, provided that, given ℓ, the set of m such that $\ell \leq L(m)$ is an interval.

The last condition is typically met by conditions like $\ell \leq X/m$ for some X. An early general version of the toy lemma is to be found in [59, Lemma 4] (see also [10, Lemma 8]). The proof progresses simply by writing the sum to be studied in the form $\sum_{m \leq M} b_m c(m)$ and using Cauchy's inequality. In the resulting sum $\sum_{m \leq M} |c(m)|^2$, open the square and invert summations and the result will follow readily. What has been gained here? If one sets $\gamma(n) = \sum_{\ell m = n} a_\ell b_m$ (ensure that $a_\ell = 0$ when $\ell > L$ and similarly for b_m and m), we see that we are studying

$$\Bigg|\sum_{n \leq N = LM} \gamma_n \, e^{2i\pi n/q}\Bigg|.$$

When the only information we have on γ_n is that it is bounded above in absolute value by 1, the best possible upper bound is $\sum_{n \leq N} |\gamma_n|$, which can be as large as $N = LM$. The above lemma uses the structure and saves a factor $2/\sqrt{q}$! Note the discreet conditions $L \geq q$ and $M \geq q^2$ that are, in fact, essential.

Here is what A. Ingham wrote in Zentralblatt on I. M. Vinogradov's paper:

> This is a fairly simple deduction from Cauchy's inequality, and the essential basis of the result has been available since 1910. It is hardly surprising, however, that its possibilities remained so long unsuspected. For double sums occurring in (1) do not appear naturally in the known treatments of the above problems, and in any case a straightforward application is liable to give only crude results owing to the loss involved in the use of Cauchy's inequality. It is, in fact, in devising ways of adapting the lemma to the various problems, and in elaborating techniques for bringing it to a successful conclusion, (...), that the author reveals his amazing powers.

This is a flexible and powerful principle. Our presentation is voluntarily naive; modern versions rely heavily on the underlying bilinear structure and on Bessel type inequalities for the relevant quasi-orthogonal system. But, again, we are ahead of the subject; let us go back to the beginning of the previous century!

6 The Eulerian approach continued

While sieves and their derivatives occupied the fore, the Eulerian approach was still under scrutiny. The main hurdle being the lack of information on the zeroes, Harald Bohr & Edmund Landau [2] decided in 1914 (somewhat before Brun's discovery) to look for regions that do not have *many* zeroes.[10] What they showed is a *density estimate*; for any $\theta > 1/2$, we have

$$\frac{\#\{\rho = \beta + i\gamma, \zeta(\rho) = 0, |\gamma| \leq T, \beta > \theta\}}{\#\{\rho = \beta + i\gamma, \zeta(\rho) = 0, |\gamma| \leq T, \beta > 0\}} \to 0.$$

This statement says that most of the zeroes have a real part $\leq \theta$ for any $\theta > 1/2$ (and, in fact, by the functional equation, almost all zeroes with positive real part have a real part close to 1/2). The Riemann hypothesis states that all these zeroes have indeed a real part *equal* to 1/2; the above statement is a statistical step in this direction. And this statistical step turns out to be a crucial one, since it started a very fecund branch of investigation that delivered new results for the next 80 years (this theory has now somewhat stalled).

H. Bohr & E. Landau studied the function $(1-\zeta(s)P_D(s))^2$, where P is a finite Euler product: $P_D(s) = \prod_{p \leq D}(1-p^{-s})^{-1}$. This is one of the striking features of analytic functions: it is possible to bound from above the number of zeroes by bounding from above some integral containing them. Here, H. Bohr & E. Landau integrated $|1-\zeta(s)P_D(s)|^2$ on a square and proved this quantity to be a multiplicative constant times larger than the number of zeroes in a smaller region, a process later improved upon by John Edensor Littlewood.

The Swedish mathematician Fritz Carlson in 1920 [8] simply replaced the product P_D by a sum $M_D(s) = \sum_{d \leq D} \mu(d)/d^s$ and considered $(1-\zeta M_D)^2$. He obtained in this manner much better bounds for the number of zeroes with real parts $> \theta$. Many authors continued this line of work: J. E. Littlewood, Alan Titchmarsh, Albert Ingham, Pál Turán, Atle Selberg, Askold Vinogradov and Enrico Bombieri, to name but a few! Since A. I. Vinogradov a *zero detection method* has been used instead of the method described above but this is not the subject here! It is relevant to note that Hermitian methods became more and more important and the large inequality (see (13) below) proved to be an essential tool. Yu Linnik is the pioneer of the line of investigation [40] that became understood as the use of Bessel type inequalities for quasi-orthogonal systems ... More will be said on this subject later!

There are two main highpoints of the theory of density estimates: the Y. Linnik theorem concerning primes in arithmetic progressions in 1944 and the Bombieri-Vinogradov theorem in 1965. In essence, these authors proved statements concerning density of zeroes and this is the major part of their work. These statements were then converted into results concerning the prime numbers, via some ad hoc explicit formula.

The Linnik theorem [41], [42] says that there exist two constants C_0 and q_0 so that, when q is larger than q_0, there exists, for each residue class a modulo q, a prime number congruent to a modulo q and below q^{C_0}.

The Bombieri-Vinogradov theorem [3], [58][11] says that, for each positive constant A, we have

$$\sum_{q\le \frac{\sqrt{x}}{(\log x)^{A+4}}} \max_{\substack{1\le a\le q,\\ \gcd(a,q)=1}} \left|\Delta(X;q,a)\right| \ll \frac{x}{(\log x)^A},$$

$$\text{where } \Delta(X;q,a) = \sum_{\substack{p\le X,\\ p\equiv a[q]}} \log p - \frac{x}{\varphi(q)}. \quad (6)$$

This theorem can be seen as a statistical Generalized Riemann Hypothesis and it serves in many situations as a replacement. And we should also recall the Guido Hoheisel theorem from 1930 [34]:[12] there exist two constants $X_0 \ge 1$ and $\delta_2 \in (0,1)$ such that every interval $[X, x+X^{\delta_2}]$ contains at least a prime number when $X \ge X_0$. This proof created some turmoil when it was published, as the existence of such a $\delta_2 < 1$ was only known under the hypothesis that no zero with real part $\ge \delta_2$ existed (to be precise, a slightly stronger hypothesis is needed) and seemed close to being equivalent to it. Nowadays, we term a *Hoheisel theorem* any theorem that proves a similar statement with some definite value of δ_2. The initial value provided by Hoheisel was very close to 1.[13]

7 The '68 generation

Patrick Gallagher [20] remarked in 1968 that the process is abnormally convoluted: the proof starts from the series $\sum 1/n^s$, retrieves in some fashion information on its zeroes and deduces from that information on the primes. Why not use a shortcut and skip the zeroes? This is easier said than done but P. X. Gallagher found such a shortcut. He simply multiplied $-\zeta'/\zeta$ by the kernel used for density estimates! This amounts to writing

$$\frac{\zeta'}{\zeta} = 2M_D\zeta' - M_D^2\zeta'\zeta + \frac{\zeta'}{\zeta}(1-\zeta M_D)^2, \quad (7)$$

where M_D is the Dirichlet polynomial we have defined above. The difficult term is the last one: it is the only one that still has a denominator. But this term is essentially of the shape identified by I. M. Vinogradov! A short dictionary is called for here: one has to convert operations on Dirichlet series into operations on their coefficients. The main rule is that the (arithmetic) convolution is trivialised when expressed in terms of Dirichlet series, i.e.:

$$\gamma_n = \sum_{\ell m=n} a_\ell b_m \Leftrightarrow \sum_{n\ge1}\frac{\gamma_n}{n^s} = \sum_{\ell\ge1}\frac{a_\ell}{\ell^s}\sum_{m\ge1}\frac{b_m}{m^s}.$$

This equality is either formal or valid in the domain of absolute convergence of the three series (lighter hypotheses are possible!). It should be pointed out specifically that this implies

$$\zeta(s)M_D(s) - 1 = \sum_{n>D}\frac{v_n}{n^s}.$$

The coefficients v_n are not important, as we noted earlier; they are bounded by a divisor function.[14] What is really important is that the variable d *cannot* be small.

The first two terms in (7) give rise to linear sums while the last one gives rise to a bilinear one. This transformation was completed by Robert Vaughan in 1975 [57] by removing a finite polynomial to ζ'/ζ; he introduced, for some parameter y typically like $X^{1/4}$, the finite Dirichlet polynomial

$$F_y(s) = \sum_{n\le y}\Lambda(n)/n^s$$

so that

$$\frac{\zeta'(s)}{\zeta(s)} + F_y(s) = \sum_{n>y}\frac{-\Lambda(n)}{n^s}.$$

Now, multiply together $(\zeta'/\zeta)+F_y$ and $(1-\zeta M_D)$ and expand as above. This product is a Dirichlet series whose coefficients are a convolution product of two sequences that both vanish when the variable is small: it is indeed of the special shape highlighted by I. M. Vinogradov!

What has been gained in the process?

- The sieve part in I. M. Vinogradov's process was *not* in most cases the main term, while the linear part is *expected* to carry the main term. More will be said on this point later.
- The method is simple and flexible: one can change the kernel; and it applies to other functions instead of the Riemann zeta functions, like the Dedekind zeta functions or Hecke L-series. Note that the multiplicativity is essential but not the functional equation. The method also applies to the Moebius function but so did I. M. Vinogradov's method, as already noticed by Harald Davenport [10]. However, H. Davenport reduced the problem to the case of primes while the present proof is direct.

On using this approach, P. X. Gallagher obtained in 1970 [21] a major theorem that unifies the Linnik and the Bombieri-Vinogradov theorems: the Gallagher prime number theorem, which is still unsurpassed in strength.

Since there has recently been a flourish of works on the Moebius function, and since many people have asked how this theory handles this case, let me be more precise here. The identity to use is a simplification of one I devised recently, as explained later in this survey. It relies on the simpler kernel $(1-\zeta M_D)$. We consider the identity

$$\frac{1}{\zeta} = \left(\frac{1}{\zeta} - M_D\right)(1-\zeta M_D) + 2M_D - \zeta M_D^2.$$

It only remains to identify the coefficients! It is best for applications to express the result in functional form. For any function f and provided $D \le X$, we have:

$$\sum_{X<n\le 2X} f(n)\mu(n) = \sum_{m\le D^2} u_m \sum_{\substack{\ell\ge1,\\ X<\ell m\le 2X}} f(\ell m) + \sum_{\substack{\ell>D, m>D,\\ X<\ell m\le 2X}} \mu(\ell)v_m f(\ell m), \quad (8)$$

where v_m has been defined above and where

$$u_m = -\sum_{\substack{hk=m,\\ h,k\le D}} \mu(h)\mu(k).$$

In the first summation on the right side of (8), we hope to be able to evaluate the summation over ℓ. For instance, in the toy lemma case, one selects $f(x) = \exp(2i\pi x/q)$ and the sum over ℓ is bounded by q, giving rise to a total contribution bounded by $O(qD^2)$. Concerning the second sum, we first note that the variable m ranges $(D, 2X/D)$. We cover this interval by at most $(\log(2X/D^2)/\log 2$ disjoint intervals of the shape

$(M, M']$ for some $M' \le 2M$. Our toy lemma applies provided that $D \ge q^2$. We note that $\sum_{m\in[M,M']} |v_m|^2 \ll M(\log M)^3$. Collecting our estimates, we have proved that

$$\sum_{X<n\le 2X} \mu(n)e^{2i\pi n/q} \ll qD^2 + (\log X)^{5/2}\frac{X}{\sqrt{q}},$$

provided $D \ge q^2$ and $D < X$. On selecting $D = q^2$ and assuming that $q \le X^{2/7}$, this gives our case study result:

$$\sum_{X<n\le 2X} \mu(n)e^{2i\pi n/q} \ll (\log X)^{5/2}\frac{X}{\sqrt{q}}. \tag{9}$$

This simple result is way beyond the power of the classical Eulerian approach! But the proof we have given requires not more than half a page!

There has recently been renewed activity around the Moebius function, as in [25] and [26], and around a conjecture due to Peter Sarnak.[15] This subject is somewhat off our main road, though we have to specify that getting to the Moebius function is done as above. Recently Jean Bourgain, Peter Sarnak & Tamar Ziegler have given in [4] another way to handle the Moebius function that follows a combinatorial path closer to that of I. M. Vinogradov.

In short, we have reached a point where the Eulerian approach has evolved sufficiently to resemble the combinatorial one! In both cases, the idea is to represent the characteristic function of the primes as a linear combination of linear forms and of bilinear forms. In fact, and this is most apparent in the identity used by Hedi Daboussi in [9] or in the ones used by John Friedlander and Henryk Iwaniec in [18, section 3], one can start from the convolution identity $\Lambda = \mu \star \mathrm{Log}$ and rearrange terms therein, having the size of the variables in mind.

A bilinear decomposition for the primes is one of the main ingredients of Christian Mauduit & Joël Rivat [44] in their proof of the 40 year old conjecture of Gelfand: there exists up to an error term as many prime numbers whose sum of digits in base 2 is odd or even. It is at the heart of the proof of Terence Tao [55] that every odd integer $\neq 1$ is a sum of at most five primes, and also at the heart of Harald Helfgott's proof that every odd integer $\neq 1$ [32], [33] is a sum of at most three primes.[16] The second paper has a final result better than the first one, of course, but T. Tao's paper develops ideas around small intervals containing sums of two primes that are of independent interest. H. A. Helfgott closed, after about 75 years, the proof of I. M. Vinogradov: we knew that the statement was true for large enough integers (and large enough meant really large) and bringing this bound down was no small achievement.

8 Sad news: There are limitations! The main term problem

Let us resume our general analysis. The problem that will be addressed in this section is that, in the initial Vinogradov method, the sieve part does not yield the main term. To understand properly why, here is a simplified presentation of the Vinogradov method I developed some years back. In the Brun sieve, the sequence $\mathcal{A}$ is the sequence of integers that do not have any prime factors less than some given bound z. This parameter is typically between a high power of $\log X$ and X to a power that tends very slowly to 0. To reach the primes from the interval $(X, 2X]$, we still need to remove all the integers that have a prime factor between z and $\sqrt{2X}$. Say that p is such a prime. The bad candidates have thus the form pm … and this is bilinear! Well, almost but not quite: m has to be required to have no prime factors below p if we want the representation pm to be unique and this ties p and m together … Before continuing, it should be noted that the process used is known as the Buchstab iteration [7]. I learned recently while reading the notes of [48] that, in the late '70s, Hans-Egon Richert had performed a similar analysis from the Selberg sieve.

The problem encountered is well identified: to get a proper bilinear form, one needs to *separate* both variables. This problem is serious but often not deadly. One can introduce here the number $\omega_{z,\sqrt{2X}}(m)$ of prime factors of m that lie within $(z, \sqrt{2X}]$, and the representation pm has multiplicity $\omega^*(pm)$, say, so it is enough to divide by this number. Now, though, p and m are tied in $\omega^*(pm)$! Well, yes, but less so. For most m's, i.e. for the ones that are not divisible by p, we have $\omega^*(pm) = 1 + \omega^*(m)$; the other ones correspond to integers of the shape p^2k and, since p is large enough, they are of a (usually) negligible quantity.

Since the sequence $\mathcal{A}$ that comes from the Brun sieve is larger than the primes, it leads to a larger main term! Hence what we treat like an error term contains, in fact, part of the main term. In the linear/bilinear approach, the linear part can in usual problems be shown to have the proper size, at least if believed conjectures do hold. But *the way* the bilinear form is treated induces a loss of precision that can be deadly! On our toy problem, for instance, (9) is a lot less than what is expected, namely at least:

$$\sum_{X<n\le 2X} \mu(n)e^{2i\pi n/q} \ll (\log X)^{100}\frac{X}{q} \tag{10}$$

for $q \le X^{2/7}$. But, if we were to prove such a statement, we would prove that there are no Siegel zeroes or, equivalently, that the class number of the imaginary quadratic field $\mathbb{Q}(\sqrt{-q})$ is at least $\gg q^{1/2}/(\log q)^{200}$. In fact "simply" improving the power of q from $1/2$ to $(1/2) + \delta$ for any $\delta > 0$ would be a major achievement. We will see below more as to where this limitation comes from. The best result to date [51, Corollary 5][17] reads

$$\sum_{X<n\le 2X} \mu(n)e^{2i\pi n/q} \ll \frac{X}{\sqrt{q}}\prod_{p|q}\Big(1+\frac{1}{\sqrt{p}}\Big) \tag{11}$$

for $q \le X^{1/9}$. The last product is just an annoying blemish. In the case of a prime modulus q, proving that the implied constant is < 1 would prove that there are no Siegel zeroes!

J. Friedlander & H. Iwaniec managed in their awesome work [18], [19] to overcome in some delicate cases this enormous difficulty.

There is another major limitation. The toy lemma has been presented with an additive character $e^{2i\pi n/q}$ but what happens with a multiplicative character? The bilinear form becomes trivial and nothing can be gained anymore. A path could be to express these multiplicative characters, modulo q say, in terms of additive ones modulo q. Such a process loses $\sqrt{q}$ due to the size of the Gauss sum, reducing the saving acquired to nil!

9 Other identities and divisors: The philosophy extends

It has been mentioned several times that this method offers flexibility but the reader has only seen two identities so far. In his state thesis in 1980, Étienne Fouvry used Vaughan's identity recursively.[18] At the time of writing the corresponding paper [15], Roger Heath-Brown had published in [30] and [29] a systematised version that E. Fouvry preferred to use. This systematised version consists of selecting the kernel $(1-\zeta M_D)^k$ for an integer parameter k to be chosen (É. Fouvry took $k = 12$, for instance, and later reduced to $k = 7$).

In 1961, Y. Linnik produced in [43] another kind of identity by considering $\log\zeta = \log(1-(1-\zeta))$ together with the Taylor expansion of $\log(1-z)$ around $z = 1$. We present the modification due to Heath-Brown in [30, Lemma 3] in which the zeta function is multiplied by the finite Euler product P_D introduced above in F. Carlson's proof. The function $\log(\zeta P_D)$ is also the Dirichlet series $\sum_{n\geq 2}^{*} \frac{\Lambda(n)}{\log n}\frac{1}{n^s}$, where the star means that n has only prime factors $> D$; it is expedient here to introduce the product Π_D of all the primes not more than D. The condition on n is then simply that n and D are coprime. On the other side the function $(1-\zeta P_D)^k$ has for any positive integer k the Dirichlet series representation $(-1)^k\sum_{n\geq 1}^{*} d_k^\sharp(n)/n^s$, where the summation is again restricted to integers n prime to Π_D and where $d_k^\sharp(n)$ is the number of ways of writing the integer n as a product of k integers, all *strictly larger than 1*. We get, when n is prime to Π_D,

$$\frac{\Lambda(n)}{\log n} = \sum_{k\geq 1}\frac{(-1)^{k+1}}{k}d_k^\sharp(n). \qquad (12)$$

On restricting n to the range $(X, 2X]$ and assuming that $D^{K+1} > 2X$, the summation above can be truncated at $k \leq K$. Moreover, if we abort the summation at an odd (or even) number of steps, we get an upper (or lower) bound, as in the inclusion-exclusion principle!

But a closer look at Linnik's identity is called for: *it transposes problems for primes into problems for divisor functions*. This is what transpires from É. Fouvry's work [15]: if one knows well enough the divisor functions, up to products of six divisors, then this implies an improved Bombieri-Vinogradov theorem of the primes: the inequality $q \leq \sqrt{x}/(\log x)^{A+4}$ could be replaced by $q \leq x^{\frac{1}{2}+\delta}$ for some positive δ. Such a theorem would be *stronger* than the Generalized Riemann Hypothesis! The reader can see rapidly how modular forms come into play here: the distribution of the divisor function is linked with the distribution of Kloosterman sums, which are in turn coefficients of modular forms.

