NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

Photo courtesy of Maryam Mirzakhani

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The "Bounded Gaps between Primes" Polymath Project

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EMS Agenda

2015

6–8 March
Executive Committee of the EMS
Prague, Czech Republic

28–29 March
Meeting of Presidents
Innsbruck, Austria

11 April
Meeting of the Committee for Developing Countries
Oslo, Norway
Contact: Michel Waldschmidt, miw@math.jussieu.fr

EMS Scientific Events

2015

7–10 January
Aspects of Lie Theory, INdAM
Rome, Italy
http://www1.mat.uniroma1.it/~bravi/indam2015.html
EMS distinguished speaker: Vera Serganova (Berkeley, CA, USA)

10–13 June
AMS-EMS-PMS Congress
Porto, Portugal
http://aep-math2015.spm.pt/

6–10 July
European Meeting of Statisticians,
Amsterdam, The Netherlands
http://www.ems2015.nl/
Bernoulli Society-EMS Joint Lecture: Gunnar Carlsson (Stanford, CA, USA)

31 August–4 September
17th EWM General Meeting
Cortona, Italy
http://www.europeanwomeninmaths.org/
EMS Lecturer: Nicole Tomczak-Jaegerman (Edmonton, Alberta, Canada)

18–20 September
EMS-LMS Joint Mathematical Weekend
Birmingham, UK

Date to be fixed
25th Anniversary of the EMS, Institut Henri Poincaré
Paris, France

2016

18–22 July
7th European Congress of Mathematics
Berlin, Germany
http://www.7ecm.de/

Editorial: The EMS – A Strong Society with a Vibrant Life

Marta Sanz Solé (EMS President 2011–2014)

At the end of this year, I step down as President of the European Mathematical Society, a position that I assumed in January 2011. It has been a great privilege and pleasure and a gratifying experience to develop an intensive and challenging programme, whose main lines were presented at the 2010 Council and trusted by the delegates. Throughout my four years in office, I have been in touch with and have reported to the EMS membership regularly. The annual *Messages from the President* (published in the EMS Newsletter or distributed by email), the reports at the annual Meetings of Presidents of EMS member societies and the reports of the Councils held in 2012 and 2014 contain a detailed account of activities that have been held, actions that have been pursued on a diversity of fronts and projects that have either been sketched, initiated or accomplished. All that is archived on the EMS website and is available, either through open access or through your societies and council delegates, and I will therefore spare you from a lengthy description repeating those points here.

Without good coordination and organisation and the hard work, enthusiasm, energy and creativity of many collaborators, nothing could have been achieved. I owe my gratitude to many people and institutions: the members of the Executive Committee, the chairs and members of the EMS committees, the Director of the EMS Publishing House, the leaders and collaborators of EMS initiatives, the efficient staff at the EMS office in Helsinki and many more. Special thanks go to the past presidents for their advice whenever it was needed.

Last June, the Council in Donostia elected Pavel Exner, Mats Gyllenberg and Sjoerd Verduyn Lunel as President, Treasurer and Secretary, respectively, for the period 2015–2018. I wish them and the whole Executive Committee every success. I am fully confident that the activity and influence of the EMS will continue to increase according to its mission.

In 2015, the EMS will celebrate its 25th anniversary. A scientific day with the theme "Challenges of the Mathematical Sciences in the 21st century" will take place at the Institut Henri Poincaré in Paris. Cédric Villani, the director, gladly offered to open the doors of the institute to the EMS on this special occasion.

The creation of the EMS 25 years ago is better understood in its historical context. In the late 1960s, there were moves toward establishing scientific organisations with the aim of strengthening European scientific and cultural unity and furthering cross-border collaboration and co-operation. This was one of the components in the much more ambitious and wider project of developing European integration, initiated in 1957 with the Treaty of Rome. The European Physical Society was founded in 1968 and the European Association for Chemical and Molecular Sciences was founded in 1970, to refer to just two scientific disciplines. With a cross-disciplinary perspective, the European Science Foundation (ESF), established in 1976, played a crucial role in that process. It has been instrumental in setting up the conditions to develop European science.

Mathematics did not remain on the sidelines during this period. In 1978, the European Mathematical Council was founded under the leadership of Sir Michael Atiyah. Its mission was to help the emergence of an identity among European mathematicians. It was only in 1990 that this embryonic structure evolved into the European Mathematical Society, by decision of 33 founding societies in a meeting in Mandralin (Poland) on 27–28 October (see http://www.euro-math-soc.eu/history-european-mathematical-society).

Throughout almost 25 years of existence, the life of the society has expanded in a rapidly changing world. Focusing on the key political events and regulations, 1990 was the year of the fall of the Eastern Bloc and 1993 was the year of the Maastricht Treaty, establishing the European Union with 12 member states. Today, the EU consists of 28 member states. This represents a major increase in weight, influence and complexity. Concerning research and innovation, the European Research Area was proposed in 2000 and the current European framework programme Horizon 2020 aims to complete and further develop this idea. As for education, the Bologna Process was born in 1999, aimed at ensuring more compatible and coherent systems of higher education in Europe. In 2010, it resulted in the European Higher Education Area.

As a professional society, the EMS has been representing the mathematical community in this landscape and lobbying for the promotion of the discipline. After years of absence, mathematics is present in the EU work programmes for research and innovation and in the calls for proposals, though not yet at the desired level. In the Autumn of 2014, the European Commission DG Connect organised a consultation followed by a workshop, specifically addressed at European mathematicians, on the role of mathematics for the future of digital sciences. This is an encouraging outcome of longstanding efforts.

In the last few decades, mathematics has been making major advances both in fundamental theory and in applications. Interactions between different mathematical areas, with other scientific and engineering disciplines, and with economics and social sciences have increased dramatically. Research in climate, finance and medicine, to mention just a few fields, is integrating mathematical sciences and computing in a substantial and essential way. Exposure of mathematics to a multidisciplinary environment has never been so strong. One of the consequences is the necessity to rethink the mathematical curriculum in high schools and universities and to provide appropriate scientific training to the new generation of mathematicians. The Internet has changed the way scientists work, collaborate and communicate their results, and also the way knowledge is transmitted. Nowadays, the mathematical ecosystem is highly complex and therefore very rich.

Throughout its committees and general structure, the EMS is providing a very appropriate forum to debate these trends and their effects, and to pursue actions on different fronts. Mathematical education, applications of mathematics, new modes of publishing, ethical issues, the training of young researchers, the general role of mathematics in society, gender balance and solidarity are among the EMS concerns and activities.

In addition to representing the European mathematical community at European institutions, including the European Parliament and the European Commission, the EMS is contributing significantly to mathematics in publication and research.

The EMS Publishing House is a landmark in the world of non-commercial mathematical publishing and the partnership with ZbMATH and EuDML shows the commitment of the society to reference, access and preservation of scientific documents. The European Congresses of Mathematics are now consolidated as the most important mathematical meetings in Europe. They are a showcase for breakthroughs and significant advances by senior and young researchers in a forum that demonstrates and promotes the invaluable unity of the discipline.

The strength of the society has its basis in its wide membership, consisting of about 60 national mathematical societies, 40 mathematical research centres and departments and about 3000 individuals. This clearly shows the success of the EMS in creating a strongly supported organisation across Europe and in building an integrated community. Rather than competing with its member societies, the EMS takes or shares leadership in initiatives and promotes debates, broad in diversity, amplitude and potential impact, to help mathematicians and their science meet the challenges of the 21st century. Our members "use" us as a platform for forming, sharing and voicing opinions, seeking advice or solidarity and, in general, as a network for synergies.

The EMS is a successful story of European collaboration of mathematicians. I feel very proud to have been one of its pilots. Thank you for your confidence and encouragement during my period of office.

European Congress of Mathematics Call for Bids for the 8ECM

Marta Sanz-Solé (EMS President) and Stephen Huggett (EMS Secretary)

Outline bids from mathematicians to organize the 2020 Congress are now invited, and should reach the EMS Secretariat by 28 February 2015. The address of the Secretariat is Department of Mathematics & Statistics, P.O. Box 68, Gustaf Hällströmink. 2b, 00014 University Helsinki, Finland (Phone: +358 9 1915 1507 Fax: +358 9 1915 1400; Email: ems-office@helsinki.fi).

The information below may be helpful to possible organizers. Informal discussions are welcomed, and may be addressed to the President of the EMS.

General information about ECMs

European Congresses of Mathematics are organized every four years. The first Congress was held in Paris in 1992, and since then they have been held in Budapest, Barcelona, Stockholm, Amsterdam, and Kraków. In 2016 the Congress will take place in Berlin.

Experience of previous Congresses suggests that around a thousand people may attend. The duration has so far been five days. Ten EMS Prizes are awarded to outstanding young European mathematicians at the opening ceremony, together with the Felix Klein and Otto Neugebauer Prizes. The Congress programme should aim to present various new aspects of pure and applied mathematics to a wide audience, to offer a forum for discussion of the relationship between mathematics and society in Europe, and to enhance cooperation among mathematicians from all European countries. The standard format of previous ECMs has been:

- about 10 plenary lectures;
- section lectures for a more specialized audience, normally with several held simultaneously;

- mini-symposia;
- film and mathematical software sessions;
- poster sessions;
- round table discussions.

An exhibition space for mathematical societies, booksellers, and so on is required. No official language is specified and no interpretation is needed. Proceedings of the Congress are published by the EMS Publishing House (http://www.ems-ph.org/).

Decision process for 8ECM

(i) Bids are invited via this notice in the EMS Newsletter, and via letters to the EMS member societies; the deadline for bids is 28 February 2015. These bids need only be outline bids giving a clear idea of the proposal and possible sources of financial and local support.

(ii) The Executive Committee of the EMS will consider the bids received. It will invite one or more of the bids to be set out in greater detail so that it can decide which bids are sufficiently serious options to be considered further. The deadline for such "worked up" bids, which will include a draft budget and a commitment to follow the conditions set up by the Executive Committee, is 31 July 2015.

(iii) The EC will then create a short-list of sites that appear to offer the best possibilities for a successful Congress, and appoint a Site Committee to visit the short-listed sites between September and December 2015 to check a range of items in connection with the development of the Congress. For example:
- Size and number of auditoriums; location and equipment
- Room for exhibitors
- Hotel rooms and dormitories; location, prices, number in different categories and transportation to lectures
- Restaurants close to Congress site, number and prices
- Accessibility and cost of travel from various parts of Europe
- Financing of the Congress, including support to participants from less favoured countries
- Financing for the EMS Prizes
- Experience in organizing large conferences
- Timing of the Congress
- Social events
- Plans to use the occasion of the Congress as publicity for mathematics

The EC may make a recommendation to the Council based on the report of the Site Committee.

(iv) In 2016, at the EMS Council prior to 7ECM, a decision will be reached.

Relations between the EC and the Organizing Committee of 7ECM

The actual Congress organization is the responsibility of the local organizers. Two committees must be appointed: the Scientific Committee and the Prize Committee.

The Scientific Committee is charged with the responsibility for conceiving the scientific programme and selecting the speakers. The Prize Committee is charged with the responsibility of nominating the EMS Prizewinners. For each of these committees the Chairs are appointed by the EC. There will be a call for suggestions of members of these two committees from the EMS member societies, after which the membership of each committee will be approved by the EC, in consultation with its Chair. In addition, the Executive Committee will appoint the Otto Neugebauer Prize Committee and the Felix Klein Prize Committee, the latter in co-operation with ECMI and the Fraunhofer Institute for Industrial Mathematics in Kaiserslautern.

The EC must be kept informed by the Chair of each committee about the progress of their work.

The local organizers are responsible for seeking financial support for the Congress and for the meetings of its committees. However, the EMS will provide some financial support for mathematicians from less favoured countries in Europe, and will also assist and advise in seeking sources of funding.

The level of the registration fee is of great importance to the success of an ECM, and the EC must be involved before a final decision on the level of fees is made; members of the EMS normally receive a reduction of some 20% on the registration fees. The EC would be pleased to offer advice to the local organizers on matters such as the scientific programme, the budget, registration, accommodation, publications, web site, and so on, but in any case the EC must be kept informed of progress at its regular meetings. Publicity for the ECM via the EMS Newsletter and the EMS site

http://www.euro-math-soc.eu/

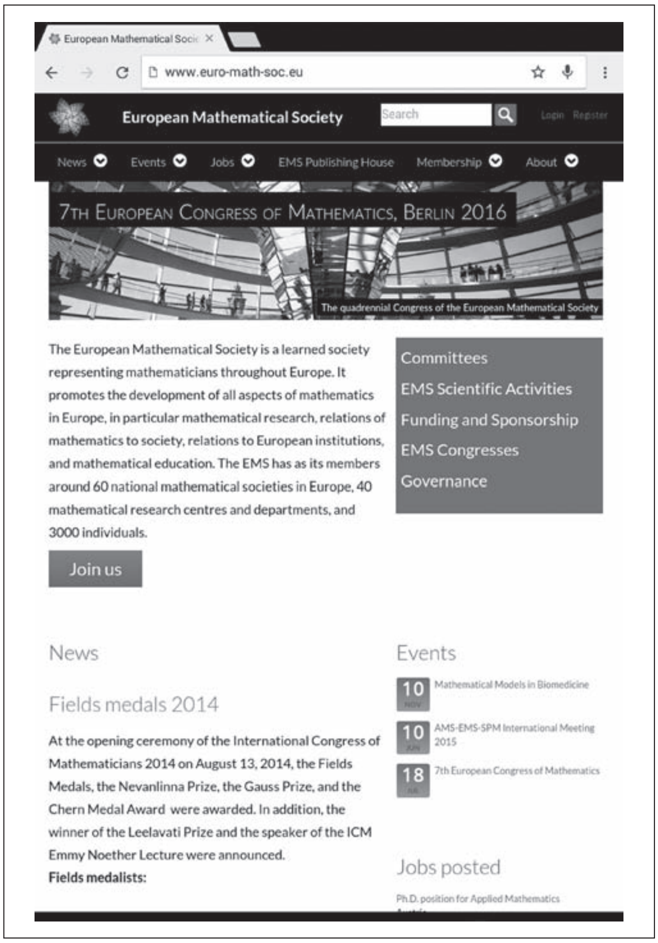
should appear regularly.

EMS Website www.euro-math-soc.eu Upgraded

Martin Raussen (Aalborg University, Denmark), EMS Vice-President and Content Administrator of the EMS Website

History

The EMS website www.euro-math-soc.eu, based at the office in Helsinki, went online in 2008. Before it was established, the EMS participated in the multi-purpose site www.emis.de, resulting in a reasonably complicated information flow. With our new independent website, it became easier to publish relevant information quickly and to use it for many different purposes.



Many aims

In fact, the EMS website has many different types of user and serves many different purposes. First of all, it is conceived as a service to the EMS membership (individuals and corporate) and to the mathematical community at large: to find information about interesting news and events, to post and to look for mathematical jobs, to post and to look for forthcoming conferences, to search for book reviews, etc. The EMS leadership, management and the EMS committees use the website to inform others about their work. Another important feature is the membership database containing information about individual and corporate EMS members. Members can pay their fees online through the site; new members can register online. Last but not least, the site collects and archives information about EMS activities, mainly for internal purposes.

New challenges

While all this had already been achieved, the technical facilities and the demands of users have developed quickly in recent years. Let me single out two particularly important trends. Firstly, mobile devices with small screens (iPads, iPhones, etc.) have become more and more popular; the EMS website was very difficult to read and to use on these platforms. Secondly, hackers try to use weak spots on all sorts of websites for their purposes, so far only with annoying and not disastrous effects for the EMS.

The EMS website is underpinned by a DRUPAL based content management system (CMS), organising an ever growing database. This CMS had been basically unchanged since it was built and put into use. It has become increasingly important to update the site's management and to bring it up to date in order to allow a better service to users and make it less vulnerable to attacks.

Development

In the Autumn of 2013, the EMS Executive Committee decided to commission a development project with a DRUPAL expert based in Scotland. After a thorough investigation of the many different uses and users of the website, he developed a "wiring" of the EMS website from scratch in the latest version of DRUPAL, which is quite different from the previous one.

New features

Apart from allowing the site to adapt to various forms of (in particular mobile) devices and making it less vulnerable to attacks, several other goals were set, among them the following: information added to the website will be much better structured (by additional tags) in order to make it easier to filter and to search for relevant information; EMS committees will be able to handle their own sub-sites autonomously under the same framework, partly for the public and partly for internal use that only committee members can access; users who wish to upload information will need to log on (using a simple procedure) in order to make abuse more difficult; users will be able to classify their main mathematical interests and will then be offered news about mathematical events, conferences, jobs and book reviews, etc., sorting under the same "tag"; and it will be easier to start online discussions on the site under suitable moderation.

Transition

On 29 June, in the midst of the EMS Council Meeting in Donostia, the newly designed website www.euro-math-soc.eu went online and replaced the previous one. Users (including this author) have to get used to the new design and the new options. Less is visible directly on the front page; you have to click a few times before you find what you are interested in. The biggest change for users is that you now need to log into a user account if you wish to comment on a particular item or if you wish to submit a news article, event, job posting or book review. Such submissions are still subject to manual review by EMS editors but it does allow contributors to be cross-referenced. Registration is straightforward, requiring name and email address; the email address is never disclosed to the public nor will it be passed on to any third party. After filling in a personal page, specific information is offered related to the main mathematical interest areas specified under the menu point "Interests". For more information and a link to a login page, go to http://www.euro-math-soc.eu/news/14/06/28/changes-ems-website.

Experiences? Problems?

It cannot be expected that a major operation like this one can be executed without problems and bugs. Everybody will have to get used to the new look and will need to learn how to make use of new ways of handling the information flow and how to profit from new features. Website users are encouraged to report their experiences and any problems they may encounter to the EMS office at ems-office@helsinki.fi – please use EMS web as the subject line.

Call for Applications EMS Schools in Applied Mathematics (ESSAM)

Jose Antonio Carrillo (Imperial College London, UK)

Currently, four series of summer schools in applied mathematics take place every year under the EMS banner. Their existence and organisation are part of the activities of the EMS Applied Mathematics Committee. The EMS considers as a priority the goal that the schools maintain a high scientific level and focus on relevant topics of impact.

The EMS helps these schools in fundraising and, in particular, contributes toward the participation of young researchers from European and Mediterranean countries.

Initially, there was a school organised by IMPAN in Bedlewo and a school organised by CIME taking place every year. In 2010, a summer school in biomathematics joined ESSAM, organised every year by the ESMTB (European Society for Mathematical and Theoretical Biology). The last ESSAM school was created in 2011: a summer school in mathematical finance, organised by the Institut Louis Bachelier.

The programme committee of each organising institution incorporates a representative of the EMS Applied Mathematics Committee for the planning of future schools and to certify their scientific level and the fulfilment of EMS requirements.

If you want to create such a series or you are in charge of an existing one and you want to become part of the ESSAM schools, please send a message to Stéphane Cordier at stephane.cordier@math.cnrs.fr.

List of ESSAM 2014 schools:

- EMS-ESMTB Summer School: "Dynamics of infectious diseases".
- IME-EMS Summer School in Applied Mathematics: "Centralized and Distributed Multi-agent Optimization: Models and Algorithms" (funded by the EMS).
- Seventh European Summer School in Financial Mathematics.
- EMS School on Stochastic Analysis with applications in biology, finance and physics (funded by the EMS).

You can find more information about these schools and their guidelines at the EMS Applied Committee's webpage: http://www.euro-math-soc.eu/ems-summer-schools-applied-mathematics-essam.

André Lichnerowicz Prize in Poisson Geometry – 2014

Rui L. Fernandes (University of Illinois at Urbana-Champaign, USA)

The André Lichnerowicz Prize was established in 2008 to be awarded for notable contributions to Poisson geometry. The prize is awarded every two years at the "International Conference on Poisson Geometry in Mathematics and Physics" to researchers who have completed their doctorates at most eight years before the year of the conference.

The prize was named in memory of André Lichnerowicz (1915–1998) whose work was fundamental in establishing Poisson geometry as a branch of mathematics. In 2014, it was awarded by a jury composed of the members of the scientific and advisory committees of the biennial Poisson Conference. The prize awarded 500 USD to each recipient, the funds provided by the host institution of the conference, the University of Illinois at Urbana-Champaign.

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The 2014 prize was awarded to
David Li-Bland and **Ioan Mărcuţ**
on 4 August 2014 at the University of Illinois
at Urbana-Champaign.

***

David Li-Bland received a PhD in mathematics at the University of Toronto in 2012 under the direction of Eckhard Meinrenken. He is currently an NSF Postdoctoral Fellow at UC Berkeley. Li-Bland has made important contributions to Dirac and Poisson geometry. In his thesis, he introduced and studied the infinitesimal counterparts of Courant groupoids and Dirac groupoids. In collaboration with Severa, he developed a theory of moduli spaces of flat connections on "quilted surfaces", with varying structure groups for different regions of the surface. They found that these moduli spaces have natural quasi-Poisson structures, and suggested a universal quantisation scheme applicable in this setup. Other accomplishments of Li-Bland include an integration procedure for exact Courant algebroids (with Severa), a classification of Dirac Lie groups (with Meinrenken) and the linear "derived" symplectic category (with Weinstein).

Ioan Mărcuţ received his PhD in mathematics from the University of Utrecht in 2013, under the supervision of Marius Crainic. He is currently a Postdoctoral Fellow at the University of Illinois at Urbana-Champaign. Mărcuţ has made fundamental contributions to the global geometry of Poisson structures, most notably through his (semi-)local forms and rigidity results. Such results range from generalisations of Conn's linearisation theorem to explicit computations of moduli spaces of Poisson structures. First of all, he extended the geometric approach of Crainic-Fernandes, proving a generalisation of Conn's theorem around symplectic leaves. He also clarified and simplified the original analytic approach of Conn, making it available for the study of other geometric structures and allowing him to prove a much more general rigidity result (around Poisson submanifolds); as an application, he provided the first explicit computation of a non-trivial Poisson moduli space. Other accomplishments of Mărcuţ include a direct geometric proof of the existence of symplectic realisations (with Crainic), the study of obstructions and deformations of log-symplectic structures (with Osorno Torres) and the study of transversals in Poisson geometry (with Frejlich).

150th Anniversary of the London Mathematical Society

Stephen Huggett (University of Plymouth, UK)

On the 16th January 1865 the London Mathematical Society held its inaugural meeting, at University College London. Augustus De Morgan was elected its first President. The Society grew rapidly, and quickly acquired a high reputation abroad: for example, in 1870 Michel Chasles commented approvingly on it, and encouraged mathematicians in France to follow suit. (Here we should also note that the Moscow Mathematical Society was founded one year earlier, in 1864: happy anniversary to our Russian colleagues!)

There were monthly meetings at which papers were read, notably by people such as Cayley, Sylvester, Clifford, Maxwell, and Kelvin. Some of these were published in the Proceedings, but this was not automatic: from the beginning, there was a strict refereeing process! Papers which survived this process included Maxwell's 1871 paper *On the mathematical classification of physical quantities*, and Turing's 1937 paper *On Computable Numbers, with an Application to the Entscheidungsproblem*.

Of the many distinguished mathematicians who have served as President, we may single out Hardy as having had a particularly significant effect: the Society was already international in outlook, but he vigorously promoted this, which we remember today in our Hardy Fellowship. He worked very hard for the Society throughout his career, and gave a considerable bequest to the Society, which has helped us to remain a strong independent voice.[1]

Today the top priority for the Society remains support for research in mathematics, through its research grants schemes, through its advocacy of good research policy to research councils and government, and of course through its publications.

On the 16th of January 2015 we will be holding a big meeting in London to launch a year of celebrations of our 150th anniversary. It will be on the theme of Mathematics: Unlocking Worlds, and it is planned to make it accessible on the web. This will be the first of many events, some of them completely new for us, and some of them things that we usually do but that we plan to make even better during our anniversary year. Please look at www.lms.ac.uk/2015-events-listing for all the details, but here are two examples.

The Society has had women members since the 1880s, so it is especially appropriate that from the 14th to the 17th of April there will be a Women in Mathematics Celebration held in the Mathematical Institute at the University of Oxford. Young mathematicians, from those just leaving school to postdoctoral researchers, will hear talks from some of the most eminent women mathematicians of recent years, demonstrating what a career in mathematics can achieve. As well as these inspiring mathematical talks, there will be opportunities for networking and socialising.

From the 11th to the 13th of September there will be an LMS/EMS Joint Mathematical Weekend held at Birmingham University. This also celebrates the 25th Anniversary of the European Mathematical Society! The emphasis of the meeting will be on Algebra, Analysis, and Combinatorics, and the interactions between these areas and probability theory. In addition, the Societies are keen to promote participation by young researchers from across Europe via a session for postdoctoral researchers and a poster session for PhD students.

Members of the European Mathematical Society are very welcome indeed to take part in our celebrations.

Stephen Huggett was the Programme Secretary of the London Mathematical Society from 2001 to 2011, and has been its General Secretary since 2012.

1 For those interested in the history of the Society, a good source is the introduction in Oakes S, Pears A, Rice A, *The Book of Presidents 1865–1965* London Mathematical Society 2005.

Call for Nominations for the Ostrowski Prize, 2015

The aim of the Ostrowski Foundation is to promote the mathematical sciences. Every second year it provides a prize for recent outstanding achievements in pure mathematics and in the foundations of numerical mathematics. The value of the prize for 2015 is 100.000 Swiss francs.

The prize has been awarded every two years since 1989. The most recent winners are Ben Green and Terence Tao in 2005, Oded Schramm in 2007, Sorin Popa in 2009, Ib Madsen, David Preiss and Kannan Soundararajan in 2011 and Yitang Zhang in 2013. See

http://www.ostrowski.ch/index_e.php?ifile=preis

for the complete list and further details.

The jury invites nominations for candidates for the 2015 Ostrowski Prize. Nominations should include a CV of the candidate, a letter of nomination and 2–3 letters of reference. The Chair of the jury for 2015 is Christian Berg of the University of Copenhagen, Denmark. Nominations should be sent to berg@math.ku.dk by April 15, 2015.

James Maynard to Receive 2014 SASTRA Ramanujan Prize

Krishnaswami Alladi (University of Florida, Gainsville, USA), Chair SASTRA Ramanujan Prize Committee

The 2014 SASTRA Ramanujan Prize will be awarded to Dr James Maynard of Oxford University, England, and the University of Montreal, Canada.

The SASTRA Ramanujan Prize was established in 2005 and is awarded annually for outstanding contributions by young mathematicians to areas influenced by the genius Srinivasa Ramanujan. The age limit for the prize has been set at 32 because Ramanujan achieved so much in his brief life of 32 years. The prize will be awarded on 21–22 December at the International Conference on Number Theory at SASTRA University in Kumbakonam (Ramanujan's hometown), where the prize has been given annually.

Dr Maynard has made spectacular contributions to number theory, especially on some famous problems on prime numbers. The theory of primes is an area where questions which are simple to state can be very difficult to answer. A supreme example is the celebrated "prime twins conjecture", which states that there are infinitely many prime pairs that differ by 2. In the last 12 months, Dr Maynard has obtained the strongest result towards this centuries old conjecture by proving that the gap between consecutive primes is no more than 600 infinitely often. Not only did he significantly improve upon the earlier groundbreaking work of Goldston, Pintz, Yildirim and Zhang but he achieved this with ingenious methods that are simpler than those used by others. Maynard's remarkable success on the "small gap problem" on primes is built upon the ideas and results he developed in his doctoral work a few years ago on some other famous problems on primes such as the Brun–Titchmarsh inequality. Maynard's results and methods have led to a resurgence of activity worldwide in prime number theory.

In his doctoral thesis of 2013 at Oxford University, written under the direction of Professor Roger Heath-Brown, Maynard obtained a number of deep results on some fundamental problems. One of the intriguing and important questions on prime numbers is how uniformly they are distributed in various arithmetic progressions of integers up to a given magnitude. There is an heuristic estimate for the number of such primes in arithmetic progressions and one usually gets a bound of the order of magnitude given by the heuristic. This Brun–Titchmarsh problem becomes difficult when the modulus or gap between the members of the progression becomes very large and results are weaker compared to the conjectured size. Earlier researchers had relied on a certain unproven hypothesis concerning the Siegel zeros of L-functions in order to treat these large moduli. But Maynard showed in his thesis (without appeal to any hypothesis) that the number of primes in arithmetic progressions is bounded by the suspected heuristic size for arithmetic progressions whose moduli do not exceed the eighth root of the largest member of the progression. This important paper appeared in *Acta Arithmetica* in 2013.

A generalisation of the prime twins conjecture is the prime k-tuples conjecture, which states that an admissible collection of k linear functions will simultaneously take k prime values infinitely often in values of the argument. In the last 100 years, several partial results toward the k-tuples conjecture have been obtained either by replacing prime values of some of these linear functions by "almost primes" (which are integers with a bounded number of prime factors) or by bounding the total number of prime factors in the product of these linear functions. Another major achievement in his doctoral thesis is his work on "Almost-prime k-tuples" (which was published in *Mathematika* in 2014) in which he obtains bounds for the number of prime factors in the product of these admissible linear functions. These bounds are superior to bounds obtained by earlier researchers, except in the case of the product of three linear functions, where his result was just as strong as the 1972 result of Porter, who had confirmed that the triple product will have no more than eight prime factors infinitely often. But in a separate paper that appeared in the Proceedings of the Cambridge Philosophical Society in 2013, Maynard broke the impasse by improving on Porter's result and showing that a product of three admissible linear functions will have no more than seven prime factors infinitely often. In establishing these fundamental results, Maynard introduced a number of new methods and techniques that enabled him to achieve a sensational result on the small gaps problem on primes within a year of completing his DPhil – when he was a post-doctoral fellow at the University of Montreal, Canada, with Professor Andrew Granville as his mentor.

The prime number theorem implies that the average gap between the n-th prime and the next prime is asymptotic to log n. Two questions arise immediately:

(i) How small can this gap be infinitely often (the small gap problem)? and
(ii) How large can this gap be infinitely often (the large gap problem)? The prime twins conjecture says that the gap is two infinitely often.

It was a sensation a few years ago when Goldston, Pintz and Yildirim (GPY) showed that over a sequence of integers n that tend to infinity, the gap between the n-th prime and the next can be made arbitrarily smaller than the average gap. It was shown by GPY that if a certain conjecture of Elliott and Halberstam on the distribution of primes in arithmetic progression holds then the gap could be made as small as 16 infinitely often.

Two years ago, Y. Zhang stunned the world by showing that infinitely often the smallest gap is no more than 70 million! This was the first time a bounded gap was established unconditionally. For this seminal work on the small gap problem, Goldston, Pintz, Yildirim and Zhang received the 2014 Cole Prize of the American Mathematical Society.

Zhang's method was quite complex. He had to circumvent the use of a hypothesis that went beyond the Bombieri–Vinogradov theorem. Terence Tao, by leading the Polymath project, reduced Zhang's bound to 4680. But within a few months of Zhang's achievement, Maynard took the world by storm by establishing, in a simpler and more elegant fashion, that the gap between the primes is no more than 600 infinitely often! This sensational paper on "Small gaps between primes" will soon appear in the *Annals of Mathematics*. Actually, in this paper, Maynard establishes a number of other deep results. For example, he shows that for any given integer m, the gap between the $n+m$-th prime and the n-th prime is bounded by a prescribed function of m. Very recently, Maynard has joined the Polymath project and now the gap between consecutive primes has been shown to be no more than 246 infinitely often, by adapting the method in Maynard's paper in the *Annals of Mathematics*. This is to appear in the journal *Research in the Mathematical Sciences*.

Within the last month, Maynard has announced a solution of the famous \$10,000 problem of Paul Erdős concerning large gaps between primes. This was also simultaneously announced by Kevin Ford, Ben Green, Sergei Konyagin and Terence Tao but Maynard's methods are different and simpler. In 1938, Robert Rankin established a lower bound for infinitely many large gaps, which remained for many years the best result on the large gap problem. The great Hungarian mathematician Paul Erdős asked whether the implicit constant in Rankin's lower bound could be made arbitrarily large and offered \$10,000 to settle this question either in the affirmative or in the negative. In the last 50 years, there have been improvements made by various leading researchers on the value of the implicit contant in Rankin's bound – by Rankin himself in 1962, by Helmut Maier and Carl Pomerance in 1990 and by Janos Pintz in 1997 – but the Erdős problem still remained unsolved. Maynard has now shown that the implicit constant in Rankin's lower bound could be made arbitrarily large. So in settling this notoriously difficult problem, an impasse of several decades has been broken. Another fundamental recent work of Maynard is a collaboration with William Banks and Tristan Freiburg on the limit points of the values of the ratio of the gap between consecutive primes and the average gap. Results on the small gaps problem imply that 0 is a limit point and the work on the large gap problem shows that ∞ is a limit point. No other limit point is known. In this joint paper it is shown that if 50 random real numbers are given then one at least of the 50 consecutive gaps between them and 0 will occur as a limit point. Thus, in the last three years, Maynard has obtained several spectacular results in the theory of primes by methods which will have far reaching implications.

