# NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

7ECM BERLIN 2016

## 7th European Congress of Mathematics

July 18-22, 2016
TU Berlin

European Mathematical Society
25 years

June 2015
Issue 96
ISSN 1027-488X

**Features**
Knotted Vortex Lines and Vortex Tubes in Stationary Fluid Flows
On Delusive Nodal Sets of Free Oscillations
Gösta Mittag-Leffler

**Anniversary**
The First Years of the EMS

**Interview**
Jacob Murre – Remembering Grothendieck


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*http://euro-math-soc.eu/newsletter*

# European Mathematical Society

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# EMS Agenda

2015

**1 – 3 September**
Meeting of the Women in Mathematics Committee
During the 17th EWM General Meeting, Cortona, Italy
Contact: Caroline Series, c.m.series@warwick.ac.uk

**5 November**
Annual Meeting of the Applied Mathematics Committee of the EMS, Frankfurt Airport, Germany
http://www.euro-math-soc.eu/committee/applied-math

**27 – 29 November**
Executive Committee Meeting, Steklov Institute, Moscow, Russia

2016

**18 – 20 March**
Executive Committee Meeting, Institut Mittag-Leffler
Djursholm, Sweden

# EMS Scientific Events

2015

**6 – 10 July**
European Meeting of Statisticians, Amsterdam, The Netherlands
http://www.ems2015.nl/
Bernoulli Society-EMS Joint Lecture: Gunnar Carlsson (Stanford, CA, USA)

**10 – 14 August**
ICIAM Congress, Beijing, China
http://www.iciam2015.cn/

**27 August**
Boole 200 Lecture, Cork, Ireland
EMS Speaker: Stanley Burris

**31 August – 4 September**
17th EWM General Meeting, Cortona, Italy
http://www.europeanwomeninmaths.org/
EMS Lecturer: Nicole Tomczak-Jaegerman (Edmonton, Canada)

**18 – 20 September**
EMS-LMS Joint Mathematical Weekend, Birmingham, UK

**22 October**
25th Anniversary of the EMS, Institut Henri Poincaré, Paris, France

2016

**16 – 20 March**
27th Nordic Congress of Mathematicians, Stockholm, Sweden
Bernoulli Society-EMS Joint Lecture: Sara van de Geer (ETH Zurich)

**18 – 22 July**
7th European Congress of Mathematics, Berlin, Germany
http://www.7ecm.de/

# Editorial: 7th European Congress of Mathematics

## Technische Universität Berlin, Germany 18–22 July 2016, www.7ecm.de

7ECM BERLIN 2016

Volker Mehrmann and Elise Grubits (both Technische Universität Berlin, Germany)

Dear Colleagues,

We cordially invite mathematicians from all over the world to participate in the 7th European Congress of Mathematics in 2016 in Berlin!

**The Organisers:** The quadrennial Congress of the European Mathematical Society is organised by the German Mathematical Society (DMV), the International Association of Applied Mathematics and Mechanics (GAMM), the Research Center MATHEON, the Einstein Center ECMath and the Berlin Mathematical School (BMS).

European Mathematical Society · Deutsche Mathematiker-Vereinigung · GAMM · Research Center MATHEON Mathematics for Key Technologies · ECMath Einstein Center for Mathematics Berlin · Berlin Mathematical School

**Scientific Programme:** The programme of the congress will cover all areas of theoretical and applied mathematics. There will be 10 plenary lectures, 31 invited lectures, several prize lectures, the Hirzebruch lecture, the Abel lecture as well as a public lecture and an outreach lecture for students. Moreover, a lecture series on "Berlin in the History of Mathematics" is scheduled. Registered participants are invited to organise mini-symposia and satellite events.

**Plenary Speakers**

Karine Chemla (CNRS, University Paris Diderot & ERC Project SAW)
Alexandr A. Gaifullin (Russian Academy of Sciences, Moscow)
Gil Kalai (Hebrew University of Jerusalem)
Antti Kupiainen (University of Helsinki)
Clément Mouhot (University of Cambridge)
Daniel Peralta-Salas (Instituto de Ciencias Matemáticas, Madrid)
Leonid Polterovich (Tel Aviv University)
Peter Scholze (Universität Bonn)
Karen Vogtmann (University of Warwick)
Barbara Wohlmuth (Technische Universität München)

**Invited Speakers**

Spiros Argyros (National Technical University of Athens)
Anton Baranov (St. Petersburg State University)
Nicolas Bergeron (Université Pierre et Marie Curie, Paris)
Bo Berndtsson (Chalmers University of Technology)
Christian Bonatti (Université de Bourgogne, Dijon)
Pierre-Emmanuel Caprace (Université Catholique de Louvain)
Dmitry Chelkak (Steklov Institute, St. Petersburg)
Amin Coja-Oghlan (Goethe Universität Frankfurt)
Sergio Conti (Universität Bonn)
Massimo Fornasier (Technische Universität München)
Christophe Garban (Université Lyon 1)
Moti Gitik (Tel Aviv University)
Leonor Godinho (Instituto Superior Técnico, Lisbon)
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Radha Kessar (City University London)
Kaisa Matomäki (University of Turku)
Bertrand Maury (Université Paris Sud)
James Maynard (University of Oxford)
Sylvie Méléard (École Polytechnique, Palaiseau CNRS)
Halil Mete Soner (ETH Zürich)
Roman Mikhailov (Steklov Mathematical Institute)
Giuseppe Mingione (Università degli Studi di Parma)
Fabio Nobile (École Polytechnique Fédéral de Lausanne)
Joaquim Ortega-Cerda (Universitat de Barcelona)
Gábor Pete (Budapest University of Technology & Economics)
Tristan Rivière (ETH Zürich)
Elisabetta Rocca (Weierstraß Institut für Angewandte Analysis und Stochastik, Berlin)
Silvia Sabatini (Universität zu Köln)
Giuseppe Savaré (Università degli Studi di Pavia)
Nikolay Tzvetkov (University of Cergy-Pontoise)
Stefaan Vaes (Katholieke Universiteit Leuven)
Anna Wienhard (Universität Heidelberg)
Geordie Williamson (Max-Planck-Institut für Mathematik, Bonn)

**Friedrich Hirzebruch Lecture:** This lecture, dedicated to Friedrich Hirzebruch (first President of the EMS). We are proud to host this event on the first congress day of 7ECM. The speaker will be Don Zagier, Director of the Max-Planck-Institute for Mathematics in Bonn.

**Abel Lecture:** For the first time in the history of the ECM one of the Abel Laureates will give a dedicated lecture to 7ECM participants.

**Prizes:** Calls for the Otto Neugebauer Prize, the Felix Klein Prize and ten EMS prizes are underway (see the March Issue of the EMS Newsletter).

**Posters, Mini-Symposia and Contributed Sessions:** All registered participants are welcome to contribute to the programme in terms of posters, mini-symposia and contributed sessions. A dedicated call will follow in July in 2015.

**Grants:** To ensure broad participation in 7ECM and reduce economic barriers, 100 grants will be offered to mathematicians from less developed countries. The grants will cover a tuition waiver and financial support to a maximum level of €400. Women are particularly encouraged to apply.

**Satellite Events:** We invite mathematicians to organise satellite events (conferences, etc.) around the congress. 7ECM participants will enjoy some privileges in registering for the satellite events. Preconditions for granting satellite event status are scientific quality, geographical proximity and temporal connection with 7ECM.

**Public Lecture:** Everyone with an interest in mathematics is invited to attend the public lecture by Helmut Pottmann (Technische Universität Wien).

**Next Generation Outreach Lecture:** Peter Scholze (Universität Bonn) will give a lecture for high school students in mathematics.

**Exhibitions:** As a publishing house or a specialised company you may want to generate excitement for your products and services and thus maximise your visibility before, during and after the congress through several sponsorship opportunities. The historical *Lichthof* in the university's main building is the ideal location for exhibitions.

**Lichthof. ©TU Berlin Pressestelle**

**Social Programme:** The conference dinner will take place at the Palais am Funkturm – a unique venue in the architectural style of the 1950s. The costs for the dinner are included in the registration fee. Moreover, a welcome reception is planned for the first day of the congress.

**Side Programme:** We have developed a broad and inspiring cultural side programme for the period of the congress and beyond. Participants of 7ECM are invited to visit the interactive exhibition IMAGINARY in the main building of TU Berlin (Lichthof). On 17 July 2016, the Opening Ceremony for Transcending Tradition – Jewish Mathematicians in German-Speaking Academic Culture will be held at the Jewish Museum Berlin. The exhibition will be on display for the period of the congress and beyond. Furthermore, the exhibition Women Mathematicians around the World – A Gallery of Portraits curated by Sylvia Paycha will be presented in the Mathematical Library at TU Berlin. Also, a mathematics film festival is planned.

**Berlin in the History of Mathematics:** Mathematics in Berlin started in the year 1700 when the Academy of Science (today known as the Berlin-Brandenburgische Akademie der Wissenschaften (BBAW)) was founded. Gottfried Wilhelm Leibniz initiated its establishment and was its first president. Leonhard Euler worked at the academy from 1741 to 1766. Joseph-Louis Lagrange became his successor as the director of the academy's "Mathematical Class". A special "History Session", organised by Martin Grötschel, with lectures by Eberhard Knobloch on Leibniz, Gerhard Wanner on Lagrange and Günter M. Ziegler on Euler will highlight the mathematical development in the 18th century. Additionally, Jürgen Sprekels will survey the work of Karl Weierstrass, who had tremendous influence on the mathematics of the 19th century.

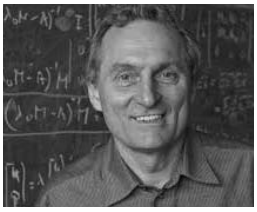
**Mathematical Berlin:** For 7ECM, the guidebook "Mathematical Berlin" by Iris and Martin Grötschel will be published. Readers will be guided through the centre of Berlin, with locations of mathematical interest highlighted and background information provided about mathematics in Berlin, mathematical institutions and many important mathematicians who have worked here. All participants will receive a hard copy as a welcome gift.

**Proceedings:** The proceedings of 7ECM will be published after the congress by the European Mathematical Society Publishing House.

Please find up-to-date information at www.7ecm.de and subscribe to our newsletter!

On behalf of the Local Organising Committee, we are looking forward to welcoming you to Technische Universität Berlin in 2016. Let Berlin inspire you with the creative atmosphere of this fascinating city!

**Contact:** Volker Mehrmann (mehrmann@math.tu-berlin.de), Chairman of the 7ECM Local Organising Committee.



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*Volker Mehrmann (mehrmann@math.tu-berlin.de), Chairman of the 7ECM Local Organising Committee, earned his PhD in 1982 at the University of Bielefeld. Since 2000, he has been a professor at the Technische Universität Berlin. In 2011, he was awarded an ERC Advanced Grant. His research interests are numerical mathematics/scientific computing, applied and numerical linear algebra, control theory and the theory and numerical solution of differential-algebraic equations. He is also Chairman of the Research Center Matheon and Vice-President of the Society of Applied Mathematics and Mechanics.*

Photo: private

*Elise Grubits (grubits@math.tu-berlin.de) is an Executive Board Assistant at the Coordinating Office of the Research Center Matheon and the Einstein Center ECMath. In this position, she coordinates the work of the Local Organising Committee.*

# New Members of the Editorial Board

**Ramla Abdellatif** currently holds a research and teaching position (AGPR or agrégée-préparatrice) at the Ecole Normale Supérieure de Lyon, where she moved after completing her PhD thesis under the supervision of Guy Henniart in Orsay (Université Paris-Sud 11). Her main research interests are p-modular and p-adic representations of p-adic groups and their behaviour in the setting of the Langlands programme, as well as their connections with p-modular representations of finite groups. Besides her research and teaching activities, she is actively involved in mentoring activities and the dissemination of mathematics and science for younger people, especially (but not only) schoolchildren and high-school pupils.

**Vladimir Salnikov** is a researcher at the University of Caen Lower Normandy (France). His work for his PhD thesis concerned two major topics: graded geometry in theoretical physics – in collaboration with Thomas Strobl (Lyon) – and constructive methods in analysis of integrable systems – in close contact with Sergey Stepanov (Moscow). His current research interests include generalised geometry, various aspects of dynamical systems and some applications in physics and mechanics. Vladimir is the Lavrentiev 2012 Prize winner, which permitted him to organise a workshop in Rouen on integrability in dynamical and control systems.

Since the early stages of his career, Vladimir has also been involved in teaching for graduate students and at high school. He also actively participates in a number of activities for a general audience aimed at the popularisation of science, such as the "Why Maths?" exhibition.

Vladimir's webpage can be found at http://www.vladimir-salnikov.org/.

# New EMS Publicity Officer

**Richard Elwes** studied mathematics at Oxford University, before completing a PhD in model theory (mathematical logic) at the University of Leeds, and holding a postdoctoral position at Albert Ludwigs Universität, Freiburg. He has worked as professional science writer, and is the author of five books on mathematics aimed at the general public: *Maths 1001* (2010), *Mathematics Without The Boring Bits* (2010), *The Maths Handbook* (2011), *Chaotic Fishponds and Mirror Universes* (2013), and *Maths in 100 Key Breakthroughs* (2013). Between them, his books are available in ten languages. He has also written for the press, notably feature articles for *New Scientist* magazine, and for several online outlets including *Plus* magazine and his own blog *Simple City*. He is an active participant in scientific discussions on social media.

He has broad mathematical interests, and has authored research papers in model theoretic algebra, analysis of complex systems inspired by social science, and applications of combinatorics to network science. He has a strong interest in mathematical outreach and education, makes occasional radio appearances, and regularly delivers masterclasses and invited talks to high school students, to undergraduate and general mathematical audiences, and to the wider public. He is currently working as a Senior Teaching Fellow at the University of Leeds, UK.

# Report from the EMS Executive Committee Meeting in Prague, 6–8 March 2015

Richard Elwes, EMS Publicity Officer

The Executive Committee was hosted in Villa Lanna by the Institute of Mathematics of the Academy of Sciences of the Czech Republic. On Friday, it was welcomed by Jiří Rákosník, Director of the Institute. On Saturday, the meeting was addressed by Bohdan Maslovski, President of the Czech Mathematical Society.

### President’s Report

Pavel Exner greeted the new Executive Committee for this first meeting of its tenure, and reaffirmed the Society’s central aim: to represent European mathematics on the global stage. He thanked the outgoing committee for its work, and observed that the new committee has high standards to sustain and challenges to meet.

### Treasurer’s Report

With the Treasurer Mats Gyllenberg absent due to ill-health, the President delivered the report. It was confirmed that the financial contributions of the University of Helsinki to the Society’s Office will be made explicit within future EMS budgets. The Executive Committee reiterated its gratitude to the University of Helsinki for its continuing support. A fuller discussion of the handling of EMS assets was postponed to the next Executive Committee meeting.

### Membership

Applications for Institutional Membership were received from the Department of Mathematical Sciences at Aalborg University and from the Faculty of Mathematics, Natural Sciences and Information Technologies at the University of Primorska. The Executive Committee approved both of these, along with a list of 80 new individual members. An application for Class 1 Membership from the Armenian Mathematical Union was also received. This will be presented at the next Council Meeting for approval.

### Publicity, and EMS on the Internet

The President welcomed Richard Elwes, the newly appointed Publicity Officer. A Web Team was appointed to manage, oversee, and develop the new EMS website. Additionally it was agreed that the society should increase its presence on social media. The Publicity Officer and the Editor-in-Chief of the Newsletter, Lucia Di Vizio, will consider strategy here.

### Scientific Meetings

The Executive Committee discussed the report of committee member Volker Mehrmann on preparations for the 7th European Congress of Mathematics (ECM) in

Berlin, 18–22 July 2016. It was agreed that preparations are proceeding well, and that an interesting meeting is in prospect.

The committee received preliminary bids to host the 8th ECM in 2020 from the University of Sevilla (Spain) and University of Primorska (Slovenia). Deeming both to be serious candidates, the committee agreed to invite both parties to prepare full, detailed bids. Representatives of both bids will be invited to the next Executive Committee meeting and a final decision will be taken by the EMS Council in 2016.

Several other future events were discussed. The committee agreed that the EMS Boole lecture will be delivered by Stanley Burris from the University of Waterloo in Cork, Ireland, in August 2015.

In order to simplify Calls for Proposals, the committee agreed that henceforth the submission deadline for events in year $n+1$ should be 30th September in year $n$.

### Society Meetings

The Executive Committee discussed the program for the meeting of the Presidents of the Member Societies in Innsbruck, Austria (28–29 March 2015; see page 8 of this Newsletter), and agreed to schedule a discussion on the practice of political lobbying for mathematics.

The committee also discussed preparations for the one-day anniversary event in celebration of the 25th Anniversary of the EMS, at the Institut Henri Poincaré in Paris, on 22nd October 2015.

### Standing Committees

The committee voted to appoint Roberto Natalini as Chair of the Committee for Raising Public Awareness of Mathematics and Patrick Foulon as Vice-Chair of ERCOM (the forum of directors of European Research Centres in the mathematical sciences).

The Executive Committee considered a report from the committee on Applied Mathematics, and noted that promising proposals have been received for a new ESSAM school (European Applied Mathematics Summer Schools in Applied Mathematics) devoted to mathematics with a modelling component (understood in a wide sense).

A report was received from the Education Committee. The President additionally reported that he had attended the previous meeting of that committee and reminded it of a prior request to prepare a broad inventory of educational methods in mathematics across Europe.

The Chair of the Electronic Publishing Committee Jiří Rákosník was present and presented his report, with a focus on the future of the European Digital Mathematics Library.

Reports were also received from the Committees on Developing Countries, Ethics, European Solidarity, Meetings, Publications, and Women in Mathematics. Regarding the last of these, the Executive Committee agreed to support the organisation of a majority female event at the Mittag-Leffler Institute in the summer of 2016.

### Publishing

The state of the EMS Publishing House was discussed, and the Executive Committee agreed a list of candidates for the House's Scientific Advisory Board.

The Editor-in-Chief of the Newsletter of the EMS, Lucia Di Vizio, then delivered her report.

It was agreed to reappoint Mireille Chaleyat-Maurel as Editor of the Society's E-News and to add the Publicity Officer to its editorial team. To better synchronise the E-News with the Newsletter, it was decided that henceforth the E-News should be sent out on the same day that the Newsletter becomes available online.

### Relations with Funding Bodies and Political Organisations

The President discussed the relations of the EMS to the Initiative for Science in Europe (ISE). Of particular interest is the ISE's campaign responding to the so-called Juncker Plan, which proposes cuts to the budget of Horizon 2020 (see 'Reaction to the Juncker Plan', p. 6, EMS Newsletter, March 2015). A possible change in the legal status to the ISE was also discussed; the Executive Committee will support such a move if it becomes necessary.

The President related new appointments and other recent developments at the European Research Council (ERC).

### Relations with Mathematical Organisations

The Executive Committee discussed its nominations to several Scientific Committees and other learned bodies. It was pleased to invite Sara van de Geer to deliver the EMS-Bernoulli Society joint lecture, to be delivered at the Nordic Congress of Mathematics in Stockholm, 16–20 March 2016.

### Conclusion

The next Executive Committee meeting will be 27–29 November 2015, at the Steklov Institute in Moscow. The President concluded the current meeting by expressing the thanks of everyone present to the Czech Institute of Mathematics, and to Jiří Rákosník in particular, for the excellent hospitality and organisation. The committee then retired to the roof terrace, where a beautiful view of the city was enjoyed in the sunshine, accompanied by a glass of slivovice.

# Report from the Meeting of Presidents of Mathematical Societies in Innsbruck, 28–29 March 2015

Richard Elwes, EMS Publicity Officer

After a welcome from Pavel Exner, President of the European Mathematical Society, the meeting got underway with a Tour de Table, in which everyone introduced themselves and their society. In total, the 37 participants represented 27 mathematical societies (including EMS). On behalf of everyone present, Pavel Exner then conveyed congratulations to the London Mathematical Society, represented by Terry Lyons, which celebrates its 150th Anniversary this year.

A short presentation about our hosts, the Austrian Mathematical Society, was delivered by its President, Michael Oberguggenberger of the University of Innsbruck. This society was founded in 1903 by Ludwig Boltzmann, Gustav von Escherich and Emil Müller, originally as the Mathematical Society in Vienna.

### EMS and Member Society Presentations

As has become traditional, Pavel Exner began the EMS President's report with a run-down of the main activities of the EMS and its standing committees, and the ways in which these benefit its corporate and individual members, and European mathematics generally.

He then drew attention to several upcoming events:

- To mark the 25th Anniversary of the EMS, a one day meeting entitled "Challenges for the next 25 years" will be held at Institute Henri Poincaré, Paris, on 22nd October 2015.
- The 8th EMS Joint Mathematical Weekend will be cohosted with the London Mathematical Society in Birmingham, UK, on 18–20th September, 2015.
- The next EMS Council meeting will be held at Humboldt University, Berlin, 16–18th July 2016 (directly before the 7th European Congress of Mathematics).

Fernando Pestana da Costa, President of the Portuguese Mathematical Society (SPM) spoke about preparations for the international meeting in Porto, 10–13th June 2015, to be hosted jointly by the SPM, EMS, and the American Mathematical Society.

The meeting then discussed a report from Volker Bach, President of the German Mathematical Society, on progress towards the 7th European Congress of Mathematics in Berlin, 18–22nd July 2016.

The EMS has received preliminary bids to host the 8th European Congress of Mathematics in 2020 from the Universities of Sevilla and Primorska. The meeting heard presentations from Antonio Campillo López and Tomaž Pisanski, respective Presidents of the Royal Spanish Mathematical Society and the Society of Mathematicians, Physicists and Astronomers of Slovenia, in support of these two bids. (The final decision will be made by the EMS Council in 2016.)

Betül Tanbay, President of the Turkish Mathematical Society, reported on the first Caucasian Mathematics Conference held in Tbilisi, Georgia, in 2014, under the auspices of the EMS and in cooperation with the Armenian, Azerbaijan, Georgian, Iranian, Russian and Turkish Mathematical Societies. A second conference is planned for 2016 in Turkey. The ensuing discussion was highly supportive of this endeavour, and especially welcomed scientific cooperation between countries with difficult political relationships.

Xavier Jarque Ribera, President of the Catalan Mathematical Society, spoke about its history and activities and introduced two recent innovations: a new journal, Reports@SCM, which aims to assist young researchers in getting published, and the establishment of the Barcelona Dynamical Systems Prize under the patronage of Carles Simó.

Bohdan Maslovski, President of the Czech Mathematical Society, then reported on the practice and politics of Research Evaluation in the Czech Republic, prompting a lively discussion of how this problematic process varies across different regions.

### Discussion on Political Lobbying

Time had been set aside for an informal discussion of the relationship between mathematics and politics at the national level. In several countries it seems difficult for mathematicians to access to the ear of government. This heightens the importance of the EMS's work at the European level, and that of those national societies which do have political influence. The ensuing discussion included arguments along the following lines:

- Lobbying is most effective when accompanied by solid evidence of the second order benefits of mathematics. As an example, the UK's Engineering and Physical Sciences Research Council published a major report commissioned from Deloitte, which estimated the contribution of mathematical science to the UK economy at 10% of all jobs and 16 per cent of Gross Value Added to the UK economy, over the year 2010.
- It is worth addressing influential individuals below the ministerial level, rather than restricting attention to senior politicians.

- Mathematicians should maintain lines of communication to the media.
- Mathematics can appear profoundly unattractive from the outside. Thus it is a continual effort to present our subject in an appealing fashion.
- Initiatives that relate to both education and research can have a greater impact than those focussed on research alone. (In certain countries this may be harder to achieve, due to budgetary/governmental separation of these domains.)
- In several countries, combined associations of societies for the natural sciences have been formed, which aim to speak to government on matters pertaining to research and education. Relatedly, it was suggested that physics and mathematics together have a stronger voice than mathematics alone.

**Concluding remarks**

Richard Elwes, newly appointed Publicity Officer for the EMS, spoke briefly about the need for the EMS and member societies to engage with social media.

Pavel Exner then brought proceedings to a close by thanking the local organisers for their faultless preparation and hospitality, and for the warm welcome we all received at the University of Innsbruck. The day then concluded with a lunch of traditional Tyrolean finger food.

# Joint Anniversary Weekend EMS-LMS Mathematical Meeting Birmingham, September 18–20, 2015

Christopher Parker (University of Birmingham, UK)

European Mathematical Society 25 years

London Mathematical Society 150 years

To celebrate the 150th year of the London Mathematical Society (LMS) and the 25th year of the European Mathematical Society (EMS) we are organizing a mathematical weekend, to be held in Birmingham from Friday September 18th to Sunday 20th, 2015. All mathematicians, from Europe and elsewhere, are warmly invited to participate.

The weekend features three themes: Algebra, Analysis and Combinatorics. There will be plenary talks by the following speakers:

Noga Alon, Tel Aviv, Princeton
Keith Ball, Warwick
Béla Bollobás, Cambridge, Memphis
Timothy Gowers, Cambridge
Stefanie Petermichl, Toulouse
Aner Shalev, Jerusalem

There will be over twenty other invited talks presented in parallel sessions. The speakers are:

*Algebra session:* Ben Klopsch, Düsseldorf; Martin Liebeck, London; Gunter Malle, Kaiserslautern; Bob Oliver, Paris; Cheryl Praeger, University of Western Australia; Donna Testerman, Lausanne.

*Analysis session:* Franck Barthe, Toulouse; Tony Carbery, Edinburgh; Tuomas Hytönen, Helsinki; Sandra Pott, Lund; Christoph Thiele, Bonn; Luis Vega, Bilbao; Julia Wolf, Bristol.

*Combinatorics session:* Jozsef Balogh, Illinois; Mihyun Kang, Graz; Michael Krivelevich, Tel Aviv; Marc Noy, Barcelona; Wojciech Samotij, Tel Aviv; Mathias Schacht, Hamburg; Benny Sudakov, Zurich.

*History session:* Niccolò Guicciardini, Bergamo

Participation by early-stage researchers is particularly welcome and some funding is available to support them. Additional sessions are planned for post-doctoral researchers to present their work, and there will be a poster session for doctoral students.

Registration for the meeting is via the website http://web.mat.bham.ac.uk/emslmsweekend./

# José Mariano Gago (16 May 1948 – 17 April 2015)

Pedro Freitas (University of Lisbon, Portugal)

José Mariano Gago passed away on Friday 17 April at the age of 66. An electrical engineer from the Instituto Superior Técnico in Lisbon who took a PhD in Physics in Paris followed by a spell at CERN, Gago went on to be heavily responsible for the great reform and growth of the Portuguese science system over the last 25 years.

Firstly as Head of the Portuguese Science Funding Agency (JNICT, now FCT) from 1986 to 1989 and then as Minister for Science and Technology (1995–2002) and Minister for Science, Technology and Higher Education (2005–2011), Gago introduced or made stable several reforms that transformed the incipient Portuguese research environment of the 1970s into a vibrant one, where a career in research became a natural thing to consider.

These measures included, for instance, evaluations by international panels and a massive programme of individual grants that allowed young graduates to pursue their PhDs and postdoctoral studies abroad, avoiding scientific inbreeding during a fourfold increase in the number of doctorates between 1987 and 2001.

Aware that it was fundamental that a taste for science be developed from an early age, Gago was also responsible for the creation of the Ciência Viva programme in 1996. This programme, whose general aim was to promote a culture of science and technology in the Portuguese population, has about 20 interactive centres throughout the country, with activities ranging from agronomy to astronomy and mathematics. The implementation of Ciência Viva in Portugal has been considered the model to be followed in other countries and has helped establish his reputation among his peers in the European Union.

He took part in several European bodies and institutions, playing an active role in the shaping of the European research landscape. Gago was, for instance, instrumental in defining the Lisbon goals and, as the first President of the Initiative for Science in Europe (ISE) in 2004, instrumental in the movement that led to the creation of the European Research Council.

More generally, Gago's lifetime work for science has been recognised not only by the Portuguese state who honoured him with the Ordem Militar de Sant'Iago da Espada in 1992 but also by other European countries such as Spain (Orden de Isabel la Católica, 2006) and Germany (Verdienstorden der Bundesrepublik Deutschland, 2009).

Throughout his ministerial career, José Mariano Gago was always very supportive of several initiatives of the Portuguese Mathematical Society, such as the popular science series Tardes de Matemática, where he made a point of always being present in the audience in spite of his heavy agenda as minister. He also played an important role in the formal creation of the Portuguese Mathematics Commission, the Portuguese link to the International Mathematical Union.

# European Girls' Mathematical Olympiad

Birgit van Dalen (Dutch Mathematical Olympiad, Zoetermeer, The Netherlands)

There has been a long tradition of mathematical Olympiads in many countries. The most prestigious competition, the International Mathematical Olympiad (IMO), will be held for the 56th time in 2015, and more than 100 countries will participate in this event. For a few years, there has also been a contest just for girls: the European Girls' Mathematical Olympiad (EGMO). This event has already grown to be one of the largest international mathematical contests.

The UK took the initiative in organising the first EGMO in 2012. They realised that girls almost never made it onto the UK team for the IMO and, in fact, only 10% of IMO contestants from around the world were female. Not believing that girls are simply unable to do mathematics at such a high level, they wanted to create more opportunities for girls to develop their mathematical skills. And so the EGMO was born. It was designed to be European to keep the travel expenses for participating teams limited. However, guest teams from outside Europe are welcome to join if they want to; for example, the USA has participated in every EGMO so far.

The goal of the EGMO is to increase female participation in both national mathematical contests and in the IMO. This will take some time, perhaps many years, but as the team leader of the EGMO team from the Netherlands, I'm already witnessing the positive effects of the EGMO. In my country, more girls currently enter the first round of our national Olympiad than a few years ago and more girls reach the finals. Also, the enthusiasm of girls who have been to the EGMO is unmistakable. After returning from the EGMO, they double their efforts in our training programme. One of the girls who went to the first EGMO in 2012 made it onto the IMO team the same year. I'm fairly sure she wouldn't have managed that without the experience of the EGMO. She participated in two more EGMOs and two more IMOs and finally managed to get a gold medal at the IMO in 2014. She is the first Dutch girl ever to win a gold medal at the IMO.

The fourth EGMO took place in April 2015 in Minsk, Belarus. There were 109 contestants from 29 different countries. One of them managed to get a full score on the problems, which were slightly (but only slightly) below IMO level. Aside from the two mornings of the contest, there were many excursions and an opening and closing ceremony. There were many opportunities for the girls to socialise and play games. At the end of the event, Facebook names were hurriedly exchanged, enabling the girls to stay in touch with each other. Hopefully these friendships will last for years and, in due time, the EGMO girls will form a substantial network of female mathematicians.

*Birgit van Dalen is the Chair of the EGMO Advisory Board and has been team leader of the Dutch delegation in every EGMO so far. In the Netherlands, she is one of the main organisers of the national mathematical Olympiad, as well as a secondary school mathematics teacher.*

# The First Years of the European Mathematical Society

Aatos Lahtinen, Treasurer of the EMS 1990–1998 (University of Helsinki, Finland)

This is the story of the creation of our society in 1990–1994. At this initial stage, we eagerly tried to move forward with quite a few things simultaneously, like jugglers. For this story, I will, for clarity, concentrate on one item at a time.

## Prologue

In 1978, in Helsinki, European mathematical societies founded an unofficial body – the European Mathematical Council (EMC) – chaired by Sir Michael Atiyah to foster cooperation. It was, however, prevented from evolving. The next attempt was made in 1986 as if with foresight of the crumbling of the Iron Curtain. After four years of toil, European mathematical societies were invited to a meeting in Madralin, 27–28 October 1990. The purpose was to replace the EMC by a legal coalition, the European Mathematical Society (EMS). The seat of the society would be in Helsinki. I was responsible for writing the draft statutes so that both the EMC and the Finnish authorities were content. The draft I presented in Madralin was the fifth.

After extensive debates, the meeting approved the statutes (with some modifications) and unanimously agreed on the establishment of the European Mathematical Society with its seat in Helsinki. Due to this, the official documents had to be in Finnish. The delegates signed the Finnish Charter, hoping that it was appropriate. Then we toasted happily to the genesis of the society. In the same context, the European Mathematical Council was dissolved, having fulfilled its purpose.

The meeting now became the council of the society. It unanimously elected the following Executive Committee: President - F. Hirzebruch, Vice-Presidents - A. Figa-Talamanca and C. Olech, Secretary - E.C. Lance, Treasurer - A. Lahtinen, Committee members - E. Bayer, A. Kufner, P.-L. Lions, L. Marki and A. St Aubyn.

Hirzebruch now took the chair and the council began to shape the society. Then, Max Karoubi made a tempting suggestion. He was preparing a large European Congress in Paris in 1992 and proposed it as the Congress of EMS, without any financial responsibility. After some consideration, the council eagerly approved the proposition. It also agreed that this would be a tradition: the society would have a congress every four years starting with the Paris congress.

When Hirzebruch finally closed the meeting, we parted with a strong feeling that the society would grow to be an influential spokesman of mathematics in Europe.

## Executive Committee

The Executive Committee began to build the society on the cornerstone laid in Madralin. We were eager and enthusiastic. The meetings lasted a long time and were full of ideas and lively discussions, which were sometimes quite colourful. We always aimed at a consensus, which was usually achieved by the skilful chairmanship of Hirzebruch. Lance and the acting secretary D. Wallace had an amazing ability to crystallize the agreements and the essence of discussions in the minutes.

The Executive Committee was too small to build the society. Therefore, for each task we nominated a separate committee, enlisted from outside but reporting to us. By the end of 1994, we had 12 committees or equivalents from Applied Mathematics to Women and Mathematics. It was not always easy to find motivated people for these. However, the committees themselves do not do anything; the people in the committees do everything.

We decided to meet twice a year. However, some enterprises could not wait six months. For these, we set up the General Purpose Committee consisting of president, secretary and treasurer. It also dealt with any other matters referred to it.

Our meetings were during the weekends at the invitation of a committee member. Many meetings had attractive surroundings, whispering: “Come here!” but the length of the agendas prevented that. One time, our lodgings were at a museum. Its alarms were activated in the evening, preventing us not only from visiting the museum but also from leaving our lodgings. No breaks during the work!

The agendas also contained small things. The abbreviation EMS was questioned because the Edinburgh Mathematical Society already used it. A small study revealed that the acronym was also used by several others, like Express Mail Service, European Monetary System, etc. We still decided, however, to use EMS as the society’s acronym.

The terms of C. Olech, E. Bayer, A. Kufner and A. St Aubun ended in 1992 and only E. Bayer was standing for re-election. The council unanimously voted to elect L. Marki as vice-president and E. Bayer, I. Laboriau, A. Pelzar and V. Solonnikov as committee members for 1993–1996.

For the period 1995–1998, there were vacancies for the posts of Hirzebruch, Figa-Talamanca, Lance, Lahtinen and Lions. By the statutes, Hirzebruch could not be re-elected. Of the others, only I was standing for re-election. The list of uncontested nominations was J.-P. Bourguignon as president, P.W. Michor as secretary and Lahtinen as treasurer. D. Wallace was elected as vice-president and A. Conte as a committee member.

Bourguignon stated that following its successful foundation, the society should move onto the second phase of activity and spoke on the major aims during his presidency. The council got a definite impression that the presidency would be in good hands also for the next four years.

### Membership

The society was founded to have both corporate and individual members. Corporate members were full, associate or institutional members. All 33 societies which had participated in the European Mathematical Council were deemed to be full members, except the Mathematical Society of DDR, which was ceasing to exist. The European Mathematical Trust was admitted as the first associate member and Atiyah as the first individual member.

