

EMS Magazine

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Learning large-scale industrial physics simulations

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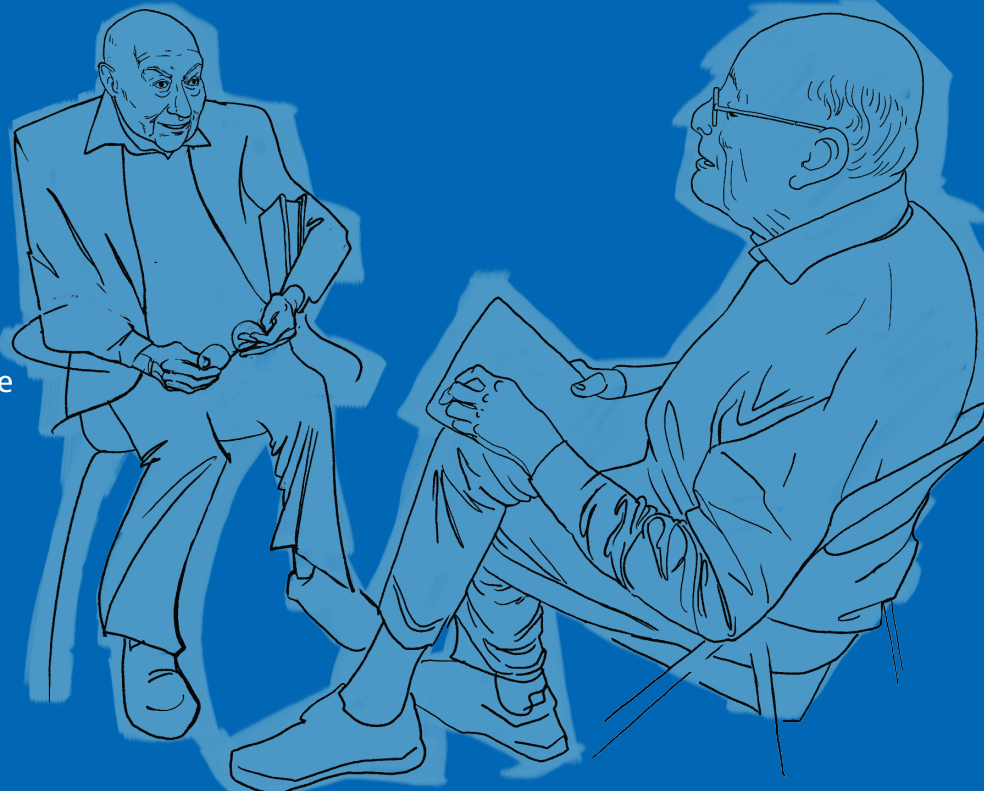
100 years of Jacques Dixmier

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and Teresa Monteiro Fernandes

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Mathematics as a private space





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The cover illustration by António B. Araújo is based on the photo depicting 100-year-old Jacques Dixmier with Jean-Pierre Serre at the anniversary dinner.

A message from the president



Photo by Jim Høyer,
University of Copenhagen.

I would like to wish all the members of the European Mathematical Society a happy new year, and I would like to extend these wishes to every reader of the EMS Magazine as well, but if you are not a member of the EMS please consider signing up.

A new year means we have to say goodbye to many people that have worked hard for the EMS throughout our many committees, both the Executive Committee and our standing committees. It of course also means welcoming all the new members. I would like, in particular, to thank our retiring EMS vice president Beatrice Pelloni who has worked tirelessly for the EMS. She has been instrumental in setting up our EMS Young Academy (EMYA), and for ensuring that this is one of the real success stories of the EMS from the past years. Beatrice was also a driver for many other activities of the EMS, such as the EMS Topical Activity Groups (TAGs). Her enthusiasm and wisdom will be missed on the Executive Committee. Beatrice will continue as officer for another EMS, namely the Edinburgh Mathematical Society, one of the member societies of the European Mathematical Society, and I am looking forward to working with her in that role. As I already mentioned in my last message, we are lucky that the council elected current EC member Victoria Gould is willing to replace Beatrice as vice president of the EMS. The EMS Executive Committee is also saying goodbye to Frédéric Hélein and Luis Narváez Macarro. Frédéric, in particular, has been very engaged with the politics of scientific publications, and I am very happy that he has agreed to stay on at our Committee for Publications and Electronic Dissemination, so our collaboration will continue. Luis has decided to devote more time to his research and some big ongoing projects. Luis is the kind of person every committee needs. Someone who stays out of heated discussions to then conclude them with decisive and insightful input. I wish Luis all the best with all his future projects. In my last message I already welcomed our new EC members María Ángeles García Ferrero (from EMYA), Adam Skalski, and Alain Valette. Maria, Adam, and Alain have already participated in our EC meetings for this fall, as in the recent great and successful meeting in Turin.

Let me also use this opportunity to thank all the outgoing members from our standing committees. The EMS is indebted to their hard work:

- Applications and Interdisciplinary Relations (CAIR): Thierry Horsin;
- Developing Countries (CDC): Bengt-Ove Turesson (chair), Sophie Dabo-Niang, and Francesco Pappalardi;

- Education: Gregoris Makrides (chair), Tinne Hoff Kjeldsen (vice chair), Jürg Kramer, Csaba Szabó, and Ayşe Berkman;
- Ethics: Patrizia Donato;
- European Solidarity: Roland Duduchava and Alice Fialowski;
- Meetings: Julio Moro Carreño (chair) and Monika Ludwig;
- Publications and Electronic Dissemination (PED): Olaf Teschke (chair), Thierry Bouche, Fatiha Alabau-Boussouira, Vittorio Coti Zelati, Tomaž Pisanski, and José Miguel Urbano;
- Mathematics Outreach and Engagement: Chris Budd, Kristóf Fenyvesi, Marianna Freiberger, and Sophia Jahns;
- Women in Mathematics (WiM): Alessandra Celletti and Shiri Artstein.

I am also very grateful that several people have accepted the non-trivial task of being new chairs of committees: Balázs Szendrői (CDC), Christophe Ritzenthaler (ERCOM, European Research Centres on Mathematics), Klavdija Kutnar (Meetings), and Evelyne Miot (PED).

On the publication side, Laure Saint-Raymond is stepping down as editor-in-chief of the EMS Surveys and François Golse has accepted to replace her and join Gerard van der Geer on the EMS Surveys editor-in-chief team. Thanks to Laure for all the amazing work she has put into making the Surveys a great success and to François for carrying on the torch.

So what are we planning to do in the EMS in the coming year? In the past few years the EMS has started several new activities: the EMS Young Academy (EMYA), Topical Activity Groups (TAGs), and Strategic Activities. The Strategic Activities were paused for several reasons, but it is our hope to restart these soon. We think, however, that the time has now come to engage more with research and educational politics. We are considering forming a few working groups to support us in this endeavor and have already started a working group to engage with the European Research Council – an EMS-ERC working group that, among other things, should support the ERC in their goal to promote diversity in the group of applicants and grantees in mathematics. I am glad that Pavel Exner, Eduard Feireisl, Sara van de Geer, Kathryn Hess, Volker Mehrmann, Piotr Nowak, Marie-Francoise Roy, Wil Schilders, Susanna Terracini, and Ulrike Tillmann have joined me in this group.

Lastly, I ask you to please mark your calendars for some of our forthcoming conferences. In June 2025, we will have the second Balkan Mathematical Congress BMC II in Thessaloniki, Greece and in less than a year from now in January 2026, the EMS will join colleagues in India to have a joint mathematics meeting in Pune, India. I hope to see many of you at these conferences.

For now, I wish you all a healthy and productive 2025.

Jan Philip Solovej, President of the EMS

Brief words from the editor-in-chief



Dear Readers of the EMS Magazine,

This issue is the last one of 2024. The balance of this year is very positive: the published articles cover a variety of topics, as they range from interviews to articles on scientific themes, education, gender, generational opportunities, and equity. Furthermore, our articles continue to attract the attention of other societies.

Some articles originally published in the Magazine have been requested for republication in other English-language publications, such as the ICIAM Newsletter, *ICIAM Dianoia*, or translated into Chinese for *Mathematical Advances in Translation*, a publication of the Chinese Academy of Sciences.

This issue will also be the last one with Michael Th. Rassias as editor. He will conclude his second four-year term as editor in charge of the “Problem corner” section on the last day of 2024. I would like to thank him for the exceptional work he has done over

the years in making this section consistently engaging and dynamic by suggesting and collecting challenging new problems, some of which were proposed by the most prominent mathematicians in the field, including several Fields medalists and Abel Prize winners.

As usual, this issue will contain many interesting articles on a wide range of topics. Without diminishing other contributions, I would like to particularly highlight the following: with this issue, we are starting a series of articles presenting the work of the 2024 winners of the ten EMS Prizes, the Felix Klein Prize, the Otto Neugebauer Prize, and of the two new prizes: the Paul Lévy Prize in Probability Theory and the EMS/ECMI Lanczos Prize. In this issue, we begin with the article “Learning large-scale industrial physics simulations” by the Felix Klein Prize winner Fabien Casenave.

Finally, I conclude by wishing all of you a *new year* full of discovery, inspiration, and sharing of knowledge.

Donatella Donatelli
Editor-in-chief

Learning large-scale industrial physics simulations

Fabien Casenave

In an industrial group like Safran, numerical simulations of physical phenomena are integral to most design processes. At Safran's corporate research center, we enhance these processes by developing fast and reliable surrogate models for various physics. We focus here on two technologies developed in recent years. The first is a physical reduced-order modeling method for non-linear structural mechanics and thermal analysis, used for calculating the lifespan of high-pressure turbine blades and performing heat analysis of high-pressure compressors. The second technology involves learning physics simulations with non-parameterized geometrical variability using classical machine learning tools, such as Gaussian process regression. Finally, we present our contributions to the open-source and open-data community.

1 Introduction

Consider an operator F that represents a complex numerical problem or an expensive experiment. Industrial design processes often require frequent evaluations of such operators in many-query tasks, such as parametric exploration, optimization, uncertainty quantification, or calibration. These tasks are intractable without modifications or simplifications, making them unsuitable for real-life industrial applications. For instance, optimizing the shape of an

aircraft wing to maximize finesse under a minimum thickness constraint involves many iterations of running a complex fluid solver, which is computationally intensive, see Figure 1.

A common approach to manage these tasks is by replacing the expensive operator F with a surrogate f . This surrogate is created by learning the behavior of F from data. A training database is constructed by selecting inputs using a design of experiments (DoE) algorithm and computing corresponding outputs by evaluating F . The surrogate, formed as a regressor trained on these input/output pairs, must be accurate enough to replace F in many-query tasks and fast enough to be practical for industrial applications.

For tabular problems where inputs and outputs are low-dimensional vectors ($\mathbb{R}^d \ni x \mapsto y = F(x) \in \mathbb{R}^p$), traditional regression methods like linear or polynomial regression, nearest neighbors, random forests, support vector regression, or Gaussian process regression can be used. This work focuses on learning physics problems, which involve complex, high-dimensional heterogeneous objects like meshes, boundary conditions, fields, or material models, unlike tabular problems. The inputs are the varying components of the physical setting in the many-query task, while the outputs are the predicted components used for decision-making.

For example, the AirFRANS physics learning problem [1] involves a mesh of the fluid domain, boundary conditions, the Reynolds-averaged Navier–Stokes equations, and the $k - \omega$ turbulence model.

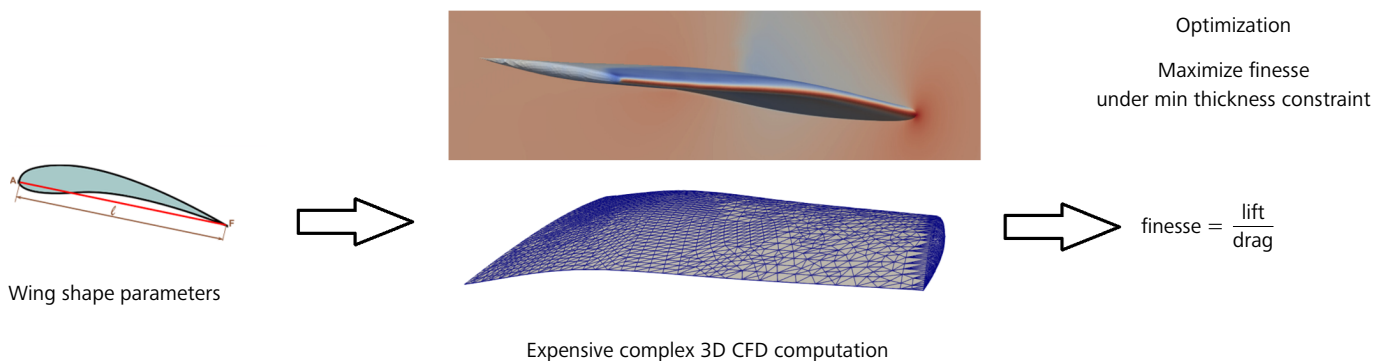


Figure 1. Optimization of the shape of an aircraft wing.

Here, the inputs include the mesh, angle of attack, and inlet velocity, while the outputs are the velocity, pressure, and turbulent velocity fields. The chosen turbulence model being fixed in the training dataset, it is not part of the inputs of the learning problem.

The advantage of physics learning problems over tabular problems with scalar inputs and outputs is the flexibility to change shapes beyond the training parameterization and the ability to modify the post-treatments of predicted solution fields after learning.

This article first discusses physical reduced-order modeling technologies developed at Safran in Section 2. Then, it introduces a machine learning method for non-parameterized geometrical variability in Section 3. Finally, it presents Safran's initiatives in the open-source and open-data communities in Section 4.

The article is a shortened version of the paper submitted to the Proceedings of the 9th European Congress of Mathematics, based on the work presented during the Felix Klein Prize Lecture on July 16th, 2024.

2 Physical reduced-order modeling

Physical reduced-order modeling (ROM) is a surrogate modeling technique that uses the underlying physics equations during the exploitation stage.

2.1 Linear data compression

During the training stage, we generate inputs using DoE algorithms and compute corresponding outputs by evaluating the operator F . We then apply a linear dimensionality reduction algorithm to these outputs to construct a reduced-order basis. In the exploitation stage, we use the same algorithms as in the high-dimension problem but restrict the solution to the subspace spanned by the reduced-order basis. For example, if F involves solving a high-dimensional initial boundary value problem, we replace the finite element basis with the reduced-order basis to keep using the Galerkin method, expressing the reduced solution as a linear combination of the reduced-order basis elements. Various methods have been proposed to adapt physical ROMs to different problems and simulation methods involved in F evaluations. One such method is the Reduced Basis method [20], which starts with a single input/output pair and enriches the data by selecting configurations where the current model makes the largest error, using fast and accurate error bounds [5] for accuracy assessments and to trigger enrichment steps.

The speed-up in physical ROMs comes from the reduced-order basis having a much smaller cardinality than the finite element basis, leading to smaller linear systems to solve in the Newton's algorithm during the exploitation stage. However, constructing these small systems can be costly, especially with non-linear problems requiring

numerical integration over the mesh. To achieve practical speed-up, precomputations and approximations (called hyperreduction) are performed during the training stage to reduce computational effort in the exploitation stage, see [4, Section 3.2] for more details, and [6, Section 1] for references on hyperreduction methods.

We focus on non-linear structural mechanics and non-linear transient heat problems, utilizing snapshot proper orthogonal decomposition (POD) for dimensionality reduction, which is parallelizable with domain decomposition. Additionally, we use a localized empirical cubature method (ECM), which derives reduced quadrature formulae independently for each subdomain. This involves solving an optimization problem to minimize the number of non-zero elements of the quadrature weights vector, using a greedy approach called non-negative orthogonal matching pursuit. This ensures the reduced operators retain the spectral properties of F , maintaining symmetry and positive definiteness of the tangent operator in the Newton's algorithm.

For non-linear structural mechanics and transient heat problems, we have demonstrated significant speed-ups, see respectively [4] and [6]. For instance, our physical ROM reduced the computation time for the lifetime prediction of 3D elastoviscoplastic high-pressure turbine blades from weeks to hours, and accelerated transient heat analysis of high-pressure compressors with complex boundary conditions, achieving errors compatible with industrial requirements.

In structural mechanics applications, quantities of interest often include dual quantities, such as plastic strain or von Mises stress. To reconstruct dual quantities over the entire mesh, the gappy POD method is used. By enforcing the reduced quadrature scheme to include well-chosen quadrature points, the gappy POD reconstruction can be made well-posed, see [4, Proposition 1]. Our numerical experiments revealed a strong correlation between the prediction error on dual quantities and the ROM-gappy-POD residual, which is defined as the error, on the reduced quadrature points, between the online computation of the corresponding quantity by the constitutive law solver and the gappy POD reconstruction. Thus, we introduced [3] an error indicator, which is a regressor trained to predict the error from the ROM-gappy-POD residual. This indicator successfully maintains the error below a chosen threshold by triggering enrichment steps.

Linear dimensionality reduction enables the application of the Galerkin method during the exploitation stage. However, this restriction can sometimes result in a reduced-order basis with a cardinality too large to achieve significant speed-up, even with hyperreduction. These situations, referred to as poorly reducible problems, will be addressed in the following section.

2.2 Piecewise linear data compression

The reducibility of the approximation of F by f linear dimensionality reduction is evaluated by the rate of decay of the Kolmogorov

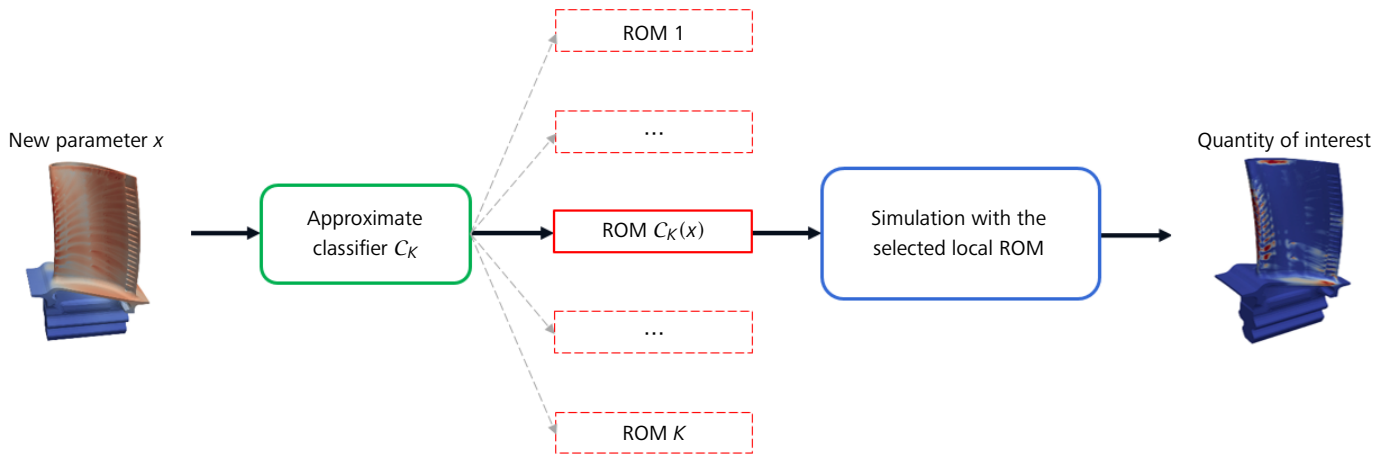


Figure 3. Local ROM recommendation.

N -width of the output solutions with respect to the dimension N of an optimal approximation vector space. The Kolmogorov N -width measures the worst approximation error of a set of outputs by the best N -dimensional subspace. If the decay rate is fast, linear data compression methods are effective. If not, the reduced-order basis becomes too large, making the reduced problems slow to solve, indicating a poorly reducible problem. This challenge is addressed in Thomas Daniel's Ph.D. work [11].

To tackle poorly reducible problems, we focus on creating a dictionary of local physical ROMs using piecewise linear data compression, by clustering the outputs fields, see Figure 2. Consider

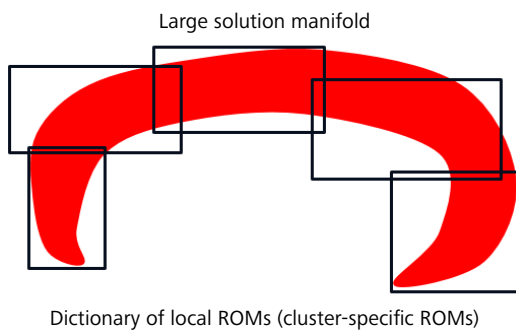


Figure 2. Piecewise linear data compression

the problem of predicting the lifetime of 3D elastoviscoplastic high-pressure turbine blades, now with the temperature loading field as input. To use a dictionary of local ROMs, we need to know in advance in which cluster of the outputs the solution is located, for us to use the adequate local ROM. To achieve this, we train a classifier to select the correct local ROM based on the 3D input temperature field, similar to how classifiers in computer graphics

recognize objects in images, see Figure 3. Such workflows are proposed in [13]. Unlike image classifiers, our situation involves few high-dimensional inputs. To improve the classifier training, we developed specialized feature selection and data augmentation techniques. This approach ensures accurate local ROM selection and effective handling of complex, high-dimensional input data, see [14].

Partitioning outputs using the L^2 -norm in the ambient solution space has been explored in the literature, but this does not guarantee low-dimensional approximation subspaces. In [12, Property 4.13], we demonstrate that the partitions of outputs minimizing the k -medoid cost function with sine dissimilarity are exactly the minimizers of the sum of a local variant of the Kolmogorov 1-width, weighted by the partition element's volume. This method, which considers the relative angles between outputs, offers a practical algorithm for optimally partitioning outputs for our purposes.

Consider a 2D advection problem where Gaussian processes with small (0.1) and large (1) amplitudes and various vertical positions ξ_2^0 move from left to right, see Figure 4. We illustrate the

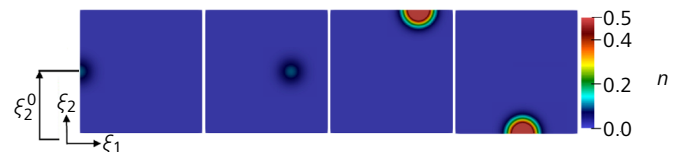


Figure 4. Some output fields for the 2D advection problem [12].

clustering of these outputs using multidimensional scaling (MDS) with five clusters, comparing L^2 -norm and sine dissimilarity. With the L^2 -norm, all outputs with the small amplitude (0.1) are tightly packed at the center, likely grouping into the same cluster. This cluster contains all the independent directions of the outputs, hence

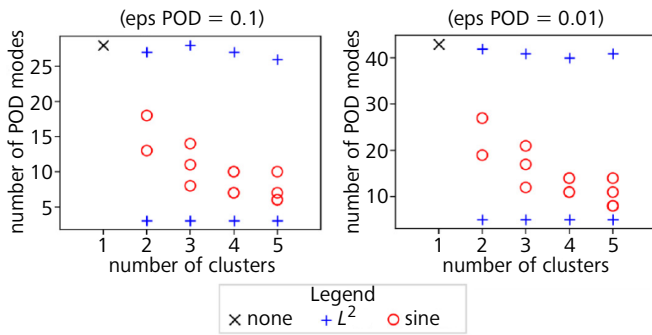


Figure 5. Number of POD modes for each local basis with respect to the number of clusters, for various accuracy criteria of the POD truncation, applied to the 2D advection problem [12].

has a Kolmogorov N -width not smaller than the one of the set of all outputs. Conversely, with sine dissimilarity, three connected components emerge, corresponding to the three considered ξ_2^0 values. At each position are located two outputs varying only by magnitude, indicating zero sine dissimilarity for outputs differing only in magnitude.

In Figure 5, we compare the effectiveness of clustering outputs using the L^2 -norm versus sine dissimilarity in constructing local ROMs of small dimensions. When using the L^2 -norm, one cluster ends up with the same dimension as a global ROM, specifically the cluster containing all small magnitude outputs. This indicates that no matter how many clusters are chosen, one cluster remains as poorly reducible as the entire set of outputs. Conversely, when using sine dissimilarity, the dimensions of all local ROMs decrease as the number of clusters increases. This demonstrates that clustering outputs using sine dissimilarity is beneficial, as it successfully reduces the ROM dimensions. Therefore, the choice of dissimilarity in output clustering is crucial, and the sine dissimilarity proves to be effective in this context.

In [15], a dictionary of local ROMs is developed to quantify the uncertainty of the plastic strain and von Mises stress in high-pressure turbine blades with respect to input temperature loading. First, we parameterize the variability of the input temperature field and construct two DoEs: one MaxProj of size 80 and one Sobol' of size 120. We evaluate the operator F for these 200 configurations, taking 7 days and 9 hours on 48 computer cores. Using the sine dissimilarity with the PAM k -medoid clustering algorithm, we partition the MaxProj DoE outputs into two clusters in 5 minutes. Next, we construct the two corresponding local ROMs using the POD-ECM technique presented in Section 2.1, which takes 5 hours on 24 computer cores. We label the 120 Sobol' DoE outputs by computing the sine dissimilarity with the two identified medoids in 5 minutes. Our feature selection algorithm takes 16 minutes on 280 computer cores, followed by training a logistic regression classifier with elastic-net regularization in 1 minute. This process results in a ROM dictionary of two local ROMs and a classifier ca-

pable of recommending the appropriate local ROM based on input temperature loading, making our surrogate f ready for use. Finally, we conduct probability density estimation of the plastic strain and von Mises stress in areas of interest on the turbine blade using 1,008 Monte Carlo evaluations of f . This yields relative errors of 1–2% and a speed-up factor of 600.

