NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

Editorial
IMAGINARY

Feature
Mathematical Knowledge Management

Discussion
Who Are the Invited Speakers at ICM 2014?

June 2014
Issue 92
ISSN 1027-488X

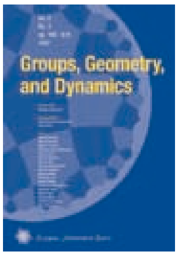
Cover picture: Herwig Hauser

Citrus $x^2+z^2 = y^3(1-y)^3$

European Mathematical Society

Journals published by the

European Mathematical Society



ISSN print 1661-7207
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2014. Vol. 8. 4 issues
Approx. 1200 pages. 17.0 x 24.0 cm
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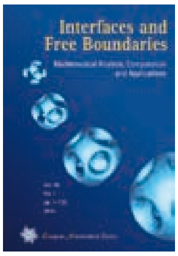
Aims and Scope
Groups, Geometry, and Dynamics is devoted to publication of research articles that focus on groups or group actions as well as articles in other areas of mathematics in which groups or group actions are used as a main tool. The journal covers all topics of modern group theory with preference given to geometric, asymptotic and combinatorial group theory, dynamics of group actions, probabilistic and analytical methods, interaction with ergodic theory and operator algebras, and other related fields.



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2014. Vol. 16. 4 issues
Approx. 500 pages. 17.0 x 24.0 cm
Price of subscription:
390 € online only
430 € print+online

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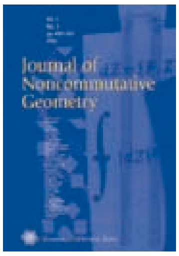
Aims and Scope
Interfaces and Free Boundaries is dedicated to the mathematical modelling, analysis and computation of interfaces and free boundary problems in all areas where such phenomena are pertinent. The journal aims to be a forum where mathematical analysis, partial differential equations, modelling, scientific computing and the various applications which involve mathematical modelling meet.



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2014. Vol. 8. 4 issues
Approx. 1200 pages. 17.0 x 24.0 cm
Price of subscription:
298 € online only
358 € print+online

Editor-in-Chief:
Alain Connes (Collège de France, Paris, France)

Aims and Scope
The *Journal of Noncommutative Geometry* covers the noncommutative world in all its aspects. It is devoted to publication of research articles which represent major advances in the area of noncommutative geometry and its applications to other fields of mathematics and theoretical physics. Topics covered include in particular: Hochschild and cyclic cohomology; K-theory and index theory; measure theory and topology of noncommutative spaces, operator algebras; spectral geometry of noncommutative spaces; noncommutative algebraic geometry; Hopf algebras and quantum groups; foliations, groupoids, stacks, gerbes.

ISSN print 1664-039X
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Approx. 800 pages. 17.0 x 24.0 cm
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238 € online only
298 € print+online

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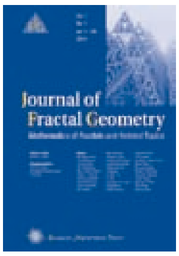
Aims and Scope
The *Journal of Spectral Theory* is devoted to the publication of research articles that focus on spectral theory and its many areas of application.



ISSN print 2308-1309
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2014. Vol. 1. 4 issues
Approx. 400 pages. 17.0 x 24.0 cm
Price of subscription:
198 € online only
238 € print+online

Editor-in-Chief:
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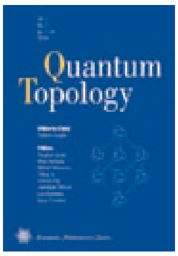
Aims and Scope
The *Journal of Fractal Geometry* is dedicated to publishing high quality contributions to fractal geometry and related subjects, or to mathematics in areas where fractal properties play an important role. The journal accepts submissions of original research articles and short communications, occasionally also research expository or survey articles, representing substantial advances in the field.



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198 € online only
238 € print+online

Editor-in-Chief:
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Aims and Scope
Quantum Topology is dedicated to publishing original research articles, short communications, and surveys in quantum topology and related areas of mathematics. Topics covered include: low-dimensional topology; knot theory; Jones polynomial and Khovanov homology; topological quantum field theory; quantum groups and hopf algebras; mapping class groups and Teichmüller space categorification; braid groups and braided categories; fusion categories; subfactors and planar algebras; contact and symplectic topology; topological methods in physics.

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EMS Agenda

2014

27 June
Executive Committee of the EMS, Donostia/San Sebastián, Spain

28–29 June
EMS Council Meeting
Auditorio Antonio Beristain, University of the Basque Country, Campus de Gipuzkoa, Donostia/San Sebastián, Spain

21–23 November
Executive Committee of the EMS, Barcelona, Spain

EMS Scientific Events

2014

30 June – 4 July
First Joint International Meeting RSME-SCM-SEMA-SIMAI-UMI
Bilbao, Spain
http://www.ehu.es/en/web/fjim2014/

28 July – 1 August
EMS-IAMP Summer School on Mathematical Relativity
http://homepage.univie.ac.at/piotr.chrusciel/SummerSchool2014/index.html

17 – 24 August
EMS-ESMTB Summer School in Applied Mathematics
"Dynamics of Infectious Diseases", Helsinki, Finland
http://mathstat.helsinki.fi/research/biomath/summerschool2014/

1 – 5 September
EMS Summer School in Applied Mathematics
Seventh European Summer School in Financial Mathematics
Oxford, UK
http://www2.maths.ox.ac.uk/euroschoolmathfi14/

1 – 5 September
EMS Summer School, Dyadic Harmonic Analysis and Related Topics, Santander, Spain
http://www.imus.us.es/SANTALO14/

5 – 6 September
Caucasian Mathematical Conference, Tbilisi, Georgia
Roland Duduchava: roldud@gmail.com (President of the Georgian Mathematical Society)
Armen Sergeev: sergeev@mi.ras.ru (Member of the Executive Committee of the EMS)

5–11 October
EMS Summer School in Applied Mathematics
Stochastic Analysis with Applications in Biology, Finance and Physics, Będlewo, Poland
http://bcc.impan.pl/14EMSschool/

2015

10–13 June
AMS-EMS-PMS Congress, Porto, Portugal
http://aep-math2015.spm.pt/

2016

18–22 July
7th European Congress of Mathematics, Berlin, Germany
http://www.7ecm.de/

Editorial: IMAGINARY – Mathematics Communication for the 21st Century

Gert-Martin W. Greuel (University of Kaiserslautern, Germany), Andreas Daniel Matt (Mathematisches Forschungsinstitut Oberwolfach, Germany) and Antonia S. J. S. Mey (Free University Berlin, Germany)

IMAGINARY is the name of a collaborative mathematics outreach project that aims to improve the image and understanding of mathematics and in this way awake an interest and fuel passion for the subject in children and adults. This goal is achieved in different ways: on the one hand by showing the beauty and art in mathematics and on the other hand through surprising applications. To best understand the project we have to go back to its beginning.

IMAGINARY was born at the Mathematische Forschungsinstitut Oberwolfach (MFO) in conjunction with the Year of Mathematics in 2008 in Germany. It started with the travelling exhibition "IMAGINARY – through the eyes of mathematics" shown in 12 German cities. Due to its tremendous success, follow up exhibitions were soon organised in Austria, Switzerland, Spain, UK and Ukraine. The program SURFER, developed for IMAGINARY, became a centrepiece of the exhibition. It teaches in a playful way the connection between formula and form, between algebra and geometry through beautiful 3D surfaces. In this way, it bridges the gap between art and mathematics. An example of such a surface, Citrus, is seen on the front cover of this issue. The visitors of the exhibition get the chance to alter the algebraic equations, see the effects on the displayed surfaces in real time and even get to take a printout back home.

IMAGINARY exhibition at the Leibniz University, Hannover, 2009.

Since 2008, the IMAGINARY exhibition has been shown in over 60 cities in Germany alone but has also travelled further afield to 4 continents, 29 countries and over 120 cities with more than 1 million visitors in total. In Europe, IMAGINARY has been presented in 17 countries with talks, workshops, media activities and, in most cases, exhibitions.

What made the exhibition unique from the beginning is its highly interactive and intuitive nature and its open access and open source philosophy. This is also reflected in the many positive comments left in the guest book:

- This already beautiful exhibition is obtaining a special liveliness by excellent leadership.
- Mathematics makes happy.
- Super, especially that you can also use the program in the school.
- A wonderful exhibition. I have spent much time here and met many beautiful things, it had to take place more often and actually as a permanent event!
- Thank you and keep it up!
- It is a fantastically beautiful exhibition.
- The magic world of mathematics is not easy to understand. But you can bring them closer.
- We were again there, because it was so fascinating.
- I should have perhaps studied math…
- Simply gorgeous, cool programs.

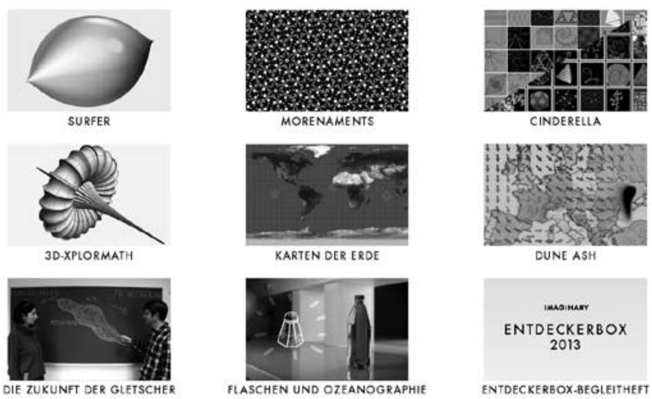
IMAGINARY exhibitions and events have been organised in more than 120 cities.

However, the original exhibition was not enough; it focused on a very beautiful yet small part of mathematics. The project needed to grow further and the Mathematics of Planet Earth Year 2013 (MPE) presented a good opportunity to do so. A competition for virtual exhibition modules themed around MPE was announced and IMAGINARY offered to provide the required web infrastructure in order to make the modules of the competition available online. At the launch of the MPE year in Europe at the UNESCO in Paris, the web interface to IMAGINARY (imaginary.org) went live, displaying entries for the competition and, of course, the winners.

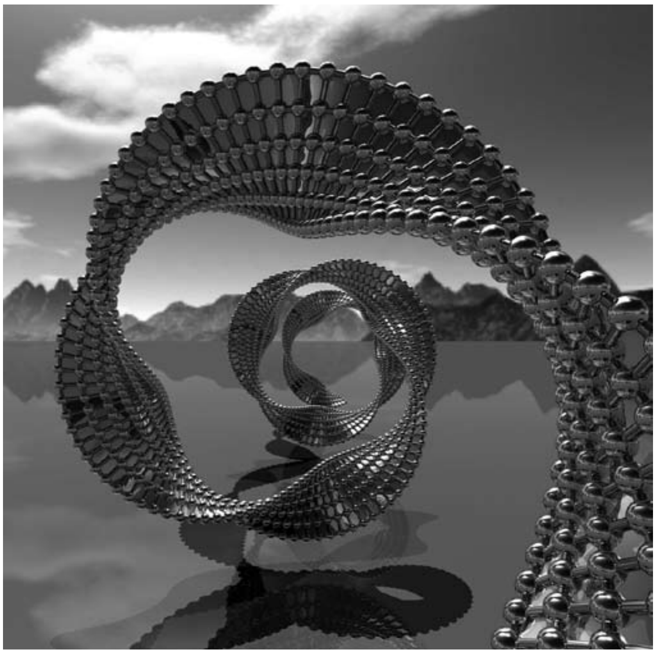
The web platform provides an online resource for mathematics outreach. One might think of it as a "pick and mix science stand", like a pick and mix sweets stand but instead of sweets one gets to choose between different mathematical tools, all for free. Everything should be easy to digest but also awake an interest to learn more about the non-trivial mathematics behind it.

The tools are grouped into different categories, such as exhibitions, galleries, films and hands-on exhibits with instructions on how to recreate them at home, school or university. Currently imaginary.org hosts two full exhibitions: the "IMAGINARY – through the eyes of mathematics" original travelling exhibition is available and free to download; at the same time, a complete MPE exhibition is also available, consisting of a series of modules with a more applied mathematics focus, such as a program that calculates the displacement of volcanic ash clouds (Dune Ash) or a film discussing how mathematical modelling of glacial movement works in order to predict the future behaviour of glaciers. Of course, exhibits from both exhibitions may be mixed. Furthermore, IMAGINARY also ventured into school education. In December 2013 the so-called ENTDECKERBOX (discovery box) was launched. It is primarily aimed at use in the classroom and provides resources for teachers in order to make mathematics lessons more interactive and interesting for the pupils.

Interactive station Dune Ash at the UNESCO in Paris, 2013, simulating volcanic ash dispersion in real time



Nine programs and films included in the IMAGINARY discovery box for schools.



The Björling minimal surface by Ulrich Pinkall – part of the "IMAGINARY – through the eyes of mathematics" exhibition.

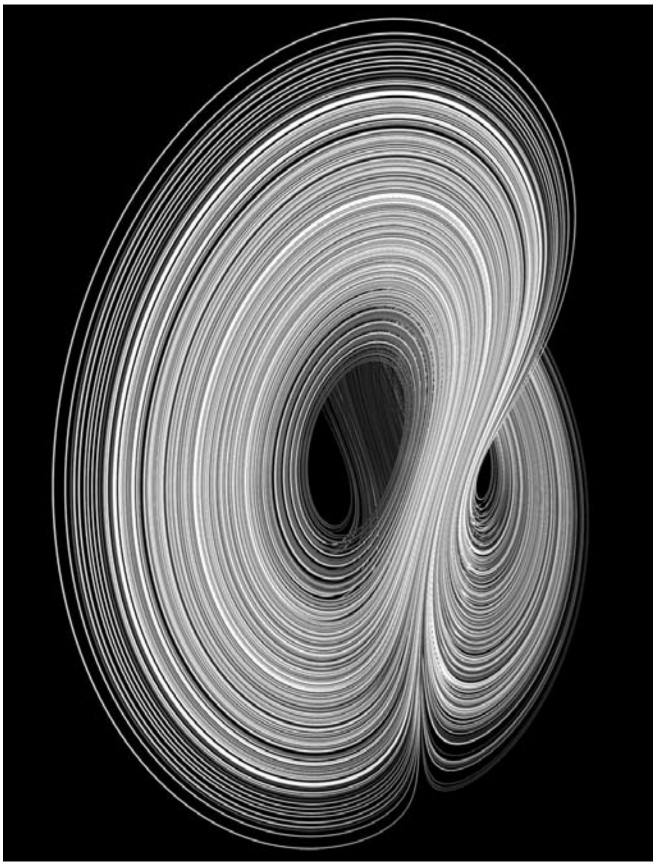
Who is behind IMAGINARY nowadays? IMAGINARY is a project by the MFO, accounted by its director Gerhard Huisken, with funding from the Klaus Tschira Stiftung. It is maintained by a committed core team (mathematicians, software engineers, graphic designers, etc.), who run the project, develop the internet platform and give advice on how to coordinate exhibitions, but also dream up new ventures of where IMAGINARY will go in the future. At the same time, and most importantly, it is community driven. This means that anyone who has an interesting piece of software, film or other type of interactive material can upload this to the website and make it available to the rest of the community. In this way, the community becomes an integral part in the communication process by not only experiencing but also creating content and thus advancing mathematics communication to the 21st century. Of course, anyone can just use the material and create a mathematics event, exhibition or workshop. Due to this community driven aspect of the website, the German news outlet "Spiegel ONLINE" called it the "YouTube of Mathematics".

As a result of its success in general, and particularly in the last two years, IMAGINARY has seen a lot of media coverage across Europe for its numerous successful exhibitions. Gert-Martin Greuel, the former director of MFO and scientific advisor of IMAGINARY, and Andreas Daniel Matt, the curator and project manager of IMAGINARY, both also initiators of the project, were awarded the German Media Prize for Mathematics in November 2013. In the first half of 2014, a new exhibition was launched at the Technische Museum in Berlin, showing three MPE related pieces until 24 June 2014. Furthermore, IMAGINARY has started a collaboration with the African Institute for Mathematical Sciences (AIMS) and, in association with AIMS, an interactive IMAGINARY event was organised for the first time in Africa

SURFER installation at the IMAGINARY exhibition in Belgrade, 2012.

Cedric Villani inaugurating the IMAGINARY exhibition in Paris, 2010.

at the 10th anniversary pi-day celebrations in Dar es Salaam, Tanzania. The event was attended by over 2000 schoolchildren and their teachers from three universities, 33 secondary schools and 14 primary schools. In November 2014, a workshop and exhibition will be organised in Cape Town to plan future mathematics communication activities with partners on the African continent. At the same time, IMAGINARY's travelling exhibitions are currently on tour or planned for the coming months in Germany, Russia, Spain, Norway, Portugal and Hungary. But most certainly, the planned highlight is an IMAGINARY exhibition organised by the NIMS institute at the



The Lorenz attractor by Jos Leys, an image submitted at the MPE competition.

ICM in Seoul, South Korea, in August, which will be the largest IMAGINARY exhibition yet, featuring all modules of previous years, as well as new software, images, films and sculptures. Following the IMAGINARY philosophy, we invite all mathematicians to send us their latest images, films or software programs to be included at the ICM exhibition.

The IMAGINARY pick and mix science stand has become a franchise (non-commercial and open), where you can taste new maths and bring your own ideas. It has grown from a single exhibition into a collaborative framework and community driven movement with IMAGINARY teams in several countries. The Royal Spanish Mathematical Society (RSME), for example, has already organised more than 15 exhibitions and several museum installations. The question now is how does IMAGINARY continue? A new project is to connect modern mathematics and current research to outreach. Mathematicians visiting the MFO are asked to write about their current work but for a general public. These so-called "snapshots of modern mathematics" are then reviewed and edited and distributed through the project. Another new idea is to increase networking between mathematics communicators. To kick start

Young IMAGINARY fans, 2008.

this, an open map of all mathematics museums on our planet was prepared – yes, there are many more than one might think!

IMAGINARY is constantly evolving and always welcomes exciting new contributions and ideas (for example the game http://2048.imaginary.org).

The Newsletter thanks IMAGINARY, and in particular Andreas Daniel Matt, for providing the cover illustration.

Gert-Martin Greuel is a professor at the University of Kaiserslautern. His research interests are algebraic geometry, singularity theory and computer algebra. He is one of the authors of the computer algebra system Singular. Since January 2012, he has been Editor-in-Chief of Zentralblatt MATH. Until March 2013, he was the Director of the Mathematisches Forschungsinstitut Oberwolfach (MFO) and at present he is the scientific adviser of IMAGINARY.

Andreas Daniel Matt is a mathematics communicator at the Mathematisches Forschungsinstitut Oberwolfach. He studied mathematics and computer science and did his PhD in mathematics in machine learning at the University of Innsbruck and the University of Buenos Aires. Since 2007, he has coordinated the project IMAGINARY. He also curates the MiMa Museum for Minerals and Mathematics, Oberwolfach.

Antonia S.J.S.Mey is a postdoctoral researcher in the Mathematics and Computer Science Department at the Free University, Berlin. She completed her PhD at the University of Nottingham in theoretical physics. Her research interests lie in applied mathematics and focus on stochastic estimators and Markov state models and their applications to biological problems. Since 2014, she has been a member of the IMAGINARY team.

AMS-EMS-SPM International Meeting 2015, 10–13 June 2015, Porto, Portugal

The meeting will bring together the American Mathematical Society (AMS), the European Mathematical Society (EMS) and the hosting Portuguese Mathematical Society (SPM) in the UNESCO world heritage city of Porto.

The scientific scope of this meeting ranges from plenary talks of general interest to special sessions focusing on current research in specific areas. There will be distinguished Invited Addresses, an evening Public Lecture and a variety of Special Sessions.

List of invited speakers:
Marcus du Sautoy (evening Public Lecture), University of Oxford, UK
Rui Loja Fernandes, University of Illinois, USA
Irene Fonseca, Carnegie Mellon University, USA
Annette Huber-Klawitter, Albert-Ludwigs-Universität Freiburg, Germany
Mikhail Khovanov, Columbia University, USA
André Neves, Imperial College London, UK
Sylvia Serfaty, Université Pierre et Marie Curie Paris 6, France
Gigliola Staffilani, MIT, USA
Marcelo Viana, Instituto de Matemática Pura e Aplicada, Rio de Janeiro, Brazil

For more information see http://aep-math2015.spm.pt/

EMS Executive Committee Meeting in London from the 28th of February to the 2nd of March 2014

Stephen Huggett (University of Plymouth, UK)

Membership

The Treasurer reported that at the Council meeting this year several Societies (hence, Full Members) would be applying to move up a class. The Executive Committee was also pleased to agree to elect the European Set Theory Society to Associate Membership of the EMS, and to approve the list of about 80 new individual members.

The Publicity Officer Dmitry Feichtner-Kozlov reported that, as planned, a selection of institutions in the UK, France, and Germany had been sent a letter and a poster, inviting them to consider joining the EMS. The President reported that she had done the same in Spain. This process would continue with Italy and Holland.

The web

The consultant, Robert Carr, presented his proposal for a redesign of the EMS web site. There would be a prototype ready soon, and he requested feedback on it. The finished site would be ready to show Council. The Executive Committee approved this process, and thanked Robert.

Lucia Di Vizio presented a paper giving detailed usage statistics for our Newsletter presence on both Facebook and Twitter. Her report was welcomed by the Executive Committee.

Scientific Meetings

Volker Mehrmann reported that in addition to the usual mathematical programme, the 7th European Congress of Mathematics would include a mathematical film festival, a public lecture on mathematics and architecture, a student conference, and historical lectures and walks. It was agreed that the membership of the Neugebauer Prize Committee can be made public, as can the Chair of the Klein Prize Committee. The Treasurer reported that Foundation Compositio Mathematica is happy to fund about half of the cost of the EMS prizes. The membership of the EMS Prize Committee was discussed. Finally, it was agreed that Michael Atiyah and Jean-Pierre Bourguignon be suggested as the EMS representatives on the Hirzebruch Lecture panel.

The Executive Committee discussed the First Joint International Meeting RSME–SCM–SEMA–SIMAI–UMI, to be held in Bilbao from June 30th to July 4th 2014, which looks very strong. Jouko Väänänen suggested that the EMS organise a lecture about (and visit to) the Richard Serra sculptures in the Guggenheim Museum.

Armen Sergeev reported on the Caucasian Mathematics Conference, to be held in Tbilisi, Georgia, on the 5th and 6th of September 2014 under the auspices of the European Mathematical Society and the cooperation of the Armenian, Azerbaijan, Georgian, Iranian, Russian and Turkish Mathematical Societies.The six invited speakers, one from each of the six participating countries, had now been finalised. Funds from the EMS Solidarity Committee would be used to support some of the invited speakers and two young mathematicians from each country.

EMS Council 2014

The President reported that local preparations for the 2014 meeting of Council in Donostia/San Sebastián were very good, and that details were almost ready to be published on the Council web site. The Executive Committee then considered whether to propose to Council any amendments to the Statutes. It was agreed that the By-Laws 10–12 might be reviewed, in order to ensure that delegates were members of the EMS. After a detailed discussion of the membership fees it was agreed to propose small increases for both corporate and individual members. The draft agenda for Council was agreed in outline, except that it should include a presentation of the web site, which would form part of the report from the Executive Committee.

Standing Committees

José Antonio Carrillo presented the report of the Applied Mathematics Committee, noting in particular that the Bedlewo Summer School is now confirmed. Then Volker Mehrmann reported on the EU-MATHS-IN project, which is applying for support under COST.

Alice Fialowski presented the report from the Chair of the Committee for Developing Countries, noting that none of the new Emerging Regional Centres of Excellence under consideration is from Africa.

The Treasurer introduced the draft position paper on Open Access from the Publications Committee. It was agreed that this was very important, but the Executive Committee would only comment in detail once a more nearly final draft had been received. Gert-Martin Greuel observed that it was unique to this paper that the role of libraries was analysed.

Gert-Martin Greuel reported that IMAGINARY would be having an exhibition at the ICM in Seoul and at the next ECM in Berlin, and the platform www.imaginary.org is now available in German and Spanish (besides English); translations into French and Catalan are on the way.

Publishing

It was noted that the Proceedings of the 6th European Congress of Mathematics had now been published. The Executive Committee agreed the details of the renewal of the Editorial Board of *Journal of the European Mathematical Society*.

The President reported that the committee awarding the EMS Monograph Prize was unanimous in deciding to make a joint award to two of the submissions:

- *Foundations of Garside Theory*, by Patrick Dehornoy with François Digne, Eddy Godelle, Daan Krammer, and Jean Michel
- *Topics in Elliptic PDEs and Measure Theory*, by Augusto Ponce

Closing

The next meeting will be a short one just before the Council in Donostia/San Sebastián. The autumn meeting will be in Barcelona from the 21st to the 23rd of November.

The President expressed the gratitude of the whole Executive Committee to the London Mathematical Society for its hospitality.

New Editorial Board of the Journal of the European Mathematical Society

The Executive Committee of the EMS has appointed the new editorial board of its flagship journal JEMS. The composition of the new editorial board is as follows:

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François Loeser, Université Pierre et Marie Curie, Paris, France

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Annalisa Buffa, CRN Pavia, Italy
Tom Bridgeland, University of Oxford, UK
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Umberto Zannier, Scuola Normale Superiore, Pisa, Italy
Tamar Ziegler, The Hebrew University of Jerusalem and Technion, Haifa, Israel

The Editor-in-Chief and the editors have an initial term of four and three years, respectively, with the possibility of an extension for a second term. From 1 June 2014, authors submitting articles to the journal will be directed to an electronic submission system on the journal's homepage and the new editorial board will handle these articles.

Created by the EMS in 1998, the first volume of JEMS, published by Springer-Verlag, appeared in January 1999, with Jürgen Jost as Editor-in-Chief. Since January 2003, the Editor-in-Chief has been Haim Brézis. With the creation of the EMS Publishing House, the publication of JEMS was moved from Springer to the EMS publisher in 2004.

Today, JEMS is one of the strongest mathematical journals, thanks to the skill and dedication of the first two editorial boards. In this transition period, the EMS would like to fully recognise and acknowledge the extraordinary work of the departing editorial board:

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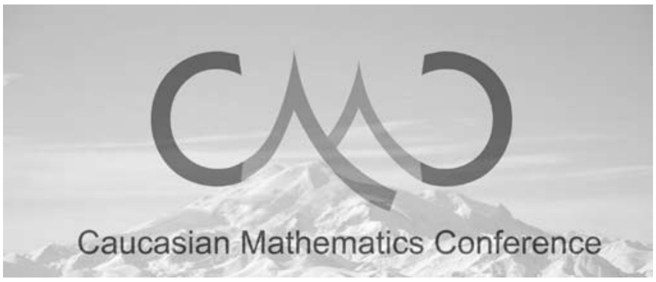
This board is responsible for the handling of papers submitted to JEMS before 1 June 2014 and will terminate its functions by September 2015.

The EMS is greatly indebted to the departing editorial board, which under the leadership of Professor Brézis has raised the journal to its current high rank. At the same time, the EMS expresses its gratitude to the new editorial board for having accepted this important responsibility and warmly welcomes all its members.

EMS Executive Committee
May 2014

The First Caucasian Mathematics Conference (CMC I)

Roland Duduchava (Ivane javakhishvili Tbilisi State University, Georgia)



How much can mathematicians contribute to peace and harmony in the world? At a time when the region is surrounded by old and new tensions, the first Caucasian Mathematics Conference (CMC I) is being organised in Tbilisi, Georgia, 5–6 September 2014, under the auspices of the European Mathematical Society and with the cooperation of the Armenian, Azerbaijan, Georgian, Iranian, Russian and Turkish Mathematical Societies. Although it is welcoming mathematicians from all over the world, the first aim of the Caucasian Mathematical Conference is to bring together mathematicians from Caucasian and neighbouring countries annually in one of these countries.

The idea of this gathering was expressed by Betül Tanbay at the EMS Presidents Meeting in Aarhus in 2013 and, encouraged by Marta Sanz-Solé, the presidents of the Georgian and Turkish Mathematical Societies Roland Duduchava and Betül Tanbay started the preliminary work with EMS-EC member Armen Sergeev. An organisational meeting in January 2014 in Istanbul gave the first shape to the idea. The three initiators mentioned were joined by the representatives of the participating countries and institutions:

Carles Casacuberta (ex-officio; Chair of the EMS Committee for European Solidarity),
Mohammed Ali Dehghan (President of the Iranian Mathematical Society),
Tigran Harutyunyan (President of the Armenian Mathematical Union),
Misir Jumayil oglu Mardanov (Representative of the Azerbaijan Mathematical Union) and
Marta Sanz-Solé (ex-officio; President of the European Mathematical Society),

forming together a Steering Committee responsible for the scientific programme of the conference. The committee has selected the invited speakers:

Maria Esteban (C.N.R.S. at the University Paris-Dauphine),
Mohammad Sal Moslehian (Ferdowsi University of Mashhad),
Garib N. Murshudov (MRC Laboratory of Molecular Biology, Cambridge),
Dmitri Orlov (Steklov Institute, Moscow),

Samson Shatashvili (Trinity College, Dublin),
Leon Takhtajan (University of Stony Brook) and
Cem Yalçin Yildirim (Boğaziçi University, Istanbul).

A list of invited young mathematicians (under 40) will be selected by the National Mathematical Societies and approved by the Steering Committee. Roland Duduchava was appointed the Chairman of the CMC I and the Local Organising Committee consists of Buchukuri Tengiz, Davitashvili Tinatin (Scientific Secretary), Eliashvili Merab, Natroshvili David (Vice Chairman) and Sigua Levan.