This feature of Y. Linnik's identity can be found again in R. Heath-Brown's identity if one forces k to be so large that $D^k > X$. In this manner, the bilinear part does not come into play for integers below X! In functional form, this reads:

$$\sum_{n\leq X}\Lambda(n)f(n) = \sum_{1\leq r\leq k}\binom{k}{r}(-1)^{r+1}\times \sum_{\substack{n_1,\cdots,n_r\leq D,\\ n_{r+1},\cdots,n_{2r},\\ n_1\cdots n_{2r}\leq X}}\mu(n_1)\cdots\mu(n_r)\log n_{2r}\, f(n_1\cdots n_{2r}).$$

This is all well and good but we already have difficulties treating products of three divisors (see the groundbreaking [36]) not to mention products of four of them, so it may be more efficient to simply consider these divisor functions as convolution products and resort to I. M. Vinogradov's bilinear form approach. In the above, one may tie some variables together, say d_1 and d_2, in a single $m = d_1d_2$ affected by the coefficient u_m defined above!

The divisor angle can, however, be made to bear with more efficiency if one aims at a result weaker than a Bombieri-Vinogradov theorem. É. Fouvry put this philosophy into practice in [16]: the quantity considered is, for some fixed a and some positive δ:

$$\sum_{q\leq x^{\frac{1}{2}+\delta}} c(q)\Delta(X;q,a),$$

with the weights $c(q)$ fairly general and, yet again, convolution products of special kinds (Δ is defined in (6)). This is at the heart of the recent breakthrough of Ytang Zhang [60]: there exist infinitely many pairs of primes p and p' such that $|p-p'| \leq 7\cdot 10^7$. The argument follows the pathway opened in 2006 by Daniel Goldston, János Pintz and Cem Yıldırım [23], [22] but the main novelty comes from the treatment of the error term or, more precisely, in curbing the proof so that it produces an error term of a special form, as already noted by A. Ingham in his assessment of I. M. Vinogradov's work. Studying this error term is also no small task! Let us note that this entails controlling bilinear terms of the form we have already seen but also some convolution of three divisors; or a three-linear form; or, as Y. Zhang puts it, *a type III sum*.

10 The combinatorial approach, revival time

While the work on identities has been going strong, a different line continued from I. M. Vinogradov's approach. We have seen that the correcting term from the sieve part contained part of the main term and that a coarse treatment via Cauchy's inequality was not enough. Some authors, however, developed a gentler treatment in some cases; such a line started in [31] where the authors obtained a strong improvement on the Hoheisel theorem (any $\delta_2 > 11/20$ is accessible; compare with Hoheisel's initial value!). Combinatorial ideas are put into effect and show their teeth! This has been amplified, developed and refined by Glyn Harman in several papers [27], [28], [1] in what this author calls his *adaptative sieve*. This is surely a very accomplished work and it led to the best results in many problems (like $\delta_2 = 0.525$).

In this section I should mention the development in [12, Section 6] and, in particular, Theorem S thereof. This subtle theorem ensures that the sequence of primes will be properly distributed in some sequences, provided one knows how to bound some linear sums as well as some bilinear ones. This step is extremely difficult in this application, as the accessible information is not enough for a usual approach! A special combinatorial treatment is required which is contained in the Theorem S mentioned, in particular for handling products of three divisors of about the same size.[19] This falls within the general philosophy we have developed so far. The novelty here is that the bilinear form arising from I. M. Vinogradov's approach is treated with more care and the main term extracted from it.

11 Treating the bilinear forms

This paper has covered at length that a bilinear structure was involved in all these representations, whether directly via Eulerian identities or after more work via combinatorial means (and now both methods mix happily!) but the way to treat this bilinear part has up to now remained extremely coarse, essentially via the toy lemma above. Even at this level of coarseness, the method yields impressive results but a better understanding is called for. And it will show again how the sieve ideas and the Eulerian approach mingle together. In most of the problems on primes, the treatment of the bilinear sum is the most difficult part and, in return, what we are able to prove at this level conditions the kind of identity one has to prove or choose.

One way to start telling this part of the story is through the Bombieri-Vinogradov theorem. In the initial proof, the one that deals with density estimates of zeroes, a major role is played by an inequality that finds its origin in the work of Y. Linnik: the large sieve inequality. It was later discovered by Hugh Montgomery [45] that this inequality could be used in a sieve context and … led to results as strong as the Selberg sieve! In some sense, this inequality is *dual* to the Selberg sieve [37] and this notion of duality has to be understood in the usual sense, i.e. when a bilinear coupling is at stake. Let me state a special case of this inequality in the strong form given by H. L. Montgomery & R. C. Vaughan [46], and at the same time by A. Selberg with a different proof:

$$\sum_{q\le Q}\sum_{\substack{1\le a\le q,\\ \gcd(a,q)=1}}\Big|\sum_{n\le X} b_n e^{2i\pi\, na/q}\Big|^2 \le \sum_{n\le X}|b_n|^2(X+Q^2), \quad (13)$$

valid for any sequence of complex numbers (b_n). Such an inequality is of course reminiscent of our toy lemma above. It should be looked upon as a Bessel type inequality for a quasi-orthogonal system. From a practical viewpoint, if we were given any single sum above, say $\sum_{n\le X} b_n e^{2i\pi\, na_0/q_0}$, Cauchy's inequality would give us the bound $\sqrt{\sum_{n\le X}|b_n|^2 X}$ and it is the best possible at this level of generality. The above inequality tells us that, for the same price, we can bound many more sums! This is provided Q^2 is less than X, which will be our case of use. So the idea is to put as much as we can on the left side and use this bound. The reader will not be surprised to find an inequality of this type in H. A. Helfgott's work.

When compared with our toy lemma result, the reader may worry about the missing $1/\sqrt{q}$ …and rightly so! But we have a summation above of length about q over a modulo q: in the toy lemma, simply split the variable ℓ according to its residue class modulo q. There remains a slight difficulty, as ℓ is not guaranteed to be coprime with q but this hurdle is easily overcome.

This principle can be pushed very far and many more sums incorporated on the left side! To prove (11), and elaborating on unpublished material by A. Selberg in 1972–73 and of Yoishi Motohashi [47], I developed in [51] a quasi-orthogonal family of identities for the primes, where the polynomial M_D is replaced by a family $M_D^{(r)}$. One of the first lemmas of the proof is the following (version of an) inequality due to Y. Motohashi [47, Lemma 3], with $R=\sqrt{N/T}$:

$$\sum_{\substack{r\le R/q,\\ \gcd(r,q)=1}}\frac{1}{\varphi(r)}\sum_{\substack{1\le a\le q,\\ \gcd(a,q)=1}}\int_{-T}^{T}\Big|\sum_{n\le N}\frac{b_n c_r(n)}{n^{it}}e^{2i\pi\frac{na}{q}}\Big|^2 dt \ll X\sum_{n\le X}|b_n|^2,$$

provided that b_n vanishes as soon as n has a factor in common with q. Here $c_r(n)$ is the Ramanujan sum. The reader should not be scared of such an inequality, for it is a gentle monster! If we take $t=0$, $a=1$ and $r=1$, the inner sum is simply $\sum_{n\le N} b_n e^{2i\pi\, n/q}$ as in the toy lemma case. But for the same price, we have added an integration over t in a large range (this part has been classical since P. X. Gallagher [21, Theorem 3]), as well as a summation over r which comes from A. Selberg.

The inequality above says that the three families of "characters" $(c_r(n))_r$, $(n^{it})_t$ and $(e^{2i\pi na/q})_a$ are quasi-orthogonal in themselves but also when mixed one with the other.

How can one put that into practice? That's more easily said than done but here are some hints: when p is a prime number prime to r, the Ramanujan function of order r takes value -1 at p, i.e. $c_r(p)=-1$. As a consequence, when $r\le X$, and for any function f, we have

$$\sum_{X<p\le 2X} f(p) = -\sum_{X<p\le 2X} f(p)c_r(p).$$

We can use this fact to introduce an average over r and, for instance, for any non-negative function g, we find that

$$\Big|\sum_{X<p\le 2X} f(p)\Big|^2 = \frac{\sum_{r\le X} g(r)\Big|\sum_{X<p\le 2X} f(p)c_r(p)\Big|^2}{\sum_{r\le X} g(r)}.$$

On using a bilinear form representation, this $c_r(p)$ will become a $c_r(\ell m)$ and if we can prove a proper Bessel inequality, only one term on the right side will contribute: we will save the denominator! A similar process is used in [14]. See also [13] and [39] for more comments on *amplification* techniques.

The summation over r can be regained by the process above but a similar process does not apply to the other "characters" and this is where the limitation comes from: we do not know that the sequence (b_n) does not conspire with some $e^{i\pi na_0/q}/n^{it_0}$ for instance to give rise to a large contribution. We would be surprised if this were to happen, of course, but, at this level, we do not know how to eliminate this possibility. We say in short that the *diagonal* contribution matters most. There has been a good amount of work to try to dispense with it. The general theme is to go back to the proof of the large sieve type inequalities we use: in these proofs, the Fourier transform has an important role, very often in conjunction with the Poisson summation formula or spectral theory (as for instance in [12]). So the idea is to introduce such a Fourier transform where a smooth variable occurs and use the Poisson summation formula. This is for instance what is used in [17]. The Linnik dispersion method [43] is another similar L^2-mechanism that eliminates the diagonal contribution (see for instance [15, Section 3]).

12 This is *not* the end!

I hope the reader now has a proper idea of the flexible tool praised so much in the introduction! There remain large parts of unexplored territory, as well as some peaks in the distance … A.-M. Legendre asked long ago whether every interval $(N^2, (N+1)^2)$ contains a prime when N is a positive integer. This is roughly equivalent to showing that an interval of length $\sqrt{x}$ around x contains a prime: the methods we have give $x^{0.525}$ but, even after all the plausible refinements, reaching $x^{0.5}$ will require a novel input. The sum $\sum_{p \le X} \mu(p-1)$ is still a mysterious entity and there are many other problems on primes that are unsolved, at present time just out of our grasp but who knows what will happen tomorrow!

Notes

1. Such observations sometimes had the status of theorems and sometimes the status of truth, or maybe "*experimental truth*", since the very notion of proof was shaky at the time.
2. Historians later discovered that René Descartes had stated this property some 50 years before. One of the modern princes of arithmetic Pál Erdös commented: "It is better that the conjecture be named after Goldbach because, mathematically speaking, Descartes was infinitely rich and Goldbach was very poor."
3. He was wrong: 6077 is an exception but it seems to be the last one!
4. This will become the "prime number theorem".
5. This not-so-easy proof is very popular, the downside being that few know that much more is accessible. For instance, there is a prime p satisfying $2n < p < 3n$ provided $n \ge 2$.
6. I am skipping here the fascinating *abc*-conjecture, which is known to hold in the case of $\mathbb{Q}[X]$ and has created some turmoil recently in the case of $\mathbb{Z}$ …
7. No. I won't give you my list!
8. This "main" term is $\prod_{p \le \sqrt{2X}}(1-\frac{1}{p})X$, which is equivalent, by one of Mertens' theorems, to $2e^{-\gamma}X/\log X$. We have $2e^{-\gamma} = 1.122\cdots$ while the prime number theorem will, almost a century later, show that it should be 1.
9. At the time, I. M. Vinogradov was also nearly drowned under the administrative work he had to cope with as director of the Steklov Institute!
10. By the way, Harald Bohr was the rising Danish star while V. Brun, two years older, was the Norwegian rising star. Harald Bohr was also an accomplished football player and was, together with his teammates, responsible for the greatest defeat of the French national team (17 to 1!).
11. The Russian mathematician Askold Ivanovich Vinogradov is not to be confused with the other Russian mathematician (the mathematical great-grandchild of Pafnuty Lvovich Chebyshev) whose work is at the heart of this paper: Ivan Matveyevich Vinogradov
12. G. Hoheisel was the mathematical grandchild of David Hilbert.
13. Any $\delta_2 > 1 - (1/33\,000)$ would do. This is really small!
14. We have $v_n = \sum_{\substack{d|n,\\ d\le D}} \mu(d)$ when $n \ge 2$ and 0 otherwise. And thus $|v_n| \le d(n)$.
15. See also [35, (13.7)]; P. Sarnak's conjecture somehow quantifies this statement. See also [24].
16. Both papers have been submitted; there are good reasons to believe in their solidity but rules are rules and the checking should be completed before the results are fully accepted!
17. As I said, rules are rules and the result I mention now has only been submitted!
18. He iterated it 12 times! The formulas obtained were so long that he printed them in landscape format …
19. For the reader who wants to follow this proof in a gentler manner, and who can read French, I recommend the book [38].

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Interview with Jean-Pierre Bourguignon: "Mathematicians must move up a gear"

Marc Herzlich (Université Montpellier 2, France)

Jean-Pierre Bourguignon, 66, retired on 31 August 2013 from the Centre National de la Recherche Scientifique (CNRS) and from his charge as Director of the Institut des Hautes Études Scientifiques (IHÉS) near Paris. He started his mathematical career in the late 1960s as a researcher and has remained a CNRS fellow ever since. He was on leave as a professor at the École Polytechnique from 1986 to 1994 and then took the reins of IHÉS in Bures-sur-Yvette. His involvement in scientific policy issues for mathematics in France and in Europe has been very important over the last 30 years. He was the second President of the European Mathematical Society from 1995 to 1998. During his presidency, the EMIS website (www.emis.de) was established in collaboration with ZentralBlatt MATH and the journal of the society (JEMS) was initiated. He was also one of the founding members of Euroscience and he contributed to every EuroScience Open Forum. On the occasion of his retirement, the newsletter has carried out a long interview with him.

Youth and Studies

Did your interest in mathematics start early as a child? Or did it come later? When did you decide you were really "into mathematics"?

The moment I got interested in mathematics is very clear in my memory. During high school, I was a good student in mathematics although I did not work much outside the class. I had a very good teacher for five years, who used to ask the best students to help others. This certainly had an impact on me as it forced me to consolidate my knowledge of school mathematics.

But, for my last year in high school, I got another teacher. He was known to be a true mathematician but not very skilled in teaching. He spoke about many things which were not supposed to be taught in class but he showed a real passion for science, astronomy, etc., although it was really difficult to understand what he was talking about. The first grade I got in mathematics this year was 0.5 and some of my classmates got 0.25 and even 0. (Note: French marks range from 0 to 20, 20 being the highest grade). But this is the moment when I realised there was indeed something to understand in mathematics and I began to work substantially harder on it to respond to the challenge of this teacher. Before that, I was more attracted by literature and philosophy.

Due to these efforts, the first year after the *baccalauréat* was a real delight, but the second one was close to hell (it was the year of the entrance competition to *Grandes Écoles*). The reason for this was that my mathematics teacher used to assess students with respect to what he expected from them. "Average" students would get good marks if they did reasonably well but, if he was expecting more from you, you had to achieve something extraordinary to get a good mark (and usually you got a much lower one!).

Jean-Pierre Bourguignon (Photo: Jean-François Dars)

Listening to you makes me think that, sometimes, one should not teach mathematics in too "smooth" a way, at least to the best students…

I am convinced that one should not hide difficulties when teaching mathematics. Young people like challenges. If the content is empty, they will not like what you teach. And you need to make them think by themselves. This was certainly the most important gift I received from that teacher in high school I spoke about. Note that, in all these classes, I had good physics teachers to reassure me of my ability to understand science but it was not enough to motivate me to study physics further.

You then entered École Polytechnique…

I entered the school in 1966, at a time when the level of some courses was quite low: a number of teachers were alumni who had not done any research. Mathematics was an exception, as the teachers were coming from outside, such as Gustave Choquet, who was the analysis professor for my year, and Laurent Schwartz. This was also partly the case in physics but not in several other sciences. Hence, we students organised ourselves to set up some "commando operations" to replace some deficient teachers. For instance, we organised reading sessions in mechanics and in probability theory. This is the way I got myself interested in mechanics as I read relevant material from many different sources: Sedov, Truesdell, etc.

Differential geometry

But you undertook PhD studies in mathematics, not in mechanics…

After finishing my studies at *École Polytechnique*, I thought I should do research in mechanics, as I felt not strong enough in mathematics when comparing myself to students of my generation who had followed the curriculum of *École Normale Supérieure*. I had already formed my own ideas: I had read Vladimir Arnold's paper linking the motion of fluids and the geodesic flow on the diffeomorphism group of a manifold; I felt this approach could be applied to solving the Euler equations. But, when I approached people from the mechanics department at the university, who were old-fashioned, they did not accept the idea that I had chosen a subject for my thesis myself. This dissuaded me from working in mechanics. (Actually, a few months later, I learned from Claude Godbillon that the outline I had suggested had been successfully put to work by David Ebin and Jerrold Marsden.) At the same time, I began to meet people in mathematics who were much more open. I then turned to the part of mathematics that was the closest to mechanics and this is the way I chose to work in differential geometry under the supervision of Marcel Berger.

I was hired as a research assistant by the CNRS in 1968, right after leaving *École Polytechnique*, at the age of 21. This was an extraordinary chance because it gave me much freedom: at that time such positions were almost permanent. This is why I could easily take advantage in 1972 of an invitation made to me by Jim Simons, then chairman of the mathematics department at Stony Brook, after he attended one of my lectures in Paris. I spent the academic year 1972-73 at Stony Brook. The department at Stony Brook was an extraordinary place for differential geometry: Jeff Cheeger, Detlef Gromoll and Wolfgang Meyer were there but also David Ebin, John Thorpe, etc. It was also the first position held by Shing-Tung Yau. Actually, we were teaching the same calculus course; this helped us become close friends and we started conducting a research project together, an attempt to disprove the Calabi conjecture that led to a joint publication. Bob Osserman invited me to spend the summer in Stanford, during which an invitation for lunch by Shiing-Shen Chern made a big impression on me. You have to realise that, at that time, differential geometry was a bit despised among French mathematicians. This stay in the US helped me a lot as, having worked in a different and very challenging environment, I came back convinced that interesting mathematics could be done in differential geometry. And I also got to know better several people who turned out to be very important for me later on, such as Jim Simons, Blaine Lawson and Shing-Tung Yau, just to name a few.

While in Stony Brook, I also witnessed the first attempt at a common seminar between physicists and mathematicians on gauge theory, co-organised by Jim Simons and Chen Ning Yang. It was a failure because mutual understanding was quite low at that time (things changed shortly after, around 1974). But that was not at all a failure for me, as I then had the hint that the fact I had learned some serious physics at *École Polytechnique* put me in a peculiar position among mathematicians. This proved to be very profitable later when, in the late 1970s,

Jean-Pierre Bourguignon and Sir Michael Atiyah (Edinburgh). (Photo: Jean-François Dars)

gauge theory became very fashionable among physicists. I had the appropriate background to tackle some of the problems of interest to physicists. An important question at that time was the stability of Yang-Mills fields: "is every weakly stable critical point of the Yang-Mills functional on S^4 an absolute minimum?" Blaine Lawson was spending a sabbatical year at IHÉS and we met regularly. One day, while checking with him the outline of an introductory course on gauge theory I was about to give to physicists, I mentioned that I knew how to prove "one half" of the much sought after theorem. And, to my surprise, he replied that he had a proof of the other half! Hence, it took us little time to check that putting the pieces together gave the result. Note that one idea came from a paper of Jim Simons in dimension 5. I like this story because it shows how things work sometimes in mathematics and also why it is so important to have open communication between people.

Working for the community and IHÉS

Was it already clear for you that you would turn from research proper to research management at some point?