2.3 Limitations

Despite the successful application of physical ROMs to many equations and configurations, two significant challenges persist. First, non-parameterized geometrical variations make it difficult to compare outputs supported on different meshes. Second, global non-linear data compression prevents the use of the Galerkin method during the exploitation stage. Some authors have developed solutions to address these limitations while still constructing a surrogate f that involves solving the physics equations. However, these approaches often involve complex algorithms that can hinder practical speed-ups.

In the following section, we propose a surrogate f that incorporates non-linear data compression and manages non-parameterized geometrical variations, but does not rely on solving the physics equations during the exploitation stage.

3 Learning physical problems with non-parameterized geometrical variability

3.1 Mesh morphing Gaussian process

In our recent work [9], we introduce the mesh morphing Gaussian process (MMGP) technique, which integrates four key components: (i) mesh morphing, (ii) finite element interpolation, (iii) dimensionality reduction, and (iv) Gaussian process regression. This approach is designed to learn solutions to partial differential equations (PDEs) that involve geometric variations not explicitly parameterized.

Figure 6 illustrates the MMGP inference process for predicting a field of interest. The figure should be read from left to right: the left side shows various geometries corresponding to different samples, while the right side displays the target fields to be predicted on these geometries. For non-parameterized meshes, we perform a shape embedding process to convert meshes into learnable objects, by considering the coordinates of the mesh vertices as continuous fields. The left column of Figure 6 depicts the continuous field of the x -component of the coordinates, showing vertical iso-values. We then apply a deterministic morphing process to transform input meshes into a common shape, such as the unit disk shown in Figure 6. Each sample is converted into a mesh of the unit disk, modifying the coordinate fields according to the input mesh shapes. We then select a common mesh of the unit disk and project the coordinate fields from each morphed mesh onto this common mesh using finite element interpolation. This process

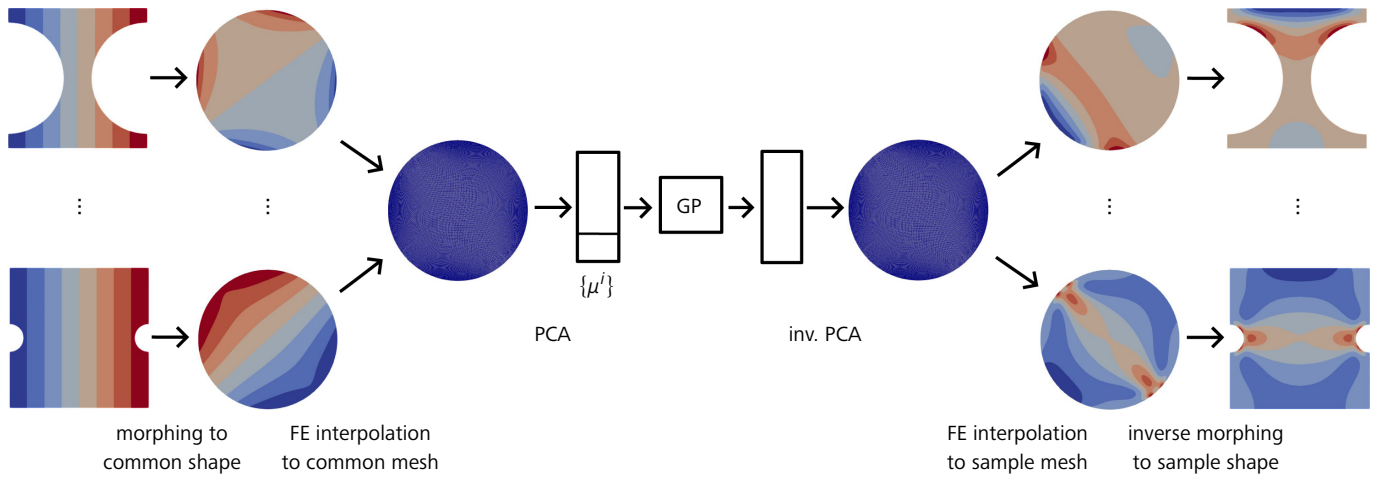


Figure 6. Illustration of the MMGP inference workflow for the prediction of an output field of interest [9].

results in all coordinate fields being represented on the same mesh, allowing us to use classical dimensionality reduction techniques, like principal component analysis (PCA), to generate low-dimensional vectors for the meshes. If scalar inputs are part of the learning problem, these are appended at the end of these vectors.

Similarly, a field embedding process is needed to convert variable-sized output fields into learnable objects. We apply the same transformations used for coordinate fields: morphing onto a common shape, finite interpolation onto a common mesh, and PCA compression. This process results in low-dimensional vectors representing our output fields of interest.

These deterministic pre-treatments effectively reduce the physics learning problem to a low-dimensional input/output tabular regression problem. The morphing and related steps act as the dimensionality reduction phase, which, while possibly highly non-linear, simplifies the machine learning task by converting large, variable-dimensional objects into smaller manageable ones. The deterministic nature of these pre-treatments is essential, as it simplifies the machine learning stage and avoids the complexity of dealing with large and variable-dimensional data directly.

We use Gaussian process regression for its high accuracy and ability to estimate predictive uncertainties effectively. For further details and numerical experiments demonstrating the efficacy of MMGP, including comparisons with recent deep learning technologies like MeshGraphNet and U-net graph convolutional networks, we refer to [9].

3.2 ML4PhySim competition

We participated in a competition [16] organized by IRT SystemX and partners, where we employed our MMGP technology. The challenge involved predicting pressure, velocity, and turbulent viscosity fields on the AirFRANS dataset [1], introduced in Section 1.

To achieve effective morphing, we first extended all 2D meshes to a common boundary box to handle irregular external boundaries, as shown in Figure 7.

We select the geometry of the first pretreated mesh from the training set as the common shape. We specify the morphing of chosen control points — on the intrados and extrados of the airfoil, the wake line, and the external boundary — and use a Radial Basis Function (RBF) morphing algorithm to compute the transformation of all other points. We project all fields, including input coordinates and output fields, onto the common mesh (the pretreated and morphed mesh from the first training sample). For dimensionality reduction, we used snapshot proper orthogonal decomposition (POD) as detailed in [9, Annex C]. The snapshot POD approach ensures that objects reconstructed from reduced representations preserve zero linear relationships, which is critical for maintaining zero boundary conditions, such as those for the velocity field at the airfoil surface. This helps to ensure that the zero boundary condition is respected in the predicted fields. We train separate Gaussian processes for each generalized coordinate associated with the snapshot POD of each output field. The Gaussian process kernel combines a constant term with an RBF kernel and a white noise term.

Some results are illustrated in Figure 8.

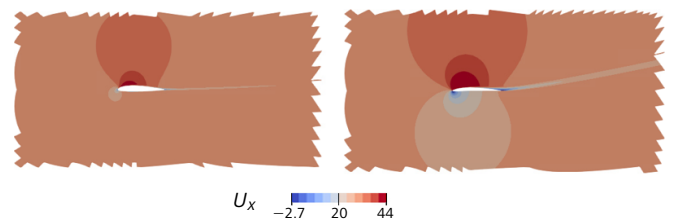


Figure 7. Illustration of the horizontal velocity for two samples of the AirFRANS dataset: the external boundaries are very irregular.

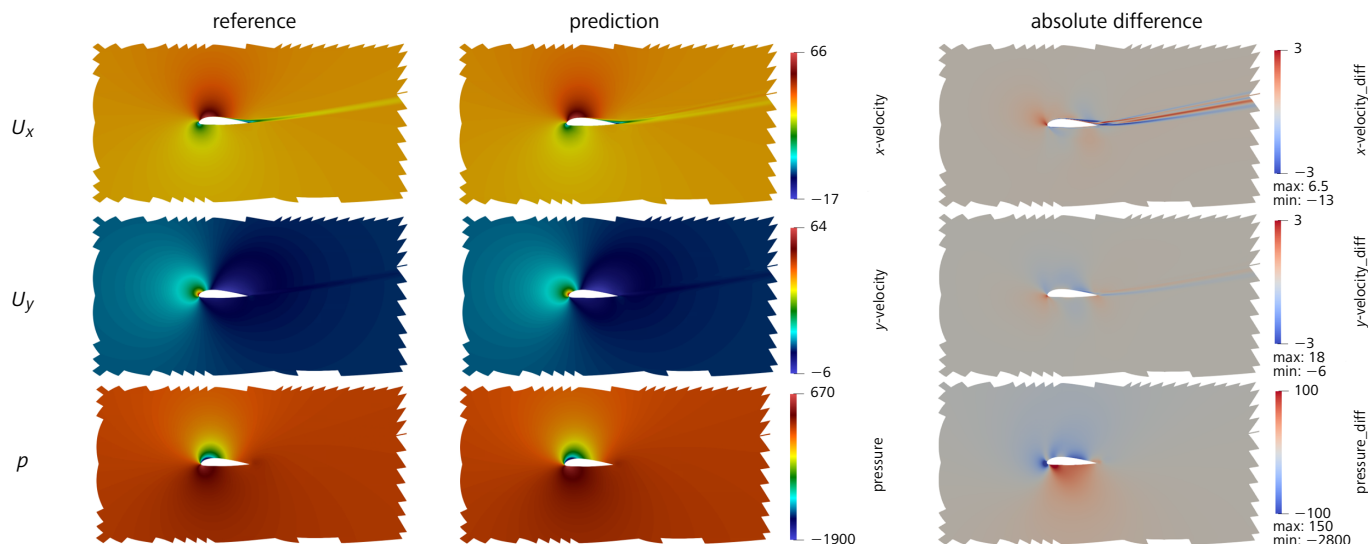


Figure 8. Illustration of the result on a test sample.

The competition score, ranging from 0 to 100, evaluates both the speed and accuracy of the surrogate model, including scalar quantities derived from the output fields (such as lift and drag coefficients) and their Spearman’s rank correlation coefficients, tested on two test sets, one of them being out-of-distribution. Table 1 shows that our method won first place, outperforming competitors who used advanced deep learning techniques, despite our approach using simpler and more classical tools.

3.3 Outlooks

MMGP is an effective method for approximating field and scalar outputs in complex physics problems with non-parameterized geometrical variability. It combines mesh morphing with finite element interpolation and uses shape embedding by reducing the dimensions of coordinate fields, simplifying the machine learning task. This approach enables efficient Gaussian process regression on reduced-dimensional data.

The method can handle large meshes, can be trained on CPU, is interpretable, and provides accurate predictions and predictive uncertainties. It is especially efficient for industrial part design, where configurations often have low intrinsic dimensions.

However, MMGP has some limitations: the morphing process must be customized for each use case, it assumes a fixed mesh topology, and speed-up is limited by the need for morphing and interpolation during inference. Future work will focus on addressing these issues by developing a generic morphing method, creating efficient approximate morphing procedures for inference [19], and deriving an optimal morphing strategy to minimize dimensionality further.

4 Open source and open data

Since the inauguration of the Safran corporate research center in 2015, we have implemented processes to make our research

rank	method	physics (30%)				ML-related (40%)					OOD generalization (30%)				global score (%)							
		physical criteria				accuracy (75%)					OOD accuracy (42%)		OOD physics (33%)									
		C_D	C_L	ρ_D	ρ_L	\bar{u}_x	\bar{u}_y	\bar{p}	\bar{v}_t	\bar{p}_s	speed-up (25%)	\bar{u}_x	\bar{u}_y	\bar{p}		\bar{v}_t	\bar{p}_s	C_D	C_L	ρ_D	ρ_L	speed-up (25%)
1	MMGP	●	●	●	●	●	●	●	●	●	27.4	●	●	●	●	●	●	●	●	●	28.08	81.29
2	GNN-FC	●	●	●	●	●	●	●	●	●	570.77	●	●	●	●	●	●	●	●	●	572.3	66.81
3	MINR	●	●	●	●	●	●	●	●	●	518.58	●	●	●	●	●	●	●	●	●	519.21	58.37
4	Bi-Trans	●	●	●	●	●	●	●	●	●	552.97	●	●	●	●	●	●	●	●	●	556.46	51.24
5	NeurEco	●	●	●	●	●	●	●	●	●	44.93	●	●	●	●	●	●	●	●	●	44.78	50.72

Table 1. ML4PhySim competition final leaderboard [17].

codes open-source and non-confidential datasets available as open-data. Our commitment to accessibility, quality, and robustness involves using DevOps tools and practices, such as Git for version control, code reviews, automated testing, deployment pipelines, and extensive documentation. We opt for permissive licenses like MIT or BSD3 for our code and CC-BY-SA for datasets. Our codes are hosted on GitLab, documentation on Read the Docs, and packages are distributed via conda-forge, when possible.

4.1 Open-source codes

One recent development is the PLAID (Physics Learning AI Data-model) library, which formalizes physics learning problems and manages complex datasets. It leverages the CGNS standard [10] for data handling, promoting consistency and ease of data sharing among practitioners.

The codes developed for the methods presented in the previous sections are open-source. Notably, Muscat [2, 29] is a li-

brary for mesh processing in finite element computations. PLAID [30] is used as a foundation for our physics learning developments. Physical ROM technologies are supported by two libraries: Mordicus [18], co-developed with the MOR_DICUS consortium, and genericROM [26], developed by Safran. The implementation of genericROM is non-intrusive, parallel with distributed memory, and can handle non-parameterized variability, see [25] for details on these notions and a description of Mordicus and genericROM. Additionally, MMGP, discussed in Section 3, is also available as open-source [28]. For tabular learning problems, Safran's open-source platform Lagun [27] supports a range of tasks including design of experiments, surrogate modeling, sensitivity analysis, and optimization.

4.2 Open-data datasets

We are in the process of making our non-confidential datasets available as open-data in the PLAID format, facilitating their use

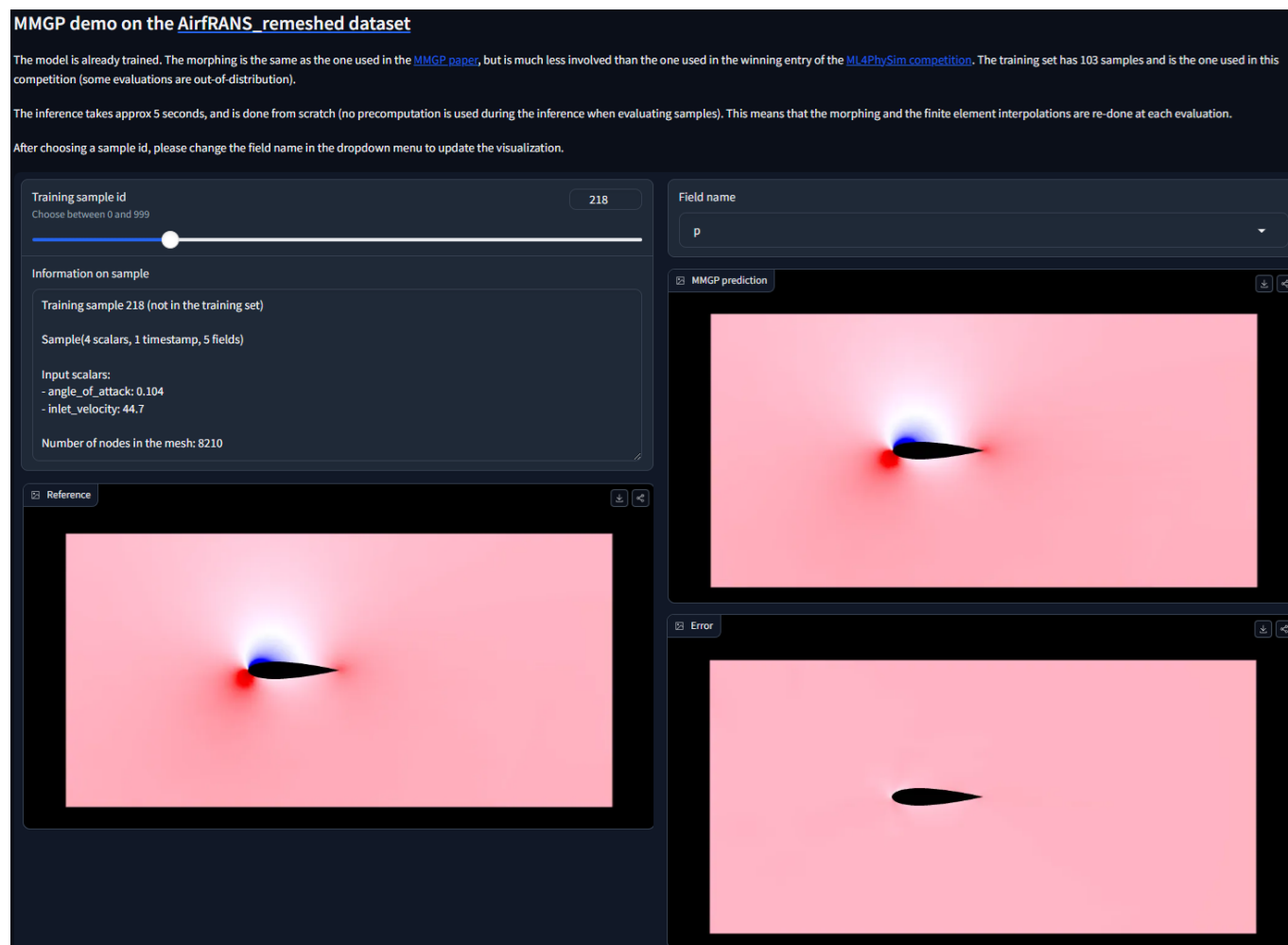


Figure 9. Hugging Face Spaces hosting a demo of our method MMGP applied on the physics learning problem of the ML4PhySim competition.

by the community. These datasets are shared through Zenodo and Hugging Face. Specifically, the three datasets used in our recent comparison of MMGP with advanced deep learning methods in [9] are accessible on Zenodo [7, 21, 22] and Hugging Face [8, 23, 24].

Additionally, we have distributed a demo of our MMGP method on Hugging Face's Spaces platform, which was applied to the ML4PhySim competition problem presented in Section 3.2, see Figure 9. In this demo, users can select a sample ID, which triggers the MMGP surrogate model, which involves: morphing, finite element interpolation, projection onto snapshot POD modes, inference using a Gaussian process regressor, and finally, inverse snapshot POD and finite element interpolation. Predicted fields, reference data, and errors are plotted. Note that for real-time performance, the demo uses a remeshed dataset with meshes reduced from approximately 180,000 nodes to around 8,000 nodes.

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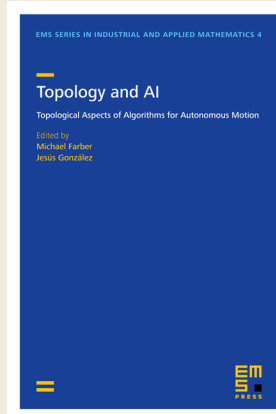
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Interview with Motoko Kotani

Jean-Pierre Bourguignon

I am in the Tokyo office of Tôhoku University, one of the Japanese national universities located in Sendai, in the Northern part of the country, to interview Motoko Kotani, a professor at this university and executive vice-president in charge of international strategy for research, for the EMS Magazine.

Jean-Pierre Bourguignon: *Dear Motoko, my first question to you is: when did you decide that you will be a mathematician? Was there a particular person responsible for this decision? Did you have to overcome some obstacles?*

Motoko Kotani: Since I was a very small child, I loved reading books. On the other hand, I was shy and not very good at communicating and participating in social activities with other kids. I enjoyed being by myself with books.

JPB: *Where were you born?*

MK: In Osaka but, when I was 10 years old, my family moved to Kamakura, a suburb of Tokyo. Therefore, I basically grew up in Tokyo. In junior high school, I asked the schoolteachers many questions after classes. The questions were not about what they taught but, since I read books, I came up with some questions and ideas on my own.

JPB: *Questions on mathematics?*

MK: Not necessarily. At the beginning, I went to many teachers, and maybe some teachers considered me difficult. Then I found a mathematics teacher who answered my questions, and I liked to ask him more often. At that time, I thought it was his nature, positive and happy, that made him answer my questions. Now that I have become a mathematician myself and look back, I realize that it's in the nature of mathematics. Indeed, in many subjects, if you are a kid, you probably cannot ask questions in a proper way to a teacher. This involves a lot of knowledge, and also understanding the reason why something being right and wrong is not just logical. But somehow about mathematics I could explain what I thought in a logical way. And the teacher could point out when I made



Professor Motoko Kotani. (Photo credit: Tôhoku University)

a mistake or I was wrong; or, if I was right, when what he said was actually wrong, I could convince him. I found the whole thing very fair and inspiring, and I became even more motivated. This is how I started to like mathematics.

JPB: *Was there one specific teacher who really answered more of your questions or were there several?*

MK: There were several ones, that's why I thought it's because of the nature of mathematics. At the beginning, I asked questions related to many different subjects, but eventually I just concentrated on mathematics, because I had good experiences with mathematics teachers, and therefore I continued to ask mathematical questions. At that time, I didn't know mathematician could be a profession.

JPB: *When did you discover there was the profession of a mathematician?*

MK: Maybe in high school or maybe even when I applied to the university and entered the University of Tokyo. When I discovered there was such a profession, I wanted to become a mathematician.

JPB: *But that you had to study mathematics was obvious to you. It was not physics, not chemistry, not biology?*

MK: Choosing mathematics was obvious. I don't know when I started thinking that way, but I trusted the famous saying that "*mathematics is the language of the universe.*" I thought mathematics lies at the core of scientific knowledge. If I had only one life, I wanted to be connected with the most essential language of our universe.

JPB: *When you thought that you may become a mathematician, did that mean immediately that you wanted to get a PhD? Or did this happen after you had gone through graduate school?*

MK: At the University of Tokyo, you first entered the Science School; after two years you had to select your major department. I did very well in my first two years of undergraduate studies, my scores in mathematics and physics were all very good in particular. When I became a student in the mathematics department, I met very talented students, for example, Kenji Fukaya and Mikio Furuta, nowadays well-known mathematicians. I was shocked, and a bit disappointed. It looked like, although I loved mathematics, mathematics did not need me. That was my immediate feeling. Still, I wanted to continue, because I was not able to consider something else as my profession. At the same time, I didn't have the confidence to achieve my goal, namely, to make some kind of contribution in mathematics.

JPB: *Why were you doubtful? Because you had the feeling that other people you were studying with, like Kenji Fukaya or Mikio Furuta, were faster or better than you?*

MK: Faster and much deeper.

JPB: *Oh, you thought so?*

MK: Yes. Completely different dimensions. In the end, I started late, later than them. They were flying in the sky, but I was crawling on the ground. So, they were 3-dimensional, while I was bound to the plane. I developed the feeling that, probably, I cannot contribute too much in my life.

JPB: *Who was your advisor?*

MK: Professor Takushiro Ochiai, an associate professor back then, was my undergraduate advisor. I didn't go to graduate school at the University of Tokyo, but at Tokyo Metropolitan University. There was no professor in differential geometry at the University of

Tokyo. Ochiai-sensei said, if I want to continue in geometry, Osaka, Tsukuba and Tokyo Metropolitan are good universities to choose from. I decided to go to Tokyo Metropolitan University because it is in Tokyo, so that I could commute easily.

JPB: *And your advisor for the PhD there was?*

MK: Professor Koichi Ogiue. Actually, he soon after became president of Tokyo Metropolitan University and got extremely busy.

JPB: *That was bad news for you.*

MK: No. I was happy. Anyway, I was quite independent in the studies. When I entered the master course, I had recovered my confidence.

JPB: *Good.*

MK: Because the way undergraduate studies go, you know, there are summits which everybody is aiming at. Since they are beautiful, nothing else seems important. When I started the master course, I found mathematics is actually much richer. There are so many problems you can consider, and, if you are motivated, you can find interesting programmes of your own. I felt there was room for me to contribute.

Actually, I was very quick to write a paper. My first paper [1], which appeared in the Tôhoku Mathematical Journal, was written in the first year of my master studies.

JPB: *And you found a problem to work on by yourself or did somebody ask you questions?*

MK: When I became Ogiue-sensei's student, he gave me a list of articles. He suggested me to read the book by Marcel Berger on the spectrum of the Laplacian, and this became my subject. I studied several articles he suggested, and I found some kind of generalization, leading to a paper.

JPB: *In a sense, at some point you really became a mathematician by yourself... when did you defend your PhD?*

MK: It was in 1990.

JPB: *You found that differential geometry was a place where there were enough problems to which you could contribute.*

MK: Geometry used to be more algebraic, the representation of groups or some kind of tensor calculus. When I entered the master course, already global or geometrical analysis was being introduced in Japan. Around Tokyo, Tokyo Metropolitan, Keio, and TiTech, there were several PhD students both in geometry and PDE,

who wanted to learn more about global analysis. We gathered every Saturday to read books, Thierry Aubin's book, and Mikhael Gromov's green book. I found them very interesting. Those topics were more like what I wanted to study. I found differential geometry or global analysis suited me.

JPB: *Was Professor Obata there?*

MK: Professor Obata already retired. This group consisted of younger people, like master and doctoral students. They were all motivated to work in global analysis.

JPB: *Did you have opportunities to travel abroad early in your career? Did any special event, a conference, a visit, a personal encounter play a critical role in the development of your career?*

MK: I don't remember when I went abroad for the first time, but one thing that was very important for me was the International Congress of Mathematicians (ICM) held in Kyoto in 1990. There I met these professors whom I knew only by their names in articles. Before the ICM, there were conferences inviting mathematicians from abroad, specialised in global analysis, geometry, etc. Participating in the ICM was however extraordinary.