The programme consists of invited speakers' talks and parallel sessions. Participants wishing to organise a parallel session are invited to send applications before 15 June 2014. The organiser of such a session should present together with its title a preliminary list of participants with at least five speakers.

The conference fee is 20 USD. The organisers will apply for financial support from different foundations and, in case of success, will try to pay accommodation for the selected young participants.

The online registration page and all information about the conference can be found at euro-math-soc.eu/cmc/ or http://www.gmu.ge/cmc and contact email addresses are cmc.tbilisi@gmail.com and cmc.tbilisi@gmu.ge.

Citizens of many countries do not need a visa for Georgia (EU countries, USA, Japan, Israel, Turkey, Canada, Armenia, Azerbaijan, Moldova, Russia, Ukraine, etc.) or else many can get a visa when entering Georgia.

Tbilisi can be reached by aeroplane from most of the big airports of Europe, Turkey and the former Soviet republics (see: http://www.tbilisiairport.com/en-EN/).

Immediately after the CMC I, the fifth International Conference of the Georgian Mathematical Union will be held in Batumi, Georgia, 8–12 September 2014 (see: http://www.gmu.ge/Batumi2014).

New Service by EU-MATHS-IN

The association EU-MATHS-IN (promoted by the EMS and the ECMI) has launched a new service: a website for advertising jobs for mathematicians in companies or institutions working on industrial contracts. The scope of the website is advertising jobs in industrial mathematics and related fields.

Please circulate this to all interested people, industrial contacts and young mathematicians looking for jobs.

Job announcements can be found or deposited at

http://www.eu-maths-in.eu/jobs

EMS Monograph Award

The EMS Monograph Award is assigned every two years to the author(s) of a monograph in any area of mathematics that is judged by the selection committee to be an outstanding contribution to its field. The prize is endowed with 10,000 Euro and the winning monograph will be published by the EMS Publishing House in the series "EMS Tracts in Mathematics".

The first EMS Monograph Award (2014) has been assigned jointly to

Patrick Dehornoy (Université de Caen, France)
with **François Digne** (Université de Picardie Jules-Verne, Amiens, France), **Eddy Godelle** (Université de Caen, France), **Daan Krammer** (University of Warwick, Coventry, UK) and **Jean Michel** (Université Denis Diderot Paris 7, France)
for their work
Foundations of Garside Theory

and

Augusto C. Ponce (Université Catholique de Louvain, Belgium)
for his work
Elliptic PDEs, Measures and Capacities – From the Poisson Equation to Nonlinear Thomas–Fermi Problems

Scientific Committee
John Coates, Pierre Degond, Carlos Kenig, Jaroslav Nesetril, Michael Roeckner, Vladimir Turaev

Submission
The second award will be announced in 2016, the deadline for submissions is 30 June 2015.
The monograph must be original and unpublished, written in English and should not be submitted elsewhere until an editorial decision is rendered on the submission. Monographs should preferably be typeset in TeX. Authors should send a pdf file of the manuscript by email and a hard copy together with a letter to:

European Mathematical Society Publishing House
ETH-Zentrum SEW A27, Scheuchzerstrasse 70, CH-8092 Zürich, Switzerland
E-mail: info@ems-ph.org

EMS Tracts in Mathematics

This series includes advanced texts and monographs covering all fields in pure and applied mathematics. Tracts will give a reliable introduction and reference to special fields of current research. The books in the series will in most cases be authored monographs, although edited volumes may be published if appropriate. They are addressed to graduate students seeking access to research topics as well as to the experts in the field working at the frontier of research.

Most recent titles:

Vol. 21 Kaspar Nipp and Daniel Stoffer: *Invariant Manifolds in Discrete and Continuous Dynamical Systems* (see also page 68)
978-3-03719-124-8. 2013. 225 pages. 58.00 Euro

Vol. 20 Hans Triebel: *Local Function Spaces, Heat and Navier–Stokes Equations*
978-3-03719-123-1. 2013. 241 pages. 64.00 Euro

Vol. 19 Bogdan Bojarski, Vladimir Gutlyanskii, Olli Martio and Vladimir Ryazanov: *Infinitesimal Geometry of Quasiconformal and Bi-Lipschitz Mappings in the Plane*
ISBN 978-3-03719-122-4. 2013. 214 pages. 58.00 Euro

The Abel Prize Laureate 2014

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2014 to

Yakov G. Sinai
of Princeton University, USA, and the Landau Institute for Theoretical Physics, Russian Academy of Sciences,

"for his fundamental contributions to dynamical systems, ergodic theory and mathematical physics".

Photo: Archives of the Mathematisches Forschungsinstitut Oberwolfach

The Ferran Sunyer i Balaguer Prize 2014

fundació FERRAN SUNYER I BALAGUER
Institut d'Estudis Catalans

The Ferran Sunyer i Balaguer Prize 2014 winners were:

Professors Véronique Fischer and Michael Ruzhansky (Imperial College London), for the work

Quantization on Nilpotent Lie Groups

Abstract: The theory of quantisation originated at the end of the 1920s in mathematical physics. The mathematical aspect of this theory concerned with the analysis of operators has been intensively developed over the past 50 years. Nowadays, it is widely accepted and used as a powerful tool to study different questions in the theory of partial differential equations. However, the settings of these studies have been mainly Euclidean and commutative in nature. This monograph presents the recent advances of the theories of quantisation and pseudo-differential operators on compact and nilpotent Lie groups. It also contains applications to non-commutative global analysis and to partial differential equations, in particular on the Heisenberg group.

This monograph will be published by Springer Basel in their Birkhäuser series *Progress in Mathematics*.

On Some Conjectures in Additive Number Theory

Preda Mihăilescu (Universität Göttingen, Germany)

Number theory is noted for problems that are easy to formulate but not as easy to solve. It is also a fact that mathematicians like to make progress. So, if involved in this field, rather than collapse under the weight of simple questions hundreds of years old, they will typically either develop interesting theories which gain a life of their own and eventually pay off with some solution of the old problems – at least partial, if not complete, as in the case of Fermat's Last Theorem – or they imagine variants. In this short presentation, these processes will be illustrated with a family of number theoretical problems concerned with the distribution of primes and prime-tuples, an area which has experienced dramatic progress in the last decade.

1 The Cramér Model and gaps between primes

The distribution of prime numbers is one of the most relevant questions of analytic number theory, and it can be a specialisation of arbitrary depth. We fix a notation and write $\mathbb{P} \subset \mathbb{N}$ for the set of rational primes and let their sequence be numerated by $p_1 = 2, p_2 = 3, \ldots, p_n, \ldots \in \mathbb{P}$. The counting function is $\pi(x) = |\mathbb{P}_{\leq x}|$ and the prime number theorem states that

$$\lim_{n\to\infty} \frac{\pi(x)}{x/\log(x)} = 1,$$

which suggests that typically we should have

$$p_{n+1} - p_n \sim O(\log(n)).$$

To try to make the meanings of the terms precise and to explain why one expects a certain *average* behaviour, Cramér used the following model, which has become a reference ever since. To quote its description from Soundararajan's presentation [9]: *The indicator function for the set of primes (that is, the function whose value at n is* 1 *or* 0 *depending on whether n is prime or not) behaves roughly like a sequence of independent, Bernoulli random variables* $X(n)$ *with parameters* $1/\log n (n \geq 3)$. *In other words, for* $n \geq 3$, *the random variable* $X(n)$ *takes the value* 1 *(n is prime) with probability* $1/\log n$, *and* $X(n)$ *takes the value* 0 *(n is composite) with probability* $1 - \frac{1}{\log n}$. *For completeness, let us set* $X(1) = 0$ *and* $X(2) = 1$. This probabilistic approach awakens some reasonable reserve: numbers are not *almost prime* or *almost composite*; they are either one or the other. However, it turned out that the model leads to some astonishing insights and suggestions concerning the behaviour of primes *at large*. One of the immediate consequences is related to the interval among consecutive primes. The following conjecture is deduced from the assumption underlying Cramér's model:

Conjecture 1. Given an interval $0 \leq \alpha < \beta$, we have

$$\frac{1}{\pi(x)} \cdot \left|\{p \leq x \ : \ \text{pnext} \in (p + \alpha \log p, p + \beta \log p)\}\right| \to \int_\alpha^\beta \exp(t) dt,$$

as $x \to \infty$, where pnext denotes the next prime larger than p.

The gaps between primes are consequently also expected to be Poisson distributed with mean value $\log(x)$; the function describing the deviation from the expected value for the gap length is

$$\delta(n) = \frac{p_{n+1} - p_n}{\log(p_n)}$$

and it has the expected value of one; the model and related conjecture also imply that

$$\liminf_{n\to\infty} \delta(n) = 0 \quad \text{and} \quad \limsup_{n\to\infty} \delta(n) = \infty.$$

After some initial work together with Erdős, Rankin could prove, for large gaps, that there is some positive constant c such that

$$\delta(n) > c \frac{\log^{(2)}(p_n) \cdot \log^{(4)}(p_n)}{\left(\log^{(3)}(p_n)\right)^2},$$

for infinitely many values of $n > 1$; here, the upper index of the (natural) logarithm indicates the *iteration count*, thus

$$\log^{(2)}(p_n) = \log(\log(p_n)).$$

This is hardly the largest size of gap that one would expect; on the contrary, the limit

$$\limsup_{n\to\infty} \frac{\delta(n)}{\log(p_n)} = 1$$

is suggested by the model. However, neither empirical computations nor rigorous proofs have been made so far to provide larger lower bounds than Rankin's. Erdős offered a prize of \$10,000 for the first person to provide an improvement of Rankin's results but the prize has not been challenged to date.

As already mentioned, the Cramér model cannot be expected to provide a completely consistent picture of the distribution of primes. According to the model, this distribution should be Poisson – however, Helmut Maier proved [7] in 1985 a theorem which reflects, in an impressively simple manner, the deviation of the set of primes from the predicted Poisson distribution:

Theorem 1. For any $\lambda > 1$, we define

$$g(\lambda, x) = \frac{\pi\left(x + (\log(x))^\lambda\right) - \pi(x)}{(\log(x))^{\lambda-1}}.$$

Then

$$\liminf_{x\to\infty} g(\lambda, x) < 1 < \limsup_{x\to\infty} g(\lambda, x).$$

Since within the Cramér model the limit should exist and be 1, the theorem makes the limitations of the model concrete.

Small gaps

Moving on to *small* gaps, we encounter one of the fascinating mathematical thrillers of the last decade. Note first that the prime gaps are

$$p_{n+1} - p_n \geq 2,$$

the value 2 corresponding to the *prime twins*. Thus proving the expected fact that there exist an infinity of prime twins would be solving one of the important conjectures concerning small prime gaps. But even

$$\liminf_{n\to\infty} \delta(n) = 0$$

was not known in 2002 when Goldston announced, on behalf of himself and Yildirim, an amazing result: he claimed the proof that

$$\delta(n)/\log(n)^{1/16} < 1$$

for infinitely many primes and thus a sharpening of the vanishing of the limes inferior, a sharpening which is much stronger than the corresponding result of Rankin for large gaps. The proof was unfortunately erroneous and was forgotten for several years, until the same authors, together with J. Pintz, made the breakthrough in 2005, giving a correct proof of

$$\liminf_{n\to\infty} \delta(n) = 0.$$

While the three authors joined forces with Graham, considering various related problems, the next major breakthrough followed in 2013 and was achieved by Y. Zhang, who refined the method of Goldson, Pintz and Yildirim, thus proving a fixed upper bound

$$\liminf_{n\to\infty} (p_{n+1} - p_n) < 7 \cdot 10^7.$$

The upper bound has since been successively improved over the whole year and, at the time of writing, had dropped below $5 \cdot 10^5$; see [1] for a tabular presentation of the detailed evolution of the proven bounds.

The repartition of primes can be considered on a more detailed basis by asking the question about repartition of primes in arithmetic sequences; concretely, given two coprime integers m, a, one wishes to gather information about the function

$$\pi(x; m, a) = |\{p \in \mathbb{P}_{\leq x} \ : \ p \equiv a \bmod m\}|.$$

Dirichlet had already established the asymptotic behaviour

$$\lim_{x\to\infty} \frac{\varphi(m) \cdot \pi(x; m, a)}{\pi(x)} = 1$$

but more subtle questions exist concerning, for instance, the asymptotics of the difference

$$|\pi(x; m, a) - \pi(x; m, b)|.$$

An important conjecture due to Elliott and Halberstam refers to these oscillations; more precisely, considering the maximal deviation

$$E(x; m) = \max_{(a,m)=1} \left| \pi(x; m, a) - \frac{\pi(x)}{\varphi(m)} \right|,$$

Elliott and Halberstam stipulate the following.

Conjecture 2. For every θ with $0 < \theta < 1$ and for every $A > 0$ there exists a constant $C > 0$ such that

$$\sum_{1\leq m\leq x^\theta} E(x; m) \leq \frac{Cx}{\log^A(x)}.$$

The conjecture has been proven for all $\theta < 1/2$ by Vinogradov and Bombieri (known as the Bombieri-Vinogradov theorem). The insight of Goldston was that the lack of small gaps between primes has some indirect impact upon the repartition of primes in arithmetic progressions, thus relating the problem to the Elliott-Halberstam conjecture. In fact, the recent improvements of Zhang [11], [1] use a partial strengthening of the Bombieri-Vinogradov theorem, which holds only for *smooth* moduli. Besides bringing the inferior limes into the constant realm, the idea opened a fruitful line of research in which successive small improvements led over more than half a year to uninterrupted improvements of the lower bound.

The question about small gaps between primes has thus come into an arena which may eventually lead to new insights on the Elliott-Halberstam conjecture and possibly, in time, to an approach for the twin prime problem too. For an extended presentation of the result of Goldston, Pintz and Yildirim, Soundararajan's presentation [9] is recommended. One finds in Quanta's column of Erica Klarreich [6] a fascinating account of the work of Zhang and the ulterior *polymath project Polymath8* of Terence Tao aimed at improving upon Zhang's bound. Both papers reward the reader with essential new information which goes beyond the scope of our small presentation.

At the time of writing, the Oxford postdoctoral student James Maynard had just announced at an October meeting in Oberwolfach that he had developed his own method for proving the existence of small gaps between primes. The paper is still under preparation but the method is claimed to allow not only for the improved bound $p_{n+1} - p_n < 468$ for infinitely many values of n but also to allow the proof of upper bounds of the type $p_{n+k} - p_n < C$, thus showing that there are infinitely many k-tuples of successive primes, for which the distance between the largest and the smallest is bounded by a fixed constant C. While the results of Maynard are being written down, Andrew Granville is in the process of writing a detailed survey [4] that goes beyond the one written by Soundararajan [9] and covers all the recent developments in the domain of small gaps between primes, including the works of Zhang and Maynard.

The Hardy-Littlewood Conjecture and other consequences of Cramér's model

Most of the questions of *additive* nature related to gaps between primes have a corresponding multiplicative question. For instance, the dual to the question about the infiniteness of the set of prime twins is the question of whether there exist infinitely many *Sophie-Germain primes*, i.e. such that $p, 2p + 1 \in \mathbb{P}$. As for the prime gap problem, one may replace this with the question of whether there are infinitely many prime pairs of the type $p, 2ap + 1$, etc. But it is time to break the "simplicity" of pairs and consider prime tuples. One natural question to ask about prime tuples is whether they can occur in the form of arithmetic progressions, and whether this is for arbitrarily large tuples. The question has been considered for a long time to be out of reach for analytic approaches, and was a topic of intense computational investigations [3], leading to record lengths of arithmetic progressions consisting only of primes. However, Terence Tao and Ben Green succeeded [5] in proving in 2004 that there exist arbitrarily long arithmetic progressions consisting only of primes. As one would expect, the method leads to new questions – one of them is related to the case when k polynomials $p_i(x) \in \mathbb{Z}[X], i = 1, 2, \ldots, k$, are given and one wishes to know if there are infinitely many integers x, m such that $x + p_i(m)$ are all simultaneously prime. The result was established in 2006 by Tao and Ziegler and it obviously implies the one on arithmetic progressions.

The problem is more intricate if one wants to count the frequency of arithmetic sequences or, more generally, of given patterns of primes. In this context, an accurate analysis based on the Cramér model leads to the famous conjecture of Hardy and Littlewood: for integer sets $\mathcal{H} = \{h_1, ..., h_k\}$ they first defined a constant $\Sigma(\mathcal{H}) \in \mathbb{R}_{\geq 0}$, which vanishes if there is no $(k + 1)$-tuple of primes with mutual distances h_i; otherwise it is a constant which is deduced from the Poisson distribution of gaps (see [9] for more details), such that the following is consistent with the Cramér model:

Conjecture 3. Let $\mathcal{H} = \{h_1, ..., h_k\}$ be a set of positive integers such that $\Sigma(\mathcal{H}) \neq 0$. Then

$$|\{n \leq x : n + h_1, ..., n + h_k \in \mathbb{P}\}| \sim \Sigma(\mathcal{H}) \frac{x}{(\log x)^k}.$$

This is what one would expect and it suggests not only that there should be infinitely many Sophie-Germain pairs but that their occurence is quite frequent, about one in every $\log(x)^2$, on average. A stronger version of this conjecture is due to Bateman and Horn and it replaces the linear polynomials $f_i(x) = x + h_i$, for which the Hardy Littlewood conjecture requires that $f_i(n) \in \mathbb{P}$ for all i, by some predetermined polynomials $f_i(x)$ of arbitrary degree. The claim of the conjecture is then similar, with the same distribution function depending only on k and a structural constant that replaces $\Sigma(\mathcal{H})$.

Particular problems and conjectures

A variety of problems related to prime gaps or to distributions of primes have been proposed, sometimes in the hope that these might represent a simpler, more accessible problem. The hope did not often come true.

The *Hypothesis H* of Schinzel [10] states:

Conjecture 4. Consider s polynomials $f_i(x) \in \mathbb{Z}[X]; i = 1, 2, \ldots, s$ with positive leading coefficients and such that the product

$$F(X) = \prod_{i=1}^{s} f_i(x)$$

is not divisible, as a polynomial, by any integer different from ± 1. Then there is at least one integer x for which all the polynomials $f_i(x)$ take prime values.

Apparently, this is only a more accessible version of the Bateman-Horn conjecture.

Dorin Andrică [2] proposed during his years of study the following conjecture, which was sufficiently supported by numerical evidence:

Conjecture 5. Let $p_n \in \mathbb{P}$ denote as usual the n–th prime. Then, for all $n \in \mathbb{N}$, the distance

$$\sqrt{p_{n+1}} - \sqrt{p_n} < 1.$$

Equivalently, the gap

$$g_n = p_{n+1} - p_n < 2\sqrt{p_n} + 1.$$

This appears as if it was an easy exercise, given that we "know" from the Cramér model that the typical gap has length $\log(p_n)$ and we even expect that no larger gaps than $\log^2(p_n)$ exist, with little to say about exponential gaps like $O(\sqrt{p_n})$. But not only does the estimate lay in the order of magnitude of what one can prove by assuming the Riemann Hypothesis but it appears that proving the one is essentially equivalent to proving the other.

An apparently even simpler conjecture was formulated by Michael Th. Rassias [8], at the age of 14, while preparing for the International Mathematical Olympiad that was held in Tokyo in 2003. Rassias' numerically supported conjecture claims:

Conjecture 6. For any prime $p > 2$ there are two other primes $p_1 < p_2$ such that

$$p = \frac{p_1 + p_2 + 1}{p_1}.$$

We have here a surprising feature of presenting a prime as a *quotient*; however, after some algebraic manipulation, we recover the expression

$$(p - 1)p_1 = p_2 + 1,$$

thus obtaining an interesting combination of a generalised Sophie Germain twin problem

$$p_2 = 2ap_1 - 1,$$

strengthened by the additional condition that $2a+1$ be a prime number too. We have seen that such questions are caught by the Hardy–Littlewood conjecture. One may ask if Rassias' conjecture is to some extent *simpler* than the general Hardy–Littlewood conjecture or its special case concerning distribution of generalised Sophie-Germain pairs $p, 2ap+1 \in \mathbb{P}$? This is not likely to be true.

A further related problem, much appreciated by amateur number theorists in search of interesting computations, is related to *Cunningham chains*, i.e. sequences of primes

$$p_{i+1} = mp_i + n, i = 1, 2, \ldots, k - 1, \quad \text{for fixed coprime } m, n \in \mathbb{N}_{>1}.$$

As for the large arithmetic progressions of primes, there are computing competitions for the longest Cunningham chain or for the one built up of the largest primes – but unlike the breakthrough of Green and Tao, there is no general result known on large Cunningham chains to date. Rassias' conjecture can also be stated in terms of Cunningham chains, namely: there exist Cunningham chains with parameters $2a, -1$ for a such that $2a - 1 = p$ is prime.

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Ideas That Will Outlast Us

To the Memory of George Kac (Georgiy Isaakovich Kac)

Leonid Vainerman (University of Caen, France)
Translated into English by Nataliya Markova

Preface

Here is a story about a person whose work and destiny were closely related to two important advances in 20th century mathematics: supermathematics and quantum groups. The first serious step towards supermathematics (the introduction of Lie superalgebras and formal Lie supergroups) was made by F.A. Berezin and G.I. Kac in [3]. Interesting memories have been recorded about Felix Alexandrovich Berezin but almost nothing has been written about Georgiy Isaakovich Kac (in fact, only [2]). Discussing the impact of [3] on mathematics and mathematical physics, Yu.A. Neretin wrote: "I have heard very little about the second author of this paper, G.I. Kac. It is worth noting that Kac's papers on 'ring groups' have to a great extent set the stage for another 'explosion', namely, the works on 'quantum groups'." See also V.G. Drinfeld's discussion in [4].

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I knew George Kac (a remarkable mathematician and a remarkable person) between 1968 and 1978. He died of a heart attack on 20 May 1978, in the prime of his talent and vitality. His friend B.I. Khatset wrote that his modesty and generosity had earned him the nickname Pierre Bezukhov (one of the principal characters of Leo Tolstoy's *War and Peace*) and, as you will see below, he was like this in mathematics, too. After G.K.'s death, whilst spending considerable time at research and academic centres where follow-up work was taking place, I observed the strong development of his ideas. That is why you will find here not just personal memories but also reflections on his mathematical ideas, their genesis, their evolution and their impact on other researchers. This is not a scientific paper so it does not claim to be rigorous or exhaustive. Nevertheless, it will be more understandable to those who are to a certain extent familiar with algebra and analysis. I am writing mainly about events that I have witnessed.

Kac algebras

In the autumn of 1968 I started attending G.K.'s seminar on operator rings. He was a remarkable speaker and his manner of setting out the material was clear and rigorous and, moreover, lively and accessible. Nothing I had heard before could match it and long after his death only a handful of speakers left a comparable impression.

We came to know each other more closely after my graduation from Kiev University, when I was hired by the Kiev Aviation Engineering Military College, where Kac served as a professor of mathematics. Even though I graduated from the university with honours, in those years, at the height of anti-Semitism, it was impossible for me to formally enrol in a PhD programme. For the same reason G. K. could not work either at the university or at the academy of sciences. Happily, Yu.M. Berezanski helped him set up his seminar at the institute of mathematics. Over the following years, I was fortunate to work with G.K., which not only guided my scientific career but also determined my future life path. I have to mention that at the same period I was doing my PhD thesis under the informal supervision of M.L. Gorbachuk, to whom I am deeply grateful.

G.K. offered me the choice of two research topics. The first was to further develop his joint work with Berezin [3], one of the pioneering works in the area now called supermathematics and dedicated to the study of mathematical structures (superalgebras, Lie superalgebras, supermanifolds, etc.) graded, in the simplest case, by the group Z/2Z. The motivation was provided by quantum mechanics, where two kinds of particles, fermions and bosons, are governed by totally different statistical laws. Both Kac and Berezin had a strong background in theoretical physics, so no wonder they became pioneers in this new area. Kac's PhD thesis, defended in 1950 (with N.N. Bogolyubov as an advisor), was dedicated to the correlation theory of electron gas. Let us also mention the superanalogue of Frobenius' theorem, obtained by G.K. and A.I. Koronkevich shortly after [3].

Nevertheless, I missed my chance to become a "supermathematician" and chose another topic, *ring groups*, introduced by G.K. around 1960. Below is a reminder of the sources of this theory.

Let G be a commutative, locally compact group and $\underline{G}$ its dual, i.e. the group of its unitary continuous characters (which is also commutative and locally compact). Pontryagin's duality says that the dual to $\underline{G}$ is isomorphic to G. However, this beautiful theory breaks down if G is non-commutative, even if it is finite, since the characters of such a group are too few and do not contain all the information about the group. To save the duality theory one could replace the characters with the irreducible unitary representations (in the commutative case they are just characters). Indeed, T. Tannaka showed in 1938 that a compact group can be restored up to isomorphism from a collection of its representations, while in 1949 M.G. Krein gave an axiomatic description of such a dual of a compact group. Nevertheless, the fact that this dual, a block-algebra, is not a group breaks the symmetry of the duality. Later on, such a non-symmetric duality theory was developed by W.F. Stinespring (1959) for unimodular groups and by N. Tatsuuma (1965–66) for general locally compact groups.

Translating Stinespring's paper into Russian for the collection "Matematika", G.K. came up with the idea to construct a new category of *ring groups* that would contain both groups and their duals with a symmetric duality acting within (i.e. an object and its dual must have the same mathematical structure, like in Potryagin's duality). A ring group is a collection (A, Γ, S, m), where A is an operator algebra, an *invariant measure m* is a *trace* on it (i.e. a positive central linear form which can be unbounded), a *coproduct* Γ is an algebra homomorphism from A to its tensor square, and an *antipode S* is an involutive anti-isomorphism of A, satisfying certain axioms.

When A is commutative, it can be realised as the algebra of essentially bounded measurable functions on a unimodular group G with respect to the invariant measure of G, $\Gamma f(x) = f(xy)$, $Sf(x) = f(x^{-1})$ (where x, y are elements of G) and m is the integral over the invariant measure on G. So, unimodular groups are included in the category of ring groups. Their duals are exactly cocommutative ring groups (i.e. Γ is stable under the permutation of factors in the tensor product). In this case, the algebra A is generated by the translation operators $L(x)$ or, equivalently, by the convolution operators $L(f)$ with f a continuous integrable function on G. Γ maps $L(x)$ to its tensor square, $S(L(x)) = L(x^{-1})$ and $m(L(f)) = f(e)$, where e is the unit of G.

In purely algebraic terms, ring groups are nothing other than *Hopf algebras* studied earlier in topology. G.K. was not aware of the existence of Hopf algebras and reinvented them when he introduced ring groups.

Lastly, G. K. gave a construction of a dual in this category. Applying this construction twice, one has an object isomorphic to the original one, like in Pontryagin's duality. These results were first announced in *Soviet Mathematical Doklady* in 1961 and later published in detail in [7]. That work used the techniques from I. Segal's paper on traces on operator algebras, also translated into Russian by G. K. for the "Matematika" collection.

The research topic I chose was the open problem formulated in G.K.'s habilitation thesis (Moscow University, 1963): to extend ring group theory in a way to cover all locally compact groups.

Fairly soon, I identified the technical tools to be prepared. Firstly, the above mentioned traces had to be replaced by possibly unbounded positive forms on operator algebras that are not necessarily central, called *weights*. Secondly, G.K. made systematic use of close ties between traces and so-called *Hilbert algebras*, so one had, passing from traces to weights, to develop an appropriate generalisation of Hilbert algebras. The idea appealed to G.K. and we started working on it with vigour. Shortly afterwards, the papers by M. Takesaki and F. Combes appeared, containing the needed techniques; they were clear to us since we had already gone halfway.

Now the coveted target was within reach but we had to hurry, since we were not alone in our pursuit. By that time, Takesaki had already written a paper on generalisation of ring groups; in addition, he had mastered all the necessary techniques. It remains a mystery to me why he did not come first in the race, being a leading specialist with a number of brilliant results to his credit. G.K. was also convinced that J. Dixmier saw the same objective. He said: "We must hurry. I'm sure that Dixmier has someone tackling this problem." As it turned out, he was absolutely right. But even in that stressful situation his integrity did not fail him: since he considered that it was me who had suggested the basic ideas, he decided to allow me the opportunity to complete the solution by myself, thus becoming the sole author, while it was he who had set the problem and had put much effort into adequately preparing me for solving it. He liked me to come to the lecture room at the end of his lectures to discuss and to walk with him from Uritski Square, where the Military College was situated, past the railway station to his home on Bolshaya Podvalnaya Street (the street names are of the period). Sometimes these discussions continued in his apartment.

Working on my own involved some risk because I had not yet fully mastered the necessary techniques and needed more time. To make matters worse, I was deeply upset by the death of my father in August 1971. Finally, seeing that I was not making much progress, G.K. understood that we could well lose the race and took over. He promptly got through a couple of issues that had baffled me and with his support I started to advance much quicker. In a joint effort, we had fairly rapidly completed a draft version. Even then G.K. remained true to himself; he suggested that I first publish part of the solution on my own, and only after that we publish the entire solution, which we did – the note [16] was submitted earlier than both papers [17].

Then we were in for an ordeal. Takesaki's paper [14] appeared, in which he took one more step towards the generalisation of ring groups. It was unavailable in Kiev, while the review on it in "Реферативный Журнал" (the Soviet analogue of *Mathematical Reviews*) indicated the construction of a duality theory that generalised the one of ring groups and covered all locally compact groups. Given Takesaki's reputation and the title of the paper, there was little doubt that we had lost. G.K., upset, dropped all his work and rushed to Moscow to read it firsthand (usually he visited Moscow several times a year in order to keep track of publications unavailable in Kiev and to socialise with colleagues, such as M.A. Naimark, F.A. Berezin, A.A. Kirillov and others). Back in Kiev with a photocopy of Takesaki's paper in hand, G.K. said that the reviewer was wrong and that the paper's results had not achieved the goal.

