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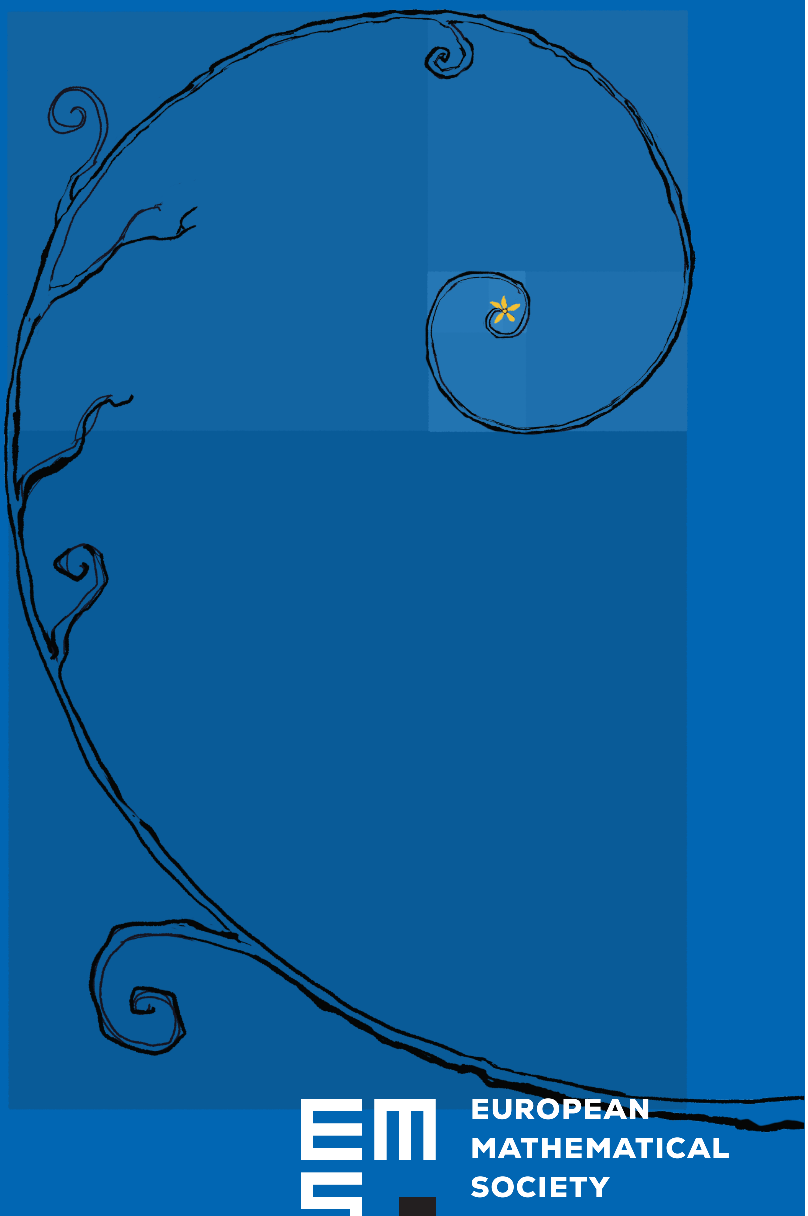

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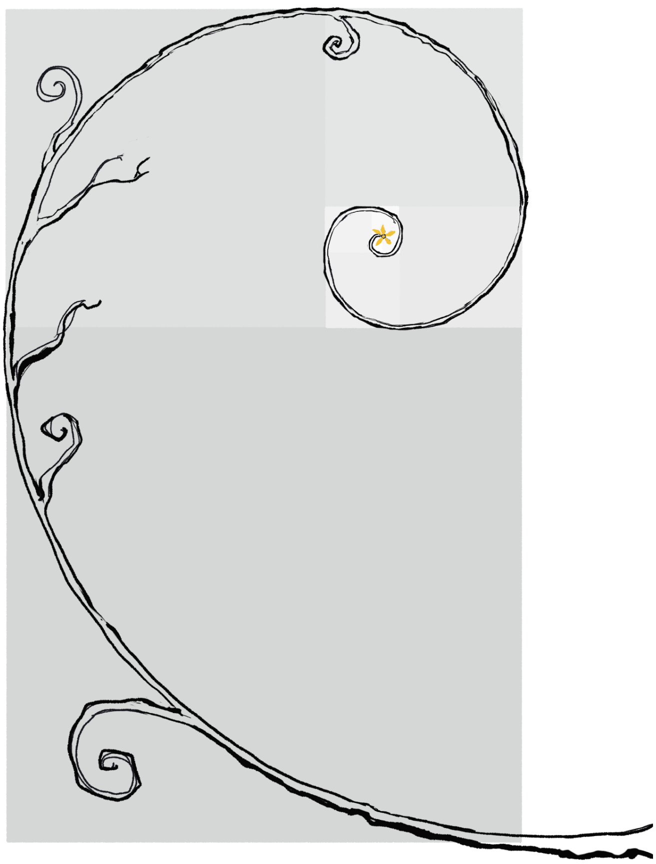

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The cover illustration by António B. Araújo is a composition on the 2026 theme of the International Day of Mathematics: 'Mathematics and Hope.'

Photo by Jim Høyer, University of Copenhagen.

The big European Mathematical Society event of the year is the EMS council meeting to be held in Lisbon, Portugal on June 27–28, 2026. This is the big biennial meeting where our members, both society and individual members, will send delegates to help to set the direction of the EMS in the next two years. I am looking forward to this chance to interact with our members, and I hope many delegates will be able to attend. At every council we elect new representatives for four-year terms on the EMS Executive Committee. This coming council is particularly important as we will be electing a new president. My term as president will be coming to an end with this calendar year, and I am looking forward to meeting the new president elected by the council. I will be working closely with the new president-elect after the council to secure a continued successful running of the EMS and a successful transition of the presidency.

The really big mathematical event of the year is the International Congress of Mathematicians (ICM) to be held in Philadelphia in July. This will be an occasion to meet mathematicians from across the world and to celebrate the great achievements of mathematics and mathematicians. As is well known, the last ICM to be held in Saint Petersburg was canceled because of the Russian war on Ukraine. Unfortunately, this year concerns have been raised once again about the political situation and the safety of participants traveling to the United States. While we fully acknowledge the validity of these concerns the EMS is supporting the IMU and the local organizers of the ICM in their continued efforts to create a safe, inclusive, and interesting congress. The EMS will have a stand at the ICM shared with EMS Press and zbMATH Open. My personal opinion is that the European Mathematical Society should be active in science politics and in the goal of achieving academic freedom worldwide, not in world politics more generally. We should work for worldwide collaborations among mathematicians. We should, when possible, break barriers and not create them. For this reason, I have decided to participate in the ICM and to represent the EMS at the IMU General Assembly to be held in New York just before the ICM. I am looking forward to going. I would be sorry to miss this or any other opportunity to celebrate mathematics.

Nevertheless, I am writing this with a bit of apprehension. The current state of the world is unfortunately very frightening and unstable. When I first became president the former president, Volker Mehrmann, told me that my term could only be easier than his because he had to deal with both COVID and the war in Ukraine. While my presidency has not been affected by something as directly life changing as a pandemic the political situation is probably worse now and even more complicated and unstable. As I am writing this message the war launched by the U.S. and Israel on Iran just started a few days ago. The situation may look very different by the time you read this and even more so as we approach summer. I sincerely hope it will not affect the congress in Philadelphia too much and that we will have a very successful gathering.

Let me end this message on a positive note. I hope many of you have enjoyed the EMS Lecture Series on Mathematics Education[1] moderated by Tom Crawford. If you could not follow the first three by Anna Stokke, Nuno Crato & Tim Surma, and Hung-Hsi Wu you can still watch them on our YouTube channel.[2] I encourage you to do so. The next lecture is by Sarah Powell on April 10.

Finally, I am looking forward to meeting many of you as delegates or guests at the council in Lisbon in June.

Jan Philip Solovej
President of the EMS

[1] https://euromathsoc.org/news/ems-lecture-series-on-mathematics-education-190
[2] https://www.youtube.com/@EuropeanMathematicalSociety/streams

Dear readers of the EMS Magazine,

In this issue, you will find many interesting contributions in the usual columns on discussions, societies, education, and book reviews, as well as another article by one of the 2024 EMS Prize winners, this time written by Tom Hutchcroft.

Moreover, with the current issue a new editor is joining the editorial board: Matteo Petrera, who will take care of the Zentralblatt column. A short biographical note about him can be found at the end of the issue. I would like to thank the former editor, Octavio Paniagua Taboada, for his superb work and dedication during his eight years of service on the Magazine's editorial board. I am also grateful to Matteo Petrera for accepting to devote part of his time to the Magazine.

Finally, it is very important for me to emphasize that with this issue the Magazine joins the celebration of the International Day of Mathematics,[3] whose theme for 2026 is: *Mathematics and Hope*. In this regard I invite you to take a look at the articles: "The Math Solidarity Platform" by Emine Yıldırım and "On the European Solidarity Committee of the EMS" by Alice Fialowski, as well as at the Magazine's cover illustration by A. B. Araújo.

Mathematics, with its universal language can be understood as a bridge: a structure that connects what seems distant, fragmented, or even incompatible. It connects cultures, disciplines, generations, and ideas. In doing so, it becomes a source of hope. Each generation builds upon the discoveries of those before it. This continuity offers hope because it situates us within a long human story of inquiry and progress. Knowledge is not lost; it is transmitted, refined, expanded. We are not starting from nothing, we are building forward. Mathematical collaboration creates communities of shared reasoning. It connects minds across borders, ideas across centuries, abstraction and reality, challenge and solution.

Hope arises wherever connection replaces isolation.

Donatella Donatelli
Editor-in-chief

[3] https://www.idm314.org

Are there non-trivial theorems about all finitely generated groups?

Tom Hutchcroft

We discuss several interpretations and potential counterexamples to the meta-mathematical question in the title of the paper, focussing on geometric group theory and probability on groups.

> *I shall not today attempt further to define the kinds of material I understand to be embraced within that shorthand description, and perhaps I could never succeed in intelligibly doing so. But I know it when I see it, and the motion picture involved in this case is not that.*
>
> Jacobellis v. Ohio, 378 U.S., p. 197 (Stewart, J., concurring)

1 Thesis

The goal of this article is to give a light-hearted and, hopefully, thought-provoking critique of the following famous aphorism:

"Theorem". *If a property is satisfied by all finitely generated groups, then it must be satisfied for trivial reasons.*

This "theorem" is often attributed[1] to Gromov, the father of the field known as *geometric group theory*, who may have intended it as a joke. In geometric group theory, one seeks to understand groups as *geometric objects*, starting with the observation that if G is a group with finite generating set S (meaning that G does not have any strict subgroups containing S), then G acts by isometries on its *Cayley graph* $\mathrm{Cay}(G,S)$, the graph with vertex set G and where two vertices x and y are connected by an edge if $x = ys$ or $x = ys^{-1}$ for some $s \in S$. (Technically this defines the *right* Cayley graph, on which G acts by *left* multiplication.) The resulting graph metric on G is called a *word metric*. Although different Cayley graphs of G need not be isomorphic, they are always *bi-Lipschitz equivalent* in the sense that for any two such metrics there exist positive constants c and C such that $c d_2(x,y) \leq d_1(x,y) \leq C d_2(x,y)$. (This is analogous to how all norms on $\mathbb{R}^d$ are equivalent.) As such, one can always consider a finitely generated group as a "geometric" object up to bi-Lipschitz equivalence. In fact, one typically works with a larger family of equivalences known as *quasi-isometries* (in which one also allows distances to be distorted by *additive* constants), which has the advantage that finitely generated groups are quasi-isometric to their finite-index subgroups and $\mathbb{Z}^d$ is quasi-isometric to $\mathbb{R}^d$. Flipping this perspective on its head (and slightly extending the scope of geometric group theory from finitely generated groups to compactly generated, locally compact groups), one can alternatively think of geometric group theory as a *language* for the study of arbitrary "homogeneous" geometric spaces, a subject that is of obvious interest to geometers and which one need not be an algebraist to appreciate or contribute to. (It is obligatory here to mention the bridge between these two perspectives provided by the Švarc–Milnor lemma, a.k.a. the fundamental observation of geometric group theory, which states that if a group acts properly discontinuously and cocompactly on a proper length space (X,d), then G is finitely generated and its Cayley graphs are quasi-isometric to X.)

Is the "theorem" not obviously false?

Leaving aside the question of how to define "trivial," the first reaction of a certain kind of mathematician upon reading the above "theorem" will be to point out various obvious "counterexamples," perhaps starting with a "property of a group" defined through some irrelevant but true and difficult-to-prove assertion; at least one of "The Riemann hypothesis is true" or "The Riemann hypothesis is false" will do. A more charitably minded mathematician might contend that one is obviously not supposed to consider "properties" of a group defined via such irrelevancies (that in this example do not even reference the group), but instead point out "counterexamples" that involve either conditional statements ("if P then Q")

[1] Ghys and de la Harpe [22] write *"On attribue à M. Gromov l'affirmation suivant laquelle un théorème valable pour tous les groupes ne peut etre que trivial ou sans importance,"* which is of course a slightly different statement. Interpreters of this quote should bear in mind that geometric group theorists will often leave the words "finitely generated" implicit and talk simply of "groups," especially in oral communication with colleagues.

or dichotomies ("P or Q"), an important example being Gromov's theorem:

If G has polynomial volume growth, then it is virtually nilpotent.

which is certainly a non-trivial statement concerning all finitely generated groups. (Here a group is said to have polynomial volume growth if the radius-r balls of one – and hence all – of its Cayley graphs have cardinality at most $(1 + r)^C$ for some constant C, and is said to be virtually nilpotent if it has a nilpotent subgroup of finite index.) This is of course just one particularly prominent example among countless others of a theorem stating that one property of a finitely generated group implies another, and since many of these theorems are clearly non-trivial (and *pas sans importance*), we should be careful to exclude such conditional statements from the "theorem." There are also important, non-trivial theorems showing that some complicated data associated to a group is equivalent to some other complicated data,[2] or similarly that one quantity associated to a group is always bounded by another quantity,[3] which do not always "feel like" "counterexamples" to the "theorem" even though they do arguably pertain to a "property" of all finitely generated groups. (What is a property?) Similarly, there are e.g., several important theorems in percolation theory that hold on general transitive graphs (e.g., the sharpness of the phase transition and the mean-field lower bound on the critical volume tail [1]), but which "feel like" properties of percolation rather than properties of the group and which I do not personally find compelling as "counterexamples" to the "theorem." (There are other theorems from percolation that do "feel like counterexamples" to me, which I discuss below.)

It is not the intention of this article to develop a logical framework specifying precise constraints to be placed on what "properties" the "theorem" applies to (if such a thing is even possible[4]). Instead, I want to ask:

- What interesting examples are there, if any, of "counterexamples" to the "theorem," that is, non-trivial properties that hold for all finitely generated groups?
- What is the value of the "theorem" as *advice* – that is, as a rule of thumb cautioning mathematicians not to waste their time working at an overly great level of generality in a way that is unlikely to lead to interesting results?

For both questions, it will be interesting to examine not just theorems that apply to *all* finitely generated groups, but also those applying to very large classes of such groups, such as all *infinite* finitely generated groups. For the first question, I hope to convince the reader that there are unambiguous counterexamples to the theorem that are of genuine mathematical interest and do not involve any "cheating" via irrelevant or conditional statements as above. For the second, the answer is more complicated: while the implicit advice of the "theorem" does usefully guard against certain naïve mathematical endeavours, it may also lead to an excessively pessimistic outlook that I hope to offer some respite from.

Dreams or nightmares?

I was motivated to write this article partly by an interesting cultural difference[5] I have observed between probabilists and group theorists regarding problems at the intersection of their interests: Many of the problems probabilists find most interesting about "probability on groups" concern very large classes of (finitely generated) groups, such as all infinite groups, all non-amenable groups, all groups of at least quadratic growth, etc. Two prominent open problems of this flavour are the Benjamini–Schramm conjecture [4], which asserts that percolation on any non-amenable Cayley graph has a non-trivial phase in which there are infinitely many infinite clusters, and the Liouville stability problem, which asks whether the Liouville property (all bounded harmonic functions are constant) is independent of the choice of generating set for all finitely generated groups; experts strongly believe the Benjamini–Schramm conjecture to be true (see [28, 29, 53] for partial results), while there is less consensus on the Liouville stability problem.

Two further important open problems coming internally from within group theory that involve putative properties of all finitely generated groups include the question whether all finitely generated groups are *sofic* [33] and Gaboriau's problems of fixed price and cost vs. Betti number [21]. For the first of these problems, it is widely believed that non-sofic groups exist, and preprints have recently appeared arguing that the closely related Connes embedding conjecture and Aldous–Lyons conjecture are false [6, 7, 32]. (Interestingly, the methods of these papers are non-constructive and do not yield explicit counterexamples.) There is less consensus regarding Gaboriau's problems.

[2] A favourite example of mine from GGT being Olshanskii's characterisation of metrics arising on G from word metrics on larger groups H via embeddings $G \hookrightarrow H$ [49, 50], up to bi-Lipschitz equivalence, as exactly the left-invariant metrics of at most exponential volume growth.

[3] This includes the various inequalities relating the volume growth, isoperimetry, heat-kernel decay, entropy, and rate of escape of the random walk as surveyed in [63]. Many of these relationships hold for arbitrary bounded-degree graphs and are not really specific to groups.

[4] Philosophers have attempted to solve similar problems using so-called *relevance logics* (a.k.a. *relevant logics*) [45].

[5] This difference appears to be orthogonal to other well-noted cultural differences between "analysts" and "algebraists" pertaining to, say, the two cultures of mathematics [23] and the manner in which they eat corn on the cob [60].

Regarding problems like this, I have often found probabilists more likely to believe in positive solutions[6] while among group theorists there is a strong tendency, informed in part by the above "theorem," to expect counterexamples to exist to any sufficiently general conjecture that has resisted serious attempts at proof for any length of time. This intuition is hard-won, and stems in part from important negative results obtained via the construction of pathological examples including Grigorchuk's groups of intermediate volume growth [24] and Gromov–Osajda monsters [52]; see also, e.g., [9, 19, 50, 51] for further examples. On the other hand, while there is undoubtedly a degree of naïvety behind the probabilist's attitude to these questions, it has also led to a number of significant results over the years including, e.g., the solutions to the Choquet–Deny problem [20], the $p_c < 1$ problem [12, 17], and the many important results proven about percolation on non-amenable groups by subsets of Benjamini, Lyons, Peres, and Schramm in the late 1990s [3, 40, 44]. My opinion is that optimism and pessimism are both valuable and that mathematicians should endeavour not to neglect either point of view.

2 Antithesis

We now discuss various possible "counterexamples" to the "theorem," i.e., interesting, non-conditional statements applying to all finitely generated groups whose only known proofs are non-trivial. The examples we give are highly biased towards my own research interests at the intersection of geometric group theory and probability.

Cayley graphs with few automorphisms

Let us begin with the following theorem of Leemann and de la Salle [38, 39], which has the advantage that its statement is very clean and easy to appreciate. Every finitely generated group acts by automorphisms on each of its Cayley graphs, but these Cayley graphs may have many further automorphisms. For example, the four-regular tree (which is a Cayley graph of the free group on two generators) has uncountably many automorphisms.

Theorem 2.1. *Every finitely generated group has a Cayley graph with countable automorphism group.*

The proof of this theorem proceeds by case analysis according to whether or not the group is virtually Abelian (i.e., has an Abelian subgroup of finite index): The theorem was proven for virtually Abelian groups (or, more generally, all groups with an element of infinite order) via an elementary argument in the earlier work [56], while for groups that are *not* virtually Abelian the authors use a probabilistic construction relying on a theorem of Tointon characterising virtually Abelian groups in terms of commuting probabilities of random walks [61]. In the latter case the authors show the stronger result that the group has a Cayley graph with no automorphisms other than the group itself; this is not true for infinite[7] Abelian groups, which always have an additional $x \mapsto -x$ symmetry.

Diffusive lower bounds on the random walk

Our next proposed "counterexample" has the caveat that it applies only to *infinite* finitely generated groups. Recall that the simple random walk on a Cayley graph (or any other graph) is a random process which, at each time step, jumps to a uniform random neighbour of its current position independently of everything it has done previously. One of the most basic questions one can ask about random walk is its *rate of escape*, i.e., the large-n asymptotics of the typical distance travelled by the random walk in n steps. This typical distance is of order $\sqrt{n}$ on infinite Abelian groups as a consequence of the central limit theorem, and is of order n on the k-regular tree for $k \geq 3$. (It is a major open problem whether the rate of escape can depend on the choice of generating set for general finitely generated groups.)

It is now known that the rate of escape for the random walk on an infinite, finitely generated group can be essentially any function between $\sqrt{n}$ and n [9]. On the other hand, it is *not* possible for the rate of escape to be slower than $\sqrt{n}$, in contrast to random walk on *fractals* (more accurately "pre-fractals," like the graphical Sierpiński gasket) that often have rate-of-escape of order n^β for $\beta < 1/2$ [36]. The following theorem appeared in the work of Lee and Peres [37] (which also establishes stronger results of the same form), but existed previously as folklore and is sometimes attributed to Virág. Here we write $(X_n)_{n \geq 0}$ for the random walk started at the identity and write $|X_n|$ for the distance between X_n and the identity.

Theorem 2.2. *Let G be a finitely generated group with symmetric generating set S, and consider the simple random walk on the Cayley graph* $\mathrm{Cay}(G,S)$. *If G is infinite, then there exists a constant c such that*

$$\mathbb{E}|X_n|^2 \geq cn$$

for every $n \geq 0$.

[6] For example, the probabilists David Aldous and Russ Lyons are some of the only mathematicians to have publicly supported the conjecture that all groups are sofic, motivated by their more general conjecture about unimodular random graphs [2].

[7] This is needed to rule out groups of the form $(\mathbb{Z}/2\mathbb{Z})^n$, where the map $x \mapsto -x$ is just the identity.

Geometric purists can expunge all reference to probability from this theorem by reformulating it in terms of solutions to the heat equation. See also [30,63] for further results and conjectures related to this theorem. The proof of this theorem relies on a case analysis according to whether or not the group is amenable. When the group is non-amenable, the stronger statement $\mathbb{E}|X_n|^2 \geq cn^2$ holds as a trivial consequence of Kesten's theorem (which states that a group is non-amenable if and only if the random walk return probabilities decay exponentially). On the other hand, it is a (very) special case of a theorem of Mok, Korevaar, and Schoen [35, 46] that if G is amenable (or, more generally, does not have Kazhdan's property (T)), then it admits a non-trivial *equivariant harmonic embedding into Hilbert space* $\Psi\colon G \to \mathbb{H}$. In this case the image $\Psi(X_n)$ is a (possibly infinite-dimensional) martingale and one has

$$\mathbb{E}|X_n|^2 \geq c_1 \mathbb{E}\|\Psi(X_n)\|^2 = c_1 \mathbb{E} \sum_{i=1}^{n} \|\Psi(X_n) - \Psi(X_{n-1})\|^2 = c_2 n$$

by equivariance of the embedding and orthogonality of martingale increments.[8] This theorem is related to the fact that every infinite Cayley graph admits a harmonic function of at most linear growth, another "counterexample" to the "theorem" that plays an important role in Kleiner's proof of Gromov's theorem [34] and can be proven via a similar case analysis.

The Burnside problem

I now describe a purely algebraic counterexample to the "theorem" pointed out to me on MathOverflow by Moishe Kohan. The *Burnside problem* asks for each integer n, must every finitely generated group G such that $g^n = \mathrm{id}$ for every $g \in G$ be finite? An infinite finitely generated group with this property is called a Burnside group of exponent n. The problem was solved for large, odd n in a breakthrough 1968 work of Novikov and Adian [48], who proved that there exist Burnside groups of exponent n for each odd $n > 4381$. While this is not a counterexample to the "theorem" since it concerns specific groups, it is also known that Burnside groups of exponent 1, 2, 3, 4, and 6 do not exist, with the cases $n = 4, 6$ being non-trivial theorems of Sanov and Hall [26, 57]. This yields purely algebraic properties (e.g., "there exists $g \in G$ such that $g^6 \neq \mathrm{id}$") that are true for all *infinite* finitely generated groups for non-trivial reasons. The cases $n = 5, 7$ of the Burnside problem remain open (among other small values).

[8] Russ Lyons has pointed out to us that an alternative simpler proof of this theorem is presented in [42, Section 13.9] that may make the theorem too easy to count as a "counterexample" to the "theorem," depending on what one considers "trivial." This proof still relies on a case analysis according to whether or not the group is amenable.

Coarse regular variation of the inverse growth

I next give a "counterexample" that, while more technical than my other examples, has the strongest credentials of non-triviality in that its proof relies on some of the deepest work in geometric group theory: Gromov's theorem [25], Pansu's theorem [55], and Hrushovski's theorem [27]. It is also arguably the more "purely geometric" of my proposed counterexamples. Recall that a function $f\colon (0,\infty) \to \mathbb{R}$ is said to be *regularly varying* if $f(x) > 0$ for all sufficiently large x and the limit

$$\lim_{x\to\infty} \frac{f(\lambda x)}{f(x)}$$

is well defined for each $\lambda > 0$. The first basic theorem about regularly varying functions is that (subject to very mild additional regularity assumptions such as measurability or Baire measurability that certainly hold for the piecewise-continuous functions we consider), this convergence must hold uniformly for compact sets of λ in $(0,\infty)$ and that there must exist a real number α, known as the *index of regular variation*, such that

$$\lim_{x\to\infty} \frac{f(\lambda x)}{f(x)} = \lambda^\alpha$$

for every $\lambda > 0$. Note that regular variation is a first-order asymptotic property, in the sense that if $f(x) \sim g(x)$ and f is regularly varying, then g is regularly varying of the same index as f. Regularly varying functions of index 0 are known as *slowly varying functions* and include functions such as $\log(1+x)$, $\log(1+\log(1+x))$, $\exp[\sqrt{\log x}\,]$, and so on. Regularly varying functions were introduced by Karamata in the 1930s with the goal of producing a unified framework for Abelian and Tauberian theorems (i.e., conditions under which the first-order asymptotics of a function and its Laplace transform or a sequence and its generating function determine one another), and are now a standard part of the language in many fields of pure mathematics with an emphasis on asymptotic analysis, including, e.g., probability theory and analytic number theory; see, e.g., [5] for an introduction.

Regular variation is one of several natural regularity properties for functions of (sub)polynomial growth; for increasing functions, a much weaker but still useful property is the *doubling property*

$$\limsup_{x\to\infty} \frac{f(\lambda x)}{f(x)} < \infty \qquad \text{for each } \lambda > 1.$$

In geometric group theory, regularity properties like doubling or regular variation often arise as hypotheses in theorems relating the asymptotics of one quantity to another, such as in the relationship between random-walk return probabilities and the spectral profile [10], or the relation between the isoperimetric profiles of two groups and their wreath product [18]. Moreover, it is common to consider a rather loose notion of asymptotic equivalence between monotone functions, where if f and g are increasing we say that $f \simeq g$ if there exist positive constants c and C such that

$cg(cx) \le f(x) \le Cg(Cx)$ for all sufficiently large x. This is natural, since many functions associated to a finitely generated group, such as the volume growth and the isoperimetric profile (defined below) are well defined (i.e., independently of the choice of finite generating set) modulo this notion of asymptotic equivalence. However, when working at a high level of generality one often comes to realise that this notion of equivalence is not very useful for functions not known to satisfy at least some weak form of regularity such as doubling. These issues came to a head for me in my paper [30], where I was able to prove a conjecture of Lyons, Peres, Sun, and Zheng [43] only for groups whose *isoperimetric profile*

$$\Psi(n) := \min\Big\{\frac{|\partial W|}{|W|} : W \subseteq G,\ |W| \le n\Big\}$$

is *coarsely regularly varying*, that is, equivalent to a regularly varying function under the equivalence relation $\simeq$. (Here ∂W denotes the set of edges of the Cayley graph with one endpoint in W and the other in $G \setminus W$.) I conjecture, but have so far been unable to prove, that this is *always* the case, for every infinite finitely generated group.