JPB: *But you didn't travel abroad before that?*

MK: Actually, I met several people there, and I said I wanted to visit. They invited me, or something like that.

JPB: *What is the first country outside Japan that you visited? Was it the U.S.? Was it Germany?*

MK: Maybe the U.S. I visited David Hoffman.

JPB: *Was he in Massachusetts at that time?*

MK: Yes. And then, in 1993, I applied to visit the Max Planck Institute, and I spent one year in Bonn. During my stay in Bonn, I also visited many other places in Europe. In 2001, I stayed at IHÉS, and École polytechnique in France for one year. These were some opportunities that changed my research scope.

JPB: *At some point in your career, you were asked to take some responsibility, in particular the directorship of a laboratory in materials science. How did this happen? What was the main challenge when you took over this?*

MK: For this, I need to explain from the beginning. The national programme "World Premier International (WPI) Research Centre Initiative" was established in 2007 to build within Japan "globally visible" research centres that boast a very high research standard

and outstanding research environment. Applications were open to universities from all around Japan. In the first round, Tôhoku University received one of the five institutes, AIMR, together with the IPMU (University of Tokyo), iCeMS (Kyoto University), IFRcC (Osaka University), and MANA (AIST [the National Institute of Advanced Industrial Science and Technology]). And at that time in Japan, there was no support for such big programmes. This one was for the first time a 10-year programme with, each year, 1.30 billion Yen given (close to 10 million Euro), a substantial amount of financial support compared to usual grants. Of course, many research universities wanted to get one. Tôhoku University was very proud to be one of them. Because it was famous for materials science, it was therefore natural that it has the Advanced Institute for Materials Research (AIMR). This programme was very ambitious, with the purpose of reaching world-leading scientific excellence and recognition by not only gathering important persons, who do excellent research, but also by creating a new scientific frontier together through interdisciplinarity and diversity.

JPB: *Then an evaluation came some two to three years after the opening?*

MK: The mid-term evaluation took place in the 5th year, but it was different from the one done by the WPI programme committee which annually monitored the progress made.

JPB: *And the WPI programme committee was external to the lab itself?*

MK: It was an international committee set by the Ministry for Education, Culture, Sports, Science and Technology (MEXT). AIMR has been told the quality of the science was very high but they should have more ambition. The university has excellent researchers in materials science who produce good papers with/without AIMR. The question was: what has been produced which would not have been done without the WPI programme?

JPB: *What should have been the effect of WPI?*

MK: In the application proposal, AIMR promised to create a new materials science by gathering leading materials scientists with different backgrounds from all around the world. When you hear materials science, you may think it is one discipline, but it is not. Materials science is interdisciplinary research, some related with physics, with chemistry, with metallurgy, or with bioengineering. AIMR researchers talked to each other and found several interesting common phenomena across different materials systems, but something more was needed to integrate those ad-hoc discoveries to create a new materials science. You know it is difficult.

JPB: *Okay, so the objective was ambitious.*

MK: As a result, the university found that mathematics could be the catalyst for activating the interaction across disciplines. At that time, I was not involved in the AIMR, but the university noticed that a mathematician led a project called “*Mathematics and Materials Science*” in the CREST programme at the Japan Science and Technology Agency (JST) and thought it could be a solution to invite this person, me, to lead the AIMR to identify its direction.

JPB: *And you became director of the Tôhoku WPI Institute?*

MK: I was invited to become the new director for the centre in 2012, after I went through the mid-term evaluation in 2011 by showing AIMR’s strategy, which was accepted well by the WPI programme committee.

JPB: *Wonderful. The main challenge there was to come up with a more ambitious and more global approach. But what was the main challenge you faced when becoming the director, because you were not an obvious choice?*

MK: It took me one month to decide whether I would like to take this position or not when I was invited to become a director by the university. After all, I am not a materials scientist, which is not physics, which is a much more logical subject. It is not such a simple thing to create materials. At that time also, there was some movement in Japan to use mathematics as a basis for interdisciplinary research. I had been claiming that mathematics can play an important role as the common language of science. If you want to really change science, you should use mathematics. I kept saying that. I thought it is an opportunity for me to prove what I have been saying. As a mathematician, I can only say yes when being challenged to prove something.

JPB: *But, in a sense, you didn’t feel there was some resistance from physicists...*

MK: I was very lucky that, because of the special nature of the institute, its members were highly motivated for collaboration.

JPB: *Yes, I remember you mentioned to me at that time, that people understood that they had to work together much more closely.*

MK: I was also very lucky that, in 2013, we published a paper in “*Science*” on materials using topology. A successful example of collaboration with mathematics by conceptualising the data materials scientists obtained in experiments.

JPB: *You with a few people?*

MK: I was one of the authors together with a young mathematician, and several materials scientists doing experiments.

JPB: *Excellent. Later you also became the president of the Mathematical Society of Japan (MSJ). How did this happen? You were the first woman to be in this position.*

MK: It was no less challenging, actually. I had been working in the MSJ Governing Board for eight years by that time. I was happy to contribute to the MSJ in any way.

JPB: *And in this function of MSJ president, what did you find the most challenging?*

MK: What the MSJ faced happened to many academic societies in Japan. The society has a long history: it started at a small scale, based on voluntary effort. It was not built like a company. Many things functioned based on implicit understandings and experiences, without written rules. It was the time academic societies, not only the MSJ, but almost all the society, started reorganising their governance. When I became president, I noticed there were very few written rules. Everything was done in a more or less non-organisational way. As a result, I and several other members of the board decided to establish written legal rules, and this was a real challenge.

JPB: *How long have you been the MSJ president?*

MK: Two years: 2015 and 2016.

JPB: *But at that time, you also were the director of the Advanced Institute for Materials Research at Tôhoku, isn’t that right?*

MK: Yes, and I was also a member of the Council on Science and Technology Policy (CSTP) in the Cabinet Office. As a result, I came to Tokyo once a week at least.

JPB: *Could you take care of the Institute, the MSJ and take part in the CSTP at the same time?*

MK: Yes. I used to come to Tokyo frequently. When I came, I stopped by the MSJ office. Actually, it didn’t increase my duty so much, but the responsibility was big because, as I said, I tried to reform the MSJ to make it an organisation governed more professionally.

JPB: *Wonderful. The next point I want to make concerns one of the structures at Tôhoku University you have been involved in, namely the Tôhoku Forum for Creativity. Was this your idea or somebody else’s idea? How did this come about and was there a special need for such a structure at Tôhoku?*

MK: As I told you, I visited the Max Planck Institute in Bonn and IHÉS in Paris. They are institutes where mathematicians can visit and stay

longer under thematic programmes. Japan has such an institute, RIMS in Kyoto, which supports long-time mathematical activities in Japan mainly by organising workshops, and collaborative research among small groups, but does not really run thematic programmes. All mathematicians in Japan wanted to have an institute like MSRI, Institut Henri Poincaré, the Newton Institute. We didn't have one.

JPB: Oh, I understand. The Tôhoku Forum for Creativity (TFC) runs several thematic programmes annually, but it is not just for mathematics but also open to all branches of science. How was it established?

MK: I wanted to have such an institute somewhere in Japan. I had the opportunity to speak to the president and executive vice-president for research, and was able to convince them to have it focused on creativity, gathering people from different backgrounds. It was not for the science which already existed, but for exploring pioneering ideas. The president was a big supporter of dreams. In the end, he liked the idea and gave me an opportunity to present this idea to the advisory board of the president, composed in half from industry people, and in half from Tôhoku University people. The CEO of the Tokyo Electron Company, which is one of the biggest semiconductor companies in the world, also liked this idea of buying dreams and decided to finance it. This is why the building has the name Tokyo Electron at the front door. This is how it was built.

JPB: I understand. As you know, I participated in the TFC Advisory Board, and I really found you managed to attract very good and diverse proposals.

MK: Thank you. Actually, professor Maeda worked very hard to achieve that. He visited as many researchers at Tôhoku University as he could, discussed with them and encouraged them to apply.

JPB: You mentioned already at some point that you were involved in the Council for Science and Technology Policy (CSTP) in the Cabinet Office. How was this? Later on, you were also involved in international relations for RIKEN [National Research and Development Agency, Japan] and so on? How did you get involved in all these more policy-oriented actions? Did you propose yourself or did people just call upon you?

MK: They invited me. I think it is related with my being successful to rebuild the WPI-AIMR. They thought I was good at taking care of different organisations, possibly international ones. I could approach things from several different angles, and that's why they invited me to take these functions.

JPB: Did the fact that, at that time, Yuko Harayama, a colleague of yours at Tôhoku, was a CSTP executive director play a role?

MK: I don't know, but I guess it was the case. The structure of the CSTP is the following: the chair is the prime minister himself, half of the members are ministers related to science and technology, and half of the members are so-called executive members, some from industry, some from academia, and one representative from the Science Council of Japan. Yuko Harayama and another person from industry were appointed full-time, other members were part-time. It meant that the two were leading the work; the prime minister and ministers taking advantage of it. The prime minister orders forming opinions or plans of actions on certain issues and this executive group works to come back with recommendations or statements on his requests. Upon that response, CSTP together with the ministers discuss the issues. The prime minister decides to move ahead. And then it becomes public, and it is financed.

JPB: It seems to be a system which does not involve too many intermediaries. You are talking directly to the prime minister. It's not filtered by many layers.

MK: It is a valuable feature of the CSTP. The members are given opportunities to speak directly to the prime minister without consulting others.

Now, I'm the science and technology co-advisor to the minister of foreign affairs, but such a system has not yet been established there. We meet the minister from time to time and speak to her/him directly. There is no mechanism however to authorise and make those conversations have financial consequences. They are recommendations to the minister. I think it is also important for the minister to have advisors to hear opinions in this way which are not intended to the public.

JPB: Wonderful. Other actions you have been involved in concern the situation of female mathematicians. You were associated, for example, with an exhibit of portraits of female mathematicians created by Sylvie Paycha, which was presented in Tokyo at the Europa House, which is the location of the European Union Embassy. You are also a member of the Committee for Women in Mathematics of the International Mathematical Union (IMU). What is your assessment of the situation of female mathematicians in Japan or more broadly? Do you see progress?

MK: There is progress, but a very, very slow one. The number of girls studying science at the University of Tokyo is not increasing but decreasing.

JPB: And the percentage is what? 20%? 30%?

MK: 20% or something like that at the University of Tokyo. At Tôhoku University it is increasing and now it is 30%.



A photo taken at a meeting of the IMU Committee for Women in Mathematics held at the ICTP in Trieste in 2023. (Photo credit: International Mathematical Union)

JPB: *I see. Do you have some actions in mind which need to be taken to improve the situation?*

MK: The year 2006 is a memorable year for female scientists. The Japan Gender Equality Bureau at the Cabinet Office was established in 2001 and started setting the Basic 5-year Plans. The first one was 2001–2005, and the second one was from 2006 on. Based on the second one, a budget was allocated to projects to encourage women in science. Tôhoku University got its project supported. I was the project leader: we changed some of the working rules to make them more flexible, established a kindergarten on campus, supported parents to work with young kids by hiring babysitters and research assistants, appointed female PhD students as role models for the younger generations and sent them out to give lectures in high schools. I think we did whatever we could do, but the problem continues. Better than nothing, the numbers are going up, but very slowly. For the gender gap index, Japan is ranked 120th among 165 countries in 2023, it is always the lowest among OECD countries.

JPB: *But is there some national effort with special support, or does each university have to decide of an action on its own?*

MK: There has been a national effort. The problem is related with culture, working styles, education, unconscious bias and many other things.

JPB: *This is continuing, or not so much?*

MK: I think it does. National universities, such as Tokyo, Kyoto and Tôhoku, have their performance evaluated and one of the criteria is the ratio of women among faculty staff and students. The rule is

that you are encouraged to hire women in line with the ratio of PhD students. It is of course welcome to hire more women, but you are not forced to hire like 50%, i.e., parity. At Tôhoku University the target of the hiring is that 30% should be women.

JPB: *But do you have your own thoughts on why so few women go to study science in universities? Is it just that families are worried if they go in this direction? What is the cause?*

MK: The reason there are very few girls or women in science is that, first of all, in Japan there are two words – *rieki* and *bunkei*. *Rieki* is natural sciences, engineering, medical sciences, etc. while *bunkei* is social sciences, literature and humanities. And there are big gaps between these two groups in relation with gender: one is for men, the other one for women. At the age of 15 at school, there is, as you know, the OECD PISA test. Japan always gets a very high score in mathematics and science literacy. There is sometimes a small gap between girls and boys, but not too big, and anyway the scores of girls are much higher than the averages. This means that there is not much of a difference at the age of 15, but in Japan it is precisely at that age that students have to choose either *rieki* or *bunkei* as their path. When you decide, you don't know much about science; you just look around: here are the guys, here are the women. If you don't have a high motivation to go into science, you follow the traditional trend...

JPB: *And you end up on the other side. But are there actions taken to change this?*

MK: For example, Tôhoku University encourages high school girls to consider going to *Rieki*. We deliver "science ambassador titles" mostly to female PhD students in science. They go to high schools and tell them [The girls in school] that there are a variety of reasons for being interested in science. Science is broad. Scientists are diverse. The girls don't have to be Marie Curie.

JPB: *But do you hope that some changes are coming up? You mentioned that figures could even be decreasing*

MK: Japan really should change. The universities should change. This new big endowment fund for excellent universities could be one of the driving forces. It's not only the situation of women, but also internationalisation is really slow in Japan.

JPB: *Among students, you mean? For faculty?*

MK: Especially for faculty. In the PhD training, especially in science and engineering, I don't know the figures for all of Japan for sure, but at the University of Tokyo, 2-3% in the undergraduate school, around 25% for the master course, around 35% in the doctor course in 2023.

JPB: *Are the figures for Tōhoku or more globally for Japan?*

MK: The figure for Tōhoku University is similar to that of the University of Tokyo. A big challenge for Japanese research universities is to have more international senior faculty members. In the WPI centres, 30% of the principal investigators, and 40% of researchers are international.

JPB: *Recently, you have been active in the International Science Council (ISC), an organisation which brings together international scientific unions and national academies as its members. Do you feel there is a special need for such structures presently? What is your vision about international cooperation in science in structures like ISC?*

MK: My motivation to participate in international science organisations came from the big earthquake in East Japan in 2011, and then more recently from COVID-19. The challenges humanity faces today are global. They should be considered more at the international level. This was even more evident for COVID-19: first, it started in Asian countries, and we didn't expect it would expand so rapidly to the whole world, and in the end it took three to four years to get it under control worldwide. Many problems humanity (as a whole) faces are closely related to science and technology. I believe that, more than before, science can contribute to solve them, at least contribute to seeking for a solution collaborating with many stakeholders. Not a single country, not a single scientific discipline can tackle them. We need to establish platforms or frameworks to form global views. We must help each other. We must also develop a vision on how to go forward to establish a sustainable inclusive society in the future.

JPB: *But do you feel that the present geopolitical tension in the world makes the need for scientists to be more engaged in this direction more necessary, or do you feel that it has always been like this?*

MK: The present tension makes it much more difficult to collaborate at the international level. I however believe the core value of science is its openness. The freedom and responsibility of scientists should be maintained.

JPB: *I see. My next question is, in your function as executive vice-president for research of Tōhoku University, you have been engaged in preparing the university's application to the new ambitious national Excellence Programme. How do you feel about the most recent developments? Tōhoku's application has been pre-selected. The previous president was very engaged in supporting the application. Now you have a new president at Tōhoku. Will he be as determined to support a project which would bring a major change in the university organisation?*

MK: Yes, I think so. Japan needs change and reform making it more international. Japan is, somehow, very conservative, and is not willing to change. We need more diversity, which is a driving force of evolution.

JPB: *Under the new president of Tōhoku University, your responsibilities as vice-president have been extended to international relations. You have explained to us why, for you, making Japanese universities more international is a very important step to be taken. Do you really feel this is an urgent action?*

MK: I think so. It is most important for Japanese research to keep its high quality. Recently I learned the change in geographical distribution of research. The number of scientific articles with authors in low- and middle-income countries (LMIC; World Bank classification) grew faster than the number with authors in high-income countries (HIC) since 1993 according to the report by Carlos Henrique de Brito Cruz from Elsevier. Now the governments establish funds for global talent mobilities and encourage researchers and students to collaborate internationally.

JPB: *More things you want to say about any new developments in mathematics?*

MK: Since I was involved in research in materials science, I'm more interested in applied questions, but nowadays I think there is no difference between pure and applied areas. They are more or less integrated. Your ideas, especially in geometry, can find applications.

JPB: *A great thank you for having given so much of your precious time for this interview.*

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After having been a fellow of the Centre national de la recherche scientifique (CNRS, France) for 44 years, Jean-Pierre Bourguignon is emeritus since 2013. He taught at the École polytechnique (1986–2012) and was director of the Institut des Hautes Études Scientifiques (IHÉS, 1994–2013). He was the second president of the European Mathematical Society (1995–1998), and the president of the European Research Council (2014–2019) and for another year by interim. Jean-Pierre Bourguignon is now Nicolaas Kuiper honorary professor at the IHÉS. His field is differential geometry and global analysis at the interface with theoretical physics. He has strong ties to Japan, having made numerous academic visits throughout the country. He also received various honours there.

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100 years of Jacques Dixmier

Alain Connes

On Sunday, 26 May 2024, we celebrated Jacques Dixmier's 100th birthday with an intimate dinner at his home, in the small flat he has occupied for many years at 11 bis rue du Val-de-Grâce in Paris.

Preparations for this event were based on discussions with Jacques, in particular to draw up the guest list, which was limited to eight guests due to the small size of his flat. Among his students, Jacques chose to invite Claire Anantharaman-Delaroche, Michel Duflo, and me. Another distinguished guest, who was particularly dear to Jacques, was Jean-Pierre Serre, who responded immediately to our invitation and traveled all the way from Switzerland, where he now lives.

In connection with Jacques' literary activities, we also invited Odile Jacob, her husband Bernard Gottlieb, and my wife Danye Chéreau; Jacques wrote two novels with Danye and me, which Odile published.

Several interesting episodes quickly led to lively discussion, which remained general throughout the meal.

One of the highlights was when everyone was invited to share an anecdote linked to their childhood. Jacques' story was that he remembered going for a walk with his parents when he was being unruly. His father had reprimanded him, advising him to behave like the little boy who walked in front of them as a family, exemplary in his discipline. Jacques complied, but shortly afterward, the model boy himself started to get into mischief. Jacques' reaction at that moment was not to rejoice that the other child was not such a good example, but to feel ashamed for his own father. This memory is indeed a perfect illustration of one of the facets of Jacques's personality.

This anecdote prompted the other guests to share their own childhood memories. Jean-Pierre Serre, for example, recalled that at the age of two he used to crawl under a chest of drawers. One day, when he stood up suddenly, he hit his head on the piece of furniture and, in a fit of childish anger, insulted the chest of drawers as if it were a person. I did not dare suggest that he'd acquired "la bosse des maths" on that occasion.¹

Another highlight of the evening was when Bernard Gottlieb recalled the example of one of his fellow students, who was admitted to the École normale supérieure (ENS) with a higher score than Poincaré, while expressing surprise that this individual then devoted his life to finance. Serre, with his incisive wit, asked him this question: "At what point did he realize that he was not a mathematician?," thus underlining the fundamental difference between success in a prestigious competitive examination and a true vocation for mathematical research. This comment gave me the opportunity to read a captivating text by Jacques on the nature of the mathematician's work and the specificities of research.

In search of a theorem

While traveling, I came across a very ancient city. Legend has it that a temple stands at its center, a temple that is splendid in its architecture, its decorations, and, what is more, the spiritual energy that emanates from it. Nobody in living memory has visited this temple (it is a divine residence), but I am willing to give it a try.

To be honest, it may not have been a legend that alerted me, but a dream. A dream so convincing that I am convinced this temple exists.

As I said, the city is very old. A mysterious silence reigns here. The streets are narrow and dark. It is not easy to get to the center, so I make my way slowly through the meandering streets. There are probably several routes, but I am not looking for the shortest; I will be quite happy if I find one.

The task is proving more difficult than expected, as the winding streets are full of dead ends. So, I often had to turn back.

In the long run, I get a bit discouraged. I have dark thoughts. Perhaps this labyrinth was once built by a genius who wanted to play tricks on future visitors, a genius so evil that all the streets around the center would be blocked off. In my worst moments

¹ A French expression meaning "a natural knack for mathematics." The word "bosse" literally means a bump, like one you might get from a physical impact. The expression playfully suggests that someone's

talent in math might be as if they've developed a "bump" for it, as if from a knock on the head. It is used to describe someone with an innate mathematical gift.

of discouragement, I sometimes even consider... that there is no temple.

No! It cannot be! The temple does exist, and there's a way in. My moment of depression is receding. My observations and calculations lead me to believe that, overall, I am closer to my goal.

A vague urban plan takes shape: avenues and squares. The ancient architect was not a hostile demon.

But obstacles of another kind arise; messages from the outside world reach me, demanding my presence, sometimes insistently.

Today, I have recorded my successes and setbacks in a manuscript. What does the future hold for me?

Suddenly, the scenery changes. I am no longer in the heart of a city, but in the heart of a virgin forest. It is night. I catch a fleeting glimpse of a flickering light, probably far away. I need to reach that light.

It is vital!

This was followed by a lively discussion on the concept of "great mathematician" between Jacques Dixmier and Jean-Pierre Serre. Jacques defended the pragmatic idea that eighty percent of mathematicians would easily agree on a list of "great mathematicians," provided one restricts to mathematics that are at least 50 years old. And this without defining precise criteria. Serre, for his part, insisted on the impossibility of defining such criteria. He gave the example of Grothendieck and Ramanujan, two undeniably important figures of the 20th-century mathematics, but who have nothing in common. In the end, the participants agreed on the idea that the community of mathematicians is not a totally ordered set in terms of mathematical value, but rather a partially ordered one.

Serre went on to explain that you do not do mathematics for fame, but simply for the overriding need to understand and that it is impossible to give up research altogether. Jacques is a perfect example of this, since he resumed his research activities two years ago, after a twenty-year hiatus, in response to a question he had asked me, on which we published a collaborative article. For Jacques, the main motivation for doing mathematics is to participate in humanity's quest for knowledge.

The most memorable moment of the dinner came at the end, after the cake. We had been waiting for the champagne, but it finally appeared, as if by magic, in another form at just the right moment: Jacques, on his feet, blossoming, launched into a carefully prepared speech, to say a few particularly relevant and kind words to each of the guests. He spoke without the slightest note or hesitation and began by recounting with humility and precision the milestones in his life as a mathematician. From his secondary studies, the entrance exam to the ENS, to his recent discoveries, he recalled each phase where, thinking at first that he would not succeed, he had been surprised to do better than he thought he could. The audience listened with rapt attention, eager to prolong the moment.



Jacques Dixmier and Jean-Pierre Serre before the dinner.

He then went round the table, paying a personal tribute to each of the guests. To Michel Duflo, he recalled that Duflo's grandfather had been his literature teacher. Then he fondly recalled a memory linked to his scout leader, Duflo's mother, while mixing technical comments with praise for his pupil. For Claire Anantharaman-Delaroche, he evoked the reading of her thesis, described as 'Roman,' since he had read most of it in Rome during a trip. This charming detail was tinged with a benevolent seriousness, typical of Jacques, who used this serious tone to evoke memories shared with his former student.

The moment when he spoke about me was special, because I had taken the initiative by expressing the exceptional importance of my meeting with Jacques. I simply said that this meeting had been the long-awaited revelation of a mathematician who understood me perfectly.

One of Jacques' most touching tributes was to Jean-Pierre Serre. Jacques explained that Serre had never made him feel inferior at any time during their long relationship, particularly during their work together in the Bourbaki group.

For Odile Jacob, Jacques had written a poem about the inauguration of the publishing house's new premises, located in the building long occupied by the family of André and Simone Weil. This text, imbued with emotion and gratitude, will serve as the final point in the account of this unforgettable evening.

To Odile Jacob

Your gracious visit, O dear Odile,
Your wondrous tales, your words so agile,
On this day, to my great delight,
Enlightened my humble room so bright.

With charm, you spoke of the grand hall,
Where now your plans and dreams will all
Flourish and grow in that noble space,
Once Weil's retreat, touched by his grace.

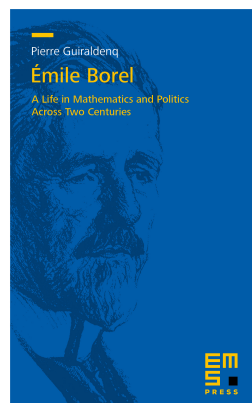
A vast amphitheater you now prepare,
A temple like Delphi, with grandeur rare,
Where the world's elite shall take the stage,
To share their wisdom, age to age.

And in this hallowed place, I see,
A frieze, or better yet, a grand tapestry,
That immortalizes, for all to view,
Your crowning as the Muse of Books so true.

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New EMS Press book



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Pierre Guiraldenq
(École Centrale de Lyon)

Translated and edited by
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Émile Borel, one of the early developers of measure theory and probability, was among the first to show the importance of the calculus of probability as a tool for the experimental sciences. A prolific and gifted researcher, his scientific works, so vast in number and scope, earned him international recognition. In addition, at the origin of the foundation of the Institut Henri Poincaré in Paris and longtime its director, he also served as member of the French Parliament, minister of the Navy, president of the League of Nations Union, and president of the French Academy of Sciences.