We completed and published our papers. But about the same time, papers came out by M. Enock and J.-M. Schwartz (who worked under Dixmier's supervision) containing equivalent results, although using a somewhat different technique. As Michel Enock later told me, Dixmier had also urged them to hurry, explaining that besides Takesaki, there had to be someone in Kiev working on the same problem with Kac. Our French colleagues used to send us their preprints and papers, while we could not respond to them, since we worked at a Military College and, bound by secrecy regulations, were not allowed to communicate with foreigners. For example, in 1975 we were invited to participate in a conference in Marseille dedicated to the subject of our studies and I rashly showed my invitation at the so-called First Department of the College. I was lucky to get away with it; the KGB men told me that if I wanted to continue my work there, I must throw the invitation away.

In view of the fundamental role of G.K. in the discovery of the new mathematical objects, the suggestion of Enock and Schwartz to name them *Kac algebras* was highly appropriate. I learned about it from G.K. himself. He had very expressive eyes and it was apparent how much he enjoyed the news. Of course, being a man of great modesty, he never actually pronounced this name. Today, an entire book [5] is dedicated to this subject.

Kac algebras were invented not just for the sake of beauty but with a view to applying them to the solution of various problems. According to G.K., ring groups had to be regarded in the same way as the ordinary groups that they generalise, while their application areas might be wider, which later proved to be the case. Nevertheless, bringing ring groups into effective use required a deeper understanding of their structure, examples and properties. Back in the early 1960s, G.K. had distinguished and started to explore special classes of ring groups: *compact, discrete and finite*.

G.K.'s habilitation thesis contained a list of open problems – some of them have been solved since (like the one examined above). One of these open problems is the fact that the existence of an invariant measure is an axiom of a Kac algebra and is not derived from other axioms, as in the case of ordinary groups (the uniqueness of this measure is easy to prove). This is justified by Weil's and Haar's theorems, which state that the existence of an invariant measure is equivalent to the existence of a locally compact topology on a group. However, it would be natural to define Kac algebras only in algebraic and topological terms and *to prove* the existence of an invariant measure, thereby generalising the Haar theorem for ordinary groups. For the above special classes of Kac algebras this was done in [11, 13].

In particular, finite Kac algebras [11] are finite-dimensional semisimple Hopf *-algebras over the field of complex numbers. Their axiomatics contains a *counit* that is the analogue of the unit in an ordinary group. The existence of an invariant measure is a theorem.

In his habilitation thesis and in [9], G. K. extended a number of classical results on finite groups to finite Kac algebras. In particular, he obtained the analogue of the Lagrange theorem stating that the order of a subgroup divides the order of a group. Later on, a stronger statement was established by V.D. Nichols and M.B. Zoeller for any finite-dimensional Hopf algebra. As for finite groups, the only Kac algebra of prime dimension is the cyclic group. This result has been generalised many times in recent works by various authors. G.K. also proved that for any irreducible representation of a finite Kac algebra, there is a basis in which its matrix elements are algebraic integers.

We saw above that commutative and co-commutative Kac algebras correspond to ordinary groups and their duals respectively. Of special interest are examples of *nontrivial* Kac algebras that do not belong to these two classes. The first such examples were built by G.I. Kac and V.G. Paljutkin [8, 10, 11]. As noted by V.G. Drinfeld [4], they were the first known examples of *quantum groups*.

Ordinary groups can be realised as set transformation groups and Kac algebras can also act, but on algebras instead of sets. G.K. repeatedly voiced the opinion, which is now generally accepted, that algebras are noncommutative counterparts of sets and ring groups are noncommutative counterparts of groups. He suggested a definition of *a ring group action* on an algebra and a construction of the crossed product of these objects. Later on, Enock and Schwartz dedicated a series of papers to that type of construction and related results.

Discussing the choice of new problems in the mid-1970s, G.K. strongly advocated an in-depth study of finite ring groups and their representations in the vein of his works. Another option was the search for new examples of nontrivial Kac algebras but that turned out to be an uphill task, and only 20 years later did I understand how to systematically construct them. However, in those days analytical aspects of the theory seemed closer to me than algebraic ones.

I decided to explore such generalisations of Kac algebras, which would cover so-called *generalised shift operators* or *hypergroups*, studied earlier by J. Delsarte, Yu.M. Berezanski, S.G. Krein and B.M. Levitan. This line of research offered challenging analytical problems and interesting applications. But, despite the fact that this problem was mentioned in his habilitation thesis and in [8], G.K. thought that such a theory would be somewhat deficient because the coproduct must be just a positive map and not a homomorphism of operator algebras like in the Kac algebra theory, so that certain essential properties of Kac algebras would be lost.

Still, the fact that due to weaker constraints the theory became far richer in applications seemed of importance to me. In the 1980s and 1990s, encouraged by Yu.M. Berezanski, I worked in this direction. But in the mid-1970s, to my great sorrow, our active collaboration with G.K. was practically over, even though we continued to see each other and discuss mathematical and non-mathematical topics. Since then, I had always kept track of Kac algebra theory, my first love in mathematics, but it was not until the mid-1990s that I returned to it.

Quantum Groups

Let us now turn to the evolution of G.K.'s ideas after his death. Kac algebras solved the problem of duality but their range of applications was not wide enough (which was the reason for me to take up a generalisation). However, the genuine breakthrough in understanding the approach to extend this circle of ideas occurred in the mid-1980s, when V.G. Drinfeld and others discovered quantum groups [4]. In purely algebraic terms, the squared antipode in a quantum group is not necessarily trivial (it is trivial in a Kac algebra) – a seemingly small matter but one that makes the fundamental difference. Many important examples and applications appeared, in particular in theoretical physics and topology. As Alain Connes put it in the preface to [5], Kac algebras have proved "not sufficiently non-unimodular" to cover new applications and therefore the necessity arose for quantum groups. Note that Kac algebras are "non-unimodular" generalisations of ring groups in their own right!

As for quantum group theories in operator algebraic framework, S.L. Woronowicz [18] built the theory of compact quantum groups that contained the theorem of existence of invariant measure (for compact Kac algebras this theorem had been proved earlier [13]) and studied their irreducible representations. Constructing concrete examples of quantum groups, he overcame numerous functional analytical difficulties which highlighted the challenges in extending Kac algebra theory to capture all interesting examples while keeping its beauty and symmetry.

In the 1950s, Stinespring noted the essential role played in the duality of unimodular groups by the unitary sending $f(x,y)$ to $f(x,xy)$. G.K. had built an analogue of this operator for arbitrary ring groups and discovered its fundamental property, the so-called *pentagonal relation*. Later on, G.K. and I, as well as Enock and Schwartz, made full use of this observation passing to non-unimodular Kac algebras. But S. Baaj and G. Skandalis [1] took it even further, pointing out that a *multiplicative unitary* (i.e. satisfying the pentagonal relation) under certain regularity conditions already allows one to build two operator algebras in duality, each of them carrying a more general structure than that of a Kac algebra.

After the end of the Soviet era, Michel Enock and I exchanged all our past publications. He could not know that I had already received everything he had sent me before, since not a single response came back from behind the Iron Curtain (the envelopes I often received had traces of being open and re-sealed, certainly by the KGB, but they contained only mathematical articles). We started to correspond and soon Michel came to Kiev. One of the sentimental events of his stay was our visit to G.K.'s tomb at Gostomelskoye cemetery.

When I came to Paris in the spring of 1992, many colleagues said that they were happy to see a representative of George Kac's school. They did not know that G.K. could never have formal graduate students, and his "school" eventually consisted of just V.G. Paljutkin, me, A.I. Koronkevich, and V. Zhuk. It was so unfair that G.K. had not enjoyed even a small part of the recognition that he so much deserved!

Besides the above mentioned difficulties of his life, his scientific results had not been properly appreciated by some leading specialists, e.g. by L.S. Pontryagin, whose duality principle had been so brilliantly extended by G.K., and by I.M. Gel'fand, who had been rather critical of G.K.'s works. In 1983, Gel'fand tried to persuade me to take up a different subject, saying: "Do not bury your talent as your teacher Kac did!" This was just before the discovery of quantum groups, for which V.G. Drinfeld received the Fields Medal and which had Kac algebra theory as a direct predecessor (see [4])!

Gradually the idea arose to get back to constructing non-trivial examples of Kac algebras and quantum groups. When Woronowicz and others built examples of quantum groups, each case was unique and presented specific difficulties. It was desirable to have constructions generating a number of different examples in a unified way. For example, Drinfeld proposed a purely algebraic way to deform the coproduct and antipode without changing the algebra of a given quantum group in order to get a new one. Applying this construction, called *twisting*, to the dual of an ordinary group one could hope to come up with interesting examples of quantum groups. I started to work on analytical aspects of twisting.

Enock came to Kiev once again in May 1994 on the occasion of G.K.'s 70th anniversary. At the meeting of the Kiev Mathematical Society, Enock and I gave presentations about different facets of G.K.'s activities and, in particular, discussed the problem of examples of Kac algebras. Afterwards, we continued our discussions and in the spring of 1995 the paper [6] was finished. Later on, I extended and reinforced these results. In particular, we constructed a series of new "quantisations" of the Heisenberg group, which is popular with physicists. All of these objects turned out to be not only Kac algebras but even unimodular ring groups in the sense of G.K.'s very first definition.

The finite-dimensional aspect of twisting was discussed in Kiev with D. Nikshych, who built deformations of classical series of finite groups – symmetric, dihedral, quasiquaternionic and alternated. The latter gave the first known examples of *simple* Kac algebras (i.e. without proper normal Kac subalgebras).

In the late 1990s, there was progress in the long-awaited generalisation of Kac algebra theory. Baaj and Skandalis, on the one hand, and Woronowicz, on the other hand, better understood the conditions to be placed on a multiplicative unitary so that it generates a pair of quantum groups in duality. A version of such a theory was proposed by T. Masuda, I. Nakagami and S.L. Woronowicz. In Leuven (Belgium), A. Van Daele discussed at his seminar new examples of quantum groups and approaches to the construction of a general theory. Among other things, he distinguished a class of quantum groups defined in a purely algebraic way such that their topological properties could be derived.

Finally, a satisfactory theory of locally compact quantum groups was proposed in 1999 by J. Kustermans and S. Vaes [17]. It was as beautiful and symmetric as Kac algebra theory was; without much exaggeration one can say that it had been modelled on the latter. A locally compact quantum group is a collection (A, Γ, m, n,), where A, Γ, m are the same as in Kac algebra theory, and n is a right invariant weight on A. The axioms do not mention antipode explicitly but imply its existence and properties and not one but two weights are present – here it is: the second non-unimodularity! It resembles an ordinary locally compact group with two invariant measures: left and right.

In 1999–2000, I spent a few months in Leuven and could familiarise myself with these things firsthand, whereupon I got an idea to generalise the *theory of extensions* [8], which G.K. used for getting non-trivial examples of Kac algebras [10, 11]. Given a commutative Kac algebra K_1, which is an algebra of functions on an ordinary group G_1, and a co-commutative K_2, which is a dual of an ordinary group G_2, the question is whether it is possible to build an extension of K_1 by K_2 – a new Kac algebra such that K_1 would be its normal Kac subalgebra and K_2 would be the corresponding quotient. G.K. showed that such an extension exists if and only if the groups G_1 and G_2 act on each other as on sets and these actions are compatible in a special way. He had described all these extensions in terms of what is referred to as *bicrossed product construction*. Later on, this construction was rediscovered by M. Takeuchi and S. Majid.

Now that we had a far wider category of locally compact quantum groups in hand, one might expect to make the most of the construction. I shared these ideas with Stefaan Vaes and in a few months the paper [15] was finished. Taking various G_1 and G_2, one could get plenty of concrete examples of Kac algebras and quantum groups. In our later work, we classified extensions with G_1 and G_2 low-dimensional Lie groups; another quantum group with surprising regularity properties was built by Baaj, Skandalis and Vaes with G_1 and G_2 coming from number theory; in yet another case, my former PhD student Pierre Fima used similar groups to construct examples of quantum groups with prescribed types of their von Neumann algebras.

Baaj, Skandalis and Vaes also extended to locally compact quantum groups another brilliant idea of G.K., who had built in [8] what was later (when the importance of this sequence in various problems became evident) called the Kac exact sequence.

I will conclude my notes with a story about Kac algebras "in action". In the postscript to [5], A. Ocneanu explained that Kac algebras must arise as non-commutative analogues of group symmetries in the subfactor theory founded by V. Jones. Indeed, in 1994, W. Szymanski and R. Longo independently of one another proved that if N is a finite index subfactor of a factor M then under certain conditions there is necessarily a Kac algebra K acting on M such that N is a subalgebra of fixed points with respect to this action. Vice versa, given a Kac algebra and its action on a factor, one can build a finite index subfactor. This is a far-reaching extension of classical Galois theory, where N and M are fields, and K is a Galois group. Later on, M. Enock and R. Nest came up with a similar result for subfactors of infinite index, in which case a Kac algebra had to be replaced by a locally compact quantum group.

Postscript

The purpose of these notes was to show, apart from personal memories, the powerful influence of Georgiy Isaakovich's works on the progress of a wide area of mathematics. He has been gone for more than 35 years but in scores of recently published works, you will effortlessly find clear evidence of their foundation in his ideas. Indeed, as Pushkin put it: "I have built a monument not wrought by hands…"

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Mathematical Knowledge Management: Transcending the One-Brain-Barrier with Theory Graphs

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We present the emerging discipline of Mathematical Knowledge Management (MKM), which studies the possibility of computer-supporting and even automating the representation, cataloguing, retrieval, refactoring, plausibilization, change propagation and in some cases even application of mathematical knowledge.

We focus on theory graph technology here, which supports modular and thus space-efficient representations of mathematical knowledge and allows MKM systems to achieve a limited mathematical literacy that is necessary to complement the abilities of human mathematicians and thus to enhance their productivity.

1 Introduction

Computers and Humans have complementary strengths. While the former can handle large computations and data volumes flawlessly at enormous speeds, humans can sense the environment, react to unforeseen circumstances and use their intuitions to guide them through only partially understood situations. In mathematics we make use of this complementarity by letting humans explore mathematical theories and come up with novel proofs, while delegating symbolic/numeric computation and typesetting of documents to computers.

There is, however, one area where the specific strengths of computers are not commonly exploited in mathematics: the *management of existing mathematical knowledge*. In contrast to the creation of new knowledge, which requires (human) insights and intuition, the management (cataloguing, retrieval, refactoring, plausibilization, change propagation and in some cases even application) of mathematical knowledge can be supported by machines and can even be automated in the near future given suitable representation formats and algorithms.

With more than 100 thousand articles published annually, the times where a human could have an overview over – let alone have working knowledge in – all of mathematics are long gone. Even web-based information systems with global reach like the Cornell e-print arXiv [1] and reviewing services like the AMS reviews [16] or Zentralblatt Math [22] cannot prevent duplication of work and missed opportunities for the application of mathematical results.

Even if they make mathematical documents available at the click of a mouse, they cannot transcend a fundamental invariant: to do mathematics with any of that knowledge, it must pass through a human brain, which has a very limited capacity – compared to the volume, breadth and diversity of mathematical knowledge currently accessible in documents. We will call this the *one-brain-barrier* (*OBB*) of mathematics. Of course the OBB applies to all areas of human knowledge but the highly networked nature and rigorous presentation of mathematical knowledge make it a particularly appealing subject for studying the possibilities of machine support in knowledge management.

The effect of the OBB is particularly noticeable for the mathematical practice of *framing*, i.e. establishing that an object a (of class A) can be viewed as an incarnation of a class B of objects (via an *interpretation mapping* $\iota : B \to A$); see [7] for an introduction. Framing is exceedingly successful in establishing seemingly new mathematical knowledge in bulk: proving a single theorem T about objects b in B yields the theorem $\iota(T)$ about objects $\iota(b)$ in A "for free". Influential interpretations (where the theory of B is particularly rich) are often referred to as "representation theorems" as they cross-fertilise between mathematical areas by bridging represenational differences. But the method of framing is useful also in the small: we use it every time we apply a theorem, only that we usually do not state necessary interpretations explicitly but leave their reconstruction to the reader. The ability to to this without conscious thought is part of every mathematical education. The utility and ubiquity of framing lets some mathematicians consider it a defining characteristic of mathematical practice.

Note that the proof that any A can be interpreted to be a B may be highly non-trivial; but the problems with finding interpretations and representation theorems begin way before that: a prerequisite is that working knowledge about both As and Bs is co-located in one brain. There are two kinds of situation where interpretations are needed:

1. We have established a new result about a class B of objects and want to apply it to all known objects a that can be interpreted to be of class B.
2. We study object b (of class B) and want to see whether there are classes A it can be interpreted as, so that we can take advantage of the knowledge about A.

Both induce a search problem for an interpretation $\iota : B \to A$, where A ranges over "all mathematical knowledge". So, even if we assume that the knowledge about B is already available – a questionable assumption in 2 – the OBB hits us with full force. Note that even though the proof of the representation theorem for ι usually requires (human) insight and intuition, the application of the frame (the transport of the knowledge from B to A) is a largely syntactical process, which can be machine-supported.

In this article, we want to survey the field of Mathematical Knowledge Management, its basic ideas and methodological basis in general (next section), then focus on a subset of methods that put the notion of framing at the centre (Section 3) and show how they can be used to transcend the one-brain barrier in Section 4. Section 5 concludes the article.

2 Mathematical Knowledge Management

Mathematical Knowledge Management (see [15, 3]) is a young research area at the intersection of mathematics, artificial intelligence, computer science, library science and scientific publishing. The objective of MKM is to develop new and better ways of managing sophisticated mathematical knowledge based on innovative technology of computer science, the internet and intelligent knowledge processing. MKM is expected to serve mathematicians, scientists and engineers who produce and use mathematical knowledge; educators and students who teach and learn mathematics; publishers who offer mathematical textbooks and disseminate new mathematical results; and librarians who catalogue and organise mathematical knowledge.

Even though MKM focuses on (MK)M (the "management of mathematical knowledge"), its methods touch on M(KM) (the "mathematics of knowledge management"). It is a fundamental tenet of MKM that important aspects of knowledge can be formalised into a state where their management can be supported by machines. Conversely, it is assumed that *only* by making such aspects of knowledge formal – e.g. by semantics extraction – will they become amenable to automation. As a consequence, semantisation is an important concern for MKM.

As the "client list" of MKM above already suggests, the notion of knowledge management is rather inclusive. Correspondingly, the aspects of knowledge studied by MKM and the "depth of formalisation" are varied, ranging from full formalisation of the mathematical content in a logical system [5, 21] to machine-readable versions of the Mathematics Subject Classification [12].

It has always been an aim of the MKM community to build a digital mathematical library – envisioned to be universal in [3]. Based on this, sophisticated mathematical software systems could help humans articulate, organise, disseminate and access mathematical knowledge to allow them to concentrate on those parts of doing mathematics that need human cognitive facilities – creating new mathematical knowledge and appreciating the beauty of the existing canon.

In this paper we focus on ways to structure a digital mathematical library into modular theory graphs and explain salient mathematical practices in this setting. Our exposition here follows [20], which introduces OMDoc/MMT, currently the strongest formulation of theory graph ideas, which were initially introduced in [4] and extended to a comprehensive representation format for mathematical knowledge and documents (OMDoc: Open Mathematical Documents [8]).

3 Theory Graphs & Computer-Supported Framing

The main idea of the theory graphs paradigm in MKM is to take the idea of interpretations seriously and use it as a structuring principle in a modular representation methodology for mathematical knowledge. Neither the modularity principle nor the use of interpretations are particularly new in mathematics – they have been the main structural mechanism behind Bourbaki's systematic redevelopment of mathematics. But the MKM community is developing ways of formalising them and is making use of these structures for machine support. We will now explore the basic notions and show how these relate to the mathematical practice of framing.

Theory graphs

build on two central notions: theories (the nodes of the graph) and theory morphisms (the edges). The former are just collections of symbols denoting indivisible mathematical objects and axioms stating the assumptions this theory makes about them.

> *Theory morphisms* are mappings from symbols in the source theory to expressions in the target theory, such that all axioms are mapped to theorems of the target theory. (*)

Figure 1 shows a simple example of a theory graph. It consists of a theory monoid of monoids, which has the symbols op for the binary composition operation and unit for its neutral element. Symbols and axioms (which we omit in Figure 1 for simplicity) have tripartite global names of the form g?t?c in OMDoc/MMT, where g is the URL of the document that contains the theory t and c is the name of a symbol in t. Note that such tripartite names (*MMT URIs*) are valid uniform resource identifiers, which is a crucial prerequisite for being used in web-based MKM systems. In Figure 1, we have assumed that all theories are in the same document and we can therefore omit the URL g throughout.

Inheritance

Now, a theory cgp of Abelian groups can be obtained from a monoid by adding an inverse operation inv. In OMDoc/MMT we can avoid duplication of representations by utilising an inclusion morphism (visualised by an arrow $\hookrightarrow$) that "copies" all symbols and axioms from the source theory monoid to cgp. Note that inclusion morphisms are trivially theory morphisms, since the target is defined to make them so. OMDoc/MMT treats incoming inclusions like symbols and names them in their target theory, so that the inclusion of the monoid in Figure 1 is globally accessible to MKM systems via the MMT URI g?cgp?mon. The inherited symbols are available in cgp via the name of the inclusion that supplied them, e.g. composition as mon/op or globally as g?cgp?mon/op. Axioms are inherited similarly.

In the next step we can obtain a theory ring via two inclusions: ring?add and ring?mul (and adding a distributivity axiom). Note that ring has two distinct binary operations: the additive ring?add/mon/op and the multiplicative ring?mul/op,

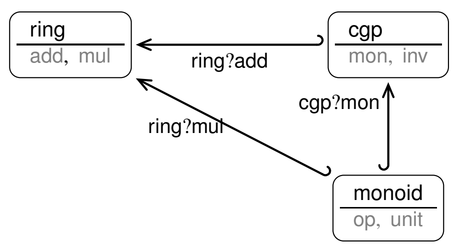


Figure 1. A simple Theory Graph

two neutral elements: ring?add/mon/unit and ring?mul/unit, and the additive inverse ring?add/inv. Note that the respective copies of the axioms of mon and cgp are inherited as well. ring inherits eight axioms: two closure, two associativity and two neutrality axioms, plus additive inverse and commutativity. We see that OMDoc/MMT can mimic the efficiency of mathematical vernacular in this respect.

Notation

Before we go on with our exploration of Figure 1, a clarification of the level of representation is in order. We have seen that the use of inclusion morphisms gives us theory structures that systematically supply URIs for the objects we expect. We should think of OMDoc/MMT as a kind of "machine language" for representing mathematical knowledge. In particular, URIs as names are good for communication of knowledge over the internet and its manipulation by machines but they are woefully inadequate for communication with humans, who expect to see, for example, $\circ$ instead of MMT URI g?cgp?mon/op in Abelian groups and may use $+$ for ring?add/mon/op and $\cdot$ for ring?mul/op in rings. To account for this, OMDoc/MMT offers a system of *notation definitions* that allow us to associate human-oriented notations with symbols and regain the familiar and helpful notations of mathematical discourse. Indeed, making notation definitions first-class citizens of the representation format allows us to model them and their effects in modular theory graphs: they are inherited along inclusions and thus available wherever the inherited symbols are visible. In MKM systems that communicate with humans, we use notation definitions to generate formula presentations for the user but keep the symbol URIs to render additional services, e.g. definition lookup.

Views

We have seen that inclusions can be used for a modular development of, for example, the elementary algebraic hierarchy. But the real power of theory graphs comes from another kind of theory morphisms: *views*. In contrast to inclusions they link two pre-existing theories and thus we have to prove that all the source axioms are theorems of the target theory to establish the theory morphism condition (*). Moreover, views usually have non-trivial symbol mappings. In Figure 2 we extend the theory graph from Figure 1 with a theory of integers and two views. v_1 maps the monoid operation to integer addition and the neutral element to the integer number zero. To establish this mapping as a view we have to establish (*) by proving closure of $\mathbb{Z}$ under $+$, associativity of $+$, and $x + 0 = x$ for all $x \in \mathbb{Z}$ (the *proof obligations* induced by v_1) from the axioms of integers.

The view v_2 is similar, only that it additionally maps the inverse operation to $-$ and has proof obligations for the inverse axiom and commutativity. Note that v_2 can be made modular by mapping the inclusion mon to v_1 instead of the direct mapping of (inherited) symbols. This also allows us to inherit/reuse the proof obligations of v_1 in v_2. A view v_3 from ring to integers can be established similarly.

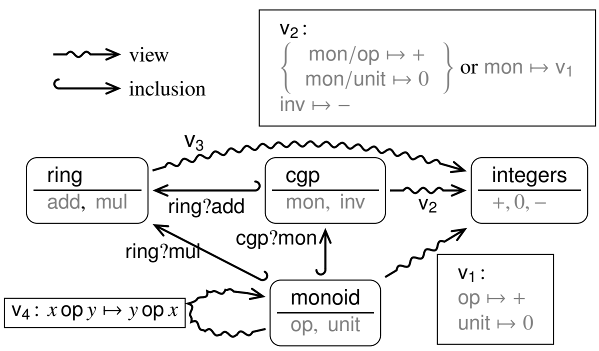


Figure 2. A Theory Graph with Views/Examples

Theory Morphisms Transport Proofs and Theorems

But more important than the mechanics of theory graphs is the realisation that in the presence of a theory morphism $S \xrightarrow{\sigma} T$ by (*) any S-theorem t becomes a T-theorem $\sigma(t)$ as the S-proof π of t can be translated along σ: any appeal to an S-axiom in π becomes an appeal to a T-theorem in $\sigma(\pi)$ by (*). In particular, $\sigma(t)$ can be referred to as $T?[v]/\mathrm{t}$, just like induced symbols, only that v is the MMT URI of the view σ.

In this context, note that the pragmatics of *structures* (inclusions with non-trivial mappings) and views are complementary. Structures are constitutive to a theory and restrict the "applicability" of a theory, since they introduce axioms that must be fulfilled by proof obligations, while views are (logically) redundant but allow us to "apply" a theory elsewhere. The subgraph induced by structures must be acyclic, while views may induce cycles. Indeed, theory isomorphisms (pairs of inverse theory morphisms) are a good way to represent equivalent theories, alternative definitions, etc.

Views at Work in Mathematics

In our example in Figure 2, this means that all group theorems apply to $(\mathbb{Z}, +)$ (and all theories that are reachable from it via theory morphisms). This already explains one mathematical practice: proving a conjecture in the greatest possible generality. In theory graph terms, this means establishing a theorem as low as possible (in terms of the pre-order induced by theory morphisms) in the graph, since the "cone of influence" gets bigger this way.

Another mathematical practice is to abbreviate proofs by meta-arguments, e.g. "(*iv*) the proof of this case follows from case (*iii*) by symmetry". In most cases, such proofs by meta-arguments can be seen as making the reader aware of a theory-endomorphism like v_4 that can be used to transport the proof of (*iii*) into one of (*iv*). Finally, mathematical examples can be interpreted as theory morphisms in theory graphs (another consequence of (*)). In our example, $(\mathbb{Z}, +, 0)$ is an example of a monoid by v_1, $(\mathbb{Z}, +, 0, -)$ is an example of an Abelian group via v_2 and $(\mathbb{Z}, +, 0, -, *, 1)$ is an example of a ring via v_3. An example becomes "non-obvious" if it has at least one view component.

Framing

In conclusion, we note that theory morphisms are *very natural candidates* for representing the structures underlying the *mathematical practice of framing* highlighted in the introduction. Indeed, we can see the condition (*) as being the essence of framing. We are currently exploring the logical and cognitive consequences of this idea. It seems that theory morphisms directly account for representation theorems like Stone's theorem or Ken Ribet's link from the Taniyama-Shimura conjecture to Fermat's last theorem, which made Wiles' proof possible. But framing also seems to be at work in less obvious places, e.g. in mathematical synonyms. The concepts of "nodes" and "edges" in a graph are often called "points" and "lines", borrowing terminology from geometry. I believe that this metaphoric usage of names is licensed by a partial view from (linear, ordered) geometry to theory graphs. A problem in the study of framing is that it is a cultural skill of mathematics and that mathematical literacy requires mastering framing to a level where it becomes almost subconscious, and therefore hard to observe via introspection.

MKM for Theory Graphs

Even our simple example in Figure 2 shows that theory graphs are very information-rich objects whose structure can be used to explain many mathematical practices. But the full power of theory graphs only comes out when they are implemented in a software system that supports these practices. OMDoc/MMT has been implemented in the MMT API [19], which supports services including notation-definition-based presentation (see above), incremental *flattening* (i.e. computation of the symbols, axioms and theorems of a theory S induced by the theory graph), as well as type- and proof-checking (an important aspect of MKM we have neglected in this article).

Such systems allow us to make use of the space efficiency afforded by the modular representation in theory graphs: we only need to represent (and store) a small subset of the knowledge available to an experienced mathematician. In our experience, the induced knowledge outweighs the represented by a factor of 50/1 (if it is finite at all) even for relatively small theory graphs. Note that the quotient of the size of the induced knowledge over the represented knowledge is a good measure not only for the space efficiency of the representation system but also for the mathematical literacy of a human. Experienced mathematicians induce much more (implicit) knowledge from what they read than inexperienced ones – because the former, it is conjectured, have more densely connected mental theory graphs at their disposal. OMDoc/MMT provides standardised identifiers for all induced knowledge items and the MMT API can compute their values on demand and reason with them. In this sense, the MMT API can be considered to possess a certain amount of "mathematics literacy" that allows it to render higher-level mathematical services.