The 1992 council accepted six full and two associate members. The application of the Israel Mathematical Union caused a debate on whether it was a European society. As a precedent, it was mentioned that Israel participated in the European Song Contest! Finally, it was accepted by vote. The 1994 council accepted four full members. At the end of 1994, the society had 42 full members and 3 associate members.

Our society chose an unusual route for membership applications. An individual member of a corporate member would apply via his or her own society and the EMS membership would begin automatically when the membership fee was paid. Applications from other individuals were treated by the Executive Committee. It was also agreed that corporate members would collect the EMS membership fees from their members and account them to us.

This procedure made the enlisting of individual members very easy for us and gave rapid results. On October 1991, we already had 1000 individual members and in September 1992, the number was 1663. Then the increase ceased. In August 1994, we only had 1526 and in December 1994, about 1600 individual members.

In addition, there were continuous oscillations in the membership. Many new members paid their fees only once or twice and then disappeared. Apparently, our society did not fulfil their expectations. In fact, we could only offer our members the Newsletter and the possibility of influencing the development of the society. Also, because an individual member did business with us via his or her national society, the relation to our society remained secondary and did not create togetherness with us.

### Finance

The office of the society was placed under my control at the University of Helsinki. It was tended by Ms Tuulikki Mäkeläinen. Her contribution to the running of the society's everyday business cannot be overestimated. I persuaded the Ministry of Education to pay her salary.

Economically, the society started with a €3,000 inheritance from the dissolved European Mathematical Council. For a long time, our income consisted entirely of membership fees and remained modest. For instance, our income in 1994 was €50,000. The fees of some East European societies were temporarily waived and some societies did not pay their fees. It was clear that the society had to find new sources of income for any new enterprise.

The Greek Mathematical Society was not paying its membership fees and was not reacting to reminders. Then, in 1993, I was invited as a speaker to its 75th anniversary meeting in Athens. When I met the president of the society, I reminded him of the unpaid fees. Next day, he gave me a thick wad of drachmas. They were enough for the unpaid fees and the fees for the next two years.

### Newsletter

The first task of the Publications Committee, chaired by S. Robertson, was to create a newsletter. Robertson swiftly enlisted joint editorial teams in Prague and in Southampton and organised production and distribution in Southampton.

The first issue of the Newsletter appeared on 1 September 1991 with 20 black and white pages of B5 size. On the front page, there was a letter from Hirzebruch inviting everyone to build the society. In addition to articles and advertisements, there was also information on the Paris congress.

From then on, the Newsletter appeared quarterly. To satisfy the increasing desires of readers and advertisers, it grew from B5 to A4 and the number of pages exceeded 30. During the period 1991–1994, the Newsletter could not yet afford colours but its popularity grew, together with the development of its contents.

### Publications

At the Madralin meeting, we had already discussed whether the society should have mathematical journals. The Executive Committee continued to consider it and the item "Publications" was on every agenda. Journals would serve the members and produce income for the society. On the other hand, some insisted that there were already enough journals and that our member societies with their own journals would not welcome us in their territory. We decided by vote, however, to have a try.

The society did not have enough capital to establish a journal. The only possibility was cooperation with a commercial publisher. In 1992, D. Wallace was authorised to conduct negotiations with Springer-Verlag. After two years, the Executive Committee could present to the council a plan for the Journal of the European Mathematical Society. After an animated discussion, the council instructed the Executive Committee to proceed. The society was finally getting a journal.

Another approach to publishing was initiated in 1994 by setting up a Committee on Electronic Publishing, chaired by P. Michor, who had presented a far-reaching memorandum on the subject.

### European Community Liaison

Because the society needed close contacts to the European Community, we established the European Community Liaison Committee, chaired by A. Figa-Talamanca. A vivid discussion on our connections to the European Community took place during all the Executive Committee meetings.

Hirzebruch and Figa-Talamanca made contacts with the commissioners for science and education. They made our activities known and discussed, among others things, the role of mathematics in science programmes. They were also influential in recommending names for the CODEST Mathematics and Computer Science Panel and in getting mathematical input in the Human Capital and Mobility Programmes.

As contacts with Brussels became more frequent, L. Lemaire (Brussels) was nominated as Liaison Officer with the European Community. Lemaire did valuable work in maintaining contacts with the bureaucracy and the politicians, as well as circulating news to EMS members.

### EMS Congresses

The fate of the Paris congress was for a while uncertain because it appeared that it did not have the support of the French Mathematical Society. For the unification of the French field, a "Haut Comité du Congrès", where all French mathematical societies and the EMS could be represented, was set up in addition to the already functioning organising committee. This finally rescued the congress but not without heated discussions.

In the end, the congress was a success. In Sorbonne, there were 1300 participants from 58 counties. The atmosphere was good, the presentations were brilliant and the discussions at the Round Tables were intense. The prizes for young mathematicians were given out by Jacques Chirac, the Mayor of Paris, at the Town Hall.

For the 1996 congress, the council of 1992 received two applications: Hungary and Barcelona. Both seemed to be possible choices. Hungary was chosen by a vote of 31 to 13. The European Mathematical Congress of the EMS had established its place in European mathematics.

### Euroconferences

Hirzebruch pointed out to the Executive Committee in 1992 that the European Science Foundation (ESF) organised European research conferences but so far none in mathematics. He was authorised to offer both pure and applied mathematics to the ESF conference series.

When ESF responded positively, we got a promise from P.-L. Lions to make a proposal on applied mathematics starting in 1994 and from L. Babai to make a proposal of two conference series in pure mathematics starting in 1995. After some twists and turns the ESF accepted these proposals. We also succeeded in getting E. Bayer nominated as our representative to the Steering Committee of the ESF.

### EMS Lectureship

In 1993, Hirzebruch suggested an EMS lectureship on special topics. The matter was considered at the General Purpose Committee and at the Executive Committee. The concept and its draft rules were accepted in 1994. H. W. Lenstra (University of California) was invited as the first EMS lecturer.

### Applications of Mathematics

The first task of the Committee on Applications of Mathematics (Chair: J. Hunt, and later A. Jami) was to organise the Round Table "Mathematics in Industry" at the Paris congress. After that, the committee took care of the Euroconference programme on applied mathematics and liaised with ECCOMAS and the ECMI.

### Education

The emphasis of the Mathematical Education Committee, chaired by W. Dörfler, was on undergraduate and upper secondary level mathematics education. During the period 1991–1994, every Newsletter issue had educational articles like "Gender and Mathematics Education" and "Computers in Teaching Initiative".

### Summer Schools

The Summer School Committee (Chair: L. Marki) gathered information on regular summer schools with an emphasis on young mathematicians. In the beginning, we could only offer them only symbolic support by declaring that a summer school would be arranged "under the auspices of EMS".

### Support for East European mathematicians

At first, we could only support East European mathematicians with small things like waiving membership fees. Thanks to donations, we could compensate the participation costs of some young East European mathematicians for the Paris congress. In practice, these funds were at a Parisian bank and I, as treasurer, paid the subventions not by money but by cheques. This was the first and probably the last time in my life that I would have a French chequebook.

In 1992, I persuaded the Finnish Mathematical Society to donate 5000 FIM to EMS for travel expenses of Estonian mathematicians. After this, the Executive Committee set up a Committee for Support of East European Mathematicians with an annual budget of €10,000. To my surprise, the chairman J.-M. Deshouillers spoke decent Finnish. We were disappointed in the small number of applications.

### Women and Mathematics

The Committee on Women and Mathematics, chaired by E. Bayer, began to collect information on the number of women mathematicians and the proportion of women mathematics students. The results were presented to the 1992 council. There were large variations between countries but the differences in educational systems made comparisons difficult. The committee also arranged a Round Table at the Paris congress. Later on, the committee decided to concentrate its activities on countries with a particularly low proportion of women mathematicians.

### Epilogue

During the period 1990–1994, the dream of the European Mathematical Society finally came true. The society was founded and its essential functions were established. The society also planted many seedlings, which were to bloom later on. Fritz Hirzebruch, with his skill, diligence and devotion, was absolutely the right person to take care of all this.

I participated in the planning of the society from 1986. As treasurer, I attended every meeting of the Executive Committee and the General Purpose Committee for eight years. I have written these memories on the basis of the references and my fading recollections.

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# Recollection of a Very Exciting Time

Jean-Pierre Bourguignon (IHÉS, France)

The early part of the 1990s was a very special time in European history, with considerable transformations that followed the fall of the Iron Curtain. For a number of us, there was a clear urge to see the construction of a more interactive Europe move forward at a faster pace. The creation of the European Mathematical Society in 1990 and its first years have to be put in this context, even if the process started earlier with the main objective of reinforcing East-West relations. On this initial front, the critical role played by Sir Michael, under the umbrella of the European Mathematical Council, must be acknowledged. The choice of Madralin in Poland to hold the EMS constitutional meeting bears some symbolic value in this respect.

At the time of the Madralin meeting, I was President of the Société Mathématique de France (SMF) and my recollection of both the preparation of the meeting and of the discussions there gives me the feeling that the prehistory of the EMS discussed by Sir Michael in the March 2015 issue of the Newsletter had ended differently from the way it started. In Madralin, if a tough discussion on the key question as to whether the EMS should have individual members took place, it did not oppose the Germans and the French but above all the British and the French. It indeed appeared to the French as evidence, in view of the transformations Europe was going through, that the EMS should look for the personal engagement of the largest possible number of our colleagues. After a day of heated debates on the nature of the EMS, "should it be a society of societies or a more integrative structure?", it took the open-mindedness and natural authority of Friedrich Hirzebruch, who had agreed on the principle of being the first EMS President, to strike a deal between defenders of these two positions after a very intense evening confrontation behind closed doors. Sir Michael gave his consent to the deal in the most spectacular way, in being the first individual member of the newly born society. It is still under this dual governance that the EMS works, showing that the compromise was well founded (one of the many legacies we owe to Fritz Hirzebruch).

I recall this event to explain why it came to me as a complete surprise when he approached me to enquire whether I would consider running as his successor. I had the feeling of having been one of the troublemakers at the constitutional meeting. In addition, the organisation of the First European Congress of Mathematics in Paris in 1992, whose idea was launched by Max Karoubi and a few colleagues before the EMS existed, did not go at all as smoothly as one would have hoped. Indeed, people in charge of both the Société de Mathématiques Appliquées et Industrielles (SMAI) and the SMF were very concerned about the financial soundness and the overall format of the enterprise, at least the way it had started. As President of the SMF, I was one of them. The image that the French mathematical community gave of itself, on this occasion, to its European counterparts was not that of serenity. After a critical arbitration by Fritz Hirzebruch and Henri Cartan (the special relationship he enjoyed with Fritz and his visionary engagement for a federal Europe being of course important reasons to call upon him at the age of 88), the congress took on a manageable format and was, in the end, a great success. I did not expect that this rather troubled sequence of events would provide me with the opportunity to play any prominent role on the European mathematical scene any time soon. However, this did not take into account Fritz's magnanimity.

Spending the first semester of 1994 at MSRI allowed me to distance myself and to reflect on what I could propose as a course of action for my mandate as President of the EMS, taking a global perspective. I had already put forward the idea of forming a team with Peter Michor, who would be running for the EMS Secretary position, as I knew about and appreciated very much his engagement for developing internet tools for the service of mathematicians. That such an effort should be made in the context of the EMS looked to me as most appropriate. This is how the EMIS (the European Mathematical Information Service) came to life as the EMS portal, a typical example of how the EMS could provide new services to the mathematical community.

In the same vein, during my mandate, after several attempts had shown that the American Mathematical Society was not ready to share the responsibility of a truly universal database, I made sure that the EMS became a partner of the bibliographical database Zentralblatt für Mathematik (ZbM) alongside the Heidelberg Academy of Sciences. This was made possible thanks to the open minds of people running the German agency in charge of the database, the FachInformation Zentrum Karlsruhe, and of the mathematical editors of Springer, who were in charge of commercialisation. It was an important step to improve the service provided to the mathematical community, and a number of mathematicians have contributed to this effort. This was, and still is, a formidable asset for Europe. I must add here that the precious help and involvement of Bernd Wegner, then Editor-in-Chief of ZbM, were also critical in that and in the birth of the EMIS.

At the time I took office, one of the issues that preoccupied a number of mathematicians was the new emphasis put in a number of countries on applied mathematics and the consequences for the mathematical community. I did not see this as a problem but as a fantastic opportunity. For me, it was completely clear that the EMS had to make substantial efforts to attract a sufficiently large proportion of applied mathematicians. This required, of course, the organisation of appropriate activities and this attention to the diversity of profiles of the events supported by the EMS was on the minds of the members of the Executive Committee all the time. I was very pleased that Rolf Jeltsch, a well recognised applied mathematician, agreed to run for president and became my successor in 1999.

Showing the interest of the EMS for interfaces, one aspect of the previous line of thought was one of the motivations for the creation of the "Diderot Mathematical Forums" as an activity of the EMS. Their format – three coordinated conferences in three European cities sharing some sessions by telecommunication – was meant to highlight the richness and diversity of mathematics in Europe. The first four were: "Mathematics and Finance" in London, Moscow and Zurich, "Mathematics and Environment with Focus on Water" in Amsterdam, Madrid and Venice, "Mathematics as a Force in the Evolution of Culture" in Berlin, Florence and Krakow, and "Mathematics and Music" in Lisbon, Paris and Vienna. Several others continued the chain. It is worth pointing out that the one on "Mathematics and Music" led to the creation of a mathematics group at IRCAM, the research institute on music created by Pierre Boulez that is located next to the Centre Pompidou in downtown Paris. This group continues to thrive and its international visibility was recently celebrated in a conference in Singapore.

The Second European Congress of Mathematics was held in Budapest in 1996. It was really the first ECM for which the EMS was directly involved in the conception, as the Paris Congress had grown fundamentally out of a local initiative. It had a number of specific features, some that continued, such as the EMS Prizes (funded by the Mairie de Paris in 1992, they were financed in 1996 by the local branch of Motorola, thanks to the efforts of Hungarian colleagues), and some that were more specific, such as a special session to celebrate Paul Erdös at l'Institut Français or the Junior ECM held in Miskolc a few days after ECM2, a remarkable achievement of very engaged colleagues that brought together enthusiastic young students from several countries. I cherish the memory of these two events as they gave me, in particular, the opportunity to witness the extraordinary ability of this exceptional mathematician in interacting with young people (unfortunately, he passed away shortly afterwards).

As my interest in seeing the EMS have individual members was known from Day 1 of the society (actually, the membership got close to 2000 at the end of my mandate, showing that the perspective of winning the participation of a significant number of colleagues was a real one), I made sure that national or regional societies felt welcome and properly involved in the development of the EMS. This was the reason for suggesting that each society should have a correspondent, to be sure that a channel of communication was open to as many of them as possible.

Making sure that mathematicians were potential partners for a number of European institutions was an important task that required better knowledge of the functioning of these institutions and, conversely, making people in charge of them aware of how the mathematical community operates and how mathematicians could contribute. Luc Lemaire, as EMS Officer in charge of these contacts, played a critical role in achieving that. This meant a number of contacts with people in charge of research at the European Commission but also the need to obtain some room for mathematics in the programmes of the European Science Foundation. Keeping in mind the key vision that scientists are the ones who make the difference, this led me to join 12 other scientists calling for the creation of Euroscience, a grass roots organisation of scientists that was finally established in 1997 at a meeting in Strasbourg and which developed into a key player on the European scene.

Thanks to the competent support of Mireille Chaleyat-Maurel, who helped the Executive Committee as Communication Officer, an active policy was established, aiming at making the EMS more visible. These efforts took many forms. One of them was the introduction of a new logo for the society, with an interesting mix of mathematical content, the Fibonacci sequence, and an aesthetic show of diversity, reflecting the reality of Europe.

Apparently, the choice was not so bad as this logo is still the one the EMS is using. An attempt was even made to create a mathematical press agency, with the general public as final target. Unfortunately, in spite of great efforts, the EMPRESSA project could not deliver what was hoped and waned away.

During my mandate, the preparation of the launch of JEMS, the Journal of the European Mathematical Society, continued with the identification of the first Editor-in-Chief and the signing of a contract with Springer Verlag. However, a more ambitious goal – coming up with an economically viable solution for the creation of a European Mathematical Publishing House – could not be finalised before I left office.

The EMS Executive Committee also made considerable efforts to respond to the call made by Jacques-Louis Lions, then President of the International Mathematical Union, to participate in the World Mathematical Year 2000 he had convinced UNESCO to establish. It also developed contacts with the Chinese Mathematical Society and the African Mathematical Union, whose existence actually preceded that of the EMS.

At this early stage of existence of the EMS, priority had to be given to developing new activities. Some of them, such as summer schools, were successful, developed further and now belong to the natural environment of European mathematicians. All this could only be achieved thanks to the hard work and the contributions of a number of colleagues: first of all, of course, the members of the EMS Executive Committee but also the chairs of the various EMS committees, whose activities played a significant role in the rapid recognition gained by the EMS. The exceptional quality of support given by Tuulikki Mäkeläinen, in charge of the EMS Secretariat in Helsinki, must also be acknowledged with gratitude. All in all, this was a very exciting time and a special moment in my professional life.

*Jean-Pierre Bourguignon is a differential geometer. He spent his whole career as a fellow of the Centre National de la Recherche Scientifique (CNRS) and he held a position as professor at the École polytechnique from 1986 to 2012. He received the Prix Paul Langevin in 1987 and the Prix du Rayonnement Français in Mathematical Sciences and Physics from the Académie des Sciences de Paris in 1997. He is a foreign member of the Royal Spanish Academy of Sciences. In 2005, he was elected honorary member of the London Mathematical Society and has been the Secretary of the Mathematics Section of the Academia Europaea. In 2008, he was made Doctor Honoris Causa of Keio University, Japan, and, in 2011, Doctor Honoris Causa of Nankai University, China.*
*He was the Director of the Institut des Hautes Études Scientifiques (IHÉS) from 1994 till 2013.*
*From 1990 to 1992, he was President of the Société Mathématique de France and he was President of the European Mathematical Society from 1995 to 1998. He is a former member of the Board of the EuroScience organisation (2002–2006) and has served on EuroScience Open Forum (ESOF) committees from 2004 to 2013. Since January 2014, he is the President of the European Research Council.*

# Cameras Among Mathematicians! Video – From Live to the Archives

**The First European Congress of Mathematics, Paris – La Sorbonne, July 1992**

François Tisseyre (Director “EcoutezVoir” Studio)

The year 1992 was for us (members of *EcoutezVoir*, a small associative audio-visual workshop working especially for the popularisation of science and in particular mathematics) strongly marked by the first European Symposium of Mathematics. The Sorbonne, in the heart of the Latin Quarter, was filled with hundreds of mathematicians from Europe and elsewhere. This was history! It was, as often, thanks to Jean-Pierre Bourguignon and Thierry Paturle (*École Polytechnique*) that we were gathered and then thanks were due to Adrien Douady, who accompanied and guided us in what turned out to be an exciting experience for the non-mathematicians that we were and that we remained.

This memorable episode began several weeks before the symposium and ended a whole year later. Two activities were involved: the dissemination of films on mathematics and the production of a documentary and set of interviews.

### “Cinemath”: a mini film festival

Among the many activities scheduled, a small working group was constituted in order to create a mini film festival about mathematics within the symposium: *Cinemath*. It was with pleasure that I joined this committee, which included Jean Brette, tireless populariser at the *Palais de la Découverte*, Thierry Paturle and Colette Loustalet, who had just published, within the association *Imagiciel*, a remarkable catalogue of 121 films for the teaching of mathematics. This catalogue was our irreplaceable and invaluable source of information. For several weeks, we were able, using this catalogue, to acquire, view and analyse many films from Europe and elsewhere, until we selected a reasonable portion to fit into the cultural activities of the symposium. The advice of Jean-Michel Kantor, and his media experience, was often enlightening.

Fifteen films were finally retained, grouped into three programmes. Productions came from various organisations: universities, research centres, producers and associations. The films would be streamed at the symposium for three days in an auditorium at the Sorbonne.

During this preparation, we frequently met the organisers of the symposium, in particular Fulbert Mignot and François Murat, busy with their tasks and the multiple problems in relation with the event. It soon became clear to us that this historic symposium, creating great passion among mathematicians, was far from the interest of the media. It is also true that mathematicians, however excellent they may be in scientific communication, had, at the time, paid little attention to more general communication. This is what motivated our desire and our proposal to cover what was clearly for us an historic event, in order to keep as alive a trace as possible, next to the traditional publications like the proceedings.

### "Mathematics, my village": a documentary

In the light of the programme of the symposium, our challenge was huge. We had to be crazy to launch such a project. And that is what happened. The green light given by the Organising Committee did not signify support, however essential it might have been in a project of this scale: weeks of preparation, a week of filming and months of editing lay ahead. Assistance came from the *Palais de la Découverte*, under the leadership of Michel Demazure, then learned societies (the SMF and the SMAI[1]) and, above all, as so often, the *École Polytechnique* and *Imagiciel*. But we knew that this support would only partially cover the foreseeable needs. We were going to invest time, lots of time.

On the side of *EcoutezVoir*, Claire Weingarten joined me. Each equipped with a small camera and microphone we dove in for a crazy week in the heart of the symposium. Our initial idea was to make a kind of standard portrait of a symposium of scholars, with its mathematical uniqueness, foraging here and there over the scientific, cultural and social activities. But from the beginning, we were caught up by the events and our cameras quickly began recording for very long hours at a time. We could not, however, record everything: this would be impossible and unusable afterwards. How to choose what to record then? This is where the irreplaceable help of Adrien Douady came in.

We had met Adrien a few years earlier, thanks again to Jean-Pierre Bourguignon, for a documentary for the Symposium Mathématiques *À Venir* (1986): *Y a-t-il un mathématicien dans la salle?* (Is there a mathematician in the room?). This meeting was the beginning of a long and loyal friendship, based on the desire to share with the largest number of people images and notions, supposedly abstract but still not out of reach. He quickly became a regular visitor to our workshop (close to the rue Mouffetard) which we already used frequently. "Is it time for a coffee?" he asked, pointing his nose through the open window. Between the nearby café and our large whiteboard, Adrien has taught me a thousand things with unfailing patience. Given the circumstances, he helped us to choose the most significant moments of the symposium, either according to interest about the topics or personalities of the speakers.

And this is how we found ourselves, on this hot Monday, 6 July 1992, at work for a week among a rare species set free in its natural environment: 1,300 mathematicians, men and women, happy to be physically reunited to discuss orally the subjects of their lives that they often share from a distance. "Like villagers celebrating the 4th of July," said our friend Douady. So, we were to discover the various aspects of this community life in this privileged moment of a symposium, like so many scenes of a film yet to be written.

The plenary lectures were the most accessible events, set in the grand auditorium Richelieu of the Sorbonne. The speakers made commendable efforts to address their talks to the largest numbers. The space was vast, the audience numerous, the attention intense. I remember two significant talks: that of David Mumford – extremely clear, obvious, human; and Vladimir Arnold – passionate and biting, featuring the work of Victor Vassiliev under the eye of the great Israel Gelfand.

A pleasant surprise was that we were allowed everywhere, thanks to our small cameras. Staying discreet, without interfering with the speakers or the audience and operating with a low light – at this price, we felt immersed and protected, being part of the thing.

The parallel conferences were clearly more challenging. Being able to understand anything was not on the agenda anymore, only the music of the words, the atmosphere and often the active participation from the audience. The audiences were a fascinating topic: all different kinds of atmospheres could be observed, from the most intense attention to complete relaxation. Many times we even found some sleepers. But it wasn't just a nap. Suddenly, we saw one of them wake up, make remarks, exchange and then go back to sleep…

But you could not sleep anywhere. At the end of Don Zagier's lecture, I met Claire, who had just filmed it, and the outgoing public: everyone had stars in their eyes, still amazed. Watching the rushes later on, I understood: Zagier had just given a performance worthy of a rock star!

The poster sessions took us by surprise: hanging around panels made with more or less art but full of meaning, mathematicians were waiting for others to discuss in a slightly cosy atmosphere. A sense of time suspended. Exotic!

Many other dimensions of the symposium emerged during the week: the aspects for younger people, with the Junior Symposium and a funny theatre play; a magnificent musical demonstration of the physicist and composer Jean-Claude Risset; an intelligent didactic exhibition to discover in the calm of the Sorbonne chapel; and the

[1] SMF: Société Mathématique de France, SMAI: Société de Mathématiques Appliquées et Industrielles.

round tables, which were often passionate and visibly difficult to manage.

A touching moment was Henri Cartan receiving an important personal message at the Germany Embassy, the great mathematician and his wife in the gardens, enveloped in a peaceful atmosphere of friendship.

Another rather hilarious moment was the City of Paris Award delivered by Mayor Jacques Chirac himself. Backstage, the over-excited politician waited for the green light then rushed to the centre of the crowd like a bull in the arena, reading with conviction a flamboyant speech in the spirit of the Third Republic, under the playful eye of one of the organisers – perhaps the malicious author of speech – that the Mayor obviously only came across on the spot.

### "Mathematics, my village": interviews

Another crucial contribution by Adrien Douady to our project was to choose and contact a certain number of mathematicians likely to accept an interview, despite their overloaded schedules. His wife Régine, a renowned specialist in the worlds of mathematical research and teaching, offered her assistance for the preparation of these interviews. The Douadys, in their hospitality, opened Adrien's office, two steps from the Sorbonne, for the recording of these interviews. This was ideal.

And then there was Adrien going fishing for us, bringing home his catch, with mathematicians delighted by the invitation from such a host. In his office, we were comfortable, quietly preparing. I aimed for a face-to-face interview, with a camera by the corner of the eye; the result is that the interviewee looks at you right in the eyes ... and is not reading a teleprompter.

In some cases, due to lack of time, we had to improvise interviews at the Sorbonne. All these interviews included Michael Berry, Jean Brette, Henri Cartan, Catherine Goldstein, Max Karoubi, Maxim Kontsevich, David Mumford, Ragni Piene, Jim Ritter, Dietmar Salamon, Victor Vassiliev, Michèle Vergne, Claude Viterbo, Jean-Christophe Yoccoz and Don Zagier. Questions dealt simply with the symposium (participation, interest, impressions and exchanges) but also their personal research practice. The answers were as varied as the personalities of our guests. In this diversity, we sought out what could be common between all these practices of mathematics. Régine Douady told us about her famous notion of change of framework: a way to reconsider problems while transposing them from one framework to another.

We especially saw temperaments of creators, far from the mechanistic picture of mathematicians. This almost artistic dimension revealed itself in the words of our interviewees, as well as in their way of expressing themselves. Michael Berry was fascinating due to the grace of his speech – magically illuminating. Ragni Piene defended the cause of children and women in mathematics with communicative conviction. Jean-Christophe Yoccoz launched into a series of drawings that usually only computers know how to do. David Mumford passionately outlined his theory of vision. Don Zagier showed a subtle precision, while stating a very personal theory on mathematicians and music…

Music is also a specialty of the Douadys: a select audience had the privilege of a private concert of the *Arpeggione Quartet*, the final note for us on this extraordinary week.

Then the spotlights were turned off and everyone returned home. A little shaken, we looked at the result of our shots. It took weeks to view, analyse, transcribe and annotate the whole considerable set of rushes of this famous week.

At that time, we began using a revolutionary video editing process: virtual editing. This technique, then new and costly, allowed us to enter into the world of digital video, which was to prove useful for other insane projects. The editing was finalised in 1993 at the studio of *École Polytechnique*.

With Adrien Douady, the adventure vividly continued that same year: "It is nice to make movies *about* mathematicians or mathematics. But when are we going to do movies *on* mathematics?" And this is how a new challenge started: *La dynamique du lapin* (The dynamics of the rabbit), a film that took more than four years of work, followed by the exhibition "*Un monde fractal*" (A fractal world), which toured the world for seven years. I finally ended up feeling at home in the holomorphic-dynamics family, all generations included.

With Jean-Pierre Bourguignon, another exciting brotherhood continued, centred on the IHÉS he directed for nearly 20 years. It was another family portrait: that of an institution of excellence dedicated to fundamental science protected from the outside world and yet in close contact with it.

Twenty-three years after the symposium, what is left of this experience?

A 26-minute documentary, a series of 15 interviews. They are archives already! Some have already received the Fields Medal and others have left us, like Adrien Douady on a nasty, mistral day.

Mathematics seems to have become more lovable, thanks to the efforts, combined or not, of the tireless actors of popularisation and their younger siblings. And we have continued to believe that the movies could contribute to their memory.

Let us hope it lasts.

Paris, 29 April 2015

*François Tisseyre was a founding member, in 1976, of Atelier EcoutezVoir. He is a film-maker as well as a director of film, documentaries, audiovisual broadcasts and exhibitions in different domains (photography, music, mathematics, astronomy and engineering). All of his work has been undertaken in the framework of EcoutezVoir, which is a place for reflection and experimentation on audiovisual and multimedia communication in different cultural and scientific domains.*

The Newsletter thanks Killian and Jean-Luc Dorier for translating the original French article.

# Institut Mittag-Leffler

## A Seat of Research Excellence Where Every Corner Reflects the Great History of Mathematics

Ari Laptev (Imperial College London, UK)

### History of the Institute

Institut Mittag-Leffler is an international centre for research and postdoctoral training in mathematical sciences. It was founded on 16 March 1916 by Professor Gösta Mittag-Leffler and his wife Signe (born Lindfors), who donated their magnificent villa with its first-class library for the purpose of creating the institute that bears their name. It is the oldest mathematics research institute in the world, which, since 1919, has operated under the auspices of the Royal Swedish Academy of Sciences and enjoys great autonomy.[1]

The mission of Institut Mittag-Leffler is to support international, high-level research in mathematics, with special attention to the development of mathematical research in Nordic countries. It also has a responsibility to serve as a contact and link between mathematicians in the Nordic countries, as well as the international research community. Major activities of the institute include research programmes, conferences, workshops and summer schools.

In 1916, the financial plans for the institute were completely viable, due to Mittag-Leffler's adequate financial resources. However, in 1922, there was a large financial crash related to the economic crises in Europe. This disaster brought Mittag-Leffler near bankruptcy and, at the time of his death in 1927, the resources of the institute did not allow him to realise his original intentions.

In 1927, the mathematicians of the Royal Swedish Academy of Sciences appointed Torsten Carleman as director of the institute, whilst allowing him to retain his professorship at Stockholm University. He lived in Mittag-Leffler's villa, maintaining the library and using it for occasional lectures. Although the publication of *Acta Mathematica* was continued, the institute was not as active as originally planned.

After Carleman's death in 1949, the academy searched for a new director. Two promising candidates were Fields Medallist Lars Ahlfors, who was at Harvard University, and Arne Beurling, who was at the University of Uppsala and later moved to the Institute for Advanced Study in Princeton. Both turned down the offer. In 1949, Fritz Carlson was appointed as the director of the institute but sadly died in 1952. Otto Frostman then served as acting director. O. Frostman lived in "Gula Villa", which was the former home of Gösta's brother Fritz.

This period was critical for the institute. The academy appointed Lars Gårding and Åke Pleijel to evaluate the situation and their suggestion was to sell Mittag-Leffler's villa. At the time, the committee members simply did not see any reason to keep the institute because it was difficult to bear the running costs of the buildings and there were no prospects of receiving funding that would enable Mittag-Leffler's dream to come true. Fortunately, not all members of the academy agreed with this suggestion and one of them was Lennart Carleson, who became Chief Editor of *Acta Mathematica* in 1956 and was elected to the academy in 1957. He refused to give up Mittag-Leffler's dream and was determined to do something about it.

By the 1960s, the endowment for the institute had still not grown sufficiently to finance mathematical activities. However, during the 1960s, when many universities around the world were expanding, Lennart Carlesson was able to sell several hundred complete sets of *Acta*. The sales greatly enhanced the institute's endowment.

In 1968, Lennart Carleson became the director of the institute and he was finally able to realise the intentions of Mittag-Leffler. With financial support from the Knut and Alice Wallenberg Foundation and insurance companies, housing for visitors was constructed and the main building was modified to provide the required office space. Grants from the Nordic countries made it possible to invite foreign mathematicians for extended visits and to support young mathematicians with fellowships. Since 1969–70, when the first scientific programme on harmonic analysis was held, the institute has been operating in essential ways as envisaged by Mittag-Leffler.

[1] I remember about two years ago that Cedric Villani gave a speech at one of the IHP conference parties, proudly saying that the Institute Henri Poincaré, built in 1928 and sponsored by the Rockfeller Foundation and Edmond de Rothschild, was the second research institute of mathematics in the world. After his speech, I teasingly asked him which institute was the first. He immediately confirmed that it was Institut Mittag-Leffler!

## Mittag-Leffler Institute today

Today, the board of the institute has 13 members. According to the testament of Mittag-Leffler, it includes four representatives from the Nordic countries – Denmark, Finland, Iceland and Norway – and nine members of the Royal Swedish Academy of Sciences. The institute has an international advisory board whose members are L. Lovasz, C. Villani and S.-T. Yau.

We now run two annual research programmes of 3.5 months during the period September–May and also one week summer conferences, workshops and school programmes over the period May–July.

The institute is an active member of ERCOM, the EMS committee of European Research Centres On Mathematics. Within ERCOM activities, the institute is a very involved member of the European postdoctoral programme (EPDI). An important part of our collaboration with the EMS are the regular female schools that the institute organises together with the EMS Committee of Women in Mathematics.

We have a number of outreach activities that are not directly connected to research in mathematics but that we believe are important for the image of the institute. Four years ago, we initiated the much appreciated Klein Days lectures for high school teachers in collaboration with the National Centre for Mathematics Education (NCM) in Gothenburg. The Klein Days are now supported financially by Brummer & Partners, which is the sixth largest hedge fund company in Europe.

Another popular event is the annual meeting of Chairmen of Mathematics Departments in Nordic Countries, when we often invite representatives of Nordic Research Councils. This forum allows us to discuss different problems in common with our countries and enables us to coordinate our various activities.

For many years the Swedish mathematical community has tried to convince the Swedish Research Council (VR) to give adequate financial support to the Mittag-Leffler Institute. Finally, after an international evaluation by VR some years ago, the evaluating committee prepared a report containing strong support for the institute. After this report in 2011, the VR decided to substantially increase the institute's funding. In comparison with VR support of six years ago, the funding from VR has now increased from 1M SEK to 10M SEK annually.

About three years ago the institute received 40M SEK for a six year period from the Wallenberg Foundation for improving the institute's infrastructure. This funding allowed us to renovate the flats built by Carleson in 1968 and has substantially improved our facilities so that we can now offer our guests excellent service.

At the moment, we have two more "building projects". One of them is the renovation of the façade of the main building which was built in 1905. The second project concerns the rebuilding of the Kuskvillan. In particular, we are planning to extend the building with a new, modern seminar room and a library for Gosta's rare collection of books. The ground floor of the Kuskvillan will be a common room where our guests will have the possibility of having informal meetings and discussions.

Next year, we are planning to celebrate the 100th anniversary of the institute. It has been agreed that this event will be attached to the 27th Nordic Congress of Mathematics, which will take place in Stockholm, 16–20 March 2016 (Wednesday to Sunday). The first day will be devoted to the institute's history. Among the speakers who have agreed to give their presentations are Arild Stubhaug, who will speak about Gösta's life, June Barrow-Green, who will give an address on *Acta Mathematica*, and Jan-Erik Björk, who will be covering "The Swedish life of Sonya Kovalevskaya".

The library of Gösta Mittag-Leffler contains not only very valuable books but also Gösta's correspondence with some of the most prominent scientists of his time. In particular, we have about 30,000 letters received by Gösta, including letters from such people as Einstein, Cantor, Weierstrass, Poincaré and Kovalevskaya, and also about 27 outgoing letters. Most of these letters are already scanned and we are planning to upload them to the internet before the institute's 100th anniversary.