James Maynard was born in Chelmsford, England, on 10 June 1987. He obtained his BA and Master's in Mathematics from Cambridge University in 2009. He then joined Balliol College, Oxford University, where he received his Doctorate in Philosophy in 2013. During 2013–14 he was a post-doctoral fellow at the University of Montreal, Canada. Within a period of three years, Maynard has startled the mathematical world with deep results in rapid succession. The SASTRA Ramanujan Prize will be his first major award in recognition of his many fundamental results in prime number theory.

The 2014 SASTRA Ramanujan Prize Committee consisted of Professors Krishnaswami Alladi – Chair (University of Florida), Roger Heath-Brown (Oxford University), Winnie Li (Pennsylvania State University), David Masser (University of Basel), Barry Mazur (Harvard University), Peter Paule (Johannes Kepler University of Linz) and Michael Rapoport (University of Bonn).

Previous winners of the prize are Manjul Bhargava and Kannan Soundararajan in 2005 (two full prizes), Terence Tao in 2006, Ben Green in 2007, Akshay Venkatesh in 2008, Kathrin Bringmann in 2009, Wei Zhang in 2010, Roman Holowinsky in 2011, Zhiwei Yun in 2012 and Peter Scholze in 2013. The award of the 2014 SASTRA Ramanujan Prize to James Maynard is a fitting recognition in the tenth year of this prize and is in keeping with the tradition of recognising groundbreaking work by young mathematicians.

Krishnaswami Alladi is a professor of mathematics at the University of Florida, Gainesville, where he was Department Chairman during 1998–2008. He received his PhD from UCLA in 1978. His area of research is number theory. He is the Founder and Editor-in-Chief of The Ramanujan Journal *published by Springer. He helped create the SASTRA Ramanujan Prize given to very young mathematicians and has chaired the prize committee since its inception in 2005.*

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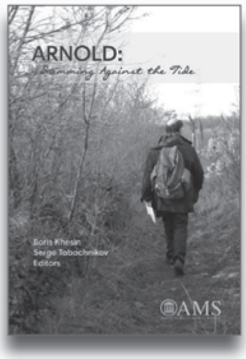

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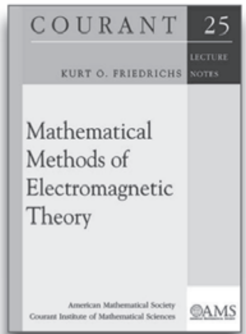

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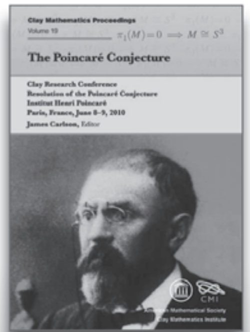

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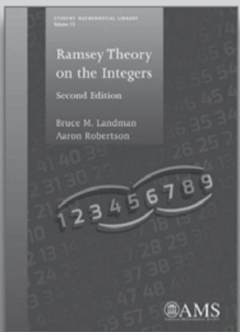

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The "Bounded Gaps between Primes" Polymath Project

A Retrospective Analysis

D. H. J. Polymath

For any $m \geq 1$, let H_m denote the quantity $H_m := \liminf_{n\to\infty} (p_{n+m} - p_n)$, where p_n denotes the n^{th} prime; thus, for instance, the twin prime conjecture is equivalent to the assertion that H_1 is equal to two. In a recent breakthrough paper of Zhang, a finite upper bound was obtained for the first time on H_1; more specifically, Zhang showed that $H_1 \leq 70\,000\,000$.

Almost immediately after the appearance of Zhang's paper, improvements to the upper bound on H_1 were made. In order to pool together these various efforts, a *Polymath project* was formed to collectively examine all aspects of Zhang's arguments and to optimise the resulting bound on H_1 as much as possible. After several months of intensive activity, conducted online through blogs and wiki pages, the upper bound was improved to $H_1 \leq 4680$. As these results were being written up, a further breakthrough was introduced by Maynard, who found a simpler sieve-theoretic argument that gave the improved bound $H_1 \leq 600$, and also showed for the first time that H_m was finite for all m. The Polymath project, now with Maynard's assistance, then began work on improving these bounds, eventually obtaining the bound $H_1 \leq 246$, as well as a number of additional results, both conditional and unconditional, on H_m.

In this article, we collect the perspectives of several of the participants of these Polymath projects, in order to form a case study of online collaborative mathematical activity and to speculate on the suitability of such an online model for other mathematical research projects.

1 Introduction

In a blog post [19] from January 2007 entitled "Is massively collaborative mathematics possible?", Timothy Gowers proposed a new format for solving mathematical research problems, based on a large number of small contributions from many mathematicians in a public online forum, as opposed to the traditional model of a small number of mathematicians collaborating privately and intensively on a single problem. After an enthusiastic online response to this blog post, Gowers soon launched the first of the "Polymath" projects. This project, now known as Polymath1, had a specific and feasible-looking goal: to take an important theorem in density Ramsey theory (the density Hales-Jewett theorem of Furstenberg-Katznelson [17]), which until this project had only a difficult ergodic-theoretic proof, and find a purely combinatorial proof of the statement. Within 24 hours of the online launch of this project, Gowers' blog post was already flooded with a large number of mathematical comments, ideas and observations. Promising approaches were soon identified and explored, reading seminars were set up to try to digest difficult papers in the literature and a wiki was also set up to maintain links to all relevant online resources, as well as lengthier arguments not easily placed in a blog comment. After six weeks of hectic activity, with contributions from dozens of mathematicians, Gowers was able to declare victory: a convincing outline of an elementary proof of the density Hales-Jewett theorem had been found. The process of formally writing up the results [39, 40] took somewhat longer (closer to six months than six weeks); nevertheless, the project was a considerable success by most reasonable standards. See [20] for a further discussion of the Polymath1 project.

After Polymath1, several further Polymath projects (and Polymath-like projects) were launched by Gowers and several other mathematicians, with varying degrees of success. For instance, Polymath4, whose aim was to find fast, deterministic methods to locate primes, ended up with a modest partial result, published[1] in [44], although the original aim of improving the fastest previously known method was not actually achieved. An online collaborative project to analyse a claimed proof of the infamous $P \neq NP$ problem by Deolilakar was not "officially" a Polymath project but followed its format and used the same wiki as the Polymath projects; it ended up with a strong consensus that the errors discovered in the argument were unlikely to be fixable (see [26, Chapter 1]). A number of smaller "mini-Polymath" projects, based around solving problems from the International Mathematical Olympiads, were also successful (with the problems chosen invariably being solved within 48 hours) and were generally enjoyed by the participants, although such projects were more of a social mathematical event than a genuine mathematical research programme (see [31] for an analysis of one of these projects). On the other hand, several Polymath projects, even after promising starts (with a level of intensity and participation comparable to that of the Polymath1 project), eventually stalled, with initial, promising ideas to attack the problem encountering seemingly insuperable obstacles and with the level of attention given by the participants falling below a critical mass. However, even the stalled projects have made some contributions toward their objectives, which (due to the open nature of these projects) are freely available for other researchers to use; for instance, some of the ideas developed in the stalled Polymath7 project to solve the "hot spots" con-

1 The original intention was to publish this article under the agreed pseudonym of "D. H. J. Polymath" but the journal requested that the authors' true names and affiliations be used instead.

jecture for acute triangles were used (with permission) in a separate publication [43] outside of the Polymath project.

This article is devoted to the most recent Polymath project, namely the Polymath8 project set up to understand, build upon and improve the breakthrough work of Zhang [47] on bounded gaps between primes. More precisely, for any $m \geq 1$, let H_m denote the quantity $H_m := \liminf_{n\to\infty}(p_{n+m} - p_n)$, where p_n denotes the n^{th} prime; thus, for instance, the twin prime conjecture is equivalent to the assertion that H_1 is equal to two. Prior to the work of Zhang, it was not known unconditionally if any of the H_m were finite, although under the strong additional assumption of the Elliott-Halberstam conjecture [12], the bound $H_1 \leq 16$ was shown in an important paper of Goldston, Pintz and Yıldırım [18], who also established the unconditional result $\liminf_{n\to\infty} \frac{p_{n+1}-p_n}{\log p_n} = 0$. By reviving and brilliantly modifying some arguments of Bombieri, Fouvry, Friedlander and Iwaniec [14, 15, 6, 7, 8] and combining[2] this with variants of the Goldston-Pintz-Yıldırım argument, Zhang established the bound $H_1 \leq 70000000$. In his paper, Zhang comments: "This result is not optimal ... to replace the [upper bound on H_1] by a value as small as possible is an open problem that will not be discussed in this paper."

This provided an irresistible challenge to many of the mathematicians following these developments. One week after the release of Zhang's preprint, it was remarked by Mark Lewko (as an offhand comment on the website MathOverflow) that by replacing a single crude upper bound in Zhang's paper, one could improve the upper bound on H_1 to 63374611; one week after that, it was observed by Trudgian [46], in a short arXiv preprint, that another "cheap" tweak to Zhang's argument could lower the bound further to 59874594. A few days later, in a blog post of Morrison [30], another slight (and computer-assisted) optimisation to Zhang's construction improved the bound further to 59470640. At this point, several other mathematicians began to join in the comments of Morrison's post, contributing further small observations and improvements – very much in the spirit of the "Polymath" enterprise. Indeed, after some quick discussion on the Polymath blog [38], it was agreed to retroactively convert Morrison's post to the first post on what would now become the eighth Polymath project, whose stated aims were to understand Zhang's argument as thoroughly as possible and then to optimise the bound on H_1 from that argument. After three months of intense activity, in which the upper bound H_1 was repeatedly lowered[3] by a variety of methods, the upper bound on H_1 eventually stabilised at $H_1 \leq 4680$.

At this point, attention began to turn to the lengthy, but necessary, process of writing up the results [41]. But, as the paper was nearing completion, news of another breakthrough in the subject emerged: by a clever modification to the original sieve-theoretic techniques of Goldston, Pintz and Yıldırım, Maynard [28] was able to circumvent many of the technicalities of the arguments of Zhang (or of Polymath8) and obtain a further significant improvement of the bound, to $H_1 \leq 600$; furthermore, bounds on H_m for larger m were now available and, under the assumption of the Elliott-Halberstam conjecture, the previous bound of $H_1 \leq 16$ was improved slightly to $H_1 \leq 12$.

With this new advance, the Polymath8 participants decided to start a second phase to the project (concurrently with finishing the writing up of the first phase). In this second phase, dubbed Polymath8b, the new methods of Maynard would be combined with the existing results of the first phase (now dubbed Polymath8a) to obtain further improvements to H_1 and H_m. After about eight months of intense activity, the results of this paper were also written up and submitted [42]; the bound on H_1 had been lowered unconditionally to 246 and the bound $H_1 \leq 6$ was obtained, assuming the generalised Elliott-Halberstam conjecture (with the latter bound being the limit of the sieve-theoretic method). Furthermore, a number of technical sieve-theoretic innovations have been discovered en route to these results, which will hopefully have value for other problems in analytic number theory beyond the bounded gaps between primes problem.

In this retrospective analysis, we record various first-hand impressions of the Polymath8 project as it unfolded, both from active participants and from casual followers, who have reported both on their experience and on their thoughts on whether the success of this particular project can be replicated for other problems. The contributions are arranged in chronological order of submission.

2 Terence Tao

Terence Tao is a professor of mathematics at UCLA.

The first that I heard about Zhang's breakthrough on bounded gaps between primes was on 13 May 2013, when I started getting emails from friends and colleagues about a rumoured breakthrough on this problem in number theory. I was intrigued but initially sceptical; the closely related twin prime conjecture, for instance, has attracted more than its fair share of arguments that ended up completely falling apart, and Yitang Zhang had published little in the area for over a decade. But then a trusted friend passed on the news that Zhang's result on bounded gaps had apparently already been refereed and accepted by the Annals of Mathematics and also reported some details of the argument (a combination of the Goldston–Pintz-Yıldırım method and the arguments of Bombieri, Fouvry, Friedlander and Iwaniec). At that point, I felt the information I had was credible enough to post a brief comment about it on my own Google+ page. By the next day, the Annals put up the preprint on its webpage, Zhang presented his work in a special seminar in Harvard and suddenly the online maths community was buzzing with the result.

Initially, I was preoccupied with other research projects and felt content to let the rest of the analytic number theory community digest the result; I sent Zhang a quick congratulatory email and returned to work. A week or two later, when the first blog post appeared, in which Zhang's original bound of 70 000 000 was lowered to 59 470 640 with some computer assistance, I again commented on this on my Google+ page, figuring that this could be an opportunity for number theory enthusiasts to make a nice contribution to this subject, and planned again to return to work … but then decided to spend just a few minutes fiddling with the problem myself.

2 For a more in-depth treatment of the mathematics of Zhang's work, see [22].

3 A timeline of improvements may be found at michaelnielsen.org/polymath1/index.php?Timeline_of_prime_gap_bounds.

Zhang arrived at the bound of 70 000 000 by constructing an "admissible k-tuple" whose diameter was bounded by this amount. An admissible k-tuple is a set of k increasing integers, with the property that it misses at least one residue class mod p for each prime p; for instance $(0, 2, 6)$ is an admissible 3-tuple but $(0, 2, 4)$ is not. These tuples play a central role in the famous *prime tuples conjecture* of Hardy and Littlewood [23], which generalises the twin prime conjecture and is still completely open, despite many partial results. In Zhang's work, k had to be sufficiently large; his analysis allowed him to take $k = 3\,500\,000$. Zhang constructed his admissible tuple by considering a block of k consecutive primes that was guaranteed to be admissible; the subsequent improvements came from finding slightly narrower blocks of consecutive primes and checking the admissibility by computer. The discussion reminded me of some work of Hensley and Richards [24], who used admissible tuples to show that the prime tuples conjecture contradicted another conjecture of Hardy and Littlewood about the prime counting function $\pi(x)$; I did a quick calculation that suggested that this result ought to lower the bound further to 58 885 998, posted this on the blog and then tried to return to "real" work again.

But something kept drawing me back. At the time, I was working on a lengthy research project which I felt would take months, if not years, to complete (and indeed, it is still far from completion at the time of writing and was, in fact, shelved as the Polymath project began to absorb more of my time). In contrast, with just ten minutes of effort, there was a chance to push the bound down a little more and claim, however briefly, the "world record" for bounded gaps between primes. Plus, it gave me an excuse to actually go through Zhang's paper in more detail. So I found myself returning again to the blog (which was now becoming quite lively with other comments and contributions) and digging further and further into Zhang's paper to try to squeeze out more improvements. The initial gains had come purely from reading the first two pages of Zhang's 56-page paper, keeping his value 3 500 000 of k intact; by reading the next 20 or so pages, we found out how this number was arrived at (by combining a sieve-theoretic argument of Goldston, Pintz and Yıldırım with a deep, new distributional estimate on primes) and began to optimise this value too. Again, we began with "cheap" numerical optimisations, for instance using numerics to lower k to 2 947 442, which in turn lowered the H_1 bound to 48 112 378, but then started looking carefully at how Zhang controlled various error terms to make more significant gains; within a few days, we had reduced the bounds for both k and H_1 by about a full order of magnitude.

By this point, it had become abundantly clear that the project could benefit from being organised as a Polymath project, with a wiki page to keep track of all the progress and links to resources, and with a separate thread to discuss administrative issues. We hastily proposed retroactively converting Morrison's blog post to such a project, with the twin goals of improving the bound on H_1 and also understanding and clarifying Zhang's argument (and related arguments in the literature). Reaction to this proposal was generally positive but there were some objections. Chief among these was the concern that the existence of such a prominent project, with many mathematicians participating, might discourage and intimidate others from trying to contribute to this development of the subject. However, it was felt that there would be a large surge of interest in the problem even if there was no formal Polymath project and that the narrow focus of the project would allow other work relating to Zhang's breakthrough. Indeed, in the months after Zhang's paper, there were a number of other preprints [32], [1], [27], [16], [2], [45] that improved Zhang's result in ways other than a direct improvement of H_1, as well as preprints [33], [28] that improved the H_1 bound beyond the record obtained by Polymath8; so it appears that the Polymath8 project did not, in fact, "crowd out" traditional research in this direction. Indeed, with the additional breakthrough of Maynard [28], the pace of new research papers in this area has even accelerated [4], [5], [25], [10], [36], [3], [29], [37], [34], [11], [35].

The Polymath8 project naturally split into three pieces: the search for narrow admissible tuples of a given cardinality k; the refinement of the value of k; and the further reading of Zhang's paper. Scott Morrison's blog post was already hosting a lively discussion of the first component of this project; in early June, I wrote two further blog posts to host the discussion of the other two components, while Scott "rolled over" his blog post (as per the usual Polymath custom) to a fresh one, in which he summarised the previous progress.

The most immediate route to improving k was to digest and then optimise the sieve-theoretic components of Zhang's work; this followed fairly closely the previous work of Goldston, Pintz and Yıldırım which I was already familiar with, so I spent most of my efforts on this part of the project, in particular writing up detailed notes on the GPY sieve on my blog. Almost immediately, high-quality mathematical comments started rolling in from many different readers: typos and other minor errors in the blog post were quickly pointed out and different aspects of the argument were discussed. It was quickly noticed that a certain cutoff function f in the definition of the sieve could be optimised to improve the value of k; this led to a calculus of variations problem which was soon solved with the aid of Bessel functions, bringing k (in principle, at least) all the way down to 34 429 (and H_1 to about 390 000). It was then quickly pointed out that this Bessel function optimisation had been previously worked out in [13] (and in earlier unpublished work of Conrey); indeed, one of the advantages of the Polymath projects, with their broad level of participation, is that connections to relevant literature are very likely to be unearthed by at least one of the participants.

By the second week of June, a couple of us had managed to complete our collaborative online reading of Zhang's paper, in particular understanding how the distributional estimates on primes that were used to bound k arise from three key estimates, which Zhang called "Type I", "Type II" and "Type III" estimates. I then prepared some technical blog posts on these estimates, as well as the combinatorial argument Zhang used to merge them together. The Polymath project had now split into *five* active and loosely interacting components: one group focused on optimising the Type I and Type II estimates (which were a lengthy but fairly straightforward application of standard analytic number theory methods, such as the Linnik dispersion method and Weil exponential sum estimates); one group focused on the Type III estimates (which were somewhat more exotic, relying on an exponential sum estimate of

Birch and Bombieri, which in turn used Deligne's work on the Riemann hypothesis on varieties); one group focused on the combinatorics of putting these estimates together to create a distributional estimate on primes; one group focused on optimising the GPY sieve to convert distributional estimates on primes into a concrete value of k; and one group focused on finding narrow prime tuples which converted values of k to values of H. We had managed to organise ourselves into a sort of factory production line: an advance in, say, the Type I estimates would be handed over to the combinatorics group to produce a new distributional estimate in primes, which the sieve team would then promptly convert into a revised value of k, which the prime tuples team would then use to update their value of H_1. Steady improvements from all of these groups led to new improvements to these values on a daily basis for several weeks (see michaelnielsen.org/polymath1/index.php?title=Timeline_of_prime_gap_bounds for a timeline of these gains).

Many important contributions came from mathematicians who were not initially involved in the Polymath project. For instance, in the first week of June, Janos Pintz released a preprint [33] in which he went through Zhang's paper and optimised all the numerical constants, leading to further improvements in the values of k and H_1; within a day or two, the Polymath team had gone through the preprint (correcting some arithmetic errors along the way), lowering k to 26 024 and H_1 to about 280 000. Later, we ran into a problem in which we became victims of our own success: the values of k had dropped so low (around 6000) that a certain error term in the sieve analysis that decayed exponentially in k became non-negligible. Pintz, who had followed developments closely, came up with a much more efficient way to truncate the sieve that rendered this error term almost non-existent (and dropping k almost immediately from 6000 to about 5000) and then shared these computations with the Polymath project. Similarly, Etienne Fouvry, Emmanuel Kowalski, Phillipe Michel and Paul Nelson, who had previously worked on using Deligne-type exponential sum estimates to obtain distributional estimates similar to Zhang's Type III estimates, found a simpler approach than Zhang to these estimates that gave significantly better numerology and they also donated these arguments to the Polymath project and began actively participating in the following discussion. Thomas Engelsma, who had made extensive numerical computations of narrow admissible tuples over a decade ago, also opened up his database to our project.

By the end of June (with k dipping below 1000 and H falling below 7000), our understanding of various components of the project had matured significantly, though the pace of discussion was still lively (with perhaps 20 comments each day, down from a peak of 50 or so each day). A database had been set up at math.mit.edu/~primegaps/ to automatically record the narrowest known prime tuples for a given value of k (up to $k = 5000$), which automated the task of converting k-values to H-values, with several of the more computationally minded participants competing to add ever narrower tuples to the database. A prime distribution estimate could similarly be converted into a value of k with a few lines of Maple code, using Pintz's version of the truncated sieve, and a further few lines of code allowed us to automatically convert any improvement in the Type I, II, III estimates to a prime distribution estimate in an optimised fashion. The Type II and Type III estimates had been optimised to such an extent that they were no longer the dominant barrier in further improvement of the prime distribution estimates. The Type I estimates were the only remaining major source of improvement but over the course of the next few weeks, several new ideas on optimising these estimates (most notably, using the q-van der Corput method of Graham and Ringrose [21], combined with the theory of trace weights arising from the work of Deligne, and carefully splitting the summations to maximise the effectiveness of the Cauchy-Schwarz inequality) were implemented, leading to the final values of $k = 632$ and $H_1 = 4680$.

Further numerical improvements looked very hard to come by and so, for the next few months, the Polymath project participants devoted their efforts to writing up the results, splitting the paper into several sections on a shared Dropbox folder, with various participants working in separate sections in parallel and coordinating their efforts through blog comments. Somewhat to our surprise, once all the various arguments, spread out over a dozen blog posts, had been collated together, the final paper turned out to be extremely large – 163 pages, to be exact[4] (for comparison, Zhang's original paper was 53 pages in length). Somehow, the Polymath format had managed to efficiently segment the research project into more manageable chunks, so that no individual participant had to absorb the entire 163-page argument at any given time while the research was ongoing.

In late October, when the Polymath paper was going through several rounds of proofreading in preparation for submission, James Maynard announced a major breakthrough [28], in which the prime gap H_1 was lowered substantially to 700 (quickly revised downward to 600), with the improvement $H_1 \leq 12$ available under the assumption of the Elliott-Halberstam conjecture. Furthermore, Maynard had found a way to modify the GPY sieve argument in such a way that the new distribution estimates on primes (the most difficult and novel part of Zhang's work and also of the Polymath project) were no longer needed and, furthermore, the methods could for the first time be used to bound H_m for larger m. (I had also begun to arrive at similar conclusions independently of Maynard, though I did not get the precise numerical results that Maynard did.) After some discussion with Maynard and with the Polymath team, it was decided that we should both publish our results separately and then join forces to see if the new ideas in Maynard's paper could be combined with those in the Polymath paper to obtain further improvements (this effort was soon dubbed "Polymath8b").

I had some mixed feelings about the continuation of the Polymath8 project; running the Polymath8a project had already consumed several months of my time and I was thinking of turning to unrelated research projects. However, the enthusiasm of the other participants and the lure of getting a quick payoff from comparatively brief snatches of mathematical thought were once again too great to resist.

4 After significant revisions in the light of the referee report and subsequent developments, the paper has since been shortened somewhat to 107 pages.

Interestingly, Polymath8b ended up moving in a rather different mathematical direction to Polymath8a; it turned out to be quite difficult to effectively combine the prime distribution estimates from Polymath8a with Maynard's sieve to bound H_1 (although they could be used to bound H_m for larger values of m) and it was soon realised that the most promising way forward was to either optimise or generalise a certain multidimensional calculus of variations problem used in Maynard's work.

The former approach was initially more fruitful; by taking the code Maynard used to obtain lower bounds for a variational problem by testing that problem on polynomials, and making it run faster and more efficiently on more powerful computers, we managed to cut down the unconditional H_1 bound from 600 to 300. However, improving the conditional bound of $H_1 \leq 12$ was far tougher; indeed, we had established an upper bound on the variational problem that Maynard used to show that we could not hope to improve upon this bound without modifying the sieve. In order to break the 12 barrier, the efficiency of our sieve (measured by a quantity that we called M_4, which arose from a four-dimensional variational problem) had to exceed 2. There was then a lengthy and frustrating "Zeno's paradox" period in which the efficiency M_4 of our sieves kept improving incrementally (from 1.845, to 1.937, to 1.951, etc.) but never quite enough to surpass the magic threshold of 2 needed to break the barrier. Finally, there was a breakthrough; after several weeks of effort, we stumbled upon an "epsilon trick" that allowed one to slightly enlarge the class of permissible cutoff functions in the variational problem, at the expense of worsening the quantity that one was trying to optimise. It turned out that this tradeoff was advantageous, allowing us to move M_4 to 2.018 and to reduce the bound to $H_1 \leq 8$ (though we had to replace the Elliott-Halberstam conjecture with a strengthening of that conjecture, which we call the generalised Elliott-Halberstam conjecture). A similar "Zeno's paradox" game then played out for the analogous three-dimensional variational problem M_3, which after several further refinements, both to the sieve and to the numerical procedure for optimising the sieve, eventually pushed M_3 to be larger than 2 as well, giving the optimal result $H_1 \leq 6$ assuming the generalised Elliott-Halberstam conjecture. (The parity obstruction of Selberg prevents any better bound on H_1 from purely sieve-theoretic considerations.) Some of this new technology also allowed a slight lowering of the unconditional bound of H_1 to 246 but further improvement beyond this point seemed to require enormous amounts of computation and by early May we were happy to "declare victory" at this point and write up the Polymath8b results.

The Polymath8 project had perhaps one or two dozen active participants but many more mathematicians and interested amateurs followed the progress online. During the project, whenever I visited another institution, I was usually asked what the latest value of H_1 was and how low I thought it would go. In that respect, we were fortunate that we had such a simple and easily understood statistic that could be used as a proxy for the degree of technical advancement we were making; it is not clear if future Polymath projects will be as easy to follow on a casual basis.

The project also forced me to think and work in different ways from what I was accustomed to. I do not have extensive experience with programming, and most of the really heavy computational work was done by others on this project, but I did manage to write up some simple Maple code to at least verify the numerical computations that others had generated. At another juncture, the way forward hinged on finding the optimal way to decompose the unit cube into polyhedral pieces; lacking sufficient geometric or algebraic intuition, I ended up having to build a cubic lattice out of my son's construction toys in order to visualise all the decompositions being proposed. I also found myself having to learn areas of mathematics I would not otherwise have been exposed to, from ℓ-adic cohomology to the Krylov subspace method. All in all, it was an exhausting and unpredictable experience but also a highly thrilling and rewarding one.

3 Andrew Gibson

Andrew Gibson is an undergraduate mathematics student at the University of Memphis.

Shortly after Zhang announced his result and you (Tao) proposed the project, my classmates and I began a small, weekly seminar with a professor devoted to studying some of the theory involved (analytic number theory, sieve methods, etc.), albeit on a much more elementary level that was within our reach. Of course, the majority of the actual proof is still mostly over our heads but at least I feel as if I've gained a bird's-eye-view of the strategy and, probably more importantly, how it fits into the larger field. (For instance, before any of this, I could never have explained the Bombieri-Vinogradov theorem or the Hardy-Littlewood prime tuple conjecture.) So for us the project was a great excuse to enter a new subject and has been immensely educational.

More than that though – reading the posts and following the 'leader-board' felt a lot like an academic spectator sport. It was surreal, a bit like watching a piece of history as it occurred. It made the mathematics feel much more alive and social, rather than just coming from a textbook. I don't think us undergrads often get the chance to peak behind closed doors and watch professional mathematicians "in the wild" like this so, from a career standpoint, it was illuminating. I get the sense that this is the sort of activity I can look forward to in grad school and as a post-doc doing research (...hopefully).

I also suspect that many other students from many other schools have had similar experiences but, like me, chose to stay quiet, as we had nothing to contribute. So, thank you all for organising this project and for making it publicly available online.

4 Pace Nielsen

Pace Nielsen is an assistant professor at Brigham Young University.

As a pre-tenure professional mathematician, I believe that a short introduction to my experiences in the Polymath8 project will be helpful to other young mathematicians deciding whether or not to participate in similar endeavours. I initially joined the project for a number of reasons. One is that I enjoy optimising numerical results. There is a certain pleasure that comes from deriving an elegant computational result. Another reason I joined is that I wanted to understand Yitang Zhang's breakthrough as much as possible. His result was quickly followed by James Maynard's (and independently Terry Tao's) multi-dimensional, probabilistically motivated sieve, which is also incredibly interesting to me. In my opinion, this improved sieve deserves a spot as one of the top advancements in analytic number theory of the past half-century.

My initial incursions into the project consisted mainly of pointing out minor corrections to the Polymath8a paper and asking questions about some of Tao's blog posts. Hence, I don't consider myself a full participant in the 8a portion of the work; it was only during the 8b half of the work that I became a contributor.

There were a few things that surprised me about the whole experience. First was the friendliness of the other participants, particularly our host Terry Tao. I want to publicly thank everyone involved in the project for the positive experience. A special thanks goes to James Maynard, whose kindness in sending me some of his original Mathematica code was what finally pushed me into full activity in the project.

Second, a large number of mathematicians I know commented (in personal communications to me) on the fact that they were "impressed with my bravery" in participating. It caught me off guard that so many people had been following the project online and that all of my comments (including my mistakes) were open to such a wide readership. I believe it is important to consider this issue before deciding to participate in a public project. Some of the mistakes I made would never have seen the light of day in a standard mathematical partnership. However, any collaboration relies on the ability for the participants to share ideas freely, even the "dumb" ones.

The third surprise was how much time I devoted to this project. I think that came from really getting excited about the work. This is also the point at which I want to mention a (mild) concern on my part. As a pre-tenure professor, I made the conscious choice going into this project that any time I spent was not necessarily going to be reflected in my tenure file. I knew that even if I contributed enough work to be noted as an official "participant" in the online acknowledgements, it wasn't clear whether this would count as co-authorship in the eyes of those on my tenure committee or even of my colleagues generally. Indeed, there are aspects of this type of cooperative mathematics which make it difficult to decide how participation in this venture should be treated by the mathematical community at large. There are so many levels of effort that it can be somewhat confusing where the line is drawn between providing comments vs. being a full co-author vs. everything in-between. Also, the Polymath projects don't follow the convention used in other sciences of having the project manager decide who is a co-author on the final paper.

On the one hand, I support the idea of collaborative mathematics without an eye towards recognition. With respect to the Polymath8b project in particular, I'm happy to give D. H. J. Polymath all the credit. On the other hand, I do consider myself a co-author on the Polymath8b portion of the project and thus want to take ownership for my part of the work.

The fourth and final surprise was that I did contribute something meaningful! Sometimes this contribution happened by making simple comments on the blog. For instance, I remember waking up one morning with the realisation that a construction we were attempting would contradict the parity barrier in sieves. This idea then led the experts to write some interesting formal mathematics on this issue. Sometimes my contribution was made in the time-consuming, busy work of writing, running and debugging computer code. And sometimes it was just in contributing my experience after thinking about the geometric picture long enough. While it is intimidating working with Fields Medallists and other experts, it is also a once in a lifetime opportunity to rub shoulders with such a wide array of mathematicians.

While I rate my involvement as extremely positive, others who are contemplating joining a massive mathematical online collaboration should keep in mind the costs in time, public embarrassment and potential lack of control of your work before fully committing to the experience.

5 James Maynard

James Maynard is a fellow at Magdalen College, Oxford.

As a graduate student who had been looking at closely related problems, it was thrilling to hear of Zhang's initial breakthrough. I didn't participate in the subsequent Polymath 8a project, although I found myself reading several of the posts whilst studying Zhang's work for myself. I intended to avoid working on anything too close to the Polymath project (to avoid any competition) but one day I was going back through some ideas I'd had several months earlier on modifications of the 'GPY sieve' and I realised I could overcome the obstacles I had faced in my original attempt. This modification (also discovered by Tao) gave an alternative, stronger approach to gaps between primes, although it didn't produce the equidistribution results which lie at the heart of Zhang's work. With some small numerical calculations, this allowed me to show that there were infinitely many pairs of primes which differ by at most 600, and allowed one to show the existence of many primes in bounded length intervals.

I knew the numerical bounds in my work hadn't been fully optimised – there was some slack in my approach and there were also several opportunities to extend the method (such as incorporating ideas from Zhang and Polymath8a, or using more careful arguments). I was therefore pleased and excited when I learnt there was the intention for a Polymath8b project which I could be part of – it was very exciting for me that there was such an interest in my work!

The style of a large collaborative project was very new to me. I had relatively little experience of research collaboration and I was used to mainly trying out ideas alone. I certainly hadn't fully appreciated quite how public the posts were (and how many mathematicians who were not active participants would read the comments). In some ways this was quite fortunate; not realising the attention posts might receive made me more willing to contribute openly. I posted several ideas which were not fully thought through, some of which were rather stupid in hindsight, but some of which I believe were useful to the project (and probably more useful than posting a fully thought out idea a few days later). The atmosphere of the project certainly helped encourage such partial contributions, which I feel was a large factor in the project's success.