I am now ready to state my next proposed "counterexample." In an underhanded effort to make the theorem I am about to state apply to all finitely generated groups rather than merely all *infinite* finitely generated groups, I will depart from the standard conventions by saying that a function $f\colon (0,\infty) \to [0,\infty]$ is regularly varying if it either takes values in $(0,\infty)$ and is regularly varying in the usual sense or satisfies $f(x) \equiv 0$ or $f(x) \equiv \infty$ for all sufficiently large x, in which case we say f is regularly varying of index $-\infty$ or $+\infty$ as appropriate. We write $\mathrm{Gr}(r)$ for the number of group elements of word length at most r.

Theorem 2.3. *Let G be a group with finite generating set S. Then the inverse growth function* $\mathrm{Gr}^{-1}(x) := \inf\{r \ge 0 : \mathrm{Gr}(r) \ge x\}$ *is coarsely regularly varying.*

Many groups have isoperimetric profile of the same order as the reciprocal of the inverse growth, with a one-sided bound always holding by a theorem of Coulhon and Saloff-Coste [11], so that this theorem supports my conjecture that the isoperimetric profile is always coarsely regularly varying.

Proof of Theorem 2.3. If G is finite, then $\mathrm{Gr}^{-1}(x) = \infty$ for all $x > |G|$, so that $\mathrm{Gr}^{-1}(x)$ is regularly varying of infinite index. If G has polynomial volume growth, it follows from Gromov's theorem [25] that G is virtually nilpotent and hence, by a theorem of Pansu [55], that there exist a constant $A > 0$ and an integer $d \ge 1$ such that

$$\mathrm{Gr}(r) \sim Ar^d \qquad \text{as } r \to \infty$$

and hence that

$$\mathrm{Gr}^{-1}(x) \sim (x/A)^{1/d} \qquad \text{as } x \to \infty,$$

which is more than sufficient for regular variation (with index $1/d$). What if G has superpolynomial growth? It is a consequence of a deep theorem of Hrushovski [27] that if G has superpolynomial volume growth then

$$\frac{\mathrm{Gr}(2r)}{\mathrm{Gr}(r)} \to \infty$$

as $r \to \infty$ (the theorem of [27] states more generally that $\lim_{n\to\infty} |A_n^2|/|A_n| = \infty$ for any exhaustion of G by finite sets $(A_n)_{n\ge 1}$). This is easily seen to imply that the inverse function $\mathrm{Gr}^{-1}(x)$ is coarsely slowly varying (that is, $\simeq$-equivalent to a slowly varying function). ■

Theorems for "most" infinite finitely generated groups

There are several important theorems concerning not quite all infinite finitely generated groups, but instead all infinite finitely generated groups that are not virtually $\mathbb{Z}$. While such theorems are arguably not true "counterexamples" to the "theorem," they are similar in spirit in that they establish non-trivial theorems concerning very large classes of groups. One example is the "$p_c < 1$" problem of Benjamini and Schramm [4], which asked whether an infinite, finitely generated group has a percolation phase transition if and only if it is not virtually $\mathbb{Z}$. This problem was solved by Duminil-Copin, Goswami, Raoufi, Severo, and Yadin [13], who proved that $p_c < 1$ for every bounded-degree graph satisfying a $(4+\varepsilon)$-dimensional isoperimetric inequality; the remaining cases have polynomial growth so that they are virtually nilpotent via Gromov's theorem and can be treated via classical methods. (E.g., using the fact they contain subgraphs quasi-isometric to $\mathbb{Z}^2$.) The proof of the high-dimensional case of the theorem relied on a highly non-trivial comparison theorem relating percolation with another more complicated model defined in terms of the Gaussian free field. A new and much simpler proof of the theorem, still relying on a case analysis between low- and high-dimensional examples, was recently given by Easo, Severo, and Tassion [17]; this new proof also establishes the purely geometric fact that an infinite finitely generated group has the *exponential cut sets property* if and only if it is not virtually $\mathbb{Z}$. Another important example in the same vein is Varopoulos's theorem [62], which states that the random walk on an infinite, finitely generated group is recurrent (returns to the identity infinitely often) if and only if the group is virtually $\mathbb{Z}$ or $\mathbb{Z}^2$, which again can be deduced as a consequence of Gromov's theorem.

Theorems requiring uniform *control of all groups*

Let us close this section by mentioning that there has recently been significant progress made on theorems concerning *uniform* properties of all finitely generated groups (or all infinite finitely generated groups, or all infinite finitely generated groups that are not virtually $\mathbb{Z}$, etc.), with constants that are either universal or depend only on, say, the size of the generating set. The problem of proving uniform theorems like this is often closely related to the problem of proving theorems about *finite* groups, where one is typically forced to work with *families* of finite groups in order to make meaningful statements about asymptotic properties. Significant results in this direction include finitary versions include $p_c < 1$ theorems for finite transitive graphs [31], "gap at 1" theorems for the critical probability on infinite vertex transitive graphs [31, 41, 54], and finite-graph versions of Varopoulos's theorem [58]. Many of these results rely on a *structure vs. expansion dichotomy* using the *finitary structure theorem* of Breuillard, Green and Tao [8] (a deep and powerful strengthening of the theorems of Gromov [25] and Hrushovski [27] stating roughly that Hrushovski's theorem holds *uniformly* over all finitely generated groups) and its extension to transitive graphs due to Tessera and Tointon [59]. Using this finitary structure theory, one can attempt to treat "high-dimensional" and "low-dimensional" cases separately and "patch the analysis together" on scales in which the dimension switches from high to low (if such scales exist). Our recent proof of Schramm's locality conjecture with Easo [15] and Easo's extension of this result to finite graphs [14] also rely heavily on these ideas, including in particular a finitary version of the fact that groups of polynomial growth are finitely presented [16].

3 Synthesis

A unifying feature of many of the examples considered in the previous section is that the proofs involve some kind of case analysis. As such, one might propose the following modification to our "theorem":

"Theorem*". *If a property is satisfied by all finitely generated groups* for the same reason*, then this reason must be trivial.*

Again I do not really claim this "theorem" is "true" in any precise mathematical sense,[9] merely that it is *more true* than the original "theorem."

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[9] Wes Wrigley has pointed out to me that intuitionistic logic [47] does not allow for proofs by case analysis, so that one could perhaps interpret "Theorem*" in intuitionistic terms. I am too ignorant of intuitionism to make an informed comment on this, but would prefer to keep the "theorem" both vague and within the framework of standard mathematics (at the cost of making it likely to be false!).

References

[1] M. Aizenman and D. J. Barsky, Sharpness of the phase transition in percolation models. *Comm. Math. Phys.* **108**, 489–526 (1987)

[2] D. Aldous and R. Lyons, Processes on unimodular random networks. *Electron. J. Probab.* **12**, article no. 54, 1454–1508 (2007)

[3] I. Benjamini, R. Lyons, Y. Peres and O. Schramm, Critical percolation on any nonamenable group has no infinite clusters. *Ann. Probab.* **27**, 1347–1356 (1999)

[4] I. Benjamini and O. Schramm, Percolation beyond $\mathbb{Z}^d$, many questions and a few answers. *Electron. Comm. Probab.* **1**, article no. 8, 71–82 (1996)

[5] N. H. Bingham, C. M. Goldie and J. L. Teugels, *Regular variation*. Encyclopedia Math. Appl. 27, Cambridge University Press, Cambridge (1989)

[6] L. Bowen, M. Chapman, A. Lubotzky and T. Vidick, The Aldous–Lyons conjecture I: Subgroup tests. arXiv:2408.00110v1 (2024)

[7] L. Bowen, M. Chapman and T. Vidick, The Aldous–Lyons conjecture II: Undecidability. arXiv:2501.00173v1 (2024)

[8] E. Breuillard, B. Green and T. Tao, The structure of approximate groups. *Publ. Math. Inst. Hautes Études Sci.* **116**, 115–221 (2012)

[9] J. Brieussel and T. Zheng, Speed of random walks, isoperimetry and compression of finitely generated groups. *Ann. of Math. (2)* **193**, 1–105 (2021)

[10] T. Coulhon and A. Grigor'yan, On-diagonal lower bounds for heat kernels and Markov chains. *Duke Math. J.* **89**, 133–199 (1997)

[11] T. Coulhon and L. Saloff-Coste, Isopérimétrie pour les groupes et les variétés. *Rev. Mat. Iberoamericana* **9**, 293–314 (1993)

[12] H. Duminil-Copin, S. Goswami and A. Raoufi, Exponential decay of truncated correlations for the Ising model in any dimension for all but the critical temperature. *Comm. Math. Phys.* **374**, 891–921 (2020)

[13] H. Duminil-Copin, S. Goswami, A. Raoufi, F. Severo and A. Yadin, Existence of phase transition for percolation using the Gaussian free field. *Duke Math. J.* **169**, 3539–3563 (2020)

[14] P. Easo, Sharpness and locality for percolation on finite transitive graphs. *Geom. Funct. Anal.* **35**, 1526–1586 (2025)

[15] P. Easo and T. Hutchcroft, The critical percolation probability is local. arXiv:2310.10983v1 (2023)

[16] P. Easo and T. Hutchcroft, Uniform finite presentation for groups of polynomial growth. *Discrete Anal.* article no. 1 (2025)

[17] P. Easo, F. Severo and V. Tassion, Counting minimal cutsets and $p_c < 1$. *Forum Math. Pi* **13**, article no. e23 (2025)

[18] A. Erschler, On isoperimetric profiles of finitely generated groups. *Geom. Dedicata* **100**, 157–171 (2003)

[19] A. Erschler, Piecewise automatic groups. *Duke Math. J.* **134**, 591–613 (2006)

[20] J. Frisch, Y. Hartman, O. Tamuz and P. Vahidi Ferdowsi, Choquet–Deny groups and the infinite conjugacy class property. *Ann. of Math. (2)* **190**, 307–320 (2019)

[21] D. Gaboriau, What is … cost? *Notices Amer. Math. Soc.* **57**, 1295–1296 (2010)

[22] É. Ghys and P. de la Harpe (eds.), *Sur les groupes hyperboliques d'après Mikhael Gromov*. Progr. Math. 83, Birkhäuser Boston, Boston, MA (1990)

[23] W. T. Gowers, The two cultures of mathematics. In *Mathematics: frontiers and perspectives*, Amer. Math. Soc., Providence, RI, 65–78 (2000)

[24] R. I. Grigorchuk, Degrees of growth of finitely generated groups and the theory of invariant means. (in Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **48**, 939–985 (1984). English translation: *Math. USSR-Izv.* **25** 259–300 (1985)

[25] M. Gromov, Groups of polynomial growth and expanding maps. *Inst. Hautes Études Sci. Publ. Math.* **53**, 53–73 (1981)

[26] M. Hall, Jr., Solution of the Burnside problem for exponent 6. *Proc. Nat. Acad. Sci. U.S.A.* **43**, 751–753 (1957)

[27] E. Hrushovski, Stable group theory and approximate subgroups. *J. Amer. Math. Soc.* **25**, 189–243 (2012)

[28] T. Hutchcroft, Percolation on hyperbolic graphs. *Geom. Funct. Anal.* **29**, 766–810 (2019)

[29] T. Hutchcroft, Nonuniqueness and mean-field criticality for percolation on nonunimodular transitive graphs. *J. Amer. Math. Soc.* **33**, 1101–1165 (2020)

[30] T. Hutchcroft, Small-ball estimates for random walks on groups. arXiv:2406.17587v1 (2024)

[31] T. Hutchcroft and M. Tointon, Non-triviality of the phase transition for percolation on finite transitive graphs. *J. Eur. Math. Soc. (JEMS)* **27**, 4283–4346 (2025)

[32] Z. Ji, A. Natarajan, T. Vidick, J. Wright and H. Yuen, MIP∗ = RE. *Commun. ACM* **64**, 131–138 (2021)

[33] K. Juschenko, *Sofic groups*. Unpublished lecture notes https://web.ma.utexas.edu/users/juschenko/files/soficgroups.pdf

[34] B. Kleiner, A new proof of Gromov's theorem on groups of polynomial growth. *J. Amer. Math. Soc.* **23**, 815–829 (2010)

[35] N. J. Korevaar and R. M. Schoen, Global existence theorems for harmonic maps to non-locally compact spaces. *Comm. Anal. Geom.* **5**, 333–387 (1997)

[36] T. Kumagai, *Random walks on disordered media and their scaling limits*. Lecture Notes in Math. 2101, Springer, Cham (2014)

[37] J. R. Lee and Y. Peres, Harmonic maps on amenable groups and a diffusive lower bound for random walks. *Ann. Probab.* **41**, 3392–3419 (2013)

[38] P.-H. Leemann and M. de la Salle, Cayley graphs with few automorphisms. *J. Algebraic Combin.* **53**, 1117–1146 (2021)

[39] P.-H. Leemann and M. de la Salle, Cayley graphs with few automorphisms: the case of infinite groups. *Ann. H. Lebesgue* **5**, 73–92 (2022)

[40] R. Lyons, Phase transitions on nonamenable graphs. *J. Math. Phys.* **41**, 1099–1126 (2000)

[41] R. Lyons, A. Mann, R. Tessera and M. Tointon, Explicit universal minimal constants for polynomial growth of groups. *J. Group Theory* **26**, 29–53 (2023)

[42] R. Lyons and Y. Peres, *Probability on trees and networks*. Camb. Ser. Stat. Probab. Math. 42, Cambridge University Press, New York (2016)

[43] R. Lyons, Y. Peres, X. Sun and T. Zheng, Occupation measure of random walks and wired spanning forests in balls of Cayley graphs. *Ann. Fac. Sci. Toulouse Math. (6)* **29**, 97–109 (2020)

[44] R. Lyons and O. Schramm, Indistinguishability of percolation clusters. *Ann. Probab.* **27**, 1809–1836 (1999)

[45] E. Mares, Relevance logic. In *The Stanford Encyclopedia of Philosophy (E. N. Zalta and U. Nodelman, eds.)*, Metaphysics Research Lab, Stanford University (2024)

[46] N. Mok, Harmonic forms with values in locally constant Hilbert bundles. In *Proc. Conf. in Honor of J.-P. Kahane (Orsay, 1993)*, *J. Fourier Anal. Appl.* (Spec. Iss.), 433–453 (1995)

[47] J. Moschovakis, Intuitionistic logic. In *The Stanford Encyclopedia of Philosophy (E. N. Zalta and U. Nodelman, eds.)*, Metaphysics Research Lab, Stanford University (2024)

[48] P. S. Novikov and S. I. Adjan, Infinite periodic groups. I. (in Russian) *Izv. Akad. Nauk SSSR Ser. Mat.* **32**, 212–244 (1968). English translation: *Math. USSR-Izv.* **2** 209–236 (1968)

[49] A. Yu. Ol'shanskii, Distortion functions for subgroups. In *Geometric group theory down under (Canberra, 1996)*, pp.281–291, de Gruyter, Berlin (1999)

[50] A. Yu. Ol'shanskii and D. V. Osin, A quasi-isometric embedding theorem for groups. *Duke Math. J.* **162**, 1621–1648 (2013)

[51] A. Yu. Ol'shanskii, D. V. Osin and M. V. Sapir, Lacunary hyperbolic groups. *Geom. Topol.* **13**, 2051–2140 (2009)

[52] D. Osajda, Small cancellation labellings of some infinite graphs and applications. *Acta Math.* **225**, 159–191 (2020)

[53] I. Pak and T. Smirnova-Nagnibeda, On non-uniqueness of percolation on nonamenable Cayley graphs. *C. R. Acad. Sci. Paris Sér. I Math.* **330**, 495–500 (2000)

[54] C. Panagiotis and F. Severo, Gap at 1 for the percolation threshold of Cayley graphs. *Ann. Inst. Henri Poincaré Probab. Stat.* **59**, 1248–1258 (2023)

[55] P. Pansu, Croissance des boules et des géodésiques fermées dans les nilvariétés. *Ergodic Theory Dynam. Systems* **3**, 415–445 (1983)

[56] M. de la Salle and R. Tessera, Characterizing a vertex-transitive graph by a large ball. *J. Topol.* **12**, 705–743 (2019)

[57] I. N. Sanov, Solution of Burnside's problem for exponent 4. (in Russian) *Leningrad State Univ. Annals [Uchenye Zapiski] Math. Ser.* **10**, 166–170 (1940)

[58] R. Tessera and M. Tointon, Sharp relations between volume growth, isoperimetry and escape probability in vertex-transitive graphs. arXiv:2001.01467v2 ([v1] 2020, [v2] 2024)

[59] R. Tessera and M. C. H. Tointon, A finitary structure theorem for vertex-transitive graphs of polynomial growth. *Combinatorica* **41**, 263–298 (2021)

[60] B. Tilly, Analysis vs algebra predicts eating corn? (2010) https://bentilly.blogspot.com/2010/08/analysis-vs-algebra-predicts-eating.html (accessed 16 April 2025)

[61] M. C. H. Tointon, Commuting probabilities of infinite groups. *J. Lond. Math. Soc. (2)* **101**, 1280–1297 (2020)

[62] N. T. Varopoulos, Théorie du potentiel sur des groupes et des variétés. *C. R. Acad. Sci. Paris Sér. I Math.* **302**, 203–205 (1986)

[63] T. Zheng, Asymptotic behaviors of random walks on countable groups. In *Proc. Int. Cong. Math. 2022,* Vol. 4, pp. 3340–3365, EMS Press, Berlin (2023)

Tom Hutchcroft is the Gordon and Betty Moore Professor of Mathematics at the California Institute of Technology. Prior to coming to Caltech, he received his undergraduate degree from the University of Cambridge and his PhD from the University of British Columbia under the supervision of Asaf Nachmias and Omer Angel. His work spans a range of topics in probability, group theory, and mathematical physics. He received the EMS Prize in 2024.

t.hutchcroft@caltech.edu

The Math Solidarity Platform

Emine Yıldırım

On February 6, 2023, a magnitude 7.8 earthquake struck southern and central Türkiye and parts of Syria, followed by a 7.5 event. The destruction was geographically vast: tens of thousands of lives were lost, universities were shut down, and students were thrown into uncertainty. The question that haunted me was how could I help those who were affected.

As a pure mathematician, my first response was a form of helplessness. I was not a doctor, nurse, cook, or professional relief worker. I could not clear rubble or repair cell towers. Yet it is a mistake to equate "immediate material necessity" with "the whole space of necessity." Human beings do not survive solely by calories and shelter; they also survive by reasons to keep a day structured, by the option of thinking beyond the next hour, and by experiencing being addressed as a human rather than merely as a victim. My only reliable resource was my mathematical expertise and a network of people who shared it.

The project that eventually emerged – *Matematik Dayanışma Platformu* (the Math Solidarity Platform) – began from that narrow but genuine premise: if we could restore even a small piece of continuity for affected students, we might offer something that is not food or medicine, but is still a lifeline.

A precursor of this platform was a program that already existed. Before the earthquake, I had been participating in the Directed Reading Program (DRP) Türkiye: an independent, online initiative run by volunteer graduate students and early-career researchers. DRP Türkiye pairs undergraduates in Türkiye with mentors around the world to read a piece of mathematical research together during summer months, typically on topics not covered in standard undergraduate courses. The program's implicit philosophy is that mathematical growth is not only additive (more techniques) but also exploratory (new objects, new questions, new styles of thinking), and that mentorship can make such exploration possible.

After the disaster, two of DRP Türkiye's founding members, Şefika Kuzgun and Feride Ceren Köse, asked a practical question that was almost a moral one: could this network, built for intellectual exploration, be repurposed – quickly and responsibly – into a form of disaster support? They reached out to me, and together with Tekin Karadağ we met to think concretely about what could be done that was neither symbolic nor naïve. That conversation produced the first iteration of the Math Solidarity Platform. Later Şefika Kuzgun, Zeynep K., and I ran a second iteration. We were all graduate students or early-career researchers at the time and voluntarily acted on this initiative.

We set two operational principles that were simple but, under the circumstances, nontrivial. First, we recruited mentors as broadly as possible: undergraduate and graduate students, postdocs, and faculty from different institutions. The variety mattered, because some students needed careful remediation while others needed companionship or actual help in problem-solving. Second, we adopted an open-acceptance policy for mentees. We accepted applicants from across Türkiye without interrogating their "degree of need," and we did not restrict participation by region, since displacement meant that "affected areas" were not a stable category. If a student applied, we treated the application itself as sufficient evidence that support was desired.

At this point, an uncomfortable constraint became obvious. We understood mathematics; we did not understand trauma response. If the platform was to be safe rather than purely well-intentioned, it had to be built with an explicit awareness of vulnerabilities: intimidation, withdrawal, shame about falling behind, and the ordinary unpredictability of life after catastrophe. To address this, we arranged a training session with a professional psychologist, and we required all mentors to attend. This was not a ceremonial add-on. It provided mentors with practical guidance on maintaining a welcoming environment, avoiding inadvertent harm, and recognizing that "participation" might look irregular even when the student's interest was genuine. The training also gave us permission to ask questions we had been afraid to ask and to admit what we did not know.

Our initial goal was to support university students affected by the earthquake in their basic mathematics courses. The aim was both to support students in their coursework and to help them return to their pre-earthquake lives as quickly as possible. For this platform, learning mathematics is a means rather than an end. Accordingly, meetings were not regarded as the primary site of mathematical instruction, but rather as a form of academic support, comparable to study sessions or office hours. Central to this approach was creating an environment where students felt comfortable expressing themselves.

The continuity between the DRP and the new platform was not merely logistical. It was structural: the basic unit was still the relationship between a student and a mathematically engaged mentor, but the aim shifted from enrichment to stabilization.

We moved toward group study, pairing small groups of students with a mentor. The reasoning was not simply pedagogical: a group lowers the activation energy required to speak, ask questions, and admit confusion. In a one-to-one setting, a student may feel watched. In a group the student can feel accompanied. Meetings were held online at times determined by each group. Despite severe infrastructure damage, mobile networks in affected areas recovered sufficiently quickly that some students were able to join intermittently via phone connections.

The practical focus of the meetings was usually homework support, but we repeatedly emphasized to mentors that the deeper aim was restoration of normality: allowing students, even briefly, to step away from the disaster and rebuild familiar routines. In other words, we wanted students to regain agency over their lives through learning. We believe that when a student can articulate an idea in their own words, they are no longer merely receiving aid but participating in a shared practice.

We urged mentors not to solicit stories about the earthquake, while remaining attentive and respectful listeners when students chose to share their experiences voluntarily. Our platform is brought together through mathematics education and instruction. Confusion may arise if the roles of scholarship provider or financial or psychological support counselor are combined with instructional or mentoring roles. Therefore, we recommend that mentors maintain their role as instructors and avoid assuming the additional roles mentioned above.

The program ran in two rounds, Spring 2023 and Fall 2023, in which approximately one hundred and sixty students applied, respectively. One fact surprised us: in the first round, we had more mentors than students.

One mentor expressed what we gradually came to believe was the platform's central benefit: *"The point was not that students needed assistance with assignments, but that they were able to talk about mathematics with fellow students and with people who remained passionate about the subject. That conversation functioned as a bridge to 'normal times,' a way to preserve continuity with the selves they had been before the disaster. In one hour of a weekly discussion where 'I can still think' becomes believable again."*

To prevent the program from collapsing into a single narrow function, we created auxiliary social structures. For mentees, we organized weekly social meetings with intentionally nontechnical topic – books, films, hiking and nature, culture, music – so that participants could relate to one another as whole people rather than only as problem-solvers. For mentors, we organized informal "tea hours" to exchange experiences, share tips, and monitor the health of the mentoring environment. These meetings also produced an unanticipated benefit: they strengthened connections within the Turkish mathematical community, domestically and abroad, precisely because participants came from many countries and professional stages.

No honest account would omit the difficulties. Motivation and attendance were fragile. Some students registered but did not attend; others attended partially; some disappeared. This was not a mystery once one considered the surrounding constraints: housing insecurity, food scarcity, internet disruptions, damaged cell towers, closed schools and universities, and the psychological labor of processing loss. In such a context, consistency is not a virtue that can be demanded. The lesson for future efforts is not that students are unreliable, but that any support program must be designed with the expectation of irregular attendance.

One of the platform's values is to show how mathematics can function as a source of motivation, and joy, as well as a tool for maintaining self-respect. This viewpoint matches my own experience as a mathematician coming from an underprivileged background. Şefika Kuzgun, who personally experienced the devastating 1999 earthquake, similarly understands the power of studying and receiving support when the world becomes unstable. None of this romanticizes catastrophe. It simply rejects the false dictum that "in a crisis, only material needs count and intellectual pursuits are a luxury." For many people, intellectual pursuits – music, art, literature, mathematics – are part of what makes life livable.

We acknowledged that our platform had its own limitations in the aftermath of the earthquake. The hope was not that such a platform would "solve" anything. The hope was more modest and realistic: that it offered a small but genuine form of continuity, one conversation at a time.

I would like to sincerely thank my amazing friends and co-organizers: For the first round (Spring 2023): Tekin Karadağ, Feride Ceren Köse, Şefika Kuzgun. For the second round (Fall 2023): Zeynep K. and Şefika Kuzgun. I also thank all of our participants – students and mentors – for their effort, care, and hope.

For reference, the platform's website (in Turkish) is: https://sites.google.com/view/matematikdayanisma

Emine Yıldırım obtained her PhD from the Université du Québec à Montréal in 2018. She held several postdoctoral positions: first as a Coleman Postdoctoral Researcher at Queen's University, Canada; then as a Simons Postdoctoral Fellow at the Isaac Newton Institute and the University of Cambridge, UK; and subsequently as a Research Fellow at the University of Leeds, UK. She is currently a researcher at the International Center for Mathematical Sciences – Sofia in the newly established research group "Quantum Groups and Cluster Algebras." Yıldırım's research interests lie in the representation theory of algebras, cluster algebras, and related combinatorics, including surface triangulations, friezes, lattices, and dynamical actions.

emineyyildirim@gmail.com

On the European Solidarity Committee of the EMS

Alice Fialowski

I served eight years on the European Solidarity Committee. It is a very important committee in the European Mathematical Society. It seems worthwhile summarizing here the mission of the committee, as it is not widely known. In a nutshell: it focuses on less developed, less wealthy European countries and mathematicians.

The *mission* of the European Solidarity Committee is the following:

- provides support to European mathematicians in need;
- supports early-career researchers and colleagues with limited resources for mathematics;
- focuses on countries or individuals affected by economic hardship, political instability, or conflict;
- offers assistance via grants and travel support;
- aims to promote equity and inclusion in mathematical communities.

In June 2025, I participated in the second Balkan Mathematics Conference in Thessaloniki and gave a talk jointly with Özlem Beyarslan on the European Solidarity Committee, with the goal of advertising the activities of the committee in Balkan countries. (The first Balkan Mathematics Conference was held in Romania two years ago.)

Poster for the Balkan Mathematics Conference II.

I was very happy that among the 75 participants there were mathematicians from Greece, Turkey, Serbia, Slovenia, India, Austria, Croatia, USA, Poland, Oman, UK, Germany, Morocco, Kuwait, Nigeria, China, Bosnia and Herzegovina, Qatar, Hungary, Canada, Australia, Ireland and Denmark. A really richly varied audience!

Unfortunately, Albania, Bulgaria, Kosovo, Montenegro and North Macedonia were not represented.

I hope that as a result of this talk the Balkan countries will take advantage of the possibilities offered by the European Solidarity Committee.