One more project concerns our two journals *Acta Mathematica* and *Arkiv för Matematik*. From 1 January 2017, we are planning to have both journals available online. For *Acta*, this means that mathematicians from all over the world will be able to have free access to the journal, starting from its first volume published in 1882. The paper copies of the journals will continue to be printed and distributed to subscribers. The cost of the free online copies will be covered by a private donation that we are expecting at the end of this year.

For me personally, the post of director of the institute has been really rewarding. For many years, my predecessors were struggling to keep a high level of mathematical activity on an extremely small budget. During the last three years, the total institute's budget finally became adequate for an institute of this status and we are now able to offer our guests an appropriate service. The institute has now become a modern, dynamic place, where the beauty of high level mathematics is combined with great history and the beauty of the institute and its idyllic environment.

Every participant of a programme or conference is always met at the door by cheerful and positive members of our staff: Inger Halvarsson, Maria Weiss, Fawzi Mourou, Mikael Rågstedt and Annika Augustsson.

Here are two comments from our visitors:

*"The Institute is a national treasure that will continue to contribute to the development of mathematics in Sweden"* – Enrico Bombieri.

*"There are two ways, of essentially equal value, in which one's life can be brightened by the Royal Swedish Academy of Sciences. One is to receive a Nobel Prize. The other is to receive an invitation to Institut Mittag-Leffler"* – Jouko Väänänen.

Ari Laptev
Director
Institut Mittag-Leffler

# Gösta Mittag-Leffler (1846–1927)

Arild Stubhaug (University of Oslo, Norway)

## A man of conviction

Speaking at a congress of Scandinavian natural scientists in Stockholm in 1898, Mittag-Leffler began his address in the following manner:

> *If I were a modern man, and if I thought that the chief wisdom of life was to clearly perceive the spirit of the times, the meaning of which is the summary of the majority's opinions, and I align my views as closely as possible in accordance with this spirit, then I would begin what I wish to say to you with a respectful but also slightly deferential bow to the lyrical enthusiasm that inspired the first meetings of natural scientists [which had taken place regularly since 1839]. Then, with a shrug, I ought to hasten to mention the Scandinavianism of the day as a well-meaning but impractical idea, and instead declare my respect for contemporary nationalism, which is so much more sensible.*
> *But, you see, I am not a modern man, I am a mathematician, and I know that a point of view that lacks truth and probability cannot stand, whether or not it be either an expression of the spirit of the age, or held by the vast majority. [1]*

Mittag-Leffler was literally aglow with his chosen science; not only had he gone his own way and chosen his own paths for advancing his field of study but he was also a central figure in the milieu of the natural sciences of his day. For him, mathematics was the foremost of the sciences; in terms of pure thought, it was the one science that reigned over all the others.

Swedish mathematics acquired a prominent standing through the work of Mittag-Leffler and this provided inspiration for Swedish cultural endeavours in a whole series of fields. In terms of Nordic mathematical research, he was a dominant figure who gave rise to understanding and cooperation. Moreover, his international position can be seen from the fact that he received honorary doctorates from six universities – Bologna, Oxford, Cambridge, Oslo, Aberdeen and St. Andrews – and that he was president and vice-president of a series of international congresses of mathematics, as well as an honorary member of almost every academy of the sciences in the world.

The foundation for his great celebrity status was laid in the 1880s. He then founded *Acta Mathematica* (1882) and some of the journal's very first contributions were the epoch-defining works of Henri Poincaré and Georg Cantor. Mittag-Leffler built up *Acta Mathematica* to become the leading periodical of the world's mathematicians; he managed to get Sonja Kovalevsky (Swedish spelling) to Stockholm as a senior reader in mathematics and she later became the world's first female professor of mathematics. He also arranged a successful mathematical competition (1885–89). Mittag-Leffler called it King Oscar II's Prize competition and on the prize committee he collaborated with Karl Weierstrass and Charles Hermite. The winner of the competition was Henri Poincaré with a paper on the Three Body Problem.

As the leading professor in mathematics in Sweden, and through his teaching, Mittag-Leffler created what in terms of mathematical history is called the Stockholm School, with several of his students from mathematical analysis.

## Upbringing and education in Sweden

Gösta Mittag-Leffler was born in Stockholm on 16 March 1846; his father, Johan Olof Leffler, was a teacher and Member of Parliament for a time; his mother, Gustava Mittag, was the daughter of a Lutheran priest and dean. Gösta, who later took his surname from both his mother and father, grew up in a home that was open to the cultural currents of the day, in a city that was marked by school and educational reforms but still with great and persistent divisions. He was often ill as a child and therefore, under his own direction, he studied the material required for the obligatory examinations. His fascination for the field of mathematics came early, "first and foremost through the discovery that something might emerge so clearly and evidently that it could be considered proven, and in consequence, there could be no doubt as to its truth" [2]. He had a very capable mathematics teacher at the Stockholm Gymnasium who drilled his students in geometric exercises and gave them what amounted to private tuition in the "newfangled" infinitesimal calculus.

As a student (1865–1872) at Uppsala University, his main subject was mathematics and he took his doctorate with a treatise on applications of Cauchy's argument principle, and thereafter became a senior reader or "Docent". But, as a demonstration of the level of Swedish mathematical research at the time, he has said that during a congress of the Scandinavian natural sciences in Copenhagen in 1873, he began to have doubts about his decision to become a mathematician. He became alarmed because it seemed to him that Danish mathematicians of his age group were much better schooled than he was and, in his own words, he felt himself "completely annihilated" by not being able to follow their mathematical reasoning. The only consolation was the fact that he had received a large stipend to enable him to study abroad and, during the coming three years, he would study mathematics in Paris, Göttingen and Berlin.

## The meetings in Paris, Göttingen and Berlin

Beginning in the Autumn of 1873, Mittag-Leffler spent half a year in Paris. Here he met all the leading mathematicians – from the young Gaston Darboux (born in

1842) to the elderly Joseph Liouville (born in 1809) and Michel Chasles (born in 1793). The most important of his contacts, however, was Charles Hermite. Mittag-Leffler felt it was difficult to follow Hermite's lectures on elliptic functions, due not only to the content, the language and the terminology but also because Hermite, at the time, had such pain in one leg that he was unable to stand at the blackboard and consequently sat at the podium and read aloud from his manuscript. For his part, however, Hermite developed an interest in the young Swede and the two of them conversed outside the auditorium about a whole series of issues. Hermite's strong Catholic convictions acted upon Mittag-Leffler's own preoccupations about the question of faith, something that was engrossing so many during this particular period of history. In his diaries and letters he gave many descriptions of Hermite, whom he acknowledged as a master whose level he would never reach in terms of his own scientific work. Hermite considered that, for the moment, German mathematics was superior to that of the French and he spoke with the greatest admiration about Bernhard Riemann, Karl Weierstrass and other German mathematicians, and recommended that Mittag-Leffler make a longer sojourn in Germany. Hermite regretted that he himself was unable to journey there due to the antagonism that still existed between the two countries after the French-German War of 1870–71.

From April until August 1874, Mittag-Leffler stayed in Göttingen to attend the lectures of Ernst Schering on "Abelian functions after Riemann". Mittag-Leffler did not consider that Schering's greatness was on a par with that of Hermite but admitted nonetheless that he learnt a great deal from Schering that he could not have learnt from Hermite. He also met Lazarus Fuchs in Göttingen; Mittag-Leffler was struck to such a degree by Fuchs' superior mathematical abilities that he crossed out the word "Docent" from his own visiting card, feeling he did not deserve such a title.

Mittag-Leffler went to Berlin in the Autumn of 1874 and, after only a short time in the city, he had struck up personal relationships with both Karl Weierstrass and Leopold Kronecker, and the relationship to Weierstrass became particularly crucial. Weierstrass was obliging and kind in every way and he must rapidly have recognised a gifted pupil in the 28-year-old Mittag-Leffler – in any case Weierstrass presented his lectures in such a way that they exhibited considerable thought for Mittag-Leffler. Weierstrass' methods and his rigorous analysis also became the star that Mittag-Leffler would follow in his own research.

## Professor in Helsinki

While he was still in Berlin, Mittag-Leffler applied for a vacancy as a professor of mathematics in Helsinki and, with letters of recommendation from Hermite, Schering, Kronecker and Weierstrass, it was impossible for the Finnish authorities to bypass his application, even though the position in Helsinki had, as a requirement, the comprehension of written Finnish, something Mittag-Leffler had not mastered.

In St. Petersburg in February 1876, on his journey from Berlin to Helsinki and with references from Weierstrass, Mittag-Leffler for the first time met Sonja Kovalevsky. He became "impressed by both her feminine goodness and superior intelligence" [3]. He wrote back home to his mother that the visit to her home was "one of the most remarkable in my life".

During his four and a half years in Helsinki, Mittag-Leffler was successful in a number of areas: he lectured in basic analysis and elliptic functions and succeeded in developing a series of students to defend their dissertations; in these ways, he provided impetus for the advance of mathematical research in Finland, Hjalmar Mellin's work being the best example. In addition, before Mittag-Leffler (35 years of age) returned to Stockholm in 1881, he became engaged to Signe af Lindfors (20 years of age), the daughter of a prosperous Finnish general and businessman. By means of this marriage, Mittag-Leffler gained access to the capital, which would provide the basis for the great fortune he would later build up through numerous investments and the management of extensive business ventures.

## Professor in Stockholm

Back in Sweden (1881), Mittag-Leffler was the first professor at the newly-founded Stockholm College [Stockholms Högskola]. In terms of its point of departure, this was a new kind of scientific institution for Sweden and was planned as an alternative in the capital city to the country's two well-established universities at Uppsala and Lund. The primary focus of Stockholm College was research and, in the beginning, emphasis was placed on the natural sciences as a stable foundation for other disciplines. But right from the inception of the college, a discussion ensued about the degree to which the institution should educate persons who would be able to step quickly into public service and government posts, that is to say, about whether or not the college should hold the public service examinations. Mittag-Leffler energetically fought for the position that the college should be an institution where the best people in their chosen fields would hold free public lectures and where there ought never to be talk of formal examinations and transcripts of marks. He wanted to create an academy, an institution that followed the pattern of that which was most esteemed abroad: the Collège de France in Paris and the Royal Institution in London. However, public service examinations were introduced to Stockholm College in 1904 and the institution developed into the University of Stockholm.

Among other things, it was in light of such pedagogical and research-related questions that Mittag-Leffler declared, as the year 1900 rolled into place, that he was not "a modern man". All the same, he had become a frontline figure in the international mathematical milieu and would remain such for the remainder of his life.

The success of *Acta Mathematica* lay in the combination of having a secure sense of determining what was new and what was on the cutting edge of mathematical research, and an ability to bring forward moral and economic support from a whole range of different sources.

The idea of having a periodical had first been raised by the Norwegian Sophus Lie and had then been conceived as a forum for Nordic mathematicians. Right from the beginning, an editorial board of Nordic mathematicians had stood behind the publication. Also, right from the start it had been Mittag-Leffler who managed and conducted *Acta Mathematica* and who made it into a scientific success where most of the world's leading mathematicians would publish their works. It was also in *Acta Mathematica* that Mittag-Leffler himself published, in 1884, the work that gave him a place among the internationally renowned mathematicians, that is, a general form of what is known as the Mittag-Leffler theorem.

The fact that Mittag-Leffler had managed to get Sonja Kovalevsky to Stockholm in the Autumn of 1883 was a victory for all involved. She participated in the work of the editorial board of *Acta Mathematica* and, in addition, made the journal into a forum in which Russian mathematicians participated. With her charm and intelligence, she also rapidly became a central figure in Stockholm's social life. Together with Mittag-Leffler's sister, the well-known writer Anne Charlotte Leffler, she published two plays about the position of women in a male-dominated society and, in this manner, broke through one of the barriers between high academic ideas and those with which the rest of the population were concerned and in which they could participate.

The international competition connected to King Oscar II of Norway and Sweden was directed by *Acta Mathematica* (1885–89) and, through this competition, Mittag-Leffler succeeded in strengthening not only the periodical but also the standing of mathematics and his own reputation. King Oscar thus appeared as the friend of both Mittag-Leffler and the journal. Mittag-Leffler sat on the jury together with Hermite and Weierstrass and thus got a little of their celebrity status, and the winner of the competition was the brilliant Poincaré.

### Financial support, teaching and gatherings

One of the important tasks of a professor at Stockholm College was to find financial support for the college, which was largely based on private funding. Mittag-Leffler was extremely good at finding such support and it was in this connection that he also had a certain degree of contact with Alfred Nobel. When it became clear from Nobel's last will and testament (1896) that there were neither funds bequeathed to the College nor a Nobel Prize for Mathematics, rumours spread that a possible conflict between Mittag-Leffler and Nobel lay behind this turn of events. However, everything indicates that such was definitely not the case but, rather, the theoretical aspects of the discipline of mathematics had dampened the enthusiasm of Nobel, who was, above all, a man of practice.

By means of his form of teaching while a professor in Stockholm, Mittag-Leffler achieved a standard of mathematics for the college that was on a level with the best abroad. Among those he attracted around him – often referred to as the Stockholm School – were Edvard Phragmén, Ivar Bendixson, Helge von Koch, Ivar Fredholm and later also Torsten Carleman.

At this time, Mittag-Leffler also built up a comprehensive mathematical library – 40,000 volumes and a significant number of brochures and individual treatises, as well as original manuscripts – at his large villa at Djursholm just north of Stockholm. It became a matter of course for men and women of science to visit Djursholm and Mittag-Leffler when they came to Sweden. In fact, he was the man who started the tradition to arrange a splendid dinner for the Nobel Prize laureates on every 11 December (and later on 12 December).

Mittag-Leffler's position and reputation in the world of international science increased with his many initiatives and with his performance at various congresses and gatherings.

One of those whose life changed after paying a visit to Mittag-Leffler at Djursholm was the Hungarian born Marcel Riesz, who, in the Summer of 1908, went to Sweden as a tourist. Three years later, at the instigation of Mittag-Leffler, Riesz was appointed as a senior reader at Stockholm College and he became a permanent resident of the country.

One of the very last mathematicians who came to visit Mittag-Leffler at Djursholm was André Weil. This occurred in March 1927, four months before the death of Mittag-Leffler. Weil later wrote about the meeting (*Acta Mathematica* 1982). The young Weil had mainly come to assist in the work of preparing Mittag-Leffler's mathematical draft on polynomial series expansions for publication. In the course of things, the plans for such a monograph slid away with the sands of time but Weil took great delight in the beauties of nature at Djursholm, meeting Riesz, Gustav Cassel and other men of science at the dinner table in the villa. Above all, he prized the late night-time hours he spent alone in Mittag-Leffler's large library, where he could simply pick up and read any of Mittag-Leffler's content-rich correspondence with all the great mathematicians of the last half-century. The 81-year-old Mittag-Leffler was the perfect host "and he knew it," wrote Weil. And, as for the appearance of the old man, Weil wrote: "He looked like a bird – a bird of prey of course, such as one could see in the Skansen [open-air museum and zoo] in Stockholm; frail, but still tough, wiry."

### The institute

To mark his 70th birthday in 1916, Mittag-Leffler and his wife established a foundation, the aim of which was to advance the standing of pure mathematics. Following the deaths of the Mittag-Lefflers, their properties were bequeathed to this foundation. Both the leadership and activities of the foundation were connected to the Royal Swedish Academy of Sciences. Down through the years, the publication of *Acta Mathematica* has been led from Institut Mittag-Leffler at Djursholm; however, Mittag-Leffler's dream that the family bequest should also provide the point of departure for an "institute for visitors" was not realised until 1969, when it was instituted by Professor Lennart Carleson. This was a time when it was common to build research institutions around a permanent staff of mathematicians and thus, even in 1969, the notion of an institute for visiting scholars was not something that was typical of the historical moment or that arose with "the spirit of the times". Today, the institute is a significant part of the worldwide mathematical milieu.

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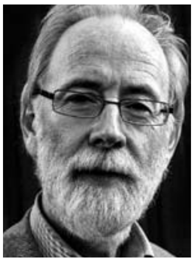
Stubhaug, Arild: Gösta Mittag-Leffler. A Man of Conviction, Translated from the Norwegian and Swedish edition by Tiina Nunnally, Springer-Verlag 2010.



*Arild Stubhaug is an acclaimed poet and biographer. He holds university degrees in mathematics, literature and the history of religion. He has also been awarded an honorary doctorate by the University of Oslo. His biography* Niels Henrik Abel and His Times *(published in English in 2000 and later in German, French and Japanese by Springer-Verlag, and Chinese) was followed by* The Mathematician Sophus Lie *(published in English in 2002 and later in German and French by Springer-Verlag, and Japanese) and the biography of Gösta Mittag-Leffler (published in Norwegian and Swedish in 2007 and in English by Springer-Verlag in 2010). Stubhaug has also published major works on Norwegian writers and statesmen.*

# Knotted Vortex Lines and Vortex Tubes in Stationary Fluid Flows

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*In this paper, we review recent research on certain geometric aspects of the vortex lines of stationary ideal fluids. We mainly focus on the study of knotted and linked vortex lines and vortex tubes, which is a topic that can be traced back to Lord Kelvin and was popularised by the works of Arnold and Moffatt on topological hydrodynamics in the 1960s. In this context, we provide a leisurely introduction to some recent results concerning the existence of stationary solutions of the Euler equations in Euclidean space with a prescribed set of vortex lines and vortex tubes of arbitrarily complicated topology.*

## 1 Introduction

The dynamics of an inviscid incompressible fluid flow in $\mathbb{R}^3$ is modelled by the hydrodynamical Euler equations

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla P\,, \qquad \operatorname{div} u = 0\,,$$

where $u(x, t)$ is the velocity field of the fluid, which is a time-dependent vector field, and $P(x, t)$ is the pressure function, which is defined by these equations up to a constant. This system of partial differential equations was first published by Leonhard Euler in 1757, one year after the birth of the great composer Wolfgang Amadeus Mozart, and still stands as a major challenge for engineers, physicists and mathematicians.

The motion of the particles in the fluid is described by the integral curves of the velocity field, that is, by the solutions of the non-autonomous ODE

$$\dot{x}(t) = u(x(t), t)$$

for some initial condition $x(t_0) = x_0$; these are usually called *particle paths*. The integral curves of $u(x, t)$ at fixed time $t$ are called *stream lines* and thus the stream line pattern changes with time if the flow is unsteady.

Another time-dependent vector field that plays a crucial role in fluid mechanics is the *vorticity*, defined by

$$\omega := \operatorname{curl} u\,.$$

This quantity is related to the rotation of the fluid and is a measure of the entanglement of the stream lines. The integral curves of the vorticity $\omega(x, t)$ at fixed time $t$, that is to say, the solutions of the autonomous ODE

$$\dot{x}(\tau) = \omega(x(\tau), t)$$

for some initial condition $x(0) = x_0$, are the *vortex lines* of the fluid at time $t$. A domain in $\mathbb{R}^3$ that is the union of vortex lines and whose boundary is a smoothly embedded torus is called a (closed) *vortex tube*. Obviously, the boundary of a vortex tube is an invariant torus of the vorticity.

In this short note, we are concerned with *stationary* solutions of the Euler equations, which describe an equilibrium configuration of the fluid. In this case, the velocity field $u$ does not depend on time and the Euler equations can then be written as

$$u \times \omega = \nabla B\,, \qquad \operatorname{div} u = 0\,,$$

where $B := P + \frac{1}{2}|u|^2$ is the Bernoulli function. This is a fully nonlinear system of partial differential equations so, a priori, it is not easy to see for which choices of the function $B$ there exist any solutions and which properties they can exhibit. It is obvious that, for stationary flows, the particle paths coincide with the stream lines.

Our goal in this article is to introduce some results in fluid mechanics whose common denominator is that the main objects of interest are the stream and vortex lines of ideal fluid flows. In particular, we shall review the recent construction of stationary solutions of the Euler equations in $\mathbb{R}^3$ describing topologically nontrivial fluid structures [4, 5]. Mathematically, these problems are extremely appealing because they give rise to remarkable connections between different areas of mathematics, such as partial differential equations, dynamical systems and differential geometry. From a physical point of view, these questions are often considered in the Lagrangian approach to turbulence and in the study of hydrodynamical instability.

In this context, a major problem that has attracted considerable attention is the existence of knotted and linked vortex lines and tubes[1] (see Figure 1). The interest in this question dates back to Lord Kelvin [20], who developed an atomic theory in which atoms were understood as stable, knotted, thin vortex tubes in the ether, an ideal fluid modelled by the Euler equations. Kelvin's theory was inspired by the transport of vorticity discovered by Helmholtz [12], which in particular implies that the vortex tubes are frozen within the fluid flow and hence their topological structure does not change with time. Vortex tubes were called water twists by Maxwell and were experimentally constructed by Tait by shooting smoke rings with a cannon of his own design. The stability required by Kelvin's atomic theory led him to conjecture in 1875 that thin vortex tubes of arbitrarily complicated topology can arise in stationary solutions of the Euler equations [21].

The mathematical elegance of Kelvin's theory, in which each knot type corresponds to a chemical element, captivated the scientific community for two decades. However, by the end of the 19th century, with the discovery of the electron and the experimental proof that the ether does not exist, it was clear that this theory was erroneous. Nevertheless, Kelvin's vortex tube hypothesis was an important boon for the de-

1 We recall that a knot is a smooth closed curve in $\mathbb{R}^3$ without self intersections, and a link is a disjoint union of knots.

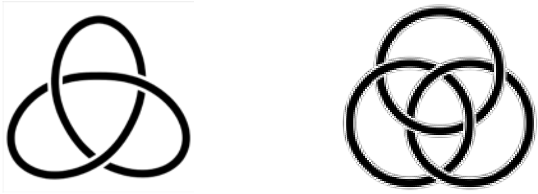

**Figure 1. The problem involves showing that there are stationary solutions of the Euler equations realising links, e.g. the trefoil knot and the Borromean rings depicted above, as vortex lines or vortex tubes.**

velopment of knot theory and fluid mechanics. In particular, Kelvin's conjecture has been a major open problem since then and has had a deep influence in mathematics.

In modern times, the study of knotted vortex tubes has been a central topic in so-called topological hydrodynamics [3], a young area that was considerably developed after the foundational works of Arnold [1, 2] and Moffatt [14]. Arnold, in his celebrated structure theorem, classified the topological structure of the stationary solutions when the Bernoulli function is not identically constant and he conjectured that a particular class of stationary solutions, called Beltrami flows, should exhibit stream lines of arbitrarily complicated topology.[2] Moffatt introduced the concept of helicity to study the entangledness and knottedness of the fluid and gave an heuristic argument supporting the existence of stationary states with stream lines of any knot type [15], leaving completely open the case of vortex lines and tubes.[3] A stronger conjecture was stated in the 1990s by R. F. Williams [22], who asked about the existence of a fluid flow with stream lines tracing out all knots. The main difficulty of these problems is that they lie somewhere between partial differential equations and dynamical systems, which explains why purely topological or analytical techniques have not been very successful in these kinds of problems.

It should be emphasised that the interest of Kelvin's conjecture is not merely academic; in fact, spectacular recent experiments by Kleckner and Irvine at the University of Chicago [13] have physically supported the validity of Kelvin's conjecture through the experimental realisation of knotted and linked vortex tubes in actual fluids using cleverly designed hydrofoils (see Figure 2). Furthermore, the existence of topologically complicated stream and vortex lines is crucial in the study of Lagrangian theory to turbulence and in magnetohydrodynamics.

This article is organised as follows. In Section 2, we explain how Helmholtz's transport of vorticity gives rise to knotted structures in the time-dependent Euler equations (for short times) and review Moffatt's heuristic argument suggesting the existence of stream lines of any knot type in stationary Euler flows. In Section 3, we state Arnold's structure theorem and introduce Beltrami fields and Arnold's conjecture in this context; we also review the geometric approach of Etnyre and Ghrist to address the existence of knotted vortex lines and tubes in the stationary Euler equations. In Sections 4 and 5, we state the realisation theorems on vortex lines [4] and vortex tubes [5], proved recently by the authors of this note, which establish Kelvin's conjecture and related conjectures; we also include readable detailed sketches of the proofs of these results.

2 In Arnold's words [1]: "Il est probable que les écoulements tels que curl $v = \lambda v$, $\lambda =$ cte, ont des lignes de courant à la topologie compliquée."

3 In Moffatt's words [16]: "there may exist steady knotted vortex tubes configurations, but no technique has as yet been found to prove the existence of such configurations."

**Figure 2. A knotted vortex tube of water obtained in the Irvine Lab at the University of Chicago (Figure courtesy of William Irvine)**

## 2 Helmholtz's transport of vorticity and Moffatt's magnetic relaxation argument

In 1858, Helmholtz [12] discovered that the vorticity is transported by ideal fluid flows, so that for different times $t_0$ and $t_1 > t_0$ the phase portraits of the autonomous vector fields $\omega(\cdot, t_0)$ and $\omega(\cdot, t_1)$ are topologically equivalent. This turned out to be a fundamental mechanism in fluid mechanics, which placed vorticity in a leading role in analysing the Euler equations.

Using the transport of vorticity, it is easy to construct time-dependent solutions of the Euler equations with vortex lines of complex topology. The basic idea is the following. Suppose that $u(x, t)$ is a time-dependent solution of the Euler equations. Then its vorticity satisfies the transport equation

$$\frac{\partial \omega}{\partial t} = [\omega, u]\,,$$

with $[\cdot, \cdot]$ the commutator of vector fields. Therefore, the vorticity at time $t$ can be expressed in terms of the vorticity $\omega_0(x)$ at time $t_0$ as

$$\omega(x, t) = (\phi_{t,t_0})_* \omega_0(x)\,,$$

where $(\phi_{t,t_0})_*$ denotes the push-forward of the non-autonomous flow of the velocity field between the times $t_0$ and $t$.

From this expression for the vorticity, it results that the vortex lines at time $t$ are diffeomorphic to those at time $t_0$. Accordingly, one can attempt to construct the initial vorticity $\omega_0$ with a prescribed set of vortex lines and tubes. This is a problem in dynamical systems where the only constraint on the vector field $\omega_0$ is that $\operatorname{div} \omega_0 = 0$, which in $\mathbb{R}^3$ implies that $\omega_0$ is exact, i.e. there exists a vector field $u_0$ such that $\operatorname{curl} u_0 = \omega_0$. The initial vorticity $\omega_0$ can be constructed as follows. let $L$ be the finite link in $\mathbb{R}^3$ that we want to realise as a set of vortex lines. As it has trivial normal bundle, a tubular neighbourhood $N_k$ of each component $L_k$ of $L$ is diffeomorphic to $\mathbb{S}^1 \times \mathbb{R}^2$. We take each neighbourhood $N_k$ so that the compact sets $\overline{N}_k$ are pairwise disjoint. Let us parameterise $N_k$ with local coordinates $\alpha \in \mathbb{S}^1 := \mathbb{R}/(2\pi\mathbb{Z})$ and $z = (z_1, z_2) \in \mathbb{R}^2$. In these coordinates, the Euclidean volume reads as

$$dx = f(\alpha, z)\, d\alpha\, dz_1\, dz_2$$

for some smooth positive function $f$. Using this parametrisation, we can define a vector field $v_k$ in each domain $N_k$ as

$$v_k := \frac{F(\rho^2)}{f}\Big(\partial_\alpha + G(\rho^2)\partial_\varphi\Big),$$

where we have used the polar coordinates $(\rho, \varphi)$ defined as $z_1 = \rho\cos\varphi$ and $z_2 = \rho\sin\varphi$, and $F$ and $G$ are smooth functions such that $F(0) = 1$ and $F = 0$ for $\rho \geqslant 1$. By construction, $v_k$ is a smooth vector field compactly supported in $N_k$ and it is straightforward to check that it is volume preserving for any choice of the functions $F$ and $G$. Moreover, $L_k$ is an integral curve of $v_k$ and, for any $\rho_0 > 0$, the domain $\{\rho < \rho_0\}$, expressed in the coordinates $(\alpha, \rho, \varphi)$, is an invariant tube of $v_k$.

Using the fields $v_k$, we can prescribe the initial vorticity as the compactly supported divergence-free vector field

$$\omega_0(x) := \begin{cases} v_k(x) & \text{if } x \in N_k\,, \\ 0 & \text{if } x \in \mathbb{R}^3 \setminus \bigcup N_k\,. \end{cases}$$

Through the Biot–Savart operator, this initial vorticity corresponds to the initial velocity

$$u_0(x) := \frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{(x-y)\times\omega_0(y)}{|x-y|^3}\, dy\,,$$

which falls off at infinity as $|u_0(x)| < C/|x|^2$.

By construction, the link $L$ is a union of vortex lines of the initial vorticity $\omega_0$. This field is integrable and nondegenerate in the sense that each tubular neighbourhood $N_k$ is filled by vortex tubes and the vortex lines are either periodic or quasiperiodic depending on whether the value of the function $G(\rho^2)$ on the invariant torus $\{\rho = \rho_0\}$ is rational or not. Therefore, the classical local (in time) existence theorem implies that there is a smooth solution of the Euler equations with initial datum $u_0$ which is defined for $t \in [0, T)$ (it is not known whether the maximal time of existence $T > 0$ is actually infinite). The solution $u$ has a set of vortex lines diffeomorphic to the link $L$ for all $t \in [0, T)$ and vortex tubes enclosing these vortex lines, as we wanted to show.

The importance of this simple argument is that it suggests the existence of stationary solutions of the Euler equations with knotted and linked vortex lines and tubes. Heuristically, one can argue as follows. If there is a smooth global solution $u(x, t)$ that evolves, for large times, into an equilibrium state, characterised by a stationary solution to Euler $u_\infty(x)$, it is conceivable, although certainly not at all obvious, that this stationary solution should also have a set of closed vortex lines diffeomorphic to $L$. Of course, these hypotheses prevent us from promoting this heuristic argument to a rigorous result.

In this direction, Moffatt [15] introduced a particularly influential scenario which was inspired by ideas of the physicists Zakharov and Zeldovitch. Moffatt's heuristic argument, based on the magnetic relaxation phenomenon, supports the existence of knotted stream lines, although making his approach precise seems to be way out of reach despite the recent rigorous results in this direction (see, for example, [10]). To explain this argument, let us consider the following magnetohydrodynamic system with viscosity $\mu$:

$$\frac{\partial u}{\partial t} + (u\cdot\nabla)u = -\nabla P + \mu\,\Delta u + H \times \operatorname{curl} H\,,$$
$$\frac{\partial H}{\partial t} = [H, u]\,, \quad \operatorname{div} u = \operatorname{div} H = 0\,.$$

In this equation, $u(x, t)$ represents the velocity field of a plasma, $H(x, t)$ is the associated magnetic field and $P(x, t)$ is the pressure of the plasma.

Just as in the case of the previous argument based on vorticity transport, the idea is to take initial conditions $(H_0, u_0)$ such that the vector field $H_0$ has a prescribed set of invariant closed lines, possibly knotted and linked. The construction of $H_0$, whose only constraint is that it is volume preserving, can be done exactly as in the case of vortex lines. Then one can argue that, if there is a global solution with this choice of initial conditions, it is reasonable that the viscous term $\mu\,\Delta u$ forces the velocity to become negligible as $t \to \infty$. If the magnetic field also has some definite limit $H_\infty(x)$ as $t \to \infty$ then this limit field satisfies

$$H_\infty \times \operatorname{curl} H_\infty = \nabla P_\infty\,, \qquad \operatorname{div} H_\infty = 0\,.$$

Formally, these equations are the same as the stationary Euler equations, so $H_\infty$ is then a stationary solution of the Euler equations. Since the magnetic field is transported by the flow of the velocity field, the same argument as above suggests that one can hope that $H_\infty$ should have a set of integral curves (i.e. stream lines) diffeomorphic to any prescribed link. The problems that appear when one tries to make this argument rigorous are similar to those appearing in the case of vortex lines, e.g. it relies on the global existence of solutions of the aforementioned MHD system, which is currently not known.

## 3 Arnold's structure theorem, Beltrami fields and the contact geometry approach

In spite of the fact that it is very challenging to make rigorous the ideas introduced in Section 2, these arguments are the main theoretical basis for the firm belief in the validity of Kelvin's conjecture and related conjectures among the physics community.

A landmark in this direction is Arnold's structure theorem [1, 2], which asserts that, under mild technical assumptions, the stream and vortex lines of a stationary solution of the Euler equations, whose velocity field is not everywhere collinear with its vorticity, are nicely stacked in a rigid structure akin to those which appear in the study of integrable Hamiltonian systems with two degrees of freedom:

**Theorem 1** (Arnold's structure theorem)**.** Let $u$ be a solution of the stationary Euler equations in a bounded domain $\Omega \subset \mathbb{R}^3$ with analytic boundary. Suppose that $u$ is tangent to the boundary and analytic in the closure of the domain. If $u$ and its vorticity $\omega$ are not everywhere collinear then there is an analytic set $C$, of codimension at least 1, so that $\Omega \backslash C$ consists of a finite number of subdomains in which the dynamics of $u$ and $\omega$ are of one of the following two types:

- The subdomain is trivially fibered by tori invariant under $u$ and $\omega$. On each torus, the flows of $u$ and $\omega$ are conjugate to linear flows (rational or irrational).
- The subdomain is trivially fibered by cylinders invariant under $u$ whose boundaries sit on $\partial\Omega$. All the stream lines of $u$ on each cylinder are periodic.

The proof of Arnold's structure theorem is based on two simple observations: the Bernoulli function $B$ is a nontrivial first integral of both $u$ and $\omega$ and, on each regular level set of $B$, the fields $u$ and $\omega$ are linearly independent and commute. For our purposes, the main consequence of Arnold's theorem is that when $u$ and $\omega$ are not collinear, there is not much freedom in choosing how the vortex lines and vortex tubes can sit in space, so it should be difficult to construct topologically complicated vortex structures. This rough idea was confirmed in [7] by showing that, under appropriate hypotheses, the rigid structure predicted by Arnold indeed leads to obstructions on admissible knot and link types for stream and vortex lines.

In fact, with suitable assumptions, it is not difficult to extend Arnold's theorem to solutions defined on the whole $\mathbb{R}^3$, so the hypothesis that $u$ is defined on a bounded domain $\Omega$ is not essential. Actually, Arnold himself emphasised that the key hypothesis is that the velocity and the vorticity should not be everywhere collinear and predicted that when this condition is not satisfied, i.e. when the velocity and vorticity are everywhere parallel, then one should be able to construct stationary solutions of the Euler equations with stream and vortex lines of arbitrary topological complexity.

Therefore, if one tries to prove Kelvin's conjecture, or to construct stationary solutions with stream and vortex lines of any link type, it is natural to consider solutions of the form

$$\operatorname{curl} u = f\, u\,, \qquad \operatorname{div} u = 0\,,$$

with $f$ a smooth function on $\mathbb{R}^3$. Taking the divergence in this equation, we infer that $\nabla f \cdot u = 0$, i.e. that $f$ is a first integral of the velocity field. As a consequence of this, the trajectories of $u$ must lie on the level sets of the function $f$. The solutions of this equation are very difficult to handle. In fact, it can be shown [6] that there are no nontrivial solutions for an open and dense set of factors $f$ in the $C^k$ topology, $k \geqslant 7$. In particular, there are no nontrivial solutions whenever $f$ has a regular level set diffeomorphic to the sphere.