It was remarkable (to me, at least) how smoothly the project went; this was partly to do with the problem having very clearly defined goals and being modular in its nature but also due to the openness and friendliness of the participants. There seemed to be a good balance amongst the participants – some had more computational expertise, whilst others had a more theoretical background and it was certainly useful to have both groups together for the effectiveness of the project. Much of the improvements in the unconditional bound came from extending the computations I had done initially and it was certainly useful to have people who were rather more experienced than me with the larger computations.

I was surprised at how much time I ended up devoting to the Polymath project. This was partly because the nature of the project was so compelling – there were clear numerical metrics of 'progress' and always several possible ways of obtaining small improvements, which was continually encouraging. The general enthusiasm amongst the participants (and others outside of the project) also encouraged me to get more and more involved in the project. Finally, the nature of the work also made it very suitable for working on in short bursts, which turned out to be very useful since I was travelling quite a lot whilst most of the project was underway.

I was aware, as a junior academic without a permanent position, that I might ultimately not receive much credit in the eyes of hiring committees for participation in an atypical project such as the Polymath, where it is difficult to gauge the merit of my contribution. In my case, the fact that the project was so closely associated with my earlier work and the fact that I found the project so interesting made me happy to accept this (although I made a conscious effort to continue to work on other projects at the same time). This is certainly something I feel any similarly junior prospective participant should be aware of, however.

Overall, I really enjoyed the Polymath experience. It was a great opportunity to work with several other mathematicians and I feel pleased with the final results and my contribution to them.

6 Gergely Harcos

Gergely Harcos is a research advisor at Alfréd Rényi Institute of Mathematics and a professor at Central European University.

I guess I am no longer a junior mathematician, which is a bad thing, but on the good side I can perhaps add a different perspective on my participation in the Polymath8 project.

Last year, I applied for some serious grants and it was on the same day when I learned that my proposals would be rejected. This was quite discouraging and I felt like I needed to take a break from my main line of research. Around the same time, Zhang's exciting paper came out and, shortly after, Terry Tao initiated a public reading seminar on his blog that later turned into the Polymath8 project. I was already familiar with the earlier breakthrough by Goldston-Pintz-Yıldırım; in fact, I had incorporated it in my courses at Central European University and advised some students on related topics. I have always found this part of number theory very beautiful, although my research interests have been elsewhere. Following Terry's clean and insightful blog entries and the accompanying comments initially served two different purposes for me. First, I hoped to understand the new developments in a field that I found appealing. Second, I hoped to get a change of air in mathematics for the reasons explained above.

The Polymath8 project developed at blazing speed and my initial goal was simply to catch up and read everything posted on the blog. This was quite challenging because I am rather slow and prefer to check every line carefully, but at least I could serve as an early referee for the project. As a bonus, I got some ideas of how to improve certain points in an argument already posted. In short, the Polymath8 project helped me to get going and feel useful, and participating was a lot of fun. At one point I embarrassed myself by posting several different "proofs" to an improved inequality that I conjectured, only to find out later that the claim was false. All this is recorded and preserved in the blog but I do not regret it as it was honest and reflects the way mathematics is done. We try and we often fail.

It was also very interesting how my colleagues reacted. Some thought that one should not devote too much effort to a paper published under a pseudonym but, in fact, my participation here got far more attention than elsewhere. For a couple of months, the first question I was asked was about the current record on prime gaps. Another benefit of a Polymath project is that there is no pressure on the participants; one is free to join for a while then leave, and ignorance is normal as in a mathematical conversation.

7 David Roberts

David Roberts is a postdoctoral fellow at the University of Adelaide.

I saw the announcement of Zhang's talk on Peter Woit's blog and posted on Google+ (13 May 2013) about the twin prime problem, about prime gaps more generally and about Zhang's talk and how big a deal it was. This post received much attention (more than I would have expected) and over the course of Polymath8, and the pre-official Polymath work, I kept posting the current records online, with explanations of what the progress meant or how it happened.

I'm not an analyst or number theorist (I work in category theory) so I was content to read the progress of the project and learn how all this business worked. I'd read through Zhang's preprint and was totally nonplussed but the careful analysis – and exposition! – of Terry Tao and the other active participants made the ideas much clearer. In particular, concepts and tools that are well-known to analytic number theorists and are used without comment were brought into the open and discussed and explained.

When more specialised experts started working on subproblems, particularly numerical optimisation, it gave me snippets I could mention in my first-year algebra class to let them know how generalisations of the things they were learning (eigenvectors, symmetric matrices, convex optimisation, etc.) were being applied at the cutting edge of research. I even showed, on projector screens, Terry's blog and the relevant comments, some made that very day. It has been a great opportunity to expose students of all stripes to the idea of research in pure mathematics, and that a problem in number theory needs serious tools from seemingly unrelated areas.

For me, personally, it felt like being able to sneak into the garage and watch a high-performance engine being built up from scratch: something I could never do but could appreciate the end result and admire the process.

8 Andrew Sutherland

Andrew Sutherland is a principal research scientist at MIT.

I first heard about Zhang's result shortly after he spoke at Harvard in May 2013. The techniques he used were well outside my main area of expertise and I initially followed developments purely as a casual observer. It was only after reading Scott Morrison's blog, where people had begun discussing improvements to Zhang's bound, that I realised there was an interesting and essentially self-contained sub-problem (finding narrow admissible tuples) that looked amenable to number-theoretic combinatorial optimisation algorithms, a subject with which I have some experience. I ran a few computations, and once I saw the results it was impossible to resist the urge to post them and join the Polymath8 project. Aside from interest in the prime gaps problem, I was curious about the Polymath phenomenon and this seemed like a great opportunity to learn about it.

Like others, I was surprised by how much time I ended up devoting to the project. The initially furious pace of improvements and the public nature of the project made a very addictive combination and I wound up spending most of that summer working on it. This meant delaying other work but my collaborators on other projects were very supportive. I certainly do not begrudge the time I devoted to the Polymath8 effort; it was a unique opportunity and I'm glad I participated.

In terms of the Polymath experience, there are a couple of things that stand out in my mind. In order to make the kind of rapid progress that can be achieved in a large scale collaboration, the participants really have to be comfortable with making mistakes in a forum that is both public and permanent. This can be a difficult adjustment and there was certainly more than one occasion when I really wished I could retract something I had written that was obviously wrong. But one eventually gets used to working this way; the fact that everybody else is in the same boat helps. Actually, I think being forced to become more comfortable with making mistakes can be a very positive thing. This is how we learn.

The other thing that impressed me about the project is the wide range of people that made meaningful contributions. Not only were there plenty of participants who, like me, are not experts in analytic number theory, there were at least a few for whom mathematics is not their primary field of research. I think this is major strength of the Polymath approach: it facilitates collaboration that would otherwise be very unlikely to occur.

It is perhaps worth highlighting some of the features of this project that made it a particularly good Polymath candidate. First, the problem we were working on was well-known and naturally attracted a lot of interested observers; this made it easy to recruit participants. Second, we had a clearly defined goal (improving the bound on prime gaps) and a metric against which incremental progress could be easily measured; this kept the project moving forward with a lot of momentum. Third, the problem we were working on naturally split into sub-problems that were more or less independent; this allowed us to apply a lot of brain power in parallel and when one branch of the project would slow down, another might speed up. Finally, we had an extremely capable project leader, one who could see the whole picture and was very adept at organising and motivating people.

I don't mean to suggest that these attributes are all necessary ingredients for a successful Polymath project but I think it is fair to say that, at least in this case, they were sufficient.

9 Wouter Castryck

Wouter Castryck is a postdoctoral fellow of FWO-Vlaanderen.

In June last year, one of my colleagues informed me about the Polymath8 project and, in particular, about the programming challenge of finding admissible k-tuples whose diameter H is as small as possible. I decided to give it a try and join "team H" of the production line. Luckily, I jumped in shortly after the project had started, at a point where there was still some low-hanging fruit. Like others, I experienced how addictive it was to search for smaller values of H, while trying new computational tricks and combining them with ideas of the other participants. Because the value of k kept decreasing as well, new challenges popped up every other day or so, which fuelled the excitement. The whole event was intense and highly interactive and progress was made at an incredible speed. (It is not academic to say so but when the other teams managed to decrease k to a size where our computational methods became superfluous, this was a bit of a disappointment. Luckily, Maynard's work on prime triples, quadruples, etc., put larger values of k back into play.)

Along the way, it became clear that the online arena in which it all took place attracted many spectators and it felt like a privilege to be in there. It did require a mental switch to post naive (and sometimes wrong) ideas on the public forum but in the end I agree with Andrew Sutherland that this is not necessarily a bad thing.

Gradually, by reading the blog posts, I also learned about the other parts of Zhang's proof. On a personal level, this I found the most enriching thing: participating in the Polymath8 project was a very stimulating way of learning and appreciating a new part of mathematics. This is definitely thanks to the clarifying and enthusing way in which Terence Tao administered the project. At the same time, I must admit that I did not grasp every detail from A to Z. From the point of view of a "coauthor", this is somewhat uncomfortable but it may be inherent to the production line model along which the Polymath projects are organised.

10 Emmanuel Kowalski

Emmanuel Kowalski is a professor at ETH.

I remember reading the first blog posts of T. Gowers concerning the Polymath idea and briefly following the first steps at that time. I was intrigued but did not participate. As far as I remember, my only public comment was a brief note after that progress was successful, which still illustrates how I feel about the way this idea turned out: it provides a striking, largely publicly available, illustration of the way mathematical research works (or at least, often works...). Although

Gowers envisioned a collaboration with a very large number of participants, what actually happened felt, in fact, closer to a "standard" collaboration, with the important difference that anybody was free to jump in at any time and become a collaborator.

I also spent a little time thinking about a later Polymath project (finding large primes deterministically) but this also did not reach the level of actual participation. Thus, the Polymath8 project is the first time I have actually actively joined in this type of mathematical work.

In a general way, I feel that the Polymath idea does not fit very well with the way I tend to work – this is not a criticism but a statement of fact. In particular, following closely a free-for-all discussion on a blog requires a type of focus and concentration which I often find difficult to keep up over more than a few days.

In the case of the Polymath8 project, however, a few coincidences led to my participation:

- I knew much of the subject material quite well, having studied the Goldston-Pintz-Yıldırım method relatively closely, as well as understanding (with varying degrees of depth) many of the ingredients of Zhang's work.
- I had the occasion, during conferences and other visits, to discuss Zhang's work intensively with some of the best experts in the first weeks following the appearance of his paper; I reported on some of these discussions on my own blog but I did not, at that time, track the Polymath8 project closely.
- In particular, with É. Fouvry, Ph. Michel and P. Nelson, we were quickly able to understand, rework and finally improve the crucial "Type III" estimates that were closely related to the work of Friedlander and Iwaniec concerning the ternary divisor function, which Fouvry, Michel and I had independently improved quite strongly a few months before; here the main point was the use of Deligne's formalism of the Riemann Hypothesis over finite fields, which we understood much better through the theory of trace functions over finite fields.
- The final point was Ph. Michel's visit to California in the Summer of 2013, during which it was formally decided to develop and incorporate this result in the Polymath8 project.

In the end, my closest involvement with the project came when the first draft of the paper was written-up (I would probably have written the section on Deligne's formalism, if Ph. Michel had not begun it first) and the time came to review and check it. Since I was formally identified as a participant to the project, I felt strongly the need to read through the whole text, especially since parts of the claims were really dependent on numerical facts for their correctness and interest. At first, I was only thinking that I would do a quick review but I find it difficult to read papers this way and I ended up doing a line-by-line check of the whole text, checking all computations and reproducing (without copy-paste) all the numerical constants involved. It turned out that I only found one rather minor mathematical mistake in the original text and, in particular, no numerical value needed to be changed. I still find this rather remarkable. Interestingly, the referee reports, which were extremely thorough, turned up a few more such slips. This illustrates how hard it is to get a long mathematical argument rigorously right!

Because this review was also done largely in public (on T. Tao's blog, where the referee reports were also made available by public links) or semi-publicly (in the Dropbox folder where the paper was written and in my own internal version-controlled setup), I think this can provide interesting insights concerning the way mistakes may arise and be corrected in long mathematical papers. I feel, however, that the number of problems was exceptionally small. In fact, I believe that a similarly long paper of Polymath style, written without a strong guiding hand like Tao's, could face serious issues or questions concerning its correctness if it turned out that sections were written very independently, with authors working in a loose organisation and not fully mastering every part of the arguments. This seems to me one important issue to keep in mind if this type of collaborative work becomes more commonplace.

Finally, I would like to emphasise a very positive point that was made by R. de la Bretéche in his report [9] on the Polymath8a project: this project unified what might have been otherwise many small-scale improvements and tweaks to Zhang's work, which would certainly have appeared much more slowly – if at all, in view of Maynard's breakthrough!

11 Philippe Michel

Philippe Michel is a professor at EPF Lausanne.

My involvement in the Polymath8 project began as a passive blog reader; when Zhang's breakthrough paper was publicly released, we, together with Paul Nelson in Lausanne, just like many other mathematicians, started to read the paper avidly. I should say that the material was not entirely a stranger to me – my Master's thesis subject was to understand the Acta Mathematica paper of Bombieri-Friedlander-Iwaniec and works related to it like the Inventiones paper of Deshouillers-Iwaniec – but during the previous years, my mathematical interests had been rather remote. It was only during the past few months that I have been leaning back towards this area of analytic number theory – the study of the distribution of arithmetic functions along primes and arithmetic progressions – due to recent joint work with Etienne Fouvry and Emmanuel Kowalski. So I was a bit rusty on Linnik's dispersion method, and reading Terry Tao's blog posts on Zhang's work and the synthesis of the subsequent progress made by Polymath8 was tremendously useful to freshen up my memories and to test my intuitions. I should say that besides this, I also found it very useful and instructive to read the comments made by other people at the end of each blog post.

My active participation in the project began only near the middle of June 2013; at that time, the Polymath8 project was already going full steam ahead and had already made remarkable advances: the most visible one was a significant improvement over Zhang's initial constant 7×10^7 (dividing it by more than 1000); another impressive result was the proof of an equidistribution theorem for primes in arithmetic progressions of large smooth moduli, going beyond the Bombieri–Vinogradov range, whose proof used "only" Weil's bound for exponential sums in one variable (Deligne's work being replaced by several applications of Cauchy inequality and of the q-van der Corput method).

One portion of the argument that was still hurting the exponents concerned the "type III sums". We[5] had realised at an early stage that our previous work on the distribution of the ternary divisor function in large arithmetic progressions might help the cause; however, it was only during the Fouvry 60 conference at CIRM in the middle of June that we made this concrete and decided that it could be worth contacting Polymath (through Terry Tao). It is highly probable that without the existence of the Polymath8 project – its openness and the chance for anybody to bring their own contribution – we would never have dared to make such a technical improvement public and, most likely, we would have forgotten about it after some time! This work was also an occasion to develop material of broader interest: one can find there a general account on ℓ-adic trace functions which will hopefully be useful to the working analytic number theorist; for instance, we provide an easy to use presentation of the q-van der Corput method for general trace functions. Another interesting outcome is the following: the quest for improvements on the numerical value of the distribution exponent has led to quite sophisticated transformations of the sums appearing in the dispersion method. The fact that the resulting "complete" algebraic exponential sums and their associated sheaves have a nice geometrical structure is a pleasant surprise that triggers a lot of interesting questions on the ℓ-adic side.

I very much like working in a collaborative manner,[6] sometimes with fairly large groups of people, and, yet, this first participation in a Polymath project was at an entirely different scale and it took me some time to adapt. One issue was to cope with the continuous flow of comments and new ideas made by the many participants (in particular, I often wondered whether I was contributing enough by comparison with others); another was to absorb the fact that the project was performed under the public eye (and the vertiginous feeling that any mistake could be known to any mathematician in the world and would stay forever). As for others, these concerns eventually disappeared and I began to fully enjoy the spontaneity of having everybody working openly through a public blog; at some point even this became slightly addictive: while attending a thesis defence (not in my area fortunately!), I found myself checking the latest development of the project and sending an email to some of the Polymath people to test an idea I had just had on how to handle some unpleasant exponential sum. All in all, this has been a fun and rewarding experience and I am very thankful to Terry Tao for setting up and conducting the project; I am also thankful to the other participants for their constructive and highly collaborative attitude.

Bibliography

[1] J. Andersson, *Bounded prime gaps in short intervals*, preprint.

[2] W. Banks, T. Freiberg, C. Turnage-Butterbaugh, *Consecutive primes in tuples*, preprint.

[3] W. D. Banks, T. Freiberg, J. Maynard, *On limit points of the sequence of normalized prime gaps*, preprint.

[4] W. D. Banks, T. Freiberg, C. L. Turnage-Butterbaugh, *Consecutive primes in tuples*, preprint.

[5] J. Benatar, *The existence of small prime gaps in subsets of the integers*, preprint.

[6] E. Bombieri, J. Friedlander, H. Iwaniec, *Primes in arithmetic progressions to large moduli*, Acta Math. 156 (1986), no. 3–4, 203–251.

[7] E. Bombieri, J. Friedlander, H. Iwaniec, *Primes in arithmetic progressions to large moduli. II*, Math. Ann. 277 (1987), no. 3, 361–393.

[8] E. Bombieri, J. Friedlander, H. Iwaniec, *Primes in arithmetic progressions to large moduli. III*, J. Amer. Math. Soc. 2 (1989), no. 2, 215–224.

[9] R. de la Breteche, *Petits écarts entre nombres premiers et polymath : une nouvelle maniére de faire de la recherche?*, Gazette des Mathématiciens, Soc. Math. France, Avril 2014, 19–31.

[10] A. Castillo, C. Hall, R. J. Lemke Oliver, P. Pollack, L. Thompson, *Bounded gaps between primes in number fields and function fields*, preprint.

[11] L. Chua, S. Park, G. D. Smith, *Bounded gaps between primes in special sequences*, preprint.

[12] P. D. T. A. Elliott, H. Halberstam, *A conjecture in prime number theory*, Symp. Math. **4** (1968), 59–72.

[13] B. Farkas, J. Pintz, S. Révész, *On the optimal weight function in the Goldston-Pintz-Yıldırım method for finding small gaps between consecutive primes*, To appear in: Paul Turán Memorial Volume: Number Theory, Analysis and Combinatorics, de Gruyter, Berlin, 2013.

[14] É. Fouvry, H. Iwaniec, *On a theorem of Bombieri-Vinogradov type*, Mathematika 27 (1980), no. 2, 135–152 (1981).

[15] É. Fouvry, H. Iwaniec, *Primes in arithmetic progressions*, Acta Arith. 42 (1983), no. 2, 197–218.

[16] T. Freiberg, *A note on the theorem of Maynard and Tao*, preprint.

[17] H. Furstenberg, Y. Katznelson, *A density version of the Hales-Jewett theorem*, J. Anal. Math. **57** (1991), 64–119.

[18] D. Goldston, J. Pintz, C. Yıldırım, *Primes in tuples. I*, Ann. of Math. 170 (2009), no. 2, 819–862.

[19] W. T. Gowers, `gowers.wordpress.com.`

[20] W. T. Gowers, M. Nielsen, *Massively collaborative mathematics*, Nature, 15 October 2009.

[21] S. W. Graham, C. J. Ringrose, *Lower bounds for least quadratic nonresidues*, Analytic number theory (Allerton Park, IL, 1989), 269–309, Progr. Math., 85, Birkhäuser Boston, Boston, MA, 1990.

[22] A. Granville, *Bounded gaps between primes*, preprint.

[23] G. H. Hardy, J. E. Littlewood, *Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes*, Acta Math. 44 (1923), 1–70.

[24] D. Hensley, I. Richards, *On the incompatibility of two conjectures concerning primes*, Analytic number theory (Proc. Sympos. Pure Math., Vol. XXIV, St. Louis Univ., St. Louis, Mo., 1972), pp. 123–127. Amer. Math. Soc., Providence, R.I., 1973.

[25] H. Li, H. Pan, *Bounded gaps between primes of the special form*, preprint.

[26] R. Lipton, K. Regan, *People, Problems, and Proofs, Essays from Gödel's Lost Letter: 2010*, Theoretical Computer Science, 2013 XVIII, Springer-Verlag.

[27] J. Maynard, *Bounded length intervals containing two primes and an almost-prime II*, preprint.

[28] J. Maynard, *Small gaps between primes*, preprint.

[29] J. Maynard, *Dense clusters of primes in subsets*, preprint.

[30] S. Morrison, "I just can't resist: there are infinitely many pairs of primes at most $59,470,640$ apart", sbseminar.wordpress.com/2013/05/30.

5 E. Fouvry, E. Kowalski, P. Nelson and I.

6 The last non-survey paper I wrote alone was back in 2004.

[31] A. Pease, U. Martin, *Seventy-four minutes of mathematics: An analysis of the third Mini-Polymath project*, In Proceedings of AISB/IACAP 2012, Symposium on Mathematical Practice and Cognition II.

[32] J. Pintz, *Polignac Numbers, Conjectures of Erdős on Gaps between Primes, Arithmetic Progressions in Primes, and the Bounded Gap Conjecture*, preprint.

[33] J. Pintz, *A note on bounded gaps between primes*, preprint.

[34] J. Pintz, *On the ratio of consecutive gaps between primes*, preprint.

[35] J. Pintz, *On the distribution of gaps between consecutive primes*, preprint.

[36] P. Pollack, *Bounded gaps between primes with a given primitive root*, preprint.

[37] P. Pollack, L. Thompson, *Arithmetic functions at consecutive shifted primes*, preprint.

[38] D. H. J. Polymath, polymathprojects.org

[39] D. H. J. Polymath, *A new proof of the density Hales-Jewett theorem*, Ann. of Math. (2) **175** (2012), no. 3, 1283–1327.

[40] D. H. J. Polymath, *Density Hales-Jewett and Moser numbers*, An irregular mind, 689–753, Bolyai Soc. Math. Stud., 21, János Bolyai Math. Soc., Budapest, 2010.

[41] D. H. J. Polymath, *New equidistribution estimates of Zhang type, and bounded gaps between primes*, submitted.

[42] D. H. J. Polymath, *Variants of the Selberg sieve, and bounded intervals containing many primes*, to appear, Research in the Mathematical Sciences.

[43] B. Siudeja, *On the hot spots conjecture for acute triangles*, preprint. `arXiv:1308.3005.`

[44] T. Tao, E. Croot, H. Helfgott, *Deterministic methods to find primes*, Math. Comp. **81** (2012), no. 278, 1233–1246.

[45] J. Thorner, *Bounded gaps between primes in Chebotarev sets*, preprint.

[46] T. S. Trudgian, *A poor man's improvement on Zhang's result: there are infinitely many prime gaps less than 60 million*, preprint.

[47] Y. Zhang, *Bounded gaps between primes*, Annals of Mathematics 179 (2014), 1121–1174.

D. H. J. Polymath is the pseudonym for research papers arising from online collaborative "Polymath" mathematical research projects. Correspondence concerning this particular Polymath project can be directed to Terence Tao at tao@math.ucla.edu and information about Polymath projects can be found at http://michaelnielsen.org/polymath1/index.php?title=Main_Page.

Anatolii Skorokhod

(10 September 1930 – 3 January 2011)

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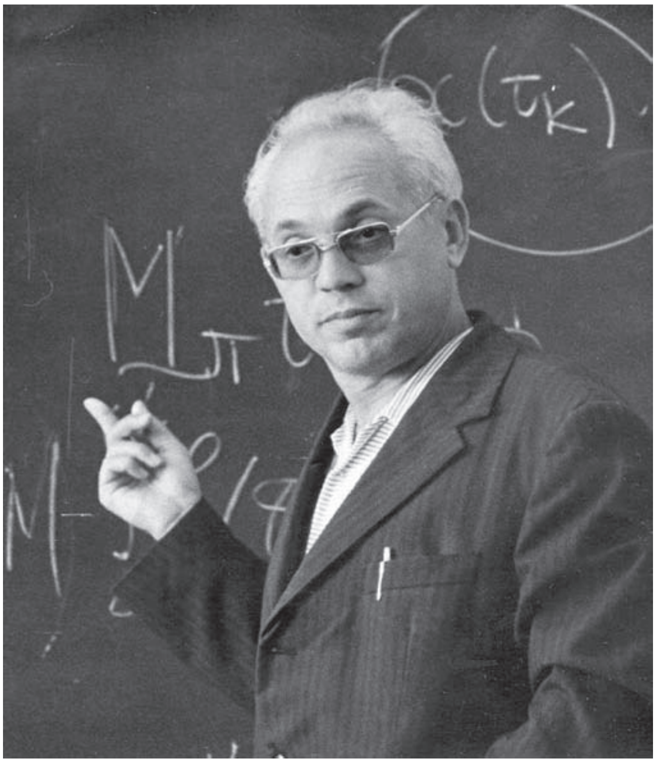

Anatolii Skorokhod

This article is devoted to an outstanding mathematician and excellent teacher A. V. Skorokhod, who recently passed away. The introductory part has been written by V. Buldygin (who died in 2012), A. Dorogovtsev and M. Portenko. A version of Skorokhod's biography is presented by I. Kadyrova, who was his wife from 1975 up to his death. M. Portenko describes his own impressions about the first book by A. V. Skorokhod, and A. Dorogovtsev presents his point of view on the evolution of the notion of the Skorokhod integral and related topics.

The name A. V. Skorokhod belongs with the few outstanding mathematicians of the second half of the last century whose efforts have imparted modern features to mathematics. His extraordinarily creative potential can be appraised by everyone who has studied contemporary stochastic analysis and realised that a considerable proportion of its notions and methods were introduced into mathematics by A. V. Skorokhod. It suffices to mention the notions of Skorokhod's topology, Skorokhod's space, Skorokhod's embedding problem, Skorokhod's reflecting problem, Skorokhod's integral and the method of a single probability space, strong and weak linear random operators, and stochastic semigroups (also proposed by him) and the power of his creative capacity becomes clear. Some of these notions (e.g. Skorokhod's integral) are now useful not only in mathematics but also in modern theoretical physics.

Graduating from the University of Kiev in 1953, A. V. Skorokhod carried on his postgraduate studies at Moscow University from 1953 to 1956, where he had the opportunity to learn from the achievements of the famous Moscow probabilistic school with academician A. N. Kolmogorov at its head. During this time A. V. Skorokhod gained authority among the scientific world when he had managed to formulate and prove the general invariance principle. A particular case of that principle was known as a result by M. Donsker (established in 1951). However, Skorokhod's result was not a simple generalisation of M. Donsker's. In order to formulate and prove it, A. V. Skorokhod introduced several new topologies into the space of functions without discontinuities of the second kind (one of those topologies is now well known as Skorokhod's topology and is useful in many branches of mathematics). Moreover, he proposed an original approach to the problem on the convergence of probability distributions (the method of a single probability space). Making use of those new tools, A. V. Skorokhod formulated and proved the general invariance principle in an accomplished form. Those results are now included in any fundamental monograph on the theory of stochastic processes. The probabilists of those days were deeply impressed by Skorokhod's new ideas and, in 1956, the most authoritative probabilist A. N. Kolmogorov published the paper "On Skorokhod's convergence", where he gave his own interpretation of the notions just introduced by A. V. Skorokhod. In one of his papers published in 2000, Professor V. Varadarajan of the University of California wrote:

> I was a graduate student in Probability theory at the Indian Statistical Institute, Calcutta, India in 1956, and still remember vividly the surprise and excitement of myself and my fellow students when the first papers on the subject by Skorokhod himself and Kolmogorov appeared. It was clear from the beginning that the space *D* with its Skorokhod topology would play a fundamental role in all problems where limit theorems involving stochastic processes whose paths are not continuous (but are allowed to have only discontinuities of the first kind) were involved.

However, not only the famous Moscow School of Probability Theory had an influence on the scientific work of A. V. Skorokhod. A significant part of his research was devoted to the theory of stochastic differential equations, originated by I. I. Gikhman, Kiev, in his works in the 1940s–1950s (independently, that theory arose in the works of K. Itô, Japan, at about the same time). With the influence of I. I. Gikhman, A. V. Skorokhod engaged in scientific investigations in that theory after coming back to Kiev in 1957. The results of those investigations obtained by him over 1957–1961 formed the basis of his doctoral dissertation and his first book "Studies in the theory of random processes", published by Kiev University in 1961. The assertions expounded in that book, as well as the methods used by A. V. Skorokhod for proving them, were fundamentally different from those that were typical in the theory of stochastic differential equations at that time: the book was full of new ideas, new methods and new results. At the beginning of the 1960s, A. V. Skorokhod published several articles devoted to the theory of stochastic differential equations that described diffusion processes in a region with a boundary. Those were pioneering works and they stimulated a real stream of investigations on the topic at many probabilistic centres around the world. It should be said that the theory of stochastic differential equations has now become one of the most essential acquisitions of the whole of mathematics in the second half of the 20th century and it is impossible to over-estimate the contribution of A. V. Skorokhod.

The full list of Skorokhod's publications consists of more than 300 articles published in various journals, and 23 monographs, some of them written jointly with co-authors (the number of monographs should be increased to 45 if translations are taken into account). Under Skorokhod's supervision, more than 50 graduate students defended their candidate dissertations and 17 of his disciples became doctors of mathematics. It should be added that A. V. Skorokhod paid considerable attention to popularising mathematics amongst schoolchildren. He was Rector of the University of Young Mathematicians which was active for 10 years at the Institute of Mathematics in Kiev. Each academic year at that university started with a lecture delivered by A. V. Skorokhod. He published 16 textbooks and popular-science books (some of them with co-authors).

A. V. Skorokhod was incessantly in search of new mathematical truth. He was able to see the gist of a problem, to find out an original unexpected approach to it and to create an adequate method for solving it. Besides, he was in the habit of thinking over problems thoroughly every day. Owing to his intense work day after day, the creative spark given to him from God became a bright shining star of the first magnitude on the mathematical frontier.

A brief biographical outline

Anatoli Vladimirovich Skorokhod was born 10 September 1930 in the town of Nikopol, Dnipropetrovsk region (previously Ekaterinoslavskaya province) to a family of teachers. Anatoli spent his childhood in Southern Ukraine. His parents taught in rural schools around Nikopol. Anatoli's childhood took place during the very difficult 1930s: the ruin after the Revolution and the Civil War of 1919–1922, collectivisation of peasants, dispossession, exile and hunger.

Anatoli's father Vladimir Alexseevich taught mathematics, physics and astronomy, primarily in high school. A great teacher, he was erudite and had a sharp analytical mind. From him, Anatoli inherited an inquisitive, analytical mind and a critical attitude toward everything. His father played a major role in the choice of his eldest son's profession. Anatoli's mother Nadezhda Andreevna taught Russian and Ukrainian literature, history, music and singing, as well as mathematics. Nadezhda Andreevna had many different talents. She was a good musician and had a vivid dramatic talent. Nadezhda Andreevna also had good writing skills. Boasting an excellent style, she wrote scripts, stories and poems.

Anatoli entered elementary school at the age of seven. His studies were interrupted by World War II. The part of Ukraine where the Skorokhods were living was occupied at the beginning of the war.

The post-war years in Southern Ukraine were years of poor harvest and, in 1946, trying to escape from the hunger, the family moved to live in Kovel, a town at Volyn in the western region of Ukraine. Their father was offered a position of school principal. Studying in high school was easy for Anatoli without any apparent effort. He was excellent in all subjects. Despite always being interested in mathematics, during his school years Anatoli did not feel any predestination to become a mathematician.

After his graduation with a gold medal from high school in 1948, Anatoli followed the advice of his father and submitted his documents to the Kiev State University (named after Taras Shevchenko) and was enrolled in the Faculty of Mechanics and Mathematics.

Skorokhod's scientific work began in his student years. Under the supervision of Boris Vladimirovich Gnedenko (at that time Chairman of the Department of Probability Theory) and Iosif Illich Gikhman (then an associate professor of the department), Anatoli started his work in probability theory. At the end of his student years Skorokhod became involved in the research related to the famous Donsker invariance principle.

During 1953–1956, Anatoli was studying in the graduate school of Moscow State University under the supervision of Eugene Borisovich Dynkin.

This period of study in this graduate school was a remarkable period in Skorokhod's life in many ways. At this time (the 1950s) in the Faculty of Mechanics and Mathematics, a broad audience of talented young people gathered around the great teachers of the older generation. These young mathematicians saw their future in the service of fundamental science. Amongst this group, Anatoli Skorokhod was distinguished by his independence in research work and the courage and originality of his approaches to problem solving. According to Anatoli, the main thing that he benefited from in Moscow graduate school was the seminar of his advisor E. B. Dynkin, called "Analysis, Algebra and Probability Theory".

Skorokhod's PhD thesis (his dissertation was defended in May 1957) contained descriptions of new topologies in the space of functions without discontinuities of the second kind and the application of them for proving limit theorems for stochastic processes. The Donsker invariance principle was generalised to the case when the limit process is a general process with independent increments. In the proofs of the theorems he used the original method invented by the author, known as the "method of a single probability space". The importance of the ideas of a very young mathematician was confirmed by the entire future development of the theory of stochastic processes. The terms "Skorokhod topology", "Skorokhod space" and "Skorokhod metric" are included in all basic books on the theory of stochastic processes.