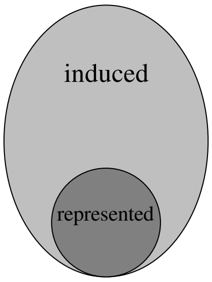


Figure 3. Knowledge Addressable in MMT

4 Overcoming the One-Brain-Barrier

We have seen above that with theory graph technology, MKM systems can achieve a certain level of mathematical literacy even without the ability for automated reasoning – another important aspect of MKM we will not cover in this article. But in the foreseeable future it seems clear that no machine will even come close to a human mathematician's creative ability of inventing new mathematics. But we can make use of the high storage capacity and cheap computation that MKM offers. It is claimed that even with limited mathematical literacy, machines can (*i*) complement human mathematicians and (*ii*) even uncover – rather than discover – novel mathematics by overcoming the OBB, if we have theory graphs that contain more knowledge than the average human mathematician. In this section, we will assume such theory graphs will become available and sketch one example for (*i*) and two for (*ii*) – which may provide motivation for organising a community effort to build such graphs. Starting points could be theorem prover libraries like the Mizar library [14], which has more than 1000 theories which together contain more than 50,000 formal theorems and definitions (see Figure 4 for the inheritance graph – the Mizar theory language only supports inclusions). We are currently in the process of exporting half-a-dozen theorem prover libraries into the OMDoc/MMT format to create a large theory graph (the Open Archive of Formalizations; OAF) to experiment on.

Complementing Humans by Searching the Induced Knowledge Space

The idea is very simple: we use a formula search engine like MathWebSearch (see [10, 11]) and instead of indexing all the represented knowledge items with their respective URIs, we extend this to the set of induced items and their MMT URIs (which the MMT API can compute by flattening). Consider the situation in Figure 5, where we are searching for the associativity formula $X+(Y+Z) = (X+Y)+Z$, and MathWebSearch returns the MMT URI http://latin.omdoc.org/math?IntAryth?assoc, which – together with the underlying theory

Figure 4. The Mizar Inheritance Graph

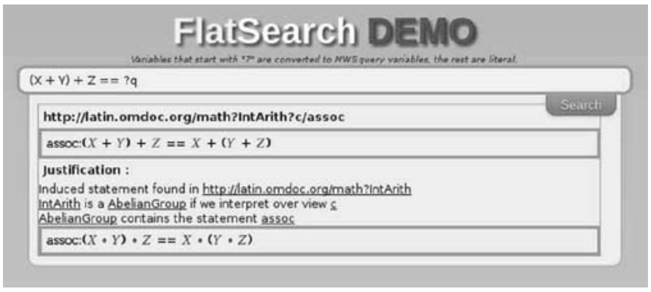


Figure 5. Searching for Induced Knowledge Items

graph – contains enough information to generate an explanation of the reason + is associative on $\mathbb{Z}$. In essence, this experimental search engine searches the space of (induced) mathematical knowledge rather than just the space of mathematical documents.

Making Bourbaki Accessible

Ideas like the ones above can solve one of the problems with the Bourbaki book series, which is written in such a very concise and modular manner that it can only be understood if one has all the previous parts in memory. We have extracted the theory graph underlying the first 30 pages of Algebra I [2]. It contains 51 theories, 94 inclusions and 10 views. The theories contain 82 Symbols, 38 axioms, 30 theorems and 17 definitions [13]. For knowledge items higher up in the graph, there is no (single) place in the book which states all their axioms or properties. With the MMT API, we can generate flattened descriptions for reference and with FlatSearch we can search for their properties. We conjecture that the simple explanation feature from Figure 5 can be extended into a "course generator" that generates a self-contained – modulo the reader's prerequisite knowledge – document that explains all aspects necessary for understanding the search hits. With theory graph technology, Bourbaki's Elements can be read by need, as a foundational treatise should be, instead of being restricted to beginning-to-end reading.

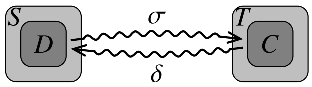
Uncovering Theorems

We can now come back to the discussion on the OBB from the introduction. One way to overcome the problem of missing interpretations is to systematically search for them – after all, machine computation is comparatively cheap and the number of potential theory morphisms is bounded by the square of the number of theories times the number of symbols in theories. In particular, the MathWebSearch formula search engine [10] can efficiently search for substitutions (which are essentially the same as the symbol-to-expression mappings of theory morphisms). We have explored this idea before the theory graph technology was fully developed, and the TheoScrutor system [17] found a considerable number of simple views in the Mizar library. While these were relatively syntactic and obvious – after all, for a *view finder* to work, the proofs for the proof obligations have to be part of the library already – they had not previously been noticed because of the OBB, even though the Mizar project has a "library committee" tasked with finding such linkages. We expect that once the OAF is sufficiently stable, a renewed experiment will yield many more, and more interesting views and possibly novel theorems – after all, the HOL Light and Isabelle Libraries contain Tom Hales' proof of the Kepler Conjecture and the Coq library contains the proof of Feit-Thomson's Odd-Order Theorem.

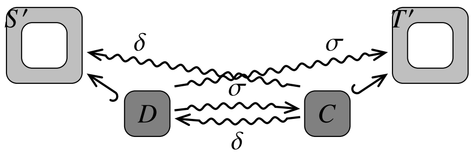
Note that once a set of views has been found by the view-finder, we can generate the induced theorems in all accessible theories and iterate the view finding process, which might find more views with the induced theorems.

Refining Theory Graphs

Finally, even partial views – which should be much more numerous than total ones in a theory graph – can be utilised. Say we have the situation below with two theories S and T, a partial theory morphism $S \xrightarrow{\sigma} T$ with domain D and codomain C, and its partial inverse δ.



Then we can pass to the following more modular theory graph, where $S' := S\backslash D$ and $T' := T\backslash C$. In this case we think of the equivalent theories D and C as the intersection of theories S and T along σ and δ. Note that any views out of S and T now have to be studied, if they can be pulled back to C and D.



We have observed that many of the lesser known algebraic structures in Bourbaki naturally arise as theory intersections between better known structures. We hope to explain the remaining ones via other category-theory-inspired theory graph transformation operations.

Note that operations like theory intersections apply theory graph technology to the problem of theory graph maintenance, which is itself a problem greatly hampered by the OBB without MKM techniques.

5 Conclusion

In this paper we have explored opportunities to lift the one-brain barrier in mathematics, which limits the application of mathematical knowledge both inside the mathematical domain as well as in other disciplines. We propose that the way forward is to employ computer systems that can systematically explore immense knowledge spaces, if these are represented in sufficiently content-oriented formats. Together with the creation, curation and application of digital mathematical libraries (DMLs), this is one of the central concerns of the new field of Mathematical Knowledge Management (MKM).

We have presented the theory graphs approach as a representation paradigm for mathematical knowledge that allows us to make its modular and highly networked structure explicit and therefore machine-actionable. We have seen that theory graphs following the "little theories" approach contain the information structures necessary to explain – and thus ultimately support by computer – many mathematical practices and culture skills. This has the potential to significantly extend the "mathematical literacy" of mathematical knowledge management systems and consequently make them more suitable as tools that complement human skills.

We have explored three exemplary mathematical applications of theory graph technologies and one MKM-internal to give an intuition of what services we can expect if we embark on the enterprise of representing large bodies of mathematical knowledge and its network structures in machine-actionable formats. The availability of such DMLs is currently the largest bottleneck for overcoming the OBB in mathematics. We are currently experimenting with establishing an open archive of formal mathematics (the OAF project [18]) by integrating theorem prover libraries. But formalisation often poses a high burden on the author and forces decisions about logical foundations that are irrelevant mathematically. Therefore, we are currently researching ways the theory graph methods presented here can be extended to representations of mathematical knowledge in which the degree of formalisation is flexible. Flexiformal representations – see [9] for a discussion – are much closer to mathematical vernacular, which mixes informal parts (natural language) with formal parts (e.g. formulae and functional markup for mathematical statements) and are therefore easier to obtain in practice. But mathematical literacy may be limited by the availability of formal/machine-actionable parts; therefore, we are additionally investigating methods for automated semantics-extraction from mathematical documents, which would greatly enhance the reach of the methods described in this paper.

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Is a Graviton Detectable?

Freeman Dyson (Institute for Advanced Study, Princeton, USA)

1 Introduction

I am enormously grateful to Dr. K. K. Phua, and to everyone else who had a hand in organizing this conference, for inviting me to visit Singapore. I am also grateful to my old and new friends who came to Singapore to help me celebrate my birthday. As a former Brit, I am delighted to see this sparkling new country, which has prospered by giving free play to Chinese enterprise while still driving on the left side of the road.

Now I come to the technical substance of my talk. It is generally agreed that a gravitational field exists, satisfying Einstein's equations of general relativity, and that gravitational waves traveling at the speed of light also exist. The observed orbital shrinkage of the double pulsar [1] provides direct evidence that the pulsar is emitting gravitational waves at the rate predicted by the theory. The LIGO experiment now in operation is designed to detect kilohertz gravitational waves from astronomical sources. Nobody doubts that gravitational waves are in principle detectable.

This talk is concerned with a different question, whether it is in principle possible to detect individual gravitons, or in other words, whether it is possible to detect the quantization of the gravitational field. The words "in principle" are ambiguous. The meaning of "in principle" depends on the rules of the game that we are playing. If we assert that detection of a graviton is in principle impossible, this may have three meanings. Meaning (a): We can prove a theorem asserting that detection of a graviton would contradict the laws of physics. Meaning (b): We have examined a class of possible graviton detectors and demonstrated that they cannot work. Meaning (c): We have examined a class of graviton detectors and demonstrated that they cannot work in the environment provided by the real universe. We do not claim to have answered the question of "in principle" detectability according to meaning (a). In Sec. 3 we look at detectors with the LIGO design, detecting gravitational waves by measuring their effects on the geometry of space–time, and conclude that they cannot detect gravitons according to meaning (b). In Secs. 4 and 5 we look at a different class of detectors, observing the interactions of gravitons with individual atoms, and conclude that they cannot detect gravitons according to meaning (c). In Secs. 6 and 7 we look at a third class of detectors, observing the coherent transitions between graviton and photon states induced by an extended classical magnetic field, and find that they also fail according to meaning (c).

In Sec. 2 we look at a historic argument used by Niels Bohr and Leon Rosenfeld to demonstrate the quantum behavior of the electromagnetic field, and explain why this argument does not apply to the gravitational field. In Sec. 8 we briefly examine the possibility of observing primordial gravitons at the beginning of the universe by measuring the polarization of the cosmic background radiation today. No definite conclusions are reached. This talk is a report of work in progress, not a finished product. It raises the question of the observability of gravitons but does not answer it. There is much work still to do.

2 The Bohr–Rosenfeld Argument

Before looking in detail at graviton detectors, I want to discuss a general theoretical question. In 1933 a famous paper by Niels Bohr and Leon Rosenfeld, [2] was published in the proceedings of the Danish Academy of Sciences with the title, "On the Question of the Measurability of the Electromagnetic Field Strengths." An English translation by Bryce de Witt, dated 1960, is in the Institute library in Princeton, bound in an elegant hard cover. This paper was a historic display of Bohr's way of thinking, expounded in long and convoluted German sentences. Rosenfeld was almost driven crazy, writing and rewriting 14 drafts before Bohr was finally satisfied with it. The paper demonstrates, by a careful and detailed study of imaginary experiments, that the electric and magnetic fields must be quantum fields with the commutation relations dictated by the theory of quantum electrodynamics. The field-strengths are assumed to be measured by observing the motion of massive objects carrying charges and currents with which the fields interact. The massive objects are subject to the rules of ordinary quantum mechanics which set limits to the accuracy of simultaneous measurement of positions and velocities of the objects. Bohr and Rosenfeld show that the quantum-mechanical limitation of measurement of the motion of the masses implies precisely the limitation of measurement of the field-strengths imposed by quantum electrodynamics. In other words, it is mathematically inconsistent to have a classical electromagnetic field interacting with a quantum-mechanical measuring apparatus.

A typical result of the Bohr–Rosenfeld analysis is their equation (58),

$$\Delta E_x(1)\Delta E_x(2) \sim \hbar |A(1,2) - A(2,1)| \,. \qquad (1)$$

Here the left side is the product of the uncertainties of measurement of two averages of the x-component of the electric field, averaged over two space–time regions (1) and (2). On the right side, $A(1,2)$ is the double average over regions (1) and (2) of the retarded electric field produced in (2) by a unit dipole charge in (1). They deduce (1) from the standard Heisenberg uncertainty relation obeyed by the measuring apparatus. The result (1) is precisely the uncertainty relation implied by the commutation rules of quantum electrodynamics. Similar results are found for other components of the electric and magnetic fields.

The question that I am asking is whether the argument of Bohr and Rosenfeld applies also to the gravitational field. If the same argument applies, then the gravitational field must be a quantum field and its quantum nature is in principle observable. However, a close inspection of the Bohr–Rosenfeld argument reveals a crucial feature of their measurement apparatus that makes it inapplicable to gravitational fields. In the last paragraph of Sec. 3 of the Bohr–Rosenfeld paper, they write: "In order to disturb the electromagnetic field to be measured as little as possible during the presence of the test body system, we shall imagine placed beside each electric or magnetic component particle another exactly oppositely charged neutralizing particle." The neutralizing particles have the following function. Suppose we have a mass carrying a charge or current J whose movement is observed in order to measure the local electric or magnetic field. The movement of the charge or current J produces an additional electromagnetic field that interferes with the field that we are trying to measure. So we must compensate the additional field by adding a second mass, carrying the charge or current $-J$ and occupying the same volume as the first mass. The second mass is constrained by a system of mechanical linkages and springs to follow the movement of the first mass and cancels the fields generated by the first mass. This cancellation is an essential part of the Bohr–Rosenfeld strategy. It is then immediately obvious that the strategy fails for measurement of the gravitational field. The test-objects for measuring the gravitational field are masses rather than charges, and there exist no negative masses that could compensate the fields produced by positive masses.

The conclusion of this argument is that the Bohr–Rosenfeld analysis does not apply to the gravitational field. This does not mean that the gravitational field cannot be quantized. It means only that the quantization of the gravitational field is not a logical consequence of the quantum behavior of the measuring apparatus. The fact that the electromagnetic field must be quantized does not imply that the gravitational field must be quantized.

3 Can LIGO Detect a Graviton?

In the LIGO experiment, if it is successful, we shall detect a classical gravitational wave, not an individual quantum of gravity. A classical wave may be considered to be a coherent superposition of a large number of gravitons. LIGO is supposed to detect a wave with a strain amplitude f of the order of 10^{-21}. According to Landau and Lifshitz [3], page 370, the energy density of this wave is

$$E = \left(\frac{c^2}{32\pi G}\right)\omega^2 f^2\,, \tag{2}$$

where G is Newton's constant of gravitation and ω is the angular frequency. For a wave with angular frequency 1 Kilohertz and amplitude 10^{-21}, Eq. (2) gives an energy density of roughly 10^{-10} ergs per cubic centimeter. A single graviton of a given angular frequency ω cannot be confined within a region with linear dimension smaller than the reduced wave-length (c/ω). Therefore the energy density of a single graviton of this frequency is at most equal to the energy of the graviton divided by the cube of its reduced wave-length, namely

$$E_s = \left(\frac{\hbar\omega^4}{c^3}\right). \tag{3}$$

For an angular frequency of 1 Kilohertz, the single graviton energy density is at most 3×10^{-47} ergs per cubic centimeter. So any gravitational wave detectable by LIGO must contain at least 3×10^{37} gravitons. This wave would be barely detectable by the existing LIGO. For a LIGO apparatus to detect a single graviton, its sensitivity would have to be improved by a factor of the order of 3×10^{37}. Even this vast improvement of sensitivity would probably not be sufficient, because the detection of weak signals is usually limited not only by the sensitivity of the apparatus but also by the presence of background noise. But to see whether detection of single gravitons is possible in principle, we disregard the problem of background noise and analyze the structure and operation of a super-sensitive LIGO detector.

For a rough estimate of the sensitivity of a LIGO apparatus required to detect a single graviton, we equate (2) with (3). This gives the strain f to be detected by the apparatus,

$$f = (32\pi)^{1/2}\left(\frac{L_p\omega}{c}\right), \tag{4}$$

where L_p is the Planck length

$$L_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.4 \times 10^{-33}\ \text{cm}\,. \tag{5}$$

The strain is derived from a measurement of the variation of distance between two mirrors separated by a distance D. The variation of the measured distance is equal to fD, so long as D does not exceed the reduced wave-length (c/ω) of the graviton. For optimum detectability we take D equal to (c/ω). Then the variation of distance is by (4)

$$\delta = (32\pi)^{1/2} L_p\,. \tag{6}$$

Up to a factor of order unity, the required precision of measurement of the separation between the two mirrors is equal to the Planck length, and is independent of the frequency of the graviton.

Is it possible in principle for a LIGO apparatus to measure distances between macroscopic objects to Planck-length accuracy? The following simple arguments give a negative answer to this question. First consider the case in which the objects are floating freely in space. The Heisenberg uncertainty relation between position and momentum of freely floating objects gives the lower bound

$$M\delta^2 \geq \hbar T\,, \tag{7}$$

for the variation of distance δ, where M is the mass of each object and T is the duration of the measurement. Now T must be greater than the time (D/c) required to communicate between the two mirrors. If δ is equal to the Planck length, (5) and (7) imply

$$D \leq \left(\frac{GM}{c^2}\right). \tag{8}$$

So the separation between the two mirrors is less than the Schwarzschild radius of each of them, the negative gravitational potential pulling them together is greater than Mc^2, and they are bound to collapse into a black hole before the measurement can be completed.

We next consider the situation that arises when the two mirrors are clamped in position by a rigid structure. In this case the precision of measurement of the distance between the two mirrors is limited by quantum fluctuations of the rigid structure. We use a simple dimensional argument to estimate the magnitude of the fluctuations. Let s be the velocity of sound in the structure, let D be the separation between the mirrors, and let M be the mass of the structure. There will be at least one mode of sound-vibration of the structure which gives a displacement affecting the measurement of D. The mean-square quantum fluctuation amplitude of the displacement in this mode will then be, up to a factor of order unity, at least as large as the zero-point fluctuation,

$$\delta^2 \geq \left(\frac{\hbar D}{Ms}\right). \tag{9}$$

The duration of the measurement must be of the order of (D/c), the time it takes the graviton to travel through the apparatus. This duration is shorter than the period (D/s) of the sound-vibration, since s cannot exceed c. Therefore the uncertainty of the measurement is at least equal to the instantaneous vibration-amplitude δ. If the uncertainty is as small as the Planck length (5), then (9) implies

$$\left(\frac{GM}{c^2}\right) \geq \left(\frac{c}{s}\right) D > D\,. \tag{10}$$

Again we see that the separation between the two mirrors is smaller than the Schwarzschild radius of the apparatus, so that the negative gravitational potential of the apparatus is greater than Mc^2 and it will again collapse into a black hole. It appears that Nature conspires to forbid any measurement of distance with error smaller than the Planck length. And this prohibition implies that detection of single gravitons with an apparatus resembling LIGO is impossible.

It is clear from Eq. (3) that we have a better chance of detecting a single graviton if we raise the frequency into the optical range and use a different kind of detector. When the frequency is of the order of 10^{15} Hertz or higher, a single graviton can kick an electron out of an atom, and the electron can be detected by standard methods of atomic or particle physics. We are then dealing with the gravitoelectric effect, the gravitational analog of the photoelectric effect which Einstein used in 1905, [4] to infer the existence of quanta of the electromagnetic field, the quanta which were later called photons. The possibility of detecting individual gravitons in this way depends on two quantities, (a) the cross-section for interaction of a graviton with an atom, and (b) the intensity of possible natural or artificial sources of high-frequency gravitons. Most of this talk will be concerned with estimating these two quantities.

4 Graviton Detectors

The simplest kind of graviton detector is an electron in an atom, which we may approximate by considering the electron to be bound in a fixed potential. We choose coordinate axes so that the z-axis is the direction of propagation of a graviton. There are then two orthogonal modes of linear polarization for the graviton, one with the wave-amplitude proportional to xy, and the other with the amplitude proportional to $(x^2 - y^2)$. We choose the x and y-axes so that they make angles of 45 degrees to the plane of polarization of the graviton. Then the matrix element for the electron to absorb the graviton and move from its ground state a to another state b is proportional to the mass-quadrupole component,

$$D_{ab} = m \int \psi_b^* xy \psi_a \, d\tau\,, \tag{11}$$

where m is the electron mass. Equation (11) is the quadrupole approximation, which is valid so long as the wave-length of the graviton is large compared with the size of the atom. The total cross-section for absorption of the graviton by the electron is

$$\sigma(\omega) = \left(\frac{4\pi^2 G\omega^3}{c^3}\right) \sum_b |D_{ab}|^2 \delta(E_b - E_a - h\omega)\,, \tag{12}$$

where E_a and E_b are the energies of the initial and final states. It is convenient to consider a logarithmic average of the cross-section over all frequencies ω,

$$S_a = \int \sigma(\omega) d\omega/\omega\,. \tag{13}$$

Integration of (12) gives the sum-rule

$$S_a = 4\pi^2 L_p^2 Q\,, \tag{14}$$

where the Planck length L_p is given by (4), and

$$Q = \int \left| \left(\frac{x\partial}{\partial y} + \frac{y\partial}{\partial x}\right) \psi_a \right|^2 d\tau \tag{15}$$

is a numerical factor of order unity. It is remarkable that the average cross-section (14) is independent of the electron mass and of the nuclear charge. The same formula (14) holds for the absorption of a graviton by a neutron or proton bound in a nuclear potential.

For simplicity we assume that the electron is in a state with zero component of angular momentum about the z-axis, with a wave-function $f(s, z)$, where s is the distance from the z-axis. Then (15) becomes

$$Q = \left(\int s^3 [f']^2 ds\, dz \right) \Big/ \left(2 \int s [f]^2 ds\, dz \right), \tag{16}$$

where f' means the partial derivative of f with respect to s. The inequality

$$\int s^3 \left[f' - \left(\frac{f}{s}\right) \right]^2 ds\, dz > 0 \tag{17}$$

implies that for any $f(s, z)$

$$Q > \frac{1}{2}\,. \tag{18}$$

On the other hand, if the electron is in an s-state

$$f(r) = r^{-n} \exp\left(-\frac{r}{R}\right), \tag{19}$$

where r is distance from the origin, then

$$Q = \left(\frac{4}{5}\right)\left[1 - \left(\frac{n}{6}\right)\right]. \tag{20}$$

From (18) and (20) it appears that for any tightly-bound s-state Q will be close to unity. The cross-section for absorption of a graviton by any kind of particle will be of the same magnitude

$$4\pi^2 L_p^2 = \frac{4\pi^2 G\hbar}{c^3} = 8 \times 10^{-65}\ \text{cm}^2\,, \tag{21}$$

spread over a range of graviton energies extending from the binding-energy of the particle to a few times the binding-energy. For any macroscopic detector composed of ordinary matter, the absorption cross-section will be of the order of 10^{-41} square centimeters per gram.

5 Thermal Graviton Generators

We have a splendid natural generator of thermal gravitons with energies in the kilovolt range, producing far more gravitons than any artificial source. It is called the sun. Stephen Weinberg long ago calculated [5] the graviton luminosity of the sun, caused by gravitational bremsstrahlung in collisions of electrons and ions in the sun's core. A later calculation [6] corrected a mistake in Weinberg's paper but does not substantially change the result. For an electron–ion collision with energy E, the differential cross-section $p(\omega)$ for producing a graviton of energy $\hbar\omega$ is divergent at low energies, so that the total cross-section has no meaning. The physically meaningful quantity is the integral of the differential cross-section multiplied by the energy of the graviton,

$$\int p(\omega)\hbar\omega \, d\omega = \left(\frac{320}{9}\right) Z^2 \alpha^2 L_p^2 E \,, \tag{22}$$

where α is the electromagnetic fine-structure constant and Z is the charge of the ion. Including a similar contribution from electron–electron collisions, (22) gives a total graviton luminosity of the sun

$$L_g = 79 \text{ Megawatts} \,, \tag{23}$$

or about 10^{24} gravitons per second with energy in the kilovolt range. This gives a flux at the earth of

$$F_g = 4 \times 10^{-4} \text{ gravitons per cm}^2 \text{ per second} \,. \tag{24}$$

If we imagine the whole mass of the earth to be available as raw material for the manufacture of graviton detectors, with the cross-section (21) per electron and the flux (24), the counting-rate is 2.4×10^{-17} per second. If the experiment continues for the lifetime of the sun, which is 5 billion years, the expected total number of gravitons detected will be 4. The experiment barely succeeds, but in principle it can detect gravitons.

According to Gould, [6] there exist in the universe sources of thermal gravitons which are stronger than the sun, namely hot white dwarfs at the beginning of their lives, and hot neutron stars. Gould estimates the graviton luminosities of a typical white dwarf and a typical neutron star to be respectively 10^4 and 10^{10} times solar. Their luminosities are roughly proportional to their central densities. But the lifetimes during which the stars remain hot are shorter than the lifetime of the sun, being of the order of tens of millions of years for the white dwarf and tens of thousands of years for the neutron star. The lifetime output of gravitons will therefore be respectively 100 and 10^5 times solar. To stretch the theoretical possibilities of detection to the limit, we may suppose the detector to have mass equal to the sun and to be orbiting around the source of gravitons at a distance of 0.01 astronomical unit with an orbital period of 8 hours. Then the expected number of gravitons detected will be of the order of 10^{13} for the white dwarf and 10^{16} for the neutron star. The detection rate is roughly one per minute for the white dwarf and 3×10^4 per second for the neutron star. The conclusion of this calculation is that graviton detection is in principle possible, if we disregard the problem of discriminating the graviton signal from background noise.

The most important source of background noise is probably the neutrinos emitted by the sun or the white dwarf or the neutron star as the case may be. These neutrinos can mimic graviton absorption events by ejecting electrons from atoms as a result of neutrino–electron scattering. The neutrinos have higher energy than the gravitons, but only a small fraction of the neutrino energy may be transferred to the electron. From the sun, about 10^{14} neutrinos are emitted for each graviton, and the cross-section for neutrino–electron scattering is about 10^{20} times the cross-section for graviton absorption (see Ref. [7]). Therefore there will be about 10^{34} neutrino background events for each graviton absorption event.

For white-dwarfs and neutron-stars the ratio of background to signal is even larger, since neutrino production and scattering cross-sections increase with temperature more rapidly than graviton production and absorption cross-sections. Without performing detailed calculations, we can assert that for all thermal sources of gravitons the ratio of neutrino background to graviton signal will be of the order of 10^{34} or greater. In all cases, the total number of detected graviton events is vastly smaller than the square-root of the number of background events. The graviton signal will be swamped by the statistical scatter of the background noise.

Before jumping to conclusions about the detectability of gravitons, we must explore possible ways in which the neutrino background events might be excluded. The first possible way is to surround the detector with a shield thick enough to stop neutrinos but let gravitons pass. If the shield is made of matter of ordinary density, its thickness must be of the order 10^{10} kilometers, and its mass is so large that it will collapse into a black hole. The second possible way is to surround the graviton detector with neutrino detectors in anticoincidence, to catch the outgoing neutrino after each scattering event. This way fails for the same reason as the shield. The neutrino detectors would need to be at least as massive as the shield. The third possible way is to build a shield or a set of anticoincidence detectors out of some mythical material with superhigh density. The known laws of physics give us no clue as to how this might be done. We conclude that, if we are using known materials and known physical processes in a noisy universe, detection of thermal gravitons appears to be impossible.

6 Nonthermal Gravitons

It is possible to imagine various ways in which energetic objects such as pulsars may emit nonthermal gravitons of high energy. One such way is a process first identified by Gertsenshtein[8], the coherent mixing of photon and graviton states in the presence of an extended classical magnetic field. The graviton emission from various celestial objects resulting from the Gertsenshtein process was calculated by Papini and Valluri [9]. Some interestingly high graviton luminosities were predicted.

The Gertsenshtein process results from the interaction energy

$$\left(\frac{8\pi G}{c^4}\right) h_{ij} T_{ij} \,, \tag{25}$$

between the gravitational field h_{ij} and the energy–momentum tensor T_{ij} of the electromagnetic field. This interaction expresses the fact that electromagnetic fields have weight, just like other forms of energy. Now suppose that h_{ij} is the field of a graviton traveling in the z direction and

$$T_{ij} = \left(\frac{1}{4\pi}\right)(B_i + b_i)(B_j + b_j)\,, \qquad (26)$$

is the energy–momentum of the photon magnetic field b_i superimposed on a fixed classical magnetic field B_i. Then the interaction (25) contains the term

$$I = \left(\frac{4G}{c^4}\right)h_{xy}B_x b_y\,, \qquad (27)$$

bilinear in the graviton and photon fields. The effect of this bilinear term is to mix the photon and graviton fields, so that a particle that is created as a photon may be transformed into a graviton and vice versa. There is an oscillation between graviton and photon states, just like the oscillation between neutrino states that causes neutrinos to change their flavors while traveling between the sun and the earth. If a photon travels a distance D though a uniform transverse magnetic field B, it will emerge as a graviton with probability

$$P = \sin^2\left(\frac{G^{1/2}BD}{2c^2}\right) = \sin^2\left(\frac{D}{L}\right), \qquad (28)$$

with the mixing-length

$$L = \left(\frac{2c^2}{G^{1/2}B}\right) \qquad (29)$$

independent of wave-length. In all practical situations, D will be small compared with L, so that

$$P = \left(\frac{GB^2D^2}{4c^4}\right). \qquad (30)$$

The quadratic dependence of P on D makes this process interesting as a possible astrophysical source of gravitons. The numerical value of L according to (29) is roughly

$$L = \left(\frac{10^{25}}{B}\right), \qquad (31)$$

when L is measured in centimeters and B in Gauss.