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Claire Voisin

Mathematics as a private space – from the unveiling of conjectures to worldwide recognition

Ana Isabel Mendes and Teresa Monteiro Fernandes

Claire Voisin (France, 1962) is considered one of the most important experts in algebraic geometry of our time and one of the most relevant mathematical personalities of the international mathematical scene.

This interview took place in November 2023. By the end of January 2024, Claire Voisin was awarded the Crafoord Prize in Mathematics, which is granted by the Royal Academy of Sciences of Sweden and the Crafoord Foundation (Lund, Sweden) to distinguish scientific areas not included in the Nobel Prize categories. As soon as we heard about this exciting piece of news, we asked her permission to append some extra questions at the end of the interview. To get to know her better, we answered with the text below to the natural question: ultimately, who is Claire Voisin?

Claire Voisin got her PhD at the Université Paris-Sud, Orsay in 1986. Alongside her career as a researcher of the CNRS (Centre national de la recherche scientifique), she worked at the Paris-Sud University, Orsay, at the Institut de mathématiques de Jussieu (IMJ), at the Institut des Hautes Études Scientifiques (IHÉS, Bures-sur-Yvette), at the École Polytechnique (Palaiseau) and, more recently, she held the chair of algebraic geometry at the Collège de France, thus becoming the first woman to hold such an honorable position. At present, she is *directrice de recherche* (CNRS) at IMJ-PRG.

Claire Voisin approached and solved problems with a long history. Her name is undoubtedly connected with famous conjectures such as Kodaira's conjecture, Hodge's conjecture, Green's conjecture and mirror symmetry. She was awarded several prizes among which we highlight the Shaw Prize (2017), known as the eastern Nobel Prize. We also highlight, among others, the European Mathematical Society Prize (1992), the Servant prize of the Académie des Sciences (1996), the Ruth Lyttle Satter Prize in mathematics (2007), the Clay Research Award (2008), the CNRS Gold Medal (2016), and, more recently, the L'Oréal-UNESCO Women in Science Award (2019). She was an invited speaker at the International Congress of Mathematicians (ICM) in 1994. In recognition of her contribution to science, she is a member of several scientific societies, including the Académie des Sciences, the German National Academy of Sciences Leopoldina, the Royal Society of London, the National Academy of Sciences (US) and the American Academy of Arts and Sciences.



Claire Voisin in her home-office in 2004 (courtesy of Claire Voisin).

Ana Isabel Mendes/Teresa Monteiro Fernandes: *Claire, we are deeply impressed to learn that you grew up in a family of 12 sisters and brothers. It is hard to imagine how your parents organized the family life and how you managed to have the quiet to become the person you are now.*

Claire Voisin: It was not exactly as you say. In practice, the 12 of us never lived at home at the same time, because the family grew up with 22 years between the eldest to the youngest (the last one was born in 1968). My elder sisters left early, and I am the tenth. In my first memories, there were only five or six siblings at home. The worst was not to have a room of my own. The house was big, but the rooms were not many, however large.

[AM/TMF]: *They were overcrowded...*

CV: In this respect I recall with much sympathy the writer Virginia Woolf who wrote about this need for private space (A Room of One's Own) as a condition for liberty and creativity. I had a room of my own only by the end of high school, of which I have excellent memories.

Nowadays, I have a large apartment in Paris which I enjoy very much and where I feel relaxed.

[AM/TMF]: *Besides the quiet as a condition for creativity, what you achieved is so impressive that we wonder if it originated somewhere in your childhood, possibly triggered by family incentives. How were your parents?*

CV: My parents were born in 1917 (my father) and 1922 (my mother). My mother had already turned 40 when I was born. With such an offspring, they were not particularly concerned about encouraging their children. However, in 1972 my father (an engineer) became unemployed. On one side it was dramatic, but on the other side this allowed him to devote time to my mathematical education. This was possible because most of my elder brothers and sisters had already left home, and my next brother had no interest in mathematics. My father had an extensive mathematical culture which he transmitted to me (geometry of triangles, some infinitesimal calculus, ellipse equation) in the old manner, since his education took place before World War II. Later, I went through a phase of criticism because at school the approach was completely different, the so-called modern mathematics, for instance, set theory, axioms, abstract operations, writing numbers in base 2 [laughs]... still the baggage learned with my father was excellent.

Another detail: my elder brother (10 years older) went to Paris to continue his studies and he left me a book on algebra, which I started studying. It was lucky.

[AM/TMF]: *So, you started by yourself..., but we can say that your father influenced you.*

On the occasion of the Shaw Prize, you also mentioned in an interview that the spirit of the sixties had a lot of weight on the family atmosphere. Could you be more precise?

CV: It was not really the sixties, but the end of the sixties, the Beatles were already the past. My parents were very intellectual, they were strongly concerned by international politics. Around 1968, we talked at table about themes like the death of Martin Luther King.

In such a way that my younger brother was named Martin. They worried much about the problems of the so-called third world: hunger, lack of freedom, etc.

On the other side, my elder brother, who was 16 at that time, was very present in the discussions because he participated in the public unrest.

My parents often talked about the philosophers of the Collège de France, about Sartre and de Beauvoir. But what they worried most about was moral implications, hence their interest in Martin Luther King Jr., but also in Abbé Pierre. Did you hear about him? He was a priest dedicated to the fight against poverty. My parents followed closely these themes and I listened to them as well as to the points of view of my elder brothers who were politically active. It was a way of drawing their attention...



Claire (third from the bottom left) with her family in 1966 (courtesy of Claire Voisin).

[AM/TMF]: *Had this social concern had an influence on you? Or had it the opposite result (as children sometimes react by opposition to their parents)?*

CV: My parents were very intense, they worried about the major world problems. There was no place for individual attention. Somehow, we had no right to complain in face of the evils of the world. We were not supposed to be important. This was not a good thing. There was a total lack of ambition for us, in view of some more important things. I was lucky to escape from this way totally devoid of ambition.

This vision was excessive in face of individual work. Mathematics was for me a way of drawing my father's attention, by contrast.

[AM/TMF]: *In the seventies, in Paris, it was usual for scientists and not scientists to attend the seminars of the Collège de France given by great thinkers of the time, such as Michel Foucault. Does this habit continue?*

CV: No, much less, that habit was lost. None of the present professors of the Collège de France has an influence like that of the philosophers of those years. In May of 1968 there was a mix and sharing of different forms of thinking that no longer exists.

[AM/TMF]: *We read that mathematics was not your first early choice. How did you make your choice?*

CV: I considered all possibilities. The director of my school advised me to enroll in the "Classes préparatoires" of the Lycée Louis-le-

Grand, which I have attended from 17 to 19 years old. It was an excellent decision. The teaching was very good. I was good at mathematics, but I did not see its depth. It was like a game, but I don't like games, I am a very serious person. I also remember having excellent literature teachers in high school.

I learned solid mathematics, but it was like a finite set of rules, one did some calculations... I was missing something; it was too practical... Of course, practice is good to understand definitions.

[AM/TMF]: *Deep down, do you consider that mathematics at school is taught as a set of rules with which you must play, but there is no place left to create new things, to be creative?*

CV: It was when doing research that I started taking mathematics more seriously. I was in front of something very promising, very serious. I understood then that there was so much to discover, so much we don't know! It is not just a game at all.

[AM/TMF]: *It is like lifting a veil. There is a world to be discovered. But for that one needs background...*

CV: When I was about 21, I wondered whether I should study philosophy. It was at a time when I didn't yet know what it was to do mathematics. Nowadays, I don't regret leaving philosophy.

[AM/TMF]: *Claire, you said in an interview something like "I am nostalgic for the time when my life was centered at home and shared between research and my children's education." For what reason?*

CV: I love my children and still more since they are adults. Now that they left home, my life is very busy, I travel more, I meet more people, I participate in more commissions. In the past, my life was more concentrated: at home. I did mathematics at home, I traveled less that maybe I regret, but I consider that traveling around the world and discussing with people is more of a distraction than anything else. What did it bring to me? That the most important thing for me is to stay inside myself. Personally, I don't think it is essential to interact with many people. So, what I meant is that, up to a certain moment, I had only mathematics and my family life. The contrast of mathematics with family life was crucial. Mathematics was my space, just mine.

[AM/TMF]: *In Portugal, at the university, one has too many teaching duties, which overloads mothers' lives. Childcare falls much upon the mother.*

CV: In France, although not so as 30 years ago, the support for childhood is very good. School is mandatory at the age of 3, and before that daycare is available from 3-4 months old. I had a lot of help! What was complicated was to convince people of the

importance of doing mathematics at home. Now and then I tried to have one of my children with me, but it was too hard.

[AM/TMF]: *So, there is nothing to regret...*

CV: Nowadays, I have more time, but I don't do more... before I only lived for the central aspects, in a focused way.



Claire and her children in the garden, in 1998 (courtesy of Claire Voisin).

[AM/TMF]: *Claire, you know that you are an example of how maternity is not a prevention for great success in research. Apart from sharing tasks with your husband, himself a well-known mathematician, have you any secret of how to make the most of your time?*

CV: Mathematics was my space. I started again quickly after each birth. Sometimes I had to fight for my mathematical life... Because it is my way of expressing myself.

It was not so hard because our life was simple, but we were also well organized. We lived in a small town not needing a car, everything was close and simple.

[AM/TMF]: *We are aware that painting and music have a special place in your life. Let us begin with painting.*

CV: When I was 19, 20, until 22 years old, I painted a lot. But just as a complement to mathematics. My social life was practically empty. It is not in my nature to relate with other people.

[AM/TMF]: *However, you met your husband!*

CV: [laughs] Meeting my husband was almost a miracle; at the time he was a professor in Orsay where I was preparing my PhD, but we

did not meet there. It was lucky. I was very lucky. In fact, throughout life, I never had to take great decisions, things just happened.

[AM/TMF]: *Now you have a greater social life, after so many prizes and the fame after so many outstanding results you achieved.*

Changing the subject, one of us has the pretension of painting... Could we see one of your favorite works and include it in the published version of this interview?

CV: Of course, I am going to show you my painting that I consider the best. I like it very much, and so I keep it. I painted it when I was very young. Now I am more focused on drawing. I draw small things like trees, simple things.

In the past, with my children, we used to do lots of things, modeling... None of them were very talented. Correction, two of them were. Those were very nice moments! I also enjoyed educating and making activities, but not for playing pretend.



At the IHÉS in 2007 (courtesy of Claire Voisin).

[AM/TMF]: *Did you help your children in mathematics?*

CV: It was not an easy task. At first, I tried to teach them, but my approach was too abstract. 9- and 10-year-old children learn easily, but they don't have a great capacity for abstraction. I ended up teaching simple things like systems of two equations with two

unknowns, for instance. Two of my 5 children are mathematicians: when they were teenagers it became hard to discuss mathematics with them, and the older they grew the worse. They wished to discover by themselves, maybe I was too fast. My other children don't have an interest in mathematics. One day, my third daughter, who was in the "classe préparatoire" for management (about 18 years old), needed help, but with no real interest. I had great trouble in convincing her to keep her mobile off. I don't have very nice memories of that time.

[AM/TMF]: *We know that each of your children plays an instrument. What was the role of music in your family life?*

CV: Yes, bassoon, oboe, viola, piano, cello. They are quite good. I started playing the violin late. I believe I don't have a nice sound; it is just for fun. I never forced them to study, I thought it had to come from them. But I encouraged them to play an instrument because it is excellent to create discipline since one must play every day. They all ended up being good musicians.

[AM/TMF]: *Is any of your children a professional musician?*

CV: No... The problem is that it is hard to have a good life. You must give concerts but also teach. Leila Schneps has four children and one of them is a professional musician, quite encouraged by her, but I did not encourage mine. I think it is a very hard life. There are many disappointments, merit is not easily recognized. While at the university you can teach and do research, and you are recognized in a fairer way.

[AM/TMF]: *You must have a passion for what you do... Could you say you love Paris?*

CV: Yes, undoubtedly! It is true. We spent 23 years in Bourg-la-Reine, which is a very nice small town, very convenient for raising children. Eleven years ago, we moved to Paris when they were finishing high school. I feel much better in Paris where I walk a lot, thinking about mathematics. Working in Bourg-la-Reine was not as effective. In Paris, I am more focused.

[AM/TMF]: *Will Paris be more exciting?*

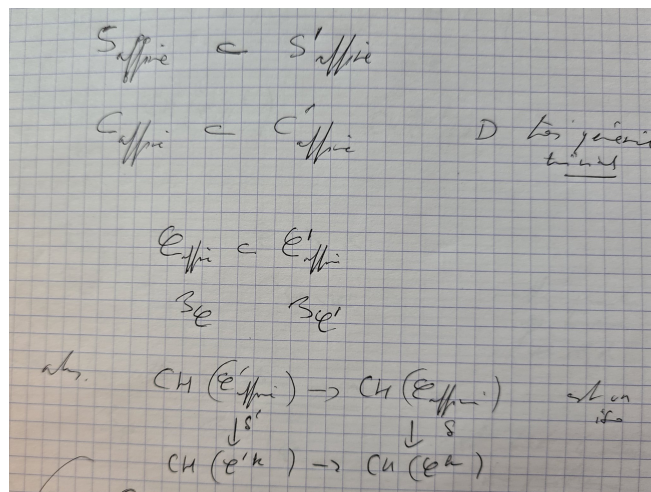
CV: It is a very intense city. The architecture, the environment, are thrilling.

[AM/TMF]: *Other interests of your youth were philosophy and poetry. What is it like in the present? Can you share with us some authors of your choice?*

CV: In poetry I really liked Mallarmé, Baudelaire, Rimbaud, Gérard de Nerval and the classics in general. More recent, René Char,



In her office in 2022 (courtesy of Claire Voisin).



Notes by herself (courtesy of Claire Voisin).

Philippe Jaccottet. I currently don't read poetry, but poetry left an impression on me. For example, Baudelaire stayed in my memory. As well as, in the case of philosophy, I remember for example Jean Cavallès. Forty years ago, my list of authors was long. In fact, nowadays, my tastes have changed, I read more novels, biographies, essays, history books. I continue to read a lot.

[AM/TMF]: *Returning to the question, we know that you did "speed dating" with young students at the Collège de France. Despite teaching not being part of your duties, do you enjoy it?*

CV: It is hard to say. I don't think I have patience to teach, especially with teenagers. Although some experiences went well. I really enjoy teaching master's courses and I had to do it when I was at the Collège de France. In this case you must be very well organized, you must try to encourage. It is very demanding to effectively introduce often sophisticated tools necessary for research. It is not like giving a talk where sometimes you must be vague.

Students must understand all from the beginning. For me, it is very demanding, but I am interested in giving courses from time to time.

[AM/TMF]: *We remarked that you don't avoid public discussion around mathematics, giving interviews, talks, etc. In one of these, we particularly appreciated one of your statements: "Tous les puissants outils théoriques que nous développons trouveront tôt ou tard une application." (All the powerful theoretical tools we develop will sooner or later find an application.)*

CV: This takes us to the question of why. Maybe because I work in pure mathematics, particularly in algebraic geometry, where there are lots of questions we would like to answer, but the applications

seem to be empty. Often, we are confronted by ourselves and others about why we do it. Honestly, I think that this question is wrong, firstly because we are deeply interested in a theme, secondly because that is the way mathematics develops.

It develops by creating tools for us, but in the future these tools will serve other purposes. I don't believe what I do is useless.

[AM/TMF]: *Of course not. We don't ask a musician why he makes music and what it's for.*

CV: That is another aspect, but that is not what I am referring to. Of course, that is a good answer because it [music] is an essential activity. Human beings like abstraction. And mathematics is the only science where you find it with the greatest rigor. I think mathematics is the most important science for the development of human thinking. However, I wanted to emphasize the importance of the mathematics we are currently doing. The theorems I am proving may not have a direct application, but the tools I am developing and creating are certainly useful for the development of science.

[AM/TMF]: *Arnaud Beauville, your PhD advisor, gave a talk about your proof of the Green conjecture at the famous Bourbaki Seminar in 2003. He was a senior and Claire a junior. It looks like a role reversal. How did you feel at the time?*

CV: I was not that junior, when he gave that talk at Bourbaki, I was already in my forties. I no longer considered myself a junior. Still, it was very nice of him to do so. I suffer with the fact that Arnaud Beauville, despite being so famous and highly regarded in France, is not a member of the Academy. He is so strong and so good!



A painting painted by herself (courtesy of Claire Voisin).

I don't understand why I am and he is not. There may be a number of political reasons why he isn't, and I got the title just because I am a woman. Nowadays, it is fine to give these titles to women. I don't like this situation and I consider it very unfair.

[AM/TMF]: *This takes us to another question. How do you deal with being the only woman who won the Shaw Prize?*

CV: As a person, I always have been lonesome, and, as I said before, I mostly lived at home without socializing. In other words, I never suffered from living in a mostly male world. Eventually, if I had been more involved with university life, I might have suffered more from these conditions. Therefore, I never thought much about being the only woman. It was really a recognition. I confess that I was, and I am, a little afraid that I have been awarded the prize exactly because of that, because of being a woman. This idea for me is painful.

[AM/TMF]: *We don't believe it. The prize is yours because you deserved it!*

CV: Look at the media... What they say, the only thing they say, when you receive a big award, is that you are a woman. They don't talk about the work. And I don't just speak for myself, I speak in general. When Karen Uhlenbeck won the Abel Prize, the only thing the newspapers referred was that she was a woman. It is absolutely dramatic that they only focus on this aspect. When you are a woman who goes through this situation, when you work,

your gender does not matter. Unfortunately, we conclude that what is most important to many people is that we are women. More than our work.

[AM/TMF]: *It is sad that we still must talk about this. About being women. We should already be in another stage...*

CV: Sometimes I feel myself caught in a trap. I became a woman mathematician. Not simply a mathematician. I am regarded as a woman mathematician. I didn't want it to be like this. I didn't want the mathematics I do to be appreciated or evaluated from the point of view that I am a woman.

[AM/TMF]: *Maybe this vision is persistent because you have a large family and family life usually is limiting in achieving great successes.*

CV: Do you know Christopher Hacon? He works in algebraic geometry and has six children! His wife has a job. For him there is no contradiction between being an excellent mathematician and his family life.

[AM/TMF]: *It is as you said: you must be focused on your work. We understood that, for you, Claire, work is more important than prizes themselves. But we can't resist asking: what work did you most enjoy being recognized?*

CV: For me, perhaps the recognition I most appreciated was the talk at the Bourbaki Seminar on my work. First, because I am proud of my work and second, because the Bourbaki Seminar is an institution in France.

I must say that, for sure, it is the best recognition I ever received. Someone you like talking about your work... Of course, receiving prizes is also very pleasant, but they don't have the same meaning. I concede that they give us more visibility, but the latter is not so easy to manage. After prizes, after their involvement and their consequences, it becomes harder to return to our corner and to do mathematics. It is somehow disturbing.

[AM/TMF]: *You have lots of students. How do you manage them? Do you give a problem to each of them? Do you follow them closely?*

CV: I follow them closely, I give them problems that are in general developments of my own work, I more or less know where they go. I have my own strategies. The best students have more freedom to do what they want, and I give others lots of clues.

Nowadays, a PhD thesis takes only three years, including a report; my students must have two or three written works. I never give them completely open problems which I don't know where they go. These three years are when they learn a lot, it is the

time when they become researchers, which they learn by writing papers, by proving theorems. Usually, I go to the university at least once a week to participate in my seminar and to follow my students.

[AM/TMF]: *Do you currently have any problems that you are most obsessed with at present? Could we suggest you talk a little about your work on mirror symmetry?*

CV: I stopped working on that long ago. I stopped for two reasons: one is that there were already lots of people working on the subject, and the other that I didn't keep up with the physicists. They worked fast, and I couldn't follow with rigorous proofs. At the same time, I felt I was straying from my own path. I preferred to follow my own questions. About the other question, I now work on a subject that interests me a lot, very beautiful, the hyperkähler manifolds. They are compact Kähler manifolds which may not be algebraic, their geometry is more abstract and, in general, they don't contain interesting complex submanifolds. Of course, we understand them better via deformation theory and come back to algebraic geometry thanks to Kodaira's theorem: many are algebraic, many are not. A bit like complex tori and abelian varieties. The theme appeals to different areas of mathematics, like complex differential geometry, the properties of Kähler metrics and, of course, algebraic geometry. Their moduli spaces are known thanks to Torelli's theorem. Their algebraic cycles are very particular. These varieties are a bit like abelian varieties, but more mysterious because they don't possess a group structure.

I have been working on that for the last two or three years. We received an ERC Synergy Grant for this project. I work with Professors Debarre, Macrì and Huybrechts. The project, financed with 8.5 million euros, consists in testing some of the fundamental conjectures of modern mathematics on hyperkähler manifolds. This is a subject on which I already wrote a number of joint papers, while I usually publish alone. It is something new to me which turned out to be quite enjoyable.

I am currently investigating the algebraic cycles. I want to understand them like in topology.

We see them as subvarieties of an algebraic variety, and we introduce some relations on the generated abelian group. This is how Chow groups appear.

[AM/TMF]: *Close to the theory of motives?*

CV: Yes, a theory very much related to motives, from the complex algebraic geometry, Hodge theory and Hodge conjecture viewpoints. Chow groups are what we compute in a motive. Of course, motives are more general.

[AM/TMF]: *To finish, who were the mathematicians that most influenced or inspired you?*



During the EMS awards ceremony. We recognize starting from the right: Labourie, Hirzebruch, Karoubi, Jacques Chirac, herself, Franke, Goncharov, Kontsevich (and other laureates) (courtesy of Claire Voisin).

CV: Clearly, when I started, Philip Griffiths. He influenced me a lot. He was working on Hodge theory. When I started my studies, I wrote a number of papers influenced by his work. I did not know him yet. I only made his acquaintance later, because we started a discussion on Chow groups. Fulton was also very influential. He wrote an amazing book called *Intersection Theory* where he formalized intersection products in the Chow groups, which were essential and valuable for solving lots of problems. This completes Griffiths' work, who always used complex geometry. Now, in Fulton's book, the approach is from algebraic geometry.

There are other mathematicians whom I consider as most important in my area, and who influenced me, like Jean-Pierre Serre and also Alexander Grothendieck, but I never met him.

[AM/TMF]: *To conclude, Claire, is there any question you were never asked despite the fact you would like to be asked, about your life as a mathematician?*

CV: In the interview proposal you sent me, there was a question I would like to answer, namely, what am I most proud of in life? I thought a lot about this question, and I would say I am most proud that I didn't lose my passion for mathematics. In fact, I think it is the opposite: the more I work, the more I do research, the more I know my area, the more interesting my studies become. Maybe things would have been a little different when I was turning 40. It was a matter of life and death. Maybe it was too much then. Currently, I deeply like what I do. It is so interesting, there are so many questions, there are so many paths to follow! I am very happy I haven't lost my passion!

Claire Voisin being awarded the Crafoord Prize

[AM/TMF]: *Claire, this is a philosophical question: do you think this prize [the Crafoord Prize] was awarded to you at the right moment?*

CV: It is difficult to answer... It is hard to say you deserve a prize. From this point of view, the sooner, the better. I don't know if it is the right moment, but this is a most important prize, and I will do my best to be at its level and to convince people that I deserve it.

[AM/TMF]: *You are already on a pedestal! Your answer seems to reveal a feminine nuance corresponding to the eternal need of women to always prove what they deserve.*

CV: This is not exactly the feminine side. The question is my area, algebraic geometry, where exceptional mathematicians like Serre, Grothendieck, Deligne, affirmed themselves. They made us feel so small... It would be different in a more recent area. Algebraic geometry and Hodge theory have a long history. I can mention, for example, Riemann and the theory of hyperelliptic integrals. In the master course I teach at present, I spend lots of energy explaining the strength of these results and their extreme importance for algebraic geometry.

[AM/TMF]: *How did your students react to your prize?*

CV: They may not be aware of it, since they don't talk about it. And I prefer not to be embarrassed with many demonstrations.

[AM/TMF]: *Picking up this question, we know that often, mathematicians give parties when they are awarded prizes. How is it going to be?*

CV: I will have a party with my children, but only after concluding the course I talked about, because it takes a lot of energy out of me.

[AM/TMF]: *Does this prize embrace all your mathematical work, or does it emphasize some particular result?*

CV: The prize refers to my work as a whole. Personally, I would highlight the paper on Green's conjecture, the paper on Kodaira's problem, but also the paper on the stable rationality of algebraic varieties by specialization methods.

Returning to the question about "the right time," sometimes awards happen when you are depressed, in an impasse, the research is not progressing, and what you do seems to be useless, but

this was not the case. I am very happy with the recent work with János Kollár which answers a problem going back to the sixties. This is an important and pleasant work, not yet published. The main result is quite easy to state, and the proof is very elegant. I include this work among my most important results.

[AM/TMF]: *Will your life change?*

CV: As I said before, I'll keep working hard but not really because of the prize. Each of us has an intimate master, with one's own orientations and criticisms.

[AM/TMF]: *Do you intend to go to Sweden to receive the prize?*

CV: Of course! I like to be polite. This prize represents a financial support to science.

We must be grateful and polite. Our governments, for the most part, are not very respectful where science is concerned. It's a duty of politeness towards those who are.