Accordingly, in order to keep things simple, we are naturally led to consider a constant proportionality factor $f$ to construct stationary solutions with complex vortex patterns. Then, we will focus our attention on *Beltrami fields*, which satisfy the equation

$$\operatorname{curl} u = \lambda u$$

for some nonzero constant $\lambda$. This equation immediately implies that $\operatorname{div} u = 0$. Notice that Beltrami fields satisfy the equation $\Delta u = -\lambda^2 u$ and hence, by standard elliptic regularity, they are real analytic. However, they cannot be in $L^2(\mathbb{R}^3)$ so they do not have finite energy. Actually, it is an open question whether the Euler equations in $\mathbb{R}^3$ admit any (nonzero) stationary solutions with finite energy. Obviously the stream lines of a Beltrami field are the same as its vortex lines, so henceforth we will only refer to the latter.

After establishing his structure theorem, Arnold conjectured that, contrary to what happens in the non-collinear case, Beltrami fields could present vortex lines of arbitrary topological complexity, which is fully consistent with Kelvin's conjecture. Indeed, there is abundant numerical evidence and some analytical results that suggest that the dynamics of a Beltrami field can be extremely complex. The most thoroughly studied examples are the ABC fields, introduced by Arnold in [1]:

$$u(x) = (A \sin x_3 + C \cos x_2,\ B \sin x_1 + A \cos x_3,\ C \sin x_2 + B \cos x_1)\,.$$

Here $A, B, C$ are real parameters. It is remarkable that all our intuition about Beltrami fields comes from the analysis of a few exact solutions, which basically consist of fields with Euclidean symmetries and the ABC family.

From an experimental viewpoint, it was observed in actual fluid flows [18] that in turbulent regions of low dissipation (and hence governed by the Euler equations) the velocity and vorticity vectors have a tendency to align, which is precisely the Beltrami condition. This is an additional support in order to consider Beltrami fields as the right solutions if one wants to construct topologically complicated vortex structures. As a matter of fact, these fields also play an important role in magnetohydrodynamics, where they are known as force-free magnetic fields. These force-free solutions model the dynamics of plasmas in stellar atmospheres, where complicated magnetic tubes, which are the analogues of vortex tubes, have been observed.

An interesting approach to the problem on the existence of knotted and linked vortex lines in stationary Euler flows is due to Etnyre and Ghrist. It hinges on the connection of Beltrami fields with contact geometry [8]. The main observation is the following. Let $u$ be a Beltrami field and $\alpha$ its dual 1-form, so that the Beltrami equation can be written using the Hodge $*$-operator as

$$*d\alpha = \lambda\alpha\,.$$

Therefore, if the Beltrami field does not vanish anywhere, we have that

$$\alpha \wedge d\alpha = \lambda |u|^2\, dx_1 \wedge dx_2 \wedge dx_3$$

does not vanish either, so that by definition $\alpha$ defines a contact 1-form. Conversely, if $\alpha$ is a contact 1-form in $\mathbb{R}^3$, there is a smooth Riemannian metric $g$ adapted to the form $\alpha$ so that this 1-form satisfies the Beltrami equation above with the Hodge $*$-operator corresponding to the metric $g$. The vector field dual to the 1-form $\alpha$ is a Beltrami field with respect to the adapted metric $g$ and is called a Reeb field in contact geometry.

The reason why this observation is useful is that the machinery of contact geometry is very well suited for the construction of contact forms whose associated Reeb fields have a prescribed invariant set, e.g. a set of closed integral curves or invariant tori. Therefore, one finds that there is a metric in $\mathbb{R}^3$ that is, in general, neither flat nor complete, such that the Euler equations in this metric admit a stationary solution of Beltrami type, with a set of vortex lines and vortex tubes of any knot and link type. The geometric properties of a metric adapted to a contact 1-form are very rigid [9], so this strategy cannot work when we consider the Euler equations for a fixed (e.g. Euclidean) metric.

## 4 A realisation theorem for knotted vortex lines

In this section we shall discuss a realisation theorem showing the existence of Beltrami fields with a set of closed vortex lines diffeomorphic to any given link [4]:

**Theorem 2.** Let $L \subset \mathbb{R}^3$ be a finite link and let $\lambda$ be any nonzero real number. Then one can deform the link $L$ by a diffeomorphism $\Phi$ of $\mathbb{R}^3$, arbitrarily close to the identity in any $C^m$ norm, such that $\Phi(L)$ is a set of vortex lines of a Beltrami field $u$, which satisfies the equation $\operatorname{curl} u = \lambda u$ in $\mathbb{R}^3$. Moreover, $u$ falls off at infinity as $|D^j u(x)| < C_j/|x|$.

We have only considered the case of finite links but the case of locally finite links can be tackled similarly, at the expense of losing the decay condition of the velocity field. In particular, taking into account the fact that the knot types modulo diffeomorphism are countable, it follows that there exists a stationary solution of the Euler equations whose stream lines realise all knots at the same time, thus yielding a positive answer to a question of Williams [22].

The closed vortex lines in the set $\Phi(L)$ are hyperbolic, i.e. their associated monodromy matrices do not have any nontrivial eigenvalues of modulus 1. Since $\operatorname{div} u = 0$, this immediately implies that these vortex lines are unstable. Notice, however, that the theorem does not guarantee that $\Phi(L)$ contains all closed vortex lines of the Beltrami field.

The $1/|x|$ decay we have is optimal within the class of Beltrami solutions, not necessarily with constant proportionality factor [17], so our solutions belong to the space $L^p(\mathbb{R}^3)$ for all $p > 3$. Notice that the $1/|x|$ decay was not proved in [4] (indeed, in this paper the Beltrami field was not shown to satisfy any conditions at infinity) but follows from the more refined global approximation theorem that we proved in [5].

We shall next sketch the proof of Theorem 2. The heart of the problem is that one needs to extract topological information from a PDE. Our basic philosophy is to use the methods of differential topology and dynamical systems to control auxiliary constructions and those of PDEs to realise these auxiliary constructions in the framework of solutions of the Euler equations. For concreteness, to explain the general idea of the proof we will concentrate on constructing a solution for which we are prescribing just one vortex line $L$, which is a (possibly knotted) curve in $\mathbb{R}^3$.

### Step 1: a geometric construction

It is well known that, perturbing the knot a little through a small diffeomorphism, we can assume that $L$ is analytic. Since the normal bundle of a knot is trivial, we can take an analytic ribbon $\Sigma$ around $L$. More precisely, there is an analytic embedding $h$ of the cylinder $\mathbb{S}^1 \times (-\delta, \delta)$ into $\mathbb{R}^3$ whose image is $\Sigma$ and such that $h(\mathbb{S}^1 \times \{0\}) = L$.

In a small tubular neighbourhood $N$ of the knot $L$ we can take an analytic coordinate system

$$(\theta, z, \rho) : N \to \mathbb{S}^1 \times (-\delta, \delta) \times (-\delta, \delta)$$

adapted to the ribbon $\Sigma$. Basically, $\theta$ and $z$ are suitable extensions of the angular variable on the knot and of the signed distance to $L$ as measured along the ribbon $\Sigma$, while $\rho$ is the signed distance to $\Sigma$.

The reason why this coordinate system is useful is that it allows us to define a vector field $w$ in the neighbourhood $N$ that is key in the proof: simply, $w$ is the field dual to the closed 1-form

$$d\theta - z\,dz\,.$$

From this expression and the definition of the coordinates it stems that $w$ is an analytic vector field tangent to the ribbon $\Sigma$ and that $L$ is a stable hyperbolic closed integral curve of the pullback of $w$ to $\Sigma$.

### Step 2: a robust local Beltrami field

The field $w$ we constructed in Step 1 will now be used to define a local Beltrami field $v$. To this end we will consider the Cauchy problem

$$\operatorname{curl} v = \lambda v\,, \qquad v|_\Sigma = w\,. \tag{1}$$

One cannot apply the Cauchy–Kowalewski theorem directly because the curl operator does not have any non-characteristic surfaces as its symbol is a skew-symmetric matrix. In fact, a direct computation shows that there are some analytic Cauchy data $w$, tangent to the surface $\Sigma$, for which this Cauchy problem does not have any solutions: a necessary condition for the existence of a solution, when the field $w$ is tangent to $\Sigma$, is that the pullback to the ribbon of the 1-form dual to the Cauchy datum must be a closed 1-form.

Through a more elaborate argument that involves a Dirac-type operator, one can prove that this condition is not only necessary but also sufficient. Therefore, the properties of the field $w$ constructed in Step 1 allow us to ensure that there is a unique analytic field $v$ in a neighbourhood of the knot $L$ which solves the Cauchy problem (1).

It is obvious that the knot $L$ is a closed vortex line of the local Beltrami field $v$. As a matter of fact, it is easy to check that this line is hyperbolic (and therefore stable under small perturbations). The idea is that, by construction, the ribbon $\Sigma$ is an invariant manifold under the flow of $v$ that contracts into $L$ exponentially. As the flow of $v$ preserves volume because $\operatorname{div} v = 0$, there must exist an invariant manifold that is exponentially expanding and intersects $\Sigma$ transversally on $L$, which guarantees its hyperbolicity.

Accordingly, $L$ is a robust closed vortex line. More concretely, by the hyperbolic permanence theorem, any field $u$ that is close enough to $v$ in the $C^m(N)$ norm, $m \geqslant 1$, has a closed integral curve diffeomorphic to $L$ and this diffeomorphism can be chosen $C^m$-close to the identity (and different from the identity only in $N$).

### Step 3: a Runge-type global approximation theorem

The global Beltrami field $u$ is obtained through a Runge-type theorem for the operator $\operatorname{curl} - \lambda$. This result allows us to approximate the local Beltrami field $v$ by a global Beltrami field $u$ in the $C^m(N)$ norm. More precisely, for any positive $\delta$ and any positive integer $m$, there is a global Beltrami field $u$ such that

$$\|u - v\|_{C^m(N)} < \delta\,.$$

The field $u$ falls off at infinity as

$$|D^j u(x)| < \frac{C_j}{|x|}\,.$$

Basically, the proof of our Runge-type theorem [5] consists of two steps. In the first step we use functional-analytic methods and Green's functions estimates to approximate the field $v$ by an auxiliary vector field $\tilde{v}$ that satisfies the elliptic equation $\Delta\tilde{v} = -\lambda^2\tilde{v}$ in a large ball of $\mathbb{R}^3$ that contains the set $N$. In the second step, we define the approximating global Beltrami field $u$ in terms of a truncation of a Fourier-Bessel series representation of the field $\tilde{v}$ and a simple algebraic trick.

To conclude the proof of the theorem, it is enough to take $\delta$ small enough so that the hyperbolic permanence theorem ensures that if $\|u - v\|_{C^m(N)} < \delta$ then there is a diffeomorphism $\Phi$ of $\mathbb{R}^3$ such that $\Phi(L)$ is a closed vortex line of $u$ and $\Phi - \mathrm{id}$ is supported in $N$ with $\|\Phi - \mathrm{id}\|_{C^m(\mathbb{R}^3)}$ as small as wanted.

## 5 A realisation theorem for knotted vortex tubes

In Theorem 2, we have used Beltrami fields to prove the existence of stationary solutions of the Euler equations with vortex lines of any link type. Let us now show that one can construct stationary solutions with knotted vortex tubes, as predicted by Kelvin, using Beltrami fields as well. To state this result, let us denote by $\mathcal{T}_\epsilon(L)$ the $\epsilon$-thickening of a given link $L$ in $\mathbb{R}^3$, that is, the set of points that are at distance at most $\epsilon$ from $L$. The realisation theorem for vortex tubes can then be stated as follows [5]:

**Theorem 3.** Let $L$ be a finite link in $\mathbb{R}^3$. For any small enough $\epsilon$, one can transform the collection of pairwise disjoint thin tubes $\mathcal{T}_\epsilon(L)$ by a diffeomorphism $\Phi$ of $\mathbb{R}^3$, arbitrarily close to the identity in any $C^m$ norm, so that $\Phi[\mathcal{T}_\epsilon(L)]$ is a set of vortex tubes of a Beltrami field $u$, which satisfies the equation $\mathrm{curl}\, u = \lambda u$ in $\mathbb{R}^3$ for some nonzero constant $\lambda$. Moreover, the field $u$ decays at infinity as $|D^j u(x)| < C_j/|x|$.

The parameter $\lambda$ in the theorem cannot be chosen freely: it must be of order $O(\epsilon^3)$. In fact, if we allow a diffeomorphism $\Phi$ that is not close to the identity, we can get any nonzero constant $\lambda'$ just by considering the rescaled field

$$u'(x) := u\Big(\frac{\lambda' x}{\lambda}\Big),$$

which satisfies the Beltrami equation $\mathrm{curl}\, u' = \lambda' u'$. However, the fact that the vortex tubes are thin, in the sense that their width is much smaller than their length, is a crucial ingredient in the proof of the theorem.

The proof of Theorem 3 also yields information on the structure of the vortex lines inside each vortex tube:

1. There are infinitely many nested invariant tori (which bound vortex tubes). On each of these tori, the vortex lines are ergodic.
2. In the region bounded by any pair of these invariant tori, there are infinitely many closed vortex lines, not necessarily of the same knot type as the curves in the link $L$.
3. There is a set of elliptic[4] closed vortex lines diffeomorphic to the link $L$ near the core of the vortex tubes. Being elliptic, they are linearly stable.
4. The vortex tubes are both Lyapunov stable and structurally stable.

4 We recall that a closed integral curve of a vector field is elliptic if its associated monodromy matrix has all its eigenvalues of modulus 1.

The proof of Theorem 3 also relies on the combination of a robust local construction and a global approximation result, as in the case of Theorem 2. In fact, this global approximation result was used in the statement of Theorem 2 to ensure that our Beltrami fields fall off at infinity. However, the construction of the robust local solution is much more sophisticated than in the case of vortex lines and requires entirely different ideas.

Basically, the robustness of the tubes follows from a KAM-theoretic argument with two small parameters: the thinness $\epsilon$ of the tubes and the constant $\lambda$. The local solution must now be defined in the whole tubes, not just on a neighbourhood of the boundary. This makes it impossible to construct the local solution using a theorem of Cauchy-Kowalewski type, as we did in Step 2 of Theorem 2. Instead, we need to consider a boundary value problem for the curl operator in which the tangential part of the field cannot be prescribed. As a consequence of this, one cannot directly take local Beltrami fields which satisfy the non-degeneracy conditions of the KAM-type theorem: these conditions must be extracted from the equation using fine PDE estimates. This is in great contrast to the prescription of the Cauchy datum that we made in Step 1 of Theorem 2, which readily ensures the hyperbolicity of the closed vortex lines and leads to very subtle problems with a deep interplay of PDE and dynamical systems techniques.

As we did in the sketch of the proof of Theorem 2, we will concentrate on constructing a solution for which we are prescribing just one vortex tube $\mathcal{T}_\epsilon \equiv \mathcal{T}_\epsilon(L)$, where $L$ is a (possibly knotted) curve in $\mathbb{R}^3$.

### Step 1: a local Beltrami field in a tube

We will obtain a local Beltrami field $v$ in $\mathcal{T}_\epsilon$ as the unique solution of a certain boundary value problem for the Beltrami equation. To specify this problem, let us fix a (nonzero) harmonic field $h$ in $\mathcal{T}_\epsilon$, which satisfies

$$\mathrm{div}\, h = 0 \quad \text{and} \quad \mathrm{curl}\, h = 0$$

in the tube and is tangent to the boundary. By Hodge theory, it is standard that there is a unique harmonic field in $\mathcal{T}_\epsilon$ up to a multiplicative constant. For concreteness, let us assume that $\|h\|_{L^2(\mathcal{T}_\epsilon)} = 1$.

The boundary problem we will then consider is

$$\mathrm{curl}\, v = \lambda v$$

in $\mathcal{T}_\epsilon$, supplemented with the boundary condition $\partial_n v = 0$ and a condition on the harmonic part of $v$ such as

$$\int_{\mathcal{T}_\epsilon} v \cdot h \, dx = 1\,.$$

Notice that, in this boundary problem, we are specifying the normal component of $v$ on the boundary (which we set to zero, to ensure that $\partial\mathcal{T}_\epsilon$ is an invariant torus) but not the tangential component. This will be important later on.

Through a duality argument, it is not hard to prove that for any $\lambda$ outside some discrete set, and in particular whenever $|\lambda|$ is smaller than some $\epsilon$-independent constant, there is a unique solution of this problem. An easy consequence of the proof is that the field $v$ becomes close to $h$ for small $\lambda$, in the sense that

$$\|v - h\|_{H^k(\mathcal{T}_\epsilon)} \leqslant C_{k,\epsilon}|\lambda|\,. \qquad (2)$$

The problem now is that, when one tries to verify the conditions for the preservation of the invariant torus $\partial\mathcal{T}_\epsilon$ under small perturbations of $v$, one realises that the above existence result is far from enough: the robustness of the invariant torus depends on KAM arguments, which require very fine information on the behaviour of $v$ in a neighbourhood of $\partial\mathcal{T}_\epsilon$.

An important simplification is suggested by the estimate (2): if we take small nonzero values of $\lambda$, it should be enough to understand the behaviour of the harmonic field $h$, since the local solution $v$ is going to look basically like this field (more refined estimates are needed to fully exploit this fact but this is the basic idea).

Therefore, our next goal is to estimate various analytic properties of the harmonic field $h$. To simplify this task, we will introduce coordinates adapted to the tube $\mathcal{T}_\epsilon$, which essentially correspond to an arc-length parametrisation of the knot $L$ and to rectangular coordinates in a transverse section of the tube defined using a Frenet frame. Thus we consider an angular coordinate $\alpha$ taking values in $\mathbb{S}^1_\ell := \mathbb{R}/\ell\mathbb{Z}$ (with $\ell$ the length of the knot $L$) and rectangular coordinates $y = (y_1, y_2)$ taking values in the unit 2-disc $\mathbb{D}$.

To extract information about $h$, we start with a good guess of what $h$ should look like: one can check that there is some function of the form $1 + O(\epsilon)$ such that the vector field

$$h_0 := [1 + O(\epsilon)]\,(\partial_\alpha + \tau\,\partial_\theta)$$

is "almost harmonic", in the sense that it is curl-free, tangent to the boundary and satisfies

$$\rho := -\operatorname{div} h_0 = O(\epsilon)\,.$$

Here $\tau$ is the torsion of the curve $L$ and $\theta$ is the angular polar coordinate in the 2-disc. The actual form of $h_0$ and $\rho$ is important but we will not write these details to keep the exposition simple.

From the above considerations, we infer that the harmonic field is given by

$$h = h_0 + \nabla\psi\,,$$

where $\psi$ solves the Neumann boundary value problem

$$\Delta\psi = \rho \quad \text{in } \mathcal{T}_\epsilon\,, \qquad \partial_n\psi|_{\partial\mathcal{T}_\epsilon} = 0\,, \qquad \int_{\mathcal{T}_\epsilon} \psi\,dx = 0\,. \quad (3)$$

When written in the natural coordinates $(\alpha, y)$, we obtain a boundary value problem in the domain $\mathbb{S}^1_\ell \times \mathbb{D}$, the coefficients of the Laplacian in these coordinates depending on the geometry of the tube strongly through its thickness $\epsilon$ and the curvature and torsion of $L$.

In the derivation of the result on preservation of the invariant torus we will need to solve approximately the boundary value problem (3), thus showing that $\psi$ is of the following form:

- $\psi = O(\epsilon^2)$,
- $D_y\psi =$ (certain explicit function) $+ O(\epsilon^4)$,
- $\partial_\theta\psi =$ (certain explicit function) $+ O(\epsilon^5)$.

The explicit expressions above are important but we will omit them so as not to obscure the main points of the proof.

To obtain these expressions, we need estimates for the $L^2$ norm of $\psi$ and its derivatives that are optimal with respect to the parameter $\epsilon$. The reason for this is that standard energy estimates of the form

$$\|\psi\|_{H^{k+2}(\mathcal{T}_\epsilon)} \leqslant C_{\epsilon,k}\|\rho\|_{H^k(\mathcal{T}_\epsilon)}$$

are of little use to us because, for the preservation of the torus, we will need to be very careful in dealing with powers of the small parameter $\epsilon$. In particular, it is crucial to distinguish between estimates for derivatives of $\psi$ with respect to the "slow" variable $\alpha$ and the "fast" variable $y$, and even to trade some of the gain of derivatives associated with the elliptic equation (3) (in some cases) for an improvement of the dependence on $\epsilon$ of the constants. Estimates optimal with respect to $\epsilon$ are also derived for the equation $\operatorname{curl} v = \lambda v$ in $\mathcal{T}_\epsilon$ to help us exploit the connection between Beltrami fields with small $\lambda$ and harmonic fields.

### Step 2: A KAM theorem for Beltrami fields

To analyse the robustness of the invariant torus $\partial\mathcal{T}_\epsilon$ of the local solution $v$, the natural tool is KAM theory. At first, it may not be immediately obvious why we can apply KAM-type arguments, as $v$ is a divergence-free vector field in a three-dimensional domain and KAM theory is usually discussed in the context of integrable Hamiltonian systems in even-dimensional spaces.

The key here is to consider the Poincaré (or first return) map of $v$. To define this map, we take a normal section of the tube $\mathcal{T}_\epsilon$, say $\{\alpha = 0\}$. Given a point $x_0$ in this section, the Poincaré map $\Pi$ associates to $x_0$ the point where the vortex line $x(\tau)$ with initial condition $x(0) = x_0$ cuts the section $\{\alpha = 0\}$ for the first positive time. The analysis in Step 1 gives that the harmonic field $h$ is of the form

$$h = \partial_\alpha + \tau(\alpha)\,(y_1\,\partial_2 - y_2\,\partial_1) + O(\epsilon)\,, \quad (4)$$

so, with a little work, one can prove that the Poincaré map is well defined for small enough $\epsilon$ and $\lambda$. Identifying this section with the disc $\mathbb{D}$ via the coordinates $y$, this defines the Poincaré map as a diffeomorphism

$$\Pi : \mathbb{D} \to \mathbb{D}\,.$$

Since the vector field $v$ is divergence-free, one can prove that the Poincaré map preserves some measure on the disc.

Notice that the invariant torus $\partial\mathcal{T}_\epsilon$ manifests itself as an invariant circle (namely, $\partial\mathbb{D}$) of the Poincaré map. To establish the robustness of the invariant torus $\partial\mathcal{T}_\epsilon$, we will resort to a KAM theorem [11] to prove that the invariant circle of $\Pi$ is preserved under small area-preserving perturbations. After taking care of several technicalities that will be disregarded here, thanks to this theorem we can conclude that the invariant torus $\partial\mathcal{T}_\epsilon$ is robust provided two conditions are met: that the rotation number of $\Pi$ on the invariant circle is Diophantine and that $\Pi$ satisfies a nondegeneracy twist condition.

We would like to emphasise that computing the rotation number $\omega_\Pi$ and the twist $\mathcal{N}_\Pi$ of the Poincaré map amounts to obtaining quantitative information about the vortex lines of $v$. This is a hard, messy, lengthy calculation that we carry out by combining an iterative approach to control the integral curves of the associated dynamical system (i.e. the vortex lines) with small parameter $\epsilon$ and the PDE estimates, optimal with respect to $\epsilon$, that we obtained for $v$ in Step 1. The final formulas are

$$\omega_\Pi = \int_0^\ell \tau(\alpha)\,d\alpha + O(\epsilon^2)\,,$$

$$\mathcal{N}_\Pi = -\frac{5\pi\epsilon^2}{8}\int_0^\ell \kappa(\alpha)^2\,\tau(\alpha)\,d\alpha + O(\epsilon^3)\,, \quad (5)$$

where $\kappa$ and $\tau$ denote, respectively, the curvature and torsion of the knot $L$. The leading term of $\omega_\Pi$ is the total torsion of the curve $L$, while the leading term of the twist $\mathcal{N}_\Pi$ is proportional to the helicity of the velocity field associated with the vortex filament motion under LIA [19]. These quantities are the first and the third constants of the motion for the LIA equation.[5]

These expressions allow us to prove that for a "generic" curve $L$ the rotation number is Diophantine and the twist is nonzero, so the hypotheses of the KAM theorem are satisfied. Hence, the invariant torus $\partial\mathcal{T}_\epsilon$ of the local Beltrami field $v$ is robust: if $u$ is a divergence-free vector field in a neighbourhood of the tubes that is close enough to $v$ in a suitable sense (e.g. in a $C^m$ norm with $m \geqslant 4$) then $u$ also has an invariant tube diffeomorphic to $\mathcal{T}_\epsilon$ and, moreover, the corresponding diffeomorphism can be taken close to the identity.

It is worth mentioning that the formula (5) provides some intuition about the question of why one needs to be so careful with the dependence on $\epsilon$ of the various estimates: the twist, which must be nonzero, is of order $O(\epsilon^2)$. Another way of understanding this is by looking at the expression (4) for the harmonic field, which implies that our local solution $v$ is an $\epsilon$-small perturbation of the most degenerate kind of vector field from the point of view of KAM theory: a field with constant rotation number.

### Step 3: a Runge-type global approximation theorem

To complete the proof of the theorem, we use the same Runge-type theorem as in Step 3 of the outline of the proof of Theorem 2 to show that there is a Beltrami field $u$ in $\mathbb{R}^3$ close to the local solution:

$$\|u - v\|_{C^m(\mathcal{T}_\epsilon)} < \delta\,,$$

falling off at infinity as

$$|D^j u(x)| < \frac{C_j}{|x|}\,.$$

Putting all three steps together, this gives the outline of the proof of Theorem 3.

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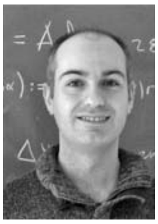


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5 This connection between the quantities measuring the nondegeneracy of the KAM argument for the vortex tubes and the LIA equation is quite surprising, and we do not see any obvious explanation for it.

# On Delusive Nodal Sets of Free Oscillations

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*In memoriam of Vladimir Arnold*

The name of Vladimir Arnold, who passed away on 3 June 2010, is well known to mathematicians all over the world. Indeed, along with the Kolmogorov–Arnold–Moser theory about the stability of integrable systems (his best known contribution to mathematics), there are several other notions associated with him, for example Arnold's conjecture on the number of fixed points of symplectic maps, Arnold's cat map, Arnold diffusion and the Arnold tongue in dynamical systems theory.

A biographical sketch of Vladimir Igorevich Arnold by O'Connor and Robertson (in the MacTutor History of Mathematics archive) is available online at http://www-history.mcs.st-andrews.ac.uk/Biographies/Arnold.html. A lot of interesting details about Arnold's life and work are presented by his colleagues and disciples in the tribute and memories published in 2012 (see [21] and [22], respectively). From these notes one gets a clear idea that everybody who maintained contact with him was greatly impressed by his extraordinary personality.

Among Arnold's numerous honours is the Dannie Heineman Prize for Mathematical Physics awarded in 2001 jointly by the American Physical Society and the American Institute of Physics. This honour is not accidental because he had a deep feeling for the unity of mathematics and natural sciences. His oft quoted remark is that mathematics is a part of physics, in which experiments are cheap.

It is therefore no wonder that one of Arnold's papers published posthumously deals with an important property of eigenoscillations in mathematical physics (see [3], submitted for publication six months before his death). In this paper,[1] Arnold, with his inherent mastery of both the subject and storytelling, describes a fascinating fact about an incorrect theorem that was announced in the classical book [8] by Courant and Hilbert. (This edition is cited in [3] but, for the reason explained below, Arnold used either the 2nd German edition [9] or, most likely, its Russian translation published twice, in 1933 and 1951.)

The theorem in question deals with nodal sets (or, for brevity, nodes) of linear combinations of some particular eigenfunctions (see the next paragraph). Such a set is simply defined as the set where a function vanishes. To make the importance of eigenfunctions clear, we just mention that they serve to describe free oscillations of strings and membranes, and nodes show where an oscillating object is immovable because, by its definition, a node separates the sets where the function is positive and negative. In one, two and three dimensions, nodal sets consist of points, curves and surfaces, respectively. Pictures of nodal curves for some modes of oscillations of the square membrane fixed along its boundary can be found in many textbooks (see, for example, [36], p. 266).

**Vladimir Igorevich Arnold in 1977**

It is amazing that there are many theorems and conjectures proved to be incorrect in this area of research. Let us list those considered in this paper and recall other renowned questions concerning the same spectral problems of mathematical physics. We begin with the theorem which is the topic of Arnold's paper [3]. It concerns nodes of linear combinations of eigenfunctions of the Dirichlet Laplacian and we illustrate the question's essence with some elementary examples. This material is presented in the first section.

What is widely known about the eigenvalue problem for the Dirichlet Laplacian is the question 'Can one hear the shape of a drum?' posed by Mark Kac in 1966 in the title of his paper [20]. However, this question is about the whole set of eigenvalues, whereas there are many subtle questions about properties of eigenfunctions corresponding to individual eigenvalues. One of them, referred to as Payne's conjecture, concerns nodes of the second eigenfunction; being more technical, it is considered in the third section.

It is worth mentioning that the negative answer to Kac's question was obtained in 1992; it is presented in a form accessible to a general audience in [14]. However, this answer, like the incorrectness of the theorem mentioned above and discussed in [3], is only a part of the story. In November 2012, S. Titarenko presented another part at the Smirnov Seminar on Mathematical Physics in St. Petersburg (http://www.

1 An item in the collection dedicated to the 75th anniversary of the Steklov Mathematical Institute in Moscow. Before 1934, when the Soviet Academy of Sciences was moved from Leningrad to Moscow, this institute was a division of the Physical-Mathematical Institute organised by V. A. Steklov in 1921 (see Steklov's recollections cited in [25]).

pdmi.ras.ru/~matfizik/seminar2012-2013.htm). The most important point of his talk entitled 'When can one hear the shape of a drum? Sufficient conditions' was that to give a positive answer to Kac's question, the boundary of the drum's membrane must be smooth. Indeed, smoothness is violated in all of the now numerous examples giving a negative answer (see, for example, [13], p. 2235; this article also contains an extensive list of references on mathematical and physical aspects of isospectrality). Unfortunately, Titarenko's result is still unpublished.

The second section deals with the well known phenomenon of liquid sloshing in containers (widely used examples of these are tea cups, coffee mugs, wine glasses, cognac snifters, *etc*.). The corresponding mathematical model – the so-called sloshing problem (it is also referred to as the mixed Steklov problem) – attracted much attention after the award of the 2012 Ig Nobel Prize for Fluid Dynamics to R. Krechetnikov and H. Mayer for their investigation of why coffee so often spills while people walk with a filled mug [30]. This effect results from the correlation between the fundamental sloshing frequency and that of the steps. Here, a property of sloshing nodes (the liquid remains immovable there during its free oscillations) is considered. The example presented demonstrates that a gap in the proof of a certain theorem describing the behaviour of nodes cannot be resolved.

Another aim of this paper is to show how the application of rather simple tools (in particular, an analysis of the behaviour of functions defined explicitly, for example, by improper integrals and even by elementary trigonometrical formulae) leads to interesting results concerning important questions that challenge both mathematical and physical intuition. It should be emphasised that such questions were among Arnold's favourites. Indeed, his unique intuition in the subject of catastrophes, for example, allowed him to guess, on the spot, the right answers when physicists and engineers asked him what kind of catastrophic effects could be expected in their problems. Many of his guesses were based on very simple models like that considered in the next section.

## Arnold on a footnote in the Courant–Hilbert book

Arnold begins his story with the following:

> topological result [...] valid on any compact manifold: an eigenfunction $u$ of the Laplace operator
>
> $$\Delta u = \lambda u \text{ with eigenvalue } \lambda = \lambda_n$$
>
> (we arrange them in order of increasing frequencies $-\lambda_1 \leq -\lambda_2 \leq -\lambda_3 \leq \dots$) vanishes on the oscillating manifold $M$ in a way such that its zeros divide $M$ into at most $n$ parts.

In its original form, the result obtained by Courant in 1923 concerns nodes of eigenfunctions of a self-adjoint second order differential operator (for example, the Sturm–Liouville operator on an interval and the Laplacian in a bounded higher-dimensional domain) with one of the standard boundary conditions (for example, the Dirichlet and Neumann conditions). Namely, Courant's theorem asserts that (see [8], p. 452):

> if [the] eigenfunctions are ordered according to increasing eigenvalues, then the nodes of the $n$th eigenfunction divide the domain into no more than $n$ subdomains. No assumptions are made about the number of independent variables.

**V. Arnold lecturing in Syktyvkar in 1977**

Two of the simplest examples illustrating this theorem are provided by the equation describing the set of possible shapes of an homogeneous string in free time-harmonic oscillations:

$$-u'' = \lambda u \quad \text{on } (0, \pi), \tag{1}$$

augmented by either the Dirichlet conditions

$$u(0) = u(\pi) = 0, \tag{2}$$

which means that the ends of a string are fixed, or the Neumann conditions

$$u'(0) = u'(\pi) = 0 \tag{3}$$

when the ends are free. It is clear that the eigenfunction $u_n = \sin nx$, $n = 1, 2, \dots$, corresponds to $\lambda_n = n^2$ under the boundary conditions (2), whereas conditions (3) give

$$u_n = \cos(n-1)x \text{ and } \lambda_n = (n-1)^2, \text{ respectively.}$$

Note that in both cases the $n$th eigenfunction divides the interval into precisely $n$ parts. Courant proves that this property remains valid for a general Sturm–Liouville problem.

Prior to proving the latter result, a footnote announcing the notorious incorrect theorem appears at the end of the proof of the theorem cited above (see the first footnote on p. 454 in [8]):

> The theorem just proved may be generalized as follows. Any linear combination of the first $n$ eigenfunctions divides the domain, by means of its nodes, into no more than $n$ subdomains. See the Göttingen dissertation of H. Herrmann, Beiträge zur Theorie der Eigenwerte und Eigenfunctionen, 1932.

Below, this assertion is referred to as Herrmann's theorem. Arnold writes about it:

> This *generalization of Courant's theorem* is not proved at all in the book by Courant and Hilbert; it was just mentioned that the proof "will soon be published (by a disciple of Courant)".

From the last sentence, we see that Arnold used either the 2nd German edition [9] published in 1931 or, more likely, its Russian translation. Then he continues:

> Having read all this, I wrote a letter to Courant: "Where can I find this proof now, 40 years after Courant announced the theorem?" Courant answered that "one can never trust one's students: to any question they answer either that the problem is too easy to waste time on, or that it is beyond their weak powers".
>
> As regards Courant and Hilbert's *Mathematical Physics*, according to Courant's published recollections, this book was nevertheless written by his students.

Of course, Arnold exaggerates the role of students but, at the beginning of the preface to [8], Courant writes that the second German edition was "revised and improved with the help of K. O. Friedrichs, R. Luneburg, F. Rellich, and other unselfish friends".

Soon after receiving Courant's reply, Arnold discovered that applying Herrmann's theorem to the eigenfunctions of the Laplacian on the sphere $S^N$ with the standard Riemannian metric, one obtains an estimate for the number of components complementing a real algebraic hypersurface of degree $n$ in the $N$-dimensional projective space (see [4]). The idea behind this is that the so-called spherical harmonics (eigenfunctions of the Laplacian on the two-dimensional sphere) are defined as follows. The set of these functions corresponding to the $n$th eigenvalue consists of restrictions to $S^2$ of homogeneous harmonic polynomials of degree $n-1$ in $\mathbb{R}^3$ (see [36], p. 263). Hence a linear combination of eigenfunctions corresponding to the first $n$ eigenvalues is also an harmonic polynomial whose degree is bounded by $n$. In [3], Arnold comments on his estimate as follows:

> [...] it turned out that the results of the topology of algebraic curves that I had derived from the generalized Courant theorem contradict the results of quantum field theory. Nevertheless, I knew that both my results and the results of quantum field theory were true. Hence, the statement of the generalized Courant theorem is not true (explicit counterexamples were soon produced by Viro). Courant died in 1972 and could not have known about this counterexample.