We may also consider the Gertsenshtein process as the basis of a graviton detector consisting of a hollow pipe of length D filled with a transverse magnetic field B. The tube must be accurately pointed at a putative source of gravitons in the sky. At the far end of the tube is a shield to block incident photons, and at the near end is a detector of photons resulting from the conversion of gravitons on their way through the tube. If D is one astronomical unit (10^{13} cm), then (30) gives

$$P = 10^{-24}B^2\,. \qquad (32)$$

The field B must be very strong to obtain a reasonable rate of conversion of gravitons to photons. A detector with the same design has been used in a real experiment to detect axions that might be created by thermal processes in the core of the sun [10]. The axion field is supposed to interact with the electromagnetic field with an interaction energy similar to (27), but with a much larger coupling constant. The experimenters at CERN in Switzerland are using a surplus magnet from the Large Hadron Collider project as an axion-detector, pointing it at the sun and looking for kilovolt photons resulting from conversion of axions into photons. The length of the magnet is 9 meters and the magnetic field is 9×10^4 Gauss. They have not yet detected any axions.

The Gertsenshtein process does not require the classical magnetic field to be uniform. For a nonuniform field, the conversion of photons to gravitons still occurs with probability given by (28), if we replace the product BD by the integral of the transverse component of B along the trajectory of the photons. Likewise, the conversion will not be disturbed by a background gravitational field, even when the field is strong enough to curve the photon trajectory, because the gravitational field acts in the same way on photons and gravitons. In a curved space–time, the photons and the gravitons follow the same geodesic paths, and the photon and graviton waves remain coherent.

7 Nonlinear Electrodynamics

However, there is an important disturbing factor which was neglected in previous discussions of the Gertsenshtein process. The disturbing factor is the nonlinearity of the electromagnetic field caused by quantum fluctuations of electron–positron pairs in the vacuum [11, 12]. The fourth-order term in the electromagnetic field energy density is (Ref. [12], page 190),

$$\left(\frac{\alpha}{360\pi^2 H_c^2}\right)[(E^2 - H^2)^2 + 7(E \cdot H)^2]\,, \qquad (33)$$

where α is the fine-structure constant and

$$H_c = \left(\frac{m^2c^3}{e\hbar}\right) = 5 \times 10^{13} \text{ Gauss} \qquad (34)$$

is the critical magnetic field at which electron–positron pair fluctuations become noticeable.

When the field in (33) is divided into classical and photon components as in (26), there is a term quadratic in both the classical and photon fields,

$$\left(\frac{\alpha}{360\pi^2 H_c^2}\right)(4(B \cdot b)^2 + 7(B \cdot e)^2)\,, \qquad (35)$$

where b and e are the magnetic and electric fields of the photon. From (35) it follows that the photon velocity v is not equal to c but is reduced by a fraction

$$g = 1 - \left(\frac{v}{c}\right) = \left(\frac{k\alpha B^2}{360\pi^2 H_c^2}\right). \qquad (36)$$

The coefficient k is equal to 4 or 7 for a photon polarized with its magnetic field or its electric field parallel to B. We consider the case $k = 4$, since that case is more favorable to the Gertsenshtein process. Since the graviton field is not affected by the nonlinear electromagnetic interaction (33), the graviton velocity is precisely c, and the photon and graviton waves will lose coherence after traveling for a distance

$$L_c = \left(\frac{c}{g\omega}\right) = \left(\frac{90\pi^2 c H_c^2}{\alpha B^2\omega}\right) = \left(\frac{10^{43}}{B^2\omega}\right). \qquad (37)$$

If the propagation distance D is larger than L_c, the Gertsenshtein process fails and the formula (30) for the photon–graviton conversion probability is incorrect. A necessary condition for the Gertsenshtein process to operate is

$$DB^2\omega \leq 10^{43}\,. \qquad (38)$$

Furthermore, even when the Gertsenshtein process is operating, the probability of photon–graviton conversion according to (30) and (38) is

$$P \leq \left(\frac{10^{36}}{B^2\omega^2}\right). \qquad (39)$$

We are interested in detecting astrophysical sources of gravitons with energies up to 100 kilovolts, which means frequencies up to 10^{20}. With $\omega = 10^{20}$, (38) and (39) become

$$D \leq \left(\frac{10^{23}}{B^2}\right), \quad P \leq \left(\frac{10^{-4}}{B^2}\right). \tag{40}$$

We consider two situations in which (40) has important consequences. First, with typical values for the magnetic field and linear dimension of a pulsar, $B = 10^{12}$ and $D = 10^6$, (40) shows that the Gertsenshtein process fails by a wide margin. The calculations of the graviton luminosity of pulsars in Ref. [9] assume that the Gertsenshtein process is producing high-energy gravitons. These calculations, and the high luminosities that they predict, are therefore incorrect. Second, in the hollow pipe graviton detector which we considered earlier, (40) shows that the Gertsenshtein process can operate with a modest field, $B = 10^5$ Gauss, and a pipe length $D = 10^{13}$ cm, but the probability of detection of each graviton traveling through the pipe is only 10^{-14}. If the field is made stronger, the length of the pipe must be shorter according to (40), and the probability of detecting a graviton becomes even smaller. The detector can work in principle, but fails for practical reasons in the real universe.

8 Conclusions

We have examined three possible kinds of graviton detector with increasingly uncertain results. First, the LIGO detector for low-energy gravitons, which we prove ineffective as a consequence of the laws of physics. Second, the gravitoelectric detector for kilovolt gravitons, which we prove ineffective as a consequence of the background noise caused by neutrino processes in the real universe. Third, the coherent graviton-conversion detector for high-energy gravitons, is ineffective only because of practical limits to the size of magnetic detectors. In addition to these three kinds of detector, there is a fourth kind which actually exists, the Planck space telescope, detecting polarization of the microwave background radiation. According to Alan Guth [13], the polarization of the background radiation in an inflationary universe could provide direct evidence of the existence of single gravitons in the primordial universe before inflation. The results of the Planck polarization measurements are not yet published, and it remains to be seen whether the observations are able to distinguish between primordial gravitons and other gravitational effects of primordial matter. The question, whether a detector of present-day microwave radiation is in principle able to detect primordial gravitons, remains open.

Many papers have been published, for example Eppley and Hannah [14] and Page and Geilker[15], claiming to demonstrate that the gravitational field must be quantized. What these papers demonstrate is that a particular theory with a classical gravitational field interacting with quantum-mechanical matter is inconsistent. Page and Geilker assume that the classical gravitational field is generated by the expectation value of the energy–momentum tensor of the matter in whichever quantum state the matter happens to be. They performed an ingenious experiment to verify that this assumption gives the wrong answer for a measurement of the gravitational field in a real situation.

In this talk I am not advocating any particular theory of a classical gravitational field existing in an otherwise quantum-mechanical world. I am raising three separate questions. I am asking whether either one of three theoretical hypotheses may be experimentally testable. One hypothesis is that gravity is a quantum field and gravitons exist as free particles. A second hypothesis is that gravity is a quantum field but gravitons exist only as confined particles, like quarks, hidden inside composite structures which we observe as classical gravitational fields. The third hypothesis is that gravity is a statistical concept like entropy or temperature, only defined for gravitational effects of matter in bulk and not for effects of individual elementary particles. If the third hypothesis is true, then the gravitational field is not a local field like the electromagnetic field. The third hypothesis implies that the gravitational field at a point in space–time does not exist, either as a classical or as a quantum field.

I conclude that the first hypothesis may be experimentally testable, but the second and third may not. Analysis of the properties of graviton-detectors, following the methods of this paper, cannot distinguish between the second and third hypotheses. Three outcomes are logically possible. If a graviton detector is possible and succeeds in detecting gravitons, then the first hypothesis is true. If graviton detectors are possible and fail to detect gravitons, then the first hypothesis is false and the second and third are open. If a graviton detector is in principle impossible, then all three hypotheses remain open. Even if their existence is not experimentally testable, gravitons may still exist.

The conclusion of the analysis is that we are still a long way from settling the question whether gravitons exist. But the question whether gravitons are in principle detectable is also interesting and may be easier to decide.

In conclusion, I wish to thank Tony Rothman and Steven Boughn, [16] for helpful conversations and for sharing their thoughts with me before their paper was published.

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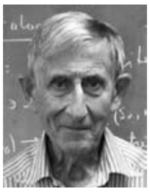


Freeman Dyson [dyson@ias.edu] has been for most of his life a professor of physics at the Institute for Advanced Study in Princeton. His most useful contribution to science was the unification of the three versions of quantum electrodynamics invented by Feynman, Schwinger and Tomonaga. He subsequently worked on nuclear reactors, solid state physics, ferromagnetism, astrophysics and biology, looking for problems where elegant mathematics could be usefully applied. He has written a number of books about science for the general public. Dyson is a fellow of the American Physical Society, a member of the U.S. National Academy of Sciences, and a fellow of the Royal Society of London. In 2000 he was awarded the Templeton Prize for progress in Religion, and in 2012 he was awarded the Henri Poincaré Prize at the August meeting of the International Mathematical Physics Congress.

This article is based on an invited talk given at the Conference in Honour of the 90th Birthday of Freeman Dyson, Institute of Advanced Studies, Nanyang Technological University, Singapore, 26–29 August 2013. It is a reprint of Is a graviton detectable? by Freeman Dyson, Int. J. Mod. Phys. A28, 1330041 (2013), c World Scientific, Singapore, 2013.

Bill Thurston (1946–2012)

John H. Hubbard (Cornell University, Ithaca, US)

Photo courtesy of the Cornell Math Department

William Thurston (30 October 1946–21 August 2012) was a geometer: he taught a whole generation a new way to think about geometry. In all the fields in which he worked, the "after Thurston" landscape is nearly unrecognisable from what it was "before Thurston".

His influence still dominates these fields, through his own results and through his many students: he supervised 33 theses and many of these former students are now major mathematicians in their own right. At last count (April 2014), he had 177 students, grand-students and great-grand-students.

Brief biography

Thurston received his bachelor's degree from New College (now New College of Florida) in 1967. For his undergraduate thesis he developed an intuitionist foundation for topology. Following this, he earned a doctorate in mathematics from the University of California, Berkeley, in 1972. His PhD advisor was Morris W. Hirsch and his dissertation was on Foliations of Three-Manifolds which are Circle Bundles.

After completing his PhD, he spent a year at the Institute for Advanced Study (where he worked with John Milnor and started Milnor's interest in dynamical systems), then another year at MIT as an assistant professor. In 1974, he was appointed as a professor of mathematics at Princeton University. In 1991, he returned to UC Berkeley as a professor of mathematics and in 1993 became Director of the Mathematical Sciences Research Institute. In 1996, he moved to University of California, Davis. In 2003, he became a professor of mathematics at Cornell University.

He did revolutionary work in foliation theory, low-dimensional topology, dynamics and geometric group theory.

Foliation Theory

His early work, in the early 1970s, was mainly about foliations. He solved so many problems that the field was "tsunamied": graduate students were discouraged from its study because it appeared that Thurston would solve all the problems. Only now is the field catching up with his contributions.

At the time, finding a codimension one foliation of a manifold was a major result in its own right. He solved the problem completely: a compact manifold has a codimension one foliation if and only if its Euler characteristic is 0.

He also found that any real number could be a Godbillon-Vey invariant, a major open problem at the time.

With John Mather, he showed that the cohomology of the group of homeomorphisms of a manifold is the same whether one gives the group the discrete topology or the topology of uniform convergence on compact sets. This result was prescient: at the time no one thought about such questions, but today they are central to a whole field.

Results involving complex analysis

In the late 1970s and early 1980s, Thurston found a collection of results that appear to be unrelated but are all closely connected; the connecting thread was *Teichmüller theory* and *complex analysis*. This led Lars Ahlfors, a professor at Harvard, arguably the greatest complex analyst of the 20th century and one of the two first Field's Medallists, to write as his complete NSF proposal in the early 1980s: "I will continue to study the work of Bill Thurston." Ahlfors was indeed awarded the grant.

And Al Marden, of Minnesota, another great complex analyst, claimed that "everyone saw that Thurston was the best complex analyst in the world", and created the Geometry Center as a playground for Thurston's ideas. Many great things came out of the Geometry Center, in particular the two videos *Not Knot* and *Inside out*, that set a whole new standard for mathematical videos.

The results that so electrified the mathematical community were the following.

1. *Homeomorphisms of surfaces.*
   Every homeomorphism of a compact surface is either of finite order, reducible or pseudo-Anosov.
2. *Hyperbolization of 3-manifolds that fiber over the circle.*
   A compact 3-manifold that fibers over the circle has a hyperbolic structure if and only if its monodromy is pseudo-Anosov.
3. *Hyperbolization of Haken manifolds.*
   A compact Haken 3-manifold admits a hyperbolic structure if and only if it contains no incompressible tori.
4. *Topological characterization of rational functions.*
   A post-critically finite branched map of the 2-sphere to itself is equivalent to a rational function if and only if it admits no Thurston obstructions.

Proving just one of these results requires an entire graduate course; each involves a multitude of totally new ideas and techniques. There are international conferences every year about just aspects of each of them.

Photo courtesy of Thurston 60th Birthday Conference, Princeton University 2007

Further, the two hyperbolization theorems inspired Thurston to make the *geometrization conjecture*: every irreducible compact 3-manifold has a natural geometry, which belongs to one of eight types. This conjecture was solved by Perelman in 2005; the mathematical community is still coming to terms with his proof.

As an interesting sideline, the famous Japanese *couturier* Issey Miyake and his clothes designer Dai Fujiwara designed their 2010 collection after Thurston's "eight geometries". Bill was invited to the presentation of the collection at Fashion Week in Paris; further, he arranged that I was also invited to the show. Bill really got into the swing of things: the set was suggestive of the fundamental domain for a Kleinian group and Bill wandered around it winding giant ropes around Dai and himself, all the while speaking about how this reflected various aspects of the geometry of knots. The press corps were mystified but much entertained, and had a field day.

This was actually far more in keeping with Bill's general outlook on mathematics (and life in general): he kept in his office and at home an extensive collection of toys and games that could be given mathematical meaning; he always thought that mathematics should be a playful activity.

Dynamical systems

Item 4 on the list above is part of a bigger picture. All of Bill's thinking was influenced by dynamical systems, which are somehow the essence of "infinite processes".

All of Bill's major results are concerned with the question: what is the right geometry underlying this problem? He would create the geometry by some infinite process, starting with some inappropriate geometry and applying a transformation coming from his problem over and over, until it converged to the right geometry. So in some sense all his work concerned dynamical systems.

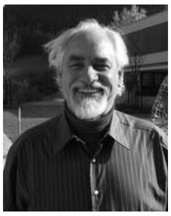
But more specifically, Milnor and Thurston wrote a paper about iteration in one dimension in the 1980s. This paper contains an enormous wealth of ideas involving kneading sequences, zeta-functions and many other things. It has inspired literally hundreds of further papers; it has inspired a whole generation of mathematicians.

Geometric group theory

Around 1980, Bill broadened his focus from groups that arise in specific geometric problems to groups in general. I well remember feeling that this was an unfortunate development: I loved the immediacy of the geometric problems and found the new direction too general and abstract.

Not so, said Bill: "A group is a very geometric object." It took me a long time to see that he was right. The book *Word processing in Groups* was a massive attempt to write in one place the many insights that Bill brought to the subject. It took the combined efforts of Epstein, Paterson, Cannon, Holt, Levy and Thurston.

The entire mathematical community will miss his deep insights. I miss him on a more personal level: Bill was a good friend. He had a wonderfully sunny outlook and he kept his good temper through the incredible hardships of his final illness.



John Hamal Hubbard was born on 6 or 7 October 1945 (the actual date is unknown). He is a professor at Cornell University in Ithaca, New York, and at the Université de Provence in Marseille, France. He is well known for mathematical contributions to the fields of complex dynamics and Teichmüller theory.
Hubbard graduated with a Doctorat d'État from Université de Paris-Sud in 1973 under the direction of Adrien Douady. He has written many influential papers on complex dynamics and he has written several books. In 2006, he completed the first volume of a series devoted to Teichmüller theory and applications to four revolutionary theorems of Bill Thurston. In his free time, he enjoys long bike rides in Ithaca and he likes collecting various mushrooms found across the Cornell campus; he can easily give the binomial nomenclature for each species.
Photo courtesy of Mathematisches Forschungsinstitut Oberwolfach.

Who Are the Invited Speakers at ICM 2014?

Martin Andler (Université de Versailles St Quentin, France)

The purpose of this study is to give an overview of the ICM 2014 speakers. My purpose is not to say something about their mathematics but to answer questions about their gender, geographic origin, where they went to school at the various stages of their lives, etc. This list of 206 excellent mathematicians provides a good sample of our community, and hence of the globalisation of higher education and of the academic job market.

In a way, the data contained in this article are not surprising – we all know, more or less, that the USA is very attractive for scientists from all over the world and we all know that a number of countries have a hard time retaining their best scientists. However, the numbers given here provide, for the first time to my knowledge, a thorough picture of the situation.

I. The Issues

Our main focus is geographic mobility and in particular the imbalances that circulate academics away from their countries. This is a very sensitive issue, which triggers conflicting views. Indeed, there are four different ways to think about it.

1. *Individual freedom*. For many of us, the foremost issue is the individual freedom to travel and to live where we want. We all remember the not so distant time when our colleagues from the Soviet Union could not come to conferences, even the most prestigious ones like ICM, let alone accept jobs in other countries. Further in the past, around World War II, America, mostly the USA, welcomed many mathematicians fleeing Europe where they could no longer work, and where their lives were threatened? But there are other compelling reasons to want to leave one's country, e.g. miserable economic conditions or completely inadequate working conditions. So, indeed, this is an essential aspect.

2. *The progress of science*. For others, the most important issue is mathematics. Following Hilbert's words: "Wir müssen wissen, wir werden wissen"[1], what matters is discovering new theorems, and our collective duty is to allow the best mathematicians to have the best intellectual conditions where they can produce the best mathematics – and the duty of those who are mathematically gifted is to devote their lives to advancing their science. As André Weil wrote in his memoirs, justifying his decision to avoid the military draft at the beginning of World War II:

> *Mon dharma (…) me semblait manifeste: c'était de faire des mathématiques tant que je m'en sentirais capable. Le pêché eût été de m'en laisser détourner.*

3. *Competition*. A very common view about the academic world is that the main factor of progress is the competition between universities to attract the best talent, both faculty and students. If the top universities can attract the best mathematicians and give them excellent working conditions, it will benefit those mathematicians and it will contribute to the progress of mathematics. But also it will encourage other institutions to improve, so that they too can attract top talent.

4. *Brain drain*. But there is a completely different approach to this issue. Indeed, one can argue that the countries whose most gifted natives emigrate to take on jobs in more advanced countries, as engineers, doctors, scientists and academics, are losing a lot; not only is the huge investment in their education wasted but the potential for future progress has evaporated. Pushing the argument further, the richer countries take the easy solution: instead of enforcing the necessary reforms and making adequate investment in their own systems of primary and secondary education, so that they can train their own badly needed scientific cadres, it is much cheaper to import partially or fully trained young people.

In the economic literature, one finds mixed conclusions about brain drain. The obvious loss to the country of origin is mitigated to some extent by the following facts:

- The possibility for young people with high potential to leave their country provides an incentive to acquire high level skills, which may have an overall positive effect.
- In some cases, there is a fairly high rate of return for emigrants who eventually return to their home country.
- Emigrants send back part of their income to their home country; the flow of money is actually higher than direct economic aid to developing countries.

These arguments might not apply well to mathematicians, whose contribution is not limited to the advancement of their science but in significant part to teaching. If the most qualified professors leave a country, the qual-

1 "We must know, we will know" – Hilbert's retirement address at the Vereinigung deutscher Naturwissenschaftler und Mediziner in the autumn of 1930.

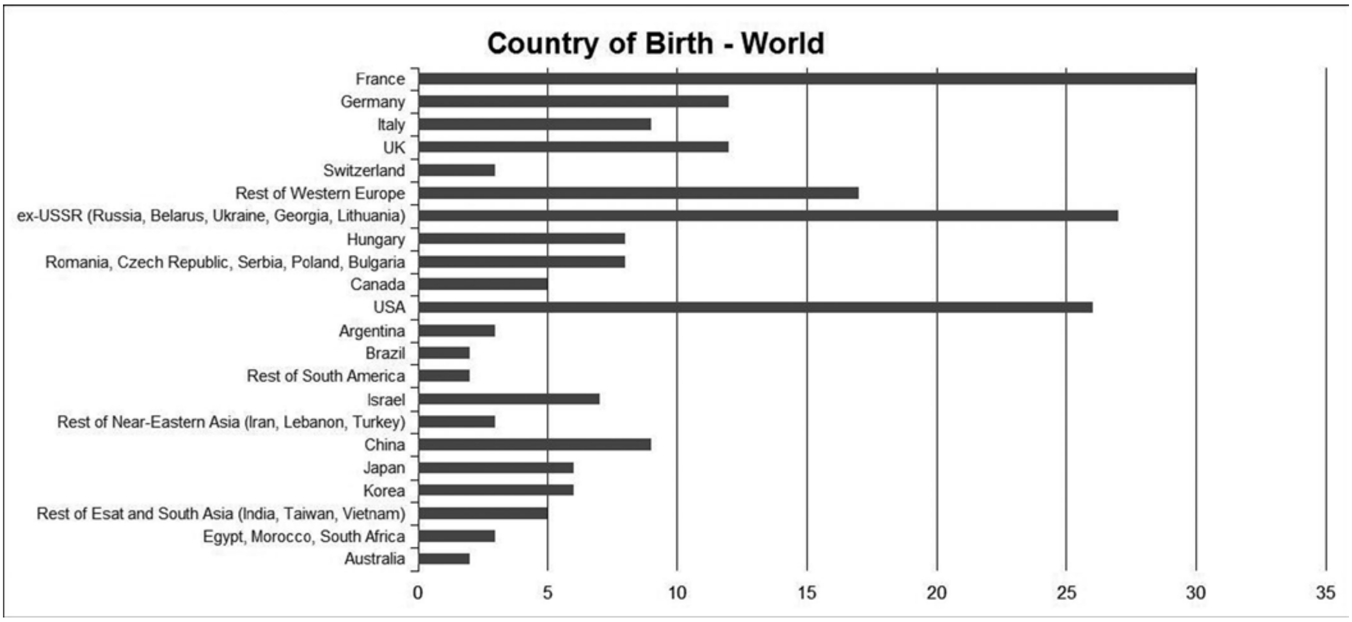


Table 1a: Country of birth – World. Number of ICM speakers per country or region.

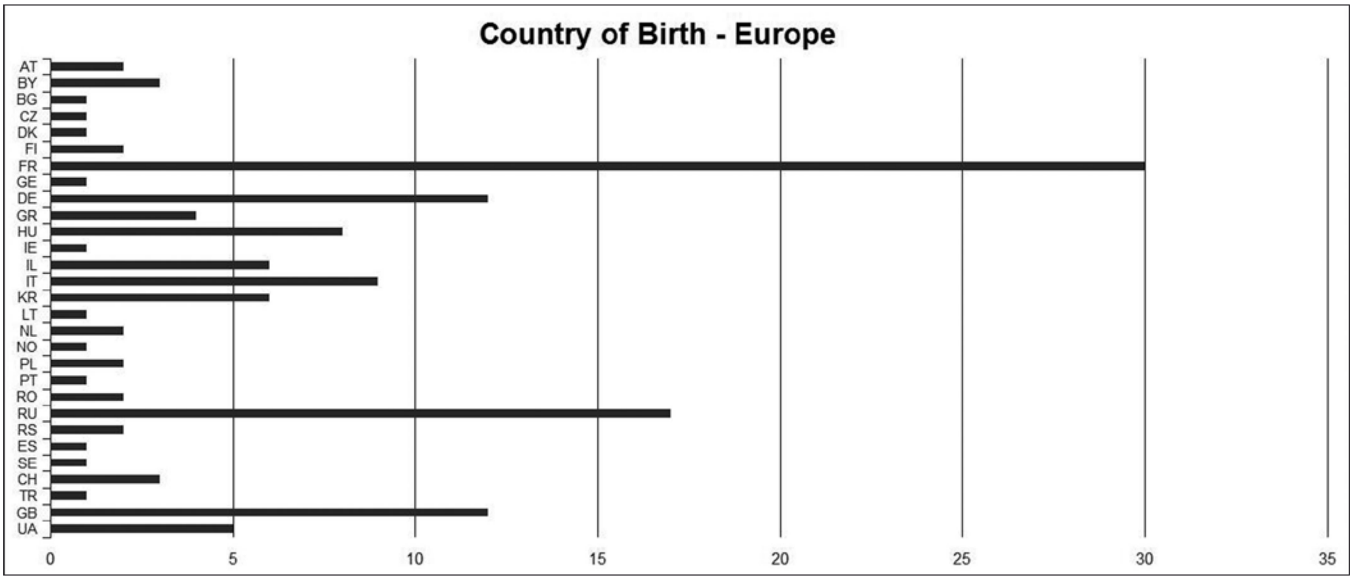


Table 1b: Country of birth – Europe. Number of ICM speakers per country in Europe.

ity of education in universities will decline, or fail to rise; hence the training of mathematics teachers will not be satisfactory; hence…

Giving some numbers may illustrate the economic aspect of the discussion. Following the OCDE publication *Education at a Glance 2013*, Tables B1.3a and B1.3b, the total average expenditure for primary/secondary education in the USA is $142,000 in purchasing power parity, and for undergraduate education it is $81,000. Hiring a foreign trained mathematician represents a saving of $223,000.

As we have briefly discussed, brain drain is indeed an intricate matter. All four approaches to the problem have a strong claim to the truth. I will not argue further – everyone should weigh the different aspects and consider how they might be reconciled. Hopefully these data can help both policymakers and our community to weigh the various arguments for the further development (and spread) of mathematics.

II. The Results

1. The median age of ICM lecturers is 46; the average is slightly higher, at 46.66 ± 1. For plenary speakers, the median is 52 and the average 51.66 ± 1, whereas for sectional speakers the median and the average are both 46. Among the speakers, there are 27 women.

2. The average time difference between PhD year and year of first degree is 5 years (for the 150 people for whom I have reliable data). Because the notion of first degree varies considerably from country to country (see item III.2 below) and because some students do not go

Western Europe	France	32
	Germany	11
	Italy	9
	UK	16
	Switzerland	3
	Rest of Western Europe	14.5
Eastern Europe	USSR (Russia, Belarus, Ukraine, Georgia, Lithuania)	22.5
	Hungary	5
	Romania, Czech Republic, Serbia, Poland, Bulgaria	8
North America	Canada	4
	USA	36
South America	Argentina	3
	Brazil	2
Near East Asia	Israel	9
	Rest of Near-Eastern Asia (Iran, Lebanon, Turkey)	2
East and South Asia	China	9
	Japan	6
	Korea	6
	Rest of East and South Asia (India, Taiwan)	3
Africa	Egypt, Morocco, South Africa	1
Oceania	Australia	2

Table 2: Country of undergraduate studies. Number of ICM speakers per country of undergraduate studies.

directly from undergraduate to PhD studies, this number should be taken with caution, and overestimates slightly the actual duration of PhD studies. However, it does indicate that the path to high level international recognition does not require finishing a PhD very quickly.

3. Most of the details for the geography of ICM speakers are given below. However, one can summarise the main conclusions in a few lines.

a. The countries giving birth to the largest number of ICM speakers are: France (30), USSR (27), USA (26), UK and Germany (12), Italy and China (9), Hungary (8), Israel (7) and Japan and Korea (6). Europe (not including the USSR) has altogether 95.
b. Geographic mobility is marginal between country of birth and country of secondary education, which is expressed, for a mathematically alert audience, by the fact that the transition matrix is almost diagonal. Between birth and undergraduate studies, this is still the case. Apart from mobility inside the USSR, the only significant flow is to the USA and to the UK: 10 persons in our sample left their country to study in the USA at an undergraduate level, the number being 3 for the UK. It would be very interesting to see how the numbers change in the next 20 years, in view of the aggressive recruiting policies of a number of universities in the US and the UK, for instance geared to International Mathematics Olympiad participants.
c. The situation is, expectedly, very different at the graduate level. The main beneficiary of the flow of students is the USA (which attracted 58.5 PhD foreign-born students, representing two thirds of the total of 84.5 ICM speakers who obtained their PhD in the USA); the UK is a distant second, with 4.5 foreign-born out of 16.5 PhD students, and Israel third, with 3 foreign-born out of 7 students. The countries that lose the most at that level are: USSR (with 10 departures out of 27 students), Korea (6 out of 6), Hungary (6 out of 8) and Israel (3 out of 7).
Interestingly, greater Europe, not including the former USSR, loses 23 individuals (who obtained their PhD in the US) out of a total of 95.
d. Finally, where do they work?
 - The USA has the lion's share, with 73 speakers working there, before France (35), the UK (19.5), Canada (10.5), Germany (8), Switzerland (7), Italy (6.5) and Japan and Korea (6).
 - In terms of mobility, the main fact emerging from the tables is that 30 individuals who obtained their PhD in the US leave the US at some point after their PhD to work in another country (often their home country), whereas 20 who obtained their PhD outside of the US work there.