In 1957–1964, Skorokhod was working as a faculty member in his "alma mater" – Kiev University. In 1961, he published his first book called "Studies in the Theory of Random Processes", which was the basis of his doctoral dissertation, defended in 1963. At the beginning of 1964, at the Institute of Mathematics of the Academy of Sciences of Ukraine, the Department of the Theory of Stochastic Processes was opened and A. V. Skorokhod became head of this department. In the same year he was awarded the title of professor. In 1967, A. V. Skorokhod was elected a corresponding member of the Ukrainian Academy of Sciences.

After Skorokhod's return from Moscow in 1957, he began a friendship, scientific cooperation and long-term and fruitful co-authorship with I. I. Gikhman. They wrote many well known books together.

A. V. Skorohod played a prominent role in the development of Ukrainian probability theory, particularly in the Kiev school. The scale and diversity of his research and teaching activities were striking. Generations of students grew up listening to his lectures and using textbooks and monographs authored or co-authored by him.

Under A. V. Skorokhod's leadership (since 1966) the national seminar on probability theory at Kiev State University has gained credibility and relevance not only in Kiev but also far beyond. A. V. Skorokhod supervised graduate students at the university, as well as at the Institute of Mathematics. He was the advisor of 56 PhD students. Among his graduate students were not only Ukrainian students but also young scientists from India, China, Vietnam, East Germany, Hungary, Nicaragua and other countries. Under his guidance, 17 doctoral theses were also written.

A. V. Skorokhod made a great impact helping raise the level of elementary mathematics teaching in Ukraine and the popularisation of mathematics.

In 1993–2011, Skorokhod worked at the Department of Statistics and Probability of Michigan State University, USA. His research areas were the investigation of the behaviour of dynamic systems under random perturbations, some problems of financial mathematics and martingale theory. In 2000, Skorokhod was elected a member of the American Academy of Arts and Sciences.

In an interview, Skorokhod was asked how he felt about social activities of scientists. To that Anatoli replied: "Negatively. I believe that a scientist should be a professional." However, under circumstances that required the demonstration of personal courage and providing support by his authority in defence of the civil rights and freedoms of citizens, he joined the protesters. In April of 1968, a group of 139 scientists, writers and artists, workers and students wrote a letter to the leaders of the former USSR expressing their concern regarding the renewed practice of closed political trials of young people from the midst of the artistic and scientific intelligentsia. Participation in this event was natural for A. V. Skorohod – a man with a sense of dignity, courageousness and independence, who was never afraid of authority and could not be indifferent to the flagrant flouting of civil rights in the country.

A V. Skorokhod was a true patriot of Ukraine. He hated everything that was part of the concept of "imperial thinking" with regard to Ukraine: denial of identity of language and culture and of the very existence of the distinctive Ukrainian nation, and the rejection of the idea of an independent Ukrainian state. That love for Ukraine made him an active participant of the national liberation movement "People's Movement of Ukraine" ("Rukh", late 1980s). Anatoli took part in all activities carried out by the initiative group when the movement was still only emerging. At that time active participation in the creation of "Rukh" was dangerous but that did not stop Skorokhod. When independence of Ukraine was proclaimed and the "Rukh" began to turn into a political, bureaucratic organisation, Skorokhod completely lost interest in it and any participation in its activities.

As a mathematics phenomenon, the extent and significance of "Skorokhod" was due not only to the mathematical talent Anatoli possessed but also an equivalent gift of personality. His mathematical talent, intuition and efficiency caused surprise and delight; his modesty, indifference for awards and titles, an absence of vanity, the independence of his judgments and his inner freedom served as moral standards in his social circle. Skorokhod's heartwarming subtlety and depth attracted people to him. Erudite in various fields of knowledge, including history and philosophy, he had a love of literature and classical music and a passion for poetry (Skorokhod knew by heart whole volumes of poems of his favourite poets: Ivan Bunin, Osip Mandelshtam, Anna Akhmatova, Boris Pasternak and Joseph Brodsky, and was able to recite them for hours). He was an inspiration for others to follow his example, involving his friends and disciples in the same areas of spiritual and aesthetic interests. Anatoli was a caring, loving son, a loyal, understanding father and a good friend, always ready to support in difficult circumstances, to listen and to help. In personal relations he was very sincere, very romantic and able to love selflessly.

In one of his articles, Anatoli wrote:

> Only a curious to oblivion person can be a good mathematician … With the help of mathematics new surprising and unexpected facts are often discovered. In fine art the beautiful creation always contains something unexpected, though not all unexpected is beautiful. Whereas in mathematics unexpected is always beautiful … there is nothing more beautiful than a simple and clear proof of a non-trivial statement.

The engagement in mathematics was for Skorokhod a way of existence as natural as breathing.

"I think about mathematics always," Skorokhod wrote in one of his letters. The hum of problems he thought about was continuous and incessant in his mind. In his work on problems Skorokhod did not dig deeply in the literature in the search of suitable tools that could be adapted or modified to suit his needs. He created his own original methods and constructions that determined new directions in the development of the theory of stochastic processes for decades. Until the very end of his creative life, Skorokhod maintained an inquisitive curiosity, always searching for harmony and beauty of mathematics.

Anatoli Vladimirovich Skorokhod died in Lansing, Michigan, 3 January 2011. Relatives and friends made a last farewell to Anatoli with the words: "A bright star has returned to the Universe." The ashes of Anatoli Skorokhod were buried 20 May 2011 at Baikove cemetery in Kiev.

A few words about the first book by A. V. Skorokhod

I was a fourth-year student at Kiev University when the first book by A. V. Skorokhod "Studies in the theory of random processes" was published (1961). By that time, I had already taken the course on the theory of stochastic differential equations taught by him and that facilitated my efforts in comprehending the book. Even so, to read it was an uphill struggle for me but I didn't give up. Moreover, I managed to read it while serving in the Soviet Army (1964–1965).

More than 50 years have passed since then. Many other books on the topic have been published by various authors (including Anatolii Volodymyrovych himself). However, for many researchers of my generation, the first book by A. V. Skorokhod still remains a spark that has stimulated their enthusiasm for the theory of stochastic processes. The power of Skorokhod's creative ability and the courage of his searching mind were displayed in that book as brilliantly as five years before in his fundamental work "Limit theorems for stochastic processes" (1956). I will consider some aspects of the book in this article.

With more than 50 years experience in reading mathematical works by A. V. Skorokhod, I should say that besides the usual difficulties felt by everyone when trying to comprehend something new, certain troubles connected with Skorokhod's manner to expound the material arose. In my opinion, Anatolii Volodymyrovych was not always irreproachable in this respect. One can sometimes come across a sentence in his texts that can be treated in several different ways and it is difficult (particularly for young mathematicians) to perceive which one he intends. I once pointed out this carelessness that occasionally existed within his style. He replied that he was not able to understand why a reader should put into a written sentence a sense other than the author did. But I was not going to give in and said: "It is the author's responsibility to structure any phrase in such a way that makes it clear for any reader what the author has meant." He replied with the question: "Do you really believe it possible to read a mathematical text without thinking over it?" I then understood his position. He showed no concern for particularising the material expounded in his works. To think over new problems was more important to him than to expound the thoughts and ideas already discovered by him or others.

There is another source of difficulties in reading the texts of A. V. Skorokhod. According to his own confession, if he made a mistake when writing a sentence or a formula, he did not notice it when re-reading: instead of what was written he saw what should be written.

In conclusion to this introductory part of the article: Skorokhod's texts are not straightforward but it is worth it to read them.

* * *

A. V. Skorokhod started lecturing at Kiev University in 1957. Over the three previous years, he was a postgraduate student at Moscow University. His studies there were a dazzling success: he had formulated and proved the most general limit theorems for stochastic processes and, moreover, he had invented an original method for proving them. In spite of his young age, he had already succeeded in gaining authority amongst experts in probability theory. His PhD thesis (candidate dissertation) had already been prepared for defence and rumours were around that the second scientific degree, i.e. doctorate of mathematics, would be conferred to him for that thesis (in reality it turned out not to be so for a reason that will not be discussed here).

In such a situation, it would be natural for everyone to take a pause in scientific research but not Anatolii Volodymyrovych. His list of publications shows that his searching mind was working incessantly. However, the main field of his scientific interests was changing: the theory of stochastic differential equations started attracting his attention. This theory had just been originated by the early 1950s. It would not be right to speak of its details here. I only say that the theory was independently created by K. Itô (Japan) and I. I. Gikhman (Kiev) in their works published during the 1940s and early 1950s.

K. Itô developed the theory of stochastic differential equations based on the notion of a stochastic integral he had introduced. His notion was a generalisation of Wiener's in two directions: firstly, the integrand in his notion was a random function (Wiener constructed the integral of a non-random one) and, secondly, he constructed integrals not only with respect to Brownian motion but also with respect to a (centered) Poisson measure. Given some local characteristics of a stochastic process to be constructed and, besides, such "simple" objects as Brownian motion and a Poisson measure, he wrote down a stochastic integral equation (it could be written as a differential one) whose solution gave the trajectories of the process desired. Under some conditions on given coefficients (the local characteristics mentioned above), he managed to prove the existence and uniqueness of a solution and to establish it as a Markov process. The set of stochastic processes that were differentiable in Itô's sense was endowed by a calculus different from the classical type (for example, the Itô formula created a new rule of differentiating a function of a stochastic process having Itô's stochastic differential). The approach to the theory of stochastic differential equations given by K. Itô turns out to be exceptionally proper: in most of the monographs on the topics, the basic notion is the notion of Itô's integral or some of its generalisations.

I. I. Gikhman did not have such a notion. Nevertheless, his notion of a stochastic differential equation was quite rigorous in a mathematical sense. It was based on the notion of a random field that locally determined the increments of the process to be constructed as a solution to the corresponding stochastic differential equation. Under some conditions on a given random field, I. I. Gikhman proved the theorem on the existence and uniqueness of a solution with given initial data. If the random field did not possess a property of after-effect then the solution was a Markov process. In the case of that field being given by a vector field of macroscopic velocities plus the increments of Brownian motion transformed by a given operator field, the corresponding solution turned out to be a differentiable function with respect to the initial data (under the assumption, of course, that the mentioned fields were given by smooth functions in spatial arguments). With this result, I. I. Gikhman managed to prove the theorem on the existence of a solution to Kolmogorov's backward equation (i.e. a second-order partial differential equation of parabolic type) without any assumptions on the non-degeneracy property of the matrix consisting of the coefficients of the second spatial derivatives (it is well known how important such a property is in the analytical theory of those equations). That was a significant result showing that some theorems in the theory of partial differential equations can be proved with the use of purely probabilistic methods.

I have just described briefly the situation in the theory of stochastic differential equations formed by the beginning of 1957. I think that A. V. Skorokhod had the opportunity to acquaint himself with Itô theory during his studies in Moscow. As far as I know, just after coming back to Kiev, his regular discussions with I. I. Gikhman started taking place and he was able to comprehend Gikhman's approach to the theory. That branch of mathematics was then quite new and A. V. Skorokhod was entirely engaged in investigations on the topic. His works published 1957–1961 (though not all of them) were related to the theory of stochastic differential equations. At the end of 1961 in the publishing house of Kiev University, the first book by A. V. Skorokhod came out. Besides the title, there was a subtitle on its cover, namely "Stochastic differential equations and limit theorems for Markov processes".

A. V. Skorokhod started the book by setting out Itô theory of stochastic differential equations (more precisely, its multidimensional version) and then he presented several of his new results that essentially influenced the further evolution of the theory.

First, he proved the theorem on the existence of a solution to a stochastic differential equation under the assumption that its coefficients were only continuous (they were also assumed to satisfy the usual growth conditions at infinity, of course), i.e. they might not satisfy the Lipschitz condition in spatial arguments, as was the case in Itô's or Gikhman's theories. That theorem was an analogue to the Peano theorem in the theory of ordinary differential equations. The uniqueness of a solution was not guaranteed. It should also be said that the solution constructed by A. V. Skorokhod in proving that theorem seemed to be of somewhat different character from those constructed by K. Itô or I. I. Gikhman. They constructed the solution on the probability space where an initial position, Brownian motion and a Poisson measure (or Gikhman's random field) were given and that solution turned out to be a functional of those objects. Solutions with this property were later called strong solutions. And A. V. Skorokhod made use of the method of a single probability space (previously invented by him) for proving his theorem and it could not be guaranteed that his solution was a strong one. An absorbing problem then arose: what conditions on given coefficients of a stochastic differential equation one should impose in order to assert that the solution of that equation was strong. A detailed investigation of that problem can be found in the fundamental article by A. K. Zvonkin and N. V. Krylov "On strong solutions of stochastic differential equations" (1974).

Second, for stochastic differential equations describing the processes with jumps, he established the differentiability of a solution with respect to the initial data. This allowed him to derive an integro-differential equation for the corresponding mathematical expectation. That equation was an analogue to the Kolmogorov backward equation for diffusion processes obtained before by I. I. Gikhman (see above).

Third, he found out the conditions under which a pair of stochastic differential equations generated two measures on the space of all functions without discontinuities of the second kind such that one of those measures was absolutely continuous with respect to the other one, and the formula for the corresponding density was written. Those formulae are very important in mathematical statistics when some unknown parameters in the coefficients are to be estimated or one of the two given competitive hypotheses about the coefficients is to be chosen.

Fourth, he formulated and proved a very interesting theorem on comparison of solutions to a given pair of stochastic differential equations on a real line whose diffusion coefficients coincided and whose drift coefficients were related with an inequality valid for all instants of time and at any point of the real line (a term containing a Poisson measure was absent in the equations under those considerations). Then it turned out that the solutions were related with the same inequality as ever, if only their initial positions did so. Making use of this theorem, A. V. Skorokhod established the uniqueness of a solution to a one-dimensional stochastic differential equation with continuous coefficients satisfying the usual growth conditions at infinity and such that the diffusion coefficient was given by a Hoelder continuous function (in spatial argument) with exponent greater than 1/2. As a matter of fact, it was the first result on the existence of a strong solution to an equation with non-Lipschitzean coefficients.

Fifth, he found out the conditions on a given sequence of Markov chains such that the stochastic processes generated by those chains were weakly convergent (in the first of the topologies introduced by him into the space of all functions without discontinuities of the second kind) to the solution of a stochastic differential equation. Some results of the kind had been known by that time for the case of a limiting process being a diffusion one.

Sixth, the last chapter of the book was devoted to the so-called "embedding problem" that could be formulated as follows: for a given one-dimensional Brownian motion, an integrable stopping time was to be constructed such that the value of the Brownian motion at that stopping time was distributed according to a beforehand given measure on a real line being centered and having finite second moment. A. V. Skorokhod solved this problem and applied it to estimating the probability that a sequence of the normalised sums of independent random variables was located inside the region bounded by two given curves. The embedding problem formulated and solved by A. V. Skorokhod in 1960 has stimulated investigations in probability theory for over 50 years. At an international conference "Skorokhod space: 50 years on" that took place in Kiev in 2007, one of its sections had the title "The Skorokhod Embedding Problem". The organiser of that section, Professor J. Obloj from the UK, published an excellent brief review of the talks on the topic: "The Skorokhod embedding problem: old and new challenges" (see Abstracts of that conference, part 1, pp. 93–97).

The five points above were Skorokhod's first steps in the theory of stochastic differential equations. It should be mentioned that there was one more step in that theory made in the years 1957–1961, namely his pioneering results related to the theory of stochastic differential equations describing diffusion processes in a region with boundary points. Those results were not included in the book. They originated the theory of stochastic processes now called "The Skorokhod Reflection Problem". A section with this title, organised by Professors P. Dupuis and K. Ramanan from the USA, also took place at the conference mentioned above.

Skorokhod's achievements described in this article show how extremely long his first steps were into the theory of stochastic differential equations and how quickly he moved from the position of beginner to one of leader. That theory became his favourite field of mathematics and he had many opportunities after 1961 to think over some of its problems again and again.

Many people knew how great he was, not only as a mathematician but also as a human being. I would like to address anyone who was close to him with the following lines:

> Don't say wistfully: "He is no more …"
> But say thankfully: "He was!"

The evolution of the Skorokhod integral

One of the important notions of modern stochastic calculus is the notion of the extended (also called Skorokhod) stochastic integral. It arose at the beginning of the 1970s.

In the original Skorokhod paper [16], the extended integral was built in as an operator on Hilbert-valued Gaussian functionals. Suppose that H is a real separable Hilbert space and ξ is a generalised Gaussian random element in H with zero mean and identity covariation. If σ-field $\mathcal{F}$ of random events is generated by ξ then every square-integrable random variable α can be expanded into the Itô-Wiener series

$$\alpha = \sum_{k=0}^{\infty} A_k(\xi, \ldots, \xi). \tag{1}$$

Here $\{A_k(\xi, \ldots, \xi)\}$ are the infinite-dimensional Hermite polynomials associated with the symmetric Hilbert–Shmidt forms on H

$$E\alpha^2 = \sum_{k=0}^{\infty} k!\|A_k\|_k^2, \tag{2}$$

where $\|A_k\|_k$ is the Hilbert–Shmidt norm on $H^{\otimes k}$. Define the stochastic derivative $D\alpha$ of α as a square-integrable random element in H which satisfies the relation

$$(D\alpha, \varphi) = \sum_{k=0}^{\infty} kA_k(\varphi, \xi, \ldots, \xi), \ \varphi \in H. \tag{3}$$

Finally, the Skorokhod integral can be defined as the adjoint operator $I = D^*$ acting from the space of the square-integrable H-valued random elements to the space of square-integrable random variables. In the terms of Itô–Wiener expansion, the Skorokhod integral can be described as follows. Let the square-integrable random element x in H be represented by the series $(x, \varphi) = \sum_{k=0}^{\infty} A_k(\varphi, \xi, \ldots, \xi)$, $\varphi \in H$. It can be checked that $A_k \in H^{\otimes k+1}$ for every $k \geq 0$. Denote by ΛA_k its symmetrisation. Then $I(x) = \sum_{k=0}^{\infty} \Lambda A_k(\xi, \ldots, \xi)$.

This algebraic approach to the definition of I and D gives a possibility of deriving many useful properties of these operators and disclosing deep relationships with quantum physics, leading to such results as hypercontractivity of the Ornstein–Uhlenbeck semigroup, the logarithmic Sobolev inequality, etc. (see [32, 35]).

The briefly described algebraic approach to the definition of the Skorokhod integral is not unique. Another one is closely related to the theory of Gaussian measures in infinite-dimensional spaces. Let the space H be embedded into another Hilbert space H' by a Hilbert–Schmidt operator. Then ξ becomes a usual random element in H'. Let ν be the distribution of ξ in H'. Note that H is the space of the admissible shifts for ν. Consider a function $f \in C^1(H')$ with bounded derivative. Then f has a bounded Fréchet derivative f'_H along H. It can be proved that the stochastic derivative $Df(\xi)$ coincides with $f'_H(\xi)$. Taking the closure we can organise the Sobolev space $W_2^1(H', \nu)$ of the functions with the Sobolev derivative along H. In this construction, the extended stochastic integral $I(x)$ arises via the integration-by-parts formula for Gaussian measures and can be interpreted as a logarithmic derivative of the initial measure ν along the vector field (H-valued) x [38, 37]. Wiener measure, the stochastic derivative and its adjoint operator were also introduced in [22]. The extended integral is used in measure theory. For example, the Radon–Nicodym density of the measure transformed by a flow of mappings can be described in terms of the extended integral [15, 14, 36]. On the other hand, the relationship with measure theory gives the possibility of constructing the Skorokhod integral in a non-Gaussian situation using logarithmically smooth measures, by differentiation along non-linear transformations and even the formal rules of differentiation on the space of random variables [7–11, 33, 34].

The Skorokhod integral got its name "extended" due to the most known case of its application, when $H = L_2([0; 1])$ and ξ is generated by the Wiener process w via the formula $\forall\, \varphi \in H: \ (\varphi, \xi) = \int_0^1 \varphi dw$. It occurs that the multidimensional Hermite polynomials $A_k(\xi, \ldots, \xi)$ are the multiple Wiener integrals

$$\int_0^1 \overset{k}{\cdots} \int_0^1 a_k(\tau_1, \ldots, \tau_k)dw(\tau_1)\ldots dw(\tau_k)$$

with the symmetric kernels a_k, satisfying the relation

$$\int_0^1 \overset{k}{\cdots} \int_0^1 a_k(\tau_1, \ldots, \tau_k)^2 d\tau_1 \ldots d\tau_k = \|A_k\|_k^2.$$

Really, let x be a random element in $L_2([0; 1])$ with finite second moment. x can be viewed as a random function. Define the filtration $(\mathcal{F}_t)_{t\in[0;1]}$ associated with the Wiener process as usual. Then suppose that x is adapted to $\mathcal{F}_t$ (as a random element in $L_2([0; 1])$). It is well known [25, 5] that under this condition x belongs to the domain of definition of the Itô stochastic integral with respect to the Wiener process. But it is very interesting that x belongs to the domain of I and $I(x) = \int_0^1 x(s)dw(s)$. So, the Itô integral is a partial case of the Skorokhod integral. This fact was found out in the 1970s by many authors. Since then, many articles have been written on the Skorokhod integral and stochastic integration. The main directions of investigation here were the following. In the Itô case and in a more general construction of the stochastic integral with respect to the semimartingale, the approximation by the integral sums can be used. In particular, the Skorokhod integral was obtained via approximation by the step functions, the Ogawa symmetric integral was created and the relationships between these integrals and Stratonowich integrals were studied (see, for example, [13–17]). In the case of the Skorokhod integral with an indefinite upper limit, the quadratic variation remains the same as in the Itô case and some interesting relations with martingales can be obtained [18–20, 32]. One of the main properties of the Itô integral is its locality. Namely, if random functions x_1, x_2 on $[0; 1]$ are integrable with respect to the Wiener process w then $\left(\int_0^1 x_1 dw - \int_0^1 x_2 dw\right)\mathbf{1}_{\{x_1=x_2\}} = 0$. This property is very important. For example, it allows the use of the stopping technique in the consideration of stochastic differential equations. The corresponding property of the Skorokhod integral was established in two different situations. In an initial Skorokhod article, the following statement was proved: if the random element x has a stochastic derivative then x lies in the domain of I. For stochastically differentiable random functions the Skorokhod integral has a locality property. This was proved in [19].

The other approach was proposed in [5, 2]. Define the smooth open subset Δ of probability space as $\Delta = \{\omega : \alpha(\omega) > 0\}$, where α is a stochastically differentiable random variable. Then it can be proved that for x_1, x_2 from the domain of I (not necessarily stochastically differentiable) the equality $(x_1 - x_2)\mathbf{1}_\Delta = 0$ implies

$$(I(x_1) - I(x_2))\mathbf{1}_\Delta = 0. \tag{4}$$

The statements mentioned above give us the possibility of defining the Skorokhod integral for the random functions that do not have finite moments. Recent results on the description of subsets of probability space that have a locality property (4) can be found in [21]. Stochastic equations with non-Itô integrals have been actively studied since the 1970s. As an example, the boundary value problems or integral equations of the second kind can be considered. Such equations were treated initially in the article [17, 1] using the algebraic definition of the Skorokhod integral. One of the interesting cases where anticipation arises is the Cauchy problem for ordinary stochastic differential equations with the initial condition which depends on the future noise. The interesting result was obtained in [29]. A linear one-dimensional equation was considered

$$\begin{cases} dx(t) &= a(t)x(t)dt + b(t)x(t)dw(t) \\ x(0) &= \alpha. \end{cases} \tag{5}$$

Here α is a functional of w and the equation is treated in the sense of the Skorokhod integral. It was proved in [29] that the solution has the form

$$x(t) = T_t\alpha \cdot \mathcal{E}_0^t, \tag{6}$$

where $T_t\alpha$ is a transformation of α corresponding to the change $w(\cdot) \to w(\cdot) + \int_0^{t\wedge\cdot} b(s)ds$, and $\mathcal{E}_0^t$ is a usual stochastic exponent (related to the case $x(0) = 1$). The reason for the appearance of $T_t\alpha$ in the solution can easily be seen if we recall the definition of the Skorokhod integral as a logarithmic derivative. It can be verified that I has a structure of the infinite-dimensional divergence operator. So (5) can be treated as a first order differential equation in infinite-dimensional space. Then (6) becomes the solution obtained via the characteristic method. This approach was introduced in [7]. This point of view on equation (6) explains the absence of the solution in a general non-linear case and the absence of the good form of a solution in the linear vector case. The last case with the commuting matrix-valued b was considered in [30]. Surprisingly, the algebraic definition of the Skorokhod integral turns out to be very helpful in the consideration of (6). The following representation of the solution was obtained in [8] $x(t) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_0^t \overset{k}{\cdots} \int_0^t D^k\alpha(s_1, \ldots, s_k) \cdot \mathcal{E}_{s_k}^t b(s_k)\mathcal{E}_{s_{k-1}}^{s_k} b(s_{k-1}) \ldots \mathcal{E}_0^{s_1} ds_1 \ldots ds_k$. In the 1990s a new reason for studying the extended stochastic integral arose. This was the development of the models from financial mathematics. In these models, fractional Brownian motion plays the role of the noise process [31]. So, integration with respect to it must be constructed. One of the possible approaches leads to the extended stochastic integral with respect to Gaussian integrators [9]. The Gaussian process η on $[0; 1]$ is called the integrator if there exists such $c > 0$ that for an arbitrary partition $0 = t_0 < t_1 < \ldots < t_n = 1$ and real numbers $a_1, \ldots, a_n$, we have

$$E\Big(\sum_{k=0}^{n-1} a_k(\eta(t_{k+1}) - \eta(t_k))\Big)^2 \le C \sum_{k=0}^{n-1} a_k^2(t_{k+1} - t_k). \tag{7}$$

In general setup, introduced by A. V. Skorokhod, the noise generated by the integrator η can be related to the Gaussian generalised element in H of the form $A\xi$ with the certain continuous linear operator A. Now, the definition of the integral with respect to η can be $I(A^*x)$, where I is the original Skorokhod integral. This construction is a particular case of the action of the random map on the random elements proposed in [5]. The corresponding Itô formula for the Skorokhod integral with respect to the Gaussian integrator was proved in [9]. Note that the integrator does not necessarily have the semimartingale property. Hence, the related stochastic calculus is purely non-Itô calculus. In various mathematical models where the anticipation arises, the type of construction of the stochastic integral is motivated by external reasons. For example, in the stochastic boundary value problem, physicists prefer the symmetric stochastic integral [33]. But the same problem can be treated with the Skorokhod integral (with a different solution of course) [1]. In spite of this, there exist mathematical problems where the extended stochastic integral arises naturally by necessity. These are the problems of filtration theory. The main property of the Skorokhod integral here is the following

$$\Gamma(A)\int_0^T x(t)dw(t) = \int_0^T \Gamma(A)x(t)d\gamma(t), \tag{8}$$

where $\Gamma(A)$ is the operator of the second quantisation in the space of Wiener functionals and the differential $d\gamma$ is the Skorokhod differential with respect to the integrator $\gamma = \Gamma(A)w$. As is well-known, $\Gamma(A)$ can be an operator of conditional expectation in the case when A is a projector [32]. So the relation (8) can be used for deriving the anticipating equation for optimal filter in the nonsemimartingale case (see [8, 10]). The last equation is a partial stochastic differential equation with anticipation. The properties of the solutions (existence, smoothness, large deviations) are studied in [13].

The ideas mentioned above and facts about the Skorokhod integral reflect the main steps in the development of this notion. More details can be found in the references. Certainly, the list of references is far from complete but the main ideas have been covered and the cited references can be used to support the study of the rich and beautiful object that is the Skorokhod integral.

Bibliography

[1] A.A. Dorogovtsev. Boundary problem for the equations with stochastic differential operators. *Theory Probab. Math. Statist.*, 40(11):23–28, 1989.

[2] A.A. Dorogovtsev. Stochastic calculus with anticipating integrands. *Ukrainian Math. J.*, 41(11):1460–1466, 1989.

[3] A.A. Dorogovtsev. Stochastic integrals with respect to Gaussian random measures. *Theory Probab. Math. Statist.*, 44:53–59, 1992.

[4] A.A. Dorogovtsev. One property of the trajectories of the extended stochastic integrals. *Siberian Math. J.*, 34(5):38–42, 1993.

[5] A.A. Dorogovtsev. *Stochastic Analysis and Random Maps in Hilbert Space*. VSP, Utrecht, The Netherlands, Tokyo, Japan, 1994.

[6] A.A. Dorogovtsev. One approach to the non-Gaussian stochastic calculus. *J. Appl. Math, and Stoch. Anal.*, 8(4):361–370, 1995.

[7] A.A.Dorogovtsev. *Anticipating stochastic equations*. Proceedings of the Institute of Mathematics of the National Academy of Sciences of the Ukraine, 15. Institut Matematiki, Kiev, 1996.

[8] A.A. Dorogovtsev. Anticipating equations and filtration problem. *Theory Stoch. Proc.*, 3 (19)(1–2):154–163, 1997.

[9] A.A. Dorogovtsev. Stochastic integration and one class Gaussian stochastic processes. *Ukrainian Math. J.*, 50(4):495–505, 1998.

[10] A.A. Dorogovtsev. Smoothing problem in anticipating scenario. *Ukrainian Math. J.*, 57(9):1218–1234, 2005.

[11] A.A. Dorogovtsev. Smoothing problem in anticipating scenario. *Ukrainian Math. J.*, 57(10):1327–1333, 2006.

[12] A. Benassi. Calcul stochastique anticipatif: Vartingales hierarchiques. *C.R. Acad. Sei., Ser. l, Paris*, 311(7):457–460, 1990.

[13] A. Millet, D. Nualart, and M. Sanz-Sole. Composition of large deviation principles and applications. *Ann. Probab.*, 20(4):1902–1931, 1992.

[14] A.S. Ustunel and M. Zakai. *Transformation of Measure on Wiener Space*. Springer-Verlag, Berlin, Heidelberg, New York, 2000.

[15] A.V. Skorokhod. *Integration in Hilbert Space*. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 79. Springer-Verlag. New York; Heidelberg, 1974.

[16] A.V.Skorokhod. One generalization of the stochastic integral. *Theory Probab. Appl.*, 20(2):223–237, 1975.

[17] A.Yu. Shevliakov. Stochastic calculus with anticipating integrands. *Theory Probab. Math. Statist.*, 22(11):163–174, 1981.

[18] C.A. Tudor. Stochastic calculus with anticipating integrands. *Bernoulli*, 10(2):313–325, 2004.

[19] E.Pardoux and D.Nualart. Stochastic calculus with anticipating integrands. *Probab. Theory Related Fields*, 78:535–581, 1988.

[20] E. Pardoux and P. Protter. A two-sided stochastic integral and its calculus. *Probab. Theory Related Fields*, 78:15–19, 1987.

[21] A.M. Gomilko and A.A. Dorogovtsev. Localization of the extended stochastic integral. *Sbornik: Mathematics*, 197(9):1273–1295, 2006.

[22] M.Hitsuda. Formula for Brownian partial derivatives. In *The Second Japan-USSR Symp. on Probab. Theory, Tbilisi; Springer-Verlag, Berlin, New York*, pages 111–114, 1972.

[23] M. Jolis and M. Sanz-Sole. Integrator properties of the Skorokhod integral. *Stoch. and Stoch. Reports*, 41(3):163–176, 1992.

[24] N.N. Norin. Extended stochastic integral for non-Gaussian measures in the locally-convex space. *Russian Math.Surveys*, 41(3):199–200, 1986.

[25] D. Nualart. *The Malliavin calculus and related topics*. Springer-Verlag, New York, 1995.

[26] O. Enchev. Stochastic integration with respect to Gaussian random measures. In *Ph.D. Thesis, Sofia Univ., Sofia*, pages 52–60, 1983.

[27] Shigeyoshi Ogawa. Quelques propriétés de l'intégrale stochastique du type noncausal. *Japan J. Appl. Math.*, 1(2):405–416, 1984.

[28] O.G. Smolyanov. Differentiable measures on the group of functions taking values in a compact Lie group. In *Abstract of the Sixth Intern. Vilnius Conf. on Probab. and Math. Statist., Vilnius*, pages 139–140, 1993.

[29] R. Buckhdan. *Quasilinear partial stochastic differential equations with out nonanticipation requirement*. Prepr. Humbolt Univ., No. 176, Berlin, 1989.

[30] R. Buckhdan, P. Malliavin, and D. Nualart. Multidimensional linear stochastic differential equations in the Skorokhod sense. *Stoch. and Stoch. Reports*, 62(1–2):117–145, 1997.

[31] S. Tindel, C.A. Tudor, and F. Viens. Stochastic evolution equations with fractional Brownian motion. *Probab. Theory Related Fields*, 127(2):186–204, 2003.

[32] B. Symon. *The $P(\varphi)_2$ Euclidean (quantum) field theory*. Princeton Univ. Press, 1974.

[33] V.I. Klyackin. *Dynamics of stochastic systems*. Phizmathlit, Moscow, 2002.