Indeed, seven years after Courant's death, Viro found an example of a real algebraic hypersurface for which Arnold's estimate does not hold, thus establishing what is incorrect about Herrmann's theorem. Namely, it is valid only under some restrictions on the number of independent variables; in particular, it is false for the Laplacian on $S^3$ and higher-dimensional spheres (see [37]).

However, Herrmann's theorem is true for eigenfunctions of the Dirichlet and Neumann problems for Equation (1). Indeed, the $n$th Dirichlet and Neumann eigenfunctions can be written in terms of the Chebyshev polynomials: $\sin nx = \sin x\, U_{n-1}(\cos x)$ and $\cos(n-1)x = T_{n-1}(\cos x)$, respectively. Also, elementary trigonometric formulae (see 1.331.1 and 1331.3 in [15]) give, for $n > 1$:

$$\sin nx = \sin x \sum_{k=0}^{[(n-1)/2]} (-1)^k \binom{n-k-1}{k} (2\cos x)^{n-(2k+1)}, \quad (4)$$

$$\cos(n-1)x = 2^{n-2}\cos^{n-1} x + \frac{n-1}{2} \sum_{k=1}^{[(n-1)/2]} \frac{(-1)^k}{k} \binom{n-k-2}{k-1} (2\cos x)^{n-(2k+1)}. \quad (5)$$

Here, $[m]$ stands for the integer part of $m$.

According to Equation (4), a linear combination of the first $n$ Dirichlet eigenfunctions is the product of $\sin x$ and a polynomial of $\cos x$ whose degree is at most $n-1$. Therefore, it has at most $n-1$ zeros and the number of nodes on $(0, \pi)$ is also less than or equal to $n-1$. A similar conclusion follows from (5) for a linear combination of the first $n$ Neumann eigenfunctions. Let us illustrate this by considering linear combinations of the first two Dirichlet and Neumann eigenfunctions, which are

$$\sin x(C_1 + 2C_2 \cos x) \quad \text{and} \quad C_1 + C_2 \cos x, \ \text{ respectively.}$$

Here $C_1$ and $C_2$ are some constants. Both linear combinations have at most one node on $(0, \pi)$. It exists when $C_2 \neq 0$ and when

$$\left|\frac{C_1}{C_2}\right| < 2 \ \text{ and } \ \left|\frac{C_1}{C_2}\right| < 1$$

for the combinations of the Dirichlet and Neumann eigenfunctions, respectively. These conditions are also necessary for the existence of a node.

In the second section of [4], Arnold turns to the following Sturm–Liouville problem:

$$-u'' + qu = \lambda u \ \text{ on } (0, \ell), \quad u(0) = u(\ell) = 0, \qquad (6)$$

where $q$ is a positive function on $[0, \ell]$. He outlines Gel'fand's idea of how to prove Herrmann's theorem for eigenfunctions of this problem. It consists of replacing:

> the analysis of the system of $n$ eigenfunctions of the one-particle quantum-mechanical problem by the analysis of the first eigenfunction of the $n$-particle problem (considering, as particles, fermions rather than bosons).

This approach so attracted Arnold that he included Herrmann's theorem for eigenfunctions of problem (6) together with Gel'fand's hint into the 3rd Russian edition of his *Ordinary Differential Equations* (see Problem 9 on the list of supplementary problems at the end of [5]).

In [3], Arnold devotes two pages to some details of Gel'fand's analysis but, in the end, he writes:

> Unfortunately, the arguments above do not yet provide a proof for this generalized theorem: many facts are still to be proved. [...]
>
> Gel'fand did not publish anything concerning this: he only told me that he hoped his students would correct [...] his theory. He pinned high hopes on V. B. Lidskii and A. G. Kostyuchenko. Viktor Borisovich Lidskii told me that "he knows how to prove all this". [...] Although [his] arguments look convincing, the lack of a published formal text with a rigorous proof of the Courant–Gel'fand theorem is still distressing.

This is still true, despite the fact that in September 2012 Victor Kleptsyn (Institut de Recherche Mathématique de Rennes) outlined his proof for all gaps remaining in the above approach in a talk entitled 'Fermions and the Courant–Gelfand theorem' at the Moscow Seminar on Dynamical Systems (see http://www.mathnet.ru/php/seminars.phtml?option_lang=eng&presentid=5644). Unfortunately, only the Russian abstract of this talk is available.

## On sloshing nodal curves

A particular case of the mixed Steklov eigenvalue problem gives the so-called sloshing frequencies and the corresponding wave modes, i.e. the natural frequencies and modes of the free motion of water occupying a reservoir. When the latter is an infinitely long canal of uniform cross-section $W$, the two-dimensional problem arises. In this case, the boundary $\partial W$ consists of $F = \{|x| < a, \ y = 0\}$ and $B = \partial W \setminus \bar{F}$ lying in the half-plane $y < 0$. The former is referred to as the *free surface* of water, whereas the latter is the *canal's bottom*.

The velocity potential $u(x, y)$ with the time-harmonic factor removed must satisfy the following boundary value problem:

$$u_{xx} + u_{yy} = 0 \quad \text{in } W, \tag{7}$$

$$u_y = \lambda u \quad \text{on } F, \tag{8}$$

$$\frac{\partial u}{\partial \mathbf{n}} = 0 \quad \text{on } B. \tag{9}$$

Here $\mathbf{n}$ denotes the exterior unit normal on $B$ and $\lambda = \omega^2/g$ is the spectral parameter to be found along with $u$ ($\omega$ is the radian frequency of the water oscillations and $g$ is the acceleration due to gravity). In order to exclude the non-physical zero eigenvalue of (7)–(9), it is usual to augment the problem's statement with the orthogonality condition

$$\int_F u \, \mathrm{d}x = 0. \tag{10}$$

The condition on $F$ is the Steklov boundary condition first introduced by Steklov in 1896 but the standard reference for the Steklov problem is the paper [35] published in 1902. Problem (7)–(10) and the three-dimensional version have been the subject of a great number of studies over more than two centuries; see [11] for a historical review, whereas early results are presented in Lamb's classical treatise *Hydrodynamics* [28].

It is well known that this problem has a discrete spectrum, that is, an infinitely increasing sequence of positive eigenvalues of finite multiplicity (the latter is the number of different eigenfunctions corresponding to a particular value of $\lambda$). The corresponding eigenfunctions $u_n$, $n = 1, 2, \ldots$, form a complete system in an appropriate Hilbert space. Unlike eigenfunctions of the Dirichlet and Neumann Laplacian, the first paper about properties of solutions to (7)–(10) had only been published by Kuttler in 1984 (see [24]). Since then, a number of interesting results concerning the so-called 'high spots' of sloshing eigenfunctions have appeared (see the recent review [26] aimed at the lay reader).

The main result of [24] is analogous to Courant's theorem. Namely, if the eigenfunctions are ordered according to increasing eigenvalues then the nodes of the $n$th eigenfunction divide the domain into no more than $n+1$ subdomains. In view of the additional condition (10), the number of subdomains is $n + 1$ instead of $n$ appearing in Courant's theorem. Kuttler's reasoning (a version of Courant's original proof) proves this assertion after omitting the superfluous reference to the following incorrect lemma.

*For every eigenfunction of problem* (7)–(10) *nodal curves have one end on the free surface F and the other one on the bottom B.*

Counterexamples demonstrating that this lemma is incorrect were constructed 20 years after publication of [24]. They provide various domains $W$ for which there exists an eigenfunction of problem (7)–(10) having a nodal curve with both ends on $F$. Let us outline the approach applied for this purpose in [23]. The example involves a particular pair velocity potential/stream function (the latter is an harmonic conjugate to the velocity potential) introduced in the book [27], § 4.1.1,

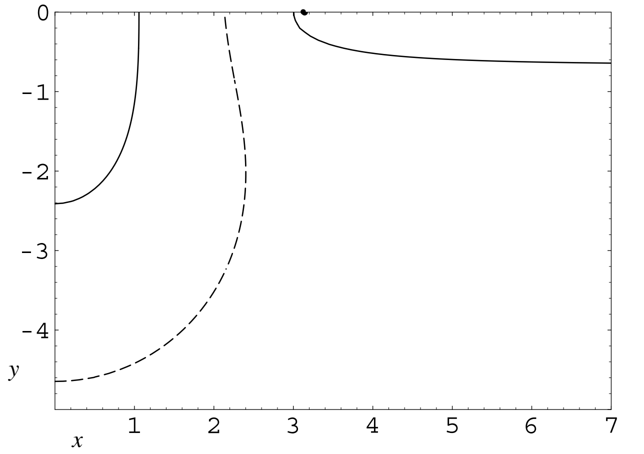


**Figure 1. Nodal lines of $u$ (solid lines) and $v$ (dashed line) given by** (11) **and** (12)**, respectively, with** $\lambda = 3/2$

namely,

$$u(x, y) = \int_0^\infty \frac{\cos k(x - \pi) + \cos k(x + \pi)}{k - \lambda} \mathrm{e}^{ky} \, \mathrm{d}k, \tag{11}$$

$$v(x, y) = \int_0^\infty \frac{\sin k(x - \pi) + \sin k(x + \pi)}{\lambda - k} \mathrm{e}^{ky} \, \mathrm{d}k, \tag{12}$$

where $\lambda = m/2$ and $m$ is odd. Then the numerators in both integrals vanish at $k = \lambda$ and so they are understood as usual infinite integrals. It is easy to verify that $u$ and $v$ are conjugate harmonic functions in the half-plane $y < 0$. Moreover, we have that

$$u(-x, y) = u(x, y) \quad \text{and} \quad v(-x, y) = -v(x, y), \tag{13}$$

which allows us to study the behaviour of nodal curves of these functions only in the quadrant $\{x > 0, y < 0\}$ in view of their symmetry about the $y$-axis.

In § 2 of [23], this behaviour is investigated in detail for $\lambda = 3/2$ and illustrated in Figure 1, where only the right half of the picture is shown in view of (13). It is proved that $v$ has a nodal curve which has both ends on the $x$-axis (dashed line). This nodal curve serves as $B$ because the boundary condition (9) is fulfilled on it in view of the Cauchy–Riemann equations holding for $u$ and $v$. Furthermore, there exists a nodal curve of $u$ (solid line) lying in $W$, defined by the described $B$. Moreover, it has both ends on the $x$-axis, thus delivering a counterexample to Kuttler's lemma.

More complicated counterexamples to Kuttler's lemma are obtained numerically for $\lambda = 5/2$; see Figure 2, where again only the right half of the picture is shown. In this case, apart from the $y$-axis, there are two nodes of $v$ (dashed lines and their images in the $y$-axis) and four nodes of $u$ (solid lines and their images in the $y$-axis). Both finite nodes of $u$ are located within the domain $W$ whose bottom $B$ is given by the whole exterior node of $v$. In another counterexample, the bottom consists of the right half of this node complemented by the segment of the $y$-axis.

Besides, taking the interior node of $v$ as the bottom, we see that the nodes of $u$ connect this bottom with the corresponding free surface. Of course, the same is true for all known cases of the sloshing problem in two and three dimensions for which separation of variables is possible, thus providing a misleading hint.

## On nodal curves of oscillating membranes with fixed boundaries

The topic of this section is the eigenvalue problem

$$u_{xx} + u_{yy} + \lambda u = 0 \text{ in } D, \quad u = 0 \text{ on } \partial D, \tag{14}$$

where $D$ is a bounded domain in $\mathbb{R}^2$. Its solutions $(u_n, \lambda_n)$, $n = 1, 2, \ldots$ (for every $\lambda > 0$ satisfying (14) the number of its repetitions is equal to its multiplicity) serve to represent pure tones that the elastic membrane $D$ can produce when fixed along its boundary. As was mentioned above, along nodal curves an oscillating membrane stays immovable. This is why they are important to study.

In [16], published after defending his dissertation discussed above, Herrmann remarked that Courant's theorem admits sharpening for eigenfunctions of problem (14). Such a refinement appeared in 1956 (see [33]) and is nowadays usually referred to as the Pleijel's nodal domain theorem. Its most interesting consequence says:

*The number of subdomains, into which the nodes of the k-th eigenfunction of problem* (14) *divide D, is equal to k only for finitely many values of k.*

In the last section of his note, Pleijel writes that "[...] it seems highly probable that the result [...] is also true for free membranes", that is, when the Dirichlet boundary condition is changed to the Neumann one in (14). This conjecture was recently proved by Polerovich [34] under the assumption that $\partial D$ is piecewise analytic. The difficulty of this case is that, along with nodal subdomains lying totally in the interior of $D$, there are subdomains adjacent to $\partial D$ where the Neumann condition is imposed. To the former subdomains, the original technique used by Pleijel and involving the Faber–Krahn isoperimetric inequality is applicable, whereas the latter ones require an alternative approach based on an estimate for the number of boundary zeros of Neumann eigenfunctions.

According to Courant's theorem, the fundamental eigenfunction $u_1$ does not change sign in $D$, whereas the node of $u_2$ divides $D$ into two subdomains. Both these cases give the maximal number of subdomains in a trivial way. A less trivial fact obtained in [33] is that only the first, second and fourth eigenfunctions give the maximal number of subdomains for a square membrane with fixed boundary.

During the past few decades, much attention has been paid to the following question. *How does the only node of $u_2$ divide D into two subdomains?* In his widely cited survey paper [31] published in 1967, Payne conjectured that the nodal curve of $u_2$ cannot be closed for any domain $D$ (see Conjecture 5 on p. 467 of his paper).[2] It happened that, like Herrmann's theorem, this conjecture is only partly true. The corresponding results are outlined below.

Six years later, Payne proved the following theorem confirming his conjecture (see [32]).

*If D is convex in x and symmetric about the y-axis then $u_2$ cannot have an interior closed nodal curve.*

Prior to proving this assertion, Payne lists some important facts about eigenvalues and nodes of eigenfunctions that follow from the theory of elliptic equations. (In particular, it yields that all solutions of (14) are real analytic functions in the interior of $D$.) These properties are as follows:

(i) If $D'$ is strictly contained in $D$ then the inequality $\lambda'_n > \lambda_n$ holds for the corresponding eigenvalues.
(ii) No nodal curve can terminate in $D$.
(iii) If two nodal curves have a common interior point then they are transversal; this also applies when a nodal curve intersects itself.

Several partial results followed Payne's theorem (see references cited in [2]) before Melas [29] proved that the conjecture is true for all convex two-dimensional domains with $C^\infty$ boundary. This happened 25 years after it had been formulated. Two years later, this result was extended by Alessandrini to the case of general convex domains in $\mathbb{R}^2$. Namely, his theorem is as follows (see [2]).

*Let D be a bounded convex domain in the plane. If u is an eigenfunction corresponding to the second eigenvalue of problem* (14) *then the nodal curve of u intersects $\partial D$ at exactly two points.*

Payne's conjecture is also true for a class of non-convex planar domains, as was recently shown in [38].

Let us turn to results demonstrating that Payne's conjecture is not true for *all* bounded domains, to say nothing of unbounded ones. The first counterexample to the general conjecture in $\mathbb{R}^2$ belongs to M. Hoffmann-Ostenhof, T. Hoffmann-Ostenhof and N. Nadirashvili [17] (see also [18]), who constructed a multiply connected domain such that the nodal set of $u_2$ is disjoint with $\partial D$.

To describe such a domain, we apply non-dimensional variables, which is natural from a physical point of view, remembering Arnold's remark about mathematics as a part of physics. Since the boundary of a domain considered in [17] involves two concentric circumferences (the origin is chosen to be at their centre), we take the radius of the smaller circumference to be of unit length. According to [17], the radius of the larger circumference, say $r \in (1, +\infty)$, must be taken so that the fundamental eigenvalue of problem (14) in the annulus with interior and exterior radii equal to 1 and $r$, respectively, lies strictly between the first and second eigenvalues of problem (14) in the unit circle. These values are well known, being equal to $j^2_{0,1}$ and $j^2_{1,1}$, respectively; here $j_{0,1} \approx 2.405$ and

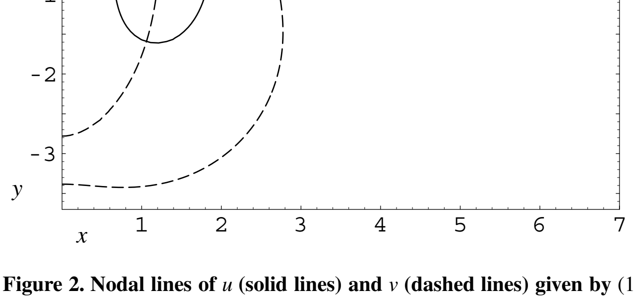


**Figure 2. Nodal lines of $u$ (solid lines) and $v$ (dashed lines) given by (11) and (12), respectively, with $\lambda = 5/2$**

2 It is worth mentioning that Yau repeated this question 15 years later but only for convex plane domains. Maybe he expected it not to be true in its full generality.

$j_{1,1} \approx 3.832$ are the least positive zeros of the Bessel functions $J_0$ and $J_1$, respectively.

A standard separation of variables gives the fundamental eigenvalue for the described annulus. It is equal to $\mu^2$, where $\mu(r)$ is the least positive root of the following equation:

$$J_0(\lambda)Y_0(\lambda r) - J_0(\lambda r)Y_0(\lambda) = 0.$$

Here, $Y_0$ is the zero-order Bessel function of the second kind. Thus, the condition imposed on $r$ can be written in the form:

$$2.405 \approx j_{0,1} < \mu(r) < j_{1,1} \approx 3.832. \tag{15}$$

The existence of $r$ such that (15) is valid is considered by the authors of [17] as an obvious fact and its natural explanation from a physical point of view is as follows. Since $\mu(r)$ is the frequency of free oscillations of an annulus with fixed boundary, it monotonically decreases from infinity to zero as the annulus width $r - 1$ increases from zero to infinity, and so inequality (15) holds when $r$ belongs to some intermediate interval. However, it is worth giving a quantitative evaluation of this interval and this can easily be done with the help of classical handbooks. The table on p. 204 in [19] gives that 2 belongs to this interval because $\mu(2) \approx 3.123$, whereas Table 9.7 in [1] shows that 5/3 and 5/2 are out of it because $\mu(5/3) \approx 4.697$ and $\mu(5/2) \approx 2.073$. More detailed information about the behaviour of $\mu(r)$ can be obtained from the graph plotted in Figure 110 on p. 204 in [19].

The next step is characterised in [18] as "carving" $N > 2$ holes in the circumference separating the unit circle from the annulus in order to obtain a single multiply connected domain; the angular diameter of each hole is $2\epsilon$, where $\epsilon \in (0, \pi/N)$. Therefore, it is convenient to use polar coordinates for this purpose: $\rho \geq 0$ and $\theta \in (-\pi, \pi]$ such that $x = \rho\cos\theta$, $y = \rho\sin\theta$. The boundary of the domain $D_{N,\epsilon}$ is as follows:

$$\partial D_{N,\epsilon} = \{\rho = r\} \cup \left\{\rho = 1, \theta \notin \cup_{k=0}^{N-1}\left(\frac{2\pi k}{N} - \epsilon, \frac{2\pi k}{N} + \epsilon\right)\right\}$$

and so $\partial D_{N,\epsilon}$ consists of $N + 1$ (at least three) components.

Now we are in a position to formulate the main result proven in [17] and [18].

*Let $r > 1$ be such that inequality* (15) *holds. Then there exists $N_0 \geq 2$ such that for $N \geq N_0$ and sufficiently small $\epsilon = \epsilon(N)$ the following assertions are true:* (i) *the 2nd eigenvalue of problem* (14) *in the domain $D_{N,\epsilon}$ is simple;* (ii) *the nodal curve of the corresponding eigenfunction $u_2$ is a closed curve in $D_{N,\epsilon}$.*

In their proof, the authors use the symmetry of the domain $D_{N,\epsilon}$. Moreover, they note

> we have not tried to get an explicit bound on the constant $N_0$ [...]. This [...] would probably lead to an astronomical number.

Then they conjecture that no simply connected domain has a closed nodal curve of $u_2$.

In 2001, Fournais [10] obtained "a natural higher dimensional generalisation of the domain" constructed in [17]. Instead of using the symmetry of a domain, he applied an alternative, and in some sense more direct, approach to "carving" evenly distributed holes in the inner sphere in order to obtain the desired conclusion.

The next step was to consider unbounded domains. In this case, Payne's conjecture does not hold even for planar domains satisfying conditions used by Payne himself when proving the conjecture for bounded domains. Namely, the following theorem was obtained in [12].

*There exists a simply connected unbounded planar domain which is convex and symmetric with respect to two orthogonal directions, and for which the nodal line of a 2nd eigenfunction does not touch the domain's boundary.*

## Brief conclusions

The above examples are taken from a rather narrow area in mathematical physics. Nevertheless, they clearly show that even incorrect and/or partly correct theorems and conjectures often lead to better understanding not only of the corresponding mathematical topic but, sometimes, a topic in a completely distinct field.

Another conclusion concerns the role of style in Arnold's papers and, especially, his books. It combines clarity of exposition, mathematical rigour, physical intuition and masterly use of pictures. Therefore, it is not surprising that he is among the world's most cited authors and No. 1 in Russia according to http://www.mathnet.ru/php/person.phtml?&option_lang=eng. Every mathematician would enjoy those of his papers aimed at a general audience, in particular [6] and [7], which show that his English was as excellent as his Russian. Unfortunately, some translations of his papers leave a lot to be desired (for example, one finds 'knots' instead of 'nodes' in [3]; see the top paragraph on p. 26).

There is a common opinion that Agatha Christie's novels are helpful for learning English (the author's own experience confirms this). In much the same way, Arnold's papers and books are helpful for both learning mathematics and learning to write mathematics.

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# Interview with Prof. Dr. Günter Pickert

Albrecht Beutelspacher (University of Giessen, Germany) and Günter Törner (University of Duisburg-Essen, Germany)

*This is an English version of an interview which appeared in Mitteilungen der Deutschen Mathematiker-Vereinigung Volume 23, Issue 1, Pages 48–58, 2015. Reprinted with permission.*

The following is based on an interview with Dr. Pickert, which was conducted by the authors on 20 April 2014. The interview has been edited on behalf of the interviewee.
Günter Pickert died on 11 February 2015.

Günter Pickert was born on 23 June 1917 in Eisenach. Due to his father's occupation, Pickert's family had to move house not just once but repeatedly; however, in 1933, Pickert was able to sit his Abitur at the age of 16, which allowed him to begin his studies during the summer semester of 1933 in Göttingen. Pickert also studied at TH Danzig; however, after three semesters, he returned to Göttingen and was awarded a doctorate under Helmut Hasse in 1938. During World War II, Pickert served as a soldier in Poland and Russia and finally as a first lieutenant in Tunisia, where he was captured and held by the Americans as a prisoner of war. In 1946, Pickert returned to Germany and started a permanent job at the Mathematical Institute in Tübingen. Pickert qualified as a university lecturer in 1948. Before being called to hold a chair at the University of Gießen in 1962, Pickert also worked in Göttingen and Heidelberg. After being the Head of the Mathematics Department in Gießen several times, Pickert was conferred emeritus status in 1985. In his old age, he still lives in Gießen, the location of his last university, and still actively takes part in developments in the field of mathematics.

**Photo: Renate Pickert-Edelmann**

***B: We thank you for granting us this wonderful opportunity of asking you some questions concerning the field of mathematics and its developments over the past decades. Thereby, we surely don't want to neglect the didactics of mathematics and its essential changes.***
I'm sure you will understand that, as a 97-year-old, I won't be able to tell you a lot about the great mathematical problems. However, I have a couple of memories and anecdotes ready for you and I have been able to recall a thing or two while preparing for this meeting.

## 1 David Hilbert's "Grundlagen der Geometrie" and the precedential works by Moritz Pasch

***T: To me, the name Günter Pickert is first of all connected to the field of geometry. I honestly regret not having attended any of your lectures on the foundation of geometry during my studies at Gießen University; however, I have enjoyed your work "Einführung in die Endliche Geometrie", which was in the making at that time.***
***Nonetheless, I have learnt from you that we shouldn't consider Hilbert's 1899 book as a completely new depiction of geometry. I want to talk about another geometer from Gießen who, professionally speaking, wasn't too far away from David Hilbert at the end of the 19th century…***
Yes, Moritz Pasch, of course. The discovery of betweenness as an axiom of order, which was missing if we regard the list of known postulates at that time, is solely thanks to him. Basically, his principle was the same as Hilbert's: the axioms, or the core premises, dictate the acceptable assumptions; the elevation, on the other hand, shouldn't contribute anything. However, it was Pasch's primary goal to work out a mathematical analysis of the perception of space.

Hilbert initially followed the same thought as Pasch; a long time ago I had a look at the postscript of his lecture, which had been made by one of the attendees before the release of his book. The postscript has been released and has been filed away in the institute library of the mathematical institute in Göttingen. It follows from this postscript that Hilbert also had close relations to a mathematical analysis of the perception of space. In this respect, the two of them weren't actually so dissimilar.

Hilbert's achievement – and this was very neatly depicted by Herman Weyl in his obituary in 1944 – was the fact that he treated geometry in the same respect as one was used to treating algebra.

It was far removed from Pasch's attempts to be able to have different opportunities in order to pinpoint the multiple non-geometries. Unquestionably, Pasch focused on logical correctness, as did Hilbert later on. Therefore, we cannot regard this attempt as the special feature in Hilbert's work.

## 2 David Hilbert

***T: Speaking of David Hilbert again, did you have the chance of getting to know him during your time in Göttingen?***
Yes, indeed. It must have been his last lecture during the winter semester of 1933/34; during the summer term he didn't lecture and then during the following winter term he gave this last lecture. He really wasn't much of an imposing figure, a small man I would rather say, unlike Heinrich Behnke, for example, whom I met repeatedly. I remember that his assistant Arnold Schmidt, who used to sit in the front row, had to help him out from time to time. I once talked about this fact to Hellmuth Kneser, who studied in Göttingen during the 1920s. He told me: "Well, that wasn't due to Hilbert's age; it was the same story when he was younger."

As a student, I had already started to study his foundations of geometry. Consequently, it was very impressive to finally meet him and listen to him in person.

***B: How would you consider the effect of Hilbert's foundations of geometry? Without any doubt it has been greater than the effect of Pasch's work. May the reason for this be rooted in the fact that, in those days, there was the chance of allowing access to different geometries?***
First of all, this was due to Hilbert as a person. He had acquired a very good reputation within the mathematical community; clearly he enjoyed a better reputation than Pasch. Pasch was more of an outsider and he wasn't really valued by his contemporaries in the same way that Hilbert was. I guess it has been due to the constellations of scientific sociology that Hilbert's work has been held in higher regard than Pasch's work. At least, that's the way I see it.

***B: There is one more thing that interests me in connection to Hilbert. There have been handed down a number of succinct phraseologies, sometimes even polemic ones. Would you say that this spirit could have been observed during his last lectures?***
No. Usually they were quite dull. Basically it was exactly what I, as a student, studied in my book and learnt by heart. His lectures were unlike those of Behnke, Walter Benz or some of our other colleagues, who used to present their insights with great emotion. You wouldn't find such emotion in Hilbert's lectures; he seemed to be rather dry and his East Prussian accent rendered his appearance even more odd.

***S: Hilbert is ascribed the remark that instead of points, lines and planes we could say tables, chairs and tankards; it wouldn't matter as long as they matched the axioms. Is it possible that he could have said something like that?***
I can imagine him saying something like this since he was said to be quite humorous outside of the lecture hall.

***T: I, for my part, am interested in the following. Hilbert's book was published in 1899 and now it is 2014, so 115 years have passed. In the following, we will talk about your book "Projektive Ebenen". Do you think that in the past 100 years there has been a book that can be compared to Hilbert's? Or do you think that at the turn of the previous century, geometry had already been discussed to death?***
The first question is rather difficult to answer and we need to specify the nature of the book in question. Concerning the second question – that's most definitely a clear 'NO'. Geometry has developed as algebra has and will develop. I originally started with algebra and, if you want to put it that way, I grew up with van der Waerden's book.

***B: Let's put it differently: do you think that "Grundlagen der Mathematik" has been the starting point for the developments within the field of geometry during the 20th century? Or has it merely been one book amongst many?***
We have to see Hilbert's book back then within a larger context. The foundation has changed insofar as we now acknowledge the plurality of structures – as we do in algebra – and this seed was sown by Hilbert. Mathematicians have followed this tradition, in algebra as well as in geometry, and this is what marks the progress.

## 3 It all started with algebra… doctorate under Hasse

***B: You have just mentioned that van der Waerden's book of Modern algebra was of great value to you.***
Yes, indeed. To be honest, this text brought me onto the right track, at the latest, I would say, when I returned to Göttingen in order to be awarded a doctorate after three semesters of studying at TH Danzig. Thereafter, I treated geometry based on this understanding of algebra.

***B: Maybe you could tell us a little more about Modern algebra, since you knew van der Waerden and Emil Artin.***
Well, I didn't really notice Artin in the literature back then. I focused on van der Waerden and, except for this particular work I mentioned earlier, I only took a look at a couple of works I had been recommended by Helmut Hasse; I even read a paper by Jean Dieudonné that he had published before the war.

I should probably mention that in 1939 I sent him a special print of my dissertation. Due to the war he obviously didn't answer this letter.

***T: So would you say that you were an algebraist with a number theorist as a supervisor?***
Yes, indeed. I made Hasse happy with a dissertation outside of his field of work (at least that's what I hope I did). Hasse then reached out for von der Waerden as an expert. It's been quite a similar situation for my fellow student Paul Lorenzen, who I got to know in one of Hasse's seminars after returning from Göttingen. Hasse took him on as a doctoral candidate as well and appointed Kohl as his expert.

***B: Is it true that Lorenzen's work was even further away from Hasse's field of work than yours?***

Yes, at that time Hasse was basically concerned with lattice theory – plain lattice theory. The Hasse Diagram is clearly a part of lattice theory; however, Hasse once told me that he wasn't very happy about such a simple thing being named after him.

***T: That's exactly what I have told my students recently; Hasse has clearly made more contributions than having invented the Hasse Diagram.***
In fact, I didn't find my way into the field of geometry due to Hilbert but later, in Tübingen, thanks to Hellmuth Kneser.

***T: Meaning after World War II?***
Exactly. While in Crossville (USA) [see Section 4], I had already started taking a closer look at dilations and reflections; however, I can't quite recall how that came about. It was only after my conversations with Hellmuth Kneser that I discovered my actual interest in geometry.

I even had my own ideas on free mobility: Helmholtz' Problem of Space. I just happened to come up with it and then I just got more and more into geometry; however, I have always been in the habit of doing algebra for myself.

## 4 Lectures as a prisoner of war in the US – the camp university

At the end of March 1943, First Lieutenant Günter Pickert became a British prisoner of war in Tunisia. Via Casablanca, he was brought to Halifax and finally reached an American shack camp in the Cumberland Plateau about 100 kilometres east of Nashville – with him were dozens of officers. After the Geneva Convention, officers being held as prisoners of war didn't need to work. From a physiological stance and for their own sake they needed to develop activities to compensate. Pickert was released from captivity at the end of May 1946.

***T: In one of your subordinate clauses, you just mentioned an American city: Crossville, which was the location of your time as a prisoner of war. What happened there exactly? Someone has told me that there was a university!?***
Exactly. In fact, we founded the university ourselves; the Americans didn't really contribute anything themselves. Later, the German ministry and a couple of professors started contributing and sent us some material. Even my wife sent me a couple of books at my personal request but basically we relied on our memories.

***B: How did that work exactly? Did you hold lectures or seminars?***
Well, among us there was a major who had been very interested in administration, so he took care of the administration, as did Graichen, a Bavarian philologist, who organised the philological fields. I was in charge of mathematics and also offered lectures. There were also a couple of interested colleagues with whom I worked: Mr Beysiegel, a meteorologist, who had been shot as a weather analyst but who had survived, and finally Mr Mangelsdorf, a deputy head teacher. Together we gave a seminar on quantum physics, basically from memory because we didn't have a lot of literature.

***B: Do you remember which lectures you gave there?***
Well, once we taught analysis based on a book by Rudolf Ernst Rothe, an applied mathematician from Berlin. Some people were very interested indeed. Lieutenant Paschen, for example, who lived a couple of shacks further up the plateau, asked me: "Master, this explanation is not sufficient. I have to ask for private tuition." To be honest, I suppose that's where I gained my first experiences concerning didactics. I, for my part, taught descriptive geometry, basically by means of pen and paper since we didn't have a blackboard, and I relied on what I had heard during the lectures at TH Danzig.

And of course there were also language classes. I myself actually took part as a student in such a class, which was being organised by an experienced export merchant. There was even a class organised by an American officer, though he was more of a non-commissioned officer performing officer's duties; however, he did have a bachelor's degree in languages and consequently taught us.

Unfortunately, they stopped the whole language related undertaking because one of us managed to escape the camp by climbing the garbage containers. However, what we did when we were by ourselves couldn't be forbidden and, to be honest, they didn't really care.

***T: How many people are we talking about? How many people, soldiers and captives, were in these different courses with their different focuses?***
I treated von der Waerden's book with two participants: one of them was Major Bäru, whom I have been able to welcome as a guest here in Gießen; the other one had already studied for a couple of semesters. But of course we also had bigger courses; in analysis there were about 10 to 20 participants, but no more than that though.

My seminar on descriptive geometry was one of the bigger courses as well and finally I have to mention the seminar on quantum physics as referred to earlier. As a matter of fact, there was also a branch dealing with those who wanted to catch up on their Abitur. Oddly enough, this undertaking worked out just fine because the Ministry of Education later acknowledged these examinations. Just take Mr Förster as an example: he had left school with an intermediate school certificate and managed to do his Abitur in this way. Based on his attendance in my seminar on descriptive geometry, one student passed on some sketches during his studies in Stuttgart. I have received words of praise and have been told that I had finally taught my students some mathematics; that is indeed a cause of pleasure. I stayed in contact with some of them but unfortunately most of them have now died.

## 5 Jean Dieudonné and Bourbaki…

***B: You mentioned the name Nicolas Bourbaki earlier…***

Yes, indeed. I came across Bourbaki in 1946, when I was working as an assistant in Tübingen; inspecting the local library, I found his volumes.

To be honest, I didn't quite like them at first; it wasn't exactly the style I had been used to with von der Waerden. Somehow it appeared too abstract to me, until Bourbaki more and more came to life for me. I met Jean Dieudonné at a conference in Oberwolfach in 1949, which had been organised by Hellmuth Kneser and a French colleague. Jean Dieudonné participated with some of his students, among them Jean-Pierre Serre.

That's where I established contact with Dieudonné. He had been in Nancy earlier and that's how Bourbaki obtained a professor's chair at Nancago. Consequently, Dieudonné was one of the key figures in the whole Bourbaki affair.

Karl Heinrich Hofmann once told me – I almost didn't recall it – that I had slowly crept up on Bourbaki. Algebra and topology had been two essential structures of mathematics for me and, in this, Bourbaki's ideas weren't too far away from mine.

I know I am repeating myself but I still have to lament the disadvantages of Bourbaki's Procrustean bed: non-associative loops, very useful in the foundation of geometry, simply dropped out of his ideas. Non-associativity was merely tolerated within Lie algebra but you weren't allowed to think any further than that.

It was basically his claim for sole representation that repelled me. I have to tell you an anecdote: Dieudonné was a man of action and sometimes he could be very emotional in his comments. I experienced this for the first time during a conference in Aarhus. Hans Freudenthal was one of the participants and he knew very well how to push Dieudonné; he thought it was entertaining.

At one point, they were talking about the question of whether there was a reasonable way to use angles of more than 360°. Dieudonné was totally against it and literally screamed to the conference room that the idea was sheer nonsense and so forth. That was shortly before lunch break.