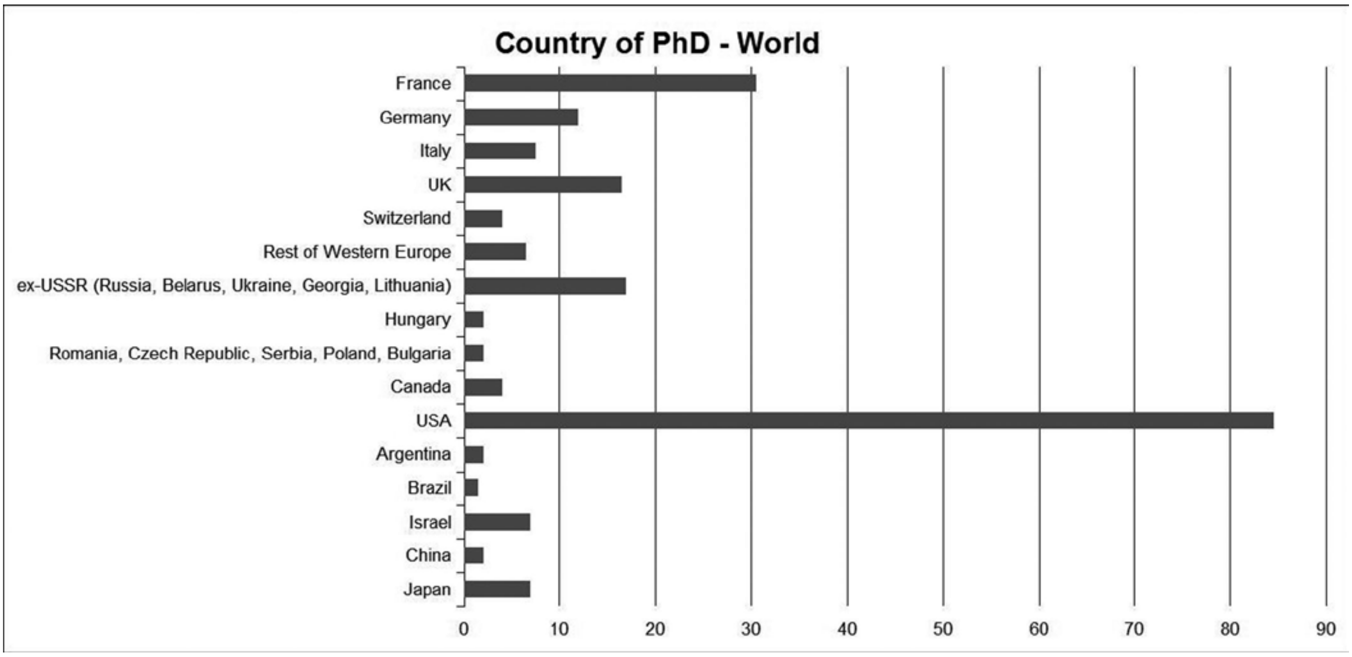
III. Some remarks on the methodology

The data that I gathered and analysed are the following:

- Gender.
- Date of birth.
- Country of birth.
- Country of primary-secondary education (K-12).
- Country and institution of undergraduate education; date of undergraduate degree.
- Country, institution and adviser of PhD training.
- Main positions held before present.
- Present position if different from the one on the official ICM programme.

My aim was to study the national origins of the 206 speakers and to describe their mobility. Obviously, not all of the information is statistically interesting: if there are 1 or 2 speakers born in country C, where they study or work does not teach us much.

I chose to restrict the study to the 206 mathematicians who are either plenary speakers or sectional speakers in one of the mathematics sessions (Sections 1 to 17), at the exclusion of the history, popularisation and education sessions – not that I think that these activities are less worthy (in view of the fact that a substantial part of my present activity revolves precisely around history of mathematics, popularisation of mathematics and mathematics education, it would be somewhat paradoxical).



Country of PhD – World. Number of ICM speakers per country or region of PhD studies.

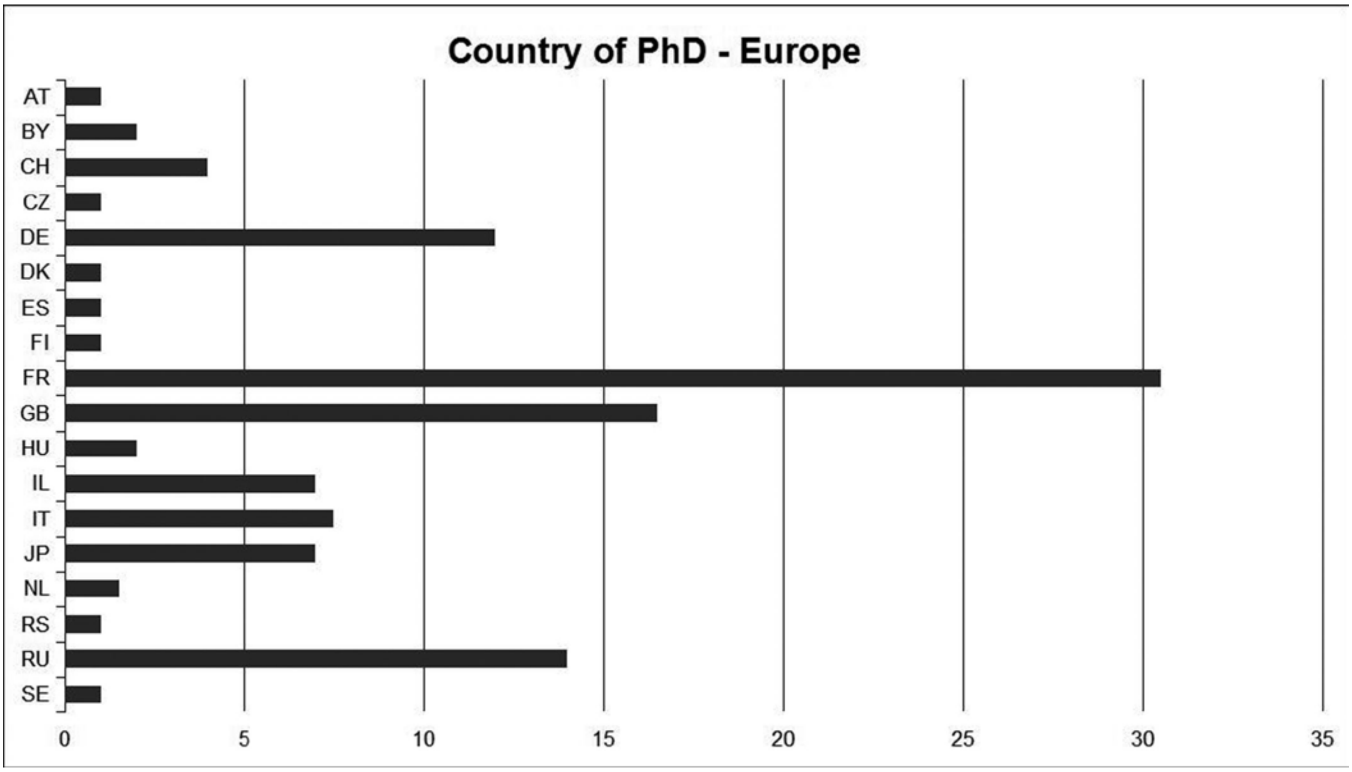


Table 3b: Country of PhD – Europe. Number of ICM speakers per country of PhD studies in Europe.

I started by searching the internet for publicly available information on each of the 206 speakers. I found more than 80 CVs on their webpages, plus other pieces of information. Of course, the mathematics genealogy project (http://genealogy.math.ndsu.nodak.edu/) proved to be extremely useful. I wrote directly to 60 of them for whom my information was lacking, asking them to fill in a short questionnaire, guaranteeing that I would use it only for statistical purposes. I received 37 responses. Further research, and some educated guesses, provided me with reasonably reliable information (again, only for statistical purposes) for all but two speakers.

The information that proved to be the most difficult to gather was birth years. Therefore I do not attempt to compute interesting information like the average age of PhD graduation (see also the remark below). However, the available data does not indicate a strong pattern of extremely quick studies, particularly at the PhD level.

There are significant national differences in the organisation of studies in the present sample. Indeed, the

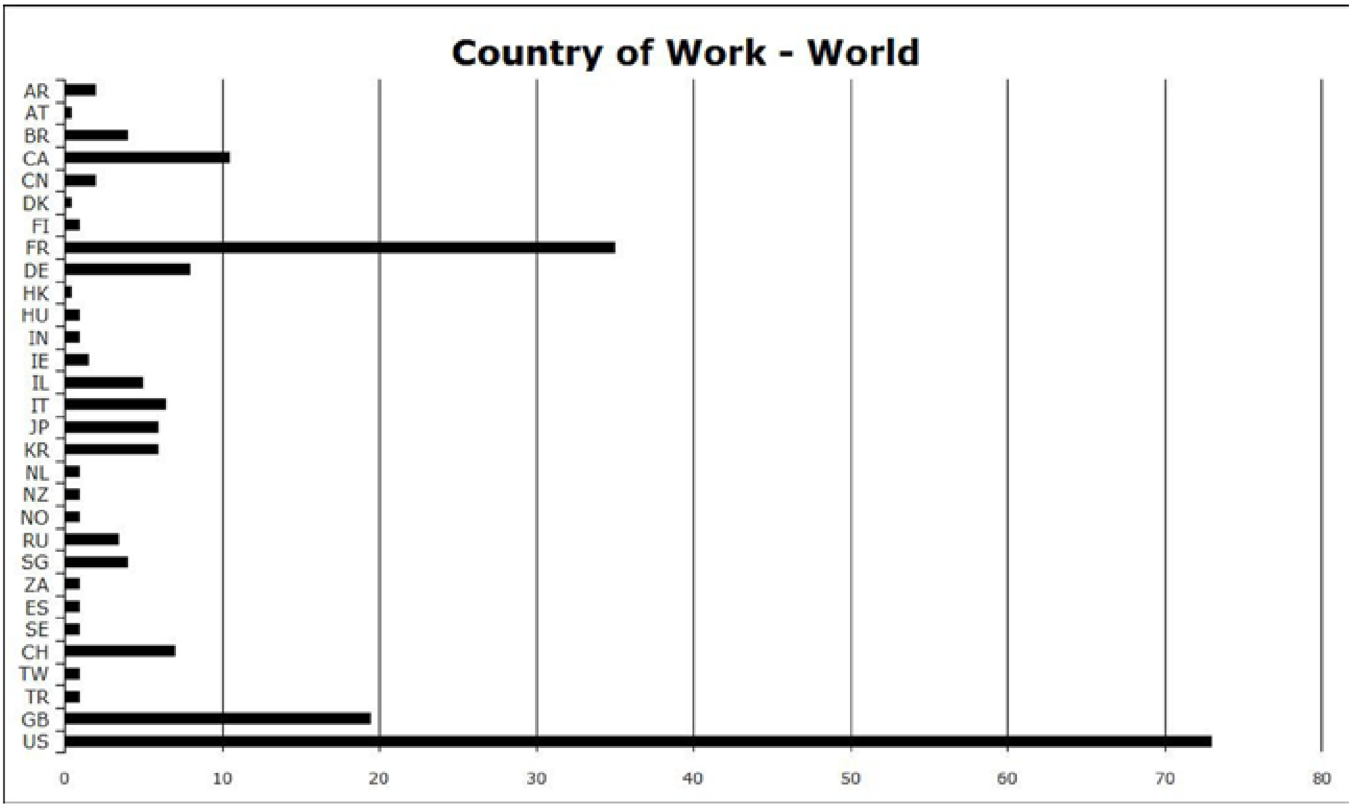


Table 4a: Country of work – World. Number of ICM speakers per country or region of employment.

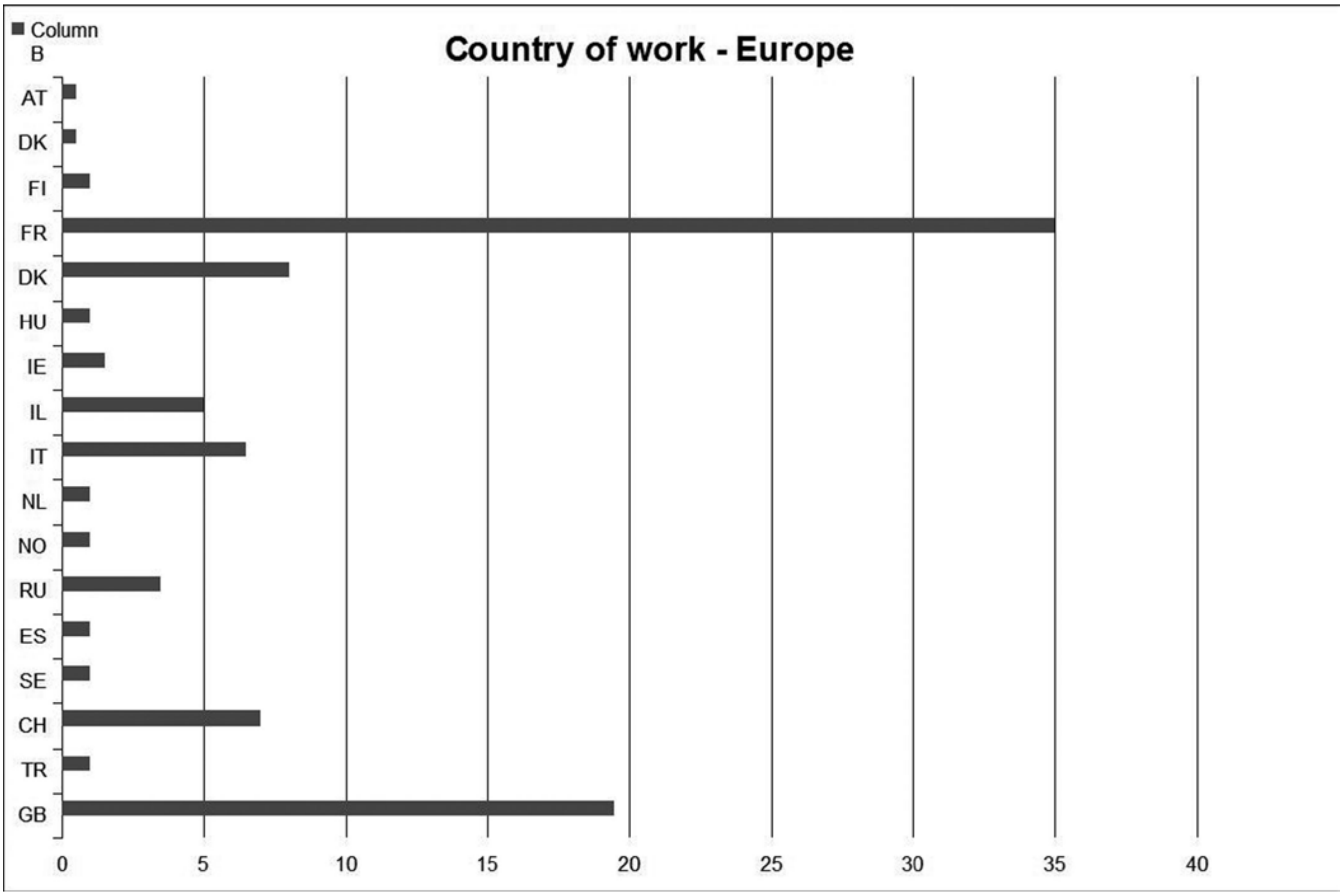


Table 4b: Country of work – Europe. Number of ICM speakers per country of employment in Europe.

4+4 US system (4 years of undergraduate study and theoretically 4 years of PhD) is not present everywhere. Many European countries did not, until recently, offer a degree below the Master's level – corresponding to 5 years of university studies. In several countries, there are systems with two doctorates: PhD and habilitation or, for people roughly over 50 in France, the thèse de 3ème cycle and the thèse d'Etat. In Italy, the PhD system is fairly recent; older colleagues do not hold degrees beyond the *laurea* – a Master's level degree. For those, I have chosen to classify them as holding an Italian PhD.

For most mathematicians in our sample from the countries belonging to the former Soviet Union, the USSR classification seems to be more relevant, since most of them studied before 1989. For this reason, I have regrouped those mathematicians in the 'USSR' category, even though it may seem dubious in terms of the political situation today. For similar reasons, I have grouped together in the tables former Eastern European countries belonging to the Eastern Bloc – of course today most of them belong to the European Union or plan to join it.

As may be expected, there is no significant mobility between birth and country of primary/secondary schooling. Therefore the tables I give omit primary/secondary schooling data altogether.

Several speakers have dual appointments: I have chosen to count them as 0.5 for each country.

It would certainly be very interesting to make a similar study for past and future ICMs.

The tables below may reveal obvious mistakes in cases where the numbers are small. Please feel free to write to me so that I can correct the data in my tables and possibly post corrected tables.

Martin Andler is a professor at the University of Versailles. Before that, he held a CNRS position, first at the Université Paris 7 and then at Ecole Normale Spérieure. He has held visiting positions at MIT, Rutgers University and IAS Princeton. His research concerns representation theory of Lie groups and history of mathematics. He is heavily involved in outreach activities as the President of Animath (www.animath.fr).

	France	Germany	Italy	UK	Switzerland	Rest of Western Europe	USSR	Hungary	Rest of Eastern Europe	Canada	USA	Argentina	Brazil	Rest of South America	Israel	Rest of Near Eastern Asia	China	Japan	Korea	Singapure	Rest of East and South Asia	Oceania
France	27				1						2											
Germany		10							1		1											
Italy	0.5		7.5								1											
UK	1			11																		
Switzerland					3																	
Rest Western Europe				1.5		7.5					8											
USSR				1			17				6				3							
Hungary		1		1				2			4											
Rest Eastern Europe									1	2	5											
Canada											5											
USA											26											
Argentina											1	2										
Brazil											0.5		1.5									
Rest South America											2											
Israel										1	2				4							
Rest Near Eastern Asia	1									1	1											
China				1							5						2	1				
Japan																		6				
Korea											6											
Singapure																						
Rest East and South Asia											5											
Africa	1			1							1											
Oceania											2											

Table 5: Birth to PhD. The number on line i and column j is the number of speakers born in country i, who obtained their PhD in country j.

	France	Germany	Italy	UK	Switzerland	Rest of Western Europe	USSR	Hungary	Rest of Eastern Europe	Canada	USA	Argentina	Brazil	Rest of South America	Israel	Rest of Near Eastern Asia	China	Japan	Korea	Singapure	Rest of East and South Asia	Africa	Oceania
France	27.5					1				0.5	1												
Germany		6		3	2				1	1													
Italy	1		6.5	0.5						1													
UK	1			8							3												
Switzerland	0.5			1	1	0.5																	
Rest Western Europe				3.5	1	4.5				0.5	6												1
USSR	2			1	1	1.5	3.5			1	13		2		2								
Hungary		2			1			1		1	3												
Rest Eastern Europe										2	6												
Canada										2	3												
USA	1			0.5						2	22.5												
Argentina											1	2											
Brazil													2										
Rest South America	1										1												
Israel											4				3								
Rest Near Eastern Asia					1						1					1							
China										0.5	4						2			2	0.5		
Japan																		6					
Korea																			6				
Rest East and South Asia											2									1	2		
Africa											1											1	
Oceania				1							1												

Table 6: Birth to work. The number on line i and column j is the number of speakers born in country i, who work in country j.

	France	Germany	Italy	UK	Switzerland	Rest of Western Europe	USSR	Hungary	Canada	USA	Argentina	Brazil	Israel	Rest of Near Eastern Asia	China	Japan	Korea	Singapure	Rest of East and South Asia	Africa	Oceania
France	26.5				1	1			0.5	1.5											
Germany		7		3	2																
Italy	1		5.5	0.5						0.5											
UK				10.5						4					1					1	
Switzerland	1.5			1	1	0.5															
Rest Western Europe						3.5			0.5	1.5											1
USSR	2				1	1.5	2.5		1	7		2									
Hungary					1			1													
Rest Eastern Europe					1					2											
Canada									2	1				1							
USA	4	1	1		1	1	1		6.5	53		0.5	2				6	2	2.5		
Argentina											2										
Brazil												1.5									
Israel										4			3								
China															1			1			
Japan																6					

Table 7: PhD to work. The number on line i and column j is the number of speakers who obtained their PhD in country i, who work in country j.

Interview with Alois Kufner on the Occasion of his 80th Birthday

Luboš Pick (Charles University in Prague, Czech Republic)

Alois Kufner (born 1 February 1934 in Pilsen, Czech Republic) is a professor of mathematics at the Mathematical Institute of the Academy of Sciences of the Czech Republic in Prague. He was the director of this institute from 1979 to 1990. He is the author of many books. His research interests include mainly the theory of function spaces and its applications. For many years he worked in the central bodies of the Union of Czech Mathematicians and Physicists. He has spent a lot of effort working for the benefit of talented young mathematicians.

Do you still remember when and why you decided to become a mathematician?
That happened against my own will. I wanted to become a high-school teacher of mathematics and physics. Hence I applied as an undergraduate to the Faculty of Mathematics and Physics at Charles University in Prague. But exactly in that very year, the study programme for teachers was carried over to a different faculty, specifically the Pedagogical Faculty of the same university. In order to follow my plan, I would have to change my application. But my teacher at the secondary school talked me out of it and I ended up studying mathematics.

The supervisor of your PhD thesis was Jindřich Nečas. Was collaboration with him exciting? Who else had influence on your choice of research area?
Let me first slightly correct you. My diploma thesis was on singular integral equations and my supervisor was Jan Potoček. He had two diploma students and only one PhD position. He offered that Ivo Babuška could take one of them over to his department of PDEs in the academy, and Babuška chose me. This resulted in my very first job, and I did the doctorate simultaneously with it. Babuška's department was quite impressive and the work was interesting. At that time, Sobolev spaces and functional-analytic solutions of PDEs were the cutting edge of research in analysis, and I happily joined in. Nečas was a member of that department and worked with me in an informal way. He pointed me towards applications of weighted spaces, which has been my principal research topic ever since. I also collaborated with several students from Charles University, where Nečas had a part-time job. In this way, I encountered nonlinear equations. Collaboration with Nečas definitely had significant impact on me. Aside from him and Potoček, I consider also Sergey Mikhailovich Nikolskii and Sergey L'vovich Sobolev my equally important (though rather remote) teachers.

You authored an impressive array of books and monographs. Do you still remember the first one of them? Which of your books is your favourite one?
I prefer books with an impact on the general public rather than monographs specialised in a narrow field. That's why I wrote three little books for the "School of Young Mathematicians". My first book was a compendium on Fourier series, written jointly with Jan Kadlec. Oddly enough, it is one of my most cited ones. It is difficult to say which of my books I like most: perhaps "Geometry of Hilbert Space" (in Czech), in which my main goal was to explain to a wide range of readers (including students and applied people) the basics of functional analysis.

You founded the "Prague school of function spaces", now quite well known in the world. How did it all start?
I first studied Sobolev spaces in connection with PDEs. When Nečas moved to the USA in 1968, we began, together with his students Oldřich John and Svatopluk Fučík, studying function spaces systematically. We established an informal seminar. Later it moved from the university to the academy and that was exactly the moment when the Prague school began. Its first generation was formed by our diploma students.

You rebuilt a once purely service Department of Mathematics at the University of Pilsen into one of the best mathematics departments in the country. How did you manage it?
I did not really build it – that is exaggerated. Several quite able people had been there from the very beginning. I got there by accident. In 1972, during my military service in Pilsen, I visited Jiří Holenda, a schoolmate of mine from university. He persuaded me to invest time and effort into reorganising their department, which then seemed rather incoherent. I organised seminars on PDEs there and I tried to attract some dedicated young people. Then the process started to run on its own. The expertise and organising abilities of other people such as Stanislav Míka, Pavel Drábek and Jiří Holenda significantly contributed to the rise and glory of the Pilsen department.

During your directorship of the Mathematical Institute of the Czechoslovak Academy of Sciences, in the middle of a politically difficult period, you kept some people

employed who would otherwise have had problems finding a job. How did you manage it?
First of all, I never had the ambition of being a director and I got into this position by sheer coincidence of circumstances. I took it as a service to my "maternal" institution. The times were not easy indeed. My principal goal was to keep a quiet atmosphere in the institute, rather calm down than provoke conflicts. Somehow, it worked.

You spent a lot of effort and energy for the benefit of the Union of Czech Mathematicians and Physicists and also of the EMS. What are your impressions about the origins and the current state of the EMS?
I was a deputy of the union at the first informal meeting dedicated to establishing a European organisation of mathematicians, held in 1978 in Helsinki. Hospitality and engagement of Finnish mathematicians were very important for the beginning of the EMS. The EMS also probably wouldn't exist without the effort of Sir Michael Atiyah and later of Professor Friedrich Hirzebruch. The Czech union contributed quite significantly, too. I was intensively working in the Extended Preliminary Committee of the EMS till 1992. The current publication activity of the EMS is formidable. My overall impression is that the original goal of establishing an analogue of the AMS has not been met yet.

You had a decisive impact on the fruitful long-term collaboration of Czech mathematicians with experts from abroad. Did your personal friendship with foreign mathematicians play a role here?
Personal contacts have always been very important. Another significant parameter is the location. For example, the Friedrich Schiller University in Jena was easy to travel to, and moreover it is not far from Prague. Collaboration with the Soviet Union was also possible through various mutual agreements. Contacts with the Royal Society in the UK were more difficult to maintain, but possible. After 1990, everything became easy. I always made some original contacts and my younger colleagues could then take advantage of them. An important factor was the organisation of conferences in our country since new contacts were built there.

The first Spring School on Nonlinear Analysis, Function Spaces and Applications (NAFSA) was organised in 1978; the 10th one is just coming. How was the organisation in times of no email, internet or cell phones?
NAFSA schools were an offspring of the Pilsen seminars. Conditions for the organising work were of course very different to the current ones. But we were able to manage it even with quite modest means such as the usual mail. And hard conditions had a good impact on the integrity of our team.

Your wife is a quite well-known translator from German and other languages. What is family life like for two professionals so deeply engaged in their work?
We have been together for over 55 years. I respect her translating job very much and I have always tried to give her maximum support. I believe that she also has understanding for my mathematics. I appreciate that she accompanies me during my travels if circumstances allow. She is however not always delighted by me leaving her on her own during conference talks.

What do you like most and what do you like least about being a mathematician?
I like the life in general and the life of a mathematician in particular. What I do not like is the sometimes negative attitude of the public towards mathematics. Too often we hear celebrities emphasising that they never liked mathematics, that they have never been capable of understanding it. Mathematics deserves better!

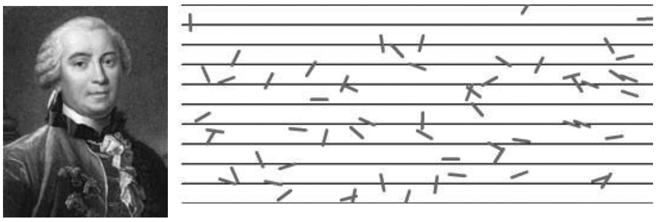
Luboš Pick is a professor of mathematical analysis at Charles University in Prague, Faculty of Mathematics and Physics. He studied at Charles University in Prague and obtained a PhD at the Mathematical Institute of the Czechoslovak Academy of Sciences in 1990. In the period 1991–1994 he was employed by the University of Wales in Cardiff, UK. He has written one monograph and over 60 research papers in the theory of function spaces. In his free time he translates books from English to Czech and plays bass guitar in several rock and folk bands.

Buffon: Did He Actually Throw Sticks?

Ehrhard Behrends (Freie Universität Berlin, Germany) and Jorge Buescu (Universidade de Lisboa, Portugal)

George-Louis Leclerc, Comte de Buffon (1707–1788) is famous for the following "experiment":

> Find a room with a plank floor, and denote the width of the planks by a. You will need a stick or some similarly shaped object, whose length $2r$ must be less than a. This condition guarantees that the stick will cross the edge of at most one plank when it is tossed on the floor.



Buffon and the "needle experiment."

> The probability P that such an event occurs (that is, that the stick does not end up entirely on a single plank) is then $4r/(\pi a)$. This formula contains the mathematical constant π, and we have, therefore, the opportunity of computing this number "experimentally." It will, to be sure, be necessary to throw the stick "very often," a number of times that we shall denote by n. If the stick lies across two boards k times out of the n throws, then according to the law of large numbers, k/n should be a good approximation of P, and by solving the equation $P=4r/(\pi a)$ for π, we will have found a good approximation of π.

(Of course, instead of planks and sticks, one could simply use paper, along with needles or matches; the only requirement is that the distance between the lines on the paper be large enough.)

It is generally accepted that Buffon's experiment represents the first Monte Carlo experiment in the history of mathematics, namely an experiment in which a random method is used to solve a problem at least approximately. Such methods can be used, for example, to compute definite integrals over multidimensional domains of definition and solve numerous counting problems.

Buffon's family became wealthy through an inheritance, and so in his youth, Buffon became financially independent. Like many of his contemporaries, he became fascinated by the rapidly developing natural sciences. Indeed, he was interested in everything. His studies led to a planned fifty-volume encyclopaedia *Histoire naturelle, générale et particulière*, thirty-six volumes of which were published. From 1739 on, he served as the administrator of the royal garden in Paris ("Jardin Royal," today "Jardin des Plantes"). An echo of his service can be seen today in the fact that in the fifth arrondissement, at the southern border of the Jardin des Plantes, near the Jussieu Campus, there is to be found a Rue Buffon.

La Rue Buffon.