[34] V.V. Baklan. One generalization of stochastic integral. *Dopovidi AN Ukraine, Ser. A*, 41(4):291–294, 1976.

[35] S. Watanabe. *Stochastic differential equations and Malliavin calculus*. Tata Inst, of Pundam. Research, Bombay, 1984.

[36] Yu.L. Dalecky and G.Ya. Sohadze. Absolute continuity of smooth measures. *Funct. Anal, and Appl.*, 22(2):77–78, 1988.

[37] Yu.L. Dalecky and S.N. Paramonova. One formula from Gaussian measures theory and estimation of stochastic integrals. *Theory Probab. Appl.*, 19(4):845–849, 1974.

[38] Yu.L. Dalecky and S.V. Fomin. *Measures and differential equations in infinite-dimensional space*. Kluwer Acad. Publ. Boston, 1983.

[39] Yu.L. Dalecky and V.R. Steblovskaya. Smooth measures: absolute continuity, stochastic integrals, variational problems. In *Proc. of the Sixth USSR–Japan Symp. on Probab. Theory and Math. Statist., Kiev – WSPC*, pages 52–60, 1991.

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Top Mathematicians of the World!

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Introduction

Essential Science Indicators (ESIs) are a product of Thomson Reuters that measure scientific performance over a 10-year period. They have published a list of Highly Cited Researchers 2014 in 21 disciplines. These people have been selected as “well-established scientists” since they are in the top 1% by total citations in a given field. It is claimed that this list identifies significant trends in the sciences and social sciences. Among 3216 scientists, there are 99 mathematicians (see http://highlycited.com/).

We should highlight that our investigation does not criticise the researchers but rather challenges the criteria and the ways of ranking.

World distribution of highly cited mathematicians

Distribution of mathematicians (counted as first or second affiliations) around the world is presented in Table 1.

Country	Number	Country	Number
USA	42	Italy	2
Saudi Arabia	26	Spain	2
China	18	Serbia	2
UK	5	South Korea	2
France	4	Norway	1
Switzerland	4	Ireland	1
Iran	4	Malaysia	1
Germany	3	Jordan	1
Australia	3	Austria	1

Table 1

A strange point is the situation of Saudi Arabia with 26 affiliations from the 99 mathematicians!

Subject distribution of highly cited mathematicians

The main mathematics subject classifications of these mathematicians are recognised as in Table 2.

MSC code	Mathematics subject	Number of mathematicians
17	Nonassociative rings and algebras	2
18	Category theory; homological algebra	1
22	Topological groups, Lie groups	1
34	Ordinary differential equations	12
35	Partial differential equations	14
37	Dynamical systems and ergodic theory	1
39	Difference and functional equations	1
41	Approximations and expansions	1
42	Fourier analysis	1
47	Operator theory	12
48	Calculus of variations and optimal control; optimization	1
57	Manifolds and cell complexes	2
60	Probability theory and stochastic processes	3
62	Statistics	28
65	Numerical analysis	7
68	Computer science	1
76	Fluid mechanics	2
86	Miscellaneous	1
90	Operations research, mathematical programming	1
92	Biology and other natural sciences	6
94	Information and communication, circuits	1

Table 2

As one can observe, the mathematicians are not uniformly distributed among the 62 main subjects of mathematics (MSC2010). This is to be expected, since ESIs use total citations and there are more papers in Differential Equations than in, for example, Category Theory. In more pure subjects, there are a few selected mathematicians. Moreover, about 28% of the 99 mathematicians work in Statistics and about 26% in Differential Equations (Partial or Ordinary). Even in a certain subject one may see that most of the mathematicians are working in a special field. For instance, 12 of the 99 mathematicians are working in “Operator Theory” (MSC47) and 10 of them do research in the field of “Fixed Point Theory”. Probably there are similar situations in other subjects. Are these special fields, like fixed point theory, really “significant trends” or “research fronts” in mathematics?

A fair way may be for the ESIs to make a partition of all mathematics subject classifications into about 6 categories and then select the top 1% of mathematicians in each category.

Citation

There has been a continual struggle between quantity and quality of research papers. It is certain that both quality and quantity are important but in an ultimate judgment quality is preferred.

Nowadays, several essential tools for assessing quality are based on academic citations. Unfortunately, whenever a measure is put in place, some people find ways to circumvent it. Among highly cited scientists, one can find people who are selected due to a lot of self-citations (i.e. citations made by the author themselves). For example, ten top cited papers in MathSciNet of one of these 99 mathematicians have received a total of 217 citations among which there are 118 self-citations. This means that 54% of the citations are self-citations, i.e. more than half of the references to these ten top papers come from the author’s own papers. Is it possible that they would still be selected as an influential researcher without considering the self-citations? One of the 99 top mathematicians even had about 30 papers in one year in an ISI-journal. It is worth noting that there are highly cited mathematicians on the list because of non-self-citations (i.e. citations of their papers by other researchers). For a fair judgment, we propose that the ESIs use non-self-citations instead of total citations for highly cited researchers.

In the developing countries, there are many mathematicians who cannot do "hard mathematics" due to the existence of a lot of serious problems in their countries as well as poor scientific training. They have to publish many papers either to obtain rewards or to avoid academically perishing! On the other hand, there are fields in which writing and publishing of papers is rather easy. To write papers in such "easily-producing-paper" fields highly increases the citations to such papers. Hence people working in such subjects have a greater chance to be selected as top mathematicians.

Although among the highly cited mathematicians there is a Fields Medallist (Terence Tao), who has played key roles in the creation of significant ideas, some of the highly cited authors have no paper in the prestigious list of top 50 journals of MathSciNet. If we assume that publications in top journals of MathSciNet determine the main stream of mathematics, one could argue how these top mathematicians influence mathematics.

We should note that any attempt to correct the system may quickly lead to an "adjustment" by people whose main goal is to have a high citation index and not the scientific quality of their research.

Impact factor

In some countries, administrators often classify journals into a few classes based merely on the impact factors of those journals when they want to evaluate researchers for hiring or promoting or give them funding. They then assign certain points to papers according to their classifications and simply count points but do not pay enough attention to the contents of papers! There are some journals, in particular business-open-access journals, whose impact factors have been greatly increased in recent years by self-citations. Such journals seek rich customers and rich governments who pay for publications in order to show that they are promoting and developing!

To illustrate this situation, we use the so-called Mathematics Citation Quotient (MCQ) computed by MathSciNet in 2013, which is the average number of times that journal papers issued from 2008 to 2012 were cited in 2013. Let us choose one of the open-access journals in mathematics. The number of items published in the journal from 2008 to 2012 was 624. The number of citations of the journal was 472 so its MCQ was 472/624 = 0.62. Among 472 items, there were at least 325 self-citations! This is a large ratio, i.e. 68% self-citations!

Conclusion

We should separate quantity and quality of papers when we evaluate researchers or research. Each of them has its worthiness for scientific assessment. Choosing one of them without having a deep look at the other is incomplete and may mislead us. A combination of both seems to be defensible.

For quantity, introducing a selected list of papers (e.g. 5 or 10 items) that are examined to be good (implicitly, this is what NSF does) would help reduce the publication of many "easy" papers as they would not be counted.

For quality, we can really base our assessment on our knowledge of the whole body of mathematical literature. More precisely, we can rely on a combination of the following items:

1. Originality of works of the mathematician together with their depth and extent.
2. The prestige of journals (and their impact factors) in which the mathematician published their papers.
3. The number of citations as well as the reputation of persons and journals citing the works of the mathematician.
4. The global view of the community of famous mathematicians about works of the mathematician.
5. Interaction with editorial boards of prestigious journals.
6. Applications of the works of the mathematician in mathematics and other disciplines.

Although some of the above factors are subjective, the assessment of several well-established mathematicians in a committee with the use of bibliometrics as objective tools can reduce the deficiency of subjectivity and provide a favourable scholarly judgment.

The developed world countries often act as above but the developing countries seldom do. As evidence, one can observe that mathematicians receiving awards and plenary speakers at the International Congress of Mathematicians (ICM) and the European Congress of Mathematics (ECM) are not necessarily people with a lot of papers or with a large number of citations.[1] In developing countries, the situation is rather complicated due to neither an established evaluation system (original scientific traditions) nor enough well known experts for assessment.

Acknowledgement

The author would like to sincerely thank Professors Vitali Milman, Peter Semrl, Daniele Struppa and Dirk Werner for their useful comments.

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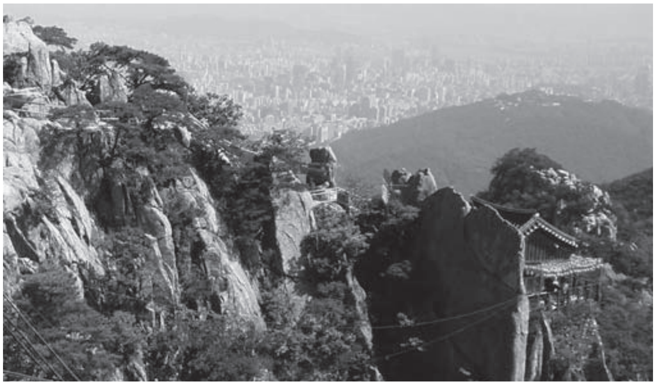
[1] See the article "Who Are the Invited Speakers at ICM 2014?", M. Andler, Newsletter no. 92, June 2014.

"Dreams and Hopes for Late Starters"– The ICM 2014 in Seoul (South Korea)

Olaf Teschke (FIZ Karlsruhe, Germany)

One of the formative developments in 21st century mathematics is certainly the massive internationalisation of research at all levels, a trend which is naturally best reflected by the International Congresses. From this viewpoint, it was a very natural decision to hold this year's ICM in the Republic of Korea, a country which has arguably undergone the most stunning development over the last 50 years, from economic figures comparable to sub-Saharan Africa after the Korean War to a high-tech nation heavily relying on cutting-edge innovation, a sophisticated educational system and a fully fledged scientific landscape. So, the ICM was again held in an Asian country (following the 2010 congress in Hyderabad), significantly marking the changing scientific world map.



View at the Gangnam district including COEX centre from Yeonjuam heritage at Gwanaksan mountain.

Advancing to the top – half a century of Korean progress

In this context, some comparisons to the previous congress in India might be natural. Hosting a congress for more than 5000 participants puts some requirements on infrastructure that are not easily fulfilled. At Hyderabad, it led to the situation that the congress formed merely a gated community in a conference centre quite distant from the city, with very few opportunities for contact with the country's everyday life. The situation in Seoul has been very different: the gigantic COEX complex hosting the congress is intimately connected to the vibrant waves of the surrounding Gangnam business district, and the excellent subway network makes other areas easily accessible, within the natural limitations of a megacity (Seoul has more than 10 million inhabitants and its metropolitan area has more than 25 million, almost half of the country's population).

Another difference was due to the different historical viewpoints: while India laid much emphasis on the famous past, reaching back to the origins of today's number system, the congress motto "Dreams and hopes for late starters" clearly showed Korea's pride of its achievements over the last few decades. While the country has its own venerable tradition of science, dating at least back to the Silla Kingdom in the 7th century, the long period of almost complete isolation during the Joseon Dynasty led to stagnation, and the following decades of Japanese colonialism and war clearly weren't very suitable for improving this situation. Therefore, the focus was on the country's rise to one of the world's leading economies, and the subsequent development of a large mathematical research community. What has been described afterwards as the "Miracle on the Han River" started at the beginning of the 1960s with a severe cut of foreign (mainly US) aid[1] after a student uprising that led to the resignation of the autocratic Rhee regime. Following a military coup in 1961, an ambivalent development took shape: in quite a centralised, almost socialist, manner, a series of five-year plans were implemented, which rapidly transformed the backward economy into a heavily industrialised, export-oriented system supported by ample government credits, import restrictions and an increasingly efficient administration. Even though the 10% annual growth rates of the next two decades were also accompanied by severe political repression and practically dictatorial powers of the president General Park Chun-Chee, the economic progress of these years is, in general, positively commemorated in the country. This nostalgia even affected the outcome of the 2012 presidential elections, which saw Park's daughter Park Geun-hye as a clear winner.

The Cheomseongdae observatory from 7th century, one of the earliest conserved monuments of Korean science, located at the ancient Silla capital Gyeongju, where the IMU assembly took place.

However the key factors of this economic success story were probably neither the central coordination nor the market power of the quasi-monopolistic *chaebols* (large, family-controlled business conglomerates with special government support, of which about a dozen form the largest part of the country's economy), but the existence of

1 In 1960, about half the national budget consisted of foreign aid.

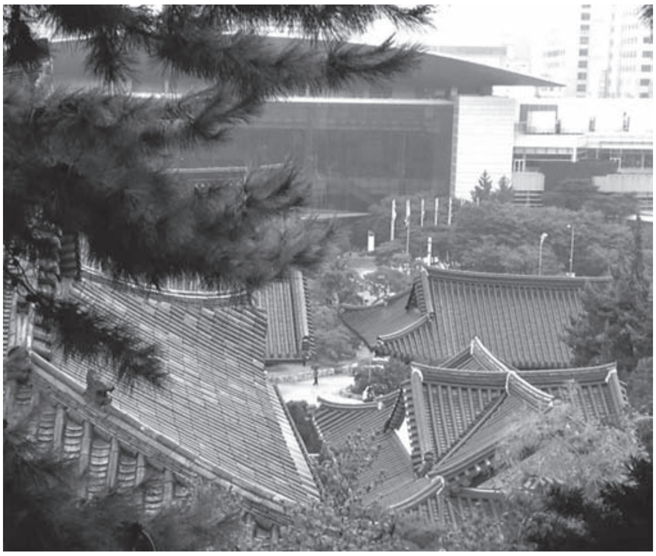
a large, diligent work-force willing to work for low wages, which has been extremely well-educated. Indeed, the importance of learning, already deeply rooted in Korean society by Confucian traditions, seems to have been brought only to extremes by the hardships experienced in the 20th century. Strong family bindings and the hope of securing the next generation's success helped to bear the burden accompanying the process of industrialisation. Despite impressive growth rates it took many years for a stable middle-class to emerge – a development which formed the basis of the democratisation process that started only in 1987, pushed along further by the opening of the country in the course of the Olympic Games the following year.

Today, Korea still bears the imprint of the rapid transformation from agricultural to high-tech society, which has hardly allowed for the social relations to cope with the changes. In effect, its inhabitants enjoy the top level in Asia with regard to average disposable incomes and civil liberties, but they suffer from a comparatively low level of life satisfaction, especially in the generation that was hit by the 1998 Asian crisis. The country is proud of its worldwide leading digital infrastructure, as well as the remnants of its traditional culture that remained unaffected by colonisation, war and progress. The popular media exercise massive cultural influence abroad (known as the *Korean wave*). The highest levels of education and innovation benchmarks are accompanied by the sad top level for suicide rate in the OECD countries.

Mathematics as a 'tiger economy'-propelled science

How does mathematics fit into this picture? On several occasions at the congress, two extreme figures were prominently mentioned: the total number of Korean mathematics publications in international journals up to 1981 (three) and up to today (more than 50,000). These numbers, based on the notorious ISI statistics, are of course somewhat distorted: there were several very good Korean mathematicians before the 1980s but they were almost exclusively working abroad (mostly in the US) and hence were not counted here; and a considerable fraction of the recent documents are generated from the well-known publication/citation mills, whose scientific value is limited to feeding such statistics. Nevertheless, there has obviously been a dramatic change.[2]

As the former minister of science and technology outlined at the MENOA meeting preceding the ICM[3], mathematics has been long been valued as a fundamental facet of education, and a tool in the industrialization process; not primarily as a subject of research. The main efforts in the first phases of industrialization went into the construction of an effective math education system and the formation of a large class of engineers (with an advanced mathematical understanding). This prevailed until the 80s, when it finally became clear that the transition from a production-oriented to a knowledge-based economy required research infrastructure beyond education. By investing the money available through the economic success, not only several doctoral programs but full mathematical research centres and institutes were founded, and – the more tricky part – staffed with competent scientists, often Korean mathematicians which formerly went abroad for doing research, but also foreign researchers who were offered attractive positions. The perhaps most astonishing fact is that this seemingly naïve blueprint really bore fruits: within only few years, a new generation of highly creative mathematicians emerged. Again, the reasons for the success may not so much be attributed to the perfect execution of a centralized scheme as rather to a favourable environment: the adaptation of mathematical olympiads fit well into the competition-oriented education system, and the much improved prestige of the subject – due to the increasing public awareness of its importance to address the challenges of modern science, as well as of the good career opportunities – resulted in figures like that now about 15% of the students of KAIST[4] choose math as major subject (compared to 2% in 1994). So it has been well-deserved that the IMU promoted South Korea, in an unprecedented move, directly from a Group 2 to a Group 4 country in 2007, and awarded it as the ICM host soon afterwards. In an act of generosity, which did not forget the country's humble past, Korea donated a substantial amount of money to the NANUM program of travel grants for 1,000 mathematicians from developing countries, in the hope of spurring a similar positive development.



The COEX centre viewed from the neighbouring Bongeunsa temple.

The ICM hosted by an emerging mathematics nation

How did the congress evolve in such an environment? As mentioned, the large COEX provided ample infrastructure; actually, just a fraction of it turned out to be sufficient – much to the relief of the organising committee (led by the unfailing Park Hyungju), which had been

[2] This is also indicated by the Math Genealogy Project, which counts the first two Korean thesis only in 1984, and 411 since then, with 33 in 2013; clearly, these data represent only a sample.

[3] See the report of M. Waldschmidt in this Newsletter issue, p. 41.

[4] Korean Advanced Institute of Science and Technology, one of the top research institutions.

surprised only a few months before the congress by the launch of a complete reconstruction of the venue.[5]

Just one of several large halls turned out to be sufficient for the ceremonies and plenary lectures (even having been cut in half after the first week), and the surrounding rooms and floors served well for the purposes of exhibitions, special sessions, information and meetings. Also, the ICM was held at the end of the monsoon season, with heavy rainfall caused by a distant typhoon just before its start, as well as for the excursion and the final day, but the unfavourable weather conditions didn't hamper the experience.

Like previous ICMs, about half the participants were local mathematicians, many of them students eager for direct contact with their heroes. The special flair of the ICMs has much benefited from the moments of tension before the prize winners are publicly announced at the opening ceremony in the presence of many thousand participants. It has been a valuable effort of previous IMU committees to preserve discretion until the ceremony, regardless of the usual rumours and leakages; so it was certainly a drawback that this time the prizes had been announced in advance on the IMU website and hence already spread by the news, to the effect that at least every participant owning a smartphone was deprived of these moments of suspense.

Security controls before the opening ceremony accounted for the only bottlenecks in a smoothly organised event.

On the other hand, the enormous efforts of the organisers helped compensate for this, with a ceremony that was opened by an impressive video show and live performances introducing the host country. There was a touching moment during the Cheo-Yong-Mu (a traditional Korean dance) where the masks seemed to scare the little daughter of Maryam Mirzakhani, who therefore spent the very moments before receiving the award taking her to the rear seats and calming her down.

Ceremonies and awards

The climax of the event was, of course, the award ceremony, which introduced the Fields Medallists and the Nevanlinna prize winner through some short but fine videos. As has often been said, the Fields Medals are distinguished among other prizes by bestowing greatness on their winners, not vice versa. They serve both well as a motivation for the next generation of mathematicians and as a link of our science to the mass media, who have gradually accepted them as an equivalent of the Nobel prizes. The diversity of the Fields Medallists could be viewed as proof of the rapid worldwide development in mathematics addressed above: Artur Avila, a Brazilian specialist of dynamical systems, working now at IMPA and CNRS; Manjul Bhargava, a number theorist born in Canada to Indian immigrants, who grew up in New York and works now at Princeton; Martin Hairer, of Austrian descent, who grew up in Geneva und works now on stochastic PDEs at Warwick; and Maryam Mirzakhani, who was the first women, as well as the first Iranian, to be awarded a Fields Medal and now works on hyperbolic geometry at Stanford.

The first female medallist contributed much to the extensive media coverage of the congress. Certainly, it has been somehow overdue – one might even say that the 78-year delay was caused by the late introduction of the medal itself, which barred Emmy Noether from the award. In any case, it should be seen as a new normality, not a singular event, since there have also been other worthy female candidates, so that it wouldn't have been a big surprise if more than one woman had received a medal. This time, the pressure of the media was fully on Maryam Mirzakhani; future female medallists may look forward to evoking less extensive curiosity.

The Fields Medallists and their work will be introduced in specific contributions of this newsletter; of course, Ingrid Daubechies, the IMU president, also announced the other prizes which have been added to the schedule of the congress over the years: Subhash Khot from NYU received the Nevanlinna prize (introduced in 1978, for mathematical aspects of computer science), Stanley Osher of UCLA received the Carl Friedrich Gauss Prize for applications in mathematics (introduced in 2006) and Phillip Griffiths from Princeton received the Chern Medal Award (introduced in 2010 for lifetime achievements). The awards were presented by the President of South Korea, the already mentioned Park Geun-hye, and the prestige of the congress is underlined not just by the presence of the head of state but also the purported anecdote that the Pope, who was visiting the country at the same time, had to postpone his audience with her by one day because she was occupied with the mathematicians.

Media, public perception, and scientific program

The high reputation of mathematics has also been underlined in the following days by the extremely intense

[5] This is only one example of the restless building activities in Seoul; likewise, the surrounding area south of the Han River (the literal meaning of *Gangnam*) used to be one of the poorest parts of the city, almost an hour of subway travel distant from downtown Seoul, until it was practically bulldozed in the 80s and emerged as a skyscraper-filled business district, making up for almost 10% of Korea's land value in the virtual currency of real estate speculation.

[6] On site, more than 250 media representatives were registered.

attention of the media,[6] naturally with a focus on the Fields Medallists. Even though the interest in the prize winners has always been a cornerstone of recent congresses (also as a vehicle to motivate the next generation of mathematicians, especially in the host country), the attention reached even higher levels here, to the effect that the medallists were frequently beleaguered by a crowd of enthusiasts. As just two indicators of the general interest, one could mention more than 20,000 participants of the public programme (mainly from schools) and long queues at the three autograph sessions of the Fields Medallists. A minor drawback was the fragmentation of the accessible areas: a hierarchy of at least seven groups of mathematicians (from the interested public and ordinary participants to VIPs, all labelled by their badges) was introduced with different rights of room access, which were enforced by a kind but firm omnipresent security. This introduced a less democratic element into the congress, somewhat contrasting its tradition of broad exchange.

Manjul Bhargava and his interviewer from the EMS Newsletter among the crowd visiting the Artur Avila autograph session in the exhibition hall.

The lectures followed the established pattern of plenary sessions in the morning and special sessions in the afternoon, the main adjustment perhaps being a tightened schedule. E. g., the original schedule sometimes lacked breaks when the participants had to walk longer distances through the winding floors between the main lecture hall and the rooms for the parallel sessions. This was, however, a minor issue and it must be emphasised that literally hundreds of volunteers displayed the famous Korean hospitality and efficient service by guiding the participants to sometimes hidden locations and solving all kinds of other problems.[7] Due to the growing number of prizes, an increasing fraction of plenary slots is fixed, which makes a representative selection of speakers even more difficult. However, the programme committee did a very good job, which makes it a question of personal taste to single out certain lectures. However, I would like to mention James Arthur's great effort of giving a survey of the Langlands programme accessible to a broad public, Manjul Bhargava's superb lecture ranging from Plimpton 322 to the Birch–Swinnerton–Dyer conjecture, and James Milnor's Abel lecture on "Topology throughout four centuries", not just packed with mathematical excellence but also a lot of hidden humour and modesty for his own contributions (this lecture was also a rare occasion to experience applause from the audience after the chair granted indefinite extra time).

James Milnor and fans after the Abel lecture.

An honorary mention should also be awarded to Yitang Zhang, who, being the only speaker not to rely on slides, fought bravely with modern technology – which insufficiently tried to electronically simulate a blackboard. In the end he succeeded triumphantly by completing his famous proof on prime pairs despite all obstacles (the congress video doesn't give the full impression of his efforts since it has been cut by some minutes).[8]

Panels, social events, and more

Beside the lectures, the congress has also evolved into an event that tries to give several impulses which may be improperly addressed as mathematical politics. While the IMU is certainly aware of the inherent dangers of being more than just a neutral umbrella (which even led to the dissolution of its predecessor in 1932), the organisation has long extended its activities beyond the traditional area of solely supervising the organisation of the congresses.[9] The mentioned activities supporting developing countries are a good example, as well as the associated International Congress of Women Mathematicians and activities related to mathematics education and the publication landscape. The IMU has been particularly successful over the last few years in raising money from different sources to support these causes. A prominent example was announced at the opening: a donation from the five Breakthrough Prize winners of a considerable fraction of their awards (the establishment

[7] As one would expect, there was also an ICM app keeping its users up to date with ongoing events and changes in the schedule, and of course a more than ample digital infrastructure available at the centre.

[8] A sad note of the congress was that due to Maryam Mirzakhani's illness, her lecture had first to be postponed, and then cancelled.

[9] David Mumford's modest statement from the 1998 congress that the IMU president holds neither an especially visible nor influential position would not be fully adequate today.

[10] One should also mention the Leelavati prize for math journalism here, which was established in 2010, and given to Adrian Paenza at the closing ceremony.

of the new prizes being another facet).[10] Inevitably, the immanent question about preserving the organisation's independence surfaced,[11] but charitable activities like the successful DonAuction event at the congress (which raised $9895 for the support of young mathematicians by auctioning mathematical objects) can certainly not be tainted by such questions. Moreover, the IMU aimed to pursue open discussions by the now well-established panels. Four years ago, for example, the misuse of bibliometric measures in science evaluation was a large issue,[12] while this time the focus was on mathematics education, popularisation and dissemination.[13]

In addition, there were a number of social events, including a number of activities related to Korean culture, the extensive mathematics popularisation programme already mentioned, a special event related to Baduk (Korean for the board game known in the West as Go) including lectures and simultaneous games, and several receptions.

A highly welcome idea was the introduction of a childcare service, which aimed at creating a family-friendly congress. Though it was still quite limited this time (the reservation became available at the website only shortly before the congress, when it became clear that the service was only offered for 4–8 year olds), this has been a starting point which will hopefully be extended in the future. One should also add that this congress was extremely well-documented and communicated, not only by the videos of the main lectures but also by social media, including an extensive Facebook site, which offered multifaceted impressions of the congress. However, the virtual media side also serves to justify the importance of the "real world meeting". Judging from the access figures, more visitors enjoyed the congress in person than in virtual form, which underlines again the importance of live encounters in our science. Thus there are enough reasons to look forward to the next congress, which will be held in Rio de Janeiro in 2018!

Maryam Mirzakhani with her family, still at limelight at the Abel prize reception.

[11] The well-deserved gratitude towards James Simons, whose foundation had supported several ICM-related activities, most notably the $ 250.000 charitable donation connected to the Chern medal which was given this time to the African Mathematics Millennium Science Initiative to strengthen the developing countries' activities, and who consequently was given a prominent role at the congress, contrasted to the condemnation of him by an U.S. senate committee for implementing a 6 billion tax avoiding scheme for his Renaissance Technologies hedge fund just three weeks before the ICM. The possibility of resulting reputation loss illustrates just one of the many difficult aspects of sponsor acquisition.

[12] In contrast, this year's Gauss Prize Lecture put heavy emphasis on citation counts, H-indexes etc. to justify the significance of the awardee's work.

[13] See also Thierry Bouche's report on the progress of WDML/GDML in this Newsletter's issue, p. 41.

Olaf Teschke is member of the Electronic Publishing Committee and the Executive Board of EuDML initiative. In the Editorial Board of the EMS Newsletter, he is responsible for the zbMATH Column.

The ICM 2014 in Seoul: At Last a Fields Medal for a Woman

Elisabetta Strickland (University of Rome "Tor Vergata", Italy), EMS-WIM Committee

The 27th International Congress of Mathematicians took place in Seoul (South Korea), 13–21 August 2014. The media coverage of this event all over the world has been such that certainly no one can have missed at least the main fact, namely that for the first time a woman has been honoured with one of the four Fields Medals, the most coveted award in the world of mathematics. The recipient was the 37-year-old Iranian Maryam Mirzakhani, full professor at Stanford University in the United States. Her research topics include hyperbolic geometry, ergodic theory and symplectic geometry.

However, even if the general focus was mainly on her, the other three winners Artur Avila (Brazilian mathematician and expert in dynamical systems and spectral theory), Manjul Bhargava (American number theorist, born in Canada and currently on the staff at Princeton) and Martin Hairer (Austrian mathematician now living in Great Britain and expert in stochastic partial differential equations) also received due attention. It was impossible not to give credit to them, brilliant, young and interesting as they were and proud to step on the podium before the fateful 40 years, the age-limit that cannot be exceeded for the award of the super-medal.

As far as the other awards are concerned:

- The Nevanlinna Prize, named in honour of Rolf Nevanlinna (the Finnish mathematician) and awarded every four years since 1982 for substantial contributions in mathematical aspects of information sciences, went to Subhash Khot, an Indian scientist from the Courant Institute of New York.
- The Carl Friedrich Gauss Prize, granted since 2006 jointly by the International Mathematical Union (IMU) and the German Mathematical Society for outstanding mathematical contributions that have found significant applications outside mathematics, went to Stanley Osher, an American mathematician.
- Finally, the Chern Medal, international award recognising outstanding lifelong achievement of the highest level in the field of mathematics, went to Philipp Griffiths, Professor Emeritus at the Institute for Advanced Study, known for his work on complex varieties in algebraic geometry. The winner announced that he had decided to hand over the $250,000 cash prize associated with the medal to mathematics in African countries: one has to remember that Phil Griffiths carried out, for many years, wide-ranging work with the IMU on the Committee for Developing Countries.

Nevertheless, in all the many articles which have been written in newspapers and magazines all over the world, in print and online, not very much has been reported about how this congress looked from the inside, to those who had the opportunity to see *in loco*, with their own eyes, the great show that the Korean people were able to stage, under the guidance of Hyungju Park, the local organiser and now a new member of the Executive Committee of the International Mathematical Union (IMU).

The proposal to organise the ICM in Seoul was presented and approved by the IMU General Assembly of Bangalore in India, shortly before the ICM in Hyderabad in 2010. The IMU General Assembly is a kind of parliament of mathematics, which gathers together the delegations of all the countries which adhere to the IMU through their national organisations before each International Congress.

Four years ago, as soon as the venue for the next ICM was official, the organising machine went into action and, while the members of the panels nominated by the IMU to choose the invited and plenary speakers started their work, the members of the organising committee launched themselves into a marathon whose goal was to find the funds and the right people to carry out the organisational program, study the location and space available, etc.

The venue chosen for Seoul was the COEX (a gigantic complex built exclusively to host meetings), which has the peculiarity of having the biggest shopping mall in Asia in its underground. Situated in the area of Gangnam, central and elegant, populated by skyscrapers flashily illuminated at night, the COEX was the perfect location for the purpose. Taking into account that over 5000 attended this ICM and that the registration fee included invitations to the welcome dinner and the opening ceremony, the huge halls of the COEX proved themselves perfect for all these people. The scheme of the programme was similar to that of previous congresses: the plenary lectures took place in the largest halls while the invited section lectures were in a series of rooms, scattered around but easy to find, thanks to the help of a legion of Korean students hired for the purpose, who were so zealous that instead of merely showing the location of the room on the map of the COEX, they took the trouble to guide visitors personally to the right place.

The architects who planned the COEX didn't have budget problems, as was evident by the first class quality of the building materials, the intelligent design and the spacious common space. In effect, the latter offered a remarkable counter-example to whosoever may worry about the usefulness of big congresses, considering them too dispersive. No, they are not, judging from the lovely sight of all those young people from 120 countries, sitting on the benches overlooking the large glass windows of the COEX during the intervals, commenting on the talks and chatting to each other, their laptops on their knees. This was truly scientific exchange – exactly what everyone was looking for.

So one can appreciate that the ICM was organised in the right place. My memory of the congress focuses on some specific issues, starting with the opening ceremony. The first memory is that on the official IMU website, contrary to tradition, the names of the Fields Medal winners appeared officially some hours in advance; not everyone had seen the announcement and someone said it was a bug. In any case, the delegates to the General Assembly, which took place three days before in Gyeongju, were informed, even if we were sceptical.

So, when Maryam Mirzakhani arrived in the main hall, we already knew that she had been awarded the Fields Medal.

For this reason, while we were all waiting for the start of the opening ceremony, I really couldn't resist reaching over to her, as she was sitting not far from me, in the front section reserved for the General Assembly delegates, the Imu Executive Committee members and the plenary speakers. I just wanted to say to this petite woman with two wonderful eyes and an expressive face how immense was the happiness of all the women in the hall for this wonderful piece of news. She saw I was moved while I was speaking to her, so she took the hand I was holding out to her, gracefully, simply, gently. There was not a bit of haughtiness in her expression, only the awareness of something enormously meaningful for someone who has devoted her entire life to coping successfully with mathematics, with all the pros and cons this involves.