The talk focused on the advantages of using the committee for mathematicians in less developed countries. We have noticed in our work that these less-developed countries in the Balkans do not apply for any support to the European Solidarity Committee, perhaps because they are not aware about its existence. The problem in those countries is that there are only a few individual members, partly because for people there it is sometimes even complicated to pay the local Mathematical Society fee, let alone the EMS fee. This part of Europe is still in a different situation from Western countries. There are also countries where the local Mathematical Society is not a member of the EMS. As an example, in Serbia there are nine public and ten private universities, and also some colleges. It is rather likely that the information about the European Solidarity Committee does not reach those places.

This might be hard to imagine for mathematicians in the West, where there are lots of individual members, and where a good communication exists between institutes, universities, allowing to make known the possibilities offered by the EMS.

Our committee aims to balance this out. To help with the lack of information, I suggested the audience to look at the EMS homepage, with News (among others, information on coming conferences, fresh information), Magazine (free quarterly journal) and calls for grant applications. This is free for everybody, so people would not feel so isolated. With the internet these days, there are also lots of free sources, like Google, Google Scholar, arXiv, zbMATH Open, MathSciNet, etc.

How can our committee help you?

Twice a year we have a call for individual mathematicians to apply for travel grants to another place – perhaps with more resources – and for organizing a conference or workshop.

Let us look at these possibilities.

1. Travel grants

Offered to mathematicians in a less wealthy country without appropriate financial support.

Priorities include, without being restricted to these:

- early-career researchers (max. ten years after PhD degree);
- colleagues beyond retirement age who are still active.

These grants are for

- presenting results at conferences;
- attending courses;
- research stays in foreign countries;
- work with colleagues and access to libraries.

Criteria include:

- scientific achievements and potential of the applicant;
- scientific value of the goals of the applicant;
- residence in a less wealthy country with fewer resources. I would underline this criterion; that has not been done well up to now in our committee. This will be addressed in greater detail later on.

How much support can you get?

- Max. 500 Euros for destinations within Europe;
- Max. 900 Euros for trips from Europe to destinations outside Europe.

Let me mention that the European Solidarity Committee does not support applications from outside Europe to Europe, but there is another committee, the Developing Countries Committee, to which one can apply for some financial help from a developing country to Europe. Another committee you can apply to is the Women in Mathematics Committee if you plan to attend a workshop or conference organized mainly for women.

What do you need to apply?

- Scientific curriculum vitae with list of publications (date of the PhD degree). This is very important: we sometimes get too short or too long curriculum vitae; prepare it very well!
- Description of the scientific content related to the proposed activity (abstract or project summary).
- Proof of the acceptance by the partner institution.

2. Event funding grants

Offered for organizing scientific events of conferences.

Who is favored to get this support?

- Qualified mathematicians from less wealthy countries without financial support.

The priorities include without being restricted to these:

- Early career researchers (max. ten years after PhD degree);

Group photo at the Balkan Mathematics Conference II in Thessaloniki. (Courtesy of Andreas Koutsogiannis)

- colleagues beyond retirement age who are still active;
- support events organized in less favored places in Europe. I would underline this priority. There are two possible ways: either you apply from a less favored country for a conference in a more favored country with some colleagues or apply for a conference in a less wealthy country. This makes it possible for people to participate in their home country without high travel costs.

Criteria:

- include scientific value of the event;
- its impact on the place where it is organized.

How much support can you get?

- Max. 3000 Euros for each activity.

As an example, I would point out the recent conference in Istanbul "Model Theory Days," May 17–20, 2025, supported by the European Solidarity Committee with 1500 Euros; although this is not a big amount, it did make possible to organize the conference. Let me also mention that if you already have some support from some source, it is easier to find other support, as for instance was done in Istanbul.

By the way, in Turkey, as Özlem Beyarslan, a committee member, mentioned, information on the European Solidarity Committee grants is distributed to all universities in the country, which is a great way to let mathematicians know about these possibilities.

Event-funding grant application:

- scientific curriculum vitae of the organizers with a list of publications;
- preliminary budget for the entire event, including expected other funding sources;
- an explanation on how EMS funding will be used;
- details about the main lines of the event;
- preliminary list of speakers and participants.

Deadlines for both grants
There are two application rounds through each year.

First round: opens on October 7, *deadline January 31*. Decisions are made in March.

Second round: opens on February 1, *deadline September 30*. Decisions are made in November.

Warning: Keep track of the decision time, the workshop should not be planned before March and November of the year, respectively!

Application process
Register an account on Google Grants and log in.

Browse the list of open calls in the "Apply section" and select the item you wish to apply for.

Complete your application and submit it for review before the deadline.

You can revise your application online as many times as you want, until the deadline.

If you have questions, feel free to contact Markus Juvonen, juvonen@euromathsoc.org.

It is a technical procedure, you will be able to submit your application, do not be afraid!

Let me show some data from our last round on individual grants. We received 19 applications, and very few of them are from less wealthy countries. I believe that the reason is that the availability of these is not yet known in those countries.

number	from	to
2	Germany	USA
2	United Kingdom	Canada
1	Finland	United Kingdom
1	France	Japan
1	Italy	Austria
1	Italy	Canada
1	Italy	Kazakhstan
1	Italy	Netherlands
1	Italy	Poland
1	Italy	Portugal
1	Italy	United Kingdom
1	Italy	USA
1	Portugal	USA
1	Romania	Belgium
1	Romania	France
1	Turkey	Greece
1	United Kingdom	France

And here is the list of 26 applications for event grants:

number	from	to
7	Italy	Italy
3	France	France
3	Portugal	Portugal
2	Netherlands	Netherlands
2	United Kingdom	United Kingdom
1	Germany	Denmark
1	Germany	Italy
1	Germany	Netherlands
1	Italy	France
1	Italy	Sweden
1	Netherlands	Germany
1	Poland	Turkey
1	Romania	Romania
1	United Kingdom	Germany

Even worse! Where are the less favored countries? People should be aware that the level of these countries is very different from that in the more favored countries. But how can you pay for a scientific trip or organize a conference when you have difficulties paying your Society fee? That is the mission of the European Solidarity Committee, to help those mathematicians.

Talking to people at the conference, I met many mathematicians who – for some reason – left their home country and now live somewhere else. A beautiful mission for them would be to organize a conference in their home country, with colleagues still living and working there. This would fertilize the mathematical life in their home country. Come back and improve the level of mathematics research and teaching in your home country!

The individual grants are also very important for those countries. Apply for a travel grant, go to a country with better resources, work with colleagues there, use the libraries, and come back to raise the level of mathematics at home.

Unfortunately, we could not achieve these objectives yet, but we are working on it. As already mentioned above, the main problem must be that in those less favored countries people are not aware of these possibilities.

To highlight the importance of these events for less wealthy countries:

- In most areas of mathematics there are not many people working on fashionable topics.
- They might be particularly important for mathematicians who might be isolated in their field to be part of a larger community that will understand and appreciate their work.

- In smaller countries, academic leaders may be unaware or not appreciate some areas of mathematics. It is natural.
- For this reason, it is important that the European Solidarity Committee supports more areas of interest than is possible in any one country. There are other gifted mathematicians who are not in the mainstream, and might feel isolated in, say, a smaller place.
- Anyone in any European country is eligible to apply, even if she/he is not a member of the EMS and/or of the local society.
- Individuals can apply on their own, not necessarily through their home university or institute.
- One of the virtues of these grants offered by the European Solidarity Committee is that they give recognition to people whose colleagues and/or local university are unaware of the value of their work. It helps everyone, not just the applicant. It helps the university, the institute, it helps everybody.

Talking to Karine Chemla, we agreed that the committee has to be more active in advertising these grants in less favored countries, and have a contact person – not necessarily the head of the local main Mathematical Institute – someone who is willing to distribute the information to all universities and also smaller institutes in their country (as Turkey started to do).

When I met people at the conference, I noticed that about half of the participants came not from the Balkans; yet from our conversations it turned out that their home country is indeed one of the Balkan countries. And they do care about their home country, that is why they came! So, you are most welcome to organize a workshop or conference with your colleagues in your home country and encourage people there to visit you or other places, to collaborate and make your home country scientifically stronger!

I wish all of you success in your mathematical career!

(Map courtesy of Mapswire.com)

Alice Fialowski is a professor in mathematics at the Eötvös Loránd University Budapest, Hungary. She has strong ties to the EMS. She served in the Executive Committee for four years, and after that in the European Solidarity Committee for eight years. Her research is in Lie theory, cohomology, deformation theory and applications to physics.

fialowski@inf.elte.hu

The hidden art in things breaking apart

Julien Meyer

This article explores the hidden structures that emerge when materials break apart, revealing unexpected connections between natural processes, mathematics, and art. In particular, it examines Alberto Burri's Grande cretto nero*, whose fracturing evokes hyperbolic geometry reminiscent of Poincaré's models and Escher's tessellations. Through this interdisciplinary journey, the article shows how close attention to seemingly trivial phenomena can inspire profound insights across science and art.*

Imagine walking through a forest. At first everything seems ordinary—the fallen leaves on the ground, the cracks in the drying mud along the path, the songs of birds overhead. But if you pause for a moment and look more closely, your perception may suddenly shift: held against the sun, a leaf reveals a delicate network of veins, the cracks, once random, now divide space into polygons and the bird calls fall into a structured rhythm. What seemed trivial a moment ago now appears deeply ordered… and somehow, beautiful.

But how do such moments emerge? And why should they matter to us, apart from leaving us momentarily amazed? Part of the answer may lie in a remarkable human ability: the capacity to hold our breath and intentionally focus on what at first appears as ordinary.

These encounters with nature, these tiny moments, spark our curiosity and set our minds in motion. Each observation begins with something common yet carries the seed of a story: a moment when someone noticed a regularity, asked a question, and sought an explanation. What begins as simple curiosity can grow into structure, classification, and, in time, theory.

In this sense, the ability to perceive order within apparent randomness lies at the root of science, and of mathematics itself.

Reading the cracks

Patterns are universal. They can be found in every object and every material, in plants and animals alike: from stripes in the fur of a tiger, structural colors in butterfly wings, or the fractal growth forms of stony corals. And even when things break apart, decompose and decay, physical laws are at play, guiding the process toward some structured outcome.

Among these diverse expressions of order, this article turns to one phenomenon in particular: the formation of cracks. When a material gives way under stress, the release of tension inscribes itself as a network of fissures. Such networks are visible in drying mud, in the glaze of ceramics, or even in the fusion crust of

Figure 1. When held against the sun, a delicate network of veins emerges in the leaf of a beech tree (left). A network of cracks forms in drying mud (center) and EcoSons Étude IV. by Sam Erpelding, an artistic-ecological exploration of natural soundscapes in Luxembourg, the Netherlands, and Austria, combining field recordings with data-driven sonification to reveal hidden patterns in habitat ambience (right).

Figure 2. Network of cracks in drying mud (left), cracks in the glaze of ceramics as a decorative element (center) and cracks in the fusion crust of a meteorite (right).

meteorites. They are at once destructive and generative, producing geometries of striking regularity.

Locally, a single crack may appear structureless and random, but the situation becomes far more interesting when two or more cracks are at play. Imagine an old concrete pavement with two cracks approaching each other from opposite directions. At first, it seems as if the cracks try to avoid and turn around one other, only to meet moments later, at nearly right angles. This pattern repeats across countless materials and situations. But why? How does one crack influence the path of another? How do they “communicate”?

Figure 3. Two cracks circulate each other and tend to meet at right angles.

The answer lies in the material itself. As a crack propagates, it releases tension along its path, leaving the remaining stress oriented perpendicular to the crack. When a second crack approaches, it tends to grow in the direction that maximizes the release of this residual stress. This naturally leads the cracks to meet at angles of roughly 90°, forming what is typically called a *T-junction*.

The story becomes more complex when cracks repeatedly heal and break again, as for instance in mud subjected to cycles of wetting and drying. In such cases, T-junctions can evolve into so-called *Y-junctions*, where the cracks meet at angles of about 120°, distributing stress evenly in three directions. Thus, the geometry of crack junctions depends strongly on the physical process behind the cracking.

A striking example with profound implications comes from planet Mars, where NASA’s Curiosity rover discovered networks of hexagonal mud cracks in Gale Crater. Captured in 2021, these patterns provide the first direct evidence of sustained wet-dry cycling on the planet [3]. Unlike the sharp T-junctions produced in single drying events, the hexagons formed only after repeated rehydration gradually softened the angles into Y-junctions. Such cyclic wetting and drying is particularly significant, as it is thought to create the conditions necessary for the chemical steps that precede life. Thus, these Martian cracks not only record ancient climate but also hint at environments once capable of supporting the chemistry of life’s origins.

The hidden geometry of things breaking apart

Apart from the physical processes that govern how cracks form, under the right conditions, their networks surprisingly follow purely mathematical rules, grounded in simple combinatorial arguments.

Imagine taking a blank sheet of paper and drawing randomly straight lines across it. The lines divide the sheet into a mosaic of regions. If you count the number of vertices of each region and then take the average, you will always find the result close to four.

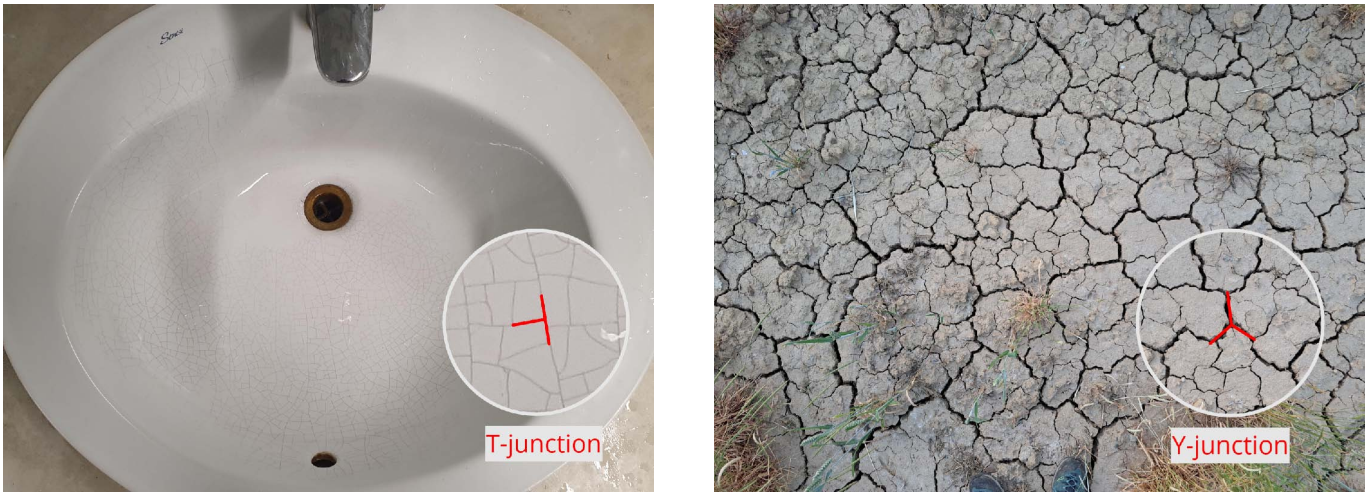


Figure 4. Network of cracks in a sink (left) meeting at a 90° angle vs. cracks in drying mud forming angles of about 120°.

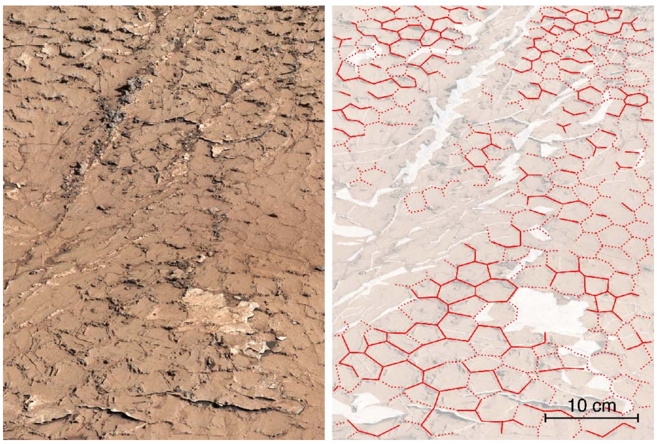


Figure 5. Hexagonal crack patterns on Mars, formed by repeated wetting and drying cycles. (Courtesy of NASA / JPL-Caltech / MSSS / IRAP)

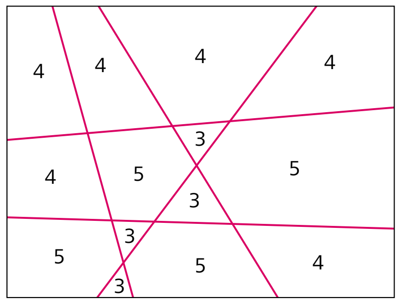
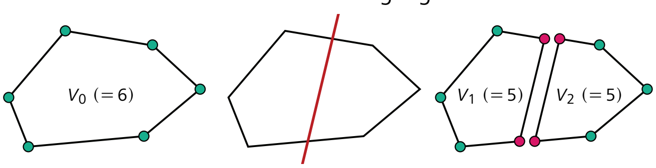


Figure 6. Drawing random lines on a sheet of paper and counting the number of vertices for each region. The average number of vertices per region converges to 4.

To see this, let V_0 be the number of vertices in a given region. When a new line divides this region into two, let V_1 and V_2 denote the numbers of vertices in the resulting regions.



Splitting the region into two, the average number of vertices is given by

$$V = \frac{V_1 + V_2}{2} = \frac{V_0 + 4}{2}$$

or equivalently $V - 4 = \frac{V_0 - 4}{2}$.

Each time the process is repeated, the distance to 4 is cut in half so that V approaches 4.

This very simple and purely mathematical reasoning applies to real world networks of cracks made of straight lines. A simple example of this are broken ice plates.

In a widely discussed article [1], Gábor Domokos and some collaborators developed and extended these ideas to more general crack networks, including those where cracks meet at Y-junctions. Since Y-junctions typically form angles of about 120°, it seems natural to expect that the average number of vertices per region should be close to six, like in a honeycomb. Turning to geology, they examined Earth's tectonic plates, which almost always meet at Y-junctions. They counted an average of 5.8. Close to six—but not quite.

At this point, Domokos recounts a telling reaction. The geologists on the team were delighted: 5.8 was "close enough" for the messy reality of Earth's surface. The mathematicians, however,

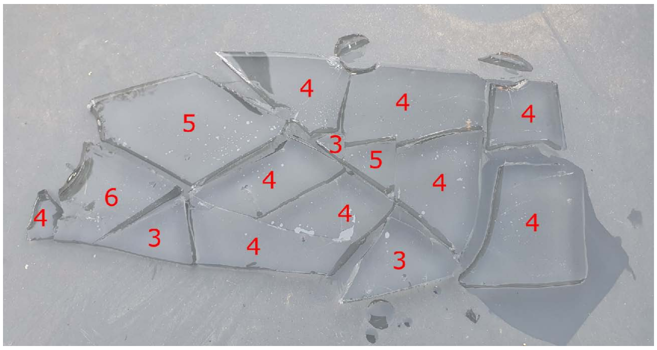


Figure 7. Try this at home! The average number of vertices by region in this broken ice plate is given by $61 : 15 \approx 4,07$.

were unsettled. Why the gap? What was missing? And then the insight struck: Earth's surface is not flat, but a giant sphere! Once they adapted their argument to a positively curved geometry, the discrepancy vanished, and the expected value shifted from exactly 6 down to 5.8, matching the observations almost perfectly.

Now, the geometer in us must feel alerted, remembering that apart from Euclidean and spherical space, there is another type of constant curvature geometry: *hyperbolic space.* Could it be that there are also real-world examples hinting to hyperbolic spaces?

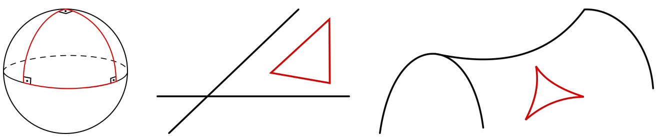


Figure 8. There are three types of constant curvature space: spherical (positive curvature), Euclidean (zero curvature), and hyperbolic (negative curvature).

Between 1882 and 1884, the French mathematician Henri Poincaré (1854–1912) introduced and extensively studied two fascinating models of hyperbolic space: the *hyperbolic upper half-plane* and the *hyperbolic disc*. In these models, geodesics, analogous to "straight lines" in Euclidean geometry, always intersect the boundary at right angles, revealing a rich and counterintuitive structure that challenged traditional notions of space.

These groundbreaking ideas in non-Euclidean geometry later became a source of inspiration for several 20th-century artists, most notably perhaps the Dutch graphic artist Maurits Cornelis Escher (1898–1972). Escher was captivated by the possibilities of infinite tessellations, impossible constructions, and spaces with non-zero curvature. He famously explored hyperbolic patterns in works such as *Circle Limit I–IV*, where repeated fish and angels and devils appear to shrink toward the boundary of a circle, visually representing the hyperbolic plane.

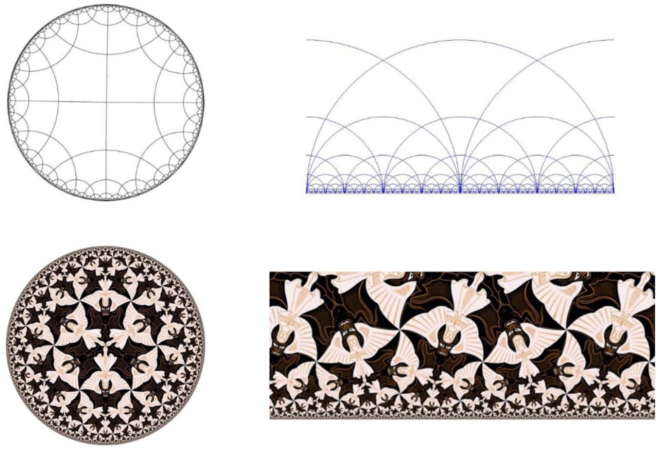


Figure 9. The Poincaré disc (left) and the upper half-plane (right) are two equivalent models of negatively curved space. The Dutch artist Maurits Cornelis Escher (1898–1972) translated these geometries into art in works such as Circle Limit IV (bottom left).

The hyperbolic structure in Alberto Burri's *Grande cretto nero*

But what about the cracks themselves? At the Centre Pompidou in Paris, among some of the most iconic artworks of the 20th century, one encounters a monumental black canvas by the Italian artist Alberto Burri[1] (1915–1995): The *Grande cretto nero* (1977). This rectangular work, measuring 1.5 by 2.5 meters, is covered with a dense mosaic of fractures. The cracks are sharply defined, spreading across the entire surface. As the eye moves toward the edges, the fractured regions become smaller and smaller, similar to the shrinking tiles in Poincaré's hyperbolic models.

This hyperbolic flavor is especially evident at the lower boundary. The fragments visibly diminish in size as they approach the edge, while the cracks themselves often follow curved lines that meet the boundary at right angles, just like geodesics in Poincaré's upper half-plane. At first this may seem puzzling, but it becomes less surprising if we recall what was said earlier about cracks: they naturally seek to release stress by meeting boundaries or other cracks at angles close to 90°. In Burri's work, the cracks formed near the edges follow precisely this principle, bending in ways that maximize the release of tension as they approach the boundary. The same kind of hyperbolic behavior can sometimes be observed in the cracked surfaces of old roads and pavements.

Seen this way, Burri's work brings to mind Poincaré's upper half-plane, especially along the lower edge where the fragments shrink, and the cracks meet the boundary at right angles. Yet unlike the half-plane, which is bounded only from below, Burri's rectangular canvas is enclosed on all four sides, and a similar hyperbolic

[1] https://www.centrepompidou.fr/fr/ressources/personne/cbq6bAe

Figure 10. Alberto Burri (1915–1995): *Grande cretto nero* 1977.
The following copyright also applies to all figures derived from this:

behavior emerges along every edge. This brings the analogy even closer to *Poincaré's disc model*, where space contracts uniformly toward all boundaries. It is therefore natural to ask *how Burri's* cretto *would look like if reimagined within a disc?*

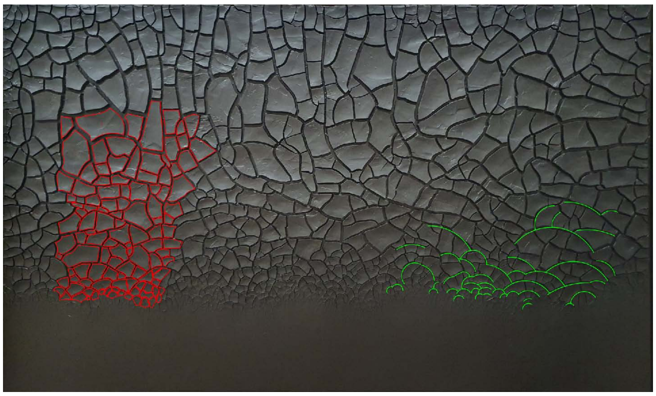


Figure 11. In the *Grande cretto nero*, fractured regions become smaller and smaller towards the boundary (red) while the cracks often follow curved paths that meet the boundary at right angles (green).

Mapping the square to the disc

Poincaré's upper half-plane and the disc are equivalent models of hyperbolic space, connected by transformations that preserve their geometry. Burri's canvas, however, is a rectangle so that these standard transformations do not readily apply. To preserve the crucial behavior of the cracks meeting at right angles (especially at the boundary), we require a transformation that preserves angles, a so-called *conformal* transformation. Fortunately, there is such a transformation which does exactly what we need: the *Schwarz–Christoffel transformation*.

More precisely, the Schwarz–Christoffel (SC) transformation provides a conformal map between the unit disc

$$D = \{w \in \mathbf{C} : |w| < 1\}$$

and polygonal regions of the plane. In the case of a square, explicit formulas can even be written down using elliptic integrals. Following Fong [2], the forward map

$$\mathrm{SC} : D \longrightarrow [-1,\ 1]^2$$

from the disc to the square is given by

$$z = \mathrm{SC}(w) = \frac{1-i}{-K_e} F\Big[\cos^{-1}\Big(\frac{1+i}{\sqrt{2}}w\Big), m = 1/2\Big] + (1-i)$$

where $F(\varphi, m)$ is the incomplete Legendre elliptic integral of the first kind with modulus $m = 1/2$, and $K_e = K(1/2)$ is the complete elliptic integral of the first kind. Explicitly,

$$F\Big(\varphi, \frac{1}{2}\Big) = \int_0^{\varphi} \frac{d\theta}{\sqrt{1 - \frac{1}{2}\sin^2\theta}},$$

$$K_e = F\Big(\frac{\pi}{2}, \frac{1}{2}\Big) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - \frac{1}{2}\sin^2\theta}} \approx 1.854.$$

Computational implementation

To actually use the Schwarz–Christoffel transformation to map Burri's work to a disc, we wrote a Python program that constructs the image in the disc pixel by pixel. The procedure can be summarized as follows:

1. Load and center-crop the source image to a square domain.
2. Construct a uniform Cartesian grid in the unit disc.
3. For each disc point w, compute its image $z = SC(w)$ in the square.
4. Convert z into pixel coordinates and assign to w the corresponding color from the square image.
5. Assemble all pixels to obtain the disc-shaped image.

Reinterpreting Burri's Grande cretto nero *as a disc*

The implementation we described above sends a square image conformally onto the disc. However, Burri's *Grande cretto nero* is *rectangular rather than square*. A direct mapping of the rectangle to the disc would inevitably produce horizontal squeezing and distortions, obscuring the geometric features we wish to preserve.