Then, after lunch break, the conference manager went to the tape recorder with a smile, switched it on and there was Dieudonné's screaming over and over again. The tension vanished into thin air and Dieudonné agreed with anything and everything; he had just needed to make his opinion known (strongly).

I also recall a later episode; it was during the 1960s at a conference in the old abbey of Echternach. Heinrich Behnke loved this abbey; he had always had a thing for this kind of prestigious historical building.

A Swiss colleague from Lausanne gave a talk on how to treat geometry. He set out different methods and was in the midst of assessing them when Dieudonné started screaming and couldn't be held back. When the chairman asked Dieudonné to express his opinion objectively, he was peeved and answered that he just couldn't catch what the Swiss colleague had said. So this incident was ironed out quite quickly.

***T: Why were Bourbaki and Dieudonné so far removed from geometry?***

We should mention that it was not about the old triangle geometry but more about geometry as a whole.

***T: Dieudonné is ascribed the quotation: "geometry is linear algebra".***

Basically, even Emil Artin used algebra in order to develop geometry. Artin is not too far from Dieudonné's ideas in that respect but the kind of geometry that is of interest when talking about projective planes didn't even exist in Artin's time. It was geometry simply deduced from the structure of the vector space.

There's another story. From time to time he was even complaisant. I saw that once. It was during a conference organised by George Papy, and Dieudonné and Marshall Stone were among the participants. Dieudonné gave a speech on the theory of integration and made a very disparaging remark about the Theorié boolean Américaine. The American Stone was in the first row and interrupted by crying out 'Je proteste'. Dieudonné answered immediately by saying: 'No, no. I didn't mean you!'

We all knew that he had been referring to Paul Halmos but not to Stone. I guess he had been lowering himself then but, as I have told you, it was the first time I met him during that conference in 1949 in Oberwolfach. It was very nice; I ran a race from the sawmill to the bridge with Serre and Martin Kneser. I came in third. Well, we were young…

***B: From your point of view, how did Bourbaki leave his mark on the field of mathematics and was it maybe in a way that was too extreme?***

Yes, as I said, it was the claim for sole representation casting a shadow over his work and, again, the Procrustean bed excluding anything that didn't conform.

***B: Today it is said that Bourbaki may be too abstract: no illustrations and so forth.***

That is certainly a way of seeing it but that is a fact that has actually never bothered me. It has been more the global principle bothering me, the almost violent system of organising mathematical fields. Non-conforming aspects simply went by the wayside.

## 6 Projective and affine geometry

***T: The richness of the internal structures of projective and affine geometry wasn't known in his time. Your book has revealed them.***

Well, in terms of geometry, Artin had already paved the way and we also have to mention the book by Wilhelm Schwan, which has been a little bit neglected. They have laid the groundwork.

***T: How did your book come about? What made you write it?***

I regret not being able to recall this in great detail. After having tackled subjects like free mobility, I simply realised that, for me, linear algebra and geometry seemed to be connected more and more closely. And then – well, I was inspired by questions coming from a student, who didn't study mathematics but philosophy and who wanted to understand more geometrico.

I started working more closely on the topic and published a small booklet in cooperation with the publishing house Otto Salle. I suppose this was the reason why I met my old teacher Karl Friedrich Schmidt and I do recall that he encouraged me to write another book following my publications "Einführung in die höhere Algebra" und "Analytische Geometrie", namely "Projektive Ebenen".

In the winter term of 1933/34, I attended one of F.K. Schmidt's lectures; he stepped into the breach as a guest lecturer. He came from Jena and gave a lecture on complex analysis. I couldn't even attend his lecture; I could only do the exercises but it worked nonetheless. F.K. Schmidt supported me then so that my book could come out. Back then, he must have known me from my time in Göttingen, even though I had only attended the seminars. In Hainberg, he took the whole crowd from his lecture to a tourist café and bought us coffee and cake, if I recall correctly. And when I introduced myself, he simply said: 'Well, so you are Mr. Pickert?!'

***B: Do you remember when you were actually writing your book "Projektive Ebenen"? I suppose you have to systemise a lot of primary literature when you write a new book. Usually, you come up with simplifications and different approaches and maybe even new results which may be added implicitly. Was writing your book such a process of systemising and knocking into shape?***

Exactly, you're hitting the nail on the head. I had read a lot, especially by Marshal Hall. His work within the Transactions of the American Mathematical Society had contributed a lot to what I could do later on, as well as the kind of geometries that could be described as a network.

***T: Was that already due to the influence of Reinhold Baer? When did you meet him?***

I believe, and I also pointed this out in a short abstract about Baer's colloquiums, that Reinhold Baer came to Tübingen as a guest lecturer following Hellmuth Kneser's invitation. That's where I met him. We had already been in contact earlier when I was dealing with the Helmholtz problem. At this point, my book was already completed in draft form so I was able to send it to him. Peter Dembowski was one of Baer's students back then and he proofread the corrections and fixed a few things.

***T: As is generally known, there are some older works by Baer, from the 1940s, in which he tries to manage the balancing act between geometry and algebra.***

Yes, yes, indeed. I have also been able to fall back upon the works of his student Hugh Gingerich. But, in fact, that had already happened earlier due to my personal contact with Baer, also while I was writing my book. I believe he had even sent me special copies. Up till then, we had known each other as scientists before we actually met each other in person.

## 7 Mathematics and its didactics

***B: Your name is inseparably connected to the creation of the didactics of mathematics. How did your interest in the didactics of mathematics come about in the first place?***

During the 1950s, it was only well-known specialist scientists who devoted themselves to didactics of the upper secondary grades (*Stoffdidaktik*). Maybe it was Hellmuth Kneser's influence; he had always been very interested in the topic. Based on his collected works, we pointed to lectures on the scientific foundations of mathematical school curricula. I believe it was in 1955 that Heinrich Behnke invited me to his annual Pentecoast Meeting. He seemed to have doted on me but I appreciated the recognition. Together with Wilhelm Schweizer, I established a seminar at the University of Tübingen. Schweizer, an honorary professor, invited classes to the university and demonstrated to his students his teaching within these classrooms.

***T: Wilhelm Schweizer is an editor of the famous series of textbooks the Lambacher-Schweizer isn't he? The books were first printed in 1945.***

Theophil Lambacher was only responsible for getting a printing licence after the Second World War, since he kept a clean sheet. Schweizer, however, had to leave school for political reasons after 1945. Later on, he was re-established as a principal. As far as I remember, Lambacher promoted some special approximations about $\pi$.

Erich Kamke teased him by calling him a proportional protestant, since people were chosen for the ministry at Stuttgart according to their affiliation to the various provinces and confessions. Lambacher himself had not contributed anything to the schoolbook mentioned above.

Schweizer was lecturing for prospective teachers at university and it happened that he said in front of his students: 'Well, as our little Pickert said yesterday…' He was pointing at my eldest son in his class, which led to loud laughter.

***B: You referred to a time when teacher students at university were instructed by well-established teachers as the Tübinger Schweiz, which you mentioned before. Their jobs were teaching at the Gymnasium but they were running courses – for a few hours – at the university. However, this picture changed and gradually professorships (for mathematics education) arose. When did this change happen?***

It was the time when the reform of mathematics' teaching at school and the change of curricula had the highest political priority. Historians in education call it the Sputnik Shock. Money was available. OECD was a major player and stakeholder. Thus, these reforms were carried out more or less at the same time in different European countries. Let me describe this transfer from the perspective of my university at Gießen.

First there were approved and established teachers, which brings in more, e.g. Gerhard Holland, Heinz

Schwartze, Elmar-Bussen Wagemann and Arnold Kirsch. The last person named (Arnold Kirsch) had, for some years, a position at the mathematical department. Qualification involved profound professional practice. By the way, I don't know whether teaching practice is still necessary for applying for a professorship in mathematics education.

***T: One of your contributions at that time, I do still remember, was a small, thin, beige-coloured booklet. You wanted to address teachers and introduce them to modern mathematics. It was printed at the Gießen mathematical department.***
You are right. I edited it together with Arnold Kirsch and it was intended to serve as material for in-service teaching courses. Although we were not experts, we included a chapter about elementary probability theory. Many years later, I met a retired religious teacher on a bus and he reminded me of that course, which was very helpful for him while teaching mathematics out in the field.

***B: Looking back 40 years, you have known many, many colleagues and researchers, as well as mathematics educators. Could you name some prominent representatives in mathematics education?***
Of course. I want to name the meritorious Hans-Georg Steiner, who died too early; however, I would like to comment that I was not content with some of his later publications. Arnold Kirsch was an excellent mathematics educator who unfortunately fell behind sometimes because he didn't push himself. Once again, I regret deeply that both are departed. Next, I remember many older colleagues, e.g. Freudenthal, Behnke et al., who served as examples and thus were influential – last, but not least, they worked as mathematicians.

***T: In the 1970s – I still remember – we read Freudenthal's "Mathematics as an Educational Task" in a seminar together. Browsing this book, I am still finding notes written in lead pencil, originating from our discussions and your comments.***
Freudenthal was on one hand an excellent mathematician and on the other hand a talented and encouraging mathematics educator. I learnt a lot through him.

***T: Freudenthal presented many times at your Didaktisches Kolloquium. However, I would also like to name some of your students, of whom some went on to teach me, e.g. Herrn Benno Artmann.***
You are right. I also want to highlight Benno Artmann – unfortunately, he died too early.

***B: You are someone who gets along with many people perfectly. How do you manage it? On the other hand, you are someone with the talent to formulate pointedly and perhaps cuttingly but I don't know of any enmity between you and anyone!?***
To be honest, I will exclude those who felt offended by my style. The harshest resonance that I came across was clearly from my colleague Helmuth Gericke. I don't remember exactly the situation. Maybe it was some syntax lapse of Gericke. I pointed out to him this deficit and I believe that I deeply offended him. He returned the letter telling me that he didn't want to possess such a letter. By the way, generally we had a normal relationship. I was reminded of a low German proverb: 'Let the farmer keep his piglet; he only possesses one'.

***B: I think that there are only a few who felt offended.***
Well, it was my style not to dispute but to clear up my position with respect to my opponent. A colleague once characterised my style as follows: if you receive a letter from Günter Pickert, the letter starts with lauding passages, followed by many pages of critical comments.

## 8 Activities during teacher training

***B: However, maybe you could tell us a little bit more: I believe that, in the 1950s, prospective teachers only attended lectures on mathematics. Can we assume that they were processed quite well? Today, teacher trainees explicitly learn didactics, not only implicitly by attending lectures held by good teachers but explicitly because they are specifically being taught. I am very interested in your opinion on this development. Do you think this is important progress or an important step?***
You both know my scepticism. I regret that many authors in publications on mathematic education restrict their insights and reports to case studies. It is not uncommon that the mathematical framework is fading or in some cases totally ignored.

***B: Let me add more. In your time, didactics was reduced to pure, nonetheless very wide-ranging, subject-matter didactics. Today, subject-matter didactics hardly has its place within the field of mathematics didactics, which merely focuses on didactic processes!?***
Not to be misunderstood, but discussing and exploring didactical processes might be relevant; however, without referring to the mathematical background didactical processes, it is fragmentary.

***T: Well, it also focused on the university education of teachers for secondary school level I and for primary education – as we would phrase it today. There was a cycle of four semesters, called 'The Scientific Foundation of Mathematics Education', offered to teacher trainees by the Independent Department of Educational Studies (AfE) at the University of Gießen.***
Yes, indeed. This was necessary and finally quite good. Here, again, I was following in the tradition of Hellmuth Kneser, since he himself had developed such a series of lectures, which you can find in his collected works.

Actually, I had been thinking in this direction and then I realised that unfortunately very few students being trained for 'Realschule' and 'Hauptschule' even enrolled for mathematics because they didn't get along with the general lectures. Therefore, I simply invented a new course, which was slightly slimmed down compared to what Kneser required for his lectures.

At times this was very deep mathematics. However, I simply compiled the most necessary elements and so designed the respective courses. Actually, they can still be found – although slightly varied – and they have inspired others to design similar courses. I designed and offered my courses off my own back; actions speak louder than words, as they say.

***T: You have commented on the question 'Is there actually such a thing as school mathematics?' to which a mathematics educator critically responded.***
Yes, I made a little bit of an effort there and I also acted in an advisory capacity to the publication of a schoolbook, Andelfinger–Nestle by Herder Press, but I have to admit that it's partly my fault that the book hasn't received the right attention.

***B: Why would that be the case?***
Well, let's put it this way: we simply planned to do too much.

***T: There is this one sentence that I remember, if I'm allowed to quote: 'The camels don't find the oases.'***
[laughing] Did I say this?

***T: Yes, indeed. In this book, there were so-called oases, in which problems were compiled.***
Yes, yes, I remember. Well, so there were Andelfinger and Nestle, students of mine at Tübingen University, and they asked me to participate in their project, in an advisory capacity. Mr Reith once put it this way: in the covering notes for teachers, Mr Pickert tells you how it should be done properly. However, I believe that I made too many suggestions which they incorporated in their book and this might have led to the failure of the book.

***T: I do remember a second remark of yours, that you sometimes made in your respectable didactic colloquiums: '…often I feel like a salesman for vacuum cleaners, a salesman who only hardly knows how to use a vacuum cleaner.'***
I really don't remember… well… I mean, it's especially difficult if he comes into a flat without sockets.

## 9 Teaching and learning mathematics

***T: We all know that mathematics is not particularly one of the best-liked subjects. Do you have a key for changing this attitude amongst students and teachers and for achieving success?***
In my opinion, it is possible to achieve success by means of a little bit of mental training, regardless of the attitude towards the respective subject. However, this might be a little bit trickier in mathematics than elsewhere.

***B: I want to stress my colleague's question even more. If I understand you correctly, you are convinced – just as we are – that especially the making of mathematics, the close reading and precise working can even create fun.***
As mentioned earlier, if I recall the Mathematikum Giessen, I believe that the reception of mathematics has changed a bit due to your commendable works and those of your fellow colleagues. However, working with mathematics is not fun per se, even if you have fun doing so.

Comparing it to sports, why do people do such things? Climbing the Himalayas and spending 40,000 euro on it… But nobody wonders why they do it. If someone enjoys doing mathematics, people start thinking you are crazy. Nonetheless, we can help bring mathematics closer to the general public. Of course, its utility doesn't necessarily mean anything, insofar as it doesn't matter to students if you tell them 'that's needed for technology'… I don't believe in these kinds of justifications.

***B: Additionally it's not always only demanding mathematics that is being used there. It can be a long journey to understand the technology before you can finally see where they used the mathematics.***
Let me put it frankly. The way you introduce people to mathematics is not trivial. In my time, they often talked about the didactic principle of joy – doing mathematics is supposed to be fun!

We should remember Zeitler's thesis of enjoying mathematics, pushing the didactics of joy. Surely it is not always easy to realise but we should try connecting routine exercises to interesting problems. In older schoolbooks, there were stories of converting formulas that filled pages and were absolute nonsense. We should connect this to quadratic functions, for example.

There are different ways of enjoying yourself and one of them is mathematics. However, mathematics polarises people either totally against it or completely supporting it.

My sons didn't inherit my enthusiasm for mathematics. The eldest became an engineer and he even had some mathematical questions from time to time; the second one works as a senior public prosecutor and believes that jurisprudence is the pearl of all science. I mean, it was never my intention to point them in a certain way towards mathematics. In my position as a father, I responded to their questions but, as I said, my youngest son always simply wanted to know the result. I insisted on working out the right result together. His reaction was: 'No, no, then I'll just copy them from someone tomorrow morning.' Nothing could be done there!

***T: But isn't this one of the central problems of mathematics?***
Yes, of course. It's difficult. On the one hand, you have to teach certain contents because they are basically the foundation of a lot of different educational contents and, on the other hand, it is supposed to be fun. That's quite hard to combine.

***B: Coming back to your occupation as a professor, I realised a very interesting tension there. On the one hand, you are someone coming from algebra, from very exact and abstract work; on the other hand, you have always been an advocate of descriptive geometry, so basically***

***very exact sketching. What I am interested in is whether you actually think in images or in mathematical formulas?***

***T: You also sketch it right? But you have never published those sketches in your works, have you?***

Well, the bin is part of a mathematician's work equipment, right?

***T: Yes, that's true.***

No, I mean I haven't burnt all of my bridges; I have even pointed them out in my works. However, I don't believe that I think in images so much. I think I see the connections with regards to mathematical formulas more clearly and I always look out for my students to have mastered the syntax as well.

***B: Are you talking about mathematical syntax or also linguistic syntax?***

Linguistic syntax is taken for granted, I believe. I was referring to mathematical syntax. You can't get the analysts to give up talking of  when referring to the circle, but never mind. As long as you know how they mean it… As long as you know how it's done properly, you can behave badly.

***T: Learning mathematics surely is active mathematical engagement, as Freudenthal put it.***

I always say that learning mathematical formulas by heart is nonsense; you should learn them by means of application. If you just use them often enough, you will eventually use them automatically. I have never memorised the formulas for solving quadratic equations and I have done so on purpose. I always do the quadratic completion myself; that's a lot easier.

***T: Regarding parabolas, I have never used the root formula for cubic functions either.***

Yes, Cardano. There's no sense in that; you're not supposed to learn something like that by heart. In case you need it, you should try and find out for yourself how to solve the cubic function. That's better. So, in that respect, I'm against learning by heart. I have never learnt anything by heart in mathematics myself. But I have always worked with it and, in time, you just do so automatically. That's the way things go in everyday life as well.

***T: So you achieve understanding whilst working and applying them?***

That's when you have to learn it. Strangely, the use of brackets is regarded as something very typical for mathematics. I remember my neighbour in Tübingen, an elderly dermatologist, who used to greet me by saying: 'So, do you want to open and close your brackets again?' That was the one thing he recalled from mathematics, even though there are styles of writing without brackets.

You don't even need variables – Bourbaki pointed this out in his book on foundations – if you connect the positions in which the same variables occur by means of an arc. You simply need to insert the same element in the respective positions. I have always fought against regarding general numbers as a special kind of number. Fortunately, it seems to have become practice to understand the term with spaces.

From my fellow student Paul Lorenzen, I learnt to be aware of mathematical syntax. I followed his operational justification of mathematics also in a very intense correspondence. We met, as I mentioned earlier, in one of Hasse's seminars.

## 10 Mathematics in society and the perspective of mathematics

***B: Maybe we can talk about another aspect which could be characterised as the social standing of mathematics. Do you think that there have been changes during the nearly 100 years of your life?***

Well, you, Mr Beutelsbacher, have clearly made a contribution to these changes.

***T: If we look into the future, how would you judge the perspective of mathematics?***

That's difficult, indeed. As Hellmuth Kneser replied around 1950 when asked what he thought about mathematics in the 20th century: 'That's very difficult, especially during the second part.' Therefore, I don't dare give a prediction. Even though I'm still following the literature, I am too branched off the topic.

***B: If you look back on the mathematics of the 20th century and maybe compare it to the mathematics of the 19th century, which was also full of great mathematics, do you think they are equal in rank or is 20th century mathematics better or did they focus more on the substantial topics during the 19th century?***

You mean in comparison?

***B: Exactly. Has mathematics changed to the same extent that we can observe for the changes during the 19th century?***

Yes of course, but maybe in a different direction. On the one hand, there is a strong tendency for formalisation, initiated by the basic research of Hilbert-Bernays, and I have also contributed my share. With some gratification, I note that my colleagues from Münster thought I had done quite a good job there.

I was especially concerned with bringing this into mathematics. Perhaps it's not that good because it keeps away the fantasy but I intended to do it as precisely as possible. So this is one of the main characteristics of the 20th century. I don't think it was as central to mathematics during the 19th century. As Freudenthal once maliciously told someone who had pointed out a gap: '…and this is an axiom!' Freudenthal was always quite cutting.

***B: I should formulate the final word. We wish you further pleasure with our field within society, wisdom for our professional group and luck for sound health and continuous mental clarity. We personally look forward to celebrating a very special birthday colloquium with you…***

I thank you for showing such great interest in my person, even though my expiry date is already by far exceeded.

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*Günter Törner is first of all a research mathematician. He is still working today, especially in the field of noncommutative algebra, and has been working for more than 30 years on noncommutative valuation rings and generalisations and discrete mathematics. Since he has been involved in secondary teacher education, he is also engaged as a researcher in mathematics education. His research interests are problem solving, belief theory, professional development of teachers and particular topics linked to epistemology of various philosophies of mathematics. Since he regards himself as a commuter between mathematics and mathematics education, he has twice been elected as Secretary of the German Mathematical Society. In addition to all this, he also runs small cooperation projects with companies in the area of optimisation and scheduling theory.*

# Remembering Grothendieck – An Interview with Jacob Murre

Ulf Persson (Chalmers University of Technology, Göteborg, Sweden)

***You may be, amongst those still alive, the one who knew Grothendieck the longest; in fact, you were almost exact contemporaries. When was the first time you met him?***
It was in the Spring of 1955 in Chicago. As to the first remark, this cannot be the case. Serre, Ribenboim and Cartier definitely met him earlier and undoubtedly knew him much better.

***Anyway, this is a pretty exclusive set. What were you doing in Chicago? Were you a post-doc?***
No, I was still a graduate student, but my advisor Kloosterman had sent me to Weil in Chicago to learn algebraic geometry.

***So what was Grothendieck doing there?***
He was actually at Kansas at the time doing functional analysis, or maybe he had already moved to homological algebra. Weil had invited him to give a lecture because he had already acquired quite a reputation as an upcoming bright mathematician.

***So what was your first impression? How did he appear? Had he already shaved his head?***
To disappoint you, I do not remember much of his visit. I went to his lecture, which was on functional analysis. I had, at the time, naturally no inkling that he would become one of the very greatest mathematicians of the 20th century. As to his appearance, I have no recollection, but if it were extreme in any way, I certainly would have remembered.

***When was the next time?***

**J. Murre, Angers, France, July 1979. Courtesy of Ulf Persson.**

That was at the ICM in 1958 in Edinburgh where he gave a famous lecture outlining his visions of the development of algebraic geometry. Unlike the first time, this made a really deep impression on me. I was even able to ask him some questions later during the congress. But our discussions were, of necessity, rather superficial; he was the centre of attention, always surrounded by people. He did give me a preprint though (written by Borel and Serre) on his work on the Riemann-Roch theorem.

***And this was when your relationship started in earnest?***
I would say that happened the following year when he wrote to me to ask whether I could generalise the key

theorem of my PhD thesis to also hold in mixed characteristic.

***By the way, what is the key theorem about?***
It is the so-called "linear connectedness theorem" saying that the total transform of a smooth point by a birational transformation is linearly connected, meaning that any two points can be joined by a sequence of rational curves inside the transform. However, I was only able to prove this over a field.

***So you mostly communicated via letters?***
In fact, that same year, Nico Kuiper, later to become Director of the IHÉS but who at the time was a professor at the Agricultural University in Wagening (NL), invited him to give a lecture there. I attended the lecture of course and, afterwards, Kuiper took us to his house and I finally got an opportunity to speak extensively with him.

***What did you talk about?***
The Picard variety, which at the time was very much at the centre of interest. Matsusaka, Weil and Chow had already constructed it algebraically but Igusa had discovered pathologies in positive characteristic. They were very mysterious. Grothendieck knew it all of course and I asked him whether his new theory of schemes would be able to explain and even remove those pathologies. Grothendieck told me that he had not yet given those matters serious thought because the theory would be treated in Chapter XII of his forthcoming EGA.

***This is a remarkable statement. He was really planning ahead; it also bespeaks great confidence in his powers.***
Yes, he was very confident that he would clarify the questions when he got around to it. Not only that, he claimed that the people just mentioned made too strong assumptions and tried to prove too little. He would make lesser assumptions and prove more.

***You must have been very impressed, or did you think he was merely bragging?***
Let me say that my attitude was one of scepticism. At the time, I wisely said very little.

***But he was not bragging?***
Of course not. He did eventually fulfil his promise three years later, if not actually in Chapter XII of his EGA but instead in his two beautiful Bourbaki lectures (232 and 236), where he constructed the Picard Scheme and thereby explained and removed all the pathologies.

***You must have been impressed?***
Very much so.

***But let us backtrack. You were brought up on Weil's foundations; what was your attitude to schemes initially?***
I had certainly made much effort to learn the language of Weil and thus I was naturally very hesitant to jettison all that effort in order to acquire yet another language. But I think the word "language" is misleading, although I know it is often used in this context. I would prefer the word "theory". In the end I decided to ask the advice of Weil. I trusted him very much and was convinced that he would give me the right advice. By that time, Weil had already left for IAS at Princeton and, in the Spring of 1960, I was at Evanston and I made a visit to the institute.

***Weil has the reputation of being a rather nasty man and many people admitted that they were afraid of him. I have also heard that Weil was rather jealous of Grothendieck and his advances in algebraic geometry as he felt dethroned. It must have been a very sensitive subject to bring up with him.***
First, let me point out that Weil has always been very kind to me and I am and will always be very thankful for all the things I have learned from him…

***…That makes perfect sense. If a great mathematician is 'nasty', it is because he does not suffer fools gladly…***
…Let me finish. I visited Weil and we took a walk in the surrounding woods, which all visitors to the institute are very familiar with. I then brought the matter up with him.

***What did he say?***
He said 'Grothendieck is very strong. He has done things none of us have been able to do'.

***Whom did he refer to specifically?***
I did not dare to press him on that point. He had made his point. The master had spoken and the message was not only clear but, as it would turn out, very great for me. So from then on I started to study schemes.

***So when did your collaboration with Grothendieck start?***
I would not call it a collaboration – it was not that close – but I think that it was in 1961 when he invited me to IHÉS and I went there in the first half of 1962.

***So you went to Bures-sur-Yvette?***
No. At the time, the famous SGA seminars were in fact still held in Paris, in the 16th arrondissement, in a building of the Fondation Thiers. But I lived out in Bures in one of the apartments the institute had acquired.

***Just to get the flavour, could you describe the scene?***
I will do my best. It was always held on a Tuesday afternoon. Arriving before the lecture, I would typically find Grothendieck and Serre engaged in a lively discussion. Dieudonné was there of course and during my term Néron was a visitor too. Then of course there were all the students of Grothendieck.

***Who were they at the time?***
I do not recall all the names but certainly Demazure, Gabriel, Verdier and Raynaud, along with Mme Raynaud. But you can see that Grothendieck was always busy – so much demand on his time – so there were few opportunities for me to speak to him.

***Are there any other things you remember? Did Grothendieck do all the lecturing?***
No. During the first few weeks Néron gave a series of lectures on his theory of the Néron model, following upon Grothendieck's regular lectures. They were, however, phrased in the language of Weil and I therefore suspect that they were a bit difficult for most of the audience to follow. As to other things, I should not forget Mlle Rolland, the secretary, who saw to all the practical things and made it all run so smoothly.

***So, nevertheless, you had few opportunities to talk to Grothendieck?***
At the seminars, yes, I had few opportunities, but Grothendieck also invited me to his home. At the time, he was living with his family in Paris, on the île de la Jatte to be precise.

***This sounds exciting. Could you please tell us what was going on?***
Luc Illusie has described them beautifully in his note 'Reminiscences of Grothendieck and his school', his experiences being very similar to my own. But to be more specific as to my own, I would arrive after lunch and be alone with him. Naturally, I took advantage of the opportunities and asked him a lot of questions, no doubt very simple ones, maybe even occasionally stupid ones but, no matter what, he would always be very patient and explain carefully, even what to him must have seemed very elementary ones.

***Illusie has also told me of this experience with Grothendieck. He was never at a loss for an answer I take it.***
Not always. Sometimes, if very rarely, he did not know of an answer.

***What would happen then?***
He would say something to the effect that he thought that he had considered the problem. Then he would turn around and open a cabinet just behind his chair. The cabinet would be crammed with handwritten manuscripts and he would take one out, glance at it and then come up with an answer.

***What kind of questions did you ask him about?***
As you surely know, the final written versions of his work are so general and overwhelming – I would even say intimidating – so mostly I asked for clarifications.

***And he was able to give those, without intimidation?***
Yes, very much so because when you discussed with him privately it was so different. He always took, as a starting point, a natural problem in order to relate it to his ideas, which consequently became so much more understandable.

***Nothing beats a personal discussion to convey mathematics.***
That is very true. To hear him explain his marvellous ideas and to see how his brilliant mind attacked problems are what I treasure most among my mathematical recollections.

***So you would have him all to yourselves during those afternoons?***
You make it sound as if it were a very regular occurrence; in fact, it did not happen that often, but often enough. And typically after our afternoon sessions, he or his wife Mireille would ask me to stay for dinner. They were very hospitable. Invariably after dinner Grothendieck would resume expounding his ideas and often I got so engrossed that I had to hurry to catch the last train out to Bures.

***So your switch from varieties to schemes turned out to be a wise investment?***
Very much so. For my generation, it was a revolution. In fact, during my first visit to his place I asked him why he had come up with the notion of a scheme, when varieties constituted, and still do of course, such a beautiful subject with lots of deep theorems and challenging problems.

***And what did he say?***
Basically, he claimed that nilpotent elements exist in algebraic geometry by nature. To neglect them, i.e. to remove them, is an artificial, not to say a brutal form of surgery, akin to amputation. They are there for a good reason. To ignore them leads only to confusion, even to pathologies. By taking them into account, not only will we rid ourselves of pathologies – we will also understand varieties better and get new powerful tools to attack classical problems involving varieties.

***And what did you think of that?***
It opened my eyes. Contrary to what many may think, Grothendieck did not develop the theory of schemes just for the sake of generalisation. The reason, or at last one of the main reasons, was that you needed schemes to understand varieties.

***And you agree?***
Of course. To give just one example, the pathologies of the Picard variety in positive characteristics appear because you should really consider the Picard scheme. Technically, a scheme is needed to represent the Picard functor. And besides, the power of the nilpotent elements is shown in his attack on the fundamental group of a curve in positive characteristics by lifting the curve to characteristic zero. I cannot emphasise enough that in Grothendieck's approach to mathematics, he was never striving for generalisations for their own sake.

***Although this is a natural conclusion when you encounter his written work.***
Yes, maybe, but the key concept is not generalisation but naturalness. He was always looking for the natural context and with his fabulous insight and intuition he was almost always able to find this context, which, however, I must admit with some regrets, required generalisations.

***So those were forced upon us?***
Very much so.

***So this is a faithful summary of his philosophy?***

Very much so. Whenever he explained something to me, I could always sense this underlying strategy of his. By the way, I would like to return to my pet topic of the Picard functor.

***By all means.***
As I have already referred to, during that term I attended his two Bourbaki lectures on the Picard functor. In his construction of the Picard scheme he followed, more or less, Matsusaka's original construction of the Picard variety, with the crucial exception of replacing Chow points with the Hilbert scheme. This relies heavily on projective methods and thus the case of a proper variety over a field was not covered. This gave me the rare opportunity to explain something to him instead, which, needless to say, made me very happy. Between his two lectures, I told him about the construction of the Picard scheme in this particular case. Of course I would never have been able to produce this construction had I not been properly instructed by him. After a long struggle, I had finally understood his results on pro-representability of functors, and the existence and comparison results of EGA III furnished me with a powerful tool to enable me – at least over a field – to characterise functors representable by a commutative group scheme from which my insight on the Picard functor in the proper case dropped out.

***This must have been a very satisfying experience to you. How did Grothendieck react?***
He saw immediately that it was all correct, and during our subsequent discussion he even suggested some simplifications which I later incorporated in my paper. I should add, though, that my results were subsequently surpassed by the work of Mike Artin on representability of algebraic spaces.

***So, at that time your collaborations with Grothendieck would start in earnest and continue throughout most of the 1960s? I take it that you were a regular visitor to IHÉS.***
As I have pointed out before, 'collaboration' is too presumptuous a word to indicate my relation to Grothendieck. As to my visits to IHÉS during the year, they were indeed several but, because of my duties at my home institution at Leiden, I was normally only able to visit for a few days, with two exceptions. In 1963, I was able to stay for a month and in 1967 for a couple of weeks. Those visits were also somewhat different as IHÉS had definitely moved to Bures in 1963, and so had Grothendieck with his family – later he would move on to Massy.

***Do you have some poignant recollections from that period?***
I have at least some that stand out. In particular, back in 1963, when I was on the train with him to attend a lecture by Hyman Bass to be given in Paris, we started to talk about what we would do when we were old. Grothendieck expressed a wish to become like Zariski, meaning following and enjoying the work of his former students.

***But it would not turn out like that.***
No, sadly not.

***Anything mathematically that stands out?***
It would be his incipient theory of 'motives'. The first time I heard about it was in the Fall of 1964, when I made a visit in preparation for a Bourbaki talk I was to present the following Spring. During a break in our intense discussions, I asked him what he was working on at the time. He disclosed that he was working on a new theory, a theory he referred to as of 'motives', which would finally explain the similarity of all cohomology theories, and he elaborated a little on his ideas. Later on, in 1967, he gave a series of lectures on his theory but unfortunately I was unable to attend them. I was later informed about them by Manin, who had been in attendance.

***You keep telling me that you did not collaborate with Grothendieck, yet there is a joint paper, not to say a monograph, with him.***
Let me put it this way. Grothendieck was always very generous in sharing his ideas. The paper to which you are referring started like this. Grothendieck and I took a walk together – I am not so sure of when, most likely in 1968 or 1969. He told me that he wanted to study the tame fundamental group of a normal point on a two-dimensional scheme, in a way similar to Mumford's classical study. He already had an idea of how to do this and had in effect solved the major part of the problem; however, there remained some technical parts he had not yet resolved. He suggested that I look into it, as he had more pressing things to attend to. On my return back to Leiden I struggled with them and, after some time, I was able to sort out the remaining parts, and of course I wrote to him. He suggested I should publish those results on my own. I protested in my next letter, pointing out that the idea, as well as a large part of the solution, was due to him. The only honourable thing would be to write a joint paper, and he agreed.

***We are now approaching the end of the 1960s and, with that, the end of the Grothendieck era. Can you report on its twilight?***
I would not use that word. It indicates a decline that was not present.

***But you could perhaps see signs?***
Signs are often more pronounced afterwards than at the time when you have no idea of what they may portend. To give an example, the last time I visited Grothendieck at his home was in 1969. He had by then moved to Massy. Formerly, he had never complained about his tasks and duties but, this time, he admitted that writing EGA and taking care of SGA took a lot of his time. As I usually did when visiting him, I asked him about an update of the status of the Weil conjecture. He said that he would not be surprised if one of these young people came up with a solution…

***…Did he mention any names?***

He mentioned Deligne and Bombieri. He thought so because he suspected that only one new idea would be needed to overcome the present deadlock.

***And he was right?***
As usual he was, although the new idea was far from what he had hoped for and expected.

***And on this we need not dwell. Was this also the last time you met with him?***
No, not quite. I remember how, in the evening of that final visit, he walked me back to the station, barefoot. I was staying at IHÉS as usual. Actually, the last time I met him in the flesh was the following year, at the ICM at Nice. By that time there was a definite difference from before. His interest had shifted from mathematics to ecology. 'Survivre' was his great preoccupation. I actually joined him at a meeting of 'Survivre'. Afterwards, I told him that I got the impression that a majority of the participants did not share his idealism and they were only struck by his celebrity status. As was to be expected, he strongly disagreed. I also pressed him about mathematics. He claimed that he was still interested but there were far more important things to do.

***Such as surviving?***
Yes. He was very pessimistic. If the world continued the way it did, there would be a time, soon in fact, when, among other things, it would be impossible to do mathematics.

***He had some major ecological disaster in mind?***
Obviously.

***But he was not right this time?***
Depends on what you mean by 'soon'.

***Did your relations end at this point?***
No, they did not. Although we would never meet again, we did keep up a correspondence.