Not at all! It came somewhat later, at the beginning of the 1980s. I spent 1980 in the US visiting the Institute for Advanced Study (IAS) for half the year and Stanford for the other half, and at the same time I was asked to run to become a member of the so-called "CNRS National Committee for mathematics". It happened that the person expected to chair the committee was not elected! As a result, I became president of that committee when I was only 33. And, since I was so young, I had to work very hard on all subjects the committee had to tackle to gain some weight vis-à-vis some much more senior scientists in mathematics and in other sciences. After that, I kept being involved in various committees and boards.

Now, let us make a giant leap in time: you have been leading IHÉS for 19 years. What would you give as your greatest success at the institute?

Let me start with something that may sound strange at first. I am very pleased with the quality of the administrative staff we managed to hire. The institute can rely on them. This might appear as not as important as the

Jean-Pierre Bourguignn with Maxim Kontsevitch (IHÉS), Mikhail Gromov (IHES), and Pierre Cartier (IHÉS).
(Photo: Jean-François Dars)

scientific side of the director's job but I really believe that it played a very significant role in the development of the institute. This is something we scientists sometimes forget, but we shouldn't.

And on other matters?
For the director of IHÉS, one of the most important tasks, and certainly the most intellectually challenging, is to keep recruiting professors at the highest level. For instance, it was not so obvious that the institute could attract Maxim Kontsevich. The preparation of such hiring is very stimulating as it requires looking in all directions and making use of many different contacts and advice. The other thing I would like to mention is the efforts to improve the financial resources, initially to repair and improve the buildings. During my first six years as director of IHÉS, support to renovate the premises was promised several times but the money never came. It took a lot of time before we realised that our methods when looking for money had to be changed. The key person who helped us achieve this change was Jean-Paul Gimon, who had been a leading figure at the Crédit Lyonnais Office in New York. He convinced us that the institute had to become professional when looking for private funds. In fact, he did more than that, as he managed to provide the money necessary for the institute to begin its first fundraising campaign through a donation of the Flora Family Foundation financed by the Hewlett family (as his wife Eleanor is the daughter of Mr William Hewlett). Then, properly advised and having trained some staff members, the institute finally conducted successfully its first campaign, raising 13 million euros in three years. Now, twelve years later, the funds gathered by IHÉS amount to nearly 38 million euros, with an endowment now close to 30 million euros.

Jim Simons played an important role in that?
Of course! The Simons Foundation chaired by his wife Marilyn gave a substantial part of this money, always through challenge-gifts: "I give some amount if you find the same amount from other sources". For the last one, 2 million euros are still pending. Of course, the quality of the institute and its international image and rooting were absolutely critical in obtaining such support but a lot of people, from many countries, also helped, e.g. Shiing-Shen Chern from China and Heisuke Hironaka from Japan, and also France - to name just three of them: André Lévy-Lang, Philippe Lacour-Gayet (who, as chief scientist of Schlumberger, developed a vision of the role that mathematics can play in engineering oriented companies) and Marwan Lahoud, one of the directors at EADS-Airbus, who reads mathematics books when he sits on a plane, and he travels a lot! (We first met at *École Polytechnique* where he was a student of mine.) I would also like to mention the involvement of the AXA group in funding research. IHÉS has been one of the beneficiaries through the AXA chair occupied by Maxim Kontsevich. All of this was obtained because we could raise the institute's profile and learn from others.

How much of this can be made to work for other centres? How can people, either public entities or private donors, be convinced to give money to research centres whose only activity is to enhance contacts between people, especially in a time when so much focus is put on electronic communications, the internet and so on?
I believe first that the local and national contexts have to be taken into account. Even in the United States, things may be difficult. If one puts aside the IAS, which serves as a model for all other institutes, other centres face difficulties. This is a frequent topic of discussion between institute directors, and the common attitude is to share contacts and information, even though the situation has become tough everywhere. But, once again, if one is to follow this path, it should be done as real professionals would do it. This is something the mathematical community should consider because the real alternative is between doing nothing and trying to watch how it is done according to the standards. IHÉS could succeed thanks to the advice and support of a few people outside mathematics (and even outside the science community) who opened our eyes. It meant learning a lot and working hard.

In Europe, we are lucky enough to have a very dense network of mathematicians…
And very diverse, too.

The strategy you describe may be valid for such prominent places as IHÉS. But what about more modest ones? And university departments of any size?
There is certainly a danger there. One must realise that public money will almost surely not grow. Hence, one has to find complementary ways of funding research and to secure positions for young people. I am very concerned by the situation for young people. Too many people behave as if it can be ignored. For instance, in France, the number of young mathematicians that might be hired in universities or research institutions in the next four to five years could be 40% lower than the current number because of the demography: the number of people expected to retire during this period will be very small. This is true also for several other European countries, and this

may lead to a disaster for the mathematical community and some other communities.

For mathematics, there is at least some hope because a significant number of private companies are becoming aware of the importance of mathematics – and of mathematicians! – for their economic development. They may then become convincing advocates for mathematics. Of course, the drawback may be that private companies attract all the bright young people with challenging opportunities and higher salaries.

But are these private companies aware that training the young people they might later hire requires keeping a wide number of university departments with a broad spectrum of research, both in the pure and applied parts of mathematics?

At least some do, as their leaders have now understood that they need a diverse population of young mathematicians, with a strong background in different subjects (not only numerics or statistics). My fear is rather that careers of mathematicians in private companies may become so attractive that almost nothing is left for academia. To avoid this, the mathematical community needs to find allies among these private companies at the European level. If European countries are not able to train the next generation of researchers, the damage will also be very important for them, although we must be aware that they could look for their recruitment in Asia or in Latin America.

Careers in private companies are important even for research in pure mathematics. A better job market in the private sector for mathematicians means less risk for students who undertake graduate or PhD studies, even in pure mathematics! The situation is much more subtle than the traditional dilemma between pure and applied research. Identifying companies with this profile and working with them, both for the scientific input or for lobbying, is likely to be really worth the effort.

Mathematics in Europe

Let us come back to the academic community. You were expressing your concern about the future of European mathematics...

I am afraid the threat is real. Some of the funding mechanisms that have been devised in the last 10 to 20 years have given a strong impetus to certain places but they have also strengthened the brain drain inside Europe or even inside a single country. To give examples, the support from the European Research Council (ERC) that goes to researchers in Central or Eastern European countries is a very small proportion of the total amount distributed. This low level of support has more to do with the infrastructures available than with the real quality of colleagues working there. The South of Europe may soon face the same type of problems because the academic job market is in such a bad shape there. You may observe this phenomenon even at the scale of Germany: by raising Berlin and Bonn to such a high position (which they certainly deserve), the Exzellenzinitiative is also contributing to brain drain in other places.

Mikhail Gromov (IHÉS) lecturing during the conference given in honor of Jean-Pierre Bourguignon's retirement on September 13th at IHÉS; on the left, Laurent Lafforgue (IHÉS), chairman of the session. (Photo: Jean-François Dars)

What could be done at the European level to fight this? The danger seems to be particularly high for medium-size centres, which cannot pretend to be at the very first rank in the world but are also too large and developed to focus their research on a limited number of subjects where they are competitive?

I believe that research networks are the answer. Unfortunately, the current funding system encourages the opposite by concentrating money on small, localised and short-term teams. This may be fruitful for certain sciences but mathematicians must urgently organise themselves to explain better that our science has different needs: besides new resources connected to modelling and big data, we need money to organise and to attend meetings, some bringing together people working in neighbouring areas on a regular basis; and people also need to meet other scientists who are scientifically close to them but not identical! And for this we have to learn how to join forces with other scientists who have a different sociology. One example which may be used as a model is the MITACS network (Mathematics of Information Technology and Complex Systems) in Canada. It was managed by the three Canadian institutes (the Centre de Recherches Mathématiques, the Fields Institute and the Pacific Institute for the Mathematical Sciences) but it really brought together all academic institutions and researchers interested in industrial mathematics.

Another important concern is the average size of projects that European institutions are willing to fund. It is currently still increasing because the larger the project, the smaller the cost, proportionally, that is dedicated to management. To fight this, we must be able to prove that mathematical networks (which often involve much less money) will also cost less in management.

But I am very sad to say that a positive evolution in the future doesn't seem likely. For instance, the ERC cannot handle this under its current rules... On the other hand, I believe we have an urgent need for more strategic thinking in the way we can work with European scientific institutions.

To be provocative, couldn't one say that Europe is also lacking purely individual grants that could foster mobility and exchanges if they are attributed to a sufficiently large number of people?

Jean-Pierre Bourguignon with Christoph Sorger (University of Nantes) and Emmanuel Ullmo, new Director of IHÉS. (Photo: Jean-François Dars)

I am not sure because this could again rapidly lead to concentration of money on a few individuals.

What about the role of learned societies, especially the EMS?
I believe it must increase its role but in an original way. Zentralblatt MATH is an exemplary case of an institution that European mathematicians have to fight for. EMS backed it up early and played a crucial role in improving it. Let us take also the example of publications. It is obvious to everybody that the publication business is undergoing major changes technically and economically. Almost all mathematical societies in Europe have publications and almost all face difficulties in keeping them healthy, the only exception being the LMS. The EMS has launched its own Publishing House (for sure a good move) but it competes with the publishing houses of national learned societies. Couldn't we try and imagine new means of cooperation, leading to shared costs, so that publications of the individual member societies are put under some kind of "European umbrella" without losing their national identities? Europe is certainly more complicated than Northern America but I am afraid it won't survive if progress is not made very soon about such issues.

One of the most important issues the mathematical community presently faces is the cost of publications. Is the European level the right level to act?
Commercial publishers have a strong influence in Brussels and they almost succeeded in convincing officials that "open access" should be understood as "gold open access", a true semantic hold up. They were strongly backed by the UK government but it may now soften its position as a recently issued report by the UK Parliament shows that supporting "gold open access" rather than "green open access" may turn out to be a very poor strategy for the future of the country. At the European level the current state of affairs is somewhat unclear. If it has now been prominently stated that open access will be the common rule for Europe, will it be gold or green, or even something else? Someone computed that turning to gold open access would multiply the total cost of publications for the French mathematics community by a factor of three. Once again, we mathematicians must explain our specific modus operandi but we cannot fight alone; we must find allies, not only among colleagues working in social sciences or humanities (who may have publishing habits that are the closest to ours) but also within the natural sciences.

Mathematics, mathematicians and society

A last question: you have also spent much time on the relations between mathematics and the general public. The way science is perceived is also a crucial issue for mathematicians. What should we do more to improve our image?
I believe that the mathematical community as a whole hasn't put enough thought into this. The situation may differ from one country to another but this is especially true in France. Here again we must become more professional. The main media now have a very high level of expectation for documents coming from the scientific community and the way they engage the discussion with a broad public. I have been confronted with this on many occasions: films, conferences or the recent Fondation Cartier pour l'art contemporain exhibit "Mathematics, a Beautiful Elsewhere". Of course, this doesn't mean that small scale operations, at the level of a few persons or a single mathematics department in a university, should be forgotten. Such initiatives are very useful but, if we really want to reach a wide audience, we must work with professionals. Another thing missing is a repository of all cultural events, films, artworks, historical documents… related to mathematics. They are plentiful but their existence often remains a well kept secret.

If we try to sum up our discussion, maybe one should say that there are a number of issues on which mathematics and mathematicians should try to work harder…
I believe that it is absolutely necessary and the work done by some along paths I mentioned in the interview should not be dismissed by the community as silly efforts or as naive submission to private economic interests. Working with private companies requires a very clear definition of the respective roles so that we shall not be dominated by them. The same is true with politicians or journalists: we have our way of thinking and our needs, they have theirs and there is no reason we should give up ours. It's not a lost battle. But mathematicians must definitely move up a gear.

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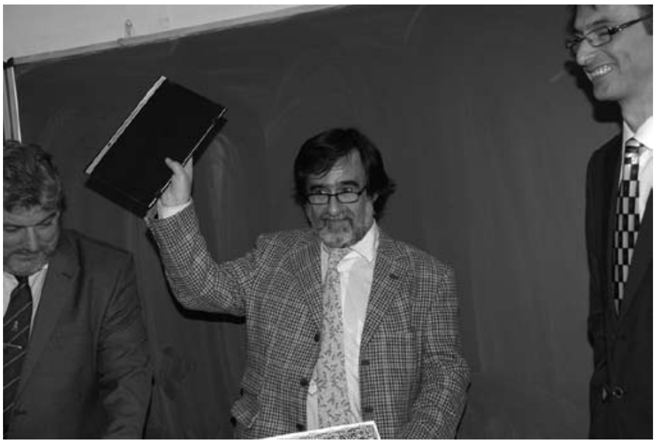
In the Mirror of Mathematics (Doamna matematică şi eu)

Preda Mihăilescu (Universität Göttingen, Germany)



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You have invited me at a time in my life when it is a little early for me to give merely retrospective talks. But you have chosen to honour me as if it were the time for counting the crops … while I am working to see that it is not; yet, I must recognise that I am deeply moved.

So I will respond to the situation with a loose series of recollections which are neither simple retrospection nor prospects for the future. They are rather bound up in questions like: "How does the world look in the mirror of mathematics? How do we see ourselves in that mirror?" and "What do we serve with mathematics" and "What makes it beautiful?".[1]

I should mention from the beginning that my relationship to *Lady Mathematics* could not have started in a more dull way – no choice or passion involved but simple being. It happened that, as a child, I had a gift for numbers; I could, for instance, perform mental multiplication of two or three digit numbers at the age of four, and there was the reciprocal discovery of the fact that many other people could not do that. This contrast marked somehow my early age: mathematics was supposed by *others* to be part of me, yet I did not know what exactly that could be. I also enjoyed reading, music and travels, often more than numbers.

The first moment of attitude towards mathematics that I recall happened in my early teenage years. At a party of elderly people (around thirty!), I happened across Horia Ene, the specialist of fluid mechanics at that time and later one of the founders of modern Romanian numerical analysis. At that party, he was explaining to an audience the platonic attitude to mathematics and opposed to this attitude his "own" point of view, of mathematics being rather a natural science. In my present understanding, I would guess that he was, at that time, arguing on the line of thought that was expressed also by major mathematical physicists of the Russian school. For the early teenager that I was, the question was interesting and I heard myself talk out loud beyond all shyness: "Of course it is a natural science, for two reasons: first, the objects it deals with, numbers and figures and all this, are very lively so they must have some kind of nature of their own. And second, it happens to fit the way nature functions, it helps describe nature, so it is a science of nature." Then I fell back into silence, scared by my own audacity. What I had done was somehow *melt* together the platonic and empirical points of view into some kind of unity.

Without ever getting really entangled in systematic philosophy, and keeping a sensibly close distance to this discipline, I may say that dancing on the various sides of this presumed unity is something that accompanied me throughout most of my life. Even today, I never cease to be surprised at how daring and consuming this project can be, when pursuing it to its consequences.

Our teenage years are a time when we form some of the leading questions that accompany us throughout life. I shall therefore mention one that preoccupied me; needless to say, it was partly influenced by its arising at the end of de-Stalinisation in a country striving for some liberalisation. I discovered later that the scope is much broader. I was very much in accord with Malraux and his *"le 21-ème siècle sera spirituel ou ne sera pas du tout"*: convinced that technical development would continue but fail to solve any major problem of life. With respect to science and the more restricted area of mathematics, I was also convinced that science had stepwise taken over the role of religion and in some sense it was natural to think of a *Church of Science*. Two hundred years of natural sciences had passed and, far from being questioned with sharp inquisitions, it had become the source of accepted understanding of Nature, Life and the Universe, while the Church had been progressively pushed into a defensive position. *This* was excessively true in communist countries but I found that it held to some extent in all Western civilizations.

As a natural consequence, like priests were once the protectors of the *living word*, but pursued this task from their very human limitations, some with grace and creative humility, some with power and despotism, we should be prepared and careful about the temptations and responsibilities of the carriers of understanding passing over to science, bringing the possibilities of *failure* to the field of science too, e.g. in the form of abuse of power or influence.

These early questions – which have meanwhile become commonplace but were less discussed at that time – had the positive impact of leading me to ponder how the mathematics I was doing *served* either practice or beauty or both. Briefly, *I was preparing myself for taking responsibility in my way of doing mathematics*.

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My evolution took a radical change when I decided to "leave" the country and stay in Switzerland, two years after Ceauşescu's *small cultural revolution* of 1971. It thus turned out that at 18 I was looking ahead to building a life in a new country and, in this situation, I was most interested in the way mathematics could be useful in practical life. I needed to know how a pragmatic and industrious country could present studies (at the ETH), so that they serve real economy in a constructive way. The question preoccupied me for decades, until I understood that it has no answer in organisations. There exists an answer only in the precious few people one may find in either field (academic or industry), who are used to looking beyond the margins of their glass.

I studied at a renowned university, the ETH, with prestigious names in its past – Minkowski, Herman Weil, Nevanlina, van der Waerden, etc. – and several current monuments – Stiefel, Eckmann, Moser, etc. The curriculum was good – compared with what I lived to see later – very much so. And I certainly learned to appreciate more of the formal-abstract and deductive approach to mathematics. I can say, without being unfair, that the passion was missing, though. And, most of all, it was missing between colleagues, who hardly shared their interest for the field, an unpleasant development which seems to have become part of the past. I came to understand that this was more than only my subjective and particular impression. This led at times to the only nostalgia I had: I thought that for my mathematical development, I had missed a lot by not studying in Romania. But, of course, I did not spend much time on this thought, the mission being to survive.

In addition, there were sufficient compensations even in my mathematical studies, with resources I would not have found in Romania. I interrupted my studies after the second year, in order to see what was being asked from mathematics in industry. My subject became numerical analysis and I followed it in work and for a PhD for half a dozen years. Along with the essential computer skills, I learned two things in this period: on a practical level, the importance of an appealing presentation and visualisation of results – computer graphics was just emerging – and writing the first graphical programs that helped visualise the results of involved numerical computations – a useful and fascinating task. Meanwhile, visual support is something that we have become so accustomed to that it cannot be imagined out of our lives.

In my year of practical work at what was then Brown Boveri AG, I worked in the department of numerical analysis, being exposed to some insights around the simple and beautiful myths about our science, which is supposed to predict natural processes precisely. Instead of this hardly believable story, I learned a few of the timeless tricks in the art of applied numerical analysis. The first thing is: never trust your tools; try them out first on some *reference data*. More importantly, even in classical problems, with well-understood differential equations as a model, the numerical search for solutions remains partly an art. Solutions are given by approximation processes which always converge to *something* but only in very favourable cases can be proved to converge to the desired solution, even in the lucky cases when a *desired solution* is known to exist. Often, the practical cases cannot be known or proved to satisfy the nice, theoretical conditions required. So there is always a reminiscence of art, in choosing appropriate starting points, imagining good validation tests to verify that the solution has the chance to correspond to a real life situation, etc.

From a point of view of school mathematics, what engineers do is thus apparently lacking rigour. From a more down-to-earth perspective, it is a very respectable and responsible art, since they must take full responsibility for the validity and interpretation given to the computed results – and this requires some additional kind of rigour. We speak, of course, of the few experienced and open-minded engineers who create the standards of their departments, the others trusting the trend.