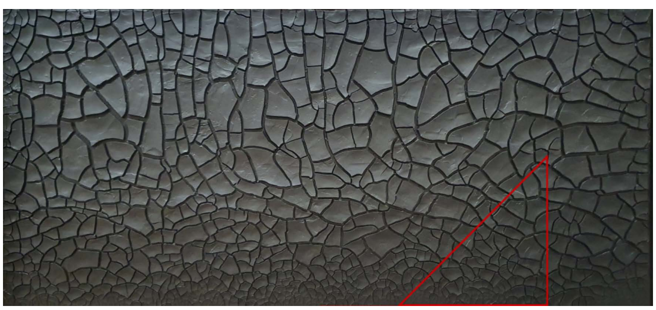
To avoid this, we first reinterpret Burri's canvas as a square. Concretely, we crop out the central region of the painting and conceptually "glue" its left and right parts together to form a square image. This step sacrifices the rectangular proportions but preserves the essential crack structure at the boundaries.

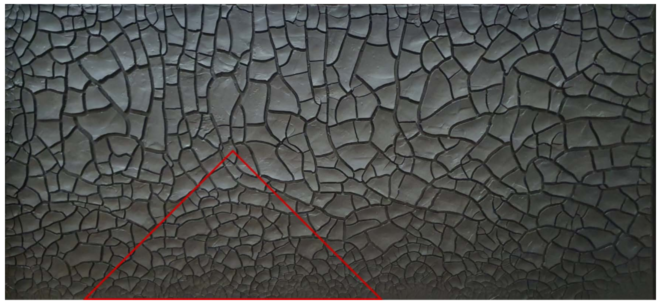
Once reinterpreted as a square, the image can be conformally mapped onto the unit disc. The crack geometry is faithfully preserved: fractures remain orthogonal to the boundary, while the overall structure acquires the characteristic hyperbolic flavor familiar from Poincaré's disc model. Beyond this, the resulting image strikingly evokes the hyperbolic patterns of Escher's *Circle Limit I–IV.* Yet there is a crucial difference: Escher's artworks are deliberate drawings, where he intentionally depicts hyperbolic geometry, whereas in Burri's work, the structure arises spontaneously from cracks, purely shaped by physical laws.

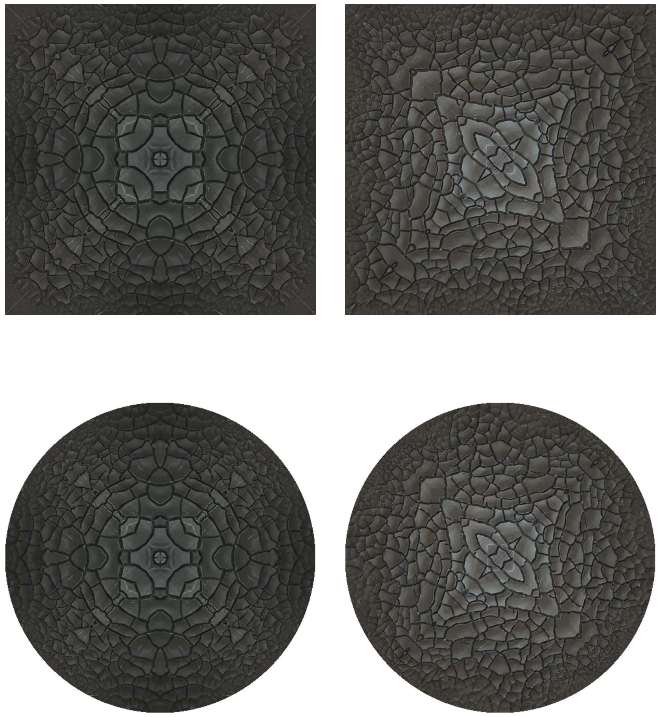
Figure 12. Burri's *Grande cretto nero* (left, central region removed) and its conformal image under the Schwarz–Christoffel transformation (right). The hyperbolic behavior is preserved, producing an image with a striking "Escher flavor."

Some more hyperbolic art from the Grande cretto nero

Beyond simply removing the central region to obtain a square image of *Grande cretto nero*, alternative constructions are possible. One such approach consists in cropping specific triangular sections from the artwork and assembling copies of them to form a square. This procedure introduces artificial symmetries which may be perceived as visually appealing. However, they depart from the purely natural crack formation that characterizes Burri's original composition, as illustrated in our earlier example (Figure 12). Below, we present several instances obtained from different triangular selections.







An invitation to the unintentional

From simple mud cracks to traces of life on Mars to hyperbolic structures in art: what may at first seem like a trivial observation, a simple curiosity, can grow into a more profound inquiry that opens unexpected connections. Careful attention reveals structures that link, in our case, material science, physics, and mathematics, and at times even unexpectedly resonate with art. At these crossings, disciplines merge with one another, leaving behind ties deeper than any single field alone.

I therefore invite you to stay curious, to pause and take time for the "little things," the unexpected and the unintentional. For it is precisely in these moments that familiar knowledge often expands into new insights.

References

[1] G. Domokos, D. J. Jerolmack, F. Kun and J. Török, Plato's cube and the natural geometry of fragmentation. *Proc. Natl. Acad. Sci. USA* **117**, 18178–18185 (2020)

[2] C. Fong, The conformal hyperbolic square and its ilk. In: *Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture*. pp. 179–186 (2016)

[3] W. Rapin, G. Dromart, B. C. Clark, J., Schieber, E. S. Kite, L. C. Kah, L. M. Thompson, O. Gasnault, J. Lasue, P.-Y. Meslin, P. I. Gasda and N. L. Lanza, Sustained wet–dry cycling on early Mars. *Nature* **620**, 299–302 (2023)

Python implementation of the Schwarz–Christoffel mapping transformation

Python code implementing the Schwarz–Christoffel transformation to map a square image conformally onto a disc. This code was used to generate the disc versions of Burri's *Grande cretto nero*.

```
import numpy as np
from PIL import Image
from mpmath import ellipf
import cmath

# -- User settings --
INPUT_PATH  = "input.png"  # square input image
OUTPUT_PATH = "output_disc.png"
RES          = 512       # output resolution (diameter)

# -- Load and square-crop input --
im = Image.open(INPUT_PATH).convert("RGB")
W, H = im.size
side = min(W, H)
left = (W - side) // 2
top  = (H - side) // 2
im_sq = im.crop((left, top, left+side, top+side))
src = np.asarray(im_sq)
S = side

# -- Constants --
Ke = 1.8540746773013719184
     # complete elliptic integral K(m=1/2)
m_param = 0.5
one_over_sqrt2 = 1.0 / np.sqrt(2.0)

# -- Output grid on unit disc --
x = np.linspace(-1, 1, RES)
y = np.linspace(-1, 1, RES)
X, Y = np.meshgrid(x, y)
R = np.sqrt(X*X + Y*Y)
mask = R <= 1.0
out = np.ones((RES, RES, 3), dtype=np.uint8) * 255

def z_to_src_xy(z):
    zx, zy = np.real(z), np.imag(z)
    px = (zx + 1.0) * 0.5 * (S - 1)
    py = (zy + 1.0) * 0.5 * (S - 1)
    return int(np.clip(round(px), 0, S-1)), \
           int(np.clip(round(py), 0,S-1))

# -- Schwarz–Christoffel map --
for iy in range(RES):
  for ix in range(RES):
    if not mask[iy, ix]:
      continue
    w = complex(X[iy, ix], Y[iy, ix])
    phi = cmath.acos((1.0+1.0j) * w * one_over_sqrt2)
    Fphi = ellipf(phi, m_param)
    z = ((1.0-1.0j)/(-Ke)) * Fphi + (1.0-1.0j)
    jx, jy = z_to_src_xy(z)
    out[iy, ix] = src[jy, jx]

Image.fromarray(out).save(OUTPUT_PATH)
print(f"Saved {OUTPUT_PATH}")
```

Copyright disclaimer

Julien Meyer obtained his PhD in mathematics from the Université libre de Bruxelles and is head of scientific mediation at the National Museum of Natural History in Luxembourg. Fascinated by the connections between science, nature and art, he explores the world from its fundamental physical laws to its living inhabitants and the traces they leave behind.

julien.meyer@mnhn.lu

Are all bets off when it comes to agreement gambles?

Bahareh Zahirodini and Moinak Bhaduri

It is well known that views on social issues are influenced by demographics. Whether similar people agree or dissimilar people disagree on touchy topics are useful insights that social science articles point to. These insights remain "internal" to the system, so to speak. In this work, using data from a Bentley–Gallup survey and the martingale mathematics of e-values, we showcase how someone "external" to the system could exploit these insights to build up a huge fortune. We achieve this through two parts. First, by showing the internal predictability will help (that is, if, for instance, similar people agree, that it is useful knowledge, as is if similar people predictably disagree). Second, by analyzing the aggressiveness or conservativeness of the "external" better (how much of the current wealth should they bet on the next issue/question) which has got nothing to do with whether similar people agree, that is, with the previous part. We recommend certain optimal ways to bet, with and without demographic knowledge of the participants.

Imagine having to build up a fortune – or lose one – based on rightly or wrongly predicting whether two people will agree on some issues pressing to society. *Whether* you would agree to be part of the scheme could depend on many factors. The types of issues being discussed (which political party should be in power is more divisive than whether we would need oxygen to live) or how many issues we are talking about. And if you do agree to play, *how* you will place your bets could depend on a further set of factors. Some external, like the two people that are being interviewed for possible agreement, and some internal, like how aggressively or conservatively you would want to bet (that is, how much of your current capital are you willing to put on stake). Amidst a charged political climate where disagreements are more common than ever, such games bring out strategies that, frivolously, help people amplify their capital, but more seriously, point to strange structures of society that might have gone, otherwise, unnoticed. And there are tools. Take the social survey that Bentley University did in collaboration with Gallup and made available to its students and faculty: 128 questions were asked, nearly all around how modern businesses function or ought to function, and demographic details of the respondents, along with political leanings were stored. It may be vital to query, as people become more and more similar, whether we can bet confidently on their agreement, or as they become more and more dissimilar, on their disagreement – and end up becoming rich. Martingales – some structures from probability theory – furnish useful answers. But first, some background!

Storms in (eight) teacups

Tales of odd coincidences, no matter how repeated, seldom cease to thrall. The setting for one was a tea party in full swing. 1935. Central London. Sir Ronald Fisher, already a well-known statistical figure, was in attendance. So was Dr. Muriel Bristol, one of his colleagues, less known. Fisher invited Muriel to join him for tea. Muriel declined, citing he was pouring tea first and then milk, while she liked it the other way. And she could tell just by tasting. Stunned, Fisher decided to test whether Muriel was bluffing. Promptly, out were laid eight cups, four with milk first, and four with tea, that looked rather similar. Muriel was invited to take sips and guess the order of composition. Now this invitation she accepted! And came off with flying colors. Each cup was rightly decoded! Much of this is the substance of Fisher's randomized trials, books and cinema that later succeeded commercially (David Salsburg's *The Lady Tasting Tea*), but in all seriousness, it points out a nagging inadequacy, an ineffectiveness of a way of thinking – an unfairness – that the contemporary statistical community is bent on distancing itself from.

It has to do with probabilities. In case Muriel was truly bluffing, what are the chances she could have rightly predicted each cup just by chance? Quite remote. 1 out of 70, in fact. This is because Fisher could have chosen the four spots out of eight for the milk-first cups in $C(8,4) = 70$ ways. And if Muriel were really bluffing – she was told it would be half and half (the order profile that is, not the milk – that is a pun courtesy of a colleague who was heckling me while I was giving a talk) – all of these 70 arrangements would have been equally likely to her. Hence, the chances of getting that specific arrangement – the right one – would have been 1/70 or 0.01428, the p-value in today's language (with Fisher's null being she is bluffing), which is less than our usual 5% or 10% levels of

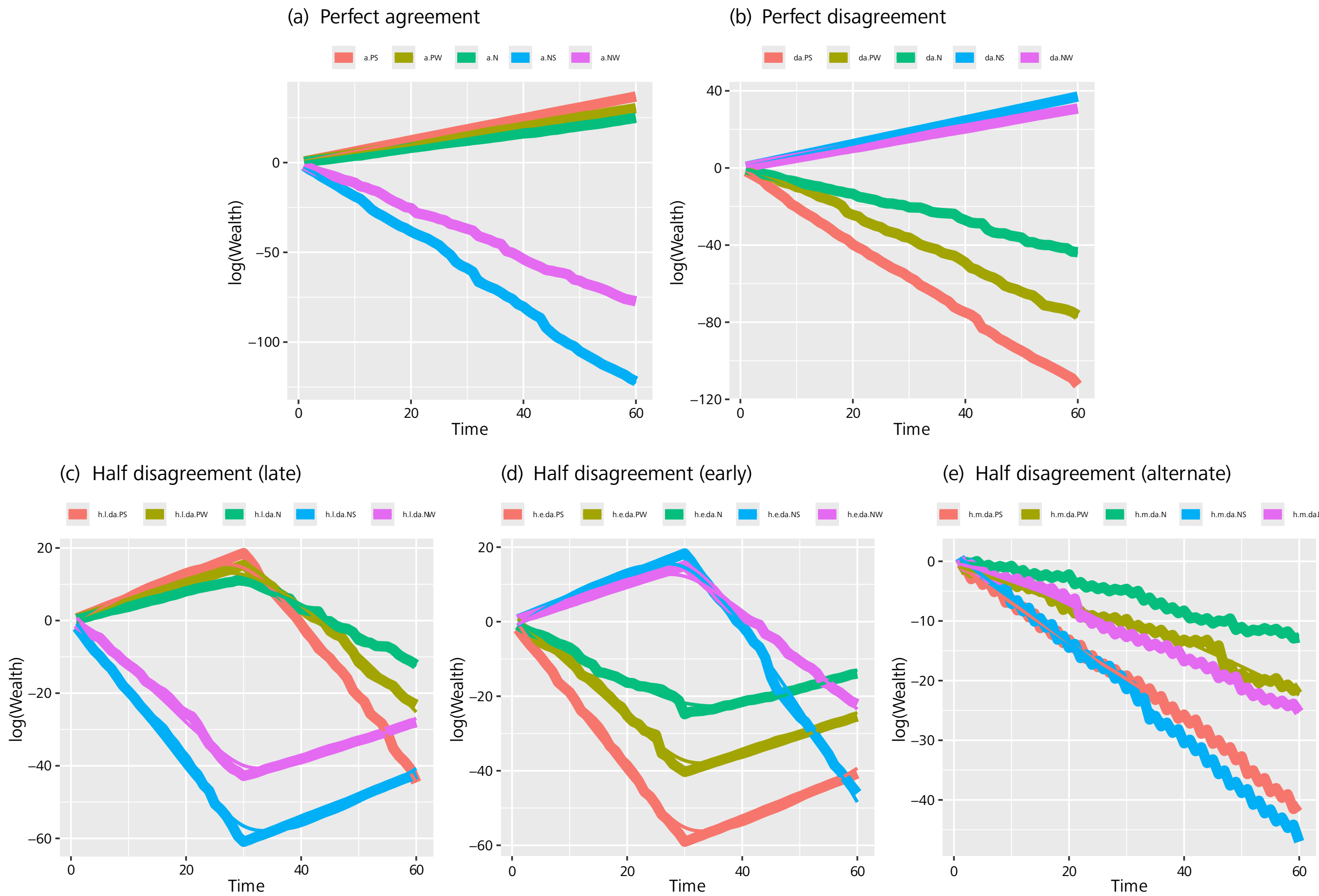


Figure 1. Simulated wealth profile scenarios under one-sided agreement or disagreement (panels (a) and (b)), and a mixture of agreements and disagreements (panels (c), (d), (e)). (e) Half disagreement (alternate)

significance. This, to Muriel, is a sweet ending because Fisher gives up his null claim and recognizes her tasting prowess.

Troubles begin if Muriel somehow got at most one wrong. The hostile statistical environment she had to perform under would have begun to show. The probability that a chance guess would trigger at most one error is $17/70 = 0.24$ (argue how) – inflating the p-value, making Fisher more prone to saying the order-detection is impossible, even though she did extremely well, getting the *majority* right.

Betting – the thrill of backing up your guess with some form of capital – rectifies this problem, summoning a fairness, through allowing Muriel not just voice her decision, but reveal how confident she feels in her verdict. Imagine Fisher conducting the experiment as a sequence, each time tossing a coin: if it lands head, he prepares $\{T, M\}$, if tails, $\{M, T\}$. Muriel, starting with an initial wealth of \$1, is expected to bet on the outcome of the toss. Table 1 describes the details.

The chief idea is this: if Muriel *is* bluffing, it would be nearly impossible for her to accumulate a huge fortune through random guessing. A running record of Muriel's wealth, then, would offer another way to test Fisher's hypothesis, only this time leveling the playing field somewhat (see the table above).

The storms spread beyond teacups

Eerily, you will find parallels everywhere. Rightly viewed, many statistical tasks we are routinely pitted against may morph into Murielesque prophecies. Predicting whether or not two respondents on the Gallup survey will agree (that is, both will respond with a 1 or a 2 or a 3 or a 4) on social issues such as whether businesses should prioritize increasing profits each year, or reduce the wage gap between executives and ordinary employees (a long list in [2, 6, 8]) may be seen as predicting the order of drink preparation (an observed agreement will, for instance, correspond to Fisher putting tea first). Guessers may want to build up a fortune through these games of predicting agreements and disagreements, instead of the one about predicting milk and tea.

toss outcome		order agreement		notations
head (H)	⇒	milk, tea $\{M, T\}$	⇒	$R = +1$
tail (T)	⇒	tea, milk $\{T, M\}$	⇒	$R = -1$

c_t: confidence; how much someone wants to bet on head at stage t ($-1 < c_t < 1$). (Muriel may choose 0.9 if she's extremely confident it landed head or -0.9 if she's extremely confident it landed tail.)
E_t: wealth at stage t ($E_t = E_{t-1}\{1 + c_t R_t\}$). (They are starting with \$1, i.e., $E_0 = 1$.)

toss outcome	Fisher prepares	Muriel		C		A	
		confidence	wealth	confidence	wealth	confidence	wealth
T (i.e., $R_1 = -1$)	$\{T, M\}$	$\{T, M\}$ ($c_1 = -0.8$)	$E_1 = 1\{1 + (-0.8)(-1)\} = 1.8$	$\{T, M\}$ ($c_1 = -0.1$)	$E_1 = 1.1$	$\{T, M\}$ ($c_1 = -0.9$)	$E_1 = 1.9$
H (i.e., $R_2 = +1$)	$\{M, T\}$	$\{M, T\}$ ($c_2 = 0.7$)	$E_2 = 1.8\{1 + (0.7)(1)\} = 3.06$	$\{T, M\}$ ($c_2 = -0.1$)	$E_2 = 0.99$	$\{T, M\}$ ($c_2 = -0.9$)	$E_2 = 0.19$
H (i.e., $R_3 = +1$)	$\{M, T\}$	$\{M, T\}$ ($c_3 = 0.9$)	$E_3 = 3.06\{1 + (0.9)(1)\} = 5.814$	$\{M, T\}$ ($c_3 = 0.1$)	$E_3 = 1.089$	$\{M, T\}$ ($c_3 = 0.9$)	$E_3 = 0.361$
T (i.e., $R_4 = -1$)	$\{T, M\}$	$\{M, T\}$ ($c_4 = 0.1$)	$E_4 = 5.814\{1 + (0.1)(-1)\} = 5.233$	$\{M, T\}$ ($c_4 = 0.1$)	$E_4 = 0.98$	$\{M, T\}$ ($c_4 = 0.9$)	$E_4 = 0.036$

Table 1. Players' wealth evolution. The players: (1) The truly great taster Muriel, (2) a person "C" who bluffs, but bluffs cautiously, and (3) a person "A" who bluffs, but bluffs aggressively.

With each question playing the role of a cup (we look at sixty questions here: the 11, 12, 13, 14 and 23 series detailed in [2, 3, 7, 8]), Figures 1 and 2 document (on a logged scale) the evolution of wealth as issues change. Skill and attitude are two organic imperatives that drive these evolutions. If the guessers are skilled (that is, can rightly predict agreements or disagreements), their wealth will increase (that is, the "cR" term in Table 1 will be positive) and if they are aggressive (that is if they put a lot at stake, that is, Cs are higher in magnitude) their wealth will get updated by huge amounts.

To test our intuitions, were erect five hypothetical guessers:

PS: who strongly (with c more than 0.7) believes that two individuals would agree,

PW: who weakly (with c more than 0.35) believes that the two individuals would agree,

N: who is neutrally positive and is unwilling to pick a side,

NW: who weakly (with c less than -0.35) believes the two individuals would disagree,

NS: who strongly (with c less than -0.7) believes that the two individuals would disagree,

and task each of them with predicting the agreement pattern under:

Case (i) (shown in Figure 1(a)): where the two people being checked for possible agreement are the same person (we took this to be the first person on the survey, but this can be anybody).

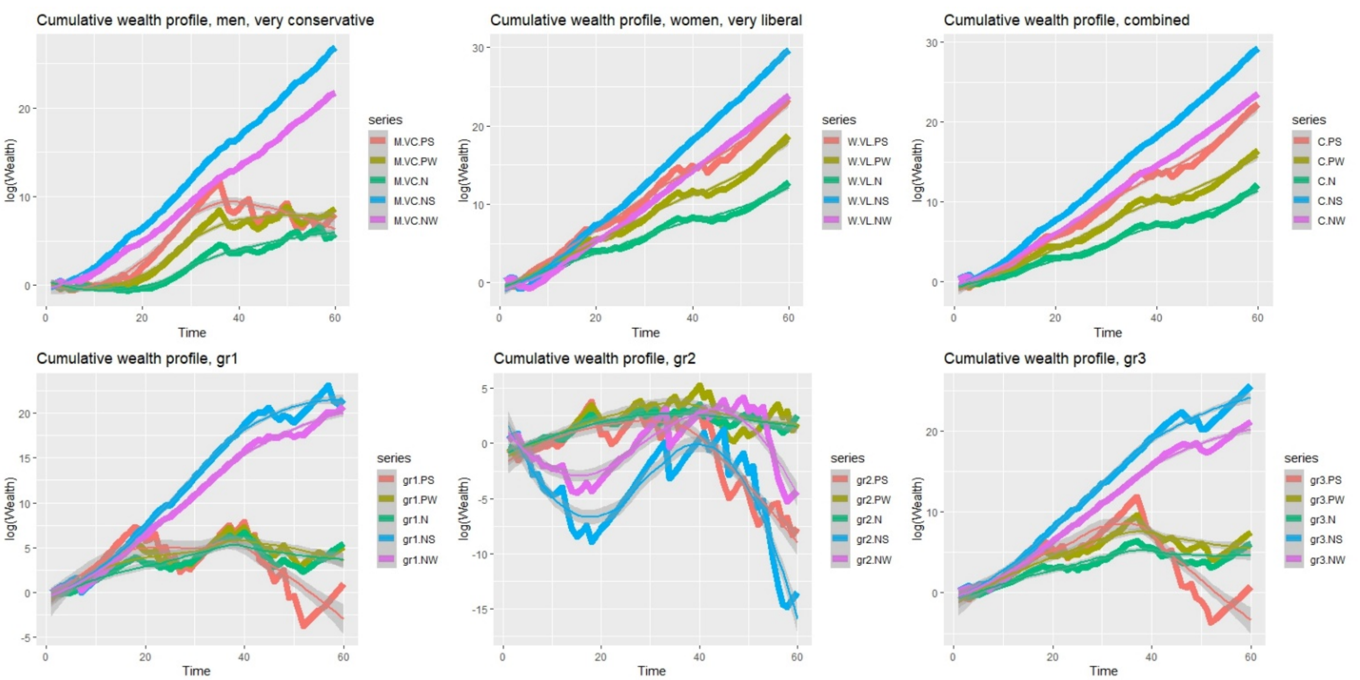


Figure 2. Wealth profiles under SM and UMM sampling schemes, PS, PW, N, NW, and NS betting strategies.

Case (ii) (shown in Figure 1(b)): where one never – not even once on the set of sixty questions – says what the other says.

Clearly, given case (i), that is, under the knowledge of perfect agreement (one, by definition, will agree with oneself consistently), the "P"-category guessers will seem to be the most skilled and among them, "PS" should amass the larger fortune, while the "N"s will seem to be unskilled and "NS" will suffer more. Figure 1(a) confirms this. The situation, expectedly, reverses perfectly under case (ii), when strongly saying people will disagree will be the most profitable (Figure 1(b)). Panels (c), (d), and (e) showcase the scenario when they agree only half the times and *when* that half happens. These are expected too.

On the fuller survey data, we extend this analysis and ask, for instance, if we knew the two people have similar demographic (similar gender, income, age, education, etc.) or political details, should we bet strongly that they would agree on these social issues? Or, if they have differing details, then bet strongly that they would disagree? However many details we record, there will generally be *many* individuals with these properties. Many pairs may, therefore, be formed (this did not happen in cases (i) or (ii) above, where, in each case, we had two people, and hence, just one pair). Depending on *how* we choose to form the pairs, we install the following sampling schemes:

Sure match (SM) scheme: when the two people picked surely have the same details (for instance, both are surely men, or both are surely women, if we are considering just one detail: gender).

Sure mismatch (SMM) scheme: when they surely have different details (surely one is a man, the other, a woman).

Unsure mismatch (UMM) scheme: when they may or may not have the same details (they could both be men, or both women, or one a man and the other a woman).

We add a general $G_k(x, y)$ guesser whose confidence on any guess never goes beyond the limit (x, y). If the gap between x and y increases, the guesser becomes chaotic (potentially changing his attitude a lot across the sequence), if it decreases, their attitudes become more stable. For instance, $G_P(0.7, 1) = \text{PS}$.

A line on Figure 2, at any time, represents the average wealth accumulated across comparing all possible pairs of people up to that point in time, according to that sampling-guesser profile (trend lines and confidence intervals are supplied for easier visualisation). The requirement that people should be similar across *many* demographic and political details (instead of a few) necessarily offers fewer potential pairs that may be formed for possible opinion agreement. This explains why the lines in the lower row on Figure 2 are more volatile than those in the first.

Generally, we may have felt that given that two people share similar details, the better who bets on their *agreement* (i.e., the G_P ones) will *accumulate* wealth and those that bet *aggressively* will accumulate *more* wealth. While this is true when the two are sampled from some sections of the population, at least initially – for instance, when both are very liberal women (Figure 2) – this is not the case in general, for instance, for very conservative-leaning men.

Figures 3 and 4 invite parameters such as income and education, in addition to gender and politics (under any mismatch scheme – UMM or SMM – we have sampled individuals from the extremes of these demographic axes to mimic the possibility of being maximally incongruous) under the chaotic (Figure 3) and more stable (Figure 4) betting attitudes (with an x-y range of 0.2). This attitude does not influence the way of wealth accumulation for certain groups (for instance, SMs for very conservative, low income, uneducated men, row 1, column 2, Figures 3 and 4), while it does for some others (for instance, SMs for very liberal, high income, educated women, row 2, column 2, Figures 3 and 4). The *amounts* of wealth variations, however, differ, even for the groups where the patterns and order of the five colors stay the same. Thus, for instance, saying very strongly that two very conservative, low income, uneducated men will keep agreeing will be a recipe for losing money. But one would lose *more* money (-40 on the log-wealth scale) under the chaotic scheme (betting 0.7 of the current capital on one question, 0.98 on the next) than (-30 on the log-wealth scale) under the stabler scheme (betting 0.7 of the current capital on one question, 0.75 on the next), for instance.

Under minimal background knowledge (Figures 3 and 4, column 1), nearly every strategy will *inflate* wealth as long as one consistently sticks to the strategy, although the amount of inflation would vary. As many details get revealed, frequently, we find results one may expect. While contrasting very *different* individuals (rows 3 and 4, column 2, Figures 3 and 4), betting (strongly or weakly) that they will *disagree* will be profitable in the long run. However, the opposite need not be true. With two people that are very *similar*, strongly betting on their *agreement* need not be prudent forever (rows 1 and 2, column 2, Figures 3 and 4). Neutrality may be recommended at these times.