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ChatGPT 101 for L^AT_EX users

George Grätzer

1 Introduction

Late in the fall of 2022, I was contemplating updating my book, *More Math into L^AT_EX*, fifth edition [1]. It was seven years old, and a lot had changed (as witnessed by [3]). Therefore, I started contemplating a sixth edition, which would be a huge undertaking. For instance, BibL^AT_EX would be a new topic for the book. Its manual alone is about 350 pages and I would have to read at least ten like that.

Then the gods smiled at me. In November 2022, ChatGPT 3.5 was released; see my article [2]. Let me illustrate what ChatGPT 3.5 provides for a L^AT_EX user.

ChatGPT in action

We start with two very simple uses of ChatGPT.

First, I ask ChatGPT: What is the end-of-line command?

```
latex what is the end-of-line command?
```

ChatGPT responds¹:

The L^AT_EX command indicating the end of a line is `\\`. This command is used to break the line at the point of insertion. It's commonly used in text environments, such as within a paragraph, or in tabular environments, like tables, to move to the next line. For example, in a paragraph:

This is some text. `\\` More text.

"More text." will appear on the next line.

In a table:

```
\begin{tabular}{cc}
Cell1 & Cell2 \\
Cell3 & Cell4
\end{tabular}
```

Remember that `\\` is a line break command and not a new paragraph command. To start a new paragraph, a blank line or the `\par` command is used.

How very nice.

Second, by mistake, you define a new command:

```
\newcommand{\vec}{...}
```

```
! LaTeX Error: Command \vec already defined.
```

You can find out the definition of the `\vec` command by entering into interactive mode and typing

```
* \show \vec
```

LaTeX responds with:

```
> \vec=macro: ->\mathaccent "017E.
```

```
<*> \show \vec
```

informing you that `\vec` is a command, and specifically, a math accent. This is not very intuitive. Now we use ChatGPT:

```
latex what is the \vec command?
```

The answer:

The `\vec` command in L^AT_EX is used to denote a vector. When this command is applied to a character or a group of characters in your L^AT_EX document, it places a small arrow above them. This is a common notation in mathematics and physics to indicate that the symbol represents a vector quantity.

For example, writing `\vec{x}` in L^AT_EX will display the letter 'x' with an arrow over it, symbolizing that 'x' is a vector.

ChatGPT goes on with some illustrations.

I also consult ChatGPT on hyphenation, looking up strange phrases, and so on. You will find many uses of ChatGPT in this article.

ChatGPT hallucinations

While my experience with ChatGPT was very positive, experts were skeptical: "You know my opinion of ChatGPT and other large language models; they can be misleading and under certain circumstances, produce entirely captivating yet nonsensical and fabricated content." This is usually called a 'ChatGPT hallucination.'

Let us ask ChatGPT what this means:

¹Another time, the response may differ.

A GPT hallucination refers to instances where a language model like GPT (Generative Pre-trained Transformer) generates text that is either factually incorrect, nonsensical, or unrelated to the input it was given. These hallucinations occur when the model makes errors in generating responses, often due to limitations in its training data or the inherent complexities of natural language processing.

Here are a few key points about GPT hallucinations:

1. *Based on patterns, not facts.* GPT models are trained on large datasets of text and learn to predict the next word in a sequence based on patterns in the data. They do not have access to external information or real-world knowledge. This sometimes leads to the generation of text that seems plausible but is factually incorrect.
2. *Context limitations.* GPT models have a limited context window, which means they can only 'see' a certain number of words at a time. If important context falls outside this window, the model might generate irrelevant or incorrect content.
3. *Inherent uncertainties.* Language can be ambiguous, and sometimes the model must guess the user's intent or the meaning of certain phrases. This can lead to hallucinations where the model fills gaps with incorrect or nonsensical information.
4. *Training data constraints.* The quality and diversity of the training data significantly impact the model's performance. If the training data has biases, inaccuracies, or lacks certain information, this can lead to hallucinatory outputs.
5. *Complexity of tasks.* More complex tasks, such as answering detailed technical questions or understanding nuanced human emotions, are more challenging for the model and can lead to higher rates of hallucination.

Thus GPT hallucination is a limitation of current AI language models, reflecting the challenges in dealing with the vast and intricate nature of human language and knowledge. It's an area of ongoing research and development in the field of artificial intelligence.

Now the situation is clear. A L^AT_EX query uses the vast data set of L^AT_EX pages on the internet, in books and articles with no biases or inaccuracies. Therefore, there are no hallucinations I could find, except in programming.

To help you to gain such benefits from ChatGPT, I decided to write this article.

2 What is ChatGPT?

Let us ask ChatGPT:

What is ChatGPT?

And here is the answer (edited for length):

ChatGPT is specifically designed to generate human-like text in a conversational context, based on a deep learning architecture known as transformers. The key characteristics and functionalities of ChatGPT include:

1. *Conversational AI.* ChatGPT is fine-tuned for dialogue, making it adept at understanding and generating text in a conversational format. It's capable of maintaining context over a series of message exchanges, allowing for more coherent and relevant interactions.
2. *Large scale language model.* ChatGPT is trained on a vast corpus of text data. This training enables it to generate text that's often coherent, contextually relevant, and surprisingly human-like.
3. *Versatility.* It can perform a wide range of language tasks, including answering questions, providing explanations, engaging in casual chit-chat, creative writing, summarizing texts, translation, and more.
4. *Contextual understanding.* ChatGPT can understand and recall information from the current conversation, allowing it to provide more relevant and continuous responses. However, it does not have access to external databases or the internet for real-time information, and its knowledge is limited to what it learned during training, up to its last update.

It is remarkably easy to start using ChatGPT. Go to [OpenAI . com](https://openai.com), scroll down, and click on:

Try ChatGPT ↗

Now² at the bottom of the new page, you see:

Message ChatGPT

ChatGPT can make mistakes. Check important info.

Type your prompt, click the up arrow. That is it.

3 What it cannot do

It should now be clear that we should ask ChatGPT questions that require the large amount of data it has acquired to answer. Otherwise, you may really get 'nonsensical and fabricated content.'

² Or you can simply load ChatGPT directly from <https://chatgpt.com>.

Donald Knuth³ illustrates this:⁴ “Since one of today’s popular recreations is to play with ChatGPT, I decided on 07 April 2023 to try my own little experiment.”

Knuth’s first question:

Tell me what Donald Knuth says to Stephen Wolfram about ChatGPT.

Clearly, the answer will not be useful. There is no large, relevant data set. You are asking the shepherd in New Zealand about elephants.

His tenth question is better:

How many chapters are in *The Haj* by Leon Uris?

ChatGPT responds, in part:

“The Haj” by Leon Uris is divided into three parts.

This is incorrect, it is divided into five parts. (Knuth thinks it is divided into four parts. One better than ChatGPT but still wrong.)

I asked ChatGPT:

Be an expert on 20th-century novels. How many parts are in *The Haj* by Leon Uris?

And the answer is:

As of my last update in April 2023, I do not have the specific titles of the parts in “The Haj” by Leon Uris. For the most accurate and detailed information, including the titles of the parts, I recommend referring to a physical copy.

4 Asking good questions

How do we ask good questions that elicit useful answers? Let us ask ChatGPT.

How do we ask effective questions?

Here is the answer (edited for length):

Asking good questions is a crucial skill that can lead to more informative and useful answers. Here are some tips for asking effective questions:

1. *Be clear and concise. Aim for clarity in your questions, and avoid overly complex or convoluted phrasing. A clear, direct question is more likely to receive a clear, direct answer.*
2. *Be specific. General questions can lead to vague answers; specific questions help in receiving detailed and relevant information. For example, instead of asking “How can I improve my health?,” ask “What are some effective low-impact exercises for improving cardiovascular health?”*
3. *Provide context. Giving context helps the person you’re asking understand your question better. For instance, if you’re asking for advice on a project, briefly explain what the project is about.*
4. *Know your objective. Understand what you want to achieve with your question. Are you looking for facts, opinions, advice, or something else? Tailor your question accordingly.*
5. *Open-ended vs. closed-ended questions. Use open-ended questions (those that can’t be answered with a simple ‘yes’ or ‘no’) to encourage a detailed response; use closed-ended questions when you need specific information or a concise answer.*
6. *Prioritize your questions. If you have multiple questions, prioritize them. Ask the most important ones first, especially in situations where you might have limited time.*
7. *Follow-up questions. Be prepared to ask follow-up questions if the response you get isn’t clear or if it leads to more queries.*

There are several more key considerations for asking effective questions (if you want to charge a lot, call this ‘prompt engineering’).

1. Specify the field of inquiry. For instance, as demonstrated in Section 3, we initiated the question with

Be an expert on 20th-century novels.

Since our focus is solely on \LaTeX , you should always begin every session with

Be an expert on \LaTeX .

Typing \LaTeX is sufficient. GPT will remember the instruction.

2. Asking ChatGPT a question is just the beginning of a conversation. For example, ask:

Define $\backslash\text{bmatrix}$

and after the long answer, respond

more examples

³The creator of \TeX .

⁴<https://www-cs-faculty.stanford.edu/~knuth/chatGPT20.txt>

ChatGPT will then provide several additional examples. Next, respond with:

All examples should be at least 4 x 4 matrices.

and now all examples are 4x4!

3. Let us think step by step. This is really amazing. Ask:

Define `\bmatrix`, let's think step by step.

It will teach you the use of the `\bmatrix` command in seven steps, so easy to understand.

To show you the responses would require several pages. Type these commands and see the responses of ChatGPT.

5 The GPT Store

Let OpenAI tell you what the GPT Store is (edited for length).⁵

Users have already created over 3 million custom versions of ChatGPT. We're starting to roll out the GPT Store to ChatGPT Plus users, so you can find useful and popular GPTs. Visit chat.openai.com/gpts to explore. The store features a diverse range of GPTs developed by our partners and the community. Browse popular and trending GPTs on the community leaderboard, with categories like DALL·E, writing, research, programming, education, and lifestyle.

There are featured GPTs every week.

We will also highlight useful and impactful GPTs. Some of our first featured GPTs include:

- 1. Expand your coding skills with Khan Academy's Code Tutor.*
- 2. Design presentations or social posts with Canvas.*
- 3. Find your next read with Books.*
- 4. Learn math and science anytime, anywhere with the CK-12 Flexi AI tutor.*

We call the items in the store special-purpose GPTs (GPT SPs).

The store also has top picks in seven categories; we select only two categories to discuss, Writing and Programming.

To get to the special-purpose GPTs, go to ChatGPT and click on Explore GPTs.

6 Style and grammar

In the Writing category, we find twelve items, most of them help to improve your writing. Here is a typical editing dialogue with the Creative Writing Coach. We ask for help—observe that the prompt

asks “list changes by sentence, give reasons for change”; this is very important:

Edit for grammar and style, list changes by sentence, give reasons for change, ignore correct sentences:

A~ypical English grammar book ranges from 400 to 1860 pages in length (the runner-up is 1779).

These are general books not particularly useful for math.

I think few mathematicians would want to spend time reading any of them.

So this book is not trying to teach grammar, but it does utilize some terminology.

What we aim to do is simple and modest: select a few easy topics for discussion.

If you follow our recommendations, you will eliminate 80% of your errors, making your papers easier to read and understand.

Note: 80%! Achieving 100% would require 600 pages!

I suggest reading a topic a day.

It's just a few minutes of easy reading.

Have fun.

Now the reply.

Corrected version with changes listed by sentence:

Original: These are general books not particularly useful for math.

Correction: These are general books that are not particularly useful for math.

Reason: Added “that are” to clarify the relationship between “general books” and “not particularly useful for math.”

Original: So this book is not trying to teach grammar, but it does utilize some terminology.

Correction: This book is not trying to teach grammar, but it does utilize some terminology.

Reason: Removed “So” at the beginning for a stronger, more direct statement.

Original: What we aim to do is simple and modest: select a few easy topics for discussion.

Correction: Our aim is simple and modest: to select a few easy topics for discussion.

Reason: Simplified the structure for clarity and conciseness.

Original: Note: 80%! Achieving 100% would require 600 pages!

Correction: Note: 80%! Achieving 100% would require 600 pages!

Reason: No change needed; the sentence is clear and effective as is.

⁵ [http://openai.com/blog/introducing-the-gpt-store](https://openai.com/blog/introducing-the-gpt-store)

Original: I suggest reading a topic a day.
Correction: I suggest reading one topic a day.
Reason: Changed "a topic" to "one topic" for consistency and clarity.
Original: It's just a few minutes of easy reading.
Correction: It's just a few minutes of easy reading.
Reason: Replaced the curly apostrophe with a straight one for consistency.

Much improved! This is really ChatGPT on steroids.

7 TeX programming

In the Programming category, there are a number of GPT SPs that do programming. We pick three: Code Copilot (Code Smarter, Build Faster), Grimoire (Code Wizard, Programming Copilot), and Code Guru (reviews code and comments existing code).

We ask them for help to code a command `\threejump`, inspired by the command `\cleardoublepage`:

Create a LaTeX command named `\verb|\threejump|` that will add `\verb|\newpage|` commands until the page number is divisible by 3.

7.1 Code Copilot

```
\documentclass{article}

\makeatletter
\newcommand{\threejump}{%
  \ifnum\numexpr\thepage\relax
    % Check if page number is divisible by 3
    \ifnum\numexpr\thepage\relax<3\relax\newpage\fi
    \ifnum\numexpr\thepage\relax=3\relax\else%
    \ifnum\numexpr\thepage\relax>3\relax%
    \ifnum\numexpr\thepage\relax<6\relax%
    \ifnum\numexpr\thepage\relax=6\relax\else
    \ifnum\numexpr\thepage\relax>6\relax%
    \ifnum\numexpr\thepage\relax<9\relax%
      \newpage%
    \else\newpage%
      \newpage%
    \fi%
  \fi%
  \fi%
  \fi%
  \fi%
}
```

```
\fi%
\else%
  \newpage%
\fi%
}
\makeatother

\begin{document}

\section{First Section}
Some text here.

\threejump

\section{Second Section}
Some more text here.

\threejump

\section{Third Section}
Even more text here.

\threejump

\end{document}
```

It does not compile. The relevant part of the log file:

```
./threejump.tex:33: Missing number, treated as zero.
<to be read again>
                \relax
l.33 \threejump
? ax
l.33 \threejump
```

7.2 Grimoire

```
\documentclass{article}
\usepackage{etoolbox}

\newcommand{\threejump}{%
  \whileboondvi{\numexpr\thepage\relax\mod 3\relax
                > 0}%
    {\newpage}%
}

\begin{document}
```

```
% Example content to test the \threejump command
Page 1
\newpage
Page 2
\newpage
Page 3
\newpage
Page 4
\threejump
Page 7 % This should be on a new page that is
      % divisible by 3

\end{document}
```

This is much more elegant, but does not typeset either. The log file:

```
l.63 \threejump
      ./threejump.tex:63:
      You can't use '\numexpr' in horizontal mode.
\threejump ->\whileboondvi {\numexpr
      \thepage \relax \mod 3\relax > 0}{\newp...
l.63 \threejump

?
```

7.3 Code Guru

```
\documentclass{article}

% Define the \threejump command
\newcommand{\threejump}{%
  \newpage
  \ifnum\numexpr\value{page}\relax\mod 3\ne 0
    \expandafter\threejump
  \fi
}

\begin{document}

% Sample content to demonstrate \threejump command

\section{Section 1}
Some content on the first page.

\threejump

\section{Section 2}
Some content on a page number divisible by 3.
```

```
\end{document}
```

This may be the most elegant code, using recursion. Unfortunately, this does not compile either. Here is the relevant part of the log file:

```
! Undefined control sequence.
\threejump ...m \numexpr \value {page}\relax \mod
      3\ne 0 \expandafter \three...
l.18 \threejump

?
```

7.4 Testing

As indicated by the log files, unfortunately, none of these three codes compile. Chalk this up to early days. When they will compile, this will open up a really easy way to be exposed to T_EX programming.

8 Hallucinating on steroids

Every week, OpenAI features four GPT SPs. In the second week after the opening of the GPT Store, it selected the GPT SP, Consensus. The next week, it listed Consensus as ‘Trending’ (one of six). Consensus is “Your AI Research Assistant. Search 200M academic papers from Consensus, get science-based answers, and draft content with accurate citations.”

So immediately, I dreamed about lists of publications with research summaries! This would be much more useful than the list of publications we usually get.

```
Be a lattice theory expert.
List of publications of George Grätzer.
```

The search for publications specifically authored by George Grätzer yielded a limited number of results. Here are some of his publications:

1. “Joint Review of General Lattice Theory (Second Edition) and the Congruences of a Finite Lattice: A Proof-by Picture Approach” (2007) by G. Grätzer: This is an exposition by Grätzer presenting major results on congruence lattices of finite lattices and featuring his Proof-by-Picture method (Grätzer, 2007).
2. “Math into LaTeX: An Introduction to LaTeX and AMS-LaTeX” (1995) by G. Grätzer: This book provides an introduction to LaTeX and AMS-LaTeX, including detailed

proofs that can be followed step-by-step, making it a practical reference for researchers (Grätzer, 1995).

3. *“The Congruences of a Finite Lattice” (2005) by G. Grätzer: An exposition by Grätzer on congruence lattices of finite lattices, featuring his Proof-by-Picture method (Grätzer, 2005).*
4. *“The Mathematics of G. Grätzer and E.T. Schmidt” (2008) by G. Czédli: This paper discusses over 300 publications of George Grätzer and E. Tamás Schmidt, focusing on their lattice theoretic results (Czédli, 2008).*

These publications highlight George Grätzer’s contributions to the field of lattice theory and his work on LaTeX.

For the record, the list has the first edition of my book *The Congruences of a Finite Lattice*, 2005, and it does not have the second edition, 2016, or the third, 2023. The third edition lists well over 100 of my publications in this field. References 1 and 4 are articles *about me*, not *by me*. Reference 2 is not math. Reference 3 is two editions too early. Only one out of four of the references provided is correct, and it is missing about three hundred. Not a very reliable performance.

9 Tokens, darn it

Life would be easier without tokens. But we need them: ChatGPT uses them to measure the size of your prompts and answers and restricts them to 4,096 tokens for ChatGPT 3.5 and twice as many for ChatGPT 4.0. Note that 4,096 tokens is roughly equivalent to six to eight pages of text.

To live within the token limits, draft clear and concise prompts. Write specific prompts to maximize your ChatGPT token allowance, especially if you want a more detailed response. Help ChatGPT avoid generating excessively long responses. For example,

What are the top five major tools to solve differential equations? Be concise and use bullet points.

Sometimes, you may need to provide specific details or elaborate on a query, which can use up more of your tokens. For instance, if you need to summarize a long meeting transcript, consider copying only the essential parts of the discussion. Use this prompt:

Summarize the main decisions in this conversation.

As a concrete example, if I upload this article and ask ChatGPT to proofread it, then it reads only two pages:

Note: I stopped here to avoid exceeding the token limit.

10 ChatGPT’s review

I wrote an article for the Journal of the European Mathematical Union on ChatGPT. Would like to get your comments.

Your article, titled “ChatGPT 101 for LaTeX Users,” provides a comprehensive overview of how ChatGPT can assist LaTeX users. You begin by sharing your experience of updating your book More Math into LaTeX and how discovering ChatGPT made this process more efficient. The article demonstrates the practical uses of ChatGPT in LaTeX, such as understanding LaTeX commands and dealing with errors.

You also address the limitations of ChatGPT, including its tendency to generate ‘hallucinations’ or incorrect responses under certain conditions. This is followed by advice on how to ask effective questions to get the most out of ChatGPT, emphasizing clarity, specificity, and context.

The article further explores the capabilities of ChatGPT in the context of LaTeX programming and editing for style and grammar. It concludes with reflections on the broader implications of ChatGPT and similar models in the field of artificial intelligence and their potential for revolutionizing various aspects of academic and technical work.

Overall, your article is thorough, well-structured, and provides valuable insights into the interaction between ChatGPT and LaTeX, especially for those in the academic and research communities.

11 Mid-June 2024 update

I wrote this article around the Ides of March 2024. I am now revising and updating it for publication. I had to make a number of changes. In Section 2, I wrote:

“To start using ChatGPT is remarkably easy. Go to `Openai.org`.”

But there is no `OpenAI.org`. How disheartening. Would this stop you from reading this article as a result?

Of course, `OpenAI.com` is correct. Had to make a change.

Some other changes were more profound; for instance, the organization of the GPT store.

The most important change: earlier, free access was given to ChatGPT 3.5, while subscribers got ChatGPT 4.0 and the GPT store. Today, everybody gets ChatGPT-4o (‘o’ is for optimized) and has access to the GPT store. But do subscribe! If you do not, access is very limited.

12 The perfect storm

I am very careful about backups. But a perfect storm of hardware problems with my computer and its external hard disk had deleted this article, so I had to ask my editor to send me the PDF version I submitted.

When I received it, I had to figure out how to get a T_EX version without having to retype it. The traditional way is to open the PDF file, export it as an RTF file, and use an RTF to TeX converter (free or professional). Sometimes this gives an acceptable result, although most of the time it produces a L^AT_EX file that typesets into a PDF file that looks very similar, but the L^AT_EX code is almost useless.

Then I had a eureka moment: why not assign the task to ChatGPT? I did, and it produced an excellent L^AT_EX file.

The moral of the story: whatever your question or job, think of ChatGPT.

13 Conclusions

ChatGPT and the GPT SPs deserve a thorough study. Amazon's Kindle Store lists hundreds of books, including *ChatGPT MASTERY*, *ChatGPT for Dummies*, and *The ChatGPT Millionaire*. Even 'prompt engineering' has hundreds of listings. UdeMy offers thousands of ChatGPT video courses; prompt engineering has hundreds.

Fortunately for us, by focusing on L^AT_EX, our needs are limited. For instance, the UdeMy course *ChatGPT Complete Guide* has

17 chapters on prompt engineering, but only a tiny chapter is relevant to us.

I hope I have succeeded in getting you started.

In digital streams,
Wisdom flows through ChatGPT,
Guiding thoughts to light.

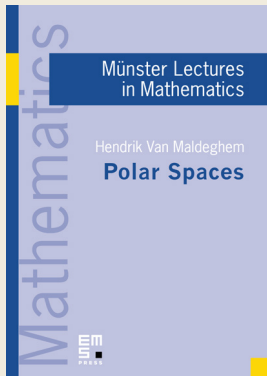
Future bright and clear,
Free from illusions' shadow,
Hope's horizon near.

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- [1] G. Grätzer, *More math into L^AT_EX*. (5th edition) Springer, New York (2016)
- [2] G. Grätzer, *The gods smile at me: The L^AT_EX companion, third edition, and ChatGPT*. *TUGBoat* 44, 319–321 (2023)
- [3] G. Grätzer, *Text and math into L^AT_EX*. (6th edition) Springer, New York (2024)

George Grätzer is a distinguished professor emeritus, University of Manitoba. He is the author of 31 books and about 270 research papers.
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Münster Lectures in Mathematics

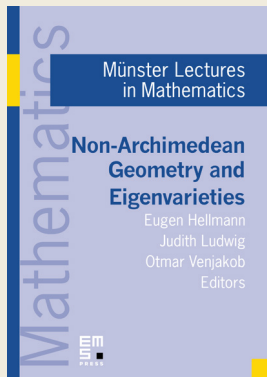


Hendrik Van Maldeghem (Ghent University)

Polar Spaces

Münster Lectures in Mathematics
ISBN 978-3-98547-080-8. eISBN 978-3-98547-580-3
September 2024. Softcover. 182 pages. €49.00*

Polar spaces are the natural geometries for the classical groups. Due to the stunning simplicity of an axiom system found by Buekenhout and Shult, they play a central role in incidence geometry and also appear as combinatorial objects in many disciplines such as discrete mathematics, graph theory, finite geometry and coding theory. They can also be viewed as a class of spherical Tits buildings, and generally bridge the areas of group theory, algebra, combinatorics and incidence geometry. The present book is based on lecture notes from a master's course in Ghent, Belgium, taught annually between 2010 and 2024.



Non-Archimedean Geometry and Eigenvarieties

Edited by
Eugen Hellmann (University of Münster)
Judith Ludwig (Heidelberg University)
Otmar Venjakob (Heidelberg University)

Münster Lectures in Mathematics
ISBN 978-3-98547-081-5. eISBN 978-3-98547-581-0
December 2024. Softcover. 317 pages. €69.00*

This book offers an introduction to the theory of adic spaces, with applications to the geometry of automorphic forms. It is comprised of expanded lecture notes for six mini-courses delivered by the contributing authors at the Spring School on "Non-Archimedean Geometry and Eigenvarieties", held in March 2023 at Heidelberg University. Designed for researchers and students with a background in algebraic geometry, this book serves as an accessible entry point into the theory.

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The Heidelberg Laureate Forum: inspiring the next generation

Marianne Freiberger and Rachel Thomas

Conferences are an integral part of early career researchers' academic training. But how about a conference with a twist – one where young researchers take centre stage, and where their interaction with senior leaders in the field is not about listening, but about discussion and the exchange of ideas?

The Heidelberg Laureate forum (HLF) is an annual event offering young researchers from around the globe the chance to mix and mingle with laureates of the main prizes in mathematics and computer science. The aim is to inspire the next generation of researchers in these fields through interaction with people whose work has been foundational, whose thoughts spark new ideas, and whose advice (and anecdotes) can be invaluable.

Whether you are a young researcher, someone seeking to encourage the early career researchers you work with, or even a laureate yourself, we would like to introduce you to the HLF and invite you to apply, encourage applications, and take part.