When I went back to my seat, I felt overwhelmed with emotion and I thought that everything else to follow would fade in comparison. But this was unfair, as more surprises were still to come. For example, there was an important novelty in that Jim Simons, the American mathematician, hedge fund manager and philanthropist, who through his Foundation supports projects in mathematics and in research in general, produced four short movies on the Fields Medallists that were really outstanding. In the films, Artur Avila appeared while doing research in his office at IMPA, Rio de Janeiro, Martin Hairer while he was strolling on the campus of the University of Warwick, Manjul Barghava with the buildings of Princeton in the background and Maryam Mirzakhani kneeling with a felt-tip pen in her hand on a large white sheet of drawing paper unrolled on the floor, full of Riemann surfaces and formulas spread all around, while her little son was playing close to her. Thanks to these short movies, we could see them in their daily lives, within the walls where their beautiful ideas came into their minds.

Moreover, one cannot discount the speech of the President of the Republic of South Korea, Park Geun-nye (a woman), given without notes or slides. This was a Korean tribute to mathematics and mathematicians, which was indeed rather realistic, judging from the enormous technological progress of the country, which has arisen from the ashes of the Korean War as a fully-developed society in which research and education are on the frontline.

Besides the vast choice of plenary and section lectures in all the main research areas of mathematics, the special lectures were also very interesting, for example the Emmy Noether Lecture, which is given by a female speaker, this time the American Georgia Benkart, who illustrated the connections between Schur–Weyl duality and McKay correspondence. There was also a range of public lectures, such as the one given by Jim Simons, which the majority of participants didn't miss, if for no other reason than to see in person the man who has made a fortune with his hedge funds and has then assigned a considerable part of it to financing structures and initiatives for mathematics. He should be made a saint!

As an aside, it's useless to hope that the next ICM will take place in a town closer to Europe, as the General Assembly in Korea has approved Rio de Janeiro as the location for 2018. Nevertheless, let's cheer up because for us Europeans the ICM will be easier to reach in 2022, as the General Assembly in Rio will hopefully approve the proposal made for an ICM in Paris.

These are not minor details. If for once in a while we could save ourselves many long hours of flight, for us Europeans it would be great. It took a degree of heroic spirit to reach South Korea from Rome, which is not around the corner and which has few direct flights to Seoul. But, strictly for the record, when the Italian delegates arrived in Gyeongju for the General Assembly, after 12 hours of flight plus three of bullet train and one of bus, we felt ashamed because we met Alice Dickenstein, the new Vice-president of the IMU, who had arrived from Argentina via Dallas, after travelling for 25 hours, but seemed in perfect shape, unlike us who were sleepy with jet-lag and starving.

The presentation of the Brazilian bid for ICM 2018 has been approved with obvious enthusiasm, not least because it will be the first time that the ICM will take place in the southern hemisphere. We couldn't prevent ourselves from smiling when, during the slide show about the architectural structures proposed for the event, we saw a picture of the Maracanino, a stadium slightly smaller than the legendary Maracana but not far from it.

Even if I risk being boring by underlining the female presence, one should say that at this ICM, as far as women speakers are concerned, the United States featured 11 women speakers (two among the 21 glorious plenary speakers: Maryam Mirzakahni and Vera Serganova), France seven and Italy three, while Canada, Germany, UK, Israel, the Netherlands and New Zealand had one each. This means that among the sectional speakers (of which there were 218) women represented around 11% of the list. This is still a small percentage but we believe that things will now improve. This ICM represented a turning point, as the award to Maryam Mirzakhani will set an impetus for change.

And perhaps it would be nice to add that one day before the Opening Ceremony of the ICM, on 12 August, the International Congress of Women Mathematicians (ICWM 2014) took place at the Ewha Womans University in Seoul and the invited speakers included Donna Testerman, Hee Oh, Gabriella Tarantello (from my own university, "Tor Vergata" in Rome – a little bit of local pride is never inappropriate!), Laura Demarco, Motoko Kotani, Jaya Iyer, Isabel Dotti and, last but not least, Ingrid Daubechies, IMU President until the end of this year, when the newly elected President Shigefumi Mori will start his term.

So think what you like. In Seoul, women mathematicians experienced great satisfaction, thanks to the masterstroke of Maryam, the marvellous heroine of this unforgettable 2014 ICM!

Elisabetta Strickland is a full professor of algebra at the University of Rome "Tor Vergata" and is Deputy President of INdAM, the Italian National Institute for Advanced Mathematics. Since 2008, she has been a delegate for individual members on the EMS Council and, since January 2014, she has been a member of the Women in Mathematics Committee of the EMS. In 2009, she co-founded the first Gender Inter-university Observatory based in Rome, Italy. She was in South Korea as Head of the Italian Delegation at the IMU General Assembly.

Short Report on ICM2014

Michel Waldschmidt (Université Pierre et Marie Curie, Paris, France) on behalf of the Committee for Developing Countries of the EMS

The ICM 2014 in Seoul has put a strong emphasis on the need to improve the geographical distribution of mathematical research. Under the NANUM programme, 1000 mathematicians from developing countries were supported and could participate in the conference. Three NANUM Regional Networking sessions were organised on the first three evenings of the conference, one for Latin America, one for Africa and Eastern Europe and one for Asia, where the participants from these regions who were supported by the NANUM programme could meet.

ICM 2014 started on 13 August. The day before, on 12 August, a symposium "Mathematics in Emerging Nations: Achievements and Opportunities" (MENAO) was organised by the International Mathematical Union.

The Committee for Developing Countries participated in these meetings. In particular, the Committee for Developing Countries had a booth during the MENAO event, where a poster and a leaflet as well as a beamer presentation were displayed.

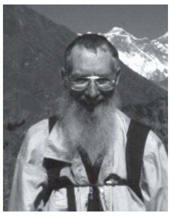
During international conferences, many publishers have a booth where they advertise and sell their recent books. ICM2014 was no exception. Among the exhibitors were many academic and publishing houses. At the end of the conference, most of them donated some of their remaining unsold books to our committee. Some representatives from African institutes packed these 164 books (with a total weight of 126 kg) and shipped them to their countries.

ICM2014 gave the opportunity for the Committee for Developing Countries and the Committee for European Solidarity to hold a joint meeting.

The website of our committee provides more information on our activities; in particular the homepage

http://euro-math-soc.eu/EMS-CDC/index.php

displays the latest news from the EMS-CDC.



Michel Waldschmidt, emeritus professor at the University Pierre et Marie Curie (Paris 6) where he has been teaching for 40 years, is an expert in the theory of transcendental numbers and diophantine approximations. From 2001 to 2004 he was President of the Mathematical Society of France. He actively promotes the idea that developing countries also need to have good mathematicians at all levels, including at a research level. From 2005 to 2009, he was Vice-president of CIMPA (Centre International de Mathématiques Pures et Appliquées). He is Chair of the Committee for Developing Countries of the European Mathematical Society. Being retired has given him more time for his hobbies: ultra marathons and trekking.

Update on Recent Global Digital Mathematics Library (GDML) Developments

Thierry Bouche (Université Joseph Fourier – Grenoble I, Saint-Martin d'Hères, France)

Since the start of the millennium, the mathematical community has been considering building a Worldwide Digital Mathematics Library, where all written mathematics would be stored and made accessible. This objective has been pursued in isolation by many people around the world but it seems that the global goal has regained momentum this year: the EuDML initiative has been formed under the auspices of the EMS, an IMU-sponsored study has been published by the US National Research Council (see this newsletter's September 2014 editorial) and a number of activities have taken place in association with the ICM.

1. A meeting to discuss the next steps towards realising the GDML was held on 17 August at ICM2014 in Seoul and attended by 14 participants in person and eight remote participants. Following a wide-ranging afternoon discussion, an eight-person working group (WG) was created, under the sponsorship of the IMU: Patrick Ion (Chair), Thierry Bouche, Bruno Buchberger, Michael Kohlhase, Jim Pitman, Olaf Teschke, Stephen Watt and Eric Weisstein.

The group began meeting immediately after the main session. It has been charged with designing a roadmap before the end of the year covering the practical next steps toward the GDML, determining its organisational structure, prioritising the different requirements for its implementation, estimating an incremental budget (both start-up and sustaining funds) and fostering the writing of proposals to funding organisations.

This group will need to coordinate the two facets of the envisioned library: content aggregation and delivery (probably the most pressing demand of academic mathematicians); and development of radically new ways to extract, datamine, compute or activate the mathematical knowledge contained in these texts (to which researchers in computer science and artificial intelligence are eager to contribute). If this coordination achieves reasonable success then we can imagine a fantastic tool to navigate the literature, following concepts, methods and mathematical objects rather than bibliographic references, which would certainly open new paths for research in a decade or so.

2. The World Digital Mathematics Library (WDML) was the topic of a panel discussion, moderated by Peter Olver, at ICM2014 in Seoul on 20 August.

The panellists were Thierry Bouche, Ingrid Daubechies, Gert-Martin Greuel, Patrick Ion, Rajeeva Karandikar and June Zhang. Further details, including links to the panel brief and background materials, can be found on the CEIC website http://www.mathunion.org/ceic/resources/icm-2014-panels/, while a videotape of the entire panel discussion appears on YouTube at https://www.youtube.com/watch?v=OERXmv2oIyU.

A summary of the panel will appear in the ICM proceedings, Vol. 1.

3. As a result of the meeting and the panel, the IMU has initiated a WDML blog, with Peter Olver serving as moderator, at http://blog.wias-berlin.de/imu-icm-panel-wdml/. The IMU encourages everyone to submit posts and to publicise the blog with the community at large.

4. The CEIC/IMU has set up a moderated email list for people interested in the WDML/GDML, to encourage additional discussions amongst experts. The address is gdml@mathunion.org but only members can post to the list without approval. To ask to join the list, send an email to Peter Olver at olver@umn.edu.

Thierry Bouche [thierry.bouche@ujf-grenoble.fr] is the Director of Cellule Mathdoc (makers of NUMDAM, CEDRAM and mini-DML) and a Member of the Institut Fourier at Université Grenoble Alpes, F-38000 Grenoble, France. He is the Chair of the EUDML initiative and a Member of the CEIC and the GDML working group (IMU).

Interview with Fields Medallist Martin Hairer

Ulf Persson (Chalmers University of Technology, Göteborg, Sweden)

This is a question I always ask the Fields Medallists. Were you surprised getting the medal?
I cannot very well answer 'No' to that question. But when I got a message from Ingrid Daubechies at the IMU about a convenient time for me to take a call, I more or less understood what it was all about. And I admit that I had heard rumours to the effect that I was being considered.

But you were never on any official shortlist, such as getting a medal from the EMS?
No. The work for which I was rewarded only took place in the last four years. I was awarded the Fermat prize though, which has done a remarkable job at predicting Fields medalists over the last few years (4 out of the last 6 recipients).

How do you think this will affect your life?
Not too much I hope, but invariably it will. I still hope that I will be able to pursue mathematics as I have always done and not simply become some sort of poster boy for mathematics…

…pinned up on the walls, you mean. After all, the original intention with the medal was to encourage young people to keep up the good work.
Exactly.

But let us go back to the beginning. What is your background? What were your parents doing?
My father was actually a professor of mathematics in Geneva, a numerical analyst as a matter of fact. And my mother was a primary school teacher but gave that up when she had children. I have two siblings by the way.

Geneva. That means that you grew up in a bilingual setting, speaking German at home and French otherwise.
That is true.

So when did you get interested in mathematics?
Early on. I was always interested in mathematics and, of course, through my father I got curious about it when very young.

Were you really interested in mathematics? There is a difference between being very good at mathematics (effortlessly catching on with the basic, elementary stuff, as most of our colleagues did) and discovering what mathematics is all about and getting a glimpse of its true fascination.

I think I did that early on. I asked my father what a differential equation was and he explained it by drawing arrows on a piece of paper – what I would later learn to be integral curves. Then we had a calculator on which you could program some simple graphics and I programmed and played around with it.

How old were you then?
I was about eleven I think.

This is definitely something that was not available to me when I was a child. It meant that you learned to program.
Yes I did.

Programming is very seductive; unlike mathematics you never get stuck.
This is not true; fairly early on I realised that programming did encounter obstructions due to the limitations of language, which I naively did not appreciate at first. I learned to program in Pascal and there was e.g. no way of dynamically updating function pointers.

This is undoubtedly true but this is a high level of frustration; I meant more in the usual run. When programming you always get immediate feedback, which you do not get in mathematics.
This is true. And in the early days of programming on personal computers – I am speaking about the mid- and late-1980s…

…Funny – this is the time I learned to program too. Although there is an age difference of a quarter of a

century between us, we apparently learned to program at the same time….
…and then you could write simple code that produced graphics as good and sophisticated as was commercially available.

And this is not true today.
Definitely not; the learning curve is much steeper. Back then, you could be self-taught and self-contained. Now you have to do it much more modularly, invoking already available programs.

This definitely takes the fun and charm away from it. Programming can be a very good pedagogical way of implementing mathematical ideas, provided you do it from scratch. If you just push buttons, invoking already available tools, it will of course be much more efficient but you will be bored. It is like doing mathematics by combining theorems which you do not understand but instead treat as black boxes – good for getting on and getting results but ultimately self-defeating as far as mathematical enjoyment is concerned. But returning to encountering mathematics: when I was young, it was classical Euclidean geometry that triggered my interest. I was fascinated by the way you could reason logically and compellingly – in many ways a moral insight. Did you have anything of that? It has been removed from Swedish curricula for a long time.
If you mean proving that different angles are equal, using the congruence theorems and such things, we were exposed to it but it was never pursued further.

So it did not make too deep an impression on you?
Not really.

Programming is about the same thing: implementing small logical steps.
But what really made an impression on me was Cauchy's residue formula. I was very impressed; it seemed to come out of the blue. Before, when studying mathematics, there were no surprises. You more or less understood what was coming and how to prove it. But this was very different – so totally unexpected.

It is only when you get to analytic functions that the element of magic enters mathematics, when you realise that there are very deep connections.
This is true.

What were your interests besides mathematics?
Physics and computer science. I actually studied physics and wrote my thesis on physics, but really it was more mathematical and my thesis advisor was more of a mathematician in spirit.

You did not pursue computer science?
No. As I indicated to you, in the past amateurs could produce codes that provided graphics on a commercial level. I simply imagined that I knew everything there was to know and saw little point in starting all over again. I know in retrospect I was being unfair.

Perhaps, but not really. There is a much richer culture in mathematics than you can find in programming. In fact, I suspect that early on your interest in programming was provoked by mathematics. Mathematics does provoke so many excuses for programming, making it tangible.
That is true. Still, I did consider a career as a commercial programmer. I knew it would provide a good salary and allow me to live a very comfortable life.

As we discussed, you do not get stuck in programming the way you do in mathematics. The challenges can always be met.
The academic world struck me as rather harsh and I worried that I might not make it. I decided to give it a chance. I applied for a post-doc grant and I used the money to attend a special year arranged at Warwick where I had previously attended a special year devoted to stochastic partial differential equations. In fact, Warwick is about the only place in England such things are being done.

Yes, I know about those special years. They have been arranged for almost 50 years I believe.
Yes, more or less since the university was founded. It is one of the very best universities in England as well.

And you got stuck there.
I would not put it that way, but it is quite interesting how I got to end up there.

You married a UK woman?
More or less. My wife is actually of Chinese origin, but she did her PhD in England on an exchange programme that was rather unusual at the time (during the late 1980s). She also spent time in the States before returning to the UK. But another reason was that my research went very well. As it turned out I and a friend ended up in some kind of competition: what we were doing was also of interest to other people, and there was a precise conjecture many different teams wanted to prove, many of them quite distinguished. But we had a good idea and felt that we had an edge.

So the competition stimulated you? This is not so common in mathematics, where people tend to fan out, according to the image formulated by von Neumann, and there is often no one else doing what you are trying to do. It makes for peace and quiet. I suspect that the physicists are far more focused.
They certainly are. The main thing about the competition was that it certainly made us finish up much faster than we would ordinarily have done without this pressure. And then it caused a minor sensation and I was hailed as an upcoming young star.

In other words, your luck was made. I guess this was the reason you got tenured only two years after your PhD.

Certainly, it was.

Competition does provide a measure of objectivity of standards, which may not always be present in mathematics. It is via competition you can measure yourself against your peers. Many mathematicians have been drawn into mathematics due to early successes in mathematical competitions and Olympiads.
I never competed in school. I would not have done well I believe. Solving those problems seems to hinge on some clever combinatorial tricks, which I would never have come up with. I simply do not feel comfortable with that kind of mathematics.

Solving problems under time pressure and doing actual research are quite different – the difference between a sprint and a marathon – and it is actually remarkable that the correlation is as high as it is.
That is true. Competition problems are also a bit artificial and combinatorial, as I just said, while real research engages in more natural and deeper theorems.

What kind of mathematics do you dislike?
Dislike?

Yes, dislike.
I do not dislike any mathematics. It is rather that I do not feel equally comfortable with all kinds of mathematics.

Such as?
Graph theory, group theory. I just would not be able to come up with proofs. I have, of course, a general overview but with many kinds of mathematics it is just very superficial.

So your mathematical career has followed a straight path since you were ten? Do you have any other interests besides your scientific?
I would not put it that way. As to other interests, I am a bit at a loss. I like to hike, I like to listen to music.

What about literature?
You mean reading. I read a lot but purely for pleasure and entertainment – nothing serious, only very light stuff.

So you do not want your reading to interfere with your mathematics? So you only read…
…trash you mean. Sure. And I do not know how to play an instrument, which I regret very much. It would have been nice to be able to play the piano. But this is probably too late now.

Could be. Learning to play an instrument involves a lot of drudgery probably having little to do with music per se – a kind of muscular coordination training best engaged in when young and malleable. But why do you think that mathematics and music are so often seen as related?
For one thing there is the physics of sound that lends itself to a mathematical treatment.

Pythagoras and all that. But this is not what we really have in mind.
Of course not. Music is the most abstract of arts and by far the most structured.

Yes, especially as it is based on recurring themes that are subtly changed, just as in mathematics where themes are never mechanically repeated. Music that is algorithmically generated is boring.
True.

How come music is never expected to be applicable, while mathematics is? And why do you have to be a mathematician to appreciate the beauty of mathematics but you can, emotionally at least, enjoy music even if you cannot compose?
I am not so sure about non-mathematicians being unable to appreciate mathematics after all. But it is true that music attracts a far greater audience than mathematics. I guess it has to do with the threshold level of mathematics.

Do you feel that mathematics ultimately has to be applied to justify itself? When the virtues of mathematics are sung to the general public, it is always the applications that are lauded. The Korean President referred to animation in her speech. Personally, I suspect that people are bored by that. Fermat's theorem and Perelman engage the public imagination much more than applications to, say, transmission between mobile telephones.
I am definitely not saying that mathematics has to be applicable to be justified. In that sense I identify myself as a pure mathematician, thinking of mathematics as an art. Also, I do not believe that there is a sharp distinction between pure and applied mathematics; there is a continuous spectrum, on which an individual may, over time, occupy different locations…

…except, of course, social. They are usually in different departments…
…Except not at Warwick, at least. This is one of the things for which I approve of my university. It is, of course, true that the kind of mathematics I do, with its ties to probability theory, is considered very applied, the reason for which is obviously that it grew out of physics.

This is true of analysis in general, while many of the harder problems of analysis concern subtleties of infinite sets, having no concrete counterpart in the real world.
Quite true. As to the public appreciation of mathematics, Fermat's theorem is something that people in general, or at least people with some interest in mathematics, are able to understand.

Let us change tack. Do you think that there are too many people going into mathematics?
I do not see how this is really a problem. True, too much is published in mathematics but I do not see how one can do anything about it. This is the way the system works.

Young people need to show something in order to get ahead and get a job.

One could make it harder to get papers published
Publishers would not like that. They thrive on greater and greater volumes. I do not see how it can be changed. And not that it is such a big problem either, if I am to be honest.

You think that quantity by itself can be a good thing?
In my field of analysis, you have results which are true under certain conditions. What those conditions are cannot be fixed in any clean canonical way, as is maybe the case in other fields of mathematics. If you change the conditions, the results become subtly different. There is certainly great value in exploring this systematically and to do so, you really need a lot of people working. Most of the results obtained may not be interesting at all but you never know what you may come up with.

This ties up with mathematics becoming more and more of a big science, something the funding agencies would definitely appreciate.
Of course they do. Big projects are what they are comfortable with. It makes it much easier to give money for one thing. Another tendency which I very much deplore is to identify winners and shower them with money, while others do not get anything at all – a clear case of winners take it all. I can see that this system may have some relevance to big science when a project needs a lot of resources and you have to prioritise in order to maintain critical masses, but in mathematics? It is so different. A mathematician does not need much money, only enough to keep him comfortable and not having to worry about basic needs, including such things as going to meetings that he may find interesting or inviting people whom he may want to talk to and learn from. We are clearly talking about peanuts.

Could it be possible that the forms of mathematical research will change, partly under the pressures of funding, and that it will be more like in big science where there are large projects involving a lot of people with a definite hierarchy and where most people are simply told what to do? You are good at a certain type of combinatorial arguments – solve this! Maybe many mathematicians might find this a relief, not having to take personal responsibility for their research.
As to a big project, the only thing I know of is the classification of finite simple groups, although Polymath provides a systematic attempt to pool the efforts of many to a common goal, where everyone puts in their piece of the puzzle but without the hierarchy you are referring to.

As one of my colleagues put it, in other fields graduate students are an asset, in mathematics a liability.
Yes, you have to come up with a good problem for them and more often than not solve it. As to the concept of large projects, it is good for some things but when it comes to the creative breakthroughs in mathematics, the kinds of things we referred to as coming out of the blue, this is solely the result of individual efforts.

Another danger with this change of the traditional culture is that mathematics may be diverted into avenues that are not intrinsically interesting from a mathematical point of view. As an example, take the calculations of the shapes of complex molecules in life sciences. Those shapes can, in principle, be derived from basic quantum physics but it seems that this will have to do more with simulations than mathematical stimulation -- no global understanding. Is there a danger that mathematics will run out of simple but powerful ideas and become inhuman, in the sense of being inaccessible to the individual mind.
It is true we mathematicians prefer to understand why something is true, not just being told using some complicated verification. But this does not only occur in applied mathematics but also in pure; I think of the notorious computer proof of the four-colour problem from the 1970s. The theorem is true just because a computer has checked a vast number of special cases. It gives no insight. As to your worry about running out of new exciting things, this is far from imminent. I am thinking in particular of the recent results on sparse matrices which were presented here in Seoul at the congress.

Have you ever read a mathematics book from cover to cover?
Come to think of it, I think only once or twice, when I was a young student. A good writer of a mathematical book knows this of course and writes in such a way that it can be disassembled into small, self-contained parts

We discussed before the impossibility of giving definite formulations to theorems. What is important in a result is not any of its various formulations but the idea that lies behind it. You cannot treat a theorem as a black box. But ideas can never be formulated.
I am not so sure about that. I think one can convey ideas, but it is true it has to be done obliquely. You can present them many times, subtly changing the formulations, saying the same thing over and over…

…This ties in with the often touted opinion that mathematics is something you get used to. You may not really understand what you are doing but believe you do…
…Another very important thing is to present the instructive example. The specific example conveys the general idea without having to formulate it. In my talk I gave an explicit example; did you go to my talk?

No, I was unable to do so. In the same vein it seems that the most effective ways of conveying mathematical understanding is through personal conversation – the time honoured method since the beginning of human time. Why is that really?
It has to do with the pacing, the possibility of direct interaction. You can ask questions. Pictures can be drawn.

Pictures are very important; although crude and specific, they convey more than a thousand words, as they say.

Proofs, when written down, are formal, although of course never adhering to the standards logicians may want to impose. In that sense, they provide a detour when it comes to communication. How do you read proofs, or do you at all?
When I encounter a result, my first reaction is: 'Can I prove this myself?' And I try. Usually, I see how it is being done but occasionally I get stuck. I look for how that difficulty is treated in the proof, so in effect I am successively bisecting the proof, zooming in on the crucial point (there could of course be several). So I either get it or discover that the proof is wrong.

This ties up with how to present mathematics to an audience. What do you think of this modern fashion of PowerPoint, beamers and all? I find them flashy.
They certainly are flashy. I prefer the blackboard, of course, but in a situation like here at the congress, you have no choice. Using a blackboard presentation, you do not really have to prepare, if you see what I mean; also, you can change your presentation in mid-flow; you can draw pictures spontaneously. Also in my published work I try to include as many pictures as is feasible.

How do you draw them? Personally, I taught myself PostScript.
I used to program in PostScript myself, or rather I generated PostScript code writing in C++. Now I have adopted TikZ, developed by the same guy who invented beamer. It makes the interface with text much more natural; you do not need to keep separate files.

Another disadvantage with this is that the speaker becomes more and more superfluous; the talk becomes a movie and the speaker can join the audience. In other words, there is a lack of presence and presence is what a personal conversation has. But I would like to pick up on a thread we lost some time ago. Time is linearly ordered but structures of ideas are not, which is part of the difficulty of conveying mathematics in time. In the past there was not this pressure to publish. What is better for the ordinary mathematical graduate student: to study and learn something which is well-known but interesting or to do something new, which even if original is uninteresting?
For the sake of culture, the first is obviously to be preferred. But it is hard to predict what is interesting, just as it is hard to predict what may be applicable.

But in many cases a thesis, or more generally a typical paper, may just be a pointless technical exercise, the purpose of which is solely to satisfy bureaucratic demands, not to be read and pondered.
As I have already said, this is the way the world works and there is little we can do about it.

In the past, there was the possibility of being a high school teacher. Most people with PhDs in mathematics in Sweden in the first half of the previous century ended up as high school teachers. Weierstrass was one for a long time. It was a prestigious position.
Not anymore. Times have changed. True, I know personally a woman who was very good and ended up as a high school teacher in Switzerland.

Finally, to return to a theme we touched upon earlier, what motivates you to solve problems? Obviously not money. There can hardly be any more difficult way of earning a million dollars than by solving one of the Millenium problems, even for mathematicians. Could it be fame?
Money – certainly not. As I noted we have modest needs and they are usually met. And as to fame, I've got my fair share of that now.

So we come back to the love of mathematics, the sweetness of the challenge and something most people simply cannot understand.

Ulf Persson [ulfp@chalmers.se] has been a member of the editorial board of the EMS Newsletter since 2006 and a professor of mathematics at Chalmers University of Technology in Gothenburg, Sweden, since 1989, receiving his Ph.D. at Harvard in 1975. He has in the past interviewed recent Fields medalists specifically for the EMS Newsletter, as well as other mathematicians for alternate assignments, in a conversational style, some published, others as of yet unpublished. There is a plan to make a selection and collect them into a forthcoming book.

Interview with Corrado De Concini – Member of Fields Medallist Committee for Hyderabad

Ulf Persson (Chalmers University of Technology, Göteborg, Sweden)

You were a member of the Fields Medal Committee for the ICM in Hyderabad 2010.
That is correct.

It is an important committee, maybe the most important one connected to the ICM, because its decisions will have historical importance. How is it formed and how long in advance?
I do not know how it is formed. I guess that the IMU Executive Committee has something to do with this. As to the second part of the question, I was contacted in September 2007.

What was your reaction? Did you accept it all at once?
I felt that it was a great honour but I also felt afraid of the responsibility. I decided to accept rather quickly.

How does the work of the committee proceed? Do you meet in person?
In fact, we only met in person twice: once in the US and once in Europe. For the rest of the time, discussions were by email. But those physical meetings were very important, at least for me.

Could you be more specific about the way you go about it? In what way does it differ from the work of a hiring committee?
Actually I have no real experience with hiring committees. We do not really have them in Italy and when I have been involved in such committees outside Italy, my position has been that of an outsider, only being asked questions at the very end. As to the Fields Medal Committee, I can only speak for the one in which I was involved. Basically, there was a list of people who had been put forward – of all kinds. There was a school teacher from the south of Italy who was pushing his own case. Some of the dossiers for the candidates contained letters and precise presentations. At the first physical meeting we made a shortlist – actually not so short but restricting to the serious contenders.

How many are we talking about?
I do not recall the exact number. It must have been somewhat more than ten but of course I could be wrong.

So what did you do with the shortlist?
We had to complement the dossiers of the candidates, mostly by requesting additional letters. The final decision was made at the last physical meeting.

You have to rely on the opinions of experts.
Yes, you are forced to. Still, I tried to read at least some of the papers of each candidate.

Was that possible, or even meaningful?
To be honest, it was very difficult to read the papers in subjects which are distant from my own.

Could you be more specific?
I prefer not to be. As to it being meaningful, this is difficult to answer. But you have to form a personal opinion, however tentative, in order to profit from the opinion of others. You hope that at least some member of the committee will be in the position to be able to explain the work.

The whole thing has to be a secret, which means that it is difficult to involve too many outsiders.
Yes. It made things more complicated. I went around to people asking them to give me some short lectures on the work of some candidates. This was a bit embarrassing, as they looked at me as if I was a madman.

They must have suspected something?
Surely. But nobody confronted me with a direct question. And presumably there were no leaks.

So the process converged?
Yes. By the time we had the last meeting, the shortlist had shrunk, and slowly it was whittled down to four.

There are, of course, no real objective criteria. The discussions that lead up to the decisions could easily lead in other directions. There must be a certain element of Brownian motion. So even with the same committee, the outcome might have been slightly different, depending on mood.
This is true and inevitable. I would say it happens every time and is quite natural. It is, after all, a meeting of human beings who bring their own inclinations and tastes. It is a matter of influencing each other. There are, of course, extreme cases, such as Perelman, when surely every committee would come to the same conclusion. The matter is, of course, sensitive so I will not mention more names.

I guess the discussions could become heated.
Actually, I was a bit surprised coming from Italy; I found the general atmosphere rather civilised. Of course there was intensity but never animosity. And this is true; I am not expressing myself diplomatically.

Maybe the discussions themselves were too diplomatic. As an Italian, you would have preferred more passion, some spitting of blood so to speak?
This I refuse to comment upon.

As I noted above, nothing really compares with the Fields Medal in mathematics, in spite of there being other much more lucrative prizes. My own explanation is that the Fields Medal bestows greatness, and rather early on in the career, while the other prizes merely confirm it. You are, in a way, making kings. Do you have any comments on this?
I find this very difficult to comment upon. Either I have to give it serious thought or just ignore it. Anyway, it is a general question, having little to do with being on a committee or not.

But you must have some thought on it. It surely must influence your attitude to the committee work.
If you insist, you must give me time and accept an answer in Italian.

How much politics enters the discussion? I am thinking of balancing things. Obviously not when it comes to geographical distribution – that would be absurd – but maybe when it comes to different mathematical disciplines or, more sensitively still, gender balance?
Obviously, as you say, there is no attempt at geographical distribution. In our case, two medals were awarded to France, albeit one recipient being of Vietnamese origin. As to the other, I am less qualified to have an opinion and would prefer to be vague and informal. I think that in recent years there has been a tendency to ‘open up’ subjects which are less ‘traditional’, such as, for example, probability and dynamical systems. I would not say that this is the result of being more ‘balanced’, only ‘fairer’ in the sense that the committees are now more willing to look seriously at a wider spectrum. And I would not call it political but cultural. That is my impression anyway. And I would not call it negative nor positive.

It makes the work of a committee more difficult when more disciplines are penetrated.
That is of course true. Still I wonder when we will see any medals awarded to a statistician or a numerical analyst.

You referred to traditional subjects. Do you think that some mathematical subjects not only carry more political or cultural prestige but are more important to mathematics as a whole?
This is a personal question.

Not if there is some consensus about it. What about the gender issue?
I cannot, of course, speak about the work of the present committee but when it came to mine, several female candidates were very seriously considered. However, even if the gender issue was considered, it was only briefly and never taken into consideration. At least not by me. In short, women were considered very seriously along with men but there never was any open pressure to try to select a woman.

And by implication, we can assume that this was the case with the present committee?
I assume so but of course I cannot speak for something I do not know.
But is this not the general attitude of mathematicians. There is absolutely no desire to suppress women, contrary to what the outside world may suspect due to the skewed distributions. In fact, most male mathematicians are delighted when female mathematicians distinguish themselves, but the delight would be tempered if it were brought about deliberately. I sincerely believe that just as mathematicians are famously indifferent to ethnic and national divisions, they are the least sexist of all scientists.
It is a very general statement and also a very strong one, to which I am not ready to subscribe.

Those are strong words but we all want to believe them.
Maybe.

Author information about Ulf Persson: see page 47.

A Brief History of the Irish Mathematical Society

Martin Mathieu (Queen's University Belfast, UK), President of the IMS

The Irish Mathematical Society (IMS) is a small and rather young mathematical society in Europe. It was founded in 1976 and has no more than 350 members, many of whom live on the island of Ireland but a good many of whom reside abroad. Unsurprisingly, it aims to foster the development of mathematics, in particular when coming with an Irish connection, and it interacts with other mathematical societies through various reciprocity agreements. This short note intends to review the history of the society and highlight its present activities.