The experience gathered in several practical projects I could bring to a conclusion – namely implementation and field applications – had a lasting effect on my mathematical taste. If I work on some subject of *applied mathematics*, I am most concerned with it being *applicable*, and this quality is estimated with the perspective of an engineer – would the theory or algorithm help make a difference? Would someone be inspired to use it? One key ingredient for *applicable* approaches is the *adequate* amount of maths for the informational complexity of the given practical problem. Too much is overkill; insufficient and one should try to do better. There are many other interesting things to learn from such an experience but let us finish by saying that I will never understand mathematicians who look down on engineers, due to their presumed lack of rigour or mathematical understanding. Sometimes one might wish that a mathematician, confronted with an engineering problem, would know so well how to select and apply the adequate mathematical knowledge through to a conclusion: knowing there is a theory which in principle solves the problem is certainly not the right approach. As rigour is to the mathematician, so is *confidence* to the practitioner; he must be able to have full confidence in the technical decisions for which he takes responsibility. The two may meet but are not identical.[2]

It happened that numerical analysis did not become my specialty, although I had completed a PhD in the subject. The reason was simply that I discovered computational instability in an algorithm that was the core of my subject, and did not accept presenting a thesis based on such an algorithm. Due

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to the departure of my PhD advisor, the work was accepted but should have come to an end, so I preferred to drop it. My attitude was received with understanding and I chose to go in the direction of computational number theory. This meant, as a living, cryptography and IT security, a practical specialty in which I worked for another dozen years. Although not much mathematics could be used here either, algorithms are always a fascinating subject. The art of programming, the love of which was made immortal by Donald Knuth, became a central attraction in this time. It was a great experience, to discover how in this crude practical art, aesthetical criteria are in perfect balance with the most desired practical requirements, like efficiency, sustainability, expandability of software, etc. Among thousands of IT specialists working in the company, I had the chance to meet two or three real specialists in the art of programming. In discussing this art with them, I was surprised that it was not only my instinct as a semi-professional but also their full- and long-term experience that led to the same kind of conclusion. A software system that sounds good, explains clearly and has friendly architecture will also have all the chances to be – after due testing of course – a reliable and expandable one. And vice-versa, a software that no one can read but is built with the most accurate and advanced flowchart tools and semantic testing has more chances to have hidden problems that will eventually be flushed out – mostly when the developer is not an employee of the company anymore, as Murphy wants it.[3]

Beyond the immediate use of applications of mathematics, in engineering, computer science or the various modellings used by the sciences and their surrogates, there is a virtue of mathematics which one starts suspecting when working in various environments – one that is becoming more widely known and is slowly revealing a new and important *attractivity for mathematics*.

This has less to do with any explicit mathematical area of knowledge but is a consequence of mathematical training and work: the mathematician is used to distinguish premises and axioms on which statements rely. They are also trained to quickly verify if a certain conclusion is *the only one* which can be drawn from a given set of premises. This is uncommon in the logic of daily life, where people tend to identify "solution" with "the only solution" or "A is not true" with "non-A must be true", etc., with disastrous consequences in communications. It is exactly these kinds of faculties which mean that one may find oneself as a mathematician in the midst of a hot social debate – or, say, controversy at work – as the only one who sees that the tension relies on some false premise or inconsistent logical deduction. Another typical example: A and B consider two equal vectors in the x-y plane. A sits on the y-coordinate, B on the x-coordinate and both consider the vector closer to them to be the largest. Stated like this, the error is obvious but in real life the coordinate systems are not as evident, and the common frame of reference may simply not be given a priori. But there is this implicit assumption that if we can talk to someone then speech itself already gives us a common frame of reference: dangerous!

To my understanding, the subject received well deserved academic interest following the successful experiments and books of Daniel Kahneman – the only psychologist who has so far received a Nobel Prize – and coauthors. Their examples are indeed very interesting and the general attitude is to reveal that there is something like an inborn predisposition to *simplification*, which is inherent to the physiology of our brain and escapes rational thinking and good- or ill-intention. Roughly, the talk is about System I and System II, the first being responsible for quick identifications of objects, and relating them in some mainframe "story" which makes up for the reality perception of the world in which we live. The quick identification is done basically on association and the efficient, yet superficial, adaptation of System I is of fundamental importance to our navigation through the challenges of daily life. When all fails, System II is asked for. This is an analytic system and it verifies if hypotheses meet facts, a process that requires effort and change. The dynamics between the two systems support the art of survival as an individual. I can only recommend to all of you the books of Kahneman, like "Quick thinking, slow thinking" and several more. For our context it is enough to underline two facts:

(i) It is relieving to consider many misunderstandings of daily and social life as consequences of almost physiological causes rather than overloading them with judgmental emotions.
(ii) As mathematicians we have trained – with fascination and, at times, actually fighting against our nature – the best resources that are given to us to escape the circle of illusion and enhance System II. As a consequence, it acts slightly faster, has a good archive of typical situations and is more willing to do its complete job for ascertaining or improving the perceptions of System I.

I should add that mathematical training does indeed help in this context but it is never a guarantee. This is work in itself, i.e. bringing the thinking experience back to matters of real life – many mathematicians are seen not to perform very well in this respect. This may have to do with a feeling of loss, when leaving the domain of well-posed, clear and challenging mathematical questions and trading it against the real world in which the bigger effort consists of bringing the issues of debate into a sustainable, logical form. On the other hand, in my experience in the practical world, I have encountered precious examples of mathematicians who mastered both the necessary social skills and succeeded in quickly highlighting the simple core issues of endless discussions.

To make a message short, experience has shown that mathematicians can, under some additional favourable conditions, help spread some of the tools of logic, analysis and deduction, which are well received by people of different skills, who discover that they can make their lives and communication aptitudes better too. Needless to say, this does not happen by explaining derivative functors or even some classical fact like the fundamental theorem of algebra but on working on examples that life offers and deducing their logical tissue.

I did not know about Kahneman when I was building my own experiences in this direction but I felt much relief and happiness when I discovered his theories (and also the more incisive speculations of Nissim Taleb, the author of *The Black Swan*). After all, I believe it is, for a mathematician, an ongoing, background preoccupation to question and verify how the generally assumed facts and theories about the functioning of the various aspects of life agree or not with facts and our experience. *This preoccupation becomes almost a source of tension that is transformed into mathematical creativity*. But it can build the set of qualities for which a mathematician may become more and more required in the various aspects of "real life", not only in the classical engineering branches.

Transforming the impulses from everyday life into valuable mathematical work is a matter of disposing freely of one's own time. No surprise, then, that it happened in the year 2000 that I chose the return to academia. There was a certain day, in Berlin, when I had the chance of a new start in both *worlds*. Choosing between a mysteriously named BATIIa position in Paderborn and one in the research lab of a major US cryptography enterprise, I chose in favour of the first and I came back to academia.

In the first year I found myself passing most of my free time (gained at the cost of a light wallet) with wondering, dreaming and drawing conclusions about the unexpected life of presumed solutions to Diophantine Equations which were expected to have none (although no one had so far succeeded in proving this). It is not easy to speak – and probably not worth trying – about the lightness and seriousness of the pursuit of bringing light on mathematical objects *that do not exist*. But I cannot deny the fact that this activity is undoubtedly one of the aspects that make people resent the *beauty* of mathematics. One thing that I can state about it with full conviction, and in a way that does not require one to be a mathematician in order to understand or guess what is meant, is the fact that this process is not really about *discovering* or, even less, *creating* something new. There may exist domains of mathematics where such an image is possibly more fit but number theory is not one of them. The real work is one of consistent thinking and freeing of *expectations and prejudice*: expectation that this or the other method should reveal so little or so much of the presumed result; and prejudices about what "simple" and what "difficult", or even "impossible", might mean or prejudices about what one knows and what others might or might not know. One accepts that all these side-thoughts are not more than guesses, which may be leading or misleading guesses. And the journey is led by the sense of life in the mysterious objects that should not exist. I wrote a poem at that time which described the "dance of ghosts"[4] waiting for the daylight to dissipate them. I have since lost the poem but its major intention, which was, of course, to strengthen my conviction that in the end the daylight will dissipate the ghosts, was achieved.

The first ray of light dissipates the existence of the presumed solutions to an equation that should not have more than one. Towards the beginning of 2002 I had thus succeeded in gathering sufficient light about possible solutions to a long-standing Diophantine Equation $x^u - y^v = 1$ that their existence dissipated in proof. *Briefly, what I wish to stress is the fact that, in some cases, mathematics is not about creation or invention; it is about elimination of misconceptions, until the necessary clarity gets revealed*. Very much like Michelangelo's work in which he was freeing from marble – it is not a nice metaphor; it is the pure truth. This is not new but I find it useful to recall.

Living through that process to the end, I could pay my ticket to stay on the side of life in which mathematics needs less to explain its right for being – it suffices that it perpetuates and enriches itself in some satisfactorily way (the environment that Latins called *Alma Mater*, with maternal protective aspects, although we are very parsimonious about the Soul/Alma).

I will be very brief about this last period of my relationship with Lady Mathematics, also because it is more difficult to have the necessary distance from it. There are several important things to say about this time. First, I came into a position to function as an emissary of Her accomplishments and settlements – teach and spread. Unsurprisingly, maybe – but for me with certain enjoyment – I discovered that there are means to transport the excitements and tremors of the game around this Mirror, to young, unknown and a priori so different minds and souls. There is something undeniably solid about it: you use the rigorous form and can transmit the passion for the subject, for the Mirror, which gives us a new understanding of life and ourselves. Second, in the hard work of years on new questions, I became free of the beginner's presumption about the existence of *those who know*. Mathematics has uncounted facets and fascinating rooms. Along with this maturing, I was also blessed with the developing of dedicated, deep relationships to several mathematicians of various tastes, countries and eye-colour (some of them are in this room now). A major gift in this period, along with the experience of teaching, was the chance to pursue an interest for some fascinating problem, without interruptions due to work in a totally different area. I hope this work leads to new fruits.

During this time and probably to the end of the loneliness, I kept wondering about what and in which way Lady Mathematics succeeds in reflecting back about, say, Nature (in lack of more precise terms). It began with rigid bodies and the movement of the planets. We have spoken about this, about Poincaré's caution that even the simplest classical trajectories should be considered as virtually unpredictable due to the capricious effects of slight variations in the limit conditions. It took many decades until this observation was understood to its full implications – this led to the development of dynamical systems, a booming discipline of mathematics these days. We have thus been taught caution in the classical and, presumedly, in most certain citadels of exact applications of mathematics to predictions concerning the physical world. Meanwhile, a request and interest for mathematics grows in most areas of cognition, from biology and other unformalised disciplines, which can yet, to some extent, be subject to experimental verification and possible confirmation of hypotheses, all the way through to *social and economical disciplines*, which run the highest risks of falling under dogmatism. In these expanding areas of application, mathematics is in a life and death struggle between a possible misleading use for creation of *accurate illusions and make-believe* and the strong and timeless use for rigorous understanding and elimination of misconceptions. Deciding which of the two takes the lead becomes more a matter of the taste and character of the involved researchers rather than of the properties of the mathematics involved. But there is certainly much movement going on – *this can become a subject for a future, really retrospective discussion we could have here*.

Notes

1. What you will hear are my opinions and experiences. I take full and temporary responsibility for them; they are not the opinions of my employer or others unnamed. I have found them time and time again confirmed in reading. So, if you hear some thought or remark that sounds known to you and connect it to some person that I might have failed to quote or mention, chances are you may be right.
2. There are, of course, several other arts in the engineering life that are of less use in mathematics – the art of avoiding responsibility and gathering praise and advantages, etc. Or real importance as it is … but this is off-topic here.
3. Or, according to some rumours, benefiting the financial interests of the developer himself.
4. Dansul nălucilor sounds softer in Romanian.

Preda Mihailescu [preda@uni-math.gwdg.de] was born in Bucharest, 1955. He studied mathematics and computer science in Zürich, receiving his PhD from ETH-Zürich. He was active during 15 years in the industry, as a numerical analyst and cryptography specialist. In 2002, Mihailescu proved Catalan's Conjecture. This number theoretical conjecture, formulated by the French mathematician E. C. Catalan in 1844, had stood unresolved for over a century. The result is known as Mihailescu's Theorem. He is currently a professor at the Institute of Mathematics of the University of Göttingen.

The European Physical Society: Past, Present and Future

John Dudley and David Lee (European Physical Society, Mulhouse, France)

Introduction

The European Physical Society (EPS) works to promote the interests of physics in Europe. Its activities revolve around the themes of promoting excellent physics research and supplying a European view on important questions relating to physics. The EPS provides a forum for the exchange of ideas and experience, bringing together physicists from different countries, working in all fields of physics.

Organisation

The EPS is a not-for-profit organisation created in 1968. The 41 EPS member societies (in 41 countries) represent over 130,000 physicists throughout Europe. Over 3000 physicists participate directly in the EPS as individual members. European research institutions such as CERN, CEA, EFDA-JET and EPFL are represented as associate members.

The secretariat of the EPS is located on the campus of the Université d'Haute Alsace, Mulhouse, France. The staff provide central services, such as accounting and IT, and centrally manage important ongoing activities in areas such as conference organisation, publication and EU projects.

The EPS is represented in cutting edge physics research through its 18 topical divisions and groups. These grass roots bodies work under the aegis of the EPS, creating strong communities of active researchers to engage in many different activities. Divisions and groups also provide expert advice in many areas such as energy and education, necessary for developing EPS policy statements. Divisions and groups are also essential to the EPS in organising outreach activities and ensuring international cooperation in specific fields of physics.

An executive committee of 12 members supports the EPS president in developing and implementing EPS activities. Various action committees are given mandates to support the executive committee in specific fields, such as equal opportunities, conferences and European integration.

One of the challenges is to ensure gender, topical and geographic balance on all governing bodies.

Conferences

EPS divisions and groups organise many of the world's leading physics conferences, allowing members of the European and global physics community to share their research and exchange with their colleagues. Conference venues often move around Europe, actively encouraging the involvement of national member societies and communities. The EPS provides professional conference organisation services to its members to efficiently manage their meetings, allowing them to concentrate on promoting scientific excellence. Among the conferences organised by EPS divisions and groups are CLEO/Europe-EQEC, the European High Energy Particle Physics Conference, the General Conference on Condensed Matter Physics and the European Physics Education Conference. The EPS conferences play an important role in establishing European communities of physicists in the various fields, communicating science and supporting career development.

The EPS has developed and administered a wide range of exchange and grant programmes that are aimed at involving early career physicists in important international events.

Publications

Publication remains a priority for the EPS, for its importance in communicating to its members and the community at large, as well as the role that scientific publication plays in the career development of researchers.

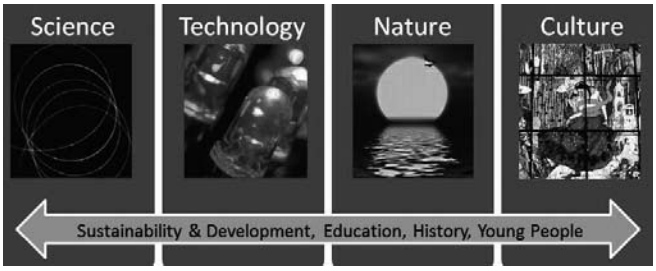
The EPS publishes e-EPS, a monthly electronic bulletin that provides timely information on research, policy developments and research in physics. While e-EPS concentrates on news related to physics, its target audience of over 50,000 regular readers includes journalists, policy makers and interested members of the general public.

Europhysics News (EPN) is published six times a year. It is the news magazine of the EPS, publishing high quality review articles of research and developments in physics, and other items of interest to EPS members. It is distributed in over 25,000 copies to EPS members around Europe.

The EPS provides the editorial office for EPL, a letters journal on the frontiers of physics. EPL publishes around 1000 manuscripts per year and is available in over 2000 institutions worldwide. EPL is a partnership, published in cooperation with IOP Publishing, the Italian Physical Society (SIF) and the French Physical Society (SFP-EDPS). EPL also counts 13 other physics learned societies among its members.

The EPS and the Institute of Physics (UK) jointly publish the European Journal of Physics (EJP). This journal publishes articles to describe and better understand physics education and teaching at university level.

The EPS is involved in policy statements in publishing in open access for example. It works with physics publishers that have strong ties to the scientific commu-



nity to discuss strategic development of publications and to promote a series of core journals: EJP, EPL, the EPJ Series, the JPhys series and NJP.

Outreach

The EPS is involved in many public awareness initiatives to advance public understanding of science. Most recently, the EPS has spearheaded the initiative to have 2015 declared the International Year of Light by the United Nations. The International Year of Light is a global initiative that will highlight to the citizens of the world the importance of light and optical technologies in their lives, for their futures and for the development of society. The International Year of Light project includes over 100 partners from over 85 countries, including scientific societies, museums, universities and other organisations. The partnership has been working since 2010 to prepare the groundwork for a coordinated series of activities throughout the world during 2015.

The International Year of Light will allow the universality of light and the variety of its applications to be appreciated via many and varied themes covering broad areas of interest, supported by cross-cutting themes addressing essential issues to be included in all activities. Actions will be implemented on national, regional and international levels. The main structure of these activities is illustrated below.

A resolution from the UN adopting 2015 as the International Year of Light is expected in November/December 2015.

Gender

Gender equality in physics deserves and requires continuous attention. The EPS recognises the importance of an appropriate gender balance in physics and attaches great importance to the representation of women in governing bodies of the EPS and conference programme committees, and equal opportunities to pursue scientific careers. The EPS also supports specific initiatives, such as the European Platform for Women Scientists (EPWS) and the International Conference of Women in Physics (ICWP), which concentrate on issues related to gender balance in physics.

Education

The next generation of physicists is important to the EPS, as they are the future researchers and innovators in science, society and economy. Through the EPS Young Minds initiative, university students are encouraged to engage in outreach activities locally and to create professional networks with other university students throughout Europe.

The EPS Education Division studies issues related to teaching physics at university level. It provides advice to the EPS on how policy developments will impact university education. The EPS has developed a series of specifications, describing the basic requirements for curricula for physics studies at bachelor's, Master's and doctoral levels.

The EPS also represents the physics community in European projects related to science education. More emphasis is being placed on inquiry based science education and the EPS is actively involved in developing standards, educational material, repositories and dissemination of IBSE methods in Europe.

Physics for Development and European Integration

Beyond its contribution to the understanding of the universe, physics plays a role in social, cultural and economic development. The EPS engages in and supports activities that make use of these features. These include: schools and conferences to create communities and user groups in developing countries, support of international projects such as SESAME, bringing together scientists in the Middle East, and community building and best practice transfer in project funding opportunities in developing regions in Europe.

Energy

The physics community and the EPS are involved in initiatives exploring important issues related to sustainable energy for Europe. This goes beyond pure research and often involves considerations such as competitiveness, security, economy and climate. The EPS, physicists and physics will provide key insights into many areas related to energy, including the future of fission energy: in providing nuclear data for alternative processes, in the development of new concepts, e.g. accelerator-driven sub-critical reactors, or in the storage, reprocessing and transmutation of waste. Physics plays an obvious role in the development of fusion energy in the form of magnetic confinement or inertial fusion. But physics is also of high importance in the techniques of renewable energy and energy saving technology. Photovoltaic electricity production is still a rich research field with many unexplored possibilities using new materials, plastic substances, material combinations and the integration of nano-technologies. Exciting discoveries and technological breakthroughs can be expected from this area. Similar arguments apply to fuel cells and techniques to store energy – specifically electricity.

Physics and the Economy

The new European research programme Horizon 2020 is being launched to reinforce the intimate link between basic science and technological applications, in order to stimulate Europe's progress in research and innovation, a major challenge for the future. In this context, the EPS commissioned a study to look at the contribution of physics to the economies of Europe. The independent economic analysis from the Centre for Economics and Business Research (Cebr) used public domain Eurostat data from 29 European countries, EU27 countries plus Norway and Switzerland. The 4-year period 2007-2010 was examined.

The 4-year snapshot of the European economy shows that the physics-based industrial sector generated over 15% of total European turnover, exceeding the contribution of the entire retail sector, and over 15 million jobs per year corresponding to over 13% of overall employment within Europe. The full report is available at: http://www.eps.org/?page=policy_economy.