There were plans for this street to play an important role in November 2013 as part of a campaign for the popularization of mathematics. The rpa committee (rpa = "raising public awareness") of the European Mathematical Society (EMS) met that month in Paris, and someone came up with the idea of "reconstructing" the Buffon experiment in the Rue Buffon. The conditions could not have been more propitious, for the relevant mathematics can be easily understood by a lay audience (while at the same time being not at all trivial), and with a bit of luck and professional preparation, one could hope for good coverage in the media and to score a triumph for the popularization of mathematics.

But the project was cancelled. When some committee members got in touch with the Parisian historian of mathematics Bernard Bru to learn more about the historical details, they learned that there exists no historical evidence that Buffon saw any relationship between his theoretical deliberations and a calculation to approximate the value of π, nor whether his famous "experiment" was in fact ever carried out. We have here, then, an interesting example of the fact that historical truth and "generally accepted knowledge" need have little relationship to each other. Let us now look more closely at this discrepancy between fact and supposition.

We begin with facts that are not in dispute.

1. In 1733, Buffon submitted an article to the Royal Academy of Sciences (of which he became a member in 1734) in which he correctly calculated the probability that a randomly tossed stick of length $2r$ would intersect one of several parallel lines an equal distance a apart (provided that $2r<a$):

> *Sur un plancher qui n'est formé que de planches égales & parallèles, on jette une Baguette d'une certaine longueur, & qu'on suppose sans largeur. Quand tombera-t-elle sur une seule planche?*[1] (See [2])

[1] Freely translated: "One tosses a stick of a certain length and of negligible thickness onto a floor consisting of parallel boards of equal width. When will it fall on only a single board?

He points out that one can use his formula to determine the value of *a* for which the probability of landing on a single board is fifty percent: "Il y a donc une certaine largeur de la planche qui rendroit le pari ou le jeu égal."[2]

2. This research was published – in a more extensive version – in 1777, in his *Histoire naturelle*. It will now become even clearer that the principal motivation for his investigations was in calculating odds for gamblers:

> *Je suppose que, dans une chambre dont le parquet est simplement divisé par des points parallèles, on jette en l'air une baguette, et que l'un des joueurs parie que la baguette ne croisera aucune des parallèles du parquet, et que l'autre au contraire parie que la baguette croisera quelques-unes des ces parallèles; on demande le sort de ces deux joueurs. (On peut jouer ce jeu sur un damier avec une aiguille à coudre ou une épingle sans tête.)*[3]

3. Buffon later carried out experiments related to the St. Petersburg paradox. This involves a game in which a fair coin is tossed repeatedly until it first lands showing heads. If this occurs at the *k*th toss, the player wins 2^k ducats. Since the expected value of the player's winnings is infinite, it makes sense that the cost to play the game should also be infinite.

Buffon describes the game in [3], beginning on page 394, and on page 399, one learns that he has made some relevant experiments:

> *J'ai donc fait deux mille quarante-huit expériences sur cette question, c'est-à-dire j'ai joué deux mille quarante-huit fois ce jeu, en faisant jeter la pièce par un enfant.*[4]

4. Laplace took up Buffon's needle problem and stated explicitly that one could use the theoretical calculations to determine an experimental approximation of π. After determining the probability that a line would be crossed, he writes:

> *Si l'on projette un grand nombre de fois ce cylindre, le rapport du nombre de fois où le cylindre rencontrera l'une des divisions du plan au nombre total des projections sera, à très peu près, la valeur de* $4r/(a\pi)$*, ce qui fera connaître la valeur de la circonférence* 2π.[5] [9]

5. Buffon's needle problem seems to have sparked interest in actual experiments from the mid-19th century onwards. Apparently the first documented one was performed in 1850 by Rudolf Wolf [13], then a professor at the University of Bern. Augustus de Morgan refers in 1859 ([11], pp. 283–4) that a certain Ambrose Smith performed the experiment in 1855 with 3204 trials and a student of his with 600 trials.

How reasonable are such experiments? The theory says that only results of the following type are to be expected. If one tosses the stick n times and hits a line k times, using k/n in calculating an approximation to π, then with some probability p, the result that one obtains is within some value ε of π. Here one may choose a value of p (close to 1) and a value of ε (small) and one can then determine a suitable value of n, for example using Chebyshev's inequality. Unfortunately, for moderately large p and not very small ε, the required n is astronomically large and convergence exceedingly slow. Buffon's method is therefore not well suited to obtaining information about the digits of the number π.

It is worth noting in this connection the experiments reported in 1901 of one Lazzerini, who maintained that after throwing 3408 sticks, he had obtained a value of π accurate to six decimal places ([10], p. 120). Papers by Gridgeman [5] and Badger [1] refute this claim as extremely unlikely and probably due to data manipulation.

In notable contrast to the historically verified evidence, there remains the impression that Buffon explicitly had in mind a determination of an approximate value of π by means of an "experiment" and that he in fact carried it out:

1. The Buffon problem is treated regularly in textbooks on stochastic theory. In books on this subject in both English and German, we have found not a single one in which the least doubt is cast on the statement that "Buffon wished to calculate an approximation to π with his experiment". (This is true, alas, of the textbook *Elementare Stochastik* of the first author.)

2. These textbook authors find themselves in good company, for even in books on the history of the theory of probability, it is maintained, without citing any sources, that Buffon performed such experiments. Here are two examples:

- "It was originally performed with a needle" ([7], p. 75).

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[2] Freely translated: "There is, therefore, a certain width of board for which the wager – that is, the game – is fair."

[3] Freely translated: "I assume that within a room in which the parquet is simply divided by parallel points, one tosses a stick into the air, and one player wagers that the the stick will not cross any of the parallels in the parquet, while the other player wagers that it will cross one of the parallels. One asks for the odds for each player. (One could also play this game on a checkerboard with knitting needles or headless pins.)" (See [3], p. 411ff.)

[4] Freely translated: I have carried out 2048 experiments with respect to this question, that is, I have played this game 2048 times, making use of a child to toss the coin."

[5] Freely translated: If one tosses this cylinder with great frequency, then the quotient of the total number of throws for which the stick lands on one of the divisions of the plane and the total number of throws will have approximately the value $4r=(a\pi)$ which will suffice to determine the value of the circumference 2π."

- Many investigators (including Buffon) used this result for the experimental determination of π" ([10], p. 120).

(Other history books at least leave open the connection of the approximation of π.) According to Bernard Bru, this situation is the result of a misunderstanding: the experiments on the St. Petersburg paradox were at some time or other extrapolated to encompass experiments on π and then the "facts" were simply repeated without verification from original sources.

3. The internet is not a big help. At the website Mac Tutor History of Mathematics (http://turnbull.mcs.st-and.ac.uk/history/), which we visit frequently and value highly, the following is said about Buffon: "His most notable contribution to mathematics was a probability experiment which he carried out calculating π by throwing sticks over his shoulder onto a tiled floor and counting the number of times the sticks fell across the lines between the tiles." (Many years ago, there was even more nonsense served up. Instead of "sticks" being tossed, it was "white loaves of bread". One does not have to look far to see how that error arose. The word "baguettes" in Buffon's original text (see above) was mistranslated as the familiar stick-shaped loaf of French bread (*la baguette de pain*). But "la baguette" has a number of meanings in French, including simply "stick", which makes considerably more sense in this context. At the time, the first author notified the manager of the website to check the translation and soon thereafter the offending word was changed.

It is also worth noting that Buffon's "experiment" lends itself to a number of interesting generalisations. Here are a few examples:

- What happens if the stick is longer than the distance between the planks? (This question was answered by Laplace. For more recent treatments, see [4] by P. Diaconis.)
- Can one replace sticks with some two-dimensional surface, such as a coaster? (One can, of course, obtain formulas for the probability but it depends on the shape of the surface whether the number π will appear in the formula. Thus, for example, square coasters are suitable for calculating approximations to π, while circular ones are not.)
- How does the situation change if one replaces the stick with a curved segment in the plane? (See in this regard "Buffon's Noodles" [12].)

Buffon's needle problem is the first in the then unknown territory of geometric probability, and opened up a whole new area of mathematical thinking. Klain and Rota state that it is "(…) the theorem leading into the heart of Geometric Probability" ([8], p. 3).

For this reason alone Buffon would rightfully deserve a place in the history of mathematics. Regarding the questions raised here, on the other hand, it is unlikely that new documents will surface that will provide conclusive information. Therefore, we recommend to all authors of future books on stochastic or probability theory not to involve Monsieur Buffon in any throwing of sticks, needles, loaves of bread or similar articles.

In closing, we would like to thank Bernard Bru (Paris) and Eberhard Knobloch (Berlin) for their help in elucidating the problems described in this work.

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The present paper is an extended version of an article that was originally published in German by the first author in the proceedings of the German Mathematical Society (March 2014; this version was translated to English by David Kramer).

Ehrhard Behrends was a professor for mathematics at Freie Universität Berlin until March 2014. For many years he was the secretary of the German Mathematical Society; at present he is the Chair of the rpa committee of the EMS. He is responsible for the development of www.mathematics-in-europe.eu, a popular mathematical webpage under the auspices of the EMS.

Jorge Buescu is a professor of mathematics at the University of Lisbon. Besides his mathematical activity, he is highly engaged in the popularisation of mathematics, being the author of several books. He is a member of the EMS rpa committee and an editor of the EMS Newsletter.

ICMI Column

Mariolina Bartolini Bussi (Università di Modena e Reggio Emilia, Italy) and Jean-Luc Dorier (Université de Genève, Switzerland)

The 23rd ICMI Study: Primary Mathematics Study on Whole Numbers
Co-Chairs: Mariolina Bartolini Bussi & Sun Xuhua

A new study will be conducted by the International Commission on Mathematical Instruction. This study, the 23rd led by the ICMI, addresses for the first time mathematics teaching and learning in primary school (and pre-school), taking into account inclusive international perspectives, socio-cultural diversity and institutional constraints. The broad area of *Whole Number Arithmetic* (*WNA*), including operations and relations and arithmetic word problems, forms the core content of all primary mathematics curricula. The study of this core content area is often regarded as foundational for later mathematics learning. However, the principles and main goals of instruction in the foundational concepts and skills in WNA are far from universally agreed upon, and practice varies substantially from country to country. An ICMI study that provides a meta-level analysis and synthesis of what is known about WNA would provide a useful base from which to gauge gaps and silences and an opportunity to learn from the practice of different countries and contexts.

Whole numbers are part of everyday language in most cultures but there are different views on the most appropriate age at which to introduce whole numbers in the school context. Whole numbers, in some countries, are introduced in pre-school, where the majority of children attend before the age of 6. In some countries, primary schooling comprises Grades 1–6; in others it comprises Grades 1–5. Thus the entrance age of students for primary school may vary from country to country. For these reasons, this study addresses teaching and learning WNA from the early grades, i.e. the periods in which WNA is systematically approached in formal schooling – in some contexts this includes pre-school.

Primary schooling is compulsory in most countries (in all Western countries), although there is considerable variation in the facilities, resources and opportunities for students. This is the uneven context where mathematics teaching and learning takes place. Mathematics is a central feature of early education and the content, quality and delivery of the curriculum is of critical importance in view of the kinds of citizens each country seeks to produce.

In the international literature, there are many contributions about primary school mathematics. In many cases, especially in the West, early processes of mathematical thinking, often observed in early childhood (i.e. 3–8 year-old children), are also investigated by cognitive and developmental psychologists. They sometimes study the emergence of these processes in clinical settings, where children are stimulated by suitable models so as to observe the emergence of aspects such as one-to-one correspondences, counting, measuring and other processes. In several countries, Piaget's theory has been very influential despite criticism. Neuroscientists have also been studying for some years the emergence of "number sense". However, recent perspectives highlight that what is still needed is serious and deep interdisciplinary work with experts in mathematics education.

The ICMI has acknowledged that it is timely to launch, for the first time in its history, an international study that specifically focuses on early mathematics education that is both basic and fundamental. When foundational processes are concerned, a strong epistemological basis is needed. This is where the involvement of the ICMI adds value with respect to analyses carried out in other fields. Such epistemological analysis was part of the classical works of professional mathematicians (e.g. Klein, Smith and Freudenthal) who played a major role in the history of the ICMI and considered mathematics teaching as a whole.

The ICMI study will be organised around five themes that provide complementary perspectives on approaches to early WNA in mathematics teaching and learning. Contributions to the separate themes will be distinguished by the theme's specific foci and questions, although it is expected that interconnections between themes will emerge and warrant attention.

The five themes are:

1. The why and what of WNA.
2. Whole number thinking, learning and development.
3. Aspects that affect whole number learning.
4. How to teach and assess WNA.
5. Whole numbers and connections with other parts of mathematics.

Themes 1 and 2 address foundational aspects from the cultural-historic-epistemological perspective and from the (neuro)cognitive perspective. What is especially needed are reports about the impact of foundational aspects on practices (both at the micro-level of students and classrooms and at the macro-level of curricular choices).

Themes 3 and 4 address learning and teaching, respectively, although it is quite difficult, sometimes, to separate the two aspects; for example, in some languages and cultures (e.g. Chinese, Japanese and Russian) the two words collapse into only one.

Theme 5 addresses the usefulness (or the need) to consider WNA in connection with (or as the basis for) the transition to other kinds of numbers (e.g. rational numbers) or with other areas of mathematics traditionally separated from arithmetic (e.g. algebra, geometry and modelling).

ICMI Study 23 is designed to enable teachers, teacher educators, researchers and policymakers around the world to share research, practices, projects and analyses. Although reports will form part of the programme, substantial time will also be allocated for collective work on significant problems in the field, which will eventually form part of the study volume. As in every ICMI study, ICMI Study 23 is built around an international conference and directed towards the preparation of a published volume. The study conference will take place in Macau, China, and will be hosted by the University of Macau (3–7 June 2015).

As is usual practice for ICMI studies, participation in the study conference will be by invitation only for the authors of submitted contributions that are accepted. Contributions have to be submitted before 15 September 2014; they will be reviewed and a selection will be made according to the quality of the work and the potential to contribute to the advancement of the study, with explicit links to the themes contained in the discussion document and the need to ensure diversity among the perspectives. The number of invited participants will be limited to approximately 100 people.

The *first product* of ICMI Study 23 is an electronic volume of the proceedings, to be made available first on the conference website and later on the ICMI website. It will contain all the accepted papers as reviewed papers in the conference proceedings (with an ISBN number).

The *second product* is a gallery of commented video-clips about practices in WNA, to be hosted on the conference website and later, possibly, on the ICMI website.

The *third product* is the ICMI study volume. The volume will be informed by the papers, the video-clips and the discussions at the study conference as well as its outcomes.

The International Programme Committee for ICMI Study 23 invites submissions of contributions of several kinds: theoretical or cultural-historic-epistemological-essays (with deep connections to classroom practice, curricula or teacher education programmes); position papers discussing policy and practice issues; discussion papers related to curriculum issues; reports on empirical studies; and video-clips on explicit classroom or teacher education practice. To ensure rich and varied discussion, participation from countries with different economic capacity or with different cultural heritage and practices is encouraged.

The ICMI Study 23 website is open at the address:
http://www.umac.mo/fed/ICMI23/.

The website contains a longer version of this discussion document and the detailed information about deadlines for submission; it will be regularly updated and used for sharing the contributions of those invited to the conference in the form of conference pre-proceedings. Further information may be requested at the following address:
icmiStudy23@gmail.com.

The members of the International Programme Committee are: Maria G. Bartolini Bussi (University of Modena and Reggio Emilia, Italy), Xuhua Sun (University of Macau, China), Berinderjeet Kaur (National Institute of Education, Singapore), Hamsa Venkatakrishnan (University of the Witwatersrand, Johannesburg, South Africa), Joanne Mulligan (Macquarie University, Sydney, Australia), Jarmila Novotna (Charles University, Prague, Czech Republic), Lieven Verschaffel (KU Leuven University, Belgium), Maitree Inpasitha (Khon Kaen University, Thailand), Sarah Gonzalez de Lora (PUC Madre y Maestra, Dominican Republic), Sybilla Beckmann (University of Georgia, Athens, GA USA), Roger E. Howe, ICMI Liaison (Yale University, New Haven, CT, USA), Abraham Arcavi, ex-officio, ICMI Secretary General (The Weizman Institute of Science, Rehovot, Israel) and Ferdinando Arzarello, ex-officio, President of the ICMI (University of Turin, Italy).

The ICMI Emma Castelnuovo Award for Excellence in the Practice of Mathematics Education

The International Commission on Mathematical Instruction (ICMI) is committed to the "development of mathematical education at all levels" and its aims are "to promote the reflection, collaboration, exchange and dissemination of ideas on the teaching and learning of mathematics, from primary to university level. The work of the ICMI stimulates the growth, synthesis and dissemination of new knowledge (research) and of resources for instruction (curricular materials, pedagogical methods, uses of technology, etc.)".

The ICMI has decided in the past to create two awards to recognise outstanding achievements in mathematics education research: the Felix Klein Award, honouring a lifetime achievement, and the Hans Freudenthal Award, recognising a major cumulative programme of research.

In order to reflect a main aspect of the ICMI (as stated above) not yet recognised in the form of an award, the ICMI has decided to create a third award to recognise outstanding achievements in the practice of mathematics education. This award will be named after Emma Castelnuovo, an Italian mathematics educator born in 1913, to celebrate her 100th birthday and honour her pioneering work. Further details are available at http://www.mathunion.org/icmi/activities/awards/emma-castelnuovo-award/.

While preparing this column, we received the very sad news that, on 13 April, Emma Castelnuovo passed away in her sleep. In the discussion lists of Italian mathematics teachers, dozens of condolence and memory messages have been posted. An interview with Emma can be downloaded from http://www.icmihistory.unito.it/clips.php.

EMF2015, Tipaza (Alger), Algeria, 10–15 October 2015

The scientific meetings of the *Espace Mathématique Francophone* have been organised every third year since 2000 and are acknowledged as a regional conference by the ICMI. The next one will take place in Tipaza (Algeria) in 2015 (10–15 October). The theme will be: '*Cultural plurality and universality of mathematics: challenges and prospects for their teaching and learning*'. A very short excerpt from the general presentation follows.

One of the strengths of mathematics lies in the universal nature of the results. However, before reaching the status of universality, each mathematical concept first emerges in a particular cultural context and is enriched by the contributions of various civilisations that have contributed to its development and diffusion. Thus, depending on the period, various civilisations have had a leading role in the mathematical discoveries and dissemination of older concepts. The Maghreb (especially the territory that is now Algeria) was, at a turning point in history, one of the important places in the development and dissemination of mathematics throughout the Mediterranean region. The most famous examples are the popularisation of the positional decimal system in the 9th century and its circulation through the Maghreb and Europe and also the development of symbolisation for scoring fractions and writing equations and the development of combinatorics.

This movement between the plurality of cultural roots and universality of mathematics is found in many aspects in teaching and learning mathematics. On the one hand, in the construction of knowledge the student meets the concept in specific situations; then a necessary contextualisation/depersonalisation leads to the institutionalisation of knowledge. On the other hand, universal knowledge is not taught directly but undergoes changes at different levels of the chain of didactic transposition. To what extent are cultural roots being taken into account in these transformations? Is it necessary to consider this cultural dimension and how? To what extent would the contextualisation of mathematical concepts in their cultural dimension possibly promote student learning? The International Programme Committee of this conference is chaired by Laurent Theis (Université de Sherbrooke, Canada) with Ahmed Djebbar (Université de Lille, France) as vice-chair and Rachid Bebbouchi (University Houari Boumediene, Alger) at the head of the local organisation. Further information is available from the website http://www.mathunion.org/ICMI > news or from Jean-Luc Dorier (Jean-Luc.Dorier@unige.ch), who is a member of the International Programme Committee.

Educating the educators: Conference on international approaches to scaling-up professional development in maths and science education
15–16 December 2014 in Essen, Germany

The European project mascil (Mathematics and Science for life!) and the German Centre for Mathematics Teacher Education (DZLM) are pleased to host a conference at the University of Duisburg-Essen in the northwest of Germany.

This is the first international conference specifically devoted to the topic of educating the educators (such as teachers, teacher educators and the educators of teacher educators as well as multipliers and institutions engaged in teacher professional development) in particular in relation to disseminating innovative teaching approaches, such as inquiry-based learning. It will serve as a lever and platform for international exchange about concepts and experiences concerning questions such as: ‘What are the features of successful concepts and professional development? What are the needs and experiences of the different target groups? Which pitfalls have to be avoided?’ This international conference will connect researchers and practitioners engaged in the fields of maths and science education in order to discuss concepts of scaling-up teacher professional development. Please visit http://educating-the-educators.ph-freiburg.de for regularly updated information about the conference. We are looking forward to receiving your contributions. Further information can also be obtained from Günter Törner (guenter.toerner@uni-due.de), Chair of the EMS-Committee of Education.

ERME Column, May 2014

Viviane Durand-Guerrier (ERME President, Université de Montpellier 2, France)

CERME 8-proceedings

The 8th Congress of European Research in Mathematics Education (CERME 8) took place in Antalya, Turkey, 6-12 February 2013. The publication of the proceedings was carried out by Maria Alessandra Mariotti (Chair of the CERME8-IPC), Behiye Ubuz (Chair of the CERM8-Loc) and Çiğdem Haser, with the collaboration of the Working Group leaders and co-leaders. The proceedings have been published by Middle East Technical University, Ankara, Turkey, on behalf of the ERME. They are available on the website of the ERME alongside the proceedings of all previous CERMEs: http://www.mathematik.uni-dortmund.de/~erme/.

News from CERME 9

The 9th Congress of European Research in Mathematics Education (CERME 9) will take place in Prague (Czech Republic), 4–8 February 2015.

Konrad Krainer (Austria) is the Chair of the IPC (International Programme Committee), which is composed of Jorryt Van Bommel (Sweden), Marianna Bosch (Spain), Jason Cooper (Israel), Andreas Eichler (Germany), Ghislaine Gueudet (France), Marja van der Heuvel-Panhuizen (the Netherlands), Uffe Jankvist (Denmark, Co-Chair), Maria-Alessandra Mariotti (Italy), Despina Potari (Greece), Ewa Swoboda (Poland), Nad’a Vondrová (Czech Republic) and Carl Winsløw (Denmark).

Nad'a Vondrová (Chair) and Jarmila Novotná (Co-Chair) are in charge of the local organisation.

The chief aims of the ERME are to promote communication, cooperation and collaboration in research in mathematics education in Europe, in order to know more about research and research interests in different European countries and to create opportunities for inter-European cooperation between researchers in collaborative projects. This conference is designed as a starting point in promoting these aims in a communicative spirit.

In order to achieve these goals, CERME deliberately and distinctively moves away from research presentations by individuals towards collaborative group work. Its main feature is the *Thematic Working Group* (TWG), whose members work together on a common research domain. TWGs will have about 12 hours over four days in which to meet and progress their work. Each Thematic Working Group (TWG) should aim to provide a good scientific debate with the purpose of deepening mutual knowledge about the problems and methods of research in the field. Conference participants are expected to work within just one group.

There will be about 20 Thematic Working Groups at CERME 9: details about the focus of each group can be accessed from the list of the TWGs on the CERME 9 website. Researchers wishing to have their work discussed at the conference should submit a paper or a poster proposal to one of these TWGs. CERME papers and posters must be about research (empirical, theoretical or developmental). In addition, there will be plenary activities and a poster session.

The deadline for submission of papers will be 15 September; the deadline for poster proposals will be 1 October. All the deadline details are available at http://www.cerme9.org/about-cerme-9/deadlines/.

In order to support researchers who might not be able to attend the CERME without financial assistance, the ERME has developed the Graham Litter Fund. A limited amount of financial support is available to researchers. Applications should be sent before 15 September 2014. Information is available at http://www.cerme9.org/about-cerme-9/financial-support/.

The ERME is deeply interested in the active participation in CERME of young researchers in mathematics education. From CERME 9 onwards, the International Programme Committee (IPC) will comprise two elected representatives of Young Researchers, and the inclusion of Young Researchers in the co-chairing of the Thematic Working Groups is considered.

I invite you to visit the website of CERME 9: http://www.cerme9.org, which will be regularly updated.

Guarding your Searches: Data Protection at zbMATH

Jens Holzkämper (FIZ Karlsruhe, Germany) and Olaf Teschke (FIZ Karlsruhe, Germany)

This year, as in the quadrennial cycles before, the mathematical community eagerly awaits the announcement of the Fields Medallists during the opening ceremony of the ICM on 13 August. Though there are as usual many speculations and educated guesses, we can be pretty optimistic that the names of the prize winners will once again remain secret up till then; indeed, discretion has worked well so far and this has been a great part of the fun.

Of course, perfect secrecy is hard to maintain and is probably harder than ever nowadays due to technical developments. It is not necessary to kidnap and interrogate the Chair of the Fields Committee; surveillance of communication of the people involved would do (including not just possible committee members but, at least at a later stage, press officers of mainstream media), since at least some of them would most likely not refrain from using insecure channels. Alternatively, the analysis of search queries as employed in trend mining could evolve into a pretty clear picture of the ongoing discussions if restricted to a sufficiently adapted user pool – which might be intricate for Google or Bing (where mathematics is just a tiny noise in the large data stream) but would surely be applicable for MathSciNet or zbMATH requests.

Fortunately, this Orwellian scenario is unlikely, but only since it appears that no one capable of doing so would be interested in spoiling the party at the ICM. However, there is much more sensitive information around. Hiring decisions are often connected to an evaluation through scientific databases. On a general level, knowing in advance what mathematical research is going on is definitely of interest both inside and outside of mathematics. Frequently, allegations of plagiarism have been made, involving claims that new results might have been copied from a colleague and (pre-)published first by someone else; on the other hand, knowing developments in many applications ahead of publication could lead to significant advantages. Cryptography is a well-known example, which directly pertains to organisations that are capable of large-scale surveillance. However, mathematical results are part of so many applications that it wouldn't make sense to restrict to this (or extend to network algorithms, data mining, pattern recognition, etc.) as it is not certain whether more impact might come from theoretical foundations of number theory or faster matrix multiplication. Mathematics is interesting as a whole, which is actually pursued on a rather transparent level by the NSA (which publicly spends a lot of money on grants, and even more on recruiting mathematicians). No doubt similar activities occur, if less prominently, in other services and regions[1]. With the information that became public from the Snowden files, one would be surprised if there were no algorithms that keep track of research activities and persons in areas like, for example, cryptography; and the uneasy feeling that these algorithms may raise alarm due to connections unknown even to the researchers themselves, or just due to false positive signals, is certainly not a good environment for independent research.

While the solution of this dilemma obviously requires efforts of the society as a whole, we can try to improve things in our small area. More than 20 million search queries every year in zbMATH are only a very tiny fraction of the world's web traffic but possibly large enough to derive sensitive information in our subject, especially when queries could be personalised (which happens, for example, when EMS member accounts are used). This confronts us with the task of taking measures for data protection – at least as far as can be done from the zbMATH supplier side. Concerning the data connection, an SSL certificate (on a non-Heartbleed[2]-affected server) has been set up over the last month to protect zbMATH queries (soon to be upgraded further with software that allows perfect forward secrecy[3]), encrypting all data exchange between your browser and our servers.

While this closes the most obvious vector of attack, the question of handling the information on our servers remains. The level of possible access by the secret services to user data stored by providers has been a central topic in recent discussions. While connection data are elusive, the only secure way to protect search logs at supply servers is their permanent deletion. On the other hand, erasure is in conflict with requests from librarians, and also possibly cripples functions of the interface. Hence there are decisions to be taken, which will be outlined below.

From the librarians' side, there is an ongoing demand for access data. At the moment, the most common standard is described by the COUNTER Code of Practice for e-Resources[4]. While the desire to evaluate the usage of the resources is highly understandable, the 90 pages (in-

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1 While there has been a discussion about the impact of mathematical finance to the banking crisis, it seems that the ethical dimension of mathematicians' contribution to global surveillance infrastructure is yet unexplored.

2 The crypto-apocalypse of April 2014: http://heartbleed.com/, https://xkcd.com/1354/.

3 https://en.wikipedia.org/wiki/Forward_secrecy.

4 http://www.projectcounter.org/code_practice.html.

cluding appendices) of the recent 4th release indicate an increasing demand for the granularity of information about the usage of journals and databases – information which can only be generated from access logs. Details in various library requests from the past included, for example, differentiation between searches, clicks, long and short views, and sessions, together with overall access numbers. The need for each single number makes sense on its own (since, for example, the total number of views or downloads for articles, reviews or profiles is not always meaningful) but there is a prospective danger that along with the legitimate wish to evaluate the relevance of a resource, a framework evolves around quantitative measures that monitors user behaviour in too detailed a way. What is lost is the awareness that such statistics simply cannot grasp fundamental aspects from the side of mathematical content. This might be best illustrated with an example. At the climax of the El Naschie scandal, one of the authors asked a librarian whether they had succeeded in eliminating Chaos, Solitons and Fractals from their Elsevier bundle. The surprising answer was: "Why should we? The access numbers skyrocketed over the last few months – this journal is obviously of highest importance for our mathematicians!" The conclusion is not just the old insight that ideally mathematicians should have the final decision on which resources they need but also that they should not overly rely on possibly treacherous statistics. What we would like to add is that the creation of detailed statistics may evolve into a privacy problem itself. This problem reaches even beyond library licensing – in the area of Open Access, the trend to justify relevance by download figures is even more pronounced; on the other hand, when such statistics are used for ranking purposes ("most popular article", etc.), the threat of manipulation is immanent. Detection and levelling of manipulation attempts would again require an overhead of user surveillance which doesn't seem desirable – hence, the preferable alternative seems to refrain from an overuse of quantitative data.