In most of the examples we have documented, at least on the larger-sized combinations (that is where many pairs could have been formed, that is, on the lines with lower volatilities) the blue NS lines stay, generally, on top. Thus, if we are not sure of the nature of the people whose agreement is being questioned, betting strongly that they would disagree seems to be a sure way of accumulating wealth. *How much* will be accumulated, however, depends on the people being compared and the way of comparing. There are other benefits to tracking the evolution this way instead of simply reporting the final wealth. Figures 1(c) and 1(d), for instance, show how the ultimate wealth state (and the amount of fluctuations) could be similar, but the *way* to reach that state could be quite different. On nearly each diagram on Figures 2, 3 and 4 made with the real survey data, we note structural shifts around question 40 (with this specific *order* of querying), signaling the onset of a group of touchy issues and divisive questions that alter substantially the ongoing pattern of wealth accumulation – whether good or bad – fracturing a neat predictability.

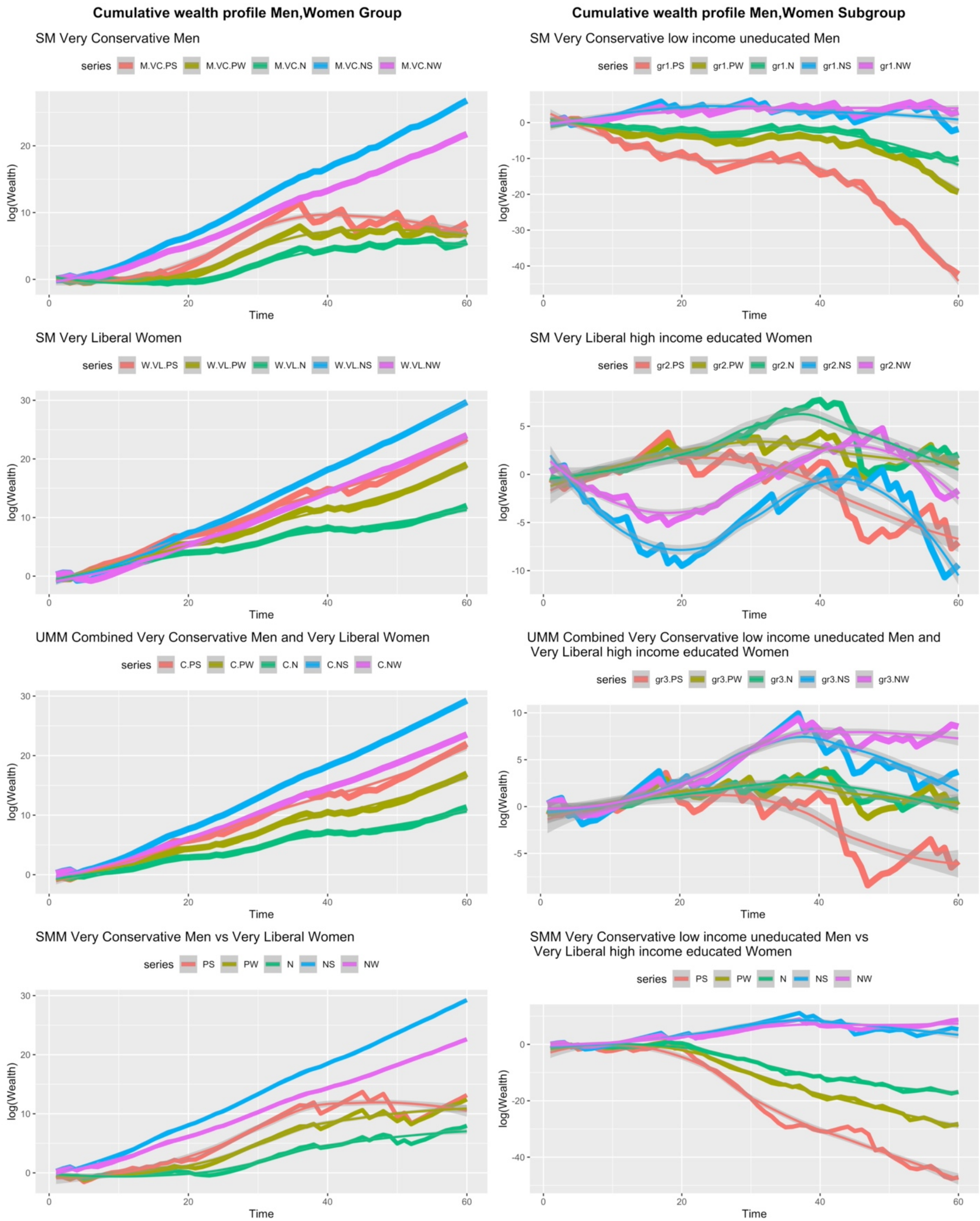


Figure 3. Wealth profiles under chaotic strategies outlined above. Many pairs go on to form column 1, fewer to form column 2 (which explains the stabler and wilder fluctuations). We recommend staying away from comparing across columns but comparing within a column.

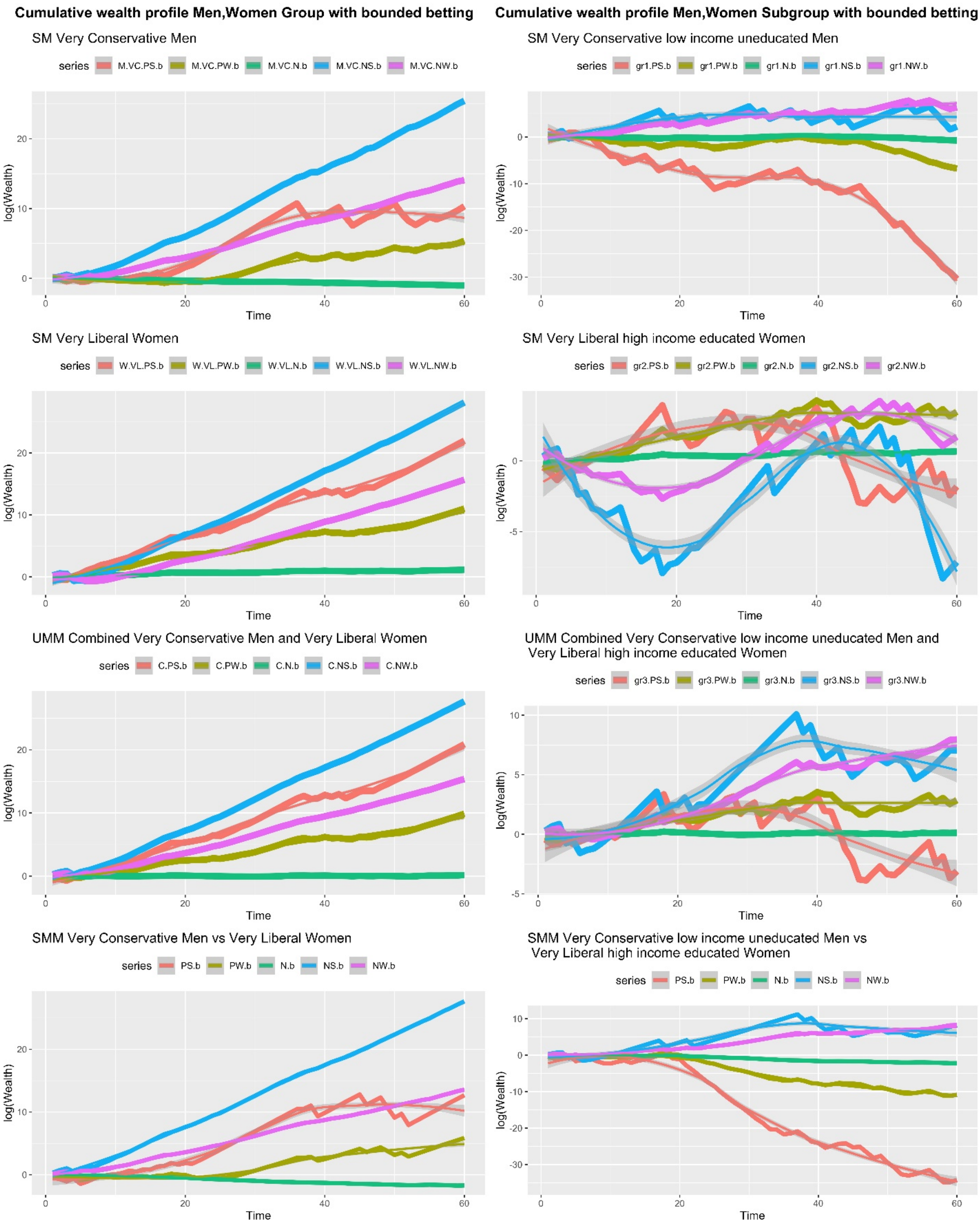


Figure 4. Wealth profiles under non-chaotic (i.e., stable) strategies (chaos range = 0.2). Many pairs go on to form column 1, fewer to form column 2 (which explains the stabler and wilder fluctuations). We recommend staying away from comparing across columns but comparing within a column.

People have checked whether two jury members would agree (a judge, for instance, may be interested in that).[1]

The "fortune" may not literally represent money, but some other version of capital: how long would the trial last, how long would a relationship thrive (between two people as long as they continue to agree on certain key things, between a researcher and a research topic as long as the researcher agrees on the interestingness of the topic). Basically, as long as guessing the agreement between two sides is of interest, all of this will go through. Or it could be literal money too. There is something called "bet-on agreement" that is similar.[2]

For relevant subtleties on gambling over public opinion, we point interested readers to [1]. Other authors have looked at the effect of demographic factors on betting tendencies too, in other ways, for other ends [4, 5].

You may not care much about betting. For frittering away time and money on pointlessly risky pursuits (*How to Gamble If You Must* may be a good read). Still, these line diagrams, through their ability of revealing the wealth you could have accrued, of pointing, more vitally, to deeper cracks in the societies' thought patterns, of showcasing fluctuating fortunes and other ramifications that result, become momentarily worth watching.

AI statement

The authors declare that no artificial intelligence was used either in the analysis behind or the writing of this manuscript.

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References

[1] D. Basak and J. Deb, Gambling over public opinion. *Am. Econ. Rev.* **110**, 3492–3521 (2020)

[2] M. Bhaduri, Do we let businesses get away with dodgy ethics and loose morals? An application of Markov random fields. *Chance (N. Y.)* **37**, 6–12 (2024)

[3] M. Bhaduri and B. Zahirodini, How businesses function. Podcast Stats + Stories, Episode 346 (2024) https://statsandstories.net/economics1/how-businesses-function

[4] S. Castrén, J. Kontto, H. Alho and A. H. Salonen, The relationship between gambling expenditure, socio-demographics, health-related correlates and gambling behaviour—a cross-sectional population-based survey in Finland. *Addiction* **113**, 91–106 (2018)

[5] T. A. Latvala, T. P. Lintonen, M. Browne, M. Rockloff and A. H. Salonen, Social disadvantage and gambling severity: a population-based study with register-linkage. *Eur. J. Public Health* **31**, 1217–1223 (2021)

[6] B. Zahirodini and M. Bhaduri, Pyramids of priorities and a poisoned omen: differing hierarchies of desired business ethics. *Nightingale* (2024) https://nightingaledvs.com/pyramids-of-priorities/

[7] B. Zahirodini and M. Bhaduri, The stuff we cannot sweep under a torn carpet: perfectly imperfect tessellations from slices of Maslow's triangle. *Nightingale* (2024) https://nightingaledvs.com/the-stuff-we-cannot-sweep/

[8] B. Zahirodini and M. Bhaduri, Marking contemporary America's attitudinal stance toward modern businesses: a Bayesian attempt to detect deviation patterns based on demographics and politics. *Chance (N. Y.)* **38**, 52–62 (2025)

[1] https://plaintiffmagazine.com/recent-issues/item/the-effect-of-jury-size-on-trial-outcomes

[2] https://www.investorinsights.asia/post/what-is-a-valuation-adjustment-mechanism-vam-bet-on-agreement

Dashboards for details and subtleties

- https://public.tableau.com/app/profile/bahareh.zahirodini/viz/Gallup-Bentley/Gallup-BentleyBusinessEthicsAnalysis
- https://moinak.shinyapps.io/GallupShinyM/
- https://moinak.shinyapps.io/Tessellations/
- https://statsandstories.net/economics1/how-businesses-function
- additional material: https://doi.org/10.6084/m9.figshare.29882372.v1

Bahareh Zahirodini (MSBA, Bentley University, 2024) is a research assistant working alongside Moinak Bhaduri at the Department of Mathematical Sciences at Bentley University, Massachusetts. Her work focuses on financial modeling and its intersection with social and demographic issues. She has served as a teaching assistant and trading room analyst at Bentley's Hughey Center for Financial Services and is a member of Beta Gamma Sigma, an international business honor society (for business major students who are in the top 20 percent of graduates).

bzahirodini@falcon.bentley.edu

Moinak Bhaduri (PhD, mathematical sciences, University of Nevada, Las Vegas, 2018) is currently an associate professor at the Department of Mathematical Sciences at Bentley University in Massachusetts. Rooted in applied probability, he studies spatio-temporal Poisson processes and others, such as the self-exciting Hawkes or log-Gaussian Cox processes that are natural generalizations. His primary interests include developing change-detection algorithms in systems modeled by these stochastic processes, especially through trend permutations. Bhaduri heads the editorial board of the NextGen column of the New England Journal of Statistics in Data Science.

mbhaduri@bentley.edu

Obituary of Peter Lax

Constantine M. Dafermos

Peter David Lax passed away on May 16, 2025, having lived a long and riveting life. He was the last of the generation of great analysts who, in the mid-twentieth century, set the current directions in the field of partial differential equations.

Peter was born to a Jewish family in Budapest on May 1, 1926. His parents were established physicians, so he and his older brother John grew up in a comfortable, intellectual family environment. Peter demonstrated extraordinary mathematical ability from an early age.

The life of the family was turned upside down by the coming of World War II. Fortunately, the American consul in Budapest was a patient of Peter's father and secured for them visas to the United States. Peter often recounted his memories as a fifteen-year old traveling in a sealed train from Budapest to Lisbon, through Germany, and boarding the last steamship to New York on October 5, 1941, just two days before the attack on Pearl Harbor.

Peter's father set up a successful practice in New York City, with notable patients such as Béla Bartók, Alexander Korda and Greta Garbo. Eventually, the family settled in an apartment house on the Upper West Side which has become the residence of three Lax generations.

Peter's reputation as a mathematical prodigy having preceded his arrival in the U.S., he was soon introduced to his famous compatriots John von Neumann and Gábor Szegő and also to Albert Einstein. On the recommendation of Szegő, who was actually a relative of Peter, he enrolled at the age of sixteen in the mathematics program of New York University, directed by Richard Courant.

Two years later, Peter's studies were interrupted when he was drafted into the U.S. Army. Fortunately, he was assigned to the Manhattan Project and spent the bulk of his army service at Los Alamos, where he had the opportunity to become acquainted with prominent physicists and applied mathematicians—both senior, including Enrico Fermi, Edward Teller and Stanisław Ulam, and of his own generation, Richard Feynman, Richard Bellman and John G. Kemeny. Peter was particularly impressed by the intellectual power and scientific breadth of von Neumann, who thence became his hero.

At Los Alamos, Peter witnessed the effectiveness of applying mathematics to real-life problems and in particular the usefulness of scientific computation. These experiences are manifested in his future teaching and research. It thus appears that in addition to changing the course of his life, the Second World War also set the directions of his scientific work.

In 1946, Peter was discharged from the army and returned to NYU in order to complete his undergraduate work and to enroll in the graduate program. As a graduate student, he was fortunate to be exposed to the teaching of world-class experts in partial differential equations, including Courant, Kurt Friedrichs, and Fritz John, and also to be in the company of classmates like Joe Keller, Cathleen Morawetz, Harold Grad, Louis Nirenberg, and Martin Kruskal, who along with him would join the ranks of the next generation's leading mathematicians. He completed his dissertation on hyperbolic equations, under the direction of Friedrichs, and was awarded the PhD in 1949.

Peter and several of his gifted classmates joined the faculty of NYU. The new appointees Lax, Nirenberg and Morawetz, together with Courant, Friedrichs and John of the older generation, rendered NYU the Mecca of partial differential equations in the U.S. throughout the mid-twentieth century.

As a researcher, Peter possessed a talent for early recognition of fertile open fields, along with the insight to discern internal, possibly simple, structures underlying complex mathematical phenomena. Illustrative examples of Peter's contributions that look deceptively simple and yet played a central role in the development of the theory of linear partial differential equations and of numerical analysis in the 1950s are the Lax-Milgram lemma and the Lax equivalence theorem. Another demonstration of Peter's insight is his work on integrable systems, in the late 1960s: A large family of partial differential equations had been derived over the years which possessed an infinite number of conserved integrals as well as soliton solutions with remarkable interaction properties. By introducing the celebrated Lax pairs, Peter solved the mystery by uncovering the abstract underlying structure shared by these diverse equations.

Another major contribution of Peter Lax, which opened a new chapter in the theory of partial differential equations, stemmed from his work on quasilinear hyperbolic systems in divergence form, which he dubbed hyperbolic conservation laws, a term that has now become standard. Numerous systems of this type, manifesting the conservation laws of classical physics, had been derived and studied over the years, beginning with the Euler equations that govern isentropic gas flow. In a seminal paper published in 1957, Peter distilled the diverse information that had been amassed over a period of two centuries and developed a systematic formalism for hyperbolic systems of conservation laws in a single spatial dimension which set the direction of research in the area up to the present time. Peter himself made further important contributions to that field by abstracting the notion of entropy and by building, in collaboration with Jim Glimm, a complete theory of BV solutions to pairs of conservation laws. He also contributed to the numerical analysis of hyperbolic conservation laws by developing, together with Burton Wendroff, the method associated with their names.

Of course, Peter Lax did important notable research in several other fields, for example in scattering theory, in collaboration with Ralph Phillips. However, his impact extends beyond his printed papers. Indeed, his views on applied mathematics circulated widely. He was an effective teacher and mentored a number of gifted students who then spread the word. He lectured extensively around the globe and participated in countless panels and governing boards.

Peter Lax also played a leading role in scientific administration, beginning with his service as Director of the Courant Institute over the period 1972 to 1980. He also served as Vice President and then President of the American Mathematical Society, as a member of the Board of Governors of the National Science Foundation, and on innumerable program evaluation committees.

Lax's contributions were amply recognized by the international scientific community and he received a great number of honors, including major ones such as the Abel Prize, the Wolf Prize and the National Medal of Science. He was a member of the National Academy of Sciences, the Soviet Academy of Sciences, the French Academy of Sciences, the Hungarian Academy of Sciences and the Academia Sinica.

The single word that would describe Peter's personality is "charm," which radiated to people of any age and gender. His charm was inseparable from his generosity, in scientific matters as well as in everyday life. He encouraged and nurtured his numerous students together with many other younger mathematicians. In fact, my first acquaintance with Peter and his work, back in 1969, set to a great extent the direction of my scientific work.

Peter was very cosmopolitan. He arrived in the U.S. young enough to become fully immersed in American culture and yet mature enough to have retained vestiges of the culture of his native Hungary, including slight traces of a Hungarian accent. His cosmopolitanism was further nurtured by his tenure at the Courant Institute, in the company of teachers, colleagues and students from all over the world. He was also a fabled storyteller about his encounters with notable people.

Peter Lax receiving the 2005 Abel Prize from His Royal Highness Crown Prince Haakon in the aula of the University of Oslo. (Photo: Scanpix, by courtesy of The Abel Prize)

Peter was a keen follower of national and international politics. He was of liberal persuasion and abhorred totalitarian regimes, left and right. My impression is that he was a pragmatist rather than an ideologue. In fact, as I recall, he once told me that "moderation" was his favorite word in the lexicon.

In 1948 Peter married Anneli Cahn, a classmate in graduate school and later a colleague at the Courant Institute. They had two sons, Johnny and Jimmy. Unfortunately, Johnny died in 1982, in a tragic traffic accident, and this was a severe blow to his parents and I could see Peter's emotional reaction whenever he visited Brown University, which was Johnny's alma mater. Jimmy became a physician and stayed close to his parents. Anneli died of cancer in 1999 and in 2006 Peter married the violist Lori Berkowitz, who was the widow of the mathematician Jerry Berkowitz and the daughter of Courant; she also preceded him in death.

It is amusing to observe the quirks of famous scientists. Since the time he was young, Peter often dozed during lectures, and lecturers who were not aware that this was habitual felt disappointed and occasionally even insulted. At the conclusion of lectures, Peter made sure to ask relevant questions to the lecturer to demonstrate that he could follow even when he was asleep!

Peter's life touched the lives of many people. Neither the man nor his work will be forgotten.

Constantine M. Dafermos is an applied mathematician working at the interface between continuum mechanics and partial differential equations. He was born in Athens, Greece. He graduated from the National University of Athens in 1964 with a diploma in civil engineering and from the Johns Hopkins University in 1967 with a PhD in mechanics. After serving as a postdoctoral fellow at Johns Hopkins in 1967–1968, and as an assistant professor at Cornell University in 1968–1971, he joined the faculty of Brown University, where he is now serving as the Alumni-Alumnae University Professor Emeritus of Applied Mathematics.

constantine_dafermos@brown.edu

The Fermat Museum in Beaumont-de-Lomagne (France), a new place to love mathematics

Maryvonne Spiesser

Since last July (2024), a new museum dedicated to mathematics and its key figures has opened in Beaumont-de-Lomagne,[1] the birthplace of Pierre (de) Fermat. The *Fermat Science Association*, founded in the 1990s and responsible for the design of this museum, has extensive experience in promoting mathematics to the general public and schoolchildren. With the opening of this museum, the association is fulfilling its initial goals: promoting access to mathematical culture for all, popularizing mathematics, and energizing the birthplace of Pierre de Fermat.

The museum is housed in the private mansion of the Fermat family. The building dates back to the 16th century, and in this historic location, architects have created an outstanding museum structure while preserving the most prestigious elements of the former residence. Beaumont-de-Lomagne is a town of about 4,000 inhabitants located in Tarn-et-Garonne, at about sixty kilometers from Toulouse. The challenges of establishing the museum in a rural area, far from cultural centers, as well as the expected audience, mainly families and school groups, led to the design of an original space divided into three sections: at the start of the visit, the focus is on the architectural heritage and 17th-century history; then the mathematics of the 17th century; and finally, contemporary mathematics.

Pierre de Fermat (engraving by François Poilly, ca. 1679).

The Fermat family mansion. (Courtesy of Fermat Science Association)

We first step into the intimate world of Pierre de Fermat. A hologram recreates the mansion as it appeared in the time when the mathematician lived there. A family tree represents the three families from which Pierre de Fermat and his wife Claire Delong came. It also shows the interactions between the two religious groups, Catholic and Protestant, as well as the evolution of society towards an increasing importance of the merchant bourgeoisie and its rise to the nobility through the judiciary. Fermat was appointed magistrate to the Toulouse Parliament in 1631. He lived between his hometown, Toulouse, and Castres (at one-hour drive from Toulouse), where he was repeatedly appointed to sit in the *Chambre*

[1] At one-hour drive North-West from Toulouse (south of France). At 45 minutes from the international airport Toulouse-Blagnac, 35 minutes from Montauban, 25 minutes from Castelsarrasin. There is no railway station in Beaumont-de-Lomagne, but there are such stations in Montauban and Castelsarrasin.

The 17th-century corridor. (Courtesy of Fermat Science Association)

de l'Édit, a chamber composed equally of Catholic and Protestant magistrates.

The major events of the 17th century, whether political, religious, or scientific, are recounted through a so-called "game of goose," created for this purpose.

In the next section, we imagine Fermat's library. We know his main readings from his correspondence. Among the key documents in this library are Apollonius of Perge's *Conics* and Diophantus' *Arithmetica* in a Greek-Latin version. Fermat owned a copy of the latter (now lost), in which he wrote in the margin the well-known conjecture that was made famous after its proof by Andrew Wiles in 1994. Also highlighted are Viète's *Ars Analytica*, Fermat's favorite book from which he gained his algebraic knowledge, and Bachet de Méziriac's *Problèmes plaisants et délectables*. Several public activities, such as magic squares, are based on this book of "mathematical recreations."

On the mathematical side, a room is dedicated to the scientific fervor of the early 17th century, to the *République des Lettres* originally fostered by Father Mersenne, who created a European communication network for scholars. A focus on the cycloid, the "queen of curves," which fascinated many of Fermat's contemporaries, illustrates this idea.

The final section of the permanent exhibition is dedicated to contemporary mathematics. Interactive devices explore some areas of modern mathematics linked to Fermat's work (probability, optimization, number theory).

The contribution on probabilities is discussed through the *Problème des partis*, which was the main subject of the correspondence between Fermat and Pascal in 1654. A game of chance between two players (with initial stakes) is interrupted before one of them wins. The question is how to fairly divide the stakes based on the number of rounds each player has accumulated ("sharing of stakes"), implying that, if the game were to resume, each player should bet the amount they received. A video game has been created to help understand this problem.

Fermat was one of the pioneers of future infinitesimal calculus with his *Method for finding maxima and minima* (heavily contested by Descartes). This area of mathematics is represented in the museum by a narrated video about optimal control, regarding the calculation of satellite trajectories (orbital transfers).

Finally, the story of *Fermat's Last Theorem*, which captivated the most eminent mathematicians for several centuries before making headlines worldwide in the 1990s, is presented through a comic book and an exclusive interview with the main author of the proof, Andrew Wiles.

A room is also dedicated to temporary exhibitions. The first of these is titled *Enter the World of AI*. It was created by Fermat Science in partnership with the House of Mathematics and Computer Science in Lyons and the Poincaré House in Paris (two museums on mathematics which the EMS Magazine has covered in recent issues). The next exhibition will start in January 2026, on mathematics and cooking (*Dans ma cuisine*).

Children aged 3–8 are also given significant consideration. A space is dedicated to them, currently organized around geometric shapes through construction games. This area is set to evolve to feature "Emmy the Fox," an activity created within the framework of the *Fermat Science Association*'s relations with its European partners, which will invite younger audiences to appreciate mathematics as soon as January 2026.

In this unique place, aimed at family, tourists, and school audiences, we sought to involve visitors as much as possible throughout the journey, which is made possible by the technological tools available today. Manipulations, puzzles, and challenges are part of the experience.

Our goal is to immerse the visitor in the life of a 17th-century family, honor the memory of Pierre de Fermat, and transmit his passion for mathematics to the public, offering "another perspective on mathematics" ("*une autre idée des mathématiques*") by providing an innovative space around scientific and mathematical culture. These are the ambitions that guided the creation of this space.

More information and details on the website museefermat.com; contact at contact@museefermat.com.

Maryvonne Spiesser was a lecturer in mathematics and history of mathematics at the University of Toulouse 3, France. Her research focuses on the history of practical arithmetic and algebra at the end of the Middle Ages, and on 17th-century mathematics, particularly on Fermat's work. She is involved in the popularization of mathematics and is vice-president of the Fermat Science Association.

maryvonne.spiesser@orange.fr

Nineteen years of ICMAT: strengthening excellence in mathematical research

Javier Aramayona and Ágata Timón García-Longoria

The Institute of Mathematical Sciences (ICMAT) in Madrid, Spain, celebrates this year its nineteenth anniversary, marking its consolidation as an internationally recognised research centre spanning all areas of mathematics. ICMAT is a joint initiative of the Spanish National Research Council (CSIC, the country's largest public institution devoted to scientific and technical research, under the Ministry of Science, Innovation and Universities) and three leading public universities in Madrid with strong traditions in mathematics: the Universidad Autónoma de Madrid (UAM), the Universidad Complutense de Madrid (UCM), and the Universidad Carlos III de Madrid (UC3M).

ICMAT was founded in 2007 and moved in 2010 to its current building, a modern facility offering excellent infrastructure for mathematical research. Despite its short history, the institute has established itself as a leading mathematics centre in Europe, both in terms of research excellence and scientific activity.