Physics and EU Policy

A key role of the EPS is to coordinate input from Europe's physics community to inform science policy debate. The EPS provides responses to EU consultations, in fields such as the Common Strategic Framework for EU Research and Innovation and Open Access, based on input from member societies. The EPS also works closely with other learned societies in the physical sciences and regularly meets with EC officials.

The past and the future

The activities of the EPS since it was founded in 1968 in Geneva have made decisive contributions to the emergence of a European physics community. Moreover, the EPS, as an actor in civil society, provides an example of a democratic bottom-up process of self-organisation. The EPS and the European physics community have been at the forefront for many joint European projects which, as a beacon of science, have attracted worldwide attention and thus demonstrated the achievements of these joint efforts far beyond the boundaries of Europe.

There are many challenges and opportunities ahead. Horizon 2020, starting in 2014, the International Year of Light in 2015 and the start up of new research facilities such as ELI in 2016 point to many new opportunities for physics and the EPS.

Erwin Schrödinger International Institute for Mathematical Physics – Restart[1]

Joachim Schwermer (University of Vienna, Austria)

1. Restart

In October 2010, when the Erwin Schrödinger International Institute for Mathematical Physics (ESI) had been in existence as an independent research institute since 1993, the scientific directorate and the international community of scholars had to learn with great distress of the intention of the government of Austria to cease funding for the ESI. Due to budgetary measures affecting a large number of independent research institutions in Austria, funding of the ESI would be terminated as of January 1st, 2011. Since its start it was the mission of the ESI to advance research in mathematics, physics and mathematical physics at the highest international level through fruitful interaction between scientists from these disciplines. An abrupt end for the scientific activities of the Institute and the closure of the ESI appeared on the horizon. Weeks of trembling uncertainty followed, mixed with signs of a solution in which the University of Vienna would be involved. In the wake of a protest action by renowned scholars and academic institutions worldwide, an agreement was achieved in January 2011 that the ESI could continue to exist but now as a research centre ("Forschungsplattform") at the University of Vienna. As a partner in this agreement the Ministry of Science and Research (BMWF) guaranteed to fund the "new" ESI through the University yearly with a reduced budget until 2015. At a time when pure research and scholarly activities are undervalued, the opportunities for scholars and young researchers that the Institute provides have never been more necessary. The University of Vienna took the chance and created a home for "one of the world's leading research institutes in mathematics and theoretical physics", as Peter Goddard, the chair of the international review committee for the Institute, commissioned by the BMWF, and its members put it in 2010 in a letter to the Ministry.

2. Stabilisation

With this new institutional framework in place since June 2011, it was the main task of the new governing board of the ESI, called the "Kollegium", to carry on the mission of the ESI and to retain its international reputation. These aims include, in particular, to support research at the University of Vienna, to contribute to its international visibility and appeal, and to stimulate the scientific environment in Austria. Setting aside all the technical issues the transition process of the ESI involved, and which had to be taken care of, the ESI could begin restoring its fundamental scientific activities. This was and still is the prevalent task: striving to be excellent and thereby keeping its position within the international scientific community of scholars as a research institute with a specific unique character. The ESI is a place that is very conducive to research and, at the same time, integrates scientific education and research in mathematics and mathematical physics.

Photos: ©Philipp Steinkellner (http://www.steinkellner.com/)

In retrospect, though the planning horizon was very short, the *Thematic Programmes,* scheduled already far ahead for 2011 and 2012, turned out in the end to be successful scientific events. Additionally, various workshops and other activities could be solicited for the year 2012 on short notice, involving, in particular, young researchers who came to the Institute for the first time as organisers. In addition, by January 1, 2012, the Erwin Schrödinger Institute had established the *Research in Teams Programme* as a new component in its spectrum of scientific activties.

The transition of the Erwin Schrödinger Institute from an independent research institute to a "Forschungsplattform" at the University of Vienna was a complicated

1 The text of this article is taken from my first two articles in the ESI 2013 booklet, published in 2013. See http://www.esi.ac.at/material/esi-booklet-2013.

process. There are far-reaching differences in operation as a consequence of the university's involvement in the running of the Institute. This includes issues concerning payments to participants, modifications to the premises and future funding prospects. However, the Institute has continued to function, even flourish, during the radical changes of its status.

The Institute currently pursues its mission in a number of ways:

- Primarily, by running four to six *thematic programmes* each year, selected about two years in advance on the basis of the advice of the International ESI Scientific Advisory Board.
- By organizing *workshops* and *summer schools* at shorter notice.
- By a programme of *Senior Research Fellows* (SRF), who give lecture courses at the ESI for graduate students and post-docs.
- By a programme of *Research in Teams*, which offers teams of two to four *Erwin Schrödinger Institute Scholars* the opportunity to work at the Institute for periods of one to four months, in order to concentrate on new collaborative research in mathematics and mathematical physics.
- By inviting *individual scientists* who collaborate with members of the local scientific community.

Even through the transition period the ESI had to go through in the years of 2010 and 2011, the ESI has remained a leading international center for research in the mathematical sciences. This position has been achieved with a minimal deployment of resources, financial and human, especially when compared with similar institutes in other countries.

3. The Institute's Scientific Management and its Resources

The arrangements that provide for the scientific direction and administration of the Institute are perhaps among the noteworthy features of the ESI. Indeed, the Institute is run in a quite minimalist fashion.

The organizational structure of the ESI is as follows: The ESI is governed by a board ('Kollegium') of six scholars, necessarily faculty members of the University of Vienna. These members of the board are appointed by the President (Rektor) of the University after consultations with the Deans of the Faculties of Physics and Mathematics. It currently consists of Goulnara Arzhantseva (Mathematics), Adrian Constantin (Mathematics), Piotr T. Chruściel (Physics), Joachim Schwermer (Mathematics), Frank Verstraete (Physics), and Jakob Yngvason (Physics). All members of the Kollegium still act as Professors at the University.

In addition, the Scientific Advisory Board of the ESI plays a crucial role in keeping this Institute alive scientifically. The members of the Scientific Advisory Board of the ESI, which currently consists of seven international scholars, have a variety of tasks: they assess the programme proposals submitted to the ESI, they point out interesting scientific developments in the area of Mathematics and Mathematical Physics, and suggest topics and possible organizers for future activities of the Institute, and, most importantly, during the yearly meeting, they review – and criticize – the scientific performance of the Institute during the past year and make suggestions for possible improvements. Though the name of the Institute only contains Mathematical Physics as the subject of concern, Mathematics plays an equally important role in its scientific activities.

On June 1, 2011, the Scientific Advisory Board of the ESI was restructured. Only scholars who are not affiliated with a scientific institution in Austria can be appointed as members. Thus, at the same time, its composition changed. For the sake of continuity, John Cardy (Oxford), Horst Knörrer (ETH Zürich), Vincent Rivasseau (Paris) and Herbert Spohn (München) were reappointed. As new members Isabelle Gallagher (Paris), Helge Holden (Trondheim) and Daniel Huybrechts (Bonn) joined the Board, starting January 2012.

The day-to-day functioning of the ESI is overseen by the Director. The Director is appointed by and accountable to the Rektor of the University. Besides the ongoing oversight of the ESI, the Director chairs the Kollegium, represents the ESI at meetings of the European Institutes and has responsibility for the budget of the ESI. The Director makes sure the ESI functions in a way manner with its mission.

The administrative staff of the Institute, currently consisting of three people, two of them working on part time basis, is also extremely lean but very efficient in handling the approximately 450 visitors per year.

Situated at Boltzmanngasse 9 in Vienna, the ESI is housed in the upper floor of a two hundred-year-old Catholic seminary. This building provides a quiet and secluded environment. By its distinctive character, the ESI is a place that is very conducive to research.

The Institute is still funded by the Austrian Federal Ministry for Science and Research, via the University of Vienna, but it works on the basis of much smaller resources financially than in the years before 2011.

4. Thematic Programmes and Workshops

The Institute's scientific activities are centred around four to six larger thematic programmes per year. Planning for these programmes typically begins two years in advance. About three quarters of the scientific budget are used for these activities each year. In addition, smaller programmes, workshops and conferences are organized at shorter notice, as well as visits of individual scholars who collaborate with scientists of the University of Vienna and the local community.

The list of research areas in mathematics, physics and mathematical physics covered by the scientific activities of the Erwin Schrödinger Institute in the years 1993 to 2013 shows a remarkable variety.

The pages of the annual ESI report, available on its web page, provide ample evidence that the high quality of the scientific programmes was sustained and, in particular, undiminished during and shortly after the radical changes the Institute had to face. Longer thematic programmes and the open approach to research they offer and encourage form a fundamental pillar of the work of the ESI. The Institute provides a place for focused collaborative research and tries to create the fertile ground for new ideas.

It is generally noted, as already the Review Panel of the ESI pointed out in its report in 2008, that over the last years the ESI has widened the range of its thematic programmes and other scientific activities from being originally more narrowly focused within mathematical physics. The Scientific Directorate has increased the scope of the activities mounted by the Institute into areas of mathematics more remote at present from theoretical physics. This process will continue in the same fashion, with special emphasis on the fruitful interactions between mathematics and mathematical physics.

The themes of the programmes which were in place in 2013 range from “The Geometry of Topological D-branes”, “Jets and Quantum Fields for LHC and Future Collidiers” over “Forcing, Large Cardinals and Descriptive Set Theory” to “Heights in Diophantine Geometry, Group Theory and Additive Combinatorics”.

In 2014 the ESI will host four thematic programmes, the first one dealing with “Modern Trends in Topological Quantum Field Theory”, followed by one centered around “Combinatorics, Geometry and Physics”. The programmes “Topological Phases and Quantum Matter” and “Minimal Energy Point Sets, Lattices and Designs” cover the second half of the year, supplemented by various workshops.

5. Senior Research Fellowship Programme

In order to stimulate the interaction of the Institute’s activities with the local community, the Institute initiated a Senior Research Fellowship Programme in 2000. Its main aim is attracting internationally renown scientists to Vienna for longer visits. These scholars would interact with graduate students and post-docs in Vienna, in particular, by offering lecture courses on an advanced graduate level. This programme enables PhD students and young postdoctoral fellows at the surrounding universities to communicate with leading scientists in their field of expertise. Currently W. Ballmann (U Bonn) gives a course, entitled “Geometry of Symmetric Spaces” and L. Dabrowski (SISSA) Trieste) lectures on “Spinors: Classical and Quantum – Elements of Non-commutative Riemannian Geometry”.

6. ESI Scholars

By January 1, 2012, the Erwin Schrödinger Institute had established the *Research in Teams Programme* as a new component in its spectrum of scientific activties. The programme offers teams of two to four *Erwin Schrödinger Institute Scholars* the opportunity to work at the Institute in Vienna for periods of one to four months, in order to concentrate on new collaborative research in mathematics and mathematical physics. The interaction between the team members is a central component of this programme. The number of proposals, on themes of topical interest, was and is still high. However, due to limited resources, the Kollegium could only accept four applications for the year 2012 resp. five for 2013.

Of course, by invitation only, the ESI continues to have individual scientists as visitors who pursue joint work with local scientists. In some cases these collaborations originate from previous thematic programmes which took place at the ESI.

7. Junior Fellows and Summer Schools

Funding from the Austrian Federal Ministry for Science and Research (BMWF) enabled the Institute to establish a Junior Research Fellowship Programme (JRF). Its purpose was to provide support for advanced PhD students and postdoctoral fellows to allow them to participate in the activities of the ESI. Grants were given for periods between two and six months. This programme was very successful and internationally held in high esteem but unfortunately it came to an end because funding by the BMWF was terminated with the end of 2010. The presence of the Junior Reseach Fellows at the Institute, together with the Fellows of the European Post Doc Institute, had a very positive impact on the ESI’s scientific atmosphere through their interaction with participants of the thematic programmes, through lively discussions with other post-docs and also through the series of JRF seminars. In conjunction with the JRF Programme, the ESI had regularly offered Summer Schools which combined series of introductory lectures by international scholars with more advanced seminars in specific research areas. However, even though the JRF programme had to be discarded, the Institute continues its long term policy of vertical integration of scientific education and research. Summer Schools are still essential components of the scientific activities of the ESI.

In 2010, a “May Seminar in Number theory” took place to introduce young researchers to exciting recent developments of current research at the crossroads of arithmetic and other fields. During the summer 2011 a school dealt with recent developments in mathematical physics. Jointly with the European Mathematical Society (EMS) and the International Association of Mathematical Physics (IAMP) the ESI organized in 2012 the “Summer School on Quantum Chaos”. This Instructional Workshop attracted more than 45 graduate students, post-docs and young researchers from all over the world. A poster session accompanied this event.

8. Conclusion

With the ESI now being a research centre within the University of Vienna, the best way of benefiting the local community is to ensure that the ESI continues to function at the highest level internationally and thus attract the world’s leading scholars to Vienna where their presence will enhance and stimulate research further. This has been and still is the approach successfully followed by the ESI.

Joachim Schwermer, Director
Erwin Schrödinger
International Institute
for Mathematical Physics

Why is University Mathematics Difficult for Students? Solid Findings about the Secondary-Tertiary Transition

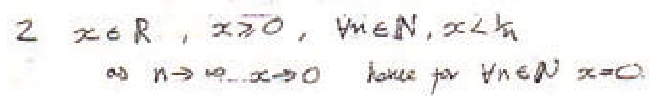
Ghislaine Gueudet (Université Européenne de Bretagne, Rennes, France) on behalf of the Education Committee of the EMS

Every mathematics teacher at university level has experienced the difficulties met by first-year students – even by students who were very successful at secondary school. Students requiring help because they feel unable even to start solving a problem or students proposing inadequate reasoning or proof (EMS Committee on Education, 2011) are just two among many possible problematic situations.

The following example is given by Nardi & Iannone (2005). First year students were asked to answer the following question:

Let $x \in \mathbb{R}$ have the following properties: $x \geq 0$, and $\forall n \in \mathbb{N}$, $x < 1/n$. What is x?

One of the students answered as follows:



Should the teacher conclude that the student was not able to answer the question because it is too abstract for him? That this student did not understand the concept of limit? Or that the student found the answer but was not able to formulate it properly? And how is it possible to help this student?

We note that the student, in his answer, only uses a few English words but many symbols. He tries to produce an answer that "looks like" a university mathematics sentence. He does not recognise the different status of n (any integer) and of x (a fixed value). He might have encountered the argument "$u_n < 1/n$", used to prove that a sequence (u_n) tends to 0, and reproduces the conclusion, wrongly applying it to x. However, he finally seems to understand that x is not a sequence but a single value, and gives the answer "$x = 0$". Some further work, devoted to exercises involving several letters with different status, might help him avoid such confusions and the teacher could also ask him to use more text and normal words in writing his solutions.

More generally, mathematics education research on the secondary-tertiary transition provides results that can contribute to interpreting novice students' difficulties and proposing possible support for these students (de Guzmán, Hodgson, Robert & Villani, 1998; Artigue, Batanero & Kent, 2007; Gueudet, 2008; Thomas et al., 2012). We offer below a short summary of some of these results and evoke some teaching devices which have been recognised as successful in overcoming transition difficulties.

Level of abstraction

Some of the research studies that address what is often called "advanced mathematical thinking" have provided evidence that the concepts encountered at university are more abstract and thus require "advanced" thinking modes (Tall, 1991). A sequence of real numbers $(u_n)_{n\in\mathbb{N}}$, for example, is already an abstract concept. Working on sequences, the students have to figure out an infinite list of numbers, labelled by integers. At university level, students not only study sequences (their nature, their possible limits and so on) but they also have to perform operations on sequences and have to consider sets of sequences and vector spaces of sequences, for example, which means a high level of abstraction.

Research results concerning specific topics

Some of the research work concerning transition focuses on specific topics and identifies features of these topics that raise delicate issues for lecturers in charge of their teaching. For example, the historical development of linear algebra evidences that this theory has been introduced to unify different areas. Several mathematicians observed similarities between problems concerning functions and problems concerning sequences, for example, and suggested that this could be explained by a single, general theory (Dorier, 2000). This very long process cannot be reproduced in a linear algebra course, where students encounter this Formalising, Unifying and Generalising theory (Robert, 1998) with no possibility of experiencing the need for it. Some students, for example, are unable to convey a meaning to the concept of supplementary subspaces because they do not associate any representation with this general notion.

Other obstacles are connected with wrong intuitions, for example when students have to deal with infinity. González-Martín (2009) has identified such difficulties about infinite sums: students consider, in particular, that the sum of an infinite number of terms should be infinite. Such difficulties are very hard to avoid but knowing them usefully informs the teacher, who can refer to this "geographical map" of difficulties to build their teaching.

New mathematical practices and norms

In a previous paper in the EMS Newsletter, we introduced the notion of sociomathematical norms, as "practices of participation and performance regarded in a mathematics lesson as proper or correct" (EMS Committee on Education, 2013). These norms are certainly different at university from what they are at secondary school.

With this perspective, one can consider that entering university is like entering a new country, with a new language and new rules. Students have to learn this language, using new signs like quantifiers and using language in new ways (Chellougui, 2009; Stadler, 2011). The example given at the beginning of this article (Nardi & Iannone, 2005) can in fact be interpreted in this way.

The change in sociomathematical norms between secondary school and university also concerns the proofs expected from students (Dreyfus, 1999; EMS Committee on Education, 2011). In many countries, schools focus more on argumentation; hence deductive proofs are new requests for the student entering university (Mariotti et al., 2004; Engelbrecht, 2010). These new requests are not always the object of explicit teaching at university (Hanna & de Villiers, 2008; Hemmi, 2008). Even the models that could be provided by university textbooks are not reliable enough: variations of rigour in proofs are observed in textbooks, as noted by Durand-Guerrier and Arsac (2003), for example about (ε, δ) proofs. Sometimes, the link between ε and δ is made explicit by a notation like δ; sometimes it remains implicit.

Some of the new mathematical practices are difficult to grasp for students, in particular because they remain implicit. Another factor of difficulties is that some practices require a kind of expertise. University mathematics resembles more and more the experts' mathematics: students have to find examples, develop a flexible use of different kinds of representations, make attempts and control these on a theoretical level, etc. (Lithner, 2000; Harel, 2008). Mathematicians often refer, for this purpose, to a repertoire of familiar situations but building such a repertoire is a long-term process (not so long, perhaps, for brilliant students but the teachers naturally need to be able to support all kinds of students!).

A difference of culture between secondary school and university

Another kind of explanation is linked to the basic observation that university and secondary school are different types of institutions (Chevallard, 2005), with different cultures (Hall 1981), thus shaping the way mathematics is taught (Artigue, 2004).

At the secondary level, given a certain topic, students often meet only a limited number of tasks, not connected to each other, each of them being associated only with a given technique used to address it (Bosch, Fonseca & Gascon, 2004). For example, in many countries, "compute the limit of a function at $+\infty$" is done at secondary school, for a given function, by referring to familiar functions (Praslon, 2000); the students cannot refer to a formal definition of limit, which is not presented at this stage. At university, the students can still be asked to "compute the limit of a function at $+\infty$" but it might be very general, for a function characterised by some properties. Moreover, the students can refer to a theory of limits, which articulates the different possible tasks and problems (Winsløw, 2012). The university culture also comprises the introduction of many new concepts and properties in a limited time and there is a need for substantial mathematical autonomy by the students: in the choice of a method, in the building of a counter-example, etc.

Supporting transition – some successful examples

The findings briefly exposed here correspond to causes of the novice students' difficulties. Naturally, in many countries these findings have informed the design of courses specifically intended to help students overcome these difficulties. A summary of many interesting initiatives can be found in Holton (2001).

These initiatives can take the form of bridging courses: additional teaching, given at the very beginning of the first university year, trying to fill the gap between secondary school and university, often by proposing exercises only requiring secondary school knowledge but asking for more autonomy (e.g. Biehler et al., 2011, develop a bridging course using an online platform).