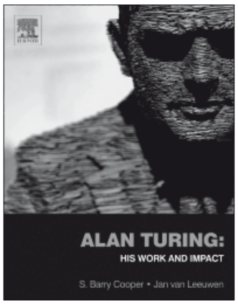
This also concerns the second issue: availability of functions based on usage data. Nowadays, we are accustomed from shopping platforms to seeing options like "most popular items" or "users interested in this also viewed…". Wouldn't it make sense to implement something similar in scientific databases? The barrier would be, again, the willingness to exploit data from users at a large scale. Though not personalised at the level of such applications, sensitive data may become available implicitly. As a very basic example, it is known (and plausible) that researchers often search for themselves to ensure the correctness of their data. Therefore, publishing "popular searches" is not fully independent of information on who uses the database to what extent, something which is certainly not of public interest. On a more sophisticated level, let us consider the example of hiring mentioned above. Institutions that have special hiring seasons tend to have significantly larger access numbers to zbMATH in these months, which may indicate that the evaluation by profiles and reviews contributes much to the database usage. A seemingly innocent function like "people searching for this person looked also for…" would be prone to inadvertently revealing competing applications in the hiring process, which is certainly not desirable. Again, even the attempt would require long-term storage and data mining of usage data, with all the known (and, most likely, many yet unknown) problems involved.

Our approach to the topic is rather straightforward. Traditionally, FIZ Karlsruhe (as provider of zbMATH) has very high standards for data protection[5], which also derives from the history of supplying not only information on scientific progress but also, for example, on patents, where usage data would reveal business-relevant development strategies. Beyond these standards (which forbid, for example, monitoring individual actions) and, of course, the requirements of German data protection law (which is generally considered to meet the highest levels in international comparison), we would assess the value of user data protection much higher than the potential benefit of applications derived from them. Therefore, our decision is to delete user logs permanently after the finalisation of the general access statistics required by the libraries and to refrain from further analysis. The deletion currently takes places at the latest a year after access.

We hope that this policy finds the acceptance of the zbMATH user community.

[5] Privacy policy available at http://www.fiz-karlsruhe.de/fiz_privacy_policy.html?&L=1

Book Reviews



Alan Turing
His Work and Impact

TS. Barry Cooper and
J. van Leeuwen, Editors

Elsevier, Amsterdam, 2013
xxi + 914 p.
ISBN print 978-0-12-386980-7
ISBN e-book 978-0-12-387012-4

Reviewer: Jean-Paul Allouche

A hundred years after his birth, who – even without being a mathematician or a computer scientist – has never heard the name Alan Turing? But who can explain his work in detail or at least describe all the directions of his work? The book under review is a monumental (almost 1000 pages) tribute to Alan Turing, which gathers together texts by Turing as well as comments and papers by 70 contributors. The "tour de force" is that it is quite possible, and even recommended, to read the book not linearly but by selecting pieces randomly, just for fun. To quote the introduction: "So we have not tried to reproduce the style of an archive, rather aiming at a book to be read, to be dipped into for pure pleasure, to be enjoyed and browsed in office, on train or bus, or accompanying the proposer to some distant scientific meeting or place of relaxation." Before trying to describe the contents more precisely, we would like to underline, as the authors of the book did, the enormous influence of Turing in many various fields. The regularly updated URL http://ftp.math.utah.edu/pub//bibnet/authors/t/turing-alan-mathison.html shows a bibliography of his work and related works.

The book is divided into four parts. The first of these parts, entitled 'How do we compute? Can we prove?' contains in particular the extraordinarily influential paper 'On Computable Numbers, with an Application to the *Entscheidungsproblem*'. Almost everybody has heard the expression 'Turing machine'. Now the question of whether or not Turing is the real father of computers can be addressed by typing the words 'Turing machine' into a search engine on a … computer, and looking at the approximately 2,200,000 answers. This first part of the book ends with a text by Turing 'The reform of mathematical notation and phraseology', where Turing indicates that his statement of the "type principle" was suggested by lectures of Wittgenstein.

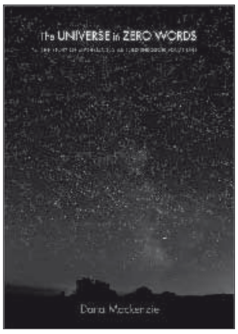
The next part, entitled 'Hiding and unhiding information: cryptology, complexity and number theory', speaks of course of Turing cracking (or seriously contributing to cracking) the *Enigma* (the most important cipher machine(s) used by the Germans during World War II). But this part also contains, in particular, two number-theoretic works of Turing *Some calculations of the Riemann Zeta-function* and (with S. Skewes) *On a theorem of Littlewood*, where the authors prove the existence of some real x greater than 2 and less than a double exponential constant such that $\pi(x) > \mathrm{li}\,(x)$ (where, as usual, $\pi(x)$ is the number of primes less than x and $\mathrm{li}\,(x)$ is the logarithmic integral of x, i.e. the principal value of $\int_0^x \mathrm{d}\,t/\log t$).

Readers interested in the question of intelligent computers or, more precisely, in the possible links between (human) intelligence and "intelligent" computers will be delighted to read the third part 'Building a brain: intelligent machines, practice and theory'. This is, of course, a vast field, whose most popular item is the so-called Turing test: a test to show the ability of a machine to have (or to simulate?) intelligent human behaviour or, more precisely, to have behaviour that cannot be distinguished from an intelligent human.

The last part is entitled 'The mathematics of emergence: the mysteries of morphogenesis'. It is devoted to the important work of Turing in biology. The main points are Turing's 1952 paper 'The chemical basis of morphogenesis' and his work (manuscripts and lectures) on the morphogen theory of phyllotaxis.

Full of enthusiasm for this book, I tried to find at least one criticism. This was not that easy! Possibly a very minor point is that I dislike the word "impact" in the title (Webster: Latin *impactus* p.p. of *impingere*, to push, strike against. See "Impinge"). The deep and longstanding influence of Turing deserves a term better and more elegant than this fashionable and somewhat hollow word.

My short description of this exciting book is probably frustrating, and for two reasons: firstly, because it only scratches the surface of Turing's work; secondly, because it does not (and cannot) give an idea of the extremely interesting contributions of the 70 contributors. We hope that this possible frustration will urge the readers of this brief review to read the book. We are sure that they will definitely enjoy it. It is quite possible that they will also find there ideas or flashes of inspiration for their own research.



Dana Mackenzie

The Universe in Zero Words
The story of mathematics as told through equations

Princeton University Press, 2012
224 p.
ISBN print 978-0-691-15282-0
ISBN e-book 978-1-400-84168-4

Reviewer: Yuriy V. Rogovchenko (Kristiansand)

The Newsletter thanks Zentralblatt MATH and Yuriy V. Rogovchenko for the permission to republish this review, originally appeared as Zbl 1273.00014.

This nice book invites readers to explore the beauty of the most influential mathematical equations. Dr. Dana Mackenzie, who became a full-time writer after being a mathematics professor for thirteen years, writes in the introduction: "Equations are the lifeblood of mathematics and science. They are the brush strokes that mathematicians use to create their art, or the secret code that they use to express their ideas about the universe. The rest of the world, outside of science, does not speak the language of equations, and thus a vast cultural gap has emerged between those who understand them and those who do not. This book is an attempt to build a bridge across that chasm. It is intended for the reader who would like to understand mathematics on its own terms, and who would like to appreciate mathematics as an art."

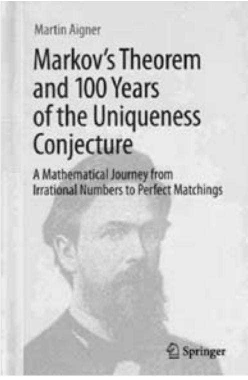
Although mathematics has numerous branches, all known for their beauty, significance, and abundance of important equations, the author decided to limit his selection only to four. "I consider the four main tributaries of mathematics to be algebra, geometry, applied mathematics, and analysis. All four of them mingle together and cooperate in a most wonderful way, and witnessing this interaction is one of the great joys of being a mathematician. Nearly every mathematician finds himself drawn more to one of these tributaries than the others, but the beauty and power of the subject undoubtedly derives from all four. For that reason, the four chapters in this book each have a theme, or "storyline" running throughout, relating the evolution of the four branches over the ages." This book tells the story of twenty four beautiful and powerful equations that may be regarded as milestones in the development of mathematics, science and society.

Recently, I had a pleasure of reviewing another nice book by Professor Stewart [I. Stewart, *Seventeen equations that changed the world*. London: Profile Books] where the stories of some equations discussed in this volume can be found. It was really interesting to compare the selection of equations by two authors. As Dr. Mackenzie explains, "the choice of equations was necessarily a matter of individual taste and preference." Some of criteria he used "to decide what makes an equation great" are listed below.

"1. It is surprising. […] [It] tells us something that we did not know before. 2. It is concise. […] It contains nothing but the essentials. 3. It is consequential. […] The equations that make the deepest impression are the ones that revolutionize mathematics, change our view of the world, or change the material possibilities of our lives. 4. It is universal. […] An equation proven today will remain true forever."

Equations picked for this volume range from elementary, $1+1=2$ known to preschoolers, to inspirational Fermat's last theorem $x^n+y^n=z^n$ still fascinating thousands of amateurs, to exquisitely sophisticated Chern-Gauss-Bonnet equation $\int_{\mathcal{M}} Pf(\Omega)\, 2\pi^n \chi(\mathcal{M})$ that allows to deduce information about the shape of the universe if the curvature at each point is given. The book is written in a very transparent and elegant manner; it is both enjoyable and informative reading. The reader will absolutely love exciting historical facts and excellent illustrations, diagrams, pictures carefully selected by the author. The volume concludes with a useful bibliography and a helpful index. A very entertaining text that appeals not only to mathematics enthusiasts, but also to a wide audience with a quite limited mathematical background.

Yuriy Rogovchenko received his Ph.D. in Differential Equations from the Institute of Mathematics, National Academy of Sciences of the Ukraine, in 1987. He held professor's positions at the Eastern Mediterranean University, North Cyprus, University of Kalmar, Sweden and Umeå University, Sweden. Since August 2012, Dr. Rogovchenko is Professor of Mathematics at the University of Agder, Norway. His research interests lie within qualitative theory of ordinary, functional, and impulsive differential equations, perturbation methods, mathematical modeling in biology, social sciences and economics. Dr. Rogovchenko was nominated a distinguished reviewer of Zentralblatt MATH in 2009 and received a Certificate of Recognition from the President and CEO of FIZ Karlsruhe Sabine Brünger-Weilandt and the President of the European Mathematical Society Ari Laptev.



Martin Aigner

Markov's Theorem and 100 Years of the Uniqueness Conjecture
A mathematical journey from irrational numbers to perfect matchings

Springer International Publishers, Cham, 2013, x + 257 pp.
ISBN print: 978-3-319-00887-5
ISBN e-book: 978-3-319-00888-2

Reviewer: Franz Lemmermeyer (Jagstzell, Germany)

The Newsletter thanks Zentralblatt MATH and Franz Lemmermeyer for the permission to republish this review, originally appeared as Zbl 1276.00006.

Let α be a real number. Dirichlet showed that there exist infinitely many fractions $\frac{p}{q} \in \mathbb{Q}$ with $|\alpha - \frac{p}{q}| \le \frac{1}{q^2}$. Now consider all positive real numbers L such that $|\alpha - \frac{p}{q}| < 1/Lq^2$ holds for infinitely many fractions $\frac{p}{q}$. The supremum of all these L is denoted by $L(\alpha)$; then $L(\alpha) = 0$ if and only if α is rational, and $L(\alpha) \ge 1$ otherwise. The set $\mathcal{L} = \{L(\alpha) : \alpha \in \mathbb{R} \setminus \mathbb{Q}\}$ is called the Lagrange spectrum. A. Markoff [Math. Ann. 15, 381–407 (1879); 17, 379–400 (1880)] showed that the Lagrange spectrum below 3 consists of all numbers of the form $\sqrt{9m^2 - 4}/m$, where m runs through the "Markoff numbers". These are defined as the set of all natural numbers x_i occurring as solutions of the Diophantine equation $x_1^2 + x_2^2 + x_3^2 = 3x_1x_2x_3$. It is easy to show that every Markoff number appears as the largest number in some Markoff triple $(x_1x_2x_3)$, and the uniqueness conjecture predicts that each Markoff number is the maximum of a unique Markoff triple.

The investigation of the uniqueness conjecture from different perspectives is the main goal of this book. After some introductory chapters, the reader is introduced to Cohn matrices, the modular group $SL(2, \mathbb{Z})$, free groups, graphs and trees, and to partial results towards the uniqueness conjecture using the arithmetic of quadratic fields. The whole discussion is very elementary, and requires no preliminaries except some familiarity with basic concepts of algebra and number theory.

The reviewer regrets that the author has given in to the temptation of keeping the book on a very elementary level throughout. Readers enjoying the section on hyperbolic geometry will be well advised to have a look at the very nice book "Fuchsian groups" by S. Katok [Fuchsian groups. Chicago: The University of Chicago Press (1992)]. Similarly, Markoff's original motivation for studying these questions, the theory of binary quadratic forms, is only briefly mentioned on pp. 36–38 even though quadratic forms cast their shadows almost everywhere in this book: continued fractions and the uni-modular group are intimately connected with Lagrange reduction of quadratic forms, and the arithmetic of ideals in quadratic number fields also is just one way of presenting Gauss composition and the class group of forms. The readers will find a beautiful introduction to the dictionary between these languages in the recent book "Algebraic theory of quadratic numbers" by M. Trifkovič [Algebraic theory of quadratic numbers. New York, NY: Springer (2013)].

This beautiful book gives readers a chance to familiarize themselves with a very simple and yet very difficult problem in number theory, and teaches them that it pays to look at a problem from many different angles. I recommend it to all students who are already hooked to number theory, and perhaps even more to those who are not.

Franz Lemmermeyer received his Ph.D. from the University of Heidelberg and is currently teaching mathematics at the gymnasium St. Gertrudis in Ellwangen. His interests include number theory and the history of mathematics.

Letter to the Editor

Massimo Ferri (Università di Bologna, Italy)

The Role of Application-oriented Mathematics

Dear Editor,

In the March Newsletter, Ciro Ciliberto, President of the Italian Mathematical Union, complained about the negative comments on a Marie Curie fellowship proposal in mathematics, concerning a lack of interdisciplinarity and socio-economic impact of the proposed project. I fully agree with him on the fact that the evaluation criteria seem not to comprehend the specificities of pure mathematical research; I hope that it will be possible to give these criteria more flexibility and also that interdisciplinarity within mathematics will be recognised as such.

Still, it seems to me that this event stresses a point that has already appeared in this newsletter in other forms: does mathematics "sell" itself conveniently to the scientific community and society at large? I think it would be necessary to do it not only for getting funds, grants, etc., but also as a fair attitude of mathematicians toward society. I'm not only speaking of popularisation – which is an important cultural issue anyway – but also of a better two-way communication between mathematics and other scientific and technological areas. There is not just "applied" mathematics but "application-oriented" mathematics, inspired by the many facets of modern technology, which is growing fast in our departments and can provide the link I am hoping for.

Unfortunately, in my opinion, application-oriented mathematics is not given the correct appreciation within mathematics when it comes to fund distribution and – above all – academic competition, at least in Italy. There may be several reasons for that: the claim that application-inspired mathematics is of a lower level; the fact that its correctness is more difficult to check; even a sort of retaliation against the attitude denounced by Professor Ciliberto (an attitude which has severe fallout in terms of funding). On the other side, mathematicians often and correctly "defend" on the media our discipline, pointing out its manifold applications.

I think it's high time to discuss this issue openly. I see two symmetric rigidities: a lack of sensitivity of some scientific environments to the specificities of pure mathematics, and a lack of sensitivity of some mathematical environments to the importance of application-oriented research. Should we go on pretending that the problem does not exist? If not, how can we face it? Is this just an Italian phenomenon or a European one? I hope that the EMS Newsletter will offer its pages to a debate which cannot be deferred any longer.

Massimo Ferri
Professor of Geometry at the University of Bologna

Personal Column

Please send information on mathematical awards and deaths to Mădălina Păcurar [madalina.pacurar@econ.ubbcluj.ro]

Awards

Christoph Aistleitner (Technische Universität Graz, Austria) has been awarded the **2013 Information-Based Complexity Young Researcher Award**.

Michèle Artigue (Université Paris Diderot) has been awarded the **2013 Felix Klein Medal** of the International Commission on Mathematical Instruction (ICMI).

Markus Bachmayr (RWTH Aachen University, Germany) has been awarded the **John Todd Award 2013** by the Oberwolfach Foundation.

Hugo Duminil-Copin (Université de Genève, Switzerland) has been awarded the **Oberwolfach Prize 2013** by the Oberwolfach Foundation.

Gerd Faltings (Max Planck Institute for Mathematics in Bonn, University of Bonn, Germany) has been awarded the **2014 King Faisal International Prize for Science**.

Gert-Martin Greuel (Technische Universität Kaiserslautern, Germany) and **Andreas Matt** (Mathematisches Forschungsinstitut Oberwolfach, Germany) have been awarded the **Medienpreis Mathematik 2013** by the Deutsche Mathematiker Vereinigung (DMV) for their work on the project IMAGINARY.

Camillo De Lellis (University of Zurich, Switzerland) and **Martin Hairer** (Warwick University, UK) have been awarded the **Fermat Prize 2014** by the Institut de Mathématiques de Toulouse.

John N. Mather (Princeton University) has been awarded the **Brouwer Medal 2014** by the Royal Dutch Mathematical Society.

Stefan Müller (University of Bonn, Germany) has been awarded the **Heinz Gumin Prize 2013** by the Carl Friedrich von Siemens Stiftung.

Maria Pe Pereira (Institut de Mathematiques de Jussieu, France) has been awarded the **2013 Rubio de Francia Prize** of the Royal Spanish Mathematical Society.

János Pintz (Alfréd Rényi Mathematical Institute of the Hungarian Academy of Sciences, Hungary) and **Cem Y. Yıldırım** (Boğaziçi University, Istanbul, Turkey) have been awarded the **2014 Cole Prize in Number Theory**, together with Yitang Zhang and Daniel Goldston.

Peter Sarnak (Institute for Advanced Study in Princeton, USA) has been awarded the **2014 Wolf Prize in Mathematics**.

Peter Scholze (University of Bonn, Germany) has been awarded the **2013 SASTRA Ramanujan Prize**.

Yakov G. Sinai (Princeton University, USA, and the Landau Institute for Theoretical Physics, Russian Academy of Sciences, Russia) will be awarded the **2014 Abel Prize** by the Norwegian Academy of Science and Letters.

Herbert Spohn (Technical University in Munich, Germany) has been awarded the **Cantor Medal 2014** by the German Mathematical Society.

Cédric Villani (Université de Lyon, France) has been awarded the **2014 Doob Prize** of the American Mathematical Society (AMS).

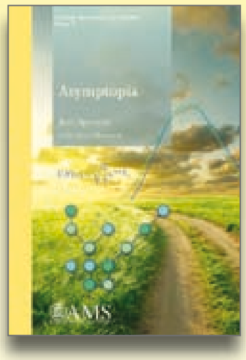
Yitang Zhang (University of New Hampshire, USA) has been awarded the **Rolf Schock Prize 2014** by the Royal Swedish Academy of Sciences and also the **Ostrowski Prize 2013** by the Ostrowski Foundation.

Deaths

We regret to announce the deaths of:

Gerhard Becker (12 July 2013, Germany)
Dan Laksov (25 October 2013, Norway)
Kurt Leichtweiss (23 June 2013, Germany)
Hanfried Lenz (1 June 2013, Germany)
Günther Nürnberger (11 May 2013, Germany)
Joel Sakarovitch (20 March 2014, France)
Wolfgang Schwarz (19 July 2013, Germany)
Christoph Scriba (26 July 2013, Germany)
Stein Arild Strømme (31 January 1014, Norway)
Piero Villaggio (4 January 2014, Italy)
Marc Yor (10 January 2014, France)
Jacqueline Ferrand (26 April 2014, France)

AMERICAN MATHEMATICAL SOCIETY



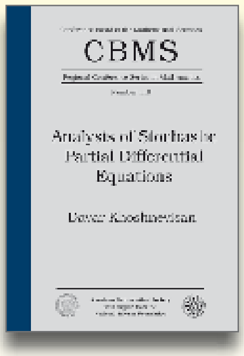
ASYMPTOPIA

Joel Spencer, *New York University*
With Laura Florescu, *New York University*

Asymptotics in one form or another are part of the landscape for every mathematician. The objective of this book is to present the ideas of how to approach asymptotic problems that arise in discrete mathematics, analysis of algorithms, and number theory. A broad range of topics is covered, including distribution of prime integers, Erdös Magic, random graphs, Ramsey numbers, and asymptotic geometry.

Student Mathematical Library, Vol. 71

Jun 2014 195pp 9781470409043 Paperback €36.00



ANALYSIS OF STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

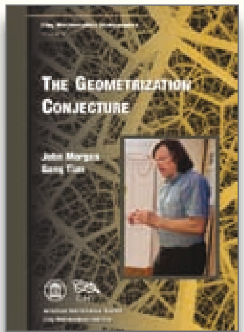
Davar Khoshnevisan, *University of Utah*

The stochastic PDEs that are studied in this book are similar to the familiar PDE for heat in a thin rod, but with the additional restriction that the external forcing density is a two-parameter stochastic process, or what is more commonly the case, the forcing is a "random noise," also known as a "generalized random field." At several points in the lectures, there are examples that highlight the phenomenon that stochastic PDEs are not a subset of PDEs.

CBMS Regional Conference Series in Mathematics, Vol. 119

Jul 2014 116pp 9781470415471 Paperback €34.00

A co-publication of the AMS and CBMS



THE GEOMETRIZATION CONJECTURE

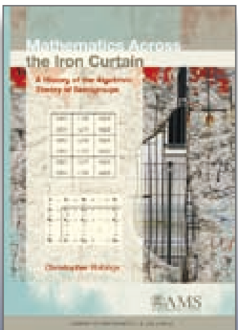
John Morgan, *Simons Center for Geometry and Physics* & Gang Tian, *Princeton University and Peking University*

Provides a complete proof of the geometrization conjecture, which describes all compact 3-manifolds in terms of geometric pieces. The method is to understand the limits as time goes to infinity of Ricci flow with surgery. The first half of the book is devoted to showing that these limits divide naturally along incompressible tori into pieces on which the metric is converging smoothly to hyperbolic metrics and pieces that are locally more and more volume collapsed. The second half of the book is devoted to showing that the latter pieces are themselves geometric.

Clay Mathematics Monographs, Vol. 5

May 2014 291pp 9780821852019 Hardback €73.00

A co-publication of the AMS and Clay Mathematics Institute



MATHEMATICS ACROSS THE IRON CURTAIN

A History of the Algebraic Theory of Semigroups

Christopher Hollings

The theory of semigroups is a relatively young branch of mathematics, with most of the major results having appeared after the Second World War. This book describes the evolution of (algebraic) semigroup theory from its earliest origins to the establishment of a full-fledged theory. Semigroup theory might be termed "Cold War mathematics" because of the time during which it developed. There were thriving schools on both sides of the Iron Curtain, although the two sides were not always able to communicate with each other, or even gain access to the other's publications.

History of Mathematics, Vol. 41

Sep 2014 449pp 9781470414931 Hardback €99.00

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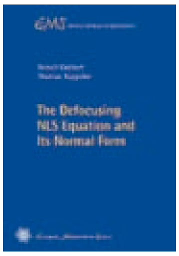


Benoît Grébert (Université de Nantes, France) and Thomas Kappeler (Universität Zürich, Switzerland)
The Defocusing NLS Equation and Its Normal Form (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-131-6. March 2014. 176 pages. Softcover. 17 x 24 cm. 32.00 Euro

The theme of this monograph is the nonlinear Schrödinger equation. This equation models slowly varying wave envelopes in dispersive media and arises in various physical systems such as water waves, plasma physics, solid state physics and nonlinear optics. More specifically, this book treats the defocusing nonlinear Schrödinger (dNLS) equation on the circle with a dynamical systems viewpoint. By developing the normal form theory it is shown that this equation is an integrable partial differential equation in the strongest possible sense. In particular, all solutions of the dNLS equation on the circle are periodic, quasi-periodic or almost-periodic in time and Hamiltonian perturbations of this equation can be studied near solutions far away from the equilibrium.
The book is not only intended for specialists working at the intersection of integrable PDEs and dynamical systems, but also for researchers farther away from these fields as well as for graduate students. It is written in a modular fashion, each of its chapters and appendices can be read independently of each other.

Emmanuel Hebey (Université de Cergy-Pontoise, France)
Compactness and Stability for Nonlinear Elliptic Equations (Zurich Lectures in Advanced Mathematics)

ISBN 978-3-03719-134-7. 2014. 304 pages. Softcover. 17 x 24 cm. 42.00 Euro

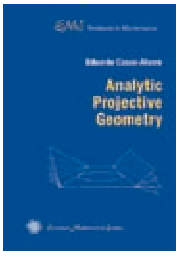
The book offers an expanded version of lectures given at ETH Zürich in the framework of a Nachdiplomvorlesung. Compactness and stability for nonlinear elliptic equations in the inhomogeneous context of closed Riemannian manifolds are investigated, a field presently undergoing great development. The author describes blow-up phenomena and presents the progress made over the past years on the subject, giving an up-to-date description of the new ideas, concepts, methods, and theories in the field. Special attention is devoted to the nonlinear stationary Schrödinger equation and to its critical formulation.
Intended to be as self-contained as possible, the book is accessible to a broad audience of readers, including graduate students and researchers.

Robert J. Marsh (University of Leeds, UK)
Lecture Notes on Cluster Algebras (Zurich Lectures in Advanced Mathematics)
978-3-03719-130-9. 2013. 132 pages. Softcover. 17 x 24 cm. 28.00 Euro

The aim of these notes is to give an introduction to cluster algebras which is accessible to graduate students or researchers interested in learning more about the field, while giving a taste of the wide connections between cluster algebras and other areas of mathematics.
The approach taken emphasizes combinatorial and geometric aspects of cluster algebras. Cluster algebras of finite type are classified by the Dynkin diagrams, so a short introduction to reflection groups is given in order to describe this and the corresponding generalized associahedra. A discussion of cluster algebra periodicity, which has a close relationship with discrete integrable systems, is included. The book ends with a description of the cluster algebras of finite mutation type and the cluster structure of the homogeneous coordinate ring of the Grassmannian, both of which have a beautiful description in terms of combinatorial geometry.

Isabelle Gallagher (Université Paris-Diderot, France), Laure Saint-Raymond (Université Pierre et Marie Curie, Paris) and Benjamin Texier (Université Paris-Diderot, France)
From Newton to Boltzmann: Hard Spheres and Short-range Potentials (Zurich Lectures in Advanced Mathematics)
978-3-03719-129-3. 2013. 150 pages. Softcover. 17 x 24 cm. 32.00 Euro

The question addressed in this monograph is the relationship between the time-reversible Newton dynamics for a system of particles interacting via elastic collisions, and the irreversible Boltzmann dynamics which gives a statistical description of the collision mechanism. Two types of elastic collisions are considered: hard spheres, and compactly supported potentials.
Following the steps suggested by Lanford in 1974, we describe the transition from Newton to Boltzmann by proving a rigorous convergence result in short time, as the number of particles tends to infinity and their size simultaneously goes to zero, in the Boltzmann-Grad scaling.
This book is intended for mathematicians working in the fields of partial differential equations and mathematical physics, and is accessible to graduate students with a background in analysis.



Eduardo Casas-Alvero (Universitat de Barcelona, Spain)
Analytic Projective Geometry (EMS Textbooks in Mathematics)

ISBN 978-3-03719-138-5. 2014. 636 pages. Hardcover. 16.5 x 23.5 cm. 58.00 Euro

This book contains a comprehensive presentation of projective geometry, over the real and complex number fields, and its applications to affine and Euclidean geometries. It covers central topics such as linear varieties, cross ratio, duality, projective transformations, quadrics and their classifications – projective, affine and metric –, as well as the more advanced and less usual spaces of quadrics, rational normal curves, line complexes and the classifications of collineations, pencils of quadrics and correlations. Two appendices are devoted to the projective foundations of perspective and to the projective models of plane non-Euclidean geometries. The presentation uses modern language, is based on linear algebra and provides complete proofs. Exercises are proposed at the end of each chapter; many of them are beautiful classical results.
The material in this book is suitable for courses on projective geometry for undergraduate students, with a working knowledge of a standard first course on linear algebra. The text is a valuable guide to graduate students and researchers working in areas using or related to projective geometry.

Matheon – Mathematics for Key Technologies (EMS Series in Industrial and Applied Mathematics, Vol. 1)
Peter Deuflhard, Martin Grötschel, Frank Schmidt (all Konrad-Zuse-Zentrum, Berlin, Germany), Dietmar Hömberg, Volker Mehrmann, Martin Skutella (all Technische Universität Berlin, Germany), Ulrich Horst, Jürg Kramer (both Humboldt-Universität zu Berlin, Germany), Konrad Polthier, Christof Schütte (both Freie Universität Berlin, Germany) and Jürgen Sprekels (Weierstraß Institut für Angewandte Analysis und Stochastik, Berlin, Germany), Editors

978-3-03719-137-8. 2014. 466 pages. Hardcover. 17 x 24 cm. 48.00 Euro

Mathematics: intellectual endeavor, production factor, key technology, key to key technologies? Mathematics is all of these! The last three of its facets have been the focus of the research and development in the Berlin-based DFG Research Center Matheon in the last twelve years. Through these activities Matheon has become an international trademark for carrying out creative, application-driven research in mathematics and for cooperating with industrial partners in the solution of complex problems in key technologies.
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