***A frequent one? And on what did you correspond?***
I would not say it was frequent. Sometimes a lapse of a year would occur between letters. While initially our correspondence had always been on mathematics, after 1971 this stopped and we confined ourselves to write about commonplace things.

***So it was a correspondence between friends, not colleagues?***
Yes. I did once broach a mathematical topic after 1971. I had sent him a reprint of a paper I had written on the motive of a surface and dedicated to him. I also asked him a few questions about motives. He acknowledged the paper as a nice one but, as to my questions, he simply wrote that he had not thought of such questions for a long time.

***So there was nothing controversial about your late correspondence?***
No, with one exception, which led to a minor crisis. He had sent me his 'Récoltes et Semailles'...

***…which he had already written in the 1980s but whose existence did not become more widely known until later…***
This is true. I read parts of the manuscript, which was painful enough – not the whole thing; that would have been impossible for me. It was painful not only because I had problems reading it in French but, more to the point, because I disagreed with him on so many points. The matter being delicate, I chose to respond only superficially. He was very disappointed by my response. I realised how depressed he must have been while writing it and I wrote back that although I could not agree with many points, I had not behaved like a friend and regretted it very much. He accepted my apology and after that our relations returned to normal.

***But not indefinitely.***
That is true. My last letter to him was dispatched in 1991. It was returned to me stamped 'undeliverable'. After that I completely lost contact with him.

***But it was not personal?***
Not in the sense that (as I subsequently learned) this was the case with all his former friends and colleagues and that he became a recluse in the Pyrenees. On the other hand, how can you experience it as not personal?

***What is your lasting impression of Grothendieck?***
Of course I admire him as being one of the greatest mathematicians of the 20th century. But I also admire him for his human qualities.

***Such as?***
His honesty and his principled stands, against the military and for the poor and the weak.

***You do not find him naive in some of his stands?***
Of course he was naive. 'Improving the world' is very different from doing mathematics. But nevertheless I admire his principled stands and his refusal to compromise his convictions. His anxiety for the future of mankind was sincere and, I am afraid, justified as well. It must have frustrated him and hurt him deeply that his mathematical friends and colleagues did not follow him and share his concerns and worries. He did not compromise, also when it came to himself and his life. He was logically consistent, not just in mathematics, and he accepted the consequences of it, also when it affected his personal life. This is what made his life so tragic in the end.

***But this is not the way you prefer to remember him?***
No, it is not. I want to, and I actually do, remember him as he was when we met in Paris and Bures. He was a genius of course but also generous and helpful, as well as being cheerful and optimistic. This is the image that endures in my mind and I find myself truly privileged not only to have met him but to have known him.

*For information on Ulf Persson please refer to issue 95 of the Newsletter, page 50.*

# Pursuing a Mathematical Career in Tokyo – Davis – Manchester – Tokyo

Yuji Nakatsukasa (University of Tokyo, Japan)

I am an untenured early-career researcher in mathematics (specifically numerical analysis) and I am an applied mathematician. I love to understand and contribute to the theory but I am also interested in what impact it has in real life. My career path is somewhat non-standard: having grown up in Japan until my Master's, I flew to California (UC Davis) to do my PhD. I then did a postdoc at the University of Manchester for almost two years and now I am back at the University of Tokyo. I therefore contribute this letter in the hope that sharing my experience and thoughts will be of some value to others, especially those who come from countries where their research field is not necessarily mainstream and who are considering studying or pursuing research abroad.

Here is an outline of my academic path so far. My flight to the US was not planned much in advance: my Master's degree supervisor had to move to another university – soon after I joined his group – and he offered me several options, including (i) changing group, (ii) changing university, and (iii) changing country. Without much thinking I chose (iii) and to make this happen I went to a workshop in Tokyo that a professor from Davis was attending. I spoke to him in broken English; though I did spend some years in the US as a kid, sadly I had lost much of it by then. I applied for Davis and got admitted. Most international students spend ages preparing for the TOEFL and GREs (Graduate Record Examinations – admission requirement for most US graduate schools) and apply for dozens of universities. Since in my foolishness I applied only for UC Davis, I do not have much advice to give here; Davis happened to be a great place for me but it is true that outstanding universities tend to have outstanding faculty members and brilliant students. However, if you already have a specific field that you intend to work in, it is important that you choose a university that has an active faculty member in that field.

Being a "typical Japanese" (some of my colleagues outside/inside Japan would disagree) who is somewhat introverted and reserved, getting used to American culture was a challenge and I don't think I have entirely overcome it. But it is indeed true that many behaviours that are considered impolite – or at least non-typical – in Asia are fine in the US. Reverse examples exist (e.g. making sounds when eating noodles) but they are arguably much fewer.

Academically also, I went through difficult and unstable times. For the first two or three years of my graduate studies, I was uncertain that I would be able to pursue a career as an academic. It took me about three years to get a grasp of the scope of the current body of knowledge in the specific field of numerical linear algebra, along with an idea of what would be a worthwhile contribution.

Meeting the right people at the right time was crucial in my career. I got to meet a leading figure in the field from Manchester when I was about to finish a paper, and getting positive comments gave me significant self-esteem and energy to work harder. I think it never hurts to be connected to brilliant established researchers, as they can share ideas and give you insights, and make connections with different fields, and even help your job-hunt.

One important aspect of being a PhD student in the US is that (quite often) you are financially semi-independent as long as your academic record is fine. An anecdote is that I did not take GREs, which verges on stupidity now that I think back. This resulted in me getting admitted to UC Davis but without any guarantee of financial assistance. Nonetheless, I ended up paying minimal tuition and survived more or less on my own during my PhD studies.

The typical student-supervisor relationship certainly varies between countries. The stereotype is that Asian education is a bit like a boss-servant relationship, whereas it is more colleague-like in Western countries. My impression is that there is much truth in this but it really depends more on the specific persons. I have had a few supervisors and they are all completely different. It is a very good idea to get to know your potential supervisor well before committing yourself. Many US universities give the students a few years to decide their supervisor.

Another important aspect is the timeframe for completing a PhD, which varies significantly from country to country. In the US, students finish when they are ready; some get a PhD in 2 years but 5–6 years is common and some spend even longer. In Japan, by contrast, almost everybody spends 3 years for a PhD, following a two-year Master's programme. In the UK, 3.5 years is the norm. Such differences can be important as having to get a job by a fixed date can be stressful.

Currently at the University of Tokyo, which traditionally attracts some of the best talents of the country, I am regularly amazed by the brilliance and talent that the students exhibit. I would nonetheless strongly encourage them to consider going abroad. To me the biggest advantage of being in the US or UK is that we get to listen to and talk to leading researchers from all around the world, as they visit to give seminar talks all the time. This is the aspect that I currently miss the most in Japan, as it is somewhat remote from many parts of the world. I believe getting to see and talk to your heroes has further benefits, in that you get to observe and inherit their work ethics: without exception the big names in my field all work very hard; seeing this had a huge impact in forming my work habits. I am also completely convinced that exchanging ideas facilitates progress significantly. And it really helps to be in touch with a few role models, not just one. You observe them, contemplate and choose your own style (e.g. I feel I am most creative in the morning in bed).

Another difference I noticed is that the classes are much more focused and detailed in the US than in Japan, sometimes involving open problems. This provides the opportunity to think deeply about one subject. Every mathematician needs to establish an area that they understand very deeply: a home ground. Without having one we cannot write papers, and mastering one subject generates confidence. Once we have one, acquiring the second is usually easier, as we start to see connections. My home field is numerical linear algebra and I don't know how many I will try to acquire in the future but I am certain that my home field(s) will provide unique guidance whenever I see a problem. It wouldn't hurt to try to have one early, perhaps before you contemplate going abroad.

*Yuji Nakatsukasa is an assistant professor at the University of Tokyo. He is a numerical analyst focusing on matrix eigenvalue problems. Originally from Japan, he obtained a PhD from the University of California at Davis in 2011 and was a postdoctoral research associate at the University of Manchester before going back to Tokyo in 2013. He was awarded the Leslie Fox Prize in 2011 and the Alston Householder Award in 2014.*

# A Presentation of the Italian Association of Mathematics Applied to Economic and Social Sciences

Marco LiCalzi (Università Ca' Foscari Venezia, Italy)

The Italian Association of Mathematics Applied to Economic and Social Sciences (AMASES) is a tightly knit mathematical society with a focused scope. It was founded in 1976 and comprises about 450 members, most of whom work or have professional collaborations in Italy. Its main goals focus on promoting theoretical and applied research, as well as general public awareness of all areas of mathematics as applied to economics, finance, insurance, management and social sciences at large.

This short note reviews the history of the society and highlights its present activities. The roots of AMASES lie in the fields of financial mathematics and actuarial sciences, where some of its intellectual forerunners used to work professionally before or whilst pursuing academic research. One was Francesco Paolo Cantelli (1875–1966), whose name graces the Borel-Cantelli lemma and the Glivenko-Cantelli theorem. He spent 20 years at the National Institute for Security Deposits and Loans, before entering academia as a professor of actuarial mathematics and founding the Italian Actuarial Institute.

Similarly, Bruno de Finetti (1906–1985) spent 15 years with Assicurazioni Generali at the beginning of his career. In recognition of its intellectual debt to him, AMASES named him Honorary President of the Association from 1983 until his death. Besides his role as a staunch promoter of subjective probability, he managed to lead outstanding careers as a statistician and as an actuary, as well as being an influential thinker on social and political issues. His combination of talents and his impact on improving society is still an inspiring example for the AMASES community.

In the 1960s, as the interplay between academia and mathematical business professions intensified, a small group of mathematicians from the faculties of economics, business administration and statistics throughout Italy realised the need for an institution devoted to coordinating and stimulating research and education in the mathematical applications for these fields. The first exploratory meeting took place in Trieste in 1966, attended by 15 distinguished applied mathematicians, including Bruno

**From the left: Bruno de Finetti, on his appointment as honorary president of AMASES (Bologna 1983), Luciano Daboni, Claudio de Ferra (both past presidents of AMASES) and Giuseppe Ottaviani.**

de Finetti himself and Giuseppe Ottaviani (1914–1994), a beloved student of Cantelli at the Faculty of Economics in Rome and his natural academic heir.

The association was officially established on 27 July 1976 by 35 founding fellows. Its first annual conference took place in Pisa on 4–5 November 1977. Since its inception, the official seat of the association has been located in Milan (currently at Bocconi University).

AMASES has been holding its annual conference since 1977, typically in early September. Every conference hosts a few invited lectures, aimed at representing the range of approaches and applications pursued within the scope of the association. This has now come to include fields as diverse as mathematical finance, economic theory, management science and decision and game theory, as well as computational techniques. The special attention of AMASES towards computation has a long history, as witnessed by the fact that the last Honorary President, Mario Volpato (1915–2000), was one of the founders and Vice-President of CINECA, the largest Italian computing centre.

**Harold W. Kuhn delivers his lecture "A Life in Optimization: Tales of Eponymy" at the 33rd AMASES Annual Conference in Parma, 1 September 2009.**

AMASES sponsors related research and actively supports satellite thematic conferences and summer schools. It has introduced special awards both for the best doctoral dissertations and for the best papers presented by young researchers at the annual conference. Under the umbrella of the Italian Federation for Applied Mathematics, it has joined forces with the Italian Association for Operations Research (AIRO) and the Italian Association for Applied and Industrial Mathematics (SIMAI) to promote a wider spectrum of activities in applied mathematics.

AMASES has been publishing a scientific journal since 1978. Until 1999, the masthead was *Rivista di Matematica per le Scienze Economiche e Sociali* (Review of Mathematics for the Economic and Social Sciences); this journal accepted papers in Italian, English and French. In 2000, AMASES expanded the scope of the journal and gave it a more international slant. The title was changed to *Decisions in Economics and Finance: A Journal of Applied Mathematics* (nicknamed DEF) and English became the only official language, while publication and technical assistance were entrusted to Springer-Verlag. The aims and scope state that DEF "provides a specialized forum for the publication of research in all areas of mathematics as applied to economics, finance, insurance, management and social sciences. Primary emphasis is placed on original research concerning topics in mathematics or computational techniques which are explicitly motivated by or contribute to the analysis of economic or financial problems".

*Marco LiCalzi holds a PhD in Decision Sciences from Stanford University. He is a professor of mathematical methods for economics at Università Ca' Foscari Venezia and has held visiting positions in France, UK and USA. He has served as secretary for Amases and as editor for its journal. His research interests lie at the interface of decision theory and game theory.*

# ICMI Column

Jean-Luc Dorier (University of Geneva, Switzerland)

## Inaugural Emma Castelnuovo Award for Excellence in the Practice of Mathematics Education

The ICMI is delighted to announce the first recipients of the Emma Castelnuovo Award for Excellence in the Practice of Mathematics Education: Hugh Burkhardt and Malcolm Swan.

We look forward to honouring them at ICME-13 in Hamburg next year with the other ICMI medallists.

The following is the full citation from the Award Committee chaired by Professor Jeremy Kilpatrick.

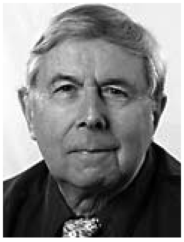
The Emma Castelnuovo Award for 2016 goes to *Hugh Burkhardt* and *Malcolm Swan*, University of Nottingham, Nottingham, UK.

It is with great pleasure that the ICMI Castelnuovo Awards Committee hereby announces that the 2016 Emma Castelnuovo Award for Excellence in the Practice of Mathematics Education goes to Hugh Burkhardt and Malcolm Swan in recognition of their more than 35 years of development and implementation of innovative, influential work in the practice of mathematics education, including the development of curriculum and assessment materials, instructional design concepts, teacher preparation programmes and educational system changes. Burkhardt and Swan have served as strategic and creative leaders of the Nottingham based Shell Centre team of developers. That team has included many talented individuals over nearly four decades, in parallel with the contributions of more recent teams of international collaborators. Burkhardt and Swan are selected because of their continuous leadership of this work. Together, they have produced groundbreaking contributions that have had a remarkable influence on the practice of mathematics education, as exemplified by Emma Castelnuovo.

Burkhardt and Swan's approach is distinguished by their efforts to address the problem of improving learning strategically and across all levels of education by:

- Designing activities for learners based on an understanding of their thinking.
- Designing lessons that promote deep learner engagement in those activities.
- Designing professional development to help teachers use the activities and lessons.
- Designing system change (e.g. in assessment, curriculum and teacher support) to promote the above.
- Encouraging educational researchers to value more highly the impact of change on the educational system.

In 1976, Hugh Burkhardt was appointed Director of the Shell Centre for Mathematical Education. Struck by the limited influence of educational research on what happens in schools, he decided to focus the centre's work on research and development aimed at having a direct impact on classroom practice. He saw tools for practitioners as key products, complemented by research papers on the insights that emerged. He saw the importance of excellence in design in determining the quality and impact of those products. Over the years, he recruited some exceptional designers of classroom materials and assessment instruments. His appointment of Malcolm Swan was a key element in the success of the many projects they have subsequently led, with Burkhardt leading the strategic design of the products and processes, and Swan leading the detailed design of the learning activities for students, the teaching approaches and the professional development programmes that support teachers in attending to a full range of mathematical practices.



**Hugh Burkhardt**

Since the late 1980s, the Shell Centre's work has been entirely dependent on funding of projects from outside the University of Nottingham. Through multiple administrative arrangements and formal name changes, the Shell Centre team has maintained a continuity of identity and purpose, built around Burkhardt and Swan, with contributions over the years from many other talented educational designers. The nature and quality of the work has appealed to funding agencies, so that funding has been continuous and has grown, building to a current team of about ten people in Nottingham and many more through collaborative projects. For example, a project that has received considerable attention is the Mathematics Assessment Project (MAP), which is a collaboration between the Shell Centre team and the University of California, Berkeley.

Its 100 Classroom Challenges, which are formative assessment lessons based on diagnostic teaching, have received over 3 million lesson downloads. Through the MAP and other projects, Burkhardt and Swan continue to have an extensive impact on mathematics teaching and assessment around the world.

Hugh Burkhardt obtained his PhD in mathematical physics in 1957 from the University of Birmingham. He served as a lecturer and then a senior lecturer in mathematical physics at the University of Birmingham from 1960 to 1976. Since then, he has been at the University of Nottingham, where he served as Director of the Shell Centre for Mathematical Education until 1992. He has subsequently led a series of international projects, particularly in the UK, USA, Australia and the European Union. He is the Project Director of the Mathematics Assessment Resource Service (MARS) and a Visiting Professor at Michigan State University. He founded the International Society for Design and Development in Education (ISDDE) to nurture a community of educational designers so that the quality of work improves through shared expertise, and he chairs the advisory board of its e-journal, Educational Designer.

Through his strategic leadership of the Shell Centre team, contributions to many of its influential products and development of its engineering research methodology, Burkhardt has made outstanding contributions to educational design and to thinking about structured educational change. He has worked on improving educational practice through the use of high quality assessment, fostering the synergy of research and development in educational design, and creating partnerships to work with educational systems, funding bodies and mathematics education experts. His initiatives often involve questioning established orthodoxies in mathematics education and design, resulting in innovations in the strategic and structural design of products that form the basis of new and more effective approaches. The impact on learning and teaching in classrooms has been his priority throughout. In 2013, he was awarded the ISDDE Prize for Educational Design for his lifetime achievement.

**Malcolm Swan**

Malcolm Swan obtained his Postgraduate Certificate in Education (PGCE) with Distinction in 1976 from the University of Nottingham and his PhD in Education there in 2005. He joined the Shell Centre in 1979 and until 2007 was a lecturer in the centre and the School of Education at Nottingham. From 2007 to 2008, he was an associate professor and reader in mathematics education, and from 2009 to the present, a professor of mathematics education at Nottingham. He directs the Centre for Research in Mathematics Education (CRME), which evolved from the Shell Centre. His research provided a basis for design research into materials for teaching and for effective programmes for professional development of teachers. His expertise is evident in the products of his role as hands-on "lead designer" for most of the Shell Centre team's projects.

Through multiple applied research projects with colleagues, Swan has led the Shell Centre work on developing and implementing tactical lesson designs and templates that enable teachers with a wide variety of personal skills to enact challenging pedagogy.

The imaginative tasks and assessments that have resulted are crafted to highlight significant points of learning on a wide range of topics. They are a testament to his creativity as well as to his understanding of mathematical learning, student engagement and the needs of teachers. In 2008, he was awarded the ISDDE Prize for Educational Design for the classic publication The Language of Functions and Graphs.

Burkhardt and Swan's educational vision for mathematical learning encompasses all strands of mathematical proficiency but focuses especially on conceptual development, mathematical modelling, problem solving and reasoning. Their vision of the classroom is one where students are active learners, learning through problem solving, discussion, reasoning and collaboration. The instructional materials, professional development materials and system changes coming out of the Shell Centre work have enhanced the mathematics education of millions of students worldwide. In summary, Hugh Burkhardt and Malcolm Swan are eminently worthy recipients of the first Emma Castelnuovo Award.

### Upcoming ICMI activities

- The XIV Interamerican Conference on Mathematics Education, 3–7 May 2015, Tuxtla Gutiérrez, Chiapas, México – http://xiv.ciaem-iacme.org/index.php/xiv_ciaem/xiv_ciaem.
- The 7th ICMI-East Asia Regional Conference on Mathematics Education (EARCOME 7), 11–15 May 2015, Cebu City, Phillipines – http://earcome7.weebly.com/.
- The ICMI Study 23 "Primary Mathematics Study on Whole Numbers", 3–7 June 2015, Macau, China – http://www.umac.mo/fed/ICMI23/.
- Psychology of Mathematics Education (PME39), 13–18 July 2015, Hobart, Tasmania, Australia – http://www.pme39.com/.
- Espace Mathématique Francophone (EMF2015), 10–14 October 2015, Alger, Algeria – http://emf2015.usthb.dz/.
- Conferencia Internacional do Espaço Matemático em Lingua Portuguesa (CIEMeLP) – a regional conference of the Espaço Matemático em Lingua Portuguesas (EMeLP – affiliated to ICMI), 28–31 October 2015, Coimbra, Portugal.

### Proceedings of ICME11 and ICME12

The first volume of the proceedings of the 12th International Congress on Mathematical Education (held in Seoul, South Korea, in 2012) has now appeared. The whole volume can be freely downloaded from http://link.springer.com/book/10.1007%2F978-3-319-12688-3.

The proceedings includes the speeches at the opening ceremony, the plenary lectures, the plenary panels, the survey team reports, lectures by the awardees, abstracts of the plenary lectures and more.

The ICMI is happy to announce that all the materials that were collected from ICME11 in Mexico can be found at http://www.mathunion.org/icmi/publications/icme-proceedings/materials-from-icme-11-mexico/.

We are publishing these materials on the ICMI website in lieu of the official proceedings for the benefit of the worldwide mathematics education community in general and the attendees of ICME11 in particular. Some of these documents have been edited but it has been a rather rough and incomplete process.

If anyone involved in preparing these documents wishes to resubmit a more polished version, we will be happy to replace the present version with its revision. We will continue to edit these pages as time and resources permit.

Should official proceedings appear in the future, they will take precedence over these documents. The ICMI invites its members and friends to send (or let us know of) revised versions, or any further materials that should be included with these materials, to Lena Koch, the ICMI Administrator: lena.koch@wias-berlin.de.

# CERME 9 in Prague: The Largest ERME Conference Ever

Konrad Krainer (Alpen-Adria-Universität Klagenfurt, Austria), IPC Chair, and Nad'a Vondrová (Charles University in Prague, Czech Republic), LOC Chair

### The goal of CERME and related ERME activities

The 9th Congress of European Research in Mathematics Education (CERME 9) took place in Prague (Czech Republic), 4–8 February 2015. These conferences are organised every other year by the European Society for Research in Mathematics Education (ERME), an affiliate organisation of the ICMI since 2010. Like at earlier CERMEs, a community called the Young European Researchers in Mathematics Education (YERME) organised a YERME day (3–4 February 2015) preceding CERME 9. Another important feature of ERME (supporting researchers entering the field) is the YERME Summer School (YESS), which takes place during even years. From 2016 on, the so-called ERME Topic Conferences will also be included. All these activities, including CERMEs, have a communicative, cooperative and collaborative nature. Thus, in contrast to most other conferences, CERMEs are organised as working conferences. They are "European" by definition but colleagues from all over the world are welcome; we are happy to learn from them and, of course, it is great when European research in mathematics education becomes better known and used abroad.

### The largest CERME ever

CERME 9 was attended by 672 people from 49 countries from all over the world (but mainly from Europe). The two biggest groups of participants came from Germany (104) and Sweden (63). The programme comprised – in addition to the plenary activities mentioned later – seven sessions of 20 parallel Thematic Working Groups (TWGs) and two additional time slots where the 20 TWGs could report their results to interested participants. Research papers and posters of TWGs had been reviewed and made available to all participants before the conference via the CERME 9 webpage to allow deep discussions to be held at the conference. During the conference, the papers and posters were expected to be further developed. This means that the pre-conference proceedings are transformed into post-conference proceedings after a thoughtful quality assurance process. The proceedings will be available by the end of 2015. The conference was organised by the Faculty of Education, Charles University in Prague, namely the Department of Mathematics and Mathematical Education. The Local Organising Committee (LOC) was chaired by Nad'a Vondrová and co-chaired by Jarmila Novotná. The International Programme Committee (IPC) was chaired by Konrad Krainer (Austria) and co-chaired by Uffe Jankvist (Denmark). Further members of the IPC were Jorryt Van Bommel (Sweden), Marianna Bosch (Spain), Jason Cooper (Israel), Andreas Eichler (Germany), Ghislaine Gueudet (France), Marja van den Heuvel-Panhuizen (the Netherlands), Maria Alessandra Mariotti (Italy), Despina Potari (Greece), Ewa Swoboda (Poland), Nad'a Vondrová (Czech Republic) and Carl Winsløw (Denmark).

### The 20 Thematic Working Groups (TWGs)

The 20 TWGs and their leaders had been selected by the ERME Board on the basis of suggestions by the IPC. Each TWG had a liaison person from the IPC and on average four co-leaders, aiming at a certain diversity regarding gender and region. Where possible, a young researcher was included in the team. The CERMEs are growing with each congress and it is a challenge to arrange the size of the groups to be manageable. This task was even more difficult this time, as 436 research reports and 106 posters were accepted for presentation at the congress. Although each TWG had its own call for papers and presented its specific focus and scope, some traditional groups had to be split, resulting in "sister groups". Examples were TWG15 'Teaching mathematics with resources and technology' (led by Jana Trgalova, France) and TWG16 'Student's learning mathematics with resources and technology' (led by Hans-Georg Weigand, Germany). There was even a triad of TWGs dealing with teacher education: TWG18 'Mathematics teacher education and professional development' (led by Stefan Zehetmeier, Austria), TWG19 'Mathematics teacher and classroom practices' (led by Despina Potari, Greece) and TWG20 'Mathematics teacher knowledge, beliefs and identity' (led by Miguel Ribeiro, Portugal). Thus, participants had to decide in which group they would work, based on a lot of interesting alternatives. Even TWGs which continued from previous CERMEs in their original form faced the same challenge. For example, the work on proof in algebra could have been submitted to TWG1 'Argumentation and proof' (led by Samuele Antonini, Italy) or to TWG3 'Algebraic thinking' (led by Jeremy Hodgen, UK) or even to TWG14 'University mathematics education' (led by Elena Nardi, UK). It could even have been connected to students' creativity and thus could have been suitable for TWG7 'Mathematical potential, creativity and talent' (led by Roza Leikin, Israel). Strong overlaps existed among other groups as well, for example TWG5 'Probability and statistics education' (led by Corinne Hahn, France) and TWG6 'Applications and modelling' (led by Susana Carreira, Portugal). If the paper was about early mathematics, it could have gone to TWG13 'Early years mathematics' (led by Mariolina Bartolini Bussi, Italy) or to TWG2 'Arithmetic and number systems' (led by Sebastian Rezat, Germany) or to TWG4 'Geometrical thinking' (led by Joris Mithalal, France). In contrast, TWG11 'Comparative studies in mathematics education' (led by Paul Andrews, Sweden) and TWG12 'History in mathematics education' (led by Uffe Thomas Jankvist, Denmark) were relatively independent. There were also some transversal groups which could include empirical studies from all the TWGs, i.e. TWG10 'Diversity and mathematics education: social, cultural and political challenges' (led by Lisa Björklund Boistrup, Sweden), TWG8 'Affect and mathematical thinking' (led by Pietro Di Martino, Italy), TWG9 'Mathematics and language' (led by Núria Planas, Spain) and TWG17 'Theoretical perspectives and approaches in mathematics education research' (led by John Monaghan, United Kingdom). This diversity of topics required the TWG leaders to cooperate intensively even before the beginning of the review process, deciding the most suitable group for the submitted work. Big thanks must go to all the TWG leaders and their co-leaders (from 23 countries!) whose hard work before, during and after the conference was indispensable for the organisation of the congress.

### Plenary activities

Presentations and abstracts of all plenary sessions can be downloaded at the CERME 9 website and full papers will be part of the proceedings.

The plenary panel "What do we mean by cultural contexts in European Research in Mathematics Education?" was organised by Barbara Jaworski (United Kingdom), in collaboration with Mariolina Bartolini Bussi (Italy) and Susanne Prediger (Germany), and moderated by Marianna Bosch (Spain). The team had been in discussion with a group of young researchers who had communicated their perspectives. On the panel, Edyta Nowinska (Germany) represented the young researchers Annica Andersson (Sweden), Mustafa Alpaslan (Turkey) and Marta Pytlak (Poland). The panel addressed several sub-questions including: 'How do cultural influences challenge the universality of research practices and outcomes?' and 'Which (hidden) values of your culture influence your research?'. The questions and related themes motivated many comments from the participants, leading to vivid discussions and interesting replies from the panel members.

The first plenary lecture "Research in teacher education and innovation at schools – Cooperation, competition or two separate worlds?" was presented by Jarmila Novotná (Czech Republic) and moderated by Andreas Eichler (Germany). The talk claimed that the field of research in mathematics teacher education has changed considerably over the years, which asks for a new definition of issues and trends. Thus, the focus of the first part of the lecture was on the main trends in current research in teacher education and practice. The second part of the lecture presented a more detailed discussion of several current research areas and their theoretical backgrounds, as well as applications of their findings in teacher education and everyday school practice.

The second plenary lecture "Understanding randomness: Challenges for research and teaching" was presented by Carmen Batanero (Spain) and moderated by Despina Potari (Greece).The talk stressed that ubiquity of randomness and the consequent need to understand random phenomena in order to make adequate decisions led many countries to include probability in the curricula from primary education to post-secondary education. The presentation reflected on the different meanings of randomness and on the different approaches to research on understanding randomness, with a particular emphasis on the European contribution. Finally, some ideas were presented to improve teaching and continue research on this topic.

These wonderful talks and discussions, as well as the poster presentations, greatly contributed to the success of the conference, allowing some very interesting scientific exchanges.

### Some further notes

Welcome and farewell addresses by representatives of Charles University in Prague (Vice-President Stanislav Štech, Vice-Dean Michal Nedělka), the EMS (President Pavel Exner) and the ICMI (former President Michèle Artigue) and of course ERME itself (President Viviane Durand-Guerrier, Vice-President Susanne Prediger) gave CERME 9 a special flavour, which was enhanced by splendid classical music during the opening and closing ceremonies, produced by students of the Faculty of Education – future music teachers. Meetings like 'ERME meets newcomers', the joint report by the ERME Board and the EMS Education Committee and the General Meeting completed an attractive programme.

The conference venue was right in the centre of Prague, near Wenceslas Square, and consisted of three buildings of the Faculty of Education, where the TWG work took place, and Hotel Ambassador, where the plenaries were held, as well as the opening and closing ceremonies and the gala dinner. The IPC, and in particular the LOC, had to meet the challenge of organising a conference for nearly 700 people instead of the planned 550. Luckily, the management of the Faculty of Education was forthcoming and allowed the conference to spread to nearly all of its buildings. The organisation was made possible by a large group of undergraduate and doctoral students who were available for any help required during the conference and the firm Guarant, as well as the hotel staff, guaranteed professional support. The weather was also helpful because the snow and frost, normally present in February, only came on the last day! Thus, it was possible for the participants to take part in several types of excursion on Friday, including visits to Karlštejn Castle (only opened for the congress participants) and a glass factory and several tours around Prague.

The participants were asked to provide their feedback on the conference (for the first time in an electronic format) and were kind enough to (mostly) praise the organisers for a wonderful conference. The critical comments will not be forgotten and will be used as feedback for further conferences, including CERME 10. It was great to hear the thanks by the President and the Vice-President in the closing ceremony; however, we would like to stress that the success of CERME 9 has a lot of fathers and mothers, including all the people and groups named above. We gratefully thank the ERME Board for trusting us to organise the conference. It was a real pleasure for us!

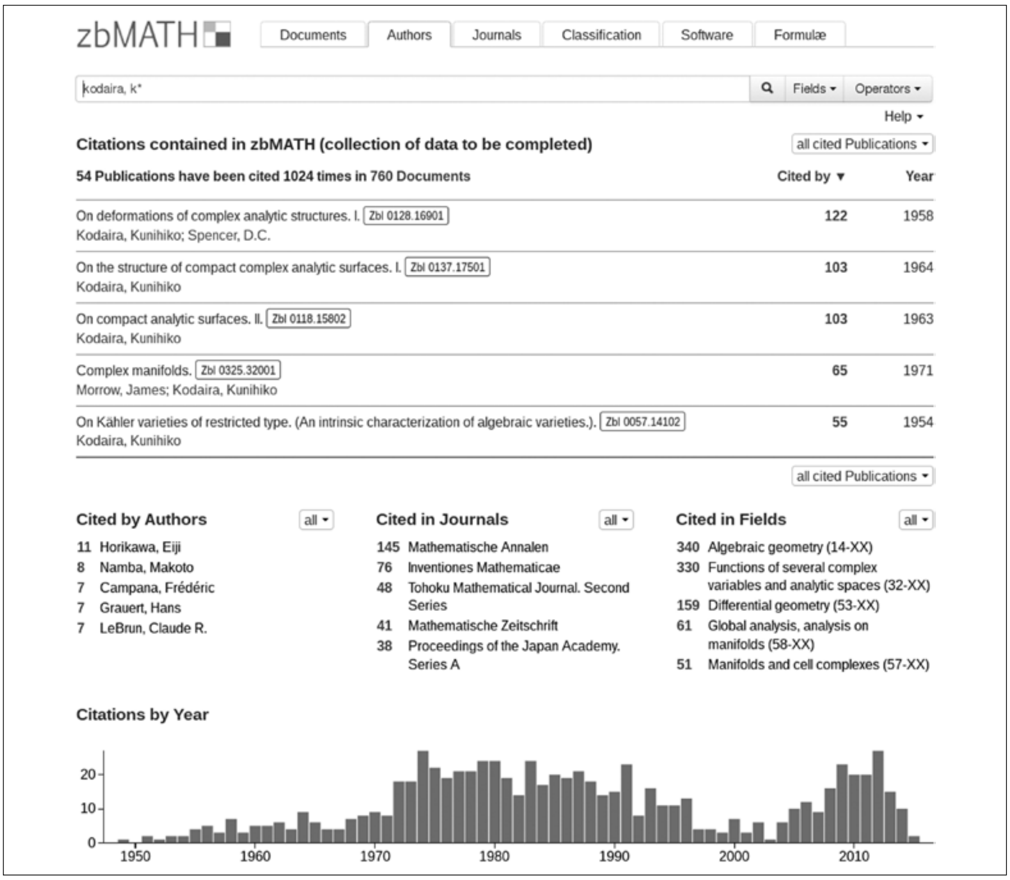
*Konrad Krainer is a full professor at the Alpen-Adria-Universität Klagenfurt (Austria), Faculty of Interdisciplinary Studies. He worked several years as a mathematics teacher and wrote his doctoral and habilitation theses in the field of mathematics education. He is the leader of the nationwide IMST project (in particular, improving mathematics and science teaching in Austria), co-editor of several books including the International Handbook of Mathematics Teacher Education, and chairman of the scientific jury of the 13th European Union Science Olympiad (EUSO 2015). He was associate editor of JMTE, a founding and board member of ERME and a member of the Education Committee of the EMS. His recent research focuses on teacher education, school development and educational system development related to mathematics and science teaching.*

*Nad'a Vondrová works as an associate professor at the Faculty of Education, Charles University in Prague (the Czech Republic), where she chairs the Department of Mathematics and Mathematical Education. She wrote her doctoral and habilitation theses in the field of mathematics education. She has been the coordinator of several national research projects on various aspects of mathematics education. She educates future mathematics teachers and leads further development courses. She has been a member of ERME since its beginning and, until 2015, she acted as Secretary of the ERME Board. She is a member of the editorial board for Educational Studies in Mathematics. Her recent research focuses on teacher education (professional vision and technological-pedagogical content knowledge) on the one hand and pupils' thinking processes in mathematics on the other.*

# Citation Profiles in zbMATH

Olaf Teschke (FIZ Karlsruhe, Berlin, Germany)

Since April 2015, author profiles in zbMATH have been enlarged with a large amount of information related to citations. The gathered information expands the now well-established profile pages. Besides a total count of citations, there is a list of all cited publications (which can be sorted by publication year or individual citation count) and a detailed analysis of their distribution with respect to citing authors, journals and subject areas, as well as a diagram illustrating their timeframe. The whole picture generally looks like in the following example:




**zbMATH citation profile for Kunihiko Kodaira**

It is hopefully clear from the structure that more emphasis has been laid on qualitative aspects than just quantitative ones. Firstly, one has to keep in mind that everything that has been said about author profiles in earlier columns[1] is even more true for the citation aspect. When using it, one should keep in mind the limitations given by scope, availability of information and data precision.