In several universities, support courses are offered for volunteer students or for students who encounter difficulties (e.g. the "Tremplin operation" at the University of Namur in Belgium, with sessions centred on questions raised by the students). In other places, the pedagogical organisation of the courses has been changed to foster more mathematical activity and involvement from the students. In Helsinki, for example, the courses are devoted to the completion of tasks by the students under the supervision of an instructor (Hautala et al., 2012). In Barcelona (Barquero, Bosch & Gascón, 2008), "study and research paths" (SRP), guided by a given question, are proposed to students (e.g. *how to predict the short- and long-term evolution of a population size*). Naturally, the needs, and possible relevant teaching approaches, are different for mathematics majors, engineering students (Jaworski & Matthews, 2011), biology students, etc.

The design of courses for novice students could provide the opportunity for rich collaborations between mathematicians and mathematics educators!

Authorship

Even though certain authors have taken the lead in each article of this series, all publications in the series are published by the Education Committee of the European Mathematical Society. The committee members are Tommy Dreyfus, Ghislaine Gueudet, Bernard Hodgson, Celia Hoyles, Konrad Krainer, Mogens Niss, Juha Oikkonen, Núria Planas, Despina Potari, Alexei Sossinsky, Ewa Swoboda, Günter Törner, Lieven Verschaffel and Rosetta Zan.

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ICMI Column

Mariolina Bartolini Bussi (Università di Modena e Reggio Emilia, Italy) in collaboration with Abraham Arcavi (Secretary General of the ICMI)

Executive Committee meeting

The new Executive Committee (EC) met in Berlin, 21–24 May 2013. A main concern of the first meeting of the new EC was to ensure that the ongoing programmes, carried out so diligently by the previous EC, would continue smoothly alongside and consistently with the planning of forthcoming events. The EC plans to meet once a year (working through email communications and Skype meetings in between). These meetings will take place in different parts of the world so that the hosting communities of mathematics educators can have the opportunity to benefit from the visiting EC members (via lectures, workshops, etc., by the EC members).

AFRICME

The 4th Africa Regional Congress on Mathematics Education (AFRICME 4) was held in Maseru, Lesotho (11–14 June). Nkosinathi Mpalami was the Chair of the Congress Steering Committee.

The meeting was attended by 53 participants from the USA, UK, France, South Africa, Malawi, Botswana, Swaziland, Kenya, Uganda and Lesotho. Many of the participants were young researchers (doctoral students or recently graduated PhDs). Michele Artigue and Jill Adler reported that the conference was very well organised, with a good scientific quality and a friendly atmosphere. The three plenary lecturers were by John Mason, Mamokgethi Setati Phakeng and Michèle Artigue. The majority of contributions referred to teacher education and practices, with a strong emphasis on linguistic diversity. Ferdinando Arzarello, ICMI's president, greeted the participants via video conference.

AFRICME and other planned activities across Africa (e.g. the planned CANP meeting in Tanzania in 2014 and the Espace Mathématique Francophone meeting in Algiers in 2015) constitute a major effort for enhancing mathematics education within the continent through supporting the creation of regional networks of practitioners and researchers.

ICMI STUDY 23: Teaching and learning whole numbers in primary mathematics classrooms

The IPC meeting of the study will be held in Berlin (19–24 January 2013) under the responsibility of the co-chairs Mariolina Bartolini Bussi and Xuhua Sun (University of Macau – China). The main aim of this meeting is the preparation of the Discussion Document to be distributed worldwide. Further information will be given in the next issue.

Archiving

The ICMI is launching a joint initiative with the International Mathematical Union (IMU) to build and maintain archives to contain existing and future records which may be of historical and practical interest for the present and future generations. These archives would consist of primary source documents, photographs, recordings and more that accumulate as the organisation functions in its everyday life. The IMU and the ICMI decided to devote efforts to undertake archiving in the most professional manner in order to select, classify and organise the records. Efforts are also invested in order to choose the most up-to-date technological infrastructure to store the records for easy and efficient access.

Some of the archives will remain secret for 70 years due to their sensitive contents (e.g. deliberations on the Awards Committees). The ICMI archive will be part of the IMU archive, which is located in the Berlin headquarters of both organisations. The curator of the IMU archive is Guillermo Curbera and the archivist is Birgit Seeliger. Bernard Hodgson was appointed as the curator of the ICMI archives and he has already started to work on it. We are confident that, given Bernard's meticulous and responsible way of working and his passion for history, the resulting archives will become a wonderful resource at the service of the ICMI community.

ERME Column

Susanne Schnell (Technical University Dortmund, Germany)

YERME and Seventh YERME Summer School (YESS-7) in Kassel, Germany http://www.mathematik.uni-dortmund.de/~erme/

The ERME is dedicated to promoting the so-called three C's of collaboration, cooperation and communication. This idea is taken seriously not only among well-renowned researchers in mathematics education but also among young researchers who are still working on establishing a profile in the field.

It is a special interest of the ERME to foster the potential of this group by providing support to developing their expertise and academic skills. Thus, young researchers are organised in a specific ERME group called YERME: Young European Researchers in Mathematics Education. This group is the result of spontaneous meetings between young researchers at the CERME-II and CERME-III conferences. The group aims to create a co-operative style of work amongst people of different countries and to help young researchers in preparing and developing their careers in the field of mathematics education. The term "young" here refers to academic age and includes, for instance, PhD students and post-doctoral researchers.

The ERME has now officially accommodated the continual growth of this group of young researchers (reflected in the number of participants at CERME). In February 2013, the bylaws were modified in the General Assembly of the CERME to include two representatives of the young researchers on the board of the ERME: C. Miguel Ribeiro from Portugal (elected for two years) and Susanne Schnell from Germany (elected for four years).

The two main events organised for the YERME are the YERME day and the YERME Summer School (YESS).

The YERME day traditionally takes place on the day before the bi-annual CERME. Here, experts hold discussions and working groups which focus on the conceptualisation of empirical research such as "Design of a research study: what are the component parts, how are they related to each other and how might they be discussed in a thesis?" and scientific working skills such as "Literature search" and "Reading and writing of reports and papers".

The YESS, which is also held every two years, is a summer school organised for PhD students and post-doctoral researchers in mathematics education and others entering mathematics education research. The 7th YESS will take place in 2014 from 4 to 11 August in Kassel, Germany.

Over the course of a week, the participants get to work in thematic working groups and have the chance to compare and integrate their experiences in the field of mathematics education research. Here, they can present research ideas, frameworks and difficulties to get constructive feedback from their peers as well as support from a variety of highly qualified experts. When the highly engaging work in the expert groups ends, participants can get to know one another in the evening events and thus gain insights into the various situations of mathematics education researchers in different European countries. Bonds that are formed here can be kept for future research life.

Participants are asked to prepare and present a paper concerning their current status of research, e.g. preliminary results, work in progress or comprehensive information about graduate studies and/or future research plans. This paper will be read by the experts and other participants as a foundation for in-depth discussions, for instance on specific challenges or common topics within the working group.

The proposed topics of the thematic working groups for the upcoming YESS-7 in Germany are:

1. Teacher knowledge and practice; teacher education and professional development.
2. Teaching and learning mathematics at primary level.
3. Teaching and learning mathematics at secondary and advanced level.
4. Information technologies in mathematics teaching and learning.
5. Cognitive and affective factors in learning and teaching mathematics.
6. Theoretical perspectives and linguistic and representational aspects of teaching and learning mathematics.

Based on the success and insights gained from prior YESS events, plus the suggestions of the participants, the programme will include:

- Ten two-hour sessions in six parallel Thematic Working Groups.
- Short lectures given by the invited experts.
- Discussion Group Sessions concerning topics of common interest.
- Informal Discussion Group Sessions, self-organised by YERME, devoted to issues that are relevant to the needs of individual YERME participants.

More details such as the names of the experts, the first announcement as well as the launch of the website will be presented at: http://www.mathematik.uni-dortmund.de/~erme/.

The deadline for applications for admission is 20 January 2014.

As experience from previous years has shown, the YERME Summer School generates a great deal of interest among researchers, particularly because of its inclusive atmosphere, as well as the high quality of the scientific discourse. Former YERME Summer Schools have taken place in Klagenfurt (Austria, 2002), Podebrady (Czech Republic, 2004), Jyväskylä (Finland, 2006), Trabzon (Turkey, 2008), Palermo (Italy, 2010) and Faro (Portugal, 2012).

The international organising programme committee for YESS-7 consists of Viviane Durand-Gerrier, João Pedro da Ponte, Paolo Boero, Susanne Schnell, C. Miguel Ribeiro and Rita Borromeo-Ferri. Paolo Boero has assumed the role of scientific coordinator. The venue of Summer School 2014 is Kassel, right in the heart of Germany. The local organisers are Rita Borromeo Ferri (Chair), Werner Blum and Maike Hagena (all from the University of Kassel).

The major lines of research in mathematics education at the University of Kassel are teacher knowledge and practices, teacher development, teaching and learning of mathematical modelling, problem solving as well as other key competencies, numbers and statistics, use of technology in the classroom, assessment of students, diagnosis and feedback in mathematics lessons. It also has a Master's programme in mathematics education for pre-service teachers and an education programme for vocational teachers.

Susanne Schnell [susanne.schnell@tu-dortmund.de] is currently post-doctoral research assistant at the Institute for Development and Research in Mathematics Education (IEEM), TU Dortmund University. She is representative of Young Researchers in the ERME Board.

"Show Me Your Data! What Publication Records in zbMATH Can Say About Women's Careers in Mathematics"

Helena Mihaljević-Brandt and Lucía Santamaría (both FIZ Karlsruhe, Berlin, Germany)

The gender gap in academia, in particular the differences in productivity for male and female scholars and their apparent persistence over time, has been a topic of interest for the last few decades. A large body of research is devoted to finding explanations for the so-called productivity puzzle and to proposing measures for alleviating the gender imbalance, especially in STEM[1] fields. Although the presence of women among graduate and postgraduate degree holders has increased over time, permanent positions in research and science, let alone the high-rank university appointments, are far from being proportionately distributed among men and women. For instance, the recent survey commissioned by the LMS shows that a meagre 6% of all professors in mathematics departments at British universities are women.[2]

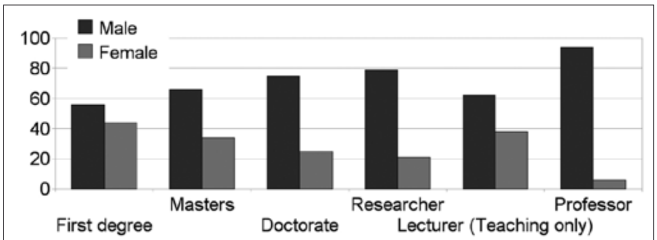


Figure 1. The percentage of women and men in mathematics departments in UK higher education institutions, as presented in the recent study commissioned by the LMS.

Undoubtedly, a key factor in achieving and sustaining a successful academic career is a solid record of scholarly contributions in the form of research papers. Several studies have thus looked at the differences in scientific output according to gender. As editors of zbMATH, we wanted to contribute to the discourse by performing a quantitative analysis on a large body of bibliographic data. The zbMATH database, which has evolved from the reviewing service Zentralblatt MATH, comprises the largest digital metadata collection of publications in pure and applied mathematics reaching back to the 19th century. Currently, over 3,300,000 entries drawn from more than 3,000 journals and serials, and 170,000 books are indexed in the service.

The attempt to extract, from a very large corpus of publications, reliable information on the scientific output of researchers and to discover possible dependences on gender automatically faces the problems of author identification and gender assignment. The task of attributing an exact set of publications to a certain author based on person name is far from trivial, due in part to common names (e.g. John Miller), the eventual omission or abbreviation of parts of the name by (some) publishers, the variability in names arising from spelling and transliterations, or even name changes after marriage. This explains why author name disambiguation is a longstanding research topic with high relevance for bibliometric studies and publication retrieval, and why many information infrastructure providers, publishers and researchers are eagerly searching for sustainable and effective global solutions.

The correspondence between person names and gender is even more involved, in particular when dealing with data gathered from heterogeneous multilingual sources, as is typical for the zbMATH corpus. In the past decades, unisex names have progressively become more popular, in particular in English-speaking countries, making it increasingly difficult to assign an unequivocal gender even within the same cultural context. Furthermore, even names that unambiguously denote a given gender within one country can be assigned to the opposite gender in another country. For instance, "Andrea" is a typical male name in Italy but is given to women in many other countries including Germany, Hungary and Spain. Sometimes, the ordering of compound first names matter, as evidenced by "María José" and "José María", and the gender information can be encoded in the surname, as happens in Russia and other former Soviet countries. Last but not least, a problem that remains near to unsolvable is the treatment of transliterated names from China and some other East Asian countries due to the high loss of information.

The service zbMATH devotes continuous efforts to correcting author identification and creation of author profiles. Since the amount of data is extremely large (and keeps growing) – currently, more than 3,300,000 publication records are related to more than 5,000,000 authorships, corresponding to approximately 800,000 author identities – most of the author identification at zbMATH is performed algorithmically.[3] Clearly, false assignments do occur and it is an ongoing work to decrease the amount of wrongly assigned publications as much as possible. For our analysis we hence excluded erroneous author profiles from our starting dataset in order to produce reliable results. For gender attribution we used a dictionary of more than 42,000 first names covering the

[1] STEM stands for science, technology, engineering and mathematics.

[2] http://www.blitzadv.co.uk/LMS-BTL-17Report.pdf.

[3] See "On authors and entities" in this column, Eur. Math. Soc. Newsl. 71, 43–44 (2009).

vast majority of first names in Europe and in overseas countries (USA, Japan, India, etc.). The dictionary, which is freely accessible on the Web,[4] is based on census information and, for some countries, manual classification by native speakers. The gender of a name is provided with a dependence on its cultural context, together with information on approximate frequency of occurrence.

We have applied this gender assignation scheme to the subset of records in zbMATH classified as pure mathematics (MSC 03 to 58) since the 1970s. The author names for which no reliable gender could be found were labelled as "unknown". The topics we have addressed in our study are the proportion of women authors in mathematics, the difference in the average scientific production between male and female researchers (productivity puzzle), the representation of women in particular subfields and their presence as authors of leading research journals. To the best of our knowledge, our analysis comprises the largest corpus of data ever used for a study on scholarly publications and gender in mathematics, and maybe even for any other scientific field. Our research is still at the beginning and we are planning to refine our results but also broaden our scope as part of an ongoing research project. Among other topics, we would like to study gender distribution among single-author papers, look for communities with a high occurrence of women and analyse gaps in careers over time. A research article presenting our findings thus far is already in preparation.

As a first communication of our work we have recently contributed a talk at the 16th General Meeting of the European Women in Mathematics (EWM) in Bonn, titled "Show me your data! What scholarly publication numbers can say about women's careers in mathematics".[5] One of our goals there was to initiate a dialogue around the question of what community-based services like zbMATH can do to raise awareness on the contributions of women to mathematics and give them visibility. Our presentation elicited the interest of the audience and led to positive feedback and fruitful discussions. Possibly the most commented result of our study was the gender distribution of the authors in several top journals: with an average of less than 5% female authors, there is no noticeable increase when looking at data from the last few decades, as Figure 2 indicates for the concrete case of the *Journal of the AMS*.

Compared to these data, the figures from the *Journal of the European Mathematical Society* almost look pleasing, although there is still room for improvement until a fair representation of female mathematicians in top journals is achieved. Results like these crave for explanations, as they might reveal an unconscious bias and need to be taken seriously by the mathematical community. This goal does not come from a capricious whim – the percentage of active female mathematicians at the graduate, post-graduate and professorial level has for some time been larger than what's seen in these top journals. It is thus a matter of equitableness that this fact be mirrored in the scholarly publication landscape.

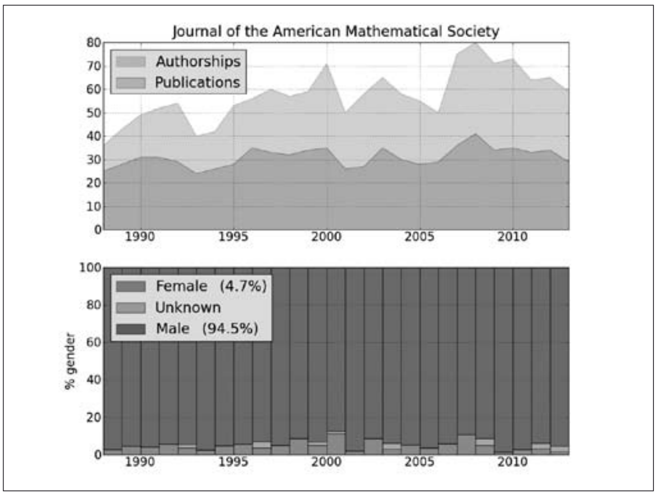


Figure 2. Top panel: number of publications and authorships (publications times number of authors) per year in the *Journal of the AMS* since the late 1980s. Bottom panel: percentage of male and female authors per year.

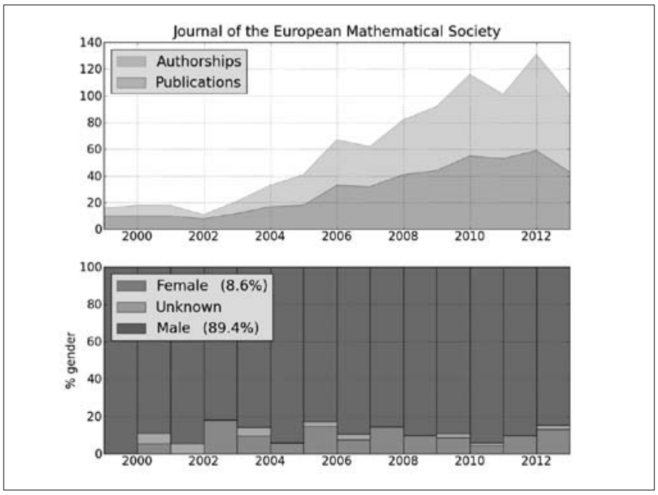


Figure 3. Figure displaying the same data described in the figure above but computed for the *Journal of the EMS*, which was founded in 1999.

We hope that our research contributes to raising awareness on the matter of advancement of women in mathematics. Furthermore, we believe that our work provides an additional and significant source of data for further discussions on this and related topics.

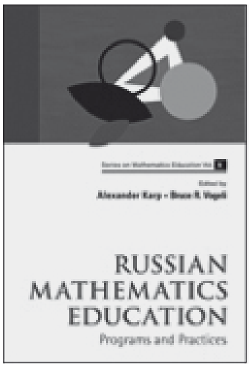
Helena Mihaljević-Brandt was born in 1982 in Sarajevo and is currently working on the editorial board of zbMATH. She studied mathematics in Göttingen and obtained her PhD from the University of Liverpool in the field of complex dynamics.

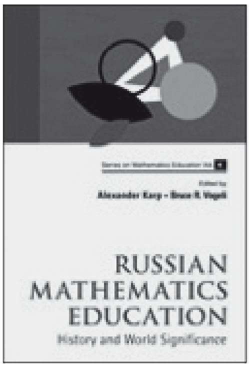
Lucía Santamaría studied theoretical physics in Salamanca, Spain and did her PhD in numerical relativity and gravitational-wave astrophysics at the AEI in Potsdam. As an editor at zbMATH she has been involved with the improvement of author disambiguation techniques.

[4] The dictionary can be downloaded directly from ftp://ftp.heise.de/pub/ct/listings/0717-182.zip

[5] Slides available at http://www.zentralblatt-math.org/static/ewm/Presentation_EWM2013.pdf.