ICMAT's international standing is reflected in several indicators. To date, its researchers have secured 12 European Research Council grants across the Starting, Consolidator and Advanced categories, amounting to about half of all ERC funding awarded in Spain within the Mathematics panel (PE1). These projects span areas such as singularity formation in the Euler and Navier–Stokes equations; geometry-driven phenomena in fluid mechanics, PDEs and spectral theory; harmonic analysis; partial differential equations and geometric measure theory; quasiconformal methods and their applications; and restriction phenomena for the Fourier transform with implications for Schrödinger and wave equations. In these and other fields, ICMAT researchers are recognised as international leaders.

In addition, the institute has strengthened its presence at the discipline's leading forum, the International Congress of Mathematicians. At the upcoming 2026 ICM in Philadelphia (USA), two ICMAT researchers will be invited speakers: David Pérez García, for his work on the mathematical classification of quantum phases of matter, and Javier Parcet, for his advances on Connes' rigidity conjecture in functional analysis and geometric group theory. This follows the invitation of Diego Córdoba as a speaker at the 2018 ICM in Rio de Janeiro.

ICMAT's development has been supported to a significant extent by the Severo Ochoa and María de Maeztu Programme for centres and units of excellence, established in 2011 by the Spanish Ministry of Science, Innovation and Universities to identify and fund research structures of outstanding international impact. Each Severo Ochoa accreditation provides €4 million over a four-year period, together with approximately ten PhD fellowships. ICMAT has been part of the programme since its inception, receiving distinctions in 2011, 2016, 2019 and 2023, making it one of only five research centres in Spain, across all disciplines, to have obtained four such awards to date. This support has enabled the institute to implement its scientific strategy, firmly grounded in research excellence, high-impact scientific activity, the training of young researchers and a sustained commitment to strengthening the presence and value of mathematics within society.

Figure 1. ICMAT is located on the UAM+CSIC Campus of International Excellence in Madrid. (Credit: Álvaro Minguito/ICMAT)

Over 200 people committed to mathematical research

ICMAT's evolution under the Severo Ochoa programme becomes evident when examining changes in personnel between 2010 (prior to the first award) and 2022. In 2010, the institute comprised 40 faculty members, 20 postdoctoral researchers, 29 graduate students

Figure 2. The ICMAT facilities make it possible to host an intense program of scientific activities.

Figure 3. A large part of the centre's activity is funded by the Severo Ochoa programme of the Ministry of Science, Innovation and Universities. (Credit: Álvaro Minguito/ICMAT)

and 5 management staff. By 2022, these numbers had risen to 65 faculty, 50 postdoctoral researchers, 45 graduate students and 11 management staff.

Permanent researchers are senior and highly influential scientists, many of whom have received prestigious distinctions. Notably, Diego Córdoba was awarded Spain's National Research Prize in 2023 and Antonio Córdoba, former director of the centre, received the same prize in 2011. In addition, Alberto Enciso was honoured with the Princess of Girona Prize in 2014, and David Ríos Insua was awarded the AXA–ICMAT Chair in Risk Analysis. Several ICMAT researchers are also members of the Royal Spanish Academy of Sciences.

The vibrant international postdoctoral community at ICMAT, comprising around 60 researchers at any given time, plays a key role in the institute's scientific dynamism. These early-career scientists are attracted through highly competitive calls and typically undertake two-year research stays, often supported by prestigious national and European fellowships. Their presence contributes to a constant renewal of research lines and fosters strong international collaborations. In parallel, ICMAT provides an outstanding training environment for young researchers: about 45 doctoral candidates are currently pursuing their PhD dissertations under the supervision of permanent staff, benefiting from close mentoring and an active programme of seminars, workshops and visiting scholars, while around ten master's students each year are offered a structured introduction to research and receive close supervision for the development of their master's theses.

Moreover, the institute further benefits from an excellent management team consisting of 11 people, working in areas such as IT services, project and grant management, outreach and communications, travel administration and the organisation of scientific events, providing essential support for the effective operation of the centre. This team has been created thanks to the Severo Ochoa programme, and its work is essential to the development and functioning of the institute's scientific activity.

In line with its commitment to nurturing talent at all career stages, ICMAT has also developed several initiatives addressing equality and gender issues within the mathematical community, a field in which a significant gender gap persists worldwide. These sustained efforts were formally recognised with the CSIC Gender Equality Accreditation Distinction, awarded to the institute in 2021.

Research structure

ICMAT's research activity is currently organised into three main scientific groups:

- Group A: Algebra and Geometry. Focused on areas such as abstract algebra, algebraic geometry, differential geometry and topology, including fundamental questions in group theory, number theory, geometric mechanics and their applications in mathematical physics.
- Group B: Mathematical Analysis and Differential Equations. Encompassing harmonic analysis, partial differential equations, geometric group theory and functional analysis, with research ranging from classical analytical problems to applications in mathematical physics.
- Group C: Applied Mathematics. Working at the interface of mathematics with data science, machine learning, quantum information and mathematical modelling for scientific and technological applications.

Research conducted at ICMAT is regularly published in leading mathematics journals, including Annals of Mathematics, Acta Mathematica, Inventiones Mathematicae, the Journal of the American

Mathematical Society, Duke Mathematical Journal and the Proceedings of the National Academy of Sciences, among others.

These achievements also translate into research projects funded through various competitive instruments, both national and international. Beyond the ERC grants, ICMAT participates in European Union schemes ranging from Marie Skłodowska-Curie Actions in Pillar I to projects within Pillar II, Global Challenges and European Industrial Competitiveness. In Spain, ICMAT researchers lead over 30 research projects annually, primarily funded by the Spanish State Research Agency (AEI), the main public funder of basic research in the country.

Intensive research activity

A significant proportion of ICMAT's scientific activities is funded through the Severo Ochoa programme. A central component of this scientific programme is the Severo Ochoa Laboratories and Distinguished Visiting Professors Programme, designed to strengthen research groups led by internationally renowned external researchers.

The ICMAT–Severo Ochoa Laboratories create local research clusters led by one or two exceptionally strong mathematicians, selected through a competitive international process. Each laboratory receives funding for scientific activities, including the organisation of a thematic programme, as well as support for predoctoral and postdoctoral positions. They provide a natural framework for collaboration among research lines aligned with the interests of the laboratory director(s), enhance ICMAT's international visibility and contribute significantly to the training of young researchers.

The fourth edition of the programme, launched in 2024, features eight laboratory directors: Martin Bridson, Ignacio Cirac, Charles Fefferman, Ngô Bảo Châu, Nigel Hitchin, Gilles Pisier, Alan Reid and Mikael de la Salle. The current distinguished visiting professors are Bruno Anglès, Elena Celledoni, Monika Ludwig, Eugenia Malinnikova, Eero Saksman and Eva Miranda.

Since its creation in 2012, the programme has included prominent laboratory directors such as Ian Agol, Simon Donaldson, Viktor Ginzburg, Marius Junge and Stephen Wiggins, and Distinguished Visiting Professors including Kari Astala, Anthony Bloch, Filippo Bracci, Anthony Carbery, Juncheng Wei, Marius Junge and Rafael de la Llave.

Fostering mathematical vocations

As part of its scientific strategy, ICMAT maintains a strong commitment to training new generations of mathematicians. Beyond its participation in standard master's and doctoral programmes, the institute develops its own initiatives aimed at fostering and identifying early mathematical talent.

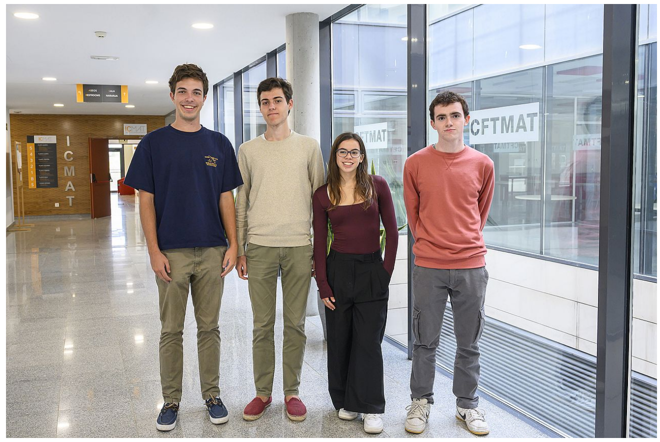

Figure 4. In 2025, four first-year undergraduate students in mathematics began the ICMAT Mathematics Intensive Programme. (Credit: Álvaro Minguito/ICMAT)

In 2025, ICMAT launched the Mathematics Intensive Programme (MIP), the first intensive training programme for undergraduate mathematics students in Spain. The aim of MIP is to offer a very small group of undergraduates with extraordinary mathematical talent a high-level complementary education in a stimulating environment. The programme fills an important gap in the Spanish system, which previously lacked a structured initiative to nurture the early development of highly capable university students, as found in other European countries.

Four students have taken part in this inaugural edition, selected from more than 70 applicants. Each has been assigned an ICMAT researcher as mentor for the duration of their degree. In addition to mentorship, MIP offers advanced minicourses and other activities, open upon registration to other undergraduates. At the end of each academic year, participants present a mathematical project before ICMAT researchers, and upon completing their undergraduate studies receive an official certificate from the Spanish Research Council (CSIC) accrediting their participation in the programme.

Through MIP, ICMAT expands its long-standing commitment to mathematical training, which already includes the Pequeño Instituto de Matemáticas (PIM). In this programme, every Friday during the school year, 150 students aged 12 to 18 gather at ICMAT to explore mathematics in an engaging, problem-solving environment that differs from standard classroom instruction. Launched in 2022–2023, PIM provides a setting for young people passionate about mathematics to develop their interest and curiosity.

ICMAT also organises the JAE School of Mathematics, held annually for senior undergraduate and master's students. Over two weeks, around 100 participants engage directly with cutting-edge research topics and interact with active researchers. In parallel, the Severo Ochoa Introductory Research Fellowships (INTRO-SO) offer 25 undergraduate students (and exceptionally, master's

Figure 5. ICMAT is committed to fostering interest in mathematics among students at different educational levels. (Credit: Álvaro Minguito/ICMAT)

students) a one-month training placement supervised by an ICMAT researcher, introducing them to the institute's research lines and scientific activity.

Beyond the mathematical community

ICMAT is also a pioneering centre in the communication and dissemination of mathematics. In 2012, it created its Mathematical Culture Unit, staffed by professionals dedicated to outreach and public engagement who work closely with researchers and partner institutions to bring mathematics to diverse audiences. In 2014, the unit was recognised as a Scientific Culture Unit by the Spanish Foundation for Science and Technology, becoming the only specialised mathematics unit with this distinction. Its head, Ágata Timón, has been invited to participate in a round table on mathematics communication at ICM 2026 in Philadelphia (USA).

The unit produces original content on ICMAT's scientific activities (newsletters, website news items and interviews, and audiovisual material for major social networks) and collaborates actively with mainstream media. For ten years, it coordinated the mathematics section Café y teoremas in El País, publishing around 400 articles by contributors from around the world and reaching more than one million readers annually. Two new sections succeed this initiative in 2025: Dimensión Fractal in elDiario.es and Entre teoremas in ABC.

In addition, the unit organises around thirty workshops, lectures and other activities each year for general and school audiences, presenting mathematical topics in an engaging and accessible manner.

ICMAT also plays an active role in the structuring of Spanish and European science. Nationally, its director, Javier Aramayona, serves as second vice-president of SOMMa, the alliance of Severo Ochoa centres and María de Maeztu units, of which ICMAT is a member. At the European level, he is vice-president of ERCOM, the committee of the European Mathematical Society that brings together Europe's leading mathematics research centres. ICMAT has been a member of ERCOM since 2016.

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Javier Aramayona is a permanent researcher at the Spanish National Research Council (CSIC) and works at the Institute of Mathematical Sciences (ICMAT), where he serves as director since October 2022. He earned his PhD from the University of Southampton in 2005 and has held academic positions in the U.K., France, Ireland, and Spain before joining CSIC in 2020. His research focuses on geometry, topology, and group theory, and his work has been widely published in leading mathematics journals. He currently holds institutional roles at the European and national levels, serving as vice-chair of the European Research Centres on Mathematics (ERCOM) and as vice-president of the SOMMa (Alliance of Severo Ochoa Centres and María de Maeztu Units), and he has a strong interest in scientific policy, particularly in the articulation and strengthening of the Spanish and international research systems and in the creation of a more cohesive and inclusive scientific system.

javier.aramayona@icmat.es

Ágata Timón García-Longoria has coordinated the Mathematical Culture Unit at the Institute of Mathematical Sciences (ICMAT) since its founding in 2012. She holds a degree in mathematics from the Complutense University of Madrid, as well as two master's degrees from the Carlos III University of Madrid: one in science, technology and environmental communication and journalism, and another in applied research in media. Throughout her career, Timón has focused on promoting mathematics through a wide range of formats, primarily written media, but also video, audio, and live events such as workshops, lectures, and theatrical performances. She is the author of five popular science books and, since 2016, has coordinated Café y teoremas, a mathematics outreach section in El País, the leading Spanish-language newspaper. She has published over 250 articles in the media, mainly on science and mathematics. Now she coordinates Dimensión fractal in elDiario.es and Entre teoremas in ABC.

agata.timon@icmat.es

Renewing doctoral education in Finland – with applied mathematics at the forefront

EMYA column regularly presented by Jesse Railo

Mikko Helenius, Tanja Tarvainen and Siiri Utriainen

In 2024, Finland's Ministry of Education and Culture announced a pilot programme for renewing doctoral education practices in Finland, with an aim at shortening graduation times and enabling faster employment of doctors by companies. This pilot programme aims to train 1000 new doctors in three-year-long pilot projects. One of the selected pilot projects is the Doctoral Education Pilot for Mathematics of Sensing, Imaging and Modelling (DREAM), which is a consortium formed by partners in seven Finnish universities, aiming to pilot doctoral training practices in the fields of applied mathematics, physics, computing and imaging sciences.

Finland – the land of Nokia, Oura, ICEYE and Supercell to name a few – has a vibrant technology industry supported by the nation's rich history in technological research. Its role in the Finnish economy is significant, accounting for 12% of Finnish employment and producing more than half of the country's exports of goods and services [1].

For years, Finland has aimed to increase its research and development (R&D) financing to four percent of its gross domestic product (GDP) by 2030. This goal was set in 2017 by the Research and Innovation Council, led by the prime minister, and again by the National Roadmap for Research, Development and Innovation in 2020 [6]. A national legislation on central government R&D expenditure, a unanimous commitment in the Parliamentary Research, Development and Innovation (RDI) working groups, and a multiannual funding plan have since followed to realise this target.

This ambitious goal also underlines the importance of increasing the number of top RDI talents in the Finnish workforce. In February 2024, the Ministry of Education and Culture (OKM) announced that it would allocate €255 million to Finnish universities to implement a doctoral pilot programme [5]. By providing funding to recruit 1000 new doctoral researchers, this initiative aims to increase the number of doctorates, enhance and develop doctoral education practices, strengthen the RDI-related talent pool, and increase Finland's international competitiveness [4].

After an open call to universities, 15 pilot projects were chosen based on an international review carried out by the Research Council of Finland. Nine of these pilot projects take place in the research fields of the Flagship Programme of the Research Council of Finland, and six pilots' research fields were chosen freely. One of the selected pilot was the *Doctoral Education Pilot for Mathematics of Sensing, Imaging and Modelling (DREAM)*.

1 Doctoral Education Pilot for Mathematics of Sensing, Imaging and Modelling

The Doctoral Education Pilot for Mathematics of Sensing, Imaging and Modelling is one of the thematic projects of the doctoral education pilot programme. It originates from the Flagship of Advanced Mathematics for Sensing, Imaging and Modelling (FAME), which serves as a foundation for the DREAM pilot's core research community, methodological expertise, and industry-driven training.

1.1 Flagship of Advanced Mathematics for Sensing, Imaging and Modelling

Since 2018, the Research Council of Finland has funded the Finnish Flagship Programme as part of the Finnish government's research and innovation goals. Supported by a long-term funding, this initiative facilitates high-quality research ecosystems, called flagships, each working on their focused areas, for example, artificial intelligence, 6G, water resources, climate change mitigation, chronic diseases, and quantum technology.

The Flagship of Advanced Mathematics for Sensing, Imaging and Modelling (FAME) is a multidisciplinary competence centre that has its roots in the Finnish inverse problems research community [2]. The flagship consortium is formed by eight partners: Aalto University, Finnish Meteorological Institute, LUT University, Tampere University, University of Eastern Finland, University of Helsinki, University of Jyväskylä, and University of Oulu, with *Tanja Tarvainen* (University of Eastern Finland) as its director, and *Nuutti Hyvönen* (Aalto University) and *Samuli Siltanen* (University of Helsinki) as vice-directors. It aims to benefit the society through cutting-edge research in applied mathematics, physics and computing. The FAME flagship currently encompasses a network of 48 principal investigators and over 350 researchers. Out of all FAME members, about

Figure 1. Distribution of the nationalities of the DREAM doctoral researchers.

a third are women and around 35% are foreigners. Members include, for example, mathematicians, physicists, engineers, biochemists, computer scientists, and medical doctors. In addition to the academic partners, the FAME ecosystem consists of collaborators from different aspects of society, for example, companies from fields such as healthcare, clean technology and process industry, university hospitals, universities, research institutes, and the Finnish education sector.

1.2 *Launch of the DREAM pilot*

Between August 2024 and January 2025, 100 new doctoral researchers started their contracts in the DREAM pilot, located at the seven partner universities of the FAME flagship. The aim of the DREAM pilot is to develop new doctoral education practices in the field of mathematics, physics, computing and imaging, and to enhance transfer of graduated doctors in these fields from universities to companies.

Imaging and sensing challenges are encountered in various applications in society and industry. For example, advanced cost-efficient solutions in imaging, diagnostics, and therapeutics are needed to enhance healthcare and ensure equal access to it. Non-destructive testing methods are lacking in the materials and process industry, where the ability to monitor and control targets without causing interference is highly desirable for safety, energy efficiency, and sustainability. Similar challenges are also encountered in environmental applications such as monitoring of biodiversity, exploring groundwater resources, and predicting effects of climate change.

These and many other timely applications of imaging and sensing present complex challenges, but also plenty of opportunities for potentially high-impact solutions. The DREAM pilot educates experts in a diverse and multidisciplinary setting, encompassing applied mathematics, physics, engineering, and applied sciences. By bringing together experts on the fields of the FAME flagship, DREAM provides a systematic form to train the next generation of professionals to answer the RDI needs of Finland's top export sector.

The DREAM pilot filled its doctoral researcher positions in 2024 with a fair and transparent recruitment process, with the commitment to equal treatment for all applicants. Job offers were formulated collaboratively following a standardised procedure and national standards, and posted on national channels and international, globally accessible web-based platforms. Eventually, the doctoral researcher base of DREAM grew to include 19 different nationalities, see Figure 1 for the illustration of the distribution of the nationalities of the DREAM doctoral researchers.

1.3 *Joining forces for training future professionals*

The Finnish inverse problems community has a long history on mutual collaboration and shared activities. This originates already in the 1990s when the first *Finnish Inverse Days* workshop was organised. This was followed by establishing *the Finnish Inverse Problems Society*, and later consolidated by three consecutive projects in the Research Council of Finland's Centre of Excellence programme. One important effort has been the joint activities in doctoral training that has included, for example, Finnish summer schools on inverse problems. Both FAME and DREAM enable development of these training activities even further, in addition to meeting the requirements of the pilot programme for faster graduation and employment of the doctors by companies.

To enable both MSc and PhD students a wider selection of studies on a faster cycle, a cross-institutional study agreement was formed between the DREAM partner universities. This agreement is coordinated by the University of Jyväskylä, and it was signed by each partner in August 2025. The agreement now includes, in addition to inverse problems courses, courses on topics including, for example, applied mathematics, numerical methods and imaging. At this moment, the agreement is being renewed to cover all Finnish universities offering training in mathematics and a wider selection of courses. In addition to joint training activities, DREAM and FAME develop ways to make use of shared infrastructure and software, and open data repositories. At this moment, a list of open data and software can be found on the FAME website, together with a description of infrastructure at different FAME sites.

Finland has a long history on organising summer schools on inverse problems, and many current professors remember taking part in such schools during their doctoral studies already in the 1990s. In the last two years, the FAME flagship has organised an *Inverse Problems Summer School* as part of the *Jyväskylä Summer School* [3]. The Jyväskylä Summer School has gathered students from all over the world since 1991, to deepen their expertise on Science, Technology, Engineering and Mathematics (STEM) subjects and to expand their professional and academic networks in

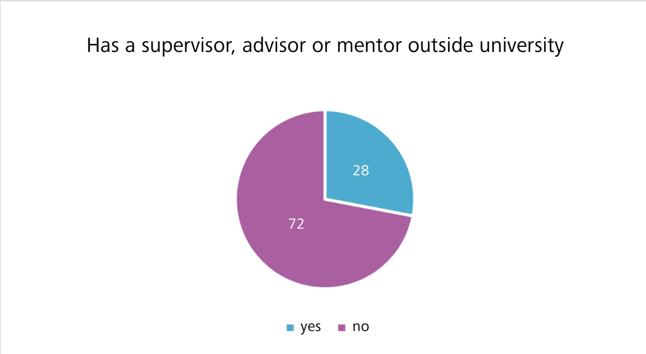

Figure 2. Percentage of the DREAM doctoral researchers who have an appointed supervisor or mentor outside academia in 2025.

an international and interdisciplinary science community. Organised by the Faculty of Mathematics and Science and the Faculty of Information Technology at the University of Jyväskylä, the Jyväskylä Summer School is one of the largest and oldest summer schools in Finland. The application period for the summer school starts typically in March. All courses of the Inverse Problems Summer School are free of charge and taught in English by esteemed guest lecturers from the international inverse problems community. In 2024, Ronny Ramlau (Johann Radon Institut for Computational and Applied Mathematics (RICAM) and Johannes Kepler University, Austria) gave a course on "Integral Equations and Compact Operators" and Felix Lucka (Centrum Wiskunde & Informatica, the Netherlands) on "X-ray Computed Tomography Inside Out: Physics, Mathematics, Imaging and Applications." Further, in 2025 Tatiana Bubba (University of Ferrara, Italy) lectured on "Mathematics of X-ray Computed Tomography" and Babak Maboudi Afkham (University of Oulu, Finland) delivered the course "Introduction to Uncertainty Quantification for Inverse Problems." In 2025, participants of the Inverse Problems Summer School gathered almost 70 course completions. In addition to intensive academic work, the Jyväskylä Summer School also offers an extensive programme of extracurricular activities such as get-togethers, picnics, and cultural events. In 2024, the FAME flagship contributed to the social programme by hosting a summer evening cruise for the participants of the inverse problems courses, and in 2025, FAME organised a game night at a bowling alley.

To boost their academic activities, the DREAM doctoral researchers are encouraged to participate in the annual Inverse Days conference, a highlight event of the year for the Finnish inverse problems research community. What started as a small workshop in the 1990s has since evolved into a prominent scientific event. For many doctoral researchers, Inverse Days has traditionally been the conference where they give their first scientific presentations. Organised in 2025 by the University of Helsinki on the week before Christmas, the conference became the largest Inverse Days event to date, with over 200 registered participants. The programme included, in addition to regular presentations, dedicated industry and AI sessions, a women in inverse problems networking event, and a gathering for young researchers.

To enable a smooth transition from academia to other parts of society after the dissertation, the DREAM pilot aims that all doctoral researchers have a supervisor, mentor, or collaborator from a company or other stakeholder. Figure 2 shows the percentage of DREAM doctoral researchers who have a supervisor or mentor appointed in the beginning of the pilot. The aim is to have a mentor assigned for each doctoral researcher by the end of the first year in the pilot. Furthermore, the DREAM doctoral researchers are also encouraged to undertake secondments during which they would work at the stakeholder's premises for 1–3 months. Through FAME, DREAM has also built a strong partnership with the business and labour market lobbying organisation *Technology Industries of Finland*, which has assisted in facilitating opportunities for doctoral researchers and company representatives to meet and mingle.

1.4 First year's follow-up

The maximum funding period provided by the Ministry of Education and Culture for each individual doctoral researcher in the pilot is three years. One of the purposes behind the pilot programme is to develop practices that make sure a greater number of doctoral researchers can complete their studies and thesis work in a faster time frame. While the ministry does not restrict the time for individual doctoral researchers to complete their degree, as they are employed by universities, the dedicated three-year funding period is meant to make it possible for doctoral researchers to focus solely on their research and make for a swift graduation [4].

To make sure that everything is moving along and identify any potential roadblocks, the DREAM pilot conducts an annual progress reporting questionnaire to all its doctoral researchers. The first such "temperature check" was conducted in September 2025. In addition to reporting their completed studies and status of their dissertation work, doctoral researchers were given an opportunity to grade on a scale of 1–5 factors such as the level of received supervision and usefulness of studies.

The desired accelerated time frame for graduation places significant importance on the quality of supervision. The FAME-DREAM ecosystem comprises a wide range of committed experts with decades worth of supervision experience of doctoral researchers. This dedicated and mentor-oriented pool of supervisors is one of the DREAM pilot's greatest assets, which was also reflected in the questionnaire's answers, see Figure 3.

In the case of doctoral researchers getting support outside universities, the DREAM is steadily on the right track, as also indicated in Figure 2. Furthermore, despite being under shared special attention and yet divided across seven different universities, it has been

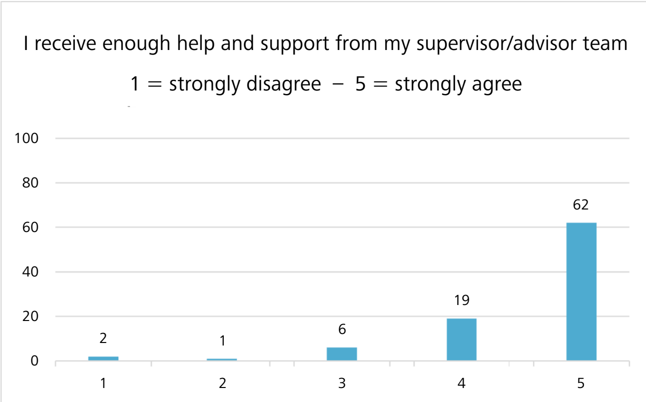

Figure 3. Doctoral researchers' experiences on supervision.

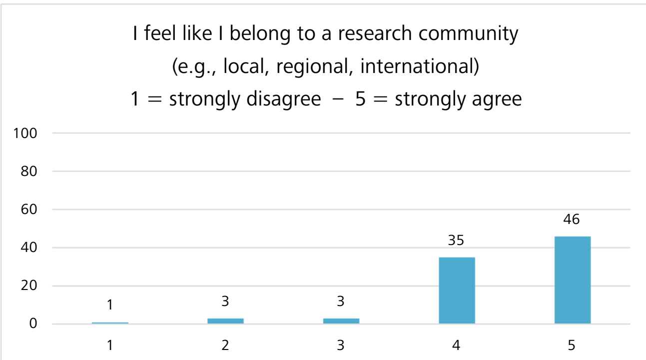

Figure 4. Doctoral researchers' experiences on belonging to a research community.

reassuring to see that most of DREAM's doctoral researchers have found a community for themselves, as demonstrated by Figure 4. In DREAM and FAME, researchers work as members of close-knit research groups, and we believe that these immediate contacts serve everyday community building, peer support, and sense of belonging also for the DREAM doctoral researchers.

Overall, the report in autumn 2025 provided valuable feedback, useful insights, and a truly inspiring snapshot of where the DREAM pilot is at this point of its mission. The next checkpoint is planned to take place in autumn 2026.