Traditionally, science in Ireland has been represented through the Royal Irish Academy (RIA), established in 1785 in Dublin and granted royal charter in 1786. One of the academy's best-known figures was Sir William Rowan Hamilton, who was president for the period 1837-1846. Until recently, the academy had a dedicated National Committee for Mathematical Sciences (now merged with Physics and Chemistry), which, until the mid 1970s, coordinated many mathematical activities such as scientific meetings with the Dublin Institute of Advanced Studies (DIAS), a research institute for Theoretical Physics and Celtic Studies founded by Éamon de Valera in 1940. Upon returning from their PhD studies in the United States, a group of young mathematicians comprising F. Holland, J. Kennedy, D. McAlister and T. T. West started a series of Summer Schools, the first of which took place in July 1969 at Trinity College Dublin focusing on Group Representations and Quantum Mechanics.

With support from all the Irish universities, as well as the IMU, further Summer Schools were organised in subsequent years and they turned out to be a great success and helped to further the development of mathematics in Ireland. Some of the proceedings of these meetings were published in the *Proceedings of the Royal Irish Academy*. In discussions at these conferences, the idea of forming a national body for mathematics in Ireland emerged. A draft constitution, which was based on that of the Edinburgh Mathematical Society, was drawn up by D. McQuillan, J. Lewis and T. T. West. At the DIAS Easter Symposium, this constitution was accepted on 14 April 1976 and thus the *Irish Mathematical Society* (or, by its Irish name, the *Cumann Matamaitice na hÉireann*) came into being.

The IMS is an all-Ireland professional organisation of mathematicians. One of its main activities is the Annual Meeting (which includes the society's Annual General Meeting), typically held in early September or late August. In 2004 at Queen's University Belfast and in 2009 at the National University of Ireland Galway it was held jointly with the *British Mathematical Colloquium*. The meeting covers all aspects of mathematics (pure and applied) as well as mathematics education, and often attracts well-known speakers from abroad. The *Fergus Gaines Cup* is awarded at the meeting to the best performer in the Irish Mathematical Olympiad.

The society's main publication is the *Bulletin* which appears twice per year. The aim of the *Bulletin* is to inform society members, and the mathematical community at large, about the activities of the society and about items of general mathematical interest. It publishes articles written in an expository style such as informative surveys, biographical and historical articles, short research articles, classroom notes, Irish thesis abstracts, book reviews and letters. All areas of mathematics are considered: pure and applied, old and new. The *Bulletin*'s first issue appeared in 1978 (then called the *Newsletter*) and the more recent issues are freely available online from the society's website.

The society supports mathematical activities such as small conferences and workshops held in Ireland and organised by its members. A list of recent events can be found on the website. Organisers of such meetings are asked to write a brief report of their event for publication in the *Bulletin*. Some of these conferences have been held jointly with other mathematical societies; for this purpose and generally to further the interaction and scientific exchange between societies, reciprocity agreements are in place with the American Mathematical Society, the Deutsche Mathematiker-Vereinigung, the Irish Mathematics Teachers Association, the Moscow Mathematical Society, the New Zealand Mathematical Society and the Real Sociedad Matemática Espanola. These agreements also include the exchange of journals published by the respective societies.

But there is not only mathematics in Ireland…

The IMS also plays a role in promoting mathematics to the general public, emphasising its fundamental importance for modern society. Often such discussions are sparked by educational issues such as unusually low performance of students in the end of school certificate exams or by (proposed) changes to the mathematical curriculum. Research funding, or rather the lack of it, has become a general problem for mathematics, in particular the less applied strands, but the small voice of the IMS is often not listened to. Recent new policies and strategic changes in the main Irish funding body, the Irish Science Foundation, has resulted in a draught for PhD student funding. This may create a serious problem for Irish mathematics in the medium term.

In compiling the above historic information, I could fortunately draw on an excellent survey by the late T.T. West: The origins of the IMS, *Irish Math. Soc. Bull.* 51 (2003), 73–75.

Martin Mathieu studied at the University of Tübingen, Germany, where he obtained his doctoral degree in 1987 and his Habilitation in 1991. He has held positions at the University in Saarbrücken, Germany, the University of Iowa, USA, the National University of Ireland, Maynooth, and since 1998 he has been at Queen's University Belfast. He was elected a Member of the Royal Irish Academy in 2010 and presently serves as President of the IMS (2013–2014). His research interests lie at the interface of analysis and algebra, notably operator theory on C-algebras and noncommutative ring theory.*

The Mediterranean Institute for the Mathematical Sciences

Sadok Kallel (MIMS Director)

The Mediterranean Institute for the Mathematical Sciences (MIMS) is a newborn in the world of mathematical institutes around the world. It was created in January 2012 in the capital city of Tunis, Tunisia, under the initiative of a number of mathematicians from a few universities around the country and outside. It held its first major activity in June of that year in the form of a conference on *Operads and Configuration Spaces*, the proceedings of which are now published.

The inaugural conference of MIMS was held in Tunis in October 2012. Close to 200 participants were present in the main amphitheatre of the *Cité des Sciences*, the largest science hub in the country. The plenary speaker, also honorary chairman of MIMS, was Sir Michael Atiyah, who spoke about *beauty and mathematics*. Since then, MIMS has been an active and important addition to the mathematical landscape south of the Mediterranean and in the MENA region.

Cité des Sciences, Tunis

Inaugural conference with Sir M.F. Atiyah

MIMS is a private and non-profit institution. It has a multifaceted mission which is articulated around two major themes: Mathematical Research and Mathematical Education.

Mathematical Research

MIMS offers logistics and a stimulating environment for collaboration and for interaction between faculty members and students. One of the main motivational factors for launching MIMS was to cater to the sizable community of North African mathematicians working abroad, many of whom look for better ways to connect back with Tunisia and the Maghreb. At this stage, MIMS is located at the *Cité des Sciences* and uses amenities consisting of two amphitheatres, various seminar rooms, a buffet style cafeteria and a student dormitory, which MIMS has been using for its junior guests.

MIMS holds several annual activities that attract a growing number of participants, from Tunisia but also from neighbouring Algeria and Morocco. The MIMS International Conference is held once a year on themes

"Operads and Configurations Spaces" conference

On the roofs of Tunis

of pure and applied mathematics. This year's conference was in memory of Mohamed Salah Baouendi, one of the most renowned complex geometers of the 20th century. Conference proceedings will appear with Springer in 2015. The next conference will be a joint event with a CIMPA[1] research school taking place in Tunis on 20–29 March 2015 on the theme of *Nonlinear Partial Differential Equations arising from Geometry and Physics*. It is possible to register for conferences and to subscribe to the institute newsletter through the MIMS website.

GGTM-MIMS Conference" in memory of M.S. Baouendi: visit to the Bardo Museum.

A geometry and topology conference is also held annually at the institute. The choice of this theme is motivated by the observation that geometry at large is one of the weaker links in mathematical education in the Maghreb. MIMS partners with the *Grouping for the development of Geometry and Topology in Maghreb* (GGTM) to try to remedy this situation. The GGTM made its debut in 2003 in Marrakech and helped in training and advising several graduate students in fields related to geometry and topology.

The applied side of MIMS research activities is ensured through collaboration with the *Laboratory of Numerical and Mathematical Modelling in Engineering Sciences* at the largest engineering school in the country, ENIT; the latest conference there was held on the field of data mining and machine learning.

Still in connection with its research activities, MIMS sponsors mathematical events around the Maghreb, either by participating in scientific committees or by providing financial support. This year alone, MIMS has sponsored half a dozen conferences in Tunisia and a school in Rabat, Morocco, in memory of Bill Thurston. By playing the role of a regional, albeit modest, sponsor, MIMS gets its inspiration from CIMPA, whose major role is to stimulate and fund mathematical activities in the developing world.

[1] Centre International des Mathématiques Pures et Appliquées (Nice, France).

Mathematical Education

MIMS organises summer lectures series for Master's level students. A month-long course was given by Abbès Bahri (Rutgers University) in 2013, which was then complemented by a two-month course at the Tunis Polytechnic School in February and March 2014. In Summer 2014, Nader Masmoudi (Courant Institute) gave lectures at the *Faculté des Sciences de Tunis.*

The MIMS summer lectures are now being expanded collaboratively with CIMPA and AMU[2] to a fully fledged two-week school, held once a year, in turn between the three main countries of the Maghreb. The AMU and CIMPA have supported for many years the organisation of thematic schools at the Master's level in sub-Saharan Africa. MIMS will work on expanding this concept to Northern Africa and to promoting collaboration between mathematicians on both sides of the Sahara divide.

MIMS organises general public conferences aimed primarily at early university students, teachers and mathematically oriented minds. These conferences are generally well-attended (key speakers were José Luis Rodriguez in 2013, Jean-Paul Delahaye in 2014 and some mathematicians from MIMS; Marie-Francoise Roy and Ahmad Djebbar are soon to visit MIMS in 2015). Another regular event run by MIMS is the *Journée des Doctorants* (JDD) held every year in September. Its purpose is to give a forum for students to present their thesis work and to train them on giving their first public talks. The first two JDDs were held in Tunis and the third this year took place at the *Faculté des Sciences* in Monastir, Tunisia.

MIMS educational outreach is expanding as more projects line up. The institute is launching a new online journal of a special kind. This is *The Graduate Student Mathematical Diary* (GSMD), a journal that intends to publish writings and notes of graduate or postgraduate students on topics of interest or on current research. Nowadays, bright students have webpages and blogs. They often post very interesting accounts of material they have learnt or records of talks they have given. The GSMD offers to polish these notes or papers and have them refereed and published. In so doing, it also serves as a pedagogical tool whereby students learn to publish their first papers. The GSMD takes its inspiration from a similar journal that was discontinued in 2000: the *Journal des Elèves de l'Ecole Normale Supérieure de Lyon*. The first issue of this online journal will appear in Spring 2015.

[2] African Mathematical Union.

MIMS "Mathematics for All" conference

"Journée des étudiants" in Monastir

A more recent initiative is to implement the concept *Les Mathématiques Itinérantes* in selected colleges and schools around the country. This idea was initiated at the University of Lille 1 (France) in 2007 and has had reasonable success. It arranges for general talks and runs small research labs with young students at the college level, introducing them to novel mathematical ideas and constructions. For MIMS, this can help attract some of the best students to mathematical research. A first conference is to be held early in 2015 at the *Institut National des Sciences Appliquées et de Technologie* (INSAT). The philosophy the institute wishes to convey to the younger generation is that *Mathematics is about observations not just about equations*!

Mathematical Landscape

Mathematics in Tunisia and in the Maghreb has relatively flourished over the last two decades. The number of first-class mathematicians from this region of the world is noticeably increasing as a result of real efforts on the ground to build higher standard mathematical education and research. Active professional mathematical societies with a growing membership exist in all of Tunisia, Algeria and Morocco. The *Tunisian Mathematical Society* SMT, an institutional partner of MIMS, is now 23 years old.

The French system of preparatory schools has had a great influence on strengthening the mathematical component in higher education. The largest preparatory school in Tunisia is the *Ecole Préparatoire* at the *University of Carthage* (IPEST), which is another institutional partner of MIMS.

Over the last decade, the Ministry of Higher Education and Scientific Research in Tunisia has encouraged the creation of mathematics laboratories, spread out amongst the universities around the country. Research topics cover harmonic analysis, partial differential equations, algebra and representation theory, topology and applied mathematics. Reasonable funds were allocated to this endeavour, which was part of the country's desire to increase its fraction of GDP in research. As a result, a spike in the number of defended theses, the number of hires and the number of published papers has been observed.

Presentation on tilings at the Cité des Sciences

The emergence of mathematical laboratories all across major universities in Tunisia has strengthened the need for an umbrella research institute like MIMS on the national level.

MIMS Structure and a Look at its Future

MIMS has a motivated team of mathematicians helping to carry out all of the existing and upcoming projects. It has an executive board renewable every three years, an advisory board and a scientific committee consisting of renowned mathematicians from around the world. MIMS has a handful of major institutional partners that help with human assistance or with logistics. A major supporting sister institution is the *Southern Mediterranean University* (SMU), a leading educational institution in Africa running a business school (MSB) and an engineering school (MedTech).

MIMS aspires to create a *postdoctoral* position and to put in place a *Research in Pairs* type project. The institute has already established a partnership with the *Pacific Institute for the Mathematical Sciences* (PIMS) in Canada with the goal of creating opportunities for North African mathematicians to visit that country and organise joint activities. Another established partnership is with the *American University of Sharjah* (AUS), one of the finest of the Gulf universities. It can be mentioned that MIMS will sponsor, alongside the AMS, a major meeting at AUS in 2–5 April 2015.

MIMS greatly welcomes collaboration with the international mathematical community. Partnerships with more established institutions are essential to the betterment of MIMS activities and to strengthening its base. In today's turbulent time in our region, financial support is lacking. For MIMS to continue supporting local and regional activities, running its programmes and setting up an electronic library, external financial and logistical support is both sought and needed. As more and more mathematicians know about MIMS and espouse its mission, we have high hopes that this institute will succeed in becoming, over time, a prominent player around the Mediterranean and in the MENA region in promoting mathematical education and research.

MIMS website: www.mims.tn

ICMI Column

Jean-Luc Dorier (Université de Genève, Switzerland)

Emma Castelnuovo – In memoriam

On 13 April 2014, shortly after the ICMI announced the launch of an award in her name (see the June 2014 Issue 92 of the EMS Newsletter), Emma Castelnuovo passed away at the age of 100 years and four months. She was born in Rome in December 1913 to the mathematician Guido Castelnuovo (1865–1952) and to Elbina Enriques, sister of the mathematician Federigo Enriques. After graduating in mathematics in 1936, she worked as a librarian at the Institute of Mathematics of the University of Rome. Between 1939 and 1943 and due to Italian racial laws (leggi razziali) she could only find work as a teacher in a Jewish school. In 1943, the family fled the Nazi roundups, taking refuge with friends, in hospitals and in religious institutions. After the war, she taught mathematics in Torquato Tasso Secondary School in Rome and worked intensively with fellow teachers to rethink and renovate teaching methods. She published several books, among them *Geometria Intuitiva* (which was also very popular in Spain in its Spanish version), *Didattica della matematica* and *La matematica nella realtà* (Mathematics in the real world).

In her books, Emma Castelnuovo wrote that a main objective is to awaken the intuition, the interest of the students in the subject and their taste for research through the observation of facts, techniques and fundamental properties of geometric figures. She believed that intuition, interest and taste are not innate but rather that they develop when students participate in creative work. Teachers need to stimulate the natural and instinctive curiosity of students, to lead them through the discovery of mathematical truths, to convey the idea of doing mathematics by themselves and to instil the need for a progressive logical reasoning.

Her work on didactics of mathematics was very influential in several countries.

The announcement and the call for nominations for the Emma Castelnuovo Award can be found at http://www.mathunion.org/icmi/activities/awards/emma-castelnuovo-award/.

For information on Jean-Luc Dorier see Newsletter issue 93 (September 2014), p. 49.

International Congress on Mathematical Education ICME-13, Hamburg, Germany – Sunday 24 July to Sunday 31 July 2016
http://www.icme13.org/

The *Society of Didactics of Mathematics* (Gesellschaft für Didaktik der Mathematik – GDM) has the pleasure of hosting ICME-13 in 2016 in Germany. The congress – to be held under the auspices of the *International Commission on Mathematical Instruction* (ICMI) – will take place at the University of Hamburg from Sunday 24 July to Sunday 31 July 2016. Hamburg is a bustling cosmopolitan port in the north of Germany and, with 1.8 million inhabitants, is its second largest city. It offers the perfect environment for a challenging congress.

We invite participants from all over the world to come to Hamburg and make ICME-13 a rich experience for all. ICME-3 took place in Germany in 1976 in Karlsruhe and we are proud to welcome mathematics educators from all over the world back to Germany. The congress attendees will experience the very special characteristics of the German mathematics education discussion, which is closely connected to European traditions of didactics of mathematics and has seen important recent developments.

The *Society of Didactics of Mathematics* represents the German speaking community of didactics of mathematics, bringing together mathematics education groups from Germany, Austria and Switzerland. Supported by the *German Mathematical Society*, the *German Educational Research Association* and the *German Association for the Advancement of Mathematics and Science Education*, we are eager to welcome ICME-13 participants to Germany.

Gabriele Kaiser, University of Hamburg Convenor of ICME-13

Rudolf vom Hofe, President of the Society of Didactics of Mathematics

CANP Tanzania

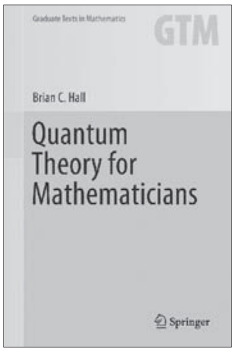
The CANP (Capacity and Networking Project) was launched some four years ago by the international bodies of mathematicians and mathematics educators – the International Mathematical Union (IMU) & the International Commission on Mathematical Instruction (ICMI) – in conjunction with UNESCO and the International Congress of Industrial and Applied Mathematics (ICIAM). The project is a response to Current Challenges in Basic Mathematics Education (UNESCO, 2011).

The CANP aims to enhance mathematics education at all levels in developing countries so that their people are capable of meeting the challenges these countries face. It seeks to enhance the educational capacity of those responsible for the preparation and development of mathematics teachers, and to create sustained and effective regional networks of teachers, mathematics educators and mathematicians, with strong links to the international community. Three CANP conferences have already been held in different parts of the world and their success is shown by the satisfaction of the participants and by the establishing of many follow-up activities.

The last CANP meeting took place in Dar-es Salam, Tanzania, 1–12 September 2014, and gathered together over 80 participants from East African countries.

For more details, see http://www.mathunion.org/icmi/activities/outreach-to-developing-countries/canp-project-2014-east-africa/.

Book Reviews



Brian C. Hall

Quantum Theory for Mathematicians

Springer New York, 2013
xvi, 554 p.
ISBN 978-1-4614-7115-8

Reviewer: Hirokazu Nishimura

The Newsletter thanks Zentralblatt MATH and Hirokazu Nishimura for the permission to republish this review, originally appeared as Zbl 1273.81001.

The mathematical culture and the physical culture are very near and very remote at the same time. This ambivalence is often irritating to both mathematicians and physicists. As is well known, Paul A.M. Dirac believed that, if there is a God, he is a great mathematician. Eugene P. Wigner has stated that the miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. Even in ancient times, Euclid held that the laws of Nature are but the mathematical thoughts of God. Nevertheless, it is not easy for a general mathematician to study physics, in particular, quantum theory. Books on quantum mechanics written by physicists are not subject to the high-level precision or exactness that mathematicians are accustomed to. Physicists often use different terminology and notation for familiar mathematical concepts, which are terribly confusing to hasty mathematicians. Although mathematicians are keenly aware of the great influence of quantum physics upon mathematics such as the Wigner-Mackey theory of induced representations, the Kirillov-Kostant orbit method and quantum groups, mathematicians who want to study quantum mechanics for the first time should choose a textbook truly adequate for them so as to avoid spending a lot of time in vain.

There are a few textbooks on quantum theory for mathematicians who are alien to the physical culture, say, [1–11] and so on, but this modest textbook will surely find its place. All in all, the book is well written and accessible to any interested mathematicians and mathematical graduates.

The organization of the book goes as follows. Chapter 1 is devoted to the historical origins of quantum theory. Chapter 2 is a hasty introduction to classical mechanics, including a short treatment of Poisson brackets and Hamilton's form of Newton's equations. The Hamiltonian approach to classical mechanics is generalized to manifolds in Chapter 21. Chapter 3 is concerned with the essentially probabilitistic nature of quantum theory, where the probabilities are captured by the position and momentum operators while the time-evolution of the wave equation is described by the Hamiltonian operator. Chapter 4 deals with several methods of solving the free Schrödinger equation in dimension one, while Chapter 5 is concerned with a particle in a square well. The author spends the succeeding five chapters on the spectral theorem. After giving perspectives on the theorem in Chapter 6, the spectral theorem for bounded self-adjoint operators is stated in Chapter 7 and established in Chapter 8. The spectral theorem for unbounded self-adjoint operators is stated and proved in terms of projection-valued measures in Chapter 10 after an introduction to unbounded self-adjoint operators in Chapter 9. The succeeding four chapters are concerned with the canonical commutation relations in some way or other. Chapter 11 deals with the harmonic oscillator, introducing the algebraic approach to quantum mechanics in place of analysis as the way to solve quantum systems. Harmonic oscillators are rampant in physics. The Heisenberg uncertainty principle and the Stone-von Neumann theorem are two important consequences of the commutation relations between the position and momentum operators, the former being dealt with in Chapter 12 and the latter being dealt with in Chapter 14. Both results are examined carefully with due regard to certain technical domain conditions. Due to Groenewold's no-go theorem [12], there is no single perfect quantization scheme, but the Weyl quantization, on which the author spends a large portion of Chapter 13, is regarded as having the best properties. The succeeding four chapters are concerned with less elementary properties of quantum theory. Chapter 15 is devoted to so-called WKB (standing for Gregor Wentzel, Hendrik Kramers and Léon Brillouin) approximation, which gives an approximation to the eigenfunctions and eigenvalues of the Hamiltonian operator in one dimension. Chapter 16 gives a brief introduction to Lie groups, Lie algebras and their representations, which is put to use in Chapter 17 for studying angular momentum and spin in terms of representation of SO(3), where it is noted that the notion of fractional spin is to be understood as a representation of the Lie algebra of SO(3), which has no corresponding representation of SO(3) itself. I note gladly in passing that the author is the author of a successful book on Lie groups and Lie algebras [13]. Chapter 18 is devoted to describing the energy levels of the hydrogen atom, including some discussion on its hidden symmetries. Chapter 19 is concerned with composite systems in terms of tensor products of Hilbert spaces, bosons and fermions, the Pauli exclusion principle, and so on. The coda of the book consists of three chapters, dealing with some advanced topics on classical and quantum mechanics. Chapter 20 develops the path integral formulation of quantum mechanics rigorously by using the Wiener measure. The chapter begins with the Trotter product formula, then turning to the heuristic formulation of Feyman himself and finally obtaining the so-called Feyman-Kac formula. Chapter 22 considers the geometric quantization program from a symplectic viewpoint, paving the way to Chapter 23, which lays out in an orderly fashion all the ingredients (bundles, connections, polarizations, etc.) needed to do geometric quantization generally.

References

[1] Takhtajan, Leon A. *Quantum mechanics for mathematicians.* Graduate Studies in Mathematics 95. Providence, RI: American Mathematical Society (2008).

[2] Ticciati, Robin. *Quantum field theory for mathematicians*. Encyclopedia of Mathematics and Its Applications. 72. Cambridge: Cambridge University Press (1999).

[3] de Faria, Edson; de Melo, Welington. *Mathematical aspects of quantum field theory*. Cambridge Studies in Advanced Mathematics 127. Cambridge: Cambridge University Press (2010).

[4] Grensing, Gerhard. *Structural aspects of quantum field theory and noncommutative geometry.* Vol. 1. Hackensack, NJ: World Scientific (2013).

[5] Grensing, Gerhard. *Structural aspects of quantum field theory and noncommutative geometry.* Vol. 2. Hackensack, NJ: World Scientific (2013).

[6] Folland, Gerald B. *Quantum field theory: A tourist guide for mathematicians*. Mathematical Surveys and Monographs 149. Providence, RI: American Mathematical Society (2008).

[7] Costello, Kevin. *Renormalization and effective field theory.* Mathematical Surveys and Monographs 170. Providence, RI: American Mathematical Society (2011).

[8] Deligne, Pierre; Etingof, Pavel; Freed, Daniel S.; Jeffrey, Lisa C.; Kazhdan, David; Morgan, John W.; Morrison, David R.; Witten, Edward (eds.). *Quantum fields and strings: a course for mathematicians.* Vol. 1, 2. Material from the Special Year on Quantum Field Theory held at the Institute for Advanced Study, Princeton, NJ, 1996–1997. Providence, RI: AMS, American Mathematical Society (1999).

[9] Zeidler, Eberhard. *Quantum field theory. II: Quantum electrodynamics. A bridge between mathematicians and physicists*. Berlin: Springer (2009).

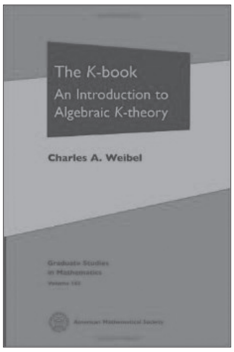
[10] Zeidler, Eberhard. *Quantum field theory III: Gauge theory. A bridge between mathematicians and physicists.* Berlin: Springer (2011).

[11] Teschl, Gerald. *Mathematical methods in quantum mechanics. With applications to Schrödinger operators.* Graduate Studies in Mathematics 99. Providence, RI: American Mathematical Society (2009).

[12] Groenewold, H.J. On the principles of elementary quantum mechanics. *Physica* 12, 405–460 (1946).

[13] Hall, Brian C. *Lie groups, Lie algebras, and representations. An elementary introduction*. Graduate Texts in Mathematics. 222 (2003).

Hirokazu Nishimura was born in Kyoto in 1953 and was raised there. He attended Kyoto University and received his Bachelor of Science with a major in biophysics in 1976. After graduation, he entered the Research Institute for Mathematical Sciences at Kyoto University as a graduate. From November 1979 through March 1986 he was enrolled as a regular member there. Since April 1986 he has been enrolled at the Institute of Mathematics at the University of Tsukuba. Axiomatic methods are now the commonplace, but differential geometry has defied its axiomatization. His current ambition is to axiomatize differential geometry by emancipating synthetic differential geometry from topos theory, thereby giving a unified treatment to various differential geometries ranging from the orthodox one to noncommutative geometry. He is the author of more than 90 papers and editor of the book A Lost Mathematician, Takeo Nakasawa. *His major speciality is surely mathematics, but his minor speciality is artistic design. He chose to present an artistic work of his in place of his picture. The title of the work is* A Great Fuss of Great Mathematicians. *He has displayed several works of his design at the National Art Centre, Tokyo, Japan.*



Charles E. Weibel

The *K*-book. An Introduction to Algebraic *K*-theory

American Mathematical Society, Providence, RI, 2013
xii, 618 p.
ISBN 978-0-8218-9132-2

Reviewer: Werner Kleinert

The Newsletter thanks Zentralblatt MATH and Werner Kleinert for the permission to republish this review, originally appeared as Zbl 1273.19001.

K-theory as an independent discipline in pure mathematics emerged about fifty years ago. From its very beginnings, *K*-theory was divided into algebraic and topological *K*-theory, according to its various conceptual contents and applications. Roughly speaking, algebraic *K*-theory deals with functorial invariants for rings, fields, schemes and varieties, algebraic sheaves and vector bundles, or more general categories with particular structure, and its role in modern algebraic geometry, algebraic number theory, algebraic topology as well as in the theory of operator algebras within functional analysis is nowadays a ubiquitous, utmost crucial one. Among the classical research monographs and textbooks in algebraic *K*-theory, most of which were published during the last thirty years of the 20th century, there are the well-known works of H. Bass [1], R. Swan [2], J. W. Milnor [3], V. Srinivas [4], J. Rosenberg [5], H. Inassaridze [6] and B.A. Magurn [7]. Each of these classics reflects its own spirit of the age and comes with its own particular focus, thereby imparting its respective individual viewpoint of the rapidly developing subject of algebraic *K*-theory.

The book under review aims to provide another comprehensive introduction to the principles of algebraic K-theory, with the special intention to combine the classical approaches with more recent topological techniques for higher algebraic *K*-theory, on the one hand, and to describe some of the topical applications of the latter in contemporary algebraic geometry and number theory, on the other hand. In this regard, the present textbook of algebraic K-theory takes the reader from the classical basics of the subject to the present state of the art, leading her/him in such a way to the forefront of current research in the field.

As the author, one of the leading experts in algebraic *K*-theory, points out in the preface to his new "*K*-book" in hand, this book project of his grew steadily since the mid-1990s, when the K-theory landscape had significantly changed, and when all the venerable classics in the field (see above) had appeared. Actually, the growing of this book could be followed up on the author's web page at Rutgers University, USA, ever since, and the complete book can still be downloaded from there under the address: http:/www.math.rutgers.edu/~weibel/.

As for the contents, the book consists of six chapters, each of which is subdivided into several sections. Chapter 1 gives an introduction to the basic objects studied in algebraic *K*-theory: projective modules over a ring and vector bundles over algebraic schemes. This includes such standard material as free modules and stably free modules, projective modules, the Picard group of a commutative ring, topological vector bundles and their Chern classes as well as the very basic facts on sheaves of modules and algebraic vector bundles over schemes in algebraic geometry. Here the presentation is rather survey-like, with many instructive concrete examples and explanations, whereas hints to proofs of crucial results are given in the exercises at the end of this introductory chapter. Several ways to construct the Grothendieck group K_0 of a mathematical object are described in Chapter 2 ranging from the group completion of a monoid to K_0 of an exact category.

Along the way, the reader meets the related groups $K(X)$, $K0(X)$, and $KU(X)$ of a topological space X, lambda-rings and their Adams operations, the K_0-group of schemes and varieties, before a section on Waldhausen categories and their K_0-theory concludes this chapter.

The classical constructions of the functors K_1 and K_2 are the topics discussed in Chapter 3, where most of the material is presented in the form of a brief overview.

Apart from the Whitehead group K_1 of a ring and its relative version, the fundamental theorems relating K_1 and K_2, H. Bass's concept of K_{-1} and K_{-2} and an overview of J. Milnor's theory of the group $K_2(\mathbb{R})$ of a ring $\mathbb{R}$, also Steinberg groups, Steinberg symbols, and Matsumoto's theorem on $K_2(F)$ of a field F, also an analogue of Hilbert's Theorem 90 for K_2-groups of cyclic field extensions are touched upon. This chapter ends with a furthergoing discussion of Milnor's general groups $K_n^M(F)$ associated to a field F.

Chapter 4 is devoted to the four standard constructions for higher groups: Quillen's BGL^+-construction for rings via homotopy groups of certain topological spaces, the group completion constructions for symmetric monoidal categories via the homotopy theory of topological H-spaces, Quillen's Q-construction for exact categories, and the so-called "Waldhausen wS-construction" for Waldhausen categories. Again, a wealth of topics, in this context, is taken up, but only very few proofs of theorems are presented, as the focus is on a panoramic, explanatory presentation of the advanced, highly abstract and sophisticated material. The subsequent Chapter 5 presents the fundamental structure theorems for the constructions in higher *K*-theory as outlined in Chapter 4 all of which give the same *K*-theory in the special case of a base ring. The author restricts the attention to exact categories and Waldhausen categories, using the extra structure of those to derive the extensions to higher *K*-theory of the various structure theorems for K_0 discussed in Chapter 2. Among the numerous applications of higher *K*-theory in algebra and algebraic geometry are several cases of the so-called Gersten conjecture on discrete valuation domains and the interpretation of the *K*-cohomology of regular quasi-projective schemes in terms of Gersten's coniveau spectral sequence for the higher K-theory of rings. Finally, Chern classes for rings and schemes are depicted in the context of higher K-theory.

The concluding Chapter 6 turns to the problem of computing the higher K-groups of fields. More precisely, the goal of this chapter is to explain what the present state of knowledge of the algebraic K-theory of (number) fields is, thereby largely illuminating the historical developments from the early 1970s until now. This includes topics such as the K-theory of algebraically closed fields, the K-theory of $\mathbb{R}$, relations to motivic cohomology, K_3 of a field, and various K-theoretic results for special number fields, local fields, and the ring $\mathbb{Z}$ of integers.

Each section of the book ends with a large set of related exercises, which are mainly of purely theoretical nature. These exercises mostly refer to additional concepts and theorems from the respective research literature, therefore requiring intensive further reading. However, ample hints to the original papers are given throughout the text, thereby referring to the sweeping bibliography with more than 230 references at the end of the book.

Indeed, Charles Weibel's "K-book" offers a plethora of material from both classical and more recent algebraic K-theory. It is a perfect source book for seasoned graduate students and working researchers, who are willing and eager to follow the author's expository path, and who are ready for a lot of additional reading and self-reliant work. The many instructive examples and clarifying remarks help the reader grasp the essentials of algebraic K-theory from a panoramic view, and the entire exposition represents a highly valuable and useful guide to the subject in all its diversity and topicality. Although barely being a textbook for neophytes in the field, despite the wealth of background material sketched wherever necessary, the book under review is certainly the most topical presentation of algebraic K-theory at this time, and an excellent enhancement of the existing literature in any case.

References

[1] Bass, Hyman. *Algebraic K-theory.* Mathematics Lecture Note Series. New York-Amsterdam: W.A. Benjamin (1968).

[2] Swan, R.G. *Algebraic K-theory*. Lecture Notes in Mathematics 76. Berlin-Heidelberg-New York: Springer-Verlag (1968).

[3] Milnor, John W. *Introduction to algebraic K-theory*. Annals of Mathematics Studies. No.72. Princeton, N. J.: Princeton University Press and University of Tokyo Press (1971).