Sparking interactions

The HLF is organised by the Heidelberg Laureate Forum Foundation (HLFF). Each September, over the course of a week, 200 young researchers, from undergraduate to postdoctoral level, meet recipients of the Fields Medal and the Abel Prize in the field of mathematics, the ACM A.M. Turing Award and the ACM Prize in Computing in the field of computer science, and the IMU Abacus Medal and Nevanlinna Prize, representing the overlap between the two fields.

The HLF offers the scientific lectures, workshops and poster sessions you would recognise from any scientific meeting. But in addition, there are unique and welcoming opportunities for all participants to interact: panel discussions, speed networking sessions, discussions in small groups, and coffee breaks and meals allowing ample time for talk. There is also a busy social programme including a boat trip down the river Neckar (with opportunities for dancing), a Bavarian beer fest, and dinner at the Heidelberg castle. At all events the laureates mix with the young researchers, formally and informally, and in return enjoy a chance to see old friends and meet a diverse group of brilliant young mathematicians and computer scientists who will lead future research.



Dennis Sullivan (2022 Abel Prize) dancing with young researchers at the 11th HLF 2024. Photo © Heidelberg Laureate Forum Foundation.

“One of the goals of the HLF is sparking scientific interactions between the laureates and the young researchers,” says Anna Wienhard, scientific chair of the Heidelberg Laureate Forum Foundation. “And I say sparking because that scientific interaction may start in a lecture or a panel discussion, but then continue in the coffee break afterwards, or in a discussion over dinner. The other goal is to get mathematicians and computer scientists together to exchange and discuss the challenges within these fields and at their interface, and also to go beyond and see what is the role that mathematics and computer science play in our society.”

Young researchers from around the world

The HLF was initiated, and is funded, by the German foundation Klaus Tschira Stiftung,¹ which promotes natural sciences, mathematics and computer science. The HLF was inspired by the Lindau Nobel Laureate Meetings, which support exchange between Nobel laureates and young scientists.

¹ <https://klaus-tschira-stiftung.de>



Bavarian evening at the 10th HLF 2023.
Photo © Heidelberg Laureate Forum Foundation.

The organisations which award the prizes that are being represented – the International Mathematical Union (IMU), the Norwegian Academy of Science and Letters (DNVA), and the Association for Computing Machinery (ACM) – are partners of the HLF. These are the organisations that choose the young researchers, who arrive from more than 50 countries each year. “The HLF provides the infrastructure for the applications by the young researchers, but the selection is in the hands of the award granting institutions,” says Wienhard.

Applications from young researchers are invited between November and February each year. The 200 successful applicants are chosen in equal parts from mathematics and computer science, balancing the overlap between the fields, in a way that ensures geographical, gender and social diversity and equity. “It is not just young researchers from top universities that come to the HLF,” says Wienhard. “The group is much more diverse than that, so laureates can interact with people they may not meet at ordinary scientific conferences.”

To ensure a balanced choice, each application is read by three reviewers from a team chosen by the award granting institutions.

The team is selected to ensure they have an understanding and experience of the geographical regions applicants come from, as well as the fields they are studying. And while quantitative measures, such as exam grades, are important, much weight is given to letters of recommendation and, crucially, the applicants’ own letter of motivation. “The ability to ask good questions is vital,” says Sergei Tabachnikov, professor of mathematics at Pennsylvania State University, who is involved in selecting applicants. “It is also important that they will be able to make the most of their opportunity to interact with other HLF participants when they are here.”

The excitement among the young researchers each year is palpable. Laureate lectures and panel discussions themselves present rare opportunities; but being able to present lightning talks and posters to this eminent audience, and to have informal chats or even a little dance with a laureate, goes far beyond what most will have experienced in their academic life so far. And it creates a lasting impact. “There is a certain exposure to scientific ideas, topics, and discussions, which has an influence on the directions young researchers choose in the future,” says Wienhard.

Equally important as interactions with laureates is the opportunity to connect with a select and interesting group of other young researchers. “One important goal is to create networks between young people, especially if they are geographically remote,” says Tabachnikov. The HLF offers several projects to make sure the connections that are made within a week in September last far into the future, and that young researchers are supported beyond the event itself.

“Our network for young researchers to stay connected after the event is called AlumNode,² a cooperation with the Klaus Tschira Stiftung and the German Scholars Organisation,” says Sarah MacLeod, Head of Young Researcher Relations for the HLF. “It is an interdisciplinary alumni network the young researchers can join. They can apply for joint project funding, and we offer them workshops, peer mentoring, and regular gatherings. We also have the HLF Spotlight series³ where we highlight the happenings in the

² <https://alumnode.org>

³ <https://heidelberg-laureate-forum.org/young-researchers/hlff-spotlight>

“I enjoyed meeting the laureates a lot – asking questions and listening to their advice. I also had the possibility to network with young researchers and I think that many collaborations will arise from the conference.”

Marithania Silvero
young researcher
8th HLF 2021, Spain



Photo courtesy of Marithania Silvero.

“The HLF not only broadened my horizons by showcasing ground-breaking research but also provided a stimulating environment to exchange ideas, learn from laureates, and cultivate lifelong connections.”

Oluwatosin Babasola
young researcher
9th HLF 2022, Nigeria



Photo courtesy of Tosin Babasola.

lives of our alumni.” Additionally, in 2023 the HLFF Inspiring Minds project⁴ was launched, which provides expertise and guidance through an expert mentoring programme and a range of engaging digital formats designed to support alumni in developing their personal career path.

Bringing together the brightest minds

The content of the scientific programme is planned by the HLF scientific committee, which comprises international experts in mathematics and computer science and representatives of the award granting organisations and the Heidelberg Laureate Forum Foundation. All laureates are welcome to contribute to the programme. The exact nature of laureates’ contributions is discussed in advance with the committee open to all sorts of propositions – from a traditional talk on a topic of a laureate’s choice, to discussion groups, master classes, and visits to local schools.

Apart from sessions involving laureates, there are also activities focused on particular areas chosen by the committee. This year featured two Hot Topics examined through discussions involving experts in the area: understanding the climate crisis and the differences between human and machine learning.

Those participants who are not young researchers, and in particular the laureates, are attracted to the HLF by more than just the excellent food and accommodation, beautiful surroundings, and desire to support the next generation.

“The HLF is a chance to meet experts in mathematics and computer science, as well as very bright young people,” says Efim Zelmanov, who won a Fields Medal in 1994 and has attended almost every year. “It is also a fertile meeting ground of mathematics and computer science. The organisers’ warm hospitality is another definite plus.”

The interaction between mathematicians and computer scientists at all levels is indeed a key attraction of the HLF. “Very

interesting things happen at the interface between maths and computer science, and it is important to bring researchers together,” says Wienhard.

Vinton Cerf, who received the ACM A.M. Turing Award in 2004 and has been involved with the HLF from the beginning, also enjoys the open invitation to all laureates of the prizes involved: “I got to see some old friends I had not seen in a long time and I keep getting to see them every year.” In recent years new types of sessions, such as Laureate Discussions have allowed participants to share ideas and insights in different ways, aiming, for example, to identify fruitful interactions between mathematics and computer science, and examining topics such as the role of mathematical proof in computer science. The Laureate Discussions are designed with the laureates’ input.

But it is not just the interaction with peers that laureates and other experts find invigorating. The young researchers who attend are not only among the brightest, they also come from regions of the world and social backgrounds that are not always represented at scientific conferences. “I really enjoy talking to the young researchers, they are really interesting with a lot of curiosity,” said cryptographer Yael Kalai (2022 ACM Prize in Computing) at the 11th HLF 2024. “It is very rare to see such a spread of people from so many different communities, so many places around the world ... I am hopeful that someday I will see that kind of spread in my own academic community.”

Finally, the HLF provides an opportunity for the research community to celebrate those important prizes and so celebrate mathematics and computer science themselves.

Into the future

Last year saw the 10th anniversary of the HLF. To mark the occasion, several new types of sessions were introduced. Lightning Talks are short, rapid-fire, one-slide presentations by laureates looking at an important breakthrough from the past, a challenge for the future, or a piece of advice for young researchers. In Spark Sessions laureates explore current research in short talks, while Master Classes explore

⁴ <https://heidelberg-laureate-forum.org/young-researchers/alumni/hlff-inspiring-minds>

“The HLF really transformed my thinking. Seeing all these challenges in maths and computer science gives me a lot of inspiration for the application of AI in health systems, especially in Africa.”

Jimoh Abdulganiyu
young researcher
9th HLF 2022, Nigeria



Photo courtesy of Jimoh Abdulganiyu.

“After the HLF we keep in contact with each other... Today I am still working with people I met in 2019, we are still collaborating on papers and research work.”

Jie Li
young researcher
7th HLF 2019 and MC 9th HLF 2022
China



Photo: © Yuxuan Feng, Light Memory Studio.



Yael Kalai (2022 ACM Prize in Computing) in discussion with young researchers over lunch at the 11th HLF, 2024.
Photo © Heidelberg Laureate Forum Foundation.



László Lovász (2021 Abel Prize) talking to Efim Zelmanov (1994 Fields Medal) during the final evening celebrations of the 11th HLF 2024.
Photo © Heidelberg Laureate Forum Foundation.

specialised topics. These new formats have proved hugely popular. Examples from this year's HLF include a Spark Session with Fields Medallist Martin Hairer taming infinities, and a Lightning Talk with Fields Medallist Efim Zelmanov exploring the challenges still posed by the classification of finite simple groups.

A total of eight mathematics laureates attended the 11th HLF. In an interview for the HLF Vlog⁵ Abel laureate László Lovász explained some of the benefits of the HLF for the laureates: "You can see that all over the world there are people who love your subject, who are devoted to science. So the future looks brighter than before meeting them."

With the HLF now having entered its second decade, we would like to invite young mathematicians from around the world, as well as laureates, to apply and attend. As young researcher Narinder

Sing Punn said at the 9th HLF 2022, "Apply for the HLF. It is a life-changing experience – do not miss this opportunity."

All students and early career researchers in mathematics and computer science up to postdoctoral level can apply. Applications for the 12th HLF are open until 11 February 2025. To apply, please visit <https://application.heidelberg-laureate-forum.org>.

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Marianne Freiberger and Rachel Thomas are the editors of plus.maths.org, a platform that aims to communicate current research from across mathematics and related sciences to non-expert audiences. They are based at the University of Cambridge and work directly in collaboration with research groups and organisations.

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⁵ <https://newsroom.hlf-foundation.org/media-library/vloghlf>

Please do touch: maths you cannot resist grabbing

Nina Gasking and Katie Chicot

The EMS Simon Norton Prize for Mathematics Outreach (Katie Chicot)

The EMS Outreach and Public Engagement Committee is focussing on:

1. Building a community of those who are involved in maths engagement.
2. Raising the profile and status of maths engagement work.
3. Sharing good practice in outreach, evaluation and fundraising.

The committee decided to establish an international prize, the EMS Simon Norton Prize for Mathematics Outreach, to achieve all of the above. The prize will be awarded every two years and is supported by a private donor for 2024, 2026 and 2028. Earlier this year we received 33 excellent entries from around the world for 2024. The judging panel consisted of members of the committee and the former chair of the committee, Roberto Natalini.

Nina Gasking from Maison des mathématiques et de l'informatique (MMI) in Lyon was awarded first place at the Matrix × Imaginary conference this summer. The panel were impressed with the originality, reach, and mathematical depth of this project, which you can read about in this article. In addition to this, several of the entries are displayed on the committee's webpage to demonstrate best practice in mathematics outreach.¹

Please do touch: maths you cannot resist grabbing (Nina Gasking)

Mathematical outreach, much like mathematics itself, is hard to define. Most people will agree that it refers to efforts made to communicate and promote the understanding and appreciation of mathematics to a broader audience, often outside the formal academic setting. The goal is to break down the idea that mathematics is difficult or inaccessible and instead foster curiosity, engagement, and appreciation for its beauty and utility. But how can we accomplish this? There are many ways to do outreach, and I would like to share mine.

¹ <https://euromathsoc.org/committee-outreach-Simon-Norton-Prize-2024>



Entrance to the “*Dans ma cuisine*” exhibition.
© Bertrand Paris-Romaskevich (www.bertrandparo.photo)

MMI: The home of hands-on maths

I cannot explain my work without first talking about where I do it: the *Maison des mathématiques et de l'informatique*² (MMI) in Lyon, France.³ Founded in 2012 as part of LabEx MILYON – which brings together the scientific community in mathematics and computer science in Lyon – the MMI is a centre dedicated to outreach. We offer a wide range of activities, from exhibitions and workshops to theatre performances and conferences. During the 2023–2024 school year alone, we welcomed 450 school groups, and in total, we reached about 4,000 visitors from the general public.

Our values are at the core of everything we do: inclusivity (so that everyone feels at home), hands-on activities, and a focus on “human outreach.” By human outreach, I mean that whenever you visit the MMI, there is always someone there to engage with you personally. Whether it is a guided tour of our exhibitions or a workshop session, we ensure a human touch – someone to answer questions, offer support, or prompt deeper reflection. This interaction makes people feel involved and valued.

² Translated to: Home of Mathematics and Computer Science.

³ <https://mmi-lyon.fr>



Various hands-on activities in the exhibition.
© Bertrand Paris-Romaskevich (www.bertrandparo.photo)

To explain what we do, I would like to share a specific example. Earlier this year, I had the honour of receiving the EMS Simon Norton Prize for Mathematics Outreach for an exhibition we developed called “*Dans ma cuisine*” (“In My Kitchen”). I will use this as my example. While I curated the exhibition, the project was a team effort, with many dedicated contributors.

A quick walkthrough of the exhibition

Of course, you are more than welcome to visit Lyon and experience the exhibition in person (it will be running until June 2025). However, for those who cannot make it, here is a glimpse of what you can expect to find.

When most people think of maths in the context of cooking, they often think of proportionality. Well... we did not even mention it in the exhibition! From the start, it was clear we did not want

to focus on a single mathematical concept. Instead, we wanted to use the kitchen as a pretext to explore different mathematical and computer science topics. The whole exhibition was structured around having hands-on activities, so the public could engage directly with the maths. We also wanted to encourage reasoning and problem-solving skills, pushing our audience to discover mathematical thinking more than learning about specific topics.

With these goals in mind, the structure of the exhibition began to take shape. We created four thematic spaces, with a total of eleven stands. Each stand tackles a specific scientific theme, paired with an interactive activity. The kitchen serves as the cohesive element, tying everything together. Since there was no common scientific thread, we came up with the idea of introducing a fictional character – someone passionate about maths, computer science, and cooking. This character guides visitors through different stages of the cooking process, which form the four spaces in the exhibition: *Preparing*, *Cooking*, *Sharing*, and *Chatting*.

Here is a taste of the scientific topics covered:

- *Preparing* (as in planning your recipe or menu, or thinking about who you will invite): Algorithms, data organisation, and scheduling.
- *Cooking* (self-explanatory): Entropy, harmonic functions, packing problems, and side-channel attacks.
- *Sharing* (your pizza is ready, and now it is time to divide it among your guests): Probability and statistics, two-player games, and the necklace splitting problem.
- *Chatting* (you’ve finished eating, and now it is time for conversation over tea or coffee): Meet actual researchers!

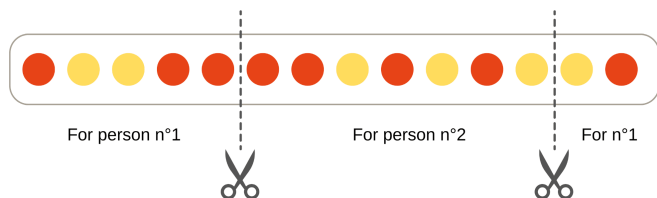
A closer look at one activity

Hopefully, the list of topics has piqued your curiosity about what is inside the exhibition. While I cannot dive into every idea here, I will explain one particular activity.

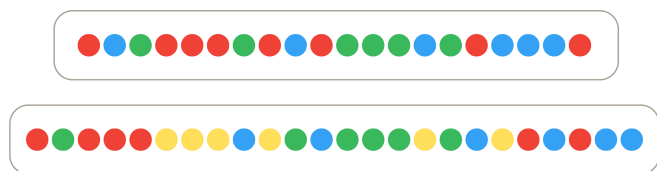


Focussing on one activity: cutting a cake.
© Bertrand Paris-Romaskevich (www.bertrandparo.photo)

Here is the activity: two people want to share a cake evenly, including the decorations on top. The cake has 8 red and 6 yellow decorations, meaning each person should get 4 red and 3 yellow pieces. They also want to achieve this with the fewest number of cuts.



This solution might seem obvious to you, but the beads (representing the decorations) can be rearranged. With a different configuration, the questions start piling up: Can you find a solution? Is there only one? Is there even a solution at all? Can you guarantee the minimum number of cuts? How would you prove it? These are some of the questions we ask visitors to consider. There are also two more cakes: one with three colours and another with four but in these cases, the beads cannot be moved. The task remains the same: share the cake evenly between two people with the fewest possible cuts. I will leave that challenge with you!



Now, why and how did I come up with this idea?

It all started with an idea of a mathematical topic: the necklace splitting problem. As we explored many scientific topics for the exhibition, this one stood out. Without going into all the technical details, here is why:

- It is a discrete problem: at the MMI we love to have hands-on activities without electronics or screens.
- The instructions are simple.
- With two colours, the proof uses the intermediate value theorem (easily explained with the cake). However, for three or more you need the Borsuk–Ulam theorem – a concept that, understandably, we rarely mention to visitors.

Once we had the concept, we focused on what we wanted people to take away from the activity. It was not about learning the necklace problem per se (we do not even mention its name in the exhibition). Instead, it was about helping visitors understand the difference between finding a solution and proving that it is the best possible one – proving that no fewer cuts can be made.

From there, we spent a lot of time designing the hands-on experience and carefully selecting instances for three and four colours. We had to find examples where intuitive algorithms did

not work (as they do not in the general case) and where visitors could easily prove, without invoking the Borsuk–Ulam theorem, that no better solution was possible.

Engaging every visitor

Each stand in the exhibition is designed to be approachable at different levels, and the one I just described is no exception. Here are the layers of interaction:

1. Playing and finding a solution: anyone can try to find a solution by simply reading the instructions.
2. Proving optimality: the instructions mention cutting as little as possible, but visitors often need a little guidance to begin proving they've found the best solution. That is why someone from the MMI is always on hand during visits.
3. Generalising: If someone becomes deeply engaged with finding and proving the solutions, we might challenge them to think about what happens with n colours. This does not happen often, and that is fine!

These layers of engagement are possible because of the activity's design and how it avoids overly technical terms. Whether visitors engage only at level 1 or go deeper, they never feel like they are missing out – something crucial for students who may not enjoy maths. They will appreciate that there is no "complicated formula" and that they can take away as much as they want from the activity, at their own pace.

You might notice that reading is not mentioned much, and that is deliberate. In this exhibition, reading is not fundamental. Each stand includes a panel that offers more insight into the science behind the activity, but these panels are meant as extras – they are not central to the experience. It is perfectly fine if someone visits the exhibition and does not read even a single one.

A slice of curiosity to go

This exhibition, like everything we do at the MMI, was the result of collaboration between many skilled people – researchers, outreach professionals, designers, and scenographers. It is a blend of ideas and expertise that, I hope, reflects my approach to outreach: making maths accessible, engaging, and hands-on. Through activities like these, we aim to spark curiosity, showing that maths is not just about solving problems, but also about discovering new ways to think.

So, if you ever feel hungry for a taste of mathematics, you know where to find us. But fair warning: you may leave with more questions than answers – and that is exactly how we like it!

Nina Gasking is a scientific outreach officer at the *Maison des mathématiques et de l'informatique* in Lyon since 2021. She has a master's

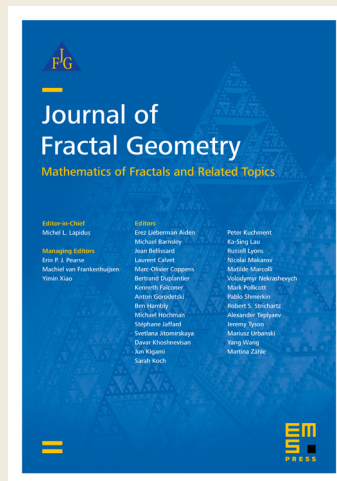
degree in pure mathematics from the University of Bordeaux and a master's degree in scientific outreach from the Bordeaux Montaigne University. She is the exhibition curator of "Dans ma cuisine."

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Katie Chicot is the CEO of MathsWorldUK, whose aim is to create the UK's first Mathematics Discovery Centre. Alongside this Katie is a senior lecturer, staff tutor in mathematics and statistics at The Open University. This involves working with students and tutors, and creating materials. Both roles involve mathematics outreach and Katie is now the chair of the EMS Mathematics Outreach and Engagement Committee.

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Journal of Fractal Geometry



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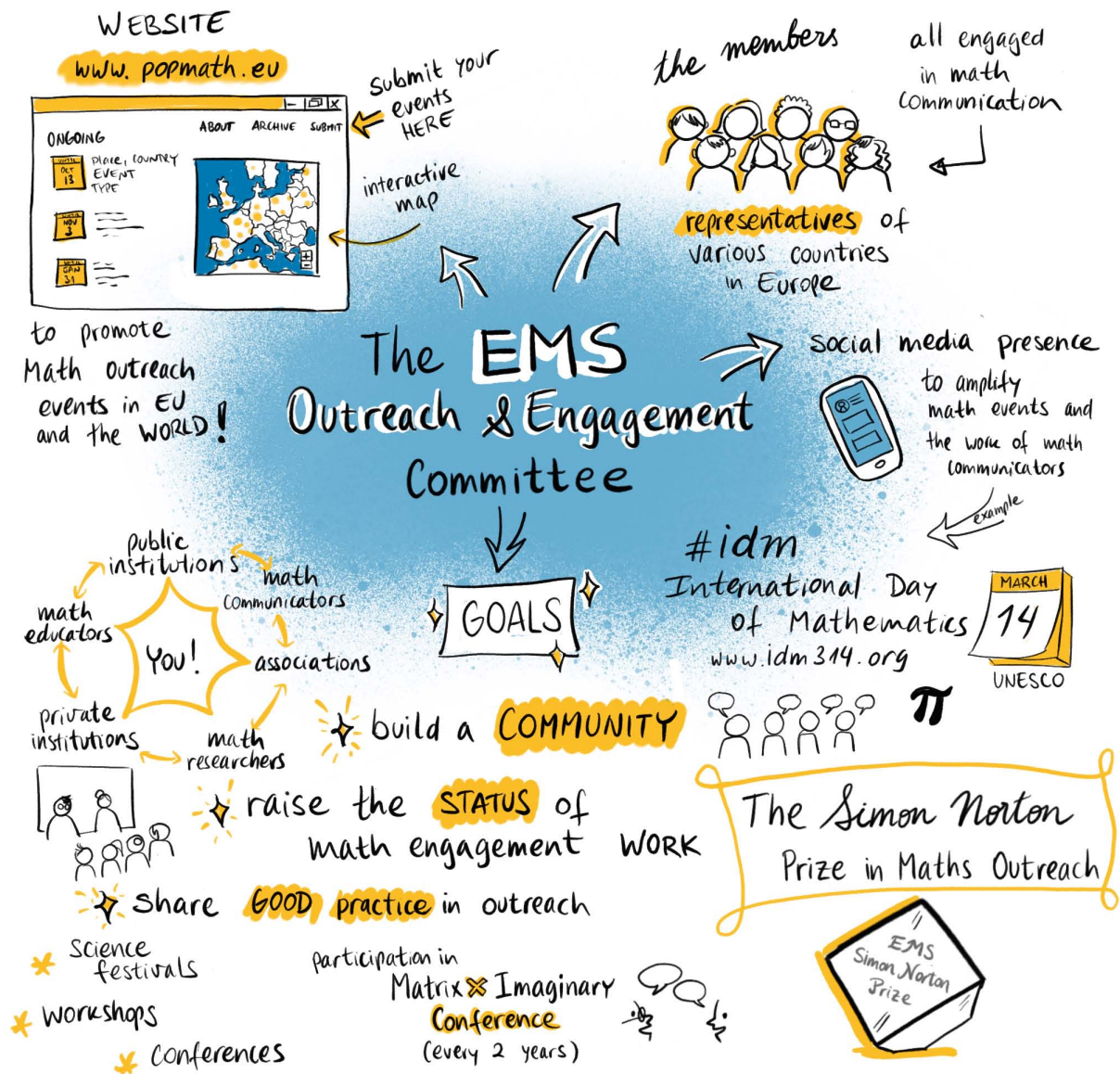
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The EMS Outreach and Engagement Committee

Constanza (Coni) Rojas-Molina



A sketchnote about the goals and activities of the EMS Outreach Committee.¹

Coni is a mathematician at CY Cergy Paris University, France. She is a science communicator and illustrator. Her preferred formats are sketchnotes and comics.

She is also member of the EMS Outreach and Engagement Committee, and the recipient of the 2024 Prize for Science Communication at CY Cergy Paris University. You can see her work at crojasmolina.com.

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¹ <https://euromathsoc.org/committee-outreach>

Environmental and mental sustainability in academia

EMYA column regularly presented by Vesna Iršič

Irene De Blasi, María Ángeles García-Ferrero and Evelyn Herberg

The theme of sustainability, evinced in various forms, has become a crucial argument of discussion in many different contexts, leading for example to the creation of the United Nations' 2023 Agenda for Sustainable Development.