2 Living the DREAM

The attention that the pilot programme has received in Finland has also brought the doctoral researchers appointed into the pilot into the public eye giving interviews, for example, in professional magazines and university news articles. It is also our pleasure to share career stories and experiences of two doctoral researchers of the DREAM pilot.

Figure 5. Doctoral researcher Aada Hakula, Aalto University, Finland.

Aada Hakula

Aada Hakula (Figure 5) works in the Inverse Problems Group at the Department of Mathematics and Systems Analysis of Aalto University. In her PhD, she studies model uncertainties in inverse problems with Nuutti Hyvönen as a supervisor, in collaboration with Antti Hannukainen, and Murata Electronics as an industrial collaborator. She started working with the Inverse Problems Group already as an undergraduate when she participated in a summer project on optimal experimental design in X-ray imaging. A few years later, an MSc thesis on model uncertainties in diffuse optical tomography followed, and eventually led to a PhD work.

Hakula tells that she has always been interested in mathematics, and that also led her to choose to study it in the university. She remembers mathematics being her favourite subject in school, and says it often felt like the easiest one as well. In addition, her parents are mathematicians, and she believes that for that reason her enthusiasm for mathematics was always understood and encouraged at home.

Hakula's PhD work includes doing her own research, studying, functioning as a course assistant, and attending different events, such as conferences and summer schools. She tells that she especially enjoys this variety of different assignments. Among the DREAM activities, she has participated, for example, in the Inverse Days and Inverse Problems Summer School, and feels that both of these events were great for networking and learning. She also enjoyed the possibility to meet other PhD students from Finland and around the world. In addition, she attended an Industry Connect and Matchmaking event where the DREAM doctoral researchers had the opportunity to meet representative companies collaborating with the doctoral education pilot. Hakula also mentions being especially happy of her colleagues with whom she can share thoughts about PhD work over lunch or coffee, as well as free time activities, for example, climbing. In addition, she talks highly of her supervisor, describing him as incredibly supportive and helpful throughout her studies.

Hakula tells that one of the challenges during the PhD has been to accept that research can be unpredictable. She is now finalising her first article, and tells that when she started the PhD work she expected a more straightforward process of conducting the work and publishing the article quickly. Regardless, she feels that the process has taught her a lot, and she is now prepared for the remaining part of her studies. This also includes a collaboration with the Murata Electronics company. According to Hakula, the acquaintance with real-life inverse problems that companies are encountering has been very interesting. Overall, she is grateful for the chance to do a PhD in Finland, and for the financial security that the pilot offers for three years. She is eager to see where the PhD studies will take her next.

Fatemeh Maleki Almani

Fatemeh Maleki Almani (Figure 6) is a doctoral researcher in the Computational Physics and Inverse Problems Group at the Department of Technical Physics of the University of Eastern Finland. The topic of her doctoral dissertation is "Computational Modelling, Optimization and Control in Industrial Processes," with Jari Kaipio and Marko Vauhkonen as supervisors and Arto Voutilainen and Marzieh Hosseini from the Rocsole company as industry mentors. She tells that she chose to pursue PhD studies at the University of Eastern Finland because of the strong reputation of its inverse problems group, and when she was searching for a PhD position, she was especially impressed by the profile and publications of the professor who later became her supervisor. The combination of academic strength, solid research infrastructure, and meaningful industrial partnership made this university the most compelling choice for her doctoral training.

Figure 6. Doctoral researcher Fatemeh Maleki Almani, University of Eastern Finland, Finland.

Maleki Almani tells that originally her interest in computation began when, while participating mathematical olympiads and a mathematical competition in 2016, she discovered her ability to solve challenging analytical problems. During her master's studies, she worked on different research projects and collaborated with different groups, which exposed her to the practical relevance of mathematical modelling and strengthened her motivation for scientific research. She feels that the topic of her PhD project, which combines mathematical analysis, computational methods, and real-world applications, is a natural continuation on her academic path.

Maleki Almani's PhD project includes an active collaboration with Rocsole, a Finnish company specialising in smart process imaging, electrical tomography, and real-time data analytics for industrial environments. This collaboration has provided direct exposure to industrial applications, strengthened the practical relevance of her research, and connected her academic development to a broader international industrial system.

Maleki Almani feels that one of the most valuable aspects of pursuing a PhD in Finland is the respectful, friendly, yet highly professional academic culture. This environment maintains a healthy balance between work and personal life, while employment-based contracts allow doctoral researchers to focus more deeply and sustainably on their research. She feels that her doctoral studies have provided a highly enriching and supportive research experience, and that the university offers a friendly and intellectually stimulating environment, complemented by supervisors who are consistently supportive, engaged, and committed to advancing the project. She feels that the structured nature of doctoral studies in Finland, with its emphasis on clarity, well-defined expectations and organised progression, as well as high-quality training courses, has been particularly valuable. Moreover, the doctoral education pilot has not only ensured financial stability but has also opened access to opportunities that would not typically be available, such as summer schools, webinars, transferable-skills training, cross-university workshops, and both national and international networking events.

Acknowledgements. The authors wish to thank Aada Hakula and Fatemeh Maleki Almani for sharing their experiences, and all doctoral researchers, supervisors and collaborators of the DREAM pilot on fruitful collaboration. This work was supported by the Finnish

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References

[1] T. Apajalahti, L. Pöntynen and H. Mikkonen, Miksi teknologiateollisuus tarvitsee niin paljon uusia osaajia? – Asiantuntijat vastaavat. (2025) https://teknologiateollisuus.fi/miksi-teknologiateollisuus-tarvitsee-niin-paljon-uusia-osaajia-asiantuntijat-vastaavat/

[2] Flagship of Advanced Mathematics for Sensing, Imaging and Modelling's website. https://fameflagship.fi

[3] Jyväskylä Summer School website. https://www.jyu.fi/en/study-with-us/summer-and-winter-schools/jyvaskyla-summer-school

[4] Ministry of Education and Culture, Pilot projects for doctoral programmes. https://okm.fi/en/pilot-projects-for-doctoral-programmes

[5] Ministry of Education and Culture, Universities receive additional funding for training a thousand new doctoral graduates. Press release (2024) https://okm.fi/en/-/universities-receive-additional-funding-for-training-a-thousand-new-doctoral-graduates

[6] Parliamentary RDI Working Group, Final report of the Parliamentary Working Group on Research, Development and Innovation. (2021) https://urn.fi/URN:ISBN:978-952-383-516-0

Mikko Helenius is a member of the Department of Mathematics and Systems Analysis of the Aalto University, where he works as a coordinator of the FAME flagship and DREAM pilot.

mikko.e.helenius@aalto.fi

Tanja Tarvainen is a professor at the Department of Technical Physics of the University of Eastern Finland. She is the director of the FAME flagship and DREAM pilot.

tanja.tarvainen@uef.fi

Siiri Utriainen is a project coordinator at the Department of Technical Physics of the University of Eastern Finland. She works as a coordinator of the FAME flagship and DREAM pilot.

siiri.utriainen@uef.fi

Thematic Working Group on Mathematics for Work, Society and Personal Development: Lifelong Learning, TWG7

ERME column regularly presented by Andreas Stylianides and Florian Schacht

In this issue presented by the group leaders Michele Giuliano Fiorentino, Peter Frejd, Pauline Vos, Javier Díez-Palomar, Oda Heidi Bolstad and Rosa Alberto

CERME Thematic Working Groups

We continue the initiative of introducing the CERME Thematic Working Groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for people working in pure and applied mathematics. Our aim is to disseminate developments in mathematics education research discussed at CERMEs and enrich the ERME community with new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.

Introducing CERME's Thematic Working Group 7 – Mathematics for Work, Society and Personal Development: Lifelong Learning

The Thematic Working Group 7 (TWG7) investigates how mathematics is encountered, learned, and used across the entire life course, in evening schools, in workplaces, communities, homes, and informal environments, and how these encounters shape people's participation in social, economic, and cultural life.

Mathematics takes different forms across communities [9]. For instance, within the international community of research mathematicians, mathematics is both the object of research, the tool, and the result of it, whereas in medical research, mathematics serves as a methodological tool for improving health care. In compulsory schools, mathematics is experienced by many students as a list of ready-made rules, as rather alienating and feared, and as a threshold to further studies [1, 6]. Likewise, in daily life and in vocational workplaces, mathematics is a tool for finding practical solutions and for adequately participating in society.

Because of the different goals, rules, language, tools, and people in these environments, the mathematics differs across environments. In university mathematics research, there is rigour and more esoteric content. By contrast, within vocational practices, numbers and variables always have concrete meaning (measured in meters, hours, euros, etc.). Also, there is responsibility associated to the mathematical answers, for instance, a nurse's calculation errors can be a matter of life and death, whereas an incorrect proof of inapplicable mathematics will be sad, but not a matter of life and death. Within daily life, mathematics is embedded in routines and is so deeply internalised that it is hidden or perceived as intuitive.

The establishment of TWG7 arose from a shared social motivation: to make mathematics socially useful and accessible to everyone beyond compulsory education, especially in fragile contexts with vulnerable groups. Across Europe and globally, deep inequalities persist in who benefits from mathematical knowledge and how it is valued [2, 3]. The group's focus reflects the conviction that mathematics education can play a transformative role in empowering individuals, supporting employability, and enabling informed citizenship.

This perspective resonates with wider educational movements for lifelong learning and social justice [4, 8]. It positions mathematics not as a gatekeeper of privilege but as a shared resource for personal and societal development. Consequently, TWG7 engages with research exploring mathematics in adult education, everyday life, at work, and in community settings, where learning and using mathematics can become an instrument for participation, dignity, and well-being. For example, a citizen needs to understand graphs and percentages to critically assess government spending proposals for community needs. Although this may look like 'simple' mathematics to research mathematicians, approximately one in four citizens worldwide struggles with this, which makes them susceptible to manipulation and can hinder meaningful participation in public debates about local budgets, health policies, or transportation planning [7]. Dignity can be upheld when marginalised individuals gain mathematical skills that open access to better employment opportunities or empower them to navigate bureaucratic systems and assert their rights. Well-being is supported when learners experience increased confidence and autonomy through understanding and applying mathematics in their daily lives, such as managing healthcare information or navigating digital technologies.

Conceptualising the domain

TWG7's research is grounded in the idea that mathematical practices are deeply intertwined with human experience and that access to them is socially distributed. Across diverse contexts, people face mathematical challenges that can lead to empowerment or exclusion. The group explores how the use of mathematics emerges in everyday situations, such as professional problem-solving, household budgeting, health decisions, and leisure activities, and how individuals make sense of these experiences. The notion of vulnerability is central, highlighting how precarious life conditions and limited learning opportunities affect engagement with mathematics, as well as how creativity and resilience emerge in such contexts.

Topics in the domain of TWG7 are researched through a variety of theoretical and methodological approaches, including critical mathematics education, socio-cultural theory, design-based and action research, all of which reveal mathematics as a lifelong and life-wide human practice.

The work of TWG7 revolves around three main themes, closely connected through the idea of lifelong engagement with mathematics.

1. Adults learning mathematics

This theme examines how adults return to mathematics education in formal and informal settings such as evening schools, community programs, or correctional institutions. Adult learners often bring diverse experiences, motivations, and emotions, including anxiety or prior failure, as well as curiosity, agency, and unexpected ways to use mathematics based on their informal experiences.

Research in this area highlights the importance of creating learning environments that are flexible, inclusive, and responsive to adult learners' personal circumstances. Studies also show that adult mathematics education contributes not only to individual empowerment but also to social well-being, intergenerational learning, and family support.

2. Mathematical literacy and numeracy

Mathematical literacy and numeracy are the mathematical capabilities essential for citizens navigating contemporary societies shaped by data, digital technologies, and quantitative information. Research presented in TWG7 emphasised how these competences influence people's employment opportunities, health-related decisions, and their ability to participate in civic life.

Studies in TWG7 explored conceptual frameworks for mathematical literacy, numeracy, large-scale assessments of adult numeracy, and ways to integrate numeracy into professional education. A recurring concern is that numeracy is often perceived only as basic arithmetic, rather than as reasoning and problem-solving situated in real-life contexts, as described in the research literature (see for example [5]). Our research reveals widening numeracy gaps (e.g., between countries, between men and women, etc.) and we observe political tensions when calls for improving numeracy is motivated by economic productivity rather than the learners' personal and social needs. This also points to the complex and sometimes conflicting interests of policymakers, balancing economic productivity with broader aims of social participation and civic inclusion.

3. Vocational and workplace mathematics

Research in vocational and workplace mathematics examines how mathematics operates within professional practices and is taught in vocational education. Across countries, vocational curricula vary widely in content and emphasis, from academically oriented to practice-based or hybrid models.

Findings indicate that learners engage more meaningfully with mathematics when they recognise its relevance to their vocational identities and future work. Collaborative teaching between mathematics and vocational educators, as well as the use of authentic artefacts and workplace scenarios, has been shown to enhance motivation and perceived usefulness.

Cross-cutting perspectives and future directions

A defining feature of TWG7 is the interconnection between its themes. Research on numeracy often overlaps with studies on vocational learning or adult education, showing that mathematical literacy, workplace mathematics, and lifelong learning are not separate fields but parts of a continuous spectrum of mathematical activity. This interconnectedness encourages not only advancement of practice but also theoretical innovation. Within TWG7, new concepts have been developed to describe how mathematics circulates between experts, workers, and citizens, how authenticity is negotiated in learning tasks, and how artefacts mediate between vocational and mathematical domains. Such conceptual work demonstrates the group's contribution to advancing both theory and practice in mathematics education.

Methodologically, TWG7 features remarkable diversity. Large-scale quantitative surveys, qualitative case studies, design experiments, and theoretical analyses coexist and complement one another. This variety reflects the richness of the research field and its openness to interdisciplinary dialogue.

Looking ahead, TWG7 recognises several directions for future work:

- developing research-based approaches to teaching mathematics for adults and lifelong learners;
- integrating numeracy and critical mathematics education into vocational and professional contexts;
- strengthening collaboration between mathematics educators, industry professionals, and policymakers;
- and deepening theoretical models that link personal development, social participation, and work through mathematics.

Concluding Remarks

Mathematics accompanies people throughout life as a tool for understanding the world, making decisions, and building identities. The research within TWG7 explores how mathematics education can respond to this lifelong presence by addressing issues of equity, relevance, and meaning.

By connecting mathematics with work, society, and personal development, TWG7 contributes to a broader understanding of what it means to learn, use, and live with mathematics in the 21st century.

References

[1] J. Boaler, Mathematics from another world: traditional communities and the alienation of learners. *J. Math. Behav.* **18**, 379–397 (2000)

[2] G. E. FitzSimons, Commentary on vocational mathematics education: where mathematics education confronts the realities of people's work. *Educ. Stud. Math.* **86**, 291–305 (2014)

[3] I. Gal, A. Grotlüschen, D. Tout and G. Kaiser, Numeracy, adult education, and vulnerable adults: a critical view of a neglected field. *ZDM Math. Educ.* **52**, 377–394 (2020)

[4] P. Gates and R. Jorgensen (Zevenbergen), Foregrounding social justice in mathematics teacher education. *J. Math. Teach. Educ.* **12**, 161–170 (2009)

[5] K. Hoogland, The changing nature of basic skills in numeracy. *Front. Educ.* **8**, article no. 1293754 (2023)

[6] D. Kollosche, Social functions of mathematics education: a framework for socio-political studies. *Educ. Stud. Math.* **98**, 287–303 (2018)

[7] OECD, Do adults have the skills they need to thrive in a changing world?: Survey of adult skills 2023. OECD Skills Studies, OECD Publishing, Paris (2024)

[8] O. Skovsmose, *An invitation to critical mathematics education*. Sense Publishers, Rotterdam (2011)

[9] E. Wenger, *Communities of practice: Learning, meaning, and identity*. Cambridge University Press, Cambridge (1998)

Michele Giuliano Fiorentino is a researcher at the University of Bari Aldo Moro, Italy. His work focuses on semiotic mediation as a tool for fostering meaningful learning across disciplines, with particular attention to upper secondary schools characterised by high dropout rates. His research explores how innovative teaching practices and interdisciplinary approaches can support students in challenging educational contexts. In addition, his interests include the integration of artificial intelligence in education to enhance teaching and learning processes, as well as research on early childhood and primary teacher education. Michele is co-leader of CERME's TWG7 on 'Mathematics for work, society and personal development.'

michele.fiorentino@uniba.it

Peter Frejd is an associate professor at the department of mathematics, Linköping University, Sweden. His research combines mathematical modelling both in- and outside of school and mathematics teaching in vocational education. He leads the project "Mathematics teaching in vocational programs: The importance of context for students' motivation and academic performance" funded by the Swedish Research Council. In 2025, he was co-chair/organiser of ICTMA22, a 2025 conference of the International Community of Teachers of Mathematical Modelling and Applications. Peter is a co-leader of CERME's TWG7 on 'Mathematics for work, society and personal development.'

peter.frejd@liu.se

Pauline Vos is a professor of mathematics education at Western Norway University of Applied Sciences. Her research concerns people who are uncomfortable with school mathematics. She leads research into inclusive maths classrooms avoid the traditional, deductive, teacher-centred approach, and instead use meaningful mathematics. Also, she studies differences between 'school mathematics' and the everyday use of mathematics by 'ordinary' people at home or in vocations. Pauline leads CERME's TWG7 on 'Mathematics for work, society and personal development.'

pauline.vos@hvl.no

Javier Díez-Palomar is a professor of mathematics education at the University of Barcelona, Spain. He was president (2019–2023) and vice-president (2023–2025) of the Multidisciplinary Association of Educational Research (AMIE). He is convenor of the NW24 Mathematics Education Research network at the European Educational Research Association (EERA) and has represented Spain at the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM) since 2012, serving as vice-president since 2023. He is a member of the OECD Numeracy Expert Group (PIAAC) and the editor-in-chief of the journal REDIMAT and associate editor of ALM International Journal.

jdiezpalomar@ub.edu

Oda Heidi Bolstad is an associate professor of mathematics education at Volda University College, Norway. She is a teacher educator and her research areas include mathematical literacy, mathematical modelling and inclusive mathematics education. She is editor of the monograph *International Perspectives on Teaching and Learning for Mathematical Literacy* published by Brill in 2025. Oda is a co-leader of CERME's TWG7 on 'Mathematics for work, society and personal development.'

oda.heidi.bolstad@hivolda.no

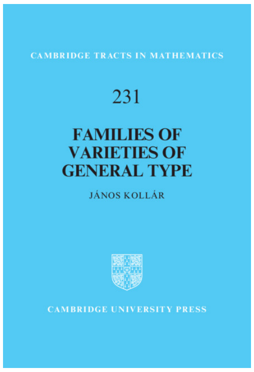
Rosa Alberto is a researcher at the University of Applied Sciences Utrecht, the Netherlands, in the Mathematical and Analytical Competence of Professionals research group. She works to improve numeracy skills and attitudes through awareness, learning materials, professional development, and research. Her focus includes informal (digital) learning contexts, family-based approaches, and adults in prison. She is a trustee of Adults Learning Mathematics and a co-leader of CERME's TWG7 on 'Mathematics for work, society and personal development.'

rosa.alberto@hu.nl

Book reviews

Families of Varieties of General Type by János Kollár

Reviewed by Stefano Filipazzi



János Kollár's *Families of Varieties of General Type*, written with the collaboration of Klaus Altmann and Sándor J. Kovács, is a monumental contribution to modern algebraic geometry. It completes a decades-long endeavor to generalize the moduli theory of algebraic curves to higher-dimensional varieties. The book concludes and brings to fruition a complete moduli theory for varieties of general type, a 30-year work by many mathematicians, where Kollár's vision and contribution stand out. The importance of this monograph has been recognized by the János Bolyai International Mathematics Award, a prestigious prize given every five years by the Hungarian Academy of Sciences.

The work takes the moduli theory of hyperbolic (marked) curves as a starting point and guides the reader through the theory needed to generalize it to their higher-dimensional counterparts, namely, stable varieties and stable pairs. Throughout the text, the main focus is the notion of a family of varieties or pairs, consistently with the modern emphasis on functorial and stack-theoretic approaches to moduli. There are many subtleties in generalizing this notion from curves to higher-dimensional varieties. Step by step, the author addresses the natural difficulties that arise in the process and thus motivates the many new notions and constructions needed to develop the moduli theory for higher-dimensional varieties.

In the case of smooth and projective algebraic curves of genus $g \geq 2$, Deligne and Mumford developed a complete and satisfactory moduli theory: To construct a compact moduli space for curves of a fixed genus $g \geq 2$, it suffices to consider families of nodal curves of (arithmetic) genus g. Furthermore, families of nodal curves over a base scheme S are simple to describe: They are flat morphisms $C \to S$ such that every geometric fiber is a nodal curve. Then, this description can be readily extended to curves of genus g with n marked points.

Already in dimension 2, if we consider smooth projective surfaces with ample canonical line bundle, it is not immediately clear what singular spaces should be allowed to compactify this moduli problem, nor what notion of family shall be used. The first question has been settled for some time and leads to the theory of stable varieties and stable pairs, which is thoroughly discussed in an earlier book by Kollár, *Singularities of the Minimal Model Program*. Yet, an answer to the first question only clarifies *what* geometric objects shall be parametrized, but not *how*. This book provides the answer to this latter question, thus completing the quest for a moduli theory generalizing the one of curves to higher dimensions.

In 1988, Kollár and Shepherd-Barron observed that the notion of family utilized for curves is not well-behaved already in the case of surfaces: relevant discrete invariants are indeed allowed to jump in a flat family. The core of the book (Chapters 2–7) discusses how the notion of flatness should be refined to obtain a well-behaved theory of families of stable varieties or pairs.

The first step is to understand what a family of stable varieties (or pairs) over a smooth curve is. In this case, since the base variety is smooth and 1-dimensional, the correct notion can be fully characterized with the language of pairs. Then, as we generalize this notion to arbitrary bases, more subtleties arise. To define a family of stable varieties $\pi\colon X \to S$ over a reduced scheme S, it suffices to ask that π is flat and the relative canonical divisor $K_{X/S}$ is $\mathbb{Q}$-Cartier. Then, if S is not reduced, it is necessary to analyze all the sheaves $\omega_{X/S}^{[m]}$ and ask that they are flat and with S_2 fibers for every $m \in \mathbb{Z}$. This suffices to define the notion of family of stable varieties; for the case of pairs, the situation is even more intricate. Indeed, if we consider a candidate family of pairs $\pi\colon (X, D) \to S$, explicit examples show that we cannot expect the support of D to be flat over S. Therefore, a refined understanding of the possible alternatives to flatness is needed; this quest then leads Kollár to introduce the notions of C-flatness (inspired by Cayley hypersurfaces), well suited for families over reduced bases, and K-flatness, which generalizes the notion of C-flatness and provides the final notion needed to complete the theory.

Once the notion of family of stable varieties and pairs is fully understood, we are ready to harvest the desired results: The book culminates with the proof that the Kollár–Shepherd-Barron–Alexeev (KSBA) stability condition for pairs leads to a well-behaved moduli theory in characteristic 0, inclusive of a projective coarse moduli space.

The book is structured into eleven chapters, each progressively building toward the main results. A summary of the key sections includes:

- Chapter 1: Provides historical background on moduli theory, tracing developments from Riemann and Cayley to Deligne and Mumford. It outlines the transition from curves to higher-dimensional varieties and highlights the main challenges therein.
- Chapter 2: Develops the theory of 1-parameter families of (locally) stable varieties and pairs, including detailed definitions of local stability and the introduction of slc singularities.
- Chapter 3: Treats the notion of a family of (locally) stable varieties over an arbitrary base. Approaches with the theory of Chow varieties (Cayley–Chow families) and Hilbert schemes (Hilbert–Grothendieck families) are compared.
- Chapter 4: Establishes the core theory of stable pairs over reduced base schemes. The notions of Mumford divisor and C-flatness are introduced here as a crucial geometric condition for managing families of divisors.
- Chapter 5: Provides numerical criteria for flatness and stability, most notably Theorem 5.1, which relates the constancy of the volume of the canonical divisor to the stability of families.
- Chapter 6: Explores moduli problems where the divisorial part is flat and compares several notions of stability, including those due to Viehweg and Alexeev.
- Chapter 7: Introduces K-flatness and its formal properties, which strengthen the framework for working with families of divisors independently of projective embeddings. This chapter provides the final notion of family of (locally) stable pairs over an arbitrary base.
- Chapter 8: Synthesizes previous developments and the work of Chapters 2–7 into a general moduli theory for stable pairs, culminating in Theorem 8.1—the main result establishing that Kollár–Shepherd-Barron–Alexeev stability yields a good moduli theory with coarse projective moduli spaces.
- Chapters 9–11: Contain auxiliary material such as hulls and husks, miscellaneous ancillary results, and a handbook about the Minimal Model Program.

To sum up, *Families of Varieties of General Type* is a landmark work that settles a central question in algebraic geometry. The clear and precise style will certainly make this book the main reference on moduli of algebraic varieties, both for learners and experienced researchers.

The exposition follows Kollár's characteristic and enjoyable writing style: Abstract constructions or definitions go hand in hand with explicit and concrete examples that provide motivation and support for the reader. Another strength of the book is its historical account of moduli theory in algebraic geometry, which allows the reader to appreciate how the theory grew over the past two centuries and provides the more curious with many references to previous works.

This book's contribution to algebraic geometry is both foundational and forward-looking. This is reflected by the multiple uses and audiences the book is suited for. On the one hand, its exhaustive content will make it an indispensable resource for researchers in moduli theory and birational geometry. On the other hand, graduate students and researchers interested in learning more about this field will find a reference rich in examples and motivations that will invite the reader to dive into moduli theory.

János Kollár, *Families of Varieties of General Type.* Cambridge Tracts in Mathematics 231, Cambridge University Press, 2023, xviii+472 pages, Hardback ISBN 978-1-009-34610-8, eBook ISBN 978-1-009-34611-5.

Stefano Filipazzi is an assistant professor at Duke University. He works in algebraic geometry. He obtained his PhD in 2019 at the University of Utah under the supervision of Christopher D. Hacon and spent time at UCLA (2019–2021) and EPFL (2021–2025).

stefano.filipazzi@duke.edu

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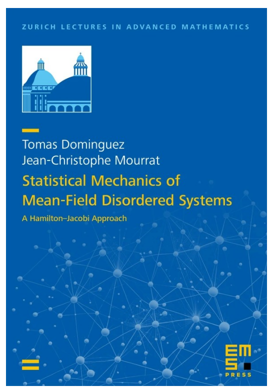
Statistical mechanics of mean-field disordered systems: A Hamilton–Jacobi approach by Tomas Dominguez and Jean-Christophe Mourrat

Reviewed by Roland Bauerschmidt



The book provides a beautifully written introduction to the topic of mean-field spin glasses from the point of view of infinite-dimensional Hamilton–Jacobi equations – a perspective that has been extensively developed by Mourrat and collaborators in the last years. The prototypical example of a mean-field spin glass is the Sherrington–Kirkpatrick model, whose free energy is defined as the limit

$$f(\beta) = \lim_{N\to\infty} -\frac{1}{N}\mathbb{E}\log \sum_{\sigma\in\{\pm1\}^N} \exp\Big(\frac{\beta}{\sqrt{N}}\sum_{i,j=1}^{N} g_{ij}\sigma_i\sigma_j\Big),$$

where the g_{ij} are independent standard Gaussian random variables. In physics terminology, the g_{ij} are an example of quenched disorder of a spin system with configurations $\sigma \in \{\pm 1\}^N$ having probability weight proportional to

$$\exp\Big(\frac{\beta}{\sqrt{N}}\sum_{i,j=1}^{N} g_{ij}\sigma_i\sigma_j\Big).$$

The term mean-field refers to the aspect that all pairs of sites $i, j \in \{1, \dots, N\}$ play an equivalent role (as opposed to a lattice system, for instance, in which the interaction depends on the distance of the sites). As prototypical examples of disordered systems, mean-field spin glasses have become a very active research area within probability theory, with motivation and connections from statistical physics to computer science and statistical inference. Some of this motivation is explained in the book without assuming any prior knowledge.