### Scope, availability, reliability: on which data are the profiles based?

The issue of scope is rather clear-cut. By restricting to mathematical publications, zbMATH can only reflect a well-defined subgraph of the citation web. Usually, this inevitably leads to significantly lower reference figures compared to generalist services (like, for example, Google Scholar, even if frequent duplications there are not taken into account), especially in "border" areas like mathematical physics, engineering or economics, which are considerably larger than mathematics (at least in terms of publications). On the other hand, a clear restriction of the scope gives a more precise meaning to the references inside mathematics, avoiding the marginalisation of mathematics in generalist information systems with respect to common quantitative measures that are biased by the different publication and citation behaviour in other disciplines.[2]

Availability of citation data is another important issue and also the key reason why the zbMATH citation profiles are, at the moment, still labelled as "to be enhanced": not from a technical viewpoint but since we are aware that a further enlargement of the underlying references is certainly desirable. This is, above all, a technical issue. Right now, about 10 million references are contained in zbMATH, connected to more than 400,000 documents since 1873 (of a total of 3,500,000). This share is, how-

[1] See, for example, O. Teschke and B. Wegner, "Author profiles at Zentralblatt MATH", *Eur. Math. Soc. Newsl.* 79, 43–44 (2011).

[2] See, for example, O. Teschke, "Negligible numbers", *Eur. Math. Soc. Newsl.* 82, 54–55 (2011).

ever, quite unevenly distributed with respect to year. Whilst references are available for almost 40% of recent documents, this rate drops to 6% for the 1960s and below 0.1% for the 19th century. Note, however, that even this is more than in most information systems, which often tend to neglect most of the historical documents. The special situation in mathematics, where decades-old publications are frequently highly relevant,[3] imposes the task on us to include as many historical documents as possible. Obviously, the main issue here is digitisation: even today, a considerable proportion of publications are only available in print or in limited digital form. Recently, of the 1497 journals with at least an issue added during the last year to zbMATH (out of overall 2193 journals with possible math content indexed in the database), references are available for 493 of them; this fraction will soon grow further and may eventually converge to a hopefully realistic figure of more than 60% of the indexed publications.

The most complicated question, however, is data precision. The question of author disambiguation is obviously the most important one for author profiles and has been discussed regularly in this column. Due to large improvements over the last two years, triggered by enhanced algorithms as well as the opportunity for community input,[4] the precision of document assignments is now at a sufficiently high level to reasonably generate derived profile information without risking an intolerable degree of error propagation. One should keep in mind that simple profile information like “author x has cited author y z times” is affected by three levels of possible errors, which may even accumulate with the number of publications. While author disambiguation is the most sensitive issue here, one also has to match a large number of references to documents in zbMATH precisely (and, importantly, to ignore false positive best matches when the publication is actually out of scope!). While this is a fairly standard problem, for which several solutions exist, the divergent shapes of references still make it demanding to identify the corresponding zbMATH entries in the long tail. This is especially true for books, which are usually among the most cited publications of an author but often cited in a non-standard way. Up till now, a conservative approach has been employed, with a preference on precision compared to the amount of matched references; this, however, certainly leaves some room for the enlargement of the data underlying the profiles.

### How could citation profiles be used?

Though the weaknesses of bibliometric measures are well-known, the creation of rankings still seems to be the most common, and least sensible, use of citation information. We take this opportunity to emphasise that this should not be the primary use-case of the zbMATH citation profiles. What is usually much more helpful is to see who cites, where the citations come from, which areas are involved and how sustainable they are – that is precisely provided by the core of the analysis given in the profile, which groups the available data into citing authors, citing journals, citing MSC subjects and the distribution at the timeline.

For instance, the breakdown according to the authors does not just include the special case of self-citations,[5] which have often led to distorted impact measures in the past, but the comparison with co-author and co-citation information from the profiles also makes it easier to detect citation rings, of which some examples gained notorious fame recently. Much more important is, of course, the positive aspect – with a few clicks, the user can easily follow the development of knowledge in the footsteps of the protagonists.

The journal facet shows a similar feature that should enable the user to value quality over quantity; often, a single reference of the *Annals* will be more meaningful than a huge number of citations from more quantity-oriented journals. On the other hand, when exploring the dark side of publication behaviour, one may explore some of the known cases of citation farming, which will frequently show a typical pattern of many references coming from a very limited number of journals (often identical to those where the articles have been published). So, this analysis may help to more quickly detect problematic behaviour than in the past.

The area analysis according to the Mathematical Subject Classification is, of course, another example of the usefulness of such a detailed analysis, especially for thematic searching. We can only suggest starting with some prominent examples and exploring the often surprising applications of the results in very different areas, often illustrating hidden connections within mathematics.[6] It may also be surprising to see the often very different patterns of the citation timeline, which reflect not only a quite different research and citation behaviour in different areas but frequently show large gaps or accumulations, reflecting stagnation or breakthroughs for a certain problem.

One should also remark that these profile details are only the standard examples of questions which can be answered by zbMATH queries. As usual within our search philosophy, all existing query results can be further refined or extended by filtering or adding logical combinations with other facets, allowing the user to formulate virtually any complex question. So feel free to explore this new feature for yourself!

*Olaf Teschke [teschke@zblmath.fiz-karlsruhe.de] is member of the Editorial Board of the EMS Newsletter, responsible for the Zentralblatt Column.*

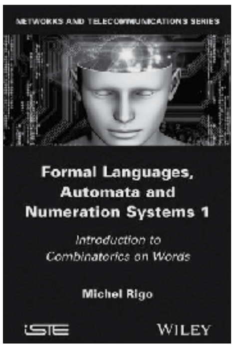
[3] T. Bouche, O. Teschke, K. Wojciechowski, “Time lag in mathematical references”, *Eur. Math. Soc. Newsl.* 86, 54–55 (2012).

[4] H. Mihaljević-Brandt, N. Roy, “zbMATH author profiles: open up for user participation”, *Eur. Math. Soc. Newsl.* 93, 53–55 (2014).

[5] An excessive example is also given by the footnotes of this column, which are not, however, counted in the zbMATH reference database.

[6] E.g. you could explore which results in mathematical finance build upon Grothendieck’s work, though he might not have been too glad about this fact himself.

# Book Reviews




Michel Rigo

**Formal Languages, Automata and Numeration Systems 1**
**Introduction to Combinatorics on Words**

Wiley 2014
ISBN 9781848216150
336 pp

Reviewer: Jean-Paul Allouche

At the beginning, the main task of computers was data processing, in particular data sorting, possibly using punch cards. The necessity of coding data by combining symbols taken from a finite set naturally implies, when it comes to coding numbers, the use of numeration bases. Expansions of numbers can be treated as "words" on an "alphabet" (i.e., a finite set) so that "combinatorics on words" is consubstantial with computers and the theory of computers. Of course, combinatorics on words was also studied without any reference to (nor knowledge of?) data encoding. One of the fathers of this subject is Thue, a famous number-theorist, who, as early as 1906–1912, asked (and answered positively) the following question: is it possible to construct an infinite binary sequence which does not contain three consecutive identical finite blocks?

Up to the end of the 1990s, not that many books were devoted to combinatorics on words. Actually, there were two collective books signed by M. Lothaire; one in 1983 (with a new edition in 1997) and one in 1990. The link with mathematics, in particular with dynamical systems, through sequences generated by automata or by substitutions, was already present: the seminal book of M. Queffélec is dated 1987 (with a new edition in 2010). The reader can consult the respective reviews in *Zentralblatt*.[1]

At the beginning of the 2000s, it became clear that the field was expanding rapidly and that links with automata on one hand and with mathematics on the other hand (in particular number theory) were developing quickly. Thus, two more books by M. Lothaire were published (in 2002 and 2005, respectively). In between, three other books became available, a collective book by N. Pytheas Fogg in 2002, a book by F. von Haeseler in 2003 which should be better known and a book by J. Shallit and the author. Of course, other books can be cited, such as a collective book with editors V. Berthé and M. Rigo that appeared in 2010. The reader can consult the respective reviews in *Zentralblatt*.[2]

Other books address combinatorics on words or automatic sequences as chapters or sections (e.g., the book by G. Everest, A. van der Poorten, I. Shparlinski and T. Ward, and the book by Y. Bugeaud)[3].

The author of the book under review has succeeded in writing a new, exciting book on the links between formal languages, automata and numeration systems. This book is very nice, maybe partly because of what the author himself confesses: "[…] Indeed the book most probably reflects what I myself prefer." The reader will first learn or recall introductory definitions and results about languages, factors, cellular automata, discrete dynamical systems and continued fraction expansion. Then, they will be gently led to morphic words, including automatic words, in a chapter finishing with Sturmian words. One of the "simplest non-trivial" morphic words is the Prouhet–Thue–Morse sequence, which can be constructed as follows. Start from 0 and then apply iteratively (and in parallel) the morphism (i.e., the rewriting rule) $0 \to 01$, $1 \to 10$. The following words are obtained:

$$0,\ 01,\ 0110,\ 01101001,\ 0110100110010110, \ldots$$

This sequence of words converges (in any reasonable sense) to the infinite sequence

$$01101001100101101001011001101001\ldots$$

which is invariant under the morphism $0 \to 01, 1 \to 10$.

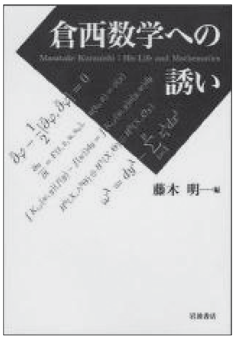
Sequences generated by morphisms can be "trivial" (i.e., periodic or ultimately periodic) but the nontrivial ones have the twofold aspect of being simply generated but having a possibly "complicated" structure. (The reader can find in the previously mentioned books how such sequences occur in number theory, harmonic analysis, iteration of continuous functions and also physics.) This structure is progressively unveiled in the third chapter of the book, more being announced for Volume 2. Of course the second volume is much awaited: not only is it announced as "A crash course on regular languages" as well as "decidability issues" but it will also contain a chapter on *abstract numeration systems*. These numeration systems were introduced in a seminal paper by P.B.A. Lecomte and M. Rigo (see Zbl 0969.68095). They are based on reverting the usual approach: instead of starting from a "classical" numeration base and looking at the expansions of integers as "words", one starts from a *regular language* on some alphabet; one sorts the words of this language lexicographically and then one enumerates these words, thus obtaining a bijection from the language to the set of integers, where the $n$th word is by definition the representation of the integer $n$ in this "abstract" numeration base.

[1] Zbl0514.20045, Zbl0874.2004, Zbl0862.05001, Zbl0642.28013 and Zbl 1225.11001.

[2] Zbl1001.68093, Zbl1133.68067, Zbl1014.11015, Zbl1057.11015, Zbl 1086.11015 and Zbl 1197.68006.

[3] Zbl 1033.11006 and Zbl 1260.11001.

The reader can happily work with this first volume, which is enriched with exercises and historical notes, not forgetting the bibliography of more than 400 items, while waiting for Volume 2, which promises to be an exciting follow-up.



Akira Fujiki

**An Invitation to Kuranishi Mathematics. (Kuranishi sugaku eno izanai.) (Japanese)**

Iwanamishoten, Tokyo 2013
ISBN 978-4-00-005272-6
189 pp

Reviewer: Hirokazu Nishimura

*The Newsletter thanks Zentralblatt MATH and Hirokazu Nishimura for the permission to republish this review, originally appeared as Zbl 06388518.*

This book, whose subtitle is "Masatake Kuranishi: His Life and Mathematics", is a companion volume of [1], though it is written in Japanese. The book is divided into two parts. The first part is concerned with Kuranishi as a person, and the second part deals with mathematical works of Kuranishi.

The first part consists of roughly 100 pages, about four fifths of which is his autobiography based on several interviews to Kuranishi by Tadashi Tomaru. The first part contains also three essays, namely, Memories of Kuranishi written by Victor Guillemin (his English essay is followed by its Japanese translation), Memories of Kuranishi written by Kuranishi's younger brother (Shigeru Kuranishi) and Memories of an apprentice under Professor Kuranishi written by Makoto Namba.

The second part consists of five reviews on the mathematics of Kuranishi. The first review, written by Tohru Morimoto, is concerned with geometric theory of partial differential equations centering Kuranishi's publications [2–4]. The second review, written by Akira Fujiki, is concerned with Kuranishi families in deformations of compact complex manifolds centering Kuranishi's articles [5–8]. The third review, written by Kimio Miyajima, is concerned with CR manifolds centering Kuranishi's articles [9–11]. The fourth review, written by Mitsuhiro Ito, is concerned with Yang-Mills connections and Kuranishi mappings centering Kuranisi's publication [6]. The fifth review, written by Ryushi Goto, is concerned with generalized complex structures and their deformation theory centering Kuranishi's papers [5] and [6].

Kuranishi was born in Tokyo in 1924, when Tokyo was still in a turmoil after the Great Kanto Earthquake in 1923. He entered Nagoya University in 1944, when the situation in the Pacific War was deteriorating day by day to Japan. Nagoya University was founded in 1939, when the Second World War erupted. Then and there he met a number of brilliant professors, say, Kosaku Yoshida, Tadashi Nakayama, Yozo Matsushima, Kiyoshi Ito and Goro Azumaya. We can find Noboru Ito and Nobuo Shimada among his peers, the first being destined to become famous in the theory of finite groups and the second being specialized in algebraic topology. After graduation, Kuranishi became an instructor of Tokyo Institute of Technology, where his first paper [12] was written and he had spent three years until he moved to Nagoya University. Until 1952, when he got his Ph.D. from Nagoya University, David Hilbert's fifth problem concerning the characterization of Lie groups had occupied a central position in the mind of Kuranishi. With respect to this, he has written two papers [13] and [14], which have contributed greatly to [15].

As is well known, it is not easy to read publications of Élie Cartan, though they are all significant contributions to mathematics. It was Yozo Matsushima who invited Kuranishi to mathematics of Élie Cartan. Kuranishi learned from Élie Cartan that the first step in the study of some mathematical structure should be the thorough study of a good model, and the structure itself should be understood as a deformation of the model. The use of differential forms in Kuranishi's later study of complex structures and CR structures is to be attributed to his encounter with publications of Élie Cartan, who has founded the theory of differential forms. Since Hilbert's fifth problem was settled, Kuranishi's main interest was then oriented towards geometric theory of partial differential equations. Élie Cartan is known to have devoted all his energy to the study of Pfaff systems or exterior differential systems and pseudogroups in the first decade of the 20th century ([16–23] and so on). Kuranishi's first work in this area is [2], which was to play a pivotal role in his study of deformations of complex structures, and which Kuranishi considers one of his most important and most fundamental works. Kuranishi's work with respect to pseudogroups is [3].

Thanks to D. Montgomery's invitation, Kuranishi was entitled to spend two years since 1954 at Institute for Advanced Study. He then spent a year and a half at Chicago University, where he met A. Weil, S. S. Chern, A. P. Calderón and A. Zygmund, and at Massachusetts Institute of Technology. Calderón and Zygmund are famous for the theory of singular integrals ([24–27]), which was developed into the theory of pseudo-differential operators by Joseph J. Kohn, Lars Hörmander and Louis Nirenberg in the 1960s. In 1961 Kuranishi spent three months at the Tata Institute of Fundamental Research, where C. L. Segal stayed at that time, Kuranishi happened to meet Henri Cartan, whose father is Élie Cartan, and Kuranishi gave a lecture entitled "On Exterior Differential Systems", whose lecture notes by Venkatesha Murthy were published there.

Kuranishi's intimate friendship with K. Kodaira began in 1954, when Kuranishi stayed at I.A.S. Kodaira and D.C. Spencer are famous for the deformation theory of complex structures ([28–29]), and Kuranishi's first contribution in this area is [30], which enticed Kodaira and Spencer to invite Kuranishi to Princeton University as a research fellow for a year since September 1960. Besides Kodaira and Spencer, S. Lefschetz, E. Artin and

S. Bochner were enrolled there at that time. The Kodaira-Spencer seminar at Princeton University, of which R. Gunning was a regular member, inspired Kuranishi to finish the paper [5]. In September 1961 Kuranishi moved from Nagoya University to Columbia University, where he had stayed until he retired at the age of 75 in 1999. Victor W. Guillemin was once an instructor at Columbia University, and has written [31] with Kuranishi. M. Namba stayed at Columbia University as a foreign student for three years and a half in the 1960s. In New York, Louis Nirenberg, younger than Kuranishi by a year, lived near Kuranishi. Among colleagues of Kuranishi at Columbia University we can find Samuel Eilenberg, who is famous for his successful books [32] and [33] and had once Daniel Kan, William Lawvere and K. T. Chen among his students, and also Richard Hamilton, who is famous for [34] leading to [35] and [36].

You can find more information in the book, and the reviewer urges strongly that the book should be translated into English.

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# Letter to the Editor

Patrizia Donato (University of Rouen, France)

## My upsetting experience of the organization of the COPDE 2015 conference (Munich 25–29, 2015)

The aim of this letter is to inform our European mathematical community about what happened at the conference, in order to avoid possible future similar problems, believing that I am also expressing the feelings of most of the participants, with whom I have deeply discussed the matter.

**Some facts:** As about 80 participants, I accepted an invitation by Dr. A.R. to give a contributed talk at the conference, presented as the successor event of COPDE 2014 in Novacella/Italy. The invitation (written by Dr A.R. on behalf of 2 other) stated that the conference fees (200 EUR) would cover all local expenses (conference material, hotel and meals during the conference). Dr. A.R. signed as Assistant Professor of TUM, the TUM logo was on the website of the conference (hosted, it seemed, by the university of one of the organizers, Arizona University) and, in particular, Prof. Bellomo (member of the scientific committee of the COPDE 2014) was indicated as a member of the scientific and local organizing committee. The location was an historical building of TUM. We later realized that Prof. Bellomo (not present), did not know to be on conference Board.

I paid the registration fees and Dr. A.R. wrote to me that he will reserve the hotel. A few days before the beginning of the event, though, it began to appear that something was wrong.

**Concerning the accommodation:** Till the opening day, almost nobody knew where he would be accommodated. After several unaswered messages about the hotel I called Dr A.R. by telephone. He gave me the name of the hotel, asking me to pay the hotel myself, since he had not yet received the money of the grant supporting the conference, saying that he would reimburse me later. I accepted but I asked him to warn the other participants of this circumstance. He did not. When I arrived at the conference, I realized that for many people the accommodation had not even yet been booked and that in any case almost all the participants had to pay everything themselves (it seems that the hotel fees of some were successively covered). People could finally find an accommodation, several by themselves.

**Concerning the conference:** Except for the conference rooms, there was no organization whatsoever for the conference: no poster about the conference in the building, no conference materials (only the program sent by email only on March 24th), no social program, not even any refreshments or coffee. Despite this situation, the conferences did begin to take place, handled by the chairmen, with the help of some organizers. Some of the participants found the conference interesting due to the quality of the speakers and the talks. For me and others it was a disaster, being not in a peaceful scientific atmosphere.

**Our action:** Supported by some other participants, I proposed a clarifying collective discussion with Dr. A.R. He explained that he still had not received an answer regarding the grant but after three days he finally admitted that the grant was refused. The matter about how to be reimboursed were not clear, neither what he had paid with the registrations fees for and why he did not cancel the conference or at least inform people of the situation before their arrival.

The Dean of the Faculty of Mathematics of TU Munich, Professor Gero Friesecke, informed about the situation, shocked and extremely sorry wrote us that he was in no way informed about this meeting, neither organized nor endorsed by TUM (whose logo has been now removed from the website, as well as the organizers names) and he would activate an internal investigation in collaboration with the TUM legal department. He explained that Dr. A.R. is currently not employed in TUM and he collected the conference fees on his own account, reserving the conference rooms as a private person. As far as I know, the hotel of several people were finally paid. I have still no news about my fees (as others, among them some of the scientific committee).

**In conclusion:** I have no words for qualifying the behavior of the main organizer of the event, responsible for wasting our time, our money and that of our institutions, although the reasons are completely obscure to me. He should never organize an event again. Some other co-organizers, at least those with whom I personally discussed had participated in good faith, trying to do their best once in Munich. But in my opinion, the organizing committee also has a responsability, since maybe they delegated the organization, without checking what is going on.

What happened (unprecedented in my 40 years of experience) has to be a warning sign for our community, in order to be careful when accepting to be organizer or speaker in a conference.

Patrizia Donato
Full Professor, University of Rouen
May 11, 2015

We made a similar experience with the COPDE 2015 conference and we agree with the conclusions of Patrizia Donato:

Darya Apushkinskaya (Universität des Saarlandes)
Maria-Magdalena Boureanu (University of Craiova)

Renata Bunoiu (Université de Lorraine-Metz)
Giuseppe Cardone (Università del Sannio)
Sandra Carillo (Università di Roma La Sapienza)
Graça Carita (University of Evora)
Krzysztof Chelminski (Warsaw University of Technology)
Bernard Dacorogna (Ecole Polytechnique Fédérale de Lausanne)
Yanghong Huang (Imperial College London)
Luisa Faella (Università di Cassino)
Andrei Fursikov (Moscow State University)
Agnieszka Kalamajska (Warsaw University)
Kristina Kaulakyte (University of Zurich)
Yana Kinderknecht (Universität des Saarlandes)
Alessia Elisabetta Kogoj (Università di Bologna)
Alexander Kurganov (University of New Orleans)
M. Rosaria Lancia (Università di Roma La Sapienza)
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Salim Aissa Salah Messaoudi (King Fahd University of Petroleum and Minerals)
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Marta Strani (Université Paris Diderot)
Maria Transirico (Università di Salerno)
Paola Vernole (Università di Roma La Sapienza)
Elvira Zappale (Università di Salerno)
Stephanie Zube (University of Zurich)

# Personal Column

*Please send information on mathematical awards and deaths to newsletter@ems-ph.org.*

## Awards

The Norwegian Academy of Sciences and Letters has decided to award the **Abel Prize** for 2015 to the American mathematicians **John F. Nash, Jr.** and **Louis Nirenberg**.

The **Adams Prize** is awarded jointly by the Faculty of Mathematics and St John's College of the University of Cambridge. This year's topic was "Algebraic Geometry" and the prize has been awarded jointly to **Arend Bayer** (University of Edinburgh) and **Thomas Coates** (Imperial College London).

The American Mathematical Society has awarded: the 2014 **Cole Prize in Number Theory** to **Daniel Goldston** (San Jose State University), **János Pintz** (Alfréd Rényi Institute) and **Cem Y. Yildirim** (Boğaziçi University in Istanbul); the 2014 **Stefan Bergman Prize** to **Sławomir Kołodziej** (Jagiellonian University and Vice-President of the Polish Mathematical Society);the 2015 **Cole Prize in Algebra** to **Peter Scholze** (Bonn University, Germany); and the 2015 **Albert Leon Whiteman Memorial Prize** to **Umberto Bottazzini** (Università di Milano, Italy).

One of the **Alexander von Humboldt Professorships** for 2015 has been awarded to **Harald Andrés Helfgott** (CNRS, France).

**Aharon Ben-Tal** (Israel Institute of Technology), **Vincent D. Blondel** (Universite catholique de Louvai), **Franco Brezzi** (Istituto Universitario di Studi Superiori di Pavi), **Per Christian Hansen** (Technical University of Denmark), **Petros Koumoutsakos** (ETH Zurich), **Rodolphe Sepulchre** (University of Cambridge) and **Halil Mete Soner** (ETH Zurich) have been designated 2015 **SIAM Fellows**.

**George Luszti** (MIT, US) has won the **Shaw Prize** in Mathematical Science, awarded by the Shaw Prize Foundation based in Hong Kong.

At the 2015 Annual Meeting of the Dutch Society for Statistics and Operations Research, **Bert Zwart** was awarded the **Van Dantzig Award**. This award is presented to a researcher younger than 40, who, over the past five years, has made an exceptional contribution to the field of statistics and operations research.

At the Dutch Mathematical Congress in Leiden, 15 April 2015, **Djordjo Milovic** received the **KWG Prize** for PhD students, which is funded by the Royal Dutch Mathematical Society.

Each year, the International Association of Applied Mathematics and Mechanics (GAMM) grants young outstanding researchers two different awards: the **Richard von Mises Prize** 2015 has been awarded to **Siddhartha Mishra** (Zurich) and the **Dr. Klaus Körper Prize** 2015 has been awarded to **Thomas Berger** (Hamburg), **Kathrin Hatz** (Heidelberg), **Julian Fischer** (Leipzig) and **Annika Radermacher** (Aachen).

**Luis Vega** (Universidad del País Vasco UPV/EHU and BCAM, Spain) has been awarded the **Blaise Pascal Medal in Mathematics** 2015 by the European Academy of Sciences.

**María Jesús Esteban** (CNRS and Université Paris-Dauphine) has become a member of Jakiunde, Basque Academy of Sciences, Arts and Letters.

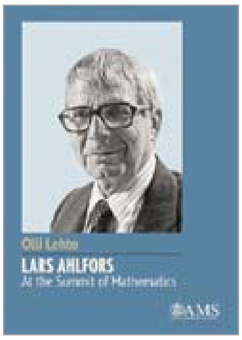
## Deaths

We regret to announce the deaths of:

**Jean-Claude Douai** (3 March 2015, Lille, France)
**Evarist Giné** (13 March 2015, Connecticut, US)

AMERICAN MATHEMATICAL SOCIETY

## LARS AHLFORS - AT THE SUMMIT OF MATHEMATICS

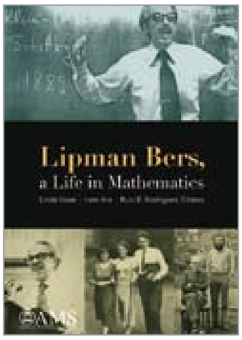



*Olli Lehto, University of Helsinki*
*Translated by William Hellberg*

Tells the story of the Finnish-American mathematician Lars Ahlfors (1907-1996). At the age of twenty-one Ahlfors became a well-known mathematician having solved Denjoy's conjecture, and in 1936 he established his world renown when he was awarded the Fields Medal. In this book the description of his mathematics avoids technical details and concentrates on his contributions to the general development of complex analysis. Besides mathematics there is also a lot to tell about Ahlfors - World War II marked his life, and he was a colourful personality, with many interesting stories about him.

Jul 2015 122pp 9781470418465 Paperback €44.00

## LIPMAN BERS, A LIFE IN MATHEMATICS




*Edited by Linda Keen, Lehman College, Irwin Kra, Stony Brook University & Rubí E. Rodríguez, Pontificia Universidad Católica de Chile*

Part biography and part collection of mathematical essays that gives the reader a perspective on the evolution of an interesting mathematical life, this book is all about Lipman Bers, a giant in the mathematical world who lived in turbulent and exciting times. It captures the essence of his mathematics, a development and transition from applied mathematics to complex analysis - quasiconformal mappings and moduli of Riemann surfaces - and the essence of his personality, a progression from a young revolutionary refugee to an elder statesman in the world of mathematics and a fighter for global human rights and the end of political torture.

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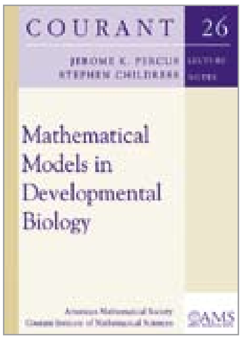
## MATHEMATICAL MODELS IN DEVELOPMENTAL BIOLOGY




Jerome K. Percus & Stephen Childress, *New York University*

The path from relatively unstructured egg to full organism is one of the most fascinating trajectories in the biological sciences. Its complexity calls for a very high level of organization, with an array of subprocesses in constant communication with each other. These notes introduce an interleaved set of mathematical models representative of research in the last few decades, as well as the techniques that have been developed for their solution. Such models offer an effective way of incorporating reliable data in a concise form, provide an approach complementary to the techniques of molecular biology, and help to inform and direct future research.

*Courant Lecture Notes, Vol. 26*

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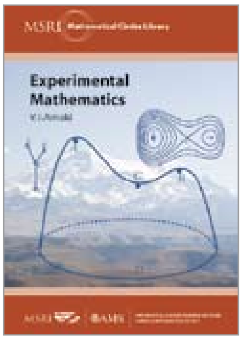
## EXPERIMENTAL MATHEMATICS




*V.I. Arnold*
*Translated by Dmitry Fuchs & Mark Saul*

Presents several new directions of mathematical research. All of these directions are based on numerical experiments conducted by the author, which led to new hypotheses that currently remain open. The hypotheses range from geometry and topology to combinatorics to algebra and number theory. Written in Arnold's unique style, the book is intended for a wide range of mathematicians, from high school students interested in exploring unusual areas of mathematics on their own, to college and graduate students, to researchers interested in gaining a new, somewhat non-traditional perspective on doing mathematics.

*MSRI Mathematical Circles Library, Vol. 16*

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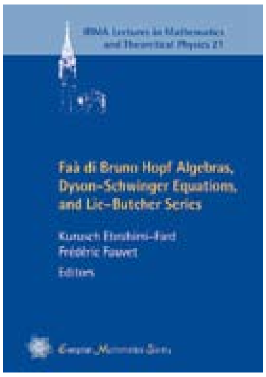
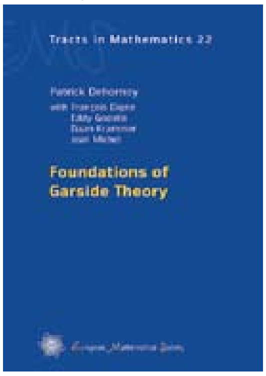



Kurusch Ebrahimi-Fard (Universidad Autónoma de Madrid, Spain) and Frédéric Fauvet (Université de Strasbourg, France)
**Faà di Bruno Hopf Algebras, Dyson–Schwinger Equations, and Lie–Butcher Series**
(IRMA Lectures in Mathematics and Theoretical Physics, Vol. 21)

ISBN 978-3-03719-143-9. 2015. 466 pages. Softcover. 17 x 24 cm. 48.00 Euro

Since the early works of G.-C. Rota and his school, Hopf algebras have been instrumental in algebraic combinatorics. In a seminal 1998 paper, A. Connes and D. Kreimer presented a Hopf algebraic approach to renormalization in perturbative Quantum Field Theory (QFT). This work triggered an abundance of new research on applications of Hopf algebraic techniques in QFT as well as other areas of theoretical physics.
The present volume emanated from a conference hosted in June 2011 by IRMA at Strasbourg University in France. Researchers from different scientific communities who share similar techniques and objectives gathered at this meeting to discuss new ideas and results on Faà di Bruno algebras, Dyson–Schwinger equations, and Butcher series.
The purpose of this book is to present a coherent set of lectures reflecting the state of the art of research on combinatorial Hopf algebras relevant to high energy physics, control theory, dynamical systems, and numerical integration methods. This volume is aimed at researchers and graduate students interested in these topics.

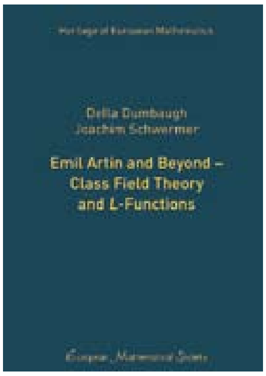
**EMS Monograph Award Winner 2014**




Patrick Dehornoy (Université de Caen, France) with François Digne (Université de Picardie Jules-Verne, Amiens), Eddy Godelle (Université de Caen, France), Daan Krammer (University of Warwick, Coventry, UK) and Jean Michel (Université Denis Diderot Paris 7, France)
**Foundations of Garside Theory** (EMS Tracts in Mathematics, Vol. 22)

ISBN 978-3-03719-139-2. 2015. 710 pages. Hardcover. 17 x 24 cm. 108.00 Euro

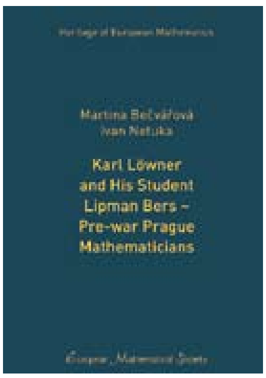
This text is a monograph in algebra, with connections toward geometry and low-dimensional topology. It mainly involves groups, monoids, and categories, and aims at providing a unified treatment for those situations in which one can find distinguished decompositions by iteratively extracting a maximal fragment lying in a prescribed family. Initiated in 1969 by F. A. Garside in the case of Artin's braid groups, this approach turned out to lead to interesting results in a number of cases, the central notion being what the authors call a Garside family. At the moment, the study is far from complete, and the purpose of this book is both to present the current state of the theory and to be an invitation for further research.
There are two parts: the bases of a general theory, including many easy examples, are developed in Part A, whereas various more sophisticated examples are specifically addressed in Part B. The exposition is essentially self-contained. It should be easy to use the text as a textbook. The first part of the book can be used as the basis for a graduate or advanced undergraduate course.




Della Dumbaugh (University of Richmond, USA) and Joachim Schwermer (University of Vienna, Austria)
**Emil Artin and Beyond – Class Field Theory and *L*-Functions** (Heritage of European Mathematics)

ISBN 978-3-03719-146-0. 2015. 248 pages. Hardcover. 17 x 24 cm. 68.00 Euro

This book explores the development of number theory, and class field theory in particular, as it passed through the hands of Emil Artin, Claude Chevalley and Robert Langlands in the middle of the twentieth century. Claude Chevalley's presence in Artin's 1931 Hamburg lectures on class field theory serves as the starting point for this volume. From there, it is traced how class field theory advanced in the 1930s and how Artin's contributions influenced other mathematicians at the time and in subsequent years. Given the difficult political climate and his forced emigration as it were, the question of how Artin created a life in America within the existing institutional framework, and especially of how he continued his education of and close connection with graduate students, is considered. In particular, Artin's collaboration in algebraic number theory with George Whaples and his student Margaret Matchett's thesis work "On the zeta-function for ideles" in the 1940s are investigated. A (first) study of the influence of Artin on present day work on a non-abelian class field theory finishes the book. The volume consists of individual essays by the authors and two contributors, James Cogdell and Robert Langlands, and contains relevant archival material.




Martina Bečvářová (Czech Technical University and Charles University, Prague, Czech Republic) and Ivan Netuka (Charles University, Prague, Czech Republic)
**Karl Löwner and His Student Lipman Bers – Pre-war Prague Mathematicians** (Heritage of European Mathematics)

ISBN 978-3-03719-144-6. 2015. 304 pages. Hardcover. 17 x 24 cm. 78.00 Euro

This monograph is devoted to two distinguished mathematicians, Karel Löwner (1893–1968) and Lipman Bers (1914–1993), whose lives are dramatically interlinked with key historical events of the 20th century. K. Löwner, Professor of Mathematics at the German University in Prague (Czechoslovakia), was dismissed from his position because he was a Jew, and emigrated to the USA in 1939. Earlier, he had published several outstanding papers in complex analysis and a masterpiece on matrix functions. In particular, his ground-breaking parametric method in geometric function theory from 1923, which led to Löwner's celebrated differential equation, brought him world-wide fame and turned out to be a cornerstone in de Branges' proof of the Bieberbach conjecture. L. Bers was the final Prague Ph.D. student of K. Löwner. His dissertation on potential theory (1938), completed shortly before his emigration and long thought to be irretrievably lost, was found in 2006. It is here made accessible for the first time, with an extensive commentary, to the mathematical community.
This monograph presents an in-depth account of the lives of both mathematicians, with special emphasis on the pre-war period. Each of his publications is accompanied by an extensive commentary, tracing the origin and motivation of the problem studied, and describing the state-of-art at the time of the corresponding mathematical field.

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