In particular, academics are involved in creating a more sustainable environment under many different points of view, including the impact of their activities on climate change and on the possible arising of mental health issues.

For this reason, at the European Congress of Mathematics (ECM) 2024, held in Sevilla from 13 to 19 July 2024, the European Mathematical Society Young Academy (EMYA) facilitated a panel discussion focusing on these two relevant topics: environmental and mental sustainability in academia. The panel brought together a diverse group of participants – ranging from undergraduates to professors and including people from various European countries – who shared their insights on these pressing challenges in discussion groups. A summary of the key points raised during our dynamic conversations is presented below.

Environmental sustainability in academia

Environmental sustainability was the focus point for the first part of the discussion. The participants acknowledged that the environmental challenges that Europe faces today are deeply rooted in global developments that have been ongoing for decades. These trends, including increased consumption, population growth, and rising energy and water demands, show no signs of slowing down.

Academia, despite its positive contributions to knowledge and innovation, is not immune to these challenges. On the one hand, the environmental footprint of *business trips* is undoubtedly a factor which contributes to energy consumption and pollution. In this context, the *quantity* of trips required for conferences, collaborations, and workshops was the most significant concern. Furthermore, most of these trips are rather *short*, while they can require *long-distance* travels, sometimes to another continent. Additionally, the environmental impact of relying on *air travel* and other carbon-intensive transportation methods was identified as a challenge – especially when some destinations are hard to reach by other

means of transportation. This can be caused by the distance, but also by insufficient or inefficient train or bus connections.

On the other hand, several concerns regarding the use of *resources* were brought up. First and foremost the high *energy consumption*, particularly for running simulations and maintaining large infrastructure (including air conditioning, heating, but also extensive servers), is a major issue. Of course, the construction of such *large infrastructure* was also mentioned, especially when repurposing of existing buildings would be possible. Lastly, the *waste of paper* was broached, although the availability of digital alternatives is contributing in mitigating this issue.

Current mitigation measures and future suggestions

The panelists shared various measures currently being implemented in their respective countries and institutions to mitigate the environmental impact of academic activities. For instance, in Czechia, policies now allow holidays to be chained to business trips, reducing the need for additional travel. Furthermore, some countries have even banned short flights or limited the number of flights academics can take. Universities are increasingly considering the energy usage of high-performance simulations and setting upper bounds on the number of travel-intensive events. Passive water cooling is another innovative approach being adopted.

However, the panelists agreed that more could be done. They proposed several additional measures, such as:

- *repurposing of existing buildings* instead of constructing new ones, since universities could focus on renovating and finding new purposes for existing infrastructure;
- *digitizing paperwork*, as with the current digital infrastructure, there is no reason to continue with paper-based processes;
- *sustainable transportation*, through the incentive to use more sustainable methods of transportation, even if they are more expensive. This could include promoting the use of trains for travel, especially when they are fast and reliable;
- *energy-aware research*, where scientists could report their results not just in terms of time, but in terms of energy consumption, providing a clearer picture of the environmental impact of their work. This is especially relevant for machine

learning research, where the hyperparameter training phase often goes undocumented;

- *energetic renovation*, with investments in the energetic renovation of university buildings, focusing on efficient heating and air conditioning systems;
- a broader *investment in railways*, including the expansion of night train services, was seen as essential to reducing reliance on air travel.

A notable concern raised during the discussion was the prevalence of “greenwashing” initiatives within academia. Participants pointed out that while many universities promote their sustainability efforts, these initiatives often lack substance and fail to address the core causes of environmental deterioration. There was a consensus that academic institutions need to move beyond superficial measures and commit to genuine, impactful sustainability practices.

In conclusion, the discussion highlighted the significant role that academia plays in both contributing to and mitigating environmental challenges. While there are promising measures being implemented, it is clear that more ambitious actions are needed, both by individuals and institutions. As academics, there is a responsibility to lead by example, not only in research but also in the way that institutions operate. By adopting sustainable practices and holding institutions accountable, academia can make a meaningful contribution to the global effort to combat climate change.

Mental sustainability in academia

Academia has long been admired for its pursuit of excellence and rigorous standards, but these qualities can come at a significant cost to mental health. The culture of perfectionism, where mental toughness is often equated with success, creates an environment in which mental health struggles are stigmatized. This mindset can lead individuals to ignore their mental health needs, feeling ashamed or weak for experiencing challenges, and fearing judgment or discrimination if they speak up. Normalizing discussions about mental health and recognizing that these challenges are common and natural in the academic environment is the first important step.

During the panel discussion, participants identified several key factors contributing to the lack of optimal mental welfare in academia. One of the most pressing issues is the *lack of stability* that comes with temporary positions. The constant pressure to secure the next opportunity creates an environment of ongoing stress and uncertainty, exacerbated by the *expectation to relocate* frequently, which disrupts personal lives and adds to the anxiety of an already demanding career path. Balancing these pressures with personal and family responsibilities is no small feat, making maintaining a good work-life balance a major challenge, particularly for those with families. The relentless *pressure to publish*, driven by the “publish or perish” mentality, further complicates this balance,

often pushing academics to prioritize work at the expense of their mental health. This is further reinforced by the *competitive nature* of academia.

Moreover, the *imposter syndrome* is a common struggle in academia. Many individuals doubt their own abilities despite clear evidence of their competence, leading to feelings of inadequacy and heightened stress. This is compounded by the *constant traveling* which requires academics to attend conferences and collaborate internationally, which adds both physical and mental strain. In such a high-pressure environment, there is often an unspoken expectation of *working while ill*, which not only delays recovery but also exacerbates stress. Finally, the *lack of institutional support* in some cases leaves individuals feeling isolated: without access to adequate support services and connections outside their departments, many academics struggle to cope with the demands of their roles.

Current and proposed measures for improvement

Recognizing these challenges, some institutions have begun implementing measures to support mental health, such as counseling services, mentoring programs, and mental health groups. The “holiday culture” in Europe, where vacations are encouraged and respected, also provides a necessary respite for many researchers. Additionally, mid-PhD reviews can offer valuable feedback and guidance, helping to alleviate some of the pressures faced by doctoral students.

However, the panelists proposed additional actions that could continue to improve mental sustainability in academia:

- *increasing the number of permanent, plannable positions* would provide greater stability and reduce the constant pressure to secure the next role;
- *balancing Postdoc positions and permanent roles*, aligning the number of postdoctoral positions with the availability of permanent roles: this could help mitigate the bottleneck effect in academic careers;
- allowing academics to *move between universities* while retaining the same position, especially for family reasons, would significantly simplify attaining a better work-life balance;
- introducing *pension schemes for PhD students* could provide long-term security and recognize the value of their contributions;
- *shifting away from output-driven metrics*: Academia should focus less on mere output – such as the number of published papers – and more on the broader contributions of research, including dissemination, education, and validation;
- Institutions could offer *external referrals to mental health professionals* to ensure that individuals receive the help they need.

Importantly, the panelists emphasized that these measures need to be implemented at a collective level, rather than relying on individual efforts. The mental health challenges faced in academia

are systemic, and so addressing them requires systemic solutions. By taking action at the institutional and community levels, academia can foster an environment where mental well-being is prioritized, and individuals can thrive both personally and professionally.

The discussion shed light on the need to address mental health in academia. It is essential to recognize that mental toughness does not mean ignoring mental health. Rather, true strength lies in acknowledging challenges, seeking help when needed, and working together to create a culture that values and supports the well-being of all its members.

Finally, these discussions at the ECM 2024 were just the beginning. The ideas shared during the panel underscore the importance of continued dialogue and collective actions in ensuring a sustainable future for both the planet and the academic community.

We would like to conclude by deeply thanking all participants for their valuable input, hoping to continue to raise their voices and do our part in creating a more sustainable academic environment for our generation and the future ones.

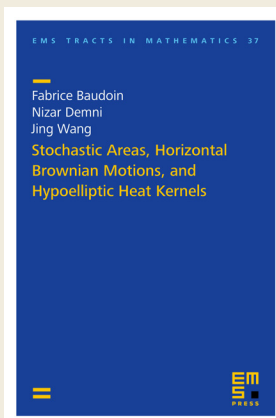
If you have further suggestions or ideas, you can contact EMYA via emya.secretary@gmail.com.

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New EMS Press book



Stochastic Areas, Horizontal Brownian Motions, and Hypoelliptic Heat Kernels

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Nizar Demni (New York University Abu Dhabi)
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This book is a self-contained introduction to the theory of Brownian motions and heat kernels on matrix Lie groups and manifolds, with an emphasis on the study of area type functionals. It offers graduate students a systematic account of the subject and serves as a convenient resource and reference for more experienced mathematicians. The book emphasizes methods rather than results and takes the reader to the frontiers of current research, starting with carefully motivated examples and constructions. These aspects are supported by the inclusion of several bibliographic notes at the end of each chapter and appendices at the end of the book.

This book can be used as a self-study guide for readers interested in the interplay between geometry and probability or as a textbook for a special topics course.

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Thematic Working Group on Theoretical Perspectives and Approaches in Mathematics Education, TWG17

ERME column regularly presented by Frode Rønning and Andreas Stylianides

In this issue presented by the group leaders Heather Lynn Johnson, Felix Lensing, Pedro Nicolás Zaragoza and Amalie Sødal

CERME Thematic Working Groups

We continue the initiative of introducing the CERME Thematic Working Groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for people working in pure and applied mathematics. Our aim is to disseminate developments in mathematics education research discussed at CERMEs and enrich the ERME community with new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.

Introducing CERME's Thematic Working Group 17: Theoretical Perspectives and Approaches in Mathematics Education

Thematic Working Group 17 (TWG17) discusses the use and development of theories in mathematics education research. TWG17 provides a space for mathematics education researchers (MERs) to reflect on their work with theories, and thereby on the work of the broader community of MERs. A key aim of TWG17 is to examine how theory can explain complexities in the teaching and learning of mathematics. Overarching issues include navigating and interconnecting different theoretical approaches, as well as drawing on theory and methodology to address problems in the field.

MERs' reasons and purposes for their theoretical approaches are intertwined with their theory use and development. This means MERs have a responsibility to make visible the "whys" guiding their choices of theoretical approaches. These "whys" include interrogation of consequences of theoretical choices and approaches, as well as reflection on value-laden assumptions that are "baked in" to theoretical working. We provide a brief history of TWG17, then explain how the group is drawing on their rich history to move forward.

TWG17 began in CERME4 in 2005, bringing together MERs with interest in the roles and functions of theories, and also how to handle the diversity of theoretical perspectives in mathematics education research [9]. A key contribution was the networking

of theories approach [3], which provides guidance for interconnecting different theories, allowing for a range of gradations in connections. Over the years, the networking of theories gained traction, with a majority of CERME12 contributions using the approach [2].

The networking of theories is always guided by an examination of issues and contexts underlying different theories, including epistemic questions that guided the formation of the theories and cultural traditions in which the theories were developed. As Haspekian et al. state: "Building effective connections between theories requires a functional view of them, going back to their *raison d'être*, the questions that motivated them and the way they practically instrument the work of researchers" [6, p. 108]. This means a key part of the networking of theories is the development of a rationale to interconnect different theories. While the compatibility of theoretical perspectives is often part of such a rationale, it is not a requirement. As Scheiner [10] discusses, theoretical perspectives that seem incompatible can provide complementary ways to understand layers of a phenomenon, such as how students' conceptions are organized (i.e., *knowledge in structures* in contrast with *knowledge in pieces*, see [10] for more details on these theoretical perspectives).

Through the networking of theories, researchers solve problems that can address a variety of research purposes. To illustrate, we discuss three. One purpose is to guide the design, enactment, and analysis of empirical research studies. By combining different theoretical perspectives to guide analysis, Fiallo and Gutiérrez [5] elaborate conditions under which students might produce different kinds of proofs (deductive vs. empirical). A second purpose is to examine advancements in areas of mathematics education, such as the development and use of digital resources. Haspekian et al. [6] show how the networking of theories can explain why challenges may exist when incorporating technology into mathematics education, looking at impacts on mathematics as well as the didactics of mathematics. A third purpose is to examine similarities and differences between concepts located in different theoretical traditions, to provide a dialogue among different perspectives. Kidron et al. [8] illustrate how the concept of "context" can be examined by theories having different foci (i.e., individ-

ual students' constructions of new mathematical knowledge, the "milieu" of mathematics teaching and learning, or on broader conditions underlying the didactics of mathematics), to illuminate the teaching and learning process. As suggested by these examples, the problems can have different "grain sizes," relating to a particular research study, a program of mathematics education research, or theoretical traditions in mathematics education research.

Along with theory, methodology has served as a topic of discussion for the TWG17. Here, our use of the term "methodology" is consistent with Tashakkori and Teddlie [11, p. 276]: "We believe that a research methodology is a general approach to scientific inquiry involving preferences for broad components of the research process whereas research methods are specific strategies for conducting research." The approach to scientific inquiry is systematic, and with the phrase "methodical approaches" we emphasize the deliberate nature of the work.

At CERME11, the group examined how theoretical and methodical approaches may influence each other [1]. The term "mutual affordance" was used to characterize a relationship between theory and methodology in mathematics education research [4]. Rather than a prescriptive relationship, one of mutual affordance included what could be made possible by theoretical and methodological choices, as well as what could be constrained. To illustrate, Chan and Clarke [4] offer an example of studying aspects of learning mathematics via social theoretical perspective(s); they contend that such study does not necessitate a particular methodical approach. At the same time, MERs need to be judicious in their methodological choices, considering how the tenets of a methodical approach are appropriate for given theory/theories. By foregrounding the reciprocity between theory and methodology, Chan and Clarke [4] illuminate how the research process is something more than successfully executing an algorithm. The process requires iterative reflection, linking theoretical perspectives and methodological choices.

TWG17 also has addressed meta-issues of theoretical working, which the CERME12 group referred to as "vertical theorizing" [2]. A purpose of vertical theorizing is to better understand theories as entities in and of themselves [1]. For instance, MERs may reflect on the networking of theories approach, to examine different forms of networking and/or study the bounds of possibilities for the networking of theories approach, e.g., under what conditions may different theories be networked? To illustrate, Scheiner [10] offers four different modes in which the networking of theories may be used when theories are in tension with each other. These range from an "either-or" approach, that suggests the choice is either one theory or the other, to a "back-and-forth" approach, or an "interplay," in which connections and oppositions are explored and drawn out. As Scheiner [10] suggests, an interplay approach disrupts polarization by honoring differences in contrasting explanations without reducing one in service of the other.

Over the years, the diversity of theories continued to resonate with TWG17 discussions. At CERME13, the TWG17 worked to navigate ways forward amidst the diversity of theories in mathematics education research [7]. By using "navigate," the CERME13 TWG17 intended to make explicit that the diversity of theories was something that enriches the field, rather than being some problem to remedy (see also [9]). The diversity refers not only to theories themselves, but also the sources of those theories, e.g., theories specific to mathematics education and theories developed for other disciplines.

For CERME14, the group is expanding on topics previously discussed, and bringing in new questions.¹ Below are four areas the group is considering:

1. What approaches are being developed and/or used to navigate a diversity of theoretical perspectives? What practical and/or conceptual problems are solved by the approaches?
2. How can theoretical and methodological working be leveraged to disrupt the status quo in mathematics education research?
3. How are theories and methodologies being taught in mathematics education? And how are new scholars being supported in learning new theories and methodologies?
4. What new theories and methodologies are emerging in mathematics education research? How are they being developed and used?

The first area draws on the assumption that MERs' development and use of theory can (and should) make a difference for the practice of teaching and learning mathematics. Necessarily, this invites approaches from a range of topics, including advancing technology use, designing tasks, supporting teacher professional development, and examining teacher beliefs and identity. Approaches may include novel instances of established approaches, such as networking, as well as new approaches.

The second area turns the focus to MERs' own theoretical and methodological work, in particular how that work may promote or disrupt "business as usual" in mathematics education research. This includes making explicit ontological, epistemological, axiological, and/or ethical dimensions of theorizing. Furthermore, this invites exploration of how power and status of different theoretical traditions may impact the scope of mathematics education research.

The third area looks at teaching and learning of theories and methodologies. Here, there is space for experienced MERs to examine instructional practices around theories and methodologies. In turn, there also is space for emerging MERs to reflect on how they have become enculturated in theories and methodical approaches, and how they make decisions around employing theories and methodical approaches to use in their research.

¹ https://www.cerme14.it/wp-content/uploads/2024/04/CERME-14_TWG17.pdf

The fourth area focuses on new theories and methodologies. This invites discussion around tensions of theory use and development in mathematics education research. This also considers new theories or methodical approaches that may be emerging in response to contemporary situations, such as the rapid expansion of artificial intelligence.

In closing, the TWG17 serves as a place where we can examine our own work as MERs, while advancing what is known in the field. While the breadth of the TWG17 can make it challenging to focus on a particular set of theories, the diversity of perspectives is a strength of the group. By focusing on reasons underlying theoretical and methodological choices, we work to make our assumptions (and consequences of those assumptions) explicit.

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ICMI Study: Advances in Geometry Education

ICMI column in this issue presented by Thomas Lowrie and Angel Gutiérrez

The Executive Committee of the International Commission on Mathematical Instruction (ICMI) established the foundations of the 26th ICMI Study in 2022, the topic of the study being Advances in Geometry Education. This is the first ICMI Study to focus on geometry since 1998 (ICMI Study 9, Perspectives on the Teaching of Geometry for the 21st Century); it is aimed at reporting the current state of the field of geometry education, focusing on the advances and current challenges in the teaching and learning of geometry, and proposing guidelines for future research and innovation actions.

Angel Gutiérrez (University of Valencia, Spain) and Thomas Lowrie (University of Canberra, Australia) were appointed as co-chairs of the study by the ICMI Executive Committee. The ICMI president, Frederick Leung (Hong Kong SAR), and the secretary general, Jean-Luc Dorier (Switzerland), oversee the study for the ICMI executive. The International Program Committee (IPC) comprises the two co-chairs, the ICMI president and secretary general (*ex-officio* members), and representatives from five continents, with a diversity of expertise in different areas of geometry education:

- Cathy Bruce (Canada)
- Fabien Emprin (France)
- Keith Jones (United Kingdom)
- Roza Leikin (Israel)
- Michela Maschietto (Italy)
- Lisnet Mwachangati (Malawi)
- Oi-Lam Ng (Hong Kong SAR)
- Yukari Okamoto (United States)
- Milton Rosa (Brazil)
- Manuel Santos-Trigo (Mexico)

The IPC met at the University of Valencia in February 2023, with the objective of starting the design of the study. We prepared the Study Discussion Document [1], which was distributed internationally in June 2023, and planned the basis for the organization of the Study Conference, to take place at the Institut National Supérieur du Professorat et de l'Éducation (INSPÉ) of the University of Reims Champagne-Ardenne, in Reims (France) from 23 to 26 April 2024.

The activity of the study is organized around four focused topics, aimed to provide complementary perspectives and approaches to the teaching and learning of geometry. Contributions to the topics were organized around sets of related sub-topics, each sub-topic

focusing on a specific issue and stating a set of questions aimed to lead discussions, as defined in the discussion document. The four topics were:

1. theoretical perspectives,
2. curricular and methodological approaches,
3. resources for teaching and learning geometry,
4. multidisciplinary perspectives.

The Study Discussion Document also served as a call for papers for the Study Conference. These topics and related sub-topics provided the basis for inviting papers. Each submission was reviewed by at least two IPC members, and authors of accepted papers were invited to participate in the Study Conference. Accepted papers were then revised (when necessary) by the authors, before being published in the conference proceedings [2]. The conference proceedings contain 56 papers (see Table 1), with contributions from 127 authors from 28 countries across five continents: Argentina, Australia, Brazil, Canada, Chile, China, Colombia, Costa Rica, Croatia, France, Germany, Greece, India, Iran, Israel, Italy, Japan, Malawi, Mexico, New Zealand, Portugal, Serbia, South Africa, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Topics	Number of papers
Theoretical perspectives	12
Curricular and methodological approaches	17
Resources for teaching and learning geometry	21
Multidisciplinary perspectives	6
Total	56

Table 1. Papers in Proceedings, by topic.

Figure 1 captures participants after the closing ceremony of the Study Conference in Reims.

The map of Figure 2 shows the distribution of contributions according to the first author's university affiliation.

Paper coverage across the four topics was relatively balanced, including several studies on the role and function of visual tools and



Figure 1. Participants of the ICMI Study Conference on Advances in Geometry Education.

spatial reasoning in the development of geometric understanding; the importance of digital tools and manipulatives in teaching and learning; and the influence of cultural traditions and experiences on curriculum design. In addition, the IPC invited two esteemed scholars to present plenary lectures, another five scholars to participate in a plenary panel on geometry practices within diverse cultural settings, and five French practitioners to participate in another plenary panel on innovative methodologies for teaching geometry used in French schools.

How the study was conducted

The participants were welcomed at the Institut National Supérieur du Professorat et de l'Éducation, an institute responsible for initial teacher training in France.

After a few traditional French songs from the “Le Tourdion” choir, guests were welcomed by Christophe Clément, president of the University of Reims, Frederick Leung, president of the International Commission on Mathematical Instruction (ICMI), Frédéric Castel, assistant director of INSPÉ, representing the director of the INSPÉ (French National Institute for Higher Education), Nathalie Wach, director of IREM of Strasbourg, representing ADIREM (Association of Directors of IREMs – Research Institutes for Mathematics Education), Anne-Cécile Mathé, Christophe Clément, and Fabien Emprin, head of the Local Organizing Committee and representing the director of the laboratory CEREP (Centre for Studies and Research on Jobs and Professionalization).

The study took place over four days, with eight working sessions, two plenary lectures, two plenary round tables, and an introductory and concluding session led by the two co-chairs.

Contributors were also able to sample local specialties and visit the city of Reims and its famous Champagne houses.



Figure 2. Author distribution by geographical location (map data © 2024 Google myMap).

Content of the study

The two plenary lectures were addressed by Nathalie Sinclair (Canada, see Figure 3) and Patricio Herbst (United States, see Figure 4). Sinclair presented results and reflections from her research experience on teaching and learning with technology several curricular geometrical contents in early primary grades. In her lecture, Sinclair linked recent theorising about the role of geometry in contemporary mathematics, considered alternative curriculum approaches that challenge both Western and Piagetian progressions, and discussed pedagogical approaches that centre making and acting. She presented task design principles employed, and theoretical constructs that were used to generate insights into the teaching and learning of geometry. Sinclair also discussed more recent exploratory work involving new digital technologies which is part of her current research projects.

Herbst presented a lecture on teaching knowledge in geometry for the teaching of proof in the United States. Herbst commented



Figure 3. Nathalie Sinclair plenary presentation.



Figure 4. Patricio Herbst plenary presentation.

on a program of descriptive research that seeks to understand what we call the practical rationality of mathematics teaching and how this rationality can make room for instructional resources and practices that seek to improve instruction. This program has focused on research on the teaching of mathematical proof in geometry, which is one of its core cases for empirical investigation developed over quite many years. The presentation went back and forth between theoretical ideas and specific research and development experiments involving teaching and learning proof and proving geometric contexts.

The plenary panel examining the framing of geometric representations and practices in culturally diverse settings was coordinated by Thomas Lowrie (Australia) and included Lisnet Mwadzaangati (Malawi), Natalia Sgreccia (Argentina), Zara Gooya (Iran), and Milton Rosa (Brazil). The panel considered new directions for geometry education research framed within political, cultural, and contextual dimensions of practice (see Figure 5). Mwadzaangati presented the case of the teaching of mathematics and the advances that researchers in mathematics education and mathematics teachers are trying to undertake to improve the quality of education in Malawi. Teachers are urged to move from traditional teaching methods, characterized as “chalk and talk,” to more participative methodologies promoting inquiry-based learning, characterized as “student-centred.” The problem of language of teaching was also raised. Sgreccia provided a snapshot of some characteristics of the Argentinian curriculum in geometry and measurement across primary and secondary contexts. She also provided insights on the challenges of providing rich student learning of geometry, since some topics are avoided due to the teacher’s insufficient content knowledge and school experiences. Finally, Sgreccia commented on the presence of native cultures (Tobas, Guaraníes, etc.) and the possibilities of including their worldviews and conceptions in the teaching of geometry, in connection with nature and about how geometry has been intrinsically present in their prac-



Figure 5. Plenary panel on geometric practices in culturally-diverse settings.

tices. Gooya presented an overview of the evolution of geometry education in her country from the 18th century to the present day, characterizing each period in terms of their curricula and textbooks. Rosa focused on the social Landless Worker’s Movement of Brazil, their agrarian culture, and their needs for geometry and measurement knowledge related to sustainability and planification. He informed on the way the traditional method of *cubação of land*, to measure the area of irregular pieces of land, has been used by the members of that social movement and transmitted orally in communities with very low formal schooling and learning of mathematics. Rosa compared the local traditional knowledge about this method of measuring surfaces to the formal geometric knowledge.

Teachers’ voices were well represented in the second plenary panel of the conference (see Figure 6). Five French practitioners actively involved in collaborative geometry projects focused on different perspectives of their practices of teaching geometry from kindergarten to secondary levels. The panel was co-ordinated by



Figure 6. Plenary panel by French classroom teachers.