Book Reviews





Alexander Karp and Bruce R. Vogeli

Russian Mathematics Education
Vol. 4: History and World Significance, 2010
Vol. 5: Programs and Practice, 2011

World Scientific, Singapore

Reviewer: Mariolina Bartolini Bussi

The quality of Russian mathematics education became a topic of interest, at the international level, some decades before Chinese mathematics education. At the end of the 1950s, the Soviet Union succeeded in launching the first artificial Earth satellite. The Soviets had won the space race, demonstrating their unsurpassed technology. This success was perceived by the American media as evidence of Soviet supremacy over science and technology education in the US and paved the way towards greater attention on the ways of teaching and learning mathematics in Russia. As Karp and Vogeli write in the introduction to the volumes to be reviewed here:

> *There has been no shortage of writing on Russian (Soviet) education in general and its mathematics component in particular. […] As far as we know, no attempt, however, has been made, pre-Gorbachev or since, to give a systematic description and analysis of the origin and development of mathematics education in Russia. This two-volume work is an attempt to provide this description (vol. 4, p. vii).*

The two-volume work contains 20 chapters, each of them addressing a specific topic. Most chapters have been prepared in Russian and then translated into English for this publication. Hence they are new to the Western readership. In this review we shall comment on some of the chapters.

The first volume is opened by four chapters on the history of mathematics education in Russia:

1. *Mathematics Education in Russia before the 1917 Revolution* (T. Polyakova).
2. *Reforms and Counter-Reforms: Schools between 1917 and the 1950s* (A. Karp).
3. *Towards a History of Mathematics Education Reform in Soviet Schools (1960s–1980s)* (A. Abramov).
4. *Challenges and Issues in Post-Soviet Mathematics Education* (M. Bashmakov).

The first period (10th to 17th century) is briefly outlined in order to show the background of where mathematics education emerged in the 18th century as a broad national concern. Peter I (the Great), after travelling to Europe, brought about the first public schools with a clear, dominant position for mathematics. The great Swiss mathematician Leonhard Euler was invited in 1727 to Petersburg Academy, founded in 1724 by Peter I. Euler himself was the founder of the Mathematical-Methodological School, which played an ever-increasing role in the development of mathematics education, over the course of the entire 18th century and later. Euler published and influenced the publication of several school textbooks with innovative ideas, for the time and, as the author subtly notes, "even for today" (vol. 4, p. 14). Since then, most famous Russian mathematicians (e.g. Lobachefsky and Chebyshev) have spent time and energy writing textbooks and defining pedagogical guidelines and syllabuses. Russian mathematicians were well represented in the movement towards reform in mathematics education that was officially organised in Rome in 1908 when the International Commission on Mathematical Instruction was founded (with Felix Klein as president).

The second period (1917–1950s) was strongly affected by the revolution of 1917. It is analysed by historians with ambiguous attitudes, criticising in some cases the works of revolutionary writers as "models of incompetence, utopianism and impracticality" and praising in other cases the liberal style introduced by them and by talented teachers. However, the literature about these early years of the Soviet schools is very limited, not only in the West but also in the original Russian. However, as is obvious, "all subsequent developments in Russian mathematics education in one way or another made use of what had been achieved during these years or attempted to reorganize and reform it" (vol. 4, p. 80).

The third period (1960s–1980s) is linked from the very beginning to the name of Andrey Nikolayevich Kolmogorov (who, together with Leo Tolstoy for general education in the former period, has won worldwide recognition for achievements and devotion to the cause of education). In Chapter 3, Kolmogorov (1903–1987) is a protagonist. After founding a great school of mathematics, he devoted the final 24 years of his life to improving mathematics education in Soviet schools. This commitment is partly related by the author to Kolmogorov's conviction that "it was his moral duty to do everything possible for science, for his homeland and for humanity" (vol. 4, p. 89). This attitude is not uncommon among Russian scientists (Chapter 5 of the book is devoted to the tradition of involvement of Russian mathematicians in mathematics education). Kolmogorov was especially interested in facilitating the development of talented children and was one of the founders (in 1935) of the first Moscow Mathematics Olympiad for schoolchildren, with the so-called "mathematics circles" for students alongside. Kolmogorov's influence was also on curricular reforms, from primary school onward. One of the most interesting ideas introduced at that time was the need to approach early algebraisation in primary school, with letter notations, formulas, simple linear equations and corresponding problems. This paved the way toward the original curriculum developed by Davydov for early algebra, which was also well known in the West (see below).

The fourth period (post-Soviet mathematics education) starts with Gorbachev's *perestroika* and illustrates school reforms that were also influenced by parallel educational practices abroad. Important events were: the emergence of a uniform standard (paired with a greater control to be granted to regional authorities); the competition to produce new textbooks; the definition of objectives in three different areas (pragmatic or utilitarian issues; intellectual issues for the development of the individual; and pedagogical issues for principles of intellectual integrity); and the dawn of the computer age. This very rich chapter ends with an interesting section about comparisons with other countries. The author starts from the study on *Comparative features of levels of education for students 16 years of age*, carried out by the European Mathematical Society on behalf of the European Union, and addresses two main issues, drawing on the final report of the study: 'How are we the same?' and 'How are we different?' I have tried to connect with the quoted website of the study but unfortunately it is no longer available.

The following two chapters (5 and 6) are related to each other.

5. *Mathematicians and Mathematics Education: A Tradition of Involvement* (A. Sossinsky).
6. *Russian Traditions in Mathematics Education and Russian Mathematical Contests* (M. Saul & D. Fomin).

In the former, several portraits appear of famous mathematicians who were involved in educational matters, mainly in the education of talented students. In the latter, some kinds of contests are analysed together with sample problems. The tension towards mathematical contests is related to the difference between Russia and its neighbouring countries ("if it can be said to have neighbours", vol. 4, p. 224). This analysis is very interesting and deserves a larger quotation, as this part of the story is not so well known in the West:

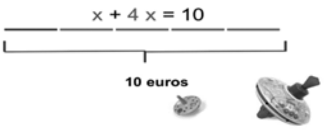
> *Before 1917 most of Russia was agrarian and illiterate. Industrialization was taking place rapidly in the early years of the 20th century, but did not yet affect the bulk of the population. Cultural life was under strong political control. The revolutions of 1917, which eventually brought the Bolsheviks to power, were accompanied by severe difficulties, but also ushered in a decade full of hopes and dreams for many Russians… the evolution of new styles of art, literature, music, dance and architecture, that has started in the last years of the Tsars, received new impetus and took unexpected turns, often supported by the newly formed Soviet state. Then the totalitarian noose started tightening around the country's intellectual life. A mass emigration of talent depleted the artistic community. The Russian scientific community became isolated from the outside world […]. Mathematics, on the other hand, offered intellectual freedom. Mathematicians needed no laboratories, and so the regime had no levers of control over their work. And the government needed this work, both ideologically and practically. Ideologically, because Communism ("Scientific Socialism") saw science and technology as building the future of humanity. Practically, because the physical and natural sciences, which found direct industrial and military application, depended on the results of mathematicians for their more immediate fruitful results. […] For all these reasons, mathematics attracted fine minds. And, because these minds had some unspoken, even unconscious anti-totalitarian agenda, the mathematical community began to assume the character of a subculture within Russian, or Soviet, society. One of the characteristics of any such subculture is a need to "reproduce", to find new and younger members. More than in most mathematical communities, researchers took an active interest in education (vol. 4, p. 224–5).*

This explains the flourishing of several kinds of mathematical contests, with forms related to the cultural circumstances. Virtually all contest questions are open-ended (and not in the form of short answers or multiple choices items). In the beginning, this style was related to the scarce diffusion of printing and duplicating equipment (for the controlling purposes of the government). Later it became the Olympiad-style. This is a beautiful example of the fact that in life, good and evil cannot always be separated in a strong way but only for ideological purposes.

Chapter 7 by J. Schmittau is devoted to *The Relevance of Russian Elementary Mathematics Education* and mainly to the Davydov curriculum for primary schools, exploiting Vygotskian theory and dialectical logic. This part of Russian research on mathematics education has been, probably, the most influential in the West, as it entered with originality into the debate about set theory as the base for the development of numbers. Davydov and his colleagues sought to eliminate the gap between elementary and secondary mathematics study, noting that the former is traditionally focused on number and arithmetic, whilst the latter on algebra. The way to fill the gap was found exploiting Vygotskian theory and dialectical logic. An example is given about the circle. Following Vygotsky, Davydov criticises the empirical way of approaching the circle, asking students to "abstract" the concept from a variety of objects such as round plates, discs, wheels and so on. A theoretical approach would ask children to fix one end of a piece of string and rotate the other, thereby producing a circle and abstracting its essence as a path which is everywhere equidistant from a fixed point. Just one experience, carefully managed, can in this way lead pupils to the meaning of circle, illustrating the Hegelian idea of the "ascent from the abstract to the concrete" (vol. 4, p. 255). Davydov sought to introduce children to theoretical thinking in mathematics as early as possible, in marked contrast to the Piagetian imperative to delay learning that required abstract reasoning until adolescence. Although this idea may seem Utopian, there is also a trend of research in the West that has developed this approach (see, for instance, Bartolini Bussi, 1996, and Bartolini Bussi et al., 1999). As far as numbers are concerned, Davydov rejected the popular Western approach through sets, instead fostering a measuring approach, drawing on quantities and developing algebraisation early at primary school level. It is not possible to enter into details here but it is worthwhile mentioning just a small idea (that is introduced in Chapter 2 of the second volume), the so-called "pictorial equations", which proves to be a very strong tool for shifting the attention from numbers to quantities and relationships between quantities. Consider, for instance, the following word problem about tops.

John has spent 10 euros to buy 2 tops. The smallest one costs ¼ of the other. How many euros does each top cost?

A pictorial equation is represented here. The first segment on the left is the unknown. This drawing is a preliminary and pictorial representation of the symbolic equation:



$$x + 4x = 10$$

In Chapter 2 of vol. 5 on *The History and the Present State of Elementary Mathematical Education* (O. Ivashova), Davydov's curriculum is placed side by side with other curricula of the same period. Davydov's curriculum was used (as was another one by Zankov) only in experimental settings and criticised by other Russian contemporary mathematics educators, as "they worked well in terms of general development but were inefficient in furnishing students with specific mathematical skills" (vol. 5, p. 59). But some of their ideas were also popularised by other textbook authors (e.g. Moro et al.). In short, it seems that Davydov's message for early algebraisation has become a standard in later Russian textbooks from the 1st grade onward (vol. 5, p. 73), with a position that is not so common in the West (for a relevant exception, see Cai & Knuth, 2011, where, however, one of the editors is of Chinese origin).

Chapter 8 concerns *The Preparation of Mathematics Teachers in Russia: Past and Present* (N. Stafanova). After a very short reference to pre-primary and elementary schools (taught by generalist teachers), the chapter is then devoted to secondary schools. At the beginning, a "trivial" statement is given: "mathematics teachers must know their subject well and to be able to teach it to their students" (p. 292). But to better define this idea, there is no reference to Western research on mathematics teacher education, which has tried to split this competence into many sub-competences to be studied separately. Without entering into details, we can just observe that, as in other countries, to enter "a mathematics teacher preparation program, every applicant must go through a competitive selection to obtain one of a limited number of available places in the program, that are paid for by an agency of the federal government, currently the Ministry of Education and Science" (vol. 4, p. 300).

The penultimate chapter of the first volume is devoted to *Russian Influence on Mathematics Education in the Socialist Countries (Poland, Hungary and Cuba)*. There is no mention of the large influence on Chinese Mathematics education (see Wang, 2013, p. 66).

Chapter 10 is devoted to an outline of the *Influence of Soviet Research in Mathematics Education in the US* (J. Kilpatrick). The author was among those responsible for the English translation of the 14 volumes of the series Soviet Studies in the Psychology of Learning and Teaching Mathematics (1969–1975) and Soviet Studies in Mathematics Education (8 volumes, 1990–2) together with other important Russian contributions.

Vol. 5 "deals mainly with the contemporary situation, although this does not rule out a historical perspective, without which it is often impossible to understand what is happening today" (vol. 5 p. vii).

Chapter 1 is *On the Mathematics Lesson* (Karp & Zvavich). To begin with, some contextual information is given, which is very useful for understanding the Russian context.

> *Perhaps the most important difference between the teaching of mathematics in Russia and, say, the United States is the fact that usually a teacher works with the same classroom for a considerable length of time – the composition of the class virtually does not change, and the class continues to have the same teacher. […] A teacher can be assigned to a 5th grade classroom*[1] *and, in principle, remain with the students until their graduation (vol. 5 p. 3).*

Elementary school teachers are generalist and teach all the subjects, whilst after grade 5 they are specialist (e.g. mathematics teachers). Classrooms were, in the past, crowded with up to 40 students but now they only have 25–30 students.

> *The mathematics classroom, a special classroom in which mathematics classes are conducted, has usually seemed barren and empty to foreign visitors. They see no cabinets filled with manipulatives, no row of computers next to the wall or in the back of the room, no tables nearby piled high with materials of some kind or other. […] The large room has three rows of double desks, and each double desk has two chairs before it. […] The front wall is fully mounted with blackboards.*

[1] Grades 1–4 are for elementary school (starting when children are 7 years old); grades 5–10 are for secondary school, with no distinction between middle and high school.

[..] Various drawing instruments usually hang beside the blackboards. […] in sum, we would say that the mathematics classroom has usually had, and indeed continues to have, a Spartan appearance not only because Russian schools are poor. […] The view is that students should not be distracted by anything extraneous during class. Class time is not a time for leisurely looking around, but for intensive and concentrated work (vol. 5, p. 10).

This text may be compared with a famous quotation from Dewey (1900), who would have criticised the Russian classroom as the prototype of "traditional education" to be contrasted with his view of active methods centred on the learners.

Some few years ago I was looking about the school supply stores in the city, trying to find desks and chairs which seemed thoroughly suitable from all points of view – artistic, hygienic, and educational – to the needs of the children. We had a great deal of difficulty in finding what we needed, and finally one dealer, more intelligent than the rest, made this remark: "I am afraid we have not what you want. You want something at which the children may work; these are all for listening." That tells the story of the traditional education. Just as the biologist can take a bone or two and reconstruct the whole animal, so, if we put before the mind's eye the ordinary schoolroom, with its rows of ugly desks placed in geometrical order, crowded together so that there shall be as little moving room as possible, desks almost all of the same size, with just space enough to hold books, pencils and paper, and add a table, some chairs, the bare walls, and possibly a few pictures, we can reconstruct the only educational activity that can possibly go on in such a place. It is all made "for listening"—because simply studying lessons out of a book is only another kind of listening; it marks the dependency of one mind upon another. The attitude of listening means, comparatively speaking, passivity, absorption; that there are certain ready-made materials which are there, which have been prepared by the school superintendent, the board, the teacher, and of which the child is to take in as much as possible in the least possible time.

In Chapter 1 of vol. 5, the scarce (if any) care in furnishing classrooms is contrasted with the great care in designing and studying the lesson structure. Spartan classrooms and care in studying the lessons are also typical of schools in the Far-East (e.g. Chinese classrooms), which have developed specific methodologies to improve the lesson structure. In a recent book on the methodology of mathematics education (Maslov, 2005, quoted by the author), 19 types of mathematics lesson are identified and carefully described.

Chapter 2 of vol. 5 concerns *The History and Present State of Elementary Mathematics Education in Russia* (O. Ivashova). We have briefly mentioned this chapter above.

The four following chapters (3–6) concern secondary school and different subject areas (3: Geometry; 4: Algebra; 5: Analysis; 6: Combinatorics, Probability and Statistics). They concern the standards and illustrate some sample problems.

The last four chapters concern:

7. *Schools with an Advanced Course in Mathematics and Schools with an Advanced Course in the Humanities* (A. Karp).
8. *Assessment in Mathematics in Russian Schools* (A. Karp & L. Zvavich).
9. *Extracurricular Work in Mathematics* (A. Marushina & M. Pratusevich).
10. *On Mathematics Education Research in Russia* (A. Karp & R. Leikin).

It is not possible to give more details but it is worthwhile to quote some observations from the very last chapter, where a comparison between Russian and Western mathematics education is sketchily outlined.

In the West work that is specifically aimed at improving how children are taught – work on textbooks and programs – is largely (even if with important exceptions) a commercial, rather than a scientific concern. As for scientific work, it is often focused on far narrower problems, which in our view pertain, strictly speaking, to psychology, not to mathematics education. The loss here is twofold: commercial textbooks lack deep methodological ideas (or sometimes any methodological ideas), while scientific works lack a sense of reality and practical applicability (vol. 5 p. 470).

This final statement could not be applied in the same way to all the Western traditions of research in mathematics education. It is my contention that, in the Italian tradition of research for innovation in mathematics education, a balance between scientific rigour and practical applicability is maintained (Arzarello & Bartolini Bussi, 1998). In a sense, it is not surprising that the attitude is similar to the Russian tradition of mathematics education, as the influence of Russian educators and mathematics educators has been very large in Italy too.

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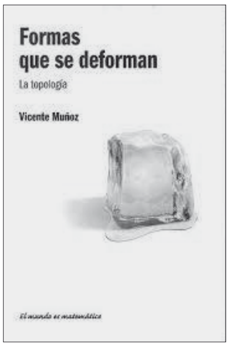


Vicente Muñoz

Formas que se deforman.
La topología
El mundo es matemático
(Spanish edition)
ISBN: 978-84-473-7439-7
RBA Coleccionables S.A.,
Barcelona, Spain, 2011

Les formes qui se déforment. La topologie
Le monde est mathématiques (French edition)
ISBN: 978-2-8152-0476-7

Distorting and Transforming Shapes. Topology
Everything Is Mathematical (English edition)

Reviewer: Zindine Djadli

The book "Les formes qui se déforment" (in French), written by Vicente Muñoz, aims to give an overview of topology from a very simple point of view. As a definition, the author proposes the following: topology is the study of objects and their deformations, considering that an object is equal to another one if it is possible to deform one into the other without breaking it. One of the purposes of this book is to explain the deep meaning of this definition in different contexts. The paradox is that, apparently, topology seems not to be part of mathematics. For many people maths comprises formulas, equations, functions and numbers but here, the material seems to be immaterial and exploring the deep nature of objects is more of an abstract matter and a jump into creativity and abstraction. That is not the usual way people see mathematics! Moreover, the author takes a lot of interesting examples from other fields of science and 'real life'.

This book is a trip toward topology aimed at non-specialists, even if specialists will also surely find a lot of pleasure in reading it. I will not call it a popularisation book. It could be seen as a book about the popularisation of mathematics but it is also much more than that. It is a travel guide, a guide for people who want to travel into a strange world, a world where intuition can be misleading, a world where objects seem familiar but are still so complicated. It is not a coincidence that the book starts with an introduction centred on some considerations about the shape of the Earth and the universe (and ends with this also); it is an invitation, as written by the author, to observe and then to understand. This starts by observations of what stands around us and what we live on.

The second chapter of the book is also a journey, a journey into a two dimensional world, a world called Flatland (a reminiscence of a novel by Edwin A. Abbott as mentioned in the book). It is also related to some parts of the book by Brian Greene "The elegant universe", where Greene tries to explain string theory and the introduction of extra dimensions starting from a world where there are only two. The third chapter is devoted to the classification of surfaces (what Vladimir Arnold thought was as important as the discovery of America).

Progressing through the book, geometry enters into the game and it is here that we see the real strength of the book. The author doesn't avoid (as many authors do when trying to explain topology to the uninitiated) the fundamental link between topology and Riemannian geometry. Riemann becomes a very important player in the book, along with Poincaré, of course, in Chapter 5, which is devoted to the study of three dimensional objects and Poincaré's conjecture. The last chapter is, to my opinion, the most stimulating one. The author comes back to his original motivation: what is the shape of the universe? And not only its shape but, more importantly, what is its future, or, if one prefers, how does its shape command its future? This chapter is not only a way of saying that topology is more than a game for mathematicians or 'food for thought'; it is also part of an answer to one of the most important questions we can ask: what is our destiny?