The non-rigorous but ingenious computation of the free energy by Parisi (using what is known as the "replica method") was a major achievement in theoretical physics that opened the way to the understanding of a broad class of disordered systems. The result is given by a somewhat mysterious variational formula that is now known as the Parisi formula:

$$-f(\beta) - \log 2 = \inf_{\zeta}\Big(\Phi_\zeta(0,0) - \beta^2\int_0^1 t\zeta(t)\,dt\Big),$$

where the infimum is over probability distribution functions ζ on $[0,1]$ and Φ_ζ is the solution on $[0,1] \times \mathbb{R}$ of a certain PDE, namely

$$-\partial_t\Phi_\zeta(t,x) = \beta^2\big(\partial_x^2\Phi_\zeta(t,x) + \zeta(t)(\partial_x\Phi_\zeta(t,x))^2\big),$$

with terminal condition $\Phi_\zeta(1,x) = \log\cosh(x)$. The Parisi formula was eventually proved by Guerra and Talagrand, using an approach different from the replica method of Parisi.

The book by Dominguez and Mourrat systematically develops another perspective on the free energy of the Sherrington–Kirkpatrick model (and more general spin glasses), which is that it also turns out to be given in terms of the solution of an infinite-dimensional Hamilton–Jacobi equation. Indeed, up to a trivial constant, the free energy $f(\beta)$ with $\beta = \sqrt{2t}$ turns out to be given by $f(t,0)$ where f solves

$$\partial_t f(t,q) = \int_0^t \partial_q f(t,q,u)^2\,du,$$

and q takes values in the space of square integrable increasing paths from $[0,1)$ to $\mathbb{R}_{\geq 0}$. As a Hamilton–Jacobi equation, the (viscosity) solution of this equation is given by the Hopf–Lax formula:

$$f(t,q) = \sup_{q'}\Big(f(0,q+q') - \frac{1}{4t}\int_0^1 (q'(u))^2\,du\Big).$$

One of the main results presented in the book under review is that this variational formula is in fact equivalent to the Parisi formula. The Hamilton–Jacobi formulation provides a conceptually natural (perhaps less mysterious) perspective on the free energy.

The main motivation for the Hamilton–Jacobi approach to spin glasses is to understand more general models for which the analogue of the Parisi formula is not yet understood. This includes the situation where the quadratic nonlinearity in the Hamilton–Jacobi equation for the Sherrington–Kirkpartrik model is replaced by a nonconvex function. The book concludes with an outlook on this topic of current research by discussing the example of a bipartitive version of the Sherrington–Kirkpatrick model.

The results in the book are presented with a lot of intuition and background along the way. In addition to the motivation of mean-field spin glasses, both from the point of view of statistical physics and from that of statistical inference, the book by Dominguez and Mourrat includes concise, yet essentially self-contained introductions to the necessary mathematical background topics. This includes chapters on convex analysis, the required background on Hamilton–Jacobi equations including the theory of viscosity solutions, and an introduction to Poisson point processes. These concepts are illustrated in examples relevant for the problem of mean-field spin glasses at hand and complemented with a number of exercises.

The book would be an excellent reference for an advanced topics course or a student seminar, by providing an introduction to the active research area of spin glasses in probability as well as introductions to various topics of general mathematical relevance. The book is a real pleasure to read and therefore also an excellent reference for anyone who would like to learn more about this fascinating subject.

Tomas Dominguez and Jean-Christophe Mourrat, *Statistical mechanics of mean-field disordered systems: A Hamilton–Jacobi approach.* Zurich Lectures in Advanced Mathematics, EMS Press, 2024, vi+361 pages, Softcover ISBN 978-3-98547-074-7, eBook ISBN 978-3-98547-574-2.

Roland Bauerschmidt is a professor of mathematics at the Courant Institute of Mathematical Sciences. His research interests focus on probabilistic aspects of statistical physics and quantum field theory.

bauerschmidt@cims.nyu.edu

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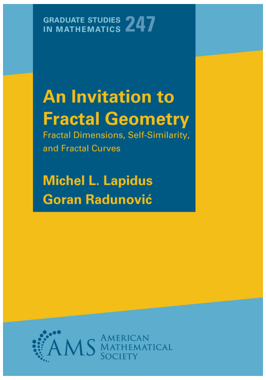
An Invitation to Fractal Geometry: Fractal Dimensions, Self-Similarity, and Fractal Curves by Michel L. Lapidus and Goran Radunović

Reviewed by Lars Olsen



This is a nice and well-written textbook on fractal geometry at the advanced undergraduate level. In order to study fractal geometry, some knowledge of measure theory is inevitably needed, and every author of a textbook on fractal geometry at the undergraduate level must decide how many technical details in measure theory he/she believes are needed and should be included. To answer this question, one is reminded of Tolstoy's short story "How much land does a man need?" in which the protagonist, in the final paragraph, learns that the answer to the eponymous question is: "Enough to get buried in." Similarly, many students have the impression that the answer to the question "How much measure theory do you need to study fractal geometry?" is also: "Enough to get buried in." It is the duty of any author of a textbook on fractal geometry aimed at undergraduate students to carefully make sure that the students are not buried in measure theoretical details before starting the study of fractal geometry. Some authors achieve this by avoiding a technical discussion of measure theory, see, for example [2, 5, 6, 11], whereas others include carefully and appropriately designed chapters discussing the technical foundations of measure theory, see, for example, [1, 3, 10, 12]. The present book belongs to the latter category and includes a thorough discussion of measure theory.

The book consists of four parts.

Part 1 ("Preliminary Material") consists of Chapters 1–3. Chapter 1 contains an interesting introduction to fractal geometry with many examples illustrating a wide variety of fractals. Chapter 2 and Chapter 3 contain technical material covering basic theory of metric space (20 pages) and measure theory including construction of measures using Carathéodory's approach (40 pages), respectively.

Part 2 ("Dimension Theory") is the core and central part of the book and consists of Chapters 4–8. Chapter 4 provides a thorough introduction to Iterated Function Systems, including symbolic dynamics. Chapters 5–7 provide an introduction to Hausdorff measures with respect to arbitrary gauge functions, including a detailed discussion of the δ-approximative Hausdorff measures with respect to different covering systems. Chapter 8 gives a thorough discussion of the Minkowski dimension and the box dimensions in addition to a detailed discussion of the Hausdorff dimension, the packing dimension and the box dimensions of self-similar sets satisfying the open set condition.

Part 3 ("Fractal Curves and Their Complex Dimensions") consists of one long chapter (120 pages), namely, Chapter 9. This chapter provides an introduction to the theory of zeta functions and complex dimensions. While the material covered in Part 2 is standard and can be found in almost all undergraduate textbooks on fractal geometry, the material in Part 3 is less standard and was developed by Lapidus and various collaborators during the past 30 years, cf. [8, 9] and the references therein. This is the first time this material is presented in a textbook for undergraduate students and this makes the present textbook unique amongst other undergraduate textbooks on fractal geometry. Because of this, it seems appropriate to explain the material in Part 3 in slightly more detail. Loosely speaking, the work presented in Part 3 says that the Minkowski dimension of a fractal string can be written as a series involving the poles of the zeta function of the fractal string. More precisely, for a bounded Borel set subset A of $\mathbb{R}^m$, the distance zeta function ζ_A of A is defined by

$$\zeta_A(s) = \int_{A_\delta} \mathrm{dist}(x, A)^{s-m}\, dx$$

for the complex variable $s \in \mathbb{C}$, where $\delta > 0$ and A_δ is the δ-neighbourhood of A (the choice of δ is, in a precise technical sense explained in the book, unimportant), and the complex dimensions of A are by definition the poles of the meromorphic extension of ζ_A. The authors are particularly interested in the following special case. Namely, fix an open bounded subset Ω of the real line $\mathbb{R}$ and let $A = \partial\Omega$ be the boundary of Ω; the set $\partial\Omega$ is called a fractal string. A fractal string is typically a fractal set and fractal geometers are interested in studying the Minkowski dimension and the Minkowski content of $\partial\Omega$. Lapidus' key thesis is that the complex dimensions of $\partial\Omega$ provides an "explicit" formula for those quantities. More precisely, for $\varepsilon > 0$, let $V(\varepsilon) = \mathrm{vol}\{x \in \mathbb{R} \mid \mathrm{dist}(x, \partial\Omega) < \varepsilon\}$ denote the (1-dimensional) volume of the ε-neighbourhood of $\partial\Omega$. The Minkowski dimension, D, of $\partial\Omega$ is defined by $D = 1 - \liminf_{\varepsilon \searrow 0} \frac{\log V(\varepsilon)}{\log \varepsilon}$, and the lower and upper Minkowski contents of $\partial\Omega$ are defined by

$$\mathcal{M}_* = \liminf_{\varepsilon \searrow 0} \varepsilon^{-(1-D)} V(\varepsilon), \quad \mathcal{M}^* = \limsup_{\varepsilon \searrow 0} \varepsilon^{-(1-D)} V(\varepsilon).$$

One of the key results in Part 3 says that (under suitable conditions on the string $\partial\Omega$) we have the following explicit formula for $V(\varepsilon)$:

$$V(\varepsilon) = \sum_{\omega} c_\omega \frac{(2\varepsilon)^{1-\omega}}{\omega(1-\omega)} + R(\varepsilon), \tag{1}$$

where the sum is over all the complex dimensions ω of $\partial\Omega$, the number c_ω is essentially the residue of $\zeta_{\partial\Omega}$ at the pole ω, and $R(\varepsilon)$ is an error term of lower order. It follows from (1) that $\varepsilon^{-(1-D)} V(\varepsilon) = g(\varepsilon) + \varepsilon^{-(1-D)} R(\varepsilon)$, where $g(\varepsilon)$ is a function defined explicitly in terms of the complex dimensions and whose oscillatory behaviour determines the values of the Minkowski contents $\mathcal{M}_*$ and $\mathcal{M}^*$ of $\partial\Omega$.

Part 4 ("Appendices") consists of two short appendices, A and B. Appendix A explain lower and upper limits, and Appendix B provides a more detailed and technical discussion of Carathéodory's extension theorems.

The presentation is clear. Useful motivations and examples are presented before important definitions and details of all proofs are given. A very large number of further interesting historical notes and references are spread out through the text. Each section ends with a fairly large collection of useful exercises. Most of the exercises are not of a computational nature, but require that the reader provides proofs of various mathematical statements. Finally, the book contains a very long (633 entries) and useful list of references including many recent entries (i.e., after 2000).

There are numerous other textbooks in fractal geometry at the undergraduate level, including, [1–7, 10–12]. The book under review is more advanced than Falconer's 1990 texts [5, 6] and the texts by Y. Pesin & V. Climenhaga [11] and Zähle [12] but less ambitious than Mattila's graduate textbook [10]. Whereas Falconer's popular textbooks [5, 6] avoids technical measure theoretical details and presents a large number of examples of fractal sets taken from many parts of mathematics, the book under review provides the reader with the proper measure theoretical foundations for the subject (but at a level that is more accessible to the beginning graduate student than the treatment found in Mattila's text [10]) and concentrates on a more limited number of topics. The book is suited for an advanced undergraduate course in fractal geometry stressing the measure theoretical foundations of the subject. If supplemented with [5], the students will learn the rigorous measure theoretical foundations for the subject and also encounter numerous interesting examples.

Michel L. Lapidus and Goran Radunović, *An Invitation to Fractal Geometry: Fractal Dimensions, Self-Similarity, and Fractal Curves.* Graduate Studies in Mathematics **247**, American Mathematical Society, 2024, xxvii+600 pages, Hardcover ISBN 978-1-4704-7623-6, Softcover ISBN 978-1-4704-7895-7, eBook ISBN 978-1-4704-7896-4.

References

[1] M. Barnsley, *Fractals everywhere*. Academic Press, Boston, MA (1988)

[2] G. A. Edgar, *Measure, topology, and fractal geometry*. Undergrad. Texts Math., Springer, New York (1990)

[3] G. A. Edgar, *Integral, probability, and fractal measures*. Springer, New York (1998)

[4] K. J. Falconer, *The geometry of fractal sets*. Cambridge Tracts in Math. 85, Cambridge University Press, Cambridge (1986)

[5] K. Falconer, *Fractal geometry: Mathematical foundations and applications*. John Wiley & Sons, Chichester (1990)

[6] K. Falconer, *Techniques in fractal geometry*. John Wiley & Sons, Chichester (1997)

[7] J. M. Fraser, *Assouad dimension and fractal geometry*. Cambridge Tracts in Math. 222, Cambridge University Press, Cambridge (2020)

[8] M. L. Lapidus, G. Radunović and D. Žubrinić, *Fractal zeta functions and fractal drums: Higher-dimensional theory of complex dimensions*. Springer Monogr. Math., Springer, Cham (2017)

[9] M. L. Lapidus and M. van Frankenhuijsen, *Fractal geometry, complex dimensions and zeta functions: Geometry and spectra of fractal strings*. Springer Monogr. Math., Springer, New York (2006)

[10] P. Mattila, *Geometry of sets and measures in Euclidean spaces: Fractals and rectifiability*. Cambridge Stud. Adv. Math. 44, Cambridge University Press, Cambridge (1995)

[11] Y. Pesin and V. Climenhaga, *Lectures on fractal geometry and dynamical systems*. Stud. Math. Libr. 52, American Mathematical Society, Providence, RI (2009)

[12] M. Zähle, *Lectures on Fractal Geometry.* Fractals Dyn. Math. Sci. Arts Theory Appl. 8, World Scientific, Singapore (2024)

Lars Olsen graduated with a master's degree (cand.scient.) in mathematics and physics from the University of Copenhagen in 1992. He then went to the University of North Texas for his doctoral studies and graduated with his PhD in 1994. Since then, he has been working at the University of St Andrews in the beautiful coastal town of St Andrews in Scotland. Lars works on fractal geometry, and is, in particularly, interested in multifractal analysis and multifractal zeta-functions. When he is not thinking about fractal geometry, he enjoys relaxing and working in the garden together with his wife and their two cats, Ronja and Klaus. In fact, Klaus belongs to Lars and his wife's daughter, but she could not take him with her when she left for university four years ago. This is not really a problem because Klaus loves to live with Lars and his wife, and enjoys catching birds in the garden.

lo@st-and.ac.uk

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Letter concerning a review of the *Handbook of the History and Philosophy of Mathematical Practice*

Stelios Negrepontis, Athanase Papadopoulos and Bharath Sriraman

This concerns a review of the *Handbook of the History and Philosophy of Mathematical Practice* edited by Bharath Sriraman which appeared in the EMS Magazine, Issue 135 (2025). In a lengthy introduction, the reviewer explains what he considers to be the history and philosophy of mathematics, a viewpoint we do not share. Concerning the present *Handbook*, we believe that he overlooks the essential point, which is the word "practice" in the title. In the following paragraphs, we would like to address only the reviewer's explicit criticisms, of which there are two; the other criticisms are vague or unsupported by anything specific and will not be addressed in this letter.

The first criticism concerns articles on Plato's philosophy, which occupy (according to the reviewer) 170 pages of the four volumes of the Handbook – a length that the reviewer finds excessive. But more importantly, he considers the articles not in the spirit of the Handbook.

The articles on Plato are written by Stelios Negrepontis, one of the signatories of the present response. These chapters appear either not to have received detailed attention from the reviewer, or their meaning and scope may not have been fully understood, much less the relevance of Plato's writings to mathematical practice. This is what we would like to address first, in simple terms.

The aim of these chapters is to show, through examples, that mathematics, and in particular the notion of periodic anthyphairesis, underlies Plato's entire philosophy. For this purpose, different themes and writings of Plato are analyzed. We do not seek to repeat what is written in these articles, but the importance of Plato in Western philosophy, especially in the philosophy of mathematics, and also in the history of mathematics, may not be fully clear to the reviewer and this compels us to reiterate certain points that should be widely known but are not. To this end, we quote some authors who have written on the subject and who know what they are talking about.

Aristoxenus of Tarentum, who was a student of Aristotle, writes in his opus magnum, Ἁρμονικὰ στοιχεῖα, referring to Aristotle, that Plato's teaching was essentially mathematical: "Such was the condition, as Aristotle used often to relate, of most of the audience that attended Plato's lectures on the Good. They came, he used to say, every one of them, in the conviction that they would get from the lectures some one or other of the things that the world calls good; riches or health, or strength, in fine, some extraordinary gift of fortune. But when they found that Plato's reasonings [logoi] were of sciences [mathemata/mathematics] and numbers, and geometry, and astronomy, and of good and unity as predicates of the finite, […]." The fact that Plato was a mathematician is also confirmed by Proclus, the major philosopher of late antiquity, whose opinion is generally considered to be reliable. Proclus includes the name of Plato in the list of mathematicians he gives in Part II of the Prologue of his *Commentary on the First Book of Euclid's Elements*, an information generally considered to be due to Eudemus, another student of Aristotle. Proclus writes: "[…] Plato, who appeared after them, greatly advanced mathematics in general and geometry in particular because of his zeal for these studies. It is well known that his writings are thickly sprinkled with mathematical ratios [mathematikois logois] and that he everywhere tries to arouse admiration for mathematics among students of philosophy." In addition to those testimonials from antiquity, we come to the modern period, namely to Bertrand Russell, who writes in his *History of Western Philosophy*: "It is noteworthy that modern Platonists, almost without exception, are ignorant of mathematics, in spite of the immense importance that Plato attached to arithmetic and geometry, and the immense influence that they had on his philosophy." Finally, we quote Bourbaki, from the *Éléments d'histoire des mathématiques*: "It has been said that Plato was almost obsessed with mathematics; without being an inventor in this field himself, he became, from a certain point in his life, acquainted with the discoveries of contemporary mathematicians (many of whom were his friends or students), and never ceased to take a direct interest in them, even going so far as to suggest new directions for research; thus, in his writing, mathematics constantly serves as an illustration or a model (and sometimes even nurtures, as with the Pythagoreans, his inclination for mysticism)."

In fact, reading the chapters by Negrepontis in this Handbook conveys the full breadth and depth of the work of Plato, as opposed to simplistically dismissing it because of a lack of awareness of its mathematical significance. The concept of "anthyphairesis" relates all of Plato's philosophy to mathematical reasoning and the method of recollection and is at the heart of the Platonic idea that learning is a process of recollection of the perfect forms. But the reviewer, who is an early-stage researcher in the history of mathematics, instead of delving deeper into what the chapters of this Handbook say about Plato, considers that the subject has nothing to do with the "philosophy of mathematical practice."

Incidentally, the total number of pages of the articles written by Negrepontis on Plato and included in this Handbook is 270 and not 170, so the reviewer, while skimming the four volumes of the Handbook, has probably missed some of them, and in any case, it seems like he did not go through the full table of contents in detail.

The second explicit criticism that seems inappropriate to us is the following.

The reviewer wonders why there is an article about Yuri Manin, written by the second signatory of the present text; he declares that the article "seems rather out of place." If he had examined the list of editors of the Handbook, the reviewer would have understood why. Indeed, Manin is one of the editors. And since he passed away while the Handbook was still in production, the editor-in-chief asked the author to write an article about him. But it must also be said that, quite apart from this, Manin is one of the quintessential working mathematician-philosophers, and it was also for this reason that an article about him, emphasizing his philosophical side, which is less well known to the mathematical community, was not only appropriate, but a duty we set for ourselves. In this regard, it should be noted that among the editors of the Handbook, apart from Yuri Manin (1937–2023), two other editors passed away before the end of the project, which obviously took several years; these were Reuben Hersh (1927–2020) and Chandler Davis (1926–2022), for whom philosophy, mathematics, and the philosophy and history of mathematics were a matter of everyday practice.

The articles that are explicitly criticized by the reviewer are written by mathematicians, who also work on the history of mathematics, but from the point of view of a mathematician plainly engaged in mathematical research. This brings us again to the word "practice" in the title of the Handbook, and this is probably the source of misunderstanding with the reviewer, who is a historian of mathematics with no professed practice in mathematics. We align ourselves with the perspective expressed by André Weil in his famous 1978 letter to the Editor of the *Archive for History of Exact Sciences* titled *Who Betrayed Euclid?*, in which he observed that when a discipline emerges at the intersection of two established fields, misunderstandings may arise from a lack of familiarity with one or both areas. Explicitly, André Weil writes: "When a discipline, intermediary in some sense between two already existing ones (say A and B) becomes newly established, this often makes room for the proliferation of parasites, equally ignorant of both A and B, who seek to thrive by intimating to practitioners of A that they do not understand B, and vice versa. We see this happening now, alas, in the history of mathematics. Let us try to stop the disease before it proves fatal."

Without wishing to engage in controversy, we suggest that certain features of the report in question may reflect the reviewer's early career stage. This, in turn, raises the question of the criteria by which reviewers are selected for the EMS Magazine. Under different circumstances, one might have hoped for an additional review offering a more extensive engagement with this huge project (114 articles involving a very large spectrum of topics). Let us end by pointing out that the publishers' website of the book includes reviews by Jean-Pierre Bourguignon, Barry Mazur and John Stillwell who all express a different perception of the Handbook.

New editor appointed

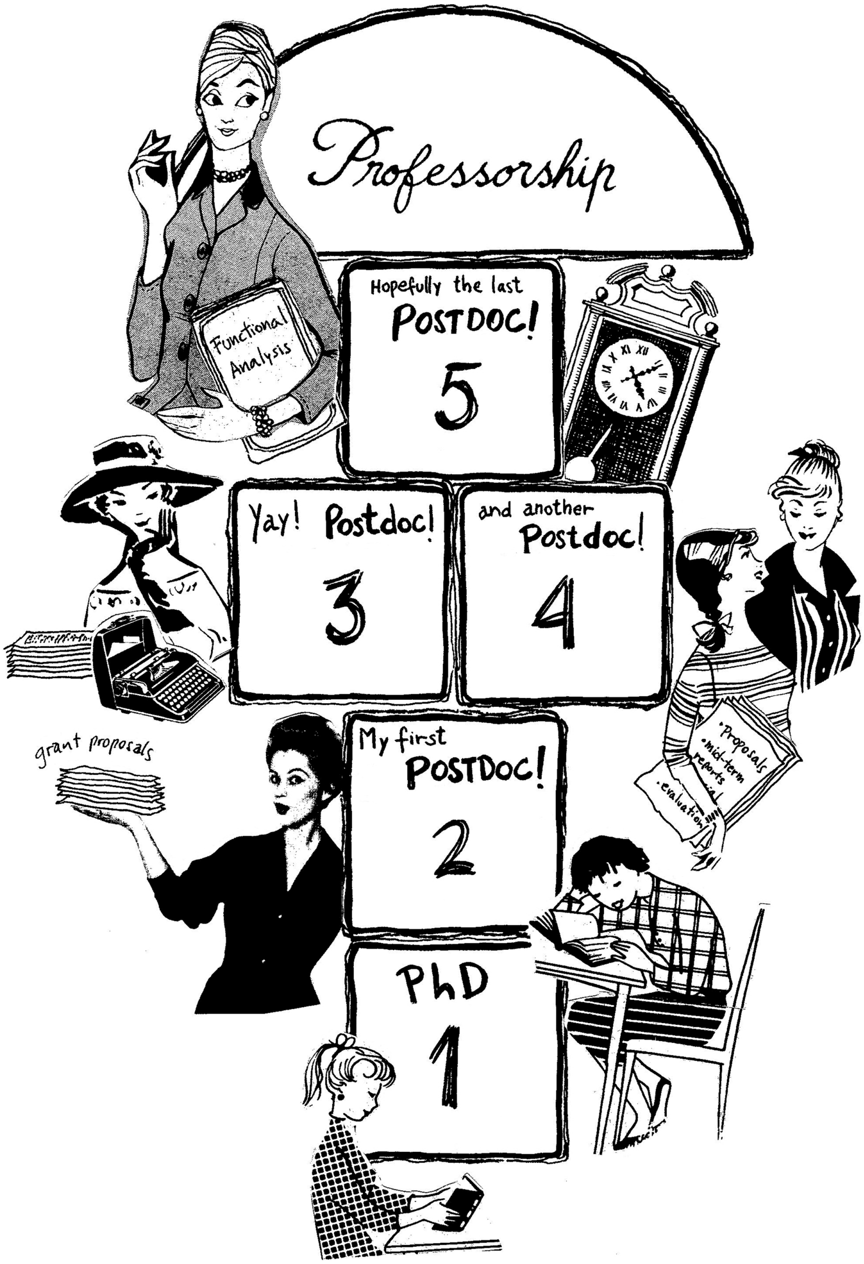
Matteo Petrera has been an editor and researcher at zbMATH Open, Mathematics Department, FIZ Karlsruhe – Leibniz Institute for Information Infrastructure, since 2020. His work focuses on editorial workflows and on the integration, cross-linking, and indexing of mathematical research data. In this role, he contributed to the transition of zbMATH to a fully open service, which became effective in January 2021.

He obtained his PhD in physics (mathematical physics) from Roma Tre University in 2007, after completing his degree in physics there. In the same year, he was awarded a Marie Curie European Fellowship to conduct research at the Technical University of Munich, Department of Mathematics. From 2009 to 2020, he held a research position at the Technical University of Berlin, Department of Mathematics, where his responsibilities included research, teaching, and the supervision of undergraduate and doctoral students. Beginning in 2012, he participated for two consecutive funding periods in the project *Discretization in Geometry and Dynamics*, funded by the German Research Foundation (DFG). His research interests were in the theory of integrable systems, both finite- and infinite-dimensional, in continuous and discrete time. His work was situated at the intersection of dynamical systems, geometry, and algebra.

EMS MAGAZINE 139 (2026) — DOI 10.4171/MAG/305

Toy model for a career in academia

Constanza (Coni) Rojas-Molina



Toy model for a career in academia, originally published at ragebb.wordpress.com.

Coni is a mathematician at CY Cergy Paris University, France. She is a science communicator and illustrator. Her preferred formats are sketchnotes and comics. She created the blog *The rage of the blackboard* at ragebb.wordpress.com where she interviewed female mathematicians and wrote articles about life in academia, from a mathematician's perspective. She is a member of the EMS Outreach and Engagement Committee, and the recipient of the 2024 Prize for Science Communication at CY Cergy Paris University. You can see her work at crojasmolina.com.

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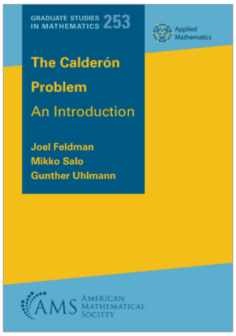

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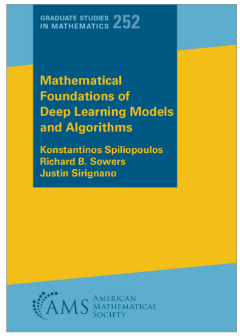

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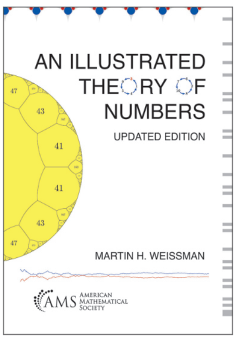

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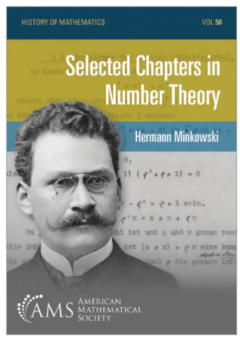

Hermann Minkowski

History of Mathematics, Vol. 50

Hermann Minkowski (1864–1909) was a prominent member of the famous golden epoch of mathematics at Göttingen University in Germany. The present book is an English translation of notes (never published) from the course on number theory taught by Minkowski in Winter 1907/08. The main topic of the course was the Fermat theorem and various methods used to prove it at that time.

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