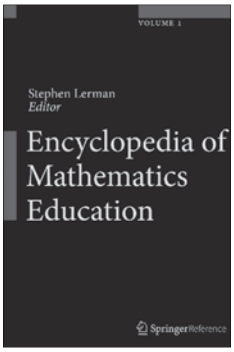
[4] Srinivas, V. *Algebraic K-theory.* 2nd ed. Progress in Mathematics (Boston, Mass.) 90. Boston, MA: Birkhäuser (1996).

[5] Rosenberg, Jonathan. *Algebraic K-theory and its applications.* Graduate Texts in Mathematics. 147. New York, NY: Springer-Verlag (1994).

[6] Inassaridze, Hvedri. *Algebraic K-theory.* Mathematics and its Applications (Dordrecht). 311. Dordrecht: Kluwer Academic Publishers (1995).

[7] Magurn, Bruce A. *An algebraic introduction to K-theory*. Encyclopedia of Mathematics and Its Applications. 87. Cambridge: Cambridge University Press (2002).

Werner Kleinert received his doctoral degree in commutative algebra in 1971. After his postdoctoral qualification (habilitation) in the field of algebraic geometry in 1979, he was promoted to university lecturer at Humboldt University in Berlin, an academic position that he held there until his retirement in 2010. His main research interests have always been the geometry of moduli spaces of algebraic curves and abelian varieties, together with related topics such as Riemann surfaces, theta functions and Teichmueller theory.



Encyclopedia of Mathematics Education

Stephen Lerman (Editor)

Springer New York, 2014
xxii, 672 p.
ISBN 978-94-007-4977-1

Reviewer: Ghislaine Gueudet

An invitation to read

The *Encyclopedia of Mathematics Education*, written under the direction of Stephen Lerman, was published in 2014 simultaneously as a traditional book and a dynamic online resource on the Springer Reference website.

This article does not claim to be a complete review of this Encyclopedia – I have not yet read all the 163 articles! Its aim is more to present the perspective retained by the Editorial Board and give a flavour of the content of the Encyclopedia, as an invitation to the potential reader.

A comprehensive resource for a large audience

The Encyclopedia project was to write a "comprehensive reference text, covering every topic in the field of mathematics education research" (Lerman, Preface, vii). Moreover, this text was to be informative, taking into account the very latest results of research, but also accessible to anyone who has an interest in mathematics education: not only researchers in mathematics education but also mathematicians, teachers, students and policymakers. For this purpose, the Editorial Board has gathered together a team of 174 authors from more than 30 countries, specialists of different aspects of mathematics education. The entries can concern specific mathematic topics (and the related learning and teaching issues) like "Algebra teaching and learning" (by Carolyn Kieran) and "Calculus Teaching and Learning" (by Ivy Kidron) or more transverse issues like "Inquiry-based mathematics education" (by Jean-Luc Dorier and Katia Mass), "Problem solving in mathematics education" (by Manuel Santos-Trigo) and "Instrumentation in mathematics education" (by Luc Trouche). They can also present theories or concepts used in mathematics education like "Activity theory in mathematics education" (by Wolff-Michael Roth) and "Didactic contract in mathematics education" (by Guy Brousseau). The content of the Encyclopedia actually represents the major results obtained in mathematics education over more than 40 years, with a variety of perspectives (epistemological, cognitive, socio-cultural, etc.) developed by its international group of authors.

Visiting the letter "*M*"

An Encyclopedia can be a useful tool for answering a precise question. It can also be viewed as a place to wander… Let's try a brief random walk within the letter "*M*" of the Encyclopedia – one of the richest letters, with 25 entries! (This is perhaps not surprising, since Mathematics is the central focus here.) The first entry is entitled "Manipulatives in Mathematics Education" (by Maria G. Bartoloni Bussi and Francesca Martignone). This article mentions various kinds of manipulatives, ranging from historical ones like Napier bones to very recent digital tools, like the Bee-bot floor robot; it discusses the differences between concrete and virtual manipulatives, from an educational perspective; it identifies critical issues, linked with the use of manipulatives, like students' autonomy or their age (why are high school teachers often reluctant to use manipulatives in class?); it also presents a theoretical approach, the semiotic mediation, which is especially relevant to studying the learning-teaching processes when manipulatives are involved.

After the "Manipulatives" entry, there starts a long list of "Mathematical…" entries: "Mathematical ability", "Mathematical approaches", "Mathematical functions teaching and learning"… Let's read this one (by Mogens Niss), which is the first article we meet in the list with a focus on a specific mathematical theme. Within the huge body of research on this topic, the author retains a focus on students' difficulties. Functions can have diverse representations: algebraic, graphical, tabular etc.; this causes several specific difficulties that have been clearly identified and has led to the design of teaching interventions using special software supporting the articulation of several representations. Another dimension of complexity is that functions have different aspects: a simple correspondence linking "every element in a given domain to one and only one element in another domain" is for the learner very different from a tool intervening in the modelling of extra-mathematical situations, for example. "Mathematical Modeling and Applications in Education" and "Mathematical representations" are other entries under the letter *M* that can usefully complement the article about functions. "Mathematical Proof, Argumentation and Reasoning" also faces the challenge of synthesising multiple research works on the subject. In this article, Gila Hanna recalls that "a proof is much more than a sequence of logical steps that justifies an assertion" and that it can play various roles in mathematics practice, like establishing connections and suggesting new hypotheses. It can also take different forms, remaining informal but providing a high level of reliability. Teachers have to introduce students to these different kinds of proofs and, at the same time, teach them the rules of reasoning as well as presenting patterns of argument. This is a delicate task, indicating the need for adequate teacher education (pre-service and in-service) – this connects us directly with the "Mathematics Teacher Education Organization" entry, a few steps further in the Encyclopedia… This article (by Jarmila Novotná, Hana Moraová and Maria Tatto) offers an international view of the multiple existing organisations for teacher education but also discusses the skills, abilities, knowledge and attitudes that students graduating from teacher preparation programmes should master. The reader interested in teacher education can go on and read the "Models of In-service Mathematics Teacher Education" and "Models of Preservice Mathematics Teacher Education" entries and can naturally switch to the letter T, where they will find, for example, "Teacher Education Development Study – Mathematics (TEDS-M)". The challenge here might be to stop reading the Encyclopedia!

Final word (or not)

Let us go back to the foreword of the Encyclopedia, written by Jeremy Kilpatrick:

"This encyclopedia represents a major step forward in the field of mathematics education, bringing to everyone with a professional interest in mathematics education access to the latest and best thinking in the field." (Kilpatrick, Foreword, vi).

Naturally, I fully support this enthusiastic statement. Moreover, this major step is not a final step, since the online version should permit regular updates and discussion between authors and readers. For all your questions about research in mathematics education, you will find elements of answers in the Encyclopedia of Mathematics Education and you can contribute with your comments to a continuous improvement of its content!

Ghislaine Gueudet is a professor of mathematics education at the ESPE Bretagne (School for Teacher Education). She has represented the French Association for Research in Mathematics Education (ARDM) on the French ICMI sub-commission (CFEM) since 2008 and also represents the CFEM on the EMS Educational Committee. Her research concerns university mathematics education and the design and use of educational resources (digital resources in particular).

Emil Artin and Helmut Hasse. The Correspondence 1923–1958

Günther Frei, Franz Lemmermeyer and Peter J. Roquette (Eds.)

Springer Basel, 2014
xx, 484 p.
ISBN 978-3-0348-0715-9

Reviewer: Javier Fresán

If *Parallel Lives* featuring mathematicians were to be written, Emil Artin and Helmut Hasse would certainly deserve one of the chapters. Both born in 1898, they completed their education at around the same time and were soon recognised as rising stars of German number theory. Already, their dissertations contained groundbreaking results: Hasse proved the local-global principle for quadratic forms over the rationals and Artin investigated hyperelliptic curves over finite fields, in particular the analogue of the Riemann hypothesis, which he was the first to consider. They did not stop there: Hasse generalised his results to any number field in his *Habilitation* and Artin developed the theory of L-functions that is nowadays named after him. Within a few years, not only were they the youngest professors in Germany but they also belonged to what Weyl once called the "honours class" of mathematicians who had solved one of Hilbert's problems.

Artin and Hasse probably met for the first time at the annual conference of the German Mathematical Society in 1922. This would be the beginning of a lifelong friendship that was to overcome the dark days of the Third Reich, when Artin was dismissed from university and forced to go into exile, whereas Hasse had – to say the least – ambiguous feelings about Nazism. Although they only co-authored two papers, they maintained an extensive scientific correspondence during the emergence of modern class field theory, sometimes with high exchange frequency. This could come as a surprise, since Hasse was an "ardent letter writer" (p. 10) but Artin was not fond of letter writing: he only wrote in reply and often had to begin his letters "with a long litany of apologies, accusations […] and promises to better myself" (p. 81). He preferred teaching and conversation, at which he excelled. His lectures have been described as "polished diamonds": even when he approached classical topics such as Galois theory or the gamma function, his clarity of presentation was so remarkable that textbooks are still based upon his ideas today.[1]

The correspondence

The volume under review assembles 73 letters and postcards, mainly from Artin to Hasse, written between 1923 and 1958. Most of them had already been published in German, first in a small edition by Günther Frei in 1981, then in collaboration with Peter Roquette and Franz Lemmermeyer in 2008. This second edition was supplemented by voluminous comments, so detailed that it will remain an "introduction to class field theory on a historical basis" written by the leading experts on the subject. In particular, their detective work almost gives the reader the impression that they would be able to reconstruct the contents of Hasse's letters, which seem to be lost. One can only welcome the completion of such an ambitious project, now available to a wider audience thanks to an English translation, which also includes some new letters from 1937 onward, although these are significantly less relevant than the previous ones.

The correspondence opens with an intense exchange of letters, dated July 1923, which led to Artin and Hasse's first joint paper on the so-called second supplementary law for odd prime exponents. Thirty years later, Hasse would melancholically refer to that period as "the old days when we bombed each other rapidly with postcards about reciprocity formulas" (p. 429). Already in these first letters, the influence of Takagi's memoirs on class field theory is manifest. After a decade of "utter scientific solitude", the Japanese mathematician had presented his theorem that all abelian extensions of number fields are class fields during the ICM held in Strasbourg in 1920. Unfortunately, Germans were not allowed to attend and Takagi's contribution went unnoticed. Back in Japan, he had the idea of sending reprints of his work to Siegel, from whom Artin borrowed them.[2] He was deeply impressed by the potential of the techniques and urged Hasse to study them. This would result in Hasse's report on class field theory (*Klassenkörperbericht*), a streamlined presentation of Takagi's work with proofs "reduced to their skeletons", through which a whole new generation of mathematicians encountered class field theory.

Artin *L*-functions

From 1926 to 1930, Artin's L-functions occupy a central place in the correspondence, but the reader will not need to wait that long to witness their birth: as early as in the first letter, Artin announces that he has found "the general L-series attached to Frobenius group characters which accomplish for general fields exactly what the usual L-series accomplish for abelian fields" (p. 50). At the time of writing, the paper *Über eine neue Art von L-Reihen*[3] was already in press. To prove that this new kind of *L*-function was well-behaved, Artin reduced to the abelian case and compared them to Dirichlet *L*-series. For this he needed to show the "general reciprocity law" that there is a canonical isomorphism between the Galois group and the ray class group under which every unramified prime ideal corresponds to its Frobenius substitution.

[1] To learn more about Artin's teaching, we refer the reader to Zassenhaus's obituary, reprinted on pages 18–24.

[2] One can still find the original reprints at the Sammlung Siegel in the MPIM Library in Bonn. I thank Anke Völzmann for this information.

[3] The title echoes Hecke's *Eine neue Art von Zetafunktion*, where *L*-functions of grössencharacters were defined.

In 1923, Artin was only able to prove the statement for composita of cyclotomic extensions or cyclic extensions of prime degree. However, he formulated it as a theorem instead of a conjecture. From this he could derive what two years later would become Chebotarev's density theorem, a former conjecture of Frobenius (also stated as a theorem). Amusingly enough, Artin could only prove his reciprocity law using Chebotarev's results. All these developments are well documented in the correspondence: on 10 February 1926, Artin asks Hasse if he knows whether Chebotarev's theorem is right,[4] since "if it is correct, we surely will have pocketed the general abelian reciprocity laws" (p. 82). In the following months, Artin and Schreier thoroughly studied the article in a seminar, which led to some simplifications. Once convinced of the validity of the result, it took some time for Artin to work out the details of the proof, but on 17 July 1927 he could inform Hasse that "this semester I gave a two hours course on class field theory and finally proved the "general reciprocity law" in the version that I have given it in my article on L-series" (p. 107).

There followed a second "bombardment of letters" in which the implications of Artin's reciprocity law were discussed. These included Hilbert's conjecture that each ideal of a number field becomes principal in the Hilbert class field (proved by Furtwängler in 1928), the question of whether a tower of successive Hilbert class fields always terminates (finally answered in the negative by Shafarevich and Golod in 1964) and possible generalisations of class field theory to arbitrary Galois extensions. Hasse was particularly interested in how to derive explicit reciprocity formulas for power residues,[5] which was the subject of Part II of his *Klassenkörperbericht*. When Artin achieved his proof, he had almost completed it but decided to rewrite it taking into account the new reciprocity law. "I am sorry that you now have to rewrite the whole report," Artin says, adding right after that: "I believe, however, that it will be worth the trouble" (p. 137). In the years to follow, Hasse's report will come up regularly in the correspondence, until the moment when Artin acknowledges the reception of the galley proofs in August 1930.

The letter dated 18 September 1930 is another remarkable historical document. The presentation of the theory of L-series in Hasse's report "tempted [Artin] to think about […] things that [he had] put aside for such a long time" (p. 268). The 1923 paper still suffered from some defects: first, Artin could only define the local factors of the L-functions at unramified primes, the main difficulty being that Frobenius automorphisms are not uniquely determined in the ramified case. This led to an unnatural detour: to get the whole L-series he needed to reduce to the abelian case, where the complete definition was known, then use the functional equation and a "well known argument due to Hecke" that was not presented in full detail before Hasse's report. Now, prompted by his friend's remarks, Artin could give a uniform definition at all places – essentially the same as that used today – so that "all relations and theorems[6] […] hold exactly right from the start (*von vornherein*)" (p. 273). This also allowed him to define the shape of the functional equation. To do so, he introduces the gamma factor at each infinite prime and gives the definition and the main properties of the conductor, including the "deep" fact that it is an integral ideal. In a subsequent letter he will confess: "I believe that everything depends on guessing. In the case of the conductor as well as that of the functional equation I had to guess everything" (p. 291). So here we have a letter where the local contributions to Artin L-functions, the gamma factors at infinity, the Artin conductor and the Artin root numbers are introduced! If that was not enough, he concludes with the remark that "the new results very much support the conjecture that the $L(\chi, s)$ are entire" (p. 277), a still unproven statement that is considered to be "one of the great challenges of number theory".

An invitation

It should be clear from the above how precious these documents are for the history of number theory in the last century. The Artin-Hasse correspondence contains many other topics that have not been addressed in this short overview. They vary from highly technical pages, where local class field theory emerges, to concrete questions concerning the distribution of the argument of cubic Gauss sums or the existence of unramified icosahedral extensions. Hasse's theory of complex multiplication and his proof of the Riemann hypothesis for elliptic curves are briefly mentioned as well. It only remains for me to invite the reader to accompany the masters on this fascinating journey through class field theory.

Javier Fresán wrote his thesis in arithmetic geometry at the University Paris 13 under the supervision of Christophe Soulé and Jörg Wildeshaus. After a year at the MPIM in Bonn, he is currently an SNF post-doctoral fellow at the ETH in Zürich. His research interests include periods, special values of L-functions and the arithmetic of flat vector bundles. He is also very engaged in science popularisation.

[4] This seems to contradict Emmy Noether's information that Artin was the referee of Chebotarev's article, since it was already published by the Mathematische Annalen when Artin asked Hasse about it.

[5] Artin himself had considered his result as "somewhat strange" since, before him, a reciprocity law was a statement about power residues in number fields containing roots of unity.

[6] As the editors explain, Artin refers here to the functoriality properties of L-series with respect to inflation and induction of representations.

Letters to the Editor

An External View on Mathematical Science

Ludwig Holzleitner (Stutensee, Germany)

Dear Editor,

As a mathematician currently working in the field of application of mathematical science, I have been following this issue of application-oriented mathematics (as was pointed out by Professor Massimo Ferri in the Newsletter from June 2014) with great interest. I do not want to comment on the case of the Marie Curie fellowship discussed there; I feel I don't know enough about this specific case. Rather, I would like to provide my colleagues at universities with an "outside" view from the partial academic and more application-oriented world as one finds it in a research centre, hoping to clarify some misunderstandings. I must clearly point out that the opinion expressed in this letter is solely mine.

There is much collaboration between my research centre and colleagues from both universities and industry; this has generally been seen as very fruitful for both sides. Colleagues from all over Europe and of different sciences (chemistry, physics, engineering, etc.) benefit from grants to work at our centre. However, I have not found any mathematician among them so far. Also, other sciences regard mathematics rather as an application of prefabricated "recipes", which in fact it should not be. When recently encountering some statistical/combinatorial problems in particle radiation detection and material balance evaluation I had no time for proper mathematical modelling, theorem proving or digging deeper into it. Therefore I mentioned this to university colleagues, trying to encourage them to collaborate or to ask younger colleagues to apply for grants to work here. I was very surprised to find them not interested at all, neither for themselves nor to pass it onto younger colleagues. After reflecting a while about that experience I realised some points which I would like to stress subsequently:

1. Looking, for example, at the board of the EMS or similar mathematical societies, it is striking that all acting members are employed at some kind of university. So the sole employer for people working in the field of mathematics seems to be universities, aside from some think-tanks, for example in France (almost negligible in terms of number of persons).
2. I do not know a single instance of a mathematician who returned to university after having spent some years in industry. However, although not the rule, it is common practice in many other sciences like chemistry, physics, engineering … Also, the fact that I, as a mathematician, am now working in the field of physics demonstrates how open other sciences are compared to mathematics.
3. Looking at awards granted to young researchers (e.g. published in the EMS Newsletter) it is striking that most of them are awarded on grounds of research in theoretical topics like the Riemann-conjecture, Poincaré-conjecture, etc., which may not be without applications, albeit very distant ones (having application in other theories, e.g. quantum physics, is not considered as a real "application" in the non-mathematical world). These awards pave the way for one of the rare opportunities nowadays of getting a position in mathematics.

The above message seems clear to me as to others: if a young mathematician wants to stay in his field, they better concentrate on getting well known in a highly theoretical topic, hoping to publish a highly-appreciated paper or receiving some award, and they better stay away from any application as far as possible. This attitude directs mathematicians away from application-oriented science, making collaboration between mathematical institutes and more practically oriented institutions difficult.

I'm afraid that in this way, mathematical science currently isolates itself by ignoring the outside scientific world and believing in its self-sufficiency. Collaboration between mathematical institutes is not regarded as "interdisciplinary" and application of mathematical results in other theories is – as has already been mentioned – not considered as a real "application" by the outside scientific world and I fully agree with that. There is clearly a need for pure mathematics. However, for a successful transition to the 21st century, mathematical science must open itself to collaboration with other sciences and applied institutions, both concerning collaboration in research projects and in opening its institutes and positions.

Ludwig Holzleitner
Stutensee, Germany

Reply to: 'An external view on mathematical science' by Ludwig Holzleitner

Volker Mehrmann (Technische Universität Berlin, Germany)

I believe that it is very good to have an external look at the situation of a society and to point out deficiencies and make constructive suggestions to improve things. I thank Ludwig Holzleitner for taking this initiative and would like to comment and clarify some points where I see that he is right and where he is not aware of the developments that have taken place in recent years.

It is true that many colleagues in other sciences consider mathematicians as providers of a toolbox of techniques for them and that mathematicians do not like to be reduced to this role of a mathematical science. Providing new methods and techniques for other sciences, however, is an essential part of mathematical research. But fundamental research in mathematics is important, even if it is not application driven. Furthermore, without new developments in theoretical mathematics the resources for producing and analysing new methods and algorithms would quickly dry up. In recent years, therefore, application driven fundamental research in mathematics has grown substantially, documented by the existence of a large number of research laboratories all over Europe. These laboratories are represented in the EMS through the Applied Mathematics Committee and also through the ERCOM institutes. There is a lot of activity sponsored by the EMS in the direction that is mentioned in L. Holzleitner's letter. This includes the recently founded European service network EU_MATHS-IN, which has the EMS and the ECMI as promoting organisations (see http://www.eu-maths-in.eu/). If L. Holzleitner would have contacted colleagues represented in the networks of EU-MATHS-IN then I am sure the reaction to a request for help would have been different and help would have been provided. Concerning the other points mentioned in the letter:

- 'So the sole employer for people working in the field of mathematics seems to be universities, aside from some think-tanks, for example in France (almost negligible in terms of number of persons).'
  Answer: This is definitely not the case; there are a large number of mathematicians working in industry and also in publically funded research centres. For example, in Austria there is the RICAM Institute and there are many Fraunhofer and other non-university mathematical institutes in Germany, France, the Netherlands, etc. It is true that not all mathematicians are open to other sciences, engineering, industry or application work but the observation in the letter must arise from a single negative experience; to contradict the observation one can easily find lots of counterexamples in the success stories book of the forward look mathematics and industry (http://www.springer.com/mathematics/book/978-3-642-23847-5)
  or several reports that can be found at http://www.eu-maths-in.eu/index.php?page=generalReports.

- 'Looking at awards granted to young researchers (e.g. published in the EMS newsletter) it is striking that most of them are awarded on grounds of research in theoretical topics like the Riemann-conjecture, Poincaré-conjecture, etc.'
  Answer: I would have to agree that it is harder to receive awards like the Fields Medal for work in applied mathematics but there are now quite a lot of applied mathematics awards, such as the ICIAM, SIAM, GAMM, etc., awards that are highly recognised and are awarded to young mathematicians in industrial and applied mathematics.

- 'The above message seems clear to me as to others: if a young mathematician wants to stay in his field, they better concentrate on getting well known in a highly theoretical topic, hoping to publish a highly-appreciated paper or receiving some award, and they better stay away from any application as far as possible.'
  Answer: I strongly disagree. Consider as an example that in the last 10 years almost 100 young researchers in application driven research from the Research Center MATHEON have received offers for professor positions.

- 'I'm afraid that in this way, mathematical science currently isolates itself by ignoring the outside scientific world and believing in its self-sufficiency.'
  Answer: I agree that there may be a tendency in this direction with some mathematicians but the overall picture is exactly the opposite. More and more mathematicians turn to collaboration with other sciences and engineering.

- 'However, for a successful transition to the 21st century, mathematical science must open itself to collaboration with other sciences and applied institutions, both concerning collaboration in research projects and in opening its institutes and positions.'
  Answer: I fully agree that this is needed and I hope it has become clear that this is already happening very strongly now.

Volker Mehrmann
Chair of the Research Center MATHEON in Berlin and Member of the EMS Executive Board

Personal Column

Please send information on mathematical awards and deaths to Mădălina Păcurar [madalina.pacurar@econ.ubbcluj.ro]

Awards

Michèle Artigue (University Paris Diderot, France) receives the **Luis Santaló Medal** 2014.

Artur Avila (Instituto Nacional de Matemática Pura e Aplicada and University Paris 7), **Manjul Bhargava** (Princeton University, USA), **Martin Hairer** (University of Warwick, UK, and New York University, USA) and **Maryam Mirzakhani** (Stanford University, USA) have been awarded the **Fields Medal** 2014.

Joseph Ayoub (University of Zürich, Switzerland) and **Moritz Kerz** (University of Regensburg, Germany) have been awarded the **K-Theory Foundation Prize** 2014.

Carlos Beltrán (Universidad de Cantabria, Spain) has been awarded the **Stephen Smale Prize** 2014 of the Society for the Foundations of Computational Mathematics.

Ángel Castro (Universidad Autónoma de Madrid/ICMAT, Spain) has been awarded the **Jose Luis Rubio de Francia Prize** 2013 of the Real Sociedad Matemática Española.

David Eelbode (University of Antwerp, Belgium) is awarded the **Clifford Prize** 2014.

Alberto Enciso (Consejo Superior de Investigaciones Científicas/ICMAT, Spain) has been awarded the **Príncipe de Girona Prize for Scientific Research** 2014.

Véronique Fischer and **Michael Ruzhansky** (Imperial College, London, UK) have been awarded the 2014 **Ferran Sunyer i Balaguer Prize**.

David Li-Bland (University of Toronto, Canada) and **Ioan Mărcuț** (University of Illinois, USA) have been awarded the **André Lichnerowicz Prize** 2014.

Maryam Mirzakhani (Stanford University, USA) and **Peter Scholze** (Bonn University, Germany) are the recipients of the 2014 **Clay Research Awards**.

Dan Petersen (University of Copenhagen, Denmark) is the winner of the **International Stefan Banach Prize** for a Doctoral Dissertation in the Mathematical Sciences for 2014.

Marcus du Sautoy (University of Oxford) is awarded the 2014 **Christopher Zeeman Medal**.

Dennis Parnell Sullivan (State University New York, USA) is awarded the **Balzan Prize** 2014.

Benedikt Wirth (Westfälische Wilhelms-Universität Münster, Germany) has been awarded the **Alfried Krupp von Bohlen und Halbach Foundation Prize** 2014.

The **London Mathematical Society** has awarded prizes for the year 2014. **Miles Reid** (University of Warwick, UK) has been awarded the Polya Prize. **Martin Hairer** (University of Warwick, UK) has been awarded the Fröhlich Prize. **Caroline Series** (University of Warwick, UK) has been awarded the Senior Anne Bennet Prize. **Daniel Freed** (Texas University, USA), **Michael Hopkins** (Harvard University, USA) and **Constantin Teleman** (University of California, Berkeley, USA) have been awarded the Senior Berwick Prize. The Whitehead Prizes were received by **Clement Mouhot** (University of Cambridge, UK), **Ruth Baker** (University of Oxford, UK), **Tom Coats** (Imperial College, London, UK), **Daniela Kühn** (University of Birmingham, UK) and **Deryk Osthus** (University of Birmingham, UK).

Deaths

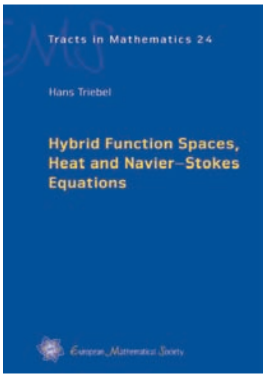
We regret to announce the deaths of:

María Jesús Bayarri (19 August 2014, Spain)
Carlos Benítez (7 March 2014, Spain)
Hans-Bjørn Foxby (8 April 2014, Denmark)
Samuel Gitler (9 September 2014, Mexico)
James Alexander Green (7 April 2014, UK)
Alexandre Grothendieck (13 November 2014, France)
Ferran Hurtado (2 October 2014, Spain)
Pierre Liardet (29 August 2014, France)
Alexander Murray MacBeath (14 May 2014, UK)
Manuel Valdivia (29 April 2014, Spain)
María Josefa Wonenburger (14 June 2014, Spain)

New books published by the

European Mathematical Society

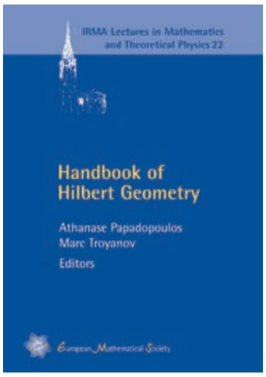
Individual members of the EMS, member societies or societies with a reciprocity agreement (such as the American, Australian and Canadian Mathematical Societies) are entitled to a discount of 20% on any book purchases, if ordered directly at the EMS Publishing House.



Hans Triebel (University of Jena, Germany)
Hybrid Function Spaces, Heat and Navier–Stokes Equations (Tracts in Mathematics Vol. 24)

ISBN 978-3-03719-150-7. 2015. 196 pages. Hardcover. 17 x 24 cm. 48.00 Euro

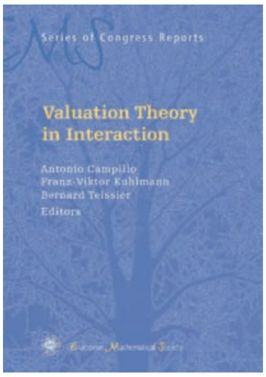
This book is the continuation of *Local Function Spaces, Heat and Navier–Stokes Equations* (Tracts in Mathematics 20, 2013) by the author. A new approach is presented to exhibit relations between Sobolev spaces, Besov spaces, and Hölder–Zygmund spaces on the one hand and Morrey–Campanato spaces on the other. Morrey–Campanato spaces extend the notion of functions of bounded mean oscillation. These spaces play a crucial role in the theory of linear and nonlinear PDEs.
Chapter 1 (Introduction) describes the main motivations and intentions of this book. Chapter 2 is a selfcontained introduction into Morrey spaces. Chapter 3 deals with hybrid smoothness spaces (which are between local and global spaces) in Euclidean n-space based on the Morrey–Campanato refinement of the Lebesgue spaces. The presented approach relies on wavelet decompositions. This is applied in Chapter 4 to linear and nonlinear heat equations in global and hybrid spaces. The obtained assertions about function spaces and nonlinear heat equations are used in the Chapters 5 and 6 to study Navier–Stokes equations in hybrid and global spaces.
This book is addressed to graduate students and mathematicians having a working knowledge of basic elements of (global) function spaces, and who are interested in applications to nonlinear PDEs with heat and Navier–Stokes equations as prototypes.



Handbook of Hilbert Geometry (IRMA Lectures in Mathematics and Theoretical Physics Vol. 22)
Athanase Papadopoulos (Université de Strasbourg, France) and Marc Troyanov (École Polytechnique Fédérale de Lausanne, Switzerland), Editors

ISBN 978-3-03719-147-7. 2014. 460 pages. Hardcover. 17 x 24 cm. 78.00 Euro

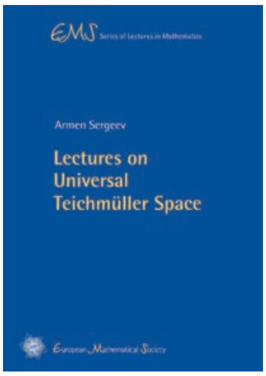
This volume presents surveys, written by experts in the field, on various classical and the modern aspects of Hilbert geometry. They are assuming several points of view: Finsler geometry, calculus of variations, projective geometry, dynamical systems, and others. Some fruitful relations between Hilbert geometry and other subjects in mathematics are emphasized, including Teichmüller spaces, convexity theory, Perron–Frobenius theory, representation theory, partial differential equations, coarse geometry, ergodic theory, algebraic groups, Coxeter groups, geometric group theory, Lie groups and discrete group actions.
The Handbook is addressed to both students who want to learn the theory and researchers working in the area.



Valuation Theory in Interaction (EMS Series of Congress Reports)
Antonio Campillo (Universidad de Valladolid, Spain), Franz-Viktor Kuhlmann (University of Saskatchewan, Saskatoon, Canada) and Bernard Teissier (Université de Paris Rive Gauche, France), Editors

ISBN 978-3-03719-149-1. 2014. 670 pages. Hardcover. 17 x 24 cm. 98.00 Euro

Having its classical roots, since more than a century, in algebraic number theory, algebraic geometry and the theory of ordered fields and groups, valuation theory has seen an amazing expansion into many other areas in recent decades. Moreover, having been dormant for a while in algebraic geometry, it has now been reintroduced as a tool to attack the open problem of resolution of singularities in positive characteristic and to analyse the structure of singularities. Driven by this topic, and by its many new applications in other areas, also the research in valuation theory itself has been intensified, with a particular emphasis on the deep open problems in positive characteristic.
The book presents high quality research and survey papers and is of interest to researchers and graduate students who work in valuation theory, as well as a general mathematical audience interested in the expansion and usefulness of the approach.



Armen Sergeev (Steklov Mathematical Institute, Moscow, Russia)
Lectures on Universal Teichmüller Space (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-141-5. 2014. 112 pages. Softcover. 17 x 24 cm. 24.00 Euro

This book is based on a lecture course given by the author at the Educational Center of Steklov Mathematical Institute in 2011. It is designed for a one semester course for undergraduate students, familiar with basic differential geometry, complex and functional analysis.
The universal Teichmüller space $\mathcal{T}$ is the quotient of the space of quasisymmetric homeomorphisms of the unit circle modulo Möbius transformations. The first part of the book is devoted to the study of geometric and analytic properties of $\mathcal{T}$. It is an infinite-dimensional Kähler manifold which contains all classical Teichmüller spaces of compact Riemann surfaces as complex submanifolds which explains the name "universal Teichmüller space". Apart from classical Teichmüller spaces, $\mathcal{T}$ contains the space $\mathcal{S}$ of diffeomorphisms of the circle modulo Möbius transformations. The latter space plays an important role in the quantization of the theory of smooth strings. The quantization of $\mathcal{T}$ is presented in the second part of the book. In contrast with the case of diffeomorphism space $\mathcal{S}$, which can be quantized in frames of the conventional Dirac scheme, the quantization of $\mathcal{T}$ requires an absolutely different approach based on the noncommutative geometry methods.
The book concludes with a list of 24 problems and exercises which can be used during the examinations.

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