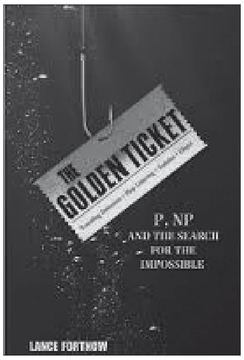
There is no need to say that I really like this book (and I confess that I don't like it as a mathematician). I like it because it is a permanent invitation to curiosity. I like it because it is a nice attempt to convince people that questions are more important than answers. I really recommend the reading of this beautiful book, especially to young people and students (not only mathematical students of course) who sometimes wonder what mathematics is for. It would surely be a very nice Christmas gift!

Zindine Djadli is a professor of mathematics at University Grenoble-Alpes (France). His fields of interest are Riemannian geometry and global analysis on manifolds.

Erratum of the book review "The Millennium Equation and the Fun of Mathematics"

Contrary to what was mentioned in the book review "The Millennium Equation and the fun of mathematics" (pages 58-59 of September 2013 issue), the authors of the comic strip "L'équation du millénaire" are Nicolas Rougerie, mathematician, Gaël Octavia, scientific journalist, and Camille Bouvard, credited for the drawings. Professors Claude Bardos and Jean-Yves Chemin have not been asked for scientific supervision of the scenario but have provided the authors with precious historical information and anecdotes.

Gaël Octavia



Lance Fortnow

The Golden Ticket. P, NP, and the Search for the Impossible

Princeton University Press, 2013
ISBN 978-0-691-15649-1

Reviewer: Andreas Maletti

The newsletter thanks Zentralblatt MATH and Andreas Maletti for the permission to republish this review, which originally appeared as Zbl 1267.68003.

"The golden ticket" is your key to the fascinating world of computational complexity. It provides an in-depth, yet easily understandable treatment of one of the most famous problems of mankind: P vs. NP. The whole book is fun to read and can be fully appreciated without any knowledge in (theoretical) computer science. Fortnow's efforts to make the difficult material accessible to non-experts should be commended. He intersperses the technical parts with many anecdotes, some of which might even be news to researchers in theoretical computer science. The book thus caters to all audiences, from novices with an interest in computational problems to experts with knowledge in theoretical computer science.

Chapter 1 introduces the basic P vs. NP problem and presents the mundane search solution. At the same time, the reader is made aware of the limitations of an exhaustive search by showing that even a moderately sized problem might yield a huge search space. Naturally, the chapter also presents the famous Clay Institute list of problems (as the relation between P and NP is on that list), of which solutions are worth $1,000,000 (per problem).

Chapter 2 presents the author's vision of a world in which P = NP. The Urbana programme, the universal solution engine for NP-complete problems, is shown to solve a variety of hard optimisation, scheduling and analysis problems. The author touches on several interesting areas of life here and shows the developments that would ensue with the help of the Urbana programme. While many improvements might be desirable, the author also cautions that the same power can be used towards less desirable goals. Overall, the vision shared here is interesting and very captivating.

Chapter 3 is the first main chapter and introduces the problem in more detail. In particular, it presents several problems set in our world and in Frenemy, in which two people hate each other unless they are best friends. The problems discussed here are: shortest distance, matchmaking, cliques, Eulerian and Hamiltonian cycles, and map colouring. The Frenemy Institute is then tasked with solving these problems and some of the problems indeed have efficient solutions. The author neatly shows that the fastest known algorithms for similar problems can behave in vastly different ways. For example, switching from min-cut, which can be solved efficiently, to max-cut, for which no efficient algorithms are known, illustrates this point. The chapter concludes with some examples of how the P vs. NP problem permeated other disciplines such as biology, physics, economics and, last but not least, mathematics.

Chapter 4 introduces the notions of reduction and NP-hardness in a way that is understandable to anyone with a basic education in mathematical logic. It recounts the history of this fundamental notion with a strong focus on the reductions between clique and SAT and the history of the name NP-complete, which is used for NP-hard problems that are in NP. In addition, it gives a multitude of sometimes well-known and ubiquitous problems that are NP-complete. This is complemented by a selection of NP problems that are seemingly difficult (not known to be in P) but not known to be NP-hard (like factorisation). It is nice to see that even this "middle-ground" is mentioned.

Chapter 5 goes back one step further. It recalls the efforts that eventually led to the famous P vs. NP problem. Since there was a strong divide between the East (Russia) and the West (USA) in those days, it recounts the history of this fascinating problem in those two parts of the world. In particular, the less well-known developments in Russia offer an excellent insight into the famous Russian mathematical research. Most mathematicians and computer scientists might be absolutely unaware of the efforts of the Russian scientists Sergey Yablonsky, Andrey Kolmogorov and Leonid Levin, although they all made important contributions to complexity theory.

Chapter 6 shows how we currently deal with NP-complete problems. While computing advances have allowed us to solve small and sometimes even medium sized problems by brute-force search, the search space of large problems quickly grows beyond all reasonable limits. It might have been similarly unimaginable for researchers living in the 1970s that medium sized problems could be solved but the exponential progress in computing (speeds) has allowed more and more complex problems to be addressed. However, the speed game stopped a while ago. The number of transistors still rises exponentially but new paradigms (like parallelism) are taking the lead. The author thus introduces techniques that guide the search (heuristics) or approximate the result. Finally, the reader is reminded that they maybe want to solve another problem altogether.

Chapter 7 demonstrates standard approaches to proving the relation between P and NP. It shows how one can, in principle, prove properties about all imaginable algorithms. In addition, it warns the reader about the common pitfalls and recalls the interesting JACM submission policy for P vs. NP papers.

Chapter 8 deals with cryptography, which uses the fact that NP-hard problems are currently difficult to solve. The chapter first recalls the highlights of the history of secret codes and then recounts ubiquitous public-key cryptography. The currently used cryptographic systems would be easy to break if P = NP, so the author discusses the remaining provably secure system (one-time pad) and how this could be implemented in a P = NP world. Finally, zero-knowledge protocols, which can be used to convince people that one knows a secret without giving it away, are introduced on the Sudoku example.

Chapter 9 gives a short overview of quantum technology, which is the basis for quantum computing. It shows how this computational model affects the P vs. NP question and illustrates that the difficulty switches from the actual computation, which is now relatively easy due to inherent parallelism, to reading off (observing or recognising) the final result. This observation problem is explained together with the important notion of entanglement, which is currently hard to maintain perfectly, only adding to the complexity.

Chapter 10 is the last chapter and provides an outlook into the reasonable future. This world is very different from the world described in Chapter 2. It showcases the upcoming challenges like "Big Data" and the new technologies that still need to be fully utilised.

Andreas Maletti received a PhD in theoretical computer science from the University of Technology Dresden, Germany, in 2006. He currently holds an Emmy-Noether grant that allows him to lead a research team at the University of Stuttgart to investigate the use of formal automaton models in machine translation. His research interests include the theory of tree automata and tree transducers and applications in natural language processing.

Personal Column

Please send information on mathematical awards and deaths to Mădălina Păcurar [madalina.pacurar@econ.ubbcluj.ro]

Awards

Assyr Abdulle (École Polytechnique Fédéral de Lausanne, Switzerland) has been awarded the 2013 **Germund Dahlquist Prize**.

Michel Balinski (CNRS, École Polytechnique/Paris Tech, France) was named the 2013 recipient of the INFORMS **John von Neumann Theory Prize.**

Andriy Bondarenko (Kiev University, Ukraine) has been awarded the 2013 **Vasil A. Popov Prize**.

Margaret Brown (King's College London, UK) has been awarded the 2013 **Kavli Education Medal**.

Alberto Enciso (Instituto de Ciencias Matemáticas, Spain) has been awarded the 2013 **SEMA Prize** of the Spanish Society for Applied Mathematics.

Joscha Gedicke (Humboldt University, Germany) has received one of the 2013 **SIAM Student Paper Prizes**.

Antoine Joux (Université de Versailles-Saint-Quentin-en-Yvelines, France) has been awarded the 2013 Gödel Prize, together with Dan Boneh and Matthew Franklin (USA).

Thomas W. B. Kibble (Imperial College London, UK), **Phillip James E. Peebles** (Princeton University, USA) and **Martin John Rees** (University of Cambridge, UK) have been awarded the 2013 **Dirac Medal and Prize**.

Daniel Kressner and **Christine Tobler** (ETH Zürich, Switzerland) have been awarded one of the 2013 **SIAM Outstanding Paper Prizes.**

Kim Larsen (Aalborg University, Denmark), **Paul Pettersson** (Mälardalen University, Sweden) and **Wang Yi** (Uppsala University, Sweden) have been named the recipients of the 2013 **Computer-Aided Verification Award**.

Nicole Megow (Technical University of Berlin, Germany) received the 2013 **Heinz Maier-Leibnitz Prize** in Discrete Mathematics/Computer Science.

Heiko von der Mosel and **Henryk Gerlach** (RWTH Aachen University, Germany) have received the 2013 **Merten M. Hasse Prize**.

Rahul Pandharipande (ETH, Zürich) received the 2013 **Clay Research Award**.

Benoit Perthame (Université Pierre and Marie Curie and Institut Universitaire de France, France) has received the 2013 **Blaise Pascal Medal** in Mathematics.

Friedrich Pillichshammer (Johannes Kepler University, Linz, Austria) received the 2013 **Prize for Achievement in Information-Based Complexity**, together with Josef Dick (University of New South Wales, Sydney, Australia).

Marcin Pilipczuk (Warsaw University, Poland) received the 2013 **Stefan Banach International Prize**.

Richard Pink (ETH, Zürich) received the 2013 **E.H. Moore Research Article Prize**, together with Michael Larsen (Indiana University, USA).

Omri Sarig (Weizmann Institute of Science) received the 2013 **Erdos Prize**.

Peter Scholze (University of Bonn, Germany) has been awarded the 2013 **SASTRA Ramanujan Prize**.

Ivan Smith (University of Cambridge, UK) has been awarded the 2013 **Adams Prize**.

Martin Strehler (Brandenburgische Technische Universität Cottbus, Germany) has received the 2013 **Klaus Tschira Prize**.

Ye Tian (Academy of Mathematics and Systems Science, Chinese Academy of Sciences, PRC) has been awarded the 2013 **Ramanujan Prize**.

Xavier Tolsa (Universitat Autonoma de Barcelona, Spain) has been awarded the 2013 **Ferran Sunyer i Balaguer Prize**.

Masato Tsujii (Kyoshu University, Japan) has received the 2013 **MSJ Autumn Prize**.

Jasson Vindas (Ghent University, Belgium) has been awarded the 2013 **ISAAC Award** of the International Society for Analysis, its Applications and Computation.

The **French Académie des Sciences** has awarded a number of prizes for 2013.
David Hernandez (Université Paris Diderot-Paris 7, Institut de Mathématiques de Jussieu, France) received the 2013 **Jacques Herbrand Prize.**
Jean-Michel Morel (École Normale Supérieure de Cachan, France) received the 2013 **Great Prize Inria**.
Sylvia Serfaty (Université Pierre et Marie Curie, France) and **Pierre Vanhove** (CEA de Saclay, France) received the 2013 **Great Prize Mergier Bourdeix**.
Albert Fathi (École Normale Supérieure de Lyon, France) received the **Great Prize Sophie Germain**.
Zoe Chatzidakis (Centre National de la Recherche Scientifique, Institut de Mathématique, Université Paris 7, France) received the 2013 **Leconte Prize**.
Frédéric Klopp (Université Pierre et Marie Curie, Institut de Mathématique de Jussieu, France) received the **Gabrielle Sand Prize.**
Emmanuel Breuillard (Université Paris-Sud à Orsay, France) received the **Charles-Louis de Saulses de Freycinet Prize**.

The **London Mathematical Society** has awarded a number of prizes for 2013.
John Thompson FRS (University of Cambridge, UK) received the **De Morgan Medal.**
Nick Trefethen FRS (University of Oxford, UK) received the **Naylor Prize and Lectureship**.
Frances Kirwan FRS (University of Oxford, UK) received the **Senior Whitehead Prize**.
Luis Alday (University of Oxford, UK), **Andre Neves** (Imperial College London, UK), **Tom Sanders** (University of Oxford and University of Cambridge) and **Corinna Ulcigrai** (University of Bristol) received the **Whitehead Prize**.

Deaths

We regret to announce the deaths of:

Robert Cauty (7 July 2013, France)
Éric Jaligot (10 July 2013, France)
Egbert Brieskorn (11 July 2013, Germany)
Vicent Caselles (14 August 2013, Spain)
Guy Roos (16 August 2013, France)
Christian Léger (21 August 2013, France)

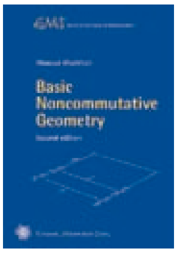
New books published by the

European Mathematical Society

Individual members of the EMS, member societies or societies with a reciprocity agreement (such as the American, Australian and Canadian Mathematical Societies) are entitled to a discount of 20% on any book purchases, if ordered directly at the EMS Publishing House.

François Labourie (Université Paris Sud, Orsay, France)
Lectures on Representations of Surface Groups (Zurich Lectures in Advanced Mathematics)
978-3-03719-127-9. 2013. 146 pages. Softcover. 17 x 24 cm. 32.00 Euro

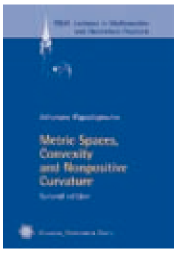
The subject of these notes is the character variety of representations of a surface group in a Lie group. We emphasize the various points of view (combinatorial, differential, algebraic) and are interested in the description of its smooth points, symplectic structure, volume and connected components. We also show how a three manifold bounded by the surface leaves a trace in this character variety.
These notes were originally designed for students with only elementary knowledge of differential geometry and topology. In the first chapters, we do not insist in the details of the differential geometric constructions and refer to classical textbooks, while in the more advanced chapters proofs occasionally are provided only for special cases where they convey the flavor of the general arguments. These notes could also be used by researchers entering this fast expanding field as motivation for further studies proposed in a concluding paragraph of every chapter.



Masoud Khalkhali (The University of Western Ontario, London, Canada)
Basic Noncommutative Geometry. Second edition (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-128-6. November 2013. 258 pages. Softcover. 17 x 24 cm. 36.00 Euro

This text provides an introduction to noncommutative geometry and some of its applications. It can be used either as a textbook for a graduate course or for self-study. It will be useful for graduate students and researchers in mathematics and theoretical physics and all those who are interested in gaining an under- standing of the subject. One feature of this book is the wealth of examples and exercises that help the reader to navigate through the subject. While background material is provided in the text and in several appendices, some familiarity with basic notions of functional analysis, algebraic topology, differential geometry and homological algebra at a first year graduate level is helpful.
Two new sections have been added to this second edition: the Gauss–Bonnet theorem and the definition and computation of the scalar curvature of the curved noncommutative two torus, and a brief introduction to Hopf cyclic cohomology. The bibliography has been extended and some new examples are presented.



Athanase Papadopoulos (IRMA, Strasbourg, France)
Metric Spaces, Convexity and Nonpositive Curvature. Second edition (IRMA Lectures in Mathematics and Theoretical Physics Vol. 6)

ISBN 978-3-03719-132-3. December 2013. Approx. 310 pages. Softcover. 17 x 24 cm. 48.00 Euro

This book is about metric spaces of nonpositive curvature in the sense of Busemann, that is, metric spaces whose distance function satisfies a convexity condition. It also contains a systematic introduction to metric geometry, as well as a detailed presentation of some facets of convexity theory that are useful in the study of nonpositive curvature in the sense of Busemann. The concepts and the techniques are illustrated by many examples, in particular from hyperbolic geometry, Hilbert geometry and Teichmüller theory. For the second edition, some corrections and a few additions have been made, and the bibliography has been updated.
From the reviews on the first edition: …Papadopoulos does a fantastic job of bringing together all sorts of themes in geometry, from introductory material for beginners to intricate properties of moduli spaces of Riemann surfaces. At times the treatment is necessarily somewhat sketchy, but still one can get some ideas, and references are given with additional information. The main body of the text is quite systematic, with digressions, examples, and notes in various directions, and should prove a valuable resource for students in particular. The historical comments are fascinating….(Bull. Amer. Math. Soc.)

European Congress of Mathematics, Kraków, 2–7 July, 2012
Rafał Latała, Andrzej Ruciński, Paweł Strzelecki, Jacek Świątkowski, Dariusz Wrzosek and Piotr Zakrzewski, Editors

ISBN 978-3-03719-120-0. December 2013. 824 pages. Hardcover. 16.5 x 23.5 cm. 108.00 Euro

The European Congress of Mathematics, held every four years, has become a well-established major international mathematical event. Following those in Paris (1992), Budapest (1996), Barcelona (2000), Stockholm (2004) and Amsterdam (2008), the Sixth European Congress of Mathematics (6ECM) took place in Kraków, Poland, July 2–7, 2012, with about 1000 participants from all over the world.
Ten plenary, thirty-three invited lectures and three special lectures formed the core of the program. As at all the previous EMS congresses, ten outstanding young mathematicians received the EMS prizes in recognition of their research achievements. In addition, two more prizes were awarded: the Felix Klein Prize for a remarkable solution of an industrial problem, and – for the first time – the Otto Neugebauer Prize for a highly original and influential piece of work in the history of mathematics. The program was complemented by twenty-four minisymposia with nearly 100 talks, spread over all areas of mathematics. Six panel discussions were organized, covering a variety of issues ranging from the financing of mathematical research to gender imbalance in mathematics.

Robert J. Marsh (University of Leeds, UK)
Lecture Notes on Cluster Algebras (Zurich Lectures in Advanced Mathematics)
978-3-03719-130-9. 2013. 132 pages. Softcover. 17 x 24 cm. 28.00 Euro

The aim of these notes is to give an introduction to cluster algebras which is accessible to graduate students or researchers interested in learning more about the field, while giving a taste of the wide connections between cluster algebras and other areas of mathematics.
The approach taken emphasizes combinatorial and geometric aspects of cluster algebras. Cluster algebras of finite type are classified by the Dynkin diagrams, so a short introduction to reflection groups is given in order to describe this and the corresponding generalized associahedra. A discussion of cluster algebra periodicity, which has a close relationship with discrete integrable systems, is included. The book ends with a description of the cluster algebras of finite mutation type and the cluster structure of the homogeneous coordinate ring of the Grassmannian, both of which have a beautiful description in terms of combinatorial geometry.

Isabelle Gallagher (Université Paris-Diderot, France), Laure Saint-Raymond (Université Pierre et Marie Curie, Paris) and Benjamin Texier (Université Paris-Diderot, France)
From Newton to Boltzmann: Hard Spheres and Short-range Potentials (Zurich Lectures in Advanced Mathematics)
978-3-03719-129-3. 2013. 150 pages. Softcover. 17 x 24 cm. 32.00 Euro

The question addressed in this monograph is the relationship between the time-reversible Newton dynamics for a system of particles interacting via elastic collisions, and the irreversible Boltzmann dynamics which gives a statistical description of the collision mechanism. Two types of elastic collisions are considered: hard spheres, and compactly supported potentials.
Following the steps suggested by Lanford in 1974, we describe the transition from Newton to Boltzmann by proving a rigorous convergence result in short time, as the number of particles tends to infinity and their size simultaneously goes to zero, in the Boltzmann-Grad scaling.
This book is intended for mathematicians working in the fields of partial differential equations and mathematical physics, and is accessible to graduate students with a background in analysis.

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