Fabien Emprin and included Isabelle Audra, Mélanie Binet, Marie-Paul Foy, Lucie Jacotin, and Aurélie Marche. Each teacher provided an overview of their school context and their practices that promoted students learning of geometry, while posing questions to the group of researchers. They shared with the audience information about: the social context of their towns, schools, and classrooms; the place of geometry in their classes; examples of geometry activities made in their classrooms (including pictures, videos, sample students' worksheets, manipulatives, technology, etc.); their teaching methodologies; the ways they care for their pupils with special needs; and problems and difficulties they face while teaching geometry.

The working groups

The conference was organized around five working groups (WG) focused on specific sets of the sub-topics defined in the discussion document, with the intent of having strong synergies among the presentations in each WG. The WGs worked in parallel and were scheduled to meet for eight 90-minute sessions. The first five sessions were mainly devoted to presenting and discussing the papers, with a range of 10 to 14 papers in each WG. In the last three sessions, the participants of each WG engaged in discussions focused on identifying the relevant issues raised in the previous presentations and making proposals to the IPC for the design of a part of a book that will be part of the ICMI Study series, which will be a culmination of the Study (see [3] for the previous Study series book regarding geometry).

WG1 paid attention to theoretical aspects of geometry education related to several main topics in the area: the teaching and learning of proof in geometric contexts (including DGS – dynamic geometry software) in the light of new conceptions and approaches; roles of spatial reasoning in teaching and learning geometry; relationships between visualization and geometry reasoning; and connections between visualization and spatial reasoning.

WG2 (Figure 7) analysed characteristics of geometric thinking across different educational levels, students' capacities – from those with learning difficulties to mathematically gifted students – and curricula – influenced by local political and cultural characteristics. This group also analysed the knowledge and competencies necessary to teach and learn geometry and the design of learning environments, from several positions like interdisciplinarity, problem solving/posing, contexts influencing the learning of geometry, and different types of tools.

WG3 discussed issues related to pre- and in-service teacher professional training, addressing issues like teachers' content knowledge of geometry, strategies for teacher training, and types of tasks suitable to be posed to teachers to promote their content and didactical knowledge for teaching geometry.



Figure 7. Working Group 2 participants.

The participants in WG4 discussed burgeoning issues related to the use of ICT (Information and Communication Technology) in teaching and learning geometry, including: resources available that are useful in the classrooms; characteristics of DGS like dynamism, embodiment, feedback; design of tasks to promote students' understanding and learning of specific geometric topics; ways of supporting reasoning with technology, like promoting deductive reasoning and proofs, supporting visualization in problem solving, or use of analogy to promote understanding of new constructs; and also constraints and limitations due to the lack of ICT resources in classrooms.

Finally, WG5 (see Figure 8) worked on issues related to the use of manipulatives and visual tools in teaching and learning geometry, in a kind of juxtaposition to the discussions in WG4. They dealt with the concept of resources (physical, digital, tasks, etc.) as a starting point to analyse issues like the specificities of resources for geometry classes, resources for teachers, and kinds of manipulation from kindergarten to university.



Figure 8. Working Group 5 participants.

Next steps

The IPC is now working on the design of the Study Volume, which will be part of the ICMI Study series. The chapters will cover, in more depth, the content presented, and questions raised in the conference, highlighting a diversity of issues and reflections related to the teaching and learning of geometry at the different educational levels, together with methodological and practical approaches to deal with them. The volume will also provide guidelines for mathematics education research and innovation for the coming decades.

Final reflections

This was a thought-provoking, energizing, and meaningful working group conference. We were able to assemble a cohort of geometry educators from across the world – from diverse backgrounds and experiences – to work harmoniously and positively for a concentrated period. We appreciate that such opportunities, where so many world-leading discipline specialists gather for a common goal, seldom occur in one's professional life. The distinctness of this Study will be the way in which this cohort has been able to capture the nexus between geometry education theory and practice, across the lifespan (from the early years of learning to advanced multidisciplinary contributions of geometry).

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Entity linking for zbMATH Open

Marcel Fuhrmann, Philipp Scharpf and Moritz Schubotz

1 Introduction

As scientific knowledge expands, specialization has become increasingly prominent, with researchers focusing on narrower fields to deepen understanding and foster innovation. While specialization enables experts to tackle complex problems precisely, it also challenges interdisciplinary collaboration, as breakthroughs often require integrating insights across diverse fields.

Even in mathematics alone, no one can be an expert in all specialized fields, each with its own ecosystem of terminologies. Every once in a while, a reader stumbles across a term or phrase in a document or abstract unfamiliar to them, or at least the specific details are not clearly stated.

zbMATH Open is an information service for mathematicians in research and teaching, with networked information on mathematical topics, authors, publications, references, and software. It provides detailed information on mathematical publications dating back to 1868. It provides access to around five million bibliographic entries with reviews or abstracts from more than five thousand journals and book series and some 200,000 books.

Therefore, providing context to all mathematical terms and phrases, proofs, lemmas, theorems, and concepts in these texts cannot be accomplished by augmenting every abstract on zbMATH Open manually. Thus, we decided on an unsupervised machine-learning algorithm.

To further support the disambiguation of mathematical terms, we are currently developing a new service to augment any abstract of a document published on zbMATH Open so that it includes links to the context of the corresponding mathematical entity.

2 What is the goal?

Generally speaking, entity linking means associating the main entity types, i.e., persons, organizations, locations, dates, and times, to a representation in a knowledge base or knowledge graph [1].

Entity linking can be challenging due to entity name variations or ambiguity. Several types exist, e.g., a single concept that can be called by multiple names (synonymous) or a single name that

can mean multiple concepts (polysemy) [2]. This requires entity linking to utilize the entity's context or additional information to disambiguate.

Mathematical entity linking can be employed to ground mathematical entities in documents semantically. Wikidata¹ can help to achieve this by storing and linking both the concept name (with a persistent identifier called QID) and the corresponding Wikipedia page (or, if applicable, a mathematical formula) [5].

QIDs are unique and persistent identifiers used in Wikidata with corresponding concept item pages (often also linked to a set of Wikipedia articles in different languages) to refer to a specific, unique concept.

An example of how an unsupervised machine learning algorithm would augment the abstract of a document shown on zbMATH Open with mathematical entity linking can be viewed in Figure 1. On the left side, the zbMATH document page is displayed, and links to the corresponding Wikipedia article are displayed on the right side.

3 Applied methodology for implementing mathematical entity linking

In this section, we explain how the algorithm of our machine-learning algorithm works.

Following the ideas of [3, 4], two steps are required for formula concept discovery and recognition: First, we need to define a list of entities, and second, we need to identify mentions of those entities. For the list of entities, we used mathematical concepts from Wikidata. We collected QIDs related to topics in mathematics from the Wikidata API.² We created our own database consisting of a list of tuples, each containing a mathematical term and its corresponding Wikidata page.

However, the content of the items stored in Wikidata may change over time. Entries get corrected, and others may be removed to minimize redundancies or other reasons. Thus, it is

¹ <https://www.wikidata.org>

² <https://www.wikidata.org/w/api.php>

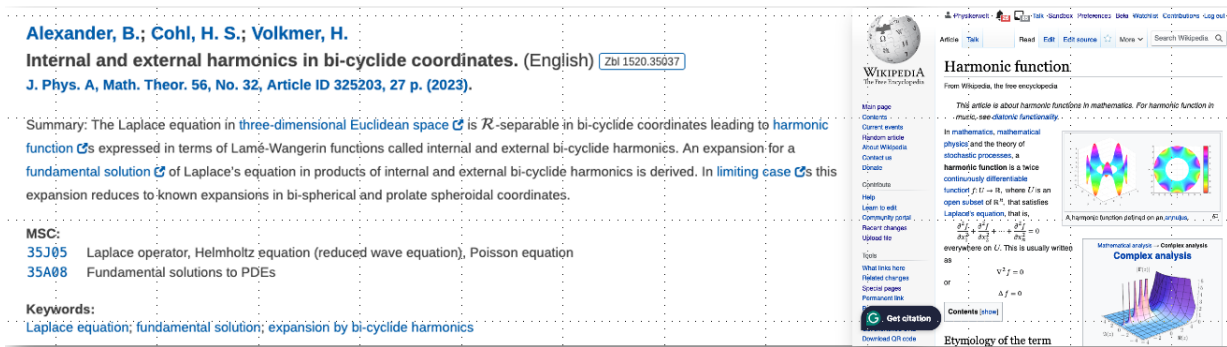


Figure 1. Example preview of the new entity linking function in zbmATH Open

important to update this augmentation database regularly to ensure its sustainability. For this purpose, an automated update system was created.

Another challenge was the creation of a robust algorithm for concept recognition that searches any given text for phrases that match the respective terms and replaces the simple text with underlying links to the corresponding Wikidata webpage or Wikipedia link if existing. This algorithm first removes any punctuation marks and removes latex code if existing. Then, natural language processing searches the text in question and identifies mathematical expressions in their variations and ambiguities, then predicts the most probable candidate based on the context.

This is going to be applied to any abstracts that are available to zbmATH Open, provided their respective licence agreements allow for it. The new service is expected to be available in Q4 2024.

4 Outlook and conclusion

To expand the usage of this technology, we plan on introducing a similar approach for mathematical formulas appearing in abstracts that correspond to a Wikidata entry.

This project is quite more complex, as each formula needs to be analysed and extracted from the text using open-source code libraries for symbolic mathematics to classify the identifiers and operators in their mathematical context correctly.

Currently, we are investigating different approaches. Each of those methods consists of analysing variable symbols and operators. They differ in how the extracted symbols are interpreted:

- we construct a knowledge graph query to find similar formulas and retrieve the best fitting candidate,
- extracts and categorizes individual parts from a formula string into identifiers and operators,
- prompting the formula retrieval via an open source large language model.

In addition, the concepts are currently used for display only on zbmATH Open. In the future, we plan to use the concepts for

navigation and search. For example, they can be used to classify articles that are more fine-grained than currently possible with the mathematical subject classification. In addition, they could be used to filter search results by concept.

To summarize, the new natural language processing will be useful for the automated augmentation of the abstract texts stored within the zbmATH Open database. It will extend the readability and findability to all publications, and therefore, open access for zbmATH Open data will continue to expand.

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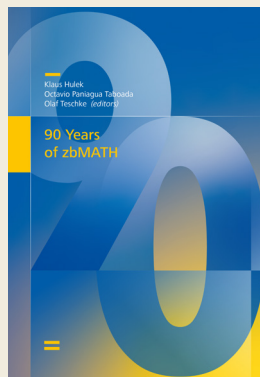
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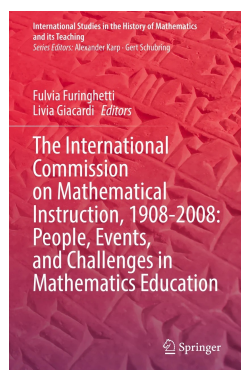
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Book review

The International Commission on Mathematical Instruction, 1908–2008: People, Events, and Challenges in Mathematics Education by Fulvia Furinghetti and Livia Giacardi (editors)

Reviewed by Carlo Toffalori

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Tension underlies every scientific field, and it is generated by conflicts between forces that bring about progress. The first conflict – one that concerns us specifically – is between the research of unexplored horizons, new ideas, and the transmission of our findings, which is the heritage we have acquired through our investigation. In other words, there is a contrast between a first creative phase and the one that follows, dedicated to

translating that discovery into teachings.

Another conflict eliciting tension – intrinsically tied to the previous one, if not even partly overlapping it – can be found between the history of that scientific field, its creation of theories and methods, and its further development. The latter cannot be a simple addition to the premises set by tradition, but at the same time it cannot completely disregard them.

All that applies to mathematics as well. Indeed, its greater abstractness accentuates this duality, making it subtler and adding its own peculiarities. As stated in 1990 by Jean-Pierre Kahane, a scholar of harmonic analysis and the president of both the Société mathématique de France and the ICMI: “*In no other science is the distance between the taught and the new so large. In no other science has teaching and learning such social importance.*” On the same subject, he also added: “*In no other science is there such an old tradition of scientists committed to educational questions.*”

The book under review perfectly depicts this duality, in its retracing a hundred years of ICMI (1908–2008). Leaving acronyms aside, the *International Commission on Mathematical Instruction* is an organization whose purpose is already stated in its name. However, Hyman Bass – American algebraist, author of the famous book *Algebraic K-Theory* – and the ICMI’s general secretary, Bernard Hodgson, have also defined it as “*a forum for promoting reflection, collaboration, exchange and dissemination of ideas, and information on all aspects of the theory and practice of contemporary mathematics education, as seen from an international perspective.*” A goal that is easy to share and support in theory, but difficult to put into practice, as it does not deal solely with the evolution of mathematics, it also deals with that of society and single individuals. Indeed, as the subtitle of this work rightly reminds us, the first century of ICMI was made up of people, events, and challenges. The book gives us an extensive and in-depth account of them.

It is first and foremost a *tale of events*, often leading to successful outcomes, when not to failures and crises. The birth of the ICMI, which coincides with the fourth International Congress of Mathematicians (ICM) held in Rome, can be considered the result, if not the culmination, of a movement that had been growing for years, bringing together mathematicians such as Felix Klein and educators the likes of David E. Smith.

Let us not forget that 1908 was also the year Klein published his classic *Elementarmathematik vom höheren Standpunkte aus* (*Elementary Mathematics from an Advanced Standpoint*). And Klein himself was the first president of an international committee founded in Rome which dealt with the teaching of mathematics. However, the member nations were limited to a few European countries – 17 in total – and the United States, although another 15, mostly non-European countries, were keen to join the organization.

However, it did not take long for the newly founded committee to experience local bias, misunderstandings, doubts, and disagreements between mathematicians who even came from the same country, so much so that Klein himself found a few opponents in Germany.

The committee had a bilingual name: *Commission Internationale de l'Enseignement Mathématique* (CIEM) in French and *Internationale Mathematische Unterrichtskommission* (IMUK) in German. From a political standpoint, it tried to assure adequate representation to all its various member states by also constituting national subcommittees, while from a scientific point of view, it discussed the teaching of mathematics in general – its organization, programs, methods – with a focus on applied mathematics too. Its mandate was renewed at the ICM every four years. The organization carried out its activities until 1914, when it dissolved due to the outbreak of World War I.

The end of the war did not suffice to revive the committee, as it needed to find a new balance between victorious nations, such as France, and defeated ones, like Germany. It was restored in 1928, at the ICM held in Bologna, but it had yet to relive its past glory, due to the political upheavals of those decades.

It was only after World War II that ICMI was reborn under its current name, as a permanent subcommittee of the *International Mathematical Union* (IMU). Its mandates were no longer determined by the ICM. As a result, the ICMI grew and gained momentum, to the point that, throughout the years, it prompted important initiatives such as a journal (*Educational Studies in Mathematics*), a collection of studies (*ICMI Studies* series), an international congress of its own – the *International Congress on Mathematical Education* (ICME) – and, in recent times, a website.¹ In 2008 the member states were over eighty, while today they are more than ninety worldwide.

However, the organization's new framework generated different scientific, political, and social problems. Indeed, the newly found ICMI needed to:

- define its relationship with the IMU itself, and acquire a precise and autonomous identity;
- open a dialogue with the other movements and organizations outside the IMU, but equally invested in the study and promotion of mathematical teaching;
- establish a similar connection with UNESCO and with global institutions that further the progress of culture;
- renew mathematics curricula and the ways of implementing them (fluctuating, during the twentieth century, between Bourbakist rigour and reliance on intuition);
- assess the very particular role of mathematics, a science in of itself, and at the same time the foundation of all the others;
- study the advent of information science and technology and the educational use of its tools;
- extend its focus to other nations and civilizations, in particular developing countries of the third and fourth world, or those, such as India and China, bearing mathematical heritages that are partly different from the traditional, Euclidean one, inspired by the classical Greek culture;

- understand and reappraise these “alternative” approaches, but also incorporate them to the most widespread methods in Western culture;
- create and support the teaching of mathematics in less developed nations, without concentrating on syllabi alone, but also on schools and places of learning.

The first century of ICMI is also marked by *challenges*.

The first, and perhaps the most relevant, arose within the mathematical world itself, and concerned the role of teaching within it. From a political standpoint, this issue was tied to the problem mentioned above regarding the identity of the ICMI and the position it held in various institutions, primarily the IMU.

It was also necessary to pinpoint the very essence of the mathematics that needed to be taught and passed down. In a period as frenetic as the twentieth century, this implied the search for the right balance between the so-called pure and applied mathematics.

Growing attention was drawn to the “vertical” development of mathematics education – from primary school to university – which was, and still is, intertwined with pedagogical issues.

Another vital matter concerned the progressive awareness of emerging nations, as well as the acknowledgement of the mathematical heritage of every single civilization and its people.

Similarly, there was a rise in awareness concerning gender issues and female representation.

ICMI dealt with these challenges by finding a common path, a synthesis of “tradition” and contributions from different cultures. To face them, it created study groups, meetings and conferences all over the world. At the same time, the didactics and history of mathematics gained further autonomy as academic disciplines.

The three chapters featured in the first part of the book deal with the events and challenges that ICMI had to face between 1908 and 2008. The authors are, from the first to the third chapter:

- Gert Schubring, who wrote on the foundation and development of CIEM/IMUK up to World War I, on the committee's dissolution in 1920, and on the attempted reconstitution in 1928;
- Fulvia Furinghetti and Livia Giacardi, who focused on the period starting from 1952, which saw the rebirth of ICMI, and on the progressive establishment of mathematics teaching as an autonomous field of research;
- Marta Menghini, who explored the “renaissance” of ICMI in the late sixties and its subsequent developments up to 2008.

Adding to these three chapters, the second part of the book, edited by Fulvia Furinghetti and Livia Giacardi, provides detailed and documented information on many of the aspects of the commission's century-old history.

However, the history of the ICMI is also undoubtedly made of – and by – *people*, starting from personalities like Felix Klein, Jean-Pierre Kahane, Hyman Bass to many others, often renowned

¹ <https://www.mathunion.org/icmi>

mathematicians not specialized in teaching, or teaching alone, but sensitive to the transmission of their science and, as such, called to give their contribution to the commission, or even lead it. Here are a few significant examples extracted from the list of ICMI presidents (1908–2008):

- Marshall Stone, an American mathematician who gained notoriety for the duality theory between Boolean algebras and Boolean spaces, the so-called Stone–Čech compactification, and was president of the IMU even before leading the ICMI;
- Hans Freudenthal, a Dutch expert in algebraic topology, and an energetic guide and leader of the development of ICMI in the 1960s;
- André Lichnerowicz, a French scholar of differential geometry and mathematical physics, president of the Société mathématique de France, as well as of the ICMI;
- Shokichi Iyanaga, a Japanese researcher in topology and functional analysis.

In more recent times, there have been female leaders at the head of the ICMI, such as the president in office in 2008, Michèle Artigue, a French mathematician specialized in teaching. Among her successors, in the period following the one under scrutiny, it is worth mentioning the South African Jill Adler. Her nationality underlines the increasingly international scope of the ICMI. It is not surprising, then, that the current president, Frederick Koon-Shing Leung, is Chinese from Hong Kong, and the current executive committee includes Mercy Kazima, a representative from Malawi.

These names are no longer those of cutting-edge scholars who have turned their attention from various fields of mathematical research to the problems of teaching. On the contrary, they belong to those who have specialized in the teaching of mathematics itself, thus proving that a change has occurred over the last decades. Indeed, learning and education have progressively become academic disciplines, with their own identities and specificities.

But now let us go back to the people who have furthered the development of the ICMI in the century around which this book revolves, that is between 1908 and 2008. The third and final part of this study provides a very accurate series of short portraits of these protagonists. There are overall 54 biographies of members of the Central/Executive Committee of the ICMI who have succeeded one another in a hundred years, as well as those of ten eminent figures.

An extensive section of the first part is devoted to introducing these personalities and furthering our knowledge about them. Indeed, we hear their voices through a collection of unpublished letters concerning the ICMI and its problems. This extraordinary collection of documents, edited by Livia Giacardi, is the result of her extensive and thorough research. It is worth noting that at the time there were no e-mails, no messaging apps, no computers equipped with an almost unlimited memory and data storage. On the contrary, old paper mail travelled between countries that were

sometimes at war and across borders often impenetrable, sealed in distinctive envelopes such as that of the air mail with its typical red and blue edges.

The documents thus recovered show exchanges of opinions, scientific and diplomatic discussions on the life of the ICMI, sometimes expressed in different languages. They involve many scholars including IMU presidents – prestigious figures such as Rolf Nevanlinna, Georges de Rham, Henri Cartan. This work also offers readers reproductions of the original letters.

The Italian contribution shines through all this documentation – a source of national pride, if we may. Not only because the two dates that mark the beginning and the end of the century under scrutiny are linked to Rome, home of the ICM congress that gave birth to the ICMI in 1908 and the same city that hosted its centennial celebration in 2008; nor because the 1928 ICMI congress that attempted, albeit unsuccessfully, to revive the commission was held in Bologna. The Italian contribution was made invaluable by its various influential Italian figures who enriched the ICMI and furthered its progress. From Guido Castelnuovo – one of the representatives of the original CIEM/IMUK, together with Enriques and Scorza, and later vice-president of the commission after its renewal in 1928 – to his daughter Emma, who was a guest speaker at the first ICMI held in Lyon in 1969. And let us not forget that in 2007 Mariolina Bartolini Bussi became a member of the Executive Committee and in 2013 Ferdinando Arzarello became president.

For the sake of brevity, we have limited ourselves to just a few names, but this book offers an extended list. Furthermore, one of the ICMI awards, established in 2013, was named after Emma Castelnuovo herself.

In conclusion, the book edited by Fulvia Furinghetti and Livia Giacardi paints a very rich and detailed picture of the first century of ICMI. It is the result of the impressive and formidable work of its two editors, and of other equally prestigious authors who collaborated with them on the single chapters of the book. It employs an historical approach, aimed at gathering news, facts, chronicles, documents, and biographies. As was said at the beginning, we strongly believe that the accurate account of the past is the main way to inspire new reflections on the impulses and ideas that those events have generated, to stimulate discussions and in-depth analysis of the present, and ultimately suggest future choices.

What this book wants to convey – structurally and thematically – is the fundamental role of history and teaching in preserving and enlivening mathematics and every other science, as well as the importance of a dialogue between different cultures. Today, history and teaching have become highly specialized, as is natural in any discipline that grows, refines, and evolves. Thus, the works of influential researchers who were also passionate about education, like Jean-Pierre Kahane, might seem outdated. It is difficult for nowadays scientists to concentrate on multiple fields, as history

and teaching require a full-time commitment from those who cultivate them.

However, the close cooperation between “mathematicians,” “historians” and “experts in mathematical education,” is one of the main assets of ICMI, as it has transpired throughout its long history, and we wish that it may carry on in new, different forms.

We hope that this work will make its way into the libraries of all departments of mathematics and that it will be frequently, extensively, and fruitfully consulted. After all, the aim of the ICMI, as finely expressed by the book and briefly summarized in these few lines is, and should be, shared by all mathematicians.

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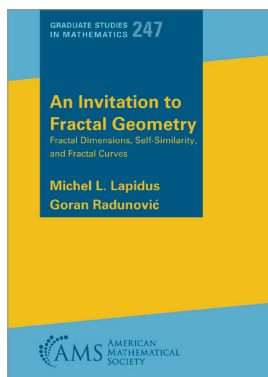
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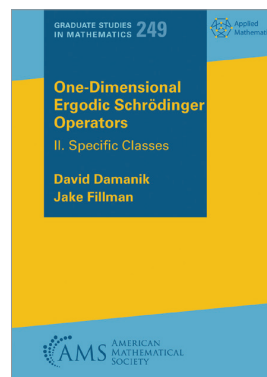
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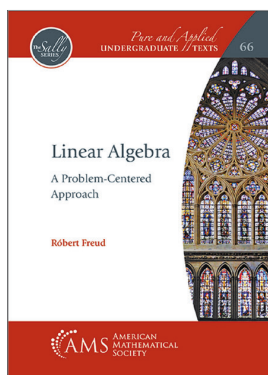
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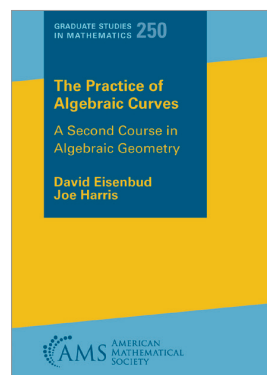
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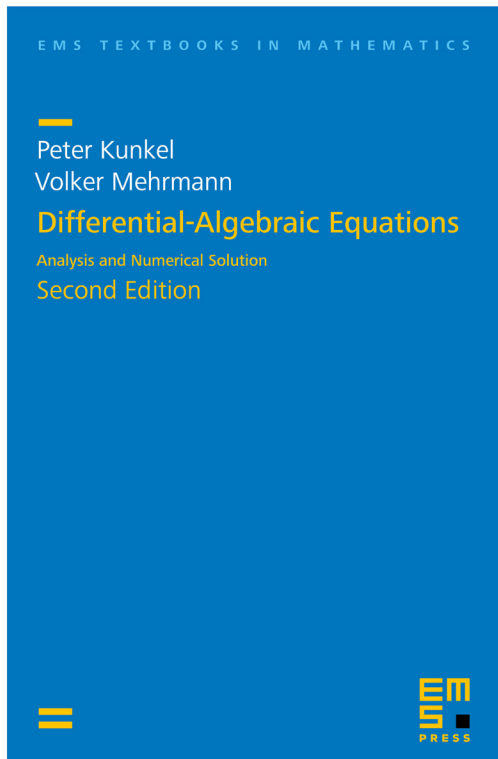
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